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### Mutual Coupling Between Loudspeakers

Robert Edward Bodenheimer  
*University of Tennessee - Knoxville*

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Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

December 10, 1958

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I am submitting herewith a thesis written by Robert Edward Bodenheimer entitled "Mutual Coupling Between Loudspeakers." I recommend that it be accepted for nine quarter hours of credit in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

J. D. Feltner  
Major Professor

We have read this thesis  
and recommend its acceptance:

Edgar S. Eaves  
\_\_\_\_\_  
C. H. McLean

Accepted for the Council:

Alan H. Henthorn  
Dean of the Graduate School

MUTUAL COUPLING BETWEEN LCUDSPEAKERS

---

A THESIS

Submitted to  
The Graduate Council  
of  
The University of Tennessee  
in  
Partial Fulfillment of the Requirements  
for the degree of  
Master of Science

---

by

Robert Edward Bodenheimer

December, 1958

## ACKNOWLEDGEMENT

The author wishes to express his appreciation to Professor J. D. Tillman for suggesting the interesting thesis topic and offering invaluable assistance during the progress of the thesis; to Dr. J. F. Pierce for his helpful suggestions; to Dr. E. D. Eaves of the Department of Mathematics and to Dr. C. H. Weaver of the Department of Electrical Engineering for their kind interest and encouragement; and to Professor P. C. Cromwell, Head of the Department of Electrical Engineering, for providing the means by which the author could continue his education.

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## INTRODUCTION

A complete system for reproduction of sound in the home consists of a chain whose first link is formed by a turntable pick-up and whose final link is made up of one or more loudspeakers. The loudspeaker is considered among the weaker links of this chain. In fact, the quality of sound from a reproducing system can be entirely dependent upon the performance of its loudspeaker system.

Only in the last few years have separate speakers been employed to reproduce a part of the frequency spectrum. In present day sound systems two or more identical speakers are often installed to improve the performance over a part of this spectrum. Since the speakers work into the same elastic medium, there will be coupling of the speakers through the medium. The literature is almost entirely void of information concerning this coupling effect. It is the purpose of this thesis to investigate the mutual coupling between identical loudspeakers and to determine whether or not this coupling can be neglected in the design of multi-unit speaker systems.



## DIRECT RADIATOR LOUDSPEAKER

The direct radiator, permanent magnet loudspeaker has a radiating element which is directly coupled to the air. In its simplest form this type of speaker may be represented as in Figure 1. The first elements observed in the usual dynamic speaker are the basket, or housing, and the specially treated paper cone, or diaphragm, mounted on the front of the basket. The voice coil is wound on a supporting former which is rigidly attached to the diaphragm. Motion of the voice coil imparts motion to the diaphragm; this in turn imparts motion to the surrounding medium, resulting in the propagation of sound waves.

A spider, or centering device, is provided to center and to retain the voice coil in the circular air gap of a permanent magnet. Proper alignment insures axial motion of voice coil in the air gap of the magnetic circuit, thus eliminating wobble. The rim of the conical diaphragm is mounted on the supporting basket by means of a mounting gasket, or flexible annular ring. The corrugations allow the cone to be elastically suspended at the rim. This insures that the cone will vibrate in a true axial direction.<sup>1</sup>

Usually the cone is sufficiently stiff at the low frequencies to move as a unit. For purposes of analysis, it is mathematically convenient to consider the loudspeaker diaphragm to be a circular piston of diameter equal to the diameter of the cone and oscillating in a large, thin, rigid wall (infinite baffle). This provides a good approximation to loudspeaker performance up to frequencies of the order of 1000 cycles.

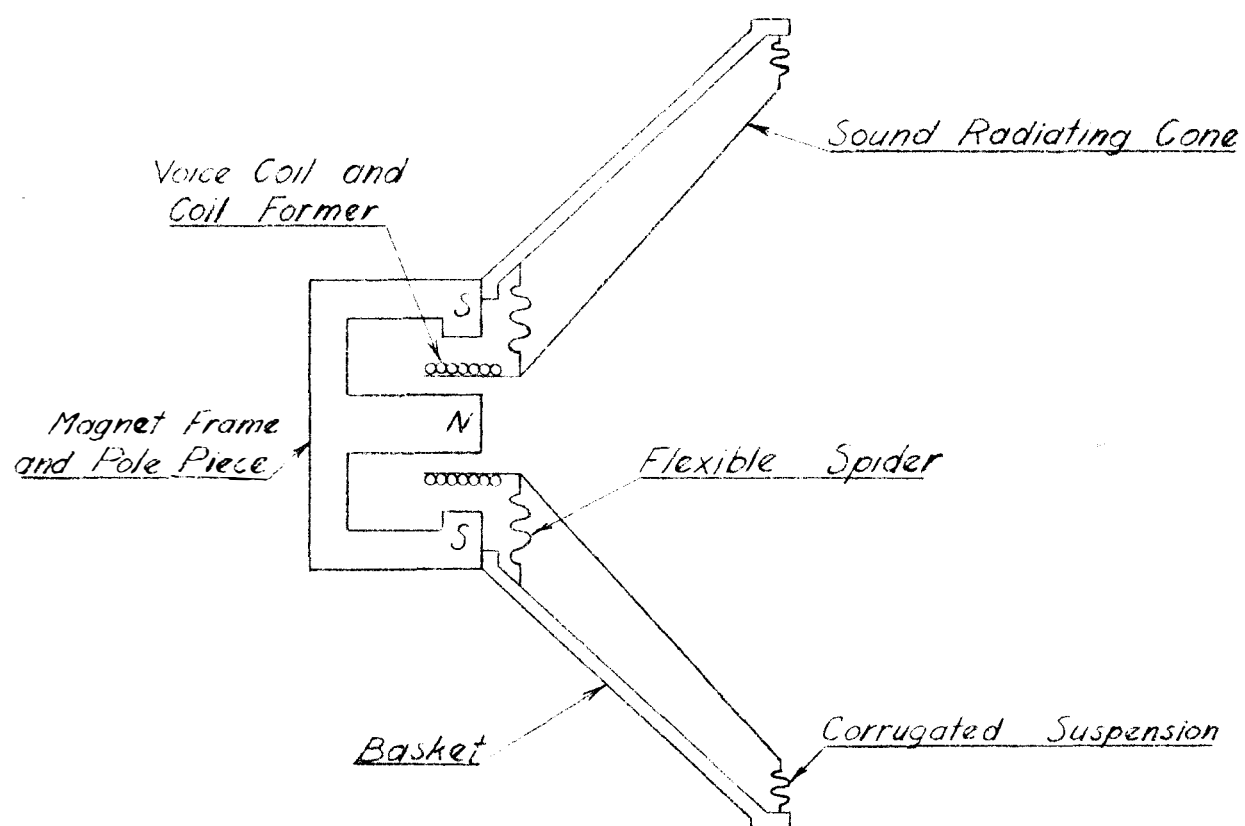


Figure 1. A permanent-magnet, direct radiator loudspeaker.

Above this frequency the diaphragm proceeds to "break up" into several modes of oscillation and ceases to behave as a rigid disk.<sup>2</sup>

An analysis of the mechanical impedance of the air load upon one side of a vibrating piston has been presented very completely in Lord Rayleigh's<sup>3</sup> analysis of sound. The air load mechanical impedance has resistive and reactive components of a rather complicated character:

$$\begin{aligned} Z_A &= R_A + j X_A \\ &= \pi R^2 \rho c \left[ 1 - \frac{J_1(2kR)}{kR} \right] + j \frac{\omega \rho \pi}{2k^3} K_1(2kR) \end{aligned} \quad (1)$$

where

$R$  = radius of piston

$\rho$  = density of air

$c$  = velocity of sound

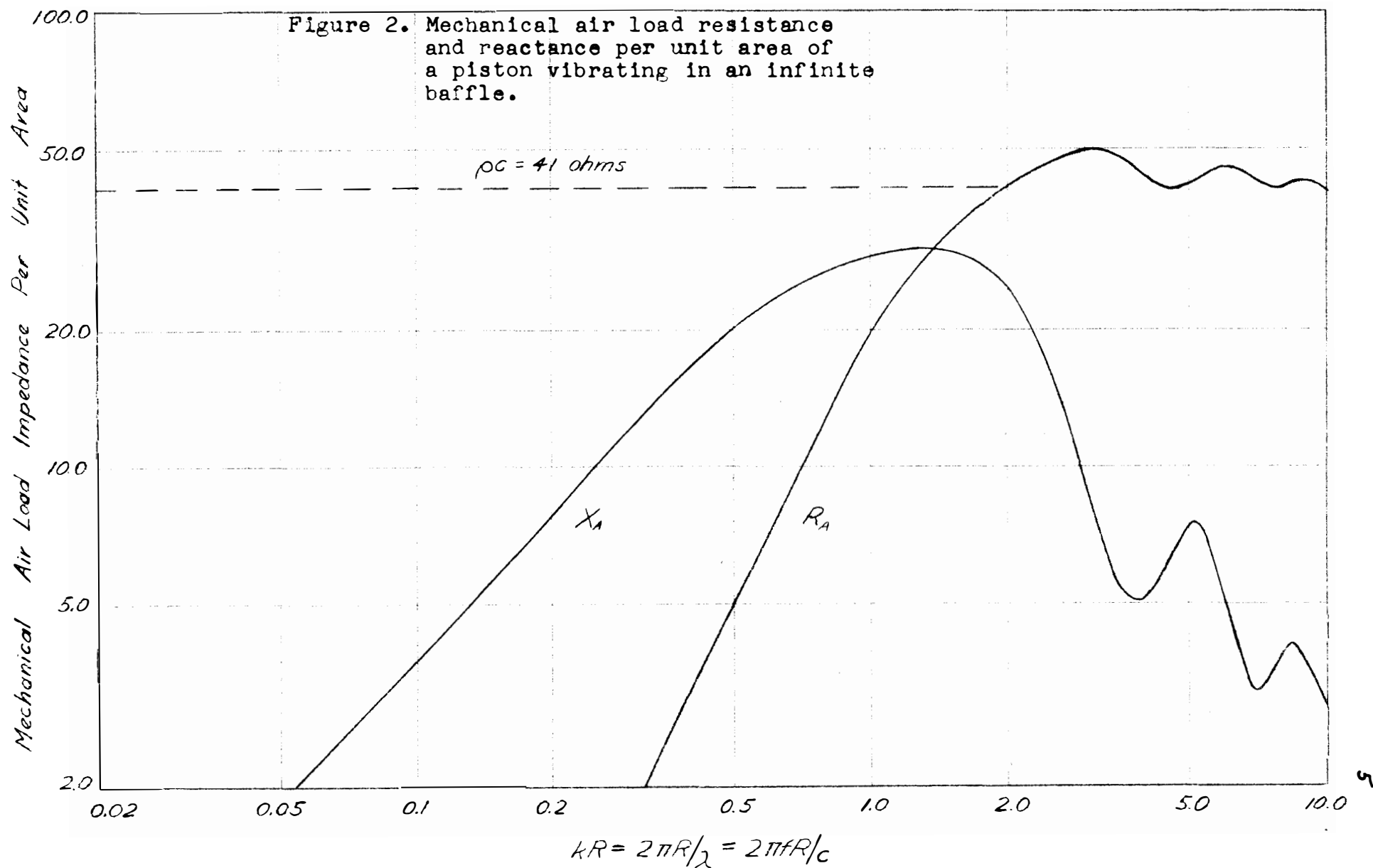
$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$f$  = frequency

$\lambda$  = wavelength

and  $J_1(2kR)$  and  $K_1(2kR)$  are Bessel functions of the first and second kind. Figure 2 shows the resistive and reactive components of the mechanical impedance per unit area of the piston. It may be seen from Figure 2 that when the wavelength of the radiated sound wave is greater than the circumference of the piston ( $\frac{2\pi R}{\lambda} < 1$ ), the radiation resistance decreases as the square of the frequency. Above the frequency where  $\frac{2\pi R}{\lambda} = 1$ ,

Figure 2. Mechanical air load resistance and reactance per unit area of a piston vibrating in an infinite baffle.



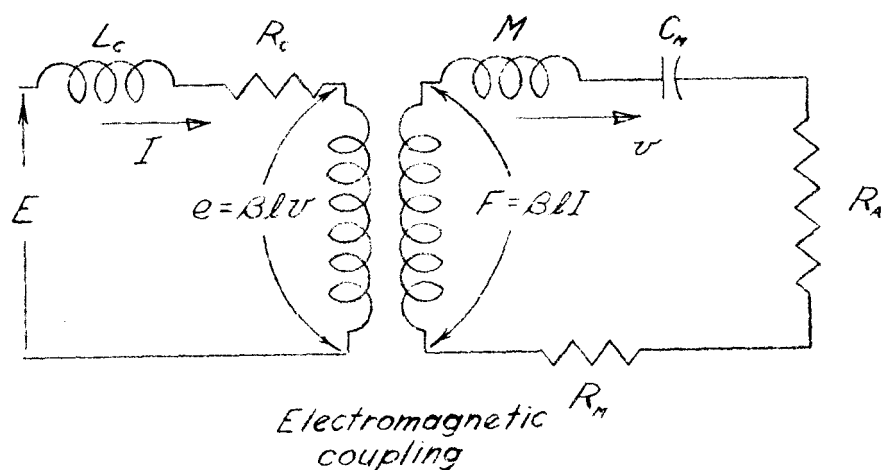
the radiation resistance is constant. To have a constant radiation resistance at 100 cycles would require a cone with a diameter of approximately 3 feet. Since there is a limit to the physical size of the cone, it is apparent that the radiation resistance will not be constant at low frequencies.

The equivalent electrical circuit<sup>4</sup> of the direct radiator loudspeaker is illustrated in Figure 3. Use of the equivalent circuit and elementary electromagnetic theory leads to an expression for the voice coil impedance<sup>5</sup> of the loudspeaker. This impedance may be written symbolically as

$$\begin{aligned} Z_{vc} &= Z_c + Z_{mo} \\ &= R_c + j\omega L_c + \frac{B^2 l^2}{R_A + R_m + j\omega M + \frac{1}{j\omega C_m}} \end{aligned} \quad (2)$$

where  $Z_c$  is the electrical impedance of the voice coil in the absence of motion, and  $Z_{mo}$  is the motional electric impedance due to the movement of the mechanical system. The motional impedance is due solely to the motion of the voice coil in the magnetic field and the emf induced in the coil. The effect of this vibrating mechanical system is to introduce a resonant condition in the voice coil impedance curve as shown in Figure 4. At the extreme high frequencies the motional impedance approaches zero, and the impedance curve is determined primarily by the reactance of the voice coil inductance.

From the electrical equivalent circuit, the sound power output<sup>5</sup> is given by



$R_c$  = voice coil resistance.

$L_c$  = voice coil inductance.

$Z_c$  = voice coil impedance.

$M$  = mass of vibrating system.

= mass of cone and coil + mass of air load.

$C_m$  = cone compliance of suspensions.

$R_m$  = mechanical resistance of system.

$Z_m$  = mechanical impedance.

$R_a$  = air load or radiation resistance.

$B$  = air gap magnetic flux density.

$e$  = length of the voice coil.

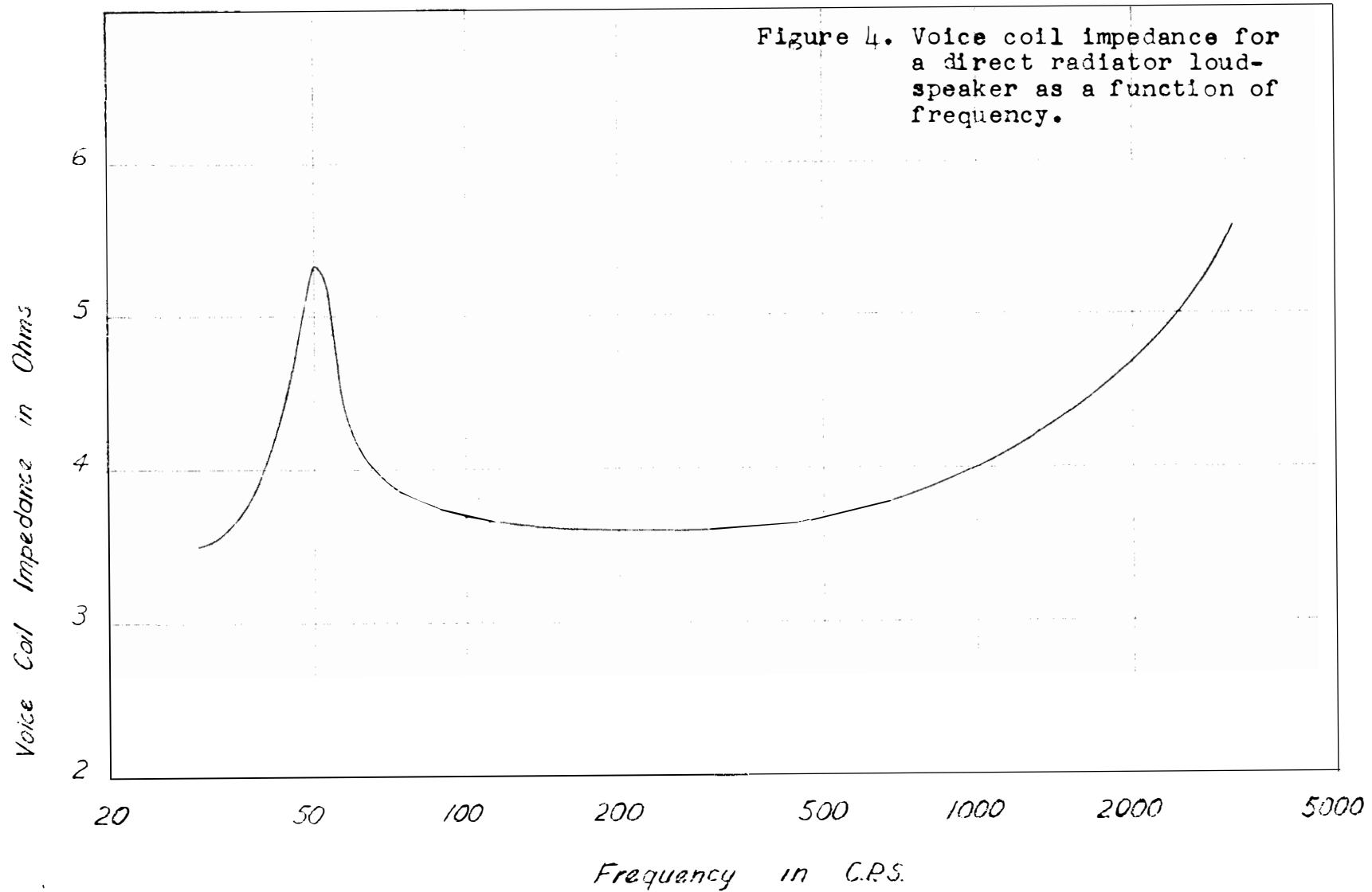
$F$  = force exerted on diaphragm due to a current in the voice coil.

$E$  = voltage applied to loudspeaker terminals.

$v$  = velocity of the cone.

Figure 3. Electrical equivalent circuit and symbols for an analytical study of the direct radiator loudspeaker.

Figure 4. Voice coil impedance for a direct radiator loudspeaker as a function of frequency.



$$P = v^2 R_A \quad (3)$$

where

$$v = \frac{F}{Z_M + Z_A} = \frac{BlI}{R_M + R_A + j\omega M + \frac{1}{j\omega C_m}} \quad (4)$$

Low sound power output at the low frequencies is primarily due to the small radiation mechanical resistance. The output at the high frequencies is limited by the mechanical mass reactance of the vibrating system. To improve the low frequency response, a large diaphragm is required. This inherently means a large mass, thus producing a poor high frequency response. For these reasons most systems built for performance over a wide frequency range consist of two or more loudspeakers, a large loudspeaker (a large radiation resistance) to reproduce the low frequencies, and a small speaker (a small mass reactance of vibrating system) to reproduce the high frequencies.

The principal disadvantages of the direct radiator loudspeaker are low efficiency and a narrow directivity pattern at high frequencies. The advantages of this type speaker, which lead to its universal acceptance, are simplicity of construction, small space requirements, and relatively uniform response characteristics.<sup>5</sup>



## MUTUAL COUPLING BETWEEN DIRECT RADIATOR LOUDSPEAKERS

In many loudspeaker applications two or more identical loudspeakers are used. Characteristics of these multi-unit systems are greater acoustic power output and wider angle of coverage. The one quantity which is common to both loudspeakers is the medium. Since a radiating loudspeaker whose surface is in contact with an elastic medium will radiate to all points of the medium, it follows that there will be mutual coupling of the loudspeakers through the medium. This mutual coupling will appear to the loudspeakers as a change in the mechanical air load impedance.

Assuming the loudspeaker can be simulated by a piston vibrating in an infinite baffle, the mechanical air load impedance for this piston can be solved by first solving the wave equation for the velocity potential  $\phi$ . The velocity potential is subject to known boundary condition which are the maximum velocity of the piston, the piston radius, the frequency of oscillation of the piston, and the velocity of the baffle. The pressure can be obtained in terms of the velocity potential. This pressure is not constant over the surface of the piston. Therefore, the total force of the air load on the piston must be found by integrating the pressure over the surface area of the piston. The mechanical air load impedance  $Z_A$  is then the ratio of the total reactive force on the piston to the velocity of the piston. The solution for  $Z_A$  is presented in the preceding section.<sup>6</sup>

There is particular interest in the derivation of the mutual air load impedance between two pistons of radius  $A$  mounted in an infinite baffle. Assume the pistons are separated a distance  $L$  as shown in Figure 5(a). If there is no motion of piston 2, a velocity potential  $d\phi$ , will be set up at any point in the medium by the motion of an elementary area  $dS$  of Piston 1. The velocity potential is given by

$$d\phi = \frac{U_0 dS}{2\pi r} e^{j(\omega t - kr)} \quad (5)$$

where

$U_0$  = velocity amplitude of elementary area  $dS$

= velocity amplitude of Piston 1.

$r$  = distance from elementary area to the point

$k$  =

$e^{j(\omega t - kr)}$  = freely traveling, simple harmonic hemispherical wave.

Since only the velocity potential on the surface of Piston 2 is of interest in determining the mutual air load impedance, the geometry will be confined to the plane of the baffle and, in particular, to a point on the surface of Piston 2. The pressure is given in terms of the velocity potential and density of the medium to be

$$p = \rho \frac{\partial \phi}{\partial t} \quad (6)$$



Using this fundamental expression for pressure, the pressure at a point on the surface of Piston 2 caused by the source element  $dS$  is

$$dp_i = j \frac{\rho c k}{2\pi r} U_0 dS e^{j(\omega t - kr)} \quad (7)$$

From Figure 5(b),  $r$  and  $dS$  are defined in terms of the plane geometry of the pistons to be

$$\begin{aligned} dS &= a da d\psi \\ r^2 &= R^2 + a^2 - 2Ra \cos(\psi - \theta). \end{aligned} \quad (8)$$

If the assumption is made that the pressure will be constant along the chords of Piston 2 parallel to the  $y$ -axis, the expression for  $r^2$  can be simplified to

$$r^2 = R^2 + a^2 - 2Ra \cos \psi. \quad (9)$$

This is true since the pressure at point  $p$  and the intersection of the chord passing through point  $p$  parallel to the  $y$ -axis and the  $x$ -axis will be the same. Integration over the surface area of Piston 1 leads to the pressure at any point on the  $x$ -axis passing through Piston 2. The total pressure is

$$P_i = j \frac{\rho c k U_0}{2\pi} \int_0^A \int_0^{2\pi} \frac{e^{j(\omega t - kr)}}{r} a da d\psi. \quad (10)$$

The expression may be simplified by examining the quantity  $\frac{1}{r}$ , where

$$\frac{1}{r} = \frac{1}{[R^2 + a^2 - 2Ra \cos \psi]^{\frac{1}{2}}} \quad (11)$$

Factoring out  $R^2$  leaves

$$\frac{1}{r} = \frac{1}{R} \left[ 1 + \frac{a^2}{R^2} - 2 \frac{a}{R} \cos \psi \right]^{-\frac{1}{2}} \quad (12)$$

Define  $h \equiv \frac{a}{R}$  and  $\mu \equiv \cos \psi$ . The expression for  $\frac{1}{r}$  now becomes

$$\frac{1}{r} = \frac{1}{R} \left[ 1 + (h^2 - 2h\mu) \right]^{-\frac{1}{2}} \quad (13)$$

If the above expression is expanded by the binomial theorem and the series rearranged in ascending power of  $h$ , the coefficients of  $h$  take on a mathematical form known as the Legendre Polynomials<sup>8</sup>.  $\frac{1}{r}$  may now be written as

$$\frac{1}{r} = \frac{1}{R} \sum_{n=0}^{\infty} P_n(\mu) h^n. \quad (14)$$

Substituting this into the expression for pressure yields

$$P_i = j \frac{\rho c k U_0}{2\pi} \int_0^A \int_0^{2\pi} \sum_{n=0}^{\infty} P_n(\mu) h^n e^{i(\omega t - kR[1+h-2h\mu]^{\frac{1}{2}})} da d\psi. \quad (15)$$

There is no available means of integrating the above expression.

Since the pressure at any point on the x-axis through Piston 2 is known, the force on a chord segment of Piston 2 can be found. Integration over the area of Piston 2 gives the total reactive force of Piston 2 by Piston 1. The ratio of the reactive force to the velocity of Piston 2 gives the mutual load impedance on Piston 2. Note that as soon as there is motion of Piston 2, the solution for  $p_1$  is no longer valid since Piston 1 is no longer in an absolute infinite baffle. However, as a first approximation, the assumption that  $p_1$  is unaffected by the motion of Piston 2 could be made.

$P_1$  could be integrated if it is assumed that the distance between pistons is large compared with the radius of the pistons. Since interest in the mutual coupling is confined to the case of close coupling, that is, the spacing between speakers is small, this derivation is not pursued.

It should now be apparent that some other scheme of determining the mutual coupling must be used since the integration for  $p_1$  is very complicated, if not impossible. The method to be used is to measure the self and mutual impedances at the voice coil terminals of one speaker due to the coupling or change of the air load impedance by the second speaker.

## MUTUAL AND SELF IMPEDANCES OF COUPLED LOUDSPEAKERS

A four terminal linear bilateral network<sup>o</sup>, no matter how complex, may be represented by Figure 6. The general voltage loop equations for this network may be written as

$$V_1 = I_1 Z_{11} + I_2 Z_{12} \quad (16)$$

$$V_2 = I_1 Z_{21} + I_2 Z_{22} . \quad (17)$$

$Z_{11}$  and  $Z_{22}$  are the self impedances of loops 1 and 2 respectively. The self impedances are defined as

$$Z_{11} = \frac{V_1}{I_1} \quad \text{with loop two open circuit}$$

$$\text{and } Z_{22} = \frac{V_2}{I_2} \quad \text{with loop one open circuit.}$$

$Z_{12}$  is the impedance reflected into loop one by a current in loop two, and  $Z_{21}$  is the impedance in loop two by a current in loop one. According to the reciprocity theorem,  $Z_{12} = Z_{21}$  if the impedances of the network are linear and bilateral.

The self and mutual impedances for most linear bilateral circuits are easily determined by open circulating the proper loop and making the appropriate measurements. It would be very difficult to measure the self and mutual impedances for a loudspeaker system in this manner since the loudspeaker must be open circuited. To open circuit a loudspeaker would require the diaphragm to be motionless, that is, the velocity of the diaphragm be equal to zero. For the velocity to be zero, it would be necessary for the speaker to work into an infinite air load impedance.

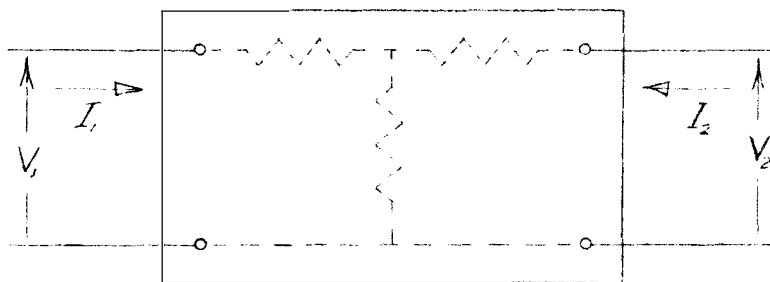


Figure 6. Representation of a four terminal network.



The method for determining the self and mutual impedances of coupled loudspeakers is to measure the symmetric and antisymmetric impedances.<sup>10,11</sup> To explain this method, define

$$V_s \equiv \frac{V_1 + V_2}{2} \quad (18)$$

$$V_a \equiv \frac{V_1 - V_2}{2} \quad (19)$$

where  $V_s$  and  $V_a$  are the symmetric and antisymmetric voltages respectively. In the same manner, define

$$I_s \equiv \frac{I_1 + I_2}{2} \quad (20)$$

$$I_a \equiv \frac{I_1 - I_2}{2} \quad (21)$$

where  $I_s$  and  $I_a$  are the symmetric and antisymmetric currents. Solving equations (18) and (19) simultaneously for the applied voltages and (20) and (21) for the loop currents yield

$$\begin{aligned} V_1 &= V_s + V_a \\ V_2 &= V_s - V_a \\ I_1 &= I_s + I_a \\ I_2 &= I_s - I_a \end{aligned} \quad (22)$$

For identical speakers  $Z_{11}$  is equal to  $Z_{22}$ , and  $Z_{12}$  is equal to  $Z_{21}$  since the loudspeaker is a linear bilateral network. Therefore, substituting equations (22) into equations (16) and (17) and regrouping the terms give

$$V_s + V_a = I_s (Z_{11} + Z_{12}) + I_a (Z_{11} - Z_{12}) \quad (23)$$

$$V_s - V_a = I_s (Z_{11} + Z_{12}) + I_a (Z_{12} - Z_{11}). \quad (24)$$

Addition of (23) and (24) yield

$$V_s = I_s (Z_{11} + Z_{12})$$

from which

$$\frac{V_s}{I_s} \equiv Z_s = Z_{11} + Z_{12}. \quad (25)$$

$Z_s$  is the symmetric impedance and is the input impedance at either of the loudspeaker terminals when equal voltages are applied in phase across the terminals of both loudspeakers. Hence,  $Z_s$  is simply the input impedance at one loudspeaker when both are driven in phase.

Subtracting equations (23) and (24) give the antisymmetric impedance to be

$$\frac{V_a}{I_a} \equiv Z_a = Z_{11} - Z_{12} \quad (26)$$

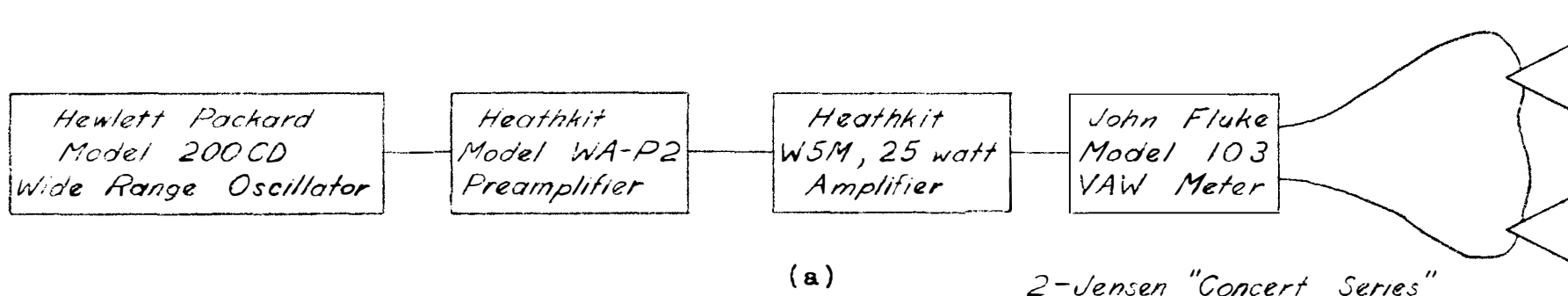
where  $Z_a$  is the antisymmetric impedance.  $Z_a$  is the input impedance at either of the loudspeaker terminals when equal voltages are applied out of phase to both loudspeaker terminals. Thus  $Z_a$  is obtained by driving the speakers out of phase. Once the symmetric and antisymmetric impedances are determined, the self and mutual impedances may be obtained by solving equations (10) and (11) simultaneously. The results are

$$Z_{11} = \frac{Z_s + Z_a}{2} \quad (27)$$

$$Z_{12} = \frac{Z_s - Z_a}{2} \quad (28)$$

The symmetric and antisymmetric impedances may be measured by use of the electrical circuit shown in Figure 7(a). The two speakers work into an infinite medium and are mounted in a 4' by 8' sheet of plywood in such a manner that the distance between the speakers is adjustable. This plywood baffle is sufficiently large to approximate an infinite baffle and to eliminate doublet action. When the effective pathlength from the rear to the front of the speaker diaphragm becomes small compared with the wavelength of the propagating sound wave, the approximation is no longer valid. The problem is even more complex due to the fact there are two speakers mounted in the baffle. However, since the paths existing from the rear, around the baffle, to the front of the speaker are of different lengths, the doublet action will not arrive in phase even at extremely low audio frequencies.

Since the impedances to be measured are complex, it is not only necessary to measure the magnitude of this impedance but also its phase angle in order to determine the resistive and reactive components of the self and mutual impedances. This may be accomplished with use of a VAW meter, which will measure the voltage, current, and power delivered directly to the speaker input terminals. The frequency range over which the measurements were made is well within the frequency



2-Jensen "Concert Series"  
P.M. Speakers, Type No. P12-T  
12" - 9 watt.

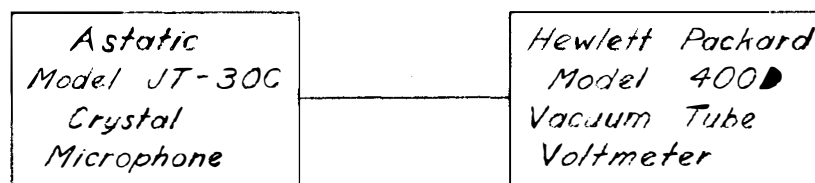
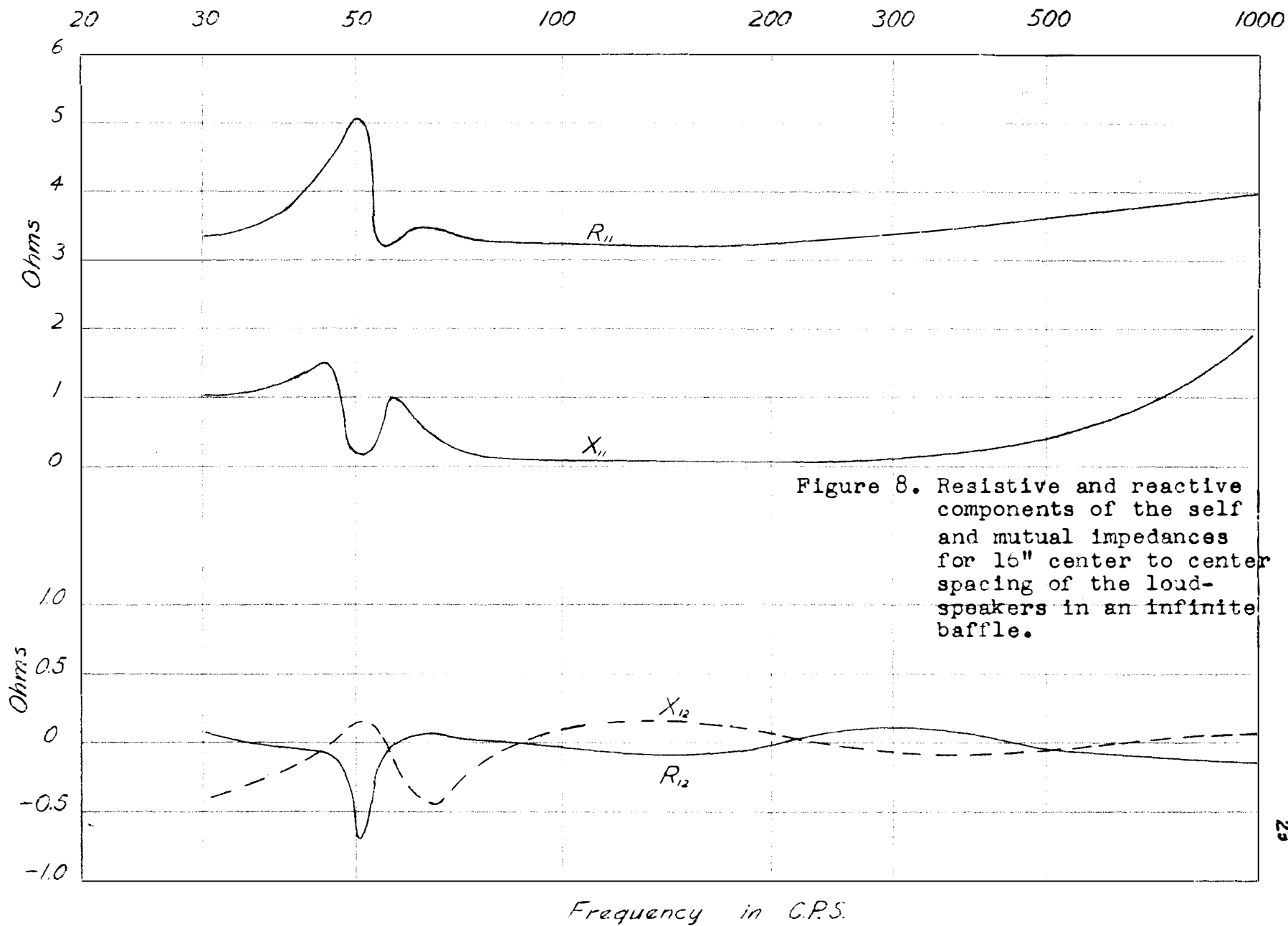


Figure 7. (a) Electrical circuit for measuring the symmetric and antisymmetric impedances.  
(b) Additional circuit required for measuring the directivity pattern.

range of all the test equipment. Meters employed in the VAW meter are within plus or minus 3% of full scale for any reading within the rated frequency range (20-20,000 cycles) of the meter.

The self and mutual impedances may be determined as a function of frequency for a given separation of the speakers by applying equations (27) and (28). The resistive and reactive components of the self and mutual impedances for the smallest speaker separation are shown in Figure 8. From Figure 8 it appears that there is a deviation in the resistive component of the self impedance as compared with that of an isolated speaker. Tests show that, although the speakers are the same model and type, there is a difference in the individual resonant frequencies.

The cone velocity of a single speaker will be controlled by the mass reactance above the resonant frequency and by the compliant reactance below the resonant frequency. If two speakers of the same type are used in the same system, and their resonant frequencies differ, then for frequencies between the resonances, the cone velocity of the speaker with the lower resonant frequency will be mass controlled. The cone velocity of the speaker with the higher resonant frequency will be controlled by the mechanical compliance. In other words, although the speakers are being driven in phase, the diaphragm of one speaker will lag that of the other. As the two pressure waves are not in phase, there will be a reduction in the pressure



corresponding to the direction of greatest phase difference. Since the spacing of the speakers is small, this effect will be observed mostly along the axis of the speakers. Verification of this fact is shown in the directivity pattern of Figure 9. The pattern is measured at 55 cps which was between the resonant frequencies of 51 cps and 60 cps for the individual speakers. A crystal microphone and vacuum tube voltmeter, shown in Figure 7(b) are used in addition to the equipment described previously, to determine the directional pattern. The response of the microphone is not critical since the measurements are made for constant frequency.

The investigation of this effect is extended by placing the two speakers in separate enclosures, where the spacing of the speakers remains unchanged. Since the enclosure completely separates the front and back of the speaker, the doublet action is now eliminated. The added series compliance, due to an eight cubic foot enclosure, raises the resonant frequencies of the speakers to 75 cps and 80 cps. The speaker with the lower resonant frequency is then tuned by decreasing its enclosure volume until its resonant frequency corresponds to that of the speaker with the higher resonant frequency.

The directivity pattern is then obtained at the resonant frequency (80 cps) of the system and is shown in Figure 10. The reduction in pressure, as measured along the axis of the loudspeakers when there is a difference in the resonant frequencies, is eliminated by tuning the speakers to the same resonant

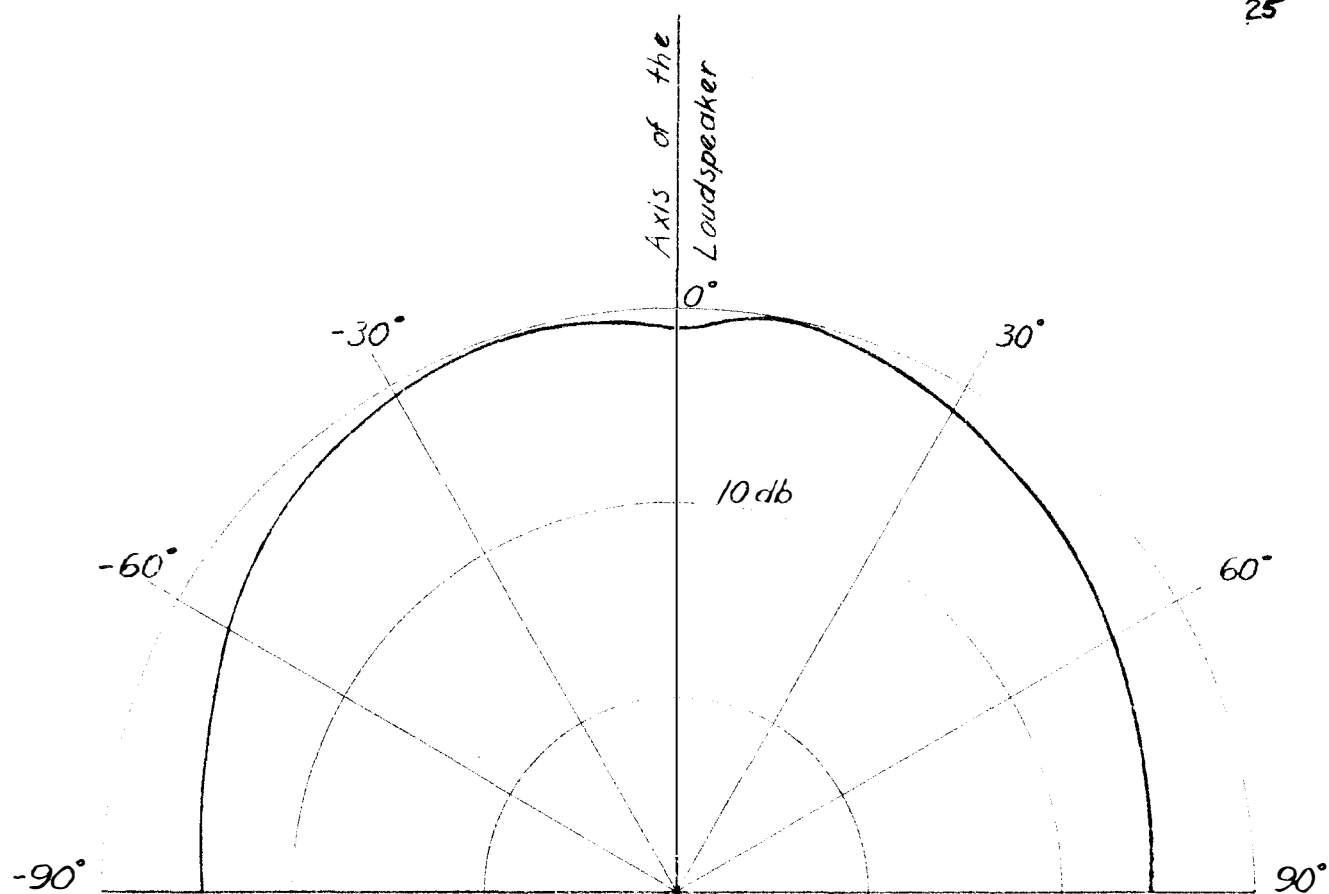


Figure 9. 55 cps directional pattern for 16" spacing of speakers located in an infinite baffle.



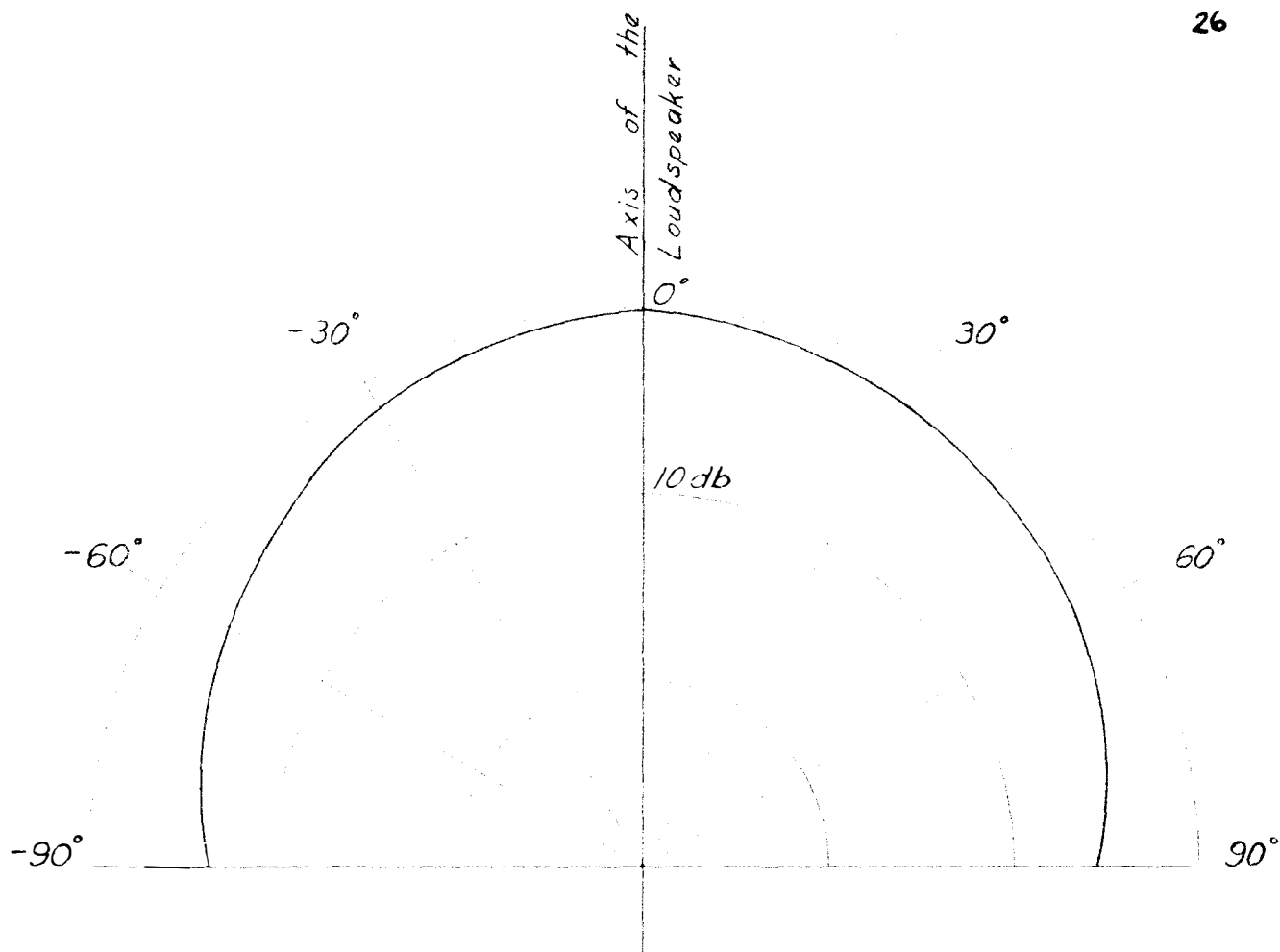


Figure 10. 80 cps directional pattern for 16" spacing of speakers located in separate enclosures.

frequency. The components of the self and mutual impedances are shown in Figure 11 for the case where the speakers are placed in separate enclosures. The irregular variation of the resistive and reactive components of the self and mutual impedances in the region of the resonant frequency is now eliminated.

There is also interest in the variation of the mutual impedance as a function of speaker separation for a fixed frequency. The resistive and reactive components of the mutual impedance are shown in Figure 12 for three different frequencies. The periodicity of the impedance variation with distance is roughly sinusoidal. As the separation varies up to a half-wavelength (60 inches) at 100 cps, the mutual impedance components vary through a half-cycle. At 200 cps the separation varies up to one wavelength (60 inches). The mutual impedance parameters have a variation of one complete cycle. It appears then that the sign of the mutual impedance for any frequency depends upon the spacing of the speakers in wavelengths for that particular frequency.

When two pistons are driven in phase, the pressure at the face of each piston is in phase. Therefore, if the spacing of the speakers is small compared with the wavelength of the sound wave, there will only be a slight phase difference in the pressures at any point. However, if there is a speaker spacing of a half-wavelength for that sound wave, the pressure arriving at Piston 1 due to Piston 2 will be out of phase with

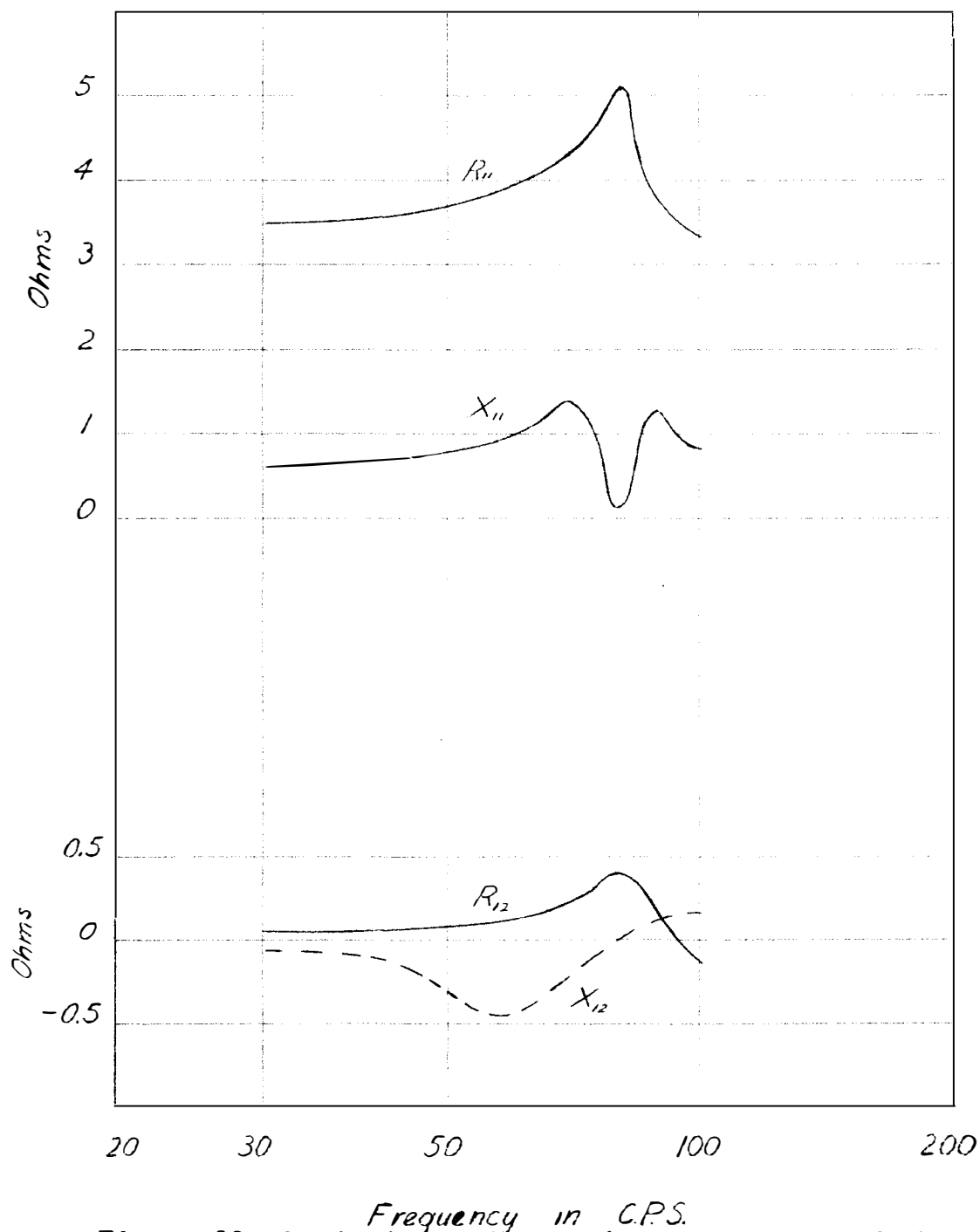


Figure 11. Resistive and reactive components of the mutual and self impedances for 16" spacing of the speakers in separate enclosures when both speakers are tuned to 80 cps resonant frequencies.

# Mutual Coupling

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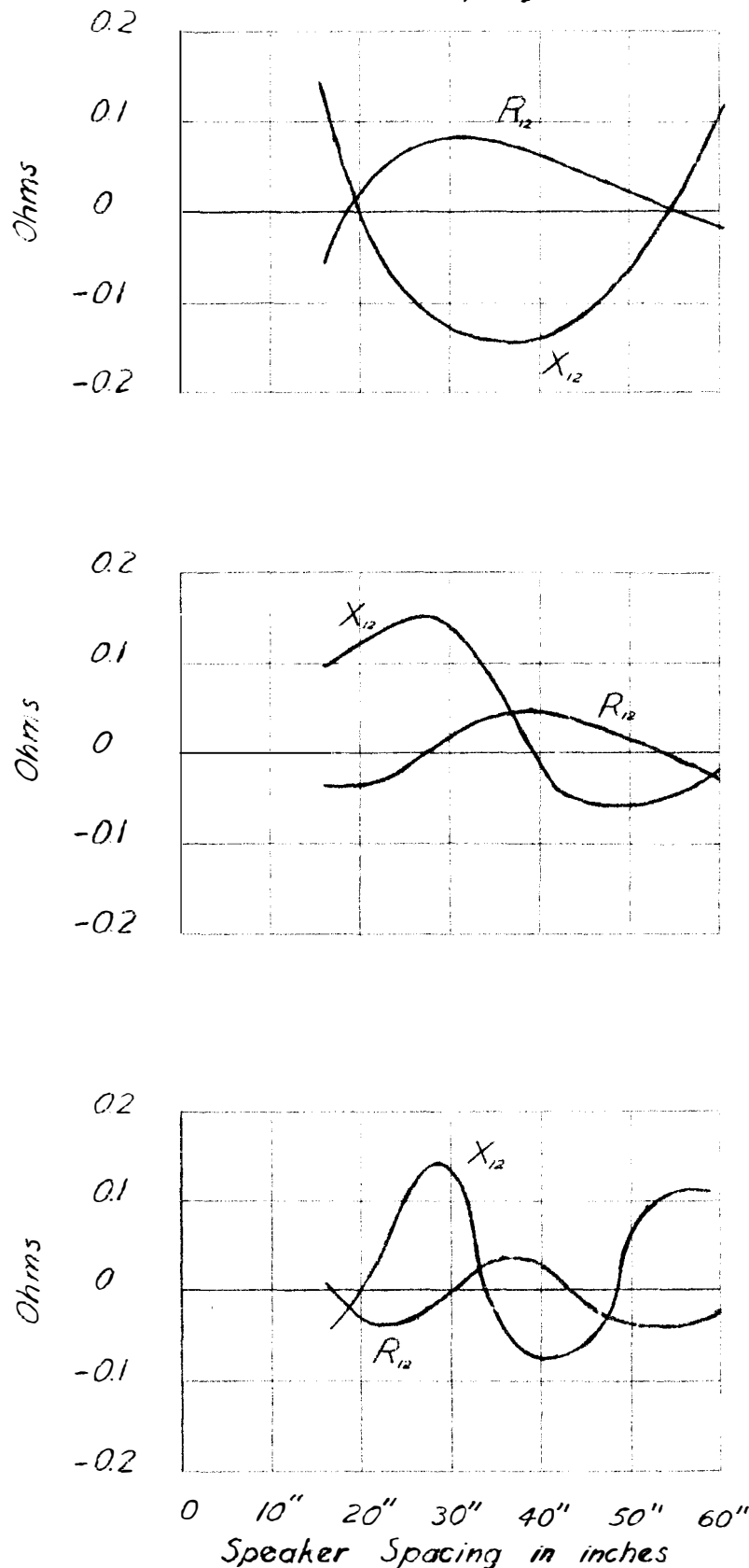


Figure 12. The resistive and reactive components of the mutual impedance as a function of speaker spacing for frequencies of 100, 200 and 400 cps. Speakers are located in an infinite baffle.

the pressure of Piston 1. Thus it seems that the variation of the mutual coupling as a function of spacing will be some type of damped sinusoid. This correlates the results as measured in Figure 12, which shows the variation in mutual impedance as a function of speaker spacing measured at the speaker terminals. In all the above cases the magnitude of the mutual impedance is small compared with the magnitude of the self impedance of the loudspeaker.

## CONCLUSIONS

A multi-unit speaker system may be designed by considering the performance of each speaker independently if their resonant frequencies are the same. The effect of the mutual coupling between two identical speakers can be entirely neglected since the mutual impedance is small compared with the self impedance of the speakers. If there is a difference in the mechanical resonances of the individual speakers, design precautions must be taken to avoid pressure reductions in the directivity patterns for frequencies between the resonances. This effect can be completely eliminated by mounting the speakers in separate enclosures and tuning them to the same resonant frequency. It is necessary that this effect be eliminated in the stereophonic reproduction of sound since the phase relationship existing between the two channels is very important.

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