3-D MHD plasma armature railgun simulations

Dmitri A. Kondrashov

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Dennis Keefer, Major Professor

We have read this dissertation and recommend its acceptance:

Remi Engels, Robert Roach, Roy Schulz

Accepted for the Council:

Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)
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We have read this dissertation and recommend its acceptance:

Roy J. Schulz
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Accepted to the Council

Associate Vice Chancellor and Dean of The Graduate School
3-D MHD Plasma Armature Railgun Simulations

A Dissertation Presented
for the Doctor of Philosophy Degree
The University of Tennessee, Knoxville

Dmitri A. Kondrashov
May 1996
ACKNOWLEDGEMENTS

I would like to thank all those who have helped me in completing this dissertation. I am grateful to Dr. Dennis Keefer and Dr. Roger Crawford for giving me the opportunity to continue my research in the United States. I have benefited greatly from working with them during the last four years, and I hope that our relationships will endure. In particular, I would like to thank Dr. Keefer for his help and guidance in my work.

I also thank Drs. Remi Engels, Robert Roach, and Roy Schulz, for reviewing my work. Their suggestions helped me greatly in preparing and improving this dissertation.

I would like to acknowledge the help and encouragement of Dr. Jaime Taylor both in academic and everyday life.

Special thanks are given to the people of Moscow High Temperature Institute (IVTAN) who encouraged me to start working in the area of numerical simulation of the plasma armature railgun. I am grateful to Dr. Valerii Zatelepin for being my tutor during my work at IVTAN.

A special note of appreciation is given to my family. All these years, I felt your love and support. I owe this work to the patience and love of my wife Veronica. She shared all challenges of it with me. Without her, this work would not be possible.
ABSTRACT

The goal of a plasma armature railgun is to accelerate the projectile to hypervelocities (i.e., to velocities beyond 5 km/s). Despite extensive research, projectile velocities achieved in the plasma armature railgun experiments were under 6-8 km/s - unfortunately, far less than predicted theoretical values. Experimental and numerical studies did not bring a full understanding of the factors limiting performance of the plasma railgun. The numerical studies so far have been limited to 1-D and 2-D computer models. In this dissertation, it is demonstrated that these models inadequately predict the main physical features of railgun plasma flow. To understand the railgun physics, 3-D magnetohydrodynamic (MHD) modelling is necessary.

To perform a 3-D MHD time-dependent computer simulation of the plasma armature railgun, a new code MAP3 (MHD Arc Plasma) was developed at University of Tennessee Space Institute (UTSI). MAP3 uses an efficient numerical method to solve Maxwell’s equations and Navier-Stokes equations to develop a complex time-dependent electromagnetic and velocity vector field distribution in the railgun. The importance of MAP3 numerical scheme that uses a staggered grid to solve Maxwell’s equations, is demonstrated.

MAP3 provides the first qualitative and quantitative understanding of 3-D physical phenomena in the plasma armature railgun. The results of the 3-D computer simulation for 1-cm and 2-cm bore railguns with ablating walls, are presented.

A profound influence of the inherently 3-D nature of the railgun electromagnetic field on the plasma flow is demonstrated. A strong spatial nonuniformity of the electromagnetic force generates a flow of plasma towards the projectile near the rail and away from the projectile along the center of the bore. This plasma flow is exactly the opposite to flow
provided by the previous 2-D numerical models. A zone of high-shear flow near the rail surfaces can increase viscous losses, which are not accounted for in the usual performance estimates.

A direct simulation of the particular experiment is the logical step to develop this work further. A satisfactory qualitative agreement was demonstrated between numerically obtained B-dot signal (armature magnetic field) and typical experimental data. MAP3 can be used to study secondary arc formations. The 3-D flow has a tendency to elongate the plasma or arc armature, creating a current conducting tail. This process may play an important role in the formation of secondary arcs. MAP3 can be extended to analyze conditions believed to be accountable for the development of the secondary arcs and to provide reliable quantitative simulation. These conditions should include current that changes in time, and higher ablation rates.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION ..................................................</td>
<td>1</td>
</tr>
<tr>
<td>1.1. Plasma Armature Railgun ...................................</td>
<td>1</td>
</tr>
<tr>
<td>1.2. Plasma Armature MHD Description ..........................</td>
<td>5</td>
</tr>
<tr>
<td>1.3. 1-D Computer Models .......................................</td>
<td>6</td>
</tr>
<tr>
<td>1.4. 2-D Rail Plane Computer Models ............................</td>
<td>7</td>
</tr>
<tr>
<td>1.5. 2-D Insulator Plane Computer Model .......................</td>
<td>10</td>
</tr>
<tr>
<td>1.6. Why 3-D MHD Modelling Is Necessary .......................</td>
<td>11</td>
</tr>
<tr>
<td>1.7. 3-D MHD Modelling ..........................................</td>
<td>13</td>
</tr>
<tr>
<td>1.7.1. 3-D MHD Modelling At UTSI ..........................</td>
<td>13</td>
</tr>
<tr>
<td>1.7.2. Other 3-D MHD Modelling .................................</td>
<td>13</td>
</tr>
<tr>
<td>1.8. Dissertation Overview .......................................</td>
<td>14</td>
</tr>
<tr>
<td>II. MAP3 ELECTROMAGNETIC SOLVER ...............................</td>
<td>15</td>
</tr>
<tr>
<td>2.1. Governing Equations for the Electromagnetic Solver ......</td>
<td>15</td>
</tr>
<tr>
<td>2.2. Transformation of Variables ..................................</td>
<td>17</td>
</tr>
<tr>
<td>2.3. MEGA Formulation ............................................</td>
<td>18</td>
</tr>
<tr>
<td>2.4. MAP3 Formulation ............................................</td>
<td>19</td>
</tr>
<tr>
<td>2.5. MAP3 Gauging Procedure .....................................</td>
<td>23</td>
</tr>
<tr>
<td>2.6. MAP3 Discretization Schemes ..................................</td>
<td>24</td>
</tr>
<tr>
<td>2.6.1. Scheme A, Non-Staggered Grid ...........................</td>
<td>24</td>
</tr>
<tr>
<td>2.6.2. Scheme B, Staggered Grid ..................................</td>
<td>27</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURES</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Physics of the railgun acceleration</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Flow by 2-D rail-to-rail MHD model</td>
<td>8</td>
</tr>
<tr>
<td>2.1 Staggered grid cell in MAP3</td>
<td>28</td>
</tr>
<tr>
<td>2.2 Computational grid, rail-to-rail plane</td>
<td>32</td>
</tr>
<tr>
<td>2.3 Computational grid in the cross-section; &quot;+&quot; marks the axial line along which the comparison is made</td>
<td>38</td>
</tr>
<tr>
<td>2.4 $B_z$ distribution, stationary problem; $t=10^5$ s</td>
<td>38</td>
</tr>
<tr>
<td>2.5 $J_z$ distribution, stationary problem; $t=10^5$ s</td>
<td>39</td>
</tr>
<tr>
<td>2.6 $B_z$ distribution, stationary problem; $t=6\cdot10^4$ s</td>
<td>39</td>
</tr>
<tr>
<td>2.7 $J_z$ distribution, stationary problem; $t=6\cdot10^4$ s</td>
<td>40</td>
</tr>
<tr>
<td>2.8 Total axial force, stationary problem</td>
<td>40</td>
</tr>
<tr>
<td>2.9 $B_z$ distribution, moving problem; $t=10^5$ s</td>
<td>42</td>
</tr>
<tr>
<td>2.10 $J_z$ distribution, moving problem; $t=10^5$ s</td>
<td>42</td>
</tr>
<tr>
<td>2.11 $B_z$ distribution, moving problem; $t=2\cdot10^4$ s</td>
<td>43</td>
</tr>
<tr>
<td>2.12 $J_z$ distribution, moving problem; $t=2\cdot10^4$ s</td>
<td>43</td>
</tr>
<tr>
<td>2.13 Armature force, moving problem</td>
<td>44</td>
</tr>
<tr>
<td>2.14 Total force, moving problem</td>
<td>44</td>
</tr>
<tr>
<td>2.15 Current density vector in the railgun cross-section by MAP3-B, moving problem, $t=2\cdot10^4$ s</td>
<td>45</td>
</tr>
</tbody>
</table>

2.16 Current density vector in the rail-to-rail symmetry plane by MAP3-A,
moving problem, $t=2\cdot10^4$ s ........................................ 46

2.17 Current density vector in the rail-to-rail symmetry plane by MEGA,
moving problem, $t=2\cdot10^4$ s ........................................ 47

3.1 Van Leer flux vector splitting ........................................ 54

3.2 MAP3 work chart ..................................................... 58

3.3 MAP3 plasma flow solver ............................................ 58

3.4 MAP3 electromagnetic solver ......................................... 59

4.1 Sketch of the plane surfaces used for data analysis ................. 66

4.2 3-D distribution of current density and magnetic field in the 1-cm bore,
T-350 plane .............................................................. 68

4.3 Axial force distribution in the 1-cm bore, T-350 plane ............. 69

4.4 Axial force distribution in the 1-cm bore, T-10 plane ............... 70

4.5 Flow vector field relative to the projectile and temperature distribution
in the 1-cm bore, R-A plan ............................................... 72

4.6 Flow vector field relative to the projectile and current density distribution
in the 1-cm bore, I-A plane ............................................... 73

4.7 Flow vector field relative to the projectile and current density distribution
in the 1-cm bore, I-8 plane ............................................... 74

4.8 Flow vector field in the 1-cm bore near the projectile, T-380 plane ... 75

4.9 Flow vector field relative to the projectile and current density distribution
in the 2-cm bore, I-A plane ............................................... 77
4.10 Flow vector field relative to the projectile and temperature distribution in the 2-cm bore, R-A plane .................................................. 78
4.11 Time evolution of the current density for the 1-cm bore along the railgun axis ................................................................. 79
4.12 Time evolution of the current density for the 2-cm bore along the railgun axis ................................................................. 79
4.13 Numerically obtained $B_y$ time evolution ................................................. 81
4.14 Numerically obtained B-dot signal ....................................................... 81
4.15 Typical form of the experimental B-dot probe (armature magnetic field) . 82
I. INTRODUCTION.

1.1. Plasma Armature Railgun.

A railgun, or electromagnetic launcher (EML), is a device for converting electromagnetic field energy to kinetic energy of a mass. In conventional railgun designs, electric current from an external power source passes through conducting rails and a moving mass called the armature (Figure 1.1). The armature can be a solid conductor, or can be a gaseous plasma column or arc connecting the rails. The induced magnetic field is trapped in the bore between the rails and back of the armature. The armature is accelerated by the resulting axial electromagnetic force. If the armature is not the projectile itself, the armature transfers its momentum to an upstream, solid, non-conducting projectile. In contrast to conventional gas-chemical accelerators, for a railgun the velocity of the projectile was thought not to be limited by a finite signal propagation velocity in the bore gas. Plasma, solid and hybrid armature designs have been used in EML applications. An advantage of the plasma armature is much smaller armature mass, leading to the possibility of applications producing hypervelocity projectile velocities (i.e. those beyond 5 km/s).

The plasma armature railgun has been the subject of active research since the experiments of Rashleigh and Marshall in 1978 [1]. However, projectile velocities obtained in their experiments were under 6-8 km/s, unfortunately, far less than theoretical optimum values. It is generally accepted that so-called primary armature
Figure 1.1. Physics of the railgun acceleration.
separation from the projectile and secondary arc formation limit the performance of the plasma armature railgun [33]. The acceleration of the projectile is determined by the surface integral of the plasma pressure over the base of the projectile, less the integral of surface drag force acting on the projectile surfaces in contact with the rails. Ideally, the armature stays in the base region of the projectile, and they both move with the same speed. Therefore, the base pressure closely follows the magnetic field pressure value in the bore, and the resulting system acceleration is the most efficient. The plasma armature in this case works as an "electromagnetic piston". However, numerous experiments have shown that such an ideal situation is very difficult to achieve. Usually, during the shot, the armature separates from the projectile [2]. This is known as "primary separation". Between the armature and the projectile is a buffer gas of low conductivity, relatively cold plasma. Acceleration of the buffer gas leads to additional momentum and viscous losses, and reduces the projectile base pressure. Primary armature separation is the reason why the base of the projectile shows very little ablation when it is recovered after a test.

Moreover, plasma motion is not connected to the speed of the armature as a current conducting structure. The plasma accelerated by the electromagnetic force may have a velocity relative to the armature. If the plasma velocity is much greater than the actual armature speed, the corresponding momentum losses further reduce the base projectile pressure.

Secondary arc formation is another main factor believed to limit performance of the railgun. Electrical breakdown of the gas behind the primary armature creates parasitic
secondary arcs. A corresponding current split reduces the electromagnetic force acting on the primary armature. Consequently, projectile acceleration decreases.

Intensive experimental efforts were made to study factors that lead to secondary arc formation and primary armature separation. Of particular interest was the role of ablation products in the armature electrodynamics. Intensive thermal radiation from the plasma, which can reach $2 \cdot 10^4 - 4 \cdot 10^4$ K in temperature, causes the bore walls to evaporate or melt. Parker et al. [3,4] suggested that part of the ablated material is trapped in the armature thus increasing the mass to be accelerated. This would act as an additional inertia force. Also, hot ablated material left behind in the bore would provide a potential source for secondary arc formation.

However, experiments of Stefani and others [5,6] provide conflicting evidence about the role of ablation in secondary formation. Stefani et al. [5] reported results with a railgun operating without ablation. An important observation of tests with ablation was a longer armature than that observed in non-ablating tests. The authors of [5] concluded that non-ablating armatures do not form secondaries or restrikes for railgun armature velocities up to 4 km/s.

By contrast, Witherspoon et al. [6] experimented with low ablation ceramic insulators and showed growth in a secondary armature "tail". These tests have shown the development of long armatures, which contradict the findings of Stefani et al. [5].

Some early observations on the formation of secondary arcs led to the conclusion that a decreasing current (negative $dI/dt$) causes formation of the secondaries. However, as more experimental data have been collected, it was concluded that $dI/dt$ is not a prime
factor in the secondary formation [7].

Summarizing, the experimental studies did not bring a full understanding of the mechanism of secondary arc formation or primary separation. Thus, intensive efforts to numerically model the plasma armature railgun were undertaken by the author. The results of the principal parts of numerical simulations are described in this dissertation.

1.2. Plasma Armature MHD Description.

It became clear during the study that the armature does not behave as a simple electromagnetic "piston", and thereafter the attention was focused on studying detailed railgun physics. With significant recent progress in the advancement of computers and numerical methods, numerical modelling became an important tool in analyzing the armature dynamics. In a typical railgun, high (~1 MPa) pressures, low conductivities, and the time scale for plasma formation and motion describe the plasma within continuum gas MHD theory. The plasma is treated as a single compressible continuous fluid. Plasma motion is governed by the time-dependent conservation equations of mass, momentum, and energy. The MHD interaction is represented by an electromagnetic body force \((J \times B)\) in the momentum equation, and a power dissipation \((J^*E)\) in the energy equation. The electromagnetic field is computed from Maxwell’s equations in the MHD limit, i.e., without the displacement current in Ampere’s law. A modified Ohm’s law is used to compute current density. Because of the relatively high plasma pressure, the Hall effect in the railgun plasma can be neglected [34].
To complete the plasma description, transport properties such as electrical and thermal conductivity, viscosity, and the equation of state are specified as functions of plasma temperature, species, and pressure, if required. For typical railgun conditions, radiation heat transfer in the plasma can be described in a diffusive Rosseland approximation [34]. That is, the heat flux is calculated via total effective thermal conductivity. The simulation of ablation is also an important part of the physics of the process included in the numerical model.

Over the past decade, MHD numerical models for railguns have evolved from 1-D and 2-D quasi-steady calculations [8,9] to 1-D unsteady calculations [10,11] and 2-D unsteady calculations [12-15]. These models are described below.

1.3. 1-D Computer Models.

The objective of the quasi-steady 1-D and 2-D models [8,9] was to achieve a qualitative understanding of plasma conditions in railguns. Several important simplifications were made to obtain approximate analytical solutions of the governing equations. The calculations neglected heat conduction and viscous losses and assumed a constant armature length. Some scaling equations were derived to predict mean values of flow variables within the arc. The 1-D unsteady MHD models were developed to understand how the plasma would respond to unsteady perturbations such as a change in current and arc mass addition. Batteh et al. obtained damped oscillations around quasi-steady solutions [10]. Under the set of 1-D approximations, the electromagnetic model
was reduced to the solution of a 1-D diffusion equation for the transverse component of the magnetic field in the bore, $B_\perp$. Unsteady and quasi-steady MHD models have been used in parametric analyses [10]. The 1-D models helped considerably in understanding some basic phenomena in the armature such as coupling between magnetic field diffusion and the plasma flow. Despite this, 2-D models were needed to study multidimensional phenomena and, in particular, ablation dynamics at the rail, surface and transport of ablation material in the plasma armature region.

1.4. 2-D Rail Plane Computer Models.

A 2-D model first evolved as a rail plane simulation [12-14]. This model implied infinitely wide and perfectly conducting rails. With this assumption, a 2-D diffusion equation for the transverse component of the magnetic field was solved. A uniform magnetic field across the bore was a boundary condition at the breech side. The rail plane models helped study the role of rail ablation products in the armature dynamics.

The same plasma flow pattern was observed in all rail plane simulations. In a frame connected to the projectile, near the rail, there is a strong plasma velocity in the $-y$ direction. In the center of the railgun bore, the plasma flows toward the projectile. Such a solution was expected to be obtained, since the ablated material appears to enter the domain at rest relative to the rails. According to the 2-D model, the magnetic pressure, and therefore the electromagnetic force, is uniform across the channel.
Therefore, the observed flow is driven mainly by the ablation process and viscous drag forces, but not the electromagnetic force. Figure 1.2 shows a sketch of a typical velocity vector field for the 2-D case. However, as this thesis will show, 3-D simulations have shown that the electromagnetic force is very non-uniform in x- and y-direction across the channel. This leads to significantly different plasma flow patterns obtained for a 3-D model. Despite the limitations of the 2-D models, these rail-to-rail MHD models were effective in broadening the knowledge about the physical factors influencing armature dynamics.

Frese [12] particularly studied how heat conduction to the rail influences ablation rate. He carried out two different sets of calculations. In the first model, all the heat flux at the wall was used to calculate ablation. In the second, a part of the total heat flux was assumed to be conducted into the rails. The results showed that heat conduction to the rail limits ablation rate and provides a more compact armature.

Keefer and Tipton [13] studied the influence of the projectile-bore interaction on the plasma flow. They obtained a primary separation by injecting a mass at the junction of the rails and the projectile. They concluded that projectile material eroded by the frictional interaction of the projectile with the bore could be responsible for the primary separation.

Boynton and Huerta [14] claimed to obtain a secondary arc formation and primary separation with a rail ablation simulation. They concluded that ablation is the primary factor that leads to development of secondary arcs. They also suggested that the ablation makes the armature longer which, in turn, initiates secondary arcs by an Ohmic heating
Figure 1.2. Flow by 2-D rail-to-rail MHD model.
instability. This argument is supported by experiments with non-ablating walls [5] which did not show any secondary arcs.

However, their simulation was very similar to that made by Frese [12] whose results did not show any secondaries. So, the question remains whether the predicted physical results obtained were influenced by the particular numerical technique used to solve the governing equations. The authors of [14] argue that their results are more reliable than those of Frese since they used an explicit rather than an implicit algorithm. However, based on research conducted by the author of this dissertation, non-physical numerical effects can appear in the explicit solution of the magnetic field diffusion equation, which has a singularity in non-conducting regions. Thus, the calculation of low-conductivity regions on the rear of the armature may lead to spurious solutions and, thus, to growth of numerically induced secondary arcs.

1.5. 2-D Insulator Plane Computer Model.

Incentive for an MHD simulation of the railgun insulator plane, i.e. plane passing through the center of the gun and perpendicular to the insulator, was provided by experimental data obtained at UTSI by Keefer et al. [16]. Spectroscopic measurements suggested a transverse flow carrying ablated rail material to the insulator walls. It was also interesting to study the cold boundary layers which form near non-conducting insulators.

Up until 1996, the only 2-D MHD calculations, in a plane containing the
insulators, were performed by Zatelepin and Kondrashov [15]. The main challenge and focus for these calculations was the development and evaluation of the electromagnetic model and, in particular, boundary conditions for the magnetic field. From these calculations in the insulator plane, the simulated magnetic field has two $x$ and $y$ components, which lie in the plane, while the electric field and current density each have one normal $z$-component.

The authors of [15] suggested an approximate electromagnetic model. A transverse component of the magnetic field and normal electric field were assumed to have a weak dependence upon the transverse $x$ coordinate. They were computed by solving a one-dimensional diffusion equation with parameters averaged across the channel. However, the current density was computed from Ohm's law using the local plasma conditions. The second (axial) component of the magnetic field was then calculated from the resulting current density solution by the Biot-Savart's law. The results have shown some predictable plasma flow features such as flow away from the projectile in the cold insulator boundary layers. Also, the armature current was split to small filaments, and an intensive secondary flow was generated due to a strong spatial nonuniformity of the electromagnetic force. No ablation was included in these computations.


There are several essential reasons why 3-D MHD modelling of the railgun arc
or plasma armature region is necessary. First, because the flow is so complex, there are no natural planes or axes of symmetry, and all cross-flow components of fluid velocity are comparable in magnitude. The electromagnetic field in the railgun bore is essentially and fundamentally three-dimensional. The approximate electromagnetic models adopted for the 2-D simulations produced results which were far from reality. The rails themselves were not included in the 2-D numerical models, and they were assumed to be perfect electrical and thermal conductors. The 3-D simulations have shown the vital role played by the rail current distribution in forming the time-dependent electromagnetic field. Therefore, the electromagnetic force distribution and, consequently, the plasma flow pattern obtained in the 2-D models cannot be reliable. For the plasma armature, the effects on plasma or arc motion of the spatial 3-D nonuniformities of electromagnetic forces and Ohmic heating are especially important to study.

The importance of 3-D electromagnetic effects for the solid armature railgun have been shown by the computer simulations of Keefer, Taylor and Crawford [17]. Three-dimensional eddy currents significantly influenced the value of total axial armature force depending on the railgun configuration.

Also, there are several important 3-D plasma phenomena which cannot be fully understood in the 1-D or 2-D simulations. Of particular interest is the plasma flow which results from nonuniform electromagnetic forces and the ablation process of both the rail and the insulator surfaces. Optical measurements of the plasma emission from within the bore [16] suggest that strong three-dimensional mixing occurs inside the plasma armature which transports rail material to the insulator surfaces.
To address these important issues, 3-D time-dependent numerical simulations of the plasma armature were needed. To provide these solutions became the objective of the present dissertation.

1.7. 3-D MHD Modelling.

1.7.1. 3-D MHD Modelling at UTSI.

A 3-D time-dependent code MHD code MAP3 (MHD Arc Plasma) was developed at UTSI based on a 3-D Navier-Stokes code developed in part by the author at the Moscow High Temperature Institute (IVTAN) [18]. The IVTAN code was extended by the author to provide a numerical simulation for the railgun plasma armature. A novel efficient method for computing the 3-D time-dependent electromagnetic field was incorporated into the IVTAN code to allow full 3-D unsteady MHD simulations of the plasma armature flow in the railgun.

The first full-bore 3-D MHD simulation of the plasma armature railgun using MAP3 was performed and reported in [19]. It forms the basis of this dissertation. These results are discussed in Chapter V.

1.7.2. Other 3-D MHD Modelling.

A 3-D analysis of EML plasma armature was performed by Esposito et al. using
an equivalent network approach [20]. The electromagnetic, mechanical, and thermodynamic governing equations are transformed into three separate equivalent electric networks. A formal analogy is assumed between fluid-mechanical and electric quantities. Such a methodology implies a linear solution of non-linear equations of motion. Therefore, some important dynamic phenomena such as shock waves cannot be described. A 3-D numerical simulation of a plasma armature railgun was obtained using this method for the case of isothermal plasma, without ablation. To obtain a solution for the electromagnetic field, somewhat arbitrary assumptions about the current distribution were made. In particular, the entire conducting railgun region is divided into subzones in which some of the current density components are set to zero. The results showed a plasma flow similar to that obtained by MAP3 in [19]. However, no physical explanation was given for the observed plasma flow features or characteristics. It is an open question whether such results are caused by the peculiar numerical technique adopted in the equivalent electric network method, or by model assumptions.


A description of the MAP3 electromagnetic solver is given in Chapter II. The results of benchmarking MAP3 against the 3-D finite-element electromagnetic code MEGA [21-22] are given at the end of Chapter II. Chapter III addresses the numerical procedures adopted in MAP3. The results of the 3-D MHD plasma railgun simulation are discussed in Chapter IV. Summary is given in Chapter V.
II. MAP3 Electromagnetic Solver.


The time-dependent Maxwell's equations for magnetic induction field $B$ (T) and electric field $E$ (V/m) are the governing equations:

\[
\frac{\partial B}{\partial t} = -\nabla \times E ;
\]

\[
\nabla \times B = \mu_0 \left( J + \varepsilon_0 \frac{\partial E}{\partial t} \right) ;
\]

\[
\nabla \cdot B = 0 ;
\]

\[
\frac{\partial \rho_e}{\partial t} + \nabla \cdot J = 0 .
\]

The first two equations of (2.1) are Faraday's law and Ampere's law, respectively. The last equation of (2.1) represents conservation of charge or current continuity equation where $\rho_e$ is the charge density and $J$ is the current density (A/m²). The dependence of current density $J$ upon the electromagnetic field is specified by a generalized Ohm's law where the Hall's effect has been neglected for high-pressure ($10^2$-$10^3$ atm) railgun conditions [34]:

\[
J = \sigma (E + v \times B) ,
\]

where $\sigma$ is electrical conductivity (Ωm) and $v$ is velocity vector (m/s).
The system of Equations (2.1) and (2.2) is simplified in the MHD approximation. The displacement current \( \varepsilon_0 \frac{\partial E}{\partial t} \) is neglected in Ampere’s law since it is much smaller (order of \( V^2/c^2 \), [34]) than the conduction current \( J \). Also, the plasma is assumed to be charge-neutral. That is, the charge density \( \rho_e \) is set to zero, which is a reasonable approximation for the high-pressure, low-temperature (\( \approx 2 \cdot 10^6 \) K) railgun plasma [34]. Thus, the system of equations to be solved is:

\[
\begin{align*}
\frac{\partial B}{\partial t} &= -\nabla \times E ; \\
\nabla \times B &= \mu_0 J ; \\
J &= \sigma (E + v \times B) ; \\
\nabla \cdot J &= 0 . \\
\nabla \cdot B &= 0 .
\end{align*}
\]  

(2.3)

Two approaches may be used to solve the system of equations (2.3). It is possible to derive a single equation for the magnetic field by combining the first three equations of (2.3). Thus, the following equation can be obtained:

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B) - \nabla \times \left( \frac{1}{\mu_0 \sigma} \nabla \times B \right) .
\]  

(2.4)

For the constant \( \sigma \), it can be simplified to:
\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \frac{1}{\mu_0 \sigma} \nabla^2 B,
\]

which is a diffusion type of equation for vector \( B \). However, for the 3-D railgun problem, boundary conditions for the magnetic field are not known in advance. Also, Equation (2.4) has a singularity in non-conducting regions, and finally, solving Equation (2.4) does not ensure that the magnetic field is divergence-free. This may lead to a non-physical solution while advancing a numerical calculation. A well known option to overcome these difficulties is to solve the governing equations in transformed variables. This procedure is widely used to compute 2-D eddy current problems, and it is the basic method adopted in the 3-D finite-element code MEGA [21].

2.2. Transformation of Variables.

A magnetic vector potential \( A \) is introduced as

\[
B = \nabla \times A.
\]  \hspace{1cm} (2.5)

The magnetic field defined in this way is automatically divergence-free. The expression for the electric field may then be derived from the Faraday's law as follows:

\[
E = -\nabla \varphi - \frac{\partial A}{\partial t},
\]  \hspace{1cm} (2.6)

where \( \varphi \) is the electric scalar potential. By substituting Equations (2.5) and (2.6) in Ampere's law and in the current continuity equation, two equations are obtained for the potentials:
\[ \sigma \mu_0 \left( \frac{\partial A}{\partial t} - \nu \times \nabla \times A \right) - \nabla^2 A = -\sigma \mu_0 \nabla \phi ; \]  
(2.7)

\[ \nabla \cdot \sigma (\nabla \phi) = \nabla \cdot \sigma \left( \nu \times \nabla \times A - \frac{\partial A}{\partial t} \right) . \]  
(2.8)

To insure a unique \( A \), a divergence of the magnetic vector potential should be specified.

In deriving Equation (2.7), a Coulomb gauge is used:

\[ \nabla \cdot A = 0 . \]  
(2.9)

2.3. MEGA Formulation.

Equations (2.7) and (2.8) are only solved in the conducting regions to save computational resources. In nonconducting regions, a scalar \( \psi \) is defined from \( B = \nabla \psi \).

Substitution into the equation

\[ \nabla \cdot B = 0 \]

gives a Laplace equation for \( \psi \) to be solved. The idea behind this approach is that, instead of three Laplace equations (2.7) for the three components of \( A \) in the nonconducting regions, only one equation for \( \psi \) is solved. At the conductor boundary, a set of interface conditions must be implemented to couple solutions from the different regions. Thus, a so-called \( A-\phi-\psi \) formulation was first used in MEGA by Rodger et al. to obtain the solution for the stationary armature railgun problem (\( v = 0 \)) [21]. This solution was obtained with the same conductivity in the rails and in the armature. In an
attempt to simplify the solution procedure for such a problem, authors of [21] proposed to let $\nabla \varphi$ be zero everywhere in the governing equations. Thus, an $A-\psi$ formulation was obtained [22]. In this case for constant conductivity, Equation (2.8) is satisfied automatically if the Coulomb gauge is satisfied. Therefore, only Equation (2.7) must be solved. The gauging procedure is the same for both $A-\varphi-\psi$ and $A-\psi$ MEGA formulations.

A numerical constraint

$$A_n = 0$$

(2.10)

is specified on the conductor boundary.

In MEGA, the constant value of the electric potential over the current source surface is calculated and defined by the specified current. However, if the conductor with the current source surface is moving, this is no longer a correct boundary condition. Therefore, the authors of MEGA adopted the $A-\psi$ formulation for the moving conductor problem ($\nu \neq 0$). In such a case, the current continuity equation (2.8) is not directly solved, and the current in the solution is no longer conserved.

2.4. MAP3 Formulation.

The choice of the MAP3 formulation was dictated by the requirement to compute a typical 3-D MHD phenomena. For example, in the EML plasma the conductivity distribution is nonuniform because of plasma motion and MHD interaction. Consequently, the boundary between the conducting and nonconducting regions changes in time and usually has a complex shape. Due to secondary flows, all components of the
Table 2.1. Summary of essential differences between MAPS and MEGA.

<table>
<thead>
<tr>
<th>MEGA</th>
<th>MAPS</th>
</tr>
</thead>
<tbody>
<tr>
<td>only spatially constant electrical</td>
<td>solves problems with a nonuniform</td>
</tr>
<tr>
<td>conductivity is solved;</td>
<td>conductivity distribution;</td>
</tr>
<tr>
<td>does not conserve the current</td>
<td>conserves the current ($\nabla \cdot J = 0$);</td>
</tr>
<tr>
<td>($\nabla \cdot J \neq 0$);</td>
<td>a complex-shape conductor boundary</td>
</tr>
<tr>
<td>only a fixed-shape conductor boundary</td>
<td>that changes in time is computed;</td>
</tr>
<tr>
<td>is possible;</td>
<td>a full 3-D MHD code (both flow and</td>
</tr>
<tr>
<td></td>
<td>electromagnetics solvers).</td>
</tr>
</tbody>
</table>

velocity vector are typically non-zero. For the code to be efficient, it is desirable that all possible velocity and conductivity distributions be computed with the same numerical scheme and boundary conditions.

The author has significantly modified the method adopted in the 3-D finite-element electromagnetic code MEGA [21,22] to develop the efficient Maxwell’s solver MAP3 for 3-D MHD calculations (Table 2.1). The MEGA code did not permit calculation of the moving armature region with a nonuniform conductivity, and it did not conserve the current. Also, the method adopted in MEGA would be difficult to apply if the conducting region boundary has a complex shape (see below). The new MAP3 code was very effective in overcoming these problems.

In the MAP3 formulation, Equations (2.7) and (2.8) for $A$ and $\varphi$ are solved in
conducting regions. The difference between MAP3 and MEGA is that, instead of a scalar magnetic field, \( A \) is computed from Equation (2.7) in non-conducting regions as well. This eliminates difficulties in implementing coupled boundary conditions between \( A \) and \( \psi \) on the conductor-nonconductor interface when it moves and has a complex shape. Also, the gauge procedure is significantly simplified in comparison with MEGA. In MAP3, the uniqueness of \( A \) is enforced by imposing \( \nabla \cdot A = 0 \) on the boundaries of the computational domain. Thus, no additional conditions are required for \( A \) on the conductor boundary. By contrast, for schemes [21] and [22] a constraint \( A \cdot n = 0 \) on the inside of the conductor surfaces has to be satisfied as a part of the gauge procedure. Such a constraint is difficult to implement when the conductor boundary has a complex shape [23].

In the MAP3 method, the normal derivative of \( \varphi \) is used as a boundary condition on the current source surface. Unlike the procedure used in MEGA, this boundary condition allows the computation of \( \varphi \) inside a moving conductor with a current source. Calculation of the electric scalar potential from the current continuity equation ensures current conservation in the solution. It must be mentioned that in MAP3 plasma armature simulations described in Chapter 4, the rail with a current source is stationary. However, in some applications it is preferable to formulate the problem so that the conductor with a current source is moving. A formulation of this type was used for the MEGA and MAP3 benchmarking.

For the Maxwell solver, two schemes were developed by the author. Scheme A was derived on a non-staggered grid by a finite-difference technique and is of first-order
accuracy in space. Scheme B employed a staggered grid and control volume method and is of second-order accuracy. Upwind differencing was used to stabilize the numerical scheme in the moving conductor case. This common CFD procedure was implemented in the method for convected components of the magnetic vector potential in Scheme A, and for components of the magnetic field in Scheme B.

The results for a 2-D railgun problem were compared between MAP3 and MEGA by Taylor [24]. For the 2-D railgun, there is a known analytical expression for the armature force. For a stationary armature, both codes underpredicted the true armature force: MAP3, by 1.6%, and MEGA, by 2.2%. For the moving armature, both MEGA and MAP3 underpredicted the armature force by 3.5%. The MAP3 solution appears to be more accurate than MEGA since it does not predict anomalous current densities. See Reference [24] for details.

The newly developed schemes were tested on a 3-D conventional railgun problem for both stationary and moving armatures. The moving armature problem was simulated by fixing the computational domain in the armature and moving the rail in the opposite direction. The results were compared with the solution obtained by the 3-D finite-element code MEGA using the $A$-$\psi$ formulation. There was good agreement between MAP3 and MEGA in the stationary armature case. The discrepancy between MAP3 and MEGA observed in the moving armature problem is believed to be due to differences in current conservation between the two codes. The staggered grid version of MAP3 eliminates some unrealistic numerical effects and provides more sensible results from a physical point of view.
In deriving Equation (2.7), the Coulomb gauge $\nabla \cdot A = 0$ was used. It is important to notice that, formally, solving Equations (2.7) and (2.8) together implies

$$\nabla^2 (\nabla \cdot A) = 0 .$$  \hspace{1cm} (2.11)

It is well known that a solution of the Laplace equation with homogeneous Dirichlet boundary conditions is zero everywhere. Therefore, to have a divergence-free $A$ inside the computational domain, it is enough to impose $\nabla \cdot A = 0$ on the boundaries. Equations for $\phi$ and $A$ are coupled through source terms on the right-hand side of Equations (2.7) and (2.8). Thus, an intermediate set of iterations on each time step is required to ensure that the Equation (2.9) is satisfied inside the domain. To obtain the values of $\phi^{n+1}$, $A^{n+1}$, on the next $n+1$ time level from the known $\phi^n$ and $A^n$, a solution $\phi^{m,n+1}$ and $A^{m,n+1}$ on the $m^{th}$ intermediate iteration is calculated. To start the iterative process, $A^{0,n+1}$ and $\phi^{0,n+1}$ are set to

$$\phi^{0,n+1} = \phi^n ;$$

$$A^{0,n+1} = A^n .$$
2.6. MAP3 Discretization Schemes.

When discretizing the governing equations, it is desirable to develop a method which ensures conservation of current. This property is important because integral characteristics such as force and Ohmic losses depend strongly upon the total current in the system. Two numerical schemes, A and B, have been developed. They use the finite-difference and finite-volume techniques, respectively, on a cartesian grid, to obtain the discretized form of Equations (2.7) and (2.8). Discretized equations contain only the y-component of velocity vector which was used in the 3-D test computations described in Section 2.7. The solution for the plasma armature railgun was obtained with all the components of velocity in the equations. Computational solutions are obtained on a non-staggered grid for Scheme A, and on a staggered grid for Scheme B. The domain is partitioned into a collection of grid cells with a grid node at the center of a cell. The location of cell faces is determined by half-grid points whose coordinates are an average of the corresponding nodes.

2.6.1. Scheme A, Non-Staggered Grid.

In this method, all dependent variables are defined at one spatial location - at the grid node. A linear interpolation function is assumed between the neighboring cells for the magnetic and electric potentials. The finite-difference equations are derived via the standard Taylor series expansion. A discretized form of Equation (2.7) for cell \((i,j,k)\) is
as follows:

\[
\mu_0 J_{x,ijk}^{m,n+1} = -\mathcal{L} A_{x,ijk}^{m,n+1}, \quad J_{x,ijk}^{m,n+1} = \sigma_{ijk} \Phi_{x,ijk}^{m,n+1} + G_{x,ijk}^{m,n+1} + F_{x,ijk}^{m-1,n+1};
\]

\[
\mu_0 J_{y,ijk}^{m,n+1} = -\mathcal{L} A_{y,ijk}^{m,n+1}, \quad J_{y,ijk}^{m,n+1} = \sigma_{ijk} \delta A_{y,ijk}^{m,n+1} + G_{y,ijk}^{m,n+1} + F_{y,ijk}^{m-1,n+1};
\]

\[
\mu_0 J_{z,ijk}^{m,n+1} = -\mathcal{L} A_{z,ijk}^{m,n+1}, \quad J_{z,ijk}^{m,n+1} = \sigma_{ijk} \delta A_{z,ijk}^{m,n+1} + G_{z,ijk}^{m,n+1} + F_{z,ijk}^{m-1,n+1};
\]

In these equations, the upper index \( n \) represents the time level; the upper index \( m \) represents the intermediate iteration level for a particular \( n \). The lower indices \( i, j, \) and \( k \) represent the grid numbering along the \( x, y, \) and \( z \) axes. \( \mathcal{L} \) is a standard finite-difference analogue of \( \nabla^2 \) of 2nd-order accuracy evaluated at the grid node (see Appendix A). Terms \( G \) and \( F \) represent contributions to Ohm's law from magnetic vector and electric potential,

\[
F_{x,ijk}^{m,n+1} = -\sigma_{ijk} l_{f,x} \Phi_{x,ijk}^{m,n+1}; \quad G_{x,ijk}^{m,n+1} = -\sigma_{ijk} \delta A_{x,ijk}^{m,n+1} + S_{x,ijk}^{m,n+1};
\]

\[
F_{y,ijk}^{m,n+1} = -\sigma_{ijk} l_{f,y} \Phi_{y,ijk}^{m,n+1}; \quad G_{y,ijk}^{m,n+1} = -\sigma_{ijk} \delta A_{y,ijk}^{m,n+1};
\]

\[
F_{z,ijk}^{m,n+1} = -\sigma_{ijk} l_{f,z} \Phi_{z,ijk}^{m,n+1}; \quad G_{z,ijk}^{m,n+1} = -\sigma_{ijk} \delta A_{z,ijk}^{m,n+1} + S_{z,ijk}^{m,n+1}.
\]

The induced current term \( \sigma v \times B \) and its finite-difference analogue \( S \) are divided in the equation for the particular component of \( A \), into two parts. One part involves the considered component \( A_x \) and \( A_z \) (if only \( v_y \neq 0 \)) and is calculated implicitly on the \( m^{th} \) iteration using first-order upwind differencing. The other part involves the remaining component \( A_y \) and is calculated explicitly with central differencing on \( (m-1)^{th} \) iteration:
Use of implicit and upwind differencing for the convected components of \( A \) \( (A_x \text{ and } A_z) \) provides a stable solution.

In the above equations, \( l_f \), \( l_c \) and \( l_u \) represent finite-difference operators with forward, centered, and upwind differencing, respectively (see Appendix A). The operator \( \delta \) is a finite-difference analogue of the time derivative with 1st-order accuracy:

\[
\delta A_{m,n+1} = \frac{A_{m,n+1} - A_n}{\Delta t}.
\]

where \( \Delta t \) is time step.

The equation for electric scalar potential is a discretized form of the current continuity equation:

\[
\Delta_x F_{m,n+1} + \Delta_y F_{m,n+1} + \Delta_z F_{m,n+1} = 0,
\]

where \( \Delta \)-operators enforce current conservation in a finite-difference form at node \((i,j,k)\):

\[
\Delta_x f_{ijk} = \frac{f_{ijk} - f_{i-1,jk}}{x_{i+\frac{1}{2},jk} - x_{i-\frac{1}{2},jk}};
\]

The calculation of \( F \) in Ohm’s law with \( l_f \) operators makes Scheme A of 1st-order accuracy.
An algorithm of 2nd-order accuracy is used to calculate the magnetic field \( \mathbf{B} \) from the solution for magnetic vector potential \( \mathbf{A} \). The nodal value of the Lorentz force \( \mathbf{J} \times \mathbf{B} \) is calculated using the nodal values of current density and magnetic field. The integral force over the conducting arc is calculated by taking a product of its cell value and the cell volume and performing a sum over the computational domain.

2.6.2. Scheme B, Staggered Grid.

In this method, the computational solution is obtained on a staggered grid; that is, dependent variables are evaluated at different space locations. The electric potential is still specified at the grid nodes, but the components of current density and magnetic vector potential are defined on separate cell faces. The magnetic field is calculated on the edges of the cell. The cell layout is shown in Figure 2.1. This alignment makes the grid suitable for a control volume discretization. Positioning of \( \mathbf{E} \) and \( \mathbf{B} \) field components over the grid cell was formulated by Yee [25] in his finite-difference time domain (FDTD) method for full Maxwell's equations wave analysis. The advantage of the method is that the discretized solution obeys the governing equations in their integral form over appropriate volumes and contours. It is worth noting that other staggered grid cells have been developed for solving full Maxwell's equations. The Yee cell, however, is the most convenient and simple way of discretizing in rectangular coordinates.

In discretizing Equation (2.7), finite-difference expressions centered at the cell faces are used, which results in 2nd-order spatial accuracy:
Figure 2.1. Staggered grid cell in MAP3.
The $\mathcal{L}$ operators are now different from those in Scheme A as they are here evaluated at cell faces rather than at the grid nodes (see Appendix A). The terms $G$ and $F$ undergo the following changes:

$$J_{x,i,j,k}^{m,n+1} = G_{x,i,j,k}^{m,n+1} + F_{x,i,j,k}^{m-1,n+1}, \quad \mu_0 J_{x,i,j,k}^{m,n+1} = -\mathcal{L}A_{x,i,j,k}^{m,n+1},$$

$$J_{y,i,j,k}^{m,n+1} = G_{y,i,j,k}^{m,n+1} + F_{y,i,j,k}^{m-1,n+1}, \quad \mu_0 J_{y,i,j,k}^{m,n+1} = -\mathcal{L}A_{y,i,j,k}^{m,n+1},$$

$$J_{z,i,j,k}^{m,n+1} = G_{z,i,j,k}^{m,n+1} + F_{z,i,j,k}^{m-1,n+1}, \quad \mu_0 J_{z,i,j,k}^{m,n+1} = -\mathcal{L}A_{z,i,j,k}^{m,n+1}.$$

The upwinding procedure on the staggered grid for the $\sigma (v \times B)$ term leads to upwinding of the convected components of the magnetic field rather than components of the magnetic vector potential, as in Scheme A. The induced current term $\sigma v \times B$ in the finite-difference equations is then written as

$$F_{x,i,j,k}^{m,n+1} = -\sigma_{i,j,k} l_{f,x} \varphi_{ijk}^{m,n+1}, \quad G_{x,i,j,k}^{m,n+1} = -\sigma_{i,j,k} l_{f,x} \delta A_{x,i,j,k}^{m,n+1} + S_{x,i,j,k}^{m,n+1},$$

$$F_{y,i,j,k}^{m,n+1} = -\sigma_{i,j,k} l_{f,y} \varphi_{ijk}^{m,n+1}, \quad G_{y,i,j,k}^{m,n+1} = -\sigma_{i,j,k} l_{f,y} \delta A_{y,i,j,k}^{m,n+1},$$

$$F_{z,i,j,k}^{m,n+1} = -\sigma_{i,j,k} l_{f,z} \varphi_{ijk}^{m,n+1}, \quad G_{z,i,j,k}^{m,n+1} = -\sigma_{i,j,k} l_{f,z} \delta A_{z,i,j,k}^{m,n+1} + S_{z,i,j,k}^{m,n+1}.$$
The cell face values of conductivity and \((\sigma v_y)\) are found by averaging the corresponding neighboring nodes. The discretized form of Equation (2.8) is obtained by a finite-volume technique applied to the grid cell:

\[
\Delta_x F_{x,i+\frac{1}{2},jk}^{m,n+1} + \Delta_y F_{y,j+\frac{1}{2},jk}^{m,n+1} + \Delta_z F_{z,k+\frac{1}{2},jk}^{m,n+1} = -\Delta_x G_{x,i+\frac{1}{2},jk}^{m,n+1} - \Delta_y G_{y,j+\frac{1}{2},jk}^{m,n+1} - \Delta_z G_{z,k+\frac{1}{2},jk}^{m,n+1}.
\]

Unlike in Scheme A, here current conservation is enforced in the integral of the discretized equation over the cell volume \((i,j,k)\). Components of the magnetic field are evaluated at the edges of the grid cell with which they are aligned:

\[
B_{x,i+\frac{1}{2},j,k}^{m,n+1} = l_{f,x} A_{x,i+\frac{1}{2},j,k}^{m,n+1} - l_{f,z} A_{z,i+\frac{1}{2},j,k}^{m,n+1},
\]

\[
B_{y,i+\frac{1}{2},j,k}^{m,n+1} = l_{f,z} A_{z,i+\frac{1}{2},j,k}^{m,n+1} - l_{f,x} A_{x,i+\frac{1}{2},j,k}^{m,n+1},
\]

\[
B_{z,i+\frac{1}{2},j,k}^{m,n+1} = l_{f,y} A_{y,i+\frac{1}{2},j,k}^{m,n+1} - l_{f,x} A_{x,i+\frac{1}{2},j,k}^{m,n+1}.
\]

To find the electromagnetic force, the corresponding components of current density and magnetic field are averaged and then multiplied to form the node value of \((J \times B)\). The integral force is calculated as in Scheme A.
2.7. 3-D Benchmarking Test.

To benchmark the developed codes, a 3-D time-dependent solution was calculated for the railgun problem using a simple square bore, copper rail and copper armature. For such a problem, no closed-form analytical solution exists. The MAP3 solution was compared with the results calculated by the 3-D finite-element electromagnetic code MEGA with $A$-$\psi$ formulation [22]. The simulated model consisted of a 1-cm square bore railgun with a 0.5-cm long solid armature, Figure 2.2. Due to symmetry, only one quarter of the cross section was computed. The outer boundaries in the air are located 4 cm away from the axis of the model. At this distance, a further increase in computational domain was found not to influence the solution inside the bore. In order to resolve regions with high gradients, the grid is nonuniform, and has 34 points in the axial direction and $20 \times 30$ points in the railgun cross section.

A problem with the armature moving in a positive $y$-direction was simulated by moving the rail in the opposite direction. Cases with the stationary rail and a moving rail with a negative velocity $v_y = -500$ m/s were computed and compared. The test may be considered a very hard benchmark because in the time-dependent solution all the components of current density and magnetic field are essential.
Figure 2.2. Computational grid, rail-to-rail plane.
2.8. Boundary Conditions.

The equations for the magnetic vector potential are solved over a closed computational domain, so on each boundary all components of \( \mathbf{A} \) need to be specified. In contrast, the equation for electric potential is solved only within the conductor. Thus, boundary conditions for \( \phi \) are required only on interfaces between the conductor and nonconducting regions and in the areas where the conductor crosses the boundaries.

In general, to derive the boundary conditions, physical and numerical conditions are used. The boundaries of the computational domain are of two types:

I -- tangent magnetic flux, \( B_n = 0, J_r = 0 \), and

II -- normal magnetic flux, \( B_r = 0, J_n = 0 \).

In the computational domain, only the rail-to-rail symmetry plane is a type II boundary. On all other boundaries the tangent magnetic flux is specified. To define the conditions for all the components of \( \mathbf{A} \) on the boundary of type I and to enforce the Coulomb gauge, a numerical constraint is imposed:

\[
\nabla \cdot \mathbf{A} = 0
\]  
(2.12)

A combination of (2.12) with a tangent magnetic flux condition leads to the following set:

\[
\begin{align*}
\frac{\partial A_n}{\partial n} &= 0 \\
A_t &= 0
\end{align*}
\]

On a boundary of type II, physical conditions result in
so the additional numerical constraint (2.12) is not needed. Normal magnetic flux boundary conditions imply

\[
\begin{align*}
  A_n &= 0 \\
  \frac{\partial \phi}{\partial n} &= 0 \\
  \frac{\partial A}{\partial n} &= 0
\end{align*}
\]

The gauging procedure, however, is not violated. The relation (2.13) can be obtained by computing the entire domain through the rail-to-rail symmetry plane. In this case all boundaries of the computational domain would be of type I. If the numerical constraint (2.12) is imposed on the boundaries, a divergence-free \( A \) inside the computational domain is obtained. The latter obviously satisfies (2.13) everywhere in the domain, including the rail-to-rail symmetry plane.

To derive boundary conditions for the electric scalar potential, physical constraints on the current density are used. A \( J_n = 0 \) condition is applied on all insulator surfaces: on the conductor boundaries, the normal magnetic flux boundary, and on the muzzle end of the rail. This condition is used in the equations for both \( A \) and \( \phi \) and is incorporated in the resulting system of linear equations by explicitly setting \( G = -F \) at the necessary points.

The current source condition for the total current \( I_o \) is specified on the breech end of the rail:
\[ I_0 = \int J_n dS. \]

In the integral, \( dS \) is an element of the rail cross-sectional area. Also, to ensure the uniqueness of \( \varphi \), a zero potential is assigned on the insulator-insulator symmetry plane, which is consistent with the \( J_z = 0 \) condition on the tangent magnetic flux boundary.

The equation for total current gives a condition for \( \varphi \) in terms of a normal derivative \( \varphi'^{(m)} \) which is a constant over the current source surface:

\[
\varphi'^{(m)} = \frac{\partial q^{m,n-1}}{\partial n} = - \frac{I_0 + \int \sigma \delta A^{m-1,n+1} dS}{\int \sigma dS} \tag{2.14}
\]

Here, the integral is taken over the inlet surface of the rail. This is equivalent to specifying a uniform source current density which is modified in the solution either by transient processes or by the movement of the conductor. With (2.14) as a boundary condition, the solution has a constant value of \( \varphi \) over the inlet surface in the stationary case. In the moving rail case, the calculated potential varies depending on the location. It is known that the potential is not constant on the current source surface of the moving conductor. Therefore, the boundary condition given by (2.14) demonstrates a correct physical behavior and can be used for both the stationary and moving rail.

During an intermediate set of iterations, the value of \( \varphi'^{(m)} \) is constantly updated according to Equation (2.14). This process is equivalent to solving the following equation:

\[ \varphi' = f(\varphi') \tag{2.15} \]
where \( f \) is some implicit function representing the right-hand side of Equation (2.14). To increase the convergence rate of the iterative process, a Newton's method is applied to Equation (2.15):

\[
\varphi^{(m+1)} = \varphi^{(m)} - \frac{f(\varphi^{(m)}) - \varphi^{(m)}}{f(\varphi^{(m)}) - f(\varphi^{(m-1)})} - 1
\]

For the moving rail problem, the validity of the \( B_n = 0 \) condition on the left boundary of Figure 2.2 should be discussed. At first, it seems that it cannot be applied since \( B_n \) that has diffused into the rail is convected to the left boundary along with the rail. However, the governing equations in the moving rail acquire properties of a mixed hyperbolic-elliptic type. Thus, the influence of the left boundary on the solution within the rail drops as \( 1/\text{Re}_m \), where \( \text{Re}_m = \sigma V L^2 \mu_0 \) is a corresponding magnetic Reynolds number. For the moving rail problem, simple calculations show that the value of \( \text{Re}_m \) is equal to \( \sim 140 \), so \( B_n = 0 \) does not significantly influence the solution inside the rail. Outside the rail, the tangent magnetic flux condition is still important since there the equations for magnetic vector potential are of the elliptic type.


After the discretization, four systems of linear equations are obtained - three for the components of \( A \) and one for \( \varphi \). The resulting coefficient matrices of these linear systems are, in general, nonsymmetric because of the non-equal spacing. Instead of
solving one matrix for all equations, the matrices are solved separately in a sequence. A separate solution of the matrices accelerates the convergence of the iterative process. The matrices for the components of A are solved first, next the current source boundary condition is updated from Equation (2.14), and, finally, the system $\varphi$ is solved. The algorithm is repeated until convergence is achieved.

The conjugate gradient - squared (CGS) method [26], along with a modified incomplete Cholesky factorization [27] as a preconditioner, is used as an iterative solver.

2.10. Computational Test Results.

The computation was performed for stationary and moving armature problems by both Method A, the simple grid, and Method B, the staggered grid. The results for the local electromagnetic field distribution and the total integrated axial force were compared with results obtained using MEGA. The total axial force is a sum of force exerted in both the rail and the armature. For the comparison, the dominant components of magnetic field and current density in the armature, i.e. $B_z$ and $J_z$, were chosen. Axial distributions of these variables were taken along the line passing through the grid point in the armature corner adjacent to the rail and air (Figure 2.3). This region of the armature has the highest gradients in the solution.

For stationary rail calculations, the corresponding graphs at time $t_1=10^{-5}$ s and $t_2=6 \cdot 10^{-4}$ s are shown in Figures 2.4 - 2.7. Figure 2.8 shows the time dependence of the total axial force. At the earliest times, the magnetic field is concentrated on the back
Figure 2.3. Computational grid in the cross-section; "+" marks the axial line along which the comparison is made.

Figure 2.4. $B_z$ distribution, stationary problem; $t=10^{-5}$ s.
Figure 2.5. $J_z$ distribution, stationary problem; $t=10^5$ s.

Figure 2.6. $B_x$ distribution, stationary problem; $t=6\cdot10^4$ s.
Figure 2.7. $J_z$ distribution, stationary problem; $t=6\cdot10^4$ s.

Figure 2.8. Total axial force, stationary problem.
surface of the armature. It is mainly the strong axial gradient of the magnetic field that results in a high current density in this region. At the later time, the magnetic field diffuses considerably into the armature. This allows a more uniform distribution of current density. At this time, the axial total and armature forces have nearly reached their steady state values, while the magnetic field continues to diffuse into the conducting structure. At the early time, both codes capture a strong magnetic field gradient.

The results of these calculations indicate that the region of the largest difference between MAP3 and MEGA is located at the rear of the armature near the rail. The MAP3 calculation gives somewhat higher absolute values of current density than MEGA at the first few points in the armature. Elsewhere, the two codes show a satisfactory agreement in the magnetic field and the current density. For resolving the total force and armature force, the Method B provides better agreement with MEGA than the Method A. For both MAP3 methods, however, the discrepancy with MEGA is less than 5%.

The results for the moving armature case are shown in Figures 2.9 - 2.14. At early times, the magnetic field has not yet diffused deep into the rail, and the velocity effects are negligible. Therefore, the axial distributions of the field variables at $t_1=10^{-5}$ s are similar to those in the stationary case at the same time. At later time $t_2=2\cdot10^4$ s, the eddy currents induced by the rail movement become substantial. They cause current to concentrate on the inside surfaces of the rail and armature. This is seen on plots of the current density vector at the rail-to-rail plane passing through the axis of the model (Figures 2.16 - 2.18). A feature noticeable in the MAP3-A and MEGA solutions but absent in the MAP3-B solution is a reverse current in the rail at the rear of the armature.
Figure 2.9. $B_z$ distribution, moving problem; $t = 10^{-5}$ s.

Figure 2.10. $J_z$ distribution, moving problem; $t = 10^{-5}$ s.
Figure 2.11. $B_x$ distribution, moving problem; $t=2\cdot10^4$ s.

Figure 2.12. $J_z$ distribution, moving problem: $t=2\cdot10^4$ s.
Figure 2.13. Armature force, moving problem.

Figure 2.14. Total force, moving problem.
Figure 2.15. Current density vector in the railgun cross-section by MAP3, moving problem; $t=2\times10^{-4}$ s.
Figure 2.16. Current density vector in the rail-to-rail symmetry plane by MAP3-A, moving problem; $t=2 \cdot 10^{-4}$ s.

Figure 2.17. Current density vector in the rail-to-rail symmetry plane by MAP3-B, moving problem; $t=2 \cdot 10^{-4}$ s.
Figure 2.18. Current density vector in the rail-to-rail symmetry plane by MEGA, moving problem; $t=2\cdot10^4$ s.
This appears to have an entirely numerical origin. Similar reverse current is also observed in the 2-D rail-to-rail MEGA simulation [24] where its non-physical nature is apparent. The method of enforcing the current conservation on the staggered grid is assumed to be responsible for the lack of such reverse current in the solution obtained by MAP3-B, the staggered grid version of MAP3.

The MEGA solution may have up to 50% error locally in the total current along the rail and the armature for the moving problem. By contrast, in MAP3 the electric scalar potential is computed directly by solving the current continuity equation and, therefore, the current conservation is enforced within machine accuracy. It appears, however, that only the staggered grid formulation can eliminate the entirely non-physical current effects by enforcing current conservation over the grid cell volume. Oscillations of the magnetic field in the MEGA results (Figure 2.11) may be caused either by the non-physical currents or by resulting non-monotonic calculation of the induced current term $v \times B$.

The axial component of the Lorentz force is defined as

$$F_y = J_z B_x - J_x B_z .$$

The main components of the current density and the magnetic field in the bore contribute to the $J_z B_x$ part of the force. As mentioned above, at steady state, the absolute value of $B_z$ at the rear of the armature is higher in the moving rail problem than in the stationary rail one. Thus, the $J_z B_z$ part of the armature axial force increases. However, the eddy currents induced in the railgun cross-section contribute negatively to the force (in the $-J_x B_z$ part). Indeed, $B_z$ is negative in the simulated quarter of the railgun cross section.
The same is true for \( J_z \) in the observed pattern of eddy currents in the armature (Figure 2.15). As the magnetic field diffuses into the armature, the absolute value of \( J_z B_z \) increases. This is a possible explanation for the decrease in armature force observed as the steady state is approached in the MAP3 solutions (Figure 2.13). The difference in the armature force between MAP3 and MEGA calculations is about 10%, and in total force, about 4%.

A solution obtained on a staggered grid by MAP3-B in the moving rail problem has a smaller value of the armature force than in the stationary case. However, with MEGA and MAP3-A, the armature force increases. It seems more reasonable to accept the MAP3-B result. Because, first, the staggered grid solution eliminates purely numerical effects such as reverse current flow in the rail at the rear of the armature. Second, the eddy currents induced by the rail movement are clearly a source of losses in the system. The eddy currents should act against the cause of their origin - the rail movement. Therefore, the decrease of the axial force and increase of the rail force are reasonable to expect if the rail is moving in the negative axial direction. The left boundary of the computational domain is located where the magnetic field has not fully diffused into the rail. Thus, only part of the total axial rail force is calculated from the solution. On the other hand, all the available armature force is calculated. The decrease in the armature force observed in the MAP3-B solution is fairly small (3%). However, this effect may be important in other problems.
2.11. Conclusions.

The 3-D time-dependent electromagnetic code MAP3 is expected to be capable of solving a broad class of common MHD problems with nonuniform conductivity and velocity distributions. A new boundary condition for $\varphi$ on the current source surface was proposed. Its use demonstrated, through the numerical solutions, a correct physical behavior, and it can be used for both moving and stationary conductors with a current source. This allows the use of electric scalar potential in problems with moving conductors, which is important for the current conservation feature of MAP3-B.

The new code MAP3 was benchmarked against the 3-D finite-element code MEGA on a conventional solid armature railgun problem, for both stationary and moving armatures. There is a satisfactory agreement between MAP3 and MEGA for the stationary problem. A discrepancy observed in the moving armature problem is believed to be due to differences in current conservation used in the two codes. Use of a staggered grid in MAP3 was found to eliminate some undesirable numerical effects.
III. MAP3 Plasma Flow Solver.


The plasma is modeled as a single continuous compressible fluid with mass density \( \rho \), velocity \( \mathbf{v} = (v_x, v_y, v_z) \), and specific internal energy \( e \). Pressure \( p \) and temperature \( T \) are calculated from the equations of state:

\[
p = f_1(\rho, e),
\]

\[
T = f_2(\rho, e).
\]

Equations governing plasma motion in the Navier-Stokes formulation include the electromagnetic force \( \mathbf{J} \times \mathbf{B} \) and the power dissipation \( \mathbf{J} \cdot \mathbf{E} \):

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0;
\]

\[
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \rho \mathbf{v} \mathbf{v} = \mathbf{J} \times \mathbf{B} + \nabla \cdot \tau - \nabla p;
\]

\[
\frac{\partial}{\partial t} \left( \rho e + \frac{\rho \mathbf{v}^2}{2} \right) + \nabla \cdot \left( \rho \mathbf{v} \left( \frac{\mathbf{v}^2}{2} + \frac{P}{\rho} + e \right) \right) = \nabla \cdot (\mathbf{v} \cdot \tau) + \nabla \cdot k_{tot} \nabla T + \mathbf{J} \cdot \mathbf{E};
\]

\[
\tau_{ij} = \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3} \mu \nabla \cdot \mathbf{v}. \tag{3.1}
\]

The equation for state functions \( f_1 \) and \( f_2 \), viscosity \( \mu \), total thermal conductivity \( k_{tot} \), and electrical conductivity \( \sigma \), are taken from tabulated data by Marshall [28].
data refer to a 10% copper-90% hydrogen equilibrium plasma at temperatures 5000 K to 60000 K and pressures 1 to 7000 atm. The value of $k_{\text{net}}$ includes a "radiation thermal conductivity" [12].


Railgun plasma conditions are characterized by strong MHD interaction. Electromagnetic force and electric power dissipation lead to intensive heat generation and heat and momentum transfer. "Conservative" numerical schemes are preferable for the numerical simulation of such plasma flows. They ensure that the solution satisfies the main conservation laws on the grid. This helps prevent occurrence of non-physical or pure numerical effects from arising in the course of the solution.

A control volume technique was used to discretize Equations (3.1) to obtain a conservative numerical scheme. In the scheme the equations are integrated over a grid cell or finite control volume. By this method, the plasma conditions averaged over the grid cell are calculated. Plasma properties computed in the iterative procedure are formally assigned to the interior node of the grid cell. The same control volumes are used for both the electromagnetic and the plasma flow solver to discretize the governing equations.

After control volume discretization, fluxes of mass, momentum, and energy at the cell faces are calculated. The convective fluxes are evaluated explicitly using Godunov's procedure [29]. The procedure is also known as a Riemann problem solver [35]. The
original Godunov's procedure [29] is iterative, and is sometimes also referred to as an exact Riemann solver. To save computational resources, so-called approximate Riemann solvers were developed, in particular, by Van Leer [35], which lack any iteration cycles. They enable a decrease in the amount of computation time by several orders without influencing the accuracy of the solution [35].

In this dissertation, a Van Leer flux vector splitting [35] with Hanel modifications was used as a Riemann solver [30]. The following formulas specify fluxes calculated at the face with a normal $n$. $L$ and $R$ represent cells adjacent to that face (Figure 3.1). A component of velocity normal to the cell face defines relative Mach numbers at the adjacent cells:

$$M_L = \frac{v_n^L}{a_L}, \quad M_R = \frac{v_n^R}{a_R};$$

The mass flux $F_m$, momentum flux $F_{mom}$, and energy flux $F_{energ}$ through the cell face with surface area $S_n$ are calculated as:

$$F_m = F_{mL} + F_{mR};$$

$$F_{mom} = F_{mL}v_n^L + F_{mR}v_n^R + (p_L + p_R)S_n;$$

$$F_{mom^\tau} = \begin{cases} F_m v_n^L, & \text{if } F_m > 0 \\ F_m v_n^R, & \text{if } F_m < 0 \end{cases};$$

$$F_{energ} = F_{mL}H_L + F_{mR}H_R;$$

where
Figure 3.1. Van Leer flux vector splitting.
\[ F_m^\pm = \begin{cases} \pm \frac{1}{4} (M - 1)^2 \rho aS_n & \text{if } |M| \leq 1 \\ \frac{1}{2} (M \pm |M|) \rho aS_n & \text{otherwise} \end{cases}; \]

\[ p^\pm = \begin{cases} \frac{P}{4} (M \pm 1)^2 (2 + M) & \text{if } |M| \leq 1 \\ \frac{P}{2} (M \pm |M|)/M & \text{otherwise} \end{cases}; \]

\[ H = e + \frac{1}{2} v^2 + \frac{p}{\rho}. \]

The discretization of viscous and heat conduction terms and the time-advancing procedure are described in details in [18]. Below is a brief summary of these numerical procedures.

To approximate spatial derivatives in the viscous and heat conduction terms of Equation (3.1), the computational space around the control volume interior grid node was divided into 48 tetrahedrons. Inside a tetrahedron, a linear distribution in x-y-z of plasma velocity and temperature in x-y-z is assumed. The tetrahedron values of derivatives then become coefficients in a linear approximation. The cell face is divided into 8 triangles, each of them belonging to a particular tetrahedron. Next the fluxes are calculated through each of the triangles and are summed over the face.

MAP3 uses a forward difference scheme for the approximation of time derivatives in Equation (3.1):
A mixed explicit-implicit scheme is used to advance the solution in time from level \( n \) to level \( n+1 \). For one particular component of the momentum equation, say \( v_n \), the viscous terms which involve that component of velocity are calculated implicitly by the ADI (alternate direction implicit) method [31]. These terms are evaluated at level \( n+1 \) and have the following form:

\[
\frac{\partial f}{\partial t} = \frac{f^{n+1} - f^n}{\Delta t}
\]

where \( \alpha_j \) are numerical coefficients. The other viscous terms are evaluated explicitly on level \( n \). Test runs have shown that the time step limit dictated by the viscous terms can be avoided by such splitting. For the energy equation, heat conduction terms (\( \nabla \cdot (k_{\eta \eta} \nabla T) \)) were calculated by ADI, and the viscous power dissipation (\( \nabla \cdot (v \cdot \tau) \)) was evaluated explicitly. The explicitly calculated MHD (\( J \times B, J \cdot E \)) and inviscid terms are entry arguments for the ADI equations. Advancing in time was performed first in the mass equation, then in the momentum equation, and, finally, in the energy equation. See [18] for additional details. Whenever the viscous terms do not impose the time-step limit they can be evaluated explicitly to save computational resources. This can be determined by test runs for the particular problem.
3.3. Coupled Solution Procedures in MAP3.

The plasma flow and electromagnetic equations are coupled through the electromagnetic force and dissipated power terms in Equation (3.1) and the values of conductivity and velocity in Equations (2.7) and (2.8). MAP3 adopts a sequential procedure to solve the coupled plasma flow equations and electromagnetic equations. After each numerical time step, necessary information is exchanged between the plasma flow solver and electromagnetic solver. The value of the time step is limited by the semi-implicit plasma flow solver through the usual Courant-Friedrich limit (CFL) criterion.

The MAP3 electromagnetic solver requires significantly more CPU work than the flow solver on one time step. It is mainly due to the extensive computational resources required for the conjugate gradients method. Therefore, subcycling is adopted to solve the electromagnetic equations less often than the flow equations. Test runs have shown that the subcycling does not noticeably influence the details of the predicted solution. The detailed MAP3 procedure and inter-solver communication procedure is described by flow charts in Figures 3.2, 3.3, and 3.4.
Figure 3.2. MAP3 work chart.

Figure 3.3. MAP3 plasma flow solver.
Solve equation for magnetic vector potential $A$.

Solve equation for electric scalar potential $\varphi$.

Update current source boundary condition for $\varphi$ by Equation (2.14).

Solution for Eq. (2.14) converges.

Figure 3.4. MAP3 electromagnetic solver.
IV. 3-D Plasma Armature Simulation.

4.1. 3-D Problem Formulation.

A conventional railgun with a plasma armature was simulated by MAP3-B, which, from now on will be simply denoted MAP3. Symmetry assumptions allow the calculations to be performed in one quarter of the railgun cross-section. The entire region from the breech to the projectile was simulated, so the problem of specifying changing boundary conditions, such as at the armature trailing edge, was avoided.

Sixty-cm long railgun with 1-cm and 2-cm square bores were simulated. The thickness of the rail was 0.5 cm. A uniform grid with 300 points in the axial (y) direction and 10×10 points in the simulated part of the cross-section (x,z-plane) was used in the plasma flow solver.

The numerical solution was calculated on a fixed grid. The base of the projectile is the right-hand boundary for the plasma flow solver inside the bore. As the projectile moves, new grid points are added to the computational domain. As soon as the projectile has reached the end of the bore, calculations stop. Inside the bore, the simulations were performed on the same grid for the electromagnetic and plasma flow solver.

The computational region for the electromagnetic solver in the cross-section extends through the rail and insulator to an outside air boundary. In the axial direction it extends beyond the base of the projectile with an additional 5 axial grid points. With the movement of the projectile, the right-hand boundary for electromagnetic solver
expands, and extra grid points are added into the computational domain. The grid spacing is chosen to resolve the diffusion of magnetic field to the rail and the armature with sufficient accuracy. For simplicity, heat transfer is not calculated in the rail, so the electrical conductivity of the rail remains constant throughout the calculations. During the run, the value of the current passing into the rail and through the armature is kept constant at \( I_0 = 120 \, \text{kA} \) (240 kA total gun current) for 2-cm bore, and at \( I_0 = 60 \, \text{kA} \) (120 kA total gun current) for 1-cm bore. Total mass of the projectile \( m_{pr} \) is taken to be 4 g for the 2-cm bore, and 1 g for the 1-cm bore.

4.2. Boundary Conditions.

For the plasma flow solver, a no-slip condition on the solid surface is used to determine viscous momentum flux on the boundary. A radiation absorbing condition is specified for heat flux on the rail, insulator, projectile, and breech surfaces. The normal component of energy flux \( q_n = \sigma T^4 \) is based on the value of temperature at the cell adjacent to the boundary, which is an optically thick plasma assumption. Also, on the rail and insulator surfaces ablation is allowed, so the corresponding convective fluxes of mass, momentum and energy are used as boundary values for interior calculations. These fluxes are obtained from localized mass and energy conservation principles applied to the solid surfaces. This will be described shortly.

The projectile velocity \( V_{pr} \) is determined from Newton's law:
\[
m_{pr} \frac{dV_{pr}}{dt} = \int p dS,
\]
where the pressure integral is taken over the base of the projectile, vacuum is assumed on the front surface of the projectile, \( m_{pr} \) is the mass of the projectile. The surface friction forces between the projectile and the bore walls are neglected to simplify the numerical model. The side of a cell adjacent to the projectile is expanding with velocity \( V_{pr} \), and new grid points are added into the computational domain as necessary. The Riemann problem algorithm for the moving boundary [29] is used for the calculation of convective fluxes at the base of the projectile. On symmetric boundaries, corresponding symmetry conditions are imposed.

For the electromagnetic solver, the boundary conditions are the same as described in Section 2.8. The only difference is that the rail is stationary; however, grid points are also added to the rail domain as the projectile proceeds down the bore.

4.3. Initial Conditions.

The problem was initiated by first computing the projectile motion from rest at \( x=3 \) cm by acceleration from the force of expansion of a cold nonconducting gas initially compressed to \( p=100 \) atm, using only the Navier-Stokes code. This simulates the condition of using a light gas gun projectile injector, and made it unnecessary to assume boundary conditions at the rear of the armature. The initial electromagnetic field is zero everywhere.
When the projectile reached the initiation position at $x \approx 10$ cm, the temperature in an axial region near the base of the projectile was increased to $T_0=10,000$ K to make the plasma conducting, simulating the action of a fuse to initiate an arc. The desired temperature increase was achieved by changing the density at constant pressure in the armature region. The initial distribution of the electromagnetic field was calculated at this point by taking a time step in the electromagnetic solver with a specified rail current. The simulation then continued by computing both electromagnetics and plasma flow. Initial length of the plasma armature was chosen to be 4 cm to avoid non-physical behavior, such as excessive heating, after starting the entire code.

4.4. Ablation Model for Rail and Insulator Surfaces.

The main goal in simulating surface ablation was to observe the 3-D flow which resulted from incoming mass from both the rail and insulator. The primary concern was the dynamics of the flow rather than the details of the surface process. A relatively simple physical model by Frese [12] was chosen to simulate the ablation rate from the solid surfaces.

Ablation was assumed to occur by melting of the surface at the melting temperature $T_m$ of the material. The ablation mass flow rate is set proportional to the radiative heat flux and is given as
\[ \dot{m} = \frac{\sigma T^4}{H}, \quad (4.1) \]

where \( H \) is the heat of fusion of the material.

The material properties for rail and insulator ablation were taken from Meger et al. [32]. The computed fluxes of mass, energy, and momentum due to ablation appear as boundary values for the corresponding fluxes in the mass, momentum, and energy equations. The ablation rate set by (4.1) was reduced by a factor of 30 at the insulator surface. It was done in order to account for the presence of cold gas radiation absorbing boundary layers near the insulator surface. The existence of the layer made of the insulator decomposition products and a reduction factor of 30 was predicted by Keefer, Sedghinasab and Crawford [16].

4.5. Presentation of Results

A summary of the final state of the simulation for 1-cm and 2-cm bore is given in Table 4.1.

<table>
<thead>
<tr>
<th></th>
<th>( t, )</th>
<th>( V_{pr}, )</th>
<th>( x_{pr}, )</th>
<th>armature length, m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>s</td>
<td>m/s</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>1-cm bore</td>
<td>1.2 \cdot 10^{-2}</td>
<td>1,772</td>
<td>0.71</td>
<td>0.12</td>
</tr>
<tr>
<td>2-cm bore</td>
<td>6.1 \cdot 10^{-3}</td>
<td>1,534</td>
<td>0.65</td>
<td>0.2</td>
</tr>
</tbody>
</table>
The solution results are analyzed from data obtained predicted on several surfaces as defined in [19] (Figure 4.1). I-A is the insulator-to-insulator plane passing through the axis of the railgun. The insulator-to-insulator plane will be denoted as I-k and is located k number of grid points away from the railgun axis. Likewise, rail-to-rail surfaces R-A and R-k are analyzed. Finally, a transverse plane is defined as T-k where k specifies the axial grid point number with respect to the breech. To have a better visualization of the 2-D plots, dimensions in the cross-section are enlarged by a factor of 10.

4.6. 3-D Electromagnetic Field Distribution in Plasma Armature Railgun.

The inherently 3-D nature of the railgun electromagnetic field has a profound influence on the plasma flow. Therefore, it is important first to understand the main features of the electromagnetic solution, which are the same for 1-cm and 2-cm bores. The 3-D magnetic field is wrapped around the rails and armature and diffuses into the railgun structure. Solutions of the magnetic field and current density are closely coupled through Ampere's law. As $B$ diffuses into the railgun structure, it changes outside and redistributes current inside. Main characteristics of the magnetic field diffusion are described below. At time $t=0$ after the power has been turned on, the current flows mostly on the conductor surfaces, and magnetic field has not yet diffused into the railgun structure. This can be thought of as transient eddy currents induced in the structure that keep the magnetic field outside the conducting material. The diffusion of the magnetic field into the rail is the fastest on the rail corners, and causes the current to concentrate
Figure 4.1 Sketch of the plane surfaces used for data analysis.
there.

As the magnetic field diffuses, the eddy currents dissipate, and the current distribution within the rail becomes more uniform. The steady-state electromagnetic solution for the stationary armature is a uniform current distribution on the breech side of the rail.

Additional eddy currents are induced in a moving armature. They are directed to oppose the change of magnetic field caused by the armature motion and produce what is usually called "velocity skin effect" [36]. No matter whether the eddy currents are produced by motion of the armature or by transient diffusion, they redistribute the current into the corners of the rail. The time required for the magnetic field to diffuse into the plasma is much shorter than for the rail. However, the armature current distribution is significantly influenced by the diffusion of the magnetic field into the rails. Consequently, the current is forced to flow from rail corner to rail corner rather than through the center of the gun.

The electromagnetic field solution for 1-cm and 2-cm bore is similar. The solution for the 1-cm bore is presented here. The connection between the magnetic field diffusion and the current density distribution is shown in Figure 4.2.

The observed 3-D electromagnetic field distribution leads to strong spatial nonuniformity of the electromagnetic force. Figures 4.3 and 4.4 show contour plots of the axial force in T-350 and I-10 planes. The peak of the force is located near the rail surface, also off-axis.
Figure 4.2. 3-D distribution of current density and magnetic field in the 1-cm bore, T-350 plane. Current density is scaled by the factor of $10^6$. 
Figure 4.3. Axial force distribution in the 1-cm bore, T-350 plane.
Figure 4.4. Axial force distribution in the 1-cm bore, I-10 plane.

The main features of the plasma flow are the same for both the 1-cm and 2-cm bores, and are presented primarily for the 1-cm bore. The following figures show the plasma velocity vector field in the projectile reference frame superimposed with the contours of temperature and current density magnitude $|J|$.

The central symmetry planes R-A and I-A are shown in Figures 4.5 and 4.6, respectively. A distinct off-axis maximum of the current distribution is clearly visible in the I-A plane (Figure 4.3). A strong circulation with a positive velocity near the rail and negative velocity on the central insulator-to-insulator plane is also observed. This circulation is caused by the nonuniformity of the electromagnetic force, which was discussed above. As a result, the plasma is accelerated mostly near the rail (see the I-8 plane, Figure 4.8). The accelerated plasma reaches the projectile with a positive velocity relative to the projectile, where it turns diagonally downward (see the T-380 plane, Figure 4.8). The plasma moves away from the projectile since the electromagnetic force is not sufficient to balance the pressure gradient in the I-A plane. This flow pattern was not predicted by 2-D models which predicted negative velocity near the rail and positive velocity in the center. It should be noted that the strong shear flow near the rail leads to higher viscous losses than predicted by the 2-D models. This effect may be very important at higher projectile velocities.

Near the insulator where the force is much smaller than in the center, a relatively cold nonconducting layer is formed due to ablation. This explains the high negative
Figure 4.5. Flow vector field relative to the projectile and temperature distribution in the 1-cm bore, R-A plane.
Figure 4.6. Flow vector field relative to the projectile and current density distribution in the 1-cm bore, I-A plane.
Figure 4.7. Flow vector field relative to the projectile and current density distribution in the 1-cm bore, I-8 plane.
Figure 4.8. Flow vector field in the 1-cm bore near the projectile, T-380 plane.
velocities of the plasma flow near the insulator as well as the drift of the plasma toward the insulator on the projectile surface. As shown in the I-8 plane, part of the plasma near the insulator is forced back toward the armature by a small circulation at the rear. Similar figures for the 2-cm bore in the I-A and R-A planes (Figures 4.9, 4.10) show that the main features of the plasma flow are same as in the 1-cm bore case.

The plasma flow directed away from the projectile on the central I-A plane carries hot material so a thermal tail behind the armature is formed (see Figure 4.5).

Figures 4.11 and 4.12 show axial plots of the current density magnitude $|J|$ along the center line at different times. The evolution of the profiles shows that the development of the current tail at the back of the armature is more prominent in the 1-cm case. The armature elongation may be a factor leading to formation of secondary arcs. Local maxima of the current are observed toward the rear of the armature (Figure 4.7) and may be precursors to secondary arc formation. The length of the armature is approximately 20 cm for the 2-cm bore, and 12 cm for the 1-cm bore. As one can see, there is a buffer of low-conducting plasma near the projectile. This agrees with the experimental observation of primary separation by Clothiaux et al. [2].

A common method for experimental study of the plasma armature motion in the railgun is by B-dot probes [37]. There are two major types of these probes which have different orientation in space, and detect either rail current or armature current. The probe records the induced voltage proportional to the time derivative of one of the components of the magnetic field in the bore, depending on the orientation of the axis of the probe loop with respect to the railgun.
Figure 4.9. Flow vector field relative to the projectile and current density distribution in the 2-cm bore, I-A plane.
Figure 4.10. Flow vector field relative to the projectile and temperature distribution in the 2-cm bore, R-A plane.
Figure 4.11. Time evolution of the current density for the 1-cm bore along the railgun axis.

Figure 4.12. Time evolution of the current density for the 2-cm bore along the railgun axis.
A probe that measures $\frac{\partial B_y}{\partial t}$ detects mainly the magnetic field from the armature current. The experimental data for this kind of probes are available in literature. The obtained 3-D solution for the plasma armature railgun allows to qualitatively compare for the first time numerically obtained B-dot signal with the available experimental data.

From the numerical solution for 1-cm bore, the time evolution of the $B_y$ axial component of magnetic field was obtained at two grid points. They are located at the central insulator-to-insulator plane, 0.6 cm from the center of the gun, 0.35 m and 0.45 m from the breech. The obtained data are shown in Figure 4.13. The value of $\frac{\partial B_y}{\partial t}$ at the particular time point was obtained by fitting the data with a 10-point parabola by a least-square method, then by differentiating the obtained analytical expression. This was done to avoid possible spikes that would occur if data in Figure 4.13 are differentiated directly. The results of the obtained $\frac{\partial B_y}{\partial t}$ are shown in Figure 4.14, and are compared with experimental data by Jamison and Burden [38] (Figure 4.15).

As shown in Figure 4.15, the numerical solution captures the characteristic form of the experimental B-dot trace: a relatively sharp negative peak followed by a wider, lower-amplitude positive peak. The smaller positive peak is a well-known evidence of the secondary arc development [6]. Jamison and Burden unfolded axial current density distribution in the armature from the obtained B-dot probe by a parametrically fitting procedure. Their result is very similar to the data obtained by MAP3 (Figures 4.11-4.12).
Figure 4.13. Numerically obtained $B_y$ time evolution.

Figure 4.14. Numerically obtained B-dot signal.
Figure 4.15 Typical form of the experimental B-dot probe (armature magnetic field)
V. SUMMARY AND CONCLUSIONS

To perform a 3-D MHD time-dependent computer simulation of the plasma railgun, a new code MAP3 (MHD Arc Plasma) was developed at UTSI. The MAP3 numerical model is discussed in this dissertation. MAP3 uses an efficient numerical method to solve the coupled Maxwell's equations and Navier-Stokes equations to develop the complex time-dependent electromagnetic and velocity vector field distribution in the railgun. Results of 3-D benchmark testing against a 3-D finite-element electromagnetic code MEGA, are presented and discussed.

The importance of the MAP3 scheme that uses a staggered grid to solve Maxwell's equations was demonstrated. Unlike MEGA, the staggered grid form of the MAP3 formulation allows current conservation in the solution, which is important in MHD computations, and eliminates some nonphysical effects found in the MEGA solution.

MAP3 can simulate a plasma armature railgun in both 2-D and 3-D approximations. It was demonstrated that 2-D computer models inadequately predict the main physical features of plasma flow in travelling arc-driven railguns. To better understand the railgun physics, 3-D MHD modelling is necessary.

The results of a 3-D MHD computer simulation using the staggered grid version of MAP3, for 1-cm and 2-cm bore railguns with ablating surfaces, are presented. MAP3 provided the first qualitative and quantitative understanding of current, magnetic field, temperature, and velocity distributions in the 3-D driven plasma armature railgun. A
profound influence of the inherently 3-D nature of the railgun electromagnetic field on the plasma flow is shown. The railgun current is forced to flow from rail corner to rail corner rather than through the center of the gun. Thus, a strong spatial nonuniformity of the electromagnetic force develops, generating a flow of plasma with positive velocity near the conducting rail and negative velocity in the central insulator-to-insulator plane. This result is exactly the opposite to results provided by 2-D MHD models.

A direct simulation of the particular experiment is the logical step to develop this work further. A satisfactory qualitative agreement was demonstrated between numerically obtained B-dot signal (armature magnetic field) and typical experimental data. MAPS can be used to study secondary arc formations, which requires further work. The 3-D flow has a tendency to elongate the plasma or arc armature, creating a current conducting tail. This process may play an important role in the formation of secondary arcs. MAPS can be extended to analyze conditions believed to be accountable for the development of the secondary arcs and to provide reliable quantitative simulation. These conditions should include current that changes in time, and higher ablation rates.
BIBLIOGRAPHY
Bibliography


APPENDIX
Appendix A.

Finite-Difference Operators.

Forward differencing of \( \frac{\partial f}{\partial x} \):

\[ l_{f,0} f = \frac{f_{i+1,j,k} - f_{i,j,k}}{x_{i+1,j,k} - x_{i,j,k}}. \]

Central differencing of \( \frac{\partial f}{\partial x} \):

\[ l_{c,0} f = \frac{f_{i+1,j,k} - f_{i-1,j,k}}{x_{i+1,j,k} - x_{i-1,j,k}}. \]

Backward differencing of \( \frac{\partial f}{\partial x} \):

\[ l_{b,0} f = \frac{f_{i,j,k} - f_{i-1,j,k}}{x_{i,j,k} - x_{i-1,j,k}}. \]

Upwind differencing of \( u \frac{\partial f}{\partial x} \):

\[ ul_{up,0} f = \begin{cases} u_{i,j,k} l_{d,0} f, & \text{if } u_{i,j,k} > 0 \\ u_{i,j,k} l_{f,0} f, & \text{if } u_{i,j,k} < 0 \end{cases} \]

Central differencing of \( \nabla^2 f \):

\[ \mathcal{L} = \mathcal{L}_x f + \mathcal{L}_y f + \mathcal{L}_z f; \]

\[ \mathcal{L}_x f = \frac{1}{h_h} f_{i+1,j,k} - f_{i,j,k} \left( \frac{1}{h_h} + \frac{1}{h_h} \right) + \frac{1}{h_h} f_{i-1,j,k} \]

\[ h_+ = x_{i+1,j,k} - x_{i,j,k} \]
\[ h_- = x_{i,j,k} - x_{i-1,j,k} \]
\[ h = x_{i+\frac{1}{2},j,k} - x_{i-\frac{1}{2},j,k} \]

\( \mathcal{L}_y \) and \( \mathcal{L}_z \) are defined similar to \( \mathcal{L}_x \).
Curriculum Vitae

Dmitri A. Kondrashov was born on May 11, 1967, in Ryazan, Russia. He graduated from High School No. 39 in Ryazan in 1984.

In Summer 1984 he went to Moscow Institute of Physics and Technology (MIPT). He attended the MIPT from 1984 to 1990. During his 3rd year, he started working part-time at the Laboratory of MHD Numerical Simulation of Moscow High Temperature Institute (IVTAN) as a research assistant. He continued to work there until his graduation in 1990 with Master of Science Degree with a major in Applied Mathematics. He then enrolled as a Ph. D. student at the MIPT while working at IVTAN. In summer 1992, he was admitted to the University of Tennessee Space Institute (UTSI) as a Ph. D. student where he worked as a graduate research assistant.

Continuing his studies at UTSI he received his Doctor of Philosophy degree with a major in Engineering Science and Mechanics in May of 1996.

Dmitri A. Kondrashov and his wife Veronica reside in Manchester, TN.