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Stability and Evolution of Planar and Concave Slopes under Unsaturated and Rainfall Conditions

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Abstract:

Natural slopes are often observed to have a concave, convex, or a combination concave/convex profile, yet constructed slopes are traditionally designed with planar cross-sectional geometry. In this paper, the stability of two planar slopes was compared with that of companion concave slopes, designed to have similar factors of safety (FOS) under gravity loading. The stability of these slopes was then investigated in response to a suction event followed by a precipitation event, and it was shown that both the planar and the concave slopes experienced similar changes in stability. Additional analyses were conducted with a simulated erosion mechanism to investigate how the planar and concave shapes would evolve under a sequence of three similar suction/precipitation/erosion cycles. The results suggest that for these slopes, the second and third simulated weather cycles reduced the stability of the slopes, yet had a lesser effect on the concave slopes than the planar slopes. This is in spite of the fact that the planar slopes became more “concave-like” due to the simulated erosion, and suggests slopes designed to be concave may perform better than the planar slopes.

Keywords: Planar slope, soil suction, unsaturated slope, soil viscosity, limit equilibrium, shear strength reduction factor.
**Introduction:**

Most constructed slopes, both cut and fill slopes, are designed with a planar cross-section geometry (uniform gradient), which while being more straightforward for analysis and construction, does not produce natural appearing earth structures. The natural, sustainable shape of a slope could be concave, convex, or a combination of concave and convex (Schor and Gray, 2007). A conceptual model of slope evolution from a planar shape to a concave shape indicates that both mass stability analysis and surficial erosion processes enable a stable shape to be created for given soil properties (Gray, 2013). The driving forces, material properties, and slope geometry are the determinant parameters in the mass stability analysis, and govern the factor of safety (FOS) of the slope. In a planar slope, the driving force increases linearly from top to bottom of the slope, and the tractive force due to erosion also increases with the distance downslope. However, as described by Schor and Gray (2007), in a concave slope, the driving force decreases from top to bottom as the angle of the slope gradually decreases, and the tractive force exerted by the runoff also decreases as the slope decreases. A concave slope having a constant rate of erosion down the slope reaching steady-state equilibrium while maintaining mechanical stability has been suggested as an optimal slope shape (Jeldes et al. 2018). A uniform rate of erosion may lead to parallel retreat of some concave slopes. Hancock et al. (2003) argued that a compound shape can be described using an area-slope relationship, which is the relationship between the drainage area and slope of a point on the slope.

Stability analyses of concave slopes have been conducted based on slip-line theory, limit equilibrium method, and limit analysis approach. Utili and Nova (2007) utilized an upper bound method of limit analysis to reach an optimal log spiral profile of a slope yielding a maximum safety factor for given average slope angle or given soil properties. Jeldes et al. (2013) simplified the Sokolovskii (1960, 1965) slip-line theory solution using an analytical approximation to reach an optimum concave slope shape based on the effective shear strength parameters, total soil unit weight and the slope height. The theoretical
mathematical solution for this method produces a sharp vertical edge at the top of the slope or “cusp” based on the tension crack depth, and it was suggested that the cusp may be an unstable and temporary component of the slope in many cases. The erosion of concave slopes was shown to be lower than that in equivalent planar slopes, and a design procedure was suggested (Jeldes et. al 2015). Vahedifard et al. (2016) used a geometric technique incorporating a limit equilibrium formulation to develop stability numbers for a wide range of circular concave slopes including the effects of the upper inclined slope surface. The inclined surfaces with different angles were considered as an upper component of the slope and they concluded that the increase of the upper slope angle or the cusp formation resulted in an unstable situation for slopes. Vo and Russell (2017) investigated the role of soil suction in unsaturated non-planar slopes in and developed a series of stability charts in dimensionless form.

In this paper, two “virtual” or contrived slopes taken from the literature, along with equivalent companion concave slopes, are evaluated by the Finite Element method to investigate the effect of shape and the evolution of slope shape during a series of weather cycles. Here, a companion slope is defined as one with the same height and material properties but with a concave shape defined according to Jeldes et al. (2013). The stability of the slopes is evaluated under gravity alone, then the application of soil suction or a drying event, followed by a constant intensity rainfall event. Specifically, the following are addressed:

- The stability and change in the FOS of both planar and concave slopes due to reduction of unsaturated soil strength caused by a precipitation event
- The evolution of both planar and concave slope geometry as a series of precipitation events reduces the surficial soil suction and strength, allowing the material to be eroded and lost from the slope.

The intent of the investigation was not to evaluate the effect of specific hydraulic characteristics (Cai and Ugai 2004) or model the erosion process (Jeldes et al. 2018). Instead, the intent was to investigate how both planar and concave companion slopes might generally respond to an arbitrary drying and
rainfall event, and how the shape of the slopes may evolve over a series of these cycles. The goal was to see if a concave shape was sustainable, and if the sharp cusp at the top of slope was likely to be a temporary feature that exists due to the mathematics of the soil tensile strength.

In addition to the above, the convergence criteria of the coupled hydro-mechanical visco-plastic model used to represent the soil in the unsaturated slopes is described, along with other details of the numerical approach.

**Numerical investigation of the stability of planar and equivalent concave slopes**

In this paper, the mechanical stability of two planar slopes with different geometries and mechanical material properties was investigated under gravity load to obtain the initial design FOS as identified by the Shear Strength Reduction Technique (SSRT) (e.g. Zienkiewicz et al. 1975; Griffiths and Lane, 1999). This method has been used in both FEM and LE methods and the FOS obtained in the SSRT corresponds to the Strength Reduction Factor (SRF) at which the slope would fail, or the ratio of the actual shear strength of the soil to the lowest shear strength at which failure occurs. The analysis is conducted using factored shear strength parameters $c'_f$ and $\phi'_f$ (and $\phi^b_f$)

$$c'_f = \frac{c'}{SRF}, \phi'_f = \arctan \left( \frac{\phi'_f}{SRF} \right) \text{ and } \phi^b_f = \arctan \left( \frac{\phi^b}{SRF} \right)$$  \(1\)

where $c'$ is the effective cohesive strength, $\phi'$ is the effective internal friction angle, and $\phi^b$ is the internal friction angle with respect to suction. Failure is reached by gradually and systematically increasing the SRF, thus obtaining the FOS. The FOS’s from the planar slopes are then used to determine the shape of companion concave slopes based on the analytical equations proposed by Jeldes et al. (2013). The coordinates of the companion concave slopes are obtained based on the density, $\gamma$, slope height, $H_s$, and the factored strength parameters $c'_f$ and $\phi'_f$ of the companion slope. This produces a concave slope with
essentially the same FOS as the original planar slope. The coordinates of the concave slope are described

\[ x = \begin{cases} 0, & -h_{cr} \leq y \leq 0 \\ A(\gamma y - p_t B (\csc \phi + 1)), & y > 0 \end{cases} \]  

(2)

where

\[ A = \frac{\cos \phi}{2\gamma(1 - \sin \phi)} \]  

(3)

\[ B = \ln \left[ \frac{\sigma_y (\frac{1 - \sin}{1 + \sin}) + 1}{\frac{\sigma_y}{p_t} K_a + 1} \right] \]  

(4)

\[ h_{cr} = \frac{2c \cos}{\gamma(1 - \sin)} \]  

(5)

with \( \sigma_y = \gamma y \) or the geo-static vertical stress, and \( p_t = c' \cot \phi \). Note that \( p_t \) is the tensile strength of the soil, \( K_a = (1 - \sin \phi)/(1 + \sin \phi) \) is the Rankine active coefficient of earth pressure, and \( h_{cr} \) is the height of the tension zone. The equation describes a slope contour in the quadrant with x-axis positive to the right and y-axis positive downward, with \( h_{cr} \) lying above the x-axis from the coordinates (0,0) to (0, \(- h_{cr}\)). Notice that this \( h_{cr} \) tension zone does not contribute to resistance, but only to destabilization.

The two slopes are designated as Slope 1 (of moderate inclination) and Slope 2 (of steep inclination) and were adapted from the literature (Le et al. 2015, and 2016; Jeldes et al. 2015, respectively). Slope 1 (Fig. 1a) was selected as an example of a slope that was only moderately stable in the absence of the soil suction (Le et al. 2015). Slope 2 (Fig. 1b) was selected as the stability and erosivity were compared with an equivalent concave slope (Jeldes 2015).

Slope 1 has a height of 10 m and an inclination angle of 26.5° while Slope 2 has a height of 15 m and an inclination angle of 41°, and both slopes have a water table surface assumed to be at a depth of 15 m from the top of the slope. The pore water pressure is assumed to be distributed hydrostatically (linearly) from the water table surface toward the lowest and the uppermost levels as positive or negative values, respectively. The negative value, referred to as suction, exists for a height of 15 m in both slopes, which
is a typical depth of the wetting and active zone in arid and semiarid regions (Nelson et al. 2001). This corresponds to an assumed suction value of 150 kPa at the ground surface in both slopes. The stability of the two slopes was investigated under the gravitational loading, followed by a 5-day period of drying or application of the suction, which was then followed by a significant precipitation event with a rate of 43.2 mm/day ($5^4$ kg/m$^2$/s) with duration of 5 days applied through the ground surface. This rainfall event is consistent with that investigated by Le et al. (2015).

The analyses were performed with the finite element code Code Bright (DIT-UPC, 2015), which couples the hydraulic and mechanical properties of the soil. The mechanical material properties were taken from the original references for each of the slopes, to allow for direct comparison with the previously published FOS results. In order to better compare the response during the drying and precipitation events, the two slopes were assumed to have identical hydraulic properties, with the majority of the hydraulic properties taken from the Slope 1 references (Le et al. 2015, 2016). The focus of the analysis was not on the effect of the hydraulic properties on slope stability, as was investigated by Cai and Ugai (2004). Instead, the focus of the paper was on the evolution of the slope shape and the differences in response between the planar and concave slopes, which is better examined if the hydraulic properties were identical. Since there may be some inconsistency between the mechanical parameters and the hydraulic parameters of the two slopes, they may best be considered “virtual” or contrived slopes chosen to compare planar and concave slopes and the evolution in shape of both.

Code Bright (DIT-UPC, 2015), employs the net mean stress and suction as main stress variables as:

\[
\bar{\sigma}_{ij} = \sigma_{ij} - \max(p_g, p_l)\delta_{ij}
\]

(6)

\[
S = \max((p_g - p_l), 0)
\]

(7)
where: $\sigma_{ij}$ is the total stress, $p_g$ is the gas or air pressure, which is assumed to be zero, $p_i$ is the water pressure, and is $\delta_{ij}$ the Kronecker delta. By assuming air pressure equal to zero, the net normal stress is defined as the total stress above the water table and the effective stress below water table.

The soil water characteristic curve (SWCC) or the relationships between negative pore water pressure/suction, $s$ and degree of Saturation, $S$ and the relationship between $S$ and unsaturated hydraulic conductivity, $k_u$ were chosen to be compatible with van Genuchten’s equation (1980). The effective saturation, $S_e$, is defined such that it varies between 0 and 1 (Fig. 2a) as

$$S_e = \frac{S - S_r}{S_s - S_r} \left[ 1 + \left( \frac{s}{s_e} \right)^{1/(1-m)} \right]^{-m}$$

where $S = \text{degree of saturation}$; $S_r = \text{residual saturation}$; $S_s = \text{maximum saturation}$; $s = \text{suction}$, $s_e = \text{air-entry suction parameter} = s_e = s_{eo} \exp[ \eta( n_0 - n) ]$

$s_{eo} = \text{reference air-entry pressure}$; $\eta = \text{parameter for the influence of porosity, } n, \text{ on the SWCC}$; $m = \text{shape function}$; and $n_0 = \text{reference porosity}$.

The unsaturated hydraulic conductivity, $k_u$ (Fig. 2b) is a function of the effective saturation, $S_e$, and porosity, $n$, and is expressed as

$$k_u = k_s k_r$$

where $k_s = \text{saturated permeability (m/s)}$ and $k_r = \text{relative permeability}$ defined as follow:

$$k_s = k_{so} \left[ \frac{n^3}{(1-n)^2} \right] \left[ \frac{(1-n)^2}{n_0^3} \right]$$

$$k_r = \sqrt{S_e} \left[ 1 - \left( 1 - S_e^{1/m} \right)^m \right]$$

where $K_{so} = \text{saturated permeability}$.

Thus, the FEM code varies the permeability as the soil porosity (or volumetric strain) and the saturation change as defined above. During a rainfall event, the initial negative pore water pressure on the upper
surface is reduced due to boundary flow, and the pressure is redistributed below the ground surface based on the hydraulic conductivity. A negative leakage coefficient $\gamma$ was used as a boundary parameter along the ground surface to maintain the pore water pressure less than or equal to zero avoiding positive pore water pressure. Thus the flux, $q_i$ is described both on the boundary and within the slope using the following flux equations (i.e. Darcy’s law):

$$q_i = q_{lo} + \gamma_i (p_i + p_{lo})$$ (12)

$$q_i = -\frac{K k_r}{\mu_t} (\nabla p_i + p_i g)$$ (13)

where $q_i$ = water flux (kg/m$^2$/s), $q_{lo}$ = reference water flux (e.g. rainfall), $K$ = intrinsic permeability (m$^2$), $k_r$=relative permeability, $\gamma_i$ = leakage coefficient, $\mu_t$=water viscosity (MPa.s), $p_i$=water density (kg/m$^3$), $p_i$=water pressure (kPa), $p_{lo}$=reference water pressure (kPa), and $g$=acceleration due to gravity (m/s$^2$).

A visco-plastic model with a Mohr-Coulomb failure criterion, as implemented in the Finite Element code Code_Bright (DIT-UPC, 2015) was used to model the time-dependent progressive failure. The incremental stress state in the soil is represented by

$$d\sigma_{ij} = D^e_{ijkl} (d\varepsilon_{kl} - \delta_{kl} \frac{ds}{K_s} - d\varepsilon^p_{kl})$$ (14)

where $d\sigma_{ij}$ = incremental stress matrix

$D^e_{ijkl}$ = elastic stiffness matrix (isotropic)

$s$ = soil suction

$dc_s$ = the incremental strain

$K_s$ = bulk modulus against suction changes

$dc^p_{kl}$ = incremental plastic strain

Yield is defined by the extended Mohr-Coulomb criterion as:
where $\theta$ is the Lode angle, $J$ is the square root of the second invariant of deviatoric stress tensor, $\phi'$ is the soil friction angle, $p$ is soil net mean stress, and $p_t$ is the soil tensile strength $= c' \cot \phi'$ with $c'$ the soil cohesive strength. The shear strength reduction technique (SSRT) was used by incrementing the reduction factor by 0.01 from 1.0 to the value resulting in failure.

The visco-plastic rate dependency is introduced by a plastic multiplier $\lambda^p$ expressed as a function of the distance between the current stress point in the soil matrix and the inviscid plastic locus:

$$d\lambda^p = (dt/\eta^v) \left( F^p \right)$$

where $dt$ is the time increment, $\eta^v$ is the soil viscosity, and $F^p$ is the Mohr-Coulomb yield function. The inviscid plastic locus ($\bar{F}^p$) is defined as follows:

$$\bar{F}^p = F^p - (\eta/ dt) d\lambda^p \leq 0$$

where $\bar{F}^p = $ Mohr Coulomb yield criterion (eq 15), and the non-associated plastic potential function $G^p$ is

$$G^p = \left( \cos \theta + \frac{1}{\sqrt{3}} \sin \theta \sin \phi' \right) J$$

was assumed to limit dilatancy. The material parameters used for the two slopes are summarized in Table 1.

**Convergence criteria for the coupled hydro-mechanical visco-plastic model**

Limit equilibrium and slip-line analyses are common methods for evaluating slope stability. The collapse zone or failure surface determined by slip-line method distinguishes this method from the limit equilibrium method, which considers the shear strength mobilized only a single failure surface. The failure zone can also be determined using a finite element approach with the SSRT. The loss of numerical
convergence or the onset of a sudden displacement are two typical methods to identify failure using a finite element approach (Griffiths and Lane, 1999; Zienkiewicz et al. 2005; Hicks and Spencer, 2010)). In solutions with coupled hydro-mechanical material models, non-convergence within a given number of FE iterations can fail to detect the actual failure zone (Le et al. 2015). However, the sudden change of nodal displacement during the gradual reduction of shear strength can be an effective method to predict the failure as long as some rational criteria are selected.

In the investigation of Slope 1, Le et al. (2015) defined convergence criteria for identifying failure and controlling the solution of the coupled hydro-mechanical visco-plastic model. Based on numerous finite element analyses, they established three displacement criteria that could be determined numerically at one or more surface nodes to identify a convergent failure condition. The FOS of the slope is then taken as the largest strength reduction factor at which all 3 criteria are satisfied. For clarity in the subsequent discussion, the three criteria identified by Le et al. (2015) are designated as follows:

1. **Relative Displacement Criterion** - The increment of either horizontal or vertical displacement during one strength reduction step \((i)\) of 0.01 exceeds 10 times the previous step \((i - 1)\); \(\Delta x_i > 10 \times \Delta x_{i-1}\) or \(\Delta y_i > 10 \times \Delta y_{i-1}\).

2. **Absolute Displacement Criterion** - Increment of total displacement > 2 mm during a strength reduction step of 0.01.

3. **Cumulative Displacement Criterion** - The cumulative vertical or horizontal displacement > 10 mm.

Le et al. (2015) suggest that the first (Relative Displacement) criterion identifies failure as a sudden increase in displacement, but criterion 2 (Absolute Displacement Criterion) assures that this does not occur at very small absolute displacements. The third (Cumulative Displacement) criterion assures that a considerable level of deformation has occurred and prevents any possible misleading reduction factor during the early stages of the solution.

In this study, strength reduction steps of 0.01 were applied over a 2.5-day time interval, with the nodes on the surface experiencing a gradual increase in displacement. The Absolute Displacement Criterion was found to be satisfied at different displacements for the two different slope geometries and material
properties. For example, for Slope 1 the Absolute Displacement Criterion of 2 mm was met at the same
strength reduction step (problem time of about 52.5 days) as the Relative Displacement Criterion
(indicated by a greater than 10 times increase in displacement with the next strength reduction step of
0.01). This is illustrated in Fig. 3a where the total displacement of Point A on the crest of Slope 1 as a
function of problem time is shown as the strength reduction factor is increased. However in Slope 2,
different geometry and material properties result in a different critical point on the surface (Point B near
the toe), and the Relative Displacement Criterion was not met until the absolute displacement was 7
mm (Fig. 3b) at a problem time of 210 days, well past the time when the Absolute Displacement
criterion was met. In both slopes, when the Relative Displacement Criterion was satisfied, the
Cumulative Displacement Criterion was met, and the FOS was taken from the SSRT. The problem time
was then continued to further enhance the displacements and better define the failure zone.

In addition to these criteria, an additional criterion, referred to as the Rate of Displacement Change
Criterion, was implemented for this paper. The solution was assumed to have converged when the
change in displacement of a point on the slope surface from one step of the SSRT to the next was less
than $10^{-5}$ m/day, and was invoked regardless of the other three criteria. In Fig. 3 and subsequent figures
of FEM results, the conditions are depicted for the last strength reduction step for which the
convergence criteria are met, and the FOS is taken as the strength reduction factor.

Factors affecting computational time

The value of the viscosity of the soil has a significant effect on both the computation time and the
convergence of the visco-plastic model. To consider the effect of the assumed value of viscosity on the
computation time, Slope 1 was analyzed with viscosity values of 1 MPa.s, 100 MPa.s, and 100,000 MPa.s.,
with each value producing a different time to obtain convergence or “problem time” as well as different
computational times. Fig. 4 compares the results for the three different assumed viscosity values to
achieve a steady or convergent solution for the displacement of Point A on the top of Slope 1, and indicates that the displacement solution is independent of the viscosity but the problem time as well as the computational time varies significantly with the value of viscosity. The results of the convergence study indicate that for a viscosity of 1 MPa.s., a convergent displacement was reached in a problem time of about 0.01 day (Fig. 4), but 14 days of computational time was required. However, if the viscosity was increased to an artificial viscosity of $10^2$ MPa.s., the identical convergent displacement could be reached in a problem time of 0.04 days but a computational time of only 4 days is required. Likewise, for an artificial viscosity of $10^5$ MPa.s., the computational time is reduced to only 0.04 days. Thus, by assuming an artificially large value of viscosity, the computational time can be reduced significantly but the problem time for displacement convergence also becomes artificial or fictitious. Since the time dependence of the solution is not be important for these problems, the use of a fictitious time to facilitate the solution may be convenient, and is consistent with that described by Zienkiewicz et al. (2005).

To place these viscosity values in context, reported experimental measurements on various soils yielded a range of viscosity from $10^{-1}$ to $10^{10}$ MPa.s (Vyalov et al. (1986). A range of viscosity between $10^{-3}$ MPa.s. and 5 MPa.s for different soils at various moisture contents was also reported (Ghezzehei and Or 2001; Or and Ghezzehei 2002). Viscosity values of $5 \times 10^{-2}$ to $3 \times 10^{-1}$ MPa.s have been suggested for clay loam soil (Karmakar and Kushwaha, 2007), while values of $10^{-4}$ to $5 \times 10^{-1}$ MPa.s have been reported for various soils below the liquid limit (Widjaja and Lee 2013).

The solution time increment is automatically controlled in CODE_BRIGHT. Because of the coupling between the hydraulic and mechanical analysis, a time interval is specified during which the mechanical forces, displacements, and hydraulic flux are calculated and allowed to come to equilibrium. The computation time also increases as the time interval increases, and the effect of varying the time interval required for the visco-plastic algorithm to converge was investigated. As shown in Fig. 5, the FOS reaches
a minimum value as the time interval is increased. Based on these analyses, a time interval of 2.5 days was found to be sufficiently large for both Slope 1 and Slope 2 to satisfy all four of the above convergence criteria.

Results and Discussion:

Results from the stability analyses conducted on the two planar slopes and their companion concave slopes are shown in Figs. 6 through 11. For both Slope 1 (Fig. 6) and Slope 2 (Fig. 8) under gravity loading, companion concave shapes were obtained using the equations proposed by Jeldes et al. (2013) in Equations 2-5. The reduced effective cohesion and effective friction angle ($c'$ and $\tan \phi'$) required for the equations were obtained from the original $c'$ and $\tan \phi'$ divided by the FOS of the planar slopes under gravity load. Finite element analysis confirmed that the concave slopes achieved essentially the same FOS under gravity alone as the planar slopes. Both Slope 1 and Slope 2 concave shapes (Figs. 6b and 8b) exhibited a cusp or cliff at the top of the slope where the largest displacements were concentrated, suggesting that the materials in these zones may be most vulnerable to erosion or disturbance.

To investigate the effects of an unsaturated condition on the stability of the slopes, both the planar and concave slopes were subjected to a 5 day period of drying or application of negative pore water pressure through the boundary conditions on the slope as mentioned earlier. As expected, the FOS for both slopes increased under the suction conditions, but the mode of potential slope failure became more deep-seated as shown in Figs. 7 and 9. The distribution of the pore water pressure and suction obtained by mass water balance equation in the slopes following drying period at $t=5$ days is shown in Fig. 10. The water flux (i.e. water rate multiplied by a cross-sectional area) due to the hydraulic head (i.e. the gravitational head plus the pore water pressure head) is shown in Fig. 11. The computation of direction and rate of water flux indicates that Slope 2 (Figs. 11 (c) and (d)) is subjected to water flow from right to the left side of the slope.
in a opposite direction (from negative pore water pressure toward positive pore water pressure) to eliminate the overland flow generating positive pore pressure on the ground surface, whereas Slope 1 (Figs. 11 (a) and (b)) displays water flow in a level below the ground surface from positive pore pressure zones to negative pore pressure zones directed from bottom to top of the water table.

Following the 5 day period of drying or application of the suction, a precipitation rate of 43.2 mm/day (5.4 kg/m²/s) with duration of 5 days was applied through the ground surface. This rainfall event is consistent with that investigated by Le et al. (2015). The loss of suction due to rainfall led to a decrease in the factors of safety for both slopes and both planar and concave shapes. Figs. 12 and 13 suggest that the failure zones after the precipitation event change little in Slope 2, but become more surficial in Slope 1, and the concave Slope 1 appears to have a concentrated zone of large displacements in the portion near the cusp.

The computed FOS from Figs. 6 – 9 and Figs. 12 and 13 are summarized in Table 2, and where applicable compared with the FOS values reported from the literature for these slopes. The results shown in Table 3 indicate that while the Slope 1 and Slope 2 responded differently to the suction and rainfall events, each of the concave slopes responded in a manner similar to their planar companion. This suggests that the concave slopes will behave in a similar manner to the weather events as their equivalent planar slope, but should maintain the advantages of the concave geometry with respect to erosion as identified by others.

**Evolution of Planar Slopes:**

To estimate how the shape of planar slopes may evolve over time due to erosion, it is assumed that the large displacement failure zones within the slopes correspond to the portions where the soil particles may be loose and detached and first susceptible to erosion during a rainfall event. It is recognized that this is a very approximate means to represent soil loss due to erosion, and the resulting sediment is not re-deposited at the toe. However, the focus was on the change in shape of the slope profile, especially in the
upper portion of the slope in the region of the mathematical or tension “cusp.” Fig. 14 illustrates the
displacement time history of selected points on the surface and slightly below the surface of Slope 1. This
figure depicts the large displacements with time on the surface of the slope after applying the final
strength reduction increment of 0.01, reaching a maximum total nodal displacement of about 500 mm for
a problem time of 1400 days. The time history of several intermediate points below the surface is also
shown, with smaller displacements trending to a point on the interior where a more stable time history
at a displacement of about 10 mm is shown. It is assumed that the shape of the eroded slope can be
approximated by removing from the FE mesh the soil in this zone of unstable displacement history.

The two planar slopes 1 and 2 with the predicted zones of high displacement due to the first rainfall event
from Fig. 12a and 13a are repeated as Fig. 15a and 15b respectively, while Figs. 15c and 15d show the
resulting shape with the elements from the “eroded” zone removed by the method described above and
in Fig. 14. The new modified FE meshes were again loaded with the same initial gravity, suction, and
rainfall loadings described previously, and Figs. 15c and 15d show the computed displacements and zones
subject to erosion from the second weather event. Figs. 15e and 15f depict the slopes with the soil eroded
from the second weather event, and the response to a third cycle of gravity, suction and rainfall. The
rainfall/erosion effects on both slopes tend to transform the planar slope into a concave slope. The more
moderate Slope 1 in Fig. 15e suggests it is reaching a steady state concave shape after the third event,
while the steeper Slope 2 in Fig. 15f appears to be following the “parallel retreat” mode of failure (Jeldes
et al. 2018). These results also suggest that each suction/rainfall/erosion event is producing a slight
decrease in the computed level of safety or FOS.

Evolution of Concave Shapes:

The approach described above was taken to investigate how the shape of concave slopes would evolve
under the same sequence of drying/rainfall/erosion events. The concave shape corresponding to Slope 1
and Slope 2 was subjected to the 5 day period of drying followed by the 5 day precipitation event, and the results from Fig. 12b and 13b are repeated as shown in Fig. 16a and 16b. The predicted zones of high displacement or erosion susceptibility due to the rainfall event were removed as described above to produce the slope shapes as shown in Figs. 16c and 16d, along with the high displacement zones due to the second weather event. The shape was further modified as shown in Figs. 16e and 16f, and the response to the third weather event is shown. It is noted that the second round of weather and slope modification led to an increase in the stability or FOS for concave Slope 1 with the failure zone concentrated around the cusp at the top of the slope, and the third round of loading and modification resulted in no change in the FOS. However, while the stability increased from the first weather event to the second in concave Slope 2, it decreased slightly due to the third event. As with the planar Slope 2, the concave Slope 2 exhibited the parallel retreat response. Both concave slopes 1 and 2 were observed to lose much of the cusp at the top of the slope due to the simulated rainfall/erosion events.

The responses in terms of the FOS’s of Slope 1 (concave and planar) and Slope 2 (concave and planar) to the three suction/rainfall/erosion weather events are summarized in Table 3. Although it is recognized that the numerical differences between the various FOS values may not have much significance, some relative differences and trends are observed. These changes in the computed FOS after the various simulated drying/rainfall/erosion events are also shown in Fig. 17. It is noted that while the second and third simulated erosion/rainfall events reduced the stability of the two planar slopes, these events had a lesser effect on the stability of the concave slopes. This is in spite of the fact that the planar slopes became more “concave-like” due to the simulated erosion, and suggests that for at least these two slopes under the simulated weather and erosion events, the slopes designed to be concave may perform better than the planar slopes.
Conclusions:

The mechanical stability of two planar slopes with different geometries and mechanical material properties were investigated using a FE simulation with a coupled hydro-mechanical visco-plastic soil model, and the Shear Strength Reduction Technique used to identify the factor of safety (FOS). Both planar slopes were taken from the literature, with Slope 1 being of moderate inclination and being only moderately stable in the absence of soil suction, and Slope 2 of more steep inclination. Companion concave slopes were created from both planar slopes using the expression suggested by Jeldes et al. (2013) to achieve slopes with approximately the same FOS under gravity. The convergence of the visco-plastic soil model was investigated, as well as the effects of time interval and the assumed value of viscosity on the solution.

The slopes were evaluated under gravity, and an arbitrary rainfall event preceded by an initial drying or evaporation condition producing soil suction that was partially dissipated by the rainfall event. The results indicate that while the two slopes responded differently to the suction and rainfall events, the two concave slopes responded in a manner similar to their planar companion. This suggests that the concave slopes will behave in a similar manner to the weather events as their equivalent planar slope, but should maintain the advantages of the concave geometry with respect to erosion.

To investigate the evolution of slope cross-sectional shape due to the suction/rainfall/erosion cycles, it was assumed that the portions of the slope with significant displacements would tend to be the areas where the soil would have a tendency to erode. Although not intended to be a rigorous representation of erosion and soil loss, this approximation should identify the soil zones with the highest degree of erodibility. These erodible soil zones were removed from the mesh, creating a slope with modified cross section. A sequence of three suction/rainfall/erosion cycles was found to transform both planar slopes into concave slopes. The more moderate Slope 1 appeared to be tending towards a steady state concave
shape after the third event, while the steeper Slope 2 appeared to be following a “parallel retreat” mode of failure.

A similar approach was taken to observe the evolution of the concave slopes due to the same sequence of three suction/rainfall/erosion cycles. The results suggested that while the stability increased from the first to the second weather event for both slopes, in concave Slope 1 the failure zone was concentrated around the cusp at the top of the slope, and the third round of weathering/erosion resulted in no change in the FOS. However, in concave Slope 2, the stability decreased due to the third event, and as observed for the planar Slope 2, the concave Slope 2 exhibited a parallel retreat response. Both concave slopes 1 and 2 were observed to lose the cusp at the top of the slope due to the simulated rainfall/erosion events.

It was noted that while the second and third simulated erosion/rainfall events reduced the stability of the two planar slopes, these weather events had a lesser effect on the stability of the concave slopes. This is in spite of the fact that the planar slopes became more “concave-like” due to the simulated erosion, and suggests that for at least these two slopes under the simulated weather and erosion events, the slopes designed to be concave may perform better than the planar slopes.

Data Availability Statement:

Some or all data, models, or code generated or used during the study are available in a repository or online in accordance with funder data retention policies. Input files used with the Code_Bright (DIT-UPC, 2015) analyses will be available at Tennessee Research and Creative Exchange (TRACE) https://www.trace.tennessee.edu/ which is the University of Tennessee’s institutional open-access repository.
References:


doi:10.2136/sssaj.03615995004400050002x.


Fig. 1. Slope geometry, water table, applied boundary conditions, and meshing:

a) Slope 1 (after Le et al. 2015, 2016) and b) Slope 2 (after Jeldes et al. 2015)

Fig. 2. Soil water characteristic curve (a) and unsaturated hydraulic conductivity permeability function (b) assumed for the slopes.

Fig. 3. Maximum total displacement evolution using shear strength reduction factor technique at (a) point A for Slope 1 and (b) point B for Slope 2. The vertical red line indicates the last strength reduction step before the Relative Displacement Criterion convergence criterion is satisfied.

Fig. 4. Problem time for the total displacement of Point A (Fig. 3) for different viscosities of the soil matrix in Slope 1

Fig. 5. Effect of time interval on the factor of safety for Slope 1 (a, b) and for Slope 2 (c, d) under gravity loading only and both gravity and suction loading, respectively

Fig. 6. Total displacement contour and factor of safety under gravity load of Slope 1 (a) planar (b) concave

Fig. 7. Total displacement contour and factor of safety under gravity and suction loads of Slope 1 (a) planar (b) concave

Fig. 8. Total displacement contour and factor of safety under gravity load of Slope 2 (a) planar (b) concave

Fig. 9. Total displacement contours and factor of safety under gravity and suction loads of Slope 2 (a) planar (b) concave

Fig. 10. Distribution of pore water pressure/suction in slopes following drying period at time = 5 days: a) Slope 1 planar, b) Slope 1 concave, c) Slope 2 planar, d) Slope 2 concave
Fig. 11. Computed hydraulic flux in slopes following drying period at time = 5 days: a) Slope 1 planar, b) Slope 1 concave, c) Slope 2 planar, d) Slope 2 concave

Fig. 12. Total displacement contours and factor of safety under gravity and suction loads followed by the precipitation event: Slope 1 (a) planar (b) concave

Fig. 13. Total displacement contours and factor of safety under gravity and suction loads followed by the precipitation event Slope 2 (a) planar (b) concave

Fig. 14. Total displacement versus time at various points in the failure zone of Slope 1, used for the identification of eroded soil zones

Fig. 15. Evolution of planar slopes from first to third rainfall events: Planar Slope 1 (figures a, c, and e) and Planar Slope 2 (figures b, d, and f)

Fig. 16. Evolution of concave slopes from first to third rainfall events: Concave Slope 1 (figures a, c, and e) and Concave Slope 2 (figures b, d, and f)

Fig. 17. Effect of drying/rainfall/erosion event on the computed factor of safety of planar and concave slopes 1 and 2
### Table 1. Mechanical and hydraulic soil properties used in Slopes 1 and 2

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter name</th>
<th>Value (Slope 1)</th>
<th>Value (Slope 2)</th>
<th>Symbol</th>
<th>Parameter name</th>
<th>Value (Slope 1)</th>
<th>Value (Slope 2)</th>
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<tr>
<td><strong>E</strong></td>
<td>Elastic Modulus (MPa)</td>
<td>100</td>
<td>20</td>
<td><strong>m</strong></td>
<td>Shape function for retention curve</td>
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<td></td>
<td>Poisson’s ratio</td>
<td>0.3</td>
<td>0.3</td>
<td><strong>η</strong></td>
<td>Parameter for porosity influence on retention curve</td>
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<td></td>
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<td></td>
<td>Bulk Modulus against suction changes (MPa)</td>
<td>10⁷</td>
<td>10⁷</td>
<td><strong>Sₙ</strong></td>
<td>Maximum saturation</td>
<td>1</td>
<td></td>
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<tr>
<td></td>
<td>Total density (kg/m³)</td>
<td>1800</td>
<td>1900</td>
<td><strong>Sᵣ</strong></td>
<td>Residual saturation</td>
<td>0.001</td>
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<tr>
<td></td>
<td>Effective cohesion (kPa)</td>
<td>5</td>
<td>15</td>
<td><strong>K</strong></td>
<td>Intrinsic permeability (m²)</td>
<td>10⁻¹²</td>
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<td></td>
<td>Effective friction angles (degree)</td>
<td>20, 18</td>
<td>35, 18</td>
<td><strong>kₘ</strong></td>
<td>Saturated permeability (m/s)</td>
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<td></td>
<td>Initial Porosity</td>
<td>0.33</td>
<td>0.296</td>
<td><strong>sₑ₀</strong></td>
<td>Reference air-entry pressure (kPa)</td>
<td>20</td>
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<td></td>
<td>Viscosity (MPa.s.)</td>
<td>10⁵</td>
<td>10⁵</td>
<td><strong>γₗ</strong></td>
<td>Leakage Coefficient</td>
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<td>Soil Tensile Strength (kPa)</td>
<td>13.7</td>
<td>21.4</td>
<td><strong>μₗ</strong></td>
<td>Water viscosity, (MPa.s.)</td>
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<tr>
<td></td>
<td>Reference water pressure (kPa)</td>
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<td>100</td>
<td><strong>ρᵢ</strong></td>
<td>Density of water (kg/m³)</td>
<td>10³</td>
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*Note: The viscosity value was not reported by Le et al. (2015 and 2016)*
### Table 2. Computed Factor of Safety for Slopes 1 and 2

<table>
<thead>
<tr>
<th>Slope</th>
<th>FOS - Gravity only, before suction</th>
<th>FOS - with suction</th>
<th>FOS - with rainfall</th>
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<tbody>
<tr>
<td></td>
<td>This study</td>
<td>Literature solution</td>
<td>This study</td>
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<tr>
<td>1 (Planar)</td>
<td>1.16</td>
<td>---</td>
<td>1.98</td>
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<tr>
<td>1 (Concave)</td>
<td>1.17</td>
<td>---</td>
<td>2.05</td>
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<tr>
<td>2 (Planar)</td>
<td>1.43</td>
<td>1.50 (Jeldes et al. 2015)</td>
<td>1.79</td>
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<tr>
<td>2 (Concave)</td>
<td>1.44</td>
<td>1.51 (Jeldes et al. 2015)</td>
<td>1.74</td>
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### Table 3. Changes in Factor of Safety for Slopes 1 and 2 due to Weather Events 1, 2 and 3

<table>
<thead>
<tr>
<th>Slope</th>
<th>FOS Gravity only</th>
<th>FOS due first drying/rainfall event</th>
<th>FOS due second drying/rainfall event</th>
<th>FOS due third drying/rainfall event</th>
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<tr>
<td>Weather Event</td>
<td>-</td>
<td>1</td>
<td>2</td>
<td>3</td>
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<tr>
<td>Slope 1 (Planar)</td>
<td>1.16</td>
<td>1.66</td>
<td>1.58</td>
<td>1.36</td>
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<tr>
<td>Slope 1 (Concave)</td>
<td>1.17</td>
<td>1.71</td>
<td>1.78</td>
<td>1.78</td>
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<tr>
<td>Slope 2 (Planar)</td>
<td>1.43</td>
<td>1.69</td>
<td>1.62</td>
<td>1.58</td>
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<td>Slope 2 (Concave)</td>
<td>1.44</td>
<td>1.64</td>
<td>1.90</td>
<td>1.62</td>
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