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## Scalar absorption by spinning D3-branes

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We discuss absorption of scalars by a distribution of spinning D3-branes. The D3-branes are multi-center solutions of supergravity theory. We solve the wave equation in various cases of supergravity backgrounds in which the equation becomes separable. We show that the absorption coefficients exhibit a universal behavior as functions of the angular momentum quantum number and the Hawking temperature. This behavior is similar to the form of the gray-body factors one encounters in scattering by black holes. Our discussion includes the problematic case of spherically symmetric distributions of D-branes, where resonances arise. We obtain the same universal form for the absorption coefficients, if the region enclosed by the D-branes is excluded from physical considerations. Non-extremal D-branes are also discussed. The results are similar to the extremal cases, albeit at half the Hawking temperature. We speculate that new degrees of freedom enter as one moves away from extremality.

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### I. INTRODUCTION

There exists ample evidence of an exact correspondence [1–3] between  $\mathcal{N}=4$  four-dimensional supersymmetric  $SU(N)$  Yang-Mills theory in the large- $N$  limit and string theory in a supergravity background representing a collection of D3-branes whose near-horizon geometry is the product of an anti-de Sitter space and a five-dimensional sphere ( $AdS_5 \times S^5$ ) [1–3]. This correspondence enables us to calculate correlation functions as well as various thermodynamic properties of super Yang-Mills theories in the large- $N$  limit using results from supergravity [4–7]. Exact results are obtained primarily due to the superconformal invariance of super Yang-Mills theory.

The metric for a stack of coincident D3-branes is

$$ds^2 = \frac{1}{\sqrt{H}} (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \sqrt{H} (dy_1^2 + \dots + dy_6^2) \quad (1)$$

where

$$H = 1 + \frac{R^4}{|\mathbf{y}|^4}, \quad |\mathbf{y}|^2 = dy_1^2 + \dots + dy_6^2 \quad (2)$$

and the dilaton field is constant. Near the horizon ( $|\mathbf{y}| \rightarrow 0$ ), one may drop the constant term in the harmonic function  $H$  and the resulting metric describes the space  $AdS_5 \times S^5$ . The  $AdS_5$  throat size  $R$  is also the curvature of  $S^5$ . We obtain exact superconformal invariance on the boundary of the AdS space.

This is a special solution of the supergravity field equations. It has zero temperature and is maximally supersymmetric. If our goal is to understand quantum gravity, we need to study a larger set of backgrounds that possess less symmetry and contain the special case (1) with  $H$  given by Eq.

(2) as a limit point. Such studies have already appeared in the literature, starting with linearized perturbations around the special solution [8] and including solutions of the full non-linear field equations [9,10]. Of particular importance are solutions that represent a collection of branes (multi-center). In this case, the harmonic function becomes

$$H = 1 + R^4 \int d^6 y' \frac{\sigma(\mathbf{y}')}{|\mathbf{y} - \mathbf{y}'|^4}, \quad \int d^6 y' \sigma(\mathbf{y}') = 1 \quad (3)$$

where  $\sigma(\mathbf{y})$  describes the distribution of branes, which can be discrete or continuous. These D-branes correspond to a broken phase of the super Yang-Mills theory where certain operators develop non-vanishing vacuum expectation values. This is the Coulomb branch of the gauge theory, because the remaining massless bosons mediate long-range Coulomb interactions. Superconformal symmetry is broken. The space of these solutions is the moduli space of the Yang-Mills theory. In this language, the special solution (1) with  $H$  given by Eq. (2) corresponds to the origin of moduli space.

From the supergravity point of view, even Eq. (1) with  $H$  given by Eq. (3) is but a special class of solutions (extreme solutions). A larger class of non-extreme solutions has been found [10–12]. It might be of interest to study this larger class of solutions, even though one expects that they correspond to unstable states in the Hilbert space of the gauge theory. They are all finite temperature configurations and they might shed some light on the thermodynamic properties of gauge theories, such as phase transitions [13].

A useful tool in these investigations is the study of interactions of branes with external probes [14–17]. In particular, the absorption cross section for a scalar in an AdS background has been shown to agree with the one obtained from superconformal field theory. This agreement has been shown to be exact in the low energy (for the scalar) limit and for all partial waves of the scalar field [14]. Extensions to higher energies have also been considered [16]. In the more general case of non-coincident D-branes, such calculations are considerably more involved, because the wave equation be-

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comes non-separable. Certain distributions of D-branes have been discussed for the scattering of  $s$ -waves.

It should also be mentioned that the case of a spherically symmetric distribution of D-branes has presented a puzzle [19]. In this case, the incident scalar field exhibits resonant behavior at certain values of its energy. These special frequencies extend all the way to infinity and are multiples of  $l/R^2$ , where  $l$  is the size of the distribution of the D-branes. Thus, they all go to zero as we approach the AdS limit ( $l \rightarrow 0$ ), but it is not clear how that limit is to be described. These resonances arise if one allows for reflection of the incident wave off of the D-branes without accounting for absorption.

Here, we extend the study of absorption of scalars by a distribution of D3-branes to include a larger set of supergravity backgrounds than previously considered and arbitrary partial waves. We solve the wave equation in the respective backgrounds for various D-brane distributions in the extremal limit. We also extend the analysis to the case of a non-extremal D3-brane. In general, the waves become singular at the positions of the D-branes. We find that the absorption coefficients exhibit a universal behavior similar to the form of the gray-body factors in the case of black-hole scattering [18].

Our calculations include the troublesome spherically symmetric D3-brane distribution where resonances arise for an infinite number of frequencies of the incident wave [19]. It is easy to understand the origin of these resonances. If the D-branes cover a closed surface in the transverse space (with coordinates  $y_1, \dots, y_6$ ), then the possibility arises of multiple reflections of the incident wave off of the D-branes. We demonstrate this for a spherical shell of D-branes as well as a long needle (cylindrical symmetry). The waves are no longer singular on the D-brane surfaces. Then it is natural to

ask what type of discontinuity the D-branes should impose on the incident wave function. This question will probably be settled by a conformal field theoretical calculation. Here we show that if reflection is forbidden, the absorption coefficients exhibit the same behavior as in the ‘‘healthier’’ cases, where no resonances arise (i.e., when the D-branes are distributed over an open surface in the transverse space). There is no reflection if the region enclosed by the D-brane distribution is excluded from physical considerations. Thus, it appears that the Schwarzschild-like coordinates one works with in supergravity are more physically relevant than the  $y_i$  ( $i=1, \dots, 6$ ) coordinates of the transverse space in Eq. (1).

Our discussion is organized as follows. In Sec. II, we introduce the metric for a collection of spinning D3-branes in the extremal limit and solve the wave equation in the respective backgrounds. In Sec. III, we extend the analysis to the case of a non-extremal D3-brane. The results are similar to the extremal cases, albeit at *half* the Hawking temperature. Our conclusions are summarized in Sec. IV.

## II. EXTREMAL D3-BRANES

In this section, we solve the wave equation for a scalar field in a supergravity background representing a distribution of D3-branes in the extremal limit. For completeness, we start with the metric for non-extremal spinning D3-branes and then take the extremal limit. In this limit, the branes are no longer spinning, but they settle to a state which is distinct from the AdS limit. These new vacua are stable due to the existence of a chemical potential. They are distinguished by quantum numbers that correspond to the angular momentum in the non-extremal regime [10].

The metric for a general distribution of spinning D3-branes in ten dimensions is<sup>1</sup> [10–12]

$$\begin{aligned}
 ds^2 = & \frac{1}{\sqrt{H}} \left( - (1 - fr_0^4/r^4) dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \right) + \sqrt{H} f^{-1} \frac{dr^2}{\lambda - r_0^4/r^4} \\
 & + \sqrt{H} r^2 \left( \zeta d\theta^2 + \zeta' \cos^2 \theta d\psi^2 - \frac{l_2^2 - l_3^2}{2r^2} \sin(2\theta) \sin(2\psi) d\theta d\psi \right) \\
 & - f \frac{2r_0^4 \cosh \gamma}{r^4} \sqrt{H} (l_1 \sin^2 \theta d\phi_1 + l_2 \cos^2 \theta \sin^2 \psi d\phi_2 + l_3 \cos^2 \theta \cos^2 \psi d\phi_3) dt \\
 & + f \frac{r_0^4}{r^4} \sqrt{H} (l_1 \sin^2 \theta d\phi_1 + l_2 \cos^2 \theta \sin^2 \psi d\phi_2 + l_3 \cos^2 \theta \cos^2 \psi d\phi_3)^2 \\
 & + \sqrt{H} r^2 \left[ \left( 1 + \frac{l_1^2}{r^2} \right) \sin^2 \theta d\phi_1^2 + \left( 1 + \frac{l_2^2}{r^2} \right) \cos^2 \theta \sin^2 \psi d\phi_2^2 + \left( 1 + \frac{l_3^2}{r^2} \right) \cos^2 \theta \cos^2 \psi d\phi_3^2 \right]
 \end{aligned} \tag{4}$$

where

<sup>1</sup>We have fixed various typographical errors in Refs. [10–12].

$$H = 1 + f \frac{r_0^4 \sinh^2 \gamma}{r^4}, \quad f^{-1} = \lambda \left( \frac{\sin^2 \theta}{1 + \frac{l_1^2}{r^2}} + \frac{\cos^2 \theta \sin^2 \psi}{1 + \frac{l_2^2}{r^2}} + \frac{\cos^2 \theta \cos^2 \psi}{1 + \frac{l_3^2}{r^2}} \right) \quad (5)$$

$$\lambda = \left( 1 + \frac{l_1^2}{r^2} \right) \left( 1 + \frac{l_2^2}{r^2} \right) \left( 1 + \frac{l_3^2}{r^2} \right) \quad (6)$$

$$\zeta = 1 + \frac{l_1^2 \cos^2 \theta + l_2^2 \sin^2 \theta \sin^2 \psi + l_3^2 \sin^2 \theta \cos^2 \psi}{r^2}, \quad \zeta' = 1 + \frac{l_2^2 \cos^2 \psi + l_3^2 \sin^2 \psi}{r^2} \quad (7)$$

and the charge of the branes is

$$R^4 = \frac{1}{2} r_0^4 \sinh(2\gamma). \quad (8)$$

The horizon is the root of  $\lambda - r_0^4/r^4 = 0$ . The parameters  $l_i$  ( $i=1,2,3$ ) are the angular momentum quantum numbers representing rotation around axes in three distinct planes, respectively, in the six-dimensional transverse space.

In the extremal limit, the horizon shrinks to zero ( $r_0 \rightarrow 0$ ) and also  $\gamma \rightarrow \infty$ , so that the charge  $R^4$  remains finite. The angular momenta also vanish and we obtain a static configuration. These configurations are still described by the three angular momentum quantum numbers. They are at finite temperature. The metric in the extreme limit becomes

$$ds^2 = \frac{1}{\sqrt{H}} (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \sqrt{H} f^{-1} \frac{dr^2}{\lambda} + \sqrt{H} r^2 \left( \zeta d\theta^2 + \zeta' \cos^2 \theta d\psi^2 - \frac{l_2^2 - l_3^2}{2r^2} \sin(2\theta) \sin(2\psi) d\theta d\psi \right) \\ + \sqrt{H} r^2 \left[ \left( 1 + \frac{l_1^2}{r^2} \right) \sin^2 \theta d\phi_1^2 + \left( 1 + \frac{l_2^2}{r^2} \right) \cos^2 \theta \sin^2 \psi d\phi_2^2 + \left( 1 + \frac{l_3^2}{r^2} \right) \cos^2 \theta \cos^2 \psi d\phi_3^2 \right] \quad (9)$$

where

$$H = 1 + f \frac{R^4}{r^4} \quad (10)$$

and the other functions,  $f, \lambda, \zeta, \zeta'$  are still given by Eq. (7). It can be shown that this metric is equivalent to the multi-center form (1) with  $H$  given by Eq. (3) through the following transformation [10,20]:

$$\begin{aligned} y_1 &= \sqrt{r^2 + l_1^2} \sin \theta \cos \phi_1 \\ y_2 &= \sqrt{r^2 + l_1^2} \sin \theta \sin \phi_1 \\ y_3 &= \sqrt{r^2 + l_2^2} \cos \theta \sin \psi \cos \phi_2 \\ y_4 &= \sqrt{r^2 + l_2^2} \cos \theta \sin \psi \sin \phi_2 \\ y_5 &= \sqrt{r^2 + l_3^2} \cos \theta \cos \psi \cos \phi_3 \\ y_6 &= \sqrt{r^2 + l_3^2} \cos \theta \cos \psi \sin \phi_3. \end{aligned} \quad (11)$$

The wave equation in a general background is complicated. We will therefore restrict attention to the special case  $l_2 = l_3$  ("cylindrical" symmetry). We do not expect our conclusions to change in the more general case, although it would take considerably more effort to prove it. The metric becomes

$$ds^2 = \frac{1}{\sqrt{H}} (-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) \\ + \sqrt{H} \left( 1 + \frac{l_2^2}{r^2} \right) \zeta \frac{dr^2}{\lambda} + \sqrt{H} r^2 \left[ \zeta d\theta^2 + \left( 1 + \frac{l_1^2}{r^2} \right) \right. \\ \times \sin^2 \theta d\phi_1^2 + \left( 1 + \frac{l_2^2}{r^2} \right) \\ \left. \times \cos^2 \theta (d\psi^2 + \sin^2 \psi d\phi_2^2 + \cos^2 \psi d\phi_3^2) \right] \quad (12)$$

where

$$H = 1 + \frac{R^4}{\left( 1 + \frac{l_2^2}{r^2} \right) \zeta r^4}, \quad \lambda = \left( 1 + \frac{l_1^2}{r^2} \right) \left( 1 + \frac{l_2^2}{r^2} \right)^2, \\ \zeta = 1 + \frac{l_1^2 \cos^2 \theta + l_2^2 \sin^2 \theta}{r^2}. \quad (13)$$

Notice that the various functions,  $H, \lambda, \zeta$ , comprising the metric tensor, are functions of  $r, \theta$  only. The ten-dimensional wave equation for a scalar field,

$$\partial_A \sqrt{-g} g^{AB} \partial_B \Phi = 0 \quad (14)$$

becomes separable for fields that are independent of the angular variables  $\psi, \phi_i$  ( $i=1,2,3$ ). Indeed, for a field of momentum  $k_\mu$  and mass  $m^2=k_\mu k^\mu$ ,

$$\Phi(x^\mu; r, \theta) = e^{ik \cdot x} \Psi(r, \theta) \quad (15)$$

after some algebra, we obtain

$$\begin{aligned} \frac{1}{r^3 \left(1 + \frac{l_2^2}{r^2}\right)} \partial_r (\lambda r^5 \partial_r \Psi) + m^2 r^2 \Psi + \frac{m^2 R^4}{r^2 \left(1 + \frac{l_2^2}{r^2}\right)} \Psi \\ - (\hat{L}^2 - m^2 l_1^2 \cos^2 \theta - m^2 l_2^2 \sin^2 \theta) \Psi = 0. \end{aligned} \quad (16)$$

We will solve this equation in the limit where the mass is small compared with the AdS curvature, and the angular momenta are also small:

$$mR \ll 1, \quad l_i \lesssim mR^2 \quad (i=1,2). \quad (17)$$

In this limit, the terms proportional to the angular momentum components,  $l_i$  ( $i=1,2$ ), can be dropped. Indeed, their contribution is  $m^2 l_i^2 \ll m^2 R^2 \ll 1$ . Therefore, they are small compared to the angular momentum ( $\hat{L}^2$  term) contribution. The wave equation becomes

$$\begin{aligned} \frac{1}{r^3 \left(1 + \frac{l_2^2}{r^2}\right)} \partial_r (\lambda r^5 \partial_r \Psi) + m^2 r^2 \Psi + \frac{m^2 R^4}{r^2 \left(1 + \frac{l_2^2}{r^2}\right)} \Psi - \hat{L}^2 \Psi \\ = 0. \end{aligned} \quad (18)$$

The eigenvalues of  $\hat{L}^2$  are  $j(j+4)$ . Therefore, the radial part of the wave equation is

$$\begin{aligned} \frac{1}{r^3 \left(1 + \frac{l_2^2}{r^2}\right)} (\lambda r^5 \Psi')' + m^2 r^2 \Psi + \frac{m^2 R^4}{r^2 \left(1 + \frac{l_2^2}{r^2}\right)} \Psi - j(j+4) \Psi \\ = 0. \end{aligned} \quad (19)$$

We will solve this equation in two regimes,  $r \gg mR^2$  and  $r \ll 1/m$ , and then match the respective expressions asymptotically.

For  $r \gg mR^2$ , we obtain

$$\frac{1}{r^3} (r^5 \Psi')' + m^2 r^2 \Psi - j(j+4) \Psi = 0 \quad (20)$$

whose solution is

$$\Psi = \frac{1}{r^2} J_{j+2}(mr) \quad (21)$$

where we dropped the solution which is not regular at small  $r$ . The normalization is arbitrary, since we only care about ratios of fluxes. At small  $r$ , the solution behaves as

$$\Psi \sim \frac{m^2}{4(j+2)!} \left(\frac{mr}{2}\right)^j. \quad (22)$$

In the regime of small  $r$  ( $mr \ll 1$ ), the wave equation becomes

$$\frac{1}{r^3 \left(1 + \frac{l_2^2}{r^2}\right)} (\lambda r^5 \Psi')' + \frac{m^2 R^4}{r^2 \left(1 + \frac{l_2^2}{r^2}\right)} \Psi - j(j+4) \Psi = 0. \quad (23)$$

To solve this equation, we distinguish between three cases, in which one, two or three components of the angular momentum are non-vanishing, respectively.

### A. One-component angular momentum

The simplest case is the one where  $l_2=l_3=0$ . The D-branes are a limiting configuration of non-extremal branes spinning around an axis in the plane defined by the coordinates  $y_1, y_2$  in the transverse space [cf. Eq. (11)]. They are uniformly distributed on a disk of radius  $l_1$  in this plane [9,10]. To see this, note that the harmonic function  $H$  [Eq. (13)] can be written as

$$H = 1 + \frac{R^4}{\zeta r^4} = 1 + \frac{R^4}{(r^2 + l_1^2 \cos^2 \theta) r^2}. \quad (24)$$

The D-branes are in the region bounded by the  $r=0$  surface, which is a disk of radius  $l_1$  in the plane  $y_3 = \dots = y_6 = 0$  [because of Eq. (11)]. Define

$$y_{||}^2 = y_1^2 + y_2^2 = (r^2 + l_1^2) \sin^2 \theta, \quad y_{\perp}^2 = y_3^2 + \dots + y_6^2 = r^2 \cos^2 \theta \quad (25)$$

in terms of which  $H$  becomes

$$H \approx 1 + \frac{R^4}{l_1^2 y_{\perp}^2} \quad (26)$$

as  $r \rightarrow 0$ . The density of D-branes is therefore independent of  $y_{||}$  and  $\sigma = 1/\pi l_1^2$ , i.e., the D-branes are uniformly distributed on a disk of radius  $l_1$  in the  $y_{||}=0$  plane.

The wave equation for small  $r$  [Eq. (23)] is

$$\frac{1}{r^3} (\lambda r^5 \Psi')' + \frac{R^4 m^2}{r^2} \Psi - j(j+4) \Psi = 0 \quad (27)$$

where  $\lambda = 1 + l_1^2/r^2$ . To solve this equation, change variables to  $u = 1/\lambda$ . Then

$$(1-u)u^2 \frac{d^2 \Psi}{du^2} + u(2-u) \frac{d\Psi}{du} + \frac{m^2 R^4}{4l_1^2} \Psi - \frac{j(j+4)u}{4(1-u)} \Psi = 0. \quad (28)$$

Next, we need to control the behavior at the singular points  $u=0,1$ . As  $u \rightarrow 0$ , we obtain

$$u^2 \frac{d^2 \Psi}{du^2} + 2u \frac{d\Psi}{du} + \frac{m^2 R^4}{4l_1^2} \Psi - \frac{j(j+4)}{4} u \Psi = 0. \quad (29)$$

Assuming  $\Psi \sim u^a$ , we obtain

$$\begin{aligned} a &= -\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{m^2 R^4}{l_1^2}} = -\frac{1}{2} + i\kappa, \\ \kappa &= \frac{1}{2} \sqrt{\frac{m^2 R^4}{l_1^2} - 1} \approx \frac{mR^2}{2l_1} = \frac{m}{4\pi T_H} \end{aligned} \quad (30)$$

where  $T_H = l_1/2\pi R^2$  is the Hawking temperature. As  $u \rightarrow 1$ , we obtain

$$\begin{aligned} (1-u)^2 \frac{d^2 \Psi}{du^2} + (1-u) \frac{d\Psi}{du} + \frac{m^2 R^4}{4l_1^2} (1-u) \Psi - \frac{j(j+4)}{4} \Psi \\ = 0. \end{aligned} \quad (31)$$

Assuming  $\Psi \sim (1-u)^b$ , we obtain  $b = (j+4)/2$ . Now set

$$\Psi = A u^{-1/2+i\kappa} (1-u)^{j/2+2} f(u). \quad (32)$$

Equation (29) becomes

$$\begin{aligned} (1-u)u \frac{d^2 f}{du^2} + [1+2i\kappa - (j+4+2i\kappa)u] \frac{df}{du} \\ - \frac{(j+3+2i\kappa)^2}{4} f = 0 \end{aligned} \quad (33)$$

whose solution is the hypergeometric function

$$f(u) = F((j+3)/2+i\kappa, (j+3)/2+i\kappa; 1+2i\kappa; u). \quad (34)$$

To obtain the behavior of  $\Psi$  for large  $r$ , note that

$$\begin{aligned} F((j+3)/2+i\kappa, (j+3)/2+i\kappa; 1+2i\kappa; u) \\ = \frac{\Gamma(1+2i\kappa)\Gamma(j+2)}{[\Gamma((j+3)/2+i\kappa)]^2} \frac{1}{(1-u)^{j+2}} + \dots \end{aligned} \quad (35)$$

where the dots represent terms that are regular in  $1-u$ . Therefore, using Eqs. (32), (34) and (35), we arrive at

$$\Psi \approx A \frac{\Gamma(1+2i\kappa)\Gamma(j+2)}{[\Gamma((j+3)/2+i\kappa)]^2} \left(\frac{r}{l_1}\right)^j. \quad (36)$$

Comparing with the asymptotic form (22), we obtain

$$A = \frac{[\Gamma((j+3)/2+i\kappa)]^2}{\Gamma(1+2i\kappa)} \frac{m^{j+2} l_1^j}{2^{j+2} (j+1)! (j+2)!}. \quad (37)$$

In the small  $r$  limit, we have  $u \approx r^2/l_1^2$  and Eq. (32) reads

$$\Psi \approx A \left(\frac{r}{l_1}\right)^{-1+2i\kappa}. \quad (38)$$

The absorption coefficient, which is the ratio of the incoming flux at  $r \rightarrow 0$  to the incoming flux at  $r \rightarrow \infty$ , is

$$\begin{aligned} \mathcal{P} &= \frac{\Im(\lambda r^5 \Psi^* \Psi')|_{r \rightarrow 0}}{\Im(r^5 \Psi^* \Psi')|_{r \rightarrow \infty}} = 4\pi\kappa l_1^4 |A|^2 \\ &= 4\pi\kappa \frac{|\Gamma((j+3)/2+i\kappa)|^4}{|\Gamma(1+2i\kappa)|^2} \frac{m^{2j+4} l_1^{2j+4}}{4^{j+2} ((j+1)! (j+2)!)^2}. \end{aligned} \quad (39)$$

This is of the same form as the gray-body factors obtained in black-hole scattering [18] for large  $j$ . Indeed, comparing

$$\mathcal{P} \sim |\Gamma((j+3)/2+i\kappa)|^4 = \left| \Gamma\left(\frac{j+3}{2} + i\frac{m}{4\pi T_H}\right) \right|^4 \quad (40)$$

with the general form of a gray-body factor [18],

$$\mathcal{P}_{b.h.} \sim \left| \Gamma\left(\frac{j+2}{2} + i\frac{m}{4\pi T_L}\right) \right|^2 \left| \Gamma\left(\frac{j+2}{2} + i\frac{m}{4\pi T_R}\right) \right|^2 \quad (41)$$

we see that we get contributions from both left- and right-moving modes at temperatures  $T_L = T_R = T_H$ .

Equation (39) also reproduces results derived earlier in the AdS (zero temperature) limit. Indeed, in the small temperature limit, we have  $\kappa \rightarrow \infty$  and

$$A \approx \sqrt{\pi} \frac{i^{j+2} R^{2j+4} m^{2j+4}}{4^{j+2+i\kappa/2} l_1^{2(j+1)} (j+2)!} \frac{\Gamma(1/2+i\kappa)}{\Gamma(1+i\kappa)} \quad (42)$$

for even  $j$ , where we used the Gamma function identities

$$\Gamma(2x) = \frac{1}{\sqrt{2\pi}} 2^{2x-1/2} \Gamma(x) \Gamma(x+1/2), \quad \Gamma(x+1) = x\Gamma(x). \quad (43)$$

Since also  $|\Gamma(\frac{1}{2}+i\kappa)|^2 = \pi/\cosh(\pi\kappa)$  and  $|\Gamma(1+i\kappa)|^2 = \pi\kappa/\sinh(\pi\kappa)$ , we obtain

$$|A|^2 \approx \frac{\pi R^{4j+8} m^{4j+8}}{4^{2j+4} \kappa l_1^4 ((j+1)! (j+2)!)^2} \quad (44)$$

and so the absorption coefficient (39) becomes

$$\mathcal{P} = 4\pi\kappa l_1^4 |A|^2 \approx \frac{\pi^2 R^{4j+8} m^{4j+8}}{4^{2j+3} ((j+1)! (j+2)!)^2} \quad (45)$$

in agreement with earlier results [14].

## B. Two-component angular momentum

Next, we turn to the case where two angular momentum quantum numbers are non-vanishing. Let  $l_1 = 0$ ,  $l_2 = l_3$ . Then the harmonic function  $H$  [Eq. (13)] becomes

$$H = 1 + \frac{R^4}{\left(1 + \frac{l_2^2}{r^2}\right) \xi r^4} = 1 + \frac{R^4}{(r^2 + l_2^2)(r^2 + l_2^2 \sin^2 \theta)}. \quad (46)$$



The branes lie in the 4-sphere bounded by the surface  $r=0$ , i.e., they lie inside a sphere of radius  $l_2$  in the  $y_1=y_2=0$  hyperplane. Define [cf. Eq. (11)]

$$y_{\perp}^2 = y_1^2 + y_2^2 = r^2 \sin^2 \theta, \quad y_{\parallel}^2 = y_3^2 + \dots + y_6^2 = (r^2 + l_2^2) \cos^2 \theta. \quad (47)$$

As  $r \rightarrow 0$ , the harmonic function becomes

$$H \approx 1 + \frac{R^4 r^2}{l_2^4 y_{\perp}^2} \quad (48)$$

and so the density of D-branes is proportional to  $r^2 \rightarrow 0$ . It follows that there can be no D-branes in the interior of the four-sphere, therefore, the D-branes are distributed on the surface of the four-sphere of radius  $l_2$  defined by  $r=0$ . This distribution is uniform by symmetry, and the density is  $\sigma = 1/2\pi^2 l_2^3$ .

The wave equation for small  $r$  [Eq. (23)] is

$$\frac{1}{r^3 \sqrt{\lambda}} (\lambda r^5 \Psi')' + \frac{R^4 m^2}{\sqrt{\lambda} r^2} \Psi - j(j+4) \Psi = 0 \quad (49)$$

where  $\lambda = (1 + l_2^2/r^2)^2$ . Changing variables to  $u = 1/\sqrt{\lambda}$ , we obtain

$$u(1-u) \frac{d^2 \Psi}{du^2} + \frac{d\Psi}{du} + \frac{R^4 m^2}{4l_2^2 u} \Psi - \frac{j(j+4)}{4(1-u)} \Psi = 0. \quad (50)$$

As  $u \rightarrow 1$ , we obtain

$$(1-u)^2 \frac{d^2 \Psi}{du^2} + (1-u) \frac{d\Psi}{du} + \frac{R^4 m^2}{4l_2^2} (1-u) \Psi - \frac{j(j+4)}{4} \Psi = 0. \quad (51)$$

Assuming  $\Psi \sim (1-u)^b$ , we find  $b = (j+4)/2$ . As  $u \rightarrow 0$ , we obtain

$$u^2 \frac{d^2 \Psi}{du^2} + u \frac{d\Psi}{du} + \frac{R^4 m^2}{4l_2^2} \Psi - \frac{j(j+4)}{4} u \Psi = 0. \quad (52)$$

Assuming  $\Psi \sim u^a$ , we find

$$a = i\kappa, \quad \kappa = \frac{R^2 m}{2l_2}. \quad (53)$$

Setting

$$\Psi = A(1-u)^{j/2+2} u^{i\kappa} f(u) \quad (54)$$

the wave equation (50) becomes

$$u(1-u)f'' + (1+2i\kappa - (j+4+2i\kappa)u)f' - (j/2+1+i\kappa)(j/2+2+i\kappa)f = 0 \quad (55)$$

whose solution is the hypergeometric function

$$f(u) = F(j/2+1+i\kappa, j/2+2+i\kappa; 1+2i\kappa; u). \quad (56)$$

In the large  $r$  limit, we have  $1-u \approx l_2^2/r^2$ ,

$$f(l_2^2/r^2) \approx \left(\frac{l_2}{r}\right)^{-2j-4} \frac{\Gamma(1+2i\kappa)\Gamma(j+2)}{\Gamma(j/2+1+i\kappa)\Gamma(j/2+2+i\kappa)} \quad (57)$$

and so

$$\Psi \approx A \frac{\Gamma(1+2i\kappa)\Gamma(j+2)}{\Gamma(j/2+1+i\kappa)\Gamma(j/2+2+i\kappa)} \frac{r^j}{l_2^j}. \quad (58)$$

Comparing with the asymptotic form (22), we obtain

$$A = \frac{\Gamma(j/2+1+i\kappa)\Gamma(j/2+2+i\kappa)}{\Gamma(1+2i\kappa)} \frac{m^{j+2} l_2^j}{2^{j+2} (j+1)! (j+2)!}. \quad (59)$$

In the small  $r$  limit, we have  $u \approx r^2/l_2^2$  and

$$\Psi \approx A \left(\frac{r}{l_1}\right)^{2i\kappa}. \quad (60)$$

The absorption coefficient is [cf. Eq. (39)]

$$\begin{aligned} \mathcal{P} &= 4\pi\kappa l_2^4 |A|^2 \\ &= 4\pi\kappa \frac{|\Gamma(j/2+1+i\kappa)\Gamma(j/2+2+i\kappa)|^2}{|\Gamma(1+2i\kappa)|^2} \\ &\quad \times \frac{m^{2j+4} l_2^{2j+4}}{4^{j+2} ((j+1)! (j+2)!)^2} \end{aligned} \quad (61)$$

in agreement with the one-component case [Eq. (39)] and with the same small-temperature ( $\kappa \rightarrow \infty$ ) limit as before [Eq. (45)].

### C. Three-component angular momentum

Next, we consider the case where all three angular momentum quantum numbers are non-vanishing. This case is problematic, because of the emergence of an infinite number of resonances. These problems arise because the  $r \rightarrow 0$  surface encloses a region of finite volume in the transverse space spanned by the  $y_i$  ( $i=1, \dots, 6$ ) coordinates [10]. In the cases previously considered, when at most two angular momentum quantum numbers were non-vanishing, the  $r \rightarrow 0$  surface enclosed a region of zero measure in the transverse space. When this region has finite volume, the possibility arises of the wave bouncing off of the branes (which are distributed on the  $r=0$  surface) an infinite number of times, hence the resonant behavior [19]. We shall discuss two cases: (a) the spherically symmetric case ( $l_1=l_2=l_3$ ), and (b) the ‘‘long needle’’ case ( $l_2=l_3 \ll l_1$ ). We will compute the absorption coefficients in both cases and show that they are in agreement with our previous results. It should be emphasized that this agreement is obtained when one performs the calculations in the Schwarzschild-like coordinates (4). The  $y_i$  ( $i=1, \dots, 6$ ) coordinates span a larger space which includes a region of finite volume surrounded by D-branes. It appears that this region, which is disconnected

from the region spanned by the Schwarzschild-like coordinates (4), should be excluded from physical considerations.

First, let us consider the spherically symmetric case,  $l_1 = l_2 = l_3$ . The wave equation (23) becomes

$$\frac{1}{r^3 \lambda^{1/3}} (\lambda r^5 \Psi')' + \frac{R^4 m^2}{r^2 \lambda^{1/3}} \Psi - j(j+4) \Psi = 0 \quad (62)$$

where

$$\lambda = \left( 1 + \frac{l_1^2}{r^2} \right)^3. \quad (63)$$

Switching variables to  $y = r \lambda^{1/6} = \sqrt{l_1^2 + r^2}$  [note that  $y^2 = y_1^2 + \dots + y_6^2$ , due to Eq. (11)], we obtain

$$\frac{1}{y^3} (y^5 \Psi')' + \left( \frac{R^4 m^2}{y^2} - j(j+4) \right) \Psi = 0 \quad (64)$$

whose solution is

$$\Psi = A \frac{1}{y^2} H_{j+2}^{(1)} \left( \frac{R^2 m}{y} \right). \quad (65)$$

We discarded the other solution,  $H_{j+2}^{(2)}$ , because we require an incoming wave as  $r \rightarrow 0$  (indeed,  $H_{j+2}^{(1)} \sim y^{-3/2} e^{iR^2 m/y}$  for small  $y$ ). In the large  $r$  limit, we have  $y \approx r$  and so

$$\Psi \approx -iA \frac{2^{j+2} (j+1)!}{R^{2j+4} m^{j+2}} r^j. \quad (66)$$

Comparing with the asymptotic form (22), we obtain

$$A = i \frac{m^{2j+4} R^{2j+4}}{4^{j+2} (j+1)! (j+2)!}. \quad (67)$$

The absorption coefficient is

$$\begin{aligned} \mathcal{P} &= \frac{16\pi R^2 m}{l_1} |A|^2 \Im(H_{j+2}^{(1)*} H_{j+2}^{(1)}) \\ &= \frac{16\pi R^2 m}{l_1} \frac{m^{4j+8} R^{4j+8}}{4^{2j+4} ((j+1)! (j+2)!)^2} \Im(H_{j+2}^{(1)*} H_{j+2}^{(1)}) \end{aligned} \quad (68)$$

where the Bessel functions are evaluated at  $2\kappa = R^2 m/l_1$  (i.e., at  $y = l_1$ ). To compare with previous results, use

$$H_{j+2}^{(1)}(2\kappa) = \frac{(-i)^{j+5/2}}{\sqrt{\pi\kappa}} e^{2i\kappa} \left( 1 + \frac{i}{4\kappa} (j+5/2)(j+3/2) + \dots \right). \quad (69)$$

Using the Gamma function identities (43), after some algebra we obtain

$$\begin{aligned} H_{j+2}^{(1)*}(2\kappa) &= \frac{2^{i\kappa} i}{\kappa^{j+2}} e^{-2i\kappa} \frac{\Gamma(j/2+5/4+i\kappa)\Gamma(j/2+7/4+i\kappa)}{\Gamma(1+2i\kappa)} \\ &+ \dots \end{aligned} \quad (70)$$

and also

$$\begin{aligned} H_{j+2}^{(1)'}(2\kappa) &= \frac{2^{-1-i\kappa} i}{\kappa^{j+3}} e^{2i\kappa} \frac{\Gamma(j/2+7/4-i\kappa)\Gamma(j/2+9/4-i\kappa)}{\Gamma(1-2i\kappa)} \\ &+ \dots \end{aligned} \quad (71)$$

Therefore, the absorption coefficient can be written as

$$\begin{aligned} \mathcal{P} &= \frac{4\pi\kappa}{((j+1)!(j+2)!)^2} \\ &\times \frac{|\Gamma(j/2+5/4+i\kappa)\Gamma(j/2+7/4+i\kappa)|^2}{|\Gamma(1+2i\kappa)|^2} \left( \frac{ml_1}{2} \right)^{2j+4} \end{aligned} \quad (72)$$

in agreement with previous results [Eqs. (39) and (61)].

Complications arise when one continues into the  $y < l_1$  region. To do that, we need to assume a certain, spherically symmetric, distribution of D-branes. Assuming the space  $y < l_1$  is empty, we obtain the wave equation

$$\frac{1}{y^3} (y^5 \Psi')' + \left( \frac{R^4 m^2}{l_1^4} y^2 - j(j+4) \right) \Psi = 0 \quad (73)$$

whose solution is

$$\Psi = A' \frac{1}{y^2} J_{j+2} \left( \frac{R^2 m}{l_1^2} y \right) \quad (74)$$

where we discarded the solution which is singular as  $y \rightarrow 0$ .

Also, we may now have an outgoing wave in the  $y > l_1$  region, as well as an incoming wave, so Eq. (65) should be replaced by

$$\Psi = A \frac{1}{y^2} H_{j+2}^{(1)} \left( \frac{R^2 m}{y} \right) + B \frac{1}{y^2} H_{j+2}^{(2)} \left( \frac{R^2 m}{y} \right) \quad (75)$$

and Eq. (67) becomes

$$A - B = i \frac{m^{2j+4} R^{2j+4}}{4^{j+2} (j+1)! (j+2)!}. \quad (76)$$

Demanding continuity at  $y = l_1$ , we can obtain all coefficients,  $A, B, A'$ . The system exhibits a resonant behavior when either the wave function or its derivative vanishes on the D-brane (at  $y = l_1$ ). In this case, we have either  $J_{j+2}(R^2 m/l_1) = 0$  or  $J'_{j+2}(R^2 m/l_1) = 0$ , both of which have an infinite number of solutions,  $m_n = x_n l_1 / R^2$ , or  $m_n = x'_n l_1 / R^2$ , where  $J_{j+2}(x_n) = J'_{j+2}(x'_n) = 0$ .

In hopes of shedding some light on this singular behavior, one may study the case where two of the components of the angular momentum are small, i.e., let us assume



$$l_2 = l_3 \ll l_1 \lesssim R^2 m. \quad (77)$$

This is a small departure from the one-component case considered above and one may hope to recover that solution in the limit  $l_2 \rightarrow 0$ . Contrary to expectations, we find that the resonances persist in the  $l_2 \rightarrow 0$  limit. Thus, even switching on small components leads to a significant departure from the one-component case. It appears that the region spanned by the Schwarzschild-like coordinates is more physically relevant than the entire transverse space spanned by the coordinates  $y_i$  ( $i = 1, \dots, 6$ ).

We will solve the wave equation by considering two regimes,  $r \gg l_2$  and  $r \ll l_1 \lesssim R^2 m$ . When  $r \gg l_2$ , we can write

$$\frac{1}{r^3} \left( \left( 1 + \frac{l_1^2}{r^2} \right) r^5 \Psi' \right)' + \frac{R^4 m^2}{r^2} \Psi - j(j+4) \Psi = 0 \quad (78)$$

which is identical to the one-component angular momentum case. Therefore, the solution is

$$\Psi = A u^{-1/2+i\kappa} (1-u)^{j/2+2} \times F(j/2+1+i\kappa, j/2+1+i\kappa; 1+2i\kappa; u) \quad (79)$$

where  $u = 1/(1+l_1^2/r^2)$ ,  $A$  is given by Eq. (37), and  $\kappa$  is given by Eq. (30). At small  $r$ , we have  $u \sim r^2/l_1^2$ , so

$$\Psi \sim A \left( \frac{r}{l_1} \right)^{-1+2i\kappa}. \quad (80)$$

In the regime  $r \ll l_1 \lesssim R^2 m$ , we can write the wave equation as

$$\frac{1}{r^3 \left( 1 + \frac{l_2^2}{r^2} \right)} \left( \left( 1 + \frac{l_2^2}{r^2} \right)^2 \frac{l_1^2}{r^2} r^5 \Psi' \right)' + \frac{R^4 m^2}{r^2 \left( 1 + \frac{l_2^2}{r^2} \right)} \Psi - j(j+4) \Psi = 0. \quad (81)$$

Switching variables to  $y = \sqrt{1+r^2/l_2^2}$ , we obtain

$$\frac{1}{y} (y^3 \Psi')' + \frac{R^4 m^2}{l_1^2} \Psi - \frac{j(j+4)l_2^2}{l_1^2} y^2 \Psi = 0 \quad (82)$$

whose solution is

$$\Psi \sim \frac{1}{y} I_{2i\kappa} \left( \frac{\sqrt{j(j+4)} l_2}{l_1} y \right). \quad (83)$$

For sufficiently small  $j$ , the argument of the Bessel function is small, so we can approximate

$$\Psi \approx C y^{-1+2i\kappa}. \quad (84)$$

At large  $r$ , we have  $y \sim r/l_2$ , so

$$\Psi \sim C \left( \frac{r}{l_2} \right)^{-1+2i\kappa}. \quad (85)$$

Matching the two asymptotic forms, we obtain  $C = A(l_2/l_1)^{-1+2i\kappa}$ , and

$$\Psi = A \left( \frac{l_2^2 + r^2}{l_1^2} \right)^{-1/2+i\kappa}. \quad (86)$$

The absorption coefficient is found to be

$$\begin{aligned} \mathcal{P} &= 4\pi\kappa l_1^4 |A|^2 \\ &= 4\pi\kappa \frac{|\Gamma((j+3)/2+i\kappa)|^4}{|\Gamma(1+2i\kappa)|^2} \frac{m^{2j+4} l_1^{2j+4}}{4^{j+2} ((j+1)!(j+2)!)^2} \end{aligned} \quad (87)$$

in agreement with the one-component case [Eq. (39)].

Again, complications arise when one considers the region  $y < 1$ . To illustrate the effect, we shall only consider the case of an  $s$ -wave,  $j=0$ . In this case, Eq. (82) is the wave equation in cylindrical coordinates. For higher  $j$ , we need to express the wave functions in terms of cylindrical harmonics instead of spherical harmonics, and calculations become increasingly involved for large  $j$ . For  $j=0$ , we obtain for  $y > 1$ ,

$$\Psi = C_1 y^{-1+2i\kappa} + C_2 y^{-1-2i\kappa}. \quad (88)$$

Assuming that the region  $y < 1$  is empty, the wave equation becomes

$$\frac{1}{y} (y^3 \Psi')' + \frac{R^4 m^2}{l_1^2} y^2 \Psi = 0 \quad (89)$$

whose solution is

$$\Psi = D \frac{1}{y} \sin \left( \frac{R^2 m}{l_1} y \right). \quad (90)$$

This exhibits resonant behavior when the wave function vanishes on the D-branes, at  $y=1$ . The resonances are at the points

$$m_n = \frac{n\pi l_1}{2R^2}. \quad (91)$$

Notice that in the limit  $l_2 \rightarrow 0$ , these resonances persist and it is not clear how one may recover the one-component solution discussed above. However, if one excludes the cylindrical region  $y < 1$  enclosed by the D-branes from physical considerations, then one obtains the same type of absorption coefficients as in the rest of the cases (with one or two non-vanishing angular momentum quantum numbers) and no resonances arise.

### III. A NON-EXTREMAL D3-BRANE

When we try to go away from extremality, the wave equation becomes very complicated, because the metric develops off-diagonal elements. Still, we would like to study the more general background of non-extremal branes and study their limit as the horizon shrinks to zero. Here we discuss the simplest case of all angular momentum quantum numbers being zero. Setting  $l_1=l_2=l_3=0$  in Eq. (4), the metric becomes

$$ds^2 = \frac{1}{\sqrt{H}} \left( -(1-r_0^4/r^4)dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \right) + \sqrt{H} \frac{dr^2}{1-r_0^4/r^4} + \sqrt{H} r^2 [d\theta^2 + \sin^2\theta d\phi_1^2 + \cos^2\theta(d\psi^2 + \sin^2\psi d\phi_2^2 + \cos^2\psi d\phi_3^2)] \quad (92)$$

where

$$H = 1 + \frac{R^4}{r^4}. \quad (93)$$

The radial part of the wave equation,

$$\partial_A \sqrt{-g} g^{AB} \partial_B \Phi = 0 \quad (94)$$

for fields that are independent of the angular variables  $\psi$ ,  $\phi_i$  ( $i=1,2,3$ ) and  $\bar{x}$ ,

$$\Phi = e^{i\omega t} \Psi(r) Y_j(\theta) \quad (95)$$

can be written as

$$\frac{1}{r^3} \left( \left( 1 - \frac{r_0^4}{r^4} \right) r^5 \Psi' \right)' + \frac{r^2 \omega^2}{(1-r_0^4/r^4)} H \Psi - j(j+4) \Psi = 0. \quad (96)$$

We will solve this equation for wavelengths (horizons) of size much larger (smaller) than the AdS scale,

$$r_0 \ll R \ll 1/\omega. \quad (97)$$

We will also assume  $r_0 \lesssim R^2 \omega$  (so that we can take the limit of the frequency being large compared to the temperature).

Away from the horizon,  $r \gg R^2 \omega$ , we can replace  $H$  by 1 [by comparing its contribution to the  $j(j+4)$  term]. Therefore,

$$\frac{1}{r^3} (r^5 \Psi')' + r^2 \omega^2 \Psi - j(j+4) \Psi = 0 \quad (98)$$

whose solution is

$$\Psi = \frac{1}{r^2} J_{j+2}(\omega r). \quad (99)$$

Next, we consider the region near the horizon,  $r \ll R$ . In this case,  $H \approx R^4/r^4$ , and so

$$\frac{1}{r^3} \left( \left( 1 - \frac{r_0^4}{r^4} \right) r^5 \Psi' \right)' + \frac{R^4 \omega^2}{r^2 (1-r_0^4/r^4)} \Psi - j(j+4) \Psi = 0. \quad (100)$$

To solve this equation, first we need to isolate the singularity at the horizon. The wave function at the horizon behaves as  $\Psi \sim (1-r_0^4/r^4)^{i\kappa}$ . It is therefore convenient to define

$$\Psi = A \left( 1 - \frac{r_0^4}{r^4} \right)^{i\kappa} f(r), \quad \kappa = \frac{R^2 \omega}{4r_0} = \frac{\omega}{4\pi T_H} \quad (101)$$

where  $T_H = r_0/\pi R^2$  is the Hawking temperature. Then Eq. (100) becomes

$$r^2 \left( 1 - \frac{r_0^4}{r^4} \right) f'' + r \left[ 5 - (1-2i\kappa) \frac{r_0^4}{r^4} \right] f' - j(j+4) f = - \frac{4R^4 \omega^2}{r^2} \frac{1 + \frac{r_0^2}{r^2} + \frac{r_0^4}{r^4}}{1 + \frac{r_0^2}{r^2}} f. \quad (102)$$

The function  $f$  has a regular limit as  $r_0 \rightarrow 0$  (as expected, since we have already isolated the singularity in the wave function). Neglecting higher-order corrections, we set  $r_0 = 0$  in Eq. (102). The result is [cf. Eq. (64)]

$$r^2 f'' + 5r f' + \frac{R^4 \omega^2}{r^2} f - j(j+4) f = 0 \quad (103)$$

whose solution is [cf. Eq. (65)]

$$f(r) = \frac{1}{r^2} H_{j+2}^{(1)} \left( \frac{R^2 \omega}{r} \right). \quad (104)$$

In the large  $r$  limit, we have

$$\Psi \approx -iA \frac{2^{j+2} (j+1)!}{R^{2j+4} \omega^{j+2}} r^j. \quad (105)$$

Comparing with the asymptotic form (22), we obtain

$$A = i \frac{\omega^{2j+4} R^{2j+4}}{4^{j+2} (j+1)! (j+2)!}. \quad (106)$$

The absorption coefficient is

$$\mathcal{P} = 8\pi\kappa r_0^4 |A|^2 |f(r_0)|^2. \quad (107)$$

Using the approximation (70), after some algebra we find that for frequencies large compared to the temperature, the absorption coefficient (107) becomes

$$\mathcal{P} \approx \frac{8\pi\kappa}{((j+1)!(j+2)!)^2} \times \frac{|\Gamma(j/2+5/4+2i\kappa)\Gamma(j/2+7/4+2i\kappa)|^2}{|\Gamma(1+4i\kappa)|^2} \left(\frac{\omega r_0}{2}\right)^{2j+4} \quad (108)$$

in line with the results we obtained in the extremal cases [e.g., Eq. (72)], but at *half* the Hawking temperature.

If we are allowed to speculate, we would like to note that this is reminiscent of the case where there are two modes at temperatures  $T_1$  and  $T_2$ , and the Hawking temperature is given by  $2/T_H = 1/T_1 + 1/T_2$  [18]. If  $T_2 \rightarrow \infty$ , then  $T_1 = T_H/2$ . Thus, when we go off extremality, it seems that the number of degrees of freedom doubles with the extra degrees living in a very hot bath. Of course, all this needs to be taken with a grain of salt, since away from extremality supersymmetry is broken and there is no guarantee that the supergravity analysis is in any way dual to the superconformal field theory on D-branes. Still, it is intriguing that similar results are obtained for both non-extremal and extremal supergravity backgrounds.

#### IV. CONCLUSIONS

We discussed the absorption of scalar fields by a distribution of D3-branes in the extremal limit. These distributions are obtained as limiting cases of spinning branes which are solutions of the full non-linear supergravity field equations [9–12]. In the extremal limit, the branes are no longer spinning, but they settle into a state which is distinct from the AdS limit and is characterized by angular momentum quantum numbers  $l_i$  ( $i=1,2,3$ ). The AdS limit is obtained when these quantum numbers approach zero. This set of supergravity solutions is dual to the Coulomb branch of the  $\mathcal{N}=4$  four-dimensional  $SU(N)$  super Yang Mills theory, which is a superconformal field theory. We solved the wave equation for scalar fields in the respective supergravity backgrounds and computed the absorption coefficient in each case. We found that the absorption coefficients exhibited a universal behavior as functions of the angular momentum quantum number of the partial wave and the Hawking temperature. This functional dependence is of the same form as

the gray-body factors associated with black-hole scattering [18].

We also discussed the problematic case of a spherically symmetric distribution of D-branes [19]. This is an example of the more general case where the D-branes are distributed on a surface that divides the transverse space, defined by coordinates  $y_i$  ( $i=1, \dots, 6$ ) [Eq. (1)] into two distinct regions. This occurs when all three quantum numbers  $l_i$  ( $i=1,2,3$ ) are non-vanishing [10]. The wave can then bounce off of the D-branes an infinite number of times and when this happens, one obtains a resonance. We obtained these resonances for both a spherically symmetric and a ‘‘long-needle’’ distribution of branes. They present a puzzle as one approaches the AdS limit. However, if one uses the Schwarzschild-like coordinates (4) instead, which only cover the region outside the D-brane shell, thereby excluding the inner region from physical considerations, we showed that one obtains the same form for the absorption coefficients as in the rest of the cases. It seems that the Schwarzschild-like coordinates (4) are more appropriate physically than the  $y_i$  ( $i=1, \dots, 6$ ) coordinates.

Finally, we went off extremality and solved the wave equation in the background of a brane with a finite event horizon. Here, too, we obtained the same form for the absorption coefficients, albeit at *half* the Hawking temperature. We speculated that off extremality, new degrees of freedom enter which must live in a hot bath. Of course, since supersymmetry is broken, there is no guarantee that a duality exists between supergravity and superconformal field theory on D3-branes. Yet, it is intriguing that similar results are obtained in both the extremal and the non-extremal cases, albeit with a twist.

It would be interesting to extend this analysis to more general supergravity backgrounds. This is important for a better understanding of the maximally supersymmetric AdS limit and its thermodynamic properties. It will shed more light on the interesting issue of the AdS conformal field theory correspondence.

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