Improvement of the Prediction of Surface Ozone Concentration Over Conterminous U.S. by a Computationally Efficient Second-Order Rosenbrock Solver in CAM4-Chem

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**RESEARCH ARTICLE**

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**Key Points:**
- The second-order Rosenbrock solver significantly reduces the positive bias of surface ozone concentration over CONUS, Europe, and East Asia.
- By refining the time step size, the first-order implicit solver used in CAM4-Chem fails to offer a statistically significant improvement.
- The second-order Rosenbrock solver is almost twice as fast as the original first-order implicit solver.

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**Abstract**
The global chemistry-climate model (CAM4-Chem) overestimates the surface ozone concentration over the conterminous U.S. (CONUS). Reasons for this positive bias include emission, meteorology, chemical mechanism, and solver. In this study, we explore the last possibility by examining the sensitivity to the numerical methods for solving the chemistry equations. A second-order Rosenbrock (ROS-2) solver is implemented in CAM4-Chem to examine its influence on the surface ozone concentration and the computational performance of the chemistry program. Results show that under the same time step size (1800 s), statistically significant reduction of positive bias is achieved by the ROS-2 solver. The improvement is as large as 5.2 ppb in Eastern U.S. during summer season. The ROS-2 solver is shown to reduce the positive bias in Europe and Asia as well, indicating the lower surface ozone concentration over the CONUS predicted by the ROS-2 solver is not a trade-off consequence with increasing the ozone concentration at other global regions. In addition, by refining the time step size to 180 s, the first-order implicit solver does not provide statistically significant improvement of surface ozone concentration. It reveals that the better prediction from the ROS-2 solver is not only due to its accuracy but also due to its suitability for stiff chemistry equations. As an added benefit, the computation cost of the ROS-2 solver is almost half of first-order implicit solver. The improved computational efficiency of the ROS-2 solver is due to the reuse of the Jacobian matrix and lower upper (LU) factorization during its multistage calculation.

**1. Introduction**
Atmospheric chemistry is important for global climate simulation because of the close coupling of transport, physical, chemical, and biological processes. The feedbacks among chemical reactions, climate equilibria, anthropogenic emissions, and land use changes provide a new dynamical perspective on global and regional climate and air quality predictions. Among the atmospheric chemical constituents, tropospheric ozone is a critical pollutant that can significantly affect ecosystems, agriculture productions, public health, and climate forcing [Stevenson et al., 1998; Fiscus et al., 2005; Karnosky et al., 2007; Cooper et al., 2010; Sun et al., 2015]. However, ozone is not directly emitted and its complex photochemical reaction mechanism makes its simulation a challenge. Significant bias in the prediction of ozone concentration exists zonally and seasonally for both single model output [Zeng et al., 2008; Lamarque et al., 2012; Val Martin et al., 2015] and multimodel ensemble mean results [Stevenson et al., 2006, 2013; Young et al., 2013]. There are also inconsistencies between the estimated ozone concentration from global climate models and the observed seasonal cycle [Fiore et al., 2014]. In the CAM4-Chem model we are using, optimizing the dry deposition scheme based on land use changes has significantly improved the simulation of summertime (June, July, and August, JJA) surface ozone concentration over the U.S. [Val Martin et al., 2014, 2015]. However, further efforts are required to reduce the remaining bias.

Few studies have investigated the numerical chemical solver itself in the performance of global chemistry-climate models. Shampine [1982] studied initial value problems for stiff systems of ordinary differential equations (ODEs) and proposed an approach to automatically select either an explicit Runge-Kutta formula...
or Rosenbrock formula at every time step. The Rosenbrock method was competitive with the backward differ-
entiation formulas (BDF) in some circumstances. The sparse matrix vectorized gear code (SMVGEAR)
developed by Jacobson and Turco [1994] was also implemented in global chemistry models such as GMI
[Rotman et al., 2001], IMPACT [Rotman et al., 2004], GATOR-GCMOM [Jacobson and Ginnebaugh, 2010; Jacob-
son et al., 2015], and GEOS-Chem [Zhang et al., 2011]. However, this solver used an iterative method to solve
the ODEs and included the reevaluation of the Jacobian matrix during the iteration, which was intrinsically
slower than other solvers using a noniterative method. Sandu et al. [1997] tested a set of box-model atmo-
spheric chemistry problems (TMK model, CBM-IV model, AL model, a NASA HSRP/AESA stratospheric model,
and an aqueous model) with different solvers, including LSODE (Livermore Solver for Ordinary Differential
Equations) and Variable-coefficient Ordinary Differential Equation solver (VODE) that were expected to per-
form similarly to SMVGEAR. The benchmark problems covered a wide range of photolytic, homogeneous
(gas phase, liquid phase) and heterogeneous (gas-liquid) reactions. The results showed that the Rosenbrock
solvers were the most cost effective and performed well for real problems with large variety of conditions
that could occur at different grid cells. Verwer et al. [1999] applied a second-order, L-stable Rosenbrock
(ROS-2) method to the three-dimension atmospheric reaction and transport problem including photochem-
istry, advection, and diffusion. Three chemistry models (RIVM, CBM-IV, and WET) using the ROS-2 method
were examined and the ROS-2 method was proven to be an excellent candidate for global air quality
modeling with large time steps on the order of minutes. Blom and Verwer [2000] examined different
operator-splitting methods for the atmospheric transport-chemistry problems and the test results revealed
that the Rosenbrock W-method, split at the linear algebra level, was a better option than Strang operator-
splitting or source splitting. The W-method avoided the artificial stiff transients during the chemistry com-
putation and boundary condition issues for integration in time, though its implementation was complex.
Long et al. [2013] developed a coupled chemistry and climate system model by linking the NCAR Commu-
nity Atmosphere Model (modal-CAM v3.6.33, the atmospheric component of Community Climate System
Model (CCSM3)) and the Max Planck Institute for Chemistry’s Module Efficiently Calculating the Chemistry
of the Atmosphere (MECCA; v2.5) to investigate the multiphase process in the atmosphere. The Kinetics Pre-
Processor (KPP) [Sandu and Sander, 2006] package was used to provide user-defined solvers and the bench-
mark intercomparison among three solvers from the Rosenbrock family, namely ROS-2, ROS-3, and RODAS-
3, showed good agreement for ozone and OH radical prediction. Note that only the atmospheric compo-
nent was considered in the work of Long et al. [2013] and the effect of different solvers in a long-term simu-
lation was not explored.

In this paper, we do not achieve improvement by a better design of the sparse matrix. Breaking the data
structure in order to introduce some external packages would disable existing optimizations. Instead, we
seek to obtain improved performance of chemistry and computation from the algorithm itself. Therefore,
based on the literature above, the numerical solvers from the Rosenbrock family are good alternatives for
full atmospheric chemistry and climate simulation. These methods have already been incorporated into
some regional models such as the Weather Research and Forecasting model coupled with Chemistry (WRF-
Chem) and the Community Multi-scale Air Quality Model (CMAQ) [Linford et al., 2009; Sarwar et al., 2013].
But their use in global models is very limited. In this study, the global climate model we use is the Commu-
nity Earth System Model (CESM) with online chemistry activated (CAM4-Chem) [Lamarque et al., 2012]. Cur-
rently the chemical solver uses a fully implicit Euler method that gives unconditional stability but only first-
order accuracy in time [Kinnison et al., 2007]. This could be part of the reason for poor performance of ozone
in previous work with CAM4-Chem. To bridge the gap between the need for better estimate of ozone from
a scientific perspective and the limitation of a low order accuracy solver in the current CAM4-Chem, the
ROS-2 solver is implemented to replace the original chemical solver and tested to see whether there is any
benefit for the global climate and chemistry simulation. Unlike the implicit Euler method, the ROS-2 method
avoids the reevaluation and redecomposition of the Jacobian matrix. These are the most time-consuming
parts in a chemical solver [Daescu et al., 2000]. Hence, we are also interested in the improvement of compu-
tational performance of chemistry with the ROS-2 solver. The mathematical formula of the ROS-2 method
and its implementation in CAM4-Chem are described in the methodology section. The predictions of sur-
face ozone concentration over the conterminous U.S. (CONUS) between the ROS-2 solver and the original
first-order implicit solver are compared, as well as their computational efficiencies with varying numbers of
processors on a massively parallel supercomputer. Finally, we discuss the major differences we have
observed, and make further recommendations.
2. Methodology

2.1. Chemistry Configuration in CAM4-Chem

The Community Earth System Model (CESM version 1.2.2) is a state-of-the-art global climate model. It consists of four components: atmosphere, land and land-ice, ocean, and sea ice. CAM4-Chem is an implementation of atmospheric chemistry in CESM and its chemistry is fully coupled with the radiative absorption processes of the Community Atmosphere Model, the atmospheric component of CESM (CAM4) [Neale et al., 2013]. The chemistry mechanism in the current CAM4-Chem version is adapted from the standard Tropospheric Model for Ozone and Related chemical Tracers (MOZART-4) [Emmons et al., 2010]. However, more chemistry-specific parameterization such as dry/wet deposition schemes and species types are expanded in the current CAM4-Chem version. The details about their differences, together with CAM4-Chem’s representation of atmospheric chemistry in the global model, are well evaluated in previous work [Lamarque et al., 2012]. In total, the chemical mechanism includes 212 reactions, with 40 photochemical reactions and 172 gas phase reactions. There are 103 chemical species: 8 species (CH$_4$, N$_2$O, CO, Rn, Pb, H$_2$, HCN, and CH$_3$CN) are solved explicitly by the first-order (forward) Euler method and the remaining 95 species are solved implicitly by a backward Euler method with a Newton-Raphson iteration for quick convergence. These 95 species include all the chemically active species such as ozone and OH radicals, which contribute most to atmospheric chemical reactions. It can be further decomposed into three components: production $P(y)$, loss $L(y)$, and independent forcing $I(y)$ terms. $P(y)$ and $L(y)$ are mainly calculated by the species mixing ratios and reaction rates, while $I(y)$ is evaluated based on the external forcing (i.e., aircraft and lightning emissions).

2.2. ROS-2 Solver Description

In the state-of-the-art CAM4-Chem version, an operator-splitting approach is used and the atmospheric chemical reactions are integrated by the stiff ODE solver separately from other processes like dynamics and physics. Thus, for the chemistry update, each control volume behaves like a box model. The equations for chemical species conservation and reaction in the atmosphere take the form of an autonomous system:

\[
\frac{Dy}{Dt} = F(y) = P(y) - L(y) + I(y)
\]  
(1)

where $y = (y_1, y_2, \ldots, y_N)^T$ is the vector of volume mixing ratios for $N$ species (here represents the 95 implicit species) at given latitude, longitude, and vertical coordinate; the source term $F(y)$ represents the atmospheric chemical reactions. It can be further decomposed into three components: production $P(y)$, loss $L(y)$, and independent forcing $I(y)$ terms. $P(y)$ and $L(y)$ are mainly calculated by the species mixing ratios and reaction rates, while $I(y)$ is evaluated based on the external forcing (i.e., aircraft and lightning emissions).

The two-stage, linear-implicit Rosenbrock scheme for the ODE above can be written as Verwer et al. [1999]:

\[
(1-h_1A)k_1 = F(y^n)
\]  
(2)

\[
(1-h_1A)k_2 = F(y^n + h_1k_1 - 2k_1)
\]  
(3)

\[
y^{n+1} = y^n + \frac{3}{2}h_1k_1 + \frac{1}{2}h_2k_2
\]  
(4)

where $I$ is an $N \times N$ identity matrix; $h$ is the time step size; $A = \frac{df(y)}{dy}$ $|_{y=y^n}$ is the Jacobian matrix at time $t = t^n$; $y^n$ and $y^{n+1}$ are the solution vectors of species mixing ratios at time $t = t^n$ and $t^{n+1}$, respectively. Vectors $k_1$ and $k_2$ are the intermediate solutions at each stage. Parameter $\gamma$ is a constant and appears in the stability function with $z = h\gamma$ for problem $y' = Ay$ as

\[
R(z) = \frac{1 + (1 - 2\gamma)z + (\frac{1}{2} - 2\gamma + \gamma^2)z^2}{(1 - \gamma z)^2}
\]  
(5)

A solver is defined as $A$-stable if $|R(z)| \leq 1$ as $z \to \infty$ and it is further called $L$-stable if $R(z) \to 0$ as $z \to \infty$. The ROS-2 solver proposed above is $A$-stable for equation (5) if and only if $\gamma > \frac{1}{2}$. Furthermore, by choosing $\gamma = 1 \pm \frac{1}{\sqrt{\epsilon}}$ this scheme becomes $L$-stable ($R(\epsilon) = 0$), which is good for simulating some chemical species with a short life span in the atmosphere. In practice, $\gamma = 1 + \frac{1}{\sqrt{\epsilon}}$ is usually chosen due to its better nonlinear stability behavior under large time steps [Verwer et al., 1999] and this value is thus used in this study. An obvious advantage to the implementation of the ROS-2 solver is that it does not require the reevaluation of the Jacobian matrix at each stage, while the Newton method requires the reevaluation of the Jacobian matrix.
matrix during each iteration. Therefore, it can utilize the same “lower upper” (LU) factorization result at each stage of the solution update. Since updating the Jacobian matrix and conducting the LU factorization are the most time-consuming operation, this benefit should speed up the chemistry update and save much computation time [Daescu et al., 2000].

2.3. Time Step Size Setting
In this work, \( h = 180 \) and \( 1800 \) s are chosen as two different time step sizes to examine whether the first-order implicit solver and the ROS-2 solver can benefit from the time refinement. It is noted that the previous literature claimed that the ROS-2 solver was able to work under time step size of \( 600 \) and \( 900 \) s [Verwer et al., 1999; Blom and Verwer, 2000]. However, those studies were either working on a simple chemical mechanism or a benchmark simulation without considering the complexity of an earth system model. Furthermore, they agreed that the necessity to form a better conditioned system, resolve the initial transients accurately and handle the nonlinear chemistry in the real atmosphere might require a more restricted time step size [Shampine, 1982; Sandu et al., 1997; Verwer et al., 1999; Blom and Verwer, 2000]. In addition, the concepts of A-stability and L-stability are defined for an idealized linear system. The real atmospheric chemistry system is nonlinear and can be very stiff with the existence of fast and slow reacting species. Thus, a small time step is usually recommended to make the chemical reaction system valid for “linearization” (keep the property of linear system) and avoid impacting the quality of numerical integration. Based on those concerns, we implement an adaptive time step method for the ROS-2 solver when the time step size equals \( 1800 \) s. If the program detects a location where a negative solution is generated by the ROS-2 solver, it refines the time step by a factor of 2 for that location and recomputes the chemistry. This procedure is repeated until the refined time step size is less than \( 180 \) s, in which case we simply set the time step size to \( 180 \) s to save computation time. We have found that the ROS-2 solver can run continuously with a fixed \( 180 \) s time step for the standard full tropospheric chemistry mechanism TROP_MOZART [Emmons et al., 2010].

2.4. Algorithm and Implementation
The steps of the algorithm are as follows:

a. Calculate the independent forcing term, which is treated as invariant as in the original first-order implicit solver design.

b. Calculate the linear and nonlinear components of Jacobian matrix to form the left-hand side system in equations (2) and (3). CAM4-Chem uses the same chemical preprocessor as MOZART-4 [Lamarque et al., 2012], which reads the specific chemical mechanism file and converts it into Fortran code to provide the input for the calculation of linear and nonlinear parts. For the TROP_MOZART mechanism, there are 737 nonzero matrix entries of the system matrix and they are computed explicitly and exactly. The sparsity pattern reveals a classic “arrow matrix” ordering with the arrow pointing up as shown in Figure 1a. If the LU factorization is performed directly on this matrix, almost all the entries in the upper right matrix will be changed from an initial zero to a nonzero value during the execution. This is known as a fill-in issue [Lee, 2010] and it will significantly increase the cost of computation, which is determined by the total number of nonzero entries rather than by the size of the sparse matrix. Therefore, a permutation operator is applied before doing the LU factorization and the system matrix is flipped over to point down (see Figure 1b). The LU factorization starts with the nearly diagonal part of the matrix and all fill-ins occur down the right-hand side column where there are already a lot of nonzero entries. Note that although the system is different between the first-order implicit solver and the ROS-2 solver, the formation of the Jacobian matrix is exactly the same. Hence, the routines from the original first-order implicit solver can be used directly with only minor modification. It is also worth noting that in other solvers like SMVGEAR, the JSPARSE algorithm developed by Jacobson and Turco [1994] is used to reorder the Jacobian matrix based on the combined production and loss terms. In addition, the partial pivoting process is removed from the decomposition process to further reduce the number of matrix calculation. Instead, the chemical preprocessor of CAM4-Chem uses a diagonal Markowitz scheme [Lee, 2010] to reorder the Jacobian matrix, which searches on diagonal elements and chooses the pivot with the smallest Markowitz weight.

c. Form the LU factorization for the system matrix. As shown in Figure 1, the system matrix is very sparse with about 90% zeroes. In the current version of CAM4-Chem, the LU factorization is hardwired with a fixed pivoting order by knowing exactly the nonzeroes of system matrix. There are a total of 824 nonzero
matrix entries after performing the LU factorization. Since the hardwired LU factorization does not need to test for pivots, the routine is highly optimized and efficient. It should be mentioned that extensive testing is done to ensure that the pivot strategy is robust even with the modification of the system in the ROS-2 formulas. We compared the accuracy of LU solution by capturing selected system matrices (examples used in section 3.1) and analyzing them in MATLAB. The results showed that either full pivoting (including both row and column pivoting) or partial pivoting (only including row pivoting) had no appreciable impact on the accuracy of solution as compared with the fixed pivoting strategy.

d. Calculate the source term $F$ on the right-hand side.

e. Solve for the first-stage solution vector $k_1$ with explicitly programmed steps of reduction.

f. Update source term $F$ for the second stage with intermediate approximation $y^{n+1}$. 

g. Solve for the second-stage solution vector $k_2$ using the same LU factorization result from the first stage.

h. Update species mixing ratio vector from $y^n$ to $y^{n+1}$ for the next time step.

3. Results and Discussion

3.1. Numerical Analysis of the First-Order Implicit Solver and the ROS-2 Solver

Before comparing the model simulation results, it is necessary to verify that both solvers converge to the same solution if the time step is small enough and they provide the numerical convergence rate as we expect. In order to verify this, a test problem is chosen for a linear system $\frac{du}{dt}=a$. Given the initial condition $u_0=1$ at $t_0=0$, it is trivial to derive the analytical solution, $u(t)=e^t$. By arbitrarily setting $t_{end}=2$, separate numerical simulations with different time step sizes are performed to integrate from $t_0$ to $t_{end}$ and the numerical results for both solvers are summarized in Table 1. Note that for the ROS-2 solver, we only use the given fixed time step size for verification tests and no refinement of time step is involved. In Table 1, $\Delta t$ is the time step size for a numerical simulation; $\text{Abs (Error1)}$ is the absolute error between the exact solution and the numerical solution generated by the first-order implicit solver at time $=t_{end}$; $\text{Rate1}$ is the ratio of current absolute error over the previous absolute error with twice time step size for the first-order implicit solver; $\text{Abs (Error2)}$ and $\text{Rate2}$ are the same but for the numerical solution generated by the ROS-2 solver. The decimal precision of numerical results is shown as six digits so that the absolute error less than 1.E-06 is not included in the table. For the time step sizes finer than 1.25E-02, the error between exact solution and the
A numerical solution generated by the first-order implicit solver is cut by half when the time step size is reduced by a factor of two, indicating a linear convergence of the first-order implicit solver and thus first-order numerical accuracy. On the other hand, a quadratic convergence (second-order accuracy) is observed for the ROS-2 solver as the error between exact solution and the numerical solution generated by the ROS-2 solver is cut by four when the time step size is reduced by a factor of two. Therefore, for a linear system, it proves that the convergence rate and accuracy of both solvers behave as we expect. Due to the quicker convergence of the ROS-2 solver, its absolute error with time step size equal to 3.91E-04 is already around 3.10E-06, while the absolute error for the first-order implicit solver with the same time step size is about 2.89E-03. However, Table 1 reveals that both solvers will eventually converge to the same solution as long as the time step size is small enough.

For the nonlinear system, the CAM4-Chem chemistry modules are first isolated such that other processes like dynamics and physics are excluded and the update of chemistry behaves exactly like a box model. Then input from some arbitrarily selected grid cells are used to provide the species concentrations, chemical reaction rates, and independent forcings as initial condition. Lastly, a series of box-model simulations are performed for one climate time step (1800 s by default) with different numerical time step sizes. We have tested input from different grid cells and they generally provide consistent results of numerical analysis. Hence, one grid at Eastern U.S. (latitude $= 34.4^\circ$, longitude $= -90.0^\circ$, level $= 26$ (bottom layer)) is used as an example for illustration (see Table 2a). Compared to Table 1, one additional column named “Ozone” is added to show the ozone concentration (units: mol/mol) from the box-model simulation after one climate time step. Note that the chemistry used in CAM4-Chem is a nonlinear system and therefore no analytical solution exists. A common approach to obtain the “exact” solution in this case is to solve the system with a very tiny numerical time step size. We find out that for the ROS-2 solver, when the time step size is about 1.76 s, the ratio of absolute difference between ozone solution calculated by 1.76 s and ozone solution calculated by 3.52 s over the ozone solution calculated by 3.52 s is smaller than 1.E-12, which is beyond the decimal precision in CAM4-Chem. Hence, 6.03432167442E-13 mol/mol is chosen as the “exact” solution here for numerical analysis. It turns out that the first-order implicit solver still converges linearly for this nonlinear system while a quadratic convergence is again observed for the ROS-2 solver. Table 2a also shows that if the time step size is small enough, the first-order implicit solver will still converge to the “exact” solution with relative error (“Abs(Error2)” column divided by “Ozone” column) smaller than 1.E-12. However, the ROS-2 solver behaves better here since the ROS-2 solver using a numerical time step size equal to 1800 s produces a solution with relative error smaller than 1.E-7.

It is worth noting that the initial condition for the convergence test above is from the initial data of CAM4-Chem. The species concentration (e.g., ozone) is a little away from the realistic chemistry state of the

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**Table 2. Numerical Analysis of the First-Order Implicit Solver and Second-Order Rosenbrock (ROS-2) Solver for a Nonlinear System With Box-Model Simulations (Units: mol/mol)**
atmosphere due to the spin-up effect. Therefore, we further examine another case with the initial condition
from the output of CAM4-Chem after 3 month simulation. It turns out that when the time step size is refined
to 3.43E-3 s, the relative error of the ROS-2 solver is smaller than 1.E-12 and thus 5.973488706086E-08 mol/
mol can be used as the “exact” solution for ozone (see Table 2b). For the ROS-2 solver, quadratic conver-
gence is observed after the time step size is refined to 7.03 s and smaller, while the first-order implicit solver
converges linearly at the similar scale. The first-order implicit solver at time step size equal to 1800 s surpris-
ingly shows smaller relative error (7.95E-4) than that for the ROS-2 solver at the time step size equal to
1800 s (2.52E-3). However, both relative errors can be thought small enough when doing the analysis of
ozone concentration in the realistic atmosphere. Overall, based on the box-model analysis here, it seems
appropriate to use the ROS-2 solver with 1800 s time step for the real simulation of atmospheric chemistry.

3.2. First-Order Implicit Solver and ROS-2 Solver at 1800 s Time Step

We focus our analysis on annual and summertime mean surface ozone concentrations generated from a
decadal simulation from 2001 to 2010. Using the 1800 s time step, the output from the ROS-2 solver (ROS-
2_1800s) and the first-order implicit solver (ORI_1800s) are compared. Figure 2a shows a nationwide
decrease of 10 year averaged annual mean surface ozone concentration between the ROS-2_1800s and the
ORI_1800s (ROS-2_1800s minus ORI_1800s). The largest and smallest differences are
\[2\text{3.12} \text{ and } 2\text{0.58 ppb}
\]
at the grid-cell level. By averaging the whole corresponding grids inside each state, the state-level differ-
ences are summarized in Table 3 and the Student’s \(t\) test suggests that the state-level differences of annual
mean surface ozone concentration between the ROS-2_1800s and the ORI_1800s are statistically significant
\(\alpha=0.05\) for 47 states except Washington. Considering the 10 year averaged surface ozone concentration
during the summer season when the photolytic reaction is the most active during the year, Figure 2b clearly
presents an even larger difference than the annual difference, especially over the Northeastern U.S. The
largest and smallest differences at the grid-cell level are \(-5.62\) and \(-0.18\) ppb, respectively. For the state
level, the ROS-2_1800s again shows a lower prediction of summertime mean ozone concentration over the
CONUS than the ORI_1800s, similar to that for the annual mean ozone concentration. According to Table 3,
the Student’s \(t\) test suggests that the differences are statistically significant over 42 out of 48 conterminous
states at \(\alpha=0.05\), ranging from \(-5.17\) to \(-0.75\) ppb. We further evaluate both model results with the
ground-level observation data obtained from the Air Quality System (AQS) archived by U.S. EPA (https://
www.epa.gov/aqs, \(\sim\)1200 observation sites over the CONUS). For the difference of 10 year averaged annual
mean surface ozone concentration between model and monitor data, Figures 3a and 3b show that both
ORI_1800s and ROS-2_1800s are likely to produce more than 20 ppb overestimate at the Eastern and West-
ern U.S. coastal areas. The ROS-2_1800s reduces the bias mainly over the Eastern U.S., consistent with the

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plot in Figure 2a. The differences of 10 year averaged summertime mean surface ozone concentration between both solvers and AQS observation data are depicted in Figures 3c and 3d. The bias is even higher and could be larger than 30 ppb, especially over the Central (e.g., West Virginia) and Northeast (e.g., Pennsylvania). The overestimate of ozone concentration in the Eastern U.S. is a well-known issue from the previous literature about global chemistry-climate models [Murazaki and Hess, 2006; Reidmiller et al., 2009; Lapina et al., 2014], and potential reasons include coarse global resolution that fails to represent the steep topographic gradients in mixing depths [Fiore et al., 2009] and disproportionate sensitivity of models at the high ozone concentration level [Hollaway et al., 2012]. The spatial distribution of surface ozone concentration for the ROS-2_1800s presents an evident reduction of ozone bias at Southeast (e.g., Georgia), Central (e.g., Illinois), and Northeast (e.g., Pennsylvania). In addition, recalling the results of Student’s t test, most of these reductions are statistically significant at $z=0.05$. This reveals that the ROS-2_1800s is likely to significantly reduce the overestimate of surface ozone concentration to some extent, especially during the summer season.

3.3. First-Order Implicit Solver With 180 and 1800 s Time Step

Since a bias of surface ozone concentration is observed between the ORI-1800s and AQS monitor data, it is worth investigating whether refining the time step size is able to reduce the bias for the first-order implicit solver. The time step size in this study is refined to 180 s and the output from first-order implicit solver (ORI_180s) is compared with that from the ORI_1800s. Figure 4a shows a nationwide decrease of 10 year averaged annual mean surface ozone concentration between the ORI_180s and the ORI_1800s (ORI_180s minus ORI_1800s). The largest and smallest differences at the grid-cell level are $2.03$ and $2.05$ ppb, respectively. The state-level differences are listed in Table 4 and the mean normalized gross error $EMNGE$ is also calculated based on equation (6):

$$EMNGE = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{|O_i - M_i|}{O_i} \right) \times 100\%$$

where $N$ is the number of observations from AQS by time and space in each state; $O_i$ and $M_i$ are the $i$th values of observation and model in each state, respectively. Albeit the ORI_180s gives a widely lower estimate of surface ozone concentration and $EMNGE$, the Student’s t test shown in Table 4 suggests that the state-level differences of annual mean surface ozone concentration over the CONUS between the ORI_180s and the ORI_1800s are not statistically significant at $z=0.05$. Considering the 10 year averaged summertime mean surface ozone concentration, Figure 4b presents that both positive and negative differences between
As stated above, the ROS-2_1800s reduces the surface ozone concentration bias between simulation and observation data. A further question is raised: Can the ROS-2 solver benefit from refining the time step size and the ORI_180s and the ORI_1800s appear, with the highest positive value 0.26 ppb and negative value −0.54 ppb. However, the Student’s t test shown in Table 4 indicates that the difference of summertime mean surface ozone concentration between the ORI_180s and the ORI_1800s is again not statistically significant at α=0.05. As stated before, the summertime mean surface ozone concentration is strongly underestimated by the current version of CAM4-Chem. By keeping the same first-order implicit solver but using the 180 s rather than 1800 s time step, the prediction bias seems to be reduced nationwide except at five states (Alabama: 0.16 ppb, Mississippi: 0.12 ppb, Georgia: 0.13 ppb, South Carolina: 0.04 ppb, and Louisiana: 0.16 ppb). Nevertheless, the benefit is less visible due to the small difference (difference of Ε_MGNE < 1% for most states) and neither the positive nor negative difference is statistically significant. Therefore, we cannot conclude that a better prediction is obtained by simply refining the time step size. On the other hand, the ROS-2_1800s does provide statistically significant reduction of both annual and summertime mean surface ozone concentration bias over most conterminous states, as described in the previous section. Furthermore, the magnitude of surface ozone concentration decrease is also much higher than that here. This reflects the fact that the improvement from the ROS-2_1800s may not follow its second-order accuracy as we supposed, but instead from its suitability for stiff problems [Shampine, 1982; Verwer et al., 1999]. Based on the results of numerical analysis in section 3.1, the first-order solver may converge to the “exact” solution at tiny time step size but the computation will be unaffordable in the real simulation of an earth system model. However, the ROS-2 solver is likely to converge to the “exact” solution with relatively large time step size and the ROS-2 solver with 1800 s time step has already provided a solution with small relative error. So we think the ROS-2_1800s solver could handle stiff system better than the original first-order implicit solver and thus provide a better estimate of surface ozone concentration, especially for the summer season.

3.4. ROS-2 Solver With 180 and 1800 s Time Step
As stated above, the ROS-2_1800s reduces the surface ozone concentration bias between simulation and observation data. A further question is raised: Can the ROS-2 solver benefit from refining the time step size?
To answer this question, another time-slice simulation is conducted by using the ROS-2 solver with 180 s time step (ROS-2_180s). The difference of 10 year averaged annual mean surface ozone concentration between the ROS-2_180s and the ROS-2_1800s (ROS-2_180s minus ROS-2_1800s) is plotted in Figure 5a. Surprisingly, the ROS-2_180s provides a nationwide higher estimate of surface ozone concentration, ranging from 0.16 to 2.15 ppb. The primary difference is located in the Eastern U.S. with the largest and smallest state-level differences in West Virginia and New Mexico, respectively (see Table 5). The Student’s t test suggests that the difference between the ROS-2_180s and the ROS-2_1800s is statistically significant over 32 states at $\alpha=0.05$, varying from 0.42 to 2.07 ppb. The difference of 10 year averaged summertime mean surface ozone concentration between the ROS-2_180s and the ROS-2_1800s is shown in Figure 5b. Instead of a consistently higher estimate of surface ozone concentration over the CONUS in Figure 5a, some slight decreases are observed at parts of Texas and New Mexico. It presents a wider range of difference at grid-cell level from $-0.19$ to 4 ppb. The state-level differences shown in Table 5 indicate that the ROS-2_180s still generates a higher estimate of summertime ozone concentration over most parts of the CONUS, similar to

![Figure 3](image-url)
the results in Figure 5a. However, the Student’s t test suggests that these differences are statistically significant over 28 states at $z=0.05$, varying from 0.41 to 3.71 ppb. As demonstrated in Figure 3, the ROS-2_1800s is still likely to overestimate the surface ozone concentration over the CONUS. Therefore, the ROS-2_180s seems to increase the surface ozone concentration bias and no benefit is obtained for the ROS-2 solver from refining the chemistry time step size in this case. However, compared to the ORI_180s and ORI_1800s (see Figures (2 and 4), and 5), the ROS-2_180s still reduces the surface ozone concentration bias to some extent (up to 2 ppb for summer season). Therefore, the ROS-2_180s at least does not negatively impact the chemistry calculation compared with the original first-order implicit solver. From section 3.1, both ROS-2_180s and ROS-2_1800s provide small relative errors and do not differ too much from each other (see Tables 1 and 2). Thus, we think some other factors may play a role here. We output the system matrices from the selected grid cells used in section 3.1 and calculate the condition numbers of the matrices using the function cond() in MATLAB. All of them are larger than $10^{12}$, which means the system matrices are strongly ill-conditioned and a minor change of left-hand side system matrix or right-hand side vector could lead to a major difference in the final solution. For the box model, the solution of the current time step will be used as the input for the next time step integration. Nevertheless, in the real simulation of CAM4-Chem, other processes like advection and diffusion will bring some additional mass from the neighboring grid cells or transfer some mass from the current grid cell, leading to a “discontinuity” of the solution between the end of the current time step and the beginning of the next time step. The effect of “discontinuity” may be amplified by the ill-conditioned system and partially contributes to the difference between the ROS-2_180s and the ROS-2_1800s. It reminds us that the earth system model is very complex and the improvement from the ROS-2 solver may not be limited to its numerical properties, but also related to its interaction with other processes (e.g., dynamics) and components (e.g., land and ocean). The exact reason can only be explored by additional case studies but that is beyond the scope of this study.

3.5. Evaluation of Surface Ozone Concentration at Global Scale

Though this work mainly focuses on the bias of surface ozone concentration over the CONUS, model comparisons for variables at the global scale are necessary since CAM4-Chem is a global chemistry-climate model. The difference of 10 year averaged global surface ozone concentration between the ROS-2_1800s and the ORI_1800s is shown in Figure 6. The clear decrease of surface ozone concentration predicted by the ROS-2_1800s is observed for both annual mean (Figure 6a) and JJA mean (Figure 6b) results. In particular, greater than 3 ppb reduction occurs in Western Europe and Northeast China during the summer season.
We further compare the model results to the observation data in Europe from the European Monitoring and Evaluation Program (EMEP, http://www.emep.int/) and Asia from the Acid Deposition Monitoring Network in East Asia (EANET, http://www.eanet.asia/). For Europe, the observed 10 year averaged annual mean ozone concentration is (mean ± standard deviation over space) 32.83 ± 8.12 ppb while those for the ORI_1800s and the ROS-2_1800s are 48.69 ± 7.83 and 47.32 ± 7.76 ppb, respectively. Considering the 10 year averaged JJA mean ozone concentration, the observation value is 37.21 ± 10.08 ppb while those for the ORI_1800s and the ROS-2_1800s are 57.74 ± 12.27 and 55.65 ± 11.76 ppb, respectively. For Asia, the observed 10 year averaged annual mean ozone concentration is 34.06 ± 11.51 ppb while the values from the ORI_1800s and the ROS-2_1800s are 44.91 ± 5.39 and 43.99 ± 5.17 ppb, respectively. For the 10 year averaged JJA mean ozone concentration, the observed value is 27.91 ± 12.43 ppb while the values from the
ORI_1800s and the ROS-2_1800s are 37.43 ± 11.01 and 36.55 ± 10.57 ppb, respectively. Similar to the U.S., both solvers overestimate the surface ozone concentration over the European and Asian continents, especially for the JJA season. However, the comparison between model simulation results and observation data suggests that the ROS-2_1800s is able to reduce the bias more at the European sites. One main reason is that most of the Asian observation stations are in Japan, Korea, and Thailand, where less difference is seen from Figure 6. It is also worth noting that the observed JJA mean ozone concentration is lower than the annual mean value. This can be explained by the fact that more than half of the available ozone observation data come from sites in Japan and the concentrations there are highest in spring but lowest in summer at all stations [Chatani and Sudo, 2011; Li et al., 2016]. This comparison reveals that the ozone concentration is globally decreased by the ROS-2_1800s instead of a random fluctuation. Therefore, the lower surface ozone concentration over the CONUS predicted by the ROS-2_1800s is not a trade-off against increasing the surface ozone concentration in other global regions.

3.6. Computational Efficiency

Since the ROS-2 solver utilizes the same Jacobian matrix and LU factorization structure between the two stages, it is expected to speed up the computation compared to the original first-order implicit solver. Using the default simulation period setting (5 days), Table 6 summarizes the global statistics of average computational time per processor as a function of different numbers of threads. The percent of saved computational time and the factor of speedup are calculated by the following equations:

\[
\text{Time per processor} = \frac{\text{Total time}}{\text{NTASKS_ATM} \times \text{NTHRDS_ATM}} \quad (7)
\]

\[
\text{Percent of saved time} = \frac{T_{OBI} - T_{ROS-2}}{T_{OBI}} \quad (8)
\]

\[
\text{Factor of speedup} = \frac{T_{OBI}}{T_{ROS-2}} \quad (9)
\]

where "Total time" means the total summed wall-clock time consumed over all the processors. "NTASKS_ATM" and "NTHRDS_ATM" are variables that set the number of MPI tasks and the number of OpenMP threads per task, respectively. The product of these two variables specifies the total computational processors for the atmospheric component of CESM. \(T_{OBI}\) and \(T_{ROS-2}\) refer to the computational time of the first-order implicit

Figure 5. Difference of 10 year averaged (a) annual and (b) summertime mean surface ozone concentration (units: ppb) over the CONUS between the ROS-2 solver at 180 s time step (ROS-2_180s) and the ROS-2_1800s (ROS-2_180s minus ROS-2_1800s).
The Titan supercomputer at Oak Ridge National Laboratory (ORNL) is used for performance testing. Titan is a hybrid-architecture Cray XK7 system with 18,688 compute nodes (16 processors per node) and a theoretical peak performance exceeding 27 petaflops. The chemistry update takes the longest time per processor when only 48 processors are requested, but it drops rapidly as the number of processors increases (see Table 6). The average speedup of computational time for the ROS-2_1800s over that for ORI_1800s is around 1.95 when the total requested processors are less than or equal to 768 but reduces slightly to 1.83 when more processors are used. Similarly, the average speedup of computational time for the ROS-2_180s over that for the ORI_180s is about 1.88 when the total requested processors are less than or equal to 768 but reduces slightly to 1.77 with more processors. For the first-order implicit solver, using one tenth of the time step size does not simply result to

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<td>WI</td>
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a slower performance by a factor of 10. This is probably due to the fact that with a finer time step, fewer iteration steps are required for the chemistry solver to converge. For the ROS-2 solver, it is similarly found that the ROS-2_180s is not 10 times slower than then ROS-2_1800s. This is mainly caused by the adaptive time step method implemented in the ROS-2_1800s, which introduces additional computation at refined time step level and impedes the performance. Generally, the computational time per processor for the ROS-2_1800s is about 48% less than that for ORI_1800s and the computational time per processor for the ROS-2_180s is about 46% less than that for the ORI_180s. The ORI_1800s and ROS-2_1800s are further used to conduct 1 month and 1 year simulations separately to examine whether more improvements are possible (see Table 6). By requesting 1536 processors, the ROS-2_1800s takes about 47% less computational time than ORI_1800s for both 1 month and 1 year simulations, indicating that the performance improvement of the ROS-2_1800s over the ORI_1800s is stable at around 47%, regardless of the simulation period. It is worth noting that we use the same routines to form the Jacobian matrix and conduct the LU factorization for both solvers, which makes the comparison of performance fair. Therefore, the faster computational speed of the ROS-2 solver should be attributed to the advantage of numerical algorithm itself that avoids the reevaluation of Jacobian matrix and LU factorization between two stages.

Table 6. Summary of Computational Time (Units: s) per Processor for Each Solver (ORI_180s, ORI_1800s, ROS-2_180s, and ROS-2_1800s) With Various Simulation Periods

<table>
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<tr>
<th>Simulation Period</th>
<th>Number of Processors</th>
<th>ORI_180s</th>
<th>ROS-2_180s</th>
<th>Percent of Saved Time</th>
<th>Speedup</th>
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<td>844</td>
<td>47.48</td>
<td>1.90</td>
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Figure 6. Difference of 10 year averaged (a) annual and (b) JJA mean surface ozone concentration (units: ppb) between the ROS-2_1800s and the ORI_1800s (ROS-2_1800s minus ORI_1800s) over the global continents.
4. Conclusion

To the authors’ knowledge, this is the first study of implementing the ROS-2 solver in CAM4-Chem and evaluating its impact on the prediction of surface ozone concentration over the CONUS. Compared to the ORI_1800s, the ROS-2_1800s is likely to provide consistently nationwide lower 10 year averaged annual mean surface ozone concentration (see Figure 2a) and the Student’s t test suggests that the difference is statistically significant over 47 states at \( \alpha = 0.05 \). For the summertime, the ROS-2_1800s similarly presents a significantly lower estimate of 10 year averaged surface ozone concentration over the CONUS. The absolute difference is much larger (up to 5.2 ppb for one state) and 42 out of 48 states are statistically significant at \( \alpha = 0.05 \). This implies that the ROS-2_1800s can help improve the performance of surface ozone concentration over the CONUS, especially during the summer season when the photochemical reactions are the most active. The lower global surface ozone concentration generated by the ROS-2_1800s also suggests that its prediction of lower surface ozone concentration over the CONUS is not a random fluctuation and the model evaluation shows that the ROS-2_1800s can reduce the bias in Europe and Asia to some extent as well.

In addition, the time step size for the original first-order implicit solver is refined to 180 s (one tenth of the default time step) to examine whether it could help improve the estimate of surface ozone concentration over the CONUS. Figure 4 shows that compared to ORI_1800s, ORI-180s does slightly reduce the surface ozone concentration but the improvement is less visible (only around 0.5 ppb as maximum) and not statistically significant, either. This reveals that just refining the time step size by a factor of 10 for the original first-order implicit solver is not able to provide statistically significant improvement of surface ozone concentration prediction. Thus, it is necessary to use other chemical solvers like the ROS-2 solver in this study. However, the outperformance for the ROS-2 solver may not come from its second-order accuracy with respect to time scale, but from its suitability for solving stiff problems. We have also applied the 180 s time step to the ROS-2 solver but the ROS-2_180s surprisingly performs worse than the ROS-2_1800s for estimating the surface ozone concentration. By studying some selected grid cells, the system matrix is found to be strongly ill-conditioned and other processes (e.g., diffusion) will cause “discontinuity” for the solution between the end of the current time step and the beginning of the next time step. The ROS-2 solvers with different time step sizes could converge to different solutions after a long-term simulation. In the real application, the CAM4-Chem may accumulate errors from other processes like dynamics and physics. Thus, the time step size for the ROS-2 solver in chemistry should be chosen carefully in order to provide a compensation of error.

The computational efficiencies of the ORI_1800s, ORI_180s, ROS-2_1800s and ROS-2_180s are also analyzed. Even if the first-order implicit solver has been optimized to solve the chemical reaction system efficiently, the ROS-2 solver takes about 47% less computational time than the original first-order implicit solver when using the same time step size. Our analysis indicates this speedup is explained by the fact that the ROS-2 solver utilizes the same Jacobian matrix and LU factorization structure during the two-stage calculations, which could consume 90% of the total computational time for the chemistry update. It is also observed that the ROS-2_180s is not 10 times slower than the ROS-2_1800s because of the adaptive time step method implemented in the ROS-2_1800s.

In future work, case studies with different configurations may be explored to examine whether the benefit of the ROS-2 solver observed in this study still persists. Other solvers from the Rosenbrock family (e.g., ROS-3) can also be implemented to see whether further bias reduction is possible for the surface ozone concentration prediction over the CONUS. Since all the constituents except water vapor are treated as radiatively inactive by default, the reduction of ozone concentration from the ROS-2_1800s is not expected to influence the longwave and shortwave cloud forcings in the current simulation but it is worth investigating the differences in a fully interactive simulation in future work. The chemistry update behaves like a box model in CAM4-Chem, which is an ideal target for parallel implementations. Either NVIDIA CUDA or OpenACC could be used as a programming model on advanced high performance computing platforms to harness the power of the GPU and further improve the computational performance.

References


