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Computer Modeling in Water Management

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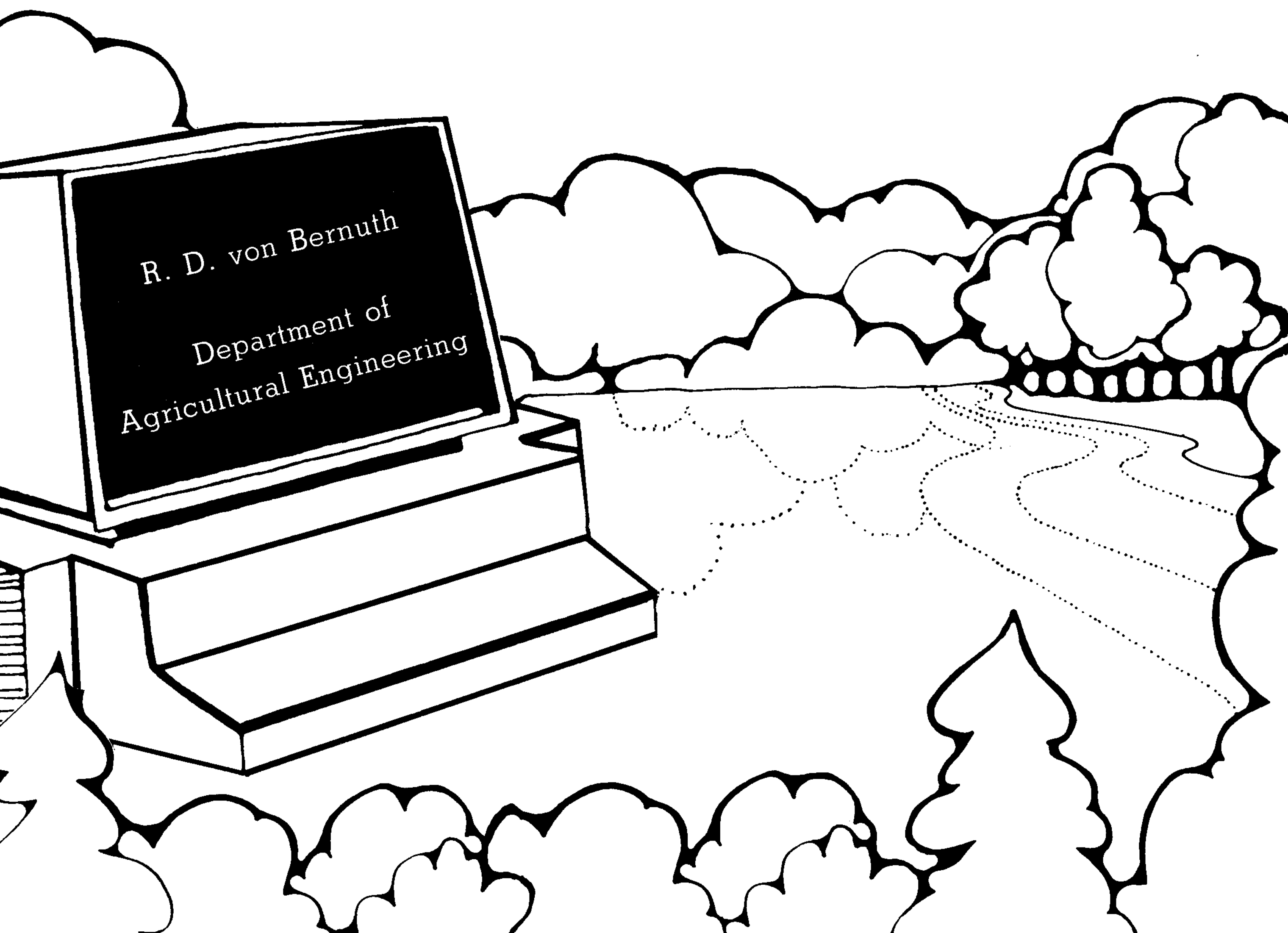

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COMPUTER MODELING IN WATER MANAGEMENT

ABSTRACT

Development of a framework for the management of surface and ground waters in areas where there is a shortage of water depends upon understanding how the two interact. Although there are cases where surface waters and ground water do not mingle, there are many more where they clearly do. The challenge in the past has been to model the interrelationship of the two. Presented here is a simplified hydrologic cycle and a survey and discussion of the numerous models that have been developed to address specific aspects of the hydrologic cycle. A brief overview of modeling is also presented to prepare the reader for the specific discussion.

INTRODUCTION

Water law in the arid western United States evolved as a result of diverting and using water, and as a means of resolving the conflicts that naturally arose when the supply was inadequate for all users. Still unresolved in many states is how to cope with the interaction between surface water and ground water. In most situations surface water and ground water do mingle and the diversion of one affects the availability of the other. Literally thousands of researchers have devoted vast amounts of time and energy to understanding how the two waters interact only to discover that their research results are highly dependent upon their particular experiment's assumptions, solution technique, or specific location.

Developed herein is a survey of modeling of the various aspects of a simplified hydrologic cycle and a discussion of each of the models. A brief discussion of modeling principles is also presented.

MODELING

What is a model? A model is a representation, generally in minature, that shows the structure or serves as a copy of something (Random House, 1980). Generally, these models involve either a physical analogy or an abstraction to mathematics. Let us presume that a model is a mathematical attempt to describe a physical phenomenon that occurs through time and space. The motivations to describe the physical phenomenon through a model may arise through an intellectual urge or a social need. The need to model the interaction of surface and ground water may still be merely an intellectual urge in many areas, but in most of the western United States it is only a matter of time until a strong social need drives science to describe that relationship.

Several approaches may be taken to modeling any phenomenon (Hooke et al., 1965), and all exist within the models of the hydrologic cycle. The most appealing mode of modeling is to formulate the laws of nature that govern the phenomenon in question. These models, known as casual or theoretical models, have the greatest appeal because they leave the least unexplained. If a theoretical cause can be found and it can be related to the actual happening through some parametric relationship, it is generally considered the most appealing and solid type of model. Such models tend to be difficult. For example, Newton's mental model for the falling apple that hit him on the head was quite complex without the simplification (but complexity) of calculus.

Frequently the person who attempts to describe a phenomenon is not as clever or fortunate as Newton and must resort to some scheme to relate what he believes to be the cause with what he believes to be the result. It may simply be that he has no idea as to cause and effect, but only through observation has discovered some measurable phenomena that are indicators of the phenomenon in question. These models are usually called descriptive or empirical models. Descriptive models may be quite simple, such as observing that the sun always rises in the east, without question of why. They may also be quite complex, such as economic models that attempt to link inflation, unemployment, and government spending.

Several approaches to empirical modeling may be taken. Commonly, the investigator will attempt to link two measurable phenonema, one of which can be determined, through a mathematical expression involving parameters. The model is known as lumped parameter when the model is global and distributed parameter when it is an assemblage of several individual and specific models (Domenico, 1972). Whatever the case may be, the model is of little use until the parameters are quantified. The usual method of quantification is statistical, and the most common statistical method is by least squares fit. An alternative approach is to synthesize the system as is often done in laboratory simulation. A disadvantage of the synthesis method is that it may not address whether the simulation is representative of the real case.

With today's high speed computers and preprogrammed analyses, resorting to a purely statistical analysis is often expedient. In the field of hydrology, the investigation of phenomena on a stochastic basis is also quite common.

APPLICATIONS OF MODELING IN WATER MANAGEMENT

Without knowing how various aspects of the hydrologic cycle interrelate, devising a workable management framework would be virtually impossible. (That is not to say it is possible even if the interrelations are known!) The simplified hydrologic cycle is treated as the total system, and the individual aspects of the cycle are treated as a distributed parameter system.

FACETS OF THE HYDROLOGIC CYCLE

The major facets of the hydrologic cycle that are frequently modeled and that could be important in determining the interrelationship of surface and ground water in the management framework of conjunctive use include: 1) precipitation; 2) evaporation; 3) infiltration and percolation; 4) plant transpiration; 5) overland flow and channel flow; 6) ground water storage, flow, and withdrawal (Davis and DeWiest, 1966).

Precipitation

Although precipitation is probably the most important variable in the long hydrologic cycle, it is also the least predictable in the short term. Even when medium length time sets of a few years of data are used for predictive purposes, the results can be in error. This is witnessed by the overappropriation of many southwestern United States rivers. Those rivers frequently were appropriated or subjected to interstate compacts based on data of a time period too short to reflect the true long term average flow. For example, the Rio Grande River Compact (Anomymous, 1938) among Colorado, New Mexico, and Texas was based on data for 1928 through 1937. Although that represented all the runoff data available at the time, other available data, such as dendrochronological data (Douglas, 1936), show that precipitation for the period of 1928-1937 was above the long term means. This introduces two of the most significant models available for precipitation prediction based on historical data.

The most common model for precipitation is a statistical model based on historical data. By analyzing past data and determining means and variances, predictions can be made for the future. The experiences in the Southwest have pointed out two very significant shortcomings. First, do the means and variance of the sample period represent that of the predictive period? No one knows, but confidence bounds should be established to aid in the use of historical data. Second, even though the long term statistics may be known, they may not be adequate as a management tool for a short time frame.

The second model available for precipitation is the dendrochronological model. However, this model only allows extension of the data base via use of tree ring correlations. This method will allow for a better estimate of the true statistics but is not much help in a predictive sense.

It should be apparent that both of these models are parametric empirical models. Considerable effort has been expended to determine causal models for weather phenomena. Some in the short term are quite good, but the term is too short for use in conjunctive management.

Two significant statistical models for predicting surface point rainfall rates have evolved from the need to predict the absorption and scattering of electromagnetic waves due to precipitation (Lin, 1978; Rice and Holmberg, 1973). These sophisticated statistical models have value as predictive probability models for conjunctive management. In addition to the traditional statistical methods used for analysis of precipitation data, several, including Yevjevich (1976), have applied stochastic processes to hydrologic data. Others have used a Markov chain simulation. Without a good causal model for predicting precipitation in the time frame of interest to conjunctive management, one must rely on historical statistics and extensions thereof by use of dendrochronology.

Evaporation

Attempts to model the evaporation phase of the hydrologic cycle have been primarily causal in nature. By use of an energy balance one can relatively accurately predict the evaporation from a surface (Jensen, 1973) given the pertinent climatological data. Evaporation pan data are frequently available to calibrate such models. In the area of plant and soil science some very sophisticated methods of measuring evaporation have been developed (Doorenbos and Pruitt, 1974; Jensen, 1973). Errors in estimation of evaporation usually do not cause excessive error in the lumped hydrologic model because of the magnitude of evaporation with respect to the other facets of the hydrologic cycle.

Infiltration and Percolation

Perhaps the second most important facet of the hydrologic cycle with respect to conjunctive use is infiltration; it is here that the first interrelationship between surface and ground water is established (presuming that the conceptual starting point of the cycle is precipitation). Water that falls must either enter the soil or run off. That which enters the soil has some probability of becoming ground water; that which does not becomes surface water.

Models being used to predict infiltration vary from simple empirical models to quite complex causal models. The Soil Conservation Service has developed a set of "family" curves for estimating infiltrations (Uhland and O'Neal, 1951). Although these curves are highly empirical, they are widely used. Other empirical models often referenced are Kostyakov (1932), Horton (1940), Holtan (1961), Kincaid et al.(1969), and United States Army Corps of Engineers (1973).

Numerous theoretically based casual models are used. Green and Ampt (1911) first developed the concept of piston flow. The Richards (1931) equation is a fundamental theoretical model. Unfortunately, the Richards equation is not directly solvable by analytical means without simplifying assumptions. (See the ground water flow section of this paper.) Another model that contributes to a solution is known as Darcy's Law (1856). (See also ground water section.) Phillip, (1975), Mein and Larson (1971), and Morel-Seytoux (1975) have also contributed theoretical models. Work by Hachum and Alfaro (1977), a refinement of Green and Ampt's work, is probably the best recent model because of its flexibility.

Plant Transpiration

When considering use of water by vegetative life, the term transpiration is used. Transpiration is generally combined with evaporation into evapotranspiration. Evapotranspiration refers to the quantity of water transpired by plants during their growth (or retained in plant tissue) plus the moisture evaporated from the surface of the soil and vegetation (Anonymous, 1949).

Jensen (1973) evaluated 16 different evapotranspiration models, and the classification of those methods is presented in Table 1. He then ranked the 16 methods and the results are presented in Table 2. As can be seen from Table 3, there is no one clearly best model, but in practice the Penman method is widely used. The Penman method does have the disadvantage of being rather complex to calculate and requires climatological data that may not be available.

Overland Flow and Channel Flow

Overland flow and channel flow have been lumped together simply because most analysis schemes for overland flow are some modification of channel flow theory. The exception to this is when a watershed is modeled as a lumped parameter system. Watershed models tend to be very site specific and too numerous to mention, but they are generally empirical in nature.

Channel flow is classified as steady or unsteady and uniform or varied according to Chow (1959). Some aspects are theoretical whereas others are empirical. Two formulae (models) quite widely used in open channel hydraulics are the Chezy formula and the Manning formula. The Chezy formula has a theoretical basis, and a discussion thereof is in Chow (1959), p. 93. (Chezy's original work was done in 1769!) Use of the Chezy equation requires estimating the resistance factor, and considerable empiricism is introduced here. In 1889 Robert Manning (see Chow, p. 98 for a discussion of the Manning equation) presented a formula that became known as the Manning formula and it is still widely used. This formula also involves considerable empiricism, and tables

Table 1

Classification of Evapotranspiration Estimating Methods of Prinicpal References

Classification	Method	References
Combination	Kohler, Nordenson and Fox	Weather Bur. Res. Paper 38, 1955, and Monthly Weather Rev. 90, 1962
	Penman	Proc. Roy. Soc., A193, 1948 and Tech. Comm. No. 53, Commonwealth Bur. of Soils, Eng., 1963
	van Bavel-Businger	Water Resources Res. Vol. 2(3), 1966 and Neth. J. Agr Sci 4, 1956
Humidity	Ivanov	WMO Tech. Note No. 97, 1968
	Ostromecki	Prace 1, Studia, Komitetu, Vol, 7 No. 1, 1965 (USDI TT 67-56052)
	Papadakis	Climates of the World, Buenos Aires, 1966
Miscellaneous	Behnke-Maxey	J. of Hydrol., 8, 1969
	Christiansen	Trans. Int'l. Comm. on Irrig. and Drain., Vol. III, 1969 and J. Irrig. and Drain. Div., Am. Soc. Civ. Engr., 94, 1968
	Olivier	Irrig. and Climate, Edward Arnold Ltd., Lonodon, 1961
Radiation	Jensen-Haise	Trans. Am. Soc. Agr. Engr., 14, 1971 and J. Irrig. and Drain. Div Am. Soc. Civ. Engr., 89, 1963
	Makkink	J. Inst. Water Eng., 11, 1957, and Am. Soc. Agr. Engr. ET Symposium, 1966
	Stephens-Stewart	J. Hydr. Div., Am. Soc. Civ. Engr. 1965, and Publ. 62 Int'l Assoc. Sci. Hydrol., 1963
	Turc	Ann. Agron. 12, 1961, and Am. Soc. Agr. Engr. ET Symposium, 1966
Temperature	Blaney-Criddle	USDA SCS Tech. Rel. 21, 1967 (Rev Sept. 1970)
	Thornthwaite	The Geographical Rev., 38, 1948, and Public, in Climat., 8 Lab. of Climat., Centerton, N. Jersey, USA, 1955

SOURCE: Jensen, 1973

Table 2

Summary of Estimated ET, Expressed as Percent of Measured ET, Mean RMS, mm/Day and Rank of Accuracy for Coastal and Inland Semiarid to Arid Areas

	Coastal			Inland-Semiarid to Arid		
	ET (%)	**RMS (mm/Day)**	**Rank**	**ET (%)**	**RMS (mm/Day)**	**Rank**
Combination						
Kohler et al.	107	0.50	3	91	1.05	3
Penman	123	.67	5*	106	.84	2
van Bavel-Businger 0.25	129	.80	7*	105	.84	1*
van Bavel-Businger 0.50	144	1.22	10	117	1.13	4
van Bavel-Businger 1.00	166	1.86	11	135	1.91	12
Humidity						
Ivanov	99	.95	4*	109	2.26	8*
Ostromecki	116	1.05	7*	147	4.67	14
Papadakis	89	1.11	6*	87	1.53	7*
Miscellaneous						
Behnke-Maxey	183	2.44	12	118	1.86	8*
Christiansen R_s	104	.34	1	82	1.26	7*
Christiansen Pan Evap.	--	--	--	83	1.27	6*
Olivier	73	1.11	9	94	1.45	5
Radiation						
Jensen-Haise	77	.83	6*	104	.97	1*
Makkink	89	.83	5*	68	2.18	11
Stephens-Stewart	80	.72	5*	80	1.22	6*
Turc	104	.60	2	74	1.78	9
Temperature						
Blaney-Criddle	96	.84	4*	67	1.77	10
Thornthwaite	85	1.19	8	55	2.55	13

*Tie.

Coastal areas include Aspendale, Copenhagen, Lompoc and Seabrook.

Inland-Semiarid to arid areas include Brawley, Davis, Kimberly, and South Park.

SOURCE: Jensen, 1973.

Table 3

The Top Five Methods of Estimating Evapotranspiration that were Evaluated in the Two Regimes

Coastal	Inland-Semiarid to Arid
1. Christiansen R_S	1. Jensen-Haise and van Bavel-Businger, 0.25
2. Turc	2. Penman
3. Kohler et al., Lake	3. Kohler et al., Lake
4. Blaney-Criddle and Ivanov	4. van Bavel-Businger, 0.50
5. Makkink, Penman, and Stephens-Stewart	5. Olivier

Estimates using pan evaporation were not evaluated because a standard pan was not used at each site, and pan evaporation data were not available at all sites.

SOURCE: Jensen, 1973.

of Manning's roughness coefficient are available. Because of the time for which Manning's equation has been known, and its widespread use, its validity and flexibility are well established.

Ground Water Storage, Flow, and Withdrawal

Ground water storage, flow, and withdrawal are here lumped into one general model even though they are usually modeled separately simply because in a conjunctive use sense it is not of much value to discuss one without the other two. The general equation that describes these interactions is derived by considering water mass balance, water momentum balance, and Darcy's law. It is a parabolic partial differential equation and is known as the diffusion equation (Davis and DeWiest, 1966; Mercer and Faust, 1980a). The pertinent variables of the fluid flow process for modeling ground water flow are summarized in Table 4. Although the same form of theoretical equation appears in theories of unsteady flow of heat and electricity, there is no known closed form analytical solution for it without simplifying assumptions or approximations. The original theory of ground water motion was developed by M. King Hubbert (1940).

Table 4

Variables of Fluid Flow Process

Dependent Variable	Application Dependent Variable
Fluid pressure	Porous media
Hydraulic head	Doubly porous media
Hydraulic potential (or drawdown)	Discrete fractured media
	Single-phase fluid
	Two-phase fluid

SOURCE: Mercer and Faust (1980b).

Simplifications

The first simplification that is usually made is to assume steady state conditions. Usually steady state simply means that acceleration (change in velocity) terms are zero. Other simplifications include homogeneity and isotropicity. With those simplifications an equation known as La Place's equation results. Analysts often assume zero flow in one or more directions to simplify the solution.

Each assumption made departs from the actual situation a little more, and the applicability of the model can only be verified by calibration or sensitivity runs. In conditions where one or more of the assumptions do not hold (for example, steady state), a series of steady state solutions at very short time intervals are run. The same type analysis can be done for non-homogeneity and non-istropicity.

There are a great many models in existence to describe ground water, but the only real differences among them come from simplifying assumptions made.

Approximations

Modern high speed, high capacity computers have made some previously known but excessively laborious approximating solution techniques possible. These techniques allow for solutions to the general ground water flow equation without making simplifying assumptions. There are two basic methods of approximating and iterating a solution to the general groundwater equation. Those two methods are known as finite-difference method (FDM) and finite-element method (FEM). The FDM approximates a differential equation with a differential approach, and the FEM approximates a differential equation with an integral approach. Although the two approaches to the solution differ in direction, they converge to the same solution. Two methods of converging to a solution are used, and the advantages and disadvantages are shown in Table 5.

A summary of the advantages of FEM and FDM approaches to approximating solutions to the ground water flow equation is shown in Table 6. It is worthy of note that there is a potential accuracy difference between the two.

Table 5

Advantages of Methods of Finite Solution

Advantages	Disadvantages
DIRECT METHODS	
Sequence of operations performed only once No initial estimates required No iteration parameters required Tolerance required	May be inefficient in terms of storage (RAM) and computation time for large problems Can have round-off errors
ITERATIVE METHODS	
Efficient in terms of storage (RAM) and computation time for large problems	Requires initial estimates Requires iteration parameter Requires tolerance Matrix must be well conditioned

SOURCE: Mercer and Faust (1980b).

Table 6

Advantages of Approximation Methods

Advantages	Disadvantages
FINITE - DIFFERENCE METHOD	
Initiative basis Easy data input Efficient matrix techniques Program changes easily	Low accuracy for some problems Regular grids
FINITE - ELEMENT METHOD	
Flexible geometry High accuracy easily included Evaluates cross-product terms better	Mathematical basis is advanced Difficult data input Difficult programming

SOURCE: Mercer and Faust (1980b).

Boundary and Initial Conditions

In order to obtain a unique solution to a partial differential equation a set of known values for the process at given times and places must be input. These conditions are called initial and boundary conditions. Steady-state problems require only boundary conditions. (By definition we have said that conditions do not change with time and therefore time dependent values, initial conditions, are not necessary.) It follows then that for unsteady-state problems both boundary conditions and initial conditions must be specified. In addition to specifying initial and boundary conditions, some key parameters must be known or approximated in order to arrive at a meaningful solution to the ground water equation. The storage coefficient, transmissivity, and dimensions of the aquifer must be known. (For a discussion of parameters, see Domenico, 1972).

Specific Applications of the General Equation

Several variations of the general ground water equation with appropriate assumptions have been presented and some of the more significant ones follow: 1) confined, artesian flow, 2) leaky artesian conditions, 3) water table conditions, 4) radial flow (Theis, 1935) solution, 5) partially penetrating wells, and 6) unconfined aquifers. For a discussion of the better known solutions, see Mercer and Faust (1980a and 1980b) or McWhorter and Sunada (1977). Toth also presented an analysis for a small drainage basin (1963).

MERGING MODELS

The Hydrologic Model

A series of distributed specific models can be integrated into one lumped hydrologic model. Without a doubt, the lumped model will be complex, expensive, and have local anomalies that will be disconcerting. However, most would agree that some information is better than none at all.

In the end, a model is useful only if yields information that either describes the real situation or gives insight toward a better understanding the real situation. By doing a good job of applying the models described herein, one could adequately describe the hydrologic cycle. This could lead to quantifying the interrelationship between surface waters and ground waters in order to develop a management framework.

In the eyes of the author, a mere description of the interaction of waters is inadequate. Of what value is a model if there is no assessment of the impacts of possible management actions? Therefore, the hydrologic model as presented thus far is insufficient for determining the feasibility of a given management framework, but must somehow incorporate the impact of the result of management action. Such was the approach taken by the state of Idaho, for example, in establishing an economic basis whereby the state engineer would appropriate ground water. Their approach was not only to develop the water relationships, but to further define legal action from an economic basis (von Bernuth, 1969).

Macro Models Including Economics

The governing factor of most management decisions is based upon economics, and therefore any model or system of models for use in management decision making must include economic considerations. The example cited above in the state of Idaho is an economic model. A more recent and comprehensive example was presented by Supalla (1982). Supalla used a macro model that included submodels of ground water, evapotranspiration, production, and economics to evaluate the effects of ground water management alternatives for the Northern United States plains. Such macro models may become integral tools for management decision making.

Economic Model Byproducts

Using an economic model to evaluate a policy gives several interesting byproducts. The direct result is the economic impact; values or costs of policy options in dollars per unit of water conserved, dollars per unit of additional life prevented, or dollars per unit of irrigated area preserved are indirect as indicated. By use of value theory and constrained maximization, shadow prices for the constrained values (if linear programming is used) evolve and inputed Lagrangian multipliers evolve (if Lagrangian constrained maximization is used) (Hillier and Liebermann, 1980; Herfindahl and Kneese, 1974).

GENERAL CAUTIONS ON THE USE OF MODELS

The numerical model in ground water simulation has greatest use for general problems involving aquifers having irregular boundaries, heterogeneities, or highly variable pumping and recharge rates. The results of a model must be interpreted both for agreement with reality and for their effectiveness in addressing the questions asked.

Factors that can cause erroneous results from a model are those of localized rapid change. For example, the ground water model is stressed by recharge surface diversions and stream flow.

Models may be misused in many ways, but the worst way is to have blind faith that the results are correct. Other misuses are over-modeling and use in situations inappropriate to the model. Frequently it is a case of too much too soon. Conditions in reality can change from those for which the model was calibrated, and models may have some built in error.

CONCLUSIONS

Computer modeling allows for the simulation of interactions of natural phenomena under conditions that may not be attainable or controllable in their true natural state. This ability to simulate allows man to study the facets of the hydrologic cycle and related use/production systems for the purpose of evaluating management alternatives. Workable models of many facets of the hydrologic cycle have been discussed. One macro model encompassing the economic impact of water management alternatives was also discussed. Because most management decisions involve economic judgment criteria, it is likely that the macro model including economics will become a widely used management decision tool.

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