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Understanding the Sources of Abnormal Returns from the Momentum Strategy.

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To the Graduate Council:

I am submitting herewith a thesis written by Yu Zhang entitled "Understanding the Sources of Abnormal Returns from the Momentum Strategy.." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mathematics.

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We have read this thesis and recommend its acceptance:

Henry Simpson, George C. Philippatos

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Understanding the Sources of the Abnormal Returns from the Momentum Strategy

A Thesis Presented for the
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ABSTRACT

This thesis studies the sources of the returns from the momentum strategy and attempts to find some hints for the heated debate on the market efficiency hypothesis over the past twenty years. By decomposing the momentum returns from a mathematical model, we investigate directly the contributors and their relative importance in generating these momentum returns.

Our empirical results support that autocorrelation of own stock returns is one of the driving forces for the momentum expected returns. The magnitude of the autocorrelation decreases as the ranking period becomes more remote. The second important source comes from the cross-sectional variation of the expected returns in the winner and loser portfolios at a given time. The third important source is the difference of the expected returns between the winner and loser portfolios. To our surprise, the cross-autocovariance does not contribute much to the momentum expected returns. Thus, the lead-lag effect can cause momentum returns, but its impact is not as significant as we had anticipated.

More importantly, by changing the weights of the winner and loser portfolios, we find that the own-autocovariance of the winner portfolio is almost negligible, compared to that of the loser portfolio. The returns of the winners are much more random than those of the losers. This asymmetric own-autocovariance found in the return decomposition provides another underlying explanation to the recent finding that the contribution of the winner and loser portfolios to the momentum returns is asymmetric, and it is the losers, rather than the winners, that drive the momentum returns.

Therefore, the market may not be as efficient as we believed before.

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CHAPTER I
INTRODUCTION

I. Background

In the 1970s the efficient market hypothesis was widely accepted among finance researchers. It has been commonly believed that information spreads in the market very quickly and, hence, the prices of the securities can quickly reflect the information with minimal delay. Thus, neither the technical analysis of past stock-price behavior nor fundamental analysis of firm specific information can help investors beat the market and earn returns higher than those of randomly selected portfolio with comparable risk. As stated in Malkiel (2003), in efficient financial markets, no investor can earn above-average returns without accepting above-average risks. This efficient market hypothesis has been engrained in much of the modern theoretical and empirical research in financial economics.

However, two decades ago, researchers found that simple investment strategies based on stocks' past returns may realize consistently positive profits. These rejections of martingale behavior of stock prices have seriously challenged the foundation of even the weak-form efficient market hypothesis.

Stock return predictability based on past returns alone, has attracted a lot of attention in finance. The literature has documented three stock trading strategies categorized in terms of time horizons: (a) short-term reversal (Jegadeesh, 1990, and Lo and Mamaysky, 1990); (b) intermediate momentum (Jegadeesh and Titman (JT), 1993); and (c) long-term reversal (Debondt and Thaler, 1985, and Fama and French, 1988). As evidence opposing the efficient market hypothesis, these stock trading strategies are typical examples of exploiting stock return predictability. The debate on the abnormal profits from the momentum strategy that sells the "losers" and buys the "winners" over a 3 to 12 month horizon is much more diverse and voluminous.

This paper focuses on momentum strategy, which—of all the strategies identified—most seriously challenges the market efficiency hypothesis (Fama, 1998). Unlike either the short-term contrarian strategy that provides too little time and requires too much cost for possible arbitrage, or the long-term contrarian strategy, that is not robust to risk adjustment (Fama and French, 1996) and is subject to measurement problems, (Ball, Kothari and Shanken, 1995), the intermediate-term momentum strategy shows strong persistence in both the U.S. and international markets (Asness, Liew and Stevens, 1997, Rouwenhorst, 1998), and continues to exist for post 1990 periods (Jegadeesh and Titman, 2001). The persistence of the momentum abnormal returns after the sample period of the original studies diminishes the possibility of data snooping bias and positions it as a more serious anomaly than other well studied anomalies such as "the small firm effect" and "the value/growth stock phenomenon", both of which disappear after the sample periods in the original studies (Jegadeesh and Titman, 2001).

Over the decades, many serious attempts have been made to explain the momentum abnormal returns from various market phenomena. The rational explanations proponents argue that the profitability of momentum strategies is explained by bearing some sort of additional risks; and, therefore, the market is at least weak-form efficient (Conrad and Kaul, 1998, Berk, Green, and Naik, 1999, Chordia and Shivakumar, 2002, and Lewellen, 2002). The behavioral explanations advocates argue that no risk factors can completely absorb the momentum abnormal returns; rather, it is the way that the irrational investors interpret the information, which causes the momentum or the pattern of stock returns (Jegadeesh and Titman 1993, 2001, Barberis, Shleifer, and Vishny, 1998, Daniel, Hirschleifer, and Subrahmanyam, 1998, and Hong and Stein, 1999). Therefore, the abnormal returns from momentum strategies constitute strong evidence that the market is not even weak-form efficient. The middle position between the above two schools of thoughts focuses on market friction explanations. Proponents of market frictions argue that parts or all of the momentum abnormal returns are justified by some kind of transaction costs in the imperfect market (Lesmond, Schill, and Zhou, 2004, Korajczyk and Sadka, 2004, Sadka, 2006, and Ali and Trombley, 2006). Nevertheless, the empirical results of the market frictions explanations are mixed with respect to market efficiency.

II. Stock Trading Strategies

Three stock trading strategies that utilize only the technical analysis and derive consistent positive profits are short-term contrarian strategy, intermediate-term momentum strategy, and long-term contrarian strategy. Of these three stock trading strategies, abnormal return from the momentum strategy is most robust and therefore is the focus of our study. These three stock trading strategies all consist of a time line of three periods: formation period, holding period and post-holding period. The strategies select stocks on the basis of returns over the past K periods (formation period) and hold them for J periods (holding period).

2.1 Short-term contrarian strategy

The short-term contrarian strategy was first documented by Jegadeesh (1990) and Lehmann (1990). It is the strategy that ranks the stocks in the past K periods, which is typically a week or a month. Then construct the portfolio by buying the past worst performing stocks and selling the past best performing stocks, and hold it for another J periods, which is also a week or a month respectively.

2.2 Intermediate momentum strategy

First documented by Jegadeesh and Titman (1993), the momentum strategy selects stocks on the basis of returns over the past K periods (formation period) and holds them for J periods (holding period). The typical length for J and K are three to twelve months. Some studies also wait S periods between the formation and holding periods to avoid microstructure effects. This is denoted as the skip period. This paper, as many other studies, measures periods in months, so J, K and S are in months. To simplify, all the momentum strategies in this paper will be described as (K, S, J). To increase the testing power, the strategy includes overlapping holding periods. Therefore, in any given month *t*, the strategy holds a series of portfolios that are selected in the current month as well as in the previous K-1 months if there are no skip months.

In the formation period, the securities are ranked in descending order on the basis of their geometric returns over this period. The long portfolio or the "winners" consists of equally weighted top P percent securities. The short portfolio or the "losers" consists of equally weighted bottom P percent securities. In much of the literature, P is 10 percent. Some studies also use value weighted (measured by market capitalization) P percent securities.

This paper will focus on the (6,0,1) equally-weighted rolling strategy and the (6,0,6) equally-weighted nonrolling strategy.

2.3 Long-term contrarian strategy

DeBondt and Thaler (1985) first documented profits from the long-term contrarian strategy. Based on the stocks' past three- five year performance, the portfolio selects the winners and losers, and holds them for another three-five year period. Since the past losers continuously outperform the past winners, this contrarian strategy of buying the past losers and selling the past winners obtains positive profits consistently.

III. Motivation

In literature, there are two ways to address the sources of the returns from the momentum strategy. One line of literature tries to determine the sources of the momentum returns by return decomposition. Expected return decomposition is important because we can find out clearly and directly how the time-series and cross-sectional variations play in generating returns from the momentum strategy. The other line of literature tries to explain why the above components can generate momentum abnormal returns. If the researchers believe the cross-sectional variation is the cause to the momentum returns, then they belong to the rational explanation proponents. They try to discover risk factors that can fully absorb the abnormal returns from the momentum strategy. On the contrary, if the researchers believe the time-series variation is the cause to the momentum returns, then they are advocates of the behavioral finance. As a result, they try to use psychological theories to explain the autocorrelation of the stock returns in the momentum strategy.

This thesis belongs to the first line of literature and tries to decompose the momentum returns and find out the major contributors to the momentum strategy. Dislike the rational explanations that reject any possibility of stock return autocorrelations in generating momentum returns, or the behavioral explanations that attribute all the momentum returns to the stock return patterns, we hypothesize that both own stock return autocovariances and cross-sectional variances generate the returns from the momentum strategy. However, the focus resides in which component is the main contributor to the momentum returns. Our paper first decomposes the momentum expected returns and then uses historical data to calculate the relative weight of each component in generating the momentum returns.

Lehmann (1990) is the first attempt in literature decomposing the returns from the momentum strategy. The weight used in Lehmann (1990) is $W_{it-k} = -[R_{it-k} - \bar{R}_{t-k}]$, where $\bar{R}_{t-k} = \frac{1}{N}\sum_{i=1}^{N} R_{it-k}$ for the contrarian strategy. Built on Lehmann (1990), Lo and MacKinlay (1990) further advanced the return decomposition. They use the weight of $w_{it}(k) = -\frac{1}{N}\,(R_{it-k} - R_{mt-k})\; i = 1, \ldots, N$. All the later studies follow Lo and MacKinlay (1990) return decomposition, such as Conrad and Kaul (1998) and Lewellen (2002).

Our return decomposition in this thesis is based on Lo and MacKinlay (1990). However, unlike all the previous studies that include all stocks in the return decomposition, our weighting scheme only picks the top winners and bottom losers in the portfolio. Our strategy reflects the most common momentum strategy that has been analyzed in literature, in which only a proportion of stocks ranked as winners and losers are weighted in the strategy. This type of strategy also takes better advantage of potential stock return patterns if there is any. Top winners and bottom losers have more tendency to retain a more stable return pattern and thus only include those stocks can avoid stock return pattern noises from the intermediate portfolio stocks. Furthermore, this type of momentum strategy reflects investors' stronger belief in the stock return continuation, thus could generate more abnormal returns and pose a greater challenge to the efficient market hypothesis. More importantly, unlike the previous return decomposition that investigates the component from the whole portfolio; our weighting scheme provides the possibility of further investigating the components from the winner and loser portfolios separately. Since the recent literature has found that the winners and losers are quite different in characteristics and their contributions to the momentum abnormal returns are asymmetric, our separate investigation of the components in the winner and loser portfolios provides us an opportunity to discover the potential cause to this recent finding in literature. This is the first research that investigate the components in the winner and loser portfolios in return decomposition.

Our empirical results indicate that both the own stock return autocovariances and cross-sectional variances are the two major contributors to the momentum returns. However, the cross-autocovariances do not play such an important role in explaining the momentum returns as other papers propose.

More interestingly, even though the own-autocovariances of the winner and loser portfolios bear the same sign, their magnitudes are quite asymmetric. Compared to the winners, the losers have much more stable return pattern and hence much larger own stock autocovariances from the ranking period to the holding period. This provides another underlying cause to the recent finding that the losers, rather than the winners, are the driving force of the momentum abnormal returns.

All these results indicate that the market may not be as efficient as we believed before.

CHAPTER II
LITERATURE REVIEW

Lehmann (1990) has suggested market inefficiency due to stock price "overreaction". He constructed a contrarian strategy by buying the past *k* period losers and selling the past *k* period winners on a weekly basis. However, this zero cost strategy earns positive returns due to the phenomenon that the past winners tend to lose and past losers tend to win in the current period. Lehmann attributes this stock price predictability to stock price "overreaction" in the previous period. For a given set of N securities over a T time periods in the portfolio, at the beginning of period t, buy w_{it-k} dollars of each security i. The weights are given by

$$w_{it-k} = -[R_{it-k} - \bar{R}_{t-k}];\ \ \bar{R}_{t-k} = \frac{1}{N}\sum_{i=1}^{N} R_{it-k}.$$

The profits for the portfolio in period $t(\pi_{t,k})$ are

$$\pi_{t,k} = \sum_{i=1}^{N} w_{it-k}\, R_{it} = -\sum_{i=1}^{N}[R_{it-k} - \bar{R}_{t-k}][R_{it} - \bar{R}_{t}],$$

so that the average profit over the T periods on this portfolio strategy is

$$\bar{\pi}_{k} = \frac{1}{T}\sum_{t=1}^{T}\pi_{t,k} = -\frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}[R_{it-k} - \bar{R}_{t-k}][R_{it} - \bar{R}_{t}].$$

Algebraic manipulation of this expression yields

$$\bar{\pi}_{k} = \frac{N}{T}\sum_{t=1}^{T}[\bar{R}_{t-k} - \bar{\bar{R}}][\bar{R}_{t} - \bar{\bar{R}}] - \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}[R_{it-k} - \bar{R}_{i}][R_{it} - \bar{R}_{i}] - \sum_{i=1}^{N}[\bar{R}_{i} - \bar{\bar{R}}]^{2},$$

where

$$\bar{\bar{R}} = \frac{1}{T}\sum_{t=1}^{T}\bar{R}_{t}\ ;\ \bar{R}_{i} = \frac{1}{T}\sum_{t=1}^{T} R_{it}$$

are the average returns of the equally weighted portfolio and of security *i* overtime, respectively. Thus, average portfolio profits depend on the autocovariances of the returns of an equally weighted portfolio, the autocovariances of the returns of the individual securities, and the cross-sectional variation in the unconditional mean returns of the individual securities.

Jegadeesh (1990) presents another empirical evidence of predictability of individual stock returns on a monthly basis. He forms ten portfolios based on returns predicted using ex ante estimates of the regression parameters.

Let $\tilde{R}_{it} = E(R_i) + \tilde{\eta}_{it}$, where $E(R_i)$ is the unconditional expected return on security *i* and $\tilde{\eta}_{it}$ is the unexpected return in month *t*, in an unconditional sense. The cross-sectional regression model is

$$\tilde{R}_{it} = a_{0t} - \sum_{j=1}^{J} a_{jt}\, R_{it-j} + u_{it}.$$

Therefore, the slope coefficients in the multivariate regression above are

$$\begin{bmatrix} a_{1t} \\ \vdots \\ a_{Jt} \end{bmatrix} = \begin{bmatrix} cov_i \begin{Bmatrix} R_{it-1} \\ \vdots \\ R_{it-J} \end{Bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} cov_i(R_{it}, R_{it-1}) \\ \vdots \\ cov_i(R_{it,} R_{it-J}) \end{bmatrix}$$

where the component of the second term is

$$cov_i\left(R_{it}, R_{it-J}\right) = cov_i\left(\eta_{it}, \eta_{it-j}\right) + var_i\left(E(R_i)\right).$$

The covariance term has two components. The first component is the average time-series of individual security returns. The second component is the cross-sectional variance of unconditional expected returns.

Jegadeesh finds that the negative first-order serial correlation (reversal) in monthly stock returns is highly significant. So if a stock's price today is higher than its average price last month, it tends to drop back below its average price the next month. The contrarian strategies that select stocks based on their returns in the previous month by buying the losers and selling the winners can generate significant abnormal returns.

Lo and Mackinlay (1990) construct a particular weekly contrarian strategy and show that, despite weakly negative autocorrelation in individual stock returns, weekly portfolio returns are strongly positively autocorrelated and are the result of important cross-autocorrelations. Therefore, they argue that the returns of the large stocks lead those of smaller stocks, or the lead-lag effect is the drive of the contrarian profits, not the stock overreaction. The particular contrarian strategy is to buy stocks at time *t* that were losers at time *t-k* and to sell stocks at time *t* that were winners at time *t-k*, where winning and losing is determined with respect to the equal-weighted return on the market. Thus, the weight for security *i* at time *t* is,

$$w_{it}(k) = -\frac{1}{N}\left(R_{it-k} - R_{mt-k}\right) i = 1, \dots, N \tag{1}$$

where $R_{mt-k} = \sum_{i=1}^{N} \frac{R_{it-k}}{N}$ is the equal-weighted market index. By construction, $w_t(k) = [w_{1t}(k), w_{2t}(k) \dots w_{Nt}(k)]'$ is an arbitrage portfolio since the weights sum to zero. Since the portfolio weights are proportional to the differences between the market index and the returns, securities that deviate more positively from the market at time *t-k* will have greater negative weight in the time *t* portfolio and vice versa. Such a strategy is designed to best take advantage of stock market overreactions. The profit $\pi_t(k)$ from such a strategy is

$$\pi_t(k) = \sum_{i=1}^{N} w_{it}(k) R_{it} \tag{2}$$

Substituting (1) into (2), we get

$$\begin{aligned} \pi_t(k) &= -\frac{1}{N}\sum_{i=1}^{N}(R_{it-k} - R_{mt-k})R_{it} \\ &= -\frac{1}{N}\sum_{i=1}^{N} R_{it-k} R_{it} + \frac{1}{N}\sum_{i=1}^{N} R_{mt-k}R_{it} \\ &= -\frac{1}{N}\sum_{i=1}^{N} R_{it-k} R_{it} + R_{mt-k}R_{mt} \end{aligned} \tag{3}$$

Then taking the expectation of (3), we get

$$\begin{aligned} E[\pi_t(k)] &= -\frac{1}{N}\sum_{i=1}^{N} E[R_{it-k} R_{it}] + E[R_{mt-k}R_{mt}] \\ &= -\frac{1}{N}\sum_{i=1}^{N}(Cov[R_{it-k}, R_{it}] + \mu_i^2 + (Cov[R_{mt-k}, R_{mt}] + \mu_m^2) \end{aligned} \tag{4}$$

Assume stock return R_t is a jointly covariance-stationary stochastic process with expectation $E[R_t] = \mu = [\mu_1, \mu_2 \cdots \mu_N]'$ and autocovariance matrices

$$E[(R_{t-k} - \mu)(R_t - \mu)'] = \Gamma_k, \tag{5}$$

where, without loss of generality, $k \geq 0$ since $\Gamma_k = \Gamma'_{-k}$

Substituting (5) into (4), we get

$$E[\pi_t(k)] = -\frac{1}{N}\, tr(\Gamma_k) - \frac{1}{N}\sum_{i=1}^{N} \mu_i^2 + \frac{\iota'\Gamma_k\iota}{N^2} + \mu_m^2$$

$$E[\pi_t(k)] = \frac{\iota'\Gamma_k\iota}{N^2} - \frac{1}{N}\, tr(\Gamma_k) - \frac{1}{N}\sum_{i=1}^{N}(\mu_i - \mu_m)^2 \tag{6}$$

where $\mu_m = E[R_{mt}] = \frac{\mu'\iota}{N}$, $tr(\cdot)$ denotes the trace operator, and ι is the identity vector with proper dimension.

Therefore, the profit of the contrarian strategy is the summation of three terms: the first term is the k^{th}-order autocovariance of the equal-weighted market index. The second term is the cross-sectional average of the k^{th}-order autocovariances of the individual securities, and the last term is the cross-sectional variance of the mean returns. Since both the first and the second terms are dependent on Γ_k, and k, they are defined as the profitability index

$$L_k = L(\Gamma_k) = \frac{\iota'\Gamma_k\iota}{N^2} - \frac{1}{N}\, tr(\Gamma_k). \tag{7}$$

The last term is constant, and is defined as

$$\sigma^2(\mu) = \frac{1}{N}\sum_{i=1}^{N}(\mu_i - \mu_m)^2.$$

Thus, the profit of the strategy is

$$E[\pi_t(k)] = L_k - \sigma^2(\mu).$$

Now, rearrange and rewrite (7) as

$$L_k = C_k + O_k$$

where $C_k = \frac{1}{N^2}\left[\iota'\Gamma_k\iota - tr(\Gamma_k)\right]$, $O_k = -\left(\frac{N-1}{N^2}\right)tr(\Gamma_k)$.

Hence, $E[\pi_t(k)] = C_k + O_k - \sigma^2(\mu)$. (8)

Written in this way, the expected profits of the contrarian strategy can be decomposed into three terms: one (C_k) depending on only the off-diagonals of the auto-covariance matrixΓ_k, the second (O_k) depending on only the diagonals, and a third $[\sigma^2(\mu)]$ that is independent of the auto-covariances. This decomposition can separate the fraction of expected profits due to the cross-autocovarainces C_k versus the own-autocovariances O_k of returns.

Equation (8) implies that the profitability of the contrarian strategy may be consistent with a positively autocorrelated market index and negatively autocorrelated individual security returns. Conversely, the empirical finding that equal-weighted indexes are strongly positively autocorrelated while individual security returns are weakly negatively autocorrelated implies that there must be significant positive cross-autocorrelations across securities. Therefore, the positive autocorrelation in weekly returns may be attributed primarily to the positive cross-autocorrelations across securities. Stock market overreaction need not be the reason that contrarian investment strategies are profitable. Alternatively, negatively autocorrelated individual returns enhance the profitability of the return-reversal strategy, but it is not required for such a strategy to earn positive expected returns.

Lo and Mackinlay (1990) also discuss the profitability of this particular contrarian strategy under different assumptions. Assume that returns R_t be both cross-sectional and serially independent. In this case, $\Gamma_k = 0$ for all nonzero *k*; hence

$$L_k = C_k = O_k = 0 \qquad E[\pi_t(k)] = -\sigma^2(\mu) \leq 0.$$

The expected profits are negative as long as there is some cross-sectional variation in expected returns. However, since $\sigma^2(\mu)$ is generally small and does not depend on the autocovariance structure of R_t, the paper focuses on L_k and ignores $\sigma^2(\mu)$ for the later studies.

If the individual stock returns are only negatively own-autocorrelated but not cross-autocorrelated with other stocks over some holding period, then

$$\Gamma_k = \begin{bmatrix} \gamma_{11}(k) & 0 & \cdots & 0 \\ 0 & \gamma_{22}(k) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_{NN}(k) \end{bmatrix}.$$

The profitability index under these assumptions for R_t is then

$$L_k = O_k = -\left(\frac{N-1}{N^2}\right) tr(\Gamma_k)$$

$$= -\left(\frac{N-1}{N^2}\right) \sum_{i=1}^{N} \gamma_{ii}(k) > 0 \quad (9)$$

The positivity of L_k follows from the negativity of the own-autocovariances, assuming N>1. Not surprisingly, if stock markets do overreact, this contrarian strategy is profitable on average.

Now, let us assume log-price X_{it} of each security *i* be given by

$$X_{it} = Y_{it} + Z_{it}$$

where, $Y_{it} = \mu_{it} + Y_{it-1} + \varepsilon_{it}$, which is random walk process, and

$Z_{it} = \rho_i Z_{it-1} + v_{it}, 0 < \rho < 1$, which is AR(1) process,

and the disturbances $\{\varepsilon_{it}\}$ and $\{v_{it}\}$ are serially, mutually, and cross-sectionally independent at all nonzero leads and lags. The k^{th}-order autocovariance for the return vector R_t is then given by the following diagonal matrix:

$$\Gamma_k = \begin{bmatrix} -\rho_1^{k-1}\left(\frac{1-\rho_1}{1+\rho_1}\right)\sigma_{v_1}^2 & \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & -\rho_N^{k-1}\left(\frac{1-\rho_N}{1+\rho_N}\right)\sigma_{v_N}^2 \end{bmatrix} \quad (10)$$

The profitability index

$$\mathrm{L}_k = O_k = -\left(\frac{N-1}{N^2}\right) tr(\Gamma_k) = \frac{N-1}{N^2} \sum_{i=1}^{N} \rho_i^{k-1} \left(\frac{1-\rho_i}{1+\rho_i}\right) \sigma_{v_i}^2 > 0$$

Since the own-autocovariances in Equation (16) are all negative, therefore may be interpreted as an example of stock market overreaction. However, the fact that the returns are negatively autocorrelated at all lags is an artifact of the first-order autoregressive process and need not be true for the sum of a random walk and a general stationary process.

However, even when stock returns follow white noise, with lead-lag relations, the contrarian strategy can also produce positive profits. The lead-lag relations are the dependence of the i^{th} security's return on a lagged common factor. Consequently, the returns to security 1 leads that of securities 2, 3, etc.; the return to security 2 leads that of securities 3, 4, etc.; and so on. But the current return to security 2 provides no information for future returns to security 1, and so on.

Let the return-generating process for R_t be given by

$$R_{it} = \mu_i + \beta_i \Lambda_{t-i} + \varepsilon_{it}, \beta_i > 0, i = 1, \dots, N \quad (11)$$

where Λ_t is a serially independent common factor with zero mean and variance σ_λ^2, and ε_{it}'s are assumed to be both cross-sectionally and serially independent. These assumptions imply that for each security *i*, its returns are white noise (with drift). This serial independence is not consistent with either the spirit or form of the stock market overreaction hypothesis. However, it is still possible to predict *i*'s returns using past returns of security *j*, where $j<i$ from the lead-lag relations. When $k < N$, the auocovariance matrix Γ_k has zeros in all entries except along the k^{th} superdiagonal, for which

$$\gamma_{ii+k} = \sigma_\lambda^2 \beta_i \beta_{i+k}$$

Therefore, the lead-lag model yields an asymmetric autocovariance matrix Γ_k and the profitability index is then

$$L_k = C_k = \frac{\sigma_\lambda^2}{N^2}\sum_{i=1}^{N-k}\beta_i\,\beta_{i+k} > 0$$

This example highlights the importance of the cross effects---although each security is individually unpredictable, a contrarian strategy may still profit if securities are positively cross-correlated at various leads and lags.

Lo and Mackinlay further study nontrading as the sole source of autocorrelation, given the same return generating process as Equation (11). In each period *t*, there is some chance that the stock i does not trade with probability p_i. If it does not trade, its observed return for period *t* is simply 0, although its true or virtual return R_{it} is still given by Equation (11). In the next period $t+1$, there is again some chance that security *i* does not trade, also with probability p_i. The probability of trading in time *t* is assumed to have no influence on the likelihood of the stock's trading in period $t+1$. If security *i* does trade in period $t+1$ and did not trade in period *t*, then its observed return R_{it+1}^oat $t+1$ is the sum of its virtual returns$R_{it+1}$and R_{it}. In fact, the observed return in any period is simply the sum of its virtual returns for all past consecutive periods in which it did not trade. This captures the essential feature of nontrading as a source of spurious autocorrelation: News affects those more frequently traded stocks first and those thinly traded stocks with a lag.

More formally, the observed returns process can be written as the following weighted average of past virtual returns:

$$R_{it}^o = \sum_{k=0}^{\infty} X_{it}(k) R_{it-k}, i = 1, \dots, N$$

where the weight $X_{it}(k)$ are defined as products of no-trade indicators:

$$X_{it}(k) = (1-\delta_{it})\delta_{it-1}\delta_{it-2}\cdots\delta_{it-k}$$

$$= \begin{cases} 1, & with\ probability\ (1-p_i)p_i^k \\ 0, & with\ probability\ 1-(1-p_i)p_i^k \end{cases}$$

for $k > 0, X_{it}(0) = 1 - \delta_{it}$, and where the δ_{it}'s are independently and identically distributed Bernoulli random variables that take on the value 1 when security *i* does not trade at time *t*, and 0 otherwise. The variable $X_{it}(k)$ is also an indicator variable, and takes on the value 1 if security *i* trades at time *t* but not in any of the *k* previous periods, takes on the value 0 otherwise.

The securities with common nontrading probability p_κ are grouped into equal-weighted portfolios. Then the observed return to portfolio κ may be approximated as

$$R_{\kappa t}^o \backsimeq \mu_\kappa + (1-p_\kappa)\beta_\kappa \sum_{\kappa=0}^{\infty} p_\kappa^k \Lambda_{t-k}$$

where the approximation becomes exact as the number of securities in the portfolio approaches infinity and where β_κ is the average beta of the securities in the portfolio. Let $R_{a\tau}^o(q)$ and $R_{b\tau}^o(q)$ be the time-aggregated observed returns of two arbitrary portfolios *a* and *b* over *q* periods. The ratio of the cross-autocorrelation coefficients are calculated as

$$\frac{\rho_{ab}^q(k)}{\rho_{ba}^q(k)} = \left[\left(\frac{1-p_b^q}{1-p_a^q}\right)\left(\frac{1-p_b}{1-p_a}\right)\right]^2 \left(\frac{p_b}{p_a}\right)^{kq-q+1} \gtreqless 1\ as\ p_b \gtreqless p_a \qquad (12)$$

Equation (12) shows that if securities in portfolio *b* trade less frequently than those in portfolio *a*, then the correlation between today's return on *a* and tomorrow's return on *b* exceeds the correlation between today's return on *b* and tomorrow's return on *a*. Therefore, portfolios with higher nontrading probabilities tend to lag those with lower nontrading probabilities.

Lo and Mackinlay later construct a plausible return-generating mechanism that consists of three components: a positively autocorrelated common factor, idiosyncratic white noise, and a bid-ask spread process.

$$R_{it} = \mu_i + \beta_i \Lambda_t + \eta_{it} + \varepsilon_{it}$$

where $E[\Lambda_t] = 0, \quad E[\Lambda_{t-k}\Lambda_t] = \gamma_\lambda(k) > 0$

$$E[\varepsilon_{it}] = E[\eta_{it}] = 0 \quad \forall i,$$

$$E[\varepsilon_{it-k}\varepsilon_{jt}] = \begin{cases} \sigma_i^2 & if\ k = 0\ and\ i = j \\ 0 & otherwirse \end{cases}$$

$$E[\eta_{it-k}\eta_{jt}] = \begin{cases} -\frac{s_i^2}{4} & if\ k = 0\ and\ i = j \\ 0 & otherwise \end{cases} \tag{13}$$

Implicit in Equation (13) is Roll's (1984) model of the bid-ask spread, in which the first-order autocorrelation of η_{it} is the negative of one-fourth the square of the percentage bid-ask spread s_i, and all higher-order autocorrelations and all cross-correlations are zero.

The autocovariance matrices for Equation (13) are given by

$$\Gamma_1 = \gamma_\lambda(1)\beta\beta' - \frac{1}{4} diag[s_1^2, s_2^2, \ldots s_N^2]$$

$$\Gamma_k = \gamma_\lambda(k)\beta\beta', \quad k > 1$$

where $\beta = [\beta_1\ \beta_2 \cdots \beta_N]'$.

Let $\beta_m = \sum_{i=1}^{N} \frac{\beta_i}{N}$, then the profitability index is given by

$$L_1 = -\frac{\gamma_\lambda(1)}{N} \sum_{i=1}^{N} (\beta_i - \beta_m)^2 + \frac{N-1}{N^2} \sum_{i=1}^{N} \frac{s_i^2}{4} \tag{14}$$

$$L_k = -\frac{\gamma_\lambda(k)}{N} \sum_{i=1}^{N} (\beta_i - \beta_m)^2 \quad k > 1 \tag{15}$$

Equation (14) shows that if the bid-ask spreads are large enough and the cross-sectional variation of the $\beta_k' s$ is small enough, the contrarian strategy may yield positive expected profits when using only one lag in computing portfolio weights. This positivity of the profitability index is due primarily to the negative autocorrelations of individual security returns induced by the bid-ask spread. Since $\gamma_\lambda(k) > 0$ by assumption, expected profits are negative for lags higher than 1.

Conrad and Kaul (1998) attempt to determine the sources of the expected profits of the entire class of trading strategies that are based on information contained in past returns of individual securities. They utilize a single framework, which uses the model in Lo and MacKinlay (1990), to decompose the profits of all strategies, both contrarian and momentum. The empirical decomposition of the profits of the strategies suggests that the cross-sectional variation in mean returns of individual securities included in the strategy is an important determinant of their profitability. The expected profit of the momentum strategy is

$$E[\pi_t(k)] = -Cov[R_{mt}(k), R_{mt-1}(k)] + \frac{1}{N}\sum_{i=1}^{N} Cov[R_{it}(k), R_{it-1}(k)] + \frac{1}{N}\sum_{i=1}^{N}[\mu_{it-1}(k) - \mu_{mt-1}(k)]^2$$

$$= -C_1(k) + O_1(k) + \sigma^2[\mu(k)]$$

$$= P(k) + \sigma^2[\mu(k)] \qquad (16)$$

where $P(k) = -C_1(k) + O_1(k)$ is the predictability-profitability index, $\mu_{it}(k)$ is the unconditional mean of security *i* for the interval {t-1, t} of length *k*, and $\mu_{mt}(k) = \frac{1}{N}\sum_{i=1}^{N}\mu_i(k)$ is the unconditional single-period mean return of the equal-weighted market portfolio at time *t*.

Under the assumption of mean stationarity of individual security returns, the above decomposition shows that total expected profits of trading strategies result from two distinct sources: time-series predictability in asset returns, measured by *P(k)*, and profits due to cross-sectional dispersion in mean returns of securities, denoted by $\sigma^2[\mu(k)]$. The first term in *P(k)*, i.e. $C_1(k)$, is the average of the first-order autocovariance of the return on the equal-weighted market portfolio, the second term, i.e., $O_1(k)$, is the average of first-order autocovariances of the *N* individual securities included in the portfolio.

Conrad and Kaul (1998) also use random walk as the return generating process of individual stocks. $R_{it}(k) = \mu_i(k) + \eta_{it}(k) \quad i = 1, \dots, N$ (17)

where $[\eta_{it}(k)] = 0 \; \forall \; i, k$, and $E[\eta_{it}(k)\eta_{jt-1}(k)] = 0 \; \forall \; i, j, k.$

Equation (17) implies that there is no return predictability in either individual securities or across different securities, therefore, the very basis of return-based trading strategies is ruled out. More importantly, further Conrad and Kaul show that combining Equation (16) and (17), the expected profit of the momentum strategy becomes$E[\pi_t(k)] = \sigma^2[\mu(k)]$. Therefore, momentum strategy is profitable even if asset returns are completely unpredictable. Conversely, contrarian strategies will generate losses of an equal amount.

However, Jegadeesh and Titman (2002) argue that the Conrad and Kaul (1998) results are subject to small sample biases in their tests and bootstrap experiments. Their unbiased empirical tests indicate that cross-sectional differences in expected returns explain very little, if any, of the momentum profits. Conrad and Kaul use the average realized return of each stock as its measure of the stock's expected return. Specifically,$\hat{\mu}_i = \frac{1}{T_i}\sum_{t=1}^{T_i} R_{i,t}$, where T_i is the number of observations available for stock i. They use the cross-sectional variance of $\hat{\mu}_i$ as the estimator of σ_μ^2. Jegadeesh and Titman argue that such design ignores the impact of the error in the estimates of $\hat{\mu}_i$ on the estimate of σ_μ^2. Let $\hat{\mu}_i = \mu_i + \varepsilon_i$, where ε_i represents estimation error. Since $\hat{\mu}_i$ is an unbiased estimator of expected returns, $E(\varepsilon_i) = 0$. However, since$\sigma_{\hat{\mu}_i}^2 = \sigma_{\mu_i}^2 + \sigma_{\varepsilon_i}^2$, the variance of the estimated expected returns overestimates the cross-sectional variance of true expected returns. They argue that the magnitude of this overestimation is exacerbated when following Conrad and Kaul (1998) and using all stocks in the sample period for the calculation of expected returns, regardless of the length of their return history.

CHAPTER III
DECOMPOSITION OF MOMENTUM RETURNS

I. Theoretical Model

To show the relative role of the cross-sectional and time-series effects in generating momentum returns, we decompose the momentum expected returns first and then discuss their profitability under different return generating processes.

Following Lehmann (1990) and Lo and Mackinlay (1990), we also use a weighted relative strength strategy (WRSS) to decompose the returns from momentum strategy. However, different from the previous literature that takes all the stocks with returns higher than the market return as winners and all the stocks with returns lower than market returns as losers, our decomposition follows the typical momentum strategy that have been analyzed most and only includes the top and bottom percentage stocks as winners and losers.

Consider a collection of N securities and denote their period *t* returns R_t a $N \times 1$ vector$[R_{it,} \cdots, R_{Nt}]'$. As Lo and Mackinlay (1990), we also assume in this section that:

Assumption 1: R_t follows a jointly covariance-stationary stochastic process with expected value $E[R_t] = \mu \equiv [\mu_1, \mu_2, \cdots, \mu_N]'$ and autocovariance matrices$E[(R_{t-1}(k) - \mu)(R_t(k) - \mu)'] = \Gamma(k)$, where $k \geq 0$, since$\Gamma(k) = \Gamma'(-k)$.

Specifically, momentum strategy buys winners and sells losers at time t, based on their performance in the time period$\{t-1, t\}$, where *k* is the length of the time interval$\{t-1, t\}$. The winning and losing outcomes are determined with respect to the equal-weighted return on the whole market. Now, first rank the stocks in descending order by their geometric mean returns over the $\{t-1,t\}$ period, i.e.,$R_1 \geq R_2 \geq \cdots \geq R_{SN} \cdots \geq R_N$, where S is the top or bottom percentage of stocks, where $0 < S < \frac{1}{2}$. Hence, top SN stocks are winners and bottom SN stocks are losers. More formally, let $w_{it}\,(k)$ denote the fraction of the trading strategy portfolio devoted to security i at time t, that is

$$w_{it}\,(k) = \begin{cases} \frac{\alpha}{SN}\left[R_{it-1}(k) - R_{mt-1}(k)\right] & if\ i = 1,2,\ldots,SN \\ 0 & if\ i = SN+1,\ldots,N-SN \\ \frac{\beta}{SN}\left[R_{it-1}(k) - R_{mt-1}(k)\right] & if\ i = N-SN+1,\ldots,N \end{cases} \quad (18)$$

where $\alpha > 0, \beta > 0$ are parameters of the weights of the winner and loser portfolios, $R_{it-1}(k)$ is the geometric mean return of security i in the time interval $\{t-1,t\}$, $R_{mt-1}(k) = \frac{\sum_{i=1}^{N} R_{it-1}(k)}{N}$ is the return of equal-weighted portfolio of all securities in the time interval $\{t-1,t\}$, and k is the length of the time interval $\{t-1,t\}$.

The weighting mechanism reflects an investor's belief that price has continuations and the success of this strategy is solely based on the time-series behavior of stock prices. This weighting mechanism allows us to decompose the returns of momentum strategy into time-series and cross-sectional variations. It also permits us to determine the relative importance of these components in generating momentum returns and answer the frequently argued question of whether the market is efficient or whether the stock prices have memories. More importantly, securities that deviate more positively (negatively) from the market mean in the time period $\{t-1,t\}$ will have greater positive (negative) weight in the time t portfolio. By taking only top and bottom S percentage of stocks in our momentum strategy, rather than all stocks, takes better advantage of stock price continuations. Because the best winners have more momentum to continue winning, and worst losers have more momentum to continue losing over an intermediate time frame.

The returns from such a strategy are simply

$$\pi_t(k) = \sum_{i=0}^{N} w_{it}(k)R_{it}. \quad (19)$$

Plugging the weight function (18) into (19) and taking expectations yields the following:

$$\begin{aligned}
E[\pi_t(k)] &= \frac{\alpha}{SN}\sum_{i=1}^{SN} E[(R_{it-1}(k) - R_{mt-1}(k))R_{it}] + \frac{\beta}{SN}\sum_{i=N-SN+1}^{N} E[(R_{it-1}(k) \\
&\quad - R_{mt-1}(k))R_{it}] \\
&= \frac{\alpha}{SN}\sum_{i=1}^{SN} E[R_{it-1}(k)R_{it}] + \frac{\beta}{SN}\sum_{i=N-SN+1}^{N} E[R_{it-1}(k)R_{it}] - \alpha E[R_{mt-1}(k)R_{wt}] \\
&\quad -\beta E[R_{mt-1}(k)R_{lt}] \\
&= \frac{\alpha}{SN}\sum_{i=1}^{SN}(Cov[R_{it-1}(k), R_{it}] + \mu_i^2) + \frac{\beta}{SN}\sum_{i=N-SN+1}^{N}(Cov[R_{it-1}(k), R_{it}] + \mu_i^2) \\
&\quad -\alpha E[R_{mt-1}(k)R_{wt}] - \beta E[R_{mt-1}(k)R_{lt}] \\
&= \frac{\alpha}{SN} tr(\Gamma_k^w) + \frac{\beta}{SN} tr(\Gamma_k^l) + \frac{\alpha}{SN}\sum_{i=1}^{SN}\mu_i^2 + \frac{\beta}{SN}\sum_{i=N-SN+1}^{N}\mu_i^2 - \alpha E[R_{mt-1}(k)R_{wt}] \\
&\quad -\beta E[R_{mt-1}(k)R_{lt}]. \qquad (20)
\end{aligned}$$

where tr($\cdot$) denotes the trace operator, Γ_k^l are the autocovariance matrix for the loser portfolio, and Γ_k^w are the autocovariance matrix for the winner portfolio. $R_{wt} = \frac{1}{SN}\sum_1^{SN} R_{it}$ is the average return of the winner portfolio at time *t*, $R_{lt} = \frac{1}{SN}\sum_{N-SN+1}^{N} R_i$ is the average return of the loser portfolio at time *t*.

Since $E[R_{mt-1}(k)R_{wt}] = E[\frac{(\sum_{i=1}^{SN} R_{it-1}(k) + \sum_{i=SN+1}^{N} R_{it-1}(k))\sum_{i=1}^{SN} R_{it}}{N*SN}]$

$$= SE\left[\frac{\sum_{i=1}^{SN} R_{it-1}(k)\sum_{i=1}^{SN} R_{it}}{S^2N^2}\right] + (1-S)E[\frac{\sum_{i=SN+1}^{N} R_{it-1}(k)\sum_{i=1}^{SN} R_{it}}{(N-SN)*SN}]$$

$$= SE[R_{wt-1}(k)R_{wt}] + (1-S)E[R_{\overline{w}t-1}(k)R_{wt}]$$

$$= S[Cov(R_{wt-1}(k), R_{wt}) + \mu_w^2] + (1-S)[Cov(R_{\overline{w}t-1}(k), R_{wt})$$

$$+\mu_{\overline{w}}\mu_w]$$

$$= S\frac{\iota'\Gamma_k^w\iota}{(SN)^2} + S\mu_w^2 + (1-S)\frac{\iota'\Gamma_k^{\overline{w}w}\iota}{(N-SN)SN} + (1-S)\mu_{\overline{w}}\mu_w$$

$$= \frac{\iota'\Gamma_k^w\iota}{SN^2} + S\mu_w^2 + \frac{\iota'\Gamma_k^{\overline{w}w}\iota}{SN^2} + (1-S)\mu_{\overline{w}}\mu_w, \quad (21)$$

where $\mu_w = \frac{\sum_{i=1}^{SN}\mu_i}{SN}$ is the average expected return of the winner portfolio, $\mu_{\overline{w}} = \frac{\sum_{i=SN+1}^{N}\mu_i}{N-SN}$, is the average expected return of the nonwinner portfolio, $\Gamma_k^{\overline{w}w}$ are the autocovariance matrix for the interaction of past nonwinners and winners, and ι is the identity matrix of corresponding dimension, for example, $\iota'\Gamma_k^w\iota = \sum\Gamma_{ij}^w$.

Similarly, $E[R_{mt-1}(k)R_{lt}] = \frac{\iota'\Gamma_k^l\iota}{SN^2} + S\mu_l^2 + \frac{\iota'\Gamma_k^{\bar{l}l}\iota}{SN^2} + (1-S)\mu_{\bar{l}}\mu_l \quad (22)$

where $\mu_l = \frac{\sum_{i=N-SN+1}^{N}\mu_i}{SN}$ is the average expected return of the loser portfolio, $\mu_{\bar{l}} = \frac{\sum_{i=1}^{N-SN}\mu_i}{N-SN}$, is the average expected return of the nonloser portfolio, $\Gamma_k^{\bar{l}l}$are the autocovariance matrix for the interaction of past nonlosers and losers.

Combining Equation (20)-(22), we get:

$$E[\pi_t(k)] = \frac{\alpha}{SN}tr(\Gamma_k^w) + \frac{\beta}{SN}tr(\Gamma_k^l) + \frac{\alpha}{SN}\sum_{i=1}^{SN}\mu_i^2 + \frac{\beta}{SN}\sum_{i=N-SN+1}^{N}\mu_i^2 - \alpha[\frac{\iota'\Gamma_k^w\iota}{SN^2} + S\mu_w^2 + \frac{\iota'\Gamma_k^{\overline{w}w}\iota}{SN^2}$$

$$+(1-S)\mu_{\overline{w}}\mu_w] - \beta[\frac{\iota'\Gamma_k^l\iota}{SN^2} + S\mu_l^2 + \frac{\iota'\Gamma_k^{\bar{l}l}\iota}{SN^2} + (1-S)\mu_{\bar{l}}\mu_l]$$

$$= \frac{\alpha}{SN}tr(\Gamma_k^w) + \frac{\beta}{SN}tr(\Gamma_k^l) - \left[\alpha\frac{(\iota'\Gamma_k^w\iota + \iota'\Gamma_k^{\overline{w}w}\iota)}{SN^2} + \beta\frac{(\iota'\Gamma_k^l\iota + \iota'\Gamma_k^{\bar{l}l}\iota)}{SN^2}\right] + \frac{\alpha}{SN}\sum_{i=1}^{SN}\mu_i^2$$

$$+ \frac{\beta}{SN}\sum_{i=N-SN+1}^{N}\mu_i^2 - S(\alpha\mu_w^2 + \beta\mu_l^2) - (1-S)(\alpha\mu_{\overline{w}}\mu_w + \beta\mu_{\bar{l}}\mu_l) \quad (24)$$

Since $\frac{\alpha}{SN}\sum_{i=1}^{SN}\mu_i^2 - S\alpha\mu_w^2 = \alpha\left[\frac{\sum_{i=1}^{SN}\mu_i^2}{SN} - \mu_w^2\right] + \alpha\mu_w^2 - \alpha S\mu_w^2$

$$= \alpha\left[\frac{1}{SN}\sum_{i=1}^{SN}(\mu_i - \mu_w)^2\right] + \alpha(1-S)\mu_w^2$$

$$= \alpha\sigma^2(\mu^w) + \alpha(1-S)\mu_w^2 \tag{25}$$

Similarly, $\frac{\beta}{SN}\sum_{i=N-SN+1}^{N}\mu_i^2 - S\beta\mu_l^2 = \beta\left[\frac{1}{SN}\sum_{i=N-SN+1}^{N}(\mu_i - \mu_l)^2\right] + \beta(1-S)\mu_l^2$

$$= \beta\sigma^2(\mu^l) + \beta(1-S)\mu_l^2 \tag{26}$$

Therefore, Equation (24) becomes:

$$E[\pi_t(k)] = \frac{\alpha}{SN}tr(\Gamma_k^w) + \frac{\beta}{SN}tr\left(\Gamma_k^l\right) - \left[\alpha\frac{(\iota'\Gamma_k^w\iota + \iota'\Gamma_k^{\bar{w}w}\iota)}{SN^2} + \beta\frac{\left(\iota'\Gamma_k^l\iota + \iota'\Gamma_k^{\bar{l}l}\iota\right)}{SN^2}\right] + \alpha\sigma^2(\mu^w) + \beta\sigma^2(\mu^l)$$

$$+\alpha(1-S)\mu_w^2 + \beta(1-S)\mu_l^2 - (1-S)(\alpha\mu_{\bar{w}}\mu_w + \beta\mu_{\bar{l}}\mu_l). \tag{27}$$

The first two terms in Equation (27) are the cross-sectional average of the weighted first-order time-series variance of the individual stock returns in the winner and loser portfolios. If the stock returns have momentum or continuation, then the first two terms are positive. If stock returns have the reverse pattern then the first two items are negative. If the market is efficient then these two terms should be equal to zero. The third and fourth terms are the average first-order autocovariance between two stocks involving a winner or loser stock and another stock. If the stocks have lead lag structure that the larger firm leads the small firm in responding to a specific risk but in the same direction, then the cross-autocovariance is positive. The fifth and sixth terms are cross-sectional variances of the mean returns in the winner and loser portfolios. The more variation of the mean returns in the winner and loser portfolio, the larger the fifth and sixth items. The rest of the items are the summation of weighted products of expected returns. The fifth to ninth items are independent of the autocovariances Γ_k. In order to measure the role of the own-autocovariances, cross-autocovariances, and cross-sectional variances separately, we further arrange the terms in Equation (27) so that we decompose the momentum returns into different parts indicated above:

$$E[\pi_t(k)] = -\frac{\alpha}{SN^2}[\iota'\Gamma_k^w\iota - tr(\Gamma_k^w) + (1-N)tr(\Gamma_k^w) + \iota'\Gamma_k^{\bar{w}w}\iota] - \frac{\beta}{SN^2}[\iota'\Gamma_k^l\iota - tr(\Gamma_k^l)$$

$$+(1-N)tr(\Gamma_k^l) + \iota'\Gamma_k^{\bar{l}l}\iota] + \alpha\sigma^2(\mu^w) + \beta\sigma^2(\mu^l) + \alpha(1-S)\mu_w^2 + \beta(1-S)\mu_l^2$$

$$-(1-S)(\alpha\mu_{\bar{w}}\mu_w + \beta\mu_{\bar{l}}\mu_l)$$

$$= -\frac{\alpha}{SN^2}[\iota'\Gamma_k^w\iota - tr(\Gamma_k^w) + \iota'\Gamma_k^{\bar{w}w}\iota] - \frac{\beta}{SN^2}\left[\iota'\Gamma_k^l\iota - tr(\Gamma_k^l) + \iota'\Gamma_k^{\bar{l}l}\iota\right]$$

$$+\frac{\alpha}{SN^2}(N-1)tr(\Gamma_k^w) + \frac{\beta}{SN^2}(N-1)tr(\Gamma_k^l) + \alpha\sigma^2(\mu^w) + \beta\sigma^2(\mu^l) + \alpha(1-S)\mu_w^2$$

$$+\beta(1-S)\mu_l^2 - (1-S)(\alpha\mu_{\bar{w}}\mu_w + \beta\mu_{\bar{l}}\mu_l) \quad (28)$$

We define $C_k = -\frac{\alpha}{SN^2}[\iota'\Gamma_k^w\iota - tr(\Gamma_k^w) + \iota'\Gamma_k^{\bar{w}w}\iota] - \frac{\beta}{SN^2}\left[\iota'\Gamma_k^l\iota - tr(\Gamma_k^l) + \iota'\Gamma_k^{\bar{l}l}\iota\right]$, as the cross-autocovariance, $O_k = \frac{\alpha}{SN^2}(N-1)tr(\Gamma_k^w) + \frac{\beta}{SN^2}(N-1)tr(\Gamma_k^l)$, is the own-autocovariance; and $V_k = \alpha\sigma^2(\mu^w) + \beta\sigma^2(\mu^l)$, is the cross-sectional variance. Thus, the expected returns of the momentum strategy can be written as

$$E[\pi_t(k)] = C_k + O_k + V_k + \alpha(1-S)\mu_w^2 + \beta(1-S)\mu_l^2 - (1-S)(\alpha\mu_{\bar{w}}\mu_w + \beta\mu_{\bar{l}}\mu_l) \quad (29)$$

Since $\mu_{\bar{w}} = \frac{\sum_{i=SN+1}^{N}\mu_i}{N-SN} = \frac{E[\sum_{i=1}^{N}R_i] - E[\sum_{i=1}^{SN}R_i]}{N-SN} = \frac{1}{1-S}\mu_m - S\frac{E[\sum_{i=1}^{SN}R_i]}{SN(1-S)} = \frac{1}{1-S}\mu_m - \frac{S}{1-S}\mu_w$,

$$\alpha(1-S)\mu_w^2 - \alpha(1-S)\mu_{\bar{w}}\mu_w = \alpha(1-S)\left[\mu_w^2 - \left(\frac{1}{1-S}\mu_m - \frac{S}{1-S}\mu_w\right)\mu_w\right]$$

$$= \alpha(1-S)\left(\frac{1}{1-S}\mu_w^2 - \frac{1}{1-S}\mu_m\mu_w\right) = \alpha\mu_w(\mu_w - \mu_m) \quad (30)$$

Similarly $\mu_{\bar{l}} = \frac{\sum_{i=N-SN+1}^{N}\mu_i}{N-SN} = \frac{E[\sum_{i=1}^{N}R_i] - E[\sum_{i=N-SN+1}^{N}R_i]}{N-SN} = \frac{1}{1-S}\mu_m - S\frac{E[\sum_{i=N-SN+1}^{N}R_i]}{SN(1-S)} = \frac{1}{1-S}\mu_m$

$-\frac{S}{1-S}\mu_l$, and $\beta(1-S)\mu_l^2 - \beta(1-S)\mu_{\bar{l}}\mu_l = \beta\mu_l(\mu_l - \mu_m)$. (31)

Let $L_k = \alpha\mu_w(\mu_w - \mu_m) + \beta\mu_l(\mu_l - \mu_m)$. Therefore,

$$E[\pi_t(k)] = C_k + O_k + V_k + \alpha\mu_w(\mu_w - \mu_m) + \beta\mu_l(\mu_l - \mu_m)$$

$$= C_k + O_k + V_k + L_k \tag{32}$$

Equation (32) shows clearly that the momentum returns can be decomposed into four parts: a) C_k depending on only off-diagonals of the autocovariance matrixΓ_k, which is the correlation between returns of two different stocks from two different time periods; b) O_k depending on only the diagonals of autocovariance matrixΓ_k, which is the correlation of own stock returns from two different time periods; c) V_k is independent of the autocovariance matrix Γ_k, which is the cross-sectional variances of the mean returns in the winner and loser portfolios for a given time period; d) L_k is also independent of the autocovraiance matrix Γ_k, which is the weighted product of winner portfolio mean return and its deviation from the mean return of the whole portfolio plus the similar weighted product from the loser portfolio.

Equation (32) also indicates the scenarios that the expected returns from the momentum strategy become positive. Since α, β, s are positive, V_k is always positive. The total number of stocks is greater than 1, so if the summation of the own-autocovariances of the stock returns in the winner and loser portfolios is positive, i.e., there is momentum in the stock returns from the winner and loser stocks, then O_k is positive. If the correlations of two different stocks from two different times are positive, then C_k is negative. L_k is positive if the expected return of the winner portfolio is positive and higher than market expected return, at the same time, the expected return of the loser portfolio is below zero, and lower than the market expected return.

II. Model Comparison

The major difference between our model and the model in Lo and MacKinlay (1990) is the weighting scheme. Instead of including all the stocks in the portfolio, our momentum strategy only takes the top winners and bottom losers, and thus, makes better use of price momentum than that of the Lo and MacKinlay (1990) model. Furthermore, our model provides the opportunity of looking into the detailed difference between the winner and loser portfolios. Thus, we can investigate the causes to the important finding in the recent literature that the contribution of the winner and loser portfolios to the momentum strategy is asymmetric, through the return decomposition. Now, we look into both models and check the difference of the expected returns with the two different weighting schemes.

2.1. The momentum expected return with Lo and MacKinlay (1990) weighting scheme

Since Lo and MacKinlay (1990) calculates the contrarian strategy returns, we first derive the expected returns for the same weighing scheme but for a momentum strategy.

Let $w_{it-1}(k) = \frac{1}{N}(R_{it-1} - R_{mt-1})$, then the expected return is

$$E\left[\pi_t^{l\&m}(k)\right] = \frac{1}{N}tr(\Gamma_k) - \frac{1}{N^2}\iota'\Gamma_k\iota + \frac{1}{N}\sum_{i=1}^{N}\mu_i^2 - \mu_m^2 = \frac{1}{N}tr(\Gamma_k) - \frac{1}{N^2}\iota'\Gamma_k\iota + \frac{1}{N}\sum_{i=1}^{N}(\mu_i - \mu_m)^2. \quad (33)$$

Let $C_k^{l\&m} = -\frac{1}{N^2}[\iota'\Gamma_k\iota - tr(\Gamma_k)]$, $O_k^{l\&m} = \frac{(N-1)tr(\Gamma_k)}{N^2}$, $V_k^{l\&m} = \sigma^2(\mu)$. Therefore, the expected returns can be rewritten in three parts, $E\left[\pi_t^{l\&m}(k)\right] = C_k^{l\&m} + O_k^{l\&m} + V_k^{l\&m}$.

2.2 The momentum expected return with our weighting scheme

The expected return of the momentum strategy with only the top winners and bottom losers is, $E[\pi_t(k)] = C_k + O_k + V_k + L_k$ in Equation (32).

2.3. Difference in returns between the two models

The difference in the expected returns of the above two models is

$E\left[\pi_t^{l\&m}(k)\right] - E[\pi_t(k)] = \Delta C_k + \Delta O_k + \Delta V_k - L_k$, where,

$$\Delta C_k = -\frac{1}{N^2}[\iota'\Gamma_k\iota - tr(\Gamma_k)] + \frac{\alpha}{SN^2}[\iota'\Gamma_k^w\iota - tr(\Gamma_k^w) + \iota'\Gamma_k^{\bar{w}w}\iota] + \frac{\beta}{SN^2}[\iota'\Gamma_k^l\iota - tr\left(\Gamma_k^l\right) + \iota'\Gamma_k^{\bar{l}l}\iota]$$

$$= -\frac{1}{N^2}\left[\iota'\Gamma_k\iota - \frac{\alpha}{S}\iota'\Gamma_k^w\iota - \frac{\beta}{S}\iota'\Gamma_k^l\iota - \frac{\alpha}{S}\iota'\Gamma_k^{\bar{w}w}\iota - \frac{\beta}{S}\iota'\Gamma_k^{\bar{l}l}\iota\right] + \frac{1}{N^2}[tr(\Gamma_k) - \frac{\alpha}{S}tr(\Gamma_k^w) - \frac{\beta}{S}tr\left(\Gamma_k^l\right)]$$

$$\Delta O_k = \frac{N-1}{N^2}tr(\Gamma_k) - \frac{\alpha}{SN^2}(N-1)tr(\Gamma_k^w) - \frac{\beta}{SN^2}(N-1)tr(\Gamma_k^l)$$

$$= \frac{N-1}{N^2}[tr(\Gamma_k) - \frac{\alpha}{S}tr(\Gamma_k^w) - \frac{\beta}{S}tr\left(\Gamma_k^l\right)]$$

$$\Delta V_k = \sigma^2(\mu) - \alpha\sigma^2(\mu^w) - \beta\sigma^2(\mu^l)$$

$$\Delta L_k = -\alpha\mu_w(\mu_w - \mu_m) - \beta\mu_l(\mu_l - \mu_m)$$

Thus,

$$E\left[\pi_t^{l\&m}(k)\right] - E[\pi_t(k)]$$

$$= -\frac{1}{N^2}\left[\iota'\Gamma_k\iota - \frac{\alpha}{S}\iota'\Gamma_k^w\iota - \frac{\beta}{S}\iota'\Gamma_k^l\iota - \frac{\alpha}{S}\iota'\Gamma_k^{\bar{w}w}\iota - \frac{\beta}{S}\iota'\Gamma_k^{\bar{l}l}\iota\right] + \frac{1}{N}[tr(\Gamma_k) - \frac{\alpha}{S}tr(\Gamma_k^w) - \frac{\beta}{S}tr(\Gamma_k^l)]$$

$$+[\sigma^2(\mu) - \alpha\sigma^2(\mu^w) - \beta\sigma^2(\mu^l)] - [\alpha\mu_w(\mu_w - \mu_m) + \beta\mu_l(\mu_l - \mu_m)] \qquad (35)$$

III. Circumstances in Generating Positive Momentum Returns

We further investigate different return generating processes that can result in positive returns from the momentum strategy.

3.1Returns follow random walk with starting point $\boldsymbol{\mu}$

Similar to Conrad and Kaul (1998), let returns R_{it} follows random walk with starting point μ: $R_{it} = R_{it-1} + e_{it}$, where e_{it} is white noise or is independently and identically distributed with 0 mean and constant variance. Thus, the stock returns R_{it} are serially independent. Now, let us further assume returns of different stocks are independent between different time periods. Therefore, Γ_k or both C_k and O_k are zero, hence, the momentum return can be written as

$$E[\pi_t(k)] = V_k + L_k = \alpha\sigma^2(\mu^w) + \beta\sigma^2(\mu^l) + \alpha\mu_w(\mu_w - \mu_m) + \beta\mu_l(\mu_l - \mu_m) \qquad (36)$$

Since V_k is always positive, if L_k is positive, then even though the stock returns do not have cross-sectional or serial dependence, the momentum strategy can still generate positive returns. However, these positive returns do not come from the stock return predictability or stock price momentum. When the winner portfolio expected returns are positive and loser portfolio expected returns are negative, the more the winners win and the more the losers lose, the higher returns the momentum strategy generates. In this scenario, even though the stock returns follow random walk or the financial market is efficient, the momentum strategy can still make positive returns. Therefore, even if the returns from momentum strategy are positive, we cannot directly conclude that they can be attributed to the stock price momentum.

CHAPTER IV
EMPIRICAL RESULTS

I. An Empirical Appraisal of Momentum Returns

To measure the relative importance of stock price predictability in generating returns from the momentum strategy which we developed in Section III, we empirically decompose the momentum returns into four parts: average cross-autocovariances (C_k), average own-autocovariances (O_k), cross-sectional variances of the expected returns in the winner and loser portfolios (V_k), and the expected returns of the winner and loser portfolios (L_k). By investigating the composition of historical momentum returns directly, we can find out the sources and their relative importance in constituting the momentum returns. All the stocks listed in NYSE & Amex, and Nasdaq markets are included in the study. The whole dataset includes a total of over 27,000 stocks that have been traded in the U.S. stock market over the past 44 years, from January 1965 to December 2009. The data on stock returns are collected from the Center for Research in Security Prices (CRSP) Monthly Stock File for NYSE, Amex, and Nasdaq stocks. Since the trading environments in NYSE and NASDAQ markets are different, stocks in NYSE and Nasdaq markets are also investigated separately to take into account of the influence from market differences.

1.1Return decomposition

In order to find potential pattern between the length of the ranking period *k* and the resulting component weights, different ranking period *k* equal to 3, 6, 9, and 12 months are examined separately. The default weight parameters for the winner and loser portfolios are set equal to 1 ($\alpha = \beta = 1$). Also, two types of momentum strategies are investigated empirically.

One is the rolling strategy, which is very similar to the most frequently investigated momentum strategy. This strategy includes overlapping holding periods. In any given month *t*, the strategy holds a series of portfolios that are selected in the current month as well as in the previous *k* months if there are no skip months. The other strategy has no overlapping holding periods and is used in the Lo and Mackinlay (1990) paper. Since the whole time span of 44 years is a rather long time period, four 10-year periods are investigated separately in order to capture any potential change of market environment. Before year 1995, there were invisibly small amounts of short-selling activities in the U. S. market. Since short sales have been argued as a necessity in correcting overpriced assets, the short-selling level can potentially affect momentum returns from the loser portfolio. It is also well-known that in the year 2005, in order to gather data and study thoroughly the effect of the uptick rule on market volatility, price efficiency and liquidity, the SEC implemented a Pilot Program from May 2, 2005 to July 3, 2007. This Pilot Program suspended the uptick rule[1] on one-third of Russell 3000 Index constituent stocks with high levels of liquidity. On July 3, 2007, the SEC finally abolished Rule 10a-1 and any rule of exchanges, including NASDAQ 3350, which applied a bid test on short sales (Bai, 2007). Thus, this Pilot Program and the abolition of these price tests may improve the trading environment for short sales, makes the correction of stock overpricing easier, and hence, affect the momentum returns from the loser portfolio. Therefore, years 1994 and 2004 are set as two cut-off points for the 10 year sub periods.

[1] The Securities and Exchange Commission (SEC) had Rule 10a-1 under the Security Exchange Act of 1934, which provided that investors must sell short a listed stock either at a price above the preceding sale price, known as the plus tick or at the last sale price if it was higher than the last different price, known as the zero plus tick. Similarly, NASDAQ Rule 3350 provided that short sales in NASDAQ stocks be either higher or at the best bid when the best bid was below the preceding best bid (Bai, 2007)

Table 1 demonstrates both the magnitudes and weights of the four components in the momentum returns. C_k depends only on cross-autocovariances that one stock's return may be correlated to another stock's return in the previous period. O_k depends only on own-autocovariances, which is also interpreted as stock price predictability or momentum. V_k is the cross-section variation of the expected returns in the winner and loser portfolios for a given time. L_k depends on the expected returns in the winner and loser portfolios. Of all these four components, only O_k directly challenges the efficient market hypothesis which states that stock price has no memory. In our empirical testing, the expected returns of the stocks are estimated by using the average returns over the whole time span. Because it is less likely that the expected stock returns keep the same over the whole time span, investigating in shorter time periods, such as 10 years, becomes very meaningful. Since the momentum returns are time-series, all the *t* tests are adjusted for potential autocorrelation and heteroskedasticity, by using Kiefer and Bogelsang (2002) method.

Table 1 shows that for rolling momentum strategies, the major contributor in the momentum return isO_k, the autocorrelation of own stock returns. This measure is significant most of times at the 10 percent level. In comparison to O_k, two different stocks' correlation between two different times, C_k, constitutes a very small amount in the momentum returns, and are much less frequently significant at the 10 percent level. Furthermore, the consistently negative sign of O_k shows that the trace of Γ_k in Equation (28) is negative. This indicates that the own stock autocorrelation is negative in the holding period. This negative own stock return autocorrelation does not necessarily mean that the past winners tend to lose or past losers tend to win. It could be caused by the fact that stock prices increase or decrease for a less amount than that of the previous period. More interestingly, the magnitude of O_k decreases as the ranking period *k* moves further away for all subperiods and the whole time span in the rolling strategy. This pattern makes sense. Stock return predictability should weaken as the reference point of time becomes more remote. Such pattern is not observed in the other three components. The second most significant component is V_k, the cross-sectional variance of expected returns in the winner and loser portfolios at a given time. This significant correlation should be caused by the common risk factor in the market. For example, when the economy is booming, all the stocks in the market benefit from this favorable market trend. L_k is significant most of times too, however, its weight is much smaller than O_k and V_k. The sub-time periods all demonstrate similar patterns. However, the components are more frequently significant and at a higher significant level in the more recent years. In the second sub-time period from year 1965 to year 1974, none of the components is significant. It should be a volatile market period where the stock returns had minimal connections. The nonrolling strategy demonstrates similar results, though at lower levels of significance. In the nonrolling strategy, both O_k and V_k are the two most important components in terms of their weights in the momentum returns. Like the rolling strategy, L_k is significant most of times, however, with a much smaller weight. To sum up, Table I tells us that stock returns do have memory to some extent and taking advantage of this phenomenon can generate profits. This does challenge the market efficiency hypothesis to some extent. Also, cross-sectional variance of expected returns in the winner and loser portfolios is another important source of momentum returns.

It is often observed in the momentum literature that when Nasdaq stocks are included in the portfolio, the returns of the momentum strategy decrease dramatically. Table 2 decomposes momentum returns for NYSE & AMEX stocks only to take advantage of market differences. Both rolling and nonrolling strategies are investigated for the whole time span from 1965 to 2009 and for different ranking period k. Similar patterns are observed as in Table 1. O_k and V_k are the two most important sources for the returns from the momentum strategy and are significant for most of the cases. The magnitude of O_k decreases as the ranking period moves remote. L_k is significant most of times but with a much smaller weight. In the nonrolling strategy, O_k and V_k are still two most important sources of the momentum returns. However, O_k is not significant at the 10 percent level.

Also, recently a few papers have noted the following: (1) the proportional contributions of the winner and the loser portfolios to the momentum abnormal returns are indeed asymmetric (Hong, Lim, and Stein, 2000, Lesmond, Schill, and Zhou, 2004); and (2) the characteristics of the loser firms are quite unique. Unlike winners, the stocks that generate the bulk of the momentum abnormal returns are the "losers" that can be characterized as small, low-price, high-beta, off-NYSE stocks. Those stocks are typically hard to sell short, and involve high trading costs (Lesmond, Schill, and Zhou, 2004). In order to investigate the different influences of the winner and loser portfolios on the momentum returns, we also change the weight parameters α or β for winner and loser portfolios one at a time to examine their different impacts on the sources of momentum returns. Table 3 presents the magnitudes and weights of the four components by increasing the weight of winner or loser portfolio monotonically while keeping the weight of the other portfolio constant. In order to investigate only the effects from the different weights of the winner and loser portfolios, the ranking period k is fixed at 6. Specifically the rolling momentum strategy (6,0,1) is used in the analysis from January 1965 to December 2009 for NYSE & AMEX stocks only. Panel A shows the results with increasing weights for the loser portfolio from β= 1 to 10 to 50, while keeping the winner portfolio weight constant. The relative contributions of the components to the momentum returns remain in proportion when the weight of the loser portfolio increases. This indicates that the magnitude of the average own-autocovariance of the winner portfolio in the holding period is negligible compared to that of the loser portfolio. Not shown in the table, both the own-autocovarinaces in the winner and loser portfolios take on a negative sign. However, the magnitude of the winner portfolio is about 20 times smaller than that of the loser portfolio. This phenomenon is observed in all the four ranking periods. For example, take *k*=6, the average own-autocovariances of the winner and loser portfolios are -0.00009765 and -0.0020 respectively. Therefore, we can tell, the returns of the winner portfolio stocks are much more random than that of the loser portfolio stocks, and thus, have much weaker pattern to track through time. Panel B increases the winner portfolio weight, while keeping the loser portfolio weight constant. The cross-sectional variance of expected returns in the winner and loser portfolios V_k increases its weight in explaining the momentum returns monotonically. It is not observed when the weight of the winner portfolio keeps constant. These results combined show that the winner portfolio performance is much more volatile than that of the loser portfolio. Hence, winners' return pattern is much less predictable or is more random than that of the loser portfolio. Or the loser portfolio has more stable return pattern in terms of own stock autocovariance. These results we obtained from the expected return decomposition clearly provide an underlying explanation to the recent finding that the loser rather than the winner portfolio is the major contributor to the momentum returns. Therefore, to buy long and take advantage of the return pattern from the winner portfolio is much less reliable than selling short and exploiting the much more stable return predictability in the loser portfolio.

1.2 Empirical comparison of our model and model in Lo and Mackinlay (1990)

The key difference of our model and the Lo & Mackinlay (1990) model is the weighting scheme. Our momentum strategy only takes the top winners and bottom losers; however, Lo & Mackinlay (1990) include all the stocks in their portfolio. According to our weighting scheme, our momentum strategy put more weights on the potential stronger stock return patterns; because the top winners and bottom losers have stronger tendency to keep the return momentum in the next period. Without including the intermediate portfolio stocks, our momentum strategy reduces the noises in the stock return patterns from the middle group stocks. Table 4 illustrates the empirical decomposition for the Lo & Mackinlay (1990) weighting scheme. Both the rolling and nonrolling momentum strategies with different ranking period k are tried for stocks listed in NYSE & AMEX over the 44 year time span from 1965 to 2009. The key difference in the empirical results of Lo & Mackinlay (1990) momentum strategy and our momentum strategy is that the average autocorrelation of own stock returns, O_k becomes much less important in explaining the momentum returns. Furthermore, it is less frequently significant at the 10 percent level. This phenomenon can be explained by the different weighting schemes. The top winners and bottom losers have stronger tendency to continue the current return patterns, and thus, by including only those top and bottom performers, the magnitude of average own stock autocorrelation tends to be higher.

Furthermore, by including all stocks in the strategy, the expected returns from the Lo & MacKinlay (1990) momentum strategy are less frequently significant and are much less in magnitude than those in our strategy which only picks the top and bottom performers. Therefore, our strategy tends to be more profitable and have more practical benefits.

CHAPTER V
CONCLUSION

Momentum strategies that take advantage of potential return predictability have puzzled the finance researchers over the past twenty years. Heated dispute about whether the market is efficient or not makes this topic even more attractive. Instead of trying to identify unknown risk factors or behavioral theories that can fully explain the momentum returns, our study attempts to decompose the momentum returns directly and use historical data to discover the sources of the momentum returns, and their relative importance in generating the momentum returns.

Lo and MacKinlay (1990) propose that the positive cross-autocovariance or the lead-lag structure, rather than the small magnitude of the negative autocorrelation, is the drive of the positive contrarian portfolio returns. Conrad and Kaul (1998) further find in their return decomposition that the positive cross-sectional variance in mean returns is responsible for the profitability of the momentum strategy. However, Lehmann (1990) and Jegadeesh (1990) argue that the first-order serial correlation in stock returns is the major contributor of the contrarian returns.

Our empirical results show that autocorrelation of own stock returns is one of the driving forces for the momentum expected returns. The magnitude of the own-autocorrelation decreases as the ranking period moves more remote. The second important source comes from the cross-sectional variance of the mean return in the winner and loser portfolios at a given time. The third important source is the difference in the expected returns between the winner and loser portfolios. To our surprise, the cross-autocovariance does not contribute much to the momentum expected returns. Thus, the lead-lag effect can generate momentum returns, but its effect is not as significant as we thought before.

Furthermore, by changing the weights of the winner and loser portfolios, we find the winter portfolio return pattern is much weaker than that of the loser portfolio. On the contrary, the loser portfolio retains a much more stable return pattern from the ranking period to the holding period. This provides another underlying reason to explain the recent finding that the loser portfolio is the major contributor to the momentum returns. Therefore, the market may not be as efficient as we believed before.

LIST OF REFERENCES

Ali, A. and M. A. Trombley (2006). Short-Selling Costs and Momentum in Stock Returns. *Journal of Business Finance and Accounting,* Vol. 33, pp. 587-615.

Asness, C. S., J. M. Liew and R. L. Stevens (1997). Parallels between the Cross-Sectional Predictability of Stock and Country Returns. *Journal of Portfolio Management*, Vol. 23, pp. 79-87.

Bai, L. (2007). The Uptick Rule of Short Sale Regulation-Can It Alleviate Downward Price Pressures from Negative Earnings Shocks? (available at http://works.bepress.com/lynn_bai/1).

Ball, R., S. P. Kothari and C. E. Shanken (1995). Can We Implement Research on Stock Trading Rules? *Journal of Portfolio Management*, Vol. 21, pp. 54-63.

Barberis, N. and A. Shleifer and R.Vishny (1998). A Model of Investor Sentiment. *Journal of Financial Economics*, Vol. 49, pp. 307-343.

Berk, J. B., R. C. Green and V. Naik (1999). Optimal Investment, Growth Options, and Security Returns. *Journal of Finance*, Vol. 54, pp. 1553-1607.

Chordia, T. and L. Shivakumar (2002). Momentum, Business Cycle and Time-Varying Expected Returns. *Journal of Finance*, Vol. 57, pp. 985-1019.

Conrad, J. and G. Kaul (1998). An Anatomy of Trading Strategies. *The Review of Financial Studies*, Vol. 11, pp. 489-519.

Daniel, K. D., D. A. Hirschleifer and A. Subrahmanyam (1998). Investor Psychology and Security Market Under- and Over-reactions. *Journal of Finance*, Vol. 53, pp. 1839-86.

DeBondt, W. F. M. and R. Thaler (1985). Does the Stock Market Overreact? *Journal of Finance*, Vol. XL, pp. 793-805.

Fama, E. and K. French (1988). Permanent and Temporary Components of Stock Prices. *Journal of Political Economy*. Vol. 96, pp. 246-273.

Fama, E. and K. French (1996). Multifactor Explanations of Asset Pricing Anomalies. *Journal of Finance*. Vol. 51, pp. 55-84.

Fama, E. (1998). Market Efficiency, Long-Term Returns, and Behavioral Finance. *Journal of Financial Economics.* Vol. 49, pp. 283-306.

Hong, H. and J. C. Stein (1999). A Unified Theory of Underreaction, Momentum Trading and Over-reaction in Asset Markets. *Journal of Finance*, Vol. 54, pp. 2143-84.

Jegadeesh, N. (1990). Evidence of Predictable Behavior of Security Returns. *Journal of Finance*, Vol. 45, pp. 881-98.

Jegadeesh, N. and S. Titman (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance*, Vol. 48, pp. 65-91.

Jegadeesh, N. and S. Titman (2001). Profitability of Momentum Strategies: An Evaluation of Alternative Explanations. *Journal of Finance*, Vol. 56, pp. 699-720.

Jegadeesh, N and S. Titman (2002). Cross-Sectional and Time-Series Determinants of Momentum Returns. *The Review of Financial Studies*, Vol. 15, pp. 143-157.

Kiefer, N.M. and T.J. Vogelsang (2002). Heteroskedasticity-Autocorrelation Robust Standard Errors Using the Bartlett Kernel without Truncation. *Econometrica*, Vol. 70, pp. 2093-2095.

Korajczyk, R. and R. Sadka (2004). Are Momentum Profits Robust to Trading Costs? *Journal of Finance*, Vol. 59, pp. 1039-82.

Lehmann, B. (1990). Fads, Martingales and Market Efficiency. *Quarterly Journal of Economics*. Vol. 105, pp. 1-28.

Lesmond, D. A., M. J. Schill, and C. Zhou (2004). The Illusory Nature of Momentum Profits. *Journal of Financial Economics*, Vol. 71, pp. 349-80.

Lewellen, J. (2002). Momentum and Autocorrelation in Stock Returns. *The Review of Financial Studies*, Vol. 15, pp. 533-564.

Lo, A. and C. Mackinlay (1990). When are Contrarian Profits due to Stock Market Overreaction, *Review of Financial Studies*, Vol. 3, pp. 175-206.

Malkiel, B. G. (2003). The Efficient Market hypothesis and Its Critics. *Journal of Economic Perspectives*. Vol. 17, pp. 59-82.

Roll, R. (1984). A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market. *Journal of Finance*, Vol. 39, pp. 1127-1139.

Rouwenhorst, K. G. (1998). International Momentum Strategies. *Journal of Finance,* Vol. 53, pp. 267-84.

Sadka, R. (2006). Momentum and Post-Earnings Announcement Drift Anomalies: The Role of Liquidity Risk. *Journal of Financial Economics*. Vol. 80, pp. 309-349.

Table 1. Return Decomposition with All Stocks in the U.S. Market

Decomposition of monthly returns from momentum strategies with a sample of all stocks in the NYSE, AMEX, and Nasdaq from January 1965 to December 2009. To capture the possible change of expected returns over the whole 44 years and to take into account of the potential change of trading environment, 10 year subperiods are also investigated. Panel A lists the magnitudes and weights of the four components and the size of the expected momentum returns for the rolling momentum strategy. Panel B lists the results for nonrolling momentum strategy. Different ranking periods (k) are examined with *k*=3, 6, 9, 12 months respectively. The default weight parameters for the winner and loser portfolios are $\alpha = \beta = 1$. The table reports *t*-statistics in parentheses, adjusted for heteroskedasticity and autocorrelation by using Kiefer and Bogelsang (2002) approach. Significance at the 1%, 5% and 10% levels is indicated by a, b and c, respectively.

Lag(k)	C_k	O_k	V_k	L_k	$E[\pi_k(k)]$	$\%C_k$	$\%O_k$	$\%V_k$	$\%L_k$
Panel A: Rolling strategy with winners and losers scheme									
a)1965~~2009									
3	-0.0001	-0.0053	0.0012	0.0001	-0.0042	2%	126%	-29%	-2%
	(-0.94)	(-6.93)[a]	(5.99)[b]	(9.42)[a]	(-8.72)[a]				
6	0.0002	-0.0038	0.0012	0.0001	-0.0027	-7%	141%	-44%	-4%
	(1.23)	(-4.64)[c]	(5.99)[b]	(14.01)[a]	(-5.14)[b]				
9	0.0004	-0.0034	0.0013	0.0001	-0.0015	-27%	227%	-87%	7%
	(4.28)[c]	(-5.09)[b]	(5.96)[b]	(10.10)[a]	(-4.34)[c]				
12	0.0003	-0.0027	0.0013	0.0001	-0.0010	-30%	270%	-130%	-10%
	(2.12)	(-3.87)	(6.01)[b]	(8.35)[a]	(-2.83)				

b)*1965-1974*

3	-0.0001	-0.0024	0.0009	0.0005	-0.0011	9%	218%	-82%	-45%
	(-0.27)	(-1.54)	(3.67)	(2.19)	(-1.56)				
6	-0.0002	-0.0012	0.0009	0.0006	0.0001	-200%	-1200%	900%	600%
	(-0.93)	(-1.26)	(3.70)	(2.08)	(0.27)				
9	0.0003	-0.0016	0.0009	0.0007	0.0003	100%	-533%	300%	233%
	(2.08)	(-2.56)	(3.69)	(2.01)	(1.90)				
12	0.0001	-0.0012	0.0009	0.0008	0.0006	5%	-40%	30%	25%
	(1.04)	(-1.97)	(3.79)[c]	(1.98)	(3.61)				

c)1975-1984

3	0.0001	-0.0071	0.0015	0.0003	-0.0052	-17%	137%	-29%	-6%
	(0.13)	(-4.56)[c]	(4.88)[b]	(3.23)	(-4.29)[c]				
6	0.0005	-0.0045	0.0016	0.0004	-0.0021	-24%	214%	-76%	-19%
	(-0.01)	(-5.18)[b]	(4.59)[b]	(5.85)[b]	(-3.14)				
9	0.0007	-0.0039	0.0016	0.0005	-0.0011	-64%	355%	-145%	-45%
	(1.07)	(-3.18)	(4.90)[b]	(6.11)[a]	(-1.52)				
12	0.0005	-0.0030	0.0016	0.0006	-0.0003	-167%	1000%	-533%	-200%
	(1.43)	(-4.44)[c]	(5.23)[b]	(5.29)[b]	(-0.81)				

d)1985-1994

3	-0.0001	-0.0089	0.0023	0.0002	-0.0065	2%	137%	-35%	-3%
	(-1.40)	(-17.81)[a]	(7.63)[a]	(6.69)[a]	(-8.64)[a]				
6	0.0004	-0.0069	0.0024	0.0003	-0.0037	-11%	186%	-65%	-8%
	(4.30)[c]	(-10.40)[a]	(7.87)[a]	(7.94)[a]	(-4.29)[c]				

9	0.0007	-0.0061	0.0024	0.0004	-0.0026	-27%	235%	-92%	-7%
	(5.32)[b]	(-10.80)[a]	(8.28)[a]	(9.21)[a]	(-3.22)				
12	0.0003	-0.0047	0.0024	0.0004	-0.0015	-20%	313%	-160%	-27%
	(2.90)	(-13.43)[a]	(7.68)[a]	(8.70)[a]	(-2.53)				
e)1995-2004									
3	0.0009	-0.0092	0.0028	0.0002	-0.0053	-17%	174%	-53%	-4%
	(2.46)	(-8.14)[a]	(6.98)[a]	(12.10)[a]	(-6.30)[a]				
6	0.0010	-0.0069	0.0029	0.0003	-0.0026	-38%	265%	-112%	-12%
	(4.66)[c]	(-10.99)[a]	(7.33)[a]	(22.19)[a]	(-5.21)[b]				
9	0.0008	-0.0059	0.0029	0.0004	-0.0018	44%	328%	161%	22%
	(5.63)[b]	(-10.92)[a]	(8.00)[a]	(23.88)[a]	(-2.93)				
12	0.0008	-0.0054	0.0029	0.0004	-0.0012	-67%	450%	-242%	-33%
	(6.41)[a]	(-9.03)[a]	(8.88)[a]	(20.31)[a]	(-1.60)				

Panel B: Nonrolling strategy with winners and losers scheme

a)1965~~2009									
3	-0.0002	-0.0017	0.0011	0.0001	-0.0007	29%	243%	-157%	-14%
	(-1.52)	(-5.52)[b]	(5.83)[b]	(8.06)[a]	(-2.95)				
6	-0.0000	-0.0009	0.0012	0.0001	0.0004	-0%	-225%	300%	25%
	(-0.29)	(-3.77)[c]	(5.51)[b]	(13.69)[a]	(1.93)				
9	-0.0003	-0.0004	0.0013	0.0001	0.0007	-43%	-57%	186%	14%
	(-2.53)	(-0.84)	(6.29)[a]	(15.89)[a]	(3.15)				
12	-0.0001	-0.0016	0.0013	0.0001	-0.0003	33%	533%	-433%	-33%

	(-2.50)	(-2.72)	(5.26)[b]	(9.73)[a]	(-1.00)				
b)*1965-1974*									
3	0.0005	-0.0015	0.0009	0.0004	0.0002	250%	-750%	450%	200%
	(0.87)	(-1.17)	(3.81)[c]	(2.27)	(0.57)				
6	-0.0001	-0.0006	0.0009	0.0004	0.0006	-17%	-100%	150%	67%
	(-0.85)	(-1.42)	(4.13)[a]	(2.18)	(2.58)				
9	-0.0001	-0.0006	0.0011	0.0008	0.0011	-9%	-55%	100%	73%
	(-0.55)	(-0.99)	(3.35)	(1.94)	(4.76)[c]				
12	0.0002	-0.0010	0.0009	0.0005	0.0005	40%	-200%	180%	100%
	(0.54)	(-1.63)	(4.51)[c]	(1.98)	(0.81)				
c)1975-1984									
3	-0.0001	-0.0031	0.0015	0.0003	-0.0014	7%	221%	-107%	-21%
	(-0.21)	(-1.95)	(5.23)[b]	(2.95)	(-1.31)				
6	0.0007	-0.0026	0.0017	0.0004	0.0002	350%	-1300%	850%	200%
	(0.85)	(-1.98)	(5.73)[b]	(4.28)[c]	(0.42)				
9	-0.0002	-0.0007	0.0016	0.0005	0.0013	-15%	-54%	123%	38%
	(-3.01)	(-2.29)	(3.90)[c]	(6.56)[a]	(5.43)[b]				
12	-0.0001	-0.0014	0.0020	0.0006	0.0010	-10%	-140%	200%	60%
	(-1.10)	(-4.40)[c]	(6.52)[a]	(5.41)[b]	(2.39)				
d)1985-1994									
3	0.0002	-0.0049	0.0024	0.0002	-0.0022	-9%	223%	-109%	-9%
	(-0.21)	(-1.95)	(5.23)[b]	(2.95)	(-1.31)				
6	0.0006	-0.0037	0.0025	0.0003	-0.0003	-200%	1233%	-833%	-100%

	(2.24)	(-4.95)[b]	(5.35)[b]	(6.90)[a]	(-0.23)				
9	-0.0004	-0.0009	0.0026	0.0004	0.0016	-25%	-56%	163%	25%
	(-4.51)[c]	(-0.96)	(5.41)[b]	(10.26)[a]	(1.16)				
12	-0.0002	-0.0029	0.0029	0.0004	0.0002	-100%	-1450%	1450%	200%
	(-1.31)	(-1.76)	(4.04)[c]	(14.04)[a]	(0.10)				
e)1995-2004									
3	0.0004	-0.0034	0.0029	0.0002	0.0001	400%	-3400%	2900%	200%
	(1.91)	(-5.83)[b]	(5.47)[b]	(12.84)[a]	(0.13)				
6	-0.0006	-0.0006	0.0034	0.0003	0.0025	-24%	-24%	136%	12%
	(-2.58)	(-1.10)	(4.38)[c]	(15.66)[a]	(3.10)				
9	-0.0001	-0.0020	0.0037	0.0004	0.0020	-5%	-100%	185%	20%
	(-0.32)	(-2.87)	(3.94)[c]	(18.52)[a]	(1.39)				
12	-0.0002	-0.0038	0.0044	0.0004	0.0008	-25%	-475%	550%	50%
	(-1.28)	(-2.51)	(3.10)	(15.50)[a]	(0.29)				

Table 2. Return Decomposition with NYSE & AMEX Stocks Only

Decomposition of monthly returns from momentum strategies with stocks listed only in NYSE, and AMEX markets from January 1965 to December 2009. Panel A lists the magnitudes and weights of the four components and the size of the expected momentum returns for the rolling momentum strategy. Panel B lists the results for nonrolling momentum strategy. Different ranking periods (k) are examined with k=3, 6, 9, 12 months respectively. The default weight parameters for the winner and loser portfolios are $\alpha = \beta = 1$. The table reports t-statistics in parentheses, adjusted for heteroskedasticity and autocorrelation by using Kiefer and Bogelsang (2002) approach. Significance at the 1%, 5% and 10% levels is indicated by a, b and c, respectively.

Lag(k)	C_k	O_k	V_k	L_k	$E[\pi_k(k)]$	$\%C_k$	$\%O_k$	$\%V_k$	$\%L_k$
Panel A: Rolling strategy with winners and losers scheme									
3	-0.0002	-0.0031	0.0006	0.0000	-0.0026	8%	119%	-23%	-0%
	(-2.04)	(-8.63)[a]	(4.31)[c]	(1.94)	(-10.46)[a]				
6	0.0002	-0.0021	0.0007	0.0000	-0.0013	-15%	162%	-54%	-0%
	(1.44)	(-3.98)[c]	(4.23)[c]	(4.92)[b]	(-4.28)[c]				
9	0.0003	-0.0019	0.0007	0.0001	-0.0008	-38%	238%	-88%	-13%
	(6.10)[a]	(-4.08)[c]	(4.22)[c]	(7.19)[a]	(-3.12)				
12	0.0002	-0.0013	0.0007	0.0001	-0.0004	-50%	325%	-175%	-25%
	(1.68)	(-2.45)	(4.21)[c]	(6.95)[a]	(-1.34)				
Panel B: Nonrolling strategy with winners and losers scheme									
3	-0.0001	-0.0009	0.0006	0.0000	-0.0004	25%	225%	-150%	-0%

	(-0.83)	(-3.01)	(4.30)[c]	(1.74)	(-1.40)				
6	0.0001	-0.0004	0.0007	0.0000	0.0004	25%	-100%	175%	0%
	(0.51)	(-2.69)	(4.13)[c]	(4.59)[c]	(2.44)				
9	-0.0001	0.0000	0.0007	0.0001	0.0007	-14%	0%	100%	14%
	(-0.63)	(0.01)	(4.45)[c]	(6.61)[a]	(2.86)				
12	-0.0001	-0.0005	0.0007	0.0001	0.0002	-50%	-250%	350%	50%
	(-1.45)	(-1.85)	(4.29)[c]	(7.35)[a]	(1.52)				

Table 3. Return Decomposition with Change of Weights

Decomposition of monthly returns from rolling momentum strategies with stocks listed only in NYSE, and AMEX markets from January 1965 to December 2009. Different combination of α, β are examined with a fixed ranking period *k*=6. Panel A lists the magnitudes and weights of the four components and the size of the expected momentum returns for the rolling momentum strategy. Panel B lists the results for nonrolling momentum strategy. The table reports *t*-statistics in parentheses, adjusted for heteroskedasticity and autocorrelation by using Kiefer and Bogelsang (2002) approach. Significance at the 1%, 5% and 10% levels is indicated by a, b and c, respectively.

(α, β)	C_k	O_k	V_k	L_k	$E[\pi_k(k)]$	$\%C_k$	$\%O_k$	$\%V_k$	$\%L_k$
Panel A: Increasing loser portfolio weight with constant winner portfolio weight									
(1, 1)	0.0002	-0.0021	0.0007	0.0000	-0.0013	-15%	162%	-54%	-0%
	(1.44)	(-3.98)[c]	(4.23)[c]	(4.92)[b]	(-4.28)[c]				
(1, 10)	0.0028	-0.0206	0.0050	-0.0001	-0.0129	-22%	160%	-39%	1%
	(2.71)	(-3.94)[c]	(3.84)[c]	(-3.77)[c]	(-4.28)[c]				
(1, 50)	0.0144	-0.0124	0.0242	-0.0008	-0.0646	-22%	19%	-37%	1%
	(2.81)	(-3.93)[c]	(3.79)[c]	(-6.08)[a]	(-4.26)[c]				
Panel B: Increasing winner portfolio weight with constant loser portfolio weight									
(1, 1)	0.0002	-0.0021	0.0007	0.0000	-0.0013	-15%	162%	-54%	-0%
	(1.44)	(-3.98)[c]	(4.23)[c]	(4.92)[b]	(-4.28)[c]				
(10, 1)	-0.0010	-0.0030	0.0024	0.0006	-0.0011	91%	273%	-218%	-55%
	(-2.36)	(-3.13)	(5.28)[b]	(9.05)[a]	(-1.48)				
(50, 1)	-0.0063	-0.0069	0.0103	0.0028	-0.0001	6300%	6900%	-10300%	-2800%

(-2.95)	(-1.86)	(5.69)[b]	(9.54)[a]	(-0.03)

Table 4. Return Decomposition following Lo & MacKinlay (1990)

Decomposition of monthly returns from momentum strategies with stocks listed only in NYSE, and AMEX markets from January 1965 to December 2009. The weighting scheme follows the Lo and MacKinlay (1990) paper with all the stocks included in the portfolio. Panel A lists the magnitudes and weights of the four components and the size of the expected momentum returns for the rolling momentum strategy. Panel B lists the results for nonrolling momentum strategy. Different ranking periods (k) are examined with k=3, 6, 9, 12 months respectively. The default weight parameters for the winner and loser portfolios are $\alpha = \beta = 1$. The table reports *t*-statistics in parentheses, adjusted for heteroskedasticity and autocorrelation by using Kiefer and Bogelsang (2002) approach. Significance at the 1%, 5% and 10% levels is indicated by a, b and c, respectively.

Lag(k)	C_k	O_k	V_k	$E[\pi_k(k)]$	$\%C_k$	$\%O_k$	$\%V_k$
Panel A: Rolling strategy with all stocks included							
3	-0.0001	-0.0002	0.0002	-0.0002	50%	100%	-100%
	(-2.97)	(-4.15)[c]	(5.66)[b]	(-8.49)[a]			
6	-0.0000	-0.0002	0.0002	-0.0001	0%	200%	-200%
	(-0.82)	(-2.59)	(5.55)[b]	(-2.66)			
9	0.0000	-0.0002	0.0002	-0.0000	-0%	2000%	-2000%
	(2.76)	(-4.33)[c]	(5.47)[b]	(-0.62)			
12	0.0001	-0.0001	0.0002	0.0000	1000%	-1000%	2000%
	(0.32)	(-1.58)	(5.41)[b]	(2.03)			
Panel B: Nonrolling strategy with all stocks included							
3	-0.0001	-0.0001	0.0002	0.0000	-1000%	-1000%	2000%

	(-0.72)	(-0.68)	(5.73)[b]	(1.29)			
6	0.0000	-0.0001	0.0002	0.0001	0%	-100%	200%
	(0.68)	(-1.24)	(5.37)[b]	(4.81)[b]			
9	-0.0000	0.0000	0.0002	0.0002	-0%	0%	100%
	(-0.22)	(0.19)	(5.46)[b]	(7.01)[a]			
12	-0.0000	-0.0000	0.0002	0.0001	-0%	-0%	200%
	(-0.69)	(-0.39)	(5.35)[b]	(8.97)[a]			

VITA

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