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Supplier Contracts with Profit Sharing, Open-book Costing and Associated Audit Rights

Hui Chen
University of Tennessee - Knoxville

To the Graduate Council:

I am submitting herewith a dissertation written by Hui Chen entitled "Supplier Contracts with Profit Sharing, Open-book Costing and Associated Audit Rights." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Business Administration.

James M. Reeve, Major Professor

We have read this dissertation and recommend its acceptance:

Bruce Behn, John T. Mentzer, Rudy Santore

Accepted for the Council:
Dixie L. Thompson

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

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Vice Chancellor and
Dean of Graduate Studies

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SUPPLIER CONTRACTS WITH PROFIT SHARING, OPEN-BOOK COSTING AND ASSOCIATED AUDIT RIGHTS

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Chen Hui
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ABSTRACT

Due to the growing trends in outsourcing and supply chain initiatives, inter-organizational control has become an increasingly important issue in today's business world. Many businesses set up the so-called "partnering" relationship with their key suppliers. The biggest challenge for this relationship is to find the best mechanisms that align the supplier's incentive with that of the buyer's and still maximize each individual firm's payoff. The purpose of this paper is to present a stylized profit-sharing contract, which is a type of contract commonly adopted to mitigate buyer-supplier incentive problems, and contrast several different types of supplier audit associated with it. Through the analysis of a dynamic Bayesian game, I find the mixed strategy audit in such a contract weakly dominates all other audit methods.

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1. Introduction

Supply chain advocates often say “collaborate or die”, however, an article in Harvard Business Review (Narayanan and Raman, 2004) reports that “companies often complain…that their supply chain partners don’t seem to want to do what is in everyone’s best interests, even when it’s obvious what’s best for the supply chain.” The authors point out that the root of such a problem is misaligned incentives between/among supply chain member companies, mainly due to certain informational asymmetries such as hidden action (e.g. the manufacturer cannot observe whether the retailer is trying hard to sell its products) or hidden information (e.g. the OEM does not know the true cost of its vendors). The article further suggests a good way to improve the situation is through better designed incentive contracts.

Why are incentive problems in supply chain context such a serious issue? In a market that is perfectly competitive, incentive problems pose a much smaller threat than in a market containing only a few players because competition can help resolve them. Perfect competition leads to an efficient price system, which significantly reduces problems associated with private information. Auction theory demonstrates the more bidders there are, the closer the bidding price will get to the bidder’ unobservable true valuation (McAfee & McMillan 1987). This means if there are plenty of suppliers associated with one buyer, their competition with each other will compete down/away the potential information rent they can earn should there be only one or a few of them.

However, the common practice for many companies today is not to buy from perfectly competitive supply market, but to set up long-term “partnering” relationship with their key suppliers, especially suppliers of highly customized products or components. The goal of this sort of relationship is to concentrate on core competency, reduce transaction cost, improve risk allocation to create "win-win" outcome. As a result, the number of suppliers significantly decreases as the level of integration between the supplier and buyer increases. For example, a study by Nishiguchi (1993) on automotive industry shows that North American automakers on average employ only 1.30 suppliers for major components, indicating a strong trend of single sourcing. The same applies to the suppliers – they typically serve only 3.6 regular customers with the biggest customer accounting for 79.9% of their total sales. The products are very customized to one major customer with the number of customers buying a particular component model being a mere 1.30. Similar examples can be easily found in government contracting, computer hardware industry, healthcare industry and so on.

Due to the significant amount of relationship-specific investment (such as R&D, marketing, human capital)[1] in supply chain context, the relationship between the buyer and supplier is effectively transformed into one that is bilateral monopolistic. As a result, the conventional market with perfect competition turns into a principal-agent relationship. A buyer faces only one or a few supplier(s) can no longer afford to ignore the existing incentive problems. She often has to take up the role of the principal and design certain contracts to incentivize the supplier(s) with private information. The optimal thing to do for a supplier that is given no incentive plan is to exert no effort or misreport cost

[1] Williamson (1985) calls it the "fundamental transformation" – once two parties start a transaction they become more likely to transact with each other in the future than with other potential candidates in the market.

information[2]. That is why contracts with a profit-sharing feature are so commonly-observed in many industries characterized with a thin supplier market. Laffont and Tirole (2002) sort the types of contracts by their “powers” of incentive: on one extreme is the fixed price contract, which makes the supplier the residual claimant and provides highest power of incentive; on the other extreme is the cost-plus contract, which makes the buyer the residual claimant and provides the lowest power of incentive; in between are various levels of profit-sharing contracts that are of intermediate incentive power.

Nishiguchi (1993) reports about 45% of the automakers in North America use profit-sharing contracts with their major suppliers; and the average sharing ratio is around 50-50. The profit/savings being shared usually comes from the difference between targeted cost and the actual cost achieved. Government contractors and many government-regulated industries have been given profit-sharing contracts for a long time. Laffont and Tirole (2002) provide a detailed survey of government procurement contracts. They state profit-sharing contracts have been growing rapidly since the 1980’s instead of the traditional “cost-plus” method. Reichelstein (1992) quote a 1987 U.S. General Accounting Office survey which claim the firms’ cost sharing ratio varies between 15% and 25%. Retail industry has also been observed to use a so-called “revenue-sharing” contract with the retailer and the supplier sharing the final revenue at an agreed ratio (Cachon 2001). The goal of such contracts is to incentivize the retailer to sell more of the

[2] Why do suppliers cheat by padding/over-reporting the costs? Laffont and Tirole (2002) summarize that there are only two causes. The first cause is the trade-off between incentive and information rent; the second being that the buyer’s desire to extract more rent from the supplier. Informational asymmetry is the key problem in profit-sharing arrangements, as the party with informational advantage can easily gain extra profit in addition to the agreed amount by not disclosing the private information. The “rent-seeking” behavior is essential to economic agents. As a result, the presence of private information often makes it impossible to achieve first-best outcome.

products supplied by the supplier while helping coordinate the inventory problem within the supply chain. Even in Hollywood, the motion picture industry has been using profit-sharing contracts traced back to the silent era (Weinstein, 1998).

Profit-sharing agreements serve to align incentives between the buyer and the supplier in a supply chain. They encourage relationship-specific investment and minimize shirking behavior. In an environment with uncertainty, profit-sharing also entails sharing of risks. As the above examples show, the actual arrangement can take different forms such as profit-sharing, savings-sharing (target costing), revenue sharing, and so on. However, they are all the same incentive scheme using but different contract parameters convenient for the industry.

A necessary condition for profit-sharing is open-book costing/pricing with associated audit/review rights. For profit-sharing between two parties to make sense, the cost data has to be transparent for the negotiation. Without the complete and "truthful" cost information, it is impossible to agree on any profit sharing ratio. Open-book costing simply grants the buyer access to the details of each cost component of the supplier's product/service, including materials, direct labor, overhead, etc. It also often gives the buyer rights to offer recommendations to the supplier on how costs can be further reduced. Of course always attached is the auditing or reviewing of the shared cost information as a mechanism to verify the accuracy of the information. The inaccuracy can come from both innocent clerical mistakes and purposeful data manipulation. Profit-sharing contracts often have detailed provisions on open-book terms and audit/review requirements. A typical such provision should include[3]: description of what is meant by "open book pricing"; relationship between pricing structure and open book pricing; umber and types of reviews/audits that may be made in any year of the open book records; reconciliation procedures if any review or audit shows that charges are not as required under the charging structure and open book provisions; period of time during which open book records required to be held; link with any benchmarking provisions in the contract.

Open-book costing/pricing with associated audit rights is such a useful tool that it even often appears in contracts that are not strictly profit-sharing per se[4]. In fact, as long as the charge is not a fixed price, open-book arrangements with audit rights are always good extra protections for the buyer. They have long been practiced by industries that adopt long-term incentive schemes with their suppliers, such as government contracting, government-regulated sectors, automotive industry, retail industry, construction industry and so on.

Supplier audit is also a topic worth some elaboration. Within accounting practice, it has always been an important component of internal auditing. *CPA Journal* (1995) provides a detailed description from a practitioner's point of view of how supplier audit can be conducted. One article titled *Outsourcing's Hidden Cost* from *Insurance and Technology (2003)* tells stories of how suppliers can inflate costs to hide secret profit for themselves and how buyers can recoup the lost profit through effective audits. The tricks the suppliers use include delaying time to pass price decreases to their buyers; buying cheaper materials and concealing the savings information; or simply taking advantage of

[3] Kindly provided by Belinda Doshi, who is a solicitor in the Technology Law Group at law firm Field Fisher Waterhouse.
[4] For example, cost-plus contracts often require open-book costing as well.

sloppily written contracts. As an example of a successful supplier audit, Hewlett-Packard recovered $30 million in 2001 that it had paid for indirect services more than agreed.

Governments have even more formal and established supplier audit agencies and procedures. Laffont and Tirole (2002) provide examples of government audit agencies such as General Accounting Office in the United States, Exchequer and Audit Department in Britain, Cour des Comptes in France and Bundesrechnungshof in Germany. They are very effective tools in monitoring government contractors and recovering excessive payments. A brief summary of DCAA (Defense Contract Audit Agency) of their 2004 work shows: "*In FY 2004, DCAA audited 10,299 pricing proposals with a total dollar value of $148.2 billion. We audited incurred costs and special audits during that same period of $112.7 billion. Approximately $2.1 billion in net savings were reported during the year. When compared to the $428.0 million expended for the Agency's operations, the return on taxpayers' investment in DCAA was $5.00 for each dollar invested.*"

As the above examples show, supplier contracts with features such as profit-sharing, open-book costing and auditing are an important business practice and worth academic investigation. The purpose of this paper is to present a stylized profit-sharing contract with random supplier audit in a game theoretical setting. I derive the optimal sharing ratios and production quantities that should be assigned to the supplier that might be of different cost types. I then contrast this contract with other feasible ones with audit probabilities being zero or one as well as evaluate their efficiency and appropriateness of use respectively. I also extend the model by adding a moral hazard element to the problem and demonstrate the resultant changes in contract parameters.

I find that under profit-sharing contracts with mixed strategy, neither the high-cost supplier nor the low-cost supplier earns any informational rent or gets a sharing ratio bigger than zero. Similar to the pure strategy incentive contract with zero auditing, the low-cost supplier is assigned with an efficient production quantity. However, the high-cost supplier's production quantity is upwardly distorted instead of downwardly distorted in the case of pure strategy incentive contracts. I also demonstrate that when the penalty imposed on the supplier increases or when the cost of audit for the buyer decreases to a certain value, the contract with random auditing dominates the pure strategy contract with zero auditing and the buyer's profit gets closer to the first-best result. When the moral hazard problem is added to the model, the optimal sharing ratios are no longer zero due to the risk-incentive tradeoff.

This study contributes to the existing literature of supply chain research by modeling and analyzing a type of commonly-seen supplier contracts involving profit sharing and supplier auditing. Unlike the well-known pure strategy incentive contracts that do not need auditing, the model in this paper employs a mixed strategy and allows cheating and ex-post auditing to occur in equilibrium. The findings of this study can provide some explanation and useful insights to the designing of supplier contracts. Furthermore, the results can also be generalized beyond the buyer-supplier context.

The rest of this paper consists of four sections. Section 2 provides a review of the related literature. Section 3 presents the basic model and characterizes the solution. Section 4 contrasts the model to the first-best scenario, the contract with zero auditing and the contract with deterministic auditing. Section 5 extends the model by adding a moral hazard element. Section 6 concludes the study.

2. Literature Review

The topic of this study relates to the following two streams of research: supply chain management and auditing.

2.1. Incentives in Supply Chain Management

Traditional research in supply chain management often assumes symmetric information. However, incentives are an important issue in supply chain practice and research. The supplier can have private information on cost or quality of the product, or the buyer can have private information on the market demand. They both could be subject to decisions on the amount of investment to make, which can also be affected by uncertainty of the environment and lead to the classic "hold-up" problem. In addition, unlike government regulating a monopolist that sells products to end-consumers or corporate authority coordinating transfer pricing between its two divisions, the buying firm in the supply chain is the principal herself. She does not have administrative power over the supplying firm and is only concerned with maximizing her own profit.

A significant problem facing the supply chain members is incomplete contracting, which means that certain variables cannot be contracted on because they are unverifiable[5]. Baiman and Rajan (2001) give a comprehensive review on the incentive problems in buyer-supplier relationship and the mechanisms to mitigate them in the framework of incomplete contracting. They demonstrate the trade-off between productive efficiency and the cost of opportunism and discuss the welfare change through different contracts.

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[5] Often these variables can be observed by both parties of the contract but cannot be verified by a third party in case of dispute. A typical example is the relationship-specific investment made by the parties involved.

Baiman and Rajan (2004) demonstrate the value of information-sharing and exchange through a buyer-supplier setting, in which the buyer has an innovation that can improve the quality of the product but the supplier may misappropriate this information if it is disclosed.

Moral hazard and adverse selection problems are also common in supply chain setting. Baiman et al. (2000) model moral hazard problems involving different types of quality costs in the supply chain. Baiman et al. (2001) further examine the contractual and incentive issues between the buyer and supplier on product quality issues. Corbett and de Groot (2000) derive the optimal quantity discount policy under asymmetric cost information.

There is also a stream of research that uses game-theoretical setting to model asymmetric information in a supply chain. Cachon (2003) and Cachon and Netessine (2004) provide examples and a review of supply chain literature on incentive conflicts and contract issues. Lim (2001) investigates a buyer's contract design problem when there is incomplete information regarding the quality of the parts delivered by the supplier using game theory.

2.2. Audit

Audit as a monitoring mechanism is used in many contexts such as monopolist regulation, procurement, financial contracting, tax audit and accounting audit. Audits in monopolist regulation setting and procurement setting are very similar. They are commonly conducted by the government (buyer) to verify the cost/quality information reported by the monopolist (supplier). Similar to procurement problem faced by private buyers such as in a supply chain, the government must design mechanisms to reduce the potential informational rent earned by the monopolistic supplier. However, unlike private buyers, the government is not the actual buyer of the product/service provided by the supplier. It is concerned with maximizing the total social welfare including the (weighted) sum of producer surplus and consumer surplus, while the principal in the supply chain problem only maximizes her own profit. The government also needs to set up correct price for the end-consumer and subsidy to the monopolist.

In financial contracting, a lender such as a bank faces a borrower with private information on his own profit/return level. Audit can help the creditor to reveal the borrower's true profit (Townsend 1978; Gale and Hellwig 1985; Williamson 1987). Tax auditing has also been analyzed in the principal-agent framework (Reinganum and Wilde 1985; Graetz, Reiganum and Wilde 1986; Scotchmer 1987; Mookherjee and P'ng 1989). In these models, income is the tax-payer's private information and cannot be freely observed. The tax authority's goal is to design optimal tax audit policy for a tax-payer that strategically chooses his reporting decision. Auditing in accounting is another important stream of research related to this paper. Unlike other audit models, the more current accounting audit models often involve three players[6]: a shareholder-principal with two agents, one is a firm manager, the other is an auditor (Demski and Sappington 1984). Often the focus of these multi-agent models is collusion between the manager and the auditor (Baiman et al. 1991).

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[6] Early auditing models in accounting did not explicitly model the auditor as a separate economic agent with his own incentives (Antle 1982), but more analytical studies seem to have changed from two-player game into three-player game.

Most of the analytical audit models mentioned above focus on pure strategy solutions. For example, Baron and Besanko (1984) set up a scenario in which the government conducts an audit to verify the monopolist's true cost. They derive an optimal result where audits are conducted in a deterministic manner as soon as a cost is reported above the cut-off value. Similar to the classic adverse selection problem as in Baron and Myerson (1982), the Revelation Principle is triggered and there is no cheating in equilibrium[7]. Audit and penalty are included in the contract to ensure truth-telling, but the agent will never cheat and no penalty will ever be collected.

Audit models with mixed strategy have the advantage that it depicts a world with cheating. Reinganum and Wilde (1985) investigate tax compliance through a mixed strategy game. Fellingham and Newman (1985) discuss an auditing game in which the auditor and client both use randomized strategies in deciding the audit procedures and the amount of effort to exert. Dunne and Loewenstein (1995) present a model including auditing with mixed strategy in an auction setting. Khalil (1997) analyzes auditing without commitment. This study also adopts the mixed strategy approach to demonstrate the optimal supplier contract in a simple dynamic Bayesian game.

[7] For a simplified version of their model, see Laffont and Martimort (2002).

3. The Model

3.1. Description of the Game

In a single period setting, a buyer has to delegate the production of certain product to a supplier. The product is highly customized, with no other potential outside buyers or sellers. They either transact with each other, or get zero payoffs. The supplier's marginal cost is c, which can only be privately observed by himself. This private cost information can be understood to come from the supplier's technology such as the productivity of his equipment, or the specific price discounts his own suppliers offer to him on certain raw materials or components. The buyer cannot freely obtain this information. There are two possible values of c, c_H and c_L, with probabilities f_H and f_L respectively and $f_L + f_H = 1$. The distribution of the probabilities is common knowledge to both the buyer and the supplier[8]. The total cost of the production is $C = c_i q_i$, where q is the quantity of the products being produced.

For simplicity, I assume the buyer does not add any extra value to the product[9]. The buyer sells the product to a market with demand q with $q'(p) < 0$, which implies the common negative relation between price and demand. The inverse demand function is $p(q)$. The price and market demand are assumed to be common knowledge for both the buyer and the supplier. The buyer and the supplier also agree on a certain "profit-sharing" arrangement. The buyer reimburses the cost the supplier reported, plus a percentage S of the total profit made. So the payment to the supplier will take the form

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[8] This assumption is reasonable because the buyer usually has enough experience or market knowledge to be able to give an estimation of the distribution of the cost even though she doesn't know the actual value taken by the supplier.

[9] The assumption sounds unrealistic, but it does not change any qualitative relation between the variables.

$Sq(p(q)-c)+cq$. The purpose of the profit-sharing arrangement is to provide incentives for the supplier to reduce the cost. The lower the cost of the product, the lower the price the buyer will set, and the more quantity will be sold to the end market.

The buyer can inspect the supplier's reported cost through an audit, and the results of the audit can be verified by a third party or a court. The audit can reveal the supplier's true cost without error, but will cost the buyer an amount of money/resource k. If the supplier is caught cheating, he will be subject to a penalty $d_C + d_U$, where d_C represents the amount of penalty that can be collected by the buyer and d_U represents uncollectible penalty such as the supplier's loss of future business and the potential damage to his reputation. However, if the audit shows the supplier being truthful, there is no bonus for his honesty.

The problem can be described in a simple dynamic Bayesian game. First nature decides the type of the cost possessed by the supplier. It is only learnt by the supplier. The buyer offers a menu of contract for the supplier to choose from. The menu includes a contract designed for the low-cost supplier and a contract designed for the high-cost supplier. Knowing his true cost, the supplier has two choices: to cheat or to be truthful. By cheating, he simply takes the contract not designed for his cost type; and by being truthful he takes the contract corresponding to his cost. Then production takes place corresponding to the contract he takes. Observing the outcome of the production, the buyer also has two choices: to audit or to not audit. If the audit is performed and the supplier is caught cheating, he has to submit the fine as stipulated. At the end of the game, the buyer sells the product to the final market and split the gain with the supplier. The timeline and the extensive form of game can be demonstrated in figure 1.

If the buyer's probability of auditing is α , then the ex-ante contract offered by the buyer to the supplier will take the form of $\{(q_H, S_H);(q_L, S_L),\alpha, d_C\}$. This menu of contracts include desired production quantities and corresponding profit sharing ratios for each type of the two costs, as well as a probability of audit and a fixed amount of penalty for supplier's cheating behavior. The penalty here is assumed to be exogenously decided[10]. The supplier will choose one contract from the quantity-sharing ratio menu, together with the associated audit and penalty.

Suppose the supplier's probability of being truthful is t . When the cost is high, he chooses to be truthful by producing quantity q_H with probability t_H , and to cheat by producing quantity q_L with probability $1-t_H$. When the cost is low, he chooses to be truthful by producing quantity q_L with probability t_L , and to cheat by producing quantity q_H with probability $1-t_L$. The principal does not know the true state of the cost, but she anticipates that the supplier might cheat. She estimates that the probability of the supplier reporting high cost and producing quantity q_H is $\pi_H = f_H t_H + f_L(1-t_L)$, and the probability of the agent reporting low cost and producing quantity q_L is $\pi_L = f_L t_L + f_H(1-t_H)$. When she performs her audit, she estimates the probability of catching the supplier with low cost pretending to have high cost is $\pi_{H|L} = \frac{f_L(1-t_L)}{\pi_H}$, and

[10] For the case of endogenous penalty with limited liability constraint from the supplier's side, please see Laffont and Martimort (2002).

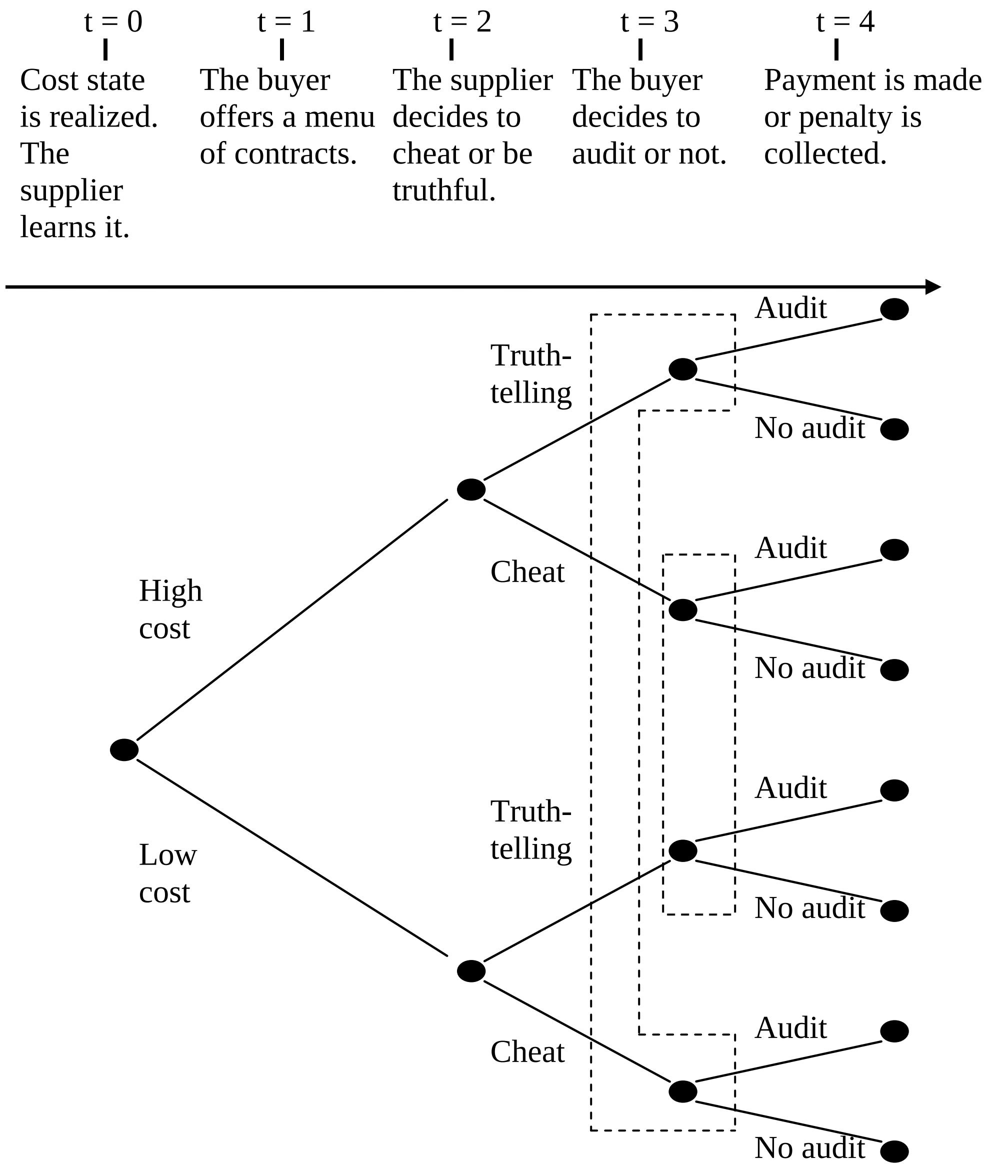


Figure 1. Game Tree

the probability of catching the supplier with high cost pretending to have low cost is

$\pi_{L|H} = \dfrac{f_H(1-t_H)}{\pi_L}$. The buyer also decides on a probability of audit α. If the audit

showsthat the supplier is being truthful, the buyer pays him his cost plus the agreed percentage of total profit; if the audit shows that the supplier cheats, he has to pay penalty d_C to the buyer and suffer an uncollectible penalty d_U.

I assume a supplier with high cost will not report low cost[11], so we can be sure that $t_H = 1$. As a result, the buyer's estimated probability of the agent reporting low cost becomes $\pi_L = f_L t_L$, and the buyer's estimated probability of the agent reporting high cost becomes $\pi_H = f_H + f_L(1-t_L)$. Again since $f_L + f_H = 1$, we know $\pi_H = 1 - f_L t_L$. It is also obvious that when a supplier reports low cost, the buyer will not need to perform an audit, so $\alpha_L = 0$.

3.2. Mixed Strategy

As pure strategies are simply degenerate forms of mixed strategy, I will first focus on the mixed strategy equilibrium of this game. The problem facing the buyer is hence:

$$\begin{aligned}
&\max_{q_H, q_L, S_H, S_L, \alpha, d_C} \pi_H\{p(q_H)q_H - (1-\alpha_H)(S_H q_H(p(q_H) - c_H) + q_H c_H) \\
&+ \alpha_H[\pi_{H|L} d_C - (1-\pi_{H|L})(S_H q_H(p(q_H) - c_H) + q_H c_H) - k]\} \\
&+ \pi_L[p(q_L)q_L - (S_L q_L(p(q_L) - c_L) + q_L c_L)] \qquad (1) \\
&= \max_{q_H, q_L, S_H, S_L, \alpha, d_C} \pi_H\{p(q_H)q_H - (S_H q_H(p(q_H) - c_H) + q_H c_H) \\
&+ \alpha_H[\pi_{H|L}(d_C + S_H q_H(p(q_H) - c_H) + q_H c_H) - k]\} + \pi_L[(1-S_L)q_L(p(q_L) - c_L)] \qquad (1')
\end{aligned}$$

subject to:

$$S_H q_H(p(q_H) - c_H) \geq 0 \qquad (\mathrm{IR_H})$$

[11] This is indeed a reasonable assumption as will be shown later in the paper.

$$t_L S_L q_L(p(q_L)-c_L)+(1-t_L)\{(1-\alpha_H)[S_H q_H(p(q_H)-c_H)$$
$$+q_H c_H - q_H c_L]-\alpha_H(d_c+d_u+q_H c_L)\}\geq 0 \qquad (\mathrm{IR_L})$$

$$t_L \in \arg\max_{t_L^*} t_L^* S_L q_L(p(q_L)-c_L)+(1-t_L^*)\{(1-\alpha_H)[S_H q_H(p(q_H)-c_H)$$
$$+q_H c_H - q_H c_L]-\alpha_H(d_c+d_u+q_H c_L)\} \qquad (\mathrm{ST})$$

$$\alpha_H \in \arg\max_{\alpha_H^*} \alpha_H^*\{\pi_{H|L}[d_C+S_H q_H(p(q_H)-c_H)+c_H q_H]-k\} \qquad (\mathrm{BA})$$

The buyer's objective function is her expected payoff under the two cost states that she estimates. When the reported cost is high, her payoff is her valuation of the delivered products minus her payment to the supplier when she does not audit and her payment to the supplier when she does audit. When she audits a supplier that reports high cost, she pays him the agreed amount if he is found to be truthful and collects a penalty d_c if he is found to be cheating. In either case, she incurs the auditing cost k. The buyer does not perform audit when the reported cost is low.

The $\mathrm{IR_H}$ condition is the high-cost supplier's participation constraint and ensures that he enters the contract. The $\mathrm{IR_L}$ condition is the low-cost supplier's participation constraint. The ST condition is the low-cost supplier's strategy about being truthful or cheating through maximizing his total payoff. The BA condition is the buyer's auditing strategy which maximizes her payoff related to the audit.

From the FOC of the ST condition, we get:

$$S_L q_L(p(q_L)-c_L)=(1-\alpha_H)[S_H q_H(p(q_H)-c_H)+q_H c_H - q_H c_L]$$
$$-\alpha_H(d_C+d_U+q_H c_L) \qquad (2)$$

It indicates the low-cost supplier's payoff if he is truthful equals his payoff if he cheats. As a result he should be indifferent between being truthful and cheating. The equation can also be written as:

$$\alpha_H = \frac{S_H q_H (p(q_H) - c_H) + q_H c_H - q_H c_L - S_L q_L (p(q_L) - c_L)}{S_H q_H (p(q_H) - c_H) + q_H c_H + d_C + d_U} \tag{3}$$

It is exactly the probability of audit the buyer will employ. Note that the larger the penalty $d_C + d_U$ is, the less frequent the buyer needs to conduct audit.

From the FOC of the BA condition, we get:

$$\pi_{H|L}[d_C + S_H q_H (p(q_H) - c_H) + q_H c_H] = k \tag{4}$$

It indicates that the buyer's gain from the audit equals the cost of the audit, so she should also be indifferent between auditing and not auditing given that the reported cost is high. Since $\pi_H = 1 - f_L t_L$, we know $\pi_{H|L} = \frac{f_L - f_L t_L}{1 - f_L t_L}$. Substituting it in equation (4), we get:

$$t_L = \frac{f_L[d_C + S_H q_H (p(q_H) - c_H) + q_H c_H] - k}{f_L[d_C + S_H q_H (p(q_H) - c_H) + q_H c_H - k]} \tag{5}$$

This is the probability the low-cost supplier chooses to be truthful based on the contract parameters. Note that when k approaches zero, t_L will approach 1, which means the supplier will not lie if the cost of audit for the buyer is infinitely small.

Further substituting the (2) and (4) into the buyer's objective function (1), the buyer's problem is simplified into:

$$\max_{q_H, q_L, S_H, S_L, d_C} \pi_H \{p(q_H) q_H - [S_H q_H (p(q_H) - c_H) + q_H c_H]\}$$
$$+ \pi_L \{p(q_L) q_L - [S_L q_L (p(q_L) - c_L) + q_L c_L]\} \tag{6}$$

$$= \max_{q_H, q_L, S_H, S_L, d_C} \frac{f_H[d_C + S_H q_H (p(q_H) - c_H) + q_H c_H]}{d_C + S_H q_H (p(q_H) - c_H) + q_H c_H - k} [(1 - S_H) q_H (p(q_H) - c_H)]$$
$$+ \frac{f_L[d_C + S_H q_H (p(q_H) - c_H) + q_H c_H] - k}{d_C + S_H q_H (p(q_H) - c_H) + q_H c_H - k} [(1 - S_L) q_L (p(q_L) - c_L)] \tag{6'}$$

subject to:

$$S_H q_H (p(q_H) - c_H) \geq 0 \qquad (\mathrm{IR_H})$$

$$S_L q_L (p(q_L -) c_L) \geq 0 \qquad (\mathrm{IR_L^{'}})$$

Proposition 1. In a contract with random audit, neither the high-cost nor the low-cost supplier receives any informational rent.

The updated problem indicates that neither high-cost agent nor the low-cost agent earns any informational rents under this contract, which means the two participation constraints are binding. The proof is simple: suppose the payoffs of the supplier under two cost states are larger than his reservation payoff, then the buyer could always lower her payment to him to achieve higher profit for herself.

With *Proposition 1,* it can also be shown that a supplier of high cost will never misreport his cost as low. If the buyer audits with probability α_H, a supplier which is low cost is indifferent between cheating and telling truth following equation (2). However, a supplier which is high cost will be strictly worse off by pretending to be of low cost. In fact, he will have a payoff that is negative even when there is no penalty imposed. That is,

$$S_H q_H (p_H - c_H) = 0 > (1-\alpha_H)[S_L q_L (p_L - c_L) + q_L c_L - q_L c_H] = (1-\alpha_H)[q_L (c_L - c_H)$$

Proposition 2. In a contract with random audit, the production quantity assigned to low-cost supplier is optimal while the production quantity assigned to high-cost supplier is upwardly distorted.

Proof. Substituting the two new constraints into the buyer’s objective function, the buyer’s problem is:

$$\max_{q_H, q_L} \frac{f_H (d_C + q_H c_H)}{d_C + q_H c_H - k}[q_H (p(q_H) - c_H)] + \frac{f_L (d_C + q_H c_H) - k}{d_C + q_H c_H - k}[q_L (p(q_L) - c_L)] \qquad (7)$$

Note that the first-order condition for q_L shows $q_L p'(q_L) + p = c_L$, which means the price and the assigned quantity associated with low-cost supplier are both efficient with $p_L = p_L^*$ and $q_L = q_L^*$, where p_L^* and q_L^* are the first-best solution. The price and quantity associated with the high-cost supplier can be given by the following implicit function:

$$\frac{kf_H c_H}{(d_C + c_H q_H - k)^2}[q_L(p(q_L) - c_L) - q_H(p(q_H) - c_H)]$$
$$+ \frac{(d_C + c_H q_H) f_H}{d_C + c_H q_H - k}[q_H p'(q_H) + p(q_H) - c_H)] = 0$$

The first term of the left-hand side of the above equation is positive, the coefficient of the second-term of the left-hand side is also positive, so $q_H p_H^{'} + p - c_H$ must be negative to satisfy the equation. It is then clear that $q_H p'(q_H) + p < c_H$, so we know that q_H must be upwardly distorted from the optimal quantity q_H^*.

3.3. Pure Strategies

There are two possible pure strategy equilibria in this game: pooling equilibrium and separating equilibrium. The pooling equilibrium requires the supplier to choose the same contract (to report the same cost) despite his true cost type. The separating equilibrium requires the supplier to always choose different contracts (to report different costs) according to his true cost type.

The only possible pooling equilibrium is when both types of supplier report high cost[12]. For this to be true, the buyer must never audit at all. The buyer will not audit only when her gain from the audit is smaller than her cost of audit. That is:

$$\pi_{H|L}[d_C + S_H q_H (p(q_H) - c_H) + q_H c_H]$$
$$= \frac{f_L(1-t_L)}{f_H + f_L(1-t_L)}[d_C + S_H q_H (p(q_H) - c_H) + q_H c_H] \leq k \quad (8)$$

If the buyer never audits, the low-cost supplier will always lie, which means $t_L = 0$. Substituting into the above equation, we get:

$$f_L[d_C + S_H q_H (p_H - c_H) + q_H c_H] \leq k \quad (9)$$

When the cost of auditing is higher than the gain from auditing, the buyer will never audit. Accordingly, the low-cost supplier will always cheat. The belief and the strategy effectively form a perfect Bayesian equilibrium.

Proposition 3. When (9) is satisfied, there exists a pooling equilibrium for the audit game. The strategies of the game are: both high-cost and low-cost suppliers report high cost and the buyer never audits; the beliefs of the game are: seeing a high-cost report, the probability of the supplier being high-cost is f_H; the probability of seeing a low-cost report is 0.

The separating equilibrium requires the high-cost supplier to report high cost and low-cost supplier to report low cost. Thus the low-cost supplier should never cheat. For this to be true, the low-cost supplier's payoff when he cheats must be smaller than his payoff when he tells truth. That is:

$$S_L q_L (p(q_L) - c_L) \geq S_H q_H (p(q_H) - c_H) + q_H c_H - q_H c_L \quad (10)$$

[12] It is not feasible for both types to report low cost, because the high-cost supplier will not be able to do it.

If the low-cost supplier does not have any incentive to cheat, we will have $t_L = 1$. Thus $\pi_{H|L} = \frac{f_L(1-t_L)}{f_H + f_L(1-t_L)} = 0$ must be true, which means that the buyer believes that the all high cost reports are from high-cost supplier. As a result, the buyer will never audit, which is a perfect Bayesian equilibrium. The problem therefore transforms into the familiar hidden information model, where the Revelation Principle is triggered and no cheating or auditing will occur in equilibrium.

Proposition 4. When (10) is satisfied, there exists a separating equilibrium for the auditing game. The strategies of the game are: the high-cost supplier always reports high cost and the low-cost supplier always reports low cost; the buyer never audits. The beliefs of the game are: seeing a high-cost report, the probability of the supplier being high-cost is 1; seeing a low-cost report, the probability of the supplier being low-cost is 1.

I also briefly present the pure incentive contract that complies with the Revelation Principle and does not involve any auditing, which has been shown above to be a pure strategy of the audit game discussed. The buyer offers a menu of contract $\{(q_H, S_H);(q_L, S_L)\}$ for the supplier to choose from. The incentive compatibility constraints in an ex-ante manner effectively prevent the supplier from cheating. The buyer's problem is:

$$\max_{q_H, q_L, S_H, S_L, \alpha, d_C} f_H\{p_H q_H - [S_H q_H(p(q_H) - c_H) + q_H c_H]\} + f_L\{p_L q_L - [S_L q_L(p(q_L) - c_L) + q_L c_L]\}$$

s.t.

$$S_H q_H (p(q_H) - c_H) \geq 0 \qquad (\text{IR}_\text{H})$$

$$S_L q_L (p(q_L) - c_L) \geq 0 \qquad (\text{IR}_\text{L})$$

$$S_L q_L (p(q_L) - c_L) \geq S_H q_H (p(q_H) - c_H) + q_H c_H - q_H c_L \qquad (\text{IC}_\text{L})$$

$$S_H q_H (p(q_H) - c_H) \geq S_L q_L (p(q_L) - c_L) + q_L c_L - q_L c_H \qquad (\text{IC}_\text{H})$$

Please note that the IC_L condition is simply the condition for the separating equilibrium to be true. The solution of the problem is also familiar: q_L is efficient while q_H is downwardly distorted. In equilibrium, low-cost supplier picks a contract with higher sharing ratio than the high-cost supplier.

One intriguing aspect of the pure strategies is that no auditing is needed. The pooling equilibrium shows that the buyer does not bother with auditing because the cost of audit exceeds the gain from the audit. The separating equilibrium shows that the buyer does not need to audit because the supplier will not cheat anyway. It is only when both conditions for pooling equilibrium and separating equilibrium are violated, the result is the mixed strategy equilibrium that involves auditing as described in section 3.2.

4. Comparison with the First-best and Deterministic Audit Contract

Now I compare the random audit contract with the first-best scenario and the deterministic audit contract to evaluate the welfare and validity of these contracts.

4.1 Comparison with the First-best

The first-best result can only be achieved when there is no informational problem. In the buyer-supplier setting described above, the buyer would know the exact cost type of the supplier and therefore offers a contract designed to him. No audit is needed. The buyer's problem is therefore:

$$\max_{q_H, q_L, S_H, S_L} f_H\{p(q_H)q_H - [S_H q_H(p(q_H)-c_H) + q_H c_H]\} + f_L\{p(q_L)q_L - [S_L q_L(p(q_L)-c_L) + q_L c_L]\}$$

subject to:

$$S_H q_H(p(q_H)-c_H) = 0 \qquad (\text{IR}_\text{H})$$

$$S_L q_L(p(q_L)-c_L) = 0 \qquad (\text{IR}_\text{L})$$

Neither high-cost supplier nor low-cost supplier earns any rent, which means S_H and S_L are both zero. The quantities assigned to the two types of supplier are also optimal. All surplus of the transaction goes to the buyer. The buyer's gain is:

$$f_H[q_H(p(q_H)-c_H)] + f_L[q_L(p(q_L)-c_L)] \qquad (11)$$

As demonstrated in section 3, the buyer's payoff in the model with mixed strategy audit is:

$$\frac{f_H(d_C + q_H c_H)}{d_C + q_H c_H - k}[q_H(p(q_H)-c_H)] + \frac{f_L(d_C + q_H c_H) - k}{d_C + q_H c_H - k}[q_L(p(q_L)-c_L)] \qquad (7)$$

It also leaves both high-cost and low-cost supplier with zero rent. However, the quantity assigned to low-cost supplier is optimal while the quantity assigned to high-cost supplier is upwardly distorted.

As demonstrated before,

$$\frac{f_H[d_C + S_H q_H (p(q_H) - c_H) + q_H c_H]}{d_C + S_H q_H (p(q_H) - c_H) + q_H c_H - k} = \pi_H \equiv f_H + f_L(1 - t_L)$$

$$\frac{f_L[d_C + S_H q_H (p(q_H) - c_H) + q_H c_H] - k}{d_C + S_H q_H (p(q_H) - c_H) + q_H c_H - k} = \pi_L \equiv f_L t_L$$

It is clear when k approaches zero, π_H will approach f_H and π_L will approach f_L. The buyer's payoff will get closer to the first-best result. We then have the next proposition:

Proposition 5. As the audit cost k decreases or as the penalty d_C increases, the buyer's payoff approaches that of the first-best result.

The intuition is easy to understand: if the hidden information can be found out through inexpensive audits, then problem of informational asymmetry will simply evaporate. Furthermore, if the penalty d_C imposed onto the supplier becomes infinitely high, the supplier will no longer "dare" cheat.

Please also note that d_U does not directly affect the buyer's payoff. The uncollectible penalty only affects the probability of the buyer's audit. However, since audit does not generate any revenue for the buyer through penalty collection, her final payoff is not changed.

From the above comparison between the mixed strategy audit contract and the first-best scenario, we see that audit contract with mixed strategy can outperform the contract with pure strategies under certain circumstances[13]. This finding has significant empirical implication – the reason we observe cheating and auditing in business reality is perhaps because people play mixed strategies and the cost to eliminate the hidden information is also low enough.

4.2. Comparison with Audit Contract with Deterministic Audit

For the completeness of audit models, I also briefly present a model with deterministic audit with probability 1. This model is first used by Baron and Besanko (1984). Its key feature is that the buyer always audits the supplier as long as she sees high-cost being reported. Since high-cost production is 100% audited, the supplier will never cheat. Hence in equilibrium, the contract induces truth-telling so the penalty will never be collected. The buyer effectively designs a contract including audit as a monitoring tool that is incentive compatible.

The problem facing the buyer is:

$$\max_{q_H, q_L, S_H, S_L, \alpha, d_C} f_H\{p(q_H)q_H - [S_H q_H(p(q_H) - c_H) + q_H c_H] - \alpha_H k\}$$
$$+ f_L\{p(q_L)q_L - [S_L q_L(p(q_L) - c_L) + q_L c_L]\}$$

s.t.

$$S_H q_H(p(q_H) - c_H) \geq 0 \qquad (\text{IR}_H)$$

$$S_L q_L(p(q_L) - c_L) \geq 0 \qquad (\text{IR}_L)$$

---

[13] The pure strategy separating equilibrium is simply the familiar incentive contract that complies with the Revelation Principle. The fact that it can be outperformed under certain circumstances by a mixed strategy contract that does not comply with the Revelation Principle shows the limitation of the Revelation Principle. One assumption of the Revelation Principle is commitment. When commitment cannot be guaranteed (as in most of the cases in real life), the Revelation Principle will not always hold. The buyer-supplier scenario presented in this paper does not assume commitment of auditing, and pure strategies can be shown to be weakly dominated by the mixed strategy.

$$S_L q_L (p(q_L) - c_L) \geq (1-\alpha_H)[S_H q_H (p(q_H) - c_H) + q_H c_H - q_H c_L]$$
$$-\alpha_H (d_C + d_U + q_H c_L) \qquad (\mathrm{IC_L})$$

$$S_H q_H (p(q_H) - c_H) \geq (1-\alpha_L)[S_L q_L (p(q_L) - c_L) + q_L c_L - q_L c_H]$$
$$-\alpha_L (d_C + d_U + q_L c_H) \qquad (\mathrm{IC_H})$$

Again we know that the buyer will not audit a supplier that reports low cost, so $\alpha_L = 0$. The IC_H condition therefore collapses into the first. Compared to the pure strategy incentive model with separating equilibrium, we see that the use of audit further relaxes the incentive compatibility constraints. The larger the penalty $d_C + d_U$ is, the easier the IC constraints are met.

The IR_H and the IC_L constraints are binding. The solution shows that the buyer should offer a menu of contracts for the supplier to choose from, in which the margin S_L for low cost supplier should be higher and the margin S_H for high cost supplier should be lower, which is also very similar to the separating equilibrium solution analyzed before.

A major flaw of this model is the issue of commitment. The buyer has to commit to conduct audit whenever she sees a report of high-cost. If she always audits, the supplier will never cheat, that is, $t_L = 1$. However, if the supplier never cheats, the buyer does not have to audit all the time. She will be better off if she deviates from her auditing policy. The perfect Bayesian equilibrium cannot exist with this contradiction[14].

Proposition 6. Supplier contract with deterministic probability of audit does not constitute a perfect Bayesian equilibrium of the audit game due to the conflict between game strategies and beliefs.

[14] Some principals might be able to commit to this ex-post inefficient audit strategy because of reputational concerns (Khalil, 1997). For example, the government might have to audit its contractors as a public policy.

5. Comparative Statistics and Numerical Illustration

Proposition 5 shows that the buyer's payoff increases as the audit cost k decreases or as the penalty d_C increases. It is also interesting to see how the optimal contract parameters change when these two exogenous variables k and d_C change. The solution to the random audit contract includes two sharing ratios S_H and S_L that are both zero and a production quantity assigned to the low-cost supplier q_L that is first-best, thus only the production quantity assigned to the high-cost supplier q_H truly varies with the exogenous variables. I perform comparative static analysis to evaluate the relationships between q_H and k and d_C.

Let $$\frac{kf_H c_H}{(d_C + c_H q_H - k)^2}[q_L(p(q_L) - c_L) - q_H(p(q_H) - c_H)] + \frac{(d_C + c_H q_H) f_H}{d_C + c_H q_H - k}[q_H p'(q_H) + p(q_H) - c_H)] = \omega(q_H, d_C, k) = 0$$, I get

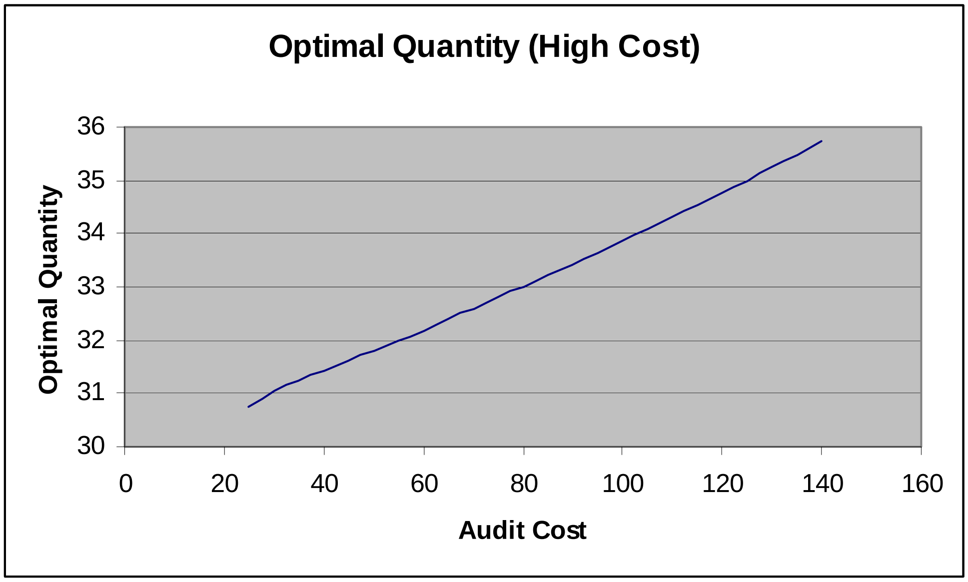
$$\frac{\partial q_H}{\partial d_C} = -\frac{\frac{\partial \omega(q_H, d_C, k)}{\partial d_C}}{\frac{\partial \omega(q_H, d_C, k)}{\partial q_H}} < 0$$ and $$\frac{\partial q_H}{\partial k} = -\frac{\frac{\partial \omega(q_H, d_C, k)}{\partial k}}{\frac{\partial \omega(q_H, d_C, k)}{\partial q_H}} > 0$$. These results imply that the optimal production quantity assigned to a high-cost supplier q_H decreases with the penalty d_C and increases with the audit cost k.

I further illustrate these relationships with a numerical example. Suppose the product unit cost can be either \$10 (high) with 30% probability or \$5(low) with 70% probability. The market demand takes a simple linear form $p = 25 - 0.25q$. I will first set the audit cost to be \$25 and collectible penalty to be \$50. These two values will be varied later to show the induced changes in optimal production quantity for high-cost supplier and the buyer's payoff. The uncollectible penalty is also $50.

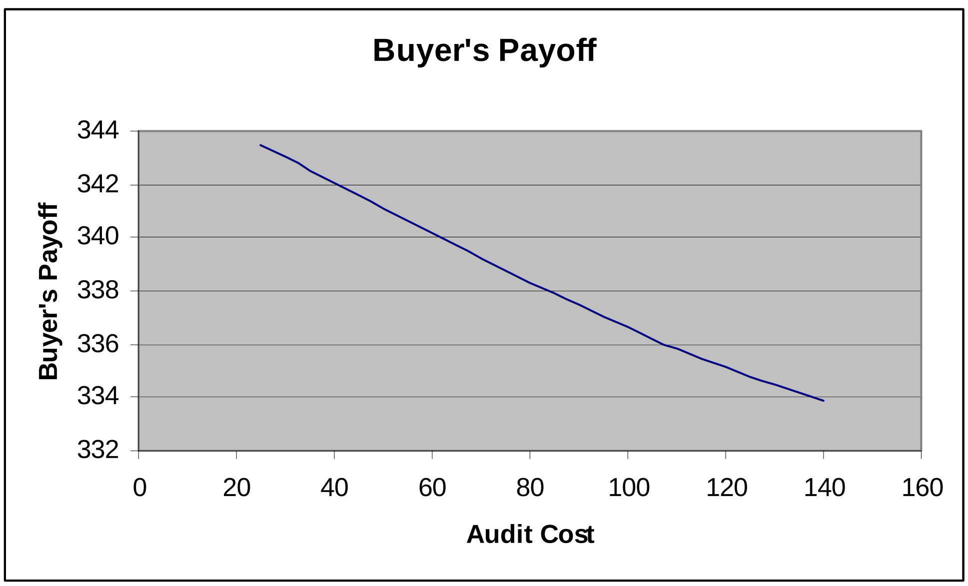
Substitute these numbers into the random audit contract, we can calculate that the optimal production quantity assigned to the low-cost supplier is 40 units and the corresponding final price is $15. From the simulation, we also get that the optimal production quantity assigned to the high-cost supplier is 30.74 units and the corresponding final price is $ 17.32. The buyer audits with a probability of around 38% and the low-cost supplier reports true cost with a probability of 97%. The total profit of the project and the payoff to the buyer are $343.51, while both high-cost and low-cost suppliers' payoffs are zero as they get zero share of the total profit.

Now I will increase the values of audit cost and collectible penalty to show the changes induced in optimal production quantity for high-cost supplier and the buyer's payoff. I will demonstrate the relations in figure 2 and figure 3.

We can see as the audit cost increases, the optimal quantity for high-cost supplier increases while the buyer's payoff decreases; as the collectible penalty increases, the optimal quantity for high-cost supplier decreases while the buyer's payoff increases. The numerical example supports the comparative static analysis.



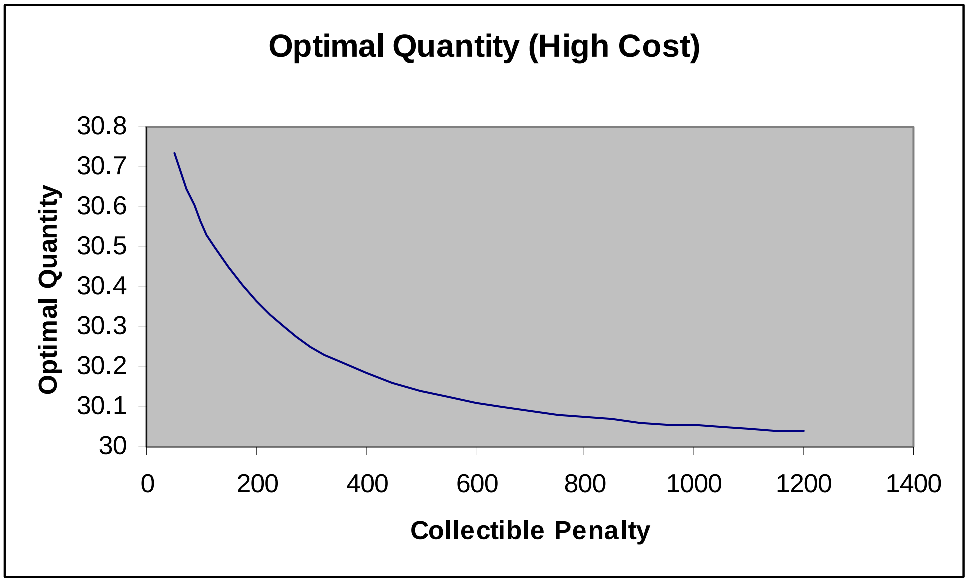
(A)



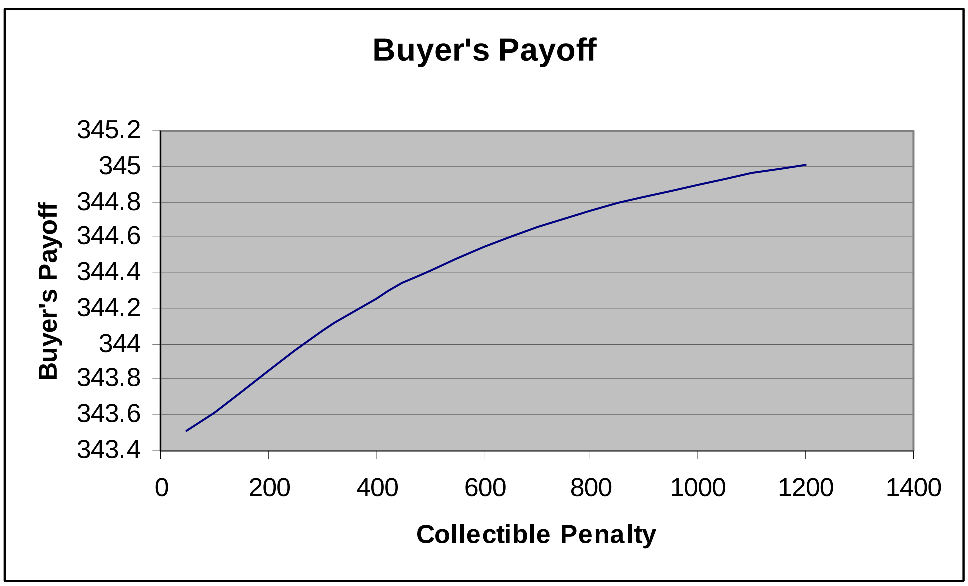
(B)

Figure 2. Graphic Illustration – Audit Cost

(A) Change in Optimal Qty. for High-cost Supplier, (B) Change in Buyer's Payoff



(A)



(B)

Figure 3. Graphic Illustration – Collectible Penalty

(A) Change in Optimal Qty. for High-cost Supplier, (B) Change in Buyer's Payoff

6. Extended Model with Moral Hazard Problem

In a setting same as of the basic model, the supplier can also exert a certain amount of effort e, which will further reduce the manufacturing cost of the product. The effort e stands for activities such as better engineering design, waste minimization or quality improvement. Different from c, e is a type of "on-the-job consumption". If the supplier chooses to shirk, the buyer cannot verify it even at a cost. The agent's disutility for effort is $v(e)$. The total cost of the production is $C = q_i(c_i - e)$, where q is still the quantity of the products being produced.

For mathematical simplicity, I assume the buyer sells the product to a market with a linear demand function $q = a - bp + \theta$ with $b > 0$ and $\theta \sim N(0, \sigma^2)$ representing the uncontrollable fluctuation of the market. The inverse demand function is simply $p = \frac{a + \theta - q}{b}$. The price and market demand are still assumed to be common knowledge for both the buyer and the supplier.

The total contribution margin (or profit as I assume there is no fixed cost) of the project is $q(\frac{a + \theta - q}{b} - c + e)$. The buyer is assumed to be risk-neutral, which means she simply maximizes expected profit. The supplier is risk averse with CARA utility function, $u(x) = -e^{-\rho x}$, $(\rho > 0)$, where ρ is the degree of risk-aversion and x is the his monetary income. The buyer and the supplier agree on a certain "profit-sharing" arrangement. The buyer reimburses the cost the supplier reported, plus a percentage S of the total profit made. The payment to the supplier therefore takes the form

$Sq(\frac{a+\theta-q}{b}-c+e)+q(c-e)$. Using the mean variance expression, we know the certainty equivalent of this payment to the supplier is

$Sq(\frac{a-q}{b}-c+e)+q(c-e)-\frac{\rho}{2}S^2q^2\sigma^2$. The buyer retains the rest of the expected profit $E[(1-S)q(\frac{a+\theta-q}{b}-c+e)]=(1-S)q(\frac{a-q}{b}-c+e)$. The purpose of the profit-sharing arrangement is to provide incentives for the supplier to reduce the cost. The lower the cost of the product, the lower the price the buyer will set, and the more quantity will be sold to the end market.

The problem facing the buyer is:

$$\max_{q_H,q_L,S_H,S_L e_H,e_L} \pi_H\{q_H\frac{a-q_H}{b}-(1-\alpha_H)[S_Hq_H(\frac{a-q_H}{b}-c_H+e_H)+q_H(c_H-e_H)]$$
$$+\alpha_H[\pi_{H|L}d_C-(1-\pi_{H|L})(S_Hq_H(\frac{a-q_H}{b}-c_H+e_H)+q_H(c_H-e_H))-k]\}$$
$$+\pi_L\{q_L\frac{a-q_L}{b}-[S_Lq_L(\frac{a-q_L}{b}-c_L+e_L)+q_L(c_L-e_L)]\} \quad (1)$$

$$=\max_{q_H,q_L,S_H,S_L,e_H,e_L} \pi_H\{q_H\frac{a-q_H}{b}-[S_Hq_H(\frac{a-q_H}{b}-c_H+e_H)+q_H(c_H-e_H)]$$
$$+\alpha_H[\pi_{H|L}(d_C+S_Hq_H(\frac{a-q_H}{b}-c_H+e_H)+q_H(c_H-e_H))-k]\}$$
$$+\pi_L\{q_L\frac{a-q_L}{b}-[S_Lq_L(\frac{a-q_L}{b}-c_L+e_L)+q_L(c_L-e_L)]\}$$

subject to:

$$S_Hq_H(\frac{a-q_H}{b}-c_H+e_H)-\frac{\rho}{2}S_H^2q_H^2\sigma^2-v(e_H)\geq 0 \quad (\mathrm{IR}_H)$$

$$t_L[S_L q_L(\frac{a-q_L}{b}-c_L+e_L)-\frac{\rho}{2}S_L^2 q_L^2\sigma^2]+(1-t_L)\{(1-\alpha_H)[S_H q_H(\frac{a-q_H}{b}-c_H+e_H)$$
$$-\frac{\rho}{2}S_H^2 q_H^2\sigma^2+q_H(c_H-e_H)-q_H(c_L-e_L)]-\alpha_H[d_c+d_u+q_H(c_L-e_L)]\}-v(e_L)\geq 0 \quad (\mathrm{IR_L})$$

$$e_H \in \arg\max_{e_H^*} S_H q_H(\frac{a-q_H}{b}-c_H+e_H^*)-v(e_H^*)-\frac{\rho}{2}S_H^2 q_H^2\sigma^2 \quad (\mathrm{IC_H})$$

$$e_L \in \arg\max_{e_L^*} t_L[S_L q_L(\frac{a-q_L}{b}-c_L+e_L^*)-\frac{\rho}{2}S_L^2 q_L^2\sigma^2]+(1-t_L)\{(1-\alpha_H)$$
$$[S_H q_H(\frac{a-q_H}{b}-c_H+e_H^*)-\frac{\rho}{2}S_H^2 q_H^2\sigma^2+q_H(c_H-e_H^*)-q_H(c_L-e_L^*)]$$
$$-\alpha_H[d_c+d_u+q_H(c_L-e_L^*)]\}-v(e_L^*) \quad (\mathrm{IC_L})$$

$$t_L \in \arg\max_{t_L^*} t_L^*[S_L q_L(\frac{a-q_L}{b}-c_L+e_L)-\frac{\rho}{2}S_L^2 q_L^2\sigma^2]+(1-t_L^*)\{(1-\alpha_H)$$
$$[S_H q_H(\frac{a-q_H}{b}-c_H+e_H)-\frac{\rho}{2}S_H^2 q_H^2\sigma^2+q_H(c_H-e_H)-q_H(c_L-e_L)]$$
$$-\alpha_H[d_c+d_u+q_H(c_L-e_L)]\}-v(e_L) \quad (\mathrm{ST})$$

$$\alpha_H \in \arg\max_{\alpha_H^*} \alpha_H^*[\pi_{H|L}(d_C+S_H q_H(\frac{a-q_H}{b}-c_H+e_H)+q_H(c_H-e_H))-k] \quad (\mathrm{BA})$$

The buyer's objective function and the $\mathrm{IR_H}$, $\mathrm{IR_L}$, ST and BA conditions remain the same as that of the basic model. The newly-added $\mathrm{IC_H}$ and the $\mathrm{IC_L}$ conditions indicate both the high-cost and low-cost supplier put in the amount of effort that makes his marginal cost equals marginal benefit.

Proposition 7. With a moral hazard element as described above, the buyer has to offer sharing ratios higher than zero to both high-cost and low-cost supplier for optimal incentive and risk-sharing tradeoff. The production quantity assigned to the low-cost supplier is still optimal, while the production quantity assigned to the high-cost supplier is still upwardly distorted.

Proof: From the FOC of the ST condition, we get:

$$S_L q_L (\frac{a-q_L}{b} - c_L + e_L) - \frac{\rho}{2} S_L^2 q_L^2 \sigma^2 = (1-\alpha_H)[S_H q_H (\frac{a-q_H}{b} - c_H + e_H)$$
$$-\frac{\rho}{2} S_H^2 q_H^2 \sigma^2 + q_H (c_H - e_H) - q_H (c_L - e_L)] - \alpha_H [d_c + d_u + q_H (c_L - e_L)] \qquad (ST')$$

It indicates the low-cost supplier's payoff if he is truthful equals his payoff if he cheats. As a result he should be indifferent between being truthful and cheating. The equation can also be written as the probability of audit the buyer will employ:

$$\alpha_H = \frac{\begin{matrix} S_H q_H (\frac{a-q_H}{b} - c_H + e_H) - \frac{\rho}{2} S_H^2 q_H^2 \sigma^2 + q_H (c_H - e_H) - q_H (c_L - e_L) \\ -[S_L q_L (\frac{a-q_L}{b} - c_L + e_L) - \frac{\rho}{2} S_L^2 q_L^2 \sigma^2] \end{matrix}}{S_H q_H (\frac{a-q_H}{b} - c_H + e_H) - \frac{\rho}{2} S_H^2 q_H^2 \sigma^2 + q_H (c_H - e_H) + d_C + d_U} \qquad (3)$$

From the FOC of the BA condition, we get:

$$\pi_{H|L} [d_C + S_H q_H (\frac{a-q_H}{b} - c_H + e_H) + q_H (c_H - e_H)] = k \qquad (BA')$$

It indicates that the buyer's gain from the audit equals the cost of the audit, so she should also be indifferent between auditing and not auditing given that the reported cost is high.

Substituting $\pi_{H|L} = \frac{f_L - f_L t_L}{1 - f_L t_L}$ into the above equation, we get:

$$t_L = \frac{f_L [d_C + S_H q_H (\frac{a-q_H}{b} - c_H + e_H) + q_H (c_H - e_H)] - k}{f_L [d_C + S_H q_H (\frac{a-q_H}{b} - c_H + e_H) + q_H (c_H - e_H) - k]} \qquad (5)$$

This is the probability the low-cost supplier chooses to be truthful based on the contract parameters. Again please note that when k approaches zero, t_L will approach 1, which means the supplier will not lie if the cost of audit for the buyer is infinitely small.

Substituting condition (ST') into (IR_L), we get:

$$S_L q_L (\frac{a-q_L}{b} - c_L + e_L) - \frac{\rho}{2} S_L^2 q_L^2 \sigma^2 - v(e_L) \geq 0 \qquad (IR_L^{'})$$

Both (IR_H) and ($IR_L^{'}$) must be binding, otherwise the buyer could always lower her payment to the supplier to achieve higher profit for herself. As a result, neither high-cost supplier nor the low-cost supplier earns any informational rents under this contract. The supplier's payoff simply equals his opportunity cost.

From the FOC of the (IC_H) condition, we get:

$$v'(e_H) = S_H q_H \qquad (IC_H^{'})$$

Substitute (ST') into (IC_L), and solve for FOC, we get:

$$v'(e_L) = S_L q_L \qquad (IC_L^{'})$$

Both FOCs show that the supplier puts in effort to make marginal effort equal his marginal gain.

Also substitute (BA')into the buyer's original objective function, her problem is simplified into:

$$\max_{q_H, q_L, S_H, S_L, e_H, e_L} \pi_H \{q_H \frac{a-q_H}{b} - [S_H q_H (\frac{a-q_H}{b} - c_H + e_H) + q_H (c_H - e_H)]$$
$$+ \pi_L \{q_L \frac{a-q_L}{b} - [S_L q_L (\frac{a-q_L}{b} - c_L + e_L) + q_L (c_L - e_L)]\}$$

$$= \max_{q_H, q_L, S_H, S_L, e_H, e_L} \pi_H \{q_H \frac{a-q_H}{b} - [S_H q_H (\frac{a-q_H}{b} - c_H + e_H) + q_H (c_H - e_H)]$$

$$+ \pi_L \{q_L \frac{a-q_L}{b} - [S_L q_L (\frac{a-q_L}{b} - c_L + e_L) + q_L (c_L - e_L)]\}$$

subject to:

$$S_H q_H (\frac{a-q_H}{b} - c_H + e_H) - \frac{\rho}{2} S_H^2 q_H^2 \sigma^2 - v(e_H) = 0 \qquad (\mathrm{IR_H})$$

$$S_L q_L (\frac{a-q_L}{b} - c_L + e_L) - \frac{\rho}{2} S_L^2 q_L^2 \sigma^2 - v(e_L) = 0 \qquad (\mathrm{IR_L^{'}})$$

$$v'(e_H) = S_H q_H \qquad (\mathrm{IC_H^{'}})$$

$$v'(e_L) = S_L q_L \qquad (\mathrm{IC_L^{'}})$$

Substitute the four new conditions into the buyer's objective function, we have:

$$\max_{q_H, q_L, S_H, S_L, e_H, e_L} \frac{f_H [d_C + v(e_H) + \frac{\rho}{2} S_H^2 q_H^2 \sigma^2 + q_H (c_H - e_H)] - k}{d_C + v(e_H) + \frac{\rho}{2} S_H^2 q_H^2 \sigma^2 + q_H (c_H - e_H) - k} \{q_H \frac{a-q_H}{b} - [v(e_H) +$$

$$\frac{\rho}{2} S_H^2 q_H^2 \sigma^2 + q_H (c_H - e_H)] + \frac{f_L [d_C + v(e_H) + \frac{\rho}{2} S_H^2 q_H^2 \sigma^2 + q_H (c_H - e_H)] - k}{d_C + v(e_H) + \frac{\rho}{2} S_H^2 q_H^2 \sigma^2 + q_H (c_H - e_H) - k} \{q_L \frac{a-q_L}{b}$$

$$-[v(e_L) + \frac{\rho}{2} S_L^2 q_L^2 \sigma^2 + q_L (c_L - e_L)]\}$$

$$\max_{q_H, q_L, e_H, e_L} \frac{f_H [d_C + v(e_H) + \frac{\rho}{2} (v'(e_H))^2 \sigma^2 + q_H (c_H - e_H)] - k}{d_C + v(e_H) + \frac{\rho}{2} (v'(e_H))^2 \sigma^2 + q_H (c_H - e_H) - k} \{q_H \frac{a-q_H}{b} - [v(e_H) +$$

$$\frac{\rho}{2} (v'(e_H))^2 \sigma^2 + q_H (c_H - e_H)] + \frac{f_L [d_C + v(e_H) + \frac{\rho}{2} (v'(e_H))^2 \sigma^2 + q_H (c_H - e_H)] - k}{d_C + v(e_H) + \frac{\rho}{2} (v'(e_H))^2 \sigma^2 + q_H (c_H - e_H) - k} \{q_L \frac{a-q_L}{b}$$

$$-[v(e_L) + \frac{\rho}{2} (v'(e_L))^2 \sigma^2 + q_L (c_L - e_L)]\}$$

The solutions (implicit solution for q_H) to the optimization problem are:

$$S_L = \frac{1}{1+\rho v''(e_L)\sigma^2}$$

$$S_H = \frac{1}{1+\rho v''(e_H)\sigma^2}$$

$$q_L = \frac{a-bc_L+be_L}{2}$$

$$\frac{1}{[d_C+v(e_H)+\frac{\rho}{2}(v'(e_H))^2\sigma^2+q_H(c_H-e_H)-k]^2}\{f_H(c_H-e_H)k\{[q_L\frac{a-q_L}{b}-[v(e_L)+$$
$$\frac{\rho}{2}(v'(e_L))^2\sigma^2+q_L(c_L-e_L)]]-[q_H\frac{a-q_H}{b}-[v(e_H)+\frac{\rho}{2}(v'(e_H))^2\sigma^2+q_H(c_H-e_H)]]\}+$$
$$(c_H-e_H)k[q_H\frac{a-q_H}{b}-[v(e_H)+\frac{\rho}{2}(v'(e_H))^2\sigma^2+q_H(c_H-e_H)]]\}$$
$$+\frac{1}{d_C+v(e_H)+\frac{\rho}{2}(v'(e_H))^2\sigma^2+q_H(c_H-e_H)-k}\{[f_H[d_C+v(e_H)+\frac{\rho}{2}(v'(e_H))^2\sigma^2+$$
$$q_H(c_H-e_H)]-k][\frac{a-q_H}{b}-\frac{q_H}{b}-(c_H-e_H)]=0$$

We see the buyer can no longer give the supplier a sharing ratio that equals zero with moral hazard problem being present. Still neither the high-cost nor the low-cost supplier earns any informational rent related to the cost information c_i , but the non-zero sharing ratio is required to mitigate the incentive problem related to the effort e_i . Again the first-order condition for q_L shows the price and the assigned quantity associated with low-cost supplier are both optimal with $p_L = p_L^*$ and $q_L = q_L^*$, where p_L^* and q_L^* are the first-best solution. The quantity associated with the high-cost supplier can be shown again to be upwardly distorted: the first term of the equation is positive as well as the coefficient of the second term, which leaves $[\frac{a-q_H}{b}-\frac{q_H}{b}-(c_H-e_H)]$ negative and shows $q_H > q_H^*$ must be true.

7. Conclusions

The purpose of this study is to present a simple profit-sharing contract involving private cost information and auditing in the context of buyer-supplier relationship within a supply chain. The audit game generates a mixed strategy and hence induces cheating and auditing in equilibrium. It is then contrasted with two other polar cases of contracts: contracts with zero audit and with deterministic audit. I find that when audit cost is sufficiently low, the contract with mixed strategy audit will approach first-best result and thus outperforms the other two contracts. There is also a distortion of production level in the different audit contracts. However, unlike the familiar downward distortion of high-cost supplier in the pure strategy incentive contract with zero audit, the high-cost supplier's equilibrium production level will be upwardly distorted in the audit contract with random audit.

When a moral hazard element is added to the problem, there is a new tradeoff to be resolved between optimal incentive and risk-sharing. The buyer has to offer a sharing ratio that is bigger than zero to both the high-cost and the low-cost supplier. The production quantity assigned to the low-cost supplier remains optimal and the production quantity assigned to the high-cost supplier remains upwardly distorted.

The major insights this study provides to supply chain practitioners are the comparison of different types of contracts. They are optimal only under certain circumstances respectively. If the buyer cannot make any credible commitment to audit, random audit leaves her best off. The results of the study can also be generalized to other contexts such as managerial compensation and corporate governance. They provide some explanation why the principals in those contexts adopt random audit mechanism to suit their needs. This paper also suggests testable hypotheses for potential empirical studies. For example, the cost of audit and the amount of penalty should both be negatively correlated with the use of audit.

This study can be extended in several ways. One potential extension is imperfect auditing. In the current model, audit reveals the true cost with 100% probability. However, the probability of truth being found is often a function of the audit cost. It would be interesting to build the audit cost as an endogenous variable instead of an exogenous one. Another possible extension is with the penalty, which is another variable that can be set as endogenous. For example, the buyer might require the supplier to return his “cheating profit” if he is found misreporting instead of an exogenously fixed amount. The third extension possibility is to consider a game with more than two cost types. It would be interesting to see whether there are new equilibria emerging such as partial pooling or partial separating equilibrium.

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