12-2009

The Path to Success: Identities that Mathematics Students develop in a Specialized Residential High School

Yan Wang
University of Tennessee - Knoxville

To the Graduate Council:

I am submitting herewith a dissertation written by Yan Wang entitled "The Path to Success: Identities that Mathematics Students develop in a Specialized Residential High School." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Education.

Vena M. Long, Major Professor

We have read this dissertation and recommend its acceptance:

P. Mark Taylor, Lynn L. Hodge, Charles R. Collins

Accepted for the Council:
Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

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THE PATH TO SUCCESS: IDENTITIES THAT MATHEMATICS STUDENTS DEVELOP IN A SPECIALIZED RESIDENTIAL HIGH SCHOOL

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Yan Wang
December 2009

DEDICATION

This dissertation is dedicated to my grandparents and parents, Zhiying, Guangfu, Minglong, and Yongzhen, who taught me to be a decent person, and my sister, Peng, and the rest of the family, for always believing in me, inspiring me, and encouraging me to reach higher in order to achieve my goals.

ACKNOWLEDGEMENTS

I would like to take this opportunity to thank some of the people who helped to make the completion of this project. My sincere thanks go first to the Mathematics Education program at the Department of Theory and Practice in Teacher Education, the University of Tennessee.

I would like to express my deepest gratitude to my advisor, Dr. Vena Long for her invaluable guidance, encouragement and support throughout my work and education. She has always been a source of inspiration during my studies. I was financially supported by graduate assistantships from the National Science Foundation (NSF) grant “The Appalachian Collaborative Center for Learning, Assessment, and Instruction in Mathematics (ACCLAIM)” and the state grant of Tennessee Governor’s Academy for Mathematics and Science awarded to Dr. Long. Without her belief in me, the completion of my education and this dissertation could be impossible. I am deeply indebted to Dr. Long for her dedication on my education, numerous and warming helps while I was studying at here.

I would also like to thank Dr. Lynn Hodge for the invaluable discussions with regard to this research. She has offered valuable thoughts and suggestion that ultimately improved the final copy of the dissertation. I would like to express my appreciation to the other members of my committee, Dr. P. Mark Taylor and Dr. Chuck Collins for their time and critical review of my work. I also would like to highlight my thanks to Dr. Terri Hopkins and Dr. Tommy Hodges for their tremendous kindly help, support and encouragement during my graduate study.

I would like to thank all student participants in this study. I cannot acknowledge them by name, however, their honest, candor, and insight are highly appreciated.

My thanks also go to my fellow doctoral friends Geri Landry, Lu Howard, Amy Sullins, Ning Yu, and Rong Ran. Thank you for your time, encouragement, and ideas. I am most grateful to the friendship.

Lastly, my husband, daughters, and parents-in-laws have offered me strengths to complete my graduate education and helped to make my time here pleasant and memorable. I do really appreciate their endless love and support.

ABSTRACT

The purpose of this study was to explore the mathematics learning practices in the lives of students in a specialized residential high school in the Southeastern region of the United States. In particular, the study explores how mathematics classroom micro-culture shapes and is shaped by students' developing identities as mathematics learners, and what mathematics learning means to them. These purposes were achieved through the lens of the theoretical framework of identity in practice (Wenger, 1998), multiple framework (Martin, 2000), and the interpretive scheme (Cobb, Gresalfi, & Hodge, 2009).

Mixed research methods were used to conduct the study. Surveys, semi-structured interviews, observations, documentation and classroom artifacts provided appropriate data sources for information collected between January and March 2009. Twenty-five junior students were invited to complete the survey at the beginning of the study. Then, six participants included three Calculus I students and three Calculus II students were selected purposely for the in-depth, one-on-one interview study.

The findings suggested the relationship between the mathematics classroom environments and students' developing relationships with mathematics. Participants expressed positively of their experiences in their classes. The nature of instructional practices contributes to students' positive views of mathematics and what it means to do and be successful in mathematics. Further, data indicated that rich residential academic community, together with teachers, and peers, enhance students' success in mathematics education. The results of this study can be used to improve practices for creating specialized residential learning environments necessary to the needs and unique

challenges of talented and gifted students, as well as to develop strong mathematical identities, both of which contribute significantly to academic persistence and achievement in mathematics.

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CHAPTER I

INTRODUCTION

Despite the fact that equity has increasingly been of interest of the mathematics education community, such work tend to focus primarily on cognition with little attention to social or cultural issues (Lubienski & Bowen, 2000; Lubienski, 2002). Although the traditional approach of documenting achievement disparities among various groups might be necessary, it is limiting to help us understand the causes of the inequitable mathematics achievement patterns or the underlying processes and issues (Ladson-Billings, 1997; Nasir, 2007; Nasir & Cobb, 2007).

To more fully understand the relationship between societal structure and the issue of equity at the level of classroom interaction, researchers have attempted to explore how students' identities as mathematical learners and doers are enabled as they participate in the activities of a mathematical classroom community in increasingly substantial ways (Anderson, 2007; Abreu & Cline, 2007; Boaler, 1999; Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009; Martin, 2007; Nasir, 2002; Cobb & Hodge, 2002; Hodge, 2006). Scholars have argued that learning is more than the acquisition of knowledge and skills. Students construct identities as they actively participate in learning practices in their mathematics classroom communities (Gresalfi & Cobb, 2006; Boaler, 1999; Cobb & Bowers, 1999; Nasir, 2002; Wenger, 1998). Therefore, not only must mathematics education address the learning of mathematics, but also how students position themselves or are positioned in the classrooms in relation to their learning of mathematics (Davis, West, Greeno, Gresalfi & Martin, 2007).

The new concerns of how students become or not become part of mathematics classroom communities have been highlighted in the past years. Recent research in mathematics education have made critical contributions toward the development of a more complex understanding of the relation between equity, identity and culture in mathematics classrooms (Boaler, 1999; Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009; Martin, 2007; Nasir, 2002; Cobb & Hodge, 2002). A number of scholars have paid serious attention to the education environment and students' perceptions about their roles from different perspectives. Boaler and Greeno (2000) reported that many high school mathematics students who are succeeding in traditional mathematics classes, surprisingly, choose not to continue to study mathematics in college due to experiencing a conflict between how they view themselves and who they want to be, and how they feel they and their decisions are viewed by other individuals in their mathematics classes. Thus, the analysis of students' interests and motivations as well as their decision whether to continue to study mathematics bring the issue of identity to the forefront due to the impact on who learns mathematics and what mathematics they learn (Anderson, 2007; Abreu & Cline, 2007; Boaler, 1999; Boaler & Greeno, 2000; Cobb & Hodge, 2002; Cobb, Gresalfi, & Hodge, 2009; Gee, 2001; Hodge, 2008; Martin, 2007; Nasir, 2002; Sfard & Prusak, 2005). That is, how students come to see themselves as mathematical thinkers and as members of mathematics communities, as well as their beliefs about mathematics learning, have influence on mathematics study.

Classroom environment plays a significant role in students' learning practices of mathematics. When I observed a class of academically disinclined math students a couple of years ago, I made a number of important discoveries. There was one student who was repeating the class for the third time. This student slept during class and would not do any seatwork. From our conversation, I knew that this student did not want to do cooperative work in the class because he felt the content was too easy for him, and he recognized that the teacher perceived him as academically deficient. On another occasion one calculus teacher with whom I communicated expressed a difficulty in getting some students involved in group work during class. For example, one student did not feel comfortable during the group work so they were uncommunicative and tended to dismiss the perspectives on mathematical problem solving strategies suggested by the group. Instead, this student expected to learn mathematics by mimicking the instructor. When this student was engaging in learning by participating in team work that seemingly involved expressing disparate ideas in front of others conflicts emerged.

As Hodge (2006) notes, students might resist and alienate themselves from the classroom community either by accepting or rejecting the perception of deficiency which they perceived that the teacher perceives in them. The first student described above had accepted the perception of deficiency from his teacher, and therefore, alienated themselves from classroom practices. The second student realized there were conflicts in the beliefs about problem solving between what he had experienced in the past and what he was currently being asked to do. Thus, he firmly rejected the variety of problem solving approaches proposed by other students. In other words, it seemed that the students chose not to take advantage of the learning setting to build strong identities in relation to mathematics classes. Students' resisting behavior and practices in their mathematics classroom communities emerged partially due to their interpretations of the classroom setting.

Statement of the Problem

The students currently in our schools represent a wide variety of cultures. They are living in places that differ widely in racial-ethnic heritage, primary language, and economic status (Campbell & Rowan, 1997; Zaslavsky, 2002). These social, cultural, and historical contexts define and shape students and their experiences and therefore, have serious implications for educational institutions (Malloy, 1997). Consequently, the issues of equity in the classrooms are becoming an increasingly salient and critical topic of discussion in education (Hodge, 2006; Ladson-Billings, 1997).

To afford students who come from diverse linguistic and cultural backgrounds, who have specific disabilities, or who possess a special talent and interest in mathematics the opportunities to be successful and to excel in an increasingly technological world, the National Council of Teachers of Mathematics (NCTM) has articulated a vision to students, school leaders, parents and other caregivers that ensures all students access to a high-quality mathematics education (NCTM 2000). NCTM (2000) regards educational equity as a core element of school mathematics education. As the first principle in the Principles and Standards for School Mathematics (PSSM), equity requires high expectations and worthwhile opportunities for all students. Teachers need to accommodate all students to learn mathematics with adequate sources and strong support.

There are two bodies of literature focusing on the issue of equity. One way of looking at equity is to study certain groups of students in terms of race, ethnicity, socioeconomic class and English language proficiency. That is, to look at the cultural or ethnic identity (Rogoff, 2003), or identity groups (Banks, 2008) in a multicultural democratic society. However, some researchers (Johnson, 2000; Rogoff, 2003) note that this categorization approach is based on the view that culture is a categorical property of individuals. Although it is important to study how people identify with the cultural aspects of individual lives, equating their identity solely with culture is no longer valid. Furthermore, the continued use of these categories "does not convey specific enough information to improve a teacher's instructional decisions" (Johnson, 2000,p. 47).

The other approach to the issue of equity has been to look at learning opportunities in the mathematics classroom. Studies from this perspective indicate that learning can be viewed as a process of participation in dynamically related cultural communities (Rogoff, 2003). Not only do students engage in deep understanding which includes the learn procedures and the concepts and mathematical relations underlying those procedures, students develop the ability to see themselves as learners and doers of mathematics and to understand how they are enabled as they participate in classroom mathematical practices (Anderson, 2007; Abreu & Cline, 2007; Boaler, 2002; Boaler & Cobb & Hodge, 2002; Cobb, Gresalfi, & Hodge, 2009; Greeno, 2000; Hodge, 2008; Martin, 2000; Nasir, 2002). These studies use situative learning theory to explore how mathematics teachers in the classroom provide opportunities for all students to negotiate competent participation in order to be successful as well as cultivate positive dispositions towards mathematics learning (Gutierrez, 1999; Gresalfi & Cobb, 2006; Lubienski, 2007). Further, research from this perspective portrays mathematics classrooms as communities of practice where the classroom culture has strong influence on students' perception of themselves as mathematical learners and doers, thereby helping to understand the dynamics of identity, culture and mathematics learning in a classroom setting (Boaler, 1999; Cobb & Hodge, 2002; Hodge, 2008; Nasir, 2002).

The construction of identity is paramount, yet identity is underdeveloped as an explanatory construct in mathematics education research arena (Cobb, 2004; Lerman, 2006). In addition, little is known about the characteristics of students' identity development in a specialized residential high school (SRHS). Students come to the new mathematics learning community, the specialized residential high school, with different experiences from the home high school (HHS). Both the home school history and the new classroom culture affect learning outcomes. Nevertheless, there has been little study of how these factors intersect in the lives of students to produce different mathematical identity. Thus, an analysis of this area is needed to understand how specialized residential high school mathematics classroom culture, learning, and identity interrelate in various ways.

Purpose of the Study

Mathematics education researchers who are interested in issues of diversity and equity point out the significant role of identity, and indicate the potentially fruitful approach of using the construct of identity to understand the interactions of culture, diversity, and mathematics learning (Arbeu & Cline, 2007; Cobb, Gresalfi, & Hodge, 2009; Hodge, 2008; Lerman, 2006; Martin, 2000). This study is about the identity development of students at a specialized residential high school in the Southeastern United States.

The study is to examine the role and meaning of classroom culture in the lives of residential high school students. In particular, the study examines how culture in the classroom shapes and are shaped by students' identities and what culture in the classroom means to students.

Research Questions

To address the purpose of the study, the study investigates two questions:

(1) What kinds of identities are developed in a specialized residential high school (SRHS) classroom?

(2) How do the students' specialized residential high school (SRHS) experiences interact with home high school (HHS) experiences to affect the development of mathematical identity?

Significance of the Study

This study of identity formation of students in advanced mathematics classrooms is critical to extending our collective knowledge about how identity impacts mathematical learning. It also aims to provide insight into students' access to significant mathematical ideas. This, in turn, should yield insight into instructional design and teaching for teachers. Ultimately, research using the construct of identity may help to better understand why there are rapidly declining numbers of United States college students pursuing bachelor's degrees in STEM careers – science, technology, engineering and math (Government Accountability Office, 2006), and why some students see math as difficult, and do not take any more classes than the required minimum (University of Michigan, 2005; University of Wisconsin – Milwaukee, 2008).

The Interpretive Scheme developed by Cobb and his colleagues (Cobb & Hodge, 2002, 2007; Cobb, Grasafi, & Hodge, 2009) will be used to guide the investigation. The

Scheme links the students' personal identities to the microculture established in the local classroom. Specifically, the classroom mathematical actions and interactions are paid primary attention. Identity of students at this point is about their views of what it means to be a learner and doer in the mathematics classroom. That is, the interpretive scheme pays attention to "students' views of what it means to know and do mathematics in the classroom, and whether, and to what extent they come to identify with classroom mathematical activity" (Cobb, Grasafi, & Hodge, 2009, p. 42).

Much research exists on identity of mathematics students with a homogenous culture and mathematical ability. Previous studies on academic communities, such as Danny Martin's (2000) African-American mathematics participants, Walker's (2006) urban students' academic communities, and Jo Boaler and Staples' (2008) case of Railside school mathematics students account for community and identity regarding mathematics learning and teaching. Very few studies concentrate on gifted and talented (G&T) students in an academic community who reside relatively close to each other, and have great interest in, and potential for, high achievement in mathematics. The importance of this study will be especially beneficial to teachers and mathematics educators as they strive to explore and examine classroom instructional design and practices, and discover effective ways to teach diverse, academically inclined students in specialized residential school classrooms. This study will also help educators rethink the issues of the achievement gap with a refreshed perspective and the knowledge that promoting equity in the mathematics classroom is a complex endeavor.

Limitations of the Study

This study has several limitations. First, the qualitative research samples are almost always purposive (Gay, Mills & Airasian, 2006). Thus, the purposive sampling approach, which will be implemented to conduct the study, will decrease the generalizability of findings. Also the findings will not be generalizable to all students in mathematics classrooms because of the unique context of a specialized residential school. Second, my unique background, experiences, and perspectives may affect the interpretation of the situation and thus impact the observations and subsequent analysis. In addition, through my work at the same school, my relationships with the students may impact the behavior of the individuals I am studying.

Delimitations

Initially, this study will try to carefully document a specialized residential high school by conducting surveys, interviews, and observations for juniors who are studying advanced mathematics in the Southeast United States. Thus, it is "someone else's job to see how it fits into the general scheme of things" (Bogdan & Biklen, 2006, p. 36). Additionally, to minimize the effects of my opinions, prejudices and biases on the data, I will be conscientiously recording detailed field notes that include reflections on my own subjectivity (Bogdan & Biklen, 2006). I will be trying to interact with participants in a natural, unobtrusive, and nonthreatening manner in order to avoid observer effect (Bogdan & Biklen, 2006). In addition, multiple data collections such as self-reported questionnaires, interviews, observations, documentations, and classroom artifacts will be used to provide additional and powerful resources.

Definition of Terms

In the following list, I describe the definition of terms that are used in the context of this study.

Identity

Identity refers to "a way of talking about how learning changes who we are and creates personal histories of becoming in the context of our communities" (Wenger, 1998, p. 5). As a result, individuals come to know who they are as learners as a result of their participation with others in the experience of life (Wenger, 1998). Identity will be viewed as a relational and a fluid construct, and thus, it is not an attribute of a person but it shapes and is shaped by the social context (Hodge, 2006; Martin, 2007; Nasir, 2002).

Mathematical identity

According to Martin (1999), mathematical identity refers to students' beliefs about "a) their ability to perform in mathematical contexts, b) the instrumental importance of mathematical knowledge, c) constraints and opportunities in mathematical contexts, and d) the resulting motivations and strategies used to obtain mathematics knowledge" (p. 19). That is, "the ways that students think about themselves in relation to mathematics and the extent to which they have developed a commitment to, and have come to see value in mathematics as it is realized in the classroom" (Cobb, Gresalfi, & Hodge, 2009, p.40). Therefore, mathematical identity encompasses such affective factors as persistence and interest in mathematics, and motivation to learn mathematics. Students' identities as doers of mathematics are developed as they participate in the activities in the mathematics classroom.

Specialized Residential High School

Specialized residential schools provide innovative learning experiences in a unique environment designed to meet the academic needs of gifted and talented high school students. They offer an enriched, well-rounded, and advanced curriculum; challenge students with extensive research programs; and integrate research, writing, critical thinking, interdisciplinary projects, and technology throughout the curriculum. Most of these schools have a strong focus on mathematics and science but also offer a broad curriculum in the arts and humanities (Stamps, 2008).

Community of Practice

"Communities of practice are groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly" (Wenger, 2008). There are three characteristics: a joint enterprise, mutual engagement, and a shared repertoire of resources to utilize in practice (Wenger, 1998).

Students' Academic Community

Students' Academic Community refers to students' webs of support for their mathematics work. These webs are complex and have many interrelated factors, including school, family, and peers, which influence their learning experiences (Walker, 2006).

Normative identity

According to Cobb and his colleagues, the normative identity "comprises both the general and the specifically mathematical obligations that delineate the role of an effective student in a particular classroom… it is a collective or communal notion rather than an individualistic notion" (Cobb, Gresalfi, & Hodge, 2009). Students have to adopt it in order to develop a sense of affiliation with mathematical activity as it was realized in their classrooms (Boaler & Greeno, 2000)

Personal identity

Personal identity is concerned with who students are becoming as they participate in particular mathematics classrooms. It is "an ongoing process of being a particular kind of person in the local social world of the classroom" (Cobb & Hodge, 2008). It includes four themes when documenting the personal identities that students develop in the mathematics classrooms: "the students' understandings and valuations of 1) their general obligations and 2) their specifically-mathematical obligations, 3) their assessments of their own mathematical competence, and 4) their assessments of other students' mathematical competence" (Cobb, Gresalfi, & Hodge, 2009)

Gifted and Talented Mathematics Students

Gifted and Talented Mathematics Students refer to those students who possess a special talent and interest in mathematics (National Council of Teachers of Mathematics [NCTM], 2000).

Equity

Excellence in mathematics education requires equity. According to National Council of Teachers of Mathematics (NCTM, 2000), equity refers to high expectations and strong support for all students. It "does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (p.12)

Culture Capital of the Mathematics Classroom

Cobb and Hodge (2002) developed the notion of the Culture Capital of the Mathematics Classroom to delineate the culture of the classroom. It consists of three aspects of the classroom microculture: Social norms, sociomathematics norms, and classroom mathematical practices. First, social norms are defined as "to explain and justify their solutions, to make sense of explanations given by others, to indicate agreement or disagreement, and to question alternatives when conflicts in interpretations had become apparent." (p. 268). The second aspect of sociomathematics norms concerns the normative aspects of classroom action and interaction with respect to domain-specific knowledge (e.g. mathematics). This norm documents "what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation" (p. 268). That is, this norm emerged concerning "how one engages in a mathematical argument". The third aspect of classroom mathematical practices focuses on "the normative ways of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas." (p. 269).

Organization of the Study

This research is organized into five chapters. Chapter one presents an introduction to this study, which includes the statement of the problem, the purpose of the study, research questions, significance of the study, limitations and delimitations, and definition of terms used in the study. Chapter II reviews the literature relevant to the study. It includes sociocultural views of mathematics students' learning in classroom settings and the identities that they develop. Given that the culture plays important roles in shaping or being shaped by students' identity formation, the literature of culture in the mathematics learning is included, followed by a description of the relationships among culture, identity, and mathematics learning in classrooms. Next, I describe the influence of contextual forces on mathematics students' learning practices and identities. To understand the gifted and talented mathematics students' learning in specialized residential settings, I include a review of literature related to their academic self-concept in a residential learning environment. Chapter III describes the methods and procedures used to conduct this study. This chapter includes the context and participants, data collection approach and procedures, data analysis procedures, and strategies I used to ensure validity and reliability. Chapter IV delineates findings of the study. The discussion of the findings, implications, recommendations, and future research are included in Chapter V.

CHAPTER II

REVIEW OF LITERATURE

In this study, I center on mathematics practices experienced by a group of students in a specialized residential mathematics classroom. I draw theoretical perspectives from the social theory of learning (Wenger, 1998), the multi-level framework (Martin, 2000), as well as Cobb, Gresalfi, and Hodge's Interpretive Scheme (2009) to account for the meanings of mathematics learning and knowledge that was developed by the students in advanced high school mathematics courses. In Wenger's framework, learning is viewed as a social practice that occurs through social participation. In this way, "learning changes who we are by changing our ability to participate, to belong, to negotiate meaning" (Wenger, 1998, p. 226). When learning occurs in the mathematics classroom, the students "construct various of forms of mathematical knowledge as they participate in socially and culturally organized activities" (Nasir & Cobb, 2007, p. 3). The capstone in Martin's (2000) research is the multilevel of sociohistorical, community, school, and intrapersonal themes. He argues that students' mathematics socialization, identity formation, and successes are influenced by these contextual forces. By making contact with the school and intrapersonal levels of Martin's framework, Cobb, Gresalfi, and Hodge (2009) propose an interpretive scheme for guiding investigations of identity in mathematics classrooms. The interpretive scheme focuses on the particular classroom's microculture and students' perceptions about "what it means to know and do mathematics in the classroom, and whether and to what extent they come to identify with classroom mathematical activity" (p. 43). More importantly, they state that the interpretive scheme effectively relates the micro-culture established in a mathematics classroom to the personal identities that students are developing in that classroom. These components are contributing to the improvement of classroom processes of teaching and learning, as well as the understanding of the issues of equity and diversity in mathematics education.

In the following discussions in the review of literature, I summarize important works pertaining to this study. I will focus on and highlight themes running through the literature that relate to mathematics students' learning situations and identities from a sociocultural perspective.

Culture in Mathematics Learning

Historically, researchers in mathematics teaching and learning have been influenced seminally by two disciplines: mathematics and psychology (Kilpatrick, 1992). As a result, such theories of learning as behaviorism or constructivism have been dominant within mathematics education for a long period of time (Cobb, Wood & Yackel, 1992). Cognitive theorists tend to concern and then investigate how the individual reorganizes its activity (Cobb, 2007). In the past twenty years however, scholars have started to (re)think about the social and cultural origins of knowledge and the nature of mathematics, and have viewed mathematical activities as inherently social and cultural in nature (Lerman, 2001; Nasir, Hand, & Taylor, 2008). In other words, whereas cognitive theorists focus upon the attributes that individuals possess, sociocultural theorists pay significant attention to "the ways in which those attributes play out in interaction with the world" (Boaler, 1999). From a sociocultural perspective, learning is viewed as a social practice that occurs through social participation (Wenger, 1998). The participation is not just in the engagement in certain activities with a certain group of people, but also “a more encompassing process of being active participants in the *practices* of social communities and constructing *identities* in relations to these communities” (p.4). Thus, learning involves “participation of the individual-in-cultural-practice” (Cobb, 2007, p. 22).

As people all over the world with diverse cultures have developed various methods to navigate in the field of mathematics and have engaged in mathematical activities to solve all kinds of problems in their daily lives (Geddes & Fortunato, 1993; Zaslavsky, 2002), mathematics education can be viewed as a cultural phenomenon (Bishop, 1988). For example, as Bishop states, mathematics is "a cultural product which has developed as a result of various activities" (p. 182), and such fundamental activities as counting, locating, measuring, designing, playing, and explaining are all part of that cultural product. Consequently, the nature and role of culture in mathematical learning is paid increasingly significant attention because of the important role in mathematics education with all its complexities and contestations (Presmeg, 2005).

There are multiple ways to conceptualize the culture (Bishop, 1988). For example, Boaler (2007) presented “habits we acquire” and “the houses we inhabit” metaphors for culture that was initially expressed by Varenne and McDermott (1999). The first metaphor is to reflect that students’ culture or class may be thought of as a set of habits that are acquired or a way of life that is characteristic of a bounded community, which individual members carry with them from place to place cohesively across time (Boaler, 2007; Cobb & Hodge, 2008; Nasir & Hand, 2006). This metaphor characterizes individuals as somewhat passive carriers of culture and thus represents the static nature of culture. As a consequence, researchers who tend to help remedy achievement gaps consider that some marginalized groups of students have not socialized into the dominant culture (Boaler, 2007).

In contrast, the second metaphor proposes the shifting and relational nature of culture (Boaler, 2007). Namely, the culture is a response rather than a characteristic. It becomes visible in some places whereas invisible in other places. As Nasir and Hand (2006) stated that "Culture is both carried by individuals and created in the moment-to-moment interactions with one another as they participate in (and reconstruct) cultural practices" (p. 458). In this sense, the culture of a group is something in a constant process of creation and recreation in moments as people "do" life. Importantly, culture not only shapes individual values and behaviors, but that individuals in turn modify and mold their culture as well (Boaler, 2007; Cobb & Hodge, 2008; Nasir & Hand, 2006; Timm, 1996). This conception of culture focuses upon people's multi-membership in a variety of small groups or communities of practice (Lave & Wenger, 1991; Wenger, 1998) to which the individual belongs. Hence, it is very important to focus on culture rather than race when addressing issues of diversity (Cobb & Hodge, 2002; Cobb & Hodge, 2008; Hodge, 2006). For example, researchers who adhere solely to achievement gaps by comparing the mathematics achievement of different racial groups could not tell the whole story. A conceptual understanding of the underlying problems and issues for the inequity is needed (Ladson-Billings, 1997; Nasir, 2002).

Views on Culture in the Mathematics Classroom

The mathematics classroom is considered, as a local "community of practice" (Wenger, 1998), a space (Cohen & Lotan, 1995), or "a figured world" (Holland, Lachicotte, Skinner, & Cain's, 1998) where students engage in the mathematical activities. The role of culture is becoming more prevalent in mathematics learning that is realized in the mathematics classroom. For instance, Boaler and Greeno borrowed the anthropologic concept of "figured worlds" (Holland, Lachicotte, Skinner, & Cain, 1998) to characterize a mathematics classroom. From their perspective, "agents come together to construct joint meanings and activities" (p. 174) in the figured worlds.

> A mathematics learning environment could be regarded as a particular figured world because students and teachers construct interpretations of actions that routinely take place there…the importance of this label for researchers of mathematics education, resides in the characterization of a mathematics classroom as an interpretable realm, in which people fashion their senses of self….the mathematics classroom may be thought of as a particular social setting- that is, a figured world-in which children and teachers take on certain roles that help define who they are. (p. 174).

As Cobb (2000) points out, "Viewed against the background of classroom social and sociomathematical norms, the mathematical practices established by a classroom community can be seen to constitute the immediate, local situations of the students' development" (p. 73). In this view, a mathematics classroom is, as a community, part of the context in which classroom interactions take place. As a result, Nickson (1992) considers cultures of a mathematics classroom as "the invisible and apparently shared meanings that teachers and pupils bring to the mathematics classroom and that govern their interaction in it" (p. 102). He further implies that these invisible or "unseen" key aspects of culture are beliefs, knowledge, and values, and the culture of a mathematics classroom will "depend on a very large extent on these hidden perspectives of teachers and pupils in relation to the subject" (p.102). This description does provide us with knowledge of what counts as mathematics classroom culture. However, this statement does not provide a concrete framework to conduct empirical analysis as mathematics educators strive to help improve instructional design and implementation in the mathematics classroom.

A line of research has investigated the different types of culture in mathematics classrooms. Particularly, a number of key features of mathematics classroom have been identified to either foster or prevent students' understanding of domain knowledge (Nasir, Hand, & Taylor, 2008). For instance, to characterize the practices established in the classroom, Cobb and Hodge (2002) developed the notion of cultural capital of the mathematics classroom that consists of three aspects of the classroom microculture: The first social norms aspect of cultural capital of the mathematics classroom focuses on how students learn to be students in school. Those students have the responsibility "to explain and justify their solutions, to make sense of explanations given by others, to indicate agreement or disagreement, and to question alternatives when conflicts in interpretations had become apparent." (P. 268). The second aspect of sociomathematics norms concerns the normative aspects of classroom action and interaction with respect to domain-specific knowledge (e.g. mathematics). This norm documents "what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation" (p. 268). That is, this norm emerged concerning "how one engages in a mathematical argument". The third aspect of classroom mathematical practices focuses on "the normative ways of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas." (p. 269).

The sociomathematics norms have salient relation with the issue of identity. Cobb and Hodge convincingly demonstrate the importance and complexity of who gets to legitimatize the mathematical solution. In general, students who participate both in-school and community mathematical practices do not consider these practices to have the same value or worth. They may feel that their everyday and home math is inferior, whereas school math is more highly valorized (de Abreu & Cline, 2007; Nasir, Hand, & Taylor, 2008). Consequently, the issues of how students negotiate the gap between cultural and domain knowledge in math, how they reposition themselves in mathematics, and what these practices mean for who they are, bring to the fore the issues of students' identity as mathematics learners (Martin, 2000, 2007). The works of Cobb and Hodge (2002) and Boaler and Greeno (2000) on classroom mathematical structures illustrate "how classrooms function as and in social cultural space to afford and constrain certain ways of doing mathematics and becoming a mathematics learner" (Nasir, Hand, & Taylor, 2008, p. 203) and how they see themselves as mathematics learners and doers (Anderson, 2007). In this view, the relationship between culture and identity becomes apparent in that the mathematics classroom cultures are "contexts in which students could display and develop their sense of mathematical competence" (Hodge, 2008).

Identity and Mathematics Learning

A range of studies on issues of identity have traditionally been located in the research area of sociology and psychology (Boaler, 1999; Martin, 2007). Researchers have studied identity from multiple of theoretical perspectives (Nasir, 2002). From sociocultural perspective, Wenger (1998) delineates identity as “a way of talking about how learning changes who we are and creates personal histories of becoming in the context of our communities” (p. 5). As he indicates, people define who they are by the ways they participate and reify themselves or are reified by others, by their community membership, by their learning trajectory—where they have been and where they are going, by reconciling our multimembership into one identity, and by negotiating a relation between the local and the global discourse communities (p. 149). This definition emphasizes the profound connection between identity, learning and practice. In his view, learning changes who we are by changing our ability to participate, to belong, to negotiate meaning in the practices of a particular community. Therefore, individuals come to know who they are as learners due to their participation with others in the experience of life (1998).

Martin (2000) noticed that no studies in these areas engage in analyses of identity development within the context of mathematics learning, consequently he and other researchers in mathematics education have become dedicated to seeking the conceptual framework in order to analyze the issues of identity in the mathematics classroom. The reason for doing this was to

> Bridge content and context by enabling us to gain a greater understanding of the process of how mathematics and its importance comes to be situated in the lives of students, how some students become marginalized, how this marginalization in mathematics is rooted in their marginalization as ‘minorities’, and how students can build on an awareness of their social struggles and history to overcome barriers imposed on them as they attempt to become doers of mathematics (Martin, 2007,p. 158).

Therefore, the notion of identity has become increasingly common in the mathematics education research literature in recent years (Boaler, 1999; Cobb, Gresalfi, & Hodge, 2009; Cobb& Hodge, 2002; de Abrea, 1995; Gee, 2001; Gustein, 2002; Martin, 2000, Stard & Prusak, 2005).

In the area of mathematics learning community, Wenger's (1998) view of identity serves as the basis for the development of different notion of identity. For instance, some researchers view identity as a relational and fluid construct whereas others define identity as narrative. However, they have agreement that identity is not an attribute of a person but it shapes and is shaped by the social context (Hodge, 2006; Martin, 2007; Nasir, 2002). Several researchers' works illustrate the first type of identity. For example, Hodge (2006) considers identity as an individual's perceptions of how others view them in interaction. She draws on Lave and Wenger's (1991) characterization of learning as participation in the activities of a particular community to contend that the identity formation and participation are co-existing. That is, students' identities as doers of mathematics are developed as they participate in the activities in mathematics classroom. Martin (2007) developed the concept of mathematical identity from his research on African American mathematics students. He claims that "the mathematics identity refers to the dispositions and deeply held beliefs that individuals develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives" (p. 150). According to him, students must have the ability to successfully "negotiate the boundaries, reposition themselves in mathematics and reconstruct their identities" (Martin, 2007, p. 148). Otherwise, students' identities might be contradictory to the expectations of the mathematics classroom community. He also determined that a range of agents such as teachers, peers, parents, community members, and societal forces come to influence students' mathematical identities through socialization processes (Martin, 2007).

Others such as Sfard and Prusak (2005) conceptualize identity as narrative. They attempted to operationalize identity in order to make it become truly useful in learning. According to them, identities are equated with stories about persons. Learning is viewed as the only way for those who wish to bridge the actual identities and designated identities. The actual identities are about the actual state of affairs, and the designated identities are future state of affairs. For instance, a mathematics student for whom being a high-ability problem solver is part of his designated identity may refuse to ignore the mathematical thinking and reasoning skills.

Relationships among Culture, Identity, and Mathematical Practices

The development of identity is linked to learning in that learning is about becoming as well as knowing (Nasir, 2002; Nasir, 2007). Mathematical learning includes "the cultural entailments of what it means to be and become a mathematics learner within a particular community" (Nasir, Hand, & Taylor, 2008,p. 194). In this sense, learning is viewed as a process of becoming as individuals engage in mathematical learning. It highlights that the formation of identity is mediated or shaped by the culture of the mathematical classroom as students participate in cultural practices. To gain greater understanding about relationships among identity, culture, and mathematical learning, mathematics education researchers have explored the extent to which students feel a sense of affiliation to mathematics in the classrooms (Cobb, Gresalfi, & Hodge, 2009; Gresalfi & Cobb, 2006; Nasir & Saxe, 2003; Nasir, Hand, & Taylor, 2008).

In Boaler and Greeno's (2000) work, they focus on how the cultural nature of the math classroom environment influences mathematics students' identity development in two contrasting classrooms. In their study, two groups of students painted the "figured world" with markedly different pictures in both reformed and conventional mathematics lessons. The first group of students presented their worlds as relational, communicative, and connected whereas the second group perceived their worlds as structured, individualized, and ritualized. For instance, the features of the reformed classroom, according to the students, include giving them the opportunity to drive meaning through discussion, to be mutually committed and accountable to each other for constructing understanding actively, to build positive relationships with the teacher and other students, and to center around a collaboration of ideas within the classroom. In contrast, students in the conventional classroom were encouraged to attend carefully to the words and practices of their teacher and to watch the teacher's demonstration of procedures instead of being given authority to discuss the mathematics and to develop and evaluate their mathematical methods. One of the striking aspects of the findings was that if students in conventional classes needed to develop identities that were compatible with a procedural teaching style then they would have to surrender the human agency. Another striking aspect was that many students in the conventional class appeared to reject mathematics because the teaching style was incompatible with their conceptions of self and they did not have the desire to negotiate such boundaries. In contrast, the students in the reformed class expressed their enjoyment of mathematics and showed their desires to continue to pursue mathematics at the advanced level.

When Boaler (2002, 2007) conducted a longitudinal study about students' participations and their math achievement in two secondary schools in England, her findings showed that the style of teaching had much to do with the types of mathematical identities with respect to gender issues. Specifically, the girls in the Amber Hill School, where teachers followed a traditional, procedural approach, attained lower mathematics grades than those of the boys at the same school. Furthermore, the in-depth interview revealed that the girls who were capable students three years prior to attending the Amber Hill School became disaffected about mathematics in the traditional classroom. On the other hand, there were no noticeable grade differences between girls and boys in the Phoenix Park School, where an open-ended, project-based approach was adopted.

These two studies showed that students' mathematical identities can be viewed as "responses to particular teaching environments" (Boaler, 2007, p. 31). This perspective clarified that identity is a relational construct and it was shaped by the culture in the mathematical community of practices.

Nasir's study provided another example of research that focused on students' constructions of identities with respect to cultural and domain knowledge in mathematics. For example, she described the striking findings when she presented average and percentage problems (see below) to middle and high school basketball players in the school mathematics worksheet and in basketball game format.

Basketball Format:

1) Say you are at the free-throw line. You take 11 shots and you make seven of them. What's your percentage from the line?

2) In the first game of the season, you score 15 points. In the second game you score 20 points. In the third game you score 10 points. What is your average score for those three games?

School Format:

1) 7/11 =——%

2) Students were shown a list of the numbers with a blank box in which to write the average of (15, 20, 10). Instructions were written as "Calculate the average for these sets of numbers and write the solution in the box".

She noticed that students did well on all of the problems when they were asked to solve the basketball problems first. Yet, when they were asked to solve the school problems first, they scored lower on two of the problems. Nasir then argued that basketball players' solutions and strategies imply a discontinuity between students' math knowledge in real world and the type of mathematics classroom activities many students are exposed to. These response patterns also point out that "culture can become salient (even if it is not recognized as so) in the math classroom" (Nasir, Hand, & Taylor, 2008, p. 189). That is, the basketball players possessed knowledge about average and percentage that was inaccessible in the math classroom in that their teachers did not know their different ways of understanding and interpretation of such knowledge.

Overall, the work highlighted above brings the relationships between culture and identity to the fore in the mathematics classroom. The recognition of both mathematics knowing as a cultural activity and the mathematics classroom as a "community of practice" (Lave & Wenger, 1991) shifts the focus of research. Namely, researchers who are interested in equity issues pay close attention to relationships among culture, identity, and school mathematical learning as well as the students' ways of acting instead of attending to the students' achievement based on race (Nasir, Hand, & Taylor, 2008). The researchers also highlight that interactions in the classroom can lead to who participates in the regeneration of cultural capital of the mathematics classroom (Boaler & Greeno's 2000; Cobb, Gresalfi & Hodge, 2009) and enhance students' successful learning.

Interpretive Scheme as Conceptual Framework

In a recent study of a group of eighth-grade mathematics students, Cobb, Gresalfi, and Hodge (2009) define the construct of interpretive scheme. They describe interpretive scheme as "an analytic approach that focuses explicitly on the relation between the microcultures established in particular classrooms and the students' developing personal identities in those classrooms" (p. 63). They argue that this "relatively fine-grained approach moves beyond global characterizations of classrooms as traditional or reform in nature" (p. 45), instead, this approach helps understand students' identification with mathematical activity and the status quo of engagement in the mathematics classrooms where the instruction is consistent with current reform recommendations. Identities in interpretive scheme consist of two aspects: normative identities as a doer of mathematics, and personal identity.

The normative identity of their interpretive scheme focuses on general and specifically mathematical obligations. As an element of the mathematics classroom social structure, mathematical classroom obligation defines the role of an effective mathematics student in that classroom. As they clarified, the general classroom obligation documents "the forms of agency" that students are able to exercise, "how authority is distributed", and thus "to whom students are accountable" (p. 63). The key point to note is that the term "authority", as applied by Cobb, Gresalfi, and Hodge to the mathematics classroom, refers to "who's in charge" in terms of making mathematical contributions in the classrooms and is closely linked to the ways in which students are able to exercise agency in the classroom. "Agency" is viewed as an "amount and focuses on the ways in which students can legitimately excise agency in particular classrooms" (p. 45) and encompasses two different forms: conceptual agency and disciplinary agency. The conceptual agency concerns the ways in which students choose methods, develop meanings and relations among mathematical concepts and principles, whereas disciplinary agency involves using established mathematical solution methods (Cobb, Gresalfi, & Hodge, 2009). These two facets of classroom general obligations are interrelated closely in that, if students had opportunities to exercise conceptual agency, they would practice at understanding the various mathematical tools that are used to solve problems, and hence, they are exercising conceptual agency. Similarly, if students are not given an opportunity to determine the correctness of a solution, they would have little experience to make justifications or refutations of particular mathematics claims, and therefore, they only exercise disciplinary agency.

Building on Lampert (1990) and Yackel & Cobb's (1996) work, Cobb et al (2009) define the specifically mathematical obligations as centering on two norms: norms for mathematical argumentation, and normative ways of reasoning with tools and written symbols. That is, the norms for what counts as an acceptable mathematical argumentation and mathematical understanding, what counts as the normative ways of reasoning with tools and written symbols, and what are the normative purpose for engaging in mathematical activity. From this perspective, students have to fulfill specifically mathematical obligations in order to be considered as mathematically competent students.

The personal identity of the interpretive scheme focuses on students' understandings and valuations of their general classroom obligations and specifically mathematical obligations and its grounds. This documentation helps one to understand why the students are making these different valuations of their classroom obligations. According to Cobb, Gresalfi, & Hodge (2009), the students' general classroom obligations and specifically mathematical obligations are formed "in the course of the ongoing classroom interactions" and students contribute to "the initial constitution and ongoing regeneration of the normative identity" (p.46). Therefore, students might come to identify with their classroom obligations, merely cooperate with the teacher, or resist engaging in classroom activities as they participate in mathematics classroom activities. It includes four themes when documenting the personal identities that students develop: the students' understandings and valuations of their general obligations, their specifically-mathematical obligations, their assessments of their own mathematical competence, and their assessments of other students' mathematical competence (Cobb, Gresalfi, & Hodge, 2009). Specifically, the analysis includes students' views about the ways in which authority is distributed and the ways in which the students can exercise agency; views about what they are accountable for mathematically; and students' perceptions of their own and other students' mathematical capabilities and underlying issues of status and power in mathematics classrooms.

Contextual Forces and Students Mathematics Learning

When students engage in the pursuit of mathematics subject matter—a socially meaningful enterprise—the community of mathematical practices then becomes resources for organizing students' learning, and the contexts in which to manifest their learning through an identity of participation (Wenger, 1998). In other words, contextual resources make their way into the classroom to affect mathematics learning. According to Martin's (2000) multilevel framework, students' persistent mathematics learning experiences and mathematics identity can be powerfully and profoundly influenced by sociohistorical, community, school, and intrapersonal forces. The sociohistorical level of his framework documents the historically based discriminatory policies and practices that have prevented African-Americans from becoming significant participants in mathematics learning.

The community level focuses on the cultural and community beliefs African-American parents and community members have about their mathematics abilities, their motivations for learning mathematical knowledge, their beliefs about the instrumental importance of mathematics, their relationships with school officials and teachers, and their socioeconomic goals and expectations for themselves and their children (p. 31). As he clarified, students' views about the importance of mathematics learning and knowledge are influenced by their parents' and other community members' mathematics learning experiences.

The school level of his framework primarily focuses on the negotiation of social and mathematical norms in the classroom. As he contends, the negotiation was influenced by teachers' beliefs, the choices of content and curricular practices, students' beliefs about mathematics, and the aspects of the dominant student culture that challenge and resist the chosen classroom and curricular practices.

The intrapersonal level of the multilevel framework delineates agency and mathematics success in light of contextual forces they encountered. It includes students' personal identities and goals; perceptions of school environment, peer groups, and teachers; beliefs about mathematics capability, and the instrumental importance of mathematics knowledge.

Walker (2005) developed the notion of students' rich academic community at the community and school level based on Martin's (2000) groundbreaking work. Walker investigated students' webs of support for their successful mathematics work which includes students' school, family and academically supportive peer groups. According to her, these interrelated factors of students' rich academic community serve as resources to help them study and understand meaningful mathematics concepts and ideas. Most importantly, her study provides evidence that students create and sustain their extensive academic communities that are effective in promoting mathematics achievement.

Gifted Students in a Residential Setting

There is an increasing emphasis on how most effectively to educate gifted-and-talented (G&T) students in residential schools. Whereas gifted and talented students have exceptional educational experiences in residential setting, these students, who are the "best" at their home high school, seem to experience a difficult adjustment when they move to residential settings where all the participants are outstanding students (Marsh, Chessor, Craven, & Roche, 1995). Therefore, they simultaneously confront the challenges of adapting to a new community (Cross & Swiatek, 2009; Visser, 2004). The adjustment may be particularly difficult if the students derive satisfaction primarily from beating everyone else rather than improving their levels of mastery and achieving new personal bests in their study (Marsh, Chessor, Craven, & Roche, 1995).

This complex situation has drawn great attention from researchers in the past decades. In particular, researchers have paid attention to the changes in *academic self-concept* (Cross & Swiatek, 2009; Goldring, 1990; Kulik & Kulik, 1991; Marsh, 1987; Marsh, Hau, & Craven, 2004) of gifted students in residential settings. Yet, the results have been inconsistent. For example, although some researchers (Goldring, 1990) found positive results of participation in G&T classes through the meta-analyses, Coleman and Fults (1985) found that participation in G&T programs led to a decline in total self-concept for those students in the bottom half of their class. Marsh and his colleagues not only confirmed Coleman and Fults' results but also found that students of all ability levels participating in G&T programs experienced a significant decline in three components of academic self-concept (reading, math, school) and no effect on nonacademic self-concept on the basis of Marsh's construct of big-fish-little-pond effect (BFLPE) (Marsh, 1987; Marsh, Chessor, Craven, & Roche, 1995). The construct of BFLPE initially developed by Marsh and his colleagues (Marsh, Chessor, Craven, & Roche, 1995) refers to a situation in which equally gifted students have lower self-perceived academic skills and academic self-concepts when they compare themselves with more talented students, and higher self-perceived academic skills and academic self-concepts when they compare themselves with less able students (Marsh, 1987). Yet, they expected further research on G&T programs specific to particular academic domain by using the expanded BFLPE paradigm. For instance, they might expect different results for mathematically able students.

In spite of the possible negative effects of G&T program on students, research has demonstrated that these students often learn coping behaviors to navigate the social and academic milieus of their residential environment. Because students have agency (Boaler, 1999; Martin, 2000), they have the capability to respond effectively to the new residential conditions by modifying their coping strategies based on their strong personal perceptions, academic and social goals (Cross & Swiatek, 2009; Martin, 2000). Marsh and his colleagues (1987, 1995) also suggested that the negative effects of the BFLPE could be counteracted by program- proposed strategies to counteract the BFLPE and maximize the benefits of gifted and talented programs:

1. Expanding the basis for selecting students to include criteria other than standardized test scores.
2. Avoiding a highly competitive environment, typical in some G&T programs, that encourages the social comparison processes underlying the BFLPE.
3. Developing assessment tasks in which students are encouraged to pursue projects which are of particular interest to them.
4. Providing students with feedback in relation to criterion reference standards and personal improvement over time rather than comparisons based on the performances of other students in the G&T class

5. Emphasizing to each student that he or she is a very able student and valuing the unique accomplishments of each individual student so that all students can feel good about themselves
6. Selecting or training teachers who are sensitive to the special needs of G&T students (p. 315).

When Cross & Swiatek (2009) conducted a study in a state-funded residential academy for academically gifted students, they found that students saw themselves as better accepted by their academy highly able classmates than they had been by peers in their previous high schools, and became slightly more humble about their academic ability. Yet, they did not significantly change their self-perception in that domain. They also revealed that these students tend to reevaluate their views of self relative to their academy peers to a small degree, to monitor their abilities and achievement relative to all of the other students who cross their paths as they attend a residential academy. In addition, these experiences extend beyond the classrooms to informal meetings among the academy students of residential settings. Through this unique 24-hour-a-day access in residential schools, the students may become more comfortable when interacting with peers academically and socially, and therefore, gain a sense of acceptance that they would never have been afforded in their home high schools (Cross, Adams, Dixon, & Holland, 2004).

Chapter Summary

I have reviewed literature in terms of culture, identity, contextual forces and mathematics learning, as well as talented and gifted students in residential settings. I discussed how macro-and micro-culture in the broader community and mathematics classroom influence the identity that was developed by mathematics students. My perspectives are based on the social learning theory by Wenger (1998), the multilevel framework by Martin (2000), and interpretive scheme by Cobb, Gresalfi, and Hodge (2009), all of which emphasize the importance of the participations, actions, and interactions in the identity development of mathematics students, and provide a solid theoretical foundation for the identity of mathematics students as well as for their learning experiences in the mathematics classroom. In order to discuss the residential learning environment of talented and gifted students that I focused on, I have included literature on the residential settings as well as students' academic self-concept in such a unique setting. The next chapter will present the methods and procedures used to collect data for this study.

CHAPTER III

METHODOLOGY

Chapter one presents the overall theoretical framework and research questions underlying the present study. A review of the main research studies in mathematics education shows a need for research that uses an integrated research design (National Math Panel, 2008). Consequently, this study employed both quantitative and qualitative methods. The purpose was to formulate a more comprehensive understanding of what factors influence the development of students' identities in the mathematics classroom. This study sought to explore the kinds of identities that mathematics students develop in a specialized residential high school (SRHS) mathematics classroom context. The natures of interactions between the home high school (HHS) and SRHS experiences and their impact on development of students' identities were also investigated. The study investigated two questions:

1) What kinds of identities are developed in SRHS classrooms?
2) How do the students' SRHS experiences interact with HHS experiences to affect the development of mathematical identity?

The quantitative and qualitative data were collected sequentially. The research consisted of two phases. The initial phase focused on the students' experiences at their HHS mathematics classrooms. These data were collected via a survey given to the students at the beginning of study (See appendix A).The latter phase examined their experiences in a SRHS mathematics classroom by utilizing such naturalistic qualitative methods (Hatch, 2002; Lincoln & Guba, 1985) as classroom observation, interview, and documents review. Therefore, rich and contextualized information will be provided to gain knowledge about identity that students develop in the mathematics classrooms. The rationale for selecting above stated methods is that the school and classroom culture must be given much attention from the perspective of the situated view on culture and identity. That is, both students' HHS and new SRHS mathematics classrooms experiences were to be taken into consideration when examining the relationships among school settings, mathematics classroom culture, and students' identity formation.

In the following sections, I discussed design elements for the study such as research design, contexts, participants, data collection strategies, data analysis procedures, and the trustworthiness of anticipated findings.

Research Design

Mixed research methods (Creswell, 2003) were used to conduct the study. The sequential transformative strategy (Creswell, 2003), which includes two distinct data collection phases, was utilized. Phase one was a quantitative study that looks at students' experiences and processes in the HHS mathematics classrooms. Following this analysis, I looked within the SRHS mathematics classroom using qualitative/case study methods to better understand the dynamics of development of students' identities. As Creswell implied

> "By using two phases, a sequential transformative researcher may be able to give voice to diverse perspectives, to better advocate for participants, or to better understand a phenomenon or process that is changing as a result of being studied" (p. 216).

There are at least two reasons for choosing the mixed approach. First, as the educational researchers noted, the survey research "determines and describes the way things are" (Gay, Mills, & Airasian, 2006, p.159). Quantitative data were gathered and then used to provide necessary and valuable information about attitudes, opinions, practices, and procedures related to mathematical learning at their HHS.

Second, in order to develop insights on how students interpret their "lives" in current SRHS mathematics classrooms, I focused on understanding students' perspectives of their mathematics education, including what they were experiencing in their current mathematics classrooms and how they would interpret these experiences. Since human behavior is significantly influenced by the setting in which it occurs (Bogdan & Biklen, 2006), the contexts, processes and meanings play a crucial role in helping find the answers to my research questions. The case study approach was used because it is a powerful, effective and special kind of qualitative work that investigates a contextualized contemporary phenomenon within specified boundaries (Hatch, 2002; Merriam, 1998; Yin, 2003).

As both Hatch (2002) and Merriam (1998) indicate, defining the boundaries, or specifying the unit of analysis is the key decision in case study design. According to Merriam (1998), one of the bounded phenomena in education was "a program, an event, a person, a process, an institution, or a social group" (p. 13). In my study, the SRHS mathematics classroom was a particular place with a specific group of high school students. Therefore, case study, again, was appropriate in this context since all students were bounded by their participation in the specialized residential school mathematics classrooms.

In summation, the major data-gathering techniques were mixed methods with sequential transformative strategy. Doing so helped me to understand participants' interpretation and construction of their identities in mathematics classroom.

Contexts and Participants

Contexts

The research was conducted over a three-month period in a specialized residential high school (SRHS), a small state-funded residential high school in the Southeast United States. The SRHS provides innovative learning experiences in a unique environment designed to meet the academic needs of gifted and talented (G&T) high school students. The school has a strong focus on mathematics and science but also offers an enriched, well-rounded, and advanced curriculum including the arts, foreign languages, and humanities. The curriculum challenges students with equivalent college courses (e.g. Calculus), extensive research programs, and integrating research, writing, critical thinking, interdisciplinary projects, and technology throughout the curriculum (Stamps, 2008). Additionally, school leaders have used regional resources to bring in a variety of extracurricular experiences and opportunities. Student population consists of those who have strong knowledge, aptitude, skill level, and proclivity toward the Science, Technology, Engineer, and Mathematics (STEM) subject areas. All of them are concurrently enrolled in their home public schools. The students live in cottages and take mostly college-level mathematics and science courses taught by both the school's faculty and university faculty. They are placed into mathematics courses after taking placement tests upon their arrival at SRHS. The students have access to university facilities, including libraries and science labs. They also take classes and engage in research projects with scientists in a national lab.

In spring 2009, the SRHS served a group of 37 students in Grades 11 through 12. Although the SRHS had a relatively small number of students, it was highly diverse. As Table 1 illustrates, the ethnic makeup of the student population for the 2008-2009 academic year was 75.6% White not Hispanic students, 15.6% Asian/Pacific Islander, and 8.9% Black not Hispanic. Table I also includes demographic information dating back to 2007, when the new residential high school began. In the spring 2009, the junior student population made up 67.6% of the SRHS total population. As a whole, Whites made up the largest group, representing the highest percentage. Table 2 illustrates the numbers and percentage of juniors in the spring semester 2009.

The SRHS initially began by drawing gifted and talented (G&T) rising juniors from the applicant pool (Twenty-four for year 2007; Thirty for year 2008) with high GPAs, High Achievements test (e.g. PLAN, ACT, &PSAT) scores, and varied school locales. Table 3 provides the PLAN Composite means, ACT Composite means, and PSAT selection index mean for SRHS 2008 juniors. All juniors of 2008 came from medium to large-sized home high school (HHS) where the school size ranges from 261 students to 2128 students.

It is important to take the issues of accessibility into consideration when making decisions about contexts (Hatch, 2002). He stated that “researchers should come clean with gatekeepers and participants about their research interests and intentions” (p. 46). The key gatekeeper is the executive director of the residential high school. I sought and

Table 1: Specialized residential high school Enrollment Summary: Number and Percentage of Students by Ethnic Group

	School Year	
Ethnic Group	2007-2008	2008-2009
	Number (%)	Number (%)
White, not Hispanic	17 (70.8%)	34(75.6%)
Asian/Pacific Islander	3 (12.5%)	7(15.6%)
Hispanic	1 (4.2%)	0(0%)
Black, not Hispanic	3 (12.5%)	4(8.9%)
Total	N=24	N=45

Source: SRHS demographic information.

Table 2: Specialized residential high school Enrollment Summary: Number and Percentage of Juniors by Ethnic Group

	School Year
	Spring 2009
Ethnic Group	**Number (%)**
White, not Hispanic	21(84%)
Asian/Pacific Islander	3(12%)
Hispanic	0(0.0%)
Black, not Hispanic	1(4%)
Total	N=25

Source: SRHS demographic information.

Table 3: Achievements Test Score Means for SRHS 2008 Juniors

	Means (Number of Students)
PLAN	23. 86 (23)
ACT	26 (7)
PSAT	176.17 (13)

Source: SRHS demographic information.

was given the permission to conduct my study by the executive director of the school and subsequently from the mathematics faculty.

Student Participants

In keeping with the goal of my study, to understand the identities that students develop in mathematics classrooms, all twenty-five junior mathematics students were invited to complete the survey for the initial phase of the three-month study. This was done under the auspices of the SRHS Internal Review Board (IRB) which allows for voluntary data collection from students for evaluation and study purposes.

Next, since case study methods must almost always focus on small samples in order to obtain the desired depth of information, sampling in a case study must be purposive (Gay, Mills, & Airasian, 2006). Therefore, I purposely selected the interview participants from the larger pool of twenty-eight students. The criteria used for selection included: 1) Students who had very different HHS mathematical learning experiences, attitudes, beliefs towards mathematical learning from those of the rest of students at SRHS. (2) The students who were "thoughtful, informative, articulate, and experienced with the research topic and setting" (Gay, Mills, & Airasian, 2006, p.114). As a result, a list of six students with diverse ethnic backgrounds was drawn from the population. Two students withdrew from the interview study after data collection was completed and their data was not used in this study. Two students who were on a waiting list were invited to the interview participant group. This resulted in only White and Asian students participating. The sample of students was predominantly male (4 out of 6 participants). Three of the students identified themselves as White not Hispanic and three identified themselves as Asian. Table 4 provides demographic information about the participating students.

Once the student sample was identified, I then explained the objectives of the study to these students. All expressed their willingness and eagerness to participate. Their parents were contacted by sending letters about the study together with an Informed Consent Form and were asked for their permission for their child to participate. These participants varied in ethnic backgrounds, neighborhoods of origin, and their HHS academic strengths; they were homogeneous in their high mathematical ability and devoted to mathematics learning.

Since I was not involved in any teaching tasks for juniors, I planned to focus solely on them. There were two different math classes for juniors. Ten students were

Table 4 Sample of SRHS mathematics Students (N=6)

Student Alias	Ethnicity	Current Class
Aidan	White, not Hispanic	Calculus I
Amanda	White, not Hispanic	Calculus I
Daniel	Asian	Calculus I
Jackson	White, not Hispanic	Calculus II
Sophia	Asian	Calculus II
Steven	Asian	Calculus II

enrolled in the Calculus II class and 18 students were currently enrolled in Calculus I class. The students in Calculus II were ready for studying advanced mathematics upon their arrival at the SRHS whereas the other group was not. To accommodate my desire to study the students' identity construction, I decided to include both classes in my observations.

Pilot Study

In the tradition of qualitative research, the qualitative researcher serves as the primary instrument for data collection and analysis. Therefore, whether the researcher is able to capture the participants' own words, behavior and affect, and letting the analysis emerge, whether the researcher can probe and clarify the participants' meanings and responses are crucial to the quality of the study (Merriam 1988; Yin, 2002). Since I am an English language learner, I may face particular language problems in carrying out my study. In this regard, conducting a pilot interview study is important to help me identify unanticipated issues, gain valuable experiences, and then refine my data collection plans.

Yin (2002) suggests that the “convenience, access, and geographic proximity can be the main criteria” for selecting the pilot interviews. In fall 2008, I conducted a pilot study with one senior student Jack (pseudonym) who was willing to share his perspectives after he was informed that the interview would be a contribution to my study, and that there was no impact on his grades or regular study time. Two questions were selected from the interview protocol. I designed interview questions guided by interpretive scheme, along with the consideration of the SRHS and HHS differences.

To increase the credibility of the pilot study conclusions, member check was solicited about interview data from Jack in that “this is the single most important way of ruling out the possibility of misinterpreting the meaning of what participants say and do and the perspective they have on what is going on…”(Maxwell, 2005, p. 111). Additionally, a fellow doctoral student from the Institute for Assessment and Evaluation at a local university agreed to join the pilot interview in order to assure that I was able to attend to student’s meanings and then interpret participants’ responses grounded in the data.

Reports from the Pilot Case

Overall, the pilot study was conducted successfully. I learned the lessons from preliminary study and I present the implications in this section. To ensure that I was able to attend to the participants’ words, behavior, and could probe and clarify the participants' meanings and interpretations (Merriam, 1998), my fellow graduate student carefully reviewed the four-page pilot interview transcripts which I had transcribed myself. The goals were to assess the accuracy of my transcription, including if I captured adolescent lexical and grammatical colloquialisms as well as nuances. We listened to ten minutes or so of audio-taped interview as we read the transcription together. The transcription did contain a few errors with word endings, and slight errors with exact replication of words the interviewee used. Therefore, we decided that the errors in transcription could be substantial enough to alter the study results, thus a transcription service would help to increase accuracy.

Additionally, we also discussed using less "yes/no" questions in the interview, particularly in the follow-up questions which are often spontaneously created. I also reminded myself that I needed to let the interviewee do more of the talking and to wait for them to speak. Informed by the results of the pilot study conducted in 2008, I designed the final study in the spring of 2009. The study was intended to gather students' mathematics learning experiences from SRHS classrooms and compare or contrast it with those of their HHS classrooms. The group of 25 juniors at SRHS was the whole study group and the group of six were participating interview study.

Data Collection

The major data-gathering techniques were survey, interview, observations, and documentation. In the following paragraphs, I describe each technique in detail.

Survey

Since the students came to the SRHS mathematics classroom with diverse mathematical background, beliefs, values, and attitudes, these factors must be identified and taken into consideration in order to compare it with new experiences. The primary survey is a modified version of Appalachia Mathematics and Science Partnership (AMSP) Partnership Enhancement Project (PEP) Mathematics Student Questionnaire (see Appendix A) that was piloted at SRHS in 2007-2008 academic year. I intended to gather key elements such as students' attitude and beliefs toward math, their perception of how teachers and family interact with them in terms of mathematics, as well as the different typical models of mathematical teaching and learning in the classrooms, which provide the image of their HHS and SRHS classroom mathematical practices. Hence, the questionnaire was a valuable tool because it gathered a large amount of data in a short time.

The questionnaire design was, again, adapted with the help of dissertation committee members in December of 2008. Questions regarding mathematics learning information were stated similarly as in PEP Questionnaire, with addition of one question concerning the students' assessment about general and specific obligations in mathematics classrooms. In addition, Committee members helped to reformulate some of the questions from the pilot study questionnaire which were found not to address certain mathematics classes (e.g. algebra II). The final version of the questionnaire included three structured questions with 46 items and two unstructured items (Gay, Mills, & Airasian, 2006). Most of the questions were Likert-like items based on a scale from "agree" to "disagree" and the informants had to circle the appropriate answer; the rest were open-ended questions.

Interview

Qualitative researchers tend to use such on-site data collection techniques as interviews, observation, and document analysis to discover the issues in the natural research setting. Although there are interviews in the survey research, the interview strategy is different in nature with those in the qualitative study (Gay, Mills, & Airasian, 2006; Hatch, 2002). According to Hatch (2002), while quantitative interviews are closed-ended questionnaires, forced choices, and Likert-scale categories and they are analyzed using statistical procedures, qualitative interviewers "create a special kind of speech event during which they ask open-ended questions, encourage informants to explain their unique perspectives on the issues at hand"(p. 23). Thus, the interview is a purposeful interaction and can be another basis for gathering the descriptive data in the participants' own words so that I can develop insights on how they interpret aspects of their math world (Bogdan & Biklen, 2007). Bogdan and Bilken further pointed out that researcher should take the time sampling into consideration when I conduct interviews with students in that the time would affect the nature of the data I collect. For example, schools are different at the start of the year than at the end; similarly, the morning and afternoon routine in the class can be quite different. Thus I interviewed with students at the same time on different days and weeks. Each student was interviewed at a location convenient for them.

In order to inquire and learn about the mathematical practices within the SRHS mathematics classroom and the influences such cultural practices have on the students' relationship with mathematics, each was interviewed once for a 35-40 minutes in a semi-structured interview conducted by using a protocol (see Appendix B). The protocol for the interview was developed by the researcher using the conceptual framework described in chapter two. Each of the interview questions underwent the revision before being used. The instrument was reviewed by each dissertation committee member to check the pertinence and efficacy of the questions. Based on their input, the instrument was then revised to ensure that the questions would be helpful in developing an image of the student in the mathematics classroom.

Observation

Observation is one of the fieldwork methods data collection tools (Hatch, 2002; Bogdan & Biklen, 2006). In order to understand the natural mathematical classroom environment as lived by student participants, observation is the most appropriate and effective way to collect data (Gay, Mille, & Airasian, 2006). According to Gay et al (2006), pairing observation and interviewing provides a substantial way to triangulate data. Although being a participant observer provides insights and develops rapport relationships with student participants that would not be possible for a nonparticipant observer, I was a nonparticipant observer in the classrooms in order to avoid losing objectivity (Gay, Mills, & Airasian, 2006).

Throughout the first three months of the semester, I observed 90-minute-long Calculus I and Calculus II classes for two class sessions per week. As a result, twelve sessions of the Calculus I and nine sessions of the Calculus II were observed and audio-taped. The purpose was to familiarize myself with the classroom environment and mathematics activities to better understand what influences students' perception of themselves as a doer of mathematics in the classroom. Hence, I was seeking to attend to the students' actions and interactions within the natural math classroom settings and to compare those behaviors with their interview responses. I had attempted to focus on teaching knowledge of content and pedagogy, and students' responses and behaviors in the classroom with the guidance of the Math Classroom Observation Checklist (National Council of Teachers of Mathematics [NCTM], 2000) (see Appendix C). It includes mathematical tasks, students' role in discourse, tools for discourse, and culture in the classroom (NCTM, 2000). While I was observing, I documented both descriptive information and reflective information in the field notes (Gay, Mills, & Airasian, 2006). Additionally, I used a laptop and notebook to record students' unspoken behaviors that audio could not catch. Immediately following each observation I reviewed and recorded my own ideas, reflections in the margin of the notebook.

Documentation

The documentary information is likely to be relevant to the current study. The documents were used in support of the survey, interviews and observation. Many researchers indicated that the documents can be categorized as personal documents, public/official documents, popular culture document, and email discussions (Bogdan & Biklen, 2006; Creswell, 2002; Merriam, 1998; Yin, 2002). Although the documents "may be incomplete or may not be authentic or accurate…it represents data that are thoughtful in that participants have given attention to compiling" (Creswell, 2003, p.186). In addition, the documentary information, as an unobtrusive source of information, can be accessed at a time convenient to the researcher even though it requires the researcher to search out the information in hard-to-find places. It saves a researcher the time and expense of transcribing as well (Bogdan & Biklen, 2006; Creswell, 2003; Merriam, 1998; Yin, 2002).

In this study, I turned to the official documents to illuminate students' background in their mathematical learning that I was trying to understand. Such students' records (e.g. initial applications to the SRHS) as writing essays, standardized test results, and teachers' recommendations provided me a glimpse of an individual history. Furthermore, it provided context and opportunities to inquire further into the development of the students' identities if the documentary evidence is contradictory rather than corroboratory (Bogdan & Bilken, 2006; Yin, 2002).

To obtain the "official perspective" (Bogdan & Bilken, 2006) of the academic achievement of the students in their home high schools, I communicated with both the school director and personnel, and tried to gain access to the copies of the students' initial applications, including course-taken sequences and grades, assigned essays, teachers' recommendations, and etc. In addition to initial application forms, classroom artifacts such as journals and mathematical work from classes were also shared by teachers, with the students' permission, to enlighten students' perceptions about their experiences in learning calculus.

Data Analysis

The analysis of the quantitative data can be described as happening in two stages. I analyzed the survey responses in two ways. First, since I had a small number of student respondents, I tabulated questionnaire mark responses to closed-ended questions on a spreadsheet. Then the percentages of respondents for each item were calculated. Second, for the other two open-ended questions, I coded answers according to patterns in the students' responses (Gay, Mills & Airasian, 2006).

The qualitative data analysis began at the beginning of data collection (Merriam, 1998). In this way, I analyzed the structured and formal data, themes about the field and student participants began to emerge. The goal of the interviews was to learn about students' views and perceptions on their beliefs about mathematics learning, mathematics coursework, classroom normative identity, and their personal identity. I relied on interpretive scheme to analyze interview and observation data to describe students' mathematical identity developed in the mathematics classroom (Bogdan & Biklen, 2006; Hatch, 2002). The Interpretive Scheme developed by Cobb et al (2009) consists of two specific issues: Normative identity, which includes general classroom obligations and specifically mathematical obligations; Personal identity, which includes students' understandings and valuations of their general obligations and specifically mathematical obligations, as well as students' assessments of their own and other students' mathematical competence (Cobb, Gresalfi, & Hodge, 2009). While the analysis of the data was guided by this Scheme, this did not limit the sources drawn from the literature.

Three phases of coding had been engaged during the process of analyzing qualitative data. The first was during data collection. I began to "cook" the raw data (Glesne and Peshkin, 1992) as I gathered it. After each observation or interview I listened to audiotapes and reviewed the field notes of sessions. I reflected on what was learned about students' mathematics learning experiences and then expanded upon them by writing down ideas, emergent themes, questions and reminders for next observation and interview session. These preliminary thoughts helped to explore the issues and determine trends. The second stage was upon completion of the data collection process. I organized all interview transcriptions, journal entries, documentation, and field notes into separate files for each participant. I began to identify major patterns relating to students' mathematics learning experiences by closely reading the interview transcripts and then developed categories broadly (Glesne and Peshkin, 1992; Strauss & Corbin, 1990). The possible themes that crossed multiple interview questions were developed. That is, students' responses about their general and specifically mathematics obligations both in HHS and SRHS, their understanding and assessment of their own and other students' mathematical competence were identified. Also, from this stage of analysis, observations audiotapes and field notes, journal entries, and documentation were used to fill in details for triangulation purposes. In the third stage, a second coder analyzed all interviews to offer alternative interpretations of students' responses from different perspectives. The coding results were compared with those of the author. When the major themes had been agreed upon, the codes were generated in order to enable categorization of students' interviews. As the interview was coded, I again read through the data sources to check for additional categories.

In all, I compared, contrasted, and evaluated the data to enlighten or contradict existing theory (Lincoln & Cuba, 1985; Yin, 2002). I organized the codes according to the research questions that guided this study: What kinds of identities were developed among students in both HHS and SRHS math classrooms? What were the impacts of nature of SRHS on students' mathematics learning? How did these experiences interact with those of HHS to affect the development of mathematical identity? In particular, I sought out comparative cases to explore surfacing issues, and constructed a revised framework for supporting development of mathematical identity.

Trustworthiness of Anticipated Findings

Validity and reliability are major concerns for any research. In a qualitative study, the trustworthiness of research results is accounted for either with what Merriam called "internal validity, reliability, and external validity" (Merriam 1998), or Gay and his colleagues referred as "credibility, dependability, confirmability, and transferability" (Gay, Mills& Airasian, 2006). According to qualitative researchers, internal validity is the extent to which research findings will accurately gauge what the researcher is trying to measure (Creswell, 2002; Gay, Mills& Airasian, 2006; Maxwell, 2004; Merriam, 1998). For this study, I enlisted a number of procedures to attain the standard of validity and reliability. First and foremost, I did "prolonged participation engagement" (Lincoln and Guba, 1985) in order to gain in-depth understanding of the mathematics classroom situation. Also, the eight week long classroom observation allowed me to make comprehensive records of all classroom activities. During this time I developed a trusting relationship with student participants to assure them that I would serve their interests and respect their inputs (Lincoln & Cuba, 1985; Merriam, 1998; Yin, 2002). In addition, multiple sources such as observations, interviews, and documents were drawn to secure the internal validity of the study. A fellow graduate student, who was involved at several coding stages, critically analyzed the coding process done by me and reviewed my interpretations. Second, in accordance with Merriam's definition of reliability, the research findings need to be examined to determine consistency with the data collected. I addressed this issue by establishing an audit trail of data archives, reflections, coded transcripts, outlines of coding scheme, and details about research site context, as well as student participants selection and descriptions. Additionally, I considered myself a nonparticipant classroom observer and thereby distanced myself from the student participants.

Lastly, the trustworthiness of this study was increased by providing for its external validity. I strived to include enough descriptive information so that other researchers might be able to judge how closely their research situations match the situation in this study, and therefore, determine the applicability of the findings (Merriam 1998). In particular, I described specifically the high ability mathematics students at SRHS compared with their counterparts in regular high school so that readers can make comparisons with their own situations. Furthermore, the purposeful sampling was used to secure the broadest diversity of student participants possible.

Chapter Summary

This chapter addressed methodology and procedures of the study. Using the knowledge of both quantities and qualitative research, I collected data through the survey of students' home high school mathematics learning experiences, interviews of students regarding SRHS mathematics learning experiences, classroom observations of the lessons at SRHS, and documentations with students' background in learning mathematics. I conducted data analyses simultaneously with data collection while maintaining flexibility in order to refine the meaningful themes. The next chapter will present the analysis of the data collected through the study.

CHAPTER IV

RESULTS

The findings from the questionnaire, observations, interviews, and documentation from both the whole and interview group are presented in this chapter. In the first two sections, I report the findings about experiences of mathematics learning in both home high school (HHS) and specialized residential high school (SRHS), respectively, which includes classroom organization, students' roles in the discourses, the kinds of mathematical tasks, and the ways in which they completed these tasks as well as the resulting students' perceptions of themselves and other students in relation to mathematics. In the following sections, I present the data regarding the six SRHS students' perceptions of their learning experiences, as demonstrated on a number of different indicators that characterize the practices in both HHS and SRHS.

The Home High School

Classroom Structure

Seventeen out of twenty-three students reported their class schedule types. According to self-reported data, about 59% percent of the students were in regular class schedule and 41% of the students were in block schedule. Students were asked to describe a typical Algebra II class period in their HHS and about 83% of the students perceived that they were learning mathematics through lectures. Specifically, the class usually began with teachers either reviewing homework for quite a long period of time or posting a daily math ACT problem and then presenting the main idea of the new lesson by going over practice examples with straightforward explanations. The students were then assigned homework and had time to work on it for the rest of the class. The following comments provided a clear indication of this teaching style:

> Usually we would start class with the teacher going over the homework if someone had a question. Then she would correct the homework and we would begin to copy that day's notes. When we were finished copying, she would explain the concept and we would do several examples as a class. Whatever time we had left, we could use to work on the homework she assigned for the next day. (Jane, Calculus I)

> Usually we would come into class and there would be a daily math practice ACT problem wrote on the board. After the bell had rang, everyone soon after finished the practice ACT problem. Once everyone was finished the teacher would go over the correct answer. After attendance and all was taken we would copy notes from the section from which we were on that day. Some days we did only one section, other days we did up to three section, once notes were taken, homework was assigned and if times remained in class we were allowed to start on it. (Ethan, Calculus I)

Among these students, 30% of the students explicitly identified that their teachers took them through the steps of problem solving with little interaction and with little thought on their part. The same group of the students also indicated that the teachers sometimes gave vague explanations and answers. That is, the teachers could not convey their ideas and mathematics principles to the students in a way that they could understand easily. Students captured the image of this regard:

> I was taking it for a week but the teacher didn't really teach that well. I mean, he's a very brilliant man but he just didn't-just uh, convey the information to the students very clearly. (Aidan)

> My math teacher was smart but horrible as a teacher and very slow. After this (homework review) we would review more or learn something new. All she would do is go over a few examples which were very boring. We always ran out of time. This was an honors class by the way… Basically we would do review problems the entire class. It was hard to understand the way she taught. All facts and no concepts. (Daniel)

Students' Roles in the Discourse

The statistics showed that for approximately 74% of the students in HHS Algebra II classes, the majority of time was spent with watching, listening and copying notes as the teacher presented information. Then they were able to practice methods in their book work by following example problems individually. Lastly, the rest of the class time was spent working on their actual homework. Perhaps because they had not experienced anything else, a relatively high percentage of students (82%) thought this was valuable to their learning. Amanda's comment is representative in this regard:

> Begin class with an easy 10 minutes warm up problem. Copy down formulas from the board. Work example problems over and over. Do homework problems exactly like the examples.

Only 13% of the students spoke about their involvement in the class. As the following remark indicates, these students were encouraged to figure out mathematical problems through group discussion and to discuss the materials occasionally in addition to listening and copying notes:

> Of course, we reviewed homework, had a lesson, took notes and all of that, but in his (the math teacher's) class, we got involved. When teaching a new concept, he worked through things with us, encouraging us to figure out how to work through problems without him. The class was very comfortable asking questions about anything they didn't understand. (Lisa)

Mathematical Tasks

Approximately 61% of the students reported that they read information from the textbook, answered questions, did problems from the textbook or a worksheet, and took short answer tests (multiple choice, fill in the blank, problem sets) the majority of the time, while 21% of the students sometimes had such experiences, around 13% of the students seldom had this type of instruction, and 6% of the students reported it was not done this way at their home high school. For example, according to Amy, "The teacher hands out a packet with partially filled in blanks for definitions, examples, and one fully-worked, or completed example". Amy noted, "He teaches the lesson by going through the packet and asking questions, and then assigns homework." Yet, around 52% of the students thought this was valuable to their learning.

Only 9% of students had engaged in hands-on activity or investigation, worked on hands-on tasks as part of a test, and discussed how math is used in real life or in different jobs as opposed to 53% of the students who had never done it. Yet, 24% of the students reported that they seldom had engaged in the above activities while 12% of the students had done it sometimes. Nonetheless, around 37% of all students thought this was, or would be valuable to their learning.

Tools for Learning

Approximately 32% of the students reported that they had never worked in a group to complete a task or assignment, and had not been afforded the opportunity to either design their own strategies or make tables or graphs for investigation. About 35% of the students seldom designed their own strategies; and used tables or graphs for investigation in groups while about 18% reported that they did so sometimes. In comparison, a relatively small percentage (15%) of the students recognized that they had opportunities to design their own way and made tables or graphs using data from investigation with fellow students to complete the tasks. Lisa was one of the students who recognized that she and her classmates benefited from group learning. As she noted: “We were allowed to communicate with one another when working on a project or homework assignment, allowing us to learn from each other. I definitely learned a lot from this class & enjoyed it more than previous math classes”. About 45% of the students thought this was valuable to their learning.

A small portion of students (13%) had prepared a written report and made a presentation to the class about their work, either in the lesson or the tests, as opposed to those (67%) who had never been afforded such an opportunity. Seventeen percent of the students had seldom engaged in this learning process and around 3% of the students had done it sometimes. However, only 29% perceived the approach as valuable to their learning.

Although half of the students (50%) indicated they used a calculator, computer, and internet to aid learning in the class, about 30% of the students had never used these tools, while 10% rarely and 11% sometimes had such experiences, respectively. Fifty percent of the students, however, thought the use of these resources would help their learning.

Students' Perception of Relationship with Mathematics

In addition to be asked the typical process of mathematics teaching and learning, the students were also given the opportunity to understand and evaluate their relationships with mathematics at HHS. According to survey, the students reported their beliefs and attitudes about their ability to perform in mathematics, the instrumental importance of mathematical knowledge, some issues in classroom contexts that affected their mathematics learning, and the resulting motivations and strategies used to obtain mathematics knowledge. Each aspect is reported in the following paragraphs.

All the students saw themselves as very intelligent, goal-oriented, and dedicated persons and had been in the advanced math courses for their grade level. They were considered by their home high school mathematics teachers as intelligent, diligent, intrinsically motivated, resourceful, and mature in abstract thinking, top-notch level and academically challenged students. About 43% of the students were actively participating in mathematics clubs and competitions at the local, state, national, and global level which shaped their fervent passion for intellectual development. Although around 30% of students disagreed with 36% of the students staying neutral, about 34% of the students agreed that only the smartest people should think about becoming mathematicians and some people just cannot learn math very well.

All students expressed high levels of confidence in their mathematics abilities. Both these students and their teachers believed that they could do well in mathematics. In addition, 94% of students agreed that their families think it is important for them to learn math and also believe that the students can get a good grade in math, with 6% of the students staying neutral. About 68% of the students recognized that they have to know a lot of facts and formulas and be able to explain how they solved a problem in order to get a good grade in math, although 18% of students disagreed, with 14% of the students staying neutral.

Students valued the instrumental importance of mathematical knowledge. Although about 48% students reported that mathematics was their favorite subject to study in home high school, they recognized that mathematics is an important element to help them attain their career plans. In particular, approximately 78% of the students agreed that knowing math is useful in everyday life and will help them obtain a good job in the future, that people need to know math, even if they are not planning to go to college. About 3% of students disagreed with the other 19% of the students staying neutral. These students also mentioned that they would like to work in a job that uses a lot of math.

In addition to understanding their own mathematical abilities and instrumental importance of mathematics, students reported some school issues that affected their mathematics learning in HHS. The majority of students (64%) agreed that their math teachers knew a lot about math, even though sometimes the teachers admitted not knowing the answer to a question. Their teachers made math interesting, asked a lot of questions, and really enjoyed teaching mathematics. About 18% of the students disagreed, with 17% of the students staying neutral. Correspondingly, about 64% of the students looked forward to coming to their math classes. Despite the fact that 12% of the students disagreed while 24% of the students remained neutral. As a result, about 56% of the students pointed out that they learned a lot in that their math class in that things they did in the class made them feel curious, and they looked forward to taking more math classes in the future. Yet, 16% of students disagreed with 27% of the students staying neutral.

Students' perceptions shed light on the nature of the mathematics teaching and the quality of mathematics teachers in HHS. For instance, about 57% of the students had a high level of appreciation towards the contributions of the teachers, which encompassed teachers' patience and caring, well-preparedness and efficiency, clear explanations, and well designed worthwhile instructional tasks. For instance, the teacher was interested in "what we needed to learn or wanted to learn." Noted Amy, "and he was always available before or after class on specific days every week." Kate spoke very highly of one HHS teacher and the ways in which this teacher was important to her mathematics achievement:

> My teacher was an amazing woman, who inspired me to work hard in math. We got along great because we both loved math. She taught us not only the basic curriculum, but also how they would be used in the real world and in future courses. She gave us homework every night and either a long quiz or a test every week. She also assigned major projects on the side, which taught me how to manage my time. She gave me the hunger to excel in math, and I will never forget that course! (Underline is original)

It should be noted that a portion of the students (20%) mentioned that peers, classroom environment affected their mathematics learning. For instance, Ethan stated that the best thing about his Algebra II class was "the environment. People in my class were mostly like me and really liked math". Daniel also mentioned that he "had some friends in that class, other than that, I was disappointed".

While the students believed and acknowledged that their teachers were certainly an important part of their mathematics learning and would have contributed to the intellectual milieu of them, they evidently realized the mathematics teachers at their HHS could not perform as effectively as they would have expected. About 28% of students adamantly declared that the class was not challenging:

> The class was not very disciplined and it was pretty chill. (Jackson)
>
> The class was honors, but the expectations for the class (or students) were not high. We were "forced" to learn some objectives on our own, and explanations were given occasionally. Basically my Algebra II class was not a Great course…The best thing about my Algebra II class was-_______. Really I taught myself and the class work was easy! (Emma)

According to Sarah, all her experiences in Algebra II and Geometry were "Teach myself thing", noted Sarah, "I had to learn by teaching myself. My teacher was not always knowledgeable of a topic". In one case, a teacher intervened with a student who was just "kind of breezing by" (Amanda, interview) in her class by playing chess. "There was little thought involved (in homework)", remarked by Amanda. Amanda continued:

> I feel that I am not being challenged; he didn't really know what to do with me because he couldn't go ahead and try to teach me these things because you have to test me on different material. So he would even-he'd just tell me, 'Go back to the back of the room,' and he'd play me in chess or something when everyone else worked on their homework, because I am-you know-get done really, really quickly.

As such, Daniel mentioned explicitly during his interview that his indifferent feeling about mathematics was due to the fact that the teacher did not know how to teach:

> Algebra 2 was pretty bad. We had a-we had a slow teacher—uh, the teacher was smart but she didn't know how to teach very well. And it was an Honors class and a block schedule and we just went really slow. Like, she spent the whole time going over one or two prob—example problems, and then that's all we learned…I mean, I was indifferent about math, so if I had to I would but—I mean that would really—it didn't really make me like math more.

Among 16% of students who took online Algebra II classes, all of them expressed that they did not like it. As the following remarks indicate, the students had difficult experiences:

> I did not like it as much as a traditional classroom because there was no immediate help from teachers and peers to help me understand things that I had trouble with. (Jessi)

> I had to learn by teaching myself. My online teacher was there to keep up with grades, and was not involved in the teaching process. (Sarah)

> The math teacher that was in there didn't know hardly anything about Algebra 2. I had to help her teach the other kids who were taking Geometry. (Aidan)

When the students were asked for a thought about the possible changes regarding academic work at their home high school math classrooms, approximately 47% of the students suggested that each teacher should be well informed about the subject that they are teaching and then substantially challenge their students in order for them to strive for excellence. For instance, Grace advocated “Raise the bar!” she continued:

> Teachers should challenge students. Push their limits…If students are given a mathematical formula tell them what it’s used for, who uses it, and what, if anything, it’s used to create. When a teacher can relate something as vague as a mathematical formula to something tangible, it can be far more easily understood. (Grace)

Amanda was also in favor of challenging courses:

> I would definitely make all the courses harder and the disciplinary consequences stricter…they would have a purpose to further the students’ education and not simply to keep them quiet. I would make more honors classes, and they would be tough and challenging. (Amanda)

Nearly 87% of the students, in talking about the effective ways of teaching, referred to their teachers’ ability to teach well in that “There are a great deal of teachers that know what they are talking about,” noted Jessi, “but when it comes to actually teaching the materials to high schoolers they do not know how to present it to them in a way that they can understand it”. Specifically, approximately 45% of the students identified the need of group activities. Steven noted that “the work-force is filled with team oriented situations and interactive learning allows students to think collectively while still challenging each other.” Meantime, these students also spoke about the teachers’ awareness of the ways of planning lessons. The following remarks exemplified students’ views in this regard:

> I would include a lot more group activities to get the class involved and help everyone to interact while also learning. (Jessi)

> I believe by allowing students to participate and get more involved with the material, a lot of students would be more open to their education. (Lisa)

> In mathematics, I would enjoy being given time to explore new material- to discover theorems and postulates before they are formally taught, and perhaps stumble upon my own contributions to the field. (Jason)

> What (mathematics) teachers should not do is to give a packet of worksheets to students just for the sake of giving a pack of worksheets. This is not a valuable teaching tool, and is a complete waste of time and paper. Students will just try to get this done and not pay attention to what they are actually doing. (Daniel)

The rest of the students (55%) suggested that teachers increase the amount of hands on activities and research projects so the students could apply what they learned to real life. As the following remarks indicate, students were expecting meaningful and deep mathematics learning:

> I don’t like always doing book work. I would like to do more hands on activities. (Jane)

> I would incorporate current journal articles and cutting-edge research into the science and math curriculum. Both of these fields are changing quickly everyday, and sometimes textbooks do not give students a real sense of recent developments. With the incorporation of recent research articles,

> students will also develop a more in-depth and intellectual understanding for innovation. (Steven)

The statistics and narrative details above showed students' perspectives on how their HHS mathematics classroom environments served as a means to aid or prevent the students' access to significant mathematical concepts and ideas, and the ways in which students identified themselves as doers and learners of mathematics in classrooms. Most of these classes, as described by students, reflect typical teaching and learning practices in mathematics classrooms in the United States (Stigler & Hiebert, 1999). The teaching and learning experience at home high school provided certain background for understanding students' past histories of learning mathematics. I gained insights from their HHS experience in order for better understanding their identity formation at specialized residential high school.

Specialized residential high school

Mathematics Curriculum

With relatively small class sizes and a cohort model, the SRHS mathematics students were expected to learn at a very high level. Based on the syllabus, students were presented with a rigorous curriculum delivered by mathematics instructors who earned Ph. D degrees in mathematics education. University faculty worked with the SRHS faculty to design college-level calculus courses that met both the state requirements for high school calculus as well as the university's requirements for the first year calculus. The calculus course is a mixture of theoretical calculus as well as applications of calculus. For example, Calculus I includes the key concepts such as limits, continuity, derivatives, differentiation rules, and applications of differentiation. Calculus II comprises such key concepts as integrals, applications of integration, infinite sequences and series, and introduction to differential equations. The theme of the calculus course is problem-solving by using such multiple representations as words, graphs, tables and symbols. Project is another way of involving students and making them actively learn. In addition, connections between calculus and physics were emphasized even though the two areas were taught in different courses.

The mathematics teachers had spent vast amounts of time discussing teaching decisions, sharing ideas in order to improve their practices. Their philosophies align closely with the National Council of teachers of Mathematics (NCTM) Principles and Standards. The mathematics at SRHS did not target the learning of different mathematical procedures; rather, the students were engaged in all kinds of activities and projects in which the need for certain mathematical reasoning and techniques became apparent. In other words, the teachers were concerned with the learning quality, rather than the quantity of the students' calculus experiences and understanding. Their priority was to give students mathematically rich practices rather than to maintain a high work rate. Procedures were not ignored but were developed through carefully selected problems and activities. To effectively present the process of action and interactions and the kind of ways of participation that students were engaging in the mathematics classrooms, it is necessary and important to give an overview of the SRHS Calculus I and Calculus II lessons.

The Calculus I Class

The Calculus I course serves as the students' first mathematics course for college credit at SRHS. In addition to the calculus textbook, the teacher utilized the textbook of activities such as "Exploring Calculus with The Geometer's Sketchpad" (Clements, Pantozzi, & Steketee, 2002). All the activities in this book afforded multiple points of access to the mathematics and "help students explore the concepts of calculus verbally, numerically, symbolically, graphically and kinesthetically" (p. V). It not only comprised short answers but also required written responses, which attempted to elicit students' ideas, hypotheses, reasoning, and explanation. The activity book is organized into chapters that mirror calculus texts such as "Exploring Change", "Exploring Limits", etc. In each chapter, a series of probing questions such as "Can you Predict the Trace?", "What Do You Expect?", "Continuous or Discontinuous?", "How Close Do You Go?", and "So What's the Function?" are organized. More importantly, this organization of the Key Curriculum provided coherence across a set of activities, which afford students the opportunity to make connections and to build concepts. For instance, students would obtain the best insight into how the derivative is affected by tracing a point representing the slope of the secant line between the two points (x, f(x)) and (x+h, f(x+h)), and then having h approach 0. Hence the secant lines become animate objects with the Geometer's Sketchpad (GSP).

According to the observation, the class usually started with going over the homework from the previous night. The whole class would discuss the homework with the teacher carefully monitoring students' responses to each other, and occasionally providing a clue to move students' thinking. The next phase was very flexible depending upon the instructional tasks of the day. Sometimes there was a pop quiz, or Sketch Pad group activity, or students could just go up to the board and start teaching the whole lesson. Sometimes the teacher gave mini-lectures to reinforce the previous concepts or to explore and make sense of a new concept. In the final phase, the class was summarizing what they learned in the class with questions provided by the teacher for reflection.

In addition to assessing students' learning based upon their responses on homework, quizzes, projects, tests, and the final exam, the students were required to create entries in their journals after finishing each chapter test. Students were expected to focus on approximately 3 to 4 ideas from the chapter that was just learned. The overall goal was to provide students with opportunities to share some key ideas from the course through description and reflection, with the addition of creating well-constructed personal documents to reference in Calculus II. The teacher demanded high levels of mathematical work in journals. That is, students were required to actively interact with group members and then logistically question each other as well as to conduct peer review of their journals. Direction was given to students in order to produce thought-provoking journals. The "Limit journal entry" is given as an example. Students could choose any topic of interest such as instantaneous velocity, "ε-δ" approximations, definition of limits, limits at a point, continuity, limits with infinity, etc. The students were informed that they needed to use a variety of representations to demonstrate what they knew within the topic and to write a detailed reflection of where they felt they stand with the material. The students were given tentative prompts such as, but not limited to, "What do you feel are aspects of this topic that are still unclear to you?", "What aspects of the topic have you completely mastered? For those topics, what would you do to explain the topic to someone else?", and "How would you place what you've learned into a larger picture of mathematics?" (Calculus I Journal Entry, spring 2009).

The Calculus II Class

The Calculus II course served as the students' second mathematics course for college credit at SRHS. As such, the difficulty of the content was heightened from students' experiences in the Calculus I class. The overarching goal of this course was to use meaningful problems and appropriate technology to develop concepts and work with applications of integral single variable calculus. The teacher considered herself a facilitator of development of knowledge rather than a dispenser of knowledge. That is, she clearly knew that her responsibility was to create lessons that actively engaged students in experimenting, hypothesizing, hypothesis-testing, and evaluating mathematics in ways in which mathematicians usually do. She believed that students would learn more deeply and retain longer knowledge that they construct rather than information that is disseminated to them. She explained, "I do not believe students should 'reinvent the wheel' but rather, explore the need for the wheel, the development of the wheel, how the wheel works, and for what applications they can use the wheel" (Calculus II Syllabus, Spring 2009).

As in the Calculus I class, a variety of methods were used in assessing students' understanding: homework, quizzes, projects/class activities, and exams. In some of the categories, students were expected to work in a team or with a partner. The students, like students in Calculus I class, were also aware that the teacher demanded high levels of work in studying mathematics and they came to appreciate that demand.

The 90-minute classroom session involved a standard order unless it was a test day. However, the overall schedule was varied. Usually, the class began with the teacher and students going over homework from the night before. They used, however, varying methods to accomplish this. Sometimes the teacher addressed students' questions when they indicated that they had experienced difficulty with a certain problem. During this phase, the teacher either explained students' questions directly or had another student explain it to some of the students who didn't get it. Sometimes homework answers were posted on the course website so that students could look them up by themselves. Sometimes the students were required to check answers with partners in the classroom. Following this discussion, the class was moved into a new lesson. Again, the length of the new lesson was pretty flexible depending upon how long the class discussion would last. Sometimes the new lesson took up almost an entire class period while the homework review didn't take up very much time. Sometimes, however, students had mathematical problems which they had time to work on alone or in a group for the rest of the class period. The homework was collected at the end of each unit on a problem-by-problem basis. In other words, assessment of homework was based on completion. Students had to show the evidence for attempting every problem at least once.

General Classroom Obligations

Two aspects of the general classroom obligations were observed in the classrooms. First, it had been noticed that the students had the ownership over their problem solving process, and the capacity to validate mathematical methods with the teacher's facilitating. Such experience was resulting from the openness of the instructional style, and the degree of option the students were given, The Calculus I classroom was taken as an example. Again, classroom observation showed that searching for the knowledge and understanding the concept to the best of their abilities was the goal of the class. Correspondingly, expectations were explicitly captured in the classroom: the students were expected to be able to work individually and collaboratively on mathematics tasks, communicate their thinking with other students and the teacher in multiple ways. When students needed help on homework or a class activity from their teacher, the teacher usually did not give the straightforward answer or offer the solutions directly. Instead, the teacher "threw" the questions back to the class by posing thought-provoking questions which led students to look for the answer. It was noticeable that the teacher made deliberate efforts not to steer the direction of solving the problem. Rather, he was able to sit down and exhibit great patience while listening to students' ideas and conjectures. Students were continuously encouraged to structure the mathematical task and had ownership to make decisions about their answers.

Second, the students were provided opportunities to engage in reinforcing conceptual understanding. That is, they were given opportunities to inquire and use their knowledge to understand the mathematical ideas and concepts, to develop their problem strategies, to present solutions, and to question other students' solution methods. The core task was to learn to "understand" rather than to "know" mathematics ideas that realized in the classrooms. In one Calculus I lesson I observed, for instance, the teacher created a task for the students to investigate in small groups with three members in each group. Students were required to find the derivative of tan(x), csc(x), sec(x), and cot(x) by using only what they knew about the derivatives of sine and cosine. Each group had the charge of finding one of four basic trigonometric rules of derivative. Students in each group developed their solution methods, found the derivatives, graphed the function and the derivative in GSP, and verified that the solution was correct. Then, they prepared a brief presentation of how they found the derivative and presented it in front of the class. One of the group members proficiently connected their laptop with Smart Board. The second member in the group wrote down and then explained how they had reached their conclusion algebraically and why their solution made sense. The third member was holding the laptop and shared the graph on GSP to clarify their conclusions as the rest of the class was trying to understand them.

Although few formal group presentations were performed in calculus class during my observation, the students often worked in small groups and tried to figure out certain math problems. They first worked on the problem individually and then got together to discuss it. Almost every student was actively choosing group and explaining what their solutions were, how the solutions were arrived, and seeking agreement from each other. The dialogues were initiated by students and rich conversations were further produced to clarify any confusion, or misconceptions. The teacher walked around and captured the images of how students were doing in order to bring some questions up in the whole group discussion. Finally, the class reflected mathematics problems by summarizing the steps.

The affordance to engage in conceptual understanding in the Calculus I class was also highlighted by students in the Calculus II class. According to Steven, “We learn integrals but we learn like, the more conceptual parts, like we can-we don’t have to know it perfectly.” He continued:

> Like we can derive it on our own like-Like derivatives for example, you know they have the long way, where you do the f, x plus h plus the original function over h or something. Well, instead of just going straight to the short cut, we learn how to do it the long way, and then we learn how to do it the short way. So we actually understood, like in case we needed to derive it on our own."

Doing Mathematics from Students' Perspective

When the focus was narrowed down on the normative aspects of classroom action and interaction with respect to mathematical knowledge, several emerging aspects were constituted in both Calculus I and Calculus II classrooms. First, students engaged in mathematical argument by using a variety of tools to reason, make connections, solve problems, present, and then generalize. It was not sufficient enough for the students to show how they had used the mathematical written symbols to reach the answer. Instead, oral language and computer tools such as Sketchpad, Excel, calculator, Maple, iTune, and the Internet were intensively used by students in the mathematical activities. Second, not only had students shared their solution methods but they also were obliged to explain their reasons and to demonstrate the process of reasoning in order to show the reasonableness of the conclusion. That is, they engaged in mathematical arguments by telling both "how" and "why". Third, students reflected on and exchanged mathematical ideas on the basis of both individual and group discussion to foster their learning. The two classroom episodes will be chosen as illustrations in this regard.

A lesson of "Limits Involving Infinity" in the Calculus I was taken as first example. The students in each group had the charge of finding the limits of polynomial functions (e.g. P/Q) as x approaches to infinity given their particular conditions such as the degree of P is less than (or “=” or “>”) the degree of Q. Each group was assigned one of these cases, and was asked to develop several examples that meet the criteria for the degree of each polynomial. The students were engaging in conversations to find the limit as x approaches infinity, to make conjecture about the end behavior of these functions, to verify that they were correct using the rule of limits, and then to prepare a brief presentation to the class. A typical presentation was taken as an instance. One of the group had the charge of finding the limits as the degree of P is greater than Q, they actively engaged in group discussion, and sought help from each other by asking “as x approaches to infinity, where does the graph go?”. Through the conversation, the students in the group finally found the limits of P/Q as x approaches to infinity by trying four functions in the Sketchpad. Then they saved the graphs on the jump drive and brought them to the teacher’s laptop. They proficiently opened the GSP files (see Figure 1 as an example) on the Smart Board screen, and gracefully presented their results and conjectures in front of the class. The class and the teacher paid close attention to what they were discussing about the case and occasionally asked questions:

…

Teacher: Your generalization would be what?

Student 1: Generally speaking, when the degree of a nominator is greater than the degree of the denominator, the limit would be the infinity.

Student 2: Does that make sense to everybody?

Teacher: Do you have something to ask them? Again, I hope you write down something for yourself…

…

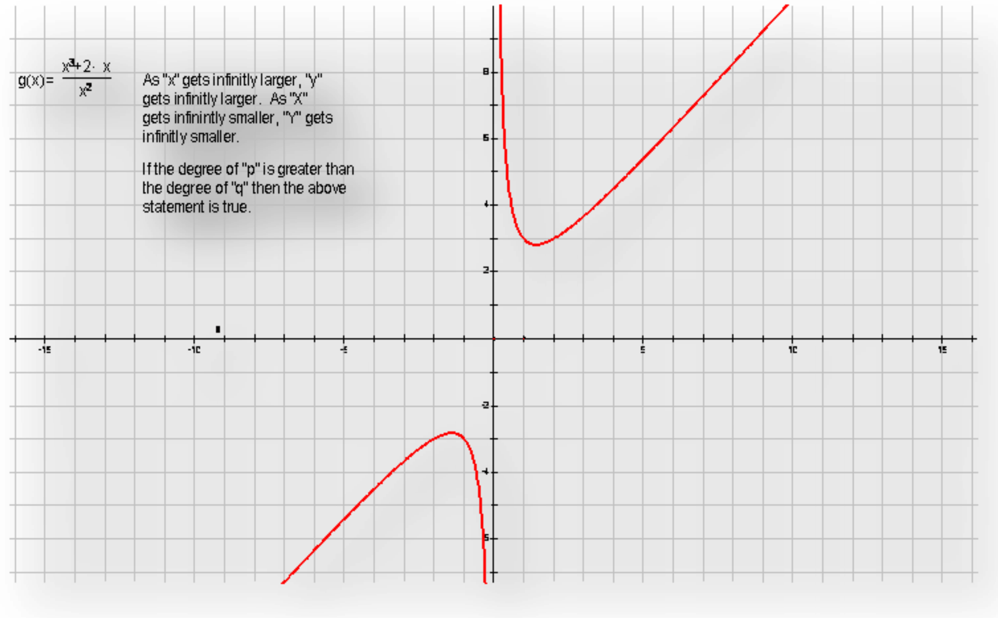

Figure 1: One of the Examples of Group Presentation

The rest of the class had agreement with the group. However, being prompted with these questions, some students came up with an important concept of "slant asymptotes" in the graph and continued to discuss this particular mathematical idea.

More meaningful discussions were undertaken as other groups presented their ways of finding the limits. Such insightful questions as "What do you mean 'so large'?", "How quickly does it get to the zero?", and "Do you understand why?" posed by the classroom audience pushed the students who were presenting to clarify and justify their reasoning.

This kind of conversation was also observed in Calculus II class as two students tried to figure out the solutions together. During one class I observed, Jason and Jackson were attempting to evaluate the indefinite integral $\int \sqrt{\cot x}\csc^2 x dx$ by substitution. On one hand they were not really clear about the substitution rules. On the other hand, notational issues were going on which made these problems more difficult than they already were. Nonetheless, Jackson and Jason started to ask questions of each other, explain their own thoughts by negotiating with each other in order to find the acceptable solution methods. The following conversation about integration by substitution was illustrative of a student interaction in this regard:

Jackson: What is the derivative of cotangent of x?

Jason: Oh, uh, it's negative cosecant square.

(Jackson wrote down $u = \cot x, du = -\csc^2 xdx$ *on the sheet)*

(about one minute later)

Jackson: What did you use of …uh, to substitute cotangent of x?

Jason: (After he took a look on Jackson's paper) Uh, oh…substitute cotangent x. Yeah, you are on the right track. Anyway it's said that, uh, you're going to have one of those issues. Oh, dx is equal to du divided by -1 because the sign of cosecant of x.

Jackson: How did you divide by 1? Would be a negative?

Jason: One second….Yeah, it's divided by -1, which means it's negative, yeah, it's the same thing.

Jackson: to write down over here*? (Jackson moved the pen to right side of* $u = \cot x, du = -\csc^2 xdx$ *and started to write down dx=)*

Jason: yeah, dx is equal to…yeah… that's right, dx is equal to, yeah….ok! (Smile with struggling)

Jackson: Negative du? Divided by…..

Jason: Yeah….we skipped things down here…

Jackson: $dx = -du$

…..

(After Jackson wrote down the $\int (\cot x)^{1/2} dx$. *He pounded a few seconds, and then asked).*

Jackson: What's got to be here *(he pointed to the cot(x))?*

Jason: We are using u in this thing, but…yeah…in this case, uh….I am trying to think how we should write it. Because I see how it works out but I am not…I can't remember how you relate the du to the dx in order for the actual equation to be correct. I am trying to think how du relates to dx since uh…ultimately, you know*…(Jackson wrote down* $\int (\cot x)^{1/2}$ $dx = 1.5u^{1.5}$*)*

Jackson: Wait…

Jason: That's okay. As I see the problems, I can see the answer. *(Jackson, again, changed the* $1.5u^{1.5}$ *into* $\frac{3}{2}u^{1.5}$ *)*

Jackson: You know the answer?

Jackson: Oh wait, all I keep thinking is that, okay, hang on….

The discussion continued as they tried to substitute the dx with $-\frac{du}{\csc^2 x}$ which seemed a big challenge to them. As Jason said, "The things that bother me here are that I see how to get the answer, and I just, uh... the actual math I need to show backed up is annoying". Their conversation of mathematical argumentation showed that they were not solely pursuing the calculational steps. Instead, they strove to make sense of the underlying concept. More importantly, these two students' engagement with mathematics had contributed to the normative ways of classroom interaction in which the students had some significant actions to do as they learned to rely upon each other, and was obliged to make sense of mathematics to others rather than to self only.

After such intensive and meaningful discussions in both classes, students were then encouraged to reflect on the mathematical knowledge. Reflecting was regarded as essential element through the process of learning in the two mathematics classrooms. In

Calculus I class, students reflected on their learning through journal entries. Not only were they reflecting on the mathematical concepts they had learned, but also they were given opportunities to reflect on how the ways in which the teacher presented the mathematical ideas changed the way they learned. Students in Calculus I class, for instance, were given chances to do reflective writing by using graphs, tables, and words in a narrative form. They went over content that they had learned, things that they didn't know and things that they knew very well. Students were spending a lot of time on it because "there's a lot to think about", Aidan noted, "You know, with what goes into it. And I mean, it sounds fairly easy as an assignment when you just hear about it. But when you're actually trying to do it, it is pretty difficult." Here is an example of how one student shared her insights on the relationship between the function and its derivative, the limit at a point, and the definition of derivative of function in the mathematical journal (See Figure 2). As such, after the teacher created the iTune movie lessons, he was seeking students' thoughts on it and made further improvements on instructional strategies.

Although students in the calculus II class had no journal entries, they were also provided opportunities to reflect on certain mathematical ideas. They used their own words to explain how they solve certain mathematics problems, which problems were not as clear as they would like, which key steps were giving them the most trouble and how they would be able to overcome it. For instance, when students were required to solve a definite integral, the teacher designed several questions that help students understand the meaning of each solving step (See Figure 3).

"…If you don't want to memorize the ways they (the derivative of the function and the original function)affect each other (which is what I had to do), try to understand the relationship among them and see a connection in which you can remember it best. Another point I want to clear up is the part about trying to find a limit where the function is discontinuous. If you have a function with point discontinuity where one point is undefined in the function, can you take the derivative of the function at that point? The answer is no because you can't calculate the slope of a tangent line through a point that doesn't exist because there would be no point for the line to be tangent to (refer back to definition of a derivative). You can take a limit at a point which is undefined, but that is because a limit is finding the value as x approaches a certain number. It is just an approximation, but derivatives are specific. There cannot be a slope value for something that doesn't exist."

Figure 2: The Example of Student Journal Writing

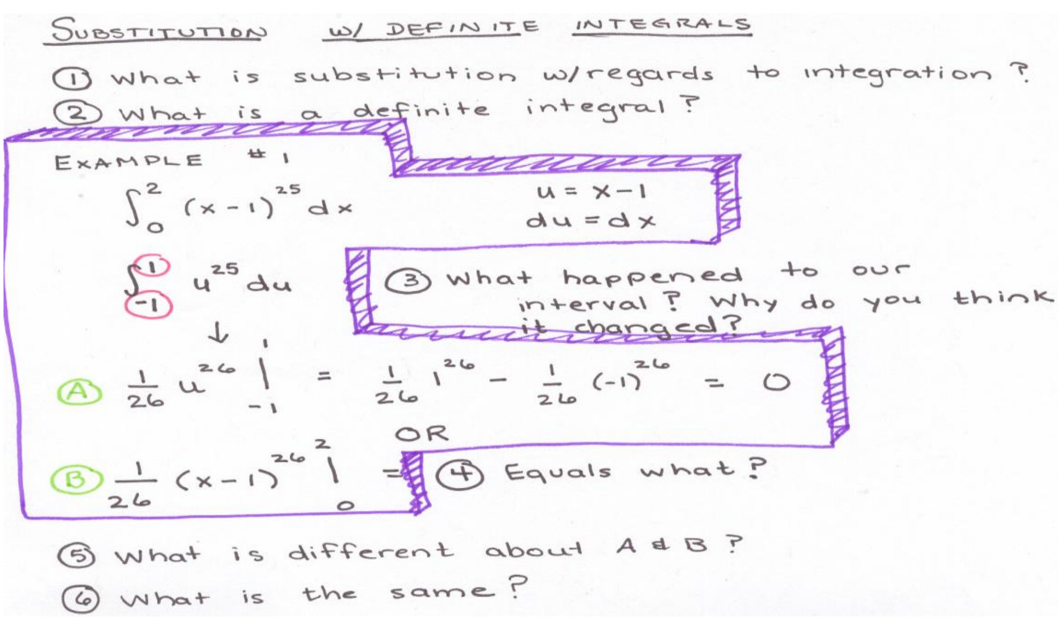

Figure 3: The Example of Reflections that Student Engaged in Calculus II class.

Students' Views of Their General Obligations

The six students who were participating in the interview identified their responsibilities in SRHS math classrooms. First, every student had to participate in the classroom mathematics learning. According to them, not only were the students enabled to discuss, present or teach in order to share their own thoughts with the class, but also they had to pay more attention to the teachers and other students' explanations, take on a new perception and then be able to ask questions if they did not understand something. Second, the students were asked to understand the relationships among the mathematical concepts and ideas rather than just mimic a procedure, and therefore they were learning meaningful mathematics. Aidan described the class:

> We're actually applying it to things that we would actually do in life so that we're not just going through the textbook and memorizing…You know, he's trying to teach us to understand *why we're doing what we're doing* and then *how to apply it.*"(Italic added)

Daniel also voiced the nature of the mathematical teaching and learning approach at calculus classrooms. They were provided the opportunity to learn mathematics in ways in which mathematicians learn:

> There's no need for memorization and that means you have to understand what you're doing. And like, you can think like, how mathematicians thought when they actually did it.

That the concepts are as important as procedures was also pointed out by Steven as evidenced in his comments on the ways in which they engaged in mathematical practice in calculus II class:

> She (the teacher) is probably one of the best math teachers I've ever had because she teaches everything perfectly. Really good. She goes—she teaches everything step by step. She doesn't just make us figure it out on our own, so I like seeing how it works in class and then trying it at home by myself.

Sophia, like Steven, also felt that she benefited from the way in which the teacher let them figure things out. She thought that if the teacher just explained to students how to do it, students' brains were not working because they did not come up with it themselves. Therefore, their experiences in the class would not be worthwhile.

Students' understanding of the normative ways of participation in the classrooms was consistent with observations. That is, both teachers and students had the capacity to use their knowledge to validate the mathematics solutions and the students had opportunities to inquire in-depth in order to foster conceptual understanding.

What It Means to Do and Learn Mathematics in Their Classrooms

When students were asked to share views on the classroom standards of mathematics reasoning, all six students recognized that explaining, interpreting, and justifying were critically valued practices in SRHS mathematics classes. These standards of mathematical arguments were established through meaningful tasks, group discussions, and usage of technologies in the classroom.

First, the students indicated that they had been engaged in higher order thinking mathematical activities and assignments. They were enabled to strive for searching and developing strategies to solve problems. During reflective writing assignments, for instance, the students researched prior knowledge that they needed to further make sense of new concepts, and deepened their understanding of mathematical ideas. Then, they developed a new strategy to help themselves eliminate some confusion about the concepts that they had earlier. For instance, Amanda at first had trouble grasping the limits provided. She was confused by which value was the “limit”, and then she developed a new strategy to figure it out:

> To avoid some of the trouble (confused which value if the ‘limit’), it helps to think of it as simply the y-value for a given x-value. Ask yourself, what is the y-value when an x-value approaches a certain number…Basically a limit is the y-value of a function as x approaches a certain number. It helps to think of it as the height of the function. If we know where x is, then we need to know where y is, and the y-value is the limit.

In addition, they were able to explain the concepts by drawing graphs to show what it means, by explaining verbally, and by proving some aspects algebraically. Not only were students obliged to understand why they were doing what they were doing with mathematics, they also explicitly discussed the ways mathematics is seen and applied in their daily lives. For example, students pointed out that derivatives have very important uses in chemistry, finance, physics, etc. Students also connected this concept to what they were currently doing in their lab project, for example, to find how much photosynthesis a plant does in a day.

Second, the students also voiced the benefits that they earned from a variety of mathematical tools they had used when developing problem solving strategies. For instance, the students had enormous opportunities to adopt a variety of resources such as laptops loaded with Sketch Pad, Excel and Maple, and graphing calculators in the process of reasoning and thinking. All of them expressed how much these powerful and substantial tools fostered their learning, and how they took advantage of them to understand and appreciate the mathematical concepts. For example, three students in Calculus I class were in favor of using Sketch Pad. They spoke about the kinesthetic nature of GSP and how it best helped them to grasp the thought process behind mathematical concepts and to make connections amongst the mathematical ideas and multiple representations as well. Daniel reported that he was unsure about the definition of the derivative and how it was formed. Nevertheless, he was able to get a grasp on this knowledge with the help of GSP:

> Sketch Pad can be used for a variety of things. And I like-the thing I like about it is like, you can see the process which happens. Like find the derivative at a point, you can actually see on Sketch Pad-like visually see the secant lines coming closer and-the points coming closer and closer until they become the tangent line. And then you understand that like, oh, as this approaches zero, this –this is what you get. And that's what-you can visually see that, for the formula and then you can see on the graph.

Furthermore, Daniel made connection between what he was currently learning with what he had learned. He indicated:

> A derivative is compromised of just the limit as the distance between two points approaches 0 to find the slope of the line tangent to the point. It just builds on what we learned before. So throughout my life, I have learned algebra to help me learn limits to help me learn derivatives.

Third, the students appreciated the value of group discussion and the ways in which they communicated mathematical reasoning to each other. According to them, group discussion had been regarded as the most valuable thing in both Calculus I and Calculus II classrooms. Participating in the group discussion and trying to grasp the knowledge to the best of their abilities were strongly encouraged. They perceived that they were allowed to express themselves openly and creatively and communicate their understanding of meanings of mathematical concepts to their group members and teachers. The core task of interacting was to learn and understand each other's ideas, realize each other's mistakes, and correct each other's mistakes. Thus, the students were accustomed to valuing, respecting, and then adopting other students' problem solving strategies for "All had their different ways of going about it", noted Amanda, "But their way is sometimes easier than mine and it's good to see it both ways". She described the nature of working in groups:

> More importantly, when students were helping and were helped in the mathematical practice, they felt comfortable waiting and then pushing group members to think and then explain why their solutions were correct.

As such, it was interesting to hear Daniel talk about the ways he was supporting other group members in the mathematics practice:

> "I always ask. Like, 'Do you think this is right?' or 'What do you think I should do?' I mean, I might even- I might already know the answer but- but I- I don't want to do that to people. Like, even if I previously knew it, I don't want to like, ruin it for them."

As with the Calculus I class students, the Calculus II students were very clear about their responsibilities when classmates presented their solutions. They knew that they were supposed to be asking questions or trying to correct their classmate's work if they did it differently. “We should like, try to find errors in their work and collaborate together to try to solve the problem”, remarked Steven, “Or fix it or if you don’t know it, ask questions and they can teach it to you”. Like Steven, Sophia voiced students’ role during the presentation, “I think we’re supposed to listen to them (when other students explain) and respect what they have to say”. She continued to describe how the norms of discussion were gradually negotiated and established as the lessons progressed:

> I think in the beginning everyone wasn’t so sure with everybody and everyone was kind of alone. But I think now, like, we have good friends and stuff so we feel more comfortable and I think people-in the beginning, everyone didn’t know anybody so they didn’t know what to expect. So like, if they were called by the teacher to the board, everyone would listen. But now there are people who are not so serious about math or there are people who are-they usually get the right answer when they solve a problem. And I think people give them more respect than they do people who they know probably don’t understand what’s going on.”

Students’ Views on Mathematical Competence that Constituted in Their Classrooms

When asked to reflect on what their own mathematical abilities were and how they perceived other students perceive them in their own mathematics classrooms. All students said that they are pretty good with mathematics in general. Yet, it is remarkable that they based their evaluations on different understandings of what counted as mathematical competence in their classrooms.

Aidan, Amanda, and Daniel gave standpoints about the nature of mathematical competence. Their comments were concerning the capability to teach and apply it to the daily life. That is, one does not know math until he or she can actually teach it.

Furthermore, They pointed out that whether they know what they were doing, whether they were responsible for each other's learning, whether they were able to handle both "what" and "why" about mathematics within the classroom where the rich discussion was undertaken showed their competence in mathematics. The following provide illustration of these comments:

> Well, I think they (fellow students) know I am a good student in math and I know what I'm doing. I kind of want to get the work done like, as quick as possible. And so sometimes they come to me for help… If you can derive the formula, there's no need for memorization, and that means you have to understand what you're doing (Daniel)

It was interesting to hear Amanda talk about the ways she was developing such ideas through the practices of teaching:

> If there's something they (her classmates) don't understand, I can teach it. And *being able to teach it like that, that's like when you really start to learn it.* If you can't explain it in words what you're doing on paper then you don't really know what you're doing." (Italic added)

In regards to the Calculus II class, the students voiced the importance of test grades. Jackson, Sophia, and Steven indicated their test grades are the most important factor to judge whether students learned the material in the time given and did it correctly on the test. As Sophia noted,

> I think that's (test performance) very important. But it doesn't mean that you are a bad math student or that you're dumb or something. But I think if you do well on the tests, it means that you're-you're on a good path and

> you're doing–you're going to eventually do-be successful, but I mean I don't know-I can't say for everybody. (Sophia)

Clearly, the mathematics classrooms were local contexts in which these students could develop and demonstrate their sense of mathematical competence. The different views about what counts as mathematics competent students in two mathematics classes were quiet different. Thus, how teacher design the classroom instruction is important to better accommodate the needs of students who value the test results.

Students' Academic Community

When asked whether they could be at the same level if they stayed in their former high school until senior year all of the students made remarkably contrasting points on the intellectual community that they embraced at both HHS and SRHS. It became clear that students' successful mathematics learning experiences were influenced by interrelated factors such as the challenge and high expectations, teachers' influences, and peers' influences on their participation and performance in mathematics.

Challenge and Expectations

All six students indicated that their teachers demanded different levels of mathematical work in HHS and SRHS classrooms. For example, in HHS, they indicated that they spent a lot of time to listening, watching, and taking notes as the teacher presented information from the textbook and demonstrated the procedure. According to Steven, for example, he did not understand the unit circle because "Our teacher just like wrote it down and said 'Memorize it.'" In her interview, Amanda recalled the general images of the ways in which four other students' home school math teachers did:

> At home school, I was just, you know, just kind of breezing by, didn't really have to study, didn't really have to try, and I was still making great grades. I have been in the most advanced math courses for my grade level, and I feel that I am not being challenged.

But at SRHS, the teachers demanded high levels of mathematical work and the students came to appreciate it. "I would have not learned the math as much as I did if I would have stayed there (HHS)." Said Steven, "This year, I learned the math a lot better. I understand it a lot more." Like Steven, Amanda said that "I came and I was, you know, really challenged." Other students also referred to high demand for understanding:

> At my old school, I didn't understand all the math I was taught. I just got enough to get by, to pass the class. But here, I actually understand it, like-I-I get the conceptual part. Like the conceptual is no problem. It's mostly procedural that gets me. (Steven)

As such, Jackson assured that he will not retake calculus in college, which he would have otherwise due to his profound grasp of mathematical knowledge:

> You know on paper, I would have been the same level (between HHS and SRHS)…I most definitely would have taken calculus again as a freshman, I –I just probably would have taken it again. Um, but coming here, I don't think I'll need to…So I won't-I won't have to start over all over again. Um, which-'cause I think that I will- I will know it for longer.

Teachers' Influence

All students indicated that the teacher stressed the importance of effort for mathematics learning. That is, mathematical accomplishment was counted as whether they have drive to learn more, and whether they had to try to be involved and put effort into it as a product of hard work and not of innate ability. “The main thing is that we’re expected to just learn more and to become better at math and better math students as the year progresses,” noted Daniel, “We just need to grow and like, be more confident with ourselves.” He continued:

> I mean, mathematics is all about persistence. I mean, you don’t have to be really smart to be good at math. You just have to try hard, and that’s what he (calculus I teacher) is looking for.

Amanda, for instance, did not consider that high grades would be a sole parameter to justify her personal mathematical ability and success since she came to SRHS. Thus, her focus was shifted from what she needs to do in order to get a high grade at HHS to what she needs to do in order to improve her learning in mathematics at SRHS. As she said:

> I didn’t do so well at first. You know, I mean like, a low, low B on the first test…I haven’t gotten 100 on a test yet. But Dr. XXX (the teacher) said that that’s good because that shows that you have room to improve…my grades went down a little bit but I mean, my learning is so much more.

In addition, to stress the importance of personal effort, the students also attributed their successful learning experiences to the ways in which the teacher taught and the particular ways of working that the students needed to engage in at SRHS. “The teachers are much, much, much better compared to my old school.” Noted Steven, “I uh, didn’t like my math teacher that much in my old school. I didn’t like the way she taught.” He continued:

> My math teacher is probably one of the best math teachers I've ever had. Because she teaches everything perfectly…we incorporate technology into our learning. We-we have-we do a very-we do the best thing. We get-learn the material, try it on the homework. (Steven)

They also noted that the teacher didn't sit up there and just "draw on the board and go, this is how you do this" at SRHS, said Aidan, "He finds ways to apply it to other situations to where we're not just going over it textbook style. We're actually applying it to things that we would actually do in life, or just uh, other ways than solving it on a problem the book gives us." As a result, he had to change the ways in which he dealt with homework:

> Because with the calculus homework right now, I mean I can't do it like I used to go and just kind of go in a matter of ten minutes, you know. I have to actually sit down and think about it. (Aidan)

As such, Daniel notably described how the class learns:

> Sometimes he (the teacher) won't tell us everything we need to know and I'll have to go look it up… He gives us problems and we try to figure out our own formulas and stuff. And like, you don't-in that class you don't have to memorize anything. You just—you can derive it.

Thus, his interest in math has been ignited as the lessons progressed:

> Yeah. Like, before I wasn't very interested in math. I just had it because the school required me to. But here I –I like math more and more motivated to study math and um, so that's changed me…'cause of my peers and Dr. XXX (teacher). The way he teaches is in-inquiry based…I've grown in math and I've come to like it more (Daniel).

The students were also very clear with teachers' expectation about the ways of working in the classrooms as well. They ought to take responsibility and support each other and access and contribute equally to mathematical tasks through the practices. Like Daniel said, "I mean, you have to work well with the group or else, you know, you won't –you'll fail". When Jackson compared his HHS experience with those of SRHS, he highlighted the insights on what counts as meaningful learning in light of his experiences in the Calculus II class:

> In calculus it's- it's- there's more of a difference between-in-in previous math classes (at HHS), if you watched the teacher do a math problem, you didn't have to do-you didn't have to write anything…you didn't have to actually solve it on your own. You could just take notes and then come back to it later that day or the next day and it would be okay. But um, I've found that starting this year, I've kind of needed to do one…you know, just actually-actually *doing something* yourself instead of just *watching them*, I think makes a bit of a difference, that's all.(*Italic added*)

It was interesting to see how Amanda thought about what it means to learn mathematics regarding her experiences. She contended that the ways of practices at her HHS were "Not really learning". Instead, "That's regurgitating what the teacher just did." In contrast, she contends "[I] Get a lot more out of my math class now than I ever did" by "Engaging in making connections and relationships between different ideas."

Peers' Influences

Unlike the lack of support from their peers inside HHS mathematics classrooms, all six students believe their SRHS peers served as academic resources. They truly appreciated the opportunity of being a member of the intellectual community at SRHS for “all the people here do care…they’re all pretty smart people.” noted Sophia. Like Sophia, Amanda found that “They are hard working, they’re intelligent and you’re-you’re not always with those kinds of people at home schools.” She continued:

> In my home school, and they would-you know, glide in on my grade and they didn’t have to do anything. It wasn’t even fair that they were allowed to be called my group. But here it’s like all those hard working kids from all different schools working together.

Students also pointed out that they worked together on mathematics projects, tests and quizzes and that they helped each other foster their learning through the group work:

> Well, we’ve done the group projects a lot. I mean, we do groups on tests, on quizzes…Well, I think you learn more from your peers and-‘cause peers teach differently, and you might understand what they’re saying. And I guess you feel more comfortable. And if you have a problem you can talk about it, you know.” (Daniel)

> I really like that we’re able to work in groups with intelligent equals and work on stuff…You can be given a huge project and if we all work together. Then we can get it done and we learn so much and get out of it because other people see things and can explain things in different ways. (Amanda)

But Aidan also added that he was pushed by peers at SRHS:

> I’d pay attention in class (HHS classroom) for like, maybe the first half and then finish the homework or just go to sleep. *But here* because I have to catch up and kind of get back on standards with the other students here, it’s kind of hard. (Aidan, Italics added)

When students commented on their own and other students' mathematics prowess, they indicated that they benefited from the immediate residential environment:

> There's like a lot of us that are good at math in that class, and so he or she could always ask one of us…Dr. XXX is just up the hill. I mean, he's—I think he's available a lot of the time. (Daniel)

> I think we're all doing just fine. We all understand it. We all work together….I'm with a different group of kids, like xxx's (name of the State) brightest, apparently…We all live together so if we need help, I can just go ask Tom or Jackson or Jason or anyone in my class. (Steven)

> "I would think probably the most valuable is just the accessibility that we have, being here at SRHS...It is more out of the way than walking across the hall or checking e-mail or something…I would say the convenience has probably been the greatest asset as far as this class goes."(Jackson)

> We talk to each other about problems and we do them on like, the board in the living room. (Sophia)

Not only had the students benefited from their own cohort class, but also they had been motivated by the members from the cohort class. For instance, Daniel's relationship with his friends in Calculus II also made contribution to his learning in Calculus I class:

> I looked up what a derivative was because my-like, I was in Pre-Calc, but all my friends were in Calculus 1. And they were talking about what a derivative was, and like, the uses and I thought it was-thought it was pretty

> cool, and like you can use it in physics. So I looked it up. And I talked with them how to do it and that's what-that's how I go to know it.

Chapter Summary

In this chapter, I analyzed the data gathered in the study of a group of high-achieving mathematics students and their activities in both HHS and SRHS classrooms. By providing both descriptive statistical analysis and narrative details, I described what kinds of experiences the students had in both HHS and SRHS mathematics classrooms, how the classroom microculture in different schools supported or hindered each of the students, what kinds of opportunities or difficulties they embraced or encountered, and how students came to see themselves as mathematical doers and learners in these mathematics classrooms.

With the rich resources at SRHS academic communities, such as teachers, peers, and technological resources, as opposed to those at HHS, the students' significant growth in mathematics learning was markedly fulfilled at SRHS, along with the new identities that students developed through the participation in the mathematics classrooms.

The analysis of the normative identity as a doer of mathematics in both HHS and SRHS classrooms illustrated different approaches of teaching and learning. In the case of their home high schools, the students perceived that they had few opportunities to learn higher level mathematical ideas, and spent more time on mimicking and memorizing mathematical procedure, and teacher mainly legitimized their work. In contrast, in the SRHS mathematics classrooms, they had the opportunity to engage in significant mathematical tasks in group, use multiple tools to explore and present their solutions, as well as make justifications jointly with peers and teachers.

The analysis of students' personal views on their classroom general and specifically obligations as well as their competence revealed significant differences regarding their experiences in both HHS and SRHS classrooms. The students perceived their primary obligation at HHS was to listen and copy notes, and then produce the correct answers by following teachers' methods. The students viewed such action as "not learning". The nature of mathematical activity in HHS mathematics classrooms entailed little thought on their part. They indicated that they were good mathematics students but they were not supported by peers and teachers. In contrast, the mathematics activities in SRHS classrooms entailed plenty of thinking, reasoning, presenting, and justifying with a variety of tools such as mathematical notations, oral languages, narrative writings, computer software, etc. Their interview responses also indicate that most of the students were intensively motivated, encouraged, and supported by both high quality peers and teachers in order to learn advanced mathematical concepts and ideas. The student and teachers jointly established the norms of participation in the classrooms. They came to see themselves and their classmates as successful mathematics learners in the SRHS classrooms based upon their ability to teach mathematical concept to other individuals, and their ability to make substantial contributions to group and class discussions.

CHAPTER V

DISCUSSION

This study sought to explore the impact of a specialized residential high school environment on talented and gifted students' mathematics learning, and to understand the role the mathematics classroom microculture played in the construction of identity. I examined how students' mathematics learning was influenced by the academic requirements; and how the classroom design and teaching influenced students' perceptions about themselves in relation to mathematics. By understanding how students were enhanced by challenging curriculums, master teachers, peers, and rich technological resources, as well as a rich residential academic community, I have shown that a strong connection exists among students' identity, mathematics classroom culture, and residential academic environment. The analyses in these connections showed that the mathematics classroom microculture within the specialized residential high school powerfully and profoundly influenced students' identities as they engaged in significant mathematics activities.

This chapter concludes the study by first giving an overview of the trajectory of the study followed by a discussion of the results. Recommendations for the field of mathematics education and for the further research into identity that students developed in the mathematics classrooms appear at the end of the chapter.

Purpose of Study

The focus of identity may be helpful in understanding the students' definitions of what it means to be learners and doers in the context of mathematics learning (Martin, 2000). From a socialcultural perspective, learning is viewed as a social practice that occurs through social participation (Wenger, 1998). The participation is not just the local events of engagement in certain activities with certain group of people, but also “a more encompassing process of being active participants in the *practices* of social communities and constructing *identities* in relations to these communities” (p.4). As a potentially fruitful approach, identity may help in the exploration of interactions between microculture and mathematics learning in classrooms, and help in the understanding of issues of diversity and equity in mathematics education (Arbeu & Cline, 2007; Cobb, Gresalfi, and Hodge, 2009; Hodge, 2006; Lerman, 2006; Martin, 2000). Identity here refers to “a way of talking about how learning changes who we are and creates personal histories of becoming in the context of our communities” (Wenger, 1998, p. 5). This definition emphasizes the profound connection among identity, learning and practice. As a result, individuals come to know who they are as learners as a result of their participation with others in the experience of life (Wenger, 1998).

When learning occurs in the mathematics classroom, students “construct various of forms of mathematical knowledge as they participate in socially and culturally organized activities” (Nasir & Cobb, 2007, p. 3). At the same time, students construct mathematical identity in relation to the mathematics learning classroom community. That is, students develop beliefs about “their ability to perform in mathematical contexts; the instrumental importance of mathematical knowledge; the constraints and opportunities in mathematical contexts; and the resulting motivations and strategies used to obtain mathematics knowledge” (Martin, 2000, p. 19). In other words, students define who they are as doers and learners of mathematics by “the ways that they think about themselves in relation to mathematics and the extent to which they have developed a commitment to, and have come to see value in, mathematics as it is realized in the classroom"(Cobb, Gresalfi, & Hodge, 2009, p. 40).

In order to understand how the classroom culture influences the learning opportunities that high-achieving students are afforded in the mathematics classrooms, this study explored mathematics education in a specialized residential high school from the perspectives of students in their calculus classrooms in the Southeastern United States. In particular, the study was to seek answers to the questions: (1) what kinds of identities are developed in both home high school (HHS) and specialized residential high school (SRHS) classrooms? (2) How do the students' SRHS experiences interact with HHS experiences to affect the development of mathematical identity?

Research Design

In order to answer the research questions, I drew the theoretical perspectives for this study from the social theory of learning (Wenger, 1998), a multi-level framework (Martin, 2000), as well as the interpretive scheme (Cobb, Gresalfi, & Hodge, 2009) to account for students' participation in advanced high school mathematics classrooms.

Students who participated in the present study were students in grade 11 at a state-funded specialized residential high school (SRHS) for academically gifted adolescents. They had strong knowledge, aptitude, skill level, and proclivity toward the Science, Technology, Engineer, and Mathematics (STEM) subject areas. The academy was made up of 37 students in the 11th and 12th grades who lived in cottages and took mostly college-level mathematics and science courses taught by both the school's faculty and university faculty. Students were placed into mathematics courses after taking placement tests upon their arrival at the SRHS. The students had access to university facilities, including libraries and science labs. They also took classes and engaged in research projects with scientists in a national lab. Although the SRHS had a relatively small number of students, it was highly diverse. All of the students were concurrently enrolled in their home public high schools.

The students in the 11^{th} grade class were asked to participate in the present study. The main source of data resulted from surveys, interviews, observations and documentation, as well as classroom artifacts. Twenty-five junior students were asked to provide data based on their home high school mathematics learning experiences early in the spring of 2009. I conducted the interviews with six students to inquire and learn about the mathematical practices within the SRHS mathematics classrooms and the influences such practices have on students' relationship with mathematics. I conducted consecutive 10-week classroom observations of the lessons in order to understand the natural mathematical classroom normative ways of actions and interactions as lived by student participants. In addition, I collected the students' mathematics work artifacts and used documents in support of the surveys, interviews and observations.

I analyzed the data from the HHS and SRHS separately. I analyzed the survey responses by reporting descriptive statistics on closed-ended questions and by coding answers according to patterns in the students' open-ended questions (Gay, Mills, & Airasian, 2006). The observation and interview data were analyzed by using strategies of "relying on theoretical propositions" (Yin, 2002) to compare and evaluate the patterns to enlighten the interpretive scheme (Cobb et al, 2009; Lincoln & Cuba, 1985; Merriam, 1998; Yin, 2002). In other words, while the analysis of the data was guided by the interpretive scheme (Cobb et al, 2009), this did not limit the themes identified from the data.

From the analysis of these data, I first presented statistics and narrative details showing students' perspectives on how their HHS mathematics classroom environments served as a means to aid or prevent the students' access to significant mathematical concepts and ideas, and the ways in which students identified themselves as doers and learners of mathematics in classrooms. Five aspects emerged which include: (1) Classroom structure; (2) Students' role in the discourse; (3) Mathematical tasks; (4) Tools for learning; and (5) Students' perceptions of relationship with mathematics. Next, I presented, in detail, the results of the experiences of the six students (Aidan, Amanda, Daniel, Jackson, Steven, and Sophia) through individual interviews, classroom observations, and their work artifacts. The data analysis was related to the nature of SRHS mathematics classroom environments and its effects on how students came to see themselves as mathematical thinkers and as members of mathematics classrooms. I described what kinds of experiences these students had, how the classroom microculture supported each of the students, and what kinds of opportunities they embraced. Again, I presented themes that related to classroom normative ways of actions and interactions, as well as how students understand and value their classroom obligations and competences in the classrooms. In addition, I presented three aspects which were consistently highlighted by students with respect to the academic community of SRHS: (1) classroom challenges and expectations; (2) teachers' influence; and (3) peer's influences.

Discussion

In this study I explored the HHS and SRHS experiences and perspectives of a group of successful mathematics students, guided by sociocultural perspectives on learning that have increasingly become of great interest in diversity and equity in mathematics education research. The theories that I drew from Wenger (1998) on learning as participation in a community of practice, from Martin (2000) on mathematics socialization and identity at the school level and intrapersonal level, as well as from Cobb, Gresalfi, and Hodge's (2009) analytic approach to analyze the identity enriched my understanding of the ways of students' participation in mathematical learning practices, and how they view who they are, as well as the academic success that they achieved.

Guided by these theories, I was interested in the students' identities and their relationships with the learning environments in which they were situated. Data showed that, at the HHS students' successful mathematics performance, in addition to the partial support from teachers, was mainly determined by the support from their family and their own diligent practices. Indeed, all students demonstrated their full interest and commitment to learning mathematics at HHS. Their high expectations of themselves and future career goals played an important role in motivating their mathematics classroom lives. Yet, these students' participation was peripheral in the classroom community (Boaler, 2000; Cobb et al, 2009; Wenger, 1998). Their learning resulted from passive reception of information and their own initiative in striving for high achievement.

In contrast, all the participant students positioned themselves as playing a central role in the community of the SRHS mathematics classrooms. The students substantially increased their confidence and competence with mathematics through the practice of the sophisticated skills of reflection and inquiry. In addition, their success in learning mathematics was, to a great extent, determined by the rich academic community (Martin, 2000, Walker, 2006). This environment encompasses meaningful and challenging mathematical practices; master teachers; good social and academic relationships with cohort and peers. In the students' view, these unique elements of the SRHS academic community profoundly enhanced their effective and successful mathematical practices. Hence, students' SRHS worlds are more dynamic and connected than those of HHS (Walker, 2006).

General Classroom Obligations

As Cobb, Gresalfi, and Hodge (2009) defined, the general classroom obligations documents "the forms of agency" that students are able to exercise and "how authority is distributed" and thus "to whom student are accountable" (p. 63). That is, who is in charge is closely linked to the ways in which students are able to exercise conceptual and disciplinary agency in the classroom. The conceptual agency concerns the ways in which students choose methods and develop meanings and relations among mathematical concepts and principles, whereas disciplinary agency involves using established mathematical solution methods (Cobb, Gresalfi, & Hodge, 2009; Gresalfi & Cobb, 2006).

Although students, through interviews and artifacts, indicated successful mathematics performance and acknowledged their high ability to learn mathematics in both schools, students identified their general obligations and what it means to learn in HHS and SRHS mathematics classes quite differently. As illustrated in the analysis of HHS experiences, the students identified their mathematics learning classroom environment as a discrete, static student world (Walker, 2006), and they saw themselves as only marginally a part of the mathematics learning community with the teacher holding absolute authority (Boaler & Staples, 2005; Cobb, Gresalfi, & Hodge, 2009; Holland, Lachiotte, Skinner, & Cain, 1998). The students perceived that they were merely complying in HHS classroom activities. Their primary obligation was to practice disciplinary agency (Boaler & Staples, 2005; Cobb, Gresalfi, & Hodge, 2009; Holland, Lachiotte, Skinner, & Cain, 1998) which involves listening and copying notes, and working independently on questions and problems from the textbook or a worksheet, reading information from the textbook, and taking short answer tests, as well as using established methods to produce the correct answers that were validated only by teachers. More importantly, these mathematics experiences, in some cases, had a negative effect.

Interesting enough, perhaps because they had not experienced other types of instruction, the majority of students thought that practicing disciplinary agency was valuable to their learning. The concept of “getting a good grade” may not be the only reason, but grades were frequently cited and may provide a sense of why they highly valuate this obligation. Presumably, the students came to learn that doing mathematics competently means that getting correct answers is more important than mathematical processes or strategies (Anderson, 2007). Hence, they might perceive that merely cooperating with the teacher by listening and copying notes would yield a good grade.

On the other hand, in the SRHS mathematics classrooms, students focused on the practices of both conceptual and calculational mathematics aspects, which encompass the exploration of the underlying rationale and explanation of particular solution methods and strategies, as well as the procedures (Thomson &Thompson, 1996). As reflected in students’ comments, the teachers gave students great opportunities to engage in significant mathematical tasks as individuals and in groups; using multiple tools to explore, question, and present their solutions; and making justifications about their own and others' solution methods jointly with peers and teachers. Hence, they came to see themselves and their peers as being a more essential part of the community of SRHS mathematics classrooms than in their HHS. Additionally, they felt comfortable in expressing their own thoughts and ideas in the mathematics classrooms with encouragement and support from peers and teachers. In Amanda's case, she asked questions because she "wants to understand," she thought "searching for that knowledge is a sign of intelligence". In this way, the core obligation was to focus on mathematical processes or strategies rather than simply giving the answers. Like Amanda, the interviews with Steven showed that they fully enjoyed the learning experiences:

> Well, the most I can do is participate and try to grasp the knowledge to the best of my ability…Participation is key in learning all these things because if you don't participate, you're not going to understand it…I'm supposed to be asking questions or trying to correct their work if they did wrong…we try to find errors in their [classmates] work and collaborate together to try to solve the problem.

These experiences did, in fact, affect students' identity, especially their motivation to learn mathematics and to continue to obtain mathematical knowledge in college. For instance, Daniel remarked on his early mathematical experiences at HHS that he "wasn't very interested in math" and from that point on he "was just still indifferent". These feelings about the mathematics did not change over the HHS years "until I came here to SRHS". He made the following comments:

> Like, before I wasn't very interested in math. I just had it because the school required me to…The teacher was smart but she didn't know how to teach very well. She spent the whole time going over one or two example problems, and then that's all we learned. And we went through like, not even half the book. It didn't really make me like math more, I was just still indifferent until I came here to SRHS…here I-I like math more and more motivated to study math and um, so that's changed me and like, I'll probably continue taking more math classes in college.

I believe that these differences in the general obligations in the HHS and SRHS mathematics classrooms have implications for the roles of mathematics teachers and students. Teachers, like those of the SRHS, should maintain high expectations of students; take an active role in organizing mathematics classrooms; and have adequate resources to support and facilitate students' potential to express themselves creatively and communicate their meanings of mathematical concepts to their peers (Anderson, 2007; NCTM, 2000). Students should consistently recognize the value of engaging with both conceptual agency and disciplinary agency. Both the teachers and students, as members of the mathematics classroom community, should jointly establish the general obligations (Yackel and Cobb, 1996).

Doing Mathematics from Students' Perspective

Specifically mathematical obligations refer to the norms for mathematical argumentation, and normative ways of reasoning with tools and written symbols (Cobb et al, 2009). From this perspective, students have to fulfill the specifically mathematics obligations in order to be considered as mathematically competent students.

What it means to know and learn in mathematics classrooms as well as how students engage in mathematical activities in both HHS and SRHS classes are very different. As indicated by students' data, using a variety of mathematical tools to conjecture, proof, make connection, discuss, and communicate mathematical ideas was not a regular part of HHS classroom. The majority of students highly valued these learning approaches; yet, acknowledged that they had very limited opportunities to engage in investigations, approach the same problem from different perspectives, prepare a written report, make a presentation to the class, or discuss how math is used in life.

Even though opportunities were limited at their home high schools, all of the students valued the types of tools and instruction they were eventually able to access at SRHS. As reflected in students' comments, the mathematics activities in SRHS classrooms entail a great amount of thinking, reasoning, presenting, justifying, and cooperating with a variety of tools such as mathematical symbols, oral language, narrative writings, and computer software. Through this learning process, the students found that they had agency to examine "what counts as an acceptable explanation and justification [which] deals with the actual process by which students contribute" (Yackel & Cobb, 1996).

Perhaps the contrasting classroom practices that students are afforded in HHS and SRHS classrooms reflect what the teacher and students value in terms of mathematics teaching and learning. This would parallel the findings in Saxe, et al's (1999) study; different practices existed in their classrooms when the teacher valued the kind of mathematical teaching and learning proposed by the NCTM Standards (NCTM, 1989, 1991, 2000), or valued a procedural approach, or valued a discovery approach, respectively.

Based on the students' perceptions of the different specific mathematical obligations, teachers would be advised to select complex mathematical tasks that allow students to explore conjectures and then develop strategies by using a variety of mathematical reasoning and proof techniques. The classroom practice should focus on the process and explanations which allow students to see mathematics as a meaningful subject that is open to conjecture, derivation and interpretation, rather than a set of procedures and rules to be memorized. In addition, teachers may want to make explicit the ways that mathematics plays a part in students' daily lives, and take an active role in including a wide variety of mathematical topics in classes that relate to occupations (Anderson, 2007).

Students' Personal Identities

Personal identity is concerned with who students are becoming as they participate in particular mathematics classrooms (Cobb & Hodge, 2007). Students develop their personal identities in relationships with school climate, teachers and peers with respect to general and specifically mathematics obligations (Anderson, 2007; Cobb, et al, 2009; Martin, 2000). It refers to the students' understandings and valuations of their general obligations, their specifically mathematical obligations, their assessments of their own mathematical competence, and their assessments of other students' mathematical competence.

Students viewed mathematics as "an interesting body of knowledge worth studying, an intellectual tool for other disciplines, and admission ticket for colleges and careers," (Anderson, 2007) and perceived themselves as able students in learning mathematics in both schools. Nevertheless, what I found surprising was the degree to which students were able to provide clear evidence of relationships between the general classroom obligations and their development of identities. That is, how students understood and valuated their classroom obligations and what it means to be competent mathematics students in HHS and SRHS classes was substantially different. Students considered themselves to be good mathematics students at their HHS classrooms, but they were merely complying with the class norms as the teacher played a main role to deliver lecture and decide the correctness of answers. They, like students in study by Boaler (2000), identified monotony and isolation as characteristics of teacher-led mathematics classrooms. They felt they were provided limited opportunities to develop interest or a sense of affiliation with mathematics, which, in turn, lead them to engage in classroom mathematical activities more deeply and effectively (Boaler & Greeno, 2000; Gresalfi & Cobb, 2006). These students learned that mathematics is a useful subject to study but not vibrant enough to attract them (Anderson, 2007). This may have resulted from the students' high expectations of more challenging and meaningful mathematical tasks, countered by teachers' lack of strong content knowledge and equally necessary pedagogy content knowledge (Shulman, 1986).

When these students came to the SRHS and were submerged in the mathematics classroom learning practices, they began to see themselves as mathematical thinkers and doers and as a valuable member of the SRHS mathematics classrooms. They played a central role in establishing the norms of participating and negotiating. In particular, through the engagement in meaningful mathematics activities, they, as members of the

SRHS academically rich mathematics community, developed a strong mathematical identity (Martin, 2000). They fulfilled an obligation-to-others rather than obligation-to-oneself as they actively made their own mathematical arguments in the classrooms (Cobb, et al, 2009). They had a strong sense of belonging at SRHS for they were deeply engaged in the practice of mathematics learning and their contributions were valued by the peers and teachers (Anderson, 2007). They had a great opportunity to develop positive dispositions towards mathematical knowledge, which includes ideas about, values of, and ways of participating in mathematical activities (Gresalfi & Cobb, 2006).

In the initial stages of the study, there was a serious question concerning my assumptions about the students' perceptions of their own competence. How would or should students assess their own and other students' mathematical ability because they were placed in Calculus I and Calculus II classes according to the replacement assessment? Would students in Calculus I class think they were less competent than those of Calculus II class? Once I began my interviews, however, I soon discovered that the students had a strong confidence about their own mathematical ability regardless of the classroom they were in. Their assessment of the mathematical abilities indicated the kinds of beliefs about what it means to be mathematically competent students. It also became apparent that the classroom experiences affected their valuation of mathematical competence. In consistency with the design experiment study of Cobb et al (2009), students valuated their own and other students' mathematical competence based on how much they were able to make significant contributions to group and whole class discussions, and whether or not they were able to teach and apply what they learned to the real world situation, which also depends on their work ethics. This criterion differs from their HHS school classrooms where getting the high grade was the sole goal. Such differences might be related to teachers' beliefs about mathematically competent students and the ways of appreciating individual differences.

Mathematics classroom is regarded as the most significant potential setting to influence students' identities; and is supposedly only one community to understand how students view themselves as doers and thinkers of mathematics ideas (Anderson, 2007; Martin, 2000). A strong identity as a mathematics learner develops both through individual agency and through a set of increasingly complex activities within which they develop corresponding competencies (Anderson, 2007; Nasir, 2007; Wenger, 1998). Therefore, teachers should carefully choose complex mathematical tasks through which students are playing an active role in approaching mathematical problems, making meaning, and generating their own solutions.

Students' Academic Community

Students' webs of support for their mathematics work were complex with many interrelated factors influencing their learning experiences (Walker, 2006). In students' views, they have been greatly benefited from the superior residential learning environment. At the SRHS their learning was greatly enhanced by the high expectations and challenges, the behaviors of members of mathematical communities, the fluid relationships with classmates, peers, and teachers, as well as the access to technological resources. Hence, the "accessibility" to all kinds of rich resources was considered to be the greatest asset by students.

As reflected in students' comments, with the small cohort class size, the teachers and students at the SRHS produce more "dynamic and connected mathematics worlds"

(Walker, 2006). Not only did the teachers created multidimensional classes by designing and implementing open mathematics activities that students could solve in different methods, but also the teachers assigned meaningful group work and journal prompts outside of class providing opportunities for students to share key ideas from the course through description and reflection. As the work of Boaler (1999) indicates, the teachers gave considerable and varying amounts of freedom to students' choices and approaches to work, the way in which they behaved in classroom activities, and the way in which students organized the work and even their work environment. Hence, the students felt more persistent, confident, positive, and motivated as they engaged in mathematical practices and had successful learning experiences. In Steven's case, he believed that "the teacher makes the difference", and considered the SRHS mathematics teacher as "one of the best math teachers I've ever had".

Peers have also played significant roles in students' learning process. Although these students viewed themselves as successful in mathematics at the HHS, these students did not perceive themselves as effective participants in the mathematics classrooms and they felt isolated. What I found surprising was that students' HHS peers, unlike the high-achieving students in Walker's (2006) study, were not considered as an important support that contributed to the students' mathematical learning perhaps due to their ordinary or even low performance in mathematics. In contrast, their interview responses indicated that their intelligent and hardworking colleagues intensively motivated, encouraged, and supported their potential to learn advanced mathematical concepts and ideas. Also upper classmates served as a resource to help them study or understand mathematical content and to understand and appreciate the instructional strategies used at the SRHS. As with

Walker's (2006) high-achieving students, the students' relationships with their peers at SRHS were fluid, and the students were able to share their interests and achievement goals in mathematics on a daily basis (Walker, 2006) for they were living in an academic home. For example, the analyses showed Amanda's feelings about being an "outlier" at the HHS and that she appreciated the opportunities to learn with high ability peers at SRHS:

> The people [at SRHS]-the people are different. There's a different class of people here. They are hard working, they're intelligent and you're-you're not always with those kind of people at home schools, public schools that kind of like that because they have everyone. And it's not just, you know, the best and the brightest…It's very mixed. Like I said seniors in my Algebra 2 class, and they still didn't get it and they're seniors. It was like, "Why are you here?" You know, it's just crazy. But I really like that we're able to work in groups with intelligent equals and work on stuff, because they can help further my understanding.

Like Amanda, Steven pointed out that learning spreads and "friends influence your decisions", as he remarked:

> Well, peer pressure-like; you normally do what your friends do. Like you want to join in on what they're doing…if your friends like, a really excited and ready to learn then you'll probably get that aura from them and you'll be really excited and ready to learn. It's like the learning just-the want to learn just spreads.

In addition to the privilege of being with high ability students, the nature and quality of the interactions also had a profound influence on their mathematical learning.

The intellectual collaboration and mathematics talk in multiple settings such as in classes, on the phone, through the email, in the bedrooms, in the living rooms, in the cafeteria, in the school bus, and in the library did substantially occur at the SRHS. Besides, these students believed that persistence and hard work made them learn better and grow in mathematics. They epitomized the mantra that "for students with the growth mindset, it doesn't make sense to stop trying" (Dweck, 2006).

The roles of parents and family members seem to be peripheral factors in supporting students' mathematics learning. Students seldom mentioned them as primary contributors to their mathematics successful learning. Nevertheless, some students indicated that their parents and family members, particularly those who have advanced degrees and are working in mathematics related fields, expressed high expectations in terms of mathematics learning. It is unclear, however, how their parents and family members served as resources to enhance students' mathematical learning.

These conclusions suggest that the rich residential academic community may enhance students' mathematically experiences and aspirations among gifted students. Since the students are new members of a specialized mathematics classroom community, it is crucial to create an equitable learning environment and adopt equitable pedagogy to help them overcome the initial social and academic challenges represented by a specialized residential setting. Students' smooth transition from the HHS learning environment to SRHS depends upon the interactions among the high-quality curriculum standards, the teachers, the students, and the residential learning environment.

Despite the differences in the students' experiences at HHS and SRHS and the differences in the identities that students developed, it would be wrong to simply dichotomize the students at the two different settings, and suggest that all the students were mathematically competent upon their arrive at SRHS, while all their HHS peers were mathematically incompetent. The situation was much more complicated than that. There were several students who developed positive affiliation with mathematics as they engaged effective practices and went beyond the procedures to connect what they had learned with real world situation at the HHS classrooms. This might imply that their peers at HHS could adopt strong academically mathematics identities "by resisting or opposing what they perceive as negative or as obstacles that stand in the way of their goals" (Martin, 2000). Likewise, there were students at SRHS who strongly stuck to the traditional demonstration of mathematics they had experienced for 10 years prior to attending the SRHS, and therefore, resisted slightly the more open approach of teaching and learning. This demonstrates that students are capable of "employing the power of their own thoughts" (Boaler, 1999) either to merely cooperate with the teacher, come to identify with their classroom obligations, or actively resist engaging in classroom activities in local classroom cultures (Cobb, Gresalfi, & Hodge, 2009). However, the majority of the students strongly identified with their classroom general and specifically mathematical obligations and developed highly positive dispositions toward mathematical knowledge as they participated in SRHS mathematics classroom activities.

Recommendations

The results of this study can be used by other residential mathematics classrooms to develop strategies for creating learning environments conducive to the needs and unique challenges of gifted mathematics students. In addition, the general mathematics classroom teachers and students could gain some insights from SRHS students' successful and effective learning experiences in the community of residential mathematics classrooms.

To adequately address the highly capable mathematics students' needs, developing a high-quality curriculum is a priority. An authentic, outcome-based, flexible and challenging curriculum is necessary to meet students' academic needs and account for individual differences. According to Hockett (2009), five principles of high-quality curriculum were identified by the experts in gifted education:

- Principle 1: *High-quality curriculum for gifted learners uses a conceptual approach to organize or explore content that is discipline based and integrative.*
- Principle 2: *High-quality curriculum for gifted learners pursues advanced levels of understanding beyond the general education curriculum through abstraction, depth, breadth, and complexity.*
- Principle 3: *High-quality curriculum for gifted learners asks students to use processes and materials that approximate those of an expert, disciplinarian, or practicing professional*
- Principle 4: *High-quality curriculum for gifted learners emphasizes problems, products, and performances that are true-to-life, and outcomes that are transformational*

- Principle 5: *High-quality curriculum for gifted learners is flexible enough to accommodate self-directed learning fueled by student interests, adjustments for pacing, and variety (Hockett, 2009).*

While curriculum plays an important part in this small but complex specialized residential system, teachers are at the heart of this system. They have considerable ability to shape the curriculum, instruction, and students' views of mathematics, and advance students' learning practice in mathematics. As students become aware of academic standard differences between the general mathematics classroom and gifted mathematics classroom, they might consciously build boundaries between different groups of mathematics students. I recommend that mathematics teachers be aware of students' potential barriers in the classroom due to the different nature of curriculum in a general classroom and a gifted classroom. I also recommend the creation of an equitable classroom microculture to accommodate students' adjustments and the development of an advanced understanding that is different from general mathematics classrooms. In addition, teachers should be explicit about students' responsibility for their own learning, create an appreciation of diverse contributions, and demonstrate different ways of teaching and learning approaches as well as high challenges in classrooms.

The visionary classrooms described in Principles and Standards for School Mathematics (PSSM) (NCTM, 2000) is:

> All students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it…Students confidently engage in complex mathematical tasks…draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from

> different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress…they value mathematics and engage actively in learning it. (NCTM, 2000, p. 3)

Although the vision is highly ambitious, it has been realized with knowledgeable teachers, high-quality curriculum, flexible and resourceful students and access to technology as exemplified at the SRHS, where both equity and excellence are enhanced and supported.

Future Research

If long-term issues in achievement and persistence for these highly able mathematics students are to be understood better and more fully, future research is needed to examine the role and importance of contextual forces such as the home high school and its mathematics classrooms, the residential high school and its mathematics classrooms, the university mathematics classrooms, and the local community (home, working places, etc.) as well as the international mathematics learning community to see how these forces contribute to students' perceptions about the meanings of mathematics learning and how they see themselves as mathematics doers and learners. Specifically, there is the need for additional research in the following areas:

1. A longitudinal study of the students as they progress through the second year at a SRHS and their college mathematics classrooms to understand the issues of academic achievement and persistence in mathematics.
2. A quantitative and qualitative study with students' mathematics teachers, both the HHS and the specialized residential school that

explores the impact of teachers and teaching practices upon gifted mathematics students' learning of mathematics.

3. Replicate this study with international specialized residential mathematics students in other countries to gain insights into how they developed the identity in their mathematics classrooms as well as how broader contexts influence their perceptions about the roles and competence in the classrooms.

Chapter Summary

This chapter presented a summary of the study, conclusions and implications, as well as recommendations. I discussed my arguments about students' mathematics learning practice and identity in a specialized residential high school advanced mathematics classroom. I drew the theoretical perspectives for this study from the social theory of learning (Wenger, 1998), a Multi-level Framework (Martin, 2000), as well as the interpretive scheme (Cobb, Gresalfi, & Hodge, 2009) to account for the meanings for mathematics learning and knowledge that are developed by the students in advanced high school mathematics courses. In Wenger's framework, learning is viewed as a social practice that occurs through social participation. In this way, participation in the mathematics classroom community would shape, and is shaped by who students think they are. Martin's framework focuses on how such multiple contexts as sociohistorial, community, and school forces influence the mathematics socializations and identities of the high-achieving students. The Interpretive Scheme (Cobb et al, 2009) is an analytic approach to exploring the identities that students develop in the mathematics classroom. Conclusions were reached that the students had successful learning experiences both at

HHS and SRHS, yet, the two experiences are different in nature and in the extent of classroom actions and interactions. Also, success was defined differently in the two environments. With support from their teachers, peers, residential learning environment, and other kinds of academic resources at the SRHS, students developed increasingly positive identities in mathematics learning and demonstrated their full passion and commitment with strong self-confidence to learn mathematics in the current classrooms as well as in near college years. In addition, I discussed recommendations for further research concerning talented and gifted mathematics students' socialization and identity development in the specialized residential classrooms. I hope that newcomer high-achieving mathematics students will build up their positive mathematical identity through the experience of successful actions and interactions with their intellectual peers and teachers, as well as the supportive residential learning environment.

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APPENDICES

Appendix A: Mathematics Student Questionnaire

I. Please mark the circle that gives your opinion about each of the following statements BASED ON YOUR HOME HIGH SCHOOL EXPERIENCES.

		Agree	Neutral	Disagree
1.	My math class is fun.	O	O	O
2.	My math class is interesting.	O	O	O
3.	Things we do in my math class make me feel curious.	O	O	O
4.	I look forward to coming to my math class.	O	O	O
5.	I learn a lot in my math class.	O	O	O
6.	I look forward to taking more math classes after this year.	O	O	O
7.	To get a good grade in math, you have to know how to explain ideas in more than one way – using equations, pictures, words, or graphs	O	O	O
8.	To get a good grade in math, you have to know a lot of facts and formulas.	O	O	O
9.	To get a good grade in math, you have to be able to explain how you solve a problem, not just give the answer.	O	O	O
10.	Only the smartest people should think about becoming mathematicians.			
11.	Knowing math will help me in the future.	O	O	O
12.	Knowing math is useful in everyday life.	O	O	O
13.	It is important to know math to get a good job.	O	O	O
14.	You need to know math, even if you are not planning to go to college.	O	O	O
15.	When I’m an adult, I’d like to work in a job that uses a lot of math.	O	O	O
16.	Some people just can’t learn math very well.	O	O	O
17.	My math teacher knows a lot about math.	O	O	O
18.	Sometimes my math teacher admits not knowing the answer to a question.	O	O	O
19.	My math teacher encourages students to think of their own explanations.	O	O	O
20.	My math teacher encourages students to ask questions.	O	O	O
21.	My math teacher really enjoys teaching math.	O	O	O
22.	My math teacher asks a lot of questions.	O	O	O
23.	My math teacher makes math interesting.	O	O	O
24.	My math teacher thinks all students can do well in math.	O	O	O
25.	My math teacher thinks I can do well in math.	O	O	O
26.	My family thinks it’s important for me to learn math.	O	O	O
27.	My family thinks I can get a good grade in math.	O	O	O
28.	I think I can get a good grade in math.	O	O	O

II. Please mark the circle that gives your opinion about each of the following statements based on your HOME HIGH SCHOOL experiences.

	English/ Language Arts	Mathematics	Science	Social Studies
29. Which one of these subjects is your <u>most favorite</u> to study in school?	O	O	O	O
30. Which one of these subjects is your <u>least favorite</u> to study in school?	O	O	O	O

III. Briefly describe a typical class period in your 9^{th} or 10^{th} grade math class.

IV. How often do you do the following things in your math class?

	We never do this	We do this a little	We do this some	We do this a lot
a. Listen and take notes as the teacher presents information	O	O	O	O
b. Watch the teacher demonstrate a concept or procedure	O	O	O	O
c. Do a hands-on activity or investigation.	O	O	O	O
d. Design our own way to do an investigation or solve a problem	O	O	O	O
e. Work in a group with other students to complete a task or assignment	O	O	O	O
f. Make a presentation to the class	O	O	O	O
g. Read information from the textbook	O	O	O	O
h. Answer questions or do problems from the textbook or a worksheet	O	O	O	O
i. Make tables or graphs using data from an activity or investigation	O	O	O	O
j. Prepare a written report from an activity or investigation	O	O	O	O
k. Take short answer tests (multiple choice, fill in the blank, problem sets)	O	O	O	O
l. Take tests requiring written responses or explanations	O	O	O	O

m. Work on hands-on tasks as part of a test	O	O	O	O
n. Discuss how math is used in everyday life or in different jobs	O	O	O	O
o. Use a calculator to help with computations	O	O	O	O
p. Use a computer or graphing calculator to make graphs or work with data	O	O	O	O
q. Use the Internet to find information	O	O	O	O

V. What is the best thing about your math class in your HOME HIGH SCHOOL?

(Adapted from AMSP PEP Observation Protocol)

THANK YOU FOR COMPLETING THIS QUESTIONNAIRE.

Appendix B: Interview Protocol

Name__________________ Date______________________

Time:_______________

Sub-items are possible follow-up questions

1. How would you describe yourself as a mathematics student?
   a. Has your opinion of your mathematical ability changed over time? If different, how so?
   b. What influenced you to take advanced mathematics? And how will you go about learning it?
   c. How do you best learn math?
   d. What do you think your fellow students think about you?
2. If it is my (someone's) first day to observe your math class, what would I (he or she) expect to see? What would the class be look like?
   a. What's the most valuable, interesting and useful part? Why?
   b. What are you expected to do? How would you fulfill to meet it?
   c. How are the other students doing in the math class?

3. Tell me about your experiences in the home school.
   a. What were your mathematics experiences like in former high school?
   b. Do you feel like you have changed in any way as a math student?
   c. Could you be at the same math level if you stay in your former high school at the junior and senior year?
   d. If different, what experiences in the courses would you contribute to the change?

4. What are your thoughts about the possibility of taking math classes in college?

Appendix C: Classroom Observation Protocol

Math Classroom Observation Checklist	
Name: ______________________ Date: ____________________ School: _____________________ Grade: 6 7 8 Class/time: ________________	
Worthwhile Mathematical Tasks	**Comments**
Students are engaged.	
Students use a variety of mathematical tools.	
Conjectures, generalizations, and “what if?” questions abound.	
Misconceptions, limited understandings, and/or flawed reasoning surface.	
Students communicate about the math tasks at hand.	
Students’ Role in Discourse	**Comments**
Students present solutions.	
Students question one another.	
Students pay attention while another student is speaking.	
Students use a variety of tools to reason, make connections, solve problems and communicate their	

thinking.	
Students make conjectures.	
Tools for Discourse	Comments
Students are using "tools" to enhance discourse.	
Four kinds of tools are: written symbols, oral language, physical materials, previously acquired skills.	
Students are using the tools to: record, communicate, and think.	
Students are presenting and modeling their work.	
Students reflect on their learning.	
Students select tools that are appropriate.	
Culture in the Classroom	Comments
Students look at problems and ideas in different ways.	
Students celebrate their Aha!s.	
Wrong answers are viewed as worthwhile.	
Students are equitable in their spoken and unspoken messages about all students' mathematical potential.	
Students respect each other student's thinking.	

(Adapted from the NCTM Teaching Standards and based on work by NO LIMIT Math Integration Specialists August 2002.)

VITA

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