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An Educational Study of Elementary Geometry

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To the Graduate Council:

I am submitting herewith a thesis written by Nathan W. S. Lamb entitled "An Educational Study of Elementary Geometry." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Education.

Joseph E. Avent, Major Professor

We have read this thesis and recommend its acceptance:

A. H. Mueller, John B. Hamilton

Accepted for the Council:
Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

June 1, 1936

To the Committee on Graduate Study:

I submit herewith a thesis written by Mr. Nathan W. Scott Lamb and entitled " An Educational Study of Elementary Geometry", and recommend that it be accepted for nine quarter hours credit in partial fulfillment of the requirements for the degree of Master of Science, with a major in Education.

Jos. Brandt

Major Professor

At the request of the Committee on Graduate Study, we have read this thesis, and recommend its acceptance.

A. D. Mueller

John B. Hamilton

Accepted by the Committee

Jos. Brandt

Chairman

AN EDUCATIONAL STUDY
OF ELEMENTARY GEOMETRY

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A THESIS

Submitted to the Graduate Committee
of
The University of Tennessee
in
Partial Fulfillment of the Requirements
for the degree of
Master of Science

by

NATHAN W. SCOTT LAMB

June 1936

FOREWORD

The writer desires to express his gratitude to Dr. Joseph E. Avent for his encouragement and for his guidance in the preparation of this thesis.

N. W. S. L.

CONTENTS

CHAPTER I

HISTORICAL BACKGROUND

I. <u>Geometry in nature.</u>

The principles of geometry are not confined to the efforts of man. They have existed in nature from the "beginning". The statement has often been attributed to Plato, that"God eternally geometrizes". Many geometric designs are only copies from nature.

> "By geometry we discover how the planets move in their respective orbits, and we demonstrate their various revolutions. By it we account for the return of the seasons. By it we discover the power, wisdom, and goodness of the Grand Artificer of the Universe".[1]

2. <u>Instinctive use of geometry by insects.</u>

The white ants of Africa build hills that may be twenty five feet high, which are ingeniously honeycombed with galleries. The Spider seems to recognize both regular polygons and similarity of figures in making a web, and the bee follows the laws of maxima and minima in constructing the hexagonal wax cells of the honeycomb.[2]

3. <u>Instinctive use of geometry by animals.</u>

Animals instinctively follow the principle that a straight

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1. Loomis, E. S., <u>The Pythagorean Proposition</u>, p. 23. Masters and Wardens Associations of the Masonic Grand Lodge of Ohio, Cleveland, 1927.
2. Stamper, A. W., <u>A History of the Teaching of Elementary Geometry</u>, p. 3. Teachers College Series, Columbia University, New York, 1906.

line is the shortest distance between two points. The beaver is as skilled in building his dome-like structure as the Esquimau is in providing his shelter.

4. Intuitive use of geometry by man.

Geometry began, like all sciences, through man's contact with nature. Man's first efforts in the development of geometry were intuitive. When a boy crosses a rectangular lot diagonally, he is not conscious that the sum of the two sides of a triangle is greater than the third side. The Indian selected the cone-shaped tee-pee for economic reasons, not from reasoning in logical geometry.

5. Geometry on higher levels of development.

From an intuitive basis as civilization developed and space and its measurement meant more to man's spiritual and physical well-being, man began to develop a rudimentary science. When the mind came to classify, to define space relations, and to summarize the product of human efforts, a second level in the experience of the race was reached.

The minds of the ancient Egyptians, Babylonians, and Chinese operated on this level.

The Greeks worked on a third and higher level in developing a system of logic which culminated in the great work of Euclid.

The fourth level was reached when the theory was put into practical use.[3] The cycle is made complete when that which arose

3. *Ibid.*, pp. 3-4.

from the practical needs returns in the form of theory to be tested and again expanded.

6. Development of geometry in Egypt.

a. Building the pyramids.

The triumph of mind over matter came in the construction of the thirty eight noted pyramids of Egypt. The pyramids were built during the fourth dynasty, antedating 3000 B. C. Professor Smyth says that the Great Pyramid of Cheops has a square base, and that it is oriented to within one fifteenth of a degree of a true north and south line. The Egyptians perhaps located it from the polar star.

The harpedonaptae or rope-fasteners obtained a right angle by use of a triangle whose sides are in proportion to 3, 4, and 5 respectively. It was formed by stretching a rope around pegs driven into the ground. The Hindus and perhaps the Chinese also used this method to find a perpendicular.

b. Using geometric designs.

The Egyptians used a form of geometry due to aesthetic influences. In their mural decorations during the figth dynasty there are evidences of geometric pringiples of symmetry. In particular the square and its diagonals, the rhombus, the isosceles trapezoid, the eight pointed star formed by two stars overlapping, and the circle divided into 4, 6, 8, and 12 parts by diameters are used.[4]

c. Surveying land.

The word geometry literally means a measure of land(from

4. Ibid., pp. 5-6.

ge, the earth or land, and metros, measure).

The king of Egypt divided the land into small squares in order to make taxation more convenient. On account of the frequent overflow of the Nile, part of these were often swept away, and the king appointed surveyors to levy the proper tax on the part of the land remaining. We are told that the benefits of the Nile resulting from its overflow were proportioned among the owners of the land, and that canals, dykes, and sluices were constructed for the purpose of irrigation. The report of the exact quantity of land irrigated, the depths of the water into plains of various levels, and the time it continued upon the surface, which determined the proportionate payment of the taxes, required scientific skill. This rudimentary surveying carried with it the necessary practical geometry.[5]

d. Early records of geometry.

1) Rhind Papyrus. The oldest written work on geometry that we now have is the Ahmes Papyrus, which was written about 1700 B. C. It seems to have been copied from at least two older works. Ahmes was a scribe of the priest caste. The manuscript was called "Directions for Knowing All Dark Things". It is known as the Rhind Papyrus, because Henry A. Rhind purchased it in Egypt about the middle of the nineteenth century. It is preserved in the British Museum.

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5. Ibid., p. 7.

The Rhind Papyrus is not a textbook, but is rather a practical handbook. It gives work in arithmetic and crude algebra as well as in geometry. Rules are given for the areas of some of the plane figures; those for the areas of isosceles triangles and isosceles trapezoids are incorrect. In finding the area of a circular field the value of π is given as 3.1604, which previously had been given as 3. The papyrus gives calculations for finding the contents of barns, and adds some examples on pyramids. These employ a rudimentary trigonometry, for the base and hypotenuse of right triangles are given to find their ratio. This ratio determined the cosine of an angle, which for all the pyramids, gives practically the same slant of the lateral faces.[6]

2) The Kahun and Illahun Papyri.[7] There are later papyri from which we gain more information of the geometry of this civilization, which began as early as we have authentic history. Those of Kahun and Illahun relate to the distribution of a given square area into two squares whose sides have a given ratio to each other. These show intelligence in number and geometrical form.

3) The Moscow Papyrus.[8] There is an account of a mathematical papyrus of the late Middle Empire, now in the

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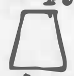
6. Ibid., pp. 8-9.
7. Karpinski, L. C., "The Parall@l Development of Mathematical Ideas, Numerically and Geometrically", School Science and Mathematics, Volume XX(December, 1920), p. 821.
8. Karpinski, L. C., "Egyptian Mathematical Papyrus in Moscow", Science, Volume LVII(May, 1923), pp. 528-529.

Museum of Fine Arts in Moscow. It gives the figure and tells how to find the volume of a truncated pyramid in a modern way:-

> "The problem is to make a 
> If it be said: ...... 4 below, 2 above: Do as follows: square this 4, which gives 16: duplicate 4, which gives 8. Do as follows: square the 2, which gives 4. Add the 16 to the 8 and the 4, which gives 28. Do as follows: take twice 28, which gives 56. This is the 56. You will find it correct."

This is the formula(The volume of the frustum of a regular pyramid is equal to one third the altitude multiplied by the sum of the bases and the mean proportional between the bases) which we would use to find the volume of a truncated square pyramid with lower base of four, upper base of two, and altitude of six.

Another problem in the Moscow Papyrus is concerned with finding the sides of a quadrilateral, when the relationship of the sides and the area of the quadrilateral are known. This problem is almost equally important, as it indicates clearly the Egyptian inspiration of a whole series of problems in Euclid's data. The problems in question are concerned with the determination of the sides of a rectangle when the sum and some other relationship of the sides are known.

4) Weaknesses of Egyptian geometry. The Egyptians failed in two essential points without which a true science of geometry cannot exist. In the first place they failed to construct a logical system of geometry based upon a few axioms and postulates. Many of their rules, especially in solid geometry, had not been proved at all, but were known to be true from abstraction or as a matter of fact. The second

great defect was their inability to generalize. Some of the simplest geometrical proofs were divided into numberless special cases each of which was supposed to require separate treatment.[9]

The Egyptians, like the Chinese, were slaves of tradition in both their government and learning. All the knowledge of geometry which they possessed when the Greek scholars visited them, six centuries before Christ, doubtless had been known to them two thousand years earlier, when they had built the pyramids.[10]

e. Babylonian contributions.

The Babylonians accomplished very little in geometry. Their interest seemed to be centered in astronomy, although they did much in arithmetic. They worshipped the heavenly bodies from the dawn of history.

They divided the circle into six parts by its radius as chords and used 360 degrees for its measure. Like the Hebrews they took π equal to 3. It is said that they possessed rules for finding the areas of squares, rectangles, right triangles, and trapezoids. They left no trace of geometric demonstration.[11]

f. Grecian contributions.

1) Egyptian influence. Greece is indebted to Egypt

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9. Cajori, Florian, A History of Mathematics, Second Edition, p. 11. The Macmillan Company, New York, 1919.
10. Ibid., p. 14.
11. Ibid., p. 15.

for its elementary geometry.[12] The Greeks thirsting for knowledge sought the Egyptian Priests, who, ambitious and anxious to acquire all knowledge which would give them a further hold upon a superstitious people, had sought to participate in the learning of the architects. Once admitted to the fraternity, they had connected the mythology of their country and their metaphysical speculations concerning the nature of God with the exclusive scientific teachings of the builders.[13] Thales, Pythagoras, Oenopides, Plato, Democritis, and Eudoxus, all visited the land of the pyramids. The Egyptians carried geometry no further than was absolutely necessary for their practical wants. The Greeks felt a craving to discover the reasons for things.

2) Sources of information. Our sources of information on Greek geometry before Euclid consists of scattered notices in ancient writers. Thales and Pythagoras left no written records. A full history of Greek geometry and astronomy was written during this time by Eudemus, a pupil of Aristotle, but it has been lost. Proclus knew of this history and gives a brief account of it.[14] This account is referred to as the Eudemian Summary.

3) Periods of Greek geometry. There are three important periods in the development of Greek mathematics: first,

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12. *Ibid.*, p. 15.
13. Loomis, *op. cit.*, p. 21.
14. Cajori, *op. cit.*, p. 15.

the one influenced by Pythagoras; second, the one dominated by Plato and his school; third, the one in which the Alexandrian School flourished in Egypt.[15]

g. The Ionian contributions.

1) Thales(640-546 B.C.), Since a history of geometry is baseless without some history of geometricians, we begin with Thales of Miletus, one of the "Seven Wise Men", who introduced the study of geometry into Greece.[16] He is said to be of Phoenician descent; but his mother, Cleobuline, bore a Greek name, while his father, Examius, is Carian. He was a merchant in his younger days, a statesman in middle life, and a mathematician, astronomer, and philosopher in his old age.[17]

As a merchant he may have accumulated the wealth that permitted him to indu ge his taste for learning and enabled him to found the Ionian School. These won for him the distinction of being enrolled as the first of the Seven Wise Men of Greece and the father of Greek geometry, astronomy, and arithmetic.[18]

His commercial pursuit took him to Egypt, where, it is said, he resided and studied the physical sciences and mathematics with the Egyptian priests. Plutarch tells us that Thales soon excelled his teachers, and amazed King Amasis by measuring the heights of the pyramids from their shadows. This was

15. Smith, David Eugene, History of Mathematics,Volume I,p. 63. Ginn and Company, Boston, 1923.
16. Cajori, op. cit., p. 15.
17. Reeve, William David, "The First of the Seven Wise Men of Greece", Mathematics Teacher, Volume XXII(February, 1930), p. 84.
18. Smith, op. cit., pp. 65-66.

done by considering that the shadow of a vertical staff of known length bears the same ratio to the shadow of the pyramid as the height of the staff bears to the height of the pyramid. According to Diogenes Laertius, Thales measured the length of the shadow at the moment the shadow of the staff was equal to its own length. He probably used both methods.[19]

Thales originated proofs of a few of the most simple theorems. He proved that the angles opposite the equal sides of an isosceles triangle are equal; that if two lines intersect, the vertical angles are equal; that the angle inscribed in a semicircle is a right angle; that a circle is bisected by its diameter; and that a triangle is determined by having its base and base angles known. He perhaps did not give proofs for the last two, for a statement from the Eudemian Summary says that Euclid thought the last one worthyof proof.[20]

The Eudemian Summary credits him with inventing a way of finding the distance of a ship at sea.[21] He was doubtless familiar with other theorems not recorded by the ancients, for it has been inferred that he knew the sum of the angles of a triangle equals two right angles, and that the corresponding sides of equiangular triangles are proportional.[22] The conception of geometrical loci is due to Thales.[23]

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19. _Ibid_., p. 66.
20. Stamper, _op. cit._, p. 11.
21. _Ibid_., p. 11.
22. Cajori, _op. cit._, p. 16.
23. Allman, G. J., _Greek Geometry from Thales to Euclid_, p.13. University Press, Dublin, 1889.

Thales founded the geometry of lines, which has ever since remained the principal part of geometry. The Egyptians had made great progress in practical geometry of areas and volumes.[24] With him we first meet with a logical geometric proof, for which reason he is looked upon as one of the great founders of mathematical science.[25] In the history of mathematics, as in the history of civilization in general, it is the setting forth of a great idea that counts. Without Thales there would not have been a Pythagoras - or such a Pythagoras; and without Pythagoras there would not have been a Plato - or such a Plato.[26]

Thales seemed to have had no teacher except while he was with the priests in Egypt. He founded a school of mathematics and philosophy at Miletus, known as the Ionic School.[27] This school for more than a hundred years continued, but after the death of Thales little was done in the progress of mathematics.

2) Anaximander, Anaximenes, and Anaxagoras. Anaximander(b. 611 B. C.) and Anaximenes(b. 570 B. C.) were the two most prominent pupils of Thales. They studied chiefly astronomy and philosophy. Anaxagoras(500-428 B. C.) was a pupil of Anaximenes and the last philosopher of the Ionian School. We know very little about him except that when he was in prison in his old age he attempted to square the circle.

24. Ibid., p. 17.
25. Stamper, op. cit., p. 11.
26. Smith, op. cit., p. 68.
27. Wentworth, George A. - Smith, D. E., Solid Geometry, p. 466. Ginn and Company, Boston, 1913.

h. Oenopides.

Oenopides of Chios was a contemporary of Anaxagoras, but he was not connected with the Ionic School. Proclus ascribes to him the solution of two problems: from a point without, to draw a perpendicular to a given line; and to draw an angle equal to a given angle.[28]

i. The Pythagorean contributions.

1) Pythagoras. Pythagoras is the most interesting figure in all the ancient history of mathematics. He easily ranks first, partly from the mystery surrounding his life, partly from his own mysticism, partly from the brotherhood he established, and partly from the unquestioned ability of the man himself.[29] He is one of those figures who impressed the imagination of succeeding times to such an extent that their real histories have become difficult to discern through the mystical haze that envelops them.[30] Authorities differ as to the place and date of his birth as they also do in regard to Euclid and Heron.[31] The writers are not entirely in agreement upon some of the other incidents of his life and works. Antiquity regards him as the successor of Thales.[32]

The father of Pythagoras, Mnessarch, obtained citizenship for services rendered the inhabitants of the Island of Samos in time of a famine. With his wife, Pithay, Mnessarch

28. Cajori, op. cit.,pp. 16-17.
29. Smith, op. cit., p. 69.
30. Cajori, op. cit., p. 17.
31. Smith, op. cit., p. 69.
32. Allman, op. cit., p. 19.

often traveled in business interests. They came to Tyre during the year 569 B. C., where Pythagoras was born. When Pythagoras was eighteen, he left Samos secretly and went to the Island of Lesbos, where he was hospitably received by his uncle. Here for two years he received instruction from Ferekid, a noted philosopher. He then went to Miletus, and studied chiefly physics and mathematics under Anaximander and Thales, the latter of whom was then already ninety years old.

Thales directed Pythagoras to Egypt as the land where he could satisfy his thirst for knowledge. He spent a year in the Phoenician priest college in Sidon in preparation, and, in the year 547 B. C., he arrived in Egypt.

Polycrates forgave him for his nocturnal flight from Samos and had a letter addressed to King Amasis in which he commended the young scholar. Yet, it cost him, as a foreigner and as one unclean, incredible toil to gain admission to the Egyptian priest caste, which only unwillingly initiated even their own people into their mysteries and knowledge.

The King in person brought Pythagoras to the priests in the temple of Heliopolis; they decided it would be impossible to receive him into their midst, but directed him to Memphis to their oldest priest, who in turn commended him to Thebes, Here somewhat severe conditions were laid upon him for his reception into the priest caste, but undaunted he performed all the rites and all the tests; and his study be-

gan under the guidance of the chief priest, Sonchis.

During the twenty one years he was in Egypt, he not only succeeded in mastering all the Egyptian learning, but also shared in the highest honors of the priests.

King Amasis died in 527 B. C. and the next year in the reign of his son, Psammenit, the Persian king, Kambis, invaded Egypt and loosed his fury upon the priests. Nearly all were carried into captivity including Pythagoras, who was taken to Babylon. Here he remained for twelve years in the center of world commerce, where Britains, Chinese, Indians, Jews, and other folk came. He also had an opportunity to acquire those learnings in which the Chaldeans were so rich.

He secured his liberty and returned to his native land when he was fifty six years of age. He found his teacher, Ferekid, still alive on the Island of Delos. He visited Greece for six months for the purpose of making himself familiar with the religious, scientific, and social conditions there.

The beginning of his teaching on the Island of Samos was extraordinarily sad. So that he might not remain without pupils, he was forced to pay his only pupil, who was also named Pythagoras, a son of Eratokles. This led him to abandon his thankless land and seek a home in the highly cultured cities of Magna Graecia(Italy).

He came to Kroton in 510 B. C., a turbulent year, for Tarquin was driven from Rome and Hippias was forced to flee from Athens. Insurrections broke out in the neighborhood of Kroton and elsewhere.

Pythagoras made his first appearance before the people of Kroton with an oration to the youth wherein he so seriously and convincingly set forth their duties that the elders of the city entreated him not to leave them without guidance. His second oration called attention to law abiding and purity of morals as the foundation of the family. In two following orations he turned to the matrons and children. As the result of the one to the matrons, in which he especially condemned luxury, thousands of costly garments were brought to the temple of Hera, because no matron could make up her mind to appear in them on the streets.

He spoke captivatingly and listeners streamed to him. Some of the worthiest men of the city, matrons, and maidens came to his evening entertainments; among whom was the young, gifted, and beautiful Theana, who became his wife.

The listeners became disciples, who formed a school in the narrow sense of the word, and the auditors formed a school in the broader sense. The disciples or mathematicians were given rigorous teaching as a scientific whole in logical succession from the prime concepts of mathematics up to the highest abstraction of philosophy. They learned to regard everything fragmentary in knowledge as more harmful than ignorance. Nothing was taught rigorously to the auditors, who subsequently formed the Pythagoreans.[33]

The Pythagorean School was a brotherhood, the members of

33. Loomis, op. cit., pp. 28-32.

which were united for life. This brotherhood had observances approaching Masonic peculiarity. They were forbidden to divulge the discoveries and doctrines of their school. The mystic and secret observances in imitation of Egyptian uses, and its aristocratic tendences caused it to become an object of suspicion. The democratic party in lower Italy revolted.[34] About the year 490 B. C., when the Pythagorean School had reached its highest splendor, a certain Hypasos who had been expelled from the school as unworthy put himself at the head of the democratic party in Kroton and appeared as accuser of his former colleagues. The school was broken up, the property of Pythagoras was confiscated, and he himself was exiled.

He lived sixteen years in Tarentum, but in 474 B. C. the democratic party gained the upper hand and Pythagoras at the age of ninety five was again forced to flee. He went to Metapontum, where he lived in poverty four years longer. Finally democracy triumphed there also; the house wherein was his school was burned, many of his disciples died a death of torture, and Pythagoras himself, having with difficulty escaped the flames, died soon after in his ninety ninth year.[35]

Pythagoras, like Thales, never wrote a treatise of his work. His theories were transmitted by word of mouth through the elect of his brotherhood. Thus his doctrines were freely made known to those who were deemed worthy to receive them. This method was adopted not only for its mysticism, but also

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34. Cajori, op. cit., p. 18.
35. Loomis, op. cit., pp. 28-32.

because of the scarcity of good writing material.[36]

The work of Pythagoras himself cannot be completely discriminated from that produced by others of the Pythagorean School.

He enunciated and demonstrated the renowned theorem known to us as the forty seventh proposition of the first book of Euclid's Elements: the square described upon the hypotenuse of a right triangle is equal to the sum of the squares described upon the other two sides. His method of proof is unknown to us, and it is not determined whether he learned the proposition from the Egyptians, Babylonians, or discovered it himself. The Egyptians, the Babylonians, and the Chinese knew how to construct the triangle whose sides are in proportion to 3, 4, and 5.[37] The general enunciation and the general demonstration were perhaps made by Pythagoras himself.[38]

Pythagoras discovered the construction of the mundane figures(the five regular solids).

Proclus says in the Eudemian Summary that he discovered the theorem: the sum of the angles of a triangle equals two right angles.

He is credited with the discovery of the problem: to construct a figure equal to a given figure and similar to another given figure.

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36. Smith, _op. cit._, pp. 72-73.
37. Loomis, _op. cit._, p. 26.
38. _Ibid._, p. 27.

It is said he learned proportion in Babylon. The one consisting of four terms is especially mentioned.

The Pythagoreans defined a point as unity having position. They considered a point as analogous to the monad; a line, to the duad; a surface, to the triad; and a body to the tetrad.

They discovered the theorem: the plane around a point is completely filled by six equilateral triangles, four squares, or three regular hexagons.

The triple interwoven triangle, or pentagram, the star-shaped regular pentagon, was used as a symbol or sign of recognition by the Pythagoreans, who called it "health".

It is said that Pythagoras considered the sphere the most beautiful of all solid figures; and the circle, the most beautiful of all the plane figures.

Proclus gives a rule formulated by Pythagoras for finding in numbers the side of a right triangle with odd numbers:-[39]

> "Pythagoras places a given odd number as the lesser of the two sides inclosing the right angle and takes the square constructed on it, and diminishes it by unity. He places half the remainder as the greater side about the right angle; and when he has added unity to this side he gets the hypotenuse. 3, side. 3x3 equals 9. 9 - 1 equals 8. 8 divided by 2 equals 4, other side. 4 plus 1 equals 5, the hypotenuse."

In the same connection Proclus gives a rule used by Plato for finding the sides of a right triangle by beginning with

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39. Allman, op. cit., pp. 24-34.

even numbers:-

> "An even number is taken as one of the sides, he divides it by two and adds unity, and this will be the hypotenuse, but by subtracting unity, the other side is found. 4, one side. 4 divided by 2 equals 2. 2x2 equals 4. 4 plus 1 equals 5, the hypotenuse. 4 - 1 equals 3, the other side."

Pythagoras used arithmetic with geometry and said that they afforded mutual aid.[40] He was the first to use definitions in his geometry.[41] He and his school made geometry a liberal science.[42]

Two centuries after the death of Pythagoras during the First Samnite War the Senate of Rome erected the statute of Pythagoras in response to the order of the Delphic Oracle to thus honor "the wisest and bravest of the Greeks", and the people called him the preceptor of King Numa, while even the great Aemilian Family was, in later years, proud to claim him as one of their honored ancestors.[43] Pythagoras was the greatest Greek mathematician and philosopher.[44]

The Pythagorean School was founded on philosophy, mathematics, and religion, wherein arose the famous ten antitheses of Pythagorean teaching, namely: 1, limited and unlimited; 2, even and odd; 3, one and many; 4, right and left; 5, male and female; 6, rest and motion; 7, straight and crooked; 8, light

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40. *Ibid.*, p. 49.
41. Smith, *op. cit.*, p. 75.
42. Stamper, *op. cit.*, p. 13.
43. Smith, *op. cit.*, pp. 96-97.
44. Burgess, E. G., "Mathematics", *School Science and Mathematics*, Volume XXIV(March, 1924), p. 265.

and darkness; 9, good and evil; 10, square and rectangle.[45] These even today are the riddles of thinkers.

The Pythagorean School continued to exist for at least two centuries after the Pythagorean fraternity was broken up.[46]

2) Philolaus. Philolaus of Kroton wrote a book on the Pythagorean doctrines, through which the world was first given the teachings of the Italian School, which had been kept secret for a hundred years.[47]

3) Archytas. The brilliant Archytas(428-347 B. C.) of Tarentum, a great statesman and general, universally admired for his virtues, was the only great geometer among the Greeks when Plato opened his school. He was probably the teacher of Plato when the latter visited Italy. Their friendship became proverbial. He saved Plato's life when he was in danger of being put to death by the younger Dionysius(cir. 361 B. C.). Horace in a beautiful ode to Archytas refers to his death in the Adriatic Sea, and he recognizes him as an eminent arithmetician, geometer, and astronomer.

The Platonic method for finding the sides of a right triangle by numbers is ascribed to Archytas, from whom Plato learned it; by publishing it, Plato was given the credit. He is said to be the first to employ scientific methods in mechanics, by introducing mathematics. He was the first to ap-

45. Loomis, op. cit., pp. 22-23.
46. Cajori, op. cit., p. 19.
47. Ibid., p. 19.

ply motion to geometry while trying to find a method for duplication of the cube. Eratosthenes relates that Plato, Archytas, Eudoxus, and Menaechmus, all found solutions to this Delian problem, with satisfactory proofs, but it was impossible to make any practical to any great extent except that of Menaechmus.[48] Archytas found a very ingenious mechanical method for the duplication of the cube. It involves a very clear notion of the generation of cones and cylinders. The problem reduces itself to finding two mean proportionals between two given lines. He advanced the doctrine of proportion.[49]

The later Pythagoreans must have exercised a strong influence on the study and development of mathematics, for the Sophists acquired geometry from Pythagorean sources. Plato bought the works of Philolaus, and he had a warm friend in Archytas.

Archytas knew and doubtless proved the following theorems:- a) If a perpendicular is drawn to the hypotenuse from the vertex of a right triangle, each side is the mean proportional between the hypotenuse and its adjacent segment. b) If a perpendicular is drawn to the hypotenuse of a right triangle from the vertex, it is the mean proportional between the segments of the hypotenuse. c) If the perpendicular from the vertex of a triangle is the mean proportional between the segments of the opposite sides, the angle at the vertex is a right an-

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48. Allman, op. cit., p. 107.
49. Cajori, op. cit., pp. 19-20.

gle. d) If two chords intersect, the rectangle of the segments of one is equivalent to the rectangle of the segments of the other. e) Angles in the same segment of a circle are equal. f) If two planes are perpendicular to a third plane, their line of intersection is perpendicular to that plane and also to their lines of intersection with the plane.[50]

j. Sophist contributions.

When Athens became the center of Greek civilization, there arose a demand for teachers. The supply came chiefly from Sicily, where Pythagorean doctrines had spread. They were called Sophists, or "wise men", who, unlike the Pythagoreans, received pay for teaching. Rhetoric was their chief subject, but they also taught geometry, astronomy, and philosophy. Athens soon became the headquarters of Greek learning. The home of mathematics among the Greeks was first in the Ionian Islands, then in lower Italy, and later at Athens.[51]

The Sophists took up the geometry of the circle,which had been almost entirely neglected by the Pythagoreans, and nearly all their discoveries were made in their innumerable attempts to solve the three famous problems of antiquity:-

a) To trisect an arc or an angle.

b) To double the cube - the Delian problem.

c) To "square the circle" - to find a square or some other rectilinear figure which would be exactly equal to the area

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50. Smith, op. cit., pp. 85-86.
51. Cajori, op. cit., p. 20.

of a given circle.

These problems have probably been discussed more and have involved probably more research than any other problems in mathematics. Mathematicians have decided their solutions impossible by means of the ruler and compasses.[52]

1) Hippias. Hippias of Elis, a contemporary of Plato, was born about 460 B. C. Finding himself unable with ruler and compasses alone to trisect an angle, he resorted to other means. He, perhaps, invented the quadratrix, a transcendental curve, by means of which an angle may be divided into any number of equal parts. The same curve was later used for the quadrature of the circle, from which it received its name.[53]

2) Hippocrates. Hippocrates of Chios, a talented mathematician, but having been defrauded of his property, was pronounced stupid. He was said to be the first to take pay for teaching mathematics. He showed that the Delian problem could be reduced to finding two mean proportionals between a given line and another twice as long. This, of course, was not done by ruler and compasses. He became celebrated by squaring lunes.[54]

The Eudemian Summary gives as the work of Hippocrates and others the following:-

a) Definitions of similar segments of circles, which are the same parts of circles, e. g.,

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52. *Ibid.*, pp. 20-21.
53. *Ibid.*, p. 21.
54. *Ibid.*, pp. 21-22.

a semicircle is similar to a semicircle,

a third part of a circle is similar to the third part of another circle.

b) Theorems, to-wit,

similar segments contain equal angles, which in all semicircles are right;

segments which are larger or smaller than semicircles contain, respectively, acute or obtuse angles;

the sides of a regular hexagon inscribed in a circle are equal to the radii;

in any triangle the side opposite an acute angle squared is less than the sum of the squares on the sides which contain the acute angle;

in any obtuse-angled triangle the square on the side subtending the obtuse angle, is greater than the sum of the squares on the sides containing it;

in an isosceles triangle whose vertical angle is double that of an equilateral triangle, the square on the base is equal to three times the square on one of the equal sides;

in equiangular triangles the sides about the equal angles are proportional;

circles are to each other as the squares on their diameters;

similar segments of circles are to each other

as the squares on their bases.

c) Problems, to-wit,

construct a square which shall be equal to a given rectilineal figure;

find a line the square on which shall be equal to three times the square on a given line;

find a line such that twice the square on it shall be equal to three times the square on a given line;

being given two straight lines, to construct a trapezoid such that one of the parallel sides shall be equal to the greater of two given lines, and each of the three remaining sides equal to the less, and circumscribe a circle about the trapezoid;

describe a circle about a given triangle;

from the extremity of the diameter of a semicircle such that the part of it intercepted between a circle and a straight line drawn at right angles to the diameter at the distance of one half the radius shall be equal to a given straight line;

describe on a given straight line a segment of a circle which shall be similar to a given one.[55]

Secrecy was contrary to the spirit of Athenian life, and

55. Allman, op. cit., pp. 76-78.

Hippocrates added to his fame by writing a textbook on geometry called the Elements.[56]

3) Zeno. The other Sophists did little for geometry, but Zeno of Elea and the Eleatic School influenced it. They claimed motion could not exist, for Achilles could not pass a tortoise; he must go half the distance and half that remaining, and there would always be some distance to go.[57] Zeno also denied multiplicity, for he said the largest division would be infinitely large, and the smallest would have no magnitude from infinite division.

4) The Atomists. The Atomists founded by Leucippus and Democritus endeavored to reconcile the Eleatic and the Ionic philosophies. The early Greek writers banished infinity from their science.[58]

k. The Platonic influence.

1) Plato. Plato(429-348 B. C.) was born in Athens and was but a youth when Sparta conquered Athens in 404 B. C.[59] He was a pupil and friend of Socrates, after whose death he traveled extensively. He studied mathematics in Cyrene under Theodorus, a Pythagorean philosopher. He went to Egypt, then to lower Italy and Sicily, where he met other Pythagoreans, becoming an intimate friend to Archytas and to Timaeus of Locri. On his return to Athens, about 389 B. C., he founded his school in the groves of the Academia and devoted the remain-

56. Cajori, op. cit., p. 23.
57. Smith, op. cit., p. 78.
58. Allman, op. cit., p. 55.
59. Cajori, op. cit., pp. 25-26.

der of his life to teaching and writing.[60]

He appreciated the value of geometry so highly that in later years he placed above the door of his school of philosophy(the Academy) the words, "Let no one ignorant of geometry enter my doors", the oldest recorded entrance requirement to college, and he spoke of God as the Great Geometer.[61]

With Plato as the head-master we do not wonder that his school produced so many mathematicians. He made valuable improvements in the logic and methods of geometry. Many of the definitions and axioms in Euclid are ascribed to the Platonic School. Aristotle credits Plato with having given us the subtraction axiom.

Analysis as a distinct method of proof in geometry was achieved by the Platonic School. This proof was used by the Greeks to discover the synthetic method, which was then given.

The Platonic School studied the sphere, the regular solids, the prism, the pyramid, the cylinder, and the cone. All of which, except the sphere and the regular solids, were until then hardly known to exist.[62]

Plato more than any of his predecessors appreciated the scientific possibilities of geometry. By his teaching he laid the foundation of the subject. He insisted upon accurate definitions, clear assumptions, and logical proof.[63]

60. Cajori, op. cit., p. 26.
61. Smith, op. cit., p. 88.
62. Cajori, op. cit., pp. 26-27.
63. Smith, op. cit., p. 90.

2) Speusippus. Speusippus, the nephew of Plato, succeeded him as head of the Academy. He wrote upon proportion. He is said to have treated with rare elegance the subject of linear, polyginal, plane, and solid numbers.

3) Leodamus. Leodamus of Thasos is said to have used the analytic method of proof. There were other minor followers of Plato, who made some additions to geometry.[64]

4) Eudoxus. Eudoxus of Cnidus, an astronomer, geometer, physician, and law-giver, was born about 407 B. C. He was the pupil of Archytas in geometry. He is said to have gone to Athens and heard Plato by whom he was received coldly. After two months he returned home. He then went to Egypt bearing a letter of recommendation from Agesilaus to Nectanabis, who recommended him to the priests. He was in Egypt a year and four months. Upon his return he founded a school at Cyzicus, which became famous in geometry and astronomy. He taught here and in the city of Propontis. At the height of his reputation he went to Athens with a great many pupils, some say to annoy Plato, who had considered him unworthy of attention. When Plato gave an entertainment, Eudoxus introduced the fashion of sitting in a semicircle, for there were many guests.

He was received with great honors when he returned to his own country, where he gave laws to his fellow citizens. He died at Cyzicus about 354 B. C.

He has been called the father of scientific astronomy. He wrote a treatise on geometry and astronomy. He also wrote

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64. Ibid., pp. 90-91.

other important works. The geometrical works of Eudoxus have been lost; we have only brief notices concerning them. It is said that he increased the number of general theorems and added three proportions to the three already existing. We are told he invented the fifth book of Euclid, which treats of proportion, that he was the discoverer of curved lines, which he used to find two proportionals between two given lines. Archimedes says that he showed that any pyramid is one third of a prism which has the same base and altitude, and that any cone is one third of a cylinder which has the same base and altitude.[65] For the measurement of the cone and cylinder he probably developed the method of exhaustion as a rigorous theory.[66]

The fame of the academy of Plato is to a large extent due to the pupils of Eudoxus from the school of Cyzicus, among whom are Menaechmus, Dinostratus, Athenaeus, and Helicon.[67]

5) Menaechmus. Menaechmus was a pupil of Eudoxus, with whom in the history of geometry an epoch closed and a new era, still in existance, began.[68] Menaechmus was an associate of Plato. He discovered the conic sections, the parabola, the hyperbola, and the ellipse. It is said that Alexander the Great, who was his pupil, asked that geometry

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65. Allman, op. cit., pp. 130-133.
66. Smith, op. cit., p. 91.
67. Cajori, op. cit., p. 28.
68. Allman, op. cit., p. 148.

be made more simple for him; whereupon Menaechmus replied: "O King, through the country there are private and royal roads, but in geometry there is only one road for all". He seems to have given two solutions to the Delian problem; he was the first to distinguish between theorems and problems. It is said that Plato blamed Eudoxus, Archytas, and Menaechmus for endeavoring to reduce the duplication of the cube to instrumental and mechanical contrivances. He said that it backslides into things of sense and does not soar and try to grasp the incorporeal, and lay hold on the eternal. Thus was mechanics separated from geometry.[69]

6) Deinostratus. Deinostratus was a brother of Menaechmus and is mentioned in the Eudemian Summary with Amyclas and Menaechmus, as having made the whole of geometry more perfect.[70] He is known chiefly for his study of the quadratrix already invented by Hippias. This curve enabled him to square a circle.[71]

7) Aristaeus. Aristaeus was a pupil of Menaechmus, and he continued and summed up the work of Philolaus, Archytas, Eudoxus, and Menaechmus.[72]

8) Theaetetus. Theaetetus of Athens was a pupil of Theodorus of Cyrene and a disciple of Socrates. He impressed both his teachers with his natural gifts and genius. He is

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69. Ibid., p. 153.
70. Ibid., p. 180.
71. Smith, op. cit., p. 92.
72. Allman, op. cit., p. 205.

represented as having greatly advanced the science of geometry. Books X and XIII of Euclid are credited to Theaetetus.[73]

9) Xenocrates. Xenocrates of Chalcidon was a friend of Plato and Aristotle. He followed Speusippus as head of the Academy and wrote a history of geometry in five books, which, like his other works, is lost.

10) Aristotle. Aristotle was a pupil of Plato, who called him the "intellect of the school". He was a teacher of Alexander the Great, and later returned to Athens and founded the Peripatetic School of philosophy. He was a voluminous writer. His chief contribution to geometry seemed to pertain to its logic. He wrote a book on invisible lines and one on mechanical problems. He advocated the separation of arithmetic from geometry. In his systematizing of logic he greatly aided Euclid. To him we owe the first definition of continuity: "A thing is continuous when any two successive parts the limits at which they touch are one and the same, and are, as the name implies, held together".[74]

1. The Alexandrian influence.

1) Importance of this period. The period from 300 B. C. to 500 A. D. marks approximately the influence of the School of Alexandria, the greatest mathematical school of ancient times. It approximately began with Euclid and

73. Ibid., pp. 206-213.
74. Smith, op. cit., p. 102.

ended with Boethius, the last great Roman mathematician, It is the most significant period of ancient mathematical history.[75]

We have seen the birth of geometry in Egypt, its transference first to the Ionian Islands, thence to lower Italy, and thence to Athens. We have noted its development in Greece. Now we return to Egypt, the land of its birth, to Alexandria.[76]

In the Nile Delta, on the site of the ancient town of Rhacotis, Alexander the Great, the famous "world conqueror" founded a city worthy to bear his name. Upon his death his vast domain was broken up, and when Antiginous his ablest general died the empire was divided into three parts. The friend and counselor of Alexander and perhaps his relative,Ptolemy Soter, came into possession of Egypt. He made Alexandria the capitol and under his reign(323-283 B.C.) this city became the center not only of world commerce but also of its literary and scientific activities. Here was established the greatest of the world's ancient libraries and its first international university. Here were trained more great mathematicians than in any other center of the ancient world. With Alexandria are connected the names of Euclid, Archimedes, Apollonius, Eratosthenes, Ptolemy the astronomer, Heron, Menelaus, Pappus, Theon, Hypatia, Diophantus, Nicomachus. No trace remains today of the famous library and museum and their exact locations are unknown.[77]

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75. *Ibid.*, p. 102.
76. Cajori, *op. cit.*, p. 29.
77. Smith, *op. cit.*, pp. 102-103.

2) Euclid. The best known of all the great names connected with Alexandria is Euclid. He is probably the most successful textbook writer in geometry the world has ever known. More than a thousand editions of his geometry have appeared in print since 1482; manuscripts of this had dominated the teaching of geometry for eighteen hundred years before 1482.[78] He is the only man of whom it can be said that he incorporated in his writings all the essential parts of the accumulated mathematical knowledge of his time.

Little is known of his life. He was born about 365 B. C., and seems to have written the Elements when he was about forty years of age. He may have been a Greek or he could have been an Egyptian who came to Alexandria to learn and teach. There is reason to believe he studied in Athens, for he seems to be of the Platonic sect and well read in its doctrines.[79] He collected the Elements, put in order much that Eudoxus had prepared, completed many things in the work of Theaetetus, and was the first who reduced to unobjectionable demonstration the imperfect attempts of his predecessors. It is said that a youth who had begun to read geometry with Euclid, when he had learned the first proposition, asked, "What do I get by learning these things?" Euclid called his slave and said, "Give him three pence, since he must make gain out of what he learns".[80]

Very little except in solid geometry has been added to the subject-matter of elementary geometry, which we are considering, since Euclid compiled his Elements. The great mathematicians

78. Ibid., p. 103.
79. Ibid., pp. 103-105.
80. Smith, op. cit., p. 105.

considered it complete and turned thier attention to other phases of mathematics. Euclid's Elements were kept alive and seemed to have been taught in the University of Alexandria after the death of the author.[81]

3) Trionometry developed. Aristarchus(about 310-230 B. C.), Hipparchus(born about 161, died about 126 B. C.), Menelaus(born about 98 A. D.), and Claudius Ptolemy(125-161 A. D.), and Richard Wallingford(1292-1335) developed trigonometry.[82]

4) Archimedes. Archimedes was born about 287 B. C. and was murdered in 212 B. C. Syracuse was the city of his birth. He was the greatest mathematician of antiquity. Plutarch tells us he was a relative of King Hieron, but Cicero is perhaps correct in the statement that he was of low birth. He is said to have visited Egypt and, since he was a great friend of Conon and Eratosthenes, he must have studied in Alexandria. Upon his return to Syracuse he invented machines to aid his friend and patron, King Hieron against Marcellus. The Romans under Marcellus finally conquered, and Archimedes was killed in the indiscriminate slaughter that followed. According to tradition he was studying the diagram of some problem in the sand, and when a Roman soldier approached he exclaimed, "Don't spoil my circles!" The soldier feeling insulted rushed upon him and killed him. It was not the wish of General Marcellus to have

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81. Cajori, op. cit., p. 30.
82. Bond, John David, The Development of Trigonometric Methods Down to the Close of the XVth Century, Isis No. 11, Volume IV(1922), pp. 297-299.

him slain, for he treated the act on the part of the soldier as murder. He admired the genius of Archimedes and raised in his honor a tomb bearing the inscription of a sphere inscribed in a cylinder. When Cicero visited Syracuse, he found the tomb buried under rubbish.

Most of his work falls without the sphere of elementary geometry, but he made valuable contributions to the circle and solid geometry. He found the value of π to be more than 3 10/71 and less than 3 1/7. He wrote two books on the sphere and the cylinder.

5) Eratosthenes. Eratosthenes was educated at Alexandria under Callimachus, whom he succeeded as custodian of the Alexandrian Library. He was interested in the duplication of the cube.

6) Apollonius. Apollonius of Perga takes second rank among the great mathematicians of antiquity. He studied at Alexandria under Euclid's successors. His chief work was on the conic sections in eight books.[83]

7) Theodosius. Theodosius lived in the reign of Trajan(98-117). He seems to have been a native of Tripoli, on the Phoenician coast. He wrote a treatise on the sphere, which was translated into Arabic and was used in Arabian schools.

8) Heron. Heron of Alexandria seemed to be an Egyptian, for his style was not Greek. He gave us the formula

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83. Cajori, *op. cit.*, pp. 34-41.

for finding the area of a triangle when the sides are known. He wrote a treatise on the section of the cylinder and one on the section of the cone.

9) Menelaus. Menelaus, a native of Alexandria, wrote a treatise on the sphere, particularly with respect to the geometric properties of spherical triangles.

10) Pappus. Pappus of Alexandria was a Greek geometer in probably the third century. Only six books of his mathematical collections have come down to us. The third book which treats of proportion, inscribed solids, and the duplication of the cube, and the sixth book which treats of the sphere, are of special interest to us.

11) Proclus. Proclus was looked upon as the successor of Plato. He studied at Alexandria and taught at Athens. To him we are indebted for much information on the history of Greek geometry.[84]

12) Theon and Hypatia. Theon of Alexandria probably used Euclid's Elements as a textbook in his teaching. Hypatia his daughter was the last great Alexandrian teacher. She has been claimed to be a greater philosopher and mathematician than her father.[85]

13) Boethius. Boethius about 500 A. D. incorporated in a work a statement of the propositions in Book I of Euclid's Elements, and he included other propositions from the second, third, and fourth books, giving at the last proofs of the first

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84. Smith, op. cit., pp. 116-137.
85. Cajori, op. cit., pp. 50-51.

three propositions of Book I. The Elements of Euclid had become known throughout Greece including Asia Minor and the Italian Colonies, and the work of Boethius proves it was not unknown in Italy.

7. Translations.

a. Translations into Arabic.

Alexandria was destroyed by the Arabs in 640 A. D., and Greek learning found a home in the Syrian cities along the Mediterranean coast. From these schools the Arabs of Bagdad obtained something of Greek learning, and the works of Euclid, Archimedes, Apollonius, and Ptolemy were translated into Arabic. Euclid was partially translated in the time of Harum al Raschid(786-809) and completed under Al Mamun. They contributed nothing to geometry, but they preserved Euclid. Thier great achievements were in arithmetic, trigonometry, and algebra. When they conquered Spain in 747, they brought their Euclid with them, but it was nearly three hundred years later when it was given to Christian Europe.[86]

b. Translations into Latin.

The Moors jealously guarded their learning, but an English monk, Adelard of Bath, while he was studying in Spain in 1120, succeeded in getting a copy of the Elements and translated it into Latin. Gherardo of Cremona made another translation about 1185, and Johannus Campanus in 1260 made a copy of the translation by Adelard and gave it out as his own. These stimulated

86. Stamper, op. cit., pp. 31-33.

the study of geometry. Leonardo of Pisa in 1220 wrote the first original work on mathematics in Europe, which was based on Euclid, Archimedes, and Ptolemy.

Euclid and Aristotle found a place in the universities of Europe, but only in Germany did there seem to be much emphasis placed upon the Elements. The study of Euclid until the invention of printing was a formal affair, but with printing it was read more and it became more common in higher institutions of learning.

c. Translations into English.

There were thirty five editions of the Elements during the thirty years following the printing by Ernst Ratdolt at Venice in 1842 of the copy by Campanus, but the first complete translation of the Elements into English was by Henry Billingsby in 1570. Solid geometry was added in 1795 in Playfair's Euclid.

8. Teaching.

The Elements of Euclid was the chief textbook used until the latter part of the eighteenth century. The first radical departure from the teaching of Euclid which was generally accepted was in the use of the text of Legendre written in 1794. This was one of the best textbooks ever used in the teaching of geometry. Legendre sought to rearrange the propositions of Euclid, separating the theorems from the problems, and simplifying the proofs without sacrificing the rigor of

the Greek methods of treatment. It is largely due to Legendre that the American schools abandoned Euclid as a textbook in geometry.[87]

The teaching of elementary geometry seems to have begun in the higher institutions of learning and has come downward. It was first taught in Europe and in America in the universities, then in the colleges, later in the secondary schools, and now the rudiments of the subject are taught to some extent in the elementary schools. Texts were first written for adult minds. As the subject has moved downward, the minds of the geometry students have become less and less mature. In Germany, in France, and in the most of the countries on the Continent of Europe the geometry taught has been more or less practical. In England the logical rigor of the Elements of Euclid has been remarkably adhered to. Their texts on geometry have been based upon the Elements of Euclid, or they have taught Euclid itself. Geometry taught in the United States comes between that taught on the Continent of Europe and the logical Euclidean geometry of England.[88]

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87. Smith, *op. cit.*, p. 488.
88. Digest of literature studied in the preparation of this thesis. There is general agreement.

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CHAPTER II

THE SOCIOLOGY OF GEOMETRY

1. Definitions of terms.

a. Sociology.

Sociology is the science that treats of the social relations of human beings. These relations may be designated by such adjectives as: conjugal, filial, sociable, political, economic, co-operative, competitive, subordinate, convivial, and philanthropic.[1]

b. Sociology of geometry.

Sociology of geometry from the standpoint of a pure science would interpret and evaluate the place of geometry in the course of human affairs without any reference to the practical results that may follow. It would include knowledge of geometry for its own sake.

Sociology of geometry from the standpoint of applied science would concern itself with those phases of geometry which are useful to society in general, those that are valuable from the standpoint of utility.

c. Socialization.

Socialization means the development of the we-feeling in

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1. Snedden, David, Educational Psychology, pp. 17-18. The Century Company, New York, 1923.

associates and their growth and capacity and will to act together.[2] It is the attainment of proper relations with other people in activities and attitudes.[3] This process is affected by a great variety of conditions and circumstances. Socialization takes place in all groups, but especially in those that are small enough to assure much personal intimacy.[4] The home, the school, the state, and the church are social institutions.

2. Specific social values of geometry.

The social values of geometry are boundless. We shall mention a few of the groups of social values.

a. Importance for understanding the universe.

In the process of conquering and subduing the earth man has been forced to learn something about the universe in which he has been placed. He has gradually learned that there are laws governing nature and he must conform to these laws in his conquering process, which is still going on. Our knowledge of the most fundamental facts with regard to the universe is very largely due to mathematical investigations,[5] and geometry has been a chief element in the mathematics involved. Geometry furnishes a scientific basis whereby the universe and the laws governing it may be studied. Without such knowledge it would be at best only guessing.

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2. Ross, E. A., Principles of Teaching, p. 375. The Century Book Company, New York, 1930.
3. Avent, Joseph E., Excellences and Errors in Teaching Methods, p. 372. Published by the author, Knoxville, Tennessee, 1931.
4. Snedden, op. cit., p. 168.
5. Moore, Charles N., "Mathematics in the Future", Mathematics Teacher, Volume XXII(April, 1929), p. 205.

The domain of mathematics is the realm of certainty. There and there alone prevail the standards by which every hypothesis respecting the external universe and all observations and all experiments must be finally judged. It is the domain to which all speculation and thought must repair for chastening and sanitation. It is the court of last resort. This may be said for all intellection whatsoever, for it is there that mind attains its highest estate.[6]

b. Importance in understanding other mathematics.

Mathematics is so connected and interwoven in its nature that one branch of it cannot well be mastered alone. Even in arithmetic many things are taken from geometry. After one has learned geometry, he can then better understand that part of arithmetic which had proved very difficult. It is also very closely connected with algebra; and trigonometry is a part of geometry. A thorough knowledge of geometry is essential to the mastery of the higher branches of mathematics. It seems to be an essential part of the foundation of a master mathematician.

c. Importance in connection with other school subjects.

If a pupil has learned geometry as he should, he will likely to some degree use similar methods to master the contents of other school subjects, since he may have learned good study habits. He will be better prepared to analyze and evaluate, as well as to use better, the processes of logical thinking.

6. Rankin, W. W., "The Cultural Value of Mathematics", Mathematics Teacher, Volume XXII(April, 1929), pp. 215-216.

He can directly apply the knowledge he has learned in geometry in some of his other school subjects, especially in science. The correct language he has learned in geometry should help him in learning to use more accurate English. He will understand many of the terms used in other subjects, since many have been taken from mathematics including geometry. Again, to the extent that geometry has elements in common with other subjects and to the extent that the teacher makes the pupil conscious of them as such,-to that extent will geometry be a real help to the pupil.

If one has succeeded in learning geometry, he will be encouraged to learn other subjects, which, no doubt, he has been reliably informed are not nearly so difficult as demonstrative geometry. His success in geometry may give him self-confidence, which is essential in learning any subject.

d. Importance in the arts and occupations.

Geometry is more important in the arts and occupations than one believes until he is led to consider the question analytically.

The field of mathematics and art have been regarded by some as unrelated. They say that geometry is the domain of logic, and that aesthetic appreciation is the realm of art. But many claim that they have extended relationship.

For one to find geometry in art, it is only necessary for him to notice the wide-spread use of simple geometric forms in certain types of repeating designs, as well as in other designs.

The figures most commonly used in repeating designs are: the equilateral triangle; the square; the circle; symetrical figures;and polygons fitted together to cover completely a plane surface.[7]

At the World's Fair, Chicago, 1933, in the mathematics exhibit, there was placed in the center of the rotunda of the Hall of Science an immense octagonal prism, on the face of which were the subdivisions of mathematics. Slides illustrating the development and history of algebra and geometry were given.

On one face was given: "Pure mathematics consists exclusively of deductions by logical principles from logical principles - Bertrand Russell".[8]

It is well to be drawn away from the idea that mathematics is a science apart, for it should be looked upon as the science which binds together all the arts of man. Well did Cicero say, "All arts which relate to mankind have a certain common bond". Whether or not we exaggerate geometry sufficiently in our minds to make it seem to be this common bond, at any rate, we may be so assured of the relation of geometry to esthetics in its varied forms, as to instill into the minds of our pupils the consciousness that such exists. If we cannot give a precise account of common characteristics, let us consider the words of Lord Balfour, one of the great philosophers

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7. Bradley, A. D., The Geometry of Repeating Design, and Geometry of Dsign for High Schools, pp. 3-57. Bureau of Publications, Teachers College, Columbia University, New York, 1933.
8. Moore, L., "Mathematics Exhibit, World's Fair, Chicago", Mathematics Teacher, VolumeXXVI(December, 1933),pp.482-6.

among modern statesmen, when speaking of the two great divisions of human emotions, as given by Smith:[9]

> "Of highest value in the contemplative division is the feeling of beauty; of the highest value in the active division is the feeling of love. ..... Love is governed by no abstract principles; it obeys no universal rules. It knows no objective standards. It is absolutely recalcitrant to logic. Why should we be impatient because we can give no account of the characteristics common to all that is beautiful, when we can give no account of the characteristics common to all that is lovable?"

We should not be impatient because we can only feel the bonds that unite mathematics and esthetics and do not have the power to express the law of union.

1) Art. Art consists of producing an effect on the observer of beauty of form, and also of beauty in color combinations. Beauty of form is the highest art. The Greeks recognized this and left the world the best that has been produced in art. Those who have been careful to draw conclusions from nature, where the Great Geometer has wrought such perfection in form and where the laws of proportion and symmetry produced forms most perfectly, have been most successful in art. Nature furnishes the basis for color combinations, and it does not happen by accident; for if it did, instead of one rainbow, we might have factorial 7, or 5,040 rainbows.

The Greeks knew the laws of symmetry and proportion. It has been said that they knew the proportion of the members of the human body and worked it into their art.

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9. Smith, D. E., "Esthetics and Mathematics", Mathematics Teacher, Volume XX(December, 1927), pp. 427-428.

In 1840, Dr. Hay made a study of Greek vases and human form. He used the composite ellipse, the ellipse with three foci, to produce curved lines of vases, which were quite similar to Greek vases. He produced curves in this way which were also quite similar to the human form.[10]

The realm of the geometry of the beautiful is so vast that we can hardly be expected to mention even the salient points of contact. The unlimited field of geometry in relation to the beautiful includes what might be designated as the fine arts, or the more expressive phrase of the French, beaux arts. Painting, for example, might be considered with reference to the works of that great genius in science, in mathematics, and in art, Leonardo de Vinci. Sculpture might well be included because of the mathematical principles employed by that majestic user of ponderous masses, Michaelangelo. Engraving, with that gifted artist of Nurnburg, Albrect Durer, who published the first modern work on curves. Decoration, with reference to the geometric designs for all ages and reaching their highest degree of perfection in the works of the Moslems, may be included. So may literature be included with reference to the mathematics of poetry and the poetry of mathematics. Indeed, we may include the beauties of nature, where geometry plays a part, of which we are usually quite unconscious.[11]

2) <u>Architecture.</u> In the broadest sense, architec-

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10. Rankin, <u>op. cit.</u>, p. 219.
11. Smith, <u>op. cit.</u>, p. 419.

ture includes all the structural designs. It may be thought of as the crystalization of human intelligence into geometric forms, or conclusions of the mind geometrized. We cannot say how much mathematics goes into it, but skill lies in the use of mathematics and then not have it visible when the work is completed. The geometry used in construction, as the finished product is observed, must not be seen with the eye but appreciated by the intellect and emotions.

An example may be found in the work of Sir Christopher Wrenn, the noted Oxford professor of mathematics, who rebuilt ecclesiastical London.

3) Transportation and communication. In transportation and communication man has called on mathematics, including geometry more heavily for conclusions than in any other human endeavor. His conquest of nature here is due largely to modern machinery, a monument to mathematics.[12]

The discovery and improvements of modern steamships, no one will dispute, involved the principles of geometry, and a knowledge of the principles of geometry has been just as essential in the construction of railroads and locomotives. The telegraph, the telephone, and the radio have likewise had their underlying geometrical and mathematical principles. The following expression of R. W. Emerson is quoted by Rankin:[13]

> "We do not listen with the best regard to the verses of a man who is only a poet, nor to his problems if he is only an algebraist; but if a man is at once acquainted with the geometrical foundation of things and with their festal splendor, his poetry is exact and his arithmetic musical."

12. Rankin, *op. cit.*, p. 221.
13. *Ibid.*, p. 218.

4) Banking and commerce. Commercial mathematics in which geometry holds a place is rapidly becoming a field of wide research. Laws of trade relations when thoroughly understood reduce to formulas. The day of the world's finance has passed that can be carried on by arithmetic only. Business is a science so far as it is capable of using mathematics.[14]

5) Medicine. In the history of mathematics it is noticeable how many great characters were interested in both mathematics and medicine. More than one hundred have been mentioned in the sixteenth century alone. Leonardo de Vinci, Robert Recorde(Royal Physician), who gave us our sign of equality, Lillio, physician to Gregory III(his suggestion being adopted for revising the calendar), are examples of noted physicians who were mathematicians.

Modern medicine is built upon conclusions drawn in the laboratory where mathematics sits on the throne and hands down the decisions.

Geometry is essential to the study of optics. Dr. Alexis Carrel of the Rockefeller Institute of Medical Research has discovered a formula for the healing of a wound involving its surface. If the wound does not heal according to this mathematical law, the doctor knows at once that complications have arisen and is thereby warned to locate the trouble and to remove it.[15]

14. Ibid., p. 222.
15. Ibid., pp. 222-223.

6) Music. Pythagoras, perhaps, the greatest name in the history of geometry, is regarded as the inventor of music. He considered music applied to numbers. Our music comes from mathematics. The following is quoted from Rankin.[16]

"Mathematics and music which are the most sharply contrasted fields of scientific research that can be found, are yet related and support each other, as if to show forth the secret connection which ties together our mind, and which leads us to surmise that the manipulations of the artists' genius are but the unconscious expressions of a mysteriously acting rationality.

"Musicians feel mathematics, mathematicians think music. Music is the dream, and mathematics is the working life."

7) Other occupations. Geometry enters to a greater or less degree into many of the other occupations of man.

Political economy and sociology and the other social sciences are becoming scientifically understood to approach a mathematical basis. As physical, mental, and moral laws become reduced more and more to a mathematical basis, the science and art of human engineering will become more and more scientific.[17]

The geometry involved in carpentry and most of the trades is obvious, and the custom is established to learn geometry as a part of the preparation for their skillful mastery.

Civil, mining, and electrical engineering are skilled occupations definitely requiring mastery of the principles of geometry.

In the occupations a knowledge of geometry seems to tend to broaden one's outlook on life in his chosen field.

e. Geometry of the environment.

The world in which we live is incurably mathematical. Ge-

16. Ibid., pp. 219-221.
17. Ibid., p. 223.

ometry is in our environment to a much greater degree than we are conscious of our geometrical surroundings. The more we learn about geometry the more we may become aware of its principles all about us. Every human being is born into a physical universe in which quantity, shape, and size play an indispensable role. The geometric principles of equality, symmetry, congruence, and similarity are implanted in the very nature of things. We cannot make or manufacture a single article without giving constant attention to its form, its dimensions, and the proper relation of its parts.[18]

The rainbow, the sunset, and the beauties of nature are very largely geometrical.

3. General social values of geometry.

a. Universal language of geometry.

The general principles and ideas of geometry remain unchanged. Since geometrical facts are universal there is no more powerful influence to develop a universal language. Due in part to the fact that the study of geometry has developed with the development of the civilization of the race, the language of geometry has been carried into other fields.

Mathematical terms and symbols including the geometrical are not confined to any rank or race of people. They defy limitation by languages, but they have a universal language of their own.

The German, the French, the English, and the Turk, all use

18. Reeve, W. D., "The University of Mathematics", Mathematics Teacher, Volume XXLII (February, 1930), p. 79.

the universal language of geometry, of course, modified to suit the needs of each. Geometry is universal.

b. Influence upon the development of civilization.

The learning of geometry and the developing of civilization seem both to have begun in Egypt, the gift of the Nile. As civilization extended and for a time centered and developed to a high degree in Greece, so did the study of geometry.

Mathematics having geometry as a sound basis seems to have developed along with civilization all down the ages. The advances made all along the way seem to have had a mathematical foundation.

The historical development of the fundamental ideas of arithmetic, algebra, and geometry presents a beautiful illustration of the process of evolution in the realm of thought. The continuity of apparently isolated problems and processes reveals the working of law in the unfolding of mathematical ideas. In algebra and in arithmetic, but to a less extent in geometry, the development of the fundamental ideas has taken place in fairly recent times, enabling one to trace the gradual unfolding of the scientific ideas involved in these subjects. The present appears as a part of a long past, directly connected with the civilizations of the Orient and the Occident.[19]

Mathematical methods and results obtained by them are so interwoven in our modern civilization that if it were possible

19. Karpinski, L. C., "The Parallel Development of Mathematical Ideas, Numerically and Geometrically", School Science and Mathematics, VolumeXX(December, 1920), p. 821.

to remove the contributions due directly and indirectly to mathematics, very little would remain.[20]

When Kepler reviewed the planetary kinematics he found the deductive geometry of the conic sections awaiting the wide range of physical applications later gathered. Then, roughly speaking, from Newton to Fourier there followed an epoch when mathematics and physical theory grew for the most part together, quite largely under the same leaders. There were germs of mathematical principles arising in physical problems and their developments reacting on physical theories.[21] The following facts are given by Gingery:[22]

> "Men have entered mathematical fields and found they could grow in them harvests of unprecidented usefulness. The field first occupied by Clark-Maxwell, Ohm, Faraday, and Hemholtz blossomed under the tillage of Morse, Bell, and Marconi into modern communication; and under Steinmetz, Edison, and Westinghouse into modern electrification of industry.
>
> "Verily the clothing we wear, the medicine we take, the radio programs we hear, the conveyances we ride in, the cosmetics we consume, and the highest intellectual experiences of our lives originate in mathematics.
>
> "The wilderness entered and subdued by Lavoisier, Avogadro, Gay-Lussac, and a host of hardy pioneers is now being cultivated by DuPont, Firestone, Goodyear, and the Ely Lilly Company. The harvests now gathered are blessing, and perhaps damning, mankind in such a multitude of ways that neither time nor intelligence will allow cataloguing them."

Volumes could be written on the geometry used in warfare and in building modern death-dealing war machines, including battleships, submarines, and airplanes. Powerful missiles, deadly explosives, and poisonous gases have mathematics including geometry as a scientific basis. So we see that man may

20. Moore, op. cit., p. 204.
21. Lunn, A. C., "Experimental Science and World Geometry", Science, Volume LXIII(June, 1926), p. 585.
22. Gingery, W. G., "How Much Mathematics Should We Teach?", School Science and Mathematics, Volume XXXIII(November, 1933), p. 830.

use and has used mathematics to destroy his fellow-man as well as to bless mankind.

4. Development of sociology of geometry.

a. Stages of development.

In Egypt the learning of geometry was confined almost exclusively to the priest caste. The priests included geometry in their mysticism. They used it to augment their power over a superstitious people. So we see geometry is social only in the group or class of the Egyptian priests.

Thales and Pythagoras visited Egypt and learned Egyptian geometry from the priests of that country. They returned to their native land and established schools or brotherhoods similar to our secret fraternities. The learning of geometry was social in these groups, but it seems not to extend outside of these groups. The school of Pythagoras seemed to be organized very much like our Masonic order.

Geometry was further socialized at Athens and later at Alexandria. The influence of the Elements of Euclid seemed to widen the social field of geometry.

The early Europeans taught Euclid in their universities. Some, of course, gave it a wider place in the course of study than others. Here geometry seemed to have a wider social value.

Geometry first found a place in the secondary schools of Europe and then in America, and, now, it seems to be studied almost universally in the secondary public schools and to some extent in the elementary schools. The social values have reached the masses of the people, especially, in America.

From the beginning of civilization geometry has had a place with the ruling and educated classes; now its social values have widened to include nearly all normal people, who are considered educated.

Teachers of geometry and those who know its social values are seeking to have its principles function more effectively in the lives of the pupils who study geometry. They are trying to make it possible for young people to learn it psychologically and in later life to apply it sociologically. Thus it would be to themselves and to others of the greatest value.

b. Studies in the sociology of geometry.

Most of the books and articles written on the teaching, learning, and applications of geometry include the sociology of geometry. The sociology of geometry is a fundamental reason for its place in the education of the race from the dawn of civilization to the present time.

The value of geometry has been pointed out by the educated from the Greek philosophers to educators of most recent times. Recently the sociology of geometry has been questioned as has been true of the other subjects taught in public schools. The outcome of the authoritative studies made in geometry has strengthened its place in the curriculum. Teachers of geometry should be able to convince the inquiring mind of youth concerning the sound sociology of geometry, so that the pupil will be assured that he will be richly repaid for the mental effort expended in learning geometry.

The studies made in the sociology of geometry are too numerous to attempt to summarize, but the reader may infer from the bibliography of this chapter and other chapters the importance of the sociology of geometry.

5. Socialization of the program of geometry.

a. Aims of the socialized program.

Subjects that can command a place in the curriculum of our modern high schools must be rich in their possibilities for social values. We shall give some of the aims of the socialized program of geometry.

1) Appreciation. This term(from the Latin, ad, to, and pretium, price) refers to the act of appreciating, setting a value on, or rating properly.[23]

There are some definite appreciations that may be acquired in geometry. The beauty of geometric forms in nature, art, and architecture should be general for all who study geometry. Learners of geometry should appreciate the symmetry, the similarity, and the regularity of figures. They should appreciate space relationships. hey should come to be able to appreciate the value of logical reasoning and the value of mathematical proof.[24]

Classes in geometry should be led to appreciate what tireless efforts and unflagging zeal have been given by great minds in the past centuries to learn geometric truth.

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23. Avent, Joseph E., The Excellent Teacher, p. 198. Published by the author, Knoxville, Tennessee, 1931.
24. Breslich, Ernest R., The Technique of Teaching Secondary School Mathematics, p. 206. The School of Education, University of Chicago, 1930.

2) Information. The socialized program in geometry should certainly include gaining information. Thorndike tells us that "Whatever exists at all exists in some amount".[25] It is just as true that whatever exists at all exists in some form or relation. Pupils should understand the fundamental concepts of geometry, the units of measurements, space forms, the mensuration formulas, the important axioms and theorems, and the relationships in space forms.[26]

Intelligent speakers and writers who wish to be exact use geometric terms and concepts so much that it is necessary for people to know the content of geometry in order to understand the architecture of buildings and the arrangements of land forms for parks and other grounds. Students should know thoroughly that the hypothesis and the conclusion are contained in a given theorem. The logical unfolding of the proof should be well understood. In fact, the information that may be obtained in geometry is used to such an extent in life that one is richly repaid for learning geometry.

3) Utilization. Geometry has always had a practical as well as a scientific value. It is useful in the mastery of other school subjects, especially other subjects in mathematics. One writer[27] believes it should be studied before algebra. The mathematician must build it well into the foundation of

25. Thorndike, E. L., The Seventeenth Year Book of the National Society for the Study of Education, Part II, p. 16. 1918. Public School Publishing Company, Bloomington, Illinois.
26. Breslich, op. cit., p. 207.
27. Judd, C. H., The Psychology of High School Subjects, p. 21. Ginn and Company, New York,

his endeavors. Its principles apply in life both consciously and unconsciously. From its incorporation into the civilization of Egypt to the present time, learners of geometry have wanted to know what will be the worth of geometry in the practical world and in their lives. In the socialization of the program of geometry the practical worth should be given a wide range. The pupils should be made to see its value in their school life and in the world of affairs.

There can be no social values unless young people can use the information they have gained. They must be able to apply the knowledge and skills learned or their lives cannot count for much in social betterment.

b. Means of socialization.

1) The socialized curriculum. The content of the curriculum and the way it is made to function in the lives of youth has much to do in bringing about socialization in general. The curriculum should contain the things needed by the pupils to enable them to accomplish most for the welfare of society. It is not enough to inspire young people to accomplish all the good they can in the world; they must be prepared in a definite way for service to humanity. The curriculum determines in a large measure what this preparation shall be. No subject that is not valuable from a social point of view should be retained in, or added to, the curriculum.

The curriculum should be so selected that the fullest preparation for service to humanity may be given the youth of the land. Their usefulness will largely depend upon their prepara-

tion. The mathematicians are convinced that the values of geometry are so socialized that it cannot be omitted from the curriculum without serious loss to youth in its equipment for life.

2) Socialized method. How best to train pupils for social usefulness has been the central problem of recent times. Much has been done in the psychology of learning, and much yet remains to be done.

A teacher's success depends very much upon his ability to employ devices and procedures in the presentation of his subject that will make the instructional materials the permanent passession of the learner. Some of these devices and procedures have been so widely adopted and successfully used that they have become known as methods of teaching. So, we speak of the lecture method, the textbook method, the drill method, the genetic method, the question-answer method, the project method, and the blackboard method. Each denotes a more or less exclusive use of a teaching procedure which, under suitable conditions, leads to superior results.

The success of a method depends upon a number of external factors, such as the age of the pupil, the former preparation of the class, the type of subject-matter, the teacher's personality, mastery of the subject, his methods, and the available physical equipment. The choice of methods is influenced by the aims to be accomplished.

Even the best method should not be used to the exclusion of all other methods. Variety is essential to successful learning. Combinations of the good features of the different methods

will more likely secure better results than the exelusive use of any one method.

a) Lecture method. The lecture method is not used to a very great extent in geometry. At times, of course, the teacher should make explanations to the class of units, assignments, and purposeful organization of material.

b) Question-answer method. the question-answer method can be used to advantage in geometry to stimulate the learner to think and to recall. It is a challenge to his best efforts and can arouse hid interest. The question should be asked the class and then the individual designated to answer.

c) Heuristic method. In the heuristic method the teacher, by means of a series of carefully worded questions, leads the pupil to understanding and discovery. It is an individual method and has little place in socialized geometry teaching.

d) Genetic method. The genetic method is very valuable in learning geometry. It is well adapted to the development of new subject-matter. The teacher guides the class as a group, giving information, teaching, supplementing the textbook, leading the pupils to make new discoveries, and teaching them how to attack and solve new problems. The pupils contribute freely. They are free to ask questions and to state their difficulties. They study and are taught at the same time.

It can be used successfully only by a skilled teacher. If correctly used, it gives the teacher his best opportunity

to arouse interest, to develop appreciation and pleasure, and to teach methods of attacking and solving problems and demonstrating theorems.

e) Laboratory method. The laboratory method is very effective, if the necessary equipment is used. It stresses the appreciation of geometry. The pupil learns by doing. He may work alone or in a small group. It is an excellent for developing the meaning of new concepts and principles.[28]

f) Problem method. The problem method is the most natural method in learning geometry. The principles of geometry to be learned may be so interwoven with the solution that it becomes most impressive upon the evolving mind of youth. Mathematics, and geometry especially, does not lend itself very well to the problem method in regard to large units. This is sometimes called the project method.

Whichever methods or combinations of methods are used, they should be employed from the social point of view. If the school is to approach the ideal, there must be socialized class procedure, which is an attempt to bring into play the social activities of the pupils. Participation of pupils is the keynote of socialized class procedure with the philosophic aims of developing social and intellectual habits, skills, and attitudes, as well as to better the pupils' mastery of subject-matter.[29] The hope in general of social progress is the

28. Breslich, op. cit., pp. 29-34.
29. Mueller, A. D., Teaching in Secondary Schools, pp. 270-271. The Century Company, New York, 1928.

substitution of co-operative effort for antagonistic conflict.[30]

Of the socialized methods available for the teaching of geometry we may say that the socialized recitation, in which all pupils participate, contribute, and mutually receive contributions and the project method which utilizes the principle of purposeful activity in groups are probably the most useful. The groups should not be large. The problem method may be either social or individual and has large place in geometry. This method is highly preparatory for life, if its technique be consciously mastered. Life is full of problems. Geometry cannot withdraw from contributing its part of the technique of meeting them.

3) The socialized school. The socialized school is one in which the school officials, the teachers, and the pupils work together in harmony to make the school serve the needs of the pupils and the community. Its purpose is to train the pupil to the extent of his ability for participation in the affairs of the community and the activities of life.

The socialized school gives the necessary training in general socialization methods, some of which are: democracy, the doctrine of shared relationships; co-operation, working together; assembly exercises; social service, which, in school, involves the help-one-another attitude. It also meets the requirements of socialization through class work, some of which

30. Beach, W. G., An Introduction to Sociology and Social Problems, p. 175. Houghton Mifflin Company, Boston, 1925.

are: free well-directed conversation, debates properly prepared and conducted, socialized recitations, in which all take part, and socialized instruction, the responsibility for which is upon both the textbook and the teacher.[31] Geometry affords opportunity for training in such life methods, and students should be made conscious of them as life methods at the time.

4) Professionalization of teaching. The teachers of antiquity, including Athens, were in general of a low social class. This does not apply to the philosophers and the teachers of higher education in Athens who held a very high rank. Oftentimes the teachers were old slaves called pedagogues.[32] Still, by the misfortunes of war a "slave" might be intellectually the peer or the superior of his master. Even in the eighteenth century the social status of teachers was universally low.[33] Their rise in society seems to have taken place simultaneously with, and in proportion to, their professional training; and government-supported schools have arisen from the charity type to those now educating the masses. The professionalization of teaching is a long story, and the climax has not yet been reached. As long as young men who are just through high school or upon leaving college use teaching as a "stepping stone" to their chosen work in life, and as long as girls do the same thing or remain teaching only until they are married, so long will the true professionalization of teaching be difficult. Our recent advance in the require-

31. Avent, op. cit., pp. 372-379.
32. Cubberley, E. P., The History of Education, p. 24. Houghton Mifflin Company, Boston, 1920.
33. Ibid., p. 446.

ments for preparation and training of teachers has done much toward making teaching a profession, for those who do not want to make teaching their life's work will not be willing to meet the requirements; and those who do will be more successful because of the professional training received. Professional training of teachers is the transmission of the experiences of successful teachers to those who wish to become teachers.[34] Professionally trained teachers who have a professional spirit seem to be accomplishing a great deal toward making teaching a profession. If geometry is to become socialized in the full meaning of that term, it must be taught by a professionally trained teacher. The teacher must have mastered geometry not only academically, but professionally as well. He must have learned something of the history of geometry, its sociology, its psychology, the main methods of teaching it, its aims and objectives in life, and the varying content of it. He must have learned, too, the ability and maturity of pupils required to master it, and their individual differences and difficulties of learning it. He must know the technique of geometry essential to success, he must carefully plan for the activities of the class period, he must start the class period promptly and eliminate waste of time, he must select and organize the subject-matter for study, he must be able to give the pupils a broad view of the unit to be studied, and he must know how to supervise the pupils' work.[35] To do these things well obviously requires the knowl-

34. Avent, Joseph E., Beginning Teaching, p. 155. Published by the author, Knoxville, Tennessee, 1931.
35. Breslich, op. cit., pp. 1-27.

edge and skill of a professionally trained teacher of geometry.

6. Place of geometry in the experience of the race.

a. Utility.

The learning of geometry probably began in Egypt, where its principles were used in surveying the land along the Nile.[36] The Egyptians studied practical geometry only. In Greece it was developed into a true science, where geometry was learned for its own sake.[37] Since the age of Grecian supremacy, civilization has found practical applications for nearly all the principles of geometry that Euclid wrote in his Elements .

The leading countries of Europe, except England,[38] study geometry from a view-point of practical utilization. In America the learning of geometry for its practical utility is gaining favor as one of the leading reasons for its mastery.

b. Convention.

Convention has had and has a very great influence upon the learning of geometry. The civilizations of Egypt and Greece have not been lost to the race, but, in fact, they have become the foundation of our civilization. Since geometry played such an important role in Egypt and Greece, other

36, Wells, Webster, and Hart, W. W., Plane Geometry, pp. 2-3. D. C. Heath and Company, Boston, 1915.
37. Smith, D. E., History of Mathematics, Volume I, p. 59. Ginn and Company, Boston, 1923.
38. Stamper, A. W., A History of the Teaching of Elementary Geometry, pp. 104-129. Teachers College Series, Columbia University, New York City, 1906.

people have studied geometry. Perhaps convention was the strongest factor for its retention in the schools of the early peoples of Europe, and, doubtless, it is a strong force in giving geometry its high place in present-day curricula. Throughout the progress of the race most people have not attempted to learn the extensive social values of geometry, and they, therefore, have turned to convention for the establishment of its importance. When a thing is valuable, if a learner is as yet unable to become conscious of such value either personal or social, if the learning of that thing may be conventionally motivated for him, convention may at the same time become utilitarian.

c. Preparation.

Geometry is the realm of absolute truth concerning form, position, and magnitude, and it has been in the possession of the race since the beginning of civilization to some degree. The Greeks developed it almost into its present completeness. So the race has had the truths of geometry as a working basis upon which to build. Geometry has been a faithful handmaiden to the great minds who have done most in giving us our civilization as it exists today. The preparation for life and for other avenues of study which it has furnished to the race is, perhaps, inestimable. It does have preparatory value for certain other mathematical studies.

d. Discipline.

The learning of geometry is claimed, as perhaps no other subject, to train the mind to reason, and we have stated else-

where that there may be transfer of training of the processes of reasoning to the extent of common elements consciously mastered. since geometry belongs in the realm of absolute truth, learning it may increase in the mind the tendency to long for and to seek for the truth in other kinds of endeavor. geometry has done much to train the race to seek for and to preserve the truth.

7. <u>The geometry of the future</u>.

Mathematicians seem to have narrowed the applications of geometry in order to hold it within the domain of mathematics. Its principles certainly operate in the field of mathematics, but they may be given a far wider range.[39] Geometry is the simplest material available for the thinking process; it is both deductive and inductive. We are told that the lost manuscripts of Euclid belonging to the *Elements* treated of fallacies. How different the teaching of geometry might have been today, if these manuscripts had been preserved. The geometry of the future will likely be interwoven with simple materials dealing with the history of thinking in general and in particular with the history of thinking as revealed in the development of geometry.[40] It will likely become more and more socialized in both content and method. Where would the builders of the Norris Dam be without geometry? The geometry of the future will be no less important in the domain of mathematics, but its

39. Blackhurst, J. H., "Geometry in the Schools of Tomorrow", *Educational Administration and Supervision*, VolumeXVIII (September, 1932), p. 459.
40. *Ibid*., p. 460.

functions will be extended to make it of even greater social value. The following may be a good example of a theorem on one phase of the future socialized geometry, which is given by Mr. Rudman:[41]

"THEOREM

"People should buy within their means when buying on the installment plan.

"Given:-
A family of five who have an income of $60 a week. The rent is $55 a month. A salesman for a car worth from $1500 to $2000 wishes to sell a car to the family and take in the trade the used four-cylinder f mily car.

"Prove:-
It is unwise for the family to make the change.

"Analysis

A major domestic venture of this sort, the purchase of a new expensive luxury, should not be embarked upon if it appears that:
1) It would be an undue strain upon the family budget.
2) It would drain the reserve funds of the family to the extent of imperiling its security.
It can be proved that it threatens both 1) and 2).

"Proof

Statements	Reasons
1. A bigger car brings with it a substantial increase in expenses.	1. Insurance, interest on loaned money, running expenses, repairs, new parts, and depreciation in the long run are about twice those in connection with the smaller car.
2. This results in an undue strain on the family resources.	2. The family has been living near its means(Inferred from Given).

41. Rudman, Barnet, "The Future Geometry", Mathematics Teacher, Volume XXV(January, 1932), pp. 27-30.

3. The lack of economic security to the individual and the family is the major curse of this industrial civilization.	3. It is at the mercy of the industrial machine, which would crush mercilessly.
4. The reserve fund is the only security.	4. It enables them to stem the tide.
5. The reserve fund would be sadly depleted.	5. There would be about $1200 to be paid the first year.
6. Security would be jeopardized.	6. The position may be lost or the one making the income may become disabled.

∴ It is unwise to make the change.

Q. E. D.

It is improbable that geometry will be discarded in education, but it is very probable that it will become more socially educative. It will perhaps be charged with the task of injecting into man's thinking and life's problems the very elements that are sadly missing. Some day man may decide to purge his processes of thinking from the prejudice, bigotry, and intellectual dishonesty, which have distorted his vision and substitute objectivity, honesty, dispassionate inquiry, open-mindedness, and steadfast refusal to accept for truth nothing but the truth. He may commission geometry to provide his new manner of thinking.[42] Man's needs may impel him to develop more fully the social possibilities of geometry.

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42. *Ibid.*, p. 32.

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16. Moore, L., "Mathematics Exhibit, World's Fair, Chicago", Mathematics Teacher, Volume XXVI(December, 1933), pp. 482-486.

17. Mueller, A. D., Teaching in Secondary Schools. The Century Company, New York, 1928.

18. Rankin, W. W., "The Cultural Value of Mathematics", Mathem tics Teacher, Volume XXII(April, 1928), pp. 215-223.

19. Reeve, W. D., "Universality of Mathematics", Mathematics Teacher, Volume XXIII(February, 1930), pp. 71-83.

20. Ross, E. A., Principles of Teaching. The Century Book Company, New York, 1930.

21. Rudman, Barnet, "The Future Geometry", Mathematics Teacher, Volume XXV(January, 1932), pp. 27-32.

22. Smith, D. E., "Esthetics and Mathematics", Mathematics Teacher, Volume XX(December, 1927), pp. 419-428.

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25. Stamper, A. W., A History of the Teaching of Elementary Geometry. Teachers College Series, Columbia University, New York, 1906.

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CHAPTER III

AIMS AND OBJECTIVES

The aims and objectives in teaching geometry are no less important than in the other subjects of the curriculum. Their utilization is perhaps more essential to the learning of geometry than in most of the other subjects, because of the generally believed inherent difficulty of geometry. If the aims and objectives are properly understood and employed, the pupil will feel that the effort to learn geometry is wisely expended; and he will thereby be encouraged to do his best work.

The function of the school is to produce desirable changes in pupils, and the dictum of the teacher is to lead the child from the place where he is to the place where we think he ought to be.[1]

Plato believed that geometry would create the mind of philosophy, because the knowledge at which geometry aims is of the eternal and not of the perishing and transient. Upon this belief he founded his famous entrance requirement, "Let no one who is ignorant of geometry enter here". The Pythagoreans held that geometry and number possessed the key to the riddle of the universe.[2]

1. Davis, Calvin O., Our Evolving High School Curriculum, p. 8. World Book Company, Yonkers, New York, 1927.
2. Winger, R. M., "Mathematical Objectives", Mathematics Teacher, Volume XXII(December, 1929), p. 463.

We quote the following from Mr Winger:[3]

"The study of mathematics fosters careful, accurate, and sustained thinking, stimulating the while thinking itself. It strengthens the reason, develops the power of generalization, cultivates the imagination, and brings one face to face with chaste but naked truth. It was perhaps Spinoza who said in substance that if mathematics-unlike history and politics - had not been independent of personal interest, the world should never have known truth".

A teaching aim is a goal of achievement in educational work, a statement of a result to be accomplished, the establishment of a point to be reached, or the determination of a task to be done. Teaching aims should be concrete, definite, specific, particular. The whole aim of education is the making of efficient persons.[4] The objectives of teaching are always either attitudes or acquired abilities. Attitudes are either attitudes of understanding or attitudes of appreciations.[5]

The excellent teacher harmonizes the immediate aims of the lesson with those of the course and the fundamental objectives of education. He selects proper materials for instruction, secures order and continuity in them, unifies the work of teaching, and provides definite standards of progress. Whatever may be the form of the specific aim, if it is made central in the teaching unit, its conservation of all efforts against

3. Ibid., p. 465.
4. Avent, Joseph E, Beginning Teaching, Eighth Edition, pp. 69-72. Published by the author, Knoxville, Tennessee, 1931.
5. Morrison, Henry C., The Practice of Teaching in the Secondary School, Second Edition, p. 19. The University of Chicago Press, Chicago, Illinois, 1931.

wastefulness is insured from the beginning.[6]

Life is a never ending process of making adjustments to the world about us and to other people. The teacher should provide many and varied opportunities for the expression of the pupil's basic tendencies, and he or she may also furnish skillful guidance that the student may express himself in ways that are socially desirable.[7]

The aims and objectives of teaching have in general ten phases. In other words they operate in ten divisions, which are as follows: physical, moral, vocational, economical, recreational, civic, industrial, commercial, and religious. The intellectual may be given as the tenth division.

Reflective thinking is a general aim or goal of the teaching process. From a psychological point of view, thinking is a period of meditation in the higher nerve centers between the reception of an impulse and its discharge in some form bringing about mental equilibrium. From a mental standpoint, it is the period of reflection intervening between stimulus and reaction. Thinking is essentially problem-solving. We can say with confidence that, if we have given material to think about, a method of thinking, and a motive for it, any normal person will think within the limitations which his inherent organic mental structure determines.[8] So, we may be sure that the superior thinking

6. Avent, Joseph E., Excellences and Errors in Teaching Methods, p. 35. Published by the author, Knoxville, Tennessee, 1931.
7. Hockett, Mrs. Ruth Manning, Teachers Guide to Child Development, p.2. California State Board of Education, 1930.
8. Morrison, op. cit., pp. 32-34.

of the educated man is due to his real and richer experience and training. It is claimed that Dr. Eliot, former President of Harvard University, said that training youth to think is the first and foremost function of our schools. He said that they should be taught to observe keenly, to reason soundly, to imagine vividly, and to express ideas clearly and forcefully.[9] It is obvious that geometry lends itself, as perhaps no other subject, to the realization of this important objective.

1. Traditional aims.

a. Stated.

Learning geometry because the educated people of the past centuries have included geometry in the list of subjects they studied is a strong reason to continue its retention in the curriculum. In every age the mastery of geometry has been believed to be of fundamental importance. The Egyptians studied it for the help it gave them in understanding their environment and for improving it. The Greeks studied it for its truth and beauty. Previous to the eighteenth century in Europe, geometry has been taught dogmatically. The students worked by rule and learned by heart. The dogmatic method was formally discredited in the nineteenth century, though it remained to an extent in England and the United States, where pupils had a tendency to learn by heart. Three aims were apparent: the practical aim, the logical aim, and the preparation for the study of advanced

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9. Davis, C. O., Our Revolving High School Curriculum, p. 72. World Book Company, Yonkers, New York, 1927.

mathematics.[10]

b. Evaluated.

Traditional aims for learning a subject may result in a drawback to the individual or society. Conditions may change and render the subject obsolete. There may be other subjects, not studied that are of far greater value, thus in a measure, wasting the time of the pupil, which he may use in pursuit of more useful information. Traditional in themselves are poor objectives in the pursuit of an education. Geometry should be studied for what it may mean to our civilization, rather than because it has been studied in the past centuries.[11]

2. Disciplinary aims.

The purpose of education according to the disciplinary theory was the strengthening of the reasoning powers, not to furnish useful information.[12] Ideals for efficient work in one subject may be carried over to other subjects, but the subject that has the most applications is the one that has the most disciplinary value.[13] The extended applications of geometry to life's situations largely give it its disciplinary value. The disciplinary theory of education has become very much restricted.

10. Stamper, A. W., A History of the Teaching of Elementary Geometry, pp. 101-103. Teachers College Series, Columbia University, New York City, 1906.
11. Herrick, C. A., "What High School Subjects Are of Most Worth?", School and Society, Volume IV (August 26, 1916), p. 309.
12. Finney, R. E., The American Public School, p.21. The Macmillan Company, New York, 1921.
13. Ibid., p. 282.

a. Disciplinary aims stated.

Breslich[14] gives the following disciplinary aims of mathematics contained in the Report of the National Committee on Mathematical Requirements. It is clearly seen that all these aims can be applied to geometry.

> "1) The acquisition in precise form of the ideas or concepts in terms of which the quantitative thinking of the world is done.
> "2) Development of the ability to think clearly in terms of such ideas and concepts. This involves training in analysis of a complex situation, recognition of logical relations, and generalizations.
> "3) Acquisition of mental habits and attitudes.
> "4) The idea of relationship and dependence."

b. Disciplinary aims evaluated.

The disciplinary value of geometry lies most of all in the exercise of reasoning powers. The reasoning in geometry is similar to that in real life.[15] We are given by James[16] that thought tends to be a part of personal consciousness, that it is always changing, that it is sensibly continuous, that it always appears to deal with objects independent of itself, and that it is interested in some part of these objects to the exclusion of others. The study of geometry is very well adapted to this stream of thought.

True mental discipline may be thought of in two ways. In the first way, a subject like geometry is disciplinary in value to the extent of control developed in the individual,

14. Breslich, op. cit., p. 197.
15. Ibid., p. 198.
16. James, William, The Principles of Psychology, Volume I, pp. 224-290. Henry Holt and Company, New York, 1918.

either over the processes of his own thinking, or over the events in his environment. The degree of personal control is yet a matter of faith or personal estimate. The degree of objective control may be observed, but not exactly measured, in the control over situations involving geometrical characteristics in the external world of planning, building, and measuring. In the second way, geometry is of disciplinary value to the extent that its own elements are identical with other elements in life processes, as of planning, building, and measuring. Without such mental discipline architects and builders would lack some of the essential necessities of their vocations.

3. Practical aims.

Our age demands action. It is no longer sufficient only to think logically. Thoughts are considered of little value, unless they are used to direct actions and to carry out movements. Modern educational leaders have turned the searchlight of practical value upon the subjects contained in the curriculum, and geometry has no reason to withdraw from the criterion proposed.

a. Practical aims stated.

Breslich[17] puts the practical aims of geometry as given by the National Committee on Mathematical Requirements in one condensed paragraph. They are:

> "Familiarity with the geometric forms common in nature, industry, and life; mensuration of these forms; development of space perception; exercise of spatial imagination."

17. Breslich, op. cit., p. 197.

Breslich[18] lists the practical uses of mathematics as given by Young. They will apply to geometry as practical aims.

"The practical aims of mathematics are: the intimate connection with everyday life, its use in occupations, its informational value, mathematics in nature."

b. Practical aims evaluated.

The practical aims of learning geometry perhaps have done more than any other kind of aims to hold geometry in its coveted place in the curriculum during the recent educational movements to evaluate critically the subjects taught in the secondary schools. The subject-matter and the methods of teaching have been given a more practical setting in geometry as well as in other subjects.

Slocum[19] sets out the following summary of the values or aims of learning geometry.

"Geometry as a science rests on universal agreement as to certain facts of experience and intuition, which gives it a basis of certainty and conviction not shared by any other subject of elementary instruction. The mental attitude which this implies is the vital feature which distinguishes geometry from experimental science like chemistry and physics, as well as from an empirical study of languages, or from one accepted on authority like geography or history. The growing mind needs to dwell on some form of absolute truth in order to develop a balanced judgment which shall be able to distinguish between the arbitrary and conflicting claims and assertions which are met in life, and no form of truth is more clear cut and distinctive than the logic of geometry.

"Upon this foundation of absolute certainty geometry erects an unassailable structure of pure thought, limited by no imperfections of measurements or other human inaccuracies. If properly taught not all is presented at once

18. Ibid., p. 199.
19. Slocum, S. E., "Geometry in the Elementary School", Educational Review, Volume LIV(October, 1917), pp. 266-267.

as a complete system, but is evolved gradually by each individual, thereby making the processes of thought an integral part of his mental equipment. The practical utility of learning geometry may be considered a by-product of sufficient value for its mastery.

"Practical utility means measurements by commercial standards and in terms of adult reality. Educational utility is measured by the vividness which a principle impresses on the mind of the pupil.

"In general utility resides in any course of instruction which serves to co-ordinate mental images, develop the reasoning ability, and add materially to the power of self-expression. Geometry is peculiarly adapted to serve these ends and should occupy a larger place in the curriculum."[20]

There are two requirements for the efficient and scientific teaching of geometry. First, it must be determined what constitutes the world of reality for the particular pupils concerned. Second, utility must be determined by the reality of the impressions conveyed.

There are many educational leaders, whose power is being felt, who are insisting that our schools should teach only those subjects which can be shown to have value from the consideration of present needs.[21] They emphasize the practical values of learning geometry as well as other subjects. In recent years the aim of education has become one of economics. The pupil is constrained to train for the work he is to do in life.[22] Thus, we see that the practical aims of learning geometry are now being seriously considered.

4. Cultural aims.

20. *Ibid.*, p. 266.
21. Herrick, *op. cit.*, p.309.
22. *Ibid.*, p. 305.

We are told by Rankin[23] that culture is an intelligent interest in the past, present, and future achievements of man. He quotes Matthew Arnold as saying, "Culture is knowing the best that has been thought and said."

Everything is of cultural value which makes life fuller and richer; which places us in greater harmony with our surroundings; which gives us a better understanding of nature; which contributes to individual growth and the development of the race.[24] Geometry has made a tremendous contribution to orientation both in nature and in life. It was said by Paul Monroe in one of his classes at Columbia University: "Any process of orientation is cultural", whether it be vocational, avocational, practical, religious, or otherwise.

When we have acquired unreserved confidence in the autonomous power of our human mind, it is then we have gained the firm fundamental upon which all deductive sciences may be built, and the independence of thought then cannot be refuted by either textbook or teacher.[25] Geometry contributes materially to this acquisition.

a. Cultural aims stated.

Breslich[26] states, from the National Committee on Mathematical Requirements, the following cultural aims:

"1. Acquisition of appreciation of beauty in geometrical

23. Rankin, W. W., "The Cultural Value of Mathematics", *Mathematics Teacher*, Volume XXII(April, 1929), p. 215.
24. Kemper, A. J., "The Cultural Value of Mathematics", *Mathematics Teacher*, Volume XXII(March, 1919), p. 127.
25. *Ibid.*, pp. 130-131.
26. Breslich, *op. cit.*, p. 198.

forms.

"2. Ideals of perfection as to a logical structure, precision of statement and thought, logical reasoning, discrimination between true and false.

"3. Appreciation of the power of mathematics."

These objectives clearly apply to geometry.

Breslich[27] classifies the objectives of teaching geometry under "powers", "appreciations", "understandings", "attitudes", "habits", and "skills".

"Geometric power:
1) To recognize forms in nature, art, and industry.
2) To understand and make two, and three dimensional drawings.
3) To use correct speech in discussions and proofs of geometric facts and principles.
4) To do neat and accurate work with geometric drawings.
5) To measure directly and indirectly.
6) To exercise spatial imagination.
7) To use symbolic notation in proofs and diagrams.
8) To analyze geometric situations.
9) To discover new geometric facts.
10) To recognize geometric relations.
11) To attack and solve problems of space.
12) To reason correctly.
13) To establish geometric facts by proofs.
14) To use a variety of methods of proof.
15) To make the fundamental geometric constructions.

"Appreciations to be acquired in geometry:
1) Of the beauty of geometric forms in nature.
2) Of symmetry, similarity, and regularity of figures.
3) Of space relationships.
4) Of the value of a logical reasoning.
5) Of the mathematical proof.

"Understandings in geometry:
1) The fundamental concepts of geometry.
2) The units of measurements.
3) Space forms.
4) The mensuration formulas.
5) Important geometric theorems and axioms.
6) Relationships in space forms.

"Attitudes:
1) A degree of interest in mathematics which will

27. *Ibid.*, pp. 203-208.

encourage the pupil to continue in the study.
2) Desire to read mathematical literature growing out of the pleasure to be derived from such reading.
3) Desire to make precise statements.
4) Desire to estimate in advance the solution of a problem.
5) Desire for thoroughness and clearness.
6) Willingness to concentrate on problems.
7) Desire to analyze complex problem situations.
8) Desire to carry tasks to completion.
9) Desire to do neat written work.
10) Desire to think logically.
11) Attitude of inquiry.
12) Desire to make discoveries.
13) Desire to grow mentally to improve former records.
14) Desire to constantly improve one's methods.
15) Desire to concentrate.
16) Desire to understand.
17) Desire to generalize.
18) Desire to assume responsibility for the assigned task.
19) Desire to pursue the study of mathematics.
20) Considering the study of mathematics a pleasure.
21) Self-confidence in studying mathematics.

"Habits and ideals:
1) Of usefulness.
2) Of persistance.
3) Of concentration.
4) Of observation.
5) Of participation.
6) Of neatness in written work.
7) Of accuracy.
8) Of thoroughness.
9) Of clearness.
10) Of precision.
11) Of interpreting results.
12) Of using good language.
13) Of checking results.

"Skills:
1) In arithmetical computations.
2) In algebraic processes.
3) Using the instruments of geometry.
4) In making graphs.
5) In solving equations and formulas.
6) In solving problems."

Under"attitudes", "habits", and "skills", Mr. Breslich includes arithmetic and algebra with geometry.

Mr. Reeve gives the following objectives in teaching[28] demonstrative geometry:

"GENERAL OBJECTIVES

I. Develop an understanding of:
   1. The need for proving statements.
   2. Difference between intuitive and demonstrative geometry.
   3. Meaning of:
      a. Axiom. c. Theorem. e. Proposition.
      b. Postulate. d. Corollary. f. Converse.
   4. Axioms, postulates, and definitions as basis of proofs.
   5. The various forms of geometric proofs as follows:
      a. Direct. c. Analytic. e. Indirect.
      b. Inductive. d. Synthetic. f. Deductive.
   6. The various steps in a geometric proof.
   7. Statement and proof of the converse of a theorem.
   8. Analysis of originals.
   9. Geometric constructions.
   10. Fundamental relationships.

II. To acquire habit and develop power for:
   1. Logical thinking.
   2. Reasoning.
   3. Induction from original problems.
   4. Critical attitude.
   5. Correct speed.
   6. Neatness and accuracy in constructions.

III. To develop an appreciation of:
   1. The human worth of rigorous thinking.
   2. The practical value of mathematics in life.
   3. The aesthetic value of mathematics."

b. Cultural aims evaluated.

Sister Alice Irene[29] outlines an experimental course in plane geometry from a cultural point of view. She had tenth grade pupils in a high school for girls.

Miss Irene had had ten years' experience mostly in

28. Reeve, W. D., "Objectives in Teaching Demonstrative Geometry", Mathematics Teacher, Volume XX(December, 1927), pp. 435-436.
29. Irene, Sister Alice, "Some Objectives to Be Realized in a Course in Plane Geometry", Mathematics Teacher, Volume XXII(December, 1929), pp. 435-446.

girls' schools. She had found girls discouraged and disappointed with mathematics; they seemed to be taking geometry only because it was required for graduation.

At the beginning of the year she at first tried to make the girls feel a fascination for geometry by telling them of its age and dignity and of its contributions to civilization. She told them about geometry in Egypt, about Euclid, and about Plato's entrance requirement. She explained its connection with the idea of good or God. She told about Pythagoras, who considered it the "sublime music of the spheres"; about Omar Kayyam, who added luster to Persian mathematics by his treatise on Euclid; about Michaelangelo and Leonardo de Vinci, who made contributions to mathematics only a little less than those to art and sculpture; about Durer, who published the first treatise on the theory of curves; and about Napoleon, who said, "The advancement and prosperity of a nation are closely joined to the perfection of mathematics".

She informed the girls that they would have the opportunity to learn something of the great subject of geometry. By careful study of its demonstrations and problems, they would develop the habit of logical thinking, and they would learn to discriminate between the true and the false. All sounded beautiful, but the learning of two propositions a day, which was necessary to cover the work required; working as many originals and applications as possible and taking tests and examinations at stated times made quite another story. June counted up its A's, few, if any; B's slightly increased; a large number of C's and D's, and, alas, too many F's. What a falling

off in registration for mathematics the following year!

She felt something should be done and went to her principal. The principal said, "Well, don't worry over the situation, for I am perfectly satisfied. Mathematics is too hard for most girls. It requires a peculiar ability, which few of them seem to have, and it has little practical value for women. .... Perhaps it is just as well a large majority of them spend their time on cultural subjects."

Her tests showed that not many could demonstrate theorems or do originals. She had no way to test for cultural values, and she had not succeeded in making the girls like geometry.

She searched every possible source to find a plan by which she could make geometry interesting. The aim of her final program was to popularize geometry by establishing a favorable attitude and by breaking down prejudice and dread. She intended to build up interest and enthusiasm as well as a desire to continue the study of mathematics. She meant to accomplish these in an informal way through contact with nature.

At the beginning of the next year after the customary formality of the first class period, without further ado about books and supplies, she told her class they were going to see a new geometrical world. The girls were willing to admit that there was geometry in houses, churches, and all other buildings; but geometry in trees, flowers, animals, the sun and stars, to them seemed ridiculous. On the way she talked of

Thales, Pythagoras, and Plato.

The first day they learned the names of the plants in the garden, the congruence of these plants, and their similarity of form. The second day questions were asked on the work of the day before. The girls took notes. Symmetry of plants was studied for about two weeks, and specimens were gathered to save. Each day notes were kept, which were in five divisions. First, new terms; second, definitions and illustrations;third, geometric ideas; fourth, informal proofs and intuitive references; fifth, historical notes or references. When they became more interested in abstract geometry, they read literature on the subject.

On an excursion they visited a Chapel, which was rich in all manner of symbolic ornaments. These were geometrical and their beauty was due to their perfect symmetry. In a rose window symmetry was pointed out, and the girls unanimously wanted to draw them. Soon they had mastered the five fundamental constructions and discovered the three conditions for the congruence of triangles.

The girls were pleased with themselves and had a good opinion of their geometric ability, and by the middle of November they were ready to begin the Second Book, which treats of circles. In the Montgomery Ward seed catalog were found interesting applications where circles figured in flowers. The daisy has a well defined circular center, and the chrysanthemum revels in concentric circles of alternating colors. There was interest even in studying the flowers themselves.

The girls discovered the horizon to be a huge circle, as

well as the outline of the sun. The teacher told them that someone had said, "The handwriting of God is the pageantry of the skies." She said that according to the nebular theory, the spiral was the first symbol to appear, the circle or its near neighbor, the ellipse, the second. The next curve was traced by a visitor, the comet, which came into our system from a far distant realm, infinity, circled around our sun and left us again, usually, never to return. The path of the comet is a parabola. She told them that if the study of mathematics is continued, much will be learned about these and other curves, a knowledge of which has given us much information about the universe. She found Andromeda and Orion to be very interesting to the girls.

The teacher assigned short themes and found the time well spent. She had a few days to spare before beginning Book Three, scheduled for the opening of the second semester.

Examples of regular polygons were pointed out in flowers during visits to the flower garden. Lilies, narcissus, jonquils, and asphodel are examples of the hexagon. Snow crystals are hexagonal. Violets, apple blossoms, four o'clocks, forget-me-nots, morning glories, and nasturtiums are examples of the pentagon; love-in-a-mist, the heptagon; cosmos, the octagon; daisy, the general polygon.

By the end of May all had been completed which were required in the minimum essentials by the National Council of 1923 and much solid geometry in addition.

Twenty four out of twenty eight had reached the median in Schorling Sanford Achievement Test.

A great majority expressed pleasure in the study of plane geometry, and a part who had been in the class wished to continue the study of mathematics.

The foregoing example of how girls can be made to see and appreciate the cultural aims and objectives in learning geometry is only one of the many possible instances. Cultural aims and objectives can be used perhaps more effectively in leraning geometry in the case of boys. The inquiring mind of youth naturally seeks to know truth, and geometry is a pure fountain for truth to which young people may come to satisfy their thirst for exact knowledge. Cultural aims and objectives, which have been such powerful factors in attracting the great minds of past ages, have not lost their magnetic power upon modern minds seeking truth.

5. Relation of geometry to the general aims of education.

If the general aims of an education can be definitely determined, then the aims of geometry must not conflict with them, but geometrical aims must be included by the general educational aims.

a. General aims of education.

The general aim of education is the end toward which the educative process is moving; and whatever may be the end, it is reached through the sum total of all the experiences of the individual.[30]

30. Hopkins, L. T., Curriculum Principles and Practices, p. 36. Benjamin H. Sanborn and Company, New York, 1929.

There are many definitions of an education stated by noted men. Some are here given.[31]

"The true aim of education is the attainment of happiness through perfect virtue" - Aristotle.

"The aim of education is the forming of a complete man, skilled in art and industry" - Rabelais.

"Education is the organization of acquired habits of action such as will fit the individual to his physical and social environment" - William James.

"We educate a child in order that he may be prepared to live a normal satisfactory life for himself, and may contribute his full share to the progress and betterment of mankind" - Eugene R. Smith.

"Education is the social agency which trains youth so that it may secure satisfaction through activities which are governed by the ideals that society thinks are valuable" - Charters.

The "Cardinal Principles of Secondary Education" formulated by the "Committee on the Reorganization of Secondary Education" under the direction of the National Educational Association are most commonly adopted as the general objectives of an education. They are ennumerated by Breslich[32] as follows:

"1. Health of individual and community.
"2. Command of the fundamental processes.
"3. Worthy home membership.

31. *Ibid.*, pp. 50-54.
32. Breslich, *op. cit.*, p. 194.

"4. Vocational guidance and preparation.
"5. Citizenship in a democracy.
"6. Worthy use of leisure time.
"7. Ethical character."

Proctor and Riccardi state:[33]

"Material should be so selected and presented by the teacher that the mastery of these materials on the part of the student will develop in him the ability and desire: first, to maintain health and develop physical fitness; second, to acquire command of the fundamentals; third, to create worthy home membership; fourth, to become an efficient citizen; fifth, to employ leisure time profitably; sixth, to prepare for a vocation; and seventh, to develop high standards of ethics."

b. Geometric contributions to general aims of education.

No one subject of the curriculum could be expected to contribute equally to each of the "Seven Cardinal Principles". Geometry contributes a great deal to some of them, but less, of course, to others.

1) Contributions of geometry to health. There is little that health owes directly to the training in geometry. The training geometry gives in logical thinking may mean much indirectly to health, for by carrying into effect thinking, illness may be prevented and health promoted.

2) Contributions to the command of the fundamental processes. Learning geometry develops useful skills, which aid in the command of the fundamental processes. The desire to be concise, definite, and accurate is a useful attitude which geometry fosters. This setting must be maintained in mastering the fundamentals of an education. The fundamental

33. Proctor, W. M. - Riccardi, Nicholas, The Junior High School, p. 155. Stanford University Press, Stanford University, California, 1930.

processes of language and measurement are aided materially by the exactness and precision required in geometrical study.

3) Contributions to worthy home membership. The love for truth has been cultivated in the study of geometry making one faithful and true to his home, which is the very foundation of civilization. The great verities of life seem to be definitely connected with the truth as learned in geometry and the ideals of the home life of the most worthy members of society. One who loves and seeks truth will most likely be a worthy member of his home. True character, which is perhaps the greatest element in society, is developed and perfected in the home.

4) Contributions to vocational guidance and preparation. In vocational guidance and preparation geometry plays a very important role. The business of the world is founded upon credit, and truth is the soul of credit. Either directly or indirectly geometry holds a very strong place in the educational preparation for nearly every calling in life. It broadens the horizon and gives the learner a deeper insight into the principles of truth upon which the work of the world is based.

5) Contributions to citizenship in a democracy. No greater qualification for good citizenship than proper regard and love for truth can be possessed. Individuals and nations must be truthful to each other if they would dwell together in peace and prosperity. The fate of a nation depends the industry, intelligence, and character of the citizens who shape its destinies. The learning of geometry has a wholesome and benefi-

cial effect upon the lives and characters of youth and manhood. Geometry was developed and appreciated by the leading citizens of Greece, that great democracy of antiquity.

6) Worthy use of leisure. Geometry does not seem to lend itself to leisure to so great an extent as history or literature, but there may be much learned in leisure time concerning the men who have developed and promoted the science of geometry. Much leisure time should be spent in intellectual pursuits and in concentrated thought, a foundation for which is laid in geometry.

7) Contributions to ethical character. Geometry has much to offer here. Again we have the influence of truth upon the forming character of youth. There is a reward for earnest application.

The study of geometry tends to train the mind in earnest, thoughtful, reflective application. It influences one to become a seeker after truth. It prepares the mind for research work. It enables one to think through a situation to a logical conclusion. It cultivates perseverence and mental endurance. It lays the foundation for the formulation and establishment of theories.

The learning of geometry influences one to become dispassionate and unselfish. Through the logical thinking one is led to consider properly the rights, opportunities, and ambitions of others. It aids individuals and groups in solving life's situations. It would provide the unimpulsive basis for people to dwell together in unity. It would become a substantial aid in enabling one to reach a plane of life upon which

he could apply the Golden Rule in his relations to others.

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CHAPTER III

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CHAPTER IV

THE CONTENT OF GEOMETRY

1. Content of old geometry.

a. Meaning of old geometry.

By "old geometry" we mean the geometry used and taught previous to the work of the great psychologists, who began the movement to modernize the curriculum, and to make the subjects contained therein psychological. We refer to the geometry taught before methods were intelligently studied and analyzed.

There are two textbooks which seem to have had the greatest influence upon the teaching of geometry from the beginning of the use of textbooks until very recent times. These were the Elements of Euclid and the textbook in geometry by Legendre.

b. Euclid.

Euclid completed and arranged in an orderly manner the work of his predecessors in his "Elements" about 300 B. C.[1] The treatise was written on parchment or papyrus and rolled up. Since it was so difficult to handle large rolls, it was divided into smaller ones called books. Euclid wrote thirteen

1. Stamper, A. W., A History of the Teaching of Elementary Geometry, P. 27. Teachers College Series, Columbia University, New York, 1906.

books, and two more books were added about one hundred fifty years later probably by Hypsides and by Damascius respectively. While Euclid selected from the material on geometry which had accumulated during the two centuries preceding himself and following the death of Pythagoras, no doubt some of the work was original with him.[2]

Sequence of subject-matter in the "Elements of Euclid": Book I, congruence, parallels, and the Pythagorean Proposition; Book II, identities, treated geometrically, now treated algebraically. The first two books treated in general of the straight line and areas; Book III, circles; Book IV, inscribed and circumscribed polygons, regular figures; Book V, the regular polygons; Book VI, similarity of polygons; Books VII, VIII, and IX, arithmetic treated geometrically; Book X, incommensurable magnitudes; Books XI, XII, and XIII, solid geometry.

The historical development of geometry does not follow the sequence of Euclid. His purpose was a minimum of logical friction. He eliminated all practical work. Mechanics was divorced from geometry largely by Plato. Euclid made no distinction between theorems and problems, and he gave no original exercises; he classes them all as propositions. He used no hypothetical constructions. He began his first book with constructions, which is of pedagogical value, but he carried it to the extreme. In his constructions he used the straight edge and the compasses only. He had no intuitive geometry as

2. Smith, D. E., History of Mathematics, Volume I, p. 105. Ginn and Company, Boston, 1923.

an introduction to the logical geometry. Euclid was a book to be read, not one to be developed; it was a philosophic treatise to be read by mature minds. We are indebted to Euclid for:[3]

1) General ennumeration of the proposition.
2) The particular statement.
3) The construction.
4) The proof.
5) The conclusion.
6) Affixing Q. E. D. or Q. E. F.

Well founded criticisms of Euclid's Elements:[4]

1) Euclid made no distinctions between propositions requiring demonstrations and those which a logician would see to be nothing but different modes of stating a previous proposition.

2) He often failed to employ generalizing notions, for example, in defining an angle as the sharp corner between two lines that meet; he did not include the reflex angle.

3) He neglected the formal accuracy with which translators have endeavored to invest the "Elements". He referred to theorems in an indirect manner by either reasserting without reference, or by saying "It has been demonstrated". He placed theorems among definitions, made assumptions not in postulates, and omitted necessary proofs.

c. Legendre's textbook in geometry.

Euclid had been preserved by the Arabs. It had undergone many translations, and other books had been written, but the

3. Stamper, *op. cit.*, p. 30.
4. *Ibid.*, p. 31.

most important one was written by Legendre near the close of the seventeenth century. It was one of the best textbooks ever written on geometry. It had wide-spread popularity in Europe, outside of England, and in the United States, where it largely supplanted the Elements. Legendre sought to separate the theorems from the problems and to simplify the proofs without lessening the rigor of the ancient methods of treatment.[5] Legendre's text differs from Euclid in sequence and in the reference to arithmetic and algebra. Hypothetical constructions were permitted. Legendre made the first logical departure from Euclid which the world recognized.[6]

2. <u>Content of modern geometry</u>.

Euclid has furnished the modern world the model for geometry texts; but with the establishment of modern school systems, the expenditure of thought and effort in building up the present day curriculum, and the concentration of great minds in seeking to make textbooks and methods psychological, there have been wrought many changes and improvements in most countries in textbooks on modern geometry.

a. <u>Euclid in England</u>.

Geometry seems to have been studied first in England in the universities by using Euclid written in Greek as a text. In 1570 Sir Henry Billingsby translated Euclid from the Greek into English. Texts written by Barrow, Gregory, and Whiston, which were translations of Euclid, were principally used until

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5. Smith, *op. cit.*, p. 488.
6. Stamper, *op. cit.*, p. 82.

Robert Simson edited Euclid in 1756. Later texts were based on his geometry. The texts of Playfair(1756) and Todhunter (1862) became very popular in England and America. The practical was not combined with the logical, as had been true on the Continent of Europe. In England Euclid reigned supreme.[7]

After the invention of differential calculus by Newton, the English universities turned their attention chiefly to the new mathematics, but the secondary schools were slow to take up the study of geometry. The secondary schools of England had not commonly adopted the study of geometry until about the middle of the nineteenth century, and then it was of the extreme Euclidean type.[8]

There are four classes of secondary schools in England, and it is not easy to distinguish between them. They are: the Public Schools, the highest class with pupils from the wealthy and aristocratic homes; the Grammar Schools with pupils from the middle classes; the Preparatory Schools, which fit boys and girls for the higher classes of the Public Schools and the Grammar Schools; the Technical Institutions, which prepare for the universities and industrial life. The Technical Institutions seem to be affiliated with the universities often as preparatory schools. Recent rigid entrance requirements caused geometry to be stressed.[9]

Geometry is also taught in some of the higher elementary schools of England, where the course is divided into Euclid

7. Ibid., pp. 62-63.
8. Ibid., pp. 87-89.
9. Ibid., p. 120.

and practical geometry. Euclid seems to be little more than introduced, but the practical including solid geometry is stressed.[10]

Recently some of the leading educators in England including Mr. Perry have been strongly recommending that the character of geometry taught in that country be changed from the Euclidean to a more practical form, but tradition in England is so powerful that the influence of Euclid upon the teaching of geometry is still very great.[11]

b. Geometry in the United States.

In the United States elementary geometry is taught principally in the high school. Oftentimes practical geometry is given in the elementary school with a view for laying the foundation for logical geometry in the high school. Geometry in the high school is strictly logical, but Euclid as such is not given.

1) As shown in textbooks.

a) Textbooks listed. There are many textbooks written and used on plane geometry, and almost as many, on solid geometry. The author of a textbook on plane geometry often writes one on solid geometry, which may be published as a separate volume or combined with the plane geometry.

Moriarty[12] examined more than seventy American textbooks on geometry for secondary schools, fifteen for junior high schools, and some foreign texts for the purpose of determining the accuracy of speech and the simplicity of demonstration.

10. Ibid., p. 121.
11. Ibid., p. 126.
12. Moriarty, M. M. S., "Geometry Notes", Mathematics Teacher, Volume XXI (May, 1928), pp. 280-291.

Under the next topic six textbooks as types are outlined:

AUTHOR	NAME OF TEXT
1. Wentworth, G. A. - Smith, D. E.	Plane Geometry
2. Wentworth, G. A. - Smith, D. E.	Solid Geometry
3. Wells, Webster - Hart, W. W.	Plane Geometry
4. Strader, W. W. - Rhodes, L. D.	Plane Geometry
5. Farnsworth, R. D.	Plane Geometry
6. Blackhurst, J. Herbert	Humanized Geometry

We think it unnecessary to use the space to list the enormous number of textbooks written on geometry. Under the "Abbreviations in the States" in this work, pp. 126-143. will be found texts used in the various states mentioned.

b. List of what is given in textbooks.

1) Content of Plane Geometry by Wentworth-Smith.[13]

a) Introduction, pp. 1-24.

Nature of arithmetic-Nature of algebra-Nature of geometry. Twenty five definitions- Instruments of geometry, straight edge and compasses. Twenty five exercises in construction with instruments. One page showing necessity for proof. Six pages of definitions, explanations, and exercises-Eleven axioms and six postulatesgiven-Six corollaries given-One page of algebraic exercises.

b) BookI. Rectilinear figures, pp. 25-92.

Theorem proved, if two lines intersect, the vertical angles are equal. Triangles, pp. 26-45: triangles classified and defined; the three theorems of congruency of triangles with special cases and exercises of applications; seven theorems involving triangles and a page of original exercises. Parallel lines, pp. 46-58: five theorems with six corollaries given;the theorem, the sum of the three angles of a triangle is equal to two right angles, is proved and three corollaries are given; two pages of originals; five theorems concerning the sides and angles of a triangle are proved. Quadrilaterals, pp. 59-67: one page of definitions;seven theorems and eight corollaries

13. Wentworth, G. A. - Smith, D. E., Plane Geometry. Ginn and Company, Boston, 1913.

on the parallelogram. polygons, pp. 59-67: one page of definitions; two theorems proved, two corollaries, and two pages of originals concerning the exterior and interior angles of a polygon; Loci, pp. 68-72: two pages of definitions and explations of loci; two theorems and a corollary on the perpendicular bisector of a line; twelve pages of originals, one page of examination questions, and one page of review questions at the end of Book I.

c) Book II. The circle, pp. 93-150.

One page of definitions; eight theorems and six corollaries on chords and arcs; one page of definitions and originals; a theorem on the tangent proved, and three corollaries given; one page of originals; three theorems, one on parallel lines intersecting equal arcs, one on three points determining a circle, and one on a tangent from an external point, are proved; one page of originals; two theorems on line of centers of two circles are proved; five pages of definitions, originals, and explanations; two theorems and two corollaries on measurement of central angles and inscribed angles; two corollaries; one page of originals; three theorems on measu ement of angles formed by intersecting chords, a tangent and a chord, and two secants, a secant and a tangent, or two tangents; one page of originals; two pages of explanations of construction; five problems of construction; six originals; two problems and one corollary on constructing an angle equal to a given angle, and dividing a line into equal parts; five originals; two theorems and two corollaries on constructing triangles; two problems and two corollaries on a circle inscribed in a triangle and circumscribed about a triangle; two problems on drawing a tangent through a point to a circle, and describing a segment of a circle in which an angle may be inscribed; a page on explanation of methods of solving problems; eight pages of originals and one page of review questions at the end of Book II.

d) Book III. Proportion, pp. 151-164. Similar polygons, pp. 165-90.

Proportion: one page of definitions and explanations; eight theorems and two corollaries on proportion; one page of originals; one theorem and two corollaries on parallel lines dividing the sides of a triangle proportionally; four original exercises; three theorems and one corollary on lines that divide the sides of a triangle proportionally being parallel, the bisectors of the interior and exterior angles of a triangle dividing the opposite sides in proportion to the adjacent sides; a page of original exercises. Similar polygons: a page of definitions and explanations; four theorems and two corollaries on similar triangles; one theorem on perimeters of similar polygons having the sa e ratio as the corresponding sides; twelve original exercises; three theorems and four corollaries on similar polygons; a page of original exercises; four theorems and one corollary on chords, secants, and tangents of a circle; a page of originals; five problems and one corollary on dividing lines proportionally; three pages of original exercises; one page of review questions.

e) Book IV. Area of polygons, pp. 191-226.

Theorems, pp. 191-210: one page of definitions and explanations; three theorems and one corollary on rectangles; a page of original exercises; three theorems and five corollaries on the area of a parallelorgam, a triangle, and a trapezoid; a page of original exercises; four theorems and four corollaries on similar polygons including the Pythagorean Proposition; a page of original exercises;two theorems on areas constructed on the sides of a triangle; a page of originals; a theorem on the sum of the squares on two sides of a triangle compared to the square constructed on the third side. Problems, pp. 211-226: a problem on computing the area of a triangle in terms of its sides; two pages of originals; eight problems on constructing polygons; two pages of problems of construction; a page of theorems; a page of review questions at the end of Book IV.

f) Book V. Regular polygons and circles, pp. 227-260.

Theorems, pp. 227-241: one page of definitions and explanations; two theorems and nine corollaries on inscribed and circumscribed polygons; a page of originals; two theorems and two corollaries on regular polygons; a page of original exercises; five theorems and six corollaries on regular polygons; a page of original exercises. Problems, pp. 242-260: three problems and five corollaries on inscribing a square, a hexagon, and a regular decagon in a circle; four original exercises; two problems on finding the sides of an inscribed polygon of double the number of sides of a given inscribed polygon, and finding the ratio of the circumference of a circle to its diameter; ten pages of original exercises including one page of examination questions.

g) Appendix, pp. 261-282.

Symmetry, pp. 261-264: a page of definitions and explanations and importance given; two theorems on symmetrical figures and the center of symmetry; a page of originals. Maxima and minima, pp. 265-272: two definitions; seven theorems and one corollary on maxima and minima, six on maxima and one on minima; a page of original exercises; the originals apply to maxima and minima. Recreations of geometry, pp. 273-276: nine recreational propositions, every triangle is isosceles, part of an angle equals the whole angle, pa t of a line equals the whole line, two perpendiculars from a point, 1 equals 0, if two sides of a quadrilateral are equal it is isosceles, History of geometry, pp. 277-281: one page of formulas.

2) Content Solid Geometry by Wentworth-Smith. In order to make connection with the plane geometry written by the same author, eleven axioms, the postulate of superposition, and seventy three theorems numbered as in the plane geometry are given.

a) Book VI. Lines and planes in space, pp. 273-316.
A page of definitions and explanations; the postulate of planes, only one plane can be drawn through two intersecting line, and three corollaries; seven theorems and seven corollaries on lines and planes in space; a page of original exercises; one theorem and two corollaries on parallel lines and planes; four original exercises; six theorems and five corollaries on perpendicular and parallel lines; a page of original exercises; two pages of definitions and explanations on dihedral angles; nine theorems and three corollaries on dihedral angles and intersecting planes; a page of original exercises; a theorem and a corollary on a perpendicular between two lines not in the same plane; five original exercises; a page of definitions and explanations on polyhedral angles; two theorems on the sums of the face angles of a trihedral angle and on any convex polyhedral angle; a page of original theorems; a page of problems of computation; a page of review questions.

b) Book VII. Polyhedrons, cylinders, and cones, pp. 317-380.
Polyhedrons and prisms, pp. 317-321: two pages of definitions and explanations; two theorems and two corollaries on sections of prisms cut by planes; a page of original exercises; Parallelopipeds, pp. 322-336: a page of definitions and explanations; three theorems and two corollaries on congruent prisms; four original exercises; a theorem on equivalent triangular prisms; a page of original exercises; four theorems and five corollaries on comparison of rectangular parallelopipeds and the volume of any rectangular parallelopiped; a page of original exercises; two theorems, two corollaries, and eleven problems on volumes of prisms; a page of original exercises. Pyramids, pp. 337-349: two pages of definitions and explanations; three theorems and three corollaries on the lateral area of a regular pyramid,edges and altitudes divided proportionally and a section similar to the base by a plane parallel to the base, and equivalent triangular pyramids with equal altitudes and equal bases; three original problems; two theorems and two corollaries on volumes of a triangular pyramid and any pyramid; two pages of original exercises; a theorem and two corollaries on the volume of three triangular pyramids formed from the frustum of a pyramid. Regular polyhedrons, pp. 350-352: two pages of definitions and explanations; a page of original exercises. Cylinders, pp. 353-361: a page of definitions and explanations; five theorems and six corollaries on cylinders; a page of original exercises. Cones, pp. 362-376: two pages of definitions and explanations; three theorems and one corollary on sections of cones and the lateral area of a cone of revolution; a page of original exercises; a theorem on the volume of a circular cone; three theorems and two corollaries on areas of cones and frustums of cones; a page of original exercises;

four pages of original propositions and one page of review questions.

c) Book VIII. The sphere, pp. 381-430.

Plane sections and tangent planes of spheres, pp. 381-391: a page of definitions and explanations; seven theorems and eleven corollaries on the sphere, a circle cut by a plane, all points on the great circle of equal distances from the pole, a point the distance of a quadrant from two points is the pole of the circle through the points, tangent plane perpendicular to a radius, a sphere may be inscribed in or circumscribed about a tetrahedron, and the intersection of two spheres is a circle; a page of original exercises; a theorem and two corollaries on the measurement of a spherical angle. Spherical Polygons, pp. 392-409: a page of definitions and explanations; two theorems on the measurement of each side in terms of the other sides and the sum of the sides of the spherical triangle; seven original exercises; three theorems on the angles and the sides of spherical triangles; a page of original exercises; a page in explanation of symmetric spherical triangles; six theorems on symmetric triangles; four original exercises; two theorems on angles of spherical triangles and the shortest line on the surface between two points; five original exercises. Measurement of spherical surfaces, pp. 410-420: a page of definitions and explanations; two theorems and four corollaries on the area of the surface revolving about an axis and the area of the surface of a sphere; a theorem on the area of a lune; a page of original exercises; a theorem and a corollary on the spherical triangle equivalent to a lune; a page of original exercises. Measurement of spherical solids, pp. 421-423: a page of definitions and explanations; a theorem and three corollaries on the volume of a sphere. Exercises, pp. 424-430: two pages of problems of computation; a page of exercises on formulas; a page on problems of loci; two pages of miscellaneous exercises; a page of review questions.

d) Appendix, pp. 431-470.

Polyhedrons, pp. 431-443: a page of definitions and explanations; four theorems and two corollaries on a truncated prism being equivalent to three pyramids of which it is composed, volumes of two tetrahedrons with one dihedral angle equal respectively being to each other as the products of the three edges, the number of the edges increased by two being equal to the number of vertices increased by the number of faces, and the sum of the face angles of a polyhedron; a page of original exercises; four theorems and four corollaries on similar polyhedrons separated into the same number of similar tetrahedrons, volumes of similar tetr hedrons to each other as the cubes of the corresponding edges, and the volume of a prismatoid; seven original exercises. Spherical segments, pp. 444-448: a theorem on the spherical volume of a spherical segment; thirteen original exercises; three pages of examination questions. Recreations of geometry, pp. 449-452: eight

recreational exercises - prove any triangle isosceles, part of an∠equal to the whole angle, part of a line equal to the whole line, show that 1 equals 0, let fall two perpendiculars from a point to a line, and if two opposite sides of a quadrilateral are equal the figure is an isosceles trapezoid. Suggestions as to beginning demonstrative geometry, pp. 453-60: a page of general suggestions; inferences as to congruent triangles; examination of an inference; inferences as to isosceles triangles; further inferences; attacking an original exercise. Applications of geometry, pp. 461-465: purpose of geometry; reason for applications; application of demonstrations; application of general theorems. History of geometry, pp. 465-468: ancient geometry; early Greek geometry; Euclid; geometry in the East; geometry introduced into Europe; solid geometry. Table of formulas, pp. 469-470: seven formulas for line values; eight formulas for plane figures; eleven formulas for areas of solid figures; twelve formulas for volumes.

3) Content Plane Geometry by Wells-Hart.[15]

a) Introduction, pp. 1-28.

History of geometry, pp. 2-3. Informal preparatory geometry, pp. 4-28: definitions and explanations of plane geometry, solid geometry, a point, a straight line, a curved line, and a broken line; it is assumed and tested by the pupils that two points determine a line, that two straight lines can intersect in only one point, and that a straight line is the shortest distance between two points; the mid-point of a line is explained and determined; the circle and lines relating to the circle are defined and explained; the angle is defined and explained; the kinds of angles and pairs of angles are defined and explained; the pupils take part in the drawings and the explanations; it is assumed and explained by drawing that the sum of all the adjacent angles around a point on one side of a line is equal to one straight angle or two right angles, that the sum of the successive adjacent angles around a point is two straight angles or four right angles, and that complements of the sa e angle or of equal angles are equal; it is assumed and explained by drawing that if two adjacent angles have their exterior sides in the same straight line, they are supplementary; that if two adjacent angles have their supplements equal respectively, they are equal; that if two adjacent angles are supplementary they have their exterior sides in the same straight line; the protractor is defined, and the pupils are shown how to use it; the pupils are given exercises for doing experimental geometry; some objections to its use are pointed out and the need of demonstrative geometry is shown; the axiom is defined and fourteen axioms are stated; the theorem, the hypothesis, and the conclusion are explained; the

15. Wells, Webster - hart, W. W., Plane Geometry. D. C. Heath and Company, Boston, 1915.

the basis of proof, definitions, axioms, hypothesis, and previously proved theorems are explained; for an illustration the demonstration of the following theorem is given: if two straight lines intersect, the vertical angles are equal; a problem, a postulate, and a proposition are defined and explained; various supplementary exercises are given.

b) Book I. PP. 29-92.

Rectilinear figures, pp. 29-86: a page of definitions and explanations; a theorem on one of the general conditions for the congruency of triangles followed by three pages of exercises and constructions; another theorem on the congruency of triangles followed by a page of exercises and explanations; a theorem on the equal angles of an isosceles triangle followed by a page of original exercises and explanations; a problem on the construction of a triangle with the sides given followed by five originals; the third theorem on the congruency of triangles and a page of original exercises and explanations; a problem on bisecting an angle and three original exercises; a theorem on the perpendicular bisector of a line and four originals; two problems on constructing the perpendicular to a line at a point in the line,and on constructing a perpendicular to a line from a point not in the line; a page of originals and explanations; a theorem on the exterior angle of a triangle being greater than either of the opposite interior angles, a corollary, and eleven originals; a page of definitions and explanations on angles formed by parallel lines being cut by a transversal; two theorems, a problem, and two corollaries on parallel lines cut by a transversal; three pages of originals and explanations; a theorem, five corollaries, and five originals on the sum of the angles of a triangle; a theorem on angles with their sides perpendicular and two originals; a theorem on the congruency of triangles having a right angle and a corollary; a page of original exercises and a page of directions for proving theorems; two theorems with the converse of each and two corollaries on a point in the perpendicular bisector of a line, and one in the bisector of an angle; a theorem on the isosceles triangle and two pages of originals; a page of exercises and definitions on polygons;four theorems, four corollaries, and six pages of originals on the parallelogram; a theorem and two corollaries on parallel lines intersecting equal segments on transversals; a problem on dividing a line into equal parts; a theorem on a line joining the mid-points of the sides of a triangle and four originals; a theorem on the median of a trapezoid; two theorems on the sum of the interior and the sum of the exterior angles of a polygon and eight originals; a page on definitions and explanations of inequalities; four theorems, three corollaries, and three originals on inequality of angles and sides of triangles; four theorems and nine originals on the bisectors of the interior angles of a triangle, the perpendicular bisectors of the sides, the altitudes, and medians of a triangle meeting in points; two pages of miscellaneous originals.

c) Book II. The circle, pp. 93-140.

A page of definitions and explanations; a problem on constructing a circle through three points not in a straight line; a page of definitions and explanations; two theorems and eight originals on central angles and subtended arcs; seven theorems, two corollaries, and fifteen originals on central angles, chords and their subtended arcs; three theorems, four corollaries, and seven originals on tangents to circles; a theorem and five originals on the line of centers of two circles bisecting their common chord; a theorem and six originals on parallel lines intercepting equal arcs on a circle; a page of definitions and explanations on measurement of angles and arcs; five theorems, four corollaries, and thirty four originals on measurement of angles; a problem and two originals on inscribing a circle in a triangle; two problems on constructing a tangent to a circle through a point on the circle and through a point without the circle; three pages of explanations and originals on loci; two pages of miscellaneous exercises.

d) Book III. Proportion - similar polygons, pp. 141-190.

Proportion, pp. 141-153: three pages of definitions, explanations, and original exercises on ratio and proportion; ten theorems and eleven originals on the fundamental theorems of proportion; a theorem, two corollaries, and three originals a line parallel with the base of a triangle dividing the other sides proportionally; a problem, a corollary, and three originals on finding the fourth proportional to three given lines; a theorem and four originals on lines dividing the sides of a triangle proportionally being parallel to the base; a theorem and two originals on the bisector of an interior angle of a triangle dividing the opposite sides; the base of a triangle is divided into parts proportional to the adjacent sides by the bisector of the vertical angle of a triangle.
Similar Polygons, pp. 153-190: a page of definitions, explanations, and originals on similar polygons; three theorems, two corollaries, and sixteen originals on similarity of triangles; a theorem and three corollaries on the homologous altitudes of similar triangles having the same ratio as any two homologous sides; a theorem and two originals product of segments intersecting chords; two theorems and nine originals on secants or a secant and a tangent drawn through a point without the circle; a theorem, a corollary, and four originals on drawing the altitude on the hypotenuse of a right tria le; a problem and two originals on constructing the mean proportional between two given sections of lines; a theorem, a corollary, and thirteen originals on the Pythagorean Proposition; three theorems, a problem, and two originals on similar polygons; three pages on supplementary topics and drawing to a scale including nine originals; four pages including nine originals on ratios and their applications; a theorem and four originals on parallel lines intercepting proportional segments on transversals; four theorems, two corollaries, and nineteen originals on the bisector of an

exterior angle of a triangle, the square of the side opposite an acute or an obtuse angle of a triangle, and the sum of the squares of two sides of a triangle; two theorems and five originals on the product of two sides equaling the product of the diameter of the circumscribed circle and the altitude upon the third side; two pages of miscellaneous original exercises and review questions.

e) Book IV. Areas of polygons, pp. 191-220.

Three pages of definitions and explanations; three theorems, one corollary, and eleven originals on rectangles; two theorems, nine corollaries, and twenty five originals on the area of a parallelogram and a triangle; a problem and two originals on constructing a triangle equivalent to a given polygon; a theorem, a corollary, and fifteen originals on the area of a trapezoid; the Pythagorean Proposition and five originals; a problem, a corollary, and three originals on constructing a square equal to the sum of two squares; two problems and two originals on areas of polygons; a theorem and two originals on triangles having equal angles; a problem, a corollary, and seventeen originals on constructing a square equal to a given parallelogram; a page of original exercises to be constructed without formal analysis; five problems and five originals on construction of a rectangle equal to a square with the sum of the base and altitude equal to a given line; to construct a square equal to a square with a given ratio to a square, and to construct a rectangle equal to a square with the difference of its base and altitude equal to a given line; two problems and five originals on constructing a polygon similar to a polygon with a given ratio to it, and to construct a polygon similar to one of two given polygons and equal to the other; two pages of miscellaneous original exercises and review questions.

f) Book V. Regular polygons, mensuration, pp. 221-306.

Regular polygons, pp. 221-237: a page of original exercises; a theorem and two corollaries on circumscribing a circle about any regular polygon; five theorems, five corollaries, and nineteen originals on the area of a regular polygon and inscribed polygons; a theorem and five originals on dividing a line into extreme and mean ratio; two problems, three corollaries, and nine originals on inscribing a regular decagon and a regular pentagon in a circle; a theorem and five originals on regular polygons of the same number of sides being similar; a theorem, a corollary, and nine originals on perimeters of polygons of the same number of sides having the same ratio as their radii or as their apothems. Mensuration of the circle, pp. 238-306: three pages of definitions and explanations on the circle; six corollaries and twenty seven originals on the area of the circle; five pages of definitions, explanations, and originals on supplementary topics; three theorems and a corollary on the circumference and area of a circle; a problem to find the perimeter of an inscribed polygon of double

the number of sides of an inscribed polygon; a problem to compute the value of π; two theorems on incommensurables; three pages with two theorems, four definitions, and fifteen originals on symmetry of plane figures; five theorems, three corollaries, two definitions, and three originals on maxima and minima; thirty three pages of supplementary original exercises, which are intended for the entire book.

4) Content Plane Geometry by Strader-Rhodes.[16]

a) Symbols and abbreviations, pp. XIV-XV. Introduction, pp.1-38.
A page of abbreviations and symbols; a page of illustrations of geometry in plants. General introduction, pp. 1-5: what you study in geometry; why you study geometry; how you study geometry; ten thought exercises; the origin of geometry. Illustrations and definitions of terms, pp. 5-25: definitions, explanations, and illustrations with oral exercises for practice. Some fundamental facts, pp. 25-28: axioms with explanations, exercises on the principles of geometry. Proof, pp. 29-31: definitions, exercises, and explanations. Formal demonstration, pp. 31-39: parts of a written demonstration with explanations and definitions; examples of three theorems demonstrated with oral exercises; a page of original exercises for practice.

b) Chapter One. Rectilinear figures, pp. 39-120.
Triangles, pp. 39-61: three pages of definitions and explanations; a theorem on general congruency of triangles with two pages of definitions, explanations, and originals; a theorem and a corollary on the general congruency of triangles with a page of originals and a page of illustrations; four pages with originals and explanations on the tools of geometry; two theorems and two corollaries on isosceles triangles; the other theorem on the general congruency of triangles and five original exercises; six pages of definitions, explanations, and originals. Angles formed by parallel lines cut by transversals, pp. 61-86: definitions, explanations, and exercises; a page of definitions and oral exercises; a page of illustrations; six theorems, nine corollaries, and four pages of originals on triangles; five pages of originals and two theorems on congruency. Quadrilaterals, pp. 87-95: two pages of definitions and oral exercises; two theorems and two corollaries on parallelograms; two theorems on a line intersecting two sides of a triangle; a page of originals on trapezoids; two theorems on the 30-60-90 triangle. Polygons, pp. 96-103: a page of definitions and a page of oral exercises on polygons; two theorems, a corollary, and a page of original exercises on the interior and esterior angles of a polygon; four theorems and a corollary on bisectors of lines and angles. Concurrent lines of a triangle,

16. Strader, W. W. - Rhodes, L. D., Plane Geometry. John C. Winston Company, Chicago, 1927.

pp. 104-110: four theorems, a page of definitions, a corollary, and three pages of definitions and originals on concurrent lines with reference to triangles. Supplementary exercises, pp. 111-116: five pages of original exercises on triangles, parallel lines, and polygons; three pages of oral exercises on Chapter One. Miscellaneous exercises, pp. 119-120.

c) Chapter Two. Circles, pp. 121-206.

Circles and their lines, pp. 121-138: five pages of definitions, oral exercises, and eight assumptions for circles; three theorems and four originals on the angles and arcs of circles; three theorems, two corollaries, and three pages of originals on the chords and arcs of a circle. Tangents, pp. 139-147: three theorems, two corollaries, three illustrations, and three pages of originals on tangents of circles. Circles and their angles, pp. 148-159: four theorems, four corollaries, two illustrations, and six pages of originals on measurements of angles. Supplementary and review exercises, pp. 159-166.

d) Chapter Three. Areas and proportion, pp. 167-206.

Areas, pp. 167-179: a page of definitions and explanations; a page of oral exercises; an illustration; five theorems, thirteen corollaries, and thirty one originals on areas of a rectangle, triangles, a parallelogram, and a trapezoid. Proportions, pp. 180-206: two pages of definitions and explanations with twelve oral exercises; four theorems, six corollaries, and twenty one originals on proportions; two theorems, three corollaries, and eight oral exercises; two theorems and one corollary on the side opposite an angle of a triangle divided harmonically; an illustration and twelve originals; nine pages of originals.

e) Chapter Four. Similar polygons. Proportional magnitudes, pp. 207-264.

Similar polygons, pp. 207-218: two pages of explanations and six originals; five theorems, two illustrations, and nineteen originals on similar triangles. Proportional magnitudes, pp. 219-264: four theorems, one illustration, and twenty originals on the sum of the antecedents in proportion to the sum of the consequents in a series of equal ratios; a theorem and eighteen originals on intersecting chords; a theorem and twenty six originals on tangents; a theorem, five corollaries, and nine originals on the Pythagorean Proposition; seven pages on spherical formulas; six pages on trigonometric ratios; two pages of exercises on drawing to a scale; a page of illustrations and ten pages of original exercises.

f) Chapter Five. Regular Polygons - their circles, pp. 265-296.

Regular polygons and their circles, pp. 265-278: three theorems, an illustration, six corollaries, and nineteen originals on inscribed and circumscribed polygons; a problem, an illustration, three theorems, and twenty one originals on the

area of a regular polygon, similarity of polygons, and the ratio of polygons to corresponding lines. Variables - constants - limits, pp. 279-296: three pages of definitions and explanations; two theorems, four corollaries, an illustration, and fifty two originals on ratios of circumferences of circles to their radii and diameters, and the area of a circle; an illustration and five pages of originals.

g) Chapter Six. Inequalities. Construction. Loci, pp. 297-350.
Inequalities, pp. 297-313: four pages of definitions, explanations, and original exercises; five theorems and three pages of originals on unequal lines of rectilinear figures; three theorems, a corollary, an illustration, and twelve originals on unequal lines and arcs of circles; a theorem on two lines drawn from a point in a perpendicular to a line. Constructions, pp. 314-340: an illustration and four pages of originals on constructions; twenty five problems of constructions, nine corollaries, and eighty nine original exercises. Loci, pp. 341-350: a page of explanations and oral exercises; eight theorems, two illustrations, and fifty nine originals on loci.

h) Supplement, pp. 351-396.
The new type of geometry examinations, pp. 351-361: an illustration and thirteen types of the new examinations and original exercises for each type. A word list for plane geometry, pp. 361-366: there are one hundred twenty five words frequently used in geometry given. Applications of geometry, pp. 367-378: seven pages of originals. Recreations, pp. 379-382: an illustration and twelve pages of originals in which geometry has been applied. A brief historical sketch of geometry, pp. 383-386: four pages of the history of geometry. Non- Euclidean geometry, pp. 387-394: two pages of explanations; six pages of definitions; two pages of formulas. Axioms and assumptions, pp. 394-396: fifty pringiples of geometry.

5) Content of Plane Geometry by Farnsworth.[17]

a) Book One, Straight line figures, pp. 1-102.
six pages on the explanation of the straight line; a page defining and explaining planes, plane geometry, solid geometry; some assumptions regarding straight lines, of which there are seven; two pages on constructions; fourteen axioms; three pages of definitions and explanations on angles, lines, and circles; seven originals and eight assumptions; four pages of explanations of congruency of triangles; five pages on a formal proof; four pages of originals; three problems of construction and nine original exercises on bisecting a line, an

17. Farnsworth, R. D., Plane Geometry. McGraw - Hill Book Company, New York, 1933.

angle, and constructing a triangle; three pages on an angle; three pages on the use of algebra in geometry; two theorems, a problem, and four originals on equality of vertical angles, right angles, and drawing a perpendicular to a line; a theorem and a problem on a perpendicular from a point to a line; two problems and two originals on congruent triangles; three theorems with the converse of each and six originals on parallel lines cut by a transversal; a problem, two theorems, a problem, and nine originals on constructing a line parallel to a given line;an exterior angle of a triangle greater than an ôpposite interior angle; a theorem on the sum of the angles of a triangle; a theorem, a page of explanations, and ten originals on congruence of triangles; a problem on drawing an angle; three theorems and four originals on equality of angles with their sides perpendicular and parallel; four theorems and seven originals on angles and sides of triangles; three pages on the uses and abuses of the indirect method of proof; seven theorems and seventeen originals on parallelograms; two pages on the form of a written demonstration; a page and eight originals on quadrilaterals; three theorems on parallel lines cutting transversals and lines parallel to the base of a triangle cutting the sides; a problem and five originals on dividing a line into equal parts; six theorems on a point in the bisector of a line and the meeting in a point of lines connected with a triangle; three pages of miscellaneous originals.

b) Book Two. Circles, pp. 103-152.

Two pages and four originals on the introduction of circles; a theorem on drawing a circle through three points not in a straight line; a problem on locating the center of an arc; fifteen theorems and thirty originals on angles, arcs, and chords of circles; four theorems and seven originals on tangents to circles; a problem and three theorems circumscribing a circle about a triangle; two pages with a theorem, five originals, and explanations on the relative positions of two circles; two pages of definitions and explanations on measurement of arcs; two originals on an inscribed angle and four theorems; a problem and seven originals on tangents; four theorems and twenty one originals on measurement of angles; ten pages of explanations with seven theorems and thirty one originals on loci; two construction problems on circles and twelve originals; four pages of miscellaneous originals.

c) Book Three. Similar figures, pp. 153-194.

Three pages of definitions and explanations on similar figures, ratio, and proportion; three original algebraic theorems on proportion; three theorems on parallel lines cutting transversals; four theorems and thirty seven originals on similar triangles; a problem and one original on drawing the fourth proportional to three lines; two theorems and twelve originals on proportional lines of circles; two theorems and five originals on the 30-60-90 triangle; eleven originals; two theorems on the bisector of the exterior and the interior

angle of a triangle; two theorems and four originals on similar polygons being divided into triangles; a problem on constructing a polygon similar to a given polygon; a theorem on the ratio of the perimeters of similar polygons; eight pages of miscellaneous originals.

d) Book Four. Areas, pp. 195-208.
Two p ges of explanations; four theorems and thirty three originals on areas of triangles and quadrilaterals; a problem on construction of a triangle equal to a given polygon; seven originals; seventeen miscellaneous original exercises.

e) Book Five. Regular polygons and circles, pp. 209-254.
A page of definitions and explanations; two theorems and sixteen originals on regular inscribed polygons; two problems on constructing a square and a hexagon inscribed in a circle; two theorems and eleven originals on inscribing figures in and circumscribing them about circles; five theorems and twenty one originals on regular polygons; four theorems and twenty nine originals on circles; fifteen pages of miscellaneous original exercises; six pages of originals on applications; a table of squares and square roots.

6) Content of Humanized Geometry by Blackhurst.[18]

The above text is intended by the author to be an introduction to thinking. It is not divided into books or chapters. It contains seventy one formal propositions and treats of some of the fundamental ideas of solid geometry in its proper connection with plane geometry. The history of geometry in connection with the study of the proposition concerned.is given when the proposition is studied.

Six pages on measuring angles with exercises and discussions; a theorem with its history, discussions, and exercises on proving vertical angles equal; three pages on perpendicular and parallel lines with seven originals; eight pages on angles and parallel lines cut by transversals with discussions, history, and originals following which is a test; a theorem on the exterior angle of a triangle equaling the sum of the opposite interior angles with five originals nd five pages of discussion; four theorems on congruency of triangles with discussions, history, and forty nine originals; seven pages on constructions containing five propositions, thirty five originals with discussions and history; a test is given; four theorems, one corollary, discussions, and seventeen originals on paralielograms; two propositions and five originals on parallel lines cut by a transversal; eleven pages on logical reasoning in which the syllogism is used; seven propositions, history,

18. Blackhurst, J. H., Humanized Geometry, Second Edition. Published by the author, DesMoines, 1934.

discussions, three corollaries, and forty three originals on angles, arcs, chords, and tangents of circles; five propositions, three corollaries, and thirteen originals on measurement of angles; two theorems, a corollary, a discussion, a test, and seven special exercises on intersecting and tangent circles; eight propositions and twenty two originals on angles, arcs, and triangles; four propositions and five originals on dividing a line into equal parts, circumscribing a circle about a triangle, drawing a tangent to a circle, and inscribing a circle in a triangle; three propositions and nineteen originals on loci; a test; three propositions, a discussion, and thirteen originals on the ratio of lines and the theory of limits; five propositions, history of the Pythagorean Proposition with an illustration, a corollary, and five originals on the areas of a triangle, a rectangle, a parallelorgam, a circle, and the pythagorean Proposition; seven propositions, discussions, four corollaries, and seventeen originals on a line parallel to the base and intercepting the other sides of a triangle, the bisector of an interior angle dividing the opposite side, and similar triangles; two propositions, a discussion, and five originals on constructing a fourth proportional to three lines and constructing a mean proportional between two lines; two theorems and two definitions in solid geometry on a line perpendicular to two other lines at their point of intersection being perpendicular to the plane of the lines, and a plane intersecting a sphere forms a circle; a test; fifty nine originals in review; a review test; twenty propositions, a corollary, and twenty originals for extra credit to superior pupils and those preparing to pass a college entrance examination.

2) Content as shown in courses of study.

a) Courses listed. Courses of study are usually made by state and city boards of education or officers appointed by them, and they are placed in the hands of the teachers to use in teaching the subjects outlined. See the "Abbreviations" in the states and cities in this work, pp. 126-144.

b) Content of courses summarized. Courses of study are usually rather intensive outlines of the work expected to be done by the pupils. They are not texts or syllabi, but they are supposed to aid the teacher in using the text and supplementary material. Oftentimes the courses of

study are made with a view to the use of a special text. They often state what shall be emphasized much and what shall be emphasized very little. Some states refuse diplomas to pupils supposed to graduate until the work outlined in the courses of study has been carefully done. The superintendent uses this method as a most effective means of having his objectives with reference to the curriculum put into effect by the teachers. The teachers may review the subjects, check the work done by the pupils, and follow the courses of study in daily work and in examinations. The courses help the teachers determine at any time what the individual pupils are accomplishing toward reaching the determined goal.

3) Content as shown in syllabi. Syllabi treat theorems and problems of construction as original exercises. They do not follow necessarily any textbook, but they have the important propositions in what is considered a psychological arrangement. It is permissible to use texts, and it is usually expected to have some well selected text and supplementary material in connection with the syllabus, but memorizing proofs is discouraged.

We are giving a proposed syllabus in plane and solid geometry by Evans:[20]

"PLANE FIGURES

Assumptions:

20. Evans, G. W., "Proposed Syllabus in Plane and Solid Geometry", Mathematics Teacher, Volume XXIII(February, 1930), pp. 87-94.

1. Two straight lines in the same plane meet if at all in one point only.
2. Not more than one parallel can be drawn to a line from a point without.
3. Not more than one perpendicular can be drawn to a line at a point in it.
4. Superposition is the real test of congruence.
5. Overturning a figure reverses its arrangement.
6. Figures are equal when their measurement numbers are equal, but figures may be equal without being congruent.
7. If in any two circles the central distance is less than the sum of their radii, and greater than their difference, the circles will meet in two points only, and those two points will be on opposite sides of the line of centers.
8. Of lines having the same extremities the straight line is the shortest.

I. Measuring distances and angles.
1. There cannot be two different numbers having the same converging approximations.
2. In any circle, if two central angles are equal, the arcs they intercept are equal.
3. In any circle, if two arcs are equal, the central angles that intercept them are equal.
4. The measurement number of a central angle is the same as that of its intercepted arc.

II. Congruent triangles.
5. Two triangles are congruent if two sides and the included angle of each are equal respectively.
6. Two triangles are congruent if a side and the angles next to it are respectively equal.
7. Two triangles are congruent if the sides of one are respectively equal to the sides of the other.

III. Perpendiculars and parallels.
8. Perpendiculars to the same straight line cannot meet.
9. Two right triangles are congruent if the hypotenuse and an angle next to it are respectively equal.
10. If a transversal is perpendicular to one of two parallel lines it is perpendicular to the other also.
11. If the figure formed by a transversal crossing two straight has these two lines parallel, the alternate interior angles are equal.
12. In the figure formed by a transversal crossing two straight lines, if the alternate interior angles are equal those two lines are parallel.

IV. Quadrilaterals.
13. In any parallelorgam the opposite sides and the opposite angles are equal.
14. In any quadrilateral, if the opposite sides are equal, or if two of the opposite sides are equal and par-

allel, the figure is a parallelogram.

15. The area of a rectangle is equal to the product of its base and altitude.

V. Measuring Triangles.

16. The area of a rhomboid is the product of either base and the altitude thereon.

VI. Angles and sides in a right triangle.

17. The area of a triangle is the product of any side and the altitude thereon.

18. In any triangle the sum of the angles is 180 degrees.

19. The two angles at the base of a triangle is equal to the exterior angle at the vertex.

20. Two right triangles are similar if an acute angle of one equals an acute angle of the other.

21. In any right triangle the sum of the two squares on the legs is equal to the square on the hypotenuse.

VII. The law of cosines.

22. The cosine of an obtuse angle is the same in value as the cosine of its supplement, but is negative in sign.

23. In any triangle ABC, the square on a equals the square on b minus 2b cosA.

VIII. Three tests of similarity.

24. If the corresponding sides of two triangles are proportional, the triangles are similar.

25. If an angle of onetriangle is equal to an angle of another triangle and the sides including those angles are proportional, the triangles are similar.

26. If two triangles have two angles of one equal respectively to two angles of the other, the triangles are similar.

IX. Ratios of similar triangles.

27. The sine of an obtuse angle is the same as the sine of its supplement.

28. The area of any triangle is given by the formula 1/2ab sinc.

29. Two similar triangles will have for their ratio the square of the ratio of similarity.

X. Similar figures in general.

30. In any two similar figures corresponding triangles are all in the same order, or all are in the reverse order.

31. Two similar polygons will have for the ratio of their perimeters the ratio of similarity, and for their ratio of areas the square of that ratio.

32. Any two similar plane figures will have the ratio of similarity for the ratio of their perimeters, and for the ratio of areas its square.

XI. Measuring the circle.

33. The perpendicular to a radius at its point on the circle is the only tangent at that point.

34. If a circle is divided into equal arcs, the chords of these arcs form a regular inscribed polygon, and the tangents at the points of division form a regular circumscribed polygon.

35. If in any circle one regular polygon of n sides is inscribed, and on it another is circumscribed, the polygons are similar and their ratio of similarity is cos θ, where θ is 180/n degrees.

36. If a series of regular polygons inscribed in or circumscribed about a circle have half as many sides as the succeeding one, their areas and their perimeters are converging approximations to the area of the circle and to its circles respectively.

37. For every circle the circumference has a constant ratio to the diameter, and the area has a constant ratio to the square of the radius.

XII. Loci.

38. The locus of points equidistant from two given points is the perpendicular bisector of the line joining them.

39. The angle between a tangent and a chord is half the central angle intercepting the same arc.

40. An inscribed angle is half the central angle intercepting the same arc.

41. Of angles all in the same order, having a fixed base and a constant vertical angle, the locus of the vertex is a circular arc.

42. The locus of the center of a circle inscribed in an angle is the bisector of the angle.

XIII. Concurrent lines.

43. Every triangle has one circumcenter only.

44. Every triangle has one incenter only.

45. The altitudes from the vertices of a triangle all pass through one point.

46. In any triangle one median meets the others two thirds of the distance from the vertex to the foot.

47. If r is the radius of the circumscribed circle of any triangle, ABC, then a equals 2r sin A, b equals 2r sin B, and c equals 2r sin C(This is the theorem known as the law of sines).

XIV. Internal and external segments.

48. In every triangle the bisector of the vertical angle divides the base into segments proportional to the sides adjoining.

49. In every triangle the bisector of the exterior angle divides the base into segments proportionally into parts that are proportional to the sides adjoining.

50. If a fixed point is within a circle every chord

through it is divided by it into segments whose products are constant.

51. If a fixed point is without a circle, every chord in line with the point is divided by it externally into segments whose product is constant and equal to the tangent from the fixed point squared.

XV. The formula for half an angle.

52. The sine of half an angle may be obtained by the formula sin θ/2 equals the square root of 1/2(1-cos θ); the cosine of half an angle can be found from the sine by the formula cos θ squared equals 1-sin θ squared."

"SOLID GEOMETRY

I. Measurement of solids.

A. A study of prisms.

1. Planes intersect in straight lines.
2. If three planes intersect in three lines, these lines either pass through one point or are parallel to each other.
3. If two intersecting lines in one plane are parallel respectively to two intersecting lines in another plane, the planes are parallel.
4. If two parallel planes are cut by a transversal plane, the intersections are parallel.
5. If two lines are each parallel to a third line, they are parallel to each other.
6. The bases of a cylinder or prism are congruent.
7. To construct a prism having given a fixed base and a fixed lateral edge.

B. Volume of a right prism.

8. All the perpendiculars to a given line at a given point lie in one plane which is perpendicular to the line at that point.
9. To draw through a given point a line perpendicular to a given plane.
10. Perpendiculars to any given plane are parallel.
11. To construct a right prism having a given base and the length of the lateral edge of the prism.
12. Two right prisms are congruent if they have congruent bases and equal altitudes.
13. The measurement number of a prism if it is a right prism and has a rectangular base is the product of the measurement numbers of the three edges meeting at any vertex.
14. The measurement number of any right prism is the product of the measurement numbers of the base and altitude.

C. Volume of a prism or cylinder.

15. The volume of an oblique prism is the product of a right section and the lateral edge.

16. The volume of any parallelopiped is the product of a base and the altitude on the base.

17. The volume of any prism is the product of its base and altitude.

18. The volume of any cylinder is the product of its base and altitude.

D. Volumes of a pyramid or cone.

19. The volume of a pyramid or cone has the same measurement number as the area under its curve of sections.

20. The sections of a pyramid or cone are similar and proportional to the squares of their distances from the vertex.

21. Pyramids having equal bases and altitudes are equal in volume.

22. The volume of any pyramid is one third the product of its base and altitude.

23. The volume of any cone is one third the product of its base and altitude.

24. The volume of a prismatoid having bases B and B', mid-section M, and altitude h, is h/6(B plus 4M plus B').

E. Volume of a sphere.

25. Any plane section of a sphere is a circle.

26. The volume of a sphere is given by the formula $4\pi r$ cubed divided by 3.

F. Areas of round surfaces.

27. The lateral area of a regular pyramid is the product of the slant height and the perimeter of the mid-section.

28. The lateral area of the frustum of a regular pyramid is the product of the slant height and the perimeter of the mid-section.

29. The lateral area of a cone of revolution is the product of the slant height and the perimeter of the mid-section.

30. The lateral area of the frustum of a cone of revolution is the product of the slant height and the mid-section.

31. The area of a sphere is given by the formula $4\pi r$ squared.

II. Similarity.

32. Two trihedral angles are congruent if their face angles are respectively equal and in the same order.

33. Two tetrahedrons are similar if the faces of one are similar respectively to the faces of the other.

34. Two similar polyhedrons will have for the ratio of their volumes the cube, and for the ratio of areas the

square, of the ratio of similarity.

35. If any two convex solids are similar they have for the ratio of volumes the cube, and for areas the square, of the ratio of similarity.

III. Loci.

36. The locus of points equidistant from two given points is the plane perpendicular to the line joining them, at its mid-point.

37. The locus of points equidistant from the vertices of a triangle is the line through its circumcenter perpendicular to its plane.

38. The locus of points equidistant from the faces of a dihedral angle is the plane bisecting the dihedral angle.

39. A sphere can be circumscribed about any tetrahedron.

40. A sphere can be inscribed in any tetrahedron.

IV. Spherical figures and solid angles.

41. Spherical triangles are congruent if the sides are respectively equal and in the same order.

42. Symmetrical spherical triangles have equal areas.

43. The area of a lune is $2\pi R$ squared where R is the critical of the angle of the lune.

44. The area of a spherical triangle is Wr squared.

45. The area of a spherical polygon is Wr squared, where W is the spherical excess of the triangle.

46. The polar angle of a dihedral angle is the supplement of its plane; the polar arc of a spherical angle is the supplement of that angle.

47. Of two spherical polygons, if the first is the pOlar polygon of the second, then the second is also the polar polygon of the first.

48. The sum of the sides of a convex spherical polygon is less than 2π.

49. To find the spherical excess of the solid angle contained in a cone of revolution.

50. The sum of any two sides of a spherical triangle is greater than the third side.

51. The shortest distance between two points on the surface of a sphere is the arc of the great circle joining these two points."

c. <u>Items most frequently emphasized</u>.

1) <u>Plane geometry</u>. The congruency of triangles seems to be most frequently emphasized. Parallel lines and the angles formed when cut by transversals receive a great deal of

emphasis. The sum of the angles of a triangle and of the interior and exterior angles of a polygon are practically always included in a text on plane geometry. In constructions, the perpendicular bisector of a line, bisecting an angle, and constructing angles of thirty, sixty, and forty five degrees are emphasized.

Congruence, especially of triangles, and similarity of figures with special treatment of similar triangles are emphasized. Areas of the triangle, the parallelogram, and the regular polygons come in for their share of emphasis. The Pythagorean Proposition is seldom omitted in plane geometry texts.

2) Solid geometry. Fixing the idea of the three dimensions in space is emphasized. The construction of planes and showing how they are determined occupy the attention of writers of texts. The cube and other regular solids are stressed. The theory of limits is taught so that solid geometry may be linked with the plane geometry but no more. The study of the sphere seems to be favored.

3. Arrangement of content.

a. Abbreviations in the states.

1) States briefly treated. Connecticut has a course of study which contains a syllabus on plane geometry[21] and one on solid geometry[22] with a great many suggestions and methods. Intuitive geometry[23] is introduced in the seventh grade for the purpose of helping pupils recognize important geometric forms,

21. A Suggested Course of Study in Mathematics, pp. 82-109. Connecticut State Board of Education, Hartford, 1930.
22. Ibid., pp. 151-172.
23. Ibid., pp. 11-17.

learn the use of instruments, make simple constructions, and use formulas. Some forms of solid geometry[24] are taught in the eighth grade. Numerical trigonometry,[25] involving the sine, cosine, and tangent, is taught in the ninth grade. Plane geometry is taught in the tenth grade, and solid geometry, followed by trigonometry, is taught in the twelfth grade. There is no state approved list of textbooks in geometry; they are selected by local committees; most of the better texts are being used.[26]

Iowa has what may be called a syllabus on plane geometry[27] and one on solid geometry.[28] Objectives, teachers' procedures, and pupils' activities are explained. Diagnostic and achievement tests are given in geometry.[29] Numerical trigonometry[30] is given in the ninth or tenth grade. Plane geometry is required in the approved schools of Iowa, but solid geometry is elective, and the textbooks in geometry are selected by the school authorities.[31]

Kentucky has an approved list of several textbooks:[32]

Name of text	Author	Publisher
Plane Geometry	Avery	Allyn and Bacon
Plane Geometry	Nyberg	American Book Company
Plane Geometry	Wentworth-Smith	Ginn and Company

24. *Ibid.*, pp. 28-31.
25. *Ibid.*, pp. 52-53.
26. Letter to the writer from Paul D. Collier, Supervisor of Secondary Education, Hartford, Connecticut, 1930.
27. *Iowa State Courses of Study in Mathematics for High Schools*, pp. 74-79. DesMoines, 1931.
28. Ibid., 97-98.
29. *Ibid.*, pp. 125-126.
30. *Ibid.*, pp. 32-33.
31. Letter to the writer from R. A. Griffin, Iowa Inspector of Consolidated Schools, DesMoines, 1934.
32. *List of Textbooks Approved by the State of Kentucky Textbook Commission*, pp. 32-33. Frankfort, 1930-1935.

Plane Geometry	Bernard	Johnson Publishing Co.
Modern Plane Geometry	Wells-Hart	D. C. Heath and Company
Plane Geometry	Hassler	Lyons and Carnahan
Beginner's Geometry	Smith	The Macmillan Company
Modern Plane Geometry	Stone-Mallory	Benjamin H. Sanborn & Co.
Modern Plane Geometry	Clark-Otis	World Book Company
Plane Geometry and Its Uses	Mirck-Newell-Harper-Mullins	Row, Peterson & Company
Solid Geometry	Stone-Millis	B. H. Sanborn & Company
Solid Geometry Revised	Palmer-Taylor-Farnum	Scott, Foresman & Co.

Most of the authors mentioned above have texts on solid geometry as well as plane geometry.

Plane geometry is required in Kentucky, but solid geometry is elective.[33]

Louisiana gives plane geometry in the third year of senior high school and solid geometry in the fourth year,[34] but geometry is not required for graduation. The texts used in both plane and solid geometry are by Wells and Hart.[35] Trigonometry is given in the last half of the fourth year, and the text is <u>Plane Trigonometry and Logarithms</u> by Simpson.[36] The following objectives are given:[37]

> "a. To give pupils a knowledge of facts and processes which are essential to the solution of present day practical problems and to the future study of mathematics.
> b. To encourage a critical attitude toward the accuracy and value of all statements.

33. Letter to the writer from Mark Godman, Public School Supervisor, Frankfort, Kentucky, 1934.
34. <u>Courses of Study for Louisiana High Schools</u>, p. 109. T. H. Harris, State Superintendent, Baton Rouge, 1933.
35. <u>Ibid.</u>, pp. 114-116.
36. <u>Ibid.</u>, pp. 116-117.
37. <u>Ibid.</u>, p. 114.

c. To develop an inquiring attitude of mind which will lead to the investigation and discovery of facts.

d. To develop a realization of the dependence of scientific and economic progress upon geometry, and an appreciation of the part geometry has played in the history and development of our civilization, particularly as exemplified by the engineer and the architect in the relationships of mathematics to art and science.

e. To provide the pupil with material for practice in accurate reasoning.

f. To develop ability to follow directions exactly, to organize material, and to appreciate the logical arrangement when setting up a proof or making a construction.

g. To promote proper study habits through planned activity designed to interest and instruct."

Michigan requires plane geometry for graduation from high school.[38] Intuitive geometry is taught in the eighth grade.[39] Plane geometry is taught in the tenth grade; solid geometry, in the eleventh; trigonometry and calculus, in the twelfth.[40] The interrelations between arithmetic, algebra, and geometry have been recognized and much progress has been made as a result in teaching mathematics.[41]

Montana uses a syllabus on plane geometry divided into courses of six weeks each with aims, attainments, and contents explained.[42] There is a short syllabus on solid geometry and an explanation of the work to be done in trigonometry.[43]

New Hampshire has what should be called a short syllabus,

38. High School Manual and Course of Study for Michigan, Bulletin No. 12, p. 42. W. H. Pierce, Superintendent of Public Instruction, Lansing, 1928.
39. Ibid., pp. 68-69.
40. Ibid., pp. 70-72.
41. Ibid., p. 67.
42. Montana Course of Study for High School Mathematics, pp. 186-199. State Board of Education, Helena, 1930.
43. Ibid., pp. 199-206.

which is for plane geometry.[44] This state also has solid geometry by syllabus.[45] A half unit of plane trigonometry is given.[46] A unit course in composite advanced mathematics, which is elective, is outlined for the twelfth grade.[47] An elective unit of fused plane and solid geometry may be taken instead of the unit in plane geometry.[48] Plane geometry may be taken in the tenth or the eleventh grade as an elective. Some of the texts are given from which a selection may be made:[49]

Name of text	Author	Publisher
New Plane Geometry	Hawks-Luby-Touton	Ginn and Company
Plane Geometry	Strader-Rhodes	John C. Winston Co.
Modern Plane Geometry	Wells-Hart	D. C. Heath and Co.
Modern Plane Geometry	Clark-Otis	World Book Co.

New Jersey makes plane geometry an elective subject.[50] Both plane and solid are electives; in fact, the New Jersey Department of Secondary Education does not prescribe the texts to be used in high school instruction.[51]

New Mexico has a syllabus on plane geometry divided into units with explanations. Some of the objectives are stated as follows:[52]

"1. Training in logical reasoning, precision of state-

44. Program of Studies Recommended for the Public Schools of New Hampshire. Mathematics and Science, pp. 51-67. Evans Printing Company, Concord, 1931.
45. Ibid., pp. 58-76.
46. Ibid., pp. 77-81.
47. Ibid., pp. 82-87.
48. Ibid., p. 47.
49. Ibid., p. 96.
50. A Manual for Secondary Schools, pp. 60-61. Trenton, 1932.
51. Letter to the writer from Howard Dare White, Assistant Commissioner of Education, Trenton, 1934.
52. Course of Study in Mathematics for Secondary Schools, Bulletin No. 5, pp. 27-45. Sante Fe, 1932.

ment and thought, and discrimination between the true and the false.

2. Appreciation of beauty in geometric forms of nature, art, and industry.

3. Familiarity with elementary properties and relations of these forms.

4. Development of space perception."[53]

There are many texts listed for use in the high schools of New Mexico:

Name of text	Author	Publisher
Plane Geometry	Wentworth-Smith	Ginn and Company
Modern Plane Geometry	Stone-Mallory	Sanborn and Company
Plane Geometry	Palmer-Taylor-Farnum	Scott, Foresman & Co.
Plane Geometry	Bernard	Johnson Publishing Co.
Plane Geometry	Avery	Allyn Bacon & Co.
Plane Geometry	Hawkes-Luby-Touton	Ginn and Company
Plane Geometry	Young-Schwartz	Henry Holt & Co.
Modern Plane Geometry	Clark-Otis	World Book Company

New York uses syllabi in teaching geometry. They are compiled under the direction of the Board of Regents of the University of the State of New York. The syllabus for solid geometry[54] is published in four books: Book Six - Lines and Planes in Space; Book Seven - Prisms and Pyramids; Book Eight - Cylinders and Cones; Book Nine - The Sphere. Plane geometry is given in Books One - Five.

North Carolina seems to require two years of mathematics in a three-teacher school, a four-teacher school, a five-teacher school, and a six-teacher school; but these two years must be intended for the study of algebra.[55] There seems not to be a list of adopted texts in geometry for North Carolina. Plane geometry is offered as an elective in the third year. Only a

53. Ibid., p. 45.
54. Syllabus in Solid Geometry. The University of the State of New York Press, Albany, 1932.
55. High School Manual, pp. 30-83. A. T. Allen, State Superintendent, Raleigh, North Carolina, 1929.

few schools teach solid geometry, which is given in the fourth year and is counted a half unit.[56]

North Dakota uses an outline for plane and solid geometry,[57] which is somewhat in the nature of a syllabus for each. Plane geometry may be given in the tenth, eleventh, or twelfth grade as a unit course. Solid geometry may be given in the eleventh or the twelfth grade as a half unit course. Trigonometry may be given in the eleventh or the twelfth grade as a half unit course. There are many texts in plane and solid geometry from which selections may be made. Some of the texts are as follows:

Name of text	Author	Publisher
Laboratory Plane Geom.	Austin	Scott, Foresman & Co.
Plane Geometry	Avery	Allyn Bacon & Co.
Plane Geometry	Morgan-Foberg-Breckenridge	Houghton Mifflin Co.
Modern Plane and Solid Geometry	Clark-Otis	World Book Company
Plane Geometry	Hassler	Lyons & Carnahan
Solid Geometry	Hassler	Lyons & Carnahan
Plane Geometry and Its Uses	Mirick-Newell	Row, Peterson & Co.
Solid Geometry and Its Uses	Mirick-Newell	Row, Peterson & Co.
Plane Geometry	Nyberg	American Book Co.
Solid Geometry	Nyberg	American Book Col
Plane Geometry	Palmer-Taylor-Farnum	Scott, Foresman & Co.
Solid Geometry	Palmer-Taylor-Farnum	Scott, Foresman & Co.
Plane Geometry	Seymour	American Book Company
Essentials of Plane Geometry	Smith	Ginn and Company
Essentials of Solid Geom.	Smith	Ginn and Company
Plane Geometry	Strader-Rhodes	John C. Winston Co.

56. Letter to the writer from A. B. Combs, Associate Superintendent of Public Instruction, Raleigh, North Carolina, 1934.
57. Administrative Manual and Course of Study for North Dakota High Schools, pp. 186-190. Department of Public Instruction, Bismarck, 1931.

Geometry is no longer a constant for graduation from the State of North Dakota, but some school boards require it for their schools.[58]

Ohio uses a syllabus in connection with the study of plane geometry. Scholarship tests combined with physics are given.[59] A vocabulary of plane geometry is given.[60]

Oklahoma gives plane geometry[61] in the tenth grade as a unit course. The following objectives are stated:

"1. To give the pupil a working knowledge of the facts and processes which are essential to the present day practical problems and to further the study of mathematics.
2. To encourage a critical attitude toward the accuracy and value of all mathematics.
3. To develop an inquiring attitude of the mind which will lead to the discovery and investigation of facts.
4. To develop a realization of the scientific and economic progress dependent upon geometry, and an appreciation of the part geometry has played in the history and development of our civilization.
5. To provide the pupil with material for practice in accurate reasoning.
6. To promote proper study habits through planned activities designed to interest and instruct.
7. To develop ability to follow directions exactly when setting up a proof or making a construction."

The basic adopted text for the high schools of Oklahoma is Plane Geometry by J. O. Hassler, published by Lyons and Carnahan.[62] A course in solid geometry is given in the eleventh or the twelfth grade as a half unit course. The text may be widely selected.[63] Trigonometry is given in the twelfth grade as a half unit course.

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58. Letter to the writer from James A. Page, Director of Secondary Education, Bismarck, North Dakota, 1934.
59. State Scholarship Bulletin No. 6, Field to Be Covered by the State Scholarship Texts in Geometry, pp. 4-5. J. L. Clifton, Director of Education, Columbus, Ohio, 1931.
60. Ibid., pp. 6-7.
61. High School Course of Study in Mathematics for the State of Oklahoma, Bulletin No. 123-A, pp. 57-72. John Vaughn, State Superintendent of Public Instruction, Oklahoma City, 1932.
62. Ibid., pp. 80-94.
63. Ibid., pp. 113-117.

Pennsylvania has a unit course in plane geometry in the form of a syllabus divided into eighteen units.[64] The syllabus has excellent explanations and eight splendid guiding principles. There is a test provided for each unit. There is a general test divided into three parts that should be given on three different days. There is a half unit course on solid geometry[65] with a syllabus similar to the one on plane geometry. A half unit course is offered in plane trigonometry[65] with a syllabus similar to the syllabi on plane and solid geometry. The individual districts select the texts used in their schools, for there is no state adoption of texts. On the whole, the texts used are those published by the standard publishing companies.[67]

South Carolina has a unit course in plane geometry[68] in the tenth grade of three year high schools and in the eleventh grade of four year high schools. The course seems to be elective. The teaching of solid geometry[69] is discouraged, for it is not required to enter any college in South Carolina; and principals are advised to offer it only when there is a justifiable demand.

Texas has a unit course in plane geometry[70] given in the second half of the tenth grade and the first half of the eleventh grade.

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64. Courses of Study in Mathematics for Senior High Schools, Bulletin No. 79, pp. 11-25. James N. Rule, Superintendent of Public Instruction, Harrisburg, Pennsylvania, 1933.
65. Ibid., pp. 65-72.
66. Ibid., pp. 73-87.
67. Letter to the writer from Walter E. Hess, State Supervisor of Secondary Education, Harrisburg, 1934.
68. High School Manual, p. 61. James H. Hope, Superintendent, Columbia, South Carolina, 1927.
69. Ibid., p. 61.
70. Teaching Mathematics, Texas High Schools, Volume IX, No. 10, pp. 35-43. State Department of Education, Austin, 1933.

A syllabus is given for the first part of the course, and it is stated that the work is to be continued in a similar treatment for the remainder of the course. The text, Plane Geometry by Bruce is adopted for all the schools except first-class four-year high schools and is on the multiple list for them. Other texts on the multiple list are:

Name of text	Author	Publisher
Modern Plane and Solid Geometry	Clark-Otis	World Book Company
Plane Geometry	Strader-Rhodes	John C. Winston Co.
Solid Geometry	Strader-Rhodes	John C. Winston Co.
Plane Geometry and Its Uses	Mirick-Newell-Harper	Row, Peterson & Co.
Solid Geometry and Its Uses	Mirick-Newell-Harper	Row, Peterson & Co.

A syllabus is given for solid geometry[71] and a short outline for trigonometry,[72] but there is no state adopted text for trigonometry.

Vermont introduces the study of geometry in the eighth and ninth grades in the form of intuitive geometry.[73] A unit course in demonstrative geometry is given in the tenth grade. The work is outlined in a syllabus. Some related solid geometry[74] may be given in the tenth grade, but a half unit course in solid geometry[75] is given in a syllabus for the twelfth grade. Numerical trigonometry is introduced in the ninth grade,[76] continued in the tenth grade,[77] and a half unit course is given in

71. Ibid., pp. 43-46.
72. Ibid., p. 46.
73. Course of Study in Mathematics for the High Schools of Vermont, Bulletin No. H. S. 1, Part X, pp. 16-17, 23-24. Vermont State Board of Education, Montpelier, 1928.
74. Ibid., p. 33.
75. Ibid., pp. 41-44.
76. Ibid., pp. 25-26.
77. Ibid., p. 32.

the twelfth grade.[78]

2) States more briefly treated. Alabama offers an elective course in plane geometry in the first or second year of high school.[79] Plane Geometry by Newell and Harper, published by Row, Peterson and Company, is the state adopted text.[80] A half unit course in solid geometry is given in the second or the third year of high school; the text is by the same authors as in plane geometry, and the course is elective. The cities are not required to use the state adopted books, but some do.[81]

California offers what seems to be an elective unit in plane geometry and a half unit in solid geometry. There are about twenty texts listed from which to select in plane geometry and almost as many in solid geometry. Many of these texts have been written very recently.[82] Each district in California selects its own textbooks, which may be adopted for a period of four years.[83]

Colorado has no state uniform textbook law and no state adopted texts.[84] Plane and solid geometry are electives, as are all other subjects in mathematics, and each local administration selects its own texts for four years. Texts vary greatly.[85]

78. Ibid., pp. 44-47.
79. Program of Studies and Adopted Textbooks for County and Rural High Schools, pp. 31-34. State Board of Education, Montgomery, Alabama, 1931.
80. Ibid., pp. 34-37.
81. Letter to the writer from W. L. Spencer, Director of Secondary Education, Montgomery, 1934.
82. State of California Department of Education Bulletin No. 6, List of High School Textbooks, pp. 36-37. Sacramento, 1932.
83. Letter to the writer from Walter H. Hepner, Chief, Division of Secondary Education for California, Sacramento, 1934.
84. Postal card to the writer from Inez Johnson Lewis, Department of Education, Denver, Colorado, 1934.
85. Letter to the writer from A. C. Cross, Colorado State High School Inspector, Boulder, Colorado, 1934.

Florida does not require geometry for graduation from high school, but a majority of the schools have a local requirement. The two state institutions of higher learning require plane geometry for entrance.[86] Florida has a state adoption of textbooks for a period of eight years. Modern Plane and Solid Geometry by Clark-Otis is adopted for the text in geometry. The plane geometry and the solid geometry may be combined or used in separate volumes.[87]

Georgia requires two units in mathematics for graduation, and plane geometry sees to be one of them. A half unit in solid geometry seems to be elective.[88]

Illinois[89] places plane geometry in the second year of high school as an elective unit subject. Solid geometry and plane trigonometry are each a half unit subject.

Indiana[90] places plane geometry in the tenth grade after intuitive has been taught in the lower grades. Modern Plane Geometry by Clark is the text used in plane geometry. Solid geometry is an elective half unit course in the eleventh grade. Introductory trigonometry may be used as a half unit elective course in the twelfth grade. The text is by the same author

86. Letter to the writer from M. R. Hinson, State Director of Public Instruction, Tallahassee, Florida, 1934.
87. State Adopted High School Textbooks for Use in the Public Schools of Florida, p. 5. Tallahassee, 1930.
88. The Accredited High Schools of Georgia, Volume XXXIV, No. 2, pp. 11-12. University of Georgia, Atlanta, 1934.
89. Standard Courses and Suggestions for Reorganized High Schools in Illinois, pp. 17-18. Francis F. Blair, Supt., Springfield, 1932.
90. Program of Studies and Digest for State Courses of Study for Indiana Schools, pp. 43-44. George C. Cole, State Superintendent of Public Instruction, Indianapolis, 1933.

as that of the plane geometry.

Kansas gives plane geometry as a unit elective course in the tenth grade. Solid geometry is an elective half unit course in the eleventh year. Trigonometry may be given in the eleventh or the twelfth year as a half unit course.[91] The State has adopted Plane and Solid Geometry by Strader-Rhodes.[92]

Mississippi[93] gives a unit course in plane geometry in the third year of high school. Solid geometry is an elective half unit course, and plane trigonometry is a half unit course. Solid geometry and trigonometry are usually given the fourth year. The state adopted texts are:[94]

Name of text	Author	Publisher
Essentials of Plane Geometry	Smith	Ginn and Company
Essentials of Solid Geometry	Smith	Ginn and Company

Tennessee[95] requires a unit course in plane geometry for graduation from high school. It is given in the third year. Solid geometry is an elective half unit course, and trigonometry is an elective half unit course. Trigonometry is an elective half unit course. Solid geometry and trigonometry are given in the fourth year. The state adopted texts are:

Name of text	Author	Publisher
Essentials of Plane Geometry	Smith	Ginn and Company
Essentials of Solid Geometry	Smith	Ginn and Company
Plane and Solid Geometry	Nyberg	American Book Co.
Plane and Solid Geometry	Strader-Rhodes	John C. Winston Co.

91. Handbook on Organization and Practices for the Secondary Schools of Kansas, pp. 19-27. W. T. Markham, Supt., of Public Instruction, Topeka, 1933.
92. Price List, p. 7. Kansas School Book Commission, Topeka, 1933.
93. Accredited High Schools and Colleges, Bulletin No. 73, p. 15. W. F. Bond, State Supt., Jackson, Mississippi, 1932-1933.
94. High School Reorganization, Bulletin No. 59, pp. 9-57. W. F. Bond, State Superintendent, Jackson, Mississippi, 1930.
95. Letter to the writer from R. R. Vance, State High School Supervisor, Nashville, Tennessee, 1934.

Virginia[96] gives a unit course in plane geometry in the second or third year of high school, but it is not required for graduation. A half unit elective course is given in solid geometry, and a half unit course is offered in trigonometry; both of which are given in the fourth year. The texts are: Plane Geometry and Solid Geometry both by Wells and Hart, and Plane Trigonometry by Robbins.

Wyoming[97] requires a unit course in plane geometry for graduation from high school. It is given in the third year.

3) States most briefly treated. Arizona[98] has no state requirements relative to plane and solid geometry, and there is no uniformity of texts used in these subjects.

Arkansas[99] does not have a uniform state textbook adoption. Plane geometry is offered as an elective unit course, solid geometry as an elective half unit course, and trigonometry as an elective half unit course.

Delaware[100] offers plane geometry as a required unit subject in the second year, and there is an elective half unit course offered in solid geometry for those needing it for college entrance.

Idaho[101] requires plane geometry for graduation from high school, but solid geometry is optional. There are no state

96. Program of Studies by the State Department of Education, Richmond, Virginia, 1931.
97. High School Standards for Wyoming, p. 14. Cheyenne, 1932.
98. Letter to the writer from H. E. Hendrix, Arizona Superintendent of Public Instruction, Phoenix, 1934.
99. Letter to the writer from W. E. Phipps, Arkansas State Commissioner of Education, Little Rock, 1936.
100. Letter to the writer from John Shillings, Assistant in Charge of Secondary Schools, Dover, Delaware, 1934.
101. Letter to the writer from J. W. Condie, Idaho State Superintendent of Public Instruction, Boise, 1934.

adopted texts; the districts select their own texts.

Maine[102] does not require plane geometry, but it is given as a unit elective. It is required for college entrance. There is no state adoption of texts. Solid geometry and trigonometry are elective half unit courses.

Maryland[103] has no formulated course of study in plane and solid geometry. The texts are chosen by the county boards of education.

Massachusetts[104] has no state course of study nor any state adopted textbooks in plane and solid geometry. The course of study and texts are left entirely to the school officials.

Minnesota[105] does not have a state adoption of textbooks. Plane geometry is offered as an elective unit course and solid geometry as a half unit elective course.

Missouri[106] has no state requirements for plane geometry, but it is offered as an elective. An elective half unit of solid geometry and an elective half unit in trigonometry is offered. There is no state adoption of textbooks.

Nebraska[107] has no state requirement of plane geometry for graduation from high school, but about ninety five per cent of the schools require it. There is no state adoption of texts.

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102. Letter to the writer from H. C. Lyseth, Agent for Secondary Education, Augusta, Manine, 1936.
103. Letter to the writer from Merle S. Bateman, Credential Secretary, Maryland State Department of Ed., Baltimore, 1934.
104. Letter to the writer from Jerome Burtt, Supervisor of Secondary Education, Boston, Massachusetts, 1934.
105. Letter to the writer from Margaret Wulf, Statistician, Minnesota State Department of Education, Saint Paul, 1934.
106. Letter to the writer from R. T. Scobee, Chief Clerk, Director High School Supervision, Jefferson City, Mo., 1936.
107. Letter to the writer from J. C. Mitchell, Director of Secondary Education and Teacher Training, Lincoln, Nebraska, 1936.

Plane Geometry by Wentworth-Smith and Plane Geometry by Wells-Hart are favored texts.

Nevada[108] has no particular texts which are adopted by the state. The selection of textbooks is left with the principal of the school or the head of the department.

Oregon[109] offers a half unit course in plane geometry and a half unit course in solid geometry, but plane geometry may be continued another half unit. They are not required for graduation from high school. The texts are Plane Geometry and Solid Geometry both by Stone-Mallory.

Rhode Island[110] has no state requirements regarding plane and solid geometry. Plane geometry is given in most schools, and solid geometry is given in some. Texts are selected by the principal or by committees.

South Dakota[111] has not required plane geometry for graduation from high school since 1932, and it is not listed as an elective, but it is probably given as an elective.

Utah[112] offers both plane and solid geometry. Plane geometry is required for college entrance, but it is not required for graduation from high school.

Washington permits the local first and second districts to make their own selections of textbooks. Until the last year

108. Letter to the writer from Amy Henson, Office Deputy, Nevada State Department of Education, Carson City, 1934.
109. Letter to the writer from James M. Burgess, School Administrator, Secondary Education, Salem, Oregon, 1934.
110. Letter to the writer from Walter E. Ranger, State Commissioner of Education, Providence, Rhode Island, 1934.
111. Letter to the writer from R. W. Kraushaar, State High School Supervisor, Pierre, South Dakota, 1934.
112. Letter to the writer from C. H. Skidmore, State Superintendent of Public Instruction, Salt Lake City, Utah, 1934.

plane geometry has been rather compulsory, but beginning in 1935 it will be a matter of personal consultation as to what subjects except English are taken.[113]

West Virginia[114] does not require geometry for graduation from high school, and there is no state adoption of textbooks. Each school is permitted to select its own texts.

Wisconsin[115] does not have a uniform state adoption of textbooks in geometry, and a great variety of texts are used. Geometry is not required for graduation from high school but it is offered as an elective.

4) <u>Summary of abbreviations in the states</u>.

a) <u>Status of plane geometry in curricula of states</u>. The following eleven states require plane geometry for graduation from high school:

Delaware	Iowa	Mississippi
Georgia	Kansas	Tennessee
Idaho	Kentucky	Wyoming
Indiana	Michigan	

All the other states except possibly South Dakota offer plane geometry as an elective unit course. South Dakota required it for graduation until 1932. Now it is not listed as an elective, but it is probably offered as such.

b) <u>Status of solid geometry in curricula of states</u>. Solid geometry is not required for graduation from high school in

113. Letter to the writer from L. O. Swenson, State High School Supervisor, Olympia, Washington, 1934.
114. Letter to the writer from A. J. Gibson, State High School Supervisor, Charleston, West Virginia, 1934.
115. Letter to the writer from John Callahan, Wisconsin State Superintendent of Public Instruction, Madison, 1934.

any of the states, but the following forty two states offer it as an elective half unit course:

Alabama
Arizona
Arkansas
California
Colorado
Connecticut
Delaware
Florida
Georgia
Idaho
Illinois
Indiana
Iowa
Kansas
Kentucky
Louisiana
Maine
Maryland
Massachusetts
Michigan
Minnesota
Mississippi
Missouri
Montana
New Hampshire
New Jersey
New York
North Carolina
North Dakota
Oklahoma
oregon
Pennsylvania
Rhode Island
South Corolina
Tennessee
Texas
Utah
Vermont
Virginia
Washington
West Virginia
Wisconsin

c) <u>Status of plane trigonometry in the states.</u> The Following twenty one states offer an elective half unit course in plane trigonometry:

Arkansas
Connecticut
Illinois
Indiana
Iowa
Kansas
Louisiana
Maine
Michigan
Mississippi
Missouri
Montana
New Hampshire
New York
North Dakota
Oklahoma
Pennsylvania
Tennessee
Texas
Vermont
Virginia

b. <u>Abbreviations in some of the largest cities.</u>

1) <u>Chicago</u>. Chicago has courses of study on plane and solid geometry in mimeographed forms. There is no adopted list of textbooks,[116] so there is a great variety of texts used in plane and solid geometry. Plane geometry is usually given in the second year of high school as an elective unit course. Solid geometry is given in the third year as an elective half unit course.[117]

116. Letter to the writer from Helen House, Assistant Secretary of the Board of Education, Chicago, 1934.
117. <u>Outline Chicago Course of Study</u>, Chicago Board of Education, Chicago, 1934.

2) New York City. New York City uses syllabi on both plane and solid geometry, which are published by the Board of Regents, University of the State of New York, Albany.[118] The texts used on plane geometry are as follows:

Name of Text	Author	Publisher
Plane Geometry	McCormack	Ginn and Company
Plane Geometry	Nyberg	D. Appleton Co.
Plane Geometry Revised	Schultz-Sevenoaks-Schuyler	Macmillan Co.
Plane Geometry	Seymour	American Book Co.
Plane Geometry	Solomon-Wright	C. Scribner's Sons
Modern Plane Geometry	Wells-Hart	D. C. Heath & Co.

3) San Francisco. San Francisco teaches a year of plane geometry and a half year of solid geometry. Both courses are elective, but plane geometry must be taken by those who are going to college or the university.[119] The texts are: New Plane Geometry by Durell-Arnold, Charles E. Merrill Company, Chicago; New Solid Geometry by the same authors and publisher.

4) Washington, D. C. Washington has an excellent outline for the teaching of intuitive and informal geometry in the seventh, eighth, and ninth grades.[120] Demonstrative plane geometry is taught in the tenth grade and solid geometry, the second half of the twelfth grade. The texts used are: Plane Geometry by Schultz-Sevenoaks-Schuyler, Macmillan Company; Solid Geometry by the same authors and publisher.[121]

118. Letter to the writer from S. J. Wilson, High School Division New York, 1934.
119. Letter to the writer from J. C. McGlade, Deputy Superintendent, San Francisco, 1934.
120. Topical Outline of the Course of Study in Mathematics for the Junior High Schools, School Document No. 4, Government Printing Office, Washington, 1933.
121. Letter to the writer from W. J. Wallis, Head of the Department of Mathematics, District of Columbia, 1934.

c. Geometry in connection with algebra and trigonometry.

Numerical trigonometry is often studied in connection with plane geometry of the triangle. In fact, this is perhaps the best method to impress upon the pupil that trigonometry is but a special application of geometry. Pupils will also see this a practical use for geometry. Of course, the trigonometry should not be unduly technical.

Algebra should be utilized in the work of plane geometry as well as solid geometry. There are many solutions in which algebra can be applied to make geometry fascinating and easily understood, which would be very difficult and confusing if treated stictly geometrically.

In many instances there are combination courses in mathematics in which algebra, geometry, and trigonometry are involved. These general courses in high school mathematics seem to be gaining in favor and importance.

4. Present trend in geometry.

a. Features formerly emphasized.

Rigorous logical proofs for all theorems based upon a few axioms, fewer postulates, and definitions were formerly insisted upon. The result of this often fostered memorizing proofs on the part of the pupils. There were fixed forms for the proof of every theorem and the solution of every problem. There was little or no history taught in connection with the study of geometry. There was discouragement to the use of practical applications. There was an extended treatment of the theory of limits of a technical nature not often referred to in studying

theorems and problems. There were few originals, and they were not well selected or well adapted to the propositions studied. Incommensurable ratios were treated to a very great extent. The technical features of geometry seemed to be emphasized.

b. Features now not emphasized but lightly touched.

Definitions are no longer given in a long list, but they are usually intended to be learned when needed by the pupil. The tendency is to treat axioms and postulates little differently from other assumptions. The theory of limits is almost omitted from plane geometry, and about the only proposition now used in solid geometry is the one stating that if two variables each approach limits and are constantly equal the limits are equal. This proposition has wide application in solid geometry. Incommensurables are not stressed any longer. The proofs of the theorems are not nearly so formal as they were a decade or more ago. Many problems and theorems are treated informally. Pupils are no longer required as a rule to prove formally that which is obvious. It is no longer considered that everything done in geometry shall be independent of algebra.

c. Features now most emphasized.

The history of geometry has been growing in favor until many authors have incorporated some very interesting and very valuable information to the pupils in the form of history. Some textbook writers give historical facts concerning propositions as they are studied. Teachers are urged by school authorities and educators to use history in order to impart the

value of geometry to pupils. The tendency to make assumptions of obvious theorems is being strongly felt. Memorizing proofs of geometric propositions on the part of pupils is being guarded against so far as possible. Geometry is being taught extensively as a course in logical reasoning. Good English is insisted upon, but the major emphasis is placed upon the thought rather than upon the form in which it is presented. Syllabi are increasing in favor, and original exercises are strongly emphasized. The practical value of geometry is being stressed, and more applications of geometry to the actual situations of life are being given in the most recent texts. The best teachers use freely the practical value of geometry in their efforts to present properly geometry to the developing mind of youth. Preceding the strictly logical study of geometry by intuitive geometry is rapidly gaining favor and has been found to be psychologically sound.

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CHAPTER V

PSYCHOLOGY OF GEOMETRY

1. Meaning of terms.

a. Meaning of psychology of geometry.

The psychology of geometry has reference to the manner in which geometry is learned. The mental processes involved and the activities of the one learning geometry are considered. How the learner grasps and retains geometrical information will be our field of study.

b. Situation and response.

A situation may be defined as a state of affairs in the life of an individual influencing him, and a response may be termed whatever the individual does because of the situation. Learning consists in establishing a connection or bond between the response and the situation.[1]

In geometry the situation may be a problem or some other geometrical task, and the response would be the solving of the problem or the doing of the geometrical task. For example, we may consider the following proposition: "The sum of the angles of a triangle is equal to two right angles". This theorem would be the situation, and the proof of it would be the response. The response or proof may occur in the mind involving

1. Thorndike, E. L., Educational Psychology, Briefer Course, p. 1. Teachers College, Columbia University, New York, 1927.

exterior-interior and alternate-interior angles of parallel lines cut by a transversal. It may take the form of circumscribing a circle about the triangle and evaluating the inscribed angles as half their intercepted arcs. It may, of course, take any of the multiple proofs of this proposition. The definition of solid geometry may be considered as a situation, then the response would be the mental conception of form in the three dimensions of space.

c. Learning and teaching.

We are not concerned with teaching geometry except in its relation to learning this subject. There is, as a matter of fact, an inseparable relation between methods of teaching geometry and the manner in which it is learned. We will focus our attention upon the processes of learning geometry from a psychological point of view.

Learning is the modification of behavior due to individual experience. Human nature in general is the result of man's original nature, the laws of learning, and the forces of nature surrounding man as he lives and learns.[2]

2. Psychological basis of learning geometry.

a. Native factors.

1) Intelligence. Perhaps there is no other subject in the public school curriculum in which native intelligence is more essential in its mastery than geometry. Since the dawn of civilization a knowledge of geometry, it seems, has been

2. Colvin, S. S., and Bagley, W. C., Human Behavior, p. 125. The Macmillan Company, New York, 1914.

proof of wisdom on the part of the person possessing it. The request of Alexander the Great and that of Ptolemy of Egypt that the learning of geometry be made easy for them with the reply of each of their famous teachers, that "There is no royal road to learning geometry", would tend to convince us that it requires intelligence, which is the capacity to learn, to understand geometry.

2) Special aptitude. Inorder to learn geometry successfully and without unnecessary drudgery and worry, a special aptitude is evidently necessary on the part of the pupil who is learning geometry. If he is slow to form a geometrical concept, is hazy in his thinking, is uncertain in what is the hypothesis of a proposition or its conclusion, or is inaccurate in his proof of a theorem or problem, he cannot hope to learn geometry easily and thoroughly.

At present there are tests of special aptitude for geometry, which may be administered students, before they even study geometry. The purpose of such tests is to ascertain in advance whether each and every student is gifted with the special aptitude in this field to justify his undertaking the study of geometry. They are ordinarily called prognosis tests in geometry.[3]

3) Sex. There seems to be little or no difference due primarily to the sex of the pupil in learning geometry. The seeming inferiority of girls is not due perhaps to sex, but to lack of motivation and poor methods of learning geom-

3. For example, Orleans's Prognosis Tests in Geometry, World Book Company, Yonkers, New York.

etry by the girls fail or dislike geometry.[4] They should free themselves from the traditional idea that geometry is more adaptable to the interests of boys and of more use to them in school and life. A great many girls who have trouble with geometry seem to try to memorize, without understanding the content including the proofs given in the textbooks, for such girls, of course, discouragement and failure await.

While there appears to be no appreciable differences between the sexes as to success in mathematics, yet small differences have been found here and there in favor of the male sex. For examples: 57% of boys reached or exceeded the median for girls in mathematics(Regents' examination and school mark);[5] 53% of boys reached or exceeded the median for girls in geometry.[6] A regular finding in all studies noting the fact is that the difference within a sex is enormously greater than it is between the average scores of the two sexes.

4) Race. Thorndike[7] tells us that a race is a group of people who to a considerable extent have a common remote ancestry, and its present descendents to a considerable extent are confined to that group.

Thorndike[8] says that on the whole the keenness of the senses seem to be about on a par in the various races of mankind.

There seem to be insufficient data to determine the men-

4. Breslich, E. R., he Technique of Teaching Secondary School Mathematics, p. 63. The School of Education, University of Chicago, 1930.
5. Thorndike, E. L., Educational Psychology, Volume III, p. 183. Teachers College, Columbia University, New York, 1925.
6. Ibid., P. 184.
7. Ibid., p. 206.
8. Ibid., p. 213.

tal alertness or intelligence of the various races. Thorndike states that the people of the Anglo-Danish District of England possess an aptitude for mathematics not shared by any other section of England. The East Anglican is a natural historian and shows little mathematical aptitude. Newton came from the Anglo-Danish area.[9]

There would in the opinion of the writer be wide differences between the races of mankind in the ability to learn geometry, since mathematics has developed with civilization, but data seem not to be available to confirm his belief. It is likely that whites may learn geometry, on the average, somewhat more easily than negroes.

5) Native factors utilized.

a) Intellectual curiosity. Geometry offers a fertile field for the intellectual inquiring young mind. The pupil is enthused at the concreteness and exactness of geometry. He may be made curious to explore its wonders and learn its mysteries. He may be led to wonder at the possibilities of exact geometric constructions, and he may be made to appreciate the indisputable logic of geometric proofs. Perhaps, no other subject in the puplic high school curriculum is more possible of satisfyingness to the ambitious young mind in its search for truth than geometry when properly taught.

b) Rivalry with other individuals. Intellectual rivalry is a wholesome incentive to youth. The desire to excel is strong, and the unbounding intellectual activities of young

9. *Ibid.*, pp. 217-218.

minds can be marshalled in this wholesome desire. Geometry is considered a very difficult subject, and the desire to excel in learning it could be in proportion to its difficulty. The pupil who outdistances his rivals in geometry is almost indisputably the intellectual leader of his group.

c) Group rivalry. Loyalty to one's group is very strong in young people, so that group rivalry can be made to function to a greater degree perhaps than individual rivalry. Since the success of the group depends upon each member, all can be led to desire to share in the victory. In the case of individuals the whole-hearted rivalry would narrow down to a few of the recognized leaders, but all can participate with hopes to win in group rivalry; so each one in the group can be expected to do his best. Learning geometry lends itself well to group rivalry, and each member of the group may be benefitted. This is especially true of several parallel sections.

b) Self-rivalry. When a pupil learns that he can improve in his work in geometry, he is pleased to the extent he considers geometry difficult. It is natural for a pupil to be elated when he finds himself able to do better work in his most difficult subjects. The feeling of success goes far toward its accomplishment. If a pupil raises his grade in geometry, has his classmates express his improvement in geometry, or has his teacher emphasize his progress, he will be much encouraged. He may keep a graph sheet of his improvement from day to day.

e) Desire for approval. There is a strong desire

in youth for approval by those of his group whom he admires. The approval of his teacher is very desirable if the members of the group are actively seeking that approval. It is a powerful aid in learning geometry when bestowed strictly upon merit. Social approval is one of the motives of mankind.

f) Desire for excellence. The tendency toward imitation in learning geometry is important. A pupil likely imitates the processes and proofs of the author of the text as well as those of the teacher and the other pupils who he believes excel in geometry. All his imitation including language may not be on purpose or conscious on his part.

The original student probably does not imitate less, but more. Instead of being a blind imitator of one master, he becomes the selective thoughtful imitator of all good models wherever found.[10] One who learns geometry must imitate the masters of geometry as well as think through many problems of his own initiation.

6) Individual differences.

a) Intelligence. A miscellaneous group will soon arrange itself in the study of geometry into three groups; the brilliant, the mediocre, and the dull students. If the same assignment is given to all the pupils one of the following will result. If the assignment fits the bright, the mediocre do nothing well, and the dull will fail. If it fits the mediocre, the dull do nothing well, and the bright are not given full development. If it fits the dull, the other groups have nothing to keep them out of mischief, and the

10. Thorndike, E. L., "Education for Initiative and Originality", Teachers College Record, XVII(Nov., 1916), 405-416.

greatest good is not done to the largest number.

We can do nothing to change the material of the brain; we can only develop it.

About the fifth the geometry class in terms of weeks from the beginning of the school year naturally divides itself into the three groups. It is a good plan for the teacher to place the bright on one side of the room, the mediocre in the center, and the dull on the other side. The teacher may give definite accumulated assignments on the board each day for each group. Each group will do the work for the dull plus its own particular assignment.[11]

Pintner[12] assumes that the distribution of individuals with reference to general intelligence approaches a normal distribution, which means a majority of individuals possess a medium or average amount of intelligence, and that an equal number of persons come above and below that average. The number of individuals decreases as the two extremes are approached until the highest and lowest intelligences are reached. He says that the general intelligence distribution may not be a normal one, but as the tests are made more scientific that distribution is approached.

The ability to learn geometry and the general intelligence of the pupil evidently have a high co-ordination. Since intelligence differs so widely among pupils, we most naturally expect their ability to learn geometry to have

11. Downing, Myrtle, "Group Teaching in Geometry", School Science and Mathematics, XXII(May, 1923), pp. 455-456.
12. Pintner, Rudolph, Intelligence Testing, pp. 71-72. Henry Holt and Company, New York, 1923.

great variation. Of course, other factors are involved besides intelligence in the learning of geometry as we shall see.

b) In aptitude. There are marked differences in the aptitudes of pupils in learning geometry. Usually aptitude runs parallel with intelligence, but this is not always true. Sometimes, pupils of noted intelligence exhibit little aptitude, however, in learning geometry. When such is the case, perhaps there are disturbing factors.

c) In accomplishment. Standard tests show as wide range in accomplishment of pupils in learning geometry as there would be shown in their general intelligence. A typical class varies from very low scores to very high scores. For examples: scores have been found to vary from as low as 5 to as high as 109 on the McMindes Achievement Test in Plane Geometry[13]; and scores vary from 24 to 95 on the Orleans Plane Geometry Achievement Test.[14]

b. Environmental factors in learning geometry.

1) Those favorable to learning geometry.

a) Skilled teacher. The skilled teacher as an environmental factor is of the greatest importance to the pupil who is learning geometry as well as in learning any other subject. The teacher should understand and be able to use the

13. McMindes, Maude, Manual of Directions for an Achievement Test in Plane Geometry, p. 2. Public School Publishing Company, Bloomington, Illinois, not dated.
14. Orleans, J. B., Plane Geometry Achievement Test, Manual Directions, p. 4. World Book Company, Yonkers, New York, 1929.

best modern methods. He should love his subject and foresee its importance to the child. He should teach geometry psychologically. He should inspire interest, initiative, and enthusiasm into the learning of geometry. He should know not only the content of geometry, but also that of connected mathematical studies and should have an understanding of a wide range in the other subjects the pupil will study.

b) Suitable textbook. A good text is an environmental aid to the learning of geometry. If geometry is psychologically presented, it will naturally aid the pupil in learning the subject. The textbook is usually the guide to the pupil. Of course, the teacher may vary from the text, but much of the pupil's time will be devoted to the textbook. The following is stated by Crowley:[15]

> "A textbook in geometry that is well organized put into the hands of capable and enthusiastic teachers and eager and alert high school pupils will transform demonstrative geometry into a thrilling adventure in the exploration of an ancient but ever new and fascinating domain of the human mind".

c) Well selected mathematical library. A good mathematical library which has geometry in the proper setting will enrich the course and make its learning purposeful. The pupil will also be less a slave to his textbook if he properly uses a good mathematical library. This, too, is a part of environment.

d) Favorable home conditions. If the home of the pupil is such that it is conducive to study and he is inclined

15. Crowley, E. B., "The High School Boy and His Geometry Textbook", Educational Review, Volume LXXIV (December, 1927), p. 274.

to work there, it is a valuable aid to learning geometry. The co-operation of the members of his family will be of the greatest value, not only in giving him direct assistance, but in making it convenient for him to study his geometry at home.

e) Proper setting of school life. The atmosphere of the school, if favorable to the learning of geometry, will make it a more pleasant undertaking. What one's classmates and friends say and do will greatly influence him in his school work. If the members of his class consider geometry important and work hard to learn it, he, too, will be encouraged in his work.

2) Those hindering the learning of geometry.

a) Unskilled or incompetent teacher. If the teacher does not see the value of geometry and does not succeed in making the pupil feel its importance, if poor and unpsychological methods of teaching are used, then it is likely the pupil will not do his best work in geometry. If the teacher is not so skilled in the subject that the pupil has the highest opinion of the teacher's ability to understand and teach geometry, then his learning geometry is likely to become drudgery and he will become discouraged and possibly fail.

b) Poor textbook. A badly and unpsychologically arranged textbook is a great handicap to learning geometry. The author of the textbook should not only know geometry, but he should be well versed in the psychology of presenting sub-

ject-matter. The pupil will most naturally imitate the author. His language and form will consciously or unconsciously partake of the language and form in the text.

c) Unsatisfactory home conditions. If there is poverty in the home rendering intellectual home work difficult or impossible, or if for any other reason the home antagonizes the pupil in his efforts to learn geometry, the bad effect will be very difficult to overcome.

d) Improper school atmosphere. Should one's classmates and the atmosphere of the school in general be critical and unfriendly toward geometry, the result would be to dampen the enthusiasm of the pupil and to discourage him. If he is made to feel that geometry is not worth the effort necessary to the mastery of it, he will make little progress.

c. Maturity.

Little children should not be asked to learn large units because the span of their attention is relatively limited. As the child becomes more mature and learns how to relate experiences, the unit which can be acquired may be steadily increased.[16] The child has had no real participation in love, and religion, and he has been hopefully shielded from Man's inhumanity to Man.[17]

Chronological age is not identical with physical maturity and neither of these two is identical with mental maturity. On

16. Judd, C. H., Psychology of Secondary Education, p. 498. Ginn and Company, Boston, 1927.
17. Ruediger, W. C., Teaching Procedures, p. 155. Houghton Mifflin and Company, Boston, 1932.

the average sixteen-year olds will differ from six-year olds because of ten years of inner growth plus the effect of training for that period.[18]

The ease and speed with which most forms of information and intellectual skills may be acquired and their permanence retained increase at a rather steady pace and reach a maximum of sixteen for average persons and possibly continue for several years longer for persons of superior minds. People probably maintain the level reached throughout most of their lives, their capacity probably gradually declining in old age. Sheer capacity to learn is appreciably greater in the period from 14-28 than from birth to 14.[19]

3. Laws of learning.

a. Laws of readiness.

Excitability of responses or readiness to act may differ one from another. Eating has at times a much higher degree of readiness than playing. The same response varies from time to time in readiness. Hunger adds to the readiness to eat. Behavior and learning are influenced by the readiness or unreadiness of responses as well as by their strength. The strength of responses determines what a mind can do, but the readiness of each determines what it will do.[20]

Readiness for any activity cannot be secured unless the pupil can succeed in that activity. He will not long show an

18. Thorndike, Briefer Course, op. cit., pp. 369-370.
19. Thorndike, E. L. - Gates, A. I., Elementary Principles of Education, pp. 195-196. The Macmillan Company, New York, 1929.
20. Thorndike, E. L., Fundamentals of Learning, pp. 328-329. Bureau of Publications, Columbia University, 1932.

interest in an activity which brings only difficulty and failure. Readiness may be developed if a pupil has enough aptitude, and in many cases it must be developed or it cannot otherwise exist. Readiness and satisfying effect are intimately related; the greater the readiness the more powerful will be the operation of effect.[21]

A river flows near the school house. It has been raining for three days intermittently, but the rain ceased in the morning, and at noon the river has reached its highest tide and begins to recede. John and Henry in the geometry class want to write their cousins the next day and tell them how high the river was and how wide it became. They go to the teacher and ask him to help them measure the stream by geometry as he had taught them a few days before. John and Henry were <u>ready</u> to do this exercise in geometry.

b. <u>Law of exercise</u>.

Exercise is a necessary condition to learning. A reaction needs to be satisfying and in most cases needs to be repeated, practiced, exercised, to be permanent. Practice alone is not enough to make it permanent; the result must be satisfying. The increase of efficiency in any complex activity involves: (1) the addition of new reactions; (2) the elimination of old ones that are not desirable; and (3) the simultaneous building up of new and the abandonment of old ones.[22]

Many theorems in geometry must be proved before the forms

21. Thorndike, E. L. - Gates, A. I., <u>op. cit.</u>, pp. 88-93.
22. <u>Ibid.</u>, pp. 93-97.

become fixed in the mind. To become permanent much exercise with them is essential.

c. Law of effect.

The individual tends to repeat and to learn those reactions quickly which are accompanied or followed by a satisfying effect, but tends to reject those bringing an annoying effect. Those are satisfying which contribute to the fulfillment of some want. The law of effect is the most fundamental principle of learning.[23] Satisfaction is the great selecter of all our behavior, geometrical or otherwise.

Annoyance associated with any response, geometrical or otherwise, tends to eliminate it. The teacher must see that the work in geometry is made satisfying, and not annoying, to the pupil. He must "sell" his subject to the learner.

One day the pupils in a certain geometry class, at the suggestion of their teacher, decided to measure the flag pole in the school yard. They measured the length of the shadow cast by the flag pole and the shadow of a boy who was five feet in height; then by proportion of the corresponding sides of similar triangles they found the height of the pole. The principal told them that the result was correct. The experience, process, and result were satisfying to them.

d. Secondary laws.

1) Law of trial and success. When one faces a new situation which influences or stimulates him at all, there is a tendency to make some sort of response to it. He tries one response, then another, and then another, and so on, till he finds

23. *Ibid.*, pp. 87-88.

the one that is successful. This is called the law of trial and success.[24]

Thid method can often be used in finding the analysis of a proposition. It is used to advantage in drawing subsidiary lines and thinking through to the solutions. Since geometric solutions are seen by the mind's eye and understood mentally, this method may often give the key to the solution. It is wasteful of time and should be reduced as much as possible.

2) Law of mind-set. Mind-sets may be emotional or intellectual, which are not mutually exclusive; both elements are frequently present in the same mind-set. They may be quite temporary as in anger, or more or less permanent as in a life ideal.[25]

When a pupil has set a determination to learn geometry and do it will, he has gone far toward accomplishing his purpose. There is much advantage to a proper mind-set in the mastery of geometry, for it insures earnest application, which is necessary in learning geometry.

A good assignment gives a favorable mind-set toward accomplishing the task. Interest and motive are sample mind-sets. The response a pupil makes to any situation is caused in large measure by his attitude or mind-set.[26]

3) The law of analogy. As an individual faces a new situation, he reacts to it in the manner he has responded or would respond to another similar situation. Since one tends by

24. Avent, Joseph E., Excellences and Errors in Teaching Methods, p. 17. Published by the author, Knoxville, Tennessee, 1931.
25. Ibid., p. 18.
26. Thorndike, "Education for Initiative and Originality", op. cit., pp. 410-411.

nature to respond to the situations of life by using a previous response to a situation somewhat like the present one, it is fundamentally important that numerous rich associations of sound analogy be made in the child's mind.[27]

In geometry, the pupil compares different figures. When he has used a method of proof and it has satisfactorily developed the theorem, he will most naturally try to use the same method of proof to demonstrate a similar proposition.

4) Law of piecemeal influence. A person does not attend monotonously to everything in a situation.[28] In viewing a landscape, some outstanding peak, some beautiful stream, or something of striking importance will hold the attention and interest of the beholder. After listening to a noted lecturer, some of the ideas are retained in the listener's mind, while most of the others are forgotten.

In studying geometry, the auxiliary lines drawn in the figure to establish the proof, or the key-idea in the discovery of the proof, usually stand out clearly in the mind. One feature of a figure at a time gets and holds attention while other features are attended to only marginally or partially.

5) Law of associative shifting. Many situations in life are very complex, and successful responses to them can not be made at first. The child may be made to begin with the things to which he can make response, and new elements may be gradually added to cause new responses by the learner until he can master the complex situations.[29]

27. Avent, *op. cit.*, 18-19.
28. *Ibid.*, p. 21.
29. *Ibid.*, p. 22.

Intuitive geometry may be begun as early as the fourth grade and the learner may by associative shifting master the elementary principles of geometry in the grades and junior high school. When he then reaches the second or third year of the senior high school, he will be most fully prepared to study demonstrative geometry. This law means that, by starting at the simplest elements and by gradually adding others, the learner may be led to make any reaction of which he is capable to any situation to which he may become sensitive.

4. Improvement in geometry.

a. Physiological conditions.

Pupils in order to do their best work in learning must have the classroom or any room where they study properly heated, lighted, and ventilated. If a pupil is not physically fit, he cannot do the work of the day. Some pupils do not get the required amount of sleep; others must do out of school physical work which is too heavy for them; and still others dissipate their energy through the many social activities. Mental fatigue is closely related to physical fatigue; mental health and ability to do mental work have as their foundation physical health, most especially, interest, enthusiasm, and freedom from worry.[30]

A pupil can not apply himself advantageously to learning geometry or any other subject, if he is handicapped physically.

30. Mueller, A. D., Teaching in Secondary Schools, pp. 80-83. The Century Company, New York, 1928.

b. Psychological conditions.

1) Identification series of conditions.

a) Ease of identifying what is to be done. A pupil cannot make progress in geometry until he learns what is given and is able to see at once what is to be done. A thorough understanding of the proposition will enable the pupil to know easily what is given and what is to be done.

b) Ease of identifying results to satisfy. If the learner has been properly studying geometry, he will have the geometrical concepts and the logical nature of geometry so strongly in mind that he can think through the steps of the proof in a most natural manner. Properly establishing the truths in geometry is perhaps the most unquestioned and most satisfying of any kind of school work. The mental ability of the pupil is, of course, a prime factor in the ease of finding satisfactory results. The work must not be too difficult for the mental age of the learner.

In computation involving arithmetic, algebra, or trigonometry the learner should weigh the results to determine whether or not they are reasonable ones to expect. He should think in round numbers and compare magnitudes mentally to be sure that the results are not unreasonable.

c) Ease of attaching satisfaction to results. There is no need for answers to be given in geometry. A pupil in most instances can be sure that the result obtained is right or wrong. The feeling of satisfaction cannot be experienced until the correct result is obtained. Geometry lends itself to a complete sense of satisfaction when the true result has been

found.

The geometric proposition called a theorem contains what is given and what is to be proved. The pupil can be led to distinguish these, if he has been taught the analysis of the theorem.

2) Interest series of conditions.

a) Interest in geometry. Attention is a state of consciousness, certain parts of which are relatively vivid, while other parts are rather indistinct. It is the selective function of consciousness, serving to emphasize some and to ignore others.[31] Passive attention is a state of consciousness in which the object attended to claims the entire interest. All attention is originally passive. Active attention is accompanied by a distinct sense of effort.[32] Such is a prerequisite to learning geometry.

In learning geometry one must have interest in the subject, but interest is assured when the pupil has the right attitude, the necessary preparation, and sufficient mental capacity. It is a teachers task to develop and to conserve the pupils interest in the subject. If the teacher is interested, enthusiastic, and masterful, it tends to beget in pupils similar attitudes.

b) Interest in improvement in geometry. Improvement if appreciated by the pupil is one good reason for his continuance in the study of the subject. As the study of geom-

31. Colvin, S. S. - Bagley, W. C., op. cit., p. 54.
32. Ibid., p. 68.

etry continues and underlying principles are discovered and used, skill is being developed in the learning of geometry. When the pupil becomes convinced that he can easily make improvement in his grades as well as his work in geometry, he is interested in doing so. His interest in improvement is a powerful stimulus to earnest application in the learning of geometry.

The pupil has perhaps heard some of his classmates in school talk unfavorably about geometry; he has likely known of others who made low grades or disliked it; he may come to the study of geometry with a prejudiced mind against it. When he gains a definite knowledge of it and finds that he can make progress, his interest to improve is greatly increased. His interest results in the accomplishment of improvement. He may graph his results and watch his learning curve rise from day to day, week to week, and from month to month.

c) Attention to the study of geometry. Attention of any kind and to any subject including geometry induces a resulting feeling of satisfaction. The more completely one has absorbed himself in the process the greater is the satisfaction of attending. What is interesting is determined by the individual's previous knowledge, by his momentary attitude, and by his instincts.[33]

One realizes from the beginning that attention given to the study of geometry is most essential in order to excel in it. When he sees he is making progress, he is more interested in giving time to his work. As his interest increases he finds him-

33. Pillsbury, W. B., Education as the Psychologist Sees It, pp. 131-136. The Macmillan Company, New York, 1925.

self more and more impelled to increased efforts.

Attention given to the other members of the class, to the carefully planned instruction of the teacher, and to the suggestions of the author of the textbook, is well repaid in the continued mastery of the work in geometry.

d) Accepting geometry as satisfying to the learner's wants. In youth there is no quest stronger than the search for truth. Since the study of geometry satisfies the learner's search for truth, as, perhaps, no other subject does, the pupil may be drawn to the learning of geometry to satisfy his want of exact knowledge.

If the pupil has learned something of its history, he may be led to want to know geometry that he may depend upon learning why there was so much in its study to involve the most gifted of the race, the ancient geometers of Greece, in its learning. Ambitious young people may be led to like to learn the things that the great men of the past studied or learned. Many great souls have desired to know geometry and by diligent application to its study have satisfied their wants. The pupil may want to know geometry for its practical as well as for its cultural values.

If the pupil plans to be a surveyor or civil engineer, he will find that geometry is essential to his needs in preparation for his vocation.

e) Intellectual desires. The young person finds so many things different from what they appear or seem that he is inclined to question them. He has been taught that there was a Santa Claus. He has learned that it is but a fancy. Some of

his friends seemed to be honest and sincere, but when the truth became known they proved to be less ideal and upright. When he comes to the study of geometry, he finds the proposition stating that a certain thing is true. His intellectual curiosity may be stimulated to spur him on to see if it is absolutely true. As he satisfies his curiosity to know the truth of the proposition, the study of geometry takes on a deeper meaning. He seeks to know why the propositions of geometry are true; a mere statement is not sufficient to satisfy his inquiring mind. Geometry furnishes an unlimited field for the activities of intellectual inquiries.

3) Motives and incentives. Although representing different phases of the same human experience the terms attention, Interest, motive, and incentive cannot be used in absolutely the same sense. One gives attention to a task because of his interest in the task or its outcome; one begins to do the task under the guidance of some motive, which in turn is excited by means of an incentive.[34]

Motives are individual and social. In individual motives one has primarily his own interests in mind. Individual motives may be classed as higher and lower. Fear of disapproval or of punishment is one of the lower motives; rivalry only to beat someone else is a lower motive; hope of reward, intellectual curiosity, satisfaction of doing right, imitation of the best,

34. Avent, Joseph E., Beginning Teaching, pp. 491-492. Published by the author, Knoxville, Tennessee, 1926.

and pride in personal achievement, are some of the higher motives. Group rivalry, mutual help, appreciation of the good name of the school, and service to others, are some of the social motives.[35]

Incentives are external influences to excite certain motives which will modify a pupil's actions along a desired line. Punishments are repressive incentives; money, medals, and prizes are material incentives; school marks, tickets for good work, honor rolls, privileges and immunities, games, contests, praise and commendation, and promotion are all worthy incentives.[36]

The higher motives and worthy incentives should be judiciously employed in learning geometry. They stimulate interest and make the learning of geometry effective. The greater the importance of the subject and the greater the value of the contents to the pupil, the more should the pupil and the teacher strive to make the subject worth the most to the pupil. Geometry challenges their most earnest and greatest efforts.

Proper motivation of geometry permits the learning of geometry to enter with least resistance into the stream of thought, which has the following characteristics: (1) every thought tends to be a part of personal consciousness; (2) thought is always changing; (3) it is sensibly continuous; (4) it always appears to deal with objects independent of itself; (5) there is interest manifested in some parts of these objects to the exclusion

35. Ibid., pp. 492-494.
36. Ibid., pp. 495-499.

of the others.[37]

c. Educational series of conditions.

1) The teacher. The teacher is himself an unavoidable condition of learning. The excellent teacher is superior to the average teacher. He has the positives and excellences in traits, virtues, attitudes, knowledges, characteristics, and relationships to a greater degree than the average teacher. While teachers remain in the realm below the excellent, boys and girls are the victims of lacks and errors, and human mistakes are thus still longer delayed in the state's attempt at correction.[38]

Much of the interest of the pupil in geometry as well as his attitude toward it and his accomplishment in learning it is due to the teacher.

2) The course of study and textbooks. Courses of study in high school should be selected with extreme care, so that the pupil may prepare for a vocation and enrich the hours devoted to leisure. They should be selected so the boy and the girl may acquire good habits and healthful interests during school life and that they may spend adult life intelligently and happily.[39] The course in geometry should be selected in the light of the needs and capabilities of the individual pupil. The courses selected and the textbooks used will largely determine the success of the pupil in geometry. Courses and text-

37. James, William, The Principles of Psychology, Volume I, pp. 224-290. Henry Holt and Company, New York, 1918.
38. Avent, Joseph E., The Excellent Teacher, pp. 467-469. Published by the author, Knoxville, Tennessee, 1931.
39. Hill, Clyde M., and Mosher, R. D., Making the Most of High School, p. 33. Laidlow Brothers, New York, 1931.

books are educational conditions for weal or woe for geometry students according to the excellence of their making and selection.

There is a growing tendency among the best teachers of mathematics to favor doing the fundamental or essential work of all courses in the classroom in order to reach all the pupils. There is a tendency toward the laboratory method, in which the teacher supervises and directs the work of the pupils, who learn by studying and doing rather than by imitating and listening.[40]

All the departments, except mathematics, of our best modern high schools seem to be well equipped. It is usual to find in the mathematics classrooms a few pieces of crayon, two or three erasers, some twine, and a few yardsticks furnished free of charge by some enterprising dealer in hardware or paints.[41]

3) Mathematical equipment. The equipment for the teaching of mathematics, including geometry, should be well selected and should be sufficiently ample to serve the needs of the pupils in studying geometry as well as in any other subject of the curriculum.

a) The ruler. The ruler is a straight edge for drawing lines; it should have a brass edge on one side and be perforated in two places to fit the rings on the pupil's notebook.

b) The protractor. The protractor is used to draw and measure angles and to check the accuracy of constructions in geometry. It may be made of cardboard, celluloid, or

40. Breslich, *op. cit.*, p. 116.
41. *Ibid.*, p. 116.

metal. Those made of metal are the best. It should be marked with black lines so that it will not strain the eye to use it. The protractor should be perforated to fit the rings of the notebook.

c) The combination triangle. A celluloid right triangle is convenient for drawing right angles and perpendicular lines.

d) The compasses. The pupil should be shown how to use this delicate instrument correctly. He should be taught how to slip the compasses over the pencil, to fasten or loose the screw, to open and close the instrument, and to open it in drawing arcs and circles.

e) The textbook. Each pupil should have his own book and he should be instructed how to read it, care for it, protect it, and use it.

f) Desks. The desk of each pupil should have enough surface space to allow him to work comfortably and to hold the open textbook and notebook.

g) Blackboards. All available wall space of a mathematics room, not too high or low, should be utilized as a blackboard. There should be enough space to accommodate at least half of the class.

h) Blackboard protractor. This instrument is used to measure and draw angles as well as check geometric constructions. There should be two or three in the classroom.

i) Blackboard rulers, pointers, and erasers. It is best to have the blackboard ruler with a handle so that

it may be pressed firmly against the board. There should be three or more in the classroom. There should be three pointers and not more than two pupils should use the same eraser.

j) Blackboard compasses. There should be about a dozen blackboard compasses in the classroom and those not in immediate use should be kept in the teacher's desk or otherwise cared for. The pupils should be taught to draw as accurate figures as possible; so it is indispensable to use the blackboard compasses in the construction of figures on the board.

4) Methods used. These are conditions of learning or failing to learn geometry. It makes all the difference in the world as to the pupil's success, in terms of the teacher's methods.

a) Lecture method. Lectures, or talks, by teachers of geometry will accomplish results only if they are carefully planned, delivered, and immediately followed by some check or test. Otherwise they waste the time of the class, and the pupils are not really benefitted.

b) Question-answer method. This a very important method when properly used. The question should be asked the class and then only one called upon to answer. It is less important in geometry than in some of the other subjects.

c) Genetic method. The teacher guides the class, giving information and leading pupils to make discoveries. If properly used the teacher can arouse interest on the part of the pupils who are studying geometry.

d) Heuristic method. The heuristic method differs from the genetic in adapting itself to individual pupils. It

can be used successfully, especially in geometry, by only skilled teachers.

e) The laboratory method. The pupil does his work in school rather than at home. Experiments are set up for the pupil to perform. This method stresses the applications of geometry. It is excellent for developing the meanings of new geometrical concepts and principles.

f) The problem method. Most of the content of geometry can be utilized by means of this method. Mathematics including geometry is in its nature made up very largely of problems. Perhaps the term itself takes its meaning from mathematics.

g) The project method. On a small scale the project method may be used to an advantage in geometry. We mean by this that it would not be desirable or practical to include extensive undertakings in geometry to the extent it may be used in some of the other subjects.

h) The Dalton method. Under the Dalton plan the classroom recitation is practically abolished; group activities take the place of recitations; each pupil is free to cover the required ground at whatever hours and pace seem best to him. The work may be assigned for a month or more ahead. The Dalton plan has been developed from the laboratory method. Its fundamental principles are: (1) freedom for individual progress and instruction; (2) time freedom with responsibility; (3) provisions for a social environment in preparation for community activities. Its purpose is the training of the pupil most ef-

fectively participation in the community life to share.[42]

If the pupil is earnest in his work in geometry, the Dalton method is an excellent one. The pupil may work at the task in the manner he finds most satisfying to him.

5) Supervision of the study of mathematics. Supervised study means that done in the classroom under the direction of the teacher, with physical and psychological conditions most favorable for study, the pupil acquiring information independently in attaining the mastery of subject-matter. Supervised study provides for individual differences, but it is not strictly an individual method. It aims to retain the advantages of mass instruction and at the same time overcome its disadvantages. It ministers directly to the individual needs at the time.[43] Supervised study is more than conducting a study period. It is directive and constructive. It keeps pupils on the alert to do their best as individuals. It is very desirable and can be made very effective in learning geometry.

a) Study habits. High school pupils need to be taught proper study habits. Success in a high school subject depends upon a variety of factors. The pupil's general and special abilities are the most outstanding, but former experiences, industry, age, and interest are very important. When a pupil has trouble in geometry it may not be the subject-matter he is trying to master, but the lack of certain study habits which he failed to acquire. It is futile to assign lessons in

42. Wilson, Lucy L. W., "The Dalton Plan", Journal of the National Educational Association, June, 1925, pp. 181-182.
43. Breslich, op. cit., p. 38.

geometry to any pupil who does not know how to study the subject. In the past the teacher left it largely to the learner to develop independently his habits of study. It was assumed by the teacher that, if he did the work assigned, he would develop naturally proper study habits. Only recently has serious attention been given to teaching pupils how to study.[44] They must be taught how to study geometry by economical methods, or they will never properly master it.

b) Study habit helps used in the University of Chicago High School.[45]

"Form a time-place habit by studying the same subject in the same place at the same time each day.

"Have proper study conditions and equipment: a quiet room, not too warm; good light at the left; a straight chair and table; the necessary books, tools, and materials.

"Study independently. Do your own work and use your own judgment, asking for help only when you cannot proceed without it, thus developing ability to think for yourself and the will power and self-reliance essential to success.

"Sit straight and go at the work vigorously, with confidence and determination, without lounging or waste of time. When actually tired, exercise a moment, open the windows, change to a different type of work.

"Arrange your tasks economically: study those requiring fresh attention, like reading, first; those in which concentration is easier, like written work, later.

"Be clear on assignment and form in which it is to be delivered. In class write the assignment down when it is made. Mark things to be carefully learned. When in doubt, consult the teacher.

"In committing material to memory, learn it as a whole; go over it quickly first, then more carefully, and then again and again until you have it. In learning forms, rules, vocabularies, etc., it will help you to repeat them aloud.

"In studying material to be understood and digested but not memorized, first go over the whole quickly, then carefully section by section; if possible, then repeat the whole quickly.

"Use judgment as well as memory; analyze paragraphs, select important points, note how minor ones are related to them; use your pencil freely to make important points, so

44. *Ibid.*, pp. 87-88.
45. *Ibid.*, pp. 94-95.

that you may learn systematically and review easily.
"Study an advance assignment promptly and review before going to class; recall memorized matter by repeating it, aloud if necessary; think through a series of points to see if you have them in order in your mind.
"Use all the material aids available: index, appendix, notes, vocabulary, maps, illustrations in your textbooks as well as other books and periodicals."

c) Study habits in geometry. Proper study habits are needed as much in geometry as in any other subject. The following are good study habits:[46]

1. The habit of expressing verbal statements in brief symbolic form.

2. The habit of connecting words and symbols with meanings.

3. The habit of making a mental summary of what is known or given in a theorem or problem and what is to be proved or to be found.

4. The habit of making drawings of abstract situations.

5. The habit of estimating in advance the approximate answer, and the deciding if the result is reasonable.

6. The habit of checking results.

The following suggestions are given by Laura Blank:[47]

"I. Copy the theorem or statement.
"II. Digest every word of the theorem, recalling or looking up every geometric term.
"III. Draw the figure carefully and go over it to see if it satisfies the theorem.
"IV. Write the hypothesis.
"V. Ask yourself what you know about every word in the hypothesis. Recall every authority that might lend meaning to the hypothesis.
"VI. Write the conclusion.
"VII. Decide the method of proof.

46. Ibid., pp. 96-97.
47. Blank, Laura, "Technique and Devices Conducive to Better Teaching of Geometry", Mathematics Teacher, Volume XXI(March, 1928), p. 172.

A. Direct
B. Superposition
C. Indirect
D. Analytic
E. Combinations of any of these
"VIII. Plan the proof mentally.
"IX. Write the proof in full.

5. Activities in geometry.

a. Smaller activities.

The pupils in the elementary grades may study to an advantage intuitive geometry, which means the study of geometric forms for the purpose of becoming acquainted with their nature by using them and experimenting with them, and making calculations as to the seeming true relations.[48] The girls in the grades may draw on paper a figure with three sides and cut along the lines obtaining triangles. They may draw similarly a figure with four equal sides and square corners and cut along the lines obtaining squares. They may draw two lines the same distance apart all along the paper, then they may draw another that cuts both of these and cut along all of these lines, and then having numbered the angles they may place them on each other. They may see which fit and replace them, and then decide which are equal. All these may be repeated with the same results. Similar exercises may be given.

The teacher may draw a straight line on the board and ask the boys to draw one on paper equal to it. They may measure it. The teacher may draw one four feet long and ask the boys to

48. Hassler, J. O., "Some Notes on the Introduction of Geometry", School Science and Mathematics, Volume XXVI (October, 1926), p. 727.

draw one equal on paper to the scale of one inch to a foot.

There are innumerable other small activities in geometry that children like to learn.

Then, it must not be overlooked that anything which the teacher or the pupil does in demonstrating a theorem or proving a problem is a small activity in the larger unit geometrical process. The psychologists are fond of saying: "It is not <u>stuff</u> which children learn; it is reactions"(or activities). Those activities which are related tend to become tied up into larger units of thought.

b. Larger activities.

The pupils may draw a rectangular solid to represent the classroom, calculate its volume, and determine the breathing space for each child in the room. They may draw a gate and explain how to brace it.

The teacher may explain how the gardener lays off rows in parallel lines in which to plant his seeds; and if he checks the rows, he forms parallelograms. He may show how the seats in a room are arranged in parallel rows, which may be straight lines or arcs of circles. He may point out how the walls of buildings are plumbed to the perpendicular; the arches of bridges and self-supporting sections of viaducts are curved in arcs of circles; the steel rails of railroad tracks are laid in parallel lines; the best form of land surveying plans the tracts in rectangles; and triangles are formed to make rigid or brace the greatest buildings and other structures.

Larger and larger geometrical activities are seen in the architects' blueprints; in the designers' plans for intricate machines; for example, the airplane. They are seen on ever enlarging scales, as in designs and executions for building the ocean liners, and in planning the Hudson River Bridge or the London Bridge. It was said of Sir Christopher Wrenn, in relation to Westminster Abbey, "If you would see his monument, look around". So everyone may see geometry's living and growing monument by looking around anywhere and everywhere.

6. Transfer of training.

a. Meaning of the term.

The transfer of training means that, if one is trained in one activity or line of thought, this training will function or carry over in large degree into other activities or lines of thought. It means that, if one is taught to reason in mathematics, he will reason well in life's situations. If he has learned skill in the study of Latin, he will use in English the skill developed in Latin.

b. The old doctrine.

The doctrine of the transfer of training came into prominence in the seventeenth century. Those who formulated and taught the doctrine of the transfer of training believed there was a perfect or almost perfect transfer. Difficult subjects were taught in order that the learner might the better overcome the difficulties in life's situations as well as mastering difficulties and technicalities in other subject-matter.[49]

49. Pintner, *op. cit.*, pp. 263-269.

c. The new doctrine.

Students of education and of psychology have found through the experimental approach that the claims laid down by the old doctrine are not altogether justified. The mind seems to be a unitary whole, made up of functions more or less closely related. Some are so closely related that improvement in one mental trait affects the action of another toward its improvement. Others are so distantly related that a change in one has little or no effect upon the other. Some seem to be antagonistic, and, therefore, an improvement in one may cause a decrease or set back in the other.[50] There seems to be a transfer to the extent of common elements in the trained functions and other functions.

d. Experiments related.

The first experiments in the transfer of training were made by psychologists in their laboratories, but they are not very helpful for school children and school purposes. Later, however, valuable information was gained with pupils in school subjects.

In experiments by James and others, it was decided that there was no transfer in memory, but improvement only in the methods. Many other experiments have been made, and in nearly all of them there was some transfer of training from the trained function to certain others.[51]

After summing up all the experiments, it seems that the

50. Jordan, A. M., Educational Psychology, pp. 192-194. Henry Holt and Company, New York, 1928.
51. Ibid., pp. 204-213.

closely allied functions show a fair amount of transfer; but, as functions become less allied, the amount of transfer may dwindle to the zero point. Learning is reacting. We learn to react to certain stimuli. Unless these stimuli appear in the new situation, we have not learned to react; and there is no transfer.[52]

The general conclusion is that there is transfer in perceptual memory, memory processes, and reasoning processes.

Transfer takes place when there are identical elements, and the amount of transfer depends upon the identity of the elements in the situations of learning and application.[53]

e. Applied to geometry.

The study of geometry definitely influences general mental ability, or the power to think abstractly. Geometry is properly learned through the processes of reasoning; by experiments we have learned that there is transfer of training in the reasoning processes; we may conclude that the idea of mental discipline, though exploded for most subjects, still has a meaning in geometry.[54]

7. Difficulties in learning geometry.

There are many difficulties in learning geometry, and some of these will now be considered.

a. Newness of the subject.

Geometry itself is indeed ancient, but the study of de-

52. Pintner, op. cit., pp. 270-277.
53. Jordan, op. cit., pp. 209-213.
54. Campbell, A. D., "Some Values of the Study of Mathematics", Mathematics Teacher, Volume XXIV(January, 1930), pp. 47-48.

monstrative geometry is so much different from the other school subjects with which the pupil has come in contact that geometry is new and strange to him. He must learn to present his ideas in a new way.[55] He must learn that memorizing, which has proved so valuable an ally in many of the other subjects, will fail him now. In order to learn geometry enthusiastically and effectively he must learn to think for himself. The author of his textbook nor his teacher can do his thinking for him if he would learn geometry.

Much depends upon a student's conception of a new subject. He should be given a true notion of the nature of the subject; he should see the possibilities of the practical uses, and he should not be plunged over his depth. There should be a contact, if possible, with other courses.[56]

b. Language difficulties.

The notation of geometry and the language used in learning geometry are naturally somewhat technical.[57] The Greeks developed geometry, but it was given to Europe largely written in Latin, having been translated from the Greek and Arabic. Latin then was the common language of the academic world. French was the language of the cultured of England for a period following the Norman Conquest. English students learned geometry from French texts. Out of an average of a hundred words in the technical vocabulary of geometry more than one half are of Latin origin. Most of the others are about equally divided

55. Nyberg, J. A., "First Month of Geometry", School Science and Mathematics, Volume XXI(January, 1921), p. 29.
56. Hassler, op. cit., p. 723.
57. Nyberg, op. cit., p.29.

between Greek and French.[58] The traditional formal language generally used, in addition to the technical Latin, Greek, and French terms, make it rather difficult for the pupil to understand clearly the language of demonstrative geometry.

In order to overcome language difficulties, accuracy in speech and simplicity in demonstration should be so learned that the pupil will have nothing to unlearn as he progresses. Nothing is so destructive of both subjective and objective confidence as unlearning. A great responsibility rests upon authors of textbooks in geometry to present faultless language and consistent logic, because youth makes both the word and thought of the book his own.[59]

c. Difficulties in logical thinking.

Logical thinking in learning geometry is of the highest importance, because the demonstration of a theorem is perhaps the purest logical process. The necessity for recognizing and overcoming the difficulties of logical thinking cannot be overemphasized.

1) Definitions, axioms, postulates. Definitions should be learned when the pupil needs them. When possible, he should be given a numerical illustration. After the definitions are studied, he should be given exercises to make their meanings clear.[60] Unless definitions are exact in the pupil's mind, he cannot understand geometry.

58. Karpinski, L. C., and Fielder, A. M., "The Terminology of Geometry", School Science and Mathematics, Volume XXIV(February, 1924), pp. 162-163.
59. Moriarty, M. M. S., "Geometry Notes", Mathematics Teacher, Volume XXI(May, 1928), p. 291.
60. Hassler, op. cit., pp. 273-274.

It is necessary for the learner to understand clearly the distinction as to meaning of an axiom, a general fact, and a postulate, the meaning of which is limited chiefly to geometry. The learner should know and be able to apply the axioms and postulates necessary to the proofs of the propositions which he studies. His merely memorizing them is not sufficient; he must be able to use them.

As demonstrative geometry is ordinarily presented, pupils often complain about the first theorems they study being obvious. They say the mental effort used to prove them is wasted. As a result many pupils develop a violent prejudice against geometry at the beginning of its logical study.

d. Dangers to guard against.

If the learning of geometry is to be most effective, there are certain dangers to guard against by both the pupil and the teacher. All pupils who study geometry do not have the best possible attitude toward the subject, and, for that reason do not get the most satisfying results. There are too many who learn to dislike geometry, and too many who fail. Some of the dangers will be considered.

1) Memorizing proofs. Nothing perhaps is more fatal to the satisfactory learning of geometry than for the pupil merely to memorize the proofs of propositions. There can be but little interest and no enthusiasm on the part of the pupil in learning geometry, when he tries to learn it in this way. Geometry is intended to be a splendid course in logical reasoning; but when the pupil tries to memorize the proofs, he makes it indeed a poor course in memory training.

The pupil who memorizes from the textbook and recites what he has learned has great difficulty with original exercises. He has not learned to reason out the geometrical processes.

To avoid the danger of memorizing proofs some teachers have discarded the textbooks and have prepared syllabi for the semester's work. The pupil must then work the propositions without seeing them proved in a book. In the syllabus method, every theorem or problem is treated as an original. Using a syllabus seems to be better than using a textbook; in the syllabus method the tendency to memorize proofs is largely removed.[61]

2) Failure to understand definitions. The pupil as well as the teacher should guard against failure to understand definitions as the terms are used. Such terms should be carefully analyzed, pictured in the mind, and illustrated by drawings and use.

3) Failure to understand logical thinking. The process of logical thinking is not always understood by the pupil. The study of geometry should lead pupils to develop, each for himself, a large part of the demonstrations in order to be able to think logically, to express clearly, and to apply the same kind of thinking to other subjects. The pupil should be warned against failure to think logically. He may be made to feel the need for it, and, at the same time, to see its value to him.[62]

61. Ryan, J. D., "Two Methods of Teaching Geometry: Syllabus Vs Textbook", Mathematics Teacher, Volume XXI (January, 1928), p. 31.
62. Barnes, H. O., "Geometry by Analysis", School Review, Volume XXVI (October, 1919), pp. 612-613.

The following suggestions made to the pupil may help him to understand logical thinking in geometry: from the theorem learn clearly the hypothesis and the conclusion. Let him ask himself mentally what he knows about every word in the hypothesis and recall every authority that might lend meaning to the hypothesis. He should write the conclusion; decide in his mind the method of proof to use; with a carefully drawn figure before him he sh uld plan the proof mentally. Finally he should write the proof in full.[63]

Miss Perry[64] made a study of the psychology of learning in geometry. She studied the comparative effects of two techniques in the solution of exercises in geometry upon students of three levels of potential abilities in reasoning. She concluded that students should have a definite technique, or outline of procedure, and have the opportunity to practice this technique. She found the results most gratifying in helping pupils understand logical reasoning.

Pupils may assume the converse to be true, reason in a circle, or use in the proof something taken from the figure not given in the theorem.[65]

Abbott[66] advocates the use of the syllogism in training the pupils in geometry to think. He says some pupils fail because

63. Blank, *op. cit.*, p. 172.
64. Perry, Winona, *A Study in the Psychology of Learning in Geometry*, Teachers College, Columbia University, 1925.
65. Sharwell, T. P., "Common Fallacies Made by Pupils in Geometry", *School Science and Mathematics*, Volume XXVII(June, 1927), p. 616.
66. Abbott, F.L., "Three Step Method of Proving Geometry", *School Science and Mathematics*, Volume XXV(April, 1925), pp. 409-411.

of apparent inability to reason; others quote propositions foreign to point in question; still others fail to observe that they have not proved the proposition.

8. Motivation of learning geometry.

A value that has the power to move one to action is called a motive. Interest is essentially the feeling of value. Children, like adults, are naturally in pursuit of experiences which will make their lives significant and meaningful; they are after values, no matter how implicit their pursuit may be.[67] Some of the many values for motivation in geometry are given.

a. Employment of social values in geometry.

The social values of geometry serve as excellent motives for its study. The pupil should be made to feel that geometry has real values in everyday life, as shown under the topic, "Geometry activities", p. 185 of this work. In the various units of study the social values should be emphasized that the pupil may be led to form the growing conviction that geometry is important in the life of the community.[68]

The geometry of design in art and decoration used in the city may be pointed out to the student, and it may be explained that to appreciate it fully one must know geometry.

b. Employment of practical values in geometry.

Young people are strongly inclined toward the practical; they are interested in the things they can do and use; they are

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67. Ruediger, op. cit., p. 316.
68. Breslich, op. cit., p. 65.

motivated by purposeful activity. They may be shown how parallels, perpendiculars, rectangles, and triangles are utilized in the construction and use of automobiles and airplanes. The practical values may be considered as the various units are studied.

The familiar device, by means of which the telephone is drawn up or pushed out of the way without changing its vertical position, is a practical application of the fact that when both pairs of opposite sides of a quadrilateral are equal they are also parallel.[69]

c. Pointing out the need of geometry in other subjects.

The need of geometry in other school subjects may make a strong appeal to high school pupils, for it may then be made to function in their experiences. Geomatry may be employed not only in other subjects in mathematics, but also to a greater or less degree in other courses. The following is an example in physics on the application of geometric construction.

A boat is being rowed across a stream thirty degrees east of north at a rate of 6 miles an hour, while the current moves it east at the rate of 9 miles an hour. Find the direction in which the boat is actually moving.

In the solution a line 6 units long is drawn to the right of a perpendicular line, and making an angle of thirty degrees with it. From the vertex a horizontal line is drawn 9 units long. From the extremities of each of these, lines are drawn parallel to each. From the point of intersection of these two auxiliary lines a diagonal of the parallelogram is drawn. This diagonal will give the direction and its length will be the distance the boat has moved in one hour.

d. Employment of cultural values.

As soon as a pupil becomes convinced that he is increasing his power and skill in mathematics by learning geometry, he is

69. Ibid., p. 66.

stimulated to more intensive work. His desire to extend his useful knowledge impels him to a greater interest. The prospect of learning work with less efforts and better results appeals to him. One of the most powerful arguments for continuing mathematics from arithmetic to geometry is the fact that some things can be done better by geometry. Many believe that the logical reasoning is the chief element in securing a continued place for geometry in the high school curriculum. The pupil may be led to appreciate the value to be derived from the practice of the type of reasoning in geometry.[70] The real stimuli to its study are intellectual growth, self-activity, personal pleasure, and understanding.

Culture is essentially inclusive of all the processes of orientation. One who never learns geometry is certainly lacking in his orientation in many of the situations of life. without it he cannot converse with his friends intelligently about many large and important facts in their environment. Culture is, too, essentially extensive beyond one's immediate vocation or other personal interests. There is a phase of such broadening processes which can never be bridged without the aid of geometry. And here is one of the places of vital transfer of training from the locus of learning geometry to that of need to use it.

9. Maintaining interest in geometry.

It is not necessary to supply motives for all that is taught in geometry, but it is very desirable to have the pupil feel that the learning of the subject is decidedly worth while

70. *Ibid.*, pp. 76-77.

and that there would be a real personal loss if geometry is not learned. We shall consider some of the ways to secure interest in the learning of geometry.

a. Employment of geometrical recreations.

Pupils as well as adults, find much pleasure in trying to solve mathematical puzzles, fallacies, and illusions.[71] There are many of these that are very interesting in geometry. The athletic field is laid off by geometry.

b. Employment of assembly programs.

Assembly programs may be furnished by the mathematics club, and geometry may be combined with other mathematical subjects, or they may be rendered by the geometry class itself. These are given in many schools at regular intervals in the presence of the entire school.

c. Using the history of mathematics.

Historical and biographical material appeal strongly to certain pupils and add interest to all. Authors of textbooks have discovered this, and many give some interesting and valuable history of the subject or contributors to it. Teachers may use the history of a subject in arousing interest. Geometry has an especial wealth of historical background which may be utilized in making geometry intensely interesting to pupils. (See Chapter I of this thesis.)

10. Results to be expected.

a. Shortcomings of the Greeks.

It is well to consider the shortcomings of the Greeks in

71. Ibid., p. 79.

geometry; for teachers may expect too much of their pupils, since so many of the fundamental concepts were not grasped and assimilated by the gifted Greeks.

They did not use the system of co-ordinate axes, and analytical geometry was practically unknown to them. They said that parallel lines could not intersect at all instead of saying that they might intersect at infinity. They did not use negative quantities, which developed historically with trigonometry. The law of sines so essential to dealing with plane and spherical triangles was not known. Both the Greeks and the Hindus divided the general triangle into right triangles for solution. When the sides were given, they used Heron's formula for its solution.

There was no formula known to the Greeks for finding the area of a general spherical triangle. The well known and convenient formula which expresses this area in terms of the spherical excess seems to have been discovered by the Englishman, Thomas Hariot(1560-1621), and its simple modern proof appears in the work of the Italian, Cavalieri(1591-1647). The ancient Greeks were also unfamiliar with the very useful process by means of which many of the theorems related to angles are translated into those related to sides, and vice versa, by means of the polar triangle.[72]

b. Expectations with America.

America has the accumulated knowledge of the past centuries

72. Miller, G. A., "Mathematical Shortcomings of the Greeks", School Science and Mathematics, Volume XXIV(March, 1924), pp. 285-286.

as a foundation upon which to build. The Greeks had very little upon which to found their structure of logical geometry. A very limited number of the Greeks in the higher classes only studied geometry. In America its truths are taught to all. In the light of what the ancient Greeks did, we may be justified in expecting much of America.

c. Results expected from the typical American classroom.

The subjective judgment of the average teacher may not vision these very accurately, or in an amount equal to the results expected by other teachers. Fortunately, there are on the American market several standard examinations in plane geometry. The Orleans Prognosis Test in Plane Geometry[73] may be given pupils before they study the subject. There is a high correlation between the success or failure on this examination and that from the course. Hence, it is possible to predict, with a large degree of reliability, what results to expect from the pupils in learning the subject. Then, there are the Columbia Research Bureau Plane Geometry Test[73] and the Orleans Plane Geometry Achievement Test[73]. Each of these examinations have "norms". It may be expected, in advance, that with good teaching the average results to be looked for from the class will be equal approximately to those represented by these "norms". There are several such standard geometry tests. Then, too, there are tests usable from week to week, as seen in Bishop-Irwin Instructional Tests in Plane Geometry[73]. The use of these "standards"enables the teacher to estimate in advance the quantity and quality of work in geometry.

73. Published by World Book Company, Yonkers, New York.

America has done much to make the learning of geometry psychological. Much has been done in this country in the art of teaching and learning geometry. We can appreciate what we have accomplished when we remember that until very recently England taught Euclid almost unchanged from the original "Elements" of Euclid.

Chiefly through experimentation and through personal experience our educational psychologists and teachers of mathematics, including geometry especially, are approaching, if not a Royal Road, at least, a safe and sane one to the learning of geometry. The fields of psychological methods are being carefully explored and mapped. It may be hopefully expected that before very much longer American youth will have the learning of geometry as interesting and as scientific as geometry itself is interesting, valuable, and scientific.

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