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## Mathematically Modeling Fetal Electrocardiograms

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## Mathematically Modeling Fetal Electrocardiograms

### Cover Page Footnote

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## Mathematically Modeling Fetal Electrocardiograms

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Some of the most common and fatal birth defects are those related to the heart. In adults, possible heart conditions are often identified through the use of an electrocardiogram (ECG). However, due to the presence of other signals and noise in the recording, fetal electrocardiography has not yet proven effective in diagnosing these defects. This paper develops a mathematical model of three-dimensional heart vector trajectories, which was used to generate synthetic maternal and fetal ECG signals. The dipole model is a useful simplification in which the electrical activity of the heart is viewed as a single time-varying vector originating at the center of the body. A system of ordinary differential equations whose numerical solution approximates this cardiac dipole vector was used. To simulate the cardiac activity of both mother and fetus, two sets of these equations must be implemented. When implementing this model, various physiological factors must be accounted for. After realistic maternal and fetal dipole vectors have been simulated, they are projected onto random unit vectors representing the lead axes of the fECG. Noise is then added to the signal. Using this model a database of realistic, synthetic fECG signals with different parameter values and noise levels has been built. Currently algorithms to extract the fetal signal from the fECG are tested on a database of clinical recordings. Our synthetic database will allow for algorithms to be tested on a broader set of data.

### Introduction

The electrocardiogram (ECG) is a test that monitors the electrical activity of the heart. This electrical activity is a series of waves that propagate through the heart causing it to contract and relax. The ECG records these waves as potential differences on the surface of the skin. Each successive cycle on the ECG represents atrial and ventricular depolarization/repolarization. A typical cycle contains five distinct peaks and troughs called the P, Q, R, S, and T waves.

Birth defects occur in about one out of every 125 infants every year (Zarzoso 2001). Monitoring of the fetal heart using electrocardiography (fECG) could be helpful in early detection of birth defects. fECG's are recorded by placing electrodes on the abdomen of the mother. The data obtained from the fECG contains useful information about the health and condition of the fetus. However, except during birth, fetal cardiac activity cannot be monitored effectively (Sameni 2007). The fetal cardiac signal is contaminated by noise from sources such as fetal movements, maternal uterus contractions, and changes in the conductivity of the biological media surrounding the fetus (Sameni 2007).

Moreover, the fetal cardiac signal is much weaker than that of an adult and can be easily dominated by noise. These factors, as well as the much stronger maternal signal, make obtaining useful information about the fetus from the fECG a very difficult task.

The challenges in separating out the desired fetal signal from the rest of the fECG have encouraged the development of new extraction algorithms. Early approaches include coherent averaging, matched filtering, auto-cross-correlation based models, adaptive filtering, and sequenced adaptive filtering (Zarzoso 2001). These methods have been largely unsuccessful. Therefore, the development of new extraction algorithms has become an area of interest for researchers.

This paper implements and extends the mathematical model found in Sameni et al. to generate a database of synthetic fECG signals. Previous papers have developed a single-channel model that simulates typical ECG morphology (McSharry 2003). In this model, each heartbeat is aligned along a unit circle. Additional work has generated a three-dimensional for the ECG (Sameni 2007). This model accounts for a variety of physiological factors, which can vary widely. These factors include baseline wander, heart rate variability, and respiratory frequency. The single dipole model is a useful simplification for mathematically modeling the propagation of electricity through the heart. This simplification allows the body to be viewed as a vector space with the fECG as a sum of the projections of the maternal and fetal dipole vectors onto the lead axes.

## Methods

The single dipole model of the heart represents cardiac electrical activity as a single, time-varying vector. The origin of this vector is the center of the heart and the end of the vector sweeps through the torso in a quasi-periodic path. The ECG signals recorded from the skin surface are linear projections of this dipole vector onto the directions of the recording electrode axes. Starting with a Mathematica (Version 9.0.1.0) notebook that implemented the single dipole model found in McSharry et al., a multi-channel dipole model containing both maternal and fetal cardiac signals based on Sameni et al. has been implemented and developed.

## General Model

The three-dimensional dynamic model for the dipole vectors of both mother and fetus found in Sameni et al. was implemented in a Mathematica notebook. This initial model with correction for upward drift, as inspired by McSharry et al., is given by

$$\begin{aligned}
 1) \quad & \theta' = \omega \\
 2) \quad & x' = -\sum_i \frac{\alpha_i^x \omega}{(b_i^x)^2} \Delta\theta_i^x \exp\left[-\frac{(\Delta\theta_i^x)^2}{2(b_i^x)^2}\right] - x \\
 3) \quad & y' = -\sum_i \frac{\alpha_i^y \omega}{(b_i^y)^2} \Delta\theta_i^y \exp\left[-\frac{(\Delta\theta_i^y)^2}{2(b_i^y)^2}\right] - y \\
 4) \quad & z' = -\sum_i \frac{\alpha_i^z \omega}{(b_i^z)^2} \Delta\theta_i^z \exp\left[-\frac{(\Delta\theta_i^z)^2}{2(b_i^z)^2}\right] - z.
 \end{aligned}$$

These equations model the dipole vector from a single source and were implemented separately to reflect both maternal and fetal cardiac signals.  $\mathbf{Q}$  describes the position of the dipole vector along a unit circle, and  $\mathcal{X}$ ,  $\mathcal{Y}$ , and  $\mathcal{Z}$  are the voltage of the heart vector in each direction. The three coordinates of the cardiac vector ( $\mathcal{X}$ ,  $\mathcal{Y}$ , and  $\mathcal{Z}$ ) are each modeled by a summation of Gaussian functions with amplitudes  $\alpha_i^x$ ,  $\alpha_i^y$ , and  $\alpha_i^z$ ; widths of  $b_i^x$ ,  $b_i^y$ , and  $b_i^z$ ; and located at the rotational angles of  $\theta_i^x$ ,  $\theta_i^y$ , and  $\theta_i^z$ . Distinct sets of maternal and fetal parameters were obtained from Sameni et al. In this equation

$$\Delta\theta_i^x = \theta \text{ Mod}(2\pi) - \pi - \theta_i^x, \Delta\theta_i^y = \theta \text{ Mod}(2\pi) - \pi - \theta_i^y, \Delta\theta_i^z = \theta \text{ Mod}(2\pi) - \pi - \theta_i^z$$

and  $\omega=2\pi f$ , where  $f$  is the beat-to-beat heart rate.

Numerical solutions to the previous equation were generated using the “NDSolve” function in Mathematica with an explicit, fourth-order Runge-Kutta method. These numerical solutions simulate the propagation of electricity through the heart over time, and can be visualized using three-dimensional parametric plots. In order to relate this model to realistic multi-channel ECG recordings, the dipole vectors needed to be projected onto the body surface and realistic noise needed to be incorporated. Therefore, the vectors of solutions for both maternal and fetal signals were projected onto a random unit vector representing the vector between two abdominal electrodes. These values were then summed with a noise signal to simulate data collection through one channel.

$$ECG(t) = R \bullet s_m(t) + R \bullet s_f(t) + N(t)$$

In this equation  $R$  is a random unit vector,  $s_m(t)$  and  $s_f(t)$  are the three components of the dipole model for the maternal and fetal cardiac vectors, respectively, and  $N(t)$  is the noise in each ECG channel at time  $t$ .

## Heart Vector Projections

A single, random, three-dimensional unit vector was generated using the “RandomReal” function in Mathematica. Maternal and fetal cardiac vectors were projected onto this random unit vector to simulate a single channel ECG recording. The randomness of this vector is assumed to account for variations in signal propagation in the body volume conductor, electrode placement, and possible rotation and scaling. To better mimic clinical use, the cardiac vectors were then projected onto four separate, randomly generated unit vectors to simulate data collection through four channels.

## Baseline Wander

Next, new terms in the equations to model baseline wander were incorporated. Baseline wander occurs in ECG recordings as a result of changing electrode placement due to breathing. The resulting model is given by

$$\begin{aligned}\theta' &= \omega \\ x' &= -\sum_i \frac{\alpha_i^x \omega}{(b_i^x)^2} \Delta\theta_i^x \exp\left[-\frac{(\Delta\theta_i^x)^2}{2(b_i^x)^2}\right] - x - c * \sin(2\pi * r * t) \\ y' &= -\sum_i \frac{\alpha_i^y \omega}{(b_i^y)^2} \Delta\theta_i^y \exp\left[-\frac{(\Delta\theta_i^y)^2}{2(b_i^y)^2}\right] - y - c * \sin(2\pi * r * t) \\ z' &= -\sum_i \frac{\alpha_i^z \omega}{(b_i^z)^2} \Delta\theta_i^z \exp\left[-\frac{(\Delta\theta_i^z)^2}{2(b_i^z)^2}\right] - z - c * \sin(2\pi * r * t) .\end{aligned}$$

Baseline wander is represented by the last term, in which  $C$  is the weight on the baseline wander function, and  $r$  represents the frequency of the baseline wander. This model for baseline wander was inspired by McSharry et al.

## Noise

Both white noise and power line noise were then implemented in the model and added to the projections separately, following equation,

$$ECG(t) = R \bullet s_m(t) + R \bullet s_f(t) + P(t) + W(t) ,$$

in which  $W(t)$  is the white noise and  $P(t)$  is power line noise.

White noise was generated as a random, interpolating, continuous function with a mean value of zero to simulate random background noise associated with data collection. Power line noise was simulated by adding a 60 Hz sinusoidal curve to the vector projections, so that

$$P(t) = \varepsilon_0 \sin(2\pi * 60 * t) ,$$

where  $\varepsilon_0$  is the weight of the power line noise. This noise represents the standard frequency for electrical interference.

## Heart Rate Variability

Although the original model defined by  $\omega=2\pi f$ , where  $f$  is the constant beat-to-beat heart rate, a more realistic ECG model would account for variations in heart rate. McSharry et al. outlines a method for producing a model with aperiodic activity based on a bimodal power distribution (Figure 1).

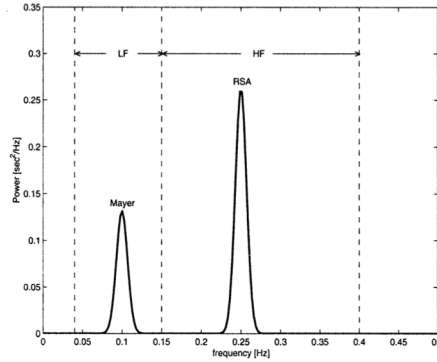


Figure 1: Power spectrum of RR intervals for one example adult ECG; figure adopted from McSharry et al.

This model inspired the following equation used to establish quasi-periodicity in the system of equations:

$$\omega = \omega_0 + \varepsilon_1 \sin(f_1 t) + \varepsilon_2 \sin(f_2 t) .$$

In this equation  $\omega_0 = 2\pi f_0$  is the original constant heart rate and  $\varepsilon_1$  and  $\varepsilon_2$  are parameters weighting the low and high frequency contribution to aperiodicity respectively. The parameters  $f_1$  and  $f_2$  represent the periodicity of these low and high frequencies. The result is a model that perturbs an initially constant value of  $\omega$  with small variations that remain bounded over time. These two sine waves account for the influence of low frequency Mayer waves and high frequency respiratory sinus arrhythmia (RSA) waves. Baroreflex regulation in the blood pressure signal creates the Mayer waves, while RSA waves are the oscillations in the RR tachogram due to the respiratory cycle.

This initial implementation of heart rate variability was expanded to more closely model the approximately normal distribution of the power spectrum for Mayer and RSA waves. This method approximates the influence of each wave as a sum of five sinusoidal curves,

$$\omega = \omega_0 + \sum_{n=-2}^2 a_n \varepsilon_1 \sin[(\omega_1 - n\Delta t)] + \sum_{n=-2}^2 a_n \varepsilon_2 \sin[(\omega_2 - n\Delta t)] .$$

$\Delta$  is a uniform step size sampling the wave at various point including the peak.

## Results and Discussion

The results include graphs of the maternal and fetal dipole vectors and their components. In addition, the projections of these dipole vectors onto the randomly generated lead axes (unit vectors) were graphed and sample points of these graphs were exported to a table. Due to the randomization of various parameters (lead axes, maternal and fetal heart rates, noise, etc.) each run of the code produced slightly different results. However, they were generally consistent both with each other and with what was expected.

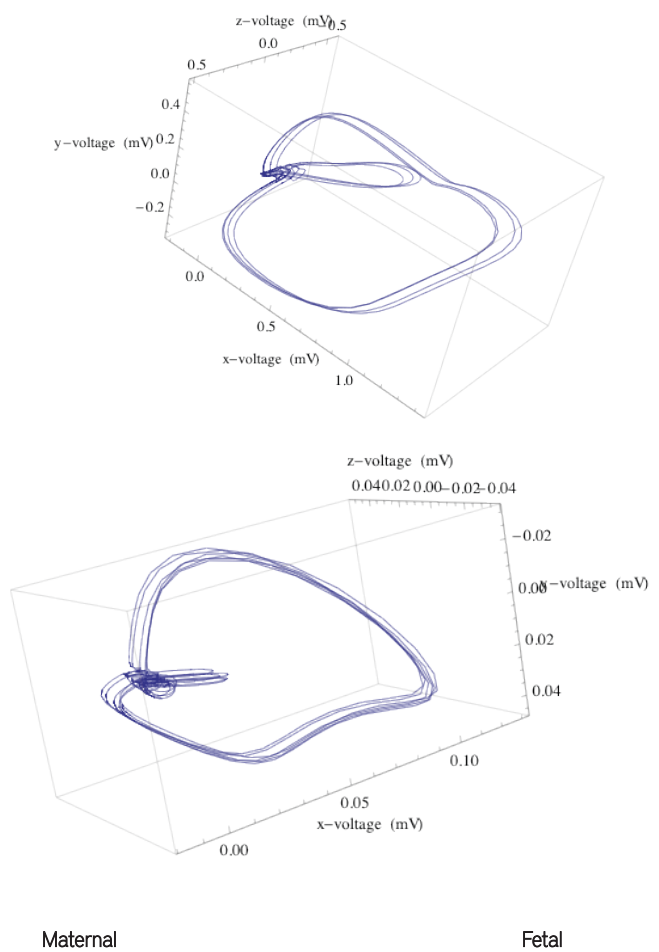


Figure 2: Heart Vector Trajectories

As seen in Figure 2, the dipole vectors are quasi-periodic, which is consistent with the vast majority of human cardiac cycles.

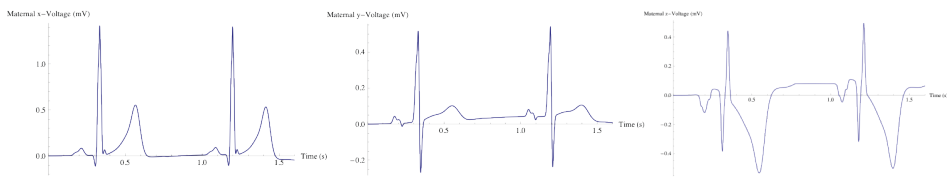


Figure 3: Maternal Synthetic ECG Components



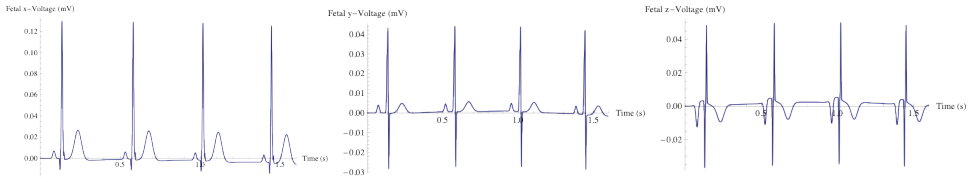


Figure 4: Fetal Synthetic ECG Components

Figures 3 and 4 illustrate the differences between the individual components of the maternal and fetal dipole vectors. The parameters corresponding to heart rate of mother and fetus were selected such that the fetal heart rate is higher than the maternal heart rate. This can be seen in the frequency of the peaks in Figures 3 and 4. Additionally, parameters were selected such that the strength of the fetal components is approximately an order of magnitude smaller than the strength of the maternal components. These results agree with clinical data.

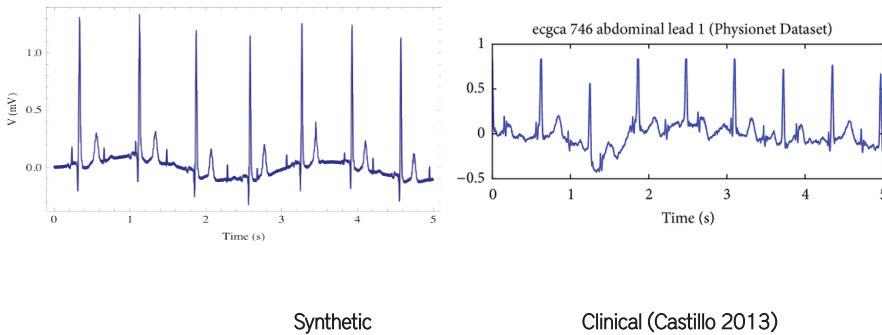


Figure 5: Comparison between a synthetic fetal ECG record (left) and a fetal ECG record collected in clinics.

Figure 5 shows a comparison of the synthetic fECG signal generated by our model and the clinical fECG recording. As can be seen, our synthetic fECG has similar morphology and features to the clinical recording. This code has been run numerous times to generate a database of synthetic fECG's. Many of the parameters were pseudo-randomly generated over realistic ranges. This randomization allows simulation of a multitude of different clinical settings. This database will allow for testing of various signal extraction algorithms.

## Conclusions

A realistic model of a multi-channel fECG was developed. This model relied on the assumption that electrical activity in the heart behaves as a dipole. The electrical activity of the hearts of both mother and fetus was simulated by the numerical solution of three differential equations each. This resulted in two functions whose inputs were time and whose outputs were three-dimensional voltage vectors. Aperiodic activity was introduced by allowing the  $\omega$  term in each equation to vary with time. This multi-dimensional equation was then projected onto several unit vectors representing the different channels of the ECG. Finally, two different types of noise, white noise and 60 Hz sinusoidal noises were added to the projections. The code was used to generate a database for signal processing algorithm testing. This will allow for the testing of signal processing algorithms in the absence of real clinical data.

Future work in this area largely consists of validating the model. Modeling baseline wander could be improved. In the current model, baseline wander assumes a constant respiratory

frequency. Future models may use a variable respiratory frequency and a more realistic, time-varying weighting factor. Additionally, noise was simulated with 60 Hz sinusoidal function representing power line interference along with a random, interpolating function for white noise. The relative influence of each of these types of noise on the fECG signal needs further study. White noise simulation now accounts for all noise unrelated to the power grid. Identifying and modeling other possible noise patterns may lead to more realistic synthetic noise. Spectrum analysis could be performed on the synthetic fECG and compared to a realistic power spectrum of the Mayer and RSA waves. This would verify realistic quasi-periodicity in the model. Parameters corresponding to the various physiological conditions of both mother and fetus could be implemented with more realistic values. Furthermore, generating a user interface in which one could easily manipulate various parameters is an area of interest. This interface could be used as an educational tool to study the influences of different noise artifacts and parameters. Finally, the principle use of the database is to allow for more effective testing of de-noising algorithms, channel reductions, fetal signal extractions, etc.

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