



University of Tennessee, Knoxville

## TRACE: Tennessee Research and Creative Exchange

---

Doctoral Dissertations

Graduate School

---

5-2014

### Essay on Firm Inventory and Innovation Behavior

Ye Gu

*University of Tennessee - Knoxville*, [ygu1@utk.edu](mailto:ygu1@utk.edu)

Follow this and additional works at: [https://trace.tennessee.edu/utk\\_graddiss](https://trace.tennessee.edu/utk_graddiss)



Part of the [Econometrics Commons](#), and the [Industrial Organization Commons](#)

---

#### Recommended Citation

Gu, Ye, "Essay on Firm Inventory and Innovation Behavior. " PhD diss., University of Tennessee, 2014.  
[https://trace.tennessee.edu/utk\\_graddiss/2696](https://trace.tennessee.edu/utk_graddiss/2696)

This Dissertation is brought to you for free and open access by the Graduate School at TRACE: Tennessee Research and Creative Exchange. It has been accepted for inclusion in Doctoral Dissertations by an authorized administrator of TRACE: Tennessee Research and Creative Exchange. For more information, please contact [trace@utk.edu](mailto:trace@utk.edu).

To the Graduate Council:

I am submitting herewith a dissertation written by Ye Gu entitled "Essay on Firm Inventory and Innovation Behavior." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Economics.

Scott M. Gilpatric, Major Professor

We have read this dissertation and recommend its acceptance:

Marianne H. Wanamaker, Georg Schaur, Adam G. Petrie

Accepted for the Council:

Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

**Essay on Firm Inventory and Innovation Behavior**

**A Dissertation Presented for the  
Doctor of Philosophy  
Degree  
The University of Tennessee, Knoxville**

**Ye Gu  
May 2014**

## **DEDICATION**

To my parents and Jing Su.

## ACKNOWLEDGEMENTS

I would never have been able to finish my dissertation without the help and support from the kind people around me.

I would like to express my sincere gratitude to my advisor, Dr. Scott Gilpatric, for his greatest support, patience and immense knowledge. He provides me great ideas, and his guidance helps me in all the time of research and writing of this dissertation.

I would also like to thank Dr. Marianne Wanamaker for her generous help during my PhD study. I feel extremely lucky to be her research assistant where I learn a lot. The good advice, support and friendship from her have been invaluable on both an academic and a personal level, for which I am extremely grateful.

I owe Dr. Shaur, Dr. Carruthers and Dr. Petrie a great deal for their kindness and willingness to serve in my committee, and their insightful thoughts and suggestions help to improve my dissertation significantly.

I am also grateful to other professors who have instructed me and my special thanks go to Dr. Vossler, the graduate director, for his support and suggestions during my dissertation.

Lastly, I would like to thank my parents, my significant other Jing Su, and many of my friends especially Zhou Yang and Youping Li, who were always supporting me and encouraging me with their best wishes.

## ABSTRACT

This dissertation studies firm's decisions on inventory investment and innovation activities. The first chapter examines firm inventory behavior. It resolves and simulates the production smoothing/buffer stock model using different sets of parameters. It shows that the relationship between a sales shock and inventory investment could be ambiguous which is different from previous predictions. The production smoothing/buffer stock model and the  $(S, s)$  model of inventory are tested using a rich Chinese firm-level dataset covering 769 manufacturing firms from 1980 to 1989, and I find that sales are positively correlated with inventory for raw materials, but negatively correlated with finished goods inventory in most cases. These findings are consistent with the theoretical predictions from the production smoothing/buffer stock model and the  $(S, s)$  model, contradicting previous test results. The second chapter examines the relationship between process innovation and market competition. I find that increased competition will shrink the demand facing each firm, and firms will have less incentive for process innovation whether the innovation outcome is deterministic or stochastic. However, when the number of firms in the market is proportional to the demand and both increase, then increased price elasticity will induce firms to devote more effort to conduct process innovation when innovation is deterministic; and under the stochastic case an inverted-U shape relation between innovation effort and market competitiveness is identified. Furthermore, when the number of firms is endogenous, the innovation incentive grows with the size of the market.

# TABLE OF CONTENTS

INTRODUCTION .....	1
CHAPTER I A Reexamination of Inventory Behavior Using Chinese Firm-Level Data ..	4
Abstract .....	5
1.1 Introduction .....	6
1.2 The model .....	11
1.3 The Data.....	17
1.3.1 Summary Statistics .....	17
1.3.2 The Applicability of the Data.....	19
1.4 Estimation Results .....	20
1.5 Estimating the $(S, s)$ model .....	25
1.6 Concluding Remarks.....	28
References .....	30
Appendix .....	33
CHAPTER II Process Innovation under Competition .....	56
Abstract .....	57
2.1 Introduction .....	57
2.2 The model .....	63
2.2.1 The deterministic innovation model .....	63
2.2.2 The stochastic innovation model .....	73
2.3 Conclusion.....	92
References.....	94
CONCLUSION.....	95
VITA .....	97

## LIST OF TABLES

Table 1 Test for the Effect of a Sales Shock on Inventory .....	21
Table 2 Survey of the Datasets Used to Examine Inventory Behavior.....	47
Table 3 Summary Statistics on Selected Variables .....	49
Table 4 Standard Deviation of Output and Sales by Firm Sizes, Locations and Industries .....	50
Table 5 Test for the Production Smoothing/Buffer Stock model by Different Firm Sizes .....	51
Table 6 Test for the Production Smoothing/Buffer Stock Model by Different Industries	52
Table 7 Test for the Effect of a Sale Shock on the Labor Force.....	53
Table 8 Summary Statistics for the Instrument to the Layoff Cost.....	53
Table 9 Test for the Effect of Layoff Costs in a Sales shock.....	54
Table 10 Test for the $(S, s)$ Model by Different Firm Sizes.....	55
Table 11 Test for the $(S, s)$ Model by Different Industries .....	55



## LIST OF FIGURES

Figure 1 Coefficients of Sales on Inventory When Changing Values of the Average Production Rate $C_4$ .....	23
Figure 2 Changes in GDP, Inventories and Sales in Billions of Current Dollars.....	39
Figure 3 Coefficients When Changing Values of the Layoff Cost $C_2$ .....	40
Figure 4 Coefficients When Changing Values of the Inventory Holding Cost $C_7$ .....	41
Figure 5 Coefficients When Changing Values of the Multiplier for the Optimal Inventory Level $C_9$ .....	42
Figure 6 Coefficients When Changing Values of the Layoff Cost $C_2$ .....	43
Figure 7 Coefficients When Changing Values of the Inventory Holding Cost $C_7$ .....	44
Figure 8 Manufacturing and Trade Inventory.....	45
Figure 9 Residue Plots for Estimating the Production Smoothing/Buffer Stock Model and the $(S, s)$ Model .....	46
Figure 10 Plot of Equation (2.1.7) .....	67
Figure 11 Plot of Equation (2.1.11) .....	69
Figure 12 The Effect of Market Competitiveness on Innovation Effort When the Number of Firms is Proportional to the Size of the Market .....	85
Figure 13 The Effect of Market Competitiveness on the Expected Output of Low Type Firms When the Number of Firms is Proportional to the Size of the Market.....	85
Figure 14 The Effect of Market Competitiveness on the Expected Output of High Type Firms When the Number of Firms is Proportional to the Size of the Market.....	86
Figure 15 The Effect of Market Competitiveness on the Expected Output of All Firms When the Number of Firms is Proportional to the Size of the Market.....	86
Figure 16 The Effect of the Market Size on the Number of Firms in Equilibrium .....	89
Figure 17 The Effect of the Market Size on Innovation Effort When the Number of Firms is Endogenous.....	90
Figure 18 The Effect of the Market Size on the Expected Output of Low Type Firms When the Number of Firms is Endogenous .....	90
Figure 19 The Effect of the Market Size on the Expected Output of High Type Firms When the Number of Firms is Endogenous .....	90
Figure 20 The Effect of the Market Size on the Expected Output of All Firms When the Number of Firms is Endogenous .....	91

# INTRODUCTION

Understanding firm's behavior is very important to both market participants and social planners. In this dissertation, I will study firm's decisions on inventory investment and innovation activities.

In chapter one, I examine inventory behavior using Chinese firm level data. Inventory investment is of great economic significance at both the macro and micro levels. At the macro level, it has been long regarded as a key determinant of business cycles, and macroeconomists often recognize inventory as a destabilizing factor which generates economic cycles that would otherwise not exist. At the micro level, however, inventories are held for various reasons to stabilize a firm's operation. For instance, manufacturers store raw materials to shorten future delivery lags and smooth production; wholesalers and retailers keep sufficient inventory to avoid running out of stock. Many firms devote a significant amount of time and effort to inventory management. Various models emerge to explain inventory behavior, and among them, the production smoothing/buffer stock model prevailed in the early literature. This chapter examines firm inventory behavior. I resolve and simulate the production smoothing/buffer stock model using different sets of parameters, and it is shown that the relation between a sales shock and inventory investment could be ambiguous which differs from predictions in the literature. The production smoothing/buffer stock model and the  $(S, s)$  model of inventory are then tested using a rich Chinese firm-level dataset covering 769 manufacturing firms from 1980 to 1989, and I found that sales are positively correlated with inventory for raw materials, but negatively correlated with finished goods inventory in most cases. These

findings are consistent with the theoretical predictions from the production smoothing/buffer stock model and the (S, s) model, but contradicts many previous empirical findings.

In chapter two, the relationship between innovative activities and market competition is modeled and analyzed. The debate over the effect of increasing market competition on firms' innovation activities has been controversial since Schumpeter (1934). Competition is one of a great many factors that affect a firm's incentive to innovate, such as the market structure, the protection of property rights, the ability to license, uncertainty regarding innovation processes and so on. Market competition can interact with these other factors, but there is no single model that captures every aspect of innovation and competition, and therefore the theoretic literature arrives at no consensus regarding the effects of competition on innovation. Generally, the theoretical literature examines the relation between innovation and competition with the level of competition being simply represented by the number of firms which is assumed to be exogenous. However, in reality the number of firms in a market is determined by market size, scale economies and other factors such as fixed costs and barriers to entry. For instance, as the size of the market grows, typically more firms will enter the market to capture available profits. Thus, I argue an important step in this literature is to allow the number of firms to be endogenous in a free entry model with zero profit-equilibrium. In this chapter, I first examine how competition affects innovation effort when the number of firms in the market is proportional to market demand. In this case, the number of firms grows at the same rate as the size of the market so that the number of customers facing each firm stays

the same. This model allows me to illustrate how competition affects innovation while neutralizing the scale effect. Then I analyze the effect of competition on innovation when there is a fixed cost involved in the production. Thus the number of firms in the market under free entry is endogenously determined by a zero-profit condition. I find that increased competition will shrink the demand facing each firm, and firms will have less incentive for process innovation whether the innovation outcome is deterministic or stochastic. However, when the number of firms in the market is proportional to the demand and they both increase, increased price elasticity will induce firms to devote more effort to conduct process innovation when innovation is deterministic; and in the case of stochastic innovation an inverted-U shape relation between innovation effort and market competitiveness is identified. Furthermore, when the number of firms is endogenous, the innovation incentive grows with the size of the market.

**CHAPTER I**  
**A REEXAMINATION OF INVENTORY BEHAVIOR USING**  
**CHINESE FIRM-LEVEL DATA**

I wish to thank my advisor, Dr. Gilpatric, for enormous help on this. I also owe to Dr. Wanamaker, Dr. Shaur, Dr. Celeste and Dr. Petrie for their insightful comments and generous support. I presented this paper on the brownbag workshop of my department, and I thank all those who provided helpful suggestions.

## **Abstract**

This paper resolves and simulates the production smoothing/buffer stock model using different sets of parameters. I show that the relation between a sales shock and inventory investment could be ambiguous which is different from previous predictions. The production smoothing/buffer stock model and the  $(S, s)$  model of inventory are tested using a rich Chinese firm-level dataset covering 769 manufacturing firms from 1980 to 1989. Two main results are found. First, the variance of the annual gross output is smaller than that of the sales revenue. In particular, small firms in heavy industry show strong evidence of using inventory to smooth production and buffer demand shocks. Second, sales are positively correlated with inventory for raw materials, but negatively correlated with finished goods inventory in most cases. These findings are consistent with the theoretical predictions from the production smoothing/buffer stock model and the  $(S, s)$  model, contradicting previous test results.

## 1.1 Introduction

Inventory investment is of great importance in economic theory at both the macro and micro levels. At the macro level, it has been long regarded as a key determinant of business cycles.<sup>1</sup> Macroeconomists often recognize inventory as a destabilizing factor which generates economic cycles that would otherwise not exist.<sup>2</sup> At the micro level, however, inventories are held for various reasons to stabilize a firm's operation. For instance, manufacturers store raw materials to shorten future delivery lags and to smooth production; wholesalers and retailers store sufficient inventory to avoid running out of stock. Many firms devote a significant amount of time and effort to inventory management.<sup>3</sup> Why does inventory that stabilizes the microeconomic activities turn out to destabilize the economy at the macro level? Various models emerge to explain the inventory behavior, and among them, the production smoothing/buffer stock model<sup>4</sup> prevailed in the early literature.

Holt. et. al. (1960) first introduce this model in which a cost minimizing firm chooses the labor force and output each period to minimize the sum of all future costs. They find that a demand shock will be absorbed by the work force, overtime hours as

---

<sup>1</sup> Blinder and Maccini (1991) show that, on average, the drop in inventory investment has accounted for 87% of the drop in GNP from 1948 to 1982 in the United States.

<sup>2</sup> Figure 2 in the appendix shows the changes in GDP, sales and total inventory in United States.

<sup>3</sup> See Larson, Olson and Sharma (2001), Defle and Van (2011) for discussions on the optimal inventory management.

<sup>4</sup> The production smoothing/buffer stock model is also referred as the L-Q model in the inventory literature because of its functional form is linear-quadratic.

well as inventory fluctuations. Therefore sales revenue is more volatile than output, and it covaries negatively with inventory investment.<sup>5</sup>

However, empirical findings in general are inconsistent with the theoretical predictions. Empirical studies show that production is more variable than sales in most industries. Moreover, sales and inventory investment are generally positively correlated.<sup>6</sup> This contradiction is considered as a fatal problem of the production smoothing/buffer stock model. Many economists attempted to modify the production smoothing/buffer stock model so that the theoretical predictions could match the data. Some modifications include adding cost shocks and targeted inventory levels, and altering the assumption of convex production cost. For example, Ramey (1988) argues that marginal cost is falling instead of rising so that manufacturing firms will bunch rather than smooth production; Fair(1989), Maccini and Rossana (1984) and Blinder (1986a) suggest that cost shocks induce firms to raise production and build up inventory when input costs are low; Blinder (1986b) argues that serial correlation in demand shocks make it inappropriate for firms to smooth production since the shocks seem to be permanent; Kahn (1987) shows that higher expected cost of running out of stock when demand increases makes firms hold more inventory, therefore inventory and sales are positively correlated. Other models also emerged to provide alternative explanations. Among them the  $(S, s)$  model is probably the most popular one. In this model, firms acquire new inventory to the upper limit of  $S$  if

---

<sup>5</sup> Holt. et. al. (1960) discuss these results in chapter 8. Blinder and Maccini (1990) also provide similar predictions solving a simple version of the production smoothing/buffer stock model.

<sup>6</sup> See Blinder (1981, 1986); Blanchard (1983); Haltiwanger and Maccini (1989).



the current inventory falls below its minimum level of  $s$ . If the inventory is anywhere between  $s$  and  $S$ , firms will not place a new order. The predictions from this model conform more closely to the stylized facts.

On the other hand, some studies explain the inconsistency between theory and data by challenging the empirical test itself instead of the theory. For example, Seitz (1993) and Schuh (1996) find that inappropriate aggregation over firms will lead to biased estimated results, and Banerjee and Mizen (2006) argue that inventory and sales are non-stationary series. They show that the closed form solution for the dynamic model could get better forecast than the existing model using the UK and US data.

From the perspective of decision makers in a firm, it is important for them to know how to adjust the inventory level with respect to a demand shock, in order to maximize profits or minimize costs. The original production smoothing/buffer stock model of Holt. et. al. (1960) does provide a very sophisticated discussion on the dynamic responses of inventory to sales fluctuations. However, when solving the model, Holt. et. al. (1960) evaluate the model at only one set of parameters and show that a positive sales shock will reduce the inventory level. In this paper, I simulate the production smoothing/buffer stock model using different sets of parameters, and then find that the managers will adjust the inventory level differently depending on the type of the firm when there is a sales shock. For example, firms whose optimal inventory level is unrelated with its sales are more likely to use inventory as a buffer, while firms holding an optimal inventory level twice as large as its expected sales will build up the inventory and increase the inventory level when there is a positive demand shock.

Moreover, in the production smoothing/buffer stock model, although labor force as one of the decision variables is essential to the inventory problem, it is generally ignored in the empirical estimations in the literature. I will show the importance of including the labor force in the estimation in the section 1.2.

This paper contributes to the literature in a significant way by simulating the production smoothing/buffer stock model with different sets of cost parameters, and then testing it with the labor force included in the estimation. Furthermore, many problems impeding earlier empirical work have been solved using the rich dataset employed in this paper. First, the dataset contains two separate types of inventory, finished goods and raw materials. According to the literature, the production smoothing/buffer stock model is a better fit for the finished goods inventory held by manufacturers which involves convex costs, while the  $(S, s)$  model may better explain the inventory of the raw materials due to the fixed cost of delivery. Estimating inventory for finished goods and raw materials respectively will give us a better understanding of inventory behavior. In the literature, only a few papers have both types of inventory in their datasets, and, even if they have, they have not utilized the data fully to test both the production smoothing/buffer stock model and the  $(S, s)$  model.<sup>7</sup> Second, the dataset is at the firm level; thus, econometric

---

<sup>7</sup> Schuh (1996) mentions in the paper that the dataset contains inventory by state of fabrication, however, the study only concentrates on finished goods inventory; Guariglia(1999) also has separate data on finished goods and raw materials, but the paper focus on examining the effect of financial constraints on the inventory investment.

problems due to aggregation could be avoided.<sup>8</sup> Third, although there are quite a few studies using firm level data, for example, Schuh (1996) uses firm level data in the U.S. manufacturing industry, and Iturriaga (2000) employs a data set from 172 Spanish firms, there is no study analyzing the problem in Chinese firms, despite China's being among the world's leading manufacturer.<sup>9</sup> A summary of the datasets that have been used to examine inventory behavior is provided in the Appendix Table 2.

Some interesting results are found in this paper. Simulation in Section 1.2 shows that the sales shock could be positively correlated with the inventory investment in some circumstances. Using the Chinese State-Owned Enterprise (SOE) dataset, it is shown that the variance of the annual gross output is smaller than that of the sales revenues at the aggregate level. In particular, small firms in heavy industry show strong evidence of using inventory to smooth production and buffer demand shocks. In addition, sales are positively correlated with investment in raw materials in all models, but negatively correlated with finished goods investment in most of the models. These findings are consistent with the predictions from the production smoothing/buffer stock model and the  $(S, s)$  model. Furthermore, the work force is found to be positively correlated with sales as well as inventory. Therefore, when estimating the effect of sales on inventory, omitting

---

<sup>8</sup> Seitz (1993) and Schuh (1996) find that inappropriate aggregation over firms will lead to biased estimated results.

<sup>9</sup> Iturriaga (2000) has a good summary on the countries that were studied in the inventory problem.

the work force will give us a positive bias in the coefficient on the sales.<sup>10</sup> This can possibly explain why most existing empirical papers obtained a positive relationship between sales and inventory which contradicts the predictions of the production smoothing/buffer stock model.

The reminder of the paper is organized as follows. Section 1.2 introduces the original versions of the production smoothing/buffer stock model, followed by the estimation models. Section 1.3 describes the dataset with summary statistics. Section 1.4 estimates the production smoothing/buffer stock model and section 1.5 tests the  $(S, s)$  model. Section 1.6 concludes this paper.

## 1.2 The model

Holt. et. al. (1960) originally introduced the production smoothing/buffer stock model in their work of *Planning Production, Inventories, and Work Force*. The analysis of inventory behavior for finished goods is based on this model, upon which the empirical model is built.<sup>11</sup>

The set up of this model is from the point of view of a production manager. Assuming the sales volume and the market price are beyond his control, the problem for him is not to maximize the profits, but to minimize the sum of all expected future costs

---

<sup>10</sup> The work force is positively correlated with both the sales and the inventory, thus if not controlling for the work force, one unit increase in sales will increase the work force, and the increase in the work force will increase the inventory as well, which will bias up the effect of the sales on the inventory.

<sup>11</sup> Many papers simplified the production smoothing/buffer stock model and get similar predictions, but the empirical results are not consistent with the theory. See Blinder and Maccini (1990) for a literature review. Analyzing the original production smoothing/buffer stock model may allow us a better estimation method.

over a horizon. In every period, the manager has some expectations on the future orders, and then he decides the number of workers he needs and how many to produce. He could hire or fire workers at some costs, and once the worker is employed, regular payroll will be paid. Workers will product the output, and the products are then stocked as inventory. When an order is received by the manager, he needs to choose between fluctuation in the inventory and the production. On one hand, a big fluctuation in the inventory level means that additional storage costs will be involved when inventory is high, and running out of stock may occur if inventory is low. On the other hand, the manager could meet the demand through flexible production. When demand is high, he could hire more workers or asked them to work overtime; and when the demand is low, layoffs or idle time may be involved. Thus, four main costs are involved in planning production and employment, which are regular payroll, hiring and firing costs, overtime costs, and inventory holding and back order costs.<sup>12</sup> The managers will consider and compare all these costs, and the firm's objective is then to minimize the sum of these costs over a horizon. The production smoothing/buffer stock model is:<sup>13</sup>

$$\begin{aligned}
 \min_{\{Y_1, \dots, Y_T\}, \{W_1, \dots, W_T\}} E(C_T) \\
 = E \left( \sum_{t=1}^T [(C_1 - C_6)W_t + C_2(W_t - W_{t-1} - C_{11})^2 + C_3(Y_t - C_4W_t)^2 \right. \\
 \left. + C_5Y_t + C_{12}Y_tW_t + C_7(I_t - C_8 - C_9S_t)^2 + C_{13}] \right) \quad (2-1)
 \end{aligned}$$

---

<sup>12</sup> For more discussions on the costs involved in planning production and employment, see chapter 2 in Holt. et. al. (1960).

<sup>13</sup> Holt. el. al.(1960) state that it is not necessary to place non-negativity restraints on the variables. A negative value, such as negative production, will be undesirable because they are expensive, and these actions will be automatically avoided in cost minimization. Also an interior cost minimum exists if  $C_2, C_3, C_4$  and  $C_7$  are positive and  $0 \leq C_{12} \leq 4C_3C_4$ .

$$\text{Subject to: } Y_t - S_t = I_t - I_{t-1} \quad (2 - 2)$$

In this model, I am minimizing the total cost which is the sum of all the future costs, where  $C_T$  is the total cost,  $C_I$ - $C_{I2}$  are the parameters,  $W_t$  is the work force,  $Y_t$  is the aggregate production,  $I_t$  is the inventory and  $S_t$  is the ordered shipments. Notations are consistent with those in Holt. et. al. (1960) so that comparisons between the solutions could be better discussed.

Among various costs, four main costs are included in the cost function and related to the decision in the production and the labor force as discussed earlier.  $C_1W_t + C_{13}$  is the regular payroll cost which is a linear function, where  $C_1$  is the average wage rate and  $C_{13}$  is some fixed cost component;  $C_2(W_t - W_{t-1} - C_{11})^2$  is the cost of hiring and layoffs which increases with both more or less workers,  $C_{11}$  makes the costs asymmetrical so that it is more general;  $C_3(Y_t - C_4W_t)^2 + C_5Y_t - C_6W_t + C_{12}Y_tW_t$  is the overtime costs, where  $C_4$  is the average production rate, and the more actual production is, the more overtime cost will be for a given size of the work force;<sup>14</sup> and  $C_7(I_t - C_8 - C_9S_t)^2$  is the inventory holding and back order costs, where  $C_8 + C_9S_t$  is the optimal inventory, and the

---

<sup>14</sup> The overtime cost is better measured as a quadratic function: if there are only a few more products needed, then only several workers will be asked to work overtime and the cost is low; but if lots of products are required, then more workers are involved and the overtime costs will increase faster. Although the approximation to the cost is poor when the production is low, the quadratic function is fit in the relevant range as Holt. et. al. (1960) stated. The other three terms  $C_5Y_t$ ,  $C_6W_t$ , and  $C_{12}Y_tW_t$  are for better approximation.

more deviation from the optimal inventory level, the more inventory holding cost will be involved.<sup>15</sup>

In every period, the inventory in the last period will be carried over to the next period, and the net of the production and the sales will be stored as inventory, so the constraint should be held, which is that the inventory in this period  $I_t$  is the inventory in the last period  $I_{t-1}$  plus the production  $Y_t$  and the current sale  $S_t$ .

The first-order conditions for this minimization problem can be obtained by differentiating  $C_T$  with respect to each decision variables, which are  $\{Y_1, \dots, Y_T\}$  and  $\{W_1, \dots, W_T\}$ . Solving a system of the first-order conditions yields  $W_t$  and  $Y_t$ . Using equation (2 – 2), I obtain the decision rules for the inventory:

$$I_t = h_1(C)W_{t-1} + (h_2(C)-1)I_{t-1} + \sum_{i=t}^T h_5(C)S_i + h_4(C) \quad (2 - 3)^{16}$$

Where  $C$  is a vector of the parameters in  $C_T$ .

Equation (2 – 3) indicates that inventory in period  $t$  is a function of the labor force in previous periods, the past inventory, and all future expected sales. Holt. et. al. (1960) evaluate inventory behavior given certain values of the cost parameters, and obtained negative sign on  $S_t$ . They also show that the change in sales will be absorbed by the inventory, work force and overtime hours, so that all three served as buffers and smooth out the production. Therefore, variance in production will be less than that in

---

<sup>15</sup> More discussions on the cost terms could be found in chapter 4 in Holt. et. al. (1960).

<sup>16</sup> See the derivation in the appendix. Equation (2-3) is the same with the equation (21) in the appendix.

sales; and inventory is negatively correlated with sales.<sup>17</sup> This conclusion has been widely used in the inventory literature, and referred as the production smoothing/buffer stock model. However, this result is derived using only one set of parameters, which is not conclusive. It is still an open area for us to determine the effects of sales on inventory robustly.

While it is difficult to obtain an analytical solution regarding the signs on sales, simulation can be used to further the analysis. Taking the values of the parameters presented in Holt. et. al. (1960), I vary one parameter, holding all others the same, and obtain the corresponding coefficients of sales on inventory, that is  $h_5(C)$  in equation(2 – 3). In particular, the cost of hiring and layoffs and inventory holding costs are of high interests, which are related to  $C_2$  and  $C_7$ . In order to examine the importance of the labor force in the regression model, I also obtain the coefficient of the sales on the labor force, which is  $f_3(C)$  in equation (18) in the appendix.

Figure 3 in the appendix shows the corresponding coefficients when varying  $C_2$ , the hiring and layoff costs, between 0.1 and 1. The upper figure is the coefficients of sales on inventory. Controlling for the initial labor force, when the current sales increase, inventory decreases. But when the expected sales in the next period increase, inventory will be built up; and the effects of the third period sales on current inventory are negligible.<sup>18</sup> The lower figure of the panel presents the coefficients of sales on the labor

---

<sup>17</sup> The dynamic response of inventory, labor force and production to sales fluctuations is discussed in Chapter 8 in Holt. et. al. (1960).

<sup>18</sup> The effects of the sales in fourth period or later on inventory are also negligible.



force, and it shows that when current sales go up, the labor force will go up, and thus more goods are produced, and inventory will be built up. From this panel, I also find that as  $C_2$  increase, that is with higher layoff costs, more inventory will be used and workforce adjustment is employed to accommodate the sales.

Figure 4 examines the corresponding coefficients when varying  $C_7$ , the inventory holding costs, between 0.1 and 1. I find that inventory also decrease as current sales increase whatever the inventory holding cost is, but when there is a high inventory holding cost, firm will prefer to hire more workers for production and inventory will not deviate from the optimal level too much.

However, when I vary the values of some cost parameter, different conclusions are obtained. Figure 5 presents how sales shock affects inventory and workers when varying the multiplier for the optimal inventory level from 0 to 1. When the optimal inventory level is more proportional to the sales, firms will tend to build up more inventory to reduce the inventory holding cost, and more workers are hired to produce the required amount. Thus, as the multiplier for the optimal inventory level increases, inventory may switch from decreasing to increasing as the current sales increases. This is a new finding from solving the production smoothing/buffer stock model, and it tells that inventory could be positively correlated with the current sales.

Figures 6 and 7 are produced when changing the value of  $C_9$ , the multiplier for the optimal inventory level, from 0 to 1. From figure 6 I find that inventory increase as current sales increase at every value of  $C_2$ , the hiring and layoff costs. This contradicts the original prediction of the production smoothing/buffer stock model. The optimal

inventory level was unrelated with the sales in the figures 3 and 4, but now is the same size as the sales. Thus, when the sales increases, the inventory will also need to be built up in order to reduce the inventory holding costs. Figure 7 shows that the inventory will be build up at a constant rate whatever the inventory holding costs  $C_7$  is. This may be due to the value of the multiplier for the optimal inventory level  $C_9$  I set up. The cost of deviating optimal inventory level is high enough to ensure the stock of the inventory will follow a pattern.

Based on the discussions above, the effect of a sales shock on inventory investment could be negative or positive depending on the assignment of the cost parameters. In addition, the labor force should be included in the estimation. Omitting the labor force variable will cause serious bias in the results because it is correlated with sales; thus bias the coefficient on the sales upward. I focus on the sales in current and next period since later periods have trivial effects on inventory. The empirical model is thus:

$$I_t = \beta_0 + \beta_1 I_{t-1} + \beta_2 W_{t-1} + \beta_3 S_t + \beta_4 S_{t+1} + \varepsilon_t \quad (2 - 4)$$

## 1.3 The Data

### 1.3.1 Summary Statistics

The dataset employed in this paper comes from a large-scale survey of Chinese State-Owned Enterprise (SOE) from 769 manufacturing firms between 1980 and 1989. Enterprises were sampled from four provinces in China: Shanxi from the north, Jilin in the northeast, Jiangsu in the coastal region, and Sichuan in the southwest. The sample

covers several industries, such as Mining & Utilities, Light Manufacturing, Chemical, Heavy Manufacturing, and others, which represents China's overall industrial structure.

A total of 160 firms were dropped due to incomplete data, of which, the operating year of 43 firms started after 1980 and thus didn't have the complete time span for the analysis. Further, firms with a zero or missing values for the key variables such as production, total inventory, sales revenue of products and total labor force, were dropped as well.

A rich set of variables are available to examine inventory behavior by stage of fabrication: raw materials and finished goods. The summary statistics of relevant variables for analysis are shown in Appendix Table 3.

According to Blinder and Maccini (1991), investment in manufacturers' finished goods inventory is the smallest components of total inventory investment, and raw materials held by manufacturers are the most volatile components of total inventory investment. In this dataset, the mean and variance as well as coefficient of variation (CV) for investment in raw materials are 564,500 RMB, 1,653,280 RMB and 2.93 respectively, compared with that for the finished goods, 137,900 RMB, 390,750 RMB and 2.83, which is consistent with the findings in Blinder and Maccini.

The standard deviation and CV of the annual gross output are smaller than that of the sales revenues of products. This result is contrary to most empirical literature, but more consistent with the prediction of production smoothing/buffer stock model, in which firms use inventory to smooth out production. However, when broken down by industries, sizes and locations, mixed results are presented in table 4. In particular, small

firms in heavy industry show strong evidence of using inventory to smooth out production and buffer demand shocks. One explanation for this is that small firms have lower production capacity and it takes longer time to produce one unit of product in heavy industry. Inventory is expected to be used especially when demand is high.

### **1.3.2 The Applicability of the Data**

Although China was a planned economy after 1949, a series of reforms of SOEs were undertaken beginning in 1978 to motivate managers and workers to be more productive.<sup>19</sup> For example, one of the policies is to allow firms to retain a portion of their profits if they fulfill the targets, or pay penalty when a loss occurs. The reforms stimulate the managers of SOEs to reduce costs in pursuing higher profits.<sup>20</sup> The government also implemented the State-owned Enterprise Cost Management Regulations on March 5<sup>th</sup> of 1984, which provided detailed guidance to induce SOEs to control the cost. It is reasonable to assume that managers of SOEs at this time had strong incentives to minimize some costs, such as the inventory holding cost, machine setup cost, shortage cost, overtime cost, and hiring and firing costs which are the components in the production smoothing/buffer stock model. Furthermore, even if firms did not minimize the regular payroll because managers in SOEs were incentive to pay higher wages or bonuses than would be costing minimizing, this would not be important of our study. All

---

<sup>19</sup> See discussions about SOE reforms in Groves et al.(1994), Lee(1999), Choe and Yin (2000), Ying(2001), Dong and Putterman (2002).

<sup>20</sup> Groves et al.(1994) show that the profit retention rates rises from a mean of 24% in 1980 to 63% in 1989.

that is required for our analysis is cost minimizing behavior regarding production and employment decision conditional on wages.

The SOE managers were also given greater flexibility to decide output and output price. As for the labor force, temporary workers were also employed for production flexibility, and managers are allowed to send the redundant workers home and pay them a very low wage.<sup>21</sup> More importantly, the cost of hiring and firing are involved in the model, so even if the firm did not have great flexibility in hiring and layoffs, the model itself has the ability to respond this constraint.

Based on the discussions above, the production smoothing/buffer stock model could be tested using this dataset.

## 1.4 Estimation Results

According to the equation (2 – 4) from the production smoothing/buffer stock model, inventory and the labor force in the last period, and current period and next period sales are used as the explanatory variables. The effects of the sales after the next period are negligible from the simulation results, so they are not included in the estimation.

Thus, the following model is used to examine inventory behavior on the finished goods.

$$\begin{aligned} \ln inventory_{it} = & \beta_0 + \beta_1 \ln inventory_{i,t-1} + \beta_2 \ln labor_{i,t-1} + \beta_3 \ln sales_{i,t} \\ & + \beta_4 \ln sales_{i,t+1} + v_t + \varepsilon_{it} \end{aligned} \quad (4-1)$$

---

<sup>21</sup> See Groves et al.(1994) and Yin (2001).

Two econometric problems may arise from estimating equation (4 – 1). First, the presence of the lagged dependent variable may give rise to autocorrelation; second, the time-invariant firm characteristics, such as firm size and location, may be correlated with the explanatory variables. To cope with the problems, Arrellano-Bond estimator is used for estimation.

The coefficient on sales is of the highest interest. In particular, I want to see what the sales shock effects will be after controlling for the labor force. The main results are summarized in the table below, which compares the results between models with different control variables. Equation (4 – 1) will also be estimated for all firms as well as by different firm sizes and industries and corresponding results could be found in tables 5 and 6 in the appendix.

**Table 1 Test for the Effect of a Sales Shock on Inventory**

Model	(1)	(2)	(3)
ln(sales)	0.047 (0.0627)	-0.251*** (0.0733)	-0.253*** (0.0721)
Labor force included	N	N	Y
FE	N	Y	Y
Fit	0.68	0.25	0.55
N	1962	1962	1962

*Note: The model fit is calculated using the square of the correlation between the observed and the predicted value.*

The first specification is the basic model, which only uses the sales as the explanatory variable. The production smoothing/buffer stock model gets a positive effect

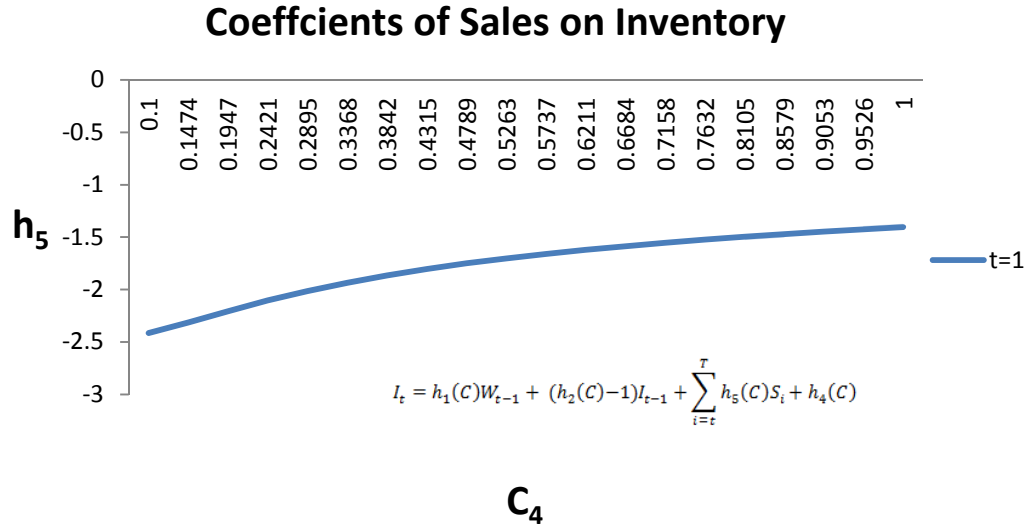
of the sales shock on inventory. Model 2 includes the time fixed effects, and the sales shock effect becomes negative. Some time trends, such as the ever-changing macro economy environment, the political or merchandise policies, will play an important role in the industry, which affect the manufacturers' performances in various circumstances. Thus, ignoring the time fixed effects in the model may cause biased results. Finally, when adding the labor force in the model, a negative effect of the sales shock on the finished goods is obtained for the production smoothing/buffer stock model as well. The coefficient for the model without the labor force is -0.251, which is greater than that for the one with the labor force, -0.253, which confirms that omitting the labor force in the production smoothing/buffer stock model will bias up the effect of the sales shock on the inventory.<sup>22</sup>

Tables 5 and 6 show us the estimation results from equation (4 – 1) by different firm sizes and industries. The coefficients on sales are negative and significant in most of these models, and the one exception is for the “other industries” category. This finding is consistent with the predictions from the production smoothing/buffer stock model; that is, sales are negatively correlated with investment in finished goods in most cases. The coefficient for the other industry is positive but insignificant. That may be due to the mixed properties of the firms. The coefficients on the sales in the next period are all insignificant which confirms that the effects of future sales on inventory investment are negligible. The mining and heavy industries have the largest impact of sales shock on

---

<sup>22</sup> See footnote 10 for more details.

inventory. As we know, the products in the mining and heavy sector are more complicated so that it takes more time to produce when a new order is placed, that is, the average production rate is relatively low. From the simulation changing the parameter of the average production rate  $C_4$ , I find that the lower the average production rate is, the more the inventory will be consumed when there is a positive sales shock, which is consistent with the empirical results for the mining and heavy industries.



**Figure 1 Coefficients of Sales on Inventory When Changing Values of the Average Production Rate  $C_4$**

*Note: This graph shows the coefficients of sales on inventory from testing the production smoothing/buffer stock model from equation (21) when changing values of the parameter of the average production rate  $C_4$ . The horizontal axis is the parameter of the average production rate,  $C_4$ .*



The lagged inventory investment is significantly correlated with inventory investment in all models except testing for the small firms, and thus Arellano-Bond estimator is reasonable for the estimation.

The effect of the labor force on inventory is also of our interest. Holt. et. al. (1960) point out that, when the labor force in the last period increases, total production will rise and inventory will be piled up. Labor can also be used to buffer demand shocks, and hence  $\beta_2$  in the equation (4 – 1) are expected to be positive. The results show the coefficients on the lag of the labor are almost all significantly positive as expected.

Table 7 examines that the effect of sales shock on the labor. Based on the equation (18) from the appendix, the following specification is employed using Arellano-Bond estimation.

$$\ln labor_{it} = \beta_0 + \beta_1 \ln labor_{i,t-1} + \beta_2 \ln inventory_{i,t-1} + \beta_3 \ln sales_{i,t} + \beta_4 \ln sales_{i,t+1} + v_i + \varepsilon_{it} \quad (4 - 2)$$

The labor force and the sales are significantly positively correlated. If not controlling for the work force, one unit increase in sales will increase the work force, and the increase in the work force will increase the inventory as well, which will bias up the effect of the sales on the inventory. Therefore, leaving the labor force in the error term will positively bias the effect of the demand shock on inventory.

Furthermore, from the simulation results in section 1.2, when layoff costs  $C_2$  increases, more inventory is used and less workers are adjusted to accommodate a positive sales shock. Equation (4 – 3) is used to test for this prediction.

$$\ln inventory_{it} = \beta_0 + \beta_1 \ln inventory_{i,t-1} + \beta_2 \ln labor_{i,t-1} + \beta_3 \ln sales_{i,t} + \beta_4 \ln sales_{i,t+1} + \beta_5 \text{layoff cost} * \ln sales_{i,t} + v_t + \varepsilon_{it} \quad (4 - 3)$$

In the data set, there is one categorical variable that represents how much flexibility the manager has to fire a worker. There are total five levels: no flexibility, a little bit, some, quite a bit and flexible. Only 1.18% of the firms have complete flexibility to fire a worker, and 8.42% of them have quite a bit flexibility, then followed by 26.6%, 41.25% and 22.56% (refer to table 8). I used this variable as a proxy to the layoff costs. The more flexibility the manager has to fire a worker, the less layoff cost it will be in this firm.

Since there are five discrete levels for the layoff costs, dummies are generated for each level, and then interact with the sales respectively. If the estimation results are consistent with the prediction, the coefficients on the interaction terms should be smaller for the higher layoff cost. The results in table 9 confirms this hypothesis. All interactions are significant and decreasing in trend as the layoff cost increases.

## 1.5 Estimating the (S, s) model

There are two measures of inventory in general, manufacturing inventory and trade inventory. Trade inventory composes an important part of total inventory, which is even more than manufacturing inventory.<sup>23</sup> The production smoothing/buffer stock model discussed above assumes a firm minimizes the expected costs when planning production and this is a better fit for manufacturing inventory, especially the finished goods they

---

<sup>23</sup> Refer to figure 8 in the appendix.

produce. However, I can't use the same model to examine trade inventory since retailers and wholesalers are not involved in production. Instead, the  $(S, s)$  model is one of the most popular models to explain trade inventory behavior.

Under the  $(S, s)$  model, a representative firm selects its inventory level to minimize the expected value of the sum of discounted costs in all future periods.<sup>24</sup> Unlike the production smoothing/buffer stock model, there are three types of costs involved, the purchasing cost, inventory holding cost and shortage costs. The model predicts that firms order a shipment if the initial inventory falls below a critical value of  $S$ , but won't buy more as long as the inventory stays above the critical value of  $s$ . Scarf (1959) finds that as long as the inventory holding and shortage costs are convex and ordering cost is some fixed costs plus the value of purchased goods, the results will always hold without any additional conditions.

Under the  $(S, s)$  policy, it is intuitive to see that firms will order a shipment and the inventory level will be restored if inventory falls below  $s$  due to a demand shock. As a result, there is a positive relationship between inventory and demand shock. In addition, output (in terms of purchasing) will be more volatile than sales. Most empirical papers found positive results for the  $(S, s)$  model.

The decision to hold raw materials by manufacturers is quite similar to retailers and wholesalers hold finished goods inventory since they all involve a fixed delivery cost. Estimating inventory for finished goods and raw materials respectively will give us

---

<sup>24</sup> Arrow, Harris and Marschak (1951) set up the problem of minimizing discounted costs using the  $(S, s)$  policy but didn't solve it. Scarf (1959) proves that the optimal inventory policy is indeed the  $(S, s)$  form.

a better understanding of inventory behavior. Following the literature, I include the sales and one lag of the inventory in the estimation.<sup>25</sup> The model specification is:

$$\ln inventory_{it} = \beta_0 + \beta_1 inventory_{i,t-1} + \beta_2 \ln sales_{i,t} + v_t + \varepsilon_{it} \quad (5-1)$$

The Arellano-Bond estimator is used for estimation as well, and the coefficient on sales is of the highest interest. The production smoothing/buffer stock model predicts it to be negative in most cases, but the  $(S, s)$  model show it is positive. Tables 10 and 11 provide us the estimation results by different firm sizes and industries. The coefficient on sales are all significantly positive in all models which is consistent with the predictions from the  $(S, s)$  model. The explanation to the opposite signs is intuitive. When there is a positive demand shock, manufacturers will place more orders of raw materials and accelerate production, so investment in raw materials will increase. However, it takes time to produce. If firms couldn't make enough products to meet the demand, inventory will be used as a buffer; hence investment in finished goods will decline.

In order to have a better sense on how well the models fit the data, I examine the model fit and the residue plot for each regression. The model fit is calculated using the square of the correlation between the observed and the predicted value, and reported in the table. The model fits are fairly strong, most of which are higher than 0.5, which indicates that the predicted value has a high correlation with the observed value, and the model fits the data well. I also do residue plot for all regressions and present two

---

<sup>25</sup>See Blinder and Maccini (1990) for a literature review. It will be also interesting to examine the effects of the delivery and purchasing cost on the inventory investment. However, since the cost function is discontinuous, the analytical solution is difficult to derive. I may resort to simulation for future work.

representative plots in the figure 9 in the appendix, which are from testing the production smoothing/buffer stock model and the  $(S, s)$  model using the full data. Residues are distributed randomly around zero against the predicted values for most regressions, and all these analyses support that both models are good fits for the data.

## 1.6 Concluding Remarks

Two theories that explain the behavior of inventory investment are the production smoothing/buffer stock model and the  $(S, s)$  model. The production smoothing/buffer stock model introduced by Holt. et. al. (1960) is a better fit for manufacturing inventory, especially the finished goods. It predicts that sales revenue is more volatile than output, and it covaries negatively with inventory investment. However, empirical findings obtained opposite results: the production is more variable than sales in most industries, and sales and inventory investment are generally positively correlated. Hence, economists began to doubt the applicability of the production smoothing/buffer stock model. By analyzing the original production smoothing/buffer stock model, ambiguous relation between a sales shock and inventory investment are obtained. In addition, the labor force is found to be a crucial component of the model and excluding it from the estimation will lead to biased results. To date, this is the first paper that considers the impact of the labor force in the empirical test of inventory behavior.

Using a rich Chinese firm-level dataset covering 769 manufacturing firms from 1980 to 1989, finished goods inventory is used to test the production smoothing/buffer stock model which involves the rising marginal cost when planning production, while inventory of the raw materials is applied to test the  $(S, s)$  model assuming a fixed delivery

cost. I find that the variance of the annual gross output is smaller than that of the sales revenues of products. In particular, small firms in heavy industry show strong evidence to use inventory to smooth production and buffer demand shocks. Moreover, sales are positively correlated with investment in raw materials, but negatively correlated with finished goods inventory in most cases. This is consistent with the predictions from both the production smoothing/buffer stock model and the  $(S, s)$  model. Furthermore, the labor force is found to be positively related with demand as well as the inventory, which indicates that excluding the labor force in the estimation will cause biased results. This explains why previous studies found contradicting results to the theoretical predictions.

While it is crucial to include the labor force in the estimation, some other variables could also be important in the analysis. For example, some literature studies the effect of cost shocks and financial constraints on inventory investment. If these variables are also correlated with both the inventory and one or more regressors, then omitted variable bias will occur. I would like to explore this further in the future, and include more variables in the estimation to see if any other factor is crucial to be included when analyzing the effect of sales shock on inventory.

## References

- Abramovitz, M.**, 1950, Inventories and Business Cycles, National Bureau of Economic Research.
- Allen, D. S.**, 1997, A multi-sector inventory model, *Journal of Economic Behavior & Organization*, Vol. 32, No. 1, pp. 55-87.
- Arrow, K. J., T. Harris and J. Marschak**, 1951, Optimal Inventory Policy, *Econometrica*, Vol. 19, No. 3, pp. 250-272.
- Banerjee, A. and P. Mizen**, 2006, A Re-interpretation of the Linear-Quadratic Model When Inventories and Sales Are Polynomially Cointegrated, *Journal of Applied Econometrics*, Vol. 21, No. 8, pp. 1249-1264.
- Blinder, A. S.**, 1981, Retail Inventory Behavior and Business Fluctuations, *Brookings Papers on Economic Activity*, Vol. 12, No. 2, pp. 443-520.
- Blanchard, O. J.**, 1983, The Production and Inventory Behavior of the American Automobile Industry, *Journal of Political Economy*, Vol. 91, No. 3, pp. 365-400.
- Blinder, A. S.**, 1986a, More on the Speed of Adjustment in Inventory Models, *Journal of Money, Credit, and Banking*, Vol. 18, No. 3, pp. 355-365.
- Blinder, A. S.**, 1986b, Can the Production Smoothing Model of Inventory Behavior Be Saved? *Quarterly Journal of Economics*, Vol. 101, No.3, pp. 431-454.
- Blinder, A. S., and L. J. Maccini**, 1990, "The Resurgence of Inventory Investment: What Have We Learned?" NBER Working Paper, No. 3408.
- Blinder, A. S. and L. J. Maccini**, 1991, Taking Stock: A critical Assessment on Recent Research on Inventories, *Journal of Economic Perspectives*, Vol 5, No. 1, pp. 73-96.
- Carpenter R., S. Fazzari and B. Petersen**, 1998, Financing Constraints and Inventory Investment: A Comparative Study with High-Frequency Panel Data, *Review of Economics and Statistics*, Vol. 80, No. 4, pp. 513-519.
- Choe, C. and X. Yin**, 2000, Do China's State-Owned Enterprises maximize profit? *The Economic Record*, Vol. 76, No. 234, pp. 273-284.
- Cuthbertson, K. and D. Gasparro**, 1993, the Determinants of Manufacturing Inventories in the UK, *Economic Journal*, Vol. 103, No. 421, pp. 1479-1492.

- Deflem, Y. and I. Van Nieuwenhuyse**, 2011, Optimal Pooling of Inventories with Substitution: A Literature Review, *Review of Business and Economics*, Vol. 56, No. 3, pp.345-374.
- Dong, X. and L. Putterman**, 2002, China's State-Owned Enterprises in the First Reform Decade: An Analysis of a Declining Monopsony, *Economics of Planning*, Vol. 35, No. 2, pp. 109-139.
- Fafchamps, M., J. W. Gunning and R. Oostendorp**, 2000, Inventories and Risks in African Manufacturing, *Economic Journal*, Vol. 110, No. 466, pp. 861-893.
- Fair, R. C.**, 1989, The Production Smoothing Model is Alive and Well, *Journal of Monetary Economics*, Vol. 24, pp. 353-370.
- Hay, D. and H. Louri**, 1994, Investment in Inventories: An Empirical Microeconomic Model of Firm Behavior, *Oxford Economic Papers*, Vol. 46, No. 1, pp. 157-170.
- Iturriaga, F.J.**, 2000, A Panel Data Study on Spanish Firms' Inventory Investment, *Applied Economics*, Vol. 32, No. 15, pp.1927-37.
- Guariglia, A.**, 1999, The Effects of Financial Constraints on Inventory Investment: Evidence from a Panel of UK firms, *Economica*, Vol. 66, pp. 43-62.
- Groves, Hong, McMillan and Naughton**, 1994, Autonomy and Incentives in Chinese State Enterprise, *the Quarterly Journal of Economics*, Vol. 109, No. 1, pp. 183-209.
- Haltiwanger, J. C. and L. J. Maccini**, 1988, A Model of Inventory and Layoff Behavior Under Uncertainty, *Economic Journal*, Vol. 98, No. 392, pp. 731-745.
- Haltiwanger, J. C, and L. J. Maccini**, 1990, The Dynamic Interaction of Inventories, Temporary and Permanent Layoffs, Johns Hopkins University.
- Herrera, A. M., I. Murtazashvili and E. Pesavento**, 2008, The Comovement in Inventories and in Sales: Higher and Higher, *Economics Letters*, Vol. 99, No. 1, pp. 155-158.
- Holt, C.C., F. Modigliani, J. F. Muth, and H. A. Simon**, 1960, Planning Production, Inventories, and Work Force, Englewood Cliffs: Prentice-Hall.
- Larson, C. E., Olson, L. J. and S. Sharma**, 2001, Optimal Inventory Policies When The Demand Distribution Is Not Known, *Journal of Economic Theory*, Vol. 101, No. 1, pp. 281-300.



**Lee Y.**, 1999, Wages and Employment in China's SOEs, 1980-1994: Corporatization, Market Development, and Insider Forces, *Journal of Comparative Economics*, No. 27, pp. 702-729.

**Kahn, J. A.**, 1987, Inventories and the Volatility of Production, *American Economic Review*, Vol. 77, No. 4, pp. 667-679.

**Maccini, L. J., and R. J. Rossana**, 1984, Joint Production, Quasi-Fixed Factors of Production, and Investment in Finished Goods Inventories, *Journal of Money, Credit and Banking*, Vol. 16, No. 2, pp. 218-236.

**McCarthy, J. and E. Zakrajsek**, 1998, Microeconomic Inventory Adjustment and Aggregate Dynamics, Staff Reports 54, Federal Reserve Bank of New York.

**Metzler, L. A.**, 1941, The Nature and Stability of Inventory Cycles, *Review of Economic Statistics*, Vol. 23, No. 3, pp. 113-129.

**Miron. J. A. and S. P. Zeldes**, 1988, Seasonality, Cost Shocks, and the Production Smoothing Model of Inventories, *Econometrica*, Vol. 56, No. 4, pp. 877-908.

**Ramey, V. A.**, 1988, Nonconvex Costs and the Behavior of Inventories, University of California at San Diego.

**Scarf, H.**, 1959, The Optimality of the (S, s) Policies in the Dynamic Inventory Problem, Applied Mathematics and Statistics Laboratory, Stanford university.

**Schuh S.**, 1996, Evidence on the Link between Firm-Level and Aggregate Inventory Behavior, Finance and Economics Discussion Series from Board of Governors of the Federal Reserve System (U.S.)

**Seitz, H.**, 1993, Still More on the Speed of Adjustment in Inventory Models: A Lesson in Aggregation, *Empirical Economics*, Vol. 18, No. 1, pp. 103-127.

**Tsoukalas, J.**, 2006, Financing Constraints and Firm Inventory Investment: A Reexamination, *Economics Letters*, Vol. 90, No. 2, pp. 266-271.

**Yin, X.**, 2001, A Dynamic Analysis of Overstaff in China's State-Owned Enterprises, *Journal of Development Economics*, Vol. 66, No. 1, pp. 87-99.

## Appendix

### Derivation of the decision rule for inventory

The production smoothing/buffer stock model is

$$\begin{aligned}
 \min_{\{Y_1, \dots, Y_T\}, \{W_1, \dots, W_T\}} E(C_T) \\
 = E\left(\sum_{t=1}^T [(C_1 - C_6)W_t + C_2(W_t - W_{t-1} - C_{11})^2 + C_3(Y_t - C_4W_t)^2 \right. \\
 \left. + C_5Y_t + C_{12}Y_tW_t + C_7(I_t - C_8 - C_9S_t)^2 + C_{13}]\right) \quad (1)
 \end{aligned}$$

$$\text{Subject to: } Y_t - S_t = I_t - I_{t-1} \quad (2)$$

where  $C_T$  is the total cost;

$W_t$  is the work force;

$Y_t$  is the aggregate production;

$I_t$  is the inventory;

$S_t$  is the ordered shipments;

$C_1W_t + C_{13}$  is the regular payroll cost;

$C_2(W_t - W_{t-1} - C_{11})^2$  is the cost of hiring and layoffs;

$C_3(Y_t - C_4W_t)^2 + C_5Y_t - C_6W_t + C_{12}Y_tW_t$  is the overtime costs;

$C_7(I_t - (C_8 + C_9S_t))^2$  is the inventory, back order and machine setup costs.

Take partial derivatives of  $E(C_T)$  with respect to  $\{W_1, \dots, W_T\}$  to obtain:

$$\frac{\partial E(C_T)}{\partial W_r} = C_1 - C_6 + 2C_2(W_r - W_{r-1}) - 2C_2(W_{r+1} - W_r) - 2C_3C_4(Y_r - C_4W_r)$$

$$+ C_{12}Y_r$$

$$= C_1 - C_6 + (4C_2 + 2C_3C_4^2)W_r - 2C_2W_{r-1} - 2C_2W_{r+1}$$

$$- (2C_3C_4 - C_{12})Y_r = 0 \text{ where } r = 1, 2, \dots, T-1 \quad (3)$$

$$\frac{\partial E(C_T)}{\partial W_T} = C_1 - C_6 + (2C_2 + 2C_3C_4^2)W_T - 2C_2W_{T-1} - (2C_3C_4 - C_{12})Y_T = 0 \quad (4)$$

Solving Equations (3) & (4) for  $Y_r$  yields:

$$Y_r = \frac{C_1 - C_6 + (4C_2 + 2C_3C_4^2)W_r - 2C_2W_{r-1} - 2C_2W_{r+1}}{2C_3C_4 - C_{12}} \text{ where } r = 1, 2, \dots, T-1 \quad (5)$$

$$Y_T = \frac{C_1 - C_6 + (2C_2 + 2C_3C_4^2)W_T - 2C_2W_{T-1}}{2C_3C_4 - C_{12}} \quad (6)$$

Then taking partial derivatives of  $E(C_T)$  with respect to  $\{Y_1, \dots, Y_T\}$  to find:

$$\begin{aligned} \frac{\partial E(C_T)}{\partial Y_r} &= 2C_3(Y_r - C_4W_r) + C_5 + C_{12}W_r + 2C_7 \sum_{i=r}^T (I_i - C_8 - C_9S_i) \\ &= 2C_3Y_r + (C_{12} - 2C_3C_4)W_r + 2C_7 \sum_{i=r}^T I_i - 2C_7C_9 \sum_{i=r}^T S_i \\ &\quad - 2(T - r + 1)C_7C_8 + C_5 = 0 \text{ where } r = 1, 2, \dots, T \quad (7) \end{aligned}$$

From equation (7) I obtain:

$$\begin{aligned} 2C_7 \sum_{i=r}^T I_i &= -2C_3Y_r - (C_{12} - 2C_3C_4)W_r + 2C_7C_9 \sum_{i=r}^T S_i + 2(T - r + 1)C_7C_8 - C_5 \\ &= 0 \text{ where } r = 1, 2, \dots, T \quad (8) \end{aligned}$$

When  $r=1$ , equation (8) turns to be:

$$2C_7(I_1 + I_2 + \cdots I_T) = -2C_3Y_1 - (C_{12} - 2C_3C_4)W_1 + 2C_7C_9 \sum_{i=1}^T S_i + 2TC_7C_8 - C_5 \quad (9)$$

When  $r=2$ , equation (8) turns to be:

$$2C_7(I_2 + \cdots I_T) = -2C_3Y_2 - (C_{12} - 2C_3C_4)W_2 + 2C_7C_9 \sum_{i=2}^T S_i + 2(T-1)C_7C_8 - C_5 \quad (10)$$

Subtract equation (10) from equation (9) yields :

$$2C_7I_1 = 2C_3(Y_2 - Y_1) + (C_{12} - 2C_3C_4)(W_2 - W_1) + 2C_7C_9S_1 + 2C_7C_8 \quad (11)$$

Similarly I obtain the following equation:

$$2C_7I_r = 2C_3(Y_{r+1} - Y_r) + (C_{12} - 2C_3C_4)(W_{r+1} - W_r) + 2C_7C_9S_r + 2C_7C_8 \quad \text{where } r = 1, 2, \dots, T-1 \quad (12)$$

When  $r=T$ , equation (8) turns to be:

$$2C_7I_T = -2C_3Y_T - (C_{12} - 2C_3C_4)W_T + 2C_7C_9S_T + 2C_7C_8 - C_5 \quad (13)$$

From equation (12) I obtain

$$\begin{aligned} 2C_7(I_r - I_{r-1}) &= 2C_3(Y_{r+1} - 2Y_r + Y_{r-1}) + (C_{12} - 2C_3C_4)(W_{r+1} - 2W_r + W_{r-1}) \\ &+ 2C_7C_9(S_r - S_{r-1}) \quad \text{where } r = 2, \dots, T-1 \quad (14) \end{aligned}$$

Combining equation (2), (11), (12), (13) & (14) yields:

$$\begin{aligned} 2C_7(Y_1 - S_1) &= 2C_3(Y_2 - Y_1) + (C_{12} - 2C_3C_4)(W_2 - W_1) + 2C_7C_9S_1 + 2C_7C_8 \\ &- 2C_7I_0 \quad (15) \end{aligned}$$

$$2C_7(Y_r - S_r) = 2C_3(Y_{r+1} - 2Y_r + Y_{r-1}) + (C_{12} - 2C_3C_4)(W_{r+1} - 2W_r + W_{r-1}) \\ + 2C_7C_9(S_r - S_{r-1}) \text{ where } r = 2, \dots T-1 \quad (16)$$

$$2C_7(Y_T - S_T) = -2C_3(2Y_T - Y_{T-1}) - (C_{12} - 2C_3C_4)(2W_T - W_{T-1}) + 2C_7C_9(S_T \\ - S_{T-1}) - C_5 \quad (17)$$

Substitute equation (5) & (6) into equations (15)-(17) to eliminate  $Y_t$ , obtaining a system of equations:

$$C_{19}W_1 - C_{20}W_2 + C_{17}W_3 = (1 + C_9)S_1 + (C_{15} + C_{17})W_0 + C_8 - \frac{C_{10}}{C_{14}} - I_0 \\ C_{17}W_{r-2} - C_{21}W_{r-1} + C_{22}W_r - C_{21}W_{r+1} + C_{17}W_{r+2} \\ = -C_9S_{r-1} + (1 + C_9)S_1 - \frac{C_{10}}{C_{14}} \text{ where } r = 2, \dots T-2 \\ C_{17}W_{T-3} - C_{21}W_{T-2} + C_{22}W_{T-1} - C_{20}W_T = -C_9S_{T-2} + (1 + C_9)S_{T-1} - \frac{C_{10}}{C_{14}} \\ C_{17}W_{T-2} - C_{21}W_{T-1} + C_{24}W_T = C_9S_{T-1} + (1 + C_9)S_T - \frac{C_{10}}{C_{14}} - \frac{C_5}{2C_7} - \frac{C_3C_{10}}{C_7C_{14}}$$

where  $C_{10} = C_1 - C_6$

$$C_{14} \equiv 2C_3C_4 - C_{12}$$

$$C_{15} \equiv 2C_2/C_{14}$$

$$C_{16} \equiv 2C_3C_4^2/C_{14}$$

$$C_{17} \equiv C_3C_{15}/C_7$$

$$C_{18} \equiv C_3C_{16}/C_7 - C_{14}/2C_7$$

$$C_{19} \equiv C_{16} + C_{18} + 2C_{15} + 3C_{17}$$

$$C_{20} \equiv C_{15} + 3C_{17} + C_{18}$$

$$C_{21} \equiv C_{15} + 4C_{17} + C_{18}$$

$$C_{22} \equiv C_{16} + 2C_{18} + 2C_{15} + 6C_{17}$$

$$C_{23} \equiv C_{16} + 2C_{15}$$

$$C_{24} \equiv C_{16} + 2C_{18} + C_{15} + 3C_{17}$$

This system has T unknown variables and T equations, and I write it in a matrix form

$$\begin{bmatrix} C_{19} & -C_{20} & C_{17} & 0 & \dots & \dots & \dots & \dots & 0 \\ -C_{21} & C_{22} & -C_{21} & C_{17} & 0 & \dots & \dots & \dots & \\ C_{17} & -C_{21} & C_{22} & -C_{21} & C_{17} & 0 & \dots & \dots & \\ 0 & C_{17} & -C_{21} & C_{22} & -C_{21} & C_{17} & 0 & \ddots & \dots \\ & & \dots & & \dots & & \dots & \dots & \dots \\ \vdots & & & & & & & & \\ & & & & & & C_{17} & -C_{21} & C_{22} & -C_{20} \\ & & & & & & C_{17} & -C_{21} & C_{24} & \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ \vdots \\ W_{T-1} \\ W_T \end{bmatrix}$$

$$= \begin{bmatrix} (1 + C_9)S_1 + (C_{15} + C_{17})W_0 + C_8 - \frac{C_{10}}{C_{14}} - I_0 \\ -C_9S_1 + (1 + C_9)S_2 - C_{17}W_0 - \frac{C_{10}}{C_{14}} \\ -C_9S_2 + (1 + C_9)S_3 - \frac{C_{10}}{C_{14}} \\ -C_9S_3 + (1 + C_9)S_4 - \frac{C_{10}}{C_{14}} \\ \vdots \\ -C_9S_{T-2} + (1 + C_9)S_{T-1} - \frac{C_{10}}{C_{14}} \\ C_9S_{T-1} + (1 + C_9)S_T - \frac{C_{10}}{C_{14}} - \frac{C_5}{2C_7} - \frac{C_3C_{10}}{C_7C_{14}} \end{bmatrix}$$

Solving this system of equations I obtain  $W_t$ . Thus, conditional on the initial values of workers  $W_{t-1}$  and inventories  $I_{t-1}$ , I find the following decision rule:

$$W_t = f_1(C)W_{t-1} + f_2(C)I_{t-1} + \sum_{i=t}^T f_3(C)S_i + f_4(C) \quad (18)$$

$$W_{t+1} = f_5(C)W_{t-1} + f_6(C)I_{t-1} + \sum_{i=t}^T f_7(C)S_i + f_8(C) \quad (19)$$

Where  $C$  is a vector of the cost parameters in  $C_T$ , and  $f_i(C)$  is a function that only depends on  $C$ .

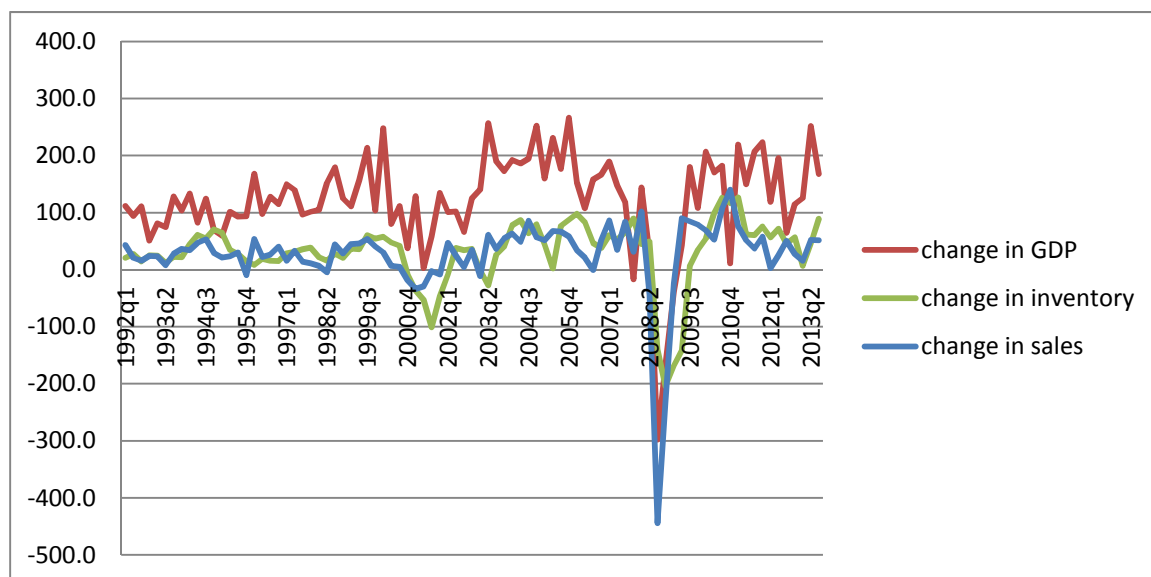
From equation (12), (18) and (19), we could see that  $W_r$  and  $W_{r+1}$  are correlated with  $I_r$  as well as  $S_r$ . Thus, if I examine the effect of sales on inventory without considering the effect of  $W_r$  and  $W_{r+1}$ , in other words, I regress inventory on sales, leaving the labor force in the error term, there will be a bias.

One way to fix this problem is to find instruments for  $W_r$  and  $W_{r+1}$ . From equation (18) and (19), we could see that  $W_{t-1}$  is correlated with  $W_r$  and  $W_{r+1}$ , and also it is predetermined. So it is good to use  $W_{t-1}$  as an instrument for both  $W_r$  and  $W_{r+1}$ . Substitute equation (18) & (19) into equation (5), I obtain

$$Y_t = h_1(C)W_{t-1} + h_2(C)I_{t-1} + \sum_{i=t}^T h_3(C)S_i + h_4(C) \quad (20)$$

Using equation (2), I could get another decision rule for the inventory:

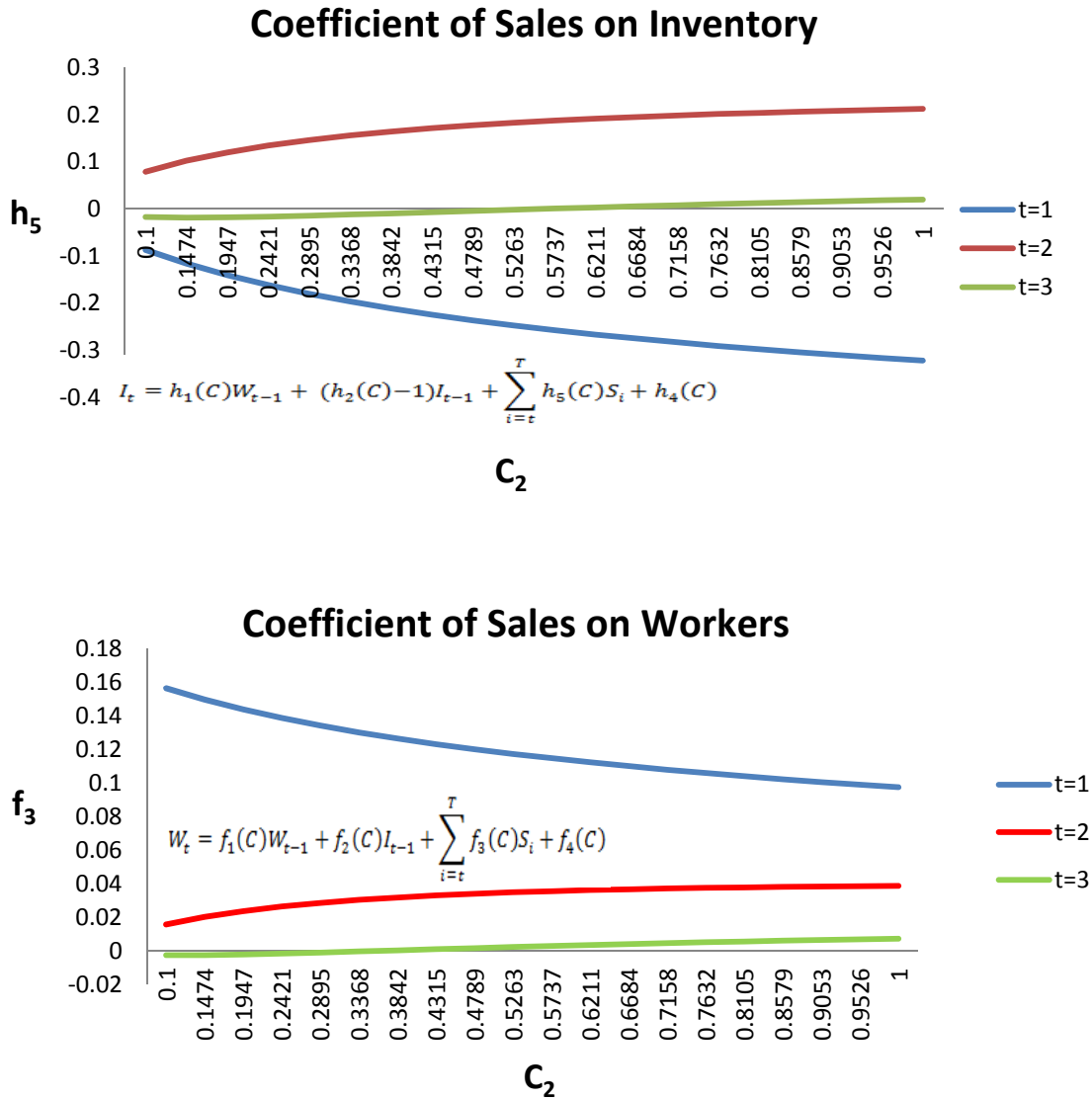
$$I_t = h_1(C)W_{t-1} + (h_2(C)-1)I_{t-1} + \sum_{i=t}^T h_5(C)S_i + h_4(C) \quad (21)$$



**Figure 2 Changes in GDP, Inventories and Sales in Billions of Current Dollars**

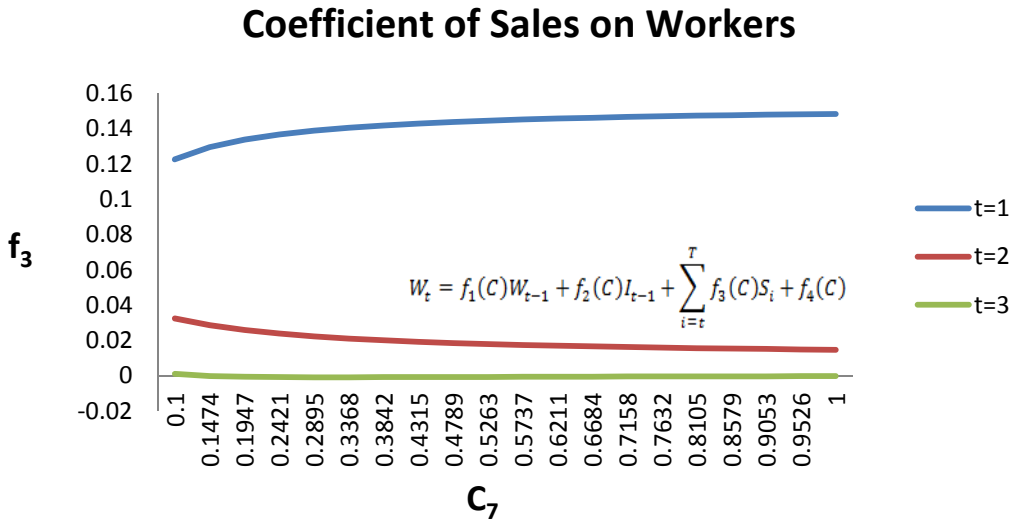
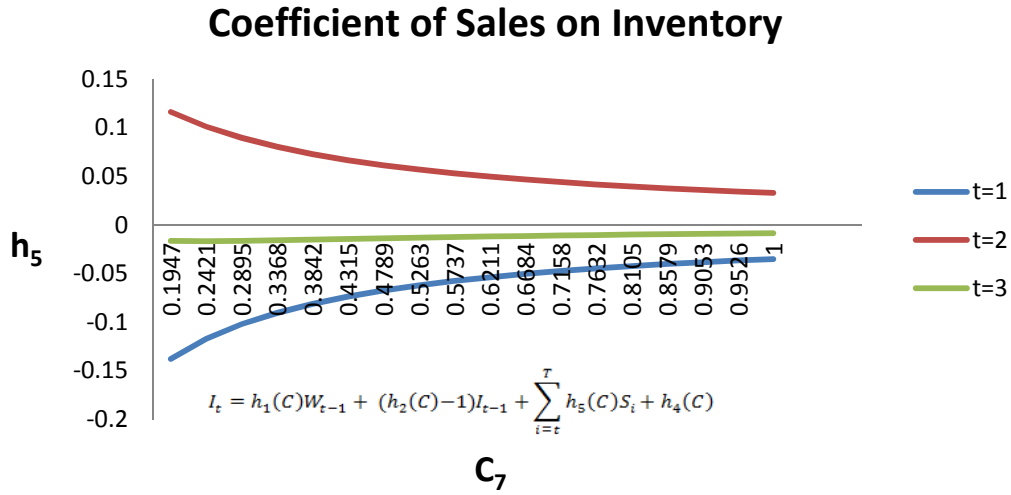
*Note: All data are seasonally adjusted and in billions of current dollars. GDP is obtained from BEA. Inventory data includes manufacturer and trade inventory, and is extracted from U.S. Census Bureau.*





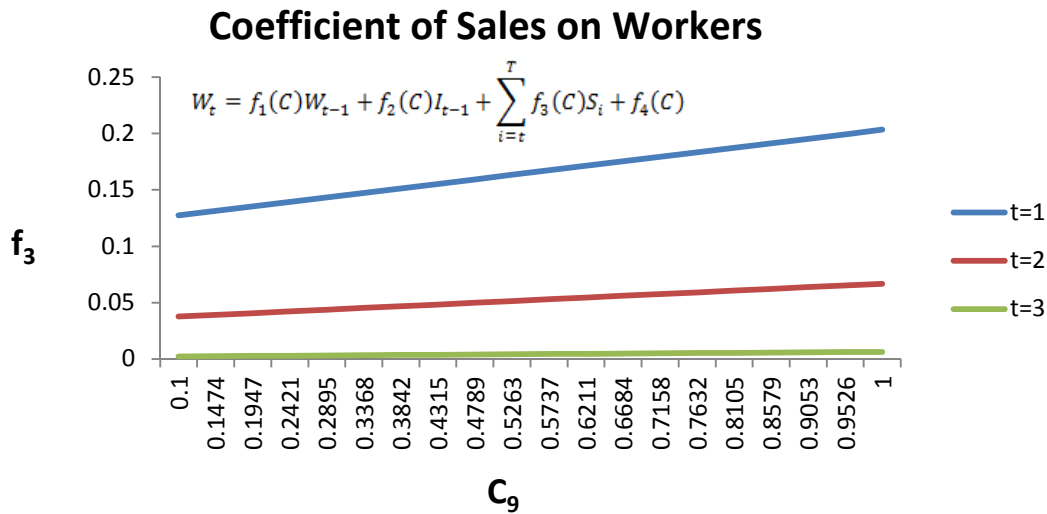
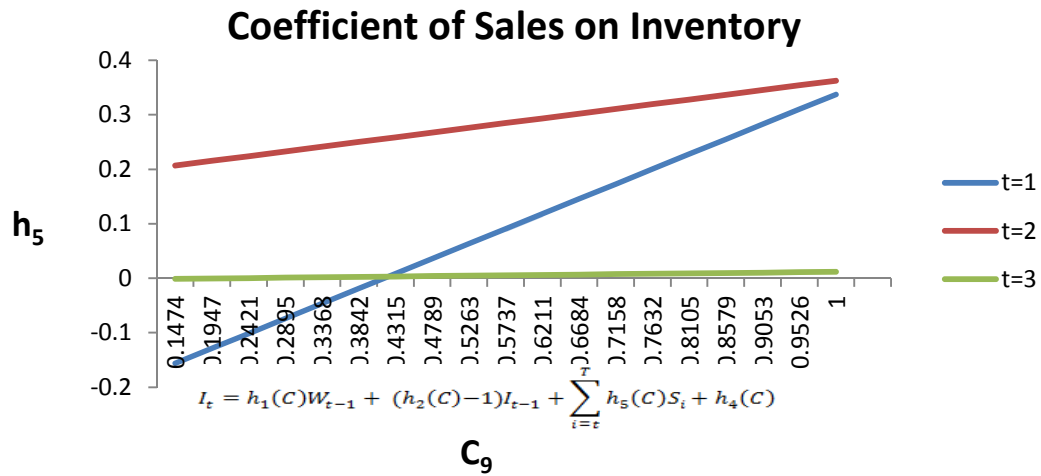
**Figure 3 Coefficients When Changing Values of the Layoff Cost  $C_2$**

*Note: These two graphs show the coefficients from testing the equation (18) and (21) when changing values of the parameter of the layoff cost,  $C_2$ . The horizontal axis is the parameter of the layoff cost,  $C_2$ . The first graph shows the coefficients of sales on inventory from equation (21), and the second graph shows coefficients of sales on workers from equation (18). The blue line is associated with the current sales, the red line with the second period sales, and the green line with the third period sales.*



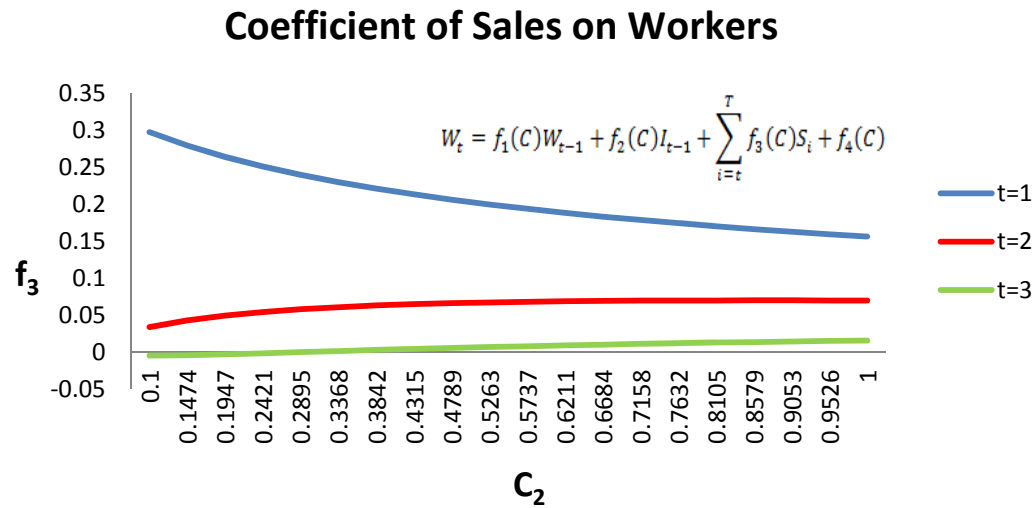
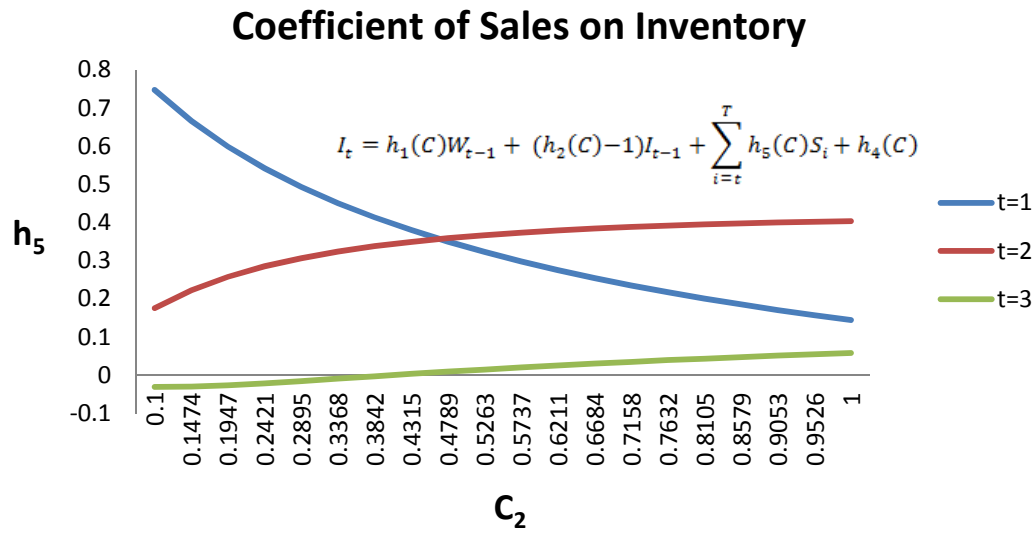
**Figure 4 Coefficients When Changing Values of the Inventory Holding Cost  $C_7$**

*Note: These two graphs show the coefficients from testing the equation (18) and (21) when changing values of the parameter of the inventory holding costs,  $C_7$ . The horizontal axis is the parameter of the inventory holding costs,  $C_7$ . The first graph shows the coefficients of sales on inventory from equation (21), and the second graph shows coefficients of sales on workers from equation (18). The blue line is associated with the current sales, the red line with the second period sales, and the green line with the third period sales.*



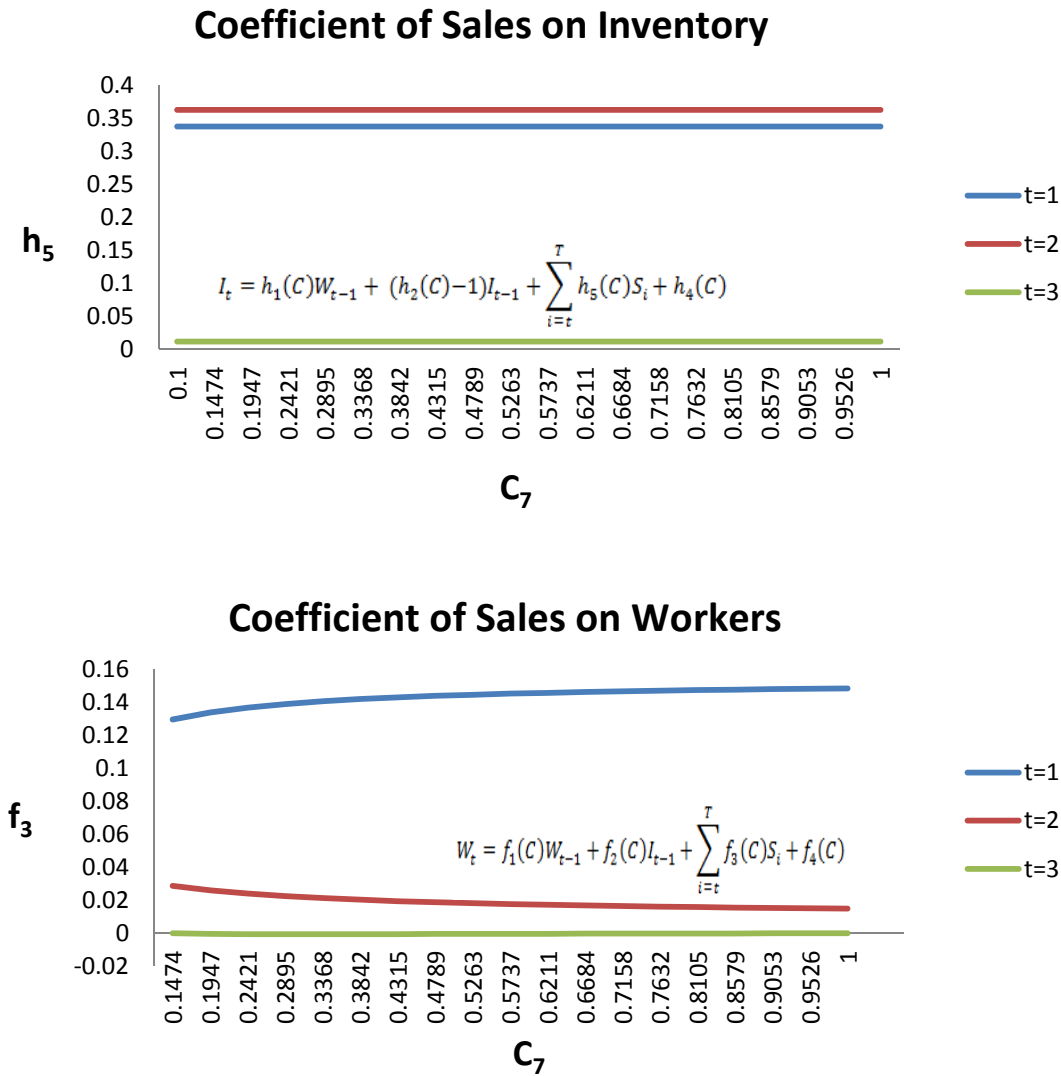
**Figure 5 Coefficients When Changing Values of the Multiplier for the Optimal Inventory Level  $C_9$**

*Note: These two graphs show the coefficients from testing the equation (18) and (21) when changing values of the multiplier for the optimal inventory level,  $C_9$ . The horizontal axis is the parameter of the multiplier for the optimal inventory level,  $C_9$ . The first graph shows the coefficients of sales on inventory from equation (21), and the second graph shows coefficients of sales on workers from equation (18). The blue line is associated with the current sales, the red line with the second period sales, and the green line with the third period sales.*



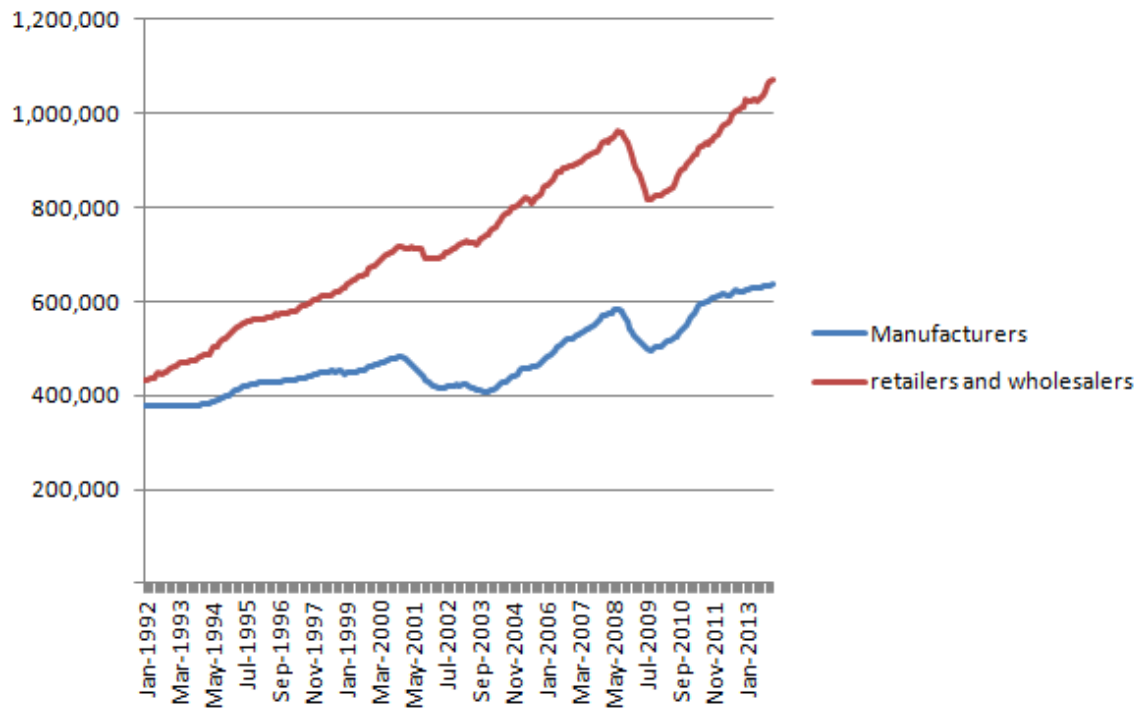
**Figure 6 Coefficients When Changing Values of the Layoff Cost  $C_2$**

*Note: This panel is comparable to the panel 1 expect that the parameter value of  $C_9$  is changed from 0 to 1. These two graphs show the coefficients from testing the equation (18) and (21) when changing values of the parameter of the layoff cost,  $C_2$ . The horizontal axis is the parameter of the layoff cost,  $C_2$ . The first graph shows the coefficients of sales on inventory from equation (21), and the second graph shows coefficients of sales on workers from equation (18). The blue line is associated with the current sales, the red line with the second period sales, and the green line with the third period sales.*



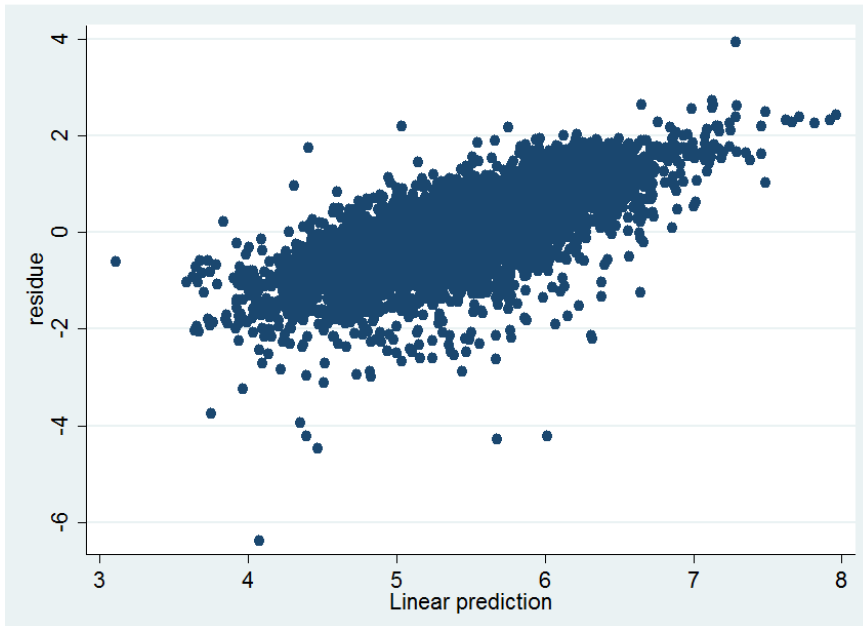
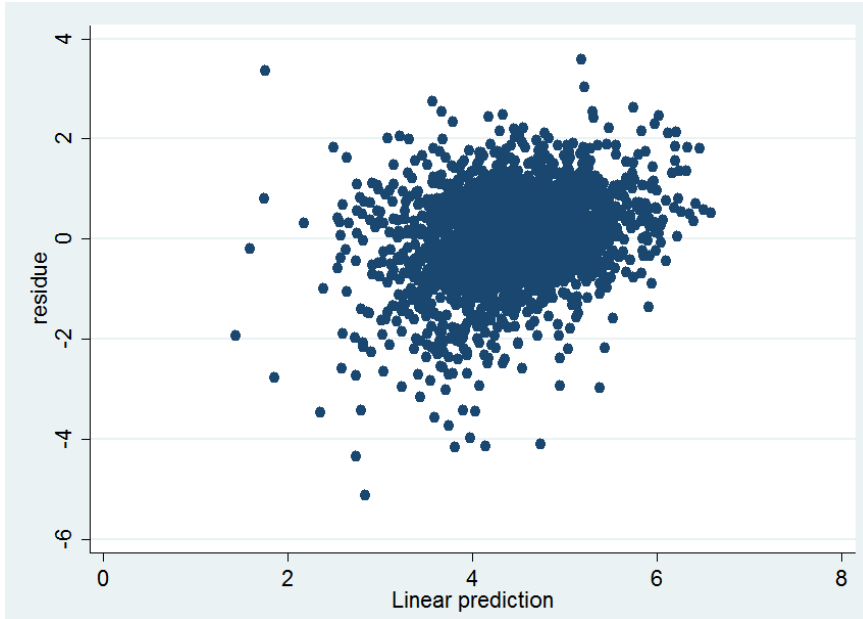
**Figure 7 Coefficients When Changing Values of the Inventory Holding Cost  $C_7$**

*Note: This panel is comparable to the panel 3 expect that the parameter value of  $C_9$  is changed from 0 to 1. These two graphs show the coefficients from testing the equation (18) and (21) when changing values of the parameter of the inventory holding costs,  $C_7$ . The horizontal axis is the parameter of the inventory holding costs,  $C_7$ . The first graph shows the coefficients of sales on inventory from equation (21), and the second graph shows coefficients of sales on workers from equation (18). The blue line is associated with the current sales, the red line with the second period sales, and the green line with the third period sales.*



**Figure 8 Manufacturing and Trade Inventory**

*Note: All data are seasonally adjusted and in millions of current dollars. Obtained from U.S. Census Bureau.*



**Figure 9 Residue Plots for Estimating the Production Smoothing/Buffer Stock Model and the  $(S, s)$  Model**

*Note: The upper figure is the residues against the predicted values from estimating equation (4-1). The lower figure is the residues against the predicted values from estimating equation (5-1).*

**Table 2 Survey of the Datasets Used to Examine Inventory Behavior**

Paper	Data	Variables
Miron and Zelds (1988)	Industrial level data for US firms from May 1967 to December 1982	Inventory, sales, interest rate, tax rate, wages, price of raw materials and energy, capital stock, monthly precipitation and temperature
Cuthbertson and Gasparro (1993)	Aggregated data from 1968:1 to 1989:4 in UK	Inventory and output, capital gearing
Hay and Louri (1994)	Annually data for UK quoted companies in 13 different industrial sector in the period 1960-85	Balance sheet and profit and loss account
Schuh (1996)	Firm-level data in U.S. manufacturing industry from 1985 to 1993.	Sales and inventories by stage-of-fabrication: finished goods, work-in process and raw materials
Allen (1997)	Firm-level data for US firms from 1981:1 to 1991:1	Inventory and sales
Carpenter, Fazzari and Petersen (1998)	Firm-level data for US firms from 1981:3 to 1992:4	Inventory, Sales, cash stock, cash flow and coverage ratio
McCarthy and Zakrajsek (1998)	Firm-level data for manufactures, retails and whole sales trade firms from 1981:4 to 1997:4	Inventory and sales
Guariglia (1999)	Annually data for 994 UK manufacturing firms from 1968 to 1991	Total, work-in-process and raw materials inventories, sales and coverage ratio
Fafchamps, Gunning and Oostendorp (2000)	Firm-level data for Zimbabwean manufacturing firms from 1993 to 1995	Inventories for raw materials, work-in-process and finished goods, sales, contractual risks, variables related liquidity constraints
Iturriaga (2000)	Firm-level data of 172 Spanish firms from July 1990 to December 1995	Total inventories, sales, finished goods variation and material variation
Banerjee and Mizen (2006)	Aggregated data for US and UK from 1982:1 to 2001:2	Inventory and sales



**Table 2. Continued.**

Paper	Data	Variables
Tsoukalas (2006)	Firm-level data for US manufacturers from 1975:1 to 1995:4.	Inventory and cash flow
Herrera, Murtazashvili and Pesavento (2008)	Firm-level data for US firms from January 1959 to March 2000	Inventory by stages of production and sales

**Table 3 Summary Statistics on Selected Variables**

Variable	Mean	Std. Dev.	CV ( Coefficient of Variation)
Annual gross output	3187.10	10314.17	3.24
Sales revenue of products	3152.54	11040.72	3.50
Total inventory	1024.1	2263.28	2.21
Raw materials	564.50	1653.28	2.93
Finished goods	137.90	390.75	2.83
Total number of workers	1902.95	5451.11	2.86

*Note: All variables are in ten thousands of RMB except the total numbers of workers.*

**Table 4 Standard Deviation of Output and Sales by Firm Sizes, Locations and Industries**

	Output (Ten thousands of RMB)	Sales (Ten thousands of RMB)
<b><i>Firm size</i></b>		
Large firm	19891.32	21249.08
Medium firm	3397.01	3311.91
Small firm	1214.09	3497.67
<b><i>Location</i></b>		
Ji Lin(northeast)	15811.28	17380.3
Jiang Su(coastal region)	5763.54	5339.38
Shan Xi(north)	9342.92	9806.05
Si Chuan(southwest)	4243.33	3977.91
<b><i>Industry</i></b>		
Chemical industry	18855.9	19380.82
Heavy industry	4881.414	7258.23
Light industry	4699.36	5642.05
Mining industry	18869.94	18322.43
Other	5994.50	5677.77

**Table 5 Test for the Production Smoothing/Buffer Stock model by Different Firm Sizes**

Variable	All firms	Small firms	Medium firms	Large firms
ln(inventory).L1	0.406*** (0.0544)	0.394*** (0.1101)	0.446*** (0.0603)	0.388*** (0.0889)
ln(sales)	-0.253*** (0.0721)	-0.331** (0.1143)	-0.103 (0.1109)	-0.444*** (0.1177)
ln(sales <sub>n+1</sub> )	-0.095 (0.0653)	-0.146 (0.1134)	-0.116 (0.1170)	0.015 (0.1040)
ln(labors).L1	0.440* (0.2665)	1.007** (0.5035)	0.291 (0.3380)	0.730*** (0.4341)
Fit	0.55	0.45	0.54	0.75
N	1962	496	936	530

*Note: This estimation is based on the equation (4-1). All standard errors are robust and reported in parentheses. The model fit is calculated using the square of the correlation between the observed and the predicted value. \*Significant at 10%. \*\*Significant at 5%. \*\*\*Significant at 1%.*

**Table 6 Test for the Production Smoothing/Buffer Stock Model by Different Industries**

Variable	Mining	Light	Chemical	Heavy	Other
ln(inventory). L1	0.144 (0.1950)	0.387*** (0.0736)	0.198** (0.0903)	0.337*** (0.0832)	0.410*** (0.1097)
ln(sales)	-0.371 (0.3814)	-0.309 (0.2479)	-0.187* (0.1051)	-0.451*** (0.1022)	0.058 (0.0938)
ln(sales <sub>n+1</sub> )	-0.012 (0.3266)	-0.110 (0.1652)	-0.266*** (0.1020)	-0.032 (0.0907)	0.051 (0.1724)
ln(labors).L1	2.332*** (0.9216)	1.044** (0.4261)	1.4255*** (0.5120)	-0.360 (0.3815)	0.119 (0.3519)
Fit	0.59	0.51	0.67	0.02	0.65
N	84	438	374	824	242

*Note: This estimation is based on the equation (4-1). All standard errors are robust and reported in parentheses. The model fit is calculated using the square of the correlation between the observed and the predicted value. \*Significant at 10%. \*\*Significant at 5%. \*\*\*Significant at 1%.*

**Table 7 Test for the Effect of a Sale Shock on the Labor Force**

Variable	All firms	Small firms	Medium firms	Large firms
ln(labor).L1	0.157 (0.0989)	0.227*** (0.0828)	0.144 (0.1533)	0.330*** (0.1017)
ln(inventory).L1	-0.002 (0.0029)	-0.004 (0.0049)	-0.0003 (0.0051)	0.004 (0.0062)
ln(sales)	0.0559*** (0.0101)	0.070*** (0.0214)	0.0673*** (0.0165)	0.035** (0.0149)
ln(sales_n+1)	0.0255*** (0.0077)	0.039*** (0.0123)	0.0042 (0.0156)	0.038** (0.0154)
Fit	0.89	0.88	0.79	0.96
N	1982	504	944	534

*Note: This estimation is based on the equation (4-2). All standard errors are robust and reported in parentheses. The model fit is calculated using the square of the correlation between the observed and the predicted value. \*Significant at 10%. \*\*Significant at 5%. \*\*\*Significant at 1%.*

**Table 8 Summary Statistics for the Instrument to the Layoff Cost**

The flexibility to fire a worker	N	freq
No	1340	22.56
A little bit	2450	41.25
Some	1580	26.60
Quite a bit	500	8.42
flexible	70	1.18

*Note: See more discussion for the instrument used for the layoff cost in the text.*

**Table 9 Test for the Effect of Layoff Costs in a Sales shock**

Variable	All firms
ln(inventory).L1	0.401*** (0.0546)
ln(sales)	-1.165*** (0.3201)
Layoff2*ln(sales)	1.139*** (0.3952)
Layoff3*ln(sales)	1.122*** (0.3320)
Layoff4*ln(sales)	0.732** (0.3301)
Layoff5*ln(sales)	0.836** (0.3403)

*Note: This estimation is based on the equation (4-3). All standard errors are robust and reported in parentheses. \*Significant at 10%. \*\*Significant at 5%. \*\*\*Significant at 1%.*

**Table 10 Test for the (S, s) Model by Different Firm Sizes**

Variable	All firms	Small firms	Medium firms	Large firms
ln(inventory).L1	0.223*** (0.0575)	0.215** (0.0879)	0.346*** (0.0565)	0.303*** (0.105)
ln(sales)	0.227*** (0.0282)	0.213*** (0.0440)	0.302*** (0.043)	0.126** (0.0620)
Fit	0.80	0.68	0.76	0.79
N	4801	1369	2297	1135

*Note: This estimation is based on the equation (5-1). All standard errors are robust and reported in parentheses. The model fit is calculated using the square of the correlation between the observed and the predicted value. \*Significant at 10%. \*\*Significant at 5%. \*\*\*Significant at 1%.*

**Table 11 Test for the (S, s) Model by Different Industries**

Variable	Mining	Light	Chemical	Heavy	Other
ln(inventory). L1	0.360*** (0.0993)	0.276*** (0.0771)	0.259** (0.0890)	-0.0195 (0.0956)	0.316*** (0.0923)
ln(sales)	0.458*** (0.1469)	0.267*** (0.0686)	0.237*** (0.0614)	0.153*** (0.0409)	0.310*** (0.0619)
Fit	0.92	0.77	0.83	0.36	0.84
N	290	1198	877	1862	574

*Note: This estimation is based on the equation (5-1). All standard errors are robust and reported in parentheses. The model fit is calculated using the square of the correlation between the observed and the predicted value. \*Significant at 10%. \*\*Significant at 5%. \*\*\*Significant at 1%.*



## **CHAPTER II**

### **PROCESS INNOVATION UNDER COMPETITION**

## **Abstract**

I analyze the relationship between process innovation and market competition. I find that increased competition will shrink the demand facing each firm, and firms will have less incentive for process innovation whether the innovation outcome is deterministic or stochastic. However, when the number of firms in the market is proportional to the demand and both increase, then increased price elasticity will induce firms to devote more effort to conduct process innovation when innovation is deterministic; and under the stochastic case, an inverted-U shape relation between innovation effort and market competitiveness is identified. Furthermore, when the number of firms is endogenous, innovation incentive grows with the size of the market.

## **2.1 Introduction**

The debate over the effect of increasing market competition on firms' innovation activities has been controversial since Schumpeter (1934). Competition is one of a great many factors that affect a firm's incentive to innovate, such as the market structure, the protection of property rights, the ability to license a patent, and uncertainty regarding innovation processes and so on. Market competition can interact with these other factors, but there is no single model that captures every aspect of innovation and competition, and therefore the theoretic literature arrives at no consensus regarding the effects of competition on innovation.

While there are many models that examine the relation between innovation and competition, two polar views co-exist in the literature. Schumpeter (1934) argues that

monopoly is temporary, so a monopolist has an incentive to keep innovating to prevent the entrance of new firms. Furthermore, innovation is costly and risky, therefore only larger firms have the ability to conduct substantial research and development (R&D). A long line of literature has followed from this argument that less competition in the market provides better breeding grounds for innovation.<sup>26</sup> On the other hand, Arrow (1962) assumes that monopolists have perfect protection of their innovations. He compares the incentives to innovate under both monopoly and competitive markets with a cost-reducing innovation model, and finds that a monopolist has less incentive to innovate compared with firms in a competitive market. This is because monopoly has more pre-innovation profit, and hence there is a negative relationship between innovation and monopoly power.

Following these two classic views, various formal models have emerged to capture the effect of competition on innovation. Dasgupta and Stiglitz (1980b) model a cost-reducing innovation in an oligopoly with  $N$  identical firms, and show that firms will invest less in innovation as the number of firms grows since the output per firm decreases. This is commonly referred as the “scale effect”. Greenstein and Ramey (1998) assume that consumers prefer new products to old ones, and a monopolist could use both new and old products to separate the customers in such a way as to earn more profits than a competitor who could only earn by selling a new product. Unlike Arrow (1962) where a monopolist loses pre-innovation profits as it replaces new products with old ones, in this

---

<sup>26</sup> See Dasgupta and Stiglitz (1980b), Gilbert and Newbery (1982), Greenstein and Ramey (1998).

model a monopolist will gain profits from both the old and new products, higher than profits in a competitive market. Thus, a monopolist will have a strong incentive to conduct innovation.

Generally, the theoretical literature examines the relationship between innovation and competition while representing the level of competition simply as the number of firms which is assumed to be exogenous. However, the number of firms in a market is determined by market size, scale economies and other factors such as fixed costs and barriers to entry. Clearly, as the size of the market grows, more firms will typically enter the market to capture available profits. Thus, I argue an important step in this literature is to allow the number of firms to be endogenous in a free entry model with zero profit-equilibrium. In this paper, I will first examine how competition affects innovation effort when the number of firms in the market is proportional to the demand. In this case, the number of firms grows proportionally with the size of the market so that the number of customers facing each firm stays the same. With this model I illustrate how competition affects innovation when neutralizing the scale effect. Then I analyze the effect of the competition on the innovation when there is a fixed cost involved in the production and thus the number of firms is endogenously determined by the zero-profit condition.

Innovation is often classified into process innovation or product innovation. Process innovation reduces production costs through enhancing the efficiency of production line or management. Product innovation entails developing a new product which may create a new uncontested market, or, an improved version of a product for which consumers have higher willingness to pay. A large part of the literature studies the

effects of competition on innovation while assuming the innovation outcome could be patented or licensed in the model.<sup>27</sup> However, unlike product innovation which is likely patentable, process innovation is likely to be an ongoing process, and very firm-specific. Each firm must exert costly effort to improve their production line and lower its marginal cost.

This cost-reducing innovation process is analogous to some models of managerial efficiency, in which managers exert costly effort to reduce the firm's marginal production cost, and this outcome is firm-specific and nontransferable. Willig (1987) illustrates that increased competition will reduce the output scale of each firm which leaves managers less incentive to put forth effort because the value of reducing the marginal cost of production falls as scale falls. However it will also increase the price elasticity of demand which induces managers to exert more effort. So the total effect is ambiguous. Schmidt (1997) finds ambiguous results as well. But instead of relying on increasing price elasticity, he argues that greater competition will increase the risk of bankruptcy so that managers are forced to work hard to prevent liquidation.

Martin (1993) develops a model to analyze the firm's managerial efficiency in a Cournot market setting. He assumes an inverse linear demand function and the managers exert costly effort to reduce the firm's marginal cost. Like Willig, he finds that increased competition will result in scale and elasticity effects in opposing directions. In his model he finds that the scale effect dominates so that the managerial effort will fall when the

---

<sup>27</sup> Kamien, Oren and Tauman (1992) study a cost-reducing innovation which could be licensed, and then be auctioned or sold at a flat fee.

number of firms increases. I employ a similar model in this paper with some revisions. Firms first exert costly effort by which the marginal cost is determined, and then play a Cournot game. A linear demand function is straightforward and is convenient for modeling the size of the market which is determined by the slope. A firm's marginal cost is an inverse function of innovation effort to represent process innovation as a cost reducing strategy.

Unlike models of managerial effort, I model the possibility that innovation is a stochastic process. A principle-agent (P-A) setting is not considered in this paper because I focus on firm-level incentives. Martin (1993) acknowledges that the P-A framework was not critical to his findings, and Bertolotti and Poletti (1996) reexamine Martin's model and show that the results hold in a complete information model, and decreasing effort with increased competition is driven by increasing return to scale rather than the asymmetric information between owners and managers.

There are two main findings. When an innovation process is deterministic, an exogenous increase in the number of firms has a dominant scale effect, and this increased competition reduces the incentive for process innovation (consistent with Martin's result). However, when the number of firms in the market grows proportionally to the demand, or when the number of firms is endogenously determined in a zero-profit equation, the scale effect is diminished and increased price elasticity dominates. In this case the increased competition resulting from a larger market will induce firms to put more effort into innovation.

If the innovation process is stochastic, somewhat different results are obtained. When the number of firms is exogenous, firms have less incentive for process innovation as before. However, when the number of firms in the market grows proportionally to the demand, I find that there will be an inverted-U shape relation between innovation effort and market competitiveness. It is intuitive to think that a moderate level of competition may best promote innovation. When there is very intense competition, the small scale of firms may lead to little benefit from a new technology, especially for one that reduces costs rather than opens a new market. However, innovation incentive may be dull in very concentrated markets because failure to innovate may not reduce the market share significantly. Most of the literature predicts that the relation between innovation and competition is monotonic with greatest innovation occurring either with monopoly or perfect competition.<sup>28</sup> However, I obtain the result that moderate level of competition produces the strongest incentive for innovation when the process is stochastic and the number of firms is proportional to market size.

Moreover, when the number of firms is endogenous due to the existence of a fixed cost, I show that the number of firms will increase but not proportionally with the size of the market. Thus this creates a stronger incentive to exert effort for innovation under increasing competition in this model than when the number of firms proportionally changes with the size of the market.

---

<sup>28</sup> Kamien and Schwartz (1976) suggest that innovation and competition has an inverted U relation. Aghion et. al.(2005) assumes that there is a spatial sequence structure finds that firms will devote more effort in innovation when more firms enter the market at low levels of competition, but they will lose incentives to innovate at high competition levels

The reminder of the chapter is organized as follows. In Section 2.2, the basic model is set up and analyzed under different assumptions regarding innovation process and the exogeneity or endogeneity of the number of firms. A summary of the results is presented in Section 2.3.

## 2.2 The model

### 2.2.1 The deterministic innovation model

#### 2.2.1.1 *The basic model*

In the model presented below and in the next few sections, I denote  $n$  as the number of firms in the market,  $P$  as market price,  $q$  as each firm's output and  $Q$  as the market output. I also use  $e$  as innovation effort,  $MC$  as firm's marginal cost and  $F$  as fixed cost. I begin by assuming a market with an exogenous determined number of firms,  $n$ . Each firm chooses his own output to maximize profit. The demand function is  $P = 1 - bQ$  where  $b$  represents the size of the market. When  $b$  doubles, the size of the market will be one half as the original size. This simple linear demand function is both trackable and facilitates examining the effect of competition on innovation when the number of firms varies with the size of the market as discussed later.

Firm  $i$  will put costly effort  $e_i$  to conduct process innovation which will reduce the marginal cost  $MC_i$ . Once the firm chooses innovation effort, the marginal cost  $MC$  is constant and inversely related with effort,  $MC = \frac{1}{1+e_i}$  where  $0 \leq e_i < 1$ . The cost for process innovation is  $c(e_i)$ , where  $c' > 0$ ,  $c'' > 0$ , denoting that the return on R&D is decreasing with effort.



The firms' problem is then to choose the output and effort to maximize their profits:

$$\max_{q_i, e_i} \pi_i = q_i \left( 1 - bQ - \frac{1}{1 + e_i} \right) - c(e_i) \quad (2.1.1)$$

The first order conditions are:

$$\frac{\partial \pi_i}{\partial q_i} = 1 - bq_i - bQ_{-i} - \frac{1}{1 + e_i} + q_i * (-b) = 0 \quad (2.1.2)$$

$$\frac{\partial \pi_i}{\partial e_i} = q_i * \frac{1}{(1 + e_i)^2} - c'(e_i) = 0 \quad (2.1.3)$$

Rearrange equation (2.1.2) to obtain

$$q_i = \frac{1 - bQ_{-i} - \frac{1}{1 + e_i}}{2b} \quad (2.1.4)$$

Since the firms are identical, I assume symmetry for  $q_i$ , that is  $q_1 = q_2 = \dots = q_i \dots = q_n$ , thus equation (2.1.4) will be

$$q_i^* = \frac{1 - \frac{1}{1 + e_i}}{2b} - \frac{(n - 1)q_i^*}{2} \quad (2.1.5)$$

Solving for  $q_i^*$  yields:

$$q_i^* = \frac{1 - \frac{1}{1 + e_i}}{(n + 1)b} \quad (2.1.6)$$

Substitute equation (2.1.6) into equation (2.1.3), and rearrange to find

$$(n + 1)bc'(e_i^*) = \frac{e_i^*}{(1 + e_i^*)^3} \quad (2.1.7)$$

To examine the effect of competition on innovation effort, I take the partial derivative of equation (2.1.7) with respect to  $n$  to find:

$$\begin{aligned}
& bc'(e_i^*) + (n+1)bc''(e_i^*) \frac{\partial e_i^*}{\partial n} \\
&= \frac{(1+e_i^*)^3 - e_i^* * 3 * (1+e_i^*)^2}{(1+e_i^*)^6} \frac{\partial e_i^*}{\partial n}
\end{aligned} \tag{2.1.8}$$

Simplifying and rearranging the above equation yields:

$$\left( \frac{1-2e_i^*}{(1+e_i^*)^4} - (n+1)bc''(e_i^*) \right) \frac{\partial e_i^*}{\partial n} = bc'(e_i^*) \tag{2.1.9}$$

From assumption,  $b$  and  $c'(e_i^*)$  are both positive, so the right hand side of equation (2.1.9) is positive. Let  $A = \frac{1-2e_i^*}{(1+e_i^*)^4} - (n+1)bc''(e_i^*)$  which is the terms in the parentheses on the left hand side, the sign of the comparative statistics of  $\frac{\partial e_i^*}{\partial n}$  is the same as the sign of  $A$ .

Since there is  $c''(e_i^*)$  in  $A$ , the Hessian Matrix of the second order condition for profit maximization will be evaluated.

$$\text{The hession matrix } H = \begin{vmatrix} \frac{\partial \pi^2}{\partial q^2} & \frac{\partial \pi^2}{\partial q \partial e} \\ \frac{\partial \pi^2}{\partial e \partial q} & \frac{\partial \pi^2}{\partial e^2} \end{vmatrix} \text{ must be negative definite for a unique}$$

max to exist, where

$$\frac{\partial \pi^2}{\partial q^2} = -b - b = -2b$$

$$\frac{\partial \pi^2}{\partial q \partial e} = \frac{1}{(1+e_i^*)^2}$$

$$\frac{\partial \pi^2}{\partial e \partial q} = \frac{1}{(1+e_i^*)^2}$$

$$\frac{\partial \pi^2}{\partial e^2} = -2q^* \frac{1}{(1+e_i^*)^3} - c''(e_i^*)$$

Thus,  $\frac{\partial \pi^2}{\partial q^2} * \frac{\partial \pi^2}{\partial e^2} - \frac{\partial \pi^2}{\partial q \partial e} * \frac{\partial \pi^2}{\partial e \partial q} = 2b(2q^* \frac{1}{(1+e_i^*)^3} + c''(e_i^*)) - \frac{1}{(1+e_i^*)^4}$  must be positive to ensure the hessian matrix is negative definite.

From  $2b(2q^* \frac{1}{(1+e_i^*)^3} + c''(e_i^*)) - \frac{1}{(1+e_i^*)^4} > 0$ , I identify an inequality condition for  $c''(e_i^*)$

$$c''(e_i^*) > \frac{1}{2b(1+e_i^*)^4} - \frac{2q^*}{(1+e_i^*)^3} \quad (2.1.10)$$

Substitute equation (2.1.10) into A to find:

$$A < \frac{1-2e_i^*}{(1+e_i^*)^4} - (n+1)b \left( \frac{1}{2b(1+e_i^*)^4} - \frac{2q^*}{(1+e_i^*)^3} \right)$$

Substitute equation (2.1.6) into the right hand side of the above equation to obtain:

$$A < \frac{1-2e_i^*}{(1+e_i^*)^4} - \frac{n+1}{2(1+e_i^*)^4} + \frac{2(1-\frac{1}{1+e_i^*})}{(1+e_i^*)^3}$$

Simplifying the above equation yields:

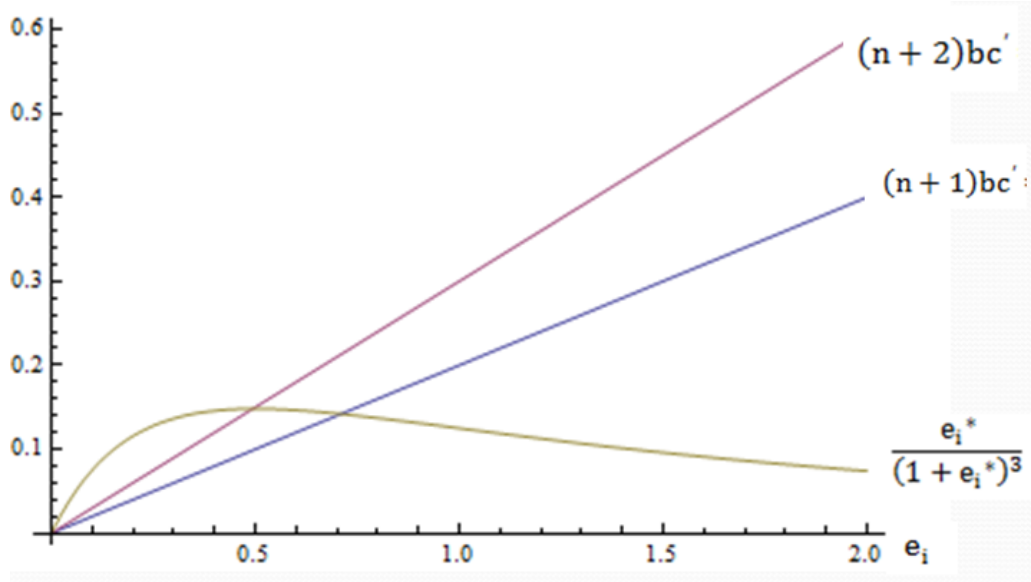
$$A < \frac{1-n}{2(1+e_i^*)^4}$$

Since there is at least one firm in the market, the right hand side is nonpositive, so A is strictly less than zero.

Thus  $\frac{\partial e_i^*}{\partial n} < 0$

This result that innovation effort declines with  $n$  can be illustrated graphically. I plot  $(n+1)bc'(e_i^*)$  and  $\frac{e_i^*}{(1+e_i^*)^3}$  on the same graph where the X-axis is effort to give us a picture of the comparative statistics of  $\frac{\partial e_i^*}{\partial n}$ . When the number of firms in the market  $n$

increases,  $(n + 1)bc'(e_i^*)$  rotates to the left, and the intersection of these two equations shifts to the left, and hence the optimal level of  $e_i$  decreases.



**Figure 10** Plot of Equation (2.1.7)

Thus, when the number of firms  $n$  increase, the increased competition will shrink the demand facing individual firms, and firms will put less effort into process innovation.

**Proposition 1:** Firms will engage in less process innovation as the number of firms in the market increases when the market size is fixed.

### 2.2.1.2 Scale neutral model

A more complete model of the competition and the incentive for innovation must account for how the number of firms is determined. A larger market supports more firms. In this section, I will examine the effect of competition on innovation when the number of firms in the market is proportional to the size of the market. This treatment neutralizes

the scale effect which is dominant in the basic model when the number of firms is exogenous.

I represent a market where the number of firms is proportional to the size of the market by employing the demand function  $P = 1 - \frac{b}{n}Q$ . Here  $b$  is a fixed parameter and the demand curve becomes flatter as  $n$  grows. In this case, the number of customers per firm is constant as the size of the market and number of firms change. Holding other assumptions the same and resolving the model, I obtain

$$\left(1 + \frac{1}{n}\right)bc'(e_i^*) = \frac{e_i^*}{(1 + e_i^*)^3} \quad (2.1.11)$$

Similarly, to examine the effect of competition on innovation effort, I take the partial derivative of equation (2.1.11) with respect to  $n$  to obtain:

$$\begin{aligned} \frac{b}{n}c'(e_i^*) + (n+1) * -\frac{b}{n^2} * c'(e_i^*) + (n+1)\frac{b}{n}c''(e_i^*)\frac{\partial e_i^*}{\partial n} \\ = \frac{(1 + e_i^*)^3 - e_i^* * 3 * (1 + e_i^*)^2}{(1 + e_i^*)^6} \frac{\partial e_i^*}{\partial n} \end{aligned} \quad (2.1.12)$$

Simplifying and rearranging the above equation to find:

$$\left(\frac{1 - 2e_i^*}{(1 + e_i^*)^4} - (n+1)\frac{b}{n}c''(e_i^*)\right)\frac{\partial e_i^*}{\partial n} = -\frac{b}{n^2}c'(e_i^*) \quad (2.1.13)$$

$b$  and  $c'(e_i^*)$  are both positive, so the right hand side of equation (2.1.13) is negative. Let  $B = \frac{1-2e_i^*}{(1+e_i^*)^4} - (n+1)\frac{b}{n}c''(e_i^*)$  which is the terms in the parentheses on the left hand side, the sign of the comparative statistics of  $\frac{\partial e_i^*}{\partial n}$  is opposite with the sign of  $B$ . Solving Hessian Matrix for this model and I obtain an inequality condition for  $c''(e_i^*)$

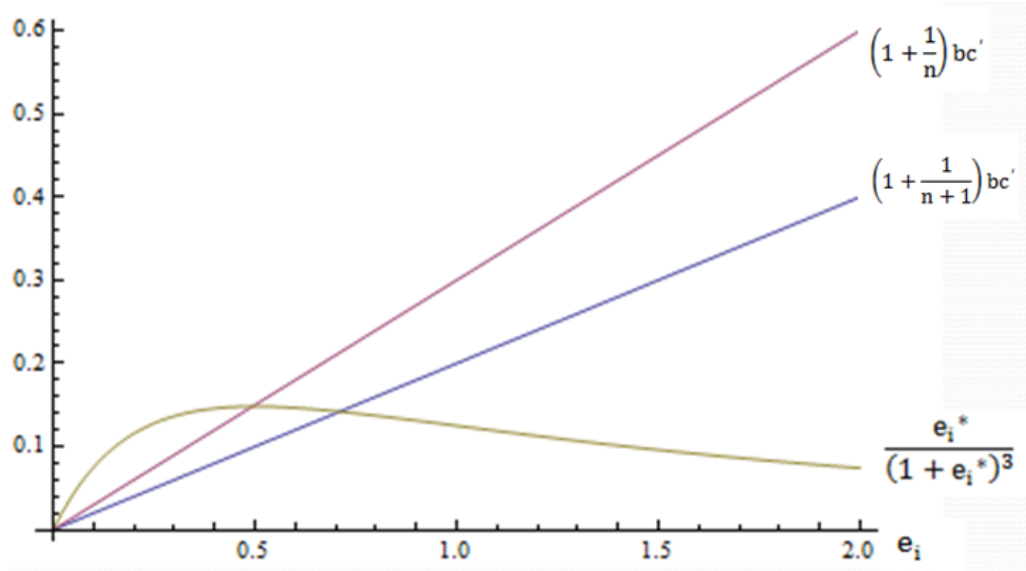
$$c''(e_i^*) > \frac{n}{2b(1 + e_i^*)^4} - \frac{2q^*}{(1 + e_i^*)^3} \quad (2.1.14)$$

Substituting equation (2.1.14) and the profit maximizing output  $q^*$  into B, and simplifying to find:

$$B < \frac{1 - n}{2(1 + e_i^*)^4}$$

This term is exactly the same as A in the basic model, and it is strictly less than zero.

Thus  $\frac{\partial e_i^*}{\partial n} > 0$



**Figure 11 Plot of Equation (2.1.11)**

Figure 11 is the plot of equation (2.1.11). When the number of firms  $n$  increases, the curve of  $\left(1 + \frac{1}{n}\right)bc'(e_i^*)$  will shift downwards, and the intersection for the optimal level of effort will increase. In this case, the number of firms in the market is proportional

to the demand, that is, the number of customers per firm remains the same. However as  $n$  grows, price elasticity increases and firms conduct greater process innovation.

**Proposition 2:** When the number of firms is proportional to demand with the customers per firm remaining the same, the price elasticity effect dominates. Firms will devote more effort to process innovation as the number of firms and market size grow.

### 2.2.1.3 Endogenous number of firms

In this section, I assume the number of firms is endogenous and constrained by nonnegative profits. A fixed cost  $F$  is introduced. Solving for nonnegative profit constraint I will find the number of firms is a function of effort and fixed cost.

$$\pi_i^* = q_i^* \left( 1 - bQ^* - \frac{1}{1 + e_i^*} \right) - c(e_i^*) - F \geq 0 \quad (2.1.15)$$

Substitute equation (2.1.6) into the above equation to obtain:

$$\frac{1 - \frac{1}{1 + e_i^*}}{(n + 1)b} \left( 1 - n * \frac{1 - \frac{1}{1 + e_i^*}}{(n + 1)} - \frac{1}{1 + e_i^*} \right) \geq F + c(e_i^*)$$

Multiplying terms, this expression becomes

$$\frac{\frac{e_i^*}{1 + e_i^*}}{(n + 1)b} \left( 1 - \frac{n}{n + 1} + \frac{\frac{n}{1 + e_i^*}}{n + 1} - \frac{1}{1 + e_i^*} \right) \geq F + c(e_i^*)$$

Summing terms in parentheses yields

$$\frac{\frac{e_i^*}{1+e_i^*}}{(n+1)b} \left( \frac{1}{n+1} - \frac{1}{1+e_i^*} \right) \geq F + c(e_i^*)$$

The left hand side then reduces to

$$\frac{\left(\frac{e_i^*}{1+e_i^*}\right)^2}{(n+1)^2 b} \geq F + c(e_i^*)$$

Isolating the term containing  $n$  gives us

$$(n+1)^2 \leq \frac{\left(\frac{e_i^*}{1+e_i^*}\right)^2}{(F + c(e_i^*))b}$$

Finally, when this expression holds with equality, this defines  $n^*$  as

$$n^* = \frac{\frac{e_i^*}{1+e_i^*}}{\sqrt{(F + c(e_i^*))b}} - 1 \quad (2.1.16)$$

This expression must hold such that  $e_i^*$  is the profit maximizing effort and  $q_i^*$  the profit maximizing output given there are  $n^*$  firms exist in the market. This defines the relationship between  $n^*$  and  $e^*$ . When the fixed cost increases, the number of firms will decrease. Moreover, when the size of the market doubles,  $n$  increases but to less than twice the original number. In this case, the number of firms will increase but not proportionally with the size of the market.



I substitute equation (2.1.16) into expression for  $q^*$ , which is equation (2.1.6), to identify  $q^*$  as only a function of fixed cost  $F$ , the size of the market  $b$ , and the effort  $e$ , and the effort cost function  $c(e)$ .

$$q_i^* = \frac{1 - \frac{1}{1+e_i}}{(n+1)b} = \frac{\frac{e_i}{1+e_i}}{\frac{\frac{e_i}{1+e_i}}{\sqrt{(F+c(e_i))b}} * b} = \sqrt{\frac{F+c(e_i)}{b}} \quad (2.1.17)$$

Insert equation (2.1.16) into (2.1.7) yields:

$$\frac{\frac{e_i}{1+e_i}}{\sqrt{(F+c(e_i))b}} bc' = \frac{e_i^*}{(1+e_i^*)^3}$$

This expression implicitly defines  $e^*$  as a function of all exogenous factors.

Simplifying this expression becomes

$$\frac{b * (c'(e_i^*))^2}{F+c(e_i)} = \frac{1}{(1+e_i^*)^4}$$

$$F+c(e_i) = (1+e_i^*)^4 * (c'(e_i^*))^2 * b \quad (2.1.18)$$

To see how the optimal effort level change in response to the size of the market  $b$  changes, taking partial derivative of equation (2.1.18) with respect to  $b$  to obtain:

$$4(1+e_i^*)^3 \frac{\partial e_i^*}{\partial b} * (c'(e_i^*))^2 * b + (1+e_i^*)^4 * 2c'(e_i^*) * c''(e_i^*) * \frac{\partial e_i^*}{\partial b} * b +$$

$$(1+e_i^*)^4 * (c'(e_i^*))^2 = c'(e_i^*) * \frac{\partial e_i^*}{\partial b}$$

Rearrange to find:

$$(4b(1+e_i^*)^3 * (c'(e_i^*))^2 + 2bc'(e_i^*) * c''(e_i^*)(1+e_i^*)^4 - c'(e_i^*)) \frac{\partial e_i^*}{\partial b} =$$

$$-(1+e_i^*)^4 * (c'(e_i^*))^2 \quad (2.1.19)$$

The right hand side of the equation (2.1.19) is negative, and let  $C = (4b(1 + e_i^*)^3 * (c'(e_i^*))^2 + 2bc'(e_i^*) * c''(e_i^*)(1 + e_i^*)^4 - c'(e_i^*))$ , the sign of the comparative statistics of  $\frac{\partial e_i^*}{\partial b}$  should be opposite with the sign of C.

Substitute the inequality condition (2.1.10) for  $c''(e_i^*)$  into C to obtain

$$C > 4b(1 + e_i^*)^3 * (c'(e_i^*))^2 + 2bc'(e_i^*) * \left( \frac{1}{2b(1 + e_i^*)^4} - \frac{2c'(e_i^*)}{1 + e_i^*} \right) (1 + e_i^*)^4 - c'(e_i^*)$$

The right hand side is then  $4b(1 + e_i^*)^3 * (c'(e_i^*))^2 + c'(e_i^*) - 4b(1 + e_i^*)^3 * (c'(e_i^*))^2 - c'(e_i^*) = 0$

So C is strictly positive, and thus  $\frac{\partial e_i^*}{\partial b} < 0$

That is to say, when the size of the market increases, the number of firms in the market increases as well but not proportionally, firms will face more demand, and put more effort in the R&D research for process innovation.

**Proposition 3:** When the number of firms is endogenous, firms will devote more effort for process innovation if the size of the market increases making it more competitive.

## 2.2.2 The stochastic innovation model

### 2.2.2.1 The basic model

The innovation function is deterministic in the basic model, but innovation is often an uncertain process, and a firm may achieve nothing despite significant effort, or

achieve success without great effort. In this section I model a stochastic innovation process.

Again, I first assume there are  $n$  identical firms in the market, and demand function is  $P = 1 - bQ$ . In period 1, firms choose their innovation effort, which, if successful, will reduce the marginal cost. For trackability, I assume there are two possible levels of marginal cost for each firm  $i$ ,  $MC_i = \{\bar{\varepsilon}, \underline{\varepsilon}\}$ , and  $0 < \underline{\varepsilon} < \bar{\varepsilon} < 1$ . The probability of being a low type is a function of effort,  $prob(MC_i = \underline{\varepsilon}) = f(e_i)$ , where  $f' > 0$ ,  $f'' < 0$ ,  $f(0) = 0$ . In period 2, firms realize their own marginal cost which is observable to all firms in the market. And in period 3, firms play an asymmetric Cournot game to choose the output to maximize their profits.

This is a sequential game so I solve this model backwards. In period 3, firm  $i$  faces a market with  $m$  low types and  $n-m-1$  high types.

Suppose that at time 3 firm  $i$  is a low type, then there are  $m+1$  low types and  $n-m-1$  high types in the market, and the firm chooses the quantity to maximize the profits:

$$\max_{q_L} \pi_L = q_L(1 - bQ - \underline{\varepsilon})$$

The first order condition is:

$$\frac{\partial \pi_L}{\partial q_L} = 1 - bQ - \underline{\varepsilon} + q_L * (-b) = 0 \quad (2.2.1)$$

Imposing symmetry,

$$Q = (m + 1)q_L + (n - m - 1)q_H \quad (2.2.2)$$

Substitute equation (2.2.2) into (2.2.1) to obtain

$$\begin{aligned}
& 1 - b[(m+1)q_L + (n-m-1)q_H] - \underline{\varepsilon} - bq_L \\
& = 1 - b(m+2)q_L - b(n-m-1)q_H - \underline{\varepsilon} = 0 \\
q_L^* &= \frac{1 - \underline{\varepsilon}}{b(m+2)} - \frac{n-m-1}{m+2} q_H^*
\end{aligned} \tag{2.2.3}$$

For high type firms, their problem is

$$\max_{q_H} \pi_H = q_H(1 - bQ - \bar{\varepsilon})$$

The first order condition is:

$$\begin{aligned}
\frac{\partial \pi_H}{\partial q_H} &= 1 - bQ - \bar{\varepsilon} + q_H * (-b) = 0 \\
1 - b[(m+1)q_L + (n-m-1)q_H] - \bar{\varepsilon} - bq_H &= 1 - b(m+1)q_L - b(n-m)q_H - \bar{\varepsilon} \\
&= 0 \\
q_H^* &= \frac{1 - \bar{\varepsilon}}{b(n-m)} - \frac{m+1}{n-m} q_L^*
\end{aligned} \tag{2.2.4}$$

Substitute (2.2.4) into (2.2.3) to find

$$q_L^* = \frac{1}{b(n+1)} [1 - (n-m)\underline{\varepsilon} + (n-m-1)\bar{\varepsilon}] \tag{2.2.5}$$

Substitute (2.2.5) into (2.2.4) yields

$$q_H^* = \frac{1}{b(n+1)} [1 + (m+1)\underline{\varepsilon} - (m+2)\bar{\varepsilon}] \tag{2.2.6}$$

when  $m \leq \frac{1-\bar{\varepsilon}}{\bar{\varepsilon}-\underline{\varepsilon}} - 1$ ,  $q_H^*$  is nonnegative. Thus I will assume  $n \leq \frac{1-\bar{\varepsilon}}{\bar{\varepsilon}-\underline{\varepsilon}} - 1$  to ensure

there is no corner solution in this Cournot game. That is, I assume innovation is not so dramatic that firms which fail to innovate will exit market.

Substitute (2.2.5) and (2.2.6) into (2.2.2) to obtain

$$Q^* = \frac{1}{b(n+1)} [n - (m+1)\underline{\varepsilon} - (n-m-1)\bar{\varepsilon}]$$

Thus,

$$\pi_L^* = \frac{1}{b(n+1)^2} [1 - (n-m)\underline{\varepsilon} + (n-m-1)\bar{\varepsilon}]^2 \quad (2.2.7)$$

$$P^* = 1 - Q^* = 1 - \frac{1}{b(n+1)} [n - (m+1)\underline{\varepsilon} - (n-m-1)\bar{\varepsilon}] \quad (2.2.8)$$

Now suppose at time 3 firm  $i$  is high type. Then there are  $m$  low types and  $n-m$  high types in the market, and the firm chooses the quantity to maximize the profits:

$$\max_{q_H} \pi_H = q_H(1 - bQ - \bar{\varepsilon})$$

The first order condition is:

$$\frac{\partial \pi_H}{\partial q_H} = 1 - bQ - \bar{\varepsilon} + q_H * (-b) = 0 \quad (2.2.9)$$

Imposing symmetry,

$$Q = mq_L + (n-m)q_H \quad (2.2.10)$$

Substitute (2.2.10) into (2.2.9) to find

$$1 - b[mq_L + (n-m)q_H] - \bar{\varepsilon} - bq_H = 1 - bmq_L - b(n-m+1)q_H - \bar{\varepsilon} = 0$$

$$q_H^* = \frac{1 - \bar{\varepsilon}}{b(n-m+1)} - \frac{m}{n-m+1} q_L^* \quad (2.2.11)$$

For low type firms, their problem is

$$\max_{q_L} \pi_L = q_L(1 - bQ - \underline{\varepsilon})$$

The first order condition is:

$$\frac{\partial \pi_L}{\partial q_L} = 1 - bQ - \underline{\varepsilon} + q_L * (-b) = 0 \quad (2.2.12)$$

$$1 - b[mq_L + (n - m)q_H] - \underline{\varepsilon} - bq_L = 1 - b(m + 1)q_L - b(n - m)q_H - \underline{\varepsilon} = 0$$

$$q_L^* = \frac{1 - \underline{\varepsilon}}{b(m + 1)} - \frac{n - m}{m + 1} q_H^* \quad (2.2.13)$$

Substitute (2.2.13) into (2.2.11) to obtain

$$q_H^* = \frac{1}{b(n + 1)} [1 + m\underline{\varepsilon} - (m + 1)\bar{\varepsilon}] \quad (2.2.14)$$

Substitute (2.2.14) into (2.2.13) yields

$$q_L^* = \frac{1}{b(n + 1)} [1 - (n - m + 1)\underline{\varepsilon} + (n - m)\bar{\varepsilon}] \quad (2.2.15)$$

Substitute (2.2.14) and (2.2.15) into (2.2.10) to find

$$Q^* = \frac{1}{b(n + 1)} [n - m\underline{\varepsilon} - (n - m)\bar{\varepsilon}]$$

Thus,

$$\pi_H^* = \frac{1}{b(n + 1)^2} [1 + m\underline{\varepsilon} - (m + 1)\bar{\varepsilon}]^2 \quad (2.2.16)$$

$$P^* = 1 - Q^* = 1 - \frac{1}{b(n + 1)} [n - m\underline{\varepsilon} - (n - m)\bar{\varepsilon}] \quad (2.2.17)$$

Combining results from these two cases , for firm i between being high and low type, given there are  $m$  other firms are low, the difference of the profits between low type and high type firms is

$$\begin{aligned} Z = \pi_L^* - \pi_H^* &= \frac{n}{b(n + 1)^2} (\bar{\varepsilon} - \underline{\varepsilon}) [2 - (n - 2m)\underline{\varepsilon} + (n - 2m - 2)\bar{\varepsilon}] \\ &= \frac{n}{b(n + 1)^2} (\bar{\varepsilon} - \underline{\varepsilon}) [2 - n\underline{\varepsilon} + (n - 2)\bar{\varepsilon}] \\ &\quad - \frac{2mn}{b(n + 1)^2} (\bar{\varepsilon} - \underline{\varepsilon})^2 \end{aligned} \quad (2.2.18)$$

Let  $p = f(e_{-i})$ , and assume the realization of the marginal cost for each firm is independent, so all combinations of the  $MC$ s for all firms follow a binomial distribution.

$$\begin{aligned}
E(Z) &= \binom{n-1}{0} p^0 (1-p)^{n-1} Z^*(m=0) + \binom{n-1}{1} p^1 (1-p)^{n-2} Z^*(m=1) \\
&\quad + \dots + \binom{n-1}{i} p^i (1-p)^{n-1-i} Z^*(m=i) + \dots \\
&\quad + \binom{n-1}{n-1} p^{n-1} (1-p)^0 Z^*(m=n-1) \\
&= \frac{n}{b(n+1)^2} (\bar{\varepsilon} - \underline{\varepsilon}) [2 - n\underline{\varepsilon} + (n-2)\bar{\varepsilon}] + \frac{2n}{b(n+1)^2} (\bar{\varepsilon} - \underline{\varepsilon})^2 [0 \\
&\quad * \binom{n-1}{0} p^0 (1-p)^{n-1} + 1 * \binom{n-1}{1} p^1 (1-p)^{n-2} + \dots + j \\
&\quad * \binom{n-1}{i} p^i (1-p)^{n-1-i} + \dots + (n-1) \binom{n-1}{n-1} p^{n-1} (1-p)^0]
\end{aligned}$$

Since  $j * \binom{n-1}{j} p^j (1-p)^{n-1-j} = j * \frac{(n-1)!}{j!(n-1-j)!} p^j (1-p)^{n-1-j} = \frac{(n-1)*(n-2)!}{(j-1)!(n-1-j)!} p * p^{j-1} (1-p)^{n-1-j}$

$p^{j-1} (1-p)^{n-1-j} = p(n-1) * \binom{n-2}{j-1} p^{j-1} (1-p)^{n-1-j}$

and  $\sum_{j=1}^{n-1} \binom{n-2}{j-1} p^{j-1} (1-p)^{n-1-j} = 1$

I find

$$\begin{aligned}
E(Z) &= \frac{n}{b(n+1)^2} (\bar{\varepsilon} - \underline{\varepsilon}) [2 - n\underline{\varepsilon} + (n-2)\bar{\varepsilon}] + \frac{2n}{b(n+1)^2} (\bar{\varepsilon} - \underline{\varepsilon})^2 p(n-1) = \\
&\frac{n}{b(n+1)^2} (\bar{\varepsilon} - \underline{\varepsilon}) [2 - n\underline{\varepsilon} + (n-2)\bar{\varepsilon} - 2p(n-1)(\bar{\varepsilon} - \underline{\varepsilon})] \quad (2.2.19)
\end{aligned}$$

In period 1, firm  $i$  chooses its effort to maximize the expected profit:

$$\max_{e_i} E(\pi^*) - e_i$$

Where  $E(\pi^*) = f(e_i)E(\pi_L^*) + (1 - f(e_i))E(\pi_H^*) = f(e_i)E(Z) + E(\pi_H^*)$

The first order condition is then:

$$f'(e_i^*)E(Z) = 1 \quad (2.2.20)$$

Substitute the expression of  $E(Z)$  into the above equation to solve for  $e_i^*$ , and impose symmetry,  $e_i^* = e_{-i}^*$ , I obtain

$$\frac{n}{b(n+1)^2} (\bar{\varepsilon} - \underline{\varepsilon}) [2 - n\underline{\varepsilon} + (n-2)\bar{\varepsilon} - 2(n-1)f(e_i^*)(\bar{\varepsilon} - \underline{\varepsilon})] f'(e_i^*) = 1 \quad (2.2.21)$$

The number of firms in the market captures the competitiveness in this case, and in order to examine the relation between innovation and competition, I take partial derivative of equation (2.2.21) with respect to the number of firms  $n$  to find:

$$f''(e_i^*) \frac{\partial e_i^*}{\partial n} E(Z^*) + f'(e_i^*) \left( \frac{\partial E(Z^*)}{\partial n} + \frac{\partial E(Z^*)}{\partial f(e_i^*)} * f'(e_i^*) * \frac{\partial e_i^*}{\partial n} \right) = 0$$

$$\text{where } E(Z^*) = \frac{n}{b(n+1)^2} (\bar{\varepsilon} - \underline{\varepsilon}) [2 - n\underline{\varepsilon} + (n-2)\bar{\varepsilon} - 2(n-1)f(e_i^*)(\bar{\varepsilon} - \underline{\varepsilon})] \quad (2.2.22)$$

Rearrange yields:

$$(f''(e_i^*)E(Z^*) + (f'(e_i^*))^2 \frac{\partial E(Z^*)}{\partial f(e_i^*)}) \frac{\partial e_i^*}{\partial n} = -f'(e_i^*) \frac{\partial E(Z^*)}{\partial n} \quad (2.2.23)$$

Since  $f'(e_i^*) > 0$  and  $f''(e_i^*) < 0$  by assumption, and I could get

$$\frac{\partial E(Z^*)}{\partial f(e_i^*)} = -\frac{n}{b(n+1)^2} 2(n-1)(\bar{\varepsilon} - \underline{\varepsilon}) < 0 \text{ from equation (2.2.22), thus } f''(e_i^*)E(Z^*) +$$

$$(f'(e_i^*))^2 \frac{\partial E(Z^*)}{\partial f(e_i^*)}, \text{ which is on the left hand side of the equation (2.2.23), is negative.}$$

To find the sign of  $\frac{\partial E(Z^*)}{\partial n}$ , I take the difference of  $E(Z^*)$  when the number of firms is  $n$

and  $n+1$ .



$$\begin{aligned}
& E(Z_n^*) - E(Z_{n+1}^*) \\
&= \frac{n}{b(n+1)^2} (\bar{\varepsilon} - \underline{\varepsilon}) [2 - n\underline{\varepsilon} + (n-2)\bar{\varepsilon} - 2p(n-1)(\bar{\varepsilon} - \underline{\varepsilon})] \\
&\quad - \frac{n+1}{b(n+2)^2} (\bar{\varepsilon} - \underline{\varepsilon}) [2 - (n+1)\underline{\varepsilon} + (n-1)\bar{\varepsilon} - 2pn(\bar{\varepsilon} - \underline{\varepsilon})] \\
&= \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{b(n+1)^2(n+2)^2} \{n(n+2)^2[2 - 2\bar{\varepsilon} + n(\bar{\varepsilon} - \underline{\varepsilon}) \\
&\quad - 2np(\bar{\varepsilon} - \underline{\varepsilon}) + 2p(\bar{\varepsilon} - \underline{\varepsilon})] \\
&\quad - (n+1)^3[2 - 2\bar{\varepsilon} + n(\bar{\varepsilon} - \underline{\varepsilon}) + (\bar{\varepsilon} - \underline{\varepsilon}) - 2np(\bar{\varepsilon} - \underline{\varepsilon})]\} \\
&= \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{b(n+1)^2(n+2)^2} [(n^2 + n - 1)(2 - 2\bar{\varepsilon}) \\
&\quad + (n^2 + n - 1)n(\bar{\varepsilon} - \underline{\varepsilon}) - (n^2 + n - 1)2np(\bar{\varepsilon} - \underline{\varepsilon}) \\
&\quad + 2n(n+2)^2p(\bar{\varepsilon} - \underline{\varepsilon}) - (n+1)^3(\bar{\varepsilon} - \underline{\varepsilon})] \\
&= \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{b(n+1)^2(n+2)^2} [(2n^2 + 2n - 2)(1 - \bar{\varepsilon}) - (n+1)^3(\bar{\varepsilon} - \underline{\varepsilon}) \\
&\quad + (n^2 + n - 1)n(\bar{\varepsilon} - \underline{\varepsilon}) \\
&\quad + (6n^2 + 10n - 1)p(\bar{\varepsilon} - \underline{\varepsilon})]
\end{aligned}$$

Since  $n \leq \frac{1-\bar{\varepsilon}}{\bar{\varepsilon}-\underline{\varepsilon}} - 1$ ,  $(n+1)(\bar{\varepsilon} - \underline{\varepsilon}) \leq 1 - \bar{\varepsilon}$

$$(n+1)^3(\bar{\varepsilon} - \underline{\varepsilon}) \leq (n+1)^2(1 - \bar{\varepsilon})$$

Thus  $E(Z_n^*) - E(Z_{n+1}^*) \geq \frac{(\bar{\varepsilon}-\underline{\varepsilon})}{b(n+1)^2(n+2)^2} [(2n^2 + 2n - 2)(1 - \bar{\varepsilon}) - (n+1)^2(1 - \bar{\varepsilon}) +$   
 $n2+n-1n\varepsilon-\varepsilon+6n2+10n-1p\varepsilon-\varepsilon=\varepsilon-\varepsilon bn+12n+22n2-31-\varepsilon+n2+n-1n\varepsilon-\varepsilon+6n2$   
 $+10n-1p\varepsilon-\varepsilon>0$

Therefore, the right hand side of the equation (2.2.23) is positive, and hence  $\frac{\partial e_i^*}{\partial n} < 0$ .

This result is the same with the deterministic case.

**Proposition 4a:** The more firms in the market, the less efforts firms will put to conduct process innovation even when the innovation result is stochastic. Scale effect dominates.

To examine the effect of market competitiveness on the expected price, I calculated the expected price and take partial derivative with respect to the number of firms  $n$ :

The expected price  $E(P^*)$

$$\begin{aligned}
&= 1 - \binom{n}{0} p^0 (1-p)^n \left[ \frac{1}{n+1} (n - n\bar{\varepsilon}) \right] \\
&- \binom{n}{1} p^1 (1-p)^{n-1} \left[ \frac{1}{n+1} (n - \underline{\varepsilon} - (n-1)\bar{\varepsilon}) \right] - \dots \\
&- \binom{n}{j} p^j (1-p)^{n-j} \left[ \frac{1}{n+1} (n - j\underline{\varepsilon} - (n-j)\bar{\varepsilon}) \right] - \dots \\
&- \binom{n}{n} p^n (1-p)^0 \left[ \frac{1}{n+1} (n - n\underline{\varepsilon}) \right] \\
&= 1 - \frac{n}{n+1} + \frac{1}{n+1} \sum_{j=0}^n \binom{n}{j} p^j (1-p)^{n-j} [j\underline{\varepsilon} + (n-j)\bar{\varepsilon}]
\end{aligned}$$

$$\text{Since } j * \binom{n}{j} p^j (1-p)^{n-j} \underline{\varepsilon} = j * \frac{n!}{j!(n-j)!} p^j (1-p)^{n-j} \underline{\varepsilon} = \frac{n*(n-1)!}{(j-1)!(n-j)!} p *$$

$$p^{j-1} (1-p)^{n-j} \underline{\varepsilon} = np\underline{\varepsilon} * \binom{n-1}{j-1} p^{j-1} (1-p)^{n-j}$$

$$\text{and } \sum_{j=1}^n \binom{n-1}{j-1} p^{j-1} (1-p)^{n-j} = 1$$

$$\text{I find } \sum_{j=0}^n \binom{n}{j} p^j (1-p)^{n-j} j\underline{\varepsilon} = np\underline{\varepsilon}$$

Analogously,  $\sum_{j=0}^n \binom{n}{j} p^j (1-p)^{n-j} (n-j) \bar{\varepsilon} = n\bar{\varepsilon} - np\underline{\varepsilon}$

Thus,

$$\begin{aligned} E(P^*) &= 1 - \frac{n}{n+1} + \frac{1}{n+1} (np\underline{\varepsilon} + n\bar{\varepsilon} - np\bar{\varepsilon}) \\ &= \frac{1}{n+1} (1 + np\underline{\varepsilon} + n\bar{\varepsilon} - np\bar{\varepsilon}) \end{aligned} \quad (2.2.24)$$

Taking partial derivative with respect to the number of firms  $n$ :

$$\begin{aligned} \frac{\partial E(P^*)}{\partial n} &= \frac{(p\underline{\varepsilon} + \bar{\varepsilon} - p\bar{\varepsilon})(n+1) - (1 + np\underline{\varepsilon} + n\bar{\varepsilon} - np\bar{\varepsilon})}{(n+1)^2} \\ &= \frac{p(\underline{\varepsilon} - \bar{\varepsilon}) + \bar{\varepsilon} - 1}{(n+1)^2} \end{aligned} \quad (2.2.25)$$

Since  $\underline{\varepsilon} < \bar{\varepsilon} < 1$ ,

$$\frac{\partial E(P^*)}{\partial n} < 0$$

That is to say, the expected price will decrease when the number of firms in the market increases.

**Proposition 4b:** The expected price will decrease as the number of firms in the market increases when the innovation result is stochastic.

#### 2.2.2.2 Scale neutral model

As I did earlier in the deterministic innovation model, I now assume the number of firms in the market is proportional to the size of the market. In order to neutralize the scale effect, the demand function then becomes  $P = 1 - \frac{b}{n}Q$ . Holding other assumptions the same with the basic model, resolving the profits maximization problem in period 3 to obtain

$$E(Z^*) = \frac{n^2}{b(n+1)^2} (\bar{\varepsilon} - \underline{\varepsilon}) [2 - n\underline{\varepsilon} + (n-2)\bar{\varepsilon} - 2(n-1)f(e_i^*)(\bar{\varepsilon} - \underline{\varepsilon})] \quad (2.2.26)$$

In period 1, firm  $i$  chooses the effort to maximize the expected profit:

$$\max_{e_i} E(\pi^*) - e_i$$

The first order condition and the comparative statistics are the same with the basic model expect the functional form of  $E(Z^*)$  is slightly different:

$$f'(e_i^*)E(Z^*) = 1 \quad (2.2.20)$$

$$(f''(e_i^*)E(Z^*) + (f'(e_i^*))^2 \frac{\partial E(Z^*)}{\partial f(e_i^*)}) \frac{\partial e_i^*}{\partial n} = -f'(e_i^*) \frac{\partial E(Z^*)}{\partial n} \quad (2.2.23)$$

As before,  $f''(e_i^*)E(Z^*) + (f'(e_i^*))^2 \frac{\partial E(Z^*)}{\partial f(e_i^*)}$  is negative. To examine the sign of  $\frac{\partial E(Z^*)}{\partial n}$ , I take the difference of  $E(Z^*)$  when the number of firms is  $n$  and  $n+1$ , but this cannot be signed in general. I conducted a simulation to show how market competitiveness affects the optimal level of innovation effort in this model.

I first calculate the expected values of  $q_L^*$ ,  $q_H^*$  and  $Q^*$ , and run simulation to examine the relation between those quantities and the number of firms.

When there are  $m$  low types firms and  $n-m$  high types firms in the market,

$$q_L^* = \frac{n}{b(n+1)} [1 - (n-m+1)\underline{\varepsilon} + (n-m)\bar{\varepsilon}]$$

$$q_H^* = \frac{n}{b(n+1)} [1 + m\underline{\varepsilon} - (m+1)\bar{\varepsilon}]$$

$$Q^* = \frac{n}{b(n+1)} [n - m\underline{\varepsilon} - (n-m)\bar{\varepsilon}]$$

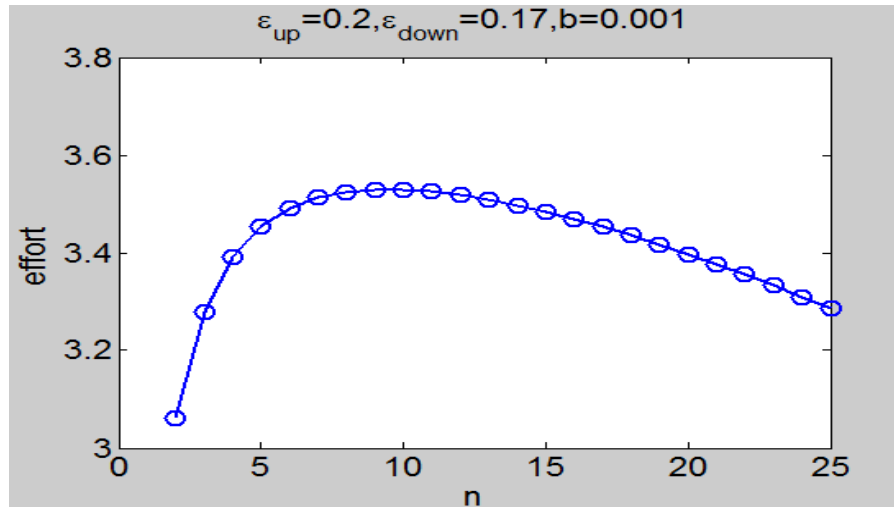
Let  $a = f(e)$ , and assume the realization of the marginal cost for each firm is independent, so all combinations of the  $MC$ s for all firms follow a binomial distribution.

$$\begin{aligned}
E(q_L^*) &= \sum_{j=0}^n \binom{n}{j} a^j (1-a)^{n-j} q_L^* \\
&= \frac{n}{b(n+1)} [1 - (n - na + 1)\underline{\varepsilon} \\
&\quad + (n - na)\bar{\varepsilon}] \\
E(q_H^*) &= \frac{n}{b(n+1)} [1 + na\underline{\varepsilon} - (na + 1)\bar{\varepsilon}] \\
E(Q^*) &= \frac{n}{b(n+1)} [n - na\underline{\varepsilon} - (n - na)\bar{\varepsilon}]
\end{aligned}$$

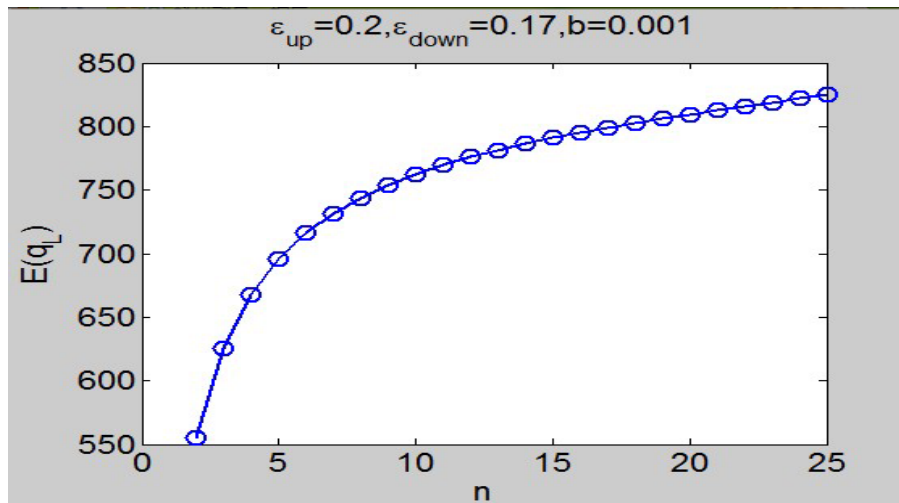
Figures 12-15 are the simulation results given the functional form  $f(e_i) = 1 - \exp(-e_i)$ . Figure 12 shows that there is an inverted-U shape relationship between the number of firms in the market and innovation effort. Starting from a small market and few firms, as market size and the number of firms grow together, firms exert more innovation effort; However, as market size and the number of firms become large, eventually firms will have less incentive to engage in process innovation because there are likely to be many successful innovations in the market and thus little realized profit even for these firms.

Figures 13-15 show that when the number of firms increases, low type firms will produce more and the market output will increase. However, high type firms will produce more when there are a few firms in the market, but as there are more and more firms in the market, the optimal quantity for the high type firms will decrease.

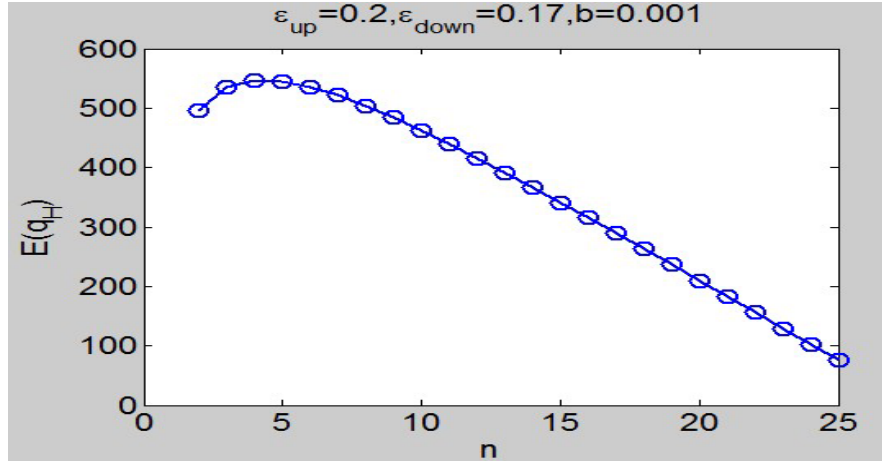
I tried different functional forms of  $f(e_i)$ , and also changed the parameter value of for  $f$ ,  $\underline{\varepsilon}$ ,  $\bar{\varepsilon}$  and  $b$  robustness check, and obtain the similar results. As is evident from simulation,  $\frac{\partial e_i^*}{\partial n}$  cannot be signed in general.



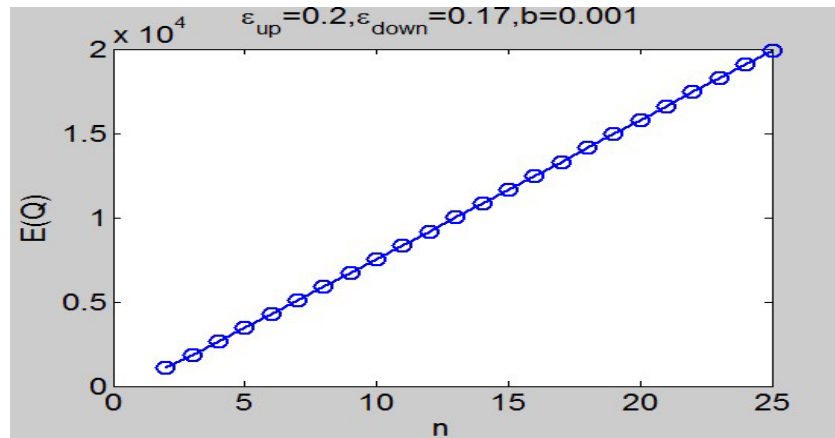
**Figure 12 The Effect of Market Competitiveness on Innovation Effort When the Number of Firms is Proportional to the Size of the Market**



**Figure 13 The Effect of Market Competitiveness on the Expected Output of Low Type Firms When the Number of Firms is Proportional to the Size of the Market**



**Figure 14 The Effect of Market Competitiveness on the Expected Output of High Type Firms When the Number of Firms is Proportional to the Size of the Market**



**Figure 15 The Effect of Market Competitiveness on the Expected Output of All Firms When the Number of Firms is Proportional to the Size of the Market**

**Proposition 5:** When the number of firms is proportional to the demand and innovation result is stochastic, there is an inverted-U shape relationship between the number of firms in the market and innovation effort. That is, firms will exert more innovation effort in a small market competing with a few firms; however, as there are

more and more firms entering the market, firms will have less incentive to conduct process innovation.

### 2.2.2.3 Endogenous number of firms

I now assume there is a fixed cost  $F$  for each firm, and the number of firms in the market is endogenous.

$$E(\pi^*) = f(e_i)E(Z^*) + E(\pi_H^*) - F$$

$$\text{where } \pi_H^* = \frac{1}{b(n+1)^2} [1 + m\underline{\varepsilon} - (m+1)\bar{\varepsilon}]^2 \quad (2.2.16)$$

$$E(Z^*) = \frac{n}{b(n+1)^2} (\bar{\varepsilon} - \underline{\varepsilon}) [2 - n\underline{\varepsilon} + (n-2)\bar{\varepsilon} - 2f(e_{-i})(n-1)(\bar{\varepsilon} - \underline{\varepsilon})] \quad (2.2.21)$$

Solving for  $E(\pi^*)$  to obtain

$$E(\pi^*) = \frac{1}{b(n+1)^2} [2f(e_{-i})(1 - \bar{\varepsilon})(\bar{\varepsilon} - \underline{\varepsilon}) + (1 - \bar{\varepsilon})^2 + [n^2 f(e_{-i}) + n - 1] f(e_{-i}) - n - 1 + 2f(e_{-i})^2] \varepsilon - \varepsilon^2 - F \quad (2.2.27)$$

Solving for nonnegative profit constraint  $E(\pi^*) - e_i \geq 0$ , I will find the number of firms is a function of effort, fixed cost,  $\underline{\varepsilon}$ ,  $\bar{\varepsilon}$  and  $b$ .

In period 1, firm  $i$  chooses the effort to maximize the expected profit:

$$\max_{e_i} E(\pi^*) - e_i$$

The first order condition is:

$$f'(e_i^*)E(Z^*) = 1 \quad (2.2.20)$$

Solving  $e_i^*$  from equation (2.2.20) to obtain  $e_i^*$  is a function of fixed cost,  $\underline{\varepsilon}$ ,  $\bar{\varepsilon}$  and  $b$ .



I couldn't obtain the analytic results for  $\frac{\partial e_i^*}{\partial b}$ , so I used simulation to see how the size of the market affect innovation effort.

Analogously, I calculate the expected values of  $q_L^*$ ,  $q_H^*$  and  $Q^*$ , and run simulation to examine the relation between quantities and the number of firms.

When there are  $m$  low types firms and  $n-m$  high types firms in the market,

$$q_L^* = \frac{1}{b(n^* + 1)} [1 - (n^* - m + 1)\underline{\varepsilon} + (n^* - m)\bar{\varepsilon}]$$

$$q_H^* = \frac{1}{b(n^* + 1)} [1 + m\underline{\varepsilon} - (m + 1)\bar{\varepsilon}]$$

$$Q^* = \frac{1}{b(n^* + 1)} [n^* - m\underline{\varepsilon} - (n^* - m)\bar{\varepsilon}]$$

Let  $a = f(e)$ , and assume the realization of the marginal cost for each firm is independent, so all combinations of the  $MC$ s for all firms follow a binomial distribution.

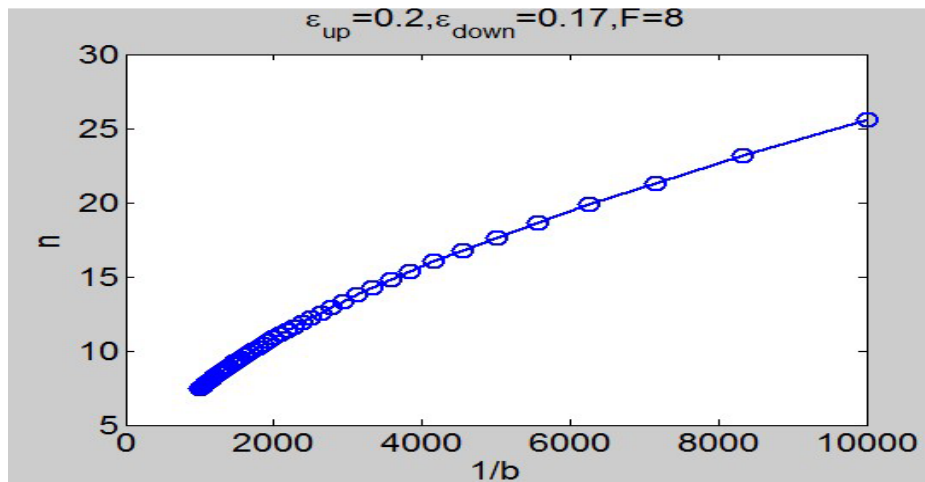
$$E(q_L^*) = \frac{1}{b(n^* + 1)} [1 - (n^* - n^*a + 1)\underline{\varepsilon} + (n^* - n^*a)\bar{\varepsilon}]$$

$$E(q_H^*) = \frac{1}{b(n^* + 1)} [1 + n^*a\underline{\varepsilon} - (n^*a + 1)\bar{\varepsilon}]$$

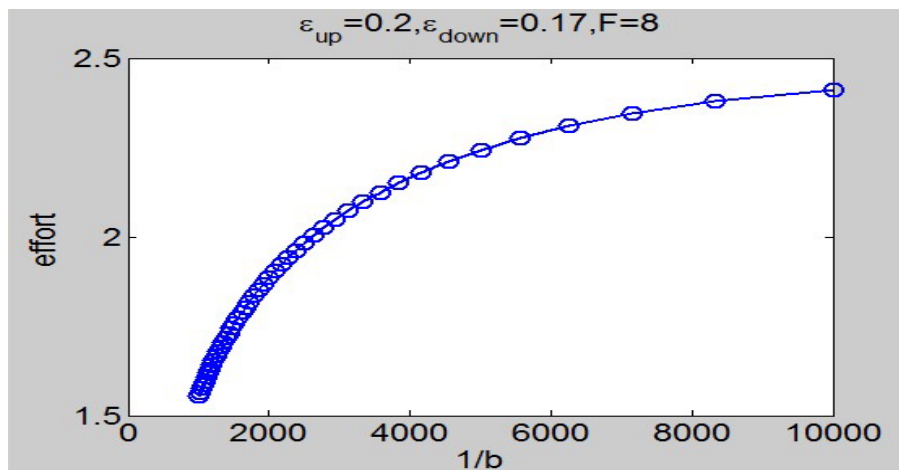
$$E(Q^*) = \frac{1}{b(n^* + 1)} [n^* - n^*a\underline{\varepsilon} - (n^* - n^*a)\bar{\varepsilon}]$$

Figures 16-20 are the simulation results assuming the functional form  $f(e_i) = 1 - \exp(-e_i)$ . Figures 16 and 17 show that when the size of the firms in the market increases, the number of firms increases but less than proportionally, and firms will put more effort for innovation. Note that fixed cost  $F$  is sufficiently large that high cost firms

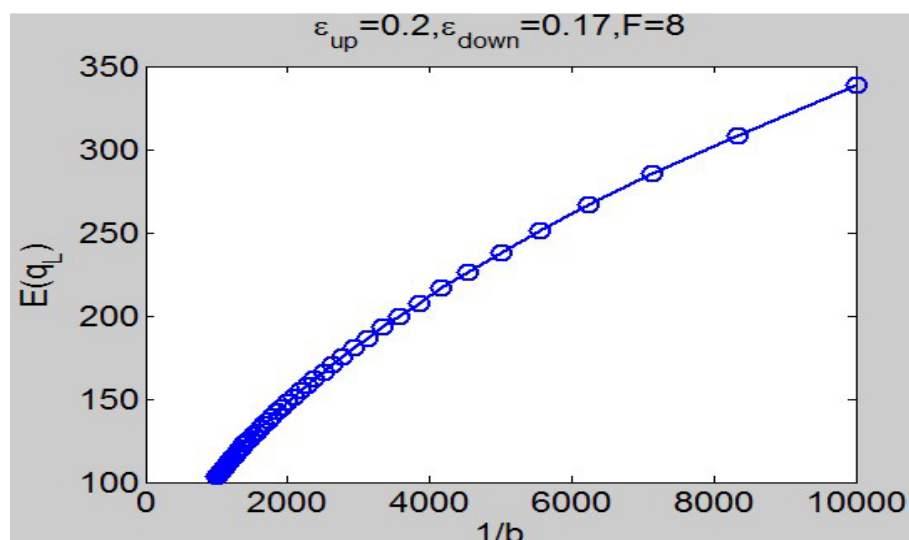
stay in the market. Figures 18-20 present the effect of market competitiveness on the expected output, and the results are similar to what I found in the last model. I did robustness check as well with alternative functional forms, and similar results are obtained.



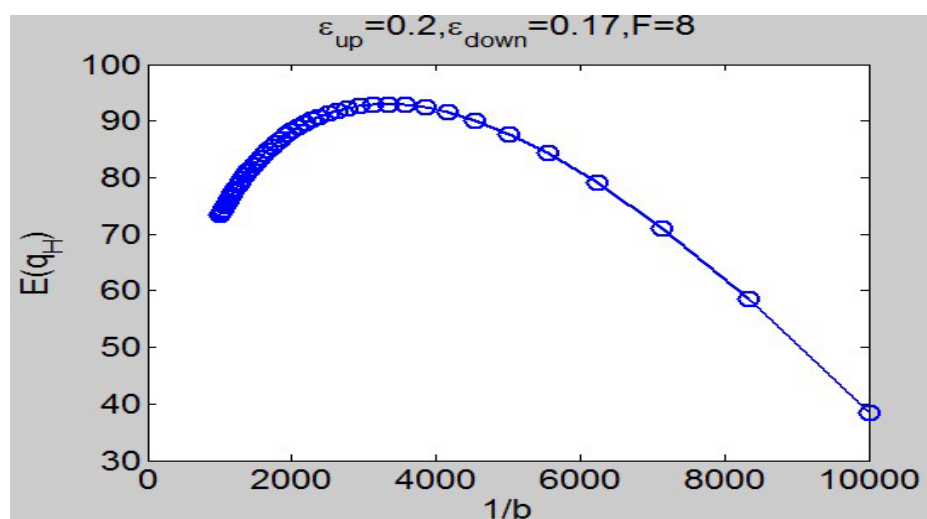
**Figure 16 The Effect of the Market Size on the Number of Firms in Equilibrium**



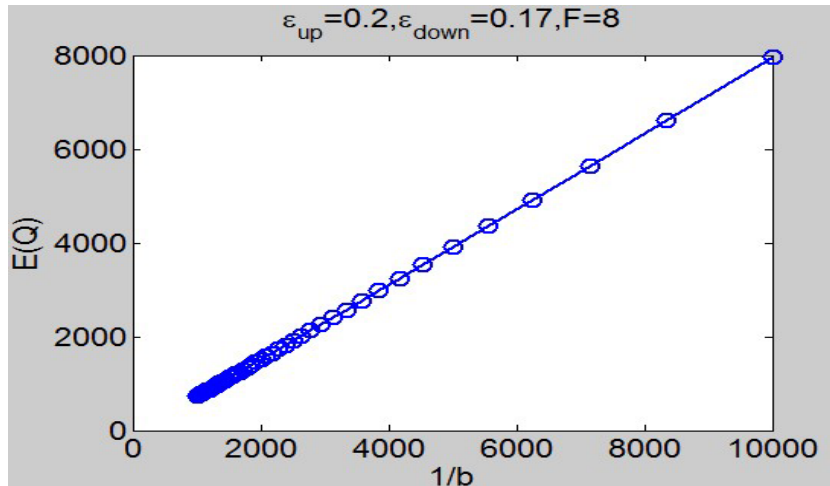
**Figure 17 The Effect of the Market Size on Innovation Effort When the Number of Firms is Endogenous**



**Figure 18 The Effect of the Market Size on the Expected Output of Low Type Firms When the Number of Firms is Endogenous**



**Figure 19 The Effect of the Market Size on the Expected Output of High Type Firms When the Number of Firms is Endogenous**



**Figure 20 The Effect of the Market Size on the Expected Output of All Firms When the Number of Firms is Endogenous**

**Proposition 6:** When the number of firms is endogenous, firms will devote more effort for process innovation as the size of the market increases when the innovation result is stochastic.

Compared with proposition 5 where firms will exert more innovation effort in a small market competing with a few firms, but have less incentive to conduct process innovation as there are more and more firms in the market, in this case, firms will definitely devote more effort for process innovation as competition increases due to increasing market size. That is because, the model that delivers proposition 5 assumes the number of firms proportionally increases with the size of the market, but in the model with proposition 6, I show that with a fixed cost, the number of firms will increase but less than proportionally with the size of the market. Holding other conditions the same, customers each firm will be more for the model with a fixed cost. Thus firms will have stronger incentive to put more effort for innovation when there is more competition for

the model with endogenous number of firms due to a fixed cost than the model where the number of firms changes proportionally with the size of the market.

## **2.3 Conclusion**

Since Schumpeter pointed out that innovative activity is related to competition, economists have developed various models to examine this relationship. However, there is no paper so far considering endogenous number of firms in the model, and deterministic innovation process is commonly assumed. In this paper, I employed a model similar to Martin's managerial efficiency model. I compared the results from the models with exogenous and endogenous number of firms, and innovation process is assumed to be either deterministic or stochastic. I find that when innovation process is deterministic, the basic results are consistent with Martin (1993). As the number of firms is exogenous, increased competition will shrink the demand facing each firm, and firms then have less incentive for process innovation. However, when the market could support more firms, such as the number of firms in the market grows proportional to the demand, or the number of firms is endogenous due to a fixed cost, the scale effect is diminished, and increased price elasticity dominates in this case which will induce firms to put more effort into innovation. Thus more competition will induce firms to devote more effort in innovation.

When innovation process is stochastic, ambiguous results are obtained when the number of firms in the market grows proportional to the demand. I find that there will be an inverted-U shape relation between innovation effort and market competitiveness. Moreover, when the number of firms is endogenous due to a fixed cost, I show that the

number of firms will increase but not proportionally with the size of the market, and firms will have stronger incentive to put more effort for innovation under increasing competition for the model with endogenous number of firms due to fixed cost than the model where the number of firms changes proportionally with the size of the market.

## References

- Aghion, P., N. Bloom, R. Blundell, R. Griffith and P. Howitt**, 2005, Competition and Innovation: An Inverted-U Relationship, *the Quarterly Journal of Economics*, MIT Press, Vol. 120, No. 2, pp. 701-728.
- Arrow, K.**, 1962, Economic Welfare and the Allocation of Resources for Invention, In "The Rate and Direction of Inventive Activity: Economic and Social Factors" (Nelson R (ed)), pp. 609-626. Princeton University Press.
- Bertoletti, P. and C. Poletti**, 1996, A Note on Endogenous Firm Efficiency in Cournot Models of Incomplete Information, *Journal of Economic Theory*, Vol. 71, No. 1, pp. 303-310.
- Dasgupta, P. and J. Stiglitz**, 1980b, Industrial Structure and the Nature of Innovative Activity, *Economic Journal*, Vol. 90, No. 358, pp. 266-293.
- Gilbert, R. and D. Newbery**, 1982, Preemptive Patenting and the Persistence of Monopoly, *the American Economic Review*, Vol. 72, No. 3, pp. 514-526.
- Greenstein, S. and G. Ramey**, 1998, Market Structure, Innovation and Vertical Product Differentiation, *International Journal of Industrial Organization*, Vol. 16, pp. 285-311.
- Martin, S.**, 1993, Endogenous Firm Efficiency in a Cournot Principal-Agent Model, *Journal of Economic Theory*, Vol. 59, pp. 445-450.
- Kamien, M. I. and N. L. Schwartz**, 1976, On the Degree of Rivalry for Maximum Innovative Activity, *the Quarterly Journal of Economics*, Vol. 90, pp. 245-260.
- Kamien, M. I., S. S. Oren and Y. Tauman**, 1992, Optimal Licensing of Cost-reducing Innovation, *Journal of Mathematical Economics*, Vol. 21, pp. 483-508.
- Schmidt, K. M.**, 1997, Managerial Incentives and Product Market Competition, *the Review of Economic Studies*, Vol. 64, NO. 2, p. 191-213.
- Schumpeter, J.A.**, 1934, The Theory of Economic Development. Cambridge, MA: Harvard University Press.
- Willig, R. D.**, 1987, Corporate Governance and Market Structure, in "Economic Policy in Theory and Practice" (A. Razin and E. Sadka, Eds.), pp. 481-494.

## CONCLUSION

In this dissertation, I examine firm's behavior from the manager's view. In chapter one, I examine inventory behavior using Chinese firm level data. By analyzing the original production smoothing/buffer stock model, ambiguous relation between sales shock and inventory investment are obtained. In addition, the labor force is found to be a crucial component of the model and excluding it from the estimation leads to biased results. To date, this is the first paper that considers the impact of the labor force in the empirical test of inventory behavior. Using a rich Chinese firm-level dataset covering 769 manufacturing firms from 1980 to 1989, finished goods inventory is used to test the production smoothing/buffer stock model which involves the rising marginal cost when planning production, while inventory of raw materials is applied to test the  $(S, s)$  model assuming a fixed delivery cost. I find that the variance of the annual gross output is smaller than that of the sales revenues of products. In particular, small firms in heavy industry show strong evidence to use inventory to smooth production and buffer demand shocks. Moreover, sales are positively correlated with investment in raw materials, but negatively correlated with finished goods inventory in most cases. This is consistent with the predictions from both the production smoothing/buffer stock model and the  $(S, s)$  model. Furthermore, the labor force is found to be positively related with demand as well as the inventory, which indicates that excluding the labor force in the estimation will cause biased results. This explains why previous studies found contradicting results to the theoretical predictions.



In chapter two, the relation between innovative activity and market competition is modeled and analyzed. I employed a model similar to Martin's managerial efficiency model. I compared the results from the models with exogenous and endogenous number of firms, and innovation process is assumed to be either deterministic or stochastic. I find that when innovation process is deterministic, the basic results are consistent with Martin (1993). As the number of firms is exogenous, increased competition will shrink the demand facing each firm, and firms then have less incentive for process innovation. However, when the market could support more firms, such as the number of firms in the market grows proportional to the demand, or the number of firms is endogenous due to a fixed cost, the scale effect is diminished, and increased price elasticity dominates in this case which will induce firms to put more effort into innovation. Thus more competition will induce firms to devote more effort in innovation.

When innovation process is stochastic, ambiguous results are obtained when the number of firms in the market grows proportional to the demand. I find that there will be an inverted-U shape relation between innovation effort and market competitiveness. Moreover, when the number of firms is endogenous due to a fixed cost, I show that the number of firms will increase but not proportionally with the size of the market, and firms will have stronger incentive to put more effort for innovation under increasing competition for the model with endogenous number of firms due to fixed cost than the model where the number of firms changes proportionally with the size of the market.

## **VITA**

Ye Gu was born in Anhui, China. She received a bachelor's degree in finance from the Central University of Finance and Economics in Beijing in July 2008. Right after graduation from the undergraduate study, she came to the University of Tennessee to start PhD study in Economics. She has great interest in manipulating and analyzing data, so she also took classes in Statistics and will obtain a Master of Science degree in Statistics in May 2014. Influenced by her father who is a businessman, she is interested in studying firm's behavior and international trade. She likes cooking, singing, reading, and skiing in the spare time.