Measurements and analytical modeling of heat transfer in flowing granular media

Mohamed Mostafa Alsharif

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I am submitting herewith a dissertation written by Mohamed Mostafa Alsharif entitled "Measurements and analytical modeling of heat transfer in flowing granular media." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Mechanical Engineering.

Edward G. Keshock, Major Professor

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W.S. Johnson, J.W. Hodgson, J.J. Perona

Accepted for the Council:

Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

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To the Graduate Council:

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[Signatures]

Accepted for the Council:

Vice Provost
and Dean of The Graduate School
MEASUREMENTS AND ANALYTICAL MODELING
OF HEAT TRANSFER IN FLOWING
GRANULAR MEDIA

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Mohamed M. Alsharif
March 1987
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ABSTRACT

An investigation was conducted of the heat transfer characteristics of a variety of flowing particulate solid media. Five different granular materials were studied in a vertical gravity driven flow through an electrically heated circular stainless steel tube test section. From measurements of wall temperature distribution, particle bulk inlet temperature, radial temperature distribution of the particle flow at the test section exit, and mass flow rate, determination of local and average heat transfer coefficients along the test section were made. Comparison of the experimental results with predictions of various models appearing in the literature were made. Agreement existed with the modified versions of the packet model. However, for correlations based upon and compared with only limited data, agreement was not satisfactory.

Consequently a more physically realistic theoretical model was developed to improve the agreement between experimental results and analytical predictions. The packet theory of heat transfer of Mickley was modified by accounting for the presence of the wall in affecting the local voidage. A model of the heat transfer processes within the media adjacent to the wall was developed that takes into account the variation of the thermophysical properties of the media in that region. A simple numerical solution to the resulting energy equation was obtained. The theory was found to correctly predict the trend of the measurements of the present study as well as the experimental data of controlled residence time available in the literature. Comparing present experimental
measurements with predictions from the model (in terms of Nusselt number versus Fourier number), the maximum difference observed was 39 percent, over a Fourier number range from 0.001 to 20. The average difference observed was estimated to be about 12.6 percent.
# TABLE OF CONTENT

<table>
<thead>
<tr>
<th>SECTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. LITERATURE SURVEY</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Previous Experimental and Theoretical Study</td>
<td>5</td>
</tr>
<tr>
<td>2.3 Overview of Phenomenology of Basic Models</td>
<td>17</td>
</tr>
<tr>
<td>2.3.1 Film model</td>
<td>17</td>
</tr>
<tr>
<td>2.3.2 Single particle model</td>
<td>18</td>
</tr>
<tr>
<td>2.3.3 Emulsion phase models</td>
<td>18</td>
</tr>
<tr>
<td>2.4 Effective Thermal Conductivity</td>
<td>19</td>
</tr>
<tr>
<td>2.5 Voidage Variation Near a Constraining Surface</td>
<td>21</td>
</tr>
<tr>
<td>3. HEAT TRANSFER ANALYSIS</td>
<td>23</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>23</td>
</tr>
<tr>
<td>3.2 Basis of Proposed Model</td>
<td>24</td>
</tr>
<tr>
<td>3.3 Formulation of the Model</td>
<td>25</td>
</tr>
<tr>
<td>3.3.1 Voidage variation</td>
<td>28</td>
</tr>
<tr>
<td>3.3.2 Effective thermal conductivity</td>
<td>29</td>
</tr>
<tr>
<td>3.3.3 Heat capacity</td>
<td>30</td>
</tr>
<tr>
<td>3.4 Theoretical Heat Transfer Predictions</td>
<td>30</td>
</tr>
<tr>
<td>3.4.1 Finite element analysis</td>
<td>31</td>
</tr>
<tr>
<td>3.4.2 Comparison with available models and experimental data</td>
<td>38</td>
</tr>
<tr>
<td>4. EXPERIMENTAL INVESTIGATIONS</td>
<td>40</td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>40</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>DESCRIPTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-1.</td>
<td>Gravity flowing particulate media in a vertical tube</td>
<td>72</td>
</tr>
<tr>
<td>3-2.</td>
<td>Local time-average heat transfer coefficient vs. axial distance</td>
<td>77</td>
</tr>
<tr>
<td>3-3.</td>
<td>Mean heat transfer coefficient vs. axial distance</td>
<td>78</td>
</tr>
<tr>
<td>3-4.</td>
<td>Local time-average Nusselt number based on bed conductivity vs. Fourier number</td>
<td>79</td>
</tr>
<tr>
<td>3-5.</td>
<td>Mean Nusselt number based on bed conductivity vs. Fourier number</td>
<td>80</td>
</tr>
<tr>
<td>3-6.</td>
<td>Local Nusselt number based on gas conductivity vs. Fourier number</td>
<td>81</td>
</tr>
<tr>
<td>3-7.</td>
<td>Mean Nusselt number based on gas conductivity vs. Fourier number</td>
<td>82</td>
</tr>
<tr>
<td>3-9.</td>
<td>Comparison of present method with Mickley and contact resistance models for local heat transfer in sand/air system</td>
<td>84</td>
</tr>
<tr>
<td>3-10.</td>
<td>Comparison of present method with other theoretical predictions for local time-average heat transfer in steel/air system</td>
<td>85</td>
</tr>
<tr>
<td>3-11.</td>
<td>Comparison of present method with predictions by Kubie [31], Mickley [11], and data from [31] for local time-average heat transfer in the glass/air system</td>
<td>86</td>
</tr>
<tr>
<td>3-12.</td>
<td>Comparison of present method with predictions by Kubie [31], Mickley [11], and data from [31] for local time-average heat transfer in the copper/air system</td>
<td>87</td>
</tr>
<tr>
<td>3-13.</td>
<td>Comparison of present method (k_o/kg=6.27) with various models (k_o/kg=6.03) of the time averaged Nusselt number</td>
<td>88</td>
</tr>
</tbody>
</table>
3-15. Comparison of present method ($k_p/k_g=4.15$) with predictions and data by Gloski [25] for local time-average Nusselt number. ........................................... 90


4-1. Particle size distribution .................................................................................. 92

4-2. Schematic diagram of apparatus used for measuring thermal conductivity. .......................................................... 97

4-3. Effective thermal conductivity ........................................................................ 98

4-4. Transient temperature rise within the line source embedded in granular media. .............................................................................................................. 103

4-5. Comparison of some of the existing correlations and data for $\varepsilon \approx 0.40$ with present measurements. .......................................................... 104

4-6. Schematic diagram of apparatus used for measuring thermal diffusivity. .......................................................... 105

4-7. Effective thermal diffusivity ........................................................................ 106

4-8. Plotting of equation (4-7). ............................................................................. 111

4-9. Variation in center temperature with time for granular media. .................. 112

4-10. Specific heat ................................................................................................. 113

4-11. Schematic diagram of the test loop. .............................................................. 118

4-12. Schematic diagram of the test section .......................................................... 119

4-13. Flowing glass beads $d=2.07$ mm .................................................................. 120

4-14. Local heat transfer coefficient along axial distance from tube inlet for the tested granular media. .......................................................... 123

4-15. Average heat transfer coefficient .................................................................. 124


4-18. Correlation of maximum flowing bed to surface coefficients against Archimedes number by [2]. .................................................. 128

4-19. Comparison of present experimental data and theory with predictions by Kubie [31], Mickley [11], and data from [31] for mean heat transfer in the glass/air system. .................................................. 129

4-20. Comparison of present experimental data and theory with other theoretical predictions and data for mean heat transfer in the steel/air system. .................................................. 130
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-1. Measured Values of Thermophysical Properties of Test Media</td>
<td>149</td>
</tr>
<tr>
<td>4-2. Summary of Uncertainty Analysis Results</td>
<td>150</td>
</tr>
</tbody>
</table>
NOMENCLATURE

A  area of heat transfer surface

A_r  Archimedes number \( \frac{gd^3\rho_g(\rho_s-\rho_g)}{\mu g^2} \)

B  gradient matrix

C  capacitance matrix

C_c  calorimeter calibration constant

c_p  specific heat

D  material property matrix

d  particle mean diameter

d_i  diameter of particles in given size range

E  rate of energy transfer by convection across a control surface

e  calorimeter millivolt output

F  global force vector

h_i  instantaneous packet heat transfer coefficient

h_x  local time average heat transfer coefficient, Equation (3-20)

h_m  surface mean heat transfer coefficient

h_x  local time average heat transfer coefficient, Equation (3-23)

h_{max}  maximum flowing packed bed to surface heat transfer coefficient

I  variational form

K  conductance matrix

k  thermal conductivity

k_g  thermal conductivity of gas

k_o  bulk effective thermal conductivity

k_s  thermal conductivity of solid particles
L  test section length
m  mass flow rate
N  shape function matrix
n  defined by Equation (3-7-d)
Num_g  surface mean Nusselt number based on gas conductivity
Num_o  surface mean Nusselt number based on bed conductivity
Nux_g  local time average Nusselt number based on gas conductivity
Nux_o  local time average Nusselt number based on bed conductivity
Nu*  modified Nusselt number
Pe_L*  modified Peclet number
Pe_L  Peclet number
Q  net power input to the test section
q  rate of energy transfer by conduction across a control surface
q'  heat input per unit length of wire heat source
q''  heat flux
q_c  energy released in the calorimeter
R  test section inner radius
R_o  radius of stationary cylinder of granular media
r  radial distance from the center of the tube
T  temperature
T_c  temperature at longitudinal axis of stationary cylinder of granular media after time t
T_i  inlet temperature to test section or initial temperature of stationary cylinder of granular media
T_L  radial temperature at exit
T_{mL}  mean exit temperature
T_{mx}  mean temperature at x
$T_s$ surface temperature of stationary cylinder of granular media

$T_{wx}$ wall temperature at $x$

$T_x$ temperature profile at $x$

$t$ time

$V$ mean velocity in axial direction

$V_s$ volume of particulate solid

$V_t$ total volume

$w_i$ weight fraction in a given size interval

$x$ axial distance from the inlet section

**Greek Symbols**

$\alpha$ effective thermal diffusivity

$\alpha$ defined by Equation (3-7-b)

$\gamma$ defined by Equations (3-7-e) and (3-7-f)

$\Delta$ increment

$\theta$ defined by Equation (3-7-c)

$\tau$ packet age

$\psi(\tau)$ frequency of occurrence of packets of age $\tau$

$\varepsilon$ voidage

$\varepsilon_o$ bulk voidage

$\pi$ 3.14159

$\Sigma$ summation

$\rho$ density

**Subscripts**

$e$ element
g  gas
r  radial direction
s  solid
x  axial direction

Superscripts

e  element
T  transpose
1. INTRODUCTION

In recent years, there has been an increased interest in the thermal behavior of granular solids because of many relatively new industrial applications. Industries involving particulate solids being immersed or carried in a "continuous" fluid phase include chemical, petroleum, metallurgical, and food and pharmaceuticals process industries. Many fluidized bed operations are physically similar in nature, as in drying, coating, granulation, coal combustion and gasification processes. Also contact dominated flows are found to occur in certain industrial heat exchange equipment designed to heat, cool or dry granular materials.

In many cases of cooling or heating granular solids it is necessary to keep the solids physically separated from the cooling or heating medium. One method of accomplishing the desired heat exchange in such cases is to allow the solid to flow in a settled condition through a single vertical pipe which is jacketed with the heating or cooling fluid. Units of this type are inexpensive, simple and cheap to operate, and are readily available. However, in packed or fluidized bed systems the design of efficient heat transfer surfaces immersed in the bed is a subject which has received much study in recent years, but yet needs additional investigation. A better understanding of the heat transfer processes involved is still required. Thus, the general intention of the present investigation is to examine the heat transfer characteristics of gravity flowing particle beds and the dependence on different flow conditions and design parameters.
Among the many recent studies of heat transfer in granular media is the work by Botterill and Desai [1] and Denloye and Botterill [2]. They have used a flowing packed bed to model the heat transfer behavior of a freely fluidized bed of particles (i.e. one having no net aggregate velocity). In [1] it was stated that the advantage of making experimental tests in a flowing packed bed was that more precise control of particle residence time was possible than that obtained in a freely fluidized bed. In another relatively recent study, Braun and Durrant [3] used sand in a gas-to-sand heat exchanger and a sand-to-water steam boiler. Although the use of phase change materials in energy storage systems is attractive because of their high latent heat of fusion, the utilization of sand as an energy storage material offers a number of significant advantages over conventional molten salt or metal liquid systems as summarized in [3]. Braun and Sarikelle [4] have also proposed sand as a novel thermal energy storage material, citing drawbacks in using molten salts; the principal advantage of a medium such as sand or silica is the high temperature levels attainable in such a working system.

Thermal analyses of the performance of equipment transferring heat to or from granular materials have been hampered by a lack of data and theoretical models for the convective heat transfer coefficients in flowing particle beds. Braun and Durrant [3], in their analysis of a gas-to-sand heat exchanger, mentioned that the range of the sand-side heat transfer coefficient is somewhat uncertain and the results reported in literature vary widely from one researcher to another. Spelt, Brennen and Sabersky [5] stated that the general problem of the flow of granular materials is extremely complex and not yet well understood. They also stated that the energy transport mechanisms and processes require further insight and investigation. Sullivan and Sabersky [6] concluded that the previously published heat transfer data indicate that the graininess of the media can, under certain conditions, directly and substantially influence convective heat transfer. Yet no definite parameter characterizing the importance of the granular nature of the
material is generally specified. Glicksman and Decker [7] reported that although there have been numerous studies of heat transfer in fluidized beds, there remain a number of unresolved questions concerning the heat transfer mechanisms. Few warnings are made concerning the limits of applicability of predictions or correlations based upon the popular homogeneous packet model. Furthermore Glicksman and Decker [7] reported that little effort has been directed at characterizing the nature of particle packing at immersed surfaces.

Heat transfer and thermal properties of a flowing granular medium, such as sand, appear to be of fundamental interest and value. The effects of particle size, ordering and thermal properties, average channel velocity and heating surface geometry appear to warrant further study.

The particular type of flow to be considered in this study is that in which the adjacent material particles are in physical contact and the interstitial fluid moves with it passively. In such a contact dominated flow the heat is generally transferred by convection between a solid surface and the granules which move relative to that surface.

The main objectives of the present research are:

1) To obtain a fundamental understanding of the heat transfer and flow characteristics of granular flowing media.

2) To generate data describing the thermal behavior and characteristics of granular media.

3) To develop a physical and analytical model of the heat transfer behavior of flowing granular materials that will be capable of providing accurate predictions and design criteria for practical systems.

To approach the mentioned objectives, the relevant literature pertaining to experimental and theoretical aspects of heat transfer to granular media were reviewed. An experimental and analytical study was undertaken and is described in the following
sections. The results obtained are then used to test existing correlations for their generality and accuracy.
2. LITERATURE SURVEY

2.1 INTRODUCTION

In this section previous experimental and theoretical analyses of convective heat transfer to flowing granular media will be reviewed. Mechanisms previously postulated for the heat transfer phenomena between a bed and a surface will be summarized. Also, available formulas and techniques for predictions of effective thermal conductivity and voidage fraction distribution near the surface are presented and discussed.

2.2 PREVIOUS EXPERIMENTAL AND THEORETICAL STUDY

The heat transfer rates between a heated solid surface and moving or fluidized beds of solid particles are much higher than for single phase gaseous flow. In order to explain this phenomenon and predict heat transfer rates for design purposes, several investigators have studied this problem experimentally and analytically.

One of the earliest investigations (1948) includes an experimental and theoretical study by Brinn, Friedman, and Gluckert [8]. They examined the factors which influence the heat transfer rates from beds of sand moving downward through vertical steam-jacketed tubes, assuming rodlike flow; Design charts were developed by them for use in predicting heat transfer rates to solids flowing in a settled condition inside vertical pipes.
Experimental measurements were also made of the heat transfer from constant temperature plates immersed in rotating packed beds of granular material by Dunsky, Zabrodsky, and Tamarin [9]. They found an increase in the average heat transfer coefficient with a decrease in the residence time or a decrease in the particle diameter. Measured values of the heat transfer coefficient were 70 percent lower than that predicted from a model based on considering heat transfer through a gas layer between a single solid sphere at constant temperature and a plane surface in contact with it.

Another experimental study, by Harakas and Beatty [10], considered an electrically heated surface immersed in a bed of fine particles (0.003 mm - 0.375 mm) contained in a rotating trough. Heat transfer coefficients were measured for a moving bed of solid particles with various interstitial gases. Velocities of the bed relative to the heat transfer surface were varied between 1.8 to 14.0 m/min. Comparison with coefficients predicted from a simple homogeneous conduction model, however, indicated discrepancies and a failure of the model as the effective thermal conductivity of the bed decreased and as the particle size increased.

In an experimental study of: (a) a pressurized system of solid particles flowing as a packed bed past a hot surface and (b) a pressurized fluidized bed by Botterill and Desai [1] it was found that the static pressure had little effect on the rates of heat transfer for flowing packed beds but an important effect on the heat transfer rates to fluidized beds of larger, closely sized particles. The maximum coefficients of heat transfer in the fluidized bed tests were generally lower by about 25 percent from those obtained in the flowing packed bed except for large, closely sized material. They concluded

No adequate model for the prediction of bed to surface heat transfer coefficient is available because the magnitude of the relevant parameters and their variations with the fluidizing conditions are not known. Until the effects of the particle packing density and their replacement rates near the transfer surface can be included, models are only likely to be useful as a basis for a posterior fitting to the experimental data.
An early theoretical model was developed by Mickley and Fairbanks [11] in which a group, or "packet", of particles was considered to be in contact with the heat transfer surface for a short residence time, then to be replaced by a fresh packet of particles from the bulk of the bed. They assumed that the thermal properties of the packets were the same as those of the quiescent bed, and derived an expression for the local instantaneous heat transfer coefficient. To support the proposed heat transfer mechanism they conducted an experimental work where heat transfer measurements were made on the same solid with several different fluidizing gases.

Botterill and William [12] introduced a simple model of conductive heat transfer to a single spherical particle immersed in a gas which comes into contact with a hot surface (at time zero). Then, by considering the number of particles in contact with a unit surface area, the overall heat transfer coefficient between the surface and the bed was estimated as a function of particle residence time at the surface. Predictions of the single particle model were lower than the experimental values. Later Botterill, Butt, Cain, and Redish [13] extended the model to consider heat transfer into a two-particle-depth layer in contact with the surface. Discrepancies between their experimental measurements and the theoretical predictions from the model were consistent with the presence of a gas film between the wall and the particles of the order of 10 percent of the particle diameter. Regarding the question of whether such gaps had any physical reality or not, Botterill [14] speculated that they did, but that it is difficult to distinguish to what extent their magnitude may differ between a flowing packed bed system and a freely fluidized bed.

Baskakov [15] introduced an additional contact resistance to the Mickley and Fairbanks model [11] to allow for the effect of the resistance to heat transfer in the region of increased voidage adjacent to the surface acting in series with the thermal resistance of the packets. The thickness of the gas layer was chosen to be approximately equal to the particle radius. Gelperin and Einstein [16] developed this approach further and
derived expressions for the overall heat transfer coefficient for a flowing granular medium in contact with a heated surface as a function of residence time, and particle and voidage thermal conductivity.

Denloye and Botterill [2] utilized the experimental heat transfer results obtained from a flowing packed bed to allow further tests of the modified packet model of heat transfer. They found that the flowing packed bed-to-surface heat transfer coefficient increased with decreasing particle residence time, with decreasing particle size, and with increasing gas thermal conductivity. They interpreted these results with the aid of a modified version of the Mickley and Fairbanks model [11]. The modified Mickley and Fairbanks model incorporated an additional wall heat transfer resistance that was given in terms of the particle diameter and the effective thermal conductivity near the wall. An empirical correlation was developed in terms of the mean particle diameter, gas conductivity and Archimedes number. In a later study, Botterill and Denloye [17] proposed a theoretical model to describe the heat transfer between an immersed heater and unrestrained packed beds. The bed was divided into two regions; (a) a region of increased voidage close to the heat transfer surface and (b) the region outside this. The voidage and gas velocity were assumed to be constant in each region. Heat transfer was assumed to take place by steady state conduction, and effective thermal conductivities were used to calculate the heat transfer in both regions. Their model overpredicted the heat transfer coefficient for particles of mean size less than 0.34 mm, but with discrepancies being less than 25 percent in general. With large particles there was an increasing discrepancy between the experimental results and the predictions of the model, with the predictions being consistently low. They suggested that the key factor in bed-to-surface heat transfer is the local voidage and the particle behavior immediately adjacent to the heat transfer surface.
Another experimental and analytical study by Sullivan and Sabersky [6], focuses upon the heat transfer from a flat plate immersed in a flowing granular medium. They showed that their experimental results corresponded well to a model in which the granular material was represented by a continuum connected to the heating surface by a special contact resistance, which was imagined to result from pure conduction through a thin gas layer, whose presence was postulated on the basis of their experimental data. The thickness of the gas layer was thus "determined" empirically. Its resistance was found to be approximately equivalent to an air gap of 1/10 of a particle diameter thickness. The correlation derived from this model agreed well with their experimental results. Their results indicated that the magnitude of the heat transfer in this configuration may vary over a large range, depending on the magnitude of the Peclet number encountered.

Spelt, et al. [5] investigated convection heat transfer for two granular materials flowing along an inclined chute. For each material and for a given depth of flow, it was found that the heat transfer coefficient first increases with velocity, then reaches a maximum and decreases as the velocity increases further. They found that the curves for smaller depths fall below those for the larger depth flows. These unexpected results clearly indicate the complexity of the heat transfer processes in a flowing granular material. Spelt, et al. related this behavior to changes in the packing density of the material caused by the flow field. The relation between velocity, density, and regimes of inclined granular chute flow has been studied recently by Campell, Brennen and Sabersky [18].

Schlunder [19] proposed a model for heat transfer between packed, agitated and fluidized beds and immersed surfaces. Within this model, the particulate material may absorb the heat supplied from the rigid wall in the sensible as well as in the latent mode. The pressure of the interstitial gas may vary from vacuum to pressures above
atmospheric. The heat transfer mechanism from the heated surface to the adjacent particles was described by: (1) wall-to-particle heat transfer by conduction, and (2) heat convection by particle motion. The combination of these mechanisms in the model leads to the introduction of several dimensionless groups, that reduce the number of variables as well as anticipating the structure of empirically based correlations. Parameters such as physical properties of the gas and solid, gas pressure and temperature, particle density, void fraction, etc. were taken into consideration. Comparison of the theory with numerous experimental data taken from various sources was found to agree within 30 percent. Predictions of heat transfer coefficients from this model (which apply to contact heating or cooling of packed beds either resting on or sliding past a stationary heated surface) agrees fairly well with the correlation by Sullivan and Sabersky [6]. Schlunder pointed out that the correlation given by Sullivan and Sabersky [6], however, does not correctly describe the effect of the particle diameter. Moreover the empirical parameter employed in the Sullivan-Sabersky correlation does not at all take into account the influence of the gas pressure, while Schlunder's studies have shown gas pressure to have a strong effect on the average heat transfer coefficient. (Botterill and Desai [1] reported that the static pressure had an important effect on the heat transfer rates to fluidized beds of large closely sized particles, but a little effect on the rates of heat transfer for flowing packed beds).

Nietert and Khalik [20] determined the heat transfer characteristics of gravity flowing particle beds at controlled rates through an electrically heated tube. The local and average convective heat transfer coefficients were evaluated as a function of the average bed velocity, particle size and heat flux. They concluded, from the results obtained by operating the experiment at different heat flux levels, that there is a strong dependence of the heat transfer coefficient on the physical properties of the particles. They indicated that additional experiments over a wide range of variables are required to
obtain a generalized Nusselt type correlation for the heat transfer coefficients in gravity flowing particle beds.

Studies of the characteristics of wall-to-bed heat transfer in a vertical tube using a fluidized particle bed were performed by Izumi and Yamashita [21]. Their vertical tube was heated electrically so as to maintain a uniform heat flux condition. Particles of glass beads were fluidized by air flowing upward. The local Nusselt number in the vertical direction was measured for different particle diameters, initial bed height, and air velocity.

Burolla, Hruby and Steele [22] investigated the heat transfer characteristics of a solar collector. They indicated that particle absorptivity, particle diameter, mass flow rate and wall temperature are important parameters indicating the final particle temperature. A simple model describing system efficiency is used to predict the effect of these parameters on the energy absorbed and the final exit temperature of the solid particles.

Decker and Glicksman [23,24] proposed a physical model of the particle-to-surface heat transfer in the emulsion phase* of a fluidized bed. They concluded that the heat transfer between the bed particles and an immersed surface is dependent on the thermal time constant of the particles and the average particle residence time on the surface. When the residence time is much greater than the time constant, the surface resistance is unimportant and a continuum model for heat transfer in the emulsion is appropriate. When the time constant is larger than the residence time, the thermal resistance between the surface must be considered and a continuum model is inappropriate. In the region of close contact between particles and the surface, a realistic model of the detailed surface geometry near points of contact must be included. When a representative model of surface roughness is assumed, the conductive heat transfer is

* Regions of high solid density.
found to be insensitive to thermophysical properties of the particles and only modestly influenced by the level of roughness. They failed to make a direct comparison of results presented by their analysis and other experimental findings because of the absence of a detailed knowledge of the contact zone geometry.

In [24], Decker and Glicksman considered and studied heat transfer to immersed surface in large particle fluidized beds. Two mechanisms of particle heat transfer at low temperatures were assumed to occur, namely (a) conduction heat transfer near the point of contact between particles and the heated surface, and (b) convection augmentation due to lateral mixing of the gas in the large voids between individual particles in the emulsion. Glicksman and Decker [7] also introduced a simplified physical model of heat transfer from a fluidized bed to an immersed surface. They concluded that packet models using continuum properties are reasonable only when particle diameters are less than 0.3 to 0.6 mm. For particles with diameters greater than 0.6 mm they stated that the thermal gradient extends only one or two particle diameters into the bed. They suggested that a discrete analysis of heat transfer to individual particles is necessary for such large particles. In the simulation of particle packing at the heat transfer surface, assuming random packing, they point out that irregular shaped particles will cover a smaller fraction of the surface than will spherical particles. This is consistent with their numerical estimates of the heat transfer coefficients for spherical particles being 50 percent greater than for irregular particles. The contribution of radiation to the total heat flux of high temperature beds is demonstrated to be a function of fluid mechanical variables as well as particle size and bed and surface temperature.

Another more recent study by Gloski, Glicksman, and Decker [25], focuses upon measuring the heat transfer between a surface and particles of diameters between 0.65 and 1.0 mm for both fixed and fluidized beds. An experimental apparatus was developed which permitted measurements of the thermal resistance between a surface
and adjacent particles during a rapid transient process. Large heat transfer coefficients were observed in the initial 20 milliseconds of contact while those during an ensuing period of about 80 milliseconds were much lower and nearly invariant with time. After that, the coefficients began to decline. They reported that this behavior is most consistent with a discrete particle model in which initial heat transfer occurs through the asperities in direct contact with the surface, followed by heat transfer through gas layers in areas adjacent to the contact points. The results obtained for fixed and fluidized beds at vertical heat transfer surfaces were virtually identical. They concluded from the absence of any significant difference between the packed bed and fluidized bed results that it is not necessary to postulate an intervening gas layer between the contact points on the surface and the particles of a fluidized bed. They further reported that one-dimensional models reflecting the local voidage distribution may be inadequate since they neglect the inhomogeneous distribution of thermal properties; Consequently, the numerical complexities of such models may not be warranted in light of this uncertainty. They call for further work in order to determine the influence of particle sphericity and the distribution of particle size.

Heat transfer in a large particle diameter fluidized beds (greater than 1 mm) with immersed surfaces have been studied by Ganzha, Upadhyay, and Saxena [26]. The model which they propose neglects radiation and regards the heat transfer coefficient as composed of gas conduction and convection components. They assumed that particles on the heat transfer surface are arranged in an orthorhombic arrangement. They further assumed that particles can be replaced by equivalent cylinders having the same volume as the actual particles and of equal height and diameter. The conduction contribution is calculated by computing the unsteady state heat transfer rate from the surface to the first layer of particles through the gas film (or lens) enclosed between the two. The convection contribution is obtained by assuming that a turbulent boundary layer on the
heat transfer surface is continuously disrupted by the solid particles, and by considering the analogy of this process with that of a flat plate immersed in a solids-free turbulent gas stream. It was found that the theoretical model predictions for the total heat transfer coefficient are in good agreement with the available experimental data on large particle systems. Comments and discussion of the paper by Ganzha, et al. [26] is presented by Kubie [27].

Yoshida, Tamura, and Kunii [28] measured effective thermal conductivity in the radial direction and the apparent film coefficient at the wall surface in moving beds of glass beads and sand. Solid particles descended downwards in a vertical tube heated externally by means of condensing steam, while air was passed upwards through the moving bed. They obtained empirical equations for the effective radial thermal conductivity and the wall film heat transfer coefficient.

Takahashi and Takeuchi [29] investigated the characteristics of the axial temperature distribution in a moving bed in which air was forced countercurrently to the gravity flow of solid particles. Both the fluid and solids were heated by an electric heater outside the tube. The axial temperature curve obtained usually had a maximum in a particular axial position.

A model for the mechanism of heat transfer from the surface of body fixed in a fluidized bed is proposed by Tasutomi and Yokota [30]. The emulsion phase was assumed to be a composite solid consisting of two layers. The first layer contacts the surface; Its void fraction is higher on account of a wall effect, while its thickness is assumed to be approximately equal to the radius of the fluidized particles. The second layer is considered to have a void fraction equal to that at minimum fluidization; It exists beyond the first layer and is assumed to have an infinite thickness. Their equation for the heat transfer coefficient is derived considering unsteady conduction from the constant temperature surface to the solids.
Kubie and Broughton [31] proposed a model for surface-to-bed heat transfer analogous to the variable wall properties approach long used in single phase heat transfer. In their model packets of material from the bed are swept to the heat transfer surface, where transient conduction occurs for the time of packet residence. Allowance is made for the variation of the void fraction within a distance of one particle diameter from the surface. The voidage variation near the surface is described from simple geometrical considerations. This important feature of the model permits the bulk density and thermal conductivity to vary according to some specified voidage distribution function in the vicinity of the wall. It was found that for the voidage distribution assumed to exist in a bed of uniform spherical particles, the thermophysical properties of the bulk changed very rapidly with distance from the surface. They concluded that the derived model agrees well with all controlled residence time data available in the literature. Further, their model was reported to require no physically unjustified concepts in order to produce agreement with experimental data. By contrast, none of the available models of heat transfer in fluidized beds which they examined were capable of producing acceptable agreement with experimental data over the full range of data without the introduction of some form of semiempirical approximation. They observe that while approximations such as (a) a gas film, (b) a region of reduced voidage or (c) a finite penetration depth, all have obvious physical significance, they are ad hoc expedients rather than solutions to the real problem of behavior close to a surface.

A modified packet model which accounts for the change of void fraction near the surface, was introduced by Ozkaynak and J.Chen [32]. Heat transfer coefficients from a vertical tube in fluidized beds were predicted by measuring emulsion packet residence times. They found that the packet model by Mickley and Fairbanks [11] was satisfactory for small particles of different physical and thermal properties but overpredicted the heat
transfer coefficients for large particles. Their modified packet model was found to successfully predict heat transfer coefficients for both small and large particles and for particles with different physical and thermal properties. Also Chandran and J.Chen [33] proposed a model for local heat transfer between a gas fluidized bed and a submerged tube based on combined dense phase and lean phase transport. The heat transfer process during dense phase contact at the tube surface is modeled by a packet renewal mechanism, while the transfer process during lean phase contact is modeled by a fluid convection mechanism. The model predictions show good agreement with experimental local and average heat transfer coefficient data for horizontal tubes.

Another study by Chandran and J.Chen [34] investigated the influence of the wall on transient conduction into packed media. Since surfaces immersed in moving packed beds and fluidized beds frequently experience a renewal type of contact with the contained media, the heat transfer during the surface renewal process was modeled by them as a transient conduction process. They maintained that this is permissible provided the contribution due to thermal radiation is negligible. They developed a simple closed form relation for the heat transfer coefficient which is applicable for use in design purposes.

Another theoretical study by P.Chen and Pei [35] was developed to describe the heat transfer between a fluidized bed and an immersed surface, based on the definition of a two phase thermal boundary layer around the surface. In considering the vigorous particle motion and the increasing bed voidage, two correlations for predicting the maximum heat transfer coefficient and the optimum flow rate were obtained. These correlations agree well over a wide range of conditions with the data reported in the literature.

A computer model for a hot fluidized bed was developed by Syamlal and Gidaspow [36]. They used a hydrodynamic model of fluidization, utilizing the continuum approach
to model fluidized bed heat transfer and the principals of conservation of mass, momentum and energy. Predictions of the heat transfer coefficients were obtained by numerical solution of the established set of nonlinear partial differential equations. They agree with measurements reported by Ozkaynak and J.Chen [32] within the accuracy of the estimated thermal conductivity of particulate aggregate solids.

2.3 OVERVIEW OF PHENOMENOLOGY OF BASIC MODELS

A review of the models proposed in the literature for heat transfer between a moving bed and a surface are given in Gelperin and Einstein [16], Botterill [14], Gutfinger and Abuaf [37], and Saxena, et al. [38]. Generally, the various models may be grouped into three basic types, as will now be summarized.

2.3.1 Film Model

The first category of models is based on a fluid film theory [37]. In this model heat transfer is assumed to occur through a hypothetical fluid film in contact with the wall to the particles moving downward outside this film; The thermal resistance of the fluid film is affected by the solid particles passing through it. As this kind of model does not take into account the influence of particle properties, the subsequent formulas are usually not in good agreement with the experimental results, as reported by Chen and Pei [35]. Also Gutfinger and Abuaf [37] indicted that because of not taking into account the influence of the solid particles on the heat transfer phenomena, the mechanism proposed cannot be considered to be complete.
2.3.2 Single Particle Model

The second category of models is based on the transient heat conduction process occurring between the immersed surface and a single particle [14]. Predictions of these models generally deviate greatly from experimental data gathered from closely controlled residence time [31]. This shortcoming was removed by introducing a hypothetical gas film between the particle and the surface [2,12,14] This model is acceptable at low Fourier numbers only [31]. To improve the model additional layers of particles were considered in an extended analyses [13,14].

2.3.3 Emulsion Phase Models

In this type of model a packet of constant voidage emulsion phase is considered to be swept into contact with the heat transfer surface for a period of time during which heat is transferred by non-steady conduction [11]. Good predictions have been obtained at large values of the Fourier number [2,31] while at small values the model fails [31,38]. Models based on this approach have been refined, however, in order to extend their validity to low Fourier numbers [16]. The method used is to introduce a time independent contact resistance at the bed surface interface to account for the increased voidage in the vicinity of the surface [2,15]. The evaluation of this contact resistance is based upon empirical considerations which enable a good data fit to be obtained. A recent modification to the packet theory has been introduced by Kubie [31] that accounts for the presence of the surface and its effect on the local voidage, and hence alters the local thermophysical properties. The variation of the voidage was assumed to be confined to the plane normal to the wall.
In the development of the analytical methods for the design of equipment for gas-solid system, the proper values of the effective thermal conductivities must be estimated. The granular medium in general consists of solid particles of various shapes and sizes in physical contact with each other, and voids of more complicated shape. A knowledge of the effective thermal conductivity of such composite materials is important for the thermal analyses of such systems. Mickley and Fairbanks [11] modeled the thermal properties of bulk material as a homogeneous mixture of the solid and fluid properties. It was shown [15,17] that the contact resistance changes due to variations with effective conductivity of the bulk material in the immediate vicinity of the wall. A number of experimental measurements and many formulae are available in the literature to predict the thermal conductivity of two component solid-gas granular systems. Generally the thermal conductivity of the bed is expressed as a function of the thermal conductivity of the gas and solid phases, the void fraction, particle size, and the pressure of the system. A review of the major published theoretical correlations of thermal conductivity of granular material and a number of proposed heat transfer models are summarized by Crane, Vachon, and Khader [39]. Both experimental techniques and theoretical models for evaluating the effective thermal conductivity of low conductivity materials are given in Pratt [40].

Effective thermal conductivity models for granular systems are generally classified as being either Ohm's law models or flux law models. Correlations for the effective thermal conductivity based on flux law models may be derived from solution of the heat conduction equation for a given system geometry. Ohm's law models assume parallel or series distribution of the components according to their arrangements with respect to the direction of the flow of heat. The effective thermal conductivity is measured utilizing
the basic standard steady state methods (guarded hot plate) and the dynamic methods (line source and probe).

Brinn et al. [8] and Mickley and Fairbanks [11] measured the effective thermal conductivity of granular media by a method similar to that described by Waddams [41], which involves measuring the unsteady radial heat flow between the surface and axis of a cylinder of material. Yagi and Kunii [42,43] performed experiments and obtained theoretical formula for effective thermal conductivities in packed beds. Willhite, Kunii, and Smith [44] and Kunii and Smith [45], derived equations for predicting the effective thermal conductivity of beds of unconsolidated and consolidated particles containing stagnant fluid; Comparison with the available experimental data indicated that their equations are satisfactory for fluids and solid particles of both high and low thermal conductivities. In [44] experimental results showed that the measured stagnant conductivity also satisfactorily represented heat transfer for flowing conditions.

An analytical study was performed by Yovanovich [46] for predicting the apparent thermal conductivity of beds of uniform diameter glass spheres. His mathematical model considered the thermal conduction resistance within the spheres, together with the conduction resistance of an effective gas gap thickness. The model is valid for gas pressures ranging from vacuum conditions to atmospheric.

Abrahamsen and Geldart [47] studied the effective thermal conductivity of fine powders fluidized with different gases. They concluded that in the range studied, the effective radial thermal conductivity does not appear to be a function of gas velocity and is most sensitive to the gas phase thermal conductivity.

The validity of existing thermal conductivity correlations for solid-gas mixtures have been examined by Kuzay [48]. He also examined the effects of porosity and radiation at higher temperatures. Dixon [49] derived formulas for the effective axial and radial thermal conductivities of packed beds from a simple thermal resistance model.
Also Yagi, Kunii, and Wakao [50] showed that the axial effective thermal conductivities in packed beds coincide with the radial conductivity when the gas flow rate approaches zero. Vortmeyer and Adam [51] analytically correlated the axial thermal conductivity at low gas flow rates. The results they obtained were in agreement with the previous work of Yagi et al. [50]. A model of the effective thermal conductivity and diffusivity of a packed bed with stagnant fluid was proposed by Jaguaribe [52]. The model requires only the void fraction and the physical properties of the fluid and solid as input; No empirical or theoretical model parameters were required.

Pande, Saxena and Chaudhary [53] measured the thermal conductivity of loose granular materials using the thermal probe method. Interstitial air pressure was varied from 0.1 to 1800 mm of mercury. A line source technique was utilized by Keshock [54] for measuring the effective thermal conductivity and thermal diffusivity of porous materials in several gaseous environments. An error analysis of the line-source technique was performed in order to estimate the resulting uncertainty of the thermal conductivity and thermal diffusivity determinations.

2.5 VOIDAGE VARIATION NEAR A CONSTRAINING SURFACE

When considering heat transfer between the particles in a packed or a fluidized bed and a surface immersed within the bed, it is necessary to know the porosity distribution at the surface across which the heat transfer occurs. Though the average void fraction within a bed may be known, the existence of a constraining surface will alter the particle packing, and hence the solid cross-sectional area, in the vicinity of that surface.

The variation of local voidage of packed beds in the vicinity of a constraining wall has been investigated by several workers. Roblee, Baird, and Tirney [55] measured the
radial variation of void fraction in randomly packed beds of spheres, cylinders, and other configurations. They utilized paraffin to fill the bed after packing and then allowed it to solidify. Results for spheres and cylinders of diameters equal to their height were such that the porosity was a minimum at one particle radius from the wall, with alternate maximums and minimums occurring at successive particle radii. The amplitude of the cycling decreased as distance from the wall increased, but significant differences were found beyond three particle diameters from the wall.

Benenati and Brosilow [56] investigated void fraction distribution in beds of uniform spheres. It was found that the radial variation of void fraction varies from unity at the wall to about 38 percent in the interior of the bed. The bed-to-particle diameter ratios were found to vary from 2.6 to infinity. Containing walls of concave and convex curvatures were studied. In another experimental work by Korolev and Syromyatnikov [57], the porosity of fixed and fluidized beds near the wall were investigated by means of x-radioscopy. According to their experimental results, the effect of the wall on the packing of the particles diminishes with increasing distance from the wall. A formula of the local voidage variation was derived in terms of the particle diameter, mean porosity, and the x-ray beam width.

Kubie et al. [31] described the local voidage variation in the vicinity of a constraining wall by a simple geometrical function for a distance of up to one particle diameter from the surface. Pilla [58] performed an experimental investigation in a two dimensional packed bed. He concluded that the geometrical function given by Kubie [31] is a very good approximation for distances of up to one particle diameter from the wall.
3. HEAT TRANSFER ANALYSIS

3.1 INTRODUCTION

An efficient approach for determining the rate of heat transfer associated with flowing granular media in smooth tubes for a uniform wall heat flux boundary condition is developed in this chapter.

The key to this proposed approach lies in the use of proper equations to evaluate the thermophysical properties of the medium in the critical region adjacent to the heat transfer surface, i.e. thermal conductivity, density, and specific heat; The voidage in this region must also be properly evaluated. Then, an efficient numerical finite element technique is used to solve the energy equation. This boundary layer type concept, which takes into account the influence of the surface on the local packing of a bed of granular material, was first introduced by Kubie and Broughton [31]. The behavior of the particles in the core of the flow field is treated as homogeneous in the present analysis.

Predictions are obtained for local time average and surface mean heat transfer coefficients for different flowing granular materials of different interstitial gases. The results of this suggested method are compared with predictions based on other theoretical methods as well as with available experimental data.
3.2 BASIS OF PROPOSED MODEL

For predictions of heat transfer coefficient of surfaces immersed in moving packed beds there exist two distinct general approaches, the homogeneous approach by Mickley et al. [11], and the discrete particle model by Botterill et al. [12]. Many investigators have suggested modifications to the basic models cited above so as to achieve good agreement with experimental data. The homogeneous model [11] has been modified recently by Kubie [31] to take into account the presence of the surface and its effect on the thermophysical properties of the bulk material in the immediate vicinity of the wall.

In the present model the packet theory of heat transfer is modified to take into account the presence of the surface and its effect on the local voidage. This is done by introducing a property boundary layer concept introduced by Kubie et al. [31] that has been found by them to be physically justified. The bed is considered as consisting of a region of higher voidage within a one particle-diameter distance from the cylindrical heat transfer surface and a core region which constitutes the bulk of the bed.

The model is based on the following assumptions:

1) Transient heat transfer takes place into a disk of the granular media flowing in contact with the hot surface (see Figure 3-1-a)*.

2) The region adjacent to the wall is characterized by greater voidage, which alters the local effective or composite thermophysical properties, such as density, specific heat, and thermal conductivity.

3) The variation of voidage is confined to within one particle diameter from the wall within any plane perpendicular to the channel's longitudinal axis.

* All Figures may be found in Appendix A
4) The core of the bed has a constant voidage and the temperatures of the gas and solid particles at any given point in the bed are of negligible difference because of the relatively high exposed particle surface area.

5) The interparticle heat transfer due to thermal radiation is negligible. (It was reported by Yagi and Kunii [42] that radiation heat transfer must be considered when the mean temperature of the packed bed is higher than 400 °C. In the present analysis the bed temperature is assumed to be much less than 400 °C. Also, in the experimental phase of the present study the maximum temperature level of the flowing media was about 100 °C).

6) A constant heat flux is maintained on the tube surface.

3.3 FORMULATION OF THE MODEL

By taking a ring shaped differential volume of length dx, thickness dr, and radius r as shown in Figure 3-1-b and making the assumptions of steady flow, no internal heat generation, no chemical reaction, and neglecting work done by body and surface forces, the energy equation reduced to

\[ E_r + q_r + E_x + q_x = E_{r+dr} + q_{r+dr} + E_{x+dx} + q_{x+dx} \]  \hspace{1cm} (3-1-a)

where \( E \) is the rate of energy convected with the bulk granular motion across the control surface and \( q \) is the rate of energy transferred across the control surface by conduction. The convective terms in the radial direction, \( E_r \) and \( E_{r+dr} \), are neglected since there is no mass flow in that direction. To elaborate upon this point, results of an experimental study of downward flowing particulate media by Davis and cited by Zenz and Othmer [59] are shown in Figures 3-1-c and 3-1-d. The experiments were carried out in a thin,
rectangular bin, with glass sides, permitting observation of the flow through a colored layer of particles. A photograph was obtained by using one piece of film exposed at 60-second intervals for 9 minutes without moving the camera. The solid lines in Figures 3-1-c and 3-1-d, taken from Zenz and Othmer [59], represents the flow contours at each exposure. The dashed lines connecting the large colored pieces were drawn to show the paths followed by the particles in down flow. Also visualisation experiments by Brinn, et al. [8] and Sullivan [60] indicated that the flow of granular material in a vertical channel is essentially rectilinear.

Additionally, it is assumed that no velocity fluctuations occur in the radial direction, analogous to those that might occur in the turbulent flow of liquids or gases, equation 3-1-a reduces to

\[ q_r - q_{r+dr} + q_x - q_{x+dx} = E_{x+dx} - E_x \]  \hspace{1cm} (3-1-b)

by noting that

\[ q_{x+dx} = q_x + \frac{d}{dx} (q_x) \, dx \]  \hspace{1cm} (3-1-c)

and a similar expression for \( q_{r+dr} \) and \( E_{x+dx} \) and by utilizing Fourier's law of conduction for \( q_x \) and \( q_r \) along with \( E_x = \rho \nu c_p T \), equation 3-1-b leads to the following partial differential equation

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ k(r) \, r \frac{\partial T}{\partial r} \right] + \frac{\partial}{\partial x} \left[ k(x) \frac{\partial T}{\partial x} \right] = \rho (r) c_p (r) V \frac{\partial T}{\partial x} \]  \hspace{1cm} (3-1-d)

The right side of this equation expresses the net rate at which heat is transferred by convection in the differential element owing to the flow into and out of the element faces in the direction parallel to the flow channel longitudinal axis. The left side represents the net rate at which heat is transferred by conduction in the radial and axial directions.
Visual experiments by Brinn, et al. [8] and Sullivan [60] not only indicated that the flow of granular material in a vertical channel is essentially rectilinear, but also the particles adjacent to the smooth wall slide along the wall at approximately the same speed as the bulk flow. Thus the velocity, $V$, in Equation (3-1-c) may be assumed constant. Equation (3-1-c) is subject to the following boundary conditions:

at the hot surface

\[ r = R, \quad \text{and} \quad 0 < x < L \]

\[ -k(r) \frac{\partial T}{\partial r} = q'' \quad (3-2) \]

at the centerline

\[ r = 0, \quad \text{and} \quad 0 < x < L \]

\[ \frac{\partial T}{\partial r} = 0 \quad (3-3) \]

at the inlet section

\[ x = 0, \quad \text{and} \quad 0 < r < R \]

\[ T = T_i \quad (3-4) \]

at the outlet section
In order to solve Equation (3-1-c) with the boundary conditions expressed by Equations (3-2)-(3-5) the functions $k(r)$, $k(x)$, and $\rho(r)$ $\epsilon_p(r)$ must be specified. The evaluation of these functions thus requires a knowledge of the voidage variation in the vicinity of the surface.

### 3.3.1 Voidage Variation

The variation of local voidage of packed beds in the vicinity of a constraining wall has been investigated by several workers as summarized in section 2.5. The geometrical function proposed by Kubie [31] for the voidage variation within one particle diameter from the wall is recommended by Pillai [58] and has been used by Chandran and Chen [34]. This function; Drived on the basis of simple geometrical considerations of single sphere in contact with plane surface, is utilized in the present analysis. Thus,

\[
\epsilon (r) = 1 - 3 (1-\epsilon_0) \left[ \frac{R-r}{d} - \frac{2 (R-r)^2}{3 d^2} \right] \quad (3-6-a)
\]

for \( (R-d) < r < R \)

and \( \epsilon (r) = \epsilon_0 \) \quad (3-6-b)

for \( r < (R-d) \)
3.3.2 Effective Thermal Conductivity

The effective thermal conductivity can be calculated from one of the models presented in section 2.4. Crane et al. [39] recommended the equation proposed by Willhite et al. [44]. Willhite indicated that their proposed equation satisfactorily represented heat transfer for flowing conditions. This equation is also valid in the region of high voidage as reported by Kubie [31] and Botterill [17]. It is as follows:

\[
\frac{k_\Theta}{k_g} = 1 + (1 - \varepsilon) \left(1 - \frac{k_g}{k_s}\right) + \frac{\gamma}{\alpha} \left(1 - \frac{k_g}{k_s}\right)^2 (1-\varepsilon)
\] (3-7-a)

where

\[
\alpha = 0.5 \left\{ \left(1 - \frac{k_g}{k_s}\right)^2 \frac{2}{\sin \theta} \right\}
\] (3-7-b)

\[
\ln \left[ \frac{k_s}{k_g} - (k_s/k_g - 1) \cos \theta \right] - (1 - \frac{k_g}{k_s})(1 - \cos \theta)
\]

\[
\sin^2 \theta = \frac{1}{n}
\] (3-7-c)

\[
n = 3 \left(5.01 - 8.42 \varepsilon_0\right) / \left(1.91 - 1.91 \varepsilon_0\right)
\] (3-7-d)

For nonspherical or high conductivity spherical particles

\[
\gamma = 0.5
\] (3-7-e)

while for spherical low conductivity particles,

\[
\gamma = \frac{2}{3}
\] (3-7-f)
The voidage $\varepsilon$ will be evaluated here by Equations (3-6-a) and (3-6-b).

As pointed out in section 2.4 the axial effective thermal conductivity coincides with the radial conductivity for stagnant interstitial gas.

### 3.3.3 Heat Capacity

The effective heat capacity in the region adjacent to the wall within one particle diameter is taken here as a weighted average of the solid and gas phases, i.e.

$$
\rho(r) c_p(r) = (\rho c_{p_s})_s (1-\varepsilon(r)) + (\rho c_{p_g})_g \varepsilon(r) \quad (3-8-a)
$$

The heat capacity of the gas phase can be neglected compared to that of the solid matter, so that

$$
\rho(r) c_p(r) = (\rho c_{p_s})_s (1-\varepsilon(r)) \quad (3-8-b)
$$

where $\varepsilon(r)$ is specified by Equation (3-6-a). In the bulk of the bed, Equation (3-8-a) or (3-8-b) is utilized with $\varepsilon(r)$ being specified by Equation (3-6-b).

### 3.4 THEORETICAL HEAT TRANSFER PREDICTIONS

Calculations of the heat transfer coefficient are obtained by solving equations (3-1-d)-(3-5) numerically using the proper equations specifying the properties of the medium in the region of high voidage adjacent to the surface as well as in the bulk of the flow field. Either finite difference or finite element methods can be utilized to
solve the partial differential equation and its boundary conditions. However, Gladwell and Wait [61] reported that the finite element method enjoys the advantages of (1) ease in modelling complex two and three dimensional geometries, and (2) its computation strategies, based on matrix handling procedures, can be highly automated.

In the present study the finite element technique is utilized to solve the governing equations of the present flow field problem.

### 3.4.1 Finite Element Analysis

The objective of this section is to determine the temperature distribution within a medium of granular material flowing under the action of gravity having a uniform velocity $V$ inside a vertical channel of circular cross section. A constant heat flux in the radial direction is maintained on the surface of the channel over the length $L$ in the axial direction.

Instead of seeking a finite element solution of equation (3-1-d) in the domain shown in Figure 3-1-e, the uniform flow velocity assumption for the interior region will be inserted in equation (3-1-d).

Since the particles move at a uniform constant velocity $V$, the $x$ coordinate, $x=Vt$, is used as a time scale. Then, equation (3-1-c) becomes

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ k(r) r \frac{\partial T}{\partial r} \right] + \frac{\partial}{\partial x} \left[ k(x) \frac{\partial T}{\partial x} \right] = \rho (r) c_p(r) \frac{\partial T}{\partial t} \quad (3-9)$$

The associated boundary conditions are given by equations (3-2)-(3-5). The initial condition is
\[ T = T_i \] \hspace{1cm} (3-10)

at time
\[ t = 0, \quad 0 < x < L, \quad \text{and} \quad 0 < r < R \]

Consider a disc with an axial thickness \( \Delta x \). The finite element idealization is shown in Figure 3-1-a, where the disc media is replaced by triangular ring elements. The equivalent variational form of equation (3-9) and the associated boundary and initial condition are given by many authors in the literature. Rao [62] and Segerlind [63] give the functional \( I \) as:

\[
I = 0.5 \int_v \left[ k(r) r \left( \frac{\partial T}{\partial r} \right)^2 + k(x) r \left( \frac{\partial T}{\partial x} \right)^2 + 2 \rho(r) c_p(r) \left( \frac{\partial T}{\partial t} \right) r T \right] dV + \int_s q T ds \] \hspace{1cm} (3-11)

The finite element equations are obtained by using the variational approach which minimizes the functional \( I \) and gives

\[
[C] \frac{\partial T}{\partial t} + [K] \{T\} + \{F\} = 0 \] \hspace{1cm} (3-12)

where \([C]\) is referred to as the capacitance matrix, \([K]\) is the conductance matrix, and \([F]\) is the global force vector. They are related to the element capacitance matrix \([C^e]\), element conductance matrix \([K^e]\), and element force vector \([F^e]\) by

\[
[C] = \sum_e [C^e] \] \hspace{1cm} (3-13-a)

\[
[K] = \sum_e [K^e] \] \hspace{1cm} (3-13-b)
\[ \{F\} = \sum_e \{F^e\} \quad (3-13-c) \]

and

\[ [C^e] = \int_v \rho c_p [N]^T [N] dv \quad (3-14-a) \]

\[ [K^e] = \int_v [B]^T [D] [B] dv \quad (3-14-b) \]

\[ \{F^e\} = \int_s q [N]^T ds \quad (3-14-c) \]

where \([N]\) is the shape function matrix, \([B]\) is the gradient matrix, and \([D]\) is the material property matrix. In the present analysis, a linear triangular axisymmetric finite element is used for which the shape function matrix \([N]\), and the gradient matrix \([B]\) are given in Appendix B. The material property matrix \([D]\), which is a function of thermal conductivity, is also given in Appendix B. All of the integrals in equations (3-14-a,b,c) are evaluated over a single element. Expressions for matrices \([C^e]\), \([K^e]\), and \([F^e]\) are given in Appendix B. Contributions from all the elements in the finite element mesh are assembled to give the overall global matrices \([C]\), \([K]\), and \([F]\).

Equation (3-12) represents a system of first order linear differential equations. There are two popular procedures for solving this system of equations to obtain the variation of the values of \(\{T\}\) at all nodal points over a period of time. The first procedure is to approximate the time derivative using a central finite difference scheme. The alternate procedure is to use finite elements defined in the time domain [63]. The finite difference technique is used in the present case. The first derivative of the nodal values between two points in time is

33
\[
d(T)/dt = 1/\Delta t \left( \{T\}_1 - \{T\}_o \right) \tag{3-15}
\]

also \{T\} and \{F\} in equation (3-12) are evaluated at the mid-point of the time interval as

\[
\{T\} = (\{T\}_1 + \{T\}_o) / 2 \tag{3-16}
\]

and

\[
\{F\} = (\{F\}_1 + \{F\}_o) / 2 \tag{3-17}
\]

Then equation (3-12) gives

\[
(2/\Delta t[C] + [K]) \{T\}_1 = (2/\Delta t[C] - [K]) \{T\}_o - ([F]_1 + [F]_o) \tag{3-18}
\]

Equation (3-18) can be solved to yield the nodal values at time \(t+\Delta t\) given the nodal values at time \(t\).

A FORTRAN program is listed in Appendix C for the solution of the present heat transfer problem. Subroutines HEATAK, DECOMP, and SOLVE are taken from Rao [62] and modified to solve the transient problem defined by Equation (3-18). Input data regarding element geometry, material properties, and boundary conditions are provided in the main program. In the subroutine HEATAK a loop over all elements is used to construct for each element in turn (1) the element conductance matrix, (2) the element capacitance matrix, and (3) the element force vector. These element matrices are then assembled directly into global conductance and capacitance matrices and a global load
vector. The subroutines DECOMP and SOLVE are then used to solve the resulting system of linear equations

\[ [A] \{T\}_j = \{b\}_j \]  \hspace{1cm} (3-19-a)

where

\[ [A] = (2/\Delta t [C] + [K]) \]  \hspace{1cm} (3-19-b)

and

\[ \{b\}_j = (2/\Delta t [C] - [K])\{T\}_{j-1} - ([F]_j + [F]_{j-1}) \]  \hspace{1cm} (3-19-c)

The subroutine DECOMP decomposes the matrix \([A]\) into \([U]^T[U]\) and the elements of the upper triangular matrix \([U]\) are stored in the array \(A\) itself. The Choleski method [62] is utilized in the subroutine SOLVE to solve equation (3-19-a) by using the decomposed coefficient matrix \([A]\). At a given time step \(\Delta t\) and known \(\{T\}_0\) equation (3-18) reduces to the form of (3-19-a) and can be solved to get \(\{T\}_1\). Then \(\{T\}_1\) values are utilized to solve equation (3-18) for \(\{T\}_2\) and so on. Considering a disc of granular media flowing over a heated length \(L\) with a uniform velocity \(V\), the total time scale, \(L/V\), is subdivided into a number of time increments, \(\Delta t\), such that \(\Delta x(=V\Delta t)\) is equal to 0.1 mm at any specified flowing velocity. The distance \(\Delta x\) will be taken as the length of the finite element side on the solid boundary as shown in Figure 3-1-a.

In the homogeneous packet theory by Mickley and Fairbanks [11] a local time average heat transfer coefficient, \(h_x\), and a surface mean heat transfer coefficient, \(h_m\), are defined as
where \( \tau \) is the time during which a packet has been in contact with the surface, \( \psi(\tau) \) is the frequency of occurrence in time of packets having age \( \tau \); That is, over a long period of time, the fraction of the total time during which the surface is in contact with packets of ages ranging between \( \tau \) and \( \tau + d\tau \) is \( \psi(\tau) d\tau \). The instantaneous packet heat transfer coefficient is given by \( h_i \), and \( A \) is the area of the heat transfer surface. For a specified surface heat flux Kubie and Broughton [31] defined the time mean heat transfer coefficient as

\[
1/h_x = 1/\tau \int_0^\tau (1/h_i) \, dt
\]  

(3-22)

The flow situation considered here, which models the actual flow, studied in the experimental phase of the present study, resembles a "slug flow" of a solid past a surface as discussed by Mickley [11]. For such a type of flow the age of all packets is always \( \tau = L/V \) and \( \psi(\tau) = 0 \), except for \( \psi(L/V) \), which is infinite, so that \( \psi(L/V)d\tau = 1 \) and the local time average heat transfer coefficient of the present analysis can be defined as

\[
h_x = q''/(T_{wx} - T_{mx})
\]  

(3-23)

where \( T_{wx} \) is the wall temperature at \( x \) (or at \( t = x/V \)) and \( T_{mx} \) is the mean disc temperature.
defined by

\[ T_{mx} = \frac{2}{R^2} \int_0^R T_x \, r \, dr \]  

(3-24)

and \( T_x(T_x(r)) \) is the temperature profile at \( x \). The surface mean heat transfer coefficient can be calculated from equation (3-21) or from an equation similar to equation (3-22) by replacing \( h_x \) with \( h_i \). The local time average Nusselt number based on bed or gas thermal conductivities and particle mean diameter, \( d \), can be defined as

\[ \text{Nux}_g = \frac{h_x d}{k_g} \]  

(3-25)

\[ \text{Nux}_o = \frac{h_x d}{k_o} \]  

(3-26)

Similarly, the surface mean Nusselt numbers are given by

\[ \text{Num}_g = \frac{h_m d}{k_g} \]  

(3-27)

\[ \text{Num}_o = \frac{h_m d}{k_o} \]  

(3-28)

Predictions of the local time-average heat transfer coefficient and surface mean heat transfer coefficient vs axial distance, \( x \), are obtained by the present analysis for flowing beds of copper/air \((k_o/k_g = 0.531/0.026 = 19.73)\), glass/carbon dioxide \((k_o/k_g = 0.100/0.016 = 6.27)\), and glass/helium \((k_o/k_g = 0.354/0.148 = 2.39)\); Results are shown in Figures 3-2 and 3-3. Also, predictions of the present analytical model of local time-averaged and mean surface Nusselt numbers based on effective bed and gas conductivity are shown in Figures 3-4, 3-5, 3-6, and 3-7 respectively.
3.4.2 Comparison With Available Models And Experimental Data

Comparisons of the local time averaged Nusselt number \( h_xd/k_g \) obtained by the present method with (1) the two-particle model [13], (2) the contact resistance model [2], and (3) Mickley's packet model [11] are shown on Figures 3-8 and 3-9. These Figures for copper/air and sand/air beds are taken from Denloye and Botterill [2]. Also shown are experimental measurements of different particle diameters. It can be seen from Figures 3-8 and 3-9 that agreement between the experimental results of [2] and present method is closed.

Figure 3-10 shows predictions of the present method of local time averaged Nusselt number vs Fourier number for a steel shot/air flowing bed. Predictions obtained from Schlunder's approximate solution [19] and Sullivan's correlation [6] are shown for comparison. Numerical solutions of Kubie [31] and a homogeneous continuum model of Mickley [11] are also shown.

In Figures 3-11 and 3-12 predictions of the present method for glass beads/air and copper shot/air beds, along with predictions of Kubie [31] and Mickley [11] are shown. In addition, data by Butt, Hampshire, Smith, Cain, and Desai are shown for comparison in Figures 3-10, 3-11, and 3-12. These three Figures are taken from Kubie [31]. In the range of Fourier numbers for the data, both Kubie [31] and present models are equally valid.

Predictions of the present method for the local time averaged and the surface mean Nusselt numbers of a glass/carbon dioxide bed \( (k_o/k_g = 6.27) \) are shown in Figure 3-13. Four other different models cited by Gloski [25] are shown for comparison, namely (1) the numerical solution of Chandran, (2) a homogeneous emulsion with surface resistance, (3) a single layer of particles, and (4) Mickley's analysis. At large
Fourier numbers (large residence time) Mickleys provides an upper bound as well as an asymptote approached by the other continuum models including the present model.

Figure 3-14 shows local time-average Nusselt numbers predicted by the present method in comparison with the Gloski [25] model and Kubie's [31] numerical solution. Also, data of Antonishin [31] for moderate to large Fourier numbers cited by Kubie are shown for comparison. The agreement of the three models are approximately the same in the range of Fourier numbers for the data given.

Local time averaged Nusselt numbers obtained by the present method for a sand/helium bed \((k_o/k_g = 4.15)\) are shown in Figure 3-15. Predictions of Chandran, those of the Gloski [25] model, and experimental measurements are also shown for comparison. Agreement between the experimental results and present method is quite valid.

Surface mean Nusselt number predicted by the present method are compared with data by Dunsky et al. [9] and predictions of the Gloski model [25] in Figure 3-16. It can be seen that the numerical solution of the present method lies reasonably close to the measurements of [9] and Gloski model [25]. Figures 3-13 to 3-16 are taken from Gloski et al. [25].
4. EXPERIMENTAL INVESTIGATIONS

4.1 INTRODUCTION

Measurements of thermophysical properties of five test media chosen for investigation are given in this section. Three of these media, namely steel shot and two sizes of glass beads, consisted of hard spherical particles with a narrow distribution of diameters about the mean. The other two materials are (1) common fine-grained sand of relatively non-uniform size distribution, and (2) Ottawa sand of a more uniform size distribution. Both types of sand are generally comprised of irregularly shaped particles.

The design and construction of the thermal loop test facility used in this study will be described in this chapter. Additionally, measurements obtained of heat transfer coefficients for all test media flowing through the vertical test section are presented in this section. The measurements obtained are compared with theoretical predictions of the method described in the previous chapter as well as with available experimental data and predictions based on other models appearing in the literature.

4.2 THERMOPHYSICAL PROPERTIES MEASUREMENTS

Before conducting any of the test transfer studies per se, measurements were first made of (1) density, (2) void fraction, (3) particle size distribution, (4) thermal conductivity, (5) thermal diffusivity, and (6) specific heat of all the materials selected for this experimental study. These measurements are described in the following sections.
4.2.1 Density

Bulk densities of a stationary bed of each of the granular media were obtained by weighing the known volume of a sample of the material contained in a graduated cylinder. Both critical state density and dense state densities were measured. The critical state is that obtained by capping the end of the graduated cylinder and causing the material to flow to the other end by slowly inverting the cylinder. The dense state is obtained by repeated striking of the cylinder until no further compaction is observed. The values obtained are presented in table 4-1.*

4.2.2 Void Fraction

The bed void fraction, \( \varepsilon_0 \), was evaluated by using the expression

\[
\varepsilon_0 = \frac{(V_t - V_s)}{V_t} \quad (4-1)
\]

where \( V_t \) and \( V_s \) are the total bed and particulate volumes respectively. The volume \( V_t \) is measured by means of graduated cylinder, while \( V_s \) is measured using a known volume of suitable liquid to fill the spaces between the particles within the bed. Void fractions for both the critical and dense states were measured. The values obtained are presented in table 4-1.

4.2.3 Particle Size Distribution

The mean particle size of each bed material was measured using a set of standard sized screens. Screen mesh numbers ranged from 8 (2362 microns) to 200 (74

* All tables may be found in appendix D.
microns). To estimate the mean particle diameter, numerous mean diameter expressions have been defined in the literature depending upon the use to which the material is being adapted. Zenz and Othmer [59] presented a summary of the defining equations of the arithmetic, geometric, logarithmic, harmonic mean, and median diameters. Also, in [59] defining expressions for the length, surface, volume, weight and volume-surface mean diameters are given. The volume-surface mean diameter equation has been suggested by Kunii and Levenspiel [64] for situations in which the sieving method is utilized for determining the particle size and distribution. Other investigators [65,66] have recommended its use in other applications also. In the present study the mean particle diameter was calculated from the following equation for the volume-surface mean:

\[
d = \frac{1}{\sum w/d_i}
\]

(4-2)

where \( w_i \) is the weight fraction in a given size interval and \( d_i \) is the diameter of the particles in a particular size range. The volume-surface diameter defines the total surface of a unit weight of a granular media of mixed particle size. Figures 4-1-a to 4-1-e illustrate the measured results. The values of the mean particle diameter obtained are also presented in table 4-1.

**4.2.4 Thermal Conductivity**

The line source technique, described and utilized in [54], was employed to measure thermal conductivity values of the porous media employed in the present tests because of its relative simplicity and speed. A similar apparatus was designed and fabricated and used in the present study; It is illustrated schematically in Figure 4-2.

The test media were poured into a container of rectangular parallelepiped shape, 15
cm by 15 cm by 20 cm. A line source of heat was generated within the test sample by passing 400 ma of current through a 0.31 mm manganin heater wire, using a d-c power supply (HP 6216 A model). A chromel-alumel thermocouple of 30 gauge was wrapped about the heater wire and connected to a potentiometric chart recorder (Tekman Electronics Ltd., TE 220 series) for recording the transient temperature behaviour.

The transient temperature rise within an infinitely long thin heat source embedded in an infinite homogeneous medium initially at equilibrium is given by [54] and [40] as

\[ T = \frac{q'}{2\pi k_0} \int_{\beta}^{\infty} \left[ \text{Exp}(-y^2) / y \right] dy \]  \hspace{1cm} (4-3)

where \( \beta^2 = \frac{r_0^2}{4\alpha t} \), \( k_0 \) = thermal conductivity of the medium, \( q' \) = heat input per unit length of the wire, and \( y \) is a dummy variable. The integral expression of Equation (4-3) may be evaluated by an infinite series, so that the temperature is given by

\[ T = \frac{q'}{2\pi k_0} \left[ -\nu/2 - \ln\beta + \beta^2 / 2 (1!) - \beta^4 / 4 (2!).. \right] \]  \hspace{1cm} (4-4)

where \( \nu \) = Euler's constant = 0.5772156. Since \( (r_0^2 / 4\alpha t) \) is small (< 0.0256) the temperature change between two times \( t_1 \) and \( t_2 \) may be approximated by

\[ T_2 - T_1 = \left( \frac{q'}{4\pi k_0} \right) \ln(t_2/t_1) \]  \hspace{1cm} (4-5)

Thus a plot of temperature rise versus the logarithm of time gives a straight line with a slope of \( q' / 4\pi k_0 \). Knowing \( q' \) and the slope, \( k_0 \) may be calculated as

\[ k_0 = \frac{q'}{(4\pi \text{ (slope)})} \]  \hspace{1cm} (4-6)
Figures 4-3-a to 4-3-e show the recorded temperature-time history of the heater wire when imbedded within a sample of each of the test media. A plot of the temperature rise versus the logarithim of time for the five chosen granular materials are shown in Figure 4-4. Figure 4-5, taken from Kuzay [48] shows a comparison of the thermal conductivity values obtained here with some of the existing correlations and available data at a porosity of $\varepsilon_0 = 0.40$. Also, values calculated from the Willhite and Kunii [44] correlation, Equation (3-7), are shown in Figure 4-5. The values obtained are also presented in Table 4-1.

4.2.5 Thermal Diffusivity

Measurements of thermal diffusivity, $\alpha$, were made using the method described by Waddams [41] and Brinn, et al [8], in which the transient heating of a cylinder of granular media of radius $R_0$ and length $2L$ ($2L > 4R_0$) at initial temperature $T_i$ is measured.

The apparatus is shown in Figure 4-6. One 30-gauge chromel-alumel thermocouple was located on the tube surface to measure the surface temperature $T_s$, and another at the center of the cylinder to measure the variation of the center line temperature, $T_c$, with time $t$. Atmospheric steam was admitted into the jacket surrounding the test medium using a Scott boiler, model no. 9058. The temperature-time history of the center line of each of the test media was recorded by a Tekman Electronics TE 220 series potentiometric chart recorder as shown in Figures 4-7-a to 4-7-e.

The theoretical equation relating $\alpha t/R_0^2$ and $(T_s - T_c)/(T_s - T_i)$ is given by Waddams [41]
\[
\frac{T_s - T_c}{T_s - T_i} = \frac{8}{\pi} \left[ \sum_i \exp\left( -\alpha \zeta_i^2 \frac{t^2}{R_o^2} \right) / \zeta_i J_1 (\zeta_i) \right] \\
\left[ \sum_{\text{odd}} \exp\left( \alpha j^2 \pi t^2 / 4L^2 \right) (-1)^{(j-1)/2} / j \right]
\]

(4-7)

where \(J_1(\zeta_i)\) is the Bessel function of the first order and \(\zeta_i\) is the ith root of equation \(J_0(\zeta)=0\) and \(j\) and \(i\) are positive integers. Equation (4-7) is shown plotted in Figure 4-8, from which the slope is determined to have a value of \(-5.72\). Plots of \((T_s - T_c)/(T_s - T_i)\) versus time, \(t\), for the five testing materials are shown in Figure 4-9. The negative slope of each curve (straight line part) is set equal to \(5.72 (\alpha/R_o^2)\), so that \(\alpha\) may be calculated from

\[
\alpha = \text{(slope)} \frac{R_o^2}{5.72}
\]

(4-8)

The values obtained are presented in table 4-1.

4.2.6 Specific Heat

Specific heat values may be obtained from the measured values of thermal diffusivity \(\alpha\), thermal conductivity \(k_o\), and bulk density \(\rho\) from the expression

\[
c_p = k_o / \rho \alpha
\]

(4-9)

It was particularly necessary to know the specific heat accurately, because this quantity is used to calculate the mean temperature of the granular material at different stations in the flow field.

The specific heats were measured by utilizing Thermonetics Seebeck Envelop Calorimeter (SEC) C-11 120-5-MC; No 101 model. The principle of the SEC is
based on the fact that all of the heat flow in the calorimeter must pass through its walls, where temperature gradient sensors are located. Thus the calorimeter envelope integrates the total heat flow in the system on either an instantaneous or long term basis. The calorimeter was calibrated to relate the heat flow (watts) to the calorimeter millivolt output signal by a calibration constant, $C_e$ determined by accurately measuring the calorimeter millivolt output signal, $e$, resulting when a constant and measured ($I^2R$) heating rate, $q_c$, is released in the calorimeter using the calibration heater supplied with the calorimeter. The constant is given by the equation

$$q_c = C_e e$$ (4-10)

To measure the specific heat the unloaded calorimeter that has come to thermal equilibrium (zero millivolt output) at a temperature level $T_1$, is raised to a different temperature level, $T_2$. The heat flow versus time trace as the transient process proceeds from the initial steady state condition to the final steady state condition was obtained with an Omega millivolt recording potentiometer (Model No. 595, Three Channel). The area under this curve is equal to the heat added to the calorimeter and its empty specimen container for the superimposed temperature perturbation. Next, the calorimeter is loaded with the granular material sample to be investigated and again exposed to the original temperature level, $T_1$ and allowed to come to equilibrium. Then the loaded SEC is exposed to the temperature level, $T_2$, allowed to reach equilibrium again, and the corresponding transient heat flow-time trace recorded. These curves are shown in Figures 4-10-a to 4-10-e. The area under a given curve is equal to the heat added to the calorimeter, its specimen container and the sample. Subtraction of the areas under the two transient traces yields the desired heat added to the sample. The specific heat, $c_p$, is determined by substituting the measured heat given to the sample, $q$, the sample
weight, \( m \), and the temperature difference for the thermal perturbation, \( T_2 - T_1 \), into the equation

\[
q = mc_p(T_2 - T_1) \quad (4-11)
\]

The results obtained for \( T_1 = 50 \, ^\circ\text{C} \) and \( T_2 = 150 \, ^\circ\text{C} \) are summarized in Table 4-1. Also the percent difference between the specific heat values measured and those calculated from Equation (4-9) are given in Table 4-1.

### 4.3 HEAT TRANSFER MEASUREMENTS

**4.3.1 Experimental Apparatus**

The flow apparatus, shown schematically in Figure 4-11, consists of five basic parts, from top to bottom: the suction fan with suitable size storage tank; the supply hopper; the test section; the regulating valve; and the heat exchanger.

The particles flow by gravity from the supply hopper through an entrance region 32 cm in length before entering the test section. The supply hopper has a considerably larger volume than the test section and enables continuous flow to persist in the vertical tube until steady state heating conditions are attained.

The test section is a stainless steel tube, 27 cm long with a 1.44 cm inside diameter and .07 cm wall thickness. Heat was added to the test section by an insulated electric heating element wrapped about the outside surface of the flow channel so as to maintain a uniform heat flux condition. The heating element was fabricated with nichrome wire and had a resistance of 41.2 ohms. A 1-140 volt variable transformer
was connected in parallel with the heater and a constant A.C. power supply. The test section and the entrance region were thermally insulated from the outside environment by a cylindrical composite wall consisting of an outer layer of fiber glass and an inner layer of high temperature organic impregnated fibrous glass. The entrance of the tube is mounted in a sleeve to allow the test section to expand freely within 5 mm in the vertical direction when it is heated. The test section is shown in Figure 4-12.

The particle flow rate through the test section was controlled by means of a sliding cone valve assembly located at the lower end of the tube exit region. The particles were continuously cooled through a high aspect-ratio rectangular cross-section heat exchanger and returned to the upper storage tank by a large suction fan located on the top of the tank.

The system as described is capable of maintaining a continuous velocity in the channel between approximately 2 and 23 cm per second, the exact limits depending on the particular material used in the flow. The lower limit on the velocity results from the inability of the flow valve to pass a steady flow without clogging at very low flow rates; the upper limit results from the difficulty in maintaining the maximum flow rate for a period of time long enough to reach steady state.

4.3.2 Measurement Techniques

The wall temperature distribution along the flow direction was measured by means of 30-gauge chromel-alumel thermocouples that were spot welded onto the outer tube surface. The heat losses in the axial and radial directions were estimated by measuring the tube surface temperature 1.5 cm before the beginning of the heated section and at the outer edge of the insulating material. A thermocouple that could be moved radially into the flowing medium was used to measure the particle inlet bulk temperature
at the inlet of the test section. These temperatures were recorded on Omega (three-channel) and Tekman (two-channel) chart recorders.

A standard voltmeter and ammeter were used to measure the AC voltage across the heating element and the current passing through it. This measurement, coupled with the known heater resistance was used to calculate the power consumed by the heater. The power input to the test section was calculated by subtracting the losses in the axial and the radial directions. The percentage losses were determined to vary between 1.5 percent to 3.5 percent.

The radial temperature distribution of the particles at the test section exit was measured by means of a chromel-alumel thermocouple wire mounted on a micrometer assembly placed upstream of the flow control valve. The thermocouple output was recorded on Tekman chart recorder. Figures 4-13-a to 4-13-c show an output chart of surface, inlet, and exit temperature of one run of glass beads of mean diameter 2.07 mm.

The mass flow rate through the channel was determined by weighing the net mass output from the channel in a fixed time interval. The mass flow rate could be converted to velocity, knowing the density of the flowing material and the known channel cross-sectional area. This process was repeated so that an average value of the velocity was determined.

The wall temperature distribution, power input, particle inlet temperature, and flow rate were used to determine the local and average heat transfer coefficients along the test section.

4.3.3 Experimental Procedure

Initially the complete inventory of particles is placed in the supply hopper. The heat exchanger cooling water is turned on, the suction fan is started, and the control
valve adjusted to its fully open position. After initiation of the flow, the power supply is
turned on and the variable transformer adjusted to obtain the desired power input to the
heating element (100 volts, corresponding ammeter reading 2.2 am). The particle radial
exit temperature profile probe is initially placed flush with the tube wall and the axial wall
temperatures monitored until steady state conditions are reached. The exit temperature
profile is then measured by moving the micrometer-mounted probe at approximately 1
mm radial intervals. The flow rate for these conditions is then measured. The flow
control valve is then changed to the next lower flow rate and the temperature balancing
process repeated. A total of seven to ten flow rates were examined for the same
material. The loop was thoroughly cleaned before the next test material was studied.

4.4 EXPERIMENTAL RESULTS

The mean temperature distribution, $T_{mx}$, of the granular media along the flow
direction has been calculated by an energy balance over the test section using the known
power input, granular flow rate, and inlet temperature. The local heat transfer
coefficient, $h_x$, at specific axial location was then calculated using equations (4-12) and
(4-13)

\[
q'' = \frac{Q}{2\pi RL} \quad (4-12)
\]

\[
h_x = \frac{q''}{(T_{wx} - T_{mx})} \quad (4-13)
\]

where $R$ is the inside radius of the tube, $L$ its length, and $Q$ is the net power input to the
tube (equal to the electric power input minus the losses in the radial and the axial
directions). The losses in the radial direction were estimated by knowing the thermal
conductivity and the temperature at the inner and outer surfaces of the insulating material. Similarly the losses in the axial direction were calculated knowing the tube surface temperature at different locations and its thermal conductivity.

An analysis of the present experimental data to estimate errors, precision, and the general validity of experimental measurements is given in Appendix F. The approach used was to perform an uncertainty analysis [67] similar to that described and used by J.P.Holman [68] and Keshock [54]. The summary of the results of this analysis is given in Table 4-2 in appendix D.

Figure 4-14 shows the measured local heat transfer coefficient along the axial direction of the test section when each of the five media flows at an approximate mean velocity 0.07 m/s. The smaller size particles of glass beads and local sand, have higher values of local heat transfer coefficient than the larger particle media.

The overall average heat transfer coefficient is calculated using the logarithmic mean temperature difference between the wall and the granular mean temperatures at inlet and exit of the test section.

Figures 4-15-a and 4-15-b show the effect of particle size and granular bulk velocity on the average heat transfer coefficient. The average heat transfer coefficient increases with decreasing particle size and with increasing average bulk velocity.

Local and mean Nusselt numbers based on particle mean diameter and bed or gas conductivity are calculated utilizing equations (3-25) to (3-28). Measurements of Nusselt number, \( h_m \frac{d}{k_g} \), of the present study for the five media tested are compared with experimental data and the theoretical correlation obtained by Sullivan and Sabersky [6] in Figure 4-16. Sullivan and Sabersky's correlation resulted in an expression relating the Nusselt number to a function of Peclet number, given in equation (4-14-a)
\[ \text{Nu}^* = \frac{1}{0.085 + 0.5 \left( \frac{\pi}{\text{Pe}^*_L} \right)^{0.5}} \]  

where

\[ \text{Nu}^* = \frac{h_m d}{k_g} \]  

\[ \text{Pe}^*_L = \left( \frac{k_d}{k_g} \right)^2 \left( \frac{d}{L} \right)^2 \text{Pe}_L \]  

\[ \text{Pe}_L = \frac{V L}{\alpha} \]

Although their Nusselt-Peclet correlation curve is seen to pass reasonably close to most of their data points, Figure 4-16 appears to reveal an inability of their model to successfully correlate data for a variety of heat transfer media.

The average heat transfer coefficient data obtained in the present study for local sand and Ottawa sand has been placed into the form of Nusselt numbers and are presented in Figure 4-17, superimposed over the results obtained for sand/air experiments by Denloye and Botterill [2]. Their results are compared with a modified Mickley-Fairbanks model (additional resistance to heat transfer at the transfer surface is included). It is seen that the present data fall reasonably close to the modified Mickley model predictions. Predictions of the present theoretical method for sand/air flowing media are shown for comparison.

Also Denloye and Botterill correlated the maximum value of the flowing bed coefficient by plotting the corresponding Nusselt number, \( h_{\text{max}} d / k_g \), against the Archimedes number. Their correlation expressed by equation (4-15)

\[ h_{\text{max}} d / k_g = 1.283 \text{ Ar}^{0.162} \quad 10^3 < \text{ Ar } < 10^6 \]  

52
is shown in Figure 4-18. The maximum average heat transfer coefficient obtained for the test media in the present study are shown for comparison. Agreement between correlation and measurements is close.

Another comparison has also been made with the predictions of the Kubie and Broughton [31]. In their model they attempted to improve upon the Mickley-Fairbanks model by taking into account the variation of thermal properties within a fluid packet in the region of the surface. Their numerical predictions for glass/air and steel shot/air media are shown in Figures 4-19 and 4-20. The present test results are compared with those predicted by Kubie-Broughton and Mickley-Fairbanks. Also, predictions of the present analytical method and Schlunder correlation [19] for glass/air flowing media are shown in Figure 4-19. In addition, data by Butt, Hampshire, Smith, Cain, and Desai [1] are shown for comparison. Predictions of the present analytical method for a steel shot/air bed as well as predictions of Schlunder's approximate solution [19] and Sullivan's [6] correlation are shown in Figure 4-20 for comparison.
5. DISCUSSION OF RESULTS

The experiments were carried out with media having the range of particle sizes given in table 4-1 (approximately from 0.26 to 2.07 mm) using air as the stagnant fluid medium in the flowing packed bed. Bed flow velocities from 0.02 to 0.23 m/s, giving packet residence times at the heat transfer surface between 1 and 13.5 second.

The effective thermal conductivity, $k_0$, was measured for the static packed bed and compared with some of the existing correlations and available data in Figure 4-5. It was not possible to know or measure the mean voidage of the granular media while flowing in order to estimate the effective thermal conductivity and bulk density. To elaborate on this matter, all of the solid particles except those adjacent to the wall (see visual results of [8]) are falling vertically under the action of the force of gravity. The particles are not being subjected to a co-current or countercurrent injection of a gaseous phase that would tend to disperse, entrain and "fluidize" the particulate media. Thus, the static packed bed values was used. However, the type of flow in the present study is considered to be a contact dominated flow, in which the adjacent material particles are in physical contact with each other, while the interstitial fluid moves with the particles passively. Thus it was anticipated that utilizing the static packed bed values of effective thermal conductivity and bulk density would be reasonably accurate for the contact dominated flow occurring within the heated flow channels.

In addition, Willhite et al. [44] have indicated that their proposed equation for the effective thermal conductivity, based upon experimental results, satisfactorily represented heat transfer for flowing gas conditions. They reported that their measured effective
thermal conductivity, when the direction of heat flow is perpendicular to the fluid motion, was found to be independent of fluid flow rate for a modified Reynold's number \( \frac{dG}{\mu g} \) where \( G \) is the mass flux flow rate) range from 0 to 6.6. Furthermore, they reported that the effect of any turbulence in the fluid is negligible. This equation is also valid in the region of high voidage as reported by Kubie [31] and Botterill [17]. Similar equations have also been utilized by Botterill [2], Colakyan and Levenspiel [65] in their heat transfer analysis of flowing packed beds.

The specific heats of the granular test media were measured directly by utilizing a Thermonetics Seebaeck Envelope calorimeter. Calculated values were also obtained using the measured values of thermal diffusivity, thermal conductivity, and bulk density and Equation (4-9). The percentage difference of the specific heat values obtained by the two methods was less than 5% for the all media tested except for Ottawa sand, where it was 20%. An average value of the specific heat of Ottawa sand and measured values of the other materials were utilized in the present study.

Figure 4-15-a and 4-15-b display the average heat transfer coefficients versus linear velocity for 0.26 mm and 0.71 mm sand particles and 0.69 mm and 2.06 mm glass beads spheres. As seen from both Figures, the average heat transfer coefficients for each size of particles increase with increasing velocity (an accompanying decrease in the particle residence time) and approached limiting maximum values. The rate of increase, however, is greater at lower solids velocities than at higher velocities. For the large 0.71 mm Ottawa sand and 2.07 mm glass beads, the heat transfer coefficient increases slower with increasing velocity. It can also be noted that, for the same linear granular media velocity, lower heat transfer coefficients are obtained with the large size particles. The same trend, i.e., lower heat transfer coefficient for increasing particle size, and higher heat transfer coefficient for increasing solids velocity have also been observed by several other investigators [1, 2, 6, 14, 20, 65].
Figures 4-14 and 4-15-b show local and average heat transfer coefficients for 1.09 mm steel shot. In previous studies of Botterill et al. [13], Botterill and Desai [1], and Harakas and Beatty [10], it was found that flowing packed beds of metals (copper-steel) gave higher heat transfer coefficients. Because of the limited kinds of test media used here, the trend observed is not especially clear; Only the data of 1.09 mm steel shot can be compared here to that of 0.69 mm glass beads (i.e. of smaller diameter) in Figure 4-15-b. Also it was noticed that the steel shot utilized in the present study was covered with an oxidized microlayer on the outer surface of the individual particles (pellets), which may significantly affect the thermal conductivity of the bed. Even with these considerations, Figure 4-15-b displays higher values of the heat transfer coefficient for steel shot than that of 0.69 glass beads for a bed velocity up to 0.06 m/s.

It is clear from the data plots of Figures 4-16, 4-17, 4-19, and 4-20 that existing methods of predicting heat transfer coefficients in the channel flow of particulate media are not sufficiently accurate. One of the intentions of the present study was to obtain heat transfer data for a wide variety of particulate media in order to test the existing predictive correlations. The results, plotted using the suggested correlation developed by Sullivan and Sabersky [6], appears to be that their proposed correlation is not sufficiently accurate.

Reasons for such inadequacy obviously lie in the simplification made in modelling the actual heat transfer and flow processes. For example, the ordered, square array of particles in the Sullivan and Sabersky model [6] is an idealization that becomes more unrealistic for (a) irregularly shaped particles, and (b) particles having a wide distribution of sizes. Also, the equivalent gas layer thickness of 1/10 of a particle diameter used to properly account for conduction through the gaseous phase is of empirical origin, suggesting an inadequacy in the modelling of the actual gaseous phase conduction process.
In order to investigate the effect of the thermophysical properties of the gas and the solid of flowing granular media; Predictions of the local time-averaged heat transfer coefficients and surface mean heat transfer coefficients obtained by the present analysis for flowing beds of copper/air, glass/carbon dioxide, and glass/helium are shown in Figures 3-2 and 3-3. The effect of the interstitial gas thermal conductivity on the heat transfer rate can be determined by comparing the relative heat transfer coefficients of the glass/helium and glass/carbon dioxide beds shown in Figures 3-2 and 3-3. It is clear that beds of higher gas thermal conductivities gives higher heat transfer coefficients.

Also the effect of the thermal properties of the solid on the heat transfer of the system is noticed from comparing the relative heat transfer coefficients of the copper/air and glass/helium beds. Even though the helium is of higher thermal conductivity than the air, the copper/air flowing bed gives higher heat transfer coefficients than the glass/helium bed. This illustrates that the solid particles of high thermal conductivity constitute the dominant contribution to the heat transfer coefficients.

The effect of gas thermal conductivity and solid thermal properties cannot be directly noticed if the predictions of the heat transfer coefficients are plotted in terms of Nusselt number (based on bed or gas effective thermal conductivities) as shown in Figures 3-4 to 3-7.

The predictions for heat transfer coefficients by the present model (for flowing copper/air and sand/air beds) are compared with (1) the two-particle model [13], (2) the contact resistance model [2], and (3) Mickleys packet model [11] on Figures 3-8 and 3-9. Also in these Figures are shown experimental results for copper/air and sand/air flowing beds [2]. It can be seen from Figure 3-8 that the two-particle model overpredicts the coefficients of heat transfer for \( t/d^2 \) values less than \( 2 \times 10^7 \) s/m\(^2\) and under predicts the coefficients for values greater than this. By introducing a gas gap of thickness equal to 10% of the particle diameter the contact resistance model reduces the discrepancy at short
resistance times but increases it at longer ones. Finally, the predictions of the packet model for copper/air and sand/air systems overestimate the experimental results. It can be seen from Figures 3-8 and 3-9 that the agreement between the present model and the experimental results is quite good.

The correlations of Sullivan and Sabersky [6], Schlunder [19] and the predictions of Mickley's method [11] and Kubie numerical model [31] are compared with Nusselt number predictions of the present model for a steel/air system in Figure 3-10. The present method's predictions and the schlunder correlation are in good agreement with the experimental data of Butt and Hampshier (presented by Kubie [31]). While the Sullivan correlation underpredicts the experimental heat transfer coefficient data, Mickley's model [11] overpredicts the values.

The predictions of the present method for glass/air and copper/air flowing beds are compared with experimental data cited by Kubie [31] in Figures 3-11, 3-12, and 3-14. Over the range of Fourier numbers for the experimental data, both the Kubie [31] and present models compare with the data equally well.

Figure 3-13 compares four models of the wall-to-bed heat transfer. The Mickley [11] homogeneous model provides an upper bound as well as an asymptote approached by the other models at large residence time (Fourier number greater than 10). Chandran's predictions [25] are almost the same as those of the present method for the local heat transfer coefficient. Gloski's [25] method gives constant Nusselt number for a value of Fourier number less than about 0.02.

A test of the present model is made by comparing its predictions of local Nusselt numbers with the experimental measurements of Gloski [25], which span the range of Fourier numbers from 0.5 down to about 0.001. The comparison is shown in Figure 3-15, where the Gloski [25] and Chandran [25] models are also shown for comparison. The predictions of the present model are in very good agreement with the experimental
data over this range of small values of Fourier number. Results at low Fourier number levels provide another indication of the validity of the method. The very good agreement of the present method's predictions with data at low Fourier numbers, as shown in Figure 3-15, is a clear indication of the validity of the proposed model. It was reported by Gloski [25] that at high Fourier numbers it is impossible to properly evaluate the relative accuracy of the various models, since they all yield similar results.

Predictions of the mean Nusselt number by the present analytical method are shown in Figures 4-17, 4-19 and 4-20 for sand/air, glass/air and steel/air flowing beds. The prediction is shown in comparison with the experimental measurements of the present study. The minimum and maximum percentage differences between the experimental measurements and theoretical predictions are -1% and -39% for local sand, 0% and 0% for Ottawa sand, 0% and -8% for small size glass beads, 15% and 32% for glass beads of large size and -3% and -28% for steel shot. The percentage difference is defined by 

\[
\frac{(N_{ex} - N_{th})}{0.5(N_{ex} + N_{th})} \times 100
\]

where \(N_{ex}\) and \(N_{th}\) are the experimental and theoretical Nusselt numbers. Quite good agreement between predicted and experimental values are obtained for Ottawa sand (\(d=0.71\) mm) and for glass beads (\(d=0.69\) mm). The maximum percentage difference between the measured and theoretically predicted values of Nusselt number for steel/air flowing media used in the present study is 28%. The low measured values might be due to an oxide film on the surface of the particles as mentioned previously.

For the local sand measurements (\(d=0.26\) mm) the heat transfer coefficient values are a maximum of 39% below the theoretical predictions. For the glass beads of mean diameter 2.07 mm, the measurements are above the theoretical predictions a maximum of 32%.

In the case of local sand the higher heat transfer predictions (compared to the experimental values) might be due to utilizing the measured mean voidage of the bed in
the static state in the Willhite [44] equation to estimate the effective thermal conductivity and density of the flowing bed. Perhaps the mean voidage under flowing conditions may be of greater importance for local sand, whose particle size distribution somewhat different from the other media in that it is very nearly uniform over a wide range of particle diameters. This larger range of size distribution may result in greater relative "packing" in a static state as compared with the flowing state.

In the case of large diameter glass beads (2.07 mm) the predictions are consistently lower than the measured values. This may suggest that near the wall region significant particle rotation could occur (as compared with small particle diameters) so that radial mixing could thereby be generated. This effect seems even more likely considering that the diameter of the heated tube is only 14.4 mm compared to the particle diameter of 2.07 mm.

In a recent study by Colakyan and Levenspiel [65] a model for heat transfer between moving bed of solids and immersed cylinders is proposed, and a predictive correlation is given. Their correlation is also compared with the experimental measurements of the present study. A maximum percentage difference of about 65% above the measured values is seen for local sand, Ottawa sand, small glass beads, and large glass beads. For the steel shot data their correlation is 104% above the measured values.

Figure 4-18 shows comparison of the measured maximum heat transfer coefficient with a correlation given by Botterill [2]. Agreement between measured values and the correlation is quite good.

From a broader point of view, however, it is very satisfying to see the existence of good agreement between the present theory and the controlled residence time data available in the literature. The previous models of heat transfer in flowing granular media introduced some form of semi-empirical approximation (such as gas-film) to produce acceptable agreement with experimental data. The model developed here uses no assumed
film properties or empirical approximation. The numerical solution of the present analytical analysis is straightforward and simple enough to be easily utilized.
6. CONCLUSIONS AND RECOMMENDATIONS

A heat transfer loop was constructed and used to determine the local and average heat transfer coefficient of gravity flowing granular media as a function of the average flowing velocity and particle size for a constant heat flux heating condition. The results obtained were compared with predictions of various models appearing in the literature. The correlation of Sullivan and Sabersky [6] in the form of a modified Nusselt-Peclet number relationship, was found to be inadequate in predicting all of the results obtained in the present study. Also, the recent correlation of Colakyan and Levenspiel [65], which assumes the existence of a gas layer between the heat transfer surface and the first row of particles, failed to correlate the present data. Therefore, an improved model was developed as part of the present study.

The model obtained was shown to agree well with the data of controlled residence time of flowing media available in the literature. Also, the theoretical model successfully correlates data measured well into the low Fourier number range. (It was reported by Gloski [25] that successful predictions in the low Fourier number range is a true test for a model.)

The predictions of the present model have also been found to agree well with the numerical method predictions of Kubie and Broughton [31] as well as those of Chandran and Chen [34]. However, the present model is simple to use by comparison and can be easily applied to different flow geometries and heating conditions.

Finally, it is important to note that in the development of the model, no physically unjustified concepts (as the gas layer of 1/10 of particle diameter) are required.
basic intent of the model was to modify the packet theory of heat transfer by accounting for the presence of the wall and its effect on the local voidage by introducing a realistic variation in the thermophysical properties of media adjacent to the wall. In effect, the model acknowledges and accounts for the particle behavior immediately adjacent to the heat transfer surface being the key factor in bed-to-surface heat transfer.

To develop the model further it would be particularly valuable to have more information about possible particle rotation and induced radial mixing in the near wall region, in addition to that radial mixing that may occur elsewhere within the bulk of the bed. Also, information about the voidage of a flowing bed and its relation to thermal transport properties appears to be a desirable goal for future research.

Additional experiments over a wide range of variables may also be desirable. Among these variables are particle size distribution and sphericity (wide and narrow particle size distribution), thermophysical properties of the solid particles, tube diameter and geometry, heating conditions (e.g. uniform wall temperature, linear variation of heat flux, etc.) Also, the effect of thermal radiation at high heat flux levels and the effect of different pressure levels of the gas-solid system should be investigated.

Finally, development of a generalized Nusselt-type heat transfer coefficient correlation for gravity flowing particle beds should be a major future goal.
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APPENDIXES
APPENDIX A

FIGURES
a. Disc of the granular media flowing in contact with the hot surface.

Figure 3-1. Gravity flowing particulate media in a vertical tube.
b. Illustration of heat balance used for driving the energy equation.

Figure 3-1. (continued)
c. Sketch of superimposed consecutive photographs of bin during solids discharge showing flow paths and profiles.

Figure 3-1. (continued)
d. Sketch of superimposed consecutive photographs of discharge from baffled bin showing solids flow paths and profiles.

Figure 3-1. (continued)
e. Division of the domain into elements.

Figure 3-1. (continued)
Figure 3-2. Local time-average heat transfer coefficient vs. axial distance.
Figure 3-3. Mean heat transfer coefficient vs. axial distance.
Figure 3-4. Local time-average Nusselt number based on bed conductivity vs. Fourier number.
Figure 3-5. Mean Nusselt number based on bed conductivity vs. Fourier number.
Figure 3-6. Local Nusselt number based on gas conductivity vs. Fourier number.
Figure 3-7. Mean Nusselt number based on gas conductivity vs. Fourier number.
Figure 3-8. Comparison of present method with contact resistance model [2], two particle model [13], and Mickley's model [11] for local heat transfer in the copper/air system.
Figure 3-9. Comparison of present method with Mickley and contact resistance models for local heat transfer in sand/air system.
Figure 3-10. Comparison of present method with other theoretical predictions for local time-average heat transfer in steel/air system.
Figure 3-11. Comparison of present method with predictions by Kubie [31], Mickley [11], and data from [31] for local time-average heat transfer in the glass/air system.
Figure 3-12. Comparison of present method with predictions by Kubie [31], Mickley [11], and data from [31] for local time-average heat transfer in the copper/air system.
Figure 3-13. Comparison of present method ($k_o/kg=6.27$) with various models ($k_o/kg=6.03$) of the time averaged Nusselt number.
Figure 3-14. Comparison of present method with predictions by Gloski [25] and Kubie [31] for local time-average Nusselt number.
Figure 3-15. Comparison of present method ($k_0/\rho g=4.15$) with predictions and data by Gloski [25] for local time-average Nusselt number.
Figure 3-16. Comparison of present method with predictions by Gloski [25] and data by Dunsky [9] for mean Nusselt number.
LOCAL SAND
\( d = 0.26 \text{ mm} \)

PERCENTAGE OF WEIGHT

PARTICLE DIAMETER (mm)

0.0 0.2 0.4 0.6 0.8 1.0 1.2

a. Particle size distribution for local sand.

Figure 4-1. Particle size distribution.
b. Particle size discrete distribution for Ottawa sand.

Figure 4-1. (continued)
c. Particle size discrete distribution for small size glass beads.

Figure 4-1. (continued)

94
d. Particle size discrete distribution for large size glass beads.

Figure 4-1. (continued)
e. Particle size discrete distribution for metallic shot.

Figure 4-1. (continued)
Figure 4-2. Schematic diagram of apparatus used for measuring thermal conductivity.
a. Recorded temperature-time history for local sand.

Figure 4-3. Effective thermal conductivity.
b. Recorded temperature-time history for Ottawa sand.

Figure 4-3. (continued)
c. Recorded temperature-time history for small size glass beads.

Figure 4-3. (continued)
Figure 4-3. (continued) d. Recorded temperature-time history for large size glass beads.
Recorded temperature-time history for metallic shot.

Figure 4-3. (continued)
Figure 4-4. Transient temperature rise within the line source embedded in granular media.
Figure 4-5. Comparison of some of the existing correlations and data for $\varepsilon_0 = 0.40$ with present measurements.
Figure 4-6. Schematic diagram of apparatus used for measuring thermal diffusivity.
a. Recorded temperature-time history for local sand.

Figure 4-7. Effective thermal diffusivity.
b. Recorded temperature-time history for Ottawa sand.

Figure 4-7. (continued)
c. Recorded temperature-time history for small glass beads.

Figure 4-7. (continued)
d. Recorded temperature-time history for large glass beads.

Figure 4-7. (continued)
e. Recorded temperature-time history for metallic shot.

Figure 4-7. (continued)
Figure 4-8. Plotting of equation (4-7)
Figure 4-9. Variation in center temperature with time for granular media.
Figure 4-10. Specific heat

Recorded calorimeter output history for local sand.
b. Recorded calorimeter output history for Ottawa sand.

Figure 4-10. (continued)
c. Recorded calorimeter output history for small glass beads.

Figure 4-10. (continued)
d. Recorded calorimeter output history for large glass beads.

Figure 4-10. (continued)
e. Recorded calorimeter output history for metallic shot.

Figure 4-10. (continued)
Figure 4-11. Schematic diagram of the test loop.
Figure 4-12. Schematic diagram of the test section.
a. Recorded tube surface temperature-time history for flowing glass beads.

Figure 4-13. Flowing glass beads \( d = 2.07 \) mm.
b. Recorded tube surface temperature-time history for glass beads.

Figure 4-13. (continued)
Glass Beads $d = 2.07$ mm
\( m = 0.988 \text{ kg/min}^2 \)
Full Scale Volt = 2 mv
Cr/Al Thermocouple

Radial Exit Temperature
Insulation Temperature

Insulation Temp.
Radial Exit Temp.
Center Line Exit Temp.
(Room Temp. 19°C)

**Figure 4-13.** (continued)

Recorded radial exit temperature glass beads $d=2.07$ mm.
Figure 4-14. Local heat transfer coefficient along axial distance from tube inlet for the tested granular media.
a. Average heat transfer coefficient vs. bulk velocity for local sand and Ottawa sand.

Figure 4-15. Average heat transfer coefficient.
b. Average heat transfer coefficient vs. bulk velocity for small glass beads, large glass beads, and metallic shot.

Figure 4-15: (continued)
Figure 4-16. Modified Nusselt number versus modified Peclet number by Sullivan [6] compared with measurements for all materials tested.
Figure 4-17. Present measurements and data by Denloye [2] compared with Mickley [11] model and modified Mickley model [2].

- Present Method
Figure 4-18. Correlation of maximum flowing bed to surface coefficients against Archimedes number by [2].
Figure 4-19. Comparison of present experimental data and theory with predictions by Kubie [31], Mickley [11], and data from [31] for mean heat transfer in the glass/air system.
Figure 4-20. Comparison of present experimental data and theory with other theoretical predictions and data for mean heat transfer in the steel/air system.
APPENDIX B

FINITE ELEMENT MATRICES
The shape function matrix $[N]$ is given by

$$
[N] = \begin{bmatrix}
N_i & N_j & N_k
\end{bmatrix},
$$

$$
N_i = L_1, \quad N_j = L_2, \quad N_k = L_3
$$

where $L_1$, $L_2$, and $L_3$ are related to the global cylindrical coordinates $(r,x)$ of nodes $i$, $j$, and $k$ as:

$$
\begin{bmatrix}
1 \\
r \\
x
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
r_i & r_j & r_k \\
x_i & x_j & x_k
\end{bmatrix}
\begin{bmatrix}
L_1 \\
L_2 \\
L_3
\end{bmatrix}
$$

or, equivalently,
\[
\begin{bmatrix}
L_1 \\
L_2 \\
L_3
\end{bmatrix} = (1/2A^e) \begin{bmatrix}
a_1 & b_1 & c_1 \\
a_2 & b_2 & c_2 \\
a_3 & b_3 & c_3
\end{bmatrix} \begin{bmatrix}
1 \\
r \\
x
\end{bmatrix}
\]  

(B-3)

where

\[
a_1 = r_j x_k - r_k x_j
\]

\[
a_2 = r_k x_i - r_i x_k
\]

\[
a_3 = r_i x_j - r_j x_i
\]

\[
b_1 = x_j - x_k
\]

\[
b_2 = x_k - x_i
\]

\[
b_3 = x_i - x_j
\]

\[
c_1 = r_k - r_j
\]

\[
c_2 = r_i - r_k
\]

\[
c_3 = r_j - r_i
\]  

(B-4)

and  \( A^e = \text{area of the triangle } ijk \)

\[
= 0.5 \left[ r_i (x_j - x_k) + r_j (x_k - x_i) + r_k (x_i - x_j) \right]
\]  

(B-5)

The gradient matrix \([B]\) is given by

\[
[B] = \begin{bmatrix}
\partial N_i / \partial r & \partial N_j / \partial r & \partial N_k / \partial r \\
\partial N_i / \partial x & \partial N_j / \partial x & \partial N_k / \partial x
\end{bmatrix}
\]

\[
= (1/2A^e) \begin{bmatrix}
b_1 & b_2 & b_3 \\
c_1 & c_2 & c_2
\end{bmatrix}
\]  

(B-6)

The material property matrix \([D]\) is given by
The capacitance matrix, \( [C^e] \), is defined by equation (3-14-a) as

\[
[C^e] = \int_v \rho c_p [N]^T [N] \, dv
\]  
(3-14-a)

By expressing \( dv \) as \( 2\pi r \, dA \), where \( dA \) is the differential area of the triangle \( ijk \), and \( [N] \) by equation (B-1);

\[
[C^e] = (\rho c_p)^e 2\pi \int_{A^e} \begin{bmatrix}
L_1^2 & L_1 L_2 & L_1 L_3 \\
L_1 L_2 & L_2^2 & L_2 L_3 \\
L_1 L_3 & L_2 L_3 & L_3^2
\end{bmatrix} \, (r_i L_1 + r_j L_2 + r_k L_3) \, dA
\]  
(B-8)

where \( r \) has been substituted as

\[
r = r_i L_1 + r_j L_2 + r_k L_3
\]  
(B-9)

By using the integration formula for natural coordinates, namely,

\[
\int_{A^e} L_1^a L_2^b L_3^c \, dA = \frac{a! \, b! \, c!}{(a+b+c+2)!} \, 2A^e
\]  
(B-10)

Equation (B-8) will be

\[
\pi (\rho c_p)^e A^e 30 \begin{bmatrix}
(6r_i + 2r_j + 2r_k) & (2r_i + 2r_j + r_k) & (2r_i + r_j + 2r_k) \\
(2r_i + 6r_j + 2r_k) & (2r_i + 2r_j + 2r_k) & (r_i + 2r_j + 2r_k) \\
(2r_i + 2r_j + 6r_k) & (2r_i + 2r_j + 2r_k) & (2r_i + 2r_j + 6r_k)
\end{bmatrix}
\]  
(B-11)
The conductance matrix, \([K^e]\), is defined by equation (3-14-b), by writing \(dV = 2\pi r\ dA\),

\[
[K^e] = 2\pi \int r[B]^T[D][B]dA
\]  
\((3-14-b)\)

utilizing equations (B-6), (B-7), (B-9), and (B-10)

\[
[K^e] = \frac{\pi k_r R^2}{2A^e} \begin{bmatrix} b_1^2 & b_1 b_2 & b_1 b_3 \\ b_1 b_2 & b_2^2 & b_2 b_3 \\ b_1 b_3 & b_2 b_3 & b_3 \end{bmatrix} + \frac{\pi k_x R^2}{2A^e} \begin{bmatrix} c_1^2 & c_1 c_2 & c_1 c_3 \\ c_1 c_2 & c_2^2 & c_2 c_3 \\ c_1 c_3 & c_2 c_3 & c_3^2 \end{bmatrix}
\]  
\((B-12)\)

where

\[
R^2 = \int_{A^e} r^2 dA = \frac{1}{12} (r_i r_j r_k) \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} r_i \\ r_j \\ r_k \end{bmatrix}
\]  
\((B-13)\)

For simplicity, if the centroidal values of \(r_k r\) and \(r_k x\), namely, \(r_c k_r\) and \(r_c k_x\) in place of \(r_k r\) and \(r_k x\) respectively in matrix \([D]\), equation (B-12) gives an approximate expression for \([K^e]\) with \(R^2\) replaced by \(r_c^2\)

where

\[
r_c = (r_i + r_j + r_k)/3
\]  
\((B-14)\)

The force vector matrix, \([F^e]\), is defined by equation (3-14-c) as

\[
[F^e] = \int q[N]^T ds
\]  
\((3-14-c)\)

135
utilizing equations (B-1) to (B-5), $ds = 2\pi r \, ds$, and the integration formula

$$\int_{s_j}^{s_i} L_1^a L_2^b \, ds = (s_j - s_i) \frac{a! \, b!}{(a+b+1)!}$$

Equation (3-14-c) gives

$$\begin{align*}
\{F^e\} &= \begin{cases} 
\pi q s_{ji} & \begin{cases} 
(2r_i+r_j) \\
(r_i+2r_j) \\
0 
\end{cases} \\
3 & \text{if the edge } ij \text{ lies on } s \\
\pi q s_{kj} & \begin{cases} 
0 \\
(2r_j+r_k) \\
(r_j+2r_k) 
\end{cases} \\
3 & \text{if the edge } jk \text{ lies on } s \\
\pi q s_{ik} & \begin{cases} 
0 \\
(2r_i+r_k) \\
(r_i+2r_k) 
\end{cases} \\
3 & \text{if the edge } ki \text{ lies on } s 
\end{cases}
\end{align*}$$

(B-16)
APPENDIX C

COMPUTER PROGRAM OF ANALYTICAL ANALYSES
FLOWING GRANULAR MEDIA; FINITE ELEMENT ANALYSIS

DETERMINATION OF TEMPERATURE DISTRIBUTION IN AXI-SYMMETRIC FIELD

integer edge(144)
real nk(146), nkz(146), lmtd, nusd, nub, nub1, nub2, numz, numzg
dimension loc(144,3), r(146), z(146), ts(146), tinf(144), h(144)
& q(144), qd(144), gs(146,4), temp(146,1), t0(146), a(144)
& roc(146)
& tri(73), rd(73), fun(73)
& velo(9), iprnt(17), tmdf(17), dis(17), his(17), hmez(17)
common /b1/dtm
data ne, nn, nb/144, 146, 4/
data iprnt /10, 20, 30, 40, 50, 60, 80, 100, 200, 300, 400, 500,
& 650, 1450, 2250, 2650, 2700/
data velo /0.001, 0.1, 1, 2, 5, 10/
hf=17600.
tin=20.
dlz=0.1e-3

physical and thermal properties of materials.

3 write(6,123)
123 format(1h1,30x, '--- glass beads of mean diameter 0.69 mm ---')
c d=0.69e-3
thks=0.850
cps=765.
dnss=2700.
void=0.41
gama=0.5
go to 7

4 write(6,124)
124 format(1h1,30x, '--- otawa sand of mean diameter 0.71 mm ---')
d=0.71e-3
thks=1.87
cps=860.
dnss=2600.
void=0.41
gama=2.73.
go to 7
7 D1=14.4E-3
OR=D1/2.
DO=15.87E-3
XAB=27.E-2
XA2=6.5E-2
XA4=14.5E-2
XA6=22.5E-2
XA7=26.5E-2
X12=8.0E-2

C-----PROPERTIES
C
C
C
C
C
THKG=0.026
DNSG=1.19
CPG=1008.

C-----PROPERTIES OF GAS (AIR).

C
C
THKG=0.016
CPG=850.
DNSG=1.79

C-----PROPERTIES OF GAS (CO2).

C
C
THKG=0.148
CPG=5200
DNSG=.165

C-----LOCALIZATION OF GLOBAL NODES TO EACH ELEMENT.

C
LOC(1,1)=1
LOC(1,2)=4
LOC(1,3)=2
LOC(2,1)=4
LOC(2,2)=1
LOC(2,3)=3
DO 10 J=1,3
DO 10 I=3,NE
JJ=I-2
LOC(1,J)=LOC(JJ,J)+2
CONTINUE
WRITE(6,92)
DO 15 I=1,NE
WRITE(6,93),I,LOC(1,1),LOC(1,2),LOC(1,3)
CONTINUE
92 FORMAT(1H1,5X,'-- ELEMENT NO. AND ITS GLOBAL NODE NO. -- ',/)
93 FORMAT(5X,4(14,5X))

C-----SPECIFYING THE RADIAL AND AXIAL COORDINATES OF NODES.

C
R(1)=0.0
R(2)=0.0
DO 20 I=3,NN,2
JJ=I-2
JK=I+1
R(I) = R(JJ) + 0.0001
R(JK) = R(I)
20 CONTINUE
DO 30 I = 1, NN, 2
Z(I) = 0.0
KK = I + 1
Z(KK) = 0.0001
30 CONTINUE
C
C----- SPECIFYING NODE AND ELEMENT BOUNDARY CONDITIONS.
C
DO 40 I = 1, NN
40 TS(I) = -1.0E+6
C
EDG(I) = 1 IF BOUNDARY CONDITION IS SPECIFIED ON EDGE 1-2
EDG(I) = 2 IF BOUNDARY CONDITION IS SPECIFIED ON EDGE 2-3
EDG(I) = 3 IF BOUNDARY CONDITION IS SPECIFIED ON EDGE 3-1
EDG(I) = 4 IF BOUNDARY CONDITION IS SPECIFIED ON EDGE 1-2 AND 2-3
EDG(I) = 5 IF BOUNDARY CONDITION IS SPECIFIED ON EDGE 2-3 AND 3-1
EDG(I) = 6 IF BOUNDARY CONDITION IS SPECIFIED ON EDGE 3-1 AND 1-2
DO 50 I = 1, NE
50 EDGE(I) = 0.0
EDGE(144) = 3
DO 60 I = 1, NE
Q(I) = 0.0
QD(I) = 0.0
H(I) = 0.0
TINF(I) = 0.0
60 CONTINUE
C
C----- SPECIFYING THERMAL CONDUCTIVITIES AND HTF. B.C.
C
DO 300 IV = 1, 9
VEL = VEL0(IV)
Q(144) = HTF
DO 65 I = 1, NN
TO(I) = 0.0
T(1) = TIN
65 CONTINUE
DTIM = DT/LZ/VEL
TIM = XAB/VEL
NTIM = TIM/DTIM + 1
TIME = DTIM
IC = 1
SN = 3.0*(((5.01-0.42*VOID)/(1.91-1.91*VOID))
SN2 = SQRT(1.0/1.0/SN)
COS = SQRT(1.0-1.0/SN)
ROCB = DNS*S*CPS*(1.0-VOID)*DNSG*CPG*VOID
C
ROCB = DNS*S*CPS*(1.0-VOID)
DO 85 ITIM = 1, NTIM
T1 = 0.5*(1.0-THK/THKS)**2*SN2
T2 = ALOG(THKS/THKG-(THKS/THKG-1.0)*COS)-(1.0-THK/THKS)**(1.0-COS)
ALBA = T1/T2
C
T3 = THK/THKG-1.0-(1.0-VOID)**(1.0-THK/THKS)
C
T4 = (1.0-VOID)**(1.0-THK/THKS)**2
C  GAMA=ALBA*T3/T4
DO 35 I=1,NN
  WRGB=0.0072-D
  IF(R(I) .GE. WRGB) GO TO 36
  ROC(I)=ROCB
  THKB=THKG*(1.+(-VOID)*(1.-THKG/THKS)+GAMA/ALBA*
          (1.-THKG/THKS)**2*(1.-VOID))
  NKR(I)=THKB
  NKZ(I)=NKR(I)
GO TO 35
36 XOD=(0.0072-R(I))/D
  VOIDX=1.0-3.0*(1.0-VOID)*(XOD-(2./3.)*X00**2)
  ROC(I)=ONSS*CPS*(1.0-VOIDX)+ONSG*CPG*VOIDX
  ROC(I)=DNSS*CPS*(1.0-VOIDX)
C  NKR(I)=THKG*(1.+(-VOIDX)*(1.-THKG/THKS)+GAMA/ALBA*
          (1.-THKG/THKS)**2*(1.-VOIDX))
  NKZ(I)=NKR(I)
35 CONTINUE
C------ SOLUTION AND PRINT OUT OF DIFFERENT VARIABLES.
C  CALL HEATAX (LOC,R,Z,NN,NE,NB,Q,TD,H,TINF,TS,EDGE,GS
&    ,TEMP,TO,A,ROC,NKR,NKZ)
  DLTR=OR/(NE/2)
  NM1=NN-1
  TWAL=(TEMP(NN,1)+TEMP(NM1,1))/2.0
  DO 210 J=1,NM1,2
    J(J+1)/2
  TR(IJ)=(TEMP(I,1)+TEMP(J+1,1))/2.0
  RD(IJ)=(R(J)+R(J+1))/2.0
  FUN(IJ)=TR(IJ)*RD(IJ)
210 CONTINUE
  NH=NN/2
  NHM1=NH-1
  SUN=0.0
  DO 220 K=2,NHM1
    SUN=SUN+FUN(K)
220 CONTINUE
  ANT=(DLTR/2.0)*(FUN(1)+FUN(NH)+2.0*SUN)
  TMEAN=(2.0/(OR+OR))*ANT
  TFILM=(TMEAN+TWAL)/2.
  HZ=MTF/(TWAL-TMEAN)
  NU1B=HZ*D/THKB
  NU1G=HZ*D/THKG
  ALFA=THKB/ROCB
  THKBOG=THKB/THKG
  FORI=ALFA*TIME/(D**2)
  TIME=TIME+TIMEP
  TIMEP=TIMEP*VEL
  IF(I*TIM .NE. 1PRNT(I)) GO TO 85
  DIS(I)=DIST
FET02210
FET02220
FET02230
FET02240
FET02250
FET02260
FET02270
FET02280
FET02290
FET02300
FET02310
FET02320
FET02330
FET02340
FET02350
FET02360
FET02370
FET02380
FET02390
FET02400
FET02410
FET02420
FET02430
FET02440
FET02450
FET02460
FET02470
FET02480
FET02490
FET02500
FET02510
FET02520
FET02530
FET02540
FET02550
FET02560
FET02570
FET02580
FET02590
FET02600
FET02610
FET02620
FET02630
FET02640
FET02650
FET02660
FET02670
FET02680
FET02690
FET02700
FET02710
FET02720
FET02730
FET02740
FET02750
FILE: FETHA FORTRAN A1 UNIVERSITY OF PETROLEUM & MINERALS, DHAHRAN

97 FORMAT(/ 'MEAN HEAT TRANS. CO. (W/M²K) = ', F10.3, /) FET02760
& 'MEAN NUSSELT NO. (HMD/KG) = ', F10.3, /) FET02770
& 'MDFIED PECLET NO. = ', F10.3, /) FET02780
& 'MEAN NUSSELT NO. (HMD/KB) = ', F10.4, /) FET02790
& 'MEAN FOURIER NO. (FTA/D²) = ', F10.4, /) FET02800
& 'RESID. TIME/D² (SEC/M²) = ', E15.7, /) FET02810

98 FORMAT(3X, 'GAS COND. = ', F8.5, 6X, 'ALFA = ', E15.7, 6X, 'ALBA = ', F7.3/) FET02820
99 FORMAT(3X, 'BED COND. = ', F8.5, 6X, 'THKB/THKG = ', F7.3/) FET02830
STOP FET02840
END FET02850

C======================================================================
C SUBROUTINE HEATAX
C======================================================================
SUBROUTINE HEATAX (LOC, R, Z, NN, NE, NB, Q, QD, H, TINF, TS, EDGE, GS
& , PLOAD, TO, A, ROC, NKR, NKZ)
INTEGER EDGE(NE)
REAL NKR(NN), NKZ(NN)
DIMENSION GS(NN, NB), PLOAD(NN, 1), LOC(NE, 3), R(NN), Z(NN), Q(NE), QD(NE)
& , H(NN), TINF(NN), A(NN), G1(3, 3), G2(3, 3), P1(3), P2(3), P3(3), TS(NN)
& , G3(3, 3), B(3, 3), TOE(3), P4(3), TO(NN), ROC(NN)
COMMON /B1/DTIM
DOUBLE PRECISION DIFF(1)
P1 = 3.1416
DO 10 1 = 1, NN
PLOAD(1, 1) = 0.0
DO 10 J = 1, NB
GS(1, J) = 0.0
CONTINUE
DO 100 I = 1, NE
N1 = LOC(I, 1)
N2 = LOC(I, 2)
N3 = LOC(I, 3)
R1 = R(N1)
RJ = R(N2)
RK = R(N3)
Z1 = Z(N1)
ZJ = Z(N2)
ZK = Z(N3)
EK1 = NKR(N1)
EKJ = NKR(N2)
EKK = NKR(N3)
EKZ1 = NKZ(N1)
EKZJ = NKZ(N2)
EKZK = NKZ(N3)
ROCI = ROC(N1)
ROCJ = ROC(N2)
ROCK = ROC(N3)
B1 = ZJ - ZK
B2 = ZK - Z1
B3 = Z1 - ZJ
C1 = RJ - R1
C2 = R1 - RK
C3 = RJ - R1
10 CONTINUE

C======================================================================
\[ A(1) = \frac{(R_1)(Z_J - Z_K) + (R_J)(Z_K - Z_I) + (R_K)(Z_I - Z_J)}{2.0} \]

\[ A(1) = \text{ABS}(A(1)) \]

\[ R_C = \frac{(R_1 + R_J + R_K)}{3.0} \]

\[ R_{OE} = \frac{(R_{OE} + R_{OE} + R_{OE})}{3.0} \]

\[ E_K = \frac{(E_{KR} + E_{KR} + E_{KR})}{3.0} \]

\[ E_K = \frac{(E_{KZ} + E_{KZ} + E_{KZ})}{3.0} \]

\[ G_{K1}(1,1) = E_K \times (81 \times B_1 \times C_1^2 + E_K \times C_1^2) \]

\[ G_{K1}(1,2) = E_K \times B_1 \times C_2 \times C_1 \]

\[ G_{K1}(1,3) = E_K \times B_1 \times C_3 \times C_1 \]

\[ G_{K1}(2,2) = E_K \times B_2^2 \times C_2^2 \]

\[ G_{K1}(2,3) = E_K \times B_2 \times C_2 \times C_3 \]

\[ G_{K1}(3,3) = E_K \times B_3^2 \times C_3^2 \]

\[ G_{K1}(2,1) = G_{K1}(1,2) \]

\[ G_{K1}(3,1) = G_{K1}(1,3) \]

\[ G_{K1}(3,2) = G_{K1}(2,3) \]

\[ 00 20 M = 1, 3 \]

\[ 00 20 N = 1, 3 \]

\[ G_{K1}(M,N) = P_1 \times R_C \times \frac{G_{K1}(M,N)}{2.0 \times A(1)} \]

\[ G_{K2}(M,N) = 0.0 \]

\[ G_{K3}(M,N) = 0.0 \]

\[ B(M,N) = 0.0 \]

\[ 20 \text{ CONTINUE} \]

\[ D = P_1 \times A(1) \times R_{OE} / 30.0 \]

\[ G_{K3}(1,1) = D \times (6.0 \times R_1 + 2.0 \times R_J + 2.0 \times R_K) \]

\[ G_{K3}(1,2) = D \times (2.0 \times R_1 + 2.0 \times R_J + R_K) \]

\[ G_{K3}(1,3) = D \times (2.0 \times R_1 + R_J + 2.0 \times R_K) \]

\[ G_{K3}(2,2) = D \times (2.0 \times R_1 + 6.0 \times R_J + 2.0 \times R_K) \]

\[ G_{K3}(2,3) = D \times (R_1 + 2.0 \times R_J + 2.0 \times R_K) \]

\[ G_{K3}(3,3) = D \times (R_1 + 2.0 \times R_J + 6.0 \times R_K) \]

\[ G_{K3}(2,1) = G_{K3}(1,2) \]

\[ G_{K3}(3,1) = G_{K3}(1,3) \]

\[ G_{K3}(3,2) = G_{K3}(2,3) \]

\[ 00 30 M = 1, 3 \]

\[ P_1(M) = 0.0 \]

\[ P_2(M) = 0.0 \]

\[ P_3(M) = 0.0 \]

\[ P_4(M) = 0.0 \]

\[ 30 \text{ TOE}(M) = 0.0 \]

\[ \text{SUM} = P_1 \times R_C \times QD(1) \times A(1) / 6.0 \]

\[ P_1(1) = \text{SUM} \times (2.0 \times R_1 + R_J + R_K) \]

\[ P_1(2) = \text{SUM} \times (R_1 + 2.0 \times R_J + R_K) \]

\[ P_1(3) = \text{SUM} \times (R_1 + R_J + 2.0 \times R_K) \]

\[ 40 \text{ IF} (E_DGE(1) \text{ EQ. 0.0}) \text{ GO TO 80} \]

\[ \text{IF} (E_DGE(1) \text{ EQ. 1.0} \text{ OR.} \text{ E_DGE(1) \text{ EQ. 4.0} \text{ OR.} \text{ E_DGE(1) \text{ EQ. 5.0}} \text{ GO TO 00} \]

\[ 40 \text{ & 70} \]

\[ 50 \text{ IF} (E_DGE(1) \text{ EQ. 2.0} \text{ OR.} \text{ E_DGE(1) \text{ EQ. 6.0}} \text{ GO TO 50} \]

\[ S_I = \text{SQR}((R_1 - R_K)^2 + (Z_I - Z_K)^2) \]

\[ G_{K2}(1,1) = G_{K2}(1,1) + C_ON(3.0 \times R_1 + R_K) \]

\[ G_{K2}(1,3) = G_{K2}(1,3) + C_ON(R_1 + R_K) \]

\[ G_{K2}(2,1) = G_{K2}(2,1) + C_ON(3.0 \times R_1 + R_K) \]

\[ G_{K2}(2,3) = G_{K2}(2,3) + C_ON(R_1 + 3.0 \times R_K) \]

\[ G_{K2}(3,1) = G_{K2}(3,1) + C_ON(R_1 + R_K) \]

\[ G_{K2}(3,2) = G_{K2}(3,2) + C_ON(R_1 + 3.0 \times R_K) \]

\[ G_{K2}(3,3) = G_{K2}(3,3) + C_ON(R_1 + R_K) \]

\[ 50 \text{ & 70} \]

\[ P_1(1) = P_2(1) + (P_1 \times O(1) \times S_I / 3.0) \times (2.0 \times R_1 + R_K) \]

\[ P_2(1) = P_2(1) + (P_2 \times O(1) \times S_I / 3.0) \times (R_1 + 2.0 \times R_K) \]

\[ P_2(1) = P_2(1) + (P_2 \times O(1) \times S_I / 3.0) \times (R_1 + 3.0 \times R_K) \]

\[ P_3(1) = P_3(1) + (P_3 \times O(1) \times S_I / 3.0) \times (2.0 \times R_1 + R_K) \]

\[ P_3(1) = P_3(1) + (P_3 \times O(1) \times S_I / 3.0) \times (R_1 + 2.0 \times R_K) \]

\[ P_3(1) = P_3(1) + (P_3 \times O(1) \times S_I / 3.0) \times (R_1 + 3.0 \times R_K) \]
P3(3) = P3(3) + (P1 * H(I) * TINF(I) * SIK/3.0) * (RI + 2.0 * RK)
GO TO 80

60 SKJ = SQRT((RK - RJ)**2 + (ZK - ZJ)**2)
CON = P1 * H(I) * SKJ / 6.0
GK2(2,2) = GK2(2,2) + CON * (3.0 * RJ + RK)
GK2(2,3) = GK2(2,3) + CON * (RJ + RK)
GK2(3,2) = GK2(3,2) + CON * (RJ + RK)
GK2(3,3) = GK2(3,3) + CON * (RJ + 3.0 * RK)
P2(2) = P2(2) + (P1 * Q(I) * SKJ / 3.0) * (2.0 * RJ + RK)
P2(3) = P2(3) + (P1 * Q(I) * SKJ / 3.0) * (RJ + 2.0 * RK)
P3(I) = P3(I) + (P1 * H(I) * TINF(I) * SKJ / 3.0) * (RJ + 2.0 * RK)
GO TO 80

70 SJ1 = SQRT((RJ - RI)**2 + (ZJ - Z1)**2)
CON = P1 * H(I) * SJ1 / 6.0
GK2(1,1) = GK2(1,1) + CON * (3.0 * RI + RJ)
GK2(1,2) = GK2(1,2) + CON * (RI + RJ)
GK2(2,1) = GK2(2,1) + CON * (RI + RJ)
GK2(2,2) = GK2(2,2) + CON * (RI + 3.0 * RJ)
P2(1) = P2(1) + (P1 * Q(I) * SJ1 / 3.0) * (2.0 * RI + RJ)
P2(2) = P2(2) + (P1 * Q(I) * SJ1 / 3.0) * (RI + 2.0 * RJ)
P3(I) = P3(I) + (P1 * H(I) * TINF(I) * SJ1 / 3.0) * (RI + 2.0 * RJ)
IF (EDGE(I) .EQ. 6) GO TO 50

80 CONTINUE
DO 85 J = 1, 3
DO 85 K = 1, 3
B(J, K) = B(J, K) - GK1(J, K) - GK2(J, K) + (2.0 / DTIM) * GK3(J, K)
85 CONTINUE
DO 87 M = 1, 3
IM = LOC(I, M)
TOE(M) = TOE(M)
87 CONTINUE
CALL MULT(3, 3, 1, B, TOE, P4)
DO 90 M = 1, 3
IM = LOC(I, M)
PLOAD(M, 1) = PLOAD(M, 1) + 2.0 * PI(M) - 2.0 * P2(M) + 2.0 * P3(M) + P4(M)
DO 90 N = 1, 3
IM = LOC(M, 1) - IM + 1
IF (IM .EQ. 0) GO TO 90
GS(M, IM) = GS(M, IM) + GK1(M, N) + GK2(M, N) + (2.0 / DTIM) * GK3(M, N)
90 CONTINUE
DO 100 M = 1, NN
IF (TS(M) .LT. -1.0E+5) GO TO 110
GS(M, 1) = GS(M, 1) + 1.0E+9
PLOAD(M, 1) = TS(M) * GS(M, 1)
100 CONTINUE
CALL DECOMP(NN, NB, GS)
CALL SOLVE(NN, NB, GS, PLOAD, DIFF)
DO 120 K = 1, NN
TO(K) = 0.0
TO(K) = PLOAD(K, 1)
120 CONTINUE
SUBROUTINE DECOMP

SUBROUTINE DECOMP(N,NB,A)
DIMENSION A(N,NB)
DOUBLE PRECISION DIFF
A(1,1)=SQRT(A(1,1))
DO 5 K=2,NB
  A(1,K)=A(1,K)/A(1,1)
  DO 25 K=2,N
    KP1=K+1
    KM1=K-1
    DIFF=A(K,1)
    DO 10 JP=1, KM1
      ICOL=K+1-JP
      IF ( ICOL .GT. NB) GO TO 10
      DIFF=DIFF-A(JP, ICOL)*A(JP, ICOL)
      10 CONTINUE
    A(K,1)=DSQRT( DIFF)
  25 CONTINUE
RETURN
END

SUBROUTINE SOLVE

SUBROUTINE SOLVE(N,NB,M,A,B,DIFF)
DIMENSION A(N,NB),B(N,M)
DOUBLE PRECISION DIFF(M)
DO 5 J=1,M
  B(1,J)=B(1,J)/A(1,1)
  DO 30 I=2,N
  30 DIFF=A(I,J)
  DO 20 J=2,NB
    IROW=I+1-K
    IF ( IROW .LT. 1) GO TO 20
    ICOL=I+1-IROW
    IF ( ICOL .GT. NB) GO TO 20
    DIFF=DIFF-A(JP, ICOL)*A(JP, ICOL)
    15 CONTINUE
    20 A(I,J)=DIFF/A(I,1)
  25 CONTINUE
RETURN
END
DO 15 J=1,M
15  DIFF(J)=DIFF(J)-A( IROW, ICOL)*B( IROW, J)
20 CONTINUE
DO 25 J=1,M
25  B(I,J)=DIFF(J)/A(I,1)
30 CONTINUE
DO 35 J=1,M
35  B(N,J)=B(N,J)/A(N,1)
 DO 60 II=2,N
 I=N+1-II
 DO 40 J=1,M
40  DIFF(J)=B(I,J)
 DO 50 K=2,NB
 K=I-1+K
 IF (IK .GT. N) GO TO 50
 DO 45 J=1,M
45  DIFF(J)=DIFF(J)-A(1,K)*B(1K,J)
50 CONTINUE
DO 55 J=1,M
55  B(I,J)=DIFF(J)/A(1,1)
60 CONTINUE
RETURN
END

C======================================================================
C .
C SUBROUTINE MULT
C===================================================================== SUBROU1INE MULT(N,M,K,A,B,C)
DIMENSION A(N,M),B(M,K),C(N,K)
 DO 10 I=1,N
 DO 10 J=1,K
 C(I,J)=0.0
 DO 10 L=1,M
 C(I,J)=C(I,J)+A(I,L)*B(L,J)
10 CONTINUE
RETURN
END

C======================================================================
C SUBROUTINE MULT
C======================================================================
<table>
<thead>
<tr>
<th>Measured Property</th>
<th>Medium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Local sand</td>
</tr>
<tr>
<td>Mean particle size (mm)</td>
<td>0.26</td>
</tr>
<tr>
<td>Bulk density (kg/m³)</td>
<td></td>
</tr>
<tr>
<td>Critical state</td>
<td>1600</td>
</tr>
<tr>
<td>Dense State</td>
<td>1800</td>
</tr>
<tr>
<td>Solid density (kg/m³)</td>
<td>2620</td>
</tr>
<tr>
<td>Void Ratio ((V_t - V_s)/V_t)</td>
<td></td>
</tr>
<tr>
<td>Critical state</td>
<td>0.39</td>
</tr>
<tr>
<td>Dense state</td>
<td>0.32</td>
</tr>
<tr>
<td>Thermal conductivity (W/m² °K)</td>
<td></td>
</tr>
<tr>
<td>Critical state</td>
<td>0.312</td>
</tr>
<tr>
<td>Thermal diffusivity (m/s²)</td>
<td></td>
</tr>
<tr>
<td>Critical state</td>
<td>0.207 x 10⁻⁶</td>
</tr>
<tr>
<td>Specific heat, (J/kg °K)</td>
<td></td>
</tr>
<tr>
<td>Equation (4-9)</td>
<td>942</td>
</tr>
<tr>
<td>Measured (SEC)</td>
<td>933</td>
</tr>
<tr>
<td>Percent difference</td>
<td>1</td>
</tr>
</tbody>
</table>
### Table 4-2. Summary of Uncertainty Analysis Results

<table>
<thead>
<tr>
<th>Property</th>
<th>Local sand</th>
<th>Ottawa sand</th>
<th>Metallic beads</th>
<th>Glass beads</th>
<th>Glass beads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean diameter using [59]:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume-surface mean</td>
<td>0.26</td>
<td>0.71</td>
<td>1.09</td>
<td>0.69</td>
<td>2.07</td>
</tr>
<tr>
<td>Arithmetic mean</td>
<td>0.16</td>
<td>0.69</td>
<td>1.05</td>
<td>0.68</td>
<td>2.03</td>
</tr>
<tr>
<td>Weight mean (mm)</td>
<td>0.41</td>
<td>0.72</td>
<td>1.11</td>
<td>0.70</td>
<td>2.09</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.128</td>
<td>0.085</td>
<td>0.139</td>
<td>0.075</td>
<td>0.207</td>
</tr>
<tr>
<td>( % )</td>
<td>49</td>
<td>12</td>
<td>12.7</td>
<td>10.8</td>
<td>10</td>
</tr>
<tr>
<td>Volume-surface mean diameter (mm)</td>
<td>0.26±0.13</td>
<td>0.71±0.17</td>
<td>1.09±0.28</td>
<td>0.69±0.15</td>
<td>2.07±0.41</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.39</td>
<td>0.88</td>
<td>1.37</td>
<td>0.84</td>
<td>2.48</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.195</td>
<td>0.54</td>
<td>0.81</td>
<td>0.54</td>
<td>1.66</td>
</tr>
<tr>
<td>Particle size range (mm)</td>
<td>0.074-1.19</td>
<td>0.42-1.19</td>
<td>0.42-1.4</td>
<td>0.42-0.85</td>
<td>1.4-2.8</td>
</tr>
<tr>
<td>Estimated uncertainty of:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured heat transfer coefficient (%)</td>
<td>±5.1</td>
<td>±4.9</td>
<td>±4.4</td>
<td>±5.0</td>
<td>±4.0</td>
</tr>
<tr>
<td>Nusselt number (based on $k_g$) (%)</td>
<td>+51</td>
<td>±24.6</td>
<td>±26.4</td>
<td>±23.2</td>
<td>±21</td>
</tr>
<tr>
<td></td>
<td>-26</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(based on $k_o$) (%)</td>
<td>+52</td>
<td>±26.8</td>
<td>±28.7</td>
<td>±25.7</td>
<td>±25.5</td>
</tr>
<tr>
<td></td>
<td>-28.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX E

COMPUTER PROGRAM FOR EXPERIMENTAL STUDY
THIS PROGRAM DESIGNED FOR PROCESSING THE EXPERIMENTAL DATA MEASUREMENTS OF PROJECT MORMEDIA/87 "THERMAL CHARACTERISTICS OF FLOWING GRANULAR MATERIALS".

VALUES OF K CORRESPONDS TO DIFFERENT TYPES OF MATERIALS:

K=1 LOCAL SAND, K=2 GLASS BEADS D2.07, K=3 GLASS BEADS D0.69
K=4 OTTAWA SAND, K=5 METALLIC BEADS.

REAL KIN5,MAS,LMTD6,LMTD7,NU30D,L6,L7,LM5(9),NUS(9),KPA,NUM(9)
DIMENSION VL(9),HL(9),Q(9),E(9),H(9),PEC(9),RT(9),F(9)
&H5(9),H50(9),H6(9),H60(9)
&DO 30 K=1,5
READ(5,*)K,NR
GO TO(1,2,3,4,5),K
WRITE(6,112)
112 FORMAT(1X,'*** VALUE OF K OUTSIDE THE RANGE')

PHYSICAL AND THERMAL PROPERTIES OF MATERIALS.

LOCAL SAND.

1 WRITE(6,121)
121 FORMAT(1H1,30X,'*** LOCAL SAND OF MEAN DIAMETER 0.26 MM ***')
D=0.26E-3
DNS=1.6E3
DNSS=2.62E3
VOID=0.39
THK=0.312
ALF=0.207E-6
CP=942
GO TO 7

GLASS BEADS OF D=2.07

2 WRITE(6,122)
122 FORMAT(1H1,30X,'*** GLASS BEADS OF MEAN DIAMETER 2.07 MM ***')
D=2.07E-3
DNS=1.46E3
DNSS=2.44E3
VOID=0.41
THK=0.22
ALF=0.163E-6
CP=942
GO TO 7

GLASS BEADS OF D=0.69

3 WRITE(6,123)
123 FORMAT(1H1,30X,'*** GLASS BEADS OF MEAN DIAMETER 0.69 MM ***')
D=0.69E-3
DNS=1.46E3
DNSS=2.44E3
VOID=0.4
THK=0.226
ALF=0.17E-6
CP=907
GO TO 7

C*****OTTAWA SAND.

4 WRITE(6,124)
124 FORMAT(1H1,30X,'*** OTTAWA SAND OF MEAN DIAMETER 0.71 MM ***')
D=0.71E-3
DNS=1.57E3
DNSS=2.61E3
VOID=0.41
THK=0.286
ALF=0.25E-6
CP=728.
CB CP=895.
CC CP=950.
GO TO 7

C*****METALLIC BEADS.

5 WRITE(6,125)
125 FORMAT(1H1,30X,'*** METALLIC BEADS OF MEAN DIAMETER 1.09 MM ***')
D=1.09E-3
DNS=4.45E3
DNSS=7.5E3
VOID=0.41
THK=0.39
ALF=0.162E-6
CP=512.
CB CP=486.
CC CP=516

C*****SPECIFICATION OF TEST SECTION.

7 D1=14.4E-3
D0=15.87E-3
XAB=27.5E-2
X2=26.5E-2
X4=14.5E-2
X6=22.5E-2
X7=26.5E-2
X12=8.0E-2
P1=3.14159
A=(P1/4.)*(D1**2)
S1=A*D1*XAB
AN=(P1/4.)*(DO**2-D1**2)
STK1=14.9
STK2=16.6
DKST=(STK2-STK1)/100.

C*****RADIAL DISTANCES INPUT.

153
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RO=D1/2.
R1=1.E-3
R2=2.E-3
R3=3.E-3
R4=4.E-3
R5=5.E-3
R6=6.E-3
R7=RO
RR1=R1/RO
RR2=R2/RO
RR3=R3/RO
RR4=R4/RO
RR5=R5/RO
RR6=R6/RO
RR7=R7/RO

C*****SPECIFICATION OF INSULATION.

DINS=11.44E-2
KINS=0.07

C*****PROPERTIES OF GAS (AIR) AT 300&400 DEG K.

AK1=0.0263
AK2=0.0338
DKA=(AK2-AK1)/100.

C*****HEAT TRANSFER CALCULATION.

DO 10 I=1,NR
READ(5,*)TBIN,VLT,AMP,MAS
READ(5,*)TS1,TS2,TS4,TS6,TS7,TINS
READ(5,*)TR1,TR2,TR3,TR4,TR5,TR6,TR7
TRAV=TR7-(TR7-TR6)*RR6**2-(TR6-TR5)*RR5**2-(TR5-TR4)*RR4**2 &
-(TR4-TR3)*RR3**2-(TR3-TR2)*RR2**2-(TR2-TR1)*RR1**2
MS(I)=MAS
VEL=(MAS/60.)/(DNS*A)
VL(I)=VEL
RT27=(XA7-XA2)/VEL
RT26=(XA6-XA2)/VEL
E27=RT27/(D**2)
E26=RT26/(D**2)
RT(1)=RT27
E(1)=E27
FOR(1)=ALF*E(1)

C*****LOSSES AND HEAT FLUX CALCULATIONS.

TSAV=(TS2+TS4+TS6+TS7)/4.
QLR=2.*PI*XAB*KINS*(TSAV-TINS)/ALOG(DINS/DO)
STK=STK1+DKST*((TS1+TS2)/2.+273.-300.)
QLX=STK*AN*(TS2-TS1)/X12
PWR=VLT*AMP
QLT=QLR+QLX
QLTP=(QLT/PWR)*100.
\[ QT = PWR-QLT \]
\[ HFLX = QT/(P1*D1*XAB) \]
\[ HF(I) = HFLX \]
\[ QL(I) = QLTP \]

**BULK TEMPERATURE CALCULATION.**

\[ CC = QT/((MAS/60.)*CP*XAB) \]
\[ TB2 = TBIN + CC*XA2 \]
\[ TB4 = TBIN + CC*XA4 \]
\[ TB6 = TBIN + CC*XA6 \]
\[ TB7 = TBIN + CC*XA7 \]

**LOCAL HEAT TRANSFER COEFFICIENT.**

\[ HX2 = HFLX/(TS2-TB2) \]
\[ HX4 = HFLX/(TS4-TB4) \]
\[ HX6 = HFLX/(TS6-TB6) \]
\[ HX7 = HFLX/(TS7-TB7) \]

**LOGARITHMIC MEAN HEAT TRANSFER COEFFICIENT.**

\[ LMTD7 = ((TS7-TB7)-(TS2-TB2))/\text{ALOC}((TS7-TB7)/(TS2-TB2)) \]
\[ LMTD6 = ((TS6-TB6)-(TS2-TB2))/\text{ALOC}((TS6-TB6)/(TS2-TB2)) \]
\[ HM7 = HFLX/LMTD7 \]
\[ HM6 = HFLX/LMTD6 \]
\[ HM = HM7 \]
\[ H(I) = HM \]

**MODIFIED NUSSELT AND PECLET NUMBERS.**

\[ TBM = (TB2+TB4+TB6+TB7)/4. \]
\[ TPROP = (TSAV+TBM)/2.+273. \]
\[ GK = AK1+DKA*(TPROP-300.) \]
\[ NUSD = (HM*D)/GK \]
\[ NUS(I) = NUSD \]
\[ NUM(I) = (HM*D)/THK \]
\[ L7 = XA7-XA2 \]
\[ L6 = XA6-XA2 \]
\[ L = L7 \]
\[ PESL = (THK/GK)**2*(D/L)**2*(VEL*L/ALF) \]
\[ FEC(I) = PESL \]

**H.T.MODEL OF W.N. SULLIVAN AND R.H. SABERSKY**

\[ KPA = 0.055 \]
\[ H6(I) = (CK/D)/(KPA+0.5*\text{SQRT}(P1/PESL)) \]
\[ H6(I) = (H6(I)-H(I))/H(I)**100. \]

**H.T.MODEL OF MICKLEY & FAIRBANKS FOR THE CASE OF HIGBIE TYPE**

\[ H(I) = 2.*\text{SQRT}((THK*DNS*CP)/(P1*RT27)) \]
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H10(I) = (H1(I) - H(I))/H(I) * 100.

C
C*****H.T.MODEL OF MICKLEY & FAIRBANKS FOR THE CASE OF HIGBIE TYPE
C*****SOLIDS FLOW WITH CONTACT RESISTANCE.
C*****REF. POWDER TECHNOLOGY, 30 (1981)175-184 O.E.J.J.M.HOELEN
C
C
RW = (D/2.)/(0.7*THK)
H2(I) = 1./(RW+0.5*SQRT((PI*RT27)/(THK*DNS*CP)))
H2D(I) = ((H2(I)-H(I))/H(I)) * 100.

C
C*****H.T.MODEL OF MICKLEY & FAIRBANKS FOR THE CASE OF HIGBIE TYPE
C*****SOLIDS FLOW WITH CONTACT RESISTANCE.
C*****REF. CH.ENGNC.SC., 1977, VOL.32, P461 A.O.O.DENLOYE&J.S.M.BOTTERILL
C
C
RW = (D/2.)/(0.7*THK)
RA = 1./((2.*SQRT((THK*DNS*CP)/(PI*RT27))))
H3(I) = 1./RA*(1.-(RW/(2.*RA))*ALOG(1.+2.*(RA/RW))
H3D(I) = ((H3(I)-H(I))/H(I)) * 100.

C
C*****H.T.MODEL OF MICKLEY & FAIRBANKS FOR THE CASE OF HIGBIE TYPE
C*****SOLIDS FLOW WITH CONTACT RESISTANCE.
C*****REF. N.I.GELPERIN&V.G.EINSTEIN P461 1971 FLUIDIZATION H4 FOR
C*****CONSTANT HETFLUX AND H5 FOR CONST. TEMP. SURFACES.
C
C
RW = (D/2.)/(0.7*THK)
RA = 1./(2.*SQRT((THK*DNS*CP)/(PI*RT27)))
RR = 2.*RA
WOR = RW/RR
ROW = 1./WOR
H4(I) = PI/RR*(1.-(WOR*PI/2.)*ALOG(1.+2.*(ROW/PI))
H4D(I) = ((H4(I)-H(I))/H(I)) * 100.
H5(I) = 2./(RR*PI/2.)*((1.-EXP((ROW/SQRT(PI)))**2)*ERFC(ROW/
&SQRT(PI)))
H5D(I) = ((H5(I)-H(I))/H(I)) * 100.

IF (I .NE. 1) WRITE(6,118)
118 FORMAT(1H1)
WRITE(6,100)MAS
WRITE(6,101)VEL
WRITE(6,102)HFLX
WRITE(6,103)0LTP
WRITE(6,104)TBIN
WRITE(6,105)TS1
WRITE(6,106)E27
100 FORMAT(/,5X,'MASS FLOW RATE (KG/MIN) = ',F6.4)
101 FORMAT(/,5X,'MEAN VELOCITY (M/SEC) = ',F6.4)
102 FORMAT(/,5X,'HEAT FLUX (WATT/M**2) = ',F9.2)
103 FORMAT(/,5X,'TOTAL LOSSES (PERCENT) = ',F6.2)
104 FORMAT(/,5X,'INLET BULK TEMP. (DEG C) = ',F6.2)
105 FORMAT(/,5X,'WALL TEMP. TS1 (DEG C) = ',F6.2)
106 FORMAT(/,5X,'RESID. TIME/D**2 (SEC/M**2) = ',E15.7)
WRITE(6,107)HM
107 FORMAT(/,5X,'MEAN H.T. COEFF. (W/M**2DEGK) = ',F6.2)
WRITE(6,108)NUSD
108 FORMAT(/,5X,'MODIFIED NUSSELT NO. = ',F6.3)
WRITE(6,109)PESL

156
FILE: EXP

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109 FORMAT(/,5X,'MODIFIED PECLET NO. = ',F7.1) EXP02760
   WRITE(6,210)CK EXP02770
210 FORMAT(/,5X,'GAS(AIR) TH. CONDUCTIVITY = ',F7.4) EXP02780
   WRITE(6,110) EXP02790
110 FORMAT(/,5X) EXP02800
   WRITE(6,111)TS2, TB2, HX2 EXP02810
   WRITE(6,111)TS4, TB4, HX4 EXP02820
   WRITE(6,111)TS6, TB6, HX6 EXP02830
   WRITE(6,111)TS7, TB7, HX7 EXP02840
111 FORMAT(/,5X,3(10X,F6.2)) EXP02850
   WRITE(6,113)TR1, TR2, TR3, TR4, TR5, TR6, TR7 EXP02860
113 FORMAT(/,5X,7(5X,F6.2)) EXP02870
   WRITE(6,114)RR1, RR2, RR3, RR4, RR5, RR6, RR7 EXP02880
114 FORMAT(/,5X,7(5X,F6.2)) EXP02890
   WRITE(6,115)TRAV EXP02900
115 FORMAT(/,5X,'MEAN OUTLET TEMP. (DEG C) = ',F6.2) EXP02910
10 CONTINUE EXP02920
   WRITE(6,116) EXP02930
116 FORMAT(1H1,///,30X,'***SUMMARY OF RESULTS***'///) EXP02940
   DO 20 J=1,NR EXP02950
      WRITE(6,117)MS(J), VL(J), HF(J), QL(J), E(J), H(J), NUS(J), PEC(J), RT(J) EXP02960
      FOR(J), NUM(J) EXP02970
   CONTINUE EXP02980
   C EXP02990
   CONTINUE EXP03000
117 FORMAT(/,1X,F6.4,1X,F6.4,1X,F6.2,1X,15.7,1X,(F7.3,1X)) EXP03010
C117 FORMAT(/,1X,F6.4,1X,13(F6.1,1X)) EXP03020
30 CONTINUE EXP03030
   STOP EXP03040
END EXP03050
UNCERTAINTY ANALYSIS

In an experiment requiring measurements of several quantities, each of which has an error or "uncertainty" associated with it, the total error that propagates into the final result may be estimated by the expression [54,67,68]

\[ e_E^2 = \sum_{n=1}^{i} \left[ \left( \frac{\partial F}{\partial n} \right) e_n \right]^2 \]  

(F-1)

where \( F \) is a function of \( n \) independent variables having errors \( e_n \). For repeated measurements of the variables, it is assumed that errors are normally distributed about the true value.

The "function" whose resultant error is to be estimated here is the Nusselt number, given by

\[ Nu = \frac{h d}{k} \]  

(F-2)

Utilizing the above expression for \( e_E \)

\[ e_{Nu}^2 = \left( \frac{\partial Nu}{\partial h} e_h \right)^2 + \left( \frac{\partial Nu}{\partial d} e_d \right)^2 + \left( \frac{\partial Nu}{\partial k} e_k \right)^2 \]

\[ = \left( \frac{d}{k} e_h \right)^2 + \left( \frac{h}{k} e_d \right)^2 + \left( \frac{hd}{k^2} e_k \right)^2 \]  

(F-3)
where \( e_h \) and \( e_k \) may be estimated as follow

\[
h = q''/(T_w - T_m)
\]

\[
e_h^2 = \left( \frac{\partial h}{\partial q''} e_{q''} \right)^2 + \left( \frac{\partial h}{\partial T_w} e_{T_w} \right)^2 + \left( \frac{\partial h}{\partial T_m} e_{T_m} \right)^2
\]

\[
= \left( \frac{1}{T_w - T_m} e_{q''} \right)^2 + \left( \frac{q''}{(T_w - T_m)^2} e_{T_w} \right)^2 + \left( \frac{q''}{(T_w - T_m)^2} e_{T_m} \right)^2
\]  \hspace{1cm} (F-4)

where \( e_{q''} \), \( e_{T_w} \), and \( e_{T_m} \) may be estimated as follows.

Since \( q'' = (VI - \text{Loss})/\pi DL \)

\[
e_{q''}^2 = \left( \frac{I}{\pi DL} e_V \right)^2 + \left( \frac{V}{\pi DL} e_I \right)^2 + \left( \frac{VI - \text{Loss}}{\pi D^2 L} e_D \right)^2
\]

\[
+ \left( \frac{VI - \text{Loss}}{\pi DL^2} e_L \right)^2 + \left( \frac{1}{\pi DL} e_{\text{Loss}} \right)^2
\]  \hspace{1cm} (F-5)

By substituting into equation (F-5) the measured values of \( V \), \( I \), \( D \), \( L \) and Losses together with the estimated errors of each of these quantities, \( e_{q''} \) was calculated.

Similarly, since

\[
T_m = T_i + q'' \pi D x/m c_p
\]
where \( m \) is the mass flow rate given by

\[
m = \frac{M}{t}
\]

and

\[
e^2_m = \left( \frac{1}{t} e_M^2 \right) + \left( \frac{M}{t^2} e_t^2 \right)^2
\]  

(F-7)

The error quantities \( e_{Tw}, e_{Ti}, \) and \( e_{cp} \) were estimated from the accuracy of the temperature measuring devices employed. Alternately, \( e_h \) may then be calculated from equation (F-4).

When the Nusselt number is based on the bed effective thermal conductivity, \( k_0 \), an expression for \( e_{ko} \) is obtained similar to that derived by [54],

\[
e_{ko}^2 = \frac{1}{16\pi^2} \frac{(\ln t/t_1)^2}{(T - T_1)^2} \left[ 1^2 e_R^2 + 4 1^2 R^2 e_I^2 \right]
\]

\[
+ \frac{q^2}{16\pi^2} \frac{(\ln t/t_1)^2}{(T - T_1)^2} \left[ e_T^2 + e_{T1}^2 \right]
\]

\[
+ \frac{q^2}{16\pi^2} \frac{1}{(T - T_1)^2} \left[ \frac{1}{t_1^2} e_{t1}^2 + \frac{1}{t_2^2} e_{t2}^2 \right]
\]

(F-8)
when $k_g$ is utilized $e_{kg}$ was estimated based on the error associated with the gas film temperature measurements.

Finally, $e_d$ was estimated by using the standard deviation formula \[ [68] \]

$$\sigma = \left[ \frac{\sum_{i=1}^{N} (d_i - d)^2}{N} \right]^{1/2} \quad (F-9)$$

where $d \pm 2\sigma$ was taken as the limit of 95 percent of the measurements to lie within. The exceptional case was that for local sand, where its size measurements are not normally distributed, a procedure proposed by Keshock \[69\] was utilized to estimate the mean particle diameter limits as $d+\sigma$ and $d-0.5\sigma$. Volume-surface mean diameters were calculated assuming that 50% errors were made in the particle diameter ranges on either end of the size spectrum (Figure 4-1-a). In one extreme case, the 50% more particles were assumed in the largest size range, while at the same time 50% fewer particles were assumed for the smallest size range. This permitted calculating an "extreme" volume-surface mean diameter on the large diameter side of the spectrum. A similar procedure was used to determine the "minimum" reasonable diameter associated with errors in measuring particle distribution size. These extreme values were roughly $+\sigma$ and $-0.3\sigma$ about the mean value.

Values $e_h$, $e_d$, and $e_k$ may then be substituted into (F-3) to obtain an estimate of $e_{Nu}$. The uncertainties in $d$, $h$, and $Nu$ for the test media utilized in the present study were estimated and given in Table 4-2 Appendix D.

The preceding errors or uncertainties assigned take into account instrumentation accuracy, errors due to lack of resolution, human and general system losses, heat leaks, etc.
VITA

Mohamed M. Alsharif was born in Mohalat Marhoom, Garpya, Egypt on April 6, 1947. He attended elementary school in that village and was graduated from Ahmedia Secondary School (Tanta) in June 1963. The following September he entered Cairo University, Faculty of Engineering, and in June 1968 he received a Bachelor of Science degree in Mechanical Engineering. In December 1968 he accepted a designe engineer position at Tanta Flax and Oil Company for five years. In September 1974 he joined the Mechanical Engineering Department at the King Fahad University of Petroleum and Minerals (KFUPM), Dhahran, Saudi Arabia, as a laboratory engineer. In the spring of 1976 he began study toward a Master degree in mechanical engineering at KFUPM. This degree was awarded in June 1979.

After three years as a lecturer in the Mechanical Engineering Department at KFUPM, he accepted a teaching assistantship at the University of Tennessee, Knoxville, USA in January 1983. He received the Doctor of Philosophy degree with a major in Mechanical Engineering in March 1987.

He is married to the former Sana Abdelhamid of Egypt, and has a daughter, Samar, and two sons, Mostafa and Abdelhamid.