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LCM-Stability of Power Series Extensions Characterizes Dedekind Domains

John T. Condo
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To the Graduate Council:

I am submitting herewith a dissertation written by John T. Condo entitled "LCM-Stability of Power Series Extensions Characterizes Dedekind Domains." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Mathematics.

David E. Dobbs, Major Professor

We have read this dissertation and recommend its acceptance:

Robert McConnel, David Anerson, Thomas Handle

Accepted for the Council:

Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

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LCM-STABILITY OF POWER SERIES EXTENSIONS CHARACTERIZES DEDEKIND DOMAINS

A Dissertation

Presented for the Doctor of Philosophy

Degree

John T. Condo

May 1991

DEDICATION

To my wife Donna, my son George, and my parents George and Alice.

ACKNOWLEDGEMENTS

As major professor and friend, Dr. David Dobbs, has always been helpful and encouraging while directing my work. His patience in guiding the development of my mathematical abilities is much appreciated.

Further, the support of the entire algebra group, including Drs. David Anderson, Robert McConnel, an d Shashikant Mulay, through courses and informal office visits, greatly aided my mathematical education. My thanks go to each one of them for their assistance.

ABSTRACT

In this dissertation, we prove the following main result in Chapter 3. A (commutative integral) domain R is a Dedekind domain if and only if $R[[X]] \subset T[[X]]$ is LCM-stable for each domain T containing R as a subring. Analogous results on flatness are given in Chapter 4. Complete statements of results cited from the literature appear in Chapter 2, while Chapter 1 places the above stated main result in historical context, as extending work of F. Richman and H. Uda.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

Suppose R is a (commutative) domain with quotient field K. A well-known result of F. Richman [12, Theorem 4] asserts that R is a Prüfer domain if and only if each overring of R (i.e., each R-subalgebra of K) is R-flat. As a consequence, it is easy to see that R is a Prüfer domain if and only if each domain T containing R as a subring induces a flat extension, T[X] over R[X], of polynomial rings. In fact, there is an ideal-theoretic variant in terms of the LCM-stability concept. (As in [8], [9], we say an R-module M is LCM-stable if $aM \cap bM = (aR \cap bR)M$ for all $a, b \in R$.) Namely, R is Prüfer if and only if T[X] is LCM-stable over R[X] for each domain T containing R as a subring. Our main interest here is to study the analogous property for power series rings. In other words, we seek to characterize domains R such that T[[X]] is LCM-stable over R[[X]] for each domain T containing R as a subring. Our main result, Theorem 3.11, is that these R are precisely the Dedekind domains.

It is interesting to record additional motivation for the above problem. Since flatness implies LCM-stability and flatness is a universal property, a flat extension $R \subset T$ of domains entails LCM-stability of $R[X] \subset T[X]$. It is natural to ask whether LCM-stablity of $R \subset T$ also entails LCM-stability of $R[X] \subset T[X]$. Several papers ([13], [15], [16]) have recently culminated in an affirmative answer [15, Corollary 3.7] in case R is a GCD-domain; later work gave an affirmative answer in case R is a Krull domain [16, Theorem 11]. These results naturally lead one to ask when LCM-stability of $R \subset T$ entails

LCM-stability of $R[[X]] \subset T[[X]]$. The easiest case arises when all such T are LCM-stable over R ; and, by the remarks of the first paragraph, this happens precisely for Prüfer domains R . To a large extent, the details of Chapter 3 amount to determining that the Dedekind domains are precisely the (Prüfer) domains with the property $R[[X]] \subset T[[X]]$ is LCM-stable for each T containing R as a subring.

Theorem 3.5 establishes that if R is a Dedekind domain, then $T[[X]]$ is LCM-stable over $R[[X]]$ for each domain T containing R as a subring. With respect to the converse, namely determining which domains R satisfy $T[[X]]$ is LCM-stable over $R[[X]]$ for all domains T containing R as a subring, Corollary 3.2 reduces the study to a class of Prüfer domains. By considering the nonArchimedean domains (in the sense of [14]) , we reduce the problem to studying one-dimensional Prüfer domains in Corollary 3.7. The local case leads to DVR's in Theorem 3.8, thanks to an appeal to work of M. Anderson and J. Watkins [1, Lemma, p. 191] . The route back from DVR's to the (global) answer is essayed by means of a globalization result and appeals to work on almost Dedekind domains (in the sense of [9]) due to J.T. Arnold [2], [4], and to J.T. Arnold and J. Brewer [3] .

Chapter 4 presents partial results on related analogous questions about flatness.

Throughout, given a symbol representing a power series, the individual coefficients are denoted by subscripting the symbol. For example, if

$\alpha \in R[[X]]$, then $\alpha = \sum_{i=0}^{\infty} \alpha_i X^i$. If $\{\alpha_j\}_{j=1}^{n}$ is a collection of power series, we write $\alpha_j = \sum_{i=0}^{\infty} \alpha_{j,i} X^i$ for each j .

We now proceed to state carefully in Chapter 2 all the known results that will be used in our proofs, as well as the results referred to above.

CHAPTER 2

BACKGROUND

We begin with some definitions. A Prüfer domain is an (integral) domain in which every nonzero finitely generated ideal is invertible. A Dedekind domain is a domain in which every nonzero ideal is invertible. A valuation domain is a quasilocal Prüfer domain. For a domain R, an R-module M is R-flat if tensoring with M is an exact functor from the category of R-modules to the category of abelian groups. An R-module M is n-flat over R if each relation $r_1 m_1 + \dots + r_n m_n = 0$ with $r_i \in R$, $m_i \in M$ is induced by suitable $f_j \in M$ $(1 \le j \le d)$ and $r_{ij} \in R$ $(1 \le i \le n, 1 \le j \le d)$ satisfying $m_i = \sum_{j=1}^{d} r_{ij} f_j$ for each i and $\sum_{i=1}^{n} r_i r_{ij} = 0$ for each j. It is shown in [5, Corollary 1, p. 27] that an R-module M is R-flat if and only if M is n-flat over R for each $n \ge 1$.

We remark that if R is a Prüfer domain and T is a domain containing R as a subring, then T is R-flat; in particular, T is 2-flat over R. D.E. Dobbs, in [8, Theorem 3.3], shows 2-flatness is equivalent to LCM-stability (for an extension of domains). Thus, if T is a domain containing a Prüfer domain R as a subring, then T is LCM-stable over R.

A domain R is regular if R is Noetherian and for each maximal ideal M of R, the height of M is equal to $\dim_{R/M} (MR_M/M^2R_M)$. A domain R is an SFT-ring if, for each ideal I of R, there is a finitely generated ideal J of R such that $J \subset I$ and there exists an integer $k \ge 1$ such that $i^k \in J$ for each $i \in I$.

We now list the results we refer to in roughly the order in which they are cited.

<u>Theorem 2.1</u> (F. Richman [12, Theorem 4]) . If A is an integral domain such that every overring of A is A-flat, then A is a Prüfer domain.

<u>Theorem 2.2</u> (H. Uda [15, Corollary 3.7]) . Let A be a GCD-domain. Then the following statements are equivalent:

(1) $A \subset B$ is LCM-stable;

(2) $A[X] \subset B[X]$ is LCM-stable.

<u>Theorem 2.3</u> (H. Uda [16, Theorem 11]) . Let A be a Krull domain. Then the following statements are equivalent:

(1) $A \subset B$ is LCM-stable;

(2) $A[X] \subset B[X]$ is LCM-stable.

<u>Theorem 2.4</u> (M. Anderson and J. Watkins [1, Lemma, p. 191]) . Let D be a rank 1 valuation domain, with valuation v . Let $s_0, s_1, s_2, \ldots$ be elements of D whose values satisfy these two conditions:

(1) $v(s_0) > v(s_1) > v(s_2) \ldots$, and

(2) $v(s_i s_{i+1}^{-1}) > v(s_{i+1} s_{i+2}^{-1})$, for all i .

Consider $f, g \in D[[X]]$ defined by

$$f = s_0 \text{ and } g = s_0 + s_1 X + s_2X^2 + \dots .$$

Suppose that $h \in (f) \cap (g)$, and consequently h may be written as bg , where $b = b_0 + b_1X + b_2X^2 + \dots \in D[[X]]$. Then $v(b_is_j) \geq v(s_0)$ for all i and j . In particular, $v(b_0) \geq v(s_0\, s_j^{-1})$, for all j .

<u>Theorem 2.5</u> (H. Uda [15, Proposition 1.7]). Let B be an overring of a domain A . Then the following statements are equivalent:

(1) $A \subset B$ is LCM-stable;

(2) $A \subset B$ is flat.

<u>Theorem 2.6</u> (N. Bourbaki [5, Corollary 1, p. 27]) . For a right A-module E to be flat, it is necessary and sufficient that the following condition hold:

If $(e_i)_{i\in I}$ and $(b_i)_{i\in I}$ are two finite families of elements of E and A respectively such that $\sum_{i\in I} e_i\, b_i = 0$, there exists a finite set J , a family $(x_j)_{j\in J}$ of elements of E and a family (a_{ji}) $(j \in J , i \in I)$ of elements of A such that $\sum_{i\in I} a_{ji}\, b_i = 0$ for all $j \in J$ and $e_i = \sum_{j\in J} x_j\, a_{ji}$ for all $i \in I$.

<u>Theorem 2.7</u> (D.E. Dobbs [8, Theorem 3.3]) . Let R be an integral domain and E an R-module. Then E is 2-flat over R if and only if E is LCM-stable over R .

Theorem 2.8 (H. Uda [15, Proposition 1.6]). For domains $A \subset B \subset C$, the following statements are equivalent:

(1) $B \subset C$ is LCM-stable;

(2) For each $M \in \mathrm{Max}(A)$, $B_M \subset C_M$ is LCM-stable.

Theorem 2.9 (c.f. I. Kaplansky [11, Theorem 188]). If R is a regular UFD, so is $R[[X]]$.

Theorem 2.10 (I. Kaplansky [11, Theorem 184]). Let R be a Noetherian domain with the property that every finitely generated module has a finite free resolution. Then R is a UFD.

Theorem 2.11 (J. T. Arnold [2, Theorem 1]). Let R be a commutative ring with identity. The following conditions are equivalent:

(1) R does not have the SFT property;

(2) There exists a prime ideal P of R such that $P[[X]] \neq \mathrm{rad}(PR[[X]])$.

Theorem 2.12 (J.T. Arnold and J. Brewer [4, Proposition 1]). Let R be a commutative ring with identity and let M be a maximal ideal of R. If Q is a prime ideal of $R[[X]]$ containing $MR[[X]]$, then $Q \subset M[[X]]$ or $Q = M + (X)$. Therefore, if $M[[X]] \supsetneq \mathrm{rad}(MR[[X]])$, then there exists a prime ideal Q of $R[[X]]$ such that $MR[[X]] \subset Q \subsetneq M[[X]]$.

Theorem 2.13 (J.T. Arnold and J. Brewer [4, Theorem 1]). Let P be a prime ideal of the domain D. If $(D[[X]])_{P[[X]]}$ is a valuation ring, then D_P is a rank one discrete valuation ring. Moreover, $(D[[X]])_{P[[X]]}$ is rank one discrete.

Theorem 2.14 (J.T. Arnold and J. Brewer [4, Theorem 3]). Let P be a prime ideal of a domain D with the property that D_P is a rank one discrete valuation ring. The following conditions are equivalent:

(1) $(D[[X]])_{P[[X]]}$ is a valuation ring;

(2) $D_P[[X]] \cap$ q.f. $(D[[X]]) \subset (D[[X]])_{P[[X]]}$.

Theorem 2.15 (N. Bourbaki [5, Theorem 3, p. 203]). Let A be a commutative Noetherian ring, M an ideal of A. Then the A-module $\hat{A}$ is A-flat.

Theorem 2.16 (N. Bourbaki [5, Proposition 4, p. 231]). Let A and B be commutative Noetherian rings, $h: A \to B$ a ring homomorphism, I an ideal of A, and J an ideal of B containing IB and contained in the Jacobson radical of B. Let $\hat{A}$ be the Hausdorff completion of A with respect to the I-adic topology and $\hat{B}$ the Hausdorff completion of B with respect to the J-adic topology ; h is continuous with these topologies and $\hat{h}: \hat{A} \to \hat{B}$ therefore makes $\hat{B}$ into an $\hat{A}$-algebra. Let M be a finitely generated B-module and $\hat{M}$ its Hausdorff completion with respect to the J-adic topology; the following properties are equivalent:

(a) $\hat{M}$ is a flat A-module;

(b) $\hat{M}$ is a flat $\hat{A}$-module.

CHAPTER 3

MAIN RESULTS

Our goal is to classify those domains R which have the property that $R[[X]] \subset T[[X]]$ is LCM-stable for all domains T containing R as a subring. We begin by showing in Proposition 3.2 that any such domain R must be a Prüfer domain. We first require a lemma.

Lemma 3.1. Suppose R is a subring of a domain S. If $R[[X]] \subset S[[X]]$ is LCM-stable, then $R \subset S$ is LCM-stable.

Proof. Let $a, b \in R$ and let $\gamma \in aS \cap bS$. Then $aS \cap bS \subset aS[[X]] \cap bS[[X]] = (aR[[X]] \cap bR[[X]])\, S[[X]]$ by hypothesis. So there exist $f_i \in aR[[X]] \cap bR[[X]]$, $\alpha_i \in S[[X]]$, $(1 \le i \le n)$ such that $\gamma = \sum_{i=1}^{n} f_i \alpha_i$. Equating constant coefficients gives $\gamma = \sum_{i=1}^{n} f_{i,0}\, \alpha_{i,0}$. Since $f_i \in aR[[X]] \cap bR[[X]]$ for each i, $f_{i,0} \in aR \cap bR$. Thus $\gamma \in (aR \cap bR)S$, and so $R \subset S$ is LCM-stable. ∎

Proposition 3.2. Let R be a domain. If $R[[X]] \subset T[[X]]$ is LCM-stable for each domain T containing R as a subring, then R is a Prüfer domain.

Proof. Lemma 3.1 shows $R \subset T$ is LCM-stable for all domains T containing R as a subring. In particular, $R \subset T$ is LCM-stable for all overrings T of R. H. Uda in [15, Proposition 1.7] shows that this implies

$R \subset T$ is flat for all overrings T of R. Hence R is a Prüfer domain by F. Richman [12, Theorem 4]. ■

We shall soon show that among the Prüfer domains, (at least) the Dedekind domains have the desired property. First, we prove two lemmas.

Lemma 3.3. Let R be a GFD and let T be a domain containing R as a subring. Then $R \subset T$ is LCM-stable if (and only if) $aT \cap bT = (aR \cap bR)T$ for all $a, b \in R$ such that g.c.d.$(a, b) = 1$.

Proof. The "only if" assertion is trivial. For the converse, let $a', b' \in R \setminus \{0\}$. Suppose $c' \in a'T \cap b'T$. Set $d = \text{g.c.d.}(a', b')$ and put $a = a'd^{-1}$, $b = b'd^{-1}$. There exist $\alpha, \beta \in T$ such that $c' = a'\alpha = b'\beta$. Thus $c' = ad\alpha = bd\beta$ and $c'd^{-1} = a\alpha = b\beta$. By hypothesis, there exist $r_j \in aR \cap bR$ and $\tau_j \in T$ $(1 \leq j \leq n)$ such that $c'd^{-1} = \sum_{j=1}^{n} r_j \tau_j$. It follows that $c' = (c'd^{-1})d = \sum_{j=1}^{n} (r_j d) \tau_j$. Since $r_j d \in a'R \cap b'R$, we have $c' \in (a'R \cap b'R)T$, and so $R \subset T$ is LCM-stable. ■

Lemma 3.4. Suppose R and T are domains, with R a subring of T. Let S be a multiplicatively closed subset of $R[[X]]$ not containing X. Then $R[[X]]_S \subset T[[X]]_S$ is LCM-stable if (and only if)

$$fT[[X]]_S \cap gT[[X]]_S = (fR[[X]]_S \cap gR[[X]]_S)\, T[[X]]_S$$

for all $f, g \in R[[X]]$ such that $f_0 \neq 0$ and $g_0 \neq 0$.

Proof. Once again, the "only if" assertion is trivial. For the converse, note that a typical element of $R[[X]]_S$ is of the form rs^{-1} where $r \in R[[X]]$ and $s \in S$. Since s is a unit of $R[[X]]_S$, $rR[[X]]_S = rs^{-1} R[[X]]_S$. Hence, to establish LCM-stability, it suffices to consider nonzero elements $f, g \in R[[X]]$. Then there exist integers $n, m \geq 0$ and $f^*, g^* \in R[[X]]$ with $f^*_0 \neq 0$ and $g^*_0 \neq 0$ such that $f = X^n f^*$ and $g = X^m g^*$. Now, suppose $\gamma \in fT[[X]]_S \cap gT[[X]]_S$. Then there exist $\alpha, \beta \in T[[X]]_S$ with $\gamma = f\alpha = g\beta$. So $X^n f^*\alpha = X^m g^*\beta$. Without loss of generality, $n \geq m$. Thus $X^{n-m} f^*\alpha = g^*\beta$. Since X is prime in $T[[X]]_S$ and X does not divide g^*, X^{n-m} divides β. So there exists $\beta^* \in T[[X]]_S$ such that $\beta = X^{n-m} \beta^*$. Hence, $\gamma X^{-n} = f^*\alpha = g^*\beta^*$. By assumption, there exist $r_j \in f^* R[[X]]_S \cap g^* R[[X]]_S$ and $\tau_j \in T[[X]]_S$ $(1 \leq j \leq n)$ with $\gamma X^{-n} = \sum_{j=1}^{n} r_j \tau_j$. Therefore, $\gamma = \sum_{j=1}^{n} (r_j X^n) \tau_j$. Since $r_j X^n \in f R[[X]]_S \cap gR[[X]]_S$ for $1 \leq j \leq n$, $R[[X]]_S \subset T[[X]]_S$ is LCM-stable. ■

Theorem 3.5. Let D be a Dedekind domain. Then $D[[X]] \subset T[[X]]$ is LCM-stable for every domain T containing D as a subring.

Proof. As shown by H. Uda [15, Proposition 1.6], it is enough to prove that $D[[X]]_{(P,X)} \subset T[[X]]_{(P,X)}$ is LCM-stable for every nonzero prime ideal P of D. Fix a prime ideal P, and set $R = D[[X]]_{(P,X)}$ and $S = T[[X]]_{(P, X)}$.

Since D is regular, so is $D[[X]]$ (c.f. [11, Exercise 5, p. 121]) and hence R is regular local. Hence, a theorem of Auslander-Buchsbaum shows R is a UFD (c.f. [11, Theorem 184]). So, by Lemma 3.3, it is enough to show $fS \cap gS = (fR \cap gR)S$ for all $f, g \in R$ such that g.c.d.$(f, g) = 1$. By Lemma 3.4, we may assume $f, g \in D[[X]]$ and $f_0 \neq 0$ and $g_0 \neq 0$.

Finally, we may assume that neither f nor g is a unit of R. To see this, note that if f, for example, is a unit of R, then $fR = R$ and $fS = S$ whence $fS \cap gS = gS = (gR)S = (fR \cap gR)S$.

This last assumption shows $(f, g) \subset (P, X)_{(P, X)}$, the maximal ideal of R. Since R is a UFD, the height 1 prime ideals of R are principal. Then, g.c.d.$(f, g) = 1$ and $\dim R = 2$ show that $(P, X)_{(P, X)}$ is the only prime ideal of R containing (f, g). Therefore $\mathrm{rad}(f, g) = (P, X)_{(P, X)}$. Hence, there exist $r_1, r_2 \in R$ and an integer $n \geq 1$ such that

$$X^n = r_1 f + r_2 g .$$

Consider $\gamma \in fS \cap gS$. There exist $\alpha, \beta \in S$ such that $\gamma = f\alpha = g\beta$. Multiplying both sides of the displayed equation by α yields $X^n\alpha = r_1(f\alpha) + r_2 g\alpha = r_1(g\beta) + r_2 g\alpha = g(r_1 \beta + r_2 \alpha)$. Now X is prime in S and $g_0 \neq 0$. Therefore, $\delta = (r_1\beta + r_2\alpha)X^{-n} \in S$. Then $\alpha = g\delta$ and $\gamma = f\alpha = fg\,\delta \in (fR \cap gR)S$, so that $fS \cap gS = (fR \cap gR)S$. ■

Results (3.6)-(3.10) will show that a Dedekind domain is the only type of (Prüfer) domain R having the property that $R[[X]] \subset T[[X]]$ is LCM-stable for each domain T containing R as a subring. The next proposition will be used

to prove that no Prüfer domain R with $\dim R \geq 2$ can have this property. First, recall from [13] that a domain R is <u>Archimedean</u> if $\bigcap_{n=1}^{\infty} d^n R = 0$ for every nonzero nonunit $d \in R$; and recall (c.f. [10]) that a valuation domain V is Archimedean if and only if $\dim V \leq 1$.

<u>Proposition 3.6</u>. Let R be a nonArchimedean domain. Then there exists an R-flat domain T containing R as a subring such that $R[[X]] \subset T[[X]]$ is not LCM-stable; it can be arranged that T is an overring of R.

<u>Proof</u>. Since R is nonArchimedean, there is a nonzero nonunit $d \in R$ such that $\bigcap_{n=1}^{\infty} (d^n) \neq 0$. Choose $0 \neq a \in \bigcap_{n=1}^{\infty} (d^n R)$. Set $f = a + \sum_{n=1}^{\infty} ad^{-2^n} X^n$. Set $T = R[d^{-1}] = R_{\{1,d,d^2,\ldots\}}$; of course, T is R-flat.

By the choice of a, we have $ad^{-2^n} \in R$ for each $n \geq 1$. Thus, $f \in R[[X]]$. Also, $f a^{-1} = 1 + \sum_{n=1}^{\infty} d^{-2^n} X^n \in T[[X]]$. In fact, since its constant term is a unit of T, $f a^{-1}$ is a unit of $T[[X]]$, and so $a = f(f a^{-1})^{-1} \in f T[[X]]$. Therefore, $a \in f T[[X]] \cap a T[[X]]$. We shall show that $a \notin (f R[[X]] \cap aR[[X]]) T[[X]]$.

First, we claim that if a and f are as above, and $r, s \in R[[X]]$ are such that $rf = sa$, then $r_0 \in \bigcap_{n=1}^{\infty} (d^n R)$. To see this, observe first that equating coefficients of X^n on each side of $rf = sa$ gives:

$$ad^{-2^n} r_0 + ad^{-2^{(n-1)}} r_1 + \ldots + ad^{-2} r_{n-1} + a r_n = as_n .$$

Next, multiplying both sides of this equation by $d^{2^{n-1}}$ leads to:

$$ad^{-2^{(n-1)}} r_0 = a(s_n d^{2^{(n-1)}} - r_1 - \dots - r_{n-1} d^{2^{(n-1)}-2} - r_n d^{2^{(n-1)}}) .$$

Thus, dividing by a yields:

$$d^{-2^{(n-1)}} r_0 = s_n d^{2^{(n-1)}} - r_1 - \dots - r_{n-1} d^{2^{(n-1)}-2} - r_n d^{2^{(n-1)}} \in R .$$

This holds for all $n \geq 1$. Hence $r_0 \in \bigcap_{n=1}^{\infty} (d^n R)$, as claimed.

Now suppose, contrary to what we wish to show, that $a \in (f\,R[[X]] \cap a\,R[[X]])\,T[[X]]$. Then there exist $h_i \in f\,R[[X]] \cap a\,R[[X]]$ and $\gamma_i \in T[[X]]$ $(1 \leq i \leq m)$ such that $a = \sum_{i=1}^{m} h_i \gamma_i$. Furthermore, for each i , there are $r_i, s_i \in R[[X]]$ with $h_i = r_i f = s_i a$. By the above claim, $r_{i,0} \in \bigcap_{n=1}^{\infty} (d^n R)$ for each i . Equating constant terms of $a = \sum_{i=1}^{m} h_i \gamma_i$ gives $a = \sum_{i=1}^{m} h_{i,0} \gamma_{i,0}$. Since $h_i = r_i f$ and $f_0 = a$, we have $a = \sum_{i=1}^{m} r_{i,0}\, a\gamma_{i,0}$. Dividing by a gives:

$$1 = \sum_{i=1}^{m} r_{i,0}\, \gamma_{i,0} .$$

Each $\gamma_{i,0} \in T = R[d^{-1}]$; so, for each i , there is an integer u_i such that $d^{u_i} \gamma_{i,0} \in R$. Let $u = \max_{1 \leq i \leq m} \{u_i\}$. Then $d^u \gamma_{i,0} \in R$ for each i . Multiplying the

last displayed equation by d^u gives $d^u = \sum_{i=1}^{m} r_{i,0} (\gamma_{i,0} d^u)$. Since $r_{i,0} \in \bigcap_{n=1}^{\infty} (d^n R)$ and $\gamma_{i,0} d^u \in R$ for $1 \leq i \leq m$, we have $d^u \in \bigcap_{n=1}^{\infty} (d^n R)$, whence we have $d = 0$ or d is a unit of R , contradicting our choice of d . Thus, $a \notin (f R[[X]] \cap a R[[X]]) T[[X]]$, and $R[[X]] \subset T[[X]]$ is not LCM-stable. ■

Corollary 3.7. Let R be a Prüfer domain such that $\dim R \geq 2$. Then there is $c \in R$ such that $R[[X]] \subset R[c^{-1}] [[X]]$ is not LCM-stable.

Proof. There exists a chain of prime ideals $0 \subsetneq P \subsetneq M$ in R . Pick $c \in M \backslash P$. Since R is a Prüfer domain, R_M is a valuation domain. Therefore $P \subset c R_M$. Hence $P \subset \bigcap_{n=1}^{\infty} (c^n R_M)$. In particular, we may choose

$$0 \neq a \in [\bigcap_{n=1}^{\infty} c^n R_M] \cap P .$$

By the choice of a, $ac^{-n} \in R_M$ for each n . So, for each $n \geq 1$, there exist $r_n \in R$ and $s_n \in R \backslash M$ such that $ac^{-2^n} = r_n s_n^{-1}$. Put $f = a + \sum_{n=1}^{\infty} r_n X^n \in R[[X]]$. Now $r_n = as_n c^{-2^n}$, so that $f = a + \sum_{n=1}^{\infty} as_n c^{-2^n} X^n$. Since $1 + \sum_{n=1}^{\infty} s_n c^{-2^n} X^n$ is a unit of $R[c^{-1}][[X]]$ and $f = a(1 + \sum_{n=1}^{\infty} s_n c^{-2^n} X^n)$, we have $fR[c^{-1}][[X]] \cap a R[c^{-1}][[X]] = aR[c^{-1}][[X]]$. However, since $f, a \in R_M[[X]]$ and R_M is nonArchimedean (R_M is a valuation domain of dimension at least two), we may apply the proof of Proposition 3.6 to get $a \notin (fR_M[[X]] \cap aR_M[[X]])R_M [c^{-1}][[X]]$. (To adapt the earlier proof, get each $r_{i,0} \in \bigcap_{n=1}^{\infty} (c^n R_M)$

since each $s_n \in R\backslash M$.) Since $R[[X]] \subset R_M[[X]]$ and $R[c^{-1}][[X]] \subset R_M[c^{-1}][[X]]$ we have $(fR[[X]] \cap aR[[X]])\, R[c^{-1}][[X]] \subset (fR_M[[X]] \cap aR_M[[X]])\, R_M[c^{-1}]\, [[X]]$. Therefore, $a \notin (fR[[X]] \cap aR[[X]])\, R[c^{-1}]\, [[X]]$, and so $R[[X]] \subset R[c^{-1}][[X]]$ is not LCM-stable. ■

It should be noted that the "Prüfer" hypothesis in (3.7) can be weakened to "locally divided" (in the sense of D.E. Dobbs [6]) . The present formulation was chosen in order to clarify the route to our main goal, Theorem 3.11.

Corollary 3.7 shows that we need only consider those Prüfer domains R such that $\dim R \leq 1$. We first consider the local case.

<u>Theorem 3.8</u> . Let (V, M) be a rank one nondiscrete valuation domain. Then there exists a domain D containing V as a subring such that $V[[X]] \subset D[[X]]$ is not LCM-stable.

<u>Proof</u>. Since V is rank one nondiscrete, there is a (surjective) valuation $v\colon q.f.(V) \to G$ where G is a dense subgroup of (the additive group of) $\mathbb{R}$. By density, we can inductively construct a sequence $\{\varepsilon_j\}_{j=1}^{\infty} \subset G$ such that $\lim_{j\to\infty} \varepsilon_j = 0$, $\varepsilon_j > \varepsilon_{j+1} > 0$ for all $j \geq 1$, and $\varepsilon_j - \varepsilon_{j+1} > \varepsilon_{j+1} - \varepsilon_{j+2}$ for all j . (Perhaps the easiest way to do this is to insure that $0 < \varepsilon_{j+1} < 2^{-1}\varepsilon_j$) . For each $j \geq 1$, choose $t_j \in V$ such that $v(t_j) = \varepsilon_j$. Put $g = \sum_{j=0}^{\infty} t_{j+1} X^j \in V[[X]]$. Let Y be an indeterminate over V , and set $D = V[Y, \{Yt_j^{-1}\}_{j=2}^{\infty}, \{Yt_j t_1^{-1}\}_{j=1}^{\infty}{}^{2}]$.

We claim $Yt_1^{-1} \notin D$. To see this, suppose, to the contrary, that $Yt_1^{-1} \in D$. Then Yt_1^{-1} can be expressed as a polynomial over V in Y, $\{Yt_j^{-1}\}_{j=2}^{\infty}$, and $\{Yt_j t_1^{-1}\}_{j=1}^{\infty}$. Working in q.f.(V) [Y], we write this polynomial as $Yt_1^{-1} = c_0 + c_1Y + c_2Y^2 + \ldots + c_nY^n$, with $c_i \in$ q.f.(V) for all i. Since Y is an indeterminate, we have $c_i = 0$ if $i \neq 1$. Hence any element of D of the form

c_1Y (with $c_1 \in$ q.f.(V)) must be a linear combination of Y, $\{Yt_j^{-1}\}_{j=2}^{\infty}$, and $\{Yt_j t_1^{-1}\}_{j=2}^{\infty}$. Thus, $Yt_1^{-1} = \sum_{r=1}^{a} d_r Yt_{j_r}^{-1} + \sum_{s=1}^{b} e_s Yt_{j_s} t_1^{-1} + cY$ where d_r, e_s, $c \in V$ for all r, s. (Note all $j_r \geq 2$.) Set $u = \max_{r,s}\{v(t_{j_r}), v(t_{j_s} t_1^{-1})\}$. Note that $0 \leq u < v(t_1)$ since $\{\varepsilon_j\}_{j=1}^{\infty}$ is a strictly decreasing sequence of positive numbers. Pick $t \in V$ such that $v(t) = u$. Then, multiplying the above linear combination by tY^{-1} leads to:

$$tt_1^{-1} = \sum_{r=1}^{a} d_r tt_{j_r}^{-1} + \sum_{s=1}^{b} e_s(tt_{j_s} t_1^{-1}) + ct.$$

By the choices of u and t, this is an element of V. But $v(tt_1^{-1}) = v(t) - v(t_1) < 0$, whence $tt_1^{-1} \notin V$, a contradiction. Hence, $Yt_1^{-1} \notin D$, as claimed.

Consider Y_g. Clearly, $Y_g \in g\, D[[X]]$. Also,

$$Y_g = \sum_{j=0}^{\infty} Y t_{j+1} X^j = \sum_{j=0}^{\infty} t_1 (Yt_{j+1} t_1^{-1}) X^j = t_1 \sum_{j=0}^{\infty} (Yt_{j+1} t_1^{-1}) X^j \in t_1 D[[X]].$$

Hence, $Y_g \in g\, D[[X]] \cap t_1 D[[X]]$. We shall show that

$Y_g \notin (g\,V[[X]] \cap t_1\,V[[X]])\,D[[X]]$, and therefore that $V[[X]] \subset D[[X]]$ is not LCM-stable.

Suppose $f \in g\,V[[X]] \cap t_1\,V[[X]]$. Then there exist $r, s \in V[[X]]$ with $f = gr = t_1 s$. By [1, Lemma, p. 191], $v(r_i) \geq v(t_1) - v(t_{j+1})$ for each i and for each j. Since $\lim_{j\to\infty} v(t_{j+1}) = 0$, we have $v(r_i) \geq v(t_1)$ for each i. Thus $r_i t_1^{-1} \in V$ for each i, so that $rt_1^{-1} \in V[[X]]$. Next, write $f = gr = gt_1(rt_1^{-1})$ and equate constant terms. Since $g_0 = t_1$, this gives

$f_0 = g_0\, t_1 (r_0\, t_1^{-1}) = t_1^2\, r_0\, t_1^{-1} = t_1\, r_0$. Thus,

$v(f_0) = v(t_1\, r_0) = v(t_1) + v(r_0) \geq v(t_1) + v(t_1) = v(t_1^2)$. Hence, there is $f_0^* \in V$ such that $f_0 = t_1^2 f_0^*$.

Now, if $Y_g \in (g\,V[[X]] \cap t_1\,V[[X]])\,D[[X]]$, then $Y_g = \sum_{k=1}^{m} f_k \alpha_k$ for some $f_k \in g\,V[[X]] \cap t_1\,V[[X]]$ and $\alpha_k \in D[[X]]$. Equating constant terms and using the result of the preceding paragraph gives $f^*_{k,0} \in V$ (for $1 \leq k \leq m$) such that

$Yg_0 = Yt_1 = \sum_{k=1}^{m} f_{k,0}\,\alpha_{k,0} = \sum_{k=1}^{m} (t_1^2 f^*_{k,0})\,\alpha_{k,0} = t_1^2 \sum_{k=1}^{m} f^*_{k,0}\,\alpha_{k,0}$. Thus,

$Yt_1 = t_1^2 \sum_{k=1}^{m} f^*_{k,0}\,\alpha_{k,0}$. Dividing by t_1^2 yields $Yt_1^{-1} = \sum_{k=1}^{m} f^*_{k,0}\,\alpha_{k,0} \in D$. But this contradicts the claim that was established above. Hence,

$Y_g \notin (gV[[X]] \cap t_1\,V[[X]])\,D[[X]]$ and $V[[X]] \subset D[[X]]$ is not LCM-stable. ■

The local case that was considered in Theorem 3.8 may now be used to eliminate any one-dimensional Prüfer domain which is not almost Dedekind. (As in [10], a domain R is said to be almost Dedekind if R_M is a DVR for

each maximal ideal M of R.) Each almost Dedekind domain is Prüfer and of dimension at most one. The converse is false.

Corollary 3.9. Let R be a Prüfer domain. If $\dim R = 1$ and R is not almost Dedekind, then there is a domain D containing R as a subring such that $R[[X]] \subset D[[X]]$ is not LCM-stable.

Proof. Since R is a Prüfer domain, R_M is a valuation domain for each prime ideal M of R. Moreover, $\dim R = 1$ shows R_M is a rank one valuation domain for each nonzero prime ideal M. Also, since R is not almost Dedekind, there is at least one prime ideal M such that R_M is nondiscrete. We fix a prime M such that R_M is a nondiscrete valuation domain.

As in the proof of Theorem 3.8, we may choose $t_j \in R_M$ such that $v(t_j) = \varepsilon_j$ for all $j \geq 1$, where the sequence $\{\varepsilon_j\}_{j=1}^{\infty}$ is as in the earlier proof. Now, for each j, write $t_j = r_j s_j^{-1}$, with $r_j \in R$, $s_j \in R \setminus M$. Since s_j is a unit of R_M, we have $v(s_j) = 0$ for $j \geq 1$. Thus $v(t_j) = v(r_j s_j^{-1}) = v(r_j) - v(s_j) = v(r_j)$ for all $j \geq 1$. So, by putting $g = \sum_{j=0}^{\infty} r_{j+1} X^j$, the proof of Theorem 3.8 shows there exists a domain D containing R_M (and hence R) as a subring such that $g\, D[[X]] \cap r_1 D[[X]] \supsetneq (g\, R_M[[X]] \cap r_1 R_M[[X]])\, D[[X]]$. But $g, r_1 \in R[[X]]$ and $g\, R_M[[X]] \cap r_1 R_M[[X]] \supset g\, R[[X]] \cap r_1 R[[X]]$. Thus, $g\, D[[X]] \cap r_1 D[[X]] \supsetneq (g\, R[[X]] \cap r_1 R[[X]])\, D[[X]]$, and so the domain D has the required properties. ■

Finally, we show the almost Dedekind, but not Dedekind, domains do not have the desired property.

Theorem 3.10 . Let R be an almost Dedekind, but not Dedekind, domain. Then there exists a domain D containing R as a subring such that $R[[X]] \subset D[[X]]$ is not LCM-stable; it can be arranged that D is an overring of R.

Proof . By a remark of Arnold [3, section 3, p. 4], R is not an SFT ring. Hence, by [2, Theorem 1], there exists a prime ideal P of R such that $P[[X]] \neq \mathrm{rad}(PR[[X]])$. Then [4, Proposition 1] shows $\mathrm{height}(P[[X]]) \geq 2$. Therefore $R[[X]]_{P[[X]]}$ is not a valuation domain; indeed, since $\dim(R[[X]]_{P[[X]]}) \geq 2$, $R[[X]]_{P[[X]]}$ is not a DVR , and so the assertion follows from [4, Theorem 1] . Since R_P is a DVR , [4, Theorem 3] shows $R_P[[X]] \cap \mathrm{q.f.}(R[[X]]) \not\subset R[[X]]_{P[[X]]}$.

Pick $\gamma \in R_P[[X]] \cap \mathrm{q.f.}\ (R[[X]]) \setminus R[[X]]_{P[[X]]}$. Since $\gamma \in \mathrm{q.f.}\ (R[[X]])$, we may write $\gamma = ab^{-1}$ with $0 \neq a, b \in R[[X]]$. Thus $a = b\gamma$ and hence $aR_P[[X]] \cap bR_P[[X]] = aR_P[[X]]$. We shall show that if $R[[X]] \subset R_P[[X]]$ is LCM-stable, then $\gamma \in R[[X]]_{P[[X]]}$, contradicting the choice of γ .

Now, if we assume $R[[X]] \subset R_P[[X]]$ is LCM-stable then $a \in (aR[[X]] \cap bR[[X]])\, R_P[[X]]$. Hence, there exist $h_i \in aR[[X]] \cap bR[[X]]$ and $\tau_i \in R_P[[X]]$ (for $1 \leq i \leq n$) such that $a = \sum_{i=1}^{n} h_i \tau_i$. Also, there exist $r_i, s_i \in R[[X]]$ (for $1 \leq i \leq n$) with $h_i = ar_i = bs_i$. Thus

$a = \sum_{i=1}^{n} h_i \tau_i = \sum_{i=1}^{n} (ar_i) \tau_i = a \sum_{i=1}^{n} r_i \tau_i$. Therefore $1 = \sum_{i=1}^{n} r_i t_i$. Since $R_P[[X]]$ is a local ring, there exists i_0 such that $r_{i_0} t_{i_0}$ is a unit in $R_P[[X]]$. Without loss of generality, $i_0 = 1$. But then r_1 is a unit in $R_P[[X]]$ and hence $r_{1,0} \in R \setminus P$. Since $r_1 \in R[[X]]$, r_1 is a unit in $R[[X]]_{P[[X]]}$. Thus, $ar_1 = bs_1$, leads to $\gamma = ab^{-1} = s_1 r_1^{-1} \in R[[X]]_{P[[X]]}$, the desired contradiction. Hence $R[[X] \subset R_P[[X]]$ is not LCM-stable and $D = R_P$ satisfies the conclusion of the theorem. ■

Hence, (3.1)-(3.10) combine to establish the following main result.

<u>Theorem 3.11</u>. A domain R is a Dedekind domain if and only if $R[[X]] \subset T[[X]]$ is LCM-stable for all domains T containing R as a subring.

The question whether the domains T in Theorem 3.11 can be restricted to be overrings of R is open. By our methods of proof, one need only settle this issue in the context of Theorem 3.8.

CHAPTER 4

RELATED RESULTS ON FLATNESS

Let R be a domain. From the definitions, an R-module M is R-flat if and only if M is n-flat over R for each integer $n \geq 1$. D.E. Dobbs has shown in [8, Theorem 3.3] that LCM-stability is equivalent to 2-flatness. Hence, Theorem 3.11 may be rephrased as follows. The domain R is a Dedekind domain if and only if $T[[X]]$ is 2-flat over $R[[X]]$ for each domain T containing R as a subring. Replacing "2-flat" with "flat", we feel it is natural to ask which domains have the property (*) that $T[[X]]$ is a flat $R[[X]]$-module for all domains T containing R as a subring.

As in the proof of Proposition 3.1, it can be shown that if $R[[X]] \subset T[[X]]$ is flat then $R \subset T$ is flat. Hence, any domain having the property (*) is a Prüfer domain, by F. Richman [12, Theorem 4]. By the above remarks, if $T[[X]]$ is not 2-flat over $R[[X]]$ then $T[[X]]$ is not flat over $R[[X]]$. Thus, Theorem 3.11 shows that any domain having the property (*) must be a Dedekind domain. That <u>some</u> domain does in fact have the aforementioned property is included in the proof of the next theorem.

<u>Theorem 4.1</u>. Let A be a subring of the domain B such that $A \subset \text{q.f.}(A) \subset B$. Then $B[[X]]$ is $A[[X]]$-flat.

Proof. For any field K, $K[[X]]$ is a DVR (and hence a Prüfer domain). Therefore $B[[X]]$ is $K[[X]]$-flat whenever $K \subset B$. Set $K = q.f.(A)$. By transitivity of flatness, it is enough to show that $K[[X]]$ is $A[[X]]$-flat.

We shall use the following characterization of flatness (c.f. [5, Exercise 2.2, p. 47]). An R-module M is flat if and only if $(I:_A a)M = (IM:_M a)$ for all ideals I of R and for all $a \in R$. Of course, $(I:a)M \subset (IM:a)$ is always true. Thus, it is enough to show $(I:a)\, K[[X]] \supset (IK[[X]]:a)$ for each nonzero ideal I of $A[[x]]$ and for each nonzero $a \in A[[X]]$.

Let I be a nonzero ideal of $A[[X]]$ and let $a \in A[[X]]$. There exists an integer $m \geq 0$ and $a^* \in A[[X]]$ such that $a = X^m a^*$ and $a^*_0 \neq 0$. Note that a^* is a unit in $K[[X]]$. Let $\gamma \in (IK[[X]]: a)$. Thus, $\gamma X^m a^* = \gamma a \in IK[[X]]$. Therefore $\gamma X^m \in IK[[X]]$ since a^* is a unit of $K[[X]]$, and hence $\gamma \in (IK[[X]]: X^m)$. So $(IK[[X]]:a) \subset (IK[[X]]: X^m)$. Now, let $\sum_{i=1}^{s} r_i k_i \in (I:X^m)\, K[[X]]$; i.e., $r_i \in (I:X^m)$, $k_i \in K[[X]]$ for $1 \leq i \leq s$. Then, for each i, $r_i a = r_i X^m a^* \in Ia^* \subset I$. Thus $\sum_{i=1}^{s} r_i k_i \in (I:a)\, K[[X]]$, and $(I: X^m)\, K[[X]] \subset (I: a)\, K[[X]]$. So, if we show $(IK[[X]]: X^m) \subset (I:X^m)\, K[[X]]$, then we will have $(IK[[X]]: a) \subset (IK[[X]]: X^m) \subset (I:X^m)\, K[[X]] \subset (I:a)\, K[[X]]$ and we will be done.

Every nonzero ideal of $K[[X]]$ is of the form $X^n K[[X]]$ for some integer $n \geq 0$. So, $IK[[X]] = (X^n)$ for some integer $n \geq 0$. Hence, there exists $d \in I$ and $d^* \in A[[X]]$ such that $d = d^* X^n$ and $d^*_0 \neq 0$. In particular, d^* is a unit of $K[[X]]$.

We have two cases. If $n \leq m$, then $d^* X^m = (d^* X^n) X^{m-n} = dX^{m-n} \in I$, and so $d^* \in (I: X^m)$. Then $1 = d^* d^{*-1} \in (I: X^m) K[[X]]$. Therefore, $K[[X]] \subset (I: X^m) K[[X]] \subset (IK[[X]]: X^m) \subset K[[X]]$, and so $(I: X^m) K[[X]] = (IK[[X]]: X^m)$. In the second case, $n > m$, and $(IK[[X]]: X^m) = (X^n: X^m) = X^{n-m} K[[X]]$. But $(d^* X^{n-m}) X^m = d^* X^n = d \in I$. Thus, $d^* X^{n-m} \in (I: X^m)$, and so $X^{n-m} = (d^* X^{n-m}) d^{*-1} \in (I: X^m) K[[X]]$, whence $(IK[[X]]: X^m) \subset (I: X^m) K[[X]]$. So, in either case, $(IK[[X]]: X^m) \subset (I: X^m) K[[X]]$, and it follows that $K[[X]]$ is $A[[X]]$-flat. ■

It follows, for example, from Theorem 4.1, that if K is a field, then $B[[X]]$ is $K[[X]]$-flat for each domain B containing K as a subring. In fact, by using [5, Exercise 17, p. 250], we see that $B[[X_1, \ldots, X_n]]$ is $K[[X_1, \ldots, X_n]]$-flat for all $n \geq 1$ whenever B contains the field K as a subring. The following proposition leads to a partial result involving arbitrary Dedekind domains.

Proposition 4.2. Let R and T be Noetherian domains such that R is a subring of T and T is R-flat. Then $T[[X]]$ is $R[[X]]$-flat.

Proof. Since T is R-flat, $T[X]$ is $R[X]$-flat. Also, since T is Noetherian, $T[[X]]$ is $T[X]$-flat. To see this, note that $T[[X]]$ is the completion of the Noetherian ring $T[X]$ with respect to the X-adic topology; now apply [5, Theorem 3, p. 203]. Transitivity of flatness therefore shows that $T[[X]]$ is $R[X]$-flat. Using [5, Proposition 4, p. 231] with $A = R[X]$, $B = T[[X]]$, $I = XR[[X]]$, $J = XT[[X]]$, $M = T[[X]]$, we therefore obtain that $T[[X]]$ is $R[[X]]$-flat. ■

Corollary 4.3 . Let R be a Dedekind domain. Then T[[X]] is R[[X]]-flat for each Noetherian domain T containing R as a subring.

Proof. Any domain T containing R as a subring is R-flat. Apply Proposition 4.2. ■

The question of characterizing domains having property (*) now reduces to determining whether "Noetherian" may be deleted from the statement of Corollary 4.3.

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