A STUDY OF INTER-CARDINALITY VOICE LEADING USING VOICE-LEADING ZONES AND THE EXTENDED 4-CUBE TRIO

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I. INTRODUCTION

Are triads and tetrachords living on such different planets, as Dmitri Tymoczko has suggested, that we cannot represent the smooth voice leading between them in an effective manner? (2011, 97–98) Transformational, neo-Riemannian, and geometric techniques have created ways to relate smooth voice leading between chords of the same number of pitch classes. However, when the cardinalities of sonorities do not agree, such as in the relationship between a triad and a seventh chord, the mathematic and graphical formulation of such smooth voice-leading processes remains problematic. Problems arise with reconciling the cardinality difference and truthfully representing the role of pitch classes and voice-leading distances involved in the process.

This work attempts to establish connections between certain types of triads and certain types of tertian seventh chords, building on the parsimonious graphs and voice-leading discussions from Jack Douthett and Peter Steinbach, Richard Cohn, and Joti Rockwell. The proposed model, which I call the Extended 4-Cube Trio, considers doublings within triads in a tetrachordal manner and introduces major seventh chords into Cohn’s original 4-Cube Trio (explained below). This model can be used to study progressions that involve chords of different cardinalities, such as the omnibus progression. Musical examples from Nikolas Medtner and popular songs from the Great American Songbook, which feature chromatic harmonic relationships involving both triads and lesser-even tetrachords, illustrate how the expanded tetrachordal universe facilitates the discussion of inter-cardinality voice leading.
II. REVIEW OF PREVIOUS WORKS

GRAPHICAL PRESENTATION OF SEMITONAL VOICE LEADING

Richard Cohn proposes that minimal voice leading plays an essential role in the syntax of chromatic harmony. Nearly even sonorities can be derived from a perfectly even sonority with minimal perturbations (2012, 8–12, 34). Among triads, the augmented triad alone is perfectly even. The nearly even chords—major and minor triads—are connected to an augmented triad, and to two other triads of the opposite mode by the moving one of the voices by a semitone. These connections are effectively illustrated by Douthett and Steinbach’s Cube Dance, reproduced in Figure 1 (1998, 253–254). In the Cube Dance, major, minor, and augmented triads are arranged according to their idealized minimal voice-leading distance (i.e. the most direct semitonal movements between the two chords) and direction. Clockwise motion indicates an upward voice-leading movement, and the counterclockwise motion signifies a downward movement.

Cohn expands the Cube dance by introducing “voice-leading zones” (2012, 102–104). Voice-leading zones are calculated by the mod-12 sum of the pitch classes in a triad. As seen in Figure 1, they are arranged as numbers on a clock-face and superposed onto the Cube Dance. The three triads in each voice-leading zone are derived from the closest augmented triad, and each triad in that zone is produced by moving one of the augmented triad’s voices by a semitone in the same direction. The algebraic difference between the voice-leading zones equals the net voice-leading distance between two triads, regardless of the actual voice-leading motion.\(^2\)

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1. Evenness is defined as the distribution of pitches within an octave. A set is maximally even when the pitches are separated by the same interval within an octave, such as a tritone for dyads and major thirds for trichords (Cohn 2013, 34; Tymoczko 2011: 62–63).

2. The hexatonic poles, such as the A minor triad in zone 1 and the C♯ major triad in zone 2, require three semitonal movements to reach each other. However, two of those movements are in opposite directions and are therefore
The same concept applies to tetrachords. Semitonal displacements from the perfectly even fully-diminished seventh chord produce four half-diminished chords and four dominant seventh chords through upward or downward shifts, respectively. However, to represent the cancelled out in the calculation of net voice-leading distance. The net result is a movement of one semitonal displacement.
voice-leading procedure of seventh chords using semitonal movements, French sixths and minor seventh chords are introduced to the graph to connect the two nearly even chords. Figure 2 reproduces Cohn’s 4-Cube Trio, with corrections by Scott Murphy, which illustrates the relationships between the five types of tetrachords (2014, 91).

These figures focus on how triads and tetrachords are each connected in their own universes, but they do not show any interactions between the two spaces. The voice-leading process between two differently-sized chords can be as smooth as the motion between triads or tetrachords, but it is problematic to explain how each voice moves using existing models.

**Works on the study of inter-cardinality voice leading**

Douthett and Steinbach 1998, as well as Rockwell 2009 use the \( \tau \) function to describe movement from one pitch class to its nearest destination in a given progression, with a maximum distance of a whole tone. The three pairs of chords in Figure 3 all relate by smooth voice-leading processes. For chords with the same number of distinct pitch classes (intra-cardinality voice leading), the model demonstrates a one-to-one relationship between pitch classes that can be described by a function, as each pitch class in the first chord is mapped onto one unique value in the second chord.\(^3\)

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\(^3\) While a one-to-one correspondence between chords of equal cardinality is possible, it is not necessarily required, as in contexts where a single voice can rest or sound more than one note. See Morris 1998, 175–208, which allows for such contexts. Douthett and Steinbach’s definition of \( \tau \)-function only deals with voice leading between two pitch class sets of the same size, and Rockwell expanded the definition to include inter-cardinality voice-leading motions.
The formulation of the function is less definitive in the case of inter-cardinality voice leading, which involves a difference in the number of distinct pitch classes between two chords. In the second example of Figure 3, it is possible to interpret both pitch classes B and D mapping onto C, as this commonly happens in the resolution of a dominant seventh chord to a tonic triad. Alternatively, resolving D to E is an equally valid mapping. Either way, two of the pitch classes in the G dominant seventh chord are mapped onto a duplicated pitch in the C major triad. In this case, the second example displays the surjective property of functions, which means the function values from different inputs can be the same.

There are multiple problems with the definition of the τ function. It is defined so loosely that the same set of input values may suggest different outcomes depending on voice-leading context. Ambiguity in the definition results in an inability to express a unique solution for any given voice-leading process. Since it takes a whole step for D to reach either C or E in the second example of Figure 3, the τ function does not offer a definite voice-leading solution, and therefore it requires some subjective interpretation to determine mappings.

Additionally, while the τ function allows many-to-one mapping works, its one-to-many inversion does not hold. In the third example in Figure 3, the τ function with input C in the C major triad is an ambiguously defined function. Mathematically speaking, a single input cannot produce multiple outputs; a parallel would be as if both $x + 2 = 3$ and $x + 2 = 4$ were...
simultaneously true. Two separate inputs are required—even if they are the same pitch class that has been split—to form a valid algebraic expression. To truthfully represent the behavior of the pitch class, the D and F need to be mapped with two distinct input values representing the pitch class to formulate an algebraic expression. With the current system, such a distinction is impossible.

Despite the limitations in formal mathematical treatment of such phenomena, we can still identify the voice-leading process as a “relation,” as suggested by Callender 1998 (223–27). A relation is more flexibly defined and, in the case of voice leading, allows the fusion and splitting of a chordal pitch or pitch class. This allows an analyst to explain voice leading between chords of different sizes, as a single pitch can split into two neighboring pitches, or two pitches can fuse into one. Yet in Callender’s work, each pitch class is still considered a unique entity, and the size of movement is confined to one semitone.

Rockwell (2009) proposes the “parsimonious voice-leading matrix” (hereafter, \( P \)-matrix) to study inter-cardinality voice-leading relation—rather than function—in terms of the number of voices moving in half steps and whole steps in both directions.\(^4\) The \( P \)-matrix is composed of four numbers and is expressed as \( P = \begin{bmatrix} u_1 & u_2 \\ d_1 & d_2 \end{bmatrix} \) where \( u_1 \) and \( d_1 \) represent the number of voices that move by a half step in upward and downward directions respectively, and \( u_2 \) and \( d_2 \) represent the number of voices that move by a whole tone. A given voice-leading relation may be reversed by exchanging the rows in the matrix. Returning to the chord pair G7 and C major in Figure 3, the \( \tau \) function can satisfactorily explain the voice-leading process from G7 to C, but not from C to G7. However, with the \( P \)-matrix, we can explain the progression from G7 to C major

\(^4\) In the following text, italicized \( P \) refers to the voice-leading relationships described by the \( P \)-matrices, to distinguish from the neo-Riemannian P.
using $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, as well as the reverse process from C major to G7 using $P = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. This method does not rely on the use of pitch classes, but on voice-leading movements, and it allows the analyst to examine voice-leading relations between pitch class sets regardless of cardinality, as found in progressions that feature smooth voice-leading.\(^5\)

Rockwell points out that the relation $P = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$, which means that two voices move by semitones in opposite directions, can be used to evaluate the voice-leading process between two dominant seventh chords whose roots are a minor third or tritone apart, and two minor triads, whose roots are a major third apart (2009: 13). Alternatively, this process might also be understood as “pure contrary motion” (Tymoczko 2011, 81–83). The most interesting application of the $P$ process, however, is the voice-leading relation between a dominant seventh chord and a minor triad whose root is a major third above that of the seventh chord. It is used in the irregular resolution of a German augmented sixth chord to a minor triad, as well as the omnibus progression, which alternates between dominant seventh chords and minor triads by moving two voices semitonally in opposite directions. Rockwell illustrates the relationship between the chords of an omnibus progression in a parsimonious graph, shown in Figure 4 (15–16). The chords are represented as nodes and the $P$-relation is represented by the edges. The graph helps us visualize the relations between lower-cardinality sonorities and the higher ones.

Rockwell further uses the $P$ relation to connect the minor triads together and form the complete birdcage graph shown in Figure 5. A birdcage graph is strictly defined as the parsimonious graph that shows both intra- and inter-cardinality voice-leading relationships using a given $P$ process. The four dominant seventh chords connected in a square and the three minor triads formed the birdcage graph (Rockwell 2009, 11).

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\(^5\) The process is identified by induction; therefore, the problem of how such process can be formulated analytically by the movements of pitches is not completely solved. See Rockwell 2009, 6–12.
triads in a triangle belong to the same voice-leading zone in Cohn’s terms. Rockwell also demonstrated the use of the same $P$-matrix using the “Ziehn Inverted Omnibus” progression, as shown in Figure 6. This progression relates major triads and half-diminished chords by the same $P$ relation (14–16).

**Figure 4:** An omnibus progression and its representation in a parsimonious graph (Rockwell 2009, Example 8)
While the birdcage graph can effectively show both intra- and inter-cardinality voice-leading motions with the same process, it cannot help us fully visualize the difference between apparently equivalent progressions that involve multiple-edge movements. As Tymoczko mentions, due to the reduction of dimensions, the voice-leading procedure from the A₇ chord to the A minor triad and from the C7 chord to the A minor triad are not the same, although the two progressions are equivalent in the birdcage graph in Figure 5 (2010, 6–7). After the tetrachord travels to the triadic universe, the graph only shows the idealized voice leading in a three-voice texture, and not the four-voice process. To truly reflect inter-cardinality voice-leading processes, one needs to consider the contrapuntal nature of voice leading in pitch space and embrace the use of multisets which allows pitch classes to appear more than once within a collection.⁶

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⁶ See Tymoczko 2010, 42. Also, Thomas Robinson 2009 lists a number of possible music applications of the multiset concept in his dissertation.
III. THE STUDY OF OMNIBUS PROGRESSIONS USING VOICE-LEADING ZONES AND THE EXTENDED 4-CUBE TRIO

In an omnibus progression, shown in Figure 7, a dominant seventh chord reaches a minor triad, and vice versa, with one voice moving upwards and another one moving downwards—both by a half step. This means that the net voice-leading distance is zero. As the chord progresses using the same $P$-relation with no net voice-leading displacement, all of the dominant seventh chords and minor triads of Figure 2 should be located in the same voice-leading zone if we allow the use of multisets in constructing the voice-leading space.
Figure 7: An example of omnibus progression

If we determine the voice-leading zones of the chords in an omnibus progression by determining the sum of a chord’s pitch class integers, an interesting discrepancy arises. The chords in an omnibus progression do not belong to the same zone as they would in accordance with Cohn’s 4-cube trio. The four dominant seventh chords in the example belong to voice-leading zone 1, while the 4 minor triads belong to zones 1, 4, 7, and 10 in triadic space. Also, the difference in voice-leading zones between a neighboring triad and tetrachord in the omnibus progression does not reflect the voice-leading distance between them, which should be 0 because of the balanced semitonal movement.

When the chords are in their own universe with no cardinality changes, it is sufficient to discuss the movement of a pitch class. However, the doubling in a triad in an inter-cardinal voice-leading process is significant. When a dominant seventh chord moves to a minor triad as part of the omnibus progression, the two voices that move in opposite directions are the root and seventh of the chord. These two voices move in contrary motion to reach the doubled fifth of a minor triad while the other two pitch classes stay in the same place—in order words, they act as the “purely parallel component.” (Tymoczko, 81–82) The progression from a minor triad to seventh chord relies on the opposite semitonal movement of the doubled fifth to the seventh
chord. The two notes of the minor chord’s doubled fifth behave differently in these situations, even though they belong to the same pitch class, and this movement results in a zero-pitch class-sum movement.

It is reasonable, then, to include the pitch class of the chordal fifth twice as we calculate the voice-leading sum of such a fifth-doubled minor triad in this voice-leading process. For instance, the voice-leading zone of a C-doubled F minor triad is 0+0+5+8 = 1 (mod 12). Table 1 shows this method’s results when calculating the voice-leading zones of the four minor triads from Figure 7’s omnibus progression. When we account for the doubled fifth, we see that all of them belong to the same voice-leading zone as the progression’s dominant seventh chords. Applying this calculation to all twelve minor triads results in all the chords used in the given omnibus progression falling into zones 1, 5 or 9, and all chords of any omnibus progression belonging to the same zone.

**Table 1**: Voice-leading zones of the minor triads used in an omnibus progression, calculated by their pitch-class sets and fifth-doubled multisets

<table>
<thead>
<tr>
<th>Minor Triads (doubled pitch class)</th>
<th>Voice-leading Zone using pitch class sets (i.e., zone in the Cube-dance)</th>
<th>Voice-leading Zone with fifth-doubling considered (triads as multisets)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F (C)</td>
<td>5 + 8 + 0 = 1 (mod 12)</td>
<td>5 + 8 + 0 + 0 = 1 (mod 12)</td>
</tr>
<tr>
<td>D (A)</td>
<td>2 + 5 + 9 = 4 (mod 12)</td>
<td>2 + 5 + 9 + 9 = 1 (mod 12)</td>
</tr>
<tr>
<td>B (F♯)</td>
<td>11 + 2 + 6 = 7 (mod 12)</td>
<td>11 + 2 + 6 + 6 = 1 (mod 12)</td>
</tr>
<tr>
<td>G♯(D♯)</td>
<td>8 + 11 + 3 = 10 (mod 12)</td>
<td>8 + 11 + 3 + 3 = 1 (mod 12)</td>
</tr>
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</table>

**Figure 8**: Two possible single semitonal voice-leading processes between a dominant seventh chord and a fifth-doubled minor triad
With this result, it is possible to conceptualize a link between triads and tetrachords on the 4-Cube Trio. In case of a dominant seventh chord and a minor triad in the omnibus progression (or the irregular resolution from a German sixth to a minor triad or a cadential 6/4), it is equivalently a dominant seventh chord, with two voices moving in opposite direction by semitone to the doubled fifth of a minor triad. The major seventh chord is one of the two steps between the two chords, like the first example in Figure 8, while the other possible sonority is the pitch class set (0367) in the second example. The root of the major seventh is then lowered by a semitone to reach the fifth-doubled minor triad. Likewise, a way to connect a root-doubled major triad—from the inverted omnibus—to a fifth-doubled minor triad—from the standard omnibus—is to raise the third and the fifth by a half step. If the latter is performed first, the result is an augmented triad with one of its chord members doubled. Looking at Figure 9, the seventh of the major seventh chord can also be raised by a half step, creating a root-doubled major triad that belongs to the same voice-leading zone as the half-diminished triads located at zones 3, 7, and 11. These are also the voice-leading zones where chord members of a Ziehn Inverted Omnibus Progressions can be found.

Figure 9: Single semitonal voice-leading process between a half-diminished chord and a root-doubled major triad

Figure 10 shows a graphical representation of these connections between different types of chords, which is referred to as the Extended 4-Cube Trio. The original members of the 4-Cube Trio—the diminished seventh chords (zones 2, 6 and 10), dominant seventh chords (zones 1, 5
and 9), half-diminished chords (zones 3, 7 and 11), French sixth chords (zones 0, 4 and 8), and minor seventh chords (zones 0, 4 and 8)—are preserved at the center of the graph. The nodes in the outer shells represent the different triads and major sevenths. There are only three types of triads represented on the graph. Root-doubled major triads are in zones 3, 7 and 11; fifth-doubled minor triads in zones 1, 5, 9; augmented triads with all possible doublings are in zones 0, 4, 8; and major seventh chords are in zones 2, 6, and 10. Triads are notated in the form of X(Y), in which X is the root of the chord and Y is the pitch class that is doubled. Major triads are represented by capitalized X while the minor triads will be in small letters. Augmented triads are notated as “Xaug.”

The major seventh chords not only connect the major triads and half-diminished chords, they also act as an alternative linking harmony to the diminished seventh chord between the dominant seventh and half-diminished chords. However, major seventh chords are unlike diminished seventh chords in that they can only relate to one dominant seventh and one half-diminished chord.

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7 For the notation of the French sixth chords, the letter indicates the bass note that French sixth chord is built upon.
8 Tymoczko commented that it is not practical to include all types of multisets in a graphical space of intercardinality voice-leading, as there are many potential multisets that can be generated, and there are multiple pathways to the same chord (cf. Figure 8). In a discrete graph, the more added nodes there are, the more difficult it is to extract a direct voice-leading path (2010, 7–8). This paper focuses on the application of the voice-leading processes in tertian chords, therefore the Extended 4-Cube Trio only includes the three types of triads, which are generated directly through semitonal voice leading from the major seventh chord. The limit may also provide voice-leading paths that are longer than the most efficient path, for example, from C7 to A half-diminished. (2011, 413–414)
9 “Bridging chords” are first discussed under this name by Douthett and Steinbach, 257–59.
Figure 10: Extended 4-Cube Trio
The voice-leading pattern of an omnibus progression is demonstrated in Figure 11. All of the chords involved in the progression are now located in voice-leading zone 1. The G dominant seventh chord reaches F#-doubled B minor triad after passing through the G major seventh chord. To move from one dominant seventh chord to another, the diminished seventh chord acts as an intermediary for single-semitonal motion.\(^\text{10}\) A similar procedure can be applied to the Ziehn Inverted Omnibus Progression in a mirrored fashion, demonstrated in Figure 12; the chords involved in the three possible inverted omnibus progressions are located in zones 3, 7, and 11.

All the members in the omnibus progression can be found within the continuous tetrahedral chord space described by Tymoczko, which is a 4-dimensional tile constructed by tetrahedra. Within this continuous space, any four-pitch collections from four entities of a duplicated single pitch class to tetrachords of four distinct pitch classes are included, and conceptually all voice-leading relationships can be presented through movements in the four-dimensional space (Tymoczko 2010: 7–8, 2011: 93–94). The Extended 4-Cube is an attempt to expand the existing 4-Cube Trio to include the lesser-even tetrachords and triads through the use of tetrachordal multisets while faithfully preserving voice-leading distances. With this addition to the graphical space, the expanded graph already permits us to apply graphing techniques in the following musical examples beyond the omnibus progression.

\(^\text{10}\) The diminished seventh serves as either an intermediary between two dominant-seventh chords, or as a substitute for one of them in at least half of the examples from the literature cited in Telesco 1998.
FIGURE 11: An omnibus progression on the Extended 4-Cube Trio
FIGURE 12: A Ziehn omnibus progression on the Extended 4-Cube Trio
IV. Analytical Applications of the Extended 4-Cube Trio

The Extended 4-Cube Trio and the modified voice-leading zones can provide a more accurate and flexible account of inter-cardinality voice-leading paths, and this is useful in analyzing passages with chromatic motions among triads and seventh chords. The following examples taken from Nikolai Medtner and the Great American Songbook all exhibit chromatic voice-leading properties, and these processes are all facilitated by the lesser-even chords. The Extended 4-Cube Trio will be used to locate close relationships between some triads and seventh chords that are otherwise not represented in other discrete graphical spaces. In the following analyses, the actual voice-leading processes notated in the musical score are preserved as much as possible, whereas some leaps to other registers are idealized with explanation.

Medtner’s “Romanza” from Forgotten Melodies II, Op. 39

Figure 13 is taken from mm. 27–33 of “Romanza” from Forgotten Melodies II, Op. 39 by Nikolai Medtner (1879/80–1951). In this excerpt, the presence of augmented triads and major seventh chords poses difficulties in applying the original 4-Cube Trio. Sonorities in this excerpt are played as arpeggiated six-note groupings. This study offers two possible interpretations of pitch relationships in the excerpt. Using the Extended 4-Cube Trio, the first reading demonstrates some symmetries in the progression, while the second reading shows how chords from the extended part of the graph play a significant role in facilitating smooth voice leading—even though they connect to fewer tetrachords and triads directly.
The excerpt is formed by nine cells of six-note arpeggiated figures, with a final cell consisting of only four notes. Each cell comprises a tetrachord formed by the first three and last two notes. The reduction is shown in Figure 13(b). Note that beginning in m. 31.3, the voicing differs from the actual music so as to preserve the voice-leading path. The first cell, for example, contains an E diminished seventh chord. The fourth note of each cell is an upper appoggiatura to a chord tone, with the exception of the last note in m. 34, whose presence is motivated by both melodic and harmonic reasons. Because of metric stress, the fourth note seems to be carried over from the previous cell and resolves downward to the fifth note between mm. 28–29 —see the
notes in dotted circles in Figure 13(a). This relationship is consistent across each of the three units of the first six cells, and in the last four cells. This suggests that the note itself might be considered an embellishment instead of a significant chord tone. The voicings in the four-voice reduction are retained, except in the cell between mm. 29.3 and 30.1 where a simple adjustment reflects the movement of the doubled A more clearly.

The progression shares some similarities with the omnibus progression, however, as there are movements that are not purely contrary. In m. 28, only the G, from the G major seventh chord resolves upward by semitone, forming a G half-diminished chord in zone 11. The only whole-step motion in this passage occurs between G and A across mm. 29–30 in the lowest voice, while B resolves to A from above. This creates an A-doubled F augmented triad in zone 0. The shift from a B minor seventh chord to a B diminished seventh chord across mm. 31–32 is the only instance where both participating voices move in the same direction (upwards, in this case) by semitone, hence moving to zone 2. After a balanced voice-leading move in mm. 31–32, another voice exchange occurs in mm. 32–33 involving the outer notes C and D between two D major seventh chords. This progression between mm. 31.3–33.2 is the same as that of mm. 27.3–30.1, only transposed up a perfect fifth. However, instead of concluding with movement by a whole step from G in chord 4 to A in chord 5, the passage ends with a motion by semitone from D to D in chord 10, forming the last chord of the section, a D half-diminished in zone 3.

11 Additionally, the inclusion of that note would result in the discussion of intercardinal pentachordal voice leading, which is regrettably beyond the scope of this paper. This is one of the greatest limits of discrete graphical representations of voice leading, as only a limited number of nodes can be presented and the number of voices is constrained by the constructed space. In the “OPC Space” presented by Callender, Quinn, and Tymoczko, the possibility of discussing voice leading with any cardinality is possible, but to create a visualized higher dimensional space “is an exercise in diminishing returns.” (Callender, Quinn, Tymoczko 2008, 348; Tymoczko 2011, 93).
The voice leading of the chords in the excerpt on the Extended 4-Cube Trio is shown in Figure 14. Chords that are not as even as those in the center of the graph possess considerable chromatic voice-leading potential. The balanced processes between the fully diminished seventh chords and major seventh chords are seen in zones 2 and 10, and the alternation between the minor seventh and the augmented triad is located in zone 0. Here, sonorities in the same voice-leading zone may differ with respect to harmonic quality and evenness, but they are treated as equivalent sonorities because they are connected to chords from other voice-leading zones through similar voice-leading motions. A diminished seventh chord can move to four half-
diminished chords as well as to four dominant seventh chords, through the voice-leading motion of a single half step. A major seventh chord can only move to one half-diminished and one dominant seventh chord, because the major seventh is less even than the diminished seventh.

The voice-leading path from E fully diminished to D half-diminished is fairly direct within the space of the original 4-Cube trio. While the E diminished seventh chord does not resolve directly to the G half-diminished chord, they are only one semitone apart, as shown by the dotted arrow. Similarly, the B fully diminished seventh chord in zone 2 only requires a half step to reach the D half-diminished chord in zone 3. However, Medtner chooses to take a detour by going through the major seventh chords and augmented triads while maintaining smooth voice leading, and this relationship can only be illuminated with the extended part of the graph.

An alternate reading of the passage that includes the fourth note in each cell also illustrates the strengths of the Extended 4-Cube Trio. Between mm. 28.2–29.2 and mm. 32.2–34.1, the passage can be evaluated by single semitonal voice-leading movements, as seen in Figure 15. In this interpretation, every voice-leading motion is tracked. The sonorities outlined in Figure 15(a) comprise the first four notes of each arpeggio, including the previously omitted appoggiaturas; they are shown as the first, third, fifth, and seventh chords in Figure 15(b). The sonorities outlined by dotted lines omit the fourth note and include the fifth note of each gesture. They are shown as the second, fourth, sixth, and eighth chords in Figure 15(b). This reading shows a more consistent voice-leading motion between voices. In the voice-leading process through chords 1 to 4, G in the top voice of chord 1 moves down by a half step to G♯ in chord 2. The lowest voice proceeds to move up a half step from F to G♯, before the top voice moves down another half step to F in chord 4. This process repeats through chords 5 to 8 beginning with a D
half-diminished chord. There is an additional half-step upshift in the lowest voice after chord 8, and the progression ends on a D half-diminished chord.

**Figure 15:** An alternative analysis of Medtner, *Forgotten Melodies* II Op. 39, “Romanza,” mm. 28–29 and mm. 32–34

(a) Alternative grouping of tetrachords

(b) Reduction

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G♭7 G♭maj7 G♭ G♭maj7 D♭7 D♭maj7 D♭ D♭maj7 D♭7
(c) An illustration of the progression from Medtner, *Forgotten Melodies II* Op. 39, “Romanza,” mm. 28–29 and mm. 32–34 on the Extended 4-Cube Trio

In Figure 15(c), the major seventh chords are located in the same voice-leading zones as the half-diminished chords. In this case, their function is similar to how diminished seventh chords serve as bridges between the dominant seventh and half-diminished chords. Instead of providing links between two nearly-even tetrachords, the major seventh chords provide links between nearly-even tetrachords and major triads.

In both interpretations, the Extended 4-Cube Trio allows us to visualize the inherent voice-leading potential of major seventh chords. Specifically, we see how they resemble diminished seventh chords in their ability to serve as links between different types of chords.
This model therefore grants major seventh chords increased analytical viability, despite their limited voice-leading potential regarding direct, one-step connections.

**TWO EXAMPLES FROM THE AMERICAN “GREAT AMERICAN SONGBOOK”**

The Extended 4-Cube Trio can be used to analyze popular songs with less common chord progressions. Excerpts from Jerome Kern’s “All the Things You Are” and David Mann’s “In the Wee Small Hours of the Morning” further demonstrate the model’s analytical utility. This section also addresses possible treatments of triadic multisets.

Jerome Kern’s “All the Things You Are” features linear chromatic motions that contribute greatly to the sound of the song. Figure 16 shows one of the song’s most highly chromatic moments.\(^{12}\) The line “The dearest things I know are what you are” starts on a G major chord, and ends on an E major chord, supported by a ii7–V7–I progression. Once the E major triad arrives, the progression departs from the tonal harmony and enters the chromatic voice-leading domain. After passing through a C augmented triad, the song then moves to an F minor seventh chord at the beginning the next phrase. The augmented triad is a sonority used to help smooth the voice leading between the E major triad and the F minor seventh chord, and its bridging function can be demonstrated by the Extended 4-Cube Trio.

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\(^{12}\) The transcription of the song is taken from Kern and Fordin 1988, 134–136.
Figure 16: Jerome Kern, “All the Things You Are,” mm. 36–41

(a) Score

(b) 4-part realization of mm. 39–41

(c) Locations of the chords in mm. 39–41 on the Extended 4-Cube Trio
Figure 16(b) shows a four-voice realization of the chord progression in Figure 16(a). In this analysis, E is doubled in both triads to facilitate the four-voice process.\(^\text{13}\) The E major triad, found in voice-leading zone 3, moves to the A\(^\flat\) augmented triad is in zone 4 through semitonal movement from B to C. Then, the doubled Es move by semitone in opposite directions to reach E\(^\flat\) and F to form an F minor seventh chord, which is also in zone 4. The economical voice leading is facilitated by an augmented triad, rather than by a tetrachord. In this example, the direct semitonal relationship between the E major triad and the A\(^\flat\) augmented triad is retained from the hexatonic cycle, but the multiset allows the augmented triad to directly connect to tetrachords.\(^\text{14}\)

In addition to the doubling in diatonic triads discussed above, other doublings in triads can facilitate tetrachordal voice leading. Figure 17 concerns the line “someday I’ll know that moment divine,” where a mostly diatonic bass line supports a succession of several chromatic chords. Figure 17(b) shows a four-voice reduction of the passage. From the D\(^\flat\) major triad to the B\(^\flat\) half-diminished chord (a D\(^\flat\) minor sixth chord in the score), two downward shifts occur.\(^\text{15}\) One of the D\(^\flat\)s from the root-doubled D\(^\flat\) major triad moves downward by semitone to the C of the D\(^\flat\) major seventh chord. The same voice then moves downward by a whole step and becomes B\(^\flat\) in the half-diminished chord. The F in the D\(^\flat\) major seventh chord also resolves by semitone to F\(^\flat\) of the B\(^\flat\) half-diminished chord.

\(^\text{13}\) The choice of doubling E is motivated by conventions of tonal harmony and voice leading. Since the E major triad acts as the tonic triad from the ii7–V7–I progression, B and D\(^\flat\) in the B7 chord will lead to E.

\(^\text{14}\) The progression might be mapped onto the Cube Dance in Figure 1 if the F minor seventh chord is reduced to a F minor triad. In the triadic universe, the movement from E major to F minor triad is in fact two semitonal upshifts. In this case, the Extended 4-Cube Trio successfully captures the voice leading process. However, adding the doubling of a triad destroys the semitonal relationships within a hexatonic cycle. The graph, therefore, can only display a single hexatonic relationship between one major or minor triad to the augmented triad when the doubled voice is not involved.

\(^\text{15}\) In Figure 17(a), the B\(^\flat\) half-diminished chord is given the lead-sheet symbol of D\(^\flat\)m6. However, in its four-voice realization, it is more appropriate to treat it as a half-diminished chord. B\(^\flat\) is treated as the bass, and D\(^\flat\) only appears at the last beat of the melody as a resolution of the escape tone E\(^\flat\).
Figure 17: Jerome Kern, “All the Things You Are,” mm. 45–48

(a) Score

(b) Four-voice realization

VLZ: 3 2 11 2 2
In Figure 17(b), the fifth of the $A_b$ Major triad is doubled instead of the root for a more accurate representation of the voice-leading movements in the excerpt. The doubled fifths result from the $D_b$ in the previous chord approaching the $A_b$ major harmony by leaping down by a minor seventh—shown in the reduction as a $D_b$ moving up by whole step—and the $F_b$ moving down by semitone. Figure 17(c) shows an additional node—marked by a plus sign (+)—in zone 2 of the Extended 4-cube trio that represents the fifth-doubled major triad. Fifth-doubled chords are typically not included on graphs because they are not generated by semitonal voice leading from the other represented chords. However, the voice leading in this passage demands this modification. Zone 2, which is the location of the added node, also houses the final chord of the
passage: A♭ diminished seventh. Thus, the last two chords are equivalent sonorities. This relationship would not be clear if a root-doubled A♭ major triad was represented on the graph.

With the inclusion of the new node, the net voice-leading distance between the B♭ half-diminished chord and the fifth-doubled A♭ triad is +3, as 2 voices move upward by a whole step (+4), and the third voice moves downward by a half step (-1). From the fifth-doubled A♭ major triad to the A♭ diminished seventh chord, the C and one of the doubled E♭s move down a half step, while the motion is balanced by the upward whole-step move from the other E♭ to F. Therefore, the voice-leading zone does not change.

The second example is taken from the beginning of the song “In the Wee Small Hours of the Morning,” shown in Figure 18. The chord symbols accompanying the first line appear to suggest significant changes in harmony. However, the chords used in the first three measures have C as their bass note. If the chords are realized in a four-part texture, only the top two voices move, while the C/E dyad remains unchanged until the arrival of D minor seventh chord.

**FIGURE 18:** David Mann, “In the Wee Small Hours of the Morning,” mm. 1–4

(a) Score

(b) Four-voice realization
Between the first four chords, most of the voice leading consists of downward motions by half step, though this trend is reversed as voices move upward starting at the C major seventh chord and ending on the D minor seventh chord. The chord tone to be doubled in the C augmented triad is difficult to determine in this situation. From the A minor seventh chord to the C augmented triad, the closest voice-leading motion is from G and A to a doubled G#. On the other hand, using smooth voice-leading principles, the two G#s in the C augmented triad will both lead to G# in the C major seventh that follows, which means that those G#s are no longer two objects with different functions. To reach the next C major seventh chord with the least voice-leading work, a C-doubled C augmented triad can keep the common tone C and reach B by a half step. The B would then move upward by a half step to return to a C augmented triad.

**Figure 19:** Chord relationships in “In the Wee Small Hours of Morning,” mm. 1–4
Figure 19 represents the voice-leading process discussed above. The dotted line separates the aforementioned substitution as an abstract process with respect to other actual voice-leading movements. The two nodes involving C augmented should be considered as equivalent sonorities. In contrast to the equivalent sonorities that belong to the same voice-leading zone and demonstrate similar voice-leading properties, the equivalent triads in this case share the same pitch classes but a different doubled pitch, and the doubled pitches function differently in tetrachordal voice-leading.

From these examples, we see that the lesser even tetrachords like major seventh chords also show voice-leading behaviors that are similar to the fully-diminished seventh chords, though they have less connectivity to other sonorities. They can also be connected to triads as multisets. The expanded universe also allows us to analyze passages that are not limited exclusively to either triads or tetrachords.

V. CONCLUSION

With the help of models and theories developed by Tymozcko, Cohn, and others, this study explores the potential of applying graphical methods to chromatic voice-leading processes between triads and tetrachords in actual musical contexts. There are more analytical and compositional applications in chromatic voice leading graphs to be discovered; however, in the process, analysts will have to carefully strike a balance between considerations of chord type, contrapuntal fidelity, and visual clarity.

While the Extended 4-Cube Trio can visualize more voice-leading relationships than Rockwell’s birdcage graph, Tymoczko notes that to truthfully graph an inter-cardinality voice-leading process, the size of the sets on the graph should be confined to the same cardinality.
Indeed, some of the voice-leading properties of the smaller chords will inevitably be unrepresented in order to preserve the contrapuntal fidelity in the space of the larger chords. In the case of the Extended 4-Cube Trio, all chords are treated as tetrachords. Therefore, not all of the triadic qualities, such as the hexatonic relationship in Douthett and Steinbach’s Cube Dance are present. Also, compromises like the use of substitution (Figure 19) are sometimes needed in order to exhibit those relationships. The graph is also flexible in numerous situations through the inclusion of additional nodes. If a certain multiset or tetrachord is deemed to be significant in a voice-leading process because of its different functions (Figure 17), a node can be added to the existing graph in its corresponding voice-leading zone.

Through the incorporation of triads as multisets and lesser-even tetrachords, the Extended 4-Cube Trio allows us to investigate inter-cardinality voice leading with more flexibility. It recognizes that the doubled voices within a triad contribute differently in tetrachordal voice-leading processes through fusion or splitting. This new graph provides a reasonable representation of potential voice leading within the two types of tertian chords, and this paper shows that there are benefits in doing so when analyzing musical passages that use both types of chords. Hopefully, the versatility of the Extended 4-Cube Trio will inspire further research, exploring the possibilities of this new voice-leading space and identifying other creative solutions to intercardinality voice leading.
WORKS CITED


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