To the Graduate Council:

I am submitting herewith a dissertation written by Joseph M. Brocato entitled "Unanticipated Inflation and Unanticipated Money Growth and Short-Run Real Output Response: Dynamic Rational Expectations Models and Empirical Tests." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Economics.

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UNANTICIPATED INFLATION AND UNANTICIPATED MONEY GROWTH AND SHORT-RUN REAL OUTPUT RESPONSE: DYNAMIC RATIONAL EXPECTATIONS MODELS AND EMPIRICAL TESTS

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To Kathy,

Jay-Jay, Lijah-Mae, and Mimi-Mae

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educational process. My mother's moral countenance was matched only by her generous financial assistance--a fact which considerably elevated our standard of living during the austere years of graduate study.

ABSTRACT

The purpose of this study is to statistically estimate, compare, and contrast two separate Phillips curve hypotheses: short-run output response is directly related to unanticipated inflation and unanticipated monetary growth rates. A related hypothesis tested, one based upon the assumption that forces endogenous to the economic system destroy the immediacy of the effects of expectational shocks, is that the output response pattern is time distributed. Of obvious interest in this regard is a comparison of the inflation and money models' distributed lag output response patterns in terms of 1) coefficient magnitudes, and, 2) lag length.

The study uses U.S. quarterly data from 1956 through 1979. Subperiod analyses, from 1956-1967 and from 1968-1979, are also undertaken. Two indices of economic activity are used, real GNP and the employment rate. Two inflation variables, the CPI and the GNP deflator, and two nominal monetary aggregates, M1 and the monetary base, are used.

Two transmission mechanisms are invoked to explain how the unanticipated variables "cause" output: 1) for the inflation models, the Lucas Hypothesis, based upon relative/absolute price confusion and information lags, is used, and, 2) for the money models, the traditional Real Balance Effect, where unanticipated increases in nominal money lead directly to more spending and output, is used. While the inflation models can appeal directly to the Lucas Hypothesis, a judgement as to which mechanism best describes the money-to-output linkage is made inferentially by comparing the inflation

and money regression results.

Expectations and expectation formation play a major role in this study. The questions addressed in this regard are: 1) how are expectations about inflation and money growth rates formed, and, 2) how do these variables introduce a dependence of present output activity on past realizations of these expectations. A chief assumption of this study is that expectations are formed "rationally" according to Muth.

The statistical methodology used here is divided into two parts: 1) the generation of the unobservable unanticipated inflation and money growth rate variables by Box-Jenkins univariate time series methods, and, 2) the use of these constructed variables as independent regressors in distributed lag equations specifically designed to examine the lag structure connecting present output activity with past (random) forecasting mistakes. The Box-Jenkins technique is appropriate here because it conforms in important respects to the basic precepts of the Muth hypothesis and because the estimated ARIMA models can be assumed to approximate the forecasting "rule" which mimics the rational forecasting behavior of economic agents. The ARIMA models employ a varying parameter technique so as to capture any structural changes in the true process generating the actual inflation and money series over the period. To further justify the use of the univariate forecasting filters, the residuals of the ARIMA inflation and money models are subjected to extensive cross-correlation tests to determine if any lead/lag covariation can be found; in all cases the hypothesis of cross-filter residual dependency is rejected.

The major findings of this study are as follows.

1. For the 1956-1979 period, unanticipated inflation or money growth is not neutral in the short-run, but is directly related to short-run output response. This finding is substantiated using either real GNP or the employment rate as the index of real economic activity. While the results of the inflation regressions support the Lucas Hypothesis, they do not support the current claim that excessive "noise" in the price signalling mechanism over the period has resulted in a positively-sloped Phillips curve.
2. A second major finding of this study is that output response due to exogenous inflation and money shocks is time distributed, with the effects on output first rising and then falling in magnitude. This finding leads this study to conclude that the conventional (i.e., comparative static) Phillips curve is more properly specified when the statistical form by which it is estimated allows for lagged or "persistence" effects in output response patterns.
3. This study finds that unanticipated money produces a longer-lasting and more pronounced effect on real output than unanticipated inflation. [The M1/GNP and monetary base/GNP lags take thirteen and sixteen quarters, respectively, to terminate; the M1/employment rate relationship requires seventeen quarters. Conversely, unanticipated CPI or deflator inflation affects GNP for only five quarters; for the employment rate, the lag is six quarters.] These results support the conclusion that the Real Balance Effect associated with unanticipated additions to nominal money growth have a more powerful influence on output than inflation-induced changes in perceived relative prices. A related conclusion is that unanticipated money is transmitted

to output primarily through unanticipated increases in liquidity in the system, with little reliance on the Lucas price confusion linkage.

4. Over the two subperiods, 1956-1967 and 1968-1979, this study finds that real output is more responsive to unanticipated inflation and unanticipated money growth in the second period than in the first period. These results, therefore, do not support the suspicion that the inflation-unemployment or the money-unemployment "tradeoff" has all but disappeared in the U.S. in the mid- to late 1970's.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

> Though the high price of commodities be a necessary consequence of the encrease of gold and silver, yet it follows not immediately upon the encrease; but some time is required before the money circulates through the whole state . . . In my opinion, it is only in this interval or intermediate situation, between the acquisition of money and rise of prices, that the encreasing quantity of gold and silver is favourable to industry . . . After the new mass of gold and silver has been digested, and has circulated through the whole state, affairs will soon return to their former situation. [David Hume, 1711, (1)]

> The process of adaptation operates, as do the adjustments of any self-organizing system, by what cybernetics has taught us to call negative feedback: responses to the differences between the expected and the actual results of actions so that these differences will be reduced. [von Hayek, 1976, (2)]

INTRODUCTION

1.1. Purpose, Methodology and Scope of this Study

The purpose of this study is to investigate empirically the hypothesis that a positive relationship exists between unanticipated inflation and unanticipated money growth rates and real output. Implicit in this hypothesis is the assumption that the effects of price and money "impulses" on current output are distributed over time. The exact form of the distributed lag structures which properly connect past unanticipated inflation and money growth rates to current levels of economic activity is an empirical issue and comprises a main part of the investigation here. That such lagged relationships do exist is an a priori belief supported by the 1) information, multiplier, and general inertia lags that characterize the economy--all factors supporting the existence of a short-run Phillips curve, and, 2) by the empirical fact that time series measures of economic activity exhibit strong positive serial autocorrelation.

Expectations and expectation formation play a major role in this study. In this regard, the questions to be addressed are the following: 1) how are expectations about inflation and momey growth rates formed, and, 2) how do these expectations introduce a dependence of present output on past realizations of these expectations. Exactly how an expectational transmission mechanism relating past to present is structured is, again, an empirical question to be addressed in this research.

The methodology used here can be divided into two parts: 1) the generation of the unobservable unanticipated inflation and money growth rate variables by Box-Jenkins time series methods, and, 2) the use of these constructed variables in regressions specifically designed to test the lag structure between expectation (or forecast) error and real output response. Regression methods will, logically, employ the use of various forms of distributed lag techniques.

A chief assumption of this study regarding the generation of the unobservable variables is that expectations are formed "rationally" as described by Muth [3]. As will be throughly explained, the Box-Jenkins modeling procedure conforms, in important respects, to the rationality requirements of the Muth hypothesis. A mild innovation here concerns the generation of the inflation and money forecasts; time series modeling will be based upon an iterative up-dating scheme designed to produce forecasts not contaminated by future observations of the data not yet experienced by economic agents. This approach is in accord with rationally formed expectations in that new realizations of the stochastic inflation and money growth rate processes are modeled only after they enter the relevant time spectrum.

Additionally, this procedure will allow the time series models to more accurately mimic any structural changes in the true parameters generating the inflation and money growth processes.

This is an historical study using quarterly data of the U.S. from 1952/1 through 1979/2. All time series and regression models will be constructed within this period. In an attempt to accurately portray the economic history peculiar to this period, no adjustment to the data as it relates to the Korean conflict or the 1972-1973 wage and price freeze has been made. This study is not an exercise in prediction--no attempt is made to forecast outside the above stated time domain.

Two price indices are used; the Consumer Price Index (CPI), and the implicit GNP deflator. The nominal money series data will utilize M1 and the monetary base (MB). Dependent variables in the regression models, as measures of short-run output response, will be composed of the deviations of real GNP (the implicit GNP deflator is used to obtain real GNP) and the employment rate from their trend rates of growth. While a positive trend rate of growth in these variables implies a positive trend in real productive capacity in the economy no distinction will be made between actual and potential real output. Additionally, no distinction will be made between price and wage inflation, although this study will concentrate solely upon the former variable.

While it is recognized that a rational forecasting horizon is probably quite lengthy, the assumption here is that the horizon is one quarter in length--that is, expectations are formed in quarter-ahead fashion from a given base period. This assumption is plausible given the related assumption that individuals operate on a spectrum of expectations horizons spanning very short to very long periods of time. The use of a quarter-ahead forecasting horizon also stems from the optimal forecasting properties inherent in the Box-Jenkins step-ahead forecasting routine.

This study assumes that expectations are homogeneous in nature, i.e., no distinction is made between the forecasting rules of different individuals or groups of individuals. Finally, while this study is in the spirit of other research concerning the Natural Rate Hypothesis, no specification will be made regarding a "natural rate" of unemployment. Other assumptions regarding expectations and their formation will be stated in the text proper as they become pertinent to the topic under discussion.

1.2. Distinctive Features of this Study

Empirical tests of the Natural Rate Hypothesis (hereafter NRH) can be characterized as comparative static analysis; evaluation of changes in output resulting from a discrepancy between actual and expected growth rates of the price level and money supply has usually been analyzed from the standpoint that a condition of "expectational equilibrium" exists. Past literature on the NRH and related subjects has tended to be couched in long-run terms, with no analysis of how expectations adjustment and learning algorithms affect short-run market disequilibrium.

In this regard there are a number of innovations which distinguish this study from previous literature on the short- and long-run Phillips curve. First, there is a specific recognition that the effects of unanticipated inflation and money growth are not instantaneous. Simply put, movement up and to the left along a negatively sloped short-run Phillips curve related to inflation and money growth takes time, and is, therefore, an inherently dynamic process requiring dynamic analysis. The static analysis characterizing past work in this area is thus replaced by time-bound models of short-run information and perception lags regarding the rate of inflation and levels of real money balances, and by models that recognize the inertia phenomena associated with the cumulative multiplier process.

A second distinctive feature here is the use of Box-Jenkins forecasting methods to quantify the unobservable inflation and money growth rate variables via variable parameter modeling. Previous empirical work utilizing the Box-Jenkins method in inflation and money growth studies have based their forecasts on fixed parameters throughout the time spectrum under study thereby violating certain precepts of the rationality hypothesis. Relatedly, the Box-Jenkins forecasting method, by specifically quantifying the unobservable variables, breaks from previous empirical work on the NRH which has only been able to <u>infer</u> the presence of expectations and expectation formation.

A third distinctive feature here, one based upon an econometric novelty, is built upon the assumption that orthogonality exists between the unanticipated money and price variables. Existing empirical work on the output-price level-money stock nexus, steeped in the monetarist-Quantity Theory, is centered around the assumption that money and prices are related. While this study does not reject the theoretical content of this assumption,. the statistical techniques used here allow money and prices, from an expectational standpoint, to be treated as separate variables in the output regressions and will thus allow an independent analysis of inflation and money growth on the level of economic activity.

A related econometic innovation is the use of free-form distributed lag regression models. Because of the particular manner in which the independent unanticipated variables are being generated, a priori lag length specification and the problems inherent in such a procedure, can be avoided.

1.3. Outline of Topics Covered

Chapter II first looks at the historical place of expectational theories of inflation in the general macroeconomic models used in economics since J. M. Keynes' *General Theory of Employment, Interest, and Money* [4]. Here special emphasis is given to the alterations that have occurred to the Hicksian IS-LM framework so that the presence of inflationary expectations could be incorporated into the model. Here also the historical development of both the short-run Phillips curve and the NRH are related to the inflationary expectations-appended IS-LM paradigm. Chapter II ends with a thorough investigation of the theoretical underpinnings of the short-run and vertical (long-run) Phillips curve. Included here is a discussion of the economic effects of price mis-information when agents have difficulty in extracting the true relative price signal from the general level of (absolute) prices.

Chapter III presents the general form of the empirical models to be tested. These models establish the cause-effect relationships between expectation mistakes and output response. These specific output models are first, however, derived from their "Phillips curve" counterparts. The output models are then defined in distributed lag form so as to account for the fact that the output-expectations response is distributed over time. Here also the fact that real output patterns traditionally display serial autocorrelation is shown to be consistent with the distributed form of the regression models. This chapter also illustrates the proposed transmission linkages that connect the unanticipated variables to output. These are the Lucas/price confusion linkage and the Real Balance

Effect. The method to test which transmission linkage best describes the money-to-output causal chain is then described.

Chapter IV deals extensively with the theoretical and empirical place of expectations generating mechanisms in past and present macroeconomic modeling. Here the two chief expectations formation hypotheses are outlined, compared and contrasted. The Rational Expectations Hypothesis (hereafter referred to as REH) is shown to be a logical evolutionary step in expectations theorizing, stemming from the neo-Classical utility-maximizing inconsistencies of the Adaptive Expectations Hypothesis (hereafter referred to as AEH). A brief review of the use of the AEH in the literature in then given. The REH is then explained. The REH is show to be a special form of the AEH when allowance is made for the fact that economic agents will up-date the algorithm by which forecasts are made. An example is then given to show the rather severe operational requirements which must be met if the rational expectations mechanism is to have any operational significance. To contrast the different effects of the learning algorithms implied by the AEH and the REH on short-run output response patterns, a short-run Phillips curve is used to demonstrate the implications of an inflation (or money growth rate) that is fully anticipated versus one that is only partially anticipated. The chapter closes with a review of the more important empirical literature dealing with the REH through 1979.

Chapter V describes the fundamental methodology that comprises the Box-Jenkins time series modeling procedure. This is a brief description intended only to present the highpoints of the Box-Jenkins philosophy as it applies to this study. This chapter also presents the requirements the inflation and money time series data must conform to if the Box-Jenkins analysis is to be applicable to producing "rational" forecasts. An important part of Chapter V concerns the justification of the assumption that forecasts produced by Box-Jenkins models are "rational" in the Muth sense of the word. This chapter also presents and deals with some important rational forecasting methodological problems encountered when the Box-Jenkins technique is used to mimic the actions of rational forecasting agents. Important here is the use of the Box-Jenkins forecasting method when observations enter the information vector in iterative fashion.

Chapter VI identifies and estimates the two inflation and money growth rate models over the 1952/1-1979/2 period. Diagnostic checks of model adequacy are also presented, as are graphical comparisons of actual and forecasted series. In order to check for structural adequacy of the form of each estimated model, subperiod estimation is performed on each series over a period of relatively mild inflation and money growth (1952/1-1967/4) and over a period of high to severe inflation and money growth (1968/1-1979-2). These subperiod estimations also provide insight into the potency of structural changes in the stochastic process generating the inflation or money time-series. Also presented in this chapter are some interesting comparisons between the Cagan expectations mechanism for inflation and that derived here using the Box-Jenkins method. It is shown that under certain very restrictive assumptions, the Cagan model can be estimated by the Box-Jenkins technique. The chapter closes with a statistical summary of the estimated time-series models.

Chapter VII subjects the estimated inflation and money models derived in Chapter VI to further statistical tests for rationality. The chief form of analysis here is the use of Theil's mean-squared-error decomposition technique. Use is also made of prediction-realization diagrams, as originated by Theil. The remaining parts of the chapter subject the residuals from the estimated inflation and money models to certain tests for causality. Correlation of residuals is checked for both contemporaneous and non-contemporaneous dependence and the justification for the use of a transfer function versus a univariate modeling procedure considered.

Chapter VIII is a synthesis of the main findings of this dissertation. The chapter brings together the previously generated unanticipated inflation and money growth rate variables and the output variables in lagged regressions. A central concern of this chapter is the determination of the proper distributed lag length for the regression equations. The Almon lag technique is applied only after some idea of the correct lag length and coefficient pattern is gleaned from the free-form distributed lag regressions. An interesting aspect of this chapter is subperiod output pattern evaluation, where the results of the regressions run over the 1956/3-1967/4 period are compared to those of the 1968/1-1979/2 period. This analysis shows that the subperiod output response patterns are not similar. The chapter also presents evidence that implies that output does not respond with equal magnitude to unanticipated inflation and unanticipated money growth over the full period. The chapter uses the comparisons of the inflation and money models to make specific inferences regarding the transmission linkages that relate the unanticipated variables to output. The chapter closes with some possible explanations of the findings and with some general conclusions pertinent to the complete study.

CHAPTER II

INFLATIONARY EXPECTATIONS AND OUTPUT RESPONSE IN MACROECONOMIC MODELS

2.1. Historical Perspective

In recent years the phenomenon of inflationary expectations has played an increasingly important role in the formulation and testing of macroeconomic models. This fact, however, has not always been the case. The purpose of this section is to present a brief outline tracing the historical development of the incorporation of inflationary expectations into macroeconomic models, using the Hicks-Keynesian IS-LM apparatus as the point of departure.[1]

The historical roots of the contemporary macroeconomic concept of the Natural Rate Hypothesis (hereafter referred to as NRH), the Classical-like theoretical pronouncement that deviations of real output from a long-run trend are of a temporary nature only, can be traced to the Hume-Cantillon [(1),(5)] idea that "transitional inflation" could result in short-run changes in output.

A more contemporary connection with the NRH can be made via Keynes' model of aggregate demand and employment, as put forth in the General Theory (hereafter referred to as GT), in which the causes of prolonged

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[1] Inflation is defined as a sustained rise in the general level of prices. A positive rate of inflation exists when the following inequality holds: $\ln(P_t/P_{t-1}) > 0$, where P_t and P_{t-1} are the level of prices in periods t and t-1, respectively.

unemployment of resources are traced directly to a multiplier-induced cumulative contraction in spending and aggregate demand. Keynes' chief analytical innovation, in this regard, is his diagnosis of what determines the level of spending and income. He hypothesized that a certain level of employment (full or otherwise) results from the amount of consumption and investment that takes place, and there is nothing inherent in the workings of the economy which will insure that the amount of spending will be sufficient to call forth a level of output necessary to employ all resources. The theoretical emphasis of the GT was thus geared toward an explanation of the under-consumption phenomenon and the fundamental issue raised concerned the stability of a system which is dynamic to automatically return to a full-employment level after an exogenous shock to the flow of spending.

The next historical stepping-stone to the modern NRH after the GT is provided by Hicks' "suggested interpretation" of Keynes' paradigm, IS-LM analysis [6]. This approach to the Keynes macro-model was aimed at rectifying the supposedly clumsy and misleading way of presenting what is a general equilibrium model in terms of a system of unidirectional causation, running from money to investment spending via the interest rate. That is, Keynes' transmission mechanism implied a one-way chain of direction in which income is defined as the sum of consumption and investment; consumption is determined by investment through the multiplier; investment is determined by the MEC and the rate of interest; and the rate of interest is determined by liquidity preference and the quantity of money. But, as Hicks reasoned, the line of causation from money to aggregate income is not unidirectional because income and the demand for money are not independent; rather they are simultaneous in nature. Thus if one interprets the GT as a description of a general equilibrium model, the one-way money-interest rate-investment-income nexus is untenable. Specifically what Hicks was objecting to was the manner in which the GT implicitly separated the goods and the money markets, and his IS-LM apparatus was aimed at combining the real and monetary sectors of the economy through a simultaneously determined level of income and rate of interest.

There has been much debate in the literature regarding the question of whether or not Hick's IS-LM interpretation of the GT has been harmful to modern macroeconomic theorizing and stabilization policy [see Leijonhufvud (7)]. The emergence of a strong coalition of support for the NRH in the 1960's is indirect proof that the original IS-LM model, as the bulwark of Keynesian "push-button" economics, has not fared well. Additionally, the persistent nature of simultaneous unemployment and inflation during this period of time has presented the model with policy problems it is ill-equipped to handle.

Some of these criticisms strike directly at the assumptions upon which the IS-LM model is constructed. For example, the model is fundamentally a static construct, while the workings of the system it attempts to describe are dynamic in nature. Thus IS-LM models force macro-analysis to be carried out in terms of an equilibrium setting, while such phenomena as the multiplier process and the cumulative contraction or expansion of spending are basically disequilibrium states.

The work of Leijonhufvud [8] and Clower [9] extend this criticism by showing how the dynamic nature of income contraction is related to the microeconomic aspects of non-clearing markets, a facet of disequilibrium analysis which is submerged in the blanket aggregation properties of the Hicks' model. By interpreting the income-expenditure model as a microeconomic disequilibrium state, these authors provide a rationale for the multiplier, based upon individual decision-making at the household level.

A problem related to the Leijonhufvud-Clower criticism of the IS-LM framework is that concerned with the informational constraints which inhibit the smooth working out of a joint price and quantity determination. The equilibrium price vector, as these authors point out, is provided only with substantial information costs. Thus trade at false prices and the resulting unemployment follow from quantity adjusting more rapidly to exogenous shocks than prices (wages), a fact which cannot be appreciated in the full-information IS-LM context. The perfect information assumption of the model thus discourages any analysis that does not rest on the comparison of one equilibrium interest rate and income position with another. The time-path of money and goods market adjustments, which depend crucially on the information about relative prices that markets in disequilibrium create, is thus confined to two points, a beginning and ending equilibrium position, with no recognition of the effects of inadequate price or interest rate information on the process of adjustment in between.

In terms of inflation and inflationary expectations, however, the above are peripheral criticisms of the Hicks model. Truly, the major shortcoming of IS-LM analysis--one that is of crucial importance in the historical development of the NRH and the short-run Phillips curve--is the assumption that the price level is fixed regardless of the level of economic activity. Of course, such an assumption is not unreasonable in a historical sense; the GT and the Hicksian interpretation were intended to analyze an economic system in which resources are idle. Thus a constant price level and zero rate of inflation are acceptable features of post-Keynesian models of real income determination, since the assumption of price rigidity can plausibly be combined with the existence of a stock of freely available (idle) productive resources to permit any level of aggregate demand to be matched by aggregate supply at the "prevailing" level of prices. Changes in the equilibrium price vector resulting from exogenous shifts in aggregate demand and/or supply are assumed to take place so that markets are cleared; the absolute level of prices is unaffected, and is, in fact, irrelevant to the determination of real output.

The major implication of the fixed price level assumption is that it is unnecessary to distinguish real from nominal magnitudes in the model; the real levels of income, money balances and interest rates are thus insulated from the effects of inflation, and the only connection between the goods and money markets is through the rate of interest. Thus, the extremely important and difficult issues concerning 1) the price level and the rate of inflation, 2) the formation and transmission of inflationary expectations, 3) the role of changing real balances on prices and employment, and, 4) the effect of changes in the nominal rate of interest on the level of real income are by-passed by assuming that the price level is an exogenous constant. The fixed price assumption is, of course, in the spirit of the time-dimensionless aspects of IS-LM analysis, especially as it regards the formation of inflationary expectations, a process which is inherently a time-bound phenomenon.

In an evolutionary sense then, it is the flexible price feature that separates post-Keynesian from contemporary macro-models of income determination. In this regard the chief point to remember is the following: to the extent that the NRH and modern economic analysis of short-run changes in output specifically incorporate inflation and inflationary expectations within the macro-model, they offer a distinct alternative to the IS-LM model in both their appeal to the contemporary problem of inflation and unemployment, and the dynamic nature of market clearing processes when inflationary expectations become an argument in the rational decision-making processes of economic agents. In a historical sense, the concept of inflationary expectations provides the missing link that connects the static general equilibrium IS-LM apparatus with the price level-dynamic properties of the short-run Phillips curve.

The combined work of Friedman [10], Patinkin [11], Bailey [12], Mundell [13], and others in the early 1960's relaxed the parametric price level assumptions of the IS-LM model in order to make it more compatible with the economic fact of inflation witnessed during this period of time. Introducing a variable price level into the model 1) provided a theoretical niche for the inflationary expectations phenomenon, and, 2) forced a distinction between nominal and real magnitudes. The latter fact meant that a given interest rate and level of income no longer determined a specific equilibrium for the money and goods sectors, but, depending upon the price level, implied that any combination of interest rates and income levels could be consistent with simultaneous goods and money market equilibrium. Only by the addition of the full-employment real level of output (as a constraining equation) was the model fully determinant--there was only one price level at which the money and goods sectors could be in equilibrium at an endogenously determined nominal rate of interest. In this role, inflationary expectations is made endogenous to the system, and represents the "forcing factor" equating any discrepancy between real aggregate supply and nominal demand.

The re-defining of the IS-LM model in terms of a variable price level provided a sound theoretical forum for the profound monetary ideas of Friedman and Patinkin, which related inflation and inflationary expectations directly to the quantity of money and the Real Balance Effect. Their basic argument, ultimately related to the Quantity Theory, is that the public's desired holdings of real cash balances cannot be manipulated by exogenous money supply growth; a variable price level will always tend to equate desired and actual real balances so that general equilibrium can be maintained even with a growing stock of nominal balances, as long as the inflation caused by such growth is anticipated.[1] Thus, if all markets are cleared with a given stock of money, M, and a price vector, P, so will they be cleared with a stock of money kM and a price vector kP (for k > 1), so long as the inflation, kP - P, is anticipated. Moreover, the disequilibrium dynamics of the economy as described by the workings of the Real Balance Effect are such that when the money supply changes from M to kM, the economy will attain a new equilibrium price vector kP, and the price level will rise by the same proportion as the change in the nominal stock of money balances. It is through the Real Balance Effect that inflationary expectations is made a uniquely monetary phenomenon in the price variable IS-LM model.

The Phillips curve and the NRH are closely connected with the theoretical advancement the inflationary expectations-appended IS-LM model represented. By introducing the expected rate of inflation as an endogenous

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[1] Of course the quantity demanded of real cash balances will not remain constant under the influence of anticipated inflation, since inflation, by raising the opportunity cost of real balances, will lower the quantity demanded. This fact accounts for the "surge" effect in the rate of inflation due to a change in the anticipated rate of inflation [see Friedman (14)].

variable in the simultaneous solution of the goods and money markets, the new Hicks format gave a theoretical foundation for both the short-run and vertical Phillips curve by providing not only a theory of the price level, but also an expectational theory of inflation and unemployment. It is for these reasons that in the 1960's, with the growing theoretical concern of macroeconomists with expectational theories of inflation, the Phillips curve, as an explanation of short-run changes in output, gained new currency in the profession.[1]

It is interesting to note that prior to the severe inflation-proned late 1960's the Phillips curve was viewed by many economists as an empirical oddity, and, in fact, used to support the basic orthodox Keynesian approach to macroeconomic policy.[2] This approach assumed that money wages and costs were generally inflexible downward and that the adjustments to real wages thought necessary to secure full employment could be achieved indirectly via policy-manipulated increases in the price level. This philosophy is most observable in the general acceptance of the 1950's presumption that "creeping inflation" was an acceptable and necessary requirement for maintenance of full employment. However, in the late 1960's, as the inflation/unemployment relationship became more tenuous, inflationary analysts (primarily Friedman and Phelps [15]) pointed out the implausibility of using the Phillips curve as a policy tool when inflationary (and monetary) expectations are introduced into the general post-Keynesian framework. It

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[1] Part of this new acceptance is founded in the fact that inflationary expectations allowed the Phillips curve to be transformed from a mere statement of association between inflation and unemployment to a statement of cause-and-effect.

[2] The Phillips curve relationship, discovered by Fisher [16] in the 1920's, bears the name of A. Phillips [17] who rediscovered the relationship in 1958.

is through the pressing criticisms of these economists and others, as expounded in the NRH, that inflation theorists have, in the 1970's revived the short-run Phillips curve by allowing explicitly for inflationary expectations. The inflationary expectations-amended IS-LM model is an essential building-block of this rehabilitation process. And, with the work of Muth and others,[1] and the idea that expectations are formed "rationally," the short-run inflation-unemployment tradeoff was reconciled with the "no tradeoff in the long-run" conclusions embodied in the vertical Phillips curve. It is the whole rational expectations philosophy, as based upon the idea that economic agents act according to an error-learning algorithm, by which the negatively sloped Phillips curve, a short-run phenomenon, can be reconciled with the NRH, a long-run phenomenon. And it is through an implicit recognition that expectations are formed rationally that the inflationary expectations-amended IS-LM framework can incorporate the chief conclusion of the NRH--in the long-run the natural rate of unemployment and the real rate of interest are fully insulted from manipulative macroeconomic policy, especially as it regards the money supply.

This historical review implies that dynamic considerations alone separate the workings of the inflationary expectations IS-LM model from the more contemporary inflation concepts embodied in the NRH and the vertical Phillips curve. The Hicks format, although up-dated to provide a place for inflationary expectations, is still static in nature and does not provide an analysis of how expectations are formed, nor how short-

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[1] For important contributions to the rational expectations literature see Lucas [18, 19, 20, 21], Sargent [22], and Sargent and Wallace [23]. Other relevant literature will be discussed subsequently.

run changes in output can be altered by the inflation-learning process of the microeconomic unit. Rather, the final effects on output brought about by a continuing inflation are described via an "inflationary equilibrium," a state where expectations have fully adjusted to equal the actual inflation rate. Alternatively, the vertical Phillips curve provides the NRH counterpart of inflationary equilibrium, but with a distinctly more dynamic explanation of how the inflationary process and changes in the anticipated rate of inflation can affect short-run resource allocation. It is these dynamic considerations that are intimately connnected with the learning processes implied by the rational expectations concept, and it is this topic which provides the theoretical rationale for this study.

2.2. A Theoretical Basis of Short-Run Changes in Output

The above historical outline of the NRH, beginning with the standard Keynesian macro model and ending with the expectations-augmented Phillips curve, has not delved into the theoretical mechanisms relating unanticipated inflation or money growth to short-run output response. It is to these issues that we now turn. After a brief discussion of the Phillips curve in terms of 1) wages and prices, and, 2) relative/absolute price signals, an expectations-oriented model of aggregate demand and supply will be illustrated.

2.2.1. The Phillips Curve and Wages and Prices.

The empirical assumption that there is no long-run trade-off between inflation and employment is based on the recognition that a simultaneous connection exists between rates of increase in wages and prices which allows expectations to affect both rates of increase. Once the economy has achieved a steady rate of inflation and, consequently, the expected and actual rates are equal, the real variables of the system--the rate of employment in particular--are determined independently of the inflation rate. The conclusion of this hypothesis follows from its theoretical underpinnings: there is only on rate of unemployment, the "natural rate" which is consistent with any steady inflation.[1] Any attempt to keep the actual rate below the natural rate will ultimately result in ever-accelerating inflation with no change in employment or output levels.

This long-run state has a short-run corollary: only unanticipated inflation has some connection to the level of employment below the natural rate. The scenario is as follows: unanticipated inflation-induced changes in output result from lags between the increase in commodity prices and real wages. These lags, reflecting a form of money illusion for both employers and workers, provide time for unexpected increases in the inflation rate to spur new production and hiring. Workers, believing that prices will remain stable, accept a rate of increase in nominal wages which they perceive (ex ante) to be equal to the rate of increase in their real wages. Employers, simultaneously, perceive a fall in real wages (ex post) due to the apparent firm-specific increase in demand for their product. Increased hiring and production result, i.e., there is a movement up and to the left along the Phillips curve. However, as nominal wage increases are translated into higher costs of production, the anticipated increase in real wages does not materialize for workers, nor does the perceived fall in

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[1] The natural rate of unemployment is not a numerical constant, but depends on the "real" factors affecting the productive capacity of labor, such as changes in skill requirements, capital growth, market information and labor mobility, as well as other incentives which can affect the labor-leisure tradeoff.

real wages occur for employers. Employment thus moves back to its original level. Since the demand for labor is a function of the real wage, inflation can increase the amount of labor demanded only if real wages are kept below their equilibrium levels. Because commodity prices adjust more rapidly to unanticipated inflation than wages do, the necessary price-wage lag occurs. This lag can persist only as long as workers incorrectly anticipate the future inflation rate, i.e., miscalculate their future levels of real wages. When correct anticipations of the future inflation rate are incorporated into nominal wage demands, actual real wages increase and the quantity of workers demanded falls back to their pre-inflation level. What matters then, in order to induce a short-run change in output is not a change in the rate of inflation per se, but a change in the portion of inflation that is anticipated. Logically, a fully anticipated inflation, one that is placed into nominal wage contracts immediately, will have no impact on output or the levels of employment.

2.2.2. Relative versus Absolute Price Information. The above description of unanticipated inflation and output response can be placed on a more neo-Classical footing. Microeconomic theory shows that it is changes in relative prices that motivate resource allocation. Unanticipated inflation amounts to a real change in relative prices that is assumed (perceived) to last into the future. However, it is changes in the absolute price level by which information regarding relative price changes are transmitted to the microeconomic unit. When the absolute level of prices has been changing at a fairly stable rate, discriminating between relative and absolute price level shifts is easy and inexpensive. However, a volatile inflation rate obscures true relative price signals leading to price confusion and misinformation. It is, in this microeconomic context, that unanticipated inflation, as a perceived increase in relative prices that leads to short-run spurts in economic activity.

Related to the absolute/relative price level argument above is the idea of inflation rate variability (as distinguished from price level variability). In this sense it is not the change in the rate of inflation, but the volatility of that rate over time that impedes the price signaling mechanism from providing a true price picture (see Logue and Willett [24] and Park [25]). To the extent that a negatively sloped Phillips curve results from a lack of price information, it follows that an increase in the variance of the inflation rate will increase the (absolute) magnitude of the Phillips curve slope, making employment levels less responsive to changes in the inflation rate. This reasoning follows from the fact that a greater variance leads individuals to discount the magnitude of a particular price observation when it is taken from a distribution whose variance is large. Such discounting means that even a particularly high inflation rate for one period will be categorized as "anticipated" if it lies within reasonable limits of a hypothesized probability distribution, as long as that distribution is characterized by a large variance.[1]

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[1] Friedman [26] takes an extreme position in this regard by showing how a positively sloped Phillips curve, the graphical counterpart of the stagflation phenomenon, can result from increased inflation rate variability over time. Excessive variability, he contends, amounts to extreme amounts of "noise" in relative price signals, thereby negating completely the informational value of individual price signals. Both employers and workers thus withdraw from the market as a result of price confusion and uncertainty.

CHAPTER III

UNANTICIPATED INFLATION AND MONEY GROWTH AND SHORT-RUN OUTPUT RESPONSE: REGRESSION MODELS TO BE TESTED AND RELATED TRANSMISSION MECHANISMS

3.1. The Phillips Curve Foundations of the Output Models

Following the empirical work of Lucas[19] and Barro [27] and others, the regression models to be tested here posit real output response to be a function of unanticipated inflation or money growth rate variables. However, these reduced-form models are, in fact, "reversed" Phillips curve constructs in which the actual inflation or monetary growth rate is interpreted as the dependent variable in equations which make the anticipated inflation or monetary growth rate a shift parameter. Therefore, before considering the specific regression models to be estimated, it is informative to derive the output models from their traditional Phillips curve counterparts.

3.2. The Inflation Phillips Curve Model

Eq. (3.2.1.) defines the general form of the short-run Phillips curve inflation model;

$$\Delta \ln P_t = a + f[(q_t)^{-1}] + c\Delta \ln P_t^e + \varepsilon_t \quad , \tag{3.2.1.}$$

where $\Delta \ln P_t$ and $\Delta \ln P_t^e$ are the actual and expected rates of inflation at time period t, respectively. The parameter a is a long-run growth factor, and q_t is an appropriate index of output response in period t (which is positive in magnitude). ε_t is assumed to obey the classical properties

$$E(\varepsilon_t) = 0 \quad \text{, and,}$$

$$E(\varepsilon_t \varepsilon_s) = 0 \text{ if } t \neq s$$
$$= \sigma^2 \text{ if } t = s \quad .$$

Note that (3.2.1.) specifies $[c\Delta \ln P_t^e]$ as a shift factor.[1]

Eq. (3.2.1.) can be rewritten as;

(3.2.2.) $$\Delta \ln P_t - c\Delta \ln P_t^e = a + f[(q_t)^{-1}] + \varepsilon_t \quad ,$$

where the right-hand side of (3.2.2.) is a measure of unanticipated inflation. Here the coefficient c takes on special significance with respect to the effect of unanticipated inflation on output. If $c = 1$, $\Delta \ln P_t = \Delta \ln P_t^e$, and inflation is "fully anticipated." Thus short-run output response is zero except for the random disturbance, ε_t, and the growth factor a. When $c = 0$, inflation is completely unanticipated which implies;

(3.2.3.) $$\Delta \ln P_t^u = \Delta \ln P_t - c\Delta \ln P_t^e = \Delta \ln P_t \quad ,$$

where $\Delta \ln P_t^u$ is the rate of unanticipated inflation in time period t. In this case $\Delta \ln P_t = f[(q_t)^{-1}]$, given the constant a and assuming $E(\varepsilon_t) = 0$. In this case output responds fully to a change in the actual rate of inflation as determined by the function $f[(q_t)^{-1}]$.

The final possibility is $0 < c < 1$, in which there is some anticipated and some unanticipated inflation. In this case the following conditions hold;

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[1] Eq. (3.2.1.) is the general form of the regression model used in tests of the NRH. If the estimate of the parameter c is close to unity and is statistically significant, the NRH is confirmed. See Lucas [19].

(3.2.4.) $\Delta \ln P_t^u = \Delta \ln P_t - \Delta \ln P_t^e > 0$, and,

(3.2.5.) $f[(q_t)^{-1}] > 0$.

Conditions (3.2.4.) and (3.2.5.) imply that the magnitude of the slope of the Phillips curve depends not only on the function $f[(q_t)^{-1}]$, but also on the coefficient c.

3.3. The Money Phillips Curve Model

The traditional Phillips curve is defined in terms of price or wage inflation only. The preceding discussion, however, has implied that short-run movements in output can result from monetary factors, specifically unanticipated money growth. While the effects of money growth on output can be inferred from the workings of the Real Balance Effect (RBE), a monetary expectations-based Phillips curve cannot be gleaned directly from the Effect since the phenomenon is not based upon expectations. The RBE, however, can be altered to include an expectations component thereby providing the rationale for a "money" Phillips curve.

Consider the following money/price level conditions that hold during an inflationary equilibrium;

(3.3.1.) $\Delta \ln M_t = \Delta \ln P_t$,

(3.3.2.) $\Delta \ln P_t = \Delta \ln P_t^e$,

and,

(3.3.3.) $\Delta \ln M_t = \Delta \ln P_t^e$, where M_t is the quantity of nominal money balances existing at time period t. These conditions allow eq. (3.2.1.), the "inflation" Phillips curve, to be written in the following form when c = 1, i.e., an inflationary equilibrium exists;

(3.3.4.) $$\Delta \ln M_t = a + f[(q_t)^{-1}] + \Delta \ln P_t^e + \varepsilon_t \quad .$$

Now, assume that expectations about the growth rate of nominal balances are formed according to the following "rule";

(3.3.5.) $$\Delta \ln M_t^e = \text{"expected money growth rate"} \quad .$$

The question here is the following: What must be the expected nominal money growth rate during an inflationary equilibrium? We know that during such a state, desired and actual real money balances are equal. Therefore, using (3.3.1.), the following equality must hold;

(3.3.6.) $$(m/p)_t^* = (m/p)_t^a = k = 1 \quad ,$$

where the lower-case letters indicate the growth rates of nominal money balances and prices, respectively and "*" indicates desired and "a" the actual "real balance ratio."

An "expected" real balance growth rate ratio can now be defined as;

(3.3.7.) $$(m/p)_t^e \quad ,$$

which implies that either $\Delta \ln M_t^e \gtreqless \Delta \ln P_t^e$. However, during an inflationary equilibrium the following equality must hold;

(3.3.8.) $$\partial (M/P)_t^a / \partial (\text{time}) = 0 \quad ,$$

where the ratio (M/P) defines the stock of real money balances at time period t. Since the level of real balances is constant during an inflationary equilibrium, the expected growth rate of nominal money balances must equal the actual growth rate of the price level. Or, since (3.3.2.) holds for an inflationary equilibrium, the following equality must hold also;

(3.3.9.) $$\Delta \ln M_t^e = \Delta \ln P_t \quad .$$

Combining (3.3.2.) and (3.3.9) gives;

(3.3.10.) $$\Delta \ln M_t^e = \Delta \ln P_t^e \quad .$$

Equality (3.3.6.) and equality (3.3.10.) can then be expressed as follows;

(3.3.11.) $$(m/p)_t^* = (m/p)_t^a = (m/p)_t^e = k = 1 \quad .$$

Equality (3.3.11.) follows from the mechanics of the RBE during an inflationary equilibrium state. That is, if real money balances are growing at a rate greater than expected, then the growth rate of the price level must be lagging the actual growth rate of the nominal money stock and the increasing level of real balances must, in turn, lead to increased spending. It is the real balance-induced inflationary disequilibrium of such spending which ultimately raises the price level growth rate and forces the expected real balance ratio into equality with the desired and actual real balance ratios, as outlined in (3.3.11.) (The transmission mechanism connecting nominal money balance growth to real output via real balance growth will be precisely outlined in Section 3.5. to follow.)

Using equalities (3.3.3.), (3.3.10.) and (3.3.11.), a short-run Phillips curve model can be defined in terms of actual and expected money balances alone;

(3.3.12.) $$\Delta \ln M_t = a' + g[(q_t)^{-1}] + c'\Delta \ln M_t^e + \varepsilon_t \quad ,$$

where,

(3.3.13.) $$\Delta \ln M_t^u = \Delta \ln M_t - c'\Delta \ln M_t^e = a' + g[(q_t)^{-1}] + \varepsilon_t \quad ,$$

and $\Delta \ln M_t^u$ defines the rate of unanticipated nominal money growth in time period t.

Eq. (3.3.12.) is a variation of eqs. (3.2.1.) and (3.2.2.) in that it makes output response a function of an expectations variable. However, (3.3.12.) is distinctive in that unanticipated money growth is transmitted directly to output without any specific reliance on price level expectations or on price level "confusion." This feature, as mentioned earlier, is in conformity with the orthogonal construction of the unanticipated inflation and money growth rate variables.[1]

Note that the coefficient c' in (3.3.12.) and (3.3.13.) has the same interpretation as does the coefficient c in the inflation Phillips curve of (3.2.1.); when c' is equal to unity, unanticipated money growth is zero and (expected) output respone is zero; when c' equals zero, the actual money growth rate is completely unanticipated, and output responds in full to a change in the growth rate of the nominal money stock as determined by the function $g[(q_t)^{-1}]$; when $0 < c' < 1$, a change in the

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[1] Again, it should be emphasized that the uncorrelated nature of the price and money expectations variables is peculiar to the time series methods employed in this study; it is not meant to deny the monetarist thesis which holds that a sustained inflation cannot be independent of nominal money stock growth. It is for this reason that the expectations money model is derived with specific reference to the conditions that hold during an inflationary equilibrium, since this state is based upon conditions unique to both expected money and price level growth rates.

actual growth rate of nominal balances is partially reflected in a change in output and a change in the anticipated growth rate of nominal balances. This format, unlike the traditional Equation of Exchange approach to the money-prices-output transmission linkage shows that only the unanticipated portion of nominal money growth can bring about a RBE.

3.4. General Form of the Output/Inflation and Output/Money Regression Models to be Tested

This section presents the "reversed" forms of the inflation and money Phillips curve models (3.2.2. and 3.3.13.) derived in the preceding section.

Before presenting these models it is necessary to state an assumption about the inflation and money growth rate variables. This study assumes that the time series realizations of the actual inflation and actual money growth rates can be divided into anticipated and unanticipated components. For the inflation rate this decomposition is;

$$\Delta \ln P_t = \Delta \ln P_t^e + [\Delta \ln P_t - \Delta \ln P_t^e] \quad , \tag{3.4.1.}$$

where $\Delta \ln P_t^e$ and $[\Delta \ln P_t - \Delta \ln P_t^e]$ are the magnitudes of the anticipated and unanticipated rates of inflation occurring in period t, respectively.

In like fashion, the actual realization of the nominal money growth rate in period t can be divided into anticipated and unanticipated components;

$$\Delta \ln M_t = \Delta \ln M_t^e - [\Delta \ln M_t - \Delta \ln M_t^e] \quad . \tag{3.4.2.}$$

Note that both $\Delta \ln P_t^e$ and $\Delta \ln M_t^e$ are formed in period t-1. Also note that equations (3.4.1.) and (3.4.2.) show that if the actual variable equals the expected variable, the unanticipated components must equal zero.

The contemporaneous forms of the output/unanticipated inflation and output/unanticipated money growth models are shown in eqs. (3.4.3.) and (3.4.4.);

$$q_t = \beta[\Delta \ln P_t - \Delta \ln P_t^e] + \varepsilon_t \quad , \qquad \beta > 0 \tag{3.4.3.}$$

$$q_t = \lambda[\Delta \ln M_t - \Delta \ln M_t^e] + \varepsilon_t \quad , \qquad \lambda > 0 \tag{3.4.4.}$$

where the unanticipated inflation or money growth rate variables occurring in period t appear in brackets. The disturbance terms are assumed to obey the classical properties. The output response variable, q_t, is stated as deviations from log-linear trend. [Note: unless otherwise stated, all subsequent references to output magnitudes for the remainder of this study are in terms of deviations from log-linear trend. The exact derivation of these output variables (real GNP and the employment rate) will be given in Chapter VIII.]

Note the following: 1) eqs. (3.4.3.) and (3.4.4.) are homogeneous of degree zero in actual and expected variables; 2) if the inflation or money growth rate is "completely anticipated" (i.e., if the forecast of the variable for period t, formed in period t-1, equals the actual variable, in an ex post sense, occurring in period t), output deviations, q_t, equal zero except for the random disturbance term ε_t; 3) if the inflation or money growth rate is unanticipated (and assuming $\Delta \ln P_t^e$ and $\Delta \ln M_t^e$ are underestimated) then $q_t > 0$ as determined by the parameters β, and λ. In this last case we can conclude that the magnitude of the short-run output response depends on two factors; 1) the magnitudes of β and λ, which are, in effect, the reciprocals of the slope coefficients seen in the traditional expectations-based Phillips curve models, and, 2) how much the actual inflation or money growth rate is "anticipated" (i.e., the magnitude of the forecast error).[1] The models thus emphasize the fact that the magnitude of output response is intimately related to how expectations are formed, a topic we turn to subsequently.

3.5. Distributed Lag Form of the Regression Models to be Tested

As was stated in the introductory remarks of this paper, an important hypothesis of this study is that the impact of the unanticipated variables on output is not just contemporaneous, but is distributed over time; the output/unanticipated variables cause-effect relationships are dynamic since it takes time for the inflation and money shocks to work through the economic system. Thus a regression structure that is distributed lag in nature is posited here as a reasonable statistical form whereby current output response is related to past inflation and money forecast error.

The forms of the distributed lag inflation and money regressions to be estimated here are given in eqs. (3.5.1.) and (3.5.2.), respectively;

$$q_t = a + \sum_0^m \beta_i [\Delta \ln P - \Delta \ln P^e]_{t-i} + \varepsilon_t \quad , \tag{3.5.1.}$$

$$q_t = b + \sum_0^m \lambda_i [\Delta \ln M - \Delta \ln M^e]_{t-i} + \varepsilon_t \quad , \tag{3.5.2.}$$

where the bracketed terms represent the magnitudes of the unanticipated variables occurring in period t-i, where i = 0 through lag m. (Note that

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[1] For the remainder of this study the terms "forecast error" and unanticipated inflation or money growth will be used interchangeably.

since output is measured as deviations from log-linear trend, the regression constants a and b are not expected to be statistically different from zero.)

There are two obvious points of interest in estimating regressions (3.5.1.) and (3.5.2.); 1) a determination of the magnitudes and signs of the lag weights λ_i and β_i, and, 2) a quantification of the correct lag length. This information will allow for a quantitative comparison of the relative strengths and time structures of the unanticipated inflation and money variables on output.

The expected signs of the lag coefficients λ and β are;

$$\partial q_t/\partial[\Delta \ln P - \Delta \ln P^e]_{t-i} = \lambda_i > 0 \quad , \text{ and } \partial q_t/\partial[\Delta \ln M - \Delta \ln M^e]_{t-i} = \beta_i > 0 \quad .$$

Since a sub-hypothesis of this study is that the effects of inflation and money forecast error on output first build up and then die off, we have a priori reason to expect that the lag weights can be approximated by a second degree polynominal of the form;

$$w_i = a + bi - ci^2 \quad .$$

Other aspects of the methodology used in determining the appropriate lag length and polynominal degree weighting scheme will be discussed in Chapter VIII.

In a statistical sense it is important to realize that deviations in output away from log-linear trend, q_t, cannot be fully explained by unanticipated inflation and money growth rates alone; many other factors, some endogenous to the multiplier process, affect output in terms of 1) the magnitude and, 2) the time lag of the unanticipated variables' impact on output.[1] This means that a straightforward statistical estimation of regressions (3.5.1.) and (3.5.2.) would be suspect because of equation misspecification. Since this study is limited to an investigation of the unanticipated inflation and money variables alone, the Cochrane-Orcutt iterative error correction technique will be employed to "proxy" these other non-expectational influences which help explain the output pattern.[2] As will be demonstrated in Chapter VIII, the use of this technique, by improving regression specification, will make the roles of the unanticipated variables more discernable and their related statistics much more reliable.

As stated earlier, the dynamic characteristics of the economic relationships under study here suggest that time lags must be used if the regression models are to properly describe the behavior of the dependent (output) variables. The dynamic nature of these economic relationships derives from the fact that output "adjustment" over time is not instantaneous but is gradual.[3] A statistical recognition that output adjustment takes time is found in the fact that most indices of real economic activity are characterized by strong first-order positive serial correlation. Some reasons supporting this serial correlation pattern

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[1] For example, expansionary fiscal policy, international trade shifts, and random supply and demand shocks can provide exogenous impulses to output. Additionally, the procyclical nature of income velocity, the dynamic workings of the multiplier process, countercyclical income policy, and the general inertia inherent in a complex industrial economy are endogenous factors which can prolong or alter exogenous impulses.

[2] An alternative estimation method which could have been used would be the Lucas supply model in which a lagged value of the dependent variable, q_{t-1}, is placed in the regressor matrix. See Lucas [18].

[3] Hall [28] has termed this phenomenon the "persistence of unemployment."

can be inferred from the dynamic characteristics associated with the cumulative multiplier and investment accelerator processes, habit persistence, positive capital adjustment costs, information and perception lags, inventory adjustment lags, and various institutional and technological constraints. These "forces" which modify output response and make it serially correlated are systematic influences, all endogenous to the dynamic workings of the economic system. Thus, while this study starts from a working hypothesis that the output regressions (3.4.3.) and (3.4.4.) should be distributed lag in nature, there is strong theoretical support for this position.

Our theory, on the other hand, as exemplified in the distributed lag regression models (3.5.1.) and (3.5.2.), suggests that output can also be influenced over time by exogenous expectational "shocks," factors which are completely random.[1] This fact raises the following pertinent question: Being random, how can the unanticipated inflation and money variables be more than just contemporaneously related to real output response, i.e., how can these non-systematic causal influences lead to systematic output response patterns which characterize q_t in the regression models? To rephrase the question in a more statistical manner, how can one hope to explain the empirical fact that q_t in eqs. (3.4.3.) and (3.4.4.) is highly autocorrelated when the expected value of the right-hand side of these two equations is zero (given that expectations of the inflation and

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[1] Of course, in this study the random nature of these shocks is derived by experimental design, a fact which is intended to mimic the effects of the unanticipated inflation/money variables on output in the real world.

money growth rates are formed rationally according to Muth)?[1]

In this study, the theoretical reconciliation of 1) random expectational shocks as an explanation for non-contemporaneous output response with, 2) the empirical fact that this response is serially correlated, is made by appealing to the following hypothesis: while inflation/money forecast error may be random, their effects are not, but persist, where they are subsequently altered (i.e., lengthened and strengthened) by the endogenous forces cited in the paragraph above. This hypothesis provides the theoretical rationale for employing a distributed lag regression form, since this structure allows one to capture (and thus, explain) some of the autocorrelated nature of the output time series and thereby link expectations with actual output patterns. The distributed lag format also allows one to quantify the cumulative "process" by which random expectational "events" can elicit a non-contemporaneous systemative output reaction.

3.6. Theoretical Basis of the Transmission Mechanisms Used in the Inflation and Money Models

In order to properly interpret and compare the regression results of eqs. (3.5.1.) and (3.5.2.), a theoretical explanation of how the impact of the unanticipated inflation and money variables is transmitted to output must be specified. This section provides these explanations. In another sense, though, the transmission mechanisms posited here are

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[1] Indeed, some economists have dismissed earlier versions of the natural rate-rational expectations models [similar to (3.4.3.) and (3.4.5.)] because they could not provide an endogenous explanation of how aggregate demand-induced fluctuations in output could persist over time. See Hall [28].

working hypotheses, the validity of which is an empirical question to be addressed by the regression comparisons made in Chapter VIII.

3.6.1. The Inflation Model and the Lucas Hypothesis. Eq. (3.5.1.), the general format of the inflation models to be estimated here, appeals to the Lucas Hypothesis (LH) to connect unanticipated inflationary shocks to real output. The LH, while a variant of the Friedman-Phelps absolute/ relative price confusion argument described earlier, is distinctive in that it makes the roll of information lags a crucial feature of short-run output response in general, and short-run output autocorrelation, in particular.

In the general form of the Lucas supply schedule,

$$q_t = \beta_0[\Delta \ln P_t - \Delta \ln P_t^e] + \beta_1 q_{t-1} + \varepsilon_t \tag{3.6.1.1.}$$

(where the output and inflation terms are as defined earlier), aggregate supply is related directly to the gap between the current and expected rates of inflation and the lagged dependent variable is used to capture the "persistence effects" of the business cycle. Unexpected increases in the inflation rate thus boost aggregate supply because suppliers (of both labor and goods) mistakenly interpret an economy-wide increase in the absolute level of prices as a permanent firm- or product-specific increase in the relative price of the product or service they are supplying. The result is that more output is forthcoming, at least temporarily, because of the transitory improvement in the terms of trade.

The LH is based upon the assumption that market participants are spatially separated and thus receive information about the prices of their own goods faster than they receive information about the aggregate price level. Thus short-run output response is explained by misinformation about the true relative price structure and by the fact that agents respond only to the perceived movements in relative prices. For example, let the perceived relative price of good "x" in market "i" in time period t be

$$P^x_{it}/P_{gt} = f_i[\Delta P_{gt}/P_{gt}] \quad ,$$

where $f_i' > 0$ and i = 1 through N markets. P_{gt} is the general price level at time period t and f_i is the function relating the relative price of good x in market i to the perceived rate of change in the general level of prices (i.e., the inflation rate). Thus an economy-wide increase in the general level of prices is seen to increase the perceived relative price of good x. In this manner, $\Delta P_{gt}/P_{gt}$ is defined as an unexpected price movement and P^x_{it}/P_{gt} is mistakenly conjectured as a rise in the relative price of good x by agents who believe that other prices are not moving when their "own prices" are (or are not moving as much). The LH posits that information about these "other prices" is not part of agents current information vectors, and that such information is received in delayed fashion.

Another aspect of the LH is that changes in nominal aggregate demand which, in fact, "generate" new information about price (and inflationary) movements in the economy, are not immediately observable even though agents are assumed to understand its probability characteristics. The notion that aggregate demand is not immediately observable provides Lucas with plausible theoretical support for the "persistence" of economic fluctuations in a rational expectations setting. By invoking the idea that changes in nominal aggregate demand are, in many settings, not contemporaneously observable, Lucas thus achieves the restriction on information sets necessary to make serially correlated forecasting errors coexist with rational forecasting behavior. Movements in nominal aggregate demand then can generate serially correlated movements in output, even though it is only the public's errors in forecasting nominal aggregate demand that cause output to respond. Thus even when agents are Muth rational, the fact that movements in aggregate demand are not immediately observable provides the LH with the capacity to explain serially correlated movements in economic activity.

A version of the LH assumed to hold for the inflation models estimated here can be demonstrated via the following system of equations;

(3.6.1.2.)
$$q_{it} = \beta_0[\Delta \ln P_{gt} - \Delta \ln P^e_{it}] + \beta_1 q_{it-1} + \varepsilon_{it} \quad , \beta_0, \beta_1 > 0$$
$$\Delta \ln P_{gt} = \lambda[n_t - \hat{n}_t] + z\hat{n}_t + u_t \quad , \lambda > 0$$
$$\Delta \ln P^e_{it} = z\hat{n}_t \quad ,$$

where q_{it} and q_{it-1} are arbitrary measures of real economic activity in particular industries or groups of industries, i = 1 through N. $\Delta \ln P_{gt}$ is the general rate of inflation and $\Delta \ln P^e_{it}$ is the firm- or product-specific expected rate of inflation in the i-th industry (formed one period earlier). The variate n_t is nominal aggregate demand, while $\hat{n}_t$ is the public's expectation of n_t in period t formed as a linear least-squares projection of n_t (i.e., an unbiased, minimum variance forecast of n_t) formed on some information set, Θ_{t-1}. The disturbance terms ε_{it} and u_t, while individually random, display cross-covariation. z is a scalar showing a linear relationship between nominal demand and the general rate of inflation. The system shows that the expected rate of inflation in industry "i" is proportional to the expected magnitude of nominal aggregate demand in time period t.

In simple terms, system (3.6.1.2.) shows that unanticipated shifts in nominal aggregate demand cause unanticipated inflation, which in turn results in an output response. Real variables, therefore, respond only to the unexpected part of the inflation rate $[\Delta \ln P_{gt} - \Delta \ln P^{e}_{it}]$, which, in turn, is directly related to the unexpected movements in nominal aggregate demand, $[n_t - \hat{n}_t]$. An anticipated increase in nominal demand causes only the expected rate of inflation to rise since $\Delta \ln P^{e}_{it} = z\hat{n}_t$, and unanticipated inflation is zero leaving real quantities unaffected (except for random error). The system demonstrates that the LH can accommodate both the short-run output/unemployment Phillips curve trade-off and the long-run NRH.[1]

It is noteworthy to point out that the LH connecting changes in nominal demand to output via unanticipated inflation is a particularly general transmission linkage in that the immediate cause of unanticipated inflation and the resulting confusion about absolute and relative prices need not be specified. Thus, while unanticipated inflation is the direct

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[1] Note that in keeping with the spirit of the LH, the output/unanticipated inflation models developed here treat economic agents as price-takers operating in a strict auction-Walrasian market setting. The advantage of this approach, admittedly a simplification, is that it greatly simplifies the task of modeling the nature of market equilibrium and market adjustment through time due to changes in nominal demand and the resulting price shocks.

cause of short-run output response, the cause of the unanticipated inflation can be related to <u>any</u> influence which might alter nominal spending.[1] The general nature of the LH thus explains its popularity in empirical investigations of the inflation/unemployment tradeoff; researchers are freed to aim their statistical efforts at discovering the relationship between unanticipated inflation and output, regardless of how the unanticipated inflation might have originated. [The inflation/ unemployment phenomenon has thus been elevated to a level where the topic is approached as a problem worth of independent study.]

3.6.2. Unanticipated Money and Output: Barro-type Models and the Lucas Hypothesis.

Recent empirical studies utilizing the LH-price confusion-output scenario have specifically related changes in nominal demand to purely exogenous monetary growth "shocks." In these models, the transmission mechanism runs from unanticipated nominal money growth to increases in liquidity and spending (i.e., increases in nominal aggregate demand). This spending is then used to explain the occurrence of of unanticipated inflation, price confusion, and, ultimately, a short-run output response. These money-oriented Phillips curve models thus posit unanticipated inflation as an <u>intermediate link</u> in the causal chain connecting unanticipated money growth to real variables in the system.

Since this study is concerned with the money/output relationship within a rational expectations setting, it is necessary to consider a

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[1] For example, supply shocks, pure fiscal policy and changes in velocity, monetary expansion, foreign trade factors, or autonomous shifts in investment or consumption expenditures.

class of reduced-form models that exemplify the unanticipated nominal money/output linkage within the confines of the LH. Clearly much of the leading theoretical and (especially) empirical work in this area is that of Robert Barro of the University of Rochester [27, 29, and Barro and Rush, 30]. The results of Barro's money/output models are of particular interest since the money/output regressions estimated here provide a viable alternative to the transmission mechanism invoked by Barro.

The following aggregate demand-aggregate supply illustration demonstrates the basic Barro-type system in which changes in the growth rate of nominal money balances affect output via a RBE-induced unanticipated inflation. Let the aggregate supply function be defined as;

(3.6.2.1.) $$q_t^s = \beta_0[\Delta \ln P_t - \Delta \ln P_t^e] + u_t^s \quad , \quad \beta_0 > 0$$

where q_t^s is the supply of output in period t and the actual and expected inflation rate variables are as defined earlier. Note that the expected rate of inflation, $\Delta \ln P_t^e$, is formed in period t-1, and u_t^s is a random error term that shifts the supply schedule because of factors not related to the inflation rate. Eq. (3.6.2.1.) thus relates deviations in real output from log-linear trend directly to the gap between the current rate of inflation and the public's prior expectations of that rate. In keeping with the spirit of the LH, this means that a positive output response occurs because the ratio

(3.6.2.1!) $$\Delta \ln P_t / \Delta \ln P_t^e > 1 \quad ,$$

meaning that information about firm- or market-specific prices is received faster than information about the aggregate price level. (In this case, a positive output response would also occur if the elasticity of the expected with respect to the actual rate of inflation was less than unity.)

Money balances are introduced into the Barro system via the aggregate demand function;

$$(3.6.2.2.) \qquad q_t^d = \beta_1[\Delta \ln M_t - \Delta \ln P_t^e] + u_t^d \quad , \qquad \beta_1 > 0$$

where $\Delta \ln M_t$ is the actual rate of growth in the nominal stock of money balances in time period t, and u_t^d is a random demand shift factor not related to money growth or expected inflation. With $\beta_1 > 1$, increases in $\Delta \ln M_t$ relative to $\Delta \ln P_t^e$ lead to a rise in aggregate demand away from trend via a RBE.

Assuming all markets clear (the Barro model abides by the auction market reasoning used by Lucas), eqs. (3.6.2.3.) and (3.6.2.4.) can be combined to give the following reduced-form simultaneous system;

$$(3.6.2.5.) \qquad \Delta \ln P_t = (1 - \beta_1/\beta_0)\Delta \ln P_t^e + (\beta_1/\beta_0)\Delta \ln M_t + (1/\beta_0)(u_t^d - u_t^s)$$

$$(3.6.2.6.) \qquad q_t = \beta_1[\Delta \ln M_t - \Delta \ln P_t^e] + u_t^d \quad .$$

Note that the current rate of inflation in the Barro system is affected by the expected inflation rate, $\Delta \ln P_t^e$, formed one period back, with the effect being positive if $\beta_0 > \beta_1$. Also note that while current changes in money growth affect the current rate of inflation, the effect is less than unity when $\beta_0 > \beta_1$ [this restriction is in keeping with inequality (3.6.2.1.) in that a perceived change in relative prices leads to a substitution effect greater than the income effect].

The Barro-type system defined by eqs. (3.6.2.5.) and (3.6.2.6.) presents a monetarist version of the transmission mechanism relating changes in the growth rate of the money stock to discrepancies between actual and expected inflation and changes in real output. Specifically, (3.6.2.5.) shows that with expectations held fixed, an increase in nominal money balances produces a less-than-proportionate increase in prices. And, as indicated by (3.6.2.6.), it is this less-than-unitary response that produces a short-run output response. Eq. (3.6.2.5.) also shows that equal increases in nominal money balances and expected inflation produce equal increases in the current rate of inflation, a situation previously described as "inflationary equilibrium." The counterpart of this expectational equilibrium state can be seen in (3.6.2.6.) where equal increases in nominal money and expected inflation leave output unaltered. Finally, the Barro system implies that short-run changes in output resulting from a given change in the growth rate of money balances can occur under two conditions: 1) if the expected rate of inflation is fixed, or, 2) if the expected rate of inflation reacts with a lag to the actual evolution of inflation caused by the monetary increases.

Since the money/output models estimated in this study utilize the RBE as a transmission link, it is important to compare the role of real balances and the RBE in the Barro-type system defined by (3.6.2.5.) and (3.6.2.6.). This system shows that it is changes in real balances that lead to price confusion and, ultimately, to an output response. Thus the Barro transmission linkage involves, in a critical manner, the Lucas information lag hypothesis. That is, while the Barro linkage places unanticipated monetary growth as the initiating causal force in the money-output nexus, the transmission chain uses the absolute/relative price confusion phenomenon as an intermediate step between the money shock and the output response.

Also, since Barro's empirical work [27, 29, 30] uses unanticipated nominal balance growth as the explanatory variable in his ouput regressions, the distinction between the transmission mechanisms associated with real and nominal money is noteworthy. Specifically, Barro's empirical model estimations rely on two forms of price mechanics; 1) the actual inflation rate must lag, momentarily, unanticipated nominal money growth in order to produce a RBE, and, 2) the increased spending resulting from the increased liquidity in the system must cause agents to misinterpret an economy-wide increase in the absolute level of prices for a perceived increase in relative prices.[1] A proper interpretation of Barro's empirical results requires that one not confuse the lag of the actual inflation rate behind nominal money growth with the <u>subsequent</u> price confusion caused by the RBE-induced increase in nominal spending; these are two distinct inflation rate phenomena, and only the latter phenomenon is directly related to output in the Barro money/output paradigm. It is for this reason that the Barro model must still be classified as "supply-oriented" in the Lucas vein.[2]

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[1] It is fair to say that this rather involved transmission mechanism has not been clearly spelled out in Barro's empirical work. However, his chief theoretical investigation of the money/output causal chain is much more precise. See Barro [31].

[2] While not empirically evaluated by Barro, his theoretical research supports the view that the variance (as opposed to the level) of unanticipated money growth is also directly related to the dispersion of relative prices and "price confusion." This variant of the Barro paradigm still appeals to the LH in that an increased variance of money reduces the informational content of observed prices and therefore makes it more difficult for individuals to respond appropriately to changing patterns of true relative prices. Thus the larger the variance of the money supply, the more likely are agents

3.6.3. The Money Models and the Real Balance Effect. Unlike the Barro-type models discussed in the preceding section, the unanticipated nominal money-to-output transmission mechanism posited in this study is based upon the monetarist interpretation of the RBE. This hypothesis is based upon the idea that increases in nominal money balances result in a larger initial stock of real money balances (and liquidity) and this real wealth leads to increased spending. The subsequent temporary increase in real money balances arises because inflation, which ultimately nullifies the rise in real balances and spending, lags nominal money growth.[1]

As stated in the introductory remarks of Section 3.3., the traditional RBE, as a monetary phenomenon, makes no distinction between anticipated and unanticipated money growth. Since the money models estimated here emphasize unanticipated monetary growth, the traditional RBE transmission mechanisms used by Friedman and Patinkin must be altered to provide a distinction between anticipated and unanticipated nominal money growth. The money-to-output transmission mechanism to be developed below will show that while increases in real balances are non-neutral, only the unanticipated part of nominal money can result in increases in

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to attribute local price movements to general inflation rather than to relative demand shifts. Barro's theory also shows that monetary variation that is within a "perceived expected distribution" is neutral with respect to price relationships.

[1] The theoretical base of this dynamic form of the RBE is obtained from the original idea by Friedman [32], where the relationship between the growth of money and subsequent inflation is not instantaneous (the "monetary overshoot" mechanism), but is described by a temporary disequalibrium in the holding of money balances. Empirical evidence supporting the existence of this lag is seen in Vogel [33] and Von Furstenburg and White [34].

real balances. This means that the lag in the actual rate of inflation necessary to activate the RBE attaches only to that part of nominal balance growth that is unanticipated. Logically, that part of nominal money growth that is anticipated results in an immediate increase in the inflation rate with no output effect. The short-run non-neutrality of unanticipated nominal money is ultimately nullified when prices begin to rise. Thus the money-to-output causal chain invoked here maintains both the short- and long-run monetarist-Quantity Theory position: in the short-run (unanticipated) nominal money growth leads to some real output response and some inflation, but in the long-run, inflation will increase by the amount of the (unanticipated) increase in the nominal money stock and output will revert to a steady-state equilibrium position.

It is apparent that the assumptions made here with regard to how the RBE connects nominal money to output, and the interpretation of the causal nature of the RBE utilized in the Barro-type models differ drastically in their explanations of why unanticipated money is non-neutral in the short-run. This difference in approach can be stated simply: the money-to-output models estimated here do not rely on the Lucas relative/absolute price confusion argument, whereas the Barro models do. While Barro's interpretation of the transmission mechanism connecting nominal money to output uses the price confusion scenario as an interim link, this study short-circuits the role of unanticipated inflation in the monetary linkage relating nominal money to output. Thus the money models estimated here are "demand oriented" in spirit, since unanticipated changes in real liquidity in the system are assumed to lead directly to temporary increases in expenditures as portfolio readjustment takes place.

Finally, since the money models appeal to the traditional workings of the RBE, only one "price" phenomenon is assumed to be operative, the lag of the actual (and expected) inflation rate behind the growth rate of nominal balances; the LH and the price confusion resulting from changes in real balances and nominal demand are not assumed to be virile arguments in the money-to-output linkage.

The unanticipated money/output regressions estimated here thus provide a test for two competing transmission mechanism hypotheses; 1) the Barro-type system which interprets changes in real balances as affecting output only through the effects of unanticipated inflation, and, 2) the traditional RBE, in which more liquidity leads to more spending without resort to price misinformation. A comparison of the regression results of the unanticipated inflation and unanticipated money models will give clues as to which hypothesis is favored by the data; if real output response to unanticipated inflation, as estimated in the inflation models (3.5.1.) is similar in magnitude and lag length to the output response elicited by the unanticipated nominal money growth models (3.5.2.), we can infer that the RBE predominately works through the price confusion linkage as posited by Barro and Lucas; if the output response is not similar, we can infer, on empirical grounds, that the RBE affects output predominately through spending-induced unanticipated increases in liquidity without much reliance on unanticipated inflation as an intermediate link. This reasoning follows from the fact that the separately estimated inflation and money regressions provide an implicit comparison of output response because they start from two different points in the causal chain connecting unanticipated money to ouput, i.e., the unanticipated inflation models start at the price confusion link, while the money models start before that link with changes in nominal (and real) balances. Thus if unanticiapted money "causes" output predominately through the LH, as posited by Barro, then the output response, as determined by the inflation models (3.5.1.), will confirm this assumption by producing similar regression results, i.e., the inflation models will, in effect, simply be measuring the same transmission mechanism but at a later point along the causal chain connecting unanticipated money to output.[1]

With the above remarks in mind, we must now specify the causal chain connecting unanticipated nominal money to real balances, the inflation rate, and output that is assumed to hold for the money models.

Starting from a state of inflationary equilibrium, the following conditions hold;[2]

$$m_t^d = m_t = [M^d/P^e]_t = [M/P] \quad , \tag{3.6.3.1.}$$

where m_t^d and m_t are the levels of real demand and supply of money balances in period t, respectively, and M^d and M are the nominal demand and supply of money in period t, respectively. P^e and P are the expected and actual

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[1] Note that while the money models tested here do not appeal to the LH price confusion argument, the empirical results observed do not completely deny that some part of the observed output response may be due to some amount of money-induced unanticipated inflation. Rather, the spirit of the competing hypothesis invoked here is that the RBE-induced clouding of relative and absolute prices is weak compared to the RBE-induced increases in liquidity and direct spending effects.

[2] Certain assumptions of the analysis to follow need be mentioned: 1) the effect of anticipated inflation on the demand for real money balances is ignored; 2) the supply of money is considered exogenous

price levels in period t, respectively.

In a steady-state equilibrium we can posit that the level of real money balances (demanded and supplied) will be proportional to the level of real output, q_t (to be consistent, since we are initially discussing the level of real balances, the level of real output rather that its deviation from trend must be used);

(3.6.3.2.) $$m_t = \beta q_t \quad , \qquad 0 < \beta < 1$$

and, from (3.6.3.1.);

(3.6.3.2!) $$\ln M_t - \ln P_t = \beta \ln q_t \quad .$$

In terms of rates of change we then have;

(3.6.3.3.) $$\Delta \ln M_t - \Delta \ln P_t = \beta \Delta \ln q_t \quad ,$$

where $\Delta \ln P_t$ is the actual rate of inflation in period t and the output measure, $\Delta \ln q_t$, is now considered as deviations from log-linear trend.

The effect on the inflation rate caused by the growth in nominal money is determined from the equation

(3.6.3.4.) $$\Delta \ln P_t = -\beta \Delta \ln q_t + \Delta \ln M_t \quad .$$

Note that if the rate of inflation is less than the rate of nominal money growth, a contemporaneous real output response, $\Delta \ln q_t$, will occur. Of course, the money transmission mechanism posited here, being short-run in nature, implies that the relationship between nominal money and

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with respect to the inflation rate; 3) a condition of inflationary equilibrium exists at the end of each period in that money holders expect that the necessary money will be supplied to satisfy the growth in their nominal demand; 4) unanticipated money growth is a random process over time, i.e., expectations about nominal balance growth are formed rationally according to Muth; 5) the rate of change in velocity is assumed to be equal to zero.

inflation is not instantaneous. However, given a long-run equilibrium condition exists, i.e., a state of inflationary equilibrium characterizes the economy, the following condition will hold;

(3.6.3.5.) $$\partial(\Delta \ln P_t)/\partial(\Delta \ln M_t) = 1 \quad ,$$

where we assume the real income elasticity of the demand for money, β, is unity.

Now dividing the actual growth rate of nominal money balances into anticipated and unanticipated components we have;

(3.6.3.6.) $$\Delta \ln M_t = \Delta \ln M_t^e + [\Delta \ln M_t - \Delta \ln M_t^e] \quad .$$

Substituting (3.6.3.6.) into (3.6.3.4.) gives;

(3.6.3.7.) $$\Delta \ln P_t = -\beta \Delta \ln q_t + \Delta \ln M_t^e + \gamma[\Delta \ln M_t - \Delta \ln M_t^e] \quad ,$$

where $0 < \gamma < 1$. Eq. (3.6.3.7.) stresses the partial adjustment of inflation to unanticipated nominal monetary shocks. Thus, while the expected growth of the money stock has an instantaneous and equiproportional effect on inflation, i.e., $\partial(\Delta \ln P_t)/\partial(\Delta \ln M_t^e) = 1$, changes that are unanticipated act only with some delay on inflation, as determined by the lag coefficient γ. If the inflation effect of unanticipated nominal balance growth is instantaneous, then $\gamma = 1$; if unanticipated nominal money growth has no impact on inflation then $\gamma = 0$. Since the unanticipated money-to-output transmission mechanism posited here invokes the RBE, it is assumed that γ lies between these two extremes. Thus, even though the parameter γ will not be statistically estimated, the assumption that inflation responds only with a lag to unanticipated changes in nominal money growth implies that $0 < \gamma < 1$. (As stated earlier, there is firm theoretical and empirical support for this position.)

Now the growth rate of real money balances in the economy is defined as;

(3.6.3.8.) $$\Delta \ln m_t = \Delta \ln M_t - \Delta \ln P_t \quad .$$

Substituting eq. (3.6.3.7.) into (3.6.3.8.) and rearranging terms gives;

(3.6.3.9.) $$\Delta \ln m_t = \beta \Delta \ln q_t + (1 - \gamma)[\Delta \ln M_t - \Delta \ln M_t^e] \quad .$$

Eq. (3.6.3.9.) thus allows for a flow disequilibrium in the level of real money balances caused by an unanticipated shock to the growth rate of nominal money balances. Note that if the adjustment parameter, γ, equals zero, real money balances will rise by the full amount of the unanticipated growth in nominal money (assuming output response is not instantaneous). If γ equals unity, then there is no adjustment to real balances and they are in long-run proportion to the level of real income as defined in (3.6.3.2.). Eq. (3.6.3.9.) shows that the dynamics of real balance adjustment, $\Delta \ln m_t$, derive from the delayed response of the inflation rate to unanticipated changes in the rate of growth of the nominal money stock. More specifically, (3.6.3.9.) shows that movements in real balances in the economy can come <u>only</u> from the unanticipated portion of nominal money growth rate activity. Note also that the adjustment parameters γ and $(1 - \gamma)$ in eqs. (3.6.3.7.) and (3.6.3.9.) sum to unity. This means that a given increase in $\Delta \ln M_t$ will be fully accounted for (initially) as either an increase in inflation or an increase in real money balances. For example, if $\Delta \ln M_t$ = 10 percent, then initially, real money balances will rise by $(1 - \gamma)10$ percent and prices by only $\gamma 10$ percent. (Recall here that we abstract from the fact that rises in the expected rate of inflation will cause decreases in the amount of real money balances demanded.)

Now restating the unanticipated nominal money/output model (3.4.4.) in terms of real balances gives;

(3.6.3.10.) $$q_t = \lambda[\Delta \ln m_t] + \varepsilon_t \quad , \qquad \lambda > 0 \quad .$$

Assuming that there is no instantaneous effect of nominal balance growth on output allows us to express real balance growth, initially, as proportional to the rate of growth of nominal balances;[1]

(3.6.3.11.) $$\Delta \ln m_t = (1 - \gamma)[\Delta \ln M_t - \Delta \ln M_t^e] \quad ,$$

and,

(3.6.3.12.) $$q_t = (1 - \gamma)[\Delta \ln M_t - \Delta \ln M_t^e] + \varepsilon_t \quad ,$$

(3.4.4.) $$= \lambda[\Delta \ln M_t - \Delta \ln M_t^e] + \varepsilon_t \quad ,$$

where $\lambda = (1 - \gamma)$.

Again, it is emphasized that the money model (3.4.4.) is based upon the assumption that the actual inflation rate lags nominal money balance growth, which increases economy-wide liquidity and, subsequently, output. For the money models, this sluggish inflation rate response is assumed to dominate any output effect resulting from nominal money growth-induced unanticipated inflation. Obviously, an individual analysis of the parameters of (3.4.4.) and (3.5.2.) cannot confirm or deny the validity of

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[1] This restriction is in keeping with the earlier assumption that an unanticipated change in the growth rate of nominal money balances is initially dissipated in more inflation and more real balance in circulation. It is plausible to assume then that nominal money shocks would not instantly increase output due to a lag between more liquidity, portfolio readjustment and more spending.

this assumption, since the money model regressions provide no way to discriminate between that part of the output response which is due purely to liquidity-induced increases in demand and that part of output response related to liquidity-induced price confusion. However, a comparison of the individual inflation and money model regressions of Chapter VIII will provide insight into the reasonableness of this assumption.

CHAPTER IV

EXPECTATIONS GENERATING MECHANISMS IN MACROECONOMIC MODELS

4.1. Introduction

In reviewing the test models of the previous chapter it is apparent that expectations about the rate of inflation and money growth play an essential role in the determination of short-run changes in output and, in general, all indices of economic activity. The modeling rationale used here implies that the effects of price and money impulses depend on whether or not such exogenous "forces" are anticipated. As was shown in Chapter III, the models are constructed so that anticipated changes have no effect on the current level of economic activity, while unanticipated movements produce transitory accelerations or decelerations in deviations of output from log-linear trend.

Noticeable in the above discussion is the lack of discussion and analysis afforded to *how* expectations are formed. This was a purposeful omission however, since aggregate expectations formation comprises a crucial part of this study and thus deserves separate and in-depth treatment. Before beginning, however, it is important to realize the following fact concerning not only this but all expectationally-based studies: the aggregate magnitudes that respresent expectations of future events are unobservable. Hence, research dealing with expectations is forced to find some proxy for this variable and some proxy for the "expectations formation generating rule." This is a delicate task: *what is required is the quantification of the "mimic" used in forming expectations of future economic* variables based upon past behavior of that variable. In all cases the validity of the results of models incorporating expectations must be judged by the plausibility of the expectation generating mechanism(s) used to quantify the unobservable variables. This task, that of finding the proper expectations mechanism mimic, however, cannot be avoided if expectations models are to be properly specified. As Jacob Mincer points out [35, p. 83]:

> This issue cannot be ignored; The use of current rather than anticipated values is equivalent to a hypothesis that expectations are largely based on current magnitudes and do not differ from them in any systematic way.

In this chapter some of the theoretical and econometric problems and implications involved in measuring expectations and expectation formation are discussed. Specific attention is given to the two main expectations generating schemes found in the literature, the Adaptive Expectations Hypothesis (AEH), and the Rational Expectations Hypothesis (REH). As will be demonstrated subsequently, the particular expectational generating method used in this study is a synthesis of both hypotheses.

4.2. The Adaptive Expectations Hypothesis

The AEH is based upon the following linear filtering scheme:[1] The expected rate of inflation is a linear weighted autoregressive function

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[1] The fact that the AEH is applied to the inflation rate in the above definition is in keeping with the literature on inflationary expectations. The filtering system, however, can be applied to any variable about which an expectation is formed. In terms of this study, while expectations generating mechanisms may, at times, stress inflation, they can be applied, with equal generality, to money growth rates as well.

of past actual rates of inflation, with the weights constrained to follow an exponential decay pattern.

In mathematical notation, using π_t to indicate either the inflation or money growth rate, the hypothesis is written;

(4.2.1.) $$\pi^e_t = \pi^e_{t-1} + \lambda(\pi_{t-1} - \pi^e_{t-1}) \quad , \quad 0 \leq \lambda \leq 1$$

Letting $\lambda = (1 - \beta)$, eq. (4.2.1.) is;

(4.2.2.) $$\pi^e_t - \pi^e_{t-1} = (1 - \beta)(\pi_{t-1} - \pi^e_{t-1}) \quad ,$$

or;

$$\pi^e_t + (1 - \beta - 1)\pi^e_{t-1} = (1 - \beta)\pi_{t-1} \quad ,$$

$$\pi^e_t(1 - \beta L) = (1 - \beta)\pi_{t-1} \quad ,$$

and,

(4.2.3.) $$\pi^e_t = (1 - \beta)[(1 - \beta L)]^{-1}\pi_{t-1} \quad ,$$

where π_t and π^e_t are the actual and expected variables in period t, respectively. L is a lag operator where;[1]

$$L^k \pi_t = \pi_{t-k} \quad .$$

Eq. (4.2.3.) can be expressed as an infinite series in the expected rate of inflation or money growth as:

(4.2.4.) $$\pi^e_t = \lambda[\pi_{t-1} + (1 - \lambda)\pi_{t-2} + (1 - \lambda)^2\pi_{t-3} + . . .] + (1 - \lambda)^{i+1}\pi^e_{t-i} \quad ,$$

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[1] In lag operator notation an infinite series (say, on y_t) can be expressed as;

$$B(L)y_t = [B(0) + B_1L + B_2L + . . .]y_t$$

$$B(0)y_t + B_1y_{t-1} + B_2y_{t-2} + ...$$ This notation is used throughout the remainder of this paper.

where $\beta = (1 - \lambda)$. As $i \rightarrow \infty$ the last term in (4.2.4.) approaches zero given the assumption that $\lambda < 1$. Thus (4.2.4.) can be rewritten as;

$$(4.2.5.) \qquad \pi_t^e = \lambda \sum_0^\infty (1 - \lambda)^i \pi_{t-1-i} \quad .$$

A number of comments are pertinent to the modeling of expectations as implied by the AEH. The above illustration shows that the expected rate of inflation can be constructed as an infinite geometric distributed lag in past actual rates of inflation with the weighting scheme determined completely by the constant β. Note that if the present is going to be reasonably related to the past, the restriction

$$0 \leq \beta \leq 1$$

must hold. This restriction simply implies that expectation formation is not explosive, i.e., the lag is stable in past actual rates of inflation.

With a fixed coefficient of learning, β, expectations are revised in each period proportionately to the difference between the actual and expected rates of inflation in period t-1. If the current rate of inflation turns out to be what was expected for the current period, then the same rate of inflation will be expected for the next period. If the current rate of inflation is greater (less) than was expected, then the currently expected rate for the next period will be revised upward (downward) by the fraction of the excess (deficiency).[1]

The amount of expectation revision is determined by the product of the coefficient of learning, λ, times the error of the forecast $(\pi_{t-1} - \pi_{t-1}^e)$,

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[1] The AEH scheme also shows the weights, $(1 - \lambda)$, $(1 - \lambda)^2$, . . . summing to unity. This is seen by wrighting the right-hand-side of (4.2.4.) as;

so the AEH format provides a crude "error-learning" algorithm. As (4.2.4.) shows, if λ is near zero the lag distribution, i.e., the "memory" of the model, is long; if λ is near unity, the lag distribution is short, implying a quicker adaptation to past forecasting mistakes. If $\lambda = 1$, then the expected rate of inflation is equal to last period's actual rate and no revision takes place. It is clear from the weighted average property of the adaptive scheme that if the actual rate of inflation becomes and remains constant, the expected rate will, in the limit, approach the actual rate.[1] However, because the coefficient of learning is fixed, the expected rate will always lag the actual rate whenever $\pi_t \neq \pi_{t-1}$ (unless $\lambda = 1$).

The AEH has very definite implications for demand management policy in terms of the real effects on output and the unemployment/inflation tradeoff. Consider the aggregate supply function (3.6.2.1.) where π_t and π_t^e stand for the actual and expected rates of inflation, respectively. Substituting (4.2.5.) into (3.6.2.1.) gives;[2]

$$(4.2.6.) \qquad q_t^s = \beta_0[\pi_t - \lambda\sum_1^{\infty}(1 - \lambda)^i\pi_{t-1-i}] + u_t^s \quad , \qquad \beta_0 > 0$$

As previously demonstrated, the fixed weighting scheme of (4.2.5.) means that π_t^e will always lag π_t if the inflation or money growth rate is rising

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$$\lambda\sum_1^{i}(1 - \lambda)^i = \lambda/[1 - (1 - \lambda)] = 1$$

assuming that i approaches infinity. Hence the distributed lag form of the AEH shows that expectations do represent a true average. Also note that the ratio of any adjacent pair of weights equals the fraction β.

[1] In terms of a money-induced inflation, this aspect of the AEH provides a strong justification for Friedman's fixed money growth rate rule, since if expectations about the rate of money growth are formed according to an adaptive scheme, a constant money growth rate would lead to the absence of systematic forecast error.

[2] Note that the π symbols in (4.2.6.) can be interpreted as either inflation or money growth rates. The implications for the output response model (4.2.6.) are the same.

over time. This, in turn, means that the bracketed term in (4.2.6.), representing the magnitude of unanticipated inflation/money growth, must always be positive. Hence output deviations must always be positive as long as the actual rate of inflation/money growth is greater in period t than it was in period t-1 [given that $E(u_t^s) = 0$].[1]

The AEH thus provides a rationale for manipulative macro policy. Specifically, because the adaptive mechanism produces biased forecasts of π_t, it implies that policy-makers, by accelerating the rate of inflation or money growth, can permanently raise the level of economic activity in a systematic fashion; inflation/money growth rates can never be completely "anticipated." For example, assume that policy-makers can constrain π_t to follow a time path indicated by;

$$(4.2.7.) \qquad \pi_t = \lambda\sum_0^\infty(1 - \lambda)^i\pi_{t-1-i} + \phi \quad , \quad \phi > 0$$

where ϕ indicates a (conscious) constant per-period intervention in the evolution of the inflation or money growth rate. By increasing ϕ by $d\phi$, the authorities can have a permanent, predictable effect on the level of economic activity of $\beta_0 d\phi$, where $d\phi$ becomes a systematic error in the expectations formation process that can never be incorporated into the extrapolative lag structure defined by (4.2.5.). Thus the AEH supports the "accelerationist" viewpoint that a permanent tradeoff exists between output and inflation/money growth and that this tradeoff can be exploited by money or fiscal policy.

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[1] Note that if the weights are constrained to sum to unity (the usual assumption made in econometric studies using the AEH), then a once-and-for-all jump in the rate of inflation or money growth implies a permanent change in the level of output. Also note that the exact amount of the lag of π_t^e behind π_t will be determined by the ratio;

$$(1 - \lambda)^i/(1 - \lambda)^{i-1} \quad .$$

4.2.1. The AEH in the Literature. The AEH has been the workhorse in empirical work dealing with expectations. While the research of many economists could be examined here, such a review would be too lengthy, and, in many respects, repetitive. Rather, this brief examination of the AEH literature will center around a discussion of the three main expectational contexts in which the hypothesis has been used; 1) interest rate studies, 2) money demand studies, and, 3) the permanent income hypothesis.[1] It is fair to state that all other uses of the AEH in the literature dealing with expectational proxies are variants of these three applications.

The effect of inflationary expectations on the nominal rate of interest provides the first setting in which expectation formation was recognized as a valid economic phenomenon needing study. Irving Fisher [36] provided the pioneering work in this area in 1933. His research was aimed at explaining the Gibson Paradox--the observation that prices and interest rates move in the same direction (that is, increases in the money supply tend to put upward pressure on prices and downward pressure on interest rates). Fisher's answer to this anomaly was that as prices rise, expectations of inflation are produced and these expectations are reflected in higher nominal interest rates as lenders attempt to hedge against repayment in future de-valued dollars.

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[1] Other writers who have employed the weak-form or extrapolative hypothesis of forecast formation include Ball [37], Roll [38], Gibson [39], Sargent [40], Andersen and Carlson [41], Feldstein and Eckstein [42], Yohe and Karnosky [43], Gordon [44], Turnovsky [45], Modigliani and Shiller [46]. Other uses of the AEH forecasting mechanism include: 1) market price expectations, Muth [47], and Nerlove [48], 2) income expectations, Zellner, Huang, and Chau [49], and 3) the term structure of interest rates, Nelson [50].

Fisher's concern with the relationship between the nominal interest rate and the expected rate of inflation is demonstrated in the following statements from his book, The Theory of Interest:

> It should be noted that insofar as there exists any adjustment of the money rate of interest to the changes in the purchasing power of money, it is for the most part (1) lagged, and, (2) indirect. The lag, distributed, has been shown to extend over several years. The indirectness of the effect of changed purchasing power comes largely through the intermediate steps which affect business profits and volumes of trade, which in turn affect the demand for loans and the rate of interest. There is very little direct and conscious adjustment through foresight. Where such foresight is conspicuous as in the final period of German inflation, there is less lag in the effects (ibid., pp. 401-402].

Fisher did not recognize economic agents as conscious processors of price information in the production of inflation forecasts. In this regard he states:

> A change in the value of money is hard to determine. Few businessmen have any clear ideas about it. . . Yet it may be true that they do take account, to some extent, at least, even if unconsciously, of a change in the buying power of money, under guise of a change in the level of prices in general [ibid., p. 495].

Fisher's belief in the subconscious recognition of an inflation trend in the economy supports the view that he did not recognize investors as gathering forms of information (e.g., money growth rates) other than historical price data to produce their forecasts of inflation. His use of the AEH then, constructed solely on past actual inflation rates, was an expectational generating mechanism well suited to his particular conception of how inflationary expectations were formed.

Fisher felt that the adjustment of the nominal interest rate to changes in the value of money was slow and indirect. He cited evidence of highly variable and frequently negative real rates of interest in support of this thesis. His explanation for this phenomenon was framed, in large part, in terms of money illusion;

> When prices begin to rise, money interest is scarcely affected. It requires the cumulative effect of a long rise, or a marked rise in prices, to produce a definite advance in the interest rate. If there were no money illusion and if adjustments of interest rates were perfect, unhindered by any failure to foresee future changes in the purchasing power of money or by custom or law or any other impediment, we should have found a very different set of facts [<u>ibid</u>., p. 416].

Hence Fisher felt that investors were plagued by a false sense of the real value of money which clouded the theoretical relationship by which the real interest rate would be adjusted upward as the rate of inflation rose. Nevertheless, he did recognize that there was a strong relationship between rates of inflation and the nominal rate of interest which was partly due to the inflationary foresight of individuals.

In equation form Fisher's hypothesis for default-free securities is;

$$i_t = r_t + p_t^e + (r_t \cdot p_t^e) \quad ,$$

where i_t and r_t are the nominal and real rates of interest, respectively, and p_t^e is the expected rate of inflation, or inflationary expectations proxy. For computational purposes Fisher ignored the last term in this equation (due to its small magnitude), which represents the expected rate of depreciation of interest payments, and estimated the parameters for the following equation;

$$i_t = \beta_0 + \beta_1 p_t^e + \varepsilon_t \quad ,$$

where the random disturbance, ε_t, is assumed to obey the Classical properties. In this equation the constant was interpreted by Fisher as representing the real rate of interest and that the nominal rate fully reflected changes in the real rate, in addition to the inflation premium, p_t^e. Hence his test amounted to an investigation of the hypothesis that $\beta_1 = 1$. Confirmation of this hypothesis implied, in turn, that inflationary expectations were fully reflected, in the long-run, in the nominal rate of interest.

Since the inflation proxy, p_t^e, was unobservable, Fisher formulated this variable according to the distributed autocorrelated lag form;

$$p_t^e = \sum_i^m v_i p_{t-i} \quad , \qquad (i,m = 0, 120)$$

where p_{t-i} was the actual rate of inflation in time period t-i. Fisher assumed the weights to decline arithmetically (in distinction to the geometric weighting pattern used in the more contemporary forms of the AEH) and to sum to unity. He then chose the v_i weights so as to give the highest R^2 .

Using both U.S. and Great Britain data he found that these specifications provided for a lag of 120 quarters or 30 years. Fisher then reasoned that because of the very long lag involved in the formation of expectations, the proxy variable resembled the level of prices more closely than the current rate of inflation. This fact, he felt, adequately explained the high correlation between the price level and the quantity of money implied by the Gibson Paradox.[1]

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[1] In an interesting article, Laffer and Zecher [51] attempted to reproduce Fisher's interest rate findings. Using his data they regressed the nominal commercial paper rate on a constant and a 120 quarter arithmetic distributed lag of actual inflation and found;

$$i_t = \underset{(4.03)}{2.013} + \underset{(6.15)}{0.854} p_t^e$$

with $R^2 = .558$, $F = 37.81$, and $DW = .313$. These findings almost duplicate Fisher's original results. The authors, however, point to the low DW statistic which, they feel "provides sufficient grounds to question these results." (*ibid*., p. 17)

Inflationary expectations have also played an important role in studies investigating the demand for money. These studies, relying upon the AEH, have incorporated inflationary expectations under conditions of both hyper- and mild inflation. Without doubt, Phillip Cagan's [52] scholarly research of hyperinflation in post-World War I Europe has been the seminal work in this area. His demand for money study during this period of time is a rare example of economic analysis being carried out under "test-tube-like" conditions.

Cagan's hypothesis was that the demand for real money balances was inversely related to the rate of inflation due to the "inflation tax" on nominal balances resulting from a rising price level. Using data from seven European hyperinflations to establish a pattern of fluctuations in the real money balance variable, he advanced the notion that the expected rate of price change was the principal variable which could account for the variation in the demand for money. He noted that real income and wealth (except that stemming from the depreciating value of nominal balances) appeared to be relatively stable during periods of hyperinflation and, therefore, he omitted these variables in his functional expression of money demand. Additionally, he supported the omission of these variables in the money demand function by reasoning that any non-money-related wealth changes would be infinitesimal in magnitude during a hyperinflation and thus could be safely overlooked.

Cagan hypothesized that the wide fluctuations which were observed in the real money balance series could be interpreted in two distinct ways. First, the observations could have been generated by shifts in the demand function for money. Alternatively, they could be viewed as changes in the quantity of money demanded, having been generated by movements along a stable demand function. Cagan constructed his model according to the latter view since it was consistent with the specification of the demand function which included the expected rate of price change as the dominant argument.

Cagan's money demand function takes the form;

(4.2.1.1) $$\ln(M/P) = -\alpha E - \delta \quad ,$$

where M is the quantity of nominal money balances, P is the price level, E is the expected rate of price level change, and α and δ are constants. Since E is unobservable, Cagan uses the following form of the AEH to generate the inflationary expectations variable;

(4.2.1.2.) $$(dE/dt)_t = \beta(C_t - E_t) \quad ,$$

where C_t is the observed rate of price change in time period t, and $\beta \leq 0$. Hence the adjustment of the expected rate of inflation over time is proportional to the difference between the observed and expected inflation rates. Here β is termed the coefficient of expectations (previously called the coefficient of learning), and indicates the speed at which the expected rate of price change adjusts to the observed rate. Cagan makes the assumption that the series of actual past inflation rates is the only set of information used in the formation of inflationary expectations--not an unreasonable assumption given the characteristics of the hyperinflationary economies Cagan is describing.

Solution of the differential equation (4.2.1.2.) yields the expression;

(4.2.1.3.) $$E_t = \frac{\beta}{e^{\beta t}} \int_{-T}^{t} (C_x e^{\beta x}) dx \quad ,$$

where the constant of integration in the solution is set equal to zero and e is the natural number. Hence, the expected rate of price change in period t is a function of past actual rates with the weights constrained to follow the familiar pattern of exponential decay.

Cagan revises the above money demand specification from continuous to discrete form since observations of money balances and the actual rate of price change are available periodically (monthly). He approximates eq. (4.2.1.3.) with the following form;

(4.2.1.4.) $$E_t = \frac{(1 - e^{-\beta})}{e^{\beta t}} \sum_{-T}^{t} C_x e^{\beta x} \quad ,$$

which is applicable in the context of discrete time and retains the characteristics of the continuous time version of expectations adjustment.

Cagan's formal regression model can be obtained by substituting eq. (4.2.1.4.) into eq. (4.2.1.1.) and adding a stochastic error term;[1]

(4.2.1.5.) $$\ln(M/P)_t = -\alpha \frac{(1 - e^{-\beta})}{e^{\beta t}} \sum_{-T}^{t} C_x e^{\beta x} - \delta + \varepsilon_t \quad .$$

Using monthly data Cagan estimated this model for the seven European economies under investigation that had experienced hyperinflation. The "period" of hyperinflation was defined as "beginning in the month the rise in prices exceeds 50 percent, and as ending in the month before the monthly rise in prices drops below that amount and stays below for at least a year." His results support the existence of a negative relationship between the quantity of real balances demanded and the expected rate of inflation. As expected, the strength of this relationship was much more pronounced in those countries that had experienced a higher rate of actual inflation.

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[1] It should be noted that Cagan assumed the actual and desired level of real balances were equal. Hence (4.2.1.5.) can be looked upon as the relationship between desired real balances and expected inflation.

The use of the AEH format to construct "average expected" or "permanent income" from actual measured income provides the third major use of the adaptive expectations scheme. The most well known piece of empiricial work in this area is Milton Friedman's work on the permanent income hypothesis [53]. Friedman's theory centers around the very plausible behavioral assumption that individuals smooth their consumption expenditures in accordance with their average expected income flow or permanent income. Hence per period consumption is not directly linked to measured income in that same period, but to one's conception of average income as computed over a long-run period of time, possibly a lifetime.

Since one's "permanent income" is an unobservable variable it must be computed for use in the long-run consumption function equation. Following the requirements of the AEH format, Friedman reasons that average expected income is a weighted average of past measured incomes. In symbols;

$$(4.2.1.6.) \qquad Y_{pt} = \beta Y_t + \beta\sum_{1}^{\infty}(1 - \beta)^{i} Y_{t-i} \quad ,$$

where Y_{pt} is permanent income in time period t, and Y_t is measured income in that same period. Eq. (4.2.1.6.) shows that transitory shocks to measured income will raise or lower permanent income, but by less than the change in the transitory part. The coefficient of adaptation, β, will determine how quickly changes in measured income are transmitted to changes in permanent income; if the coefficient is large, permanent income adjusts rapidly; if the coefficient is small the adjustment is slower. In keeping with the AEH mechanism described earlier, (4.2.1.6.) indicates that permanent income will always lag current measured income since permanent income is an average of the current measured magnitude and earlier observations.

Friedman's research using this AEH form for computing permanent income showed that long-run consumption expenditures were proportional to permanent income (thereby breaking with the strict Keynesian interpretation of the aggregate consumption function). Using (4.2.1.6.) Friedman defines the long-run consumption function in these terms;

$$C_t = \alpha Y_{pt} \quad , \qquad (4.2.1.7.)$$

where α is the long-run MPC and represents the degree of proportionality between consumption and long-run income.

Like earlier writers using the AEH format, Friedman determined the proper β weight coefficient of the permanent income series by choosing the coefficient value that produced the highest R^2 in regressions of the form of (4.2.1.7.). Friedman found the highest R^2 to be given by the weighting scheme using $\beta = 0.44$, and the best predictions about permanent income to be made by averaging measured income of the past four periods.

4.2.2 Limitations of the AEH Generating Mechanism. The chief appeal of the AEH mechanism stems from its econometric tractability--given a past time series of actual observations about which an "expected" series is desired, a plausible coefficient of learning, and a reasonable number of observations, the weighting pattern is automatically specified and the unobservable magnitude can be obtained for regression analysis. However, this benefit must be tempered by a number of serious problems inherent in the mechanical nature of the AEH generating scheme--all related to possible improper specification of behavioral parameters.

Perhaps the most unsatisfactory feature of the AEH is the fact that the mechanism forces the expectations variable to conform to an ad hoc weighting scheme and lag structure. Once the coefficient of adjustment is specified, the proxy variable is forced to follow a particular rate of exponential decay whether or not such a pattern truly represents the way in which expectations are being formed. This negative aspect of the AEH is probably less pronounced when the actual variable being modeled has exhibited a fairly stable past. In this case an exponential decay form might reasonably represent the true expectations formation process.[1] However, during periods when the variable being modeled has exhibited a high degree of volatility the fixed weighting scheme must be questioned.

A related problem concerns the linear nature of the weighting format. The arbitrary assignment of fixed coefficients to past magnitudes implies that the impact of an observation in any given period in the past will be registered with a constant weight depending upon its distance into the past, not upon the magnitude of the variable itself.

Another problem in regressions that incorporate the AEH is that one is, in effect, testing two hypotheses: 1) the fixed weighting format itself, and 2) the causal effect relating the dependent variable to the hypothesized proxy. For example, if the researcher is interested in testing the influence of price expectations on some dependent variable, he/she substitutes his proxy model for the theoretical price expectations variable and estimates the complete regression. However the weights of the expectations model are then confounded with the coefficient of price expectations in the structural model, and, unless some assumption about the sum of the weights is made a priori, it is impossible to derive the estimated coefficients of interest. Usually the assumption is made that the weights sum

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[1] The fact that measured personal income has exhibited a fairly stable past about a long-run trend lends much credence to Friedman's use of the AEH in his permanent income hypothesis.

to unity. Hence the model is testing the adequacy of this proxy restriction in addition to the underlying theory. Positive results will support the modeled relationship, but negative results cannot be used to reject it. Thus the adequacy of regression models utilizing the strict AEH cannot be judged in terms of their ability to produce high R^2 statistics. The highly autocorrelated nature of many time series can lead to statistically robust regressions, but this fact should not necessarily be interpeted as evidence that expectations are being formed in the hypothesized manner.[1]

The fact that the AEH expectations generating mechanism tends to produce statistically biased forecasts is another fault of the fixed coefficient format. As was demonstrated in Section 4.2., when the adjustment coefficient is less than one and when the actual rate of inflation (say) is rising over time, the expectations proxy will always underestimate the actual rate. The difference between actual and expected, or "forecasted," values represents an error in forecasting. The AEH suggests that a systematic relationship between forecast errors can exists over time; the result is biased forecasts.

The informational implications of systematic forecast bias can be examined in a more microeconomic setting. For example, the forecast error is;

$$(\pi_t - \pi_t^e) = u_t \quad , \tag{4.2.2.1.}$$

where π_t and π_t^e are the actual and expected rates of inflation occurring in period t, respectively. If π_t is rising over time, u_t, the forecast error, will always be positive. Formation of the unobservable expectations proxy via the AEH means the forecaster is constrained to use a restricted

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[1] Sargent [54] has shown that proxies that force the weights to sum to unity in inflation studies lead to upwardly biased estimates.

information vector when forming inflation forecasts, one consisting of past actual inflation rate magnitudes only. However, given the fact that forecasting errors are costly, it is plausible to assume that continuing serial correlation of errors in prediction (caused, for example, by a structural change in the stochastic process generating the inflation), would not be overlooked in forming expectations since the pattern of forecast mistakes is a very cheap and useful form of information about the true distributional properties of the inflation series.

For example, assume the error term in (4.2.2.1.) can be described by the following autocorrelation structure;[1]

$$u_t = \sum_{1}^{\infty} v_i u_{t-i} \quad . \qquad (4.2.2.2.)$$

The utility-maximizing forecaster will not be oblivious to the v_i weights, but would become conscious of the pattern by which the actual inflation rate continually leads the forecasts. Specifically, the autocorrelation structure implied in the v_i pattern would be incorporated in the forecasting rule. The AEH does not provide for such a "learning" process on the part of economic agents.[2]

A final defect of the AEH is that the relevant information vector denies the forecaster any knowledge of other exogenous forces which might influence the expectation formation process. This idea, as demonstrated

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[1] In terms of (4.2.2.2.) the AEH implies that $v_i = 0$ except for $i = 1$.

[2] While the AEH implies that individuals continually make incorrect predictions it should be pointed out the hypothesis will not necessarily produce biased forecasts. Forecasts will be unbiased if the variable being predicted is generated in precisely the same way as the weighting scheme chosen. In this (rare) case forecast errors would not contain a systematic pattern.

by Nelson [55] and Rutledge [56], states that purely extrapolative forecasting techniques, since they are constructed exclusively upon the past history of the variable being forecasted, exclude information which might reasonably be expected to influence the forecasting process. This position implies that the information vector should include the past histories of endogenous as well as present values of exogenous variables. The statistical implications of excluding exogenous variables means that forecasts are not minimum mean square, since the constrained information vector leads to forecast variances which are greater than they would be if the full information vector were utilized.

This indictment of the AEH forecast generating mechanism has merit if collection and assimilation costs of information are zero or negligible. However, before an extrapolative scheme can be faulted on this point, one must make certain assumptions about the costs of obtaining a "complete" information vector relative to the benefits to be received from the more accurate forecasting such a vector would provide. Depending upon the assumptions made regarding microeconomic behavior under utility maximization postulates, it is not at all apparent that forecasters faced with a limited budget and positive information costs will attempt to compile a complete listing of all exogenous factors from which more accurate predictions would follow.

Since the techniques used in this study to generate the expectations variables are intimately concerned with the assumptions made regarding the marginal costs and benefits of added information collection, further comment on this topic should wait until these assumptions are spelled out in more detail in Section 5.4.

4.3. The Rational Expectations Hypothesis

The REH is a theory about expectations formation which is aimed at correcting the faults of the AEH; in terms of modern expectational theories of inflation, the idea that inflationary expectations are formed "rationally" has replaced the strict extrapolative methods in inflation research.

Four main factors account for the popularity of the REH in contemporary inflation models and theory: 1) the inability of earlier macroeconomic theories to explain simultaneous inflation with unemployment witnessed during the 1970's, 2) the continuing monetarist claim that long-run output is invariant to sustained policy stimulation, 3) the increasing acceptance by theorists that inflationary expectations are endogenous to the inflationary phenomenon, and, 4) the general desire of economists to build inflation models that are in accord with the idea that individuals behave in their own best interests. The purpose of this section is to briefly discuss these topics and to illustrate the theoretical foundations underlying the REH.

The concept of rational expectations put forth by John Muth [ ] in 1961 is a profound theoretical pronouncement about economic behavior.[1] The essence of the hypothesis is built upon the idea that the manner in which expectations are formed depends specifically upon the structure of the relevant system describing the economy, i.e., expectations that are "rational" are the same as the true mathematical expectation of the future variables

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[1] A fact that is often overlooked in the literature on rational expectations is that a 1960 article by Muth [47], "Optimal Properties of Exponentially Weighted Forecasts," laid the statistical groundwork for his pathbreaking 1961 article. It should also be noted that the concept was suggested as early as 1954 by Modigliani and Brumberg [57].

conditional upon all variables in the model which are know to the public up to the time the expectation is formed. As Muth suggests:

> Expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory . . . It is sometimes argued that the assumption of rationality in economics leads to theories inconsistent with, or inadequate to explain, observed phenomena, especially changes over time. Our hypothesis is based on exactly the opposite point of view; that dynamic economic models do not assume enough rationality [*ibid*., p. 316].

This statement suggests that the subjective probability distribution of possible "outcomes" (for example, the distribution of possible inflation or money growth rates in period t) tend to be distributed, for a given information set, about the "average" or mean outcome of the objective probability distribution of outcomes.[1] Hence, expectations are "rational" when they concur with the expected value as implied by the relevant economic model.

Muth's hypothesis is constructed upon a combined statistical and utility-maximizing paradigm: since the rational expectation of a variable is the unbiased estimator of the actual variable, the REH imparts a statistical form of maximizing behavior to economic agents. Consider, for example an objective discoverable model of the market or economy--an uncertainty model with a specified stochastic structure. The statistician would base his forecast, or anticipation of the value of variable X by calculating the minimum-variance unbiased forecast implied by the model. Muth asserts that rational agents in the market do the same; that the market's anticipated value of X equals the model's expected value of X, where "expected" is looked

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[1] The term "objective probability distribution" is identical to the "true" probability distribution.

upon in the statistical rather than the psychological sense. The important aspect of this type of statistical decision theory for the REH is that expectations will not be formed in a manner that is inconsistent with the true properties and relationships underlying the economic model.

The REH provides a distinct alternative to the AEH on a number of fronts. As discussed above, the most unsatisfactory feature of the adaptive mechanism is the implication that in times of accelerating inflation, market participants consistently under-predict the rate of inflation because they can only refer to a limited information vector. While it is plausible that under-prediction could happen over a short period of time, in the long-run market participants will learn from past mistakes and come to anticipate inflation in an unbiased manner by basing expectations on a more complete information set, one that includes a history of past forecasting error. Additionally, because past forecasting error is a continually occurring event (in a time series sense), the information set is always being updated. It is in this way that prediction behavior is made endogenous to the expectation formation process. The REH implies then that agents won't continually persist in making inaccurate forecasts when (more) complete information is available (information that is ultimately conditioned by the structure of the economy and the policies applied).

Another point of distinction between the rational and adaptive expectations philosophies is that rational expectation schemes allow for structural change in the relationships describing simultaneous macroeconomic models. This fact has particular relevance for the validity of the behavioral parameters produced by these models. As Lucas explains;

> Given that (1) the structure of an econometric model consists of optimal decision rules of economic agents,

> and that (2) optimal decision rules vary systematicially with changes in the structure of the series relevant to the decision-maker, it follows that (3) any change in policy will systematically alter the structure of econometric models [21, p. 64].

Hence if expectations are not formed in an adaptative manner, but are based upon the way policy instruments, stabilization efforts, and other exogenous variables evolve, the coefficients of reduced-form models should be allowed to change whenever the processes governing these policy instruments and exogenous variables change. The rational expectations philosophy provides this flexibility and means that the behavioral parameters produced by simultaneous models are more correctly specified.

Another problem indigenous to adaptive models is skirted with the rational format. As shown above, the AEH requires the imposition of an identifying restriction on the lag weights so that the expectations coefficient will be econometrically identifiable. This ad hoc restriction is avoided with rational expectations models since the only identifying requirement is provided by taking the mathematical expectation of the model under study. Hence no assumptions about the length of the lag distribution or upon the weight to be given past observations in forming current expectations need by specified a priori.

Another point of distinction between rational and adaptive schemes is based on the fact that the REH reconciles expectations theory with the general notion that individuals act in their own best interest, i.e., forecasts will not be systematically off-target. In this respect the REH philosophy is in the spirit of neo-Classical utility maximization under perfect knowledge. In microeconomic theory it is changes in relative prices that initiate changes in how resources will be allocated, and the information set of relative prices is assumed known to all. Individuals are thus able to obtain first- and second-order conditions based upon the true relative values of the goods they produce and consume. However, to the extent that a sustained inflation chronically clouds the relative price information vector, inter-temporal utility maximization must be denied. Hence an individual's need to alter his/her forecasting rule can be directly attributed to the inflation-caused uncertainty about the relative values of one's future portfolio.

The idea that the REH and neo-Classical utility theory are in close conformity can be made more precise by placing it in the context of the Walrasian auctioneer and intertemporal utility maximization under full information. In Walrasian terms this means that "Monday contracts" cover not only all trades for the coming week, but for an infinite horizon also, i.e., there is a complete futures market. Introducing an unbalanced inflation into such a setting (caused, say, by a non-random "money rain") would not lead to intertemporal price misinformation however, since recontracting would eliminate any future false trading. Thus the omniscient auctioneer could provide true relative price signals even with an inflationary trend, i.e., the auctioneer via recontracting could distill relative prices from a changing absolute price level and accurate price information could be disseminated before future contracts were let. Such an inflationary setting would not stop the system from grinding through the demand and supply equations to reach an (inflated) equilibrium price vector. The outcome of this scenario is that inflation would not cause future wealth uncertainty with the result that there would be no need to form expectations of future relative prices--the auctioneer would collect this information and distribute it to all market participants costlessly.[1]

In the real world, however, neither the Walrasian auctioneer nor complete futures markets exist. An unbalanced inflation can lead to false trades because future relative price information is not provided instantaneously without cost. It is precisely because of the increased possibility of intertemporal false trading, which is "costly" in a neo-Classical utility maximizing sense, that inflationary expectations are formed. And, in this light, it is apparent that the whole idea of the rational expectations concept is in conformity with the Classical idea of self-interest. In this way the REH has returned inflation theory to the fold of neo-Classical utility maximization by providing a theoretical rationale whereby individuals can approach perfect information in a state where knowledge is not perfect. This rationale also means that utility maximization over time can only be achieved if the "rule" by which forecasts are made can be altered to reflect changing exogenous factors. This fact pinpoints the different informational requirements of utility maximization within a rational expectations framework: under the REH individuals must grope for perfect future price level knowledge; in neo-Classical theory such knowledge is provided instantaneously.[2] The chief

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[1] Note that Walrasian recontracting under unbalanced inflation leads to the same conclusions as does an "inflation equilibrium," discussed earlier, i.e., there is no alteration of the "real" aspects of the economy.

[2] One caveat, of course, must be cited in comparing utility maximization under rational expectations and neo-Classical postulates. Specifically in neo-Classical theory price is deterministic; under rational expectations price is stochastic. Therefore the attainment of first- and second-order conditions under rational expectations comes about because "on average" the expected and actual price in period t will be equal. The presence of a random disturbance term can, therefore, deny utility maximization--but not in a systematic manner.

conclusion, in terms of resource allocation, follows directly from the preceding comments: in the limit, and except for random error, the relative price information vectors provided by the rational expectations and neo-Classical frameworks are the same and will lead to identical patterns of resource allocation.[1]

A difference in the effects of policy accounts for the final point of distinction between the two expectations hypotheses. As illustrated earlier, the AEH can lead to a permanent inflation/money growth-output tradeoff via stimulative monetary or fiscal policy. With the REH however, agents know the model and the parameters triggering the inflationary/money growth process; any change in policy leads not only to a change these variables, but also to changes in the expected values of these variables. Therefore rational expectations means there is no exploitable tradeoff between inflation/money growth and output in any sense pertinent for carrying out counter-cyclical policy. The following example is illustrative.

Imposing rationality means (4.2.5.) is replaced with;

$$E(\pi_t) = \pi_t^e / I_t \quad . \tag{4.3.1.}$$

Given the nature of the conditional expectation of (4.3.1.), a stochastic disturbance term must be added to give;

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[1] Assuming, of course, that information collection and dissemination are costless, an assumption which is in keeping with the spirit of the Muth hypothesis. It is also apparent from the above comments about the REH that the AEH is at variance with neo-Classical utility maximization postulates. This conclusion follows from the fixed weighting format employed by the adaptive mechanism and means that no updating of the expectations algorithm can take place. Hence, an inflation must lead to systematic intertemporal false trading.

(4.3.2.) $$\pi_t = E(\pi_t) + \varepsilon_t \quad .$$

Since, by assumption ε_t has a zero mean and is uncorrelated with the variables comprising the current information vector, I_t, i.e., with the variables actually generating the inflationary or money growth rate process, (4.3.2.) can be written as;

(4.3.3.) $$\pi_t^e/I_t = \pi_t - \varepsilon_t \quad .$$

Substituting (4.3.3.) into (4.1.6.) gives the output model;

(4.3.4.) $$\begin{aligned} q_t^s &= \beta_0[\pi_t - \pi_t - \varepsilon_t] + u_t^s \\ &= -\beta_0\varepsilon_t + u_t^s \quad , \end{aligned}$$

and output is seen to follow a completely random pattern, with its expected value equal to zero.

An important aspect of the rational expectations philosophy is thus demonstrated: while deviations of output from trend can occur, they cannot be related to policy-induced (unanticipated) inflation or money growth rates. This fact can be demonstrated by the following example. Suppose that policymakers are able to constrain the inflation or money growth rate variable to follow a feedback rule defined by;

(4.3.5.) $$\begin{aligned} \pi_t &= (1 + \rho)\pi_{t-i} \quad , \quad 0 < \rho < 1 \\ &= (1 + \rho)^k\pi_{t-k} \quad , \quad i = 1 \ldots k \end{aligned}$$

where ρ represents the magnitude of the policy instrument. If expectations about the inflation or money growth rate are rationally formed then they will become part of the forecaster's information set. Taking the expected value of (4.3.5.) gives;

(4.3.6.) $$E(\pi_t) = (1 + \rho)^k \pi_{t-k} = \pi_t \quad .$$

Substituting eqs. (4.3.5.) and (4.3.6.) into the output model (4.3.4.) shows that the feedback rule, because it is known, has no effect on output. The probability distribution of possible levels of output is, therefore, independent of the policy value chosen for ρ. Only the random disturbance u_t^s can alter output from its trend, and only because it is unpredictable "noise."

This example emphasizes the fact that the REH provides the theoretical backbone of the NRH and the vertical Phillips curve described earlier. The conclusion is the same: deliberate and sustained demand management policy is ineffective in the long-run in altering the level of economic activity.

4.4. Operational Requirements of Rational Expectations Mechanisms: An Example

The theoretical basis of the REH is appealing when compared to the crude expectations modeling implied by the strict adaptive mechanism. However, empirical studies involving the rational expectations concept must deal with the difficult practical application aspects of econometric model-building. This fact is often overlooked in the more theoretically esoteric illustrations of the REH in the literature.

This section provides a brief example of the requirements that a "forecasting rule" must meet if it is to produce rational forecasts. In terms of the modeling aspects of this study, the analysis of a rational forecasting rule is important in two respects; 1) to highlight the quantification problems which must be addressed if the rational expectations idea is to have operational significance, and, 2) to demonstrate the dynamic "learning" requirements an aggregate rational expectation formation algorithm must meet.

Mathematically, the Muth REH is as follows;

$$E(p_t) = p_t^e/I_t \quad , \tag{4.4.1.}$$

where,

E = expectations operator

p_t = the actual rate of inflation in period t as generated by the "true" inflation model.

p_t^e = the rate of inflation in period t as generated by the "rational forecasting model."

I_t = the information vector existing at time period t.

Eq. (4.4.1.) states that the mathematical expectation of the rate of inflation as produced by the "true" inflationary process is equal to the expectation of inflation produced by a hypothesized rational forecasting rule.

Assume now that the specific forms of the right- and left-hand sides of (4.4.1.) are as follows;

$$p_t = \alpha(L)p_t + \beta(L)p_t^e + \delta(L)x_t + u_t \tag{4.4.2.}$$

$$p_t^e = A(L)p_t + B(L)e_t + C(L)x_t \quad , \tag{4.4.3.}$$

where x_t represents an exogenous variable common to both models (for example, x_t might represent a particular monetary or fiscal policy regime), e_t is an inflation forecasting error occurring in time period t, and u_t is an autocorrelated error term. The expressions $\alpha(L)$, $\beta(L)$, $\delta(L)$, $A(L)$, $B(L)$, and $C(L)$ represent lag operator polynomials of arbitrary order. Since, by assumption, current values do not affect expectations, the lag operators conform to the restriction that $\alpha_0 = \beta_0 = \delta_0 = A_0 = B_0 = C_0 = 0$. u_t has an arbitrary autocorrelated time structure which can be expressed as a function of independent and identically distributed random errors;

$$u_t = \sum_0^\infty \phi_i \varepsilon_{t-i} = \phi(L)\varepsilon_t \quad ,$$

where,

$$E(\varepsilon_t) = 0 \quad ,$$

$$E(\varepsilon_t \varepsilon_s) = 0 \text{ if } t \neq s \quad ,$$

$$= \sigma^2 \text{ if } t = s \quad .$$

Notice that eqs. (4.4.2.) and (4.4.3.) are determined simultaneously, implying that the expected rate of inflation is an endogenous variable influencing the actual inflationary process.

The reduced form of the model is;

(4.4.4.)
$$p_t = \alpha(L)p_t + \beta(L)[A(L)p_t + B(L)e_t] + [\beta(L)C(L) + \delta(L)]x_t + \phi(L)\varepsilon_t \quad ,$$

and,

(4.4.4.')
$$p_t - \alpha(L)p_t - \beta(L)A(L)p_t = \beta(L)B(L)e_t + [\beta(L)C(L) + \delta(L)]x_t + \phi(L)\varepsilon_t \quad .$$

Adding p_t to both sides of (4.4.4.') and rearranging terms gives;

(4.4.5.)
$$p_t + \phi^{-1}(L)p_t - \phi^{-1}(L)\alpha(L)p_t - \phi^{-1}(L)\beta(L)A(L)p_t = p_t + \phi^{-1}(L)\beta(L)B(L)e_t + \phi^{-1}(L)[\beta(L)C(L) + \delta(L)]x_t + \varepsilon_t \quad .$$

Now since $\phi_0 = 1$ (i.e., full weight is given to the current disturbance term in the error structure), eq. (4.4.5.) can be expressed entirely in terms of the observable variables p_t, e_t, x_t, and the current uncorrelated disturbance term ε_t;

$$(4.4.6.)\qquad p_t = [1 - \phi^{-1}(L) - \phi^{-1}(L)\alpha(L) - \phi^{-1}(L)\beta(L)A(L)]p_t$$
$$+ [\phi^{-1}(L)\beta(L)B(L)]e_t$$
$$+ [\phi^{-1}(L)[\beta(L)C(L) + \delta(L)]]x_t$$
$$+ \varepsilon_t \quad .$$

Eq. (4.4.6.) is a description of the actual evolution of p_t over time as a function of the appropriate infinite polynomial lagged series on each of the respective observable variables. It should be emphasized that this evolutionary inflation process is time dependent and has economic meaning only if it is a convergent expression in past p_t, e_t, and x_t.[1] In terms of the practical aspects of modeling time series, convergence must be assumed if coefficient parameters are to be estimated and subjected to the standard statistical examinations. In terms of computer search procedures then, convergence is a mathematical prerequisite.[2] However, from a more pragmatic standpoint convergence is a necessary requirement if rational forecasts are to be produced from past observations of the actual rate of inflation. That is, if the past is going to be sensibly related to the present, convergence of the inflation rate time series must be assumed.[3]

Applying condition (4.4.1.) to eqs. (4.4.6.) and (4.4.3.) and given the assumption that $E(\varepsilon_t) = 0$, the following equalities must hold if expectations are formed rationally;

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[1] Non-convergence of rational expectations algorithms is a distinct theoretical possibility and has been examined by Taylor [58], Brock [59], Cyert and DeGroot [60], and others.

[2] The mathematical requirement for convergence of a time series forces certain restrictions on the parameters the model can entertain. See Box and Jenkins [61, p. 90].

[3] Empirically the inflation and money growth rate series being modeled here do not exhibit explosive behavior, hence convergence assumptions are not unreasonable.

a) $A(L) = [1 - \phi^{-1}(L) - \phi^{-1}(L)\alpha(L) - \phi^{-1}(L)\beta(L)A(L)]$,

b) $B(L) = [\phi^{-1}(L)\beta(L)B(L)]$,

c) $C(L) = [\phi^{-1}(L)[\beta(L)C(L) + \delta(L)]]$.

Conditions a), b), and c) imply that the rational forecasting generator, as shown in eq. (4.4.3.) must correspond exactly to the way the inflation rate is evolving over time, when expectations are endogeneous to that evolution (except for the unpredictable error term, ε_t). This, in turn, implies that the equalities of conditions a), b), and c) hold <u>only</u> for the lag polynomial specifications A(L), B(L), and C(L), which specify the autocorrelated pattern of the rational forecasting algorithm--for any other specification, eq. (4.4.6.) will not match the forecasts produced by the rational forecasting rule as embodied in eq. (4.4.3.).

The above model presents the requirements necessary for rationally formed expectations and forcefully demonstrates the tremendous informational and computational problems which must be overcome if economic agents are going to produce rational forecasts in a dynamic setting when the forecasted variable is endogenous to the expectation formation process. Whether, in fact, agents have the requisite information or computational techniques and skills necessary to solve the simultaneous model is another question. The blunt mathematical statement of the REH as portrayed in (4.4.1.) implies that agents possess the required capabilities. However, this is possibly too strict a requirement when the theory of rationally formed expectations is placed in a real world setting. A more plausible middleground (and the approach taken in this study) is to assume that actual expectations formation only approximates the strict rationality requirements outlined above. Once a rational forecasting mechanism is specified then, it is logical to question how good an approximation it is and whether rational forecasts will be the ultimate outcome of the process of prediction. The answer to this question ultimately depends on what range of information is available to agents and what the computational requirements are, in addition to the general properties of the stochastic forces governing the evolution of the time series under scrutiny.

Another implication of equalitites a), b), and c) is that the parameters of the rational forecasting generator are instaneously updated to reflect the parameter changes of the true inflation model, i.e., there is no lag in the learning process. This implication, of course, is implausible in a real world setting characterized by information lags. When the strucural parameters of the true model change, therefore, the evolution of the actual inflation rate must differ from that of the forecasting model until the parameters A(L), B(L), and C(L) are updated. Such a process of iterative error correction means that equalities a), b), and c) will hold only in the limit and interim forecasts will not be "rational" in the strict sense of the definition. This fact, however, should not be considered a serious problem, especially in empirical analysis, where the "limit" is always a moving target anyway. One can correctly conclude then that the "expectational equilibrium" implicit in the Muth hypothesis is an analytical fiction designed to highlight the important problems in inflation theory.

The more pressing question is: Can the expectation formation algorithm be modified so as to lead agents <u>closer</u> to rationality in a world where information lags and positive information costs exist? This question implies that the truly interesting aspects of the rational expectations philosophy is not the final attainment of rational forecasts, but the in-between learning process that agents must master to produce unbiased forecasts.

The focus of this study on the short-run should now be apparent. Strict rationality implies that no distinction exists between the short- and long-run; expectations adjustment is instantaneous. It is for this reason that the Muth hypothesis, by its exclusion of a learning procedure, explicitly assumes a situation of knowledge on the part of economic agents that casts models in which it is used in a mold most commonly associated with equilibrium economics. The approach here cannot take such an extreme position. As the above example demonstrates, unbiased forecast estimates can be expected to hold only in the limit; in the short-run forecasts will be off-target because of 1) the presence of random error, and, 2) the presence of natural impediments to learning. Only by recognizing that learning is a on-going process of error correction can long-run "rationality" be reconciled with short-run forecast bias. Hence, what is required for short-run "rationality" is the specification of a method of learning which reduces short-run prediction bias. Such a specification, to be outlined in Chapter V, will give the rational expectations concept short-run operational significance.

4.5. The REH in the Literature

The literature dealing with rational expectations is voluminous, with the concept appearing in a myriad of settings since its introduction in the early 1960's. Included in this listing are its applications to those studies which have previously relied on some form of adaptive filtering system to generate forecasts; in many instances these re-specified adaptive models have provided more valid and economically plausible results.

Research employing the REH has followed both theoretical and empirical directions, and while much of this work is impressive, it is the analysis having to do with the NRH and short-run Phillips curve that is most relevant to this study.[1] Accordingly, the review of the literature here will be confined to the more important theoretical and applied research dealing with the REH as it pertains to the relationship between expectations and the level of economic activity. This eclectic approach is well justified since the NRH and the Phillips curve are the two economic phenomena which

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[1] The literature dealing with the REH not directly related to a test of the NRH has produced a rich menu of investigations dealing with a diverse number of topics. For example, Nelson [55] and Rutledge [56] have analyzed the structural requirements necessary to produce rational inflation forecasts and have concluded that strict adaptive models conform to rationality requirements only under special conditions. Frankel [62] and Audenhimer [63] have shown that a rational forecast can be decomposed into an adaptive and regressive component, with the adaptive part causing a transitory shift in the expectations function and the regressive part tending to damp this shift. The effect of expectations feedback on expectations formation has been studied by Mill [64] and by Modigliani and Brumberg [57]. Brock [59] and Cyert and DeGroot [60] have shown that self-fulling expectations can be consistent with a stochastic learning process leading to rationally formed expectations. Taylor [58] and Phelps and Taylor [65] have demonstrated the non-neutral effects of monetary policy on expectation formation during a transition to rational expectations formation. The theoretical prerequisites necessary for expectations convergence has been studied by Cooley and DeCanio [66] and by Mussa [67]. Stability of rational expectations mechanisms has been investigated by Peel [68]. The relationship between money growth and expectations formation instability has been examined by Sargent and Wallace [69]. In this particular study, Sargent and Wallace imposed rationality requirements upon Cagan's model of herinflation and found that expectations exhibited a stable time path even with a money-induced hyperinflation. Lucas [20] has developed an equilibrium model of the business cycle which shows that rationally formed expectations are compatible with cyclical movements in output. Black [70] has questioned the existence of stable expectations functions by showing that a unique price level, in the Arrow-Hahn sense of the word, may not be compatible with rationally formed expectations. Turnovsky [71] has studied the theoretical implications of structural change on expectation formation, and Gray and Turnovsky [72] have demonstrated the effect of information lags on the accuracy of expectations formation. And, of course, the Phelps volume [73] must be recognized as an important addition to the theoretical aspects of rational expectations and the microeconomics of search and labor market disequilibrium.

have provided the broadest base for empirically validating the REH.[1]

One of the earliest pieces of empirical work testing the existence of a vertical Phillips curve is provided by Lucas and Rapping [74]. Using U.S. data from 1900 to 1965, they show that their formulation of an expectations-based Phillips curve is not stable over three subperiods of the 1900-65 period, with the slope of the curve tending to become more vertical when estimated over later periods. The form of their model is;

$$U_t = \alpha + \beta p_t^e + \varepsilon_t \quad , \tag{4.5.1.}$$

where U_t is the unemployment rate and p_t^e the expected rate of price level change (the authors also used the expected rate of wage increase as a proxy for the rate of inflation; results were approximately the same). The unobservable expected rate of inflation was obtained via a Koyck transformation where the unemployment rate was regressed on the actual rate of inflation and a lagged value of the dependent variable; the expected rate was then gleaned from the estimated coefficient on the lagged dependent variable applied to

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[1] In terms of empirical research, the term structure of interest rates and spot and forward market speculation in international exchange ranks a close second to the NRH in terms of providing an empirical testing ground for the REH. These important applications of the REH have come to be known under the general heading of the "efficient markets hypothesis." This hypothesis is based on the general notion that financial markets reflect fully and almost instaneously new information concerning the future values of presently traded financial instruments. One aspect of the efficient markets hypothesis having to do with interest rates is the "expectations hypothesis" of the yield curve. This theory states that expectations are rational in that investors' expectations of future rates are unbiased estimators of those rates. Empirical tests have verified that, for competitive financial markets and excluding brokerage costs, forward rates of interest do equal the short rates that investors expect to prevail in subsequent periods. See Meiselman [75], Fama [76], Nelson [77], and Shiller [78] for support of this hypothesis. Another variant of the efficient markets hypothesis has to do with speculation in foreign exchange. Empirical work by Einzig [79] and Grubel [80] has shown that the current forward rate on international exchange is an unbiased predictor of subsequent spot rates.

actual past rates of inflation. It should be noted that since a Koyck transformation is used to obtain the expected inflation rate, the inflation proxy is generated according to adaptive rather than rational modeling. However, since individual Koyck estimates are obtained for each subperiod and since these estimates imply a quicker adaptation to the rate of inflation for subperiods closer to 1965 (i.e., a more vertical Phillips curve), the modeling procedure maintains the spirit of the REH. Additionally, given the statistical robustness of their regressions, one can plausibly conclude that because the actual inflation rate was mild over this period of time the Koyck/adaptive filtering system provides a reasonable proxy for a rational forecasting rule. Given the severe inflation experienced in the U.S. economy since 1965, however, it would be interesting to apply the Lucas-Rapping procedure to current data to see if the Koyck transformation would perform as well.

Lucas has also provided important theoretical work on the REH. In his "Expectations and the Neutrality of Money," [18] he shows that even if expectations are formed rationally, output fluctuations can be caused by monetary impulses that cause prices to convey incorrect information in spatially separate markets. This fact forces agents to speculate on whether a particular price movement is the result of a relative demand shift or a monetary one. Such uncertainty results in a short-run Phillips curve. However, the non-neutrality of money and resulting tradeoff vanishes over time as knowledge of conditions in other markets is obtained.

Lucas has also used the rational expectations philosophy to attack much of the existing econometric literature which purports to account for optimally formed forecasts. In his "Econometric Testing of the Natural

Rate Hypothesis," [81] he explains that many simultaneous macroeconomic models used for simulation purposes are misspecified since the expectations function is characterized by fixed coefficients. This fact precludes the possibility of simulated policy shocks having repercussions on the other endogenous variables. He thus uses the rational expectations concept as the basis for a forceful restatement of the position that the use of existing macromodels for policy evaluation can be misleading since the parameters of such models can be expected to change with policy.[1]

Fisher [82] has investigated another important theoretical aspect of rational expectations by considering output fluctuations when long-term (implicit as well as explicit) labor contracts are involved. His labor model shows that active monetary policy can affect the behavior of real output even under a rational expectations regime. His reasoning follows from the fact that because labor contracts interject an element of wage adjustment stickiness into labor market adjustments, stimulative monetary policy has the ability to affect the short-run behavior of output since it can be changed much more quickly than the nominal wages locked in by contract.

Possibly the greatest indictment of active stabilization policy stemming from the REH has been provided by Sargent and Wallace [23, 83]. Investigating the theoretical aspects of money policy, they show that with an

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[1] This criticism by Lucas is aimed at models that cannot account for structural shifts in parameters. Cooley and Prescott [84] have attempted to remedy this problem by developing estimation techniques which allow for variable parameters. These parameters are subject to permanent and transitory changes over time and can thus account for econometric relationships that vary systematically over time due to policy shifts. Koot [85] has also applied the varying parameter technique to rational expections models of the demand for money.

expectational Phillips curve, the behavior of the economy is <u>immediately</u> invariant to the preannounced systematic part of a money policy rule. They conclude that it is impossible to construct an anti-cyclical stabilization policy by means of money policy, which produces non-neutrality only through a random disturbance. Admittedly, their position regarding the effects of conscious stabilization policy is a bit extreme, since it completely denies the existence of a short-run tradeoff, regardless of how temporary.

Sargent has tended to modify this extreme anti-policy position in some of his empirical contributions to the rational expectations literature and the inflation/money-output tradeoff question. Possibly his best known empirical contribution, in which he does admit the possibility of a short-run Phillips curve is, "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment." [86]. This model decomposes the rate of inflation into systematic (predictable) and unexpected parts. The model permits the random or unexpected part of the inflation rate to have an effect on the unemployment rate, but denies the influence of the predictable part on unemployment. He then tests this model by regressing the unemployment rate against lagged values of itself and the random and systematic parts of the rate of inflation. The magnitudes of the coefficients in his regressions support the conclusion that the surprise part of the inflation rate has a much larger effect on the unemployment rate than does the systematic part. However, his work is marred to some extent by the fact that many of the coefficients on the surprise part of the inflation rate are statistically insignificant. He concludes that there is less evidence for a stable relationship between unemployment and unexpected inflation than between unemployment and the expected inflation rate.

Lucas is one of the few writers who has attempted to investigate the REH and the existence of a long-run vertical Phillips curve from the standpoint of the influence of the variation in the rate of inflation on output. Lucas' study, "Some International Evidence on Output-Inflation Tradeoffs," [19] is based on inflation and unemployment data for eighteen countries for the period 1952-67. The model is built on the premise that economic agents do not have enough information to distinguish relative from general price movements. As a result, suppliers of labor services and goods respond to price changes by changing quantity to a degree dependent on their past experience with the portion of price change in their market that typically represents a specific relative demand shift, as opposed to a general shift in the price level. Lucas tests this model by using the variance of the inflation rate as an argument in his output/employment regressions. His results show that the greater the variance of the general price level relative to the variance of market-specific price changes, the less economic agents will be prompted into responding to nominal aggregate demand changes. Lucas' reasoning and empirical results stongly support the a priori assumption that economic agents place less reliance in perceived changes in the price level (as indicators of true changes in relative prices) the greater the variation over time of the general price level, i.e., the output-inflation tradeoff deteriorates as the variance of the inflation rate increases. On a per country basis, Lucas found that those economies which had a history of highly volátile inflation rates experienced more unemployment than those countries with less volatile inflation rates. Lucas' results imply that rational agents learn to discount perceived changes in market-specific relative prices if those changes come from a distribution characterized by a large variance.

In a quite novel approach to the modeling of rational expectations, Anderson has addressed the criticism that standard econometric policy evaluation permits policy rules to change but doesn't allow expectations mechanisms to respond as economic theory predicts they should. In his article, "Rational Expectations Forecasts from Nonrational Models," [87], the author develops a method for simulating the standard "non-rational" St. Louis Federal Reserve model and the Federal Reserve-MIT-Penn model to simulate the effects of different constant money growth policies under the assumption that expectations are formed rationally. What Anderson does is take these models' own price equations as "rational estimates" of the expected rate of inflation. He then simulates the simultaneous solution by running a sequence of multi-period predictions using the forecasts from one entire simulation as the expectation for the next simulation. He then interprets the results in terms of the strict Muthian rationality assumption, i.e., agent's expectations for future periods are the same as the model's own predictions. Using the non-rational St. Louis model, for example, his simulated regressions show that an exploitable tradeoff between inflation and unemployment does exist, and higher money growth rates fed into the simulations not only increase the rate of inflation, but also decrease the unemployment rate substantially. However, when the rational expectations adjustment is fed into the model along with the increases in the money growth rate, the simulations show that the tradeoff virtually disappears.

All of the above empirical works cited treat expectations formation inferentially; the exact numerical magnitude of the expectations variable is not explicitly quantified. Barro, in a highly influencial piece of empirical work takes a different approach by using regression models to quantify the expectations variable. In his "Unanticipated Money Growth and Unemployment in the U.S.," [27] he tests the hypothesis that only the unanticipated movements in nominal money growth affect real economic variables. In order to quantify the money growth rate process he regresses actual nominal money growth rates (M1) on two lagged values of the dependent variable, federal government expenditures, and the unemployment rate. He then interprets the difference between the actual and fitted values of this regression as "anticipated" and "unanticipated" money growth rates, respectively. This procedure is in keeping with rationally formed expectations about money growth in the economy, since the expected value of a (properly specified) regression equation can legitimately be looked upon as an unbiased predictor of the true value.[1] One exception, however, in this regard, is the fact that the measure of anticipated money growth is not made conditional upon the information available to economic agents when expectations are formed (a problem this thesis will specifically address). Barro then uses the regression residuals as unanticipated money growth explanatory variables in his unemployment equation. His results support the existence of a "money-induced" short-run Phillips curve. Specifically, over the 1946-73 period, using annual data, unanticipated money growth, as he defines it, has a statistically significant negative effect on unemployment. Barro then goes on to test the corollary of

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[1] One caveat in Barro's method of quantifying expected and unexpected money growth rates is obvious. Only if the regression equation by which he determines the money growth rate process is correctly specified will the fitted and residual values truly reflect anticipated and unanticipated portions of the money stock growth rate. Although the arguments in his money equation do seem somewhat arbitrary, the statistical robustness of his work would imply that this criticism has little merit.

the unanticipated money growth-output thesis--fully anticipated money growth should have no statistically significant effect on the employment rate. Over the same period, Barro is unable to reject this hypothesis.

Other REH literature will be reviewed in Section 5.4. since it pertains directly to some of the specific expectational modeling problems encountered in this study.

CHAPTER V

STOCHASTIC TIME SERIES MODELS OF INFLATION AND MONEY GROWTH RATES

5.1. Introduction

As was mentioned earlier, the chief problem involved with expectational models is the generation of the unobservable expectational variables. As was demonstrated by the REH literature survey, this problem has been approached, in some instances, by utilizing models that only infer, rather than quantify the expectations variable. Such an inferential approach forces the modeling format to be carried out in a comparative static, "expectational equilibrium" fashion. Since this study is concerned with short-run expectations-induced output disequilibrium, and with the dynamic adjustment of output to unanticipated inflation and money growth rate "shocks," the expectations variables cannot be inferred, but require explicit quantification. To accomplish this task, Box-Jenkins (hereafter referred to as BJ) time series techniques will be utilized. As will be demonstrated subsequently, this statistical method is well suited for this purpose.

Since the BJ technique is a crucial part of this study, it is appropriate to give a brief summary of the methodology. This is done in Section 5.2. Section 5.3. will describe the statistical nature of the variables being modeled and will also provide a general outline of the modeling rationale involved in using the unanticipated variables. Section 5.4. will deal with some important methodological problems involved in the modeling of the inflation and money growth rate time series and justifies the use of the BJ technique in terms of the rationality requirments outlined earlier. Section 5.5. outlines the constrained information updating procedure to be used in forecasting, and Section 5.6. states some assumptions pertinent to expectational behavior in this study.

5.2. The Box-Jenkins Time Series Technique[1]

Modern time series analysis is usually associated with G. E. P. Box and G. M. Jenkins and their book Time Series Analysis for Forecasting and Control [61], published in 1970.[2] These techniques, initially inspired by engineering-oriented spectral analysis and used in non-business applications, have found increasing use in economic contexts.

The BJ time series method is a non-linear statistical estimation procedure that models a time series by disecting it into an autoregressive (AR) and a moving average (MA) part. In this way the technique separates the deterministic from the stochastic components of the observations. Unlike simple extrapolative methods the procedure presumes that the series to be forecasted has been generated by a stochastic process. The modeling process attempts to describe and characterize certain aspects of the series' stochastic nature so that an explanation of the types of forecast errors to be expected can be ascertained. A model's success is ultimately judged by its ability to reduce the raw series to random error.

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[1] A time series is defined as an autocorrelated list of observations of a variable drawn at equally spaced time intervals, the general assumption being that the sequence of observations is a realization of jointly distributed random variables.

[2] Autoregressive time series models were first introduced by Yule in 1926, and later generalized by Walker in 1931. Moving-average models were first used by Slutsky in 1937. However it was the work of Wold in 1939 that provides the modern theoretical foundations of combined AR and MA (ARMA) processes upon which the BJ technique is based.

An objective in time series analysis is to use the (implied) joint probability distribution of observations to make probability statements about future observations. In terms of this thesis, the forecasting objective involves identifying the underlying process which has generated the past inflation and money growth rates and then use that process (model) to generate forecasts for one-quarter-ahead time spans. The forecasts can properly be interpreted as the "expected" or "anticipated" part of the variable under study.

The basic assumption of the BJ approach to time series modeling is that the value of a variable in the current period, z_t, is generated by a linear stochastic process which characterizes the series as it moves through time. For example, the time series z_t can be produced by random shocks or "white noise," a_t, passing through a linear filter. This filter is characterized by a set of weights for the current and past a_t's. Thus in equation form we have;

$$z_t = a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \ldots + \psi_j a_{t-j} + \ldots \tag{5.2.1.}$$

$$= (1 + \psi_1 B + \psi_2 B^2 + \ldots)a_t$$

$$= \psi(B)a_t \quad , \tag{5.2.1.'}$$

where (B) is an infinite polynomial in the backshift operator and a_{t-j} are independent identically distributed random variables, or;

$$E(a_t) = 0 \tag{5.2.2.}$$

$$E(a_t, a_s) = 0 \text{ if } t \neq s \quad , \tag{5.2.3.}$$

$$= \sigma_a^2 \text{ if } t = s \quad .$$

Conditions (5.2.2.) and (5.2.3.) are synonomous with the time series term that the disturbances are "white noise." For the process defined by (5.2.1.') to be convergent, the roots of the polynomial $\psi(B) = 0$ must lie outside the unit circle.

The general stochastic process can also be represented in purely autoregressive form;

$$(5.2.4.) \qquad z_t = \pi_1 z_{t-1} + \pi_2 z_{t-2} + \ldots + a_t$$

$$(1 - \pi_1 B - \pi_2 B^2 - \ldots)z_t = a_t$$

$$\pi(B)z_t = a_t \quad .$$

This representation requires that the polynomial $\pi(B)$ be invertible, where $\psi(B) = [\pi(B)]^{-1}$. The polynomial $\psi(B)$ is invertible if $\pi(B)$ is stationary, or equivalently, the roots of $\pi(B)$ lie outside the unit circle.

The above MA and AR forms have an infinite number of terms. The inflation and money models under consideration here are ones which can be adequately represented by a finite number of terms. For example, if the model of the process is a weighted sum of q of the previous random shocks, we denote it as a MA model of order q, or MA(q). An AR model of order p, AR(p), is one in which z_t can be expressed as a weighted sum of p past values of z_t plus a_t, the current shock. Hence the representation of a stochastic process can include both AR and MA terms. A mixed model, denoted ARMA(p,q), is written;

$$(5.2.5.) \qquad z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \ldots + \phi_p z_{t-p} + a_t - \theta_1 a_{t-1}$$

$$-\theta_2 a_{t-2} - \ldots -\theta_q a_{t-q} \quad .$$

Using the lag operator the model becomes;

$$(5.2.5.') \qquad (1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p)z_t = (1 - \theta_1 B - \ldots \theta_q B^q)a_t \quad ,$$

or;

$$\phi(B)z_t = \theta(B)a_t \quad ,$$

and;

$$z_t = \frac{\theta(B)}{\phi(B)} a_t = a_t + \omega_1 a_{t-1} + \omega_2 a_{t-2} \cdot \cdot \cdot \quad ,$$

where the process is now represented by the ratio of two finite polynomials, both of which have roots that must lie outside the unit circle so that the $\omega(B)a_t$ series will be convergent. It is apparent that the mixed model, since it can be represented by a finite series, is more tractable from an econometric standpoint.

Stationarity of a time series requires that the mean and variance of the process be invariant with respect to time. Many processes, especially of an economic nature, are stationary in some difference. That is, while the level of the raw series is non-stationary, the first or second difference many be stationary. Thus the most general class of models are ones in which AR, MA, and difference operators appear. These models have been named by Box and Jenkins as autoregresssive integrated moving average processes, or, ARIMA(p,d,q), where p and q are as defined above, and d is the minimum difference required for stationarity (adding the differenced series after modeling has been completed accounts for "integration" in the above expression). In equation form a general ARIMA(p,d,q) model is represented as;

(5.2.6.) $$\phi(B)(1 - B)^d z_t = \theta(B)a_t \quad .$$

This differenced model can be expressed as a sum of past a_t's since it can be written as;

$$z_t = \frac{\theta(B)}{\Phi(B)} a_t = \Psi(B)a_t \quad ,$$

as long as the model is stationary. Similarly (5.2.6.) can be written in term's of past z_t's as;

$$\frac{\phi(B)}{\theta(B)} z_t = \delta(B)z_t = a_t \qquad ,$$

as long as the model is invertible.

Some time series exhibit significant seasonal patterns. A detailed discussion of seasonal time series and how the BJ approach models them is not in order here (see BJ, ibid., Chapter 9). It is sufficient to say that a seasonal model attempts to capture the deterministic elements of the process that are consecutively related in addition to any periodicity relating observations separated by set intervals of time.

Since our interest in this study is based upon the "rational" forecasting capabilities of ARIMA models, it is important that these forecasts be minimum mean square error forecasts. This is a statistical requirement of rational forecasts and means that the probability distribution of possible forecasts about the "true" forecast should be characterized by the smallest variance possible. To demonstrate this attribute, let us write the ARIMA model of (5.2.6.) as;

(5.2.7.) $$\Psi(B)p_t = \theta(B)a_t \qquad ,$$

where the transformation $\phi(B)(1 - B)^d$ is represented by the polynomial $\Psi(B)$, and the variable p_t is now interpreted as either the actual inflation or money growth rate. Eq. (5.2.7.) can be rewritten to show that the observed series is the realization of a stochastic process of past random shocks;

(5.2.8.) $$p_t = a_t + \beta_1 a_{t-1} + \beta_2 a_{t-2} + \cdot \cdot \cdot \qquad .$$

Required here is the set of weights placed on the past random shocks, the a_t's, which will give the minimum expected mean square error forecasts of future inflation/money growth rates for any lead time. The forecasted p_t variable for k periods ahead, at time origin t, is denoted $p_t^f(k)$. Now assume that the "best" forecast in terms of past shocks is;

$$p_t^f(k) = \beta_k^* a_t + \beta_{k+1}^* a_{t-1} + \beta_{k+2}^* a_{t-2} + \cdot\cdot\cdot \quad , \tag{5.2.9.}$$

where the β^*'s are the unknown optimal set of weights that minimize the mean square error of forecast. Now the realization of p_t in period t+k will be;

$$\begin{aligned} p_{t+k} = a_{t+k} + \beta_1 a_{t+k-1} + \beta_2 a_{t+k-2} \cdot\cdot\cdot + \\ \beta_{k-1} a_{t+1} + \beta_k a_t + \beta_{k+1} a_{t+1} + \cdot\cdot\cdot \quad . \end{aligned} \tag{5.2.10.}$$

Now the expected mean square forecast error in period t+k is;

$$\begin{aligned} E[p_{t+k} - p_t^f(k)]^2 = (1 + \beta_1^2 + \cdot\cdot\cdot + \beta_{k-1}^2)\sigma_a^2 \\ + \sum_{j=0}^{\infty}(\beta_{k+j} - \beta_{k+j}^*)^2 \sigma_a^2 \quad , \end{aligned} \tag{5.2.11.}$$

where σ_a^2 is the variance of the random disturbances. It can be seen that (5.2.11.) is minimized when $\beta_{k+j} = \beta_{k+j}^*$ for $j = 0, \infty$. Using (5.2.10.) the realization of p_t in period t+k can be decomposed into the optimal forecast and the forecast error, or;

$$p_{t+k} = e_t^f(k) + p_t^f(k) \quad , \tag{5.2.12.}$$

where $e_t^f(k)$ is the potential forecast error in period t+k. Since $E[p_t^f(k)] = p_{t+k}$ (since $E_t[a_{t+k}] = 0$ for $k > 0$), then $E_t[e_t^f(k)] = 0$, and the forecast is unbiased.

Rearranging eq. (5.2.12.) gives;

(5.2.13.) $$e^f_t(k) = p_{t+k} - p^f_t(k)$$

$$= \sum_{j=0}^{k-1}\beta_j a_{t+k-j} \quad .$$

It follows from (5.2.13.) that the expectation of p_{t+k}, conditional on the values of p_i (i = 0 through k) up to and including time period t is;

(5.2.14.) $$E[p_{t+k}|p_t,p_{t-1},p_{t-2}, \ . \ . \ . \ ] = p^f_t(k) \quad .$$

Thus the conditional expectation of p_{t+k} is the minimum mean square error forecast of p_{t+k} made at time period t, given the observed past realization of the inflation or money growth rate.

The forecast for any lead time k may also be written in the form of a difference equation containing AR and MA parameters;

(5.2.15.) $$p^f_t(k) = \phi_1[p_{t+k-1}] + \ . \ . \ . + \phi_{p+d}[p_{t+k-p-d}]$$

$$- \theta_1[a_{t+k-1}] - \ . \ . \ . \ . - \theta_q[a_{t+k-q}]$$

$$- [a_{t+k}] \quad ,$$

where the brackets denote conditional expectations. Since this study is particularly concerned with step-ahead quarterly forecasts, a result of importance is provided by the case where the forecasts are for a lead time of only one period (quarter). In this case the forecast error is given by;

(5.2.16.) $$e^f_t(1) = p_{t+1} - p^f_t(1) \quad ,$$

or, from (5.2.13.);

(5.2.16.) $$e^f_t(1) = a_{t+1} \quad .$$

Hence the disturbances of the stochastic process are also one-step-ahead forecast errors. In practice the true parameters of the ARIMA model describing the inflation or money growth rate series are not known but must be estimated. If the least-squares criterion of minimizing the sum of squared residuals is followed, then the BJ forecasting technique is minimizing the sum of squared errors of the one-quarter-ahead forecasts.

An important feature of the BJ methodology as it pertains to this study is the ability of the modeling procedure to revise forecasts when the time origin shifts and new information (in this case, new forecast errors) becomes available. For example, if the forecast of the inflation or money series, expressed in terms of past disturbances, in time period t+k+1 made in time period t is;

$$p_t^f(k+1) = \beta_{k+1}a_t + \beta_{k+2}a_{t-1} + \cdot\cdot\cdot \quad ,$$

when the origin moves to t+1, the forecast of p_{t+k+1} becomes,

$$p_{t+1}^f(k) = \beta_k a_{t+1} + \beta_{k+1}a_t + \beta_{k+2}a_{t-1} + \cdot\cdot\cdot \quad .$$

Thus the update of the forecast is;

$$p_{t+1}^f(k) - p_t^f(k+1) = \beta_k a_{t+1} \quad ,$$

or,

$$p_{t+1}^f(k) = p_t^f(k+1) + \beta_k a_{t+1} \quad .$$

Note that the shock $a_{t+1} = p_{t+1} - p_t^f(1) = e_t(1)$, i.e., it is the currently experienced one-quarter-ahead forecast error. The updating formula can therefore be viewed as an error-learning or "memory" mechanism. In this simple example the forecast of one period ahead is revised by adding a proportion of the most recently experienced forecast error, with the exact proportion being given by the estimated β weight (note that this is the identical form of the fixed weighting scheme of the AEH described earlier). However, in the more complex memory filter systems that will be used in this study, the forecast revision will be a function of the combined weighting schemes implied by errors occurring in more distant periods.

The forecasting properties of the ARIMA inflation and money models can also be examined in terms of their "forecast function." This function gives the forecasted value for any lead time $k > q-p-d$, where p, d, and q, refer to the "order" of the estimated model--as a function of lead time alone. The generalized ARIMA(p,d,q) model can be written;

$$(1 - \phi_1 B - . . . . - \phi_p B^p)(1 - B)^d p_t = (1 - \theta_1 B - . . . . - \theta_q B^q) a_t \quad .$$

This model shows that the value of p_{t+1}, for example, will be;

$$p_{t+k} = \hat{\phi}_1 p_{t+k-1} + \hat{\phi}_2 p_{t+k-2} + . . . + \hat{\phi}_p p_{t+k-p} + a_{t+k} - \hat{\theta}_1 a_{t+k-1} - . . . . - \hat{\theta}_q a_{t+k-q} \quad , \tag{5.2.17.}$$

where the " ^ " notation indicates the estimated parameter of the model after carrying out the d-th difference on the raw series.[1] [<u>Note: in (5.2.17.) and subsequent notation the number of AR terms, as indicated by "p," should not be confused with the raw series notation using the same subscript.</u>]

Since the unbiased forecast of p_{t+k} made at time period t is the conditional expectation of the realization in time period t+k, for $k > q$, the a_t terms disappear since $E_t[a_{t+j}] = 0, j > 0$. Thus for $k > q$, the forecasting function of (5.2.17.) becomes;

$$p_t^f(k) = \hat{\phi}_1 p_t^f(k-1) - . . . . - \hat{\phi}_{p+d} p_t^f(k-p-d) \quad ,$$

or,

$$p_t^f(k)(1 - \hat{\phi}_1 B - . . . . - \hat{\phi}_{p+d} B^{p+d}) = 0 \quad ,$$

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[1] From (5.2.17.) it is seen that the one-step-ahead forecasted value, P_{t+k}, is equal to the fitted value of the model.

where B now operates on the k-th lag. This difference equation has the general solution;

$$(5.2.18.) \qquad p_t^f(k) = b_0^t f_0(k) + b_1^t f_1(k) + \ . \ . \ . + b_{p+d-1}^t f_{p+d-1}(k) \qquad ,$$

where the b's are constants for the given time origin but change when the origin changes, and the f's are the inverses of the roots of the auxillary equation $(1 - \hat{\phi}_1 B - \ . \ . \ . \ . - \hat{\phi}_{p+d} B^{p+d}) = 0$, raised to the k-th power. Eq. (5.2.18.) shows that, given a fixed time origin and a convergent series, the forecasted value will approach a constant value as the time span of forecasts increases in length.

This illustration shows that the "memory" of the estimated time series model becomes ineffective when the number of forecasts into the future from a given time origin becomes greater than the number of MA terms. Hence the final form of the forecasting function is determined by the AR terms and by the difference operator only. However, this feature of the forecasting function is not encountered when forecasting is "within" the given observations of the time series, i.e., within a given time span t through t+k. In this study all step-ahead forecasts take place "within" the designated time span of 1952/1 through 1979/2 and, therefore, all memory terms in the estimated models are operative for all forecasts.

The identification, estimation, and diagnostic checking of ARIMA models follows a standard set of guidelines [see BJ,ibid., Chapter 5]. It would be inappropriate to give a detailed description of these steps here, but a brief account of the modeling sequence is in order.

The first step in modeling is to reduce the raw series to one that is stationary. As mentioned earlier, this requires making the mean and variance of the series invariant with respect to time by appropriate differencing. The number of differences required is determined by the nature of the raw series, with the behavior of the sample autocorrelation (ACF) and partial autocorrelation functions (PACF) associated with the various degrees of differencing serving as the major guidelines in determining the exact degree. In addition, the behavior of the sample variance as successive differences are taken can be of use in the analysis, since a difference is usually justified if it reduces the variance of the series.

After stationarity is achieved, the model identification process begins by examining the sample ACF and PACF of the differenced series. Statistically significant autocorrelation coefficients suggest the order of the AR and/or MA process to be used.[1] Assuming stationarity has been achieved, an AR process of order one will have an ACF which falls off exponentially and a PACF which cuts off after lag one. A MA process of order one will exhibit an ACF that cuts off after lag one and a PACF that decays exponentially as the lags increase. A ARMA model will have an ACF which exhibits a mixture of damped sine waves and exponentials after lag q-p, while its PACF has a similar pattern after lag p-q. Seasonal time series will characteristically have ACF's which have large values at the appropriate periodicity.

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[1] The sample ACF is a series of estimates of autocorrelations at various lags where the individual k-th lag autocorrelation is defined by the sample statistic;

$$r_k = \sum_1^{t-k}(z_t - \bar{z})(z_{t+k} - \bar{z})[\sum_1^{t}(z_t - \bar{z})^2]^{-1} \quad .$$

This statistic is the estimated autocovariance function at lag k divided by the estimate of the variance of the stochastic process which is equivalent to the autocovariance at lag = 0. Note that the statistic has meaning only if the variance of the process is finite, i.e., the time series does not exhibit explosive behavior.

It should be noted that the model which is initially identified by the above procedures should be considered, at best, as a tentative form for characterizing the nature of the time series of interest. Since a unique model is not always indicated by the identification procedure, the sample ACF's and PACF's do not always present a pattern sufficiently clear to determine the "best" model precisely. This is due to sampling error and the fact that the observed series is only one realization of the true underlying ARIMA process. One partial remedy for this problem, once a tentative model has been identified, is to "overfit" the model with additional AR and/or MA terms by either adding them directly to the model or implying them via some multiplicative term. The overfitted p and/or q terms can then be judged appropriate or not depending upon the significance of the additional estimated parameters. The objective here is parsimony--build a model with no more terms than necessary.

Once the order of the model has been tentatively identified it is estimated via maximum likelihood techniques so as to minimize the sum of squared residuals from the model.[1] Since the general equation form;

(5.2.19.) $$\theta^{-1}(B)\phi(B)z_t = a_t$$

is non-linear in the parameters, estimation must be non-linear. Typically, time series estimation is performed using the Marquardt [88] iterative search routine, in which the first two terms of (5.2.19.) are linearized via a Taylor expansion and OLSQ estimates obtained. These estimates are then used again and another iteration taken so as to reduce the sum of squared residuals.

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[1] The assumption that errors are normally distributed is, of course, required for valid use of the maximum likelihood estimation technique.

This iterative process continues until convergence on the "best" parameters occurs.[1]

Another aspect of model estimation is the use of "backforecasting." This technique can best be explained with a simple example. Assume that the model to be estimated contains a first-order MA term;

$$z_t = (1 - \theta_1 B)a_t \quad ,$$

or,

$$z_t = a_t - \theta_1 a_{t-1} \quad ,$$

where z_t has been properly differenced to achieve stationarity. For t = 1, the relationship is;

$$z_1 = a_1 - \theta_1 a_0 \quad .$$

Since z_1 is the first observation in the time series, a_0 cannot be estimated

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[1] The convergence process requires that initial values of the estimates be provided. It is wise to use more than one set of initial values to insure that a global rather than local minimum sum of squares has been achieved. Non-convergence can occur and indicates that either 1) stationarity has not been achieved, or 2) the initial estimate guesses are not precise enough. In either case the model estimation must be repeated after the problem(s) have been corrected. It should be noted that convergence can occur and yet the model still be marginally non-stationary. In this case the non-stationarity is not severe enough to halt the iterative search routine. However, the estimates of such models are suspect. For this reason, all seemingly convergent models should be tested to see if the roots of their characteristic equations lie outside the unit circle. An alternative convergence test is also available if the AR or MA process is of second- or third-order: if the roots of the characteristic equation lie outside the unit circle then certain restrictions are placed on the values that the parameters the characteristic equation can entertain; if these restrictions are met, then convergence is assured. See BJ [*ibid.*, p. 90 and p.91] for the exact nature of these restrictions. (I wish to express my appreciation to Professor Randall Cline, Department of Mathematics and Department of Computer Science, The University of Tennessee, Knoxville, for patiently explaining to me the rather complex mathematical reasoning underlying these restrictions.)

from the above form, and, without backforecasting, the value of a_0 would be set equal to its expected value of zero. However, a_0 is obviously an important factor in determining z_1. Ignoring it by setting it equal to zero will introduce a transient value (zero) into the series which may have noticeable effects upon the estimation process and parameter estimates. However, the value of a_0 may be "backforecasted" by applying the model of the stochastic process to previous observations. In essence the technique forecasts values occuring prior to z_1 from which more accurate estimates of a_0 can be obtained. The backforecasting technique, therefore, results in minimizing an "unconditional" sum of squared residuals, whereas neglect of the backforecasted residuals implies minimization of a "conditional" sum of squares.[1] Estimation based upon the latter procedure may be viewed as suspect. It is obvious also that the more MA terms a model contains the more important the backforecasting procedure is for parameter estimate accuracy. (All of the ARIMA models presented in this study are estimated by an incorporates the backforecasting procedure.)

A diagnostic check of the residuals produced by the estimated model provides the final step in estimation. Recall that a crucial assumption underlying the ARIMA model form is that the error terms associated with the process being modeled are independent and identically distributed random variables., i.e., "white noise." A properly specified model, then, is one that will possess these same properties, since the residuals are estimates of the true disturbances.

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[1] A comprehensive discussion of backforecasting and the difference between the conditional and unconditional sums of squares is given in BJ [*ibid*., pp. 209-220].

The first diagnostic test in this regard consists of comparing the sample autocorrelation of residuals with their respective standard error; a sample autocorrelation which is more than twice its standard error is evidence of model inadequacy. A second test uses the sample ACF to examine whether the sample autocorrelation of residuals, taken together, indicates an inadequate model. Here the Box-Pierce "Q-statistic" tests whether the error terms have been reduced to white noise [see BJ, ibid., p.290]. This statistic for (say) T autocorrelations of the residuals of an ARIMA(p,d,q) model is;

$$Q = (N - d)\sum_{1}^{T} r_k^2 \quad ,$$

where,

r = the autocorrelation at lag k,

N = the number of data observations in the series,

d = the number of differences required for stationrity.

Box and Pierce have shown that this statistic is approximately distributed as χ^2 with (T-p-q) degrees of freedom.[1] The null hypothesis that the residuals have been reduced to white noise would be rejected if the value of Q is greater than the table x^2 value for (T-p-q) degrees of freedom at a selected significance level.

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[1] Ljung and Box [89] revised this statistic in 1978 when tests at the University of Wisconsin showed that Q values were suspiciously low, implying that the distribution of the statistic could deviate substantially from a χ^2(T-p-q) distribution. Their revision of the statistic, called the "adjusted Box-Pierce Q-statistic," appears below and is used exclusively in judging model adequacy in this study;

$$Q = T(T + 2)\sum_{1}^{T}(T - k)^{-1} r_k^2 \quad .$$

In terms of the expectational modeling objectives of this thesis, i.e., to accurately mimic rationally formed expectations about the rate of inflation and money growth, it is important to restate the purpose of the diagnostic check on residuals: the purpose of time series modeling is to capture from the raw process all the systematic (deterministic) information contained in the data, leaving only random shocks or discrete white noise. A good model will achieve this objective, and its residuals will exhibit no statistically significant residual autocorrelation pattern.

5.3. The Time Series Data and Statistical Assumptions

The BJ method will be applied to four quarterly data series, the Consumer Price Index (CPI), the implicit Gross National Product deflator (DFT), currency plus demand deposits (M1), and high-powered money, or the monetary base (MB), consisting of currency and banks' deposits at the Federal Reserve Banks.[1]

The CPI is a fixed weight measure that gauges the change in the cost of a typical "market basket" of retail goods and services (about 400 items)

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[1] The choice of M1, rather than M2, as the measure of nominal money balances was made for the following reason. One of the effects of inflationary expectations may be to cause agents to hedge by substituting interest-earning assets, such as time deposits at banks, for money defined as M1. While the possibility of this action is remote in periods of hyper-inflation, it must be considered for the inflation rate levels encountered in the U.S. over the period of this study. Hence a study of the relationship between output and expected money growth rates, as measured by M2, would be less meaningful than for money measured by M1, since the output response resulting from "unanticipated" M2 growth would be confounded with the growth in currency and demand deposits (M1), and that portion of M2 growth (time deposits) induced by changing expectations regarding the inflation rate. The increasing use of NOW accounts is another fact that could conceivably blur the output-M1 growth rate hypothesis under investigation here. Fortunately the widespread use of these interest-bearing checking accounts (particularly in the New England states) did not commence until after the termination date of this study.

relative to a base year cost.[1] The weights attached to each component of the market basket are determined by the importance of the item in the consumer's budget and are, in part, determined by budget surveys. The weights are changed periodically, but not year-by-year. The GNP deflator is a shifting weight index measuring the change in the nominal value of goods and services produced in the economy per period of time relative to a base year. The weights are based on proportions from the national income accounts and may vary each year. The CPI and GNP deflator are chosen here as the price indices for investigation because they measure different inflation influences; the CPI is concerned strictly with items at the retail level, while the deflator is a more comprehensive index, measuring the price change in both consumer and producer goods.[2]

Whether output is more responsive to unanticipated rates of growth of the CPI or deflator is an empirical question to be addressed in Chapter VIII. Relatedly, looking at M1 and MB will provide insight into how responsive output is to the unanticipated portions of these two money stock aggregates, and to the influence of the money multiplier.

Since this study is interested in the discrete rates of growth of the above price and money series, percentage changes in these magnitudes are calculated from 1952/1 to 1979/1; these quarterly percentage rates of change

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[1] The base year used throughout this study is 1967.

[2] The wholesale price index was initially considered as a time series candidate in this study. However, the decision was made not to use this index since it is gathered from standard producer's price lists, an information source which is a notoriously inaccurate measure of the true changes in wholesale prices. For a detailed description of these indicies and the problems associated with the WPI see "Business Statistics," supplement to the _Survey of Current Business_, U.S. Department of Commerce, 1971. For a comparison of fixed versus shifting weighting schemes for the GNP deflator, see A. H. Young and C. Harkins, "Alternative Measures of Price Change for GNP," _Survey of Current Business_, March, 1969, pp. 47-52.

become the raw data for the time series modeling. That is;

(5.3.1.) $$z_t = [Z_t - Z_{t-1}](100)[Z_{t-1}]^{-1} \quad ,$$

where Z_t and Z_{t-1} represent the relevant price index or nominal money stock variable in periods t and t-1, respectively.[1]

If a time series is to be modeled, certain statistical assumptions must be made regarding the stochastic nature of the process. The following assumptions are assumed to hold for the two inflation and two money growth rate series under investigation here: 1) An observed set of data points $z_1, z_2, \ldots, z_t$ represent a particular outcome of the joint probability distribution function $p(z_1, z_2, \ldots, z_t)$, i.e., the data set $z_1, z_2, \ldots, z_t$ represents one particular realization of the stochastic process represented by the probability distribution $p(z_t, z_2, \ldots, z_t)$; 2) A future observation of z_t, z_{t+k}, is being generated by a conditional probability distribution function $p(z_{t+k}|z_1, z_2, \ldots, z_t)$. Hence the following equality holds;

(5.3.2.) $$p(z_t, \ldots, z_{t+k}) = p(z_{t+m}, \ldots, z_{t+k+m}) \quad ,$$

and,

(5.3.3.) $$p(z_t) = p(z_{t+m}) \quad ,$$

for any t, k, and m; 3) Since the series to be modeled has been rendered

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[1] An approximation of (5.3.1.) is, of course, provided by taking the log rate of change of the Z_t values, $\ln(Z_t/Z_{t-1})$. However, the initial intention of this study was to use annualized rates of change, which would have made the log transformation a much less precise approximation of the true percentage rate of change (since annualizing the quarterly figures would tend to increase the difference $Z_t - Z_{t-1}$). Although quarterly rather than annualized data was ultimately used, the transformation (5.3.1.) had already been computed and was thus used.

stationary, the following properties about the mean and variance hold;

(5.3.4.) $$\mu_z = E(z_t) = E(z_{t+m}) \quad ,$$

(5.3.5.) $$\sigma_z^2 = E[(z_t - \mu_z)^2] = E[(z_{t+m} - \mu_z)^2] \quad ,$$

for any t and m; 4) For any lag k, the following property regarding the covariance of the series holds;

(5.3.6.) $$\gamma_k = \text{cov}(z_t, z_{t+k}) = E[(z_t - \mu_z)(z_{t+k} - \mu_z)] = \text{cov}(z_{t+m}, z_{t+m+k}) \quad .$$

5.4. The BJ Technique and Rational Expectations: Justification and Methodological Problems

In Section 4.4. it was demonstrated that if knowledge about the rates of inflation or money growth is not instantaneous, a method of learning to form and improve forecasts about these variables must be specified if the rational expectations concept is to have operational significance, i.e., 1) be econometrically tractable, and 2) provide a forecasting rule that reasonably mimics the way rational agents form and update expectations. The purpose of this section is to justify the use of the BJ technique in terms of rational expectations and to show that the method conforms, in important respects, to the precepts of the rationality hypothesis. The section will also present some important methodological problems that must be addressed in the modeling procedure.

The justification for using the BJ method to mimic rationally formed expectations is as follows. Recall that an implication of the rational expectations philosophy is;

(5.4.1.) $$E[(\pi_t - \pi_t^e)] = E(e_t) = 0 \quad ,$$

where π_t and π_t^e are the actual and expected growth rates of the price level or money stock, respectively. Eq. (5.4.1.) means that rationally formed expectations will result in forecast errors, e_t, which have no systematic relationship over time, i.e., errors will exhibit no serial correlation. As was demonstrated in Section 5.2., a properly specified BJ model, by fitting parameters to the error structure of the time series, removes the serial correlation of past forecasting mistakes. This procedure is what we assume "rational" agents do when making step-ahead forecasts, i.e., they incorporate into their information vector the "weighted knowledge" of the structure of past prediction error. Hence, a BJ model that reduces the error to white noise can be considered a rational forecasting "rule" since such error is composed of pure random (non-systematic) disturbance only. The justification for viewing the BJ technique as a plausible approximation of rational forecasting then, is that <u>the identification-estimation-residual diagnosis feedback scenario outlined above is a statistical surrogate for the error-learning process rational agents undergo</u>. And, it is this same rationale that allows remaining (random) error not captured by the model to be interpreted as an "unanticipated" change in the actual time series.

An important methodological problem of this study concerns the specification of the information vector used by rational agents when forecasting. Since the BJ technique is extrapolative in nature, the assumption, implicit in the modeling procedure, is that rational forecasts can be made from an information vector containing only two types of knowledge: 1) the past history of the variable itself, and, 2) the past history of the error made in forecasting the variable. Is this a plausible assumption? Should the information vector be constrained to include just the past history of inflation and money growth rates, or should other non-price and non-money influences be admitted to the pool of knowledge upon which rational agents base their forecasts? If the answer is in the affirmative, then univariate time series analysis cannot be used to generate forecasts; the implication that follows is that rational forecasting is not compatible with extrapolative modeling.[1]

The position taken here in regards to this issue is that rational forecasts can be based upon past data of a particular variable alone, without explicit recognition of exogenous factors which might conceivably affect the forecasting rule. Specifically, while it is recognized that outside forces besides past rates of inflation and money growth influence expectations about these variables, it is assumed that these exogenous factors are fully reflected in the history of the actual series upon which the expectations are based. In effect, this stance amounts to an assumption that the evolution of these variables has accurately proxied the non-price and non-money growth determinants which might affect expectation formation.

Justification for this position rests on both theoretical and empirical grounds. As inferred earlier, the theory of utility maximization when

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[1] This is indeed a debatable issue. Nelson (55) has supported the "full information" position by illustrating that knowledge of the economic system should always reduce the forecasting error of any estimate and as a result, the history of all available time series should be utilized by agents in their determination of expectations. As a result, the empirical estimation of expectational variables, if expectations are to be rational, should include all available series. One cannot argue with this point of view in its strictest statistical interpretation--the inclusion of additional explanatory variables must, if they are truly additional knowledge, raise the adjusted R-squared statistic in empirical tests using an OLSQ procedure. However, this hardly seems a reasonable position in either an applied econometric sense, or in terms of the costs of information or the learning skills of agents. The inclusion of all potentially relevant information in rational estimation 1) assumes the data could be obtained at zero cost and 2) would tend to make unreasonable demands on an individual's knowledge of the economy.

information costs are positive can lead to a conscious decision to constrain the size of the information vector. In terms of the position taken here, the more pertinent question is: What cost considerations would lead economic agents to rely predominantly upon the past history of a variable in forming expectations about its future and willingly exclude other sources of information? This is ultimately a marginal cost-marginal benefit decision--information "inputs" enter the utility function and thus there is a "cost" which must be paid for being "rational." This fact puts definite limitations on how rational an individual will choose to be since the behavior of rational individuals should, at the least, be consistent with utility-maximizing behavior. Specifically, when information collection and assimilation is not free, expectations will be rational only up to the point at which the cost of utilizing new information is equal to the benefits to be derived from its use.

This distinction in how the Muth hypothesis is interpreted is important; it forces a redefining of what the optimal information set is and requires a discrimination between information which is available and utilized, and information which is available but not utilized because it is either, 1) too expensive, or 2) the costs of non-utilization are negligible and can be ignored. The approach used here is based upon the latter interpretation of "rational"--when information costs are positive and the agent faces a limited budget, the "economically rational" agent will make forecasts with less information (of exogenous factors) than he/she might have (this same rationale and terminology is used by Feige and Pearce [90]). Because some information is discarded there is a conscious acceptance of biased forecasts, and the (economically) optimal information vector will contain a limited quantity (and) quality) of information inputs.

A number of "cost" considerations can be used to justify the use of an information vector containing only the past history of the variable being forecasted. In terms of the price level, for example, consider the fact that most individuals enter the marketplace quite frequently and are thus "exposed" to the rates of change in the general level of prices. This frequent exposure provides agents with a relatively inexpensive "feel" for inflationary trends in the economy, and it is logical to assume that this type of cheap and easily assimilated knowledge would provide much of the information upon which forecasts are based. Additionally, to the extent that the price level is an accurate barometer of other exogenous market influences (such as a persistent increase in the money stock), it implicitly provides agents with "outside" information.

Another low-cost informational variable related to the past history of a time series is systematic forecast bias. Consider, for example, a shock to the inflation rate that initially "appears" as random but later is recognized to be a serially correlated event. As such, this event becomes a very cheap source of information because of its sequential (positive) autocorrelated nature and is thus added to the information vector.

There is also strong empirical support for the view that perfectly rational forecasts can be approximated by forecasts made from a limited information, cost-constrained base of knowledge. Some of this research demonstrates the comparable (and in some cases, superior) forecasting ability of univariate models utilizing just the past history of a variable, and multi-equation structural systems utilizing both endogenous and exogenous informational variables. For example, in conducting experiments on the Wharton econometric model, Naylor, Seaks, and Wichern [91] found that BJ univariate methods estimated investment, prices, unemployment, and GNP with a lower average absolute error than the Wharton model over the period 1963/1 to 1967/4. In fact, the error in the Wharton estimates was nearly twice as great as that found in the BJ model for all the variables forecasted, with the exception of GNP (which was only marginally superior).[1]

Nelson [92] has also provided a rigorous comparison of ARIMA and structural models by comparing the predictive ability of the Federal Reserve Board-MIT-Penn (FMP) model of the U.S. economy with ARIMA models of the FMP's endogenous variables. The FMP model consisted of 171 equations and his evaluation focused on one-quarter-ahead predictions of 14 endogenous variables of general interest, namely, nominal GNP, the unemployment rate, two price indices, and three interest rates. Predictions from the FMP model were obtained by ex post simulation where the actual values provided the ex post information for the model. In this research Nelson's methodology is in accord with the precepts of a rational expectations model. Specifically, with ex post simulation, forecasts can be thought of as being made by a user of the model who is endowed with perfect foresight with respect to the future values of exogenous variables. Hence predictions computed from the FMP model are the conditional expectations of future realizations implied by the structure of the system and the information set available to it. The fact the the FMP model utilizes the histories of all variables in the system, as well as the actual future values of exogenous variables, subsumes the set available to the ARIMA models built just on the past history

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[1] Results similar to these have been found by Zellner and Palm using a variation of the Wharton model [93].

of the variable being forecasted. One might conclude therefore, to the extent that the economy behaves "as if" it were being generated by the FMP model, that the larger information set should provide more rational, i.e., more accurate, forecasts than ARIMA models. However, Nelson found this not to be the case. While the FMP model did provide more accurate predictions, the difference between the mean error of prediction between the two methods was minor. Additionally, the ARIMA models' errors displayed relatively less autocorrelation than those of the FMP model, implying that the ARIMA predictions embodied information which was omitted by the FMP predictions, in particular, information available from the history of the variables themselves. Finally, Nelson found that the ARIMA models tracked turning points as well as the FMP model. The main implication of Nelson's analysis, in terms of rationally formed expectations, is that predictions based solely upon a "past history" information vector are not necessarily less accurate than predictions derived from a more comprehensive information set.[1]

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[1] An interesting bit of empirical work by Pearce [94] supports this conclusion from a slightly different view. Using Livingston survey data to obtain a bi-annual time series of expected inflation rates from 1947 to 1959, he compared the SSE's of this series with those produced by a univariate BJ model of the actual CPI inflation rate. His rationale for making this comparison is as follows: the Livingston survey predictions, since they are obtained from informed businessmen and economists, should be rational in the Muth sense, since the predictions are based on a non-restricted information vector. A logical basis for comparing which method is "more rational" would be to compare the forecasting accuracy of the survey and ARIMA techniques; if the survey predictions are less accurate it would suggest that this method is not "rational"; if the survey predictions are more accurate, it would suggest that agents do employ more information than just past rates of inflation in forming expectations. Pearce found the SS forecast errors to be significantly larger with the survey data than with BJ ARIMA models. He concludes that fully informed survey respondents did not efficiently utilize all available information in making forecasts and that ARIMA model forecasts, based on past information of the inflation rate only, produced "more rational" forecasts than those based on information vectors containing (potentially) much more information.

While the above discussion has given empirical support to the assertion that approximately rational forecasts can be based upon the past history of the forecasted variable alone, little attention has been given to the question of what the nature or characteristics of this past history might be like. With this concern in mind, it is pertinent here to present one important pattern of price level history of the 1952-1979 period under study here--evidence based upon the dissimilar autocorrelated structures of the inflation rate.[1] The important implication of this evidence is that the different pattern of time dependency in the inflation rate over the period has, potentially, provided forecasters with significant and inexpensive clues about the nature of the inflationary process over time.

Table 5.1. shows the autocorrelations of the quarterly rates of inflation for the CPI, Wholesale Price Index (included here for comparative purposes), and the GNP deflator for the 1952/1 through 1979/1 subperiods. Each series has been lagged eight quarters. The full period has been subdivided into fifty-four and fifty-five quarters, respectively, so as to cover time periods that are characterized by different degrees of inflation intensity (the 1952-65 period experienced none to mild inflation, while the latter period is cha acterized by high to severe inflation). All three indices show that the serial correlation of the inflation rate is 1) much stronger, and, 2) more persistent in the latter that in the former period. Since (for the most part) the inflation rate is positive over both subperiods, this observation means that the measure of association between quarterly rates of

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[1] In the interest of brevity, the autocorrelations of M1, M2, and the MB have not been presented. They show basically the same subperiod autocorrelation pattern as that displayed by the inflation measures.

Table 2.1. Autocorrelation Between Quarterly Rates of Inflation for Three Different Price Indices; 1952/1 through 1965/2 and 1965/3 through 1979/1.

	Consumer Price Index		Wholesale Price Index		GNP Deflator Index	
	1952/1-1965/2	1965/3-1979/1	1952/1-1965/2	1965/3-1979/1	1952/1-1965/2	1965/3-1979/1
	1	2	3	4	5	6
Lag						
1	.267	.775	.206	.461	.029	.664
2	-.018	.586	.153	.402	.008	.560
3	.208	.531	.379	.278	.201	.503
4	.314	.400	.200	.400	.003	.338
5	.041	.237	-.021	.224	.002	.170
6	-.174	.119	.061	.181	.258	.090
7	-.015 *	.019 *	.144	-.027 *	-.234	.020
8	-.054	-.112	.043 *	-.031 *	-.083 *	-.125 *

Notes: The symbol "*" indicates autocorrelation not significantly different from zero at the .95 confidence level. The asymptotic standard error of each sample autocorrelation is $1/\sqrt{n}$, where n is the number of observations in each time period under the null hypothesis that the true autocorrelations are zero, [cf., BJ, ibid., ch. 2]. The addition of an extra observation in the latter subperiod does not significantly affect the autocorrelations.

tion is much greater in the latter than in the former period, i.e., a high rate of inflation (higher than mean value) in one period tends to be followed by a higher (than average) rate of inflation in the following and subsequent periods much more strongly in the 1965-79 period than in the 1952-65 period.

The informational content of this finding is important in terms of expectation formation and price level predictability. Specifically, upon comparing columns one and two, three and four, and five and six, respectively, it is seen that the pattern of price change in the latter period is a much better "indicator" of what the rate of price change will be in the immediate future than is the pattern in the earlier period, where the current/future inflation rate association is much weaker. Obviously, to those agents desirous of keying in on the "true" nature of the inflationary process, such a shift in the pattern of the autocorrelation structure is an obvious (and thus, inexpensive) piece of worthwhile information. Also, this illustration indicates that, historically, the two periods are characterized by different inflationary processes. In terms of rationally formed expectations, it is plausible to assume that individuals would not be oblivious to this structural change in the inflationary mechanism, nor is it plausible to assume that they would use the same forecasting rule in both periods.

Of course, the period subdivision chosen here is arbitrary and the difference in autocorrelation structure would not have been as pronounced if the full period would have been further subdivided. The fact remains, however, that the actual past behavior of the inflation rate can provide cheap and accurate information for those wishing to more accurately predict its future. Conversely, agents ignoring the different autocorrelated structures, i.e., using the same forecasting rule over both periods, would be ignoring this information and their forecast errors could be expected to reflect this fact.

A controversial piece of empirical work studying the correlation between nominal money growth and inflation rates by Feige and Pearce [90] is also strongly supportive of the view that rational forecasts can be closely approximated by univariate models based on the past history of a variable. The issue to be raised here, one particularly relevant to this study, is the following: Since there is strong theoretical and empirical support asserting the existence of a (lagged) relationship between the growth rate of the money supply and the rate of inflation, the predictive ability of an ARIMA model of the inflation rate could conceivably be improved by enlarging the information set to include knowledge of the money supply. If this is, in fact the case, the rational forecasting ability of ARIMA inflation models utilizing the past history of inflation only are suspect.

Feige and Pearce tested this assertion. Their methodology consisted of constructing adquate ARIMA models of the inflation and money growth rates (they used monthly and quarterly data from 1953/1 to 1971/6), and then subjecting the white noise residuals of the inflation and money models to the BJ transfer function identification technique to test for post- and pre-lag cross correlations.[1] Their reasoning was that if statistically significant cross correlations exist between the residuals of the univariate models, then forecasts from the inflation model could be improved by incorporating knowledge of money growth rates into the inflation information vector. Their research indicated no cross correlation pattern which would suggest causality running from monetary aggregates to inflation. The chief implication of this finding is that information contained in the monetary series would not yield

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[1] This transfer function identification procedure is described in BJ. See BJ, ibid, Chapter 10.

more accurate predictions of inflation than a properly specified ARIMA model.

Their findings can be reconciled with received doctrine relating money to prices by recognizing the fact that the inflation rate is a very accurate proxy for past money growth rates, a fact mentioned earlier. Hence the findings of Feige and Pearce should not be interpreted as implying that a change in the money supply growth rate does not affect the rate of change of prices; rather the proper interpretation is that knowledge of monetary changes are not helpful in forecasting inflation once the information contained in past price activity is committed to a properly specified ARIMA inflation model. The Feige-Pearce analysis also suggests that an information vector containing knowledge of the history of the money stock, in addition to the inflation rate, would not provide a more accurate basis upon which to prognosticate future inflation rates than a vector containing just past price level activity.

5.5. Rational Forecasts and the Constrained Information Updating Procedure

The previous section devoted considerable attention to the specification of the proper information vector rational agents have at their disposal when making forecasts. There is a particular aspect of the ARIMA modeling in this study, however, which presents a different facet of the information specification problem. The procedure used to address this problem is described subsequently.

As described earlier, ARIMA inflation and money models will be estimated using the methods outlined in Section 5.2. for the 1952/1 through 1979/1 period. Quarter-ahead forecasts, which are the same as the fitted values of the model, will be made within this time period. Herein lies the problem as it concerns rationally produced forecasts: the step-ahead forecasts produced within the 1952-79 time frame will be determined by the AR and MA coefficients which are estimated from the full list of observations--a fact which violates the informational requirements of rational expectations modeling. That is, the inflation and money time series from period t = 1952/1 to period t + k = 1979/1 will simultaneously have an effect on the estimated coefficients. This feature is inconsistent with rationally formed expectations because it allows agents to have too much information upon which to base their forecasts. For example, an individual in period t + 50 forming a forecast for period t + 51 should not be privy to the actual values of the observations after period t + 50, i.e., he should not have knowledge of facts which have not yet come to pass. Univariate modeling using the full set of data implicitly violates this requirement.

The step-ahead forecasting feature of the BJ method is particularly appropriate for correcting this problem since the information vector upon which forecasts are made can be easily constrained to include only those observations that have occurred (ex post). First, the estimated model will be used to provide an initial step-ahead forecast using a constrained set of observations. A new observation will then be added an a new forecast obtained. This iterative updating method will continue until a full list of "constrained" forecasts are obtained.

In terms of rational behavior, it is assumed that agents use the ARIMA process that has been estimated from past data to make one-quarter-ahead forecasts of the inflation and money growth rate series; when they learn the actual value of the variable in the subsequent time period, they re-estimate their equation incorporating the new observation of the time series and again calculate a one-quarter-ahead forecast. This procedure will insure that the information set available to agents in making forecasts is not polluted by any future realizations of the time series. Additionally, it will allow agents to change their estimates of the stochastic process, but only as fast as new data becomes available. This aspect of the modeling procedure will be reflected in the fact that the coefficients which determine the value of the step-ahead forecasts will be allowed to evolve over the time period as new pieces of data enter the relevant time spectrum.[1]

The general form of the coefficient evolution model is;

(5.5.1.) $$\phi_t(B)z_t = \theta_t(B)a_t \quad ,$$

or;

(5.5.2.) $$z_t = \phi_{1t}z_{t-1} + \phi_{2t}z_{t-2} + \cdot\cdot\cdot + \phi_{pt}z_{t-p} - a_t - \theta_{1t}a_{t-1} - \theta_{2t}a_{t-2} - \cdot\cdot\cdot\cdot - \theta_{qt}a_{t-q} \quad ,$$

where the t subscript attached to the AR and MA coefficients imply they are time dependent. Letting the forecasted value of the series, z_t, be indicated by $\hat{z}_t$, the measure of unanticipated inflation or money growth rate in period t will be;

(5.5.3.) $$z_t - \hat{z}_t = z_t^u \quad ,$$

where the unanticipated variable, z_t^u, will become the independent variable(s) in the output regression models (note, the forecast $\hat{z}_t$ is formed in t-1).

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[1] It should be noted that the modeling procedure allows only for coefficient evolution; the form of the model, i.e., (p,d,q), will be determined from the full list of observations. Whether the form, in addition to the coefficients, should be allowed to evolve as new data is added is a question which will be addressed in the next section. There it will be shown that the form remained stable for the time series studied.

5.6. Inflation and Money Growth Rate Forecasts: Other Assumptions About Behavior

In the introductory remarks of this thesis the statement was made that the forecasting horizon is assumed to be one quarter in length. Given this assumption, it is necessary to state some related behavioral assumptions required for properly interpreting the use of the unanticipated growth rate variables to be used in the output regressions. The following comments apply equally to the inflation and money growth rate expectations (as indicated by the variable z).

Since $\hat{z}_t$ is the anticipated (i.e., unbiased) estimate of the variable z_t formed in period t-1, then;

$$E_{t-1}(z_t) = \hat{z}_t \quad . \tag{5.6.1.}$$

Since the forecasting horizon is one quarter in length only, a specific behavioral assumption made in this study regarding the proper time dimension interpretation of z_t is that agents "expect" $\hat{z}_t$ to hold for period t+1 also, or;

$$E_t(z_{t+1}) = \hat{z}_t \quad . \tag{5.6.2.}$$

Hence, at the <u>end</u> of period t, the <u>expected change</u> in the rate of growth of inflation or money is;

$$\hat{z}_t - E_t(z_{t+1}) = 0 \quad , \tag{5.6.3.}$$

since $E_t(e_{t+1}) = E_t(z_{t+1} - \hat{z}_{t+1}) = 0$, i.e., agents do not expect any change in $\hat{z}_t$. However, because of the actual evolution of the time series <u>during</u> period t+1, it is assumed that agents re-estimate their quarterly expectation of $\hat{z}_{t+1} = \hat{z}^*_{t+1}$. Because of the discrete nature of the data, however, this interim re-estimation process is not directly observed. The difference between adjacent (revised) forecasts is, in discrete terms;

$$(5.6.4.) \qquad \hat{z}_t - \hat{z}^*_{t+1} \quad ,$$

and represents the unanticipated portion of the time series existing at the end of period t+1 due to the presence of random forecast error.

In continuous terms, if there is some forecasting revision required during period t+1, the magnitude of the difference given by (5.6.4.) amounts to a non-zero first derivative of $\hat{z}_t$ with respect to time, or;

$$(5.6.5.) \qquad d\hat{z}_t/dt = \text{"shock to expected value of } \hat{z}_t \text{ in period t"} \quad .$$

Of course, there is nothing to preclude (5.6.5.) from equalling zero in any particular period. Thus it is assumed here that the step-ahead residuals of the ARIMA models represent the discrete counterpart of the quarterly sequence of first derivatives indicated by (5.6.5.). It is these magnitudes that provide the "shocks" to expectations, and, ultimately to output.[1]

It is also assumed here that expectations adjustement per period is complete in the sense that expectation alteration due to changes in the actual evolution of the variable during the period is completed in that same period, and not carried over into subsequent periods. If this behavioral assumption were not made, residual autocorrelation would arise, a fact which would violate rationality postulates.

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[1] This assumption regarding the time dimensional aspects of the residuals is in the spirit of Friedman's assertion that not only the first derivative, but second, and third, etc., of expectations with respect to time provide "shocks" to the prediction process [10]. Of course, Friedman is describing a continuing acceleration of the rate of inflation from a given point in time. The interpretation above is in terms of a change in expectations from a shifting point of reference.

CHAPTER VI

IDENTIFICATION AND ESTIMATION OF THE INFLATION AND MONEY GROWTH RATE MODELS: 1952/1 TO 1979/1

6.1. The CPI Inflation Model

As Figure 6.1.1. shows, the raw CPI inflation series (z_t) appears nonstationary. Figure 6.1.2., showing the ACF for the series, confirms this fact since the autocorrelations are reluctant to die off. Differencing is required and Figure 6.1.3., the ACF for the $(1 - B)z_t$ series, shows that this transformation achieves stationarity.[1] Figure 6.1.4. shows an ARMA model should be tried. Figure 6.1.5., the plot of $(1 - B)z_t$, shows that differencing has removed the trend from the data rendering the mean of the series invariant with respect to time.[2] Since published price indices are seasonally unadjusted, a fourth difference of $(1 - B)z_t$ was taken in order to check for seasonality. While the CPI does exhibit some seasonality, the ACF of $(1 - B)(1 - B^4)z_t$ did not reveal any periodicity, and, in fact, distorted the raw series as compared to the first-differenced series (the standard deviation of $(1 - B)z_t$ was .447, while that of $(1 - B)(1 - B^4z_t$ was .581).

The significant negative spikes at lags one, two, six and fourteen, and the significant positive spikes at lags four and sixteen of Figure 6.1.3.

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[1] Note that the transformation $(1 - B)z_t$ amounts to taking a second difference of the raw price level.

[2] In this case, differencing provides the same result as fitting a time trend to the data. Box and Jenkins [BJ, ibid, p. 74] have shown that repeated differencing removed a polynomial time trend of degree equal to the number of differences taken.

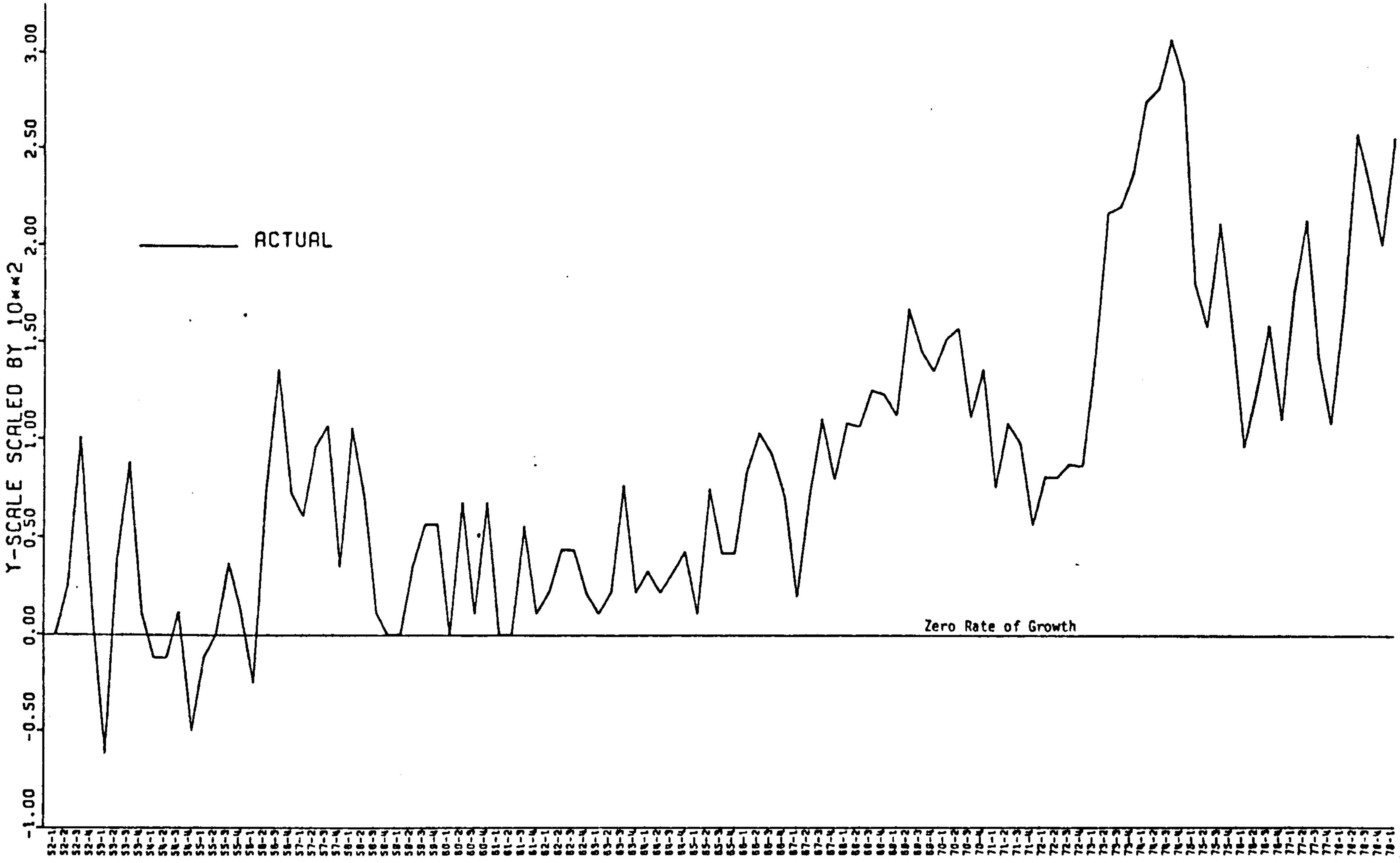


Figure 6.1.1. Actual Quarterly Inflation Rate, CPI, 1952-1/1979-1.

```
                  S.E.
            AUTO- RANDOM
ORDER CORR.  MODEL -1  -.75 -.50 -.25  0   .25  .50  .75  +1   ADJ.B-P
                    :----:----:----:----:----:----:----:----:
  1   0.815 0.094                   +   :   +           *      74.46
  2   0.706 0.094                   +   :   +         *        130.8
  3   0.710 0.094                   +   :   +         *        188.4
  4   0.656 0.093                   +   :   +        *         238.0
  5   0.544 0.093                   +   :   +      *           272.5
  6   0.474 0.092                   +   :   +    *             298.8
  7   0.463 0.092                   +   :   +    *             324.3
  8   0.405 0.091                   +   :   +   *              344.0
  9   0.342 0.091                   +   :   +  *               358.1
 10   0.349 0.090                   +   :   +  *               373.0
 11   0.388 0.090                   +   :   +   *              391.6
 12   0.387 0.090                   +   :   +   *              410.2
 13   0.394 0.089                   +   :   +   *              429.7
 14   0.378 0.089                   +   :   +   *              448.0
 15   0.429 0.088                   +   :   +    *             471.6
 16   0.478 0.088                   +   :   +     *            501.4
 17   0.430 0.087                    +  :  +     *             525.8
 18   0.398 0.087                    +  :  +    *              546.9
 19   0.381 0.086                    +  :  +    *              566.4
 20   0.355 0.086                    +  :  +   *               583.5
 21   0.297 0.085                    +  :  +  *                595.6
 22   0.253 0.085                    +  :  + *                 604.5
 23   0.239 0.084                    +  :  + *                 612.6
 24   0.194 0.084                    +  :  +*                  617.9
 25   0.146 0.083                    +  :  *                   621.0
                    :----:----:----:----:----:----:----:----:
                   -1  -.75 -.50 -.25  0   .25  .50  .75  +1

                    * : AUTOCORRELATIONS
                    + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.1.2. Autocorrelation Function, CPI, 1952-1/1979-1.

```
                S.E.
         AUTO-  RANDOM
ORDER    CORR.  MODEL   -1 -.75 -.50 -.25   0   .25  .50  .75  +1    ADJ.B-P
                        :----:----:----:----:----:----:----:----:
  1    -0.187  0.095                    *   :   +                    3.864
  2    -0.356  0.094                 *  +   :   +                    18.10
  3     0.127  0.094                    +   :  *+                    19.91
  4     0.220  0.094                    +   :   *                    25.46
  5    -0.043  0.093                    +  *:   +                    25.68
  6    -0.198  0.093                    *   :   +                    30.26
  7     0.063  0.092                    +   :*  +                    30.73
  8     0.031  0.092                    +   :*  +                    30.84
  9    -0.173  0.091                    +*  :   +                    34.43
 10    -0.130  0.091                    +*  :   +                    36.48
 11     0.126  0.090                    +   :  *+                    38.42
 12     0.011  0.090                    +   *   +                    38.44
 13     0.024  0.089                    +   *   +                    38.51
 14    -0.237  0.089                   *+   :   +                    45.63
 15     0.032  0.088                    +   :*  +                    45.75
 16     0.274  0.088                    +   :   +*                   55.47
 17    -0.095  0.088                    + * :   +                    56.66
 18    -0.041  0.087                     + *:  +                     56.88
 19     0.021  0.087                     +  *  +                     56.94
 20     0.096  0.086                     +  : *+                     58.17
 21     0.000  0.086                     +  *  +                     58.17
 22    -0.080  0.085                     +* :  +                     59.05
 23     0.071  0.085                     +  :* +                     59.75
 24     0.095  0.084                     +  : *+                     61.03
 25    -0.056  0.084                     + *:  +                     61.48
                        :----:----:----:----:----:----:----:----:
                        -1 -.75 -.50 -.25   0   .25  .50  .75  +1

                        * : AUTOCORRELATIONS
                        + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.1.3. Autocorrelation Function of (1 - B)z_t, CPI Inflation Rate, 1952-1/1979-1.

```
          PARTIAL
ORDER    AUTO COR
                  -1  -.75 -.50 -.25   0   .25  .50  .75   1
                   :----:----:----:----:----:----:----:----:
    1    -0.1866  -                *   :
    2    -0.4053  -            *       :
    3    -0.0526  -                   *:
    4     0.1208  -                    : *
    5     0.1015  -                    : *
    6    -0.0771  -                  * :
    7    -0.0187  -                    *
    8    -0.1139  -                  * :
    9    -0.2203  -                *   :
   10    -0.2729  -               *    :
   11    -0.1097  -                  * :
   12    -0.0931  -                  * :
   13     0.1702  -                    :  *
   14    -0.2054  -                *   :
   15    -0.1491  -                 *  :
   16     0.0050  -                    *
   17    -0.0963  -                  * :
   18     0.0397  -                    :*
   19    -0.0654  -                   *:
   20    -0.0399  -                   *:
   21    -0.0088  -                    *
   22    -0.0775  -                  * :
   23    -0.0518  -                   *:
   24     0.0102  -                    *
   25     0.1050  -                    : *
```

Figure 6.1.4. Partial Autocorrelation Function of $(1 - B)z_t$, CPI Inflation Rate, 1952-1/1979-1.

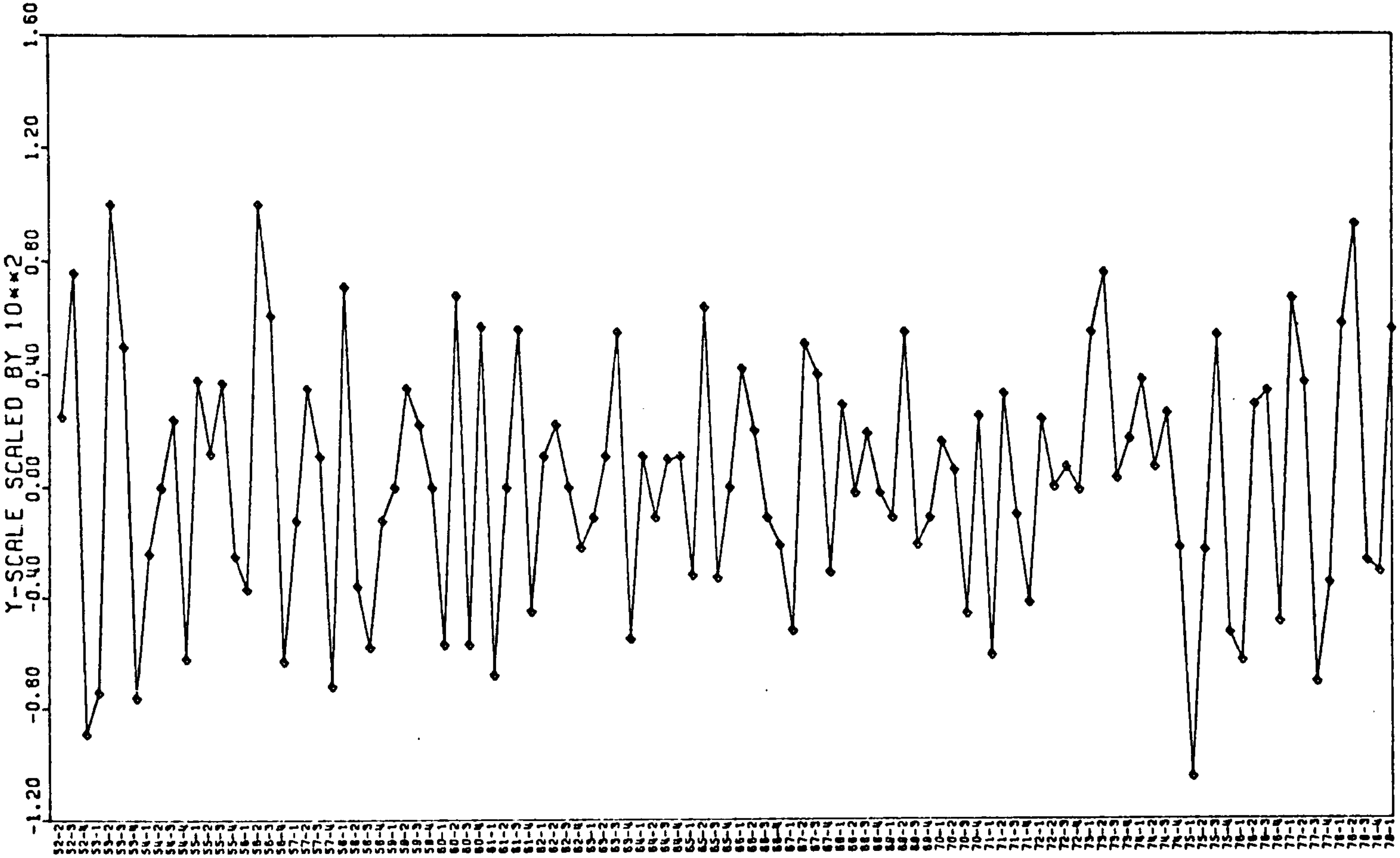


Figure 6.1.5. Plot of $(1 - B)z_t$, CPI Inflation Rate, 1952-1/1979-1.

indicate a mixed model may be appropriate. The PACF of the raw series, as shown in Figure 6.1.4. supports this view. Two mixed models were initially entertained, a (1,1,1) and a (2,1,1). In both models the AR coefficients proved to be insignificant, while the MA term was highly significant. A (0,1,2) model was therefore tried but did not satisfactorily reduce the residuals to a random state. A (0,1,3) form did improve the ACF of residuals, but the adjusted Box-Pierce (BP) Q statistic, Q(33) = 46.7[47.4], was only marginally acceptable.[1] All MA terms of this model, however, were highly significant indicating that the MA form is a proper filter specification.

Overfitting was then employed in an attempt to better model the residual pattern of the (0,1,3) model. While forms (0,1,4) and (0,1,5) did bring the BP statistic into an acceptable range, the fifth MA term of the (0,1,5) model proved to be insignificant. While the (0,1,4) model produced a good BP statistic, Q(32) = 39.9[46.2], visual inspection of the ACF revealed spikes at lags 12 and 16. In order to capture this time dependency in the residuals a multiplicative form was tried. Two models, a (0,1,3)x(0,0,1)x3, and a (0,1,3)x(0,0,1)x4 were estimated.[2] While both forms further improved the ACF of residuals, the former model produced a better BP statistic, Q(32) = 31.5[46.2], indicating the residuals have been reduced to a random state. This form was thus chosen as the final model of the CPI inflation

---

[1] The value in brackets following the BP statistic is the table chi-square value for the indicated number of degrees of freedom. All BP statistics are computed at the 95 percent level of confidence. Because of photograhic reduction constraints encountered in reproducing the ACF figures some lags were reduced from 36 to 25 or 20. Lag length, of course, does not affect the significance of the BP Q statistic.

[2] The last term in this notation indicates the order of multiplication while the second parenthesis shows to what AR or MA component the power will apply.

rate series. The estimated coefficients and related statistics appear in Table 6.1.1. as does the forecasting function.[1] Figure 6.1.6., the ACF of residuals of the final model, shows the forecast errors to be white noise.

Table 6.1.1. Estimated ARIMA CPI Inflation Model, Step-Ahead Forecasting Function, and Related Statistics; 1952/1-1979/1.

$$(1 - B)z_t = (1 - \underset{(4.92)}{.325B} - \underset{(3.13)}{.250B^2} + \underset{(10.49)}{.754B^3})(1 - \underset{(5.89)}{.554B^3})a_t$$

$Q(32) = 31.5[46.2]$

$MSE = .148$ with 104 d.f. $SSE = 15.39$

$\hat{\sigma}_a = .385$ $\bar{x}_{z_t} = .891$ $\sigma_{z_t} = .792$

$R^2 = .765$ $\bar{x}_{\hat{z}_t} = .856$ $\sigma_{\hat{z}_t} = .734$

$$\hat{z}_t = z_{t-1} + a_t - .325a_{t-1} - .250a_{t-2} + .200a_{t-3} + .180a_{t-4} + .139a_{t-5} - .418\, a_{t-6}$$

Notes: --t-statistics appear below the estimated coefficients and are significant at the .05 level.

--All R-squared statistics are "adjusted" for degrees of freedom.

--The symbols $\bar{x}_{z_t}$, $\bar{x}_{\hat{z}_t}$, and σ_{z_t}, $\sigma_{\hat{z}_t}$, represent the mean of the actual and prefiltered series and standard deviation of the actual and prefiltered series, respectively. This notation holds throughout the remainder of this chapter.

---

[1] Note: Two computer packages were used in the ARIMA model estimation: "Interactive Data Analysis," University of Chicago, 1977, and "A Computer Program for the Analysis of Time Series Using the Box-Jenkins Philosophy," by D. J. Pack, Automatic Forecasting Systems, Hatboro, Penn., 1978.

Note: At present there is much controversy in the statistical literature regarding the proper interpretation of the R-squared statistic in ARIMA modeling. In OLSQ this statistic is computed from a model that is linear in the parameters. In nonlinear estimation, however, the R-squared

```
             S.E.
       AUTO- RANDOM
ORDER  CORR. MODEL  -1      -.75    -.50    -.25    0       .25     .50     .75     +1   ADJ.B-P
                    :-------:-------:-------:-------:-------:-------:-------:-------:
    1 -0.031 0.094                            +    *:      +                           0.1089
    2 -0.093 0.094                            +  *  :      +                            1.092
    3  0.033 0.094                            +     :*     +                            1.219
    4  0.028 0.093                            +     :*     +                            1.307
    5 -0.168 0.093                            +*    :      +                            4.581
    6  0.048 0.092                            +     :*     +                            4.854
    7  0.000 0.092                            +     *      +                            4.854
    8 -0.075 0.091                            +    *:      +                            5.520
    9 -0.129 0.091                            +*    :      +                            7.521
   10 -0.091 0.090                            +  *  :      +                            8.532
   11 -0.024 0.090                            +     *      +                            8.601
   12 -0.069 0.090                            +    *:      +                            9.195
   13  0.000 0.089                            +     *      +                            9.195
   14 -0.189 0.089                            *     :      +                            13.72
   15  0.035 0.088                            +     :*     +                            13.88
   16  0.222 0.088                            +     :      *                            20.29
   17 -0.033 0.087                              +  *:    +                              20.44
   18  0.009 0.087                              +   *    +                              20.45
   19  0.039 0.086                              +   :*   +                              20.65
   20  0.008 0.086                              +   *    +                              20.66
   21  0.031 0.085                              +   :*   +                              20.79
   22  0.047 0.085                              +   :*   +                              21.09
   23  0.106 0.084                              +   :   *+                              22.66
   24  0.040 0.084                              +   :*   +                              22.89
   25  0.005 0.083                              +   *    +                              22.89
                    :-------:-------:-------:-------:-------:-------:-------:-------:
                    -1      -.75    -.50    -.25    0       .25     .50     .75     +1

                    * : AUTOCORRELATIONS
                    + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.1.6. Autocorrelation Function of Residuals, CPI Inflation Rate Model, 1952-1/1979-1.

As Table 6.1.1. shows the model has a memory of six periods. Economically this might imply that agents, in forming their optimal forecasts, look not only to the most recent forecast error, but also to last period's actual rate of inflation and the past five forecast mistakes. The form of the forecasting function is interesting in that the first, second and sixth errors show that the forecast revision is in the opposite direction of the forecast error, while the third, fourth and fifth errors show the forecast to be revised in the same direction as the error. A check of the roots of the characteristic equation of the model show that invertibility conditions are met.

Since the exponentially decaying weight filtering scheme implied by the adaptive model is so prevalent in the literature on expectations, it is interesting at this point to digress a moment and compare the rational forecasting properties of the (0,1,3)x(0,0,1)x3 model with those of the simple one-period error-learning adaptive model used by Cagan and others.

Expressing the Cagan model in discrete term gives;

$$z^e_{t+1} - z^e_t = \delta(z_t - z^e_t) \quad , \quad 0 \leq \delta \leq 1$$

---

value has questionable meaning since the statistic is calculated from the <u>last</u> linearization of the nonlinear routine and indicates how much of the observed variation in the variable is explained by the last estimate of the parameters only. Thus even with a low R-square for the last iteration, it is possible that the full nonlinear model has considerable explanatory power. Of course, the reverse reasoning also holds. The R-square statistics will be presented here with this caveat in mind. Some current statistical research attempting to address this problem is, D. A. Pierce, "R-square Measures for Time Series," <u>Special Studies Paper No. 93</u>, Division of Research and Statistics, Federal Reserve Board, Washington, D.C., 1978, and C. R. Nelson, "The Interpretation of R-square in Autoregressive-Moving Average Time Series Models," <u>American Statistician</u>, No. 4, Nov. 1976.

and,

$$z^e_{t+1} = z^e_t + \delta(z_t - z^e_t) \quad ,$$

where z^e_{t+1} is the expected value of z in time period t+1, and δ is the coefficient of expectation. If the expected values are considered to be forecasts, the above form may be expressed as;

(6.1.1.) $$\hat{z}_t(1) = \hat{z}_{t-1}(1) + \delta[z_t - \hat{z}_{t-1}(1)] \quad ,$$

where $z_t(1)$, [t = 1 . . . n], is the one-period- ahead forecast based upon information available through time period t. It is seen from (6.1.1.) that forecasts are equal to the forecast in period t-1 plus some part of the most recent forecast error.

Consider now the ARIMA (0,1,1) form;

(6.1.2.) $$(1 - B)z_t = (1 - \theta_1 B)a_t \quad ,$$

or, equivalently;

(6.1.3.) $$z_t = z_{t-1} + a_t - \theta_1 a_{t-1} \quad .$$

Rearranging (6.1.1.) in terms of observable forecasts gives;

(6.1.4.) $$z_t = z_{t-1} + a_t - \theta_1[z_{t-1} - \hat{z}_{t-2}(1)] \quad ,$$

and, via the stationarity property;

(6.1.5.) $$z_{t+1} = z_t + a_{t+1} - \theta_1[z_t - \hat{z}_{t-1}(1)] \quad .$$

Taking conditional expectations of (6.1.5.) at time period t, yields;

(6.1.6.) $$\hat{z}_t(1) = z_t - \theta_1 z_t + \theta_1 \hat{z}_{t-1}(1) \quad .$$

Now letting $\theta_1 = 1 - \delta$, and substituting gives;

(6.1.7.) $$\hat{z}_t(1) = \hat{z}_{t-1}(1) + \delta[z_t - \hat{z}_{t-1}(1)] \quad ,$$

which is identical to the forecasting format given by the AEH mechanism in

(6.1.1.). Thus the Cagan adaptive filter, when expressed in terms of discrete time can be considered an ARIMA (0,1,1) form for forecasting purposes.

The exponentially declining weights implied by the (0,1,1) form are as follows; we have from (6.1.2.);

$$(1 - \theta_1 B)^{-1}(1 - B)z_t = a_t \quad .$$

This implies that z_t can be expressed as an infinite AR form as;

$$\begin{aligned}(1 - \theta_1 B)^{-1}(1 - B) &= \pi(B)\\ &= (1 - \pi_1 B - \pi_2 B^2 - . . .) \quad .\end{aligned}$$

Equating coefficients on like powers of B gives the following weighting scheme;

$$\begin{aligned}\pi_1 &= \quad (1 - \theta_1) = \delta\\ \pi_2 &= \theta_1(1 - \theta_1) = \delta(1 - \delta)\\ \pi_3 &= \theta_1^2(1 - \theta_1) = \delta(1 - \delta)^2\\ \pi_4 &= \theta_1^3(1 - \theta_1) = \delta(1 - \delta)^3\\ &\vdots\\ \pi_j &= \theta_1^{j-i}(1 - \theta_1) = \delta(1 - \delta)^{j-i} \quad ,\end{aligned}$$

with the constant, $\delta < 1$, determining the amount of decay per period.

Table 6.1.2. shows the Cagan/adaptive (0,1,1) model applied to the raw CPI inflation series over the 1952/1 to 1979/1 period. The estimate of the learning coefficient, θ_1 = .415, implies the following numerical weights hitting the past actual rates of inflation;

Table 6.1.2. The Cagan/Adaptive (0,1,1) Model Applied to the CPI Inflation Series; 1952/1 to 1979/1.

$$(1 - B)z_t = (1 - \underset{(4.72)}{.415B})a_t$$

$Q(35) = 79.18[49.8]$

$MSE = .182$ with 107 d.f.

$SSE = 19.47$

$\hat{\sigma}_a = .426$

$R^2 = .712$

(6.1.8.) $\hat{z}_t = .585z_{t-1} + .243z_{t-2} + .101z_{t-3} + .042z_{t-4} + . . .$

This exponentially declining weighting scheme can be compared to that of the more complex (0,1,3)x(0,0,1)x3 filter of Table 6.1.2., where we have;

$$[(1 - .325B - .250B^2 + .754B^3)(1 - .554B^3)]^{-1}(1 - B) = \pi(B) \quad .$$

Again, equating coefficients on like powers of B gives the weighting scheme expressed in terms of past inflation rates (for twelve lags only).

(6.1.9.) $$\hat{z}_t = .680z_{t-1} + .011z_{t-2} + .353z_{t-3} + .115z_{t-4} + .099z_{t-5}$$
$$- .490z_{t-6} + .049z_{t-7} - .163z_{t-8} + .172z_{t-9} - .019z_{t-10}$$
$$+ .150z_{t-11} - .165z_{t-12} + . . .$$

It is apparent that the weights of the more complex model (6.1.9.) do not decline in exponential fashion, but follow a saw-toothed pattern. Comparing filters (6.1.8.) and (6.1.9.) we see that the actual CPI inflation rate series requires a considerably more complex filter than the Cagan model.

An important item to check here is whether the adaptive form, (0,1,1), fitted to the CPI inflation series, is "adequate" in a rational expectations sense. That is, since the BJ identification and estimation procedure lead to a more complex model, we might expect that the (0,1,1) filter would be underparameterized. This suspicion is confirmed by comparing the results of Tables 6.1.1. and 6.1.2. The Q statistics of the two models show that the (0,1,1) filter has not reduced the residuals to discrete random error, whereas the (0,1,3)x(0,0,1)x3 filter has. Likewise, the MSE statistics show the adaptive form provides an inferior fit. Additionally, a simple correlation of .61 was found between the errors of the (0,1,1) filter and past values of the dependent variable, $z_t - z_{t-1}$; this relatively high correlation coefficient would not be found in an adequate model [the (0,1,3)x(0,0,1)x3 filter had a correlation of .13]. We can conclude then that the Cagan/adaptive filter based upon geometrically declining weights does not predict z_t as well as the more complex filter of Table 6.1.1., and is not the best estimator. The use of the adaptive model then, to filter the CPI inflation rate series, is inappropriate as a rational expectations estimator, since it does not use the information embodied in past rates of inflation as efficiently as the alternative model.

While this study is concerned with rates of change of the price level and money stock, a logical question which arises here is this: Might it be possible to more accurately model the absolute value of the price level and money stock variables than their rates of change? If so, more precise forecasts of the absolute level of the variables in question could be obtained and then these forecasts transformed into more accurate rates of change. The assumption implicit in this reasoning is that expectations of future growth rates may be formed implicitly from the process of forming expectations about future price level and money stock levels. Of course, since expectations regarding rates of change can be mathematically derived from the absolute levels of the variables, a "true" model built upon one should provide as precise expectations estimates as the other. However, in a statistical sense, this may not be the case because one does not know the "true" model, but can only estimate it. Which form then, provides more accurate forecasts of the rate of change is an empirical question requiring statistical testing.

Table 6.1.3. presents the results of a ARIMA filter fit to the absolute level of the CPI over the 1952/1 to 1979/1 period. As the statistics indicate, the model adequately reduces the series to white noise. Note that second differencing is required; this is a reflection of the fact that first differencing was needed to render the rate of change of the CPI inflation rate series stationary (cf., fn. 1., p. 132).

Table 6.1.3. Estimated ARIMA Filter for the Absolute Level of the CPI; 1952/1 to 1979/1.

$$(1 - B)^2 z_t = (1 - \underset{(2.98)}{.266B^2} + \underset{(3.76)}{.339B^3} + \underset{(3.59)}{.349B^4})a_t$$

$Q(32) = 35.66[47.4]$

$MSE = .220$ with 105 d.f. $\qquad$ $SSE = 23.10$

$\hat{\sigma}_a = .469$

$R^2 = .699$

Now the criterion that should be adopted in order to select the appropriate method of generating expectations of the inflation rate from the two options should be based upon forecasting accuracy. Following this guideline, the sum of squared forecast errors from the estimated inflation rate model were compared to those yielded by the estimated absolute price level model. The procedure used was to generate quarter-ahead forecasts of the price level using the filter in Table 6.1.3. and then transform these forecasts into rates of change. This procedure was followed by summing the squared difference between these rate of change transformations and the actual rate of inflation for the same period. This numerical magnitude was then compared with the sum of squared residuals obtained from the inflation filter of Table 6.1.1., p. 139. The MSE's of the two approaches to determining the forecasted inflation rate appear in Table 6.1.4. It is evident that the method of forming inflationary expectations from forecasts of the inflation rate is superior to the forecasts derived from the absolute price level, although the superiority is not overwhelming. Possibly the other price and money stock series under investigation here would have produced different results. However, since this study is concerned with rates of change, and since it was shown that the CPI can be more

Table 6.1.4. A Comparison of MSE's Using the CPI Inflation Filter and the CPI Absolute Price Level Filter.

	MSE
Inflation Model (Table 6.1.1.)	.148 with 104 d.f.
Absolute Price Level Model (Table 6.1.3.)	.180 with 105 d.f.

adequately modeled as a rate than as an absolute level, the decision here is to model the rates of change for the other time series as well.

In keeping with the desire to produce forecasts not contaminated by future realizations of the rate of inflation not yet experienced by agents, the (0,1,3)x(0,0,1)x3 filter and related forecasting function was applied to the constrained information updating routine as described in Section 5.5.[1] The initial set of observations upon which the quarter-ahead forecasting commences is 1952/1 through 1956/2. Thus the first constrained forecast is made in 1956/3 and the last in 1979/2, providing a forecasted and an error series of 92 observations. It is over this period that the output regression models will be estimated.

A plot of the actual and iteratively updated forecasts appears in Figure 6.1.7. It is apparent that even with the information vector constrained to past realized values, the iterative updating procedure tracks the actual rate of inflation quite well. The pattern of coefficient evolution of the (0,1,3)x(0,0,1)x3 filter as new observations are added and the model re-estimated is shown in Table B.1. of Appendix B. Comparing Figure 6.1.8., the ACF of the iteratively produced forecasts, with Figure 6.1.2., p. 134, the ACF of the raw series, provides visual proof that the updating procedure is producing forecasts that mimic closely the actual series.

Of course, the most important piece of diagnostic analysis to check the adequacy of the updating routine is the ACF of residuals. It is essential that these residuals meet the proper statistical tests for randomness if the forecasts are to conform to the rationality criteria outlined earlier.

---

[1] The phrase "constrained information updating" and "iterative updating" are used synonomously throughout the remainder of this study.

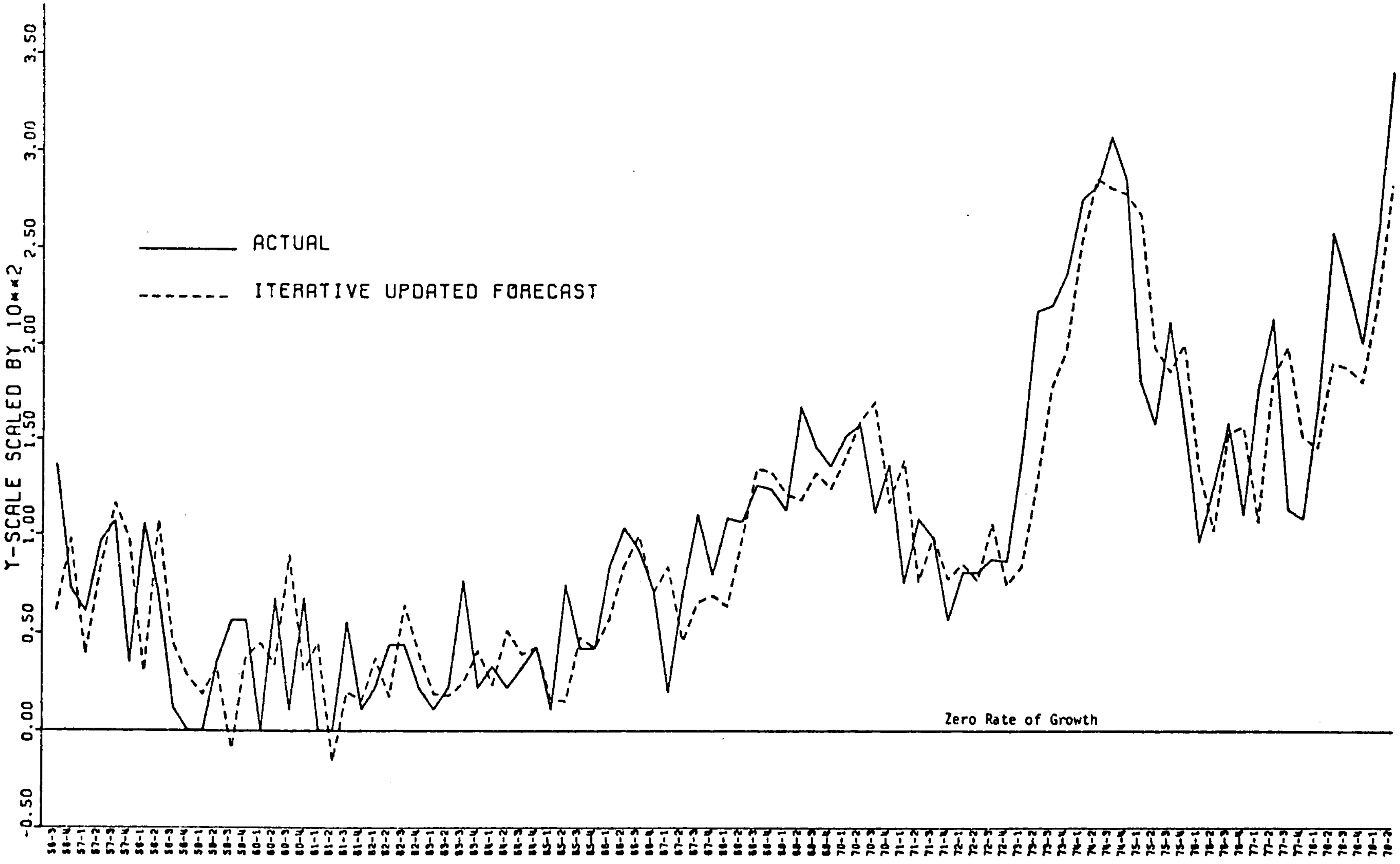


Figure 6.1.7. Actual and Iteratively Updated Forecasts, CPI Inflation Rate Model, 1956-3/1979-2.

```
             S.E.
      AUTO-  RANDOM
ORDER CORR.  MODEL  -1  -.75  -.50  -.25    0    .25   .50   .75   +1   ADJ.B-P
                    :-----:-----:-----:-----:-----:-----:-----:-----:
  1   0.855  0.103                     +    :    +               *      69.51
  2   0.791  0.102                   . +    :    +             *        129.6
  3   0.733  0.101                     +    :    +            *         181.7
  4   0.651  0.101                     +    :    +          *           223.4
  5   0.528  0.100                     +    :    +       *              251.1
  6   0.521  0.100                     +    :    +      *               278.3
  7   0.487  0.099                     +    :    +      *               302.4
  8   0.429  0.099                     +    :    +    *                 321.4
  9   0.399  0.098                     +    :    +   *                  338.0
 10   0.398  0.097                     +    :    +   *                  354.7
 11   0.387  0.097                     +    :    +   *                  370.7
 12   0.380  0.096                     +    :    +   *                  386.3
 13   0.428  0.096                     +    :    +    *                 406.4
 14   0.442  0.095                     +    :    +    *                 428.0
 15   0.457  0.094                     +    :    +    *                 451.5
 16   0.469  0.094                     +    :    +    *                 476.6
 17   0.471  0.093                     +    :    +    *                 502.1
 18   0.422  0.093                     +    :    +   *                  523.0
 19   0.384  0.092                     +    :    +   *                  540.5
 20   0.337  0.091                     +    :    +  *                   554.1
                    :-----:-----:-----:-----:-----:-----:-----:-----:
                    -1  -.75  -.50  -.25    0    .25   .50   .75   +1

                    * : AUTOCORRELATIONS
                    + : 2 STANDARD ERROR LIMITS (APPROX.)
```

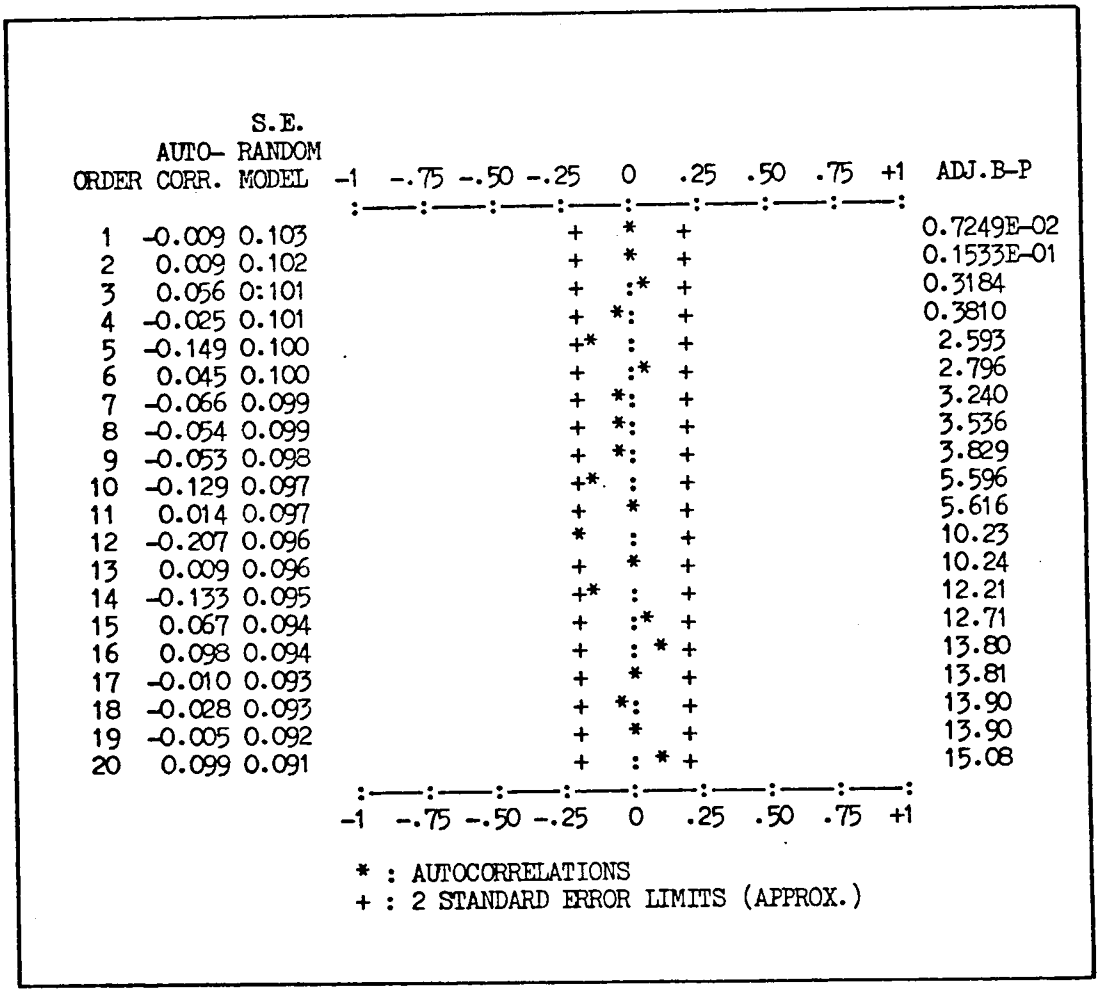
Figure 6.1.8. Autocorrelation Function of Iterative Updated Forecasts, CPI Inflation Rate Model, 1956-3/1979-2.

A potential problem in this respect is that the iterative forecasts, and associated residuals, are being produced from a model that does not have a full set of observations upon which the maximum likelihood search routine is taking place. Hence some autocorrelation of residuals and a somewhat higher SSE might be expected, as compared to the ACF of residuals obtained from estimation over the full period. However, as Figure 6.1.9. shows, this concern is unfounded for the CPI inflation model, as the ACF of iterative residuals appears to meet the requisite statistical and visual tests of model adequacy; all autocorrelations lie within the 1.96 standard error limits, and the BP statistic is satisfactory at Q(20) = 15.08[31.4].[1] To further substantiate the randomness of the residuals, a "runs" test on the median of the errors was performed and produced a Z-value of -.084, which is well within the .05 significance level limits of the standard normal distribution. A final bit of visual analysis supporting the adequacy of the iteratively-produced residuals is provided by Figure 6.1.10. which shows that the errors of the model are randomly distributed about the mean.

Given the limited information base upon which the iteratively-produced forecasts and residuals are estimated, it is pertinent at this point to compare some selected statistics of the iterative model with those produced by estimation using the full list of observations. Table 6.1.5. presents these comparative statistics. The Table shows that while both methods of forecasting produce similar statistical profiles, the residuals from the full

---

[1] Note that the use of a BP Q-statistic here is not strictly applicable in terms of a judgement of the adequacy of the updating model per se, since the number of observations upon which the model is forecasting from, and upon which the statistic is based, is not constant. However, the statistic is relevant in judging whether this particular series of observations, however produced, is white noise or not. According to this interpretation of the statistic, the residuals do meet the randomness test.



```
             S.E.
       AUTO- RANDOM
ORDER  CORR. MODEL  -1 -.75 -.50 -.25   0   .25  .50  .75  +1  ADJ.B-P
                    :----:----:----:----:----:----:----:----:
    1 -0.009 0.103                  +   *   +                  0.7249E-02
    2  0.009 0.102                  +   *   +                  0.1533E-01
    3  0.056 0:101                  +   :*  +                      0.3184
    4 -0.025 0.101                  +  *:   +                      0.3810
    5 -0.149 0.100                  +*  :   +                       2.593
    6  0.045 0.100                  +   :*  +                       2.796
    7 -0.066 0.099                  +  *:   +                       3.240
    8 -0.054 0.099                  +  *:   +                       3.536
    9 -0.053 0.098                  +  *:   +                       3.829
   10 -0.129 0.097                  +*  :   +                       5.596
   11  0.014 0.097                  +   *   +                       5.616
   12 -0.207 0.096                  *   :   +                       10.23
   13  0.009 0.096                  +   *   +                       10.24
   14 -0.133 0.095                  +*  :   +                       12.21
   15  0.067 0.094                  +   :*  +                       12.71
   16  0.098 0.094                  +   : * +                       13.80
   17 -0.010 0.093                  +   *   +                       13.81
   18 -0.028 0.093                  +  *:   +                       13.90
   19 -0.005 0.092                  +   *   +                       13.90
   20  0.099 0.091                  +   : * +                       15.08
                    :----:----:----:----:----:----:----:----:
                    -1 -.75 -.50 -.25   0   .25  .50  .75  +1

                    * : AUTOCORRELATIONS
                    + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.1.9. Autocorrelation Function of Iterative Residuals, CPI Inflation Rate Model, 1956-3/1979-2.

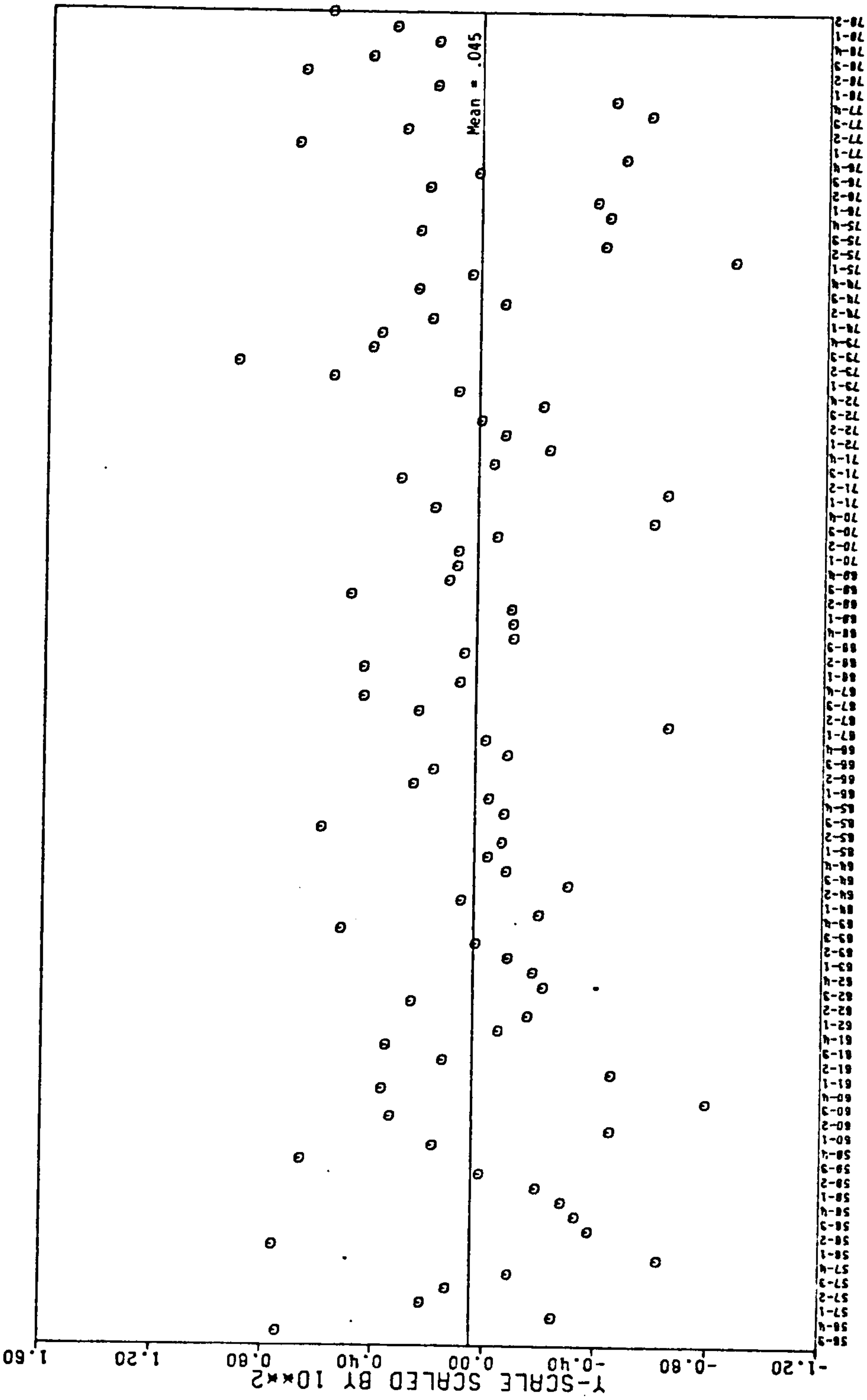


Figure 6.1.10. Plot of Iterative Model Residuals About the Mean, CPI Inflation Rate, 1956-3/1979-2.

period forecasting model do have a slightly smaller standard deviation and SSE than the residuals from the iterative updating model. This fact is to be expected since the iterative model is producing forecasts based only upon a limited set of information. The behavioral analogy of this statistical fact is that agents with perfect knowledge of the future (i.e., complete knowledge of future realizations of the rate of inflation) will forecast much more accurately than those with access to the "limited" information provided by the past alone.[1]

Recall that the form (i.e., the p,d,q) of the inflation model is not re-estimated during the updating routine, but is determined from the full data set. Implicit then in the production of the constrained forecasts is the assumption that the form of the CPI inflation model is the proper

Table 6.1.5. Comparative Statistics for the Full and Iterative CPI Inflation Rate Forecasting Models; 1956/3 to 1979/2.

	CPI Rate (Actual)	Prefiltered Forecasts (Full)	Prefiltered Errors (Full)	Prefiltered Forecasts (Iterative)	Prefiltered Errors (Iterative)
$\bar{X}$	1.063	1.030	.036	1.022	.045
σ_X	.797	.746	.391	.743	.413
SSE			14.420		15.760
MSE			.157		.171

---

[1] See Appendix A.1. for a listing of the full and iterative data. Appendix A also presents full and iterative data for the other ARIMA models.

fication for the complete time spectrum of the actual inflation rate series. Since the time period under study exhibits periods of both mild and severe inflation, this assumption should be investigated. If the fitted (p,d,q) form should require "alterations" as new observation points enter the estimation process, the relevancy of the forecasts is called into question.

In order to check the propriety of using the fixed (0,1,3)x(0,0,1)x3 form over the re-estimation period, the full 1952/1 through 1979/1 period was split into two subperiods of 54 and 55 observations, respectively. The model was then estimated over each period. The results, which appear in Table 6.1.6., indicate that the form of the model is adequate for both subperiods. The MSE's and residual standard errors are comparable, as are the numerical values of the coefficients. It should also be noted that since the residual standard error is approximately the same in both periods, heteroscedasticity is not present. The only question about model form adequacy comes from the slightly high Q-statistic in the second subperiod. However, this fact is not pressing enough to infer that the form of the model should be altered.[1]

---

[1] Note: Application of the Chow test in this case would not be appropriate because the Chow test investigates the possibility of equality of coefficients in two identical models based on two different data sets. What is trying to be determined here is whether or not the same (p,d,q) is consistent with both subsets of observations.

Table 6.1.6. Estimated CPI Inflation Model for Two Adjacent Subperiods; 1952/1 to 1965/2, and 1965/3 to 1979/1.

1952/1 to 1965/2

$$(1 - B)z_t = (1 - \underset{(5.73)}{.424B} - \underset{(2.13)}{.208B^2} + \underset{(12.13)}{.825B^3})(1 - \underset{(5.69)}{.626B^3})a_t$$

$Q(6) = 9.29[12.6]$

MSE = .165 with 50 d.f. SSE = 8.10

$\hat{\sigma}_a = .406$

$\bar{x}_{z_t} = .329$ $\sigma_{z_t} = .399$

$\bar{x}_{\hat{z}_t} = .328$ $\sigma_{\hat{z}_t} = .341$

1965/3 to 1979/1

$$(1 - B)z_t = (1 - \underset{(6.45)}{.329B} - \underset{(4.93)}{.291B^2} + \underset{(21.50)}{.905B^3})(1 - \underset{(5.34)}{.630B^3})a_t$$

$Q(6) = 14.33[12.6]$

MSE = .150 with 51 d.f. SSE = 7.52

$\hat{\sigma}_a = .387$

$\bar{x}_{z_t} = 1.44$ $\sigma_{z_t} = .689$

$\bar{x}_{\hat{z}_t} = 1.33$ $\sigma_{\hat{z}_t} = .625$

6.2. The GNP Deflator Inflation Model

Figure 6.2.1., a plot of the actual GNP deflator inflation series, z_t, indicates the process may be non-stationary over the 1952/1 through 1979/1 period. Figure 6.2.2., the ACF of the series substantiates this fact. Figure 6.2.3., the ACF of the $(1 - B)z_t$ transformation, shows one large spike at lag one. Thus, an MA model is indicated. This fact is confirmed by Figure 6.2.4., where it is seen that the PACF of the first-differenced series trails off exponentially.

Based on the clues provided by the ACF and PACF of the raw series, a (0,1,1) filter and a (0,1,2) filter were initially entertained. While the first MA term in both models was highly significant, the second MA term in the latter filter was found to be insignificant. While the (0,1,1) model did produce an acceptable BP statistic, Q(35) = 36.30[49.8], a visual check of the ACF of residuals from this form showed significant spikes at lags four, seven, and eight. While the CPI inflation series showed no seasonal pattern, the spikes at lags four and eight of the (0,1,1) model fit to the GNP deflation series presented the possibility that the series did contain some seasonal elements. Two seasonal filters were fit in an attempt to model the suspected seasonal pattern in the residuals of the (0,1,1) filter. Models of the form (0,1,1)x(0,0,1)x7 and (0,1,1)x(0,0,1)x8 were estimated, and, while both filters improved the BP statistic from that of the simple (0,1,1) form, the seasonal MA term was barely significant. Additionally, neither model was able to reduce the large spikes at lags seven and eight.

The seasonal form was thus replaced by the following non-seasonal filters;

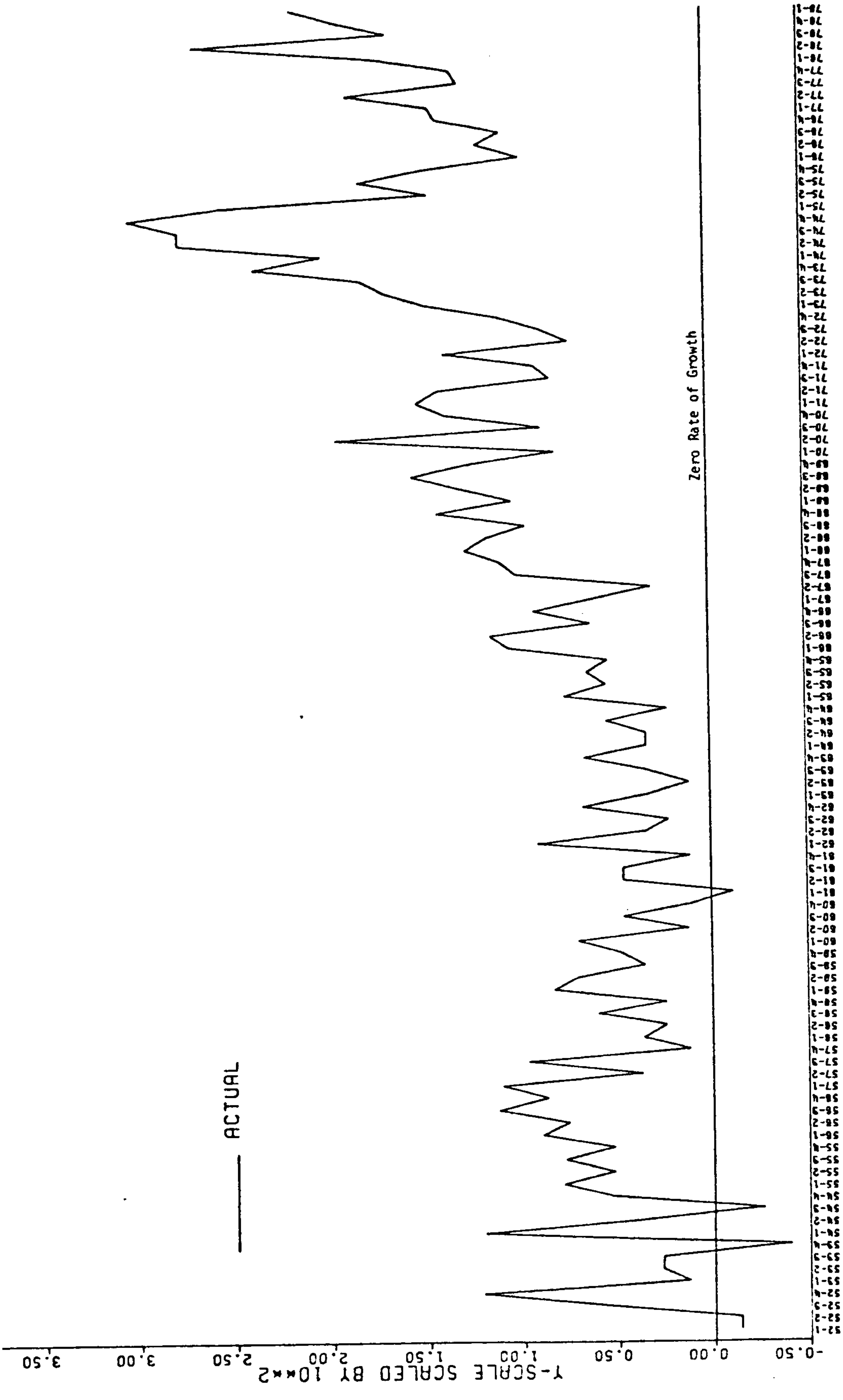


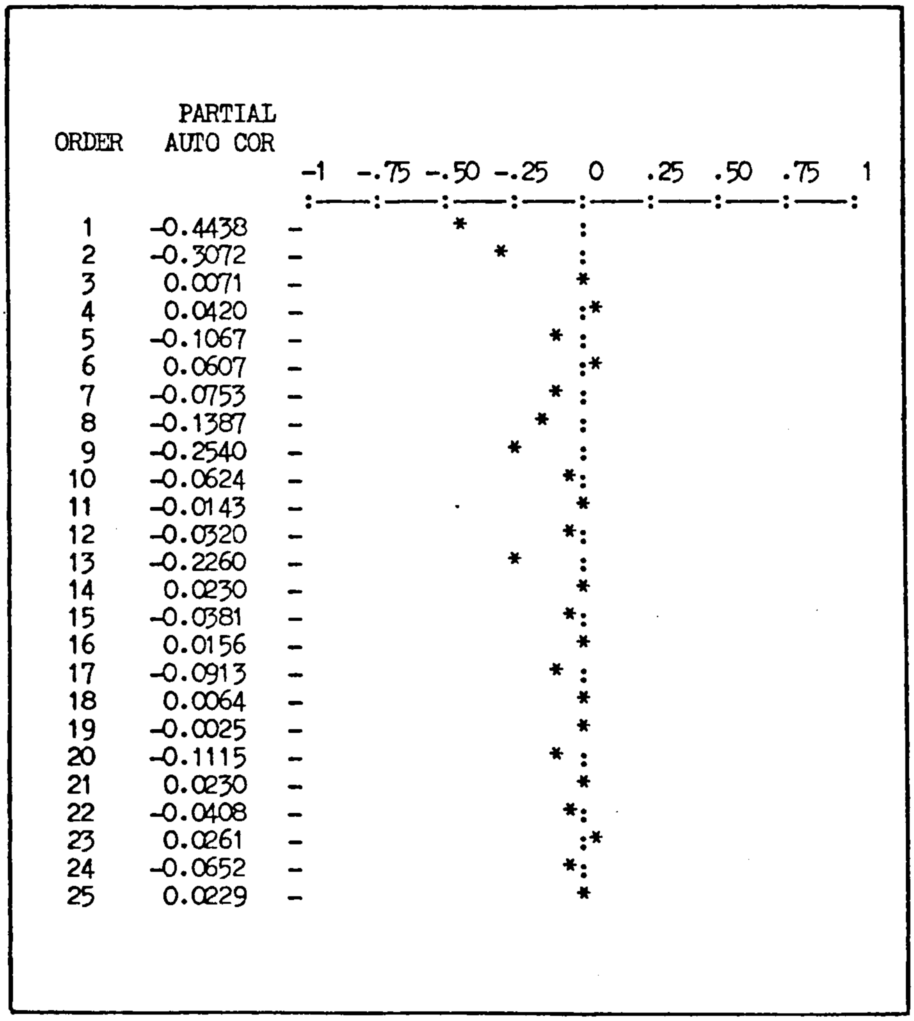
Figure 6.2.1. Actual Quarterly Inflation Rate, GNP Deflator, 1952-1/1979-1.

```
             S.E.
      AUTO- RANDOM
ORDER CORR. MODEL  -1  -.75 -.50 -.25   0   .25  .50  .75  +1  ADJ.B-P
                    :----:----:----:----:----:----:----:----:
  1   0.727 0.094                   +   :   +          *       59.22
  2   0.677 0.094                   +   :   +         *        111.1
  3   0.673 0.094                   +   :   +        *         162.8
  4   0.573 0.093                   +   :   +      *           200.7
  5   0.500 0.093                   +   :   +     *            229.8
  6   0.496 0.092                   +   :   +     *            258.7
  7   0.407 0.092                   +   :   +   *              278.4
  8   0.360 0.091                   +   :   +  *               293.9
  9   0.381 0.091                   +   :   +   *              311.5
 10   0.405 0.090                   +   :   +   *              331.5
 11   0.374 0.090                   +   :   +  *               348.8
 12   0.387 0.090                   +   :   +   *              367.5
 13   0.411 0.089                   +   :   +   *              388.8
 14   0.469 0.089                   +   :   +    *             416.8
 15   0.428 0.088                   +   :   +    *             440.4
 16   0.449 0.088                   +   :   +    *             466.6
 17   0.412 0.087                    +  :  +    *              488.9
 18   0.376 0.087                    +  :  +    *              507.7
 19   0.332 0.086                    +  :  +   *               522.6
 20   0.305 0.086                    +  :  +  *                535.2
 21   0.290 0.085                    +  :  +  *                546.8
 22   0.220 0.085                    +  :  +*                  553.5
 23   0.214 0.084                    +  :  +*                  559.9
 24   0.191 0.084                    +  :  +*                  565.1
 25   0.161 0.083                    +  :  *                   568.8
                    :----:----:----:----:----:----:----:----:
                   -1  -.75 -.50 -.25   0   .25  .50  .75  +1

                    * : AUTOCORRELATIONS
                    + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.2.2. Autocorrelation Function, GNP Deflator Inflation Rate, 1952-1/1979-1.

```
               S.E.
       AUTO- RANDOM
ORDER  CORR.  MODEL           -1  -.75 -.50 -.25  0   .25  .50  .75   +1  ADJ.B-P
                              :----:----:----:----:----:----:----:----:
    1  -0.444  0.095                     *    +   :   +                   21.87
    2  -0.050  0.094                          +  *:   +                   22.15
    3   0.170  0.094                          +   :  *+                   25.43
    4  -0.056  0.094                          +  *:   +                   25.79
    5  -0.115  0.093                          + * :   +                   27.31
    6   0.174  0.093                          +   :  *+                   30.82
    7  -0.143  0.092                          +*  :   +                   33.25
    8  -0.052  0.092                          +  *:   +                   33.56
    9  -0.037  0.091                          +  *:   +                   33.73
   10   0.097  0.091                          +   : * +                   34.88
   11  -0.063  0.090                          +  *:   +                   35.37
   12   0.000  0.090                          +   *   +                   35.37
   13  -0.104  0.089                          + * :   +                   36.72
   14   0.191  0.089                          +   :   *                   41.34
   15  -0.118  0.088                          + * :   +                   43.13
   16   0.079  0.088                          +   : * +                   43.93
   17  -0.033  0.088                          +  *:   +                   44.08
   18   0.042  0.087                           +  :* +                    44.31
   19  -0.047  0.087                           + *:  +                    44.60
   20   0.019  0.086                           +  *  +                    44.65
   21   0.063  0.086                           +  :* +                    45.20
   22  -0.080  0.085                           +* :  +                    46.09
   23   0.002  0.085                           +  *  +                    46.09
   24   0.033  0.084                           +  :* +                    46.24
   25  -0.004  0.084                           +  *  +                    46.24
                              :----:----:----:----:----:----:----:----:
                              -1  -.75 -.50 -.25  0   .25  .50  .75   +1

                              * : AUTOCORRELATIONS
                              + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.2.3. Autocorrelation Function of $(1 - B)z_t$, GNP Deflator Inflation Rate, 1952-1/1979-1.



```
          PARTIAL
  ORDER    AUTO COR
                    -1    -.75    -.50    -.25      0      .25     .50     .75      1
                    :-------:-------:-------:-------:-------:-------:-------:-------:
  1    -0.4438    -                   *             :
  2    -0.3072    -                       *         :
  3     0.0071    -                                 *
  4     0.0420    -                                 :*
  5    -0.1067    -                               * :
  6     0.0607    -                                 :*
  7    -0.0753    -                               * :
  8    -0.1387    -                             *   :
  9    -0.2540    -                         *       :
 10    -0.0624    -                                *:
 11    -0.0143    -                                 *
 12    -0.0320    -                                *:
 13    -0.2260    -                         *       :
 14     0.0230    -                                 *
 15    -0.0381    -                                *:
 16     0.0156    -                                 *
 17    -0.0913    -                               * :
 18     0.0064    -                                 *
 19    -0.0025    -                                 *
 20    -0.1115    -                               * :
 21     0.0230    -                                 *
 22    -0.0408    -                                *:
 23     0.0261    -                                 :*
 24    -0.0652    -                                *:
 25     0.0229    -                                 *
```

Figure 6.2.4. Partial Autocorrelation Function of $(1 - B)z_t$, GNP Deflator Rate, 1952-1/1979-1.

(6.2.1.) $$(1 - B)z_t = (1 - \theta_1 B - \theta_7 B^7 - \theta_8 B^8)a_t$$

and,

(6.2.2.) $$(1 - B)z_t = (1 - \theta_1 B)(1 - \theta_7 B^7 - \theta_8 B^8)a_t$$

Upon estimation it was found that both models performed well, pulling the troublesome residual spikes at lags seven and eight into an acceptable range and producing good BP statistics. Additionally, all coefficients on both filters were significant. However, visual inspection of the ACF of residuals produced by (6.2.1.) indicated that while the Q-statistic was good at Q(34) = 32.09[48.6], there remained a discernible pattern of positive residuals between lags five and twelve. This pattern was not detected in the ACF of residuals for the (6.2.2.) filter because of the multiplicative nature of the model. As an additional check on the adequacy of (6.2.2.) the model was overfit in the following manner;

(6.2.3.) $$(1 - B)z_t = (1 - \theta_1 B)(1 - \theta_7 B^7 - \theta_8 B^8 - \theta_9 B^9)a_t \quad .$$

The ninth term, θ_9, however, proved to be insignificant and was dropped from further consideration. Thus (6.2.2.) was chosen as the final model for the GNP deflator series. Figure 6.2.5. presents the ACF of residuals from this filter. It is seen that all autocorrelations lie within $\pm 2\sigma$, and no discernible pattern is evident. The roots of the characteristic equation were checked and were found to lie outside the unit circle. Table 6.2.1. presents the estimated model, related statistics, and step-ahead forecasting function. The forecasting function shows the memory of the GNP filter to be longer than that associated with the CPI inflation filter, with errors occurring nine periods past affecting the current forecast. Also, the negative signs on three of the coefficients indicate the strong tendency of the model to adjust

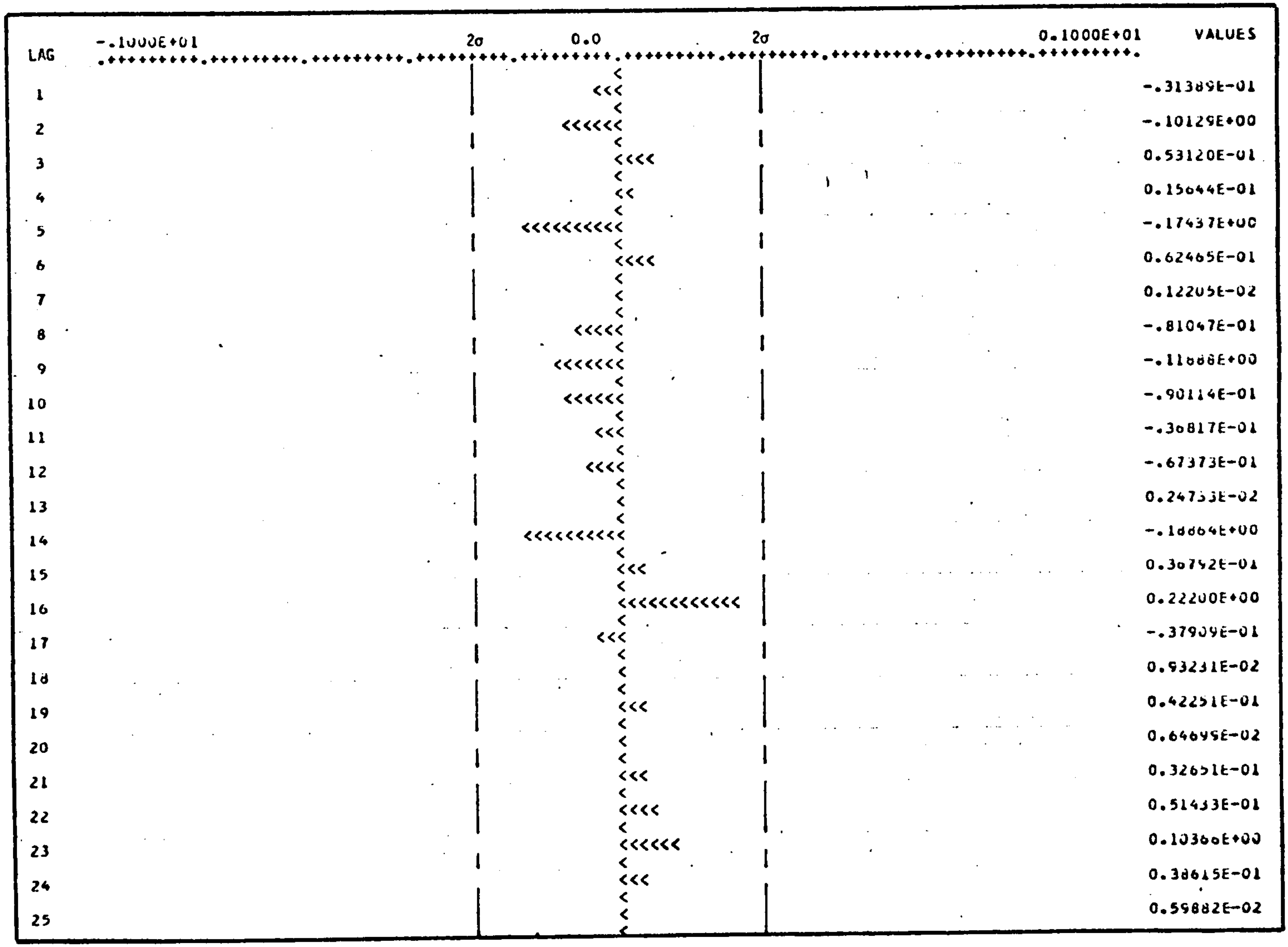


Figure 6.2.5. Autocorrelation Function of Residuals, GNP Deflator Inflation Rate Model, 1952-1/1979-1.

Table 6.2.1. Estimated ARIMA GNP Deflator Inflation Model, Step-Ahead Forecasting Function, and Related Statistics; 1952/1-1979/1.

$$(1 - B)z_t = (1 - \underset{(7.54)}{.596B})(1 - \underset{(2.28)}{.218B^7} - \underset{(3.10)}{.290B^8})a_t$$

$Q(33) = 30.03[47.4]$

$MSE = .168$ with 105 d.f. $SSE = 17.10$

$\hat{\sigma}_a = .403$ $\bar{x}_{z_t} = .951$ $\sigma_{z_t} = .687$

$R^2 = .664$ $\bar{x}_{\hat{z}_t} = .870$ $\sigma_{\hat{z}_t} = .567$

$$\hat{z}_t = z_{t-1} - .596a_{t-1} - .218a_{t-7} - .160a_{t-8} + .173a_{t-9}$$

---

casts in the opposite direction of the forecast error. Having estimated both the CPI and GNP inflation filters it is now important to point out an important feature of both models: the forecasting functions of both ARIMA processes show that last period's actual value of the inflation rate is given full weight (i.e., has a coefficient equal to unity) in the computation of the quarter-ahead forecast. This fact supports the earlier theoretical assertion that the most recent past actual value of the rate of inflation plays a significant role expectation formation.

Figure 6.2.6 shows a plot of the actual inflation series and the iteratively updated forecasts over the 1956/3 through 1979/2 period. Visual inspection shows the forecasting model tracks the series very well. The evolution of the coefficients as the model is re-estimated is presented in Table B.2. of Appendix B. The ACF of residuals produced from the updating routine is presented in Figure 6.2.7. The BP statistic associated with this series is Q(20) = 14.97[31.4], indicating that the iterative procedure has

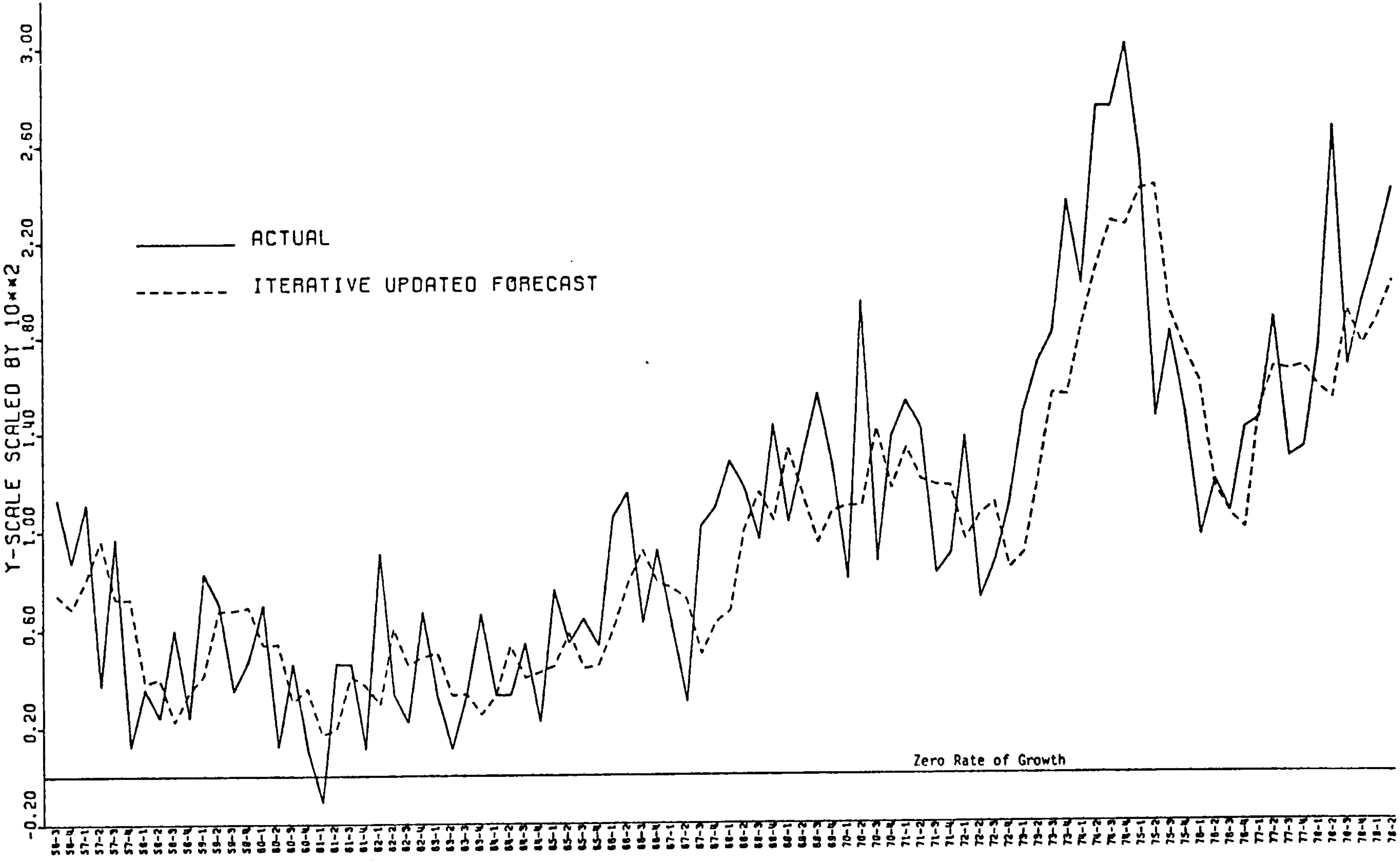


Figure 6.2.6. Actual and Iteratively Updated Forecasts, GNP Deflator Inflation Rate Model, 1956-3/1979-2.

```
              S.E.
       AUTO- RANDOM
ORDER  CORR.  MODEL         -1  -.75 -.50 -.25   0   .25  .50  .75  +1   ADJ.B-P
                             :----:----:----:----:----:----:----:----:
  1  -0.013  0.103                           +   *   +                   0.1573E-01
  2   0.077  0.102                           +   : * +                       0.5799
  3   0.081  0.101                           +   : * +                        1.225
  4  -0.004  0.101                           +   *   +                        1.226
  5  -0.178  0.100                           *   :   +                        4.392
  6  -0.002  0.100                           +   *   +                        4.393
  7   0.024  0.099                           +   *   +                        4.450
  8  -0.024  0.099                           +   *   +                        4.510
  9  -0.142  0.098                           +*  :   +                        6.615
 10   0.056  0.097                           +   :*  +                        6.951
 11  -0.096  0.097                           + * :   +                        7.937
 12  -0.128  0.096                           +*  :   +                        9.697
 13  -0.071  0.096                           +  *:   +                        10.26
 14   0.168  0.095                           +   :  *+                        13.39
 15  -0.018  0.094                           +   *   +                        13.43
 16   0.037  0.094                           +   :*  +                        13.59
 17  -0.001  0.093                           +   *   +                        13.59
 18   0.092  0.093                           +   : * +                        14.58
 19   0.016  0.092                           +   *   +                        14.61
 20   0.055  0.091                           +   :*  +                        14.97
                             :----:----:----:----:----:----:----:----:
                            -1  -.75 -.50 -.25   0   .25  .50  .75  +1

                             * : AUTOCORRELATIONS
                             + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.2.7. Autocorrelation Function of Iterative Residuals, GNP Deflator Inflation Rate Model, 1956-3/1979-2.

reduced the residuals to discrete white noise. A median runs test on the residuals supports this finding with a Z-value = .42. Although not presented here, a visual inspection of the distribution of residuals about the mean shows they are randomly distributed.

To check the adequacy of the form of the model over the complete period subperiod estimation was performed as in the CPI model. The results of this analysis appear in Table 6.2.2. and implies that the (p,d,q) form is a proper specification for both subperiods, although the form does provide a slightly more accurate fit in the first period. Both models produce residuals which are well within the random range and the MSE's of both models are comparable.

Table 6.2.2. Estimated GNP Deflator Inflation Model for Two Adjacent Subperiods; 1952/1 to 1965/2, and 1965/3 to 1979/1.

1952/1 to 1965/2

$$(1 - B)z_t = (1 - \underset{(6.66)}{.699}B)(1 - \underset{(2.17)}{.312}B^7 - \underset{(5.56)}{.079}B^8)a_t$$

Q(33) = 41.21[47.4]

MSE = .144 with 50 d.f. SSE = 7.22

$\hat{\sigma}_a = .380$

$\overline{x}_{z_t} = .460$ $\sigma_{z_t} = .364$

$\overline{x}_{\hat{z}_t} = .480$ $\sigma_{\hat{z}_t} = .209$

1965/3 to 1979/1

$$(1 - B)z_t = (1 - \underset{(3.89)}{.481}B)(1 - \underset{(1.14)}{.062}B^7 - \underset{(1.99)}{.301}B^8)a_t$$

Q(33) = 44.20[47.4]

MSE = .188 with 51 d.f. SSE = 9.62

$\hat{\sigma}_a = .434$

$\overline{x}_{z_t} = 1.412$ $\sigma_{z_t} = .608$

$\overline{x}_{\hat{z}_t} = 1.352$ $\sigma_{\hat{z}_t} = .516$

6.3. The M1 Growth Rate Model

Figure 6.3.1. presents a plot of the raw M1 growth rate series, z_t, over the period 1947/2 through 1979/1.[1] The ACF of the series is presented in Figure 6.3.2. Since the ACF promptly dies off, the raw series is stationary and differencing is not required. In order to insure that no periodicity of four remained in the seasonally adjusted data, the ACF of the transformation $(1 - B^4)z_t$ was computed. No seasonal non-stationarity was discernible in this transformation.

The prompt decay of the ACF of z_t and the significant spike at lag one of the PACF of z_t, as seen in Figure 6.3.3., indicate an AR or mixed AR/MA model may be appropriate. With these facts in mind, two filters, a (1,0,0)

---

[1] Note: Unlike the inflation models, initial estimation of the M1 and MB models over the 1952/1 to 1956/2 period proved impossible since the non-linear estimation routine would not converge using such a small number of observations. To remedy this problem the iterative money models are estimated over the 1947/2 to 1979/1 period, thus providing a sufficient number of initial observations to produce the first forecast in 1956/3.
Note: Performing an autocorrelation on the seasonally unadjusted M1 and MB growth rate series showed, as expected, strong non-stationarity at periodicity of four. For the following reasons, however, the money models are estimated using seasonally adjusted data. First, modeling the seasonal growth spurts of the money series would have required filters with more complex and lengthy memories, thereby exacerbating the convergence problems cited above. Secondly, the use of seasonally adjusted money data is justified since seasonal fluctuations in the money stock are "expected." That is, it can reasonably be assumed that, because of the long history of seasonal changes in the growth rate of the money stock, economic agents, either consciously or unconsciously adapt to seasonal money patterns when forming their growth rate forecasts. Modeling unadjusted data 1) would have amounted to a statistical assumption that the market is ignorant of these seasonal growth rate regularities, and 2) would

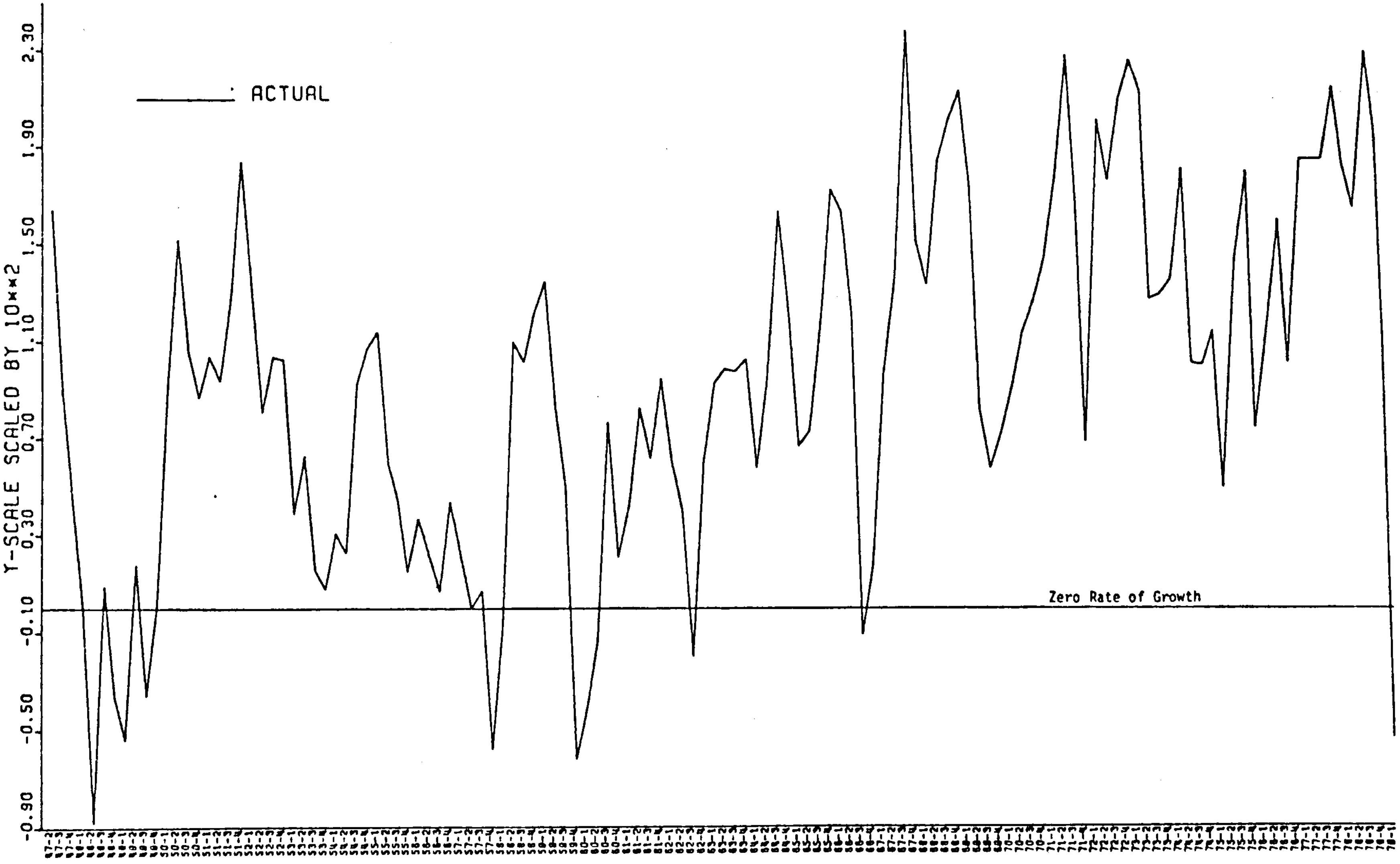


Figure 6.3.1. Actual Quarterly M1 Growth Rate (Seasonally Adjusted), 1947-2/1979-1.

```
             S.E.
      AUTO- RANDOM
ORDER CORR. MODEL  -1  -.75 -.50 -.25  0   .25  .50  .75  +1   ADJ.B-P
                   :----:----:----:----:----:----:----:----:
  1   0.713 0.087                   +  :  +          *         66.68
  2   0.486 0.087                   +  :  +      *             97.92
  3   0.378 0.087                   +  :  +    *               116.9
  4   0.282 0.086                   +  :  +  *                 127.5
  5   0.290 0.086                   +  :  +  *                 138.9
  6   0.284 0.086                   +  :  +  *                 149.9
  7   0.187 0.085                   +  :  +*                   154.7
  8   0.124 0.085                   +  : *+                    156.8
  9   0.157 0.085                   +  :  *                    160.3
 10   0.230 0.084                   +  :  + *                  167.8
 11   0.257 0.084                   +  :  + *                  177.1
 12   0.240 0.083                   +  :  + *                  185.4
 13   0.200 0.083                   +  :  +*                   191.1
 14   0.155 0.083                   +  :  *                    194.7
 15   0.220 0.082                   +  :  +*                   201.8
 16   0.331 0.082                   +  :  +   *                218.1
 17   0.340 0.082                   +  :  +   *                235.5
 18   0.308 0.081                   +  :  +  *                 249.8
 19   0.259 0.081                   +  :  + *                  260.0
 20   0.216 0.081                   +  :  +*                   267.2
 21   0.237 0.080                   +  :  + *                  276.0
 22   0.225 0.080                   +  :  + *                  283.9
 23   0.157 0.079                   +  :  *                    287.8
 24   0.081 0.079                   +  : *+                    288.9
 25   0.108 0.079                   +  : *+                    290.7
                   :----:----:----:----:----:----:----:----:
                   -1  -.75 -.50 -.25  0   .25  .50  .75  +1

                   * : AUTOCORRELATIONS
                   + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.3.2. Autocorrelation Function, M1 Growth Rate, 1947-2/1979-1.

```
        PARTIAL
ORDER  AUTO COR
                      -1    -.75    -.50    -.25      0       .25     .50     .75     1
                      :-------:-------:-------:-------:-------:-------:-------:-------:
   1     0.7134   -                                   :                     *
   2    -0.0459   -                                  *:
   3     0.0967   -                                   :  *
   4    -0.0273   -                                  *:
   5     0.1774   -                                   :     *
   6     0.0075   -                                   *
   7    -0.1151   -                                *  :
   8    -0.0008   -                                   *
   9     0.1516   -                                   :    *
  10     0.1407   -                                   :    *
  11    -0.0124   -                                   *
  12    -0.0076   -                                   *
  13     0.0195   -                                   *
  14    -0.0054   -                                   *
  15     0.1552   -                                   :    *
  16     0.1603   -                                   :    *
  17     0.0007   -                                   *
  18     0.0084   -                                   *
  19    -0.0049   -                                   *
  20     0.0104   -                                   *
  21     0.0371   -                                   :*
  22    -0.0673   -                                  *:
  23    -0.0445   -                                  *:
  24    -0.0359   -                                  *:
  25     0.1542   -                                   :    *
```

Figure 6.3.3. Partial Autocorrelation Function, M1 Growth Rate, 1947-2/1979-1.

and a (2,0,0) were estimated with a constant.[1] The second AR term of the latter model, however, proved to be insignificant. Investigation of the ACF of the (1,0,0) filter showed a mildly significant negative spike at lags four and eight of -.140 and -.147, respectively. Hence, even though quarterly non-stationarity is not indicated in the seasonally adjusted data, the residual spikes at lags four and eight of the (1,0,0) filter indicate that possibly not all of the seasonal pattern has been removed from the adjusted data. To confirm or deny this suspicion a number of mixed seasonal models were fit (note: all of the following filters are estimated with a constant term);

(6.3.1.) (1,0,0)x(1,0,0)x4

(6.3.2.) (1,0,1)x(1,0,1)x4

(6.3.3.) $(1 - \phi_1 B)(1 - \phi_4 B^4)z_t = (1 - \theta_1 B - \theta_4 B^4)a_t$

(6.3.4.) $(1 - \phi_1 B)(1 - \phi_4 B^4)z_t = (1 - \theta_1 B - \theta_4 B^4 - \theta_8 B^8)a_t$

(6.3.5.) (1,0,0)x(1,0,1)x4

(6.3.6.) (1,0,0)x(2,0,1)x4

(6.3.7.) (1,0,0)x(1,0,2)x4

(6.3.8.) (1,0,0)x(1,0,3)x4

Model (6.3.1.) produced a marginally acceptable BP statistic, but left a significant negative spike at lag eight in the ACF of residuals. Additionally, the forth AR seasonal of this form was found to be insignificant. Since the first AR coefficients in both (6.3.1.) and (6.3.2.) were highly

---

[1] The use of a constant is indicated when the series is not differenced.

significant, models (6.3.3.) and (6.3.4.) were estimated, since they retain the AR form of the previous two models but replace the multiplicative form of the MA portion with a polynomial of order four and eight, respectively. Upon estimation, however, the first MA coefficient in both forms was shown to be insignificant. Thus they were revised to the form indicated by (6.3.5.), which omits the MA term of order one. This model was more satisfactory and produced a good BP statistic of Q(32) = 29[46.2], with all coefficients highly significant. As a check on the seasonal AR portion of (6.3.5.), the model was overfitted with (6.3.6.). The second AR coefficient was found to be insignificant. Model (6.3.5.) was then overfitted with a second and third seasonal MA term as indicated by (6.3.7.) and (6.3.8.), respectively. While both filters reduced the residuals to white noise, the third seasonal coefficient of (6.3.8.) was found to be insignificant and (6.3.7.) was thus retained as the final model for the M1 series.

Figure 6.3.4., the ACF of residuals of (6.3.7.) shows that the filter has reduced the series to random error. The roots of the AR and MA characteristic equations were found to lie outside the unit circle. Table 6.3.1. presents the results of the estimated model along with related statistics and the step-ahead forecasting function. It is seen that the statistical significance of the coefficients and constant term are beyond question. The forecasting function shows that the forecasted value of M1 exhibits a strong positive autocorrelation with the value of M1 occurring in the previous period and a slightly autocorrelated association with M1 occurring four and five periods back. Error terms four and five periods back affect the current forecast also.

```
            S.E.
      AUTO- RANDOM
ORDER CORR. MODEL -1  -.75 -.50 -.25   0   .25  .50  .75  +1   ADJ.B-P
                   :----:----:----:----:----:----:----:----:
    1 -0.015 0.087                  +  *  +                    0.2771E-01
    2 -0.110 0.087                  +* :  +                    1.620
    3 -0.019 0.087                  +  *  +                    1.668
    4  0.014 0.086                  +  *  +                    1.694
    5  0.022 0.086                  +  *  +                    1.760
    6  0.164 0.086                  +  :  *                    5.411
    7 -0.070 0.085                  + *:  +                    6.083
    8 -0.008 0.085                  +  *  +                    6.092
    9 -0.025 0.085                  +  *  +                    6.178
   10  0.068 0.084                  +  :* +                    6.828
   11  0.072 0.084                  +  :* +                    7.567
   12  0.057 0.083                  +  :* +                    8.039
   13  0.048 0.083                  +  :* +                    8.374
   14 -0.076 0.083                  +* :  +                    9.223
   15 -0.068 0.082                  + *:  +                    9.896
   16  0.123 0.082                  +  : *+                    12.15
   17  0.135 0.082                  +  :  *                    14.88
   18  0.058 0.081                  +  :* +                    15.38
   19  0.024 0.081                  +  *  +                    15.47
   20 -0.076 0.081                  +* :  +                    16.35
   21  0.093 0.080                  +  : *+                    17.70
   22  0.099 0.080                  +  : *+                    19.24
   23  0.036 0.079                  +  :* +                    19.45
   24 -0.147 0.079                  *  :  +                    22.90
   25  0.030 0.079                  +  :* +                    23.04
                   :----:----:----:----:----:----:----:----:
                  -1  -.75 -.50 -.25   0   .25  .50  .75  +1

                   * : AUTOCORRELATIONS
                   + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.3.4. Autocorrelation Function of Residuals, M1 Growth Rate Model, 1947-2/1979-1.

Table 6.3.1. Estimated ARIMA M1 Growth Rate Model, Step-Ahead Forecasting Function, and Related Statistics; 1947/2-1979/1.

$$\underset{(21.05)}{(1 - .884B)}\underset{(16.42)}{(1 - .197B^4)}z_t = \underset{(295.3)}{.886} + (1 - \underset{(259.5)}{.519B^4} - \underset{(69.00)}{.138B^8})a_t$$

$Q(31) = 27.11[45.0]$

$MSE = .241$ with 123 d.f. $SSE = 29.67$

$\hat{\sigma}_a = .491$ $\bar{x}_{z_t} = .932$ $\sigma_{z_t} = .732$

$R^2 = .578$ $\bar{x}_{\hat{z}_t} = .936$ $\sigma_{\hat{z}_t} = .596$

$$\hat{z}_t = .886 + .884z_{t-1} - .197z_{t-4} + .174z_{t-5} - .519a_{t-4} - .138a_{t-8}$$

Figure 6.3.5. plots the actual and iterative updated forecasts for the M1 series over the 1956/3 to 1979/2 period. The evolution of the coefficients is presented in Table B.3. of Appendix B. The ACF of the residuals produced from the updating routine is presented in Figure 6.3.6. The BP statistic associated with this residual series is Q(20) = 14.69[31.3], and indicates the residuals are random. A median runs test on these residuals produced a Z-value = -.74, and, a visual inspection of the distribution of the residuals about the mean shows they are evenly distributed.

The results of adjacent subperiod estimation appear in Table 6.3.2. Except for the low t-values of the seasonal AR terms, the model produces a comparable fit for both periods.

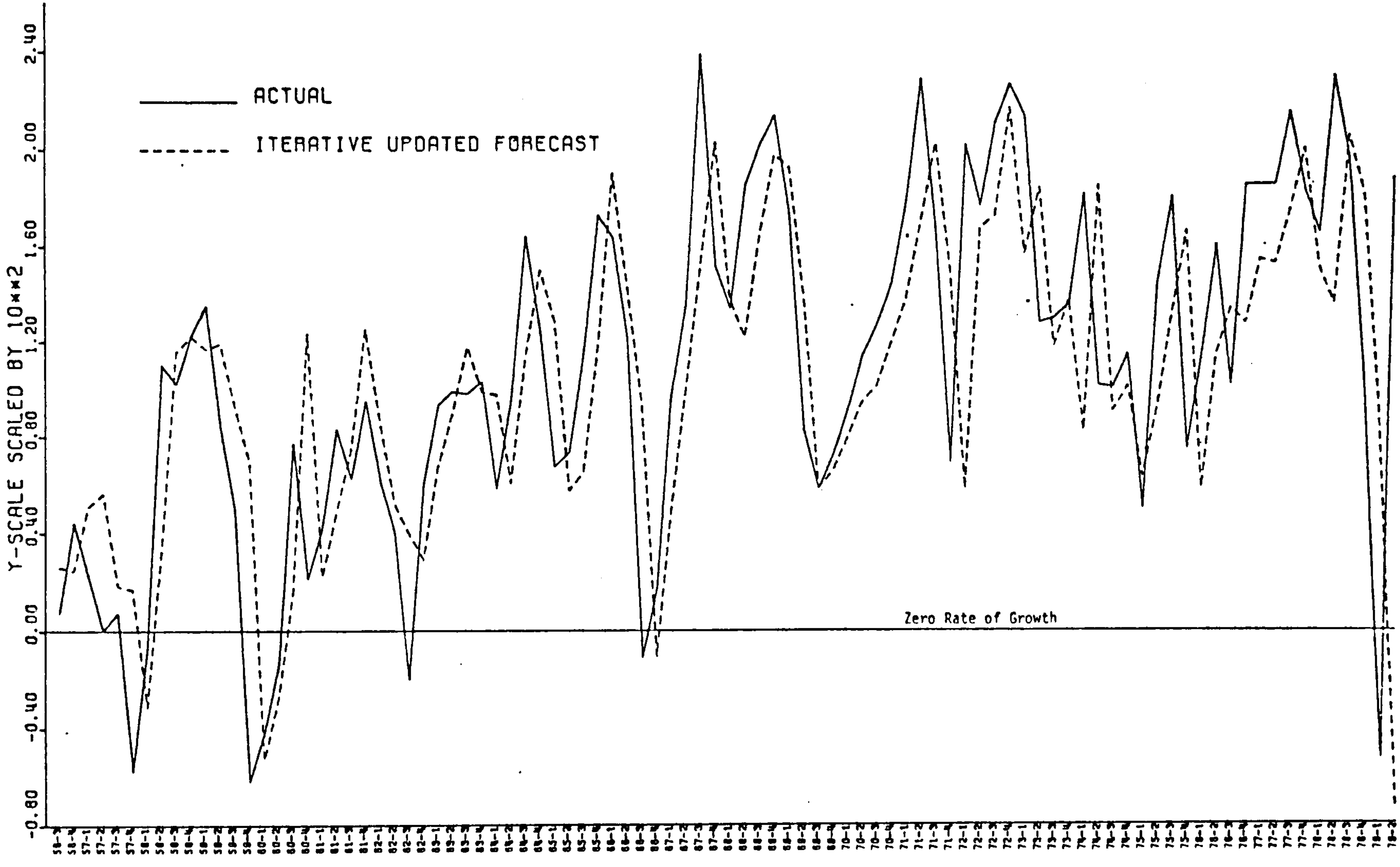
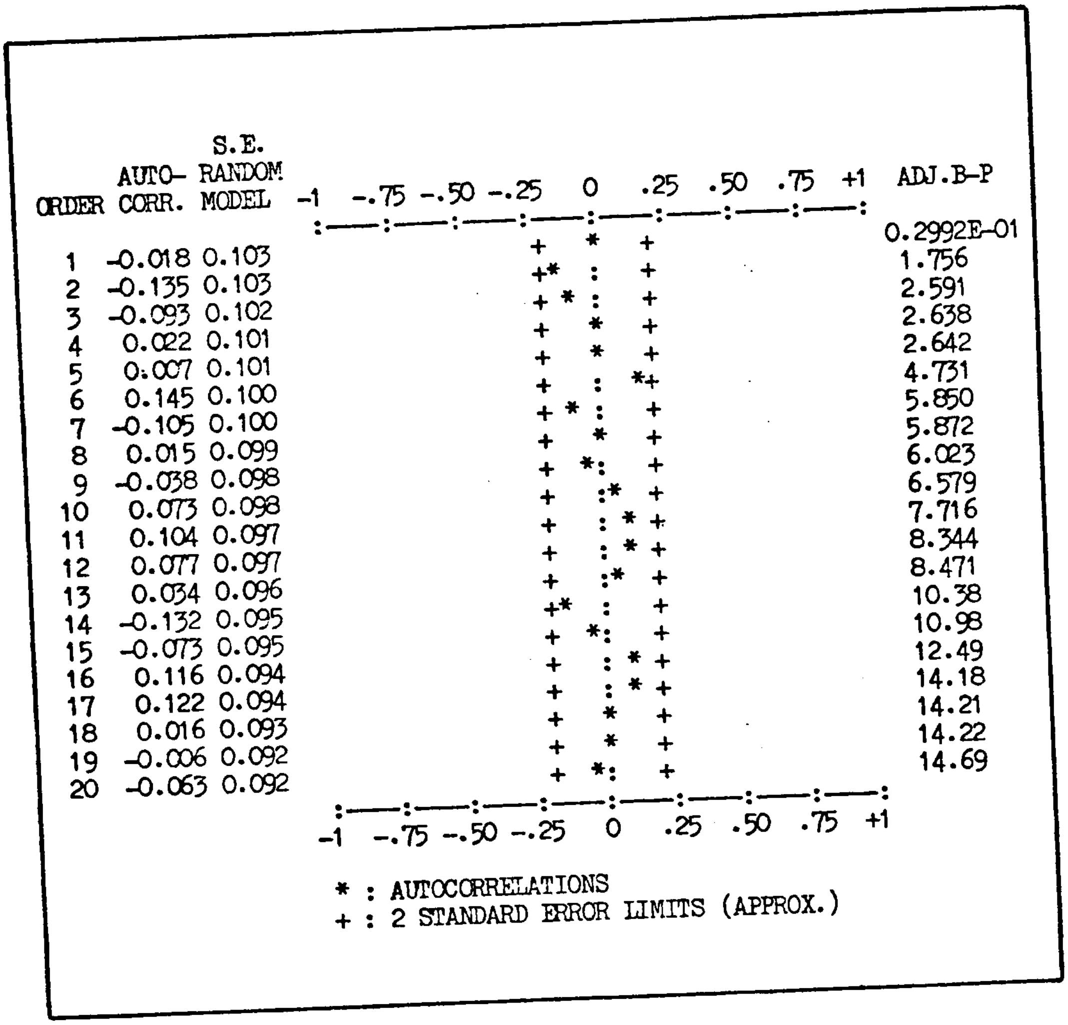


Figure 6.3.5. Actual and Iteratively Updated Forecasts, M1 Growth Rate Model, 1956-3/1979-2.



```
                S.E.
        AUTO-  RANDOM
ORDER   CORR. MODEL     -1      -.75    -.50    -.25    0       .25     .50     .75     +1  ADJ.B-P
                        :-------:-------:-------:-------:-------:-------:-------:-------:
    1  -0.018  0.103                              +     *     +                            0.2992E-01
    2  -0.135  0.103                              +*    :     +                            1.756
    3  -0.093  0.102                              + *   :     +                            2.591
    4   0.022  0.101                              +     *     +                            2.638
    5   0.007  0.101                              +     *     +                            2.642
    6   0.145  0.100                              +     :    *+                            4.731
    7  -0.105  0.100                              +  *  :     +                            5.850
    8   0.015  0.099                              +     *     +                            5.872
    9  -0.038  0.098                              +    *:     +                            6.023
   10   0.073  0.098                              +     :*    +                            6.579
   11   0.104  0.097                              +     :  *  +                            7.716
   12   0.077  0.097                              +     :  *  +                            8.344
   13   0.034  0.096                              +     :*    +                            8.471
   14  -0.132  0.095                              +*    :     +                            10.38
   15  -0.073  0.095                              +    *:     +                            10.98
   16   0.116  0.094                              +     :  *  +                            12.49
   17   0.122  0.094                              +     :  *  +                            14.18
   18   0.016  0.093                              +     *     +                            14.21
   19  -0.006  0.092                              +     *     +                            14.22
   20  -0.063  0.092                              +    *:     +                            14.69
                        :-------:-------:-------:-------:-------:-------:-------:-------:
                        -1      -.75    -.50    -.25    0       .25     .50     .75     +1

                        * : AUTOCORRELATIONS
                        + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.3.6. Autocorrelation Function of Iterative Residuals, M1 Growth Rate Model, 1956-3/1979-2.

Table 6.3.2. Estimated M1 Growth Rate Model for Two Adjacent Subperiods; 1947/2 to 1963/1, and 1963/2 to 1979/1.

1947/2 to 1963/1

$$(1 - \underset{(67.60)}{.882}B)(1 - \underset{(1.10)}{.252}B^4)z_t = \underset{(12.64)}{.531} + (1 - \underset{(2.56)}{.592}B^4 - \underset{(1.36)}{.249}B^8)a_t$$

$Q(11) = 15.02[19.7]$

MSE = .190 with 59 d.f. SSE = 11.20

$\hat{\sigma}_a = .435$

$\bar{x}_{z_t} = .460$ $\sigma_{z_t} = .364$

$\bar{x}_{\hat{z}_t} = .570$ $\sigma_{\hat{z}_t} = .524$

1963/2 to 1979/1

$$(1 - \underset{(80.54)}{.886}B)(1 - \underset{(0.706)}{.187}B^4)z_t = \underset{(24.36)}{1.341} + (1 - \underset{(2.03)}{.543}B^4 - \underset{(1.27)}{.274}B^8)a_t$$

$Q(11) = 18.97[19.7]$

MSE = .296 with 59 d.f. SSE = 17.45

$\hat{\sigma}_a = .543$

$\bar{x}_{z_t} = 1.356$ $\sigma_{z_t} = .605$

$\bar{x}_{\hat{z}_t} = 1.379$ $\sigma_{\hat{z}_t} = .508$

6.4. The Monetary Base Rate of Growth Model

Figure 6.4.1. presents a plot of the raw MB growth rate series, z_t, over the 1947/2 through 1979/1 period. Figure 6.4.2. shows that the ACF of the raw series does not die off and, therefore, differencing is required. The ACF and PACF of $(1 - B)z_t$ appear in Figures 6.4.3. and 6.4.4., respectively. Because the ACF of $(1 - B)z_t$ decays in exponential fashion, an AR form is indicated. The PACF of $(1 - B)\ z_t$ confirms this finding and furthermore, because of the highly significant negative spikes at lags one and two, implies that the AR form should be of second order.

Following these clues, a (2,1,0) model was estimated. As suspected, both AR coefficients were highly significant. The ACF of residuals, however, was unacceptable as indicated by a BP statistic of Q(34) = 50.29[48.6]. Upon visual inspection, this ACF showed significant spikes at lags four (-.104), eight (-.197), and twelve (-.180). Considering this seasonal pattern in the residuals and the fact that some seasonal movement remained in the deseasonalized M1 series, the ACF of $(1 - B^4)z_t$ was computed and examined. No periodicity at four and multiples thereof was detected however. Nevertheless, the series was still suspected of containing some mild seasonal influence and in order to model the residual pattern left by the (2,1,0) form, a seasonal model of the form;

$$(2,1,0)x(0,0,1)x4 \qquad (6.4.1.)$$

was estimated. The seasonal MA coefficient proved to be significant and the BP statistic was acceptable at Q(33) = 41.13[47.4]. However, visual inspection of the ACF of residuals showed significant spikes remaining at lags eight (-.218) and twelve (-.198).

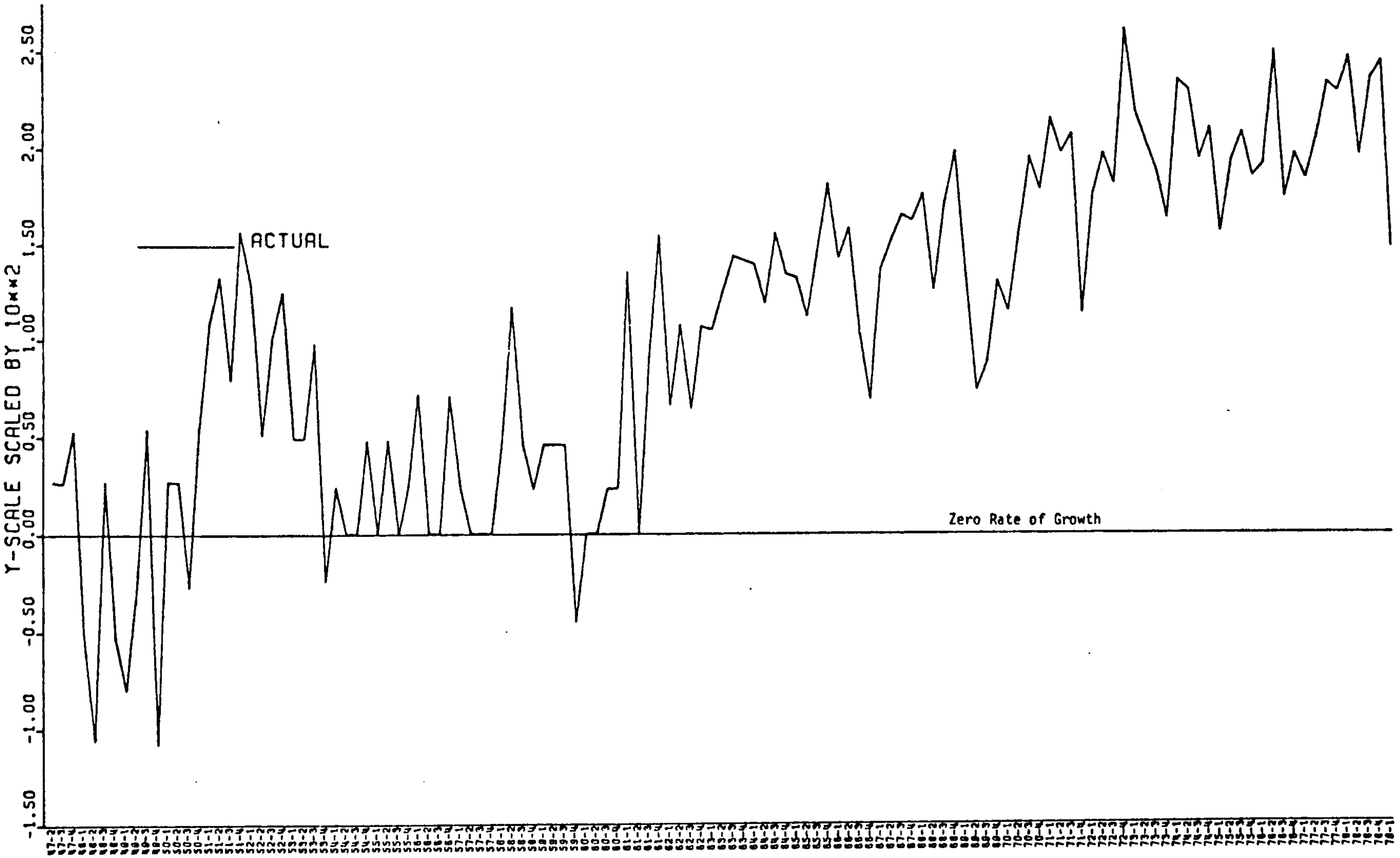
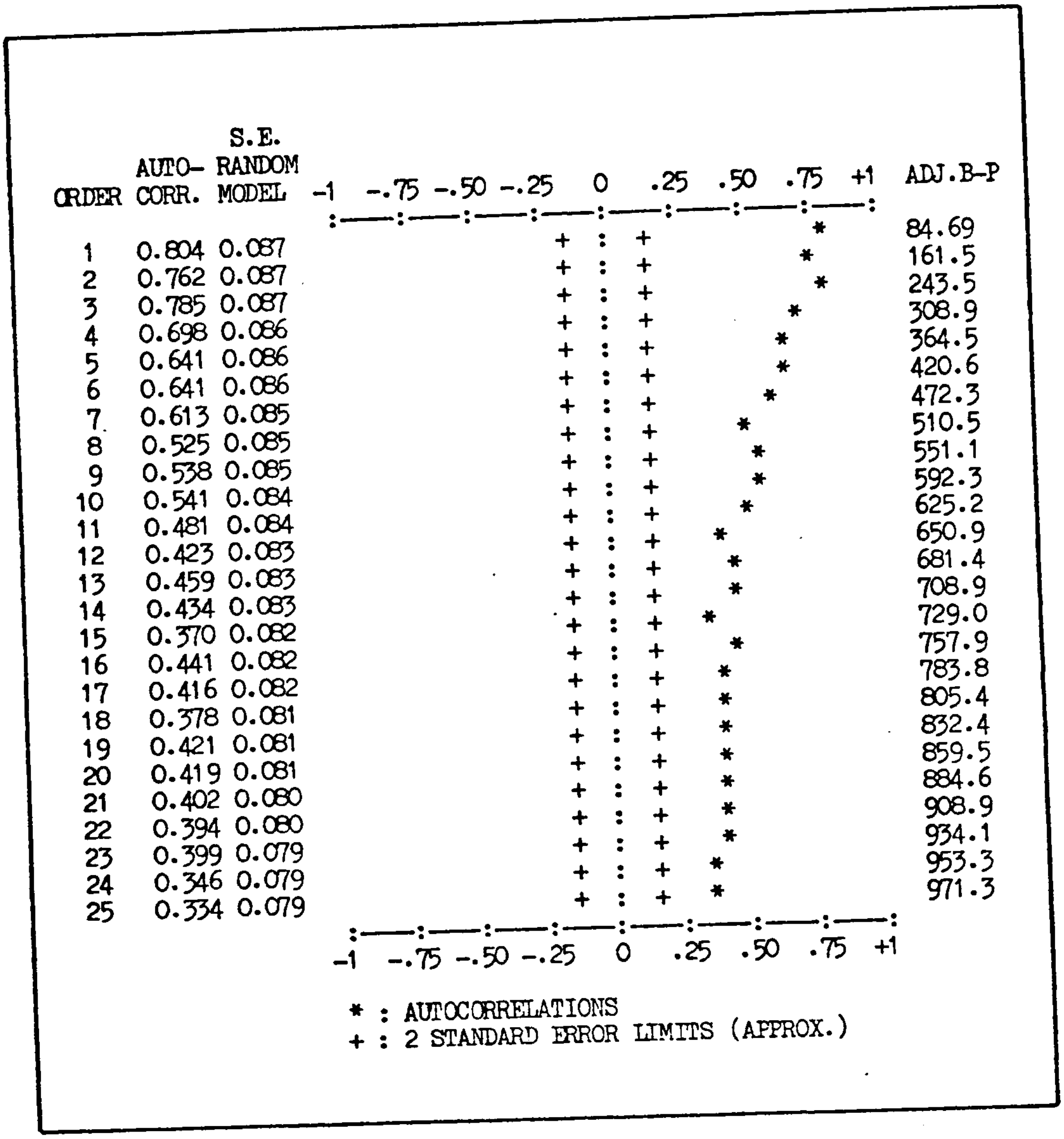


Figure 6.4.1. Actual Quarterly MB Growth Rate (Seasonally Adjusted), 1947-2/1979-1.



```
             S.E.
      AUTO-  RANDOM
ORDER CORR.  MODEL      -1    -.75    -.50    -.25      0      .25     .50     .75     +1   ADJ.B-P
                        :-------:-------:-------:-------:-------:-------:-------:-------:
  1   0.804  0.087                                 +    :    +                    *         84.69
  2   0.762  0.087                                 +    :    +                  *           161.5
  3   0.785  0.087                                 +    :    +                   *          243.5
  4   0.698  0.086                                 +    :    +                *             308.9
  5   0.641  0.086                                 +    :    +               *              364.5
  6   0.641  0.086                                 +    :    +               *              420.6
  7   0.613  0.085                                 +    :    +              *               472.3
  8   0.525  0.085                                 +    :    +           *                  510.5
  9   0.538  0.085                                 +    :    +           *                  551.1
 10   0.541  0.084                                 +    :    +           *                  592.3
 11   0.481  0.084                                 +    :    +         *                    625.2
 12   0.423  0.083                                 +    :    +        *                     650.9
 13   0.459  0.083                                 +    :    +         *                    681.4
 14   0.434  0.083                                 +    :    +        *                     708.9
 15   0.370  0.082                                 +    :    +      *                       729.0
 16   0.441  0.082                                 +    :    +        *                     757.9
 17   0.416  0.082                                 +    :    +       *                      783.8
 18   0.378  0.081                                 +    :    +      *                       805.4
 19   0.421  0.081                                 +    :    +       *                      832.4
 20   0.419  0.081                                 +    :    +       *                      859.5
 21   0.402  0.080                                 +    :    +       *                      884.6
 22   0.394  0.080                                 +    :    +       *                      908.9
 23   0.399  0.079                                 +    :    +       *                      934.1
 24   0.346  0.079                                 +    :    +     *                        953.3
 25   0.334  0.079                                 +    :    +     *                        971.3
                        :-------:-------:-------:-------:-------:-------:-------:-------:
                        -1    -.75    -.50    -.25      0      .25     .50     .75     +1

                        * : AUTOCORRELATIONS
                        + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.4.2. Autocorrelation Function, MB Growth Rate, 1947-2/1979-1.

```
             S.E.
      AUTO-  RANDOM
ORDER CORR.  MODEL -1    -.75    -.50    -.25      0      .25     .50     .75     +1    ADJ.B-P
                   :-------:-------:-------:-------:-------:-------:-------:-------:
  1 -0.416 0.088                      *      +     :     +                              22.50
  2 -0.161 0.087                              *    :   +                                25.88
  3  0.267 0.087                              +    :   +   *                            35.29
  4 -0.097 0.087                              +*   :   +                                36.55
  5 -0.118 0.086                              +*   :   +                                38.43
  6  0.055 0.086                              +    :*  +                                38.83
  7  0.154 0.086                              +    :   *                                42.08
  8 -0.252 0.085                           *  +    :   +                                50.84
  9  0.044 0.085                              +    :*  +                                51.12
 10  0.131 0.085                              +    :   *                                53.51
 11  0.012 0.084                              +    *   +                                53.53
 12 -0.237 0.084                           *  +    :   +                                61.53
 13  0.148 0.083                              +    :   *                                64.67
 14  0.116 0.083                              +    :  *+                                66.61
 15 -0.338 0.083                        *     +    :   +                                83.36
 16  0.261 0.082                              +    :   +   *                            93.41
 17  0.014 0.082                              +    *   +                                93.44
 18 -0.192 0.082                             *+    :   +                                98.99
 19  0.105 0.081                              +    :  *+                                100.7
 20  0.023 0.081                              +    *   +                                100.8
 21 -0.004 0.080                              +    *   +                                100.8
 22 -0.033 0.080                              +   *:   +                                100.9
 23  0.130 0.080                              +    :   *                                103.6
 24 -0.108 0.079                              +*   :   +                                105.4
 25 -0.056 0.079                              +   *:   +                                105.9
                   :-------:-------:-------:-------:-------:-------:-------:-------:
                   -1    -.75    -.50    -.25      0      .25     .50     .75     +1

                   * : AUTOCORRELATIONS
                   + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.4.3. Autocorrelation Function of $(1 - B)z_t$, MB Growth Rate, 1947-2/1979-1.

```
          PARTIAL
ORDER     AUTO COR
                  -1     -.75    -.50    -.25      0      .25     .50     .75      1
                   :-------:-------:-------:-------:-------:-------:-------:-------:
   1   -0.4160   -                    *            :
   2   -0.4036   -                    *            :
   3    0.0075   -                                 *
   4   -0.0044   -                                 *
   5   -0.0994   -                              *  :
   6   -0.1387   -                            *    :
   7    0.1177   -                                 :  *
   8   -0.1250   -                              *  :
   9   -0.1165   -                              *  :
  10   -0.0535   -                                *:
  11    0.1852   -                                 :     *
  12   -0.1662   -                            *    :
  13   -0.1104   -                              *  :
  14    0.0408   -                                 :*
  15   -0.1698   -                            *    :
  16    0.0043   -                                 *
  17   -0.0381   -                                *:
  18   -0.0865   -                              *  :
  19   -0.0065   -                                 *
  20   -0.1266   -                            *    :
  21    0.0014   -                                 *
  22    0.0554   -                                 :*
  23    0.1196   -                                 :  *
  24   -0.0568   -                                *:
  25   -0.0894   -                              *  :
```

Figure 6.4.4. Partial Autocorrelation Function of $(1 - B)z_t$, MB Growth Rate, 1947-2/1979-1.

Continuing on, model (6.4.1.) was revised to include a second seasonal MA term. This model;

(6.4.2.) $(2,1,0)\times(0,0,2)\times 4$

produced an improved BP statistic of Q(32) = 29.06[46.2], with a highly significant MA term of order eight. Additionally, the residual spike at lag eight was reduced to an acceptable autocorrelation of .026. However, the residual autocorrelation at lag twelve remained marginally high at -.147. In response, the following model was estimated;

(6.4.3.) $(2,1,0)\times(0,0,3)\times 4$.

While (6.4.3.) did reduce the autocorrelation of residuals at lag twelve (to .041), it did not improve the BP statistic with Q(31) = 41.33[45.0]. More importantly, the model rendered the forth MA seasonal insignificant.

The following models were estimated in an attempt to deal with these problems;

(6.4.4.) $$(1 - B)(1 - \phi_1 B - \phi_2 B^2)z_t = (1 - \theta_4 B^4)(1 - \theta_8 B^8)a_t$$

(6.4.5.) $$(1 - B)(1 - \phi_1 B - \phi_2 B^2)z_t = (1 - \theta_8 B^8 - \theta_{12} B^{12})a_t$$

Strangely, with the multiplicative model (6.4.5.), the negative spike at lag twelve reappeared. Model (6.4.4.), on the other hand, while reducing this spike, left a distinctive negative pattern in the ACF of residuals. For this reason, and because the BP statistic of (6.4.4.) was almost equal to that of (6.4.2.), the latter model was chosen as the final form for the MB series after the roots were checked for convergence requirements.

The ACF of residuals of (6.4.2) appear in Figure 6.4.5. Table 6.4.1. presents the estimated coefficients of the model, summary statistics and

```
              S.E.
      AUTO-  RANDOM
ORDER CORR.  MODEL  -1  -.75 -.50 -.25   0  .25  .50  .75   +1   ADJ.B-P
                    :----:----:----:----:----:----:----:----:
  1  -0.039  0.087                   + *:  +                     0.2014
  2  -0.009  0.087                   +  *  +                     0.2128
  3  -0.048  0.087                   + *:  +                     0.5171
  4   0.018  0.086                   +  *  +                     0.5597
  5  -0.078  0.086                   +* :  +                      1.382
  6  -0.017  0.086                   +  *  +                      1.423
  7  -0.008  0.085                   +  *  +                      1.431
  8   0.019  0.085                   +  *  +                      1.481
  9   0.022  0.085                   +  *  +                      1.548
 10   0.045  0.084                   +  :* +                      1.829
 11  -0.043  0.084                   + *:  +                      2.095
 12  -0.149  0.083                   *  :  +                      5.288
 13  -0.003  0.083                   +  *  +                      5.290
 14   0.028  0.083                   +  :* +                      5.401
 15  -0.198  0.082                  *+  :  +                      11.17
 16   0.105  0.082                   +  : *+                      12.80
 17  -0.037  0.082                   + *:  +                      13.01
 18  -0.095  0.081                   +* :  +                      14.37
 19   0.017  0.081                   +  *  +                      14.41
 20   0.026  0.081                   +  :* +                      14.52
 21   0.101  0.080                   +  : *+                      16.09
 22   0.047  0.080                   +  :* +                      16.43
 23   0.021  0.079                   +  *  +                      16.49
 24  -0.105  0.079                   +* :  +                      18.27
 25  -0.083  0.079                   +* :  +                      19.37
                    :----:----:----:----:----:----:----:----:
                    -1  -.75 -.50 -.25   0  .25  .50  .75   +1

                    * : AUTOCORRELATIONS
                    + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.4.5. Autocorrelation Function of Residuals, MB Growth Rate Model, 1947-2/1979-1.

related forecasting function. As the forecasting function shows, the current forecast of the series is positively autocorrelated with the past three values of z_t, while errors occurring four and eight periods past pull the current forecast in the opposite direction of the forecast error. Hence forecast errors tend to moderate the upward pull of past actual values of the variable on the current forecast. The function, therefore, has a plausible economic interpretation: past forecasting errors can be looked upon as causing a reversion of expectations toward "normality"--normality, in this case being defined as an average of the past three values of z_t, while the positive autocorrelation of the current expectation with past actual values of the variable can be interpreted as a revision of expectations upward from one level of normality to another.

Figure 6.4.6. presents a plot of the actual and iterative updated forecasts for the MB series over the 1956/3 to 1979/2 period. The evolution

Table 6.4.1. Estimated ARIMA MB Growth Rate Model, Step-Ahead Forecasting Function, and Related Statistics; 1947/2-1979/1.

$$(1 - B)(1 + \underset{(6.87)}{.584B} + \underset{(4.50)}{.383B^2})z_t = (1 - \underset{(2.81)}{.250B^4} - \underset{(3.07)}{.267B^8})a_t$$

$Q(32) = 29.06[46.2]$

$MSE = .177$ with 123 d.f. $\quad$ $SSE = 21.83$

$\hat{\sigma}_a = .421$ $\quad$ $\bar{x}_{z_t} = 1.053$ $\quad$ $\sigma_{z_t} = .854$

$R^2 = .761$ $\quad$ $\bar{x}_{\hat{z}_t} = 1.028$ $\quad$ $\sigma_{\hat{z}_t} = .792$

$$\hat{z}_t = .416z_{t-1} + .201z_{t-2} + .383z_{t-3} - .250a_{t-4} - .267a_{t-8}$$

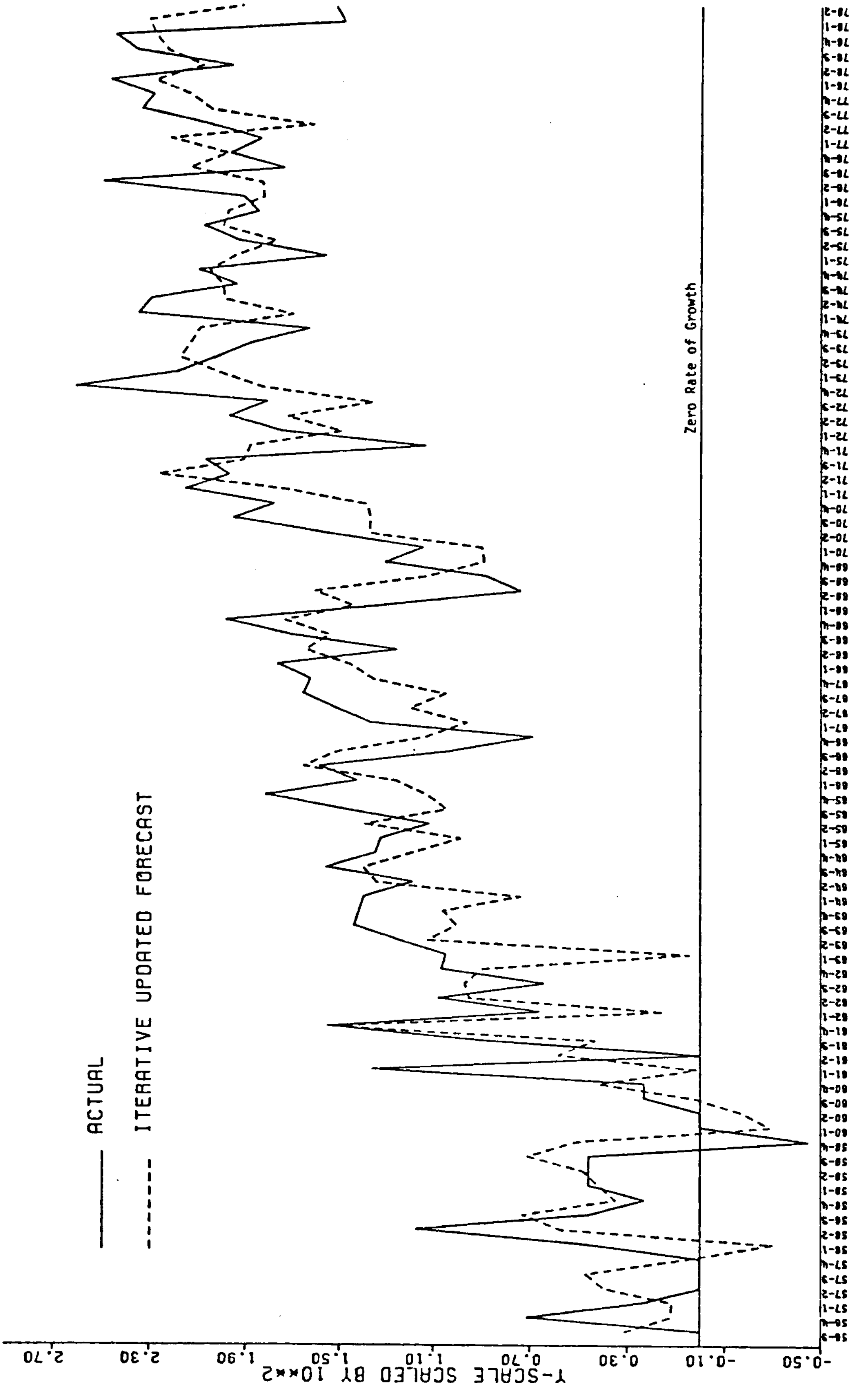


Figure 6.4.6. Actual and Iteratively Updated Forecasts, MB Growth Rate Model, 1956-3/1979-2.

tion of the coefficients is presented in Table B.4. of Appendix B. The ACF of residuals produced from the updating routine is presented in Figure 6.4.7. The BP statistic associated with this series is Q(20) = 23.77[31.4]. While the BP statistic is well within acceptable bounds in terms of the ACF of residuals taken as a group, a visual inspection of Figure 6.4.7. does reveal a spike (of -.215) at lag three. Some autocorrelation of the iterative residuals, as suggested earlier, is expected, and it is not felt that this autocorrelation presents a serious infraction of the rationality precepts outlined earlier. A median runs test on the iterative residuals produced a Z-value of -0.42, and a visual inspection of the residuals about their mean showed them to be evenly distributed.

The results of adjacent subperiod estimation appear in Table 6.4.2. These results indicate that the form of the model fits both periods adequately.

```
                S.E.
         AUTO-  RANDOM
ORDER    CORR.  MODEL   -1  -.75 -.50 -.25  0   .25  .50  .75  +1   ADJ.B-P
                         :----:----:----:----:----:----:----:----:
  1     0.027  0.103                   +    :*   +                  0.6884E-01
  2    -0.069  0.102                   +   *:    +                  0.5317
  3    -0.215  0.101                   *    :    +                   5.031
  4    -0.045  0.101                   +   *:    +                   5.227
  5    -0.040  0.100                   +   *:    +                   5.386
  6    -0.056  0.100                   +   *:    +                   5.705
  7    -0.072  0.099                   +   *:    +                   6.227
  8     0.092  0.099                   +    : *  +                   7.094
  9     0.056  0.098                   +    :*   +                   7.426
 10     0.109  0.097                   +    : *  +                   8.685
 11     0.091  0.097                   +    : *  +                   9.565
 12    -0.063  0.096                   +   *:    +                   9.989
 13    -0.040  0.096                   +   *:    +                  10.17
 14    -0.018  0.095                   +    *    +                  10.20
 15    -0.113  0.094                   + *  :    +                  11.64
 16     0.057  0.094                   +    :*   +                  12.01
 17    -0.041  0.093                   +   *:    +                  12.20
 18    -0.095  0.093                   + *  :    +                  13.24
 19     0.062  0.092                   +    :*   +                  13.70
 20    -0.026  0.091                   +   *:    +                  13.78
                         :----:----:----:----:----:----:----:----:
                        -1  -.75 -.50 -.25  0   .25  .50  .75  +1

                         * : AUTOCORRELATIONS
                         + : 2 STANDARD ERROR LIMITS (APPROX.)
```

Figure 6.4.7. Autocorrelation Function of Iterative Residuals, MB Growth Rate Model, 1956-3/1979-2.

Table 6.4.2. Estimated MB Growth Rate Model for Two Adjacent Subperiods; 1947/2 to 1963/1, and 1963/2 to 1979/1.

1947/2 to 1963/1

$$(1 - B)(1 + \underset{(5.77)}{.692B} + \underset{(3.89)}{.459B^2})z_t = (1 - \underset{(1.74)}{.218B^4} - \underset{(3.78)}{.454B^8})a_t$$

$Q(11) = 10.11[19.7]$

MSE = .228 with 59 d.f. SSE = 13.47

$\hat{\sigma}_a = .447$

$\overline{x}_{z_t} = .376$ $\sigma_{z_t} = .585$

$\overline{x}_{\hat{z}_t} = .346$ $\sigma_{\hat{z}_t} = .416$

1963/2 to 1979/1

$$(1 - B)(1 + \underset{(3.56)}{.459B} + \underset{(2.36)}{.312B^2})z_t = (1 - \underset{(3.17)}{.434B^4} - \underset{(2.41)}{.058B^8})a_t$$

$Q(11) = 12.23[19.7]$

MSE = .121 with 59 d.f. SSE = 7.12

$\hat{\sigma}_a = .347$

$\overline{x}_{z_t} = 1.730$ $\sigma_{z_t} = .443$

$\overline{x}_{\hat{z}_t} = 1.692$ $\sigma_{\hat{z}_t} = .378$

6.5. Statistical Summary of the Estimated ARIMA Models

Table 6.5.1. presents the means and variances of the inflation and money growth rate time series models for the full period and for the period covered by the iterative updating routine. In a number of respects these statistics indicate that the estimated models are correctly specified and the residuals are discrete random error. First, both the full and iterative forecasting models have means that closely approximate the means of the actual series being modeled. Second, the means of the residual series are all close to zero--this is expected since a properly specified model should produce errors that approximate the zero mean assumption made about the true disturbances. Third, the soundness of the models is implied by the fact that the variances of both the full and iterative forecasts is smaller than the variances of their respective actual series. Fourth, the variances of the residual series are all less than the variances of the forecasts. A final note about the variances in Table 6.5.1. is that in every case, except the GNP deflator, the residual variances from the iterative forecasting models are slightly greater than their full forecasting model counterpart. This fact was discussed earlier and is related to the greater forecasting accuracy of the full model as compared to the iterative model.

Table 6.5.2. concludes the statistical summary of the ARIMA models by presenting the simple correlations between the iteratively produced forecasts, errors, and actual series over the 1956/3 through 1979/2 period, the time span over which the short-run output models will be estimated. The error correlations show that inflation rate forecasting mistakes (unanticipated inflation) and money growth rate forecasting mistakes

Table 6.5.1. A Statistical Summary of the Estimated ARIMA Models.

Variable	Mean	Variance
1. CPI Inflation Model		
Actual Series (1952/1-1979/1)	.891	.890
Full Forecasts	.856	.857
Residuals	.003	.613
Actual Series (1956/3-1979/2)	1.063	.893
Iterative Forecasts	1.022	.862
Residuals	.045	.643
2. GNP Deflator Inflation Model		
Actual Series (1952/1-1979/1)	.940	.831
Full Forecasts	.870	.753
Residuals	.006	.625
Actual Series (1956/3-1979/2)	1.055	.835
Iterative Forecasts	.979	.758
Residuals	.007	.612
3. M1 Growth Rate Model		
Actual Series (1947/2-1979/1)	.932	.856
Full Forecasts	.936	.772
Residuals	-.0003	.694
Actual Series (1956/3-1979/2)	1.076	.853
Iterative Forecasts	1.040	.771
Residuals	.003	.716
4. MB Growth Rate Model		
Actual Series (1947/2-1979/1)	1.053	.924
Full Forecasts	.998	.877
Residuals	.005	.641
Actual Series (1956/3-1979/2)	1.359	.857
Iterative Forecasts	1.262	.844
Residuals	.009	.643

Table 6.5.2. Selected ARIMA Model Correlations; 1956/3 to 1979/2.

Error Correlations

	CPI	DFT	M1	MB
CPI	1.000			
DFT	.318	1.000		
M1	-.011 *	.167 *	1.000	
MB	-.191 *	-.038 *	.324	1.000

Forecast and Error Correlations

CPI	DFT	M1	MB
-.067	.033	-.259	-.248

Forecast and Actual Series Correlations

CPI	DFT	M1	MB
.881	.843	.644	.834

Note: --"*" indicates not significant at the .10 level.

(unanticipated money growth rates) are orthogonal--no inflation/money error correlations are significant at the .10 level. This finding, of course, is based upon contemporaneous association; a test for non-contemporaneous correlation between the inflation and money errors will be carried out in Chapter VII.

Among the desirable properties for ARIMA forecasts are that they be uncorrelated with step-ahead forecast errors; failure to meet this requirement implies some degree of model inadequacy and underutilization of information. Table 6.5.2. presents the correlations between the iteratively produced step-ahead forecasts (forecasts produced in period t-1), and errors occurring in period t. As shown, the inflation models meet this test. The money models do not do quite as well, with some mild negative correlations appearing between forecasts and errors. However, there is no theoretical reason why this correlation should be negative.

The last set of correlations in Table 6.5.2. are provided for reference. They indicate the degree of association between the raw inflation and money series and their respective ARIMA step-ahead forecasts. As expected, these associations are positive and strong.

This chapter has presented the BJ time series method as a technique to quantify "anticipated" or forecasted inflation and money growth rates. Rather than specifying an a priori model of expectation formation, anticipations of future inflation and money growth were assumed to correspond to optimal forecasts based upon the information contained in the past history of the variable. An iterative updating routine was used to produce forecasts not contaminated by future realizations of the variable. The residuals from these updating models are then properly interpreted as "unanticipated" inflation/money growth.

CHAPTER VII

FORECASTS AND ERRORS OF THE ESTIMATED ARIMA MODELS: FURTHER STATISTICAL TESTS OF THE RATIONALITY HYPOTHESIS

In the preceding chapter the "rationality" of the time series models was judged in terms of whether or not the forecast errors had been reduced to a purely random state by the filtering procedure. As such, these tests for rationality were a by-product of the overall BJ univariate modeling procedure. Now that the ARIMA models have been derived and have been shown to be adequate, it is instructive to subject the forecasts and errors of the inflation and money models to further statistical tests for rationality--alternative tests different from those pertaining directly to the estimation and diagnostic checking procedures indigenous to the BJ technique.

In this regard, this chapter will discuss the following topics: 1) how well the step-ahead forecasts meet OLS regression requirements for rationality, 2) how the nature of forecasting mistakes can be analyzed via statistical decomposition, 3) the use of the prediction/realization diagram to provide a pictorial account of the accuracy of the inflation and money model forecasts, 4) the relationship of forecast revision to forecast error, and, 5) tests for causality between unanticipated money growth rates and unanticipated inflation rates.

7.1. Decomposition of the Mean-Squared-Forecast Error of the ARIMA Models

Theil [95] has shown that forecasts (however derived) can be analyzed from the standpoint of bias and efficiency by decomposing the forecast error into distinct statistical components. The procedure is based upon the OLS regression of an actual magnitude of an observation at time period t, A_t, on the forecasted or "predicted" magnitude of A_t, made at time period t-1, or P_t.[1] The decomposition procedure is based upon the fact that the residuals from such a regression can be transformed into a mean-squared-forecast error (MSFE) and statistically decomposed so that the degree of "rationality" in the forecasted series can be ascertained.

Consider the relation;

(7.1.1.) $$A_t \equiv P_t + u_t \quad .$$

While (7.1.1.) is an identity, an OLS regression of actual on predicted magnitudes produces the equation;

(7.1.2.) $$A_t = \alpha + \beta P_t + \varepsilon_t \quad ,$$

where OLS "regression rationality" implies that the following (joint) conditions must hold if a forecasted series is to be considered rational;

(7.1.3.) $$\alpha = 0 \quad ,$$

(7.1.4.) $$\beta = 1 \quad .$$

Note that the dual criteria for rationality implied by conditions (7.1.3.) and (7.1.4.) are necessary requirements for rationality in the strict interpretation of the Muth hypothesis aired earlier. This can be seen by taking

---

[1] Note that the forecasts produced from the iterative updating BJ routine described in the preceding chapter are formulated in period t-1, and therefore, are "predictors" of the actual realizations occurring in period t. Given this fact, the actual observations can properly be interpreted as functions of the forecasts. It is for this reason that in the equations estimated here, P_t and A_t are the independent and dependent variables, respectively. Note also that the notation A_t and P_t refers to the actual and forecasted magnitudes used in the previous chapter. The notation has been altered to conform to Theil's terminology.

the mathematical expectation of (7.1.2.) given (7.1.3.) and (7.1.4.). We have then;

$$E(A_t = P_t + \varepsilon_t) \quad ,$$

$$E(A_t - P_t) = 0 \quad ,$$

since $E(\varepsilon_t) = 0$.

Now the MSFE is formally defined as;

$$\text{(7.1.5.)} \qquad \text{MSFE} = n^{-1}[\sum_{1}^{n}(P_t - A_t)^2] \quad .$$

Substitution of (7.1.2.) into (7.1.5.) produces a MSFE = 0 if the strict Muth conditions (7.1.3.) and (7.1.4.) are met. Otherwise, a non-zero MSFE results, its magnitude depending upon how much the forecasts, P_t, stray from the actual realizations, A_t.[1] Statistically it is evident that the MSFE is a measure of "average" forecast accuracy and is based upon the same consideration given to the variance as a measure of dispersion in conventional analysis.

The MSFE can be decomposed in the following manner;

$$\text{(7.1.6.)} \qquad \text{MSFE} = (\overline{P} - \overline{A})^2 + (S_P - rS_A)^2 + (1 - r^2)S_A^2 \quad ,$$

where $\overline{P}$ and $\overline{A}$ are the means of the forecasted and actual series, respectively, and S_P and S_A are the standard deviations of the forecasted and actual series, respectively. r is the simple correlation coefficient between the forecasted and actual series. In what follows, it is useful to redefine the three components of (7.1.6.) as follows;

---

[1] Note that the MSFE gives more weight to larger than to smaller errors.

$$(\bar{P} - \bar{A})^2 = \text{bias component (MC)}$$

$$(S_P - rS_A)^2 = \text{regression component (SC)}$$

$$(1 - r^2)S_A^2 = \text{residual variance component (RC)}$$

Each decomposition gives some information on how well the forecast meets the rationality requirements (7.1.3.) and (7.1.4.). If MC = 0, then forecasts are unbiased, i.e., on average the predicted value equals the average realized value. Errors in prediction leading to a positive value for the MC component are errors in central tendency and amount to a consistent under- or over-estimation flaw in the forecasting model. Obviously such models cannot be optimal forecasting rules in the Muth sense of rationality. In a dynamic sense, a model that correctly mimics the true process generating the series about which expectations are being formed will be characterized by a bias component that approaches zero over time, since biased forecasts will be reduced by adding or subtracting the observed average error to the expectations (predictions) of future periods.

The regression component, SC, sheds additional light on forecast rationality. Consider fitting regression (7.1.2.) in a (A,P) two-space, with A on the vertical axis (since it is the dependent variable). If conditions (7.1.3.) and (7.1.4.) are met, then (7.1.2.) plots as a 45° line out of the origin of the two-space (excluding random error) and means that;

$$\hat{A}_t = A_t = P_t \quad ,$$

where $\hat{A}_t$ is the fitted value of the dependent variable in the regression (7.1.2.). In this case, the regression produces perfect forecasts, all lying on the 45° line. Concentrating, then, just on the slope, β, in (7.1.2.) it is seen that the larger the deviation of the regression slope from unity the less efficient the forecasts will be (in the sense that the residual variance, σ_ε^2, will be greater than it would be if all forecasts lay on the 45^0 line). Therefore, to the extent that expectations are characterized by errors concentrated in the slope component, these errors will be associated with forecasts which under- (over-) estimate large changes in A_t, and over- (under-) estimate small changes in A_t, depending upon whether the intercept, α, is greater or less than zero. From (7.1.2.) it is seen that the regression component SC → 0 as $\beta \rightarrow 1$, since the regression coefficient component in (7.1.6.) takes the form;

$$(P_t - \bar{P})(A_t - \bar{A})[\Sigma_1(P_t - \bar{P})^2]^{-1} = rS_A/S_P = \beta \quad .$$

Like the MC component, we would expect SC → 0, since, over time, rational forecasters will reduce slope error because it is a systematic and predictable part of the process producing the forecast error.

The third component of MSFE, the residual variance, RC, represents the dispersion of residuals obtained from regressing the actual on forecasted magnitudes in (7.1.2.). Since the variance S_A^2 is positive, RC will equal zero only if A_t and P_t are perfectly correlated. Given the random nature of the disturbance term, ε_t, in (7.1.2.), this is unlikely. RC can be viewed as a measure of the unsystematic component affecting the accuracy of the forecasts, with accuracy being inversely related to the variance of the probability distribution of residuals. Also, since the RC component represents the unsystematic part of MSFE, it is synonomous with the unpredictable error term in the Muth rationality hypothesis. Because SC represents random noise in the regression of actual on predicted magnitudes, it would not be affected by increased forecasting accuracy. Hence, even if improved forecasting did reduce the magnitudes of MC and SC to zero, MSFE can still be expected to be positive since forecasting accuracy will always be subject to random error.

It is useful to redefine the three components in terms of their proportion of MSFE;

MC% = MC/MSFE = bias proportion

SC% = SC/MSFE = regression proportion

RC% = RC/MSFE = residual variance proportion ,

where MC% + SC% + RC% = 1. These proportions shed light on the degree of rationality embodied in the forecasting models. If MC% is large then the average predicted magnitudes deviate substantially from average realized magnitudes. If forecasts are rational we would expect such deviations to be a small percent of total forecast error. Likewise, we would expect SC% to be a small part of MSFE if forecasts are rational. Finally, even with the attainment of "regression rationality" as defined by (7.1.3.) and (7.1.4) we can expect RC% > 0, since even "fully informed" rational forecasters will be affected by random disturbance. Given a rational forecasting rule, then, we can expect, a priori, that both MC% and SC% will be small and RC% will be large, i.e., the largest percent of the discrepancy between actual and forecasted values will occur in the residual variance catagory of MSFE *if* the forecasting rule is rational. The extent to which a forecasting rule stays from rationality can then be judged by comparing the relative sizes of the three proportions making up the MSFE.

A related statistic of interest here is Theil's "U-statistic" [95]. It is composed of the MSFE and provides a measure of how worthwhile forecasting activity is, compared to no forecast at all. The U-statistic is defined as;

$$U = \sqrt{}[MSFE/n][(\sum_{1} A_t^2/n)]^{-1}$$

$$= \sqrt{\frac{1/n\sum_{1}(P_t - A_t)^2}{1/n\sum_{1} A_t^2}} \quad .$$

This statistic ranges from U = 0, when forecasts are perfect, i.e., when $P_t = A_t$, to U = 1, when no forecasting activity is undertaken, i.e., when $P_t = 0$. Obviously, the closer the U-statistic is to zero the more accurate are the forecasts and the more beneficial it is to engage in forecasting activity. While the U-statistic, as such, is not directly related to the idea of rationality per se, it can be used as an index to compare the accuracy of two or more alternative forecasting rules. To the extent that forecast accuracy follows from rationality in the formation of expectations, the statistic can then be used as a gauge as to the degree of rationality in alternative forecasting rules.[1]

Table 7.1.1. presents the results of regression (7.1.2.) for the inflation and money growth rate forecasts developed from the ARIMA models over the 1956/3 through 1979/2 iterative forecasting period. As expected, the signs of all estimated forecast coefficients are positive and significantly different from zero. It is interesting that the adjusted R-squared and residual variance statistics are comparable in all but the M1 regression. No autocorrelation is detected in any of the regression models. Referring to the M1 regression in Table 7.1.1. specifically, it is seen that the residual variance is about twice that of the residual variances of the other regressions.

---

[1] Note that the square root of the MSFE or RMSFE transforms the MSFE statistic into one comparable with the dimensions of the predictions and realizations themselves. It is seen that the RMSFE is identical to the numerator of Theil's U-statistic.

Table 7.1.1. Regression $A_t = \alpha + \beta P_t + \varepsilon_t$ for the ARIMA Models; 1956/3 through 1979/2.

$$(CPI)_t = \underset{(1.28)}{.087} + \underset{(17.69)}{.959}(FORCPI)_t$$

$R^2 = .774$ F = 313.12 n = 92

$\hat{\sigma}^2_\varepsilon = .143$ DW = 1.89

$$(DFT)_t = \underset{(0.68)}{.053} + \underset{(14.85)}{1.022}(FORDFT)_t$$

$R^2 = .707$ F = 220.67 n = 92

$\hat{\sigma}^2_\varepsilon = .142$ DW = 2.03

$$(M1)_t = \underset{(2.63)}{.301} + \underset{(7.98)}{.758}(FORM1)_t$$

$R^2 = .408$ F = 63.68 n = 92

$\hat{\sigma}^2_\varepsilon = .314$ DW = 1.79

$$(MB)_t = \underset{(2.18)}{.275} + \underset{(14.35)}{.855}(FORMB)_t$$

$R^2 = .692$ F = 205.96 n = 92

$\hat{\sigma}^2_\varepsilon = .164$ DW = 1.81

Notes: --The dependent variable is the actual value of the indicated series occurring in time period t.

--The independent variable is the forecasted value of the indicated series made in period t-1 using the iterative ARIMA procedure.

--all R-squared statistics are adjusted for degrees of freedom.

--all Durbin Watson statistics lie within the range du = 1.68 to (4 - du) = 2.32, hence we accept Ho: of no autocorrelation (at the .05 level of significance).

Referring back to Table 6.5.1., pg. 193, we see that the variance of residuals of the M1 ARIMA model is also higher for both the full and iterative updating period than the residual variances of the other ARIMA models. It is this fact that accounts for the poorer regression fit of the M1 regression model (in terms of both R-square and residual variance) as compared to the other regressionsin Table 6.5.1. However, this is a qualitative indictment of the M1 ARIMA model and will not invalidate later use of the M1 ARIMA model residuals, since diagnostic checking showed these residuals to qualify as white noise.

It should be remembered that the forecasted variables in Table 7.1.1. are being interpreted as unbiased predictors of the actual realizations of the variables. This interpretation amounts to taking the position that the null hypothesis that $\alpha = 0$ and $\beta = 1$ is true. Visual inspection of the regressions in Table 7.1.1. would tend to support at least part of this null hypothesis since the magnitudes of the forecast coefficients are fairly close to unity. Additionally, the intercept terms are all close to zero. However, these interpretations are non-statistical in nature. Since the actual/forecated regressions estimated deal with limited samples, the regression statistics of (7.1.2.) are subject to sampling error. Thus even if the rationality requirements (7.1.3.) and (7.1.4.) hold for the population, it is expected that sample results will not coincide exactly with the null hypothesis of $\alpha = 0$ and $\beta = 1$. Hence to ascertain if the forecasts of the inflation and money ARIMA models meet OLS regression rationality it is necessary to test <u>jointly</u> the null hypothesis;

$$\text{(7.1.7.)} \qquad \text{Ho: } \alpha = 0, \underline{\text{and}}\ \beta = 1$$

$$H_1\text{: do not accept Ho} \quad .$$

The F-test can be used to test the null hypothesis of regression rationality (7.1.7.). Table 7.1.2. presents these computations. Since the computed F-statistics are less than the critical value F-statistic for stated degrees of freedom, we cannot reject the null hypothesis that $\alpha = 0$ and $\beta = 1$

Table 7.1.2. F-test for the Rationality of the Regression Models of the Form $A_t = \alpha + \beta P_t + \varepsilon_t$; 1956/3 through 1979/2.

	Ho: $\alpha = 0$ and $\beta = 1$
Regression Model	F-value
CPI	.979
DFT	1.933
M1	3.620
MB	4.316

Note: The F-statistic used here is as follows:

$$F = \frac{[\text{Constrained SSE - Unconstrained SSE}]/n}{(\text{Unconstrained SSE})/T\text{-}K} ,$$

where the value of the Unconstrained SSE is sum-of-squared errors from the full regression (7.1.2.). The magnitude of the Constrained SSE is obtained from (7.1.2.) after imposing the joint conditions $\alpha = 0$ and $\beta = 1$. This means that the fitted value of A_t is forced to equal the forecast F_t. Hence the Constrained SSE is;

$$\text{Constrained SSE} = \sum_i (A_i - F_i)^2 \quad .$$

In this case the critical value of F is: $F(n,T\text{-}K) = F(2,90) = 4.81$, where n refers to the number of restrictions imposed in obtaining the Constrained SSE. The critical value of F is computed at the 1 percent level.

at the 1 percent level for all the regression models. Hence the "rationality" of the ARIMA models is supported from the standpoint of OLS "regression rationality." Note, however, that the null hypothesis would have been rejected for the M1 and MB regressions at the .95 percent level of confidence (with a critical F-value = 3.10). Hence, one can infer that, in a qualitative sense, the money ARIMA models are not quite "as rational" as the inflation ARIMA models. This observation is supported by comparing the MSE's of the inflation and money models seen in Tables 6.1.1.,6.2.1., 6.3.1., and 6.4.1., pp. 139, 164, 176, 187, where the money models have larger residual variances than the inflation models (adjusted for degrees of freedom).

Table 7.1.3. presents the results of decomposing the MSFE into its bias, regression and residual variance components in both an absolute and percentage fashion for the 1956/3 through 1979/2 period. The RMSFE and Theil's U-statistic are also presented in columns (5) and (9), respectively. Comparing columns 2, 3, and 4, and columns 6, 7, and 8, it is apparent that, for the most part, the bias and regression components of the regression models contribute very little to the average squared forecast error. Most of the forecasting error, therefore, must be attributed to non-systematic residual variance. In terms of percentages, this component accounts for at least 90 percent of the forecast error for all four models. This fact implies that the forecasts are rational in the Muth sense, since the bulk of the discrepancy between actual and forecasted magnitudes is placed in the unpredictable error category. Likewise, columns 2, and 3, and 6, and 7, show that only a small part of the discrepancy between actual and forecasted magnitudes is due to any type of systematic influence in the forecasting

Table 7.1.3. Mean-Square-Forecast Error Decomposition of OLS Regression Residuals; 1956/3 through 1979/2.

	(1) MSFE	(2) MC	(3) SC	(4) RC	(5) RMSFE	(6) %MC	(7) %SC	(8) %RC	(9) U
CPI	.145	.002	.001	.142	.381	.014	.007	.979	.288
DFT	.147	.006	.000	.141	.383	.041	.001	.958	.304
M1	.347	.012	.023	.312	.589	.035	.066	.899	.452
MB	.153	.005	.004	.144	.391	.033	.026	.941	.254

models. Specifically, the small magnitudes of columns 2 and 6 mean that the forecasts are unbiased estimators of the actual magnitudes, and the small values of columns 3 and 7 indicate the regression slopes are close to unity indicating forecasts are efficient. These results support also the acceptance of the null hypothesis (7.1.7.)--except for sampling error, the regression models meet the rationality conditions of (7.1.3.) and (7.1.4.). Finally, the low values of the U-statistic appearing in column (9) indicate the accuracy of the ARIMA forecasts.

The above are general comments about the MSFE decomposition. A few specific comments are in order, however. First, absolute size the the MSFE of M1 is almost twice that of the other variables. The relatively large value in column (7) for M1 implies that the slope component, rather than forecast bias, is responsible. Relatedly, a lower percentage of the discrepancy between actual and forecasted magnitudes of M1 falls into the random error catagory. The M1 regression also produces a higher U-statistic than the other variables. The MB regression also produces some very mild evidence of systematic influence in the forecasts, with a slighlty larger percentage being accounted for in bias (.033) than in efficiency (.026). A second interesting fact about column (1) in Table 7.1.3. is the near-equal magnitudes of the MSFE for all variables (except M1). Recalling that the absolute magnitudes of the different MSFE's are related to the ARIMA estimation procedure, it is noteworthy that these magnitudes would be so close in value given the fact that the time series models themselves differed so much in form.

In summary, it would appear from the regression MSFE decompositions that:

1) the time series models are producing forecasts that track the actual variables quite well, since the MSFE's are, in an absolute sense, very small.

2) the presence of systematic error in the regression models is a negligible part of forecast error, and, therefore, the ARIMA filters meet regression rationality requirements (subject to sampling error).

3) in general, the diagnostic check of the time series models that showed the residuals to be discrete random error is supported by regression analysis of the forecasts.

4) the "goodness of fit" of the M1 ARIMA model is mildly inferior to that of the other models.

7.2. Prediction/Realization Diagrams of the ARIMA Forecasts

The forecasting pattern of the ARIMA time series models estimated in Chapter VI can be subjected to a visual technique to judge the nature and seriousness of forecasting error. Theil's "prediction/realization" method allows for a concise diagrammatic comparison of the ARIMA models' actual forecasting capabilities relative to a perfect forecasting pattern [95, p. 40].

A brief explanation of the prediction/realization diagram is as follows. Consider a cross centered at the intersection of a vertical and horizontal axis. Let the actual rate of inflation or money growth (z_t) and the forecast or predicted rate ($\hat{z}_t$) be represented by the vertical and horizontal axises, respectively. In a 2-space this cross defines four quadrants in which a point is defined by the coordinate ($z_t, \hat{z}_t$). A positively sloped 45^0 line emanating from the origin, and thus bisecting the upper-right and lower-left quadrants represents the line of perfect forecasts (or a perfect forecasting pattern). A coordinate in the upper-right quadrant amounts to an increase in both the actual and forecasted values. If the coordinate is below and to the right of the 45° line, the forecasted value of z_t overestimates the actual value. If the coordinate is above and to the left of the 45° line in the upper-right quadrant then the forecast underestimates the actual magnitude. Similarly, the lower-left quadrant represents decreases in both z_t and $\hat{z}_t$, and is divided into an area in which the forecast underestimates the actual magnitude and an area where the forecast overestimates the actual magnitude. The upper-left and lower-right quadrants represent cases in which the forecasted and actual magnitudes have opposite signs; these cases represent turning point errors in the forecast.

Figures 7.2.1., 7.2.2., 7.2.3., and 7.2.4. plot the actual/forecast coordinates for the CPI, GNP deflator, M1, and MB ARIMA models over the iterative updating period 1956/3 through 1979/2, respectively. Note the following: 1) the "origin" of each diagram is the intersection of the vertical and horizontal axises at the zero rate of growth, 2) the predicted magnitude is the same as the iterative updated forecast formed in period t-1, 3) the diagrams have no time dimension, and, 4) the axises are scaled in terms of a quarterly rate of growth. As can be seen in Figure 7.2.1., the CPI inflation model produces more under- than overestimates, with most of the coordinates lying above the 45° line. This result is to be expected from the iterative updating procedure used with the step-ahead forecasting method of the BJ technique. Note, however, that 1) the coordinates do cluster about the 45° line, indicating a high degree of accuracy of the CPI inflation forecasts, and, 2) most of the coordinates lie in the upper-right quadrant, indicating that the forecasted and actual magnitudes moved in the same direction, i.e., the rates of change had the same sign. The diagram shows only two turning point errors, where $\hat{z}_t$ was negative while z_t was

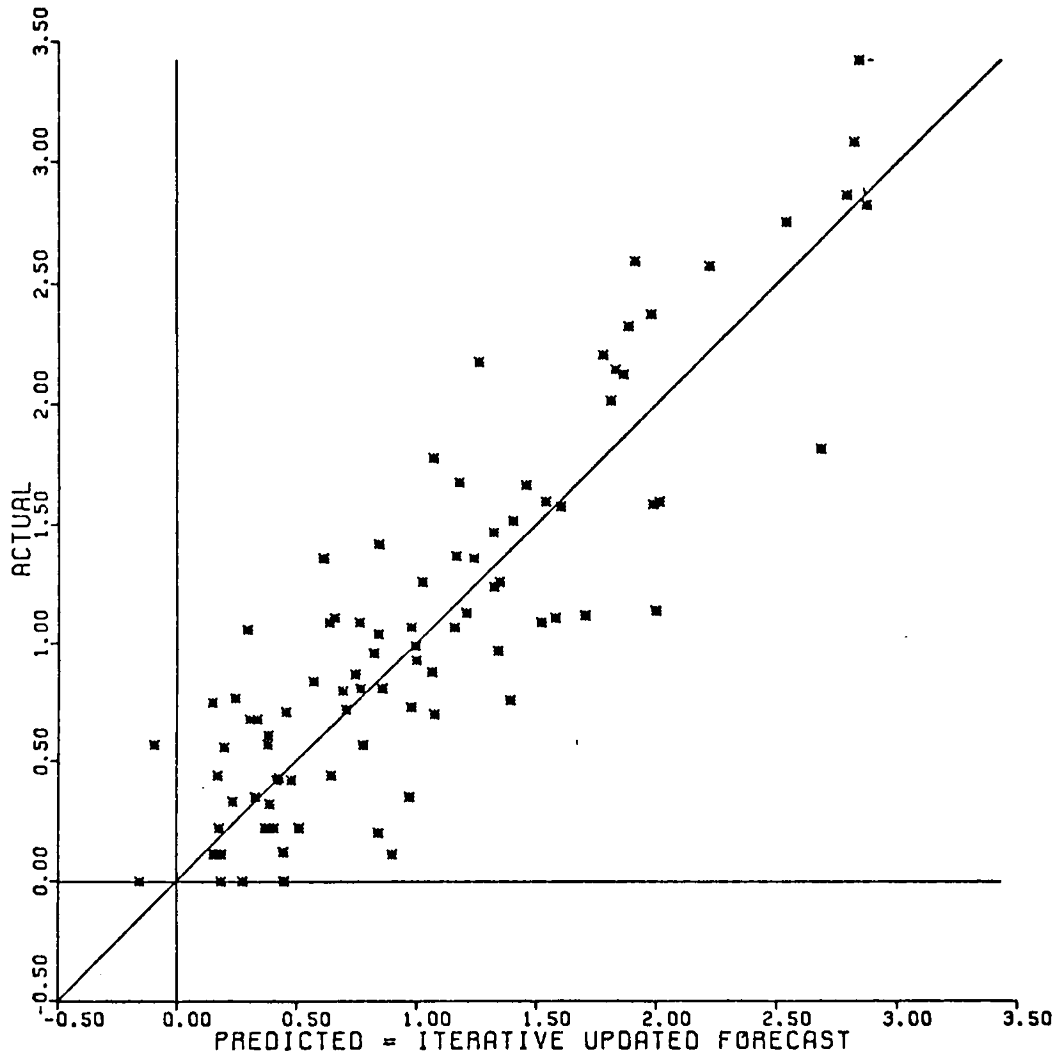


Figure 7.2.1. Prediction/Realization Diagram, CPI Inflation Rate Model, 1956-3/1979-2.

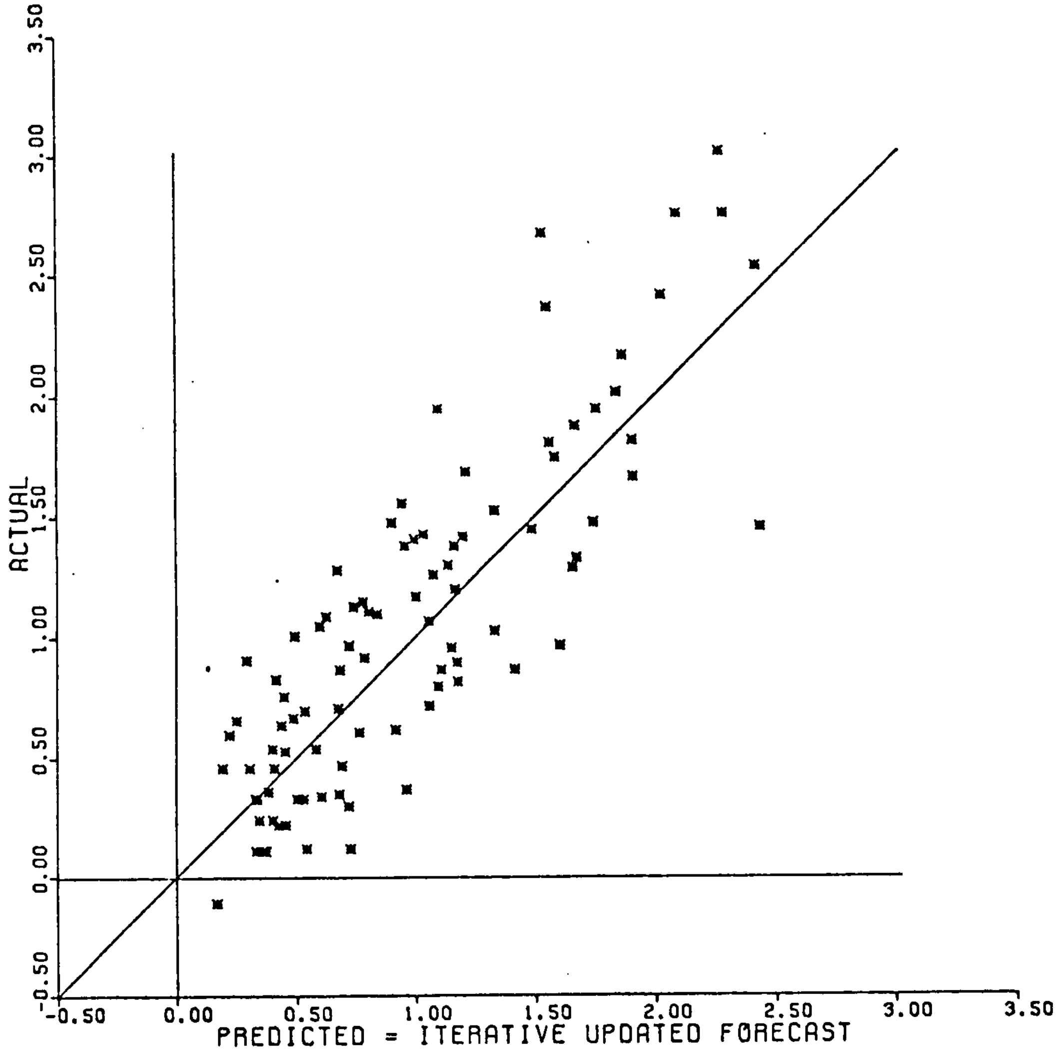


Figure 7.2.2. Prediction/Realization Diagram, GNP Deflator Inflation Model, 1956-3/1979-2.

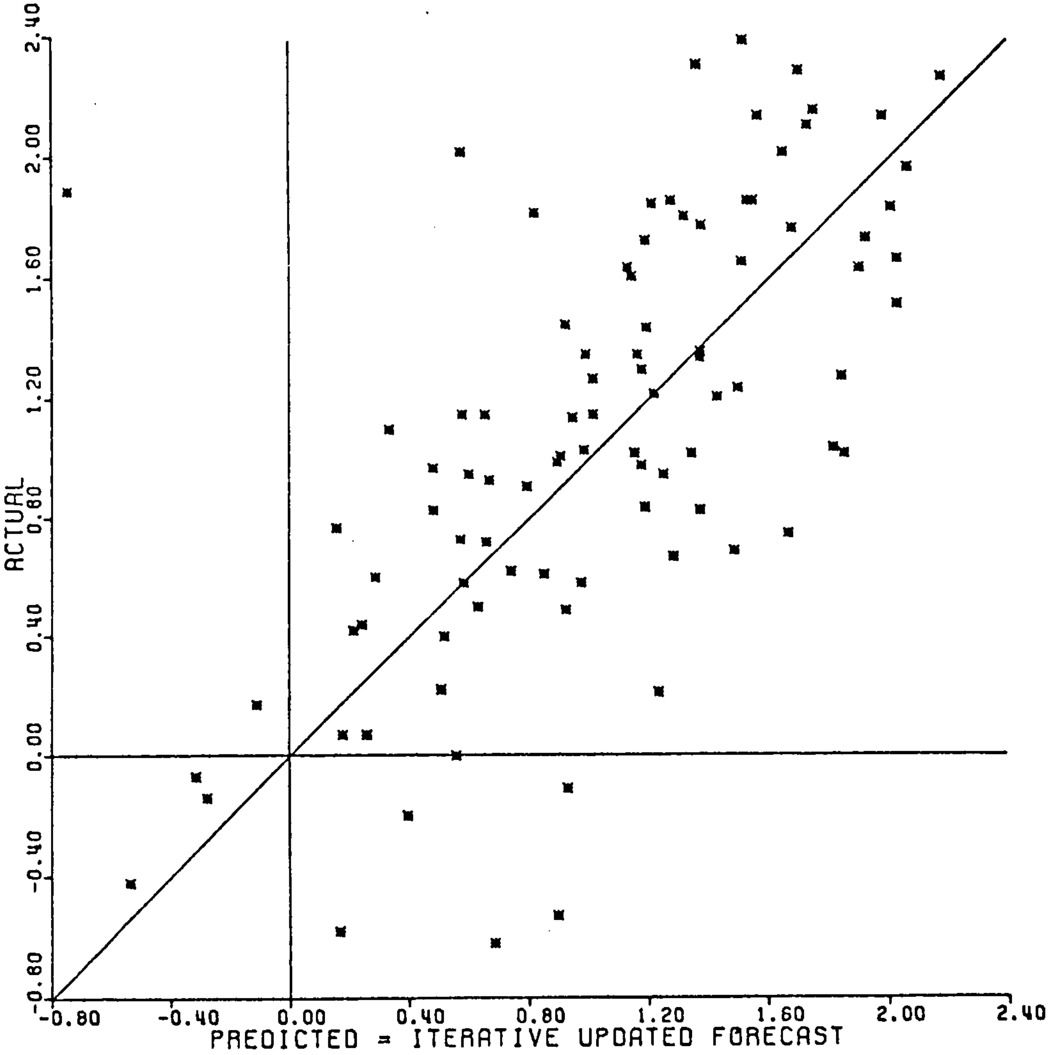


Figure 7.2.3. Prediction/Realization Diagram, M1 Growth Rate Model, 1956-3/1979-2.

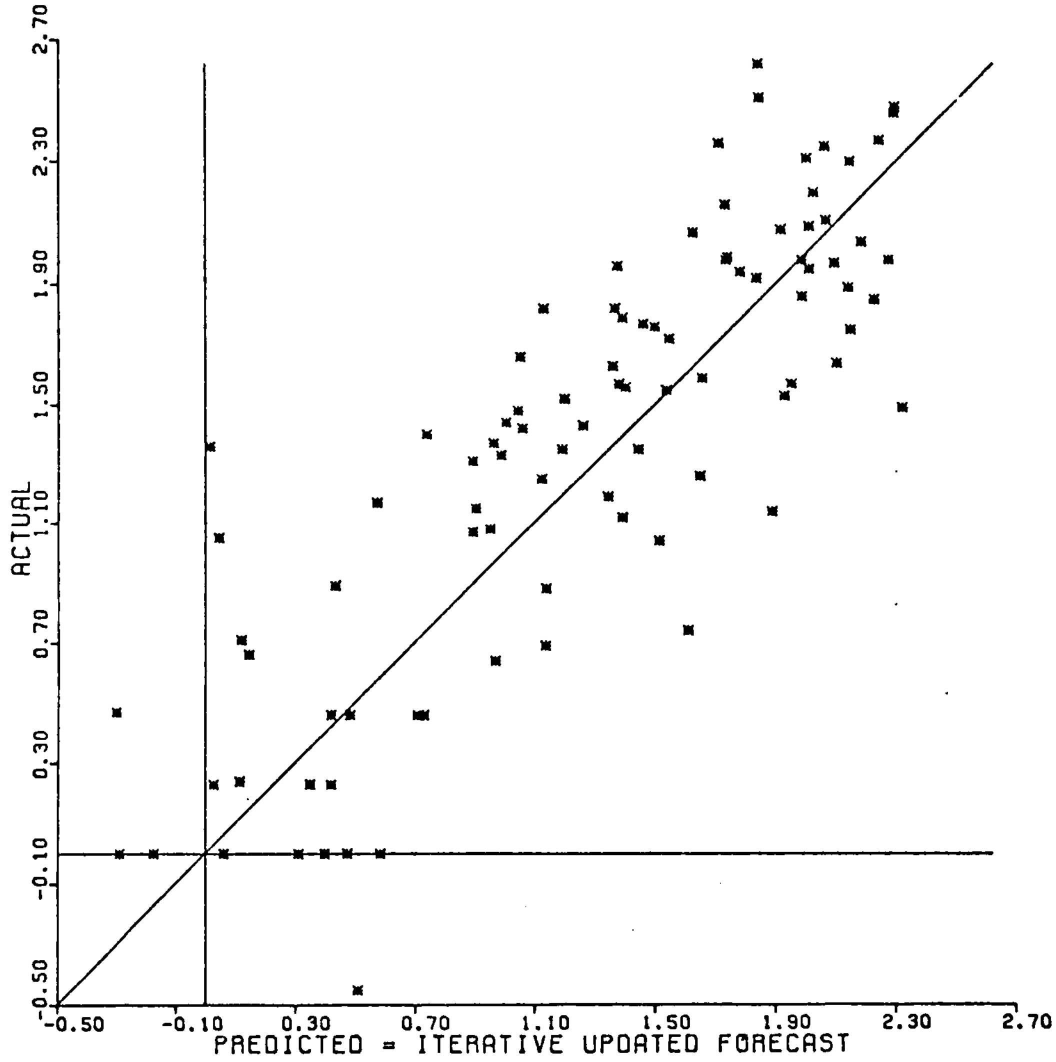


Figure 7.2.4. Prediction/Realization Diagram, MB Growth Rate Model, 1956-3/1979-2.

positive (or zero). Referring to Figure 6.1.7. (page 149), it is seen that these turning point errors occur in 1959/3 and 1961/2.

Figure 7.2.2. plots the actual and forecasted coordinates for the GNP deflator model. Again, it is evident that the coordinates cluster around the line of perfect forecasts, with the predominant number of forecasts producing underestimates of the actual rate of inflation. There is only one turning point error (in 1961/1), where the actual rate of inflation is negative but a positive value is forecasted.

Figure 7.2.3. presents the actual and forecasted coordinates for the M1 growth rate ARIMA model. The diagram forcefully pictorializes the earlier comments made in Sections 6.5. and 7.1. regarding the mildly inferior fit of the M1 model. In particular, the coordinates do not cluster about the 45° line as closely as the previous two inflation models. This fact is reflected in the greater variance of residuals produced by the iterative updating routine for the M1 model (.716) than produced by the CPI and GNP deflator models (.643 and .612, respectively). Also, the diagram shows that the M1 model does not do as good a job in predicting turning points as the two previous models, with seven such errors occurring. However, like the other inflation models, the M1 model does produce more under- than overestimates.

Figure 7.2.4. plots the actual/predicted coordinates for the MB growth rate ARIMA model. The spread of forecasts about the 45° line is considerably less than that of the M1 model, a fact reflected in a smaller variance of residuals (.643). Again, underestimates are greater than overestimates. Five turning point errors are recorded.

Table 7.2.1. summarizes the results of the prediction/realization

grams by separating the forecasts into percent underestimates, overestimates and turning point errors. The results of this decomposition show that underestimates account for a greater proportion of step-ahead forecast errors than overestimates in all models. This fact reflects, to some extent, the limited information vector upon which the iterative forecasts are made. Table 7.2.1. also shows that the money models are not as accurate as the inflation models in predicting turning point errors.[1] This evidence, when combined with the money model statistics appearing in Table 6.5.1. (p. 193) support the general conclusion that the inflation models do a slightly more accurate job in forecasting.

Table 7.2.1. Percent Decomposition of Under-, Over-, Turning Point Estimate Errors for the Inflation and Money Growth Rate ARIMA Models; 1956/3-1979/2.

	Percent Underestimated	Percent Overestimated	Percent Turning Point
CPI	53.3	44.6	2.2
DFT	58.7	40.2	1.1
M1	56.5	35.9	7.6
MB	58.7	35.9	5.4

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[1] In time series analysis it is usually much easier to predict the continuation of a rise or fall, than changes in direction, of the data. To the extent that turning points are predicted, errors for subsequent observations will be reduced. More importantly, it is the magnitude of the error which happens to be produced by a turning point, not the turning point per se, by which the adequacy of an ARIMA model should be judged. Since a BJ model, as an "optimal predictor" is one that minimizes MSFE, turning points are of no special significance. On this point, see Nelson [92 , p. 211].

7.3. Forecast Error and Forecast Revision of the ARIMA Inflation and Money Models: Implications for Rationality

The presence of error in forecasting using the estimated ARIMA models implies that while agents may be forecasting according to rationality precepts, their forecasts are still subject to mistakes. Because of the stringent diagnostic analysis of the time series models' residuals, we concluded that such forecasting error was a reflection of the unpredictable random component of the time series--it was this component that caused rational agents to experience less than perfect accuracy in prediction.

Because of the iterative forecasting procedure used in generating forecasts (and errors), it is instructive here to consider the relationship between forecast error in period t-1, $(z_{t-1} - \hat{z}_{t-1})$, and forecast revision occurring in period t, $(\hat{z}_t - \hat{z}_{t-1})$, given that errors qualify as random noise. In a strict sense, there would seem to be no relationship between such errors and forecast revision, since, as we explained earlier, there is no systematic component of random error upon which an agent might form an opinion about the direction a forecast revision should take. This position, of course, is a reflection of the Muth form of the strict rationality hypothesis and, as such, is time dimensionless. If, however, errors are produced iteratively, as in this study, and we draw a distinction between 1) "ex-post" errors (errors occurring after a forecast has been made) and, 2) "ex-ante" errors (errors which will occur in the future), random error, which is ex-ante "unknowable" becomes part of the ex-post information vector. In an ex-post sense, then, a rational forecasting rule would require that forecast revision and forecast error be related, since the aim of agents is to minimize the sum of squared forecast error. That is, if agents are to act rationally they would be expected to utilize knowledge of ex-post error in revising their forecasts. More specifically, there should be a positive and statistically significant relationship between forecast revision and ex-post forecast error if forecasting is to be considered rational, since agents can minimize long-run forecast error by revising their forecasts in the same direction as past error of which they had become aware.

To test this hypothesis using the inflation and money ARIMA models, consider the simple stock adjustment model;

$$(7.3.1.) \qquad (\hat{z}_t - \hat{z}_{t-1}) = \gamma(z_{t-1} - \hat{z}_{t-1}) \quad ,$$

where $\hat{z}_t$ and $\hat{z}_{t-1}$ are the step-ahead forecasted values in period t and t-1, respectively, and z_{t-1} is the actual value of the variable in period t-1.[1]

The following notation is applicable;

$(\hat{z}_t - \hat{z}_{t-1}) = R_t$ = forecast revision occurring in period t.

$(z_{t-1} - \hat{z}_{t-1}) = E_{t-1}$ = forecast error occurring in period t-1.

In this format, the coefficient γ in (7.3.1.) is to be estimated, and is not given a value a priori. In regression form (7.3.1.) can be written as;

$$(7.3.2.) \qquad R_t = a + bE_{t-1} + e_t \quad ,$$

where e_t is an error term assumed to obey the Classical properties.[2]

Table 7.3.1. presents the results of regression (7.3.2.) for the ARIMA inflation and money models for the iterative forecasting period

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[1] Here it is assumed that the forecast of z is formed one period prior to the subscript as indicated. Hence the forecast of z formed in period t-1 is $\hat{z}_t$. As stated in Section 5.5. this forecast is expected to hold for t.

[2] It would be illogical to estimate $R_t = a + bE_t + e_t$, since errors occurring in time period t do not enter the information vector of the B-J model until time period t+1.

Table 7.3.1. Forecast Revision as a Function of Forecast Error; ARIMA Inflation and Money Model, 1956/3-1979/2.

CPI	$R_t = -.012 + .764E_{t-1}$	$R^2 = .693$	F = 78.83
	(-0.40) (8.87)	$\sigma^2_\varepsilon = .204$	DW = 2.35
DFT	$R_t = -.013 + .391E_{t-1}$	$R^2 = .636$	F = 158.40
	(-1.17) (12.59)	$\sigma^2_\varepsilon = .110$	DW = 1.91
M1	$R_t = -.032 + .943E_{t-1}$	$R^2 = .818$	F = 405.11
	(-1.36) (20.13)	$\sigma^2_\varepsilon = .227$	DW = 1.64
MB	$R_t = -.039 + .593E_{t-1}$	$R^2 = .410$	F = 63.50
	(-1.26) (7.97)	$\sigma^2_\varepsilon = .293$	DW = 2.90

1956/3 through 1979/2. As required for ex-post rationality the slope coefficients are all positive, meaning that forecasts are revised in the same direction as the errors occurring in the previous periods. These results also show the intercept coefficients for all four models are not significantly different from zero--reflecting the fact that a rational agent would not be expected to revise his forecast, if the error committed in period t-1 was zero (assuming that only the past history of the variable enters his information vector). The strength of forecast revision is determined by the magnitude of the slope coefficient. Interestingly, the large slope coefficient for the M1 regression can be related to earlier comments about the slightly inferior fit of the M1 ARIMA model. Specifically, the poorer fit (and thus larger residual variance) of the model means that the time series residuals are more pronounced and thus play a much more prominent role in the step-ahead forecast revision as the iterative routine is executed.

Hence, past errors play a stronger role in forecast revision than in those ARIMA models where forecast errors are not as pronounced.

Considering regression (7.3.2.) again, it can be hypothesized that forecast revision of the ARIMA models is not only related to errors occurring in period t-1, but to previous periods as well. In order to test this hypothesis, the following alterations of (7.3.2.) were estimated;

$$R_t = a + bE_{t-i} + e_t \qquad ,$$

where i = 2, 3, 4, and 5. The four models, however, did not support this hypothesis, with the slope coefficients being very small and statistically insignificant. Likewise, the R-squared statistics for these regressions indicated either a very weak or non-existant relationship between forecast revision and errors occurring further than one period back.

7.4. The Estimated ARIMA Inflation and Money Models; Rationality and Tests for Non-contemporaneous Causality

Chapter VI concluded that the univariate inflation and money models could be classified as rational forecasting mechanisms since the diagnostic checks of model residuals showed that they had been reduced to random noise. It is now necessary to investigate the rationality of the univariate models from another standpoint, one concerned with the possible existence of dependence (causality) between the raw money growth rate series and the ARIMA-produced forecasts of the inflation models. As will be demonstrated, such a causality test is symmetrical in that the possible dependence between the raw inflation series and the ARIMA money model forecasts is also provided.

The need to investigate this dimension of causality stems from the following related considerations. The ability of an inflation or money model to produce rational forecasts will become suspect if it can be shown that the forecasts could have been improved by including the knowledge of another variable into the model (in this case, either the raw money or inflation series), in addition to the own past history of the series. Since a strong theoretical relationship exists between money and prices, a relationship that has strong empirical support, the rationale that forecasting models of inflation should include not only the past rates of inflation, but also the past rates of money growth as arguments in the forecasting mechanism, is theoretically appealing. And, while the theoretical connection between rates of inflation and money growth is not quite as strong as the money-to-prices argument, the same conclusion applies: possibly the money growth rate forecasts can be improved by inclusion of an exogenous variable, knowledge of the inflation rate, into the money forecasting model.

This issue of causality has specific bearing on the degree of rationality the inflation and money univariate models' possess; if a statistically significant contemporaneous or non-contemporaneous dependence between the forecasted money or inflation series and the exogenous raw inflation or money series is found to exist, the general presumption that the estimated ARIMA models comply with rationality criteria will come under question, even though the individual univariate models passed the diagnostic checks for model adequacy outlined earlier. Hence, the question to which this section is addressed is: Will the forecasting accuracy of the inflation models be improved by supplementing them with knowledge of monetary growth rate history?[1] The tools of analysis will be based upon causality tests

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[1] The converse of this question, that knowledge of inflation may improve ARIMA money forecasts, will simultaneously be addressed also.

geared to establishing any lead-lag correlations which may exist between the univariate money and inflation model residuals. These correlation tests will serve an additional purpose: since the money and inflation model residuals will be used as regressors in some of the output regression models, the lead-lag correlations will show whether or not the money and inflation regressors can be considered orthogonal.

The issue of inflation forecasting accuracy and exogenous money series knowledge has been researched by Feige and Pearce, as alluded to earlier [Feige and Pearce, (90),p.91]. They reason that since a properly specified ARIMA inflation model efficiently utilizes all information contained in the raw inflation series, a necessary condition for a contemporaneous money series to provide additional information relevant to future inflation forecasts is that it reduce the expected mean square forecast error below that of the univariate inflation model. Their conclusion is that a bivariate inflation and money model (a transfer function) will not yield more accurate forecasts of future inflation than a univariate inflation model alone. In Section 5.4. of this study, the Feige-Pearce conclusion was used to justify the assertion that inflation and money forecasts constructed from the past history of the variable alone is enough to satisfy rationality conditions, even though potentially useful exogenous money knowledge is excluded from the information vector. While it would be expedient to assume the Feige-Pearce findings hold for the ARIMA models estimated in this study, such an assumption could questioned on two fronts: 1) this study is concerned with a longer period of time, one characterized by much more severe inflation and money growth rates than in the Feige-Pearce study, and, 2) Feige and Pearce do not use the iterative forecasting routine utilized here. Thus the Feige-Pearce finding of no statistically significant cross-correlation between their ARIMA inflation and money model residuals cannot be assumed to hold for the residual patterns produced here. For these reasons, causality tests specific to this study are undertaken and will use residual data from the iterative forecast period 1956/3 through 1979/2.

7.4.1. Causality and Causality Tests. The advances in time series analysis provided by Box and Jenkins and others have created much interest and controversy about the general topic of bivariate time series dependency or "causality," and a voluminous econometric and statistical literature on the topic and on a "correct" definition of causality has evolved.[1] However, it is not within the scope of this study to subject the time series models developed earlier to the many different statistical tests for causality, nor to engage in a philosophical argument over appropriate definitions of causality. Rather, since the concern of this section is whether one economic variable (the rate of growth of the nominal money stock) can be used to improve ARIMA forecasts of another economic variable (the rate of inflation), the study and definition of causality will be limited to the work of Granger [96], Sims [97], Pierce [98], and Haugh [99]. The tenor of the research of these authors is relevant here because it deals exclusively with testing for the causal relationships between economic time series. The general title,"'Granger-Sims' tests for causality," is usually used to classify this particular category of causality analysis.

The Granger-Sims definition of causality states that X "causes" Y if present and past values of X can be used to obtain better forecasts of future

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[1] See Pierce and Haugh [100] for an excellent survey of the causality literature in temporal systems.

Y than could be obtained by only using present and past values of Y alone. A decision as to whether a univariate or bivariate model produces more accurate (i.e., more rational) forecasts can then be made by comparing the mean-squared-forecast error of the two models.

A more formal statement of the Granger-Sims definition of causality may be summarized as follows: Let $[A_t, t = 0, 1, 2, . . .]$ be the given information vector which is of potential use in forecasting future values of variable Y. This set contains at least $[(X_t, Y_t)]$, the time series of the X and Y variables, respectively. Let $\bar{A}_t = (A_s | s < t)$, and $\hat{A}_t = (A_s | s \leq t)$. Now define $\bar{X}_t$, $\bar{Y}_t$, $\hat{X}_t$, and $\hat{Y}_t$ in a similar manner. Finally, let $\sigma^2(Y|B)$ be the minimum MSFE of $F_t(Y|B)$, the step-ahead forecast of Y_t, given information vector B. The Granger-Sims definition of causality then is:

1) X "causes" Y if: $\sigma^2(Y|\bar{A}) < \sigma^2(Y|\bar{A} - \bar{X})$

2) Y "causes" X if: $\sigma^2(Y|A) > \sigma^2(Y|\bar{A} - \bar{X})$

3) X "causes" Y instantaneously if:

$$\sigma^2(Y|\bar{A}, X) < \sigma^2(Y|\bar{A}) \quad .$$

If both X causes Y and Y causes X, there is "feedback" between the two series. Note that definitions 1) and 2) are concerned with non-contemporaneous causality. Also note that 1) and 2) are neither necessary nor sufficient conditions for 3).

In terms of the above definitions, two points are noteworthy: 1) the Granger-Sims approach implies it is the improvement in forecasting accuracy (i.e., a smaller variance of forecasts about the raw time series) which is the "pay-off" from the discovery and incorporation of money growth patterns into inflation forecasts, and, 2) this method of causality analysis provides a basis for testing an assertion made earlier: knowledge of selected monetary time series should be included in the forecasting models only if the added benefits from more accurate inflation forecasts outweigh the additional costs associated with collecting the monetary information.

7.4.2. Causality and a Simultaneous Bivariate Money Growth/Inflation Rate Model. The Granger-Sims definition of causality, cited above, is quite general and, therefore, is not directly amenable to a statistical investigation of whether or not money growth rates "cause" inflation and vise versa. What is required here is the construction of a simultaneous system in which the money and inflation time series are potentially non-contemporaneously related and where the presence of "feedback" can be detected if it exists. Tests for the acceptance or rejection of causality can then be undertaken indirectly by testing for bivariate money and inflation time series simultaneity.

The following notation will be used in this subsection;

P_t = actual rate of inflation in time period t.

M_t = actual rate of money growth in time period t.

If there exists a one-way causal relationship between M and P, i.e., no feedback or reverse causality, we may describe the relationship in terms of a dynamic regression model of the form;

$$P_t = V(B)M_t + N_t \quad , \tag{7.4.2.1.}$$

where V(B) is an infinite polynomial in the backshift operator and N_t is a disturbance term (not necessarily random). Note that, except for the exclusion of reverse causation, model (7.4.2.1.) is in conformity with the theoretical and empirical literature in monetary economics which posits a transmission mechanism running from money to prices. Model (7.4.2.1.) is also known as the "transfer function" relating the input series, M_t, to the output series, P_t [see BJ, ibid., Chapter 11].[1]

Following Box and Jenkins, (7.4.2.1.) can be written as;

$$P_t = \frac{\omega(B)}{\delta(B)} M_t + \frac{\theta(B)}{\psi(B)} a_t \quad , \tag{7.4.2.2.}$$

where a_t is a white noise disturbance and $\omega(B)$ and $\delta(B)$ are finite polynomials of degree s and r, respectively. Rearranging, we have;

$$\psi(B)\delta(B)P_t = \psi(B)\omega(B)M_t + \delta(B)\theta(B)a_t \quad , \tag{7.4.2.3.}$$

Combining the products of the respective polynomials yields;

$$\gamma(B)P_t = \alpha(B)M_t + \beta(B)a_t \quad . \tag{7.4.2.4.}$$

The forecasting formula for an arbitrary lead time ℓ, is given by;

$$\begin{aligned} P_t(\ell) = {} & \gamma_1[P_{t+\ell-1}] + \ldots + \gamma_{p+r}[P_{t+\ell-p-r}] + \\ & \alpha_0[M_{t+\ell}] - \ldots - \alpha_{p+s}[M_{t+\ell-p-s}] - \\ & \beta_1[a_{t+\ell-1}] - \ldots - \beta_{q+r}[a_{t+\ell-q-r}] \quad . \end{aligned} \tag{7.4.2.5.}$$

It is seen that forecasts obtained from the transfer function model use information contained in the time series histories of both P and M.[2]

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[1] It should be noted that in order to correctly identify and estimate a transfer function model, it is necessary that the direction of causation, in a time-ordering sense, be one-way only. That is, the transfer function is only appropriate if current M is associated with future P and current P is not associated with future M. If this latter condition does not hold, the two series exhibit a "feedback" relationship.

[2] Note that in the case where the M_t series does not help in predicting P_t, the coefficients $\alpha_0, \ldots, \alpha_{r+s} = 0$. In this case the "best" forecast of P_t can be provided by just the past history of P alone, i.e., a univariate model of inflation is adequate.

Eqs. (7.4.2.4.) and (7.4.2.5.) can be placed into autoregressive form;

(7.4.2.6.) $$\frac{\gamma(B)}{\beta(B)} P_t - \frac{\alpha(B)}{\beta(B)} M_t = a_t \quad ,$$

or;

(7.4.2.7.) $$V_{PP}(B)P_t - V_{PM}(B)M_t = a_t \quad ,$$

where $V_{PP}(B)$ and $V_{PM}(B)$ are the lag operators hitting the P and M series, respectively.

In the same spirit as (7.4.2.1.), it may be plausible to posit a one-way relationship between the rate of money growth and the rate of inflation. In this case the following transfer function relating M to its past history and the past history of P may be written in the following manner;

(7.4.2.8.) $$M_t = Z(B)P_t + H_t \quad ,$$

where H_t is an autocorrelated error term. Model (7.4.2.8.) can be redefined as;

$$M_t = \frac{\omega^*(B)}{\delta^*(B)} P_t + \frac{\theta^*(B)}{\psi^*(B)} b_t \quad ,$$

$$\psi^*(B)\delta^*(B)M_t = \psi^*(B)\omega^*(B)P_t + \delta^*(B)\theta^*(B)b_t \quad ,$$

$$\lambda(B)M_t = \nu(B)P_t + \xi(B)b_t \quad ,$$

$$\frac{\lambda(B)}{\xi(B)} M_t - \frac{\nu(B)}{\xi(B)} P_t = b_t \quad ,$$

and rearranging terms gives the following autoregressive form;

(7.4.2.9.) $$V_{MM}(B)M_t - V_{MP}(B)P_t = b_t \quad .$$

Committing functions (7.4.2.7.) and (7.4.2.9.) to matrix notation and ignoring signs gives;

(7.4.2.10.)
$$\begin{bmatrix} V_{PP}(B) & V_{PM}(B) \\ V_{MP}(B) & V_{MM}(B) \end{bmatrix} \begin{bmatrix} P_t \\ M_t \end{bmatrix} \begin{matrix} = \\ = \end{matrix} \begin{bmatrix} a_t \\ b_t \end{bmatrix}$$

or;

(7.4.2.11.)
$$\begin{bmatrix} P_t \\ M_t \end{bmatrix} \begin{matrix} = \\ = \end{matrix} \begin{bmatrix} V_{PP}(B) & V_{PM}(B) \\ V_{MP}(B) & V_{MM}(B) \end{bmatrix}^{-1} \begin{bmatrix} a_t \\ b_t \end{bmatrix}$$

where a_t and b_t are random, serially independent disturbances.[1] System (7.4.2.11.) is a simltaneous model composed of moving-average terms in which the endogenous variables, P and M, are <u>potentially cross-related via their disturbances</u>.

System (7.4.2.11.) provides a convenient taxonomy of possible forms of unidirectional or bi-directional causality which may relate M to P and P to M. Specifically, four different classification possibilities may be identified:

1) If $V_{MP}(B) = 0$, while $V_{PM}(B) \neq 0$, a transfer function relating P_t to M_{t-i} $(i = 0, . . ., n)$ is appropriate.
2) If $V_{MP}(B) \neq 0$, while $V_{PM}(B) = 0$, a transfer function relating M_t to P_{t-i} $(i = 0, . . ., n)$ is appropriate.
3) If both $V_{MP}(B) \neq 0$, and $V_{PM}(B) \neq 0$, then feedback between the two series is indicated.
4) If both $V_{MP}(B) = 0$, and $V_{PM}(B) = 0$, then the two series are not related.

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[1] It is assumed that the two series M and P are jointly covariance stationary, and the matrix in (7.4.2.11.) has a non-zero determinant.

It is important to note that if classifications 1) or 2) should hold, then the univariate inflation and money models constructed earlier can not be considered "optimal" forecasting mechanisms in the rational expectations sense of the word. This reasoning follows from the fact that transfer functions, by construction, relate a variable to its own past, in addition to the past history of another variable [as seen in (7.4.2.5.)], while univariate time series models are built upon the series' past history, alone.[1] Also, if classification 3) should hold, the univariate models, since they posit a unidirectional form of causation, would not qualify as optimal forecasting models. In this case, money growth rates would "cause" inflation and inflation would "cause" money growth rates and, therefore, a simultaneous equation model would be indicated. Hence, only if classification 4) holds can the ARIMA inflation and money models be considered optimal forecasting mechanisms. That is, inflation can be adequately explained by its past history, without regard to past money history, and vise versa.

7.4.3. The Haugh Test for Residual Causality. In an applied econometric sense, a determination of the nature and direction of causal possibilities outlined in system (7.4.2.11.) would be based upon the statistical significance of the estimated coefficients of the V..(B) lag operators. Assuming, however, that these lags are probably quite lengthy and complex, a practical solution to the system would be difficult or impossible because of 1) the number of coefficients to be estimated, and, 2) the system may not be properly identified. However, if (7.4.2.11.) is to provide clues about causality,

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[1] Note that classification 2) is similar to the "reverse causation" phenomenon as applied to the standard quantity theory; that is, prices "cause" money.

some method of assessing the nature of the lag operators must be specified. Such a procedure has been developed by Haugh [99] and Haugh and Box [101], and is based upon a test for the existence and direction of non-contemporaneous causality between two covariance stationary time series.

The rationale of the Haugh test as applied to the money-inflation nexus is as follows. Since theory indicates that inflation and money growth rate series may be non-contemporaneously correlated, and, given the Granger-Sims definition of causality which has been adopted here, it would seem logical to examine the cross-correlation function (CCF) of the two series to see in which direction causation appeared to go.[1] For instance, if a particular cross-correlation, $r_{XY}(k)$, had a statistically significant non-zero value for X_t correlated with future Y_t at lead k, but not the reverse, one might deduce that causation ran from X_t (money growth rates) to Y_t (the inflation rate). However, it has been shown that in many cases concerning economic time series, the CCF based upon two raw series is misleading because of the serial correlation which is usually present in both series.[2]

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[1] The CCF for both lead and lag values of two series X and Y is defined as:

$$r_{XY}(k) = \begin{cases} n^{-1} \sum_{1}^{n-k} (X_t - \bar{X})(Y_{t+k} - \bar{Y})/S_X S_Y & k = 0, 1, 2, \ldots \\ n^{-1} \sum_{1}^{n+k} (Y_t - \bar{Y})(X_{t-k} - \bar{X})/S_X S_Y & k = 0, -1, -2, \ldots \end{cases}$$

where k = length of the lag or lead, S_v = the standard deviation of X, S_Y = the standard deviation of Y, and n = the number of observations.

[2] This cross-correlation phenomenon associated with two time series is similar to the "spurious regression" problem discussed in Granger and Newbold [102]. The problem, however, was first isolated and analyzed by Barlett [103].

Haugh has proposed a two-stage test which eliminates this problem. First, univariate models are identified and estimated for the two series. The residuals from these filters are the series "purged" of any serial correlation. Second, the two pre-whitened residual series are cross-correlated and the resulting CCF can then be analyzed to determine if causation is present and what directional possibilities may plausibly be entertained.[1] Intuitively, what the Haugh technique examines is whether a current shock (error) to the X_t series (the money growth rate) is transmitted to future Y_t (the inflation rate) by influencing the shocks driving the Y_t series. Earlier it was noted that the shock series driving the inflation rate was also the series of step-ahead forecast errors; that is, the part of the inflation rate not predictable from information contained in the past history of the variable itself. If the shock series driving the growth rate of the money stock is correlated with the inflation forecast errors, knowledge of the money series should enable a rational agent to improve on the forecasts of future inflation. Thus the Haugh test provides a method for determining whether knowledge of the history of a money series can improve the forecasting accuracy of a univariate inflation filter (and vise versa) by analyzing ARIMA residual cross-correlation.

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[1] The theoretical cross-correlation between two pre-whitened series, a_t, and b_t, for lead time k, or lag time -k, is given by the statistic:

$$\rho_{ba}(k) = \frac{\nu_{ba}(k)}{[\sigma_b^2 \sigma_a^2]^{\frac{1}{2}}}$$

where $\nu_{ba}(k)$ is the cross covariance between the series a_t and b_t for lead (lag) time k.

Haugh has shown that even though two prefilter series, Y_t and X_t, may be causally related in some fashion, they can still be expressed as univariate linear processes. Consider the following prefiltered series;

(7.4.3.1.)
$$\pi_1(B)Y_t = a_t \quad ,$$
$$\pi_2(B)X_t = b_t \quad .$$

Allowing for the possibility that the two series in (7.4.3.1.) may be co-determined allows us to place the two univariate filters in matrix form, similar to (7.4.2.10.);

(7.4.3.2.)
$$\Pi(B)\begin{bmatrix} Y_t \\ X_t \end{bmatrix} = \begin{bmatrix} A(B) & H(B) \\ C(B) & D(B) \end{bmatrix}\begin{bmatrix} Y_t \\ X_t \end{bmatrix} = \begin{bmatrix} a_t \\ b_t \end{bmatrix}$$

However, if the two series are not related, (7.4.3.1.) can be placed in an autoregressive matrix form, as follows;

(7.4.3.3.)
$$\begin{bmatrix} \pi_1(B) & 0 \\ 0 & \pi_2(B) \end{bmatrix}\begin{bmatrix} Y_t \\ X_t \end{bmatrix} = \begin{bmatrix} a_t \\ b_t \end{bmatrix}$$

Systems (7.4.3.2.) and (7.4.3.3.) now provide the basis of Haugh's statistical test to determine if, in fact, the off-diagonal elements are zero; if this is the case, then unidirectional causation is implied and the series can be correctly expressed as univariate linear processes. The null hypothesis, in this case, would be [expressed in terms of (7.4.3.2.)];

(7.4.3.4.)
$$\text{Ho:} \quad A(B) \neq 0, \; D(B) \neq 0, \; C(B) = H(B) = 0 \quad .$$

The Haugh residual cross-correlation test provides a method of testing the null hypothesis (7.4.3.4.), and is based on the fact that a joint model of univariate residuals can be derived from (7.4.3.3.) which is of the following form;[1]

$$\Pi(B)\begin{bmatrix} u_t \\ v_t \end{bmatrix} = \begin{bmatrix} \alpha(B) & \beta(B) \\ \delta(B) & \zeta(B) \end{bmatrix}\begin{bmatrix} u_t \\ v_t \end{bmatrix} = \begin{bmatrix} a_t \\ b_t \end{bmatrix} \tag{7.4.3.5.}$$

where α, β, δ, and ζ are of the same form as the operators in A, H, C, and D in (7.4.3.2.). It is also seen that the various operators in (7.4.3.2.), (7.4.3.3.) and (7.4.3.5.) have the following relationship;

$$\begin{bmatrix} A(B) & H(B) \\ C(B) & D(B) \end{bmatrix} = \begin{bmatrix} \alpha(B) & \beta(B) \\ \delta(B) & \zeta(B) \end{bmatrix}\begin{bmatrix} \pi_1(B) & 0 \\ 0 & \pi_2(B) \end{bmatrix} . \tag{7.4.3.6.}$$

We would expect that an analysis of (7.4.3.5.) to yield information on any possible causality patterns concerning the series X_t and Y_t, as u_t and v_t are components of the X and Y series that cannot be predicted from their own pasts. This fact implies that (7.4.3.5.) is a causality-preserving transformation of (7.4.3.2.), and, therefore, any lead or lag cross-correlations of the residual series a_t and b_t should reveal if the lag polynomials H(B) and C(B) in (7.4.3.2.) are statistically different from zero, and whether or not univariate models are appropriate. In terms of the ARIMA inflation and

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[1] The derivation of the relationship between the residuals of the Y and X autoregressive form in (7.4.3.1.) and (7.4.3.2.) and (7.4.3.3.) and (7.4.3.5.) is given in Haugh [99] and Haugh and Pierce [104].

money growth forecasting functions, the estimated residuals derived from (7.4.3.1.), $\hat{a}_t$ and $\hat{b}_t$, can be viewed as that part of the inflation or money series which is not captured by the respective models. If these residual series display any statistically significant lead or lag correlation, then the forecasts of both univariate models are interdependent and the univariate forecasting functions are not optimal.

Haugh and Pierce [Haugh and Pierce, ibid., p. 17] have classified the principle cross-correlation possibilities which may exist betwen two series, as determined by the values of the theoretical pre-whitened cross-correlation coefficients, ρ_{ba}. Table 7.4.3.1. presents the five possible causality events of interest here.

Table 7.4.3.1. Causality Patterns Indicated by the Theoretical Cross-Correlations of X and Y.

1.	X causes Y	$\rho_{ba}(k) \neq 0$ for some k 0.
2.	Y causes X	$\rho_{ba}(k) \neq 0$ for some k 0.
3.	Instantaneous causality	$\rho_{ba}(k) \neq 0$.
4.	Feedback	$\rho_{ba}(k) \neq 0$ for some k 0 and for some k 0.
5.	X and Y are independent	$\rho_{ba}(k) = 0$ for all k.

In practice, the true parameters of the linear filters in (7.4.3.1.) are not known, but must be estimated. Thus the cross-correlations of the residuals from the estimated models must be used as estimates of the true cross-correlations shown in Table 7.4.3.1. These cross-correlations are, therefore, subject to sampling error, and statistical tests based upon the residual cross-correlations must be used to determine the statistical significance of the estimated cross-correlations.[1] Haugh has provided two tests. One test of causality between Y and X consists of comparing individual cross-correlation estimates, $r_{ba}(k)$, with their standard error. Individual $r_{ba}(k)$ can be considered significant if they are at least twice the value of $[\sqrt{n}]^{-1}$, where n is the sample size.[2] Haugh has shown that the set of residual cross-correlations are asymptotically normally distributed and independent with zero mean and standard deviation of $[\sqrt{n}]^{-1}$. This distribution is valid under the null hypothesis of independence between the Y and X series.

A second test proposed by Haugh for determining the existence of causality between the residual series is similar to the Box-Pierce-Q-statistic in that it assesses the overall significance of a group of cross-correlation estimates at both negative and positive lags. This statistic is;

$$S_M^*(\pm) = n^2 \sum_{k=-M}^{M} (n - |k|)^{-1} r_{ba}^2(k) \quad , \tag{7.4.3.7.}$$

where n = the number of observations on each pre-whitened series, M = the number of cross-correlations estimated for either negative or positive lags, and $r_{ba}(k)$ = square of the estimated cross-correlation coefficient at lag ±k for the pre-whitened b_t and a_t series. On the null hypothesis of no causality,

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[1] The residual cross-correlations, as estimates of ρ_{ba}, are given by the statistic;

$$r_{ba}(k) = \Sigma\hat{b}_{t-k}\hat{a}_t/[\Sigma\hat{b}_t^2\ \Sigma\hat{a}_t^2]^{\frac{1}{2}}$$

[2] Derivation of the statistic $[\sqrt{n}]^{-1}$ is provided by Box and Jenkins [BJ, <u>ibid</u>., p. 382].

Haugh has shown that the statistic S_M^* is asymptotically distributed as $\chi^2(2M+1)$. The hypothesis of independence between the two series Y and X would thus be rejected if the calculated S_M^* exceeds the critical χ^2 value at a given level of significance. If the null hypothesis of no correlation between the residual series is rejected, then construction of a univariate model is inappropriate in terms of rationality criteria.

Since the direction of causality at k = 0 is indeterminate, Haugh considers an analog test of (7.4.3.7.) for either non-positive or non-negative lags only. The case where;

$$(7.4.3.8.) \qquad S_M^*(+) = n^2 \sum_{k=1}^{M} (n - |k|)^{-1} r_{ba}^2(k) > \chi_\alpha^2(M) \quad ,$$

at a given α level would indicate that X causes Y (i.e., Y leads X). Alternatively, the case where;

$$(7.4.3.9.) \qquad S_M^*(-) = n^2 \sum_{k=-1}^{-M} (n - |k|)^{-1} r_{ba}^2(k) > \chi_\alpha^2(M) \quad ,$$

would indicate that Y causes X (i.e., Y lags X). If both (7.4.3.8.) and (7.4.3.9.) are greater than the selected critical χ^2 value, feedback would be indicated.

7.4.4. The Inflation and Money Filters and the Haugh Test for Residual Causality. Following the Haugh procedure, the residuals from the estimated univariate inflation and money models were subjected to the tests for causality. These results appear in Table 7.4.4.1. for k = 12 quarters. Note that in keeping with received monetary doctrine, the money model residuals (unanticipated money growth rate) are treated as the input or "innovation" variable, while the inflation model residuals (unanticipated inflation) are

Table 7.4.4.1. Unanticipated Money Growth/Unanticipated Inflation: Haugh Residual Cross-correlation Tests for Causality; 1956/3-1979/2.

Unanticipated Monetary "Innovation" Variable	Unanticipated Inflation "Response" Variable					
	CPI Residuals			DFT Residuals		
	(1) $S^*_{\pm 12}$	(2) S^*_{+12}	(3) S^*_{-12}	(4) $S^*_{\pm 12}$	(5) S^*_{+12}	(6) S^*_{-12}
Unanticipated Monetary "Innovation" Variable						
M1 Residuals	29.0[37.7]	6.9[21.0]	15.2[21.0]	22.7[37.7]	10.4[21.0]	9.8[21.0]
MB Residuals	19.9[37.7]	4.6[21.0]	17.8[21.0]	23.5[37.7]	12.3[21.0]	11.0[21.0]

Notes: The degrees of freedom used in obtaining the χ^2 critical value for $S^*_{\pm 12}$, S^*_{+12}, and S^*_{-12}, are 25, 12, and 12, respectively. The critical χ^2 values appear in brackets following the S^* statistics and are quoted at the .05 level.

treated as the output or "response" variable. Of course, the statistical procedure of lead-lag correlation involved in computing statistic (7.4.3.7.) (as seen in columns 1 and 4 of Table 7.4.4.1.) forces each residual series to assume first a lead and then a lag relationship with the other series. The statistics appearing in columns 2 and 3 of Table 7.4.4.1. provide independent tests for money "causing" (leading) CPI inflation, and CPI inflation "causing" (leading) money growth rates, respectively. Columns 5 and 6 of the Table provide the same one-way tests for the GNP deflator.

As the results of Table 7.4.4.1. clearly indicate, the null hypothesis of no causality (in either a lead or lag fashion) between the money and inflation residuals cannot be rejected, as all the S_M^* statistics are well within the critical χ^2 values at the .95 level. Also, these results indicate that the construction of a transfer function relating changes in money growth rates to future movements in the rate of inflation (and visa versa) would be inappropriate since, in no case, is there evidence of residual causality. Hence, system (7.4.3.3.) with the zero off-diagonals represents the proper matrix interpretation of the raw inflation and money series.

Table 7.4.4.2. provides an analysis of individual residual cross-correlations at both positive and negative lags (lag zero is omitted since it has no bearing on non-contemporaneous causality). Examination of these statistics reveals only a few above the critical value of $2\sigma = .203$. And, while there is a very mild pattern of cross-correlations at positive and negative lags 3 and 4 for the CPI and DFT residuals, clearly this pattern is not strong enough to suggest causality running from unanticipated M1 growth to unanticipated inflation.

Table 7.4.4.2. Estimated Cross-correlations for Money and Inflation Residual Series; 1956/3-1979/2.

Positive Lags*

Output Series	Input Series	1	2	3	4	5	6	7	8	9	10	11	12
CPI	M1	.01	-.01	.14	.14	.07	-.07	.09	.06	.02	.06	-.06	-.01
	MB	.08	.05	.07	-.06	.14	-.03	.03	-.04	.02	.05	-.01	.03
DFT	M1	-.04	-.01	.22	.09	-.05	-.05	.08	.07	.08	.04	.12	-.05
	MB	.02	.21	.07	.05	-.15	-.09	.02	.11	.14	-.02	.02	-.02

Negative Lags**

Output Series	Input Series	1	2	3	4	5	6	7	8	9	10	11	12
CPI	M1	-.06	-.14	.14	.10	.02	.07	.15	.08	.19	.14	.01	.08
	MB	-.03	-.20	-.16	-.15	.05	.09	.09	-.09	.04	-.01	.01	-.01
DFT	M1	-.05	-.03	-.12	.12	.04	-.07	.06	.06	-.07	.20	.04	.01
	MB	-.05	-.11	-.19	-.03	.01	-.03	.03	.08	-.15	.08	-.09	.10

Notes: --The null hypothesis is: Ho: $r_{ba}(\pm k) = 0$. Individual cross-correlations are significant if they they are greater than $2\sigma = [\sqrt{92}]^{-1} = .203$. "*" means inflation residuals lagging money residuals. "**" means money residuals lagging inflation residuals.

7.4.5. Conclusions. The results of this section firmly support the conclusion that knowledge of the input series (either money growth rates or inflation rates) will not improve the ARIMA forecasts of the output series (either the inflation rate or money growth rate). Therefore, for the 1956/3 through 1979/2 time period, the estimated ARIMA money and inflation models' iteratively updated forecasting accuracy would not have been improved by inclusion of an exogenous money or inflation variable in the information vector containing just the past history of the time series itself. These findings support those of the Feige-Pearce study cited in Section 5.4. regarding the "rationality" of ARIMA models: "the relevant information set for a rational forecaster is the past history of the variable, and forecasts from properly specified univariate models can be considered 'economically rational' " [Feige and Pearce, *ibid*., p. 519].

These findings should not be interpreted to mean that a change in money growth rates does not affect the rate of inflation, but rather that knowledge of monetary changes is not helpful in forecasting inflation once an efficient ARIMA model has taken into account the inflation information contained in past price history. This same point is applicable to the ARIMA money models. Likewise, the fact that the residuals are not significantly cross-correlated should not lead to a rejection of the hypothesis that unanticipated monetary growth does not lead unanticipated inflation. Rather, the proper interpretation of the absence of residual corrolation found here supports the conclusion that the ARIMA inflation filters are successfully capturing any monetary shocks which could potentially lead unanticipated inflation. This means that the ARIMA inflation forecast errors cannot be attributed to any systematic shocks in the monetary growth rate.

In Chapter III the general forms of the inflation and money models were constructed so that the unanticipated variables could be looked upon as exogenously determined, i.e., unanticipated inflation and unanticipated money growth were used "as though" they were contemporaneously and non-contemporaneously uncorrelated. The justification for this novel approach is now clear: since the money and inflation residuals have been shown to be orthogonal, they can rightfully be looked upon as providing exogenous shocks to output.

CHAPTER VIII

REAL OUTPUT RESPONSE AS A FUNCTION OF UNANTICIPATED INFLATION AND MONEY GROWTH RATE VARIABLES: DISTRIBUTED LAG REGRESSION MODELS; ESTIMATION AND ECONOMIC ANALYSIS

8.1. Introductory Remarks and Outline of the Chapter

Chapter VIII presents, analyzes, and compares the main findings of this study. The chief purpose here is to statistically quantify the cause-effect relationships which exist between the (previously) generated unanticipated inflation/money growth rate time series variables and two selected measures of real output response in the U.S. economy over the full period 1956/3-1979/2, and for the subperiods 1956/3-1967/4, and 1968/1-1979/2. The two statistical findings of obvious interest here are 1) a determination of the proper lag length of the distributed lag regression models, and, 2) a determination of the magnitude and pattern of the lag weights that relate past inflation/money growth rate rational forecast error to contemporaneous real output response. Also of interest is an analysis of the implicit transmission mechanisms connecting unanticipated inflation and nominal money growth to output.

The following topics comprise the chapter. Section 8.2. provides a numerical listing of the inflation and money regressions to be estimated. The notation used throughout the chapter is also stated. Derivation of the detrended output variables is described in Section 8.3. Section 8.4. explains, in depth, the methodology used in constructing the distributed lag regression models. Of importance here is the description of the quasi-search procedure used in determining lag length and coefficient weighting pattern. As this section explains also, the Almon method is used only after some idea of lag length and polynomial degree is gleaned from the OLS and GLS regressions. Thus the lag and polynomial restrictions used in the Almon models are not arbitrary, a feature which is distinctly appealing from an econometric point of view. Section 8.5. presents the OLS regression results and analyzes the nature of the residual autocorrelation. Section 8.6. discusses possible approaches to deal with the autocorrelation problem. Section 8.7. will present the GLS regressions; the method of first differences and the Cochrane-Orcutt iterative error-correction method will be compared. This section will also present the Almon regression results along with subperiod regression estimates. Using the results from the Almon/GLS regressions, Section 8.8. will present elasticity and other comparative measures of output responsiveness. Section 8.9. will give a comprehensive economic analysis of the Almon distributed lag regression results for both the full and subperiod time spans. This section will also compare and contrast the competing transmission mechanisms by which the observed differences in the inflation and money regression results might be reconciled. It should be noted that only statistical analysis will be presented in Sections 8.2. through 8.8.; economic analysis and interpretation of the results will be reserved for Section 8.9. Section 8.10. will close the chapter and the dissertation with a brief summary of the thesis and findings.

8.2. Regression Models to be Estimated: A Numerical Listing and Related Notation

The following notation will be used throughout this chapter. The integer m indicates maximum lag length (to be estimated).

C_{t-i} = magnitude of unanticipated (CPI) inflation rate occurring in period t-i, where i = 0, . . ., m.

D_{t-i} = magnitude of unanticipated (GNP deflator) inflation rate occurring in period t-i, where i = 0, . . ., m.

M_{t-i} = magnitude of unanticipated (M1) money growth rate occurring in period t-i, where i = 0, . . ., m.

B_{t-i} = magnitude of unanticipated monetary base growth rate occurring in period t-i, where i = 0, . . ., m.

G_t = deviation of real GNP (base year, 1967) from log-linear trend occurring in period t.

E_t = deviation of the employment rate from log-linear trend occurring in period t.

The general forms of the distributed lag output regressions were given in Sections 3.4. and 3.5. of Chapter III. Since there are two inflation and two money regressions for each of the two output variables, there are a total of eight distributed lag equations to be estimated. Since during the course of this chapter individual equations will be discussed and compared, it is convenient to number the models for reference;

$$G_t = \Sigma\alpha_i C_{t-i} + \varepsilon_t \quad (8.2.1.)$$

$$G_t = \Sigma\alpha_i D_{t-i} + \varepsilon_t \quad (8.2.2.)$$

$$G_t = \Sigma\alpha_i M_{t-i} + \varepsilon_t \quad (8.2.3.)$$

$$G_t = \Sigma\alpha_i B_{t-i} + \varepsilon_t \quad (8.2.4.)$$

$$E_t = \Sigma\alpha_i C_{t-i} + \varepsilon_t \quad (8.2.5.)$$

$$E_t = \Sigma\alpha_i D_{t-i} + \varepsilon_t \quad (8.2.6.)$$

(8.2.7.) $$E_t = \Sigma\beta_i M_{t-i} + \varepsilon_t$$

(8.2.8.) $$E_t = \Sigma\beta_i B_{t-i} + \varepsilon_t$$

For the above regressions, the "i" subscript for the coefficients α, and β refers to the particular weight assigned the unanticipated variable in period t-i.[1]

8.3. Measures of Real Output Response: Construction of the Dependent Variables[2]

The measures of real output response used in this study are: 1) real GNP, and, 2) the percent of the labor force employed.[3] Real GNP is obtained by deflating nominal GNP by the implicit Price Deflator (base year, 1967). The employment rate is determined by constructing the ratio of total employment to the total population of working age (18 through 65 years of age). These employment figures primarily reflect the degree of economic activity in the industrial sector since they exclude military and agricultural employment.[4]

The two indices of real output response are obtained by regressing the natural log of real GNP and the employment rate on a time trend and a constant. These regressions appear in Table 8.3.1. The residuals from

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[1] These regression functional forms are assumed to be linear in the independent variables.

[2] Two computer packages are used in regression estimation here: 1) Time Series Processor, Double Precision Version, TSP Econometric Programming, Stanford, CA, and 2) Statistical Analysis Systems, SAS Institute, Inc., Raleigh, N.C.

[3] Seasonally adjusted GNP and employment data are used.

[4] While output response, in terms of labor activity, is usually measured by the unemployment rate, the employment rate was used here since it is a more accurate measure of labor force participation. The author was advised by committee member, C. B. Garrison that Bureau of Labor Statistics on the unemployment rate can be quite inaccurate due to imprecise downward adjustments made to the total employment base.

these equations are then transformed via the natural number, e. This transformed time series provides the quarterly measure of deviation of output away from trend. That is, letting the log-form of the residuals from the GNP and employment rate time trend equations be denoted as X_t^{GNP} and X_t^{EMP}, respectively, we have;

$$G_t = e^{X_t^{GNP}} - 1 \text{ , and,}$$

$$E_t = e^{X_t^{EMP}} - 1 \text{ ,}$$

as the transformed residuals entering the output regression models as dependent variables. As Table 8.3.1. shows, real GNP over the 1956-79 period has been growing at an annual rate of about 3.5 percent, while the

Table 8.3.1. Real GNP and Employment Rate Time Trend Regressions: 1956/3-1979/2.

	GNP Regression	Employment Regression
	ln[GNP/deflator] = 6.24 + .0083t (934.0) (69.5)	ln[emp. rate] = 54.13 + .0074t (307.3) (10.4)
$\hat{\sigma}_\varepsilon$	.003	.083
$\bar{R}^2$	.985	.741
$F_{(k-1,n-k)}$	483.5	108.3
DW	.105	.121
n	92	92

Notes: --GNP is measured in billions of dollars.
--t-values appear below their respective coefficients.

employment rate has been growing at an annual rate of almost 3 percent. (Note: The R-squared statistic appearing in Table 8.3.1. and subsequent R-squared statistics appearing in table or discussion form have been "adjusted" for degrees of freedom.)

8.4. Regression Methodology: Determination of Lag Length and Polynomial Degree Restrictions

The purpose of this section is to describe the quasi-search method that is used to determine the correct lag length of the distributed lag regressions. Some idea of the polynomial degree that mimics the coefficient weighting pattern will also be obtained as a by-product of this search procedure.

Three estimation methods will be used: 1) OLS, 2) GLS, and 3)Almon/GLS. The desire here is to ultimately submit the data to the Almon polynomial lag technique, since it is from this estimation method that elasticity measurements will be obtained and upon which the economic analysis will be based. However, for reasons which will be explained below, Almon estimation will be carried out only after OLS and GLS methods have provided some basic insights about the structure of the relationships under study and about some statistical problems involved in parameter estimation.

The chief attributes of the Almon method are three: 1) the distributed lag estimates maintain all the desirable properties of OLS estimation, i.e., unbiasedness, consistency and efficiency, if the conventional Classical assumptions hold, 2) the method increases the precision of the estimates and their variances by reducing the number of parameters to be estimated (if the degree of polynomial used in coefficient estimation is less than the lag length) and, 3) the technique is quite flexible in terms of the forms of admissable structures and weighting patterns which can be entertained. However, the procedure is particularly beset with two difficulties which can lead to regression misspecification: 1) improper lag length and/or 2) improper polynomial degree.[1] Both of these problems are associated with the fact that the Almon method requires an a priori statement that the lag weights lie on a function that can be approximated arbitrarily well on a closed interval by a polynomial function of sufficiently high degree and that the true lag length is known. Therefore the Almon method requires that certain "restrictions" be imposed upon the estimation of the parameters, restrictions which if not proper can lead to biased and inconsistent estimates and to invalid hypothesis tests.[2] And, except in a

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[1] A number of studies investigating the effects of misspecifying lag length and polynomial degree in Almon models are; Frost [105], Schmidt and Waud [106], Trivedi [107], Terasvirta [108], Harper [109], Griffiths and Kerrison [110], Schmidt and Sickles [111], and Trivedi and Pagan [112].

[2] There are other peripheral problems related to the method when the true lag length and polynomial degree are not known. First, there is little distribution theory associated with the Almon method. Thus when there is no prior knowledge of the true lag specification, choosing a polynomial degree and lag length specification is reduced to an amalgam of comparisons of adjusted R-squares and an assessment of the significance of coefficients. The validity of these statistics, however, is predicated on the fact that the correct lag pattern has been chosen. This means that a prior statement of lag length and polynomial degree poses a dilemma in that 1) the presence or absence of a lag length is not a testable proposition when the Almon technique is used a priori, and, 2) the choice among alternative lag lengths cannot be made on the basis of t-tests, since these tests are valid only upon the assumption that the correct lag length has been imposed on the estimation. Simply put, the a priori affirmation that the effect of an independent variable on a dependent variable is "distributed" in a certain fashion over time cannot be validly inferred, in a statistical sense, when the Almon technique is used to generate the statistics by which such an inference is made. Secondly, since the magnitudes of the estimated coefficients are, in part, determined by the length of the lag in conjunction with the polynomial specification, an improper Almon specification can lead to erroneous regression estimates even though they may appear statistically significant.

very limited manner, economic theory is of little help in determining what these restrictions should be. By using OLS and (especially) GLS estimation to provide clues about lag and polynomial restrictions, this study can capitalize on the beneficial aspects of the Almon method (the chief benefit here is the reduction in the number of parameters to be estimated in each lagged regression), while avoiding the above stated problems.

The OLS and GLS regression fitting procedure is as follows: lagged regressors are added to the output regressions in step-wise fashion until the explanatory power of the regression can no longer be improved and the signs of all coefficients (excluding the constant) are positive.[1] The following two equation format is illustrative of this step-wise method;[2]

$$y_t = \sum_0^{m-1} \beta_i X_{t-i} + \varepsilon_t \quad , \text{ and,} \tag{8.2.17.}$$

$$y_t = \sum_0^{m-1} \beta_i X_{t-i} + \beta_m X_{t-m} + \varepsilon_t \quad , \tag{8.2.18.}$$

where y_t and X_{t-i} are the dependent and lagged independent variables, respectively, and X_{t-m} is the last added lagged regressor in the sequence of regressors. The true lag length, m-1 or m, is to be determined by examining selected statistical properties of the last added lagged regressor along with a comparison of other summary statistics associated with eqs.

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[1] As will be explained subsequently, this criterion for judging lag length is not specifically dependent upon whether or not the last added regressor is statistically different from zero according to conventional usage of the t-test.

[2] The step-wise addition of variables is justified by our theory; once the effect of the unanticipated variable(s) commences, its effect is continuous until it (permanently) dies out. Thus the leg lengths to be estimated here are assumed to be finite.

(8.2.17.) and (8.2.18.). It should be noted that this methodology is based upon a comparison of these two equations and on a statistical examination of the marginal regressor given the fact that eq. (8.2.17.) has already been shown to include at least regressors 0 through m-1. The sequential addition of regressors will continue until the proper lag length is determined. Knowledge of the polynomial which is appropriate for the coefficient weighting pattern will also emerge as the lag length is determined.

Three statistics, one based upon statistical inference, are used to judge whether the last added lagged regressor belongs in the distributed lag, i.e., is adding explanatory power to the regression: 1) the adjusted R^2 statistic, 2) individual regressor t-values, and, 3) a constrained F-test to test for linear restriction. As will be demonstrated in most of the GLS regressions, the addition of a lagged regressor that does not belong in the lag causes the regression to "fail" all three of these statistical criteria simultaneously, i.e., the adjusted R^2 statistic falls from its previous value, the t-value of the added lagged regressor becomes less than unity, and the constrained F-test shows the reduction in SSE resulting from the addition of the last regressor to be due to sampling error only.

Since the intent here is to construct the lagged regression which contains the greatest amount of explanatory power, the objective in the sequential procedure will be to maximize adjusted R^2, and this rule is quite different from the rule of keeping variables only if their t-values are significant at some specified level of confidence. This reasoning follows from the (exact) statistical relationship between adjusted R^2

and the t-statistic: the addition of an independent variable to a regression will always increase the adjusted R^2 magnitude as long as its t-value is greater than unity.[1] Thus in the lagged regressions to follow, the generally accepted criterion of keeping variables only if their t-values are statistically significant at the 90 or 95 percent level of confidence will be relaxed somewhat.[2] This approach assumes then that the regression with the highest adjusted R^2 will be also be the regression with the correct lag length. Hence the fact that some subsequent regressions will display initial and final lagged coefficients which, while having t-values greater than unity, don't appear to be "significant," should not be viewed with suspicion.

A constrained F-test statistic will also be computed to help determine whether or not the marginal regressor belongs in the lag.[3] In this setting the constrained F-test is a test of the hypothesis that the lag length should be (linearly) restricted to m-1 lags only, where m-1 is the length of the true lag describing the time relationship between past unanticipated variables and current output response. Consider again eqs. (8.2.17.) and (8.2.18.). Here the null hypothesis is;

Ho: $\beta_m = 0$, and the alternative hypothesis is,

Ha: $\beta_m > 0$.

In terms of eq. (8.2.17.), eq. (8.2.18.) contains a "linear restriction,"

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[1] See Haitovsky [113] for a proof of this statement.

[2] In most cases the t-values of the GLS and Almon estimates meet or exceed conventional t-test significance levels. For the most part, the "greater-than-unity" criterion is applied to the last regressor.

[3] This F-test is used in conjunction with the R-squared and t-tests because it is felt that at least one of the methods used in determining lag length should be based upon statistical inference.

that is, $\beta_m = 0$. The alternative hypothesis implies no linear restriction should be placed on eq. (8.2.17.). If the null hypothesis is true then, the SSE's for both (8.2.17.) and (8.2.18.) should not be statistically different. Given the assumption that Ho: is true, the following F-statistic will be used to determine whether the null hypothesis should be accepted (i.e., that the true lag length is of order m-1 only);

$$F_{(r,n-k)} = \frac{[SSE(r) - SSE(u)]/r}{SSE(u)/n-k} ,$$

where r = the number of linear restrictions (one). SSE(r) and SSE(u) are the error-sums-of-squares for the restricted regression [eq. (8.2.17.)] and the unrestricted regression [eq. (8.2.18.)], respectively. If the computed F-value is less than the critical F-value (at the 95 percent level), the m-th regressor will be discarded from eq. (8.2.18.). If the m-th regressor produces an F-value greater than the critical F-value, this term will be added to the lag and the m+1 regressor estimated and then tested in similar fashion. This sequential use of the F-test will continue until an incremental regressor fails the test.[1] This occurrence will help identify the correct termination point of the lag.

The fact that the sequentially added lagged regressors are orthogonal provides two additional statistical benefits when using the GLS regressions to help establish lag length. First, the fact that the last added regressor is uncorrelated with all previously included regressors enhances the discriminatory power of the adjusted R^2 statistic in determining if the <u>last</u>

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[1] Because of the autocorrelation present in the OLS regressions, the constrained F-test will be used in GLS esimation only.

added regressor belongs in the equation. This reasoning follows from the fact that because the regressors are uncorrelated, the R^2 statistic is measuring the separate contribution of the marginal explanatory variable only. Thus if the R^2 value rises when a regressor is added to a particular output regression there is evidence that the variable is providing a unique contribution to the explanatory power of the equation and, therefore, belongs in the lag.[1] Secondly, because of the uncorrelated nature of the regressors, the lagged equations will be free from multicollinearity. Typically, distributed lag regression analysis is plagued by severe multicollinearity due to the high serial correlation characterizing lagged independent variables; this leads to imprecise coefficient variance estimates. However, it will be recalled from the time series models of Chapter VI, that, by experimental design, the unanticipated inflation and money growth rate variables are nearly mutually orthogonal, i.e., as residuals from the ARIMA univariate models they qualify as "white noise" processes.[2] This fact will allow for a more definitive analysis

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[1] Consider, for example, the case in which a dependent variable is being regressed on two explanatory variables. If the regressors are even mildly correlated, which is usually the case when the variables are time series, the explained portion of the variation in the dependent variable can be placed into three categories; variation unique to the first variable, variation unique to the second variable, and variation which is common to both variables. However, when the independent variables are uncorrelated, as is the case here, this third type of variation does not exist. In this case, the additional explanatory power provided by the added lagged regressor is <u>uniquely</u> measured by the adjusted R-square statistic.

[2] In this regard it is important to note that the combined inflation and money models, eqs. (8.2.9.) through (8.2.16.), will also be free from multicollinearity. Section 7.4. provided statistical evidence that no contemporaneous or non-contemporaneous correlation exists between ARIMA inflation and money model residuals.

of lag length since regression statistics used in judging the lag will be free from multicollinearity influences.

Finally, the determination of lag length via the methods discussed above will be made primarily from the GLS regressions, due to the autocorrelation characterizing the OLS regression residuals. Hence, the OLS estimates are presented only as a first step in "feeling out" the data and to highlight the statistical nature of the residual correlation.

8.5. OLS Regression Results and Residual Analysis

Table 8.5.1. presents the OLS regressions for the GNP output equations (8.2.1.) through (8.2.4.). Table 8.5.2. presents the OLS regressions for the employment equations (8.2.5.) through (8.2.9.). (For easier reference the regression number provided in Section 8.2. and the unanticipated regressor variable is placed above the relevant column of regression estimates in each table.) Since the OLS regressions are presented only to provide some initial idea of the lag structure and to highlight any statistical problems, individual regression evolution as lagged regressors are added is not presented. Rather, the OLS tables present the equation with the "optimum" lag, i.e., the regression with the highest adjusted R^2.

Except for the monetary base equations, the results show that most of the coefficients have the expected sign and are significant (in terms of the conventional t-test, or the greater-than-unity criterion). While the unanticipated CPI inflation variable produces a humped-shape coefficient pattern for both the GNP and employment rate equations, the deflator exhibits a Koyck-form of exponential decay. On the other hand, the unanticipated M1 growth regressions (8.2.3.) and (8.2.7.) produce an output response which is characterized by a distributed lag in which the weights follow the

Table 8.5.1. OLS Regressions: Dependent Variable, Real GNP, G_t: 1956/3 through 1979/2.

Equation	(8.2.1.) C_{t-i}	(8.2.2.) D_{t-i}	(8.2.3.) M_{t-i}	(8.2.4.) B_{t-i}
constant	-0.920 (0.98)*	-0.595 (1.09)*	-1.422 (2.57)	-2.033 (1.82)
period t	4.593 (3.97)	3.855 (2.73)	0.101 (0.11)*	-2.008 (1.13)*
t-1	4.780 (3.80)	3.035 (2.15)	0.520 (0.49)*	-0.710 (0.42)*
t-2	3.918 (3.11)		1.625 (1.43)*	-0.209 (0.12)*
t-3	2.520 (2.04)		3.194 (2.89)	0.104 (0.06)*
t-4			3.419 (3.09)	0.458 (0.26)*
t-5			3.043 (2.78)	1.235 (0.69)*
t-6			3.017 (2.75)	1.770 (0.99)*
t-7			2.449 (2.25)	1.540 (0.90)*
t-8			1.525 (1.40)*	2.135 (1.22)*
t-9				1.210 (0.69)*
t-10				1.375 (0.79)*
t-11				1.613 (0.92)*
t-12				1.828 (1.06)*
t-13				1.598 (0.92)*
t-14				1.433 (0.84)*
$\hat{\sigma}_{\varepsilon}$	4.29	5.04	4.69	5.69
R^2	.478	.283	.432	-.108
$F_{(K-1,n-K)}$	21.17	18.77	7.90	0.50**
n	89	91	84	78
DW	.390	.268	.196	.180

Notes: --t-values appear beside the estimated coefficients. All t-values are reported at their absolute value.
--"*" indicates t-values not significant at the 10 percent level. A two-tailed test is used.
--"**" indicates F-value not significant at the 5 percent level.
--coefficients are stated in billions of dollars.
--eq. (8.2.3.) was estimated without the first two terms. The results were not materially affected.

Table 8.5.2. OLS Regressions: Dependent Variable, the Employment Rate, E_t; 1956/3 through 1979/2.

Equation	(8.2.5.)	(8.2.6.)	(8.2.7.)	(8.2.8.)
	C_{t-i}	D_{t-i}	M_{t-i}	B_{t-i}
constant	-0.180 (2.68)	-0.133 (1.61)*	-0.222 (2.75)	0.089 (0.47)*
period t	0.629 (3.53)	0.492 (2.34)	-0.141 (1.03)*	-0.560 (2.12)
t-1	0.638 (3.53	0.469 (2.23)	-0.211 (1.39)*	-0.362 (1.35)*
t-2	0.608 (3.37)	0.376 (1.78)	-0.100 (0.61)*	-0.243 (0.86)*
t-3	0.551 (3.05)		0.144 (0.87)*	-0.409 (1.43)*
t-4	0.431 (2.42)		0.326 (1.98)	-0.328 (1.23)*
t-5			0.406 (2.55)	-0.206 (0.76)*
t-6			0.519 (3.23)	-0.132 (0.48)*
t-7			0.512 (3.24)	-0.036 (0.14)*
t-8			0.494 (3.07)	0.116 (0.43)*
t-9			0.366 (2.30)	-0.048 (0.18)*
t-10			0.237 (1.48)*	-0.019 (0.07)*
t-11				0.105 (0.40)*
t-12				0.070 (0.26)*
t-13				0.055 (0.21)*
t-14				0.074 (0.28)*
t-15				0.065 (0.29)*
t-16				0.043 (0.17)*
t-17				-0.018 (0.07)*
$\hat{\sigma}_{\varepsilon}$	.615	.750	.671	.836
R^2	.481	.276	.492	-.135
$F_{K-1,n-K)}$	17.14	12.20	8.13	0.51**
n	88	90	82	75
DW	.317	.226	.198	.186

Notes: --the statistical legend used here is the same as that used in Table 8.5.1.

--coefficients are stated in terms of one-hundredths percentage points.

--eq. (8.2.7.) was estimated without the first four terms. The results were not materially affected.

expected humped-shape second degree polynomial form. The monetary base regressions (8.2.4.) and (8.2.8.) indicate a very weak or non-existant relationship between current measures of economic activity and the lagged unanticipated monetary base growth rate variables.[1]

All OLS regressions show the R^2 statistics to be relatively low. As stated in the introductory remarks of this paper, this result is expected since deviations of real GNP and the employment rate from trend are influenced by many factors in addition to unanticipated inflation or money growth. With regard to the introductory remarks made in Section 8.1. of this chapter, the low R^2's should be viewed in the light that, while the distributed lag regression models are capable of generating serially correlated movements in economic activity in response to serially uncorrelated inflation/money expectational "shocks," the lagged regressors are capable of explaining only some of this dependent variable variation.

The chief negative feature of the OLS regressions is the low Durbin-Watson (DW) statistics, indicating positive autocorrelation in the residuals.[2] This means that while the OLS estimates of the regression coefficients are unbiased and consistent, they are not efficient or asymptotically efficient. More importantly, with positive autocorrelation the sampling variances of the estimates are understated meaning that the conventional use of the t and F hypotheses tests are suspect, i.e., there

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[1] Because of the presence of autocorrelation, this conclusion is not definitive at this point in the analysis.

[2] Obviously, some of the residual autocorrelation can be explained by the fact that quarterly data are being used; the shortness of the interval of observations means that random effects in the disturbances linger on in the quarterly data observation. However, the autocorrelation is much too high to be fully explained by this reason.

is a bias towards rejection of the null hypothesis that the coefficients are not statistically significant either individually or as a group. The conclusion is that a determination of lag length from the OLS results is difficult since the statistics generated by the regressions, and upon which the lag length determination is based, are suspect.

Clearly the autocorrelation of the OLS residuals poses a threat to the accuracy of the distributed lag models and needs to be addressed directly if the lag length determination process is to possess statistical acceptability. However, before proceeding to correct the autocorrelation problem, an analysis of the nature of the residual dependence is indicated. The question to be resolved before proceeding is to determine the order of the autocorrelation process, since 1) the nature of the time dependency in the OLS residuals must be known before proper corrective measures can be prescribed, and, 2) the DW statistic does not provide conclusive evidence that the autocorrelation is of first-order.

Table 8.5.3. presents both the theoretical and actual ACF's for the OLS regression residuals for five lags. Since the time dependency in the residuals is suspected to be characterized by a first-order Markov process, the theortical ACF of residuals is constructed on the assumption that the residual correlation is described by the following difference equation with residual;

$$e_t = \rho e_{t-1} + v_t \quad , \tag{8.5.1.}$$

where $|\rho| < 1$ and v_t satisfies the assumptions,

$$E(v_t) = 0$$

$$\begin{aligned} E(v_t v_{t-s}) &= \sigma_v^2 \quad \text{when } s = 0 \\ &= 0 \quad \text{when } s \neq 0 \quad . \end{aligned}$$

Table 8.5.3. Theoretical and Actual ACF for the OLS Regression Residuals.

GNP Regressions

	(8.2.1.)		(8.2.2.)		(8.2.3.)		(8.2.4.)	
	C_{t-i}		D_{t-i}		M_{t-i}		B_{t-i}	
	T	A	T	A	T	A	T	A
Lag 1	.81	.81	.87	.86	.90	.89	.91	.91
2	.66	.61	.75	.70	.81	.85	.83	.80
3	.53	.55	.66	.66	.73	.71	.75	.71
4	.43	.51	.57	.58	.66	.65	.69	.64
5	.35	.46	.50	.51	.59	.58	.62	.59

Employment Regressions

	(8.2.5.)		(8.2.6.)		(8.2.7.)		(8.2.8.)	
	C_{t-i}		D_{t-i}		M_{t-i}		B_{t-i}	
	T	A	T	A	T	A	T	A
Lag 1	.86	.87	.88	.87	.87	.86	.91	.90
2	.74	.75	.77	.78	.75	.71	.83	.83
3	.64	.63	.68	.69	.66	.64	.75	.73
4	.55	.57	.60	.55	.57	.54	.69	.65
5	.47	.48	.53	.48	.50	.50	.62	.60

Note: --The notation "T" and "A" stand for the "theoretical" and "actual" ACF's, respectively.

Since $E(e_t) = 0$, and since the v_t are serially independent, continued substitution into (8.5.1.) gives the theoretical value of ρ at s lags;

(8.5.2.) $$E(e_t e_{t-s})/\sigma_e^2 = \rho^s \quad .$$

Since the true value of ρ is not known, an estimate of this parameter is provided by the relationship;[1]

(8.5.3.) $$\hat{\rho} = \Sigma e_t e_{t-1}/\Sigma e_{t-1}^2 \quad , \text{ where } t = 1, \ldots, n \ .$$

(The actual ACF of the residuals is given by the statistic in fn. 1, p. 91.)

When ρ is generated by a first-order Markov process, the actual ACF of OLS residuals, e_t, should exhibit a geometrically declining decay as lags further removed from time period t are encountered, and should closely mimic the theoretical decay magnitudes given by relations (8.5.2.) and (8.5.3.). Table 8.5.3. shows this to be the case; the actual decay magnitudes follow closely the values of their theoretical counterparts. It is safe to assume therefore that the time dependency in the OLS residuals is truly described by a first-order Markov scheme.[2] This finding will facilitate later regression estimation aimed at correcting the autocorrelation problem.[3]

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[1] Rao and Griliches have shown that while the distribution of ρ is consistent, it is biased by the magnitude of $[(\rho + \lambda)/n]$, where n = number of observations and λ = the degree of serial correlation in the independent variables. Since it was shown earlier that $\lambda = 0$, the bias is $\simeq \rho/n$.

[2] In order to insure that the first-order scheme is fully capturing the time dependency in the residuals, a second-order AR process was fit to the residual time series. The second-order process was found to be inappropriate since $\hat{\rho}_2$ in the following equation was statistically insignificant.

$$e_t = \hat{\rho}_1 e_{t-1} + \hat{\rho}_2 e_{t-2} + v_t$$

[3] Since the autocorrelation of OLS residuals could also indicate the violation of two other Classical assumptions, $E(e_t) = 0$, and cov $(X_t, e_t) = 0$, these assumptions were examined. The mean values of the OLS residuals

8.6. Possible Approaches to Correct for Autocorrelation of OLS Residuals

Two methods are available to deal with the autocorrelation problem: 1) respecifying the matrix of regressors in the OLS equations, or, 2) submitting the data to GLS estimation.

Earlier it was explained that the measures of output response being used in this study (and in general) exhibit strong serial correlation because of the endogenous effects of the multiplier, habit persistence, positive adjustment costs, etc. It is plausible to reason here that these factors, which produce a systematic influence in the dependent variables, are not being fully explained by the lagged independent variables, and this fact is being reflected in the autocorrelated disturbances. Hence, to the extent that the autocorrelation of residuals is resulting from the presence of some unexplained systematic influence on the output variables, a possible approach aimed at correcting the residual dependency would be to reestimate the OLS regressions with an extra independent variable to account for this systematic influence. An obvious candidate would be a variable highly correlated with the dependent variable(s).[1] However, while such a respecification of the regressor matrix would improve the statistical properties of the OLS regressions by reducing residual dependency, this approach would violate the spirit of this research in that the effects of the unanticipated variables on economic

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[for eqs. (8.2.1.) through (8.2.8.)] were found to be insignificantly different from zero. Also, the correlation between the residuals and lagged va ue of the independent variables was not statistically different from zero (five lags for each regression were used in the test).

[1] For example, the detrended Index of Industrial Production, or National Disposable Income are time series which are highly correlated with detrended real GNP or the employment rate.

activity would no longer be unique to the output models, but would be confounded with the new explanatory variable as well. It would thus be impossible to isolate the lagged relationship between the output variables and the unanticipated independent variables. However, the desire to keep the effects of the unanticipated variables "unpolluted" cannot be followed at the expense of impugning the statistical acceptability of the estimation methods employed. With this reasoning in mind, a GLS approach is used. GLS estimation is appropriate since 1) it will correct the autocorrelation problem while allowing an unencumbered analysis of the unanticipated variable alone, and, 2) because we have certain knowledge of the nature of (and parameters describing) the time dependency of residuals.

8.7. GLS Regression Results and Statistical Analysis

Two GLS methods are entertained as potential estimation methods to correct for autocorrelation: 1) the method of first differences, and 2) the Cochrane-Orcutt iterative procedure (hereafter referred to as C-O). Note that to justify the use of these two GLS methods we must argue that the serial dependency of the "omitted" independent variable (i.e., one that is highly correlated with the dependent variable) that is producing the autocorrelation in the OLS residuals is uncorrelated with the included lagged unanticipated variables. Since the unanticipated variables qualify as white noise processes, we can confirm this argument.

8.7.1. First-Differenced Data Regressions. Justification of the possible use of first-differenced data is based on the fact that the estimated ρ coefficients of the first-order autocorrelation pattern are near unity. The first-differenced regression results appear in Tables 8.7.1.1. and 8.7.1.2. The tables present the equation with the optimum lag length only.

Table 8.7.1.1. GLS Regressions; First-Differenced Data; Dependent Variable, G_t^*; 1956/3 through 1979/2.

Equation	(8.2.1.) C_{t-i}^*	(8.2.2.) D_{t-i}^*	(8.2.3.) M_{t-i}^*	(8.2.4.) B_{t-i}^*
constant	-0.701 (0.91)*	-0.466 (0.02)*	-3.126 (0.90)	-1.128 (0.11)*
period t	0.910 (1.76)	0.568 (2.16)	0.811 (1.76)	0.212 (0.51)*
t-1	1.147 (2.22)	0.452 (1.88)	1.250 (1.79)	0.517 (1.41)*
t-2	1.562 (2.40)		2.079 (2.31)	0.414 (1.33)*
t-3	1.500 (2.10)		4.811 (3.39)	0.625 (1.97)
t-4	0.807 (1.76)		5.277 (4.55)	0.970 (2.96)
t-5			5.198 (4.22)	1.421 (3.03)
t-6			4.782 (3.81)	1.400 (3.00)
t-7			4.201 (2.98)	0.757 (2.76)
t-8			3.199 (2.08)	0.642 (1.08)*
t-9			2.411 (1.77)	0.789 (1.01)*
t-10			1.827 (1.05)*	0.501 (0.76)*
t-11			0.901 (1.00)*	
$\hat{\sigma}_\varepsilon$	2.52	3.54	2.97	3.99
R^2	.249	.183	.189	.132
$F_{(K-1,n-K)}$	6.71	10.94	2.53	2.10
n	87	90	80	81
DW	1.45	1.11	1.39	1.28

Notes: --the data is transformed as follows:

$C_{t-i}^* = (1 - L)C_{t-i}$

$D_{t-i}^* = (1 - L)D_{t-i}$

$M_{t-i}^* = (1 - L)M_{t-i}$

$B_{t-i}^* = (1 - L)B_{t-i}$

$G_t^* = (1 - L)G_t$, where L is the lag operator.

--except for the data transformations, the statistical legend used here is the same as that used in Table 8.5.1.

Table 8.7.1.2. GLS Regressions; First-Differenced Data; Dependent Variable, E^*_t, 1956/3 through 1979/2.

Equation	(8.2.5.) C^*_{t-i}	(8.2.6.) D^*_{t-i}	(8.2.7.) M^*_{t-i}	(8.2.8.) B^*_{t-i}
constant	-0.100 (0.01)*	-0.077 (0.19)*	-1.102 (0.98)*	0.544 (0.34)*
period t	0.321 (3.41)	0.300 (2.00)	0.201 (0.77)*	0.072 (0.83)*
t-1	0.379 (3.53)	0.350 (2.12)	0.255 (1.20)*	0.016 (0.14)*
t-2	0.391 (3.55)	0.261 (1.73)	0.475 (2.11)	-0.005 (0.04)*
t-3	0.411 (4.01)	0.197 (1.31)*	0.491 (2.80)	-0.074 (0.43)*
t-4	0.292 (2.80)		0.859 (3.78)	0.029 (1.51)*
t-5			1.204 (4.03)	0.101 (0.46)*
t-6			1.221 (5.11)	0.186 (0.79)*
t-7			1.157 (5.00)	-0.253 (1.03)*
t-8			1.019 (4.80)	-0.298 (1.22)*
t-9			0.782 (2.76)	0.212 (0.84)*
t-10			0.701 (1.98)	0.170 (0.75)*
t-11			0.599 (1.89)	-0.168 (0.80)*
t-12			0.321 (1.74)	0.046 (0.24)*
t-13			0.290 (1.65)	-0.144 (0.85)*
t-14			0.116 (1.65)	
$\hat{\sigma}_\varepsilon$	.311	.351	.300	.751
R^2	.174	.152	.259	-.106
$F_{K-1,n-K)}$	4.62	4.90	2.77	0.47**
n	87	88	77	78
DW	1.45***	1.66***	1.29	0.58

Notes: --the employment rate variable is transformed as follows:

$E^*_t = (1 - L)E_t$, where L is the lag operator.

--except for the data transformations, the statistical legend used here is the same as that used in Table 8.5.2.

--"***" indicates the DW statistic is in the upper portion of the indeterminant range.

However, the previously described step-wise procedure of adding lagged regressors is utilized to obtain the regression with the greatest explanatory power. (The constrained F-test is not used here.)

The first-differenced regression results show all coefficients to have the expected positive sign [except eq. (8.2.8.) in Table 8.7.1.2.]. And, except for eq. (8.2.8.), all coefficients are statistically significant at the 90 percent level (or exceed the greater-than-unity criterion). The adjusted R^2 values fall from those of the OLS regressions. However, since the dependent variables in the first-differenced and OLS regressions are not the same, these statistics are not comparable. The statistical explanation for the lower R^2 values rests with the fact that the differencing has reduced the variation in the dependent variable and therefore there is less for the independent variables to "explain."

Importantly, the first-differenced estimation procedure significantly reduces the strength of the autocorrelation of residuals from that of the OLS regressions--all DW statistics rise substantially.[1] And, while some of the DW statistics in eqs. (8.2.1.) and (8.2.7.) are in the high uncertain range, the fact that the differencing has damped the autocorrelation can be inferred from the lower estimates of $\hat{\rho}$ in all regressions[2] [for example, $\hat{\rho} = .81$ in the OLS estimation of (8.2.1.), while $\hat{\rho} = .28$ in the first-

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[1] First-differencing is also responsible for the lower SSE's (not shown) and standard error of the regression.

[2] It is expected that the constants should not be statistically different from zero since $\rho = 1$; the results confirm this expectation. Also, the insignificant constants confirm the idea that the regressions are properly specified without a time trend as an explanatory variable. (Note: the regressions were estimated without a constant and the statistical results were not materially altered. The constant was used in the estimation however since the computer program would not print out the R-square and F-statistics unless a constant was requested.)

differenced estimation of (8.2.1.)].

The differencing procedure, while not altering the second-degree polynomial weighting pattern for the coefficients, does change the lag length of some regressions. For example, the unanticipated CPI inflation regression (8.2.1.) in Table 8.7.1.1. maintains a humped-shape coefficient form. The unanticipated M1 regressions for both real GNP and the employment rate, while continuing to display the second-degree humped-shaped polynomial form, become longer.

Comparing the monetary base regressions (8.2.4.) in Tables 8.5.1.[p.255] and 8.7.1.1. shows that the relationships become somewhat stronger with differencing (in terms of coefficient t-values). One possible explanation for this is that the transformation, by reducing the strength of the residual dependency, allows the (weak) relationship to surface. The same conclusion however, cannot be applied to the monetary base-employment regressions (8.2.8.) as seen in Tables 8.5.2. [p. 256] and 8.7.2.1.; here the differencing provides no reduction in autocorrelation nor any improvement in regression statistics over their OLS counterparts.

8.7.2. The Cochrane-Orcutt Transformed Regressions. Implicit in the first-differencing method is the assumption that the true parameter relating adjacent residuals is equal to unity--based upon the residual analysis of the OLS regressions, this was a plausible assumption. However, while the first-differencing transformation is a step in the right direction, it is clear that some residual autocorrelation remains.

In an attempt to further reduce the residual dependency, the regressions are estimated using the C-O iterative procedure. This method allows more precise estimation since it does not arbitrarily force ρ to equal unity.

Rather, the method involves a series of estimation iterations, each producing a better estimate of ρ and a more accurate modeling of the true nature of the time dependency in the residuals.[1]

The C-O regression results appear in Tables 8.7.2.1. through 8.7.2.8.[2] Again, lag length determination is carried out via the step-wise-addition-of-regressors procedure described in Section 8.2. Here the constrained F-test to test for linear restriction is also used (this statistic, which was described in Section 8.2 also, appears at the bottom of each regression). Here also more emphasis is given to the greater-than-unity criterion for judging the importance of marginal regressors, and not the conventional t-test. (However, some initial coefficients with t-values slightly less than unity are retained if their deletion does not improve the regression results; these cases are referred to in the footnotes accompanying each set of equations. Also, because the greater-than-unity criterion for

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[1] Note that the C-O iterative method is only indicated if the serial correlation of the residuals is of first order. Earlier analysis of the residual dependency shows this is the case.

[2] In the following C-O regressions, the iteration process halts after a new estimate of ρ differs from the previous estimate by less than .005. In most cases ten to twelve iterations are required before the coefficient estimates converge on minimum mean square parameters. A possible problem with the C-O procedure is that the iterative technique may lead to a local rather than a global minimization of sum-of-squared residuals (although the possibility of this occurring is quite small when the autocorrelation is of first order). Nevertheless, in order to check for this possibility, the Hildreth-Lu "grid search" maximum likelihood procedure was used on a number of the regressions. The initial "guesses" for ρ were spaced between 0.5 and 1.0 at intervals of 0.1. In all cases the Hildreth-Lu procedure to correct for autocorrelation produced estimates of ρ which were almost identical to those produced by the C-O method. Hence we can safely infer that the C-O procedure is obtaining a minimum sum-of-squares that is global rather than local. (The Hildreth-Lu procedure was not used exclusively because of the substantial increase in computer time that was required.)

Table 8.7.2.1. Regression 8.2.1., $G_t = \Sigma\alpha_i C_{t-i} + \varepsilon_t$, Cochrane-Orcutt Transformation; 1956/3 through 1979/2.

	(1)	(2)	(3)
constant	- .808 (0.37)	- .870 (0.43)	- .985 (0.49)
period t	.813 (1.48)	1.035 (1.86)	1.080 (1.91)
t-1	1.070 (1.69)	1.610 (2.35)	1.738 (2.46)
t-2	.918 (1.58)	1.720 (2.50)	1.963 (2.60)
t-3		1.053 (1.92)	1.420 (2.00)
t-4			.427 (0.76)
$\hat{\rho}$	.900 (19.52)	.897 (19.09)	.893 (18.60)
SSE/n-k	4.18	4.10	4.16
$\hat{\sigma}_\varepsilon$	2.06	2.04	2.05
R^2	.843	.845	.841
$F_{(K-1,n-K)}$	159.42	119.63	92.23
n	89	88	87
DW	1.61	1.65	1.65
constrained F-test	4.04*	3.71*	0.67**

Notes: --DW statistics are very close to the "no autocorrelation" boundary of the indeterminant range.

--t-values appear beside the estimated ρ coefficients.

--"*" indicates constrained F-test values are significant at the 5 percent level.

--"**" indicates constrained F-test values are not significant at the 5 percent level.

--The statistical legend pertaining to the constrained F-test applies to subsequent C-O regressions (through Table 8.7.2.8.).

Table 8.7.2.2. Regression 8.2.2., $G_t = \Sigma\alpha_i D_{t-i} + \varepsilon_t$, Cochrane-Orcutt Transformation; 1956/3 through 1979/2.

	(1)	(2)	(3)
constant	- .763 (0.32)	- .785 (0.32)	- .820 (0.33)
period t	.583 (1.06)	.658 (1.14)	.655 (1.13)
t-1	.462 (0.71)	.703 (1.20)	.728 (0.98)
t-2	- .047 (0.09)	.360 (1.50)	.418 (0.54)
t-3		.513 (1.91)	.608 (0.82)
t-4			.118 (0.12)
$\hat{\rho}$	.906 (20.29)	.908 (20.33)	.907 (21.17)
SSE/n-k	4.29	4.35	4.45
$\hat{\sigma}_\varepsilon$	2.08	2.10	2.12
R^2	.835	.839	.830
$F_{(K-1,n-K)}$	154.42	111.43	85.12
n	89	88	87
DW	1.58	1.67	1.69
constrained F-test	0.14**	0.85**	0.66**

Notes: --DW statistics are very close to the "no autocorrelation" boundary of the indeterminant range.

Table 8.7.2.3. Regression 8.2.3., $G_t = \Sigma\alpha_i M_{t-i} + \varepsilon_t$, Cochrane-Orcutt Transformation; 1956/3 through 1979/2.

	(1)	(2)	(3)
constant	-3.720 (1.37)	-4.765 (1.12)	-4.338 (1.06)
period t	1.040 (2.65)	1.173 (2.94)	1.228 (2.85)
t-1	1.980 (3.16)	2.245 (3.46)	2.260 (3.27)
t-2	3.090 (3.76)	3.650 (4.13)	3.618 (3.85)
t-3	4.863 (4.90)	5.658 (5.23)	5.528 (4.79)
t-4	5.225 (4.77)	6.015 (5.12)	5.935 (4.63)
t-5	5.240 (4.60)	6.450 (5.04)	6.198 (4.53)
t-6	5.413 (4.75)	6.670 (5.21)	6.335 (4.57)
t-7	4.743 (4.38)	6.028 (4.70)	5.683 (4.09)
t-8	3.803 (3.83)	5.028 (4.22)	4.625 (3.41)
t-9	2.800 (3.28)	3.909 (3.60)	3.513 (2.80)
t-10	1.653 (2.46)	2.627 (2.88)	2.233 (1.98)
t-11	.543 (1.17)	1.378 (1.97)	1.008 (1.08)
t-12		.700 (1.48)	.375 (0.53)
t-13			- .291 (0.61)
$\hat{\rho}$	.927 (21.26)	.950 (27.19)	.945 (25.77)
SSE/n-k	3.46	3.26	3.34
$\hat{\sigma}_\varepsilon$	1.87	1.81	1.84
R^2	.877	.888	.886
$F_{(K-1,n-K)}$	48.01	47.40	42.95
n	80	79	78
DW	1.39	1.51	1.47
constrained F-test	1.86**	5.98*	0.44**

Notes: --DW statistics are within the indeterminant range.

Table 8.7.2.4. Regression 8.2.4., $G_t = \Sigma\alpha_i B_{t-i} + \varepsilon_t$, Cochrane-Orcutt Transformation; 1956/3 through 1979/2.

	(1)	(2)	(3)
constant	-1.460 (0.34)	-3.318 (0.72)	-2.753 (0.54)
period t	.610 (0.91)	.522 (0.79)	.553 (0.83)
t-1	1.248 (1.39)	1.450 (1.61)	1.480 (1.62)
t-2	1.288 (1.19)	1.533 (1.41)	1.363 (1.21)
t-3	1.965 (1.53)	2.292 (1.78)	2.088 (1.56)
t-4	1.845 (1.31)	2.500 (1.74)	2.345 (1.59)
t-5	2.243 (1.47)	3.080 (1.97)	2.960 (1.80)
t-6	2.748 (1.72)	3.640 (2.21)	3.498 (2.00)
t-7	1.828 (1.15)	3.065 (1.81)	2.890 (1.61)
t-8	1.618 (1.07)	2.818 (1.75)	2.815 (1.59)
t-9	.890 (0.63)	2.048 (1.35)	2.170 (1.29)
t-10	.673 (0.52)	1.898 (1.34)	2.053 (1.30)
t-11	.151 (0.14)	1.523 (1.21)	1.653 (1.15)
t-12	.315 (0.36)	1.425 (1.36)	1.523 (1.18)
t-13	.555 (0.85)	1.510 (1.74)	1.633 (1.54)
t-14		1.128 (1.75)	1.388 (1.59)
t-15			.227 (0.36)
$\hat{\rho}$	.939 (24.23)	.943 (25.01)	.947 (25.91)
SSE/n-k	4.65	4.51	4.58
$\hat{\sigma}_\varepsilon$	2.17	2.14	2.16
R^2	.838	.846	.841
$F_{(k-1,n-K)}$	29.60	28.76	26.52
n	78	77	76
DW	1.18	1.23	1.26
constrained F-test	0.13**	4.00*	0.91**

Notes: --DW statistics indicate some positive autocorrelation remaining in the residuals.

--The above equations were estimated without the first term. Since the statistical results remained almost identical the decision is made to present the regressions without this term deleted.

Table 8.7.2.5. Regression 8.2.5., $E_t = \Sigma\beta_i C_{t-i} + \epsilon_t$, Cochrane-Orcutt Transformation; 1956/3 through 1979/2.

	(1)	(2)	(3)
constant	- .160 (0.50)	.036 (0.08)	.113 (0.23)
period t	.246 (3.23)	.297 (4.19)	.290 (4.02)
t-1	.337 (3.56)	.333 (3.72)	.344 (3.78)
t-2	.357 (3.50)	.338 (3.50)	.345 (3.38)
t-3	.436 (4.29)	.421 (4.30)	.426 (4.12)
t-4	.322 (3.44)	.335 (3.47)	.344 (3.31)
t-5	.179 (2.44)	.181 (2.02)	.205 (1.96)
t-6		.056 (1.81)	.084 (0.87)
t-7			.042 (0.58)
$\hat{\rho}$	.910 (20.38)	.943 (26.37)	.946 (26.76)
SSE/n-k	.070	.059	.059
$\hat{\sigma}_\varepsilon$	.265	.243	.245
R^2	.8733	.8944	.8943
$F_{(K-1,n-K)}$	98.65	102.62	88.78
n	86	85	84
DW	1.26	1.47	1.42
constrained F-test	6.10*	16.64*	1.16**

Notes: --The DW statistics for eqs. (2) and (3) are very close to the "no autocorrelation" boundary of the indeterminant range.

Table 8.7.2.6. Regression 8.2.6., $E_t = \Sigma\beta_i D_{t-i} + \varepsilon_t$, Cochrane-Orcutt Transformation; 1956/3 through 1979/2.

	(1)	(2)	(3)
constant	- .067 (0.15)	- .076 (0.17)	- .096 (0.22)
period t	.071 (0.94)	.097 (1.24)	.097 (1.22)
t-1	.081 (0.89)	.153 (1.56)	.162 (1.59)
t-2	.051 (0.76)	.169 (1.72)	.183 (1.70)
t-3		.147 (1.90)	.165 (1.64)
t-4			.021 (0.26)
$\hat{\rho}$	.927 (23.43)	.930 (23.85)	.927 (23.11)
SSE/n-k	.084	.083	.085
$\hat{\sigma}_{\varepsilon}$	.292	.289	.292
R^2	.8559	.8585	.8475
$F_{(K-1,n-K)}$	130.18	178.94	96.60
n	89	88	87
DW	1.68	1.72	1.79
constrained F-test	0.64**	3.75*	0.11**

Notes: --The DW statistics indicate no autocorrelation in the residuals.

Table 8.7.2.7. Regression 8.2.7., $E_t = \Sigma\beta_i M_{t-i} + \varepsilon_t$, Cochrane-Orcutt Transformation; 1956/3 through 1979/2.

	(1)	(2)	(2')[1]	(3)
constant	-1.055 (2.05)	-1.310 (2.10)	-1.245 (2.12)	-1.495 (2.20)
period t	.007 (0.12)	.019 (0.33)		.025 (0.44)
t-1	.077 (0.82)	.109 (1.14)	.084 (1.42)	.129 (1.30)
t-2	.222 (1.74)	.274 (2.05)	.241 (2.61)	.304 (2.20)
t-3	.535 (3.29)	.583 (3.50)	.548 (4.21)	.624 (3.61)
t-4	.769 (4.17)	.809 (4.20)	.775 (4.74)	.851 (4.27)
t-5	.949 (4.76)	1.002 (4.72)	.968 (5.22)	1.048 (4.73)
t-6	1.130 (5.52)	1.197 (5.42)	1.166 (5.81)	1.253 (5.34)
t-7	1.145 (5.52)	1.241 (5.45)	1.209 (5.83)	1.305 (5.35)
t-8	1.157 (6.63)	1.277 (5.64)	1.249 (5.95)	1.357 (5.49)
t-9	.997 (4.95)	1.131 (5.05)	1.103 (5.30)	1.221 (4.96)
t-10	.854 (4.38)	1.004 (4.64)	.980 (4.80)	1.099 (4.58)
t-11	.657 (3.55)	.806 (3.19)	.786 (3.99)	.902 (3.94)
t-12	.492 (2.96)	.655 (3.39)	.640 (3.42)	.747 (3.46)
t-13	.377 (2.60)	.529 (3.10)	.521 (3.11)	.630 (3.16)
t-14	.231 (1.93)	.383 (2.55)	.372 (2.54)	.476 (2.67)
t-15	.122 (1.33)	.261 (2.11)	.251 (2.10)	.356 (2.27)
t-16	.033 (0.53)	.154 (1.63)	.148 (1.61)	.239 (1.87)
t-17		.106 (1.67)	.102 (1.65)	.177 (1.83)
t-18				.061 (0.96)
$\hat{\rho}$	.941 (24.18)	.951 (26.73)	.951 (26.53)	.955 (27.53)
SSE/n-k	.054	.053	.052	.054
$\hat{\sigma}_\varepsilon$	.234	.232	.231	.234
R^2	.9109	.9134	.9148	.9129
$F_{(K-1,n-K)}$	45.52	43.78	47.09	40.73
n	75	74	74	73
DW	1.35	1.49	1.49	1.34
constrained F-test	.436**	.442**	.441*	.439**

Notes: --DW statistics are near the "no autocorrelation" boundary of the

Table 8.7.2.7. (continued)

indeterminant range.

--Because of the insignificant first term of eq. (2), the regression was reestimated with this regressor deleted. These results, seen in eq. (2'), show only a marginal improvement in the adjusted R-square statistic and are presented for comparative purposes only.

Table 8.7.2.8. Regression 8.2.8., $E_t = \Sigma\beta_i B_{t-i} + \varepsilon_t$, Cochrane-Orcutt Transformation; 1956/3 through 1979/2.

	(1)	(2)	(3)
constant	.544 (0.33)	.866 (0.46)	.833 (0.47)
period t	- .072 (0.83)	- .111 (1.26)	- .107 (1.08)
t-1	.016 (0.14)	- .022 (0.19)	- .018 (0.15)
t-2	- .005 (0.04)	- .026 (0.18)	- .021 (0.14)
t-3	- .074 (0.47)	- .104 (0.60)	- .099 (0.55)
t-4	.029 (0.15)	- .021 (0.11)	- .015 (0.07)
t-5	.101 (0.46)	.044 (0.20)	.042 (0.19)
t-6	.186 (0.79)	.147 (0.63)	.144 (0.60)
t-7	.253 (1.03)	.252 (1.02)	.247 (0.98)
t-8	.298 (1.21)	- .316 (1.28)	- .304 (1.19)
t-9	.202 (0.84)	- .217 (0.88)	- .201 (0.79)
t-10	- .170 (0.75)	.249 (1.04)	.230 (0.92)
t-11	.168 (0.80)	.282 (1.25)	.259 (1.06)
t-12	.046 (0.24)	.168 (0.80)	.146 (0.64)
t-13	- .144 (0.85)	- .263 (1.38)	.243 (1.15)
t-14	.142 (1.02)	.265 (1.56)	- .243 (1.25)
t-15	.117 (1.03)	- .220 (1.60)	.195 (1.13)
t-16	- .045 (0.52)	- .173 (1.51)	.153 (1.09)
t-17		.129 (1.51)	.115 (0.98)
t-18			- .017 (0.19)
$\hat{\rho}$	.983 (46.60)	.985 (50.36)	.984 (48.54)
SSE/n-k	.079	.076	.079
$\hat{\sigma}_\varepsilon$	.283	.279	.284
R^2	.596	.609	.566
$F_{(K-1,n-K)}$	19.15	29.45	20.09
n	75	74	73
DW	.688	.681	.677
constrained F-test	0.50**	3.60**	0.08**

Notes: --The DW statistics indicate positive autocorrelation.
--The above equations were also estimated without lags t through

Table 8.7.2.8. (continued)

t-4; this did not improve the regression statistics nor materially alter the estimated coefficients.

judging individual marginal regressors is being emphasized, the statistical significance of individual coefficients in the C-0 equations will not be indicated along side of the coefficient as in earlier regression results. (And, in most cases, the coefficients are statistically significant according to the t-test at the 10 percent level.) The tables showing the C-0 results also present the mean sum-of-squared residuals (SSE/n-k). Since the SSE magnitudes are not comparable between regressions with different lag lengths, this statistic is intended to provide a standardized measure of the "goodness of fit" of each equation as lagged regressors are added. The equation with the optimum lag length will be associated with the lowest (SSE/n-k) value.

Because the C-0 regressions will be used exclusively in determining the final lag length and polynomial degree specifications for the Almon regressions, the C-0 results are presented to purposely illustrate the evolution of regression coefficients and related statistics as lagged regressors are added in sequential fashion. Accordingly, in the C-0 tables each model is illustrated via three equations, (1), (2), and (3), each one differing only in terms of the addition of one lagged regressor variable. In these tables, equation (2) is deemed the "best" regression in terms of the adjusted R-squared, t-, and constrained F-test criteria discussed in Section 8.2. Equations (1) and (3) and their related statistics thus provide an idea of how the equation is altered by the addition of one extra variable.

Other pertinent information regarding the estimation of the C-0 regressions is as follows. The constant and its associated standard error have been appropriately adjusted by $(1 - \hat{\rho})$, where $\hat{\rho}$ is the final estimate of the correlation between adjacent residuals after the regression sums-of-squares has been minimized. It is seen that in most regressions the adjusted constant is not significantly different from zero. Theory would imply this result since the dependent variable represents a short-run deviation of output from a long-run trend path.[1] Also, since the first observation is lost during the C-O estimation, the transformation is not exactly the same, in a statistical sense, as a GLS procedure. To remedy this situation the Prais-Winsten procedure could have been utilized to construct the lost observation. However this option was not followed here because with $\hat{\rho}$ being so close to unity the constructed value would have been quite close to zero and thus would have provided little additional information.

Earlier it was stated that the disturbance terms were not correlated with the lagged independent variables in the OLS regressions (see fn. 3, p. 259). This statistical fact however is a necessary but not sufficient condition for the absence of heteroscedasticity. With this in mind, eqs. (2) of Table 8.7.2.1., 8.7.2.3., and 8.7.2.5. [regressions (8.2.1.), (8.2.4.) and (8.2.5.), respectively] were subjected to the Golfeld-Quandt test for homoscedasticity. At the 5 percent level, the null hypothesis of homoscedasticity could not be rejected. This conclusion is expected to hold for the other regressions as well.[2]

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[1] These results would indicate that a constant should not be used in the estimation. All C-O equations were estimated without a constant and statistical and coefficient results were not appreciably changed. The constant was used in order to retrive important statistics from the computer program.

[2] In the Goldfeld-Quandt test of these regressions only one independent variable is used (the regressor with the largest numerical magnitude). Fifteen central observations are omitted from the data assembled by ascending magnitude.

For the most part the C-O regressions have the same lag length as the first-differenced equations and longer lags than the OLS regressions [this is especially true for the money equations (8.2.3.), (8.2.4.), (8.2.7.), and (8.2.8.)]. In all C-O regressions the humped-shape second degree polynomial weighting scheme emerges as the pattern characterizing the coefficient magnitudes [except for the GNP deflator equation (8.2.2.), which exhibits a third-degree polynomial weighting pattern--there is no economic reason why this should be the case]. The coefficient magnitudes and peak lag period of the C-O regressions are also comparable to those of the first-differenced equations.

Since equation (2) in the C-O tables represents the regression with the optimum lag length, the adjusted R-squared value is seen to reach a peak [a corollary statistic is (SSE/n-k), which is seen to "bottom out" at equation (2)]. The adjusted R-squared values appear at acceptable levels [except for eq. (8.2.8.)][1], and the conventionally computed F-values show all coefficients, taken as a group, to be statistically significant [again, except for eq. (8.2.8.)]. Except for the longer-lagged regressions, the constrained F-test supports the judgement that equation (2) is the optimum lag length.

It is evident from the higher DW values appearing in the C-O tables that the iterative estimation of $\hat{\rho}$ in the C-O procedure has reduced the

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[1] The adjusted R-squared values appearing in the C-O regressions are, of course, not comparable to those associated with the first-differenced equations since the dependent variable in the latter regressions is a differenced series. [The R-squared values shown in the C-O tables are computed with the raw (undifferenced) series being used as the dependent variable.] For comparative purposes the C-O-produced R-squared values were computed using the semi-differenced dependent variable; these values were markedly higher than the simple first-differenced R-squared values appearing in Tables 8.7.1.1. and 8.7.1.2.

strength of the residual autocorrelation from that achieved by simple first-differencing. However, while the iterative method has pushed the DW value for some of the regressions into the "no autocorrelation" range, the statistic for other equations remains in the indeterminant range (although in most of these cases the statistic is near the indeterminant-"no autocorrelation" boundary; these cases are indicated in the footnotes accompanying each table).

Since the C-O transformation was undertaken specifically to reduce the remaining autocorrelation in the first-differenced regressions, and since the DW statistic, by itself, is not a good indicator of the virility of the residual dependency, the ACF's for the C-O regression residuals have been computed (for five lags). These results appear in Table 8.7.2.9. For comparative purposes the ACF's for the residuals of the simple first-differenced regressions are also presented. The table shows that the C-O transformation has reduced the strength of the residual dependency from that of the first-differenced equations since the autocorrelations die off quickly. We can conclude that the remaining first-order autocorrelation is quite mild and does not appreciably affect the precision of the C-O estimates nor their respective variances.

The outstanding negative feature of the C-O regressions relates to the monetary base equation (8.2.8.). The continued low DW statistic and the presence of strong autocorrelation along with coefficients that have incorrect signs and insignificant t-values indicate that no lagged relationship between unanticipated monetary base growth rate and the employment rate can be identified. For this reason this model will be dropped from consideration in the single regression analysis to follow. (However, the

Table 8.7.2.9. ACF's for the C-0 and First-differenced Regression Residuals.

GNP Regressions

Equation	(8.2.1.) C_{t-i}		(8.2.2.) D_{t-i}		(8.2.3.) M_{t-i}		(8.2.4.) B_{t-i}	
	C-0	(1-L)	C-0	(1-L)	C-0	(1-L)	C-0	(1-L)
Lag 1	.27	.33	.29	.35	.25	.29	.39	.41
2	.15	.21	.22	.28	.11	.22	.26	.30
3	.11	.15	.14	.17	.03	.10	.27	.21
4	-.06	.11	-.04	.08	-.10	-.01	-.06	.09
5	-.09	.05	-.10	-.01	-.15	-.05	-.13	.05

Employment Regressions

Equation	(8.2.5.) C_{t-i}		(8.2.6.) D_{t-i}		(8.2.7.) M_{t-i}		(8.2.8.) B_{t-i}	
	C-0	(1-L)	C-0	(1-L)	C-0	(1-L)	C-0	(1-L)
Lag 1	.28	.32	.33	.35	.28	.29	.52	.54
2	.11	.22	.17	.17	.05	.19	.29	.31
3	.06	.17	.10	.10	.13	.08	.30	.31
4	-.10	.05	-.03	.02	-.06	-.01	.12	.22
5	-.03	.00	-.16	-.05	-.24	-.10	-.07	.12

Notes: --The lag operator (1-L) indicates the ACF for the first-differenced equations.

monetary base-employment relationship will be analyzed further in the combined inflation/money models to follow.) While the monetary base-GNP relationship appears weak [regression (8.2.4.)], it will be presented so that comparisons between it and the M1-GNP models can be made later in the analysis.

8.7.3. The Almon Regression. The Almon polynomial distributed lag regressions incorporating the C-O iterative method are presented in Tables 8.7.3.1. and 8.7.3.2. The fitting methodology is straightforward in that lag length and polynomial degree specifications gleaned from the C-O equations are applied directly to the Almon regressions. In order to determine if the computations involved in the polynomial estimation and the increased degrees of freedom (in those equations where lag length is greater than t-3) have altered lag length, the "best" Almon equation [that is the Almon equation with the same lag length as the "best" C-O regression, eq. (2)] is "over-" and "underfit" by one more and one less lagged regressor. The previously described statistical analysis is used to assess the propriety of these over- and underfitted Almon equations. In the interest of brevity these results are not presented, and the Almon tables only illustrate the regressions having the optimum lag length. Except for regressions (8.2.1.), (8.2.2.), and (8.2.6.), which have one additional lagged term, the Almon lag lengths are the same as those of their C-O counterparts.

The Almon results show that the regression coefficients mimic closely those derived using the C-O technique alone [one exception is the M1-employment equation (8.2.7.) where the Almon coefficients are somewhat lower than those produced using the C-O method]. All coefficients possess the

Table 8.7.3.1. Almon Distributed Lag Regressions with Cochrane-Orcutt Transformation, Dependent Variable, G_t; 1956/3-1979/2.

Equation	(8.2.1.)	(8.2.2.)	(8.2.3.)	(8.2.4.)
	C_{t-i}	D_{t-i}	M_{t-i}	B_{t-i}
constant	- .988 (0.50)	- .827 (0.34)	-3.657 (1.13)	-2.779 (0.71)
period t	1.077 (1.93)	.680 (1.18)	1.016 (2.60)	.225 (0.39)
t-1	1.773 (2.91)	.726 (1.15)	2.281 (4.07)	.701 (1.09)
t-2	1.896 (2.76)	.645 (1.09)	3.311 (4.35)	1.115 (1.38)
t-3	1.446 (2.38)	.438 (1.07)	4.105 (4.42)	1.468 (1.50)
t-4	.423 (1.87)	.106 (1.02)	4.665 (4.42)	1.761 (1.56)
t-5			4.989 (4.40)	1.992 (1.61)
t-6			5.077 (4.38)	2.162 (1.65)
t-7			4.931 (4.34)	2.271 (1.71)
t-8			4.549 (4.27)	2.319 (1.77)
t-9			3.932 (4.16)	2.306 (1.86)
t-10			3.080 (3.92)	2.323 (1.97)
t-11			1.992 (3.30)	2.097 (2.14)
t-12			.669 (1.44)	1.901 (2.36)
t-13				1.644 (2.59)
t-14				1.326 (2.39)
$\hat{\rho}$	.893 (18.59)	.906 (20.04)	.932 (22.91)	.935 (23.20)
SSE/n-k	4.11	4.43	3.51	4.15
$\hat{\sigma}_{\varepsilon}$	2.03	2.12	1.87	2.04
R^2	.845	.833	.879	.860
$F_{(K-1,n-K)}$	157.38	144.03	189.47	156.69
n	87	88	79	77
DW	1.66*	1.68*	1.69*	1.44***
mean lag	1.75	1.45	5.88	7.86
Σ of coef's.	6.62 (2.72)	2.60 (1.93)	44.60 (4.28)	25.52 (1.91)
polynomial	-00.19 +09.33i $-10.31i^2$	+00.51 +01.41i $-02.27i^2$	-00.48 +22.64i $-23.05i^2$	-00.31 +09.07i $-07.81i^2$

Table 8.7.3.1. (continued)

Notes: --t-values are adjacent to the sum-of-coefficients estimate.
--Mean lag is expressed in terms of quarters
--Eq. (8.2.4.) was estimated without the first term; results were not appreciably altered.
--"*" indicates no autocorrolation at the 5 percent level.
--"***" indicates mild first-order autocorrolation is present.

Table 8.7.3.2. Almon Distributed Lag Regressions with Cochrane-Orcutt Transformation, Dependent Variable, E_t, 1956/3-1979/2.

Equation	(8.2.5.) C_{t-i}	(8.2.6.) D_{t-i}	(8.2.7.) M_{t-i}
constant	- .022 (0.54)	- .101 (0.24)	- .739 (1.29)
period t	.307 (4.39)	.102 (1.30)	.027 (0.49)
t-1	.367 (5.18)	.182 (2.09)	.172 (2.29)
t-2	.387 (4.67)	.194 (1.97)	.298 (2.94)
t-3	.367 (4.15)	.139 (1.60)	.408 (3.22)
t-4	.306 (3.68)	.017 (1.22)	.501 (3.36)
t-5	.206 (2.90)		.576 (3.45)
t-6	.065 (1.98)		.635 (3.51)
t-7			.676 (3.56)
t-8			.699 (3.60)
t-9			.706 (3.63)
t-10			.695 (3.66)
t-11			.667 (3.69)
t-12			.622 (3.71)
t-13			.560 (3.73)
t-14			.480 (3.73)
t-15			.383 (3.66)
t-16			.269 (3.34)
t-17			.137 (2.15)
$\hat{\rho}$	.941 (25.68)	.925 (22.85)	.942 (24.20)
SSE/n-k	.059	.084	.063
$\hat{\sigma}_{\varepsilon}$	.242	.289	.232
R^2	.895	.850	.899
$F_{(K-1,n-K)}$	242.08	164.29	217.50
n	85	87	74
DW	1.71*	1.61**	1.58**
mean lag	2.44	1.67	8.87
Σ of coef's.	2.00 (4.49)	.635 (1.99)	8.52 (3.56)

Table 8.7.3.2. (continued)

polynomial	+0.20 +0.96i $-1.28i^2$	-0.04 +1.08i $-1.21i^2$	-0.13 +3.23i $-3.11i^2$

Notes: --The statistical legend used in this table is the same as that used in (the preceding), Table 8.7.3.1.

--"**" indicates the DW statistics are near the "no autocorrelation" boundary of the indeterminant range.

--Eq. (8.2.7.) was estimated without the first term; the results were not appreciably affected.

correct sign, and, using the "greater-than-unity" criterion, all regressors add to the explanatory power of their respective regressions. It is also observed that, in most cases, the Almon regressions register higher t-values than their C-O counterparts. The mean error-sum-of-squares and standard error of the regression statistics are comparable to those of the C-O regressions. The adjusted R-squared and F-value statistics are seen to be at acceptable levels also. Again, the humped-shape second degree polynomial pattern emerges as the coefficient weighting scheme (it is observed also that the third-degree weighting pattern characterizing eq. (8.2.2.) in the C-O estimation takes on the second-degree scheme when estimated via the Almon method). In all but one case [eq. (8.2.6.)], the Almon estimation increases the DW values indicating that autocorrelation has been further reduced from the C-O results and is at minimal levels.

At this point in the statistical analysis, this study can conclude that for the period under investigation, given the rational expectations generating filters derived earlier, the Almon/C-O regressions illustrated in Tables 8.7.3.1. and 8.7.3.2. represent the correct lag length relating contemporaneous real GNP and employment rate response to the unanticipated inflation and money variables.[1]

The Almon regressions are estimated without endpoint constraints. Since the use of endpoint restrictions in the Almon method are a controversial statistical topic (a topic which has obvious economic implications)[2],

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[1] It is pertinent to note at this juncture that simultaneous equation bias is not theoretically indicated. Since it was demonstrated in the OLS equations that $E(X_{t-1},e_t) = 0$, this theoretical position is supported statistically.

[2] See Schmidt and Waud [106] for an insightful discussion of this topic.

some justification must be tendered for the "no endpoints" approach followed.

As with an a priori statement of lag length and polynomial specification, the imposition of endpoint constraints will increase the efficiency of the estimation only if the endpoint restrictions are appropriate; if endpoint constraints do not belong in the true distributed lag pattern, their use leads to biased and inconsistent estimates.[1]

As with lag length and polynomial degree restrictions, the C-O results do furnish prior information that endpoint constraints are not justified. First, in most of the C-O regressions, the estimated coefficient in time period t is of sufficient magnitude to allow us to infer that the effect of the independent variables on the dependent variable is significantly greater than zero somewhere <u>between</u> periods t and t-1. This fact implies that the length of the initial quarterly data interval is not small enough to capture the immediacy of the output response. Like reasoning applies to the last term in the lagged equations: while the lags are considered to be finite, the magnitudes of most of the last regressors are large enough to make one uncertain that the time dependent effects of the last regressor is exactly zero at the end of the designated lag.

The above stated reservations about the use of endpoint constraints in the lagged regressions however, are of a non-statistical nature and, as such, are inconclusive. To provide statistical support for the "no end-

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[1] The regression misspecification implicit in the improper use of endpoint constraints can be particularly misleading when a second-degree polynomial is being used since the restrictions tend to nudge the shape of the lag distribution towards a near-symmetrical inverted U-shape.

point" position advocated here, the Almon regressions (8.2.1.) through (8.2.7.) were estimated with and without endpoint constraints so that a decision for or against their use could be made based upon statistical inference. Table 8.7.3.3. presents the statistical results. Here the F-test is used to test the null hypothesis that the same equations (i.e. those with the same lag length) with and without endpoint constraints are not statistically different. The results of these tests allow us to accept the null hypothesis. Thus the "no-endpoint" approach is justified.

Table 8.7.3.3. F-test for the Almon Lag Endpoint Constraints Restriction.

Regression		computed F-value
(8.2.1.)	$G_t = \Sigma\alpha_i C_{t-i} + \varepsilon_t$	0.73*
(8.2.2.)	$G_t = \Sigma\alpha_i D_{t-i} + \varepsilon_t$	0.25*
(8.2.3.)	$G_t = \Sigma\alpha_i M_{t-i} + \varepsilon_t$	1.87*
(8.2.4.)	$G_t = \Sigma\alpha_i B_{t-i} + \varepsilon_t$	1.21*
(8.2.5.)	$E_t = \Sigma\beta_i C_{t-i} + \varepsilon_t$	3.50**
(8.2.6.)	$E_t = \Sigma\beta_i D_{t-i} + \varepsilon_t$	0.65*
(8.2.7.)	$E_t = \Sigma\beta_i M_{t-i} + \varepsilon_t$	2.53*

Notes: --"*" indicates not significant at the 5 percent level.
--"**" indicates not significant at the 5 percent level, but significant at the 10 percent level.
--The F-statistic used here is computed as follows:

$$F_{(2,n-k)} = [[SSE(w) - SSE(w\text{-}o)]/2]/[SSE(w\text{-}o)/n-k] \quad ,$$

where SSE(w) and SSE(w-o) are the error-sums-of-squares with and without the endpoint restrictions, respectively.

8.7.4. The Almon Subperiod Regressions. It is of peripheral interest to compare the distributed lag output response models over the two (equal) adjacent subperiods, 1956/3-1967/4 and 1968/1-1979/2. The objective of this analysis is to determine if the unanticipated inflation/money-output response pattern is similar over two different periods of U.S. economic history, one characterized by mild inflation and monetary growth rates (the 1956-1967 period), and one characterized by relatively severe and volitile inflation and money growth rates (the 1968-1979 period). [Note that the subperiod regression independent (unanticipated) inflation and money variables have been obtained from separate ARIMA models estimated over each subperiod. The purpose of using these separately computed forecasting functions is to allow for any structural shifts in the parameters describing the inflation or money growth process over the two periods of time. This, in turn, will allow for a more definitive appraisal of any differences in output response due to rational forecast error.][1]

Estimation of the Almon subperiod equations is identical to the step-wise sequential addition-of-lagged regressors method described in Section 8.2. and used in estimating the full period Almon regressions. In the interest of brevity, only the optimum lag length Almon subperiod equations will be illustrated.[2]

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[1] Since estimating the GNP and employment rate time trend equations over the two periods produced almost identical time trend coefficients and intercepts, the dependent variables used in the subperiod equations are those obtained from the full period time trend equations appearing in Table 8.3.1.

[2] Note that since the number of observations has been halved, the Almon method results in greater efficiency of the subperiod estimates than would otherwise be the case.

The subperiod results appear in Tables 8.7.4.1. through 8.7.4.7. As expected, the coefficients have the correct (positive) sign, and the second-degree polynomial coefficient weighting pattern emerges in the C-O regressions (not shown) as the proper weighting scheme and is used in the Almon equations. Based upon previous considerations, endpoint restrictions are not used in the subperiod regressions. Note also that while some of the subperiod results appear weak, they are presented for comparative purposes if the adjacent subperiod equation in each set appears strong enough to identify a relationship. However, in some cases no subperiod relationship can be identified; in these instances this fact is indicated in the appropriate table. It is also seen that in some subperiod regressions the initial coefficients have t-values which are less than unity. In these cases the insignificant coefficients are retained in the estimation if their subperiod coefficient counterpart appears significant; to have reestimated these regressions without these insignificant terms would have destroyed the basis for comparability.

For the subperiod results it is apparent that the question of coefficient stability between the two periods can be raised (for those cases in which relationships can be identified for both periods); the magnitudes of the coefficients for the 1968-79 period are noticeably larger than those of the 1956-67 period in Tables 8.7.4.1., 8.7.4.3., 8.7.4.5., and 8.7.4.7. Unfortunately the Chow Test is inapplicable here to statistically examine the possibility of inter-subperiod coefficient instability. This is so since the statistical inappropriateness of computing the Chow

Table 8.7.4.1. Almon Subperiod Estimation; Regression (8.2.1.), $G_t = \Sigma\alpha_i C_{t-i} + \varepsilon_t$.

	1956/3-1967/4	1968/1-1979/2
constant	- .536 (0.15)	-1.268 (0.42)
period t	.051 (1.08)	2.357 (2.43)
t-1	1.195 (1.56)	2.409 (2.36)
t-2	1.855 (2.06)	2.104 (2.07)
t-3	2.030 (2.27)	1.442 (1.50)
t-4	1.720 (2.32)	
t-5	.926 (1.54)	
$\hat{\rho}$	.941 (17.73)	.873 (11.60)
SSE/n-k	2.81	3.48
$\hat{\sigma}_\varepsilon$	1.36	2.49
R^2	.858	.841
$F_{(K-1,n-K)}$	79.76	73.29
n	40	42
DW	1.79	1.71
mean lag	2.89	1.36
Σ of coef.'s	7.78 (1.92)	8.31 (2.57)
polynomial	$-1.57 + 13.09i - 11.86i^2$	$1.94 + 2.93i - 4.45i^2$

Notes: --DW statistics indicate no autocorrelation at the 5 percent level.

Table 8.7.4.2. Almon Subperiod Estimation; Regression (8.2.2.), $G_t = \Sigma\alpha_i D_{t-i} + \varepsilon_t$.

	1956/3-1967/4	1968/1-1979/2
constant	-1.731 (1.02)	no relationship is identified for this period.
period t	1.733 (2.72)	
t-1	3.093 (3.87)	
t-2	3.345 (3.56)	
t-3	2.488 (3.08)	
t-4	.523 (1.89)	
$\hat{\rho}$	.882 (12.01)	
SSE/n-k	1.68	
$\hat{\sigma}_\varepsilon$	1.29	
R^2	.872	
$F_{(K-1,n-K)}$	92.29	
n	41	
DW	1.69	
mean lag	1.73	
Σ of coef.'s	11.18 (3.63)	
polynomial	$-0.73 + 18.13i - 19.95i^2$	

Notes: --DW statistic indicates no autocorrelation at the 5 percent level.

Table 8.7.4.3. Almon Subperiod Estimation; Regression (8.2.3.), $G_t = \Sigma\alpha_i M_{t-i} + \varepsilon_t$.

	1956/3-1967/4	1968/1-1979/2
constant	- .192 (0.11)	-9.437 (2.53)
period t	.642 (1.44)	1.448 (2.40)
t-1	1.529 (2.70)	3.635 (3.74)
t-2	2.222 (3.07)	5.386 (3.93)
t-3	2.720 (3.19)	6.701 (3.97)
t-4	3.023 (3.23)	7.580 (3.98)
t-5	3.131 (3.25)	8.022 (3.98)
t-6	3.045 (3.25)	8.029 (3.90)
t-7	2.764 (3.23)	7.598 (3.96)
t-8	2.289 (3.13)	6.732 (3.95)
t-9	1.619 (2.77)	5.430 (3.86)
t-10	.753 (1.56)	3.691 (3.45)
t-11		1.516 (1.80)
$\hat{\rho}$	.880 (11.01)	.844 (9.20)
SSE/n-k	1.46	5.45
$\hat{\sigma}_\varepsilon$	1.21	2.33
R^2	.901	.876
$F_{(K-1,n-K)}$	103.57	79.22
n	35	34
DW	1.61	1.58
mean lag	5.05	5.51
Σ of coef.'s	23.74 (3.17)	65.77 (4.01)
polynomial	$-0.43 + 14.14i - 14.01i^2$	$-1.17 + 36.94i - 36.86i^2$

Notes: --DW statistics are near the "no autocorrelation" boundary of the indeterminant range (at 5 percent level).

Table 8.7.4.4. Almon Subperiod Estimation; Regression (8.2.4.), $G_t = \Sigma\alpha_i B_{t-i} + \varepsilon_t$.

	1956/3-1967/4	1968/1-1979/2
constant	-2.323 (0.41)	no relationship is identified for this period.
period t	.357 (1.40)	
t-1	.831 (1.46)	
t-2	1.212 (1.71)	
t-3	1.498 (1.81)	
t-4	1.691 (1.86)	
t-5	1.790 (1.92)	
t-6	1.795 (1.98)	
t-7	1.707 (2.06)	
t-8	1.525 (2.15)	
t-9	1.249 (2.20)	
t-10	.879 (1.84)	
$\hat{\rho}$	.963 (21.30)	
SSE/n-k	1.61	
$\hat{\sigma}_\varepsilon$	1.27	
R^2	.890	
$F_{(K-1,n-K)}$	93.05	
n	35	
DW	1.31	
mean lag	5.40	
Σ of coef.'s	14.53 (2.01)	
polynomial	$-0.20 + 7.37i - 6.74i^2$	

Notes: --DW statistic is in the indeterminant range (at 5 percent level).

Table 8.7.4.5. Almon Subperiod Estimation; Regression (8.2.5.), $E_t = \Sigma\beta_i C_{t-i} + \varepsilon_t$.

	1956/3-1967/4	1968/1-1979/2
constant	- .183 (0.42)	.131 (0.17)
period t	.188 (1.83)	.401 (4.01)
t-1	.267 (2.13)	.483 (5.14)
t-2	.307 (2.00)	.494 (4.51)
t-3	.309 (1.88)	.433 (3.95)
t-4	.272 (1.78)	.301 (3.21)
t-5	.196 (1.59)	.098 (1.00)
t-6	.081 (1.16)	
$\hat{\rho}$	.921 (14.85	.949 (19.08)
SSE/n-k	.046	.066
$\hat{\sigma}_\varepsilon$	.214	.256
R^2	.798	.927
$F_{(K-1,n-K)}$	51.24	165.99
n	39	40
DW	1.69	1.57
mean lag	2.69	2.02
Σ of coef.'s	1.62 (1.99)	2.21 (4.61)
polynomial	$0.07 + 1.09i - 1.24i^2$	$0.24 + 1.32i - 1.74i^2$

Notes: --DW statistic for the first subperiod indicates no autocorrelation; DW statistic for the second subperiod is near the "no autocorrelation" boundary of the indeterminant range (5 percent level).

Table 8.7.4.6. Almon Subperiod Estimation; Regression (8.2.6.), $E_t = \Sigma\beta_i D_{t-i} + \varepsilon_t$.

	1956/3-1967/4	1968/1-1979/2
constant	- .360 (1.40)	no relationship is identified for this period.
period t	.083 (1.70)	
t-1	.349 (2.23)	
t-2	.430 (2.73)	
t-3	.326 (2.69)	
$\hat{\rho}$	.861 (10.99)	
SSE/n-k	.055	
$\hat{\sigma}_\varepsilon$	.233	
R^2	.827	
$F_{(K-1,n-K)}$	3.55	
n	42	
DW	1.55	
mean lag	1.84	
Σ of coef.'s	1.19 (2.59)	
polynomial	$-0.36 + 2.71i - 2.31i^2$	

Notes: --DW statistic is within the indeterminant range (5 percent level).
--Computed F-value is significant at the 5 percent level, but not significant at the 1 percent level.

Table 8.7.4.7. Almon Subperiod Estimation; Regression (8.2.7.), $E_t = \Sigma\beta_i M_{t-i} + \varepsilon_t$.

	1956/3-1967/4	1968/1-1979/2
constant	- .272 (1.15)	1.706 (0.61)
period t	.073 (0.92)	.042 (0.57)
t-1	.074 (0.69)	.260 (2.18)
t-2	.129 (0.89)	.438 (2.59)
t-3	.179 (0.96)	.578 (2.75)
t-4	.223 (1.03)	.670 (2.83)
t-5	.278 (1.19)	.725 (2.90)
t-6	.349 (1.47)	.740 (2.96)
t-7	.421 (1.82)	.714 (3.00)
t-8	.455 (2.01)	.647 (3.06)
t-9	.409 (1.79)	.539 (3.09)
t-10	.269 (1.08)	.391 (2.96)
t-11	.091 (1.07)	.202 (1.98)
t-12	.067 (1.05)	
$\hat{\rho}$	.836 (8.76	.984 (32.32)
SSE/n-k	.035	.089
$\hat{\sigma}_\varepsilon$	.200	.298
R^2	.824	.912
$F_{(K-1,n-K)}$	22.47	116.19
n	33	34
DW	1.84	1.75
mean lag	6.63	5.85
Σ of coef.'s	3.02 (2.33)	5.95 (2.89)
polynomial	$-0.30 + 6.24i - 5.58i^2$	$-0.21 + 3.62i - 3.44i^2$

Notes: --DW statistics indicate no autocorrelation at the 5 percent level.
--Both subperiod equations were estimated without the first term; statistical results were not appreciably affected.

F value stems from the fact that 1) the subperiod equations have different lag lengths, 2) the regressor matrices are different, and 3) the error variance of the second subperiod is larger than the error variance of the first subperiod. Nonetheless, even without the benefit of statistical inference, the substantial difference in coefficient magnitudes strongly supports the suspicion that the "true" coefficients of the second period are larger than those of the first period.

8.8. Elasticity and Other Comparative Measures of Output Responsiveness

Tables 8.8.1. and 8.8.2. present full- and subperiod elasticity measurements, respectively, of real GNP and employment rate responsiveness with respect to the unanticipated inflation and money growth rate variables. Only the Almon results of regressions (8.2.1.) through (8.2.7.) are used in the elasticity figures.

The elasticity measurements are computed in a slightly different manner than the conventional computation of elasticities using distributed lag regression parameter estimates. Usually, the average values of the dependent and independent lagged regressors are used in the elasticity computation. However, since the output variables are detrended, and since the unanticipated variables are random noise processes, both measurements have mean values quite close to zero; their ratio is economically meaningless. To skirt this problem, the absolute values of the dependent and independent variables are used in constructing the ratio of mean values. This is a plausible approach since interest here is in the magnitude of the output response, not in the direction (i.e., sign) of the response. The elasticity results are presented in the form of total or long-run figures and annualized figures. Since the total elasticity

Table 8.8.1. Elasticity of Output with Respect to the Unanticipated Inflation and Money Growth Rate Variables; 1956/3-1979/2.

	Total (Long-run) Elasticity	Annualized Elasticity
$\eta_{G/C}$	.434	.347
$\eta_{G/D}$	.181	.145
$\eta_{G/M}$	4.212	1.296
$\eta_{G/B}$	1.946	.519
$\eta_{E/C}$	.904	.517
$\eta_{E/D}$	.305	.244
$\eta_{E/M}$	5.542	1.232

Note: --Absolute mean values for unanticipated and output variables are:

C = .2957

D = .3141

M = .4257

B = .3436

G = 4.5075

E = .6544

Table 8.8.2. Elasticity of Output with Respect to the Unanticipated Inflation and Money Growth Rate Variables: Subperiod Measurements.

	1956/3-1967/4		1968/1-1979/2	
	Total	Annualized	Total	Annualized
$\eta_{G/C}$	.556	.371	.509	.509
$\eta_{G/D}$	.756	.604	-----	-----
$\eta_{G/M}$	2.366	.860	5.954	1.985
$\eta_{G/B}$	1.394	.507	-----	-----
$\eta_{E/C}$	.742	.424	.989	.659
$\eta_{E/D}$	.514	.514	-----	-----
$\eta_{E/M}$	1.927	.593	3.932	1.311
$\eta_{E/B}$	-----	-----	-----	-----

Note: --Absolute mean values for the unanticipated and output variables over the two subperiods are:

	1956/3-1967/4	1968/1-1979/2
C	.2743	.3171
D	.2593	.3689
M	.3825	.4688
B	.3681	.3190
G	3.8370	5.1780
E	.5998	.7091

measurements are based upon lagged regressions with different lengths, the elasticity coefficients are not strictly comparable. To arrive at elasticity measurements that are comparable the total figures have been "annualized" by multiplying the total elasticity magnitude by 4/m, where m is the number of lags in the particular regression under consideration.

Table 8.8.3. presents partial elasticity measurements for the 1956-1979 period. These elasticities are computed in the same manner as the total elasticities in Table 8.8.1. except that 1) the slope coefficient peculiar to a specific lagged period replaces the sum of all lagged coefficients for the total slope measurement, and 2) the mean values of the dependent and independent variables are recomputed as the lag length is increased (thus taking into account the loss of one observation per lag in the data matrix). Because individual slope coefficients have been used in the partial elasticity computations, the elasticity magnitudes follow the familiar humped-shape second degree polynomial pattern.

Table 8.8.4. presents the percent cumulative response of the GNP and employment rate output variables effected by the unanticipated variables. Each magnitude is computed by taking the ratio of the slope coefficient at a particular point in time relative to the total sum of the response over the full lag (i.e., the sum of the coefficients of the regression). These percentages are then added period-by-period to give a cumulative output response pattern over the complete lag length.

8.9. Economic Analysis of Results: Interpretation and Opinion

This section examines the economic aspects and implications of the statistical results of the Almon distributed lag regressions and elasticity measurements derived in Sections 8.7.3., 8.7.4., and 8.8. This analysis

Table 8.8.3. Partial Elasticity of Output with Respect to the Unanticipated Inflation and Money Growth Rate Variables; 1956/3-1979/2.

	$\eta_{G/C}$	$\eta_{G/D}$	$\eta_{G/M}$	$\eta_{G/B}$	$\eta_{E/C}$	$\eta_{E/D}$	$\eta_{E/M}$
Period t	.0707	.0474	.0959	.0171	.1387	.0490	.0176
t-1	.1163	.0506	.2153	.0534	.1658	.0874	.1119
t-2	.1244	.0450	.3126	.0850	.1749	.0931	.1938
t-3	.0949	.0305	.3875	.1119	.1658	.0667	.2654
t-4	.0277	.0074	.4404	.1342	.1383	.0082	.3259
t-5			.4710	.1518	.0931		.3747
t-6			.4793	.1647	.0294		.4131
t-7			.4655	.1731			.4397
t-8			.4294	.1767			.4547
t-9			.3712	.1757			.4593
t-10			.2908	.1770			.4521
t-11			.1880	.1598			.4339
t-12			.0632	.1449			.4046
t-13				.1253			.3643
t-14				.1010			.3122
t-15							.2491
t-16							.1750
t-17							.0891

Table 8.8.4. Percent Cumulative Output Response Pattern; 1956/3-1979/2.

	Cumulative GNP Response				Cumulative Employment Response		
	C_{t-i}	D_{t-i}	M_{t-i}	B_{t-i}	C_{t-i}	D_{t-i}	M_{t-i}
Period t	.1628	.2619	.0228	.0088	.1534	.1606	.0032
t-1	.4308	.5416	.0739	.0363	.3368	.4472	.0234
t-2	.7174	.7901	.1482	.0800	.5302	.7528	.0583
t-3	.9360	.9588	.2402	.1375	.7136	.9717	.1062
t-4	1.0000	1.0000	.3448	.2065	.8666	1.0000	.1650
t-5			.4567	.2845	.9695		.2327
t-6			.5705	.3693	1.0000		.3072
t-7			.6811	.4582			.3865
t-8			.7831	.5491			.4686
t-9			.8713	.6394			.5515
t-10			.9403	.7305			.6331
t-11			.9850	.8126			.7114
t-12			1.0000	.8871			.7844
t-13				.9515			.8501
t-14				1.0000			.9064
t-15							.9514
t-16							.9830
t-17							1.0000

is divided into four parts: Section 8.9.1. will state some introductory remarks which will set the stage by which the regression results should be interpreted. While these comments have previously been stated by implication, it is now necessary to make explicit the ground rules by which the regression results will be judged and, more importantly, compared. Section 8.9.2. provides a statement of the chief findings, conclusions, and supporting economic interpretation of the results. Section 8.9.3. gives a follow-up explanation of the major findings by examining, in depth, the results in terms of the transmission mechanisms that were described in Chapter III. Here the previously unresolved issues concerning the competing transmission hypotheses (especially with respect to the money models) will be resolved by comparing the inflation and money models' regression results. Section 8.9.4. will compare and contrast selected statistical findings of the inflation and money models and offer opinion and theoretical justification for the different results obtained from the regression equations.

8.9.1. Economic Interpretation: Introductory Remarks. An important feature of the economic analysis provided in this research is that we can compare the output response due to two different forms of exogenous shocks, unanticipated inflation and unanticipated money growth. Such comparisons are, understandably, of scientific (and policy-making) interest. The legitimacy of making such comparisons is hightened when viewed from the vantage point of recent trends in empirical investigations involving expectations-oriented output behavior. Clearly the culmination of recent research trends involving the expectations/output nexus points in the direction of investigating unanticipated inflation and unanticipated money (on output) as economic phenomena worthy of separate empirical analysis and investigation. Statistically this trend is reflected in research which goes directly to the estimation of reduced-form models, without resorting to (even) a verbal description of the structural system from which the reduced-form regressions might have come. This trend in inflation and money research should not (automatically) be labeled as "theoretical oversight," as it simply reflects the desires of economists to be as expedient as possible in statistically investigating the new expectations-based theories of real economic activity. Thus the resurgence of monetarism and monetary explanations of output and the vexing problem of simultaneous inflation and unemployment has naturally lead researchers to rationalize reduced-form modeling of the inflation and money Phillips curves because they qualify as single topics worthy of individual, rigorous analysis.

The economic models analyzed in this section reflect these current trends in expectations research and modeling in that they are reduced-form structures. Thus an interesting feature of this study is that by using the same output data, time period, and expectations-generating procedures, the model estimations will allow for meaningful comparisons of these two "separate" economic phenomena; much of the spirit of the results will be lost if one does not remember that the inflation and money models, while conceivably part of a larger structural system, are viewed here as independent phenomena meriting independent analysis. This approach thus provides for objective comparisons of the effects of unanticipated inflation and unanticipated money on economic activity, in addition to a comparative analysis of possible transmission mechanisms which can plausibly be entertained in explaining any observed differences in inflation and money output patterns.

8.9.2. General Findings and Economic Interpretations. The following topics represent the general findings of this study. Specific elaborations of the particulars of these general comments will be presented in Section 8.9.4.

1. For the 1956-1979 time period investigated, aggregate rational forecast error made in predicting the inflation or monetary growth rate is not neutral in the short-run; innovations in the anticipated inflation/money growth rate variables are directly related to short-run output response patterns (as measured by deviations of real GNP and the employment rate from log-linear trend). These conclusions are supported by statistical results which are robust--the unanticipated inflation/money growth rate-output response relationship is substantiated using either real GNP or the employment rate as the index of economic activity.

These findings empirically support the current theoretical concensus of the literature surrounding the auction market-inflation/unemployment paradigm: a short-run Phillips curve will emerge when the rational forecasts of economic agents do not concur with ex post realizations of the inflation or money growth rate.[1] (This consensus, of course, excludes

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[1] Unfortunately because of the autocorrelated nature of the quarterly anticipated inflation/money growth rate variables which have been generated from the ARIMA forecasting functions, it is impossible to test the theoretical corollary implicit in the expectations-based Phillips curve, i.e., movements in output from trend are insensitive to that portion of the inflation or money growth rate that is anticipated. Note however, in contradiction to some current statements in the literature, this writer does not feel that the inability to verify the "invariance of output to anticipated variables" hypothesis econometrically renders the testing of the unanticipated variable/output relationship a meaningless statistical exercise. See "Comments" by Robert Gordon to Barro and Rush, in Fischer [30].

proponents of the implicit contract and obligational markets schools, which rely on non-market clearing explanations of expectations-induced output response (See Fischer [82]).

The finding that an expectationally-based "inflation" Phillips curve can be identified over this period supports the Lucas information lag/relative-absolute price confusion hypothesis, i.e., general increases in the inflation rate can incite increased levels of economic activity because 1) the unanticipated portion of the inflation rate is mistakenly interpreted by agents as a true increase in relative prices, and, 2) knowledge of the true structure of relative prices is received only with a lag.

Additionally, since a positive output response does occur, we can conclude that a positively-sloped inflation Phillips curve does not characterize this period of time. Thus, while the period is marked by an inflation rate with a large variance (over the 1956-1979 period, for example, the quarterly CPI has a mean inflation rate of 1.063 with a variance of .893; the GNP deflator variable exhibits similar mean and variance magnitudes), this excessive volatility is apparently not severe enough to cause the amount of price level confusion and uncertainty which would lead agents to withdraw completely from market activity. These results imply that the empirical fact of "stagflation" which characterizes the latter part of the 1956-1979 period cannot be completely explained by the LH and the price confusion argument so prevalent in current auction market literature investigating the stagflation phenomenon. Thus the claim that excessive "noise" in the price signalling mechanism is responsible for recent high unemployment with high inflation is not borne out by this study.

These results also show that a short-run "money" Phillips curve can be identified over the 1956-1979 period. However, while these statistical findings do support the theory that says unanticipated nominal money "causes" output, we cannot say more about the nature of the relationship, since two competing transmission mechanisms are involved. Whether the observed output response supports the traditional monetarist paradigm surrounding the RBE, with unanticipated money leading directly to more liquidity and spending, or the RBE-induced price confusion-to-output hypothesis (the LH), is an analytical question to be addressed in Section 8.9.3. However, we can state at this juncture that unanticipated nominal money growth "causes" output because it initially adds to real balances and liquidity (as outlined in Section 3.6.3.).

Indirectly, these results also substantiate one part of the NRH: the level of economic activity can be temporarily pushed above the "natural level" by rational forecasting error, whether that error is due to mistakes in predicting inflation or money growth. Note though that any reference between this study and the validity of the NRH is only cursory since 1) no "natural rate" of economic activity is specified in the output models (although the log-trend rate of growth of the GNP and employment rate variables might reasonably be interpreted as a proxy for the traditionally defined natural rate of growth), and, 2) there is no mechanism in the inflation or money regression equations which allows the estimated short-run Phillips curves to shift to the right over time.

2. A second major finding of this study is that certain forces endogenous to the economic system destroy the immediacy of the inflation/money growth-to-output response tradeoff; output response is shown to be time distributed, with the effects of the unanticipated variables first rising and

then falling in magnitude. The humped-shape pattern of the lag coefficients thus reflect the endogenous workings of the cumulative multiplier process triggered by exogenous expectational inflation and money shocks.

The time distributed response of output leads to the conclusion that the conventional (i.e., comparative static) expectationally-based short-run Phillips curve is more properly specified when the statistical form by which it is estimated allows for the lagged, or "persistence," effects of expectational shocks on output. This study thus demonstrates that economically plausible results can be obtained by viewing the Phillips curve phenomenon in a more dynamic setting than has been seen in past empirical studies of the short-run inflation/money tradeoff phenomenon (note that a similar study by Barro and Rush [30] investigating the distributed lag effects of unanticipated nominal money growth on output and employment was published after this study was conceived and finished). The empirical finding that output lags do exist can thus be reconciled with the underlying theory of the models since the lagged format allows for the fact that information, perception, adjustment costs, multiplier and other "inertia constraints" do alter the strict contemporaneous relationship between economic activity and unanticipated inflation or monetary growth. Additionally, the distributed lag results verify the maintained (Lucas) hypothesis that some part of the autocorrelated nature of output deviations from trend, i.e., the "persistence of unemployment," can be explained by random (i.e., rational) expectational shocks.[1]

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[1] It is interesting to point out the subtle movement in macrotheory literature from the use of the rational expectations concept as an explanation of short-term movements in economic activity to the

3. The third major conclusion of this study is that unanticipated monetary growth produces a longer-lasting and more pronounced effect on real output than unanticipated inflation over the time period studied (i.e., in a comparative static sense the "money" Phillips curve is less steeply sloped than the "inflation" Phillips curve).[1] Given the cumulative effects of the multiplier and the fact that the magnitudes of the regression coefficients and lag lengths are directly related, this result is expected. That is, the longer-lagged money regressions simply reflect the fact that money shock impacts (as reflected in the money regression coefficients), being greater than their inflation regression counterparts, take longer to work through the system once the multiplier process is triggered; coefficient magnitudes and lag lengths in the regressions are not independent.

The finding that money shocks have a greater impact on output than inflation shocks would imply, as least superficially, that the RBE associa-

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use of the rational expectations idea as an explanation of the "persistence" of unemployment and the business cycle. This synthesis of expectations theory, mostly due to the work of Lucas, as an explanation of the business cycle is not so mysterious when one realizes that when we cumulate expectations-induced short-term movements in economic activity (in a time series sense), we have a "business cycle." The Lucas use of the rational expectations idea as an explanation of the business cycle then is simply a coherent and logical evolutionary step in the return of cycle theory to a microeconomic explanation of general disequilibrium when information lags are combined with the cumulative multiplier process.

[2] The relative weakness of unanticipated inflation on output found here concurs with other related investigations in the literature. Hall [28] for example, using comparative static techniques within a rational expectations framework found the inflation-unemployment tradeoff to be weak also. Using quarterly data for the U. S. from 1954 through 1974, he found less than 1.7 percent of the variation of the employment rate from the natural rate to be attributable to unanticipated inflation. Other studies have found the effects of unanticipated monetary growth on output to be considerably stronger. See Barro [27 , 29].

ted with unanticipated additions to the nominal growth rate of the money stock are more powerful than inflation-induced changes in perceived relative prices. (Note that this statement is made without reference to the competing transmission mechanisms which can plausibly be entertained in explaining the different lag lengths observed in the inflation and money models; that topic is treated in Section 8.9.3.; what is being stated here is that the initial impact of a money shock is greater than the initial impact of an inflation shock.)

Two opinions can be offered to support this finding. First, it is plausible to reason that unanticipated monetary growth represents a very tangible alteration to the wealth position of economic agents, whereas unanticipated inflationary shocks, being composed of two inflation rate components (the actual and anticipated rates), may be more illusory and thus more difficult for agents to assess. An attendant consideration is that unanticipated inflation, once perceived, may be given different weight by different agents, i.e., unanticipated inflation may be interpreted differently by different individuals. Conversely, unanticipated increases in money and real liquidity in the system may be more easily and consistently recognized than inflation shocks and, therefore, when they occur produce more vigorous and predictable output response patterns. Note that this reasoning does not violate the Lucas information lag explanation of the short-run Phillips curve; both unanticipated inflation and nominal money growth can still be received with a lag. In this context however, the stronger reaction of output to unanticipated money would imply that agents place more "content" in money signals (i.e., liquidity signals) than price signals even though both types of "information" are received with a lag.

A second reason supporting the relatively greater initial strength of the unanticipated money shocks can be explained by the fact that the actual M1 and monetary base growth rate series exhibit less variation over the 1956-1979 period than the inflation rate series (see Table 6.5.1.). This reasoning follows that ellucidated by Lucas [19] in his study of the inflation-unemployment tradeoff over different countries: the inflation-unemployment tradeoff deteriorates as the variance of the inflation rate increases. That is, during a given period of time, the actual variation of the inflation or money growth rate series is inversely related to the magnitude of the output response.[1] Given the fact that the volatility of the inflation rate series is greater than that associated with the money series, we can posit, under rational expectations, that economic agents' perceived distribution of "anticipated rates of inflation" has a greater variance than that associated with the distribution of "anticipated rates of monetary growth." Thus an actually observed rate of inflation over this period of time has a greater probability of falling within a range of values which would not be interpreted as indicative of a "true" increase in relative prices and an improvement in the terms of trade. Thus no output response would appear. Conversely, the probability of an actually observed monetary growth rate falling within the "unanticipated" range (an thus inducing an output response via increases in unanticipated liquidity

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[1] While Lucas' comparisons are confined to inflation rate variation across different (Latin American) countries, his reasoning can be applied with equal generality to monetary growth rate variation and output response.

is much greater. Consequently, on average, a given inflation rate forecast error over this period of time would not be expected to elicit as great an output reaction as that associated with a given monetary growth rate forecast error.

Tables 8.8.1., 8.8.3., and 8.8.4. (pp.301 -05) provide elasticity measurements to support the observed unanticipated variables/output response patterns discussed above. Figures 8.9.2.1. and 8.9.2.2. give a pictorial summary of the length and magnitude of each of the unanticipated variable coefficient weighting patterns as they apply to the GNP and employment rate output variables. The weights have been normalized at unity to make them comparable. (Note that the monetary base and M1 variables in Figures 8.9.2.1. and 8.9.2.2., respectively, have been normalized at period t-1 to remove the abnormally low coefficient values of the first period.)

8.9.3. Analysis and Comparison of the Regression Results in Terms of the Hypothesized Transmission Mechanisms. Two forms of transmission mechanisms, the Lucas Hypothesis (LH) and the RBE, have been nominated as candidates to explain how the unanticipated inflation and money growth rate impulses are transmitted to the GNP and employment rate variables. As outlined in Section 3.6., in order to properly accommodate competing explanations of the unanticipated money-to-output linkage, the RBE has been subdivided into two alternative linkages; a) the RBE-direct-to-output linkage in which unanticipated money leads directly to more spending and output via more liquidity, without resort to the LH and price confusion [hereafter this variant of the RBE will be referred to as the real balance-direct-effect (RBDQ)]; b) the RBE-induced price confusion linkage in which unanticipated money affects output indirectly via unanticipated inflation, without resort

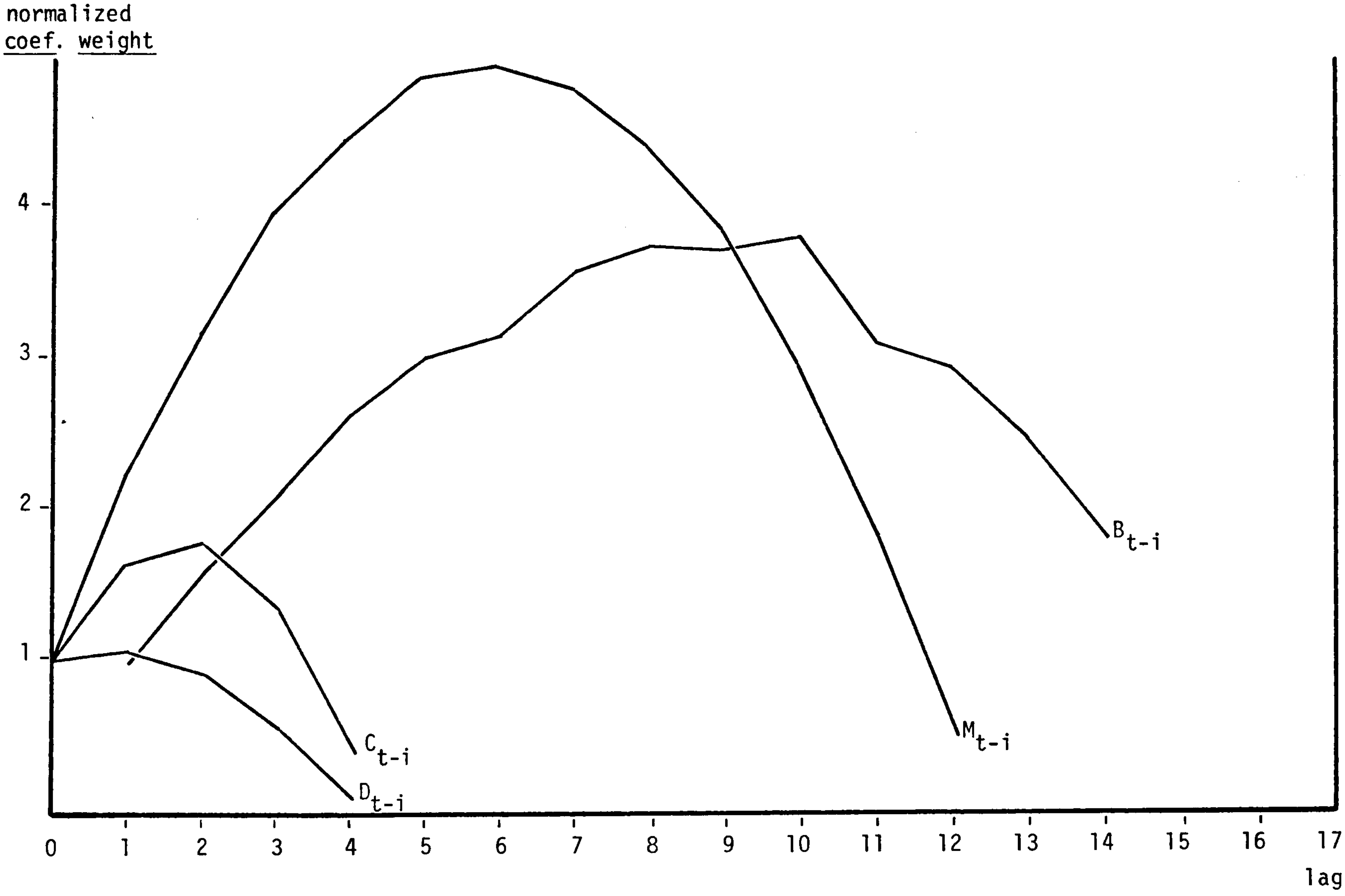

Figure 8.9.2.1. Normalized Unanticipated Variables Coefficient Weighting Patterns; Dependent Variable, Real GNP, G_t; (Almon Regressions).

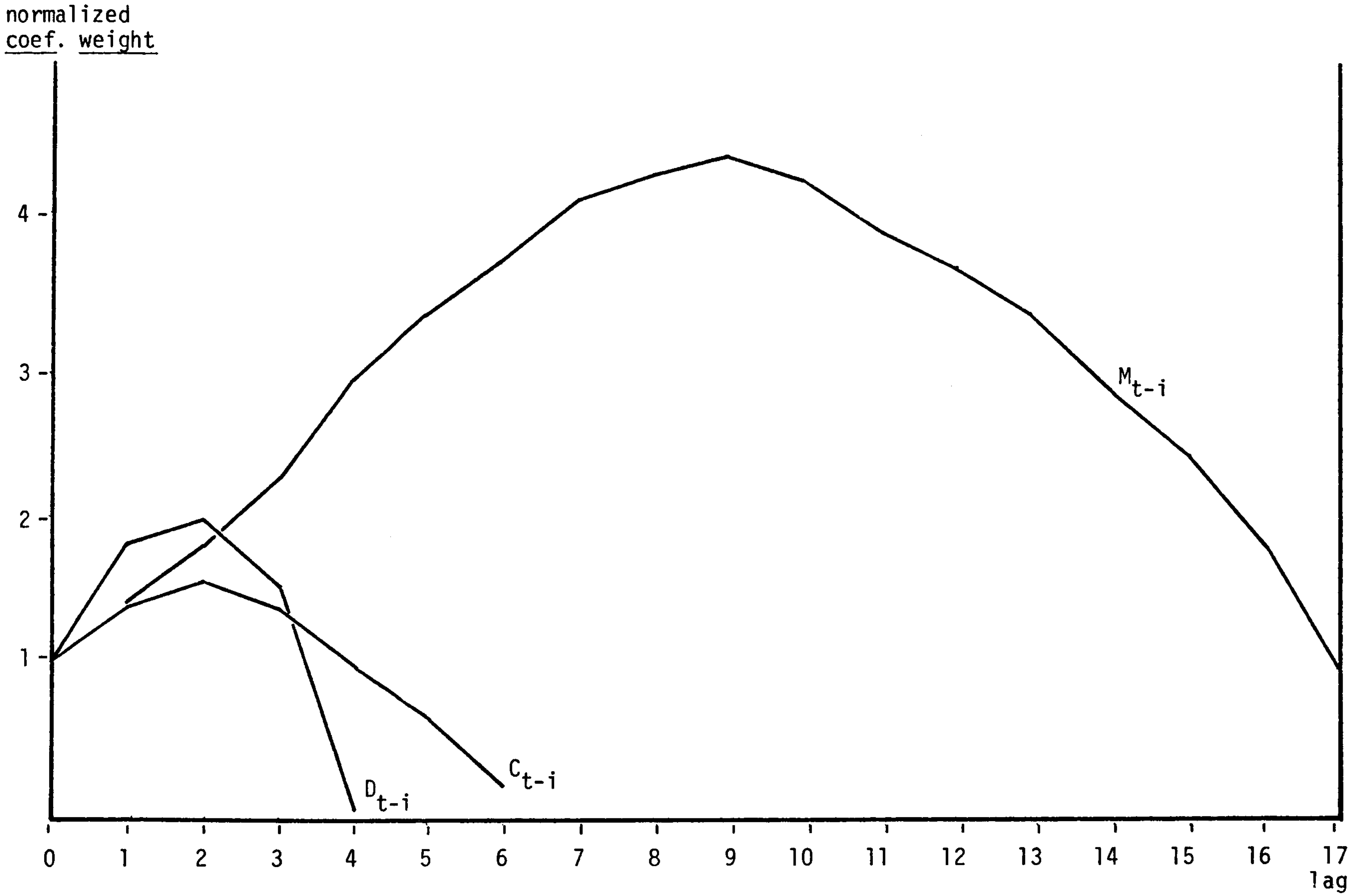

Figure 8.9.2.2. Normalized Unanticipated Coefficient Weighting Patterns; Dependent Variable, Employment Rate, E_t; (Almon Regressions).

to a pure liquidity effect. Here the LH is used as an intermediate link in the money-to-output causal chain (hereafter this variant of the RBE will be referred to as the RBLHQ effect so as to identify the presence of the LH in the money-to-output linkage).

The regression results of the inflation models as seen in Tables 8.7.3.1. and 8.7.3.2. support the LH as the appropriate transmission mechanism relating unanticipated inflation to deviations of real GNP and the employment rate from log-linear trend. As such, the LH provides a straightforward and theoretically sound explanation of why unanticipated inflation "causes" output; inflation shocks cause the required price confusion, as described in Section 3.6.1., which leads agents to temporarily misconstrue relative and absolute price movements, given the accompanying assumption that price information is not instantaneously disseminated throughout the economy.

Note however that even though the inflation regressions are statistically sound, the results should not be interpreted as a direct statistical "proof" that the LH is "the" transmission mechanism relating the exogenous inflation shocks to output. Making such a strict interpretation here would obviously be strained given the fact that the inflation models and related analysis provide no method to directly observe price confusion over the period studied. However, we can appeal to previous empirical work to indirectly verify the LH as the "true" process connecting unanticipated inflation to output. Statistical analysis of the U.S. economy (Park [25]) and at the international level (Logue and Willett [24]) shows a very strong positive correlation between the magnitude of unanticipated inflation and the variance of relative prices. Thus even though this study does not provide specific statistical measurement of relative price variation, we can appeal to the above-mentioned studies to verify that the unanticipated inflation time series measurements used here have resulted in price confusion via increased relative price variation over the 1956-1979 period. In this manner, we can say that the measured unanticipated inflation is a proxy for the unmeasured relative price confusion over the period.

Again, it should be stressed that a distinctive feature of the LH as it pertains to the regression results of the inflation models is that it is a very general linkage mechanism, i.e., the source of shifts in nominal demand which cause the unanticipated inflation are not identified. Moreover, given the fact that the inflation-unemployment question is approached as a topic worthy of "independent investigation," these "causes" need not be specified in order for the price confusion and information lag explanations of the negatively-sloped Phillips curve to have both theoretical and operational acceptability. Hence this inflation research can rightfully plead agnosticism as to the causes of price confusion, but can still remain "scientific" within the confines of accepted inflation-unemployment statistical investigation.

The analysis of the money models' regression results, however, cannot remain agnostic in terms of the transmission mechanism linking unanticipated nominal money growth to output, as there are two competing hypotheses capable of explaining how unanticipated money causes output, the RVDQ efffect, and the RBLHQ effect.[1]

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[1] It is noted that the RBLHQ effect is the transmission mechanism used by Barro's money models, as described in Section 3.6.2.

Since the money regressions do not provide a way to statistically discriminate between real balance-induced price confusion (the RBLHQ effect), and real balance-induced spending (the RBDQ effect), the task here is to utilize the empirical comparisons of the inflation and money regression results to *infer* which transmission hypothesis can most reasonably be entertained. Thus, while Section 3.6.3. posited the RBDQ effect as the (predominant) linkage connecting unanticipated money to output, this transmission mechanism is, at this point, still considered an assumption, and thus open to rejection.

As stated in Section 8.9.2. (and illustrated in Tables 8.7.3.1. and 8.7.3.2. and Figures 8.9.2.1. and 8.9.2.2.), unanticipated nominal money growth produces a stronger (i.e., larger coefficient magnitudes) and a more sustained (i.e., longer lags) output response than that produced by unanticipated inflation. *This comparative regression evidence leads this study to conclude that the unanticipated money-to-output transmission mechanism is best described by the RBDQ effect and not the Barro-type mechanism illustrated by the RBLHQ effect.* That is, the evidence supports the conclusion that, over the 1956-1979 period, money shocks affect output primarily through unanticipated increases in liquidity which lead directly to increases in spending and output. These results also imply that the money-to-output process does not rely strongly on the LH nor on a money-generated unanticipated inflation to cause an output response.[1]

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[1] Note that whether one appeals to the RBDQ or the RBLHQ effect, the longer lags of the money regressions can be justified from another standpoint; the inflation regression measurements "enter" the causal chain at the price confusion link, which is much closer to output than the (initial) unanticipated money link.

The reasoning supporting this inference stems from the inability to reconcile the observed differences in the coefficient magnitudes and lag lengths of the inflation and money regressions using the RBLHQ effect alone. That is, if unanticipated money does "cause" output by first generating a RBE-induced unanticipated inflation, the inflation and money regressions would display similar coefficient magnitudes and lag lengths, since the inflation regressions would supposedly be measuring the same transmission chain (as described by the RBLHQ effect) as the money regressions except at the (later) unanticipated inflation link. Since the results are obviously dissimilar, we are left with the task of reconciling the observed regression discrepancies using the RBDQ effect transmission linkage instead of the RBLHQ linkage. The inference then is that unanticipated increases in liquidity are strong enough to short-circuit the price confusion link and affect, in direct fashion, spending and output; the price confusion link is not (or only weakly) involved. By using the inflation models as a point of reference then, this study concludes that the money models estimated here do not rely on the LH, but are better characterized by the workings of the traditional RBE as described in eqs. 3.6.3.7. through 3.6.3.12. of Chapter III.

One caveat is in order with respect to this conclusion, however. Acceptance of the RBDQ effect as an apt description of the money-to-output linkage does not mean we can completely deny the RBLHQ effect; based upon the differences in the inflation and money regressions alone, one cannot completely reject the fact that some portion of the output response may be due to some RBE-induced clouding of relative and absolute prices. Since the money analysis does not specifically identify an inflation response per se, it is impossible to be more exact on just how much of the output response is due purely to price confusion. However, given the substantially stronger and more prolonged response of the money regressions, we can conclude that any movement in real economic activity caused by RBE-induced price confusion is quite weak compared to the alternative transmission mechanism.

The conclusion that the money linkage is better described by the RBDQ effect also carries with it implications for the price mechanics associated with unanticipated nominal money growth and inflation. The two price mechanisms involved here are 1) the lag of the actual inflation rate behind the rate of nominal money growth [as described by eq. (3.6.3.7.)], and, 2) the discrepancy between the actual and anticipated rates of inflation. While both price effects are probably operative in the money-to-output linkage, the strength of the RBDQ effect over the RBLHQ effect implies that perceived movements in relative prices are submerged in the sluggish nature of the actual inflation rate responding to increases in the growth rate of nominal money. Thus even though there may be some RBE-induced discrepancy between actual and anticipated inflation caused by nominal money shocks, the effect on output is less than a pure RBE caused by the actual inflation rate lagging nominal money growth.

It should also be pointed out that the relatively longer lags of the money regressions tacitly support the argument that the sluggish inflation response implicit in the RBDQ effect dominates any output effect which might be related to price confusion. This support is based upon the realities of price information dissemination in a modern economy. It is difficult to justify the empirical fact of the long output lags of the money regressions based upon the LH-price confusion argument, since certainly the information necessary to know the true relative structure of prices would not be disseminated that slowly in an economic system where communication facilities are so advanced. On the other hand, there is strong empirical verification that an inflation response to nominal money growth takes anywhere from twelve to twenty-three quarters [33, 29]. This empirical fact implies that a sluggish inflation response-induced RBE is active for a longer period of time than confusion about the true structure of relative prices and, therefore, the observed persistence of output behind nominal money growth is simply reflecting the partial adjustment of inflation to money--a price process that has nothing to do with price confusion or the dissemination of price information [see eq. 3.6.3.7.) for a description of this lagged price adjustment mechanism].

With the above comments in mind, it is relevant here to discuss the results of Barro and Rush [30] in a (similar) study of the relationship between unanticipated nominal money growth rates and real output response. The Barro/Rush research is pertinent here because their results, being somewhat contradictory, support the RBDQ effect transmission linkage accepted for the money regressions estimated here. Like this study, these authors use quarterly data in a distributed lag format to gauge the effects of unanticipated nominal money growth on output. The transmission mechanism assumed by Barro and Rush to connect unanticipated nominal money to output is the RBLHQ effect, and they estimate a price equation to show that the price level does lag unanticipated nominal money growth thereby causing real balances to rise. From 1947/3 though 1978/1, for example, they find real GNP lagging unanticipated money growth by eight quarters. However, their price equation, estimated over the same time period, shows a sluggish price response (to unanticipated nominal money growth) of twenty-three quarters. The eight quarter output lag and the twenty-three quarter price lag are contradictory if one trys to reconcile the results within the confines of the RBLHQ effect. Since the output lag is only eight quarters, one must assume that the RBE-induced price confusion (the LH link) is fully resolved within this period of time, yet the price equation shows that a RBE is active for a much longer period of time. One must conclude that the Barro/Rush price equation strongly undermines the theoretical rationale of the LH upon which their money regression results are based, since they use the price equation to show that a RBE is activated by unanticipated nominal money growth, but then assume the RBE is transmitted to output via spending-induced price confusion. Simply put, it is impossible to reconcile the Barro/Rush findings using the RBLHQ linkage, and it is possible that their results might be better described by the RBDQ effect linkage.[1] However, since the Barro/Rush study does not estimate the effects of unanticipated inflation on real GNP over the same period, no comparisons between inflation and output and money and output can be made (as is being done in this study).

8.9.4. <u>Comparisons and Economic Analysis of Selected Statistical Results</u>. Tables 8.7.3.1. and 8.7.3.2. show that in terms of either the two inflation variables or the two money variables, the employment response is more prolonged than the GNP response (except for the GNP deflator inflation variable,

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[1] Note that Barro and Rush do admit that the price equation may be misspecified.

which has the same lag length for either GNP or employment). The elasticity, partial elasticity, and cumulative output response measurements seen in Tables 8.8.1., 8.8.3., and 8.8.4. support this general observation, as do the mean lag measurements from the Almon regressions.

The comparisons of the GNP and employment rate lag lengths support other statistical work investigating the nature of lagged covariation between the level of GNP and the employment rate that finds the employment rate to lag (nominal) GNP from one to four quarters. Thus the longer employment lags seen in both the unanticipated inflation and money regressions can be explained by the fact that the employment rate typically lags GNP. Therefore a longer linkage is involved in those regressions where the employment rate is used as the dependent variable.

While the coefficient magnitudes of the GNP and employment rate regressions of Tables 8.7.3.1. and 8.7.3.2. are not comparable, the normalized coefficient magnitudes illustrated in Figures 8.9.2.1. and 8.9.2.2. can be directly compared; these lag weight comparisons show that although the GNP lag lengths are shorter than their employment rate counterparts, their peak impact is stronger and occurs sooner than the weights associated with the employment regressions. For example, for the M1 variable, the peak response occurs in period t-6 in the GNP regression (with a total lag of 12 quarters), while the peak response occurs in period t-9 for the employment regression (with a total lag of 17 quarters). (For the CPI and GNP deflator variables, the peak lag is the same for either GNP or employment, at t-2 lags. For the GNP regressions, both the CPI and GNP deflator variables require a total lag of four quarters, while for the employment rate regressions the CPI and GNP deflator variables require a total of four and six lags, respectively.) Note also that while the M1-GNP lag length of 12 quarters seems reasonable, the M1-employment rate lag of 17 quarters may seem a bit long. However, given the fact that movements in GNP tend to lead movements in the employment rate from one to four quarters, the 17 quarter M1-employment rate lag is consistent with the nature of the GNP-employment rate lead/lag pattern.[1]

A comparison of the slope coefficient magnitudes in eqs. (8.2.1.) and (8.2.2.) in Table 8.7.3.1. shows real GNP to be more strongly affected by unanticipated CPI inflation than unanticipated GNP deflator inflation. This same observation is made for the employment regressions (8.2.5.) and (8.2.6.) in Table 8.7.3.2. also. This finding is supported by the elasticity measurements of Table 8.8.1., which show GNP and the employment rate response to be more sensitive to unanticipated CPI than unanticipated deflator inflation.

It is interesting to speculate why economic activity would be more strongly affected by CPI inflation forecast error than GNP deflator inflation forecast error, given 1) that both measures of unanticipated inflation are formed according to rationality postulates via the BJ time series filters, and, 2) that the variation of forecast error for both series is almost identical (see Table 6.5.1.). Since the actual variation of the CPI

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[1] Using annual data, Barro [29] has simulated a Phillips curve using nominal money growth and unemployment which shows that M1 money shocks drive unemployment below its natural rate for a total of 36 quarters! This result is questionable though since such simulations are subject to the Lucas criticism that expectations and endogenous coefficient magnitudes, which are fixed throughout the simulation estimation, may, in fact, not be independent of the initial shock that produced the simulated estimates. In terms of Barro's simulated unanticipated M1-unemployment findings, it is possible that the rapidity of error-learning may quicken to make the "true" lag less than 36 quarters.

inflation series is greater than that associated with the deflator inflation series over the 1956-1979 period (the variances of the actual CPI and deflator inflation rates over the period are .893 and .835, respectively), we can speculate that the greater output response associated with the CPI inflation variable is indicative of a general reaction of agents to the increased volatility of prices making up the CPI and, therefore, to a greater degree of perceived change in the relative structure of prices (i.e., to greater price confusion).[1] This explanation is particularly appealing if we make the accompanying (plausible) assumption that the CPI, rather than the GNP deflator, is the price index most known to economic decision-makers and the index upon which agents fix when extrapolating the future inflation rate.[2] Hence, even though the CPI is "better known"

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[1] Note that this conclusion does not violate the Lucas argument that the output response deteriorates as the variance of the inflation rate increases [19]. The Lucas reasoning is based upon cross-country comparisons of actual inflation rate variation assuming that all countries in his sample face a similar "perceived distribution of anticipated rates of inflation." However, the Lucas study from which this conclusion is drawn does not provide an index of this "perceived distribution." In the study here we do have such an index in the form of the variance of the generated anticipated rates of inflation. For example, using the CPI over the 1956-1979 period, a ratio of the actual to the anticipated variation in the inflation rate is 1.10; for the GNP deflator for the same period this ratio is 1.01. Therefore, relative to the anticipated volatility of the inflation rate, the CPI is seen to produce actual rates of inflation that are, on average, outside the anticipated inflation rate distribution more often than the deflator inflation rate time series. From this finding we can rationalize why the increased volatility of the CPI rate over that of the deflator would cause more, not less, output reaction--relative to what is expected, there is more price confusion associated with movements in the CPI than with movements in the GNP deflator.

[2] For example, the CPI is the most closely watched price index primarily because it is the chief measure of changes in the "cost of living," e.g., it is the primary index used in wage escalation and other "inflation indexed" contracts. Also, the public's familiarity with the CPI can be explained in terms of low acquisition costs.

cause it is more closely watched and because information about this index is more effectively disseminated to the public, its great volatility over the 1956-1979 period (relative to what is expected) is reflected in more price confusion and, therefore, more output response. Over this period of time we have, therefore, the ironic situation in which increased efficiency in communication technology in the economy has resulted in more, not less, price confusion.

A second opinion as to why output response is less pronounced with the unanticipated deflator inflation rate centers around the fact that the CPI, being a less diffused index than the deflator, provides for less substitution within the index than the deflator when relative prices are perceived to increase due to a rise in the general level of prices. Thus a "shock" to the anticipated CPI inflation rate would not appear as pronounced when included within the broader deflator measure of price level inflation; perceived increases in relative prices would, therefore, appear to be smaller and output response less.

In Table 8.7.3.2. and Figure 8.9.2.1. it is observed that the total effect of real GNP elicited by unanticipated M1 growth is greater than that caused by unanticipated monetary base growth (the sum of the coefficients from the Almon regresssions for the M1 and monetary base regressions are 44.60 and 25.22, respectively). Possibly some explanation for this finding can be attributed to the effects of the money multiplier; since the multiplier is greater than unity (in 1978 it was 2.60), a given percentage change in unanticipated M1 growth results in a greater absolute change in unanticipated real liquidity in the system with M1 than with the monetary base variable. The rise in M1 income velocity over the

1956-1979 period (from 2.70 to 6.60) would also tend to accentuate the output response caused by the absolute rise in unanticipated M1 growth over the period.

Substantive findings of this study are provided by comparisons of the subperiod output response patterns as seen in Tables 8.7.4.1. through 8.7.4.7. in Section 8.7.4. of Chapter VIII. The statistical results of this Section provide evidence that the unanticipated variable coefficient magnitudes of the 1968-1979 subperiod are greater than those coefficient magnitudes associated with the 1956-1967 subperiod. Specifically, the subperiod regression comparisons show (in those cases where both subperiod equations can be identified) that for either the GNP or the employment rate output variables, unanticipated inflation or monetary growth produces a more pronounced and longer-lasting effect on these two measures of economic activity in the second than in the first subperiod. These findings are supported by the elasticity measurements appearing in Table 8.8.2.

A point of importance to note in considering these subperiod results is that the unanticipated variables have been generated by ARIMA forecasting functions estimated separately over each subperiod. (In the interest of brevity these time series models are not presented here; they are quite similar to the subperiod ARIMA models presented in Tables 6.1.6., 6.2.2., 6.3.3., and 6.4.3.) Because the two subperiods represent eras of different inflation and monetary growth history, the estimation of separate time series models unique to each period is undertaken to allow for any structural change in the aggregate forecasting behavior due to systematic alterations in the true process generating the actual inflation or money growth rate series. This procedure is in conformity with rationality postulates because it allows for a more precise analysis of output response elicited by forecast error over different periods of inflation and money growth history.

Justification for the greater second period unanticipated inflation-output response pattern is based upon a comparison of "anticipated inflation rate volatility" indices over the two subperiods. These indices are provided by constructing a ratio of the actual CPI or deflator inflation rate variance to the anticipated inflation rate variance over the two subperiods, where the anticipated inflation rate variance is provided by computing the variance of the ARIMA CPI and deflator inflation subperiod forecasts (this method parallels that outlined in fn. 1, p. 327). For example, during the 1956-1967 subperiod, the ratio of the actual CPI inflation rate variance to the anticipated variance was 1.02, and during the 1968-1979 period it was 1.12. For the deflator the ratio of the actual inflation rate variance to the anticipated variance during the 1956-1967 period was 1.21, while it was 1.33 during the 1968-1979 period. Thus in terms of either the CPI or the GNP deflator measure of inflation, the actual amount of inflation rate volatility rose proportionately more than the anticipated inflation rate volatility. This means that during the second period, in terms of either the CPI or the deflator, an actually observed inflation rate fell, on average, outside the anticipated distribution of inflation rates more often during the second than during the first period, i.e., in terms of either the CPI or the deflator, there were more inflation rate "surprises" during the 1968-1979 period. It is in this manner that we can reconcile the higher indices of economic activity during the second period with the greater actual variance of inflation (either CPI or deflator) during the second period. Thus it is reasonable to offer the opinion that the greater output response witnessed during the 1968-1979 period reflects the fact that even though inflation forecasts are constructed to mimic any structural change in the inflation generating process, the increased volatility of the actual inflation rate during the second period leads to increased random noise in the price structure, to perceived increases in relative prices, and, to temporarily greater levels of economic activity. Note that this conclusion should not be interpreted as meaning that agents are not forecasting inflation "rationally," but means instead that a larger portion of the actual inflation rate variation observed during the 1968-1979 period qualifies as random variation to rational agents attempting to forecast inflation.[1]

Subperiod comparisons of the actual to anticipated ratios of inflation rate variances over the two time spans can also be used to analyze stagflation. These results show that over the 1968-1979 period the increased variability of the actual (to the anticipated) inflation rate resulted in a more sensitve inflation-unemployment tradeoff than during the 1956-1967 period (i.e., a less steeply sloped Phillips curve characterizes the earlier period). This finding does not support other empirical studies in the literature which proclaim the short-run inflation-unemployment tradeoff has all but disappeared in the U.S. in the mid- to late 1970's.

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[1] Given rational expectations, the logical question here is, "Why don't rational agents adjust their "perceived distribution of anticipated inflation rates" as the Muth theory would predict, so as to closer approximate the greater actual CPI and deflator inflation rate variances of the latter period?" A plausible reply to this seeming inconsistency in aggregate expectations behavior is that while agents are making the requisite "adjustments," the behavior of the actual inflation rate (especially the CPI) has been so volatile that the public is still in the process of learning the new (larger) inflation rate variance parameters.

Note however, that this conclusion does not deny the possibility that the Phillips curve may have shifted some over the two periods of time; rather, the point here is that any such shift which could potentially lead to more inflation with no reduction in unemployment was outweighted by increases in real output due to increased inflation rate variability over the latter period. This study thus concludes that quantity adjustments were more sensitive to the inflation rate variability characterizing the 1968-1979 period.[1]

There are two instances in which the unanticipated monetary growth-output relationship can be compared over the two subperiods. Tables 8.7.4.3. and 8.7.4.7. confirm the fact that, in terms of either real GNP or the employment rate, M1 forecast error occurring in the 1968-1979 period produces a greater output response than in the 1956-1967 period, i.e., in a comparative static sense, the "money" Phillips curve for the former period is less steeply sloped that that characterizing the latter period.

Since the actual variation of the M1 growth rate series over the second period ($\hat{\sigma}^2$ = .588) is less than that associated with the first

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[1] It is important to point out here that the indexing of labor (and other contracts) during the 1968-1979 period quickened considerably over the first period. This fact is partially responsible for the more sensitive unanticipated inflation-unemployment tradeoff characterizing the 1968-1979 period, because what we normally think of as price-inflexible "contract" markets tended to act more in accord with the characteristics of price-flexible auction markets. In terms of the inflation-unemployment tradeoff, quicker contract indexing would mean that both prices and quantities would tend to adjust more rapidly to (perceived) movements in demand since inflation can be more rapidly incorporated into obligational arrangements. Thus, given 1) quicker contract indexing, and, 2) the fact that contract renegotiation is staggered throughout the economy, the Lucas auction market assumption upon which the information lag negatively sloped Phillips curve is based would seem to be holding up well.

iod ($\hat{\sigma}^2 = .646$), we cannot appeal to the argument that the observed increases in economic activity witnessed during the second period reflect increased monetary variation. Also, since we concluded earlier that the unanticipated money-to-output relationship does not rely predominantly on the LH, the whole idea of unanticipated movements (in this case, increased variance) in liquidity "causing" increased price confusion and output movements is not a consistent argument.

The greater second period M1-output response can, however, be justified in two other ways, both having to do with the workings of the RBE. First, the mean growth rate of the M1 variable quadrupled over the two periods (from .694 in the 1956-1967 period to 2.999 in the 1968-1979 period). Therefore, strictly in terms of absolute additions to the M1 money stock, the second period witnessed considerably higher levels of unanticipated liquidity than the first period. A second factor which can be cited as a cause of the greater second period M1-output observations is velocity. During the 1956-1967 period, average annual income velocity was 3.70; during the 1968-1979 period this figure climbed to 5.54. The increased second period income velocity, when combined with the significantly higher mean level of the M1 growth rate, liquidity, spending, and the multiplier, can be offered as a plausible explanation for the observed greater sensitivity of output to unanticipated M1 growth seen in the second period.[1]

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[1] The possible role of income velocity as it relates to fiscal policy should not be overlooked in this conclusion. While this study does not specifically consider velocity, it is a fact that fiscal activity was more intense in the 1968-1979 period than in the 1956-1967 period. The implication here is this: to the extent that fiscal actions are correlated with higher interest rates and increases in velocity, the "pure impact" of unanticipated money on output may not be so "pure," but may be overstated during the latter period.

8.10. Concluding Remarks

The idea that inflation and monetary growth, either as covariant or independent forces, "cause" the level of economic activity to rise is firmly entrenched in the history of economic thought.

The observation that movements in prices and changes in the quantity of money are related to output and employment, in a lagged fashion, is also part of the picture comprising the money/prices/output nexus. In terms of this aspect of macroeconomic theorizing, Hume is probably the first economist to distill the essential idea that the level of economic activity would be altered, in a lagged fashion, by increases in the amount of money in circulation. Hume must also be cited as one of the first economists to recognize that a lagged relationship existed between the quantity of money and the general level of prices.

Fisher's empirical study of the relationship between wages, prices and employment gave further credence to the importance of adjustment lag phenomena in economic activity. Under the general phrase, the "Phillips curve," Milton Friedman rejuvenated the ideas of Fisher to provide a contemporary explanation of the inflation-unemployment tradeoff after the general Keynesian income determination model was unable to provide a sound expectations-based explanation of inflation per se, and inflation with unemployment in general. Friedman has drawn heavily upon the initial insights of Fisher in two respects: 1) a negative corrolation characterizes inflation and unemployment in the short-run, and, 2) expectations and expectation formation, as an expression of utility maximizing behavior, play an essential role in a) any explanation of why output lags behind changes in prices, wages, and monetary growth, and b) explaining how and why resource allocation can be altered when expectations are introduced into economic decision-making at the microeconomic level.

Truly the major theoretical advance in inflation theory in the last decade has been the idea that output responds only to the discrepancy between the actual current aggregate price level and its expected value. Likewise, with the rise of monetarism and renewed interest in monetarism and expectations-based theories of money and output, a related idea has gained currency: output responds only to the discrepancy between the actual and expected growth rates of the money stock. It is the theoretical explanations which have been offered by modern inflation researchers as to why this actual-expected price level/monetary growth rate discrepancy might arise and how information and adjustment lags might be capable of generating short-run changes in the level of real economic variables which has raised so many empirical questions in modern macroeconomics. Typically the "how" and "why" of information adjustment lags and output have centered around questions concerning the transmission linkage of inflation/money and output. Although approached in an inferential fashion, this study has attempted to address these important empirical issues by investigating the dynamic nature of the cause-effect relationships which characterize inflation, money, and real economic variables.

Finally, the reader should not need to be reminded that the objective here is ultimately concerned with modeling human economic behavior. While a major portion of this paper has been concerned with statistical technique, this fact is not (and should never be) an "end" in itself. Expectations modeling is, understandably, a very delicate issue in economics, and the rather sophisticated expectations-generating methods employed here (even though firmly supported by economic theory) can only be considered the "means" to achieve a better understanding of how "things work" when human expectations are introduced into macroeconomic modeling.

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APPENDICES

CPI Data; Actual, Step-ahead Forecasts and Residuals

	(1) Actual Quarterly Rate of Inflation	(2) Full Model Step-ahead Forecasts	(3) Full Model Step-ahead Residuals	(4) Iterative Model Step-ahead Forecasts	(5) Iterative Model Step-ahead Residuals
1956-2	0.7500	0.1795	0.5705		
-3	1.3600	0.5581	0.8019	0.4078	0.9522
-4	0.7300	0.8212	-0.0912	1.0122	-0.2822
1957-1	0.6300	0.8193	-0.2093	0.6836	-0.0536
-2	0.9600	1.0400	-0.0799	1.3977	-0.4377
-3	1.0700	0.9982	0.0718	1.0609	0.0091
-4	0.3500	0.9750	-0.6250	1.1868	-0.8368
1958-1	1.0600	0.5266	0.5334	0.5134	0.5466
-2	0.7000	1.0380	-0.3380	0.8789	-0.1789
-3	0.1200	0.4418	-0.3218	0.2684	-0.1484
-4	0.0000	0.5346	-0.5346	0.5437	-0.5437
1959-1	0.0000	0.1254	-0.1254	0.2936	-0.2936
-2	0.3500	0.0322	0.3178	0.2548	0.0952
-3	0.5700	0.0409	0.5291	-0.0275	0.5975
-4	0.5700	0.2547	0.3153	-0.0341	0.6041
1960-1	0.0000	0.4687	-0.4687	0.4395	-0.4395
-2	0.6800	0.3185	0.3615	0.0562	0.6238
-3	0.1100	0.7862	-0.6762	0.6667	-0.5567
-4	0.6800	0.0780	0.6020	0.5310	0.1490
1961-1	0.0000	0.7653	-0.7653	0.6545	-0.6545
-2	0.0000	-0.1404	0.1404	0.1407	-0.1407
-3	0.5600	0.3724	0.1876	0.3421	0.2179
-4	0.1100	0.1421	-0.0320	0.1000	0.0100
1962-1	0.2200	0.1377	0.0823	0.0263	0.1937
-2	0.4400	0.2755	0.1645	0.2112	0.2288
-3	0.4400	0.3486	0.0914	0.2560	0.1840
-4	0.2200	0.4040	-0.1840	0.3492	-0.1292
1963-1	0.1100	0.3344	-0.2244	0.3318	-0.2218
-2	0.2200	0.2708	-0.0508	0.3099	-0.0899
-3	0.7700	0.2125	0.5575	0.2755	0.4945
-4	0.2200	0.4864	-0.2664	0.4372	-0.2172
1964-1	0.3300	0.1694	0.1606	0.1967	0.1333
-2	0.2200	0.5640	-0.3440	0.4763	-0.2563
-3	0.3200	0.1973	0.1227	0.2724	0.0476
-4	0.4300	0.4201	0.0099	0.3854	0.0446
1965-1	0.1100	0.2556	-0.1456	0.3035	-0.1935
-2	0.7500	0.2115	0.5385	0.2347	0.5153
-3	0.4200	0.5922	-0.1722	0.4909	-0.0709
-4	0.4200	0.3001	0.1199	0.2891	0.1309

	(1)	(2)	(3)	(4)	(5)
1966-1	0.8400	0.6395	0.2005	0.5466	0.2934
-2	1.0400	0.6688	0.3712	0.6177	0.4223
-3	0.9300	0.9103	0.0196	0.7907	0.1393
-4	0.7200	0.9225	-0.2025	0.8865	-0.1666
1967-1	0.2000	0.9432	-0.7432	0.9642	-0.7642
-2	0.7100	0.5227	0.1873	0.6586	0.0514
-3	1.1100	0.7258	0.3842	0.8055	0.3055
-4	0.8000	0.6196	0.1804	0.7422	0.0578
1968-1	1.0900	0.7259	0.3641	0.7129	0.3771
-2	1.0700	1.0770	-0.0074	0.9443	0.1257
-3	1.2600	1.0660	0.1945	0.9100	0.3500
-4	1.2400	1.3430	-0.1028	1.1138	0.1262
1969-1	1.1300	1.2300	-0.1002	1.1442	-0.0142
-2	1.6800	1.2700	0.4096	1.2014	0.4786
-3	1.4700	1.5120	0.0417	1.4624	0.0076
-4	1.3600	1.3510	0.0091	1.3478	0.0122
1970-1	1.5200	1.5370	-0.0166	1.4934	0.0266
-2	1.5800	1.5070	0.0734	1.5048	0.0752
-3	1.1200	1.5610	-0.4410	1.5357	-0.4157
-4	1.3700	1.2560	0.1139	1.3427	0.0273
1971-1	0.7600	1.4590	-0.6989	1.4776	-0.7176
-2	1.0900	0.8036	0.2864	1.0235	0.0665
-3	0.9900	1.1920	-0.2016	1.2061	-0.2161
-4	0.5700	0.7075	-1.1375	0.8953	-0.3253
1972-1	0.8100	0.7844	0.0256	0.7923	0.0177
-2	0.8100	0.7479	0.0621	0.7857	0.0243
-3	0.8800	0.7245	0.1555	0.6991	0.1809
-4	0.8700	0.8204	0.0496	0.7796	0.0904
1973-1	1.4200	0.8428	0.5772	0.7907	0.6293
-2	2.1800	1.2650	0.9153	1.1120	1.0680
-3	2.2100	1.7390	0.4708	1.5604	0.6496
-4	2.3800	2.0730	0.3068	1.9508	0.4292
1974-1	2.7600	2.5440	0.2165	2.4644	0.2956
-2	2.8300	2.8080	0.0217	2.7947	0.0353
-3	3.0900	2.9010	0.1891	2.9026	0.1871
-4	2.8700	3.1070	-0.2368	3.1061	-0.2361
1975-1	1.8200	2.9220	-1.1020	2.9583	-0.1383
-2	1.5900	2.3510	-0.7607	2.4153	-0.8253
-3	2.1300	2.0150	0.1145	1.9858	0.1442
-4	1.6000	1.8020	-0.2019	1.7971	-0.1971
1976-1	0.9700	1.3310	-0.3607	1.3333	-0.3633
-2	1.2600	1.1940	0.0663	1.1824	0.0776
-3	1.6000	1.2330	0.3665	1.2279	0.3721
-4	1.1100	1.3020	-0.1924	1.3026	-0.1926
1977-1	1.7800	1.1240	0.6556	1.1533	0.6267
-2	2.1500	1.7390	0.4112	1.7260	0.4240
-3	1.1400	1.7730	-0.3332	1.7939	-0.3539
-4	1.0900	1.7400	-0.6504	1.7932	-0.7032

	(1)	(2)	(3)	(4)	(5)
1978-1	1.6700	1.5720	0.0984	1.5888	0.0812
-2	2.6000	1.6430	0.9574	1.6176	0.9824
-3	2.3300	1.9610	0.3691	1.9186	0.4114
-4	2.0200	2.0210	-0.0013	2.0185	0.0015
1979-1	2.5800	2.3350	0.2446	2.3268	0.2532
-2	3.4300	2.6450	0.7858	2.6450	0.7850

GNP Deflator Data; Actual, Step-ahead Forecasts and Residuals

	(1) Actual Quarterly Rate of Inflation	(2) Iterative Model Step-ahead Forecasts	(3) Iterative Model Step-ahead Residuals
1956-2	0.7600		
-3	1.1300	0.7423	0.3877
-4	0.8700	0.6848	0.1852
1957-1	1.1100	0.8050	0.3050
-2	0.3700	0.9631	-0.5931
-3	0.9700	0.7236	0.2464
-4	0.1200	0.7260	-0.6060
1958-1	0.3600	0.3839	-0.0239
-2	0.2400	0.4026	-0.1626
-3	0.6000	0.2230	0.3770
-4	0.2400	0.3461	-0.1061
1959-1	0.8300	0.4160	0.4140
-2	0.7100	0.6772	0.0328
-3	0.3500	0.6806	-0.3306
-4	0.4700	0.6920	-0.2220
1960-1	0.7000	0.5368	0.1632
-2	0.1200	0.5423	-0.4223
-3	0.4600	0.3064	0.1536
-4	0.1100	0.3603	-0.2503
1961-1	-0.1100	0.1673	-0.2773
-2	0.4600	0.1939	0.2661
-3	0.4600	0.4085	0.0515
-4	0.1100	0.3720	-0.2620
1962-1	0.9100	0.2938	0.6162
-2	0.3400	0.6051	-0.2651
-3	0.2200	0.4550	-0.2350
-4	0.6700	0.4873	0.1827
1963-1	0.3300	0.5041	-0.1741
-2	0.1100	0.3319	-0.2219
-3	0.3300	0.3373	-0.0073
-4	0.6600	0.2507	0.4093
1964-1	0.3300	0.3299	0.0001
-2	0.3300	0.5303	-0.2003
-3	0.5400	0.4013	0.1387
-4	0.2200	0.4254	-0.2054

	(1)	(2)	(3)
1965-1	0.7600	0.4505	0.3096
-2	0.5400	0.5825	-0.0424
-3	0.6400	0.4387	0.2013
-4	0.5300	0.4532	0.0768
1966-1	1.0500	0.5987	0.4513
-2	1.1500	0.7828	0.3672
-3	0.6200	0.9191	-0.2991
-4	0.9200	0.7876	0.1324
1967-1	0.6100	0.7652	-0.1552
-2	0.3000	0.7189	-0.4189
-3	1.0100	0.4934	0.5166
-4	1.0900	0.6261	0.4639
1968-1	1.2800	0.6743	0.6057
-2	1.1700	1.0040	0.1665
-3	0.9600	1.1530	-0.1933
-4	1.4300	1.0360	0.3938
1969-1	1.0300	1.3340	-0.3045
-2	1.3000	1.1390	0.1611
-3	1.5600	0.9475	0.6125
-4	1.2600	1.0780	0.1823
1970-1	0.8000	1.0980	-0.2983
-2	1.9500	1.0980	0.8518
-3	0.8700	1.4161	-0.5460
-4	1.3800	1.1650	0.2146
1971-1	1.5300	1.3340	0.1962
-2	1.4200	1.2010	0.2191
-3	0.8200	1.1790	-0.3593
-4	0.9000	1.1760	-0.2761
1972-1	1.3800	0.9579	0.4221
-2	0.7200	1.0600	-0.3396
-3	0.8700	1.1100	-0.2404
-4	1.1000	0.8416	0.2584
1973-1	1.4800	0.9039	0.5761
-2	1.6900	1.2140	0.4761
-3	1.8100	1.5620	0.2480
-4	2.3700	1.5520	0.8179
1974-1	2.0200	1.8410	0.1788
-2	2.7600	2.0930	0.6667
-3	2.7600	2.2860	0.4740
-4	3.0200	2.2690	0.7513
1975-1	2.5400	2.4210	0.1194
-2	1.4600	2.4380	-0.9781
-3	1.8200	1.9070	-0.0871
-4	1.4800	1.7450	-0.2648
1976-1	0.9700	1.6050	0.6352
-2	1.2000	1.1690	0.0307
-3	1.0700	1.0570	0.0129
-4	1.4100	0.9979	0.4121

	(1)	(2)	(3)
1977-1	1.4500	1.4890	-0.0388
-2	1.8800	1.6690	0.2105
-3	1.2900	1.6570	-0.3670
-4	1.3300	1.6750	-0.3448
1978-1	1.7500	1.5850	0.1653
-2	2.6800	1.5340	1.1460
-3	1.6700	1.9100	-0.2401
-4	1.9500	1.7580	0.1922
1979-1	2.1700	1.8670	0.3030
-2	2.4200	2.0290	0.3910

M1 Monetary Growth Rate Data; Actual, Step-ahead Forecasts and Residuals

	(1) Actual Quarterly M_1 Growth Rate	(2) Iterative Model Step-ahead Forecasts	(3) Iterative Model Step-ahead Residuals
1956-2			
-3	0.0700	0.2454	-0.1754
-4	0.4400	0.2274	0.2125
1957-1	0.2200	0.4943	-0.2743
-2	0.0000	0.5447	-0.5447
-3	0.0700	0.1629	-0.0929
-4	-0.5800	0.1495	0.7295
1958-1	-0.0700	-0.3313	0.2613
-2	1.1000	0.3212	0.7787
-3	1.0200	1.1415	-0.1215
-4	1.2200	0.2070	0.0129
1959-1	1.3500	1.1511	0.1988
-2	0.8400	1.1767	-0.3367
-3	0.4900	0.9102	-0.4202
-4	-0.6200	0.6721	-1.2921
1960-1	0.4200	-0.5526	0.1326
-2	-0.1400	-0.2924	0.1524
-3	0.7700	0.1429	0.6270
-4	0.2100	1.2242	-1.0142
1961-1	0.4200	0.1968	0.2232
-2	0.8300	0.4666	0.3633
-3	0.6200	0.7253	-0.1053
-4	0.9500	1.2423	-0.2923
1962-1	0.6000	0.8345	-0.2245
-2	0.4000	0.5024	-0.1024
-3	-0.2000	0.3813	-0.5813
-4	0.6000	0.2759	0.3240
1963-1	0.9300	0.6530	0.2769
-2	0.9900	0.8802	0.1097
-3	0.9800	1.1648	-0.1848
-4	1.0300	0.9745	0.0554
1964-1	0.5800	0.9593	-0.3793
-2	0.9500	0.5852	0.3647
-3	1.6400	1.1192	0.5207
-4	1.2400	1.4916	-0.2516

	(1)	(2)	(3)
1965-1	0.6700	1.2723	-0.6023
-2	0.7300	0.5564	0.1735
-3	1.1500	0.6408	0.5091
-4	1.7300	1.1807	0.5492
1966-1	1.6400	1.8934	-0.2534
-2	1.2100	1.4197	-0.2097
-3	-0.1100	0.9158	-1.0258
-4	0.1700	-0.1251	0.2951
1967-1	0.9700	0.4685	0.5014
-2	1.3500	0.9775	0.3725
-3	2.3900	1.5056	0.8843
-4	1.5200	2.0204	0.5004
1968-1	1.3400	1.3633	-0.0233
-2	1.8500	1.2002	0.6497
-3	2.0200	1.6374	0.3825
-4	2.1400	1.9709	0.1691
1969-1	1.7400	1.9173	-0.1773
-2	0.8300	1.3639	-0.5339
-3	0.5800	0.5712	0.0087
-4	0.7200	0.6460	0.0739
1970-1	0.9100	0.7839	0.1260
-2	1.1400	0.9344	0.2055
-3	1.2700	1.0027	0.2672
-4	1.4400	1.1847	0.2552
1971-1	1.7800	1.3690	0.4109
-2	2.2900	1.6900	0.5999
-3	1.6700	2.0198	-0.3498
-4	0.6900	1.4793	-0.7893
1972-1	2.0200	0.5615	1.4584
-2	1.7700	1.6706	0.0993
-3	2.1100	1.7224	0.3875
-4	2.2700	2.1679	0.1020
1973-1	2.1400	1.5549	0.5850
-2	1.2800	1.8376	-0.5576
-3	1.3000	1.1690	0.1309
-4	1.3600	1.3677	-0.0077
1974-1	1.8200	0.8061	1.0138
-2	1.0200	1.8469	-0.8269
-3	1.0100	0.8959	0.1140
-4	1.1500	1.0069	0.1430
1975-1	0.5000	0.6141	-0.1141
-2	1.4500	0.9140	0.5359
-3	1.8100	1.3088	0.5011
-4	0.7500	1.6608	-0.9108
1976-1	1.1500	0.5626	0.5873
-2	1.6100	1.1343	0.4756
-3	1.0200	1.3346	-0.3146
-4	1.8600	1.2694	0.5905

	(1)	(2)	(3)
1977-1	1.8600	1.5346	0.3253
-2	1.8600	1.5237	0.3362
-3	2.1600	1.7420	0.4180
-4	1.8400	2.0013	-0.1613
1978-1	1.6600	1.5023	0.1567
-2	2.3100	1.3528	0.9572
-3	1.9700	2.0538	-0.0838
-4	1.0400	1.8131	-0.7731
1979-1	-0.5300	0.8816	-1.4116
-2	1.8900	-0.7643	2.6543

Monetary Base Data; Actual, Step-ahead Forecasts and Residuals

	(1) Actual Quarterly Rate of Inflation	(2) Full Model Step-ahead Forecasts	(3) Full Model Step-ahead Residuals	(4) Iterative Model Step-ahead Forecasts	(5) Iterative Model Step-ahead Residuals
1956-3	0.0000	0.2688	-0.2688	0.3098	-0.3098
-4	0.7100	0.1786	0.5314	0.1202	0.5898
1957-1	0.2400	0.2660	-0.0260	0.1141	0.1259
-2	0.0000	0.3297	-0.3297	0.3974	-0.3974
-3	0.0000	0.4593	-0.4593	0.4719	-0.4719
-4	0.0000	-0.0503	0.0503	0.0598	-0.0598
1958-1	0.4700	-0.0991	0.5691	-0.3014	0.7714
-2	1.1700	0.4158	0.7542	0.5716	0.5984
-3	0.4600	0.7781	-0.3181	0.7292	-0.2692
-4	0.2300	0.4500	-0.2200	0.3482	-0.1182
1959-1	0.4600	0.4921	-0.0321	0.4187	0.0413
-2	0.4600	0.3111	0.1489	0.4817	-0.0217
-3	0.4600	0.5839	-0.1239	0.7081	-0.2481
-4	-0.4500	0.5023	-0.9523	0.5073	-0.9573
1960-1	0.0000	-0.0701	0.0701	-0.2897	0.2897
-2	0.0000	-0.1708	0.1708	-0.1732	0.1732
-3	0.2300	-0.0479	0.2779	0.0279	0.2030
-4	0.2300	0.4014	-0.1714	0.4184	-0.1884
1961-1	1.3600	0.1347	1.2250	0.0141	1.3459
-2	0.0000	0.6149	-0.6149	0.5833	-0.5833
-3	0.8900	0.3334	0.5566	0.4317	0.4583
-4	1.5500	1.1920	0.3585	1.5360	0.0140
1962-1	0.6600	0.4972	0.1628	0.1464	0.5136
-2	1.0800	1.0420	0.0384	0.9524	0.1276
-3	0.6400	0.9482	-0.3082	0.9700	-0.3300
-4	1.0700	0.6954	0.3746	0.8927	0.1773
1963-1	1.0500	0.5990	0.4510	0.0450	1.0050
-2	1.2500	1.0630	0.1872	1.1244	0.1256
-3	1.4400	1.0640	0.3762	1.0054	0.4346
-4	1.4200	1.0580	0.3624	1.0597	0.3603
1964-1	1.4000	1.1990	0.2008	0.7384	0.6616
-2	1.1900	1.3600	-0.1705	1.3442	-0.1542
-3	1.5600	1.3100	0.2499	1.4008	0.1592
-4	1.3500	1.2250	0.1250	1.1924	0.1576
1965-1	1.3300	1.1560	0.1741	0.9885	0.3415
-2	1.1200	1.4120	-0.2917	1.3924	-0.2724
-3	1.4800	1.0810	0.3994	1.0453	0.4347
-4	1.8200	1.2150	0.6050	1.1303	0.6897

	(1)	(2)	(3)	(4)	(5)
1966-1	1.4300	1.3860	0.0443	1.2610	0.1690
-2	1.5900	1.6530	-0.0626	1.6548	-0.0648
-3	1.0400	1.4710	-0.4305	1.5131	-0.4731
-4	0.6900	1.1110	-0.4210	1.1368	-0.4468
1967-1	1.3700	1.0400	0.3295	0.9626	0.4074
-2	1.5200	1.2020	0.3178	1.2003	0.3197
-3	1.6600	1.1760	0.4843	1.0528	0.6072
-4	1.6300	1.4600	0.1697	1.3600	0.2700
1968-1	1.7700	1.4980	0.2719	1.4601	0.3099
-2	1.2600	1.6360	-0.3756	1.6489	-0.3889
-3	1.7200	1.5020	0.2183	1.5454	0.1746
-4	1.9900	1.7170	0.2727	1.7389	0.2511
1969-1	1.3500	1.4980	-0.1478	1.4443	-0.0943
-2	0.7400	1.6290	-0.8895	1.6097	-0.8697
-3	0.8800	1.1450	-0.2645	1.1393	-0.2593
-4	1.3100	0.9097	0.4003	0.8926	0.4174
1970-1	1.1500	0.9681	0.1819	0.9035	0.2465
-2	1.5700	1.4150	0.1547	1.3797	0.1903
-3	1.9600	1.3910	0.5685	1.3737	0.5863
-4	1.7900	1.3960	0.3942	1.3913	0.3987
1971-1	2.1600	1.7380	0.4223	1.7308	0.4292
-2	1.9800	2.2170	-0.2374	2.2746	-0.2946
-3	2.0800	1.8750	0.2053	1.9164	0.1636
-4	1.1400	1.8760	-0.7360	1.8887	-0.7487
1972-1	1.7600	1.4920	0.2680	1.4973	0.2627
-2	1.9800	1.7680	0.2117	1.7332	0.2468
-3	1.8200	1.4070	0.4130	1.3660	0.4540
-4	2.6200	1.9090	0.7110	1.8402	0.7798
1973-1	2.2000	2.0260	0.1740	2.0245	0.1755
-2	2.0400	2.1570	-0.1173	2.1842	-0.1442
-3	1.8900	2.1280	-0.2376	2.1402	-0.2502
-4	1.6400	2.0620	-0.4216	2.1024	-0.4624
1974-1	2.3600	1.7230	0.6375	1.7091	0.6509
-2	2.3100	2.0040	0.3058	2.0013	0.3087
-3	1.9500	2.0140	-0.0642	2.0103	-0.0603
-4	2.1100	2.0880	0.0223	2.0660	0.0440
1975-1	1.5700	1.9400	-0.3696	1.9522	-0.3822
-2	1.9400	1.7800	0.1603	1.7813	0.1587
-3	2.0900	2.0100	0.0803	2.0098	0.0802
-4	1.8600	1.9760	-0.1161	1.9868	-0.1268
1976-1	1.9200	1.8540	0.0660	1.8363	0.0837
-2	2.5100	1.8440	0.6655	1.8431	0.6669
-3	1.7500	2.1400	-0.3905	2.1491	-0.3991
-4	1.9800	1.9960	-0.0158	1.9865	-0.0065
1977-1	1.8500	2.2180	-0.3675	2.2278	-0.3778
-2	2.0700	1.6240	0.4459	1.6241	0.4459
-3	2.3500	2.0680	0.2821	2.0609	0.2891
-4	2.3000	2.1410	0.1594	2.1447	0.1553

	(1)	(2)	(3)	(4)	(5)
1978-1	2.4800	2.3000	0.1802	2.2939	0.1861
-2	1.9700	2.0930	-0.1229	2.0937	-0.1237
-3	2.3700	2.2370	0.1327	2.2421	0.1279
-4	2.4600	2.2910	0.1686	2.2912	0.1688
1979-1	1.4900	2.3140	-0.8242	2.3218	-0.8318
-2	1.5300	1.9290	-0.3990	1.9290	-0.3990

Coefficient Evolution Pattern for the CPI Inflation Rate Model, 1952-1/1979-1

$$(1 - B)z_t = (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)\ (1 - \Delta_3 B^3)a_t$$

	θ_1	θ_2	θ_3	Δ_3
1956-2	-.3725	-.3289	.9293	-.7391
-3	-.4121	-.3969	.9274	-.7585
-4	-.3709	-.3460	.8994	-.6642
1957-1	-.3200	-.3200	.8900	-.6600
-2	-.3871	-.4258	.8523	-.3898
-3	-.3740	-.4323	.8435	-.3879
-4	-.4757	-.4961	.8545	-.7697
1958-1	-.4671	-.4792	.8635	-.7728
-2	-.4740	-.4495	.9019	-.7880
-3	-.4500	-.4500	.9000	-.7800
-4	-.4767	-.4411	.9319	-.8059
1959-1	-.4806	-.4553	.9498	-.8309
-2	-.4800	-.4800	.9000	-.8000
-3	-.4800	-.4800	.9000	-.8000
-4	-.4994	-.4811	.9171	-.8078
1960-1	-.5484	-.5188	.9522	-.7988
-2	-.5050	-.4832	.9279	-.4686
-3	-.4997	-.4012	.9000	-.4996
-4	-.4928	-.4497	.9054	-.4981
1961-1	-.4860	-.4500	.9028	-.5333
-2	-.4800	-.4500	.9000	-.5301
-3	-.3937	-.4025	.9018	-.5387
-4	-.3962	-.4101	.9374	-.6001
1962-1	-.3900	-.4100	.9000	-.6000
-2	-.3746	-.3468	.8927	-.5913
-3	-.4797	-.1030	.8257	-.5769
-4	-.4744	-.1001	.8216	-.5687
1963-1	-.4560	-.1303	.7975	-.5004
-2	-.4500	-.1299	.7900	-.5600
-3	-.4500	-.1200	.7901	-.5601
-4	-.4684	-.1184	.8243	-.5931
1964-1	-.4600	-.1100	.8200	-.5900
-2	-.4000	-.1100	.8300	-.5901
-3	-.4430	-.1350	.8451	-.6001
-4	-.4400	-.1300	.8400	-.6000
1965-1	-.4189	-.1833	.8304	-.6241
-2	-.4100	-.1800	.8300	-.6200
-3	-.4100	-.1800	.8300	-.6200
-4	-.4100	-.1800	.8300	-.6200
1966-1	-.4100	-.1799	.8301	-.6200
-2	-.4100	-.1700	.8300	-.6200
-3	-.4115	-.1658	.8488	-.6213
-4	-.3881	-.2020	.8552	-.6247

	θ_1	θ_2	θ_3	Δ_3
1967-1	-.4602	-.1025	.8416	-.6210
-2	-.4758	-.0847	.8254	-.6194
-3	-.5576	-.2951	.6459	-.5213
-4	-.4259	-.1486	.7291	-.5728
1968-1	-.4337	-.1029	.8531	-.5978
-2	-.4309	-.1067	.8532	-.5954
-3	-.4272	-.0999	.8529	-.5903
-4	-.4294	-.1037	.8537	-.5949
1969-1	-.2866	-.2871	.9909	-.6248
-2	-.2815	-.2776	.9931	-.6285
-3	-.2622	-.3071	.9509	-.6288
-4	-.2702	-.2983	.9653	-.6284
1970-1	-.2900	-.2888	.9892	-.6230
-2	-.2630	-.2979	.9598	-.6228
-3	-.3717	-.1501	.8766	-.6217
-4	-.3840	-.1249	.8287	-.6048
1971-1	-.4147	-.0892	.8075	-.6004
-2	-.4267	-.0825	.7981	-.6160
-3	-.4247	-.0787	.8090	-.6131
-4	-.5124	-.2338	.5680	-.4436
1972-1	-.5118	-.2364	.5684	-.4448
-2	-.5115	-.2350	.5656	-.4428
-3	-.5117	-.2361	.5665	-.4441
-4	-.5100	-.2327	.5655	-.4448
1973-1	-.4948	-.2308	.5788	-.4535
-2	-.4559	-.2438	.6152	-.5162
-3	-.4167	-.2387	.6188	-.5164
-4	-.3841	-.1876	.6177	-.4940
1974-1	-.3563	-.1456	.6865	-.5298
-2	-.3563	-.1476	.6824	-.5279
-3	-.3653	-.1562	.6696	-.4998
-4	-.3668	-.1584	.6643	-.4948
1975-1	-.4170	-.0925	.8544	-.6062
-2	-.4028	-.1069	.8664	-.6206
-3	-.4104	-.0974	.8592	-.6155
-4	-.4114	-.0895	.8547	-.5982
1976-1	-.4072	-.0890	.8575	-.6009
-2	-.3577	-.1469	.8023	-.5720
-3	-.3553	-.1540	.7852	-.5684
-4	-.3897	-.1310	.8852	-.6071
1977-1	-.3903	-.1222	.8793	-.5785
-2	-.3506	-.1495	.7613	-.5285
-3	-.3477	-.1630	.7690	-.5025
-4	-.3409	-.1737	.7636	-.5296
1978-1	-.3448	-.1726	.7668	-.5306
-2	-.3384	-.2066	.7788	-.5826
-3	-.3275	-.2225	.7541	-.5624
-4	-.3344	-.2178	.7459	-.5525
1979-1	-.3254	-.2050	.7549	-.5544

Coefficient Evolution Pattern for the GNP Deflator
Inflation Rate Model, 1952-1/1979-1

$$(1 - B)z_t = (1 - \theta_1 B)(1 - \theta_7 B^7 - \theta_8 B^8)a_t$$

	θ_1	θ_7	θ_8
1956-2	-.8350	-.5731	-.1191
-3	-.8341	-.5638	-.1181
-4	-.8126	-.5529	-.1072
1957-1	-.8061	-.5100	-.1091
-2	-.7962	-.5079	-.1293
-3	-.7586	-.5142	-.1127
-4	-.7212	-.5139	-.1001
1958-1	-.7760	-.5038	-.1000
-2	-.7660	-.4929	-.0831
-3	-.7510	-.4100	-.0829
-4	-.7501	-.4100	-.0731
1959-1	-.7654	-.4339	-.0735
-2	-.7777	-.4202	-.0872
-3	-.7762	-.4200	-.0982
-4	-.7661	-.3981	-.1013
1960-1	-.7570	-.3900	-.0813
-2	-.7540	-.3940	-.0909
-3	-.7343	-.3878	-.1079
-4	-.7300	-.4001	-.1232
1961-1	-.7300	-.3842	-.1110
-2	-.7232	-.3779	-.1580
-3	-.7349	-.3378	-.1581
-4	-.7301	-.3210	-.1679
1962-1	-.7235	-.3100	-.1502
-2	-.7225	-.3010	-.1370
-3	-.7225	-.2999	-.1378
-4	-.7145	-.3524	-.1328
1963-1	-.7143	-.3424	-.1427
-2	-.7432	-.3782	-.1424
-3	-.7698	-.3777	-.1019
-4	-.7221	-.3575	-.1028
1964-1	-.7223	-.3576	-.1127
-2	-.7000	-.3575	-.1019
-3	-.7011	-.3028	-.1010
-4	-.2012	-.3129	-.0777
1965-1	-.6990	-.3128	-.0792
-2	-.6985	-.3133	-.0802
-3	-.6998	-.2922	-.0836
-4	-.6990	-.2922	-.0805
1966-1	-.6881	-.3126	-.0926
-2	-.6682	-.3129	-.1156
-3	-.6837	-.2995	-.1138
-4	-.6872	-.4706	-.3014

	θ_1	θ_7	θ_8
1967-1	-.6898	-.4703	-.3201
-2	-.7000	-.5012	-.3103
-3	-.7001	-.5211	-.3303
-4	-.7020	-.5210	-.4703
1968-1	-.7001	-.5121	-.4803
-2	-.7123	-.5000	-.4801
-3	-.7220	-.5561	-.4805
-4	-.7200	-.5560	-.4305
1969-1	-.7239	-.5610	-.4305
-2	-.7224	-.5612	-.4302
-3	-.6973	-.3779	-.2167
-4	-.6896	-.3288	-.1747
1970-1	-.6989	-.3373	-.2115
-2	-.6804	-.3503	-.2007
-3	-.6986	-.3622	-.1941
-4	-.6957	-.3697	-.1905
1971-1	-.6909	-.3554	-.2043
-2	-.6967	-.3076	-.1712
-3	-.6918	-.3399	-.2199
-4	-.6918	-.3440	-.2513
1972-1	-.7020	-.2931	-.2364
-2	-.7031	-.2847	-.2734
-3	-.7067	-.2815	-.2612
-4	-.7090	-.2814	-.2449
1973-1	-.7199	-.2351	-.2266
-2	-.7183	-.2343	-.1945
-3	-.7023	-.2465	-.2034
-4	-.6742	-.2174	-.2328
1974-1	-.6602	-.2272	-.2254
-2	-.6195	-.2546	-.2562
-3	-.5924	-.2509	-.2750
-4	-.5660	-.2214	-.2554
1975-1	-.5643	-.2199	-.2524
-2	-.5921	-.2276	-.2944
-3	-.5899	-.2321	-.2969
-4	-.5839	-.2387	-.3162
1976-1	-.5704	-.2577	-.3363
-2	-.5738	-.2553	-.3325
-3	-.5757	-.2510	-.3297
-4	-.5826	-.2382	-.2937
1977-1	-.5833	-.2365	-.2955
-2	-.5820	-.2344	-.3058
-3	-.5850	-.2357	-.3109
-4	-.5833	-.2308	-.3135
1978-1	-.5861	-.2284	-.3175
-2	-.5950	-.2104	-.2936
-3	-.6027	-.2157	-.2985
-4	-.6009	-.2160	-.2889
1979-1	-.5958	-.2160	-.2907

Coefficient Evolution Pattern for the M1 Growth Rate Model, 1947-2/1979-1

$$(1 - \phi_1 B)(1 - \phi_4 B^4)z_t = \mu_t + (1 - \theta_4 B^4 - \theta_8 B^8)a_t$$

	ϕ_1	ϕ_4	θ_4	θ_8	μ
1956-2	-.7564	-.1687	-.3125	-.5937	.6316
-3	-.7496	-.1478	-.2981	-.6305	.5725
-4	-.7517	-.3619	-.0763	-.8408	.6386
1957-1	-.7829	-.3603	-.0861	-.8434	.5693
-2	-.7662	-.0474	-.6288	-.3698	.6328
-3	-.7774	-.1405	-.4719	-.5208	.6106
-4	-.8525	-.2672	-.4212	-.5723	.5960
1958-1	-.8400	-.4476	-.2592	-.8862	.4780
-2	-.8124	-.4448	-.0153	-.8727	.4674
-3	-.8064	-.4683	-.0456	-.8552	.5313
-4	-.8057	-.4262	-.0091	-.8229	.5415
1959-1	-.8257	-.4422	-.0181	-.8520	.5638
-2	-.7843	-.0719	-.7578	-.2395	.6058
-3	-.7940	-.0721	-.7677	-.2322	.6100
-4	-.8574	-.4964	-.0545	-.8598	.5378
1960-1	-.8550	-.5813	-.0347	-.9024	.5657
-2	-.8000	-.5000	-.0299	-.9001	.4994
-3	-.7718	-.6308	-.0480	-.8532	.5810
-4	-.7745	-.5962	-.0440	-.8536	.5277
1961-1	-.7486	-.1247	-.3815	-.5895	.5421
-2	-.7447	-.2930	-.2659	-.7050	.5843
-3	-.7852	-.1477	-.3030	-.5931	.5073
-4	-.7356	-.0965	-.4158	-.4746	.5309
1962-1	-.7176	-.0395	-.4298	-.4258	.5251
-2	-.7442	-.1564	-.5455	-.3730	.5428
-3	-.7057	-.1594	-.5476	-.3308	.5166
-4	-.7024	-.1603	-.5312	-.3494	.5337
1963-1	-.7252	-.1497	-.5185	-.3298	.5255
-2	-.7043	-.3160	-.7727	-.2209	.5475
-3	-.6874	-.2772	-.7861	-.2087	.5238
-4	-.6938	-.3251	-.7859	-.2074	.5608
1964-1	-.7134	-.2793	-.7603	-.2316	.5474
-2	-.7106	-.1513	-.5371	-.3576	.5673
-3	-.7479	-.1907	-.5781	-.3664	.5780
-4	-.7026	-.1356	-.5250	-.3292	.5713
1965-1	-.7024	-.1360	-.5222	-.3404	.5721
-2	-.6882	-.2411	-.7110	-.2828	.5583
-3	-.6941	-.2840	-.6822	-.3103	.5767
-4	-.7350	-.3353	-.7382	-.2579	.5710
1966-1	-.7238	-.1899	-.5503	-.3488	.6116
-2	-.7258	-.1842	-.5366	-.3652	.6168
-3	-.7187	-.1593	-.5338	-.3688	.6085
-4	-.7135	-.1664	-.5342	-.3665	.6051

	ϕ_1	ϕ_4	θ_4	θ_8	μ
1967-1	-.7180	-.1611	-.5275	-.3736	.6121
-2	-.7030	-.2296	-.6364	-.3557	.5960
-3	-.7787	-.3329	-.8381	-.1580	.6175
-4	-.7849	-.1165	-.6171	-.3781	.6300
1968-1	-.7811	-.1091	-.6187	-.3767	.6317
-2	-.7916	-.1545	-.6311	-.3660	.6566
-3	-.8229	-.0920	-.5326	-.4628	.6651
-4	-.8470	-.1405	-.5004	-.3645	.7052
1969-1	-.8572	-.1302	-.5111	-.3640	.7153
-2	-.8305	-.1186	-.5590	-.4371	.6684
-3	-.8461	-.1613	-.5245	-.3466	.7182
-4	-.8759	-.2573	-.7134	-.2826	.8164
1970-1	-.8506	-.2053	-.6800	-.3140	.6713
-2	-.8232	-.2242	-.5771	-.3060	.7230
-3	-.8000	-.2001	-.5001	-.3001	.7231
-4	-.8001	-.1998	-.5000	-.3000	.7307
1971-1	-.8815	-.1255	-.4787	-.3237	.7831
-2	-.9165	-.2100	-.2022	-.4948	.7423
-3	-.9104	-.2346	-.1785	-.5144	.7323
-4	-.8940	-.2073	-.2104	-.4925	.7100
1972-1	-.9100	-.1508	-.2449	-.4960	.7548
-2	-.9114	-.0867	-.2914	-.4538	.7736
-3	-.9264	-.3402	-.0791	-.5690	.7859
-4	-.9270	-.3398	-.0808	-.5689	.7863
1973-1	-.9348	-.1199	-.2370	-.4762	.8143
-2	-.9158	-.2447	-.6091	-.3841	.7816
-3	-.9203	-.0618	-.2815	-.4568	.7886
-4	-.9127	-.0399	-.2985	-.4309	.7811
1974-1	-.8616	-.3877	-.1432	-.3485	.8376
-2	-.8405	-.2070	-.0028	-.3044	.8132
-3	-.8283	-.4269	-.2331	-.3280	.8167
-4	-.8373	-.2615	-.0579	-.2960	.8254
1975-1	-.8494	-.1055	-.1222	-.2840	.8178
-2	-.8806	-.0235	-.2918	-.2927	.8378
-3	-.8691	-.1871	-.0686	-.3280	.8522
-4	-.8410	-.1811	-.0664	-.2829	.8199
1976-1	-.8321	-.2260	-.0175	-.2507	.8372
-2	-.8411	-.1709	-.0650	-.2611	.8546
-3	-.8389	-.1780	-.0640	-.2648	.8452
-4	-.8550	-.1680	-.1129	-.2629	.8639
1977-1	-.8549	-.1958	-.0793	-.2697	.8696
-2	-.8619	-.1349	-.1382	-.2542	.8831
-3	-.8700	-.1405	-.1416	-.2491	.8975
-4	-.8640	-.1627	-.1238	-.2435	.8918
1978-1	-.8646	-.0478	-.1282	-.2394	.8945
-2	-.8732	-.1690	-.1100	-.2388	.9279
-3	-.8685	-.1602	-.1184	-.2309	.9001
-4	-.8493	-.1700	-.3489	-.1001	.8411
1979-1	-.8849	-.1973	-.5190	-.1381	.8862

Coefficient Evolution Pattern for the MB Growth Rate Model, 1947-2/1979-1

$$(1 - B)(1 - \phi_1 B - \phi_2 B^2)z_t = (1 - \theta_4 B^4 - \theta_8 B^8)a_t$$

	ϕ_1	ϕ_2	θ_4	θ_8
	-.7234	-.3973	.4104	.5869
1956-3	-.7014	-.3746	.4370	.5561
-4	-.6926	-.4558	.4395	.5435
1957-1	-.7077	-.4533	.4335	.5505
-2	-.7140	-.4571	.4491	.5400
-3	-.6733	-.4516	.4106	.5812
-4	-.6936	-.4649	.3879	.5937
1958-1	-.6889	-.4580	.4051	.5867
-2	-.6824	-.4467	.3922	.5947
-3	-.6377	-.4763	.4027	.5777
-4	-.6928	-.4515	.3880	.5928
1959-1	-.6358	-.4447	.3776	.6044
-2	-.6408	-.4641	.3679	.6184
-3	-.6211	-.4122	.4415	.5482
-4	-.5971	-.3939	.4420	.5472
1960-1	-.6495	-.4531	.3952	.5899
-2	-.6245	-.4416	.4080	.5805
-3	-.6423	-.4539	.4000	.5955
-4	-.6618	-.4634	.3979	.5995
1961-1	-.6368	-.4652	.3895	.6051
-2	-.6750	-.4769	.3928	.5995
-3	-.7199	-.4897	.3619	.6280
-4	-.7223	-.4694	.3331	.6465
1962-1	-.7136	-.4527	.3323	.6619
-2	-.6764	-.4448	.3378	.6572
-3	-.7211	-.4334	.3292	.6596
-4	-.7198	-.4530	.3439	.6448
1963-1	-.7207	-.4727	.3141	.5930
-2	-.6755	-.4446	.3618	.6304
-3	-.6864	-.4379	.2128	.4501
-4	-.6763	-.4498	.3817	.6062
1964-1	-.6827	-.4268	.2403	.5267
-2	-.6824	-.4247	.2391	.5282
-3	-.6789	-.4314	.1899	.4491
-4	-.6772	-.4323	.1874	.4469
1965-1	-.6790	-.4333	.2001	.4429
-2	-.6825	-.4399	.1913	.4387
-3	-.6906	-.4350	.1993	.4404
-4	-.6750	-.4443	.1886	.4243
1966-1	-.6695	-.4393	.1794	.4130
-2	-.6690	-.4404	.1782	.4123
-3	-.6588	-.4248	.2016	.4179
-4	-.6460	-.4160	.2256	.4232

	ϕ_1	ϕ_2	θ_4	θ_8
1967-1	-.6559	-.4289	.2112	.4014
-2	-.6458	-.4270	.2155	.4049
-3	-.6309	-.4048	.2319	.3808
-4	-.6246	-.4030	.2362	.3676
1968-1	-.6234	-.4008	.2250	.3606
-2	-.6215	-.4031	.2306	.3452
-3	-.6318	-.4051	.2196	.3606
-4	-.6309	-.4104	.2109	.3633
1969-1	-.6325	-.4121	.2131	.3663
-2	-.6219	-.4150	.1962	.3734
-3	-.6110	-.4047	.1996	.3823
-4	-.6101	-.4177	.1955	.3710
1970-1	-.6022	-.4118	.1949	.3644
-2	-.6008	-.4067	.2025	.3671
-3	-.6000	-.4118	.2018	.3607
-4	-.5889	-.4061	.1933	.3576
1971-1	-.5854	-.3948	.1861	.3558
-2	-.5836	-.3940	.1868	.3386
-3	-.5907	-.3972	.1807	.3460
-4	-.5950	-.3842	.1955	.3645
1972-1	-.5995	-.3801	.1851	.3587
-2	-.5973	-.3886	.1958	.3570
-3	-.5878	-.3759	.1902	.3475
-4	-.5799	-.3709	.2166	.3244
1973-1	-.5748	-.3687	.2145	.3214
-2	-.5750	-.3727	.2159	.3172
-3	-.5704	-.3680	.2231	.3186
-4	-.5663	-.3630	.2429	.3061
1974-1	-.5775	-.3708	.2337	.2959
-2	-.5671	-.3702	.2394	.2915
-3	-.5678	-.3722	.2389	.2928
-4	-.5685	-.3728	.2399	.2913
1975-1	-.5692	-.3668	.2521	.2949
-2	-.5718	-.3662	.2490	.2992
-3	-.5714	-.3673	.2485	.2990
-4	-.5722	-.3692	.2500	.2967
1976-1	-.5726	-.3696	.2516	.2922
-2	-.5700	-.3763	.2496	.2861
-3	-.4826	-.3838	.2489	.2893
-4	-.5825	-.3840	.2489	.2893
1977-1	-.5800	-.3716	.2547	.2897
-2	-.5833	-.3701	.2409	.2909
-3	-.5782	-.3700	.2456	.2883
-4	-.5764	-.3665	.2457	.2898
1978-1	-.5764	-.3662	.2485	.2870
-2	-.5768	-.3655	.2509	.2900
-3	-.5776	-.3648	.2492	.2920
-4	-.5771	-.3657	.2485	.2921
1979-1	-.5837	-.3831	.2502	.2672

VITA

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Between receipt of the M.B.A. degree and re-entry into the M.S. graduate program at Louisiana State University, he worked in the corporate planning division of a large U.S. natural resources conglomerate. In 1976 the author entered the Ph.D. program in Economics at The University of Tennessee, Knoxville and was awarded the degree in the Summer of 1981.

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