An Experimental Investigation of the Air Drag on Monofilament Fibers and Flowfield Behavior Resulting from Two Convergent High Velocity Jets: Melt Blowing Application

Bryan David Haynes

University of Tennessee, Knoxville

Recommended Citation

To the Graduate Council:

I am submitting herewith a thesis written by Bryan David Haynes entitled "An Experimental Investigation of the Air Drag on Monofilament Fibers and Flowfield Behavior Resulting from Two Convergent High Velocity Jets: Melt Blowing Application." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Aerospace Engineering.

Mancil W. Milligan, Major Professor

We have read this thesis and recommend its acceptance:

William S. Johnson, James A. Euler, Harvey J. Wilkerson

Accepted for the Council:

Dixie L. Thompson

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)
To the Graduate Council:

I am submitting a thesis written by Bryan David Haynes entitled "An Experimental Investigation of the Air Drag on Monofilament Fibers and Flowfield Behavior Resulting from Two Convergent High Velocity Jets: Melt Blowing Application." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Aerospace Engineering.

Mancil W. Milligan, Major Professor

We have read this thesis and recommend its acceptance:

[Signatures]

Accepted for the Council:

[Signature]

Vice Provost
and Dean of the Graduate School
STATEMENT OF PERMISSION TO USE

In presenting this thesis in partial fulfillment of the requirements for a Master degree at The University of Tennessee, Knoxville, I agree that the Library shall make it available to borrowers under rules of the library. Brief quotations from this thesis are allowable without special permission, provided that accurate acknowledgment of the source is made.

Permission for extensive quotation from or reproduction of this thesis may be granted by my major professor, or in his absence, by the Head of Interlibrary Services when, in the opinion of either, the proposed use of the material is for scholarly purposes. Any copying or use of the material in this thesis for financial gain shall not be allowed without my written permission.

Signature  

Date  

8-6-87
AN EXPERIMENTAL INVESTIGATION OF THE AIR DRAG ON MONOFILAMENT FIBERS AND FLOWFIELD BEHAVIOR RESULTING FROM TWO CONVERGENT HIGH VELOCITY JETS: MELT BLOWING APPLICATION

A Thesis
Presented for the
Master of Science Degree
The University of Tennessee, Knoxville

Bryan David Haynes
August 1987
ACKNOWLEDGMENTS

The author would like to express his appreciation to Dr. Harvey J. Wilkerson, Dr. William S. Johnson, and Dr. James A. Euler for the helpful suggestions they made in review of this thesis. The author would especially like to express his appreciation to Dr. Mancil W. Milligan for his guidance and assistance during this investigation and in preparation of this thesis.

This work was supported by the Exxon Chemical Company, Plastic Technology Division, Baytown, Texas.
ABSTRACT

An extensive experimental investigation has been conducted in order to better understand the air drag phenomena of a monofilament fiber in a free jet flowfield. The resulting flowfield was generated from two convergent high velocity jets. This particular arrangement is similar to the meltblown configuration used in the production of nonwoven materials. The experimental data in the results show how the fiber drag varies with stagnation pressure, air injection angle, fiber length, test section orientation, die setback, and exit velocity. A Fanno flow analysis was used to model the flow in the injection nozzles to determine the exit velocity.

The free jet flowfield was also investigated by measuring velocity distributions and by flow visualization. The velocity profiles were obtained using a hot film anemometer. Two optical systems and a smoke system were used in the flow visualization. The shadowgraph and schlieren systems were the optical systems used in the investigation. The jet spread angle and velocity distributions are compared with a classical solution.
### TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Meltblown Process</td>
<td>1</td>
</tr>
<tr>
<td>Statement of Problem</td>
<td>4</td>
</tr>
<tr>
<td>II. LITERATURE REVIEW</td>
<td>5</td>
</tr>
<tr>
<td>Experimental Investigations</td>
<td>5</td>
</tr>
<tr>
<td>Theoretical Investigations</td>
<td>10</td>
</tr>
<tr>
<td>Flowfield Investigation</td>
<td>13</td>
</tr>
<tr>
<td>III. EXPERIMENTAL DRAG INVESTIGATION</td>
<td>16</td>
</tr>
<tr>
<td>Air Delivery System</td>
<td>17</td>
</tr>
<tr>
<td>Instrumentation</td>
<td>19</td>
</tr>
<tr>
<td>Determination of Mass Flow Rate</td>
<td>21</td>
</tr>
<tr>
<td>Test Sections</td>
<td>24</td>
</tr>
<tr>
<td>General Test Procedures</td>
<td>30</td>
</tr>
<tr>
<td>Experimental Uncertainty</td>
<td>32</td>
</tr>
<tr>
<td>IV. EXPERIMENTAL FLOWFIELD INVESTIGATION</td>
<td>34</td>
</tr>
<tr>
<td>Instrumentation</td>
<td>35</td>
</tr>
<tr>
<td>Calibration</td>
<td>36</td>
</tr>
<tr>
<td>Flow Visualization</td>
<td>38</td>
</tr>
<tr>
<td>Determination of Entrained Mass Flow Rate</td>
<td>40</td>
</tr>
<tr>
<td>General Test Procedures</td>
<td>45</td>
</tr>
<tr>
<td>Experimental Uncertainty</td>
<td>47</td>
</tr>
<tr>
<td>V. ANALYSIS TO MODEL THE FLOW IN THE INJECTION NOZZLE</td>
<td>48</td>
</tr>
<tr>
<td>Fanno Model</td>
<td>49</td>
</tr>
<tr>
<td>Flow Process</td>
<td>51</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Determination of Entrance and Exit Mach Numbers</td>
<td>54</td>
</tr>
<tr>
<td>Determination of Injection Nozzle Exit Velocity</td>
<td>60</td>
</tr>
<tr>
<td>VI. RESULTS</td>
<td>63</td>
</tr>
<tr>
<td>Drag Investigation</td>
<td>63</td>
</tr>
<tr>
<td>Flowfield Investigation</td>
<td>78</td>
</tr>
<tr>
<td>Empirical Correlations</td>
<td>89</td>
</tr>
<tr>
<td>Conclusions</td>
<td>93</td>
</tr>
<tr>
<td>Recommendations</td>
<td>94</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>96</td>
</tr>
<tr>
<td>APPENDIX</td>
<td>100</td>
</tr>
<tr>
<td>A Description of the Computer Program</td>
<td>101</td>
</tr>
<tr>
<td>VITA</td>
<td>113</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1. Cross Sectional View of a Typical Meltblown System</td>
<td>2</td>
</tr>
<tr>
<td>3.1. Schematic Diagram of Air Delivery System</td>
<td>18</td>
</tr>
<tr>
<td>3.2. Measuring Head and Pulley Arrangement</td>
<td>20</td>
</tr>
<tr>
<td>3.3. Schematic Diagram to Define the Geometry of the Meltblown Configuration</td>
<td>25</td>
</tr>
<tr>
<td>3.4. Schematic Diagram of Test Section &quot;A&quot;</td>
<td>27</td>
</tr>
<tr>
<td>3.5. Pipe Cap Test Sections</td>
<td>28</td>
</tr>
<tr>
<td>3.6. Test Section &quot;B&quot;</td>
<td>29</td>
</tr>
<tr>
<td>4.1. Calibration Curve for the TSI Model 1266 Probe</td>
<td>37</td>
</tr>
<tr>
<td>4.2. Schematic Diagram of the Smoke Visualization System</td>
<td>39</td>
</tr>
<tr>
<td>4.3. Schematic Diagram of the Shadowgraph and Schlieren Systems</td>
<td>41</td>
</tr>
<tr>
<td>4.4. Isometric View of the Control Volume Used to Determine the Entrained Mass Flow Rate</td>
<td>43</td>
</tr>
<tr>
<td>4.5. The Evaluation of Volumetric Flow Rate by Numerically Integrating U vs. Y</td>
<td>44</td>
</tr>
<tr>
<td>5.1. Cross Sectional Diagram of the Injection Nozzle</td>
<td>50</td>
</tr>
<tr>
<td>5.2. Temperature Entropy Diagram to Show the Fanno Lines for Different Stagnation Pressures</td>
<td>52</td>
</tr>
</tbody>
</table>
5.3. Diagram to Show Wave Phenomena for Underexpanded Flow ........................................... 55
5.4. Schleiren Photograph of the Flowfield ................................................................. 56
6.1. The Variation of Drag with Fiber Length .............................................................. 64
6.2. The Variation of Drag with Fiber Length .............................................................. 65
6.3. The Effects of Stagnation Pressure on Fiber Drag .................................................. 67
6.4. The Effects of Stagnation Pressure on Fiber Drag .................................................. 68
6.5. The Effects of Stagnation Pressure on Fiber Drag .................................................. 69
6.6. The Variation of Drag with Injection Angle .......................................................... 70
6.7. The Variation of Drag with Injection Angle .......................................................... 71
6.8. Comparison Between the Experimental Test Sections .............................................. 72
6.9. The Effect of Setback on Fiber Drag ....................................................................... 74
6.10. The Variation of Drag with Test Section Orientation .............................................. 75
6.11. Drag As A Function of Exit Velocity ....................................................................... 76
6.12. Drag As A Function of Injection Nozzle Slot Height for Different Stagnation Pressures ................................................................. 77
6.13. The Effect of Nozzle Mass Flow Rate on Fiber Drag ............................................. 79
6.14. Velocity Profiles Taken at Several Axial Positions ................................................. 80
6.15. Comparison of Classical Solution and Experimental Jet Boundary ....................... 81
6.16. Comparison of Classical Free Jet Velocity Solution with Experimental Results 82
6.17. Velocity Profiles for Several Stagnation Pressures 84
6.18. The Variation of Mass Flow Ratios with Stagnation Pressure 85
6.19. The Variation of Mass Flow Ratios with Axial Displacement 86
6.20. Drag As A Function of the Axial Momentum Issuing From the Injection Nozzles 91
6.21. A Summary of the Linear Curve Fits for Various Lengths 92
A.1. Computer Flowchart 109
## LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Fiber radius, 0.0025 in.</td>
</tr>
<tr>
<td>A</td>
<td>Aspect ratio of the test section</td>
</tr>
<tr>
<td>$A_E$</td>
<td>Exit area of the injection nozzles</td>
</tr>
<tr>
<td>b</td>
<td>Jet boundary half width</td>
</tr>
<tr>
<td>C</td>
<td>Orifice coefficient of discharge</td>
</tr>
<tr>
<td>d</td>
<td>Orifice inside diameter</td>
</tr>
<tr>
<td>$d_s$</td>
<td>Die setback</td>
</tr>
<tr>
<td>D</td>
<td>The fiber drag</td>
</tr>
<tr>
<td>f</td>
<td>The average friction coefficient of the injection nozzle</td>
</tr>
<tr>
<td>F</td>
<td>The velocity of approach factor for the orifice</td>
</tr>
<tr>
<td>$F_a$</td>
<td>The thermal expansion factor for the orifice</td>
</tr>
<tr>
<td>$g_c$</td>
<td>A constant that relates force, length, mass and time</td>
</tr>
<tr>
<td>G</td>
<td>The jet momentum evaluated at the exit plane of the injection nozzles</td>
</tr>
<tr>
<td>h</td>
<td>The injection nozzle slot height</td>
</tr>
<tr>
<td>$h_m$</td>
<td>Manometer deflection</td>
</tr>
<tr>
<td>i</td>
<td>An index used to denote the lateral ($Y$) position and corresponding axial velocity</td>
</tr>
<tr>
<td>l</td>
<td>The length of the injection nozzle</td>
</tr>
<tr>
<td>L</td>
<td>The length of the test fiber</td>
</tr>
<tr>
<td>$m$</td>
<td>The air delivery system mass flow rate</td>
</tr>
<tr>
<td>$m_e$</td>
<td>The entrained mass flow rate</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$m_{IN}$</td>
<td>The mass flow rate issuing through the injection nozzles</td>
</tr>
<tr>
<td>$m_t$</td>
<td>The total mass flow rate through a control volume at any axial (X) location</td>
</tr>
<tr>
<td>$M$</td>
<td>The local Mach number</td>
</tr>
<tr>
<td>$M_E$</td>
<td>The Mach number at the injection nozzle exit plane</td>
</tr>
<tr>
<td>$M_I$</td>
<td>The Mach number at the injection nozzle inlet plane</td>
</tr>
<tr>
<td>$N$</td>
<td>The number of partitions used to numerically integrate the velocity distribution to obtain the volumetric flow rate</td>
</tr>
<tr>
<td>$p$</td>
<td>The static pressure</td>
</tr>
<tr>
<td>$p_E$</td>
<td>The static pressure of the flow at the injection nozzle exit plane</td>
</tr>
<tr>
<td>$p_\infty$</td>
<td>The ambient pressure</td>
</tr>
<tr>
<td>$P$</td>
<td>The stagnation pressure</td>
</tr>
<tr>
<td>$P_E$</td>
<td>The stagnation pressure of the flow at the injection nozzle exit plane</td>
</tr>
<tr>
<td>$P_I$</td>
<td>The stagnation pressure at the inlet of the injection nozzle</td>
</tr>
<tr>
<td>$P_{ox}$</td>
<td>Test section stagnation pressure</td>
</tr>
<tr>
<td>$Q$</td>
<td>Volumetric flow rate evaluated in the control volume</td>
</tr>
<tr>
<td>$R$</td>
<td>Gas constant used for air</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$s$</td>
<td>Entropy</td>
</tr>
</tbody>
</table>
t  Static temperature

$t_E$  The static temperature at the injection nozzle exit plane

$T_0$  The stagnation temperature of the flow downstream of the orifice

$U$  The axial component of velocity

$U_{max}$  The centerline ($Y = 0$) axial component of velocity

$V_E$  The exit velocity parallel to the centerline of the injection nozzle

$W$  Injection nozzle width ($Z$)

$X$  Longitudinal (axial) coordinate

$X_1, X_2$  Arbitrary axial positions

$Y$  Lateral coordinate perpendicular to the jet axis

$Z$  The spanwise coordinate perpendicular to the $X Y$ plane

Greek Characters

$\alpha$  Angle between the $X$ axis and horizontal reference plane

$\beta$  Air injection angle relative to the $X$ axis

$\gamma$  Ratio of specific heats

$\delta$  Proportionality constant

$\Delta$  Denotes a portion of the width ($Z$) of the control volume

$\varepsilon$  Equivalent sand grain diameter
η  Nondimensional parameter defined as $7.67Y/X$

θ  The angle between the X axis and the jet boundary

ξ  Diameter ratio of the orifice

ρ_p  Air density in the air supply system upstream of the orifice

ρ_∞  Ambient air density

ω_m  Experimental uncertainty in the mass flow rate

ω_{h_m}  Experimental uncertainty in the manometer deflection

ω_{ρ_∞}  Experimental uncertainty in the ambient pressure

ω_{T_o}  Experimental uncertainty in the pipe temperature

ψ(X_1,X_2)  The ratio of the total mass flow ratios between any two axial positions in the control volume

**Superscripts**

k  A constant used in the empirical correlations

*  Denotes sonic conditions ($M = 1$)
CHAPTER I

INTRODUCTION

The current research was undertaken to investigate the drag on a monofilament fiber and the flowfield characteristics associated with the meltblown process. A literature survey indicated no previous experimental investigations utilizing the meltblown configuration to investigate the drag on a cylinder in axial flow. The current investigation was unique in experimentally modeling the meltblown configuration to obtain drag and flowfield data.

I. Meltblown Process

The meltblown process is an unique production technique used to fabricate nonwoven fabrics. A schematic cross sectional view of a typical meltblown configuration can be seen in Figure 1.1. The center section is referred to as the "die". The die has many extrusion holes. A typical die is 26 inches in length with 20 holes per inch. These holes are small (0.3 millimeters in diameter).

A brief explanation of the meltblown process will next be presented. A molten polymer such as polypropylene is extruded through the die. As the polymer exits the orifice, hot streams of air, between 500° and 700° from
Figure 1.1. Cross Sectional View of a Typical Meltblown System.
above and below the die converge on the polymer to pull and form a fiber. The air is injected at an angle of 30°. The fiber is drawn out and the diameter of the fiber reduces by 200 times the original orifice diameter. The fibers entangle and form a web of material. The fibers are held together by the entangling and thermal bonding in the flowfield downstream of the die. One advantage of the meltblown process is the capability of forming fibers with small diameters. On the average, the meltblown fibers range between 0.5 to 2 micrometers. A major disadvantage of the meltblown process is the high production cost. A large airflow rate is necessary to achieve the required exit velocities at the die. Heating this air to the operating temperature requires a large amount of energy. The high production cost is the result of the energy necessary to heat and compress the air. Because of this high production cost, meltblown materials are expensive.

It would be advantageous for the textile industry if the production cost of meltblown materials could be reduced. If the fiber characteristics could be expressed as a function of the meltblown geometry, stagnation pressure, stagnation temperature, etc., the optimum configuration could be determined for a more efficient production technique. However, there is limited experimental data to relate the fiber characteristics to different meltblown geometries, stagnation pressure, etc.
II. Statement of Problem

The major purpose of this investigation was to determine the drag dependency of one monofilament fiber on fiber length, stagnation pressure, mass flow rate, die setback, gravity die orientation, flow injection angle, and injection nozzle exit velocity. The injection nozzle exit velocity was formulated analytically using a Fanno flow model. An experimental investigation provided the data for the drag dependency on the variables mentioned above. A flowfield investigation using flow visualization and hot wire anemometry was used to determine the nature of the flow downstream of the injection nozzle.
CHAPTER II

LITERATURE REVIEW

The research material from past investigations was separated into three major subject areas. The topics covered in these subject areas are helpful in better understanding the current investigation. The three major subject areas are: experimental investigations, analytical investigations, and flowfield investigations.

I. Experimental Investigations

The results of a brief literature survey in this area indicates no significant investigations into the air drag phenomena associated with the meltblown process. However, there have been several investigations conducted which deal with the determination of drag on a monofilament fiber using geometrically different configurations. Because of the similarities in the physics of the configurations, i.e., monofilament in an axial flowfield, the trends in the experimental data can be compared. However, the numerical results, such as Reynolds number and drag coefficient, cannot be compared due to differences in the configurations.

The major differences between the meltblown configuration and the other experimental configurations
used by investigators can be summarized as follows. The fiber in the meltblown process is in a free jet where no solid boundaries contain the flowfield except at the fiber origin for positive setback. The fiber in this case is free to move. A majority of the investigations, to be covered in more detail later, used a fiber contained in a cylinder attached to a measuring device at one end and fixed at the other end. The fiber in this case is not free to move; exceptions to this occurred when investigators wished to determine the effects of allowing the fiber to move freely.

The flowfield for the meltblown configuration resembles that of a free jet issuing from a rectangular nozzle. Many investigators used circular test sections which produced a symmetrical flowfield around the fiber. Lastly, the flow is injected at an angle in the meltblown configuration while the other experimental configurations produced parallel flow along the entire fiber.

Gould and Smith (1) conducted an extensive experimental investigation on monofilaments in axial flows with velocities up to 300 m/s. They used a variety of

---

Numbers in parenthese refer to similarly numbered references in the bibliography.
fiber diameters with lengths up to 80 centimeters. A 25 millimeter diameter test section was used. They showed that as the velocity increases so does the fiber drag.

Another important question dealing with the meltblown process is the fiber spacing. If the extrusion holes were too close, the fiber drag decreased due to the interaction of the boundary layers. Gould and Smith found that for a fiber spacing greater than 670 microns there was not sufficient interaction of the boundary layers to reduce the drag. The die used in The University of Tennessee Textile Department has 20 extrusion holes (0.3 millimeters diameter) per inch which results in an initial fiber spacing of 970 microns.

Anderson and Stubbs (2) conducted similar research as Gould and Smith; however, their range of velocities were low in comparison to Gould and Smith. Using a cylindrical test section with a diameter of 5.5 millimeters, the fiber diameters ranged between 20 to 50 microns. The upper limit of the test section velocity was 17.7 m/s. The results obtained by Anderson and Stubbs agreed with those of Gould and Smith. Anderson and Stubbs found the same drag behavior when changing the test section air velocity, fiber diameter, or fiber length.

Both of the previously mentioned investigations used a similar arrangement to collect data. The fiber was contained within a cylinder. At one end the fiber was
attached to a measuring device and restrained at the other end. The flow passed along the entire length of the fiber.

The velocity at the outer edge of the monofilament boundary layer was treated as a constant along the entire length of the monofilament in the investigations mentioned so far. Ackroyd (3) pointed out two problems with this model. One problem being the boundary layer growth along the walls of the test section which causes the center line velocity to increase. The other problem is the presence of a favorable pressure gradient which is associated with the increasing velocity.

The velocity of the flow along the fiber in the meltblown process varies. At the position 10 inches from the injection nozzle, the center line velocity will be only 20% of the injection nozzle velocity (4). The presence of this nonuniform velocity acting along the length of the fiber results in difficulty when comparing drag coefficient versus Reynolds number data with other investigations.

Chen et al. (5) conducted an air drag investigation on the spunbonding process. Their experimental test section was comprised of an open tube (aspirator). The fiber was placed vertically through the aspirator which had two major sections. The first section was a narrow guide tube. Air was injected (parallel to the fiber) in the second section to pull the fiber.

The filament used in the investigation was one meter long with a diameter of 86 microns. The filament used was
polypropylene. One of the most interesting results was the effects of stagnation pressure on the fiber drag as a function of length. As the stagnation pressure increased, the drag experienced a sharp increase in the second section of the aspirator. On the plot, this increase appears as a spike on the curve.

Selwood (6) also conducted experiments on the axial drag of monofilaments. Selwood's work was not as extensive as the previous investigations. Selwood's experimental procedure gave results for fibers which were pulled through still air. He observed that his drag values were one-half of those reported by other investigators using short static fibers in turbulent flow.

Sir Geoffrey Taylor (7) conducted an extensive investigation on the swimming of narrow animals in water. Experimentally, Taylor obtained normal and longitudinal forces on a 3/8 inch diameter cylinder in a wind tunnel at a velocity of 40 ft/s. The cylinder was positioned at different incidence angles to the free stream.

Taylor modeled a flexible cylinder with waves propagating down the cylinder. The waves moving towards the rear of the cylinder produces a forward velocity of the cylinder. Taylor calculated the amplitude of the waves necessary to produce the greatest forward velocity. The solution he obtained was compared to photographs of a snake swimming. The comparison was good. In the current investigation observations show that the fiber drag
increases as the frequency of the vibrating fiber increases.

II. Theoretical Investigations

There have been numerous theoretical investigations conducted on cylinders in axial flows. These investigations were carried out under a variety of different flows such as turbulent compressible, laminar incompressible, etc. Numerical and integral techniques were implemented in solving the governing differential equations for the boundary layer axial flow over a cylinder. The numerical results of these investigations are not appropriate for comparison with the meltblown configuration, but the solution techniques and physical conclusions are important in gaining a better understanding of axial flow over a cylinder.

In addition to the velocity solution in the boundary layer, several investigators have analyzed the dynamic response of the cylinder in the flowfield. Paidoussis et al. (8, 9, 10) and Busby et al. (11) have extensively analyzed the instability and dynamic motion of flexible cylinders in uniform axial flows. Their investigations developed the equations of motion for a flexible cylinder in axial flow.

In developing the equations of motion for the cylinder, Paidoussis (8) assumed the angle of incidence of the
cylinder with the flow was small. This assumption insures separation does not occur. The external forces acting on the fiber were modeled as form and viscous drag. The inertial force of the cylinder was neglected.

Paidoussis et al. (9) found that a cylinder in axial flow is unstable at low velocities. The cylinder experiences a yawing motion at these low velocities. The motion is induced by random perturbations in the mean flow. At higher velocities, the instabilities are in the form of flutter which can be oscillatory or nonoscillatory (10).

Busby et al. (11) analyzed the motion of a towed wire (either elastic or rigid) behind re-entry vehicles. The arrangement is somewhat analogous to the experimental arrangement used in the present investigation because the cylinder is restrained at one end and free to vibrate in the flowfield. The amplitude of the cylinder motion was found to be proportional to the initial deflection. The initial deflection of a monofilament fiber could be caused from velocity perturbations in the turbulent flowfield.

The drag on a cylinder can easily be determined by calculating the shear stress at the surface of the cylinder knowing the velocity distribution. Glauert and Lighthill (12) used the Polhausen method and an asymptotic series to obtain the solution for an incompressible laminar boundary layer over a cylinder in axial flow. White (13) used an integral form of the momentum equation along with the assumption of the cylindrical law of the wall to solve for
the turbulent incompressible boundary layer over a cylinder in axial flow.

Cebeci et al. (14, 15) solved for the laminar and turbulent compressible and incompressible boundary layers on cylinders in axial flow. The method of solution was numerical. The finite-difference method used was found to be fast and accurate in determining the solution.

Sparrow et al. (16), Eckert (17), Sakiadis (18), and Ackroyd (3) obtained solutions for turbulent flow over a cylinder in uniform axial form. Sparrow investigated the velocity and thermal boundary layers. He solved for the velocity boundary layers in the following way. The key to Sparrow's solution was in the shear stress distribution as a function of nondimensional distance and velocity using the friction velocity. This term also contained the eddy diffusivity for momentum. If the shear stress distribution were known, the solution could easily be obtained by integrating the expression developed for the shear stress distribution. However, the shear stress distribution was not known. Therefore, Sparrow developed a shear stress distribution which was consistent with other experimental data. Using this distribution and expressions for the diffusivity terms, the shear stress distribution was integrated resulting in a differential equation in terms of the nondimensional velocity. His results compared within 8 to 9% with experimental data.
Eckert studied what effects the curvature of the cylinder had on the boundary layer. Eckert found that for incompressible flow the curvature had no effect on surface friction if the ratio of boundary layer thickness to cylinder radius was less than one. The diameter of meltblown fibers range between 0.5 to 2 microns in diameter. He used an integral form of the momentum equation along with a 1/7 power law for the velocity distribution to solve for the boundary layer thickness.

Sakiadis used an integral form of the momentum equation along with velocity profiles which satisfied the boundary conditions. His solution involved the laminar and turbulent boundary layers on a continuous cylindrical surface moving in a fluid at rest.

Ackroyd used the same solution technique as White. Ackroyd showed the results obtained for uniform flow over a cylinder are not significantly different from the results obtained for an extruding cylinder in a fluid at rest.

III. Flowfield Investigation

Another important aspect of this investigation was to gain a better understanding of the meltblown jet flowfield. The flowfield data obtained using the coldflow models was compared with numerous investigations on turbulent free jet flowfields. The comparison between the flowfield data
obtained using the cold flow models and data from other investigations is good.

Two and three dimensional turbulent rectangular jets have extensively been studied. Narain (19) investigated the momentum flux development issuing from rectangular, circular, and elliptic nozzles. Narain found that the jet issuing from a slender rectangular nozzle with an eccentricity less than one exhibits three zones of development. A potential core region, characteristic decay region, and axisymmetric decay region make up the three zones. The initial flow region in the meltblown process may differ from the rectangular jet because the flow is injected at an angle in the meltblown process.

Sfeir (20) found that three dimensional effects increases with axial displacement, X. Also, Sfeir speculated that the saddle-back velocity profiles in the X, Z plane is the result of elliptical vortex rings surrounding the jet. The saddle-back shape is where the velocity increases toward the outer edges of the spanwise boundary. The centerline velocity is not the maximum velocity, but there exists two maximums at the outer edges.

Goldschmidt et al. (21) investigated the apparent flapping motion of turbulent jets. He found that the lateral oscillatory motion is hidden within the turbulent field but does exists. From the investigation, Goldschmidt concluded that the flapping frequency increased with decreasing axial position from the nozzle. The flapping
motion could be the driving force in exciting the fibers to vibrate in the meltblown process.

Other important factors of free jet flowfields were of interest in the current investigation. Mean flow properties such as centerline velocity decay, half-width boundary layer growth, and velocity profiles have been studied by many investigators. Sforza et al. (4, 22), White (23), and Schlichting (24) have extensively covered these mean flow properties. The solutions agree well with the experimental data and were helpful in understanding the free jet flowfield. The results of the flowfield investigation will be discussed later.
CHAPTER III

EXPERIMENTAL DRAG INVESTIGATION

In the preliminary stages of the investigation, the experiments were developed such that pertinent information would be obtained to better understand the drag phenomena of a monofilament fiber using the meltblown configuration. The experimental data served as the only basis to draw conclusions since an analytical method was not available to determine the drag on a cylinder in a free jet flowfield. The investigation was extensive and explored different aspects of the fiber drag phenomena and flowfield associated with the meltblown process.

One of the most important factors associated with this research was designing a test section which would duplicate the meltblown configuration. Several small cold-flow test sections were developed for the investigation. The designs of these test sections will be covered more extensively later in this chapter.

The experimental investigation will be covered in two parts. Part one of the experimental investigation covers the equipment and procedures used for the investigation of the fiber drag. The flowfield investigation will be covered in the following chapter, part two.
I. Air Delivery System

The compressed air was supplied by the house compressor rated at 250 cfm (cubic feet per minute). The compressor was capable of delivering this volumetric flow rate at 110 psig. The compressor was more than adequate to meet the demands of the experimental investigation. Figure 3.1 is a schematic diagram of the air delivery system contained within the laboratory.

A valve was located where the main line from the compressor entered the laboratory. This valve was used to shut the flow off to the entire system in order to make repairs. A filter and pressure regulator were installed in the upper portion of the air delivery system. The regulator was installed to remove any pressure fluctuations in the flow. The regulator was set to maintain a pressure of 50 psig downstream of the regulator.

The flow rate was determined by measuring the pressure drop across an orifice. An orifice was chosen over a flow nozzle and venturi tube because the orifice was less costly to fabricate and easier to install. The design and location of the orifice was in accordance with the ASME standards on flow measurement (25).

The valve downstream of the orifice was used to throttle the flow in the test section.
Figure 3.1. Schematic Diagram of Air Delivery System.
II. Instrumentation

The drag on the monofilament fiber was measured using a Rothschild electronic tensiometer. The tensiometer utilizes a capacitance probe to detect tension on a fiber. A four gram measuring head was used. The tensiometer was calibrated and checked for reproducibility. The instrument was calibrated by attaching weights to a fiber connected to the measuring head. A plot was generated which showed a linear correlation between the fiber tension (drag) and scale deflection.

Gould and Smith (1) discussed several difficulties when trying to measure the tension on a moving fiber. At very high speeds, the frictional forces on the tensiometer pegs are the same order of magnitude as the actual tension on the fiber. Gould and Smith chose to use a stationary fiber for this reason. A stationary fiber was used in the present investigation.

Figure 3.2 is a photograph of the measuring head and pulley arrangement. This arrangement was used in order to obtain drag data while varying the length of the fiber without turning the flow off.

A U-type manometer and differential pressure gage were used to measure the pressure drop across the orifice. Mercury or water was used in the manometer, depending on the airflow rate. The pressure drop across the orifice
Figure 3.2 Measuring Head and Pulley Arrangement.
increases with the flow rate. Mercury was used for high flow rates while water was used for small flow rates.

III. Determination of Mass Flow Rate

The air velocity in the air delivery pipe for the maximum mass flow rate is approximately 60 ft/s. The mean temperature of the flow in the pipe is usually in the neighborhood of 80 °F. Using the value for the temperature and the maximum velocity in the pipe, the corresponding Mach number is 0.053. The flow in the pipe can be assumed to be incompressible.

The mass flow rate of air through the orifice was determined using the technique presented in Reference (25).

\[ \dot{m} = 359CFd^2F_a \sqrt{h_m \rho_p} \]

where:  
\( \dot{m} \) = mass flow rate of air, lbm/hr  
C = coefficient of discharge  
d = diameter of orifice, in  
h_m = differential pressure, in  
\( \rho_p \) = density of fluid at orifice inlet, lbm/ft^3  
F_a = thermal expansion factor  
F = velocity of approach factor

The thermal expansion factor takes into account the expansion of the orifice. In other words, at high
temperatures the orifice opening will increase in size due to the expansion of the material used for the orifice. The thermal expansion factor will be assumed to be unity because of the low operating temperatures, e.g., the thermal expansion factor for 430 stainless at 500 °F is only 1.005. The diameter ratio of the orifice to inside pipe diameter was established such that there would be a measurable manometer deflection for low flow rates. This would insure that the same orifice could be used for measurements taken over a wide range of flow rates. The velocity of approach factor can be determined knowing the diameter ratio and is given by

\[ F = (1 - \xi^4)^{-\frac{1}{4}} \]  

(3.2)

For a diameter ratio of 0.5, the velocity of approach factor, from Equation (3.2), is 1.0327. The coefficient of discharge varies slightly over a range of Reynolds numbers for a given diameter ratio. The coefficient of discharge was taken to be the average value, \( C = 0.61625 \). Inserting the values for \( F, Fa, C \) and \( d \) into Equation (3.1) gives

\[ \dot{m} = 245.68 \sqrt{\frac{h_m}{\rho_p}} \]  

(3.3)

Converting the units in Equation (3.3) from lbm/hr to lbm/s gives

\[ \dot{m} = 0.06825 \sqrt{\frac{h_m}{\rho_p}} \]  

(3.4)

The density of the air in the pipe can be determined knowing the pressure and temperature of the air in the pipe.
upstream of the orifice. There is a negligible temperature
difference across the orifice. The temperature measured
downstream of the orifice can be used in the calculations.
The ideal gas equation is given by
\[ p_p = \frac{P}{RT_0} \] (3.5)

The pressure in the pipe is the sum of the gage
pressure of the air in the pipe and the ambient pressure.
The pressure in this section of the pipe is set by the
regulator. The ambient pressure often changed and must be
treated as a variable in the correlation. The regulator
was set to maintain the flow at 50 psig. Equation (3.5)
can be rewritten as
\[ p_p = \frac{50 + p_\infty}{R(T_0 + 460)} \] (3.6)

Using the gas constant for air, Equation (3.6) becomes
\[ p_p = 2.6997 \frac{(50 + p_\infty)}{T_0 + 460} \] (3.7)

where:  
\( p_\infty = \) ambient pressure psi  
\( T_0 = \) pipe temperature, °F  
\( p_p = \) pipe density, lbm/ft

Inserting Equation (3.7) into Equation (3.4) finally
gives
\[ \dot{m} = 0.1121 \left( h_m \frac{(50 + p_\infty)}{(T_0 + 460)} \right) \] (3.8)
IV. Test Sections

The experimental investigation eventually used small cold-flow models to duplicate the meltblow configuration. Initially, several designs were studied to determine the most effective way to vary the geometrical parameters associated with the meltblown configuration. These parameters included: injection angle, injection nozzle height, and set-back. Figure 3.3 shows the parameters mentioned above.

Instead of using 20 fibers per inch, a single fiber was extended from the test section into the flowfield. The interaction of the fibers would be negligible using the same fiber spacing that is used in actual production configurations. Another constraint of the design was that the fiber extended from the flowfield through the test section directly to the measuring head in a straight line. This would reduce binding of the fiber in the guide tube giving more accurate drag measurements.

It was decided to conduct the investigation in two phases because of the complexity in trying to vary all the parameters in one design of a test section. In phase one, a test section would be used which would change the injection angle only. In phase two, the test section would be capable of varying the set-back and injection nozzle height.
Figure 3.3. Schematic Diagram to Define the Geometry of the Meltblown Configuration.
A simple means of changing the injection angle was developed. The method consisted of interchanging pipe caps. The pipe cap attached to a cross pipe section. The guide tube passed directly from the cap and exited from the other end. The air flow entered the side, and the pressure gage was attached to the opposite side. The different pipe caps had parallel slots approximately 1.4 inches long cut at an angle resembling the meltblown configuration. The normal height of these slots were 0.032 inches. Figure 3.4 is a cross sectional view of the pipe cap test section, referred to as test section "A". Three different test sections were fabricated with injection angles of 15°, 30°, and 45°. Figure 3.5 is a photograph of these test sections. These test sections were inexpensive and easy to fabricate.

A more sophisticated design was needed for the second phase of the investigation. Figure 3.6 shows several photographs of test section "B". This test section incorporated a movable center section which allowed different set-backs. The face plates were movable to change the injection nozzle height. The injection nozzle width was 2.75 in. The center section was contoured on the side the flow entered. Only one of these test sections were fabricated. The injection angle of test section "B" was 30°.
Figure 3.4. Schematic Diagram of Test Section "A".
Figure 3.5. Pipe Cap Test Sections.
Figure 3.6. Test Section "B". $A = 37.21$. 
The test sections were designed by the investigator and fabricated in the Department of Mechanical and Aerospace Engineering Shop.

V. General Test Procedures

The tensiometer was turned on several hours before the data collection. The tensiometer reading would drift initially, but the reading would stabilize after the instrument was allowed to warm up. The measuring head was positioned in the same orientation behind the test section for each run.

The test fibers were under tension overnight to insure the wrinkles would be removed where it had been wound on the spool. Before the fiber was threaded into the guide tube, the guide tube was flushed to remove any foreign particles. This insured that the test fiber could freely pass through the tube without hanging. Alcohol was used to flush the guide tube periodically. The alcohol would also dissolve oil deposits which originated from the compressor.

Before the main valve was opened, the manometer valves were checked to insure they were both open. Also, the pressure regulator was set to the operating pressure. The test section assembly was then checked to insure that all the connections were tight. If test section "B" was used, the set back and injection nozzle height were set. A pulse of air was passed through the system several times to check
the reproducibility of the tensiometer reading returning to zero. The guide tube would be cleaned again if the tensiometer reading would not return to zero. The pulses of air would also remove the residual stiffness of the test fiber due to its mechanical properties.

After the preliminary preparation mentioned above was complete, the experiment was now ready to proceed. The ambient temperature and pressure were recorded. The geometry of the test section was also documented, i.e., set-back, injection angle, etc. Five parameters were recorded for a particular test condition. These parameters included: manometer deflection, stagnation pressure, fiber length, tensiometer reading, and stagnation temperature. The stagnation temperature did not vary greatly during the experiment.

The throttling valve was slowly opened to the desired stagnation pressure. The pressure gage and manometer were checked to insure that pressure fluctuations were not present in the system. The readings would then be taken. For very high stagnation pressures (45 psig), the pressure downstream of the regulator was monitored to insure the pressure remained constant. If not, the main valve was opened further.

During the run, several data points would be checked for reproducibility. The condition of the fiber was monitored throughout the experiment. Because of the violent behavior the fiber experienced in the flowfield,
the end of the fiber could become frayed and deformed at the end. If this occurred, the fiber was replaced.

When test section "B" was used, special attention was given to the setting of the face plates. If the injection nozzles were not the same height, the flow would not be symmetric. The fiber would then be deflected either up or down depending on the situation.

VI. Experimental Uncertainty

The experimental uncertainty was determined using the analysis developed by Kline and McClintock (26). The most important parameters in this investigation are mass flow rate and fiber drag. The uncertainty of the mass flow rate was more involved because it is a function of several measurable quantities. The mass flow rate is a function of the manometer deflection, ambient pressure, and stagnation temperature of the pipe, Equation (3.8). The uncertainty in the mass flow rate is given by

\[
\omega_m = \left[ \left( \frac{\partial m}{\partial \omega_{m}} - \omega_m \right)^2 + \left( \frac{\partial m}{\partial \omega_{\omega}} - \omega_{\omega} \right)^2 + \left( \frac{\partial m}{\partial \omega_{T}} - \omega_{T} \right)^2 \right]^{\frac{1}{2}}
\]

After evaluating the partial derivatives using Equation (3.8), Equation (3.9) becomes
\[
\omega^*_m = 0.05607 \left( \frac{h_m(50 + p_w)}{T_0 + 460} \right)^{\frac{1}{4}} \left( \frac{50 + p_w}{T_0 + 460} \omega_{h_m} \right)^2 + \left( \frac{h_m}{T_0 + 460} \omega_{p_w} \right)^2 + \left( \frac{h_m}{(T_0 + 460)^2} \omega_{\eta_0} \right)^2
\]  

The uncertainty of the ambient pressure is estimated to be +/- 0.005 psi. The uncertainty of the temperature is estimated to be +/- 1 °F, and the uncertainty of the manometer deflection was estimated to be +/- 0.05 inches of either mercury or water.

The uncertainty predicted by Equation (3.10) is two orders of magnitude smaller than a typical flow rate, e.g., Equation (3.10) predicts an uncertainty of +/- 0.000579 lbm/s for a flow rate of 0.069 lbm/s.

Due to the linearity of the tensiometer calibration, the uncertainty was determined to be +/- 0.0000885 lbs. The uncertainty in the scale reading is +/- 0.1. The uncertainty of the drag reading increased as the fiber length decreased due to the increased oscillations of the drag at short lengths.
An experimental investigation was conducted on the free jet flowfield produced by the meltblown configuration. It is believed that the flowfield affects the characteristics of the fibers produced by the meltblown process. The fibers are exposed to varying center line velocities because of the development of the jet. Also, the fibers experience a phase change as it cools in the flowfield. The change in air temperature is believed to be primarily the result of entrained ambient air mixing with the original heated air issuing from the injection nozzles. A better understanding of the flowfield could result in a more precise means of controlling the fiber characteristics.

The flowfield investigation was carried out in two major areas. A major portion of the investigation consisted of mapping the flowfield using the hot wire anemometer. Velocity profiles were determined at several axial (X) locations for a variety of stagnation pressures. The total mass flow rate in the control volume as shown on page 43 could be determined at any axial location where the velocity profile was known. Knowing the mass flow rate issuing from the injection nozzles and the total mass flow
rate at any axial location, the amount of entrained flow can be determined.

In addition to the flowfield mapping, an extensive flow visualization investigation was conducted. Smoke visualization was used to determine the jet boundary and to visualize the degree of flow entrainment. A shadowgraph and schlieren system were used to see the wave patterns in the flow for large stagnation pressures when the exit Mach number of the injection nozzles were unity. This will be covered in more detail in Chapter 5.

I. Instrumentation

The flowfield data was collected using a TSI Model 1266 hot film sensor. This probe was used to collect the mean flow data. The probe was rugged and not susceptible to damage from the flowfield. A filter was placed in the air delivery system, but contaminants in the piping could still pass through the test section. A hot wire probe was broken initially in the investigation before the hot film sensor was utilized.

The TSI Model 1050-2C Dual Channel Linearized Research System was used to monitor the hot film sensor. A TSI Model 1125 Probe Calibrator was used to calibrate the hot film sensor. The calibrator consisted of two different flow chambers and a small opening. The first chamber was used to obtain velocities between 0.05 ft/s to 3 ft/s.
The next chamber was used to obtain velocities between 20 ft/s to Mach one.

The probe was positioned at different Y locations in the flowfield using a traverse controlled by a Slo-Syn preset indexer. The indexer could move the traverse in increments of 0.0005 in.

II. Calibration

The probe was positioned in the desired chamber of the calibrator to obtain a certain range of flow velocities. The probe was oriented during the calibration the same way it would be oriented during the flowfield investigation. A throttling valve was then opened to give a particular manometer deflection. A calibration curve of chamber velocity versus manometer deflection was supplied with the calibrator.

The calibration covered velocities from 0 to 480 ft/s. The output voltage was recorded for each data point taken during the calibration. The output voltage was the voltage required to keep the sensor at a constant temperature. A calibration curve shown in Figure 4.1 was generated which gives the velocity as a function of the voltage reading from the anemometer.
Figure 4.1. Calibration Curve for the TSI Model 1266 Probe.
III. Flow Visualization

The flow visualization investigation was undertaken in order to gain a better understanding of the flow characteristics associated with the meltblown configuration. The results of this investigation helped to quantify the magnitude of the entrained flow and to establish the flow boundaries. Also, the results gave further insight on the supersonic flow which results from high operating stagnation pressures. Three different methods were used for the flow visualization. These methods consisted of a smoke system, shadowgraph system, and schlieren system.

The smoke system consisted of an air supply and smoke generator. The air supply would pump a small quantity of air through the smoke generator. The smoke generator was a closed glass container with two openings. The flow from the air supply would enter one opening and the flow leaving the other opening would contain a large amount of smoke. Because of the messy films caused by burning oils, incense was used as the burning medium to supply the smoke particles. The flow leaving the generator was then pumped into the flowfield to be viewed by the observer. A schematic diagram of the smoke system can be seen in Figure 4.2.

The shadowgraph system used consisted of a collimated light source and a screen on which the image would be
Figure 4.2. Schematic Diagram of the Smoke Visualization System.
viewed. A shadowgraph is an optical system which detects the change in the density gradient by sensing the change in the light index of refraction of the flow (27). A schlieren system detects changes in density (27). In other words, the schlieren system detects the first derivative of density while the shadowgraph detects the second derivative of density. At large enough stagnation pressures, the exit Mach number of the injection nozzles becomes unity. As the stagnation pressure is increased above this point, wave formations can be seen in a small region near the injection nozzle exit. The shadowgraph was used primarily to determine when the injection nozzles choked. A schematic diagram of the shadowgraph and schlieren systems used can be seen in Figure 4.3.

IV. Determination of Entrained Mass Flow Rate

The total mass flow rate at any axial position \((X)\) is the sum of the mass flow through the injection nozzles and the entrained mass flow and is given by

\[
\dot{m}_t = \dot{m}_{IN} + \dot{m}_e
\]  

(4.1)

The entrained mass flow rate can be found directly from Equation (4.1). Rearranging Equation (4.1) gives

\[
\dot{m}_e = \dot{m}_t - \dot{m}_{IN}
\]  

(4.2)

The mass flow through the injection nozzles is the mass flow rate through the air delivery system. This flow rate can be determined knowing the pressure drop across the
Figure 4.3. Schematic Diagram of the Shadowgraph and Schlieren Systems.
orifice. If the total mass flow rate was known, the entrained mass flow rate could be calculated from Equation (4.2).

The total mass flow rate was determined by numerically integrating the velocity profile at the desired axial location. A slice was used in the flowfield for the control volume, Figure 4.4. The mass flow entering the control volume from the injection nozzle is given by

$$m_{IN} = \frac{\dot{m} \Delta W}{W}$$

The mass flow rate on the right hand side of Equation (4.3) is given by Equation (3.8). The $\Delta W$ term is the chosen width of the control volume. The flow can be treated as two dimensional if $\Delta W$ is small enough so that spanwise changes in velocity do not occur in the control volume.

From a plot of $U$ versus $Y$, the volumetric flow rate at any axial position is given by

$$Q = 2\Delta W \int_Y^b UdY$$

providing that the jet is symmetrical. The integral in Equation (4.4) can be expressed as a Riemann sum

$$Q = 2\Delta W \sum_{i=1}^N \Delta Y_i U_i$$

where $N$ is the total number of partitions used in the integration, Figure 4.5. The mass flow rate is the product of the volumetric flow rate and density. The density was evaluated using the ambient pressure and temperature. Equation (4.5) becomes
Figure 4.4. Isometric View of the Control Volume Used to Determine the Entrained Mass Flow Rate.
Dashed lines indicate partitions used in the numerical procedure.

Figure 4.5. The Evaluation of Volumetric Flow Rate by Numerically Integrating U vs. Y.
\[
\dot{m}_t = 2\Delta W \rho_\infty \sum_{i=1}^{N} \Delta Y_i U_i
\]

where:
\[
\begin{align*}
\dot{m}_t &= \text{lbm/s} \\
\Delta W &= \text{ft} \\
\rho_\infty &= \text{lbm/ft}^3 \\
\Delta Y_i &= \text{ft} \\
U_i &= \text{ft/s}
\end{align*}
\]

Inserting Equation (3.8) into (4.3) and substituting Equations (4.3) and (4.6) into (4.2) finally gives
\[
\dot{m}_e = 2\Delta W \rho_\infty \sum_{i=1}^{N} \Delta Y_i U_i - \frac{\Delta W}{W} \left( \frac{hm(50 + \rho_\infty)}{To + 460} \right)^{\frac{1}{2}}
\]
for determination of the entrainment mass flow rate.

V. General Test Procedures

Several steps were taken before the flowfield data was collected. First, the test section was positioned so the centerline of the jet would be parallel with the floor. Secondly, the traverse was positioned so the probe was initially at the centerline of the injection nozzles.

The throttling valve was opened to set the desired stagnation pressure. The traverse was then moved in increments of 0.125 inches using the indexer. At each position the voltage output was recorded from the anemometer. The flow rate through the test section was determined using the same procedure discussed in section 3.5.
The probe was moved in the Y direction until the voltage output became constant. The traverse was then moved to another axial position were the same procedure was performed again. The centerline positioning of the probe could be checked in the following way. If the probe was moved in the positive or negative Y direction from the centerline and the voltage reading decreased, then the probe was originally at the centerline position. The maximum velocity occurs at the centerline position.

The shadowgraph, schlieren, and smoke visualization systems were simple to operate. Care had to be taken in positioning the collimated light source so the image would be focused on the viewing screen. The throttling valve was slowly opened until the waves in the flowfield appeared. The stagnation pressure was reduced by 1 psi. This value of stagnation pressure was recorded as the choking stagnation pressure. The stagnation pressure was reduced because the initial waves are weak and would not be visible using the shadowgraph.

The smoke system was operated using the following procedure. The incense was ignited and placed in the smoke generator. The pump was turned on and the smoke was injected into the flowfield.
VI. Experimental Uncertainty

The uncertainty of the positioning of the probe is estimated to be $\pm 0.015$ in. The uncertainty of the velocity is estimated to be $\pm 5$ ft/s. The uncertainty of velocity was deduced from the uncertainty of the output voltage reading of the anemometer. The uncertainty for determining the choking stagnation pressure is estimated to be $\pm 1$ psig.
CHAPTER V

ANALYSIS TO MODEL THE FLOW
IN THE INJECTION NOZZLE

In order to better understand the monofilament drag as a function of the injection nozzle exit velocity, a model was developed to describe the flow through the injection nozzles.

It was desirable to model the flow using measurable parameters, i.e., stagnation pressure, ambient pressure, flow injection angle, and injection nozzle geometry. The surface friction of the injection nozzle is believed to be important in describing the flow in the injection nozzle.

From preliminary shadowgraph results, the flow was observed to choke at a stagnation pressure of 33.4 psia. The pressure ratio was

\[
\frac{p_o}{p_{ox}} = 0.4311
\]

The corresponding Mach number for this pressure ratio, assuming isentropic flow, is \( M_E = 1.16 \). The maximum Mach number in the exit plane is \( M_E = 1 \). If a stagnation pressure drop of 18.39% occurred through the injection nozzle, the exit plane pressure ratio would be:

\[
\frac{p_o}{p_E} = 0.5283
\]

which corresponds to an exit Mach number of \( M_E = 1 \). It is
logical to assume there exists a stagnation pressure drop through the injection nozzle. This decrease is believed to be the result of the surface friction mentioned earlier. Therefore, the flow through the injection jet is modeled as steady one-dimensional flow with friction, Fanno flow (28).

I. Fanno Model

The Fanno analysis is valid if the following assumptions are valid:

1. Flow is steady and one-dimensional.
2. Flow cross sectional area of injection nozzle is constant.
3. There is no work or heat transfer.
4. There are no obstructions within the flow.
5. Stagnation pressure drop is caused from surface friction only.

The system chosen for the analysis is shown in Figure 5.1. The system is comprised of an infinite constant pressure reservoir, entrance region, one-dimensional constant area duct, and an exit plane. The region before the entrance region can be assumed to be an infinite constant pressure reservoir for the following reasons. For the maximum attainable flow rate, the velocity in the air supply pipe is approximately 60 ft/s. The pressure in the region can be assumed to be the stagnation pressure because of the low pipe Mach numbers, i.e., the pressure
Infinite Reservoir = Constant Entrance to Inlet

Figure 5.1. Cross Sectional Diagram of the Injection Nozzle. a. Actual, b. Fanno Model.
ratio is approximately one. In addition, the stagnation pressure will not decrease with time.

The flow in the entrance region to the inlet is assumed to be isentropic. The flow will accelerate isentropically from the reservoir to the injection nozzle entrance. The before mentioned assumption avoids empirical correlations for losses due to sharp contractions in flows.

The major axis is in the spanwise direction, and the minor axis is in the Y direction. The ratio of the major to minor axis of the test sections are 43 and 86. For both test sections, the minor axis is much smaller than the major axis. The frictional effects will be the most dominant along the minor axis. The flow behavior along the major axis will be neglected. The flow velocity along the minor axis will be assumed to be constant. This assumption is reasonable if one takes into consideration the turbulent nature of the flow and the high Reynolds numbers which tend to flatten out a velocity profile.

II. Flow Process

A brief explanation of the flow process will now be presented. Figure 5.2 shows a temperature entropy diagram for several different stagnation pressures. The flow at the exit plan can be characterized either of three ways: subsonic, choked, or underexpanded.
Figure 5.2. Temperature Entropy Diagram to Show the Fanno Lines for Different Stagnation Pressures.
The subsonic case is shown on Fanno lines 1 and 2 in Figure 5.2. For a given stagnation pressure, the flow expands isentropically to the inlet. Because of the friction, the flow accelerates up to the exit. Since the flow is subsonic, a shock wave or expansion wave can not exist at the exit or in the duct. Therefore, the static pressure at the exit must equal the ambient pressure. Notice that as the stagnation pressure, \( P_{ox} \), increases, the pressure and temperature ratios decrease meaning the exit Mach number is increasing.

Fanno line 3 on Figure 5.2 shows the sonic case. The stagnation pressure has increased enough so that at the exit sonic conditions are achieved. The stagnation pressure no longer has an effect on the exit Mach number.

The underexpanded case is shown on Fanno line 4, Figure 5.2. Because the stagnation pressure exceeds the necessary pressure to choke the injection nozzles, the static pressure of the flow in the injection nozzle exit plane is greater than the ambient pressure. The flow will expand to the ambient pressure by passing through expansion waves. During the expansion, the flow will turn outward from the jet centerline. The normal component of the velocity at the centerline of the jet must be zero because of the jet symmetry. The flow will be parallel to the jet centerline. The flow will turn down parallel to the centerline after passing through another expansion wave. The static pressure of the flow after passing through the second
expansion wave is lower than the ambient pressure. The expansion wave will reflect from the boundary in the form of an oblique shock to raise the static pressure of the flow to the ambient pressure. After passing through the oblique shock wave, the flow will turn inward. The flow will turn back parallel to the jet centerline by passing through a second oblique shock wave. The static pressure of the flow is now greater than the ambient pressure. The entire process mentioned above will repeat itself until the viscous mixing at the jet boundary dissipates the wave phenomena. Figure 5.3 shows a schematic drawing of the wave phenomena. Figure 5.4 is a schlieren photograph showing the wave phenomena. The waves are present only in the first 0.5 inches of the flowfield.

In reality, there probably exists a more complicated flow process at the exit plane. For instance, a shock wave may be present at the centerline of the jet due to the two impinging jets. If the wave phenomena is described by Prandtl Meyer expansion waves and oblique shock waves, the Mach number of the flow downstream of the exit can be calculated. For a stagnation pressure of 30 psig, the maximum Mach number downstream of the exit is 1.44.

III. Determination of Entrance and Exit Mach Numbers

In this section the relationship used to determine the inlet and exit Mach numbers of the injection jet will be
Figure 5.3. Diagram to Show Wave Phenomena for Underexpanded Flow.
Figure 5.4. Schlieren Photograph of the Flowfield. 
$P_{ox} = 45$ psig, $\beta = 30^\circ$, $d_s = 0.0$. 
presented. The solution is not explicit for subsonic exit Mach numbers. An iterative solution technique was used to determine the Mach numbers. The criteria for the solution depends on the stagnation pressure at the exit plane of the injection jet. If the injection nozzle is choked, the Mach numbers can be determined directly. The equations used in this section are taken from John (28) and Zucrow (29).

The stagnation pressure ratio between any two points on a Fanno line is given by

$$\frac{p_2}{p_1} = \frac{M_1}{M_2} \left[ \frac{1 + \frac{y-1}{2} M_2^2}{1 + \frac{y-1}{2} M_1^2} \right]^{\frac{y+1}{2(y-1)}}$$

(5.1)

Point one "1" in Equation (5.1) can be referenced as the sonic point, $M = 1$. This casts the equation in a more convenient form. Denoting sonic conditions by an asterisk (*) Equation (5.1) now becomes.

$$\frac{p}{p^*} = \frac{1}{M} \left[ \left( \frac{2}{y+1} \right) \left( 1 + \frac{y-1}{2} M^2 \right) \right]^{\frac{y+1}{2(y-1)}}$$

(5.2)

The stagnation pressure at the exit plane must somehow be formulated as a function of the inlet stagnation pressure. The following equation results from the definition of stagnation pressure.

$$P_E = \left( \frac{P_E}{P^*} \right) \left( \frac{P^*}{P_I} \right) P_I$$

(5.3)

Since the entrance region losses are neglected, the inlet stagnation pressure $P_I$ is equal to the test section absolute stagnation pressure, $Pox$ (psia). The Mach number
of the flow is implicitly given by
\[
\frac{4f_l^*}{h} = \frac{1 - M^2}{\gamma M^2} + \left( \frac{\gamma + 1}{2\gamma} \right) \ln \left[ M^2 \left\{ \left( \frac{2}{\gamma + 1} \right) \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \right\}^{-1} \right]
\]
if the \(4f_l/h\) parameter is known. The \(4f_l/h\) parameter can be represented in the following form
\[
\left. \frac{4f_l}{h} \right|_{IN} = \left. \frac{4f_l^*}{h} \right|_{\Gamma} - \left. \frac{4f_l^*}{h} \right|_{E}
\]
The term on the left of the equality in Equation (5.5) is evaluated using the geometry of the injection nozzle. The average friction coefficient \(f\) is a function of the relative roughness of the injection nozzle and the Reynolds number. The relative roughness, \(\varepsilon/h\), depends on the equivalent sand grain diameter, \(\varepsilon\), of the material and the injection jet height, \(h\). Assuming the material in the injection nozzle is commercial steel and with the injection nozzle height \(h = 0.032\) in, the relative roughness is 0.05625. The average friction coefficient can be treated as a constant for large values of Reynolds number, \(Re > 8 \times 10^5\) and for \(\varepsilon/h > 0.004\). The average friction coefficient for this analysis was taken to be 0.02. The value was obtained directly from a Moody chart (29).

The first step in the solution involves guessing an inlet Mach number. The exit Mach number is obtained using Equations (5.4) and (5.5). The \(4f_l/h\) parameter for the exit is determined using (5.6)
\[
\left. \frac{4f_l^*}{h} \right|_{E} = \left. \frac{4f_l^*}{h} \right|_{\Gamma} - \left. \frac{4f_l}{h} \right|_{IN}
\]
Next, the exit Mach number can be implicitly determined using the result from Equation (5.6) in Equation (5.4). Knowing the inlet and exit Mach numbers the stagnation pressure at the exit plane can be determined. Implementing Equations (5.2) and (5.3) the following relationship is obtained

\[
P_E = p_\infty \frac{M_I}{M_E} \left[ \frac{1 + \frac{\gamma - 1}{2} M_I^2}{\frac{\gamma + 1}{2(\gamma - 1)}} \right]
\]

By using a value of 1.4 for the ratio of specific heat in Equation (5.7) gives

\[
P_E = p_\infty \frac{M_I}{M_E} \left[ \frac{1 + 0.2 M_I^2}{1 + 0.2 M_I^2} \right]^3
\]

The following relationship can be used to relate the ambient and stagnation pressure

\[
\frac{p_\infty}{P_E} = \left( 1 + \frac{\gamma - 1}{2} M_E^2 \right)^{-\gamma - 1}
\]

For exit Mach numbers up to choked conditions, the static exit pressure is the ambient pressure. Equation (5.9) can be rewritten as

\[
P_E = p_\infty \left( 1 + \frac{\gamma - 1}{2} M_E^2 \right)^{-\gamma - 1}
\]

Again, using 1.4 as the ratio of specific heat, Equation (5.10) becomes

\[
P_E = p_\infty \left( 1 + 0.2 M_E^2 \right)^{3.5}
\]
The criteria of the solution is satisfied if the results from Equation (5.8) and (5.11) agree. In the case the results do not agree, a new inlet Mach number is assigned and the entire process is repeated until the results converge. A computer program was written to determine the inlet and exit Mach numbers. A listing of the program along with a flowchart for this process is presented in Appendix A.

IV. Determination of Injection Nozzle Exit Velocity

The mass flow rate at the exit is given by

\[ \dot{m} = \rho_E A_E V_E \]  

(5.12)

The mass flow rate of the system can be obtained knowing the manometer deflection. The exit velocity can be determined knowing the exit density and exit area

\[ V_E = \frac{\dot{m}}{\rho_E A_E} \]  

(5.13)

The exit area is the total of the two injection nozzles.

The exit density can be written as a function of the static pressure and static temperature using the ideal gas equation

\[ \rho_E = \frac{p_E}{R t_E} \]  

(5.14)

The ratio of static to stagnation temperature for isentropic flow is given by
Using Equation (5.15) and (5.9), Equation (5.14) can be written as

$$\frac{t_E}{T_o} = \left(1 + \frac{\gamma - 1}{2} M_E^2\right)^{-1}$$

Writing the exit stagnation pressure as a function of the inlet and exit Mach numbers from Equation (5.8), Equation (5.16) becomes

$$\rho_E = \frac{P_E}{RT_o} \left(1 + \frac{\gamma - 1}{2} M_E^2\right)^{-1}$$

Substituting in a R value for air and manipulating the units to the desired form, Equation (5.17) becomes

$$\rho_E = \frac{M_I}{M_E} \frac{P_{ox}}{RT_o} \frac{(1 + 0.2 M_I^2)^3}{(1 + 0.2 M_E^2)^2}$$

The total area of the injection jet is given by

$$A_E = 2hw$$

Inserting Equations (5.18) and (5.19) into (5.13) gives

$$V_E = 26.67 \frac{\dot{m}}{h_w M_I} \frac{M_E}{P_{ox}} \frac{(1 + 0.2 M_I^2)^3}{(1 + 0.2 M_E^2)^2}$$

where: $h$(in), $w$(in), $V_E$(ft/s), $\dot{m}$(1bm/s), $P_{ox}$(psia).

Inserting the expression for the mass flow rate, Equation (3.8) into Equation (5.20) gives

$$V_E = \frac{2.99 (h_m (50 + p_{ox}) (T_o + 460)^\delta)}{h WP_{ox}} \frac{M_E (1 + 0.2 M_E^2)^3}{M_I (1 + 0.2 M_I^2)^3}$$

Equation (5.21) represents the exit velocity parallel to
the center line of the injection nozzle. The variation of Mach numbers and exit velocity as a function of stagnation pressure can be seen in Table 5.1.

Table 5.1 The Variation of Inlet and Exit Mach Numbers and Exit Velocity With Stagnation Pressures

<table>
<thead>
<tr>
<th>Pox (psig)</th>
<th>M_I</th>
<th>M_E</th>
<th>V_E (ft/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.4</td>
<td>0.477</td>
<td>644.85</td>
</tr>
<tr>
<td>10</td>
<td>0.474</td>
<td>0.672</td>
<td>876.64</td>
</tr>
<tr>
<td>15</td>
<td>0.497</td>
<td>0.82</td>
<td>1029.31</td>
</tr>
<tr>
<td>20</td>
<td>0.503</td>
<td>1</td>
<td>1241.84</td>
</tr>
</tbody>
</table>
CHAPTER VI

RESULTS

The results associated with the experimental investigation of the monofilament fiber drag will be presented first. Results of the flowfield investigation will be presented next. Following the results of both investigations, the empirical correlations relating the fiber drag as a function of mass flow rate, fiber length, etc., will be presented.

I. Drag Investigation

The variance in drag with fiber length and stagnation pressure is shown in Figure 6.1. The local maximum which occurs for fiber lengths around 1 inch prompted additional tests to generate more data points to verify the existence of this phenomena. Figure 6.2 shows the results of the additional tests. The magnitude of the local maximum increases with an increase in stagnation pressure.

If the fiber is held by hand, instead of running through the tensiometer, the local maximum mentioned above can be felt. The fiber was observed to vibrate with larger amplitudes for shorter fiber lengths (L < 1.5 inches). The tip of the fiber could actually be seen leaving the flowfield and pointing in the opposite direction. As the
Figure 6.1. The Variation of Drag With Fiber Length. $\beta = 30^\circ$. 

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>$P_{OX}$ (psia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>□</td>
<td>24.27</td>
</tr>
<tr>
<td>□</td>
<td>34.42</td>
</tr>
<tr>
<td>△</td>
<td>44.42</td>
</tr>
<tr>
<td>O</td>
<td>49.27</td>
</tr>
</tbody>
</table>
Figure 6.2. The Variation of Drag with Fiber Length. $\beta = 30^\circ$. 

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>$P_{Ox}$ (psia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle$</td>
<td>30</td>
</tr>
<tr>
<td>$\bullet$</td>
<td>40</td>
</tr>
</tbody>
</table>
fiber length was decreased further, the fiber would collide with the face plate of the test section.

One of the most promising results found in the investigation was the effect of the air injection angle on the fiber drag. Figure 6.3, 6.4, and 6.5 show how the fiber drag varies with fiber length and stagnation pressure for injection angles of 15°, 30°, and 45° respectively.

Two important features can be seen in these plots. First, the slope dD/dPox increases with filament length. Secondly, the drag increases with a decrease in the flow injection angle. From a momentum analysis view point, the drag should be proportional to the velocity component in the axial direction parallel to the fiber. As the injection angle decreases, the axial component of the velocity increases.

Initially, the axial component of the velocity is the product of the injection velocity and the cosine of the injection angle. Figures 6.6 and 6.7 show how drag varies with injection angle for two different fiber lengths. The largest drag forces were achieved using the 15° injection angle and the smallest with the 45° injection angle.

The data presented in Figures 6.1 through 6.7 were obtained using test section "A." For the remaining results, test section "B" was used to obtain the experimental data. Figure 6.8 shows a comparison of the two different test sections under the same flow and
Figure 6.3. The Effects of Stagnation Pressure on Fiber Drag. \( \beta = 15^\circ \), \( d_s \) 0.0.
Figure 6.4. The Effects of Stagnation Pressure on Fiber Drag. $\beta = 30^\circ$, $d_s = 0.0$. 

The table shows the SYMBOL and their corresponding $L$ in inches:

- Triangle (△): 2 inches
- Circle (○): 4 inches
- Circle (○): 6 inches
- Circle (○): 8 inches
- Circle (○): 10 inches

The graph plots $D \times 10^3$ (lb) against $P_{ox}$ (psia) for different values of $L$.
Figure 6.5. The Effects of Stagnation Pressure on Fiber Drag. $\beta = 45^\circ$, $d_s = 0.0$. 

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>L (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>△</td>
<td>2</td>
</tr>
<tr>
<td>○</td>
<td>4</td>
</tr>
<tr>
<td>□</td>
<td>6</td>
</tr>
<tr>
<td>○</td>
<td>8</td>
</tr>
<tr>
<td>○</td>
<td>10</td>
</tr>
</tbody>
</table>
Figure 6.6. The Variation of Drag With Injection Angle.
\( d_s = 0.0, L = 4 \) inches.
Figure 6.7. The Variation of Drag With Injection Angle.
\[ d_s = 0.0, \quad L = 8 \text{ inches.} \]
Figure 6.8. Comparison Between the Experimental Test Sections. $L = 10$ inches, $\beta = 30^\circ$. 
geometric conditions. There seems to be good agreement between the two different test sections.

The effect of die setback can be seen in Figure 6.9. For large positive or negative setbacks, the fiber drag increases. This effect is reduced with a decrease in stagnation pressure. The exit area was held constant for positive and negative setbacks. The injection nozzle slot height was constant for negative setbacks; however, the slot height increased for positive setbacks.

The effect of changing the test section orientation with respect to gravity can be seen in Figure 6.10. Changing the orientation has no measurable effect on the fiber drag. The viscous and pressure forces far exceed the gravitational forces for the flow conditions which were investigated. It is anticipated that the gravitational forces would be important with low air velocities.

The variation of fiber drag with the calculated injection nozzle exit velocity can be seen in Figure 6.11. The injection nozzle exit velocity does not act along the entire length of the fiber due to the velocity decay in the free jet. Once the injection nozzle chokes, the exit velocity is constant.

The effect of changing the injection nozzle slot height can be seen in Figure 6.12. The mass flow rate must be increased in order to maintain a constant stagnation pressure with an increase in slot height. The fiber drag increases with an increase in mass flow rate. This can be
The Effect of Setback on Fiber Drag. $L = 10$ inches, $\beta = 30^\circ$. 

Figure 6.9.
Figure 6.10. The Variation of Drag With Test Section Orientation.
Figure 6.11. Drag As A Function of Exit Velocity.
Figure 6.12. Drag As A Function of Injection Nozzle Slot Height for Different Stagnation Pressures.
seen in Figure 6.13. There exist a difference in the
classical slopes of the curves between regions A and
B. The transitions, or inflection point, occurs where the
nozzle chokes. Also, as previously mentioned, the
magnitude of the slopes vary for different fiber lengths.

II. Flowfield Investigation

Velocity profiles measured at several positions are
presented in Figure 6.14. Notice how the centerline
velocity decreases and the jet boundary spreads as the
axial position increases. Figure 6.15 shows the jet
boundary as a function of axial position. The jet boundary
was defined to be where the velocity was 10% of the
centerline velocity at a given axial position. As can be
seen in the plot, there is good agreement between the
experimental data and the classical solution of the
boundary for a plane turbulent free jet. The measured half
angle from Figure 6.15 is 13.13°. The classical solution
is 13 degrees.

The experimental data presented in Figure 6.14 is shown
in nondimensional form on Figure 6.16. Figure 6.16 shows
a comparison between the classical free jet solution and
experimental data. The experimental data is in good
agreement with the classical solution. The effect of
increasing the stagnation pressure is presented in Figure
<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>L (in)</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle$</td>
<td>2</td>
<td>$30^\circ$</td>
</tr>
<tr>
<td>$\diamond$</td>
<td>10</td>
<td>$30^\circ$</td>
</tr>
<tr>
<td>$\circ$</td>
<td>10</td>
<td>$15^\circ$</td>
</tr>
</tbody>
</table>

Figure 6.13. The Effect of Nozzle Mass Flow Rate on Fiber Drag.
Figure 6.14. Velocity Profiles Taken At Several Axial Positions. $P_{ox} = 10$ psig, $\beta = 30^\circ$, $d_s = -0.2$ inches.
Figure 6.15. Comparison of Classical Solution and Experimental Jet Boundary.
Figure 6.16. Comparison of Classical Free Jet Velocity Solution With Experimental Results.
6.17. The jet boundary stays at the same position. This was validated by measurements and smoke visualization.

The remainder of the results focuses on the entrained flow. The entrained flow which depends on stagnation pressure accounts for between 80% and 90% of the total flow rate at a location of 10 inches from the die as shown in Figure 6.18. The jet characteristics also seem to be independent of die setback.

The same mass flow ratios presented in Figure 6.18 can be seen as a function of axial location in Figure 6.19. The percentage of the flow that is entrained increases with axial displacement. These entrainment results fall within the limits given by Sforza et al. (4). A portion of the momentum issuing from the jet exit is used to entrain the ambient air. This entrainment is a result of the viscous interaction between the high speed flow in the jet and the surrounding air.

Since the results from the experimental flowfield investigation agreed well with the classical solution, the total mass flow ratio between any two axial positions was determined using the classical solution and was compared with the experimental data. Recalling Equation (4.4), the volumetric flow rate is given by

\[ Q = 2\Delta W \int_0^b UdY \]  

(6.1)

The classical solution for a free turbulent jet (24) is given by
Figure 6.17. Velocity Profiles for Several Stagnation Pressures. $\beta = 30^\circ$, $X = 10$ inches, $d_s = 0.2$ inches.
Figure 6.18. The Variation of Mass Flow Ratios With Stagnation Pressure. $X = 10$ inches, $\beta = 30^\circ$. 
Figure 6.19. The Variation of Mass Flow Ratios With Axial Displacement. $\beta = 30^\circ$, $P_{ox} = 10$ psig.
\[ \frac{U}{U_{\text{max}}} = \text{sech}^2(\eta) \]  

(6.2)

where

\[ \eta = 7.67 \frac{Y}{X} \]  

(6.3)

For simplicity, the integral will be written in one variable, \( \eta \). The differential can be rewritten as

\[ dY = \frac{X}{7.67} \, d\eta \]  

(6.4)

Next, the limits are transformed. For \( y = b \),

\[ \eta = 7.67 \frac{b}{X} \]  

(6.5)

Equation (6.5) can be simplified knowing the relationship between the jet half width and the spread angle given by

\[ \frac{b}{X} = \tan(13^\circ) \]  

(6.6)

Equation (6.5) now becomes

\[ \eta = 1.7708 \]  

(6.7)

Equation (6.1) now becomes

\[ Q = \frac{2\Delta WX U_{\text{max}}}{7.67} \int_{0}^{1.7708} \text{sech}^2(\eta) \, d\eta \]  

(6.8)

The above integral can be evaluated using a standard mathematics table (30)

\[ \int \text{sech}^2(\eta) \, d\eta = \tanh(\eta) \]  

(6.9)

Equation (6.8) can finally be written as

\[ Q = \frac{2\Delta WX U_{\text{max}}}{7.67} \left( \tanh(1.7708) - \tanh(\omega) \right) \]  

(6.10)

After evaluating the hyperbolic tangent in the above equation, Equation (6.10) becomes

\[ Q = 0.2461 \Delta WX U_{\text{max}} \]  

(6.11)

If \( W \) and \( X \) are to be inputed in inches, Equation (6.11)
becomes

\[ Q = 0.00171 \Delta W X U_{max} \]  \hspace{1cm} (6.12)

The total mass flow rate is given by

\[ \dot{m} = \rho \dot{Q} \]  \hspace{1cm} (6.13)

The total mass flow ratio between any two axial positions is given by

\[ \Psi(X_1, X_2) = \frac{\dot{m}_{X = X_1}}{\dot{m}_{X = X_2}} \]  \hspace{1cm} (6.14)

Using Equation (6.12) and (6.13), Equation (6.14) becomes

\[ \Psi(X_1, X_2) = \frac{X_1 U_{max}}{X_2 U_{max}} \]  \hspace{1cm} (6.15)

The same mass flow ratio can be determined directly from Figure 6.19,

\[ \Psi(X_1, X_2) = \frac{\dot{m}_{IN}}{\dot{m}_{IN}} \]  \hspace{1cm} (6.16)

A comparison between Equations (6.15) and (6.16) is shown in Table 6.1. The values used in Equation (6.15) were obtained directly from Figures 6.14 and 6.17.

<table>
<thead>
<tr>
<th>X (in)</th>
<th>( \Psi(8, X) ) Classical Solution</th>
<th>Experimental</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.9636</td>
<td>0.9333</td>
<td>3.17</td>
</tr>
<tr>
<td>15</td>
<td>0.7852</td>
<td>0.8571</td>
<td>9.15</td>
</tr>
<tr>
<td>26</td>
<td>0.6795</td>
<td>0.6426</td>
<td>5.43</td>
</tr>
<tr>
<td>36</td>
<td>0.5889</td>
<td>0.5000</td>
<td>15.09</td>
</tr>
</tbody>
</table>
The results of this portion of the investigation indicate that the entrainment of the ambient air is the principal factor effecting the average air flowfield temperature in a typical meltblown process.

III. Empirical Correlations

A curve was fitted to the experimental data presented in Figures 6.3, 6.4, and 6.5. From the results, the drag was expected to be a function of stagnation pressure, injection angle, and fiber length. A power curve fit was used to determine the drag in functional form of the previously mentioned variables. The following relationship was used

\[ D = \delta \cos(\beta) P_{ox}^k \]  

(6.17)

The coefficients \( \delta \) and \( k \) were determined. Equation (6.17) is given by

\[ D = (2.2 \times 10^{-5} L + 2.64 \times 10^{-4}) \cos(\beta) P_{ox}^{0.8231} \]  

(6.18)

where: \( D \) (lbs), \( L \) (inches), \( P_{ox} \) (psig), \( \beta \) (degrees). The fiber length appeared in the proportionality term. Equation (6.18) fits the data well except for a few isolated cases. The maximum error is 20%. This error occurred for a two inch fiber length with a 45° injection angle.

The slope of Equation (6.18) is given by

\[ \frac{dD}{dP_{ox}} = (1.81 \times 10^{-5} L + 2.17 \times 10^{-4}) \cos(\beta) P_{ox}^{-0.177} \]  

(6.19)
The local value of slope decreases with an increase in stagnation pressure for a fixed length. The slope increases with length, and the slope decreases with an increase in injection angle. The slope behavior can be seen in the figures.

An analytical expression was also developed to model the fiber drag using momentum considerations. Figure 6.20 is a plot to show how the fiber drag varies with G for a fixed fiber length of 2 inches. G is the flow momentum at the exit plane of the injection nozzles. The exit density was calculated using the Fanno model discussed in Chapter 5. Figure 6.21 is a summary to show how the fiber drag varies with G of all of the lengths used. These are linear curve fits of the data like shown in Figure 6.20.

By analyzing Figures 6.20 and 6.21, the fiber drag can be recognized to be a linear function of the momentum or

\[ D \sim G \] (6.20)

The drag was modeled using the final form

\[ D = \delta \cos(\beta) \frac{G}{g_c} \] (6.21)

Using the experimental data and computer output, the proportionality constant was determined. The final form of Equation (6.21) is given by

\[ D = (4.34 \times 10^{-6} L + 4.18 \times 10^{-5} \cos(\beta)) G \] (6.22)

where: D(lbs), L(inches), G(lbm - ft/s²).

The maximum error using Equation (6.22) is 30%.

Instead of having isolated maximums, as in Equation (6.18),
Figure 6.20. Drag As A Function of the Axial Momentum Issuing From The Injection Nozzles. $\beta = 30^\circ$, $L = 2$ in.
Figure 6.21. A Summary of the Linear Curve Fits for Various Lengths.
large errors are distributed throughout the investigated region. The accuracy of Equation (6.22) is somewhat reduced due to the analytical method used to calculate the exit density.

Equations (6.18) and (6.22) are valid for a particular injection nozzle slot height, fiber diameter, and setback. These parameters were as follows: \( h = 0.032 \) inches, \( d = 0.005 \), \( W = 1.36 \) inches and \( d_s = 0 \).

IV. Conclusions

Several reasonable conclusions can be made from the results presented in this investigation. The conclusion concerning the drag investigation and flowfield investigation are as follows:

1. The drag increased with a decrease in injection angle with no change in the other parameters.
2. The drag increased with an increase in stagnation pressure with no change in the other parameters except for the associated increase in the mass flow rate.
3. A local maximum drag phenomena occurred for particular short filament lengths.
4. The drag increased with an increase in filament length with no change in the other parameters.
5. For the velocities of interest in meltblown applications, the orientations of the die,
respect to gravity, has no significant effect on drag.

6. Die setback, both positive and negative, appear to slightly increase drag.

7. The flowfield associated with the meltblown process is very similar to the classical free jet flowfield.

8. A supersonic flowfield extends a short distance from the injection nozzles when operating at high stagnation pressures.

9. The injection air entrains a large amount of ambient air, up to four or five times the injected mass flow rate.

V. Recommendations

Several areas still need to be investigated thoroughly to completely understand the fiber drag associated with the meltblown process. The areas to be investigated are as follows:

1. The effect of the injection nozzle entrance geometry on fiber drag. The entrance region would be rounded as modeled in Chapter 5. Theoretically, this would reduce the stagnation pressure loss through the entrance region.

2. Supersonic air injection velocities would be used to pull the fiber. This would be accomplished by
using a converging diverging injection nozzle. This research would reveal how supersonic exit velocities affect the fiber drag.

3. A numerical solution of the momentum equations would be performed. The equations would be modeled for a cylinder in a free jet flowfield. Once the velocity distribution was known, the cylinder drag could be determined. There would be two equations with two unknowns, mainly the X momentum equation and continuity equation, and U and V velocity components. The problem is simplified greatly by uncoupling the equations from the energy equation.

4. A thorough flowfield investigation would be conducted using an actual meltblown facility. The flowfield would be mapped out. In addition, infrared photography could be used to evaluate how the gas temperature varies in the flowfield.
BIBLIOGRAPHY


A brief description on how the computer program determines the injection nozzle exit velocity will be presented. A numerical scheme was implemented to determine the solution for the inlet and exit Mach numbers because there is no explicit solution.

The solution to the problem was obtained when the results from Equations (5.8) and (5.11) agreed within a predetermined tolerance. A bisectional search method was used to determine the solution. The method works in the following way. The inlet Mach number is incremented in large steps until a sign change is detected in the residual. The residual is the difference between Equations (5.8) and (5.11). Because a sign change occurred, there must exist a zero for the residual. The program then assigns new limits for the search and a smaller stepping increment. This process continues until the residual is within the predetermined tolerance; in other words, the results converge to an acceptable error.

A more exact solution could be determined if the convergence criteria was adjusted for smaller errors. This resulted in more computational time. If the pressures cannot be accurately measured below 0.2 psi, there is no point in setting the tolerance to 0.00001 psi. This applies directly to the Mach numbers when searching for the
solution. For instance, there is negligible difference between an inlet Mach number of 0.43 and 0.43001.

The same bisectional search method mentioned above was implemented when solving for the Mach number from Equation (5.4). The equations used in the program are the same as the equations presented in Chapter 5. A flowchart of the computer program is shown in Figure A.1 page 109. The program was written in BASIC language. The actual computer program follows the presentation of the symbols used in the computer code.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A simple Mach relation using M, ME</td>
</tr>
<tr>
<td>AT</td>
<td>Total exit area of the injection nozzles</td>
</tr>
<tr>
<td>B</td>
<td>A simple Mach relation using M, ME</td>
</tr>
<tr>
<td>C</td>
<td>A simple Mach relation using M, ME</td>
</tr>
<tr>
<td>CD</td>
<td>Drag coefficient of the fiber based on exit conditions</td>
</tr>
<tr>
<td>CHK, CHKI</td>
<td>Parameters used to detect a change in the sign of the residuals (RES, RESI)</td>
</tr>
<tr>
<td>COUNT</td>
<td>A counter used for the number of data used</td>
</tr>
<tr>
<td>DEL, DELI</td>
<td>Increments used in the bisectional search</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>DELTA</td>
<td>Correlation constant used in the momentum analysis</td>
</tr>
<tr>
<td>DIAM</td>
<td>Diameter of the test fiber</td>
</tr>
<tr>
<td>DR</td>
<td>The fiber drag</td>
</tr>
<tr>
<td>F</td>
<td>The term given by Equation (5.4)</td>
</tr>
<tr>
<td>FLAG</td>
<td>A condition to signify subsonic or sonic exit conditions</td>
</tr>
<tr>
<td>FLD</td>
<td>A parameter used in the Fanno analysis</td>
</tr>
<tr>
<td>F1</td>
<td>The term determined by the difference of F and FLD</td>
</tr>
<tr>
<td>F2</td>
<td>The term given by Equation (5.4)</td>
</tr>
<tr>
<td>G</td>
<td>An exit momentum flux term</td>
</tr>
<tr>
<td>H</td>
<td>The injection nozzle slot height</td>
</tr>
<tr>
<td>HW</td>
<td>Manometer deflection</td>
</tr>
<tr>
<td>L</td>
<td>The fiber length</td>
</tr>
<tr>
<td>M</td>
<td>Entrance Mach number</td>
</tr>
<tr>
<td>MDOT</td>
<td>Mass flow rate</td>
</tr>
<tr>
<td>MDOTJ</td>
<td>Mass flow rate per unit exit area</td>
</tr>
<tr>
<td>ME</td>
<td>Exit Mach number</td>
</tr>
<tr>
<td>M1, M2</td>
<td>Numerical limits for Mach number when determining the solution for exit Mach number</td>
</tr>
<tr>
<td>M3, M4</td>
<td>Numerical limits for Mach number when determining the solution for inlet Mach number</td>
</tr>
<tr>
<td>PAMB</td>
<td>Ambient pressure</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>PHRO</td>
<td>Density of the air at the injection nozzle exit</td>
</tr>
<tr>
<td>POX</td>
<td>Stagnation pressure at the test section</td>
</tr>
<tr>
<td>PSTAR</td>
<td>The exit stagnation pressure for choked conditions (ME = 1)</td>
</tr>
<tr>
<td>PST2</td>
<td>Equation (5.11)</td>
</tr>
<tr>
<td>PST3</td>
<td>Equation (5.8)</td>
</tr>
<tr>
<td>Q</td>
<td>The flow injection angle relative to the X axis</td>
</tr>
<tr>
<td>REA</td>
<td>Reynolds number based on exit conditions for a given fiber radius</td>
</tr>
<tr>
<td>REH</td>
<td>Reynolds number based on exit conditions for a given injection nozzle slot height</td>
</tr>
<tr>
<td>REL</td>
<td>Reynolds number based on exit conditions for a given fiber length</td>
</tr>
<tr>
<td>RES, RESI</td>
<td>The residuals used in determining the solution</td>
</tr>
<tr>
<td>SR</td>
<td>Tensiometer scale reading</td>
</tr>
<tr>
<td>TH</td>
<td>Axial thickness of the injection nozzle</td>
</tr>
<tr>
<td>TOX, TPA</td>
<td>Stagnation temperatures</td>
</tr>
</tbody>
</table>
VE  Injection nozzle exit velocity parallel to the centerline of the injection nozzle

W  The width (Z) of the injection nozzle

Y  A counter

The listing of the computer program follows
10 REM ENTERING THE VALUES FOR THE VARIABLES
20 PRINT "ENTER DATE"
30 INPUT A$
40 PRINT "ENTER FLOW INJECTION ANGLE"
50 INPUT Q
60 PRINT "INPUT JET WIDTH, IN"
70 INPUT W
80 PRINT "INPUT JET HEIGHT, IN"
90 INPUT H
100 PRINT "INPUT AXIAL THICKNESS OF DUCT, IN"
110 INPUT TH
120 PRINT "INPUT DIAMETER OF FIBER, IN"
130 INPUT DIAM
140 PRINT "ENTER NUMBER OF DATA POINTS FOR ONE PARTICULAR LENGTH"
150 INPUT COUNT
160 PRINT "PRINT AMBIENT PRESSURE, IN HG."
170 Y=0
180 INPUT PAMB
190 PAMB=PAMB*.48131
200 PRINT "ENTER STAGNATION TEMPERATURE, F"
210 INPUT TOX
220 TPA=TOX+460
230 PRINT "ENTER FIBER LENGTH, IN"
240 INPUT L
250 LPRINT
260 LPRINT "DATA";A$
270 LPRINT
280 LPRINT "FLOW INJECTION ANGLE =";Q
290 LPRINT "AMBIENT PRESSURE =";PAMB; "PSOA"
300 LPRINT "FIBER LENGTH =";L; "IN"
310 LPRINT "STAGNATION TEMPERATURE ";TPA; "R"
320 LPRINT
330 LPRINT
340 PRINT "ENTER STAGNATION GAGE PRESSURE, PSIG"
350 INPUT POX
360 PRINT "ENTER MONOMETER DEFLECTION, IN"
370 INPUT HW
380 POX=POX+PAMB
390 PRINT "ENTER TENSIOMETER SCALE READING"
400 INPUT SR
410 Y=Y+1
420 DR=.000885*SR
430 AT=2*H*
440 REM FRICTION FACTOR USED (.08)
450 FLD=TH*.08/(H*COS(.0174533*Q))
460 REM FLD IS THE 4FL/D PARAMETER USED IN THE FANNO ANALYSIS
470 REM INITIALIZE THE FLAG PARAMETER. THE FLAG PARAMETER DIFFERENTIATES
480 REM THE DIFFERENT CHARACTERISTICS OF SUBSONIC AND CHOKED CONDITIONS.
490 FLAG=0
500 M3 = .2
**510** M4 = .6  
**520** DELI = .1  
**530** REM  
__________________________________________________________
**540** REM ENTERING THE MAIN SECTION OF THE PROGRAM  
**550** FOR M = M3 TO M4 STEP DELI  
**560** PSTAR=POX*(((1/M)*((.833 + .167 * M^2)^(3))^(1))  
**570** F1=((1-M^2)/(1.4*M^2))+.8570999*LOG(M^2*  
**570** (.833001+.167* M^2)^(1))  
**580** F1=F-FLD  
**590** IF E1<0 THEN FLAG=1  
**600** IF FLAG=0 GOTO 660  
**610** MJ=.2  
**620** M2=.6  
**630** DEL=.1  
**640** F1=FLD  
**650** GOTO 720  
**660** M1=.2  
**670** M2=1  
**680** DEL=.1  
**690** REM  
__________________________________________________________
**700** REM THIS SECTION DETERMINES THE EXIT MACH  
**710** REM NUMBER KNOWING  
**720** FOR ME = M1 TO STEP DEL  
**730** F2=((1-ME^2)/(1.4*ME^2))+.8570999*LOG(ME^2*  
**730** (.833001+.167* ME^2)^(1))  
**740** RESI=ABS (F1-F2)  
**750** CHK=F1-F2  
**760** IF CHK>0 GOTO 850  
**770** IF RESI<.001 GOTO 850  
**780** GOTO 830  
**790** M1=ME-DEL  
**800** M2 = ME  
**810** DEL=DEL/10  
**820** GOTO 720  
**830** NEXT ME  
**840** REM  
__________________________________________________________
**850** IF FLAG=0 GOTO 890  
**860** M=ME  
**870** ME=1  
**880** GOTO 1020  
**890** PRINT "RUNNING"  
**900** PST2=PAMB*(((1+.2*ME^2) 3.5)  
**910** PST3=(1/ME)*((.83300+.167*ME^2)^(3))PSTAR  
**920** RES=ABS(PST2-PST3)  
**930** CHK1 = PST2 - PST3  
**940** IF CHK1>0 GOTO 970  
**950** IF RES<.1 GOTO 1020  
**960** GOTO 1010  
**970** M3=M-DELI  
**980** M4 = M
990  DELI = DELI/10
1000  GOTO 550
1010  NEXT M
1020  MDOT = .1121*((HW*(50+PAMB)/TPA^.5)
1030  MDOTJ=MDOT*144/AT
1040  A=(1+.2*(M/2))^.5
1050  B=(1+.2*(ME/2))^.5
1060  C=ME/M
1070  PHRO=2.6997*B*POX/(C*A*TPA)
1080  VE=144*MDOT/(PHRO*AT)
1090  G=(MDOT/2)*144/(AT*PHRO)
1100  REL=967741.93#*MDOT*L/AT
1100  REA=REL*DIAM/L
1120  CD=2.6103*(SR/PHRO)*(1/(L*DIAM))*(1(VE^2))
1140  DELTA=DR*W*32.174/(COS(.0174533*Q)*MDOT*VE*DIAM)
1150  REM**********************************************
1160  REM PRINTING THE RESULTS
1170  LPRINT "MONOMETER DEFLECTION =" ;HW; "IN"
1180  LPRINT "FIBER LENGTH =" ;L; "IN"
1190  LPRINT "TENSIOMETER READING =" ;SR
1200  LPRINT "FIBER DRAG =" ;DR; "LBS"
1210  LPRINT "FIBER DRAG =" ;DR; "LBS"
1220  LPRINT "FIBER DRAG =" ;DR; "LBS"
1230  LPRINT "FIBER DRAG =" ;DR; "LBS"
1240  LPRINT "EXIT MACH NUMBER =" ;M
1250  LPRINT " Entrance Mach Number" ;M
1260  LPRINT "EXIT VELOCITY =" ;VE; "FT/S"
1270  LPRINT "MASS FLOW RATE =" ;MDOT; "LBM/S"
1280  LPRINT "MASS FLOW PER UNIT EXIT AREA =" ;MDOTJ;
1280  "LBM/SEC-FT2"
1290  LPRINT "REL =" ;REL
1290  LPRINT "REL =" ;REL
1290  LPRINT "REL =" ;REL
1300  LPRINT "REH =" ;REH
1310  LPRINT "G PARAMETER =" ;G; "LBM-FT/S2"
1320  LPRINT "EXIT DENSITY =" ;PHRO; "LBM/FT3"
1330  LPRINT "EXIT DENSITY =" ;PHRO; "LBM/FT3"
1340  LPRINT
1350  IF Y = COUNT GOTO 1370
1360  GOTO 340
1370  END
\[ Y = 0 \]
\[ PAMB = PAMB \times 0.49131 \]
\[ TPA = TOX + 460 \]

\[ Q, PAMB, L, TPA \]

\[ POX, HW, SR \]

\[ Y = Y + 1 \]

\[ DR = 0.000885 \times SR \]

\[ AT = 2 \times H \times W \]

\[ FLD = \frac{TH \times 0.08}{H \times \cos(0.0174533 \times Q)} \]

\[ M3 = 0.2 \]
\[ M4 = 0.6 \]
\[ DEL1 = 0.1 \]

Figure A.1. Computer Flowchart
Figure A.1. (Continued)
Figure A.1. (Continued)
Determine VE, G, RE

Printng Results
HW, L, SR, DR, POX
ME, M, VE, MDT
MDOTJ, REL, REA, REH
CD, G, PHRO, DELTA

Figure A.1. (Continued)
Bryan David Haynes was born in Knoxville, Tennessee on February 16, 1963. He attended elementary schools in Knox County and was graduated from Doyle High School in June 1981. The following September he entered the University of Tennessee, Knoxville, and in August 1985 he received a Bachelor of Science degree in Aerospace Engineering.

In the fall of 1985 he accepted a teaching assistantship at the University of Tennessee, Knoxville, and began study toward a Masters degree. He began working as a research assistant in January 1986. He married the former Miss Connie Jane Murrell on March 21, 1987.

The author is a member of Tau Beta Pi, Sigma Gamma Tau, Order of the Engineer, Academy of Model Aeronautics, and the American Institute of Aeronautics and Astronautics. He earned his private pilots license in November, 1980. Mr. Haynes will begin study toward a Doctor of Philosophy degree with a major in Aerospace Engineering at the University of Tennessee, Knoxville after receiving the Masters degree in August, 1987.