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Does the Grade Level at Which Algebra I is Completed Affect Future Mathematics Performance

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To the Graduate Council:

I am submitting herewith a dissertation written by Jamie Kay Fugitt entitled "Does the Grade Level at Which Algebra I is Completed Affect Future Mathematics Performance." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Teacher Education.

Vena Long, Major Professor

We have read this dissertation and recommend its acceptance:

Lynn Hodge, Gary Skolits, Carl Wagner

Accepted for the Council:

Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

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MATHEMATICS PERFORMANCE?

A Dissertation

Presented for the

Doctor of Philosophy

Degree

University of Tennessee, Knoxville

Jamie Kay Fugitt

December 2008

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of whom I am extremely proud.

Abstract

According to analysis of 2003 NAEP data, the percentage of students enrolling in Algebra I prior to ninth grade continues to increase, up to 42% in 2003. This current study is designed to examine the benefits of acceleration into algebra by exploring four major questions regarding timing of algebra. The first question examines relationships between student characteristics and timing of algebra. Relationships between school characteristics and timing of algebra are examined by the second question. Questions three and four explore relationships between timing of algebra and mathematics achievement and course taking, respectively.

Information was gathered on 449 students matriculating at a small liberal arts college, located in the Midwest, during 2007-2008. Students were grouped according to the grade level at which they completed Algebra I. Eighty-two students completed Algebra I prior to ninth grade, 288 during ninth grade, and 79 after ninth grade. Statistical tests utilized to analyze the data include the chi square test of independence, one way between group analysis of variance, and multinomial logistic regression.

A significant positive relationship between SES and enrollment in Algebra I prior to ninth grade and a significant negative relationship between SES and enrollment in Algebra I after ninth grade was found. No significant relationship was found between gender or race-ethnicity and timing of algebra.

Relationships between school type (home, private, public) and timing of algebra were significant. Home educated students were less likely than other students to complete Algebra I prior to ninth grade. Both home and private school students completed Algebra I after ninth grade more often than students from public schools. No significant difference in timing of algebra was found with regard to school size or school locale (rural/non-rural).

While early entrants into algebra did not complete more mathematics classes, they did complete more advanced mathematic classes and were more likely to study calculus. However, early entrants into algebra were more likely to drop out of the mathematics pipeline after tenth grade than other students. Early entrants into algebra had higher mathematics achievement as measured by Algebra II grades, mathematics grade point averages, and ACT Mathematics scores.

Table of Contents

Chapter 1	1
Background of the Study	1
<i>Significant Research Regarding Acceleration into Algebra</i>	2
<i>Personal Connection to the Study</i>	4
<i>Problem Statement</i>	6
<i>Significance of the Study</i>	6
<i>Theoretical Implications.</i>	6
<i>Practical Implications.</i>	12
<i>Limitations and Delimitations of the Study</i>	13
<i>Limitations.</i>	13
<i>Delimitations.</i>	15
<i>Definitions</i>	15
<i>Conclusion</i>	18
Chapter 2	20
Literature Review	20
<i>Non-Research Publications</i>	21
<i>Public Opinion.</i>	21
<i>Research Briefs.</i>	24
<i>Research Publications</i>	26
<i>Early Research Reporting Positive Outcomes.</i>	26
<i>Current Research Reporting Positive Outcomes.</i>	28
<i>Research Reporting Negative Outcomes.</i>	33
<i>Other Factors Affecting Mathematics Achievement</i>	36
<i>Racial-Ethnic Trends.</i>	37
<i>Socioeconomic Trends.</i>	38
<i>Gender Trends.</i>	39
<i>School Size, School Locale, and Curricular Offerings.</i>	40
<i>Conclusion</i>	42
Chapter 3	44
Methodology	44
<i>Setting</i>	45
<i>Data</i>	46
<i>Race-Ethnicity.</i>	48
<i>Student Socioeconomic Status (SES).</i>	48
<i>High School Information.</i>	48
<i>ACT Mathematics Score.</i>	49
<i>Summary.</i>	49
<i>Research Questions</i>	50
<i>Question 1.</i>	50
<i>Question 2.</i>	51
<i>Question 3.</i>	52

<i>Question 4</i>	53
<i>Conclusion</i>	54
Chapter 4	56
Findings	57
<i>Preparing the Data</i>	57
<i>Student Characteristics and Timing of Algebra</i>	60
<i>Gender and Timing of Algebra</i>	62
<i>Race-Ethnicity and Timing of Algebra</i>	62
<i>Socioeconomic Status and Timing of Algebra</i>	64
<i>Predicting Timing of Algebra</i>	67
<i>School Characteristics and Timing of Algebra</i>	72
<i>School Type and Timing of Algebra</i>	72
<i>School Size and Timing of Algebra</i>	73
<i>School Locale and Timing of Algebra</i>	77
<i>Timing of Algebra and Mathematics Course-Taking Patterns</i>	79
<i>Timing of Algebra and All Mathematics Courses</i>	80
<i>Timing of Algebra and Advanced Mathematics Courses</i>	81
<i>Timing of Algebra and Persistence in the Mathematics Pipeline</i>	82
<i>Timing of Algebra and Calculus</i>	84
<i>Timing of Algebra and Mathematics Achievement</i>	86
<i>Timing of Algebra and Algebra II Grades</i>	87
<i>Timing of Algebra and Mathematics Grade Point Average</i>	89
<i>Timing of Algebra and ACT Mathematics Scores</i>	91
<i>Conclusion</i>	95
Chapter 5	96
Conclusions	96
<i>Summary of the Study</i>	97
<i>Findings</i>	98
<i>Student Characteristics and Timing of Algebra</i>	98
<i>School Characteristics and Timing of Algebra</i>	99
<i>Course Taking and Timing of Algebra</i>	100
<i>Mathematics Achievement and Timing of Algebra</i>	101
<i>Discussion</i>	102
<i>Student Characteristics and Timing of Algebra</i>	102
<i>School Characteristics and Timing of Algebra</i>	102
<i>Course Taking and Timing of Algebra</i>	103
<i>Mathematics Achievement and Timing of Algebra</i>	105
<i>Conclusion</i>	105
<i>Recommendations for Further Study</i>	107
References	111
Vita	120

List of Tables

Table	Page
Table 1: <i>Locale Code</i>	17
Table 2: <i>Questions Addressed in the Study</i>	55
Table 3: <i>Summary of Data</i>	61
Table 4: <i>Gender*Algebra Group Cross-Tabulation</i>	63
Table 5: <i>Income Levels for Student SES</i>	65
Table 6: <i>SES Level*Algebra Group Cross-tabulation</i>	66
Table 7: <i>Likelihood Ratio Test</i>	69
Table 8: <i>Parameter Estimates for Gender+Ethnicity+SES Model</i>	70
Table 9: <i>School Type*Algebra Group Cross-tabulation</i>	73
Table 10: <i>School Size Codes</i>	76
Table 11: <i>District Size Code</i>	77
Table 12: <i>School Locale*Algebra Group Cross-tabulation for All Students</i>	78
Table 13: <i>Descriptive Statistics for 9-12 Mathematics Courses</i>	81
Table 14: <i>Algebra Group*Mathematics Persistence Cross-tabulation</i>	83
Table 15: <i>Algebra Group*Calculus Credit Cross-tabulation</i>	85
Table 16: <i>Point Value of Letter Grades</i>	88
Table 17: <i>Descriptive Statistics for Algebra II Grades</i>	89
Table 18: <i>Descriptive Statistics for Mathematics GPA</i>	90
Table 19: <i>Descriptive Statistics for ACT Math Scores</i>	93

List of Figures

Figure	Page
Figure 1: <i>Positive Relation Between SES Level and Membership in Algebra Group 1</i>	68
Figure 2: <i>Negative Relation Between SES Level and Membership in Algebra Group 3</i>	68
Figure 3: <i>Relation Between Membership in Algebra Group 3 and School Type</i>	74
Figure 4: <i>Relation Between Membership in Algebra Group 1 and School Type</i>	74
Figure 5: <i>Persistence in the Mathematics Pipeline by Algebra Group</i>	85
Figure 6: <i>Calculus Enrollment by Algebra Group</i>	86

Chapter 1

Background of the Study

Should all students study algebra? Although this question is frequently debated, there is considerable agreement among educators today that the study of algebra is critical to students' futures (Bottoms & Carpenter, 2003; NCTM, 2000; USDE, 1995). Algebra has been identified as a gatekeeper in that successful completion of algebra is required for further study in mathematics as well as other school subjects. Success in algebra has also been linked with success in college and with many career opportunities (Algebra Project, n.d.; Silver, 1997). As of 2006, 28 states require all students to complete Algebra I before graduating from high school (Public Education in Arkansas, 2006). This requirement is closely aligned with work of the National Council of Teachers of Mathematics (NCTM). In the NCTM's *Principles and Standards for School Mathematics (PSSM)*, published in 2000, the case is made that *all* students should study algebra throughout their school careers. In *PSSM*, a view of algebra as a content strand spanning the pre-kindergarten through secondary school curriculum is promoted. Significant emphasis on algebra in the middle grades is also recommended, so that a strong foundation in algebra is in place by the end of eighth grade to accommodate a challenging high school mathematics curriculum for all students.

This recommendation of algebra for all brings up another recurrently debated set of questions. When should students take Algebra I? Should all students take Algebra I at the same point in their school careers? If so, should this be seventh, eighth, or ninth grade? These questions have been discussed at school, district, state, and national levels for at least the last two decades. In 1987, Usiskin, in an article printed in the *Mathematics Teacher*, proposed that the majority of students in the United States should be enrolled in an algebra course in the eighth

grade. Over a decade later, in 1999, an article by Steen appeared in *Middle Matters* questioning why we rush students to take algebra in eighth grade. Steen suggested we continue to offer algebra to students only when they are ready to take it – some in eighth, some in ninth, and some in tenth grade.

Significant Research Regarding Acceleration into Algebra

During the 90s several opinion pieces were published regarding early entry into algebra and the effects on mathematics achievement. The majority of these articles indicate that acceleration into algebra has a positive correlation with mathematics achievement. An early research study on the topic was conducted by Smith and was published in *Educational Evaluation and Policy Analysis* in 1996. Based on analysis of the High School and Beyond data, Smith (1996) concludes that “early access to algebra has a sustained positive effect on students, leading to more exposure to advanced mathematics curriculum and, in return, higher mathematics performance by the end of high school” (p. 48).

Also in 1996, Dulaney analyzed success rates of students in the Wake County Public School System in North Carolina to provide advice to parents about the benefits and concerns associated with early entry into algebra. The results of his study paint a much bleaker picture of early entry into algebra. Dulaney's analysis indicates that many students who take algebra in eighth grade find it necessary to repeat the course in high school. His data also show that a large number of early entrants into algebra drop out of the mathematics pipeline prior to their senior year, not enrolling in the mathematics courses designed to be accessible because of early entry into algebra. Dulaney warned parents and others that “There is a risk that students who enter Algebra I too early may experience frustration or failure, may get “turned off” by the challenges

of higher mathematics courses, or may find themselves forced to enroll in courses that are more advanced than their interests require” (Dulaney, 1996, p. 2).

A more recent study, conducted by Partenheimer and Miller in Indiana with results presented at the American Education Research Association Annual meeting in 2001, also questions the benefits of early entry into algebra. The results of this case study are similar to the results of Dulaney in North Carolina, with many early entrants into algebra repeating Algebra I in high school and with many of these students exiting the mathematics pipeline in tenth grade.

Over the past five years Ma, Department of Curriculum and Instruction, University of Kentucky, has published several articles relating directly to the topic of acceleration into algebra. These articles report research conducted by Ma in which he analyzes data from the Longitudinal Study of American Youth (LSAY). The purpose of his research is to examine the effects of enrollment in formal algebra in middle school on such factors as mathematics course-taking patterns and mathematics achievement. These analyses indicate that regardless of mathematics achievement at the beginning of middle school, students who are accelerated into algebra in seventh or eighth grade experience more mathematical growth than students who are not accelerated. In regard to mathematics course-taking patterns, Ma's research shows that students completing Algebra I in eighth grade take more advanced mathematics and science courses throughout high school.

This brief reporting of significant research related to acceleration into algebra indicates that no consensus has been reached regarding the benefits of completing a formal algebra class in middle school. While the recent research conducted by Ma consistently reports positive benefits of early entry into algebra, the research is somewhat limited by the data set under consideration, with the entirety of this research based on analysis of the LSAY data. Further research should be

conducted using a variety of data sets to see if similar conclusions are reached. Additionally, research should be conducted to determine the availability of early entry into formal algebra for various groups of students in a variety of locales as well as the effects of early entry into algebra on mathematics course taking and mathematics achievement.

Personal Connection to the Study

I personally have experienced this dilemma regarding the most advantageous placement of a first formal algebra class within a child's course of study. My son was not allowed to take pre-algebra in seventh grade, thus putting him off track to take algebra in eighth grade. Believing he was developmentally and mathematically ready to study algebra in eighth grade, I contended with the school administrators for permission for him to enroll in Algebra I during eighth grade. This turned out to be a good choice for him, as he understood the material well and early entry into algebra allowed him to study calculus in high school, creating a good foundation for study of college level mathematics. In subsequent years, the requirements for early entry into algebra in our local school system became less stringent with more students enrolling in Algebra I during middle school. Subsequently, I have engaged in numerous conversations with parents questioning their decisions to enroll their children in algebra in middle school due to the intellectual struggles and frustrations their children experienced.

I have also experienced the dilemma regarding acceleration into algebra from a mathematics professor's point of view. At the college where I teach we have a significant percentage of students with weak mathematics skills. Virtually all of these students have completed a formal course in high school algebra, but many are required to complete a remedial course in algebra before they are allowed to enroll in College Algebra. The algebraic concepts which are covered in this remedial course are the same concepts which are typically covered in a

first course in formal algebra in middle school or high school. Often students comment that these algebraic concepts are completely foreign to them, or they may vaguely remember hearing about the concepts, but have very little understanding of them. Even among the students who are not required to complete a remedial algebra course, weak algebra skills are often evident. Frequently these students comment that they remember studying these algebraic concepts, but that so much time has elapsed that they don't remember them. Some indicate that they completed Algebra I in middle school and believe they have not used algebra much since then and cannot remember many of the basic concepts of introductory algebra. Would students remember more of the concepts of a first algebra course if they took Algebra I a year later and perhaps had a higher level of cognitive development and could understand abstract concepts more completely? Would other students, if allowed to take algebra earlier, receive a greater challenge while in middle school and thus develop a more positive attitude towards mathematics resulting in persistence in the mathematics pipeline and higher mathematics achievement at the end of high school? Does the time at which a first formal course in algebra is completed affect mathematics course taking and mathematics achievement?

As indicated previously, research has shown that a solid foundation in algebra is critical for future academic and career success. It is also the case that there are conflicting opinions about the optimal time for students to complete a first formal algebra course. These two factors sparked an interest for the researcher in conducting a study to attempt to determine if the time at which students complete Algebra I is related to students' future mathematics course-taking patterns and mathematics achievement. The study also examines the availability of early entry into algebra to students in rural and non-rural locales and in schools of varying sizes.

Problem Statement

The purpose of this study is to examine relationships between timing of algebra and students' mathematics achievement and mathematics course-taking patterns. Characteristics of early entrants into algebra are also examined. Additionally the availability of acceleration into algebra for students who attend different size schools in different locales will be explored. The major research questions for this study are stated below. They are further delineated in chapter three.

1. What student characteristics are associated with the grade level at which a first course in formal algebra is completed?
2. What are the relationships between school characteristics, such as school size and locale, and the grade level at which a first course in formal algebra is completed?
3. What are the relationships between mathematics course-taking patterns and the grade level at which a first formal course in algebra is completed?
4. What are the relationships between mathematics achievement and the grade level at which a first formal course in algebra is completed?

Significance of the Study

Theoretical Implications. If one looks into a typical mathematics classroom in the United States, various patterns emerge. Young children learn to count and write numbers. They study basic addition facts of whole numbers and then relate addition and subtraction. In second or third grade, children are introduced to multiplication and are encouraged to memorize basic multiplication facts before the concept of division is introduced. Children then progress through many mathematical topics including decimals, fractions, and ratios. It has long been believed that

after mastery of these and other mathematical topics, children are ready to enroll in a first formal course in algebra.

Although education generally, and mathematics education specifically, has varied greatly over the past few centuries, this widely accepted sequencing of topics in mathematics has been in existence since the early days of formal education in the United States. According to Cohen (2003), arithmetic knowledge in eighteenth century America was learned from textbooks usually imported from England. These books usually contained a collection of rules which children dutifully copied into their own copybooks. These textbooks first introduced students to numeration and place value. This was followed by addition, subtraction, multiplication, and division of whole numbers. After these topics were introduced there was some variation in texts. Some texts then covered the basic operations with fractions and decimals while others took up weights, measurements, and commercial calculations. Also included in most texts was the study of ratios and the four basic operations with multi-digit numbers. Despite the presence of this very familiar order of topics, many of the eighteenth century arithmetic texts contained claims that arithmetic had no sequential nature to it (Cohen, 2003). "The Order in which the different Rules should be taught is a matter entirely arbitrary, and therefore no directions could be given for it; however, they are so disposed as to have but little dependence on each other, and consequently every teacher is left to his own choice in that respect" (Bonycastle, 1786, preface). Despite such a disclaimer regarding the sequential nature of mathematics, the very familiar sequence of arithmetic topics studied in elementary classrooms today seems to have been firmly established since the eighteenth century.

What is vastly different between mathematics education in the eighteenth and twenty-first centuries is the target audience of mathematics instruction. During the eighteenth century,

arithmetic was regarded as a topic which was too difficult for young children and thus, formal study of mathematics was usually undertaken by twelve to fourteen year old boys. Arithmetic was viewed primarily as a vocational subject with direct application to commerce. In light of this, typically only boys engaged in the study of arithmetic. The study of arithmetic during the eighteenth century was actually more vocation specific than gender specific. However, with most women appointed to a domestic role in life, arithmetic and especially more advanced areas of mathematics were viewed as completely useless to women. Actually, because of the belief that the study of mathematics developed one's capacity for logical thought, some believed that the study of mathematics would undermine "women's essential intuitive and imaginative nature," damaging their femininity (Cohen, 2003, p. 67).

In addition to this very familiar sequencing of arithmetic topics, other familiar sequencing patterns within mathematics have been present in the U.S. educational system for well over a hundred years. According to Angus and Mirel (2003), "The main components of what became the traditional series of high school mathematics courses -- beginning algebra, geometry, advanced algebra, and trigonometry -- were in place by the late nineteenth century" (p. 443). However, this traditional series of courses has not continued without much debate and questioning. Early in the twentieth century critics began raising questions about the importance and the nature of mathematics in high school. Some argued that the traditional course of study of mathematics in high school was elitist because it allegedly neglected the needs of non-college-bound students, focusing instead on preparing students for college. In order to better understand the changes occurring in high school mathematics during the twentieth century, Angus and Mirel (2003) divide the century into four distinct periods. They refer to these four eras as "the vocationalizing high school" (1910-1930); "the custodial high school" (1930-1960); "the

shopping mall high school" (1960-1980); and "toward the twenty-first century high school" (1980-2000). Each of these periods experienced changes in the high school curriculum as a whole, and specifically in the high school mathematics curriculum.

The focus of "the vocationalizing high school" era was on devising a vocational program alongside the traditional college-preparatory program. This new curriculum was intended to make high school more attractive, thus engaging a much broader cross section of students. To make room for additional vocational classes, other subjects, including mathematics, had to play a less prominent role in the overall curriculum offerings. Basic arithmetic, algebra, and geometry courses all lost ground in enrollment percentages during this time. Also, differences in mathematics course-taking patterns increased both across gender and class lines during this era.

With the onset of the Great Depression and the decline of job opportunities for teenagers, American high schools experienced the largest increase in enrollment relative to population size in history. This brought about "the custodial high school" era in which the focus was to expand the general curriculum track and make the curriculum more interesting and immediately applicable to the needs of students. Throughout this period the enrollment rate in mathematics classes continued to decline reaching an all time low of 55% in 1948-1949 (Angus & Mirel, 2003, p. 463). A large number of general and practical mathematics courses were offered as alternative courses to algebra, geometry, and trigonometry.

During the 60s, America's high schools did not escape the effects of Americans' infatuation with individualism and personal freedom. A plethora of specialized alternative courses were made available to students hence initiating the label "the shopping mall high school." During this era, enrollment in mathematics courses remained fairly steady with a slight shift toward non-college-preparatory mathematics courses.

Sharp increases in mathematics course taking began to occur in the late 70s. Much of this growth occurred in basic and consumer mathematics classes, but increases were also present in enrollment in elementary and pre-algebra courses. This growth continued throughout the 80s and 90s with much of the growth concentrated in mathematics classes that had previously been considered strictly college preparatory. "The percent of high school graduates earning credit for elementary algebra increased from 54 percent to 67 percent" from 1982 to 1994 (Angus & Mirel, 2003, p. 474). Despite this impressive increase in mathematics enrollment after 1980, many questions about the nature of these courses and the availability of these courses to all students still remain.

Questions regarding the most advantageous sequencing of mathematics topics and courses continue to be debated. Recent publications by the NCTM bring light to this debate. For example, in the 2000 publication of *Principle and Standards of School Mathematics (PSSM)*, algebra is presented as a topic which, rather than reserved for high school, should be incorporated in all levels from pre-kindergarten through twelfth grade. Despite these recent recommendations, the majority of U.S. students complete a first formal algebra class in ninth grade. In the late 90s about 25% of U.S. students completed Algebra I in eighth grade (Riley, 1997). Recent NAEP data indicate that the percentage of students enrolling in Algebra I prior to ninth grade has increased significantly since this 1997 report. Currently some estimates suggest that as high as 42% of U.S. students enroll in Algebra I prior to ninth grade (Mathews, 2007). This particular report does not indicate how many of these students successfully *complete* Algebra I prior to high school, moving on to more advanced classes in ninth grade.

This current study has the potential of providing insights into the sequential nature of mathematics education. The findings of this study may add confirmation to the benefits of the

sequencing of mathematical topics that currently exists in the majority of U.S. schools. Perhaps, on the other hand, the findings of the study will raise questions regarding this sequencing of topics or even indicate that this sequencing limits mathematics achievement for many students.

Results of this study may also raise questions regarding the commonly accepted belief among mathematics educators that the study of formal algebra should be reserved until one has a good grasp of many basic arithmetic concepts. Most educators believe that the most efficient way for a child to learn a language is for the child to be immersed in the language. Children are not required to master the recognition and writing of all the letters of the alphabet before they are allowed to use words and even form complete sentences. Children are not required to know the spelling of words and the dictionary definitions of words before they are encouraged to use the words to communicate with others. Eventually children do study language in a formal sense, with this process made much more meaningful and accessible because of early immersion into the language. Algebra is often referred to as a language, and some educators compare the learning of algebra and the learning of language. Perhaps the study and acquisition of algebraic knowledge should be approached in a manner similar to the study and acquisition of language. Perhaps, as is being posited by the NCTM, algebraic concepts should be introduced to children during pre-kindergarten days with continual exposure to the topic throughout school. If this study finds that middle school children, even those who have not mastered many arithmetic concepts, benefit from exposure to formal algebra, new insights may be provided into the manner in which children learn algebraic concepts and the sequencing of mathematics that is practiced in so many U.S. schools.

The results of this study also have potential to contribute to the broader discussion regarding equity issues in mathematics education. Over the past several years, much research has

been conducted to analyze relationships between mathematics achievement and course-taking patterns and factors such as gender, race-ethnicity, and student SES. Although consensus has not been reached among researchers, results of much of this research do indicate that some gaps in mathematics achievement may be attributable to the aforementioned student factors. As a part of this study, these student characteristics will be examined in light of the time at which a first formal course in algebra is completed. Findings regarding relationships between these student characteristics and the opportunity to enroll in formal algebra prior to high school may be significant for equity issues, especially if early entry into algebra is shown to be related to increased mathematics achievement.

Practical Implications. Decisions about when students complete a first formal algebra class are important to academic outcomes for students. Parents often have access to publications such as the ACT Policy Report, *College Readiness Begins in Middle School*, which indicates that middle school students who complete Algebra I are more likely to enroll in advanced mathematics courses in high school and are more likely to attend a four-year college (Wimberly & Noeth 2005). However, parents may at the same time be provided contradictory information. For example, parents could access Dulaney's aforementioned research, which paints a less than positive outcome for many students who enroll in algebra in eighth grade. According to this study, a fairly significant number of students studying Algebra I in eighth grade receive a grade of C or lower with many students repeating the course in ninth grade. The primary reason given for early entry into algebra for students in Dulaney's study was to allow them to pursue Advanced Placement Calculus during their high school careers. The report indicates that only 41% of the students who accelerated into algebra actually took AP Calculus during high school. Parents are cautioned in the report that early entry into algebra does not necessarily indicate that

students will take more advanced mathematics in high school or that these students will have attained greater mathematics achievement by the end of their senior year (Dulaney, 1996).

If, in this current study, the effects of timing of completion of Algebra I on mathematics course-taking patterns and future mathematics achievement can be established, this result will be of great interest to students, parents, guidance counselors, mathematics teachers, school administrators, and state and national educational personnel. The information may be used to better advise students regarding mathematics course taking and how decisions about mathematics course taking may affect their future academic and career choices. Also, if relationships are found between school size and locale and timing of completion of Algebra I, school teachers and administrators may be challenged to rethink course offerings and course sequencing, providing more equitable mathematics education opportunities for all students.

Limitations and Delimitations of the Study

Limitations. The data set under consideration contains very limited information about student mathematics achievement prior to enrollment in formal algebra. Thus determination of the effects of early entry into algebra on rates of growth in mathematical achievement was not feasible. Additionally, information about the nature and content of the mathematics courses taken by students involved in the study was not available. Courses which share similar names but are offered by different school districts may vary greatly in quality and content. The available data did not allow the research to determine these differences.

Due to the nature of the data set under consideration it was difficult to ascertain Algebra I letter grades earned by students who completed the course prior to ninth grade. This was due to the lack of access to middle school transcripts. If Algebra I was not included on a high school transcript, but a higher level course, dependent on completion of Algebra I, was included, the

assumption was made that Algebra I was completed prior to high school and thus the student was classified as an early entrant into algebra. While this technique for identifying early entrants into algebra is common in research related to the topic, it does raise some concerns. Some research indicates that a significant number of students who attempt a first formal course in algebra prior to high school repeat the course in high school (Dulaney, 1996; Partenheimer & Miller, 2001). However, when Algebra I was included on a high school transcript, determining if the class was being repeated was not possible.

There has been much discussion and disagreement during the past few decades regarding an appropriate definition of mathematics achievement and how mathematics achievement can best be measured. Because the mathematics portion of the ACT exam is used as a measure of mathematics achievement in this study, a somewhat limited definition of mathematics achievement is utilized. Since the ACT Mathematics exam contains only multiple-choice questions, concern exists that mathematics achievement may be too narrowly defined by this exam. Despite this concern, due to the significance of ACT scores to college acceptance and placement in college courses, its use as a measure of mathematics achievement in this study is justified. The use of additional measures of mathematics achievement helps to further alleviate this concern.

Another major concern of the study was with the interaction of the variables. For example, if students who complete algebra in eighth grade score higher on a mathematics achievement exam administered at the end of high school, can one determine if any of these gains in achievement are attributable to acceleration in algebra? Perhaps the achievement should instead be attributed to parental support, SES level, prior mathematics ability, etc. Information related to some of these contributing factors was available in the data under consideration. While

this information was invaluable in determining which factors affect timing of algebra, mathematics course-taking patterns, and mathematics achievement, care was taken to limit the confounding of the variables as much as possible.

Currently in some schools, students have the option of completing Algebra I over a two year period. These two courses involved are often titled Algebra IA and Algebra IB. If a student completed Algebra IA in eighth grade and Algebra IB in ninth grade, this complicated the issue of whether or not the student should be classified as an early entrant into algebra. In the case where Algebra I was completed over two years, the first year was designated as pre-algebra and completion of the second year was designated as completion of Algebra I. Also, the sequencing and names of some mathematics courses made classification of the course as dependent on algebra or not dependent on algebra difficult in some cases.

Delimitations. This study is limited to students enrolled in a small liberal arts college in Southwest Missouri. The majority of students enrolled at the college are Caucasian and reside in the Midwest. Therefore, the results of the study will be limited in generalizability. Although data included in the study is limited to students who entered the college during the 2007-2008 academic year, the data provides a randomly selected and adequately sized sample for the study.

Definitions

ACT Mathematics Exam -- "The ACT Mathematics Test is a 60-question, 60-minute test designed to measure the mathematical skills students have typically acquired in courses taken by the end of 11th grade" (ACT, Inc., 2008a, ¶ 1). The test includes three sub-scores which are based on the following six content areas: pre-algebra, elementary algebra, intermediate algebra, coordinate geometry, plane geometry, and trigonometry.

Advanced Mathematics Courses -- For the purpose of this study, classes beyond a first course in geometry and Algebra II are considered advanced mathematics courses. Thus classes such as college algebra, trigonometry, pre-calculus, and calculus are classified as advanced. Classes such as applied mathematics, technical mathematics, pre-algebra, Algebra I, geometry, and Algebra II are not classified as advanced.

Algebra Group -- For the purpose of this study, students were placed in Algebra Groups based on the time at which they completed Algebra I. Students who completed Algebra I prior to ninth grade were placed in Algebra Group 1. Algebra Group 2 is comprised of students who completed Algebra I in ninth grade, and students who completed Algebra I after ninth grade were placed in Algebra Group 3. Only one student had not completed Algebra I, and this student was dropped from the study.

Early Entrant into Algebra – This phrase refers to a person who completes Algebra I prior to ninth grade.

Early Entry into Algebra -- This phrase refers to the completion of Algebra I prior to ninth grade. For most early entrants into algebra the course is completed in the eighth grade, and for a smaller number it is completed in the seventh grade. The phenomenon is also referred to as acceleration into algebra.

First Course in Formal Algebra -- Algebra I, combination of Algebra IA and Algebra IB, or a similar combination is considered a first course in formal algebra. The phrase, “first formal course in algebra” is used interchangeably with this phrase.

Locale Codes -- Locale codes were first used by the National Center for Education Statistics (NCES) in the 1980s to classify a school's location ranging from "large city" to "rural." The codes were redesigned in 2005 and 2006. The new locale codes, based on an address's proximity

to an urbanized area, classify an area into four major types: city, suburban, town, or rural. Each of these major types has three subcategories. For city and suburb these are based on population size differences while towns and rural areas are further delineated by their distance from an urban area. The 2005-2006 codes are used to define locales in this study and are provided in Table 1. In this study city, suburb, and town are classified as non-rural, while locale codes 41, 42, and 43 are classified as rural (Identification of rural locales, n.d.).

Mathematics Achievement -- Mathematics achievement has been measured in many different ways. For this particular study, mathematics achievement is measured in a variety of ways, including scores on the ACT Mathematics exam. Further measures, such as grades earned in Algebra II and high school mathematics grade point averages, are also considered as measures of mathematics achievement in this study.

Mathematics Course-Taking Patterns -- This phrase refers to the sequence of mathematics courses that a student takes during his or her high school career.

Table 1: *Locale Code*

Category	Locale Code
City	11, 12, 13
Suburb	21, 22, 23
Town	31, 32, 33
Rural	41, 42, 43

Prepared from information from the National Center for Educational Statistics Website -- http://nces.ed.gov/ccd/Rural_Locales.asp

Mathematics Pipeline – This phrase indicates an organization of transcript data into levels based on the normal progression and difficulty of courses within mathematics. This phrase has specific meaning in certain data sets such as the National Educational Longitudinal Study data set, but is used more generically in this study.

Non-rural High School -- High schools with a locale code of 11, 12, 13 (City); 21, 22, 23 (Suburb); or 31, 32, 33 (Town); as defined by the National Center for Education Statistics (NCES) are classified as non-rural for the purpose of this study.

Rural High School -- High Schools with a locale code of 41, 42, or 43 (Rural), as defined by NCES, are classified as rural for the purpose of this study.

Student Socioeconomic Status (SES) -- For the purpose of this study, student socioeconomic status is based on information obtained from the Free Application for Federal Student Aid (FAFSA). Family adjusted gross income and family size are compared to information provided by the U.S. Department of Agriculture regarding qualifications for public school free and reduced price meals (USDA, 2008). This information is used to assign each student an SES Code of one to four, with one indicating lowest SES and four indicating highest SES.

Conclusion

In this chapter, the importance of mastery of basic algebraic concepts by all students during high school has been established. The sequential nature of mathematical topics and mathematical course-taking patterns, currently apparent in U.S. schools, has been discussed with a brief history of this sequencing presented. This analysis led to the major questions of this study which examine relationships between the grade level at which students complete Algebra I and various components of mathematical performance. Theoretical and practical implications of the study were then established, based partially on the lack of agreement among researchers on the

effects of early entry into algebra. Finally limitations and delimitations of the study, along with definitions important to the study were presented.

Chapter 2

Literature Review

While there is general agreement among mathematics educators that *all* students should study formal algebra there is ongoing debate about the optimal timing for an introduction into a first formal course in algebra. Some argue that virtually all students should study Algebra I in eighth grade, while others assert that the nature and quality of the course is significantly more critical than the timing of the course and that students should enroll in Algebra I at different times based on motivation and maturity levels.

In what follows, arguments both in favor of and against early entry into algebra are examined. First, opinions found in non-research publications such as popular magazines and educational journals are reviewed. Attention is then given to examination of research briefs which are connected to various national data sets but often are devoid of rigorous statistical analysis. Following a review of these publications, both past and current research on the effects of timing of algebra on mathematics achievement and mathematics course-taking patterns are examined. This research is grouped into early research (prior to 2000) reporting positive outcomes of early entry into algebra, current research (2000 to present) reporting positive outcomes of early entry into algebra, and early and current research reporting negative outcomes of acceleration into formal algebra.

Following a review of research directly related to early entry into algebra a brief review of research on other factors which have been linked with mathematics achievement is included. Such factors as socioeconomic status (SES), gender, and race-ethnicity are considered in relation to mathematics achievement and course-taking patterns. Additionally, literature related to relationships between school size and locale and mathematics course-taking patterns is briefly

examined. The review of these various bodies of literature reveal gaps in current research, indicating a need for further research in the area of relationships between early entry into algebra and mathematics achievement and course taking.

Non-Research Publications

Public Opinion. One of the most popular water cooler topics in America today is education. Virtually everyone has an opinion on what is right and what is wrong with public schools in the United States. Perhaps this is because nearly all Americans play a role which involves them directly in education -- a student, a parent or grandparent of a student, an employer hoping to hire a recent graduate, a public school teacher. If one engages in a discussion regarding the challenges facing education in the United States it is common for the discussion to turn to mathematics education. Some opine that the study of any mathematics beyond basic arithmetic is a waste of time. After all, one may argue, "I am a well educated, productive citizen and I have never used algebra or geometry a day in my life." Others decry the state of mathematics education in the United States and disparage the performance of U.S. students on international mathematics exams. These individuals may believe that the study of higher level mathematics by all school age students is a necessity if the United States is to remain competitive in a global economy.

Over the past two decades there has been much debate about the merits of studying mathematics in general and algebra specifically. In the 1980s, with the advent of such significant publications as *A Nation at Risk*, *Everybody Counts*, and *Curriculum and Evaluation Standards for School Mathematics*, many began to propose that algebra, a subject which at one time had been reserved for the elite group of college-bound students, be studied by *all* students. There is evidence that over the past few years this idea has gained acceptance. As of 2006, 28 states

require successful completion of Algebra I for graduation from high school (Public Education in Arkansas, 2006). Despite the large number of students studying algebra prior to graduation, a consensus regarding high school algebra requirements has not been reached, even among educators. For example, in the April 2000 issue of *Mathematics Education Dialogues*, Noddings, Emeritus Professor of Child Education, Stanford University, and Professor of Philosophy and Education at Teachers College, Columbia University asserts that the requirement that all students take courses designed for college-bound students is shameful. She argues that requiring all students to successfully complete a formal course in algebra forces students into courses suitable for only some future careers. "Instead of showing truly democratic respect for all honest work, we treat everyone as college-bound. Students who take jobs without going to college do so by default because they are considered 'not good enough' for college" (p. 2). Noddings concludes that students should have the option to choose college or non-college curricula, doing so with pride and with both yielding an excellent and valued education.

More recently, Cohen (2006), a self-acclaimed hater of mathematics, writes in *The Washington Post* of a girl who dropped out of school after failing algebra six times. He claims that in Los Angeles, "more kids drop out of school on account of algebra than any other subject" (§ 4). He states that although he was forced to study algebra in high school he has never once in his life used algebra and the idea that the study of algebra is beneficial for strengthening reasoning skills is "a lie propagated by, among others, algebra teachers" (§ 7).

Despite an abundance of opinion pieces decrying algebra, the majority of U.S. students successfully complete at least one year of formal algebra prior to receiving a high school diploma. As more and more educators have come to agree with the idea that all students should study algebra, attention has turned to the optimal time for students to complete a first formal

algebra course. For over a century Algebra I has been most commonly studied during the freshman year of high school. Over the past two decades parents, school administrators, guidance counselors, public school teachers, and college professors have engaged in an ongoing debate regarding the most advantageous time for students to be introduced to formal algebra. A significant number of people have been involved in this debate, with many well-respected educators arguing for acceleration into algebra for all, while others caution against such a practice. In 1987, Usiskin, a well-respected professor of mathematics education, posited that most students should be taught algebra in eighth grade. He provides several reasons why he believes algebra should be taught in eighth instead of ninth grade. He reasons that algebra would be beneficial to most eighth grade students since in traditional middle school mathematics classes very little new material is explored in seventh and eighth grades. Additionally he argues that it is probably easier for children to learn algebra at age thirteen than at age fourteen, and that taking algebra in eighth grade reduces pressure on students in grades 9 through 12, allowing them to pursue the study of more advanced mathematics.

About a decade later, another well-respected educator, Steen (1999), weighed in on the issues. Steen, who teaches mathematics at St. Olaf College, argues strongly that algebra for all in eighth grade is not a viable option for reaching the goal of algebra competence for all students. He provides four reasons why he supports this stance. First he argues that few students finish seventh grade with the maturity, motivation, and preparation to successfully study formal algebra. He also states that few eighth-grade teachers are prepared to teach algebra and few algebra textbooks are properly designed to assist eighth-grade students as they progress from arithmetic to the abstract reasoning required in algebra. Finally he asserts that most teachers do not believe that all students can learn algebra in eighth grade and that this belief influences

student learning. In conclusion he states that "the most effective way to make 'algebra for all' a reality is for students to take it when they are ready -- some in eighth grade, some in ninth, and some in tenth. What matters is not when students study algebra but that they learn it well" (Steen, 1999, p. 7).

Research Briefs. Since the publication of these two hallmark pieces by Usiskin and Steen, many other articles and reports, some connected with national data sets such as National Assessment of Educational Progress (NAEP), National Education Longitudinal Study (NELS), and ACT, have been published. Authors of these reports analyze the particular data set under consideration to determine factors which affect mathematics achievement. Many of these reports examine the effects of early entry into algebra, with virtually all indicating positive benefits of the practice. For example, in the February 1996 issue of *NAEPFACTS* in which the 1992 NAEP results are examined, the claim is made that "eighth graders who were enrolled in algebra courses had consistently higher average proficiencies than students enrolled in pre-algebra, who in turn had higher proficiencies than students taking general eighth-grade mathematics courses" (Shakrani, 1996, p. 2). In a comparable analysis published by the Southern Regional Educational Board in 2003, in which an exam referenced to the NAEP is used, similar conclusions are drawn. The researchers conclude that "Students who take algebra before grade nine have higher mathematics achievement scores than students who do not take algebra in the middle grades" (Bottoms & Carpenter, 2003, p. 13). Based on this study, which specifically examines mathematics achievement in rural schools, the recommendation is made that more students should complete "real" Algebra I in the middle grades.

In a 1999 research brief, published by the National Center for Education Statistics, the NELS:88 data is examined in an attempt to determine if gatekeeper courses such as algebra and

foreign language expand educational opportunities for students. While this report does not focus on the relationship between algebra in eighth grade and mathematics achievement, other factors are connected with taking algebra in eighth grade. "Enrollment in gatekeeper courses, such as algebra and foreign language in eighth grade helps students reach higher levels in the mathematics and foreign language pipelines" (Atanda, 1999, p. 1). Also reported in this research brief is a link between studying algebra in eighth grade and an increased likelihood of applying to a four-year college.

In an ACT policy report published in 2005, a strong argument is made for college readiness beginning in middle school. Based on this conjecture, parents, teachers, and guidance counselors are urged to encourage middle school students to enroll in rigorous courses such as Algebra I and foreign language while in middle school. The report indicates that middle school students who complete courses such as Algebra I have the opportunity to enroll in advanced classes in high school which in turn results in a higher likelihood that they will apply to a four-year college (Wimberly & Noeth, 2005).

Both public opinion pieces, whether printed in education journals or in popular magazines, and research briefs, have the potential to greatly influence mathematics education in the United States. Parents, students, school board members, teachers, and other school personnel may rely on these articles for guidance on how to best advise students in decisions related to mathematics course taking. While these publications may be very beneficial to many they may also be somewhat misleading. The opinion pieces are generally not based on research, but often upon the experiences of the author and may not be generalizable. The aforementioned research related articles, often fail to take into account other important factors which may affect mathematics achievement. The most obvious seems to be prior mathematics achievement. For

example, students who enroll in formal algebra early often do so because of exceptional previous mathematical achievement. If these students already possess superior mathematics achievement levels prior to early enrollment into algebra, it would only seem logical that their level of mathematics achievement would remain superior after completion of algebra. With that being the case one must be cautious concluding that early entry into algebra *causes* increased mathematics achievement for these students. These opinion pieces and research briefs also often fail to consider other student factors such as SES and parental support and school factors such as school SES, teacher quality, and curricular materials which may also affect mathematics achievement. Since the 1990s, research which includes an analysis of the effects of many of these factors on mathematics achievement has been conducted. After a brief examination of research on the effects of acceleration into algebra, research regarding other factors affecting mathematics achievement is reviewed

Research Publications

Early Research Reporting Positive Outcomes. In a study published in 1996 in *Educational Evaluation and Policy Analysis*, Smith investigates the effects of taking algebra prior to high school on students' mathematics achievement in high school. The major question explored in this study is, "Does early entry into the advanced mathematics 'pipeline' give students an added advantage in high school math attainment?" (p. 142). Transcript files from the High School and Beyond data, including transcripts of 9,158 high school sophomores (1980) who stayed in school until graduation in 1982, were analyzed in this study. The students were considered in two groups: students who had taken algebra prior to high school ($n = 1,076$) and students who had taken an algebra course in high school ($n = 5,818$). Students who never took algebra were dropped from the study. Ordinary least squares regression methods were used to

examine the effects of taking algebra prior to high school on the number of years of advanced mathematics taken by the senior year and mathematics achievement measured during the senior year. Other factors, such as student background characteristics and tenth grade mathematics achievement were also considered in the analysis.

The results of the research reveal that there are differences in students who have early access to algebra. Students who experienced early entry into algebra came from families of higher socioeconomic status and were less likely to be from minority groups. The study also found that students who completed algebra prior to high school continued in the mathematics pipeline longer and took more advanced mathematics classes than those who took algebra in high school. Smith (1996) hypothesizes that early access to algebra may “socialize” a student into taking more mathematics, regulating access both to advanced coursework and increased achievement in high school” (p. 141).

Limitations of the study include the manner in which students were grouped. If a high school transcript included Algebra I, the student was grouped with those who had not taken algebra prior to high school. Thus students who attempted algebra prior to high school but chose to retake it in high school were not included in the early access group. If a significant number of students unsuccessfully attempted algebra prior to high school this is likely to affect the results of the study. Another limitation of the study is the lack of information on the quality of the algebra courses taken both prior to high school and in high school.

This research appears to be the most significant piece regarding early entry into algebra which was published in the 90s. However, as mentioned above, during this same period many reports and opinion pieces did appear on this topic, with the majority also suggesting positive

outcomes for algebra-taking in middle school and recommending that more students be placed in algebra during middle school.

Current Research Reporting Positive Outcomes. In a study published in *Contemporary Educational Psychology* in April 2005, Ma analyzed data from the Longitudinal Study of American Youth (LSAY) to examine if early entry into formal algebra in middle school promotes growth in mathematics achievement. The LSAY data followed over 2000 students for six years as they progressed from seventh grade through twelfth grade. Each year the students completed a student questionnaire and took a mathematics achievement test. The test measured outcomes on four mathematical subscales: basic skills, algebra, geometry, and quantitative literacy.

The key independent variables in the study were early acceleration in mathematics and initial mathematics achievement at the beginning of middle school. Other independent variables considered in the study include age, family structure and size, gender, language spoken at home, parent socioeconomic status, and race-ethnicity. School-level variables, provided by student, teacher, and principal questionnaires were considered. These include factors such as school size and location, student-teacher ratio, parental involvement, and extracurricular activities.

Hierarchical linear (HLM) models were employed to quantify the amount of growth (from Grades 7 to 12) in each of the four mathematical areas and examine the degree of stability of growth in achievement across the four mathematical areas” (Ma, 2005b, p. 445). Rates of growth were examined in four ways in each of the four mathematical areas tested. Initially, unconditional rates of growth were examined without any adjustment. The analysis indicates that statistically significant growth occurred in each of the four mathematical areas, with students

growing fastest in algebra and geometry. The data also indicate that students who experienced more mathematical growth in one area, such as algebra, also grew faster in the other three areas.

Following this examination of rates of growth without any adjustment, “conditional rates of growth were then estimated with early acceleration (together with initial mathematics achievement and their interaction), student characteristics, and school characteristics sequentially and accumulatively added to the unconditional model” (Ma, 2005b, p. 447). These analyses indicate that regardless of mathematics achievement at the beginning of middle school, students who were accelerated into algebra in seventh or eighth grade experienced more mathematical growth than students who were not accelerated. Also reported, is that students with low initial achievement benefited more from early acceleration than students with high initial achievement in mathematics. Analysis also indicates that school and student characteristics did not make much difference in the rates of growth in any of the four mathematical areas.

One concern related to this study is how students who repeated algebra in ninth grade were classified. It is not clear if a student who took algebra in eighth grade and then repeated algebra in high school was classified as receiving early entry into algebra. If a significant number of students retook algebra in ninth grade, how they were classified may affect the results of the study. Additionally, the author uses the expression “grew faster,” without clear definition, to describe relative student achievement growth. Careful examination of the results, lead the researcher to conclude that this expression is used to describe differences in point gains. The use of the expression “grew faster” complicates the interpretation of the findings. The study is somewhat limited by an inadequate amount and depth of information related to school curricular and instructional characteristics available in the LSAY data.

In a second article published from analysis of this same data set, Ma (2005a) grouped students according to ability and examined the correlation between early acceleration into algebra and mathematics growth across ability levels. Students scoring above the 90th percentile in mathematics in the seventh grade were classified as gifted; students scoring between the 65th and 90th percentiles were classified as honors; and students scoring below the 65th percentile were classified as regular. The results of the study indicate that early acceleration in mathematics had little advantage in mathematics achievement for gifted students, small advantage for honors students, and large advantage for regular students. The major limitation of this study is the small number of accelerated students, especially regular accelerated students. The author reports that statistical measures (pooled multilevel analyses) were taken to alleviate this concern.

An article published in 2000 by Gamoran and Hannigan provides some counterpoints to Ma's conclusions. This research, in which the National Educational Longitudinal Study (NELS) data are analyzed, was designed not so much to examine the effects of early entry into algebra as the benefits of algebra for all students. Students were placed into one of four groups according to whether they had not taken algebra, had taken algebra only in high school, had taken algebra only in eighth grade, or had taken algebra in both eighth grade and high school. The dependent variable for this study was mathematics achievement as measured on a 40-item multiple choice exam. Information gathered from parent and student surveys allows factors such as gender, race-ethnicity, and socioeconomic status (SES) to be considered. The research first examines the main effects of taking algebra. Secondly the linear interaction of eighth-grade mathematics scores and taking algebra is considered. Thirdly, statistical measures are taken "to consider the possibility that algebra has comparable positive effects for most students, but not for students with very weak skills" (p. 245).

As a result of this research, the conclusion is drawn that "all students, regardless of prior math skills, benefit from taking high school algebra" (p. 250). While achievement growth is evident for each of the four student groups, "differences in achievement growth between those who did take algebra and those who did not are much greater than the differences among those who took algebra at different points in time" (p. 246). In contrast to Ma's findings reported above, in which students with lower initial mathematics achievement benefited most from early entry into algebra, Gamoran and Hannigan (2000) found that "the benefits of taking algebra in high school are weaker for students whose initial scores were at the 20th percentile or lower" (p. 246).

Further conclusions, directly related to the effects of early entry into algebra, are also reported by Gamoran and Hannigan. They found that after controlling for eighth-grade mathematics scores, students who took algebra only in eighth grade gained more in achievement than students who never took algebra. "However, those who took algebra only in eighth grade, but still entered high school with very low test scores, received no boost to tenth-grade scores from their eighth-grade algebra course" (p. 246). Once again this seems to differ from Ma's findings that students with lowest initial mathematics achievement may benefit most from early entry into algebra.

Despite the fact that the evidence of this research points to benefits of taking algebra, the authors point out that it is still difficult to predict the effects of moving all students who previously have not taken algebra into high school algebra. Since it is impossible to simulate a situation in which all students enroll in algebra, the researchers compare schools with a more homogeneous population of algebra-takers to schools with a more diverse population of algebra-takers. They conclude that "the data give no reason to believe that increasing the cognitive

heterogeneity among students enrolled in algebra would weaken the impact of algebra courses on growth in student achievement" (Gamoran & Hannigan, 2000, p. 250).

Gamoran and Hannigan's analysis of the NELS data includes a category of students who take algebra in eighth grade and then retake the class in high school, eliminating the concern which was present in Ma's analysis about the classification of this group of students. However, a limitation of this study is the timing of the mathematics test used to measure mathematics achievement. This assessment was administered during tenth grade, with students having considerable more opportunities to study high school mathematics. It is not clear if the benefits of taking algebra would still be present at the end of high school and if the timing of the study of algebra might have more or less of an impact on mathematics achievement measured near the end of twelfth grade. Additionally, as with the previous study, this study is somewhat limited by an inadequate amount and depth of information related to school curricular and instructional characteristics available in the NELS data itself.

Other analyses conducted within the last five years suggest similar positive effects of early entry into algebra. For example, a group of researchers from University of California at Davis followed the high school experience of over 3,500 students from five urban high schools (Paul, 2003). The students were placed in five groups depending on if/when they took Algebra I and whether it was completed in one or two years. This research found that students completing Algebra I in eighth grade took more advanced mathematics and science courses throughout high school, received better grades in these courses, and were more likely to complete the core college prep courses required by several state universities in California. Using a logistic regression model, the researchers also found that after controlling for variables such as gender, SES, and race-ethnicity, location of a student within an Algebra Group was still a strong predictor of

success in mathematics attainment. A very significant limitation of this study is that no information is available about mathematics achievement of students prior to placement into Algebra I. Thus it is impossible to determine if the academic success of the eighth-grade algebra group is related to their accelerated placement in algebra.

The amount of research comparing acceleration into algebra and various aspects of mathematics attainment has increased since the turn of the century, with much of it indicating positive outcomes for students experiencing early entry into algebra; however, research reporting negative outcomes of acceleration into algebra is also available and is examined in the following section.

Research Reporting Negative Outcomes. The results of a case study of the effects of accelerating eighth-grade students into algebra appear in a non-published paper presented at the American Educational Research Association Annual meeting in 2001 (Partenheimer & Miller, 2001). This longitudinal study involves examining mathematics course-taking patterns and transcripts of 49 students as they progressed from eighth grade to twelfth grade, graduating in 1992. These 49 students, constituting 27% of the class, were selected from a group of 179 students to take algebra in eighth grade. During the course of the study, three students transferred out of the school system so that data from 46 participants were analyzed. Numerous research questions were posed in the study with two of particular interest to this review.

The focus of one question of interest is on the relationship between acceleration into algebra and further mathematics course taking. Accelerating students into algebra in this Indiana school appears to have produced somewhat negative results as indicated by the following findings: (1) nineteen of the 46 students (41%) chose to retake Algebra I in ninth grade; (2) sixteen of the 46 students (35%) chose to exit the mathematics pipeline after taking Algebra II in

tenth grade; and (3) only fourteen of the 46 students (30%) completed five years (including Algebra I) of advanced mathematics.

The second research question of interest focuses on the effects of early acceleration into algebra on mathematics achievement. In order to analyze achievement, grades from advanced mathematics classes were examined for the 46 participants. The number of accelerated students receiving a C grade in specific classes is as follows: Algebra I (17/46) 37%; Ninth grade Algebra I (repeat) (10/19) 53%; Geometry (29/46) 63%; and Algebra II (27/46) 59%. Perhaps even more disconcerting is the fact that 35 of the 46 students (76%) received a grade of C or below in Geometry with 37% of those who attempted Geometry in ninth grade receiving a D or F in at least one semester of the course. The grades seem surprisingly low considering the fact that students were identified to accelerate into algebra based on high IQ scores, success in seventh grade pre-algebra, and recommendation from a seventh grade teacher.

This study has some limitations, the most obvious of which is related to external validity. The reader is not able to conclude if specific characteristics of the school, students, teachers, or mathematics curriculum might affect these results so that they are not generalizable to other settings. It is worth noting that the authors cited other, some what dated literature (1981 & 1985), that reports similar conclusions about the attrition rate of accelerated algebra students in the mathematics pipeline (Dockweiler & Prevost, as cited in Partenheimer & Miller, 2001).

A second limitation of the study is that no control group was present in the study. No doubt it would have been infeasible to randomly select half of the students who were identified as candidates for mathematical acceleration and place them in a non-accelerated group; however, examining the course-taking patterns and grade point averages of the students in the school who

were not accelerated into algebra might have provided some valuable comparisons and strengthened internal validity.

In 1996, Dulaney analyzed success rates of students in the Wake County Public School System in North Carolina to provide advice to parents about the benefits and concerns associated with early entry into algebra. The results of his study are similar to those of the previously reported study. In 1990-91, about 21% of the eighth-grade students who took Algebra I received a grade of C or below in the class and 12% repeated the class. The following year eighth-grade algebra enrollment increased substantially (the report does not include enough data to determine the exact percentage increase) and the percentage of students who received a grade of C or below increased to 34% with 16% of the students repeating the class.

According to Dulaney's article, the primary reason for allowing enrollment in eighth-grade algebra in the Wake County School System is to allow for enrollment in Advanced Placement Calculus during the senior year without necessitating enrollment in two advanced mathematics courses during a single year. However, only 41% of the students who were accelerated in algebra actually took Advanced Placement Calculus.

It is important to note that the information reported for these students from North Carolina does not come from a carefully planned research study. The information is simply reported as an analysis of data to aid in course-taking decisions. Many factors such as motivation, family support, teacher characteristics, and SES may have contributed to the results of this data and those factors were not considered in this analysis. However, it is informative that the results gathered from this analysis of data from North Carolina parallel in many ways the findings reported for the Indiana students. Thus as indicated in this section, some research, although limited, seems to indicate negative outcomes associated with early entry into algebra.

Other Factors Affecting Mathematics Achievement

Research indicates that factors other than timing of mathematics course work affect mathematics achievement. Significant among these are student factors such as socioeconomic status (SES), race-ethnicity, and gender, as well as broader factors such as school SES and teacher qualifications. Over the past several years much funding has been dedicated to research designed to determine relationships between these factors and mathematics achievement. In a meta-analysis published in the *Journal for Research in Mathematics Education* in 1997, Tate examined quantitative research literature from the previous fifteen years "to determine trends in mathematics achievement of various social groups defined along lines of race, class, gender, ethnicity, and language proficiency" (p. 652). After examining general population trends, Tate analyzed research based on the following data sources: NAEP data gathered from 1973 to 1992; NELS data collected in 1988, 1990, and 1992; College Entrance Exams; and Advanced Placement Calculus Exams.

More recent information regarding factors influencing mathematics achievement was presented in 2003 at the annual meeting of the American Educational Research Association in Chicago. In this presentation, Lubienski and Shelley reported on "trends related to race and socioeconomic status (SES) and mathematics achievement, beliefs, class room experiences, course-taking patterns, and teachers' educational backgrounds" (abstract). For this study, NAEP data collected from the fourth, eighth, and twelfth grade mathematics achievement exams in 1990, 1992, 1996, and 2000 were used. Data from teacher and student surveys administered as part of NAEP assessments were also included.

The findings of these two studies concerning mathematics achievement and race-ethnicity, SES, and gender are examined below, allowing for an analysis of what changes

occurred in relationships between these variables from 1973 to 2000. Other recent research and analyses is reviewed as well.

Racial-Ethnic Trends. Tate's (1997) analysis of various research publications indicates that from the 1970s to the 1990s all racial-ethnic groups at each grade level made statistically significant gains in mathematics achievement. His findings indicate that African American and Hispanic students made greater gains than White students, slightly closing the mathematics achievement gap. With several recent studies indicating mathematics course-taking differences between racial-ethnic groups, Tate's reporting of a strong relationship between mathematics achievement and mathematics course taking is significant. Some of the research he reviewed suggests "course taking has the potential to equalize achievement levels across racial-ethnic groups" (p. 662).

In 2003 Lubienski and Shelley reported good news similar to that reported by Tate -- between 1990 and 2000 mathematics achievement, as measured by NAEP scores, increased for all racial-ethnic groups at all grade levels. However, unlike Tate, Lubienski and Shelley reported that in some instances the mathematics achievement gap between certain racial-ethnic groups appears to be growing. The Hispanic-White and Black-White achievement gaps measured at the eighth-grade level increased during the decade from 1990 to 2000, indicating that middle grades are a critical time for focus on mathematics achievement for all students. The nature of the NAEP data allowed the authors to conclude that "on average, black students are leaving high school with less mathematical knowledge than white 8th graders possess" (p. 15). Further analysis indicates that racial-ethnic achievement gaps in mathematics vary according to mathematical strand. The largest racial-ethnic achievement gaps were reported in the areas of measurement and data analysis with the smallest gaps in algebra and geometry.

Socioeconomic Trends. Both of these studies also examined relationships between student SES and mathematics achievement. Tate reviewed several studies which focused on relationships between mathematics achievement and socioeconomic status of students. These studies used various measures, such as parents' education levels, family income, and community types in which the students reside, to define socioeconomic status. Consistently, the studies indicate a very strong relationship between SES and mathematics achievement. The findings "demonstrate the need to raise the mathematics achievement of low-SES students as a whole and, even more urgently, of low-SES minority students. Moreover, these findings suggest the need for an intervention in the two geographic regions with the highest poverty levels--urban and rural communities" (p. 667).

The NAEP data, as analyzed by Lubienski and Shelley, also indicate that SES-related differences in achievement are large, with students in the highest-SES quartile significantly outscoring students from the lowest-SES quartile. The SES-related gaps are similar in size to the mathematics achievement gaps related to race-ethnicity. Lubienski and Shelley further analyzed the data to determine interactions between race-ethnicity and SES on mathematics achievement. They concluded that "SES related differences do not account for substantial portions of the race-related gaps ... [evidenced by the fact that] the lowest-SES white 12th graders score more similarly to the highest-SES black students (3-point gap) than to their lowest-SES black counterparts (22-point gap)" (pp. 15-16). In an attempt to explain the race-ethnicity and SES related achievement gaps, Lubienski and Shelley examined factors such as student beliefs about mathematics, instructional practices in mathematics classes, student course-taking patterns, and teacher educational backgrounds. Differences in instructional practices, such as methods of

assessment, curricular materials, and access to calculators were found to be related to students' race-ethnicity as well as their SES.

Gender Trends. In addition to race-ethnicity and SES, Tate reviewed studies which included analysis of NAEP, NELS:88, College Entrance Exams, and AP Calculus data in regard to relationships between gender and mathematics achievement. These studies suggest "that no significant gender achievement differences exist on items measuring basic skills" with the exception being 17 year olds on the 1992 NAEP trend assessment where males significantly outscored females (Tate, 1997, p. 670). Tate's findings also suggest that the small mathematics achievement differences that do exist between genders emerge in secondary school. Some of the researchers thus conclude that these differences in achievement levels may be explained by differences in mathematics course-taking patterns between male and female high school students.

While the 2003 Lubienski and Shelley publication does not focus on gender and mathematics achievement, in a 2006 publication, McGraw, Lubienski, and Strutchens describe gender gaps in mathematics achievement appearing in the NAEP data from 1990 to 2003. The analysis regarding gender achievement gaps reveals "relatively small but consistent gaps favoring males across reporting years (1990-2003)" (p. 145). However the analysis suggests that the gender gap is not evenly distributed across mathematical content strands or across percentiles. At each grade level, gaps favoring males were largest in the measurement strand and at the upper end of the percentile range. An analysis of interactions of gender disparities within racial-ethnic groups indicates a significant gender gap, favoring males for White and Hispanic students but no significant differences for Black students. Because of the lack of appropriate data

at the time of the analysis, interactions among gender, race-ethnicity, and SES could not be analyzed. In a 2001 study, Lubienski states, "Any existing gender gaps pale in comparison to the highly significant gaps between Black and White students, as well as the gaps between the lowest- and highest-SES students" (Lubienski, 2001, p.8).

A 2007 statistical analysis report of course-taking patterns of U.S. students, prepared by staff of NCES, corroborates many of the aforementioned findings. While the research indicates that persistence in the mathematics pipeline and enrollment in advanced mathematics courses increased for students from all subgroups from 1982 to 2004, significant disparities were found between racial-ethnic groups. White and Asian students were significantly more likely to persist in the mathematics pipeline and enroll in advanced mathematics courses than students in all other racial-ethnic groups. Similar relationships were found between SES and mathematics course taking. From 1982 to 2004 enrollment in advanced mathematics courses increased for students from all SES groups; however, disparities in enrollment in pre-calculus and calculus between the highest and lowest SES quartiles increased during this time period. In regard to gender, the report indicates that no substantive differences were found between males and females in percentages completing advanced mathematics courses (Dalton, Ingles, Downing, & Bozick, 2007).

School Size, School Locale, and Curricular Offerings. Since the 1950s a considerable amount of research has been conducted regarding ideal school size. Many of these studies have focused on school size and the relationship to the economics of education. In addition, and of significant interest to this study, is research which examines relationships between school size and curricular offerings, as curricular offerings are likely to affect course-taking patterns. Repeatedly, over the past half century, various studies have revealed a strong positive

relationship between school size and the number of distinct course offerings. For example, in a study published by Barker in 1985, personnel from over 1300 public high schools in the United States responded to surveys providing information about 105 different course offerings. The collected data reveal that over 35% of these courses were offered at significantly higher frequencies in larger schools. Included in these more frequently offered courses were four mathematics courses -- advanced geometry, calculus, probability and statistics, and computer math. Another study, examining relationships between school district size and high school course offerings in Connecticut, was published by Melnick, Shibles, and Gable (1987) during the same time frame. The results of this study also confirm that large high schools consistently offer more advanced courses than small or medium high schools. However, the results of this study differ from those reported by Barker in that small, medium, and large high schools did not differ significantly with respect to course offerings in calculus or computer programming.

Despite the increased curricular course offerings in large schools, research by Melnick et al (1987), finds that there is no significant difference in the percentage of students in need of additional help in math, reading, or writing with respect to school size. Similarly, with respect to school size, there are no significant differences on mean scores or percentages of students scoring above proficiency level on Connecticut's state exams. These, and similar findings, cause one to question whether student achievement is influenced more by number of courses offered or by the caliber of instruction offered in these courses.

Although research has repeatedly shown a positive relationship between school size and the number of courses offered, Monk (1990) believes size is not always the predictor of course offerings. With this in mind he conducted research to determine whether other variables such as community characteristics, allocation of resources, and school working environment might serve

as predictors of course offerings. While Monk concluded that the background characteristics examined in his research were very limited in their ability to explain differences in curricular offerings, he also concluded that high school size is not determinative of how broad the range of course offerings will be for a school. It appears that in addition to school size, other factors affect the variation in the number of curricular offerings of a school.

Monk and Haller (1990) conducted further research to determine other factors which predict course offerings. Using *High School and Beyond* data, and controlling for school size, they compared curricular offerings across urban, suburban, and rural schools. The results of this research indicate that among like-sized high schools, rural schools offer fewer unduplicated full year courses than do urban and suburban schools. The magnitude of this difference varies across the curriculum and is more pronounced in academic offerings than in vocational offerings. These differences in the number of academic offerings between rural and non-rural schools seemed to be most pronounced in "normal" courses. The negative effect of ruralness on the number of remedial courses and the number of advanced courses offered was not significant.

Because of the availability of courses, it is likely that a relationship exists between curricular offerings available in a school and course-taking patterns of students enrolled in that school. Research has established relationships between school size and school locale and curricular offerings. As indicated earlier, the timing of a first formal algebra class is likely to affect later course-taking decisions. Thus the factors of school size, school locale, and curricular offerings are of interest and significance to this current study.

Conclusion

As indicated by the aforementioned research, despite recent reforms in mathematics education, mathematics achievement gaps based on race-ethnicity, SES, and gender continue to

persist. Research indicates that factors affecting mathematics achievement are numerous and interconnected making it difficult to determine exactly which factors most significantly affect mathematics achievement. Some factors which have been shown to affect mathematics achievement, such as race-ethnicity, student SES, and gender can not be controlled by school personnel. However, certain instructional practices, which may have an effect on mathematics achievement, can be affected by educational personnel. One such factor is the timing of students' introduction to the formal study of algebra. If early entry into algebra is found to increase mathematics achievement for all students, then all students should have access to the formal study of algebra in middle school. If instead, mathematics achievement is increased for most students by delaying the formal study of algebra until high school, then students should not be pushed to complete Algebra I prior to high school. If the optimal timing for study of formal algebra is dependent on such factors as cognitive development, mathematical maturity, and motivation, students should be allowed to study algebra at different times throughout their school careers. Through this study the researcher will attempt to answer questions related to the most advantageous timing of a first course in formal algebra so as to provide information which may help increase opportunities for the successful study of mathematics by all students.

Chapter 3

Methodology

This study is designed to utilize a variety of quantitative statistical tests to examine several issues related to the time at which a first course in formal algebra is completed. These issues are examined through a variety of sub-questions posed to further elucidate the four following major questions:

1. What student characteristics are associated with the grade level at which a first course in formal algebra is completed?
2. What are the relationships between school characteristics, such as school size and locale, and the grade level at which a first course in formal algebra is completed?
3. What are the relationships between the grade level at which a first formal course in algebra is completed and future mathematics course taking?
4. What are the relationships between the grade level at which a first formal course in algebra is completed and mathematics achievement?

To investigate these questions, information about 459 students who matriculated at College of the Ozarks during the 2007-2008 academic year was collected. The data were entered in Statistical Package for the Social Sciences (SPSS) Version 16.0 software and appropriate statistical tests were utilized to explore the research questions. In the remainder of this chapter the methodology used in this research is further explained. Included is a description of the setting from which the data were obtained, a listing of the specific data which were collected, the sub-questions which were examined to answer each of the major questions, and the statistical tests used to address each question.

Setting

The data used in this study were obtained from students who matriculated at College of the Ozarks (C of O) during the 2007-2008 academic year. These data contain information about 459 students, the majority of whom are traditional-age, first-semester college students from the Midwest.

College of the Ozarks was founded as a high school in 1906, became a junior college in 1956, and a four-year college in 1965. This fully accredited college offers more than 31 majors which lead to either a Bachelor of Arts or Bachelor of Science degree. The majority of the approximately 1350 students who attend College of the Ozarks come from Missouri and Arkansas, although 42 states and 20 foreign countries are represented among the student body. A 14:1 student-faculty ratio exists at College of the Ozarks, with over 80 full-time faculty members, the majority of whom hold doctoral degrees. The middle 50% of first year college students at C of O have composite ACT scores ranging from 21 to 26 (Admissions, 2007).

The mission of College of the Ozarks, since its inception, has been to "Provide a Christian education for youth of both sexes, especially those found worthy, but who are without sufficient means." In accordance with this mission, ninety percent of the entering class must demonstrate financial need. According to the College of the Ozarks website, a combined parent and student adjusted gross income of \$48,000 for a family of four with one in college is expected to demonstrate financial need. The College of the Ozarks guarantees to meet the entire tuition cost for each full-time student through a combination of a work program and federal, state, and institutional funds. All full-time students are enrolled in the campus work program, working at one of the over 80 campus work areas. During the academic semester students work 15 hours per week and also work two additional 40-hour work weeks during semester breaks. Many students

are awarded room and board scholarships where they participate in the on-campus summer work program and are awarded funding to cover room and board during the academic year. Thus College of the Ozarks students have the opportunity to obtain four-year degrees with very little out-of-pocket expense, with the majority of the students graduating debt free (Cost of Education, 2007).

The 1000-acre campus of College of the Ozarks is located in Southwest Missouri near Branson. Branson, Missouri, with a population of about 6000, is located in Taney County near Table Rock Lake and Lake Taneycomo. Branson, with more than 100 live music shows, several amusement parks, numerous championship golf courses, museums, and wilderness areas, has become a tourist destination for several million people each year (Branson, Missouri, 2008). Many College of the Ozarks students supplement their income through employment in the tourism industry in Branson.

Data

As previously indicated, information on 459 students who matriculated at College of the Ozarks during the 2007-2008 academic year was collected. For each of these students, when available, the following information was collected:

- Gender
- Race-Ethnicity
- SES Information
 - Family Size
 - Family Adjusted Gross Income
 - Expected Family Contribution (EFC)
- ACT Score Information

- Overall Mathematics Score
- Pre-Algebra/Elementary Algebra Sub-score
- Intermediate Algebra/Coordinate Geometry Sub-score
- Plane Geometry/Trigonometry Sub-score
- High School Information
 - School Size
 - District Size
 - Size of Average High School Grade Level
 - School Locale (Rural/Non-rural)
 - School Type (Home, Private, Public)
- Mathematics Transcript Information
 - Grade Level at Which Algebra I Was Completed
 - Letter Grade Earned in Algebra II
 - Number of Mathematics Courses Taken in High School
 - Number of Advanced Mathematics Courses Taken in High School
 - Grade Level at Which Last Mathematics Class Was Taken
 - High School Mathematics Grade Point Average (GPA)

The majority of this information was available for all students in the study, with the most notable exception being ACT Mathematics sub-scores. In several cases, a student's ACT scores had been sent from the high school the student attended rather than directly from ACT. When this was the case, sub-scores were usually not available. Also, accurate and up-to-date information about school and district size was not easily retrievable for some of the private

schools involved in the study. Finally, the race-ethnicity of four students in the study was reported as unknown.

Race-Ethnicity. Individuals who live in the Ozarks Plateau region, which includes several counties in Missouri, Arkansas, Kansas, and Illinois, are given preference for consideration for acceptance at College of the Ozarks (Admissions, 2007). Partly due to the limited racial-ethnic diversity of the population of this region, the majority of students at College of the Ozarks are White students. Approximately 93-94% of the student body is classified as White (StateUniversity.com, 2008). Because of the small percentage of non-White students at College of the Ozarks, the race-ethnicity of each student involved in the study was classified in one of two categories -- White or non-White.

Student Socioeconomic Status (SES). Students seeking admission to College of the Ozarks must complete a Free Application for Federal Student Aid (FAFSA). This form is provided by the U.S. Department of Education and is used to determine a family's Expected Family Contribution (EFC) to a dependant's cost of education. Information is gathered about both the parents' and student's income and assets to complete this needs analysis. Factors such as family size and number of family members enrolled in college are also considered in determining EFC (Student Financial Aid, 2008). For the purposes of this study, Family Adjusted Gross Income, Family Size, and Expected Family Contribution were all obtained and considered as contributors to a student's SES level. Decisions about determining appropriate SES categories, such as low, middle, and high SES, were made after the data were collected.

High School Information. High school transcripts for all students involved in the study were examined. All mathematics courses taken, the grade levels at which they were taken, the letter grades obtained in the classes, and the credits awarded for the classes were recorded. This

information allowed the researcher to classify students according to the grade level at which they completed Algebra I. Using this transcript information, the researcher was also able to determine the following for each student in the study: total number of mathematics courses taken during high school, number of advanced mathematics courses taken during high school, grade level at which the last high school mathematics course was taken, and grade point average for high school mathematics courses.

Also available from the transcripts were the names and locations of the schools from which the students graduated. This information allowed the sizes of the high schools, as well as information about the communities in which the schools are located, to be obtained. Locale codes available from NCES were used to identify schools as rural or non-rural.

ACT Mathematics Score. The ACT Mathematics exam is a 60 minute exam which is comprised of 60 multiple choice questions. A student's ACT Mathematics composite score is calculated from three sub-scores which are based on six content areas: pre-algebra, elementary algebra, intermediate algebra, coordinate geometry, plane geometry, and trigonometry. The three sub-scores range from 1 (low) to 18 (high) and the composite Mathematics score ranges from 1 (low) to 36 (high). In 2006 the ACT exam was taken by more than 1.2 million 2006 high school graduates, earning a national average score of 20.8 (2006 ACT National News, 2006). The ACT Mathematics composite score was used as a measure of mathematics achievement in this study. In addition, when available, the ACT Mathematics sub-scores were collected and used as a measure of mathematics achievement.

Summary. Much of the information utilized in this study was collected from students through the application and admissions process and was available through College of the Ozarks Computer Center and Registrar's Office. Much of the data was stored electronically in

spreadsheet format which was available to the researcher. It was necessary for the researcher to inspect student paper files to obtain high school mathematics transcript information. High school transcripts also contained information to allow schools to be classified by type and locale. Each student file was assigned a number so that all identifying information was removed from the data prior to the analysis of the data.

Research Questions

The data were collected, compiled, and coded for each student in the study. Each student was assigned an Algebra Group Code based on the time of completion of Algebra I. Data were checked for accuracy and relevant descriptive statistics were examined, including the number of students within various categories, maximum and minimum values, means, and standard deviations. A number of inferential statistical tests were then performed to examine the four major research questions in the study. The major questions, along with sub-questions and statistical tests appropriate to answer those questions, are described below.

Question 1. The first major question examines relationships between student characteristics and the time at which a student completes Algebra I. Relationships between student factors of gender, race-ethnicity, and socioeconomic (SES) and timing of algebra were examined. Several sub-questions, along with the associated null hypotheses, and appropriate statistical tests are stated below.

Major Question 1: What student characteristics are associated with the grade level at which a first course in formal algebra is completed?

Sub-question 1.1: What is the relationship between gender and the grade level at which Algebra I is completed?

Sub-question 1.2: What is the relationship between race-ethnicity and the grade level at which Algebra I is completed?

Sub-question 1.3: What is the relationship between student SES and the grade level at which Algebra I is completed?

Sub-question 1.4: Which factors--gender, ethnicity, and student SES--can accurately predict the time at which a student completes Algebra I?

For sub-questions 1.1 through 1.3 the null hypotheses state that there is no relationship between timing of algebra and the characteristic under consideration. For each of these sub-questions, cross-tabulations were examined and a chi-square test of independence was performed and analyzed. To address sub-question 1.4, multinomial logistic regression was used to determine which, if any, of the explanatory variables are predictive of early entry into algebra.

Question 2. The second major question is designed to examine certain characteristics of the school from which a student graduated, and their relationships with the time at which Algebra I is completed. These school factors include school size, school type, and school locale. With much of the current research indicating positive student outcomes associated with completion of a formal algebra course prior to high school, an important concern is whether students in all schools have equal access to algebra while in middle school. Thus this second major question examines the relationships between school characteristics and timing of algebra. Major Question 2: What are the relationships between school characteristics such as school size and locale and the grade level at which a first course in formal algebra is completed?

Sub-question 2.1: What is the relationship between the type of high school from which a student graduates (public school, private school, home school) and the grade level at which Algebra I is completed?

Sub-question 2.2: What is the relationship between the size of the school from which a student graduates and the grade level at which Algebra I is completed?

Sub-question 2.3: What is the relationship between the locale of the school from which a student graduates and the grade level at which Algebra I is completed?

A chi-square test of independence was performed for each of these sub-questions with the null hypothesis stating in each case that there is no relationship between the grade level at which Algebra I is completed and the school characteristic under consideration. Through analysis of question two, relationships between timing of algebra and school characteristics were explored.

Question 3. The third major question investigates relationships between the time at which students complete Algebra I and mathematics course-taking patterns. The researcher attempted to determine if acceleration into algebra affects the number or type of mathematics courses taken by students while in high school. The sub-questions which were used to analyze the results of early entry into algebra on mathematics course-taking patterns are listed below.

Major Question 3: What are the relationships between the grade level at which a first formal course in algebra is completed and future mathematics course taking?

Sub-question 3.1: What is the relationship between the grade level at which Algebra I is completed and the mean number of mathematics courses taken in high school?

Sub-question 3.2: What is the relationship between the grade level at which Algebra I is completed and the mean number of advanced mathematics courses taken in high school?

Sub-question 3.3: What is the relationship between the grade level at which Algebra I is completed and persistence in the mathematics pipeline?

Sub-question 3.4: What is the relationship between the grade level at which Algebra I is completed and high school Calculus enrollment?

For sub-questions 3.1 and 3.2, one-way between-groups analyses of variance were conducted to explore relationships between timing of completion of Algebra I and mathematics course-taking patterns. The null hypothesis for sub-question 3.1 is that there is no significant difference in the mean number of mathematics courses taken in high school for students who complete Algebra I at different grade levels. The null hypothesis for sub-question 3.2 is similar, focusing on advance mathematics courses taken during high school. Sub-questions 3.3 and 3.4 were analyzed by examining cross-tabulations and by performing chi-square tests of independence. The null hypothesis for sub-question 3.3 states that there is no relationship between timing of algebra and persistence in the mathematics pipeline. The null hypothesis for sub-question 3.4 states that there is no relationship between timing of algebra and enrollment in Calculus while in high school.

Question 4. The final major question examines relationships between early entry into algebra and students' mathematics achievement. Mathematics achievement was assessed in a number of ways including Algebra II grades, high school mathematics grade point averages, ACT Mathematics scores, and ACT Mathematics sub-scores. While the ACT scores are the only measures which are standardized for all students involved in the study, examination of the other measures of mathematics achievement were also instructive. The major question and sub-questions used to assess mathematics achievement are listed below.

Major Question 4: What are the relationships between the grade level at which a first formal course in algebra is completed and mathematics achievement?

Sub-question 4.1: What is the relationship between the grade level at which Algebra I is completed and mathematics achievement as measured by Algebra II grades?

Sub-question 4.2: What is the relationship between the grade level at which Algebra I is completed and mathematics achievement as measured by high school mathematics grade point averages?

Sub-question 4.3: What is the relationship between the grade level at which Algebra I is completed and mathematics achievement as measured by ACT Mathematics scores?

For each sub-question designed to address major question four, the null hypothesis states that there is no difference in the mean mathematics achievement score for the three Algebra Groups. In each case a one-way between-groups analysis of variance was performed to determine differences in means. Where appropriate, eta squared was calculated to determine effect size. For sub-question 4.3, when available, the three ACT Mathematics sub-scores were also considered as a measure of mathematics achievement. Table 2 includes the four major questions considered in this study, the related sub-questions, and the statistical tests performed to address each of the questions.

Conclusion

As established in chapters one and two of this study, there is much debate about the optimal timing of a first course in formal algebra. Most recent research indicates that completing Algebra I in middle school is related to positive outcomes such as increased enrollment in advanced mathematics courses and increased mathematics achievement. However, due to the limited nature of this research, additional research regarding the significance of the grade level at which Algebra I is completed and the benefits of early entry into algebra should be conducted. Conjointly, research to determine characteristics of students who successfully complete Algebra I prior to high school, along with characteristics of the schools they attend, should be conducted.

Table 2: *Questions Addressed in the Study*

Major Question	Sub-question	Tests Used
1. What student characteristics are associated with the grade level at which a first course in formal algebra is completed?	1.1: What is the relationship between gender and the grade level at which Algebra I is completed?	Cross-tabulation Chi-Square Test of Independence
	1.2: What is the relationship between race-ethnicity and the grade level at which Algebra I is completed?	Cross-tabulation Chi-Square Test of Independence
	1.3: What is the relationship between student SES and the grade level at which Algebra I is completed?	Cross-tabulation Chi-Square Test of Independence
	1.4: Which factors--gender, ethnicity, and student SES--can accurately predict the time at which a student completes Algebra I?	Multinomial logistic regression
2. What are the relationships between school characteristics such as school size and locale and the grade level at which a first course in formal algebra is completed?	2.1: What is the relationship between the type of high school from which a student graduates (public school, private school, home school) and the grade level at which Algebra I is completed?	Cross-tabulation Chi-Square Test of Independence
	2.2: What is the relationship between the size of the school from which a student graduates and the grade level at which Algebra I is completed?	Cross-tabulation Chi-Square Test of Independence
	2.3: What is the relationship between the locale of the school from which a student graduates and the grade level at which Algebra I is completed?	Cross-tabulation Chi-Square Test of Independence
3. What are the relationships between the grade level at which a first formal course in algebra is completed and future mathematics course taking?	3.1: What is the relationship between the grade level at which Algebra I is completed and the mean number of mathematics courses taken in high school?	One-way Between Groups Analysis of Variance Eta squared
	3.2: What is the relationship between the grade level at which Algebra I is completed and the mean number of advanced mathematics courses taken in high school?	One-way Between Groups Analysis of Variance Eta squared
	3.3: What is the relationship between the grade level at which Algebra I is completed and persistence in the mathematics pipeline?	Cross-tabulation Chi-Square Test of Independence
	3.4: What is the relationship between the grade level at which Algebra I is completed and high school Calculus enrollment?	Cross-tabulation Chi-Square Test of Independence
4. What are the relationships between the grade level at which a first formal course in algebra is completed and mathematics achievement?	4.1: What is the relationship between the grade level at which Algebra I is completed and mathematics achievement as measured by Algebra II grades?	One-way Between Groups Analysis of Variance Eta squared
	4.2: What is the relationship between the grade level at which Algebra I is completed and mathematics achievement as measured by high school mathematics grade point average?	One-way Between Groups Analysis of Variance Eta squared Independent Samples T-test
	4.3: What is the relationship between the grade level at which Algebra I is completed and mathematics achievement as measured by ACT Mathematics scores?	One-way Between Groups Analysis of Variance Eta squared

The questions posed in this chapter, examined using data from students currently enrolled in a small liberal arts college in the Midwest, provide valuable information about the availability of formal algebra to students prior to high school. The study also provides information about relationships between the grade level at which a first formal course in algebra is completed and mathematics achievement and mathematics course-taking patterns. This information is important in informing broader issues regarding equity in mathematics education. Additionally, the findings of this research are useful for students, parents, teachers, and other educational personnel as decisions are made regarding mathematics course taking. Findings of the study also serve to challenge mathematics educators to continue to strive to provide excellent mathematics opportunities for all students.

Chapter 4

Findings

The purpose of this study is to explore and answer research questions and test null hypotheses related to the grade level at which students complete a first formal course in algebra. Information about the data and analyses of the data are presented in this chapter. The analyses and this chapter are organized around four major research questions. The first question examines student characteristics related to the grade level at which Algebra I is completed. Question two inspects certain school characteristics that may affect timing of completion of algebra. The third question explores relationships between timing of algebra and mathematics course taking. Finally, relationships between the grade level at which a first formal algebra class is completed and mathematics achievement are explored in question four.

Preparing the Data

Staff from the College of the Ozarks Computer Center provided information for the 502 students who matriculated at College of the Ozarks during the 2007-2008 academic year. Demographic information such as gender, race-ethnicity, and family adjusted gross income was provided. High school graduation date, high school class rank, high school grade point average, and high school name and location were provided. ACT overall and subject scores were provided when available.

After receiving this data, individual files for all students involved in the study were reviewed by the researcher. This review facilitated gathering of additional data such as ACT Mathematics sub-scores. Also, high school transcripts were analyzed to determine all mathematics courses taken in grades 9 through 12, the grade level at which each course was completed, the letter grade obtained in each course, and the credit value of those courses. As data

were gathered several decisions were made regarding how to record the data. For example, if more than one ACT record was available in a student file, sub-scores from the most recent exam were recorded.

Significant variation was apparent among high school transcripts. Differences included a variety of methods for recording grades. Most frequently letter grades, including plus and minus grades, were recorded for each semester. In this case the two semester grades were averaged to assign a course grade. However, occasionally one letter grade was issued for the entire course or grades were recorded as percentages. Percentage grades, when present, were converted to letter grades.

Additionally, variations were found in the naming of courses. To provide for uniformity, courses were placed in one of eight categories – Pre-Algebra, Algebra I, Geometry, Algebra II, Advanced Algebra, Trigonometry/Math Analysis, Calculus, and Other. Courses with titles such as Algebra III, College Algebra, and Pre-Calculus were placed in the Advanced Algebra category. Courses such as Business Math, Consumer Math, and Transition Math were the titles most commonly placed in the “Other” category. If credit for Algebra I was earned over two years, with course titles such as Algebra IA and Algebra IB, the first year was classified as Pre-Algebra with completion of the second year classified as completion of Algebra I. When credit for Algebra I was absent from a high school transcript, but classes for which Algebra I serves as a prerequisite were present, completion of Algebra I was recorded as occurring prior to ninth grade.

Several student files were missing data necessary for this study. Fifty-three student files were ultimately eliminated from the study. Forty-nine of these files were removed from the study due to missing or incomplete transcript data. Thirty-seven of these 49 students were transfer

students whose admittance into College of the Ozarks was dependant upon previous college performance so that high school transcript information was not available. The remaining students with incomplete transcript data were home educated students, dual enrolled students, or students attending foreign schools. In addition to the 49 students removed from the study due to missing transcript information, three students were removed from the study due to lack of SES information. One more student was removed from the study as his transcript reflected no credit for Algebra I or any course for which Algebra I is required as a prerequisite. Thus after all data were gathered and incomplete files were removed, data for 449 of the 502 students matriculating at College of the Ozarks during the 2007-2008 academic year remained in the study. These files all contained demographic information, high school transcript information, and ACT Mathematics scores. However, only 357 of the files contained ACT Mathematics sub-scores. Analyses which rely on ACT Mathematics sub-scores make use of the 357 files containing this information. Of the 449 students in the study, 346 attended public school. Some analyses in the research are based on all students involved in the study ($N = 449$) and some, when appropriate, are based on data obtained from the 346 students in the study who attended public school.

Using the Statistical Package for the Social Sciences (SPSS) Version 16.0 both descriptive and inferential analyses were conducted. To ready the data for this statistical testing, the data were analyzed in a variety of ways to check for errors. This included sorting the data, examining maximum and minimum values, inspecting various plots of the data, and analyzing descriptive statistics provided by SPSS. Where appropriate, box plots were examined to identify outliers. For most variables, no outliers were present. When outliers were present, original means were compared to 5% trimmed means. In each case the two values were very similar, indicating

that the extreme values had very little influence on the means. Thus outliers were not removed from the data set. Some of the data utilized in the study are summarized in Table 3.

When appropriate, such as the case of an ANOVA, the data were checked to determine if the variability of scores for each group was similar. The Levene test for equality of variance, provided by SPSS, was used to test for homogeneity of variance. In all but one case the associated significance level for the Levene statistic was greater than .05, indicating that the assumption of homogeneity of variance had not been violated. In the case where violation of homogeneity of variance raised a concern an appropriate non-parametric test was employed. Due to the size of the data set, possible violations of normality did not raise concerns and were not explored in great detail. All hypotheses were tested at the .05 level of significance.

Student Characteristics and Timing of Algebra

To examine the first major question: What student characteristics are associated with the grade level at which a first course in formal algebra is completed?, three student factors--gender, race-ethnicity, and socioeconomic status--were considered. Sub-questions designed to examine relationships and interactions between these three factors and timing of completion of Algebra I, were formed and are investigated below.

Students were placed into one of three categories based on the grade level at which they received credit for Algebra I. Approximately 18.3% of the students in the study ($n = 82$) completed Algebra I prior to high school and were classified as early entrants into algebra and were placed in Algebra Group 1. About 64.1% of the students in the study ($n = 288$) completed Algebra I in ninth grade and formed Algebra Group 2. The remaining 79 students in the study (17.6%) completed Algebra I later than ninth grade and were placed in Algebra Group 3.

Table 3: *Summary of Data*

Factor	Categories	Algebra Prior to 9 th	Algebra During 9 th	Algebra After 9 th	Total
		Grade Group 1	Grade Group 2	Grade Group 3	
Gender	Female	41 (16%)	164 (65%)	48 (19%)	253 (56%)
	Male	41 (21%)	124 (63%)	31 (16%)	196 (44%)
SES Level	Lowest SES	13 (9%)	91 (65%)	36 (26%)	140 (31%)
	Low SES	14 (19%)	44 (59%)	17 (23%)	75 (17%)
	Medium SES	23 (22%)	67 (64%)	15 (14%)	105 (23%)
	High SES	32 (25%)	86 (67%)	11 (9%)	129 (29%)
School Type	Home School	4 (7%)	26 (44%)	29 (49%)	59 (13%)
	Private School	8 (18%)	26 (59%)	10 (23%)	44 (10%)
	Public School	70 (20%)	236 (68%)	40 (12%)	346 (77%)
Total		82 (18%)	288 (64%)	79 (18%)	449 (100%)
Public School Locale	Non-Rural	30 (22%)	91 (67%)	15 (11%)	136 (39%)
	Rural	40 (19%)	145 (69%)	25 (12%)	210 (61%)
Public School Size by Average High School Grade Size (S)	S < 50	9 (13%)	49 (73%)	9 (13%)	67 (19%)
	51 < S < 85	18 (25%)	47 (65%)	7 (10%)	72 (21%)
	86 < S < 160	10 (15%)	51 (75%)	7 (10%)	68 (20%)
	161 < S < 300	19 (26%)	45 (62%)	9 (12%)	73 (21%)
	S ≥ 300	14 (21%)	44 (67%)	8 (12%)	66 (19%)
Public School Size by District Size (D)	0 < D ≤ 750	11 (13%)	61 (73%)	12 (14%)	84 (24%)
	751 < D < 1400	25 (27%)	62 (67%)	6 (7%)	93 (27%)
	1401 < D < 3700	16 (17%)	66 (70%)	12 (13%)	94 (27%)
	D > 3700	18 (24%)	47 (63%)	10 (13%)	75 (22%)
Total (Public School)		70 (20%)	236 (68%)	40 (12%)	346 (100%)

Note –Values are rounded to the nearest percent, causing some categories to have a total of 101%.

Gender and Timing of Algebra. To explore sub-question 1.1: What is the relationship between gender and the grade level at which Algebra I is completed?, the following null hypothesis was tested.

Hypothesis 1.1: There is no relationship between gender and the grade level at which Algebra I is completed.

A cross-tabulation of gender and Algebra Group membership was performed to allow for exploration of the number of students in each of the six categories. This analysis shows that 196 males and 253 females are included in the study. The ratio of males to females included in this study is consistent with the male to female ratio of the entire student body at College of the Ozarks (StateUniversity.com, 2008). Also apparent from the cross-tabulation, which is shown in Table 4, is that a larger percentage of male students completed Algebra I prior to ninth grade and a smaller percentage completed Algebra I after ninth grade as compared to female students. The chi-square test of independence was used to further examine Hypothesis 1.1. The decision was made to fail to reject the null hypothesis, $\chi^2(2, N = 449) = 2.010, p = .366$. Therefore, the timing of Algebra I for female students is not significantly different from the timing of Algebra I for male students. In other words, the proportion of male students who complete a first formal course in algebra prior to high school is not significantly different from the proportion of female students who complete a first formal course in algebra prior to high school. Similarly, the percentage of students who delay completion of Algebra I until after ninth grade is not significantly different for male and female students.

Race-Ethnicity and Timing of Algebra. Information about student race-ethnicity is obtained from student data sheets which are completed by College of the Ozarks students as part of the admissions process. Reporting of race-ethnicity is voluntary and optional; however, most

Table 4: *Gender*Algebra Group Cross-Tabulation*

		Algebra Group Code				
		Algebra I Prior to 9th Grade	Algebra I in 9th Grade	Algebra I After 9th Grade	Total	
Gender	Male	Count	41	124	31	196
		% within gender	20.9%	63.3%	15.8%	100.0%
	Female	Count	41	164	48	253
		% within gender	16.2%	64.8%	19.0%	100.0%
Total		Count	82	288	79	449
		%	18.3%	64.1%	17.6%	100.0%

$$\chi^2(2, N = 449) = 2.010, p = .366$$

students provide the requested information. The race-ethnicity of only four students in the study was reported as unknown. Of the 449 students included in the study, 426 indicated their race-ethnicity as "White non-Hispanic." Thus only about 5.1% of the students involved in this study indicated a race-ethnicity other than White. This percentage includes the four students whose race-ethnicity was listed as unknown. This percentage of non-White students is slightly lower than the percentage in the entire College of the Ozarks student body which includes 6.8% non-White students (StateUniversity.com, 2008). This lower percentage in the study may be attributable to the fact that several foreign students were eliminated from the study due to missing or incomplete high school transcript information.

To explore sub-question 1.2: What is the relationship between race-ethnicity and the grade level at which Algebra I is completed?, the following null hypothesis was tested.

Hypothesis 1.2: There is no relationship between race-ethnicity and the grade level at which Algebra I is completed.

Despite the small number of non-White students in the study, a cross-tabulation of race-ethnicity and the year of completion of Algebra I was performed to allow for examination of the percentages of students in each of the six categories. One-third of the six cells had expected outcomes less than five. The values obtained from the chi-square test of independence indicate that there is not sufficient evidence to reject the null-hypothesis; however, because of the low expected values, this information is not statistically meaningful and no attempt was made to interpret the results.

Socioeconomic Status and Timing of Algebra. To explore sub-question 1.3: What is the relationship between student SES and the grade level at which Algebra I is completed?, financial information was retrieved from Free Application for Federal Student Aid (FAFSA) forms completed by students. The pertinent student SES information obtained from the form includes family adjusted gross income and number of family members. Each research participant was assigned an SES Level of 1, 2, 3, or 4 based on income eligibility guidelines for free or reduced price meals for the 2007-2008 school year, as provided by the U.S. Department of Agriculture (United States Department of Agriculture, 2008). Students who qualify for free meals were classified as "Lowest SES" and were assigned SES Level 1. Students who do not qualify for free meals but do qualify for reduced price meals were assigned SES Level 2 and classified as "Low SES." Classifications of "Medium SES" and SES Level 3 were assigned to students who do not qualify for free or reduced price meals, but whose family income is less than or equal to 142% of the upper limit allowable for qualification for reduced price meals. Students with a family income greater than 142% of the upper limit allowable for qualification for reduced price meals were assigned SES Level 4 and a classification of "High SES." A 42% differential between the top two levels was chosen to provide consistency with the percentage difference between Lowest

and Low SES Levels and between Low and Medium SES Levels, as determined by the U.S. Department of Agriculture.

Table 5 provides specific allowable family incomes for families of various sizes in each of the SES Levels. The SES Level with the largest membership is Level 1--Lowest SES with 140 students, about 31.2% of the students in the study. Level 2—Low SES, with 75 students or about 16.7% of the students in the study, is the smallest group. One hundred five students (23.4%) comprise Level 3—Medium SES and 129 students (28.7%) are classified High SES Level. With each student assigned an SES level, the following null hypothesis was examined to determine relationships between student SES and timing of completion of Algebra I.

Hypothesis 1.3: There is no relationship between student SES level and the grade level at which Algebra I is completed.

Table 5: *Income Levels for Student SES*

Household Size	Maximum Annual Income for SES Level 1	Maximum Annual Income for SES Level 2	Maximum Annual Income for SES Level 3
1	13,273	18,899	26,822
2	17,797	25,327	35,964
3	22,321	31,765	45,106
4	26,845	38,203	54,248
5	31,369	44,641	63,390
6	35,893	51,079	72,532
7	40,417	57,517	81,674
8	44,941	63,955	90,816
For each additional family member add:	4,524	6,438	9,142

Based on information from the U.S. Department of Agriculture website <http://www.fns.usda.gov/cnd/governance/notices/iegs/iegs.htm>

A cross-tabulation of SES and the year of completion of Algebra I was performed to allow for examination of percentages of students within each of the twelve categories. The results of this cross-tabulation, which indicate some variation in the percentages of students within the various Algebra Groups with respect to student SES, are reported in Table 6.

In addition to cross-tabulations, the chi-square test of independence was used to test null hypothesis 1.3. The decision was made to reject the null hypothesis, $\chi^2(6, N = 449) = 23.492, p = .001$. Thus, the results indicate a significant difference in timing of completion of Algebra I with regard to student SES. As SES Levels increase, the percentage of students completing a first course in formal algebra prior to ninth grade increases. Conversely, as SES Levels increase the number of students delaying completion of Algebra I until after ninth grade decreases. Stated

Table 6: *SES Level*Algebra Group Cross-tabulation*

		Algebra Group Code			Total	
		Algebra I Prior to 9th Grade	Algebra I in 9th Grade	Algebra I After 9th Grade		
SES Status	Lowest SES	Count	13	91	36	140
		% within SES Level	9.3%	65.0%	25.7%	100.0%
	Low SES	Count	14	44	17	75
		% within SES Level	18.7%	58.7%	22.7%	100.0%
	Medium SES	Count	23	67	15	105
		% within SES Level	21.9%	63.8%	14.3%	100.0%
	High SES	Count	32	86	11	129
		% within SES Level	24.8%	66.7%	8.5%	100.0%
Total		Count	82	288	79	449
		% of Total	18.3%	64.1%	17.6%	100.0%

$$\chi^2(6, N = 449) = 23.492, p = .001$$

differently, the higher the student SES level, the more likely the student to complete Algebra I prior to ninth grade and the less likely the student to delay completion of Algebra I until after ninth grade. The positive relationship between SES Level and completion of Algebra I prior to ninth grade is displayed in Figure 1. The negative relation between SES Level and delay of Algebra I until after ninth grade is displayed in Figure 2.

Predicting Timing of Algebra. To examine sub-question 1.4: Which factors--gender, ethnicity, and student SES--can accurately predict the time at which a student completes Algebra I?, the following null hypothesis was examined.

Hypothesis 1.4: Knowing gender, ethnicity, and student SES level makes no difference in predicting the time at which a student will complete Algebra I.

To test this null hypothesis, multinomial logistic regression was utilized. Prior to performing logistic regression, checks were made of its appropriateness. The ratio of valid cases to independent variables of approximately 150 to 1 is significantly larger than the minimum preferred ratio and indicates an appropriateness to continue with logistic regression. After ensuring the appropriateness of multinomial logistic regression the analysis was performed with the results indicating the existence of a relationship between timing of algebra and the independent variables under consideration. The results were then checked for numerical problems such as multicollinearity among the independent variables, which could cause the regression model to be inappropriate. None of the independent variables in this analysis had a standard error greater than 2.0. This indicates the existence of no numerical problems and the appropriateness of continuing with the logistic regression.

The results produced by multinomial logistic regression indicate that this model with independent variables gender, ethnicity, and student SES significantly out performs the null

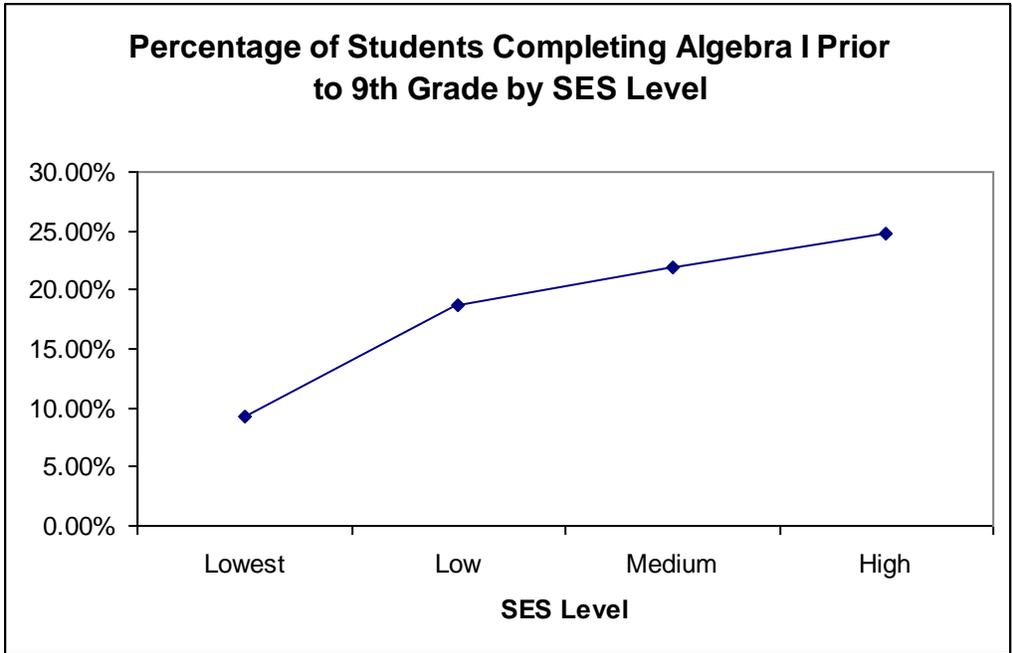


Figure 1: *Positive Relation Between SES Level and Membership in Algebra Group 1*

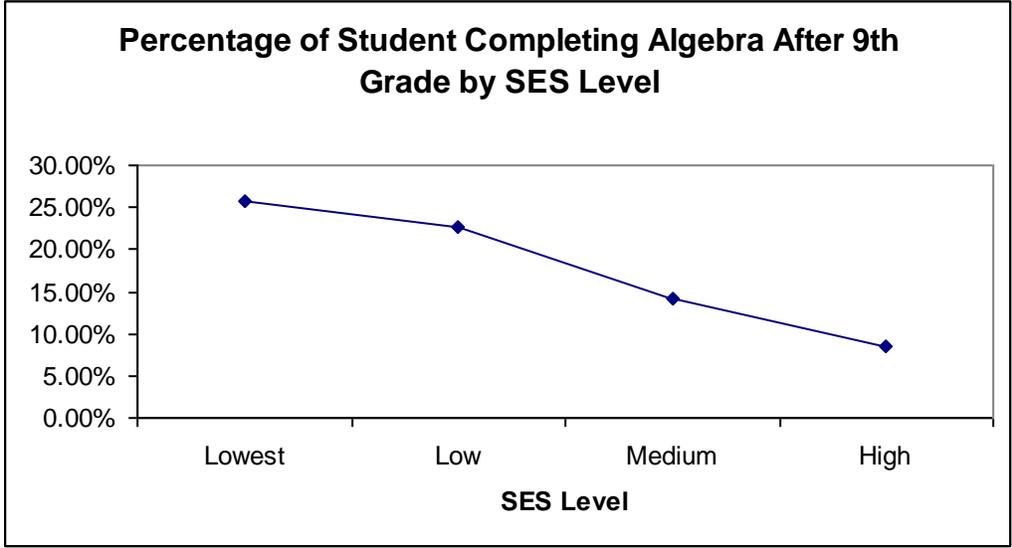


Figure 2: *Negative Relation Between SES Level and Membership in Algebra Group 3*

model for predicting the grade level at which a student will complete Algebra I, $\chi^2(10, N = 449) = 29.278, p = .001$. The likelihood ratio test presented in Table 7 indicates that student SES contributes significantly to the model ($p < .001$) but gender ($p = .504$) and ethnicity ($p = .237$) do not. Thus the null hypothesis is rejected with statistically significant evidence that student SES can be used to partially predict the time at which a student will complete a first formal course in algebra.

Information contained in Table 8 allows for further comparison of various groups of students. The first half of the table compares students who completed Algebra I prior to ninth grade with students who completed Algebra I during ninth grade. Algebra-completion patterns of students in the various SES levels are compared with algebra-completion patterns of students in the High SES Level. Students in the Lowest SES Level are significantly less likely than students in the High SES Level to complete Algebra I prior to ninth grade. Stated conversely, High SES

Table 7: *Likelihood Ratio Test*

Effect	Model Fitting Criteria		Likelihood Ratio Tests		
	-2 Log Likelihood of Reduced Model	Chi-Square	df	Sig.	
Intercept	92.898 ^a	.000	0	.	
ethnicity	95.773	2.875	2	.237	
gender	94.268	1.370	2	.504	
SES	117.859	24.961	6	.000	

The chi-square statistic is the difference in -2 log-likelihoods between the final model and a reduced model. The reduced model is formed by omitting an effect from the final model. The null hypothesis is that all parameters of that effect are 0.

a. This reduced model is equivalent to the final model because omitting the effect does not increase the degrees of freedom.

Table 8: *Parameter Estimates for Gender+Ethnicity+SES Model*

Algebra Group ^a		B	Std. Error	Wald	df	Sig.	Exp(B)	95% Confidence Interval for Exp(B)	
								Lower Bound	Upper Bound
Algebra I Prior to 9th Grade	Intercept	-1.128	.244	21.389	1	.000			
	Non-White/White	.524	.513	1.041	1	.308	1.688	.617	4.617
	Male/Female	.233	.256	.828	1	.363	1.262	.764	2.084
	Lowest SES/High SES	-.958	.364	6.930	1	.008	.384	.188	.783
	Low SES/High SES	-.167	.372	.201	1	.654	.846	.408	1.755
	Medium SES/High SES	-.081	.319	.064	1	.801	.923	.493	1.725
Algebra I After 9th Grade	Intercept	-1.967	.339	33.701	1	.000			
	Non-White/White	-.828	.771	1.152	1	.283	.437	.096	1.981
	Male/Female	-.145	.267	.296	1	.586	.865	.513	1.459
	Lowest SES/High SES	1.133	.377	9.011	1	.003	3.104	1.482	6.501
	Low SES/High SES	1.110	.431	6.639	1	.010	3.034	1.304	7.056
	Medium SES/High SES	.558	.430	1.685	1	.194	1.747	.753	4.054

a. The reference category is: Algebra I in 9th Grade.

Level students are about two and one-half times more likely to complete Algebra I prior to 9th grade than Lowest SES Level students, $OR = 2.604$ (95% CI 1.277 to 5.319) $p = .008$.

The second half of Table 8 compares students who completed Algebra I after ninth grade with students who completed Algebra I during ninth grade. As with the first half of the table, algebra-completion patterns of students in the various SES levels are compared with algebra-completion patterns of students in the High SES Level. When compared to High SES Level students, students in the Lowest SES Level are significantly more likely to delay completion of Algebra I until after ninth grade, $OR = 3.104$ (95% CI 1.482 to 6.501) $p = .003$, as are student in the Low SES Level, $OR = 3.034$ (95% CI 1.304 to 7.056) $p = .010$. The odds ratios indicate that students from both the Lowest and Low SES Levels are about three times more likely than students in the High SES Level to delay completion of Algebra I until after ninth grade. Stated conversely, students in the High SES Level are significantly less likely to delay completion of Algebra I until after ninth grade when compared with students in the two lowest SES levels.

Although this model adequately fits the data, with student SES contributing as a predictor of the time at which a student will complete Algebra I, only about 7.6% of the variation in timing of completion of algebra is explained by this gender + ethnicity + SES model. This is indicated by a Nagelkerke pseudo r-square value of .076. Other models, including only two of the independent variables, were tested using multinomial logistic regression. None of these models were found to more accurately predict Algebra Group membership than the gender + ethnicity + SES model just presented and discussed.

School Characteristics and Timing of Algebra

To examine the second major question: What are the relationships between school characteristics, such as school size and locale, and the grade level at which Algebra I is completed?, three school factors--school type, school size, and school locale--were considered.

Of the 449 students in the study 59 were home educated, 44 attended private school, and 346 attended public school. The type of school attended was determined by examination of students' high school transcripts. It is possible that some students may have graduated from one type of school but also attended a different type of school. However, close examination of high school transcripts indicate that most students in the study attended the same type of school from which they graduated at least the majority of their high school years. Although insufficient information was available to determine the nature of all private schools represented in the data, the majority are small, religious schools.

School Type and Timing of Algebra. To explore sub-question 2.1: What is the relationship between the type of high school from which a student graduates (public school, private school, home school) and the grade level at which Algebra I is completed?, the following null hypothesis was tested.

Hypothesis 2.1: There is no relationship between the type of high school from which a student graduates and the grade level at which Algebra I is completed.

A cross-tabulation of high school type and the year of completion of Algebra I was performed to allow for examination of percentages of students within each of the nine categories. The results of this cross-tabulation, which indicate some variation in the percentages of students within Algebra Groups with respect to school type, are reported in Table 9.

Table 9: *School Type*Algebra Group Cross-tabulation*

		Algebra Group Code				
		Algebra I Prior to 9th Grade	Algebra I in 9th Grade	Algebra I After 9th Grade	Total	
School Type	Home	Count	4	26	29	59
	School	% School Type	6.8%	44.1%	49.2%	100.0%
Private	School	Count	8	26	10	44
	School	% School Type	18.2%	59.1%	22.7%	100.0%
Public	School	Count	70	236	40	346
	School	% School Type	20.2%	68.2%	11.6%	100.0%
Total		Count	82	288	79	449
		% of Total	18.3%	64.1%	17.6%	100.0%

$$\chi^2 (4, N = 449) = 50.982, p < .001$$

In addition to cross-tabulations, the chi-square test of independence was used to test null hypothesis 2.1. The decision was made to reject the null hypothesis, $\chi^2 (4, N = 449) = 50.982, p < .001$, with the analysis indicating a significant difference in the timing of completion of Algebra I with regard to school type. Only 6.8% of home educated students completed Algebra I prior to ninth grade compared to 18.3% of all students in the study. A larger percentage of both home educated students (49.2%) and private school students (22.7%) delayed completion of Algebra I until after ninth grade than did public school students (11.6%). Relationships between school type and the grade level at which Algebra I is completed are illustrated in Figures 3 and 4.

School Size and Timing of Algebra. To examine relationships between school size and timing of algebra, information about school size was collected from the National Center for Education Statistics website (NCES, n.d.a; NCES, n.d.b). Both district size and school size were available for all public schools and some private schools in the study. In addition, information

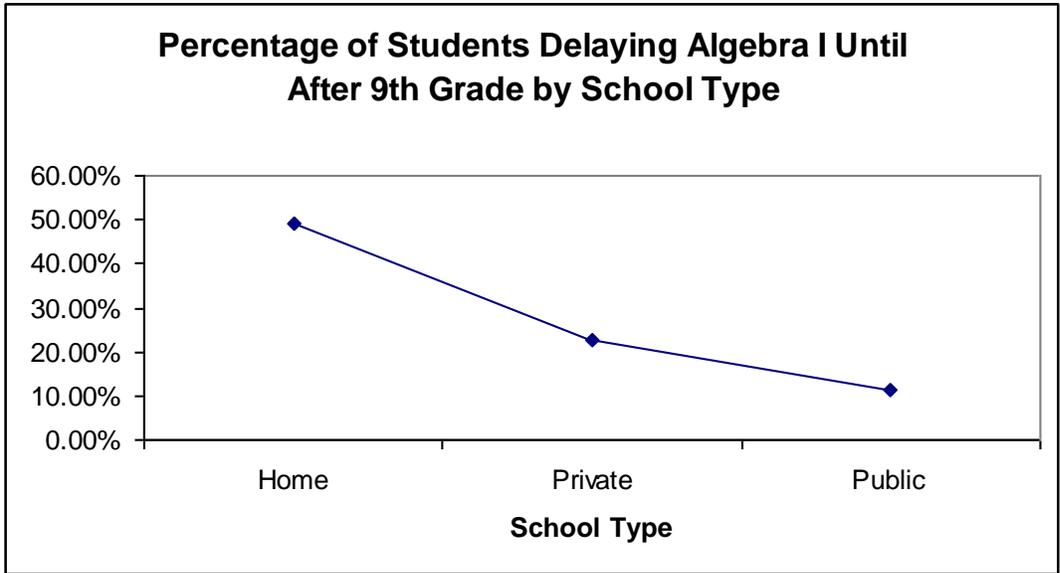


Figure 3: *Relation Between Membership in Algebra Group 3 and School Type*

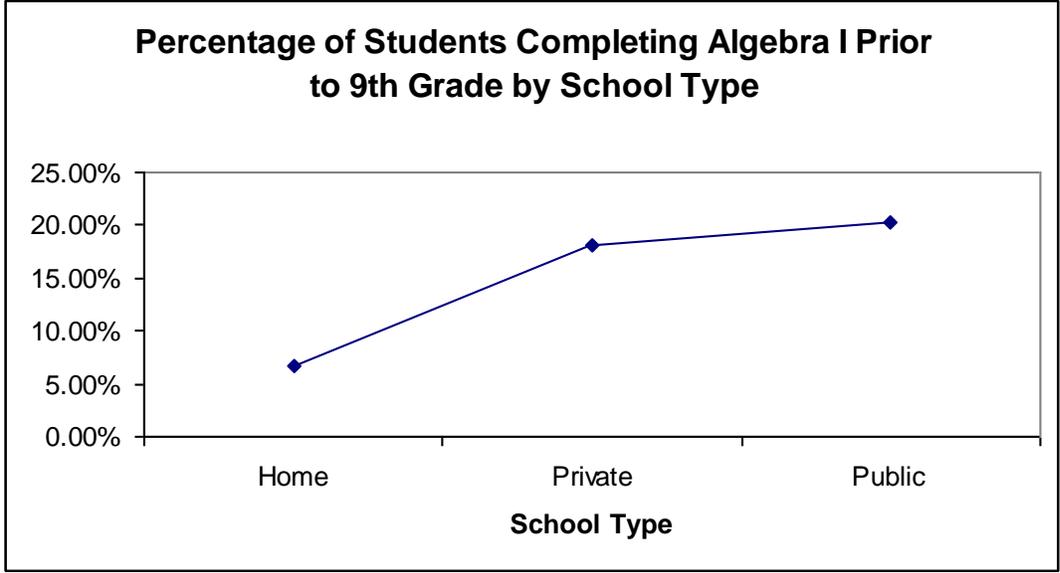


Figure 4: *Relation Between Membership in Algebra Group 1 and School Type*

about the grade band included in the high school was available. Most often high schools were organized into a 9-12 grade band structure, but 6-12, 7-12, 8-12, and 10-12 high school organizational structures were also present in the data. To determine one measure of school size, the student population of the high school was divided by the number of grades in the high school. For example, a high school including grades 9 through 12 with a student population of 420 was classified as having an average high school grade size of 105 ($420/4$). Each school was then assigned a School Size Code based on this average grade size. Because of missing size information for many private schools and the lack of relevance of school size for home educated students, only the 346 students attending public schools were included in this analysis. Schools were assigned one of five School Size Codes based on average high school grade size. A School Size Code of 1 represents the smallest schools in the study while a School Size Code of 5 represents the largest schools in the study. The maximum and minimum numbers specified for boundaries for the five codes were selected so as to divide the data into five groups of roughly equivalent size. Information on School Size Codes, along with the number of students within each category is shown in Table 10.

To explore sub-question 2.2: What is the relationship between the size of the school from which a student graduates and the grade level at which Algebra I is completed?, the following null hypothesis was tested.

Hypothesis 2.2: There is no relationship between the size of the school from which a student graduates and the grade level at which Algebra I is completed.

A cross-tabulation of School Size Code and Algebra Group membership was performed to allow for exploration of the number of students in each of the 15 categories. The results of the cross-tabulation and the chi-square test of independence support a decision to fail to reject the null

Table 10: *School Size Codes*

Average High School Grade Size (S)	School Size Code	Number of Student	Percent
$0 < S < 50$	1	67	19.4%
$51 < S < 85$	2	72	20.8%
$86 < S < 160$	3	68	19.7%
$161 < S < 300$	4	73	21.1%
$S \geq 300$	5	66	19.1%
Total		346	100%

hypothesis, $\chi^2(8, n = 346) = 6.446, p = .597$. Therefore, the grade level at which Algebra I is completed does not vary significantly with changes in school size as measured by the number of students in the average high school grade level.

To further examine possible relationships between school size and timing of algebra, the sizes of the districts containing the high schools from which students in the study graduated were examined. The school district attended by each public school student was assigned a District Size Code using the scale provided in Table 11. These boundaries for District Size Code were chosen due to natural breaks in the data and to roughly divide the data into quartiles.

Once again sub-question 2.2 and null hypothesis 2.2 were examined, this time using District Size Code as a measure of school size. A cross-tabulation of District Size Code and Algebra Group membership was performed to allow for exploration of the number of students in each of the 12 categories. The results of the cross-tabulation and the chi-square test of

Table 11: *District Size Code*

District Student Enrollment (D)	School Size Code	Number of Students	Percent
$0 < D \leq 750$	1 -- Smallest	84	24.3%
$751 < D \leq 1400$	2 -- Small	93	26.9%
$1401 < D \leq 3700$	3 -- Medium	94	27.2%
$D > 3700$	4 -- Large	75	21.7%
Total		346	100%

independence support a decision to fail to reject the null hypothesis, $\chi^2(6, n = 346) = 8.779, p = .186$. Therefore, the grade level at which Algebra I is completed does not vary significantly with changes in school size as measured by the size of the school district in which the high school from which a student graduated is located.

School Locale and Timing of Algebra. To determine relationships between school locale and timing of algebra, the data set containing all 449 students was first examined. Students were placed in one of the following four locale codes: home school, private school, non-rural public school, and rural public school. To assign a rural or non-rural code to public schools, information was obtained from the National Center for Education Statistics (NCES) website. The NCES locale codes are based on the school's physical address and its proximity to an urbanized area. The codes contain four major levels, 1-City, 2-Suburb, 3-Town, and 4-Rural. Each of these levels is then further divided into three sub-levels. For example, Rural schools are further classified as 41-Rural, Fringe; 42-Rural, Distant; and 43-Rural, Remote (Identification of rural codes, n.d.). For this study these three classifications, 41, 42, and 43, were identified as rural with all other public schools classified as non-rural.

To explore sub-question 2.3: What is the relationship between the locale of the school from which a student graduates and the grade level at which Algebra I is completed?, the following null hypothesis was tested.

Hypothesis 2.3: There is no relationship between the locale of the school from which a student graduates and the grade level at which Algebra I is completed.

A cross-tabulation of school locale and the year of completion of Algebra I was performed for all students in the study to allow for examination of percentages of students within each of the twelve categories. The results of this cross-tabulation, which indicate some variation in the percentages of students within the various Algebra Groups with regard to school locale, are reported in Table 12.

Table 12: *School Locale*Algebra Group Cross-tabulation for All Students*

			Algebra Group Code			
			Algebra I Prior to 9th Grade	Algebra I in 9th Grade	Algebra I After 9th Grade	Total
Locale	Home	Count	4	26	29	59
	School	% within locale	6.8%	44.1%	49.2%	100.0%
	Private	Count	8	26	10	44
	School	% within locale	18.2%	59.1%	22.7%	100.0%
	Non-rural	Count	30	91	15	136
	Public	% within locale	22.1%	66.9%	11.0%	100.0%
	School					
	Rural Public	Count	40	145	25	210
	School	% within locale	19.0%	69.0%	11.9%	100.0%
Total		Count	82	288	79	449
		% within locale	18.3%	64.1%	17.6%	100.0%

$$\chi^2 (6, N = 449) = 51.486, p < .001$$

In addition to cross-tabulations, the chi-square test of independence was used to test null hypothesis 2.3. The decision was made to reject the null hypothesis, $\chi^2(6, N = 449) = 51.486, p < .001$. Thus, the analysis indicates a significant difference in the timing of completion of Algebra I with regard to school locale. Home educated students appear to complete Algebra I prior to ninth grade much less frequently than students in the other three school locale categories. Home educated students and private school students appear to be much more likely to delay completion of Algebra I until after ninth grade when compared with students in the other locale categories.

Since it appears that much of the variation in school locale occurs between home and private school students and public school students, and since school locale seems less relevant to home school and private school students, similar analyses were conducted on the data set containing information on only the 346 public school students in the study. Each of these 346 public school students was identified as attending either a rural or non-rural school as described previously. Of these students, 136 graduated from non-rural schools and 210 graduated from rural schools. With only public school data included in the analyses, the decision was made to fail to reject the null hypothesis, $\chi^2(2, n = 346) = .480, p = .787$. Thus there is no significant difference for time of completion of a first formal course in algebra for students attending rural public high schools when compared to students attending non-rural public high schools.

Timing of Algebra and Mathematics Course-Taking Patterns

To explore relationships between the grade level at which Algebra I is completed and mathematics course-taking patterns, high school transcripts of students in the study were reviewed. All mathematics courses taken during grades 9 through 12 were recorded, along with

the credit awarded for each class. Most of the mathematics courses were two semesters in length and one credit was awarded for the course. In some cases, courses were one semester in length and 0.5 credit was awarded for completion of the course. Transcript information was then used to determine the total credits of mathematics earned by each student during grades 9 through 12. Additionally for each student, the number of advanced mathematics credits earned in high school was determined. For the purpose of this study, any mathematics course beyond Geometry and Algebra II, for which Algebra I is a prerequisite, was counted as an advanced mathematics course. The most frequently occurring course titles include Advanced Algebra, College Algebra, Pre-Calculus, Trigonometry/Math Analysis, and Calculus. Courses such as Consumer Math and Business Math, for which Algebra I was most likely not a prerequisite, were not counted as advanced mathematics courses even if they were taken after successful completion of Algebra I. These data were then analyzed in a variety of ways to provide insights into major question 3: What are the relationships between the time at which a first formal course in algebra is completed and future mathematics course taking?

Timing of Algebra and All Mathematics Courses. To explore sub-question 3.1: What is the relationship between the grade level at which Algebra I is completed and the mean number of mathematics courses taken in high school?, the following null hypothesis was tested.

Hypothesis 3.1: There is no difference between the mean numbers of mathematics courses taken in high school for the three Algebra Groups.

A one-way between-groups analysis of variance was conducted to explore relationships between timing of completion of Algebra I and the number of mathematics courses taken during high school. Relevant descriptive statistics for the three Algebra Groups are reported in Table 13. A decision was made to fail to reject the null hypothesis as there is no statistically significant

Table 13: *Descriptive Statistics for 9-12 Mathematics Courses*

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Algebra I Prior to 9th Grade	82	3.488	.6333	.0699	3.349	3.627	2.0	5.0
Algebra I in 9th Grade	288	3.526	.6235	.0367	3.454	3.598	2.0	6.0
Algebra I After 9th Grade	79	3.563	.7310	.0822	3.400	3.727	2.0	5.0
Total	449	3.526	.6443	.0304	3.466	3.585	2.0	6.0

$F(2,446) = .275, p = .759$

difference in the mean number of mathematics courses taken in high school for the three groups, $F(2,446) = .275, p = .759$.

Timing of Algebra and Advanced Mathematics Courses. To explore sub-question 3.2: What is the relationship between the grade level at which Algebra I is completed and the mean number of advanced mathematics courses taken in high school?, the following null hypothesis was tested.

Hypothesis 3.2: There is no difference between the mean number of advanced mathematics courses taken in high school for the three Algebra Groups.

A one-way between-groups analysis of variance was conducted to explore relationships between the time at which Algebra I is completed and the mean number of advanced mathematics courses taken in high school. However, in this case the Levene statistic was less than .05 indicating that homogeneity of variance had been violated, making interpretation of the results of the ANOVA inappropriate. Thus the Kruskal-Wallis Test, the non-parametric alternative to an ANOVA, was

run. The decision was made to reject the null hypothesis as there was a statistically significant difference in the mean ranks of advanced mathematics courses taken by students in the three Algebra Groups, $\chi^2(2, N = 449) = 146.574, p < .001$. The mean rank of 350.01 for the students who completed Algebra I prior to ninth grade is significantly higher than the mean rank of 217.35 for students completing Algebra I during ninth grade. This in turn is significantly higher than the mean rank of 123.14 for students delaying Algebra I until after ninth grade. The Games-Howell Post-hoc test, which assumes neither groups of equal size nor equality of variance, was performed. This test confirms that the mean number of advanced mathematics courses taken by students is different for the three Algebra Groups. The findings indicate that students who enroll in algebra early complete more advanced mathematics courses in high school. Students who delay completion of Algebra I until after ninth grade enroll in significantly fewer advanced mathematics courses than students who complete Algebra I in ninth grade or prior to ninth grade.

Timing of Algebra and Persistence in the Mathematics Pipeline. To explore sub-question 3.3: What is the relationship between the grade level at which Algebra I is completed and persistence in the mathematics pipeline?, the following null hypothesis was tested.

Hypothesis 3.3: There is no relationship between the grade level at which Algebra I is completed and persistence in the mathematics pipeline.

Each student in the study completed his or her final year of high school mathematics in tenth, eleventh, or twelfth grade, with the majority persisting in the mathematics pipeline through twelfth grade (61.9%). Students were assigned an Exit Code of 10, 11, or 12 based on the grade level of their final high school mathematics class.

A cross-tabulation of the Exit Code and the year of completion of Algebra I was performed for all students in the study to allow for examination of percentages of students within

each of the nine categories. The results of this cross-tabulation, which indicate some variation in the percentages of students within the various Exit Groups with regard to timing of completion of Algebra I, are reported in Table 14.

In addition to cross-tabulations, the chi-square test of independence was used to test null hypothesis 3.3. The decision was made to reject the null hypothesis, $\chi^2(4, N = 449) = 11.687, p = .020$, with the analysis indicating a significant difference in the time at which a student exits the mathematics pipeline with regard to timing of completion of Algebra I. While only 5.8% of the students exited the mathematics pipeline after tenth grade, the highest percentage of students in this category had completed Algebra I prior to ninth grade (8.5%). About one-third of the students exited the mathematic pipeline after eleventh grade, with the highest percentage of these students having completed Algebra I in ninth grade (36.5%). About 62% of the students

Table 14: *Algebra Group*Mathematics Persistence Cross-tabulation*

			Grade Level of Final Math Class			
			10	11	12	Total
Algebra Group	Algebra I Prior to 9th Grade	<i>Count</i>	7	24	51	82
		<i>% within Algebra Group</i>	8.5%	29.3%	62.2%	100.0%
	Algebra I in 9th Grade	<i>Count</i>	17	105	166	288
		<i>% within Algebra Group</i>	5.9%	36.5%	57.6%	100.0%
	Algebra I After 9th Grade	<i>Count</i>	2	16	61	79
		<i>% within Algebra Group</i>	2.5%	20.3%	77.2%	100.0%
Total		<i>Count</i>	26	145	278	449
		<i>% within Algebra Group</i>	5.8%	32.3%	61.9%	100.0%

$$\chi^2(4, N = 449) = 11.687, p = .020$$

persisted in the mathematics pipeline through twelfth grade, with the highest percentage of these students delaying Algebra I until after ninth grade (77.2%). Relationships between persistence in the mathematics pipeline and timing of algebra are illustrated in Figure 5.

Timing of Algebra and Calculus. Often a reason given for early entry into algebra is to allow students to study calculus while in high school. To study relationships between timing of algebra and Calculus enrollment sub-question 3.4: What is the relationship between the grade level at which Algebra I is completed and high school Calculus enrollment?, was considered. The following null hypothesis was formed.

Hypothesis 3.4: There is no relationship between the grade level at which Algebra I is completed and high school enrollment in Calculus.

Examination of the data indicates that only 29 of the 449 students, or about 6.5%, enrolled in Calculus while in high school. Of these 29 students, 20 had completed Algebra I prior to ninth grade and nine had completed Algebra I during ninth grade. None of the students who completed Algebra I after ninth grade enrolled in Calculus while in high school.

To further investigate relationships between timing of algebra and Calculus enrollment, a cross-tabulation of Calculus enrollment and the year of completion of Algebra I was performed for all students in the study to allow for examination of percentages of students within each of the six categories. The results of this cross-tabulation, which indicate some variation in the percentages of Calculus enrollment with regard to timing of completion of Algebra I, are reported in Table 15.

In addition to cross-tabulations, the chi-square test of independence was used to test null hypothesis 3.4. The decision was made to reject the null hypothesis, $\chi^2(2, N = 449) = 54.393, p < .001$. Thus, the analysis indicates a significant difference in enrollment in Calculus during high

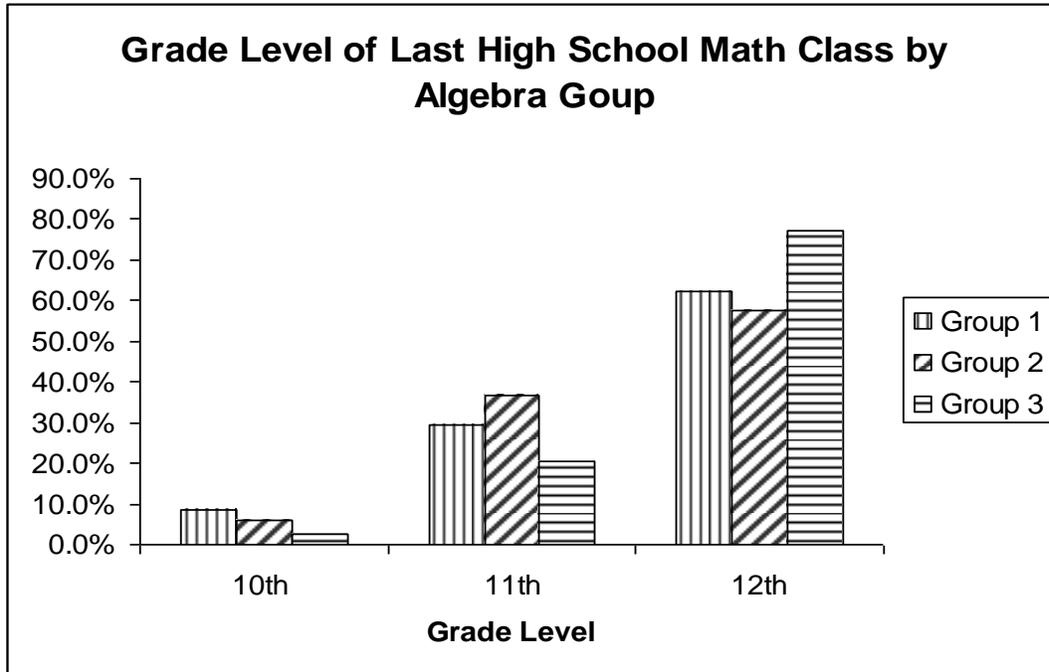


Figure 5: Persistence in the Mathematics Pipeline by Algebra Group

Table 15: Algebra Group*Calculus Credit Cross-tabulation

			Calculus Credit		
			No Calculus	Calculus	Total
Algebra Group	Algebra I Prior to 9th Grade	Count	62	20	82
		% within Algebra Group	75.6%	24.4%	100.0%
	Algebra I in 9th Grade	Count	279	9	288
		% within Algebra Group	96.9%	3.1%	100.0%
	Algebra I After 9th Grade	Count	79	0	79
		% within Algebra Group	100.0%	.0%	100.0%
Total		Count	420	29	449
		% within Algebra Group	93.5%	6.5%	100.0%

$$\chi^2 (2, N = 449) = 54.393, p < .001$$

school with regard to timing of completion of algebra. Almost one-fourth of the students who completed Algebra I prior to ninth grade enrolled in Calculus while in high school. Only 3.1% of the students completing Algebra I during ninth grade enrolled in Calculus during high school and no students who delayed Algebra I until after ninth grade enrolled in Calculus while in high school. Relationships between timing of algebra and Calculus enrollment are illustrated in Figure 6.

Timing of Algebra and Mathematics Achievement

Relationships between the grade level at which Algebra I is completed and mathematics achievement are of supreme interest in this research. To examine these potential relationships the following question was explored: What are the relationships between mathematics achievement and the grade level at which a first formal course in algebra is completed?

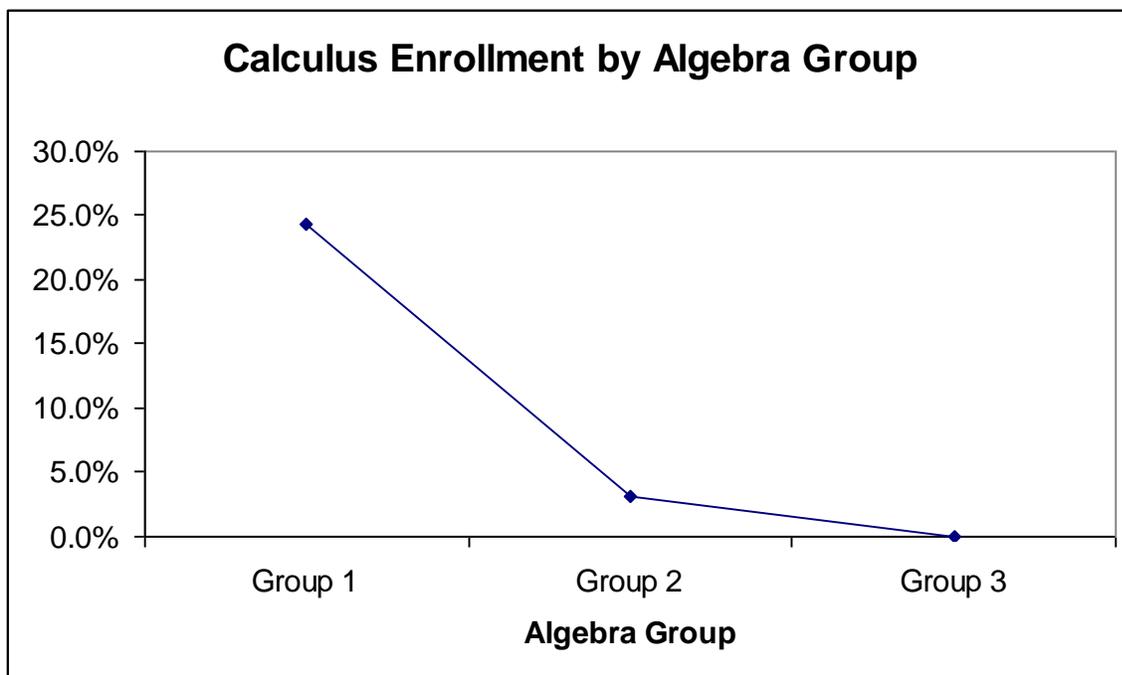


Figure 6: *Calculus Enrollment by Algebra Group*

Several measures were considered to quantify mathematics achievement. The use of Algebra I grades was not an option as Algebra I grades were not available for most of the students in Algebra Group 1 – those who completed Algebra I prior to ninth grade. Since Algebra II content builds on material learned in Algebra I, the decision was made to use Algebra II grades as one measure of mathematics achievement. In addition, students' grade point averages in all mathematics classes were considered as a measure of mathematics achievement. Only about one-half of the students in the study had completed advanced mathematics classes ($n = 241$), defined in this study as classes beyond Geometry and Algebra II. Of these students only seven had delayed Algebra I until after ninth grade. Thus the decision was made to consider grade point averages from advanced mathematics classes as a measure of mathematics achievement, including only students from Algebra Groups 1 and 2 who had completed advanced mathematics courses. Additionally, ACT Mathematics scores and, when available, ACT Mathematics sub-scores were also used to measure mathematics achievement of students.

Timing of Algebra and Algebra II Grades. To explore sub-question 4.1: What is the relationship between the grade level at which Algebra I is completed and mathematics achievement as measured by Algebra II grades?, the following null hypothesis was tested.

Hypothesis 4.1: There is no difference between mean Algebra II grades for the three Algebra Groups.

To assess Algebra II grades, student grades were retrieved from high school transcripts. Letter grades were converted to numerical grades using the conversion information provided in Table 16. When two semester grades were assigned to Algebra II, the two grades were averaged to obtain a grade for the course. Four hundred twelve of the 449 study participants had completed Algebra II.

Table 16: *Point Value of Letter Grades*

Letter Grade	Point Value
A	11
A-	10
B+	9
B	8
B-	7
C+	6
C	5
C-	4
D+	3
D	2
D-	1
F	0

A one-way between-groups analysis of variance was conducted to explore relationships between the grade level at which Algebra I is completed and grades earned in Algebra II. The decision was made to reject the null hypothesis as there is a statistically significant difference in the Algebra II grade means for the three Algebra Groups, $F(2,409) = 7.237, p = .001$, with these results reported in Table 17. Despite reaching statistical significance, the actual difference in Algebra II grade means between the groups is quite small. The effect size, calculated using eta squared, is .034.

Post-hoc comparisons using the Tukey HSD indicate that the Algebra II grade mean for students completing Algebra I prior to ninth grade ($M = 9.024, SD = 2.147$) is significantly different from the Algebra II grade mean for students completing Algebra I in ninth grade ($M = 7.916, SD = 2.441$) and the Algebra II grade mean for students completing Algebra I after ninth

grade ($M = 7.830$, $SD = 2.516$). Algebra II grade means for students completing Algebra I during ninth grade and those completing Algebra I after ninth grade do not differ significantly.

Timing of Algebra and Mathematics Grade Point Average. To examine sub-question 4.2: What is the relationship between the grade level at which Algebra I is completed and mathematics achievement as measured by high school mathematics grade point averages?, the following null hypothesis was tested.

Hypothesis 4.2: There is no difference between high school mathematics grade point average means for the three Algebra Groups.

This null hypothesis was tested using two different sets of grade point averages. First, grades for all mathematics courses were obtained from high school transcripts and mathematics grade point averages were calculated using the 11-point scale shown in Table 16. Mathematics grade point averages were available for all 449 students in the study. A one-way between-groups analysis of variance was conducted to explore relationships between the grade level at which Algebra I is

Table 17: *Descriptive Statistics for Algebra II Grades*

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval		Minimum	Maximum
					Lower Bound	Upper Bound		
Algebra I Prior to 9th Grade	82	9.024	2.1472	.2371	8.553	9.496	3.0	11.0
Algebra I in 9th Grade	274	7.916	2.4413	.1475	7.626	8.206	1.0	11.0
Algebra I After 9th Grade	56	7.830	2.5195	.3367	7.156	8.505	1.5	11.0
Total	412	8.125	2.4330	.1199	7.889	8.361	1.0	11.0

$F(2,409) = 7.237, p = .001$

completed and high school mathematics grade point averages. The decision was made to reject the null hypothesis as there is a statistically significant difference in the mathematics grade point average means for the three Algebra Groups, $F(2,446) = 6.700, p = .001$. These results are reported in Table 18. Despite reaching statistical significance, the actual difference in mathematics grade point average means between the groups is quite small. The effect size, calculated using eta squared, is .029. Post-hoc comparisons using the Tukey HSD indicate that the mean mathematics grade point average for students completing Algebra I prior to ninth grade ($M = 8.911, SD = 1.805$) is significantly higher than the mean mathematics grade point average for students completing Algebra I in ninth grade ($M = 8.037, SD = 1.954$) and the mean mathematics grade point average for students completing Algebra I after ninth grade ($M = 8.153, SD = 1.885$). Mathematics grade point average means for students completing Algebra I during ninth grade and those completing Algebra I after ninth grade do not differ significantly.

Table 18: *Descriptive Statistics for Mathematics GPA*

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Algebra I Prior to 9th Grade	82	8.91122	1.804716	.199298	8.51468	9.30776	5.000	11.000
Algebra I in 9th Grade	288	8.03688	1.954269	.115156	7.81022	8.26354	3.000	11.000
Algebra I After 9th Grade	79	8.15299	1.884822	.212059	7.73082	8.57517	3.500	11.000
Total	449	8.21699	1.940064	.091557	8.03706	8.39693	3.000	11.000

$F(2,446) = 6.700, p = .001$

To further explore relationships between the timing of algebra and mathematics grade point averages, grade point averages for advanced mathematics courses were calculated. Only 241 study participants had completed classes beyond Geometry and Algebra II. Seventy-four of these students completed Algebra I prior to ninth grade, 160 completed Algebra I in ninth grade, and only seven delayed completion of Algebra I until after ninth grade. Because of the small number of students from Algebra Group 3 who had completed advanced mathematics classes, the decision was made to consider grade point averages from advanced mathematics classes for the 234 students from Algebra Groups 1 and 2 for whom an advanced mathematics grade point average could be calculated. Once again, null hypothesis 4.2 was examined, this time using data from 234 students.

Since only two Algebra Groups were involved in this analysis, an independent-samples *t*-test was used to compare advanced mathematics grade point averages of students who completed Algebra I prior to ninth grade and students who completed Algebra I in ninth grade. The decision was made to reject the null hypothesis as there is a significant difference in advanced mathematics grade point average means for Algebra Group 1 members ($M = 8.549$, $SD = 2.517$) and Algebra Group 2 members ($M = 7.742$, $SD = 2.609$), $t(232) = 2.224$, $p = .027$. Despite reaching statistical significance, the magnitude of the differences in the means is small with an eta squared value of .021.

Timing of Algebra and ACT Mathematics Scores. To examine sub-question 4.3: What is the relationship between the grade level at which Algebra I is completed and mathematics achievement as measured by ACT Mathematics scores?, the following null hypothesis was tested.

Hypothesis 4.3: There is no difference between ACT Mathematics score means for the three Algebra Groups.

ACT Mathematics scores were available for the majority of the 449 students in the study. These scores were stored electronically and were made available to the researcher by staff from the C of O Computer Center. Some of the students had submitted more than one set of ACT scores. The policy of College of the Ozarks Admissions Office is to electronically record the highest ACT scores received prior to acceptance to the college. If, after acceptance into the college, a student submits additional ACT scores, these scores are sometimes not reflected in the data base. For the students for whom ACT Mathematics scores were not available, SAT Mathematics scores were available and were converted to comparable ACT scores using the following conversion formula:

$$ACT \text{ Math Score} = \left(\frac{SAT \text{ Math Score} - 518}{115} \right) 5.5 + 20. \quad (1)$$

This formula is used by the faculty of the Mathematics Department at College of the Ozarks to convert SAT Mathematics scores to comparable ACT Mathematics scores, for use in student placement in entry level mathematics courses. The formula was obtained by equating z-score formulas for the ACT and SAT Mathematics exams (C. Haile, personal communication, June 29, 2008). The output obtained from this formula was checked against an ACT to SAT Mathematics score conversion table accessed online (Grove, 2008). The converted scores contained in this table, which were obtained using percentiles, were comparable to scores obtained by the researcher using Equation 1. On the extremely low scores, the ACT Mathematics score obtain by

applying Equation 1 was in some cases a point lower than that reported on the conversion chart. The ACT Mathematics sub-scores were not available in electronic format but were retrieved by the researcher from students' paper files. When multiple ACT scores were available for a student, the ACT Mathematics sub-scores from the most recent testing date were recorded and used for the analyses. This decision was made so as to utilize data obtained nearest the end of high school.

A one-way between-groups analysis of variance was utilized to explore relationships between the grade level at which Algebra I is completed and ACT Mathematics scores. ACT Mathematics scores, or converted SAT Mathematics scores, were available for all 449 students in the study. A summary of Mathematics ACT score information is presented in Table 19. The decision was made to reject the null hypothesis as there is a statistically significant difference in the ACT Mathematics score means for the three Algebra Groups, $F(2,446) = 29.555, p < .001$.

Table 19: *Descriptive Statistics for ACT Math Scores*

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Algebra I Prior to 9th Grade	82	23.35	3.553	.392	22.57	24.13	15	31
Algebra I in 9th Grade	288	20.51	3.548	.209	20.10	20.92	13	31
Algebra I After 9th Grade	79	19.34	3.297	.371	18.60	20.08	11	27
Total	449	20.82	3.723	.176	20.48	21.17	11	31

$F(2,446) = 29.555, p < .001$

The effect size, calculated using eta squared, is .117, indicating a moderate effect size. Post-hoc comparisons using the Tukey HSD indicate that the mean ACT Mathematics score for students completing Algebra I prior to ninth grade ($M = 23.35$, $SD = 3.553$) is significantly different from the mean score for students completing Algebra I in ninth grade ($M = 20.51$, $SD = 3.548$) and the mean score for students completing Algebra I after ninth grade ($M = 19.34$, $SD = 3.297$). ACT Mathematics score means for students completing Algebra I during ninth grade and those completing Algebra I after ninth grade also differ significantly according to the Tukey HSD comparison.

To further explore relationships between the grade level at which Algebra I is completed and mathematics achievement as measured by ACT scores, the ACT Mathematics sub-scores were examined. Each of the three mathematics sub-scores – Pre-Algebra/Elementary Algebra; Intermediate Algebra/Coordinate Geometry; and Plane Geometry/Trigonometry – range from a low of 1 to a high of 18. These three sub-scores are then combined to determine a student's ACT Mathematics score; however, according to ACT, there is no direct arithmetic relationship between these sub-scores and the overall ACT Mathematics score (ACT, Inc., 2008b).

For each of the three ACT Mathematics sub-scores, a one-way between-groups analysis of variance was conducted using data for the 356 students in the study for whom mathematics sub-scores are available. For each of the ACT Mathematics sub-scores, the inferences provided by the respective analysis of variance are very similar to those provided by the analysis of the overall ACT Mathematics scores. In each case there is a statistically significant difference in the ACT Mathematics sub-score means for the three Algebra Groups, but in each case the effect size, calculated using eta squared, is small. Post-hoc comparisons using the Tukey HSD indicate that for each of the sub-tests the ACT Mathematics sub-score mean for students completing

Algebra I prior to ninth grade is significantly different from the means for students completing Algebra I in ninth grade and students completing Algebra I after ninth grade. No significant differences are present for any of the three ACT Mathematics sub-score means for students completing Algebra I during ninth grade and those completing Algebra I after ninth grade.

Conclusion

Statistical procedures and results were presented in chapter 4. These were organized around four major questions, all of which examined factors relevant to the timing of completion of Algebra I. The first question and analysis provided insights into relationship between certain student characteristics and the time at which a student completes Algebra I. The second question inspected certain school characteristics with regard to timing of completion of algebra. The third question and analysis focused on relationships between the grade level at which a student commences study of formal algebra and high school mathematics course-taking patterns. The final major question examined relationships between timing of algebra and mathematics achievement. Several statistically significant relationships were confirmed and reported.

Chapter 5

Conclusions

In a recent article published in *The Chronicle of Higher Education*, Charles M. Vest, president of the National Academy of Engineering is quoted with regard to the 2008 Presidential Election, "The next president will be hit between the eyes by the full force of global competition and the realization that 21st-century jobs will follow knowledge, innovation, and expertise wherever it is found in the world" (Fischer, 2007, ¶4). The author further champions the importance of all students at all levels acquiring the right set of skills for success, especially in the areas of science and mathematics. Many state governments are acting on this belief, with ten states currently in the process of increasing the number of years of mathematics required for graduation. About half the U.S. states already require students to complete specific mathematics classes to earn a diploma. The vast majority of these states require completion of Algebra I or a course with comparable content (Reys, Dingman, Nevels, Teuscher, 2007). This requirement of completion of Algebra I by all students is due in part to the fact that Algebra I is often viewed as a gatekeeper for further study in mathematics and science.

This increased focus on mathematics course taking, and specifically completion of Algebra I, prompts one to reflect on the most advantageous time for students to complete a first course in formal algebra. Should all students take Algebra I at the same point in their school careers? If so, should this be seventh, eighth, ninth, or tenth grade? These questions have been discussed at school, district, state, and national levels for at least the past two decades. In 1987, Usiskin, in an article printed in the *Mathematics Teacher*, proposed that the majority of students in the United States should enroll in an algebra course in the eighth grade. Over a decade later, in 1999, an article by Steen appeared in *Middle Matters* questioning why we rush students to take

algebra in eighth grade. Steen suggested we continue to offer algebra to students only when they are ready to take it – some in eighth, some in ninth, and some in tenth grade. The debate continues into the twenty-first century.

A main purpose of this study is to determine the significance of the grade level at which Algebra I is completed. Does completion of Algebra I prior to ninth grade increase the likelihood that more advanced mathematics courses will be taken in high school? Is mathematics achievement likely to be increased if students delay completion of Algebra I until after ninth grade? Does the grade level at which Algebra I is completed have any relationship with mathematics achievement or mathematics course-taking patterns?

Some recent research indicates positive benefits of acceleration into algebra (Ma, 2005a; Ma, 2005b; Smith, 1996). If it is indeed the case that acceleration into algebra provides positive benefits for students, do all students have equal access to early enrollment in algebra? Does early access to algebra vary with gender, race-ethnicity, or SES? Do students from all sizes and types of schools have the same opportunity to enroll early in Algebra I? Questions such as these motivated the second main purpose of this study -- to describe student and school characteristics which are significant predictors of the grade level at which a student is likely to complete a first formal course in algebra.

Summary of the Study

A quantitative study was conducted to determine relationships between the grade level at which a first formal course in algebra is completed and mathematics course-taking patterns and achievement. Also important to the analysis was the examination of relationships between timing of algebra and certain student and school characteristics. To examine these relationships, information was gathered on students who matriculated during 2007-2008 at College of the

Ozarks, a small liberal arts college in the Midwest. Information essential for the study was available for 449 of the 502 matriculating students. This information included gender, race-ethnicity, student SES, high school from which the student graduated, high school transcript information, and ACT scores. Using high school transcript information, students were placed in groups according to the grade level at which they completed Algebra I. Eighty-two of the students completed Algebra I prior to ninth grade and were placed in Algebra Group 1. Algebra Group 2 was comprised of the 288 students in the study who completed Algebra I during ninth grade. The remaining 79 students completed Algebra I after ninth grade and formed Algebra Group 3. Relationships between placement in Algebra Groups and various independent variables were tested using a variety of statistical techniques, including the chi-square test of independence, one-way between-groups analysis of variance, and multinomial logistic regression. The Statistical Package for the Social Sciences (SPSS) Version 16.0 was used to analyze the data. All hypotheses were tested at the .05 level of significance.

Findings

In this section findings of this study are briefly summarized. The results of the study are organized around the four major questions of the study, beginning with an examination of relationships between student characteristics and timing of algebra.

Student Characteristics and Timing of Algebra. The first major question of the study examined the student characteristics of gender, race-ethnicity, and SES and their relationships to timing of algebra. Because of a lack of racial-ethnic diversity among the students in the study, the results related to race-ethnicity are inconclusive. No significant difference is apparent in placement in Algebra Groups with regard to gender, with males and females equally likely to enroll in algebra prior to ninth grade. To consider SES, each student was placed in an SES Level

Group based on family size and adjusted gross income. A significant difference in placement in Algebra Groups with regard to SES is evident. For each increase in SES Level the percentage of students experiencing early entry into algebra increases. Conversely, for each increase in SES Level the percentage of students delaying study of algebra decreases. Thus the lower the SES Level the less likely the student to study formal algebra prior to ninth grade and the more likely the student to delay study of formal algebra until after ninth grade. Multinomial logistic regression confirms that the student characteristic of SES is a significant predictor of the grade level at which a student will complete Algebra I, while gender was not shown to be significant in predicting the grade level at which a student will complete Algebra I.

School Characteristics and Timing of Algebra. The second major question in the study examined certain school characteristics and their relationships to the grade level at which students complete Algebra I. School types of home, private, and public were examined. Significant differences in membership in Algebra Groups with regard to school type are evident. Home educated students are less likely than public school students to complete Algebra I prior to ninth grade. Both home school and private school students are more likely to delay completion of Algebra I until after ninth grade when compared to public school students.

Relationships between school size and timing of algebra were examined by considering both average high school grade size and district size. In both cases no significant difference in membership in Algebra Groups with regard to school size is indicated. Thus students in both large and small schools have equal access to early entry into algebra and are also equally likely to delay completion of algebra until after ninth grade. Only public school students were included in this analysis, as school size is not a factor in home schools and information about the size of the private schools was unavailable in many cases.

The third school characteristic, school locale, and relationships to timing of algebra were first examined by placing students into one of four groups -- home school, private school, non-rural public school, and rural public school. With this categorization, a significant difference in membership in Algebra Groups with regard to school locale is confirmed. However, much of the difference seems to occur between home educated students and students who attended public schools. Thus a similar analysis was conducted utilizing only data obtained from public school students. Using this data, the analysis indicates no significant difference in Algebra Group membership with regard to school locale. Thus students attending rural public schools are as likely to enroll in algebra prior to ninth grade as student attending non-rural public schools. Similarly, students attending non-rural public schools and students attending rural public schools delay completion of algebra until after ninth grade in roughly equivalent proportions.

Course Taking and Timing of Algebra. Often a reason given for allowing students to complete Algebra I prior to ninth grade is to facilitate enrollment in additional mathematics courses or more advanced mathematics courses. Alternately, others argue that students should not be encouraged to complete Algebra I prior to high school as they may then "get their math requirements out of the way" and exit the mathematics pipeline well before graduation. In response to these concerns, the results of this study indicate that there is no significant difference in the number of mathematics courses completed in high school by students in the three Algebra Groups. However, the results of the study indicate that there is a significant difference in the number of advanced mathematics courses taken by students in the three Algebra Groups. Students who delay Algebra I until after ninth grade take significantly fewer advanced mathematics courses than students who complete Algebra I in ninth grade. Furthermore, students who complete Algebra I in ninth grade take significantly fewer advanced mathematics classes

than early entrants into algebra. Calculus enrollment was specifically examined and students who accelerate into algebra are significantly more likely to enroll in Calculus in high school than students in the other two Algebra Groups.

To further examine timing of algebra and mathematics course-taking patterns, the time at which students exit the mathematics pipeline was also examined. The results indicate a statistically significant difference in the grade level at which students in the Algebra Groups exit the mathematics pipeline. Although the percentage is small, students who complete Algebra I prior to high school are most likely to exit the mathematics pipeline at the end of tenth grade. Students who delay Algebra I until after ninth grade persist in the mathematics pipeline until the end of twelfth grade at a higher rate than students in the other two Algebra Groups.

Mathematics Achievement and Timing of Algebra. The final major question of the study explored relationships between timing of algebra and mathematics achievement. For the purpose of this study, mathematics achievement is measured using high school mathematics grade point averages, grades in Algebra II, and ACT Mathematics scores. For each of these measures, statistically significant differences between mathematics achievement for the three Algebra Groups are observable. In each case students who complete Algebra I prior to ninth grade exhibit higher mathematics achievement than students in both of the other two groups. On some mathematics achievement measures, students who complete Algebra I in ninth grade show significantly higher mathematics achievement than students who delay completion of algebra until after ninth grade. When using ACT Mathematics scores as a measure of mathematics achievement a medium effect size is present. In all other cases the effect size is small.

Discussion

Several of the previously reported findings warrant further reflection and discussion. The following discussion is again organized around the four major questions of the study, starting with a discussion of findings regarding student characteristics and timing of completion of algebra.

Student Characteristics and Timing of Algebra. Results indicating no significant relationship between gender and timing of formal algebra are encouraging and supportive of recent research reporting the mathematics achievement gender gap is small, some times favoring females and some times favoring males (Hopkins, 2004; Lubienski, 2001; McGraw, Lubienski, & Strutchens, 2006). If, as this current study suggests, male and female students complete Algebra I prior to ninth grade in equal proportions, then students of both genders should have equal access to advanced mathematics courses which may lead to greater mathematics achievement. In regard to relationships between student characteristics and timing of algebra, the alarming result is the lack of representation of low SES students among early entrants into algebra. As found in this study, lack of early access to Algebra I is significantly related to future course-taking patterns leading to completion of fewer advanced mathematics courses, including Calculus. Although disturbing, these results are not surprising as they corroborate other research indicating mathematics achievement gaps along SES levels (Lubienski & Shelley, 2003; Rowley & Wright, 2007; Tate, 1997). This finding of a relationship between low SES and delay of completion of Algebra I may be an important factor to partially explain the mathematics achievement gap that exists between students of different SES levels.

School Characteristics and Timing of Algebra. The lack of significant relationships between several school characteristics and timing of algebra is encouraging. Research conducted

over the past several decades has shown that students in small and/or rural schools may have access to fewer curricular offerings (Barker, 1985; Melnick et al., 1987; Monk 1990). It seems reasonable that a lack of availability of curricular offerings could affect students' opportunities to enroll in Algebra I prior to high school. However, results of this study indicate that students in small schools and in rural schools enroll in Algebra I prior to ninth grade as often as students from non-rural schools and large schools. However, with less than 20% of the students in this current study completing Algebra I prior to ninth grade, a focus on providing greater access to the formal study of algebra prior to high school in schools of all sizes and in all locales is appropriate and important.

The finding regarding school type and timing of algebra which reached statistical significance and poses a challenge is the small proportion of home educated students completing Algebra I prior to ninth grade and the large proportion delaying completion of Algebra I until after ninth grade. A recent report prepared by the National Center for Education Statistics estimates that 1,096,000 students in the United States were home educated in 2003. This represents a 29% increase from the estimated 850,000 who were homeschooled in 1999 (NCES, 2004). Most likely the number of home educated students has continued to increase in the five years since this report. Although these homeschooled students represented only about 2.2% of the 2003 K-12 student population, the under representation of home educated students among early entrants into algebra raises a concern which is deserving of the attention of researchers and educators.

Course Taking and Timing of Algebra. The results of this current study indicate that there is no significant difference in the number of mathematics courses that students complete in high school in relation to timing of algebra. With all students in the study completing an average of

3.526 mathematics credits in high school, this result may not be surprising. Where the significant difference occurs is with advanced mathematics course taking. Although the number of students in the study enrolling in Calculus in high school is small, students who complete Algebra I while still in middle school are significantly more likely to enroll in Calculus in high school. In fact this trend is true for advanced mathematics courses in general. Students who delay Algebra I until after ninth grade take significantly fewer advanced mathematics classes than students who complete Algebra in ninth grade. These students in turn complete significantly fewer mathematics courses than early entrants into algebra.

As our society continues to advance scientifically and technologically and the demand for advanced mathematical knowledge and understanding continues to increase, all students must be provided the opportunity to study advanced mathematics. The results of this study suggest that one strategy for making advanced mathematics courses a viable option for students is to encourage formal study of algebra prior to ninth grade.

A result regarding timing of algebra and mathematics course taking that raises a concern is the statistically significant relationship between timing of algebra and persistence in the mathematics pipeline. Although a small percentage, early entrants into algebra exit the mathematics pipeline at the end of tenth grade at a greater rate than students in the other two Algebra Groups. This may be predictable as students who delay completion of Algebra I until later in high school may be required to enroll in a mathematics class in eleventh or twelfth grade to satisfy district or state requirements. However, a concern is raised when even a small percentage of students who most likely excel in mathematics and thus are accelerated into algebra, choose, during their junior and senior years of high school, to not enroll in mathematics

classes. Students should be encouraged to continue to study advanced mathematics even after they have met district and state minimum graduation requirements.

Mathematics Achievement and Timing of Algebra. Significant relationships were found between the grade level at which Algebra I is completed and mathematics achievement using a variety of measures. Students completing Algebra I while still in middle school exhibit significantly greater mathematics achievement than students in the other two groups. However, factors other than timing of completion of algebra very likely contribute significantly to students' mathematics achievement. With data available in this study, it was not possible to identify all of the factors contributing to this increased achievement for early entrants into algebra. Key among the unavailable data is information about students' mathematics achievement prior to enrollment in Algebra I, as this is likely to affect their later mathematics achievement.

With Algebra I providing a foundation for further study of advanced mathematics, it is critical that students are introduced to the formal study of algebra at a time that will provide maximum benefits. Hence it is encouraging that students who enrolled in algebra early achieved at the highest level among the three groups. This may provide support for enrolling more students in Algebra I prior to ninth grade, as well as help to dispel concerns that middle school students are not mature enough for the formal study of algebra.

Conclusion

The findings of this study suggest that completion of Algebra I prior to high school may increase students' opportunities to enroll in a greater number of advanced mathematics courses while in high school. This research and other studies suggest that increased enrollment in advanced mathematics classes may in turn result in higher mathematics achievement while in high school. Thus this current study adds to the research base in factors affecting mathematics

achievement and course-taking patterns. Past research indicates that several student factors, including gender, race-ethnicity, SES, and attitude towards mathematics, may be important in predicting course-taking patterns and mathematics achievement during high school. Results of this current study suggest that the time at which a student completes Algebra I may also be a significant factor to consider when predicting mathematics achievement and course taking. More specifically, this study, by establishing a relationship between student SES and the grade level at which Algebra I is completed, further confirms the existence of inequities in mathematics education with regard to SES. Perhaps increased attention to the placement of students in Algebra I will contribute to more equitable mathematics course taking and begin to eliminate achievement gaps between students of different economic levels.

Additionally, this study adds to the somewhat limited research base regarding the significance of the timing of completion of Algebra I. According to NAEP data, the proportion of eighth-grade students enrolling in Algebra I has grown significantly over the past few years. In certain areas, such as Arlington County near Washington DC, middle schools are reporting that up to one-half of students are successfully completing Algebra I prior to high school (Mathews, 2007). The percentage of students experiencing early enrollment in algebra is likely to continue to grow in upcoming years. This current study supports previous research indicating positive outcomes for students who experience acceleration into algebra.

Finally this current research adds to the research base concerning home educated students. During the past decade there has been a significant increase in the number of students who are receiving a formal education at home. Thus the findings of this study are timely and may provide valuable information that will be helpful in meeting some of the challenges presented to

families as they seek to provide a quality education for their children outside the familiar context of public and private schools.

Recommendations for Further Study

While this current study provides some insights regarding relationships between timing of completion of Algebra I and several issues important in mathematics education, it also suggests many other questions and areas of study. There are many closely related questions which are worthy of additional research, as well as some more indirectly related questions which have arisen as a result of this research. Some of the limitations of the data set available in this study point to further questions which could be explored under slightly different circumstances or with access to additional data. Several recommendations for additional study are briefly discussed below.

1. The study could be replicated using data from another college in the Midwest or from a college in a different geographical area. This diversity of settings might contribute to the generalizability of the findings of this study.
2. The same data set could be extended to include information about mathematics course taking during the first year of college. This would allow examination of relationships between timing of algebra and collegiate study of mathematics.
3. A similar study could be conducted using a more robust definition of SES. For example, information about parents' careers and educational levels could be included. Inclusion of information about whether study participants are first generation college students would be of special interest.
4. The study could be replicated including a measure of students' mathematics achievement prior to enrollment in a first formal course in algebra. This would allow the researcher to

examine relationships between timing of algebra and growth in mathematics achievement. A study such as this might provide valuable information that was not accessible in this study due to limitations of the available data.

5. A similar study could be conducted with the inclusion of additional school factors. Factors such as school SES, school grade level structure, mathematics curriculum, student/teacher ratio, school climate, etc could be included to examine additional relationships between timing of algebra and school characteristics.
6. Some research has shown that schools of different sizes vary in curricular offerings. A study examining relationships between curricular offerings and timing of algebra could be conducted to examine relationships between school size, curricular offerings, and the time at which students complete Algebra I.
7. A qualitative study could be conducted so as to examine detailed information about the nature and content of Algebra I classes in connection to timing of algebra.
8. Some research indicates that a large percentage of students who enroll in Algebra I in middle school repeat the course in high school. A study focusing specifically on students who enroll in Algebra I in middle school and then repeat the course in high school would provide additional information about timing of algebra and mathematics achievement. These findings might provide information currently missing from the research base and prove to be very beneficial when making decisions regarding placement of students in Algebra I.
9. Further research concerning homeschooling and mathematics course taking should be conducted. Factors such as parents' educational levels, primary reasons for homeschooling, parents' views of mathematics, and mathematics curricular materials,

might provide some insights into challenges facing parents who choose to education their children at home.

10. Less than 20% of the students in this study completed Algebra I prior to ninth grade.

Recent reports based on 2003 NAEP data suggest that 42% of students enroll in Algebra I prior to ninth grade (Mathews, 2007). In light of this disparity a study could be conducted to determine relationships between geographic locales and percentages of students enrolling early in Algebra I.

11. In light of the fact that very limited racial-ethnic diversity existed among the students involved in this study, a similar study could be conducted with a more diverse student population. Of special interest would be inclusion of students for whom English is spoken as a second language.

12. An issue related to completion of Algebra I in middle school, is the awarding of course credit. Some states and districts award high school credit for completion of Algebra I regardless of the grade level at which it is completed, while others award high school credit for the course only if it is completed during ninth grade or later. Research examining relationships between enrollment numbers in Algebra I during middle school, the granting of high school credit for completion of Algebra I in middle school, and mathematics achievement would be of interest.

These recommendations are representative of the many issues regarding timing of completion of Algebra I that warrant further research and consideration. With the findings of this study indicating significant positive outcomes for students who complete Algebra I while in middle school, acceleration into algebra should become a reality for more students. Increased access to the formal study of algebra in middle school could then move the mathematics

education community in the direction of ensuring that a strong mathematical foundation is in place for *all* students by the end of eighth grade. This will then provide a foundation for a challenging high school mathematics curriculum for all students.

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Vita

Jamie Fugitt grew up on a farm in the Missouri Ozarks, attending the same small rural school from kindergarten through twelfth grade. She attended School of the Ozarks (currently College of the Ozarks) where she earned a degree in Mathematics with emphases in Computer Science and Secondary Mathematics Certification. After teaching junior high mathematics for two years, Jamie decided to depart from teaching to spend time with her son. Quickly realizing her desire to continue to teach and learn mathematics, she began teaching at a local university and enrolled in graduate courses. In 1989 she earned a Master of Science in Mathematics Degree from the University of Missouri at Kansas City. Since that time Jamie has taught college level mathematics at University of Central Missouri, Campbell University, and College of the Ozarks.

In 1992 Jamie was given the opportunity to return to her alma mater, College of the Ozarks. There she currently serves as an Associate Professor of Mathematics and Chair of the Division of Mathematical and Natural Sciences. She teaches a variety of subjects including remedial algebra, trigonometry, geometry, and methods of teaching secondary mathematics. Her favorite classes to teach are Introduction to the History of Mathematics and Mathematics for Elementary and Middle School Teachers. Jamie has provided numerous workshops and presentations at professional meetings.

Jamie is married to Jeff and they have two children. Jeff has earned degrees in religion and is currently pursuing a PhD in Religious Studies. Son Johnathan has a Bachelor of Arts Degree in Political Science and has completed the requirements for a Master of Science Degree in Anthropology. Daughter Elizabeth is currently pursuing a Bachelor of Science Degree in Dietetics. Elizabeth is married to Blaine, who recently completed a Bachelor of Science Degree in Exercise and Movement Science.