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Examining the Process of Identification in the Mathematics Classroom and the Role of Students' Academic Communities

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To the Graduate Council:

I am submitting herewith a dissertation written by Richard J. Robinson entitled "Examining the Process of Identification in the Mathematics Classroom and the Role of Students' Academic Communities." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Teacher Education.

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Examining the Process of Identification in the Mathematics Classroom and the Role of Students' Academic Communities

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Dedication

Dedicated to the memory of my father, Dr. Franklin E. Robinson:

the best father, teacher, and friend I will ever know.

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Thanks to Dr. Lynn Hodge for her years of unfailing support, without which this would not have been possible. Thanks to my committee for their thoughtful comments and time dedicated to this project. Thanks also to Ms. Mason for allowing me to come into her classroom and conduct this study.

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Abstract

The primary purpose of this research was to provide insight into the identities students develop as they interact in a high school mathematics classroom. A *normative divide* developed which eventually split the classroom into two distinct academic factions: those who resisted the emerging local definition of what it meant to do mathematics and those who did not resist (i.e. complied or identified). A secondary purpose of this research was to understand the role of students' academic communities in mathematics identity development. Student narratives helped uncover mathematical spaces outside the classroom that each developed their own unique definition of what it meant to do mathematics (i.e. normative identity). As a result, these spaces provided students with additional *opportunities to identify* with mathematics. Implications for both theory and practice are discussed, along with future possible lines of research.

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Chapter 1

Introduction

The primary purpose of this research is to provide insight into the identities students develop as they interact in a high school mathematics classroom. A secondary purpose of this research is to understand the role of students' academic communities in mathematics identity development. As a result, this study has the possibility of providing practitioners with much needed direction into how to best support students in achieving the vision of quality mathematics teaching and learning set forth in the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

The Common Core State Standards for Mathematics (CCSSM) include a proposed set of classroom practices that describe what students should be doing in the mathematics classroom in order to become mathematically proficient. Two of these practices are to "Make sense of problems and persevere in solving them" and to "Construct viable arguments and critique the reasoning of others" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 6). Notice that neither of these practices is likely to happen spontaneously in the mathematics classroom, given the typical descriptions of many U. S. mathematics classrooms as following a script with the teacher specifying procedures and students following these procedures (Stigler & Hiebert, 1999). Moreover, these practices seem to require something extra from students that had not been previously required with "drill and kill" mathematics. This "something extra" is the centerpiece of this study and has

gone by many names, including disposition, motivation, and engagement, all of which can be understood through the lens of identity.

In the seminal report by the National Research Council entitled “Adding It Up,” the authors define a productive disposition as “the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that the steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics” (National Research Council, 2001, p. 131). The construct of personal identity subsumes not only student beliefs, but also students’ views of their own competence and valuations of mathematics as it is enacted in their classroom. As noted in Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000), “students learn mathematics through the experiences that teachers provide. Thus, students’ understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school” (p. 17). The experiences that teachers provide and what students make of these experiences are directly related to the construct of identity on which I focus in this study. I draw on the constructs of personal identity and normative identity to investigate the process of identification in the context of a high school Algebra II classroom.

My journey toward the study of identity began with John Dewey. I can still remember the first time I read through Experience and Education (Dewey, 1938) and how the description of teaching seemed to ring so true. Dewey faithfully paints a picture of education based on experience. As a teacher this was a watershed moment. As I looked

back at teaching through Dewey's eyes I saw the importance of every learning experience as part of a larger continuum of experience. As noted by Dewey: "…the principle of continuity of experience means that every experience both takes up something from those which have gone before and modifies in some way the quality of those which come after" (Dewey, 1938, p. 35). I saw how every choice a teacher makes stands to not only weigh heavily on a child's ability to understand, but also their ability and desire to use that understanding later. I saw that what I came to remember most about those teachers that had been influential during my years as a mathematics student, was not how much they taught me, it was the way in which they taught me: with particular attention to creating high quality experiences.

George Polya, another giant in education, marked the next milestone on my journey to the study of student identity. I read "How to Solve It" (Polya, 2004) after about five years of teaching, while working at a STEM high school for accelerated mathematics students. The author's central concern was the importance of creating students who could successfully tackle novel problems. To that end, Polya provides a pragmatic list of questions meant to prompt students as they problem solve. Not only did his pedagogical recommendations reaffirm many of the teaching practices I had come to employ, they also struck the same chord Dewey had years before. In fact, the scaffolding techniques Polya was espousing were merely one instantiation of what it means to create a truly educative experience.

Some years later I read Cobb and Yackel's (1996) article on sociomathematical norms for an article review in a graduate mathematics education class. As I look back

now I remember this article being among the most salient and inherently plausible articles I ever read in graduate school. I now realize that this is greatly due to the fact that the sociocultural view of teaching seamlessly blends together the theory of experience proposed by Dewey (1938) with the pragmatic approach to practice of Polya (2004). Moreover, I soon realized that one way to get at what it means to create a truly educative experience was by studying identity.

In order to fulfill the vision of the CCSSM, practitioners will need a clear picture of not only what students need to be able to do mathematically but also how best to support them. The new standards, as the NCTM Standards did, emphasize the importance of both positive dispositions and substantial understanding. This is aligned with the definition of mathematical literacy as including both competence and motivation (Moses & Cobb, 2001). Therefore, this study is urgently needed if we are to provide practitioners with both the *what* and *how* of successful CCSSM implementation. In the paragraphs to follow, I explain why this study also addresses another need in math education research, which includes coordinating different frameworks that investigate math identity. Therefore, the study has implications for teaching and scholarship.

Issues and Problem Statement

A substantial body of research has addressed issues of motivation in math education using four major theoretical orientations (see Table 1): behavioral, which describes motivations as rewards or punishments that stimulate behavior; attribution theory, which states motivation is influenced by students' views of what creates success; goal theory, describes why people engage in activities by categorizing their personal

goals; and personal construct theory, which describes motivation as a set of "rational cognitive processes" (Middleton & Spanias, 1999, p. 74). In order to compliment this predominantly quantitative literature base, a growing number of studies in the past 15 years or so have used identity from a situated perspective to examine motivation (Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009; Hodge, 2009; Nasir, 2002; Walker, 2012) .

In spite of this literature base, we still know little about why and how students come to identify, resist, and comply with mathematics in individual math classrooms. Therefore, there have been recommendations for a) more qualitative studies that attempt to understand students' experiences from their perspective (Gutiérrez, 2008), b) studies that examine a range of different cases of identification in math class (e.g. resistance, compliance, identification) (Cobb, et al., 2009), and c) studies that coordinate different identity frameworks by examining aspects of the classroom, school, and local communities that contribute to math identity development (Bishop, 2012; Cobb, et al., 2009).

In order to address these concerns, this study focuses on identity formation within the mathematics classroom, relying on student interviews as a way of including students' understanding and valuation of their classroom experiences. In the hopes of documenting a wide range of cases of identification, this study includes students from both honors and non-honors classrooms. This study will also explore ways to coherently integrate the macro perspective of Walker (2006) which focuses on the effects of academic

communities outside the classroom and the micro perspective of Cobb, Gresalfi, & Hodge (2009) which focuses on identity formation within the classroom community.

Purpose of the Study

The purpose of this study is to understand the identities that students develop in their mathematics classroom and the role of academic communities in their identity development. The underlying premise is that students engage in what they come to value or identify with. By understanding the process of identity formation in the mathematics classroom the hope is to provide researchers and practitioners with actionable insight into how to best support students as they attempt to engage in meaningful mathematics. Moreover, by including an exploratory look at academic communities outside the classroom setting, I hoped to gain an understanding into how such communities mediate the process of identity formation.

The following research questions will be used to address the purpose of my study:

1. How does the process of identity formation play out within the mathematics classroom?
    a. How is the normative identity as a doer of mathematics constructed and negotiated within the classroom?
    b. How do students' personal identities form in relation to the normative classroom identity?
2. How do students describe their academic communities both inside and outside of school (peer, family, and school relationships)?

3. What are the relationships between the academic communities students describe and student identity formation?

I will use the questions to bookend my study, returning to them in Chapter 4 in an effort to provide preliminary answers to these questions.

Significance

This study has significance for both theory and practice. With respect to theory, this study informs identity research in two ways. First, this study contributes to our understanding of how different cases of identification play out in the classroom. This is important because as a research community we still do not have a complete picture of how students come to identify with mathematics as it is enacted in the classroom. Second, this study examines the relationship between students' math identities and the role of their supportive academic communities. This is important if we are to successfully leverage the possible benefits of supportive academic communities for student learning and motivation.

With respect to practice, this study can offer practical and actionable suggestions to teachers on how to support their students' positive relationships with mathematics. With the introduction of the Common Core State Standards there is an increased emphasis on motivation and affiliation. These issues have also been a perennial concern among math teachers (Middleton & Jansen, 2011; Middleton & Spanias, 1999). This study can inform how classroom practices afford or constrain the identities students develop, and hence their motivation and affiliation within the mathematics classroom.

Students’ motivations to engage with math have implications for their access to future educational and professional opportunities (Moses & Cobb, 2001) .

Limitations

In this study, certain steps will be taken to ensure credibility of my results. Since a qualitative researcher is often seen as the main data gathering tool, steps must be taken to help the process remain as salient as possible. To this end, I must to clarify any researcher bias or tacit assumptions from the outset (Creswell, 2007). One possible source of bias comes from the fact that I have chosen to collect data from classrooms in the school at which I currently teach. Moreover, the teachers of these classrooms and I have served on committees together and created cordial relationships both inside and outside of school. As noted by Maxwell (2013),

> …the value of a qualitative study may depend on its lack of generalizability in the sense of being representative of a larger population; it may provide an account of a setting or population that is illuminating as an extreme case or ideal type.” (p.137)

I defend my choice of cases by noting that these teachers are well-respected by both their colleagues and administration, and consistently score high on measures of student achievement and value-added. Hence these Algebra II classrooms can be considered positive cases insofar as they consistently provide students with opportunities for success.

In qualitative research the onus is on the reader to determine transferability. To aid in this process a rich thick description of the participants and context will be included to allow the reader to determine if their situation shares enough common qualities to

allow transference (Creswell, 2007). For their part Paulus, Horvits, & Shi (2006) describe the transference of a *working hypothesis,* stating that as a result of thick description, “readers are given the opportunity to decide on the degree of congruence and applicability of our working hypothesis to their own teaching and learning” (Paulus, Horvits, & Shi, 2006, p. 363). Such transference is as vital aspect of this study’s utility to both theory and practice.

Delimitations

The primary purpose of this study is to provide insight into the identities students develop as they interact in a high school mathematics classroom and to examine the relationship between these math identities and students’ academic communities. I have chosen not to focus on the resources that are part of the larger school community. This delimitation is in no way meant to discount the great work of researchers such as Horn (2008), who provided evidence of identity resources that are created by teachers working together outside the classroom. Most importantly it seems, is how teachers within the same department consistently reflected together on what it means to be competent in mathematics. These conversations seemed to have a real affect on how they came to understand and describe student success and failure.

> They would narrate these troubles employing a number of concepts—maturity, determination, readiness, organization, and especially student-skills—but notably absent from their explanations were the common ones that make success seem beyond the reach of a teacher; namely, those of innate ability or inadequate home support. (p.233)

Although this research shows great promise, for clarity and focus I have chosen not to investigate identity resources that emerge from teacher interactions within the mathematics department.

Definition of Terms

I am using the following terms to help describe the underlying social structure within the mathematics classroom (normative identity) and how students come to value mathematics as it is defined locally (personal identity).

basis for discourse: The basis for discourse in a classroom encompasses the taken-as-shared mathematical objects that students and the teacher use to communicate, the nature of those objects (procedural, conceptual, or metaphorical), and the meanings those objects have for the teacher and students (experientially real or meaningless symbols).

discourse: Defined by (Gee, 2001b) as “language-in-use or stretches of language (like conversations or stories)” (p.26). This is discourse with a “small-d,” as opposed to “Big D” Discourses.

Discourses: Defined by (Gee, 2008) as “ways of being ‘people like us.’ They are ‘ways of being in the world’; they are socially situated identities” (p.3).

mathematical spaces: Defined by (Walker, 2012) as “sites where mathematics knowledge is developed, where induction into a particular community of mathematics doers occurs, and where relationships or interactions contribute to the development of a mathematics identity” (p.67).

normative identity (as a doer of mathematics): the collection of obligations and expectations that the teacher and students have for other students, obligations that are specifically related to how an effective mathematics student is locally defined.

participation structure: Lampert (1990) defines a participation structure as "the consensual expectations of the participants about what they are supposed to be doing together, their relative rights and duties in accomplishing tasks, and the range of behaviors appropriate within the event" (p.34).

personal identity: the degree to which a student comes to identify with mathematics as it is enacted in the classroom, simply complies with the teacher, or resists participating in mathematical activity.

social norms: "Social norms are characteristics of the classroom community and document regularities in classroom activity that are jointly established by the teacher and students" (Cobb, Stephan, McClain, & Gravemeijer, 2001, p. 122)

sociomathematical norms: "sociomathematical norms deal with obligations and expectations that are specific to mathematical activity" (Bowers et al., 1999, p.40)

taken-as-shared: Two students are said to have a taken-as-shared interpretation of the current situation if "each seemed to assume that his personal interpretations was shared by the other" (Bowers, Cobb, & McClain, 1999, p. 43) .

Theoretical Framework

From a situated perspective, identity is seen to develop as one participates in a specific context. In this way, studying identity involves documenting both the context and the individual's ideas about the context and his or her role within this community (Mead,

1934). From a situated perspective, aspects of the classroom can be viewed as supporting or constraining the identities that students develop in class.

I draw on Cobb et al.'s (2009) framework that examines students' identities as they develop in the mathematics classroom. This framework includes two central constructs. One of these is personal identity and the other is normative identity. Normative identity documents the classroom culture and practices that are part of life in that particular classroom. Personal identity documents students' ideas about the classroom culture, their roles as math students in the class, and their valuations about the classroom and their roles. These two constructs allow me to describe in detail what doing math means in the math classroom and what students think about the class and their obligations as members of the class.

Chapter 2

Review of the Literature

This review of literature is meant to situate the current study within the terrain of mathematics education research. To this end, first, I uncover some assumptions of fundamental importance to the current study. Second, I overview the literature related to the perennial problem of student motivation. Next, I introduce more recent work that uses identity as means of understanding students' motivation and engagement. Then, I discuss the specific identity framework that serves as the theoretical framework for the current study. I end this section with a discussion of students' academic communities and the possible resources for students' identities they provide.

Uncovering Assumptions

When conducting an educational study it can be helpful to define key terms and highlight fundamental assumptions at the outset. These definitions and assumptions greatly influence what the researcher will attend to at every point in the research process, from design, to data collection, to data analysis. Learning can be thought of as having an individual dimension and a social dimension. What learning looks like, and even what you value in education, depends in large part on which of these dimensions you choose to view as primary. It's analogous to taking a picture with a camera, in that, the same scene can look quite different depending on whether you choose to focus on the foreground or the background. In this section I discuss the interactionist perspective on learning I have chosen for this study and its associated assumptions. Inspired by Cobb and Yackel (1996), we can place perspectives on learning into one of three broad categories:

sociocultural, psychological, and interactionist (see Figure 1). I now consider each in turn, briefly highlighting one of the relevant educational theorists commonly associated with each category.

Psychological perspective: perturbation-driven individual development. From a psychological perspective, learning can be viewed as perturbation-driven development in which the individual dimension is primary. Educational theorists commonly associated with a psychological perspective include Piaget and Von Glaserfeld. Von Glaserfeld (1983) focuses on a specific type of knowledge:

> …apart from faith and dogma, a knowledge that fits observations. It is knowledge that human reason derives from experience. It does not represent a picture of the "real" world but provides structure and organization to experience. As such, it has an all-important function: it enables us to solve experiential problems (p.38).

This concept of "viability" is central to Von Glaserfeld's psychological variety of constructivism. This relates to the current study in that I assume that what it means to do mathematics is not an objective truth. On the contrary, I view what it means to do mathematics as highly context dependent and something students derive from their own experiences.

Sociocultural perspective: development as participation. From a sociocultural perspective, learning can be viewed as participation-driven development in which the social dimension is primary. The educational theorist most commonly associated with a sociological perspective is Vygotsky, with more recent work by Rogoff, Baker-Sennett, Lacasa, and Goldsmith (1995).

The “planes of analysis” framework by Rogoff et al. (1995) provides an overarching structure for the current study. The author outlines the framework as follows:

> “The *community plane of analysis* focuses on people participating with others in culturally organized activity, with institutional practices and development extending from historical events into the present, guided by cultural values and goals…The *interpersonal plane of analysis* focuses on how people communicate and coordinate efforts in face-to-face and side-by-side interaction as well as more distal arrangements of peoples activities…The *personal plane of analysis* focuses on how individuals change through their involvement in one or another activity, in the process of becoming prepared for subsequent involvement in related activities” (Rogoff, 1995,p.46)

In this study, I consider the interpersonal plane of analysis by focusing on normative identity: how students and the teacher work together to locally define, through ongoing negotiations, what it means to do mathematics in their classroom. I consider the personal plane of analysis when focusing on how students’ personal identities develop and change through their involvement in classroom activities. I focus on the community plane of analysis by investigating the specific resources upon which students draw as they form their personal identities.

The most important contribution of the sociocultural perspective to the current study, is the idea of “development as participation”:

> A person is a part of an activity in which he or she participates, not separate from it. Our perspective discards the idea that the social world is external to the

> individual and that development consists of *acquiring* knowledge and skills. Rather, a person develops through participation in an activity, changing to be involved in the situation at hand in ways that contribute both to the ongoing event and to the person's preparation for involvement in other, similar events." (Rogoff, 1995, p.54)

This sort of coupling between person and activity is central to my view of learning in the mathematics classroom. What is important for this study is that I assume that when a student participates in the mathematics classroom they help shape not only the normative identity (what it means locally to do mathematics) but also their own personal identity as a doer of mathematics.

Interactionist perspective: development as an interactive achievement. From an interactionist perspective, learning is an interactive achievement best viewed in terms of both its psychological and sociocultural dimensions, giving neither primacy over the other. More importantly,

> … individual's efforts and sociocultural institutions and practices are constituted by and constitute each other and thus cannot be defined independently of each other or studied in isolation (Bauserfeld, 1992, p.45).

This sets up my rationale for looking at both normative and personal identity. To truly understand learning in a classroom we must look at both how "doing mathematics" is locally defined in that classroom and how students come to identify with, merely comply, or resist this local definition.

Another aspect of the interactionist perspective crucial to the current study is its emphasis on the negotiation of meaning. Voigt (1995) notes “This interactionist approach views negotiation of meaning as the mediator between cognition and culture” (p.163). Put another way, negotiation of meaning acts as the mediator between the psychological and sociocultural perspectives. In my view such negotiations are not limited to the meanings of specific mathematical terms, they also include the definition of mathematics itself. This underscores the importance of context in the study of teaching and learning; hence, my choice of qualitative case study methodology.

Motivation: The Perennial Problem in Mathematics Education

A substantial body of research has addressed issues of motivation in math education using four major theoretical orientations (see Table 1): behavioral, which describes motivations as rewards or punishments that stimulate behavior; attribution theory, which states motivation is influenced by students’ views of what creates success; goal theory, which describes why people engage in activities by categorizing their personal goals; and personal construct theory, which describes motivation as a set of “rational cognitive processes” (Middleton & Spanias, 1999, p. 74). While a complete review of the motivation literature is beyond the scope of this project, in the remainder of this section I discuss two lines of work that are paradigmatic of motivation research in mathematics education: the goal theory work of Maehr & Midgley (1991) and the expectancy-value theory of Wigfield & Eccles (2000) .

One way to describe student motivation is in terms of the goals which drive student action (e.g. mastery goals vs. performance goals) (Dweck & Leggett, 1988;

Maehr & Midgley, 1991). Goal theory is described by Maehr & Midgley (1991) as being composed of four main elements:

> (a) a focus on goals as the primary antecedent of motivation, (b) a belief that the psychological environment of the classroom determines qualitative differences in the goals adopted by students, (c) the identification of key dimensions in the classroom that are associated with the development of goal stresses, and (d) a systematic attempt to translate these essential theoretical propositions into concrete strategies for organizing and maintaining classroom activities. (p.402)

The main point here is that student motivation is seen as highly related to the classroom environment. Furthermore, it is possible to empirically identify key aspects of the classroom that relate to student motivation. The current study shares these same basic assumptions, focusing on the classroom environment and its relation to student motivation.

In contrast to a goal theory perspective, some researchers choose to view motivation in terms of students' expectations of success or failure and their valuation of the activity at hand (Dweck & Leggett, 1988; Wigfield & Eccles, 2000). In a review of their own work, Wigfield & Eccles (2000) note that children form beliefs about what they are good at and what they value as early as first grade. Moreover, such beliefs were strong predictors of student achievement and continued study of mathematics. These findings are methodologically important, in that they underscore the need to attend to students' perspectives.

In summary. Research on student motivation has a long history within mathematics education. This substantial body of motivation research includes among its areas of focus: the classroom context and its relation to student motivation (Maehr & Midgley, 1991) and students' perspectives on their own mathematical abilities and what they value (Dweck & Leggett, 1988; Wigfield & Eccles, 2000). The current study shares these points of focus, foregrounding both classroom context and student perspectives.

A growing number of studies over the past 15 years or so have used identity from a situated perspective to examine motivation (Boaler & Greeno, 2000; Cobb, et al., 2009; Hodge, 2009; Nasir, 2002; Walker, 2012). The next section provides a brief introduction to the construct of identity.

Identity from a Situated Perspective

James Gee is often regarded as one of the preeminent voices in both qualitative research and methodology (specifically discourse analysis) (Gee, 2001b, 2008). With respect to identity, Gee (2001b) has provided the educational research community with a macro-level theoretical framework for investigating identity. As part of the sociocultural tradition in identity, Gee (2001b) foregrounds context and views identity as an "interactional achievement," as opposed to an innate personal trait. The centerpiece of Gee's identity framework is the idea of recognition:

> The 'kind of person' one is recognized as 'being,' at a given time and place, can change from moment to moment in the interaction, can change from context to context, and, of course, can be ambiguous or unstable. (p.99)

Thus identity is temporal, contextual, and can take on different levels of ambiguity and stability. This study is situated within the sociocultural tradition, and as such I choose to view identity as an interactional achievement that is highly context-dependent.

Sfard and Prusak (2005) present a compelling case for the use of identity as an analytic tool in qualitative research, with one caveat: the construct must be more clearly defined. As they search for a suitable definition, the authors discuss the weaknesses of related terms including *beliefs* and *attitudes*, weaknesses firmly rooted in each term's lack of an operational definition. To address a similar weakness with identity, the authors propose their own definition of identity with a major emphasis on the fact that it be operationalized in such a way as to become a useful analytic tool. Using the construct of narratives as a basis, the authors propose that "…identities may be defined as collections of stories about persons…narratives about individuals that are *reifying, endorsable,* and *significant*" (p.16). Framed in this way, identity becomes tangible to the researcher. In order to investigate identity, one needs only focus on narratives as discursive constructs. Moreover, the concepts of endorsability and significance foreground the individual agency involved in identity formation. As a result, the narratives which constitute an identity are likely to be those narratives that have the most potential to influence future action (i.e. engagement). This relates to the current study, in that, I will ask students to discuss narratives related to their personal identities as mathematics students with the hopes of learning more about how these stories relate to their choices of when and how to engage in mathematics.

Sfard and Prusak (2005) describe the potential of identity to influence action in

terms of a perceived gap between actual and designated (or expected) identities. The actual identity of a person is comprised of those reifying, significant narratives which describe the person as they actually are, often using present tense and stated as assertions of fact. On the other hand, the designated identity of a person is comprised of those reifying, significant narratives which describe the person as they are expected to be, either now or sometime in the future. Large gaps that appear between actual and designated identities, if they persist, can likely cause a person to be quite unhappy. This unhappiness is more likely if the gap involves those "critical stories" which undergird a person's identity. Changing such stories would cause a person to "lose her ability to determine, in an immediate, decisive manner, which stories about her were endorsable and which were not"(p.18). The only way most people close such gaps is through learning (a necessary condition of which is engagement). Relatedly, the current study will seek to understand how normative and personal identities interact to influence action.

Students see patterns in the everyday workings of the classroom. These patterns of interaction make up a large part of what it means to do mathematics in this classroom. How do students make sense of these patterns? Boaler (1999) explains this sense-making process in terms of *affordances and constraints*. Specifically, one way to view learning within a classroom community is to consider the affordances and constraints that community provides. This entails a recasting of what it means to be a successful mathematics student. Success no longer means having a larger store of knowledge than someone else, it means being more in tune with, more attuned to, the resources provided by the current environment. It means that knowledge can be qualitatively different,

depending on the original learning environment. If the affordances and constraints of the original environment closely match those of the current environment in which a student is asked to perform, he or she will be able to successfully apply the mathematics he or she has learned. Hence the way knowledge is cast in its original environment can have real effects on a student's ability to use that knowledge at a later time, in a different environment. This relates to the current study, in that, I will examine patterns of classroom interaction (using the construct of normative identity) and how students make sense of the patterns (using the construct of personal identity).

If there is a persistent lack of match between the affordances and constraints provided by in-school mathematics environments and out-of-school mathematics environments, students may see this as a disconnect between the nature of mathematics in these two environments. Whereas school mathematics may be seen as something teacher-led and textbook supported, outside school mathematics is an individual endeavor with no teacher or standard textbook on which to rely. In her analysis Boaler (1999) describes a school in which

> …students worked hard and completed a great many textbook exercises, but many of the students developed the perception that school mathematics was made up of numerous rules, formulas and equations, that needed to be memorized. They believed that their role in the mathematics classroom was to memorize the different rules they had been given and many believed that school mathematics was incompatible with thought. (Boaler, 1999a, p. 263)

The current study will incorporate student interviews in an effort to uncover students'

perspectives on how they come to understand and value the nature of mathematics as it is enacted in the classroom.

In her analysis Boaler (1999) explained how affordances and constraints help provide insight into the long-standing educational debate on transfer of knowledge. Students who were unable to see a match between affordances in school and those outside of school were left to create their own mathematical methods and disregard those learned in school, since they did not appear to be pertinent to “real world” mathematics. This means that the mathematics learned in school did not transfer easily to new situations. What is important for the current study is that students recognize patterns in the everyday classroom and use those patterns to discover affordances. By focusing on identity the current study foregrounds these patterns of classroom interaction (i.e. normative identity) and how students interpret them (i.e. personal identity).

Much like the identity framework used in this study (Cobb, et al., 2009), Boaler’s(1999) work is heavily based on the idea of “expectations.” In her analysis Boaler noted common student expectations including: expecting to use a method they were just taught when attacking new problems, expecting math problem solving to rely only on mathematical knowledge, and expecting to use all parts of the given information on a problem. These expectations are all related to what Boaler calls ‘cue based’ behavior. But more importantly, students came to view these expectations *as affordances* upon which they would rely on their way to becoming successful mathematics students. These results are important methodologically, in that, a situated perspective helped uncover evidence that even when students fail to learn what is intended, they always

learn something, even if that something is simply the normative ways of acting within a particular classroom.

In their seminal study on student identity, Boaler and Greeno (2000) interviewed forty-eight high school Advanced Placement Calculus students from six different classrooms. Although students used similar textbooks, their experiences within their classroom communities were quite different, especially when the researchers considered between-class differences. Drawing on the work of Holland, Jr., Skinner, & Cain (1998), the concepts of *figured worlds* and *positioning* were used to interpret these differences. According to Holland, et al.(1998), a figured world is defined as:

> …a socially and culturally constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others. (p.52)

Whereas figured worlds highlight collective aspects of the classroom microculture, positioning focuses on an individual's personal perception of that microculture.

> The dialect we speak, the degree of formality we adopt in our speech, the deeds we do, the places we go, the emotions we express, and the clothes we wear are treated as indicators of claims to and identification with social categories and positions of privilege relative to those with whom we are interacting...Positional identity… is a person's apprehension of her social position in a lived world. (Holland, et al., 1998, p. 127)

This discussion of positional identity and figured worlds serves to emphasize that in my

view, student engagement is best understood using a framework which concurrently highlights the collective and the individual.

In her seminal article Lampert (1990) defines a *participation structure* as "the consensual expectations of the participants about what they are supposed to be doing together, their relative rights and duties in accomplishing tasks, and the range of behaviors appropriate within the event"(p.34). As a participant researcher Lampert vividly illustrated how a teacher, through ongoing negotiations with students, can create a participation structure that more closely resembles that of working mathematicians. In her analysis Lampert warns that, if left to their own devices, students will often negotiate a participation structure that includes some very "nonmathematical" ways of knowing mathematics. Specifically, students create their own forms of mathematical argument and justification including treating rules as arguments, exerting physical or political power, and face-saving behavior. With respect to the latter, Lampert explains "They act as if they believe that admitting that there is something wrong with their reasoning is an admission that there is something wrong with *them*"(p.57). This analysis makes contact with my theoretical framework in the construct of normative identity (what Lampert would call the participation structure).

In summary. The current study is situated within the sociocultural tradition, and as such I choose to view identity as an interactional achievement that is highly context-dependent. I will examine the process of identity formation using the constructs of normative and personal identity, constructs that are grounded in mathematics education research. I will ask students to discuss narratives related to their personal identities as

mathematics students with the hopes of learning more about how they come to understand and value the nature of mathematics as it is enacted in the classroom. In the following section I discuss the theoretical framework for the current study.

Identity Framework

Cobb, Gresalfi, & Hodge (2009) propose an interpretive scheme in the hopes of shedding light on the process of identity formation; specifically: "how do students come to understand what it means to do mathematics as it is realized in their classroom and whether and to what extent they come to identify with that activity" (Cobb, et al., 2009, p. 42). To answer this question Cobb et al. (2009) build their framework around the sociological construct of obligations.

Consider the following thought experiment. A colleague of yours who is new to teaching has asked you to observe her class in the hopes that you, a veteran teacher, might give her some teaching advice. You walk in the room shortly after the bell rings. After spending little to no time in the classroom you can quickly begin to see what students are obliged to do as a member of this classroom community. For example, if it is a teacher-centered classroom, students might be obliged to take copious notes and ask the teacher for clarification when needed. In a more student-centered classroom we would expect obligations for students to be qualitatively different. Such obligations might include justifying one's reasoning when presenting solutions and asking other students for clarification when needed. The collections of obligations that help to locally define what it means to be a competent mathematics student are said to comprise the normative identity. In order to better understand the fundamental constructs of normative and

personal identity it is important to discuss previous research that provides a foundation for this framework.

Foundations of the identity framework. Cobb & Yackel (1996) utilize an emergent perspective, a coupling of social interactionism and psychological constructivism. Whereas a social perspective foregrounds regularities in communal practice and taken-as-shared ways of participating, a psychological perspective foregrounds an individual's interpretation of communal practice and his/her related beliefs and values. The coupling of these perspectives is based on the apparent reflexivity that exists between their constituent constructs. For example, "classroom social norms are seen to evolve as students reorganize their beliefs, and, conversely, the reorganization of these beliefs is seen to be enabled and constrained by evolving social norms" (Cobb & Yackel, 1996, p. 178). A more complete view of the emergent perspective is provided in Table 2. This table is meant to highlight the reflexivity between classroom norms and the individual's beliefs and values, and between classroom mathematical practices and the individual's mathematical conceptions and activities.

One important aspect of the classroom microculture is the set of established norms that shape the interactions of its members. Social norms within the classroom include normative ways of acting, including raising one's hand to speak, persisting in mathematical tasks, and providing reasons to support one's answer. These norms can be analyzed in terms of their related obligations and expectations (Cobb, et al., 2009; Cobb, Yackel, & Wood, 2011). To illustrate, if a teacher expects students to persist in mathematical tasks, that teacher is obligated to provide students with ample time to work

on those tasks. These expectations and obligations shape students' beliefs about their roles within the classroom, the teacher's role within the classroom, and beliefs about what it means to do mathematics. For example, the expectation of persistence means that mathematics is an activity worthy of sustained mental focus. This is yet another example of the reflexive relationship between the social perspective (communal norms) and the psychological perspective (individual beliefs).

One of the strengths of viewing classroom culture in terms of obligations and expectations is that it makes classroom norms empirically tractable, especially when the researcher attends to students' emotional acts (Cobb, Yackel, & Wood, 2011). To illustrate, consider a classroom where students are expected to allow one another ample time to think when a new problem is presented. If a student happens to transgress this norm by blurting out an answer, another student is likely to express a negative emotional act, which helps to regenerate the classroom norm. The teacher can further strengthen the class norm by affirming the negative emotional act or explicitly addressing the transgression. To reiterate, an emotional act is just an outward expression of how the student inwardly construes the situation, and the teacher's reaction (or lack thereof) to that act says as much to the class about the classroom norms as the act itself. Cobb et al.(2011) argue that one of a teacher's most important duties is to consistently "renegotiate the social context within which children attempt to solve mathematical problems and thus influence their beliefs about their own and the teachers roles and the nature of mathematical activity"(Cobb, et al., 2011, p. 67). These authors now prefer to

speak of identity, which is often seen to encompass many of the affective aspects of classroom interaction, including beliefs.

Some norms within the classroom culture are more than just social, they are inherently mathematical; these are sociomathematical norms (Cobb, et al., 2009; Cobb, et al., 2001; Cobb & Yackel, 1996; Yackel & Cobb, 1996). For example, it may be normative within a classroom to give an explanation for an answer, which falls under the category of social norms. But what it means to be an “acceptable” mathematical explanation is also something that is constantly negotiated and interactively constituted by the students and teacher, i.e. a norm.

The concept of sociomathematical norms emerged within a line of research focused on creating and sustaining a classroom culture of inquiry and problem solving (Cobb & Yackel, 1996; Yackel & Cobb, 1996). As the teacher participants in this research prompted student thinking in an effort to further productive classroom discourse, they would often ask if anyone had solved the problem in a “different way.” As a result, the teachers and students had to negotiate what it meant for two solutions to be mathematically different. Soon a related group of inherently mathematical classroom norms emerged. These sociomathematical norms included mathematical difference, mathematical sophistication, and what counts as an acceptable mathematical explanation. The first two norms relate to taken-as-shared understandings of *when* it is appropriate for someone to contribute to a discussion, whereas acceptability relates to *how* one should contribute.

Sociomathematical norms can help elucidate the role of the teacher in an inquiry-based classroom. Rather than acting as passive observers, as is sometimes thought, teachers in an inquiry based classroom have the opportunity to act as "representatives of the mathematical community," actively participating in the ongoing, often implicit negotiations of what it means to do mathematics in their particular classroom community. Specifically,

> …the teacher's role while negotiating with students is characterized as that of proactively supporting both students' individual constructions and the evolution of classroom mathematical practices so that students increasingly become able to participate effectively in the mathematical practices of the wider society. (Cobb & Yackel, 1996, p. 186)

And now we turn to the major components of the identity framework: normative and personal identity.

Normative identity. Normative identity as a doer of mathematics is defined as "both the general and specifically mathematical obligations that delineate the role of an effective student in a particular classroom… a collective or communal notion rather than an individualistic notion"(Cobb, et al., 2009, p. 43). Hence normative identity subsumes what were previously referred to as social norms and sociomathematical norms. This concept foregrounds the importance of local context. Even if a teacher has not thoughtfully worked to shape a normative identity as a doer of mathematics within her classroom, one still exists, even if it is created by happenstance. As a teacher, you can choose to play an active role in the formation of the normative identity by consistently

negotiating, legitimatizing, or censuring. Otherwise you can simply allow a normative identity to form; in which case you are still pedagogically responsible.

Personal identity. Continuing with a focus on obligations, Cobb et al. (2009) describe personal identity as concerning the degree to which an individual student identifies with, only complies with, or resists obligations within the classroom (either general or specifically mathematical). Thus identification occurs when "students turn obligations-to-others into obligations-to-oneself. In contrast, they remain obligations-to-others for students who merely cooperate with the teacher" (p.47). From an empirical standpoint, it becomes important to attend to how students come to understand and value their obligations within the mathematics classroom.

The Cobb et al. (2009) interpretive scheme is an important contribution to identity research in education because it frames the process of learning at a level of detail that is useful in both theory and practice. Its theoretical utility is rooted in its ability to provide researchers with direction as they search for evidence of identity formation (norms, obligations, and other well-operationalized constructs). Moreover these authors vividly describe what identity-related terms look like within the specific context of a mathematics classroom (i.e. authority and agency). The scheme's practical utility is rooted in its ability to provide teachers with an alternative lens with which to view their everyday practices, a view which focuses on the classroom context (which teachers largely help to shape) and its possible connections to student learning.

Students' valuations of obligations. Recall that personal identity, as I use the term, concerns the degree to which an individual student identifies with, only complies

with, or resists classroom obligations. If a student sees value in their classroom obligations they will identify with mathematics. In order to build on an already existing body of literature, I have chosen to view students' valuations of their local obligations in terms of significance.

The concept of significance is rooted in a well-developed body of research on how minority students come to resist schooling. Two dominant theories of minority student resistance are the cultural difference and castelike minority theories (D'Amato, 1992). Cultural difference theorists explain the resistance of minority students in terms of cultural differences that exist between the school community and other communities of which the student is a part (i.e. home, ethnic community, or peer groups). Castlike minority theorists, led by Ogbu (1992), distinguish between immigrant minorities (those who voluntarily immigrate, often in search of social opportunities) and castelike minorities (those who unwillingly immigrated, often due to historical factors). Researchers have found that castelike minorities often have higher levels of resistance, often explained in terms of significance.

Students are less likely to resist schooling if they find some significance in it, either structural or situational. Structural significance is related to the possible future benefits of schooling including college admission and future employment. As noted by D'Amato, "When children neither fear nor value the structural implications of school, they appear free to confront the premises and politics of school openly and directly" (1992, p.197). Structural significance is something that is often out of the purview of the teacher, deeply rooted in outside factors such as socioeconomic status or immigrant

status. I claim that structural significance is related to the other “mathematical spaces” (Walker, 2012) in which students find themselves. As a result, I address this point in my last research question.

In contrast, situational significance is related to the present benefits of schooling, including “identities, relationships, and other states of affairs within the flow of school life itself” (D’Amato, 1992, p.191). Here the term “situational” refers to the social situation in which a student finds oneself, which, as anyone who works with school-age children knows, is often seen by the child as encompassing the whole of their very existence. The concept of situational significance is where D’Amato’s resistance framework and Cobb’s identity framework make contact: both frameworks view certain pedagogical choices as either hindering or supporting resistance. For D’Amato, some pedagogical choices that help ward off resistance are small-group instruction, a more open participation structure, and making praise and blame more even across the class. For Cobb, it’s about uncovering pedagogical choices that help to create a local definition of what it means to do mathematics with which students can come to identify.

Agency and authority. General classroom obligations are often closely related to how authority is distributed within the classroom and how students are allowed to express themselves mathematically (agency). Within the mathematics classroom authority can be defined as “the degree to which students are given opportunities to be involved in decision making about the interpretation of tasks, the reasonableness of solution methods, and the legitimacy of solutions” (Cobb, et al., 2009, p. 44). We can imagine a traditional mathematics classroom in which agency is found primarily in the teacher, who transmits

knowledge to students who act as passive receivers. Such a classroom would primarily afford students the opportunity to exercise disciplinary agency (using solution methods established by the teacher). In contrast, reform-oriented classrooms seek a more egalitarian distribution of authority where students are likely afforded more opportunities to exercise conceptual agency (choosing their own solution methods and participating in meaning making). The concepts of authority and agency are useful when describing

Specifically mathematical obligations within a classroom are divided into two categories (see Figure 2): "norms or standards for mathematical argumentation and normative ways of reasoning with tools and written symbols" (Cobb, et al., 2009, p. 45). These mathematical obligations are inextricably linked to the constructs of agency and authority: students are accountable to certain members of the classroom community (distribution of authority) for specific markers of mathematical competence (mathematical obligations) and display that competence by expressing agency (conceptual and disciplinary agency). Thus normative ways of acting and participating in a classroom provide evidence of tacit, or not so tacit, obligations.

Unpacking the Process of Identity Formation

Previous studies in mathematics education have produced insightful and detailed accounts of the obligations that often make up the microculture of the mathematics classroom (Bishop, 2012; Boaler & Greeno, 2000; Cobb, 1995; Cobb et al., 2009; Cobb & Hodge, 2011; Cobb, Stephan, McClain, & Gravemeijer, 2001; Cobb et al., 2011; Gresalfi, 2009; Hodge, 2009; Sfard & Prusak, 2005; Yackel & Cobb, 1996). That being said, these accounts are largely situated within the elementary grades and provide only a

static snapshot of classroom obligations, as opposed to a dynamic description of how those obligations are constructed and negotiated over time.

The Identification Triangle. The process of identity formation is likely affected by a host of factors including previous classroom experiences, socioeconomic status, and previous mathematics achievement, just to name a few. Building on the work of (Cobb et al., 2009), I have chosen to better understand this complex and highly situated process by narrowing my investigation to include only a few key factors: student competence, classroom obligations, and student understanding and valuation of those obligations. The ways in which these factors relate can be summarized in the *Identification Triangle* (see Figure 3).

In Figure 3, the bottom part of the triangle depicts the relation between classroom obligations and student competence. The obligations within a specific classroom define what it means to be locally competent insofar as a student is deemed competent only to the degree to which they are able to fulfill their classroom obligations. Reciprocally, the levels of local competence students are able to achieve informs the ongoing negotiation of those obligations. For example, if the majority of her students are unable to fulfill a specific classroom obligation, this may indicate to the teacher that the obligation is not appropriate or well-suited for the current group of students and may need to be renegotiated or removed altogether. Hence the bottom part of the triangle represents *teacher-initiated negotiations* in response to current levels of student competence. This underscores an important point in the current model of identity formation: obligations are

not fixed rules provided by the teacher at the beginning of the semester, but instead, are continuously negotiated throughout a large portion of the semester.

The left side of the triangle denotes the *student-initiated negotiations* that are often the result of students failing to understand or see value in a classroom obligation. For example, if a student fails to see structural significance in the mathematics at hand they may explicitly negotiate their mathematical obligations by asking the age old question: “When am I ever gonna use this?” This is critical and often mistaken assumption regarding classroom obligations bears repeating: obligations are not fixed, but are continually contested parts of the classroom culture.

The right hand side of the triangle shows that the level of competence a student is able to attain within a classroom informs their understanding and valuation of the classroom obligations and vice versa. For example, one might expect that if I am consistently deemed locally incompetent I might come to see little value in classroom obligations that consistently define me as such. Hence, the right side of the triangle highlights the psychological aspects of identity formation whereas the other parts of the triangle highlight the sociological aspects.

The Process of Identity Formation: a model. By taking the components of the Identification Triangle and adding in a time component, we get the model for the *Process of Identity Formation* as seen in Figure 4. To clarify, consider a student who enters a new mathematics classroom for the first time. The far left column denotes time, beginning just before the student enters the class at time t_0 (initial time) and ending when the student leaves the class at time t_f (final time). At time t_0, the student has had previous experiences

in which they have identified, complied, or resisted some local definition of mathematics within a previous classroom. These experiences are part of what he or she brings with them as they enter the new mathematics classroom. Within the time interval (t_0, t_f), certain key points in the Process of Identity Formation are labeled as t_1, t_2, and t_3.

At time t_1 the teacher proposes an initial set of classroom obligations. This may take place all at once as the teacher discusses her syllabus, or may occur over the first few class meetings as the teacher outlines her initial expectations for the class. As soon as obligations are introduced into the classroom, the teacher and students begin to negotiate. Evidence of student-initiated negotiation includes intentionally failing to fulfill classroom obligations or other forms of resistance. Evidence of teacher-initiated negotiation includes taking out class time to provide explicit rationale obligations or even publicly censure students who have failed to meet their obligations. Moreover, at time t_1 students' initial understandings of the classroom obligations are based largely on previous classroom experiences.

By time t_2 the student has had their first assessment in the new classroom, be it a formal quiz or test, or an informal classroom discussion. The important point is that the assessment provides the student with an initial sense of their local competence, which in turn informs their valuation of their local obligations. Also notice at t_2 negotiations regarding classroom obligations are still ongoing.

At time t_3 the classroom obligations and students' understandings of them have largely solidified. Here I use "solidified" to mean that negotiations are largely at an end. While obligations *can* be negotiated all the way up until the moment the student leaves

the classroom for the last time, this is likely not the case. At some point the teacher stops providing a detailed rationale for classroom obligations and students who intentionally fail to fulfill their classroom obligations are no longer seen as negotiating, but merely as noncompliant. Until classroom obligations are solidified it is hard for a classroom to function smoothly. Alternatively, student competence can vary greatly from assessment to assessment and is not solidified until the student completes the final exam, the last indicator of how they measure up against local standards of what it means to be mathematically competent. Thus, classroom obligations solidify due to an end in negotiations, whereas competence solidifies due to an end in assessments.

This Process of Identity Formation repeats itself as students move from classroom to classroom over the years. In other words, the level of identification a student ends with in one class is the level of identification they begin with in another.

Classroom Discourse

In the defining work in the field of discourse analysis, Gee (2001b) eloquently outlines the role of discourse (language-in-use) in the classroom:

> Language has a magical property: when we speak or write we craft what we have to say to fit the situation or context in which we are communicating. But, at the same time, how we speak or write creates that very situation or context. It seems, then, that we fit our language to a situation or context that our language, in turn, helped to create in the first place (Gee, 2001b, p. 11).

Hence, the way in which the teacher and students use language in the classroom shapes and is shaped by the classroom context itself.

In his classic work, Mehan (1979) describes the nature of discourse in a first grade classroom by distinguishing between “information seeking questions” and “known information questions.” The former are what most people likely think of when they think of questioning: one person seeking needed information from the other. The later is a peculiar type of question used almost exclusively in educational settings (formal or otherwise). Referred to by Mehan as “elicitations,” these questions are posed by a teacher who is trying to elicit known information from a student in an effort to test their knowledge. As a result of such a question the student “is placed in the position of trying to match the questioner’s predetermined knowledge, or at least fall within the previously established parameters” (Mehan, 1979, p.286). This can lead to “cue based” behavior (Boaler, 1999) where students base their answers on implicit discursive signals provided by the teacher, instead of mathematical reasoning.

Students are often unaware of their own attunement to these tacit educational cues provided by the teacher. All of these cues are based on expectations that students have for the way in which their teacher acts during class discussions. For example, if in a particular classroom correct student responses are consistently followed by teacher affirmation, students will come to expect such affirmation in the future. Thus, if a student provides an answer and a teacher waits in silence, students often read this discursive cue as a sign of an incorrect response.

Interpretive frameworks for characterizing discourse. There are several frameworks for characterizing the nature of discourse in the mathematics classroom, including those of Bauserfeld (1992), Boaler and Greeno (2000), and Thompson (1994).

Each of these frameworks focuses on a different aspect of the classroom microculture, including: what students produce, teacher images of what it means to do mathematics, and who gets to verify knowledge claims. As a result of differing foci, each framework has a different set of implications for practice (see Table 3). I now discuss each framework in turn.

Bauserfeld (1992) distinguished between two types of teaching: *construction-oriented teaching* in which the primary educational aim is to produce students who can construct solutions to novel problems with a particular emphasis on meaning making, and a *process-oriented teaching* in which the primary educational aim is to produce students who can reproduce the teacher's solution procedure, irregardless of the meaning it may or may not have for the student. Bauserfeld clearly lays out his concerns regarding the later type of teaching:

> How could one generate flexibility in mathematizing any subject matter—the flexibility that is required later for problem solving—when the student's ascription of meaning is forcibly narrowed to ritualized interpretations, by social conventions rather than by mathematical necessities?"(p. 468).

Bauserfeld provides examples of pedagogical moves which support construction-oriented teaching, including: the discussion of previously solved tasks, the use of contrasting examples, and so called "underdetermined" tasks.

In their analysis, Boaler and Greeno (2000) describe two distinct types of figured worlds that emerged within the classrooms under study: the worlds of "didactic teaching" and "discussion-based teaching" (see Table 3). The former resembles a traditional

mathematics classroom in which teachers, and sometimes textbooks, validate knowledge claims. The later more closely resembles a reform-oriented classroom in which students, with measured support from the teacher, actively seek to verify each others' mathematical claims. The authors argue that understanding such distinctions is important since the figured world one finds himself or herself in can have real consequences for identity formation, and in turn mathematical success. For example, a student might come to understand and identify with a figured world of didactic teaching. Hence his or her identity as a learner and doer of mathematics will be as a passive receiver of information. If he or she does not identify, or at least comply, with these didactic norms the student may soon become unsuccessful in the classroom. This relates to the current study, in that, its primary focus will be on providing insight into the process of identity formation. But instead of figured worlds and positional identity, the current study will utilize the constructs of normative and personal identity, constructs which are grounded in mathematics education research.

Thompson (1994) chooses to characterize the nature of discourse in a particular mathematics classroom in terms of how the students and teacher orient to mathematics. Specifically, a teacher with a conceptual orientation to mathematics bases her classroom actions on certain idealized "images" of what it means to do mathematics in the classroom. These include "an images of a *system of ideas* and *ways of thinking* she intends the students to develop" and "an image of *how these ideas and ways of thinking can develop"* (Thompson, 1994,p.6). In contrast, a teacher with a calculational orientation to mathematics bases her actions on "a fundamental image of mathematic as

the application of calculations and procedures for deriving numerical results" (Thompson, 1994, p.7). These images have very real implications for the nature of classroom discourse. This distinction in orientations has been extended to describe different types of discourse that can take place in mathematics classroom (cite). This distinction in discourse is helpful in understanding the opportunities that are created for students to gain access to important ideas in mathematics.

Next I consider two lines of math education research which draw attention to resources for identity formation which appear outside the classroom: the "funds of knowledge" research of Moll (1997) and Civil (1998), and Walker's (2006, 2012) work with "mathematical spaces" and "academic communities."

Academic Communities

One group of researchers has focused on a specific academic community that they argue is central to student learning.

> …the most important resources for changing classrooms were to be found in the school's immediate cultural community and that making principled contacts with local households, critical settings with any school's community, and their resources would be central to any pedagogical effort. (Moll, 193)

In two related research projects, participant teachers visited the households of selected students in order to better understand what affordances those households provided (Civil, 1998; Moll,1997). These affordances are referred to as "funds of knowledge," or "those bodies of knowledge that underlie household activities" (Moll,1997, p.192). To illustrate, funds of knowledge might include a jobs like farming, ranching, or construction; the

creation of a family budget; or home remedies and other folk medicines. Students were also seen to contribute to the ongoing reconstitution of these funds of knowledge by acting as language interpreters in the home and providing childcare for younger siblings (Civil, 1998). In his analysis Moll (1997) is forthcoming with the fact that while these programs were quite successful in broadening teachers' understandings of their students, any specific connection to improved instruction or student outcomes remained "elusive." The current study seeks to extend this work by examining how academic communities outside the classroom support identity formation inside the classroom.

For Walker (2012), "What remains underexplored in mathematics education research is how the mathematically talented in the United States are socialized to do mathematics outside of school—how do they develop their mathematics skills, interests, and dispositions" (p.67). In her analysis Walker (2012) used the concept of engagement to operationalize mathematical socialization and the process of identity formation. Based on a literature review by Fredricks, Blumenfeld, & Paris (2004), Walker described engagement as comprised of three components: behavioral engagement (related to participation), emotional engagement (related to personal ties), and cognitive engagement (related to investment and persistence in difficult tasks). By asking mathematicians to describe their formative experiences, the resulting narratives provided evidence of engagement (i.e. narratives of participation, personal ties, or academic investment). Moreover, these narratives provided insight into the spaces where these formative experiences take place. These "mathematical spaces" are defined as

> …sites where mathematics knowledge is developed, where induction into a

> particular community of mathematics doers occurs, and where relationships or interactions contribute to the development of a mathematics identity (p.67).

The current study hopes to build on this work by analyzing both students' identity development within the classroom and their views on how outside mathematical spaces contribute to that development.

Within the field of education there are many taken-as-shared views regarding how academic communities outside the classroom impact student achievement. Walker (2006) clearly spells out some of these educational grand narratives and sets out to critically analyze their validity in her study of high achieving minority students in a small urban school. Some of these commonly held notions include: parents of minority students are chronically disengaged; minority students shy away from academic success due to possible social sanctions; and that the academic sphere of influence for minority students is largely negative and requires constant resistance. Walker (2006) provides compelling evidence against many of these supposed truisms.

First, many students in the Walker (2006) study reported viewing their parents as positive influences. Many said their parents held them to high academic standards, often times in spite of the parents' own lack of mathematical success. Parents were described as helping students with their homework for as long as they were able, even creating extra practice problems for students to attempt, later transitioning to a more motivational role as the mathematics became too difficult. Second, students in the Walker (2006) study provided evidence in direct opposition to the notion that peer academic influence is mostly negative and difficult to combat. Students described working both inside and

outside of school with their peers. Moreover, not only did the students fail to mention social sanctions from their peers, they even described positively collaborating on mathematics with lower achieving peers. Lastly, the academic sphere of influence for the high achieving students in this study included groups that are not often considered. Most notably, “near peers” is a group which was seen to include influential voices such siblings, cousins, and friends from other schools. This analysis is important, in that, the current study will explore ways to coherently integrate the macroperspective of Walker (2006) which focuses on academic communities outside the classroom and the microperspective of Cobb et al. (2009) which focuses on identity formation within the classroom.

Chapter 3

Methodology

Rationale for a Case Study Design

Within the research community there are two major perspectives on how to determine the appropriateness of a case study research strategy. Each perspective takes as its main focus a different aspect of case study research strategy, one focusing on the research *process* (Yin, 2009) and the other on the *unit of analysis* (Flyvbjerg, 2011; Merriam, 2009; Stake, 1995).

With a focus on the research process, Yin (2009) provides the following sufficient conditions for deciding when to utilize a case study research strategy: if "…a 'how' or 'why' question is being asked about a contemporary set of events over which the investigator has little or no control" (p.13). Notice how each part of these criteria refers to a different aspect of the research process: question type, historical vs. contemporary data, and investigator control. In this study I will be asking "how" questions (the research questions) about a contemporary set of events (student identity formation) over which I as the investigator will have no control (I will be using naturalistic techniques of inquiry). As a result Yin (2009) would likely assert that case study research strategy is appropriate for this study.

A case study strategy "copes with the technically distinct situation in which there will be many more variables of interest than data points" (Yin, 2009, p. 18) by using triangulation of multiple data sources and the development of theoretical propositions at the outset of the study (Yin, 2009). Since there are many variables in play, theoretical

propositions should be created to “direct attention to something that should be examined within the scope of the study” (Yin, 2009, p. 28), thereby providing focus for a study that would otherwise be unmanageable. As with all qualitative research strategies, case study strategies are used when a researcher wants to intentionally include context in the study of a phenomenon. This is in contrast to experimental research strategies, which are used to intentionally exclude the influence of context through the use of “control” techniques. In this study I will have many more variables than data points and utilize a situated perspective. As a result, case study research strategy is appropriate for this study.

With a focus on unit of analysis, Merriam (2009) provides a simple applicability criterion for case study strategies: “If there is no end, actually or theoretically, to the number of people who could be interviewed or to observations that could be conducted, then the phenomenon is not bounded enough to qualify as a case” (p.41). Stake (1995) and Creswell (2007) echo this view by describing a case as an “integrated system” and “bounded system” respectively. Typical examples of cases include a phenomenon, a person, a group of people, a process, or an activity (Creswell, 2007; Merriam, 2009; Stake, 1995; Yin, 2009). Because the case under study here is an intact Algebra II classroom, the boundedness criterion is easily met. As a result, Merriam (2009), Stake (1995), and Creswell (2007) would likely assert that case study research strategy is appropriate for this study.

Participants and Context

The participating teacher in this study, Ms. Mason, was chosen as a positive case with respect to her ability to consistently produce positive student outcomes in Algebra 2,

a course widely regarded as one of the more difficult for students within the high school curriculum. Since it is a graduation requirement in her state, Ms. Mason is under an inordinate amount of pressure to ensure student success (or at least minimize student failures). She has even been recruited in recent years to act as one of her schools' "data coaches," taking one class period per day to aggregate and analyze everything from assessment data to daily attendance numbers. All evidence seems to indicate that Ms. Mason, at least with respect to quantitative measures, is living up to the pressures of teaching Algebra 2, consistently scoring high on student achievement and growth measurements with respect to Algebra 2. By choosing Ms. Mason the hope was that her Algebra 2 class might provide greater insight into identity development than would otherwise be gained by studying a classroom whose teacher was less consistently successful in producing positive student outcomes. Ms. Mason has been teaching high school mathematics for seven years and has her primary state certification in Social Studies and a secondary certification in Mathematics.

Ms. Mason's class was chosen due to its unique ability to serve a longitudinal case (Yin, 2009). Specifically, Ms. Mason was willing to provide virtually unlimited access to her classroom over the entirety of the 2013 fall semester, something very few teachers are willing to do. Such access was crucial to gain insight into the process of normative and personal identities over time. The class was located within a large suburban high school that enrolled approximately 1888 students at the time of this study. According to information provided by the school district, 22.89% of the school's students

qualified for free and reduced lunch and the teacher to student ratio was approximately 26:1.

The students participating in the study were all members of the same standard level Algebra 2 class (i.e. non-honors). Students were placed into the class as part of the school's usual scheduling procedures. When students arrived on the first day they were asked if they would like to participate in the current study. By the time of the date of the first videotaped classroom session all students in the class had returned the necessary ascent and parental consent forms to participate in the study. The class consisted of 12 Caucasian females, 9 Caucasian males, 2 African American females, 2 African American males, and one male who fell into neither of these categories. Hence the class fell just shy of the school's overall students to teacher ratio of 26:1.

Data Sources and Collection Methods

The primary data sources for this study are video recordings of classroom sessions, field notes made by the researcher, audio recordings of interviews conducted with eight focal students, and student responses to a mathematical biography assignment and a *Student Map of Influences on Mathematical Success* (Walker, 2006). Complete classroom sessions were recorded using a laptop computer on the following dates: August 29th, September 6th, September 12th, September 20th, October 1st, October 10th, October 25th, November 5th, November 13th, and November 22nd. The laptop was placed in the front right corner of the room facing the students in order to best capture the subtle nuances of the classroom culture. As a logistical tradeoff the teacher's writings on the

were not captured by the video. Thankfully, Ms. Mason was quite consistent in describing what she wrote on the board as she wrote it.

All students in Ms. Mason's class were invited to participate in the paired interviews. By the time interviews began eight students had agreed to participate. While not ideal, this sort of self-selection was necessary. I could have sent an open invitation to all students taking Algebra 2 at the school and then chosen the class whose possible participants varied the most along some sort of identity dimension. But, as stated earlier, I prioritized the choice of classroom under study due to this specific teacher's proven track record of effective teaching and the wonderfully unfettered access she provided.

I chose to interview students in pairs in an effort to promote a richer set of student responses than would likely have been garnered from individual interviews (Cobb et al., 2009). In fact, students I interviewed seemed to find it easy discuss their common classroom experiences with one another, often requiring little prompting to get them open up. This may also have been due to the fact that I intentionally spent time taking field notes during most classroom observations in an effort to become a common fixture in their classroom. The interviews were conducted on December 11th (Janet and Tori), December 16th (Daneille and Ellen), December 17th (Adam and Sarah), and December 18th (Laura and Rebecca).

Each student who agreed to be interviewed was asked to complete two written prompts before the day of the interview: the *Student Map of Influences on Mathematical Success* (Walker, 2006) and a Math Autobiography (see Appendices). Using a form of stimulated recall (Calderhead, 1981), students were asked to discuss their answers both

prompts in an effort to uncover student narratives regarding their academic communities and possible relations to identity formation.

It is important for the researcher to clarify how specific pieces of data are directly connected to the research questions (Vincent A. Anfara, Kathleen M. Brown, & Mangione, 2002). Table 7 is meant to clarify these connections, particularly for interview questions. The interview protocol for this study was designed to provide insight into students' personal identity and engagement. As a result, the interview protocol draws on previous studies related to identity (Boaler & Greeno, 2000; Cobb, et al., 2009; Cobb & Hodge, 2011; M. S. Gresalfi & Cobb, 2011), motivation (Wigfield & Eccles, 2000), and engagement (Fredricks, et al., 2004). A complete interview protocol can be found in Appendix P.

Field notes were used to augment the video recordings of the classroom observations and later to triangulate any findings. The initial observations took the form of a "grand tour" observation (Spradley, 1980), guided by Merriam's (2009) general recommendations for conducting observations. These grand tour observations are meant to provide an overview of the social situation. As data collection continued I consistently referred to the research questions to guide my observations. As noted by Hatch (2002),

> The goal is not to find a definitive answer to your questions on a particular visit, but to give you a point of reference from which to decide where you will place yourself in the research scene and what you will look for as you go about your observation. (p.80)

By continually returning to the research questions I helped focus the specific observations and overall study. A complete observation protocol can be found in Appendix U.

Data Analysis

Overall approach. To get an initial feel for the observation data, I watched at the first video (August 29th) and the last video (November 22nd) in their entireties, documenting any patterns that began to emerge. I was specifically looking to see if classroom norms (and their associated obligations) seemed to change over time. I had expected there to be a stark contrast between the first and last class session as the norms are negotiated and become more established. Next, I considered a video in the middle, looking to see if the themes held up or if new themes emerged, using a constant comparison. Then I worked my way through each and every video, marking down specific lines numbers of data that were related to pre-existing a theme or seemed to hint at a new theme; especially if the data contained particularly illuminating discourse.

In the next phase of data analysis the themes themselves became the data. I looked to see if themes could be condensed, some themes could be retained while others may be thrown out. A theme was discarded if it had little supporting data, failed to condense with another category, and/or failed to substantially contribute to my understanding of the normative identity of the classroom under study. During this time I made it a point to complete several teacher evaluations that were part of my responsibilities as a lead teacher at the school under study (all outside of the mathematics department). This forced me to look at other classrooms, other teachers, and other microcultures. The hope was that by watching several different classrooms I would gain a

fresh perspective on my research classroom data. I also worked concurrently on the literature review.

As I began to write up the results from the observation data I concurrently began to make my way through the interview data. First I listened to all the interviews, transcribing sections that seemed of particular interest to personal identity. I focused on those comments that produced a vivid and vibrant account of what “doing mathematics” means of these students. Next I read through all that I had transcribed, looking for themes.

After transcription, I went through the data and commented on sections as I read. I began to have a conversation with the data, attending to the nuanced messages that lied below the surface of the data. I looked not at simply at what people said, but why and how they said it, all the while considering the context of each exchange. At the same time, I highlighted particularly powerful words or phrases that seemed to imply in vivo codes. Next, I took out each comment and the associated text and began to group them thematically. Table 9 provides a look at the final categories that emerged from the analysis process. It also includes details related to category origination, verification, and nomination (Constas, 1992).

Documenting Normative Identity. The unit of analysis for this study was an episode (set of interactions which all involve the same activity or idea). My goal for analyzing the data generated from the classroom observations was threefold. First, I sought to document the students’ general classroom obligations by specifically attending to how authority is distributed within the classroom and what forms of agency students

are able to legitimately exercise. General classroom obligations are highly related to social norms which are comprised of the social obligations and expectations that members of a community have for one another (Cobb et al., 2009; Cobb & Hodge, 2011; Cobb et al., 2001; Yackel, Cobb, & Wood, 1991)). While teachers often explicitly initiate the negotiation of social norms at the outset of the semester, such norms are constantly being renegotiated and reinstantiated by the members of the classroom community (Bowers et al., 1999; Cobb & Hodge, 2011; Gresalfi, 2009; Yackel et al., 1991).

To an observer, social norms (or any type of norm for that matter) are highly visible in the breach. This is argued strongly by Voigt (1995):

> One can never be sure that two persons are thinking similarly if they collaborate without conflict, especially if they agree about formal statements and processes. One of the characteristics of formal mathematics is that people can coordinate their actions smoothly although they are actually ascribing different meanings to objects. (p.172)

Thus it was necessary to attend to instances in which a student's activity appeared to break an established classroom norm and check to see if the responses of the teacher or other students serve to legitimize or delegitimize that activity (Cobb et al., 2001). Hence, the negotiation that followed a breach indicated if the norm is being challenged or further strengthened. Moreover it is in within these instances of negotiation that personal identity is formed (Cobb et al., 2009).

Second, I sought to document the students' specifically mathematical obligations by attending to norms for mathematical argumentation and the standard forms of

reasoning with tools and symbols. These sociomathematical norms are comprised of the mathematical obligations along with the expectations that members of the classroom community have for one another. Specific sociomathematical norms that I attended to included norms for what counts as a mathematically acceptable argument (Cobb, et al., 2009; Cobb, et al., 2001; Cobb, Wood, Yackel, & McNeal, 1992), normative ways of reasoning with tools and written symbols (Cobb, et al., 2009; Cobb, et al., 2001), and norms for what counts as mathematical understanding (Cobb, et al., 2009; Cobb, et al., 1992; Cobb, et al., 2011). To illustrate, when documenting norms of symbol use I searched for evidence of students either "creating and acting on experientially real mathematical objects" (Cobb, 1995, p. 104) or simply pushing around meaningless symbols in an effort to follow prescribed procedures.

These sociomathematical norms are the observable instantiation of what it means to be competent within the mathematics classroom (Gresalfi, 2009). For example, consider Cobb, Gresalfi, & Hodge (2009) who describe a classroom in which a student presents an incorrect solution and as a result is positioned as not having understood a procedure. In this class the mathematical obligation of understanding procedures is related to a local definition of mathematical competence based on a procedural understanding of the mathematics at hand. Furthermore, mathematical obligations and the related construct of mathematical competence appear to greatly influence the ways in which students come to identify with mathematics as it is locally constructed in the classroom, simply comply with the teacher, or resist participation mathematics activities (Cobb, et al., 2009; Gresalfi, 2009).

Documenting Personal Identity. My goal in the semi-structured student interviews was to document students' classroom obligations as they understood them, their valuations of those obligations, and students' views regarding their own and other students' competence in the classroom. Interview data regarding students' views of their general classroom obligations was compared with classroom observation data regarding norms established in the classroom. In previous studies utilizing the Cobb et al. (2009) interpretive framework, students' views of their obligations and the obligations indicated by observed classroom norms were quite consistent. Yet, instances of student resistance were not documented in these studies. To this end Cobb et al. (2009) remark: "In the case of resisting students, we would not be surprised if their understanding of official classroom obligations differed from those of cooperating students" (p. 63). This study hopes to shed light on this conjecture by presenting documented instances of student resistance and resisting students' understandings of the official classroom obligations.

Students who come to identify with mathematics as it is enacted within a specific classroom can be described as having "obligations-to-oneself" as opposed to "obligations-to-others" (Cobb et al., 2009; Cobb et al., 2001). During data analysis, once obligations were uncovered, it is how students spoke about those obligations that provided evidence of identity formation:

> The vehemence with which they talked about the importance of fulfilling some of the obligations that defined the role of an effective mathematics student in this classroom indicates that these obligations were on longer merely directed towards others but were becoming obligations-to-oneself. In the process the students were

developing an affiliation with mathematical activity as it was realized in this classroom. (Cobb, et al., 2009, p. 62)

As one might expect this level of “vehemence” was not observed in the classroom descriptions of merely compliant students (Boaler, 1999; Boaler & Greeno, 2000).

Ensuring Quality and Rigor

After the first few drafts of this dissertation I began to look back at my coursework in qualitative methods and the literature I had read. I soon came across a set of criteria proposed by Merriam (2002) which can be used to assess the quality of a study. Taken as a whole, Merriam’s suggestions provide a comprehensive framework for ensuring that the author makes a clear argument over the course of the entire study. In an effort to assess the quality of the current study, I have reprinted Merriam’s list in Table 5 and note that all 21 criteria have been satisfied. The hope is that this dissertation might someday serve to provide neophyte researchers in mathematics education with a specific example of research that meets Merriam’s high standards of quality and rigor.

Validity and reliability. There are two main indicators of rigor in qualitative research methodology: validity and reliability (analogous to the quantitative terms of the same names). Table 6 shows eight strategies proposed by Merriam (2002) meant to promote validity and reliability, and the extent to which seven of those eight were utilized in the current study. In the remainder of this section I discuss the specifics of how these strategies were implemented.

Certain steps were taken to ensure validity of my results (sometimes called credibility). Since a qualitative researcher is often seen as the main data gathering tool,

steps must be taken to help the process remain as salient as possible. To this end, I have tried to clarify any researcher bias or tacit assumptions from the outset (Creswell, 2007). Since in qualitative research the impetus is on the reader to determine transferability, rich thick description was also used. This thick description of the participants and context allows others to determine if their situation shares enough common qualities to allow transference (Creswell, 2007, p. 209). Paulus et al. (2006) describe the transference of a *working hypothesis,* stating that as a result of thick description, "readers are given the opportunity to decide on the degree of congruence and applicability of our working hypothesis to their own teaching and learning"(Paulus, et al., 2006, p. 363).

Peer debriefing has taken place, similar to quantitative interrater reliability, where a colleague of mine asked questions regarding methods, meanings, and interpretations. Moreover, by keeping a written record of these meetings I was able to add to the data corpus (Creswell, 2007). I also engaged in an external audit by periodically emailing current work to a trusted colleague outside the project. This person acted as an auditor and examined "both the process and the product of the account, assessing their accuracy" (Creswell, 2007, p. 209). Notice that while both the peer debriefer and auditor are both in some sense external to the process, the debriefer is closer to a co-analyst who was involved with meaning making. In contrast, the interactions with the auditor were sparser and yielded a more holistic critique of the project.

It should be noted that within the confines of the chosen Algebra 2 class I did try to ensure what Merriam (2002) calls "maximum variation" by purposefully seeking a wide range of student mathematical identities (i.e. compliance, resistance, and

identification). This was done by repeatedly prompting all students over the course of several weeks to consider being a part of the study. These efforts proved successful with students from each category agreeing to participate in the study. Their diverse set of perspectives added to the richness of the data available for the study.

Ethical considerations. Every study comes with some amount of risk for those who participate. The nature of the current study was to examine students' identities in the context of the mathematics classroom. Students may have encountered psychological stress, feelings of embarrassment, or loss of self-esteem as they reflected on and described previous classroom experiences. These feelings may have arisen during interviews or when filling out survey instruments. In order to reduce the possibility of stress students were constantly reminded of their anonymity and ability to opt out of the study at any time.

As part of this study, students were interviewed and their classroom was observed. This can often pose an inconvenience for the participants, but every effort was made to accommodate their schedules and obligations. Interviews were conducted at times that did not interfere with instructional time in the classroom. Information collected through the study of has been kept confidential.

Efforts have been made to protect participants' identity, such as using pseudonyms. No reference will be made in oral or written reports that could identify the results or comments of individual participants. Data collected have been stored securely in the office of the researcher.

Chapter 4

Findings

As promised in Chapter 1, I now return to the research questions used to address the purpose of my study, answering them using the findings that emerged from the analytic process described in the previous chapter. A discussion of the implication of these findings for both theory and practice will come in the next chapter. Without further ado, let’s consider the first research question.

Research Question 1a

How is the normative identity as a doer of mathematics constructed and negotiated within the classroom?

Throughout their time in a particular classroom students are constantly negotiating and redefining, along with the teacher, what it means to do mathematics. For the students this local definition is not simply one possible instantiation of what mathematics might look like, for them it *is* mathematics. As a result, this local definition of mathematics is what students will come to identify with, comply with, or resist *as* mathematics (Yackel & Cobb, 1996). This local definition is what I refer to as normative identity.

Charting the development of classroom obligations. The construction and negotiation of the normative identity can be described in terms of the specific obligations that help define what it means locally to “do mathematics.” Recall that these obligations can be broken into two types: general obligations and specifically mathematics obligations.

In order better understand the process of normative identity formation, I have selected an obligation and charted its development throughout the semester by highlighting certain "critical moments" that took place during the videoed class sessions. In searching for these critical moments, I re-viewed the entirety of the video data, noting moments that seemed to be of special importance to the process of identity formation. Using the previous discussion of the Identification Triangle (Figure 3) as a guide, I noted moments that included: (1) when the obligation was initially proposed by the teacher; (2) teacher-initiated negotiations in response to students failing to meet the obligation; or (3) student-initiated negotiations in response to students failing to understand or value the obligation. For clarity, in the following discussion I focus solely on teacher-initiated negotiations.

I have chosen the following mathematical obligation due to the richness of data associated with it. After charting the development of this obligation I briefly discuss the entire list of obligations that came to define the normative identity in Ms. Mason's classroom, providing illustrative examples of each. What follows is a chronology of the mathematical obligation: "*Calculator and the formula sheet acted as primary mathematical tools in solving problems*."

8-29-2013, Test Review

This lesson includes the first documented negotiations as to the role of the formula sheet in doing mathematics. Ms. Mason returns a quiz from the day before and asks the students to go over it in preparation for the next day's test.

M:OK. You get to use that note sheet tomorrow on your test. So if there's anything on here (quiz) that you're like…hey, ya know, I should of known that but I don't, I still need a little help. Maybe you need to write down that example…ON YOUR NOTE SHEET, so that you can reference that on the test tomorrow.

From the beginning of the semester the reference sheet acts as more than a light memory scaffold. Instead, it is used to store entire worked examples, or anything else a student might need for a test. Notice how the teacher explicitly negotiates the role of the reference sheet. This is something she would continue to do throughout the rest of the semester, repeatedly pleading with her students to take full advantage of formula sheet as a mathematical tool. Such pleading is partly attributable to the enormous pressure put on Algebra 2 teachers to produce high scores on state mandated end of course exams.

9-6-2013, Introduction to factoring

This lesson includes the first documented negotiations as to the role of the calculator in doing mathematics. The way in which the teacher and students choose to use mathematical tools in the classroom contributes directly to the local definition of what it means to do mathematics (Cobb, 2009). One tool that became central to doing mathematics in Ms. Mason's classroom was the calculator. Again it's not what mathematical tool is being used, but the way in which that tool is being used that contributes to the local definition of what it means to do mathematics. Near the beginning of the lesson Ms. Mason discusses how to factor the trinomial $x^2 - 5x - 14$.

M: What are the factors of -14 that add up to -5?

S: 2 and 7

M: 2 and 7? Negative 7 plus 2. Guys this is, if you can divide in you calculator, you can build a factor tree and figure out which ones add up. If you can go through your calculator and say:

$14 \div 1 = 14$

$14 \div 2 = 7$

$14 \div 3$ is a decimal.

$14 \div 4$ is a decimal…

…until you get all of your factors, you can figure out which ones are going to add up. You can do that. So then all you have to do is write those factors.

At this point in the semester the student is using the calculator as a tool, and has authority over it. Ms. Mason is using the calculator to bridge gaps in student knowledge, in the hopes of providing them access to Algebra 2 content, even if their requisite knowledge and skills may be lacking. We will see as the semester progresses that the calculator acquires more and more of the mathematical authority in Ms. Mason's classroom.

9-20-2013, Binomial expansions

This is the first lesson where we see mathematical tools gain a disproportionate amount of authority in the classroom. Let me illustrate with the following quote from Ms. Mason:

M: What is 2 to the 6^{th}? You can do that in your calculator, right? It's just 2 carrot 6. K?

Yes, one would hope that students could do that on their calculator. But also, with a small amount of number sense, we should expect students to be able to quickly do this calculation mentally. The fact that Ms. Mason appears to preempt student reasoning by defaulting to a reliance on the calculator serves to give a larger portion of the mathematical authority to the calculator.

10-1-2013, Introduction to rational functions

In this lesson, mathematical tools continue to gain a larger portion of the mathematical authority. The class spends the majority of the period discussing a reference sheet created by the teacher that lists all the properties of rational functions, and how to find them algebraically. Among the properties listed are vertical asymptotes and holes. The class does several examples, with Ms. Mason consistently referring to the reference sheet as a means of guiding her discussion. In one of the final examples of the day, the class has located a discontinuity in a rational function by setting the denominator equal to zero and solving. They are now trying to ascertain what type of discontinuity they have found.

> M: How do we know if it (the discontinuity) is a hole specifically? (long pause) Flip it back over (reference sheet). What does it say about holes? Very first thing we talked about. (long pause)
>
> S: If a factor cancels…
>
> M: IF A FACTOR CANCELS. K?

By asking students to look to the graphic organizer first, the teacher is positioning it as the mathematical authority in this situation. You ask it a question and it will answer. This

is also an example where Ms. Mason has preempted student reasoning, defaulting to an overreliance on a tool. The calculator also continues to gain authority, as seen in the next example.

A student had asked Ms. Mason to work an example on the board. The question asks the reader to graph a rational function. Ms. Mason decides to simply the rational function first, by factoring the numerator and denominator.

M: So I've got $\frac{x^2-3x-4}{x-3}$. If you don't know how to factor, what should you do?

S: (inaudible)

M: Put this where?

S: y equals…

M: In "y=" and look for what?

S: Intercepts.

M: Look for intercepts. So what are the zeros, or intercepts for this?

Here the teacher and students are negotiating when it is appropriate to relinquish authority to the calculator. I agree that tools like the calculator can be used to temporarily bridge gaps in student understanding, but they can also hide these gaps, causing even larger learning difficulties later. Also, while this use of multiple representations displays a certain amount of teacher knowledge, there is no evidence during the discussion that students viewed the same mathematical connections as the teacher.

10-10-2013, Simplifying radical expressions

This lesson further solidifies the role of the calculator as a key mathematical

authority in Ms. Mason's classroom. In the following episode, a student asks the teacher how to simplify the expression $\sqrt[5]{-32x^6y^7}$.

T: How would I do that?

S: Oh! You would go to your calculator…

T: …go to your calculator. Get it out. Practice this! You have to practice this if you wanna be able to do it when it comes time to…to assess it, right? So you type in a 5, you type your index first. Type in a 5, and then you do to math, and you go down to number…5…XROOT, right? XROOT. And then put in negative 32. And what do you get?

C: Negative 2

T: Negative 2. That means that negative 2 to the fifth power…is negative 32. Right? Ok.

As students listen to the teacher math begins to sound like another language, what some teachers call "calculator speak." This is often deemed a pedagogically acceptable trade off if the calculator is being used to relieve some portion of the mathematical burden from students who would otherwise not have access to the mathematics, due to a lack of motivation, lack of ability, or both. This trade off is weighed against the overwhelming pressure Algebra 2 teachers feel to produce results on high stakes standardized tests.

10-25-2013, Test review

This lesson provides a clear example of how the shift in authority from student to mathematical tool was often accompanied by missed pedagogical opportunities.

M: After you put this in your calculator what type of regression do you think this is?

C: (inaudible)

M: It is exponential regression. It doesn't matter if it is growth or decay, your calculator will figure that out. You calculator is…decently smart. It can figure out that it is decreasing in value, that it is decay. And so, it will figure out what that decay factor is for you.

Notice that Ms. Mason missed an opportunity to discuss how to approximate the decay/growth factor using simple division of consecutive terms. This brings up an important point: while the calculator is a tool which provides access to problems that would be otherwise out of reach, students can still be responsible for some amount of the mathematical burden. This would allow them to retain some mathematical authority rather than relinquishing it all to the calculator. As seen in this example, the calculator does all the mathematical "figuring out" that is necessary, with little to no work left for the student.

11-5-2013, Complex arithmetic

The class is discussing homework problems involving division of complex numbers.

M: Question 8 you can put right in your calculator. In fact, write down on number 11… cal-cu-lator. Same with 12… cal-cu-lator. The trick is that you need to put all of the numerator in the parenthesis, all of the denominator in parenthesis.

Here Ms. Mason is ostensibly giving the authority to do complex arithmetic solely to the calculator, and concurrently, taking it away from her students. As a result, the students are relegated to providing the calculator with problems to work. Relatedly, Ms. Mason missed an opportunity to discuss conjugates and the idea of multiplying by a well-chosen one.

11-13-2013, Test review

Even as we near the end of the semester, Ms. Mason still pleads with her students to use the reference sheet as a tool for improving test scores.

> M: Can you write that down? Since you get to use your notes you might want to write that down. HELLO?
>
> S: What are we writing down?
>
> M: Conterminal angles complete the circle. If one is negative, the other is positive. If one is positive, the other is negative. They're opposites.
>
> M: The cosine of 30^{o} equals $\frac{\sqrt{3}}{2}$. How can I verify that? Well, I go to my calculator, and I put in $\cos(30^{o})$, and I'm gonna get a decimal. I put in $\frac{\sqrt{3}}{2}$, and I should get the same decimal. That's how you check it…ok? So if you want to write on your paper: "check your answer with a calculator…"

The calculator is given the mathematical authority to verify solutions. The teacher missed an opportunity to allow students to retain some measure of authority. She could have asked them to recall their 30^{o}-60^{o}-90^{o} triangle and use its side lengths to derive the same result.

In summary. While the tools like the calculator can be used to bridge gaps in student understanding, there are resulting tradeoffs. First, there is the tradeoff of missed pedagogical opportunities, opportunities to discuss the conceptual basis for the processes being performed by the calculator. Second, there is the tradeoff of student authority, as the calculator changed throughout the semester from supporting role to mathematical authority.

General Obligations. Although there were negotiations between Ms. Mason and her students throughout the semester regarding classroom social norms, the general classroom participation structure remained relatively stable over time. The three general obligations that came to define the participation structure were:

1. *Students were obligated to take copious notes.*
2. *Students were obligated to pay attention (or at least appear to pay attention).*
3. *Students were obligated to ask questions of procedures.* While the teacher did most of the mathematical work in this classroom, students were obligated to ask questions in order to clarify the procedures she modeled.

Each of these obligations became an integral part of what it meant to do mathematics in Ms. Mason's classroom.

On a typical day, students would arrive in class and settle into their seats. The first phase of the lesson was often devoted to going over any questions from the previous day's homework assignment (Obligation 3). These questions were almost always procedural, with students asking for clarification about a specific step in a procedure or a repeated description of a procedure altogether. Hence students were largely afforded

opportunities to exercise disciplinary agency, choosing from previously discussed solution methods.

The second phase of the lesson was devoted to teaching new material. During this phase of the lesson students were expected to take notes and pay attention to procedures provided by the teacher (Obligations 1 & 2). This is the phase of the lesson when students were most often censured for not fulfilling one of the first two obligations. This is also the phase of the lesson in which the authority in the classroom was almost exclusively distributed to the teacher.

The last phase of a typical lesson was for students to begin working on the homework assignment for the next day. During this time students again exercised disciplinary agency, asking clarifying questions regarding teacher-presented procedures (Obligation 3). Students often worked in groups in which they sometimes looked to one another as a mathematical authority, asking for clarification of procedural steps. But often, at the first sign of trouble, students asked Ms. Mason for help in completing the procedure at hand, recognizing her place as the sole mathematical authority in the classroom. In the following this section I present three episodes meant to illustrate how these general obligations became a part of the microculture in Ms. Mason's classroom.

General obligations: Illustrative episodes. Taking notes and paying attention were the two primary student obligations in Ms. Mason's classroom. The teacher wanted students to take notes so that they would be able to reproduce the mathematical results she was modeling. This was an overall well-behaved class, in that they rarely challenged the teacher in any way and the class appeared to move on without incident. As a result,

the teacher rarely had to use her authority as a teacher to attend to discipline problems. That being said, the teacher did often use her authority in the classroom to directly prompt students to take notes. This consistent use of authority underscored the value that the practice of note taking had for this teacher. In Ms. Mason's class the teacher directly prompted students to take notes for one of three reasons: elaborating procedures, sanctioning students, or responding to student difficulties.

Ms. Mason sometimes required students to write down notes as she elaborated on the specifics of a procedure, as in the following episode from 10-1-2013.

Teacher: So, let's look at our "Rules for Graphing" for a second…Rules for Graphing. What's the first thing that we should do?

Student: Factor and cancel if possible.

T: Factor and cancel if possible. Ok, so if the factor cancels there's a hole in the graph, created at that number, we set the factor equal to zero. Right next to that I want you to put "SET FACTOR EQUAL TO ZERO"…on number one…on number one, at the end of number one, put "SET FACTOR EQUAL TO ZERO…AND SOLVE." Alright then number 2, to find the zeros…

This interaction reinforced the expectation that students were to pay attention and take copious notes. In line 3, the students showed that they were fulfilling the obligation of paying attention by reading aloud the appropriate line from the list of procedures when prompted by the teacher. Then in lines 5-8 the teacher makes students negotiates the specifics of students' obligation to take notes. It's highly likely that this interaction looks quite different from the student and teacher perspectives. While the teacher likely sees the

mathematical foundations of the process she is teaching, the students likely see the process as mathematics.

Other times the teacher prompted students to take notes as she sanctioned them for transgressing the note-taking norm and not fulfilling their social obligation.

T: So where's your homework Dustin? …Alan?

Dustin: I don't have it.

T: So what should you be doing right now? Daniel? What should you be doing right now Brandon?

Jeremiah: …I'm on it!

T: Jeremiah's ON it!

Jeremiah: I'm on it!

T: …I love it!

You should be writing it down guys…You may not have it but you should at least have something out writing it down so that on the day that I take up homework you've got something to give me. Every point in this class counts. Every point counts.

In this episode from 10-10-2013, the teacher took students' failure to complete their homework as an opportunity to discuss student obligations, what they "should" be doing. In line 1, the teacher's question "so where's your homework" created space for the students to explain rather than defend their actions. After Dustin acknowledged his failure to complete the homework the teacher quickly shifted to the real reason she started this interaction, to negotiate the social norm of note taking. In line 3 the teacher again

uses questioning to create space, providing the students the opportunity to recognize their breach of the note-taking norm, which Jeremiah does, exclaiming “I’m on it!” In the last few lines of the episode the teacher tries to renegotiate the breached norm by providing specific reasons why the students should fulfill their note-taking obligation.

The most common reason that Ms. Mason would instruct students to take notes was to address an apparent student mathematical difficulty. Ms. Mason’s common response to student difficulties was to ask them to write down the appropriate procedure; the hope being that, the next time the students encountered a similar problem, they would have access to the appropriate procedure. The implication is that writing down procedures is a necessary step toward mathematical understanding. The next episode helps to illustrate how Ms. Mason used note taking to remedy student difficulties.

In the following episode from 8-29-2013, Ms. Mason and her students consider the following question: A ball is dropped from the top of a building. The distance in meters above the ground can be modeled by the equation: $y = -9.8t^2 + 100$.

T: How do I find the y-intercept?

S: Plug something in.

S: Plug it in!

T: Plug in what?

S: The vertex. The x.

S: um…

T: Ok. So we need to write this down in our notes. K? To find the y-intercept let x equal zero. On the y-axis, the x value is always zero. If I’m on the y-axis the

x-value is zero. If I plug in a zero right here, what do I get?

S: zero

S: zero

T: zero, plus…

S: a hundred

T: …a hundred is what?

S: one hundred

S: one hundred

T: one hundred. That's the y-intercept. That's the y-intercept.

The students begin their search for the answer by blurting out several possible choices for the teacher to choose from. When the teacher fails to find the correct response, she immediately provides her students with a procedure for finding the y-intercept. Again, the hope is that when students come across a similar problem in the future, they will now have access to the appropriate procedure for finding the y-intercept. This sort of procedural reaction to student difficulties was seen consistently across observations throughout the semester.

Although note taking was mainly used to record procedures, Ms. Mason did occasionally attempt to use note taking as a means of highlighting mathematical connections.

> T: There are two times when we're gonna have restrictions on domain. So this is key. You need to write this down, big box around it, highlight it, star it…a little confetti...I dunno, whatever makes it stand out to you. Ok? (9-12-2013)

Notice that her discourse here is doing two things. First, it is a way of helping those who are taking notes to easily identify what she viewed as the most important mathematical ideas of the discussion. Secondly, it is part of an ongoing negotiation to encourage other students in the class to take notes at all. Here is another example of this dual-purpose discourse:

> T: So here's the big picture that I want you to take away from this. I want you to write down at the bottom underneath your picture. The linear factors of a polynomial represent …you with me? The linear factors of a polynomial represent the lines where the polynomial crosses the x-axis (9-20-2013).

Again, "here's the big picture" is a way of highlighting what the teacher believed to be the big mathematical ideas, while "you with me?" is a clear call to action for some students to begin taking notes.

Mathematical obligations. There are certain aspects of a mathematics classroom microculture that are inherently mathematical and would likely not be observed in, for example, an English classroom. These obligations involve what students are accountable for in the class and what counts as mathematical activity. The mathematical obligations identified in the analysis were:

1. *Arithmetic or procedural answers counted as mathematically sufficient* (others have used the phrase mathematically sophisticated). Listing procedures was deemed a mathematically acceptable response. While the teacher did sometimes try to negotiate toward a more substantive form of mathematical discourse ("tell

me why”), the participation structure with respect to this obligation was highly resistant to negotiation.

2. *Calculator and the formula sheet acted as primary mathematical tools in solving problems.* Each was used in such a way as to offload cognitive demand from the student onto the tool, as opposed to the use of tools for cognitive scaffolding so that the child is still required to do his/her fair share of the work.
3. *The basis for discourse used by the teacher and students was largely procedural.* Teacher and students used procedural objects as the primary objects of reasoning in the classroom, with no indication that these objects had any meaning beyond their surface-level, procedural utility.

In Ms. Mason’s class, a student’s answer was deemed a mathematically acceptable argument if it contained a correct series of procedural steps. Moreover, when discussing examples, the teacher often used scaffolding to the point of prompting students for one-word answers. Notice how what students were accountable for directly relates to the way in which students were able to participate in the classroom, in that, short procedural responses limited students to exercising procedural agency with authority resting almost solely with the teacher.

As mentioned above, much of the talk in Ms. Mason’s classroom centered around procedures. For this reason, questions posed to students usually allowed them to exercise only disciplinary agency by choosing from previously discussed solution methods, as opposed to exercising conceptual agency in which the students might need to adapt known procedures to the current situation and/or come up with their own procedure. But

choosing the correct procedure is often not enough to solve a problem: a student must also remember said procedure and be able to execute it arithmetically. In reaction to these additional memory and computational burdens, Ms. Mason consistently encouraged her students to use two specific mathematical problem-solving tools: the formula sheet and the calculator. It is the way in which these tools were used and the implications their use had for student identity formation that are of particular interest to the current study.

During class discussions, Ms. Mason and her students discussed procedural objects that seemed disconnected from any substantive mathematical ideas. Moreover these objects did not seem to be experientially real for the students, and remained largely abstract mathematical symbols. While it is important for students to become adept at mathematical procedures, this should be part of a larger conception of mathematical competence that includes conceptual understanding and connections between big ideas.

Mathematical obligations: Illustrative episodes. In the this section I present three episodes meant to illustrate how these specifically mathematical obligations became a part of the microculture in Ms. Mason's classroom.

This episode from 8-29-2013 occurred near the beginning of a review day for an upcoming test on quadratic functions. A student, Adam, who I would later interview, asks a question. During the interviews several students would describe Adam as a "math person."

A: On this test when we're doing anything with domain is it always going to be "all Real numbers"?

T: That is a *fantastic* question. So he asked the question is the domain always

going to be real…all reals? (very short pause) It *is* unless it's in context. Ok?

Notice that Adam's initial question in line 1 is meant to clarify the procedure of finding the domain of a function. The teacher's enthusiastic response let the students know that she valued the question that was asked. The teacher also sends the signal that asking questions is a valued type of mathematical activity in this classroom. In line 3 she underscored the question's importance by repeating/revoicing it for the students. However, the students were left to infer what part of the student's question the teacher valued. The teacher continues:

T: So let me show you in context on number 22 what I'm talking about. It doesn't ask for domain and range but we're gonna put the domain and range, so that we understand what's happening in context. Domain and range are restricted in the real world. Ok? So, 22 says "A ball is dropped from the top of a building. The distance in meters above the ground can be modeled by, and there's the equation: $y = -9.8t^2 + 100$. Immediately my visual self says this parabola opens which way?

S: down

S: up

S: down

T: Why does it open down Madeline?

M: Because it's negative.

T: Because he's negative in the front. Ok? (inaudible)

The teacher's choice of words in line 5 "let me show you" is doing more work that just signaling to students that an example is coming. It is also part of the ongoing negotiation of what it means to do mathematics in this classroom and more importantly who will be doing the mathematics. Consistently throughout the course of the semester the teacher used the personal pronouns "I" and "me" when discussing the doing of mathematics. This type of teacher-centered discourse let students know that she was doing the mathematics and their role was to watch mathematics being done and provide procedural and arithmetic help when prompted. It may seem like semantic hairsplitting, noting the pronoun of choice of the teacher, but when repeated over the course of approximately 120 hours of seat time in a semester (80 days x 1.5 hours per day), such small distinctions potentially have large effects.

The students respond to the teacher's question from line 10 by blurting out several possible choices, trying to match the answer that the teacher has in mind. In line 15, the teacher legitimizes this type of student response as a valid form of mathematical activity by calling on a student who gave the correct answer. Moreover, by repeating Madeline's answer in line 17 the teacher not only lets the class know that Madeline's answer is in fact correct, she also lets them know that they are only required to convince one person of the validity of their mathematical arguments: the teacher. This is a much lighter burden of proof than having to convince the entire class, since the teacher is already quite familiar with the mathematics content at hand (Gresalfi, 2009). That being said, the teacher's utterance in line 17 of "Ok?" hints that the students do have an obligation to attempt to understand, or at least acknowledge, the contributions of others. The teacher continues:

T: So the ball is dropped from the top of a building. I wanna know what's the y-intercept. How do I find the y-intercept?

Madeline: Uh, negative b over 2a?

T: The "go to" answer. That's how I find the vertex. That's vertex. So Madeline's got the vertex down.

M: Oh.

This exchange came early in the semester as the teacher and students were just beginning to negotiate what it means to be an acceptable mathematical solution. By labeling the student's response as a "go to" answer, the teacher is letting the class know that it is somehow unacceptable. Moreover, the comment in line 21, "She's got the vertex down," is an attempt by the teacher to legitimize some part of the student's solution and assert that student responses do not have to be completely acceptable or completely unacceptable. In this class it is possible to give a partially acceptable mathematical solution. Often students come to a mathematics class with the preconceived notion that every answer is either completely right or completely wrong, a likely remnant from grade school arithmetic. The teacher must actively negotiate against this false dichotomy.

T: How do I find the y-intercept?

S: Plug something in.

S: Plug it in!

T: Plug in what?

Madeline: The vertex. The x.

S: (pause) um…

30 T: Ok. So we need to write this down in our notes. K? To find the y-intercept let x
31 equal zero. On the y-axis, the x value is always zero.

In line 29 a student in the room notices that the teacher has failed to repeat Madeline's answer and correctly interprets this as a negative evaluation. The teacher then proceeds to have students write down a procedure in their notes; a mathematical activity that would come to dominate much of what it meant to do mathematics in this classroom. Specifically, doing mathematics in this classroom entailed watching the teacher's demonstration of procedures, documenting procedures, and then practicing procedures. The teacher continues:

32 T: If I'm on the y-axis the x-value is zero. If I plug in a zero right here, what do I
33 get?
34 S: zero
35 S: zero
36 T: zero, plus…
37 S: a hundred
38 T: …a hundred is what?
39 S: one hundred
40 S: one hundred
41 T: one hundred. That's the y-intercept. That's the y-intercept.

Lines 32-40 are representative of a semester long negotiation to determine what mathematical activity should look like in this classroom. Specifically, the teacher's questions created only enough space for the students to think about the next step in the

arithmetic procedure. This type of questioning by Ms. Mason was emblematic of the participation structure that came to define much of what it meant to do mathematics in this classroom.

The next episode comes from 9-6-2013, a day in which the teacher is discussing one of the most notoriously difficult topics in the Algebra 2 curriculum: factoring. In the previous day's lesson Ms. Mason has already provided students with a macro-level procedure for how to factor a given polynomial expression having 2,3, or even 4 terms. At this point in the lesson students are working through specific examples with the help of the teacher. Or more correctly, the teacher is working through examples with the help of her students. While this may appear to be only a minor difference in agency, the implications for what it means to do mathematics in this classroom were quite substantial.

1 T: Ok, so *is there* a GCF?

2 S: yes..

3 Class: Nooooo…

4 S: yes

5 S: yes

6 C: (murmuring)

This is another example of the brute force discourse that became an acceptable form of mathematical activity in this class. Just as a mathematician might use the "brute force method" to solve a problem, students in Ms. Mason's class often used brute force discourse to answer the teacher's questions: simply shouting out several possible answers

and allowing Ms. Mason to pick the right one. In this case she chooses from the answers provided by the students and quickly shifts to the next question.

7 T: So if there's not a GCF then I look to see how many terms are there? There's
8 two terms. So there's two terms I look to see is this the difference of two squares.
9 Tori: Oh! Yeah.
10 T: *Yes!* What is the square root of 16?
11 C: four…four…
12 T: Good. What is the square root of 9?
13 C: 3..3…
14 T: *Three!* One is positive, one is negative, it doesn't matter which one.
15 Tori: I thought you didn't do that unless it was an equals sign…but ok.
16 T: If there was an equals sign then I could set it equal to zero and actually solve
17 forthose factors. When you're factoring, you're factoring an expression, you don't
18 need an equals sign.

Notice that the mathematical activity in this episode is almost entirely dominated by the teacher. Also the teacher's discourse continues to provide little space for students to do any substantive mathematical thinking. The students' role in this discussion was simply to help the teacher by providing procedural and arithmetic assistance when cued. Notice that Tori's question in line 15 is meant to clarify the procedure being discussed, as opposed to a more conceptually-oriented question.

A few minutes later in the same lesson on factoring (9-6-2013):

1 T: Ok Number 2. Jerimiah, is there a *common* factor in number 2?

2 Jerimiah: …..yes…no…

3 T: Well which one is it, yes or no?

4 J: It's one 'a those.

5 T: It's one 'a those! It's fifty fifty! Let's commit to one.

6 J: No.

7 T: *No.* Why not?

8 J: Because it's nonfactor—er…

9 T: Josh can you help us?

First, the student acknowledges that he changed his answer based on social cues. Then with "Let's commit to one" the teacher indicates that in this class social cues are not an acceptable basis for mathematical explanation. Lastly, the question "Why not?" is used by the teacher to create space to negotiate the sociomathematical norm of what it means to be an acceptable mathematical explanation. Specifically, explanations require mathematical justification. While the teacher did at times ask students to justify their answer, justification failed to become a normative part of the classroom microculture, and as a result failed to become a part of what it meant to do mathematics for these students. The episode continues:

10 T: Marcus can you help us?

11 Marcus: No.

12 S: Like *1* is but…

13 M: I can do it I just can't explain it.

14 Jeremiah: Yeah see, that's how I'm feelin'…

Class: [laughter]

In lines 10-14 Marcus and Jeremiah provide further evidence of the fact that justification is not part of this classroom's microculture. Their comments are also doing work in the ongoing negotiations of what it means to be an acceptable mathematical explanation. Moreover, by using humor the students create space for the teacher to negotiate and also lessening the chance that they will be directly censured for not living up to any possible teacher expectations of justification. The use of humor by Marcus and Jeremiah is also a way of signaling to the teacher and other students their recognition that they may be falling short of meeting classroom expectations. The teacher responds by taking up the problem and continuing with the procedure herself.

T: So I'm looking to see, is there something *shared*, by all three terms.

S: no

C: noo…

T: *No!* There's no GCF. There's *nothing* shared by all three terms, other than *one*. One doesn't help me as a greatest common factor. I can factor one outta everything all day long, it doesn't help me. Um, so then I look to see, what, because the leading coefficient is one, one times six is six. What are factors of six that add up to seven?

S: one and six.

T: one and six. *Because* the leading coefficient is one, I don't have to bust up b, I don't have to go through the process of factoring by grouping, I can just write my factors…m plus one, m plus six. If I multiplied this times this, I will get this

28 polynomial.

This last section of the episode was typical of the nature of student and teacher mathematical activity in Ms. Mason's class. The teacher had control of the mathematical activity, letting her students know by consistently discussing mathematics in the first person, in terms of what *she* was doing: "I look to see…" or "I can factor…". This type of modeling is a powerful tool in a teacher's pedagogical arsenal, but when used exclusively, can change the nature of mathematical activity in a classroom from student-centered to teacher-centered. In contrast, the students' mathematical activity was relegated to one word answers and arithmetic support.

Discourse as a context for negotiation. As I looked over the video data from Ms. Mason's classroom, patterns in discourse began to emerge. One lens that helped me better understand these patterns was Mehan's (1979) seminal discussion of IRE patterns. Moreover, I began to focus on what types of mathematical objects were being discussed and the meanings those objects appeared to hold for the teacher and students. These objects can be understood from either a psychological or sociological perspective:

> "When [researchers] take a sociological perspective, they talk of taken-as-shared mathematical objects and describe them as social accomplishments that emerge via a process of interactive constitution. When they take a psychological perspective, they talk of experientially real mathematical objects and describe them as personal constructions that emerge via a process of active conceptual self-organization" (Cobb & Bauersfeld, 1995, p. 3).

Rather than one or the other, it is the reflexive relationship between the psychological and sociological perspectives that appears to be the most fruitful vantage point for understanding student learning in the mathematics classroom.

> In this [interactionist] view, individual students are seen as actively contributing to the development of both classroom mathematical practices and the encompassing microculture, and these both enable and constrain their individual mathematical activities. (Cobb and Bauserfeld, 1995, p.9)

This reflexivity is vividly illustrated by what I will call the *basis for discourse* (BFD) in the classroom. The BFD in a classroom encompasses the taken-as-shared mathematical objects that students and the teacher use to communicate, the nature of those objects (procedural, conceptual, or metaphorical), and the meanings those objects have for the teacher and students (experientially real or meaningless symbols). BFD can be thought of as a set of taken-as-shared cultural tools that both enable and constrain individuals' mathematical activities, the practical instantiation of the reflexive relationship between the sociological and psychological aspects of the mathematics classroom. Hence the concept of BFD might be a way to bridge theory and practice by helping to make visible for practitioners the way in which taken-as-shared mathematical objects enable or constrain their students' personal mathematical constructions. Next, I discuss three types of bases for discourse that were used in Ms. Mason's classroom: mathematical, procedural, and metaphorical.

At any point in time classroom discourse is about things and the connections between those things. The hope is that in the mathematics classroom discourse revolves

around mathematical things and the mathematical connections between them. I will refer to such a situation as one in which the teacher and students have a *mathematical basis for discourse*.

Sometimes classroom discourse can revolve around procedural objects that have little meaning for students. Such discussions do not necessarily point back to mathematical concepts in any meaningful way. I will refer to such a situation as one in which the teacher and students have a *procedural basis for discourse*.

Lastly, teachers sometimes use metaphors to convey mathematical ideas. During these discussions students focus on metaphorical objects and connections between them (i.e. the metaphor itself). Such discussions do not necessarily point back to mathematical objects in any meaningful way. I will refer to such a situation as one in which the teacher and students have *a metaphorical basis for discourse*.

To clarify the notion of BFD, consider the following example. Imagine a class discussion about how to graph rational functions, a topic common to Algebra II, College Algebra, PreCalculus, and Calculus classes in the United States. Within this discussion a teacher will need to mention horizontal asymptotes. Let's consider how the local instantiation of this mathematical topic might differ depending on the basis for discourse used in the classroom.

If a mathematical basis for discourse is used by the teacher and students the discussion will involve mathematical connections among experientially real mathematical objects. In this case horizontal asymptotes might be described as $\lim_{x\to\infty} f(x)$. In calculus, students come to view this limit as making $|f(x) - L| < \varepsilon$ for large x. Hence,

horizontal asymptotes can be discussed in terms of a mathematical connection (limit) between mathematical objects (x, f(x), and L).

If a procedural basis for discourse is used by the teacher and students, the discussion will involve procedural connections among procedural objects. In this case horizontal asymptotes might be described as occurring when the degree of the numerator is less than or equal to the degree of the denominator. The students are given a procedure for how to find the horizontal asymptotes: if the degree of the bottom is bigger then y=0 is a horizontal or if degrees are equal then y=(quotient of leading coefficients). Notice that just because the word “degree” is used in a classroom discussion does not make it a mathematical object. “Degrees” can merely be numbers on a page that act as procedural cues for students. Hence horizontal asymptotes can be discussed in terms of procedural connections (the procedure itself) among procedural objects (the degrees of the numerator and denominator).

If a metaphorical basis for discourse is used by the teacher and students the discussion will involve metaphorical connections between metaphorical objects. In this case horizontal asymptotes might be described in terms of a “rocks and balloons metaphor,” as was the case in Ms. Mason’s class on 10-1-2013.

T: So then we go to horizontal asymptotes, this is where it gets a little bit different. Ok? We are going to have to look at the degree of the numerator and the degree of the denominator and try to figure out if there is a horizontal asymptote. So here’s where I’m gonna give you a little story. Ok? Alright. So for horizontal asymptotes I’m gonna tell you the story about the rocks and the balloons. Ok?

Balloons go up and rocks weight you down, right? Ok. So, if we think about p(x) being the balloons and q(x) being the rocks we are going to look at the degree of the function, ok? For example, let’s say I had an x cubed over an x squared. Which exponent is bigger? The x cubed. Ok? Because the x cubed, the exponent in the numerator is bigger, what’s gonna happen? I’m gonna have more balloons than rocks and what’s gonna happen? I’m gonna float up forever and if that happens then there are no… horizontal… asymptotes.

Teachers often use “stories” like these to introduce concepts that are notoriously difficult for students to grasp. Ms. Mason next goes on to discuss the case where degrees are the same, without discussing the mathematical basis behind her analogy. This means that for students it is likely the case that, for them, the analogy *is* the mathematics. She finishes her analogy:

T: What’s gonna happen when I add rocks?
S: It’ll go down.
T: Let’s say my denominator is greater than my numerator. What’s gonna happen?
S2:You’re gonna drop like a rock.
T: I’m gonna drop like a rock to the what?
S: Ground.
T: To the ground! Y=0 is gonna be my horizontal asymptote. K? That’s the hardest one and if you kinda have an analogy to kinda help you remember what’s gonna happen, if I have more balloons that rocks, numerator’s greater than my

denominator, I go up forever there's no horizontal asymptote. There's nothing blocking me. K? If they're the same I'm gonna hover somewhere and I have to figure out where that is, by looking at the leading coefficients. And if I have more rocks than balloons, I have more exponents in my denominator, I'm gonna come all the way back down to the ground and I have a horizontal asymptote at y=0. K?

We can see that students as well as the teacher provide evidence that they are using a metaphorical basis for discourse. Moreover, we never see evidence of a change of basis toward a more mathematical basis for discourse.

In Ms. Mason's class, while there were lessons in which all three types of BFD were used, the default basis for discourse within her classroom was procedural. This choice of BFD had a direct impact on the nature of mathematical activity that took place and ultimately what it meant to do mathematics in this classroom. Moreover, I assert that this choice of BFD likely resulted in limited student access to forms of mathematical reasoning that "have clout" (Bruner, 1986). The students in the class spent the majority of their time creating or solidifying procedural connections between procedural objects. In fact, students often seemed to simply be pushing around symbols that appeared to have no real meaning for them outside of their procedural context. This means that students were denied access to the underlying structure and powerful mathematical ideas that might have been a part of the classroom discourse. There are several reasons why this can be the case, from the pressures of standardized testing to the teacher's perceptions of student competence.

Research Question 1b:

How do students' personal identities form in relation to the normative classroom identity?

To answer this question I began with interview data since it provided the most direct access to students' personal identities. Students' narratives were then used to tease out possible relationships between these personal identities and the normative identity of the classroom. Lastly I returned to the video data in search of critical moments that appeared to highlight the reflexive relationship between the normative and personal identities. The result of this analytic process was the discovery of what one student called a "big division" within Ms. Mason's class. This normative divide had an influence on and was influenced by students' personal identities as doers of mathematics.

Narratives of Compliance: Ellen, Danielle, Sarah, Laura. Student narratives of compliance often included discussions of understandings and valuations of classroom obligations. For example, students were asked to respond the following prompt: "What do you have to do to be successful in this class?"

> Ellen: Take notes. Because our teacher emphasizes on taking notes and she may say uh a certain thing, that we need to remember later, she might not write on the board but she wants you to write down almost everything she says, so we can reference to it later. So when we need to study it might help, or it will help, in the long run, to write stuff down, make sure you do take notes, cuz that will help you with your homework.

Sarah: Pay attention, take notes, well, yeah when we took notes you need to like, not just take em but like, read it and like understand it.

Interviewer: Mh Hmm

Sarah: Like highlight the main points because it's, like usually what Miss Mason did would be like on the test I guess, like what we did in our notes, that's what would like. Or like the review packets were like the tests basically, but with different numbers.

I: Mh hmm.

Sarah: So just make sure you know your study guide.

As noted in a previous section, note taking was one of the most visible aspects of the culture in Ms. Mason's class. This is largely due to the fact that she reminded her students of its importance on a daily basis, devoting class time each day to explicitly negotiating this obligation and censuring students who failed to fulfill it. Both of these students seemed to not only understand this general classroom obligation, but also saw value in it. They viewed note taking as a valuable part of what it meant to do mathematics, since it helped them perform well on homework and tests. As a result, these students can be said to have identified with certain aspects of mathematics as it was locally constituted. That being said, Sarah and Ellen were seen to comply more often than identify. For example, Ellen made it clear that she did not identify with certain aspects of mathematics when she stated:

Ellen: I don't like remembering, like formulas and knowing, you have to know so much stuff for certain sections, in math. Like they're like you need to remember

> like ten formulas for, a certain, for a certain section and sometimes you can be like, you just get confused, sometimes. Not for everyone, but for me sometimes I do. So just having to remember so much, I don't like that. That's what I don't like.

So while Ellen identifies with the general obligations of note taking and paying attention, she simply complies with mathematical obligations relating to the use of formulas. Another compliant student, Laura, echoed these feelings in her interview.

> Interviewer: What do you mean you're not a math person?
>
> Laura: I dunno, I, well, I struggle with math. I don't really enjoy it, I haven't ever really enjoyed it. And I'd much rather be doing other stuff, focusing on other things, instead of just doing the quadratic formula again, for the millionth time.

Laura was a classic example of a high achieving compliant student. From her seat in the front row, she seemed to always be focused during class time as she attentively took notes. She rarely asked questions in class, choosing instead to quietly go about her business. As we hear in her narrative, she sees math largely as a collection of time consuming busy work. Yet, she worked diligently day in and day out, complying with all classroom obligations.

Structural significance and the added motivation it provides were also consistent themes in student narratives of compliance. In the following interview excerpt Danielle was asked to elaborate on her answers to the Walker prompt (see Appendix S).

> Danielle: Like that's how I have it down, cuz like that's how I, cuz sometimes, it'll be like a long day in math and I'm like, I have to graduate. I'm almost to

graduation.

Interviewer: Oh that's fantastic.

D: And then it's like sometimes, it's like, ok I'm not gonna do this with my animals but hey, I gotta do it on the EOC so, I just have to learn it.

D: And sometimes I think of like, if my dog breaks his foot, I have to know how to fix it so. I need this.

I: right. Ok.

This excerpt provides insight into how recognizing the structural significance in mathematics can support student motivation. Specifically, by repeatedly elaborating and reminding herself of the ways in which mathematics is significant in relation to her chosen future, this student continually re-identified with mathematics, especially during those days when she was tempted to resist. In contrast, students who resist mathematics likely elaborate and remind themselves of the ways in which mathematics is not significant in relation to their chosen futures, continually re-resisting mathematics, especially during those times when doing mathematics is particularly burdensome.

Recall that significance relates to students' perceptions of current or future benefits of engaging in the mathematics. I have also described competence in terms of a locally defined set of obligations that one would need to meet in order to be recognized as an effective mathematics student. As a result, a student deemed locally incompetent in mathematics might find it hard to recognize the structural significant of mathematics. This can in turn affect the student's ability to identify with mathematics. We see that this was in fact the case in Ms. Mason's class, as seen in the following interview excerpt.

Danielle: I dunno, math is like, I just think because I never got it, so then I don't like it. Spanish I took when I was in elementary school. So, I kindof like learned it, early, so I got the hang of it. But math I'm still like really iffy about.

Danille's comments indicate that initial failures can have long-lasting effects for students' ability to identify with mathematics. In line 1 the comment "because I never got it, so then I don't like it" clearly link her previous failures and current compliance. She contrasts this in line 2 with her early success in Spanish that paved the way for later successes.

Narratives of Resistance: Rebbeca. One of the students interviewed was able to provide first person narratives of resistance in Ms. Mason's class. Her narratives of resistance included discussions of understandings and valuations of the classroom obligations. This is significant since her narratives provided answers to questions previously posed by Cobb et al. (2009), who wondered if resistant students had the same understanding of classroom obligations as other students.

> Rebecca: I don't really understand taking notes in math class. I don't think it helps because it doesn't show like step by step, it's just number after number after number… It's like if you don't understand addition, which is weird, but um, say you have 7+3. You just put 7+3=10. But there's no way to tell you how that happened. So that's kindof my problem. There's no way to tell me how I got from that to that.

Rebecca's differing understanding of the note taking obligation is likely a source of resistance. Compare this with the previous narratives of compliance from Danielle and Ellen who described note taking as an important part of doing mathematics.

When analyzing the data from Ms. Mason's class, two additional themes emerged in students' narratives of resistance: significance and failure. These findings mirror those of Ekert (1989):

> …schooling teaches children both their place in society and how to behave in that place. The gradual accumulation of differential experience in the early years of schooling leads mainstream children to believe that education will ultimately bring rewards and success, while non-mainstream children frequently come to view education as a humiliating and fruitless pursuit (Eckert, 1989,p.7)

The "rewards and success" of Eckert are what I refer to as structural significance and the view of education as a "humiliating and fruitless pursuit" is clearly a description of failure. I next describe the findings from Ms. Mason's classroom related to significance and failure.

Rebecca echoed this connection between past failures and current resistance.

> Interviewer: How important is it for you to be good in math?
>
> Rebecca: I would like to be good in math…but…I guess if you really try hard at something for a long time, you just kinda start giving up if it doesn't work. And so I kindof changed what I wanna do after high school. And so it doesn't really involve math. I don't think.
>
> I: So what do you want to do?

R: I wanted to go into criminal justice and I don’t know how much math that involves but I’m hoping not a lot.

Rebecca may have identified with mathematics at some point in the past: “I would like to be good in math.” Unfortunately, due to repeated instances of failure she soon began resisting mathematics and eventually “giving up.” As a result, she changed her future career plans into something that “doesn’t really involve math.” It appears that Rebecca realizes the innate tension caused by viewing mathematics as structurally significant while at the same time being recognized as mathematically incompetent. To resolve this tension Rebecca changes her plans for the future so that, for her, mathematics is no longer structurally significant. For Rebecca, there is no tension between being viewed as mathematically incompetent if you have a future in which mathematics will not play a significant role.

Rebecca further described the relationship between competence and resistance:

Rebecca: I feel that mostly the reason people don’t try is because they don’t think they can do it.

Interviewer: Why do you think that is?

R: Because of past test grades or past times trying to understand it and you just can’t. Like I know that Miss Mason says that she wants us to go through all the homework and try it, because there might be something we understand and something that we don’t, but I guess, it kinda gets old failing, and you’re just so used to doing it that you’re just gonna keep doing it. You know that you can’t really change it. Or you feel like you can’t.

The comment in line 7, “it kinda gets old failing,” viscerally and succinctly captures the relationship between incompetence (an inability to fulfill classroom obligations) and resistance. Thus, from a resistant student’s point of view, it is resistance that eventually causes a student to stop trying. Moreover, incompetence is viewed with a sense of permanence for this student: “You know that you can’t really change it.” Therefore, incompetence leads to feelings of permanence, which leads a student to stop trying. These findings align with previous motivation research related to students’ expectations of success or failure and their valuation of the activity at hand (Dweck & Leggett, 1988; Wigfield & Eccles, 2000).

Another student described how even Ms. Mason can succumb to the feeling of hopelessness that often accompanies a lack of student competence:

Tori: But it’s gotten to a point where like she realizes it, and so she’s like, like in
the beginning she was always like “I’m gonna help you and I don’t care if you
don’t want to, you’re gonna pass my class” kinda thing. But now she’s like, at
this point in the semester there’s not really hope for the people that haven’t tried.

It should be noted that during classroom observations, Tori was one of the students in the class who consistently interacted with the teacher during whole class discussions. Moreover, her comments during paired interviews indicated that she identified with mathematics as it was locally constructed. In contrast to Rebecca (a resistant student) who viewed lack of effort as an inevitable result of previous failures, Tori (an identifying student) seemed to indicate in line 4 that students who were failing were somehow blameworthy: “people who haven’t tried.” According to Tori, a lack of effort led to

mathematical incompetence, and not the other way around. In the next excerpt Rebecca describes incompetence and failure as not only permanent, but also somewhat "inevitable":

> Rebecca: ...I didn't want to, but it was kindof inevitable.
>
> Interviewer: What do you mean?
>
> R: like, it was the same thing with Algebra 1, is that at the first of the year I got everything, and then it slowly started getting worse and worse and worse and I just kinda gave up. So…
>
> I: Did you feel better after you gave up?
>
> R: I was less stressed. But now the stress is back because it's finals and my grade is bad <laughs>.

Previous failures in Algebra 1 appeared to set Rebecca up for failure in Algebra 2. Specifically, when she started the year off in Algebra 2, and then began to do poorly when the material became more difficult, she saw a pattern beginning to repeat, leading to feelings of inevitability. Soon Rebecca "gave up," a choice which temporarily allowed her to feel "less stressed." In Ms. Mason's class, the students who gave up (resistant) and those who persisted (compliant or identifying) would go on to form two distinct categories of students.

Narratives of Identification: Janet, Tori, & Adam. Student narratives of identification often included discussions of understandings and valuations of the classroom obligations. The additional spaces in the lines below denote overlap in conversation between the speakers.

Janet: I like how the same kind of problem has the same steps every single time. Like, I don't know, when you distribute something and then you have to like subtract to solve for x, like it's all the same. It's usually like the same process.

I: mm-hm. Ok.

Tori: I was gonna say whereas geometry, every proof was different. So you didn't really have like the same thing. Like you know what I mean?

Janet: A set in stone process. Whereas like in algebra you can kinda like go through the same thing. Ya know?

Here we see evidence that Janet and Tori identified the local definition of mathematics as highly procedural and ritualized. Put another way, these students identified with the specifically mathematical obligations in Ms. Mason's class.

Students in Ms. Mason's class who identified with mathematics described the role that classroom microculture played in their identification. For these students, doing mathematics became a group activity in Ms. Mason's class. As a result, student narratives of identification included discussions of situational significance.

Tori: Like last year I remember it was just like, I don't know. I just didn't feel like I could ask other people in the class. Whereas like this year I feel like we are all pretty much are like on the same level. Like we all understand it and so we, just, maybe like one problem I don't understand but like Janet would. So she would help me and like vice versa.

Janet: And I feel like that just because of the way she teaches. Because like the way she teaches it's like, for you to interpret how to like solve the problem

yourself.

I: mm-hm

J: And so like, if Tori interpreted it a different way, than I did, well maybe the way I did will help her if I told her…

Students who identify with mathematics in Ms. Mason's class seemed to create their own subculture in the classroom. Within this subculture students bring different perspectives to the problem solving process, perspectives that, when shared, help other students better understand the problem. Janet and Tori continued to elaborate:

Janet: I've never been like uh, I don't wanna go to math.

I: Right.

J: I like look forward to it. I feel like especially since it's fourth period, I would expect myself to be like: "Ok I'm ready to leave. Ok, when is this over?" But I don't do that, cuz I like enjoy it, cuz I have friends and I feel like, I dunno we're not necessarily like a fam-i-ly, but we are like a family.

Tori: <laughs> That made no sense.

Janet: <laughs> That sounds like cheesy but we kind of are, because like we help each other and stuff.

Interviewer: What do you mean, tell me more about that.

J: Just like the, when you're taking notes and you're like "Did you get the same answer as me?" or like sharing how you understand things.

In lines 22-23 Janet describes being accountable to other students for sharing correct answers and their developing understandings of the mathematics at hand. This obligation

was not part of the larger normative identity of the classroom, but did come to define how this subgroup of students interacted with one another. The description of this subgroup as a "family" who discuss their mathematical work together is similar to findings from Boaler (2000) when describing discussion-oriented figured worlds. Notice that this is quite different from the figured world of didactic teaching that seemed to describe Ms. Mason's classroom as a whole.

Structural significance as a source of division. During the paired interviews students discussed their plans for life after high school while also candidly commenting on the plans of others. The resulting narratives are more than just hopes and dreams for the future, they provided insight into how students view schooling and it's relation to future life plans. The following interview excerpt begins as Sarah recalls a conversation she had with another student in Ms. Mason's class about life after high school.

Sarah: One told me what he wanted to be in life and…it's not right.

I: What do you mean?

S: We got put in a group together and like we're, it was when we were doing one of the study guides…

I: mm-hm

S: …as so, I was like "Do you know how to do this?" And he was like "No." And I was like "Do you wanna learn how to?" and he was like "No. " I was like "Do you not care?" He was like "No. " I was like "Why not?" He was like "I know what I wanna be" And he's like "I wanna move to Colorado and I'm gonna grow weed and I'm gonna sell it" And I was like "Good luck with that" <laughs>

11 He was like "Well I'm either gonna end up richer than you, or in jail, so…"

12 And I was like "I'm gonna take my way"

Sarah starts off by indicating that the other student's life plans were not simply a value-neutral choice, they were "not right." When she asks the other student if he wants to learn she is surprised at his answer of "No." As the student proceeds to discuss his future plans to enter the drug trade, he appears to be a clear example of the "Burnout" category from the literature on school cultures (Eckert, 1989). Furthermore, notice how the discussion of whether or not to learn mathematics ends on an adversarial note. In lines 11 and 12 each student finishes their portion of the interactions by making a value judgment of the other's choice of future plans. Thus, when students choose to identify, comply, or resist mathematics they are concurrently defining who they are and who they are likely to become. As a result, conversations about the value of learning mathematics can quickly become conversations about self-worth. We find evidence in line 11 that the student Sarah is talking to appears to have a low self-worth since one viable option for his future is serving jail time. In contrast, Amanda indicates that she values herself too highly to make similar choices with her comment "Good luck with that."

These findings echo those of Eckert (1989) in her "Jocks and Burnouts" high school case study:

> Peer groups now incorporate concrete aspirations into their identities, and the differences between groups take on a clear relevance of future adult status. A childhood dislike for schooling is elaborated by an adolescent belief that school is

> unnecessary for the job that looms ahead; childhood success in school becomes clear preparation for college" (p.11)

When students fail to find a structural or situational rationale for schooling they are forced to look elsewhere for future plans. Sometimes these future plans can seem quite far-fetched, especially to those students who have found a rationale for schooling, as seen in the following two excerpts.

Danielle: I feel like they are set like "Oh, I'm not goin' to college." Cuz like alotta people are like "I'm gonna become a millionaire." And they're like, "Bill Gates didn't finish high school or didn't go to high school so I'm not going to."

Ellen: Oh yeah they like…they use other peoples' success as an excuse like, well so and so didn't go to college, uh the guy of Tumblr who's like a multimillionaire didn't even finish high school. And you're like...he got lucky (laughs).

D: It's luck. (laughs)

E: He got lucky. (laughs)

As with the previous excerpt, this discussion takes a decidedly value-laden tone. In line 4, Ellen's comment that "they use other people's success as an excuse" is clearly a value judgment, especially since "excuse" connotes a lack of agency and a shirking of one's responsibilities.

The mere fact that students failed to find a structural significance for schooling is likely to cause individual resistance. But the way in which structural significance is locally defined can create a type of collective resistance, as it did in Ms. Mason's class.

Specifically, part of this local definition included not only the future benefits of identifying with schooling, but also future risks. The students and teacher openly elaborated on what someone's life might look like if they failed to identify with schooling.

> Adam: Well school's like a major importance, like that sets your life basically. If you don't try in that it's like basically giving up on a portion of your life. And you don't wanna look back at it and say "Oh, well, I didn't bother doing anything in class soo..I work at McDonalds now. " <they both laugh>

The comment that school "sets your life" is a strikingly obvious indication that this student has a structural rationale for school. But there is also an undertone of judgment in the student's comments: "I work at McDonalds now." These judgments are related to the wide spread social stigma in the U.S. against minimum wage jobs and the people who work them. The teacher made similar comments in class as she explicitly negotiated a structural rationale for schooling:

> T: Ok. Let me draw your attention back up to the board, and let's walk through this together. Ok? Let's walk through this together. So alotta times in math, when we're confronted with a situation, it looks complicated and we say…I'm just gonna wait 'till Miss Mason goes through it. Right?
>
> S: Yeeah.
>
> T: You can't do that on an assessment. You can't do that on the real deal when you're being tested, and allota times when you're being tested it's something you've never seen before, it's application, and it's problem solving…and guess

what, that's what the real world is. You leave this room and you go out…you're not gonna be presented with a worksheet that says fill in these blanks. Right? You're gonna be presented with a real world problem that you have to dig in, piece out, and figure out 'Where do I start?', 'What do I do?'. So…when we're working through a lot of these problems I want you to at least give it a chance. Read through it. Try to figure out what it's asking you for. If you still don't understand then we'll walk through it. At least give it a chance, don't just give up on it and say...oh I'm just gonna wait because I don't know how to do this. Because the real world doesn't let you do that. Alright? The real world doesn't let you sit by and wait for somebody else to come show you what to do. Ok? That'll happen if you want a minimum wage job. Right? They'll hand you a task card and say "do this, follow these instructions." And if you have a problem you know you go up and ask. But that's what the minimum wage job gets. Right? You don't wanna be there. Right? You wanna be up higher, being able to say "I know how to fix this problem." Problems solvers get big bucks.

Notice the mixed signals Ms. Mason is sending. In lines 14-15 she asks students to "try to figure it out" and if that doesn't work "we'll walk through it together." These are seemingly contradictory signals of independence and dependence. But with respect to the structural significance of schooling, her message is quite clear: the life of the minimum wage worker is something to be avoided. This lower rung of society is something to rise above so that you can earn the "big bucks." While this is one common way of negotiating

the structural significance of schooling, it may have the unintended consequence of alienating some students whose friends or family currently hold minimum wage jobs. For these students, identifying with the local definition of mathematics might mean rejecting a preexisting piece of their identity outside the classroom. Such conflicts have been documented previously in the literature on student identity (Ladson-Billings, 2009; Walker, 2006).

A Normative Divide. Much like any other classroom, as students in Ms. Mason's reacted to the normative identity some students resisted while others chose to comply. But what was especially noteworthy about Ms. Mason's class was the normative divide that surfaced between two emerging groups in the classroom: *the right kind of people* and *the "wrong kind of people"* (quotes denote students' choice of words). In the sections that follow I will discuss how students' understanding of mathematical competence and the resulting normative divide were related to students' personal identities as doers of mathematics.

The interview question "What does it take to be successful in this math class?" was chosen to uncover students' narratives about competence. This question would produce unexpected insights into the dualistic nature of the classroom microculture that formed in Ms. Mason's classroom.

1 Adam: Um, in a way you could say like, surround yourself with the right kind of
2 people. Cuz in our classroom you see there's like a big division..
3 Sarah: Oh yeah
4 Adam:..where on one side of the room it's like those who will actually do a lot of

effort, like put a lot of effort into things and score high on tests, and then on the
other side it's like the kids who don't try or fill out worksheets, and just spend
alota time goofing off in class. So if you surround yourself with the wrong kind of
people you'll end up being like them and you wont try.

Adam's description of the "right kind of people" is reminiscent of Gee's conception of identity as being recognized as a certain type of person (Gee, 2001a) and provides insight into the identity work being done through Adam's narratives. Moreover, the "big division" that formed in Ms. Mason's classroom was just as real for the students as if someone had painted a line down the center of the room separating the "right kind of people" from the "wrong kind of people." This is partially due to the fact that, for these students, their narratives related to the big division were reifying, endorsable, and significant (Sfard & Prusak, 2005). Adam continued his description of the classroom divide:

Interviewr: What would the "right kind of people" to surround yourself be like?
Adam: Like those who work hard, and put time into school, and have a positive
attitude about it.
I: mm-hm
A: And those kind who are like "Oh, school work, let's just go to a part and
get drunk"
I: Right.
Sarah: That's like half our class. <laughs>
A: yeaaahhh.

S: That's like all they talk about in class.

A: Yep!

S: That one side of the room.

I: Really?

S: Yeah

A: Oh, the last time I went to a party, this this and this happened, and I accidently called my mom.

There is work being done in this discourse that goes beyond mere description. Phrases like "those kind" and "the other half" are examples of how language can be used to indicate status and solidarity (Gee, 2008). Adam was not the only student I interviewed that noticed the big division in Ms. Mason's class. Consider the narratives Janet and Tori provided when they were asked how they compared to their peers in the class.

Janet: I would say that we are definitely in the top half of the class, just because there's like a half of the class doesn't care to pay attention at all.

Interviewer: mm-hm

J: And then there's the other half.

Tori: That actually tries.

I: <laughs> Well there ya go.

J: I feel like if the other half of the class tried, we would just be in the top half, but because they don't, we're like at the very top.

I: mm-hm

J: And I don't wanna say that like, we're the best of the best, because

T: Because there are people in our class that are smarter than us

J: Yeah.

I: mm-hm

J: But because we try, like out of the people we try with, we’re probably in the middle.

I: Tell me about this group you “try with,” what do you mean?

J: Like they actually like care about their grades and want to learn and stuff like we do.

I: mm

T: Like when we are sitting in class it’s kinda like divided.

J: There like literally is a half line in the seats

T: literally there’s a half line. And like half of the class sleeps during class.

I: mm-hm

J: And the other half like does the work, does the homework. Pays attention.

T: Takes notes.

I: mm-hm

T: Asks questions.

Janet and Tori describe the real implications of the big division: “literally there’s a half line.” Also notice the reoccurring theme of students who are “trying” versus those who are “not trying.” These finding were further verified by other students I interviewed who described their divided classroom in similarly vivid ways, as in the following excerpt.

Danielle: There is, it’s like it’s literally like a divide like right in the class. The left side of the class, if you’re facing this way, doesn’t do anything, and the right side does like everything.

Ellen: We try.

D: Yeah.

E: The right side of our class tries. Or we at least, we at least try something, but the left side it’s usually just they don’t start their homework, they just talk.

D: Some kids don’t even bring a pencil. That, kindof annoys me. Cuz it’s like, cuz like this is what I think: If I don’t like a subject I’m gonna do whatever it takes so I don’t have to retake it. I think that’s dumb if you don’t like it and you’re like “Oh I don’t, I don’t even like this class anyway,” then you're gonna retake it. You like somethin’ about it, cuz you're takin’ it again. It’s annoying when others don’t try!

So we begin to see that for the students in Ms. Mason’s class the normative divide had very real implications for who they were as doers of mathematics and how they came to view others in the classroom. In fact, the longer that students on the “right side” of the room continued to identify with the local definition of mathematics while the students on the “wrong side” of the room chose to resist mathematics, the further they grew apart from one another. In fact, when I looked back through the video data, I noticed that over the course of the semester students actually changed their seats from class session to class session, steadily migrating towards students with similar views towards mathematics.

This continued until the end of the year, by which time the normative divide had become an actual physical divide.

Also, notice the two groups of students have such vastly different personal identities as doers of mathematics that, within confines of the classroom, one group appears to not like the kind of person the other group of students have become. In fact, they make choices that are “dumb” (line 10) and “annoying” (line 12).

When asked what was necessary to be successful in this class, Adam said that it was important to stay away from the “wrong kind of people.”

1 I: Hm. Ok. What is the difference? Can you describe the “wrong kind of people,”
2 What do you mean?
3 Adam: Like a lack of effort or determination, like they don’t have a common goal
4 in their mind.
5 I: mm-hm.
6 A: They just see the class as worthless and a waste of time.

Adam’s comments in line 3 that the wrong kind of people “don’t have a common goal” would be the logical antithesis of the subculture described by Janet and Tori (the right kind of people). This type of dual category analysis is typical in school cultures (see Table 8). Furthermore, the comment in line 6 that “They just see the class as worthless” supports a conjecture by Cobb (2009) that students who do not identify likely have a different understanding of the normative identity of the classroom. In fact, I spoke with one student who didn’t identify with the local definition of mathematics who provided further support to Cobb’s conjecture:

Rebecca: I don’t really understand taking notes in math class. I don’t think it helps because it doesn’t show like step by step, it’s just number after number after number.

She goes on to say…

R: It’s like if you don’t understand addition, which is weird, but um, say you have 7+3. You just put 7+3=10. But there’s no way to tell you how that happened. So that’s kindof my problem. There’s no way to tell me how I got from that to that.

Cobb (2009) conjectured that students who resist may not have the same understanding of their classroom obligations as identifying or comply students. This resisting student cannot “understand taking notes.” This is in stark contrast to other students’ understanding of their classroom note taking obligations, as noted in previous sections.

Negotiation of normative identity. As seen above, teachers and students continually negotiate various aspects of the normative identity, including the value of note taking. Because of the important role that negotiation plays in the creation and evolution of the classroom microculture (Cobb et al., 2009; Voigt, 1995), it’s important to look at some specific examples of what this process might look like in the context of a mathematics classroom. We now consider several instances in which Ms. Mason and her students were negotiating the social norm of note taking. Sometimes these negotiations were quite explicit, with the teacher being very direct and insistent.

> T: Write down example A. Write this down. Write it down. Brandon write it down. (10-10-2013)

Notice the way in which the teacher repeatedly and emphatically tells this student what is expected of them, leaving no room for miscommunication. She uses discourse to create a sense of tension that can only be relieved by the student finally complying with her request to write the example down. Here the teacher's discourse also includes a tacit appeal to authority, an unspoken "because I said so". To further support her argument within the ongoing negotiation, Ms. Mason occasionally provided specific reasons in addition to her appeals to authority, telling students not only what to write down but why they need to write it down.

> T: "n" is the index and "a" is the rad-i-cand. Write this down in your notes, so that when I say these words you know what I'm saying. Math is a foreign language. If you don't know my language…it's gonna be hard to understand me. (10-10-2013)

> T: …Jerimiah, I need you with me. Write down this last example. You will more than likely see this type of question on the EOC. Because it combines like four different concepts. They're really big about that, putting concepts together…combining and testing. Because they can't test you on every single concept that you've learned, unless they start combining concepts together, right David? Yeah, so write this example down, let's do it together. (10-10-2013)

Notice how in the last example the teacher has moved away from a blanket request aimed at the entire class and now uses specific students' names. Here the teacher is making it clear that she is negotiating directly with certain students who have chosen to breach aspects of the normative identity. Later we will see that a rift emerged in Ms. Mason's

classroom between those who chose to comply/identify with mathematics as it is enacted in this classroom and those who openly resisted. What part, if any, did teacher discourse play in the creation of this rift?

In summary. In this class an interesting pattern emerged. Students with similar understandings and valuations of the classroom obligations were drawn toward one another. Students not only described the class in terms of the "big division" that arose between students, they also described themselves as being affiliated with one side of the class or the other. In fact, the classroom eventually divided into two distinct groups, with students even changing seats over the course of the semester to be seated near students with similar personal identities. It is not that such grouping happened, but the extent to which it happened in Ms. Mason's class and the critical role it played in students' narratives regarding both their personal identities and the normative identity of the classroom that makes this particular class noteworthy.

Research Question 2

How do students describe their academic communities both inside and outside of school (peer, family, and school relationships)?

To answer this question I relied on student interview data. As noted earlier, steps were taken to promote rich data from students, including interviewing students in pairs and stimulated recall.

Family Communities: Shoot for the A's. For Ms. Mason's students, family communities acted as either a ***resource for*** student success or a ***reason why*** students were motivated to succeed. This distinction arose during stimulated recall discussions of

student responses to the Map of Student Influences. Some students described their parents as a resource for success: able to provide mathematical scaffolding or insights outside of the classroom. While other students described their parents as reasons why they were motivated to succeed, often relation to possible punishment if parental expectations were not met.

The following two excerpts illustrate the ways in which parents can provide *reasons why* their students should be successful in mathematics.

S: Yeah. I feel likes it's a job and like my parents expect me to do well. And they like always have been like "Oh it's important" so I've just like always thought that way.

I: Mm Hm

S: So if I try not to, like, I dunno. I just like can't, not think that way. And like, I don't know, I like seeing A's. Like, It bothers me when I go online and it's like B.

For students, *reasons why* they should succeed are often wrapped up in parental expectations and the related student obligations (line 1). These obligations truly drive action when they become obligations-to-oneself instead of simply obligations-to-others (Cobb, 2009); like with Sarah who said that a grade of B "bothers" her (line 6). Adam, a student who switched out of his honors-level Algebra 2 class into Ms. Mason's standard-level Algebra 2 class at the beginning of the year, provided a similar example of obligations-to-onself.

Interviewer: Why did you make the decision to go to college prep Algebra 2?

Adam: Cuz um, the honors teacher liked to give allota work. And I didn't do so

hot on the first few assignments. So I didn't wanna spend my semester trying to raise the grade up from that. Because to my parents GPA is alota what matters. And if I don't do very well with my GPA and they see like a B or a C, it's not exactly the best thing for me.

I: What do you mean? Tell me more. What do you mean "not the best thing"? What would happen? What's the worst that could happen?

A: I mean, I don't really know consequentially <laughs> because I've only gotten one B and that was by one point.

I: mm-hm. Right.

A: And my, I got alotta stern talking to for that <laughs>

I: Wow.

A: yeah.

I: Wow. Ok.

A: I was more upset with myself than anything though because my parents had always told me "shoot for the As."

As with the previous excerpt, Adam's parents had certain expectations for their son and related obligations (lines 4-6,12,and 17). He eventually turned his obligations-to-others, "shoot for the A's," into obligations-to-onself (line 16)

Parents who have the mathematical background to be able to answer content-related questions can be great *resources for* mathematics.

Sarah: He's in engineering so he's always like "Ask me questions! Ask me questions!"

I: Really?

S: He likes to answer questions and he’s good at math, so. And he can like make it simpler for me, usually.

I: mm-hm

S: He just likes to simplify it down… actually no…he really complicates it. And then he’ll simplify it.

I: Right.

S: It works out in the end.

I: Wow. Well that’s awesome. Has he been helping out for a long time with math?

S: uh-hu. Ever since I was like, tiny…just like, elementary school.

Notice that Sarah’s father helps provide an expanded local context for doing mathematics and an expanded definition of what it means to do mathematics. Specifically, her father helps define doing mathematics as something exciting and enjoyable: “Ask me questions! Ask me questions!” (line 1). He also makes mathematics a subject that can be made “simpler” and more conceptually accessible (lines 4-9). I claim that this expanded definition gave Sarah an additional opportunity to identify with mathematics, an opportunity that was not readily available to her other classmates.

Danielle provided another example of a parent who acted as *resource for* doing mathematics.

D: Well my mom’s a math person.

I: Ok.

D: She’s really good at it. And she ‘s like, when she sees something she can like

figure it out like doin' different like formulas and stuff. She can just figure it out and then she can explain it to me simpler than like what my teachers can do and stuff.

Similar to Sarah's father, Danielle's mother was able to make math simpler and more accessible. Sarah and Danielle's narratives are in line with previous findings from Walker (2006) who described parents acting as a *resource for* their students as long as they are to do so competently (although she doesn't use that exact terminology). Unfortunately, many parents are unable to help their students with mathematics after middle school and are (Cobb et al., 2001) forced to transition into a role where they instead provide *reasons why* their student should be good at mathematics.

Peer Communities: Backup teachers and club teams. In the following excerpt Tori and Janet explain their responses to the Walker prompt. Specifically, Tori explains why her classmates were placed so close to the center in her diagram (see Appendix T).

Interviewer: Why is "your classmates" listed right next to the teacher?

T: Because I kindof feel like they are my backup teacher in a way. Whereas like, sure my parents are like involved and like they're, one of my reasons I do well in math, but they haven't been there that day. <loud intercom announcement>

I: I'm sorry can you say that one more time?

T: Like, your parents, sure they could probably like help you if you really needed it, but they weren't in that class that day to be like "Oh yeah…"

Janet: to like hear the way she explained it. And maybe put it a different way.

T: cuz they .. Yeah, they were like "Oh well when I was in school, we learned it this way" and you're like "Well no we have to do it this way." So I feel like that's why your classmates are kinda like your backup teacher, because they…were there for it.

One resource for students in Ms. Mason's class were their classmates who acted as a "backup teachers." These backup teachers are often a better resource than more traditional teachers in the classroom or at home. As Tori explains, parents "haven't been there that day" (line 4) and as a result aren't as helpful when it comes to mathematics as her classmates who "were there for it" (line 13).

Sometimes students look outside their immediate group of classmates to find a peer academic support, as was the case with Sarah.

Sarah: Like on my soccer team, like, I have to spend all my time with my club team. So, I have allota girls on there are really good at math and so they always help me. It's like, "Oh I got a hundred and five in that class, let me do it." So like they always help me, cuz, like, I can like ask them questions and be like "I don't get that!" and get like frustrated around them and they don't care.

Interviewer: Right.

S: So that's easy for me to like talk to them about.

I: Do they all go to this school?

S: No.

I: Really?

S: yeah. Like, we have some Memphis girls and like Johnson City and Nashville.

I: Wow. When do you guys get to work on math together?

S: Um, we have tournaments like every other weekend usually, so, that’s usually when it is. Or I can like call them, if I need something.

This excerpt is similar to the findings of Walker (2006) related to “near peers,” a group which was seen to include influential voices such siblings, cousins, and friends from other schools. Sarah continued to explain:

S: That’s when I usually like try to understand my math is like with them. Cuz they can put it in like…since they don’t know like more, than like, there is to be known I guess, if that makes sense,

I: mm-hm

As: They can just be like, “oh this is exactly how you do it. And that’s all you need to know. “

I: Right.

One benefit of a peer academic community is actually their lack of mathematical content knowledge as compared to the teacher or other more educated adults. As a result of their limited knowledge and experience, backup teachers often offer concise, pointed help when assisting a classmate and are unlikely to offer up any seemingly unrelated information (line 19).

Research Question 3

What are the relationships between the academic communities students describe and student identity formation?

The Identification Triangle (Figure 3) provides insight as to where we should look for possible connections between academic communities and student identity.

Opportunities to identify. As students discussed the various academic communities available to them, differing normative identities began to emerge. For example, student narratives describing what it meant to do mathematics with peers outside of school were sometimes quite different from narratives describing what it meant to do mathematics in Ms. Mason's class. This implies a different set of student obligations when working with peers, which in turns means a different definition of competence. As a result, some students experienced a level of competence in the peer academic community not experienced in Ms. Mason's class. Thus, academic communities informed student identity formation. The general argument can be outlined as follows:

1. Academic communities have their own normative identity made up of classroom obligations, which in turn define **student competence**.
2. **Student competence** informs understanding and valuation of obligations; that is to say, student identity.
3. Therefore, academic communities inform student identity formation.

Hence, **student competence** appears to be a logical starting point when searching for relationships between academic communities and student identity formation. This was in fact the case when I began to analyze the student interview data.

One benefit of having a set of "near peers" is the role they play in supporting identification with mathematics outside the classroom by helping create an expanded

local context for doing mathematics and an expanded definition of what it means to do mathematics.

> Interviewer: So that's where you go to "really understand it," what do you mean really understand it? I liked how you said that.
>
> Sarah: I'll be like, like if I have a question, usually if I have a question I'll ask my dad and he'll be like "Ok, I got that answer," like for that question. But then with like the girls on my club team I'll like ask a question, I'll be like "Ok, but what if this number was here? Like, what if this happened?" I just like go more in depth with it, like, so I completely understand it.

When students work on mathematics in a very organic, natural setting they can be seen to ask more conceptually oriented questions like "what if this number was here?" This differs from previous studies that provide descriptions of student-created participation structures which include very nonmathematical ways of knowing mathematics (Boaler, 1999; Lampert, 1990). Moreover, a mathematics classroom often appears to be a non-natural setting for problem solving in that, the problems of the class are really the teacher's problems that students feel coerced into considering. In other words, classroom obligations remain obligations-to-others and never become (Cobb et al., 2009; Cobb et al., 2001). In contrast, Sarah described doing mathematics with her backup teacher in terms of obligations-to-oneself: "…if I have a question…" and "…I'll ask a question…" I claim that this expanded definition of what it means to do mathematics gave Sarah an additional opportunity to identify with mathematics, an opportunity that was not readily available to her other classmates.

Chapter 5

Discussion

Theoretical Implications

Basis for discourse. Prior to this study, several frameworks for interpreting the microculture of a mathematics classroom had been suggested (see Table 3). These frameworks differ from the proposed basis for discourse (BFD) framework in both their focus and resulting implications. For example, whereas previous frameworks have focused on what students produce (Bauersfeld, 1988), teacher's images of mathematics (Thompson, 1994), and who validates knowledge claims (Boaler & Greeno, 2000), the BFD framework focuses on the nature of discursive objects and connections between them. Existing frameworks also differ from BFD in their implications for teachers, including suggesting that teachers should strive to produce students who can attack novel problems (Bauersfeld, 1988); teachers should thoughtfully reflect on their understanding of what it means to do mathematics and how that understanding influences the way they teach (Thompson, 1994); and that teachers should allow students more opportunities to validate knowledge claims (Boaler & Greeno, 2000). In contrast to these previous studies, the BFD framework would suggest that teachers should thoughtfully consider what mathematics looks like from the viewpoint of their students, paying special attention to the nature of the objects and connections discussed in the classroom.

With respect to theory, the concept of basis for discourse has the potential to provide insight into why students often say that they understand math when they are in

the classroom but have trouble when trying to do homework later that night. This could be the result of conflicting bases for discourse where students learn mathematics using one BFD and are asked to perform mathematical tasks using a different BFD. For example, if homework questions provided by the textbook require students to make mathematical connections between mathematical objects, but students have been using a procedural BFD in class, then students will likely not be able to perform well. Thus, BFD could be thought of as an affordance to which students can become attuned (Boaler, 1999).

Normative Divide. There have been several studies documenting the social divisions that occur along socioeconomic and class lines within schools across the world. Among the most notable of these macro-level studies are the Jocks and Burnouts study of Eckert (1989) in the U.S. and the Eroles and Lads study of Willis (1977) in England. Both of these studies produced a dual category analysis, focusing on issues of class, various forms of competence, and even drug use. Most importantly, students in both studies often described themselves in reaction to the opposing category within the school. For example, a student in the Jock category would describe themselves as *not* being part of the burnout crowd. Similarly, students in Ms. Mason's class often described themselves in reaction to the opposing category, what Adam called the "wrong kind of people." But, in contrast to previous studies, the current study provides evidence of a micro-level division within the mathematics classroom (as opposed to a macro-level, school-wide division). This study is the first of its kind to provide insight into the role

such a normative divide can play in relation to students' personal identities as doers of mathematics.

Opportunities to identify. Recall that Walker(2012) defines as "sites where mathematics knowledge is developed, where induction into a particular community of mathematics doers occurs, and where relationships or interactions contribute to the development of a mathematics identity" (p.67). Three types of mathematical spaces that seemed to influence identification for Ms. Mason's students were the classroom, peer-delimited mathematical spaces, and family-delimited mathematical spaces. Notice how each of these spaces has varying criteria for membership (hence "delimited") and as a result some of Ms. Mason's students had access to these spaces while others did not. With differential access came differential *opportunities to identify (OTI)* with mathematics. This is because each mathematical space had it's own local definition of what it meant to do mathematics (i.e. normative identity). Hence, the *quantity of OTI* was seen to vary from student to student, depending on factors such as: parental expectations (*Adam*), parents' level of education (*Danielle),* and involvement in extracurricular activities (*Sarah*). Hence, this study contributes to the literature on identity by drawing attention to students' opportunities to identify with mathematics, opportunities based on the various normative identities to which a student has access.

Obligation Development and the Identification Triangle. Many studies have documented classroom norms and obligations within the mathematics classroom (Boaler & Greeno, 2000; Cobb, 1995; Cobb & Hodge, 2011; Cobb et al., 2001; Cobb et al., 2011; Gresalfi, 2009; Sfard & Prusak, 2005; Voigt, 1995; Yackel & Cobb, 1996; Yackel et al.,

1991). Yet none had explicitly tracked the development of classroom obligations within a mathematics classroom over the course of an entire semester. As a result, the current study marks a substantive contribution to the literature on identity in the mathematics classroom. Relatedly, the *Identification Triangle* (Figure 3) represents one possible model for how identity plays out in the mathematics classroom. While this model is rooted in the literature and carries with it a great deal of face-validity, it is not meant to be the definitive take on identity formation, but a starting place to begin a conversation.

Implications for Practice

Basis for discourse. Algebra 2 is a required course for all high school graduates in the state where this study took place. All students, no matter their incoming mathematical proficiency, previous mathematical experiences, or future mathematical plans, must take the course. As a result, an Algebra 2 classroom (especially at the standard level) is often filled with a sizable group of students who don't want to be there in the first place. These students can be hard to motivate, easily distracted, and even insubordinate. To provide such students with success, teachers often distill mathematics down to a set of procedures that can be easily memorized and robotically applied with little-to-no conceptual understanding. As a result, these teachers often use a procedural basis for discourse.

By teaching with a procedural BFD students are no longer expected to understand the mathematical why in the background, but only the procedural how in the foreground. A student no longer has to make connections between big mathematical ideas, they simply need to memorize a series of procedures. It is all about lifting a perceived

mathematical burden from the student who would otherwise not be able to cope. Notice how these expectations not only reflect who a teacher believes her students to be mathematically, it also concurrently helps define who her students are mathematically. Teachers in Ms. Mason's school district are actually incentivized to default to this type of procedural teaching since teacher bonuses and teacher retention are both directly tied to student test scores and related growth measures.

Teachers should be aware of the basis for discourse they are using at any given moment and question if that BFD is the most pedagogically advantageous. As I hinted in an earlier section with the topic of horizontal asymptotes, many topics can be approached using any of the three types of BFD. That being said, the choice of BFD is not without it's consequences. For example, the limit concept can be taught using a metaphorical basis for discourse including proximity metaphors (if the points x_1 and x_2 are close together, then the function values $f(x_1)$ and $f(x_2)$ will also be close together) or approximation metaphors (related to the idea of minimizing an error). Unfortunately, these metaphors can cause conceptual problems later in a students' development (Oehrtman, 2009).

Future work can include investigating how a teacher might go about choosing which BFD to use when teaching a specific topic. It is likely the case that some topics lend themselves naturally to either a procedural, mathematical, or metaphorical BFD. Moreover, when teaching, it is often necessary to change from one basis to another. For example, you might start out teaching a topic showing students how to do a few examples (procedural basis) and then move onto why the examples work the way they do

(mathematical basis). But this begs the question: is there an optimal ordering for bases of discourse? One possible answer was given by Whitehead (1929), who described the "rhythm of education" in terms of three phases: romance (falling in love with a topic), precision (learning associated procedures), and generalization (understanding the conceptual underpinnings). This may be a hint that when changing bases for discourse, it is often best to move from metaphorical, to procedural, to mathematical.

Normative Divide. The work that teachers do as they negotiate the normative identity for their classroom can have unintended consequences on the way in which students relate to mathematics and one another. In fact, it is possible for a teacher to create a normative divide within the classroom, as different groups of students are held to different level of accountability with respect to classroom obligations. This is reminiscent of the "Jocks and Burnouts" study by Eckert (1989) in which the school is described as a vehicle for perpetuating the current social order. That is to say, students from lower class backgrounds are treated in ways that will prepare them for their future lot in life, a life with little authority or agency. Whereas students from middle and upper class backgrounds are prepared for a life in which they will be leaders in their communities, and are therefore provided more authority and agency within the class.

In Ms. Mason's classroom students often defined themselves in relation to the other group of students. This is reminiscent of Eckert's (1989) description of the relation between the Jock and Burnout categories:

> …they do not exist separately as inward-looking categories, but in a state of intense mutual awareness and thus of continual mutual influence: each category

> defines itself very consciously as what the other is not...The resulting Jock-Burnout split is a function of competition among adolescents for control over the definition, norms, and values of their life-stage cohort (p.5).

This state of "intense mutual awareness" was also seen in Ms. Mason's class. Eckert also found that early academic failures had direct implications for students' identities and the formation of peer groups. Discussing the transition from junior high to high school, the author notes:

> Peer groups now incorporate concrete aspirations into their identities, and the differences between groups take on a clear relevance of future adult status. A childhood dislike for schooling is elaborated by an adolescent belief that school is unnecessary for the job that looms ahead; childhood success in school becomes clear preparation for college (p.11)

This means that early remediation during the K-8 years may be key to future mathematical success in high school and beyond. This is because by the time students have reached high school, they have likely begun to surround themselves with people who have similar views towards schooling in general and mathematics in particular. This creates the type of collective resistance that is often seen on a school-wide scale and in the case of Ms. Mason's class, sometimes on a more micro-level scale

Opportunities to identify. One student in this study described how she often worked outside of class on mathematics with members of her club soccer team (see *Excerpt 27*). The student-athletes would get together for impromptu study sessions during down time on away soccer tournaments. Not only did they do math, but this mathematical

space seemed to have it's own normative identity. Whereas the normative identity of Ms. Mason's classroom largely consisted of taking notes and practicing procedures, the normative identity of the peer-delimited space described included asking conceptual questions and mathematical risk taking. This space allowed students the freedom to explore mathematics in ways that were simply not possible in Ms. Mason's room. This alternative normative identity provided another chance for Sarah to identify with mathematics, a chance that many of her peers did not have.

I have been consistently surprised over the last decade of teaching that many students fail to realize the benefits of doing mathematics with others. First, group work is often a more natural method of problem solving. When a person is faced with a problem outside of mathematics they will enlist the help of others to bring more resources to bear on the situation. Second, when you get to upper level courses in the high school curriculum the concepts involved are often best understood through thoughtful discussions with others (peers, a teacher, a parent, etc). Lastly, when working in groups there'll likely come a time when one person will have to explain a mathematical concept to someone else in the group. The knowledge needed to unpack a mathematical concept for someone often fosters a deeper level of understanding than is possible working alone. These results imply that teachers should explicitly stress the importance of doing mathematics in groups, instead of leaving such collaborations to chance.

Future work: Big-D Discourses and Equity

Big-D Discourses. I agree with Cobb and Hodge (2002) who describe Discourses as "an empirically grounded alternative to the unreflective use of institutionalized

categories of race and ethnicity," but that "one of the challenges when conducting an analysis will be to identify the relevant Discourse communities to which students belong" (p. 263). Addressing these challenges will be part of my future research. Future work will include uncovering other Discourses at work in the high school mathematics classrooms and understanding the role that teachers play in mediating the effects of these Discourses on students' identities as doers of mathematics.

Big-D Discourses are cultural resources that the teacher and students draw upon as they negotiate what it means to do mathematics in the classroom. These Discourses can be viewed as preliminary models that each side brings to the table as the process of negotiation begins. Every cultural model is value-laden. There is some sense of "oughtness" that goes along with accepting a cultural model, so that those participating in the model are bound by certain obligations (Gee, 2001b, 2008). As a result, the normative identity of the classroom is greatly influenced by the specific set of Discourses to which the teacher and students have access. For example, the *Math as Preparation Discourse* obligates teachers to value preparation for some distant future above all else; sometimes even above conceptual understanding. The following exchanges illustrate this point:

> T: Different textbooks, different organizations use different annotation. So it's important that you understand how it's read in several different ways, because I don't know how you're going to be presented with function operations on the state test. (9-12-2013)
>
> T: "You need to be able to do this. This is the kind of question they could ask you on the EOC. You need to be able to do this. You need to be able to look at a table

> of data, calculate statistics on it, and then interpret those statistics, in an applied situation. Ok? (11-22-2013)

The state in which this study took place was still in transition to the Common Core State Standards at the time of the study. This meant the high states tests for Ms. Mason's class was still the state-issued End of Course Exams (EOCs). But with so much pressure to perform well on the EOCs, every exam becomes relatively high stakes. This topic comes up often during class sessions and the following is illustrative:

> T: So this concept of domain and range is going to come up over and over and over again. You've seen it on the last two tests, you're gonna see it on the next test. (9-12-2013)

From this quote we see that certain math topics are important because they routinely show up on tests. In other words, math draws its importance from tests that asses it, not the other way around. A related message conveyed through classroom discourse is that a mathematical topic is less important if it rarely shows up on exams, as the following excerpt shows.

> T: So binomial expansion I can tell you there's only one question on the test for binomial expansion but it should be a "gimmie" problem. It shouldn't be hard to do. Ok? (9-20-2013)

Why is this important? Because if math becomes about preparing for exams, and students don't care about what grade they make on the exam, then they will fail to identify with mathematics as it is enacted in this classroom. And, over time, they may resist

opportunities to learn mathematics further. Sometimes it's a combination of preparation for assessments and the reality of the "real world" that follows.

> T: When I assess you, I'm going to be assessing you on "what's the trend line?" If I say "what's the trend line?" I'm looking for the linear regression. Ok? If I say "What's the model for this quadratic?" then I'm looking for quadratic regression. So you've got to read your instructions carefully and understand what it is that you're looking for. Ok? As we progress and understand this regression further and the correlation coefficient further, I won't even tell you what kind of model it is. Cuz out there in the real world, they don't come up to you and say " Hey, hey, this is linear" or "Hey this is quadratic" or "Hey this is exponential." You have to figure that out. You have to figure out what that data's doing. (9-12-2013)

Again, by focusing on teacher discourse we are not squabbling over mere semantics or wordsmithing. As noted by (Gee, 2008),

> Arguing about what words (ought to) mean is not a trivial business—it is not "quibbling over mere words," "hair splitting," "just semantics." Such arguments are what lead to the adoption of social beliefs and values and, in turn, these beliefs and values lead to social action and the maintenance and creation of social worlds. Such arguments are, in this sense, often a species of moral argumentation. (p.23)

Hence, discursive choices have real consequences for student identity as identities are constructed through actions, choices, reflection, and discourse. For example, some students may see the "real world" of their future as not involving mathematics at all (as we will see in the interview data).

Equity. Gutiérrez (2012) describes equity in mathematics education in terms of its four dimensions: access, achievement, identity, and power. Future work can include the use of this equity framework to augment analysis of normative and personal identities. For example, in the Gutierrez framework access includes “a classroom environment that invites participation, reasonable class sizes and supports of learning outside of class hours” (p.19). Thus a normative identity that “invites participation” can also be said to provide access. Gutierrez relates other dimensions to normative identity, including achievement as “participation in a given class” (p.19) and power as “voice in the classroom (e.g. who gets to talk, and who decides).” Lastly, Gutierrez discusses identity in a similar fashion to Cobb et. al's (1996) notion of personal identity, albeit with a slightly more critical bend: “Identity incorporates the question of whether students find mathematics not just ‘real world’ as defined by textbooks or teachers, but also as meaningful for their lives”(p.20). In an era of ever-increasing teacher accountability, I would argue that equity deserves consideration for inclusion in observation instruments. Not only are their ethical reasons to include equity as part of formalized teacher observations, it is reasonable to expect that increasing equity and narrowing the achievement gap might go hand in hand.

My own learning

As I reflect on the experience of completing this study I quickly realize both how far I have come as a researcher and how far I have to go. I have learned a great deal about what it means to do a qualitative study, from the nuts and bolts of how to gain access to a site, to experiencing the thrill seeing a finding emerge from the data. I look forward to

spending the next few years of my career working to better understand the art and science of qualitative case study research.

I also have realized through this experience the wonderful interplay between theory and practice. While I knew that one likely informed the other, I did not know to what extend until I began to do research for myself. Specifically, I found myself thinking about research as I planned for my day's teaching and thinking about my own teaching as I reflected on how to interpret my research. As a result, I will try to continue to teach mathematics in some capacity no matter my future research role.

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APPENDICES

Appendix A

Table 1

Theories of motivation in mathematics education research (Middleton & Spanias, 1999)

Theory	Synopsis of Theory	Associated good teacher practices
behavioralist	Describes motivations as rewards or punishments that stimulate behavior.	-Use success as a motivator. -Use group incentives to allow students to attribute failure to the group and success to themselves. -Don't let an extrinsic reward be the sole motivation for a student. Once the reward is gone, the associated behavior will follow.
attribution theory	Explains acquisition of motivations. Students' future mathematical success is influenced by their view of what creates success.	Dispel belief that one is either born with mathematical prowess or not. Allow children to struggle with challenging problems. Avoid *learned helplessness* by associating effort with success.
goal theory	Explains why people engage in activities by describing their personal goals.	Create inquiry based classrooms to increase mastery goals. Clearly state academic goals for your class.
personal construct theory	Constructs associated with motivation are arousal (cognitive stimulus), personal control (includes appropriate difficulty), and interests. Motivations are individual (hence differentiated instruction required).	Be aware of students' individual constructs as you structure your classroom.

Appendix B

Table 2

Relating the social and psychological perspectives (Cobb & Yackel, 1996)

Social Perspective	**Psychological Perspective**
Classroom social norms	Beliefs about own role, others' roles, and the general nature of mathematical activity in school
Sociomathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical conceptions and activity

Appendix C

Table 3

Frameworks for Interpreting Mathematics Microcultures

Author & Focus	**Categories**	**Implications for Practice**
Bauserfeld (1992) *What students produce*	**construction-oriented teaching:** produce students who can construct solutions to novel problems **process-oriented teaching:** produce students who can reproduce teacher's solution procedure	Teachers should strive to produce students who can attack novel problems
Thompson (1994) *Teacher's images of mathematics*	**conceptual orientation:** "images of a *system of ideas* and *ways of thinking* she intends the student to develop*"* (p.6) **calculational orientation:** "image of mathematics as the application of calculations and procedures for deriving numerical results" (p.7)	Teachers should thoughtfully reflect on their understanding of what it means to *do mathematics* and how that understanding influences the ways in which they teach.
Boaler and Greeno (2000) *Who validates knowledge claims*	**discussion-based teaching:** students actively seek to verify knowledge claims **didactic teaching:** teacher validates knowledge claims	Teachers must allow students more opportunities to validate knowledge claims.
Current Study *Nature of discursive objects and connections between them*	**Conceptual basis for discourse:** classroom discourse revolves around mathematical things and the mathematical connections between them **metaphorical basis for discourse:** classroom discourse revolves around metaphorical things and metaphorical connections **procedural basis for discourse:** classroom discourse revolves around procedural objects and procedural conncetions	Teachers should thoughtfully consider what mathematics looks like from the viewpoint of the students.

Appendix D

Table 4

Description of Participants

Pseudonym	*Interview Date*	*Sex*	*Race*	*Valuation*
Janet	*12/11/13*	*F*	*Caucasian*	*identified*
Tori	*12/11/13*	*F*	*Caucasian*	*identified*
Daneille	*12/11/13*	*F*	*African American*	*compliant*
Ellen	*12/16/13*	*F*	*Caucasian*	*compliant*
Adam	*12/17/13*	*M*	*Caucasian*	*identified*
Sarah	*12/17/13*	*F*	*Caucasian*	*compliant*
Laura	*12/18/13*	*F*	*Caucasian*	*compliant*
Rebecca	*12/18/13*	*F*	*Caucasian*	*resistant*

Appendix E

Table 5

Assessing the Quality of the current study. Adapted from (Merriam, 2002).

A. Problem	1. Is the problem appropriate for qualitative inquiry? Is the question one of meaning, understanding, or process? 2. Is the problem clearly stated? 3. Is the problem situated in the literature? That is, is the literature used to put the problem in context? 4. Is the relationship of the problem to the previous research made clear? 5. Is the researcher's perspective and relationship to the problem discussed? Are assumptions and biases revealed? 6. Is a convincing argument explicitly or implicitly made for the importance or significance of this research? Do we know how it will contribute to the knowledge base and practice?
2. Methods	1. Is the particular qualitative research design identified and described? 2. Is the sample selection described? 3. Are the data collection methods described including the rationale for criteria used in selection? 4. How were the data managed and analyzed? 5. What strategies were used to ensure for validity and reliability? 6. What ethical considerations were discussed?
3. Findings	1. Are the participants in the study described? 2. Are the findings clearly organized and easy to follow? 3. Are the findings directly responsive to the problem of the study? That is, do they "answer" the question(s) raised by the study? 4. Do the data presented in support of the findings provide adequate and convincing evidence of the findings?
4. Discussion	1. Are the findings "positioned" and discussed in terms of the literature and previous research? 2. Are the study's insights and contributions to the larger body of knowledge clearly stated and discussed? 3. Are implications for practice discussed? 4. Do the study's implications follow from the data? 5. Are there suggestions for future research?

Appendix F

Table 6

Promoting Validity and Reliability. Adapted from (Merriam, 2002).

Strategy	Description	Notes regarding current study
Triangulation	Using multiple investigators, sources of data, or data collection methods to confirm emerging findings	Multiple data sources including observations, interviews, and written artifacts
Member Checks	Taking data and tentative interpretations back to the people from whom they were derived and asking if they were plausible	Teacher was not used as a member check due to possible bias related to the fact that she is a coworker. Students were not used as a member check due lack of availability during the summer months.
Peer review/examination	Discussions with colleagues regarding the process of study, the congruency of emerging findings with raw data, and tentative interpretations	Discussions included frequent and substantive discussions with a research group consisting of graduate students and dissertation advisor.
Researcher's position and reflexivity	Critical self-reflection by the researcher regarding assumptions, worldview, biases, theoretical orientation, and relationship to the study that may affect investigation	Included in Introduction and Literature Review sections.
Adequate engagement in data collection	Adequate time spent collecting data such that the data become "saturated"; this may involve seeking *discrepant* or *negative* cases of the phenomenon	Observation period spanned the entire semester and included over 15 hours of video footage.

Appendix F

Table 6. Continued.

Promoting Validity and Reliability. Adapted from (Merriam, 2002).

Strategy	Description	Notes regarding current study
Maximum variation	Purposefully seeking variation and diversity in sample selection to allow for greater range of application of findings by consumers of research	With respect to interview participants, the class as a whole was approached several times in an attempt to gain volunteers. The resulting group of interviewees varied along several factors including race, sex, and socioeconomic status.
Audit trail	A detailed account of the methods, procedures, and decision points in carrying out the study	Provided in the Methods section..
Rich, thick description	Providing enough description to contextualize the study such that readers will be able to determine the extent to which their situation matches the research context, and hence, whether findings can be transferred.	Provided in the Methods section.

Appendix G

Table 7

Research Questions in Relation to Data Sources

Research Question	Interview question	Math Autobiography Question	Map of Student Influences	Observation
1a. How is the normative identity as a doer of mathematics constructed and negotiated within the classroom?	B1, B2, B3	Q1, Q2		✓
1b. How do students' personal identities form in relation to the normative classroom identity?	A1, A2, A3, C1, C2, C3	Q3, Q4		✓
2. How do students describe their academic communities both inside and outside of school (peer, family, and school relationships)?	B1, B2, B3		✓	
3. What are the relationships between the academic communities students describe and student identity formation?	C2, C3		✓	

Appendix H

Table 8

Dual-category analysis

	Positive category	**Negative category**	**Macro or micro?**
Ms. Mason's classroom (current study)	**Right kind of people:** *-teacher approval* *-accept structural rationale for school and related future benefits* *-access to several mathematical spaces including peer- and family- delimited* *-reaction and opposition to the wrong kind of people*	**Wrong kind of people:** *-teacher disapproval* *-reject structural rationale for school due to lack of future benefits* *-access to one mathematical space: the classroom* *- acknowledged drug use*	**Micro-level findings:** Description of normative divide within a mathematics classroom
American high school (Eckert, 1989)	**Jocks:** *-upper/middle class* *-reaction and opposition to Burnouts* *-school related competence* *-teacher approval* *-polo shirts and "competitive dressing"*	**Burnouts:** *-working class* *-competence outside school* *-teacher disapproval* *-rock concert t-shirts* *-acknowledged drug use since junior high*	**Macro-level findings:** Description of divide within a school
English high school (Willis, 1977)	**Eroles:** *- upper/middle class* *-school related competence* *-teacher approval*	**Lads:** *- working class* *-reaction and opposition to Eroles* *- competence outside school*	**Macro-level findings:** Description of divide within a school

Appendix I

Table 9.

Components of Categorization (Constas, 1992)

Components of Categorization	*Temporal Designation: A priori*	*Temporal Designation: A posteriori*	*Temporal Designation: Iterative*
Origination: Where does the authority for creating categories reside?			
Participants			*(1*) (3)*
Programs			
Investigative		*(2*)(4)*	
Literature	*(5*) (6*)*		
Interpretive			
Verification: On what grounds can one justify a given category?			
Rational (logic)		*(1*) (2*) (3) (4) (5*) (6*)*	
Referential (literature)	*(1*) (2*) (3) (4) (5*) (6*)*		
External (experts)		*(1*) (2*) (3) (4) (5*) (6*)*	
Empirical (coverage)		*(2 *) (3) (5*) (6*)*	
Technical (methods)		*(1*) (2*) (3) (4) (5*) (6*)*	
Nomination: What is the source of the name used to describe a category?			
Participants		*(3)*	
Programs			
Investigative			*(1*) (2*) (4)*
Literature	*(5*) (6*)*		
<u>Category Label Key:</u> *1. Normative Divide* *2. Basis for Discourse* *3. Backup Teachers & Club Teams*		*4. Opportunities to Identify* *5. General Obligations* *6. Mathematical Obligations*	*<u>Data Source Key</u>* *( ) Interviewing* ** Observation*

Appendix J

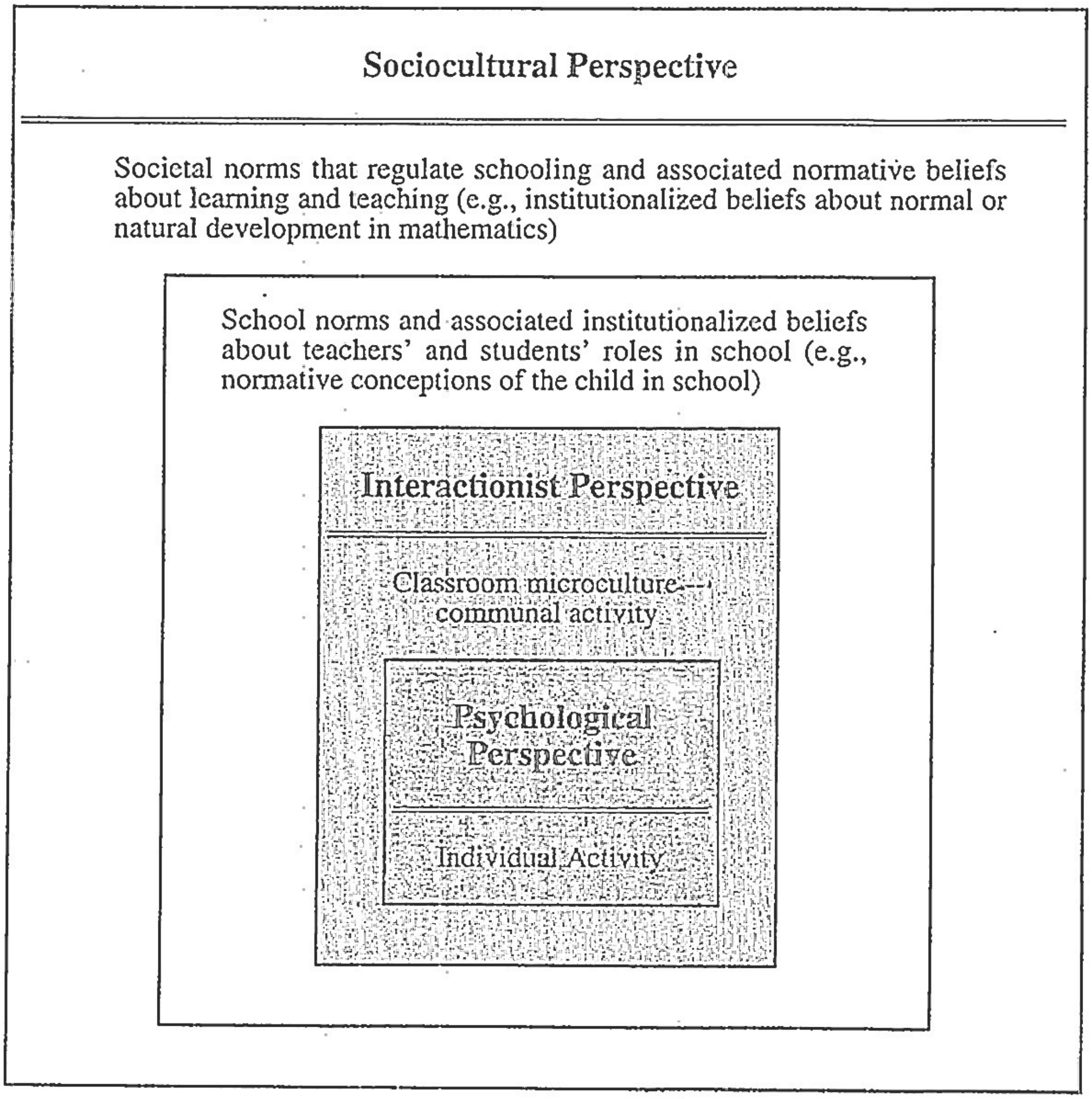


Figure 1. Three presepctives on learning, figure from Cobb and Yackel (1996)

Appendix K

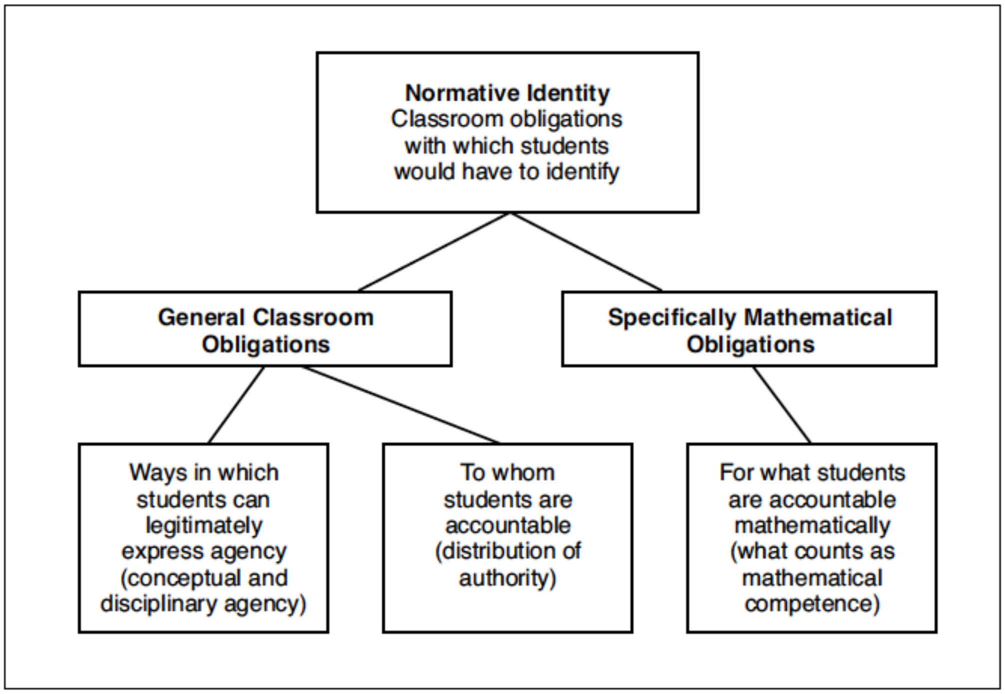


Figure 2. Normative Identity Framework (Cobb, Gresalfi, & Hodge, 2009)

Appendix L

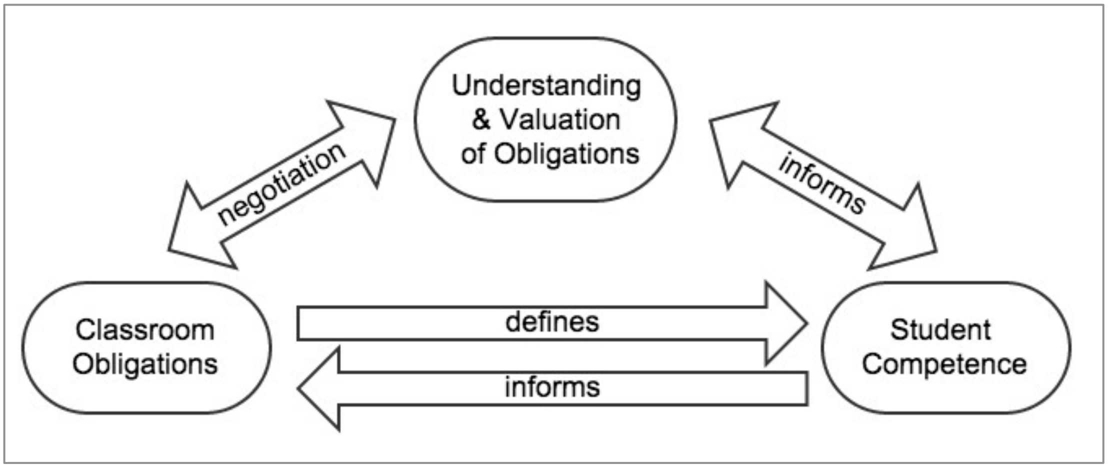


Figure 3. Identification Triangle

Appendix M

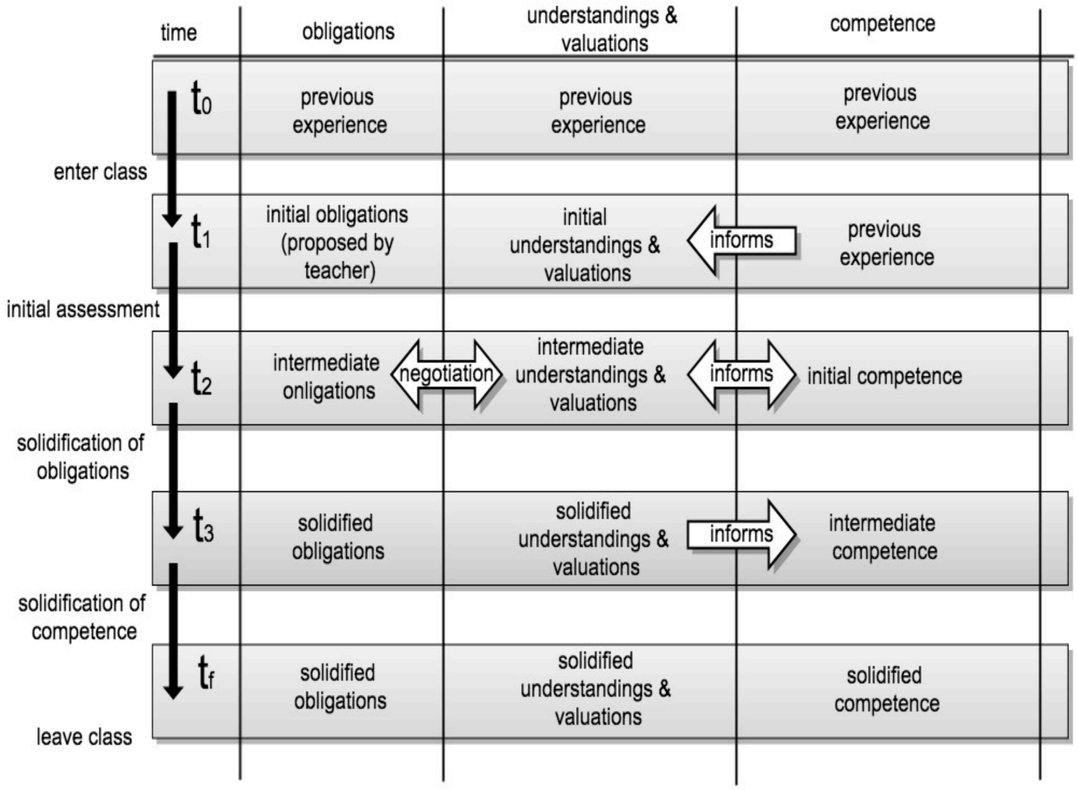


Figure 4. The Process of Identity Formation

Appendix N

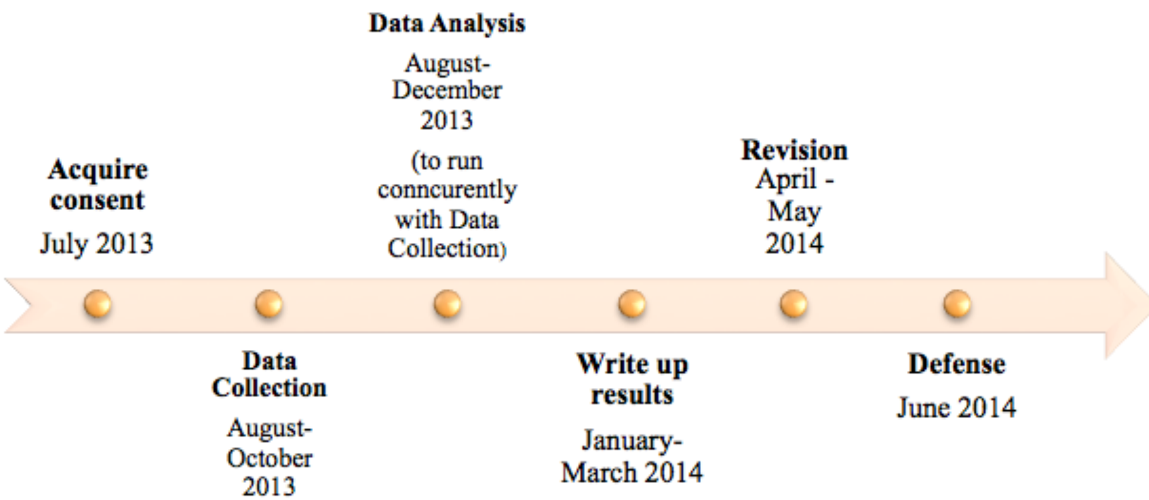


Figure 5. Timeline for current study

Appendix O

Math Autobiography

Please take a moment to tell me a little about yourself and your past experiences in math classes. This autobiography will help me better meet your needs throughout the semester. Thanks!

---

1. Imagine a new student has transferred into your class. You want to tell her about a typical day in math class. Describe in detail what she can expect, from the moment she walks in the door, to the moment the bell rings at the end of class.

2. What does it take to be successful in mathematics (Boaler and Greeno, 2000)?

3. Do you like math? Why or why not?. (Boaler and Greeno, 2000)

4. How does this math class compare to other math classes you've had?

Appendix P

Interview Protocol

INSTRUCTIONS

Good morning (afternoon). My name is Richard Robinson. Thanks for coming. I am looking for students' views on high school mathematics. My research project focuses on the improvement of teaching and learning, with particular interest in understanding how students view their high school math experiences. My goal is not to evaluate you as a student. Rather, I am trying to learn more about teaching and learning, and hopefully learn about how others can do to support student learning.

If it's okay with you, I would like to record our conversations today using this digital audio recorder. The purpose of this is so that I can get all the details but at the same time be able to carry on an attentive conversation with you.

I have planned for this interview to last about an hour. I have several questions that I would like to cover. If time begins to run short, I may need to interrupt you in order to push ahead and complete the interview.

CONSENT FORM INSTRUCTIONS

Before we get started, please take a few minutes to read and sign the release form. As a reminder, only researchers on this project will have access to these recordings which will be destroyed after they are transcribed. In addition, I ask that you sign a form meant to meet our human subject requirements. This document states that: (1) all information will remain confidential, (2) your participation is voluntary and you may stop at any time, and (3) we do not intend to inflict any harm. Thank you so much for agreeing to participate. (TURN DIGITAL RECORDER ON.)

QUESTIONS (Cobb, 2009,2011b) and (Boaler and Greeno, 2000)

A. Competence

1. How good in math are you?

2. If you were to list all the students in your class from the worst to the best in math, where would you put yourself? Why?

3. Some kids are better in one subject than in another. For example, you might be better in math than in English. Compared to most of your other school subjects, how good are you in math?

Competence Probes: How do you know?

B. Understanding of Classroom Obligations

1. Suppose a new student arrives at your school, what do you tell them they have to do to be successful in ***this*** math class?

2. Is what you have to do to be successful in ***this*** class different from other math classes you've had? Why or why not?

3. On a typical day, what do you have to do in this math class?

Understanding Probes: Do you have to do that? Is that your job? What about showing your work?

C. Valuation of Classroom Obligations

1. Would you recommend this class to other students? Why or why not?

2. Do you like mathematics? Why or why not?

3. How important is it for you to be good in math? Why?

Valuation Probes: Is that important to you?

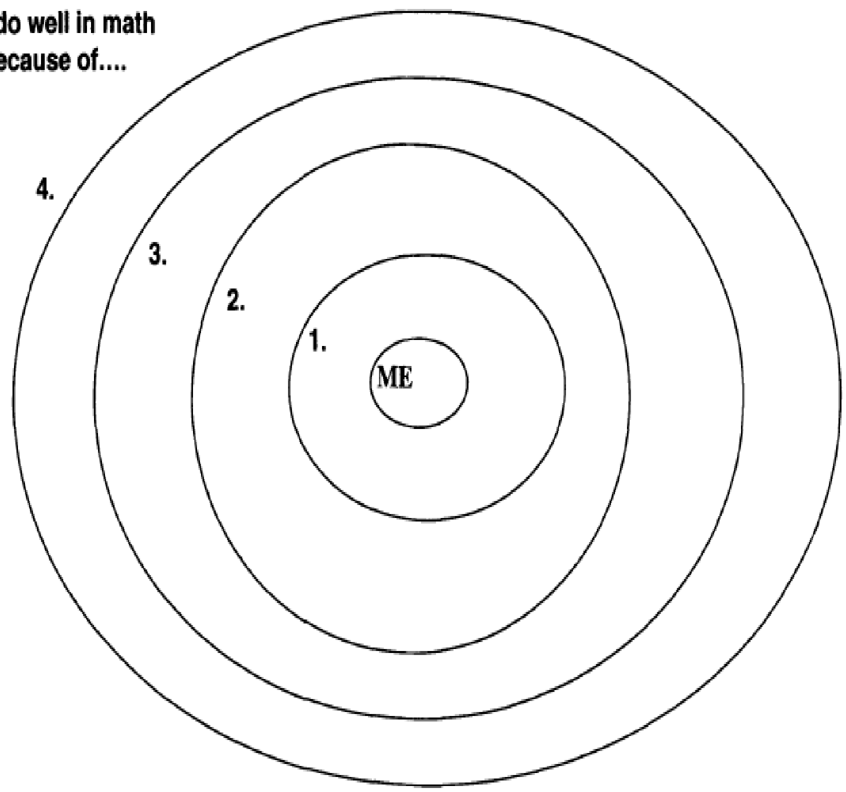
General Probes: Can you tell me more about that? What do you mean? Take me through that experience.

DEBRIEFING:
Again, thank you for coming this morning (afternoon). I really appreciate your time and your comments have been quite helpful. The purpose of this interview is to better understand students' perceptions of their experiences inside and outside of the classroom. I am interested in your opinions and your reactions. This interview is not designed to evaluate you as a student. The results of this research will provide useful information for teachers, in helping them to better support students. You will be kept anonymous during all phases of this study including any writings, published or not.
Is there anything else that you think would be useful for me to know?
Again, thank you for participating. (TURN DIGITAL RECORDER OFF.)

Appendix Q

Student Map of Influences on Mathematical Success

I do well in math because of....



Please label each circle with the reasons you think you do well in mathematics (the circle closest to you would be the reason you think is most important). These reasons can include people, activities, clubs, etc. Include in the circle a short description of how each reason is related to your math success. [If you need additional room, please write on the reverse.]

REASONS FOR MY MATH SUCCESS

1.

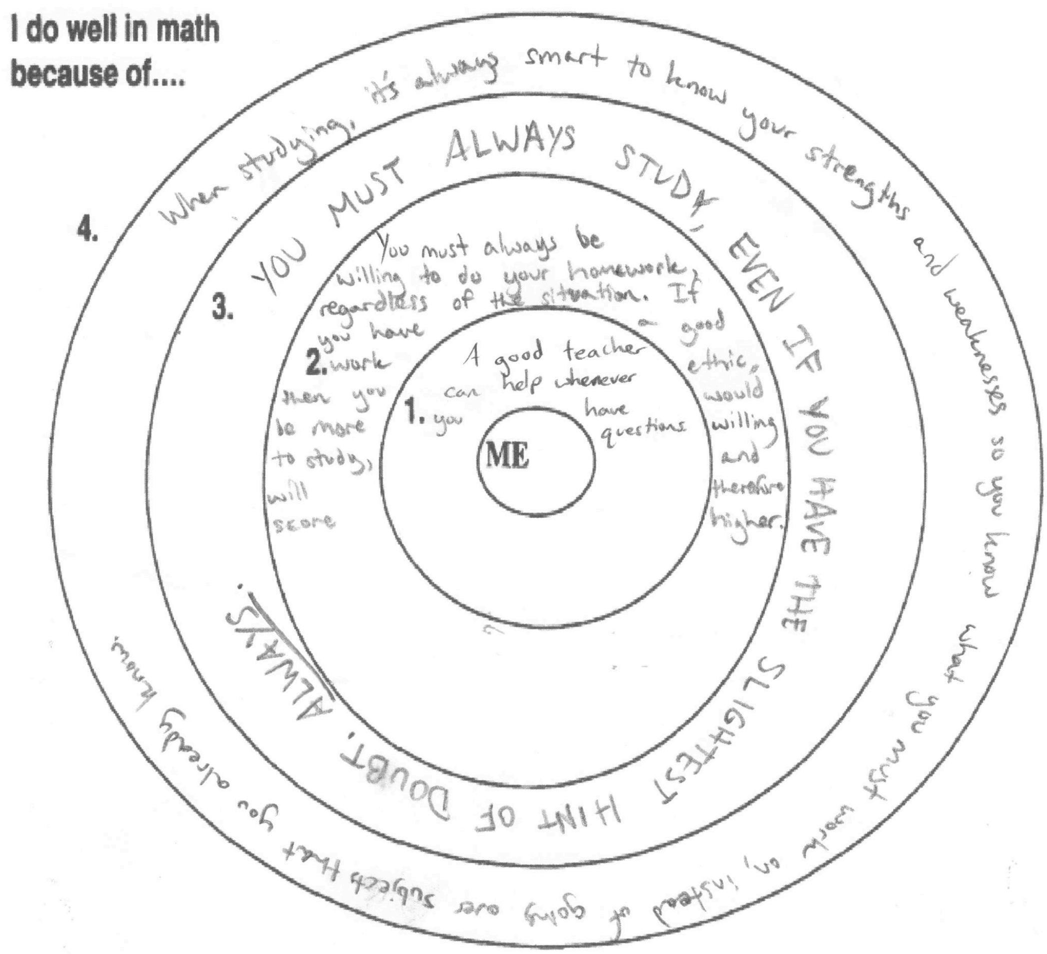
2.

3.

4.

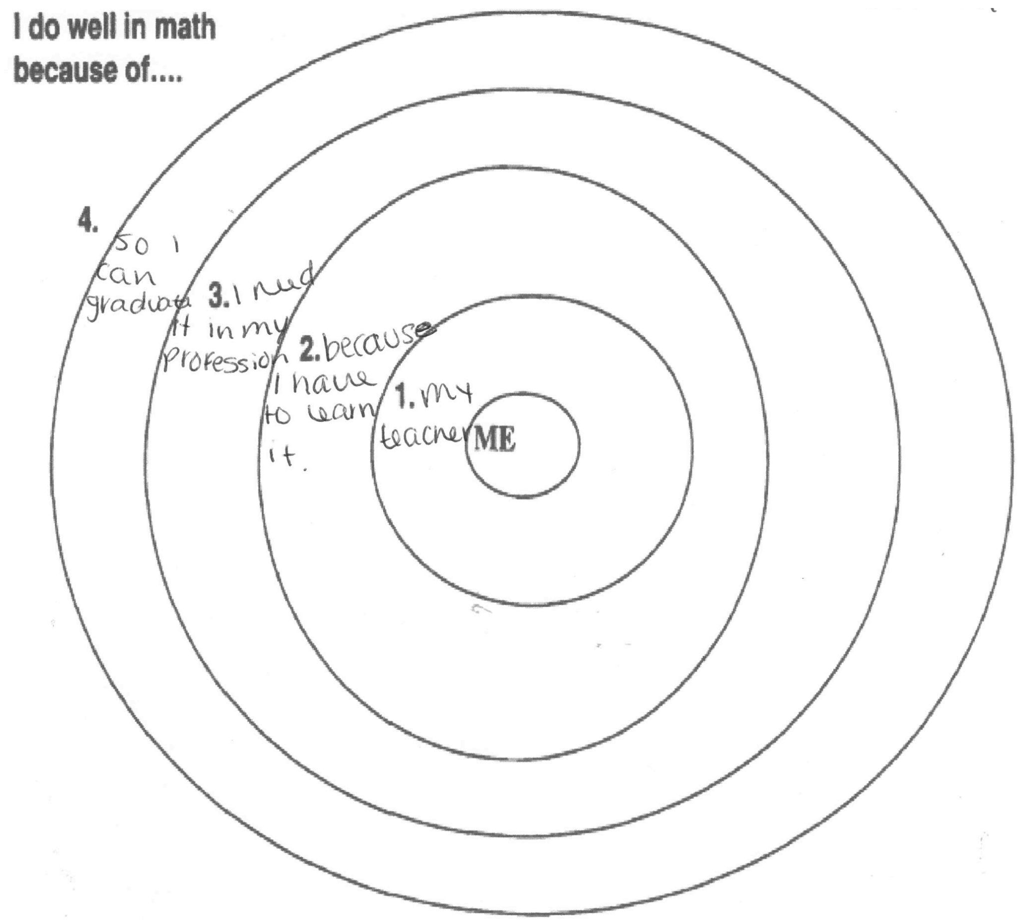
Appendix R

Adam: Student Map of Influences on Mathematical Success



Appendix S

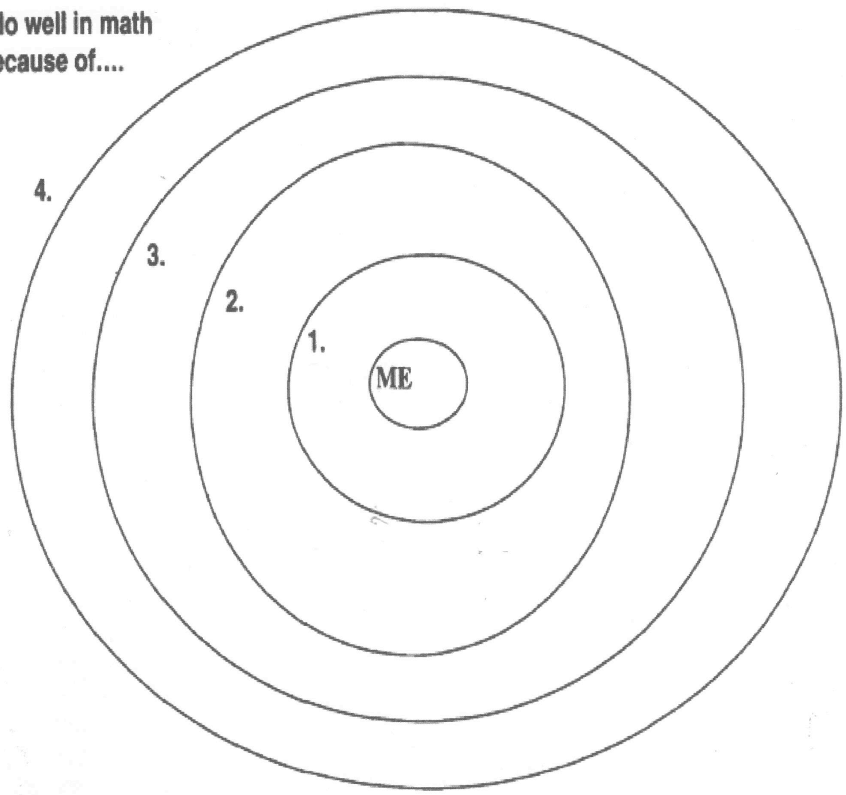
Danielle: Student Map of Influences on Mathematical Success



Appendix T

Tori: Student Map of Influences on Mathematical Success

I do well in math because of....



Please label each circle with the reasons you think you do well in mathematics (the circle closest to you would be the reason you think is most important). These reasons can include people, activities, clubs, etc. Include in the circle a short description of how each reason is related to your math success. [If you need additional room, please write on the reverse.]

REASONS FOR MY MATH SUCCESS

1. My teachers teaching skills
2. My classmates helping me when needed
3. Staying focused in class
4. My parents

Appendix U

Observation Protocol

Date:
Observation #:
Start time:
End time:
Location:
Purpose:

<table>
<tr><td></td><td colspan="2">Diagram of where everyone is in the room:

Setting/context/participants notes:</td></tr>
<tr><td>Time:</td><td>Descriptive notes</td><td>Reflective notes (OC)</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

Vita

Richard Robinson was born and grew up in Kentucky. He earned his Bachelor of Arts in Mathematics with a minor in Economics in 2000 from Murray State University (MSU). After earning a Master of Science in Mathematics from MSU in 2002, he served as a Lecturer in the MSU Department of Mathematics and Statistics for two years. He later earned a second Master of Science in Mathematics at the University of Tennessee at Knoxville in 2006. Richard currently teaches at Bearden High School in Knoxville. Upon acceptance of this dissertation, Richard will have graduated with a Ph.D. in Teacher Education with a concentration in Mathematics Education from the University of Tennessee at Knoxville in 2014.