Effective Multi-echelon Inventory Systems for Supplier Selection and Order Allocation

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Dedication

I dedicate this dissertation to my mom and dad (Xuejuan Xie and Jinming Guo), for their love, support and encouragement.

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Last, but certainly not the least, I would like to acknowledge the commitment, sacrifice and support of my parents, who have always motivated me.

Abstract

Successful supply chain management requires an effective sourcing strategy to counteract uncertainties in both the suppliers and demands. Therefore, determining a better sourcing policy is critical in most of industries. Supplier selection is an essential task within the sourcing strategy. A well-selected set of suppliers makes a strategic difference to an organization's ability to reduce costs and improve the quality of its end products. To discover the cost structure of selecting a supplier, it is more interesting to further determine appropriate levels of inventory in each echelon for different suppliers. This dissertation focuses on the study of the integrated supplier selection, order allocation and inventory control problems in a multi-echelon supply chain.

First, we investigate a non-order-splitting inventory system in supply chain management. In particular, a buyer firm that consists of one warehouse and N identical retailers procures a type of product from a group of potential suppliers, which may have different prices, ordering costs, lead times and have restriction on minimum and maximum total order size, to satisfy stochastic demand. A continuous review system that implements the order quantity, reorder point (Q, R) inventory

policy is considered in the proposed model. The model is solved by decomposing the mixed integer nonlinear programming model into two sub-models. Numerical experiments are conducted to evaluate the model and some managerial insights are obtained with sensitivity analysis.

In the next place, we extend the study to consider the multi-echelon system with the order-splitting policy. In particular, the warehouse acquisition takes place when the inventory level depletes to a reorder point R, and the order Q is simultaneously split among m selected suppliers. This consideration is important since it could pool lead time risks by splitting replenishment orders among multiple suppliers simultaneously. We develop an exact analysis for the order-splitting model in the multi-echelon system, and formulate the problem in a Mixed Integer Nonlinear Programming (MINLP) model. To demonstrate the solvability and the effectiveness of the model, we conduct several numerical analyses, and further conduct simulation models to verify the correctness of the proposed mathematical model.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction and Overview

1.1 Introduction

In today's circumstance of the global economic crisis, companies are facing increasing challenges to reduce operational costs, enlarge profit margins and remain competitive. People are forced to take advantages of any opportunity to optimize their business process and improve the performance of the entire supply chain. For most industrial firms, the purchasing of raw material and component parts from suppliers constitutes a major expense. For example, pointed out by Hayes et al. (2005) and Wadhwa and Ravindran (2007), it is expected that more and more manufacturing activities will be outsourced. Hence, among the various strategic activities involved in the supply chain management, the purchase decision has critical impacts on costs.

An essential task within the purchasing decision is the supplier selection. The traditional approach of supplier selection has been to select suppliers on the basis of price (Degraeve and Roodhooft, 1999). However, depending on the purchasing situation nowadays, selecting the right suppliers is affected by various of factors. A single criterion for supplier selection is not efficient, researchers and companies have turned into to a more comprehensive multi-criteria approach.

According to (Burke et al., 2007), a firm's supplier selection strategy is characterized by three key decisions: (a) criteria for establishing a supplier base; (b) methodology for selecting suppliers (a subset of the base) who will receive an order from the firm; and (c) the quantity of goods to order from each selected supplier. The first decision process is usually necessary since today's collaborate environment requires a low number of strategic suppliers so that the company can efficiently manage the suppliers. The purpose for it is to eliminate the inefficient supplier candidates and reduce the set of suppliers to a small range of potential suppliers.

From the potential supplier base, the specific supplier selection decision should be made to determine which supplier should receive an order to fill the demand for a specific product. Usually the suppliers in the base meet the quality, delivery and other criteria of the firm, the decision for the final supplier selection is primarily on cost considerations. Once the selected suppliers are resolved, the firm should allocate the product quantity among different selected suppliers. The focus of this dissertation is on the latter two decisions, i.e., supplier selection and order allocation. Therefore, this work reviews the supplier selection and order allocation literature concerning existing models and methodologies, identifies some important opportunities, and presents new and efficient decision-making tools aimed at helping companies select the most efficient suppliers.

For a typical supplier selection and order allocation problem, it is critical to determine which supplier to order and how much to order from each selected supplier. Thus, another relevant problem is to determine the best time to place the order. This motivates us to study the integration of supplier selection and inventory control models to derive optimal inventory policies that simultaneously determine how much, how often, and from which suppliers.

The management of inventory systems is another crucial business function for a company. This dissertation mainly concentrates the study on the multi-echelon inventory system for the supplier selection. Multi-echelon inventory systems are common in supply chains, in both the distribution and the production. In distribution, we study such systems when products are distributed over large geographical areas. To provide good service, product shipments are first stored at a central facility (warehouse). These central facilities are the internal suppliers to the customer-facing locations (retailers). This is a common distribution model for many supply chains as well as for large distributors and manufacturers. In production, inventory of raw materials, components and finished products are incorporated to each other in a similar way.

The complexities of managing inventory increase significantly for a multi-echelon distribution network with multiple tiers of locations. Generally, the overall goal for the multi-echelon distribution network is to minimize the costs for ordering, for capital tied up in the supply chain, and for providing an adequate customer service. According to (Axsäter, 2003), the successful to efficiently control the multi-echelon inventory systems has increased substantially during the last two decades. One reason is the progress in research, which has resulted in new techniques that are both more general and more efficient. Another reason is the development of new information technologies, which have dramatically increased the technical possibilities for supply chain coordination.

In this chapter, we first present an overview for supply chain basics. Then we briefly introduce the sourcing in supply chain, supplier selection, and the multi-echelon inventory control systems. Section 1.6 describes the major contributions of this research and provides an overview of this dissertation.

1.2 Supply Chain Management

A supply chain is a set of business units involved directly or indirectly in fulfilling a customer request (Chopr and Meindl, 2006). In a typical supply chain, raw materials are usually purchased from the upper suppliers and items are manufactured at the factories. The finished products are shipped to warehouse centers for storage, and then transported to retailers. Accordingly, effective supply chain strategy should consider interactions at the various level of the supply chain to reduce cost and improve service levels. Figure 1.1 shows a typical structure of supply chain.

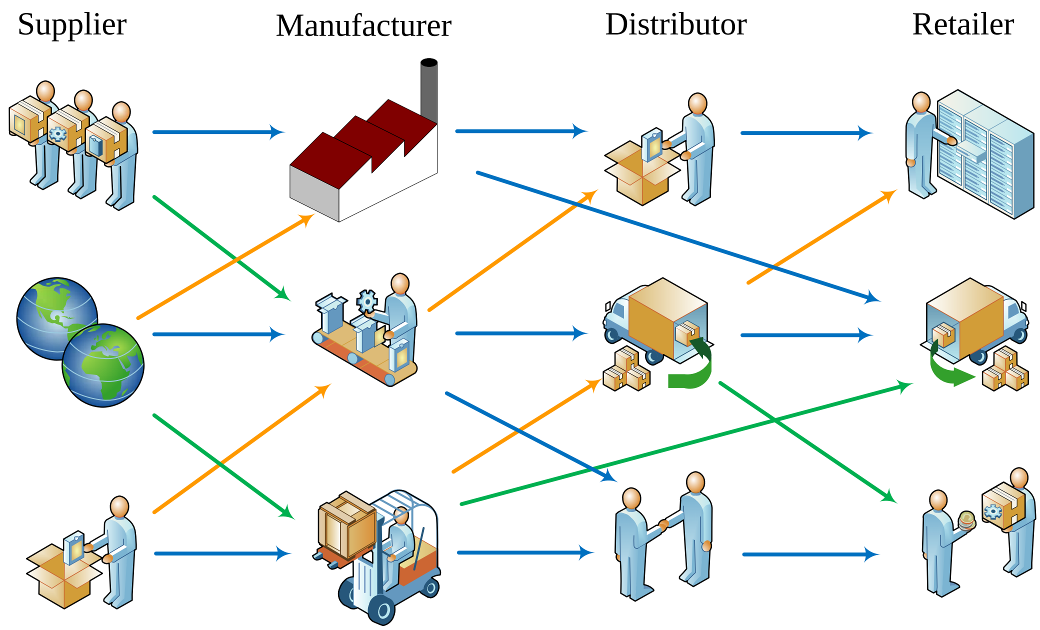

Figure 1.1: Illustration of a typical supplier chain: suppliers, manufacturers, warehouse and distributors, retailers, as well as raw materials, finished products, and intermediate inventory flow between the facilities

Simchi-Levi et al. (2008) formally define the concept of supply chain management to be a set of approaches utilized to efficiently integrate suppliers, manufactures, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system wide costs while satisfying service level requirements. According to this definition, the objective of supply chain management is to maximize the overall value generated throughout the entire system, and efficiently integrate the resource among suppliers, manufacturers, warehouses, and retailers. Optimal supply chain performance relies on the design and management of the processes, assets, and flows of material and information required to satisfy customers demand, along each echelon of the entire supply chain.

To satisfy customer demand in a supply chain, raw materials flow through a series of production and distribution stages until the final customer obtains a finished product. This is what typically represents the flow of materials. In contrast, in order to efficiently coordinate the physical flows in a supply chain, the flow of information plays an important role. For example, information about downstream customer demand must be available at each upper stage involved in the production and distribution process. To illustrate this, Figure 1.2 displays a multi-echelon supply chain. The traditional "push" strategy, represented by "make-to-stock" (MTS) in which the production is not based on actual demand, is shifting to the pull strategy, represented by make-to-order (MTO) in which the production is based on actual demand, thanks to the advances of information technology.

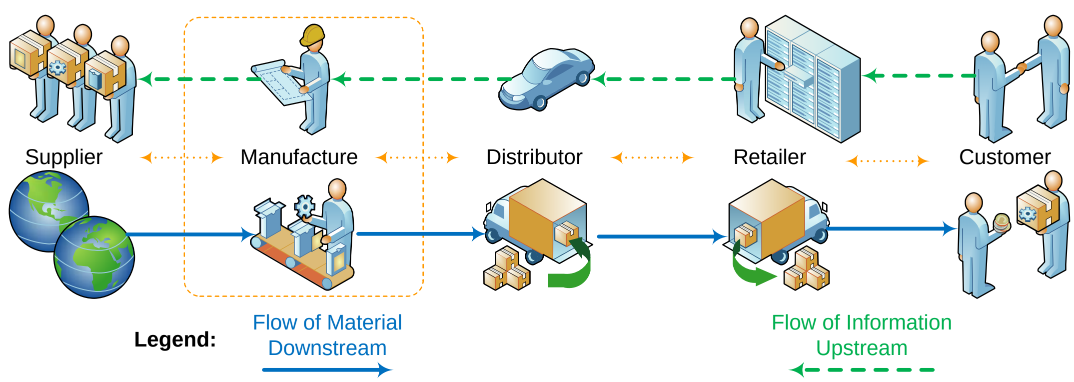

Figure 1.2: Illustration of a multi-echelon supply chain flow

There are mainly two challenges to efficiently design and operate the supply chain. One is due to uncertainty in each facility of the supply chain. This uncertainty happens in the customer demand, delivery lead time between each echelon of the supply chain, the suppliers and manufacturer capacity due to the breakdown of machines, and so on. An efficient supply chain model needs to eliminate the effects from these uncertainties as much as possible. Another important challenge is how to take into account of the whole supply chain system so that total systematic costs are minimized. The complexity increases quickly when considering the system-wide strategy. According to (Simchi-Levi et al., 2008), there are mainly several factors that increase the complexity and difficulty to globally discover the optimal solution in supply chain management. We summarize a few as the following:

1. The supply chain is a complex network of facilities dispersed over a large geography, especially because of the circumstance that the world is moving further toward multi-polarization and economic globalization.

2. Different facilities in the supply chain frequently have different and conflicting objectives. Each of supply chain members is primarily concerned with optimizing its own objectives and such self-serving focus may results in poor performance. For instance, a distributor may be concerned with its inventory cost while a retailer may be concerned with high availability and transportation costs. In fact, even within one echelon, like a manufacture, different departments may have objectives and it is imperative to make coordinated decisions to achieve a system-level optimization.

3. The supply chain is a dynamic system that evolves over the time, not only for the customer demand, but also for the supplier and manufacturer capacities. Besides, the planning process for the demand and cost parameters varies over

the time due to the impact of seasonal fluctuations, advertising and promotions, competitor's pricing and so on. This kind of variation is barely able to precisely predict, which increases the challenge to globally optimize the supply chain.

Due to the above discussed challenges, supply chain management typically concentrates on a variety of key issues. These issues span a large spectrum of a firm's activities, from the strategic through the tactical to the operational level (Simchi-Levi et al., 2008). There are plenty of literatures that study these activities. The key issues include locating facilities and configuring transportation flows to set up a supply chain distribution network, determining the appropriate levels of inventory and ordering policy at the various stages, building strategic partnership between suppliers and buyers to design and implement a globally optimal supply chain, coordinating outsourcing and procurement strategies to choose efficient suppliers, implementing critical information technology and decision-support systems to enable the efficiency of supply chain management, and so forth. For the detailed discussions and case studies, reader can refer to recent books by Chopr and Meindl (2006) and Simchi-Levi et al. (2008).

This dissertation mainly studies two key issues in supply chain management, including the supplier selection and strategy in outsourcing and procurement, and inventory control models to determine the specific ordering time and order allocation amount from the selected suppliers. In particular, our work focuses on a typical multi-echelon distribution network, and tries to develop an analytical process of finding the best system-wide supplier selection strategy. Thus, more details that are related to these two issues are demonstrated throughout the remaining sections.

1.3 Outsourcing and Procurement

Procurement and outsourcing are one of the major costs driven in supply chain. Nowadays, in order to increase efficiency, companies start outsourcing numerous parts of their business processes - from IT to raw material to customer service to logistics and transportation. A recent survey carried out by Accenture demonstrate that 80% of the companies surveyed use some form of outsourcing and a majority of these companies are spending close to 45% of their total budget on outsourcing (Accenture Consulting, 2005). According to (Johnson et al., 2010), a typical manufacturing firm spends 55% of earned revenue on purchased materials. For the US automotive industry, Wadhwa and Ravindran (2007) mentioned that the cost of components and parts from outside suppliers may exceed 50% of sales. Chopr and Meindl (2006) summarize the following benefits of outsourcing: (a) Achieve the economies of scale. (b) Improve forecasting and planning via better integration with suppliers. (c) Share risks and transfer demand uncertainty to the contract manufacturers. (d) Reduce capital investment. (e) Focus on core competencies. Consequently, in today's competitive operating environment, it is significant to determine a competitive outsourcing and procurement strategy.

In addition, economic globalization and trade liberalization enables the possibilities of global sourcing, which extends the local procurement to a worldwide scale. However, this brings not only opportunities for development but also challenges. For instance, although global sourcing offers notable cost reductions and an expanded market access, it also increases the variety and magnitude of risks faced by a local supply chain. Handfield and McCormack (2007) discuss the scenarios that global sourcing amplifies supply chain disruptions. The reason for the increasing risk is that the number of "hand-offs" required to ship products through multiple carriers, multiple ports, and multiple government check points increases, so does the probability of poor communication, human error, and missed shipments. Thus, risk management is also critical in outsourcing and procurement, especially for global sourcing and global operations.

Outsourcing and Procurement within an organization usually encompasses all activities related to the buying process. According to Aissaoui et al. (2007), there are six major purchasing decision processes: (1) 'make or buy', (2) supplier selection, (3) contract negotiation, (4) design collaboration, (5) procurement, and (6) souring analysis. The increasing importance of supply chain management motivates companies to fit purchasing and sourcing strategies into their supply chain objectives.

The first process step is to decide whether a certain component should be manufactured internally or outsourced. Typically, this decision is related to whether this product is the core competency or not. In the process (2), a pool of suppliers is usually pre-identified for the procurement based on a set of key criteria. Then the supplier selection strategy and methodology are developed to evaluate and select suppliers based on required specifications. In the stage (3) and (4), the buyer and supplier work together to build procurement contracts, and design parts/services that meet quality standards and customer specifications. The process (5) is to guarantee the supplier could deliver the product on time with the negotiated prices. Finally, the stage (6) is necessary so that the efficiency of the current purchase decision strategy can be assessed and re-designed.

Although there is extensive literature that studies the purchase decision making process and the outsourcing strategy, Aissaoui et al. (2007) discovered that the majority of the analytical studies on outsourcing decisions focus on processes (2), (5), and (6). Besides, among all of the purchasing process, the supplier selection process has received great attentions (Weber et al., 1991; Jayaraman et al., 1999; Feng, 2012). In what follows, we generally introduce the supplier selection problem.

1.4 Supplier Selection

The Supplier Selection Problem (also referred to Vendor Selection Problem) is usually a multi-criteria decision making process depended on a wide range of factors which involve both quantitative and qualitative ones (such as quality, cost, capacity, delivery, and technical potential). There are three major decisions that related to the supplier selection problem:

- *Which supplier should be selected?* Supplier selection models can be classified into two categories, single sourcing and multiple sourcing models. In single sourcing models, only one supplier is able to fulfill the buyer's demand. Thus, ranking techniques may generally applied to identify the "best" supplier. This strategy wins for the partnership between buyers and suppliers to maintain cooperation and achieve shared benefits. For multiple sourcing models, it is adopted either when none of the suppliers is able to satisfy the buyer's total demands or when procurement strategies aim at avoiding dependency on a single source to protect from shortage and maintaining steady competition among suppliers (Aissaoui et al., 2007).

- *How much should be ordered?* Regarding the issue how much quantity should be ordered, people considered it together with the order allocation problem (Sharafali and Co, 2000). Several criteria, including supplier's capacity, quality, delivery, price, and etc, may be considered to select efficient suppliers and properly allocate orders among selected suppliers.

- *When the order should be occurred?* The inventory control model and supplier selection choices are closely interrelated. Incorporating decisions to trigger orders over time with the supplier selection and order allocation may significantly reduce costs, especially in a long planning horizon. One important problem related to this area is the integration of inventory lot-sizing and supplier selection, which discusses situation where buyers can simultaneously select the most suitable suppliers for each period and optimize the lot size of each product.

The quality of the final set of suppliers largely depends on the quality of the steps involved in the selection process. According to (Monczka et al., 2005), the supplier selection process is can be addressed as follows: Step 1: recognize the need for supplier selection. Step 2: identify key sourcing requirements and criteria. Step 3: determine sourcing strategy. Step 4: identify Potential Supply Sources. Step 5: limit suppliers in selection pool. Step 6: determine method for final selection. Step 7: select suppliers and reach agreement.

1.5 Inventory Management

Carrying inventories is necessary to sustain operations within an economy. The importance of inventory management that determines policies, creates and distributes the most effectively inventories, has long been evident. Some important questions in inventory management are: how much should be ordered (i.e., order quantity), and when an order is placed (i.e., ordering policy)?

To address these questions, people have developed several mathematical models. In this section, we briefly introduce some of the well-known inventory policies. Besides, there are a number of key factors affecting the analytical models and inventory policy decisions. To illustrate the main assumptions that are adopted in this dissertation, we first introduce the major factors that affect inventory policy decision making.

- *Supply chain structure*: First and foremost in the supply chains structure. The structure indicates the manner in which both materials and information flow in a supply chain system. As mentioned earlier, the supply chain system contains many stages or echelon. To conduct appropriate inventory policies for supply chains, a system structure should be considered in the first place. The supply chain system under this study consists of a central inventory facility (referred to as the warehouse) serving several downstream stock points (referred to as the retailers). In the literature, this structure is known as one-warehouse multi-retailer or distribution system. This one-warehouse multi-retailer inventory system is widely studied in the literature. More discussion about this can be found in chapter 2.3.

- *Demand*: Demand is another important characteristic in inventory management. Demand may be known in advance, or in most commercial cases, demand is random. In this case, some forecasting tools could be implemented and historical data are available to estimate the demand rates and variability of customer demand. In this dissertation, we assume the customer's demand to be stochastic, and following Poisson process. This demand assumption is an extensively adopted assumption when considering supply chain inventory systems (such as in (Axsäter, 2003; Lee and Schwarz, 2007)). More importantly, Poisson process demand assumption is a good approximation for the arrival demand process at the retailers. Tijms (2003) demonstrate conditions under which the Poisson process is a good approximation of the demand arrival process.

- *Replenishment lead times*: Supply chain lead times greatly affect stock levels. In general, lead times measure the time delay between the placement of an order and its receipt. Typically, people consider the lead time as a measure of the responsiveness of a supplier. The longer the lead time, the more uncertainty of the downstream members, and therefore, the more requirements for inventory are necessary. This study considers lead time as the main characteristic of the different supplier, which affects the priority to choose the supplier.

- *Costs*: Common cost considered in the literature typically includes purchase cost, fixed ordering cost, inventory holding cost, backorder cost, lost sale cost, and etc. Purchase cost is critical especially when the purchase volume is large. Fixed ordering cost is the cost incurred independently of the quantity purchased, which is mainly due to the transportation cost. Holding cost is the cost to carry product in stock, and may consist of the cost of this capital invested in inventory, insurance, taxes, warehouse operating costs, and the cost of obsolescence. For backorder cost, it is assumed that customers wait for the inventory to arrive and eventually have their orders satisfied. Shortage costs may be calculated in either of two ways. (1) There may be a penalty cost incurred given that a demand arises and cannot be met from stock within a customers desired response time. This cost is charged independently of how long a customer could wait before receiving the ordered item. (2) The penalty cost may be charged as a function of the length of time a customer may wait to receive the products. Thus, this kind of costs are charged from the time an order is received (or due) until it is finally satisfied in this case. For the lost sales cost, if inventory is not available to meet the customer demand, a penalty cost will be charged in proportion to the number of sales that are lost. This work considers purchase cost, fixed ordering cost, inventory holding cost, and unit time backorder cost as the main cost criteria to select among various of suppliers.

In addition to the above discussed factors in inventory management, this work concentrates on the inventory planning over multiple time periods. As mentioned in (Aissaoui et al., 2007), even though there are many advantages to consider multi-period inventory problem in supplier selection, the majority of models that have been proposed in the literature treat supplier selection without considering multiple periods. This dissertation implements a continuous review inventory policy to study long term decision making.

From the preceding paragraphs, it is apparent the importance of incorporating inventory replenishment decisions into the supplier selection problem. In the next section, we mainly summarize the current research in this area, and present the main objectives and motivations of this dissertation.

1.6 Research Objectives and Document Organization

According to the above discussed supplier selection and inventory management basis, we could easily discover that the supplier selection and order allocation problem is closely related to inventory management. To derive optimal inventory policies that simultaneously determine how much, how often, and from which suppliers to order, typical inventory costs should be considered. Consequently, this dissertation considers holding, backorder, ordering, and purchasing cost in a multi-echelon inventory system. Additionally, criteria relevant to supplier selection (quality and capacity) are incorporated.

Although there is plenty of research for the supplier selection model, only limited studies focused on the inventory control policies integrated with supplier selection, especially under stochastic demand. However, considering the cost issue, supplier selection decision is actually highly correlated with some major logistics issues within a company such as inventory (stock level, delivery frequency, etc.) Incorporating the decisions to schedule orders over time with the supplier selection may significantly reduce costs over the planning horizon (Aissaoui et al., 2007). For example in the recent article (Mendoza and Ventura, 2010), the authors studied both supplier selection and inventory control problems under a serial supply chain system. A mathematical model was proposed to determine an optimal inventory policy in different stages and allocate proper orders to the selected suppliers. This paper extended the contributions to the research for the integration of supplier selection and inventory control problems in multi-level systems. However, the mathematical model built in this paper was based on a stationary inventory policy with a constant demand. Moreover, the constant lead time, no backorder allowed and the same order quantity for different suppliers were assumed in the paper. These assumptions could be restrictive in reality, and it may not be appropriate to order the same quantity each time from different suppliers due to the different ordering cost and replenishment lead time. Thus, in this work, we want to consider the stochastic demand and lead time for this problem, which adopts various replenishment policies for different suppliers. Besides, according to some more literature reviews given in Section 2.2, the decision model for supplier selection and inventory control policies in multi-level supply chain system requires further studies.

We plan to consider both supplier selection and inventory control problems in a serial supply chain system. A two-echelon distribution system with a central warehouse and N retailers is considered to procure from a set of suppliers. The supplier selection process is assumed to occur in the first stage of the serial supply chain, and the decision is made by a single decision maker (i.e., centralized control) who wants to reduce the total cost associated with the entire supply chain. Capacity, quality, ordering cost, unit price, holding and backorder cost are considered as the criteria for the supplier selection. For the inventory control policy, a continuous review system which applies the order quantity, reorder point (Q, R) policy is adopted to determine the inventory level held at each echelon of the supply chain. We separately consider two types of inventory assumptions for the multiple scouring inventory model. One is to consider no-order-splitting assumption at the warehouse, i.e., the warehouse places orders from different suppliers one after another, and won't order the same product from different suppliers at the same moment. The other is to assume the orders at the warehouse can be split among different suppliers. Further details will be discussed in Chapter 3 and 4. The objective of the proposed integrated model is to coordinate the replenishment decision with the inventory at each echelon while properly selecting the set of suppliers which meets capacity restrictions.

The reminder of this proposal is organized as follows. Chapter 2 reviews the literatures on supplier selection and multi-echelon inventory control problems. Chapter 3 presents the non-order-splitting model for the supplier selection and order allocation problem, including mathematical model formulation and numerical examples. In Chapter 4, the assumption for the order splitting model and the analytical model for the warehouse inventory level is conducted. Finally, Chapter 5 addresses the significance and expected contributions of this work.

Chapter 2

Literature Review

2.1 Introduction

In this chapter, the decision support models for the supplier selection, as well as the inventory control models for the multi-echelon supply chain system are reviewed. The focus of the review in this chapter is on the quantitative techniques that have been applied to supplier selection, order allocation, and inventory control models. These quantitative and operations research models offer a range of techniques that may support the purchasing decision-maker in dealing with the increased complexity and importance of supplier selection process (Boer et al., 2001).

This chapter is organized as follows. In section 2.2, several categories of decision support techniques that have been implemented to supplier selection process are discussed. Section 2.3 reviews related literature for one-warehouse multi-retailer system. Finally, the conclusions drawn from existing literature and the research Opportunities are presented in Section 2.4.

2.2 Supplier Selection

2.2.1 Previous Literature Reviews of Supplier Selection

The supplier selection problem has attracted great attentions of a number of researchers who proposed various decision models and solutions. Some previous review works for these decision methods have been presented in the literature. Weber et al. (1991) classified 74 related articles published from 1966 to 1990 which have addressed supplier selection problems based on different criteria and analytical methods. It was found that price, delivery and quality were the most discussed factors. Later in 2000, Degraeve et al. (2000) adopted the concept of Total Cost of Ownership (TCO) as a basis for comparing supplier selection models. They illustrated their model through a case study, and concluded that from a TCO perspective, mathematical programming models outperformed rating models and multiple item models generated better results than single item models. Recently, Ho et al. (2010) surveyed the literature of the multi-criteria decision making approaches for supplier evaluation and selection based on 78 international journal articles gathered from 2000 to 2008, which were classified based on the applied approaches and evaluating criteria. They observed that price or cost is not the most widely adopted criterion. Instead, the most popular criterion used for evaluating the performance of suppliers is quality, followed by delivery, price or cost, and so on. For some other review articles of the supplier selection problems, please refer to Boer et al. (2001) and Aissaoui et al. (2007). The following part of this section summarizes the contribution in the literature related to this dissertation.

2.2.2 Mathematical Programming Techniques

Various types of mathematical programming models have been formulated for the supplier selection problem, such as linear programming, mixed integer programming and multi-objective programming. In what follows, we briefly review some of the related literature that adopts these techniques.

- *Linear programming*: We first review some papers which adopted linear programming. Ghodsypour and O'Brien (1998) proposed an integration of an analytical hierarchy process and linear programming to consider both qualitative and quantitative factors in choosing the best suppliers and placing the optimum order quantities. Later in (Talluri and Narasimhan, 2003), a unique approach called 'max-min' for vendor selection was proposed by incorporating performance variability into the evaluation process. The authors built two linear programming models to maximize and minimize the performance of a supplier against the best target measures. Ng (2008) developed a weighted linear program for the multi-criteria supplier selection problem with the goal to maximize the supplier score, and studied a transformation technique to solve the proposed model without an optimizer.

- *Mixed-integer programming*: As for the mixed-integer programming technique, Kasilingam and Lee (1996) proposed a mixed-integer model to select vendors and determine the order quantities based on the quality of supplied parts, the cost of purchasing and transportation, the fixed cost for establishing vendors, and the cost of receiving poor quality parts. Tempelmeier (2002) developed a single item supplier selection and order sizing model for dynamic deterministic demands. Two versions of mixed-integer optimization model were built separately for the cases of all-units discounts and the incremental quantity discounts. Later in (Hong et al., 2005), the model which can determine the optimal number of suppliers, and the optimal order quantity so that the revenue could be maximized was built in a mixed-integer linear programming formation, followed by three steps: preparation, pre-qualification, and final selection. Recently, Hammami et al. (2012) developed a mixed-integer programming model for the supplier selection problem that took into account of inventory decisions, inventory capacity constraints, specific delivery frequency and a transportation capacity based on multiple products and multiple time periods.

- *Multi-object programming*: Due to the multi-criteria nature of the supplier selection problem, more and more researchers began to adopt multi-object programming since 2005. Narasimhan et al. (2006) developed a multi-objective model to choose the optimal suppliers and determine the optimal order quantity, which considered the following criteria: cost minimization, transaction complexity minimization, quality maximization and delivery-performance maximization. Xia and Wu (2007) studied the situation of price discounts on total business volume and proposed a multi-objective mathematical model to minimize total purchase cost, reduce the number of defective items, and maximize total weighted quantity of purchasing. The model was also built to simultaneously determine the number of suppliers to employ and the order quantity allocated to these suppliers in the case of multiple sourcing, multiple products, with multiple criteria and with supplier's capacity constraints. In (Demirtas and Ustun, 2009), to evaluate the suppliers and to determine their periodic shipment allocations given a number of tangible and intangible criteria, a two-stage mathematical approach was proposed by a multi objective mixed integer linear programming model. Some other recent works which adopted multi-objective model can be found in (Amid et al., 2009; Feng et al., 2011).

2.3 Multi-echelon Inventory Models

Efficient control of multi-echelon inventory systems is a challenging issue that has received a lot of research attentions from both practitioners and academicians over the years. Research on multi-echelon inventory systems started more than several decades ago. One of the earliest models in this topic was implemented for recoverable item in (Sherbrooke, 1968). The author presented a mathematic model based on this framework in which item demand is compound Poisson with a mean value estimated by a Bayesian procedure. The objective of this mathematical based-depot supply system model was minimizing expected backorders subject to budget constraints while setting optimal inventory policy parameters.

Deuermeyer and Schwarz (1981) presented an analytical model for estimating the expected performance measures of a one-warehouse, m identical retailers, and non-repairable spare parts inventory system. They examined a system that involves m identical retailers facing stationary Poisson demand and operating under (R, Q) replenishment policies. Later, Svoronos and Zipkin (1988) proposed several refinements based on (Deuermeyer and Schwarz, 1981), and achieved more simple and robust model. They developed an approximation model for a two-level distribution system under stochastic Poisson demand, which adopted mixture of two translated Poisson distributions (MTP) for the warehouse lead time demand. Using the MTP, they estimated the performance measures at the warehouse such as the expected number of backorders. Then Axsäter (1993) derived a recursive procedure to solve the same problem from another perspective, and demonstrated how to use their proposed method for the exact or approximation evaluations.

Bodt and Graves (1985) presented a multi-echelon inventory model with the failures generated by the compound Poisson process and deterministic shipment time from the repair depot to each site for a repairable item with one-for-one replenishment. He proposed an exact model for finding the steady-state distribution of net inventory level at each location. Axsäter (1900) proposed a simple solution procedure for a two-echelon inventory system with one-for-one replenishment system. In this paper, constant lead-time, and independent Poisson demand at retailers are assumed. The author implemented simple recursive procedures for determining the holding and shortage costs of different control policies.

Hopp et al. (1997) studied a single location problem, and formulated a constrained optimization model that utilizes (R, Q) policies, with the objective of minimizing overall inventory investment at the distribution center subject to constraints on customer service and order frequency. Because of the nonconvexity make this problem intractable to exact analysis, three heuristic algorithms that approximate the inventory policy parameters are developed. Using some approximations and the theory of Lagrange multipliers, they derived simple expressions for the inventory policy parameters. Then, Hopp et al. (1999) extended the model to address a two-echelon distribution system. They derived closed-form expressions for the inventory control parameters, and approximated the parameters in the closed-form expressions.

In Ganeshan (1999), the authors proposed a near-optimal ordering policy for a similar distribution network by considering inventory, transportation and transit components of the supply chain. Axsäter (2003) considered a two-echelon distribution inventory system consists of a central warehouse and a number of retailers controlled by continuous review installation stock (R, Q) policies. He presented a simple method that uses normal approximations for the retailer demand and the demand at the warehouse in order to approximate optimization of the reorder points. Recently, Yang et al. (2011a) implemented economies of scale and continuous-state approximation to the two-stage inventory system. A heuristic algorithm was proposed to find a near optimal policy. In Topan and Bayindir (2012), the authors studied a multi-item two-echelon spare part inventory system in which the central warehouse operates under an (nQ,R) policy and the local warehouses implement order-up-to S policy. A compound Poisson demand is considered in this paper. Four alternative approximations for the steady state performance of the system are proposed. For more work about the multi-echelon distribution systems, please refer to (Chen and Zheng, 1997; Axsäter, 2000; Al-Rifai and Rossetti, 2007).

2.4 Conclusions and Research Opportunities

Although the supplier selection decision is closely related to inventory models, Hammami et al. (2012) indicated that only a few of models incorporated the inventory management related issues. Here we want to point out some recent papers from the integration of supplier selection and inventory control perspective.

In (Haq and Kannan, 2006), the authors developed an integrated supplier selection and multi-echelon distribution inventory model. The inventory cost considered in this paper was based on deterministic demand in a given time period so that no inventory control policies are required to be considered. Similar inventory management models in supplier selection can also be found in (Demirtas and Ustun, 2009; Mendoza and Ventura, 2010; Hammami et al., 2012). For stochastic demand supplier selection model, most of research focused on a single-period demand. For instance, in (Yang et al., 2007), the authors considered a buyer who faces a single-period stochastic demand and multiple suppliers with yield uncertainty. A solution algorithm was proposed to solve the developed nonlinear mathematical model. In addition, Zhang and Zhang (2011) developed a mathematical model to implement the newsvendor inventory model for a single firm with fixed selection cost and limitations on minimum and maximum order size under stochastic demand. Some other similar papers which considered a single-period demand can be found in (Awasthi et al., 2009; Yang et al., 2011b).

Obviously, a single-period problem intended for short term planning does not necessarily consider any inventory policy for continuous replenishment over an infinite planning horizon. More importantly, extant literature showed little work on multi-stage systems, where only focus on the performance of a single buyer. Nonetheless, the inventory policies in supply chain management not only impact a single stage but also will affect the whole supply chain. These are the basic motivations of this research. Thus, as discussed in Section 1.6, this work is to implement multi-echelon multi-period inventory models for supplier selection problem to extend existing literatures.

Chapter 3

A No-order-splitting Inventory System with Supplier Selection and Order Allocation

3.1 Problem Definition and Assumptions

We model the supplier selection and order quantity allocation problem based on a two echelon inventory system. We assume that the inventory decision is made by a single decision maker (i.e., centralized control), who wants to purchase a single type of product from a set of potential suppliers. Figure 3.1 depicts a serial supply chain system under consideration of three levels, where raw materials and products flow sequentially through the supply chain to satisfy the customer demand. A single warehouse replenishes its inventory from a set of S selected suppliers with given lead times. It is assumed that all the suppliers in this identified set at level 1 satisfy the buyer's qualitative criteria (service, delivery, maintenance, etc.) and the final decision will be made based on the item price, the fixed ordering cost and the inventory cost regarding choosing of the particular supplier. The warehouse then supplies the items to N independent identical retailers, where demand occurs based on a Poisson process, which is an extensively adopted assumption when considering supply chain inventory systems (such as in (Axsäter, 2003; Lee and Schwarz, 2007)). All stockouts are considered as backorders. Therefore, supplier selection and purchasing costs only occur at the warehouse, while the product is transferred through the entire system, incurring costs like inventory costs, backorder cost, etc.

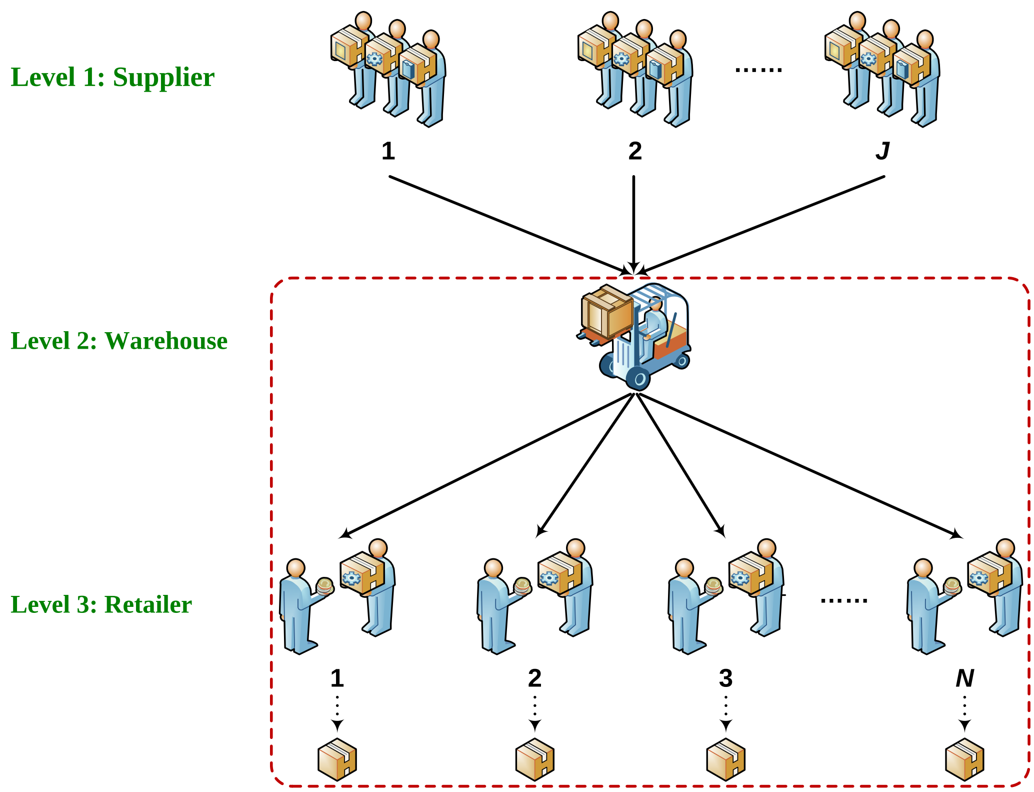

Figure 3.1: Illustration of the multi-level supplier selection system

The above supply chain system is assumed to implement the continuous review (Q, R) policy at the warehouse and the retailer. Hence, when the demand occurs at the retailer, it is satisfied from the retailer's available stock. Otherwise, the demand is backordered. Under this policy, the inventory position is checked continuously, when it declines to the reorder point R, a batch size Q is ordered at the warehouse. The inventory position is defined as the on hand inventory plus stock on order minus the number of outstanding backorders. After an order is placed with the warehouse, an effective lead time L takes place between placing the order and receiving it. After receiving the replenishment order, the outstanding backorders at the retailer are immediately satisfied based on a first-come-first-serve (FCFS) policy.

For the warehouse, the retailer replenishment orders are satisfied if the on-hand inventory at the warehouse is greater than or equal to the retailer's order size. That is, a partial replenishment of an order at the warehouse is not allowed. This is a reasonable assumption when we consider a fixed order cost k associated with each delivery from the warehouse to the retailer. The inventory policy at each retailer follows the same one as at the warehouse, i.e., the continuous review (Q, R) policy. We also adopt the widely-used two-echelon inventory system assumption, that is the batch size and reorder point of the warehouse are the integral number of that of the retailer (also can be found in (Bodt and Graves, 1985; Axsäter, 2003)). According to Chen and Zheng (1997), this integer-ratio order policy can facilitate quantity coordination among different facilities, and simplify packaging, transportation and stock counts.

When the warehouse receives the replenishment order from the selected suppliers, any outstanding backorders are fulfilled according to the FCFS policy as well.

In addition to determine the inventory policy for the warehouse and the retailer, we will not model any inventory process at the supplier. Instead, the decision maker should replenish the inventory for the warehouse and the retailer from different suppliers by performing a selection process so as to determine which supplier to be selected, the total expected quantity that are to be procured from the selected suppliers, and the frequency in which the orders are to be received. We assume that each supplier locates in different places, then prices, selection costs, transportation costs, and replenishment lead times are diverse from each other. Define S different suppliers, for each supplier j, let O_j be the fixed ordering cost each time (i.e., selection cost, transportation cost, etc.), and p_j be the price of one item from this supplier. Moreover, μ_j and v_j are respectively denoted as the mean and variance of transition time from supplier j to the warehouse, which is assumed to be known in advance. Supplier j has limited maximum capacity M_j and a restriction of the minimum total order size m_j if the supplier is selected. The objective of the proposed model is to coordinate the purchase, holding, backorder, and capacity in order to maximize the expected profit.

Before we introduce the mathematical model, we now define the notations that are used throughout the paper as the following:

Constants

S: number of suppliers

N: number of retailers

T: number of time span, in days

λ_r: demand rate at the retailer per day

L: replenishment lead time between the warehouse and the retailer, in days

p_j: net purchase cost per unit from supplier j

h: holding cost per unit per day

b: backorder cost per unit per day

k: retailer's fixed ordering cost per order

r: selling price per unit at the market

O_j: warehouse's fixed ordering cost per order for supplier j

μ_j: mean replenishment lead time between supplier j and the warehouse, in days

v_j: variance of replenishment lead time between supplier j and the warehouse

m_j: minimum average total order size of supplier j during T time units

M_j: maximum average total order size of supplier j during T time units

Decision Variables

y_j: binary variable, set to be 1 if supplier j is selected

x_j: average total ordering quantity (expected) from supplier j, in units

Q_{rj}, R_{rj}: retailer's order quantity and reorder point if supplier j is chosen, in units

Q_{wj}, R_{wj}: warehouse's order quantity and reorder point if supplier j is chosen, in units of retailer batches

Intermediate Variables

U_r: retailer's retard time, in days

D_r: retailer's demand during the delay time, in units

λ_w: demand rate at the warehouse per day

D_w: warehouse's demand during the delay time, in units

$I_{rj}(R, Q)$: expected on-hand inventory at retailer during the time supplier j is chosen, in units

$I_{wj}(R, Q)$: expected on-hand inventory at warehouse during the time supplier j is chosen, in units of retailer batches

$B_{rj}(R, Q)$: expected backorders at retailer during the time supplier j is chosen, in units

$B_{wj}(R, Q)$: expected backorders at warehouse during the time supplier j is chosen, in units of retailer batches

3.2 Model Formulation

3.2.1 Mathematical Model

In this section, the model of supplier selection and order quantity allocation under multi-echelon inventory system with stochastic Poisson demand is presented. Let x_j be the average total ordering quantity from the supplier j and y_j be a binary variable, where $y_j = 1$ means that supplier j is selected. Recall that each selected supplier j should satisfy the capacity constraint. Thus, we may select different sets of suppliers when demand rate at the retailer changes. Note here since the demand is stochastic, x_j is the expected total quantity to order from the supplier j. By knowing this value, the firm can share this information to the selected supplier, which could help the supplier to efficiently arrange its manufacturing.

The system is studied based on time span T. At any moment, only one supplier is asked to provide the replenishment orders, i.e., no-order-splitting is considered every time the warehouse places the order. If there are multiple suppliers which are selected in the final decision, each supplier is assumed to serve the warehouse separately for some continuous time. If we denote t_j as the expected time to purchase orders from supplier j, following one selected supplier (supplier a) that finishes its service time (t_a), then the warehouse may place the order from another one (supplier b) with some additional time (t_b).

The no-order-splitting assumption represents the cases where the warehouse places orders from different suppliers one after another, and will not order the same product from different suppliers at the same moment. If we were to consider order-splitting for the problem under study, different models and approaches would be needed. In this work, the different supplier lead times and ordering costs could require diverse $(Q,\ R)$ policies for both the warehouse and the retailer. However, to apply the order-splitting assumption, the retailer needs to implement the identical $(Q,\ R)$ policy no matter which supplier is selected. Thus, to avoid this dilemma, we have applied the no-order-splitting assumption.

The objective function of the proposed model consists of several parts to maximize the total expected profit. The first part is used to calculate total sales income. The second term corresponds to the purchasing cost incurred by all the units purchased from selected suppliers. The third part accounts for the total holding and backorder cost. The last term represents the fixed ordering cost for both echelons. The insight of the model is to examine the trade-offs among price, ordering, holding, and backorder costs to choose the best supplier(s) and decide the ordering policy. Based on the above-discussed assumptions and variables, the following mathematical model is developed:

$$\begin{aligned} Maximize\ \ C \ &= \ \sum_{j=1}^{S} r x_j - \sum_{j=1}^{S} p_j x_j - \sum_{j=1}^{S} \frac{x_j}{N\lambda_r}[h(NI_{rj} + Q_{rj}I_{wj}) + b(NB_{rj} + Q_{rj}B_{wj})] \\ & \quad - \sum_{j=1}^{S} (\frac{x_j}{Q_{wj}Q_{rj}} O_j + \frac{x_j}{Q_{rj}} k). \end{aligned} \tag{3.1}$$

Subject to

$$m_j y_j \leq x_j \leq M_j y_j \qquad j = 1, \ldots, S, \tag{3.2}$$

$$\sum_{j=1}^{S} x_j \leq N\lambda_r T \qquad j = 1, \ldots, S, \tag{3.3}$$

$$R_{rj} \geq -Q_{rj} \qquad j = 1, \cdots, S, \tag{3.4}$$

$$R_{wj} \geq -Q_{wj} \qquad j = 1, \cdots, S, \tag{3.5}$$

$$Q_{rj}, Q_{wj} \geq 0 \qquad j = 1, \cdots, S, \tag{3.6}$$

$$Q_{rj}, R_{rj}, Q_{wj} \& R_{wj} : Integers \qquad j = 1, \cdots, S, \tag{3.7}$$

$$y_j \in \{0, 1\} \qquad j = 1, \ldots, S. \tag{3.8}$$

Constraint (3.2) defines the capacity constraint for the selected suppliers. As for constraint (3.3), it ensures the total expected ordering quantity from all selected suppliers should be no larger than the total expected demand. This is a necessary constraint to guarantee the ordering is performed only when demand is confirmed such that there is no extra inventory cost. Besides, for standard cost structures (i.e., linear holding and backorder costs), it can be shown that the optimal R satisfies $R \geq -Q$. Therefore, it is assumed to satisfy in equations (3.4) and (3.5), which can limit the computation efforts. Constraints (3.6) and (3.7) are necessary, since there is no partial or fractional requests during the whole process and the minimum allowable size is zero.

The mathematical model requires calculations of the expected inventory and backorder level for the warehouse and the retailer. We elaborate the calculations in the following parts of this section.

3.2.2 The Retailer Inventory Analysis

Since the calculation for all the selected suppliers is identical, for notational ease, we ignore the supplier's subscript j for the notations in this section. The following analysis is identical for all the different supplier cases.

The retailer inventory position decreases with demand, and when the level reaches R_r, an order of Q_r is placed at the warehouse. Recall that in the steady state, the inventory position is uniformly distributed over $(R_r+1, R_r+2, \cdots, R_r+Q_r)$. Under a (Q, R) policy, the expected on-hand inventory for retailers is modeled as:

$$I_r = \frac{Q_r + 1}{2} + R_r + B_r - E[D_r], \tag{3.9}$$

where $E[D_r]$ is the retailer's expected demand during the delay time. The delay time consists of two parts: the replenishment lead time between the warehouse and the retailer, and the time between the placement of an order by the retailer and the release of a batch by the warehouse. The first part is denoted as L, which is assumed to be deterministic. While the second part is usually called as the retard time, which is entirely due to the warehouse stockout. We denote the retard time for the retailer as U_r. It is given as follows: the number of arrival orders in the waiting system is precisely the warehouse's backorders, and the sojourn time is equal to the retailer's retard time (Svoronos and Zipkin, 1988). Thus, the expected retard time at the retailer is calculated as

$$E[U_r] = \frac{Q_r B_w}{N\lambda_r}, \tag{3.10}$$

then we have

$$E[D_r] = \lambda_r(L + \frac{Q_r B_w}{N\lambda_r}). \tag{3.11}$$

To simplify the calculation, as illustrated by (Hopp and Spearman, 2001), we adopt normal approximation for the retailer's demand during the delay time. It is worthwhile to note that even though the demand process is the Poisson process, the demand during the delay time for the retailer does not exactly follow the Poisson distribution, due to the variability of the retard time. In this paper, to simplify the calculation, we approximate the variance of the retailer's expected demand during the delay time to be the same as its mean value, i.e., $V[D_r] \cong E[D_r]$.

Hence, using the normal approximation, expected backorders at the retailer B_r can be computed as follows (see (Hopp and Spearman, 2001)):

$$B_r = \frac{1}{Q_r}[\beta(R_r) - \beta(R_r + Q_r)], \tag{3.12}$$

$$\beta(x) = \frac{\sigma^2}{2}\{(z^2+1)[1-\Phi(z)] - z\phi(z)\}, \tag{3.13}$$

$$z = \frac{x-\theta}{\sigma}, \tag{3.14}$$

where Φ and ϕ represent the cumulative distribution function (cdf) and probability density function (pdf) of the standard normal distribution, respectively. Additionally, θ and σ are the mean and standard deviation of the demand during the delay time. Note here for equation (3.13), it defines the continuous analog to the second-order loss function $\beta(x)$. Thus, substituting θ and σ in the above equations, B_r can be computed in a function based on the variables Q_r, R_r, and B_w.

3.2.3 The Warehouse Inventory Analysis

To calculate the expected inventory level and backorder level at the warehouse, the demand process at the warehouse needs to be analyzed first. Recall that the demand process at each retailer is the Poisson process, and the replenishment order for the warehouse is Q_r each time. However, the interval between any two orders is stochastic and depends on Q_r. Hence, the demand process at the warehouse is a superposition of the retailer's ordering processes. Specifically, it is a superposition of independent renewal processes (i.e., the time between orders from each retailer is independent and identically distributed random variables), each with an Erlang interrenewal time with Q_r stages and rate per state λ_r (Deuermeyer and Schwarz, 1981). Therefore, under the assumption of identical retailers, it is straightforward to get the demand rate at the warehouse:

$$\lambda_w = \frac{N\lambda_r}{Q_r}. \tag{3.15}$$

When considering the multi-echelon problem, the determination the effective demand pattern at the upstream is always a difficult task. For this problem, it is almost impossible to find an exact distribution for the demand process at the warehouse. However, Ganeshan (1999) showed that when N is greater than 20, the Poisson process is an excellent approximation for the warehouse demand pattern. In our research, N is a sufficiently large number in each scenario, so that each retailer's order arrives approximately according to the Poisson process. This assumption is fairly reasonable, since for large retail corporation like Wal-Mart, the distribution center usually serves more than 20 stores.

Because of the independence of the superimpose process, given the mean and variance of the warehouse replenishment lead time μ and v, we can compute the mean and variance of the warehouse demand during the replenishment lead time as

$$E[D_w] = \mu\lambda_w = \frac{N\lambda_r\mu}{Q_r}, \tag{3.16}$$

$$V[D_w] = \mu\lambda_w + v\lambda_w^2 = \frac{N\lambda_r\mu}{Q_r} + \frac{N^2\lambda_r^2 v}{Q_r^2}. \tag{3.17}$$

Then we can obtain I_w similar to equation (3.9):

$$I_w = \frac{Q_w + 1}{2} + R_w + B_w - E[D_w]. \tag{3.18}$$

Due to the assumption of Poisson demand process at the warehouse when N is sufficiently large, we also approximate the warehouse's demand during the lead time as normal distribution. This is a reasonable approximation since this approximation improves as the rate of Poisson distribution increases, while the rate at the warehouse is sufficiently large. This approximation is also used in the literature, such as in (Al-Rifai and Rossetti, 2007). Then Eqs. (3.12) to (3.14) can also be adopted to calculate the expected backorder level B_w at the warehouse. Thus, B_w is a function of Q_w, R_w, and Q_r.

3.2.4 Solution Procedure

The above multi-echelon supplier selection optimization model is a large-scale non-linear integer optimization problem. Considering the case of 5 potential suppliers, the model then may contain 30 integer decision variables, which takes a lot of computational time to solve. Moreover, the inventory analysis of each echelon requires modeling and solving both echelons simultaneously. In order to model the warehouse, the retailer's order batch size must be decided as a priori. On the other hand, the retailer's calculation requires a known expected number of backorders at the warehouse. Thus, each echelon of the systems is tightly connected with each other, and to solve a large-scale non-linear integer optimization problem can be computationally intensive.

By examining the objective function (3.1), the expected inventory and backorder level at the warehouse and the retailer are independent of the expected total ordering quantity x_j. This implies that the model can be solved with two decomposed levels: one is to solve the optimal (Q, R) policy for each potential suppliers with the objective to minimize the expected holding, backorder and ordering cost (Model 1); the other (Model 2) is to choose the best suppliers with larger profit and allocate the expected order size for different suppliers. We now express the models as the following:

Model 1: Since the $(Q,\ R)$ policy applied by different potential suppliers is independent from each other, we formulate the optimization problem based on a single supplier j as minimizing the total unit system cost, denoted as E_j, which includes inventory, backorder and ordering cost as follows:

$$Minimize \qquad E_j = h(NI_{rj} + Q_{rj}I_{wj}) + b(NB_{rj} + Q_{rj}B_{wj}) + (\frac{N\lambda_r}{Q_{wj}Q_{rj}}O_j + \frac{N\lambda_r}{Q_{rj}}k). \tag{3.19}$$

Subject to

$$R_{rj} \geq -Q_{rj}, \tag{3.20}$$

$$R_{wj} \geq -Q_{wj}, \tag{3.21}$$

$$Q_{rj}, Q_{wj} \geq 0, \tag{3.22}$$

$$Q_{rj}, R_{rj}, Q_{wj} \& R_{wj} : Integers. \tag{3.23}$$

Note in this model I_{rj}, I_{wj}, B_{rj} and B_{wj} can be formulated as the equations discussed in section 3.2.2 and 3.2.3. For simplicity, we ignore those equations in this mathematical model.

Model 2: After achieving the minimized cost value E_j for each possible supplier, the mathematical model described in section 3.2.1 can be updated to a simplified model by substituting E_j in the objective function as follows:

$$Maximize \qquad C = \sum_{j=1}^{S} r x_j - \sum_{j=1}^{S} p_j x_j - \sum_{j=1}^{S} \frac{x_j E_j}{N\lambda_r}. \tag{3.24}$$

Subject to

$$m_j y_j \le x_j \le M_j y_j \qquad j = 1, \ldots, S, \tag{3.25}$$

$$\sum_{j=1}^{S} x_j \le N\lambda_r T \qquad j = 1, \ldots, S, \tag{3.26}$$

$$y_j \in \{0, 1\} \qquad j = 1, \ldots, S. \tag{3.27}$$

Model 1 is a non-linear integer optimization model with two pairs of (Q, R) decision variables to be solved, while Model 2 is an integer programming model. Apparently, this decomposition makes the multi-echelon supplier selection model solvable in a more efficient way, since the sub-models provided above reduce the scale of the problem. Thus, we implement a solution procedure which can be summarized as the following steps: (1) for each potential supplier j, solve Model 1 to achieve best (Q_{rj}, R_{rj}) and (Q_{wj}, R_{wj}) values with the minimized values of E_j; and (2) given the values of E_j for all potential suppliers, calculate the optimal supplier selection policy.

3.3 Illustrative Example and Analysis

In this section, the numerical experiments are conducted with the proposed mathematical model. The model is coded in GAMS Integrated Development Environment, and solved by Knitro commercial package for the mixed integer nonlinear model (Model 1) and Cplex for the integer programming model (Model 2), in a desktop computer with an Intel Core(TM) 2 CPU(2.00 GHz) and 4GB RAM. The main purpose for the experiments in this section is to show the solvability and the

Table 3.1: Parameter values assigned in the experiment

Parameter	T	N	λ_r	r	h	b	k	L
Value	90	20	10	100	1	3	100	1

effectiveness of the model and to demonstrate how to adopt the model for the supplier selection decision making in different scenarios.

3.3.1 Parameter Setting

In this section, the system is simulated quarterly (i.e., $T = 90$). As mentioned earlier, the number of retailers is assumed to be a sufficiently large number; we set N to be 20. There are six potential suppliers to be chosen, which may locate in different regions of the world. As a result, we study the supplier selection problem based on one firm which consists of one warehouse and twenty identical retailers. The demand rate is set to be 10 units per day. Moreover, we set the product's selling price r to be 100. The unit holding cost and the backorder cost for both the warehouse and the retailer are assumed to be 1 and 3 respectively. The fixed ordering cost from the warehouse to each retailer is considered as 100. Also the deterministic replenishment lead time from the warehouse to the retailer is set to be 1 (day). All the parameters are summarized in Table 3.1.

Table 3.2 shows additional data for each potential supplier. It is assumed that all the suppliers satisfy the buyer's qualitative criteria, each with its own various purchasing prices, fixed ordering cost, total expected order size constraint, and lead

Table 3.2: Other parameter settings related to potential suppliers

Supplier candidate j	1	2	3	4	5	6
Price (p_j)	83.0	84.0	83.5	85.0	82.5	82.0
Fixed ordering cost (O_j)	2000	1200	1000	500	3500	4500
Min. total order size (m_j)	1500	1000	2700	700	3700	4000
Max. total order size (M_j)	9500	10100	8800	21300	12920	13200
Mean of supplier lead time (μ_j)	3	2	5	2	6	7
Variance of supplier lead time (v_j)	0.2	0.5	0.5	0.1	1.5	2.0

time. Two types of potential suppliers are considered: short-range suppliers (suppliers 1-4) and long-distance suppliers (suppliers 5 and 6). As illustrated in Table 3.2, the long-distance suppliers charge less unit purchasing cost, but require more fixed ordering cost, and larger variability of the replenishment lead time.

3.3.2 Results Analysis

According to the above parameter settings, Table 3.3 displays the final selection decision. The optimal inventory policy for each level and the selected supplier order allocation along with the total expected profit are also presented. Under such settings, suppliers 1 and 3 are selected. Recall that the decision to choose a supplier or not depends not only on the cost structures (i.e., unit cost, fixed ordering cost, inventory cost, and backorder cost), but also on the restrictions for the minimum and maximum total expected order sizes. In this scenario, although the unit purchasing cost of the long-distance suppliers (suppliers 5 and 6) is the least, nothing is ordered from them because of the high ordering cost and the large mean and variance of the replenishment lead time.

Table 3.3: Decision-making variable solutions

Final selection list	Supplier 1	Supplier 3
Expected total order (x_j) (units)	9200	8800
Warehouse policy (Q_w, R_w) (units of Q_r)	(23, 7)	(19, 18)
Retailer policy (Q_r, R_r) (units)	(48, 0)	(46, 1)
Total profit ($/quarter of a year) C = 158550.1		

The model we built mainly focuses on the expected values for the selected suppliers. It does not intend to calculate the accurate total quantity to make orders from the selected suppliers, but can indicate the priority to select suppliers. Since our model is based on stochastic demand, in the real scenarios, the total order size from selected suppliers may not necessarily be the expected values calculated in Table 3.3. Besides, by calculating the average value, the company could share this information with the selected supplier so that the supplier may improve its demand forecasting accuracy. For instance, according to the results displayed in this table, supplier 3 owns higher priority to be ordered since the expected order quantity reaches its upper limit of the capacity. In stochastic demand cases, we need to purchase from supplier 3 until the total order size reaches its maximum value, and the total order size from supplier 1 will depend on the real time demand. Hence, $x_1 = 9200$ is only an average value, and it implies that supplier 1 is the secondary choice to choose when supplier 3 is available.

Considering the optimal (Q, R) policies of each selected supplier displayed in Table 3.3, we find out that the retailer's ordering policies among different selected suppliers are very similar. The main reason for this is that the replenishment quantity Q_r

affects cycle stock (i.e., inventory that is held to avoid excessive replenishment costs), and the demand rate, retailer's fixed ordering cost, and the replenishment lead time at the retailer are fixed no matter which supplier is chosen. While for the warehouse's inventory policy, different suppliers adopt diverse (Q, R) strategies since they face different lead times and fixed ordering costs which require to maintain different safety stocks and cycle stocks.

3.3.3 Simulation Verifications

The model presented in Section 3.2.2 and 3.2.3 adopts a number of simplifying assumptions. This section briefly describes a computer simulation model that is used to evaluate the accuracy of the proposed approximate model.

The simulation model is implemented in the Arena simulation software v13.9. We adopt the same assumptions and parameter settings as illustrated in Section 3.3.1. The ordering quantity and reorder points of each retailer and the warehouse inputted to the simulation model are determined through the analytical model. The purpose of the simulation is to obtain and verify the correctness of calculating the expected holding, backorder, and ordering cost in the system using the analytical method. We will not model any supplier selection process in the simulation model since our current mathematical model is more straightforward and accurate for that.

In the simulation model, the initial inventory level and inventory position are arbitrarily set to be 50 for retailers and 10 for the supplier. Such settings prevent the initial inventory status from being unrealistically empty and idle. We then warm

Table 3.4: Fit of the model: analytical vs. simulation results

Supplier	(Q, R) Policy		System cost		Relative error
	Retailer (units)	Warehouse (units of Q_r)	Analytical ($)	Simulation ($)	
1	(48, 0)	(23, 7)	1662.28	1614.72	2.86%
2	(48, 0)	(19, 4)	1507.01	1468.80	2.54%
3	(46, 1)	(19, 18)	1513.30	1474.49	2.56%
4	(46, 0)	(13, 6)	1283.15	1241.13	3.28%
5	(47, 2)	(33, 18)	1997.35	1964.06	1.67%
6	(47, 3)	(37, 21)	2151.26	2144.97	0.29%

up the simulation model to remove the influences from the initial condition. This warm up period for the simulation is determined by observing the moment when the average time-persistent inventory level begins to stabilize. In the experiments, the warm up period for the simulation is set to be 30 days in the system, while the model is run for 90 days. To obtain the time-persistent average total holding, backorder, and fix ordering cost, the model is run with 20 replications.

Table 3.4 displays the optimal (Q, R) policy calculated for each potential supplier as well as its total system costs (holding, backorder and ordering cost) according to the analytical model. Besides, the replenishment policies along with the parameter settings serve as inputs to the simulation so that the total costs based on the simulation runs are also demonstrated in the table. Observing the results in this table, the relative error between the analytical model and the simulation model is small (less than 5%). This demonstrates that the approximation adopted in the mathematical model is reasonable and acceptable.

3.3.4 Demand Rate Analysis

To study the scenarios under different demand rate, we solve four additional problem instances based on different value of λ_r, i.e., $\lambda_r = 15$, 20, 25, and 30. In this paragraph, all the parameters remain the same as the settings in section 3.3.1 except for the retailer's demand rate. The demand rate value and the optimal policy for each instance are summarized in Table 3.5. It is obvious that the expected profit increases due to the increase of demand. As the demand rate increases, more suppliers are selected since the lower cost suppliers reach their maximum capacity. This implies that we should order up to capacities of the suppliers with lower costs when we selected at least two suppliers as supply partners. This also confirms the correctness of Proposition 1. Moreover, for most of the instances, the long-distance suppliers own the least priority to be selected even though their unit price is smaller. This indicates that the demand rate does not influence a lot to the cost structure of the suppliers, and the long-distance suppliers here account for more system costs which prevent them to be selected. The table also displays the CPU times to solve the problem for each case. The time is an average value based on 10 runs. For this case with six potential suppliers, it is possible to solve the model with the commercial software package in a short amount of time.

Table 3.6 displays the optimal inventory policy in different instances, which directly corresponds to Table 3.5. Clearly, even if the same suppliers are selected in different instances, their optimal inventory policies vary a lot. The retailer's

Table 3.5: The optimal policy and expected profit based on different demand rate instance

Instance	Demand rate (λ_r)	Expected ordering quantity (x_j) (unit)	Optimal profit (C) ($)	CPU time (second)
1	10	x_1 = 9200, x_3 = 8800	158550.1	3.48
2	15	x_1 = 9500, x_2 = 8700, x_3= 8800	270959.9	3.32
3	20	x_1 = 9500, x_2 = 10100, x_3 = 8800, x_4 = 7600	385900.2	3.28
4	25	x_1 = 9500, x_2 = 10100, x_3 = 8800, x_4 = 16600	502063.0	3.39
5	30	x_1 = 9500, x_2 = 10100, x_3 = 8800, x_5 = 12920, x_6 = 12680	622634.6	4.15

replenishment order quantity and reorder point increase notably due to the increment of demand. This is due to the fact that a larger demand requires more cycle stock and safety stock to maintain low cost. Based on the running instances, it can also be observed that the retailer's order quantity in each instance is very similar among different selected suppliers. However, the retailer's reorder point may vary among different suppliers, especially for the selection of long-distance supplier (suppliers 5 and 6) in instance 5. This is because long-distance supplier with larger replenishment lead time variance which needs to hold more safety stock to avoid stockouts.

To clearly display the changes of (Q, R) policy at the warehouse for different instances, Figure 3.2 is created to display the (Q, R) policy at the warehouse when suppliers 1 and 3 are selected under different instances. Note that, in this figure, the values of order quantity and reorder point at the warehouse are based on the units of retailer's order quantity. It is undoubted to observe the increasing trend in both the order quantity and the reorder point due to the increment of demands. It can also be seen that R_w changes considerably larger than Q_w.

Table 3.6: The optimal inventory policy for selected suppliers in different demand rate instances

Instance	Selected suppliers (j)	Retailer (Q_r, R_r) (units)	Warehouse (Q_w, R_w) (units of Q_r)
1	1	(48, 0)	(23, 7)
	3	(46, 1)	(19, 18)
2	1	(57, 4)	(24, 10)
	2	(59, 3)	(19, 6)
	3	(56, 4)	(20, 23)
3	1	(68, 7)	(24, 12)
	2	(66, 7)	(20, 8)
	3	(65, 8)	(20, 27)
	4	(68, 6)	(13, 9)
4	1	(73, 11)	(25, 15)
	2	(76, 10)	(20, 9)
	3	(74, 12)	(21, 32)
	4	(74, 10)	(13, 11)
5	1	(81, 15)	(25, 17)
	2	(82, 14)	(20, 11)
	3	(79, 16)	(22, 36)
	5	(80, 19)	(36, 38)
	6	(79, 21)	(41, 45)

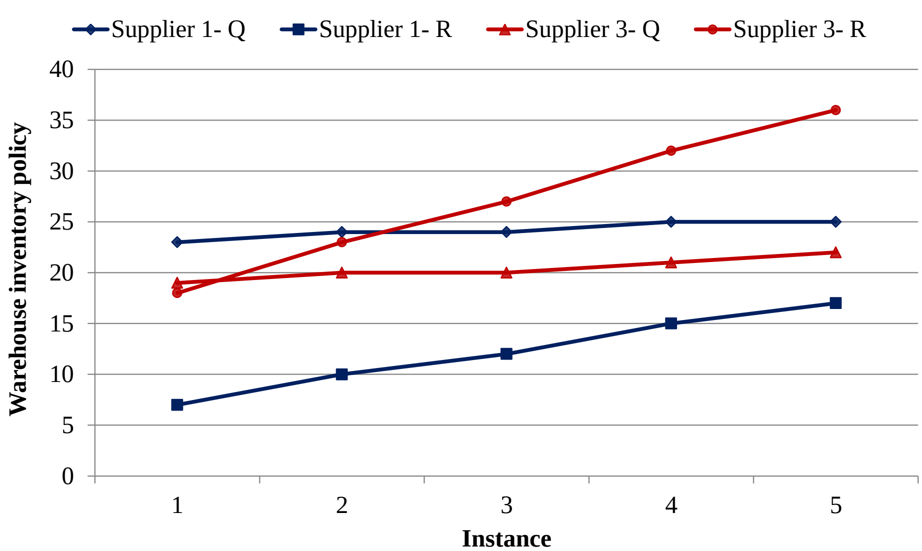

Figure 3.2: Illustration of the trend for inventory policy at the warehouse when supplier 1 and 3 are selected under different instances

3.3.5 Long-distance Supplier Analysis

Here we want to analyze the issue when selecting low-cost long-distant suppliers. In addition to high transportation cost (ordering cost), a long-distance supplier is often characterized by high delivery lead time and uncertainty. As shown in previous experiments, high lead time uncertainty results in large safety stock levels so that the expected inventory level will be high. To study the scenarios when a distant supplier should be selected and what quantity should be ordered, we conduct the experiments based on parameter changes in the long-distance supplier (price (p_j), fixed ordering cost (O_j), mean supplier lead time (μ_j) and variance of the delivery lead time (v_j) are considered). According to the parameter settings in section 3.3.1, we choose one long-distance supplier (supplier 5) to analyze, modify one parameter once at a time (other parameters remain the same), and want to analyze the impacts of the parameter on both expected ordering size x_j ($j = 5$) and total expected revenue C for different values of the demand rate instances ($\lambda_r = 10$, 15, and 20 are adopted in this paragraph). Since supplier 5 is not chosen in the final decision of these instances, by doing so, we could examine the threshold value of each parameter to involve this distant supplier in our final selection.

We first study the scenario when the variance of delivery lead time changes. Figure 3.3 illustrates the changes of expected order quantity of supplier 5 and total expected profit when its delivery lead time variance varies from 0 to 1.4 under diverse demand rates. Observing this figure, one can see that the expected total quantity purchased from the long-distance supplier decreases with increasing lead time uncertainty. The figure shows that the long-distance supplier should not be selected in some cases, especially when its lead time is highly uncertain. The lower side of figure displays the total expected profit under the same experimental settings. Clearly, it can be observed that as the variance of long-distance supplier increases, the expected profit decreases. And when the variance is big enough, the lead time variance changes will not affect the total expected profit since this long-distance supplier will not even be chosen.

Keeping variance of lead time constant, it is demonstrated in Figure 3.4 the changes of expected order quantity and total expected profit when delivery lead time (μ_5) varies from 4 to 5.5 under diverse demand rates. Note here the starting point value $\mu_5 = 4$ is set due to the assumption of positive lead time (i.e., when mean and variance of normal distribution are separately 4 and 1.5, there is less than 0.05% probability that the lead time is negative). As displayed in this figure, a similar trend can be observed as the one in Figure 3.3, which is predicable.

Figures 3.5 and 3.6 also illustrate a similar trend and pattern for the impact of unit purchasing cost and fixed ordering cost, respectively. For the unit price analysis that is displayed in Figure 3.5, it is straightforward to imagine that when the long-distance supplier reduces its unit price to a certain level, this supplier will be selected with a higher priority. In the cases when demand rate is 10, one can note that this distant supplier is selected when its unit price reduces below 81.3. Observing Figure 3.5, we can find that unit price of the long-distance supplier is very sensitive to the

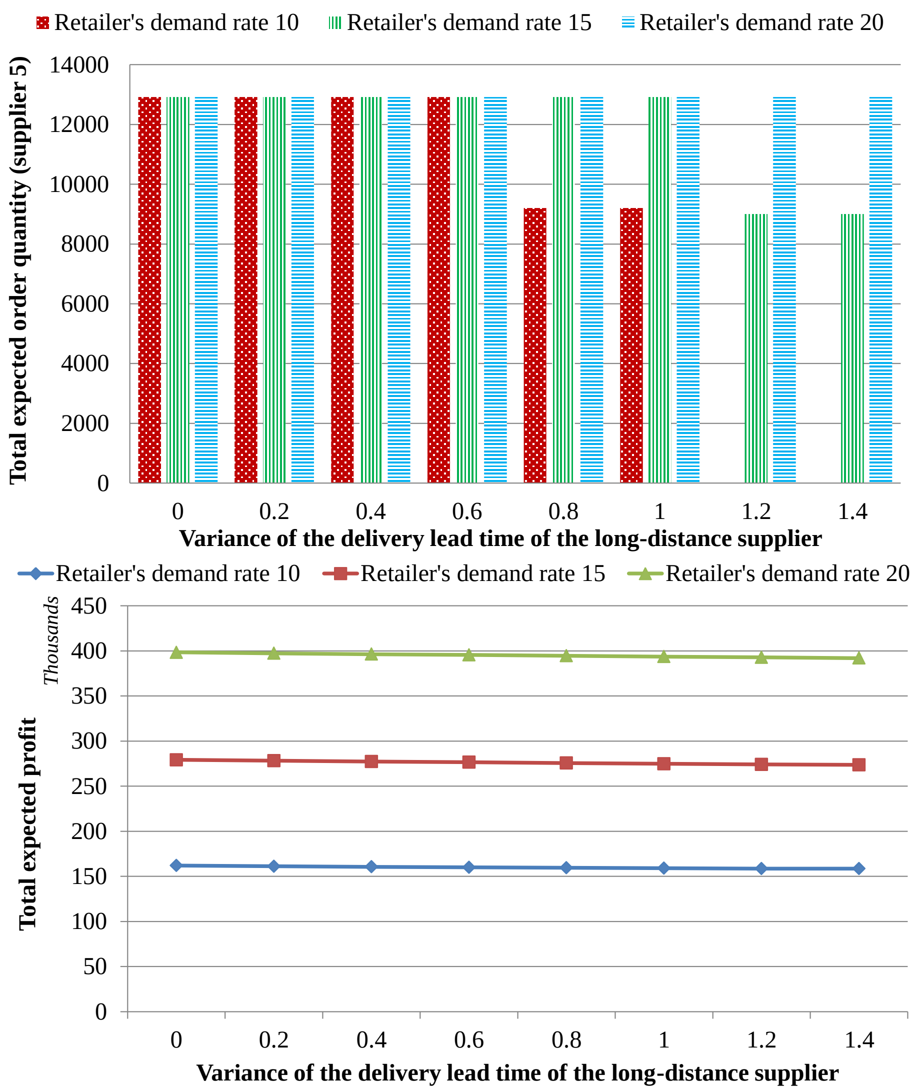

Figure 3.3: Illustration of the impact of delivery lead time variance for a long-distance supplier: total order quantity purchased from the distant supplier and the expected profit decrease with the increasing delivery lead time uncertainty

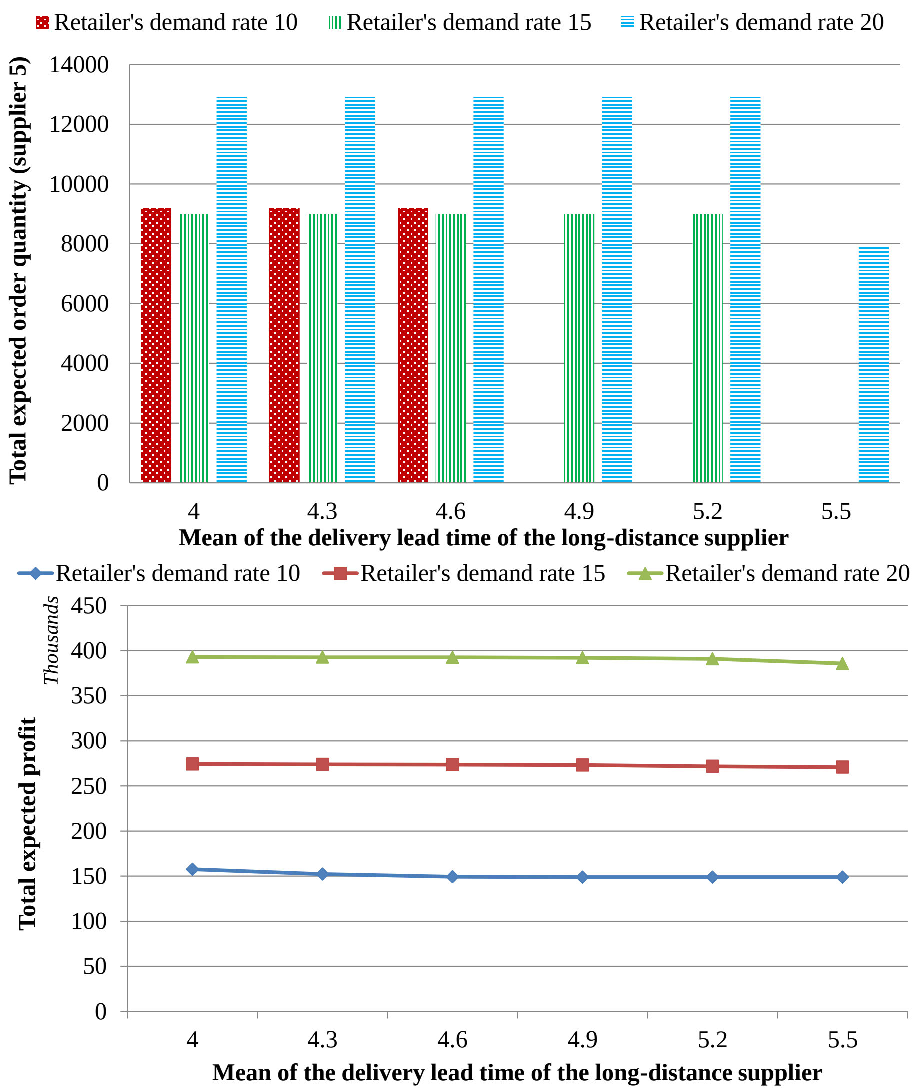

Figure 3.4: Illustration of the impact of mean delivery lead time for a long-distance supplier: total order quantity purchased from the distant supplier and the expected profit decrease with the increasing mean delivery lead time

expected profit. While for the fixed ordering cost, a very low value of fixed ordering cost will result in a more notable increase on the total expected profit than the other variables. For the instance when the demand rate is 20, a low level of fixed ordering cost where $O_5 = 500$ leads to more than \$400,000 expected profit, while a zero lead time variance case ($v_5 = 0$, see Figure 3.3) yields less profit.

3.4 Conclusions

In this chapter, we investigate a supplier selection and order allocation problem in a multi-echelon system under stochastic demand. Both the supplier selection decisions among potential suppliers and inventory control policies among one warehouse and N identical retailers are considered simultaneously. Capacity, ordering cost, unit price, holding and backorder cost are considered as the criteria for the supplier selection. A mixed integer non-linear programming (MINLP) model is proposed to select the best suppliers and determine a coordinated replenishment inventory policy at each echelon of the supply chain so that the total expected profit is maximized. To solve the model more efficiently, we decompose the mathematical model into two sub-models. Our experiments demonstrate the solvability and the effectiveness of the model. Moreover, we further investigate some issues regarding the selection of long-distance suppliers. Then, sensitivity analysis for the long-distance suppliers is conducted.

There are two limitations for the current work in this paper. First, we have adopted the no-order-splitting assumption, which requires ordering from a certain

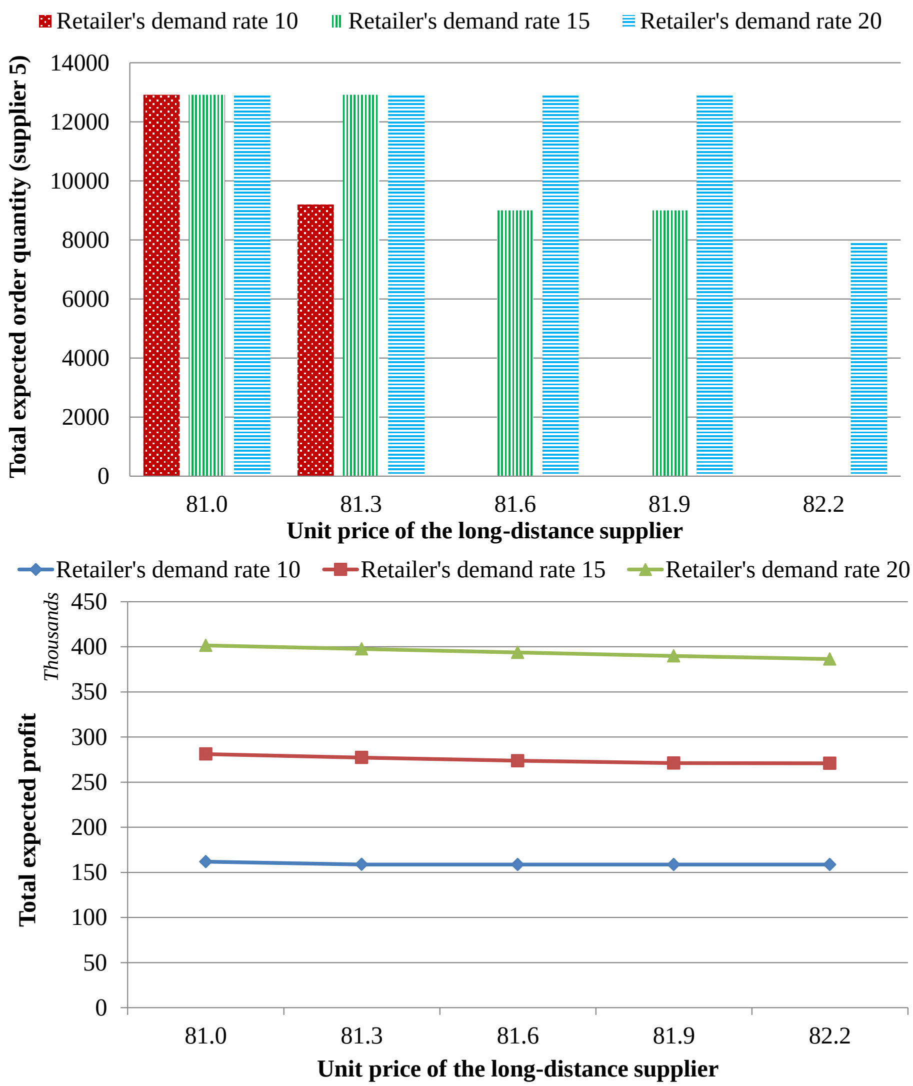

Figure 3.5: Illustration of the impact of unit price for a long-distance supplier: total order quantity purchased from the distant supplier and the expected profit decrease with the increasing unit price

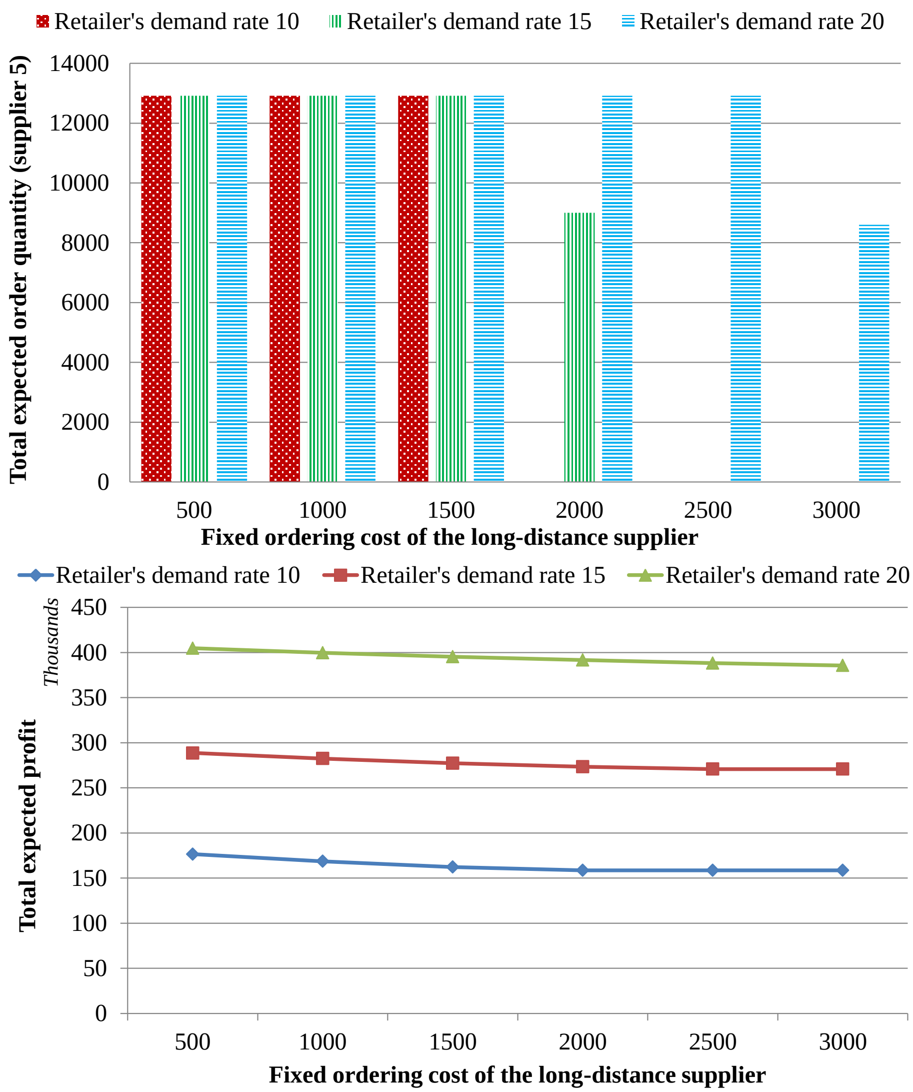

Figure 3.6: Illustration of the impact of fixed ordering cost for a long-distance supplier: total order quantity purchased from the distant supplier and the expected profit decrease with the increasing fixed ordering cost

supplier for some continues time span. In reality, a company could use the proposed model to choose strategic suppliers (a major supplier and several backups). The model can then be applied to decide the priorities to select suppliers and estimate the size to order from such suppliers. If we were to consider order-splitting for the problem under study, different models/approaches would be needed. We adopted the widely-used two-echelon inventory system assumption: i.e., the batch size and reorder point of the warehouse are the integral number of that of the retailer. Under the order-splitting setting, different suppliers may have various replenishment lead time and ordering cost, which could require different ordering policy for these retailers. This will inevitably violate the above integer-ratio policy. Thus, to avoid this dilemma, we have applied the no-order-splitting assumption. Nevertheless, the results in our experimental examples at Section 3.3.4 demonstrate similar (Q, R) polices are assigned to the retailer for different suppliers even when we considered the no-order-splitting assumption. This implies that it is possible to apply the order splitting model at the warehouse, and implement consistent replenishment policy at retailers for all the selected suppliers. Thus, future work may focus on extending order splitting model at the warehouse and the same ordering policy for the retailer. Second, to implement the stochastic demand assumption, we mainly focus on calculating the expected values of the total ordering size. The intention to use these expected values is not for signing the contract and ordering the computed amount from the selected suppliers, but for deciding the priorities to select suppliers and estimating the size to order from each supplier. Our model offers more insights to choose suppliers and allocate orders among the suppliers when considering the integration of both the supplier selection and inventory control problems in the multi-echelon system under stochastic demand.

The model has several important managerial implications. (1) Strategic partnership: the manager can use our model to select strategic suppliers based on the quantitative criteria, which provides more insights of the expected quantity to order. (2) Inventory policy: management can use the model to decide the inventory policy for the cycle, safety and transition stocks. This would give a clear indication of the amount of safety stock that needs to be hold at each location. (3) What-if analysis: the model is very flexible for sensitivity analysis for cost structures when making changes to supplier lead times, fixed ordering costs and price. Such analysis is useful when future changes are made by the suppliers.

There are several directions for the future work. First, as we mentioned above, the order-splitting model seems to be a promising direction to work on. Second, in this paper we assume the (Q, R) continuous review policy. Future work may consider a periodic review system. Moreover, our model can be extended to consider multiple products and joint replenishment costs. Finally, since the supplier selection is a typical multi-criteria decision problem, this work could be extended to multi-objective models where the trade-offs associated with these criteria can be analyzed.

Chapter 4

Optimal Order-Splitting Model for Supplier Selection

4.1 Introduction

Successful supply chain management requires an effective sourcing strategy to counteract uncertainties in both the suppliers and demands. Therefore, determining a better sourcing policy is critical in most of industries. Most of the models developed during the last decades consider the single sourcing policy, i.e., an inventory item is replenishment from a single vendor. However, there are some instances in which more than one supplier is necessary to improve the customer service time. This strategy of pooling lead time risks by splitting replenishment orders among multiple suppliers simultaneously is an attractive sourcing policy that has captured the attentions of academic researchers and corporate managers (Sazvar et al., 2014). This policy is called "order splitting".

The potential benefits for the order splitting are related to the concept of risk pooling in supply chain management (Simchi-Levi et al., 2008). Generally, risk pooling is an efficient and promising strategy to meet the challenge in supply chain management by reducing the underlying demand uncertainty through aggregation. Similarly, when the supply lead time is highly uncertain, multiple supplier model is necessary to sustain a desirable service standard. This is because multiple-supplier sources can facilitate splitting an order to counter the variability of item arrivals. Thus a significant reduction in the inventory carrying cost or shortage cost is expected, especially when lead time variability is significant (Sedarage et al., 1999). Existing literature already studied this benefit. For example, Kelle and Silver (1990) considered an n-supplier system where the lead time of each supplier has an identical Weibull distribution. The advantage of n-supplier systems compared with single-supplier systems was demonstrated, i.e., for a given safety stock level, a higher service level or a lower carrying cost can be achieved in the order-splitting model.

As discussed in Chapter 3, the experimental results imply that it is possible to apply the order splitting model at the warehouse, and implement consistent replenishment policy at retailers for all the selected suppliers. Therefore, this chapter extends the model developed in Chapter 3 by adopting the order-splitting assumptions.

4.2 Problem Definitions

4.2.1 Model Assumptions

Recall that in Figure 3.1, we assume the inventory decision is made by a single decision maker (i.e., centralized control), who wants to purchase a single type of product from a set of potential suppliers. A serial supply chain system that consists of three levels is also studied in this chapter, where raw materials and products flow sequentially through the supply chain to satisfy the customer demand. A single warehouse replenishes its inventory from a set of S selected suppliers with given lead times. It is assumed that all the suppliers in this identified set at the upper level have been pre-screened by the firm, and thus satisfy the buyer's qualitative criteria (such as service, delivery, maintenance, etc.).

We assume that this type of product is being supplied to the market at a unit price w. For each supplier j, we assume the firm has information on the unit price p_j, fixed ordering cost o_j, the unit time capacity c_j, and the reliability non-defective rate q_j representing the historical percentage of "perfect" units (i.e., $0 < q_j \leq 1$) received from the supplier. Besides, since the buyer firm would not wish to choose any supplier which has very poor historical reliability data, it is assumed that the non-defective rate is significantly larger than zero. Accordingly, the final decision will be made based on the item price, the fixed ordering cost, capacity, quality, and the inventory cost regarding choosing of the particular supplier.

Consider the case where m of S total suppliers are selected in our final decisions, as illustrated in Figure 4.1, for the warehouse, the replenishment order of the stock is made to all the selected suppliers; then the order is split simultaneously among m suppliers, and the total order quantity Q_w is given as $\sum_j^m Q_{wj}$, where Q_{wj} denotes the split order size of the selected supplier j. The warehouse then supplies the items to N independent identical retailers, where demand occurs based on a Poisson process.

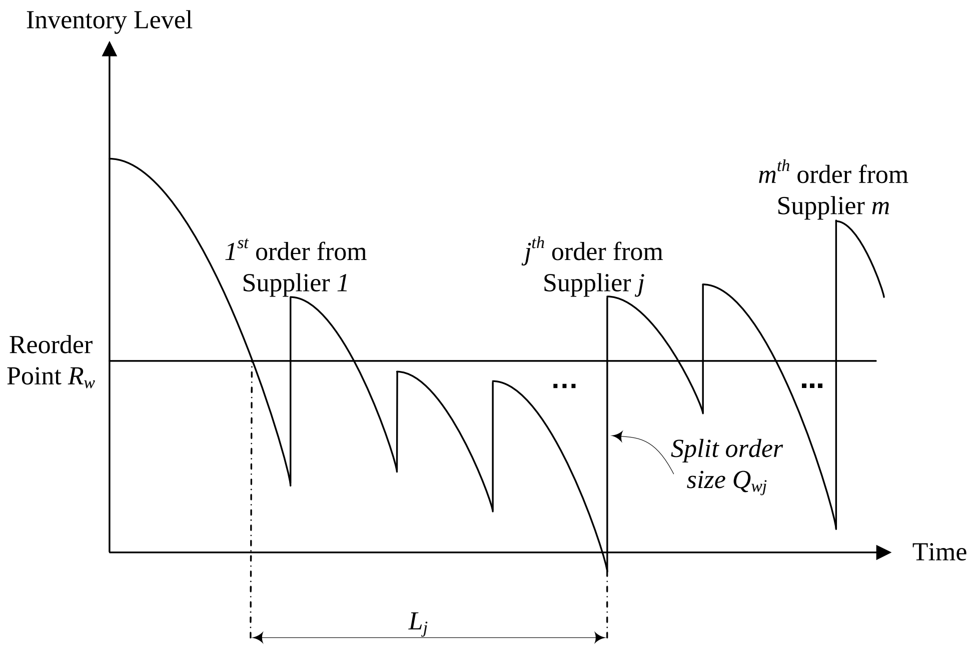

Figure 4.1: Illustration of the order split model for the warehouse

We consider an N-retailer and S-supplier system in this chapter. The multi-echelon order-splitting inventory model is built with the following assumptions.

1. The supply chain system is assumed to implement the continuous review (Q, R) policy at both the warehouse and the retailer, where the retailer adopts the same replenishment policy no matter which supplier is chosen.

(a) When the demand occurs at the retailer, it is satisfied from the retailer's available stock. Otherwise, the demand is backordered. Under this policy, the inventory position is checked continuously, when it declines to the reorder point R_r, a batch size Q_r is ordered at the warehouse. The inventory position is defined as the on hand inventory plus stock on order minus the number of outstanding backorders.

(b) After an order is placed with the warehouse, an effective lead time l_r takes place between placing the order and receiving it.

(c) After receiving the replenishment order, the outstanding backorders at the retailer are immediately satisfied based on a first-come-first-serve (FCFS) policy.

2. The warehouse maintains its own inventory based on the order-splitting continuous review (Q, R) policy.

(a) The retailer replenishment orders are satisfied if the on-hand inventory at the warehouse is greater than or equal to the retailer's order size. That is, a partial replenishment of an order at the warehouse is not allowed. This is a reasonable assumption when we consider a fixed order cost k associated with each delivery from the warehouse to the retailer.

(b) The order at the warehouse is placed when reorder point R_w is reached and will be split among m selected suppliers, and the total order quantity Q_w

is given as $\sum_j^m Q_{wj}$, where Q_{wj} denotes the split order size of the selected supplier j.

(c) The on-hand inventory level just after the final (mth) delivery from a supplier exceeds the reorder point R. This assumption is made for the classical continuous review (Q, R) system (Hariga, 2010), otherwise the system cannot form a renewal process and the decomposition analysis based on a cycle does not work.

(d) We adopt the widely-used two-echelon inventory system assumption, that is the batch size and reorder point of the warehouse are the integral number of that of the retailer (also can be found in (Bodt and Graves, 1985; Axsäter, 2003)). According to Chen and Zheng (1997), this integer-ratio order policy can facilitate quantity coordination among different facilities, and simplify packaging, transportation and stock counts.

(e) When the warehouse receives the replenishment orders from the selected suppliers, any outstanding backorders are fulfilled according to the FCFS policy as well.

3. In addition to determine the inventory policy for the warehouse and the retailer, we will not model any inventory process at the supplier. Instead, the decision maker should replenish the inventory for the warehouse and the retailer from different suppliers by performing a selection process so as to determine which

supplier to be selected, the total expected quantity that are to be procured from the selected suppliers, and the frequency in which the orders are to be received.

(a) It is assumes that each supplier locates in different places, then prices, selection costs, transportation costs, and replenishment lead times are diverse from each other.

(b) Define S different suppliers, for each supplier j, let o_j be the fixed ordering cost each time (i.e., selection cost, transportation cost, etc.), and p_j be the price of unit item from this supplier.

(c) The supplier lead time L_j is constant.

(d) Supplier j has maximum capacity c_j, and non-defective reliability rate q_j.

The objective of the proposed model is to properly select the set of suppliers which best meets capacity limits and quality requirements, allocate order-split quantity for each supplier and determine the inventory policy for stocked items, which minimize the expected total cost, consisting of the fixed ordering cost, procurement cost, inventory holding cost, and shortage cost. To solve the above problem, we build a nonlinear programming model and optimize the problem by some commercial solver.

The proposed order-splitting model is different from the study in Chapter 3. Recall that the limitation of the current non-order-splitting model is to require ordering from a certain supplier for some continues time span. The order-splitting model not only avoids this, but also considers the unit time capacity and quality constraints, which is more practical and applicable in the real world supplier selection scenarios.

4.2.2 Notations

Before we introduce the mathematical model, we now define the notations that are used throughout the paper as the following:

Constants

S: number of suppliers

N: number of retailers

m: number of selected suppliers

λ_r: demand rate at the retailer per day

l_r: replenishment lead time between the warehouse and the retailer, in days

h: holding cost per unit per day

b: backorder cost per unit per day

k: retailer's fixed ordering cost per order

q_a: minimum acceptable non-defective rate

w: selling price per unit at the market

o_j: warehouse's fixed ordering cost per order for supplier j

p_j: net purchase cost per unit from supplier j

c_j: capacity of the jth supplier per day

q_j: non-defective rate of the jth supplier

L_j: replenishment lead time between supplier j and the warehouse, in days

Decision Variables

y_j: binary variable, set to be 1 if supplier j is selected

Q_r, R_r: retailer's order quantity and reorder point, in units

R_w: warehouse's reorder point, in units of retailer batches

Q_{wj}: warehouse's split order quantity if supplier j is chosen, in unit s of retailer batches

Intermediate Variables

U_r: retailer's retard time, in days

D_r: retailer's demand during the delay time, in units

λ_w: demand rate at the warehouse per day, in units of retailer batches

t: time slot starting from the beginning of each replenishment cycle

D_w: warehouse's demand during the time t, in units

I_r: expected on-hand inventory at retailer, in units

I_w: expected on-hand inventory at warehouse, in units of retailer batches

B_r: expected backorders at retailer, in units

B_w: expected backorders at warehouse, in units of retailer batches

4.3 Model Formulation

4.3.1 The Retailer Inventory Analysis

We start with the analysis of the retailer's expected backorder level, denoted by B_r. This part is very similar as the non-order-splitting model that is illustrated in Chapter 3.2.2. Thus, we will briefly present the formulations in this section, for more details, please refer to the previous chapter.

Notice that the retailer's demand during the delay time consists of two parts: the replenishment lead time between the warehouse and the retailer, and the time between the placement of an order by the retailer and the release of a batch by the warehouse. Denote D_r as the retailer's demand during the delay time, we have

$$E[D_r] = \lambda_r(l_r + \frac{Q_r B_w}{N\lambda_r}). \tag{4.1}$$

To simplify the calculation, as elaborated in Chapter 3.2.2, we adopt normal approximation for the retailer's demand during the delay time. Thus, we have

$$V[D_r] \cong E[D_r] = \lambda_r(l_r + \frac{Q_r B_w}{N\lambda_r}). \tag{4.2}$$

Using the normal approximation, expected backorders at the retailer B_r can be computed as follows:

$$B_r = \frac{1}{Q_r}[\beta(R_r) - \beta(R_r + Q_r)], \tag{4.3}$$

$$\beta(x) = \frac{\sigma^2}{2}\{(z^2+1)[1-\Phi(z)] - z\phi(z)\}, \tag{4.4}$$

$$z = \frac{x-\theta}{\sigma}, \tag{4.5}$$

where Φ and ϕ represent the cumulative distribution function (cdf) and probability density function (pdf) of the standard normal distribution, respectively. Additionally, θ and σ are the mean and standard deviation of the demand during the delay time, which can be calculated by equation (4.2). Note here for equation (4.4), it defines the continuous analog to the second-order loss function $\beta(x)$. Thus, substituting θ and σ in the above equations, B_r can be computed in a function based on the variables Q_r, R_r, and B_w.

For the retailer's expected on-hand inventory level I_r, the same calculation is used as in equation (3.9), that is

$$I_r = \frac{Q_r+1}{2} + R_r + B_r - E[D_r]. \tag{4.6}$$

4.3.2 The Warehouse Analysis

As mentioned in Chapter 3.2.3, when N is a sufficiently large number, the Poisson process is an excellent approximation for the warehouse demand pattern. In our research, N is a sufficiently large number in each scenario, so that each retailer's order arrives approximately according to the Poisson process. This assumption is fairly reasonable, since for large retail corporation like Wal-Mart, the distribution center usually serves more than 20 stores. Under the assumption of identical retailers, it is straightforward to get the demand rate at the warehouse:

$$\lambda_w = \frac{N\lambda_r}{Q_r}. \tag{4.7}$$

In order to calculate the expected unit inventory level and backorder level, we calculate the total holding and backorder cost in a single complete replenishment cycle. As illustrated in Figure 4.2, we define a replenishment cycle as the length of time between two successive points in time where orders are placed and splitted. Since the on-hand inventory level just after the mth delivery from the final delivered supplier exceeds the reorder point R, there are no orders outstanding at the time when the inventory position reaches the reorder point R. Then, the cycle begins when the inventory position is $Q + R$, and ends when the inventory position reaches R. Therefore, the replenishment cycles can be treated as renewal cycles, and the order triggered times constitute the regeneration points of the renewal process.

In order to calculate the expected inventory and backorder level in a cycle, we study the interval between two consequent orders, and divide one cycle into a number of segments. As displayed in Figure 4.2, the defined segment [j-1, j] is the span between (j-1)th and jth arrival. Thus, if there are m suppliers to be selected, we divide the span of a replenishment cycle into m+1 segments. We then calculate the total holding/backorder level over each segment, and sum up all the segments to get the average holding/backorder level in one cycle.

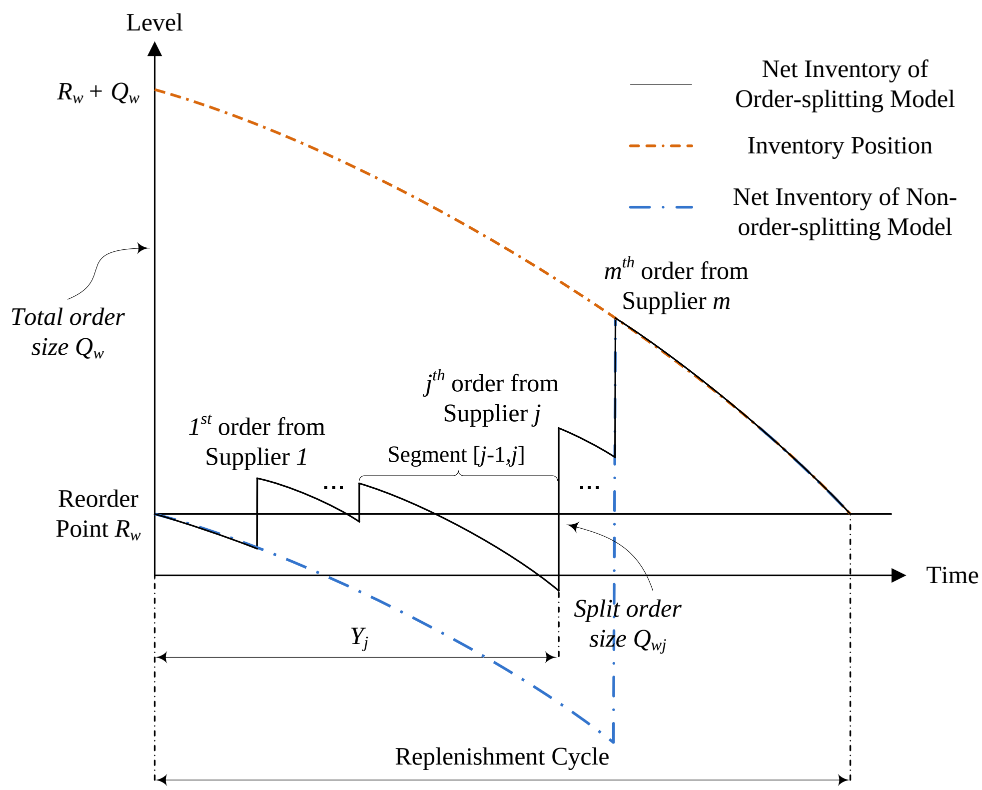

Figure 4.2: Illustration of the replenishment cycle for the order-splitting model

We first analyze the total warehouse backorder level in each segment, denoted by $T[B_{w(j-1,j)}]$, where $j = 1, ...S$. We study the scenarios where all the S suppliers are selected, so that the most general case could be considered. Let $Y_1 < Y_2 < ... < Y_S$ be the order of lead times L_1, $L_2, ..., L_S$, then Y_j is the replenishment lead time of the supplier for the jth delivery, or the time duration from the moment when the order is placed until the moment when the jth delivery is made. Thus, the expected backorder level at any given time point t between the (j-1)st and the jth deliveries, denoted by $E[B_{w(j-1,j)}(t)]$, is expressed as the follows:

$$E[B_{w(j-1,j)}(t)] = E\Big[D_w - (R_w + \sum_{k=1}^{j-1} Q_{wk})\Big]^+, \tag{4.8}$$

where t is the time slot starting from the beginning of each replenishment cycle when the warehouse place the order to the selected suppliers, and D_w denotes the warehouse demand during the time interval t. Then let $P(D_w)$ be the probability mass function (pmf) for warehouse demand during the time t, that is

$$P(D_w) = \frac{e^{-\lambda_w t}(\lambda_w t)^{D_w}}{D_w!} \tag{4.9}$$

Thus, equation (4.8) is equivalently:

$$E[B_{w(j-1,j)}(t)] = \sum_{D_w = R_w + \sum_{k=1}^{j-1} Q_{wk}}^{\infty} \Big[D_w - (R_w + \sum_{k=1}^{j-1} Q_{wk})\Big] P(D_w). \tag{4.10}$$

Let us denote $G(D_w)$ to be the cumulative distribution function (cdf) for the Poisson demand during the time interval t. Then, a simplified expression for equation (4.10) is given as

$$\begin{aligned} E[B_{w(j-1,j)}(t)] &= \Big[\lambda_w t - (R_w + \sum_{k=1}^{j-1} Q_{wk})\Big] + \lambda_w t\Big[P(R_w + \sum_{k=1}^{j-1} Q_{wk}) - G(R_w + \sum_{k=1}^{j-1} Q_{wk})\Big] \\ &\quad + (R_w + \sum_{k=1}^{j-1} Q_{wk}) G(R_w + \sum_{k=1}^{j-1} Q_{wk}). \end{aligned} \tag{4.11}$$

where λ_w is the demand rate at the warehouse, and can be computed by equation (4.7). The detailed derivations is as the following:

Proof.

$$
\begin{aligned}
E[B_{w(j-1,j)}(t)] &= \sum_{D_w=R_w+\sum_{k=1}^{j-1}Q_{wk}}^{\infty} \Big[D_w-(R_w+\sum_{k=1}^{j-1}Q_{wk})\Big]P(D_w). \\
&= \sum_{D_w=0}^{\infty}\Big[D_w-(R_w+\sum_{k=1}^{j-1}Q_{wk})\Big]P(D_w) \\
&\quad - \sum_{D_w=0}^{R_w+\sum_{k=1}^{j-1}Q_{wk}-1}\Big[D_w-(R_w+\sum_{k=1}^{j-1}Q_{wk})\Big]P(D_w) \\
&= \lambda_w t-(R_w+\sum_{k=1}^{j-1}Q_{wk}) - \sum_{D_w=0}^{R_w+\sum_{k=1}^{j-1}Q_{wk}}\Big[D_w-(R_w+\sum_{k=1}^{j-1}Q_{wk})\Big]P(D_w) \\
&= \lambda_w t-(R_w+\sum_{k=1}^{j-1}Q_{wk}) - \sum_{D_w=1}^{R_w+\sum_{k=1}^{j-1}Q_{wk}}\frac{e^{-\lambda_w t}(\lambda_w t)^{D_w-1}}{(D_w-1)!}\lambda_w t \\
&\quad +(R_w+\sum_{k=1}^{j-1}Q_{wk})\sum_{D_w=0}^{R_w+\sum_{k=1}^{j-1}Q_{wk}}P(D_w) \\
&= \lambda_w t-(R_w+\sum_{k=1}^{j-1}Q_{wk}) - \sum_{D_w=0}^{R_w+\sum_{k=1}^{j-1}Q_{wk}-1}\frac{e^{-\lambda_w t}(\lambda_w t)^{D_w}}{(D_w)!}\lambda_w t \\
&\quad (R_w+\sum_{k=1}^{j-1}Q_{wk})G(R_w+\sum_{k=1}^{j-1}Q_{wk}) \\
&= \Big[\lambda_w t-(R_w+\sum_{k=1}^{j-1}Q_{wk})\Big]+\lambda_w t\Big[P(R_w+\sum_{k=1}^{j-1}Q_{wk})-G(R_w+\sum_{k=1}^{j-1}Q_{wk})\Big] \\
&\quad +(R_w+\sum_{k=1}^{j-1}Q_{wk})G(R_w+\sum_{k=1}^{j-1}Q_{wk}). \qquad (4.12)
\end{aligned}
$$

□

The total expected warehouse backorder level in each segment can be computed by integrating the expected backorder level at any time t over the whole segment.

Thus, the following equation is developed:

$$T[B_{w(j-1,j)}] = \int_{Y_{j-1}}^{Y_j} \Big(\lambda_w t\Big[P(R_w + \sum_{k=1}^{j-1} Q_{wk}) - G(R_w + \sum_{k=1}^{j-1} Q_{wk}) + 1\Big] + (R_w + \sum_{k=1}^{j-1} Q_{wk})\Big[G(R_w + \sum_{k=1}^{j-1} Q_{wk}) - 1\Big]\Big)\ \mathrm{d}t. \quad (4.13)$$

Proposition 1. *Let* $x = R_w + \sum_{k=1}^{j-1} Q_{wk}$*, for any given parameter* $x > 0, L > 0$*, the definite integral of the expected backorder level* $E[B_{w(j-1,j)}]$ *with the upper boundary* L*, denoted by* $F(x, L)$ *is*

$$F(x, L) = \frac{e^{-\lambda_w L}(\lambda_w L)^{x+1}(\lambda_w L - x) + \big[x + (x - \lambda_w L)^2\big]\gamma(x + 1, \lambda_w L)}{2\lambda_w \Gamma(x + 1)}, \quad (4.14)$$

where Γ*, and* γ *stand for the gamma function, and the lower incomplete gamma function, respectively.*

Proof. According to (4.11), and substitute $x = R_w + \sum_{k=1}^{j-1} Q_{wk}$, we get

$$\begin{aligned} F(x, L) &= \int_0^L E[B_{w(j-1,j)}(t)]\ \mathrm{d}t \\ &= \int_0^L \Big[(\lambda_w t - x) + \lambda_w t P(x) + xG(x) - \lambda_w t G(x)\Big]\ \mathrm{d}t \end{aligned} \quad (4.15)$$

The equation (4.15) has four parts, in what follows, we separately derive each of them:

$$F_1(x, L) = \int_0^L (\lambda_w t - x)\ \mathrm{d}t$$

$$= \frac{1}{2}\lambda_w t^2 - xt.$$

For $\int_0^L \lambda_w t P(x)\mathrm{d}t$, we have

$$\begin{aligned}
F_2(x, L) &= \int_0^L \lambda_w t \frac{e^{-\lambda_w t}(\lambda_w t)^x}{x!}\ \mathrm{d}t \\
&= \int_0^{\lambda_w L} T \frac{e^{-T}T^x}{\lambda_w x!}\ \mathrm{d}T \\
&= \frac{1}{\lambda_w x!}\int_0^{\lambda_w L} e^{-T}T^{x+1}\ \mathrm{d}T \\
&= \frac{\gamma(x+2, \lambda_w L)}{\lambda_w x!}.
\end{aligned}$$

The third part $\int_0^L xG(x)\mathrm{d}t$, denoted as $F_3(x, L)$, can be addressed as

$$\begin{aligned}
F_3(x, L) &= \int_0^L x \sum_{k=0}^{x} \frac{e^{-\lambda_w t}(\lambda_w t)^x}{k!}\ \mathrm{d}t \\
&= \frac{x}{\lambda_w}\int_0^{\lambda_w L} \sum_{k=0}^{x} \frac{e^{-T}T^k}{k!}\ \mathrm{d}T.
\end{aligned}$$

Since for any positive integer x, we have

$$\Gamma(x+1, T) = x! \sum_{k=0}^{x} \frac{e^{-T}T^k}{k!},$$

thus, we have

$$F_3(x, L) = \frac{x}{\lambda_w}\int_0^{\lambda_w L} \frac{\Gamma(x+1, T)}{x!}\ \mathrm{d}T.$$

Using integration by parts, we then have

$$\begin{aligned} F_3(x, L) &= \frac{x}{\lambda_w x!}\Big[T\Gamma(x+1, T) - \Gamma(x+2, T)\Big]\Big|_0^{\lambda_w L} \\ &= \frac{x}{\lambda_w x!}\Big[\lambda_w L\Gamma(x+1, \lambda_w L) - \Gamma(x+2, \lambda_w L) + \Gamma(x+2)\Big]. \end{aligned}$$

Then the last part, $\int_0^L -\lambda_w t G(x)\ \mathrm{d}t$, can derived similarly as $F_3(x, L)$, thus

$$\begin{aligned} F_4(x, L) &= -\int_0^L \lambda_w t \sum_{k=0}^{x} \frac{e^{-\lambda_w t}(\lambda_w t)^x}{k!}\ \mathrm{d}t \\ &= -\frac{1}{\lambda_w}\int_0^{\lambda_w L} \frac{T\Gamma(x+1, T)}{x!}\ \mathrm{d}T \\ &= -\frac{1}{2\lambda_w x!}\Big[T^2\Gamma(x+1, T) - \Gamma(x+3, T)\Big]\Big|_0^{\lambda_w L} \\ &= -\frac{1}{2\lambda_w x!}\Big[\lambda_w^2 L^2\Gamma(x+1, \lambda_w L) - \Gamma(x+3, \lambda_w L) + \Gamma(x+3)\Big]. \end{aligned}$$

Finally, sum the four parts together, and convert each gamma function based on the parameter (x+1), so that equation (4.14) can be obtained. We omit the details for brevity. □

Using the above proposition, the total expected backorder level at the warehouse in each segment can be written as

$$T[B_{w(j-1,j)}] = F(R_w + \sum_{k=1}^{j-1} Q_{wk}, Y_j) - F(R_w + \sum_{k=1}^{j-1} Q_{wk}, Y_{j-1}). \tag{4.16}$$

Similarly, the total expected warehouse backorder level between the moment when the order is placed and the first delivery in this cycle, is given as

$$T[B_{w(0,1)}] = \int_0^{Y_1} E[B_{w(0,1)}(t)] \, \mathrm{d}t = F(R_w, Y_1). \tag{4.17}$$

As for the scenario of the final segment which starts from the last delivery in a cycle, and ends in the final of this cycle (or equivalently the beginning of the next cycle), we could also get

$$T[B_{w(m,0)}] \approx \int_{Y_m}^{T} E[B_{w(m,0)}(t)] \, \mathrm{d}t = F(R_w + Q_w, T) - F(R_w + Q_w, Y_m) \tag{4.18}$$

Note that this equation approximates the calculation since the cycle time at the warehouse is a random variable and we estimate its value by adopting the expected cycle time, denoted by T. Besides, as mentioned in Chapter 4.2.1, it is assumed that the on-hand inventory level just after the mth delivery exceeds the reorder point R. This indicates that when $R_w \geq 0$, the inventory level after the mth delivery is a positive number, or equivalently there is no backorder after the final delivery in a cycle, i.e., $T[B_{w(m,0)}] \approx 0$, when $R_w \geq 0$.

Recall that we approximate the demand process at the warehouse as the Poisson process, which is a renewal process with the exponentially distributed inter-arrival time, and the sum of Q_w independent random variables with the common exponentially distributed functions follows an Erlang-Q_w distribution. Thus, the replenishment cycles are the independent renewal processes, each with an Erlang inter-renewal time with Q_w stages and rate per state λ_w. Therefore, T can be calculated as

$$T = \frac{Q_w}{\lambda_w} = \frac{\sum_{k=1}^{m} Q_{wk} Q_r}{N\lambda_r}. \tag{4.19}$$

Hence, given equations (4.13) to (4.19), the expected backorder level is given as follows

$$\begin{aligned} B_w &= \frac{T[B_{w(0,1)}] + \sum_{j=2}^{m} T[B_{w(j-1,j)}] + T[B_{w(m,0)}]}{T} \\ &= \frac{N\lambda_r}{\sum_{k=1}^{m} Q_{wk} Q_r} \Bigg\{ F(R_w, Y_1) + \sum_{j=2}^{m} \Big[F(R_w + \sum_{k=1}^{j-1} Q_{wk}, Y_j) - F(R_w + \sum_{k=1}^{j-1} Q_{wk}, Y_{j-1}) \Big] \\ &\quad + F(R_w + Q_w, T) - F(R_w + Q_w, Y_m) \Bigg\}. \end{aligned} \tag{4.20}$$

Similarly as equations (4.8) to (4.11), the expected inventory level at any given time point t between the (j-1)st and the jth deliveries, denoted by $E[I_{w(j-1,j)}(t)]$, is expressed as the follows

$$\begin{aligned} E[I_{w(j-1,j)}(t)] &= E\Big[(R_w + \sum_{k=1}^{j-1} Q_{wk}) - D_w\Big]^+ \\ &= \sum_{D_w=0}^{R_w + \sum_{k=1}^{j-1} Q_{wk}} \Big[(R_w + \sum_{k=1}^{j-1} Q_{wk}) - D_w\Big] P(D_w) \\ &= \lambda_w t \Big[P(R_w + \sum_{k=1}^{j-1} Q_{wk}) - G(R_w + \sum_{k=1}^{j-1} Q_{wk}) \Big] \\ &\quad + (R_w + \sum_{k=1}^{j-1} Q_{wk}) G(R_w + \sum_{k=1}^{j-1} Q_{wk}). \end{aligned} \tag{4.21}$$

As a result, the total expected inventory level in each segment, denoted by $T[I_{w(j-1,j)}]$ can be calculated by integrating time t over the whole segment. Then, the calculation for the expected inventory level at the warehouse can be analogously derived as equation (4.20). However, in this paper, to simplify the calculation, we use the following Property to calculate the warehouse's expected inventory level I_w.

Proposition 2. *For the expected inventory level and backorder level at the warehouse, the following equality holds:*

$$I_w = B_w + \frac{Q_w}{2} + R_w - \lambda_w Y_m + \frac{\lambda_w}{Q_w} \sum_{j=2}^{m} \sum_{k=1}^{j-1} Q_{wk}(Y_j - Y_{j-1}) \tag{4.22}$$

Proof.

Method 1: According to equations (4.11) and (4.21), it is straightforward to get the following:

$$E[I_{w(j-1,j)}(t)] = (R_w + \sum_{k=1}^{j-1} Q_{wj}) - \lambda_w t + E\Big[B_{w(j-1,j)}(t)\Big].$$

The expected inventory level I_w can be expressed as

$$\begin{aligned} I_w \quad = \quad & \frac{1}{T}\Bigg\{ \int_0^{Y1} \Big(R_w - \lambda_w t + E\Big[B_{w(0,1)}(t)\Big]\Big)\, \mathrm{d}t \\ & + \sum_{j=2}^{m} \int_{Y_{j-1}}^{Y_j} \Big(R_w + \sum_{k=1}^{j-1} Q_{wj} - \lambda_w t + E\Big[B_{w(j-1,j)}(t)\Big]\Big)\, \mathrm{d}t \\ & + \int_{Y_m}^{T} \Big(R_w + Q_w - \lambda_w t + E\Big[B_{w(m,0)}(t)\Big]\Big)\, \mathrm{d}t \Bigg\} \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{T}\left\{ \int_0^T (R_w - \lambda_w t)\ \mathrm{d}t + \sum_{j=2}^{m} \int_{Y_{j-1}}^{Y_j} \sum_{k=1}^{j-1} Q_{wj} \mathrm{d}t + \int_{Y_m}^{T} Q_w \mathrm{d}t \right\} + B_w \\
&= \frac{1}{T}\left[R_w T - \frac{1}{2}\lambda_w T^2 + \sum_{j=2}^{m}\sum_{k=1}^{j-1} Q_{wj}(Y_j - Y_{j-1}) + Q_w(T - Y_m) \right] + B_w.
\end{aligned}
$$

By substituting $T = \dfrac{Q_w}{\lambda_w}$, we have

$$
\begin{aligned}
I_w &= R_w - \frac{1}{2}Q_w + Q_w - \frac{Q_w}{T}Y_m + \frac{1}{T}\left[\sum_{j=2}^{m}\sum_{k=1}^{j-1} Q_{wj}(Y_j - Y_{j-1}) \right] + B_w \\
&= R_w + \frac{1}{2}Q_w - \lambda_w Y_m + \frac{\lambda_w}{Q_w} \sum_{j=2}^{m}\sum_{k=1}^{j-1} Q_{wk}(Y_j - Y_{j-1}) + B_w.
\end{aligned}
$$

Method 2: As illustrated in Figure 4.2, the blue dotted line represents the net inventory level when the non-order-splitting model is applied to the system, where replenishment lead time is equal to Y_m in the order-splitting model. Recall that in the inventory management, the net inventory level is the on-hand inventory level minus backorder level, while the inventory position is defined as the net inventory level plus replenishment orders. Clearly displayed in Figure 4.2, the average net inventory different between the order-splitting model and the non-order-splitting model in each cycle, defined as $\triangle N[I_w]$, can be calculated as

$$
\begin{aligned}
\triangle N[I_w] &= \frac{1}{T} \sum_{j=2}^{m}\sum_{k=1}^{j-1} Q_{wk}(Y_j - Y_{j-1}) \\
&= \frac{\lambda_w}{Q_w} \sum_{j=2}^{m}\sum_{k=1}^{j-1} Q_{wk}(Y_j - Y_{j-1}).
\end{aligned}
$$

Since for the non-order-splitting model, the net inventory $N[I_w]'$ is

$$N[I_w]' = \frac{Q}{2} + R_w - \lambda_w Y_m,$$

$$\begin{aligned} N[I_w] &= \frac{Q}{2} + R_w - \lambda_w Y_m + \triangle N I_w \\ &= \frac{Q}{2} + R_w - \lambda_w Y_m + \frac{\lambda_w}{Q_w} \sum_{j=2}^{m} \sum_{k=1}^{j-1} Q_{wk}(Y_j - Y_{j-1}). \end{aligned}$$

Thus, the expected inventory level for the order-splitting model at the warehouse is expressed as

$$I_w = R_w + \frac{Q}{2} - \lambda_w Y_m + \frac{\lambda_w}{Q_w} \sum_{j=2}^{m} \sum_{k=1}^{j-1} Q_{wk}(Y_j - Y_{j-1}) + B_w.$$

This completes the proof. □

In this paper, to simplify the calculation and save the computational efforts, we use the above equality to compute the expected inventory level at the warehouse.

4.3.3 Mathematical Model

In this chapter, the order-splitting model for supplier selection and order allocation under multi-echelon inventory system with stochastic Poisson demand is presented. Denote y_j be a binary variable, where $y_j = 1$ means that supplier j is selected. Recall that each selected supplier j should satisfy the capacity constraint and the quality constraint. Thus, we may select different sets of suppliers when demand rate at the retailer changes.

The objective function of the proposed model is to maximize the expected total profit per time unit, denoted by C. It consists of several parts: the first part corresponds to the total expected revenue (i.e., the sell value w minus the purchase cost p_j) incurred by all the units purchased from selected suppliers. The second part accounts for the total *holding* and *backorder* cost. We can substitute the equations that are derived in sections 4.3.1 and 4.3.2 to get the detailed formulations. The third term is used to calculate the unit fixed *ordering* cost for the retails. While the last part represents the unit fixed ordering cost for the warehouse. Notice that the fixed ordering cost is obtained by dividing the total set up cost per order cycle, by the length of the cycle. The insight of the model is to examine the trade-offs among price, ordering, holding, and backorder costs to choose the best supplier(s) and decide the ordering policy. Based on the above-discussed assumptions and variables, the following mathematical model is developed:

$$\begin{aligned} Maximize\ \ C \ \ = \ \ & N\lambda_r\Big(w - \frac{\sum_j^S Q_{wj}p_j}{\sum_{j=1}^S Q_{wj}}\Big) - \Big[h(NI_r + Q_rI_w) + b(NB_r + Q_rB_w)\Big] \\ & -\frac{N\lambda_r}{Q_r}k - \frac{N\lambda_r}{Q_r\sum_{j=1}^S Q_{wj}}\sum_{j=1}^S y_jO_j \end{aligned} \quad (4.23)$$

Subject to

$$\frac{N\lambda_r}{\sum_{j=1}^S Q_{wj}}Q_{wj} \leq c_jy_j \qquad j = 1, \ldots, S, \quad (4.24)$$

$$\frac{\sum_{j=1}^{S} Q_{wj} q_j}{\sum_{j=1}^{S} Q_{wj}} \geq q_a, \tag{4.25}$$

$$R_r \geq -Q_r, \tag{4.26}$$

$$R_w \geq -\sum_{j=1}^{S} Q_{wj}, \tag{4.27}$$

$$Q_{rj}, Q_{wj} \geq 0 \qquad j = 1, \cdots, S, \tag{4.28}$$

$$Q_{rj}, R_{rj}, Q_{wj} \& R_{wj} : Integers \qquad j = 1, \cdots, S, \tag{4.29}$$

$$y_j \in \{0, 1\} \qquad j = 1, \ldots, S. \tag{4.30}$$

Constraint (4.24) defines the capacity constraint for the selected suppliers. As for equation (4.25), it ensures the quality constraint, where the average non-defective rate offered by suppliers should meet the minimum acceptable non-defective rate q_a. Besides, for standard cost structures (i.e., linear holding and backorder costs), it can be shown that the optimal R satisfies $R \geq -Q$. Therefore, it is assumed to satisfy in equations (4.26) and (4.27), which can limit the computation efforts. Constraints (4.28) and (4.29) are necessary, since there is no partial or fractional requests during the whole process and the minimum allowable size is zero.

4.4 Illustrative Example and Analysis

In this section, the numerical experiments are conducted with the proposed mathematical model. To solve the above constrained mixed integer nonlinear programming model, we code the mathematical model in the Matlab interactive environment, and solve the problem by the Knitro 9.0.1 commercial package, in a desktop computer with an Intel Core(TM) 2 CPU(2.00 GHz) and 4GB RAM. In particular, we implement the "active-set" algorithm in Knitro. It solves a sequence of subproblems based on a quadratic model of the problem, and implements a sequential linear-quadratic programming (SLQP) algorithm, similar in nature to a sequential quadratic programming method but using linear programming subproblems to estimate the active set (Ziena Optimization LLC, 2014).

The main purpose for the experiments in this section is to show the solvability and the effectiveness of the model and to demonstrate how to adopt the model for the supplier selection decision making in different scenarios. Subsection 4.4.1 presents the parameters that used for the experiments. In subsection 4.4.2, we elaborate the results based on the parameter setting. To evaluate the accuracy of the proposed analytical model, we develop the computer simulation model to validate the correctness of the mathematical model in subsection 4.4.3. In subsection 4.4.4, we analyze the model based on different demand rate instances. Finally, subsection 4.4.6 conduct a few sensitivity analyses for long distant supplier.

4.4.1 Parameter Setting

In this section, as mentioned earlier, the number of retailers is assumed to be a sufficiently large number; we set N to be 20. There are six potential suppliers to be chosen, which may locate in different regions of the world. As a result, we study the

Table 4.1: Parameter values assigned in the experiment

Parameter	N	λ_r	h	b	w	k	l_r
Value	20	10	1	5	100	500	1

supplier selection problem based on one firm which consists of one warehouse and twenty identical retailers. The demand rate is set to be 10 units per day. Moreover, we set the product's selling price w to be 100. The unit holding cost and the backorder cost for both the warehouse and the retailers are assumed to be 1 and 5 respectively. The fixed ordering cost from the warehouse to each retailer is considered as 500. Also the deterministic replenishment lead time from the warehouse to the retailer is set to be 1 (day). All the parameters are summarized in Table 4.1.

Table 4.2 shows additional data for each potential supplier. It is assumed that all the suppliers satisfy the buyer's qualitative criteria, each with its own various purchasing prices, fixed ordering cost, replenishment lead time, unit time capacity, and non-defective rate. Two types of potential suppliers are considered: short-range suppliers (suppliers 1-4) and long-distance suppliers (suppliers 5 and 6). As illustrated in Table 4.2, the long-distance suppliers charge less unit purchasing cost, but owns higher defect rate, charges more fixed ordering cost, and require longer replenishment lead time.

Table 4.2: Other parameter settings related to potential suppliers

Supplier candidate (S_j)	1	2	3	4	5	6
Price (p_j)	84.0	85.0	83.0	83.5	82.8	82.5
Ordering cost (o_j)	1500	1000	2000	800	4000	4800
Maximum capacity (c_j)	180	160	150	190	180	210
Supplier lead time (L_j)	2	3	3	4	6	7
Non-defective rate (q_j)	0.970	0.975	0.945	0.955	0.950	0.945

Table 4.3: Decision-making variable solutions

Final selection list	Supplier 2, Supplier 3
Warehouse splitting order quantity (Q_{wj}) (Units of Q_r)	$Q_{w1} = 6$, $Q_{w4} = 10$
Warehouse reorder point (Units of Q_r)	$R_w = 2$
Retailer (Q_r, R_r) ordering policy (Units)	(Q_r, R_r) =(101, -5)
Expected profit ($/day)	$C = 540.05$

4.4.2 Results Analysis

According to the above parameter settings, table 4.3 displays the final selection decision. The optimal inventory policy for each level and the selected supplier order allocation along with the total expected profit are also presented. Under such settings, suppliers 2 and 3 are selected. Recall that the decision to choose a supplier or not depends not only on the cost structures (i.e., unit cost, fixed ordering cost, inventory cost, and backorder cost), but also on the capacity and quality constraints. In this scenario, although the unit purchasing cost of the long-distance suppliers (suppliers 5 and 6) is the least, nothing is ordered from them because of the high ordering cost, the large replenishment lead time and the low non-defect rate.

4.4.3 Simulation Verification

The model presented in this chapter adopts a number of simplifying assumptions. This section briefly describes a computer simulation model that is used to evaluate the accuracy of the proposed analytical model.

The simulation model is implemented in the Arena simulation software v13.9. We adopt the same assumptions and parameter settings as illustrated in Section 4.4.1. The ordering quantity and reorder points of each retailer and the warehouse inputted to the simulation model are determined through the analytical model. The purpose of the simulation is to obtain and verify the correctness of calculating the expected holding, backorder, and ordering cost in the system using the analytical method. We will not model any supplier selection process in the simulation model since our current mathematical model is more straightforward and accurate for that.

In the simulation model, the initial inventory level and inventory position are arbitrarily set to be 50 for retailers and 10 for the supplier. Such settings prevent the initial inventory status from being unrealistically empty and idle. We then warm up the simulation model to remove the influences from the initial condition. This warm up period for the simulation is determined by observing the moment when the average time-persistent inventory level begins to stabilize. In the experiments, the warm up period for the simulation is set to be 30 days in the system, while the model is run for 360 days. To obtain the time-persistent average total holding, backorder, and fix ordering cost, the model is run with 20 replications.

Table 4.4: Fit of the model: analytical vs. simulation results

Scenario	(Q, R) Policy		System cost		Relative error
	Retailer (units)	Warehouse (units of Q_r)	Analytical ($)	Simulation ($)	
1	(50, 5)	(15, 14, 50, 0, 0, 10; 10)	4911.20	4955.21	0.90%
2	(70, -20)	(5, 10, 0, 22, 10, 0; -4)	4020.80	3978.6	1.05%
3	(80, 3)	(10, 10, 10, 0, 0, 0; 4)	3693.40	3708.13	0.40%
4	(101, -5)	(6, 0, 0, 10, 0, 0; 2)	2722.40	2712.30	0.37%
5	(100, -20)	(0, 24, 35, 20, 12, 8; 36)	9941.90	10069.01	1.28%
6	(20, 40)	(23, 15, 10, 8, 32, 28; 5)	6964.50	7039.90	1.08%

Table 4.4 displays the optimal (Q, R) policy calculated for each potential supplier as well as its total system costs (holding, backorder and ordering cost) according to the analytical model. Besides, the replenishment policies along with the parameter settings serve as inputs to the simulation so that the total costs based on the simulation runs are also demonstrated in the table. Observing the results in this table, the relative error between the analytical model and the simulation model is small (less than 5%). This demonstrates that the approximation adopted in the mathematical model is reasonable and acceptable.

4.4.4 Demand Rate Analysis

To study the scenarios under different demand rate, we solve three additional problem instances based on different value of λ_r, i.e., $\lambda_r = 15$, 20, and 25. In this paragraph, all the parameters remain the same as the settings in section 4.4.1 except for the retailer's demand rate. The demand rate value and the optimal order splitting policy for each instance are summarized in Table 4.5. The table also displays the CPU times

Table 4.5: The optimal policy and expected profit based on different demand rate instance

Rate (λ_r)	Warehouse ordering policy (R_w, Q_{wj}) (Units of Q_r)	Retailer ordering policy (Q_r, R_r) (Units)	Expected profit (C) ($)	CPU time (Second)
10	(R_w=2; Q_{w1}=6, Q_{w4}=10)	(Q_r=101, R_r=-5)	540.05	115.91
15	(R_w=3; Q_{w1}=7, Q_{w4}=11)	(Q_r=121, R_r=-3)	1521.82	59.68
20	(R_w=4; Q_{w1}=7, Q_{w4}=8, Q_{w6}=14)	(Q_r=142, R_r=-1)	2405.65	297.04
25	(R_w=4; Q_{w1}=8, Q_{w4}=10, Q_{w6}=13)	(Q_r=158, R_r=2)	3559.40	50.95

to solve the problem for each case. The time is an average value based on 10 runs. For this case with six potential suppliers, it is possible to solve the model with the commercial software package in a reasonable amount of time.

Observed in this table, it is obvious that the expected profit increases due to the increase of demand rate. As the demand rate increases, more suppliers are selected since the lower cost suppliers reach their maximum capacity. Besides, an interested finding is that when the demand rate increases, the chance to select the long-distance suppliers rises. The reason for this is that the demand rate is the multiplier to calculate the total expected revenue (see the first part in the objective function 4.23) so that the supplier with less unit price value takes greater advantage when the demand rate increases. Thus, the long-distance supplier will be preferable to be chosen when the system meets more customer demand.

In addition, it can be observed that even if the same suppliers are selected in different instances, their optimal inventory policies vary a lot. The retailer's replenishment order quantity and reorder point increase notably due to the increment

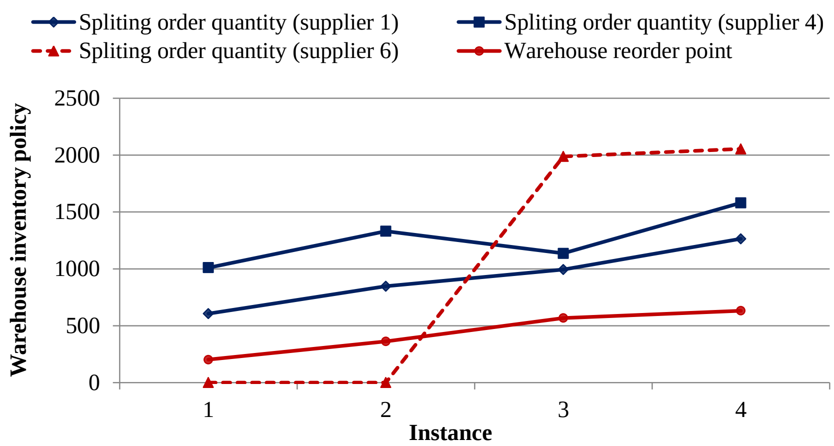

Figure 4.3: Illustration of the trend for inventory policy at the warehouse under different instances

of demand. This is due to the fact that a larger demand requires more cycle stock and safety stock to maintain low cost.

To clearly display the changes of $(Q,\ R)$ policy at the warehouse for different instances, Figure 4.3 is created to display the $(Q,\ R)$ policy at the warehouse under different instances. It is undoubted to observe the increasing trend in both the order quantity and the reorder point due to the increment of demands. It can also be seen that R_w changes considerably larger than Q_w.

4.4.5 Single Versus Multiple Sourcing

In this subsection, we want to investigate the scenario when the multiple sourcing is a dominant strategy versus single sourcing, i.e., when the order should be splitted? To illustrate this, we study the scenario where the suppliers' capacity constraints are not considered, and the single sourcing strategy is possible. Thus, except for the

Table 4.6: Investigation the effective of demand rate when suppliers is not restricted by capacity

Rate (λ_r)	Warehouse ordering policy (R_w, Q_{wj}) (Units of Q_r)	Retailer ordering policy (Q_r, R_r) (Units)	Profit (C) ($)
10	(R_w=6; Q_{w4}=14)	(Q_r=100, R_r=-5)	590.38
20	(R_w=4; Q_{w1}=7, Q_{w4}=13)	(Q_r=140, R_r=-1)	2587.73
35	(R_w=5; Q_{w1}=10, Q_{w4}=17)	(Q_r=182, R_r=8)	6019.66
100	(R_w=18; Q_{w3}=13, Q_{w4} = 13, Q_{w6}=22)	(Q_r=294, R_r=62)	23654.21
200	(R_w=21; Q_{w1}=6, Q_{w3}=17, Q_{w4}=13, Q_{w5}=15, Q_{w6}=22)	(Q_r=400, R_r=149)	52547.47

maximum capacity constraint, all the other parameters is set to be the same as the previous paragraphs in this section.

To illustrate the general trends of sourcing strategy more clearly, we investigate the effect of different demand rates. Table 4.6 displays the final supplier selection and the replenishment inventory decisions when the supplier is not under capacity constraint. It demonstrates that even if the single sourcing is possible, when the demand rate is large, more orders are splitted to more suppliers. Similar findings are also found in the literature (such as (Sedarage et al., 1999; Abginehchi and Farahani, 2010)).

4.4.6 Long-distance Supplier Analysis

Similarly as the previous chapter, we want to analyze the issue when selecting long-distant suppliers. In addition to high transportation cost (ordering cost), a long-distance supplier is often characterized by high delivery lead time. As mentioned earlier, we also assume the long-distant supplier charges less unit purchase price, but owns high defect rate. As shown in previous experiments, high lead time uncertainty results in large safety stock levels so that the expected inventory level will be high. To study the scenarios when a distant supplier should be selected and what quantity should be ordered, we conduct the experiments based on parameter changes in the long-distance supplier (price (p_j), fixed ordering cost (o_j), supplier lead time (L_j), and non-defective rate (q_j)). According to the parameter settings in section 4.4.1, we choose one long-distance supplier (supplier 6) to analyze, modify one parameter once at a time (other parameters remain the same), and want to analyze the impacts of the parameter on both splitting order quantity Q_{wj} ($j = 6$) and total expected revenue C for different values of the demand rate instances.

We first study the scenario when λ_r is 10 and 15. Since supplier 6 is not chosen in the final decision of these instances, by doing the sensitivity analysis, we could examine the threshold value of each parameter to involve this distant supplier in our final selection. We first study the scenario when the unite price changes. It is illustrated in Figure 4.4 the changes of total splitting order quantity of supplier 6 and the expected profit per unit time when its unit price changes from 80.0 to 82.0 under different demand rates. Observing this figure, it is not surprise to see that the splitting order quantity for the distant supplier decrease with the increase of unit price. The figure shows that the long-distance supplier should not be selected in some cases, when its unit price is higher than 81.5. The lower side of the figure displays the unit expected profit under the same experimental settings. Clearly, it can be observed that as the unit price of the long-distance supplier increases, the expected profit decreases. And when the unit price exceeds the threshold value so that the long-distance supplier will not be selected, the expected profit keeps the same.

Keeping other parameters constant, Figure 4.5 demonstrates the changes when the fixed ordering cost (i.e., O_6) varies from 1000 to 3500 under the instances where the demand rate is 10 and 15. Different from Figure 4.4, one can discover that the splitting order quantity increases when the fixed ordering cost rises. And when it reaches the threshold value, no more order will be splitted and assigned to this long-distance supplier. The reason for this is due to the fact that the larger fixed ordering cost requires more order quantity to avoid the total fixed ordering cost. And when the fixed ordering cost is highly large, the long-distance supplier will not necessarily be selected.

To conduct the sensitivity analysis for the non-defective rate of the long distant supplier 6, we first consider the scenario where the demand rate is 10 and 15. Nevertheless, no changes have been observed for the decisions of the proposed model. This is reasonable since the non-defective rate is not the key parameter to determine the total cost for selecting suppliers. Instead, it only affects the constraint function. Thus, when the demand rate is relatively small, the lower cost of supplier will be selected in the first place. In this subsection, in order to analyze the impact of non-defective rates, we further study the instances when the demand rate is 20 and 25. Figure 4.6 demonstrates the impact of non-defective rate for a long-distance supplier. On the upper side of the figure, it can be discovered that the less non-defective rate results in the smaller splitting order quantity. While the lower part of the figure

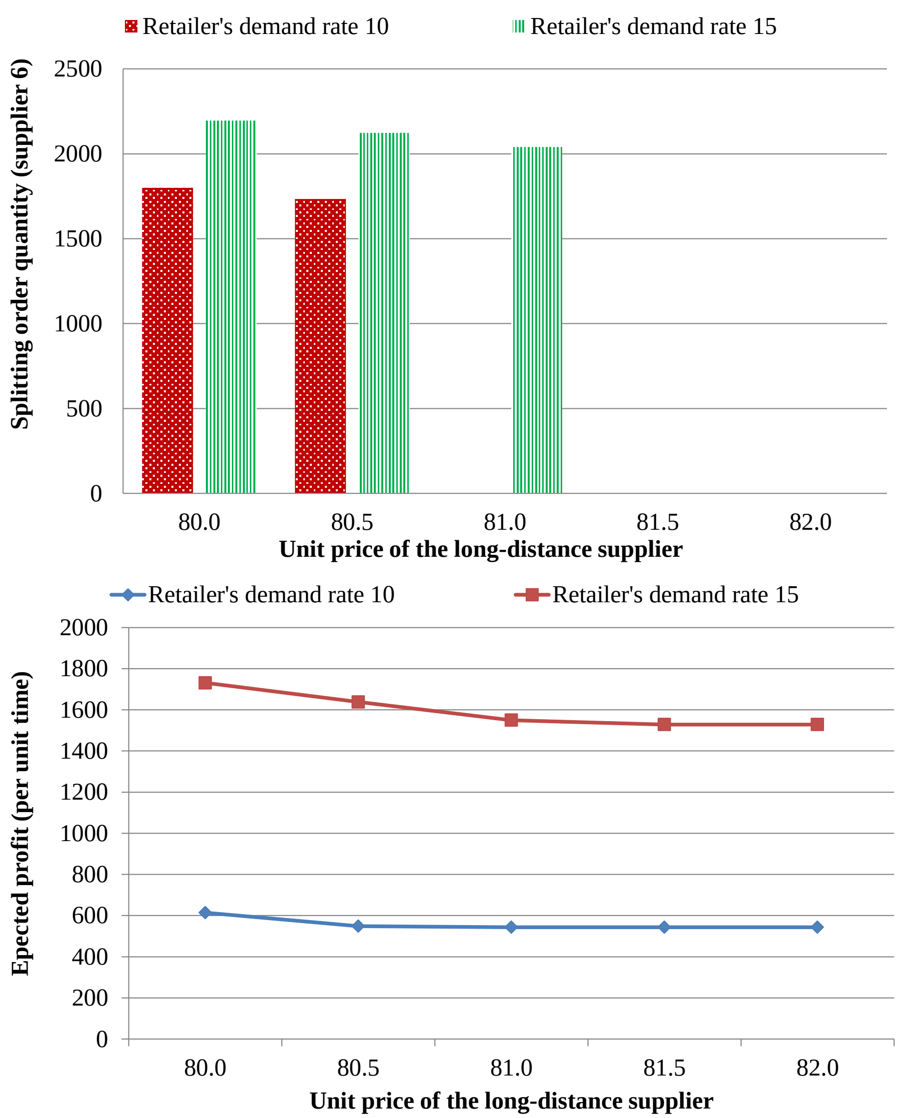

Figure 4.4: Illustration of the impact of unit price for a long-distance supplier (supplier 6): splitting order quantity for the distant supplier and the unit expected profit decrease with the increasing unit price

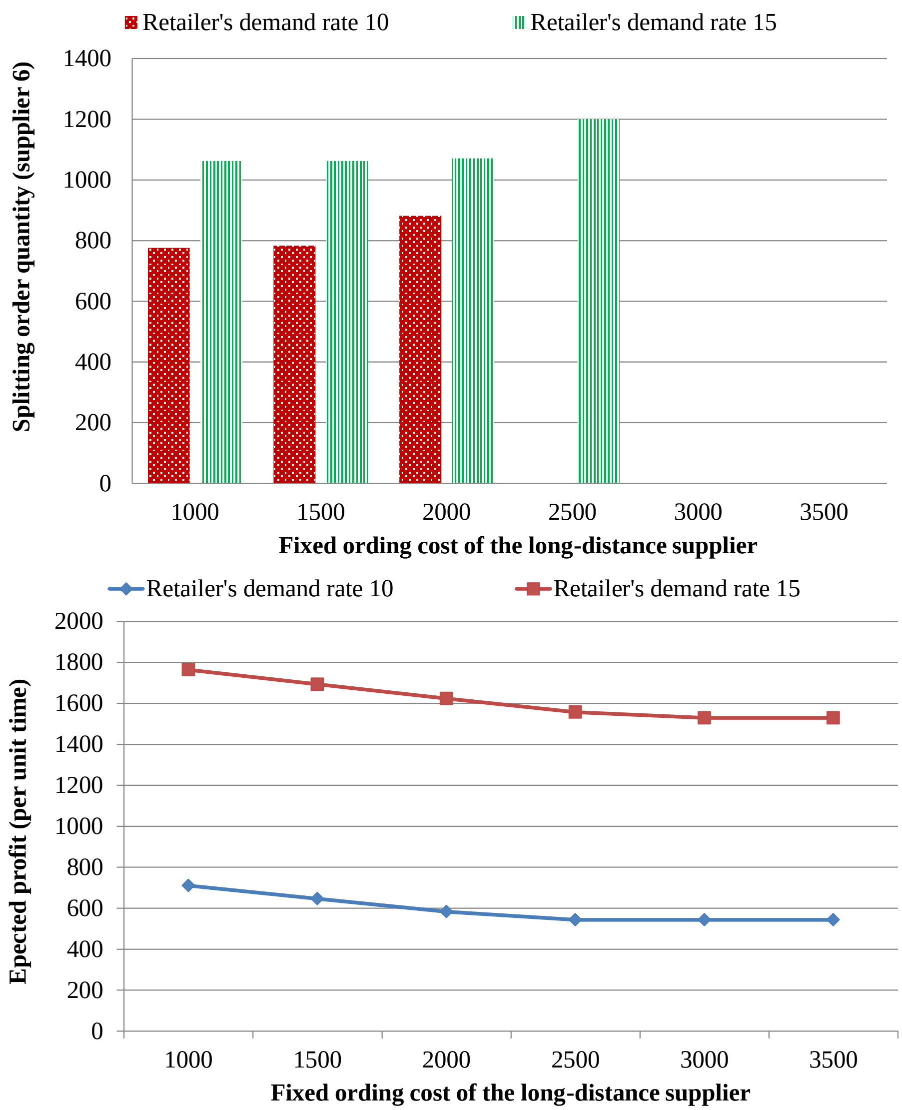

Figure 4.5: Illustration of the impact of fixed ordering cost for a long-distance supplier (supplier 6): splitting order quantity for the distant supplier increases with the increasing fixed ordering cost, and the unit expected profit decrease with the increasing fixed ordering cost

displays the similar trend as previous figures. And it is straightforward to catch the fact that the non-defective rate is not sensitive for the expected profit.

4.5 Conclusions

In this chapter, a supplier selection and order allocation problem in a multi-echelon system under stochastic demand is investigated. Both the supplier selection decisions among potential suppliers and inventory control policies among one warehouse and N identical retailers are considered simultaneously. We adopted order-splitting assumption, in which the warehouse order is split simultaneously among all the selected suppliers. In addition to the system cost, such as ordering, holding, and backorder, this chapter takes into account of capacity and reliability rate as the criteria for the supplier selection. We develop a mixed integer non-linear programming (MINLP) model to select the best suppliers and determine both the reorder point and the order-split quantities simultaneously at each echelon of the supply chain so that the total expected profit is maximized. Besides, unlike most of works in the literature, the proposed model is multi-echelon, and multi-period. To the best of our knowledge, a supplier selection model that considers all of these aspects is missing in the literature. We conduct extensive numerical experiments, which demonstrate the solvability and the effectiveness of the model. Moreover, we further investigate some issues regarding the selection of long-distance suppliers. Then, sensitivity analysis for the long-distance suppliers is conducted.

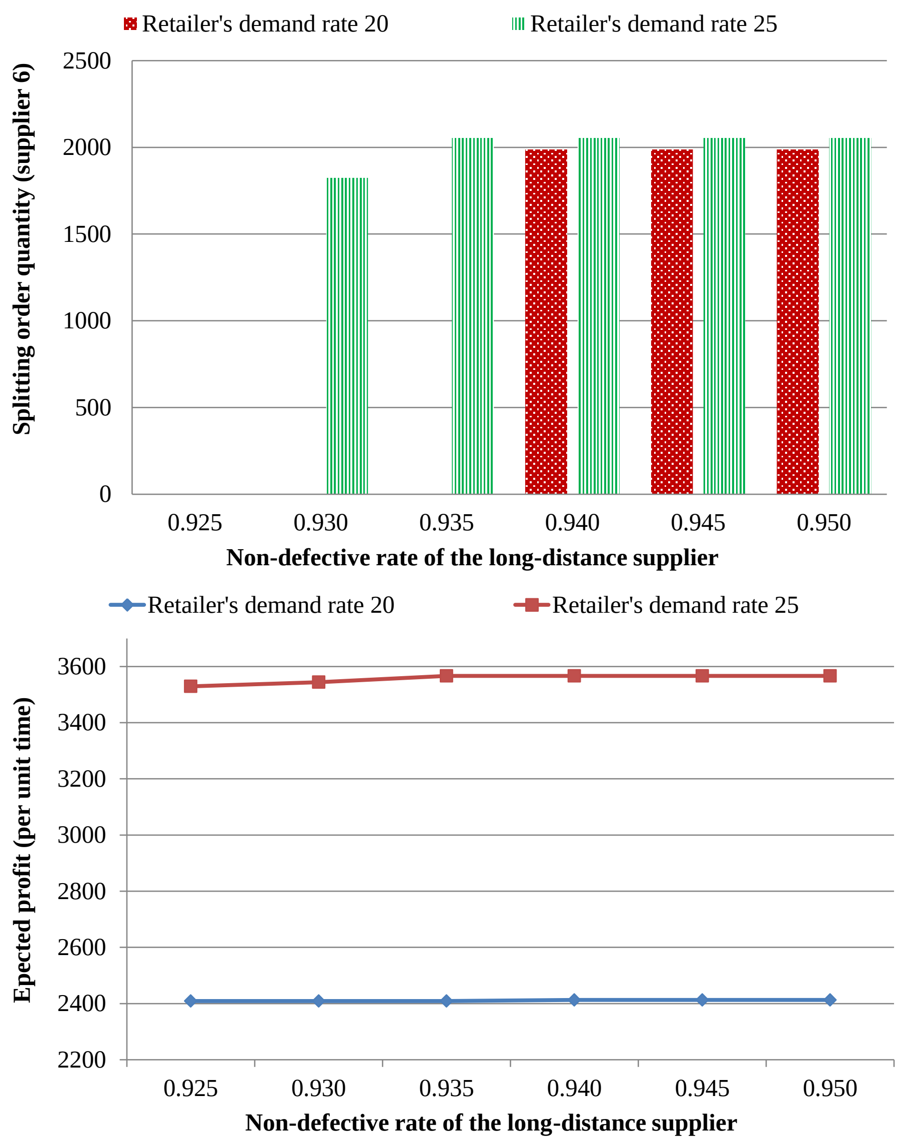

Figure 4.6: Illustration of the impact of non-defective rate for a long-distance supplier (supplier 6): splitting order quantity for the distant supplier and the unit expected profit increase with the increasing non-defective rate

There are several promising areas for future research. First, in this research, we did not consider any lead time variations between each supplier to the warehouse. Thus, one possible area of research is to consider stochastic lead times. Second, this work can also be extended to situations with multi-products, joint replenishment costs, and quantity discounts. Finally, it is also very important to develop efficient methods to solve the large size instances of the model for which the computational times with commercial software packages may be prohibitively large.

Chapter 5

Summary of the Research and Future Directions

5.1 Summary of the Research

This research addresses the importance of supplier selection and order allocation, by considering the integrated inventory control models with the supplier selection. Both the supplier selection decisions among potential suppliers and inventory control policies among one warehouse and N identical retailers are considered simultaneously. Key contributions of this research can be summarized as follows:

1. We develop a multi-echelon inventory model for the supplier selection and order allocation problem under stochastic demand. Current literature shows little research in either multi-period or multi-stage problems. To the best of our knowledge, this is the first work to tackle such a problem.

2. We study the inventory models for the non-order splitting settings, and propose a mixed integer non-linear programming (MINLP) model to solve both the supplier selection decision among potential suppliers and inventory control policies among one warehouse and N identical retailers. To solve the model more efficiently, we develop a decompose procedure.

3. We further consider the order-splitting assumption for the supplier selection and order allocation problem in a multi-echelon system. An exact analytical model is developed to select the best suppliers and determine both the reorder point and the order-split quantities simultaneously at each echelon of the supply chain so that the total expected profit is maximized.

4. To demonstrate the solvability and the effectiveness of the model, we conduct extensive numerical analysis, and further conduct simulation models to ensure and verify the correctness of the proposed mathematical model.

The first two chapters of this dissertation act as a general introduction to supply chains, inventory control models, supply selections, and analytical models used in improving supplier selection and order allocation decisions. During the literature review, we discover that even though there are many advantages to consider multi-period inventory problem in supplier selection, very limited research has been able to settle this problem. Moreover, existing papers mainly consider one stage system, nonetheless, a supply chain is a set of business that contains interactions at the various levels. Thus, the supplier selection and order allocation problem in multi-stage supply chain requires more study.

In chapter 3, we first investigate a supplier selection and order allocation problem in a multi-echelon system based on the assumption of non-order-splitting, which requires ordering from a certain supplier for some continues time span. Capacity, ordering cost, unit price, holding and backorder cost are considered as the main criteria for the supplier selection. A mixed integer non-linear programming (MINLP) model is proposed to select the best suppliers and determine a coordinated replenishment inventory policy at each echelon of the supply chain so that the total expected profit is maximized. Results in the experimental examples demonstrate similar (Q, R) polices are assigned to the retailer for different suppliers even when we considered the no-order-splitting assumption. This implies that it is possible to apply the order splitting model at the warehouse, and implement consistent replenishment policy at retailers for all the selected suppliers.

Chapter 4 extends the model to the order-splitting assumptions. We develop an exact analysis for the two-echelon multiple-supplier single-item inventory system with supplier selection, where the demand arrives according to Poisson process at the downstream retailers. The problem is to determine the reorder point and the splitting order quantity for each selected supplier so that the expected total unit profit, consisting of the fixed ordering cost, procurement cost, inventory holding cost and shortage cost, is maximized. In addition to the supplier capacity constraint, we take into account of the reliability non-defective rate as another criterion to select supplier and allocate orders among selected suppliers.

There are several important managerial implications for the models we have developed. (1) Strategic partnership: the manager can use our model to select strategic suppliers based on the quantitative criteria. (2) Inventory policy: management can use the model to decide the inventory policy for the cycle, safety and transition stocks. This would give a clear indication of the amount of safety stock that needs to be hold at each location. (3) What-if analysis: the model is very flexible for sensitivity analysis for cost structures when making changes to supplier lead times, fixed ordering costs and price. Such analysis is useful when future changes are made by the suppliers.

5.2 Future Directions

There are several directions for the future work. First, in this dissertation, we mainly assume the $(Q,\ R)$ continuous review policy. Future work may consider a periodic review system. Second, this work can also be extended to situations with multi-products, joint replenishment costs, and quantity discounts.

For the problem in Chapter 4, we mainly solve the model in the commercial software package, thus, it is also very important to develop efficient methods to solve the large size instances of the model for which the computational times with commercial software packages may be prohibitively large. Moreover, how to consider some more stochastic issues, and uncertainty risks in the order-splitting model is very promising. We have considered the fixed capacity and lead time assumptions, incorporation uncertainties in capacity, such as uncertain supply yields, is fundamental for companies to be able to develop alternative supply strategies in case of disruptions.

Finally, as a supplier selection problem, it is always interesting to study both the quantitative and qualitative methodologies, and incorporate more criteria for the problem. Additionally, since the supplier selection is a typical multi-criteria decision problem, this work could be extended to multi-objective models where the trade-offs associated with these criteria can be analyzed.

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