Capstone Design: Energy Efficient Windows Triple Pane Window Analysis

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Capstone Design: Energy Efficient Windows

Triple Pane Window Analysis

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SUMMARY

The objective of this energy efficient window test was to analyze the heat transfer through triple pane windows by determining the U-factor and solar heat gain coefficient (SHGC) values for different window configurations in order to make recommendations about window setups to reduce heat transfer. Our theoretical analysis was to include a comparison of window configurations consisting of different gas fills and Low-E glazing placements. The experimental test procedure included developing a testing apparatus and procedure to analyze the solar heat gain coefficient through triple pane windows made from windows provided by JELD-WEN Windows & Doors.

In order to complete the objectives described, we first familiarized ourselves with the heat transfer theory applicable to triple pane window systems, which included an understanding of convection, radiation, conduction. Next, we learned about the equations governing the WINDOW program in order to develop our own simulation model and to compare our results of the SHGC and U-Factor values through Newton-Raphson iteration. Finally, we constructed a test apparatus to experimentally determine the SHGC values for a triple pane window system.

When we first began running window simulations utilizing our code we noticed that different gas fills provided better thermal resistance than others. The following gas fills are ranked in order of best to worst thermal resistance performance: Xenon, Krypton, Argon, air. It should be noted that a high thermal resistance is desired in order to reduce the transfer of heat between the environments surrounding the glass system and that a low U-factor indicates a high resistance value. The U-factor values vary for each simulation test conditions and cavity gap widths, but the best U-factor value simulated for Xenon was 0.346 W/ (m²K) at a gap width 10 mm and no wind velocity. Our results indicated that the optimum gap width for different gas fills to generally be higher than those used by JELD-WEN. Next, in order to check our code results, we compared our U-factor calculations to the WINDOW program and we noticed that our calculations were within 1% to 5% of those reported by the WINDOW program. Utilizing a Newton-Raphson method, we managed to determine the SHGC values for different glazing system configurations. Simulating a glazing system utilized by JELD-WEN with coatings on the inner cavity surfaces closest to the outside ambient air, we determined the SHGC to be approximately 0.203. We also ran a simulation with only one surface coated and we determined the SHGC to be 0.296. This value indicated that only coating a single surface might by the best glazing system if coating cost is a major factor of glass manufacturing. This SHGC value is in comparison to 0.25 and 0.233, which were produced by the Window Profiler tool and the WINDOW program. Finally, we ran our experimental setup to determine the SHGC value for an assembled triple pane window. Our data reduction procedure yielded a SHGC of 0.651 for a clear triple pane window and a SHGC of 0.03 for a triple pane window with 2 inner surfaces coated. Although these values are not equal to the simulation calculations, they do show the trend that Low-E coatings reduce the transmission of radiant energy.

We were able to conclude that the technologies analyzed do reduce heat transfer through window systems, but one must also consider costs in determining the best window setup. The experimental and simulation methods could be adapted to other window technologies, but this study is limited by data acquisition uncertainty and our inability to develop an experimental method to determine U-factor.
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<td>Aspect ratio</td>
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<td>c&lt;sub&gt;p&lt;/sub&gt;</td>
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<td>g</td>
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<td>q</td>
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INTRODUCTION

OBJECTIVE

The purpose of this analysis is to determine the heat transfer across a triple pane glazing system with given dimensions for the air gaps and glass panes, as well as differing outside and inside temperatures. This report compares the U-factor and solar heat gain coefficient (SHGC) values through experimental and theoretical means and analyzes the results to determine the optimum gap widths and glazing scenarios in terms of minimizing heat transfer across fenestration products.

BACKGROUND

In today’s world, economical energy has become a major driving force of modern civilizations. With the recent population growth, concerns have emerged on whether or not modern energy generation methods can meet the demand. As a result efforts are being made to fully implement alternate methods of energy generation such as wind, solar, and nuclear. However, these methods are not the sole solution to the energy crisis. Energy efficient technologies have proven to optimize energy consumption in homes and businesses and are an excellent compliment to alternate methods of energy generation. A major category of this research and development is energy efficient windows. Windows are ascetically pleasing and allow visible light into homes and buildings; however, windows are also major enablers of heat transfer to and from the environment. Energy efficient windows work to minimize environmental effects and optimize energy consumption. The following will describe how heat is transferred across a fenestration system, define window performance values, and introduce many of the technologies behind energy efficient windows.

The term fenestration system refers to a window, door, or skylight and includes the interior and exterior components. Fenestration systems can be single pane or consist of multiple panes depending on the application at hand. The figure below shows a cutaway view of a typical triple pane fenestration window.
Heat transfer through a fenestration product is classified into three groups: temperature differential, solar gain, and infiltration. Temperature differential is all heat transfer caused by a difference in temperatures across the fenestration product including conduction, convection, and radiation; it is measured with the U-factor. Solar gain is heat due to direct or indirect solar radiation, and the amount of solar radiation that passes the fenestration product is measured by the Solar Heat Gain Coefficient (SHGC). Infiltration is a measure of the amount of air that passes through the fenestration product per unit area and lends to multiple causes of heat transfer.

Temperature driven heat transfer consists of a combination of three modes of heat transfer. Conduction is transfer through a solid; in the fenestration product this includes the glazing, spacer, and frame. The main factor in this type of heat transfer is the insulating value of the window. This is the measure of resistance to heat flow through a solid. The conduction is directly related to the heat transfer by convection.

Convection is heat transfer caused by the movement of gases. This can have two effects on a fenestration product. First a cold interior window surface can cause convection to start inside the room; this is often perceived as a draft because it causes the movement of air in the room. The other effect is convection on the outside. This is related to conduction as convection causes a heat transfer at the surface and the conduction transfers the heat inside or out, depending on direction of flow. This will also affect the inside of multi-paned windows as the air in that space can create convection currents which will decrease the insulation the space of air provides.
The final mode of temperature differential heat transfer is radiation. It is the transmittance of heat without a medium. Objects will emit invisible thermal radiation and colder objects will absorb this heat from warmer objects. Therefore, in the winter a cold window will leach heat from the hot room. The diagram below is a visual representation of these three types of heat transfer.

![Diagram](image)

**Figure 2:** Three modes of heat transfer in a fenestration system: conduction, convection, and radiation

All of these values of heat transfer are quantified using the U-factor or the overall coefficient of heat transfer. It is the reciprocal of the resistance to heat transfer, therefore the larger this number the greater the heat transfer by all three modes will be.

The solar gain is heat caused by radiation waves from the sun being absorbed or transmitted through the fenestration product. Depending on the current climate, solar gain may or may not be desired. In cold areas it is useful for heating a building, however if the building is trying to maintain a low temperature it is not helpful. An important part of solar gain is the visual transmittance. This is a measure of the visible light allowed through a window. By decreasing this value you decrease some solar gain, but this makes a room darker which can be undesirable. This value does not directly affect the efficiency of a window but is important to the purpose of a window. The method of measuring solar gain is the solar heat gain coefficient, SHGC. It is the solar heat gain through a system relative to the incident solar radiation. The higher the number the greater the heat gain and vice versa.
Solar gain is greatly affected by the transmittance, reflectance, and absorbance of the glazing. These are the amount of radiation a window transmits through, reflects away, and absorbs respectively. These values can be altered by using different types of glazing. The main method is by using window tints that accept only desired wavelengths of radiation. Therefore it may be possible to let light in but not much heat from radiation. The following drawing shows a representation of these three properties and how they affect solar heat gain.

![Diagram](image)

**Figure 3:** Diagram showing what happens to direct solar radiation on a fenestration system

Infiltration is the amount of air that enters a building through a window per unit area of window. It can directly affect the heating costs of a building because air coming in is the opposite of temperature of the desired air inside. Normally infiltration plays a very small role in window efficiency as compared to U-factor and SHGC. The most important factors in good infiltration values are window design, sealing, and weather-stripping. The figure below is a label typically found on energy efficient windows showing performance values.
Major window technologies currently utilized include glazing types, gas fills, and frame types. Glazing products can be altered by changing the glazing material, adding coating, or by combining various layers of glazing. Low-E coating is a recently developed technology that can be used to improve both heating and cooling season performance. The coating consists of metallic oxides deposited on the window surface and is virtually invisible. The coating reflects infrared radiant energy keeping heat on the side from which it originates. Other types of coatings included tinted and reflective.

Gas fills are important in controlling the spacing properties between window panes. Various gases have different properties which affect conduction and convection heat transfer. As mentioned before, enclosures experience what is known as natural convection. During the process the enclosed fluid is heated by the warm surface and rises as a result. At the top the fluid is cooled by the cold surface and falls where the process is repeated. A fluid’s tendency to begin natural convection is dependent on its associated properties. For this reason various gas fills are used in efficient windows to try and prevent the phenomenon from occurring. Commonly used gas fills include air, argon, and krypton.

Frames play a major role in the energy performance of fenestration products, as well as govern the physical characteristics. These physical characteristics include frame thickness, weight of the product, and its durability. Wood frames are traditionally chosen for residential buildings because it is attractive and performs well from a thermal point of view. It is typical to clad wood frames with vinyl or metal. This provides weather resistance and durability, yet still has the attractive wood exterior. Aluminum frames tend to have a high thermal conductance, therefore raising the U-factor, and putting it at a disadvantage to other frame materials. Also, in cold weather, the aluminum frame can become cold enough to condense moisture on the inside of the
product frames. This can be reduced or eliminated by splitting the frame into interior and exterior components and joining them with a less conductive material such as a plastic. Vinyl frames, fiberglass or engineered thermoplastics, wood composites and hybrid frames represent various other frame materials with relatively good thermal performance.

**WINDOW SOFTWARE**

Experimentally acquiring window performance values can be a tedious and time consuming process. For this reason, analytical methods for quickly obtaining and comparing the performance of various window designs are essential. Public software is available which can be used to analyze window thermal and optical performance. Window is a free computer program developed by Lawrence Berkley National Laboratory to calculate total window performance values. Window provides a full heat transfer analysis according to rating procedures developed by the National Fenestration Rating Council. The program is useful in designing and analyzing new products, educating in heat transfer across fenestration systems, and aiding building officials in meeting building codes.

The program contains directly accessible libraries of thousands of glazing layers, numerous gas fills, and environmental conditions. Individual glazing systems can be created with customizable options for number of panes and gap fills. Once the glazing system is created the program calculates performance values and displays them accordingly. Below is a screenshot of a triple pane window modeled and analyzed in Window 5.2.

![Figure 5: Screenshot showing the glazing library interface in WINDOW 5.0](image)

The software is user friendly and powerful in nature. Learning the software does not take extensive effort and doing so allows users to quickly model, analyze, and compare various window designs.
VISUAL BASIC MODEL: U-FACTOR EQUATIONS AND THEORY

THERMAL RESISTANCE

A diagram of the thermal resistances of a triple pane glazing system is shown in Figure 6. For the gas fill and surface resistances, radiation and convection resistances have been combined into a single resistance. The method for achieving this is detailed in the Effective Ambient Temperature section.

![Diagram of thermal resistances of a triple pane window]

Figure 6: Thermal resistance of a triple pane window

INDIVIDUAL GAS FILLS PROPERTIES

We used the following equations and tables to calculate the properties of different gasses according to temperature. Use Equation 1 to determine the film temperature of a gas in a window cavity; \( T_i \) and \( T_{i+1} \) are the temperatures of the window pane surfaces surrounding the gas. Use Equation 2 to determine the film temperature of air on the exterior surfaces of the glass if there is no wind velocity, the temperatures used in this calculation are temperatures of the surface of the window and the exterior gas. If a surface of the window is experiencing wind velocity, no film temperature is needed and the air properties do not need to be determined. Use Equations 3 through 9 to determine the properties of different gases for both cavities and exterior surfaces.

\[
(1)
\]
\[ T_f = T_{x_0} + 0.25(T_i - T_{x_0}) \]  
\[
\beta = \frac{1}{T_f}
\]  
\[ k = a + b \cdot T(K) \]  
\[ \mu = a + b \cdot T(K) \]  
\[ c_p = a + b \cdot T(K) \]  
\[ \rho = \frac{p \cdot M}{R_u \cdot T_f} \]  
\[ \nu = \frac{\mu}{\rho} \]  
\[ \alpha = \frac{k}{\rho \cdot c_p} \]  

Tables 1 through 4 give constants to use with the correlations given in Equations 3 through 9. The constants are those given in ISO-15099.

Table 1: Thermal conductivity equation constants for window gas fills

<table>
<thead>
<tr>
<th>Gas</th>
<th>a (W/m-K)</th>
<th>b (W/m-K(^2))</th>
<th>k at 0°C (W/m-K)</th>
<th>k at 10°C (W/m-K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>2.873E-3</td>
<td>7.760E-5</td>
<td>0.0241</td>
<td>0.0248</td>
</tr>
<tr>
<td>Argon</td>
<td>2.285E-3</td>
<td>5.149E-5</td>
<td>0.0163</td>
<td>0.0169</td>
</tr>
<tr>
<td>Krypton</td>
<td>9.443E-4</td>
<td>2.826E-5</td>
<td>0.0087</td>
<td>0.0089</td>
</tr>
<tr>
<td>Xenon</td>
<td>4.538E-4</td>
<td>1.723E-5</td>
<td>0.0052</td>
<td>0.0053</td>
</tr>
</tbody>
</table>

Table 2: Dynamic viscosity equation constants for window gas fills

<table>
<thead>
<tr>
<th>Gas</th>
<th>a (N-s/m(^2))</th>
<th>b (N-s/m(^2)-K)</th>
<th>(\mu) at 0°C (N-s/m(^2))</th>
<th>(\mu) at 10°C (N-s/m(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>3.723E-6</td>
<td>4.940E-8</td>
<td>1.722E-5</td>
<td>1.771E-5</td>
</tr>
<tr>
<td>Argon</td>
<td>3.379E-6</td>
<td>6.451E-8</td>
<td>2.100E-5</td>
<td>2.165E-5</td>
</tr>
<tr>
<td>Krypton</td>
<td>2.213E-6</td>
<td>7.777E-8</td>
<td>2.346E-5</td>
<td>2.423E-5</td>
</tr>
<tr>
<td>Xenon</td>
<td>1.069E-6</td>
<td>7.414E-8</td>
<td>2.132E-5</td>
<td>2.206E-5</td>
</tr>
</tbody>
</table>
Table 3: Specific heat equation constants for window gas fills

<table>
<thead>
<tr>
<th>Gas</th>
<th>a (J/kg-K)</th>
<th>b (J/kg-K²)</th>
<th>(c_p) at 0°C (J/kg-K)</th>
<th>(c_p) at 10°C (J/kg-K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1002.737</td>
<td>1.2324E-2</td>
<td>1006.103</td>
<td>1006.227</td>
</tr>
<tr>
<td>Argon</td>
<td>521.9285</td>
<td>0</td>
<td>521.929</td>
<td>521.929</td>
</tr>
<tr>
<td>Krypton</td>
<td>248.0907</td>
<td>0</td>
<td>248.091</td>
<td>248.091</td>
</tr>
<tr>
<td>Xenon</td>
<td>158.3397</td>
<td>0</td>
<td>158.340</td>
<td>158.340</td>
</tr>
</tbody>
</table>

Table 4: Molar mass values of different gas fills

<table>
<thead>
<tr>
<th>Gas</th>
<th>M (kg/kmol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>28.97</td>
</tr>
<tr>
<td>Argon</td>
<td>39.95</td>
</tr>
<tr>
<td>Krypton</td>
<td>83.80</td>
</tr>
<tr>
<td>Xenon</td>
<td>131.30</td>
</tr>
</tbody>
</table>

MIXTURE PROPERTIES

We used the following equations to calculate the properties of a gas mixture. Use Equation 10 to calculate the molar mass of the mixture; it is a function of the sum of the individual molar masses and the mass fraction of each gas. Use Equation 11 to calculate the density of the mixture using pressure, molar mass of the mixture, the universal gas constant, and the film temperature of the cavity.

\[
M_{MIX} = \sum_{i}^{n} m_i M_i
\]  
\[
\rho_{MIX} = \frac{p \cdot M_{MIX}}{R_u T_f}
\]

In order to calculate the specific heat of the mixture, first calculate the molar specific heat of each gas using Equation 12. Next calculate the molar specific heat values using Equation 13. The specific heat of the mixture is a function of the molar specific heat of the mixture and the molar mass of the mixture, shown by Equation 14.

\[
\hat{c}_{p,i} = c_{p,i} M_i
\]
\[
\hat{c}_{p,MIX} = \sum_{i}^{n} m_i \hat{c}_{p,i}
\]
\[
c_{p,MIX} = \frac{\hat{c}_{p,MIX}}{M_{MIX}}
\]

To calculate the viscosity of the mixture, first each gas must be related to the other gases in the cavity using Equation 15. One must notice that this relationship must be done for each gas relative to the others and vice versa. For example, if there are three gases within a cavity,
Equation 15 must be used six times. To calculate the final viscosity of the mixture, use Equation 16.

\[
\varphi_{i,j}^\mu = \frac{1 \left( \frac{\mu_i}{\mu_j} \right)^5 \left( \frac{M_j}{M_i} \right)^{2.5}}{2 \sqrt{2} \left[ 1 + \left( \frac{M_i}{M_j} \right)^5 \right]}
\]  

\[
\mu_{MIX} = \sum_{i=1}^{n} \frac{\mu_i}{1 + \sum_{j=1}^{n} \varphi_{i,j}^\mu \left( \frac{m_j}{m_i} \right)}
\]  

Determination of the monatomic thermal conductivity is similar to determining the viscosity. Use Equation 17 similarly to the use of Equation 15 to relate each component gas to the other component gases. Use Equations 18 and 19 to calculate the monatomic thermal conductivity.

\[
\varphi_{i,j}^{k'} = \frac{1 \left( \frac{k_{i}'}{k_{ij}} \right)^5 \left( \frac{M_i}{M_j} \right)^{2.5}}{2 \sqrt{2} \left[ 1 + \left( \frac{M_i}{M_j} \right)^5 \right]}
\]  

\[
k_{i}' = \left( \frac{15}{4} \right) \left( \frac{R_i}{M_i} \right) \mu_i
\]  

\[
k_{MIX}' = \sum_{i=1}^{n} \frac{k_{i}'}{1 + \sum_{j=1}^{n} \varphi_{i,j}^{k'} \left( \frac{m_j}{m_i} \right)}
\]  

Use Equations 20 through 22 to calculate the additional energy moved by the diffusional transport of internal energy in polyatomic gases.

\[
\varphi_{i,j}^{k''} = \frac{1 \left( \frac{k_{i}''}{k_{ij}} \right)^5 \left( \frac{M_i}{M_j} \right)^{2.5}}{2 \sqrt{2} \left[ 1 + \left( \frac{M_i}{M_j} \right)^5 \right]}
\]
Finally, the thermal conductivity of the cavity mixture can be found using Equation 23.

\[ k_{\text{MIX}}' = k_{\text{MIX}}' + k_{\text{MIX}}'' \]  

CAVITY CONDITIONS

We used the following equations to determine the Rayleigh and Nusselt numbers for the cavities of a window system. These equations are dependent on the gas properties determined above for either a single fill within a cavity or a mixture of gases. Use Equation 24 to determine the Rayleigh number for the cavity and then calculate the Nusselt number 1 using the appropriate equation from Equations 25 through 27. Next, determine Nusselt number 2 using Equation 28. Next take the maximum Nusselt number calculated and use Equation 30 to calculate the convection coefficient.

\[ Ra_W = \left( \frac{g \beta}{\alpha_v} \right) (T_i - T_{i+1}) W^3 \]  

\[ Nu_1 = 0.0674Ra_W^{1/3} \quad 5 \times 10^4 < Ra \]  

\[ Nu_1 = 0.0282Ra_W^{0.4134} \quad 10^4 < Ra \leq 5 \times 10^4 \]  

\[ Nu_1 = 1 + 1.760 \times 10^{-10}Ra_W^{2.298} \quad Ra \leq 10^4 \]  

\[ Nu_2 = 0.242 \left( \frac{Ra_{W}}{H} \right)^{272} \]  

\[ Nu_{CAV} = (Nu_1, Nu_2)_{\text{MAX}} \]  

\[ h_c = Nu_{CAV} \left( \frac{k}{W} \right) \]  

SURFACE CONDITIONS (NO FORCED CONVETION)

For a surface facing ambient air with no wind velocity, use the following equations to determine the convection coefficient. Use the appropriate air properties to determine the Rayleigh and Nusselt number for a surface using Equations 31 through 34. Calculate the convection coefficient for a window surface with no forced convection, no wind velocity, using Equation 35.

\[ Ra_H = \left( \frac{g \beta}{\alpha_v} \right) (T_i - T_\infty) H^3 \]
\[ Ra_{CV} = 2.5 \times 10^5 \left( \frac{e^{72\gamma}}{\sin \gamma} \right)^{1/5} \]  

(32)

\[ Nu_S = 0.56 (Ra_H \sin \gamma)^{25} \quad Ra_H \leq Ra_{CV} \]  

(33)

\[ Nu_S = 0.13 (Ra_H^{1/3} - Ra_{CV}^{1/3}) + 0.56 (Ra_{CV} \sin \gamma)^{1/4} \]  

\[ Ra_H > Ra_{CV} \]  

(34)

\[ h_C = Nu_S \left( \frac{k}{\pi} \right) \]  

(35)

SURFACE CONDITIONS (FORCED CONVECTION)

If there is wind velocity associated with any exterior surface, the following equation can be used to relate wind velocity to the convection coefficient. Equation 36 is the relationship between the window surface convection coefficient relative to the wind velocity for forced convection. Wind velocity is in units of meters per second.

\[ h_C = 4.7 + 4 \cdot V_s \]  

(36)

HEAT TRANSFER COEFFICIENTS DUE TO RADIATION

In order to determine the heat transfer coefficient due to radiation, use Equations 37 and 38. Equation 37 is used for cavities and Equation 38 is used for window surfaces facing exterior air.

\[ h_{R,G} = \left( \frac{\sigma}{1 + \frac{1}{\varepsilon_i \varepsilon_{i+1}} - 1} \right) \left[ (T_i^2 + T_{i+1}^2)(T_i + T_{i+1}) \right] \]  

(37)

\[ h_{R,S} = \sigma \varepsilon_i \left[ (T_i^2 + T_\infty^2)(T_i + T_\infty) \right] \]  

(38)

EFFECTIVE AMBIENT TEMPERATURE

The environmental conditions consist of the ambient air temperature, which determines the convection heat transfer rate, and the surrounding surface temperature, which determines the radiation heat transfer rate. In order to simplify the U-factor calculation, it is beneficial to create an effective ambient temperature so that the radiation and convection resistances can be combined. A diagram illustrating the conversion of the thermal network is given in Figure 7.
Figure 7: Conversion of surface thermal resistance network

Equation 39 is used to solve for the effective ambient temperature. The same amount of heat will be transferred between both thermal networks in Figure 2.

\[ \text{(39)} \]

WINDOW SYSTEM HEAT TRANSFER

To determine the total heat transfer coefficient of a gas, combine the heat transfer coefficients due to radiation and convection using Equation 40. In order to determine the resistance of the system, use Equations 41 and 42 to determine the thermal resistance of gas fills within the cavities, glass panes, and interior and exterior surfaces. Equations 43 and 44 are used to determine the overall heat transfer through the window system. Equation 45 is used to calculate U-factor, which is an inverse of the sum of resistances and a measure of the effectiveness of a window system resisting heat transfer.

\[ \text{(40)} \]

\[ \text{(41)} \]

\[ \text{(42)} \]

\[ \text{(43)} \]

\[ \text{(44)} \]

\[ \text{(45)} \]
ITERATION PROCESS

Given a set of interior and exterior conditions, the sum of thermal resistances must be found to calculate the heat transfer. However, the resistance of the glazing cavity gas fill depends on the surface temperatures which are initially unknown. To begin the iteration process, all Nusselt numbers are approximated as being equal to one. With this approximation, all resistances can be solved for. Once the heat transfer rate is determined, Equation 46 can be used to calculate new surface temperatures. With the new surface temperatures, new resistances for the glazing cavity gas fills can be determined, and the process can be repeated. This process is iterated until the surface temperatures converge with a maximum temperature change between iterations of 0.01K.

\[ T_i = T_{\infty,0} - Q'' \sum^n_i R''_i \]  (46)

VISUAL BASIC MODEL: SHGC EQUATIONS AND THEORY

THREE PANE TRANSMISSION AND REFLECTION: RAY TRACING METHOD

We used the following equations to calculate the transmission and reflectivity of a three pane window. Equation 47 was used to calculate the transmissivity of a layer of glass and the absorption coefficient was assumed to be 12 m\(^{-1}\). Equations 48 and 49 were used to determine the transmission and reflection values of a glass pane. The subscript \(n\) denotes which pane of the system is being analyzed, and the subscript numbers denote which surface of the pane is being analyzed. It should be noted that Equations 48 and 49 are used to analyze a surface which has incident radiation on surface 1; refer to Figure 8 for a representation of the system being analyzed. Equation 51 is used to calculate the reflectivity of the pair with incident radiation on surface 2. We used Equations 50 and 52 to determine the reflectivity of the first pair of glass and then we used Equations 53 and 54 to calculate the reflectivity and transmission of the system. The absorption of the system is calculated using Equation 55 and the SHGC value is calculated using Equation 56.

\[ \tau_n = e^{-a_n t_g} \]  (47)
\[ T_n = (1 - \rho_{n1})(1 - \rho_{n2})\tau_n^2 \]  (48)
\[ R_{n,1} = \frac{\rho_{n1} + \rho_{n2}(1 - 2\rho_{n1})\tau_n^2}{1 - \rho_{n1}\rho_{n2}\tau_n^2} \]  (49)
\[ R_{PAIR,1} = R_{1,1} + \frac{R_{2,1}T_{1,1}^2}{1 - R_{1,2}R_{2,1}} \]  (50)
\[ R_{PAIR,2} = R_{2,2} + \frac{R_{1,2}T_{2}^2}{1 - R_{2,1}R_{1,2}} \]  
(51)

\[ T_{PAIR} = \frac{T_1T_2}{1 - R_1R_2} \]  
(52)

\[ R_{3PLATES} = R_{PAIR,1} + \frac{R_3T_{PAIR}^2}{1 - R_{PAIR,2}R_3} \]  
(53)

\[ T_{3PLATES} = \frac{T_{PAIR}T_3}{1 - R_{PAIR}R_3} \]  
(54)

\[ A_{3PLATES} = 1 - T_{3PLATES} - R_{3PLATES} \]  
(55)

\[ SHGC = T_{3PLATES} + \left(\frac{1}{2}\right) A_{3PLATES} \]  
(56)

**THREE PANE TRANSMISSION AND REFLECTION: NET RADIATION METHOD**

The following equations were assembled using the net radiation method which is exemplified by Figure 8. Equations 57 through 62 are the equations used to analyze the energy flux through the system. It should be noted that several of the energy flux equations into a surface are equal to the energy flux equations leaving another surface. In order to calculate the resistivity and transmissivity of the system, we employed a Newton-Raphson iteration method to solve the system of equations. Equations 55 and 56 were used to calculate the absorption and solar heat gain coefficient of the window system. Figure 8 included in the following section displays the method used to derive the equations used in the net radiation method. It should be noted that these equations can be used to accomplish the same calculations in the previous section.

\[ R_{3PLATES} = q_{o,11} = R_{11} + q_{o,21}T_1 \]  
(57)

\[ q_{i,12} = q_{o,21} = q_{o,12}R_{21} + q_{o,31}R_{21} + q_{o,31}T_2 \]  
(58)

\[ q_{i,21} = q_{o,12} = q_{o,21}R_{12} + T_1 \]  
(59)

\[ q_{i,31} = q_{o,22} = q_{o,12}T_2 + q_{o,31}R_{22} \]  
(60)

\[ q_{i,22} = q_{o,31} = q_{o,22}R_{31} \]  
(61)

\[ T_{3PLATES} = q_{o,32} = q_{o,22}T_3 \]  
(62)
VISUAL BASIC MODELING RESULTS: SHGC

In order to compare Low-E coating combinations, we used the previously described equations to calculate the reflection, transmission, absorption, and solar heat gain coefficient for different glazing systems. Figure 8 is a diagram of the incident radiation and the subsequent reflection and transmission of solar radiation between the glass panes. The figure consists of 3 window panes and a total of 6 surfaces. Surfaces 1 and 6 face inner and outer environments of the glazing system. These surfaces can be coated with Low-E coating to reduce the SHGC value and thus reducing the transmission of solar radiation through the glass system. Table 5 is a comparison of different combinations of glazing systems; each combination is a different run. A surface that is coated with Low-E is denoted with the symbol E and surfaces that remain clear and have no coating are denoted by C. The table displays the coating for the different surfaces and the surface labels correspond to the numbered surfaces in Figure 8.

Figure 8: Net radiation analysis of a three pane window fenestration
It should be noted that the following table makes use of the following properties for surface reflectivity. When a surface is simulated to be coated with Low-E, the surface reflectivity is set to 0.549 and the surface on the reciprocating side is set to 0.429. If both surfaces are coated with Low-E, both surface reflectivity values are set to 0.549.

Table 5: Low-E coating placement comparisons

<table>
<thead>
<tr>
<th>Run</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>Reflection</th>
<th>Transmission</th>
<th>Absorption</th>
<th>SHGC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>0.298</td>
<td>0.600</td>
<td>0.101</td>
<td>0.651</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>E</td>
<td>C</td>
<td>C</td>
<td>E</td>
<td>C</td>
<td>0.750</td>
<td>0.156</td>
<td>0.094</td>
<td>0.203</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>E</td>
<td>E</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>0.762</td>
<td>0.163</td>
<td>0.075</td>
<td>0.200</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>E</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>0.670</td>
<td>0.262</td>
<td>0.068</td>
<td>0.296</td>
</tr>
<tr>
<td>5</td>
<td>C</td>
<td>C</td>
<td>E</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>0.640</td>
<td>0.264</td>
<td>0.096</td>
<td>0.312</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>E</td>
<td>C</td>
<td>C</td>
<td>0.632</td>
<td>0.264</td>
<td>0.105</td>
<td>0.316</td>
</tr>
<tr>
<td>7</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>E</td>
<td>C</td>
<td>0.607</td>
<td>0.262</td>
<td>0.130</td>
<td>0.328</td>
</tr>
<tr>
<td>8</td>
<td>C</td>
<td>E</td>
<td>C</td>
<td>E</td>
<td>E</td>
<td>C</td>
<td>0.794</td>
<td>0.111</td>
<td>0.095</td>
<td>0.159</td>
</tr>
<tr>
<td>9</td>
<td>C</td>
<td>E</td>
<td>E</td>
<td>C</td>
<td>E</td>
<td>C</td>
<td>0.798</td>
<td>0.111</td>
<td>0.091</td>
<td>0.157</td>
</tr>
<tr>
<td>10</td>
<td>C</td>
<td>E</td>
<td>E</td>
<td>E</td>
<td>E</td>
<td>C</td>
<td>0.804</td>
<td>0.103</td>
<td>0.093</td>
<td>0.149</td>
</tr>
<tr>
<td>11</td>
<td>E</td>
<td>E</td>
<td>E</td>
<td>E</td>
<td>E</td>
<td>E</td>
<td>0.824</td>
<td>0.087</td>
<td>0.089</td>
<td>0.131</td>
</tr>
</tbody>
</table>

Our calculations indicated without coating any surfaces, the solar heat gain coefficient is 0.651 and this value indicates that approximately 65.1% of the solar radiation is transmitted through the glass system. Run 2 is a display of the coating system currently employed by the JELD-WEN company; their coating method involves coating surface 2 and 5. Run 2 indicated a SHGC value of 0.203, which is a significant decrease in solar heat gain when compared to the first run. Run 3 involved a shift in the coating locations, but the SHGC value was very similar to the results from the second run. Our next comparison included glazing only a single surface of the window system. Runs 4 through 7 only involved one coating, but the location was switched throughout the window system. These runs demonstrated similar results with a SHGC value of approximately 0.3. These set of calculations indicate that there is not much improvement in reducing heat transfer due to radiation through the system if only one surface is glazed in comparison with two Low-E surfaces. Depending on the cost of coating a window surface, the fourth run might be the ideal glazing system situation. Run 4 shows that a single glazing on surface two can significantly reduce the SHGC value, which also reducing the cost of coating a window. Runs 7 through 11 display the SHGC calculations for an increasing number of surface coatings within the system. Despite increasing the amount of coated surfaces within the system, we determined that the SHGC improvement was not significant. The solar heat gain coefficient values range from 0.159 to 0.131, which is not a significant improvement in comparison with runs 2, 3, and 4. We would not recommend that more than 2 surfaces be coated because the improvement in SHGC values is not significant enough to justify the cost of coating more
surfaces. It should be noted that run 11 is never recommended, because the window surfaces adjacent to the environment will experience deterioration due to weather.

Following the results indicated by our calculations included in the previous table, we determined that runs 2 and 4 are the optimum glazing system scenarios. It should be noted that there are other factors to be considered when choosing a glazing system to use in a triple pane window. One must consider the cost of Low-E glazing and the transmittance of light through the glass. The cost of complex glazing system should always be considered because one would not want to develop a costly Low-E window that would negate any energy savings. One must also consider the visibility through the glass system. If a window has too many Low-E coatings, the visibility will be reduced to a point where one will not be able to see through it and thus ruin the ascetics and purpose of a window. Following these considerations, one should be able to determine the most efficient glazing system possible with consideration to glazing system cost and visibility.

**VISUAL BASIC MODELING RESULTS: U-FACTOR**

**EXCEL CODE INPUTS**

Table 6 gives the glazing system properties and environmental conditions used when comparing the thermal performance of different glazing systems.

Table 6: Excel U-factor code inputs

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Window Height</td>
<td>1 m</td>
</tr>
<tr>
<td>Glass Thickness</td>
<td>3 mm</td>
</tr>
<tr>
<td>Glass Thermal Conductivity</td>
<td>0.960 W/m-K</td>
</tr>
<tr>
<td>Low-E Glass Emissivity</td>
<td>0.022</td>
</tr>
<tr>
<td>Clear Glass Emissivity</td>
<td>0.840</td>
</tr>
<tr>
<td>Outdoor Air Temperature</td>
<td>35°C</td>
</tr>
<tr>
<td>Outdoor Surrounding Temperature</td>
<td>35°C</td>
</tr>
<tr>
<td>Indoor Air Temperature</td>
<td>20°C</td>
</tr>
<tr>
<td>Indoor Surrounding Temperature</td>
<td>20°C</td>
</tr>
</tbody>
</table>

**GAP WIDTH COMPARISON**

Gap spacing between window panes is a very influential factor on the thermal performance of a triple pane window. The gas fill in the cavity will remain stationary as the gap increases until a critical width is reached. One the gap is wider than the critical width, natural convection causes the gas fill to begin flowing in a circular pattern. The gas fill acts as a much better insulator when it is stationary than when it is flowing. In the stationary width range, the heat transfer is treated
as conduction, and the resistance is proportional to gap width. After the gas fill begins moving, increasing the gap causes a higher flow velocity and higher convection coefficient. Theoretically, there is an optimum gap width at which the combination of the two heat transfer effects is minimized.

Modeling results with varying gap widths matches theoretical expectations that the U-factor can be minimized with an optimal gap width. Figures 9 through 12 are plots of the U-factor as a function of gap width for various glazing systems and environmental conditions. The optimal gap width for each system is given beside the legend. Table 7 compares the optimal gap spacing for windows with Low-E and clear glass.

![Diagram of U-factor vs. gap width for different gas fills and glazing systems.](image)

Figure 9: Overall U-factor for different gas fills with Low-E coating and no wind velocity
Figure 10: Overall U-factor for different gas fills with Low-E coating and 2 m/s wind velocity

Figure 11: Overall U-factor for different gas fills with Low-E coating and 5 m/s wind velocity
Figure 12: Overall U-factor for different gas fills with no coating and no wind velocity

Table 7: Comparison of optimal gap width for Low-E (coated on surfaces 2 and 5) and clear glass windows

<table>
<thead>
<tr>
<th>Gas Fill</th>
<th>Clear Glass, No wind</th>
<th>Low E Glass, No wind</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimal Gap (mm)</td>
<td>U-factor² (W/m² K)</td>
</tr>
<tr>
<td>Air</td>
<td>27</td>
<td>1.541</td>
</tr>
<tr>
<td>Argon</td>
<td>25</td>
<td>1.491</td>
</tr>
<tr>
<td>Krypton</td>
<td>17</td>
<td>1.466</td>
</tr>
<tr>
<td>Xenon</td>
<td>12</td>
<td>1.450</td>
</tr>
<tr>
<td>80/20 Krypton/Argon at 85%</td>
<td>18</td>
<td>1.495</td>
</tr>
<tr>
<td>60/40 Krypton/Argon at 85%</td>
<td>19</td>
<td>1.499</td>
</tr>
</tbody>
</table>
When comparing the U-factor for windows with clear glass versus Low-E glass, the Low-E glass had a U-factor of approximately one-third that of the clear glass for the same gas fill. This means that for a given gas fill, a window with Low-E glass will have approximately one-third of the heat transfer of a comparable clear glass window.

For the clear glass window, the minimum U-factor changes very slightly when the gas fill is changed. The maximum difference between the U-factors is 5.9% when comparing air to xenon. When constructing a window of clear glass, if the optimal gap width is used, then the gas fill should be air because the thermal benefit of using another gas are outweighed by the additional construction cost. Although the change in thermal performance is slight, the optimal gap width changes significantly between gases. The optimal gap for air is over double the width of the optimal gap for xenon. In conclusion, the only advantage of using an expensive gas for a clear glass triple pane window is a reduced overall window thickness.

For the window with Low-E glass with the low emissivity coatings on surfaces 2 and 5, the gas fill has a significant impact on the minimum U-factor. The U-factors range from 0.577 for air to 0.346 for xenon, a 40% difference. Using an expensive gas fill instead of air for a Low-E triple pane window not only reduces the optimal window width, but also significantly improves the thermal performance of the window. Another important finding is that both krypton/argon mixtures had thermal performance worse than argon or krypton alone. Adding a complex mixture of krypton/argon is not only expensive, but inefficient. The optimal gap width for a Low-E window is 2mm smaller than that of the corresponding clear glass window.

**JELD-WEN GAP SPACING**

The JELD-WEN Corporation requested a thermal analysis of windows with several specific gap widths. The gap widths specified by JELD-WEN are listed in Table 8.

**Table 8: Gap spacing specified by JELD-WEN**

<table>
<thead>
<tr>
<th>Gap Spacing (inches)</th>
<th>Gap Spacing (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.512</td>
<td>13</td>
</tr>
<tr>
<td>0.375</td>
<td>9.5</td>
</tr>
<tr>
<td>0.312</td>
<td>7.9</td>
</tr>
<tr>
<td>0.250</td>
<td>6.4</td>
</tr>
</tbody>
</table>
Results from window modeling using our code for the gap widths listed in Table 8 are shown below in Table 9 and Table 10.

Table 9: Performance of clear glass window with JELD-WEN spacing

<table>
<thead>
<tr>
<th>Gas</th>
<th>Gap Width (mm)</th>
<th>Program Ufactor (W/m-k)</th>
<th>Minimum Ufactor (W/m-k)</th>
<th>Ufactor/Umin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>13</td>
<td>1.669</td>
<td>1.541 (27mm)</td>
<td>1.083</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>1.765</td>
<td>1.541 (27mm)</td>
<td>1.145</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>1.826</td>
<td>1.541 (27mm)</td>
<td>1.185</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.911</td>
<td>1.541 (27mm)</td>
<td>1.240</td>
</tr>
<tr>
<td>Argon</td>
<td>13</td>
<td>1.576</td>
<td>1.491 (25mm)</td>
<td>1.057</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>1.649</td>
<td>1.491 (25mm)</td>
<td>1.106</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>1.696</td>
<td>1.491 (25mm)</td>
<td>1.137</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.763</td>
<td>1.491 (25mm)</td>
<td>1.182</td>
</tr>
<tr>
<td>80/20 Krypton/Argon at 85%</td>
<td>13</td>
<td>1.519</td>
<td>1.495 (18 mm)</td>
<td>1.016</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>1.574</td>
<td>1.495 (18 mm)</td>
<td>1.053</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>1.610</td>
<td>1.495 (18 mm)</td>
<td>1.077</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.663</td>
<td>1.495 (18 mm)</td>
<td>1.112</td>
</tr>
<tr>
<td>60/40 Krypton/Argon at 85%</td>
<td>13</td>
<td>1.535</td>
<td>1.499 (19mm)</td>
<td>1.024</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>1.595</td>
<td>1.499 (19mm)</td>
<td>1.064</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>1.634</td>
<td>1.499 (19mm)</td>
<td>1.090</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.692</td>
<td>1.499 (19mm)</td>
<td>1.129</td>
</tr>
</tbody>
</table>
Table 10: Performance of Low-E glass window with JELD-WEN spacing

\[ T_{0,\text{air}} = T_{0,\text{surr}} = 35 \, ^\circ\text{C}, \quad T_{i,\text{air}} = T_{i,\text{surr}} = 20 \, ^\circ\text{C} \]

<table>
<thead>
<tr>
<th>Gas</th>
<th>Gap Width (mm)</th>
<th>Program Ufactor (W/m-k)</th>
<th>Minimum Ufactor (W/m-k)</th>
<th>Ufactor/Umin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>13</td>
<td>0.813</td>
<td>0.577 (24mm)</td>
<td>1.409</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>1.011</td>
<td>0.577 (24mm)</td>
<td>1.752</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>1.143</td>
<td>0.577 (24mm)</td>
<td>1.981</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.305</td>
<td>0.577 (24mm)</td>
<td>2.262</td>
</tr>
<tr>
<td>Argon</td>
<td>13</td>
<td>0.612</td>
<td>0.453 (23mm)</td>
<td>1.351</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>0.769</td>
<td>0.453 (23mm)</td>
<td>1.698</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>0.878</td>
<td>0.453 (23mm)</td>
<td>1.938</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.015</td>
<td>0.453 (23mm)</td>
<td>2.241</td>
</tr>
<tr>
<td>80/20 Krypton/Argon at 85%</td>
<td>13</td>
<td>0.492</td>
<td>0.463 (16mm)</td>
<td>1.063</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>0.605</td>
<td>0.463 (16mm)</td>
<td>1.307</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>0.692</td>
<td>0.463 (16mm)</td>
<td>1.495</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>0.806</td>
<td>0.463 (16mm)</td>
<td>1.741</td>
</tr>
<tr>
<td>60/40 Krypton/Argon at 85%</td>
<td>13</td>
<td>0.525</td>
<td>0.474 (17mm)</td>
<td>1.108</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>0.652</td>
<td>0.474 (17mm)</td>
<td>1.376</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>0.746</td>
<td>0.474 (17mm)</td>
<td>1.574</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>0.868</td>
<td>0.474 (17mm)</td>
<td>1.831</td>
</tr>
</tbody>
</table>

All of the gap widths specified by JELD-WEN are smaller than the optimal gap width. For the clear glass window, the Ufactor/Umin ratio ranged from 1.016 to 1.240. The Ufactor/Umin ratio for the Low-E window ranged from 1.063 to 2.262. The clear window U-factor was much closer to optimal than the U-factor from the Low-E window. As was expected, a gap width closest to the optimal gap width produced a U-factor closest to the minimum U-factor. If the JELD-WEN manufacturing facility is only equipped to make windows with the four specified gap thicknesses, the largest gap of 13mm should be chosen for each gas fill.

### COMPARISON OF UTK MODEL WITH WINDOW PROGRAM

The WINDOW program developed by Lawrence Berkeley National Laboratory was used to calculate the U-factor for comparison with the UTK model. A range of gas fills, gap widths, and window types were tested. Tables 11 and 12 show a comparison between these two models.
Table 11: Comparison of UTK model to WINDOW U-factor

Clear Glass and No Wind
\(T_{o,\text{air}} = T_{o,\text{surr}} = 35 \, ^\circ\text{C}, \, T_{i,\text{air}} = T_{i,\text{surr}} = 20 \, ^\circ\text{C}\)

<table>
<thead>
<tr>
<th>Gas</th>
<th>Gap Width (mm)</th>
<th>Program Ufactor (W/m-k)</th>
<th>WINDOW Ufactor (W/m-k)</th>
<th>Percent Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>13</td>
<td>1.669</td>
<td>1.721</td>
<td>-3.02</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>1.765</td>
<td>1.825</td>
<td>-3.29</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>1.826</td>
<td>1.896</td>
<td>-3.69</td>
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<td></td>
<td>6.4</td>
<td>1.911</td>
<td>1.993</td>
<td>-4.11</td>
</tr>
<tr>
<td>Argon</td>
<td>13</td>
<td>1.576</td>
<td>1.621</td>
<td>-2.78</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>1.649</td>
<td>1.699</td>
<td>-2.94</td>
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<td></td>
<td>7.9</td>
<td>1.696</td>
<td>1.754</td>
<td>-3.31</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.763</td>
<td>1.831</td>
<td>-3.71</td>
</tr>
<tr>
<td>80/20 Krypton/Argon at 85%</td>
<td>13</td>
<td>1.519</td>
<td>1.554</td>
<td>-2.25</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>1.574</td>
<td>1.612</td>
<td>-2.36</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>1.610</td>
<td>1.655</td>
<td>-2.72</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.663</td>
<td>1.715</td>
<td>-3.03</td>
</tr>
<tr>
<td>60/40 Krypton/Argon at 85%</td>
<td>13</td>
<td>1.535</td>
<td>1.572</td>
<td>-2.35</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>1.595</td>
<td>1.636</td>
<td>-2.51</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>1.634</td>
<td>1.683</td>
<td>-2.91</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.692</td>
<td>1.747</td>
<td>-3.15</td>
</tr>
</tbody>
</table>
Table 12: Comparison of UTK model to WINDOW U-factor

<table>
<thead>
<tr>
<th>Gas</th>
<th>Gap Width (mm)</th>
<th>Program Ufactor (W/m-k)</th>
<th>WINDOW Ufactor (W/m-k)</th>
<th>Percent Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>13</td>
<td>0.892</td>
<td>0.879</td>
<td>1.48</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>1.135</td>
<td>1.115</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>1.287</td>
<td>1.279</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.499</td>
<td>1.498</td>
<td>0.07</td>
</tr>
<tr>
<td>Argon</td>
<td>13</td>
<td>0.656</td>
<td>0.646</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>0.841</td>
<td>0.827</td>
<td>1.69</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>0.960</td>
<td>0.954</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>1.129</td>
<td>1.13</td>
<td>-0.09</td>
</tr>
<tr>
<td>80/20 Krypton/Argon at 85%</td>
<td>13</td>
<td>0.522</td>
<td>0.493</td>
<td>5.88</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>0.650</td>
<td>0.627</td>
<td>3.67</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>0.741</td>
<td>0.726</td>
<td>2.07</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>0.876</td>
<td>0.864</td>
<td>1.39</td>
</tr>
<tr>
<td>60/40 Krypton/Argon at 85%</td>
<td>13</td>
<td>0.559</td>
<td>0.535</td>
<td>4.49</td>
</tr>
<tr>
<td></td>
<td>9.5</td>
<td>0.704</td>
<td>0.683</td>
<td>3.07</td>
</tr>
<tr>
<td></td>
<td>7.9</td>
<td>0.804</td>
<td>0.79</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>0.949</td>
<td>0.939</td>
<td>1.06</td>
</tr>
</tbody>
</table>

The UTK model matched WINDOW results very closely. Percent differences for both window systems were in the 0 to 5% range with a maximum percent difference of 5.88%. This gives confidence to the UTK model because the U-factor values are very similar.
EXPERIMENTAL RESULTS

APPARATUS

Materials:

- plywood
- 2x4 boards
- wood screws
- wood staples
- 1/4” bolts with washers and wing nuts
- JB Weld
- wires (for the thermocouples)
- reflective and heat resistant paint
- black fabric
- reflective blanket
- extruded polystyrene

Figure 13: Test apparatus and equipment
Table 13: Legend for test apparatus and equipment diagram

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Test chamber containing aluminum plate and thermocouple attachments</td>
</tr>
<tr>
<td>2</td>
<td>Thermocouple wires</td>
</tr>
<tr>
<td>3</td>
<td>Window placement</td>
</tr>
<tr>
<td>4</td>
<td>Lamp</td>
</tr>
</tbody>
</table>

The apparatus used for testing was our original design concept with a few minor adjustments made during the construction stage. The wood box we constructed was framed with the 2” by 4” boards, while the bottom, front and back faces were covered with plywood. A 10” by 10” square was cut in the front face for the light to shine through to the glazing system. The box was essentially divided in half by a piece of plywood with a hole cut out of the center, again for the light to shine through to the glass. It is behind this sheet of plywood that the glazing system is placed. A ledge was screwed to the back of this dividing piece of wood for the glass to rest on so that it is centered over the hole, while another piece of plywood with the same size hole was placed behind the glass. The hole in the second piece of wood is so the light can pass through the panes of glass and reach an aluminum plate fixed to the interior back face of the apparatus. The plywood pieces and glazing system were held together with four bolts, one in each corner, and wing nuts to make changing out the various glazing systems quick and easy. The three different glazing systems tested were the clear glass and two low-e coatings with different sized spacing. The clear system and one of the low-e systems had an air gap spacing of .013, the largest spacing tested. Each pane of glass was 12” by 12” with a thickness of .00762”. The 6” by 6” aluminum plate was held in each corner by small pieces of scrap wood screwed to the interior of the back face. Two pieces of ¾” extruded polystyrene insulation were pressed against the back of the aluminum plate and held in place by a single strip of wood bolted to the back side of the back face. The thermal emittance of plate was .89 and the R-value of the insulation, which was dependent on its thickness, was 4. Wing nuts were used to tighten the wood against the insulation and hold the plate in place. The front “compartment” was painted with silver reflective paint, while the back “compartment” was painted with black with paint engineered to withstand high temperatures. Also the front and back faces were painted with the same heat resistance paint, only silver instead of black. Prior to painting, the light was turned on and allowed to shine on the wood for some time before charring it. Therefore the decision for heat resistant paint was made for the front face, while the back face was merely painted for aesthetics. Finally, the sides and top of the portion of the apparatus painted black was covered in a black cotton piece of fabric and the entire box was swathed in a reflective blanket. The fabric and blanket were stapled to the top and one side of the box, while the remaining side was held down with several strips of Velcro. This made the glazing system and other components accessible.

Once the actual apparatus was built, the testing components were installed. Seven thermocouples were made and attached throughout the portion of the box painted black. One was attached to the back of the last pane of glass, while three were placed on the bottom, top, and side respectively. Another thermocouple was placed inside a ping pong ball that had been painted black and hung from the top. The ball acted as a shield from radiation given off by the lamp. A thermocouple was attached to the back of the aluminum plate, while another one was attached to the back of
the first piece of insulation. Once all the thermocouples had been attached, they were all wired to a data acquisition board and ready for testing.

**EXPERIMENT PROCEDURE**

With the thermocouples wired to the data acquisition board and the board connected to a laptop, quickDAQ software was opened and run to collect data every second over the course of an hour. The apparatus was positioned on a table and the light was brought to the height of the cutout. Just several inches from the front face, the lamp was turned on and data was collected from each of the seven thermocouples. By the end of the hour each thermocouple had reached steady-state. Only one test was conducted each meeting to allow the apparatus to cool before testing again so as not to have any effect on future temperature readings. Once finished testing with the thermocouples, a pyrometer was used to compare values with our “poor boy” pyrometer. A data reduction program was written and implemented into our code to compile and interpret the collected data. Finally, the Windows Energy Profiler, which uses an infrared beam to measure performance values of a window already in-frame, was used on each of the three glazing systems to give an SHGC value. This data reduction gave values of SHGC that were used to compare to values given by the other methods of measurement.

**DATA REDUCTION**

During our test, we wished to build confidence in any data we retrieved. By measuring the same value through multiple techniques, we develop that confidence. If each method shows similar values, you can assume it is accurate. In our setup, we use three instruments to measure SHGC: the “Poor-Boy” pyranometer, the pyranometer, and the Windows Energy Profiler. By comparing the results of each device to one another, we will get a better assessment of the data. The first implement used is the “Poor-Boy” pyranometer. The following figure gives a brief picture of its operation.

![Figure 14: “Poor-Boy” pyranometer control surface diagram](image)
Table 14: “Poor-Boy” pyranometer variable inputs

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Absorptivity coefficient</td>
<td>0.958</td>
<td>--</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Boltzmann constant</td>
<td>$1.381 \times 10^{-23}$</td>
<td>(m$^2$kg/s)/(sec$^2$K)</td>
</tr>
<tr>
<td>$T_{surr}$</td>
<td>Temperature of enclosure surfaces</td>
<td>INPUT</td>
<td>K</td>
</tr>
<tr>
<td>$I_s$</td>
<td>Incident radiation</td>
<td>INPUT</td>
<td>W/m$^2$</td>
</tr>
<tr>
<td>$h$</td>
<td>Convection Coefficient</td>
<td>INPUT</td>
<td>W/(m$^2$K)</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Temperature of plate</td>
<td>INPUT</td>
<td>K</td>
</tr>
<tr>
<td>$T_{oo}$</td>
<td>Enclosure air temperature</td>
<td>INPUT</td>
<td>K</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>Plate emissivity</td>
<td>0.89</td>
<td>--</td>
</tr>
<tr>
<td>$t$</td>
<td>Insulation thickness</td>
<td>0.01905</td>
<td>m</td>
</tr>
<tr>
<td>$k_{ins}$</td>
<td>Insulation thermal conductivity</td>
<td>0.2703</td>
<td>W/(mK)</td>
</tr>
<tr>
<td>$T_{ins}$</td>
<td>Insulation temperature</td>
<td>INPUT</td>
<td>K</td>
</tr>
</tbody>
</table>

Raw thermocouple data comes from the test as temperatures measurements. In order to ensure steady state, we measure the temperature once every second and only use the data from the last minute. We average the data from each thermocouple and use one mean temperature at each thermocouple. For the purpose of radiation heat transfer, we calculate a $T_{surr}$ from the average of the temperatures of the test setup walls, demonstrated in this equation.

$$T_{Surr} = \frac{T_1 + T_2 + T_3 + T_4 + T_5}{5}$$  \hspace{1cm} (63)

$T_{surr}$ is then used to calculate radiation heat flux as demonstrated in the next equation.

$$q_{Rad} = \varepsilon_s \cdot \sigma (T_s^4 - T_{Surr}^4)$$  \hspace{1cm} (64)

We have this greatly simplified equation from painting the inside of the setup black to create blackbodies it removes concerns for surface properties and view factors. Next, we compute convection on the surface of the plate via the equation

$$q_{Conv} = h(T_6 - T_4)$$  \hspace{1cm} (65)

where $h$ is defined earlier with Nusselt correlations. The final mode of heat transfer in the plate, conduction, is lost through the insulation. It is calculated by way of this equation.

$$q_{Cond} = \frac{k(T_6 - T_7)}{t_{ins}}$$  \hspace{1cm} (66)
With all of the heat fluxes, it is a simple step to computing the Irradiation incident on the surface of the plate by this equation.

\[ I_s = \frac{(q_{\text{Con}} + q_{\text{Conv}} + q_{\text{Rad}})}{a_s} \]  

(67)

We then compute SHGC with this irradiation.

\[ \text{SHGC} = \frac{I_s}{I_{\text{Incident}}} \]  

(68)

For the standard pyranometer it reads a voltage which is converted into an irradiation measurement with the Calibration Constant via this equation.

Voltage (2,000) = \( I_{ps} \)  

(69)

Then the SHGC calculation is simple.

\[ \text{SHGC} = \frac{I_{ps}}{I_{p,\text{Incident}}} \]  

(70)

Where \( I_{ps} \) is Irradiation as measure behind the window and \( I_{p,\text{Incident}} \) is Irradiation measured without the window.

The final SHGC measurement is taken using the Windows Energy Profiler. It is a commercial tool designed for quick and easy measurement of SHGC and other window performance values. There is no calculation to make the tool reads it directly.

RESULTS

The first data collected is the temperatures. This table displays the average of the temperatures from the last minute of simulation for each thermocouple.

Table 15: Average temperatures used in the data reduction process

<table>
<thead>
<tr>
<th>Temperatures (^{\circ}\text{F})</th>
<th>Window</th>
<th>Sides (T1)</th>
<th>Bottom (T2)</th>
<th>Top (T3)</th>
<th>Air (T4)</th>
<th>Window (T5)</th>
<th>Plate Surface (T6)</th>
<th>Back of Insulation (T7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LowE 1/2&quot;</td>
<td>71.2</td>
<td>66.1</td>
<td>79.7</td>
<td>76.7</td>
<td>77.5</td>
<td>81.1</td>
<td>70.2</td>
<td></td>
</tr>
<tr>
<td>LowE 5/16&quot;</td>
<td>73.7</td>
<td>67.4</td>
<td>83.6</td>
<td>82.1</td>
<td>83.3</td>
<td>94.8</td>
<td>78.1</td>
<td></td>
</tr>
<tr>
<td>Clear 1/2&quot;</td>
<td>94.5</td>
<td>81.7</td>
<td>107</td>
<td>110</td>
<td>116</td>
<td>245</td>
<td>71.7</td>
<td></td>
</tr>
</tbody>
</table>

The temperatures for the two Low-E coating are similar in most respects. It lends confidence to the measurements, as the two coating should have similar temperatures, as the amount of irradiation going thought should be similar. The clear temperatures are much higher due to the
larger amount of irradiation that can pass through. Another fact that adds credibility to the data is the bottom of the setup has the lowest temperature. This is due to the cold air remaining on the bottom and the floor, which has a colder temperature than the air will cause greater heat loss out of the bottom than the sides.

With the temperatures, simple data reduction will render the Irradiation for the “Poor-Boy” Pyranometer. The normal pyranometer reading is also a simple matter to convert to Irradiance. In order to generate a SHGC value we needed a value of total radiation received. As the units cause the area to be a factor, we decided to take the total irradiance at the plate with no window. In theory, this space will receive irradiance over the same area, but without a window to stop irradiance.

Table 16: Plate irradiation comparison using the “Poor-Boy” Pyranometer and the actual pyranometer

<table>
<thead>
<tr>
<th>Plate Irradiation (W/m²)</th>
<th></th>
<th>Pyranometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Window</td>
<td>“Poor-Boy” Pyranometer</td>
<td>Pyranometer</td>
</tr>
<tr>
<td>LowE 1/2&quot;</td>
<td>9.99</td>
<td>19.6</td>
</tr>
<tr>
<td>LowE 5/16&quot;</td>
<td>69.8</td>
<td>121</td>
</tr>
<tr>
<td>Clear 1/2&quot;</td>
<td>1370</td>
<td>1310</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Plate Irradiation with no Window</th>
</tr>
</thead>
<tbody>
<tr>
<td>LowE 1/2&quot;</td>
</tr>
<tr>
<td>LowE 5/16&quot;</td>
</tr>
<tr>
<td>Clear 1/2&quot;</td>
</tr>
</tbody>
</table>

The results of this part of the experiment seem skewed at first glance. You will notice how one test had irradiances much lower than the others did. Looking at the no window irradiation case, you will notice that it is also much lower for that test. This is because for that test the lamp was moved away from the test apparatus. It caused a dispersion of the light over a greater surface decreasing the irradiance. Ideally, it should not affect the results.

The SHGC is the desired measurement from this experiment. It is listed below with every method available to us for calculating SHGC in order to compare the values.
Figure 15: Graphical comparison of SHG coefficients determined using different methods

Table 17: SHG coefficients collected determined using various methods

<table>
<thead>
<tr>
<th>Window</th>
<th>“Poor-Boy”</th>
<th>Pyranometer</th>
<th>Window Energy Profiler</th>
<th>Our Code</th>
<th>Window</th>
</tr>
</thead>
<tbody>
<tr>
<td>LowE 1/2&quot;</td>
<td>0.030</td>
<td>0.059</td>
<td>0.250</td>
<td>0.203</td>
<td>0.233</td>
</tr>
<tr>
<td>LowE 5/16&quot;</td>
<td>0.033</td>
<td>0.058</td>
<td>0.250</td>
<td>0.203</td>
<td>0.234</td>
</tr>
<tr>
<td>Clear 1/2&quot;</td>
<td>0.651</td>
<td>0.622</td>
<td>0.650</td>
<td>0.646</td>
<td>0.679</td>
</tr>
</tbody>
</table>

A number of trends are immediately apparent from the chart. First, all the methods of measuring SHGC agree on the values for the clear window. This lends credibility to the accuracy of the data. In fact, our test setup matches values with Windows, a professional window performance product. This also means our experimental procedure has merit as it can reproduce reliable data. The chart also displays a flaw in our experiment. The experimental results are quite far from the modeled results for the SHGC of Low-E coated windows. However both experimental methods, the “Poor-Boy” and normal pyranometer, have very similar results, this and the fact that they matched the clear window suggest that there is some unaccounted for error in testing the Low-E glass. Error in the measurement of the Irradiance is the most likely cause. Apparatus error is not a percent but a set number. Therefore when measuring the Irradiance through the Low-E coating we are dealing with much smaller numbers than the clear glass situation. For example, if our devices have an error of 30 W/m² it will not matter much when measuring quantities of 1300 W/m² the error is merely 2.3% of the measurement; however, when measuring values near 90 W/m² the error is 33%, much higher.

In order to adjust for this error, the intensity of the light must be increased. It could be moved close to the window or a higher watt bulb could be used. The problem with this solution is that already we have had problems with over heating of the setup. The insulation is very close to its maximum allowable temperature, any hotter and it will melt. In addition, the setup, being made of wood, will char and smoke with increased temperatures. The final and worst problem is that
the heat has a risk of shattering the windows. The temperature differential across the window creates stresses resulting in cracks and shattering. Already we have lost two windows to this phenomenon.

CONCLUSIONS AND RECOMMENDATIONS

After completing this test, which included both simulated and experimental heat transfer through triple pane windows to determine the SHGC and U-factor values, we were able to make several conclusions about the study. We successfully developed a code that utilized WINDOW equations to analyze the U-Factor of different window configurations and our results were comparable to those of WINDOW with at most a 5% difference. Next we successfully employed a Newton-Raphson method to determine the SHGC of different glazing configurations. Although these values were not directly comparable to the SHGC’s determined by WINDOW, they showed the same trend that glazing a surface with Low-E lowers the transmitted radiation through the glass. It should be noted WINDOW SHGC’s are not comparable because WINDOW determines the value by an unknown value.

After consideration, we were able to make the following recommendations for window manufacturers to reduce heat transfer through windows and thus improve the thermal efficiency. First, in consideration to the U-factor, we would recommend that JELD-WEN increase their current gap widths to reduce heat transfer. JELD-WEN could use our tabulated results and code to optimize the gap width for various gases. Our results also indicated that Xenon was the best choice for reducing heat transfer. If it is economically feasible, we would recommend that Xenon be used as the primary fill in all window products because it is the insulator and it is a nontoxic gas. Finally, we would also recommend that JELD-WEN continue to utilize their current method of Low-E glazing. We would not recommend that more surfaces be coated because it reduces visibility and increases costs. In addition, we would also recommend that only one surface be coated in order to reduce the SHGC if cost is a major factor in manufacturing. If only one surface was coated, the SHGC would still be reasonably good and relatively cheap.

If we could improve our test setup, we would improve the data acquisition method by increasing the amount of thermocouples that could be utilized for temperature data. If possible, we would also perform our tests in a controlled test chamber to manage the temperature surrounding the test area to reduce data uncertainty.
REFERENCES


Miller, William A. University of Tennessee, Knoxville Capstone Design Lectures, 2009-2010

APPENDIX A: Visual Basic Code
Sub Window()
    wmin = Sheets("Total Window").Range("B26").Value / 1000
    wmax = Sheets("Total Window").Range("B27").Value / 1000
    interval = Sheets("Total Window").Range("B28").Value / 1000

    Do While wmin <= wmax
        counter = 0
        Toinf = Sheets("Total Window").Range("B5").Value 'Deg C
        Tiinf = Sheets("Total Window").Range("B7").Value 'Deg C
        Toinf = Toinf + 273.15 'K
        Tiinf = Tiinf + 273.15 'K
        Tfilm = (Toinf + Tiinf) / 2 'Deg C

        Height = Sheets("Total Window").Range("B32").Value 'm
        tg1 = Sheets("Total Window").Range("B33").Value / 1000 'm
        tg2 = Sheets("Total Window").Range("C33").Value / 1000 'm
        tg3 = Sheets("Total Window").Range("D33").Value / 1000 'm
        g = 9.81 'm/s2

        windext = Sheets("Total Window").Range("B9").Value 'm/s
        gammaaz = Sheets("Total Window").Range("B10").Value 'deg
        gammaWN = Sheets("Total Window").Range("B11").Value 'deg

        gemiss1 = Sheets("Total Window").Range("B35").Value
        gemiss2 = Sheets("Total Window").Range("B36").Value
        gemiss3 = Sheets("Total Window").Range("C35").Value
        gemiss4 = Sheets("Total Window").Range("C36").Value
        gemiss5 = Sheets("Total Window").Range("D35").Value
        gemiss6 = Sheets("Total Window").Range("D36").Value
gref1 = Sheets("Total Window").Range("B37").Value

gref2 = Sheets("Total Window").Range("B38").Value

gref3 = Sheets("Total Window").Range("C37").Value

gref4 = Sheets("Total Window").Range("C38").Value

gref5 = Sheets("Total Window").Range("D37").Value

gref6 = Sheets("Total Window").Range("D38").Value

Tsky = Sheets("Total Window").Range("B6").Value + 273.15
Tintsurf = Sheets("Total Window").Range("B8").Value + 273.15

If Sheets("Total Window").Range("B15").Value = 1 Then
    gas1 = "air"
End If
If Sheets("Total Window").Range("B15").Value = 2 Then
    gas1 = "argon"
End If
If Sheets("Total Window").Range("B15").Value = 3 Then
    gas1 = "krypton"
End If
If Sheets("Total Window").Range("B15").Value = 4 Then
    gas1 = "xenon"
End If
If Sheets("Total Window").Range("B16").Value = 1 Then
    gas2 = "air"
End If
If Sheets("Total Window").Range("B16").Value = 2 Then
    gas2 = "argon"
End If
If Sheets("Total Window").Range("B16").Value = 3 Then
    gas2 = "krypton"
End If
If Sheets("Total Window").Range("B16").Value = 4 Then
    gas2 = "xenon"
End If
If Sheets("Total Window").Range("B17").Value = 1 Then
    gas3 = "air"
End If
If Sheets("Total Window").Range("B17").Value = 2 Then
    gas3 = "argon"
End If
If Sheets("Total Window").Range("B17").Value = 3 Then
    gas3 = "krypton"
End If
If Sheets("Total Window").Range("B17").Value = 4 Then
    gas3 = "xenon"
End If

gas1percent = Sheets("Total Window").Range("C15").Value / 100

gas2percent = Sheets("Total Window").Range("C16").Value / 100

gas3percent = Sheets("Total Window").Range("C17").Value / 100

w = wmin
kglass1 = Sheets("Total Window").Range("B34").Value 'W/mK
kglass2 = Sheets("Total Window").Range("C34").Value 'W/mK
kglass3 = Sheets("Total Window").Range("D34").Value 'W/mK
'Air Thermal Conductivity
  \[ K_{air} = \frac{(418.3709 \times (6.325 \times 10^{-6} \times T_{film}^{1.5}))}{(T_{film} + 245.4 \times 10^{-12} / T_{film})} \] W/m K

\[ h_1 = 1 \times \frac{K_{air}}{\text{Height}} \]
\[ h_3 = 1 \times \frac{K_{air}}{w} \]
\[ h_5 = h_3 \]
\[ h_7 = h_1 \]

\[ R_1 = 1 / h_1 \]
\[ R_3 = 1 / h_3 \]
\[ R_5 = 1 / h_5 \]
\[ R_7 = 1 / h_7 \]
\[ R_2 = \frac{tg_1}{k_{glass1}} \]
\[ R_4 = \frac{tg_2}{k_{glass2}} \]
\[ R_6 = \frac{tg_3}{k_{glass3}} \]
\[ R_{sum} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 \]

\[ q = \frac{(T_{o\inf} - T_{i\inf})}{R_{sum}} \]

\[ T_{1\old} = T_{o\inf} - q \times R_1 \]
\[ T_{2\old} = T_{1\old} - q \times R_2 \]
\[ T_{3\old} = T_{2\old} - q \times R_3 \]
\[ T_{4\old} = T_{3\old} - q \times R_4 \]
\[ T_{5\old} = T_{4\old} - q \times R_5 \]
\[ T_{6\old} = T_{5\old} - q \times R_6 \]
\[ T_{7\old} = T_{6\old} - q \times R_7 \]

\[ T_{1\old} = 40 + 273.15 \]
\[ \text{MaxDelT} = 1 \]
\[ \text{Do While MaxDelT} > 0.01 \]

'Calculate Air Gap Resistances
  \[ R_{a1} = RaSurf(T_{1\old}, T_{o\inf}, \text{Height}) \]
  \[ R_1 = \text{SurRes}(R_{a1}, \text{Height}, T_{1\old}, T_{o\inf}, 1, \text{windext}, \gamma_{az}, \gamma_{wN}, \gamma_{miss1}, T_{sky}) \]
  \[ R_{a3} = RaCav(T_{2\old}, T_{3\old}, w, \text{gas1}, \text{gas1percent}, \text{gas2}, \text{gas2percent}, \text{gas3}, \text{gas3percent}) \]
  \[ R_3 = CavRes(R_{a3}, \text{Height}, w, T_{2\old}, T_{3\old}, \text{gas1}, \text{gas1percent}, \text{gas2}, \text{gas2percent}, \text{gas3}, \text{gas3percent}, 1, \gamma_{miss2}, \gamma_{miss3}) \]
  \[ R_{a5} = RaCav(T_{4\old}, T_{5\old}, w, \text{gas1}, \text{gas1percent}, \text{gas2}, \text{gas2percent}, \text{gas3}, \text{gas3percent}) \]
  \[ R_5 = CavRes(R_{a5}, \text{Height}, w, T_{4\old}, T_{5\old}, \text{gas1}, \text{gas1percent}, \text{gas2}, \text{gas2percent}, \text{gas3}, \text{gas3percent}, 2, \gamma_{miss4}, \gamma_{miss5}) \]
  \[ R_{a7} = RaSurf(T_{6\old}, T_{i\inf}, \text{Height}) \]
  \[ R_7 = \text{SurRes}(R_{a7}, \text{Height}, T_{6\old}, T_{i\inf}, 2, \text{windext}, \gamma_{az}, \gamma_{wN}, \gamma_{miss6}, T_{sky}) \]

\[ h_{r1} = \gamma_{miss1} \times 5.6697 \times 10^{-8} \times (T_{sky}^2 + T_{1\old}^2) \times (T_{sky} + T_{1\old}) \]
\[ h_{r7} = \gamma_{miss6} \times 5.6697 \times 10^{-8} \times (T_{intsurf}^2 + T_{6\old}^2) \times (T_{intsurf} + T_{6\old}) \]
\[ h_{cv1} = 1 / R_1 \]
\[ h_{cv7} = 1 / R_7 \]
\[ R_{1\eff} = 1 / (h_{r1} + h_{cv1}) \]
\[ R_{7\eff} = 1 / (h_{r7} + h_{cv7}) \]
'Calculate q
Rsum = R1eff + R2 + R3 + R4 + R5 + R6 + R7eff
q = (Tno - Tni) / Rsum

'Calculate Temperatures
T1new = Tno - q * R1eff
T2new = T1new - q * R2
T3new = T2new - q * R3
T4new = T3new - q * R4
T5new = T4new - q * R5
T6new = T5new - q * R6
T7new = T6new - q * R7eff

'Temperature differences from last iterations
DelT1 = Abs(T1new - T1old)
DelT2 = Abs(T2new - T2old)
DelT3 = Abs(T3new - T3old)
DelT4 = Abs(T4new - T4old)
DelT5 = Abs(T5new - T5old)
DelT6 = Abs(T6new - T6old)
DelT7 = Abs(T7new - T7old)

'Get max temperature difference
MaxDelT = 0
If MaxDelT < DelT1 Then
    MaxDelT = DelT1
End If
If MaxDelT < DelT2 Then
    MaxDelT = DelT2
End If
If MaxDelT < DelT3 Then
    MaxDelT = DelT3
End If
If MaxDelT < DelT4 Then
    MaxDelT = DelT4
End If
If MaxDelT < DelT5 Then
    MaxDelT = DelT5
End If
If MaxDelT < DelT6 Then
    MaxDelT = DelT6
End If
If MaxDelT < DelT7 Then
    MaxDelT = DelT7
End If

'Set new temperatures as old temperatures
T1old = T1new
T2old = T2new
T3old = T3new
T4old = T4new
T5old = T5new
T6old = T6new
T7old = T7new

counter = counter + 1

Loop

Sheets("Total Window").Range("G19").Value = q
Sheets("Total Window").Range("G4").Value = T1old - 273.15
Sheets("Total Window").Range("G5").Value = T2old - 273.15
Sheets("Total Window").Range("G6").Value = T3old - 273.15
Sheets("Total Window").Range("G7").Value = T4old - 273.15
Sheets("Total Window").Range("G8").Value = T5old - 273.15
Sheets("Total Window").Range("G9").Value = T6old - 273.15
Sheets("Total Window").Range("G10").Value = T7old - 273.15
Sheets("Total Window").Range("G11").Value = R1eff
Sheets("Total Window").Range("G12").Value = R2
Sheets("Total Window").Range("G13").Value = R3
Sheets("Total Window").Range("G14").Value = R4
Sheets("Total Window").Range("G15").Value = R5
Sheets("Total Window").Range("G16").Value = R6
Sheets("Total Window").Range("G17").Value = R7eff
Sheets("Total Window").Range("G20").Value = Tno - 273.15
Sheets("Total Window").Range("G21").Value = Tni - 273.15

Ufactor = 1 / Rsum
Sheets("Total Window").Range("G18").Value = Ufactor
Sheets("Total Window").Range("K8").Value = Ra1
Sheets("Total Window").Range("K9").Value = Ra3
Sheets("Total Window").Range("K10").Value = Ra5
Sheets("Total Window").Range("K11").Value = Ra7

'Archive Values
If gas1 = "air" Then
If gas2 = "air" Then
If gas3 = "air" Then
n = Sheets("Air").Range("V2").Value
Sheets("Air").Range("B" & n).Value = q
Sheets("Air").Range("C" & n).Value = Ufactor
Sheets("Air").Range("D" & n).Value = Toinf - 273.15
Sheets("Air").Range("E" & n).Value = T1old - 273.15
Sheets("Air").Range("F" & n).Value = T2old - 273.15
Sheets("Air").Range("G" & n).Value = T3old - 273.15
Sheets("Air").Range("H" & n).Value = T4old - 273.15
Sheets("Air").Range("I" & n).Value = T5old - 273.15
Sheets("Air").Range("J" & n).Value = T6old - 273.15
Sheets("Air").Range("K" & n).Value = T7old - 273.15
Sheets("Air").Range("L" & n).Value = w
Sheets("Air").Range("M" & n).Value = Sheets("Total Window").Range("K12").Value
Sheets("Air").Range("N" & n).Value = Sheets("Total Window").Range("K13").Value
Sheets("Air").Range("O" & n).Value = Sheets("Total Window").Range("K14").Value
Sheets("Air").Range("P" & n).Value = Sheets("Total Window").Range("K15").Value
Sheets("Air").Range("Q" & n).Value = Sheets("Total Window").Range("K8").Value
Sheets("Air").Range("R" & n).Value = Sheets("Total Window").Range("K9").Value
If gas1 = "argon" Then
If gas2 = "argon" Then
If gas3 = "argon" Then
n = Sheets("Argon").Range("V2").Value
Sheets("Argon").Range("B" & n).Value = q
Sheets("Argon").Range("C" & n).Value = Ufactor
Sheets("Argon").Range("D" & n).Value = Toinf - 273.15
Sheets("Argon").Range("E" & n).Value = T1old - 273.15
Sheets("Argon").Range("F" & n).Value = T2old - 273.15
Sheets("Argon").Range("G" & n).Value = T3old - 273.15
Sheets("Argon").Range("H" & n).Value = T4old - 273.15
Sheets("Argon").Range("I" & n).Value = T5old - 273.15
Sheets("Argon").Range("J" & n).Value = T6old - 273.15
Sheets("Argon").Range("K" & n).Value = T7old - 273.15
Sheets("Argon").Range("L" & n).Value = w
Sheets("Argon").Range("M" & n).Value = Sheets("Total Window").Range("K12").Value
Sheets("Argon").Range("N" & n).Value = Sheets("Total Window").Range("K13").Value
Sheets("Argon").Range("O" & n).Value = Sheets("Total Window").Range("K14").Value
Sheets("Argon").Range("P" & n).Value = Sheets("Total Window").Range("K15").Value
Sheets("Argon").Range("Q" & n).Value = Sheets("Total Window").Range("K8").Value
Sheets("Argon").Range("R" & n).Value = Sheets("Total Window").Range("K9").Value
Sheets("Argon").Range("S" & n).Value = Sheets("Total Window").Range("K10").Value
Sheets("Argon").Range("T" & n).Value = Sheets("Total Window").Range("K11").Value
n = n + 1
Sheets("Argon").Range("V2").Value = n
End If
End If
End If

If gas1 = "krypton" Then
If gas2 = "krypton" Then
If gas3 = "krypton" Then
n = Sheets("Krypton").Range("V2").Value
Sheets("Krypton").Range("B" & n).Value = q
Sheets("Krypton").Range("C" & n).Value = Ufactor
Sheets("Krypton").Range("D" & n).Value = Toinf - 273.15
Sheets("Krypton").Range("E" & n).Value = T1old - 273.15
Sheets("Krypton").Range("F" & n).Value = T2old - 273.15
Sheets("Krypton").Range("G" & n).Value = T3old - 273.15
If gas1 = "xenon" Then
    If gas2 = "xenon" Then
        If gas3 = "xenon" Then
            n = Sheets("Xenon").Range("V2").Value
        Else
            n = n + 1
            Sheets("Xenon").Range("V2").Value = n
        End If
    End If
End If

If gas1 = "xenon" Then
    If gas2 = "xenon" Then
        If gas3 = "xenon" Then
            n = Sheets("Xenon").Range("V2").Value
        Else
            n = n + 1
            Sheets("Xenon").Range("V2").Value = n
        End If
    End If
End If

n = Sheets("Xenon").Range("V2").Value
Sheets("Xenon").Range("B" & n).Value = q
Sheets("Xenon").Range("C" & n).Value = Ufactor
Sheets("Xenon").Range("D" & n).Value = Toinf - 273.15
Sheets("Xenon").Range("E" & n).Value = T1old - 273.15
Sheets("Xenon").Range("F" & n).Value = T2old - 273.15
Sheets("Xenon").Range("G" & n).Value = T3old - 273.15
Sheets("Xenon").Range("H" & n).Value = T4old - 273.15
Sheets("Xenon").Range("I" & n).Value = T5old - 273.15
Sheets("Xenon").Range("J" & n).Value = T6old - 273.15
Sheets("Xenon").Range("K" & n).Value = T7old - 273.15
Sheets("Xenon").Range("L" & n).Value = w
Sheets("Xenon").Range("M" & n).Value = Sheets("Total Window").Range("K12").Value
Sheets("Xenon").Range("N" & n).Value = Sheets("Total Window").Range("K13").Value
Sheets("Xenon").Range("O" & n).Value = Sheets("Total Window").Range("K14").Value
Sheets("Xenon").Range("P" & n).Value = Sheets("Total Window").Range("K15").Value
Sheets("Xenon").Range("Q" & n).Value = Sheets("Total Window").Range("K8").Value
Sheets("Xenon").Range("R" & n).Value = Sheets("Total Window").Range("K9").Value
Sheets("Xenon").Range("S" & n).Value = Sheets("Total Window").Range("K10").Value
Sheets("Xenon").Range("T" & n).Value = Sheets("Total Window").Range("K11").Value
End If
End If
End If
If gas1 <> gas2 Or gas2 <> gas3 Or gas1 <> gas3 Then
n = Sheets("Mixture1").Range("V2").Value
Sheets("Mixture1").Range("B" & n).Value = q
Sheets("Mixture1").Range("C" & n).Value = Ufactor
Sheets("Mixture1").Range("D" & n).Value = Toinf - 273.15
Sheets("Mixture1").Range("E" & n).Value = T1old - 273.15
Sheets("Mixture1").Range("F" & n).Value = T2old - 273.15
Sheets("Mixture1").Range("G" & n).Value = T3old - 273.15
Sheets("Mixture1").Range("H" & n).Value = T4old - 273.15
Sheets("Mixture1").Range("I" & n).Value = T5old - 273.15
Sheets("Mixture1").Range("J" & n).Value = T6old - 273.15
Sheets("Mixture1").Range("K" & n).Value = T7old - 273.15
Sheets("Mixture1").Range("L" & n).Value = w
Sheets("Mixture1").Range("M" & n).Value = Sheets("Total Window").Range("K12").Value
Sheets("Mixture1").Range("N" & n).Value = Sheets("Total Window").Range("K13").Value
Sheets("Mixture1").Range("O" & n).Value = Sheets("Total Window").Range("K14").Value
Sheets("Mixture1").Range("P" & n).Value = Sheets("Total Window").Range("K15").Value
Sheets("Mixture1").Range("Q" & n).Value = Sheets("Total Window").Range("K8").Value
Sheets("Mixture1").Range("R" & n).Value = Sheets("Total Window").Range("K9").Value
Sheets("Mixture1").Range("S" & n).Value = Sheets("Total Window").Range("K10").Value
Sheets("Mixture1").Range("T" & n).Value = Sheets("Total Window").Range("K11").Value
n = n + 1
Sheets("Mixture1").Range("V2").Value = n
End If

wmin = wmin + interval
Loop

'SHGC Calcs
an = 12 'm^-1
tau1 = Exp(-an * tg1)
tau2 = Exp(-an * tg2)
tau3 = Exp(-an * tg3)

T1 = (1 - gref1) * (1 - gref2) * tau1 / (1 - gref1 * gref2 * tau1 ^ 2)
T2 = (1 - gref3) * (1 - gref4) * tau2 / (1 - gref3 * gref4 * tau2 ^ 2)
T3 = (1 - gref5) * (1 - gref6) * tau3 / (1 - gref5 * gref6 * tau3 ^ 2)
R11 = (gref1 + gref2 * (1 - 2 * gref1) * tau1 ^ 2) / (1 - gref1 * gref2 * tau1 ^ 2)
R12 = (gref2 + gref1 * (1 - 2 * gref2) * tau1 ^ 2) / (1 - gref1 * gref2 * tau1 ^ 2)
R21 = (gref3 + gref4 * (1 - 2 * gref3) * tau2 ^ 2) / (1 - gref3 * gref4 * tau2 ^ 2)
R22 = (gref4 + gref3 * (1 - 2 * gref4) * tau2 ^ 2) / (1 - gref3 * gref4 * tau2 ^ 2)
R31 = (gref5 + gref6 * (1 - 2 * gref5) * tau3 ^ 2) / (1 - gref5 * gref6 * tau3 ^ 2)
R32 = (gref6 + gref5 * (1 - 2 * gref6) * tau3 ^ 2) / (1 - gref5 * gref6 * tau3 ^ 2)

Ttoppair = T1 * T2 / (1 - R12 * R21)
Rtoppair1 = R11 + (R21 * T1 ^ 2) / (1 - R12 * R21)
Rtoppair2 = R22 + (R12 * T2 ^ 2) / (1 - R21 * R12)

T3plates1 = Ttoppair * T3 / (1 - Rtoppair2 * R31)
R3plates1 = Rtoppair1 + (R31 * Ttoppair ^ 2) / (1 - Rtoppair2 * R31)

T3plates2 = T3 * qout22
R3plates2 = R11 + T1 * (R21 * ((T1 + qout22 * R31 * T2 * R12) / (1 - R12 * R21)) + qout22 * R31 * T2)

Sheets("Total Window").Range("P4").Value = R3plates1
Sheets("Total Window").Range("P5").Value = R3plates2
Sheets("Total Window").Range("P6").Value = T3plates1
Sheets("Total Window").Range("P7").Value = T3plates2
Sheets("Total Window").Range("S4").Value = T1
Sheets("Total Window").Range("S5").Value = T2
Sheets("Total Window").Range("S6").Value = T3
Sheets("Total Window").Range("S7").Value = T4
Sheets("Total Window").Range("S8").Value = T5
Sheets("Total Window").Range("S9").Value = T6
Sheets("Total Window").Range("U4").Value = R11
Sheets("Total Window").Range("U5").Value = R12
Sheets("Total Window").Range("U6").Value = R21
Sheets("Total Window").Range("U7").Value = R22
Sheets("Total Window").Range("U8").Value = R31
Sheets("Total Window").Range("U9").Value = R32

qout11 = 1
qout21 = 1
qout12 = 1
qout22 = 1
qout31 = 1
qout32 = 1
Count = 0
Maxdiff = 1

Do While Maxdiff > 0.0001
qout11old = qout11
qout21old = qout21
qout12old = qout12
qout22old = qout22
qout31old = qout31
qout32old = qout32
qout32old = qout32
qout11 = R11 + qout21 * T1
qout21 = qout12 * R21 + qout31 * T2
qout12 = qout21 * R12 + T1
qout22 = qout12 * T2 + qout31 * R22
qout31 = qout22 * R31
qout32 = qout22 * T3

R3plates = qout11
T3plates = qout32

qout11new = qout11
qout21new = qout21
qout12new = qout12
qout22new = qout22
qout31new = qout31
qout32new = qout32

delta11 = Abs(qout11new - qout11old)
delta21 = Abs(qout21new - qout21old)
delta12 = Abs(qout12new - qout12old)
delta22 = Abs(qout22new - qout22old)
delta31 = Abs(qout31new - qout31old)
delta32 = Abs(qout32new - qout32old)

Maxdiff = 0
    If Maxdiff < delta11 Then
        Maxdiff = delta11
    End If
    If Maxdiff < delta21 Then
        Maxdiff = delta21
    End If
    If Maxdiff < delta12 Then
        Maxdiff = delta12
    End If
    If Maxdiff < delta22 Then
        Maxdiff = delta22
    End If
    If Maxdiff < delta31 Then
        Maxdiff = delta31
    End If
    If Maxdiff < delta32 Then
        Maxdiff = delta32
    End If
Count = Count + 1
Loop

Sheets("Total Window").Range("P17").Value = R3plates
Sheets("Total Window").Range("P18").Value = T3plates
Sheets("Total Window").Range("P19").Value = Count

End Sub
Function CavRes(Ra, He, d, T1, T2, gas1, gas1percent, gas2, gas2percent, 
gas3, gas3percent, gap, gemiss1, gemiss2)

\[ \sigma_{12} = 5.6697 \times 10^{-8} \]

'Aspect Ratio
A = He / d

'Calc 2 Nusselt Numbers
If 5 \times 10^4 < Ra Then
   Nu1 = 0.06738 \times Ra^{1/3}
End If
If 10^4 < Ra And Ra <= 5 \times 10^4 Then
   Nu1 = 0.028154 \times Ra^{0.4134}
End If
If Ra <= 10^4 Then
   Nu1 = 1 + 1.7596678 \times 10^{-10} \times Ra^{2.2984755}
End If

Nu2 = 0.242 \times (Ra / A)^{0.272}

'Find Max
If Nu1 < Nu2 Then
   Nu = Nu2
Else
   Nu = Nu1
End If

'Calc Thermal Conductivity of Air
Tf = (T1 + T2) / 2

akair = 2.873 \times 10^{-3}
bkair = 7.76 \times 10^{-5}
amuair = 3.723 \times 10^{-6}
bmuair = 4.94 \times 10^{-8}
acpair = 1002.737
bcpair = 1.2324 \times 10^{-2}
Mair = 28.97

akargon = 2.285 \times 10^{-3}
bkargon = 5.149 \times 10^{-5}
amuargon = 3.723 \times 10^{-6}
bmuargon = 6.451 \times 10^{-8}
acpargon = 521.9285
bcpargon = 0
Margon = 39.948

akkrypton = 9.443 \times 10^{-4}
bkkrypton = 2.826 \times 10^{-5}
amukrypton = 2.213 \times 10^{-6}
bmukrypton = 7.777 \times 10^{-8}
acpkrypton = 248.0907
bcpkrypton = 0
Mkrypton = 83.8
akxenon = 4.538 \times 10^{-4}
bkxenon = 1.723 \times 10^{-5}
amuxenon = 1.069 \times 10^{-6}
bmuxenon = 7.414 \times 10^{-8}
acpxenon = 158.3397
bcpxenon = 0
Mxenon = 131.3

If gas1 = "air" Then
   ak1 = akair
   bk1 = bkair
   amu1 = amuair
   bmu1 = bmuair
   acp1 = acpair
   bcp1 = bcpair
   M1 = Mair
End If
If gas2 = "air" Then
   ak2 = akair
   bk2 = bkair
   amu2 = amuair
   bmu2 = bmuair
   acp2 = acpair
   bcp2 = bcpair
   M2 = Mair
End If
If gas3 = "air" Then
   ak3 = akair
   bk3 = bkair
   amu3 = amuair
   bmu3 = bmuair
   acp3 = acpair
   bcp3 = bcpair
   M3 = Mair
End If

If gas1 = "argon" Then
   ak1 = akargon
   bk1 = bkargon
   amu1 = amuargon
   bmu1 = bmuargon
   acp1 = acpargon
   bcp1 = bcpargon
   M1 = Margon
End If
If gas2 = "argon" Then
   ak2 = akargon
   bk2 = bkargon
   amu2 = amuargon
   bmu2 = bmuargon
   acp2 = acpargon
   bcp2 = bcpargon
   M2 = Margon
End If
If gas3 = "argon" Then
   ak3 = akargon
   bk3 = bkargon
amu3 = amuargon
bmu3 = bmuargon
acp3 = acpargon
bcp3 = bcpargon
M3 = Margon

End If

If gas1 = "krypton" Then
  ak1 = akkrypton
  bk1 = bkkrypton
  amu1 = amukrypton
  bmu1 = bmukrypton
  acp1 = acpkrypton
  bcp1 = bcpkrypton
  M1 = Mkrypton
End If

If gas2 = "krypton" Then
  ak2 = akkrypton
  bk2 = bkkrypton
  amu2 = amukrypton
  bmu2 = bmukrypton
  acp2 = acpkrypton
  bcp2 = bcpkrypton
  M2 = Mkrypton
End If

If gas3 = "krypton" Then
  ak3 = akkrypton
  bk3 = bkkrypton
  amu3 = amukrypton
  bmu3 = bmukrypton
  acp3 = acpkrypton
  bcp3 = bcpkrypton
  M3 = Mkrypton
End If

If gas1 = "xenon" Then
  ak1 = akxenon
  bk1 = bkxenon
  amu1 = amuxenon
  bmu1 = bmuxenon
  acp1 = acpxenon
  bcp1 = bcpxenon
  M1 = Mxenon
End If

If gas2 = "xenon" Then
  ak2 = akxenon
  bk2 = bkxenon
  amu2 = amuxenon
  bmu2 = bmuxenon
  acp2 = acpxenon
  bcp2 = bcpxenon
  M2 = Mxenon
End If

If gas3 = "xenon" Then
  ak3 = akxenon
  bk3 = bkxenon
  amu3 = amuxenon
  bmu3 = bmuxenon
  acp3 = acpxenon
  bcp3 = bcpxenon
End If
bmuxenon = bmu3
acpxenon = acp3
bcpxenon = bcp3
Mxenon = M3
End If

'Viscosity Calc
mugas1 = amu1 + bmu1 * Tf
mugas2 = amu2 + bmu2 * Tf
mugas3 = amu3 + bmu3 * Tf

'Thermal Conductivity Calc
kgas1 = ak1 + bk1 * Tf
kgas2 = ak2 + bk2 * Tf
kgas3 = ak3 + bk3 * Tf
R = 8.314472

kprimegas1 = (15 / 4) * mugas1 * R / M1
kprimegas2 = (15 / 4) * mugas2 * R / M2
kprimegas3 = (15 / 4) * mugas3 * R / M3

kdprimegas1 = kgas1 - kprimegas1
kdprimegas2 = kgas2 - kprimegas2
kdprimegas3 = kgas3 - kprimegas3

kprimephi12 = ((1 + (kprimegas1 / kprimegas2) ^ 0.5 * (M1 / M2) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M1 / M2)) ^ 0.5)
kprimephi13 = ((1 + (kprimegas1 / kprimegas3) ^ 0.5 * (M1 / M3) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M1 / M3)) ^ 0.5)
kprimephi21 = ((1 + (kprimegas2 / kprimegas1) ^ 0.5 * (M2 / M1) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M2 / M1)) ^ 0.5)
kprimephi23 = ((1 + (kprimegas2 / kprimegas3) ^ 0.5 * (M2 / M3) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M2 / M3)) ^ 0.5)
kprimephi31 = ((1 + (kprimegas3 / kprimegas1) ^ 0.5 * (M3 / M1) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M3 / M1)) ^ 0.5)
kprimephi32 = ((1 + (kprimegas3 / kprimegas2) ^ 0.5 * (M3 / M2) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M3 / M2)) ^ 0.5)

kdprimeterm1 = kdprimegas1 / (1 + kprimephi12 * (gas2percent / gas1percent) + kprimephi13 * (gas3percent / gas1percent))
kdprimeterm2 = kdprimegas2 / (1 + kprimephi21 * (gas1percent / gas2percent) + kprimephi23 * (gas3percent / gas2percent))
kdprimeterm3 = kdprimegas3 / (1 + kprimephi31 * (gas1percent / gas3percent) + kprimephi32 * (gas2percent / gas3percent))

kdprime = kdprimeterm1 + kdprimeterm2 + kdprimeterm3

kprimepsi12 = kprimephi12 * (1 + 2.41 * ((M1 - M2) * (M1 - 0.142 * M2)) / ((M1 + M2) ^ 2))
kprimepsi13 = kprimephi13 * (1 + 2.41 * ((M1 - M3) * (M1 - 0.142 * M3)) / ((M1 + M3) ^ 2))
kprimepsi21 = kprimephi21 * (1 + 2.41 * ((M2 - M1) * (M2 - 0.142 * M1)) / ((M2 + M1) ^ 2))
kprimepsi23 = kprimephi23 * (1 + 2.41 * ((M2 - M3) * (M2 - 0.142 * M3)) / ((M2 + M3) ^ 2))
kprimepsi31 = kprimephi31 * (1 + 2.41 * ((M3 - M1) * (M3 - 0.142 * M1)) / ((M3 + M1) ^ 2))
kprimepsi32 = kprimephi32 * (1 + 2.41 * ((M3 - M2) * (M3 - 0.142 * M2)) / ((M3 + M2) ^ 2))

kprimeterm1 = kprimegas1 / (1 + kprimepsi12 * (gas2percent / gas1percent) + kprimepsi13 * (gas3percent / gas1percent))
kprimeterm2 = kprimegas2 / (1 + kprimepsi21 * (gas1percent / gas2percent) + kprimepsi23 * (gas3percent / gas2percent))
kprimeterm3 = kprimegas3 / (1 + kprimepsi31 * (gas1percent / gas3percent) + kprimepsi32 * (gas2percent / gas3percent))

kprime = kprimeterm1 + kprimeterm2 + kprimeterm3
Kair = kprime + kdprime

'Calc Conduction Coefficient and Convert to Resistance
H = Nu * (Kair / d)

hrad = 1 / ((1 / gemiss1) + (1 / gemiss2) - 1) * sigma12 * Abs(T1 ^ 2 + T2 ^ 2) * Abs(T1 + T2)
R = 1 / (hrad + H)
CavRes = R

If gap = 1 Then
    Sheets("Total Window").Range("K13").Value = Nu
    Sheets("Total Window").Range("K5").Value = Kair
End If
If gap = 2 Then
    Sheets("Total Window").Range("K14").Value = Nu
    Sheets("Total Window").Range("K6").Value = Kair
End If
End Function

Function SurRes(Ra, He, Ts, Ta, gap, V, gammaaz, gammawN, gemiss, Tsurr)
    sigma12 = 5.6697 * 10 ^ -8
    y = 90
    Rac = 2.5 * 10 ^ 5 * (Exp(0.72 * y) / 1) ^ (1 / 5)
    'Calc Nusselt
    If Ra <= Rac Then
        Nu = 0.56 * (Ra * 1) ^ (1 / 4)
    End If
    If Ra > Rac Then
        Nu = 0.13 * (Ra ^ (1 / 3) - Rac ^ (1 / 3)) + 0.56 * (Rac * 1) ^ (1 / 4)
    End If
    'Calculate Thermal Conductivity of Air
    Tf = Ta + (1 / 4) * (Ts - Ta)
ak = 2.873 * 10^-3
bk = 7.76 * 10^-5
amu = 3.723 * 10^-6
bumu = 4.94 * 10^-8
acp = 1002.737
bcp = 1.2324 * 10^-2

Kair = ak + bk * Tf

' Calc Conduction Coefficient and Convert to Resistance
If gap = 2 Then
    H = Nu * (Kair / He)
End If

If V > 0 Then
    If gap = 1 Then
        gammaw = gammaaz + 180 - gammawN
        If Abs(gammaw) > 180 Then
            gammaw = 360 - Abs(gammaw)
        End If
        If Abs(gammaw) <= 45 Then
            surf = "windward"
        Else
            surf = "leeward"
        End If
        If surf = "leeward" Then
            Vs = 0.3 + 0.05 * V
        End If
        If surf = "windward" Then
            If V > 2 Then
                Vs = 0.25 * V
            Else
                Vs = 0.5
            End If
            H = 4.7 + 7.6 * Vs
        End If
    End If
    Else
        H = Nu * (Kair / He)
End If

' 
hrad = gemiss * sigma12 * (Tsurr ^ 4 - Ts ^ 4) / (Tsurr - Ts)

R = 1 / H
SurRes = R

If gap = 1 Then
    Sheets("Total Window").Range("K4").Value = Kair
    Sheets("Total Window").Range("K12").Value = Nu
End If
If gap = 2 Then
Function RaCav(T1, T2, L, gas1, gas1percent, gas2, gas2percent, gas3, gas3percent)

'Used Therm Equations
  \(a_{kair} = 2.873 \times 10^{-3}\)
  \(b_{kair} = 7.76 \times 10^{-5}\)
  \(amu_{air} = 3.723 \times 10^{-6}\)
  \(bmu_{air} = 4.94 \times 10^{-8}\)
  \(a_{cpair} = 1002.737\)
  \(b_{cpair} = 1.2324 \times 10^{-2}\)
  \(M_{air} = 28.97\)
  
  \(a_{kargon} = 2.285 \times 10^{-3}\)
  \(b_{kargon} = 5.149 \times 10^{-5}\)
  \(amu_{argon} = 3.723 \times 10^{-6}\)
  \(bmu_{argon} = 6.451 \times 10^{-8}\)
  \(a_{cpargon} = 521.9285\)
  \(b_{cpargon} = 0\)
  \(M_{argon} = 39.948\)
  
  \(a_{kkrypton} = 9.443 \times 10^{-4}\)
  \(b_{kkrypton} = 2.826 \times 10^{-5}\)
  \(amu_{krypton} = 2.213 \times 10^{-6}\)
  \(bmu_{krypton} = 7.777 \times 10^{-8}\)
  \(a_{cpkrypton} = 248.0907\)
  \(b_{cpkrypton} = 0\)
  \(M_{krypton} = 83.8\)
  
  \(a_{kxenon} = 4.538 \times 10^{-4}\)
  \(b_{kxenon} = 1.723 \times 10^{-5}\)
  \(amu_{xenon} = 1.069 \times 10^{-6}\)
  \(bmu_{xenon} = 7.414 \times 10^{-8}\)
  \(a_{cpxenon} = 158.3397\)
  \(b_{cpxenon} = 0\)
  \(M_{xenon} = 131.3\)

If gas1 = "air" Then
  \(a_{k1} = a_{kair}\)
  \(b_{k1} = b_{kair}\)
  \(amu_{1} = amu_{air}\)
  \(bmu_{1} = bmu_{air}\)
  \(a_{cp1} = a_{cpair}\)
  \(bcp_{1} = bcpair\)
  \(M_{1} = M_{air}\)
End If
If gas2 = "air" Then
  \(a_{k2} = a_{kair}\)
  \(b_{k2} = b_{kair}\)
  \(amu_{2} = amu_{air}\)
  \(bmu_{2} = bmu_{air}\)
  \(a_{cp2} = a_{cpair}\)
  \(bcp_{2} = bcpair\)
M2 = Mair
End If
If gas3 = "air" Then
    ak3 = akair
    bk3 = bkair
    amu3 = amuair
    bmu3 = bmuair
    acp3 = acpair
    bcp3 = bcpair
    M3 = Mair
End If
If gas1 = "argon" Then
    ak1 = akargon
    bk1 = bkargon
    amu1 = amuargon
    bmu1 = bmuargon
    acp1 = acpargon
    bcp1 = bcpargon
    M1 = Margon
End If
If gas2 = "argon" Then
    ak2 = akargon
    bk2 = bkargon
    amu2 = amuargon
    bmu2 = bmuargon
    acp2 = acpargon
    bcp2 = bcpargon
    M2 = Margon
End If
If gas3 = "argon" Then
    ak3 = akargon
    bk3 = bkargon
    amu3 = amuargon
    bmu3 = bmuargon
    acp3 = acpargon
    bcp3 = bcpargon
    M3 = Margon
End If
If gas1 = "krypton" Then
    ak1 = akkrypton
    bk1 = bkkrypton
    amu1 = amukrypton
    bmu1 = bmukrypton
    acp1 = acpkrypton
    bcp1 = bcpkrypton
    M1 = Mkrypton
End If
If gas2 = "krypton" Then
    ak2 = akkrypton
    bk2 = bkkrypton
    amu2 = amukrypton
    bmu2 = bmukrypton
    acp2 = acpkrypton
    bcp2 = bcpkrypton
    M2 = Mkrypton
End If
End If
If gas3 = "krypton" Then
    ak3 = akkrypton
    bk3 = bkkrypton
    amu3 = amukrypton
    bmu3 = bmukrypton
    acp3 = acpkrypton
    bcp3 = bcpkrypton
    M3 = Mkrypton
End If

If gas1 = "xenon" Then
    ak1 = akxenon
    bk1 = bkxenon
    amu1 = amuxenon
    bmu1 = bmuxenon
    acp1 = acpxenon
    bcp1 = bcpxenon
    M1 = Mxenon
End If

If gas2 = "xenon" Then
    ak2 = akxenon
    bk2 = bkxenon
    amu2 = amuxenon
    bmu2 = bmuxenon
    acp2 = acpxenon
    bcp2 = bcpxenon
    M2 = Mxenon
End If

If gas3 = "xenon" Then
    ak3 = akxenon
    bk3 = bkxenon
    amu3 = amuxenon
    bmu3 = bmuxenon
    acp3 = acpxenon
    bcp3 = bcpxenon
    M3 = Mxenon
End If

acp = gas1percent * acp1 + gas2percent * acp2 + gas3percent * acp3
bcp = gas1percent * bcp1 + gas2percent * bcp2 + gas3percent * bcp3
M = gas1percent * M1 + gas2percent * M2 + gas3percent * M3
ak = gas1percent * ak1 + gas2percent * ak2 + gas3percent * ak3
bk = gas1percent * bk1 + gas2percent * bk2 + gas3percent * bk3

'Density Calculation
P = 101300
M = M / 1000
R = 8.314472
'Through Therm
Tf = (T1 + T2) / 2
deltaT = Abs(T1 - T2)
Beta = 1 / Tf
Denair = P * M / (R * Tf)
'Viscosity Calc
mugas1 = amu1 + bmu1 * Tf
mugas2 = amu2 + bmu2 * Tf
mugas3 = amu3 + bmu3 * Tf
muphi12 = ((1 + (mugas1 / mugas2) ^ 0.5 * (M2 / M1) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M1 / M2)) ^ 0.5)
muphi13 = ((1 + (mugas1 / mugas3) ^ 0.5 * (M3 / M1) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M1 / M3)) ^ 0.5)
muphi21 = ((1 + (mugas2 / mugas1) ^ 0.5 * (M1 / M2) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M2 / M1)) ^ 0.5)
muphi23 = ((1 + (mugas2 / mugas3) ^ 0.5 * (M3 / M2) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M2 / M3)) ^ 0.5)
muphi31 = ((1 + (mugas3 / mugas1) ^ 0.5 * (M1 / M3) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M3 / M1)) ^ 0.5)
muphi32 = ((1 + (mugas3 / mugas2) ^ 0.5 * (M2 / M3) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M3 / M2)) ^ 0.5)
muterm1 = mugas1 / (1 + muphi12 * (gas2percent / gas1percent) + muphi13 * (gas3percent / gas1percent))
muterm2 = mugas2 / (1 + muphi21 * (gas1percent / gas2percent) + muphi23 * (gas3percent / gas2percent))
muterm3 = mugas3 / (1 + muphi31 * (gas1percent / gas3percent) + muphi32 * (gas2percent / gas3percent))
Muair = muterm1 + muterm2 + muterm3
'Cp calc
Cpair = acp + bcp * Tf
'Thermal Conductivity Calc
kgas1 = ak1 + bk1 * Tf
kgas2 = ak2 + bk2 * Tf
kgas3 = ak3 + bk3 * Tf
kprimegas1 = (15 / 4) * mugas1 * R / M1
kprimegas2 = (15 / 4) * mugas2 * R / M2
kprimegas3 = (15 / 4) * mugas3 * R / M3
kdprimegas1 = kgas1 - kprimegas1
kdprimegas2 = kgas2 - kprimegas2
kdprimegas3 = kgas3 - kprimegas3
kprimephi12 = ((1 + (kprimegas1 / kprimegas2) ^ 0.5 * (M2 / M1) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M1 / M2)) ^ 0.5)
kprimephi13 = ((1 + (kprimegas1 / kprimegas3) ^ 0.5 * (M3 / M1) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M1 / M3)) ^ 0.5)
kprimephi21 = ((1 + (kprimegas2 / kprimegas1) ^ 0.5 * (M1 / M2) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M2 / M1)) ^ 0.5)
kprimephi23 = ((1 + (kprimegas2 / kprimegas3) ^ 0.5 * (M3 / M2) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M2 / M3)) ^ 0.5)
kprimephi31 = ((1 + (kprimegas3 / kprimegas1) ^ 0.5 * (M1 / M3) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M3 / M1)) ^ 0.5)
kprimephi32 = ((1 + (kprimegas3 / kprimegas2) ^ 0.5 * (M2 / M3) ^ 0.25) ^ 2) / (2 * 2 ^ 0.5 * (1 + (M3 / M2)) ^ 0.5)
kdprimeterm1 = kdprimegas1 / (1 + kprimephi12 * (gas2percent / gas1percent) +
    kprimephi13 * (gas3percent / gas1percent))
kdprimeterm2 = kdprimegas2 / (1 + kprimephi121 * (gas1percent / gas2percent) +
    kprimephi23 * (gas3percent / gas2percent))
kdprimeterm3 = kdprimegas3 / (1 + kprimephi131 * (gas1percent / gas3percent) +
    kprimephi32 * (gas2percent / gas3percent))

kdprime = kdprimeterm1 + kdprimeterm2 + kdprimeterm3

kprimepsi12 = kprimephi12 * (1 + 2.41 * ((M1 - M2) * (M1 - 0.142 * M2)) /((M1 + M2) ^ 2))
kprimepsi13 = kprimephi13 * (1 + 2.41 * ((M1 - M3) * (M1 - 0.142 * M3)) / ((M1 + M3) ^ 2))
kprimepsi21 = kprimephi21 * (1 + 2.41 * ((M2 - M1) * (M2 - 0.142 * M1)) /((M2 + M1) ^ 2))
kprimepsi23 = kprimephi23 * (1 + 2.41 * ((M2 - M3) * (M2 - 0.142 * M3)) / ((M2 + M3) ^ 2))
kprimepsi31 = kprimephi31 * (1 + 2.41 * ((M3 - M1) * (M3 - 0.142 * M1)) /((M3 + M1) ^ 2))
kprimepsi32 = kprimephi32 * (1 + 2.41 * ((M3 - M2) * (M3 - 0.142 * M2)) / ((M3 + M2) ^ 2))

kprimeterm1 = kprimepsi12 * (gas2percent / gas1percent) +
    kprimepsi13 * (gas3percent / gas1percent)
kprimeterm2 = kprimepsi21 * (gas1percent / gas2percent) +
    kprimepsi23 * (gas3percent / gas2percent)
kprimeterm3 = kprimepsi31 * (gas1percent / gas3percent) +
    kprimepsi32 * (gas2percent / gas3percent)

kprime = kprimeterm1 + kprimeterm2 + kprimeterm3

Kair = kprime + kdprime

'Property Rearrangements
Nuair = Muair / Denair
Alphaair = Kair / (Denair * Cpair)

'Calc Ryleigh
Ra = ((9.81 * Beta) / (Alphaair * Nuair)) * deltaT * L ^ 3
RaCav = Ra

End Function

Function RaSurf(Ts, Ta, L)

'Used Therm Equations
ak = 2.873 * 10 ^ -3
bk = 7.76 * 10 ^ -5
amu = 3.723 * 10 ^ -6
bmu = 4.94 * 10 ^ -8
acp = 1002.737
bcp = 1.232 * 10 ^ -2
M = 28.97

'Density Calculation
P = 101300
M = M / 1000
R = 8.314472
'From Therm
Tf = Ta + (1 / 4) * (Ts - Ta)
deltaT = Abs(Ts - Ta)
Beta = 1 / Tf
Denair = P * M / (R * Tf)

Muair = amu + bm * Tf
Cpair = acp + bcp * Tf
Kair = ak + bk * Tf

'Property Rearrangements
Nuair = Muair / Denair
Alphaair = Kair / (Denair * Cpair)
'Calc Ryleigh
Ra = ((9.81 * Beta) / (Alphaair * Nuair)) * deltaT * L ^ 3
RaSurf = Ra

End Function
APPENDIX B: Tabular Results for Figures
Sub Test()
'Variables
alpha = Sheets("Sheet1").Range("K2").Value
Thickness = Sheets("Sheet1").Range("K5").Value
epsilon = Sheets("Sheet1").Range("K3").Value
Row = Sheets("Sheet1").Range("K6").Value
Boltz = 5.6697 * 10 ^ -8
Height = Sheets("Sheet1").Range("K7").Value
Boxheight = Sheets("Sheet1").Range("K9").Value
k = Sheets("Sheet1").Range("K4").Value
T1 = Sheets("Sheet1").Range("A" & Row).Value
T2 = Sheets("Sheet1").Range("B" & Row).Value
T3 = Sheets("Sheet1").Range("C" & Row).Value
T4 = Sheets("Sheet1").Range("D" & Row).Value
T5 = Sheets("Sheet1").Range("E" & Row).Value
T6 = Sheets("Sheet1").Range("F" & Row).Value
T7 = Sheets("Sheet1").Range("G" & Row).Value

T1 = 5 / 9 * (T1 - 32) + 273.15
T2 = 5 / 9 * (T2 - 32) + 273.15
T3 = 5 / 9 * (T3 - 32) + 273.15
T4 = 5 / 9 * (T4 - 32) + 273.15
T5 = 5 / 9 * (T5 - 32) + 273.15
T6 = 5 / 9 * (T6 - 32) + 273.15
T7 = 5 / 9 * (T7 - 32) + 273.15

Taverage = (T1 + T1 + T2 + T3 + T5) / 5
Tback = T7
Tsurr = Taverage
Tinf = T4
Ts = T6

'Properties
Ral = RaSurf(Ts, Tinf, Height)
H = Convection(Ral, Height, Ts, Tinf)

'Conduction Through Plate
qcon = k * (Ts - Tback)

'Radiation incident on plate
I = (qcon + H * (Ts - Tinf) + Boltz * (epsilon * Ts ^ 4 - alpha * Tsurr ^ 4)) / alpha

Sheets("Sheet1").Range("H" & Row).Value = I
Sheets("Sheet1").Range("O1").Value = H * (Ts - Tinf)
Sheets("Sheet1").Range("O2").Value = qcon
Sheets("Sheet1").Range("O3").Value = Boltz * (epsilon * Ts ^ 4 - alpha * Tsurr ^ 4)

End Sub
Function RaSurf(Tsurface, Ta, L)
'Used Therm Equations
  ak = 2.873 * 10^-3
  bk = 7.76 * 10^-5
  amu = 3.723 * 10^-6
  bmu = 4.94 * 10^-8
  acp = 1002.737
  bcp = 1.232 * 10^-2
  M = 28.97

'Density Calculation
  P = 101300
  M = M / 1000
  R = 8.314472
  'From Therm
  Tf = Ta + (1 / 4) * (Tsurface - Ta)
  deltaT = Abs(Tsurface - Ta)
  Beta = 1 / Tf
  Denair = P * M / (R * Tf)

  Muair = amu + bmu * Tf
  Cpair = acp + bcp * Tf
  Kair = ak + bk * Tf

'Property Rearrangements
  Nuair = Muair / Denair
  Alphaair = Kair / (Denair * Cpair)
  'Calc Ryleigh
  Ra = ((9.81 * Beta) / (Alphaair * Nuair)) * deltaT * L^3
  RaSurf = 'Ra

End Function

Function Convection(Ra, He, Tplate, Ta)

  sigma12 = 5.6697 * 10^-8

  'Another Ra Corr from Therm
  y = 90
  Rac = 2.5 * 10^5 * (Exp(0.72 * y) / 1)^(1/5)

  'Calc Nusselt
  If Ra <= Rac Then
    Nu = 0.56 * (Ra * 1)^(1/4)
  End If
  If Ra > Rac Then
    Nu = 0.13 * (Ra^(1/3) - Rac^(1/3)) + 0.56 * (Rac * 1)^(1/4)
  End If

  'Calculate Thermal Conductivity of Air
  Tf = Ta + (1 / 4) * (Tplate - Ta)

  ak = 2.873 * 10^-3
  bk = 7.76 * 10^-5
  amu = 3.723 * 10^-6
  bmu = 4.94 * 10^-8
  acp = 1002.737
  bcp = 1.2324 * 10^-2
$K_{air} = a_k + b_k \times T_f$

'Calc Conduction Coefficient and Convert to Resistance
   Convection = Nu \times (K_{air} / H_e)$

End Function
APPENDIX C: Contributions

REPORT CONTRIBUTIONS

Drew Hughes: Introduction, Background, WINDOW, ray tracing diagram, report formatting, References

Michael Kee: U-factor Results, updated SHGC equations and explanations (Visual Basic section), resistance diagram

Amanda Lee: Apparatus, Experimental Procedure, report formatting

Michael McMillan: Summary, Conclusion, SHGC results (comparison), “Poor-Boy” Pyranometer diagram, SHGC and U-Factor equations and theory, report formatting, printing and binding

Jake Plewa: Data Reduction Procedure, Results

**Everyone had input into acknowledgement**
## PROJECT CONTRIBUTIONS

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<thead>
<tr>
<th>Member</th>
<th>Visual Basic Code</th>
<th>Apparatus</th>
<th>Weekly Updates</th>
<th>Test</th>
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<td>Drew Hughes</td>
<td>Automated graph generation</td>
<td>All aspects of construction and data acquisition setup</td>
<td>Scheduled meetings; Delegated tasks and prepared final weekly presenting</td>
<td>Tester</td>
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<td>SHGC iteration</td>
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<tr>
<td>Michael Kee</td>
<td>Primary algorithm development</td>
<td>All aspects of construction and data acquisition setup</td>
<td>Weekly additions to code</td>
<td>Primary tester</td>
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<td>Automated graph generation</td>
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<td>Performed data reduction</td>
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<td>Developed user-friendly interface</td>
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<tr>
<td>Amanda Lee</td>
<td>Assisted theory development</td>
<td>Developed design used for experiment</td>
<td>Adjustments to design; PowerPoint integration</td>
<td>Tester</td>
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<tr>
<td>Michael McMillan</td>
<td>Assisted theory development</td>
<td>All aspects of construction and data acquisition setup</td>
<td>Wrote weekly updates and equations that were eventually integrated into the report</td>
<td>Tester</td>
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<td>SHGC comparison table</td>
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<tr>
<td>Jake Plewa</td>
<td>U-factor function development</td>
<td>All aspects of construction and data acquisition setup</td>
<td>Various assignments and tasks</td>
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<td>Result storage method</td>
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<td>Developed data reduction code</td>
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