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Advantages of "Function Domain Sets" Confidence Intervals Over Hypotheses Comparison Tests of One Mean Residual Life (MRL) Function Dominating an Improved Baseline and of Two MRL Functions Comparisons with Applications in Modern Engineered Composite Wood Products: One Sample and Two Sample Cases. Also Exploring General Theory, Insights, and Applications of MRL Functions

Jonathan Cody Steele
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To the Graduate Council:

I am submitting herewith a thesis written by Jonathan Cody Steele entitled "Advantages of "Function Domain Sets" Confidence Intervals Over Hypotheses Comparison Tests of One Mean Residual Life (MRL) Function Dominating an Improved Baseline and of Two MRL Functions Comparisons with Applications in Modern Engineered Composite Wood Products: One Sample and Two Sample Cases. Also Exploring General Theory, Insights, and Applications of MRL Functions." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Statistics.

<div style="text-align:right">Timothy M. Young, Major Professor</div>

We have read this thesis and recommend its acceptance:

Ramon V. Leon, Frank M Guess

<div style="text-align:right">Accepted for the Council:
Dixie L. Thompson</div>

<div style="text-align:right">Vice Provost and Dean of the Graduate School</div>

(Original signatures are on file with official student records.)

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Accepted for the Council:

_**Anne Mayhew**_____

Vice Chancellor and Dean of Graduate Studies

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ABSTRACT

In this thesis, we analyze mean residual life (MRL) functions and unique "function domain sets" confidence intervals to identify important opportunities for improving quality of medium density fiberboard (MDF). We stress these tools have tremendous potential for many other forest products (e.g., various composites, natural woods), not just MDF.

These "function domain sets" confidence intervals can assess variation in quality where one MRL function dominates an industrial baseline. Assessments of the internal bond of MDF illuminate opportunities for helpful improvements, plus perform valid statistical comparisons of different types of MDF. For example, these MRL methods detect a new, higher-valued MDF product that represents an opportunity for an MDF producer to increase revenues or reduce costs due to excess MRL for a subgroup. These MRL methods can be used as diagnostics of a MDF manufacture process needing adjustments, etc. We provide MAPLE 10 code to implement these MRL procedures.

Typical traditional confidence intervals for a MRL function are centered about the function. "Function domain sets" intervals, however, produce novel statements like: "we are 95% confident that the MRL function, e(t), is greater than another function for all t in the domain set $[0, \hat{\theta})$." We study "function domain sets" intervals on internal bonds (tensile strengths) for various MDF products.

The values of MRL analyses have been demonstrated in a variety of applications beyond MDF production. The usefulness of the MRL function in other areas suggests that it has considerable potential value for the forest products industry. Recent, MRL

applications vary from modern accelerated stress testing using proportional MRL modeling, to fuzzy set engineering modeling, to maintenance and replacement of bridges in Europe, to better decision making on materials in nuclear power plants, to general applications in evaluating "degrading" systems. We anticipate that varied analyses of MRL functions and "function domain sets" confidence intervals will furnish practitioners useful tools in many fields. Applications to different areas are highlighted to demonstrate the increasing usefulness and potential of MRL methods in many industries, government agencies, and future academic research.

Keywords: Mean Residual Life, Confidence Intervals, Medium Density Fiberboard, Improvements, Engineering Systems.

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PART 1: INTRODUCTION

Mean residual life (MRL) functions measure the amount of remaining stress an item can sustain before failing. There is considerable variation in what measurement of stress may be used for different items, though time is among one of the most familiar examples. The amount of time an item will continue to be useful for is a common question of interest. In cases where time in service is not a good predictor of survival, the MRL function can be used with other stresses, such as pounds of force applied, temperature or magnetic signature. The MRL function gives results that form intuitive statements for many situations. As such, it merits consideration with many other measurements of reliability, such as hazard rate functions, that allow intelligent inferences to be made about the reliability of a product, group or individual.

See Young and Guess (2002) for how such data is stored and used in a real time data base with regression modeling to predict the strength of medium density fiberboard (MDF). This provides quick feedback to the manufacturer in order to minimize process inputs and maximize product quality within specified limits. The key metric used by manufacturers for estimating the quality or reliability of MDF is internal bond (IB). Samples from a cross section of the MDF are tested by being pulled apart. The IB at failure is then measured in pounds per square inch (p.s.i.) or corresponding metric units (kilograms per cubic meter).

We briefly explore in Part 2 literature that pertains to topics discussed in this thesis in five sections. The first section encompasses some key reliability references. Next, the second section addresses the general theory of MRL functions plus connections

to various reliability measures. Tersely in the third section we comment on classes of MRL and related citations for those highly niched specialists interested, but MRL classes are not the focus of this thesis. Section four contains some more detailed references on MRL functions applications ranging from proportional MRL modeling engineering systems, to fuzzy set modeling, to bridges, to nuclear power plants, to general application to degrading systems. In the fifth and final section we review some topics related to research on MDF.

Different readers may be more interested in one topic than another, and an in depth knowledge of all five will not be necessary to understand all the information in this thesis. At the same time, all five topics are valuable and having some familiarity with each will assist the reader in understanding the motivations and usefulness of this thesis.

Techniques are presented in Parts 3 and 4 that allow comparisons of MRL functions, either to other functions or to a baseline. The thesis presents these techniques using data on the internal bond of MDF. MDF has become an increasingly important modern engineered wood product as improvements in quality continue to widen its usefulness to consumers worldwide. Capturing properties of MDF with statistically sound reliability measurements provides opportunities for improvement that allow manufacturers to gain competitive advantages.

A key metric of quality used globally by MDF manufacturers is internal bond (IB). IB describes to some degree, the usefulness of a piece of MDF. If we consider the extreme weakest case, we can see that a piece of MDF used as a shelf at least needs to be able to support its own weight across an unsupported interval. In reality, the internal bond of a piece of MDF used as a shelf would have to be higher than this so that it can

sustain a load placed on it. Because it is natural to think about the amount of remaining force a MDF product can withstand before destruction, exploring the mean residual life function may provide opportunities for improving the quality of MDF production while minimizing cost.

Typical confidence intervals for a mean or MRL are centered about the mean or MRL. We discuss "function domain sets" confidence intervals that produce statements for example like: "we are 95% confident that the MRL function, e(t), is greater than a pre-specified μ_O for all t in the domain interval [0, $\hat{\theta}$)" where $\hat{\theta}$ is determined from the sample data, confidence level, and μ_O.

Also, we can have two sample statements like: "we are 95% confident that the MRL of population 1, namely $e_1(t)$, is greater than the MRL of population 2, $e_2(t)$, for all t in the domain interval [0, $\hat{\theta}$)" where $\hat{\theta}$ is determined from the sample data and confidence level. Other domain sets are possible in both the two sample and one sample cases. We illustrate these one and two sample "function domain sets" confidence intervals on internal bonds (tensile strengths) for MDF.

Part 3 demonstrates two-sample "function domain sets" confidence intervals that allow comparisons to be made between the MRL functions of two populations. We use data from two commonly produced product types of MDF to illustrate the confidence intervals. The analysis of the confidence intervals suggests the existence of a higher-valued product that represents a new opportunity for the manufacturer to sell for increased profit. Important to these types of confidence intervals is the capability to detect differences between groups in a range of stresses.

We show in Part 4, the use of "function domain sets" confidence intervals for comparing MRL functions to a baseline. An example is provided that uses data from MDF. There are natural applications for the type of statement this allows, indicating the additional stress that may be applied to a unit after it reaches a given stress point. If, for example, we know a steel drum is pressurized to a certain level, we could use the current pressurization level to find a "function domain sets" confidence interval that would describe a range of remaining pressures we would expect the drum to sustain before rupturing. In the case of MDF, the demonstrated confidence interval could be used to provide a margin of safety where the baseline would be the reported amount of strength of the fiberboard and the estimated "function domain sets" confidence interval for the MRL function would show the range of further stress that could be applied to the board before it would fail.

Discussion in Part 5 focuses on a specific area of interest with respect to the MRL function: the MRL of systems. In a time where increasingly high reliability is needed to ensure the proper functioning of important features of life, systems are often employed so that principles of redundancy protect features where failure of a system could have severe consequences. In particular, parallel systems where a system does not fail as long as a single component is operating are of great interest for ensuring the proper functioning of a system. Predicting the MRL functions of systems so that intervention prevents intolerable decay or outright failure has many important applications in modern life.

In Part 6, we consider some future lines of research suggested by the topics considered in earlier portions of the thesis. There are many obvious paths for the extension of "function domain sets" confidence intervals that remain to be explored.

Some thoughts on multiple comparison techniques and suggestions for other ways to apply "function domain sets" confidence intervals are mentioned. Additionally, there are a number of pathways for current research into the MRL functions of different systems. Some discussion on non-identical hazard functions for components of parallel systems and consideration of failure definitions when degradation of components is poorly understood is presented.

Appendix A discusses some of the mathematical properties used in the "function domain sets" confidence intervals described in Parts 3 and 4. It is not expected that these details will be of interest to all readers, but the technical details are provided to assist those with an interest in the mathematical and statistical properties of the confidence intervals. The chief topic of the details provided deal with asymptotic normality.

Last, we give Appendix B which provides the Maple 10 code used to perform calculations and create figures for the thesis. Comments in the introductory portion to the appendix and in the code itself are intended to assist interested readers in actually doing the calculations, multiple graphs, and confidence intervals for both small and large sample cases. Some helpful references for Maple 10 are mentioned. Figures are provided after the code that produces them to illustrate the plotting capabilities of Maple 10.

PART 2: LITERATURE REVIEW

MATHEMATICS, STATISTICS, AND RELIABILITY

The increasing study of and application of principles of statistical reliability will be of increasing importance to MDF producers as competition for business demands improved quality of product while decreasing production costs. As such, the importance of understanding and the ability to apply reliability principles will be practiced by successful MDF producers in the near future. The field of reliability is broad and a thorough review is, admittedly, not provided here. A wide number of topics are presented that should provide some highlights from the field and some topics that will be useful for understanding and making useful the techniques discussed in this thesis. Several texts may be useful to the reader. The thoughtful tome by Meeker and Escobar (1998a) should prove indispensable for most readers. Krishnaiah and Rao (1988) provide another good overview of reliability and the chapter contributed by Guess and Proschan (1988) provides good information about the MRL function. O'Connor (1985) is a classic reliability resource based on real world experience, with a balanced international perspective. He comments on reliability databases that range from military to non-military, electronic to non-electronic, etc. For more review of reliability see the classic book and Barlow and Proschan (1981), and this link on reliability http://web.utk.edu/~leon/rel/overview/rellinks.html. The National Institute on Standards and Technology, along with an industrial consortium, have helpfully provide the *NIST/SEMATECH e-Handbook of Statistical Methods*,

http://www.itl.nist.gov/div898/handbook/ which has more on reliability applications and tools. Many other cites are available but not listed.

Although not related to the MDF examples provided here, many processes for which the MRL is measured are stochastic processes. For a review of stochastic processes the work by Karlin and Taylor (1975) will be useful. For the reader especially interested in stochastic processes, Karlin and Taylor (1981) and Taylor and Karlin (1994) are good, but more advanced, resources. Many other types of processes that are useful in reliability and with respect to MRL are described in Csörgő, Csörgő, and Horváth (1986).

It is intended that the "function domain sets" confidence intervals described in Parts 3 and 4 be non-parametric in nature. A good text for the study of non-parametric statistics is Hollander and Wolfe (1999). Many of the details covered in Appendix A rely on asymptotic convergence. The text by Serfling (1980) may be useful to those readers interested in further exploration of approximation and convergence.

There may well be no end to the useful papers that cover various topics in reliability. Guess and Walker (1992) discuss the effect of burn-in on different measures of reliability, including the MRL function. Chan and Meeker (1999) discuss modeling population lifetimes when wear-out and early failure are both important. Doganaksoy, Hahn, and Meeker (1999) provide a useful consideration of analyzing data when multiple modes of failure are active and important in a population.

Meeker and Hamada (1995) give a good overall view of ways of quickly evaluating the reliability of products that do not fail often. An important part of this is accelerated testing, which is discussed in Meeker and Escobar (1993) and Meeker and Escobar (1998b).

There are countless other papers that could be included in a review of reliability literature, and we will mention only a few more here. The article by Hahn, Doganaksoy, and Meeker (1999) discusses the role that statistics may play in reliability improvement. Peña, Strawderman, and Hollander (2001) provide some non-parametric discussion of reliability data. The work by Doganaksoy, Hahn, and Meeker (2000) includes a helpful case study analyzing reliability data.

THE MEAN RESIDUAL LIFE FUNCTION

In this thesis, we discuss using the MRL function to find opportunities to improve production of MDF. Mean residual life functions (or, synonymously, mean remaining life functions, as they are sometimes called in literature) and tables have been studied and commented on by many individuals over the years. Guess and Proschan (1988), Chiang (1968), and Deevey (1947), cite the use of the MRL for annuities via expected life tables in ancient Roman culture. More recently, a wide host of papers covers many other aspects of MRL, for example, Guess, Zhang, Young, and Leon (2005), Zhao and Elsayed (2005), Anis, Basu, and Mitra (2004), Bradley and Gupta (2003), Asadi and Ebrahimi (2000), Na and Kim, (2000), Lim and Park (1998), Guess, Nam, and Park (1998), Guess, Walker, and Gallant (1992), Abouammoh (1988), Oakes and Dasu (1990), Berger, Boos, and Guess (1988), Guess and Park (1988), Guess, Hollander and Proschan (1984). These citations are a brief list of the many excellent papers written on MRL.

TESTING CLASSES OF THE MEAN RESIDUAL LIFE FUNCTION

Although not a focus of this thesis, much work has been completed to define classes of distributions which have MRL functions that exhibit certain behaviors, each

class possessing different properties that make different inferences about the MRL function possible. The development of tests to determine when a MRL function belongs to a certain class has also generated a great deal of scholarship. By way of explanation, the most natural classification is probably the Decreasing MRL function, for which whenever the stress applied to an individual increases the expected residual life of an individual decreases. Guess and Hollander (1986) discuss testing whether a MRL function exhibits a change in individual lifetime distributions or if the lifetimes fit an exponential distribution. Ebrahimi (2001) explores a test for the goodness of fit of MRL functions to any known distribution. Hendi and Abouammoh (2001) take it upon themselves to provide a test for the "new better (or worse) than renewal in expectation" class of MRL function against the exponential distribution. Anis and Mitra (2005) pursue this same goal and demonstrate that their test is consistent. Ahmad and Kayid (2005) propose a new class of MRL function they call "new better than used in total time on test transform ordering" and discuss its usefulness for modeling the residual life of series and k-out-of-n systems. Sen and Jain (1991) offer some discussion on identifying a class they call "decreasing variance residual life."

APPLICATIONS OF THE MEAN RESIDUAL LIFE FUNCTION

In this work, we provide examples of using the MRL function to improve MDF production. The value of the MRL function has been demonstrated in a number of applications beyond MDF production. The usefulness of the MRL function in other areas suggests that it has considerable potential value for the forest products industry. Because of the breadth of this area, some time is taken in Part 5 to note some further work

respecting parallel systems. Here we make note of several more general examples. Whittle and Tennakoon (2005) discuss using the MRL function to predict the time to replacement of plastic pipes in a sewer system. Tandler, Vehovar, et al (2003) and Kovpak (1981) concern themselves with using the MRL function to estimate the remaining usefulness of metallic components. Ray, Tiwari, et al. (2002) estimate the MRL function of steel tubes used in a power plant. These are somewhat industrial applications, but the breadth of fields that make use of MRL functions is wide. Estesen, Buck, et al. (1992), for example, discuss the effect of insecticides used on cotton on exposed honey bees and note use the MRL function to estimate the length of time the insecticides are dangerous to the bees. In cases where insecticides are used to defeat tree pests, this may have some direct application to production of raw materials for MDF. Zweifel, Felder, et al. (1999) make note of how health care expenditures depend much more on the mean remaining life of a human population rather than on its age.

MEDIUM DENSITY FIBERBOARD

The uses and topics of interest of to improvement of MDF are many and varied. According to the "U.S. Forest Products Annual Market Review and Prospectus, 2002-2006" (Howard, 2006) 2004 saw an increase of 32.3% in domestic production of MDF, and an increase of 22.9% in imports of MDF. Continued increases in production, imports, consumption, and exports of MDF are anticipated to be observed in the subsequent years. While consumption of MDF is expected to continue to increase, Howard (2006) also notices that real prices for forest products have declined since 1998 and are expected to continue to do so. Lower prices for products will increase the

pressure on producers to lower production costs, particularly as competition from import markets increases.

Interest in improving MDF drives considerable research into different input and process variables involved in the creation of MDF. The statistics involved in studies of the mechanisms of MDF can be complicated, especially because several variables are taken into account in determining the quality of MDF. The Composite Panel Association (2006) summarizes six properties by which MDF grade is determined by the American National Standards Institute: modulus of rupture, modulus of elasticity, internal bond, thickness swell, screw holding face, and screw holding edge. All but thickness swell are measures of the strength of MDF. Often variables that improve one property have little effect on, or even an adverse effect on, other properties important to the quality of MDF.

The use of statistical techniques to increase quality and reduce costs for producers of MDF will be increasingly vital to American producers. Research into the application of old and new statistical techniques to MDF production will achieve heightened importance. Valuable work has already been done in this area. Chen (2005) explores the estimation of extremely small percentiles in strength date, with the intention of applying the practice to MDF. Chen's (2005) work makes use of a forced censoring technique that can produce models that provide useful estimates for early failures of MDF. Unusually early failures may suggest a failure mode resulting from a problem in production that produces substandard units of MDF. Reliable detection of unusual early failure modes indicates an opportunity to improve the production process. The estimation technique used by Chen (2005) is also described in the published work of Chen, Leon, Young, and Guess (2006) and Guess, Leon, Chen, and Young (2004).

Edwards (2004) also demonstrates applying reliability techniques to improving production of MDF. Edwards (2004) takes time to discuss graphical techniques that are for practitioners to apply and understand. Edwards (2004) also provides insightful discussion about information criteria (e.g. Bozdogan's Information Complexity Criteria) that can assist in model selection and understanding what distributional assumptions are safe to apply to a given product. Because data may be sparse in the tails of non-normal distributions, Edwards (2004) also discusses when bootstrap techniques may be useful for accurately estimating lower percentiles of strength data.

Guess, Zhang, Young, and Leon (2005) discuss particular insights provided by the MRL function for strength data. An example is included using MDF data demonstrating how a graphical plot of the MRL function may reveal sources of variation insensitive to an analysis that compares the sample mean of raw data to the sample mean of data screened for outliers. The MRL technique identifies a product opportunity that could be missed if it is concluded that the outliers do not contain significant information about the MDF production process.

Research into the properties of production components and the capabilities of MDF are also expected to provide some competitive advantages for MDF producers. Wang, Winistorfer, Young, and Helton (2001a) examine the effects of a new way of pressing MDF during manufacture on the density of the layers within MDF. In Wang, Winistorfer, Young, and Helton (2001b) they extend their exploration of "step-closing pressing" of MDF to other mechanical properties of MDF. In both works, the authors find that their new pressing technique allows them to manipulate the mechanical properties of MDF under exploration.

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Suchsland and Woodson (1976) examine properties of MDF such as the relationship between increasing resin and increasing internal bond of MDF. Kartal and Green (2003) consider the advantages of different kinds of MDF with respect to fungi and termite resistance, especially as compared to woods traditionally used in building materials. Kelley, Elder, and Groom examine the chemical effects on MDF that arise from the age of the wood at the time of processing and from some processing conditions. Groom, Mott, Shaler, and Pesacreta (1999) examined earlier the effect of age of wood on the surface of fibers used in the generation of MDF believing that characterization of fiber surface would predict MDF properties. Lee, Shupe, and Hse (2004) consider the properties of MDF formed from different mixtures of bagasse and Chinese tallow tree with different resin systems. Short, Woodson, and Lyon (1978) made note of the differences in certain types of MDF made using "green" wood chips with high moisture content as opposed to partially dried wood chips.

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PART 3: TWO SAMPLE "FUNCTION DOMAIN SETS" CONFIDENCE INTERVALS

INTRODUCTION

Note that for a random lifetime X, the MRL is defined as the conditional expectation E(X - t | X > t) with the restriction that t ≥ 0. This can be further represented and simplified using the reliability function R(t) = P(X>t) = 1 – F(t) as

$$e(t) = E(X - t \mid X > t) = \left(\int_t^\infty R(x)dx \right) / R(t)$$

where we assume R(t) > 0 for e(t) to be well defined. Also note, the empirical MRL is easily calculated by substituting the standard empirical reliability function into the formula of e(t) for R(t) or by substituting the quantity of one minus the Kaplan-Meier estimate of the cumulative distribution function. Compare Guess and Proschan (1988). In this paper we will calculate the empirical MRL at the order statistics, and then linearize it in between the order statistics.

Recall the MRL function can exist, while the failure rate function might not exist, or, vice a versa, the failure rate function can exist without the MRL function existing. The reliability function can be represented as a function of the MRL, as

$$R(t) = \frac{e(0)}{e(t)} \exp\left(-\int_0^t \left[\frac{1}{e(x)} \right] dx \right)$$

Compare, for example, Guess and Proschan (1988) for additional comments and insights on MRL. For more information on special classes of distributions connected with MRL

that have been widely studied and tested in a variety of situations see the helpful paper by Hollander and Proschan (1984), plus the classic book by Barlow and Proschan (1981).

Remember that when we typically speak of a confidence interval for a function, we are describing a range of dependent variable that we believe will contain the true function mean at a particular level or particular range of an independent variable. These types of confidence intervals allow us to make statements like "we are 95% confident that the MRL function, e(t), is in the interval ($\hat{\theta}_1$, $\hat{\theta}_2$) when at stress level, t" where $\hat{\theta}_1$, and $\hat{\theta}_2$ are determined from the sample data, confidence level, and t. We suggest that for purposes of detecting variation it will also be useful to examine "function domain sets" confidence intervals. The type of statement we propose making based on the "function domain sets" confidence interval is one like "we are 95% confident that the MRL of population 1, namely $e_1(t)$, is greater than the MRL of population 2, $e_2(t)$, for all t in the interval [0, $\hat{\theta}$)" where again $\hat{\theta}$ is determined from the sample data and confidence level. Other types of confidence statements are possible based on the chosen interval, but this type of statement is most natural for analyzing MDF. We analyze two types of MDF with different densities, with the a priori expectation that the denser type will have a stronger internal bond. That is to say that, when we begin, we expect the mean residual life function of the denser MDF to dominate the less dense type over the entire range of stresses where individuals survive.

See Berger, Boos, and Guess (1988) and Balgopal (1989) for more on these types of MRL intervals. For the interested reader, proofs related to using "function domain sets" confidence intervals taken from Balgopal (1987) and Berger, Boos, and Guess

(1988) are provided in Appendix A. For more on MDF see Guess, León, Chen, and

Young, (2004), Guess, Edwards, Pickrell, and Young (2003), and Young and Guess

(2002).

In Section 2, we discuss the helpful two sample case of confidence intervals using

MRL functions. We apply these confidence intervals to real data from tests of tensile

strength of MDF. We emphasize that these intervals can be used even more broadly, not

just for regular life data. The intervals can be used for any time or stress to response data,

plus financial data, etc.

In addition to the empirical MRL function, the calculation of the confidence

intervals also uses a standard deviation function and a sample size function. The standard

deviation function may be expressed as:

$$s_n(t) = s(X - t | X > t), t \geq 0$$

where s indicates taking the standard deviation of a sample formed from the failure

stresses larger than t minus t. The sample size function may be expressed as:

$$n_d(t) = n - I(X \leq t)$$

where I represents a counting function for the number of failures that have occurred by

stress level t and n is the original sample size before any failures. Using the same

notation, an equivalent representation can be given that counts the number that survive

after time t as:

$$n_d(t) = I(X > t)$$

Recall that Weibull's original reliability function was developed by him studying and fitting strengths for various materials (see Weibull 1939, 1951). Product "life" for MDF can be measured in terms of the strength to failure, as opposed to the time to failure. The strength (e.g., internal bond) or pounds per square inch (p.s.i.) to failure is a crucial reliability parameter of the product. It naturally allows the producer to make assurances to customers about the quality, safety, and useful "life" range of the product. We will write MRL where we understand it is actually mean remaining pressure (measured in p.s.i.) until failure.

In Section 3 we have concluding comments and recommendations on these confidence intervals and future work.

TWO SAMPLE MEAN RESIDUAL LIFE CONFIDENCE INTERVALS ON MODERN ENGINEERED WOOD

We use $\hat{e}_{46}(t)$ to denote the empirical MRL of an MDF product with density of 46 pounds per cubic foot (lbs/ft^3), thickness of 0.625 inches, and width of 61 inches. We employ $\hat{e}_{48}(t)$, to denote the empirical MRL of an MDF product with density of 48 pounds per cubic foot (lbs/ft^3), thickness of 0.625 inches, and width of 61 inches. Naturally, the corresponding population MRL's are written $e_{46}(t)$ and $e_{48}(t)$, respectively. Our sample size for density of 48 lbs/ft^3 is $n_{48} = 108$ units, while the sample size for density of 46 lbs/ft^3 is $n_{46} = 974$ units. A priori, MDF workers would conjecture that a higher density of 48 lbs/ft^3 would yield a greater average IB and MRL. This turned out to be mostly true, but surprisingly was not always the case.

We discuss three figures that provide different insights into the MRL's, into the corresponding novel confidence intervals, and into the specific statistical functions that are used to create these confidence intervals. The figures are all produced in Maple 10. The empirical MRL is plotted at each unique failure then linearized between points as seen in the figures. In Figure 1 we graph the empirical MRL's for both $\hat{e}_{46}(t)$ and $\hat{e}_{48}(t)$. It is natural to conjecture for these products a decreasing empirical MRL. Recall this suggests the classical DMRL (Decreasing MRL) class tested in Hollander and Proschan (1975), plus other helpful DMRL tests by other authors. Note for density 48 lbs/ft^3 there is graphical evidence of a DMRL, but this is not the case for density 46 lbs/ft^3 (Figure 1).

As expected, the MRL for $\hat{e}_{48}(t)$ is much higher, starting with a sample mean of $\hat{e}_{48}(0) = 185.7$ p.s.i., corresponding to mean residual pressure to failure in p.s.i. (which again will be understood when we use the standard abbreviation of MRL). This is higher, as would be conjectured, than a lower density MRL's sample mean of $\hat{e}_{46}(0) = 122.7$ p.s.i. Also as conjectured, $\hat{e}_{48}(t)$ dominates $\hat{e}_{46}(t)$ for all t > 0 until around t = 150 p.s.i., when they surprisingly switch roles after a crossing. Also, note that $\hat{e}_{46}(t)$ has a later peak at 151.4 with the MRL being $\hat{e}_{46}(151.4) = 49.3$, while the other MRL (which workers expected to be higher) is actually lower instead with $\hat{e}_{48}(151.4) = 34.3$.

Mean Residual Life Functions
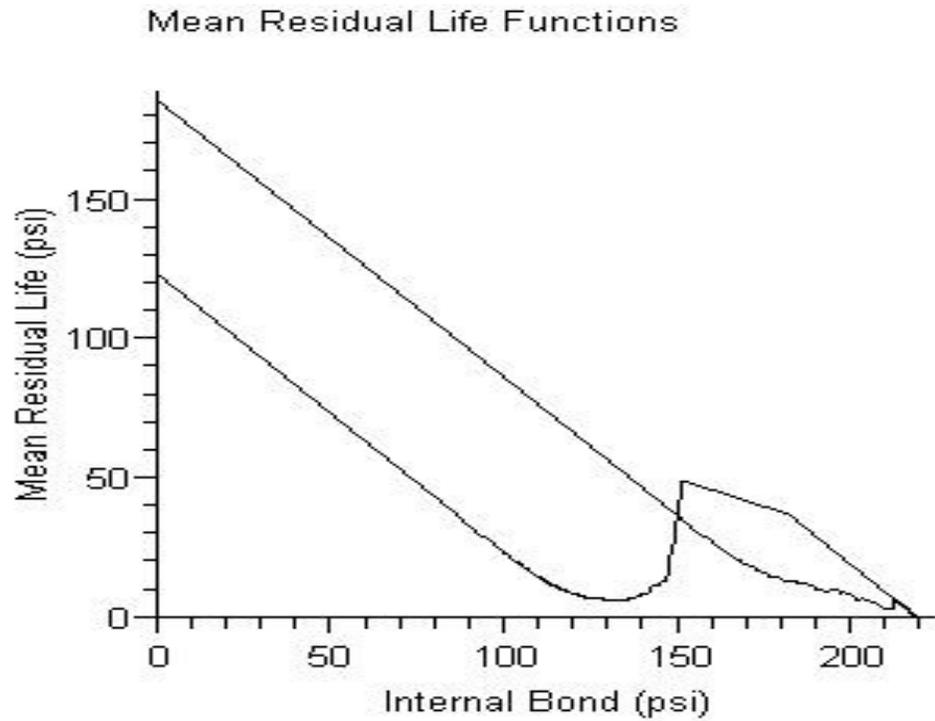
Figure 1. The empirical MRL functions (expected remaining pressure till failure in p.s.i.). Starting top graph is the MRL $\hat{e}_{48}(t)$ for MDF with density 48, while the starting bottom is $\hat{e}_{46}(t)$ the empirical MRL for MDF with density 46. Also, see that $\hat{e}_{48}(0)$ = 185.7 psi, while $\hat{e}_{46}(0)$ = 122.7 psi. Note the crossing and later peak for $\hat{e}_{46}(t)$.

Recall that a priori one would not conjecture this crossing. This suggests that for a density of 46 lbs/ft^3 some units may be produced with unnecessarily high raw material set points, i.e., slow production transition from a density of 46 lbs/ft^3 to 48 lbs/ft^3, producing an intermediate type of product misclassified and possibly "under-priced" as having a density of 46 lbs/ft^3. The upper turning in the MRL of $\hat{e}_{46}(t)$ is unusual and may yield higher product costs for the density of 46 lbs/ft^3 product that does not require the stronger IB. The MRL provides an interesting rubric for product classification and continuous improvement.

The increase in MRL above 135 p.s.i. for the product with a density of 46 lbs/ft^3 was a surprise. This may show a setup change by the manufacturer to a higher targeted strength product, i.e., the manufacturer produces a higher strength product with higher resin and wood at a slower line speed, and is unable to instantaneously meet target specifications during setup change from the nominal strength to higher strength product.

It is obvious to practitioners from the MRL graph in Figure 2 that a hybrid product, or medium-strength product, is likely being produced. The MRL graph in Figure 1 reveals an opportunity cost for the manufacturer, e.g. improve setup change time to minimize resin usage, optimize line speed targets during product change, etc. The MRL graph in Figure 1 may have, also, identified a new higher-valued product opportunity for the manufacturer. These empirical MRL behaviors can be powerful diagnostic tools to facilitate training, continuous improvement, and ultimately cost savings.

Mean Residual Life Functions and Z Function
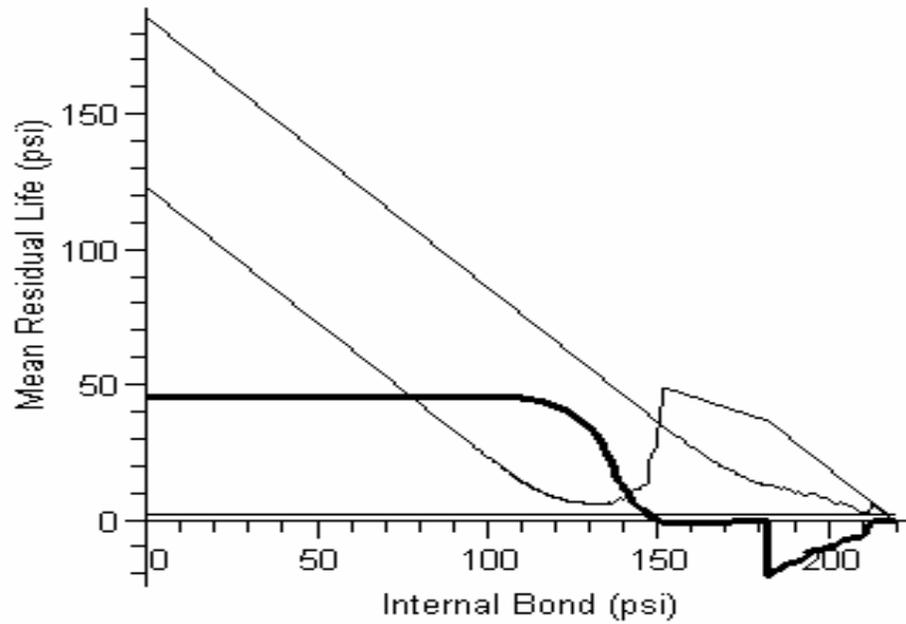
Figure 2. The same MRL functions are shown as in Figure 1, but now we have additionally the discerning statistical difference test function Zmn(t) as the darker line. It yields a crossing from above on the critical threshold line $z^{\alpha} = 2.33$ at $\hat{\theta} = \inf \{t > 0: Zmn(t) < z^{\alpha}\} = 147.98$. This produces the 99 % one sided confidence interval we discuss.

Figure 2 has two empirical MRL functions plus the statistical function $Z_{mn}(t)$ described in Berger, Boos, and Guess (1988), where

$$Z_{mn}(t) = \frac{\hat{e}_{48}(t) - \hat{e}_{46}(t)}{\sqrt{\dfrac{\left(s_{48}(t)\right)^2}{n_{48}(t)} + \dfrac{\left(s_{46}(t)\right)^2}{n_{46}(t)}}}$$

Figure 2 also shows the critical threshold z value straight line of z = 2.33 for determining a 99% confidence band. Note that $Z_{mn}(t)$ is essentially a two sample statistic on the difference between two population means, but here it is for the MRL functions at time t. Also, note that m and n are the respectively sample sizes m = n_{48} = 108 and n = n_{46} = 974. The thicker line is the statistical test function $Z_{mn}(t)$. $Z_{mn}(t)$ crosses from above the critical threshold line of z = 2.33 at the point $\hat{\theta}$ = 147.98. This implies, "we have 99% confidence that the population MRL for density 48, $e_{48}(t)$, dominates as statistically significant the population MRL for density 46, $e_{46}(t)$, for the entire interval [0, 147.98)." These can be thought of as lower confidence bounds of the form C = $1 - \alpha$, with $e_{48}(t) > e_{46}(t)$ for all t in $[0, \hat{\theta})$ where the $\hat{\theta}$ = inf {t ≥ 0: $Z_{mn}(t) \leq z_{\alpha}$} where z_{α} is the standard normal upper α quantile, z_{α} = 2.33, for the one sided lower 99% confidence interval. For a 95% we would use z_{α} = 1.645, while for 90% confidence we would use z_{α} = 1.28, etc. See Berger, Boos, and Guess (1988) for more comments and other types of these novel confidences intervals.

Lastly we give insight into the need to adjust a two sample procedure when the remaining units (i.e., remaining sample sizes) are small (Figure 3). Figure 3 has all of the Figure 2 functions, and also has the "t" value adjusted for use in a two sample procedure using the adjusted "t" and adjusted degrees of freedom. We replace the z percentile with the appropriate adjusted "t" percentile. Recall our initial large sample sizes n48 = 108 and n46 = 974. Typically, MRL is a large sample approach, but we need to stress the need for care in the later tails of the MRL when the remaining samples might be small. Note that the t values jump above 2.33 or below -2.33 to the "t" heights in Figure 3. Also, there is more need to pre-specify particular aspects there. For additional specific details see Berger, Boos, and Guess (1988).

Obviously other techniques such as TTT plots (see, Klefsjö 1991), box plots, histograms, as well as MRL plots, are helpful for process improvement and training. We stress the helpfulness of the graphs, but especially these novel confidence intervals for comparisons that are statistically valid in the two sample case and, as seen next, the one sample case.

SUMMARY AND CLOSING REMARKS

We recommend using these "function domain sets" confidence intervals for both the two sample setting and the one sample setting demonstrated in Part 4. They provide additional insights that can be used to quantify aspects suggested by graphical
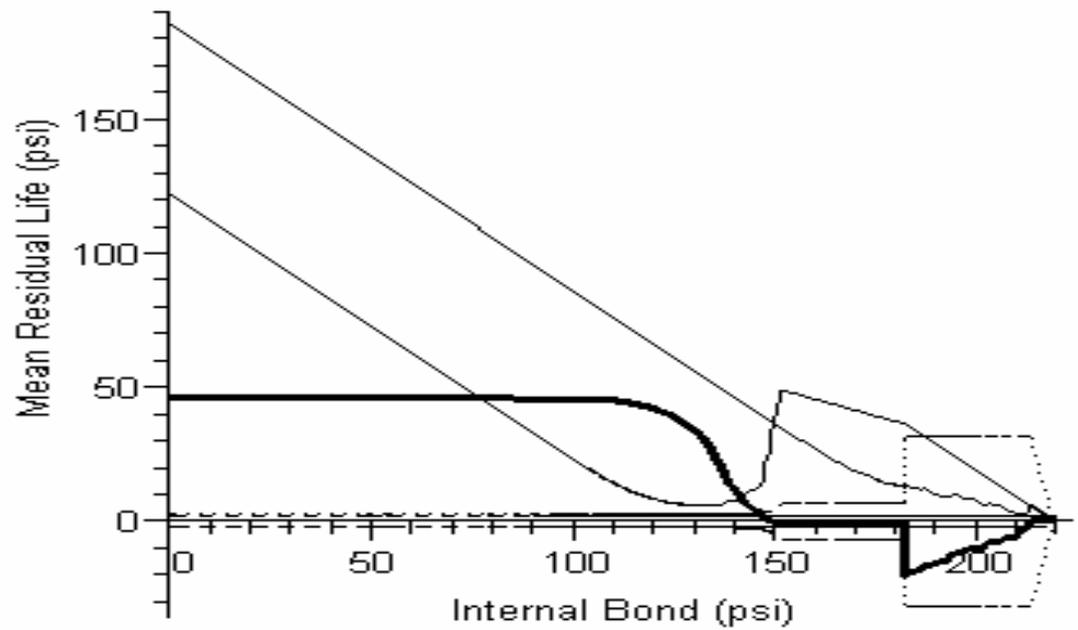
Figure 3. Similar to Figure 2, but now we have added another critical z = - 2.33 and the appropriately adjusted "t" values for when the remaining sample sizes are small. Note the jump in the "t" values as seen beyond 150 psi.

comparisons, which are also useful. These statistical intervals can be used to explore the data and find key thresholds. Recall that the "function domain sets" confidence intervals we discuss for the MRL function actually give a confidence interval for the level of stress where two MRL functions are different rather than for the MRL function itself. Note how unique, striking behaviors can be identified by comparative MRL plots and these novel confidence intervals on real word data sets on tensile strength measured by IB. Also remember the usefulness of these confidence intervals in conjunction with other methods. We would expect the MRL function of a sample of MDF to be almost strictly decreasing, and the simple act of plotting the MRL functions of the samples above draws our attention to irregularities in the manufacturing process. This may facilitate training, process improvement and cost savings.

The calculations and graphs were done in Maple, version 10. Code from Maple can be outputted in Matlab and C++. Other languages naturally are also available, for example R or S+. Copies of our code in Maple format are available by emailing Research Associate Professor Timothy M. Young at tmyoung1@utk.edu or Cody Steele at steelejc@go.com. The text of the code appears in Appendix B.

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PART 4:ONE SAMPLE MRL CONFIDENCE INTERVALS ON MODERN ENGINEERED WOOD

INTRODUCTION

We now illustrate the one sample versions of our "function domain sets" confidence intervals. These are akin to the two sample case if one of the MRL functions is constant across all stress levels. The one sample versions are advantageous because it is often naturally useful to discuss the MRL past a baseline for applications of safety and warranty. As compared to the previously described case, the statement we will make using "function domain sets" confidence intervals for the one sample case is akin to "we are 95% confident that the MRL function, e(t), is greater than a pre-specified μ_o for all t in the interval [0, $\hat{\theta}$)" where $\hat{\theta}$ is determined from the sample data, confidence level, and μ_o.

ONE SAMPLE MEAN RESIDUAL LIFE CONFIDENCE INTERVALS ON MODERN ENGINEERED WOOD

Using the internal bond data for MDF from Part 3, for the population MRL $e_{48}(t)$ of density 48 lbs/ft^3 a one sided 99% confidence interval for $e_{48}(t)$ to be, for example, above 80 p.s.i. would yield $\hat{\theta}$ = 102.57. This implies we can say with 99% confidence that the population MRL $e_{48}(t)$ is larger than 80 p.s.i. for all t in the entire interval [0, 102.57). A plot of $e_{48}(t)$ against its baseline function is shown in Figure 4.

For the population MRL $e_{46}(t)$ of density 46 lbs/ft^3, a one sided 99% confidence interval for $e_{46}(t)$ to be above 80 p.s.i. would yield instead $\hat{\theta}$ = 41.97. This implies we can say with 99% confidence that the population MRL $e_{46}(t)$ is larger than 80 p.s.i. for

Graph of density= 48 medium density fiberboard against a baseline of 80

Figure 4. A graph of $e_{48}(t)$ against a baseline of 80. Note that the first crossing of the functions occurs after the upper side of the 99% "function domain sets" confidence interval value of 102.57 for internal bond. In this sense, we know that our MRL function exceeds 80 p.s.i until we apply 102.57 p.s.i. of stress to the fiberboard.

all t in the entire interval [0, 41.97). A plot of $e_{46}(t)$ with its baseline function is shown in Figure 5.

Note the MRL function may help in determining actual minimum safety standards and extra safety thresholds that are statistically valid. MRL may also lead to economic benefits for the manufacturer from reduced rework and improved efficiency. Performance standards, safety standards, and advertising claims will tend to be phrased in terms of constants and make natural examples of baselines. Comparing the MRL function to a baseline while provides the opportunity to make statistically valid claims about the margin by which MDF would tend to exceed these baselines. In the above examples, 80 psi could be a performance standard for the types of MDF. The interval over which the MRL dominates the baseline function represents a degree of assurance the manufacturer may have that its MDF exceeds minimum standards.

Alternatively, the "function domain sets" confidence interval could also be used to detect special sources of variation in production of MDF. If the MRL function for an MDF type fails to dominate a minimal baseline function over a pre-defined interval, it would indicate failure of the process to meet production standards, indicating an opportunity to improve the process. By the same token, if the MRL function dominates a baseline over a longer than expected interval, a product opportunity would present itself that would present the opportunity to save money by more efficient production or to increase revenue of sales by setting an appropriate price for material that could be sold as having a higher internal bond than production variable settings were intended to produce.

Graph of density= 46 medium density fiberboard against a baseline of 80

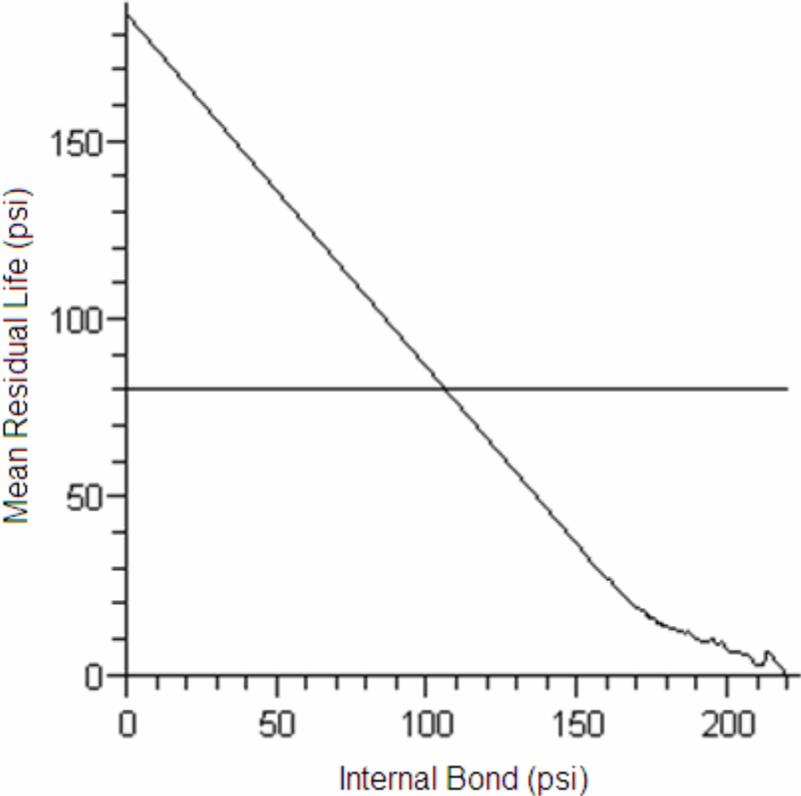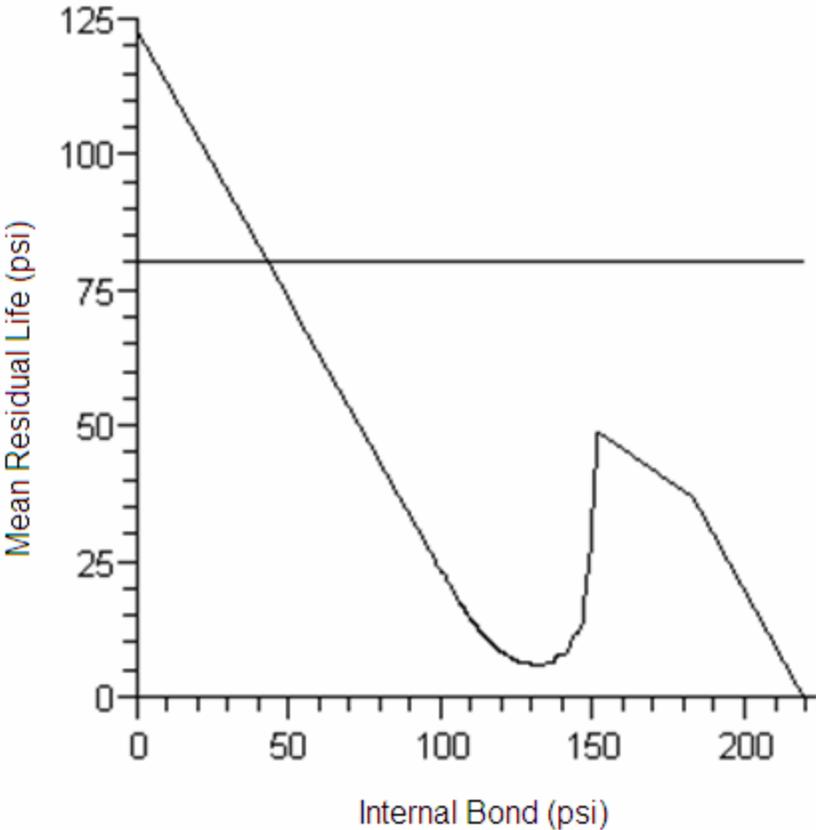**Figure 5**. Plot of the MRL function for $e_{46}(t)$ against a baseline of 80. Notice that the first crossing of these two functions occurs after the upper side of the 99% "function domain sets" confidence interval value of 41.97 for internal bond. Note that in this sense we know that our MRL function exceeds 80 p.s.i. until we apply 102.57 p.s.i. of stress to the fiberboard.

SUMMARY AND CLOSING REMARKS

We recommend using these novel confidence intervals for both the one sample setting described here in Part 4 and the two sample setting described previously in Part 3. They provide additional insights that can be used to quantify aspects suggested by graphical comparisons, which are also useful. These statistical intervals can be used to explore the data and find key thresholds. Note how unique, striking behaviors can be identified by comparative MRL plots and these novel confidence intervals on real word data sets on tensile strength measured by IB. Note too, how the one sample comparison has many natural applications. Here, with respect to comparing the internal bond of MDF to a baseline, the endpoint of the confidence interval may be thought of as a safety factor beyond the strength that will be claimed for the MDF product type. "Function domain sets" confidence intervals may facilitate improved operator awareness, process improvement, product differentiation, and cost savings.

The calculations and graphs were done in Maple, version 10. Code from Maple can be outputted in Matlab and C++. Other languages naturally are also available, for example R or S+. Copies of our code in Maple format are available by emailing Research Associate Professor Timothy M. Young at tmyoung1@utk.edu or Cody Steele at steelejc@go.com or by download from www.spcforwood.com. The text of the code is provided in Appendix B.

PART 5: MODERN MEAN RESIDUAL LIFE THEORY AND APPLICATIONS

INTRODUCTION

The MRL function has long been used for a variety of applications extending beyond evaluating the reliability of products as they are manufactured. Of particular interest recently have been the MRL functions of parallel systems, which are used in many instances when high-reliability is important, as in the components of aircraft and nuclear power plants. In addition, MDF can be machined for use as a component in a system when one considers systems of furniture, cabinets, ceiling molding, automobile parts, etc. In particular with respect to internal bond, a unit composed of multiple pieces of MDF should be constructed with consideration given to whether the failure of a single piece will constitute failure of the entire system and which components require the most strength. Discussion of the MRL of systems is relevant to understanding the performance of parallel systems with MDF components.

MEAN RESIDUAL LIFE FUNCTION THEORY RELATED TO PARALLEL SYSTEMS

Rojo and Ghebrernichael (2006) show that they can make narrower confidence interval estimates for the MRL function under the assumption that a bounding function, known or unknown, can be determined. The situation applies when an improvement in, for example, equipment or a medical treatment makes it reasonable that the MRL function of the improved system should strictly dominate the MRL function of the original system. Rojo and Ghebrernichael (2006) define a bivariate interpretation of the

MRL function, that may be extended as a multivariate MRL function, which considers the lifetimes of two individual systems. Whether known or unknown, the determination that one of these systems bounds the other allows for more efficient estimation of the MRL function of the other. The usefulness of this bounding result is of particular importance to parallel systems, which can often be bound if the class of system is known.

Myotyri, Pulkkinen, and Simola (2006) consider residual lifetime prediction in a stochastic degradation process. For illustration, they examine crack growth in a system where crack length predicts residual lifetime. Crack growth can reasonably modeled with the assumptions that the cracks strictly increase in length and are measured at specific times. This allows Myotyri, Pulkkinen, and Simola (2006) to model crack growth as a stochastic Markov process, for simplicity of computation, though they also demonstrate that numerical methods exist to allow estimation of more complex systems. The modeling of crack growth as a stochastic process is relevant to MDF performance as part of a system, i.e., MDF is a wood composite that experiences cracks in its use with fractures generally occurring parallel to the fiber wall that makes up the fibers of MDF. By observing degradation data, Myotyri, Pulkkinen, and Simola (2006) achieve residual lifetime information without observing failures. By extension, the residual lifetime predictions achieved could be used to estimate a MRL function from which inferences about a system of multiple components could be drawn.

Chinnam and Baruah (2004) discuss the extension of Myotyri, Pulkkinen, and Simola (2006) to the MRL function in the respect that they consider the use of expert knowledge for arriving at fuzzy definitions for failure. In this case, not only are actual failures not observed, but the mechanics of failure may also not be well understood. For

illustration, Chinnam and Baruah (2004) discuss a drill bit used to make holes in a stainless steel plate. Their term for this type of system is "condition-based maintenance" because the condition of the drill bit determines a great deal about the quality and efficiency of system, and must be properly maintained (in this case, replaced at the correct time) in order to cost-effectively operate the system. A fuzzy inference model, in this case trained from a neural network using the degradation data, is used to estimate the MRL function. Chinnam and Baruah's (2004) drill bit again is very relevant to MDF. One of the high-valued uses of MDF is for machining for kitchen cabinet doors, molding, furniture, etc. The interaction between diamond-tipped drill bits used for machining MDF and the tensile strength or IB reliability of MDF may directly affect drill bit reliability.

THE MEAN RESIDUAL LIFE FUNCTION OF PARALLEL SYSTEMS

A broad category of systems, in practice and in statistical thought, can be thought of as k-out-of-n systems. K-out-of-n systems are those that function correctly as long as a certain number of the original components of the system continue to operate correctly. There are numerous classes of k-out-of-n systems, some of which are: new-better-than-used, new-better-than-used-in-expectation, new-better-than-used-in-convex-ordering, new-better-than-used-in-second-stochastic-dominance, new-better-than-used-in-Laplace-order, increasing-failure-rate, decreasing-mean-residual-life, and new-worse-than-used. In theory, any of these classes could be a parallel system. Parallel systems are those that

operate as long as a single component is still functioning. Mathematically, the operational lifetime function of a parallel system, L, may be presented as:

$$L = \max\{X_1, X_2, X_3, ..., X_n\}$$

Since parallel systems are a special case of k-out-of-n systems, many properties of the MRL function of k-out-of-n systems apply to parallel systems. The MRL of a parallel system, specifically, may be defined as:

$$e^1_{(n)}(t) = E(X_{n:n} - t \mid X_{n:n} > t)$$

The above representation is for the entire parallel system. The system lasts until the failure of the last component, so the measured residual life lasts until the nth component of an n component system fails. The failure of individual components is not represented in the formula above.

The properties of parallel systems as they apply to classes of systems are discussed in numerous works. Li and Zuo (2002) provide some properties of classes of systems. Li and Zuo (2002) also show that class determination can be used to predict the behavior of the MRL functions. In particular, they show that if the lifetime function as defined above strictly dominates the MRL function then the parallel system is new-better-than-used-in-second-stochastic-dominance, which implies that the properties of new-better-than-used-in-expectation also apply to the system. Other discussions of the

properties of the MRL function with respect to classes and parallel systems may be found in Abouammoh and Elneweihi (1986) and Belzunce, Franco, and Ruiz (1999), and many others.

Asadi and Bayramoglu (2005) discuss several properties of the MRL function of a parallel system. Asadi and Bayramoglu (2005) show that under the assumption that component lifetimes have independent, identical distributions, the more working components a system has, the longer the MRL of the system will be. This means that the MRL function increases as a function of n, the number of components in the system. Although the extension is intuitive, Asadi and Bayramoglu (2005) also show that the MRL of a system decreases as the number of failed components increases, e.g. aircraft, space shuttle, bridge, etc. An upper bound for the MRL function of a parallel system with n components is also defined in the work of Asadi and Bayramoglu (2005) as:

$$e^1_{(n)}(t) \le n e_1(t) - (n-1)e_2(t)$$

Here,

$$e_1(t) = \frac{\int_t^\infty \overline{F}(x)dx}{\overline{F}(t)}$$

and

$$e_2(t) = \frac{\int_t^\infty \overline{F}^2(x)dx}{\overline{F}^2(t)}$$

The upper bound proposed by Asadi and Bayramoglu (2005) suggests that for components with independent, identically distributed lifetimes that a parallel system of n components should have a shorter MRL than n single-component systems.

Finkelstein (2001) directs his attention more generally than simply to parallel systems, considering an n-component system when he compares two kinds of statistical residual life. He terms statistical residual life as the residual life function with the independent variable being time, and terms the information-based residual life, as the residual life function which considers the number of satisfactorily operating components remaining in the system as the independent variable. Finkelstein's (2001) work makes a practical point by noticing that a parallel system has an expected residual life as soon as it is put into operation, but that residual life may be different from the expected residual life of the system when the number of failed components is known. Finkelstein (2001) considers systems with components with independent, identically distributed lifetimes and monotonic hazard functions to illustrate his point. Finkelstein (2001) illustrates that for a two-component system the information-based residual life function declines more steeply than the statistical residual life function. The information-based residual life function is lower than the statistical residual life function when only one component is operational; but the opposite is true when both components are operating. Finkelstein (2001) notes that when the component lifetimes are identically distributed, time does not affect the information-based residual life.

Finkelstein (2001) also examines the case when the component lifetimes are not identically distributed, and notes that in this case the relationship between the information-based residual life function (that considers the number of failed systems as

the independent variable) and the statistical residual life function (that considers time as the independent variable) is much more complex. When the components are not identically distributed, the information-based residual life function may depend both on the hazard functions of the components and the time at which observations are made. Li and Chen (2004) also consider the effect of assuming that the life distributions of the components in the system are independent but not identical. Note that in many cases, a single MDF item may incorporate pieces of different thicknesses and pieces that are intended to sustain different amounts of stress (e.g. furniture arms and legs) so the consideration of different lifetime distributions for the components of MDF systems is natural.

Gebraeel, Lawley, Li, and Ryan (2005) formulate Bayesian updating methods for condition-based systems monitored in real time. In this case again, actual failures are rarely observed because the condition of the system strongly determines the quality of production. Gebraeel, Lawley, Li, and Ryan (2005) provide an illustration of their Bayesian updating in accelerated testing of bearings. Degradation of bearings is measured by the amount of vibration they produce in the test, the more vibration the more degradation the bearings are believed to have experienced. First, a model for vibration as a function of time is determined with a prior assumed. The prior is updated by the data to form the posterior, which is then assumed to provide the distribution of the MRL function before the time when degradation to the point of failure has occurred. The MRL distribution thus determined makes predictions that provide an improvement over the empirical MRL distribution as evaluated for the bearing data.

Chen, Jewell, Lei and Cheng (2005) discuss the estimation of the proportional MRL function proposed by Oakes and Dasu in the presence of censoring. Here again there is a motivation to make predictions about the MRL function without observing actual failures. Chen, Jewell, Lei, and Cheng (2005) use a semi-parametric technique to allow prediction of the MRL without using failure data. Chen, Jewell, Lei, and Cheng (2005) provide an illustration using data from postoperative lung cancer patients. In Chen and Cheng (2005) a similar technique is used that results in a regression analysis of the proportional MRL function. Since destructive testing is expensive the possibility of predicting MRL without observing failures would provide advantages to MDF producers.

APPLICATIONS OF THE MEAN RESIDUAL LIFE FUNCTION IN PARALLEL SYSTEMS AND ENGINEERING

Zhang and Wang (1993) demonstrate prediction of residual life of aircraft engines using data from oil samples taken from engines. Zhang and Wang (1993) are motivated by the difficulty of accurately predicting failure from oil sample data. They cite an experiment by Aghjagan (1989) in which indications of a problem in an oil sample were used to decide when to send locomotive gear boxes for reconditioning/overhaul, but only approximately half of the gear boxes indicated by the oil samples required such work. Zhang and Wang (1993) notice that not all the data gathered from an oil sample are useful for predicting residual life, so they reduce the variables they use principal component analysis before they begin to predict residual life. By extension, estimation of the MRL function would be possible based on the predicted residual lives. With this

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ability, changes in oil routines between different gear boxes could be checked for statistical significance over several samples.

Van Noortwijk and Klatter (2004) fit the Weibull distribution to data that considers the age of existing bridges and the age of demolished bridges at the time of demolition to predict when bridges will need to be replaced. In this case, residual life is the time to replacement of a bridge. In the case of the study, data for all bridges and viaducts of interest are known and the estimation of the cost of replacing all Dutch bridges and viaducts as a function of time can be made. The estimation of the MRL function from the bridge data would allow the model to make estimations that took into account additional bridges that might be built in the future. Fryba and Pirner (2001) also consider the usefulness of the MRL function in working with the lifetimes of bridges. Fryba and Pirner (2001) document data gathered from tests of bridges in the Czech and Slovak Republics under their normal conditions of use. Monitoring the stresses of the bridges allows estimation of the residual life of the bridges. Inferences about how often inspection and repair are needed on the bridges are made based on the expected remaining life of a particular bridge.

Qian and Yan (2004) discuss using fuzzy theory to assess whether the ageing of a power transformer is proceeding in an expected or unexpected fashion. Dividing the power transformers into two groups justifies separate calculations of the residual lives of transformers that are ageing as expected from transformers that are not aging as expected. The division also highlights the need for determining the cause of failure of the transformers that are showing faster degradation than planned. The division helps to

exclude transformers operating under abnormal conditions from being used in assessing the performance of the production of all transformers.

De Backer, Schoss, and Maussner (2001) discuss the frequency with which components in nuclear power plants experience stresses unlike those planned in their design. Because of the safety issues related with the operation of nuclear power plants, De Backer, Schoss, and Maussner (2001) recommend that methods of monitoring components for stress while they are in operation should be used to update predictions about the original remaining life of the components based on its exposure to design basis conditions. De Backer, Schoss, and Maussner (2001) demonstrate changes in magnetic properties of stainless steel that occur when it is placed under stress at high temperature as a means of reevaluating the residual life of safety significant systems in use in nuclear power plants. In practice, continual monitoring of most MDF systems is not performed, but the idea that previous stress may affect the MRL is useful in terms of MDF use. For example, if an MDF board cracks under great strain such that its internal bond is weakened; there is advantage to having the ability to update the MRL function. As MDF is used for more applications that involve semi-permanent use (e.g. construction), there will be increased interest in updating stress to replacement

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PART 6: CONCLUDING REMARKS AND FUTURE RESEARCH

In this thesis, we have demonstrated the use of "function domain sets" confidence intervals for identifying opportunities for MDF producers. In particular, where more than one kind of MDF is produced, we are able to recognize a hybrid product that occurs when machine settings are switched from production of one type of MDF to another. This hybrid product represents an opportunity to the producer as a new product to sell--at a higher price than would have been claimed by MDF with a lower internal bond. This demonstration emphasizes the usefulness of the MRL function and "function domain sets" confidence intervals as a reliability tool to help increase manufacturing profits.

The work in this thesis suggests numerous avenues of pursuit for future research. As a very practical matter, continuous improvement in achieving stronger internal bonds of MDF will make the product suitable for even more applications. Already, improvements in fire resistance, termite resistance, and mold resistance have been achieved that suggest that MDF has a strong future in construction of homes. Increased understanding of the mechanisms of manufacturing MDF has a high potential to improve its usefulness to society.

There are also many pathways for future research into the MRL function and the "function domain sets" confidence intervals of the MRL discussed here. In many cases, it is desirable to estimate the MRL function without observing outright failures. There is a great deal of opportunity in specific industries to improve understanding of mechanisms of failure so that the MRL function can be more accurately modeled early enough to provide useful information about when items can be replaced before they fail. There is

also a corresponding need to identify methods of generating fuzzy definitions of failure, so that items can be replaced or repaired when such action will improve process efficiency and the quality of finished products, rather than simply avoiding failure of the item. Training neural networks has shown some promise in the area of describing failure before it can be detected as a failed item.

The "function domain sets" confidence intervals demonstrated herein are dependent on asymptotic normality to provide useful comparisons. Some conditions will arise where they could be more usefully applied if developed for use with bootstrapping or other re-sampling techniques. The application of "function domain sets" confidence intervals to other types of functions, such as the hazard rate function, may also reveal useful information about processes. It may also be worthwhile to consider the possibility of developing multiple comparison techniques for MRL functions that grow out of "function domain sets" confidence intervals. For example, multiple wood sources could be compared at the same time to discern a best type for internal bond after production with specific variable settings. This may be very important for reducing the costs associated with MDF manufacturing.

As the applications of MDF grow, the incorporation of MDF components into systems will also become more and more common. Estimation of the MRL functions of MDF and other systems will have numerous applications for describing the reliability of such systems. Since the residual lives of systems are dependent on the residual lives of their components, increased modeling of systems where the component residual lives are correlated or where component lifetimes are non-identically distributed will increase the usefulness of MRL functions in predicting the failure of complicated systems. The

estimation of the mean residual lives of parallel systems captures high importance because using a parallel system often indicates that failure of the system could have severe consequences.

APPENDICES

APPENDIX A: MAPLE CODE

INTRODUCTION

Calculations and figures in this thesis were generated principally with the Maple 10 software package. This appendix provides a few notes about the coding done in Maple 10 for this thesis. Maple is just one of many software packages that are suitable for these calculations, a short list of which could include: Matlab, R, S-Plus, and SAS. Maple was chosen because of my personal familiarity with it. Maple 10, in fact, claims the ability to export code to Matlab and C, but this function has not been evaluated for the code that follows.

In the process of converting the code to text for this document, Maple 10 removed the line ending portions of the code. In general, these are a colon, :, if one does not wish to see the result of that line of code printed in Maple and a semicolon, ;, if one does wish to view the result of that line of code. Keep in mind that the code that follows in this section will not function if a "cut and paste" operation is used to move the code from the thesis directly into Maple 10. Placing the code portion in a file with the .txt extension and then opening it in Maple 10 will also not produce satisfactory results without additional labor. If possible, the recommended course of action is to obtain an original file of the code by emailing Research Associate Professor Timothy M. Young at tmyoung1@utk.edu, by e-mailing Cody Steele at steelejc@go.com or by downloading the code from www.spcforwood.com.

When running the code with your own data sets, keep in mind that the import matrices will have to be identified properly for your own files. In the current form of the code they are saved as "x" and "x2." The current version of the code demonstrates importing from a tab-delimited file with a Microsoft Excel file extension (.xls). Many other types of files can be imported and can be found in Maple's help files. The Z and T scores in the code may be manipulated, but are written in as positive values, so the data with the MRL function expected to dominate should be entered as "x."

There are portions of the code that are included with the intention that they be helpful to individuals in their own programming efforts. The variable "printlevel," for example, tells Maple 10 how many operations to display as it runs a nested loop. It is not necessary to set the print level for the correct execution of the code, but various instances are retained to suggest places where investigation of a nested loop may be of interest. Some plot options that were not used to produce graphs in the thesis are also included for users to review. Comments and explanations are also included and may be identified by the presence of what Maple documentation tens to call a "sharp character" but which may also be familiar as a "number sign" to many readers: the # symbol. In the code text below comments have been italicized to differentiate them from operable code. Values that are shown in the code text are those obtained using the data demonstrated in the thesis.

As Maple software advances in design, many features have become accessible from push buttons, menus, and have added interactive capability. These kinds of capabilities in Maple 10 will not be adequately demonstrated by reading the code and examining the plots below. In fact, one of the more useful interactive capabilities Maple

possesses is in its plots, which can usually be accessed by right-clicking a plot displayed in the Maple 10 environment. For further assistance with Maple 10 and succeeding versions, we recommend the files available at the "Documentation Center" portion of the Maplesoft website, http://www.maplesoft.com/documentation%5Fcenter/. Programming guides and introductory manuals are available for download at no cost for people who sign up as members, also free. The Maplesoft website will also search for books on Maple published privately. During my own work here, although I used very few of the operations directly, I found Calclabs with Maple for Stewart's Single Variable Calculus, 5th Edition (Yasskin, Barrow, Belmonte, Boggess, Morgan, Rahe, Smith, and Stecher, 2003) to be well-organized and to have examples that were clear enough to be modified easily. Also, although for an older version of Maple than demonstrated below, I found Harichandran (1999) provides many useful points on operations and syntax for the beginning programmer. Harichandran's instructions are available for download from his website at http://www.egr.msu.edu/~harichan/.

CODE

```
> #The first four lines invoke Maple Command libraries used
in the rest of the code and are necessary for the correct
execution of the code.
> with(Statistics):

> with(plots):
Warning, the name changecoords has been redefined
> with(LinearAlgebra):
Warning, the name Rank has been rebound
> with(CurveFitting):
Warning, the name LeastSquares has been rebound
```

> *#The first portion of code performs the one−sample comparison of the mean residual life function to a baseline.*

> *#The following line imports data from a text file that is tab delimited. See Maple's Help files for ways to import other kinds of files. For correct operation, all failures in the data should take place when thte stress is greater than zero and a zero should be added to the data so the mean residual life function will be computed at stress = 0.*

> $x := ImportMatrix\ ("48density.txt", delimiter = "\backslash t"):$

> *#Some data manipulation steps are easier to make when data is defined in a certain way in Maple. The next line converts the original data a list so the "sort" command may be used.*

```
> p:=convert(x,list):
>
> q:=sort(p):
```

> *# In preparation for later mathematical operations, the list is converted to an array.*

> $t := Array(q):$

>

> *# c sets the number of rows that will be in the matrix where calculations of the standard errors, mean residual life, etc. will be calculated. Because the first row will represent stress = 0, the sample size is c − 1. a is a counting variable. The number of columns in the matrix, K, are predefined based on the number of calculations that will be recorded. In this case, the seven columns are : 1. row number, 2. mean residual life, 3. standard error of the sample used to calculate the mean residual life, 4. sample size, 5. stress level for each failure, 6. z − value at beginning of failure stress interval, 7. z − value at end of failure stress interval.*

>   # *for ease of checking, some lines end in a semi−colon. In Maple, lines that end in a colon do not display their result when the code is running and lines that end in semi−colons do. Occassionally lines are inserted specifically to provide an opportunity to check a value.*

> $c := ArrayNumElems(t)$;

$$c := 109$$

> $a := 1$:

> $K := Matrix(c, \, 7)$:

>   # *The 4 lines below are redone in the loop below, but are provided here as simple examples of the second, third, and fourth columns. m takes the positive values of t, which is our original data. Here, m is all the failure times. Inside the actual loop, m takes the values of the times remaining after the time of the most recent failure.*

> $m := Select(y \rightarrow is(0 < y), \, t)$:

> $K[a, 4] := c - 1$:

> $K[a, 3] := StandardDeviation(m)$:

> $K[a, 2] := Mean(m)$:

> a;

<div align="center">1</div>

>   # *This loop writes the remaining mean residual lives, standard errors, and sample sizes for the samples after each failure.*

> **for** a **to** c **do**
      $r := t - t_a;$
      $m := Select(y \rightarrow is(0 < y), r);$
      $if \ (ArrayNumElems(m)) > 1 \ then \ K[a, 3] :=$
      $StandardDeviation(m); end \ if;$
      $if \ (ArrayNumElems(m)) > 0 \ then \ K[a, 2] := Mean(m)$
      $; \ end \ if;$
      $K[a, 4] := ((c - 1) - (a - 1));$
   end do:

> $K[1, 4];$

$$1$$
$$08$$

>

> *#This loop writes the failure times into the matrix K.*

> $counter2 := 1 :$

> $for \ counter2 \ to \ c \ do \ K[counter2, 5] := t[counter2]; \ end \ do$
    $:$

> *# In cases where more than one individual fails at the*
   same stress, the mean residual life function only takes one
   value. The following loop counts the number of ties so
   that the correct value $-$ the one that considers all the
   failures that take place at a stress level $-$ can be left
   in the matrix, K.

> $nties := 0 :$

> $for \ counter4 \ from \ 1 \ to \ c - 1 \ do$
   $if \ K[counter4, 5] = K[counter4$
   $+ 1, 5] \ then \ nties := nties + 1; \ end \ if; \ end \ do :$

> $nties :$

>

> $counter3 := 1 :$

> *#The following loop removes rows that are not needed because of tied failure stresses.*

> *for counter3 from 1 by 1 to $(c - nties - 1)$ do if $K[counter3, 5] = K[counter3 + 1, 5]$ then $K := DeleteRow(K, counter3)$; counter3 := counter3 − 1; end if; end do :*

> *#The new number of rows after the tied rows have been deleted is defined below as "Krows."*

> *Krows := $c - nties$:*

>

> *#The next loop adds row numbers to matrix K for identification numbers. Although positioned in the first column of the matrix, it needs to be added at the end after the tied rows are deleted to make sense.*

> *counter7 := 1 :*

> *for counter7 from 1 to Krows do $K[counter7, 1] := counter7$; end do :*

> *#The baseline may be chosen as any number desired. The code has not been tested with the baseline defined as a function, but this may be possible.*

> *baseline := 80 :*

> *#The loop below computes a z−score for the mean residual life function vs. the baseline. This is the one sample procedure defined in Part 4 of the thesis.*

> *counter5 := 1 :*

> *for counter5 from 1 by 1 to Krows − 2 do*
$$K[counter5, 6] := \left(\frac{(K[counter5, 2] - baseline)}{\left(\frac{(K[counter5, 3])^2}{K[counter5, 4]} \right)^{.5}} \right); \ end \ do :$$

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> #*The loop below checks the endpoint of the $z-$function for the mean residual life function vs. the baseline. When the $z-$score for the desired confidence level is between the end point and the $z-$score we can find the mean residual life function "function domain sets" confidence interval endpoint. These steps follow the loop below.*

> *counter6 := 1 :*

>

> *for counter6 from 1 by 1 to Krows − 2 do*
> $K[counter6, 7] := K[counter6, 6]$
> $+ \left(\dfrac{K[counter6, 5] - K[counter6 + 1, 5]}{\left(\left(\dfrac{(K[counter6, 3])^2}{K[counter6, 4]} \right)^{.5} \right)} \right); end \quad do :$

>

> *# Desired confidence levels may be manipulated as desired. Three are provided, so different levels can be compared as useful.*

>

> *desiredconfidence1 := 90 :*

> *desiredconfidence2 := 95 :*

> *desiredconfidence3 := 99 :*

> *# Note that $z-$ values for confidence are for, in this case, one−sided confidence intervals. Adjustments should be made in the "desiredconfidence" variables above if a two−sided interval is of interest.*

> *zalpha1 :=*
> *Percentile (Normal (0, 1), (desiredconfidence1), numeric);*

$$zalpha1 := 1.281551566$$

> *zalpha2 :=*
> *Percentile (Normal (0, 1), desiredconfidence2, numeric);*

$$zalpha2 := 1.644853627$$

>

> $zalpha3 :=$
$Percentile(Normal(0, 1), \ desiredconfidence3, \ numeric);$

$$zalpha3 := 2.326347874$$

> *# The loop below finds the interval where the MRL z−score*
function crosses the z−value of the desired confidence level.
Immediately below, the failure time is interpolated that gives
the upper bound of the "function domain sets" confidence interval
.

> $counter8 := 1 :$

> *for counter8 from 1 to Krows do if $K[counter8, 7] <$*
$zalpha1$ and $K[counter8, 6] > zalpha1$ then
$interpolaterow1 := counter8;$ $counter8 := counter8 + Krows;$
end if; end do :

> *interpolaterow1 :*

> *#The lines below provide the upper end of the calculated*
"function domain sets" confidence intervals. These are not
output to the matrix, but are the main output of this portion
of the code.

> $upper90 :=$
$PolynomialInterpolation([[K[interpolaterow1, 6],$
$K[interpolaterow1, 5]],$
$[K[interpolaterow1, 7], K[interpolaterow1$
$+ 1, 5]]], zalpha1);$

$$upper90 := 103.9752291$$

> *for counter8 from 1 to Krows do if $K[counter8, 7] <$*
$zalpha2$ and $K[counter8, 6] > zalpha2$ then
$interpolaterow2 := counter8;$ $counter8 := counter8$
$+ Krows;$ end if; end do;

> *upper95* :=
    PolynomialInterpolation([[*K*[*interpolaterow2*, 6],
    K[*interpolaterow2*, 5]],
     [*K*[*interpolaterow2*, 7], *K*[*interpolaterow2*
    + 1, 5]]], *zalpha2*);

$$upper95 := 103.4875929$$

> *for counter8 from* 1 *to Krows do if K*[*counter8*, 7] <
    zalpha3 and K[*counter8*, 6] > *zalpha3 then*
    interpolaterow3 := *counter8*; *counter8* := *counter8*
    + *Krows*; *end if*; *end do*;

> *upper99* :=
    PolynomialInterpolation([[*K*[*interpolaterow3*, 6],
    K[*interpolaterow3*, 5]],
     [*K*[*interpolaterow3*, 7], *K*[*interpolaterow3*
    + 1, 5]]], *zalpha3*);

$$upper99 := 102.5728684$$

> *#This file may be renamed. It contains the numbers saved in matrix K as described earlier.*

> *ExportMatrix* ("first.xls", *K*) :

> *#The following lines of code repeat those previously for a second data sample. As before, zero should be entered into a data set of failure times so that the MRL function will be correctly computed beginning from the stress = 0 point.*

> *x2* := *ImportMatrix* ("46density.txt", *delimiter* = "\t"):

```
> p2:=convert(x2,list):
>
> q2:=sort(p2):
```

> *t2* := *Array*(*q2*) :

>

>

> *c2* := *ArrayNumElems*(*t2*);

$$c2 := 975$$

> $a2 := 1 :$

> $K2 := Matrix(c2, 7) :$

> $m2 := Select(y \rightarrow is(0 < y), t2) :$

> $K2[a2, 4] := c2 - 1;$

$$K2_{1, 4} := 974$$

> $K2[a2, 3] := StandardDeviation(m2) :$

> $K2[a2, 2] := Mean(m2) :$

>

> *for $a2$ to $c2$ do*
> $r2 := t2 - t2[a2];$
> $m2 := Select(y \rightarrow is(0 < y), r2);$
>   *if $ArrayNumElems(m2) > 1$ then $K2[a2, 3] :$*
>   *$= StandardDeviation(m2)$; end if;*
>   *if $ArrayNumElems(m2) > 0$ then $K2[a2, 2] := Mean$*
>   *$(m2)$; end if;*
> $K2[a2, 4] := ((c2 - 1) - (a2 - 1));$
>   **end do**:

> $K2[1, 4];$

$$\frac{9}{74}$$

> $counter2 := 1 :$

> *for $counter2$ to $c2$ do $K2[counter2, 5] := t2[counter2]$; end*
>   *do :*

>

> $nties2 := 0 :$

>   *for counter4 from 1 to c2 − 1 do*
     if K2[counter4, 5] = K2[counter4
    + 1, 5] then nties2 := nties2 + 1; end if; end do :

>   *nties2;*

$$6$$
$$27$$

>

>   *counter3 := 1 :*

>   *for counter3 from 1 by 1 to (c2 − nties2 − 2) do if*
    K2[counter3, 5] = K2[counter3
    + 1, 5] then K2 := DeleteRow(K2, counter3); counter3 :=
    counter3 − 1; end if; end do :

>   *Krows2 := (c2 − nties2);*

$$Krows2 := 348$$

>

>   *counter7 := 1 :*

>

>   *for counter7 from 1 to Krows2 do K2[counter7, 1] :=*
    counter7; end do :

>   *baseline2 := 80 :*

>   *counter5 := 1 :*

>   *for counter5 from 1 by 1 to Krows2 − 2 do*
    $K2[counter5, 6] := \left(\dfrac{(K2[counter5, 2] - baseline2)}{\left(\dfrac{(K2[counter5, 3])^2}{K2[counter5, 4]} \right)^{.5}} \right);$ *end*
    do :

>

>   *counter6 := 1 :*

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>

> for counter6 from 1 by 1 to Krows2 − 2 do
$$K2\,[\,counter6,\,7\,] := K2\,[\,counter6,\,6\,]$$
$$+ \left(\frac{K2\,[\,counter6,\,5\,] -\; K2\,[\,counter6 + 1,\,5\,]}{\left(\left(\dfrac{(\,K2\,[\,counter6,\,3\,]\,)^2}{K2\,[\,counter6,\,4\,]} \right)^{.5} \right)} \right); end \;\; do:$$

> counter7 := 1 :

> for counter7 from 1 to Krows2 do K2[counter7, 1] := counter7; end do :

>

> desiredconfidence1b := 90 :

> desiredconfidence2b := 95 :

> desiredconfidence3b := 99 :

> zalpha1b :=
   Percentile(Normal(0, 1), (desiredconfidence1b), numeric);

$$zalpha1b := 1.281551566$$

> zalpha2b :=
   Percentile(Normal(0, 1), desiredconfidence2b, numeric);

$$zalpha2b := 1.644853627$$

>

> zalpha3b :=
   Percentile(Normal(0, 1), desiredconfidence3b, numeric);

$$zalpha3b := 2.326347874$$

> counter8 := 1 :

> for counter8 from 1 to Krows2 do if K2[counter8, 7] <
   zalpha1b and K2[counter8, 6] > zalpha1b then
   interpolaterow1b := counter8; counter8 := counter8 + Krows2;
    end if; end do :

> interpolaterow1b :

> *upper90b :=*
   PolynomialInterpolation([[K2[interpolaterow1b, 6],
   K2[interpolaterow1b, 5]],
    [K2[interpolaterow1b, 7], K2[interpolaterow1b
   + 1, 5]]], zalpha1b);

$$upper90b := 42.31469156$$

> *for counter8 from 1 to Krows2 do if K2[counter8, 7] <*
   zalpha2b and K2[counter8, 6] > zalpha2b then
   interpolaterow2b := counter8; counter8 := counter8
    + Krows2; end if; end do;

> *upper95b :=*
   PolynomialInterpolation([[K2[interpolaterow2b, 6],
   K2[interpolaterow2b, 5]],
    [K2[interpolaterow2b, 7], K2[interpolaterow2b
   + 1, 5]]], zalpha2b);

$$upper95b := 42.19533335$$

> *for counter8 from 1 to Krows2 do if K2[counter8, 7] <*
   zalpha3b and K2[counter8, 6] > zalpha3b then
   interpolaterow3b := counter8; counter8 := counter8
    + Krows2; end if; end do;

> *upper99b :=*
   PolynomialInterpolation([[K2[interpolaterow3b, 6],
   K2[interpolaterow3b, 5]],
    [K2[interpolaterow3b, 7], K2[interpolaterow3b
   + 1, 5]]], zalpha3b);

$$upper99b := 41.97143715$$

> *ExportMatrix ("second.xls," K2) :*

> # *This begins the portion that explores the two − sample case, using the data from the matrices named K and K2 in the code. To begin with, the data from those two matrices are combined into a single matrix. The number of rows for the new matrix are determined immediately below and then the new matrix is defined on the next line. The number of columns for the new matrix, 11, is fixed based on the information that will be recorded in the new matrix.*

> *twosamprows* := *Krows* + *Krows2*;

$$twosamprows := 448$$

> *twosamp* := *Matrix*(*twosamprows*, 11) :

> *countera* := 1 :

> *counterb* := 1 :

> *#This loop enters the failure stress values from the first imported data set from the matrix named K into the first column of the two − sample matrix. It also enters the two − sample z − value for the failure times present in the first column from the first data set. Both values will be reordered for presentation.*

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> *for countera from 1 by 1 to Krows do twosamp*[*countera*, 1]
:= *K*[*countera*, 5];
if K[*countera*, 5] > *K2*[*Krows2*, 5] *then counterb* :=
Krows2 + 1 *elif K*[*countera*, 5] < *K2*[*Krows2*, 5] *then*
counterb := 1; *while K2*[*counterb*, 5] <= *K*[*countera*, 5]
do counterb := *counterb*
+ 1; *end do*; *end if*;
if K[*countera*, 4] = 0 *or K2*[*counterb* − 1, 4] = 0 *or*
(*K*[*countera*, 3] = 0 *and K2*[*counterb* − 1, 3] = 0) *then*
twosamp[*countera*, 2] := 0;
else twosamp[
countera, 2] := (*K*[*countera*, 2]
− (*K2*[*counterb* − 1, 2]

$$- (K[countera, 5] - K2[counterb - 1, 5]))\Big/\Big($$

$$sqrt\left(\left(\frac{K[countera, 3]^2}{K[countera, 4]}\right) + \left(\frac{K2[counterb - 1, 3]^2}{K2[counterb - 1, 4]}\right)\right);$$

end if; *end do* :

> *counterb*;

$$\frac{3}{49}$$

> *countera*;

$$\frac{1}{01}$$

> *counterc* := *countera*;

$$counterc := 101$$

> *counterd* := 1;

$$counterd := 1$$

> *countere* := 1;

$$countere := 1$$

> *# The loop below enters the failure stresses from the second data set, matrix K2. It also calculates and enters the two − sample z − values for the failure stress values from the second data set. Both values will be reordered for presentation.*

> *for counterc from countera by 1 to twosamprows do twosamp[counterc, 1] := K2[counterd, 5]; if K2[counterd, 5] > K[Krows, 5] then countere := Krows + 1 else countere := 1; while K[countere, 5] <= K2[counterd, 5] do countere := countere + 1; end do; end if; if K2[counterd, 4] = 0 or K[countere − 1, 4] = 0 or (K2[counterd, 3] = 0 and K[countere − 1, 3] = 0) then twosamp[counterc, 2] : 0 else twosamp[counterc, 2] := ((K[countere − 1, 2]*
− (K2[counterd, 5] − K[countere − 1, 5])) −
$K2[counterd, 2]$ $\Big/ \Big(sqrt\Big(\Big(\dfrac{K2[counterd, 3]^2}{K2[counterd, 4]} \Big)$
$+ \Big(\dfrac{K[countere − 1, 3]^2}{K[countere − 1, 4]} \Big) \Big) \Big)$*; end if; counterd := counterd + 1; end do :*

>

> *counterc;*

$$\frac{4}{49}$$

> *counterd;*

$$\frac{3}{49}$$

> *#The next four lines order the combined failure times from the two data sets. These will be put in the third column of the matrix in the next loop.*

> *failures := Column(twosamp, 1) :*

> *fail := convert(failures, list) :*

> $fails := sort(fail) :$

> $failu := Array(fails) :$

>

>

>

> $counterf := 1;$

$$counterf := 1$$

> $for\ counterf\ from\ 1\ to\ twosamprows\ do$
> $\quad twosamp[counterf,\ 3] := failu[counterf];\ end\ do :$

> $counterg := 1 :$

> $counterf := 1 :$

> $printlevel := 4;$

$$printlevel := 4$$

> $\#\ The\ loop\ below\ matches\ the\ unordered\ z-values\ from$
> $\quad earlier\ with\ the\ correct\ failure\ value\ and\ prints\ them\ in\ the$
> $\quad fourth\ column\ of\ the\ matrix.$

> $for\ counterg\ from\ 1\ by\ 1\ to\ twosamprows\ do\ counterg;$
> $\quad counterf := 1;\ if\ twosamp[counterg,\ 1] =$
> $\quad twosamp[counterf, 3]\ then\ twosamp[counterf, 4] :=$
> $\quad twosamp[counterg, 2];\ else\ while\ twosamp[counterf, 3] <$
> $\quad = twosamp[counterg, 1]\ and\ counterf < (twosamprows)$
> $\quad do\ counterf := counterf + 1;\ end\ do;$
> $\quad twosamp[counterf - 1, 4] := twosamp[counterg, 2];\ end\ if;$
> $\quad end\ do :$

> $\#The\ next\ loop\ ensures\ that\ any\ cases\ where\ the\ failure\ times\ are$
> $\quad the\ same\ have\ the\ same\ z-value.$

> $counterf :$

> $if\ twosamp[1, 3] = twosamp[2, 3]\ then\ twosamp[2, 4] :=$
> $\quad twosamp[1, 4];\ end\ if;$

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$$twosamp_{2,\,4} := 45.56163144$$

> *counterg* :

> #*The loop below repeats checking that any cases where the failure times are the same have the same z−value.*

> *counterz* := 1 :

> *for counterz from 1 by 1 to counterg − 2 do if twosamp [counterz, 3] = twosamp [counterz + 1, 3] then twosamp [counterz, 4] := twosamp [counterz + 1, 4]; end if; end do;*

$$twosamp_{1,\,4} := 45.56163144$$

> #*The next loop takes the number surviving from the first data set and puts them into the new matrix in the fifth column.*

> *counteraa* := 1 :

> *for counteraa from 1 by 1 to Krows do twosamp [counteraa, 5] := K [counteraa, 4]; end do :*

> *counterbb* := Krows + 1 :

> #*The next loop places the number surviving in column 6 of the new matrix beginning in the row after the last number surviving from the first data set.*

> *countercc* := 1 :

> *for counterbb from Krows + 1 by 1 to twosamprows do twosamp [counterbb, 6] := K2 [countercc, 4]; countercc := countercc + 1; end do :*

> *countergg* := 1 :

> *counterff* := 1 :

78

\> *#The row below puts the number surviving from the first data into the seventh column set for the ordered survival times in the third column. These will be compared to the number surviving for the second data set* (*placed in the eighth column*) *to determine the degrees of freedom for the two* − *sample t* − *statistic.*

\> *for countergg from* 1 *by* 1 *to Krows do counterff* := 1;
  if twosamp [*countergg*, 3] = *twosamp* [*countergg*, 1] *then*
 twosamp [*counterff*, 7] := *twosamp* [*countergg*, 5];
  else while twosamp [*counterff*, 3] <=
 twosamp [*countergg*, 1] *and counterff* <= *twosamprows*
 do counterff := *counterff*
 + 1; *end do*; *twosamp* [*counterff* − 1, 7] :=
 twosamp [*countergg*, 5]; *end if*; *end do* :

\> *counterdd* := 1 :

\> *counteree* := *Krows* + 1 :

\> *#The next loop places the number surviving from the second data set into the eighth column that correspond to the ordered survival times in the third column.*

\> *for counteree from Krows* + 1 *to twosamprows do counterdd*
  := 1; *if twosamp* [*counterdd*, 3] = *twosamp* [*counteree*, 1]
 then twosamp [*counterdd*, 8] := *twosamp* [*counteree*, 6]; *else*
  while twosamp [*counterdd*, 3] <= *twosamp* [*counteree*, 1]
 and counterdd <= *twosamprows do counterdd* :=
 counterdd + 1; *end do*; *twosamp* [*counterdd* − 1, 8] :=
 twosamp [*counteree*, 6] *end if*; *end do* :

\> *#The next loop makes corrections to sample sizes so that the number surviving is shown correctly at the top of the matrix where there are two zero times, since each data set had one initially.*

\> *counterhh* := 1 :

> *for counterhh from 1 by 1 to twosamprows do if twosamp[counterhh, 7] = 0 and twosamp[counterhh, 8] = 0 and twosamp[counterhh, 3] = 0 then twosamp[counterhh, 7] := twosamp[counterhh − 1, 7]; twosamp[counterhh, 8] := twosamp[counterhh − 1, 8]; end if; end do;*

$$twosamp_{2,\,7} := 108$$

$$twosamp_{2,\,8} := 974$$

> *counterii := 1 :*

> *for counterii from 1 by 1 to twosamprows − 1 do if twosamp[counterii, 7] = 0 and twosamp[counterii, 8] = 0 then twosamp[counterii, 7] := twosamp[counterii + 1, 7]; twosamp[counterii, 8] := twosamp[counterii + 1, 8]; end if; end do;*

$$twosamp_{447,\,7} := 0$$

$$twosamp_{447,\,8} := 0$$

> *#The loop below makes corrections to columns where a zero number surviving was left because no failure time matched for that column. The line immediately afterwards corrects where the zero is overwritten that should belong there.*

> *counterjj := 1 :*

> *twosamp[Krows, 1];*

$$2$$
$$19.6999969$$

> *for counterjj from 1 by 1 to twosamprows do if twosamp[counterjj, 7] = 0 and twosamp[counterjj, 3] < twosamp[twosamprows, 1] then twosamp[counterjj, 7] := twosamp[counterjj − 1, 7]; end if; if twosamp[counterjj, 8] = 0 and twosamp[twosamprows, 1] < twosamp[Krows, 1] then twosamp[counterjj, 8] := twosamp[counterjj − 1, 8]; end if; end do :*

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>

> *twosamp* [*twosamprows*, 8] = 0;

$$1 = 0$$

> *#This loop calculates the t statistic that is appropriate for comparison to the t − score of the two − sample confidence level of interest. This alerts the user to cases where differences may appear large but are not statisitically significant because of small sample sizes. Here, the confidence level is 99 %, but this may be adjusted at the convenience of the user.*

> *counterkk* := 1 :

> *for counterkk from 1 by 1 to twosamprows do tdf := min (twosamp [counterkk, 8], twosamp [counterkk, 7]); if tdf > 1 then twosamp [counterkk, 9] := Percentile (StudentT (tdf − 1), 99, numeric); end if; end do :*

> *#A matrix is output here with the information that has been calculated for the two sample case.*

> *ExportMatrix* ("check.xls", *twosamp*) :

> *#A new matrix is created that should neatly output all the numbers calculated in the previous code. A new row is added so that column labels can be added. At the end, it is output as a Microsfot Excel file. See Maple's help for directions on outputting to other types of files.*

> *outputmatrix* := *Matrix* (*twosamprows* + 1, 11) :

> *outputmatrix* [1, 1] := "unordered failure times":

> *outputmatrix* [1, 2] := "unordered z scores":

> *outputmatrix* [1, 3] := "failure times":

> *outputmatrix* [1, 4] := "z scores":

> *outputmatrix* [1, 5] := "unordered number surviving from sample1":

> $outputmatrix [1, 6] := $"unordered number surviving until, sample"

> $outputmatrix [1, 7] := $"number surviving until, sample"

> $outputmatrix [1, 8] := $"number surviving until, sample"

> $outputmatrix [1, 9] := $"$t$ $scores$":

> $outputmatrix [1, 10] := $"mrl, sample 1":

> $outputmatrix [1, 11] := $"mrl, sample 2":

> $outputcounter := 1 :$

> $outcounter := 2 :$

> *for outputcounter from 1 to 9 do for outcounter from 2 to twosamprows do*
    $outputmatrix [outcounter, outputcounter] :=$
    $twosamp [outcounter - 1, outputcounter]; end do; end do :$

> $ocounter := 1 :$

> $pcounter := 1 :$

> $printlevel := 1 :$

> # *Not included in the "check" matrix, the next loop adds the MRL values to the "output" Microsoft Excel file.*

> *for ocounter from 1 to twosamprows do for pcounter from 1 to Krows do if* $outputmatrix [ocounter, 7] =$
    $K[pcounter, 4]$ *then* $outputmatrix [ocounter, 10] :=$
    $K[pcounter, 2]; $ *end if; end do; end do :*

> $ocounter := 1 :$

> $pcounter := 1 :$

> *for ocounter from 1 to twosamprows do for pcounter from 1 to Krows2 do if* $outputmatrix [ocounter, 8] =$
    $K2[pcounter, 4]$ *then* $outputmatrix [ocounter, 11] :=$
    $K2[pcounter, 2]; $ *end if; end do; end do :*

> $ExportMatrix ($"output.xls"$, outputmatrix);$

> *#Calculations for the two sample confidence intervals are prepared below. The desired confidence levels may be adjusted as desired by the user.*

> $desiredtwosampconfidencea := 90 :$

> $desiredtwosampconfidenceb := 95 :$

> $desiredtwosampconfidencec := 99 :$

> $zalphatsa :=$
> $Percentile(Normal(0, 1), (desiredtwosampconfidencea),$
> $numeric);$

$$zalphatsa := 1.281551566$$

> $zalphatsb :=$
> $Percentile(Normal(0, 1), desiredtwosampconfidenceb,$
> $numeric);$

$$zalphatsb := 1.644853627$$

>

> $zalphatsc :=$
> $Percentile(Normal(0, 1), desiredtwosampconfidencec,$
> $numeric);$

$$zalphatsc := 2.326347874$$

> $twosampa := 1 :$

> $counterh := 1 :$

> *# The loop below finds where the two−sample z−function crosses the desired confidence level z−value and saves it as the row number as the variable "twosampa." Immediately afterwards, linear interpolation is used to find the endpoint of the "function domain sets" confidence interval for which the greater MRL function dominates the lower one. The process is repeated for for the remaining two confidence levels.*

> *for counterh from 1 by 1 to twosamprows do if twosamp[counterh, 4] < zalphatsa then twosampa := counterh; counterh := counterh + twosamprows; end if; end do;*

> *twosampa;*

$$3$$
$$44$$

> *upper90ts :=*
> *PolynomialInterpolation([[twosamp[twosampa − 1, 4], twosamp[twosampa − 1, 3]], [twosamp[twosampa, 4], twosamp[twosampa, 3]]], zalphatsa);*

$$upper90ts := 147.4675343$$

>

> *for counterh from 1 by 1 to twosamprows do if twosamp[counterh, 4] < zalphatsb then twosampb := counterh; counterh := counterh + twosamprows; end if; end do;*

> *twosampb;*

$$3$$
$$43$$

> *upper95ts* :=
>   *PolynomialInterpolation* ([[
>   *twosamp* [*twosampb* − 1, 4], *twosamp* [*twosampb* − 1, 3]],
>   [*twosamp* [*twosampb*, 4], *twosamp* [*twosampb*, 3]]],
>   *zalphatsb*) ;

$$upper95ts := 147.3940803$$

> *for counterh from* 1 *by* 1 *to twosamprows do if*
>   *twosamp* [*counterh*, 4] < *zalphatsc then twosampc* :=
>   *counterh* ; *counterh* := *counterh* + *twosamprows* ; *end if* ; *end*
>   *do* ;

> *twosampc* ;

$$3$$
$$42$$

> *upper99ts* :=
>   *PolynomialInterpolation* ([[
>   *twosamp* [*twosampc* − 1, 4], *twosamp* [*twosampc* − 1, 3]],
>   [*twosamp* [*twosampc*, 4], *twosamp* [*twosampc*, 3]]],
>   *zalphatsa*) ;

$$upper99ts := 147.9844381$$

> # *The next lines repeat the confidence interval process with*
>   *the t distribution instead of with the normal distribution,*
>   *to check for any problems with sample size making the z−*
>   *confidence intervals less useful.*

> *for counterh from* 1 *by* 1 *to twosamprows do if*
>   *twosamp* [*counterh*, 4] < *twosamp* [*counterh*, 9] *then*
>   *twosamp99student* := *counterh* ; *counterh* := *counterh* +
>   *twosamprows* ; *end if* ; *end do* ;

> *twosamp99student* ;

$$3$$
$$41$$

85

>

> $upper99tint :=$
  $PolynomialInterpolation([[$
  $twosamp[twosamp99student - 1, 4],$
  $twosamp[twosamp99student - 1, 3]],$
  $[twosamp[twosamp99student, 4],$
  $twosamp[twosamp99student, 3]]],$
 $twosamp[twosamp99student, 9]);$

$$upper99tint := 146.4622854$$

> **#Vectors are declared below to use in plots of the results from the code above. For those interested in the Maple code process used, "failweight" and "tvals" are declared first to make the failure times and t values into vectors. Each is then converted into a list so that they can easily be changed into a single vector of points, below named "fbys3b." A similar process creates a vector of points with the MRL function values.**

> $failweight := Column(twosamp, 3):$

> $tvals := Column(twosamp, 9):$

> $failweightlist := convert(failweight, list):$

> $tvalslist := convert(tvals, list):$

> $for\ i\ from\ 1\ by\ 1\ to\ twosamprows\ do\ fbys3b[i]$
  $:= ([failweightlist[i], tvalslist[i]]);\ end\ do:$

> $twosampzvals := Column(twosamp, 4):$

> $mrl1 := Column(K, 2):$

> $mrl1list := convert(mrl1, list):$

> $mrl1fails := Column(K, 5):$

> $mrl1failslist := convert(mrl1fails, list):$

> $for\ i\ from\ 1\ by\ 1\ to\ Krows\ do\ failbymrl1[i]$
  $:= ([mrl1failslist[i], mrl1list[i]]);\ end\ do:$

> $failbymrl1list := convert(failbymrl1, list):$

> $mrl2 := Column(K2, 2)$:

> $mrl2fails := Column(K2, 5)$:

> $for\ \ i\ \ from\ \ 1\ \ by\ \ 1\ \ to\ \ Krows2\ \ do\ \ failbymrl2[i]$
> $\quad := ([mrl2fails[i],\ mrl2[i]])$; $end\ \ do$:

> $failbymrl2list := convert(failbymrl2,\ list)$:

> $for\ \ i\ \ from\ \ 1\ \ by\ \ 1\ \ to\ \ Krows\ \ do\ \ baseline1[i]$
> $\quad := (mrl1failslist[i],\ baseline)$; $end\ \ do$:

> $for\ \ i\ \ from\ \ 1\ \ by\ \ 1\ \ to\ \ Krows2\ \ do\ \ baseline2b[i]$
> $\quad := (mrl2fails[i],\ baseline2)\ \ end\ \ do$:

> $baseline1a := convert(baseline1,\ list)$:

> $baseline2a := convert(baseline2b,\ list)$:

> # A number of plots are demonstrated below, some to suggest variations of plots that may be useful in Maple. Polynomial interpolation is also demonstrated as a means of solving for the mean residual life function value at a specific stress level.

> #This is a simple plot in maple that shows the MRL values of the second sample data.

> $pointplot(failbymrl2list)$;

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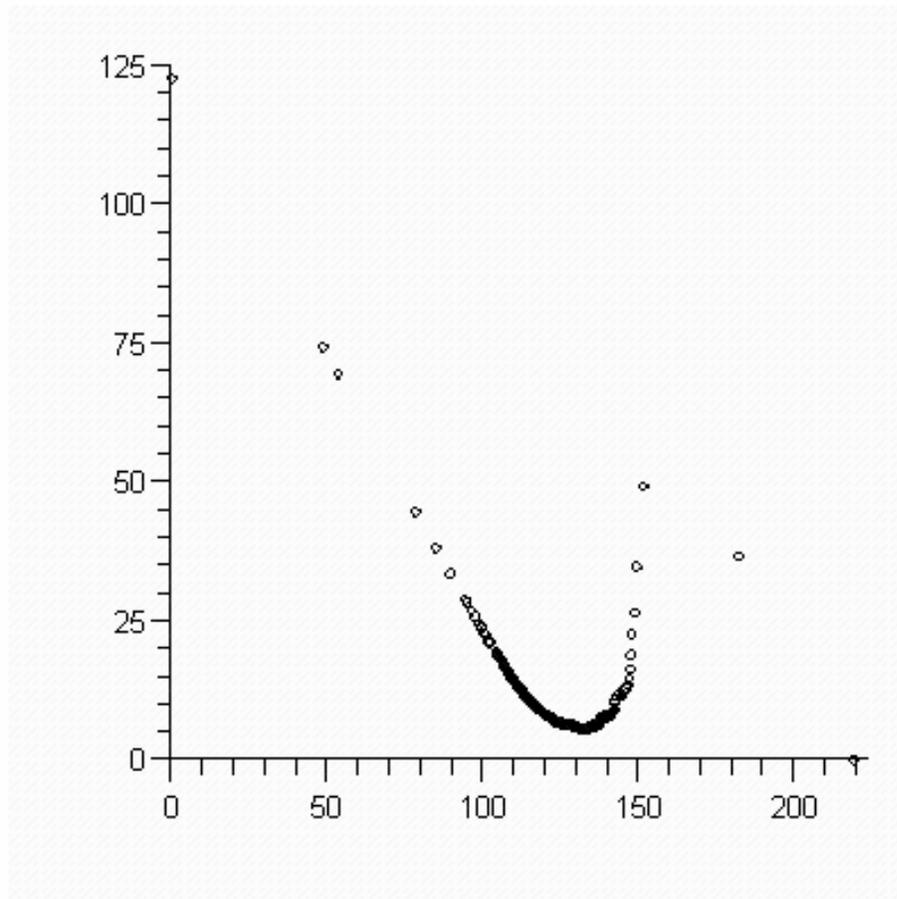
> *#Polynomial interpolation is demonstrated below. "w" has not been declared as a variable previously.*

> $blue := PolynomialInterpolation(failbymrl1list, \ w);$

> *whattype* (*blue*) ;

$$+$$

> *#The line below demonstrates solving for a specific value of the MRL function with polynomial interpolation. The second value is the stress value where the desired value lies.*

> *blue2* := *PolynomialInterpolation*(*failbymrl2list*, 0) ;

$$blue2 := 122.7357289$$

> *# The plot from above is repeated here with the connect = true option. This option tends to be more common in graphs of MRL functions and stresses that the function is often assumed to be continuous. Below is the plot of the first sample's MRL function. It also includes the one − sample baseline functions.*

> *multiple* (*pointplot*, [*failbymrl2list*, *connect* = *true*],
[*baseline2a*, *connect* = *true*],
title =" *Graph of density* = 46 *medium density fiberboard against a baseline of* 80", *labels* = [*IB*, *MRL*],
*labeldirections* = [*horizontal*, *vertical*]) ;

Graph of density= 46 medium density fiberboard against a baseline of 80

>
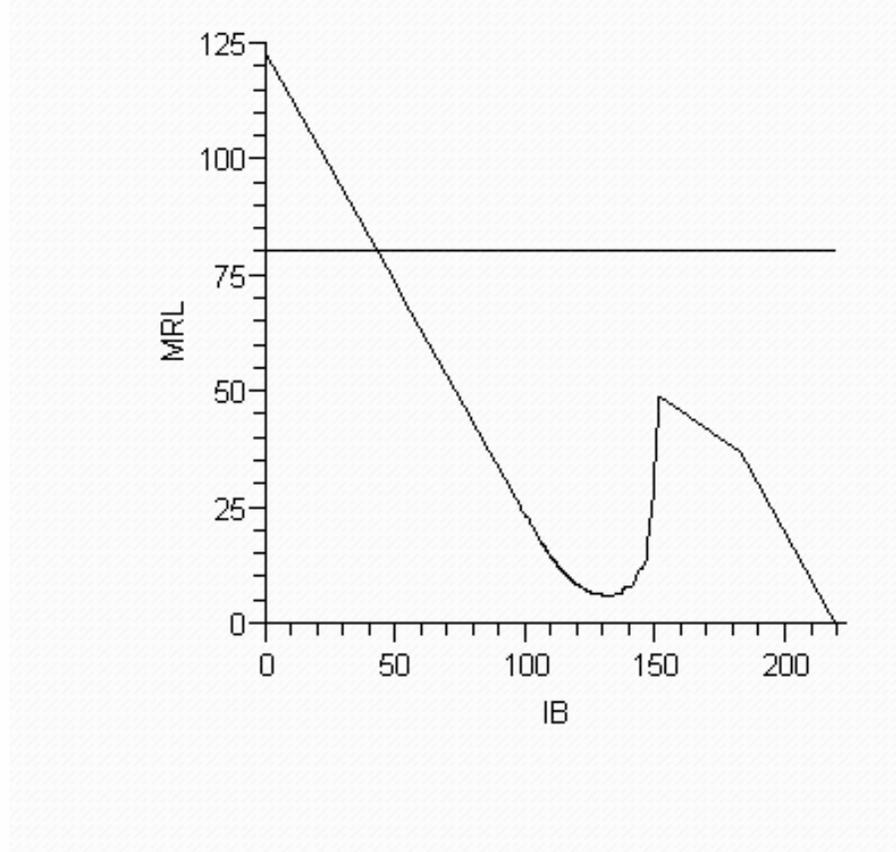
> $multiple(pointplot, [failbymrl1list, connect = true],$
> $[baseline1a, connect = true]$
> $, title =" Graph \ of \ density = 48 \ medium \ density$
> $fiberboard \ against \ a \ baseline \ of \ 80", labels = [IB, MRL],$
> $labeldirections = [horizontal, vertical]);$

Graph of density= 48 medium density fiberboard against a baseline of 80
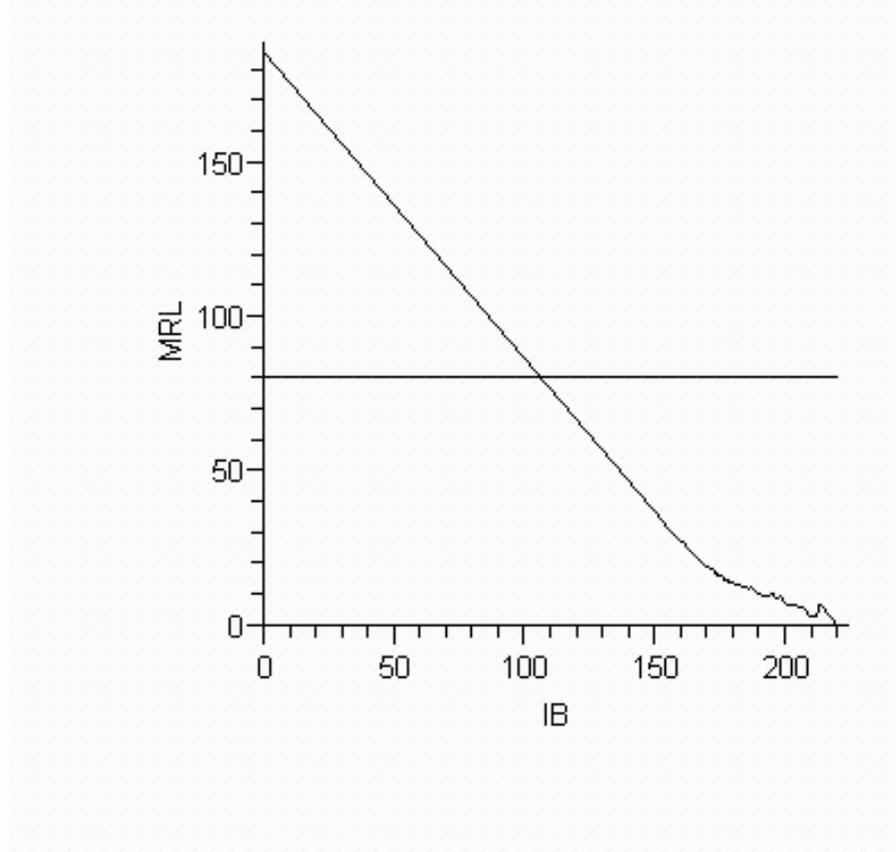
> *whattype* (*failbymrl*);

$$s$$
$$ymbol$$

> *fbys3bl* := *convert* (*fbys3b*, *list*) :

> #Below is a point plot of the calculated t − values by stress level based on the minimum surviving from the two samples. At the end, the function has been forced to be zero. Infinite or Does Not Exist would be more technically correct, but are not convenient for plotting.

> *pointplot* (*fbys3bl*);

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>
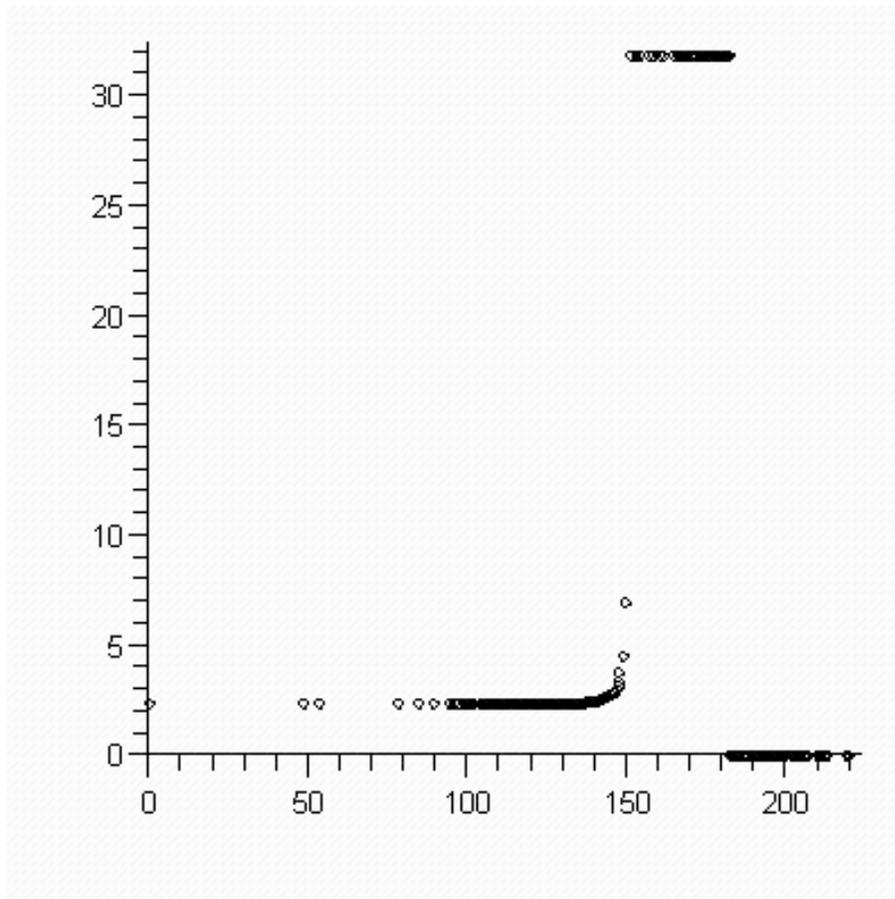
> #lists of points are created below for the zfunction and for $z-$
>   scores for the 99 % confidence level.

> $twosampzvalslist := convert(twosampzvals, \ list):$

> $for \ \ i \ \ from \ \ 1 \ \ by \ \ 1 \ \ to \ \ twosamprows \ \ do \ \ failbyz[i]$
> $:= ([failweightlist[i], \ twosampzvalslist[i]]); \ end \ \ do:$

> $failbyzlist := convert(failbyz, \ list):$

> $for \ \ i \ \ from \ \ 1 \ \ by \ \ 1 \ \ to \ \ twosamprows \ \ do \ \ ztest[i]$
> $:= ([failweightlist[i], \ zalphatsc]); \ end \ \ do:$

> $ztestlist := convert(ztest, \ list):$

> $for \ \ i \ \ from \ \ 1 \ \ by \ \ 1 \ \ to \ \ twosamprows \ \ do \ \ negztest[i]$
> $:= ([failweightlist[i], \ -zalphatsc]); end \ \ do:$

> *negztestlist* := *convert* (*negztest*, *list*) :

> #*A multiple point plot is demonstrated below containing the two MRL functions. Color options are also demonstrated. See Maple's help for a complete list of available colors.*

> *multiple* (*pointplot*,
   [*failbymrl1list*, *color* = *black*, *connect* = *true*]
   , [*failbymrl2list*, *color* = *orange*, *connect* = *true*]);

>
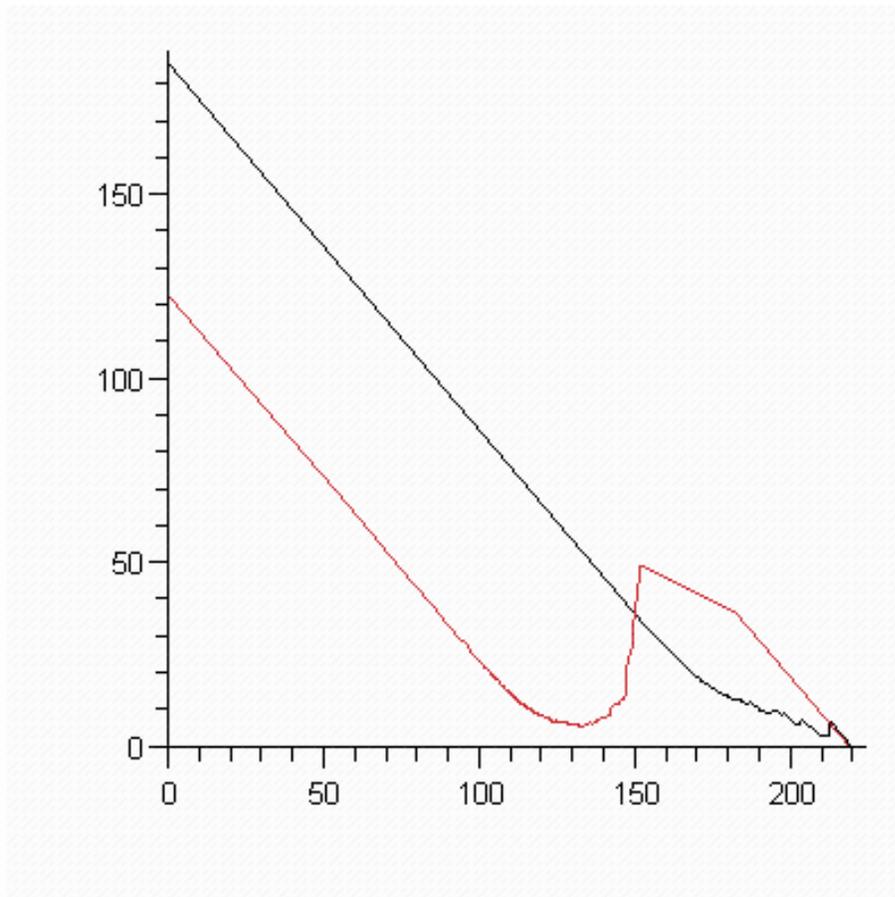
> *for i from 1 by 1 to twosamprows do negfbys3b*[*i*]
   := ([*failweightlist*[*i*], − *tvalslist*[*i*]]); *end do* :

**>** *negfbys3blist* := *convert* (*negfbys3b*, *list*) :

>

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> *# More complicated plots are below that includes multiple functions. Line thickness options and line style options are demonstrated. Plot title and axis labels are also included.*

> *multiple(pointplot,*
    [negztestlist, connect = true, linestyle = DASH],
    [ztestlist, connect = true, linestyle = DASH],
    [fbys3bl, connect = true, linestyle = DOT],
    [failbyzlist, connect = true, thickness = 2],
    [failbymrl1list, connect = true, thickness = 1]
    , [failbymrl2list, connect = true, thickness = 1],
    [negfbys3blist, connect = true, linestyle = DOT]
    , title =" Mean Residual Life Functions, Z Function, and 't'
    Critical Values", labels = [indepaxis, depaxis],
    labeldirections = [horizontal, vertical]);
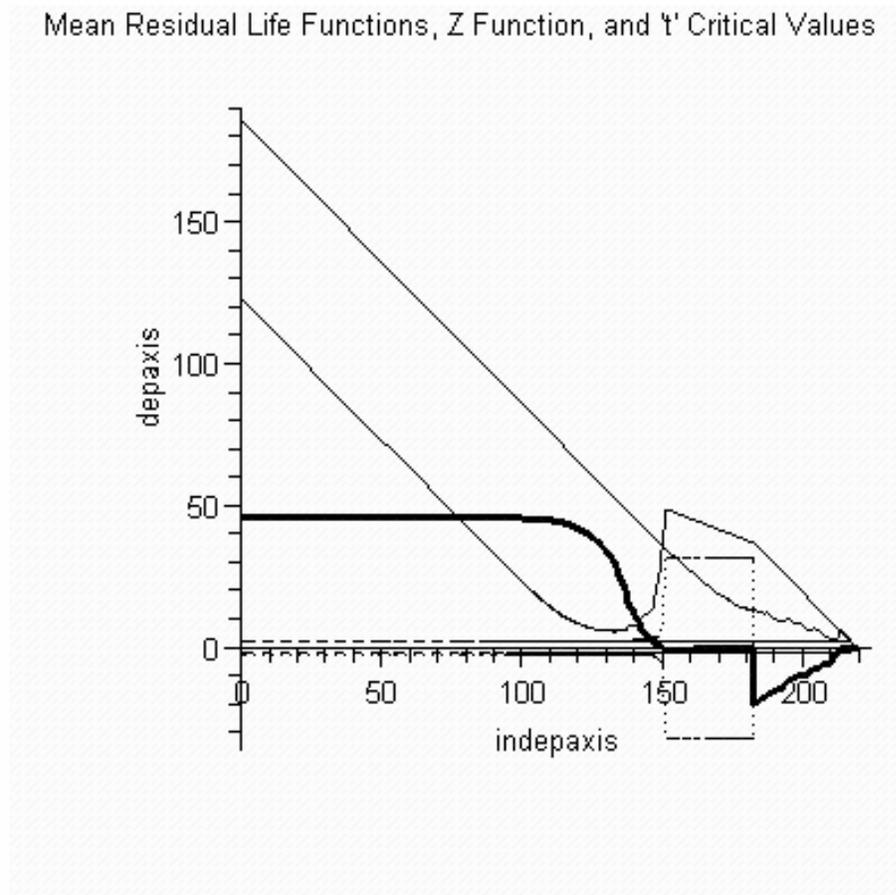
Mean Residual Life Functions, Z Function, and 't' Critical Values

> *multiple* (*pointplot*,
     [*failbymrl1list*, *connect* = *true*, *thickness* = 1]
     , [*failbymrl2list*, *connect* = *true*, *thickness* = 1], *title* =" *Mean*
      Residual Life Functions", *labels* = [*indepaxis*, *depaxis*],
   *labeldirections* = [*horizontal*, *vertical*]);

Mean Residual Life Functions
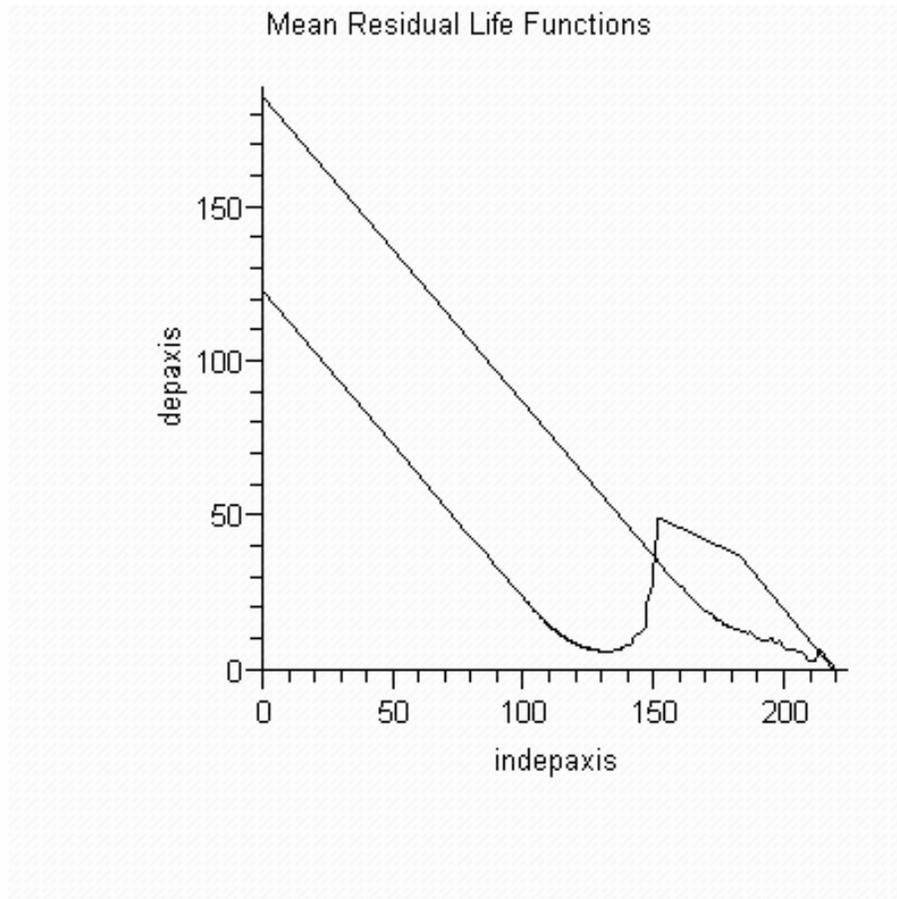
> *multiple* (*pointplot*, [*failbyzlist*, *connect* = *true*, *thickness* = 2],
     [*ztestlist*, *connect* = *true*],
     [*failbymrl1list*, *connect* = *true*, *thickness* = 1]
     , [*failbymrl2list*, *connect* = *true*, *thickness* = 1]
     , *title* =" *Mean Residual Life Functions and Z Function*",
     labels = [*indepaxis*, *depaxis*], *labeldirections*
     = [*horizontal*, *vertical*]);

Mean Residual Life Functions and Z Function
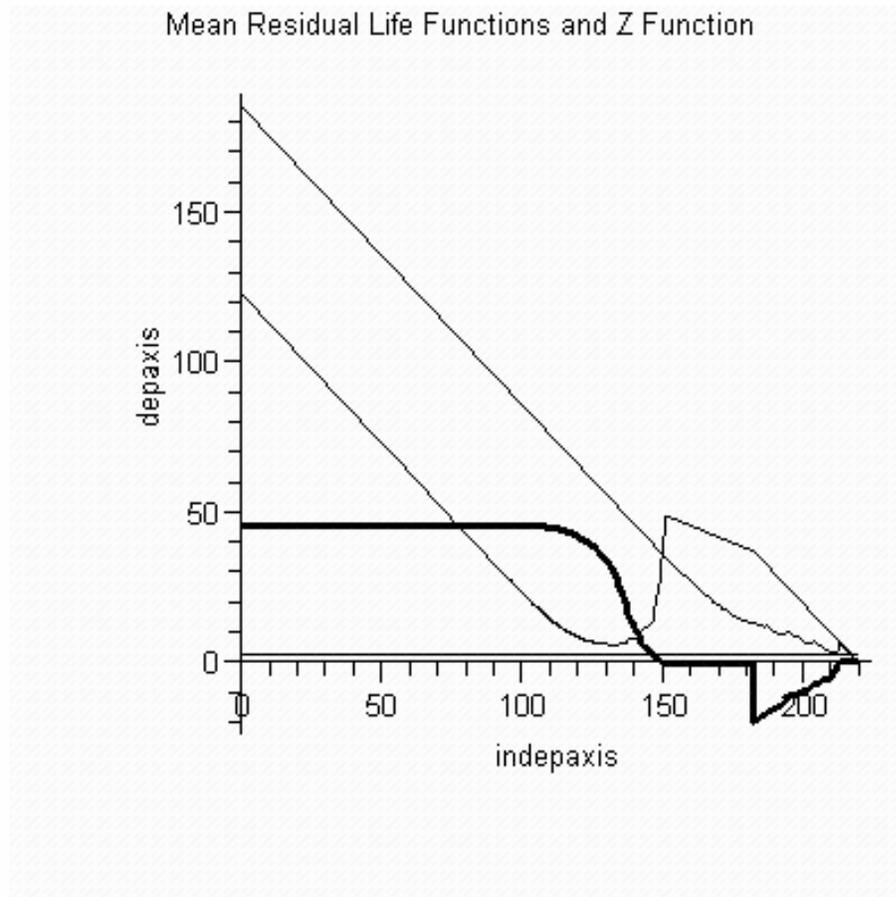
> *#This final plot shows the gridlines option in a plot.*

> $multiple(pointplot, \ [failbymrl1list, \ thickness = 1]$
> $, [failbymrl2list, \ color = red], \ gridlines);$

>
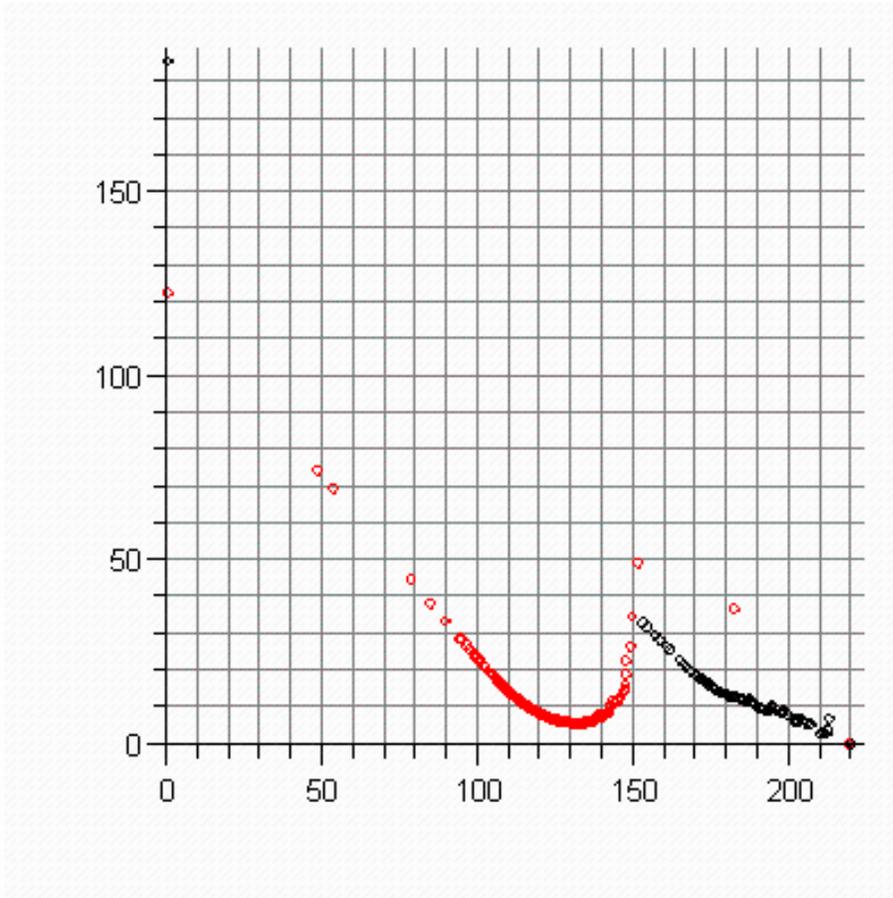
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Monagan, M. B., K. O. Geddes, K. M. Heal, G. Labahn, S. M. Vorkoetter, J. McCarron and P. DeMarco (2005). *Maple Introductory Programming Guide*. Canada, Maplesoft.

APPENDIX B: PROOFS RELATED TO "DOMAIN FUNCTION SETS" CONFIDENCE INTERVALS

INTRODUCTION

For completeness in this thesis, we summarize some key theorems, techniques, and proofs from Balgopal (1987) and comments from Berger, Boos, and Guess (1988). Some useful results from other papers are provided in detail here to assist the reader interested in developing a deeper understanding of the MRL function and the techniques described in the previous chapters. We begin with a theorem that appears in the work of Berger, Boos, and Guess (1988) and in greater detail in Balgopal (1987). Theorem 1 in this paper is also Theorem 1 in Balgopal (1987) and in Berger, Boos, and Guess (1988).

THEOREM 1

Let $X_{1,...,} X_n$ be an independent sample of failure stresses. Let $E[X_i^2]$ represent the expectation of the square of the failure stresses and $\overline{F}(t)$ be the reliability function defined by $1 - F(t)$ where $F(t)$ is the cumulative distribution function. If

$$E[X_i^2] < \infty \text{ and } \overline{F}(t) > 0$$

Let $s_n^2(t)$ denote the sample variance of the failure stresses of the samples with lives greater than t and let $\sigma_F^2(t)$ represent the variance of the samples with lives greater than t. Then,

$$(1) \quad s_n^2(t) \xrightarrow{wp1} \sigma_F^2(t) \text{ as } n \to \infty$$

Additionally, let $m(t)$ represent the number of surviving individuals at time t, let $\hat{e}(t)$ represent the empirical MRL function, let $e(t)$ be the true MRL function, and let $e_0(t)$ be the hypothesized MRL function. Then

$$(2) \quad \frac{\hat{e}(t) - e(t)}{\sqrt{s_n^2(t)/m(t)}} \xrightarrow{d} N(0,1) \text{ as } n \to \infty$$

PROOF OF THEOREM 1

Begin the proof of (1) by considering the order statistics $X_{(1)}, X_{(2)}, ... X_{(n)}$. $s_n^2(t)$ is a continuous function of four parts:

    i. $\sum_{i=1}^{n} \dfrac{I(X_i > t)X_i^2}{n}$,

    ii. $\sum_{i=1}^{n} \dfrac{I(X_i > t)X_i}{n}$,

    iii. $\sum_{i=1}^{n} \dfrac{I(X_i > t)}{n}$,

    and

    iv. $I(X_{(n)} > t)$

where the function I defines the individuals surviving past stress t. In iii, I is a counting function and as a function of t converges to the reliability function. With this result in mind, we can see that as t increases: iv converges to 1, ii converges to $E[I(X_1 > t)X_1^2]$, and i converges to $E[I(X_1 > t)X_1]$. Using the forms of convergence described in the preceding, we can write

$$s_n^2(t) = \left\{ \sum_{i=1}^{n} \frac{I(X_i > t)X_i^2}{n\overline{F}(t)} - \left(\sum_{i=1}^{n} \frac{I(X_i > t)X_i}{n\overline{F}(t)} \right)^2 \right\} I(X_{(n)} > t)$$

(1) follows from the above definition of $s_n^2(t)$.

It should be noted that the converged forms of i, ii, may also be expressed as:

i. $\displaystyle\int_t^\infty x^2 dF(x)$

ii. $\displaystyle\int_t^\infty x dF(x)$

We must keep (1) in mind to reach the proof of (2). Consider the left hand side of (2) as

$$\frac{\left(\sqrt{n}\left(\hat{e}(t) - e(t)\right) + \sqrt{n}\left(e(t) - e_0(t)\right) \right)}{\sqrt{\dfrac{s_n^2(t)}{\overline{F}(t)}}}$$

And consider separately the part

$$\sqrt{n}\left(\hat{e}(t) - e(t)\right)$$

From the definition of the empirical MRL, we can rewrite the above as a composite function substituting for $\hat{e}(t)$ as

$$\sqrt{n}\left(\int_t^\infty \frac{\overline{F}_n(x)dx}{\overline{F}_n(t)} - e(t) \right) \text{ where some X>t}$$

and

$$\sqrt{n}\left(-e(t)\right) \text{ if } X \leq t.$$

Since we are principally interested in the case where at least one item survives past stress t, we will ignore the latter case, which, in any case, converges to zero as $n \to \infty$. Considering just the first case of our separated part of (2), we may write:

$$A = \frac{1}{\overline{F}_n(t)} \left(\int_t^\infty \sqrt{n} \left((\overline{F}_n(x) - \overline{F}(x)) \right) dx - \left(\overline{F}_n(t) - \overline{F}(t) \right) e(t) \right)$$

$$= \frac{1}{\overline{F}_n(t)} \left(-\int_t^\infty \sqrt{n} \left((F_n(x) - F(x)) \right) dx - \left(F_n(t) - F(t) \right) e(t) \right)$$

If we consider just the first portion of the integral described above, we find:

$$-\int_t^\infty \sqrt{n} \left(F_n(x) - F(x) \right) dx$$

$$= -\sqrt{n} \left((F_n(x) - F(x)) x \right) \Big|_{x=t}^\infty + \sqrt{n} \int_t^\infty x \, d \left(F_n(x) - F(x) \right)$$

$$= \sqrt{n} \left(F_n(t) - F(t) \right) t + \sqrt{n} \left(\frac{1}{n} \sum_{i=1}^n I(X_i > t) X_i - \int_t^\infty x \, dF(x) \right)$$

Returning to the full equation, we may rewrite A as:

$$A = \frac{1}{\overline{F}_n(t)} \sqrt{n} \left(\frac{1}{n} \sum_{i=1}^n \left(I(X_i > t) X_i - E(I(X_i > t)) \right) - \left(I(X_i > t) - \overline{F}(t) \right) (e(t) + t) \right)$$

$$= \frac{1}{\overline{F}_n(t)} \sqrt{n} \left(\frac{1}{n} \sum_{i=1}^n \left(I(X_i > t)(X_i - t) - E(I(X_i > t)(X_i - t)) \right) - \left(I(X_i > t) - \overline{F}(t) \right) (e(t)) \right)$$

$$= \frac{1}{\overline{F}_n(t)} \sqrt{n} \left(\frac{1}{n} \sum_{i=1}^n Q_i(t) \right)$$

Where we make the definition that

$$Q_i(t) = I(X_i > t) \left(X_i - t - e(t) \right)$$

It is not complicated to arrive at the results where

$E(Q_i(t)) = 0$

and

$$Var(Q_i(t)) = \bar{F}(t)\sigma_F^2(t)$$

Using Slutsky's Theorem and the Central Limit Theorem we can now show that A is normal as $n \to \infty$ with a distribution characterized by

$$A \xrightarrow{d} N\left(0, \frac{1}{\bar{F}^2(t)}\left(\bar{F}(t)\sigma_F^2(t)\right)\right)$$

Which can be simplified further for the case when $n \to \infty$ as:

$$A \xrightarrow{d} N\left(0, \frac{\sigma_F^2(t)}{\bar{F}(t)}\right)$$

This shows that the left hand term of the top of (2) has a normal distribution. A similar proof could be applied to the right-hand side of the top of (2). Since A converges we may use Slutsky's Theorem to show that (2) converges to a standard normal distribution.

THEOREM 2

Theorem 2 is described in Balgopal (1987) as Theorem 2 and is also described in Berger, Boos, and Guess (1988) as Theorems 2 and 3 with some description that applies to a two-sample case. Theorem 2 states that if the conditions of Theorem 1 hold then

$$P\left(\frac{\hat{e}(t) - e(t)}{\sqrt{s_n^2(t)/m(t)}} + \frac{e(t) - e_0(t)}{\sqrt{s_n^2(t)/m(t)}} > Z_\alpha\right) \to 1, \alpha, or\, 0 \text{ where}$$

$\to 1$ if $e(t) > e_0(t)$

$\to \alpha$ if $e(t) = e_0(t)$

$\to 0$ if $e(t) < e_0(t)$

PROOF OF THEOREM 2

Consider as a definition that

$$B = \frac{e(t) - e_0(t)}{\sqrt{s_n^2(t)/m(t)}}$$

It is simple to see that if $e(t) = e_0(t)$ that B=0, which is to say that if the MRL function is the same as a hypothesized MRL function that B is zero. When B is zero we have from (2) of Theorem 1 that the left-hand side of Theorem 2 has a standard normal distribution. When the left-hand side of Theorem 2 has a standard normal distribution, it is straightforward to see that the probability that it is greater than z_α is simply α.

We may also show the remaining convergence probabilities by considering B. If we allow that e(t) > e$_o$(t) then we are adding positive number to the standard normal distribution included in the left-hand side of Theorem 2. When this happens, we necessarily increase the probability that the left hand side of Theorem 2 is greater than z_α, and the probability converges to 1. The opposite holds when e(t) < e$_0$(t), we subtract a positive number from the standard normal distribution included in the left-hand side of Theorem 2 and we necessarily decrease the probability that the left-hand side of Theorem 2 is greater than z_α. As e(t) becomes increasingly small compared to e$_0$(t), the probability that the left-hand side of Theorem 2 is greater than z_α converges to zero. Theorem 2 will be important for the application of confidence intervals.

THEOREM 3

Theorem 3 appears as Theorem 5 in Balgopal (1987) and deals directly with how confidence statements may be made. Balgopal's reasoning follows similar reasoning used in Berger, Boos, and Guess (1985). Theorem 3 makes statements about the probability that a described confidence procedure makes an incorrect statement. In Balgopal (1987), three confidence statements are considered:

1. $e(t) > e_0(t) \forall t : t \in [T, \hat{\theta}_1)$

2. $e(t) > e_0(t) \forall t : t \in (\hat{\theta}_2, T]$

3. $e(t) > e_0(t) \forall t : t \in \left(\max\left(0, t - \hat{\delta}\right), t + \hat{\delta} \right)$

The conditions for Theorem 3 are that we consider an independent sample of failures from F and we assume that

$$E[X_1^2] < \infty.$$

If we then assume that

$$e(t) \le e_0(t)$$

we can say that

$$\lim_{n \to \infty} \text{P[procedure 1 makes an incorrect statement]} \le \alpha$$

In addition when F is a continuous function, then

$$\lim_{n \to \infty} \text{P[procedure i makes an incorrect statement]} \le \alpha \text{ for i = 2 and 3.}$$

PROOF OF THEOREM 3

Let F and F_0 be fixed distributions satisfying the condition of our Theorem 3. For each of the three procedures if $e_F(T) \leq e_{F_0}(T)$ then by Theorem 2

$$\lim_{n \to \infty} P[\text{procedure 1 makes an incorrect statement)}$$

$$= \lim_{n \to \infty} P[Z_n(t) > z_\alpha] \leq \alpha .$$

Now assume $e_F(T) > e_{F_0}(T)$. For procedure 1 define

$$\theta_1 = \inf \ (t \geq T : e_F(t) \leq e_{F_0}(t))$$

If $\theta_1 = \infty$ then as we have helpfully

$$P[\text{procedure 1 makes an incorrect statement}] = 0.$$

For the case $\theta_1 < \infty$ recall first that MRL functions are right continuous, similar to cumulative distribution functions being right continuous. Thus, the combination $e_F(t) - e_{F_0}(t)$ is also right continuous; and hence we will have, $e_F(\theta_1) \leq e_{F_0}(\theta_1)$. Note that Procedure 1 will make an incorrect statement only if $\hat{\theta}_1 > \theta_1$. Thus, by Theorem 2,

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$$\lim_{n \to \infty} P[\text{procedure 1 makes an incorrect statement)}$$

$$\leq \lim_{n \to \infty} P[\theta_1 > \theta_1]$$

$$\leq \lim_{n \to \infty} P[Z_m(\theta_1) > z_\alpha] \leq \alpha.$$

For procedures 2 and 3, the proofs are similar except for the requirement of F to be continuous. Compare similar reasoning in Berger, Boos, and Guess (1985).

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VITA

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