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J. F. Pierce, Major Professor

We have read this thesis and recommend its acceptance:

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Carolyn R. Hodges

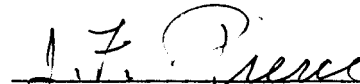
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December 1965

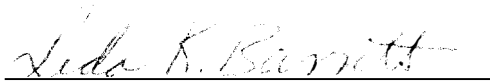
To the Graduate Council:

I am submitting herewith a thesis written by William Parker Finch entitled "Analysis of The White Emitter-Follower Amplifier". I recommend that it be accepted for nine quarter hours of credit in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.


Major Professor

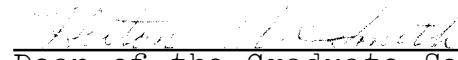
We have read this thesis and
recommend its acceptance:





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Accepted for the Council


Dean of the Graduate School

ANALYSIS OF THE WHITE EMITTER-FOLLOWER AMPLIFIER

A Thesis
Presented to
The Graduate Council of
The University of Tennessee

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

---

by
William Parker Finch
December 1965

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INTRODUCTION

A common-collector amplifier, ordinarily known as an emitter-follower, is shown in Figure 1. The desirable characteristics of this configuration are its high input impedance and its low output impedance¹. Although the voltage gain is less than unity, the circuit does have a reasonable power gain due to the high current gain of the circuit.

The applications of the emitter-follower are similar to those of the cathode-follower in vacuum-tube circuits². The emitter-follower permits the load to appear as a high impedance to the driving source so that only a small fraction of the driving voltage will be dropped across the source impedance¹. The input-signal voltage is developed across a high input impedance, whereas the output equivalent circuit has a small looking-back impedance. Thus the emitter-follower acts like an impedance transformer³. Power amplification is obtained while at the same time the impedance level is reduced.

However, the emitter-follower has inherent limitations and undesirable characteristics. The input impedance is limited to a maximum value of several hundred kilohms by the transistor and the load⁴. The output impedance is limited by the transistor to a minimum value of a few ohms⁴. The emitter-follower usually

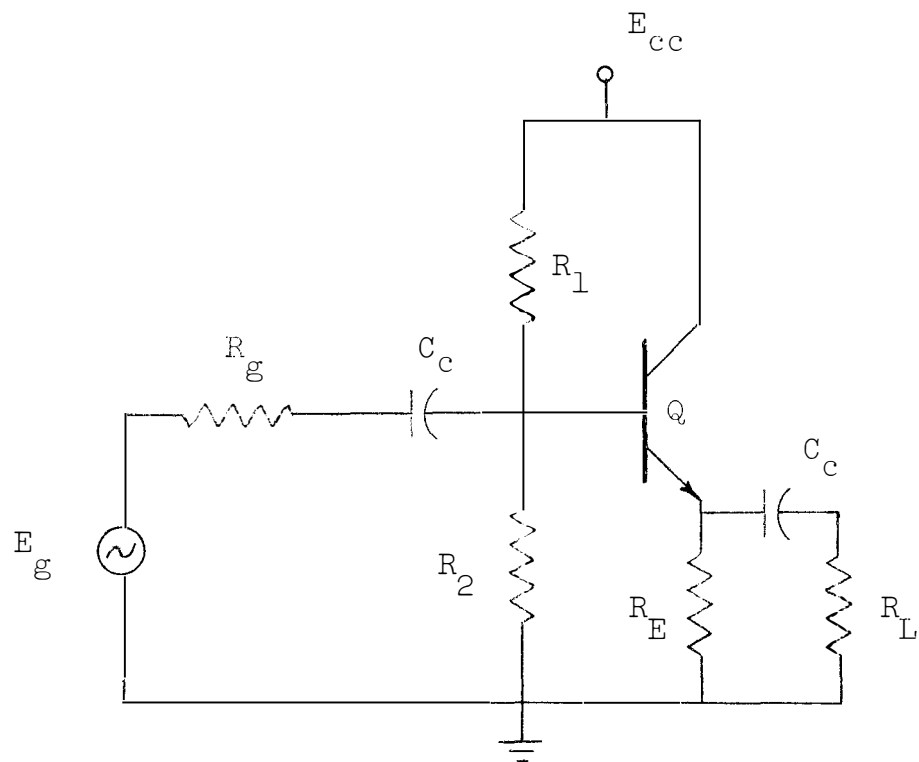


Figure 1. Circuit of The Emitter-Follower

amplifies one polarity of a bipolar pulse more than the other due to variations in the current gain (β) for large variations of current. In addition the linearity and rise time for negative pulses are different than those for positive pulses. Because of these limitations and disadvantages, it is apparent that circuits having higher input impedances, lower output impedances and better linearity for bipolar pulses would be desirable. Preliminary studies on the White emitter-follower indicated that it is better than the emitter-follower in these areas.

The White emitter-follower circuit shown in Figure 2 was named after Eric L. C. White who patented the vacuum-tube version of the circuit in 1944⁵. The ability of this circuit to amplify pulses of both polarities with nearly equal linearity and rise time is accomplished with feedback. The total feedback from the output to the emitter of Q_1 cancels out most of the input signal and leaves only a very small portion of the input signal as a driving signal for Q_1 . Effectively the feedback makes the input characteristics of Q_1 almost linear. The input impedance of the White emitter-follower is 30 times larger than that of the emitter-follower when the circuits are operated with equal load and generator impedances; the output impedance is 100 times smaller than that of the emitter-follower. When the gains of the two circuits are compared, the White

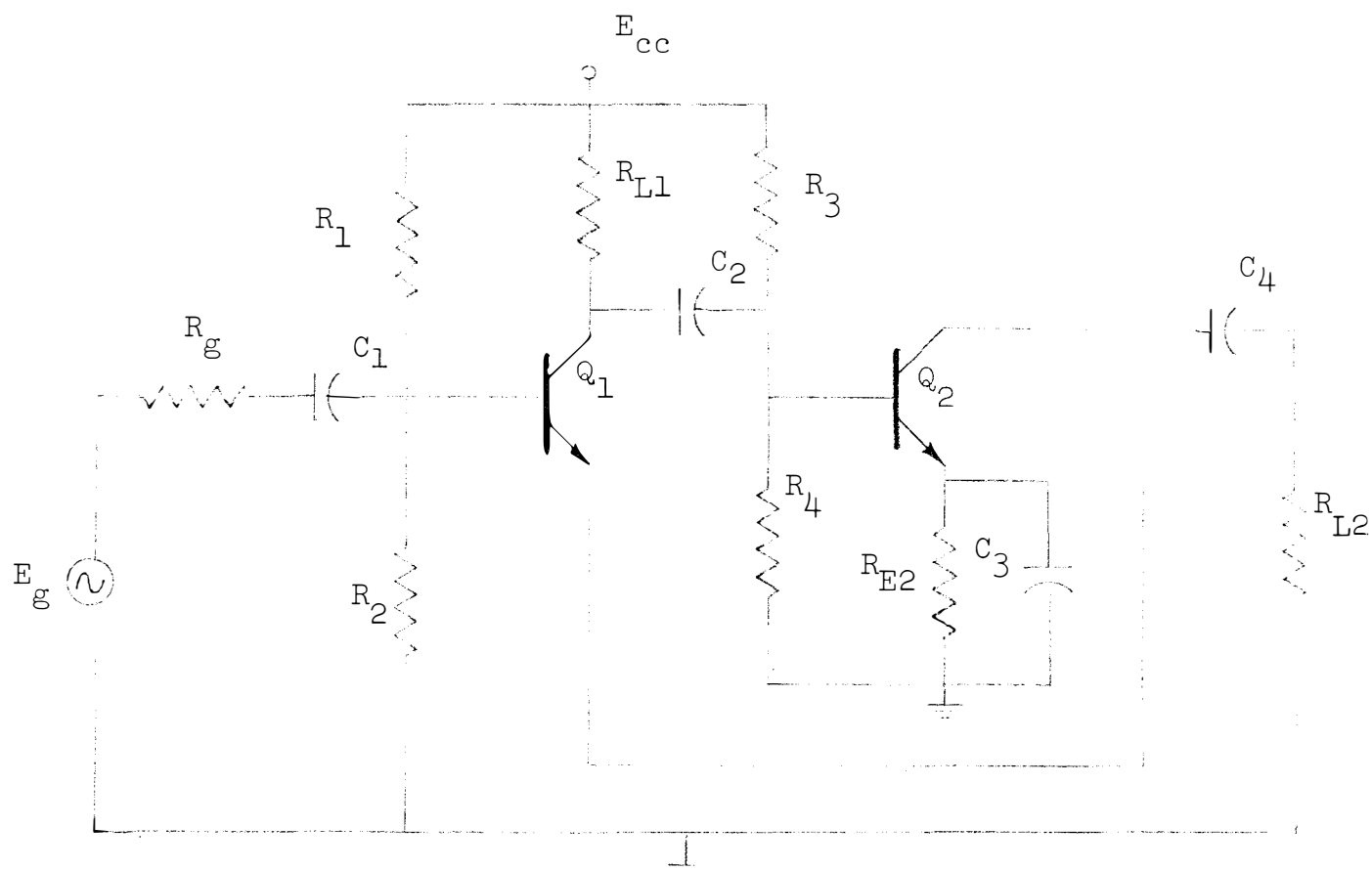


Figure 2. Circuit of The White Emitter-Follower

emitter-follower has a gain much closer to unity than the emitter-follower.

The purpose of this study was to analyze the White emitter-follower over a wide frequency range. An attempt was made to consider the effect of changing significant circuit parameters on the performance of the circuit. Calculated results are compared with experimental data.

The next section contains a discussion of the d-c and a-c operation of the circuit. First, two d-c biasing configurations are discussed, and circuit limitations imposed by the bias resistors are pointed out. The a-c operation is discussed from two points of view. The part that feedback plays in the operation of the circuit is pointed out; then an a-c signal is traced through the circuit to further clarify the operation of the circuit.

The third section contains a mid-frequency analysis of the White emitter-follower. Expressions for the gain, input impedance, and output impedance in the mid-frequency range are derived and checked experimentally. A brief comparison of the emitter-follower and White emitter-follower is made with respect to gain, input impedance, and output impedance in the mid-frequency range.

A low-frequency analysis is presented in the fourth section. The analytical methods used to obtain the calculated low-frequency response are discussed. The calculated

response is compared with an experimental response.

The fifth section contains an analysis of the transient response of the White emitter-follower. A high-frequency equivalent circuit is obtained, and the transfer function of this circuit is derived. Overshoot and rise time as a function of circuit parameters are considered. The high-frequency response of the circuit is also determined in this section.

The sixth and last section summarizes the most important results of this study and indicates the areas in which additional work can contribute useful information.

CIRCUIT OPERATION

D-C OPERATION

The d-c configuration of the White emitter-follower is unique in that the two transistors are in series. Therefore, for high beta transistors, approximately the same d-c current flows through each transistor as shown in Figure 3. The operating point is determined by specifying the collector currents and the collector-to-emitter voltages. These d-c values must be selected so that the largest a-c variation will not cause the instantaneous operating point to leave the linear region. For linear operation assume that the collector current, I_{C1} , is approximately 3 ma and the collector-to-emitter voltage, V_{CE} , for each transistor is 5.86 volts. The collector current may be expressed as

$$I_C = \alpha I_E + I_{CO} \quad . \quad (1)$$

Since the I_{CO} current for most silicon transistors is only a few nanoamps, I_{CO} may be neglected. Then Equation (1) becomes

$$I_C = \alpha I_E \quad . \quad (2)$$

Using Equation (2) the emitter current, I_{E1} , for the circuit shown in Figure 3, is equal to 3.02 ma. The emitter current, I_{E1} , and the collector current, I_{C1} , are equal. Therefore, using Equation (2) the emitter current, I_{E2} , is equal to 3.04 ma making the voltage level at the emitter of

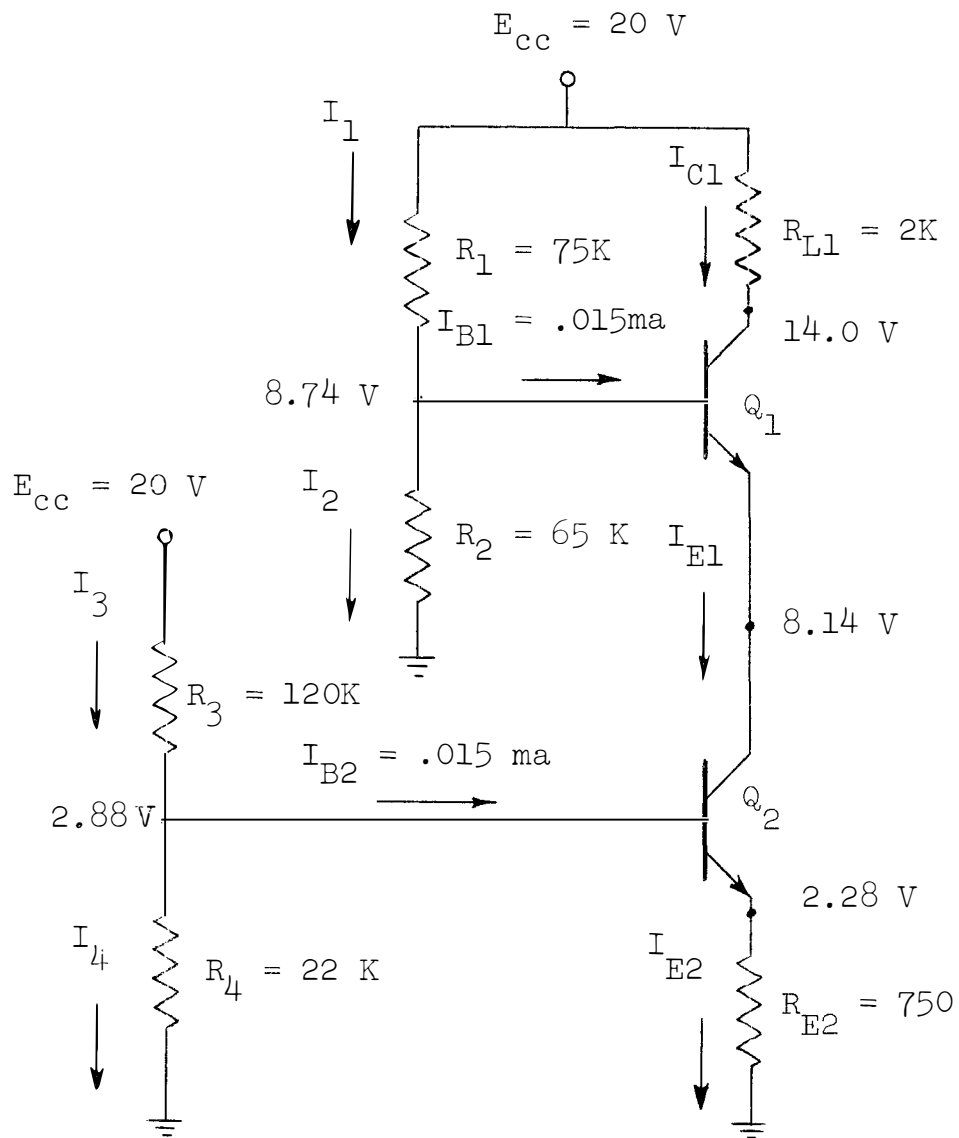


Figure 3. D-C Circuit of The White Emitter-Follower

Q_2 equal to 2.28 volts. Since V_{CE} for each transistor is 5.86 volts, then the voltage levels at the collectors of Q_2 and Q_1 respectively are 8.14 volts and 14.0 volts as shown in Figure 3. Therefore, E_{CC} should be approximately 20 volts.

Many combinations of bias-resistor values will supply the proper base current and voltage to the transistor for linear operation. But in order to reduce the effect of I_{CO} due to temperature variation, the base-bias voltage should not be a function of base current. Therefore, the choice of bias resistors should allow the current through R_1 and R_3 shown in Figure 3 to be approximately 10 times the base current. Although the I_{CO} current for silicon transistors is very small, transistors with large I_{CO} currents may require the bias configuration shown in Figure 3 to improve temperature stability. In this case, the following may be used to determine the bias resistance R_1 and R_2 . Assume a V_{BE} voltage of 0.6 volts (silicon transistor) and a beta (current gain) of 200 for each transistor. Next, consider the calculation of the bias resistors R_1 and R_2 . Since the emitter current and voltage of Q_1 is 3.02 ma and 8.14 volts, the base current and voltage values are approximately 0.015 ma and 8.74 volts. If I_1 is assumed to be 10 times the base current, then $I_1 = 0.150$ ma, and

$$R_1 = \frac{11.26 \text{ volts}}{0.150 \text{ ma}} = 75 \text{ kilohms} \quad (3)$$

Since the base current is 0.015 ma, then I_2 is approximately 0.135 ma, and R_2 should be approximately 65 kilohms.

The resistance values of R_3 and R_4 may be calculated in a similar manner. Since the emitter current and voltage of Q_2 is 3.04 ma and 2.28 volts, then the base current and voltage values are 0.15 ma and 2.88 volts. If I_3 is 0.15 ma, then R_3 should be 114 kilohms. Since I_{B2} is 0.015 ma, then I_4 is 0.135 ma, and R_4 should be 21 kilohms. The actual values of R_3 and R_4 used in the experimental circuit were 120 kilohms and 22 kilohms respectively.

The very high input impedance available with the White emitter-follower is limited due to the shunting action of the input bias resistors R_1 and R_2 . This shunting action may be minimized by using base injection bias (R_1) on Q_1 as shown in Figure 4. Since I_{CO} is very small, the I_{CO} effect due to temperature variation does not significantly effect the bias stability. Of course the value of R_1 must be calculated in order that the proper base current and voltage is available for Q_1 . For example, assume that I_{CO} current is negligible and that the base current and voltage is 0.015 ma and 8.74 volts. Then R_1 is approximately 743 kilohms. Base injection biasing allows the input impedance of the amplifier to be much higher. The actual value of R_1 that produced the desired operating conditions was 676

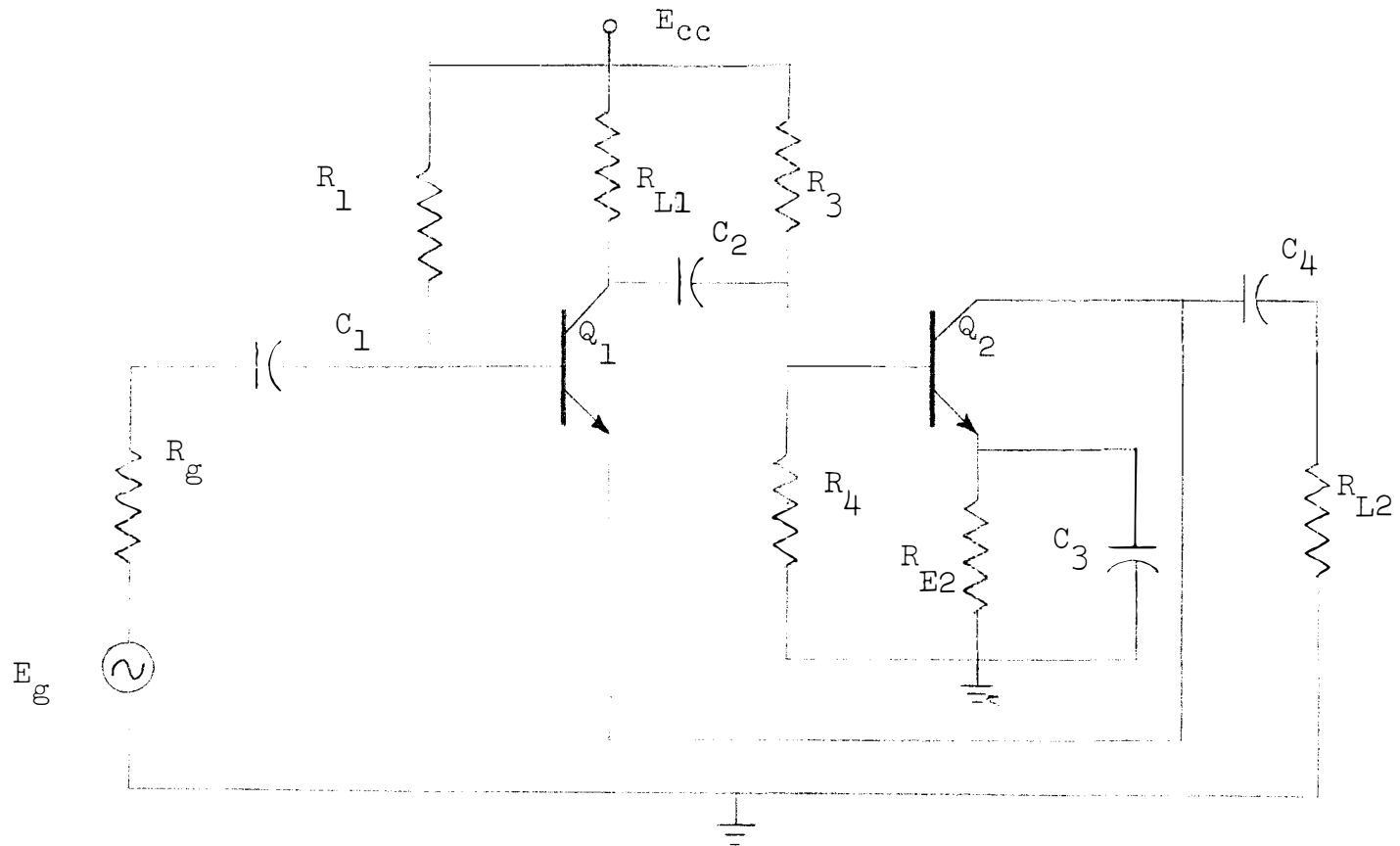


Figure 4. White Emitter-Follower With Base Injection Bias on The First Transistor (Q_1)

kilohms. A higher input impedance may be obtained by increasing R_1 and using a separate voltage supply.

A-C OPERATION

The a-c circuit for the White emitter-follower is shown in Figure 5. This figure was obtained from Figure 4 by considering the battery and capacitors as short circuits to a-c currents.

Perhaps the best way of understanding the a-c operation of this circuit is to consider the input impedance of Q_1 . To develop the input impedance for Q_1 , first consider the impedance at the emitter of Q_1 . The a-c currents I_{c2} and I_{e1} flow through R_{L2} so that

$$V_{e1} = R_{L2} (I_{e1} + I_{c2}) \quad . \quad (4)$$

In order to determine the impedance at the emitter of Q_1 , I_{c2} must be expressed as a function of I_{e1} . Since

$$I_{b2} = \frac{R_{eq1}}{R_{eq1} + R_{i2}} I_{c1} \quad (5)$$

(R_{eq1} is the parallel combination of R_{L1} , R_3 and R_4) and

$$I_{c2} = A_{i2} I_{b2} \quad , \quad (6)$$

then I_{c2} may be expressed as a function of I_{c1} as

$$I_{c2} = A_{i2} \frac{R_{eq1}}{R_{eq1} + R_{i2}} I_{c1} \quad . \quad (7)$$

I_{c1} may be expressed as a function of I_{e1} by

$$I_{c1} = \alpha_1 I_{e1} \quad . \quad (8)$$

Therefore, I_{c2} may be expressed as

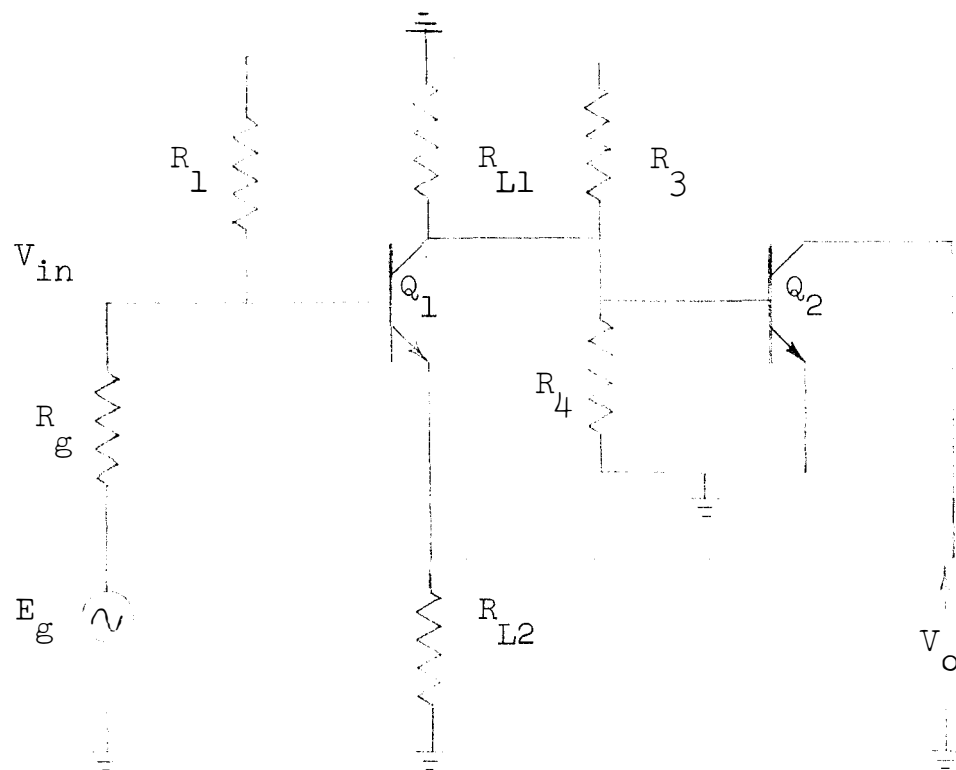


Figure 5. A-C Circuit of The White Emitter-Follower

$$I_{c2} = A_{i2} \frac{R_{eq1}}{R_{eq1} + R_{i2}} \alpha_1 I_{e1} \quad . \quad (9)$$

V_{e1} may now be expressed as

$$V_{e1} = (I_{e1} + I_{e1} \frac{A_{i2} R_{eq1}}{R_{eq1} + R_{i2}} \alpha_1) R_{L2} \quad . \quad (10)$$

Dividing Equation (10) by I_{e1} , the impedance at the emitter of Q_1 is

$$\frac{V_{e1}}{I_{e1}} = (1 + \frac{A_{i2} R_{eq1}}{R_{eq1} + R_{i2}} \alpha_1) R_{L2} \quad . \quad (11)$$

Therefore, the input impedance at the base of Q_1 is approximately

$$R_{i1} = h_{ie} + A_{i1} (1 + \frac{A_{i2} R_{eq1}}{R_{eq1} + R_{i2}} \alpha_1) R_{L2} \quad , \quad (12)$$

where A_{i1} and A_{i2} are the current gains of Q_1 and Q_2 respectively. Without the feedback from the collector of Q_2 the input impedance is

$$R_i = h_{ie} + A_{i1} R_{L2} \quad . \quad (13)$$

Thus the feedback of the White emitter-follower has increased the input impedance by the amount K , where K is

$$(1 + \frac{A_{i2} R_{eq1}}{R_{eq1} + R_{i2}} \alpha_1) R_{L2} \quad .$$

When R_{L2} is greater than several hundred ohms, the

input impedance is high (megohms). The voltage gain from the base to the collector of Q_1 is

$$A_{vt1} = \frac{A_{i1} R}{R_{i1}}, \quad (14)$$

where R is the total load impedance of Q_1 and R_{i1} is the input impedance at the base of Q_1 expressed in Equation (12). Since R is only a few hundred ohms and R_{i1} is very large, the voltage gain of Q_1 is much less than unity. Thus the driving voltage for Q_2 is much less than V_{in} .

The voltage gain of Q_2 is

$$A_{vt2} = \frac{A_{i2} R_{L2}}{R_{i2}}, \quad (15)$$

where R_{i2} is the input impedance of Q_2 . The output voltage is

$$V_O = A_{vt1} A_{vt2} V_{in}. \quad (16)$$

The base-to-emitter voltage of Q_1 , V_{bel} , is

$$V_{bel} = V_{in} - V_O. \quad (17)$$

The value of V_O can approach but cannot exceed V_{in} in the mid-frequency range assuming zero phase shift. In order for V_O to exceed V_{in} , V_{bel} must go through zero. However, if V_{bel} (the a-c driving signal for Q_1) is zero, then the output voltage, V_O , is zero. With an input signal present, the output voltage is not zero. Therefore, since V_{bel} amplifies any difference between V_{in} and V_O and drives

V_o toward V_{in} , the operating state of V_o is a value slightly less than V_{in} .

A further understanding of the circuit operation may be obtained by tracing a signal from the generator to the output. Consider E_g to be a positive going signal. Then the positive signal appearing at the base of Q_1 , V_{in} , may be defined as

$$V_{in} = \frac{R}{R + R_g} E_g, \quad (18)$$

where R is the parallel combination of R_1 and R_{i1} . Since the output voltage is fed back to the emitter and is slightly less than the input signal V_{in} , the output voltage cancels out most of the input voltage leaving only a small portion as a driving signal. The very small positive driving signal V_{be1} causes a slight increase in the base current from its d-c value. Thus the portion of the collector current, I_{c1} , flowing through R_{L1} increases slightly, causing a small decrease in the voltage at the collector of Q_1 . This small negative going signal, V_{be2} , is the driving signal for Q_2 . The small decrease in V_{be2} causes a decrease in I_{b2} from its d-c value. Thus, the collector current of Q_2 decreases. This decrease in I_{c2} causes a positive voltage, V_o , at the emitter of Q_1 slightly less in magnitude than V_{in} .

Now consider a negative going portion of E_g . The very small negative driving signal decreases the base current, I_{b1} , slightly from its d-c value. Thus the collector current, I_{c1} , decreases. The decrease in the portion of the collector current, I_{c1} , through R_{L1} generates a small positive driving voltage, V_{be2} . V_{be2} increases the base current of Q_2 , and thus the collector current increases. The increase in the collector current, I_{c2} , generates a negative voltage, V_o , across R_{L2} slightly less in magnitude and in phase with the negative input voltage V_{in} .

The feedback forces small signal operation of Q_1 . The output signal, V_o , is compared to the input signal, V_{in} , and any difference, V_{bel} , is amplified to bring the two together; the input characteristics of Q_1 are effectively made linear by the feedback signal. Non-linearity in V_o caused by Q_2 beta changes (due to large I_c variations) will be minimized by the effect of the feedback. For example, if beta of Q_2 decreases during a portion of the cycle, the output voltage, V_o , will also decrease producing an increase in V_{bel} . V_{bel} is amplified causing an increase in the collector current of Q_2 and V_o . Thus V_o is maintained slightly less than V_{in} by the driving signal, V_{bel} . Therefore, the White emitter-follower is able to amplify bipolar pulses with approximately equal amplitudes due to the feedback.

A voltage gain from the base of Q_1 to the collector of Q_2 greater than or equal to unity may be obtained by placing a resistor in the feedback loop as shown in the a-c circuit of Figure 6. This feedback resistor increases the voltage gain of Q_2 , since it allows only a portion of the output voltage to be fed back to the emitter of Q_1 . The effect of the feedback voltage on Q_1 is the same as that explained earlier in this section. The value of R_f should be limited to a few kilohms. Large values of R_f may present d-c bias problems and reduce the desirable feedback effects due to the large voltage drop across R_f .

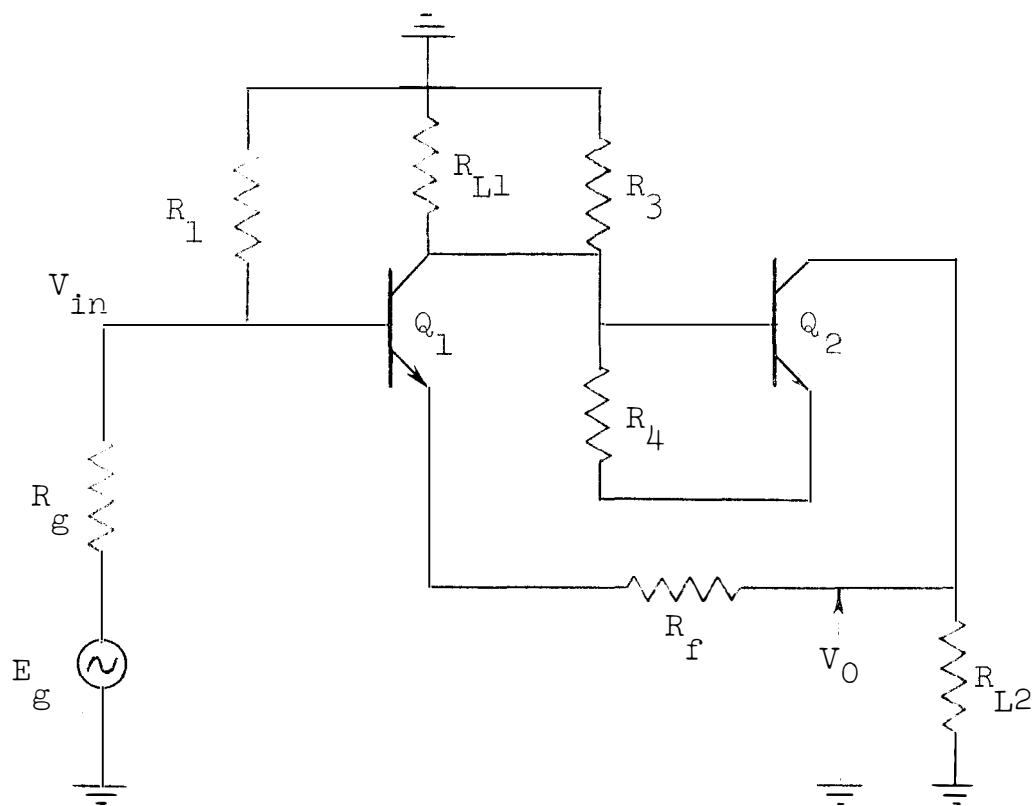


Figure 6. A-C Circuit of The White Emitter-Follower
With A Feedback Resistor R_f

MID-FREQUENCY ANALYSIS

GAIN

The mid-frequency gain, input impedance, and output impedance equations for the White emitter-follower will be derived from the equivalent circuit. The mid-frequency equivalent circuit may be constructed by using the hybrid- π equivalent circuit for each transistor. The hybrid- π equivalent circuit for a transistor is shown in Figure 7. The resistance from b' to C , $r_{b'c}$, has been neglected since the coupling that it produces between input and output circuits is usually negligible. Using equivalent hybrid- π circuits for each transistor and neglecting all capacitive reactances, the equivalent mid-frequency circuit for the White emitter-follower has the form shown in Figure 8.

The general expression for the gain from V_{in} to V_o (see Appendix II), A' , is

$$A' = \frac{\frac{1}{r_{b'1}} \left(\frac{1}{r_{b'e1}} + g_{m1} \right) \left(\frac{1}{R_{eq1} r_{b'2}} + \frac{1}{R_{eq1} r_{b'e2}} + \frac{1}{r_{b'2} r_{b'e2}} \right)}{\left[\frac{1}{r_{b'1}} \left(\frac{1}{r_{b'e1}} + g_{m1} \right) + \frac{1}{r_{b'1} R_{L2}} + \frac{1}{r_{b'e1} R_{L2}} \right] \times \frac{\frac{g_{m1} g_{m2}}{r_{b'1} r_{b'2}}}{\left(\frac{1}{R_{eq1} r_{b'2}} + \frac{1}{R_{eq1} r_{b'e2}} + \frac{1}{r_{b'2} r_{b'e2}} \right) + \frac{g_{m1} g_{m2}}{r_{b'1} r_{b'2}}} \cdot (19)}$$

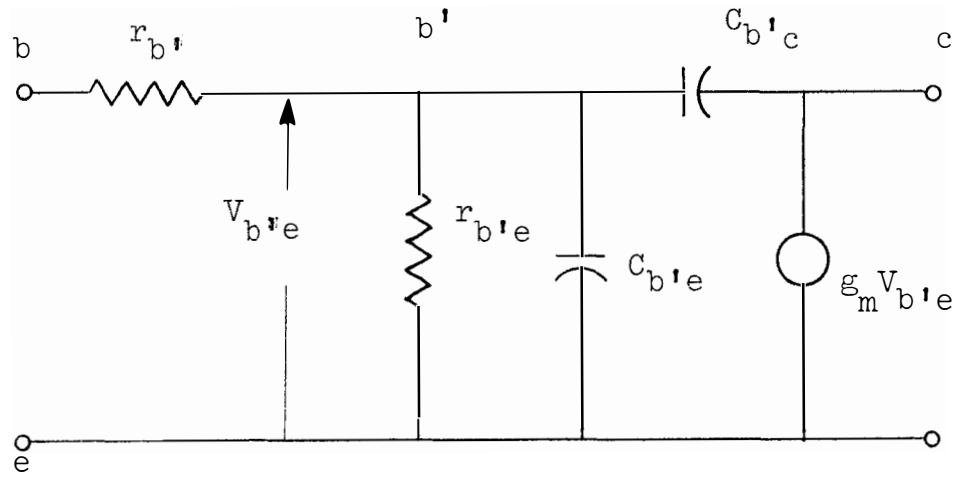


Figure 7. Hybrid - π Equivalent Circuit For a Transistor

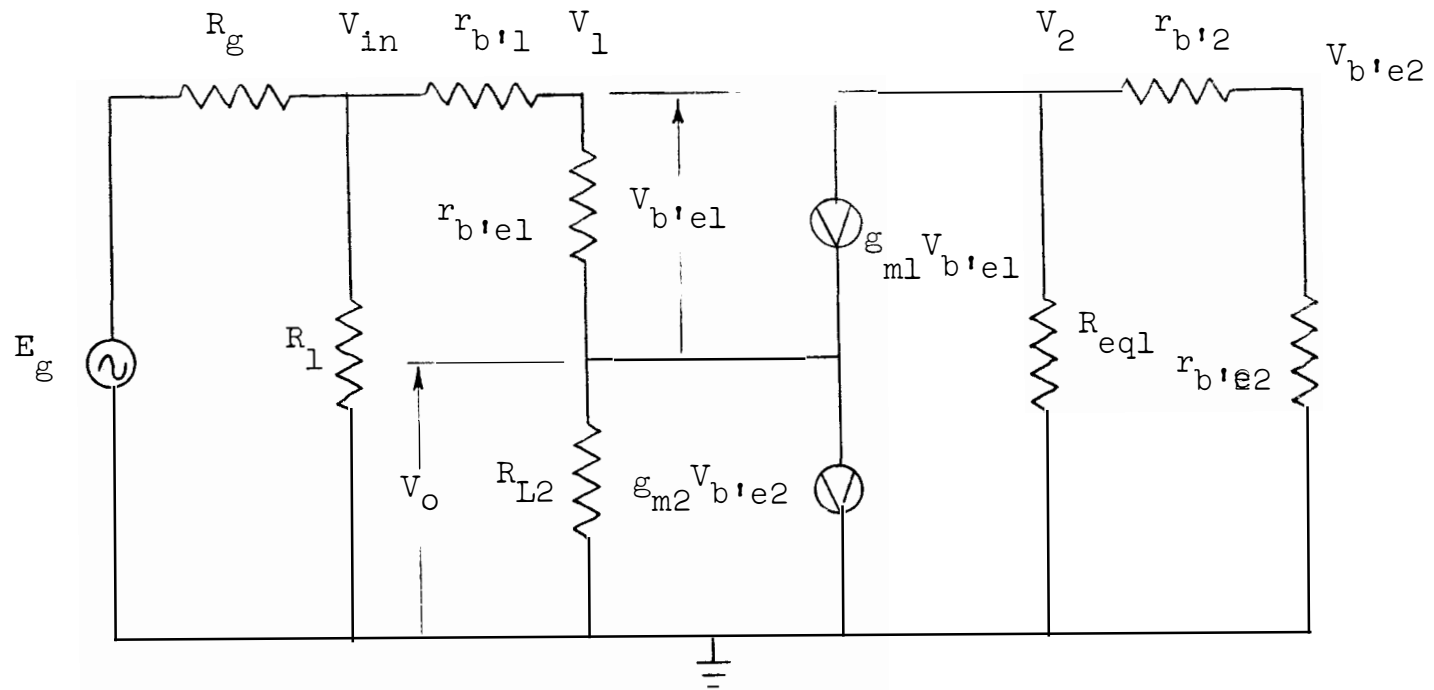


Figure 8. Mid-Frequency Equivalent Circuit of The White Emitter-Follower

where R_{eq1} is the parallel combination of R_3 , R_4 and h_{ie2} .

If the transistors are assumed to have the same parameter values, the gain equation for A' is simplified somewhat.

After some algebraic manipulation (see Appendix II) the gain equation (A'), assuming identical transistors, is

$$A' \cong \frac{1}{1 + \frac{\frac{h_{ie}}{R_{L2}(\beta+1)} \left[\frac{r_e}{R_{eq1}} + \frac{r_{b'}}{R_{eq1}(\beta+1)} + \frac{1}{(\beta+1)} \right]}{\left[\frac{r_e}{R_{eq1}} + \frac{r_{b'}}{R_{eq1}(\beta+1)} + \frac{1}{(\beta+1)} \right] + 1}} \quad . \quad (20)$$

If the equivalent load on the first transistor, R_{L1} , is approximately 1000 ohms or greater, Equation (20) simplifies to

$$A' \cong \frac{1}{1 + \left[\frac{h_{ie}}{R_{eq1}(\beta+1)} \left(\frac{r_e}{R_{eq1}} + \frac{r_{b'}}{R_{eq1}(\beta+1)} + \frac{1}{(\beta+1)} \right) \right]} \quad . \quad (21)$$

The total voltage gain is the output voltage, V_o , divided by the generator voltage, E_g . The input voltage, V_{in} , was expressed as a function of the generator voltage,

E_g , in Equation (18). Since

$$\frac{V_{in}}{E_g} = \frac{R}{R + R_g} \quad (22)$$

and

$$\frac{V_o}{V_{in}} = A' \quad , \quad (23)$$

the total gain, A , is

$$A = \frac{V_o}{E_g} = \frac{R}{R + R_g} A' \quad . \quad (24)$$

From Equations (24) and (21) it is apparent that the circuit gain is a function of three circuit parameters: R_g , R_{L1} and R_{L2} . The test circuit used to determine the measured value of the gain is shown in Figure 9. The gain variation as a function of the load resistance R_{L2} for both calculated and measured values is shown in Figure 10. The deviation of the two curves for low values of R_{L2} is attributed to the difficulty in measuring low level signals. The transistor Q_2 will saturate if all the standby current is switched into the load. When R_{L2} is reduced the input voltage must be reduced to prevent saturation in the output. With a small input voltage, the gain is difficult to determine accurately using an oscilloscope and differential amplifier. The variation in gain as a function of R_{L1} is shown in Figure 11. As shown in Figures 10 and 11, when

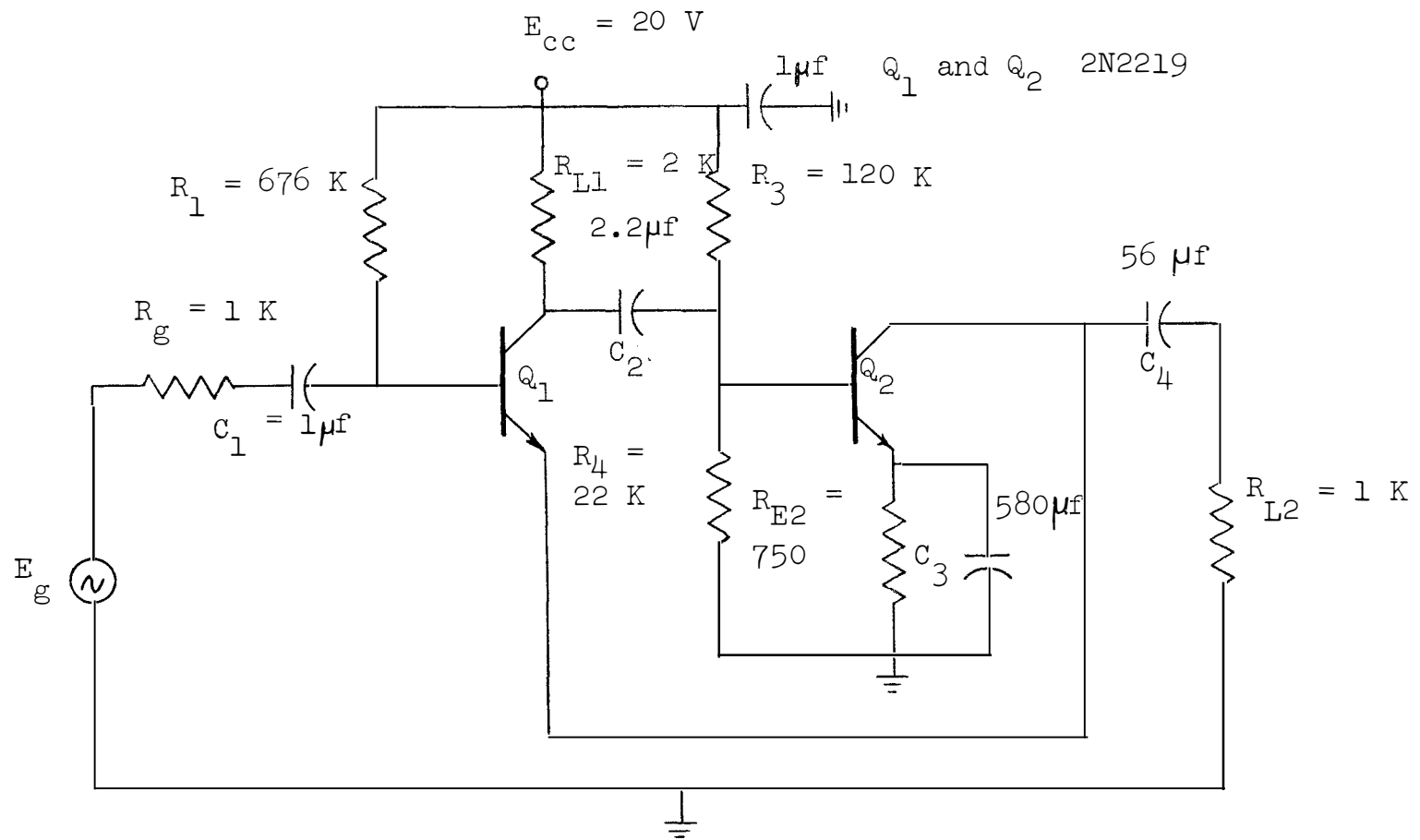


Figure 9. Mid-Frequency Test Circuit of The White Emitter-Follower

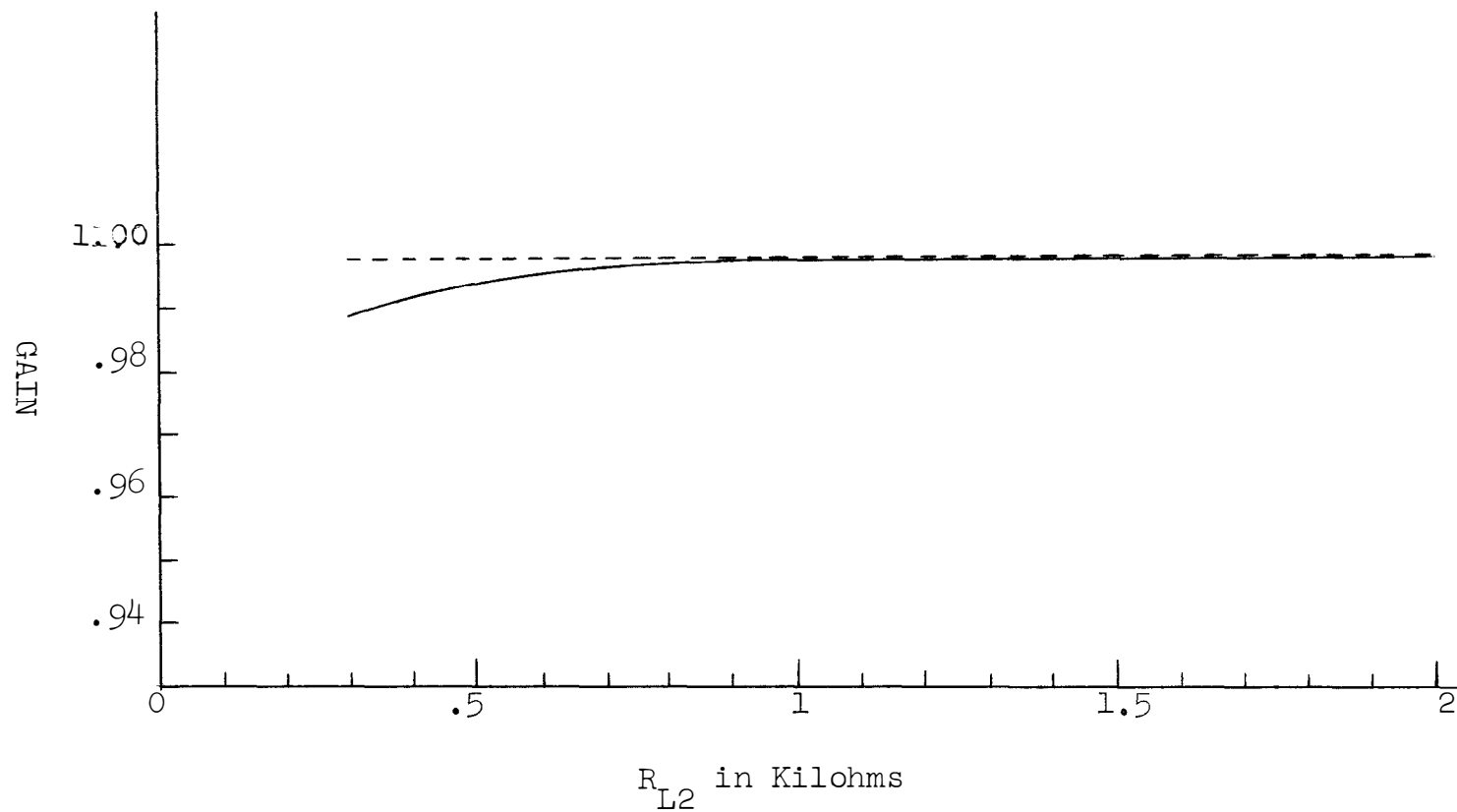


Figure 10. Gain as a Function of Load Resistance R_{L2} .
 from Measured Data, Dashes Indicate Calculated Data

Solid Curve

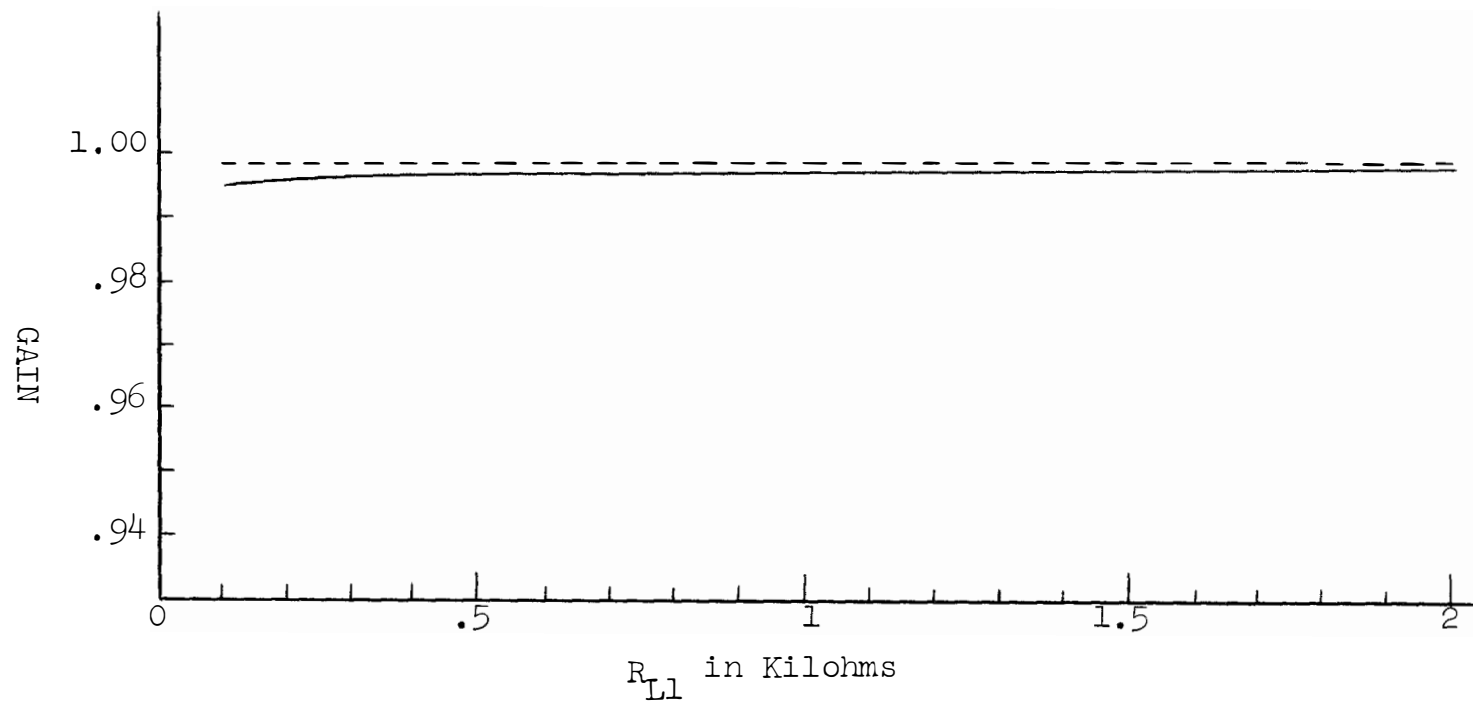


Figure 11. Gain as a Function of Load Resistance R_{L1} . Solid Curve from Measured Data, Dashes Indicate Calculated Data

R_g is small and R_{L1} and R_{L2} are greater than several hundred ohms, the gain is almost unity.

INPUT IMPEDANCE

One of the desirable characteristics of the White emitter-follower is its high input impedance. The expression for the input impedance developed in the last section may be used with some refinement and explanation. The expression for the input impedance at the base of Q_1 from Equation (12) is

$$R_{i1} = h_{ie1} + A_{i1} \left(1 + \frac{A_{i2} R_{eq1}}{R_{eq1} + R_{i2}} \alpha_1 \right) R_{L2} \quad . \quad (25)$$

Since h_{ie1} is much smaller than the second term in Equation (25), it may be neglected. Then the input impedance may be expressed as

$$R_{i1} = A_{i1} \left(1 + \frac{A_{i2} R_{eq1}}{R_{eq1} + R_{i2}} \alpha_1 \right) R_{L2} \quad , \quad (26)$$

where A_{i1} and A_{i2} are the current gains of Q_1 and Q_2 respectively. The exact equation for the current gain of Q_1 is

$$A_{i1} = \frac{r_c}{r_c (1 - \alpha) + R_L + r_e} \quad . \quad (27)$$

Since the effective load impedance at the emitter of Q_1 is large with respect to $r_c (1 - \alpha)$, the normal approximation

$(\beta + 1)$ cannot be used. The emitter resistance, r_e , may be neglected because it is very small when compared to R_L and $r_c (1 - \alpha)$. The exact current gain for Q_2 is

$$A_{i2} = \frac{\alpha r_c - r_e}{r_c (1 - \alpha) + R_L + r_e} \quad (28)$$

Since $r_c (1 - \alpha)$ is much larger than r_e and the load impedance R_{L2} , then the current gain for Q_2 may be approximated by

$$A_{i2} \cong \beta \quad (29)$$

Therefore, using the approximations developed for the current gains (A_{i1} and A_{i2}), the input impedance at the base of Q_1 may be approximated by

$$R_{i1} = \frac{r_c R_{E1}}{r_c (1 - \alpha) + R_{E1}} \quad (30)$$

where R_{E1} is

$$R_{E1} = \left(1 + \frac{\beta_2 R_{eq1}}{R_{eq1} + R_{i2}} \alpha_1 \right) R_{L2} \quad (31)$$

The variation in the input impedance at the base of Q_1 as a function of R_{L2} is shown in Figure 12. Only one experimental test point was obtained due to circuit changes necessary to make one measurement. The test circuit used was that shown in Figure 9 with $R_1 = 4.8$ megohms and

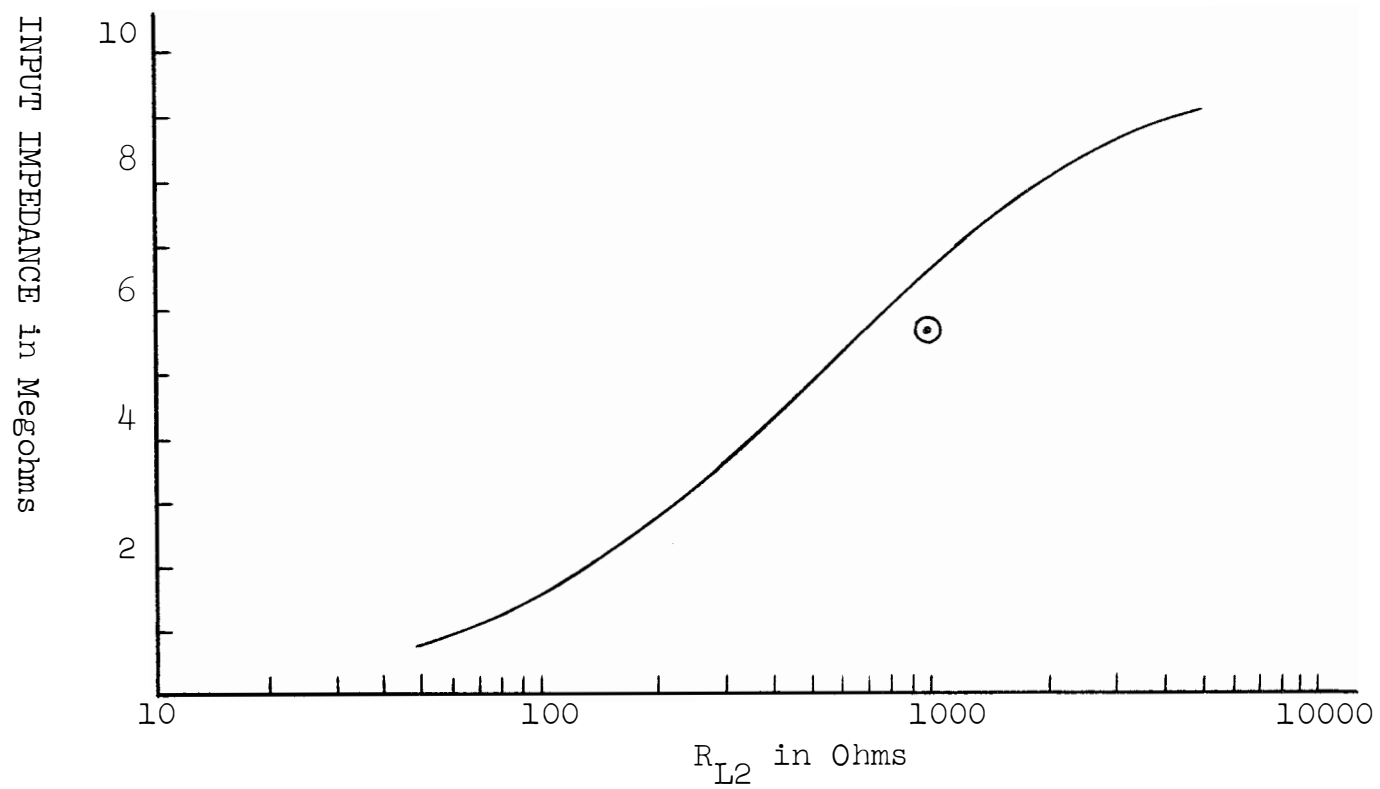


Figure 12. Input Impedance at The Base of Q_1 as a Function of Load Resistance R_{L2} . Solid Curve from Measured Data, Dot Indicates Calculated Data Point

$R_{L2} = 1$ kilohm. The calculated input impedance was 6.53 megohms; the measured value was approximately 5.67 megohms. Errors in determining beta and r_c could easily account for this difference.

Although the input impedance at the base of Q_1 is large, the total input impedance is the parallel combination of R_1 and R_{i1} . With $R_1 = 4.8$ megohms and $R_{L2} = 1$ kilohm, the total input impedance value measured was 2.6 megohms. The calculated value was 2.77 megohms. Therefore, to have a high total input impedance, R_1 should have as high a value as possible.

OUTPUT IMPEDANCE

The impedance made up of the parallel combination of the output impedance of the White emitter-follower, R_o , and R_{L2} may be found from the equivalent circuit shown in Figure 13 as follows: Assume that a voltage V_o is applied to the output terminals of the amplifier in the absence of an external input signal. The combined impedance of R_o and R_{L2} is defined as

$$R_C = \frac{V_o}{I_o} \quad , \quad (32)$$

where I_o is the current flowing into the circuit as a result of V_o . This current is

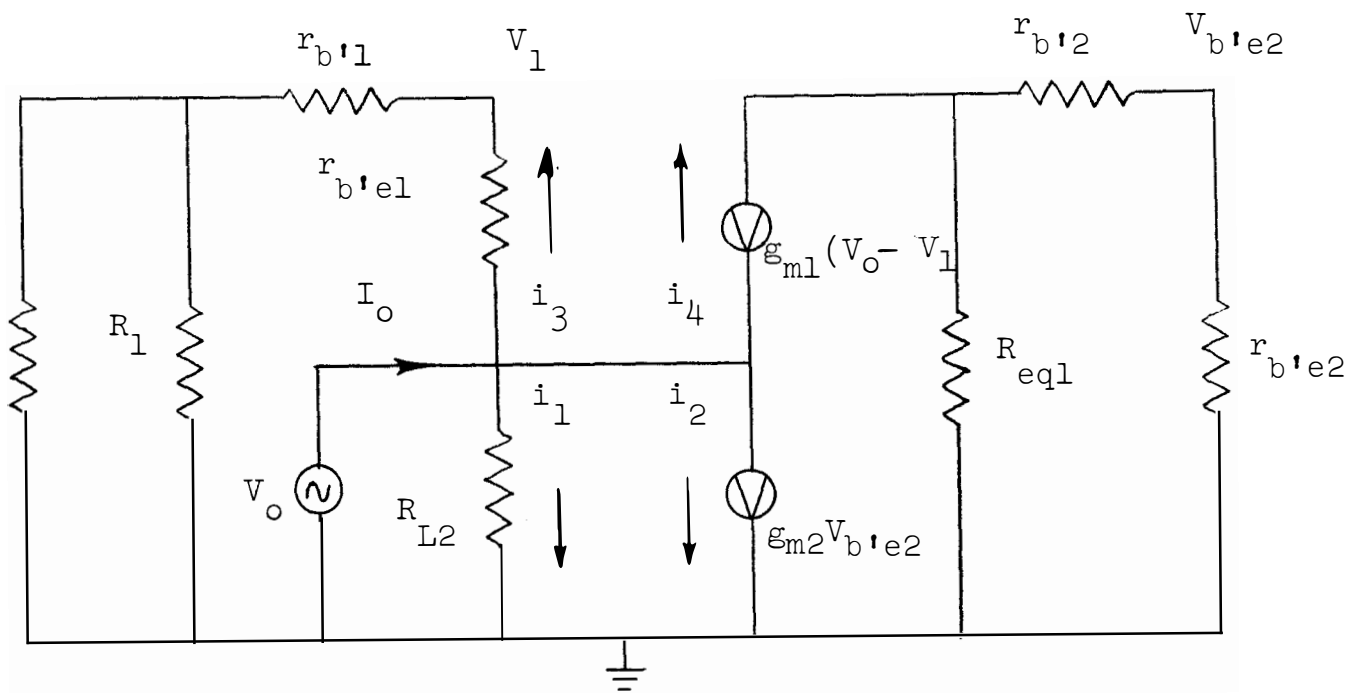


Figure 13. Output Impedance Equivalent Circuit of The White Emitter-Follower at Mid-Frequency

$$I_0 = (V_0 - V_1) \left(\frac{1}{r_{b'e1}} + g_{m1} \right) + \frac{V_0}{R_{L2}} + g_{m2} V_{b'e2} \quad (33)$$

The current flowing through $r_{b'e1}$ is

$$i_3 = \frac{V_0}{r} \quad , \quad (34)$$

where r is h_{ie1} plus the parallel combination of R_1 and R_g . Therefore, the voltage $(V_0 - V_1)$ is

$$(V_0 - V_1) = \frac{V_0}{r} r_{b'e1} \quad . \quad (35)$$

The voltage $V_{b'e2}$ is

$$V_{b'e2} = g_{m1} (V_0 - V_1) \left(\frac{R_{eq1}}{R_{eq1} + h_{ie2}} \right) r_{b'e2} \quad . \quad (36)$$

Substituting Equation (35) and (36) into Equation (33) and simplifying,

$$I_0 = V_0 \left[\frac{(\beta_1 + 2)R_{L2} + r}{rR_{L2}} + \frac{r_{b'e1} R_{eq1} r_{b'e2} g_{m1} g_{m2}}{r (R_{eq1} + h_{ie2})} \right] \quad . \quad (37)$$

Substituting Equation (37) into Equation (32),

$$R_C = \frac{1}{\left[\frac{(\beta_1 + 2)R_{L2} + r}{rR_{L2}} \right] + \frac{1}{\left[\frac{r_{b'e1} r_{b'e2} R_{eq1} g_{m1} g_{m2}}{r (R_{eq1} + h_{ie2})} \right]}} \quad (38)$$

For all practical values of load and generator impedance the term in Equation (38) containing R_{L2} is negligible compared to the other term in the equation. For example, if the load impedance, R_{L2} , and generator impedance, R_g , range from 50 ohms to 1000 ohms and from 0 to 10 kilohms respectively, then the term not containing R_{L2} in Equation (38) is at least 80 times larger than the term containing R_{L2} . Therefore, the output impedance is practically independent of load impedance, and may be expressed as

$$R_O \cong \frac{r(R_{eq1} + h_{ie2})}{r_{b'e1} r_{b'e2} R_{eq1} g_{m1} g_{m2}} \quad (39)$$

For identical transistors Equation (39) becomes

$$R_O \cong \frac{r(R_{eq1} + h_{ie2})}{(r_{b'e})^2 (g_m)^2 R_{eq1}} \quad (40)$$

The only circuit parameters that affect R_O

significantly are R_g and R_{eq1} . Curves of the measured and calculated values of R_o as a function of R_{eq1} are shown in Figure 14; the curves of R_o as a function of R_g are shown in Figure 15. As shown in both Figure 14 and Figure 15, the measured and calculated curves are in close agreement. From these curves it is apparent that R_{eq1} should be larger than several hundred ohms for low output impedance.

It is interesting to note that the same transistor used in an emitter-follower configuration has a calculated total voltage gain of 0.984 for R_L and R_g each equal to 1000 ohms, while the White emitter-follower has a gain of 0.997. The calculated output impedance is approximately 15 ohms for the emitter-follower; the White emitter-follower has an output impedance of less than 0.30 of an ohm. The calculated input impedance at the base of the emitter-follower is 201 kilohms at mid-frequency, and the White emitter-follower has an input impedance of 6.5 megohms. Thus, the White emitter-follower has definite advantages over the emitter-follower with respect to gain, input impedance, and output impedance at mid-frequency.

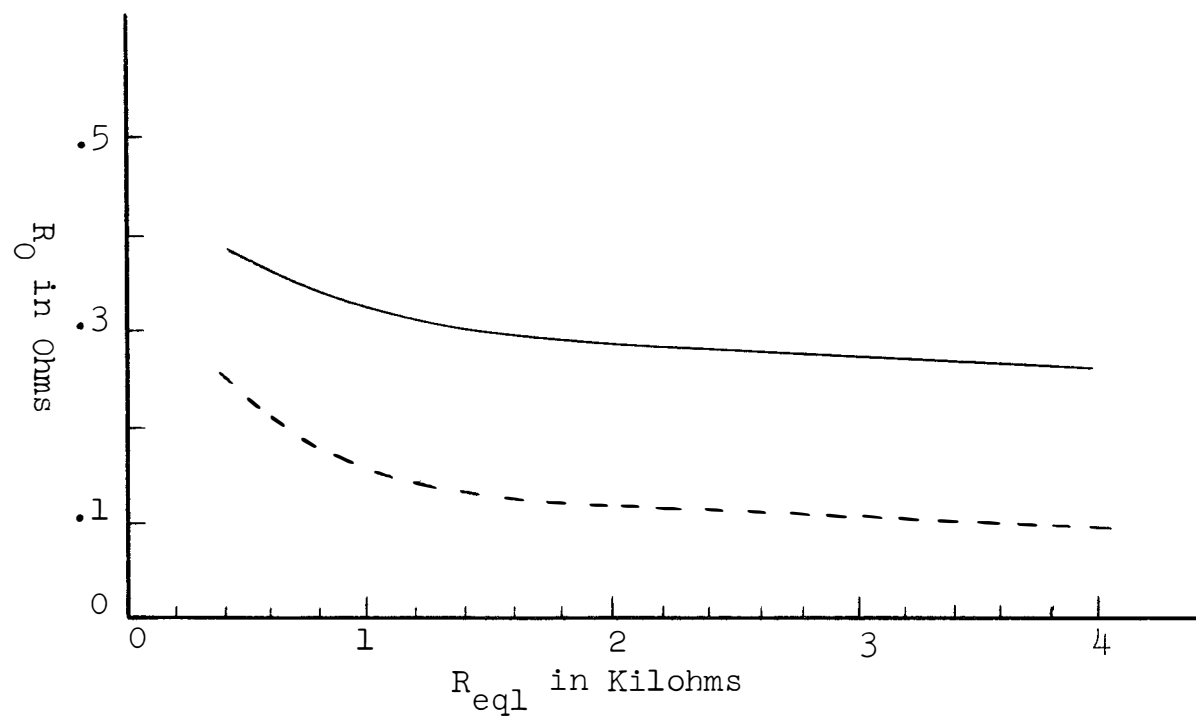


Figure 14. Output Impedance, R_O , as a Function of R_{eq1} at Mid-Frequency. Solid Curve from Measured Data, Dashes Indicate Calculated Data

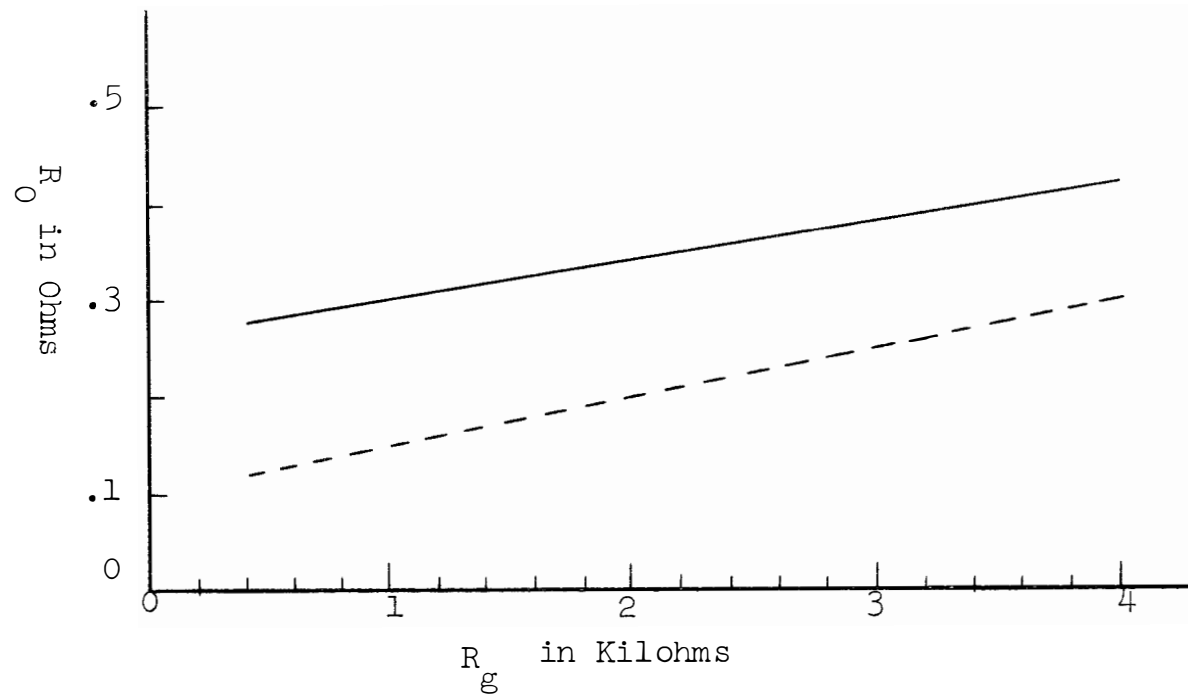


Figure 15. Output Impedance, R_O , as a Function of Generator Resistance, R_g , at Mid-Frequency. Solid Curve from Measured Data, Dashes Indicate Calculated Data

LOW-FREQUENCY ANALYSIS

The low-frequency response of an R-C coupled circuit like the one shown in Figure 16 may be normalized to the mid-frequency value as

$$A_{\text{Low}} = A_{\text{Mid}} \frac{1}{1 - \left(j \frac{f_L}{f} \right)}, \quad (41)$$

where f is any frequency, and f_L is the frequency at which the magnitude of the reactance of C is equal to the effective resistance across C ⁶. Equation (41) in terms of a magnitude and an angle is

$$A_{\text{Low}} = A_{\text{Mid}} \frac{1}{\sqrt{1 + \left(\frac{f_L}{f} \right)^2}} \angle \tan^{-1} \left(\frac{f_L}{f} \right). \quad (42)$$

When f is equal to f_L , in Equation (42), the low-frequency response is 3db down from its mid-frequency value, and the phase angle is 45 degrees relative to the mid-frequency value. The frequency, f_L , may be expressed in equation form as

$$f_L = \frac{1}{2 \pi RC}, \quad (43)$$

where C may be a coupling or bypass capacitor, and R is the effective resistance across C .

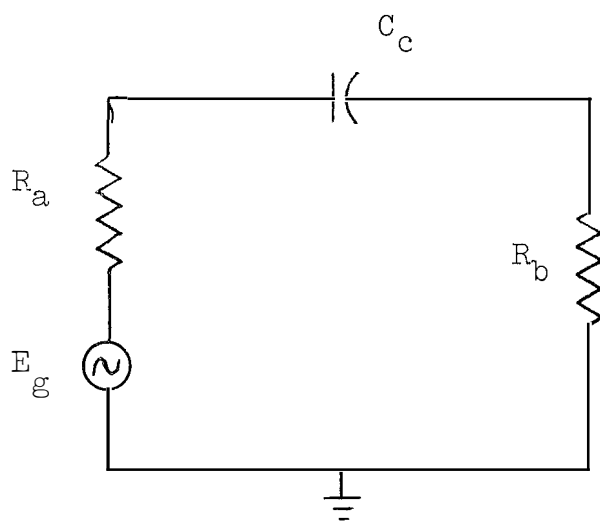


Figure 16. Simple R-C Coupled Circuit

For a circuit containing more than one coupling or bypass capacitor, the overall low-frequency response may be obtained by combining the individual responses due to each coupling or bypass capacitor. The individual response may be determined by considering each capacitor one at a time while all other capacitors are a-c short circuits. This analysis assumes that the impedance of all the other capacitors is not a function of frequency over the same range as the particular capacitor being considered. This may not be completely correct. For example, the low-frequency response due to C_2 and associated resistance shown in Figure 17 depends on the impedance in the emitter circuit of Q_2 . Since this impedance and the response due to C_2 may be functions of frequency over the same range, the response due to C_2 may only be a reasonable approximation.

Considering no interaction between responses as discussed above, the equivalent circuit for each low-frequency response is shown in Figure 18. In each case the individual responses were determined by using Equation (43). The calculations for the responses (see Figure 16) are given in Appendix III. The overall low-frequency response was obtained by algebraically adding the ordinates of the individual responses. The calculated and measured overall low-frequency responses are shown in Figure 19. Obviously the calculated response is a pessimistic approximation of

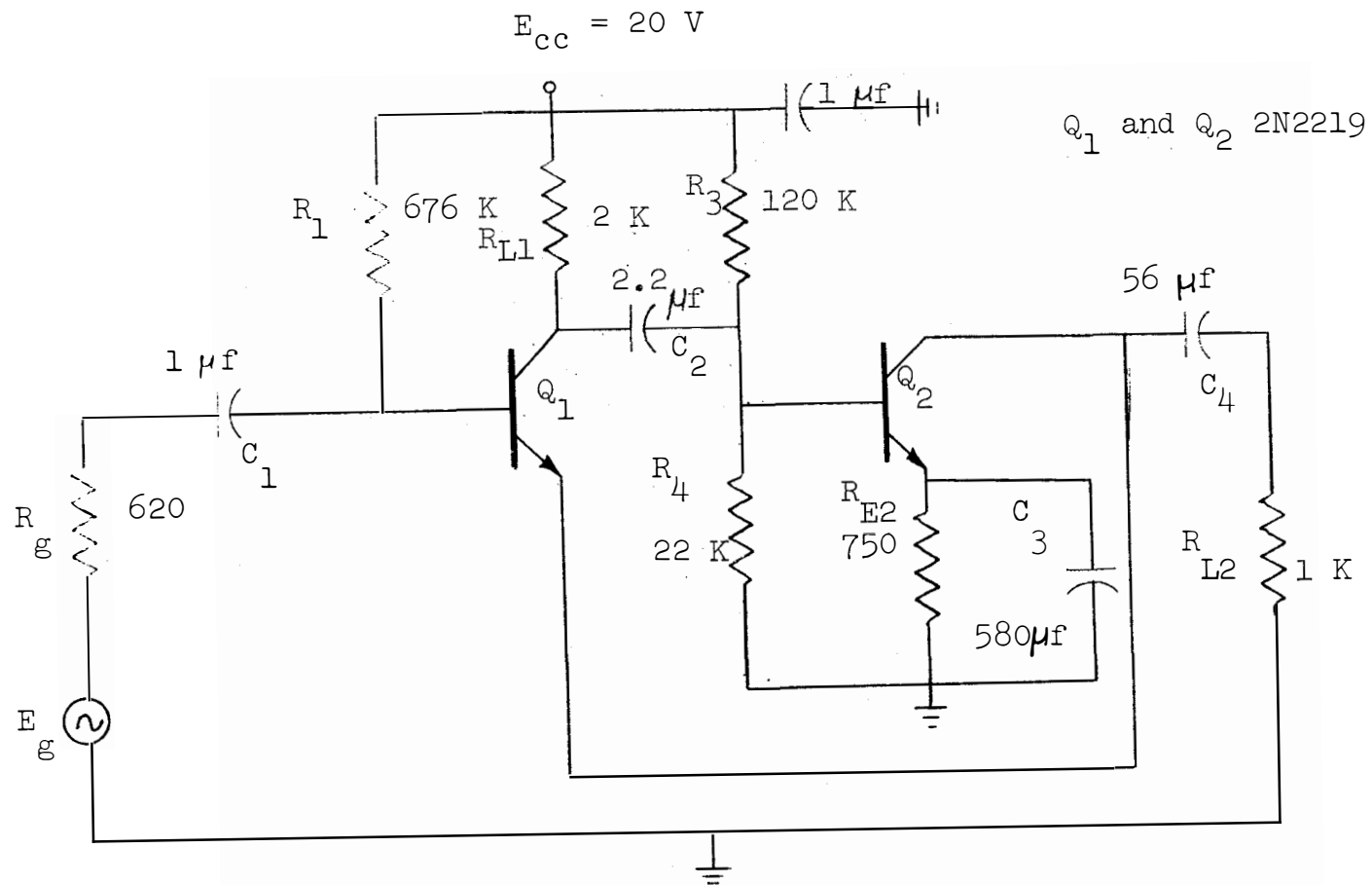
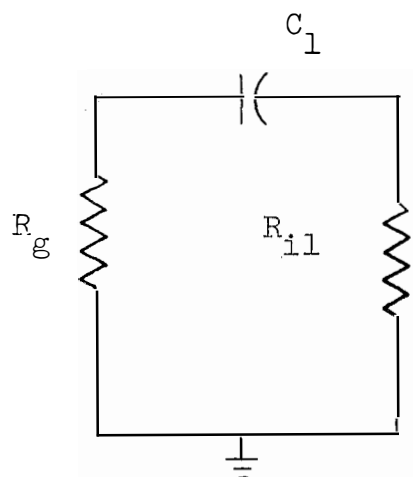
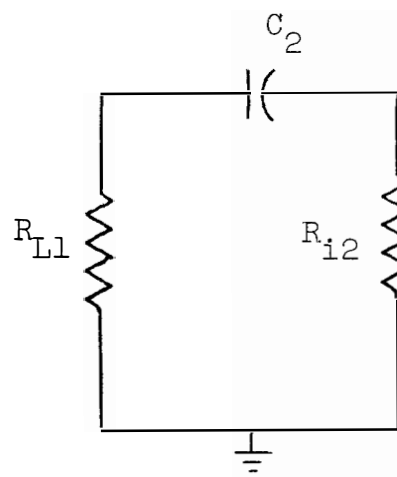


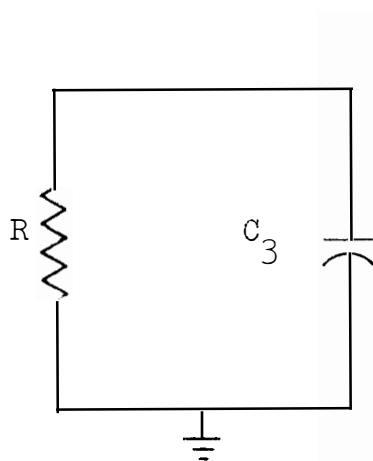
Figure 17. Low-Frequency Test Circuit of The White Emitter-Follower



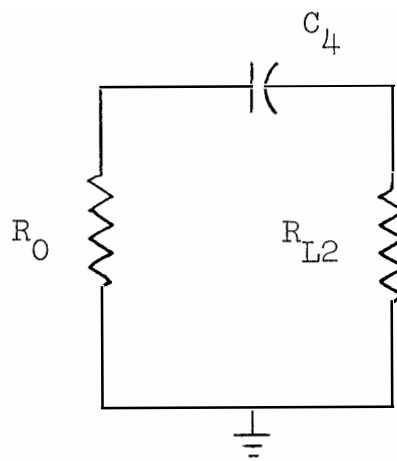
(a) C_1 And Associated Resistance



(b) C_2 And Associated Resistance



(c) C_3 And Associated Resistance



(d) C_4 And Associated Resistance

Figure 18. Equivalent Low-Frequency Circuits of Coupling And Bypass Capacitors With Associated Resistance

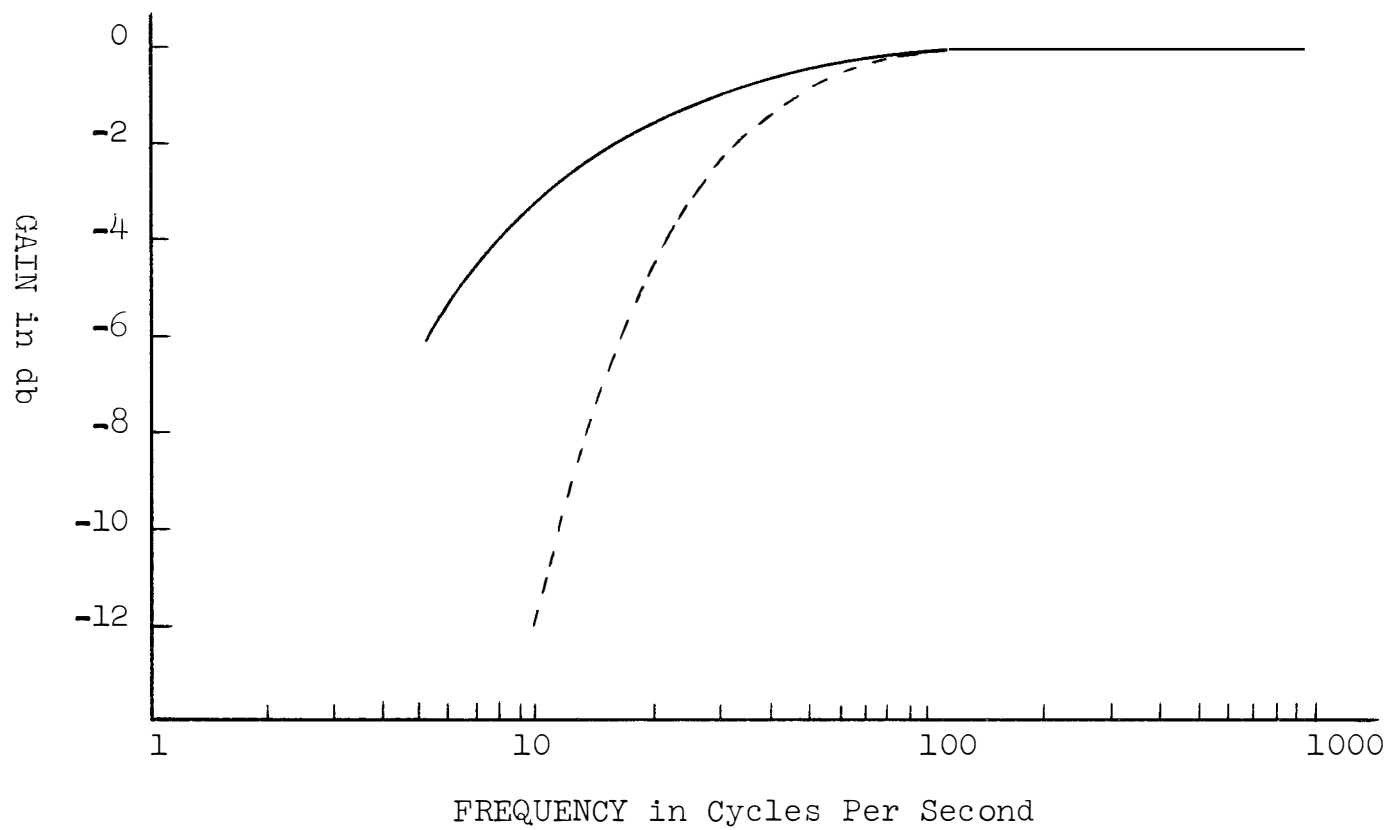


Figure 19. Gain as a Function of Low-Frequency. Solid Curve from Measured Data, Dashes Indicate Calculated Data

the overall response; the actual response is lower than predicted by calculation. According to the calculations (see Appendix III), the individual responses due to C_2 and C_3 dominated the overall low-frequency response. As discussed above, the response due to C_2 was calculated assuming that C_3 was an a-c short circuit. Actually C_3 probably has some impedance over this frequency range. Hence, the actual impedance across C_2 is probably larger than calculated. Consequently, the calculated response due to C_2 should be lower than calculated (see Appendix III). Likewise, the response due to C_3 has some interaction with the impedance of C_2 . Therefore, the calculated overall low-frequency response is only a reasonable approximation. The overall low-frequency response may be lowered by increasing the capacitance values of C_2 and C_3 .

TRANSIENT ANALYSIS

The transient response of an amplifier may be determined by applying a step function to the input of the amplifier. The output of the amplifier will not jump to its final value instantaneously; it will have a finite rise time. The rise time is usually defined as the time it takes for the output to increase from 10 per cent to 90 per cent of its final value. The output may reach a value greater than its final value. The amount by which the output initially exceeds its final value is known as overshoot, which is usually expressed as a percentage of the final value.

A rectangular wave or pulse is a combination of a positive unit step and a delayed negative unit step. This wave form is fairly easy to generate, and it reveals a great deal about the response of an amplifier. For test purposes a repetitive pulse is most useful for measuring the rise time and overshoot.

In this section a calculated and an experimental transient response is determined for the White emitter-follower circuit. The calculated response was determined for the transfer function of the equivalent circuit. To solve the transfer function for more than one set of circuit values, it was necessary to solve the transfer function with an analog computer. A step input function was used, and the output function was produced graphically. The experimental

response was obtained by observing the output of the circuit shown in Figure 20 when the input was a repetitive pulse. The equivalent circuit and transfer function governing the calculated response will be derived first. The calculated response is then compared with the experimental response.

EQUIVALENT CIRCUIT

To develop a high-frequency equivalent circuit for the White emitter-follower, each transistor was represented with an equivalent a-c circuit. A particularly useful representation is provided by the hybrid- π equivalent circuit shown in Figure 7. The high-frequency equivalent circuit for the White emitter-follower may be constructed by replacing each transistor in Figure 20 with the hybrid- π equivalent circuit; the resulting high-frequency equivalent circuit is shown in Figure 21. The equivalent circuit may be reduced because the effects of certain circuit elements are small in comparison with others. For small values of R_g (Less than 1 kilohm) and large values of R_1 (1 megohm), the shunting effect of R_1 on the input may be neglected. Neglecting R_1 decreases the number of nodal equations necessary to determine the transfer function from five to four equations. The capacitance $C_{b'c1}$ may be placed from b' to ground as the Miller-effect capacitance $C_{b'c1}(1+A_1)$ where A_1 is the gain from b'_1 to b_2 . From Section III the gain from b_1 to b_2 is less than unity. Therefore, the gain

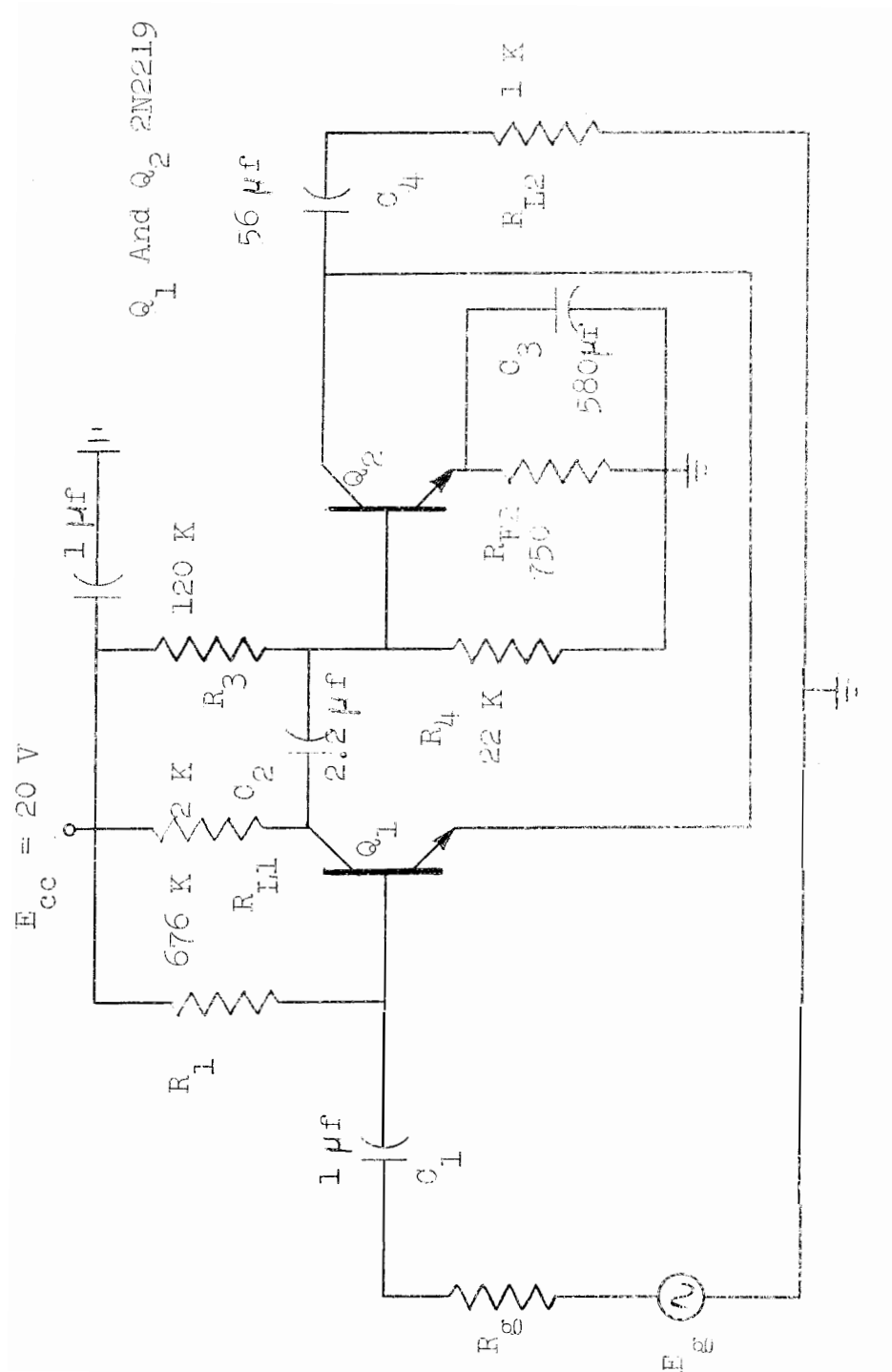


Figure 20. Test Circuit of The White Emitter-Follower Used to Determine The Transient Response

from b'_1 to b_2 is less than unity, and the Miller-effect capacitance from b'_1 to ground may be approximated by

$C_{b'c1}$. Considering R as the sum of R_g and $r_{b'1}$ and using the above simplifications, the initial circuit reduces to the equivalent circuit shown in Figure 22.

TRANSFER FUNCTION

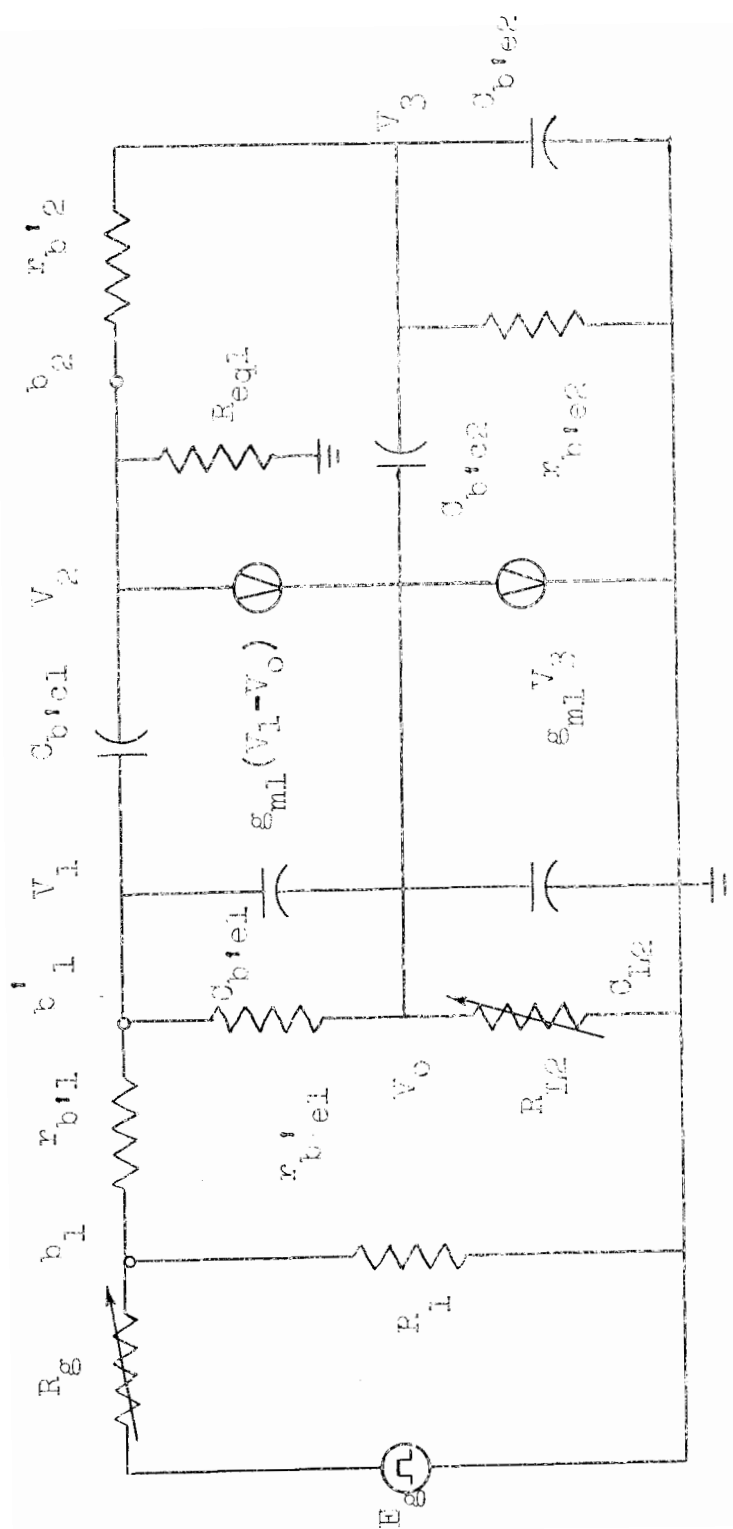
The nodal equations and the derivation of the transfer function for the equivalent circuit shown in Figure 22 are given in Appendix IV. The transfer function has the form

$$\frac{V_0}{E_g} = \frac{a_1 s^2 + a_2 s + a_3}{b_1 s^3 + b_2 s^2 + b_3 s + b_4}, \quad (44)$$

where the coefficients (a_1 , b_1 , etc.), given in Appendix IV, are functions of the transistor and circuit parameters.

Once the transistor for the circuit has been chosen, the transistor parameters are fixed for a specified d-c current level. The variable circuit parameters are R_g , R_{L2} , and R_{L1} . R_{L1} is normally not a circuit variable after the circuit has initially been designed, but R_g and R_{L2} may be varied to suit particular applications. Therefore, only the effects of the circuit parameters R_g and R_{L2} were considered.

An amplifier is unstable if non-negative real roots or non-negative real parts of complex roots of the characteristic equation exist⁷. By applying Routh's criterion



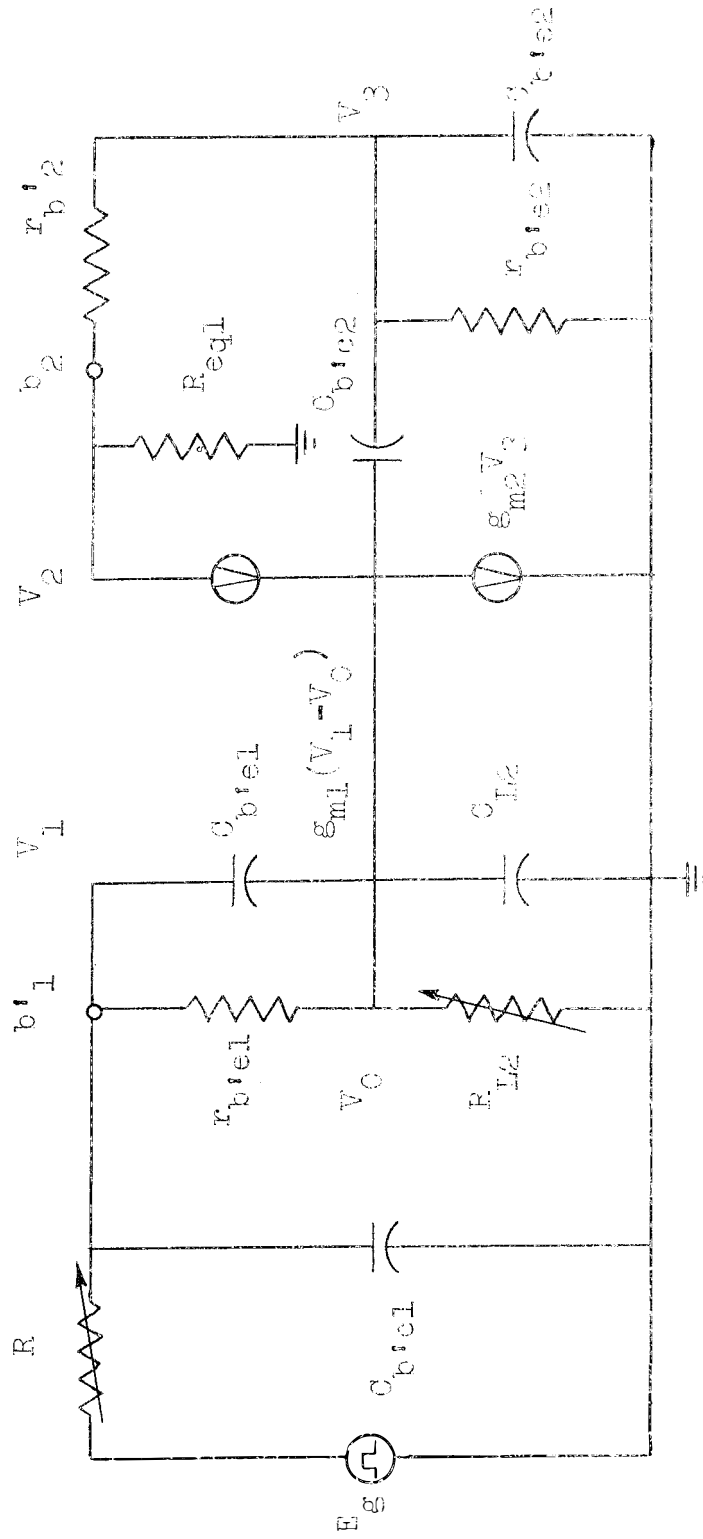


Figure 22. Reduced Equivalent Circuit of The White Emitter-Follower

to the cubic characteristic equation given in Equation (44), the amplifier is stable or no non-negative real roots exist if $b_1 b_4 < b_2 b_3$. The values of b_1 , b_2 , b_3 , and b_4 as a function of R and R_{L2} for the equivalent circuit shown in Figure 22 are:

$$b_1 = 1.556 R R_{L2} \quad , \quad (45)$$

$$b_2 = 1.08 \times 10^{11} R + 1.22 \times 10^{11} R_{L2} + 1.98 \times 10^9 R R_{L2} \quad , \quad (46)$$

$$b_3 = 2.55 \times 10^{20} R_{L2} + 3.43 \times 10^{18} R + 2.08 \times 10^{21} + 2.01 \times 10^{18} R R_{L2} \quad , \quad (47)$$

$$b_4 = 5.01 \times 10^{29} R_{L2} + 2.47 \times 10^{25} R + 4.23 \times 10^{28} \quad , \quad (48)$$

where R is $r_{b'c} + R_g$. Using Routh's criterion for the values given in Equation (45) through (48), the characteristic equation is always stable for positive values of R_g and R_{L2} . Using the experimental circuit shown in Figure 22 with a pulse input, the circuit was stable for all values of R_g and R_{L2} except values of R_g less than approximately 50 ohms. When the value of R_g was less than

approximately 50 ohms, the circuit would oscillate. The oscillation was caused by the negative input resistance of the circuit present at high frequencies; a certain amount of generator resistance was necessary to cancel this negative input resistance.

In most pulse applications, excessive overshoot cannot be tolerated. Therefore, considering useful applications of this circuit, overshoot up to 15 per cent was considered. The experimental data shown in this section was obtained using the circuit shown in Figure 20 with a pulse input. The variables, R_g and R_{L2} , were plotted as a function of the per cent overshoot for the calculated and experimental responses. The critically damped, 5 per cent overshoot, and 15 per cent overshoot cases are shown in Figures 23, 24, and 25 respectively. From these curves, the value of R_g and R_{L2} for a required amount of overshoot can be determined. For example, considering the experimental results, if R_{L2} is 1000 ohms, then R_g must be 600 ohms in order for the output response to be critically damped. If R_g is larger than 600 ohms when R_{L2} is 1000 ohms, the response will be underdamped. If R_g is smaller than 600 ohms, overshoot results. The deviation between the calculated and experimental data will be discussed at the end of this section.

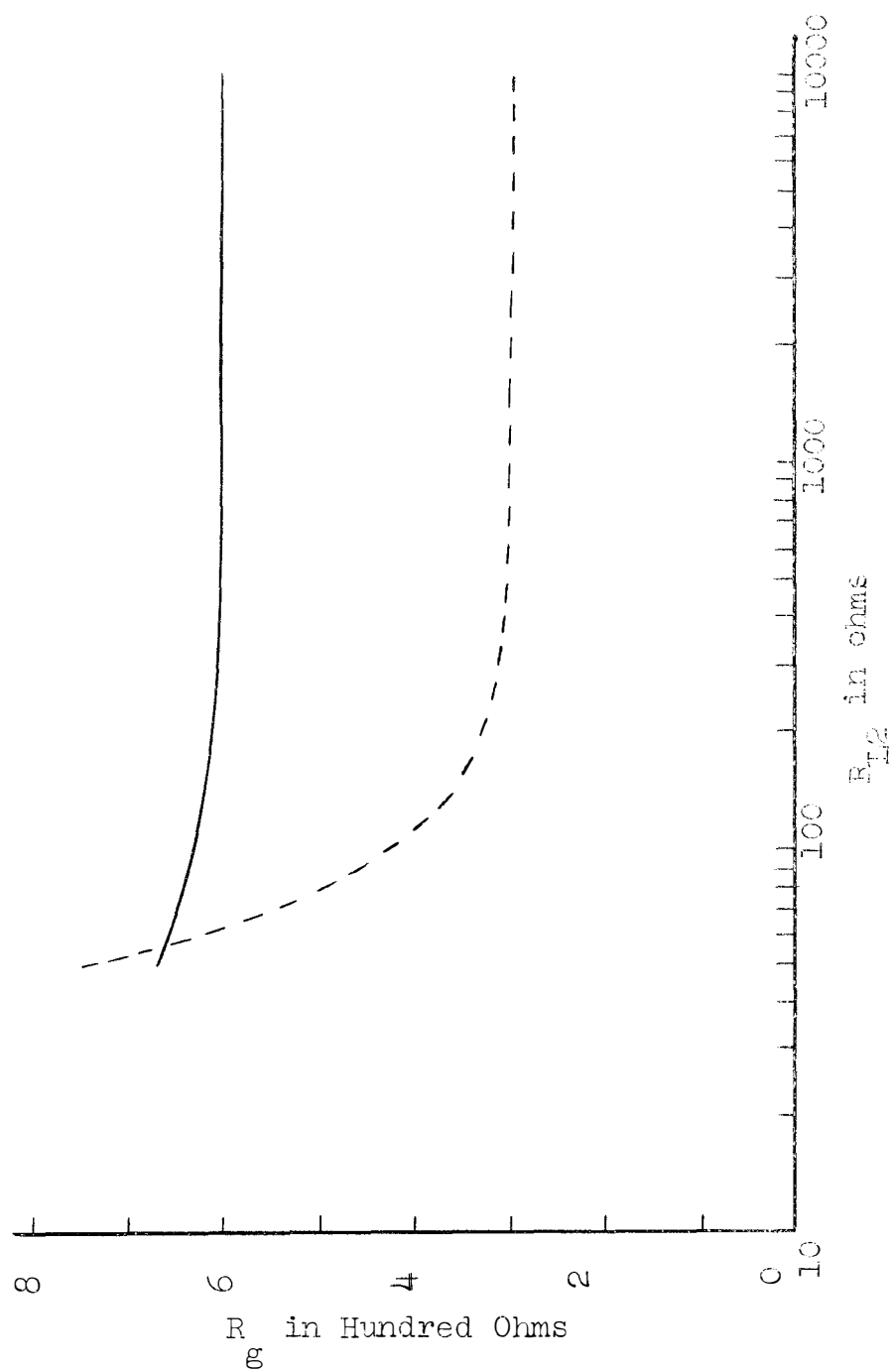


Figure 23. Generator Impedance, R_g , as a Function of Load Impedance, R_{L2} , For The Critically Damped Case. Solid Curve from Measured Data, Dashes Indicate Calculated Data.

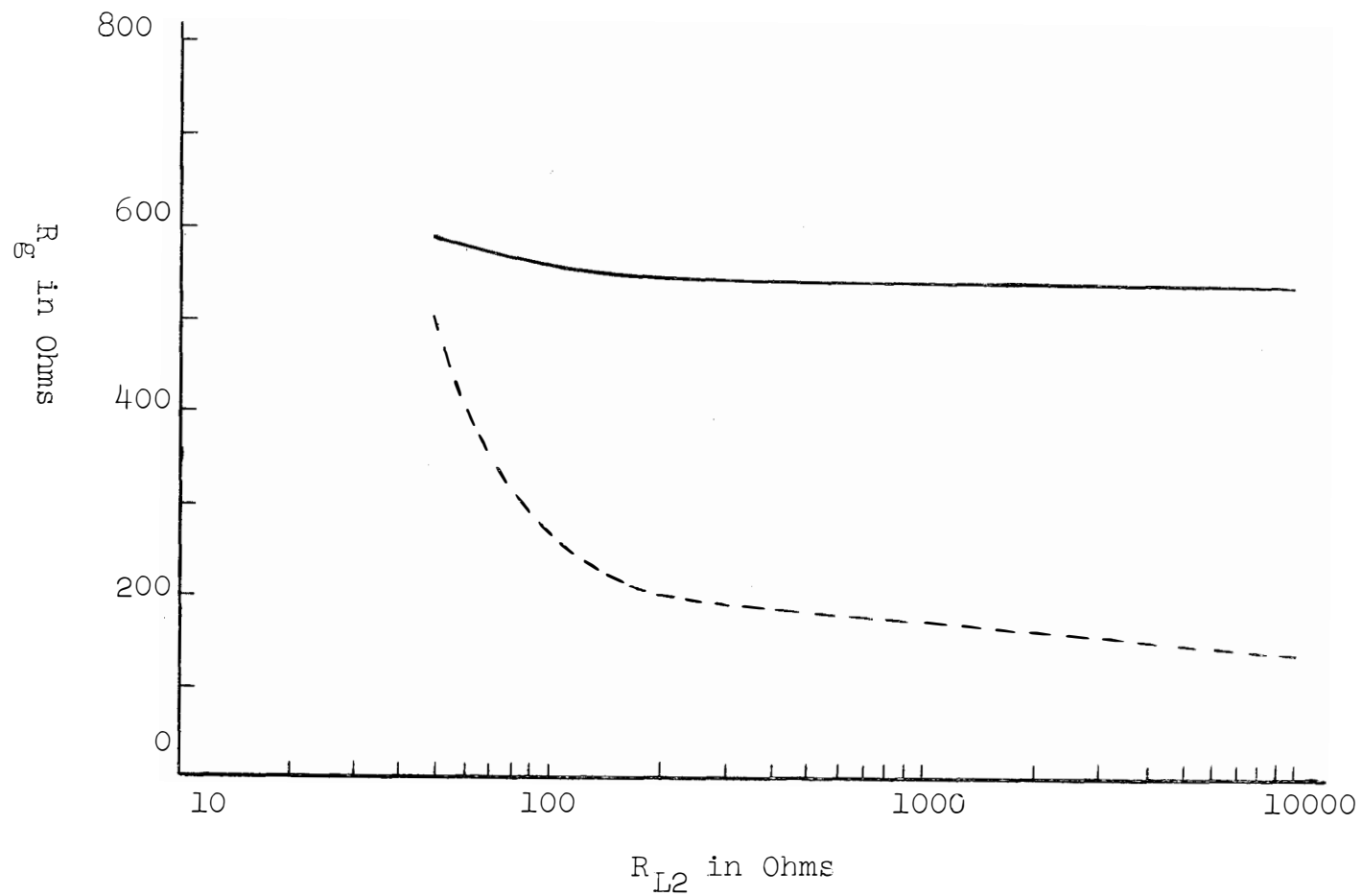


Figure 24. Generator Impedance, R_g , as a Function of Load Impedance, R_{L2} , For The Five Per Cent Overshoot Case. Solid Curve from Measured Data, Dashes Indicate Calculated Data

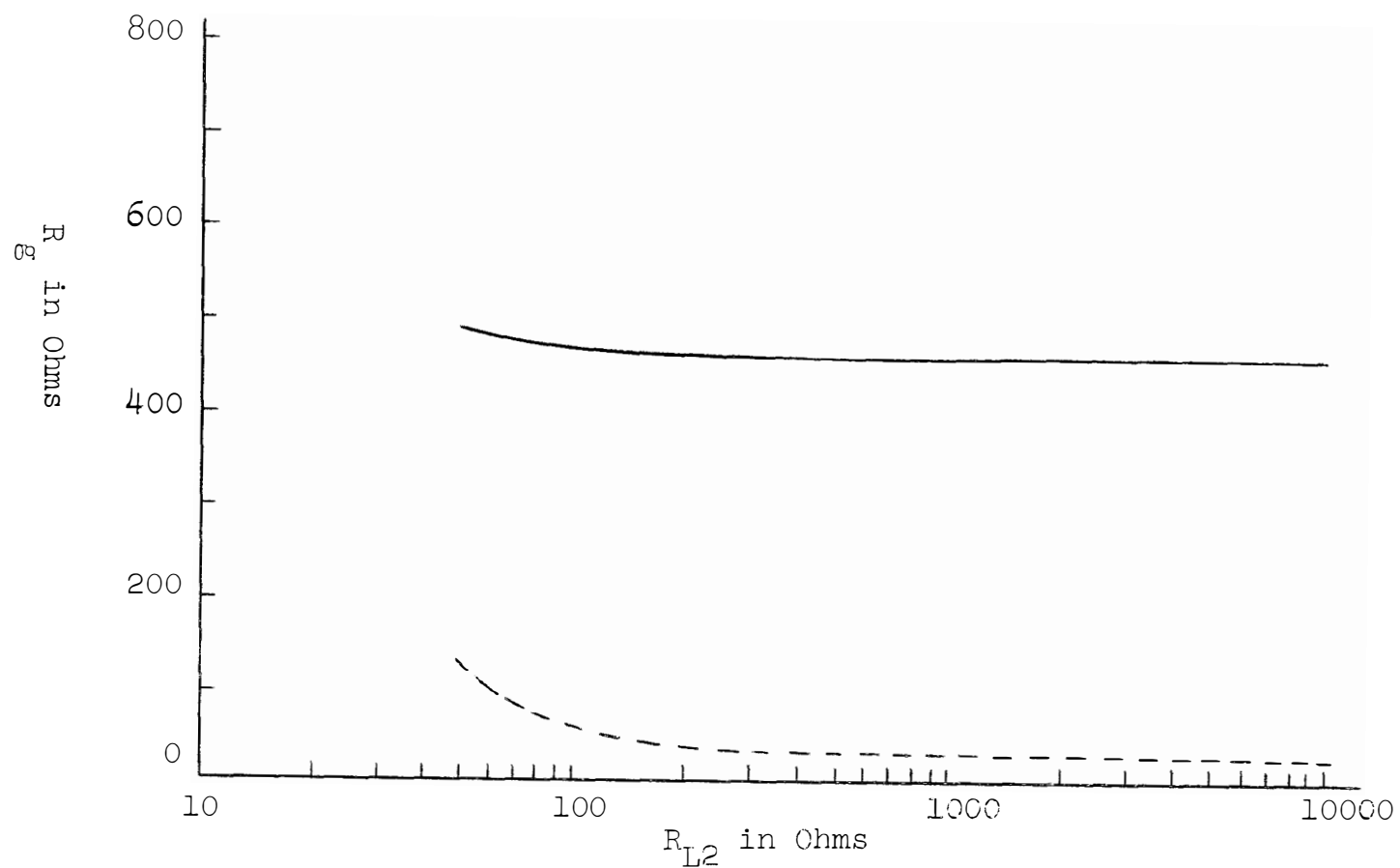


Figure 25. Generator Impedance, R_g , as a Function of Load Impedance, R_{L2} , For The Fifteen Per Cent Overshoot Case. Solid Curve from Measured Data, Dashes Indicate Calculated Data

The rise time as a function of R_g and R_{L2} for both the calculated and experimental response is shown in Figure 26. The load resistance, R_{L2} , does not have an appreciable effect on the rise time as shown in Figure 26. The difference in rise time between $R_{L2} = 1000$ ohms and $R_{L2} = 50$ ohms for values of R_g less than 1000 ohms is only a fraction of a nanosecond. Therefore, the rise time may be determined by R_g when R_g is less than 1000 ohms.

FREQUENCY RESPONSE

The bandwidth, B , of an amplifier is determined by the frequencies for which the response is 3db below the mid-frequency response. Consider the calculated critically damped case for which R_{L2} is 1000 ohms and R_g is 300 ohms.

The transfer function is

$$\frac{V_0}{E_g} = \frac{6.8 \times 10^6 s^2 + 2.54 \times 10^{17} s +}{s^3 + 1.477 \times 10^9 s^2 + 1.577 \times 10^{18} s +} - - - - - \frac{5.43 \times 10^{26}}{5.56 \times 10^{26}} \quad . \quad (49)$$

Factoring the numerator and denominator and rearranging the constants, the transfer function has the following form:

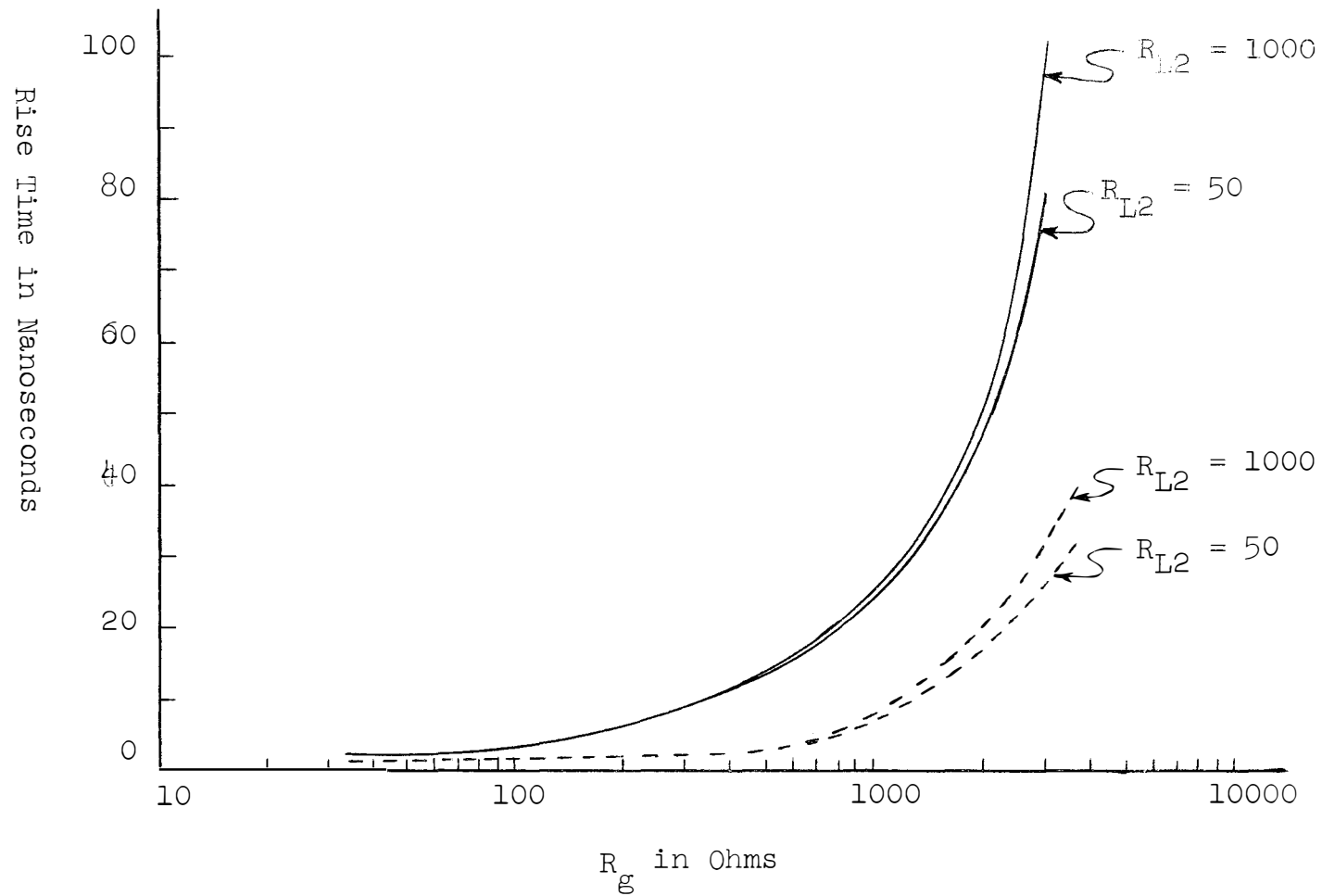


Figure 26. Rise Time as a Function of Generator Impedance, R_g , And Load Impedance, R_{L2} . Solid Curve from Measured Data, Dashes Indicate Calculated Data

$$\frac{V_0}{E_g} = \frac{1.252 \times 10^{-20} s^2 + .467 \times 10^{-9} s +}{(2 \times 10^{-9} s + 1)(.918 \times 10^{-18} s^2 + \frac{1}{.897 \times 10^{-9} s + 1})} \quad (50)$$

Substituting $j\omega = s$ it follows that:

$$\frac{V_0}{E_g} = \frac{1.252 \times 10^{-20} (j\omega)^2 + .467 \times 10^{-9} j\omega +}{(2 \times 10^{-9} j\omega + 1) \left[.918 \times 10^{-18} (j\omega)^2 + \frac{1}{.897 \times 10^{-9} j\omega + 1} \right]} \quad (51)$$

Standard frequency response techniques are available to determine the frequency response of the transfer function given by equation (51)⁷. The quadratic factors in the numerator and denominator can be arranged to have the form $\left[1 + 2\zeta(j\omega\tau) + (j\omega\tau)^2 \right]^{\pm 1}$. The response obtained from this term is a function of both τ and the damping ratio ζ . Graphical solutions for the factor $\left[1 + 2\zeta(j\omega\tau) + (j\omega\tau)^2 \right]^{\pm 1}$ as a function of ζ are found in most servo texts.⁷ For the numerator of Equation (51), ζ is greater than unity, and the roots are real and unequal. In the denominator of Equation (51) ζ is less than unity, and the roots are complex. Factoring the numerator of Equation (51) and rearranging the denominator, the transfer function has

the form

$$\frac{V_o}{E_g} = \frac{(.287 \times 10^{-10} j\omega + 1)(4.35 \times 10^{-10} j\omega + 1)}{(2 \times 10^{-9} j\omega + 1)(9.57 \times 10^{-10} j\omega)^2} \cdot \frac{1}{.897 \times 10^{-9} j\omega + 1} \quad (52)$$

The high-frequency response may be constructed using the transfer function given in Equation (52). The response due to each factor of Equation (52) may be plotted; the total high-frequency response is a combination of the individual responses of the terms in Equation (52). The total high-frequency response is shown in Figure 27. The calculated high-frequency response is 3db below the mid-frequency response at approximately 100 megacycles.

The rise time of an amplifier is inversely proportional to the bandwidth. If the overshoot is small, less than 5 per cent, the rise time can be approximated³ with good accuracy by the relation

$$t_r = \frac{0.35}{B} \quad (53)$$

The calculated bandwidth, B, for the critically damped case is 100 megacycles. Therefore, using Equation (53), the rise time is approximately 3.5 nanoseconds. From Figure 24 the rise time is also 3.5 when R_g is 300 ohms. The equal

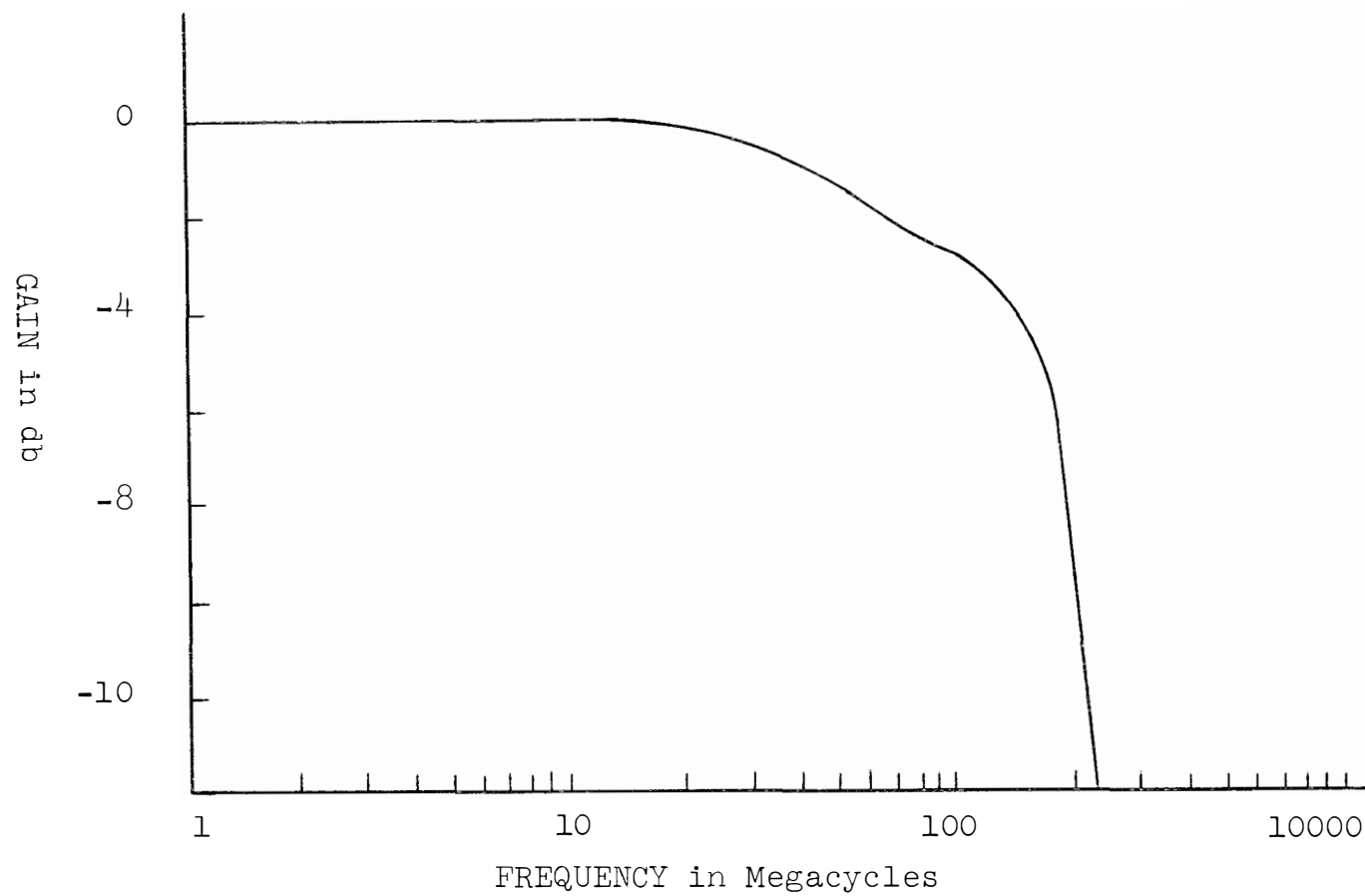


Figure 27. Calculated High-Frequency Response

values of rise time were determined by two different methods: One value of rise time was determined by the analog computer, and the other value was determined by the calculated frequency response. Using Equation (53), the measured high-frequency response should be 3db below the mid-frequency response at approximately 40 megacycles.

A significant deviation between the measured and calculated responses was noted. Two probable sources of this deviation are equivalent circuit simplifications and uncertainty in the transistor parameters. Two parameters were measured, β and h_{ie} ; the remaining parameters were taken from manufacturer's data sheets and calculated from approximate equations. The more exact equivalent circuit shown in Figure 21 should be used for better circuit approximation. Some stray and wiring capacitance should also be assumed on the output of the first stage. The transfer function of the proposed equivalent circuit would be more complicated, but more accurate calculations should result. Although the calculated results of this section deviated from the measured values; they do exhibit the same characteristics for changes in the variables.

CONCLUSION

The analysis of the White emitter-follower, indicated that this circuit exhibits definite advantages over the conventional emitter-follower in four areas: gain, input impedance, output impedance, and amplification of bipolar pulses. The feedback in the White emitter-follower accounts for the advantages.

A major disadvantage of the emitter-follower is the variation in beta, for large collector current swings, which results in unequal amplification of bipolar pulses. It has been shown that the White emitter-follower does not exhibit this non-linear beta characteristic due to the compensation of the feedback loop. A decrease in beta in the White emitter-follower will result in an increase in the driving signal, V_{be1} , which will tend to compensate for the beta reduction. The feedback yields an effective linear beta not possible in the emitter-follower configuration. Therefore, the White emitter-follower is able to amplify bipolar pulses with approximately equal amplitudes.

High values of input impedances (in the megohm range) can be obtained because of the feedback in the White emitter-follower. The collector current of Q_2 flowing through the load, R_{L2} , contributes a factor to the input impedance that is not available with the emitter-follower. For values of R_g and R_{L2} equal to 1000 ohms, the White

emitter-follower investigated in this study had an input impedance approximately 30 times that of the emitter-follower. For load variations, R_{L2} , ranging from 50 ohms to 5 kilohms, the input impedance at the base of Q_1 varied from 0.83 megohms to 9.1 megohms. Obviously the bias resistor must be considered in the determination of the circuit input impedance because of its shunting effect. For values of the load, R_{L2} , greater than several hundred ohms the bias resistor, R_1 , usually limits the input impedance. If a separate power supply is used for R_1 , the value of R_1 may be increased, and thus the circuit input impedance could be increased.

The very low output impedance available with the White emitter-follower was shown to be practically independent of the load impedance, R_{L2} . Some slight variation was noted for changes of R_{eq1} (parallel combination of R_{L1} , R_3 and R_4) and R_g . For variations of R_{eq1} and R_g ranging from 400 ohms to 4 kilohms, R_o varied from 0.4 ohms to 0.27 ohms and 0.27 to 0.43 ohms respectively. Therefore, the output impedance of this circuit is less than 0.44 ohms for any value of R_g and R_{eq1} between the values of 400 ohms and 4 kilohms. When R_g and R_{L2} equal 1000 ohms, an emitter-follower configuration has a calculated output impedance of 15 ohms, whereas the White emitter-follower has an output impedance equal to

0.30 ohms.

The gain of the White emitter-follower was shown to be practically independent of any variations of R_{L1} and R_{L2} for low values of R_g (less than 1 kilohms). For variations of R_{L1} and R_{L2} from 300 ohms to 3 kilohms the gain varied from 0.996 to 0.998 and from 0.990 to 0.998 respectively. When R_g and R_{L2} are equal to 1000 ohms, the emitter-follower has a calculated gain of 0.984, whereas the White emitter-follower has a gain of 0.998.

The low-frequency response of the White emitter-follower is limited by the value of capacitors C_2 and C_3 . Increasing these values from the values used in this study will lower the low-frequency response. Since the value of C_3 (580 μ f) was quite large, it will ultimately become the limiting factor in the low-frequency response.

For pulse applications, overshoot and rise time are a major consideration; R_g and R_{L2} are the controlling circuit parameters of overshoot and rise time. R_g is the primary consideration when fast rise time is important; variations of R_{L2} do not significantly alter the rise time. If fast rise times (on the order of a few nanoseconds) are required, then low values of R_g are necessary; however, low values of R_g contribute to overshoot. Therefore, a compromise between rise time and overshoot must be determined to arrive at a suitable value of R_g . Having determined R_g for a suitable rise time, R_{L2} may be determined

for a particular value of overshoot. For example, if a rise time of 15 nanoseconds is required, R_g must be limited to a value of 600 ohms. Then R_{L2} should be 500 ohms or greater for the output response to be critically damped. However, if five per cent overshoot is allowable, the value of R_g may be reduced, and thus a faster rise time is available.

Other suggested areas of future study are: 1) a linearity analysis of the circuit, 2) a temperature stability design, 3) an investigation of the oscillatory tendencies, with a circuit stability plot, and 4) an extended transient analysis using a more exact equivalent circuit.

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APPENDICES

APPENDIX ITRANSISTOR PARAMETERS

The transistor parameters for the 2N2219 transistor biased at $I_E = 3$ ma are

$$\beta = 200 \quad (\text{Measured})$$

$$h_{ie} = 2 \text{ Kilohms} \quad (\text{Measured})$$

$$f_t = 400 \text{ MC} \quad (\text{Manufacturer's data sheet})$$

$$C_{b'c} = 4 \text{ pf} \quad (\text{Manufacturer's data sheet})$$

$$r_c = 10 \text{ megohms} \quad (\text{Manufacturer's data sheet})$$

$$C_L = 7 \text{ pf} \quad (\text{Estimated})$$

$$r_e = \frac{K_1}{qI_E} = \frac{25 \text{ mv}}{I_E} = 8.34 \text{ ohms} \quad (54)$$

$$g_m = \frac{1}{r_e} = 120 \times 10^{-3} \text{ mhos} \quad (55)$$

$$r_{b'e} = (\beta + 1)r_e = 1.70 \text{ Kilohms} \quad (56)$$

$$r_{b'c} = h_{ie} - r_{b'e} = 290 \text{ Ohms} \quad (57)$$

$$C_{b'e} = \frac{g_m}{2\pi f_t} = 48 \text{ pf} \quad (58)$$

APPENDIX II   DERIVATION OF GAIN AROUND FEEDBACK LOOP

The four nodal equation for the mid-frequency equivalent circuit shown in Figure 8 are as follows:

$$V_{in} \frac{1}{r_{b'e1}} = \left(\frac{1}{r_{b'e1}} + \frac{1}{r_{b'1}} \right) V_1 - \frac{1}{r_{b'e1}} V_o \quad (59)$$

$$0 = - \left(\frac{1}{r_{b'e1}} + g_{m1} \right) V_1 + \left(\frac{1}{R_{L2}} + \frac{1}{r_{b'e1}} + g_{m1} \right) V_o + (g_{m2}) V_{b'e2} \quad (60)$$

$$0 = (g_{m1}) V_1 - (g_{m1}) V_o + \left(\frac{1}{R_{eq1}} + \frac{1}{r_{b'2}} \right) V_2 - \frac{1}{r_{b'2}} V_{b'e2} \quad (61)$$

$$0 = - \frac{1}{r_{b'2}} V_2 + \left(\frac{1}{r_{b'2}} + \frac{1}{r_{b'e2}} \right) V_{b'e2} \quad (62)$$

Solving Equations (59) through (62) for the voltage gain,

$\frac{V_o}{V_{in}}$, using Cramer's rule, the gain is

$$A' = \frac{V_o}{V_{in}} = \frac{\frac{1}{r_{b'1}} \left(\frac{1}{r_{b'e1}} + g_{m1} \right) \left(\frac{1}{R_{eq1} r_{b'2}} + \frac{1}{R_{eq1} r_{b'e2}} + \frac{1}{r_{b'1} R_{L2}} \right)}{\left[\frac{1}{r_{b'1}} \left(\frac{1}{r_{b'e1}} + g_{m1} \right) + \frac{1}{r_{b'1} R_{L2}} + \frac{1}{r_{b'2} r_{b'e2}} \right] + \frac{\frac{1}{R_{eq1} r_{b'2}} + \frac{1}{R_{eq1} r_{b'e2}} + \frac{1}{r_{b'1} R_{L2}}}{\frac{g_{m1} g_{m2}}{r_{b'1} r_{b'2}} + \frac{1}{r_{b'2} r_{b'e2}}} \cdot \quad (63)$$

If the transistors are assumed to have the same parameter values, then the gain equation is

$$A' = \frac{V_o}{V_{in}} = \frac{\frac{1}{r_{b'1}} \left(\frac{1}{r_{b'e}} + g_m \right) \left(\frac{1}{R_{eq1} r_{b'}} + \frac{1}{R_{eq1} r_{b'e}} + \frac{1}{r_{b'1} R_{L2}} \right)}{\left[\frac{1}{r_{b'1}} \left(\frac{1}{r_{b'e}} + g_m \right) + \frac{1}{r_{b'1} R_{L2}} + \frac{1}{r_{b'e} R_{L2}} \right] \left(\frac{1}{R_{eq1} r_{b'}} + \frac{1}{R_{eq1} r_{b'e}} + \frac{1}{r_{b'1} R_{L2}} \right) + \frac{\frac{1}{R_{eq1} r_{b'}} + \frac{1}{R_{eq1} r_{b'e}} + \frac{1}{r_{b'1} R_{L2}}}{\frac{g_m^2}{(r_{b'})^2} + \frac{1}{r_{b'1} r_{b'e}}} \cdot \quad (64)$$

Equation (64) may be rewritten as

$$A' = \frac{1}{1 + \left| \frac{\frac{1}{R_{L2}} \left(\frac{1}{r_{b'}} + \frac{1}{r_{b'e}} \right) \left(\frac{1}{R_{eq1} r_{b'}} + \frac{1}{R_{eq1} r_{b'e}} + \frac{1}{r_{b'}} \left(\frac{1}{r_{b'e}} + g_m \right) \left(\frac{1}{R_{eq1} r_{b'}} + \frac{1}{R_{eq1} r_{b'e}} + \frac{1}{r_{b'}} \left(\frac{1}{r_{b'e}} + g_m \right) \right) \right.}{\left. \frac{1}{r_{b'}} \left(\frac{1}{r_{b'e}} + g_m \right) \left(\frac{1}{R_{eq1} r_{b'}} + \frac{1}{R_{eq1} r_{b'e}} + \frac{1}{r_{b'}} \left(\frac{1}{r_{b'e}} + g_m \right) \right) + \frac{g_m^2}{(r_{b'})^2}} \right|} . \quad (65)$$

since $r_{b'e} \cong r_e(\beta+1)$ Equation (65) can be rewritten as

$$A' \cong \frac{1}{1 + \left| \frac{\frac{h_{ie}}{R_{L2}(\beta+1)} \left[\frac{r_e}{R_{eq1}} + \frac{r_{b'}}{R_{eq1}(\beta+1)} + \frac{1}{(\beta+1)} \right]}{\left[\frac{r_e}{R_{eq1}} + \frac{r_{b'}}{R_{eq1}(\beta+1)} + \frac{1}{(\beta+1)} \right] + 1} \right|} . \quad (66)$$

If R_{eq1} is greater than approximately 1000 ohms, then

Equation (66) can be simplified to

$$A' \cong \frac{1}{1 + \frac{h_{ie}}{R_{L2}(\beta+1)} \left[\frac{r_e}{R_{eq1}} + \frac{r_{b'}}{R_{eq1}(\beta+1)} + \frac{1}{(\beta+1)} \right]} . \quad (67)$$

APPENDIX III   CALCULATIONS OF LOW-FREQUENCY RESPONSES

Low-frequency calculations of the individual responses due to the coupling and bypass capacitors and their associated resistance are given in this Appendix. The general equation is

$$f = \frac{1}{2 \pi RC} \quad , \quad (68)$$

where C is the value of the coupling or bypass capacitor and R is the effective value of the resistance across C.

$$f_1 = \frac{1}{2 \pi (R_g + R_{i1}) C_1} = 0.02 \frac{\text{cyc.}}{\text{sec.}} \quad (69)$$

$$f_2 = \frac{1}{2 \pi (R_{L1} + R_{i2}) C_2} = 19.0 \frac{\text{cyc.}}{\text{sec.}} \quad (70)$$

$$f_3 = \frac{1}{2 \pi \left(\frac{R_{e2} R_{ib2}}{R_{e2} + R_{ib2}} \right) C_3} = 14.4 \frac{\text{cyc.}}{\text{sec.}} \quad (71)$$

where

$$R_{ib2} = \frac{h_{ie2}}{\beta_2} + \frac{R_{B2}}{\beta_2} \quad . \quad (72)$$

R_{B2} is the parallel combination of R_{L1} , R_3 and R_4 .

$$R_{B2} = \frac{\left(\frac{R_{L1} R_3}{R_{L1} + R_3} \right) R_4}{\left(\frac{R_{L1} R_3}{R_{L1} + R_3} \right) + R_4} \quad (73)$$

$$f_4 = \frac{1}{2\pi (R_0 + R_{L2}) C_4} = 2.84 \frac{\text{cyc.}}{\text{sec.}} \quad (74)$$

The four nodal equations for the high-frequency equivalent circuit of the White emitter-follower shown in Figure 22 are given in Equations (75) through (78). These equations are written in the Laplace transformed arrangement:

$$\frac{E_g}{R} = \left[\frac{1}{R} + \frac{1}{r_{b'e1}} + s(C_{b'e1} + C_{b'c1}) \right] V_1 - \left(\frac{1}{r_{b'e1}} + sC_{b'e1} \right) V_0, \quad (75)$$

$$0 = (g_{m1})V_1 + \left(\frac{1}{R_{eq1}} + \frac{1}{r_{b'2}} \right) V_2 - \left(\frac{1}{r_{b'2}} \right) V_3 - g_{m1} V_0, \quad (76)$$

$$0 = -\left(\frac{1}{r_{b'2}} \right) V_2 + \left[\frac{1}{r_{b'2}} + \frac{1}{r_{b'e2}} + s(C_{b'e2} + C_{b'c2}) \right] V_3 - (sC_{b'c2})V_0, \quad (77)$$

$$0 = -\left(\frac{1}{r_{b'e1}} + g_{m1} + sC_{b'e1} \right) V_1 + (g_{m2} - sC_{b'c2})V_3 + \left[\frac{1}{r_{b'e1}} + \frac{1}{R_{L2}} + g_{m1} + s(C_{b'e1} + C_L + C_{b'c2}) \right] V_0. \quad (78)$$

By the use of Cramer's rule with the above equations, the transfer function has the following form:

$$\frac{V_0}{E_g} = \frac{a_1 s^2 + a_2 s + a_3}{b_1 s^3 + b_2 s^2 + b_3 s + b_4} \quad (79)$$

The coefficients defined in terms of the transistor and circuit parameters are given below:

$$a_1 = \frac{1}{R} \left[\left(\frac{1}{R_{eq1}} + \frac{1}{r_{b'2}} \right) (C_{b'e2} + C_{b'c2}) C_{b'e1} \right] \quad (80)$$

$$a_2 = \frac{1}{R} \left[\left(\frac{1}{R_{eq1}} + \frac{1}{r_{b'2}} \right) \left[\left(g_{m1} + \frac{1}{r_{b'e1}} \right) C_{b'e2} + \frac{C_{b'c1}}{r_{b'e1}} + \frac{C_{b'e1}}{r_{b'e2}} \right] + \frac{C_{b'e1}}{R_{eq1} r_{b'2}} + \frac{g_{m1} C_{b'c2}}{R_{eq1}} \right] \quad (81)$$

$$a_3 = \frac{g_{m1} g_{m2}}{R r_{b'2}} + \frac{1}{R} \left(g_{m1} + \frac{1}{r_{b'e1}} \right) \left(\frac{1}{r_{b'e2} R_{eq1}} + \frac{1}{r_{b'e2} r_{b'2}} + \frac{1}{R_{eq1} r_{b'2}} \right) \quad (82)$$

$$b_1 = \left(\frac{1}{R_{eq1}} + \frac{1}{r_{b'2}} \right) \left[(C_{b'e1} + C_{b'c1}) \left[C_{b'e2} (C_L + C_{b'c2}) + C_{b'c2} C_L \right] + C_{b'e1} C_{b'c1} (C_{b'e2} + C_{b'c2}) \right] \quad (83)$$

$$\begin{aligned}
b_2 = & \left(\frac{1}{R_{eq1}} + \frac{1}{r_{b'2}} \right) \left[\left(C_{b'e1} + C_{b'c1} \right) \left(\frac{C_{b'c2} + C_L}{r_{b'e2}} + \right. \right. \\
& \left. \left. \frac{C_{b'c2} + C_{b'e2}}{R_{L2}} + g_{m2} C_{b'c2} \right) + \frac{C_{b'c1} C_{b'e1}}{r_{b'e2}} + \right. \\
& \left(\frac{1}{R} + \frac{1}{r_{b'e1}} \right) \left(C_{b'e2} C_{b'c2} \right) + \left(C_{b'e2} + \right. \\
& \left. C_{b'c2} \right) \left(\frac{C_{b'e1} + C_L}{R} + \frac{C_{b'c1} + C_L}{r_{b'e1}} \right) + \\
& \left. g_{m1} C_{b'e2} C_{b'c1} \right] + \\
& \frac{1}{R_{eq1}} \left[\frac{(C_{b'e1} + C_{b'c1})(C_{b'e2} + C_{b'c2})}{r_{b'2}} + \right. \\
& \left. \frac{C_{b'c1} C_{b'e1}}{r_{b'2}} + g_{m2} C_{b'c1} C_{b'c2} \right] , \tag{84}
\end{aligned}$$

$$\begin{aligned}
b_3 = & \left(\frac{1}{R_{eq1} r_{b'2}} + \frac{1}{R_{eq1} r_{b'e2}} + \frac{1}{r_{b'2} r_{b'e2}} \right) \left[\left(\frac{1}{R} + \right. \right. \\
& \left. \frac{1}{R_{L2}} \right) c_{b'e1} + \left(\frac{1}{R} + \frac{1}{r_{b'e1}} \right) (c_L + c_{b'c2}) + \left(\frac{1}{r_{b'e1}} + \right. \\
& \left. \frac{1}{R_{L2}} + g_{m1} \right) c_{b'c1} \left. \right] + \left(\frac{1}{R_{eq1}} + \frac{1}{r_{b'2}} \right) \left(\frac{1}{R r_{b'e1}} + \right. \\
& \left. \frac{1}{R R_{L2}} + \frac{1}{r_{b'e1} R_{L2}} + \frac{g_{m1}}{R} \right) (c_{b'e2} + c_{b'c2}) + \\
& g_{m2} c_{b'c2} \left(\frac{1}{r_{b'e1} r_{b'2}} + \frac{1}{R R_{eq1}} + \frac{1}{r_{b'e1} R_{eq1}} \right) + \\
& \frac{g_{m1} g_{m2}}{r_{b'2}} c_{b'c1} \quad , \tag{85}
\end{aligned}$$

$$\begin{aligned}
b_4 = & \frac{g_{m1} g_{m2}}{R r_{b'2}} + \left(\frac{1}{R_{eq1} r_{b'2}} + \frac{1}{R_{eq1} r_{b'e2}} + \right. \\
& \left. \frac{1}{r_{b'2} r_{b'e2}} \right) \left(\frac{1}{R r_{b'e1}} + \frac{1}{R R_{L2}} + \frac{1}{R_{L2} r_{b'e1}} + \frac{g_{m1}}{R} \right). \tag{86}
\end{aligned}$$

Solving the above equations in terms of R and R_{L2} gives

$$a_1 = \frac{6.24 \times 10^{-25}}{R} \tag{87}$$

$$a_2 = \frac{2.33 \times 10^{-14}}{R} \tag{88}$$

$$a_3 = \frac{4.98 \times 10^{-5}}{R} \quad (89)$$

$$b_1 = 1.556 \times 10^{-34} \quad (90)$$

$$b_2 = \frac{1.08 \times 10^{-23}}{R_{L2}} + \frac{1.22 \times 10^{-23}}{R} + 1.98 \times 10^{-25} \quad (91)$$

$$b_3 = \frac{2.55 \times 10^{-14}}{R} + \frac{3.43 \times 10^{-16}}{R_{L2}} + \frac{2.08 \times 10^{-13}}{RR_{L2}} +$$

$$2.01 \times 10^{-16} \quad (92)$$

$$b_4 = \frac{5.01 \times 10^{-5}}{R} + \frac{2.47 \times 10^{-9}}{R_{L2}} + \frac{4.23 \times 10^{-6}}{RR_{L2}} \quad (93)$$

Multiplying the numerator and denominator of the transfer function by 10^{34} yields the following values for the coefficients:

$$b_1 = 1.556 \text{ } RR_{L2} \quad (94)$$

$$b_2 = 1.08 \times 10^{11} R + 1.22 \times 10^{11} R_{L2} +$$

$$1.98 \times 10^9 \text{ } RR_{L2} \quad (95)$$

$$b_3 = 2.55 \times 10^{20} R_{L2} + 3.43 \times 10^{18} R + 2.08 \times 10^{21} +$$

$$2.01 \times 10^{18} \text{ } RR_{L2} \quad (96)$$

$$b_4 = 5.01 \times 10^{29} R_{L2} + 2.47 \times 10^{25} R + 4.23 \times 10^{28} \quad (97)$$

$$a_1 = \frac{6.24 \times 10^9}{R} \quad (98)$$

$$a_2 = \frac{2.33 \times 10^{20}}{R} \quad (99)$$

$$a_3 = \frac{4.98 \times 10^{29}}{R} \quad (100)$$