



8-2013

# A Multi-echelon Inventory System with Supplier Selection and Order Allocation under Stochastic Demand

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## Recommended Citation

Guo, Cong, "A Multi-echelon Inventory System with Supplier Selection and Order Allocation under Stochastic Demand." Master's Thesis, University of Tennessee, 2013.  
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To the Graduate Council:

I am submitting herewith a thesis written by Cong Guo entitled "A Multi-echelon Inventory System with Supplier Selection and Order Allocation under Stochastic Demand." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Industrial Engineering.

Xueping Li, Major Professor

We have read this thesis and recommend its acceptance:

James Ostrowski, Mingzhou Jin

Accepted for the Council:

Dixie L. Thompson

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

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# **A Multi-echelon Inventory System with Supplier Selection and Order Allocation under Stochastic Demand**

A Thesis Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Cong Guo

August 2013

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# Acknowledgements

I would like to express my sincere gratitude to my advisor, Dr. Xueping Li, for his guidance into my research life. I sincerely thank him for giving me the opportunity to be part of his research group and for his persistent support and understanding. I would also like to thank my thesis committee members: Dr. Mingzhou Jin and Dr. James Ostrowski, for helping me complete this thesis through all my hurdles.

I am grateful to all my colleagues in the Department of Industrial Systems Engineering who have assisted me in the course, and shared their graduate school life with me.

Last, but certainly not the least, I would like to acknowledge the commitment, sacrifice and support of my parents, who have always motivated me.

# Abstract

This article addresses the development of an integrated supplier selection and inventory control problems in supply chain management by developing a mathematical model for a multi-echelon system. In particular, a buyer firm that consists of one warehouse and  $N$  identical retailers wants to procure a product from a group of potential suppliers, which may require different price, ordering cost, lead time and have restriction on minimum and maximum total order size, to satisfy the stochastic demand. A continuous review system that implements the order quantity, reorder point  $(Q, R)$  inventory policy is considered in the model. The objective of the model is to select suppliers and to determine the optimal inventory policy that coordinates stock level between each echelon of the system while properly allocating orders among selected suppliers to maximize the expected profit. The model has been solved by decomposing the mixed integer nonlinear programming model into two sub-models. Numerical experiments are conducted to evaluate the model and some managerial insights are obtained by performing some sensitivity analysis.

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# Chapter 1

## Introduction

In today's circumstance of the global economic crisis, companies are facing increasing challenges to reduce operational costs, enlarge profit margins and remain competitive. People are forced to take advantages of any opportunity to optimize their business process and improve the performance of the entire supply chain. For most industrial firms, the purchasing of raw material and component parts from suppliers constitutes a major expense. For example, pointed out by [Hayes et al. \(2005\)](#) and [Wadhwa and Ravindran \(2007\)](#), it is expected that more and more manufacturing activities will be outsourced. Hence, among the various strategic activities involved in the supply chain management, the purchase decision has critical impacts on costs.

According to [Aissaoui et al. \(2007\)](#), there are six major purchasing decision processes: (1) 'make or buy', (2) supplier selection, (3) contract negotiation, (4) design collaboration, (5) procurement and (6) sourcing analysis. Among all of them, the supplier selection problem has received great attentions ([Jayaraman et al., 1999](#); [Liao and Rittscher, 2007](#); [Weber et al.,](#)

1991). Generally, the supplier selection problem is to define which supplier(s) should be selected and how much order quantity should be assigned to each selected supplier. It is a multi-criteria decision making process depended on a wide range of factors which involve both quantitative and qualitative ones (such as quality, cost, capacity, delivery, and technical potential).

Another relevant problem in supply chain management is to determine appropriate levels of inventory in each echelon. It is important to determine the quantity to order, the selections of suppliers and the best time to place an order. To derive optimal inventory policies that simultaneously determine how much, how often, and from which suppliers, typical ordering costs, purchasing costs, and holding costs should be considered. Although there are plenty of research for the supplier selection model, only limited studies focused on the inventory control policies integrated with supplier selection, especially under stochastic demand. However, considering the cost issue, supplier selection decision is actually highly correlated with some major logistics issues within a company such as inventory (stock level, delivery frequency, etc.) Incorporating the decisions to schedule orders over time with the supplier selection may significantly reduce costs over the planning horizon (Aissaoui et al., 2007). For example in the recent article (Mendoza and Ventura, 2010), the authors studied both supplier selection and inventory control problems under a serial supply chain system. A mathematical model was proposed to determine an optimal inventory policy in different stages and allocate proper orders to the selected suppliers. This paper extended the contributions to the research for the integration of supplier selection and inventory control problems in multi-level systems. However, the mathematical model built in this paper was based on a stationary inventory

policy with a constant demand. Moreover, the constant lead time, no backorder allowed and the same order quantity for different suppliers were assumed in the paper. These assumptions could be restrictive in reality, and it may not be appropriate to order the same quantity each time from different suppliers due to the different ordering cost and replenishment lead time. Thus, according to some more literature reviews given in Section 2, the decision model for supplier selection and inventory control policies in multi-level supply chain system requires further studies.

In this paper, we consider both supplier selection and inventory control problems in a serial supply chain system. A two-echelon distribution system with a central warehouse and  $N$  retailers is considered to procure from a set of suppliers. The supplier selection process is assumed to occur in the first stage of the serial supply chain, and the decision is made by a single decision maker (i.e., centralized control) who wants to reduce the total cost associated with the entire supply chain. Capacity, ordering cost, unit price, holding and backorder cost are considered as the criteria for the supplier selection. For the inventory control policy, a continuous review system which applies the order quantity, reorder point  $(Q, R)$  policy is adopted to determine the inventory level held at each echelon of the supply chain. The objective of the proposed integrated model is to coordinate the replenishment decision with the inventory at each echelon while properly selecting the set of suppliers which meets capacity restrictions. Thus, a mixed integer non-linear programming (MINLP) model is established to determine the best policy for the supplier selection and replenishment decisions.

The main contribution of this paper is to develop a multi-echelon inventory model for supplier selection problem under stochastic demand. To the authors' best knowledge, existing literature does not provide any model similar to the one proposed in this article. The remainder of this article is organized as follows. In Section 2, previous work related to the supplier selection is summarized. In Section 3, we present our problem definition and assumptions. In Section 4, the development and formulation of the proposed multi-echelon inventory model with supplier selection is presented. We implement the model by conducting some numerical experiments in Section 5. Finally, a summary of our work and an outlook on future research directions are presented in Section 6.

# Chapter 2

## Literature Review

The supplier selection problem has attracted great attentions of a number of researchers who proposed various decision models and solutions. Some previous review works for these decision methods have been presented in the literature. [Weber et al. \(1991\)](#) classified 74 related articles published from 1966 to 1990 which have addressed supplier selection problems based on different criteria and analytical methods. It was found that price, delivery and quality were the most discussed factors. Later in 2000, [Degraeve et al. \(2000\)](#) adopted the concept of Total Cost of Ownership (TCO) as a basis for comparing supplier selection models. They illustrated their model through a case study, and concluded that from a TCO perspective, mathematical programming models outperformed rating models and multiple item models generated better results than single item models. Recently, [Ho et al. \(2010\)](#) surveyed the literature of the multi-criteria decision making approaches for supplier evaluation and selection based on 78 international journal articles gathered from 2000 to 2008, which were classified based on the applied approaches and evaluating criteria. They

observed that price or cost is not the most widely adopted criterion. Instead, the most popular criterion used for evaluating the performance of suppliers is quality, followed by delivery, price or cost, and so on. For some other review articles of the supplier selection problems, please refer to [Aissaoui et al. \(2007\)](#) and [de Boer et al. \(2001\)](#). The following part of this section summarizes the contribution in the literature related to this paper.

Various types of mathematical programming models have been formulated for the supplier selection problem, such as linear programming, mixed integer programming and multi-objective programming. We first review some papers which adopted linear programming. [Ghodsypour and O'Brien \(1998\)](#) proposed an integration of an analytical hierarchy process and linear programming to consider both qualitative and quantitative factors in choosing the best suppliers and placing the optimum order quantities. Later in ([Talluri and Narasimhan, 2003](#)), a unique approach called 'max-min' for vendor selection was proposed by incorporating performance variability into the evaluation process. The authors built two linear programming models to maximize and minimize the performance of a supplier against the best target measures. [Ng \(2008\)](#) developed a weighted linear program for the multi-criteria supplier selection problem with the goal to maximize the supplier score, and studied a transformation technique to solve the proposed model without an optimizer.

As for the mixed-integer programming technique, [Kasilingam and Lee \(1996\)](#) proposed a mixed-integer model to select vendors and determine the order quantities based on the quality of supplied parts, the cost of purchasing and transportation, the fixed cost for establishing vendors, and the cost of receiving poor quality parts. [Talluri \(2002\)](#) studied a buyer-seller game model for evaluating alternative supplier bids with multiple performance criteria. To

select an optimal set of bids by matching demand and capacity constraints, a binary integer programming formulation was developed to determine the optimal set of suppliers. In (Hong et al., 2005), the model which can determine the optimal number of suppliers, and the optimal order quantity so that the revenue could be maximized was built in a mixed-integer linear programming formation, followed by three steps: preparation, pre-qualification, and final selection. Recently, Hammami et al. (2012) developed a mixed-integer programming model for the supplier selection problem that takes into account of inventory decisions, inventory capacity constraints, specific delivery frequency and a transportation capacity based on multiple products and multiple time periods.

Due to the multi-criteria nature of the supplier selection problem, more and more researchers began to adopt multi-object programming since 2005. Narasimhan et al. (2006) developed a multi-objective model to choose the optimal suppliers and determine the optimal order quantity, which considered the following criteria: cost minimization, transaction complexity minimization, quality maximization and delivery-performance maximization. Xia and Wu (2007) studied the situation of price discounts on total business volume and proposed a multi-objective mathematical model to minimize total purchase cost, reduce the number of defective items, and maximize total weighted quantity of purchasing. The model was also built to simultaneously determine the number of suppliers to employ and the order quantity allocated to these suppliers in the case of multiple sourcing, multiple products, with multiple criteria and with supplier's capacity constraints. In (Demirtas and Ustun, 2009), to evaluate the suppliers and to determine their periodic shipment allocations given a number of tangible and intangible criteria, a two-stage mathematical approach was proposed

by a multi objective mixed integer linear programming model. Most recently, [Amid et al. \(2009\)](#) formulated a multi-objective model in adopting a fuzzy weighted additive and mixed integer linear programming method to consider the imprecision of information and determine the order quantities. The problem focused on minimizing three objective functions: the net cost, the net rejected items, and the net late deliveries, while satisfying capacity and demand requirement constraints.

Although the supplier selection decision is closely related with inventory model, [Hammami et al. \(2012\)](#) indicated that only a few of models incorporated the inventory management related issues. Here we want to point out some recent papers from the inventory control perspective. In ([Haq and Kannan, 2006](#)), the authors developed an integrated supplier selection and multi-echelon distribution inventory model. The inventory cost considered in this paper was just based on deterministic demand in a given time period so that no inventory control policies are required to be considered. Similar inventory management models can also be found in ([Demirtas and Ustun, 2009](#); [Hammami et al., 2012](#); [Mendoza and Ventura, 2010](#)). Moreover, [Awasthi et al. \(2009\)](#) developed a mathematical model for a single firm with fixed selection cost and limitation on minimum and maximum order size under stochastic demand. Then [Zhang and Zhang \(2011\)](#) extended their work to implement a newsvendor inventory model based on such a problem. The inventory models considered in these two articles were only based on a single period of time intended for short term planning. [Yang et al. \(2011\)](#) extended their work on multiple items and solved the problem by Genetic Algorithm, which also considered a single period with a salvage value at the end of the selling season. Obviously, a single-period problem does not necessarily consider

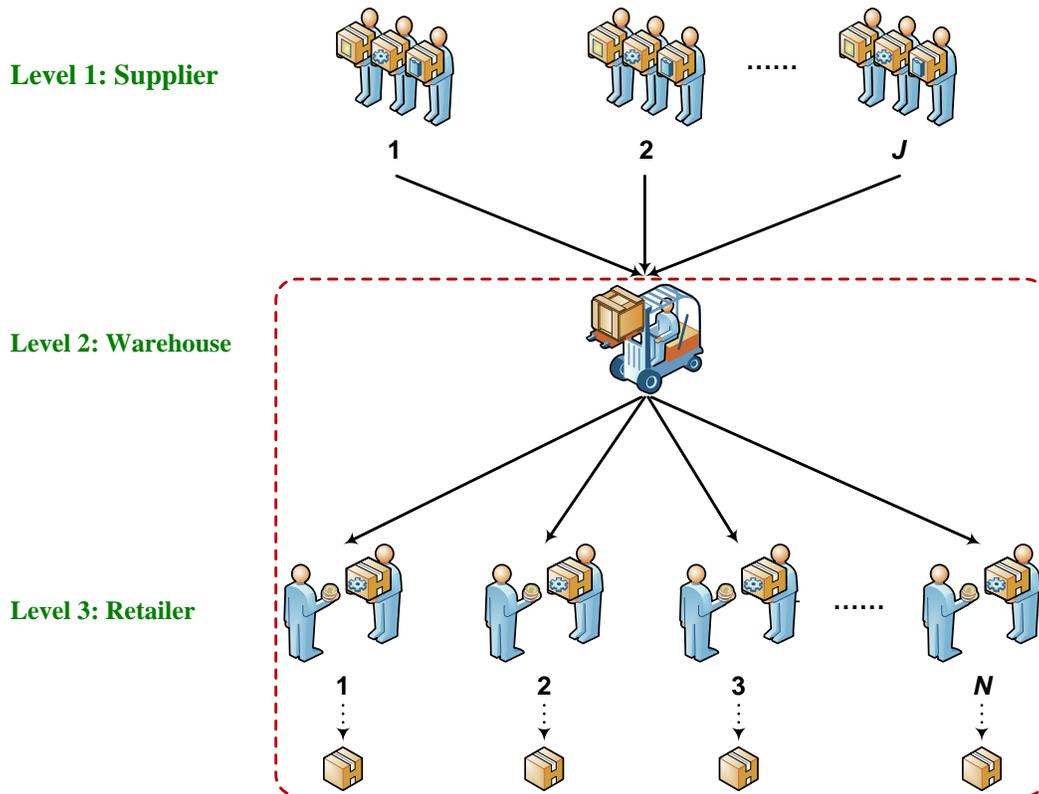
any inventory policy for continuous replenishment over an infinite planning horizon. More importantly, currently literature showed litter work on multi-stage systems, where only focus on the performance of a single buyer. Nonetheless, the inventory policies in supply chain management not only impact a single stage but also will affect the whole supply chain. These are the basic motivations of this research.

# Chapter 3

## Problem Definition and Assumptions

We model the supplier selection and order quantity allocation problem based on a two echelon inventory system. We assume the inventory decision is made by a single decision maker (i.e., centralized control), who wants to purchase a single product from a set of potential suppliers. Figure 3.1 depicts a serial supply chain system under considering of three levels, where raw materials and products flow sequentially through the supply chain to satisfy the customer demand. A single warehouse replenishes its inventory from a set of  $S$  selected suppliers with given lead times. It is assumed that all the suppliers in this identified set at level 1 satisfy the buyer's qualitative criteria (service, delivery, maintenance, etc.) and the final decision will be made based on the item price, fixed ordering cost and the inventory cost regarding of choosing the particular supplier. The warehouse then supplies the items to  $N$  independent identical retailer, where demand occurs based on a Poisson process, and all stockouts are considered as backorders. Therefore, supplier selection and purchasing costs only occur at

the warehouse, while the product is transferred through the entire system, incurring costs like inventory costs, backorder cost, etc.



**Figure 3.1:** Illustration of the multi-level supplier selection system

The above supply chain system is assumed to implement the continuous review  $(Q, R)$  policy for the inventory system at the warehouse and the retailer. Hence, when the demand occurs at the retailer, it is satisfied from the retailer's available stock. Otherwise, the demand is backordered. Under this policy, the inventory position is checked continuously, when it declines to the reorder point  $R$ , a batch size  $Q$  is ordered at the warehouse. The inventory position is defined as the on hand inventory plus stock on order minus the number

of outstanding backorders. After an order is placed with the warehouse, an effective lead time  $L$  takes place between placing the order and receiving it. After receiving the replenishment order, the outstanding backorders at the retailer are immediately satisfied based on a first-come-first-serve (FCFS) policy.

For the warehouse, the retailer replenishment orders are satisfied if the on-hand inventory at the warehouse is greater than or equal to the retailer's order size. That is, a partial replenishment of an order at the warehouse is not allowed here. This is a reasonable assumption when we consider a fixed order cost  $k$  connected to each transport from the warehouse to the retailer. The inventory policy is followed as the same at the warehouse, i.e., the continuous review  $(Q, R)$  policy. We also adopt the widely-used two-echelon inventory system assumption, that is the batch size and reorder point of the warehouse are the integral number of that of the retailer (such as in (Axsäter, 2003; Bodt and Graves, 1985)). According to Chen and Zheng (1997), this integer-ratio order policy can facilitate quantity coordination among different facilities, and simplify packaging, transportation and stock counts. When the warehouse receives the replenishment order from the selected suppliers, any outstanding backorders are also fulfilled according to the FCFS policy.

In addition to determine the inventory policy for the warehouse and the retailer, we will not model any inventory process at the supplier. Instead, the decision maker should replenish the inventory for the warehouse and the retailer from different suppliers by performing a selection process so as to determine which supplier to be selected, the total expected quantity that are to be procured from the selected suppliers, and the frequency in which the orders are to be received. We assume that each supplier locates in different places, then prices, selection

costs, transportation costs, and replenishment lead times are diverse from each other. Define  $S$  different suppliers, for each supplier, let  $O_j$  be the fixed ordering cost each time (i.e., selection cost, transportation cost, etc.), and  $p_j$  be the price of one item from this supplier. Moreover,  $\mu_j$  and  $v_j$  are respectively denoted as the mean and variance of transition time from each supplier  $j$  to the warehouse, which is assumed to be known in advance. Supplier  $j$  has limited maximum capacity  $M_j$  and a restriction of the minimum total order size  $m_j$  if the supplier is selected. The objective of the proposed model is to coordinate the purchase, holding, backorder, and capacity in order to maximize the expected profit.

Before we introduce the mathematical model, we now define the notations that are used throughout the paper as the following:

*Constants*

$S$ : number of suppliers

$N$ : number of retailers

$T$ : number of time span, in days

$\lambda_r$ : demand rate at the retailer per day

$L$ : replenishment lead time between the warehouse and the retailer, in days

$p_j$ : net purchase cost per unit from supplier  $j$

$h$ : holding cost per unit per day

$b$ : backorder cost per unit per day

$k$ : retailer's fixed ordering cost per order

$r$ : selling price per unit at the market

$O_j$ : warehouse's fixed ordering cost per order for supplier  $j$

$\mu_j$ : mean replenishment lead time between supplier  $j$  and the warehouse, in days

$v_j$ : variance of replenishment lead time between supplier  $j$  and the warehouse, in days<sup>2</sup>

$m_j$ : minimum total order size of supplier  $j$  during  $T$  time units

$M_j$ : maximum total order size of supplier  $j$  during  $T$  time units

#### *Decision Variables*

$y_j$ : binary variable, set to be 1 if supplier  $j$  is selected

$x_j$ : expected total ordering quantity from supplier  $j$ , in units

$Q_{rj}, R_{rj}$ : retailer's order quantity and reorder point if supplier  $j$  is chosen, in units

$Q_{wj}, R_{wj}$ : warehouse's order quantity and reorder point if supplier  $j$  is chosen, in units of retailer batches

#### *Random Variables*

$t_j$ : expected time to purchase orders from supplier  $j$ , in days

$\lambda_w$ : demand rate at the warehouse per day

$U_r$ : retailer's retard time, in days

$I_{rj}(R, Q)$ : expected on-hand inventory at retailer during the time supplier  $j$  is chosen, in units

$I_{wj}(R, Q)$ : expected on-hand inventory at warehouse during the time supplier  $j$  is chosen, in units of retailer batches

$B_{rj}(R, Q)$ : expected backorders at retailer during the time supplier  $j$  is chosen, in units

$B_{wj}(R, Q)$ : expected backorders at warehouse during the time supplier  $j$  is chosen, in units of retailer batches

# Chapter 4

## Model Formulation

### 4.1 Mathematical Model

In this section, the model of supplier selection and order quantity allocation under multi-echelon inventory system with stochastic Poisson demand is presented. Let  $x_j$  be the expected total ordering quantity from the supplier  $j$  and  $y_j$  be a binary variable, where  $y_j = 1$  means that supplier  $j$  is selected. Recall that each selected supplier  $j$  should satisfy the capacity constraint, as a result, we may select different sets of suppliers when demand rate at the retailer changes. The system is considered based on time span  $T$ . At this time, we assume the demand rate at the retailer is constant. Moreover, at any moment, only one supplier is asked to provide the replenishment orders, i.e., no order splitting is considered every time the warehouse places the order. If there are multiple suppliers which are selected in the final decision, each supplier is assumed to serve the warehouse separately for some continuous time. We denote  $t_j$  as the time for supplier  $j$ , which is the expected time to

purchase orders from supplier  $j$ . Thus, following one selected supplier (supplier  $a$ ) that finishes its service time ( $t_a$ ), the warehouse may place the order from another one (supplier  $b$ ) with some additional time ( $t_b$ ).

The no-order-splitting assumption represents the cases where the warehouse places orders from different suppliers one after another, and won't order the same product from different suppliers at the same time. The reason to make this assumption is mainly due to the fact that the different supplier lead time and ordering cost could require diverse  $(Q, R)$  policies for both the warehouse and the retailer. However, it is untrackable and impractical to apply different  $(Q, R)$  policies for different suppliers at the same moment. More specifically, to apply the order-splitting assumption, the retailer may need to implement the same  $(Q, R)$  policy no matter which supplier is selected. Moreover, it is still unclear that in which way (adopt this assumption or not) the system could be more efficient. Thus, we want to study the no-order-splitting situation first, and another model which violates this assumption will be considered in our future research.

The objective function of the proposed model consists of several parts to maximize the total expected profit. The first part is used to calculate total sale income. The second term corresponds to the purchasing cost incurred by all the units purchased from selected suppliers. The third part accounts for the total holding and backorder cost. The last term represents the fixed ordering cost for both echelons. The insights of the model is to examine the trade-offs among price, ordering, holding, and backorder costs to choose the best supplier(s) and decide the ordering policy. Based on the above discussed assumptions and variables, the

following mathematical model is designed:

$$\text{Maximize } C = \sum_{j=1}^S rx_j - \sum_{j=1}^S p_j x_j - \sum_{j=1}^S t_j [h(I_{rj} + Q_{rj} I_{wj}) + b(B_{rj} + Q_{rj} B_{wj})] - \sum_{j=1}^S \left( \frac{x_j}{Q_{wj} Q_{rj}} O_j + \frac{x_j}{Q_{rj}} k \right).$$

Subject to

$$\sum_{j=1}^S t_j \leq T, \quad (4.1)$$

$$m_j y_j \leq x_j \leq M_j y_j \quad j = 1, \dots, S, \quad (4.2)$$

$$x_j = N \lambda_r t_j \quad j = 1, \dots, S, \quad (4.3)$$

$$R_{rj} \geq -Q_{rj} \quad j = 1, \dots, S, \quad (4.4)$$

$$R_{wj} \geq -Q_{wj} \quad j = 1, \dots, S, \quad (4.5)$$

$$Q_{rj}, Q_{wj} \geq 0 \quad j = 1, \dots, S, \quad (4.6)$$

$$Q_{rj}, R_{rj}, Q_{wj} \& R_{wj} : \text{Integers} \quad j = 1, \dots, S, \quad (4.7)$$

$$y_j \in \{0, 1\} \quad j = 1, \dots, S. \quad (4.8)$$

Constraint (4.1) ensures that the total expected time to purchase orders from all selected suppliers should be no longer than the time span  $T$ . Constraint (4.2) defines the capacity constraint for the selected suppliers. As for constraint (4.3), it ensures the expected ordering quantity from supplier  $j$  should satisfy the expected demands at the retailer during its supplying period. For standard cost structures (i.e., linear holding and backorder costs), it

can be shown that the optimal  $R$  satisfies  $R \geq -Q$ . Therefore, it is assumed to satisfy in equations (4.4) and (4.5), which can limit the computation efforts. Additionally, constraints (4.6) and (4.7) are necessary, since there is no partial or fractional requests during the whole process and the minimum allowable size is zero.

The mathematical model requires calculations of the expected inventory and backorder level for the warehouse and the retailer. We primarily elaborate the calculations in the following parts of this section.

## 4.2 The Retailer Inventory Analysis

Since the calculation for all the selected suppliers is identical, for notational ease, we ignore the supplier's subscript  $j$  for the notations in this section. The following analysis is identical for all the different supplier cases.

The retailer inventory position decreases with demand, and when the level reaches  $R_r$ , an order of  $Q_r$  is placed at the warehouse. Recall that in the steady state, the inventory position is uniformly distributed over  $(R_r + 1, R_r + 2, \dots, R_r + Q_r)$ . Under a  $(Q, R)$  policy, the expected on-hand inventory for retailers is modeled as:

$$I_r = \frac{Q_r + 1}{2} + R_r + B_r - E[D_r], \quad (4.9)$$

where  $E[D_r]$  is the retailer's expected demand during the delay time. The delay time consists of two parts: the replenishment lead time between the warehouse and the retailer, and

the time between placement of an order by the retailer and the release of a batch by the warehouse. The first part is denoted as  $L$ , which is assumed to be deterministic. While the second part is usually called as the retard time, which is due entirely to the warehouse stockout. We denote the retard time for the retailer as  $U_r$ . It is given as follows: the number of arrival orders in the waiting system is precisely the warehouse's backorders, and the sojourn time is equal to the retailer's retard time (Svoronos and Zipkin, 1988). Thus, the expected retard time at the retailer is calculated as

$$E[U_r] = \frac{Q_r B_w}{N\lambda_r}, \quad (4.10)$$

then we have

$$E[D_r] = \lambda_r \left( L + \frac{Q_r B_w}{N\lambda_r} \right). \quad (4.11)$$

To simplify the calculation, as illustrated by (Hopp and Spearman, 2001), we adopt normal approximation for the retailer's demand during the delay time. It is worth to note that even though the demand process is Poisson process, the demand during the delay time for the retailer does not exactly follow the Poisson distribution, due to the variability of the retard time. In this paper, we ignore the variability of the retard time, and approximate the variance of the retailer's expected demand during the delay time to be the same as its mean value, i.e.,  $V[D_r] \cong E[D_r]$ .

Hence, using the normal approximation, expected backorders at the retailer  $B_r$  can be computed as follows :

$$B_r = \frac{1}{Q_r} [\beta(R_r) - \beta(R_r + Q_r)], \quad (4.12)$$

$$\beta(x) = \frac{\sigma^2}{2} \{(z^2 + 1)[1 - \Phi(z)] - z\phi(z)\}, \quad (4.13)$$

$$z = \frac{x - \theta}{\sigma}, \quad (4.14)$$

where  $\Phi$  and  $\phi$  represent the cdf and pdf of the standard normal distribution, respectively. Additionally,  $\theta$  and  $\sigma$  are the mean and standard deviation of the demand during the delay time. Note here for equation (4.13), it defines the continuous analog to the second-order loss function  $\beta(x)$ . Thus, substituting  $\theta$  and  $\sigma$  in the above equations,  $B_r$  can be computed in a function based on the variables  $Q_r$ ,  $R_r$ , and  $B_w$ .

### 4.3 The Warehouse Inventory Analysis

To calculate the expected inventory level and backorder level at the warehouse, the demand process at the warehouse need to be analyzed first. Recall that the demand process at each retailer is Poisson process, and the replenishment order for the warehouse is  $Q_r$  each time. However, the interval between any two orders is stochastic and depends on  $Q_r$ . Hence, the demand process at the warehouse is a superposition of the retailer's ordering processes. Specifically, it is a superposition of independent renewal processes (i.e., the time between orders from each retailer are independent and identically distributed random variables), each with an Erlang interrenewal time with  $Q_r$  stages and rate per state  $\lambda_r$  (also can see

(Deuermeyer and Schwarz, 1981)). Therefore, under the assumption of identical retailers, it is straightforward to get the demand rate at the warehouse:

$$\lambda_w = \frac{N\lambda_r}{Q_r}. \quad (4.15)$$

When considering the multi-echelon problem, to determine the effective demand pattern at the upstream is always a difficult task. For this problem, it is almost impossible to find an exact distribution for the demand process at the warehouse. However, Ganeshan (1999) showed that when  $N$  is greater than 20, the Poisson process is an excellent approximation for the warehouse demand pattern. In our research,  $N$  is always a sufficient large number in each scenario, so that each retailer's order arrives approximately according to a Poisson process. This assumption is fairly reasonable, since for large retail corporation like Wal-Mart, the distribution center usually servers more than 20 stores.

Because of the independence of the superimpose process, given the mean and variance of the warehouse replenishment lead time  $\mu$  and  $v$ , we can compute the mean and variance of the warehouse demand during the replenishment lead time as

$$E[D_w] = \mu\lambda_w = \frac{N\lambda_r\mu}{Q_r}, \quad (4.16)$$

$$V[D_w] = \mu\lambda_w + v\lambda_w^2 = \frac{N\lambda_r\mu}{Q_r} + \frac{N^2\lambda_r^2\mu^2v}{Q_r^2}. \quad (4.17)$$

Then we can obtain  $I_w$  similar to equation (4.9):

$$I_w = \frac{Q_w + 1}{2} + R_w + B_w - E[D_w]. \quad (4.18)$$

Due to the assumption of Poisson demand process at the warehouse when  $N$  is sufficient large, we also approximate the warehouse's demand during the lead time as normal distribution. This is a reasonable approximation since this approximation improves as the rate of Poisson distribution increases, while the rate at the warehouse is sufficient large. This approximation is also used in the literature, such as in (Al-Rifai and Rossetti, 2007). Then equations (4.12) to (4.14) can also be adopted to calculate the expected backorder level  $B_w$  at the warehouse. It is not hard to find out  $B_w$  is a function of  $Q_w$ ,  $R_w$ , and  $Q_r$ .

## 4.4 Solution Procedure

The above multi-echelon supplier selection optimization model is a large-scale non-linear integer optimization problem. Considering the case of five potential suppliers, the model then may contain 30 integer decision variables, which takes a lot of computational time to solve. Moreover, the inventory analysis of each echelon requires to model and solve both echelons simultaneously. In order to model the warehouse, the retailer's order batch size must be decided as a priori. On the other hand, the retailer's calculation requires a known expected number of backorders at the warehouse. Thus, each echelon of the system is tightly connected with each other, and the model is difficult to solve in an efficient way.

Before we move forward, the following proposition gives some properties of the optimal decision.

**Proposition 1.** *In an optimal solution, at most one supplier may not satisfy the following:  $x_j \in \{0\} \cup \{m_j, M_j\}$ .*

*Proof.* Assume that in an optimal solution, the suppliers  $a$  and  $b$  satisfy the condition  $m_a < x_a < M_a$  and  $m_b < x_b < M_b$  respectively, so that the condition of Proposition 1 is violated. Denote  $IB_j = h(I_{rj} + Q_{rj}I_{wj})/N\lambda_r + b(B_{rj} + Q_{rj}B_{wj})/N\lambda_r$ , and  $K_j = O_j/Q_{wj}Q_{rj} + k/Q_{rj}$ . Then  $IB_j$  and  $K_j$  are independent from  $x_j$ . The objective function for  $C$  can be expressed as  $\sum_{j=1}^S rx_j - \sum_{j=1}^S (p_j + IB_j + K_j)x_j$ .

Without loss of generality, for suppliers  $a$  and  $b$ , we assume that  $p_a + IB_a + K_a < p_b + IB_b + K_b$ . Also let  $\vartheta = \min\{x_b - m_b, M_a - x_a\}$ ,  $x'_a = x_a + \vartheta$ ,  $x'_b = x_b - \vartheta$ , and  $x'_j = x_j$ ,  $\forall j \neq a, b$ . Then  $\sum_{j=1}^S (p_j + IB_j + K_j)x'_j \leq \sum_{j=1}^S (p_j + IB_j + K_j)x_j$ . Since  $\sum_{j=1}^S x'_j = \sum_{j=1}^S x_j$ ,  $(x'_1, x'_2, \dots, x'_S)$  would not violate constraint (4.1). Thus we have  $C' \geq C$ . This means  $(x'_1, x'_2, \dots, x'_S)$  is a better solution for this model, which violates the assumption. Continue this process till at most one supplier supplies neither minimum nor maximum. This completes the proof.  $\square$

As mentioned in the proof of the above proposition, the expected inventory and backorder level at the warehouse and the retailer are independent from the expected total ordering quantity  $x_j$ . This implies that the model can be solved with two decomposed levels: one (Model 1) is to solve the optimal  $(Q, R)$  policy for each potential suppliers with the objective to minimize the expected holding, backorder and ordering cost; the other (Model 2) is to

choose the best suppliers with larger profit and allocate the expected order time for different suppliers. We now express the models as the following:

*Model 1:* Since the  $(Q, R)$  policy applied by different potential suppliers is independent with each other, we formulate the optimization problem based on a single supplier  $j$  as minimizing the total unit system cost, denoted as  $E_j$ , which includes inventory, backorder and ordering cost as follows:

$$\text{Minimize} \quad E_j = h(I_{rj} + Q_{rj}I_{wj}) + b(B_{rj} + Q_{rj}B_{wj}) + \left(\frac{N\lambda_r}{Q_{wj}Q_{rj}}O_j + \frac{N\lambda_r}{Q_{rj}}k\right).$$

Subject to

$$R_{rj} \geq -Q_{rj}, \quad (4.19)$$

$$R_{wj} \geq -Q_{wj}, \quad (4.20)$$

$$Q_{rj}, Q_{wj} \geq 0, \quad (4.21)$$

$$Q_{rj}, R_{rj}, Q_{wj} \& R_{wj} : \text{Integers}. \quad (4.22)$$

Note in this model  $I_{rj}$ ,  $I_{wj}$ ,  $B_{rj}$  and  $B_{wj}$  can be formulated as the equations discussed in section 4.2 and 4.3. For simplicity, we ignore those equations in this mathematical model.

*Model 2:* After achieving the minimized cost value  $E_j$  for each possible supplier, the mathematical model described in section 4.1 can be updated to a simplified model by

substituting  $E_j$  in the objective function as follows:

$$\text{Maximize} \quad C = \sum_{j=1}^S rx_j - \sum_{j=1}^S p_j x_j - \sum_{j=1}^S t_j E_j.$$

Subject to

$$\sum_{j=1}^S t_j \leq T, \quad (4.23)$$

$$m_j y_j \leq x_j \leq M_j y_j \quad j = 1, \dots, S, \quad (4.24)$$

$$x_j = N \lambda_r t_j \quad j = 1, \dots, S. \quad (4.25)$$

Model 1 is a non-linear integer optimization model with two pairs of  $(Q, R)$  decision variables to be solved. While model 2 is an integer programming model which is easy to solve. Apparently, this decomposition makes the multi-echelon supplier selection model to be solved in a more efficient way, since the sub-models provided above reduce the scale of the problem. Thus, we implement a solution procedure which can be summarized as the following steps: (1) for each potential supplier  $j$ , solve Model 1 to achieve best  $(Q_{rj}, R_{rj})$  and  $(Q_{wj}, R_{wj})$  values with the minimized values of  $E_j$ ; (2) given the values of  $E_j$  for all potential suppliers, calculate the optimal supplier selection policy.

# Chapter 5

## Illustrative Example and Analysis

In this section, the numerical experiments are conducted with the proposed mathematical model. The model is coded in GAMS Integrated Development Environment, and solved by Knitro commercial package for mixed integer nonlinear model (Model 1) and Cplex for integer programming model (Model 2), in a desktop computer with an Intel Core(TM) 2 CPU(2.00 GHz) and 4GB RAM. The main purpose for the experiments in this section is to show the solvability and the effectiveness of the model and to demonstrate how to adopt the model for supplier selection decision making in different scenarios.

### 5.1 Parameter Setting

In this section, the system is simulated quarterly (i.e.,  $T = 90$ ), since the demand rate usually doesn't change a lot during a season. As mentioned earlier, the number of retailer is assumed to be a sufficient large number; we set  $N$  to be 20. There are six potential suppliers

**Table 5.1:** Parameter values assigned in the experiment

Parameter	$T$	$N$	$\lambda_r$	$r$	$h$	$b$	$k$	$L$
Value	90	20	10	100	1	3	100	1

to be chosen, which may locate in different regions of the world. As a result, we study the supplier selection problem based on one firm which consists of one warehouse and twenty identical retailers, and faces Poisson demands with the same arrival rate at the retailer. The demand rate is set to be 10 units per day. Moreover, we set the product's selling price  $r$  to be 100. The unit holding cost and backorder cost for both the warehouse and the retailer are assumed to be 1 and 3 respectively. The fixed ordering cost from the warehouse to each retailer is considered as 100. Also the deterministic replenishment lead time from the warehouse to the retailer is set to be 1 (day). All the parameters are summarized in Table 5.1.

Table 5.2 shows additional data for each potential supplier. It is assumed all the suppliers satisfy the buyer's qualitative criteria, and own various purchasing price, fixed ordering cost, total expected order size constraint, and lead time. Two types of potential suppliers are considered: short-range suppliers (supplier 1, 2, 3, and 4) and long-distance suppliers (supplier 5 and 6). As illustrated in Table 5.2, the long-distance suppliers charges less unit purchasing cost, but requires more fixed ordering cost, and larger variability of the replenish lead time.

**Table 5.2:** Other parameter settings related to potential suppliers

Supplier candidate $j$	1	2	3	4	5	6
Price ( $p_j$ )	83	85	82	87	78	78
Fixed ordering cost ( $O_j$ )	2000	1200	800	500	3500	4500
Min. total order size ( $m_j$ )	1500	1000	2700	700	3700	4000
Max. total order size ( $M_j$ )	9200	10100	8100	21300	12920	13200
Mean of supplier lead time ( $\mu_j$ )	3	2	5	2	6	7
Variance of supplier lead time ( $v_j$ )	0.2	0.5	0.5	0.1	1.5	2.0

## 5.2 Results Analysis

According to the above parameter settings, Table 5.3 displays the final selection decision. The optimal inventory policy for each level and the selected supplier order allocation along with the total expected profit are also presented. Under such settings, supplier 1, 3 and 4 are selected. Recall that the decision to choose a supplier or not depends not only on the cost structures (i.e., unit cost, fixed ordering cost, inventory cost, and backorder cost), but also on the restriction for the minimum and maximum total expected order sizes. In this particular example, although the unit purchasing cost of the long-distance suppliers (supplier 5 and 6) is the least, nothing is ordered from them because of the high ordering cost and large variance of the replenishment lead time. Conversely, supplier 1 is selected due to its low lead time value and variation. Furthermore, supplier 4 is also chosen because of its low minimum total expected ordering quantity.

The model we built mainly focuses on the expected values for the selected suppliers. It does not intend to calculate the accurate quantity to make orders from the selected suppliers, but can indicate the priority to select suppliers. Since our model is based on stochastic

**Table 5.3:** Decision-making variables solution

Final selection list	Supplier 1	Supplier 3	Supplier 4
Expected total order ( $x_j$ ) (units)	9200	8100	700
Expected ordering time ( $t_j$ ) (days)	46.0	40.5	3.5
Warehouse policy ( $Q_w, R_w$ ) (units of $Q_r$ )	(23, 7)	(21, 20)	(12, 5)
Retailer policy ( $Q_r, R_r$ ) (units)	(50, 1)	(51, 3)	(50, -1)
Total profit (\$/quarter of a year) C = 148734.6			

demands, in the real scenarios, the total order size and order time from selected suppliers may not necessarily be the expected values calculated in Table 5.3. For instance, according to the results displayed in this table, supplier 1 and 3 own higher priority to be ordered since the expected order quantity reaches its upper limit of the capacity. In stochastic demand cases, we need to purchase from supplier 1 and 3 until the total order size reaches its maximum value, and the total order size from supplier 4 will depend on the real time demand. Hence,  $x_4 = 700$  is only an expected value, and it implies that supplier 4 is the last choice to choose when supplier 1 and 3 are available.

Considering the optimal ( $Q, R$ ) policies of each selected supplier displayed in Table 5.3, it is not surprising to find out that the retailer's ordering quantity among different selected suppliers are very similar. The main reason for this is that the replenishment quantity  $Q_r$  affects cycle stock (i.e., inventory that is held to avoid excessive replenishment costs), and the demand rate, retailer's fixed ordering cost, and the replenishment lead time at the retailer are fixed no matter which supplier is chosen. Even the retard time may vary for different supplier, it only affects safety stock (i.e., inventory held to avoid stockouts), which determines the reorder point  $R_r$ . While for the warehouse's inventory policy, different suppliers adopt

**Table 5.4:** The optimal policy and expected profit based on different demand rate instance

Instance	Demand rate ( $\lambda_r$ )	Expected ordering quantity ( $x_j$ ) (units)	Optimal profit ( $C$ ) (\$)	CPU time (seconds)
1	10	$x_1 = 9200, x_3 = 8100, x_4 = 700$	148734.6	3.47
2	15	$x_1 = 9200, x_2 = 9700, x_3 = 8100$	245937.1	3.42
3	20	$x_1 = 9200, x_2 = 10100, x_3 = 8100, x_4 = 8600$	341029.9	3.18
4	25	$x_1 = 9200, x_2 = 10100, x_3 = 8100, x_4 = 17600$	437324.8	3.29
5	30	$x_1 = 9200, x_2 = 10100, x_3 = 8100, x_4 = 21300, x_5 = 5300$	534539.2	4.13

diverse  $(Q, R)$  strategies since they face different lead time and fixed ordering cost which require to maintain different safety stock and cycle stock.

### 5.3 Demand Rate Analysis

To study the scenarios under different demand rate, we solve four additional problem instances based on different value of  $\lambda_r$ , i.e.,  $\lambda_r = 15, 20, 25,$  and  $30$ . In this paragraph, all the parameters remain the same as the settings in section 5.1 except for the retailer's demand rate. The demand rate value and the optimal policy for each instance are summarized in Table 5.4. It is obvious that the expected profit increases due to the increase of demand. As the demand rate increases, more suppliers are selected since the lower cost suppliers reach their maximum capacity. This implies that we should order up to capacities of the suppliers with lower costs when we selected at least two suppliers as supply partners. This also confirms the correctness of Proposition 1. Moreover, for all the instances, the long-distance suppliers own the least priority to be selected even though their unit price is smaller. This indicates that the demand rate doesn't influence a lot to the cost structure of the suppliers, and the

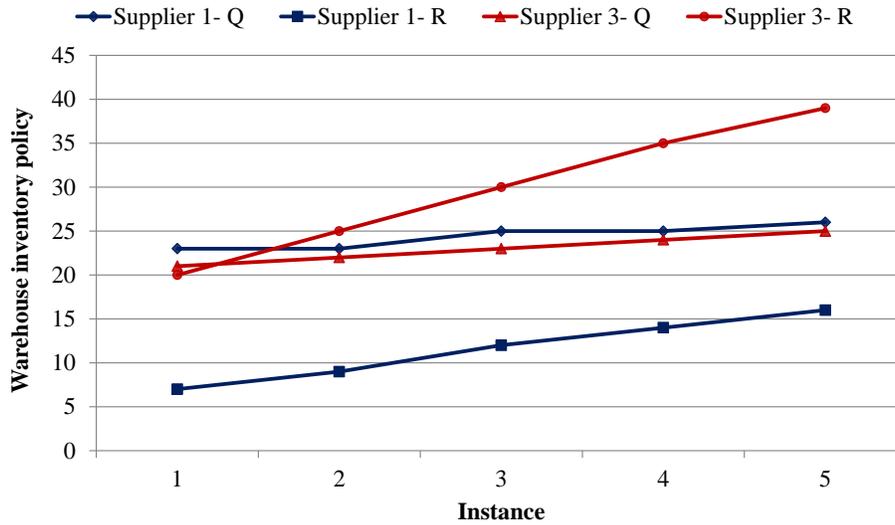
long-distance suppliers here take more system costs which prevent them to be selected. The table also displays the CPU times to solve the problem for each case. The time is an average value based on 10 times run. For this case with six potential suppliers, it is possible to solve the model with the commercial software package in a short amount of time.

Table 5.5 displays the optimal inventory policy in different instances, which directly corresponds to Table 5.4. Clearly, even if the same suppliers are selected in different instances, their optimal inventory policies vary a lot. The retailer's replenishment order quantity and reorder point increase notably due to the increment of demand. This is authentic since larger demand requires more cycle stock and safety stock to maintain low cost. Based on the running instances, it can also be observed that the retailer's order quantity in each instance is almost the same among different selected suppliers. However, the retailer's reorder point may vary among different supplier, especially for the selection of long-distance supplier (supplier 5) in instance 5. This is because long-distance supplier owns larger replenishment lead time variance which needs to hold more safety stock to avoid stockouts.

To clearly display the changes of  $(Q, R)$  policy at the warehouse for different instances, Figure 5.1 is created to display the  $(Q, R)$  policy at the warehouse when supplier 1 and 3 is selected under different instances. Note in this figure, the values of order quantity and reorder point at the warehouse are based on the units of retailer's order quantity. It is undoubted to observe the increasing trend in both the order quantity and reorder point due to the increment of demands. It can also be inspected that  $R_w$  changes more considerable than  $Q_w$ .

**Table 5.5:** The optimal inventory policy for selected suppliers in different demand rate instances

Instance	Selected suppliers ( $j$ )	Retailer ( $Q_r, R_r$ ) (units)	Warehouse ( $Q_w, R_w$ ) (units of $Q_r$ )
1	1	(50, 1)	(23, 7)
	3	(51, 3)	(21, 20)
	4	(50, -1)	(12, 5)
2	1	(64, 3)	(23, 9)
	2	(62, 3)	(20, 6)
	3	(62, 8)	(22, 25)
3	1	(71, 7)	(25, 12)
	2	(70, 8)	(21, 8)
	3	(73, 13)	(23, 30)
	4	(71, 5)	(12, 9)
4	1	(80, 11)	(25, 14)
	2	(81, 11)	(21, 9)
	3	(80, 19)	(24, 35)
	4	(81, 8)	(12, 10)
5	1	(87, 16)	(26, 16)
	2	(86, 16)	(22, 11)
	3	(87, 25)	(25, 39)
	4	(88, 12)	(13, 11)
	5	(88, 42)	(51, 51)



**Figure 5.1:** Illustration of the trend for inventory policy at the warehouse when supplier 1 and 3 are selected under different instances

## 5.4 Long-distance Supplier Analysis

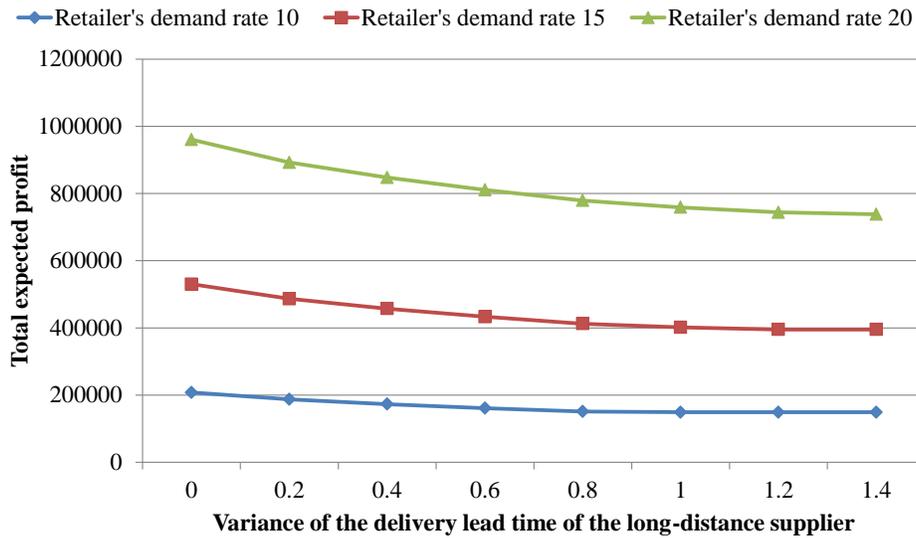
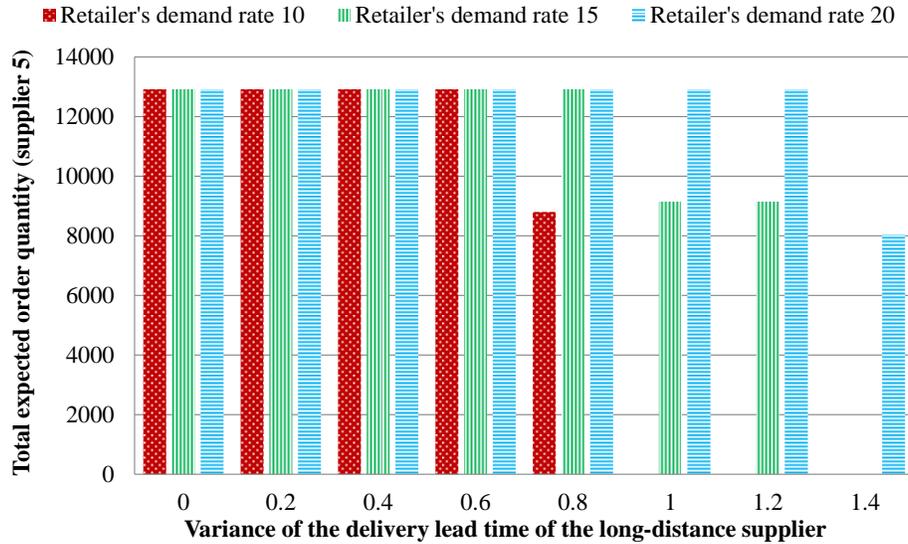
In this paragraph, we want to analyze the issue when selecting low-cost long-distant suppliers. In addition to high transportation cost (ordering cost), a long-distance supplier is often characterized by high delivery lead time and uncertainty. As shown in previous experiments, high lead time uncertainty results in large safety stock levels so that the expected inventory level will be high. To study the scenarios when a distant supplier should be selected and what quantity should be ordered, we conduct the experiments based on parameter changes in long-distance supplier (price ( $p_j$ ), fixed ordering cost ( $O_j$ ), mean supplier lead time ( $\mu_j$ ) and variance of the delivery lead time ( $v_j$ ) are considered). According to the parameter settings in section 5.1, we choose one long-distance supplier (supplier 5) to analyze, modify one parameter once at a time (other parameters remain the same), and want to analyze the impacts of the parameter on both expected ordering size  $x_j$  ( $j = 5$ ) and total expected revenue  $C$  for different values of the demand rate instances ( $\lambda_r = 10, 15,$  and  $20$  are adopted in this paragraph). Since supplier 5 is not chosen in the final decision of these instances, by doing so, we could examine the threshold value of each parameter to involve this distant supplier in our final selection.

We first study the scenario when the variance of delivery lead time changes. Figure 5.2 illustrates the changes of expected order quantity of supplier 5 and total expected profit when its delivery lead time variance varies from 0 to 1.4 under diverse demand rates. Observed in this figure, one can note that the expected total quantity purchased from the long-distance supplier decreases with the increasing lead time uncertainty. The figure shows that the

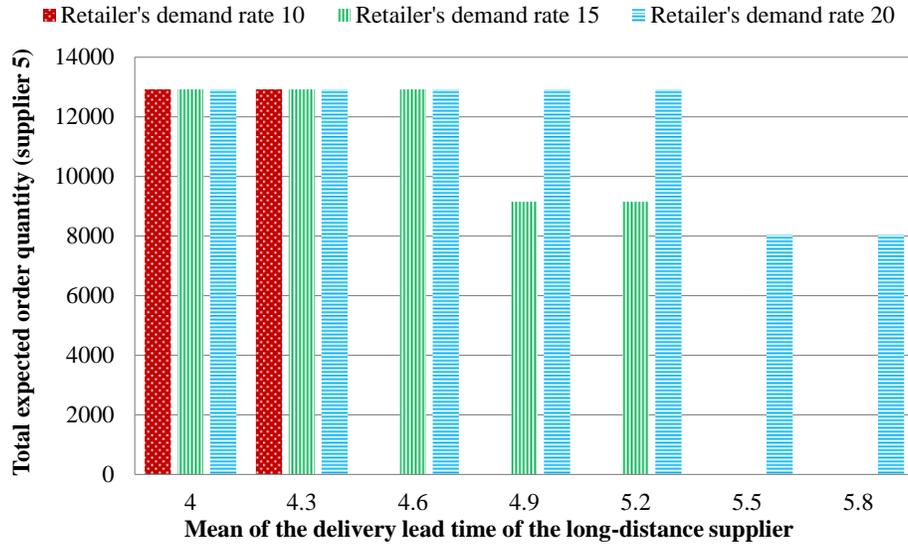
long-distance supplier should not be selected in some cases, especially when its lead time is highly uncertain. The right side of figure displays the total expected profit under the same experimental settings. Clearly, it can be observed that as the variance of long-distance supplier increases, the expected profit decrease. And when the variance is big enough, the lead time variance changes will not affect the total expected profit since this long-distance supplier will not even be chosen.

Keeping variance of lead time constant, Figure 5.3 demonstrates the changes of expected order quantity and total expected profit when delivery lead time ( $\mu_5$ ) varies from 4 to 5.8 under diverse demand rates. Note here the starting point value  $\mu_5 = 4$  is set due to the assumption of positive lead time (i.e., when mean and variance of normal distribution are separately 4 and 1.5, there is less than 0.05% probability that the lead time is negative). As displayed in this figure, a similar trend can be observed as the one in Figure 5.2, which is predicable. Comparing the total expected profit values in these two figures, one can note a lower lead time variance may let us earn more profit than a lower lead time mean. For instance when demand rate is 20, a low delivery lead time where  $\mu_5 = 4$  leads to almost \$800000 expected profit, however, a low lead time variance where  $v_5 = 0.4$  can result in a profit more than that.

Keeping variance of lead time constant, it is demonstrated in Figure 5.3 the changes of expected order quantity and total expected profit when delivery lead time ( $\mu_5$ ) varies from 4 to 5.8 under diverse demand rates. Note here the starting point value  $\mu_5 = 4$  is set due to the assumption of positive lead time (i.e., when mean and variance of normal distribution are separately 4 and 1.5, there is less than 0.05% probability that the lead time is negative).

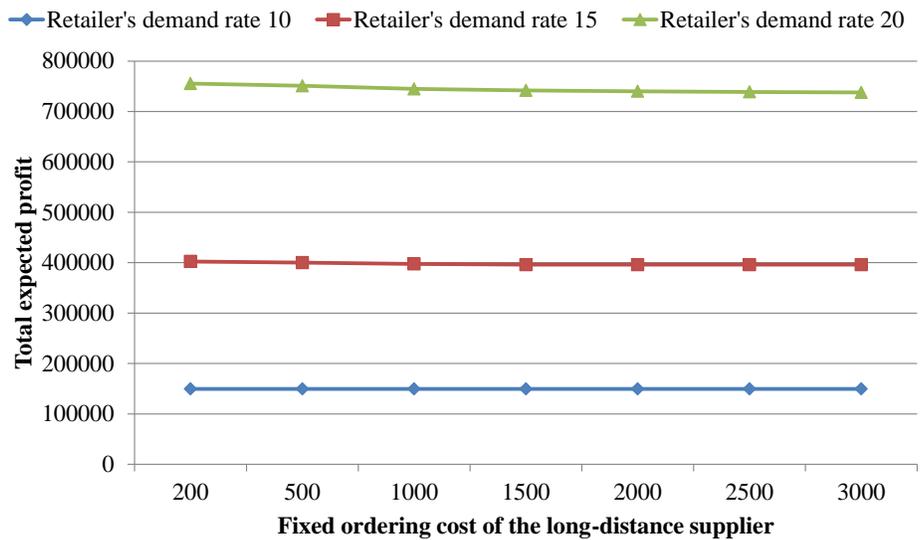
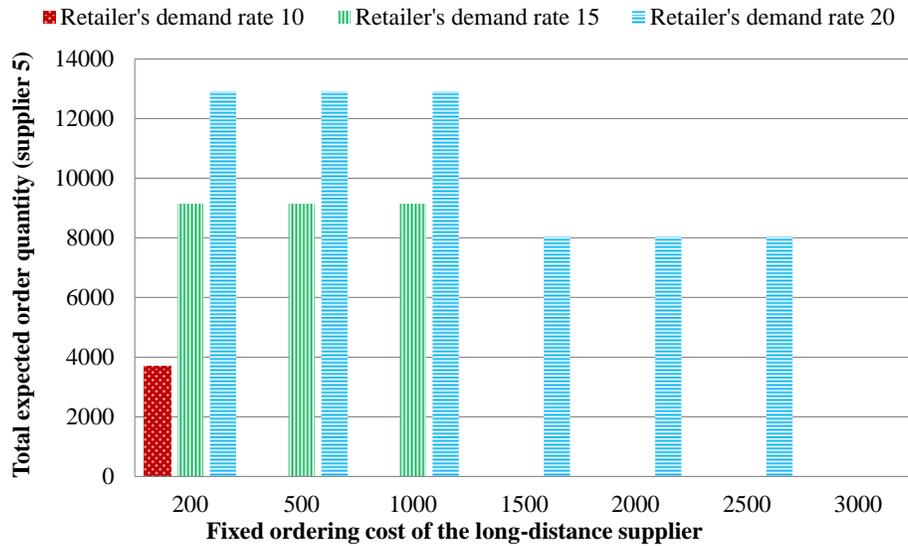


**Figure 5.2:** Illustration of the impact of delivery lead time variance for a long-distance supplier: total order quantity purchased from the distant supplier and the expected profit decrease with the increasing delivery lead time uncertainty

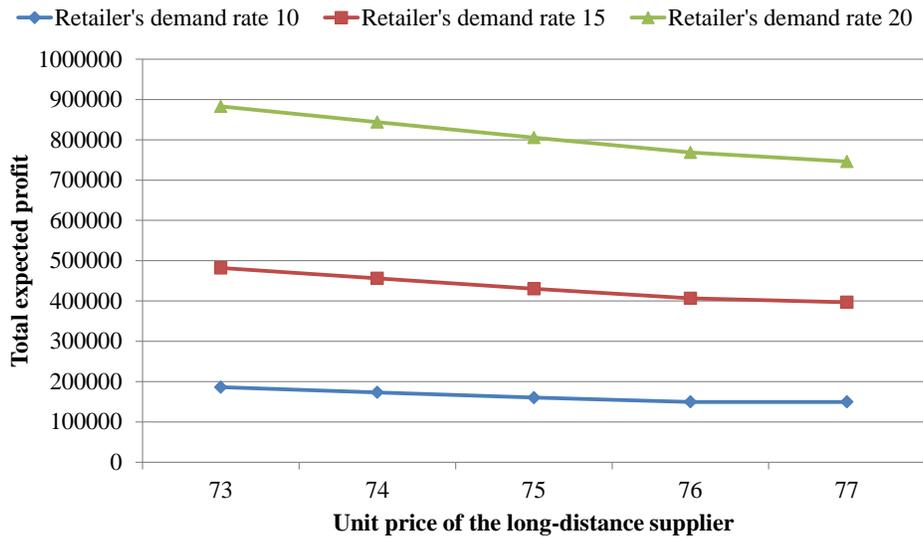
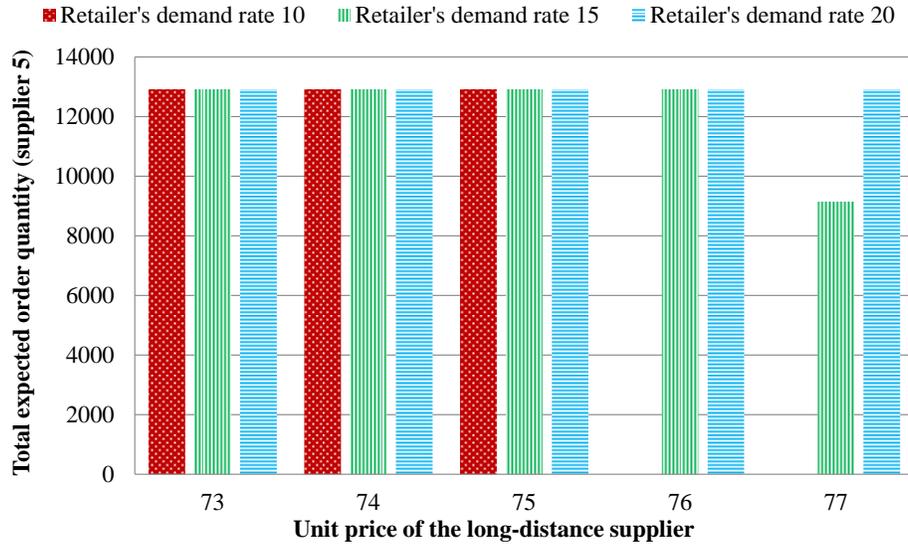


**Figure 5.3:** Illustration of the impact of mean delivery lead time for a long-distance supplier: total order quantity purchased from the distant supplier and the expected profit decrease with the increasing mean delivery lead time

As displayed in this figure, a similar trend can be observed as the one in Figure 5.2, which is predictable. Comparing the total expected profit values in these two figures, one can note a lower lead time variance may let us earn more profit than a lower lead time mean. For instance when demand rate is 20, a low delivery lead time where  $\mu_5 = 4$  leads to almost \$800,000 expected profit, however, a low lead time variance where  $v_5 = 0.4$  can result in a profit more than that.



**Figure 5.4:** Illustration of the impact of fixed ordering cost for a long-distance supplier: total order quantity purchased from the distant supplier and the expected profit decrease with the increasing fixed ordering cost



**Figure 5.5:** Illustration of the impact of unit price for a long-distance supplier: total order quantity purchased from the distant supplier and the expected profit decrease with the increasing unit price

# Chapter 6

## Conclusions

In this paper, we study a supplier selection and order allocation problem in a multi-echelon system under stochastic demand, where a buyer firm that consists of one warehouse and  $N$  identical retailers focuses its attention to analyze both supplier selection decisions and inventory control policies to optimize the entire system. Capacity, ordering cost, unit price, holding and backorder cost are considered as the criteria for the supplier selection. A mixed integer non-linear programming (MINLP) model is proposed to select the best suppliers and determine a coordinated replenishment inventory policy at each echelon of the supply chain so that the total expected profit is maximized. To solve the model more efficiently, we decompose the mathematical model into two sub-models. Our experiments demonstrate the solvability and the effectiveness of the model. Moreover, we have been interested in some issues regarding the selection of the long-distance supplier. Thus, some sensitivity analysis for the long-distance supplier is also conducted.

There are two limitations for the current work in this paper. First, we have adopted the no-order-splitting assumption, which requires ordering from a certain supplier for some continuous time span. The reason to make this assumption is mainly due to the different  $(Q, R)$  inventory policy adopted at both the warehouse and retailers for different selected suppliers. However, observed in our experimental examples in Section 5.3, similar  $(Q, R)$  policies are assigned to the retailer for different suppliers. This implies that it is possible to apply the order splitting model at the warehouse. Thus, future work may focus on extending order splitting model at the warehouse and the same ordering policy for the retailer. Second, to implement the stochastic demand assumption, we mainly focus on calculating the expected values of the total ordering size and the time. The intention to use these expected values is not for signing the contract and ordering the computed amount from the selected suppliers, but for deciding the priorities to select suppliers and estimating the size and time to order from each supplier. Our model offers more insights to choose suppliers and allocate orders among the suppliers when considering the integration of both the supplier selection and inventory control problems in the multi-echelon system under stochastic demand.

There are several directions for the future work. First, as we mentioned above, the order-splitting model is a good direction to work on. Second, in this paper we assume the  $(Q, R)$  continuous review policy. Future work may consider periodic review system. Moreover, our model can be extended to consider multiple product and joint replenishment costs. Finally, since the supplier selection is a typical multi-criteria decision problem, this work could be extended to multi-objective models where the trade-offs associated with these criteria can be analyzed.

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