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Flow and Heat Transfer in a 2-D Channel Partially Filled With Porous  
Medium

A thesis

Presented for the

Master of Science Degree

The University of Tennessee, Knoxville

Srinivas Kuppa

August, 2006

Dedicated

To

My parents Sri Bhaskara Durga Prasad and Smt. Lakshmi Kameshwari

And

My graduate advisor Dr. R.V. Arimilli

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## Abstract

Graphite foam, developed by the Oak Ridge National Laboratory, has shown excellent thermal properties and has potential applications for compact heat exchangers. The flow and heat transfer characteristics of a 2-D channel partially filled with graphite foam as the porous medium are determined. A steady, incompressible and constant-property water flow is modeled using the commercial software COMSOL<sup>®</sup>. Flow is uniform at the inlet and the base of the foam is subjected to a constant heat flux boundary condition. The permeability ( $7 \times 10^{-8}$  to  $1.4 \times 10^{-6} \text{ m}^2$ ) and the dimensionless height of the porous block are varied. Heat fluxes of  $1 \times 10^5$  and  $3 \times 10^5 \text{ W/m}^2$  are considered for Reynolds numbers of 1000 and 2000.

For the range of parameters considered in this study, the results show that

- (1) The friction factor decreases with increase in Reynolds number and permeability. It increases almost exponentially at moderate to higher values of dimensionless block height.
- (2) The Nusselt number increases with Reynolds number, permeability and dimensionless block height.
- (3) The maximum surface temperature of the heater decreases with increasing Reynolds number and dimensionless block height. It decreases with increasing permeability for both Reynolds number and heat flux cases considered.
- (4) A dimensionless parameter effectiveness is introduced and it increases linearly with heat flux and decreases rapidly with increase in Reynolds number. For example, when Reynolds number is doubled effectiveness is shown to drop by a factor of about eight.

- (5) For the two values of heat fluxes and for all permeability values considered in this study, the maximum effectiveness is found to occur at  $h_p/H = 0.15$  and  $0.10$  for  $Re_{Dh}=1000$  and  $2000$ , respectively.

## Table of Contents

| Chapter                                                 | Page |
|---------------------------------------------------------|------|
| 1. Introduction                                         | 1    |
| 2. Literature Review                                    | 2    |
| 2.1 Graphite foam                                       | 2    |
| 2.2 Heat transfer in channels filled with porous medium | 3    |
| 3. Problem Statement                                    | 6    |
| 3.1 Introduction                                        | 6    |
| 3.2 Modeling details                                    | 7    |
| 3.3 Assumptions                                         | 8    |
| 4. Equations                                            | 9    |
| 4.1 Porous media momentum equations                     | 9    |
| 4.1.1 Darcy equation                                    | 9    |
| 4.1.2 Forchheimer equation                              | 10   |
| 4.1.3 Brinkman equation                                 | 11   |
| 4.2 Background of porous media energy equation          | 12   |
| 4.2.1 Local thermal equilibrium                         | 12   |
| 4.2.2 Thermal dispersion                                | 12   |
| 4.3 Performance parameters                              | 13   |
| 4.3.1 Flow parameters                                   | 13   |
| 4.3.2 Friction factor                                   | 14   |
| 4.3.3 Relative energy error                             | 15   |

|        |                                                           |    |
|--------|-----------------------------------------------------------|----|
| 4.3.4  | Heat transfer coefficient and Nusselt number calculations | 15 |
| 4.3.5  | Efficiency and effectiveness calculations                 | 16 |
| 5.     | Problem Setup in COMSOL®                                  | 19 |
| 5.1    | Introduction                                              | 19 |
| 5.2    | Governing equations                                       | 20 |
| 5.2.1. | Incompressible Navier-Stokes equation                     | 20 |
| 5.2.2. | Brinkman equation                                         | 20 |
| 5.2.3  | General heat transfer equation                            | 22 |
| 5.3    | Boundary conditions                                       | 22 |
| 5.3.1  | Incompressible Navier-Stokes Application mode             | 22 |
| 5.3.2  | Brinkman-Forchheimer mode                                 | 23 |
| 5.3.3  | General heat transfer mode                                | 23 |
| 5.4    | Subdomain settings                                        | 24 |
| 5.5    | Mesh parameters                                           | 25 |
| 5.6    | Solution algorithm                                        | 26 |
| 5.7    | Numerical values                                          | 29 |
| 5.7.1  | Table of properties                                       | 29 |
| 5.7.2  | Range of parameters                                       | 29 |
| 5.8    | Convergence Study                                         | 31 |
| 6.     | Discussion of Results                                     | 33 |
| 6.1    | Variation of friction factor                              | 33 |
| 6.2    | Variation of percent of total flow through porous block   | 33 |
| 6.3    | Variation of Nusselt number                               | 37 |

|     |                                                    |    |
|-----|----------------------------------------------------|----|
| 6.4 | Variation of maximum surface temperature of heater | 43 |
| 6.5 | Variation of efficiency                            | 48 |
| 6.6 | Variation of effectiveness                         | 53 |
| 7.  | Conclusions and recommendations                    | 59 |
| 7.1 | Conclusions                                        | 59 |
| 7.2 | Recommendations                                    | 60 |
|     | References                                         | 61 |
|     | Vita                                               | 64 |

## **List of Tables**

|                                                                                            |    |
|--------------------------------------------------------------------------------------------|----|
| Table 5.1 Initial guess for Phase-1 velocity fields for Navier-Stokes and Brinkman domains | 28 |
| Table 5.2 Thermo physical properties of water used in this study                           | 29 |
| Table 5.3 Thermo physical properties of porous matrix used in this study                   | 30 |
| Table 5.4 Range of parameters used                                                         | 30 |
| Table 5.5 Parameters held constant for the convergence study                               | 31 |
| Table 5.6 Momentum convergence results                                                     | 32 |
| Table 5.7 Energy convergence results                                                       | 32 |

## List of Figures

|              |                                                                                                              |    |
|--------------|--------------------------------------------------------------------------------------------------------------|----|
| Figure 3.1:  | Schematic of the two-dimensional partially filled porous channel                                             |    |
|              | Configuration                                                                                                | 6  |
| Figure 3.2:  | Schematic of the configuration with an appended upstream section                                             | 8  |
| Figure 5.1:  | COMSOL <sup>®</sup> model with subdomains labeled                                                            | 19 |
| Figure 5.2:  | COMSOL <sup>®</sup> model with Navier-Stokes boundaries                                                      | 23 |
| Figure 5.3:  | COMSOL <sup>®</sup> model with Brinkman-Forchheimer boundary conditions                                      | 24 |
| Figure 5.4:  | COMSOL COMSOL <sup>®</sup> model mesh configuration with a maximum element size of 'H/100' for $h_p/H = 0.5$ | 26 |
| Figure 5.5:  | Schematic showing the solution process                                                                       | 27 |
| Figure 6.1:  | Variation of $f$ with $K$ for $Re_{Dh}=1000$                                                                 | 34 |
| Figure 6.2:  | Variation of $f$ with $h_p/H$ for $Re_{Dh}=1000$                                                             | 34 |
| Figure 6.3 : | Variation of $f$ with $K$ for $Re_{Dh}=2000$                                                                 | 35 |
| Figure 6.4:  | Variation of $f$ with $h_p/H$ for $Re_{Dh}=2000$                                                             | 35 |
| Figure 6.5:  | Variation of $F_{QP}$ with $K$ for $Re_{Dh}=1000$                                                            | 36 |
| Figure 6.6:  | Variation of $F_{QP}$ with $h_p/H$ for $Re_{Dh}=1000$                                                        | 36 |
| Figure 6.7:  | Variation of $F_{QP}$ with $K$ for $Re_{Dh}=2000$                                                            | 38 |
| Figure 6.8:  | Variation of $F_{QP}$ with $h_p/H$ for $Re_{Dh}=2000$                                                        | 38 |
| Figure 6.9:  | Variation of $Nu_{Dh}$ with $K$ for $Re_{Dh}=1000$ and $q'' = 10^5 \text{ W/m}^2$                            | 39 |
| Figure 6.10: | Variation of $Nu_{Dh}$ with $h_p/H$ for $Re_{Dh}=1000$ and $q'' = 10^5 \text{ W/m}^2$                        | 39 |
| Figure 6.11: | Variation of $Nu_{Dh}$ with $K$ for $Re_{Dh}=1000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$                   | 40 |

|               |                                                                                                  |    |
|---------------|--------------------------------------------------------------------------------------------------|----|
| Figure 6.12:  | Variation of $Nu_{Dh}$ with $h_p/H$ for $Re_{Dh}=1000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$   | 40 |
| Figure 6.13:  | Variation of $Nu_{Dh}$ with $K$ for $Re_{Dh}=2000$ and $q'' = 10^5 \text{ W/m}^2$                | 41 |
| Figure 6.14 : | Variation of $Nu_{Dh}$ with $h_p/H$ for $Re_{Dh}=2000$ and $q'' = 10^5 \text{ W/m}^2$            | 41 |
| Figure 6.15:  | Variation of $Nu_{Dh}$ with $K$ for $Re_{Dh}=2000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$       | 42 |
| Figure 6.16:  | Variation of $Nu_{Dh}$ with $h_p/H$ for $Re_{Dh}=2000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$   | 42 |
| Figure 6.17:  | Variation of $T_{s,max}$ with $K$ for $Re_{Dh}=1000$ and $q'' = 10^5 \text{ W/m}^2$              | 44 |
| Figure 6.18:  | Variation of $T_{s,max}$ with $h_p/H$ for $Re_{Dh}=1000$ and $q'' = 10^5 \text{ W/m}^2$          | 44 |
| Figure 6.19:  | Variation of $T_{s,max}$ with $K$ for $Re_{Dh}=1000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$     | 45 |
| Figure 6.20:  | Variation of $T_{s,max}$ with $h_p/H$ for $Re_{Dh}=1000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$ | 45 |
| Figure 6.21:  | Variation of $T_{s,max}$ with $K$ for $Re_{Dh}=2000$ and $q'' = 10^5 \text{ W/m}^2$              | 46 |
| Figure 6.22 : | Variation of $T_{s,max}$ with $h_p/H$ for $Re_{Dh}=2000$ and $q'' = 10^5 \text{ W/m}^2$          | 46 |
| Figure 6.23:  | Variation of $T_{s,max}$ with $K$ for $Re_{Dh}=2000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$     | 47 |
| Figure 6.24:  | Variation of $T_{s,max}$ with $h_p/H$ for $Re_{Dh}=2000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$ | 47 |
| Figure 6.25:  | Variation of $E$ with $K$ for $Re_{Dh}=1000$ and $q'' = 10^5 \text{ W/m}^2$                      | 49 |
| Figure 6.26:  | Variation of $E$ with $h_p/H$ for $Re_{Dh}=1000$ and $q'' = 10^5 \text{ W/m}^2$                  | 49 |
| Figure 6.27 : | Variation of $E$ with $K$ for $Re_{Dh}=1000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$             | 50 |
| Figure 6.28:  | Variation of $E$ with $h_p/H$ for $Re_{Dh}=1000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$         | 50 |
| Figure 6.29:  | Variation of $E$ with $K$ for $Re_{Dh}=2000$ and $q'' = 10^5 \text{ W/m}^2$                      | 51 |
| Figure 6.30:  | Variation of $E$ with $K$ for $Re_{Dh}=2000$ and $q'' = 10^5 \text{ W/m}^2$                      | 51 |
| Figure 6.31:  | Variation of $E$ with $K$ for $Re_{Dh}=2000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$             | 52 |

|              |                                                                                                   |    |
|--------------|---------------------------------------------------------------------------------------------------|----|
| Figure 6.32: | Variation of $E$ with $hp/H$ for $Re_{Dh}=2000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$           | 52 |
| Figure 6.33: | Variation of $\varepsilon$ with $K$ for $Re_{Dh}=1000$ and $q'' = 10^5 \text{ W/m}^2$             | 55 |
| Figure 6.34: | Variation of $\varepsilon$ with $hp/H$ for $Re_{Dh}=1000$ and $q'' = 10^5 \text{ W/m}^2$          | 55 |
| Figure 6.35: | Variation of $\varepsilon$ with $K$ for $Re_{Dh}=2000$ and $q'' = 10^5 \text{ W/m}^2$             | 56 |
| Figure 6.36: | Variation of $\varepsilon$ with $hp/H$ for $Re_{Dh}=2000$ and $q'' = 10^5 \text{ W/m}^2$          | 56 |
| Figure 6.37: | Variation of $\varepsilon$ with $K$ for $Re_{Dh}=1000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$    | 57 |
| Figure 6.38: | Variation of $\varepsilon$ with $hp/H$ for $Re_{Dh}=1000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$ | 57 |
| Figure 6.39: | Variation of $\varepsilon$ with $K$ for $Re_{Dh}=2000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$    | 58 |
| Figure 6.40: | Variation of $\varepsilon$ with $hp/H$ for $Re_{Dh}=2000$ and $q'' = 3 \times 10^5 \text{ W/m}^2$ | 58 |

## Nomenclature

|           |                                                                                                                                                         |
|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------|
| $C_F$     | Form coefficient                                                                                                                                        |
|           | Form drag coefficient                                                                                                                                   |
| $C_{p,f}$ | Specific heat capacity of fluid                                                                                                                         |
| $D_h$     | Hydraulic diameter of the channel                                                                                                                       |
| $E$       | Efficiency = $\frac{\text{Rate of heat transfer to fluid}}{\text{Pumping power}} = \frac{q'' L}{Q  \Delta p }$                                          |
| $f$       | Friction factor = $\frac{2  \Delta p }{\rho_f U_{in}^2}$                                                                                                |
| $F$       | Body force                                                                                                                                              |
| $F_x$     | X component of body force                                                                                                                               |
| $F_y$     | Y component of body force                                                                                                                               |
| $F_{QP}$  | Percent of total flow through the porous block at mid-length of the porous block = $\left( \frac{Q_{porous}}{Q_{porous} + Q_{non\ porous}} \right) 100$ |
| $h$       | height of the porous block                                                                                                                              |
| $h_{ave}$ | Average heat transfer coefficient                                                                                                                       |
| $h_{min}$ | Minimum heat transfer coefficient (occurs at trailing edge of porous block)                                                                             |
| $H$       | Height of the channel                                                                                                                                   |
| $K$       | Permeability of the porous material                                                                                                                     |
| $k_f$     | Thermal conductivity of the fluid                                                                                                                       |
| $k_p$     | Thermal conductivity of the porous block                                                                                                                |
| $L$       | Length of the porous block                                                                                                                              |

|                       |                                                                                  |
|-----------------------|----------------------------------------------------------------------------------|
| $L_d$                 | Length of section downstream the porous block                                    |
| $L_u$                 | Length of section upstream of the porous block                                   |
| $\dot{m}$             | Mass flow rate of fluid per unit depth of channel                                |
| $Nu_{Dh}$             | Nusselt number = $\frac{h_{ave} D_h}{k_f}$                                       |
| $P$                   | Pressure                                                                         |
| $q''$                 | Heat flux supplied to heater                                                     |
| $Q$                   | Volumetric flow rate per unit depth = $U_{in} H$                                 |
| $Q_s$                 | Rate of heat generation per unit volume                                          |
| $Re_{Dh}$             | Reynolds number based on hydraulic diameter = $\frac{\rho_f U_{in} D_h}{\mu}$    |
| $\langle T \rangle$   | Local-volume-averaged-Temperature                                                |
| $\langle T \rangle^s$ | Local-volume-averaged-Temperature of solid                                       |
| $\langle T \rangle^l$ | Local-volume-averaged-Temperature of fluid                                       |
| $T_{b, ave}$          | Bulk average temperature                                                         |
| $T_{in}$              | Inlet temperature                                                                |
| $T_{b, out}$          | Outlet bulk temperature                                                          |
| $T_s$                 | Temperature on the surface of the heater                                         |
| $T_{s,ave}$           | Average surface temperature of the heater                                        |
| $T_{s,max}$           | Maximum temperature of the heater surface (at trailing edge of the porous block) |
| $\vec{u}$             | Velocity vector                                                                  |

|                  |                                                                                                 |
|------------------|-------------------------------------------------------------------------------------------------|
| $u_{ns}, v_{ns}$ | X and Y components of velocity vector in the Navier-Stokes application mode (non-porous domain) |
| $u_{br}, v_{br}$ | X and Y components of velocity vector in the Brinkman application mode                          |
| $U_{in}$         | Inlet flow velocity based on the Reynolds number                                                |
| $u_{b, out}$     | Bulk outlet velocity of channel                                                                 |

### **Greek Symbols**

|                |                                                                                                                          |
|----------------|--------------------------------------------------------------------------------------------------------------------------|
| $\theta_{max}$ | Dimensionless maximum surface temperature of the heater surface<br>$= \frac{T_{s,max} - T_{b,in}}{T_{b,out} - T_{b,in}}$ |
| $\rho_f$       | Density of fluid                                                                                                         |
| $\varepsilon$  | Effectiveness = $\frac{E}{\theta_{max}}$                                                                                 |
| $\mu$          | Dynamic viscosity of the fluid                                                                                           |
| $\tilde{\mu}$  | Effective viscosity in a porous-fluid matrix                                                                             |
| $\eta$         | Dynamic viscosity of the fluid ( as defined in COMSOL <sup>®</sup> )                                                     |

# **1.Introduction**

The recently developed high thermal conductivity graphite foam has shown a very good promise for use in many electronic cooling and automobile radiator applications. With a specific thermal conductivity greater than copper and density one fifth that of aluminum, it has the capability to solve thermal management problems in a very efficient manner. Many recent studies have focused on the usage of carbon foam in heat exchangers and sinks. The work presented in this thesis is one such attempt to study the flow and heat transfer in a channel, which is partially filled with a carbon foam block. A constant heat flux is being supplied to the foam on one of its surfaces. Water is used as a coolant for the dissipation of heat from the heated surface. The heat transfer and flow characteristics of the problem are studied by varying the flow velocity, permeability of the porous material and the height of the block. The hydrodynamic and energy results are studied by analyzing the variation of parameters such as pressure drop, maximum surface temperature of the heated surface etc., Results are analyzed, conclusions and recommendations are made.

## **2. Literature Review**

This literature review is divided into two sections. In the first section, studies that developed the high thermal conductivity graphite foam are discussed. Subsequently, its thermal properties, novel methods for manufacturing, and applications are discussed. In the next section, the problem of flow and heat transfer in a channel filled with porous medium is reviewed.

### **2.1 Graphite foam**

A novel manufacturing process for making carbon foam was invented by researchers at the Oak Ridge National Laboratories. It has been patented as number '6033506' by the United States Patent and Trademark Office (Klett, 2000). Klett et al. (2000) explain that this carbon foam is inexpensive and takes less time to fabricate as some of the traditional steps in the manufacturing process have been removed. The foam structure and graphitic morphology were studied using optical microscopy techniques. At 2800°C, densities of the graphite foam were found to vary from 0.2 to 0.6 g/cm<sup>3</sup>. Thermal diffusivity measurements were performed revealing that the bulk thermal conductivity varied with density from 40 to 150 W/m.K.

Klett et al. (2000) have shown that when graphite foam is used as a prime material for thermal management in electronics, such as heat exchangers and heat sinks, the effective heat transfer can be increased by at least an order of magnitude, while reducing its size and weight. A once-through-foam heat exchanger has been fabricated for testing purposes. Four sandwich foam panels two with aluminum facesheets and two with copper are fabricated. Flexural and Compression testing was conducted on these samples. The

flexure specimens exhibited shear failure with only a slight de-lamination of the foam from face sheet. Compression tests revealed that the foam core crushes with a uniform load over a large displacement. The thermal conductivity testing of sandwich panels indicated that through the thickness thermal conductivity ranged between 50 and 65 W/m.K with little difference between aluminum and copper panels. It is stated that because of the open cellular structure of graphite foam, it is a prime candidate for use as a porous media heat exchanger for a power electronic substrate. In another test, heat transfer coefficients for a shell and tube heat exchanger with graphite foam as the core were measured as high as 11,000 W/m<sup>2</sup>.K. It was concluded that through further experiments graphite foam could eventually replace honeycomb in highly demanding thermal applications.

## **2.2 Heat transfer in channels filled with porous medium**

In this section, the problem of flow and heat transfer in a channel filled with porous medium is discussed.

Poulikakos and Renken (1987) investigated the problem of forced convection in a channel filled with a fluid saturated porous medium. The effects of flow inertia, variable porosity and Brinkman friction were taken into account for the study. Two channel configurations are considered, parallel plates and circular pipe. For both cases the channel wall is maintained at a constant temperature. The problem is mathematically formulated and the momentum and energy equations are solved numerically using Gauss elimination and Keller box methods, respectively. The local heat flux at the channel wall was evaluated numerically. The numerical solution is compared with the Darcy flow model solution. It was summarized that compared to the predictions of the Darcy flow model,

the general flow model predicted enhancement in heat transfer between the fluid saturated porous medium and the walls. The increase in the value of Nusselt number reflected this enhancement.

Poulikakos and Kazmierczak (1986) made a theoretical study of fully developed forced convection in a channel partially filled with a porous matrix. Both the parallel plate and circular tube channel configurations are studied. Again, for each configuration, both the case of constant wall temperature and constant heat flux are considered. A novel feature of this study is that it determines the effect of flow inside the porous region on the heat exchange between the wall and fluid. Brinkman flow equations were used to model the flow inside porous region. It documented the effect of Darcy number, thickness of porous region, ratio of effective thermal conductivities of porous medium and fluid on the flow field and on heat transfer inside the channel. This investigation led to an interesting finding that the Nusselt number variation on the thickness of porous medium was not monotonic.

Alam et al. (2002) tested a Carbon-Carbon composite single flow channel heat exchanger in a flow loop to determine the heat transfer coefficient and the pressure drop characteristics of plate-fin geometry. Heat flux is applied to the top and bottom of the heat exchanger core. Quantities such as heat flux, heat transfer coefficient and friction factor are calculated using data acquisition tools. Results are then compared with data for a standard plate fin heat exchanger of similar geometry. It was found that the C-C heat exchanger had lower friction factor and Colburn heat transfer factor than a similar standard metal plate-fin heat exchanger, but had higher surface temperature effectiveness. The variation of surface temperature effectiveness with thermal conductivity of the fin is

studied. The results predicted a huge improvement in the total surface temperature effectiveness when the thermal conductivity is changed from 10 to 300 W/m.K. This shows that a significant improvement in heat transfer performance is possible with this composite heat exchanger.

Hadim (1994) performed a detailed numerical study of forced convection in a porous channel with heat sources flush mounted on the bottom wall. The heat transfer enhancement and pressure drop in a partially filled porous channel are evaluated and compared with a fully porous channel case. The momentum equation for the porous region consists of the Brinkman-Forchheimer-Darcy terms, which accounts for both the inertia and viscous effects. The continuity, momentum and energy equations are first transformed into dimensionless governing equations using vorticity –stream function formulation. Subsequently, these were transformed into algebraic, finite-difference equations using the control volume formulation outlined by Gosman et al. (1969). These algebraic equations were solved using Gauss-Siedel point iterative scheme. Results for the partially porous channel indicate that almost the same increase in heat transfer with a significantly reduced pressure drop could be obtained compared to a fully porous channel. This suggests that the partially porous channel configuration is a more attractive heat transfer augmentation technique for electronic equipment cooling. The partial porous channel configuration concept taken from this investigation has been the motivation for the current study.

### 3. Problem Statement

#### 3.1 Introduction

Compact heat exchangers are a special class of heat exchangers that are used to obtain very high heat transfer surface area to volume ratios. Extended surfaces, usually in the form of fins are used for this purpose. In addition to finned surfaces, the usage of highly conductive porous medium is the central idea of several recent studies.

One such study is the problem of two-dimensional forced convection of a fluid passing through a partially filled porous channel as shown in Figure 3.1. Fluid flows through a channel, which has a porous block located on its lower surface. The porous block partially obstructs the fluid flow. A constant heat flux is maintained at its lower surface. The porous material would impose certain frictional resistance in the direction opposite to the flow. Because of which, a pressure drop is created along its length. Fluid must be pumped across the porous block to overcome this existing pressure drop. Apart from the flow velocity, the geometry of the block also significantly affects this pressure drop.

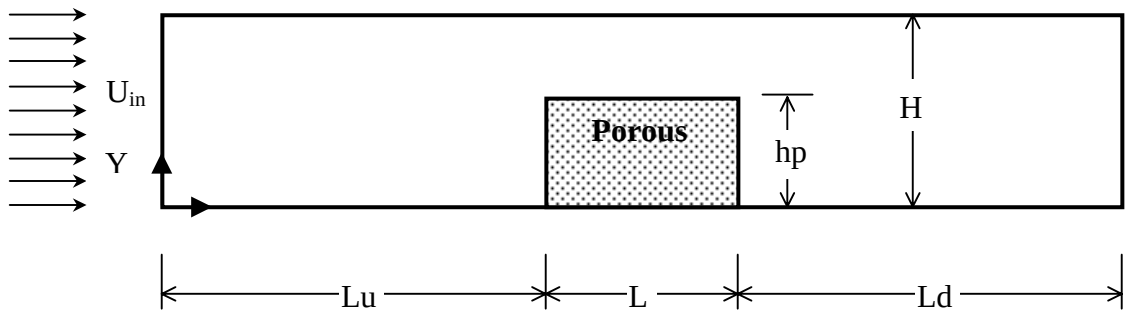


Figure 3.1 Schematic of the two-dimensional partially filled porous channel configuration

Here,  $L$  is the length of porous block. In this study, all other dimensions such as  $L_u$ ,  $L_d$ ,  $H$ , etc., are made dependent on this parameter.

The operational cost associated with the pumping of the fluid depends on the overall pressure drop in the system. Needless to say the operational costs are to be kept minimum. Higher flow velocities yield higher heat transfer rates. However, high flow velocities will result in higher pressure drop and pumping power. So a compromise has to be made between the operational cost and heat transfer capability. A parametric study will be conducted to optimize this pressure drop without severely affecting the heat transfer performance.

### **3.2 Modeling details**

For the above problem, uniform inlet velocity is specified in the  $x$  direction. The Reynolds number, based on the hydraulic diameter of the channel, is small enough for the flow to be maintained laminar ( $Re_{Dh} < 2300$ ). The hydraulic diameter of the 2-D channel is  $2H$ .

The parametric study is conducted using the finite-element analysis (FEA) package COMSOL 3.2. The geometry is defined by the primary parameter ' $L$ '. All other dimensions are expressed as ratios of this parameter. To facilitate a numerical solution, an upstream section of length  $H$  and height  $H$  with "symmetry" boundary conditions on either sides are appended to the inlet, as shown in the Figure 3.2 .

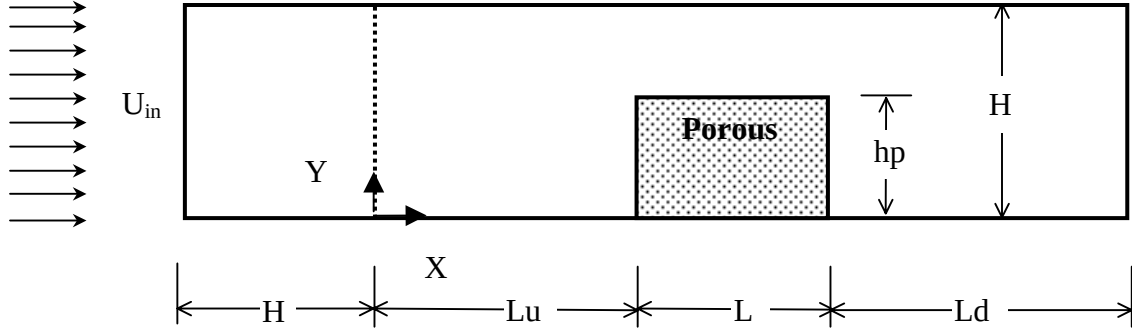


Figure 3.2 Schematic of the configuration with an appended upstream section

Once the geometry is modeled, assignment of boundary conditions, meshing and choosing proper solution parameters is done. These aspects are explained in detail in the next chapter.

### 3.3 Assumptions

The following assumptions were made within the model to solve the problem:

1. Flow is steady and two-dimensional.
2. The thermophysical properties of fluid are assumed constant.
3. The porous material is homogenous and isotropic.
4. Fluid and porous medium are in local thermal equilibrium (LTE).
5. No thermal dispersion.

## 4. Equations

This chapter contains the description and explanation of the equations used throughout the body of the thesis. In sections 4.1 and 4.2, the porous media momentum and energy equations are briefly explained. Section 4.3 deals exclusively with explanation of the performance parameters that are used in the post-processing of the results.

### 4.1 Porous media momentum equations

This section discusses various forms of the porous medium momentum equations, which are analogous to the Navier-Stokes momentum equations (5.1).

#### 4.1.1 Darcy equation

Henry Darcy in 1856 investigated the hydrology of water supply of Dijon and conducted experiments on steady state unidirectional flow in a uniform medium. Experiments revealed proportionality between flow rate and applied pressure difference. This is expressed as,

$$\vec{\nabla} P = -\frac{\mu}{K} \vec{u} \quad (4.1)$$

The coefficient  $K$  (units  $m^2$ ) is specific permeability or intrinsic permeability of the medium. It is independent of the nature of the fluid but depends upon the geometry. For single-phase flows, we abbreviate this to permeability. The term on the right of equation 4.1 is referred to as Darcy term. Darcy term accounts for the viscous shear stresses exerted by the solid pores of the matrix on the fluid. The contribution of Darcy term on the pressure drop becomes significant for low permeability values. For the present study, the permeability range considered is moderately low and hence the Darcy term

contribution to the pressure drop is of significant value. As Darcy equation does not contain higher order derivatives of velocity, it cannot satisfy the no-slip boundary condition.

#### 4.1.2 Forchheimer equation

Darcy's equation (4.1) is linear in the velocity  $u$ . It holds when  $u$  is sufficiently small. As  $u$  increases, there is a breakdown in the linearity because the form drag now becomes comparable with shear drag. Joseph et al. (1982) suggested a modification to Darcy's law as

$$\vec{\nabla}P = -\frac{\mu}{K}\vec{u} - C_F K^{-1/2} \rho_f |\vec{u}| \vec{u} \quad (4.2)$$

Where  $C_F$  is a dimensionless form drag constant. Equation 4.2 is a modification of an equation associated with the names of Dupuit (1863) and Forchheimer (1901). The last term in equation 4.2 is referred to as Forchheimer term, but the dependence of  $K^{-1/2} \rho_f$  is a modern discovery (Ward, 1964). Ward initially thought that  $C_F$  was a universal constant, with a value of 0.55, but later it was found that  $C_F$  changes with the porous material. A generalized tensor form applicable to anisotropic permeability was derived by Knupp and Lage (1995). A momentum equation with a Forchheimer correction was obtained using the method of volume averaging by Whitaker (1996). For further discussion on Forchheimer equation, see Barak (1987) and Hassanizadeh and Gray (1980).

As stated earlier, when the flow rates are higher, the form drag caused by the pressure difference around a submerged body becomes relatively important compared to the

viscous drag. Reynolds number range used in the present study is high enough to include the Forchheimer term in the momentum equation.

### 4.1.3 Brinkman equation

Brinkman equation is an alternative to Darcy equation. It takes the form

$$\vec{\nabla}P = -\frac{\mu}{K}\vec{u} + \tilde{\mu}\vec{\nabla}^2\vec{u} \quad (4.3)$$

where  $\tilde{\mu}$  is the effective viscosity as described in Nield and Bejan (1999).

It has two viscous terms. The first is the usual Darcy term and the second term is analogous to the Laplacian term that appears in the Navier stokes equation (equation 5.1).

Brinkman set  $\mu$  and  $\tilde{\mu}$  equal to one another, but in general this is not true. Bear and

Bachmat (1990) found that  $\frac{\tilde{\mu}}{\mu}$  depends upon the geometry of the medium. Tam (1969)

found that whenever the spatial length scale is much greater than  $\sqrt{\frac{\tilde{\mu}K}{\mu}}$ , the  $\nabla^2\vec{u}$  term

in the equation (4.3) can be neglected, so that Brinkman equation reduces to Darcy equation. It could be stated that for many practical purposes, there is no need to include the Laplacian term. But, when a no slip boundary condition is to be satisfied, it is indeed required, but its effect is significant only in a thin boundary layer whose thickness is of

order  $\sqrt{\frac{\tilde{\mu}K}{\mu}}$ .

## 4.2 Background of porous media energy equation

### 4.2.1 Local thermal equilibrium

Porous media energy equations are generally solved under the assumption of Local thermal equilibrium (LTE). According to LTE assumption both the solid and fluid portions of the porous matrix are modeled as a single medium. A single equation for the conservation of energy is derived using a local volume averaging technique. “ *One motivation for the use of a single energy equation is that similarities can be deduced from solutions to problems involving only fluid flow (i.e., with no porous matrix present).*”

(Norton, Jr., 2003).

Kaviany (1994) defined LTE as:

$$\langle T \rangle = \langle T \rangle^s(x) = \langle T \rangle^f(x) \quad (4.4)$$

where s, and f denote solid, and fluid respectively. Also  $\langle \rangle$  represents a volume-averaged quantity. Equation (4.4) suggests that the local volume averaged temperature of the effective single medium is equal to the local volume average temperature of each phase.

### 4.2.2 Thermal dispersion

Thermal dispersion is the heat transfer due to the hydrodynamic mixing of the fluid at the pore scale. This heat transfer is in addition to the molecular diffusion of heat. Thermal dispersion is a complex phenomena that could arise because of many reasons such as existence of interstitial tortuous paths, recirculation caused by local regions of reduced pressure etc., Angirasa (2002) has shown that thermal dispersion phenomena could be neglected for porous media flows with low permeabilities. For the present study for which the permeability range is quite low, thermal dispersion effects are not considered.

## 4.3 Performance parameters

This section deals with the derivation of all the required parameters for post processing operations.

### 4.3.1 Flow parameters

The Reynolds number of the flow is a key parameter in determining the flow and is hence discussed first in this section. Reynolds number for this study was based on the hydraulic diameter of the channel, which is calculated as,

$$D_h = \frac{4A_{cs}}{P_w} \quad (4.5)$$

Where  $A_{cs}$  and  $P_w$  represent the cross sectional area and wetted perimeter of the flow passage. For the present 2-D study, hydraulic diameter  $D_h$  reduces to

$$D_h = 2H \quad (4.6)$$

The Reynolds number of the flow based on the hydraulic diameter is defined as,

$$Re_{Dh} = \frac{\rho_f U_{in} D_h}{\mu} \quad (4.7)$$

Hence the inlet velocity of the flow  $U_{in}$  is calculated from the Reynolds number and other physical and geometrical parameters. The flow rate per unit depth of the channel (in  $m^2/s$ ) at the mid section of the porous medium is calculated by integrating the velocity component  $u_{br}$  through the height of the boundary 15.

$$Q_{porous} = \int_0^h u_{br} dy \quad (4.8)$$

Similarly the flow rate per unit depth of the channel (in m<sup>2</sup>/s) at the mid section of the non-porous or free flow domain is calculated by integrating the velocity component  $u_{ns}$  through the height of the boundary 14.

$$Q_{non\ porous} = \int_h^H u_{ns} dy \quad (4.9)$$

The percentage of flow passing through the porous medium as a fraction of total flow is computed as,

$$F_{QP} \equiv \left( \frac{Q_{porous}}{Q_{porous} + Q_{non\ porous}} \right) 100 \quad (4.10)$$

### 4.3.2 Friction factor

The friction factor is determined by calculating the pressure drop between sections located at a distance of  $L$  upstream from the front and  $L$  downstream of the back of the porous block.

Pressure at a cross section is calculated by integrating the pressure through the height of the section.

$$P = \int_0^H p_{ns} dy \quad (4.11)$$

Once the upstream and downstream pressure quantities,  $P_u$  and  $P_d$  are calculated, the pressure gradient is calculated as

$$|\Delta p| = \frac{P_u - P_d}{H} \quad (4.12)$$

Then the friction factor,  $f$  is calculated as,

$$f = \frac{2 |\Delta p|}{\rho_f U_{in}^2} \quad (4.13)$$

### 4.3.3 Relative energy error

Thermal energy flow rates at the inlet and outlet boundaries 1 and 8, respectively are calculated and used to determine the relative energy error. Since inlet conditions are uniform, energy flow rate at inlet section per unit depth of channel could be directly computed as

$$\dot{E}_{inlet} = \rho_f U_{in} C_{p,f} T_{in} H \quad (4.14)$$

For the outlet boundary of the channel, velocity and temperature distribution vary across its height, so energy flow rate at outlet section per unit depth of channel is calculated as

$$\dot{E}_{outlet} = \rho_f C_{p,f} \int_0^H u_{ns} T dy \quad (4.15)$$

The total rate of heat supplied for unit depth of channel in terms of heat flux is given by,

$$\dot{Q}_{total} = q'' L \quad (4.16)$$

The relative energy error is computed as,

$$Energy\ Error = \left( \frac{\dot{E}_{outlet} - \dot{E}_{inlet} - \dot{Q}_{total}}{\dot{Q}_{total}} \right) \times 100 \quad (4.17)$$

### 4.3.4 Heat transfer coefficient and Nusselt number calculations

In order to calculate the heat transfer coefficient and Nusselt number, the bulk average temperature of the channel and the average surface temperature of the heater surface are needed. The bulk outlet velocity is calculated as,

$$u_{b,out} = \frac{1}{H} \int_0^H u_{ns} dy \quad (4.18)$$

Then the bulk outlet temperature and subsequently the bulk average temperature of the channel are calculated as

$$T_{b,out} = \frac{1}{u_{b,out} H} \int_0^H u_{ns} T dy \quad (4.19)$$

$$T_{b,ave} = \left( \frac{T_{in} + T_{b,out}}{2} \right) \quad (4.20)$$

The average surface temperature of the heater is calculated as,

$$T_{s,ave} = \frac{1}{L} \int_{L_U}^{L_U+L} T_s dx \quad (4.21)$$

Once the two temperatures are known we could compute the average heat transfer coefficient and the Nusselt number of the flow as,

$$h_{ave} = \frac{q''}{(T_{s,ave} - T_{b,ave})} \quad (4.22)$$

$$Nu_{Dh} = \frac{h_{ave} D_h}{k_f} \quad (4.23)$$

#### 4.3.5 Efficiency and effectiveness calculations

The performance of the model is gauged by two parameters, Efficiency and the Effectiveness. Efficiency (  $E$  ) is defined as the ratio of rate of heat transferred from the heater to fluid per unit depth of the channel and the pumping power required to transport the fluid. It is written as

$$E \equiv \frac{q'' L}{Q |\Delta p|} \quad (4.24)$$

Using the definitions of friction factor (4.13) and Reynolds number (4.7), the efficiency can be expressed as

$$E = \frac{16(\rho_f H)^2 L}{\mu^3} \frac{q''}{\text{Re}_{Dh}^3 f} \quad (4.25)$$

For a constant  $\text{Re}_{Dh}$  and  $q''$ ,  $E$  would exclusively depend upon friction factor  $f$ , as all other terms in equation 4.25 remain constant. As a low value of friction factor is preferred, high values of  $E$  are desirable.

In electronic cooling applications, the maximum temperature on the heated surface is an important parameter. Therefore, it is desirable to define a dimensionless maximum surface temperature parameter  $\theta_{\max}$  as,

$$\theta_{\max} \equiv \frac{T_{s,\max} - T_{in}}{T_{b,out} - T_{in}} \quad (4.26)$$

As a constant heat flux  $q''$  is provided to the heater surface,  $T_{s,\max}$  would always exceed  $T_{b,out}$ . Hence  $\theta_{\max}$  is greater than 1 for all cases. The lower this value, the longer the life of the component and better reliability.

Total heat dissipated per unit depth of the channel to the fluid is calculated as,

$$q'' L = h C_{p,f} (T_{b,out} - T_{in}) \quad (4.27)$$

Substituting the value of  $(T_{b,out} - T_{in})$  from equation 4.27 back into equation 4.26,

$\theta_{\max}$  reduces to,

$$\theta_{\max} = \frac{T_{s,\max} - T_{in}}{\left( \frac{q'' L}{n C_{p,f}} \right)} \quad (4.28)$$

Another useful performance characteristic for electronic cooling applications would be  $\varepsilon$ , the ratio of efficiency  $E$  and  $\theta_{\max}$  which is defined as Effectiveness.

$$\varepsilon \equiv \frac{E}{\theta_{\max}} \quad (4.29)$$

As mentioned before, a high Efficiency and low maximum surface temperature values are preferable. Or in overall terms a high values of effectiveness,  $\varepsilon$  is desirable.

Substituting the expressions for  $E$  and  $\theta_{\max}$  from equations 4.25 and 4.28, into 4.29, we get

$$\varepsilon = \frac{32 (\rho_f L H)^2}{\mu^4 C_{p,f}} \frac{(q'')^2}{(T_{s,\max} - T_{b,out}) Re_{Dh}^4 f} \quad (4.30)$$

As the fluid properties,  $L$  and  $L/H$  are kept constant in the study,  $\varepsilon$  can be expressed as

$$\varepsilon \propto \frac{q''}{Re_{Dh}^4 f} h_{\min} \quad (4.31)$$

where  $h_{\min}$  is local minimum heat transfer coefficient at the trailing edge of the porous block. The local minimum heat transfer coefficient is proportional to the average heat transfer coefficient. Therefore, equation 4.31 can also be written as

$$\varepsilon \propto \frac{q''}{Re_{Dh}^4 f} Nu_{Dh} \quad (4.32)$$

## 5. Problem setup in COMSOL<sup>®</sup>

### 5.1 Introduction

The problem that was defined in section 3.1 is solved using COMSOL<sup>®</sup> 3.2 software. The model is divided into two major sub domains. One such domain is where the fluid flows and porous material fills the other domain. Due to the modeling considerations as discussed in 3.2, and keeping in mind the upcoming post processing requirements, these major sub domains are further divided into minor subdomains. The fluid and the porous sub domains are governed by the equations of the Incompressible Navier-Stokes and Brinkman application modes, respectively. A general heat transfer application mode encompasses both the fluid and porous regions. This application mode is used exclusively to solve the energy equations. Figure 5.1 shows the various

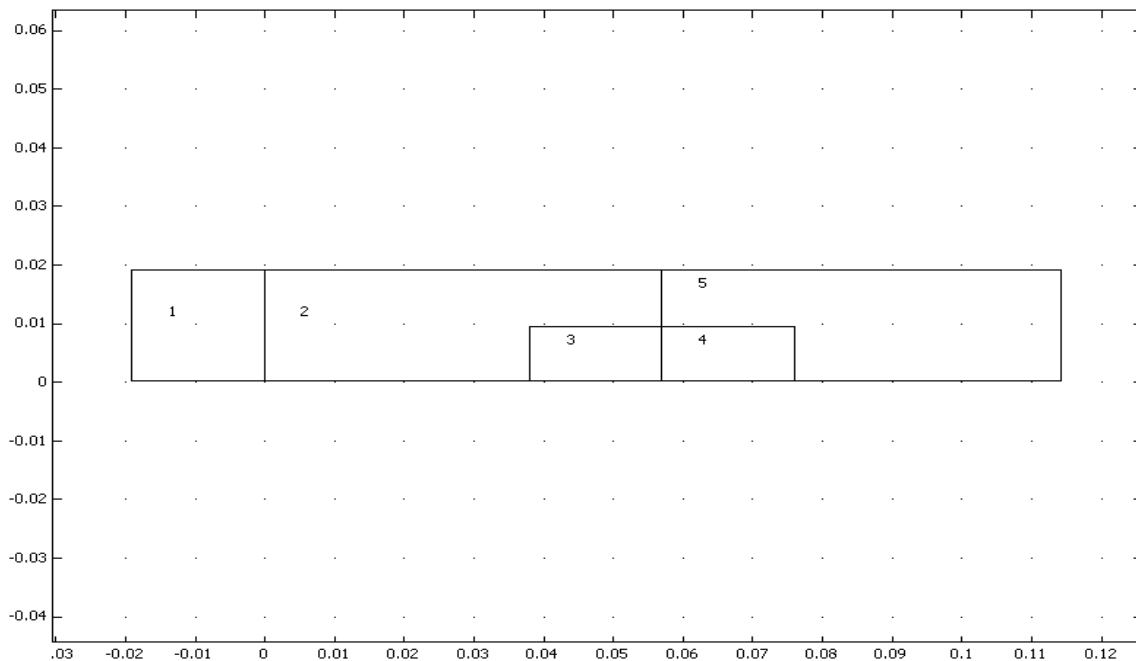


Figure 5.1 COMSOL<sup>®</sup> model with subdomains labeled

subdomains of the model, where subdomains 1, 2 and 5 belong to the Incompressible Navier-Stokes application mode and 3, 4 are associated with the Brinkman application mode.

## 5.2 Governing equations

The governing equations for each of the two different application modes are discussed in this section.

### 5.2.1. Incompressible Navier-Stokes equation

The steady state forms of the Incompressible Navier-Stokes and continuity equations used by COMSOL<sup>®</sup> (COMSOL<sup>®</sup> user's guide, 2005) are:

$$\text{Momentum:} \quad -\vec{\nabla} \cdot \eta (\vec{\nabla} \vec{u} + (\vec{\nabla} \vec{u})^T) + \rho (\vec{u} \cdot \vec{\nabla}) \vec{u} + \vec{\nabla} \cdot p = \vec{F} \quad (5.1)$$

$$\text{Mass:} \quad \vec{\nabla} \cdot \vec{u} = 0 \quad (5.2)$$

Where  $\vec{F}$  = Body force  
 $\eta$  = Dynamic viscosity  
 $\vec{u}$  = Velocity  
 $\rho$  = Density of fluid  
 $p$  = Pressure

### 5.2.2. Brinkman equation

There are two methods for solving the porous momentum equations in COMSOL<sup>®</sup>. One method is to use Darcy momentum equation and the other is usage of Brinkman momentum equations.

Within a porous material, pore walls oppose the momentum transport to fluid outside the individual pores. In most engineering cases, a detailed study of individual pores is not practical. So the porous and fluid regions are studied together by combining

them in to a single medium. Darcy's equations are based upon this approach. When the momentum transport is dominated by shear stresses, then Brinkman equations are required to describe the problem. *"Brinkman model extends Darcy's law to include a term that accounts for the viscous transport, in the momentum balance"*. (COMSOL<sup>®</sup> 3.2 user's guide, 2005).

The steady state, constant viscosity Brinkman equations used by COMSOL<sup>®</sup> (COMSOL<sup>®</sup> user's guide, 2005) are:

$$-\vec{\nabla} \cdot \eta (\vec{\nabla} \vec{u} + (\vec{\nabla} \vec{u})^T) + \frac{\eta}{k} \vec{u} + \vec{\nabla} p = \vec{F} \quad (5.3)$$

Where  $\vec{F}$  = Body force  
 $\eta$  = Dynamic viscosity  
 $\vec{u}$  = Velocity  
 $\rho$  = Density of fluid  
 $p$  = Pressure

In (4.3),  $\vec{\nabla} \cdot \eta (\vec{\nabla} \vec{u} + (\vec{\nabla} \vec{u})^T)$  represents the viscous (Brinkman) term. This term is only required to satisfy the no-slip condition on the wall. Rewriting (5.3) exclusively for pressure gradient,

$$\vec{\nabla} p = -\frac{\eta}{k} \vec{u} + \vec{\nabla} \cdot \eta (\vec{\nabla} \vec{u} + (\vec{\nabla} \vec{u})^T) + \vec{F} \quad (5.4)$$

To accommodate the effects of form drag in (5.4) the body force,  $F$ , is equated to

$$F = -C_F K^{-1/2} \rho_f \left| \vec{u} \right| \vec{u} \quad (5.5)$$

Therefore the porous medium equation becomes,

$$\vec{\nabla} p = -\frac{\eta}{k} \vec{u} + \vec{\nabla} \cdot \eta (\vec{\nabla} \vec{u} + (\vec{\nabla} \vec{u})^T) - C_F K^{-1/2} \rho_f \left| \vec{u} \right| \vec{u} \quad (5.6)$$

### 5.2.3 General heat transfer equation

To model heat conduction and convection through a fluid, the energy equation is used.

The energy equation used in COMSOL<sup>®</sup> General heat transfer application mode is (COMSOL<sup>®</sup> user's guide, 2005)

$$\rho C_p \frac{\partial T}{\partial t} + \vec{\nabla} \cdot (-k \vec{\nabla} T + \rho C_p T \vec{u}) = Q_s \quad (5.7)$$

## 5.3 Boundary conditions

### 5.3.1 Incompressible Navier-Stokes Application mode

The boundary condition settings of COMSOL<sup>®</sup> for the Incompressible Navier-Stokes domain are explained in this section. The boundaries are numbered sequentially as shown in Figure 5.2. A uniform inlet velocity is specified for the inlet boundary 1.

Symmetry boundary conditions were set on boundaries 2 and 3, which means the normal component of velocities on those boundaries is zero. No slip condition is imposed on boundaries 5, 6, 7 and 9. Boundaries 4, and 14 are being defined as neutral boundaries.

*“The neutral boundary condition states that transport by shear stresses is zero across a boundary”* (COMSOL<sup>®</sup> user's guide, 2005). A boundary is defined neutral when it does not put any constraints on the velocity and states that there are no interactions across it.

The pressure was set to zero on the outlet boundary 8. The interfacial boundary conditions (boundaries 10, 11, 12 and 13) are specified by setting the fluid pressure of the Incompressible Navier-Stokes domain equal to the porous pressure of the Brinkman side of the boundary. This completes the boundary condition specification of the Navier-Stokes mode.

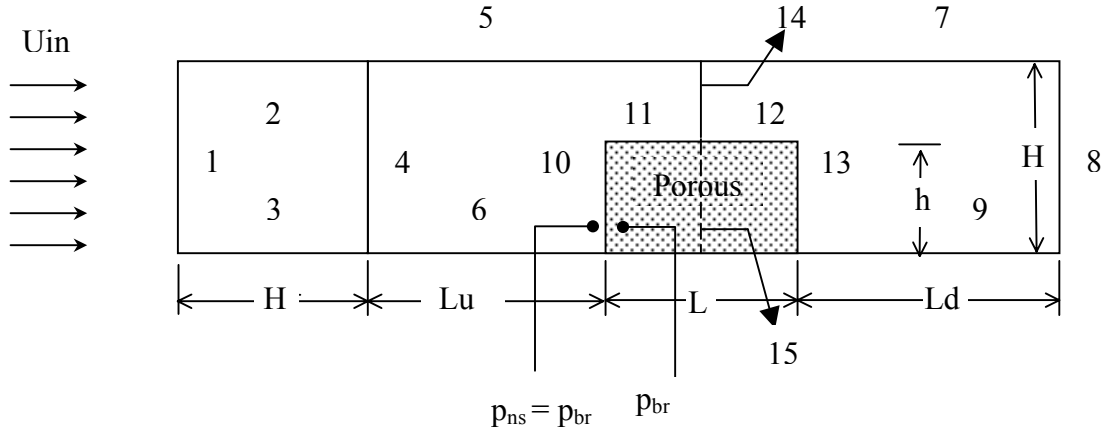


Figure 5.2 COMSOL<sup>®</sup> model with Navier-Stokes boundaries

### 5.3.2 Brinkman-Forchheimer mode

The lower surface of the porous block is assigned a no-slip boundary condition. The interfacial boundary conditions of the porous domain are specified by setting the fluid velocity of the porous medium equal to the fluid velocity in Navier-Stokes side of the boundary .

Boundary 15 is specified with a neutral boundary condition. This completes the boundary condition specification for the Brinkman-Forchheimer equations.

### 5.3.3 General heat transfer mode

The energy equation is applicable for the entire model. The boundary conditions for this mode are discussed in this section. Refer to Figure 5.3.

A constant temperature,  $T_{in}$  is set for the inlet boundary. Boundaries 2, 3, 5, 6, 7 and 9 are all thermally insulated. A constant heat flux,  $q''$ , is provided on the lower surface of the porous block. Convective heat flux condition is set for the outlet boundary 8. “*The convective flux boundary condition assumes that all energy passing through a boundary*

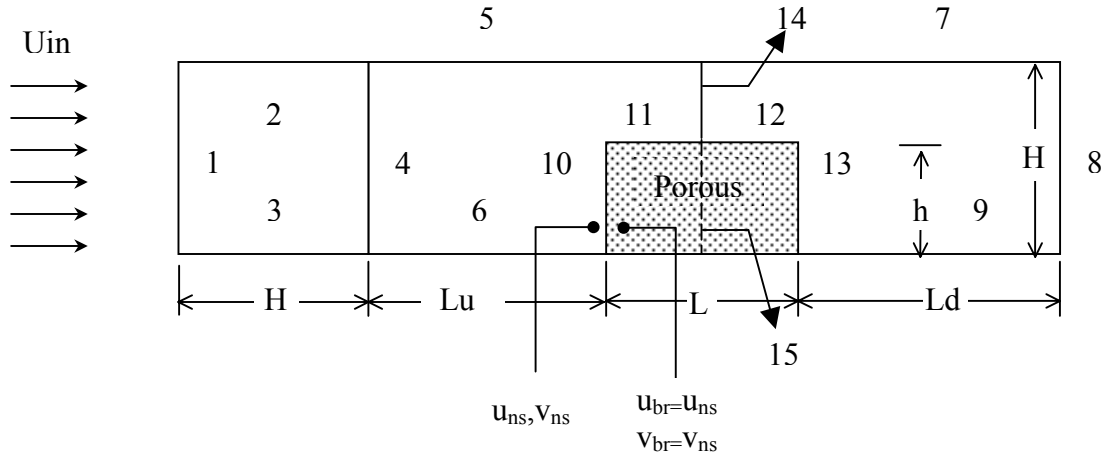


Figure 5.3 COMSOL<sup>®</sup> model with Brinkman-Forchheimer boundary conditions

does so through the convective flux mechanism. This condition thus first assumes that any heat flux due to conduction across this boundary is zero. This is a useful boundary condition, particularly in convection-dominated energy balances, where the outlet temperature is unknown". (COMSOL<sup>®</sup> user's guide, 2005). For all other internal boundaries, thermal continuity condition is prescribed. This means heat flux across an interface is unchanged.

## 5.4 Subdomain settings

For the Incompressible Navier-Stokes application mode, the density and molecular viscosity of the fluid,  $\rho$  and  $\eta$  are supplied for the subdomains 1, 2 and 5. (in Figure 5.1. The values are shown in Table 4.7.1). For the Brinkman-Forchheimer application mode (subdomains 3 and 4), the density and molecular viscosity of the porous matrix are given the same values as the fluid, as the corresponding values of the porous matrix are very small compared to the fluid. The permeability and the Forchheimer terms (4.5) are specified as  $K$  and body force components  $F_x$ ,  $F_y$ . In the general heat transfer application

mode, the thermal conductivity of fluid  $k_f$ , specific heat of fluid,  $C_{p, f}$  are specified for subdomains 1, 2 and 5. The solution of velocity field obtained from the Navier-Stokes and Brinkman equations  $u_{ns}$ ,  $v_{ns}$ ,  $u_{br}$ ,  $v_{br}$  are supplied to the energy equations. A detailed explanation of this process is given in section 4.6. Similarly, for subdomains 3 and 4,  $K$ ,  $C_{p, f}$  and  $k_{eff}$ , are used as permeability, specific heat and effective thermal conductivity of the porous medium.

## 5.5 Mesh parameters

Meshing is the partitioning of the geometry into smaller, refined elements. A coarse mesh will mostly produce a solution that is unreasonable. A highly refined mesh usually yields better solutions, but requires higher memory and computational time. A balance has to be established keeping these two factors and convergence criteria (depends on tolerance chosen) in mind. So, meshing the geometry becomes a crucial aspect for such simulations, more particularly when a series of such cases are to be solved to conduct a parametric study. For the present study, the mesh size for the entire model is based on the mesh parameters of boundary 1. All boundaries are assigned a maximum mesh element size of ' $H/100$ ', where ' $H$ ' is the height of the channel. Also a mesh element growth rate of ' $1.05$ ' is prescribed.

*“The element growth rate determines the maximum rate at which the element size can grow from a region with small elements to a region with larger elements”*

(COMSOL<sup>®</sup> user's guide, 2005). The same maximum element size is used for all other boundaries in this model. A constant mesh element growth rate of ' $1.05$ ' is maintained throughout the model. The mesh distribution of the entire model is shown in Figure 5.4.

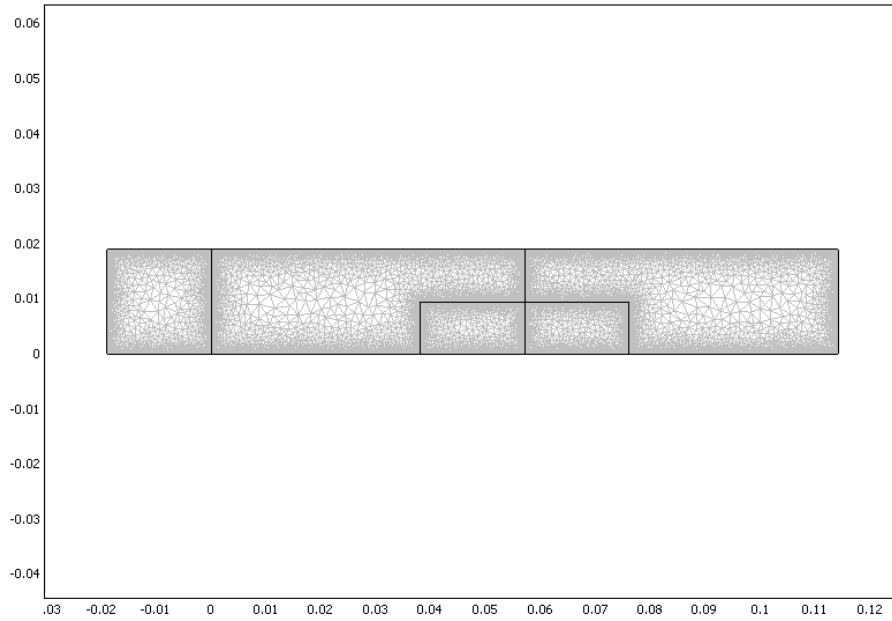


Figure 5.4 COMSOL<sup>®</sup> model mesh configuration with a maximum element size of ‘H/100’ for  $h_p/H = 0.5$

For all the cases considered in this study, solutions of Navier-Stokes and Brinkman equations converged. However, for each Reynolds number,  $Re_{Dh}$ , about 10% of the cases did not converge for the temperature solution. For these cases, converged solutions were successfully obtained using a maximum mesh element size of ‘H/125’.

## 5.6 Solution algorithm

The process of solving the model is divided into two phases, Phase-1 and 2 as shown schematically in Figure 5.5.

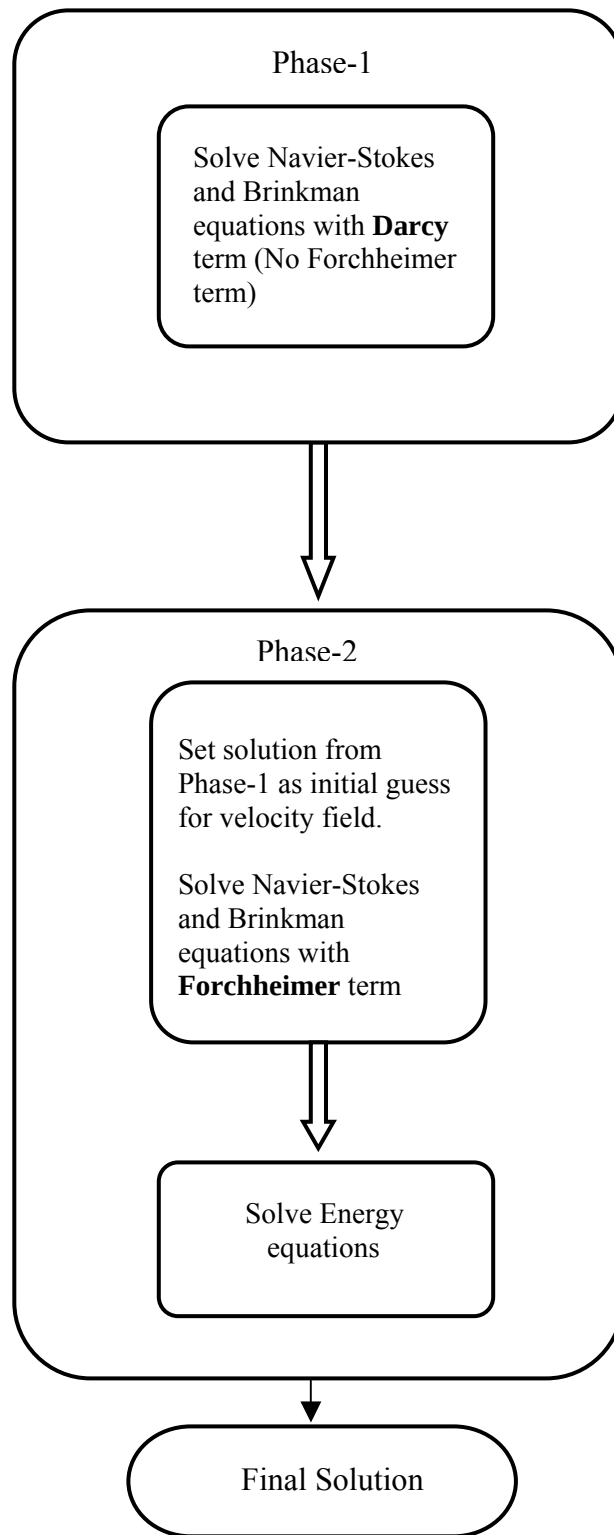


Figure 5.5 Schematic showing the solution process.

In Phase-1 the Navier-Stokes and Brinkman equations without the Forchheimer term are solved first to determine the velocity and pressure distributions. An initial velocity guess is given for both these application domains separately as shown in Table 5.1. The Reynolds number of the flow,  $Re_{Dh}$ , drives the inlet velocity,  $U_{in}$ .

Phase-2 deals with solving the Navier-Stokes and Brinkman equations with the Forchheimer term and energy equations. From equation 5.5, it is clear, that in order to calculate the pressure drop by including the Forchheimer term as body force term,  $u_{br}$  and  $v_{br}$ , must be already known. It is for this very purpose that the entire solution scheme was divided into two phases. Solution from phase-1 is used as an initial guess to Phase-2, so that  $u_{br}$  and  $v_{br}$  are known and Forchheimer term is computed. Once it is known, Navier-Stokes and Brinkman momentum equations are solved iteratively to determine the velocity and pressure field. Finally, the energy equation is solved utilizing the known velocity field to determine the temperature distribution.

Table 5.1 Initial guess for Phase-1 velocity fields for Navier-Stokes and Brinkman domains.

| Application mode | Velocity | Initial guess value |
|------------------|----------|---------------------|
| Navier-Stokes    |          |                     |
|                  | $u_{ns}$ | $0.9*U_{in}$        |
|                  | $v_{ns}$ | $0.01*U_{in}$       |
| Brinkman         |          |                     |
|                  | $u_{br}$ | $0.01*U_{in}$       |
|                  | $v_{br}$ | $0.001*U_{in}$      |

## 5.7 Numerical values

### 5.7.1 Table of properties

Water properties are evaluated at a temperature of 20° C or 293.15 K. The form coefficient of graphite was reported by Osgood (2001) and effective thermal conductivity Klett et al. (2000). See Tables 5.2 and 5.3 for properties of water and porous matrix.

### 5.7.2 Range of parameters

The inlet velocity of the problem is determined from the important parameter Reynolds number, based on the hydraulic diameter of the channel,  $Re_{Dh}$ .

Varying the Reynolds number varies the inlet flow velocity. The heat flux supplied and the permeability of the porous material that significantly affect the solution are varied over a range as shown in Table 5.4. This range was developed with the help of a preliminary study of the model. Another geometric parameter of interest is the ratio of height of porous block to the height of channel,  $h_p/H$ . The range of variation of  $h_p/H$  is also shown in Table 5.4. For all cases in this study, the length of porous block, is held a constant at  $L = 0.0381\text{m}$  and the value  $L/H$  is held constant at 2.

Table 5.2 Thermo physical properties of water used in this study.

| Property             | Symbol     | Value    | Units           |
|----------------------|------------|----------|-----------------|
| Fluid: Water         |            |          |                 |
| Density              | $\rho$     | 998.2    | $\text{kg/m}^3$ |
| Molecular Viscosity  | $\eta$     | 0.001002 | $\text{kg/m-s}$ |
| Specific Heat        | $C_{p, f}$ | 4182     | $\text{J/kg-K}$ |
| Thermal Conductivity | $k_f$      | 0.59     | $\text{W/m-K}$  |

Table 5.3 Thermo physical properties of porous matrix used in this study.

| Property                       | Symbol     | Value                    | Units             |
|--------------------------------|------------|--------------------------|-------------------|
| Porous matrix                  |            |                          |                   |
| Density                        | $\rho$     | 998.2                    | kg/m <sup>3</sup> |
| Molecular Viscosity            | $\eta$     | 0.001002                 | kg/m-s            |
| Specific Heat                  | $C_{p, f}$ | 4182                     | J/kg-K            |
| Permeability                   | K          | Values<br>in Table 5.7.2 | m <sup>2</sup>    |
| Form Coefficient               | $C_F$      | 0.5243                   |                   |
| Effective Thermal Conductivity | $k_{eff}$  | 150                      | W/m-K             |

Table 5.4 Range of parameters used

| Parameter                      | Symbol            | Range                                                                                                             |
|--------------------------------|-------------------|-------------------------------------------------------------------------------------------------------------------|
| Permeability (m <sup>2</sup> ) | K                 | [0.77 to 1.33 in increments of 0.07] *10 <sup>-7</sup> ,<br>[0.14 to 1.4 in increments of 0.14] *10 <sup>-6</sup> |
| Reynolds number                | Re <sub>Dh</sub>  | [1000 2000]                                                                                                       |
| Height Ratio                   | h <sub>p</sub> /H | [0.05 to 0.85 in increments of 0.05]                                                                              |
| Heat flux (W/m <sup>2</sup> )  | q''               | [10 <sup>5</sup> 3*10 <sup>5</sup> ]                                                                              |

## 5.8 Convergence Study

A convergence study is performed to validate the momentum and convergence tolerance values used for this study. The parameters that were held constant during this study are listed in Table 5.5.

For the momentum convergence study, the maximum element size and the momentum tolerance were varied as shown in Table 5.6. For each of these values, the percentage mass error and  $F_{QP}$  are calculated. It could be observed in Table 5.6, that the percentage mass error and  $F_{QP}$  do not change significantly with the momentum tolerance. For establishing a balance between the necessity for faster computational time and tighter tolerances, a maximum element size of  $H/100$  and momentum tolerance of  $1.00E-006$  are selected for all flow distribution and friction factor results presented in this study.

For the energy convergence study, the energy tolerance and maximum element size are varied as shown in Table 5.7. A fixed momentum tolerance of  $1.00 E-6$  is kept unchanged for this study. For each of these values, the percentage energy error is calculated. As it can be observed from the table, the percentage energy error remains constant for all values of energy Tolerances. But for values of parameters other than those in Table 5.5, the energy errors may vary with energy tolerances. So, in order make the percent energy error as low as possible, the maximum energy tolerance of  $1.00E-08$  is selected for all thermal performance results presented in this study.

Table 5.5 Parameters held constant for the convergence study

| Constant    |              | Paramaters               |             |                                |
|-------------|--------------|--------------------------|-------------|--------------------------------|
| <b>ReDh</b> | <b>H (m)</b> | <b>K (m<sup>2</sup>)</b> | <b>hp/H</b> | <b>Qflux (W/m<sup>2</sup>)</b> |
| 1,000.00    | 0.01905      | 1.40E-06                 | 0.5         | 1.00E+05                       |

Table 5.6 Momentum convergence results

| Max<br>element<br>size (m) | Momentum<br>Tolerance |          |                       |          |                       |          |                       |          |
|----------------------------|-----------------------|----------|-----------------------|----------|-----------------------|----------|-----------------------|----------|
|                            | 1.00E-04              |          | 1.00E-05              |          | 1.00E-06              |          | 1.00E-07              |          |
|                            | Percent<br>mass error | $F_{QP}$ | Percent<br>mass error | $F_{QP}$ | Percent<br>mass error | $F_{QP}$ | Percent<br>mass error | $F_{QP}$ |
| H/10                       | -0.973558             | 12.038   | -0.973559             | 12.038   | -0.973559             | 12.038   | -0.973559             | 12.038   |
| H/25                       | -0.968094             | 12.079   | -0.968095             | 12.079   | -0.968095             | 12.079   | -0.968095             | 12.079   |
| H/50                       | -0.965870             | 12.077   | -0.965871             | 12.077   | -0.965871             | 12.077   | -0.965871             | 12.077   |
| H/75                       | -0.948666             | 12.077   | -0.948666             | 12.077   | -0.948666             | 12.077   | -0.948666             | 12.077   |
| H/100                      | 0.958240              | 12.078   | 0.958240              | 12.078   | 0.958240              | 12.078   | 0.958240              | 12.078   |
| H/125                      | 0.972620              | 12.078   | 0.972620              | 12.078   | 0.972620              | 12.078   | 0.972620              | 12.078   |

Table 5.7 Energy convergence results

| Max<br>element<br>size (m) | Momentum Tolerance = 1.00E-6 |                            |                            |
|----------------------------|------------------------------|----------------------------|----------------------------|
|                            | Energy Tolerance             |                            |                            |
|                            | 1.00E-06                     | 1.00E-07                   | 1.00E-08                   |
|                            | Percent<br>Energy<br>Error   | Percent<br>Energy<br>Error | Percent<br>Energy<br>Error |
| H/10                       | -0.561888                    | -0.561888                  | -0.561888                  |
| H/25                       | -0.041367                    | -0.041367                  | -0.041367                  |
| H/50                       | 0.021294                     | 0.021294                   | 0.021294                   |
| H/75                       | 0.022821                     | 0.022821                   | 0.022821                   |
| H/100                      | 0.032583                     | 0.032583                   | 0.032583                   |
| H/125                      | 0.040926                     | 0.040926                   | 0.040926                   |

## 6. Discussion of Results

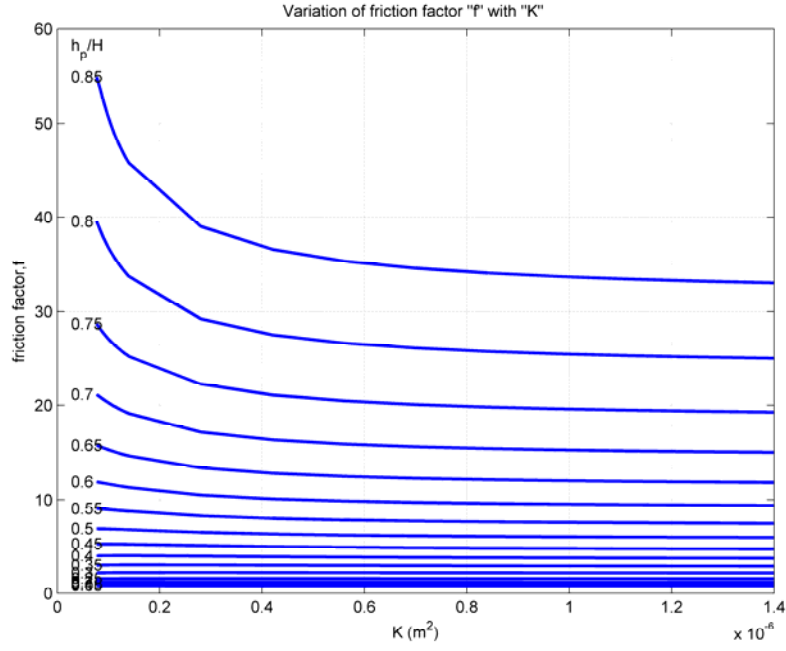
### 6.1 Variation of friction factor

For a fixed Reynolds number, the variation of friction factor  $f$ , with permeability  $K$  and  $hp/H$  is studied. For  $Re_{Dh}=1000$ , this variation is shown in Figure 6.1. It shows that at low values of  $hp/H$ ,  $f$  is approximately independent of  $K$ . As  $hp/H$  is gradually increased,  $f$  tends to decrease with  $K$ . The friction factor reduction is more rapid at low permeability values than at the higher permeabilities. As shown in Figure 6.2, for all values of  $K$ ,  $f$  increases exponentially with  $hp/H$ .

As  $Re_{Dh}$  is increased to 2000, the corresponding friction factor values at all  $hp/H$  values decreased as shown in Figure 6.3. This reduction is more pronounced at higher values of  $hp/H$ . But the overall trends of variation of friction factor with  $hp/H$  and permeability, as shown in Figure 6.4, are similar to those at  $Re_{Dh}=1000$ . Therefore, it could be concluded for the range of Reynolds numbers considered,  $f$  decreases with increasing Reynolds number for all  $hp/H$  values.

### 6.2 Variation of percent of total flow through porous block

For a constant  $Re_{Dh}$  the variation of percent of total flow through the porous block at the cross section located at the mid length of the porous block,  $F_{QP}$ , with permeability and  $hp/H$  is studied. For  $Re_{Dh}=1000$ , Figure 6.5 clearly shows that  $F_{QP}$  increases with permeability for all  $hp/H$ . For a given  $hp/H$ , the increase is more rapid at low permeability values than at the higher permeabilities. As shown in Figure 6.6, for a particular permeability, when  $hp/H$  is increased,  $F_{QP}$  increases exponentially. For the remaining permeabilities also, the same exponential increase of  $F_{QP}$  is observed.



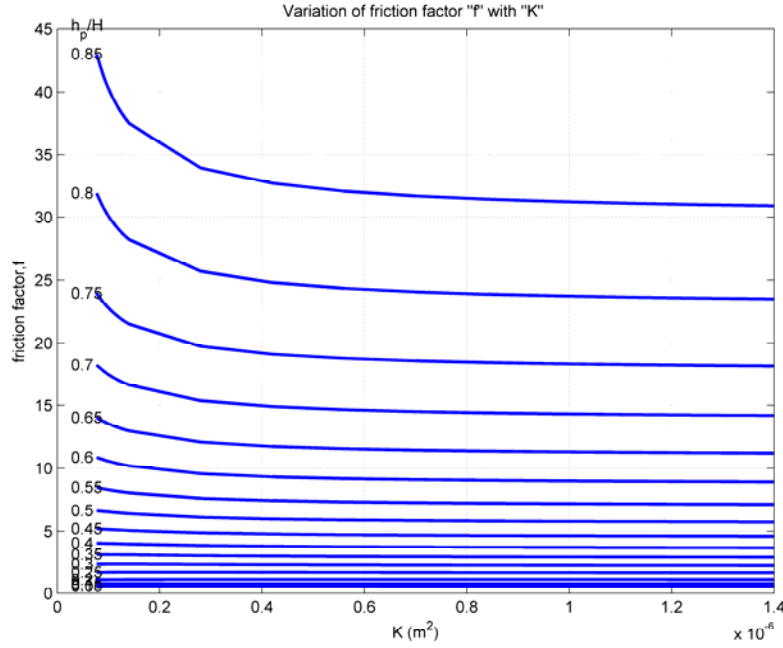


Figure 6.3 Variation of  $f$  with  $K$  for  $Re_{Dh}=2000$

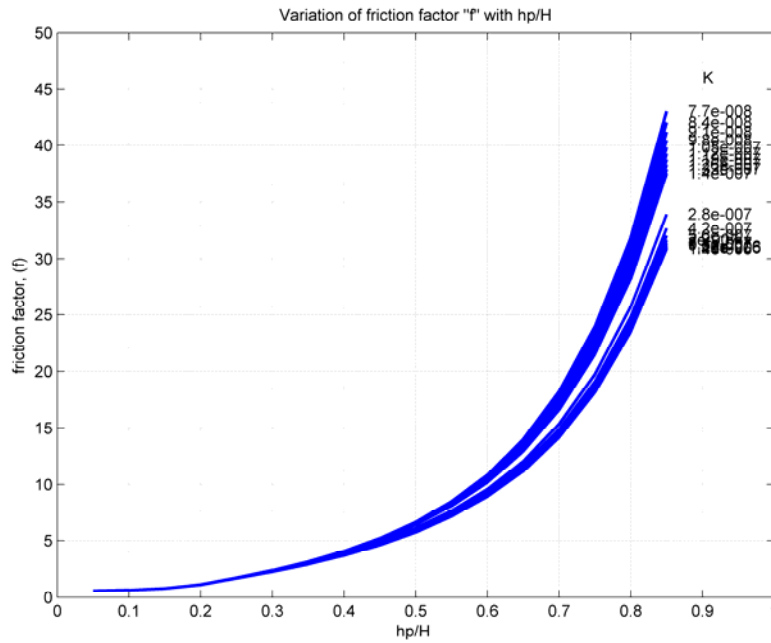


Figure 6.4 Variation of  $f$  with  $h_p/H$  for  $Re_{Dh}=2000$ . Two ranges of  $K$  (m<sup>2</sup>) are [0.77 to 1.33 in increments of 0.07] x 10<sup>-7</sup> and [0.14 to 1.4 in increments of 0.14] x 10<sup>-6</sup>



As the  $Re_{Dh}$  is increased to 2000, the corresponding  $F_{QP}$  for all  $hp/H$  increases as shown in Figure 6.7. This variation could also be verified through Figure 6.8. The overall trend of variation of  $F_{QP}$  with  $hp/H$  and permeability remained the similar to those at  $Re_{Dh} = 1000$ . In summary, it could be observed that as  $K$  decreases, the flow through porous block decreases, and as a consequence the ability to remove heated fluid is reduced.

### 6.3 Variation of Nusselt number

Nusselt number,  $Nu_{Dh}$  behaves in the same manner as  $F_{QP}$  with regards to its variation with permeability and  $hp/H$ . For  $Re_{Dh}=1000$  and  $q'' = 10^5 \text{ W/m}^2$ , referring to Figures 6.9 and 6.10 it is observed that  $Nu_{Dh}$  increases with permeability for all values of  $hp/H$ . For a given  $hp/H$ , the increase is more rapid at low  $K$  values than at the higher permeabilities. It is clear through Figures 6.11 and 6.12 that for the same Reynolds number, when heat flux is increased to  $3 \times 10^5 \text{ W/m}^2$ ,  $Nu_{Dh}$  variation with permeability and  $hp/H$  remained the same. Therefore, it is concluded that at  $Re_{Dh}=1000$ ,  $Nu_{Dh}$  is independent of  $q''$ , at least for these two fluxes. As  $Re_{Dh}$  is increased to 2000, with  $q'' = 10^5 \text{ W/m}^2$ , the corresponding  $Nu_{Dh}$  values at all  $hp/H$  values increase as shown in Figures 6.13 and 6.14. Maintaining the same value of  $Re_{Dh}$  and increasing the  $q''$  to  $3 \times 10^5$  does not affect the corresponding Nusselt number values. This is verified through Figures 6.15 and 6.16. In summary  $Nu_{Dh}$  is independent of heat flux as it should be and is proportional to  $Re_{Dh}$ .

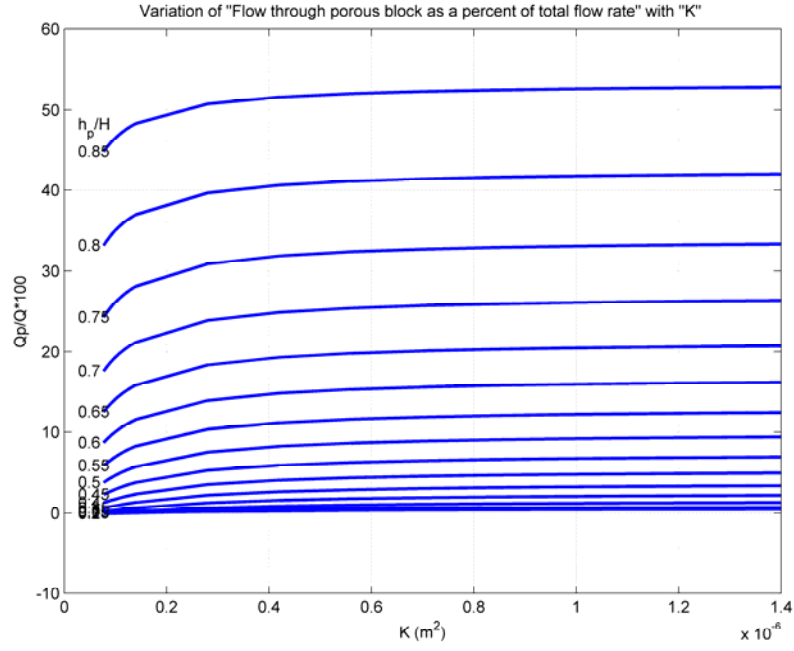


Figure 6.7 Variation of  $F_{QP}$  with  $K$  for  $Re_{Dh}=2000$

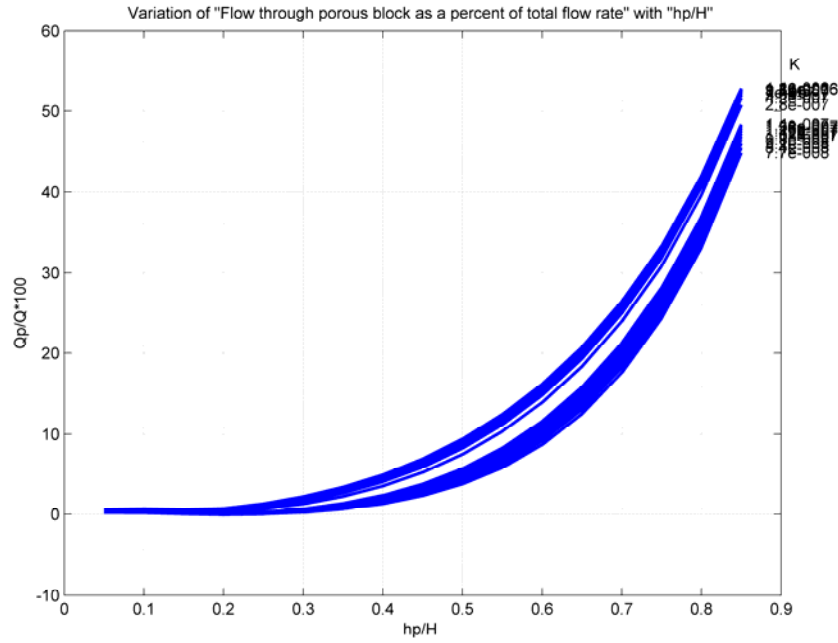


Figure 6.8 Variation of  $F_{QP}$  with  $h_p/H$  for  $Re_{Dh}=2000$ . Two ranges of  $K$  ( $m^2$ ) are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

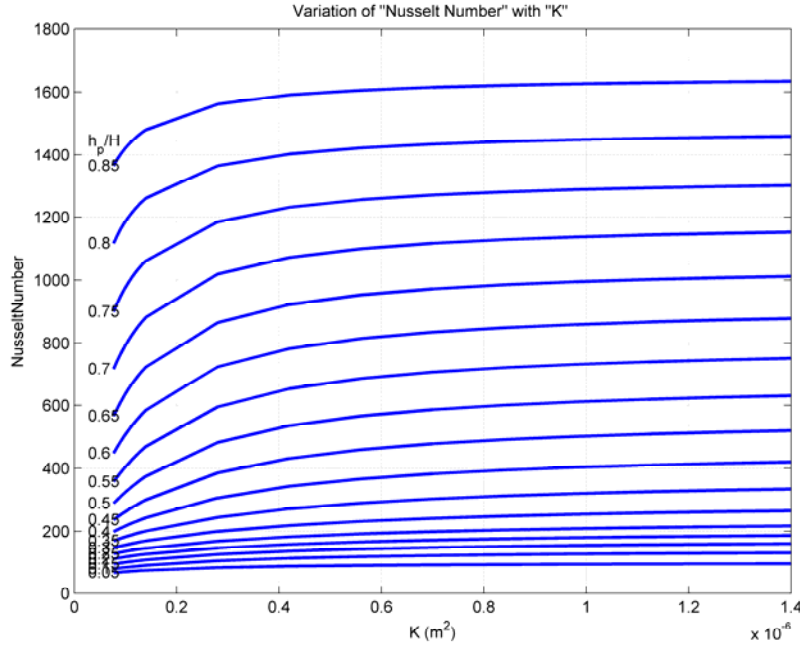


Figure 6.9 Variation of  $Nu_{Dh}$  with  $K$  for  $Re_{Dh}=1000$  and  $q'' = 10^5 \text{ W/m}^2$

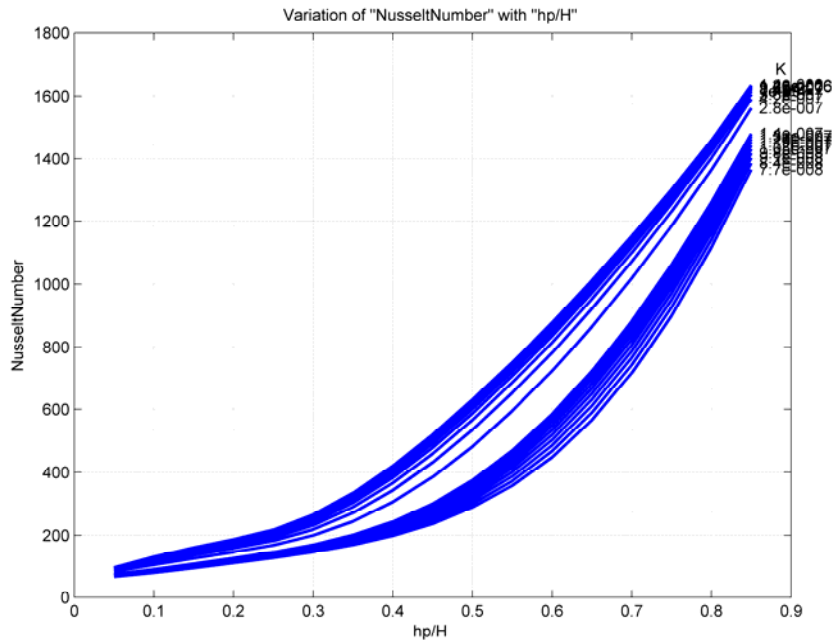


Figure 6.10 Variation of  $Nu_{Dh}$  with  $h_p/H$  for  $Re_{Dh}=1000$  and  $q'' = 10^5 \text{ W/m}^2$ . Two ranges of  $K \text{ (m}^2\text{)}$  are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

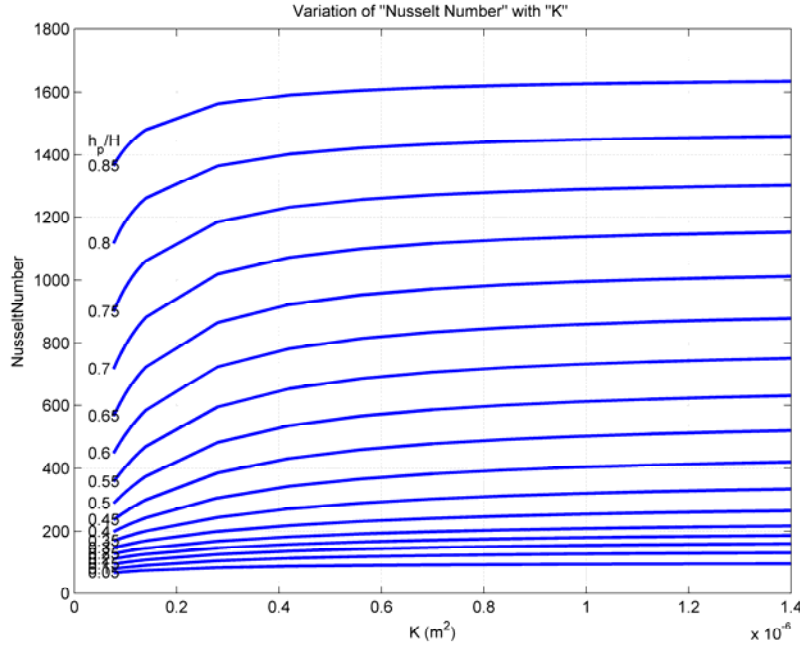


Figure 6.11 Variation of  $Nu_{Dh}$  with  $K$  for  $Re_{Dh}=1000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$

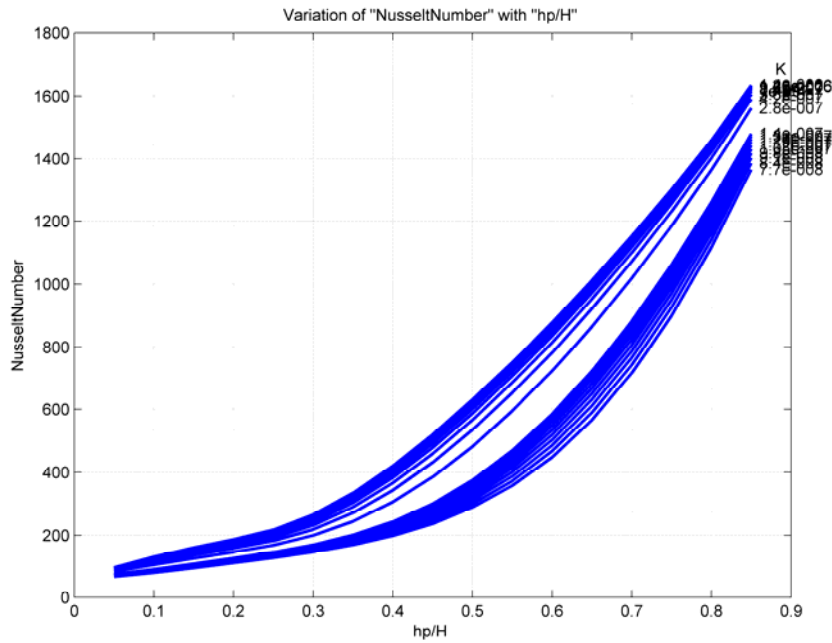


Figure 6.12 Variation of  $Nu_{Dh}$  with  $h_p/H$  for  $Re_{Dh}=1000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$ . Two ranges of  $K \text{ (m}^2\text{)}$  are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

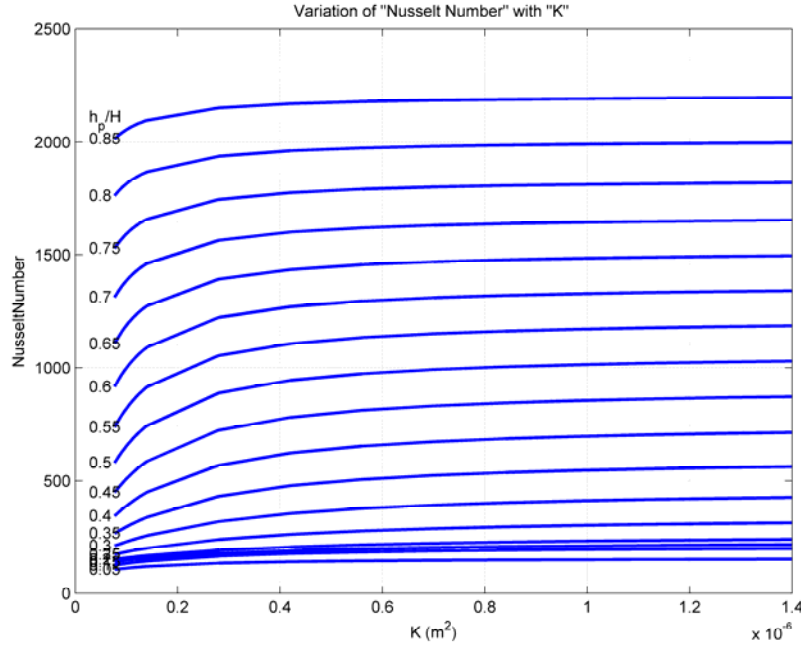


Figure 6.13 Variation of  $Nu_{Dh}$  with  $K$  for  $Re_{Dh}=2000$  and  $q'' = 10^5 \text{ W/m}^2$

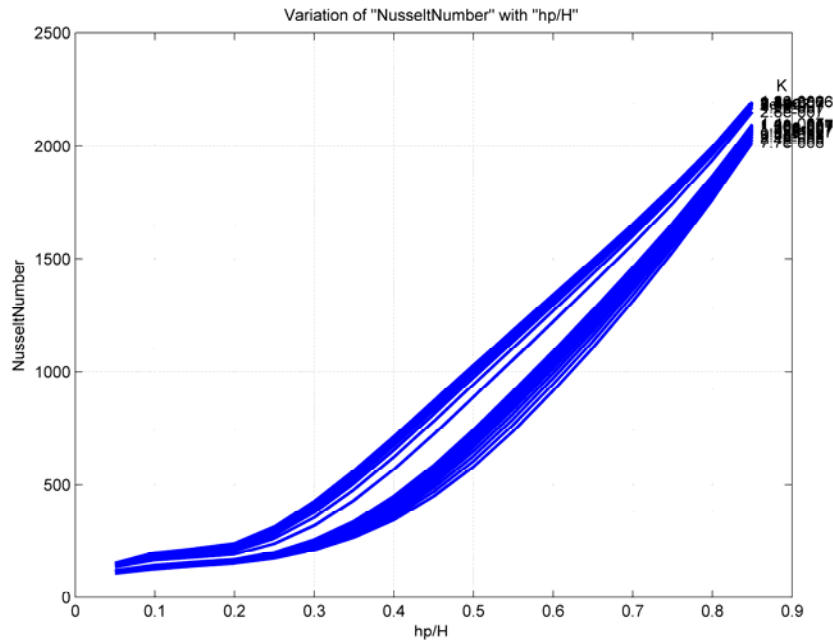


Figure 6.14 Variation of  $Nu_{Dh}$  with  $h_p/H$  for  $Re_{Dh}=2000$  and  $q'' = 10^5 \text{ W/m}^2$ . Two ranges of  $K \text{ (m}^2\text{)}$  are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

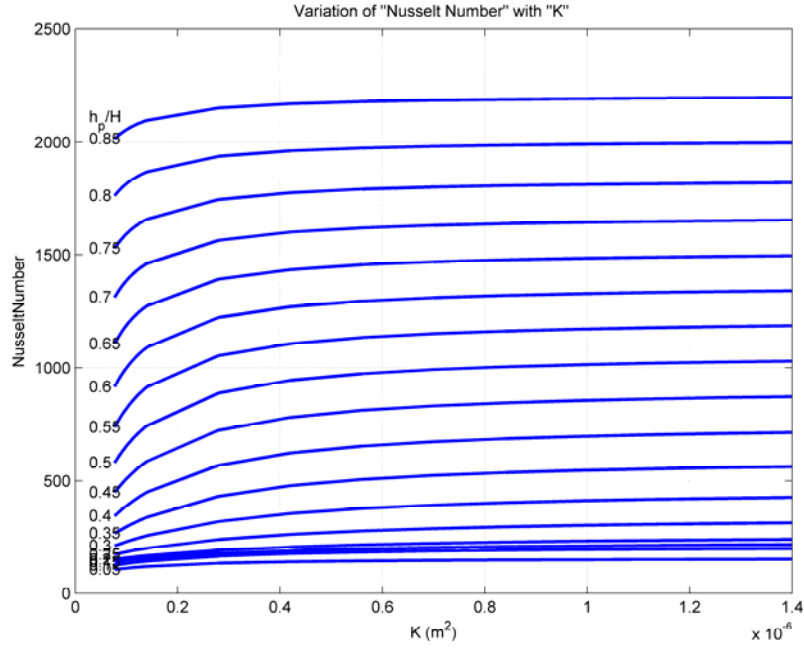


Figure 6.15 Variation of  $Nu_{Dh}$  with  $K$  for  $Re_{Dh}=2000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$

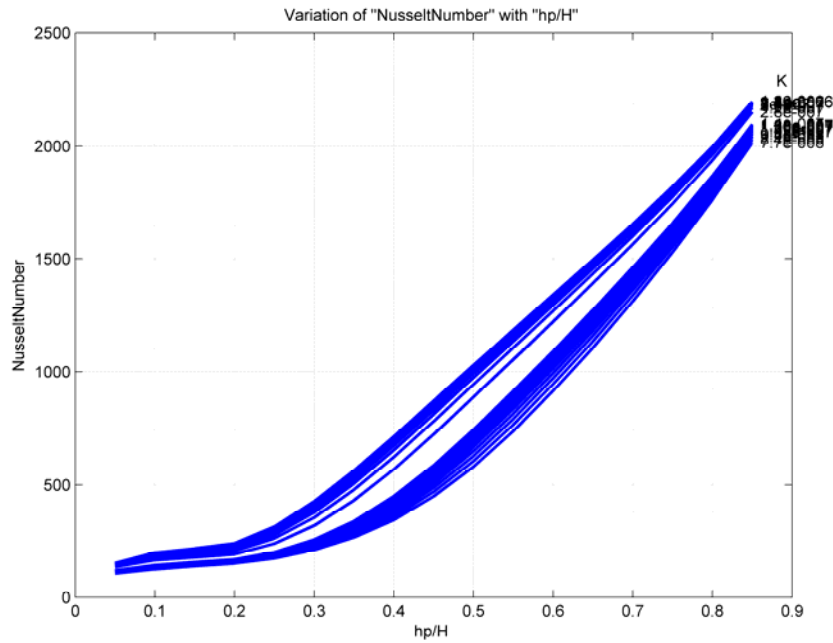


Figure 6.16 Variation of  $Nu_{Dh}$  with  $h_p/H$  for  $Re_{Dh}=2000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$ . Two ranges of  $K \text{ (m}^2\text{)}$  are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

## 6.4 Variation of maximum surface temperature of heater

Figure 6.17 shows the variation of maximum surface temperature of heater,  $T_{s, max}$  with  $K$  and  $hp/H$ . At every  $hp/H$  level,  $T_{s, max}$  decreases with increasing  $K$ . In Figure 6.18 it is shown that for each value of  $K$ , as  $hp/H$  is increased  $T_{s, max}$  is found to decrease. As Figure 6.17 indicates, for  $hp/H = 0.05$  and for most low permeability values for  $hp/H = 0.1$ ,  $T_{s, max}$  is greater than boiling point of water (as shown by the horizontal line at 373.15 K), which leaves all cases above the boiling temperature line unsuitable for use with water in a single-phase heat exchange applications. Figures 6.19 and 6.20 shows the variation of  $T_{s, max}$  with  $K$  and  $hp/H$  for the same  $Re_{Dh}$  and for a heat flux of  $3 \times 10^5$  W/m<sup>2</sup>. The values of  $T_{s, max}$  are much higher in this higher flux case for all levels of  $hp/H$ . For this heat flux case, once again, all cases above the boiling temperature line will not be suitable for use with water in a single-phase heat exchange applications. There is a much wider range of  $hp/H$  and  $K$  cases that fall in this category.

By increasing  $Re_{Dh}$  to 2000 and with  $q'' = 10^5$  W/m<sup>2</sup>, the  $T_{s, max}$  values are found to decrease as shown in Figures 6.21 and 6.22. So, for a constant heat flux  $T_{s, max}$  varies inversely with  $Re_{Dh}$ . One important consequence of increasing  $Re_{Dh}$  is that for almost all  $hp/H$  levels,  $T_{s, max}$  is below the boiling point of water. For the same  $Re_{Dh}$  when heat flux is increased to  $3 \times 10^5$ , the corresponding temperatures are further increased as shown in Figures 6.23 and 6.24. It should also be noted that  $T_{s, max}$  decreases with increasing  $K$ .

For both Reynolds numbers and heat flux cases considered,  $T_{s, max}$  decreases with increasing  $K$ . This is primarily a consequence of increased flow ( $F_{QP}$ ) through the block with increasing permeability. At low permeabilities, the flow through the block is

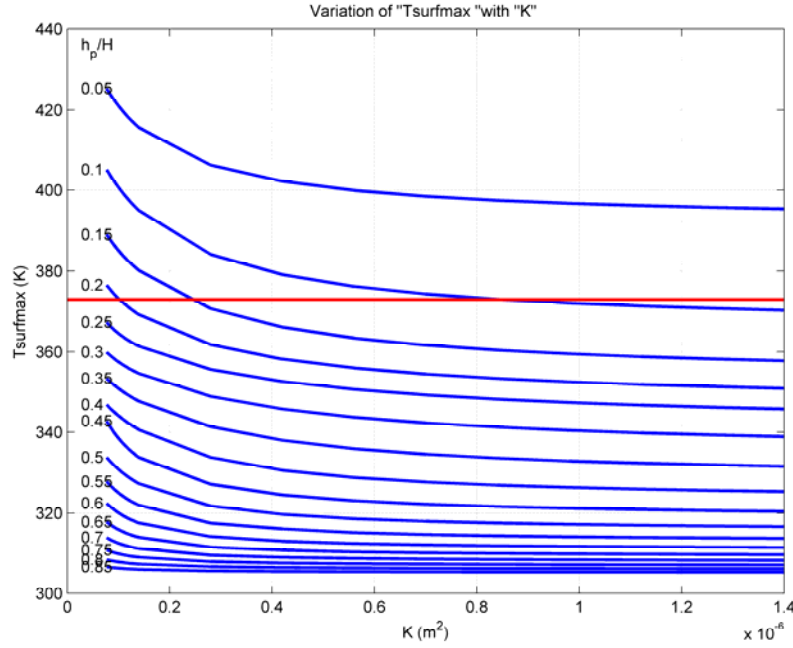


Figure 6.17 Variation of  $T_{s,max}$  with  $K$  for  $Re_{Dh}=1000$  and  $q'' = 10^5 \text{ W/m}^2$

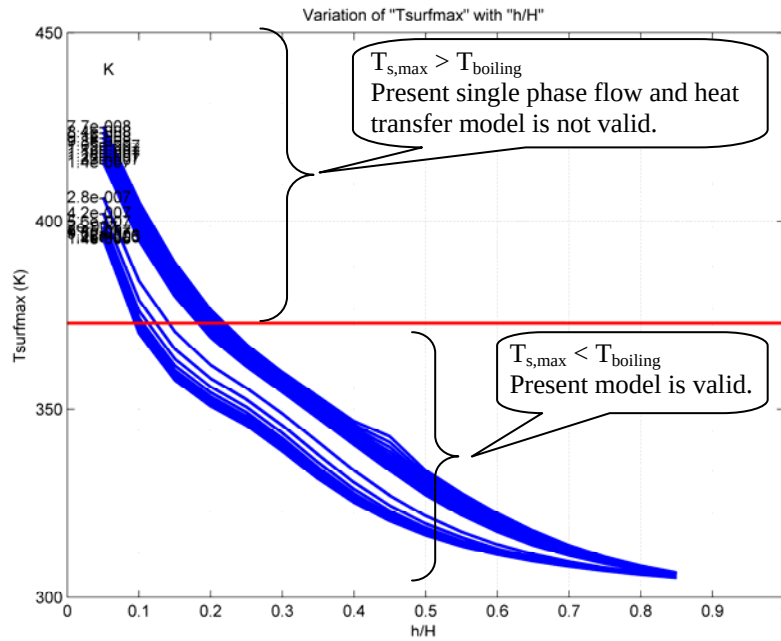


Figure 6.18 Variation of  $T_{s,max}$  with  $hp/H$  for  $Re_{Dh}=1000$  and  $q'' = 10^5 \text{ W/m}^2$ . Two ranges of  $K \text{ (m}^2\text{)}$  are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

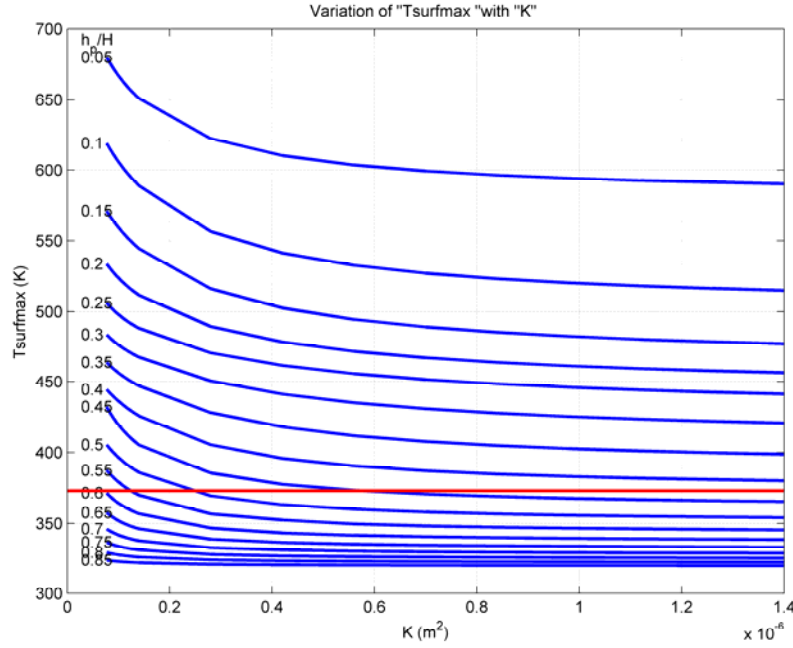


Figure 6.19 Variation of  $T_{s,max}$  with  $K$  for  $Re_{Dh}=1000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$

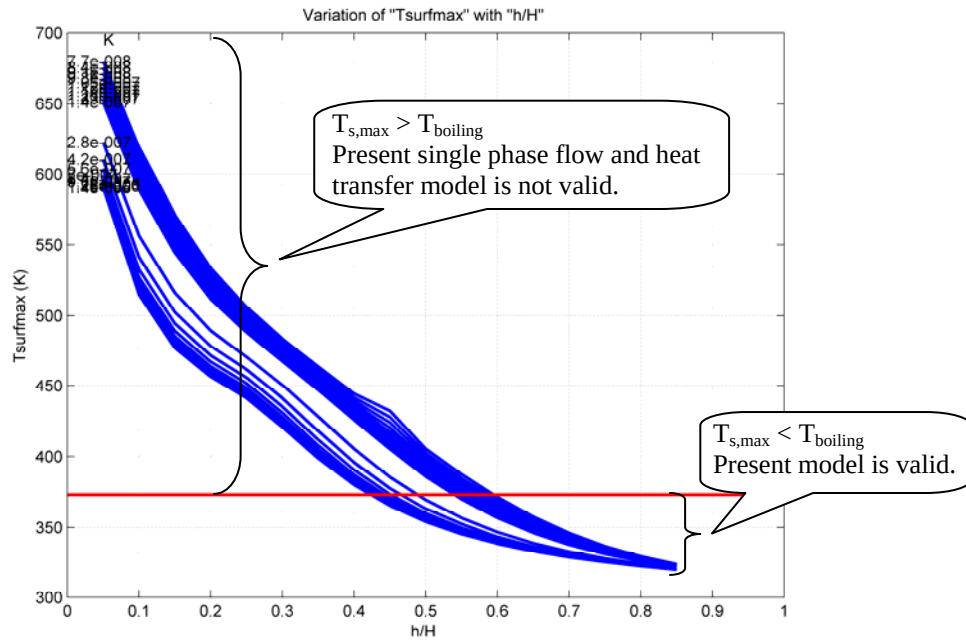


Figure 6.20 Variation of  $T_{s,max}$  with  $h/H$  for  $Re_{Dh}=1000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$ . Two ranges of  $K$  (m²) are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

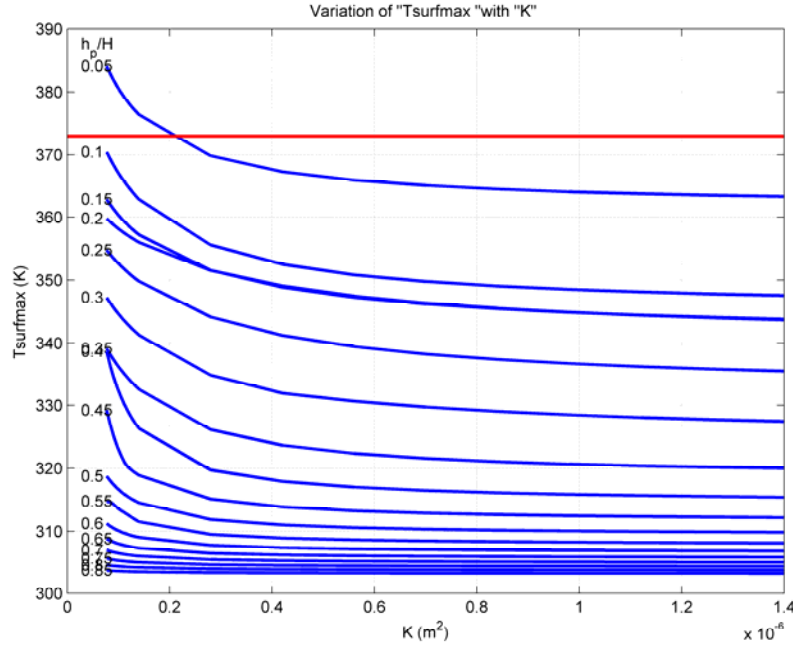


Figure 6.21 Variation of  $T_{s,max}$  with  $K$  for  $Re_{Dh}=2000$  and  $q'' = 10^5 \text{ W/m}^2$

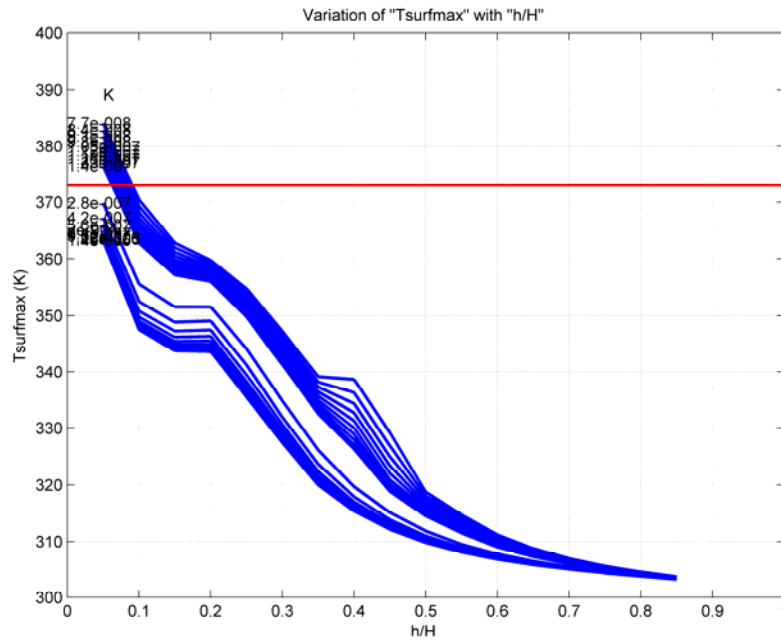


Figure 6.22 Variation of  $T_{s,max}$  with  $h/H$  for  $Re_{Dh}=2000$  and  $q'' = 10^5 \text{ W/m}^2$ . Two ranges of  $K \text{ (m}^2\text{)}$  are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

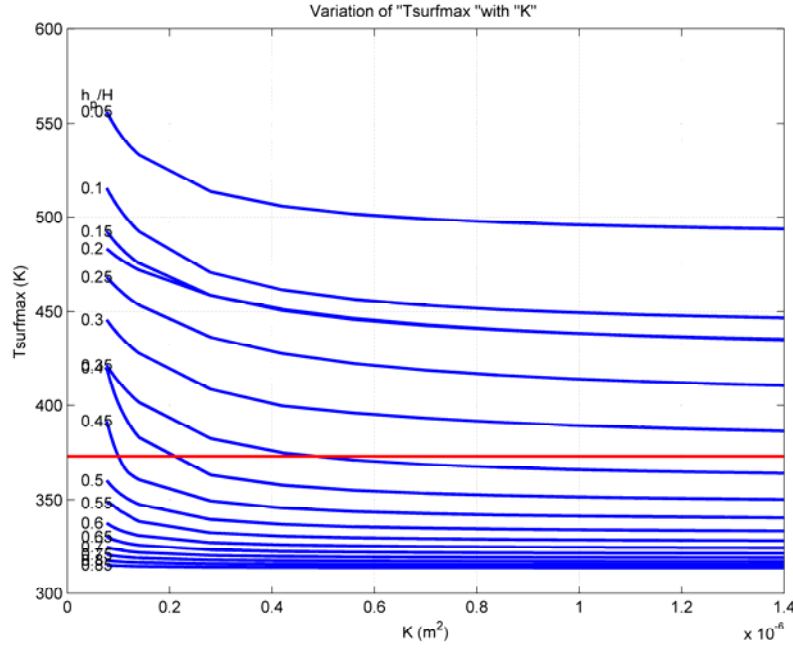


Figure 6.23 Variation of  $T_{s,max}$  with  $K$  for  $Re_{Dh}=2000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$

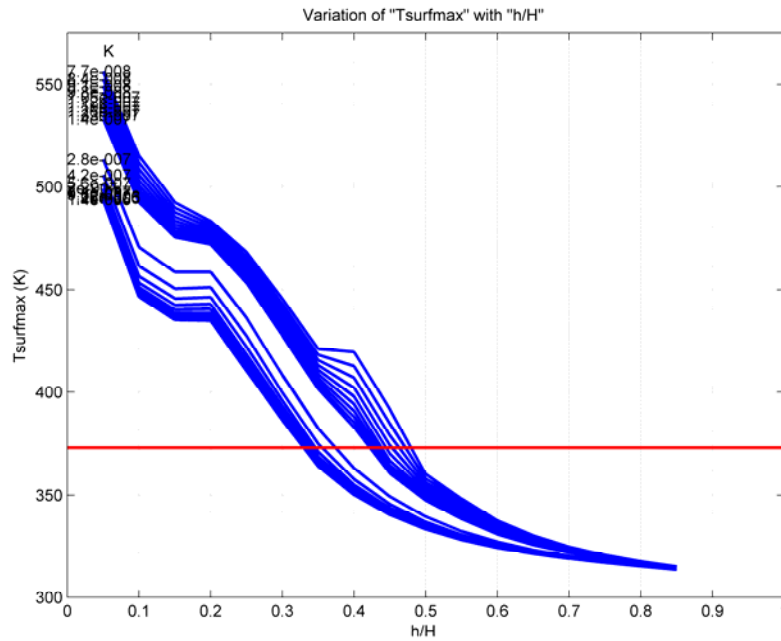


Figure 6.24 Variation of  $T_{s,max}$  with  $h/H$  for  $Re_{Dh}=2000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$ . Two ranges of  $K \text{ (m}^2\text{)}$  are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

reduced and maximum surface temperatures,  $T_{s,max}$ , increase. Increasing flux and/or decreasing Reynolds number tend to increase  $T_{s,max}$ . These trends suggest that the product of  $T_{s,max}$  and  $F_{QP}$  might be approximately constant. But this has not been verified.

## 6.5 Variation of efficiency

Efficiency is a performance measure; a ratio of the rate of heat transfer to the power input required for driving the coolant flow. It should be pointed that because of the manner in which efficiency is defined (equation 4.25), the numerical values tend to be quite high, of the order of  $10^7$ , for the range of parameters considered in this study.

The variation of efficiency,  $E$  with permeability and  $hp/H$  is studied for  $Re_{Dh} = 1000$  and  $q'' = 1 \times 10^5 \text{ W/m}^2$ . From Figure 6.25,  $E$  is found to remain approximately independent of  $K$  for all values of  $hp/H$ , and the values of  $E$  decrease with increasing  $hp/H$ . From the efficiency definition (equation 4.25), we note that it is not explicitly dependent on permeability, but permeability can have an influence through the friction factor. But the results in Figure 6.26 indicate that the dependence through  $f$  is not significant. For all values of  $K$ , Figures 6.25 and 6.26 show that when  $hp/H$  is increased,  $E$  decreases exponentially. This is primarily a consequence of friction factor increasing exponentially with  $hp/H$  (Figures 6.1 and 6.2).

The effect of heat flux on efficiency is described next. For  $Re_{Dh}=1000$ , Figures 6.25 and 6.27 or Figures 6.26 and 6.28 show that increasing  $q''$  from  $1 \times 10^5$  to  $3 \times 10^5 \text{ W/m}^2$  proportionately increases  $E$ . Similarly for  $Re_{Dh} = 2000$  tripling heat flux triples  $E$  for all values of  $hp/H$ . This is evident from Figures 6.29 and 6.31 or 6.30 and 6.32. These trends are consistent with equation (4.25).

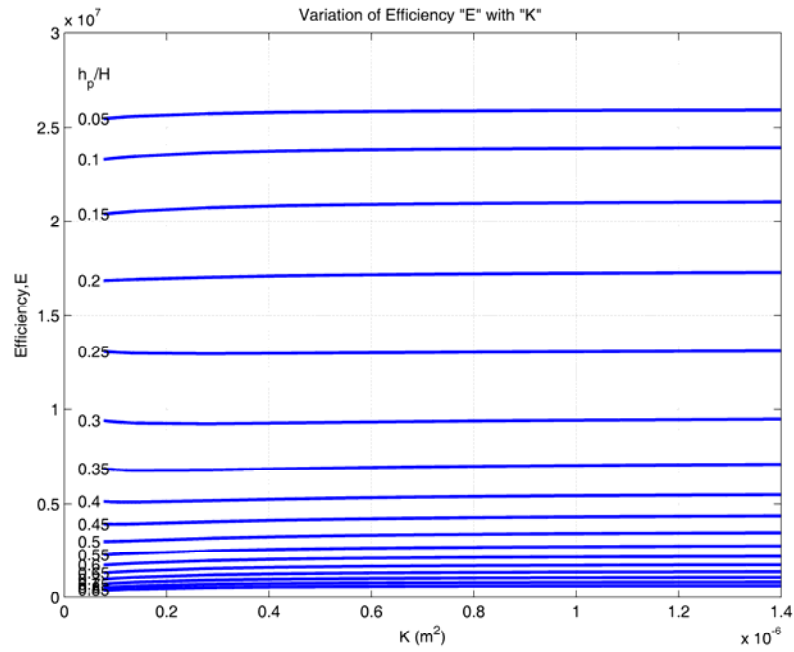


Figure 6.25 Variation of  $E$  with  $K$  for  $Re_{Dh}=1000$  and  $q'' = 10^5 \text{ W/m}^2$

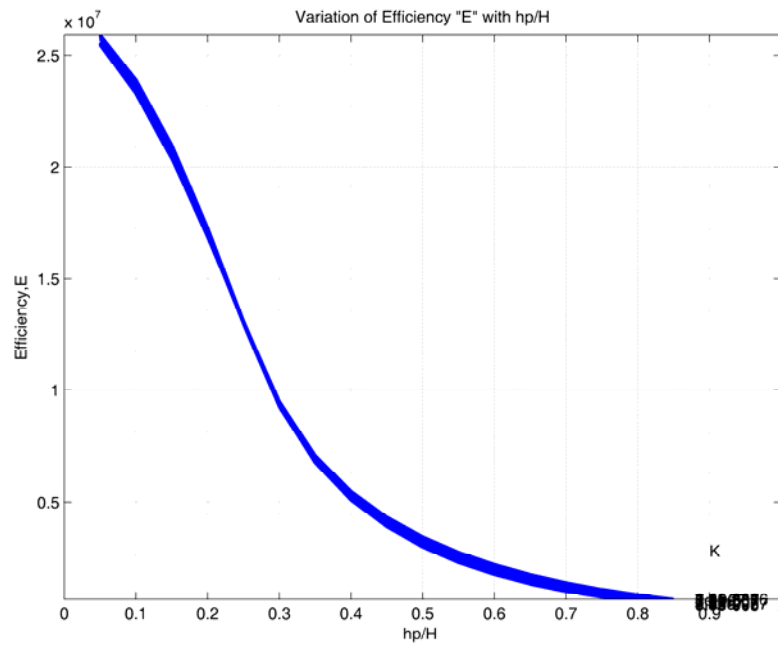


Figure 6.26 Variation of  $E$  with  $h_p/H$  for  $Re_{Dh}=1000$  and  $q'' = 10^5 \text{ W/m}^2$ . Two ranges of  $K (m^2)$  are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

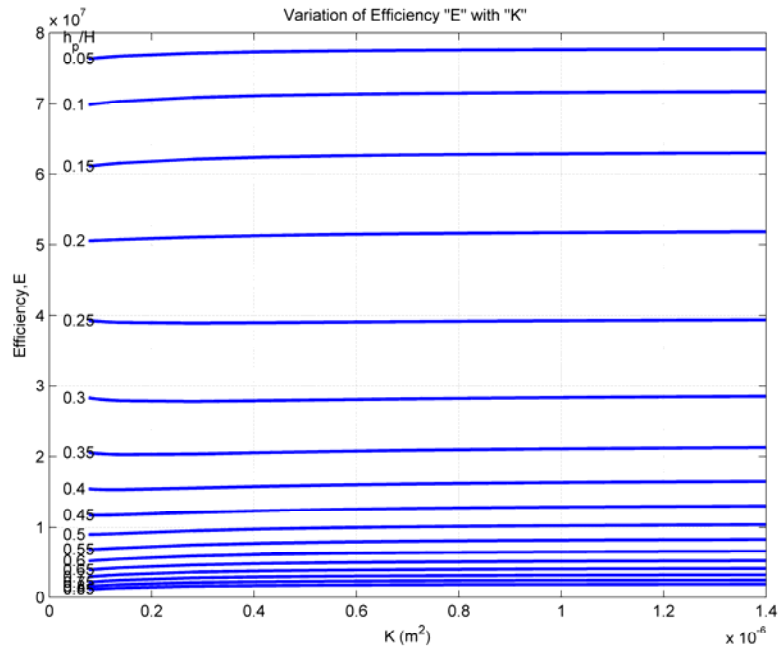


Figure 6.27 Variation of  $E$  with  $K$  for  $Re_{Dh}=1000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$

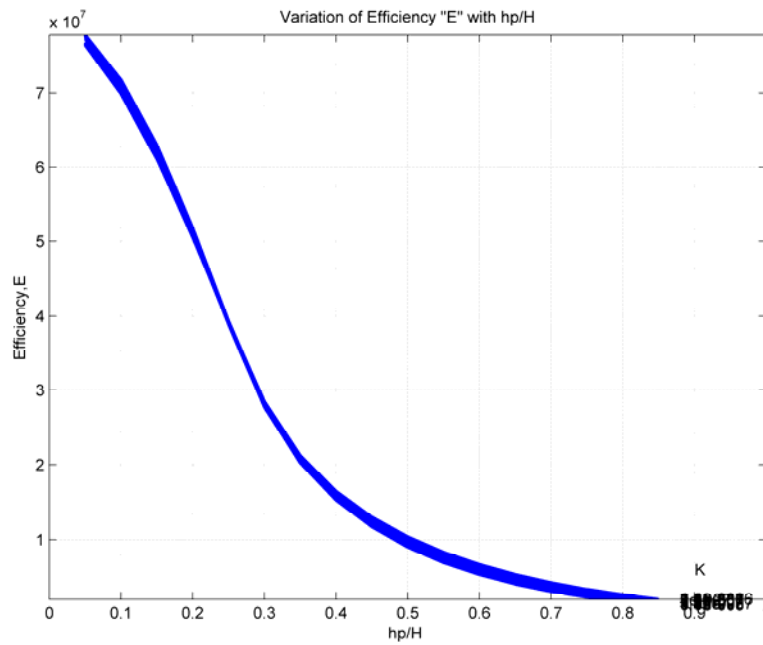


Figure 6.28 Variation of  $E$  with  $hp/H$  for  $Re_{Dh}=1000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$ . Two ranges of  $K \text{ (m}^2\text{)}$  are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

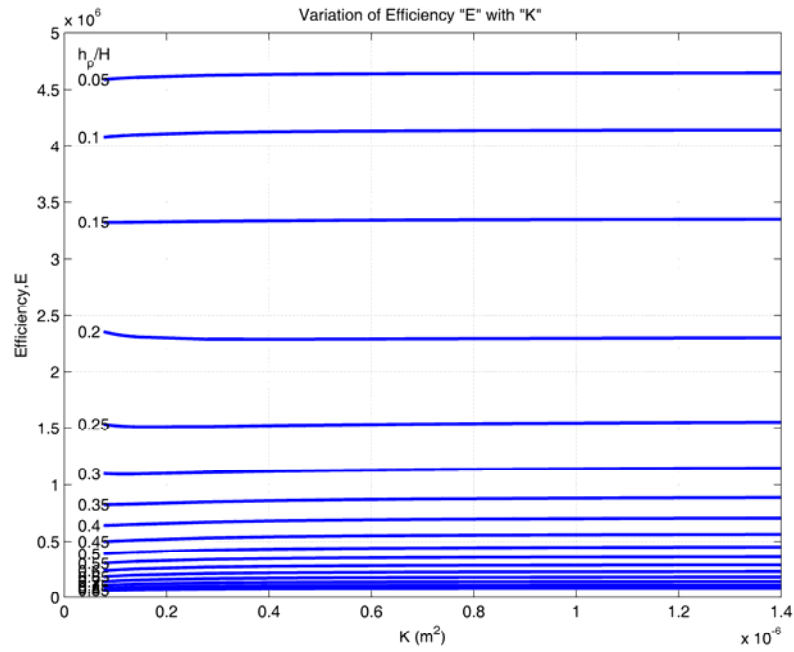


Figure 6.29 Variation of  $E$  with  $K$  for  $Re_{Dh}=2000$  and  $q'' = 10^5 \text{ W/m}^2$

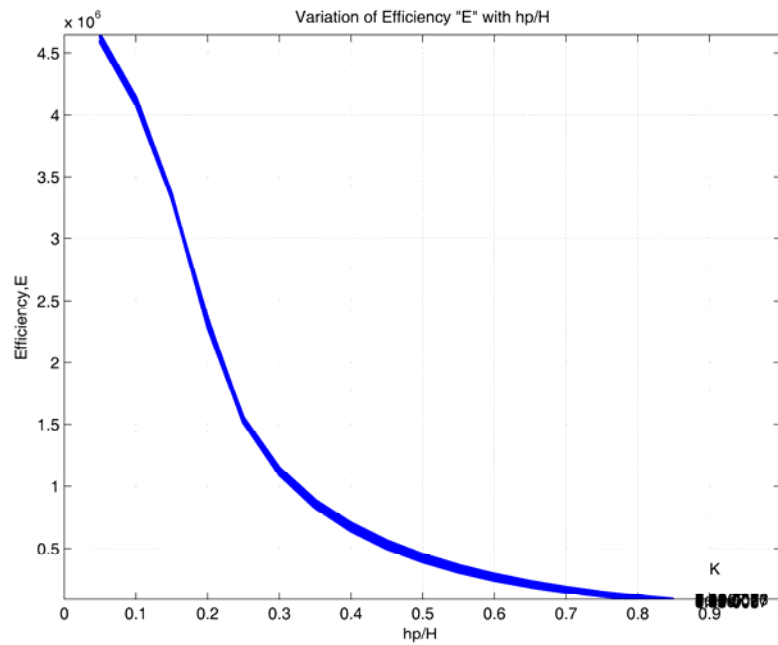


Figure 6.30 Variation of  $E$  with  $K$  for  $Re_{Dh}=2000$  and  $q'' = 10^5 \text{ W/m}^2$ . Two ranges of  $K$  (m<sup>2</sup>) are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

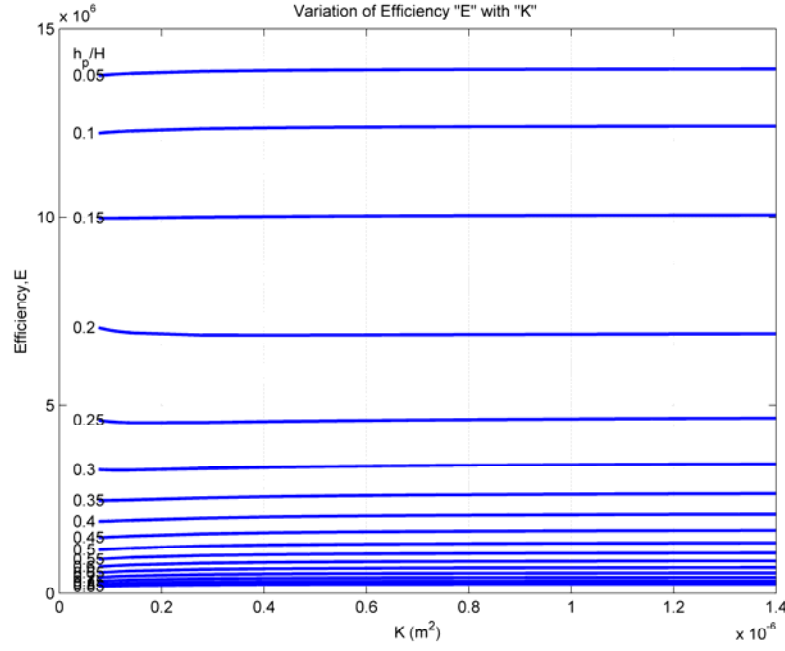


Figure 6.31 Variation of  $E$  with  $K$  for  $Re_{Dh}=2000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$

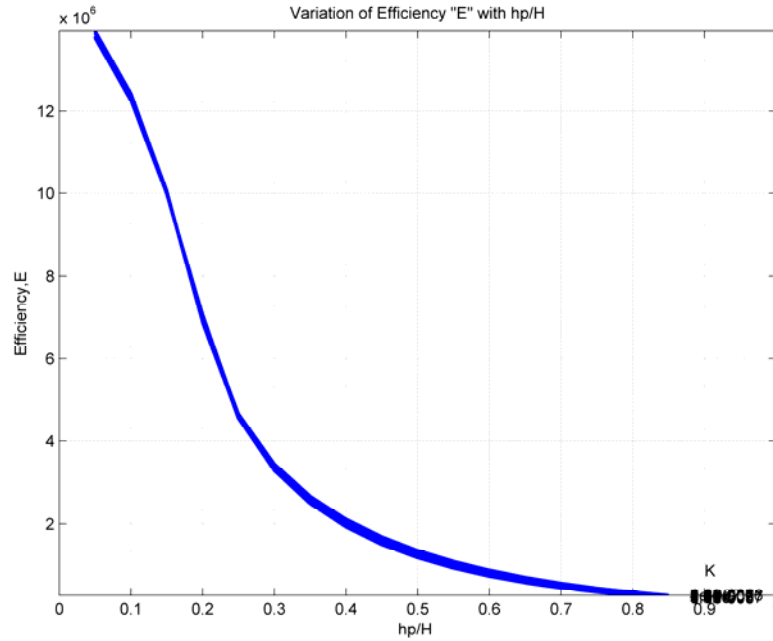


Figure 6.32 Variation of  $E$  with  $hp/H$  for  $Re_{Dh}=2000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$ . Two ranges of  $K \text{ (m}^2\text{)}$  are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

The effect of Reynolds number on efficiency is described here. For  $q'' = 10^5 \text{ W/m}^2$  and increasing  $Re_{Dh}$  from 1000 to 2000, the corresponding  $E$  values at all  $hp/H$  are decreased by a factor of about seven as shown in Figures 6.25 and 6.29 or Figures 6.26 and 6.30. Doubling the Reynolds number alone would reduce the efficiency by a factor of eight as can be seen from equation 4.25. At the same time friction factor decreases fractionally ( Figures 6.1 and 6.3) resulting in a corresponding increase in  $E$  (equation 4.25). So, these two effects together explain the reduction of efficiency by a factor of about seven.

## 6.6 Variation of effectiveness

Effectiveness is another performance measure that takes into account efficiency and the maximum surface temperature of the heated surface. The latter usually has a practical upper limit in electronic cooling applications. Effectiveness is defined as a ratio of efficiency to the dimensionless maximum surface temperature. Higher values are more desirable while meeting the practical temperature limit of a particular application. It should be pointed that because of the manner in which effectiveness is defined (equations 4.29 and 4.30), the numerical values tend to be quite high, of the order of  $10^7$ , for the range of parameters considered in this study.

The results shown in Figure 6.33 for  $Re_{Dh} = 1000$  indicate that for any fixed value of  $hp/H$ ,  $\varepsilon$  increases with  $K$ . Increasing  $K$  increases the flow through porous block and reduces the maximum surface temperature of the heater as described earlier and at the same time decreases the friction factor. Both of these factors contribute to an increase in effectiveness with increase in  $K$ . The dependence on  $hp/H$  is more clearly indicated in Figure 6.34. It shows that for  $Re_{Dh} = 1000$ , effectiveness increases with  $hp/H$  and reaches a maximum at  $hp/H = 0.15$  and decreases with increasing  $hp/H$ .

The effect of Reynolds number on effectiveness is described here. For  $q'' = 10^5$  W/m<sup>2</sup> and increasing  $Re_{Dh}$  from 1000 to 2000, the corresponding Effectiveness values at all  $hp/H$  are decreased by a factor of about eight as shown in Figures 6.33 and 6.35 or Figures 6.34 and 6.36. Doubling the Reynolds number alone would reduce the effectiveness by a factor of sixteen as can be seen from equation 4.32. At the same time friction factor decreases fractionally (Figures 6.1 and 6.3) resulting in a corresponding increase in effectiveness (equation 4.32). So, these two effects together explain the reduction of effectiveness by a factor of about eight.

The effect of heat flux on effectiveness is described next. For  $Re_{Dh} = 1000$ , Figures 6.33 and 6.37 or Figures 6.34 and 6.38 show that increasing  $q''$  from  $1 \times 10^5$  to  $3 \times 10^5$  W/m<sup>2</sup> proportionately increases effectiveness. Similarly for  $Re_{Dh} = 2000$  tripling heat flux triples effectiveness for all values of  $hp/H$ . This is evident from Figures 6.35 and 6.39 or 6.36 and 6.40. These trends are consistent with equation (4.32). Since  $Nu_{Dh}$  is independent of heat flux.

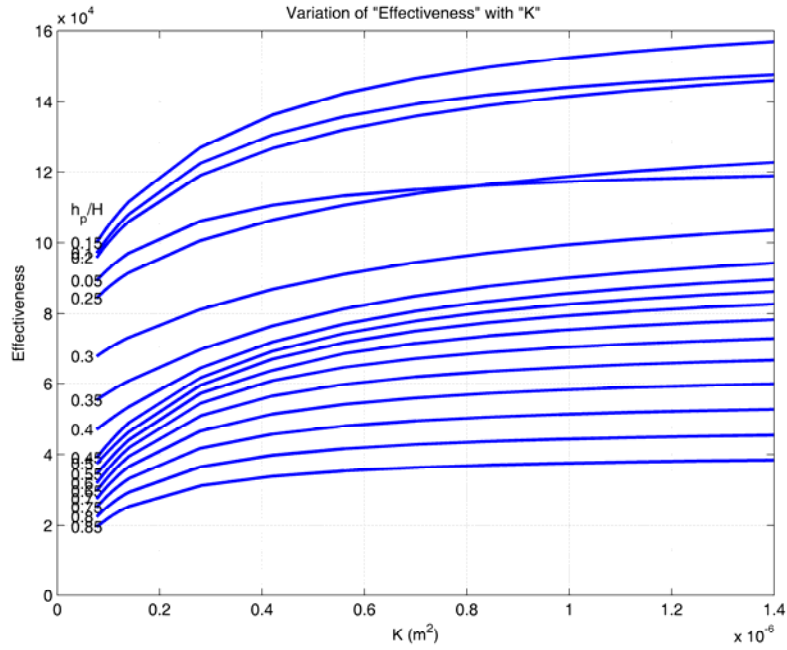


Figure 6.33 Variation of  $\varepsilon$  with  $K$  for  $Re_{Dh}=1000$  and  $q'' = 10^5 \text{ W/m}^2$

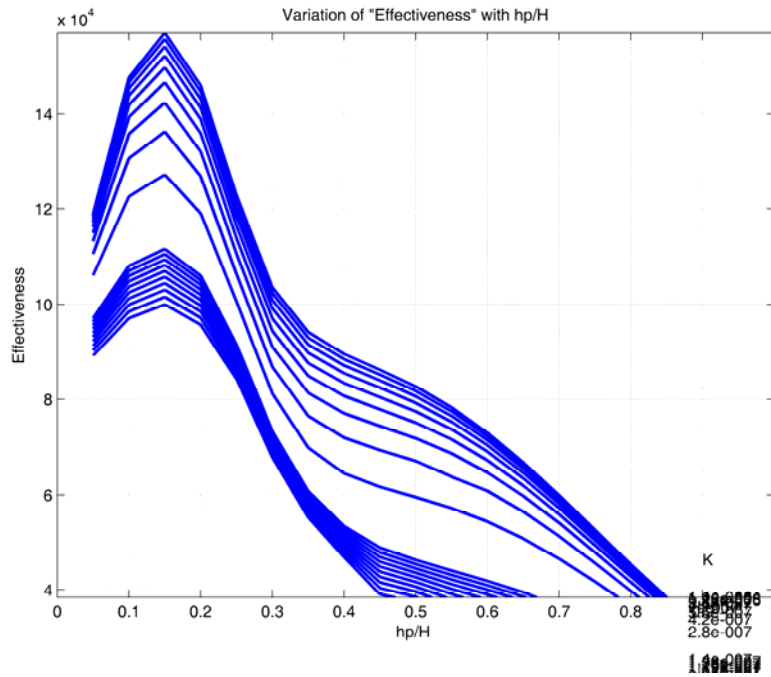


Figure 6.34 Variation of  $\varepsilon$  with  $hp/H$  for  $Re_{Dh}=1000$  and  $q'' = 10^5 \text{ W/m}^2$ . Two ranges of  $K \text{ (m}^2\text{)}$  are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

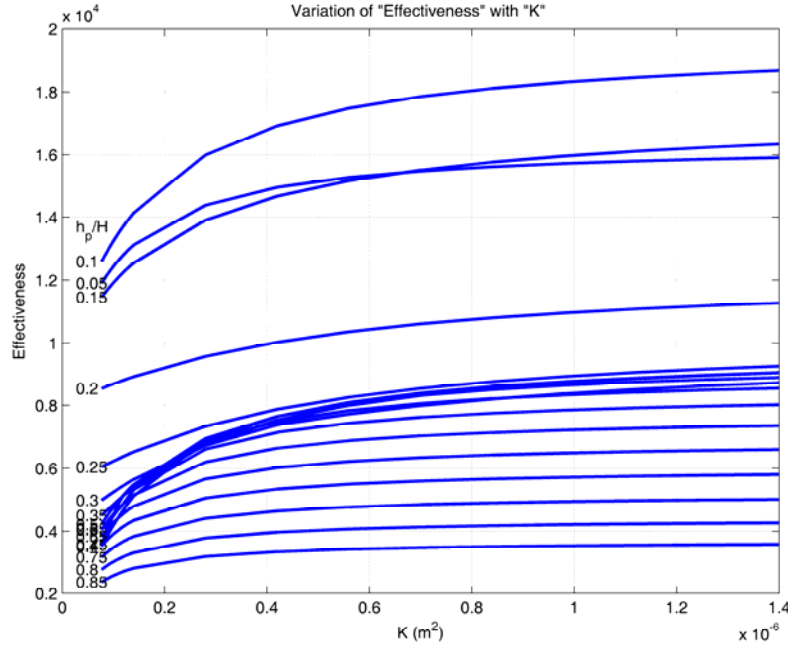
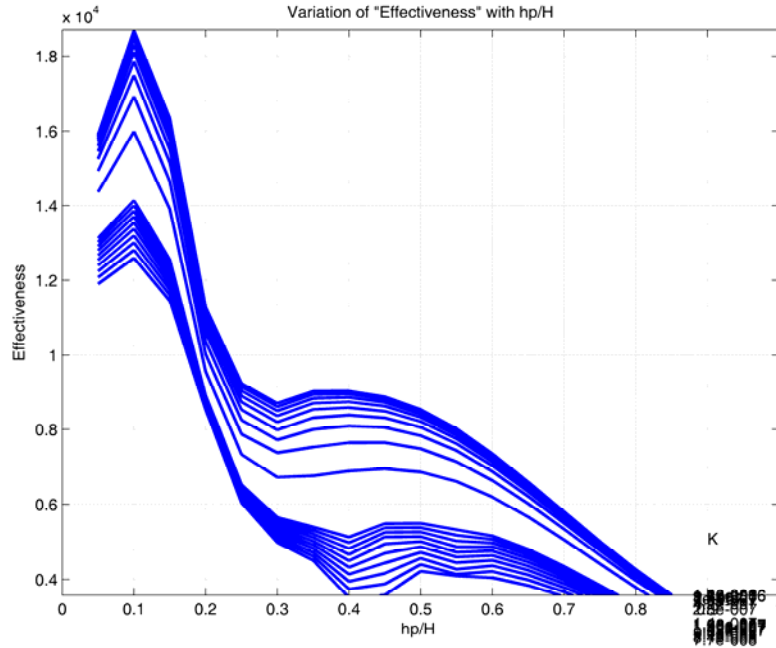


Figure 6.35 Variation of  $\varepsilon$  with  $K$  for  $Re_{Dh}=2000$  and  $q'' = 10^5 \text{ W/m}^2$



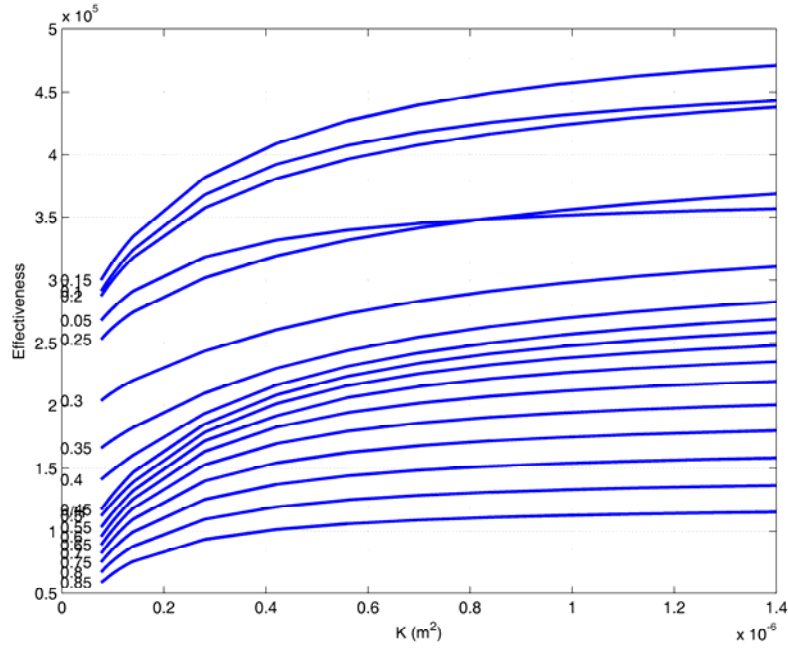


Figure 6.37 Variation of  $\varepsilon$  with  $K$  for  $Re_{Dh}=1000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$

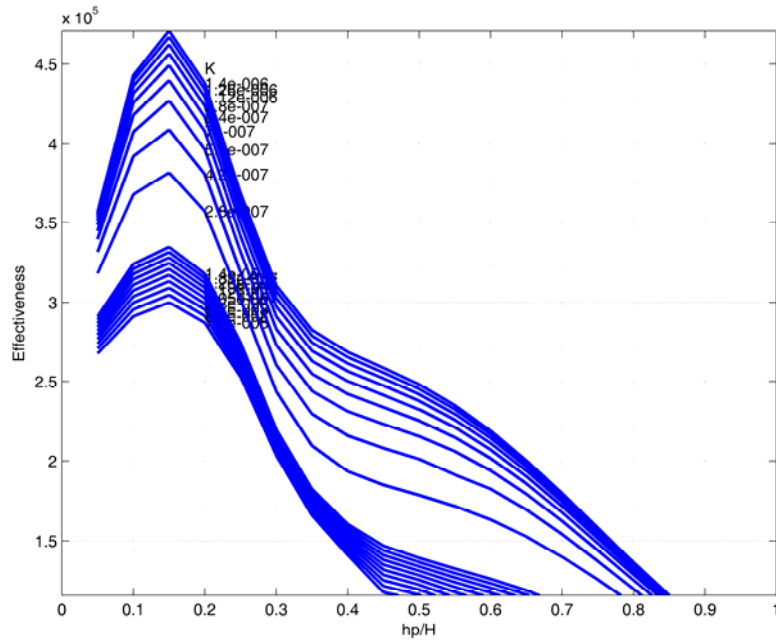


Figure 6.38 Variation of  $\varepsilon$  with  $hp/H$  for  $Re_{Dh}=1000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$ . Two ranges of  $K \text{ (m}^2\text{)}$  are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

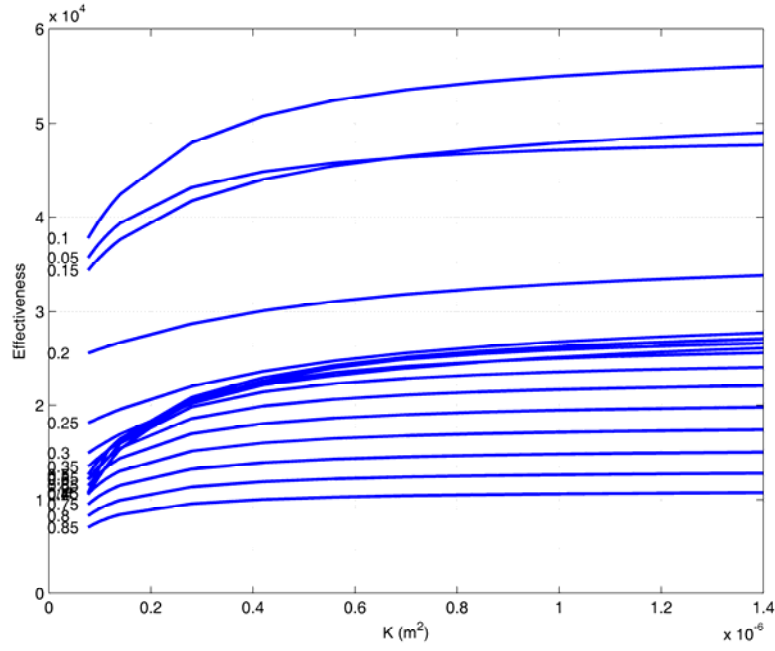


Figure 6.39 Variation of  $\varepsilon$  with  $K$  for  $Re_{Dh}=2000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$

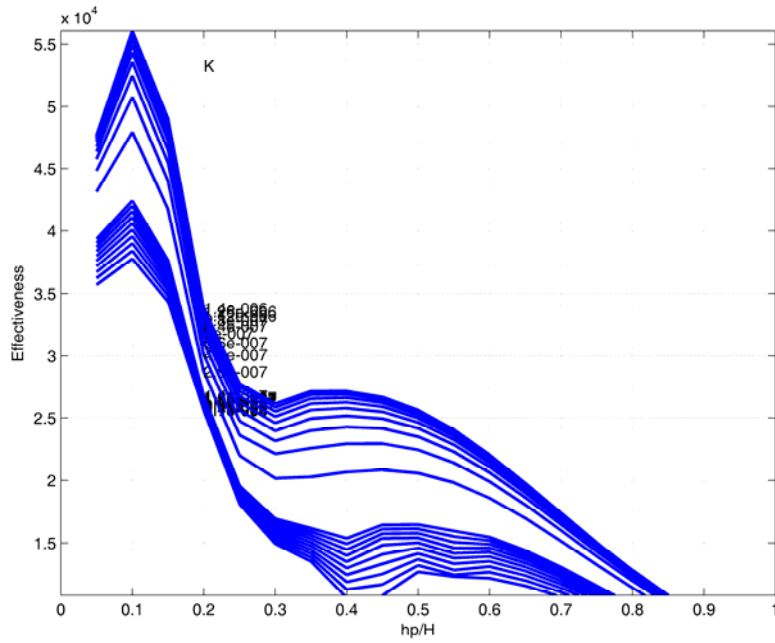


Figure 6.40 Variation of  $\varepsilon$  with  $hp/H$  for  $Re_{Dh}=2000$  and  $q'' = 3 \times 10^5 \text{ W/m}^2$ . Two ranges of  $K \text{ (m}^2\text{)}$  are  $[0.77 \text{ to } 1.33 \text{ in increments of } 0.07] \times 10^{-7}$  and  $[0.14 \text{ to } 1.4 \text{ in increments of } 0.14] \times 10^{-6}$

## 7. Conclusions & Recommendations

### 7.1 Conclusions

The flow and heat transfer characteristics of a 2-D channel partially filled with porous medium are determined. For the range of parameters considered in this study, the following conclusions are drawn

1. The friction factor,  $f$ , decreases with increase in  $Re_{Dh}$  and  $K$ . The friction factor,  $f$ , increases almost exponentially at moderate to higher values of  $h_p/H$ .
2. The Nusselt number  $Nu_{Dh}$  increases with  $Re_{Dh}$ ,  $K$  and  $h_p/H$ .
3. The maximum surface temperature of the heater decreases with increasing  $Re_{Dh}$  and  $h_p/H$ . It decreases with increasing  $K$  for both of the  $Re_{Dh}$  and  $q''$  cases considered. The situations where it exceeds the boiling temperature of water are clearly noted.
4. Effectiveness increases linearly with heat flux and decreases rapidly with increase in  $Re_{Dh}$ . When  $Re_{Dh}$  is doubled effectiveness dropped by a factor of about eight.
5. For the two values of heat fluxes and for all permeability values considered in this study, the maximum effectiveness is found to occur at  $h_p/H = 0.15$  and  $0.10$  for  $Re_{Dh}=1000$  and  $2000$ , respectively.
6. In general, effectiveness increases with increasing  $K$ .

## 6.2 Recommendations

Based on the results of this study, it is recommended that:

1. Further study be conducted to explore the reasons for maximum effectiveness occurring at a fixed value of  $h_p/H$  and its dependence on  $Re_{Dh}$ .

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## Vita

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