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The Value of Soil Testing for Potassium Fertilizer in Tennessee Cotton Production

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I am submitting herewith a thesis written by Xavier Lee Harmon entitled "The Value of Soil Testing for Potassium Fertilizer in Tennessee Cotton Production." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Agricultural Economics.

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The Value of Soil Testing for Potassium Fertilizer in Tennessee Cotton Production

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ABSTRACT

This thesis presents two separate studies focusing on the value of soil test information for potassium (K) fertilization of upland cotton. The objective of the first study was to determine the value of soil test information for available K in upland cotton production using the linear response plateau (LRP) and linear response stochastic plateau (LRSP) functions. This study uses dynamic programming to solve for optimal K fertilizer rates that maximize NPV when K carryover was and was not considered by a producer. This study extends the existing literature by comparing the value of soil testing information using a stochastic and deterministic yield response plateau functional form. Including carryover decreased the optimal K application rate and the K carryover level, while yield was optimal regardless of whether the producer considered carryover. The LRSP model Using K carryover information for K application decisions increased net present value and helped maintain steady levels of soil K. The LRSP function fit the data better than the LRP, and the value of soil testing was $11 per acre lower over ten years using the LRSP.

The objective of the second study was to determine the K fertilizer application rate and temporal frequency for obtaining K soil test information that maximizes NPV to K fertilizer in cotton production in the southeastern US. This study used a dynamic programming model to determine the optimal K application rates over time. Monte Carlo simulation was used to determine NPV for cotton production using five soil test schedules ranging from soil testing annually to every fifth year. On average, optimal K application rates for all temporal frequencies varied slightly. The range of optimal K application rates increased as the producer waited longer periods of time to update their soil test information. As the temporal frequency increased, the lower bounds of their carryover levels and yields decreased due to yield limiting levels of K.

NPV of returns to K was maximized at $7,580 per acre when producers updated soil testing information every two years, which was $2 per acre per year greater than annual soil testing.

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INTRODUCTION

In the production of upland cotton (*Gossypium hirsutum* L.), optimal management of potassium (K) fertilizer can be complicated by the dynamics of soil nutrient carryover levels. There have been frequent reports of late-season K deficiencies in the southeastern United States (U.S.) resulting in lower yields and lower profits (Maples, Thompson, and Varvil, 1988; Mullins, Burmester, and Reeves, 1997). However, research suggests that when soil K carryover is not considered by producers, K may be over applied relative to the needs of the plant, thereby decreasing profits (Harper et al., 2012). Quantifying the available K in the soil from year to year was important to establish K fertilizer recommendations that maximized profits.

Economists have developed dynamic programming models to determine profit-maximizing nutrient application rates when soil nutrient carryover levels are considered. Numerous studies have applied dynamic programming models to various crops and nutrients, where crop yield response to nutrient levels is commonly characterized using a deterministic yield response plateau functional form. Recently, the deterministic yield response functions have been extended to incorporate a normally-distributed year-random effect in the plateau. The plateau random effect emphasizes the impact of stochastic events such as insects, disease, and weather on crop yield response to fertilizer. These models have been found to more accurate estimates of optimal fertilizer rates in some circumstances (Boyer et al., 2013; Tembo et al., 2008; Tumusiime et al., 2011). Thus, information is lacking on the difference in optimal nutrient rates and net returns from stochastic and deterministic plateau yield response functions when carryover is considered.

Furthermore, an important question that remains unanswered is the optimal number of years a producer should wait to update soil test information. The literature has commonly assumed producers update their soil testing information annually, but annual soil testing might not be optimal given that some extension services recommend producers soil test anywhere from annually to every three years (Kissel and Sonon, 2011; Mylavarapu, 1997; Savoy and Joines, 2013). Further evidence that annual soil testing may not be optimal can be found in a survey of southern cotton producers, which suggests that on average producers update their soil testing information every two and a half years (Lambert et al., 2014). However, there have been no empirical studies that determine the optimal number of years producers should wait to update soil testing information.

This thesis presents two studies on the economics of soil testing for K fertilization of upland cotton in Tennessee. The objective of the first study was to determine the value of soil test information for available K in upland cotton production using the linear response plateau (LRP) and linear response stochastic plateau (LRSP) yield response functional forms. This research extends the literature by introducing the LRSP function into a dynamic programming model, and presenting an analytical solution for optimal fertilizer rates using a LRSP functional form in a dynamic programming framework.

The objective of the second study was to determine the K fertilizer application and the temporal frequency for obtaining K soil test information that maximizes net present value to K fertilizer in cotton production. Incorporating the temporal frequency of obtaining soil test information in a dynamic programing framework to determine optimal K rates and expected returns for cotton extends the literature. This research could also benefit producers through better informed use of soil testing and more efficient management of K fertilizer in cotton production.

References

Boyer C.N., J.A. Larson, R.K. Roberts, A.T. McClure, D.D. Tyler, and V. Zhou. Stochastic Corn Yield Response Functions to Nitrogen for Corn After Corn, Corn After Cotton, and Corn After Soybeans. *Journal of Agriculture and Applied Economics* 45(2013):669-681.

Harper, D.C., D.M. Lambert, J.A. Larson, and C.O. Gwathmey. "Potassium Carryover Dynamics and Optimal Application Policies in Cotton Production." *Agricultural Systems* 106(2012):84-93.

Kissel, D.E., and L. Sonon. "Soil Test Handbook for Georgia." Athens, Georgia: The University of Georgia Cooperative Extension, 2011.

Lambert, D.M., B.C. English, D.C. Harper, S.L. Larkin, J.A. Larson, D.F. Mooney, R.K. Roberts, M.Velandia, and J.M. Reeves. "Adoption and Frequency of Precision Soil Testing in Cotton Production." *Journal of Agricultural and Resource Economics* 39(2014):106-123.

Maples, R.L., W.R. Thompson, Jr., and J. Varvil. Potassium Deficiency in Cotton Takes on a New Look. *Better Crops* 73(1988):6–9.

Mullins, G.L., C.H. Burmester, and D.W. Reeves. Cotton Response to In-Row Subsoiling and Potassium Fertilizer Placement in Alabama. *Soil and Tillage Research* 40(1997):145-154.

Mylavarapu, R.S. "Soil Sampling for Precision Farming." Clemson, SC: Clemson University Cooperative Extension Service, Publication No. IL 63. 1997.

Savoy, H.J., Jr., and D. Joines. "Soil Testing." Knoxville, TN: University of Tennessee Agricultural Extension Service, Publication No. PB1061. 2013.

Tembo, G., B.W. Brorsen, F.M. Epplin, and E. Tostão. "Crop Input Response Functions with Stochastic Plateaus." *American Journal of Agricultural Economics* 90(2008):424-434.

Tumusiime, E., B.W. Brorsen, J. Mosali, J. Johnson, J.Locke, and J.T. Biermacher. Determining Optimal Levels of Nitrogen Fertilizer Using Random Parameter Models. *Journal of Agricultural and Applied Economics* 43(2011):541-552.

CHAPTER I
COMPARING THE VALUE OF SOIL TEST INFORMATION USING DETERMINISTIC AND STOCHASTIC YIELD RESPONSE PLATEAU FUNCTIONS

Abstract

We determined the value of soil test information for potassium (K) in upland cotton production using the linear response plateau (LRP) and linear response stochastic plateau (LRSP) functions. A stochastic dynamic programming model was used to determine the net present value to K fertilizer when optimal K was applied with knowledge about K carryover. Using K carryover information for K application decisions increased net present value and helped maintain steady levels of soil K. The LRSP function fit the data better than the LRP, and the value of soil testing was \$11 acre^{-1} lower over ten years using the LRSP.

Introduction

Procedures to assess the levels of soil potassium (K) readily available for consumption by field crops (available K) were developed more than fifty years ago (Mehlich, 1953), and crop response to K fertilizer has been well-documented in long-term experiments (Cope, 1981). However, attention to K management in upland cotton (*Gossypium hirsutum* L.) production grew in the late 1980s and 1990s after frequent reports of late-season K deficiencies in the southeastern United States (U.S.) resulting in lower yields (Maples, Thompson, and Varvil, 1988; Mullins, Burmester, and Reeves, 1997). These reports led to numerous agronomic studies in the U.S. Cotton Belt to recalibrate K fertilizer recommendations using soil test data to avoid late-season K deficiencies that produced negative impacts on cotton lint yield and fiber quality (Essington et al., 2002; Howard et al., 1998; Mullins, Schwab, and Burmester, 1999). While soil tests do not provide information on slowly available soil K or unavailable soil K, which can become available over time (Bertsch and Thomas, 1985), researchers concluded that quantifying readily available K in the soil from year to year (i.e. carryover K) was important to establish K fertilizer recommendations and circumvent late-season deficiencies. Today, soil tests are commonly used to inform producers on available soil K prior to planting.

Economists have developed models that consider soil fertilizer carryover to determine profit-maximizing K application rates over time in crop production (Heady and Dillon, 1961; Fuller, 1965; Kennedy et al., 1973; Stauber, Burt, and Linse, 1975). A carryover function is used to estimate the amount of available K in soils, given total K (applied and carryover) in previous periods. Dynamic programming is a common modeling approach to determine fertilizer rates that maximize net present value (NPV) (Kennedy, 1986). The difference between the NPV earned by a producer who considers carryover information and the NPV earned by a producer who does not consider carryover information determines the value of the carryover information (Harper et al., 2012).

Several studies have applied dynamic programming to the management of K fertilizer in crop production (Harper et al., 2012; Lanzer and Paris, 1981). Lanzer and Paris (1981) determined an economically optimal K rate over a nine-year planning horizon for double-cropped wheat and soybean in Brazil. The economically optimal K rate was 38 lb acre^{-1}, which was higher than the K recommendations for Brazil at that time. Harper et al. (2012) used three years of data from a cotton K fertilization experiment in Tennessee to analyze the value of soil test information under multiple information scenarios. They concluded that producers could increase NPV by \$653 acre^{-1} over five years when considering soil test information.

The selection of a functional form to model yield response to fertilizer is important for accurately determining optimal fertilizer rates and the value of soil test information (Ackello-Ogutu, Paris, and Williams, 1985; Kennedy, 1986). The quadratic response function has been frequently used to model cotton yield response to K fertilizer (Adeli and Varco, 2002; Bennett et al., 1965; Lombin and Mustafa, 1981; Pervez, Ashraf, and Makhdum, 2007). However, agronomists and economists alike have suggested that plateau-type response functions better describe yield response to fertilizer than the quadratic response function (Bullock and Bullock, 1994; Cerrato and Blackmer, 1990). A plateau-type function has either a linear or a polynomial relationship between crop yield, and the input until yield reaches a plateau, beyond which the input no longer limits yield; yield is limited by either another input affecting production or the plant reaches its natural maximum. Plateau functions, such as the linear response plateau (LRP) (Berck and Helfand, 1990; Paris, 1992) and quadratic-plus-plateau yield response functions have been used to determine optimal fertilizer rates in the dynamic programming framework (Ackello-Ogutu, Paris, and Williams, 1985; Harper et al., 2012; Jomini et al., 1991; Lanzer and Paris, 1981).

Tembo et al. (2008) extended the LRP function by incorporating a normally-distributed year-random effect in the plateau. The plateau random effect emphasizes the impact of stochastic events such as insects, disease, and weather on crop yield response to fertilizer. The linear stochastic plateau model (LRSP) was more appropriate than deterministic functions for several crops, resulting in more accurate estimates of optimal fertilizer rates (Boyer et al., 2013; Tembo et al., 2008; Tumusiime et al., 2011). Zhou et al. (2015) used the LRSP function in a dynamic programming framework to determine optimal nutrient application rates and evaluate alternative biofuel feedstock production subsidies. However, optimal fertilizer rates have never been evaluated using a stochastic plateau function, such as LRSP, in a dynamic programming framework. Using a stochastic plateau yield response function in a dynamic programming model could improve K fertilization recommendations for cotton production and offer a more accurate estimate of the value of soil test information because uncertainty around the plateau is incorporated into evaluating optimal rates.

The objective of this research was to determine the value of soil test information for available K in upland cotton production using the LRP and LRSP functions. We follow Kennedy's (1986) dynamic programming framework to solve for K fertilizer rates that maximize NPV when K carryover was and was not considered by a producer. The conceptual and econometric frameworks extend previous research by incorporating a stochastic plateau yield response function in a dynamic programming model, and presenting the analytical solution to optimal fertilizer rates using a LRSP function in a dynamic programming model.

Data

Data on cotton yield response and soil K fertility levels were collected from a nine-year field study (2000−2008) conducted at the University of Tennessee, West Tennessee Research and Education Center at Jackson (35.63°N; 88.85°W). The soil type was Loring-Calloway silt loam (thermic Oxyaquic Fragiudal and thermic Typic Fragiaqualf). The plots were not tilled. Each year, K fertilizer (muriate of potash, 0-0-60) was broadcast by hand to individual plots prior to planting at rates of 0, 25, 50, 75, 100, 125, and 150 lb K acre^{-1}. Treatments were applied to the same plots each year, starting five years prior to the first year of this study (2000) through the last year (2008). Plots were arranged in a randomized complete block design. Fertilizer treatments were replicated five or six times.

Cotton was planted using a 4-row John Deere MaxEmerge planter between April 30 and May 15 of each year. The cultivar 'PM1218BG/RR' was planted on all plots from 2000 to 2002. From 2003 to 2008, two contrasting cultivars were planted in a factorial arrangement relative to the K-fertility plots. The cultivars ‘PM1218BG/RR’ and DP555BG/RR’ were planted from 2003 to 2005, the cultivars ‘FM960BR’ and DP555BG/RR’ were planted from 2006 to 2007, and the cultivars ‘ST455B2RF’ and ‘ST5327B2RF’ were planted in 2008. Plots were 30 by 12.66 ft, containing four rows spaced 38 inches apart. Shortly before or after planting each year, nitrogen fertilizer (ammonium nitrate, 34-0-0) was uniformly drop-spread to all plots at a rate of 80 lb acre^{-1}. Ground limestone and phosphorus fertilizer were uniformly applied according to the recommendations of the University of Tennessee Extension Service (Savoy and Joines, 2001). Supplemental irrigation was used during dry spells in all years except 2002 and 2003. Thus, all other fertilizer inputs were assumed to be non-yield limiting. Table 1 summarizes monthly growing season rainfall by year at Jackson, Tennessee (National Oceanic and Atmospheric Administration, 2014). All other production practices followed the Tennessee Agricultural Extension Service (2001) guidelines for cotton production.

Seedcotton was harvested from the two interior rows of each plot twice each year using a modified John Deere 9930 spindle picker. First harvest occurred from September 7 to October 8, with a second harvest occurring fourteen to twenty-eight days later. Seedcotton weights, gin turnouts, and plot areas harvested were used to calculate lint yields. Observed lint yields from 2000 to 2008 were used to estimate yield response functions. Average annual lint yields by K rate are displayed in Table 2. Improved biotechnology from the different cultivars may have increased yields over time. Therefore, cotton lint yields were tested with a deterministic quadratic time response function (Just and Weninger, 1999). Similar to cotton in Oklahoma (Boyer, Brorsen, and Tumusiime, 2015), a time trend was not present.

Within six weeks after harvest, soil samples were collected from all plots at the 0-6 inch depth using the Mehlich I extraction method (Howard et al., 2001). The samples were tested at the University of Tennessee Soil and Forage Test Laboratory in Nashville, Tennessee. Data from Mehlich I soil tests were used to provide information on the amount of available K in the soil. Pre-planting soil test levels from 2001 to 2009 were used to estimate the carryover function. Average soil test levels across all plots and years were in the medium soil fertility range (Savoy and Joines, 2001) (Table 3). A review of observed soil test values indicated that the variance of the carryover data may increase at higher levels of total available K. Therefore, soil test levels were tested and corrected for heteroskedasticity across years.

Average annual cotton lint and elemental K prices (\$ lb^{-1}) from 1994 to 2013 were used to determine the K fertilization rates that maximized NPV over a ten-year planning horizon. Nominal prices were adjusted to reflect real prices in 2013 using the Gross Domestic Product implicit price deflator (U.S. Bureau of Economic Analysis, 2015). Real cotton prices varied from \$0.38 to \$1.07 lb^{-1}, and real elemental K prices varied from \$0.20 to \$0.91 lb^{-1} (U.S. Department of Agriculture Economic Research Service, 2013; 2014). Real lint and K prices were not correlated over time. The total cost of soil testing included the cost of obtaining the soil sample and the chemical analysis. The cost of obtaining the soil sample was \$6.57 $acre^{-1}$ $year^{-1}$, which was based on University of Tennessee Custom Rate Survey (Bowling, 2013). The cost of the chemical soil analysis was \$0.70 $acre^{-1}$ $year^{-1}$. This cost assumes a producer soil tests on a 4 ha grid, which follows University of Tennessee recommendations for soil testing (Savoy and Joines, 2013). A 5% discount rate was used to represent the opportunity cost of land in cotton production similar to previous dynamic programming literature (Harper et al., 2012; Kennedy et al., 1973; Park et al., 2007; Segarra et al., 1989; Watkins, Lu, and Huang, 1998).

Conceptual and Econometric Models

We find the value of the soil test when a LRP function and LRSP function was used to model yield response to K. Therefore, four scenarios are modeled: (1) NPV of returns to K fertilizer using the LRP yield response function considering K carryover; (2) NPV of returns to K fertilizer using the LRP yield response function when K carryover was not considered; (3) NPV of returns to K fertilizer using the LRSP yield response function considering K carryover; and (4) NPV of returns to K fertilizer using the LRSP yield response function when K carryover was not considered.

Dynamic programming model

An optimizing producer manages total K availability for continuous cotton production by applying K fertilizer at the beginning of each production year to maximize the NPV of returns to K over a planning horizon. The optimal fertilization rates are conditioned on some measure of K carryover between production periods (Kennedy, 1986; Kennedy et al., 1973):

$$\begin{aligned}
&\max_{A_t,\ldots,A_T} NPV = \sum_{t=1}^{T} \delta^{t-1} NR_t \\
&\text{Subject to:} \\
&A_t, C_t \geq 0 \\
&C_{t+1} = a_0 + a_1(A_t + C_t) \\
&C_1 \text{ given,}
\end{aligned} \tag{1}$$

where NPV is the sum of discounted returns over T years (t =1,…,T); A_t is applied K fertilizer (lb acre^{-1}); NR_t is the cotton lint net returns ($ acre^{-1}) to K fertilizer; δ is a discount factor reflecting the time value of money $1/(1+r)^t$, where r is the discount rate; C_t is carryover K (lb acre^{-1}) obtained from soil testing; C_{t+1} is the carryover level (lb acre^{-1}) obtained from soil test information prior to planting in year t+1, which is a function of applied K fertilizer A_t (lb acre^{-1}) and carryover soil K C_t (i.e., total K available (lb acre^{-1})); a_0 and a_1 are estimated parameters for the linear carryover function; and C_1 is the soil K level before fertilizer K is applied in the first period of production.

Single-period cotton lint net returns to K fertilizer are:

$$NR_t = \delta p_t^c y_t\{A_t + C_t\} - p_t^K A_t - s\,, \tag{2}$$

where p_t^c and p_t^K are cotton lint ($ lb^{-1}) and K fertilizer ($ lb^{-1}) prices, respectively; $y_t\{A_t + C_t\}$ is expected cotton lint yield (lb acre^{-1}); and s is the cost of soil testing ($ acre^{-1}), which includes purchasing the soil test and obtaining the soil samples. Soil K from previous applications accumulates into current-period soil K levels; thus, the only relevant soil K carryover level is for the current period. Residual soil K levels were determined using a carryover function. When the producer does not consider K carryover in making current-period K fertilization decisions, the cost of the soil test was zero (s = 0) and the carryover level was assumed to be zero (C_t = 0). Thus, K carryover has no influence on current-period K fertilizer application (Harper et al., 2012; Kennedy, 1986).

We follow Kennedy's (1986) dynamic framework to determine optimal fertilizer rates that maximize NPV:

$$\begin{aligned}
&V_t\{C_t\} = \max_{A_t}[NR_t + \delta V_{t+1}\{C_{t+1})\}] \\
&\text{Subject to} \\
&A_t, C_t \geq 0 \\
&C_{t+1} = a_0 + a_1(A_t + C_t) \\
&V_{T+1}\{C_{T+1}\} = 0 \\
&C_1 \text{ given,}
\end{aligned} \tag{3}$$

where $V_t\{C_t\}$ is the present value of net returns (\$ acre^{-1}) from applying the profit-maximizing K application in year *t*; and $V_{T+1}\{C_{T+1}\} = 0$ is the terminal condition stating that the producer does not receive any economic value from the K remaining in the soil at the end of the planning horizon since the producer will not get to utilize the remaining soil K after the planning horizon ends (Chiang, 1992). When maximizing NPV the economic optimality principle of marginal value product (MVP) equals marginal factor cost (MFC) is complicated by intertemporal factors such as the time value of money (opportunity cost) and fertilizer carryover (Kennedy et al., 1973; Kennedy, 1986). In this framework, K fertilizer is applied at the beginning of each production year to manage total K available to the plant. The inter-temporal optimization of this dynamic program determines fertilization rates through recursion using first order conditions (Bellman, 1957). The profit-maximizing K application strategy exists when the initial state variable (C_1) is given. The optimality conditions are solved by differentiating equation (3) with respect to the decision variable A_t:

(4) $$\frac{\partial V_t}{\partial A_t} = \delta p_t^c \frac{\partial y_t}{\partial A_t} - p_t^K + \delta \frac{dV_{t+1}}{dC_{t+1}} a_1 = 0,$$

which can be rearranged:

(5) $$\delta p_t^c \frac{\partial y_t}{\partial A_t} = p_t^K - \delta \frac{dV_{t+1}}{dC_{t+1}} a_1.$$

By the envelope theorem (Léonard and Van Long, 1992), differentiating equation (3) with respect to the state variable C_t gives:

(6) $$\frac{\partial V_t}{\partial C_t} = \delta p_t^c \frac{\partial y_t}{\partial A_t} + \delta \frac{dV_{t+1}}{dC_{t+1}} a_1.$$

Equation (5) can be substituted into equation (6) to get:

(7) $$\frac{\partial V_t}{\partial C_t} = p_t^K .$$

Equation (7) indicates K carryover at the beginning of year *t* is valued at the price of K in year *t*. A similar result can be found for year *t* + 1:

(8) $$\frac{\partial V_{t+1}}{\partial C_{t+1}} = p_{t+1}^K ,$$

which can be substituted into equation (5) to get:

(9) $$\delta p_t^c \frac{\partial y_t}{\partial A_t} = p_t^K - \delta p_{t+1}^K a_1 .$$

Equation (9) is the optimal condition for the single period K application rate when a producer has information about K carryover (Kennedy, 1986). This condition indicates the optimal K application rate in any year *t* is where the discounted MVP of K (left hand side of equation (9)) is equal to the MFC of K, which is the current year price of K less the discounted savings from K fertilizer carried over to the next year (right hand side of equation (9)).

If a producer does not consider carryover, then $C_t = 0$ and optimal condition for single-period K fertilization becomes:

(10) $$\delta p_t^c \frac{\partial y_t}{\partial A_t} = p_t^K .$$

In equation (10) the discounted savings of K fertilizer remaining in the soil until the next period is not considered.

K carryover function

A linear functional form is commonly used to model carryover in the literature (Harper et al., 2012; Jomini et al., 1991; Lanzer and Paris, 1981; Segarra et al., 1989). We adapt the linear carryover function by including a year random effect in the intercept. We estimated parameter for the carryover function using the actual measured total K available. The deterministic and stochastic models included identical linear carryover functions, which is:

$$C_{t+1,i} = a_0 + a_1(A_{t,i} + C_{t,i}) + \tau_t + u_{t,i}, \tag{11}$$

where $\tau_t \sim N(0, \sigma_\tau^2)$ is an intercept year random effect isolating the variation in carryover across years and $u_{t,i} \sim N(0, \sigma_u^2)$ a random error term for plot *i*. The two stochastic terms are assumed independent. The intercept, a_0, represents some constant amount of available K that remains in the soil over the planning horizon; the slope, a_1, is the proportion of total K from the current year readily available to the next crop estimated using observed carryover K. Maximum likelihood estimates for equation (11) were obtained using the MIXED procedure in SAS 9.3 (SAS Institute, Inc., 2011).

Yield response function

The LRP function is:

$$y_t = \min(\beta_0 + \beta_1(A_t + C_t), \mu) + w_t + \varepsilon_t, \tag{12}$$

where β_0 and β_1 are the yield response parameters; μ is the expected plateau yield parameter (lb acre^{-1}); $w_t \sim N(0, \sigma_w^2)$ is the intercept year random effect; and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ is the random error term. Independence is assumed across the two stochastic terms. Similarly, the LRSP function is:

$$y_t = \min(\beta_0 + \beta_1(A_t + C_t), \mu + v_t) + w_t + \varepsilon_t, \tag{13}$$

where $v_t \sim N(0, \sigma_v^2)$ is a plateau random effect. The three random effects are independent. Parameter estimates for the LRP and LRSP yield response functions were estimated using the observed K application rates and observed carryover levels from the experiment. Since the LRP and LRSP are nested response functions, Tembo et al. (2008) and Tumusiime et al. (2011) used the likelihood ratio test to determine whether the LRP or the LRSP model best describes the data. We follow this approach to determine whether the LRP or the LRSP model best describes cotton response to K. The maximum likelihood parameter estimates for equations (12) and (13) were obtained using the NLMIXED procedure in SAS 9.3 (SAS Institute, Inc., 2011).

Optimal K fertilizer rate

For the LRP function, the profit-maximizing K rate is the rate required to reach the plateau if the MVP below the plateau is greater than the MFC. Conversely, if the MVP of K below the plateau is less than the MFC, a profit-maximizing producer would apply zero K (Lanzer and Paris, 1981). Thus, the optimal K rate is a corner solution (zero or the plateau rate). When carryover is considered, the optimal K rate in year *t* is the corner solution less the carryover K in year *t* (C_t), given $A_t \geq 0$. However, the optimal K rate when carryover is not considered is the corner solution, assuming the carryover level or the savings associated with carryover is zero.

To solve for the optimal K rate for the LRSP considering carryover, the yield response function (equation 13) is differentiated with respect to the decision variable A_t and substituted into the optimality condition considering carryover (equation 9) to produce:

(14) $$\delta p_t^c [\beta_1 (1 - \Phi)] = p_t^K - \delta p_{t+1}^K a_1,$$

where $\Phi = \Phi[(\beta_0 + \beta_1(A_t + C_t) - \mu)/\sigma_v]$ is the standard normal cumulative distribution function and $0 \le \Phi \le 1$ (Tembo et al., 2008). By equation (14), the optimal K application rate was expressed as:

$$A_t^* = \frac{\left[\Phi^{-1}\left(1 - \frac{p_t^K - \delta p_{t+1}^K a_1}{\delta p_t^c \beta_1}\right)\right]\sigma_v + \mu - \beta_0}{\beta_1} - C_t . \tag{15}$$

When stochastic variation is considered in the plateau, the optimal application decision will be dependent upon the ratio of K and cotton prices (Tembo et al., 2008). Similarly, when a producer considers carryover the savings associated with K carryover must be accounted for as well. When a producer does not consider carryover, equation (15) becomes:

$$A_t^* = \frac{\left[\Phi^{-1}\left(1 - \frac{p_t^K}{\delta p_t^c \beta_1}\right)\right]\sigma_v + \mu - \beta_0}{\beta_1} . \tag{16}$$

Because carryover is not considered by the producer, the saving associated with K carryover is assumed to be zero. In reality, assuming K carryover has a zero value may not be realistic of producers; however, the assumption allows us to accurately benchmark the value of soil test information for the two response functions. Derivation of the optimal application rate for the LRSP is provided in Appendix A.

Monte Carlo simulation

The estimated response functions and analytical solutions for optimal fertilizer and yield are substituted into a simulation model to find a distribution of NPVs over a 10-year period. A Monte Carlo simulation model is developed for each of the four scenarios. Figure 1 summarizes the general process used to solve the dynamic programming model. Shaded boxes correspond to stochastic parameters in the model.

Uncertainty surrounding prices of cotton lint and K is introduced into the model through bootstrapping the real prices of cotton lint and K during each period of the ten-year planning horizon. To introduce uncertainty in the expected yield response, the yield response coefficients assumed a multivariate normal (MVN) distribution:

$$(17)\qquad \begin{bmatrix} \beta_0^* \\ \beta_1^* \\ \mu^* \\ \sigma_v^{2*} \end{bmatrix} \sim MVN\left(\begin{bmatrix} \beta_0 \\ \beta_1 \\ \mu \\ \sigma_v^2 \end{bmatrix}, \begin{bmatrix} \sigma_{\beta_0}^2 & \rho_{\beta_0,\beta_1}\sigma_{\beta_0}\sigma_{\beta_1} & \rho_{\beta_0,\mu}\sigma_{\beta_0}\sigma_{\mu} & \rho_{\beta_0,\sigma_v^2}\sigma_{\beta_0}\sigma_{\sigma_v^2} \\ \rho_{\beta_1,\beta_0}\sigma_{\beta_1}\sigma_{\beta_0} & \sigma_{\beta_1}^2 & \rho_{\beta_1,\mu}\sigma_{\beta_1}\sigma_{\mu} & \rho_{\beta_1,\sigma_v^2}\sigma_{\beta_1}\sigma_{\sigma_v^2} \\ \rho_{\mu,\beta_0}\sigma_{\mu}\sigma_{\beta_0} & \rho_{\mu,\beta_1}\sigma_{\mu}\sigma_{\beta_1} & \sigma_{\mu}^2 & \rho_{\mu,\sigma_v^2}\sigma_{\mu}\sigma_{\sigma_v^2} \\ \rho_{\sigma_v^2,\beta_0}\sigma_{\sigma_v^2}\sigma_{\beta_0} & \rho_{\sigma_v^2,\beta_1}\sigma_{\sigma_v^2}\sigma_{\beta_1} & \rho_{\sigma_v^2,\mu}\sigma_{\sigma_v^2}\sigma_{\mu} & \sigma_{\sigma_v^2}^2 \end{bmatrix} \right),$$

where the mean of the distribution is a vector of the estimated coefficients for each yield response function (equations 12-13); the variance of the distribution is a three by three matrix of the covariance for the estimated coefficients in each yield response function (equations 12-13), where ρ is a correlation coefficient; and an asterisk (*) indicates a randomly drawn coefficient for the simulation (Cuvaca et al., 2015). The plateau variance (v_t) was stochastic following Tembo et al.'s (2008) standard normal distribution. The coefficients of the carryover function assumed a similar multivariate normal distribution:

$$(18)\qquad \begin{bmatrix} a_0^* \\ a_1^* \end{bmatrix} \sim MVN\left(\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}, \begin{bmatrix} \sigma_{a_0}^2 & \rho_{a_0,a_1}\sigma_{a_0}^2,\sigma_{a_1}^2 \\ \rho_{a_1,a_0}\sigma_{a_1}^2,\sigma_{a_0}^2 & \sigma_{a_1}^2 \end{bmatrix} \right).$$

Making the parameter estimates in the carryover function stochastic is a unique contribution to the literature.

Uncertainty surrounding the initial carryover level (C_1) was introduced into the model by bootstrapping the observed carryover levels. The optimal K application rate in year one was found by substituting yield response and carryover parameter estimates, prices of cotton and K, and the initial carryover level into the equation for optimal K application rates. The soil K level after harvest in year t became available for use in year t+1 (equation 11). Therefore, after the first year decision, K applications for the remaining years (t=2,…,10) were influenced by K carryover from the previous season.

One-thousand iterations of the ten-year planning horizon are simulated to generate output distributions for the K application rate, K carryover level, lint yield, and NPV for each scenario. The Monte Carlo simulation is conducted using @Risk (Palisade, 2015). The expected NPVs from the simulation of the four scenarios were used to find the value of information from soil testing. The LRSP model captures the unexpected year-to-year variability in the plateau yield; thus, providing a hypothesized lower estimate for the value of information from soil testing than the LRP model.

Results

Yield response and carryover functions

The estimated yield response and carryover functions are presented in Table 4. The parameter estimates for the LRP and LRSP functions had the expected positive signs, except the intercepts were negative. Watkins, Lu, and Huang (1998) and Stauber, Burt, and Linse (1975) also found negative intercepts for wheat, barley, and seeded grasses yield response to nitrogen when carryover was considered. Like these studies, the negative intercepts were not of concern since carryover K was always greater than zero (Table 3); thus, zero total available K was not present in the data. The slope parameter estimate for the LRSP function was greater than the slope of the LRP function. Tembo et al., (2008) attributed attenuation bias to explaining the smaller slope parameter estimate for the LRP function. The expected plateau yield for the LRSP was also higher than the expected yield for the LRP, which matches previous studies (Boyer et al., 2013). The likelihood ratio statistic ([4977-4923.8] = 53.2) was greater than the critical value ($X^2_{1,0.05} = 3.84$), indicating the LRSP function described yield response to total available K better than the LRP function (Table 4), which is similar to Boyer et al. (2013), Tembo et al. (2008), and Tumusiime et al. (2011) observed.

The intercept for the carryover function indicated that 26 lb K acre^{-1} found in the soil did not come from the K application in the previous year, but remained available to the plant over the planning horizon. The carryover coefficient of 0.72 implies that 72% of the total K available in the current year will be carried over to be available for use in the next year. The carryover coefficient in Table 4 was similar to Harper et al.'s (2012) K carryover coefficient of 0.72.

Simulation results

Table 5 provides simulation results for K application, K carryover, and yield for each year of the ten-year time horizon as well as the ten-year average. For the LRP function, the optimal annual K application rate when carryover was considered ranged from 8 to 26 lb acre^{-1} with an annual average rate of 20 lb acre^{-1}. When K carryover was not considered, the optimal annual average application rate was 186 lb acre^{-1}, an increase of 166 lb acre^{-1}. Harper et al. (2012) reported optimal K application rates and carryover levels for cotton production in Tennessee higher than what we find in our study. We report results using data from a longer time-series and on a different soil type than Harper et al. (2012), which likely explains the different findings. Optimal K carryover was on average 331 lb acre^{-1} year^{-1} less when K soil test information was considered in the choice of the K fertilization rate. Fertilizer K carryover declined from an initial level of 271 lb acre^{-1} to a steady state level of 160 lb acre^{-1} when soil test information was considered, whereas, soil K increased each year when K carryover was not considered in the choice of a K fertilization rate. Lint yields were the same for both K carryover scenarios. Therefore, a producer using soil K carryover information would optimize lint yields, lower K fertilization rates and costs, and consistently lower the amount of fertilizer K remaining in the soil. These results match the existing literature on the use of soil test information in making optimal fertilizer decisions (Harper et al., 2012; Kennedy et al., 1973; Park et al., 2007; Segarra et al., 1989; Watkins, Lu, and Huang, 1998).

For the LRSP function, optimal K fertilization rates ranged from 10 to 28 lb acre^{-1} in each year with a ten-year average of 22 lb acre^{-1}. A producer that did not consider soil K information applied 162 lb acre^{-1} more K fertilizer annually than a producer who considered carryover to determine the K fertilization rate. K carryover was on average 322 lb acre^{-1} year^{-1} lower when carryover was considered. Lint yields were the same for the two fertilizer carryover scenarios. Thus, a producer who considers soil test information could achieve optimal lint yields while reducing K fertilizer application each year compared to a producer who does not consider soil test information. These findings illustrate the potential of soil test information to reduce over-application of K fertilizer while maintaining optimal lint yields in cotton production.

Comparing the LRP and LRSP results, the optimal application rates and carryover levels were higher and lint yields were lower when the plateau was stochastic. The slope parameter estimate β_1 found in the LRSP results in a higher average MVP of K, which explains why the LRSP has a higher optimal application rate of K than the LRP (Tembo et al., 2008). The higher optimal K application rate also increased the optimal K carryover rate relative to the LRP. The average K carryover levels obtained from the LRP and LRSP functions when considering soil test information were classified as medium soil test ratings according to the guidelines set by the University of Tennessee Extension Service (Savoy and Joines, 2001). However, when carryover was not considered the K carryover levels were classified as very-high soil test ratings, which may lead to nutrient imbalances (Savoy and Joines, 2001). Maintaining a medium soil test rating would be beneficial for a producer to minimize deficiency symptoms (Savoy, 2009). At medium soil test levels, the University of Tennessee Extension recommended K fertilization rate was higher than the optimal K rate determined in this study. Finally, the optimal yield was lower with the LRSP function than the LRP. Tumusiime et al. (2011) stated that the LRP function can overestimate yield potential in years when climate conditions are not suitable for production. Thus, the LRSP function has a lower optimal yield because the variation in the yield was considered.

The value of soil test information

Table 6 shows the NPV at the optimal K rates for each of the four scenarios and the value of soil test information. Using the LRP function, the NPVs for a producer who considers and does not consider carryover were \$7,526 acre^{-1} and \$7,039 acre^{-1}, respectively, giving a value of soil test information of \$487 acre^{-1} or \$63 acre^{-1} year^{-1}. The respective NPVs with and without carryover information were \$7,152 acre^{-1} and \$6,676 acre^{-1} using the LRSP function, giving a value of soil test information of \$476 acre^{-1} or \$62 acre^{-1} year^{-1}.

Given that the LRSP function described yield response to total available K better than the LRP function, the LRP function overestimated the value of soil testing by \$11 acre^{-1} (\$2 acre^{-1} yr^{-1}). Overall, testing for K carryover and using the soil test information to make K application decisions in cotton production was profitable and helped maintain a steady level of soil K. By capturing variation in the yield plateau, the LRSP function provided a lower value of information from soil testing for K in cotton than its deterministic counterpart.

Conclusion

We determined the value of information from soil testing for K in upland cotton production using the LRP function and the LRSP function. Cotton yield response and soil testing data were obtained from a nine-year experiment in Jackson, Tennessee. We follow Kennedy's (1986) dynamic programming framework to find the K fertilizer rate that maximizes NPV using the LRP and LRSP functions when carryover was and was not considered. Simulation models were used to find the expected NPVs, which were compared to find the value of testing for K in cotton production.

We build on previous research by incorporating the LRSP model into a dynamic programming model to find the value of soil testing when the plateau was uncertain. The results of this study provide information on the difference in the value of soil testing when the yield plateau is deterministic and when the yield plateau is stochastic. The results provide estimates of the value of soil testing and K recommendations for cotton production under the two plateau assumptions. A limitation of this study is that the results are specific to monoculture cotton, but future research could focus on extending the model to include crop rotations.

Regardless of the response function, including carryover in the simulation model decreased the optimal K application rate and the K carryover level, while yield remained optimal in all scenarios. Producers are often thought to over apply fertilizer; however, real-life producer decisions often involve more uncertainty than what is modeled. The value of the soil test information was \$63 acre^{-1} year^{-1} when the LRP function was used and \$61 acre^{-1} year^{-1} when the LRSP function was used. The value of soil test information was greater than cost of soil testing for both response functions. This conclusion is especially true when one considers that soil test results, typically including information on other available crop nutrients, is perhaps higher than these estimates, which are based solely on the value of the K levels.

Future research could examine the optimal frequency of soil testing K. Furthermore, we assume that producers have an expected value of K carryover equal to zero when producers do not consider carryover. However, this may not be realistic since producers typically expect some level of K carryover. Future research could investigate how the value of soil testing is affected by producers having some knowledge of available soil K levels when they do not consider carryover in making K application decisions.

References

Ackello-Ogutu, C., Q. Paris, and W.A. Williams. "Testing a von Liebig Crop Response Function against Polynomial Specifications." *American Journal of Agricultural Economics* 67(1985):873-880.

Adeli, A., and J.J. Varco. "Potassium Management Effects on Cotton Yield, Nutrition, and Soil Potassium Level." *Journal of Plant Nutrition* 25(2002):2229-2242.

Bellman, R.E. 1957. *Dynamic Programming*. Princeton, NJ: Princeton University Press.

Bennett, O.L., R.D. Rouse, D.A. Ashley, and B.D. Doss. Yield, Fiber Quality and "Potassium Content of Irrigated Cotton Plants as Affected by Rates of Potassium." *Agronomy Journal* 57(1965):296-299.

Berck, P. and G. Helfand. "Reconciling the Vonliebig and Differentiable Crop Production Functions." *American Journal of Agricultural Economics* 72(1990):985-996.

Bertsch, P.M., and G.W. Thomas. "Potassium Status of Temperate Region Soils." In R.D. Munson, ed., *Potassium in Agriculture*, Madison, WI: American Society of Agronomy, 1985. 131-163.

Bowling, B. "Custom Rates Survey, 2013." Publication AE13-06. University of Tennessee, Department of Agricultural and Resource Economics, Knoxivlle, TN> 2013. Available online at http://economics.ag.utk.edu/extension/pubs/CustomRates2013-rev.pdf.

Boyer, C.N., B.W. Brorsen, and E. Tumusiime. "Modeling Skewness with the Linear Stochastic Plateau Model to Determine Optimal Nitrogen Rates." *Agricultural Economics* 46(2015):1-10.

Boyer C.N., J.A. Larson, R.K. Roberts, A.T. McClure, D.D. Tyler, and V. Zhou. "Stochastic Corn Yield Response Functions to Nitrogen for Corn After Corn, Corn After Cotton, and Corn After Soybeans." *Journal of Agriculture and Applied Economics* 45(2013):669-681.

Bullock D.G., and D.S. Bullock. "Quadratic and Quadratic-Plus-Plateau Models for Predicting Optimal Nitrogen Rate of Corn: A Comparison." *Agronomy Journal* 86(1994):191-195.

Cerrato, M.E., and A.M. Blackmer. "Comparison of Models for Describing Corn Yield Response to Nitrogen Fertilizer." *Agronomy Journal* 82(1990):138-143.

Chiang, A.C. 1992. *Elements of Dynamic Optimization*. New York: McGraw Hill.

Cope, J.T. "Effects of 50 Years of Fertilization with Phosphorus and Potassium on Soil Test Levels and Yields at Six Locations." *Soil Science Society of America Journal* 45(1981):342-347.

Cuvaca, I.B., D. Lambert, F. Walker, M. Marake, and N. Eash. “Economically Optimal N Fertilizer Rates for Maize Produced on Vertisol and Inceptisol Soils under No-Till Management: A Case Study in Maphutseng Lesotho.” *International Journal of Plant & Soil Science* 8(2015):1-12.

Essington, M.E., D.D. Howard, H.J. Savoy, and G.M. Lessman. “Potassium Fertilization of Cotton Produced on Loess-Derived Soils.” *Better Crops* 86(2002):13-15.

Fuller, W.A. “Stochastic Fertilizer Production Functions for Continuous Corn.” *American Journal of Farm Economics* 47(1965):105-119.

Harper, D.C., D.M. Lambert, J.A. Larson, and C.O. Gwathmey. “Potassium Carryover Dynamics and Optimal Application Policies in Cotton Production.” *Agricultural Systems* 106(2012):84-93.

Heady, E.O., and J.L. Dillon. 1961. *Agricultural Production Functions.* Iowa: Iowa State University Press.

Howard, D.D., M.E. Essington, R.M. Hayes, and W.M. Percell. “Potassium Fertilization of Conventional- and No-Till Cotton.” *Journal of Cotton Science* 5(2001):197-205.

Howard, D.D., C.O. Gwathmey, R.K. Roberts, and G.M. Lessman. “Potassium Fertilization of Cotton Produced on a Low K Soil with Contrasting Tillage Systems.” *Journal of Production Agriculture* 11(1998):74-79.

Jomini, P.A. “Modeling Stochastic Crop Response to Fertilisation When Carry-Over Matters.” *Agricultural Economics* 6(1991):97-113.

Just, R.E., and Q. Weninger. “Are Crop Yields Normally Distributed?” *American Journal of Agricultural Economics* 81(1999):287-304.

Kennedy, J.O.S. 1986. *Dynamic Programming: Applications to Agriculture and Natural Resources.* London: Elsevier Applied Science.

Kennedy, J.O.S, I.F. Whan, R. Jackson, and J.L. Dillon. “Optimal Fertilizer Carryover and Crop Recycling Policies for a Tropical Grain Crop.” *American Journal of Agricultural Economics* 17(1973):104-113.

Lanzer, E.A., and Q. Paris. “A New Analytical Framework for the Fertilization Problem. 1981 *American Journal of Agricultural Economics* 63(1981):93-103.

Léonard D., and N. van Long. 1992. *Optimal Control Theory and Static Optimization in Economics.* Cambridge University Press, Cambridge.

Lombin, G., and S. Mustafa. “Potassium Response of Cotton on Some Inceptisols and Oxisols of Northern Nigeria.” *Agronomy Journal* 73(1981):724-729.

Maples, R.L., W.R. Thompson, Jr., and J. Varvil. "Potassium Deficiency in Cotton Takes on a New Look." *Better Crops* 73(1988):6–9.

Mehlich, A. 1953. "Determination of P, Ca, Mg, K, Na and NH_4." Memo. Raleign NC. North Carolina Soil Testing Division.

Mullins, G.L., C.H. Burmester, and D.W. Reeves. "Cotton Response to In-Row Subsoiling and Potassium Fertilizer Placement in Alabama." *Soil and Tillage Research* 40(1997):145-154.

Mullins, G.L., G.J. Schwab, and C.H. Burmester. "Cotton Response to Surface Applications of Potassium Fertilizer: A 10-Year Summary." *Journal of Production Agriculture* 12(1999):434-440.

National Oceanic and Atmospheric Administration National Climatic Data Center. 2015. Climate Data Online Search. http://www.ncdc.noaa.gov/cdo-web/search.

Palisade Corporation. 2014. @ Risk 6. Ithica, NY.

Paris, Q. "The von Liebig Hypothesis." *American Journal of Agricultural Economics* 74(1992):1019-1028.

Park, S.C., A. Stocker, J.A., Hattey, and J.C. Turner. Long-Term Profitability of Animal Manure Using Optimal Nitrogen Application Rate. Paper presented at the Southern Agricultural Economics Association Annual Meeting, Mobile, Alabama, 2007.

Pervez, H., M. Ashraf, and M.L. Makhdum. Influence of Potassium Rates and Sources on Seed Cotton Yield and Yield Components of Some Elite Cotton Cultivars. *Journal of Plant Nutrition* 27(2005):1295-1317.

SAS Institute, Inc. *SAS/STAT 9.3 User's Guide.* Cary, NC: SAS Institute, Inc., 2011. Available online at: http://support.sas.com/documentation/cdl/en/statug/63962/HTML/default/viewer.htm#titlepage.htm.

Savoy, H.. Interpreting Mehlich 1 and 3 Soil Test Extractant Results for P and K in Tennessee. University of Tennessee Agricultural Extension Service, Knoxville, TN. Publication W229. 2009.

Savoy, H.J., Jr., and D. Joines. Lime and Fertilizer Recommendations for the Various Crops of Tennessee. University of Tennessee, Agricultural Extension Service, Knoxville, TN. BEES Info 100. 2001.

Savoy, H.J., Jr., and D. Joines. Soil Testing. University of Tennessee Agricultural Extension Service, Knoxville, TN. Publication PB1061. 2013.

Segarra, E., E.D. Ethridge, C.R. Deussen, and A.B. Onken. "Nitrogen Carry-Over Impacts in Irrigated Cotton Production, Southern High Plains of Texas." *Western Journal of Agricultural Economics* 14(1989):300-309.

Stauber, M.S., O.R. Burt, and F. Linse. "An Economic Evaluation of Nitrogen Fertilization of Grasses When Carry-Over is Significant." *America Journal of Agriculture Economics* 57(1975):463-471.

Tembo, G., B.W. Brorsen, F.M. Epplin, and E. Tostão. "Crop Input Response Functions with Stochastic Plateaus." *American Journal of Agricultural Economics* 90(2008):424-434.

Tennessee Agricultural Extension Service. 2001. Cotton Production in Tennessee. University of Tennessee Agricultural Extension Service. Publication PB1514.

Tumusiime, E., B.W. Brorsen, J. Mosali, J. Johnson, J.Locke, and J.T. Biermacher. "Determining Optimal Levels of Nitrogen Fertilizer Using Random Parameter Models." *Journal of Agricultural and Applied Economics* 43(2011):541-552.

United States Bureau of Economic Analysis. 2015. Gross Domestic Product: Implicit Price Deflator. http://research.stlouisfed.org/fred2/series/GDPDEF/downloaddata.

United States Department of Agriculture Economic Research Service. 2013. Table 7. Average U.S. farm prices of selected fertilizers, 1960-2013. http://www.ers.usda.gov/data-products/fertilizer-use-and-price.aspx.

United States Department of Agriculture Economic Research Service. 2014. Table 11. U.S. upland cotton farm, spot, and mill prices, 1970/71-2013/14. http://ers.usda.gov/data-products/cotton,-wool,-and-textile-data/cotton-and-wool-yearbook.aspx.

Watkins, B., Y. Lu, and W. Huang. "Economic and Environmental Feasibility of Variable Rate Nitrogen Fertilizer Application with Carry-Over Effects." *Journal of Agricultural and Resource Economics* 23(1998):401-426.

Zhou, X., C.D. Clark, D.M. Lambert, B.C. English, J.A. Larson, and C.N. Boyer. "Biomass Supply and Nutrient Runoff Abatement under Alternative Biofuel Feedstock Production Subsidies." *Agricultural Systems* 139(2015):250-259.

Appendix A

Tables and figures

Table 1. Total Monthly Precipitation Levels for the Growing Season of Upland Cotton in Jackson, TN from 2000 to 2008.

	Precipitation Totals (inches)									
Month	2000	2001	2002	2003	2004	2005	2006	2007	2008	Average
March	3.93	2.81	13.00	3.56	2.50	4.10	1.77	1.15	9.75	4.83
April	5.22	2.48	1.10	2.34	9.08	8.54	5.42	3.25	8.23	5.07
May	3.52	4.87	5.90		6.23	0.36	3.60	0.86	6.86	4.02
June	3.99	4.82	2.45	6.06	2.90	6.87	4.94	2.71	2.81	4.17
July	2.46	4.71	0.85	2.42	4.74	5.46	2.12	1.76	6.28	3.42
August	2.92	4.65	5.35	3.43	4.93	7.27	3.53	0.77	2.55	3.93
September	3.27	2.28	13.09	2.79	0.69	3.95	2.89	6.28	0.79	4.00
October	0.86	7.37	6.41	4.16	7.99	0.14	2.62	8.97	3.15	4.63
Total	26.17	33.99	48.15	24.76	39.06	36.69	26.89	25.75	40.43	33.54

Source: National Oceanic and Atmospheric Administration, 2014

Table 2. Average Annual Cotton Lint Yield by K Application Rate in Jackson, TN from 2000 to 2008.

K rate	Yield (lb $acre^{-1}$)									
(lb $acre^{-1}$)	2000	2001	2002	2003	2004	2005	2006	2007	2008	Average
0	880	827	475	809	960	871	695	903	597	780
25	1092	1209	756	1208	1519	1305	1242	1312	1300	1216
50	1117	1242	835	1387	1873	1487	1427	1314	1419	1345
75	1191	1368	1003	1403	1812	1384	1408	1274	1595	1382
100	1171	1392	1072	1451	1999	1536	1462	1275	1581	1438
125	1173	1366	1069	1370	1857	1390	1310	1129	1447	1346
149	1184	1402	1038	1375	1920	1430	1317	1120	1383	1352

Table 3. Average Pre-Planting K Carryover Levels by K Application Rate in Jackson, TN from 2001 to 2009.

K rate	Carryover K (lb acre^{-1})									
(lb acre^{-1})	2001	2002	2003	2004	2005	2006	2007	2008	2009	Average
0	119	110	139	98	88	104	93	102	90	117
25	154	147	177	150	123	130	123	142	145	161
50	178	186	204	219	177	195	188	222	222	223
75	245	222	249	297	229	238	252	271	305	287
100	325	270	301	395	307	337	298	318	372	364
125	395	347	335	469	350	405	363	335	459	430
149	459	521	376	528	440	458	467	491	533	524

Table 4. Parameter Estimates for the Linear Response Plateau and Linear Response Stochastic Plateau Yield Response to Total Available K and the Linear Carryover Function.

Parameter[a,b]	Deterministic Plateau	Stochastic Plateau	Carryover[c]
Intercept (β_0, a_0)	-13.26 (78.41)	-60.27 (90.69)	25.46*** (6.78)
Slope (β_1, a_1)	7.51*** (0.59)	7.95*** (0.54)	0.73*** (0.02)
Plateau Yield (μ)	1373.34*** (14.31)	1397.05*** (14.36)	-
Plateau Random Effect (σ_v^2)	-	31996*** (4172.66)	-
Year Random Effect $(\sigma_w^2, \sigma_\tau^2)$	48867*** (4766.99)	33787*** (5197.98)	235.71
Random Error $(\sigma_\varepsilon^2, \sigma_u^2)$	30952*** (2275.42)	25416*** (1909.13)	2.74
-2 Log-Likelihood	4977	4923.8	

[a] Single, double, and triple asterisks (*, **, ***) represent significance at the 10%, 5%, and 1% level.
[b] Standard errors are in parentheses.
[c] Carryover data was corrected for heteroscedasticity.

Table 5. Monte Carlo Simulation Results for Average K Application Rate, K Carryover, Yield by Year for A 10 Year Planning Horizon.

	Linear Response Plateau (LRP)		Linear Response Stochastic Plateau (LRSP)	
Year	K Carryover is Considered	K Carryover is Not Considered	K Carryover is Considered	K Carryover is Not Considered
	Optimal K Application Rate (*lb acre*$^{-1}$)			
Year 1	17	186	19	184
Year 2	8	186	10	183
Year 3	11	186	14	183
Year 4	16	186	19	183
Year 5	21	186	24	185
Year 6	24	186	28	185
Year 7	25	186	27	184
Year 8	26	186	28	184
Year 9	26	186	26	183
Year 10	26	186	27	183
Average	20	186	22	184
	Pre-planting K Carryover Level (*lb acre*$^{-1}$)			
Year 1	270	270	270	270
Year 2	234	356	235	355
Year 3	201	419	203	416
Year 4	179	464	183	460
Year 5	167	497	172	493
Year 6	162	521	167	517
Year 7	160	539	166	535
Year 8	160	551	166	548
Year 9	160	561	166	557
Year 10	160	568	165	563
Average	174	505	179	501
	Optimal Cotton Lint Yield (*lb acre*$^{-1}$)			
Year 1	1374	1374	1312	1312
Year 2	1374	1374	1308	1308
Year 3	1374	1374	1306	1306
Year 4	1374	1374	1305	1305
Year 5	1374	1374	1310	1310
Year 6	1374	1374	1310	1310
Year 7	1374	1374	1309	1309
Year 8	1374	1374	1312	1312
Year 9	1374	1374	1300	1300
Year 10	1374	1374	1314	1314
Average	1374	1374	1309	1309

Table 6. Net Present Value (NPV) for the 10-year Period when K Carryover was and was not Considered using the Linear Response Plateau and Linear Response Stochastic Plateau Yield Response Function and the Value for Soil Test Information.

Value	Linear Response Plateau (LRP)	Linear Response Stochastic Plateau (LRSP)
NPV ($ acre^{-1}) when K Carryover was Considered	$7,526	$7,152
NPV ($ acre^{-1}) when K Carryover was not Considered	$7,039	$6,676
Value of Soil Test Information over 10 years ($ acre^{-1})	$487	$476
Annual Value of Soil Test Information ($ acre^{-1})	$63	$62

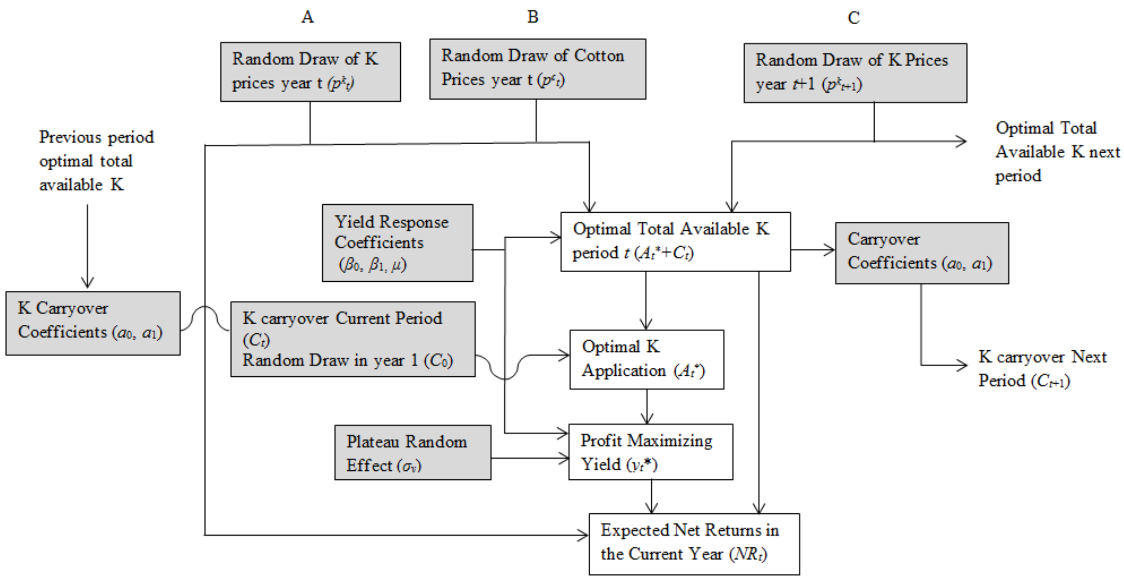

Figure 1. Flow chart depicting the process of solving the dynamic programming model and simulation for a single period (*t*).

Deriving optimal K rates

We follow Kennedy's (1986) dynamic programming approach to derive the optimal K application rates when producers' application decisions are conditioned on some knowledge of K carryover. When producers consider carryover, the optimality condition for a profit-maximizing producer is

$$\text{(A1)} \qquad \delta p_t^c \frac{\partial y_t}{\partial A_t} = p_t^K - \delta p_{t+1}^K a_1 .$$

The producer's optimal K application rate is derived by updating equation (A1) with the first-order condition of the linear response stochastic plateau (LRSP) yield response function, where the LRSP functional form is

$$\text{(A2)} \qquad y_t = (1-\Phi)(\beta_0 + \beta_1(A_t + C_t)) + \Phi\left(\mu - \frac{\sigma_v \phi}{\Phi}\right).$$

where $\Phi = \Phi[(\beta_0+\beta_1(A_t+C_t)-\mu)/\sigma_v]$ is the cumulative normal distribution function and $\phi = \phi[(\beta_0+\beta_1(A_t+C_t)-\mu)/\sigma_v]$ is the standard normal probability density function, both evaluated at the total available K level ($A_t + C_t$) in period t. Tembo et al. (2008) derived the first-order condition of the LRSP with respect to the decision variable:

$$\text{(A3)} \qquad \frac{\partial y_t}{\partial A_t} = \beta_1(1-\Phi) = 0 .$$

By substituting equation (A3) into equation (A1) we obtain

$$\text{(A4)} \qquad \delta p_t^c \beta_1 (1-\Phi) = p_t^K - \delta p_{t+1}^K a_1 .$$

We can rearrange equation (A4) to show that

$$\text{(A5)} \qquad \Phi = 1 - \frac{p_t^K - \delta p_{t+1}^K a_1}{\delta p_t^c \beta_1} .$$

If we recall that $\Phi = \Phi[(\beta_0+\beta_1(A_t+C_t)-\mu)/\sigma_v]$, we can update equation (A5) to

(A6) $$\Phi\left[\frac{\left(\beta_0+\beta_1\left(A_t+C_t\right)-\mu\right)}{\sigma_v}\right]=1-\frac{p_t^K-\delta p_{t+1}^K a_1}{\delta p_t^c \beta_1},$$

which can be rearranged to

(A7) $$\frac{\left(\beta_0+\beta_1\left(A_t+C_t\right)-\mu\right)}{\sigma_v}=\Phi^{-1}\left(1-\frac{p_t^K-\delta p_{t+1}^K a_1}{\delta p_t^c \beta_1}\right).$$

The closed-form solution for the producer's optimal K application decision can be obtained by solving equation (A7) for the decision variable A_t:

(A8) $$A_t^*=\frac{\left[\Phi^{-1}\left(1-\frac{p_t^K-\delta p_{t+1}^K a_1}{\delta p_t^c \beta_1}\right)\right]\sigma_v+\mu-\beta_0}{\beta_1}-C_t.$$

When the producer does not consider carryover, we repeat the same process using the optimality condition for a producer who does not consider carryover (equation 10).

CHAPTER II
TEMPORAL FREQUENCY OF SOIL TEST EFFECTS ON RETURNS TO POTASSIUM FERTILIZATION IN COTTON PRODUCTION

Abstract

Little research exists on the optimal temporal frequency between soil tests, given empirical data on potassium (K) carryover and its interaction with cotton yield. We evaluate how decreasing the temporal frequency between obtaining K soil test information affects the net present value (NPV) of cotton production. Monte Carlo simulation was used to determine NPV for cotton production using five soil test schedules ranging from soil testing annually to every fifth year. NPV of returns to K was maximized at \$7,580 acre^{-1} when producers updated soil testing information every two years, which was \$2 acre^{-1} yr^{-1} greater than annual soil testing.

Introduction

Potassium (K) is an important nutrient for upland cotton (*Gossypium hirsutum* L.) but can be difficult to manage over time (Howard et al., 2001). In the late 1980s and early 1990s there were frequent reports of late-season K deficiencies in the United States (U.S.) Cotton Belt (Maples et al., 1988; Mullins, Burmester, and Reeves, 1997), resulting in numerous agronomic studies on cotton response to K while considering K levels in the soil that are readily available for plant consumption (soil K) from the previous production year (i.e., carryover) (Essington et al., 2002; Howard et al., 1998; Mullins, Schwab, and Burmester, 1999). Findings from these studies triggered extension personnel in several southeastern states to recalibrate K fertilizer recommendations to maintain adequate soil K levels for cotton production and circumvent yield loss due to late-season K deficiencies. Methods to analyze soil K levels were developed more than 60 years ago (Mehlich, 1953), but gathering this information using soil tests and applying the information to make a K applications is still a growing practice among cotton producers (Zhou et al., 2015).

If a producer considers information on soil K levels before applying profit-maximizing K rates, then the producer's decision framework changes from maximizing net returns in a given year to maximizing the present value of net returns (NPV) over a planning horizon. The switch in the producer's decision framework occurs because application decisions in a given year are based on their application rates from the previous year. Economists developed dynamic programming models to determine nutrient application rates that maximize NPV with nutrient carryover (Fuller, 1965; Heady and Dillon, 1961; Kennedy et al., 1973; Kennedy, 1986; Stauber, Burt, and Linse, 1975). The dynamic programming approach separates the producer's planning horizon into discrete sequential maximization problems, solved by deriving optimality conditions for each period (Bellman, 1957). The single period application rates are conditioned on some knowledge of soil carryover levels prior to planting.

These dynamic models have been adapted to different crops and nutrients (Ackello-Ogutu, Paris, and Williams, 1985; Jomini et al., 2001; Kennedy et al., 1973; Lambert, Lowenberg-DeBoer, and Malzer, 2007; Lanzer and Paris, 1981; Park et al., 2007; Schnitkey, Hopkins, and Tweeten, 1996; Segarra et al., 1989; Watkins, Lu, and Huang, 1998). However, Harper et al. (2012) were first to use a dynamic programming model to determine K application rates that maximizes NPV for upland cotton production. They developed an application for valuing the information from soil testing in cotton production by considering multiple information scenarios in the dynamic programming framework. Harper et al. (2012) used three years of data on K soil fertility and cotton lint yields in Tennessee to determine the applied K rate that maximized NPV when soil K levels were considered and were not considered in the applied K rate decision. They found that using the soil K level information annually would increase NPV relative to not considering the soil K level information over a five year horizon.

Additionally, using soil K carryover knowledge reduced the amount of annual K that was applied and the soil K carryover levels, which may be helpful for reducing off-site K leaching.

An important factor many producers consider when using soil nutrient information to make application decisions is the length of time the producer waits until they update soil testing information (Lambert et al., 2014). The existing dynamic programming literature assumes producers annually soil test to update their soil nutrient information (Ackello-Ogutu, Paris, and Williams, 1985; Jomini et al., 2001; Kennedy et al., 1973; Lambert, Lowenberg-DeBoer, and Malzer, 2007; Lanzer and Paris, 1981; Park et al., 2007; Schnitkey, Hopkins, and Tweeten, 1996; Segarra et al., 1989; Watkins, Lu, and Huang, 1998). However, gathering information about soil nutrient variability on an annual basis might not increase the NPV (e.g., by reducing fertilizer costs) enough to pay for the cost of soil nutrient information, especially for K since it is immobile in the soil profile (Walworth, 2011). Currently, state extension recommendations encourage cotton producers to update soil K information from annually to every three years in the southeast (Kissel and Sonon, 2011; Mylavarapu, 1997; Savoy and Joines, 2013). These recommendations vary across states because production factors such as cropping intensity, soil type, tillage practices, and weather conditions play an important role in determining the length of time to wait until retesting soil nutrient levels. Nonetheless, the common assumption in the literature of soil testing annually may not be appropriate for all nutrients and crops to maximize NPV.

Lambert et al. (2014) used survey data of cotton producers in 13 southern states to determine the factors affecting the length of time between updating soil test information (temporal frequency) for precision soil sampling. They found that farm size, land ownership, farm location, and farming experience were correlated with the temporal frequency producers tested soils. Cotton producers who adopted precision soil sampling indicated they retested soils on average every two and half years, which is within the range encouraged by southeast extension agronomists. From the perspective of profit-maximizing producers this research indicates that annual soil testing might not be a profit-maximizing frequency.

The objective of this research was to determine the K fertilizer application and temporal frequency for obtaining K soil test information that maximizes NPV to K fertilizer in cotton production in the southeastern US. Optimal K rates and NPV were determined *ex ante* for five soil testing schedules of varying temporal frequencies using Kennedy's (1986) dynamic programming framework. The conceptual modeling of this study extends the literature by considering the temporal frequency of obtaining soil test information in a dynamic programming framework to determine optimal K rates and expected returns for cotton production. The results can guide producers and extension on optimal K rates and length of time between soil tests for upland cotton growers.

Empirical Framework

Dynamic programming model

A risk-neutral, profit-maximizing cotton producer chooses an amount of K fertilizer to apply (A_t) at the beginning of each production year t (t =1,…,T), conditioned on some knowledge of soil K carryover (C_t), that maximizes the NPV of returns to K over a planning horizon (Kennedy, 1986; Kennedy et al., 1973). This producer also selects the optimal temporal frequency j of soil testing by choosing some discrete number of years between retesting soils. This study used five soil testing schedules of varying temporal frequency from annually to every five years (j = 1,…,5), where j is the number of years between soil testing.

For each soil testing schedule, the producer used information from the most recent soil test to update their knowledge of carryover and apply a profit-maximizing amount of K. The optimal temporal frequency was the soil testing schedule that provides the greatest NPV over a 10-year planning period. Therefore, the maximized NPV is calculated as:

$$
(1) \qquad \begin{aligned}
&\max_{A_{t,j},\ldots,A_{T,j}} NPV_j = \sum_{t=1}^{T} \delta^{t-1} NR_{t,j} \\
&\text{Subject to:} \\
&A_{t,j}, C_{t,j} \geq 0 \\
&C_{t+1,j} = a_0 + a_1(A_{t,j} + C_{t,j}) \\
&C_{t,j} = \lambda_j Q_{t,j} + (1-\lambda_j) Q_{t-1,j} \\
&Q_0 \text{ given,}
\end{aligned}
$$

where NPV_j is the sum of discounted net returns (\$ acre^{-1}) over T years for a producer following soil testing schedule j; $A_{t,j}$ is applied K fertilizer (lb acre^{-1}); $NR_{t,j}$ is the net returns (\$ acre^{-1}) to K fertilizer for cotton production; δ is a discount factor reflecting the time value of money $(1+r)^{-t}$, where r is the discount rate; $C_{t,j}$ is the producer's knowledge of carryover K (lb acre^{-1}) in time period t; $Q_{t,j}$ is the actual carryover K (lb acre^{-1}) in time period t; $C_{t+1,j}$ is the producer's knowledge of carryover K (lb acre^{-1}) prior to planting in year t+1, which is a function of applied K fertilizer and the producers knowledge of soil K carryover (i.e., total K available (lb acre^{-1})) in year t; λ_j is an indicator variable that is equal to one in the year a producer updates their knowledge of soil K by soil testing and zero otherwise; a_0 and a_1 are estimated parameters for the linear carryover function; and Q_0 is the actual soil K level before fertilizer K is applied in the first production period. Partial budgeting was used to calculate the single period net returns for a risk-neutral profit-maximizing producer, where single period net returns (NR) are:

$$
(2) \qquad NR_{t,j} = \delta p_t^c y_{t,j}(A_{t,j} + C_{t,j}) - p_t^K A_{t,j} - \lambda_j s,
$$

where p_t^c and p_t^K are the prices of cotton lint (\$ acre^{-1}) and K fertilizer (\$ acre^{-1}) respectively; $y_{t,j}$ is cotton lint yield (lb acre^{-1}) in period t; and s is the cost of soil test information (\$ acre^{-1}), which only occurs in years when producers soil test ($\lambda_j = 1$), otherwise is zero.

We assume the producer conducts a soil test prior to production in the first year. Therefore, the producer knows the soil K level at the beginning of production year one and uses this information in selecting an optimal application rate in year one ($A_{1,j}$). After the first production year, the producer chooses to update their knowledge of soil K at the beginning of each period, following five soil test schedules j defined as:

(3) $$j = 1, \quad \lambda = \begin{cases} 1 & if \;\; t = 1,\ldots,10 \\ 0 & otherwise \end{cases},$$

(4) $$j = 2, \quad \lambda = \begin{cases} 1 & if \;\; t = 1,3,5,7,9 \\ 0 & otherwise \end{cases},$$

(5) $$j = 3, \quad \lambda = \begin{cases} 1 & if \;\; t = 1,4,7,10 \\ 0 & otherwise \end{cases},$$

(6) $$j = 4, \quad \lambda = \begin{cases} 1 & if \;\; t = 1,5,9 \\ 0 & otherwise \end{cases},$$

and

(7) $$j = 5, \quad \lambda = \begin{cases} 1 & if \;\; t = 1,6 \\ 0 & otherwise \end{cases}.$$

When a producer did not soil test in a given year, the producer's knowledge of carryover K ($C_{t,j}$) was assumed to be the actual carryover K obtained from the most recent soil test $Q_{t\text{-}1,j}$. However, when a producer did soil test, the producer's knowledge of carryover K ($C_{t,j}$) was still assumed to be the actual carryover K obtained from the most recent soil test $Q_{t,j}$. Table 7 shows how the producer's knowledge of K carryover is updated with actual soil test information by temporal frequency.

When maximizing NPV, the economic optimality principle of marginal value product (MVP) equals marginal factor cost (MFC) is complicated by inter-temporal factors such as the time value of money (opportunity cost) and fertilizer carryover (Kennedy et al., 1973; Kennedy, 1986). Therefore, a dynamic optimization technique was required to determine optimal total K levels in each period when using soil test information. This study extends Kennedy's (1986) dynamic programing framework to determine optimal total available K levels in each period:

$$
\begin{aligned}
&V_{t,j}\{C_{t,j}\} = \max_{A_{t,j}}[NR_{t,j} + \delta V_{t+1,j}\{C_{t+1,j}\}] \\
&\text{Subject to:} \\
&A_{t,j}, C_{t,j} \geq 0 \\
&C_{t+1,j} = a_0 + a_1(A_{t,j} + C_{t,j}) \\
&C_{t,j} = \lambda_j Q_{t,j} + (1-\lambda_j)Q_{t-1,j} \\
&V_{T+1,j}\{C_{T+1,j}\} = 0 \\
&Q_0 = \text{ given,}
\end{aligned} \tag{8}
$$

where $V_{t,j}\{C_{t,j}\}$ is the present value of net returns (\$ acre^{-1}) from applying the profit-maximizing K application in year t; and $V_{T+1,j}\ \{C_{T+1,j}\} = 0$ is the terminal condition stating the producer does not receive any value from the available K remaining in the soil after the last period of the planning horizon since the producer will not be able to utilize available soil K levels remaining after the last period of their planning horizon (Chiang, 1992).

The optimal single period applied K level was determined using Bellman's (1957) recursive equation. The optimality conditions were solved by differentiating equation (8) with respect to the decision variable $A_{t,j}$:

$$
\frac{\partial V_{t,j}}{\partial A_{t,j}} = \delta p_t^c \frac{\partial y_{t,j}}{\partial A_{t,j}} - p_t^K + \delta \frac{dV_{t+1,j}}{dC_{t+1,j}} a_1 = 0 \ , \tag{9}
$$

which can be rearranged:

(10) $$\delta p_t^c \frac{\partial y_{t,j}}{\partial A_{t,j}} = p_t^K - \delta \frac{dV_{t+1,j}}{dC_{t+1,j}} a_1 .$$

Using the envelope theorem (Léonard and Van Long, 1992), differentiating equation (8) with respect to the state variable $C_{t,j}$ (K carryover) gives:

(11) $$\frac{\partial V_{t,j}}{\partial C_{t,j}} = \delta p_t^c \frac{\partial y_{t,j}}{\partial A_{t,j}} + \delta \frac{dV_{t+1,j}}{dC_{t+1,j}} a_1 .$$

Substituting equation (10) into equation (11) and simplifying gives:

(12) $$\frac{\partial V_{t,j}}{\partial C_{t,j}} = p_t^K ,$$

which indicates soil carryover at the beginning of year t is valued at the price of K in year t. This result can be updated to year t + 1:

(13) $$\frac{\partial V_{t+1,j}}{\partial C_{t+1,j}} = p_{t+1}^K ,$$

which can be substituted into equation (10) to obtain the optimality condition for single period K application rate when using soil test information:

(14) $$\delta p_t^c \frac{\partial y_{t,j}}{\partial A_{t,j}} = p_t^K - \delta p_{t+1}^K a_1 .$$

Equation (14) indicates the current period optimal total available K level occurs where the MVP (left hand side of equation (14)) was equal to the MFC less the discounted savings associated with K fertilizer carried over to the next year (right hand side of equation (14)).

K carryover function

Soil K carryover was estimated as a linear function of total K available (i.e., applied K and actual carryover K), which is a commonly used functional form (Harper et al., 2012; Jomini et al., 1991; Lanzer and Paris, 1981; Segarra et al., 1989). We estimated parameters for the carryover function using the actual measured total K available. However, depending on the producer's temporal frequency of soil testing, the producer's knowledge of carryover K in the dynamic programming model was updated in each time period with the actual soil K level or the soil K level from the previous soil test. We included a year random effect in the intercept:

(15) $$C_{t+1,i} = a_0 + a_1(A_{t,i} + Q_{t,i}) + \tau_t + u_{t,i},$$

where $\tau_t \sim N(0, \sigma_\tau^2)$ is a random effect capturing the variation in carryover levels across years, and $u_{t,i} \sim N(0, u_\tau^2)$ is a random error term for plot i. The two error terms were assumed to be independent. The intercept, a_0, represents some constant amount of available K that remains in the soil over the planning horizon, the slope, a_1, is the proportion of total K from the current year readily available to the next crop estimated using observed carryover K. Soil K from previous applications accumulates into current-period soil K levels; thus, the only relevant soil K carryover level is for the current period for each of the schedules. Maximum likelihood parameter estimates for equation (15) were obtained using the MIXED procedure in SAS 9.3 (SAS Institute, Inc., 2011).

Yield response function

The selection of a functional form to characterize cotton lint yield response to K is important for determining application rates that maximize NPV (Ackello-Ogutu, Paris, and Williams, 1985). Plateau-type response functions, such as the linear response plateau or quadratic-plus-plateau, have been suggested to be more appropriate for characterizing yield response to fertilizer than polynomial or other nonlinear specifications (Ackello-Ogutu, Paris, and Williams, 1985; Bullock and Bullock, 1994; Cerrato and Blackmer, 1990). Plateau-type functional forms assume yield responds to an input in a linear or polynomial manner until it reaches a plateau, beyond which the input no longer limits yield. Tembo et al. (2008) extended the linear response plateau by including a normally-distributed random effect in the plateau to capture variation in the plateau from stochastic events such as insects, weather, and disease. The linear response stochastic plateau (LRSP) developed by Tembo et al. (2008) has been found to be more appropriate than similar deterministic functional forms to model yield response to nutrient applications for several crops and provide more accurate economically optimal nutrient rates (Boyer et al., 2012; Biermacher et al., 2009; Tumusiime et al., 2011). Harmon et al. (2016) found the LRSP to be more appropriate than a deterministic plateau function for this data. Therefore, cotton lint yield response to K applied and actual measured soil K was estimated using the LRSP function:

$$y_{t,i} = \min(\beta_0 + \beta_1(A_{t,i} + Q_{t,i}), \mu + v_t) + w_t + \varepsilon_{t,i}, \tag{16}$$

where β_0 and β_1 are the yield response parameters estimated using observed yields; μ is the expected plateau yield parameter (lb acre^{-1}); $v_t \sim N(0, \sigma_v^2)$ is a normally distributed plateau random effect; $w_t \sim N(0, \sigma_w^2)$ is the intercept year random effect; and $\varepsilon_{t,i} \sim N(0, \sigma_\varepsilon^2)$ is the random error term. Independence is assumed across the three random effects. Maximum likelihood parameter estimates for equation (16) were obtained using the NLMIXED procedure in SAS 9.3 (SAS Institute, Inc., 2011).

To solve for the optimal applied K rate, the optimality condition (equation 14) was updated with the first order condition for the LRSP yield response function with respect to applied K, defined as

(17) $$\delta p_t^c[\beta_1(1-\Phi)] = p_t^K - \delta p_{t+1}^K a_1,$$

where $\Phi = \Phi[(\beta_0 + \beta_1(A_{t,j} + C_{t,j}) - \mu)/\sigma_v]$ is the cumulative normal distribution function (Tembo et al., 2008). Equation (17) is rearranged to obtain the optimal K application rate for period *t*:

(18) $$A_{t,j}^* = \frac{\left[\Phi^{-1}\left(1 - \frac{p_t^K - \delta p_{t+1}^K a_1}{\delta p_t^c \beta_1}\right)\right]\sigma_v + \mu - \beta_0}{\beta_1} - C_{t,j},$$

The optimal application decision depends on the ratio of the per-unit K cost and cotton price and variation in the plateau (Tembo et al., 2008). Additionally, the optimal application rate will depend on the temporal frequency of soil testing and the accuracy of the producer's knowledge of their soil K levels. For the producer who soil tests in every year *j* = 1, the producer's knowledge of carryover K is equal to the actual K carryover in each year. However, for a producer who soil tests every other year *j* = 2, the producers knowledge of carryover K is equal to the actual soil K levels in years when a soil test occurs, but the producer's knowledge of carryover K may be higher or lower than the actual K carryover level in periods when the producer does not update soil testing information. Therefore, the producer who updates soil K levels less frequently may over- or under-apply K depending on the variability of soil K carryover levels between years. Given that yield responds to the amount of applied and carryover K, the producer's decision of which soil testing schedule to follow will affect yield and the subsequent returns achieved in each period. The NPV for the different soil testing schedules indicates the how often a producer needs to update their information of soil K to maximize NPV.

Monte Carlo simulation

A Monte Carlo simulation was used to introduce uncertainty into the dynamic programming model. One-thousand iterations of a 10-year planning period were simulated to generate output distributions of NPV for each of the five scenarios. The prices of K and cotton lint yield, as well as the yield response and carryover coefficients, were assumed to be stochastic, providing an *ex ante* analysis of the NPV of returns to K fertilizer for each scenario. Figure 2 summarizes the general process used to solve the dynamic programming model. Shaded boxes correspond with stochastic parameters in the model.

Price uncertainty was introduced into the model by bootstrapping the observed real prices of cotton lint and K for each period of the 10-year planning horizon. To introduce uncertainty in the expected yield response, the yield response coefficients were simulated as multivariate normal (MVN) random variables:

$$(19)\qquad \begin{bmatrix} \beta_0^* \\ \beta_1^* \\ \mu^* \\ \sigma_v^{2\,*} \end{bmatrix} \sim MVN\left(\begin{bmatrix} \beta_0 \\ \beta_1 \\ \mu \\ \sigma_v^2 \end{bmatrix}, \begin{bmatrix} \sigma_{\beta_0}^2 & \rho_{\beta_0,\beta_1}\sigma_{\beta_0}\sigma_{\beta_1} & \rho_{\beta_0,\mu}\sigma_{\beta_0}\sigma_{\mu} & \rho_{\beta_0,\sigma_v^2}\sigma_{\beta_0}\sigma_{\sigma_v^2} \\ \rho_{\beta_1,\beta_0}\sigma_{\beta_1}\sigma_{\beta_0} & \sigma_{\beta_1}^2 & \rho_{\beta_1,\mu}\sigma_{\beta_1}\sigma_{\mu} & \rho_{\beta_1,\sigma_v^2}\sigma_{\beta_1}\sigma_{\sigma_v^2} \\ \rho_{\mu,\beta_0}\sigma_{\mu}\sigma_{\beta_0} & \rho_{\mu,\beta_1}\sigma_{\mu}\sigma_{\beta_1} & \sigma_{\mu}^2 & \rho_{\mu,\sigma_v^2}\sigma_{\mu}\sigma_{\sigma_v^2} \\ \rho_{\sigma_v^2,\beta_0}\sigma_{\sigma_v^2}\sigma_{\beta_0} & \rho_{\sigma_v^2,\beta_1}\sigma_{\sigma_v^2}\sigma_{\beta_1} & \rho_{\sigma_v^2,\mu}\sigma_{\sigma_v^2}\sigma_{\mu} & \sigma_{\sigma_v^2}^2 \end{bmatrix} \right),$$

where the mean of the distribution was a vector of the estimated coefficients for the yield response function (equation 16); the variance of the distribution was a four-by-four matrix of the robust covariance estimator of the parameter estimates, where ρ is the correlation coefficient; and an asterisk ("*") denotes a randomly drawn coefficient for the simulation (Cuvaca et al., 2015). The pre-planting carryover levels after the initial year were estimated by the linear carryover function (equation 15), where the carryover coefficients followed a MVN distribution:

$$\begin{bmatrix} a_0^* \\ a_1^* \end{bmatrix} \sim MVN\left(\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}, \begin{bmatrix} \sigma_{a_0}^2 & \rho_{a_0,a_1}\sigma_{a_0}\sigma_{a_1} \\ \rho_{a_1,a_0}\sigma_{a_1}\sigma_{a_0} & \sigma_{a_1}^2 \end{bmatrix}\right). \tag{20}$$

For each iteration, new coefficients and prices were randomly sampled to determine, *ex ante*, the total available K, yield, and NPV.

Uncertainty surrounding the initial carryover level (Q_0) was introduced into the model by bootstrapping the observed carryover levels. In year one, prices of K and cotton were randomly drawn along with parameter estimates for the carryover and yield response function. The producer was assumed to soil test in year one, so the pre-planting carryover level was determined using the initial carryover level. The yield and carryover parameter estimates, prices of K and cotton, and the estimated pre-planting K carryover level were substituted into equation (18) to obtain the optimal application rate in year one. Subsequent yield and single period net returns were calculated at the optimal application rate and the carryover level. In time period two, prices of K and cotton were randomly drawn along with parameter estimates for the carryover and yield response function. After the first period of production, the producer's decision to soil test in each period is determined by the soil testing schedule they follow. If the producer tested soil K levels, then the optimal K rate was determined using the actual carryover K rate in time period two. If a producer did not soil test, then the producer's knowledge of carryover K was used to determine the optimal K application rate. The subsequent yield and NPV were determined using the application rate and actual carryover levels, and the soil testing decision process was repeated for the remaining years of the planning period for each soil testing schedule. Therefore, after the first year of production, K applications, yield, and NPV for years $t = 2,\ldots,10$ were influenced by the temporal frequency with which producers updated soil test information.

For each scenario, output distributions were generated for the annual and 10-year average applied K, carryover K, lint yield, and NPV for each scenario. The Monte Carlo simulation was conducted using @Risk (Palisade, 2015). The expected NPVs of each scenario were compared to determine the soil testing temporal frequency that provided the greatest NPV of returns to K fertilizer.

Data

Data on cotton yield response and soil K fertility levels were collected from a nine-year field study (2000 to 2008) conducted at the University of Tennessee, West Tennessee Research and Education Center at Jackson (35.63°N; 88.85°W). The soil type was Loring-Calloway silt loam (thermic Oxyaquic Fragiudal and thermic Typic Fragiaqualf). The plots were not tilled. Each year, K fertilizer (muriate of potash, 0-0-60) was broadcast by hand to individual plots prior to planting at rates of 0, 25, 50, 75, 100, 125, and 149 lb acre^{-1} of elemental K. These treatments were applied to the same plots each year, beginning five years prior to the start of the study (2000) through the last year (2008). Plots were arranged in a randomized complete block design, with five or six replications of the fertilizer treatments.

Cotton was planted between April 30 and May 15 of each year using a 4-row John Deere MaxEmerge planter. From 2000 to 2002, the cultivar ‘PM1218BG/RR’ was planted on all plots. From 2003 to 2008, two contrasting cultivars were planted in a factorial arrangement relative to the K-fertility plots. The cultivars ‘PM1218BG/RR’ and DP555BG/RR’ were planted from 2003 to 2005, the cultivars ‘FM960BR’ and DP555BG/RR’ were planted from 2006 to 2007, and the cultivars ‘ST455B2RF’ and ‘ST5327B2RF’ were planted in 2008. Plots were 30 by 12 ft, each containing four rows spaced 38 inches apart. Shortly before or after planting each year, nitrogen fertilizer (ammonium nitrate, 34-0-0) was uniformly drop-spread to all plots at a rate of 80 lb acre $^{-1}$. University of Tennessee Extension Service recommendations (Savoy and Joines, 2001) were used to apply ground limestone and phosphorus fertilizer. Thus, all other fertilizer inputs were non-yield limiting. Supplemental irrigation was used during dry spells in all years except 2002 and 2003. Monthly growing season rainfall for Jackson, TN is summarized in Table 8 (National Oceanic and Atmospheric Administration, 2014). All other production practices followed the Tennessee Agricultural Extension Service (2001) guidelines for cotton production.

Seedcotton was harvested from the two interior rows of each plot twice each year using a modified John Deere 9930 spindle picker. First harvest occurred from September 7 to October 8, with a second harvest occurring 14 to 28 days later. Lint yields were calculated using seedcotton weights, gin turnouts, and plot areas harvested. Yield response functions were estimated using observed lint yields from 2000 to 2008. Average annual lint yields by K rate are displayed in Table 9. Yields may have increased over time due to improved biotechnology from different cultivars. Therefore, cotton lint yields were tested with a deterministic quadratic time response function (Just and Weninger, 1999). Similar to cotton yields in Oklahoma (Boyer, Brorsen, and Tumusiime, 2015), a time trend was not present.

Within six weeks after harvest in each year, soil samples were collected from all plots at the 0-6 inch depth using the Mehlich I extraction method (Howard et al., 2001). The samples were tested at The University of Tennessee Soil and Forage Test Laboratory in Nashville, Tennessee. Pre-planting soil test levels from 2001 to 2009 were used to estimate the carryover function. The average soil test level for the experiment was characterized by the medium soil fertility range (Savoy and Joines, 2001) (Table 9). Soil test levels were corrected for heteroscedasticity across years.

Average annual cotton lint and elemental K prices ($ lb^{-1}) from 1994 to 2013 were used to determine the optimal K fertilization rates that maximized NPV over a 10-year planning horizon. The Federal Reserve implicit price deflator (U.S. Bureau of Economic Analysis, 2015), was used to adjust nominal prices to reflect real prices in 2013 using. Real cotton prices varied from $0.38 to $1.07 lb^{-1}, and real elemental K prices varied from $0.20 to $0.91 lb^{-1} (U.S. Department of Agriculture Economic Research Service, 2013; 2014). Real cotton and K prices were not correlated over time. The real cost of soil testing included the cost of obtaining the soil sample and the chemical analysis. The cost of obtaining the soil sample was $7.27 $acre^{-1}$ $year^{-1}$, which was based on University of Tennessee Custom Rate Survey (University of Tennessee Agricultural and Resource Economics Department, 2013). The cost of the chemical analysis was $0.70 $acre^{-1}$ yr^{-1}, which assumes a producer soil tests on a 10 acre grid, following University of Tennessee recommendations for soil testing (Savoy and Joines, 2013). A 5% discount rate was used to represent the opportunity cost of land in cotton production, similar to previous dynamic programming literature (Harper et al., 2012; Kennedy et al., 1973; Park et al., 2007; Segarra et al., 1989; Watkins, Lu, and Huang, 1998).

Results

Yield response and carryover

The parameter estimates for the yield response and carryover functions are presented in Table 10. The intercept of the LRSP function was insignificant and negative, indicating lint yield would be negative when total available K was zero. However, total available K was always observed to be greater than zero (Table 9), thus, a negative yield would be outside the range of the data. Similarly, Watkins, Lu, and Huang (1998) and Stauber, Burt, and Linse (1975) found a negative estimated intercepts in their yield response to nitrogen when carryover was considered. The remaining parameter estimates had the expected positive signs and were significant at the 1% level.

The estimated K carryover function had positive estimates for the intercept and slope ($p \leq 0.01$). The intercept indicated that 25.46 lb K acre^{-1} of soil K did not come from the amount of total K in the previous year, but remains available to the plant over the planning period (Lanzer and Paris, 1981). The estimated slope indicated that 73% of the total K available in the current period (t) will be carried over to the next period (t + 1). The carryover coefficient was similar to Harper et al.'s (2012) estimated K carryover coefficient of 0.72.

Simulation

Monte Carlo simulation results for the annual and 10-year average K application, K carryover, and yield are presented in Table 11. The 10-year average profit-maximizing K application rates for all temporal frequency of soil testing varied from 29 to 31 lb acre^{-1} yr^{-1}. However, the range of optimal K application rate across the years varied the by the temporal frequency of soil testing. The optimal K application in years when a producer did not update their soil K information with a soil test was lower than the optimal K application rate when a producer did update their information of soil K carryover. These lower applications were offset by applying a higher rate in years when soil testing information was updated to correct for K deficiencies in soil carryover levels. For example, in year two, the producer that soil tested every year applied a higher K rate to maximize NPV than the producer that soil tested every other year. However, the optimal K application rate for producer that soil tested every other year was higher in year three than the optimal K rate for the producer that soil tested every year, which was necessary to rebuild total available K levels to maximize NPV. The longer the producer waited to update their information on soil K carryover with a soil test the greater the range of optimal K application rate increased.

The 10-year average optimal K carryover levels were 204 lb acre $^{-1}$ yr^{-1} when a producer updated soil testing information annually. However, when a producer soil tested every other year and every third year soil carryover levels were reduced by 2 lb acre $^{-1}$ yr^{-1}, while carryover levels were 3 and 4 lb acre $^{-1}$ yr^{-1} lower than annual soil testing when a producer waited four and five years to update soil testing information respectively. Similarly to optimal K application rate, the soil K carryover levels across temporal frequency of soil testing. As temporal frequency of soil testing decreased, the lower bound of the range of K carryover level decreased. For instance, when a producer waited five years to soil test, the carryover K level dropped to 159 lb acre $^{-1}$, which required the producer to apply a higher rate of K to increase the soil K levels in the year they soil tested. This result shows that when a producer does not know or use soil K carryover information in managing K applications, the optimal strategy was to draw down soil K levels. This finding also resembles what southeastern cotton producers experienced in the late 1980s and early 1990s with K deficiencies (Maples et al., 1988; Mullins, Burmester, and Reeves, 1997), which reiterates the importance of soil testing for K levels in cotton production in the southeastern U.S.

The 10-year average K carryover levels reported in this study would be classified in the medium soil fertility range for each soil testing schedule according to the University of Tennessee Extension Service guidelines (Savoy and Joines, 2001). However, the optimal applied K rate recommended by the University of Tennessee Extension Service for medium testing soils are higher than what we find as optimal in this study. Moreover, the optimal total K levels for all temporal frequency of soil testing were lower than what Harper et al. (2012) reported for cotton production in Tennessee. We use data from a longer time-series and different soil type as well as a different yield response function, which might explain the differences in results. Our results suggest that recommended K application rates in cotton production based on soil test levels could be decreased.

Annual soil testing produced the highest 10-year average yield of 1384 lb acre $^{-1}$ yr^{-1} and the 10-year average lint yields decreased as the producer's temporal frequency of soil testing decreased. By waiting until every fifth year to soil test, the producer decreased their yield by 32 lb acre $^{-1}$ yr^{-1} relative to the producer that soil tests annually. The longer a producer waited to update soil testing information the lower the annual yields decreased, which might be attributed to the deficient soil carryover levels limiting yield.

Optimal temporal frequency

The expected NPV increased as temporal frequency increased from soil testing every fifth year (\$7,435 acre^{-1}) to soil testing every other year (\$7,579 acre^{-1}) (Figure 3). This indicates that the additional information on soil K carryover had a greater value than the cost of soil testing. The expected NPV increased \$82 acre^{-1} when a producer went from soil testing every five years to every four years, \$48 acre^{-1} when a producer went from soil testing every four years to every three years, and \$14 acre^{-1} when a producer went from soil testing every three years to every other year (Figure 3). Thus, the findings further indicating that the value of soil testing increased at a decreasing rate when the temporal frequency of soil testing increased. Annual soil testing provided the producer with the most accurate knowledge of soil K carryover variability over time. However, we report findings that show the additional value from having the most accurate information on soil K carryover was less than the cost of soil testing to gather that information. When a producer soil tested annually, the expected NPV decreased by \$12 acre^{-1} from soil testing every other year (Figure 3).

Pairwise comparisons were made between the distributions of simulated NPVs for all temporal frequencies of soil testing. We found that the NPV of soil testing every other year was statistically different than the NPVs from soil testing every fourth (p-value = 0.04) and fifth year (p-value = <0.0001) at 0.05 level. However, the NPV of soil testing every other year was not statistically different than the NPVs from soil testing annual and every third year. While producers who use soil testing information to manage K fertilizer in cotton production can maximize their expected NPV by updating soil testing information every other year, the optimum soil testing temporal frequency maybe found soil testing annual to every three years. The conclusion about optimal temporal frequency of soil testing was supported by what Lambert et al. (2014) found in their survey of cotton producers in the southern U.S.

Conclusion

The objective of this study was to determine the temporal frequency of soil testing for K that maximized NPV of returns to K in cotton production. Cotton lint yield and soil testing data were obtained from a nine-year experiment in Jackson, Tennessee. Cotton lint yield was characterized by a LRSP yield response function, and soil testing information was characterized by a linear carryover function. The producer’s objective was to choose the temporal frequency of soil testing and the subsequent annual K application rates that maximized their NPV of returns to K over a 10-year planning period. Kennedy’s (1986) dynamic programming framework was applied and a Monte Carlo simulation was used to introduce uncertainty into the model.

Previous studies that investigate optimal nutrient application to maximize NPV assume producers soil test on an annual basis; however, this assumption does not appear to follow producer practices (Lambert et al., 2014) or extension recommendation in the southeast (Kissel and Sonon, 2011; Mylavarapu, 1997; Savoy and Joines, 2013). This study expands previous literature by incorporating the temporal frequency of soil testing into a dynamic programming model and results will assist producers make better informed decisions regarding their use of soil testing as a tool to manage K fertilizer in cotton production.

On average over the 10-year production horizon, the profit-maximizing K application rates for all temporal frequency of soil testing varied slightly from 29 to 31 lb acre^{-1} yr^{-1}. The longer the producer waited to update their information on soil K carryover with a soil test, the greater the range of optimal K application rate increased. As producers decreased their temporal frequency of soil testing, the lower bounds of the carryover levels and yields decreased due to yield limiting levels of K. The expected NPV increased at a decreasing rate as temporal frequency increased from soil testing every fifth year to soil testing every other year, indicating that the value of the additional information from soil testing was greater than the cost of soil testing. However, the expected NPV decreased when a producer increased their temporal frequency from every other year to annually, thus, the additional value from having the most accurate information on soil K carryover was less than the cost of soil testing to gather that information.

References

Ackello-Ogutu, C., Q. Paris, and W.A. Williams. “Testing a Von Liebig Crop Response Function against Polynomial Specifications.” *American Journal of Agricultural Economics* 67(1985):873-880.

Bellman, R.E. *Dynamic Programming*. New Jersey: Princeton University Press, 1957.

Biermacher, J.T., B.W. Brorsen, F.M. Epplin, J.B. Solie, and W.R. Raun. “The Economic Potential of Precision Nitrogen Application with Wheat Based on Plant Sensing.” *Agricultural Economics* 40(2009):397-407.

Boyer, C.N., B.W. Brorsen, and E. Tumusiime. “Modeling Skewness with the Linear Stochastic Plateau Model to Determine Optimal Nitrogen Rates.” *Agricultural Economics* 46(2015):1-10.

Boyer, C.N., D.D. Tyler, R.K. Roberts, B.C. English, and J.A. Larson. “Switchgrass Yield Response Functions and Profit-Maximizing Nitrogen Rates on Four Landscapes in Tennessee.” *Agronomy Journal* 104(2012):1579-1588.

Bullock D.G., and D.S. Bullock. “Quadratic and Quadratic-Plus-Plateau Models for Predicting Optimal Nitrogen Rate of Corn: A Comparison.” *Agronomy Journal* 86(1994):191-195.

Cerrato, M.E., and A.M. Blackmer. “Comparison of Models for Describing Corn Yield Response to Nitrogen Fertilizer.” *Agronomy Journal* 82(1990):138-143.

Chiang, A.C. *Elements of Dynamic Optimization*. New York: McGraw Hill, 1992.

Cuvaca, I.B., D.M. Lambert, F.R. Walker, M. Marake, and N.S. Eash. “Economically Optimal N Fertilizer Rates for Maize Produced on Vertisol and Inceptisol Soils Under No-Till Management: A Case Study in Maphutseng Lesotho.” *International Journal of Plant and Soil Science* 8(2015):1-12.

Essington, M.E., D.D. Howard, H.J. Savoy, and G.M. Lessman. “Potassium Fertilization of Cotton Produced on Loess-Derived Soils.” *Better Crops* 86(2002):13-15.

Fuller, W.A. “Stochastic Fertilizer Production Functions for Continuous Corn.” *American Journal of Agricultural Economics* 47(1965):105-119.

Harper, D.C., D.M. Lambert, J.A. Larson, and C.O. Gwathmey. “Potassium Carryover Dynamics and Optimal Application Policies in Cotton Production.” *Agricultural Systems* 106(2012):84-93.

Heady, E.O., and J.L. Dillon. *Agricultural Production Functions.* Iowa: Iowa State University Press, 1961.

Howard, D.D., M.E. Essington, R.M. Hayes, and W.M. Percell. “Potassium Fertilization of Conventional- and No-Till Cotton.” *Journal of Cotton Science* 5(2001):197-205.

Howard, D.D., C.O. Gwathmey, R.K. Roberts, and G.M. Lessman. “Potassium Fertilization of Cotton Produced on a Low K Soil with Contrasting Tillage Systems.” *Journal of Production Agriculture* 11(1998):74-79.

Jomini, P.A., R.D. Deuson, J. Lowenberg-DeBoer, and A. Bationo. “Modeling Stochastic Crop Response to Fertilisation when Carry-over Matters.” *Agricultural Economics* 6(1991):97-113.

Just, R.E., and Q. Weninger. “Are Crop Yields Normally Distributed?” *American Journal of Agricultural Economics* 81(1999):287-304.

Kennedy, J.O.S. *Dynamic Programming: Applications to Agriculture and Natural Resources.* London: Elsevier Applied Science, 1986.

Kennedy, J.O.S, I.F. Whan, R. Jackson, and J.L. Dillon. “Optimal Fertilizer Carryover and Crop Recycling Policies for a Tropical Grain Crop.” *American Journal of Agricultural Economics* 17(1973):104-113.

Kissel, D.E., and L. Sonon. “Soil Test Handbook for Georgia.” Athens, Georgia: The University of Georgia Cooperative Extension, 2011

Lambert, D.M., B.C. English, D.C. Harper, S.L. Larkin, J.A. Larson, D.F. Mooney, R.K. Roberts, M.Velandia, and J.M. Reeves. “Adoption and Frequency of Precision Soil Testing in Cotton Production.” *Journal of Agricultural and Resource Economics* 39(2014):106-123.

Lambert, D.M., J. Lowenberg-DeBoer, and G. Malzer. “Managing Phosphorus Soil Dynamics over Space and Time.” *Agricultural Economics* 37(2007):43-53.

Lanzer, E.A., and Q. Paris. “A New Analytical Framework for the Fertilization Problem.” *American Journal of Agricultural Economics* 63(1981):93-103.

Léonard D., and N. Van Long. *Optimal Control Theory and Static Optimization in Economics.* , Cambridge: Cambridge University Press. 1992.

Maples, R.L., W.R. Thompson, Jr., and J. Varvil. “Potassium Deficiency in Cotton Takes on a New Look.” *Better Crops* 73(1988):6–9.

Mehlich, A. “Determination of P, Ca, Mg, K, Na and NH_4.” Raleigh, NC: North Carolina Soil Testing Division Memo. 1953.

Mullins, G.L., C.H. Burmester, and D.W. Reeves. “Cotton Response to In-Row Subsoiling and Potassium Fertilizer Placement in Alabama.” *Soil and Tillage Research* 40(1997):145-154.

Mullins, G.L., G.J. Schwab, and C.H. Burmester. "Cotton Response to Surface Applications of Potassium Fertilizer: A 10-year Summary." *Journal of Production Agriculture* 12(1999):434-440.

Mylavarapu, R.S. "Soil Sampling for Precision Farming." Clemson, SC: Clemson University Cooperative Extension Service, Publication No. IL 63. 1997.

National Oceanic and Atmospheric Administration National Climatic Data Center. *Climate Data Online Search.* Internet site: http://www.ncdc.noaa.gov/cdo-web/search. (Accessed January 11, 2016).

Palisade Corporation. 2014. @ Risk 6. Ithica, NY.

Park, S.C., A. Stocker, J.A., Hattey, and J.C. Turner. "Long-Term Profitability of Animal Manure Using Optimal Nitrogen Application Rate." Paper presented at the Southern Agricultural Economics Association Annual Meeting, Mobile, Alabama. February 4-7, 2007.

SAS Institute, Inc. *SAS/STAT 9.3 User's Guide.* 2011. Internet Site:http://support.sas.com/documentation/cdl/en/statug/63962/HTML/default/viewer.htm#titlepage.htm. (Accessed January 11, 2016).

Savoy, H.J., Jr., and D. Joines. "Soil Testing." Knoxville, TN: University of Tennessee Agricultural Extension Service, Publication No. PB1061. 2013.

Savoy, H.J., Jr., and D. Joines. "Lime and Fertilizer Recommendations for the Various Crops of Tennessee." *Chapter 2 Agronomic Crops.* Knoxville, TN: University of Tennessee, Agricultural Extension Service, Publication No. BEES Info 100. 2001.

Schnitkey, G.D., J.W. Hopkins, and L.G. Tweeten. "An Economic Evaluation of Precision Fertilizer Applications on Corn-Soybean Fields." Proceedings of the Third International Conference on Precision Agriculture, Minneapolis, Minnesota. June 23-26, 1996.

Segarra, E., E.D. Ethridge, C.R. Deussen, and A.B. Onken. "Nitrogen Carry-Over Impacts in Irrigated Cotton Production, Southern High Plains of Texas." *Western Journal of Agricultural Economics* 14(1989):300-309.

Stauber, M.S., O.R. Burt, and F. Linse. "An Economic Evaluation of Nitrogen Fertilization of Grasses When Carry-Over is Significant." *American Journal of Agricultural Economics* 57(1975):463-471.

Tembo, G., B.W. Brorsen, F.M. Epplin, and E. Tostão. "Crop Input Response Functions with Stochastic Plateaus." *American Journal of Agricultural Economics* 90(2008):424-434.

Tennessee Agricultural Extension Service. "Cotton Produciton in Tennessee." Knoxville, TN: University of Tennessee Agricultural Extension Service, Publication No. PB1514. 2001.

Tumusiime, E., B.W. Brorsen, J. Mosali, J. Johnson, J.Locke, and J.T. Biermacher. "Determining Optimal Levels of Nitrogen Fertilizer Using Random Parameter Models." *Journal of Agricultural and Applied Economics* 43(2011):541-552.

United States Bureau of Economic Analysis. "Gross Domestic Product: Implicit Price Deflator." 2015. Internet site: http://research.stlouisfed.org/fred2/series/GDPDEF/downloaddata. (Accessed January 11, 2016).

United States Department of Agriculture Economic Research Service. "Table 7. Average U.S. farm prices of selected fertilizers, 1960-2013." 2013. Internet site: http://www.ers.usda.gov/data-products/fertilizer-use-and-price.aspx. (Accessed January 11, 2016).

United States Department of Agriculture Economic Research Service. "Table 11. U.S. upland cotton farm, spot, and mill prices, 1970/71-2013/14." 2014. Internet site: http://ers.usda.gov/data-products/cotton,-wool,-and-textile-data/cotton-and-wool-yearbook.aspx. (Accessed January 11, 2016).

University of Tennessee Agricultural and Resource Economics Department. "Custom Rates Survey, 2013." 2013. Internet site: http://economics.ag.utk.edu/crop.html. (Accessed January 27, 2016).

Walworth, J.L. "Soil Sampling and Analysis." Publication AZ1412. The University of Arizona Cooperative Extension, Tucson, AZ. 2011.

Watkins, B., Y. Lu, and W. Huang. "Economic and Environmental Feasibility of Variable Rate Nitrogen Fertilizer Application with Carry-Over Effects." *Journal of Agricultural and Resource Economics* 23(1998):401-426.

Zhou, X., B.C. English, C.N. Boyer, R.K. Roberts, J.A. Larson, D.M. Lambert, M.Velandia, L.L. Falconer, S.W. Martin, S.L. Larkin, K.P. Paudel, A.K. Mishra, R.M. Rejesus, C.Wang, E. Segarra, J.M. Reeves. "Precision Farming by Cotton Producers in Fourteen Southern States: Results from the 2013 Southern Cotton Farm Survey." Research Series 15-001 Knoxville, TN: Department of Agricultural and Resource Economics, The University of Tennessee, 2015.

Appendix B

Tables and figures

Table 7. Producer's Knowledge of K Carryover by Temporal Frequency of Soil Testing and Year

	Temporal Frequency				
Year	j =1	j =2	j =3	j =4	j =5
$t = 0$	Q_0	Q_0	Q_0	Q_0	Q_0
$t = 1$	$C_{1,1}$[a] $= Q_{1,1}$	$C_{1,2} = Q_{1,2}$	$C_{1,3} = Q_{1,3}$	$C_{1,4} = Q_{1,4}$	$C_{1,5} = Q_{1,5}$
$t = 2$	$C_{2,1} = Q_{2,1}$	$C_{2,2} = Q_{1,2}$	$C_{2,3} = Q_{1,3}$	$C_{2,4} = Q_{1,4}$	$C_{2,5} = Q_{1,5}$
$t = 3$	$C_{3,1} = Q_{3,1}$	$C_{3,2} = Q_{3,2}$	$C_{3,3} = Q_{1,3}$	$C_{3,4} = Q_{1,4}$	$C_{3,5} = Q_{1,5}$
$t = 4$	$C_{4,1} = Q_{4,1}$	$C_{4,2} = Q_{3,2}$	$C_{4,3} = Q_{4,3}$	$C_{4,4} = Q_{1,4}$	$C_{4,5} = Q_{1,5}$
$t = 5$	$C_{5,1} = Q_{5,1}$	$C_{5,2} = Q_{5,2}$	$C_{5,3} = Q_{4,3}$	$C_{5,4} = Q_{5,4}$	$C_{5,5} = Q_{1,5}$
$t = 6$	$C_{6,1} = Q_{6,1}$	$C_{6,2} = Q_{5,2}$	$C_{6,3} = Q_{4,3}$	$C_{6,4} = Q_{5,4}$	$C_{6,5} = Q_{6,5}$
$t = 7$	$C_{7,1} = Q_{7,1}$	$C_{7,2} = Q_{7,2}$	$C_{7,3} = Q_{7,3}$	$C_{7,4} = Q_{5,4}$	$C_{7,5} = Q_{6,5}$
$t = 8$	$C_{8,1} = Q_{8,1}$	$C_{8,2} = Q_{7,2}$	$C_{8,3} = Q_{7,3}$	$C_{8,4} = Q_{5,4}$	$C_{8,5} = Q_{6,5}$
$t = 9$	$C_{9,1} = Q_{9,1}$	$C_{9,2} = Q_{9,2}$	$C_{9,3} = Q_{7,3}$	$C_{9,4} = Q_{9,4}$	$C_{9,5} = Q_{6,5}$
$t = 10$	$C_{10,1} = Q_{10,1}$	$C_{10,2} = Q_{9,2}$	$C_{10,3} = Q_{10,3}$	$C_{10,4} = Q_{9,4}$	$C_{10,5} = Q_{6,5}$

Note: This table shows how the constraint $C_{t,j} = \lambda_j Q_{t,j} + (1-\lambda_j) Q_{t-1,j}$ is updated in Equation 1.

Table 8. Total Monthly Precipitation Levels for the Growing Season of Upland Cotton in Jackson, TN from 2000 to 2008.

	Precipitation Totals (inches)									
Month	2000	2001	2002	2003	2004	2005	2006	2007	2008	Average
March	9.98	7.14	33.02	9.04	6.35	10.41	4.50	2.92	24.77	12.27
April	13.26	6.30	2.79	5.94	23.06	21.69	13.77	8.26	20.90	12.89
May	8.94	12.37	14.99	-	15.82	0.91	9.14	2.18	17.42	10.22
June	10.13	12.24	6.22	15.39	7.37	17.45	12.55	6.88	7.14	10.60
July	6.25	11.96	2.16	6.15	12.04	13.87	5.38	4.47	15.95	8.69
August	7.42	11.81	13.59	8.71	12.52	18.47	8.97	1.96	6.48	9.99
September	8.31	5.79	33.25	7.09	1.75	10.03	7.34	15.95	2.01	10.17
October	2.18	18.72	16.28	10.57	20.29	0.36	6.65	22.78	8.00	11.76
Total	66.47	86.33	122.3	62.89	99.21	93.19	68.30	65.41	102.7	85.20

Source: National Oceanic and Atmospheric Administration, 2014

Table 9. Average Annual Cotton Lint Yield and Post-Harvest K Carryover Level by K Application Rate in Jackson, TN from 2000 to 2008.

K rate (lb acre^{-1})	2000[a]	2001	2002	2003	2004	2005	2006	2007	2008	Average
					Yield (lb acre^{-1})					
0	880	827	475	809	960	871	695	903	597	780
25	1092	1209	756	1208	1519	1305	1242	1312	1300	1216
50	1117	1242	835	1387	1873	1487	1427	1314	1419	1345
75	1191	1368	1003	1403	1812	1384	1408	1274	1595	1382
100	1171	1392	1072	1451	1999	1536	1462	1275	1581	1438
124	1173	1366	1069	1370	1857	1390	1310	1129	1447	1346
149	1184	1402	1038	1375	1920	1430	1317	1120	1383	1352
					Pre-Planting K Carryover Levels (lb acre^{-1})					
0	-	119	110	139	98	88	104	93	102	104
25	-	154	147	177	150	123	130	123	142	144
50	-	178	186	204	219	177	195	188	222	196
75	-	245	222	249	297	229	238	252	271	251
100	-	325	270	301	395	307	337	298	318	319
124	-	395	347	335	469	350	405	363	335	375
149	-	459	521	376	528	440	458	467	491	468

[a]Pre-planting K carryover levels were not measured for the year 2000.

Table 10. Parameter Estimates for the Linear Response Stochastic Plateau Yield Response to Total Available K and the Linear Carryover Function.

Parameter[a,b]	Stochastic Plateau	Carryover[c]
Intercept[d] (β_0, a_0)	-60.27 (90.69)	25.46*** (6.78)
Slope[d] (β_1, a_1)	7.95*** (0.54)	0.73*** (0.02)
Plateau Yield[d] (μ)	1397.05*** (14.36)	-
Plateau Random Effect (σ_v^2)	31996*** (4172.66)	-
Year Random Effect $(\sigma_w^2, \sigma_\tau^2)$	33787*** (5197.98)	235.71
Random Error $(\sigma_\varepsilon^2, \sigma_u^2)$	25416*** (1909.13)	2.74

[a] Single, double, and triple asterisks (*, **, ***) represent significance at the 10%, 5%, and 1% level.
[b] Standard errors are in parentheses.
[c] Carryover data was corrected for heteroscedasticity.
[d] Units are reported in lb acre^{-1}

Table 11. Monte Carlo Simulation Results for the Optimal K Application Rate, Potassium Carryover, and Yield by Year for a 10-Period Planning Horizon.

	Temporal Frequency				
Value	Every Year	Every 2nd Year	Every 3rd Year	Every 4th Year	Every 5th Year
			K application (*lb acre*$^{-1}$)		
Period 1	28	28	28	28	28
Period 2	20	10	10	10	10
Period 3	27	33	21	21	21
Period 4	31	20	41	13	13
Period 5	33	42	28	54	18
Period 6	34	29	36	27	63
Period 7	35	38	36	43	29
Period 8	34	34	37	33	52
Period 9	34	36	36	35	35
Period 10	27	27	26	26	39
Average	30	30	30	29	31
			K Carryover (*lb acre*$^{-1}$)		
Period 0	293	293	293	293	293
Period 1	238	238	238	238	238
Period 2	218	218	218	218	218
Period 3	199	191	191	191	191
Period 4	189	188	180	180	180
Period 5	185	176	186	166	166
Period 6	185	184	181	185	159
Period 7	185	180	183	179	186
Period 8	184	184	184	187	182
Period 9	184	183	186	185	195
Period 10	184	184	186	185	193
Average	204	202	202	201	200
			Cotton Lint Yield (*lb acre*$^{-1}$)		
Period 1	1389	1389	1389	1389	1389
Period 2	1388	1378	1378	1378	1378
Period 3	1386	1383	1347	1347	1347
Period 4	1383	1371	1382	1285	1285
Period 5	1383	1379	1375	1380	1212
Period 6	1383	1376	1369	1376	1380
Period 7	1382	1380	1382	1376	1378
Period 8	1382	1380	1382	1372	1386
Period 9	1382	1381	1380	1383	1383
Period 10	1379	1377	1381	1376	1375
Average	1384	1379	1376	1366	1352

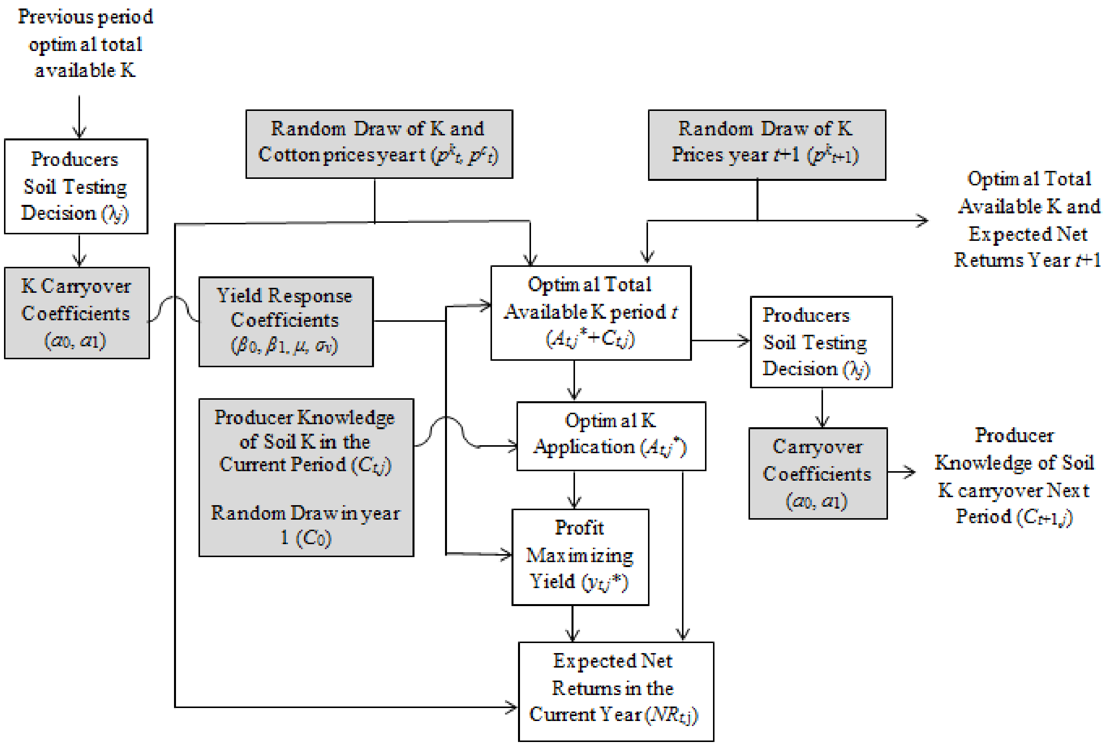

Figure 2. Flow chart of the dynamic programming model and simulation process of solving for optimal K rates.

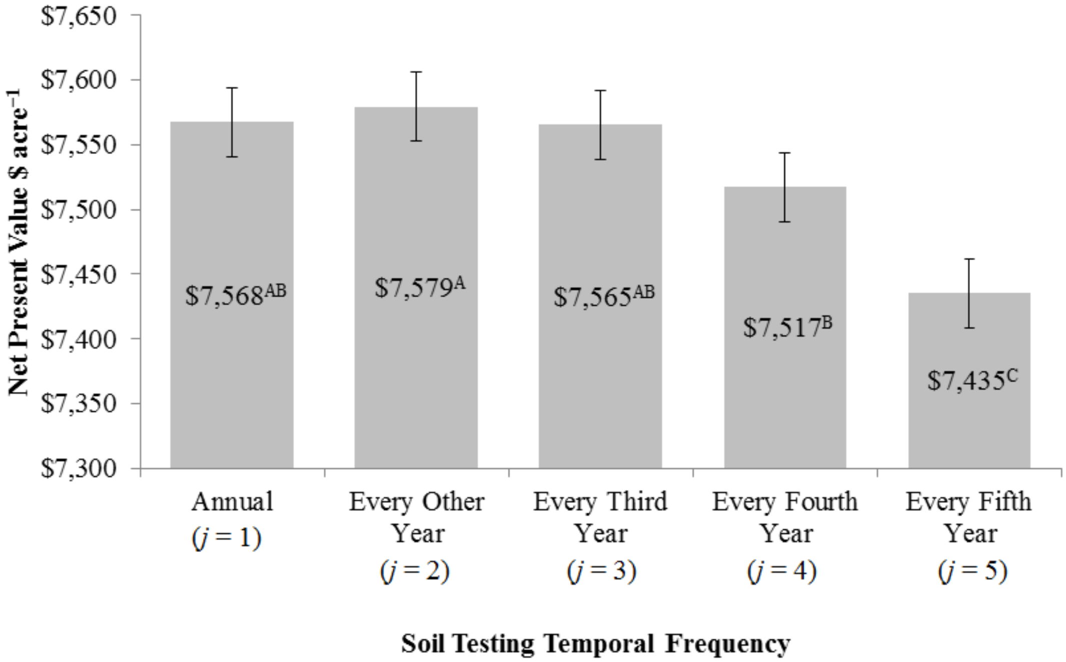

Figure 3. Net present value from applying optimal K rates over a 10-year planning period for five soil testing schedules.

CONCLUSION

This thesis presented two studies that focused on the economics of soil testing for potassium (K) fertilization of upland cotton (*Gossypium hirsutum* L.) in Tennessee. The conceptual and econometric modeling of each study provide several contributions to the literature, and the results can help producers make better informed decisions and more efficiently manage K fertilizer applications over time.

The objective of the first study was to determine the value of soil test information for available K in upland cotton production using a linear response plateau (LRP) and a linear response stochastic plateau (LRSP). This study modeled net present value (NPV) of returns to K fertilizer under four scenarios (1) K carryover was considered using a LRP, (2) K carryover was not considered using a LRP, (3) K carryover was considered using a LRSP, and (4) K carryover was not considered using the LRSP. The value of soil test information was calculated as the difference between the NPV of returns when K carryover was considered and when K carryover was not considered.

Goodness of fit criteria identified the LRSP as the better fitting model to characterize lint yield response to total available K, and thus the LRSP provided more accurate estimates of the optimal K application rates and the value of soil testing. For both the LRP and LRSP functions, considering carryover in the simulation model decreased the optimal K application rate and carryover level, while yield remained optimal regardless of if the producer considered carryover or not. Using the LRP, the estimated value of soil testing was \$63 acre^{-1} year^{-1}, while the estimated value of soil testing for the LRSP was \$61 acre^{-1} year^{-1}. Given that the LRSP was the better fitting yield response functional form, results suggest that the LRP overestimated the value of soil test information by \$2 acre^{-1} year^{-1}. Therefore, by capturing stochastic variation in the yield plateau, the LRSP function provided a lower and more accurate estimate of the value of soil testing compared to the LRP function.

The objective of the second study in this thesis was to determine the K fertilizer application and temporal frequency for obtaining K soil test information that maximizes NPV to K fertilizer in cotton production. Soil testing temporal frequency was defined as the discrete number of years producers waited to update their soil testing information. This study five soil testing schedules that varied in temporal frequency from soil testing every year to soil testing every fifth year. The optimal temporal frequency was the schedule that provided the greatest NPV over a 10 year planning horizon.

This study found that on average there was not much difference in the producer's optimal application rates as they varied from 29 to 31 lb acre^{-1} yr^{-1}. However, variation in the annual application rates increased as producers waited longer to update soil testing information. When producers updated soil test information less frequently, total available K levels were anywhere from 1 to 16% lower than optimal total available K levels for producers who soil tested every year. The result of lower total K levels was yield decreases of 1 to 12% relative to annual soil testing. The NPV of soil testing was maximized by soil testing every other year. Annual soil testing produced a NPV that was \$12 acre^{-1} lower than soil testing every other year, which indicates that the value of having the most accurate information about soil K carryover levels are not great enough to offset the cost of obtaining the additional information.

VITA

Xavier L. Harmon was born in Andersonville, TN to Archie and Gail Harmon. He attended Anderson County High School in Clinton, TN. After graduation he attended the University of Tennessee, Knoxville, where he pursued Bachelor of Science in Agriculture and Resource Economics at the University of Tennessee, Knoxville. Xavier continued his education at the University of Tennessee, Knoxville as he pursued a Master of Science degree in Agricultural Economics. He graduated in August 2016.