

Measurement of parity violation in polarized neutron capture on ^3He

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In memory of Carolyn Brooks Gottlieb.



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Abstract

The weak interaction between nucleons remains an outstanding topic of theoretical and experimental research. At low energies, where nucleons bind into nuclei, non-perturbative QCD is so far unable to make accurate calculations of the "Hadronic Weak Interaction." [1] An alternative approach to studying these systems is the meson-exchange model proposed by Desplanques, Donoghue, and Holstein, which describes hadronic weak interactions in terms of meson-nucleon coupling constants [2]. An experimental solution to the problem of measuring weak interactions between strongly interacting particles is to search for parity-violating observables. This work presents the results of the measurement of the parity-violating directional asymmetry from the capture of polarized neutrons on gaseous ^3He . This measurement was conducted with a pulsed neutron beam at the Spallation Neutron Source at Oak Ridge National Laboratory. In this experiment, neutrons are polarized, captured on ^3He , and then the spin-dependent asymmetry is measured. One year of data was recorded and analyzed to eliminate false asymmetries and extract the precise value of the proton asymmetry. This experiment was optimized through simulation of the detector signals and calculation of the geometry factors. A multi-variable, weighted custom simulation was written for this experiment and run on a computing cluster. The inputs used to construct the simulation came from measurements of the neutron phase space, ENDF cross-sections, and PSTAR ionization data. Here, we present an independent calculation of two nuclear asymmetries in $n+^3\text{He}\rightarrow p+t$: $\alpha_{pv} = (7.8 \pm 10.3 \pm 1.6) \cdot 10^{-9}$ and $\alpha_{pc} = (4.2 \pm 0.5) \cdot 10^{-7}$. This calculation has the highest precision of any measured nuclear asymmetry. These results are consistent with existing few-body experiments, and provide a unique constraint on the phase space for any future theoretical analysis of the hadronic weak interaction.

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1 INTRODUCTION

The weak force has been well-explored in the leptonic and semi-leptonic sectors. However, the study of the weak interaction between hadrons has proceeded more slowly due to the difficulty of measuring weak interactions separately from the strong backgrounds present in hadronic interactions. Studies of the Hadronic Weak Interaction include both heavy and light nuclei. The study of experimental systems with $A \leq 4$ have additional appeal since their interaction terms may be exactly calculated [1]. The weak force maximally violates parity, which enables indirect observation of weak interactions through measurements of parity violation. This is the experimental tool that is used to measure the sensitivities of the weak meson-nucleon couplings in $n + {}^3\text{He} \rightarrow p + t$. The Spallation Neutron Source is an ideal facility for this measurement, due to its high neutron flux and well-defined neutron energy through time of flight measurement. The $n^3\text{He}$ experiment aims to measure the parity-violating asymmetry to a precision of 10^{-8} .

Chapter 2 gives a brief description of the weak nucleon-nucleon interaction, the DDH model of meson exchange, and the observables in the $n^3\text{He}$ experiment. Chapter 3 describes the accelerator and detector apparatus. Chapter 4 gives a detailed explanation of the construction and outputs of the simulation used to optimize the experiment and calculate the geometry factors. Chapter 5 contains the analysis procedure for extraction the asymmetries, and Chapter 6 summarizes the results.

2 BACKGROUND PHYSICS

2.1 Parity Transformation

In the Schrödinger formulation of quantum mechanics [3], physical systems are represented by wavefunctions, $|\Psi\rangle$, which are elements of a vector space. A state which is a function of a Cartesian coordinate could be represented as:

$$\begin{aligned}\vec{r} &= \{x, y, z\} \\ \langle \vec{r} | \Psi_1 \rangle &= \Psi_1(\vec{r}) = \Psi_1(x, y, z)\end{aligned}\tag{2.1}$$

Wavefunctions contain information about the state they represent, and may evolve in time. The inner product of a wavefunction with itself will give the probability density of that state. If the wavefunction is normalized, this value must be 1:

$$\langle \Psi_1 | \Psi_1 \rangle = 1\tag{2.2}$$

In order to perform useful calculations, operators, $\hat{A}$, are used to manipulate these states, and ultimately to predict observables. One such operator is called the parity operator, $\hat{\Pi}$. This operator reverses the sign of the position coordinates in a state:

$$\hat{\Pi} \langle \vec{r} | \Psi_1 \rangle = \Psi_1(-\vec{r}) = \Psi_1(-x, -y, -z)\tag{2.3}$$

If a state is an eigenvector of the parity operator, it will return the original state and one of the eigenvalues of Π , which are ± 1 :

$$P = \langle \Pi \rangle = \langle \Psi | \Pi | \Psi \rangle\tag{2.4}$$

If $\Pi|\Psi\rangle = |\Psi\rangle$, the state is defined to have even parity. If $\Pi|\Psi\rangle = -|\Psi\rangle$, the state is defined to have odd parity. Parity is conserved in strong and electromagnetic interactions, but is violated in weak interactions.

2.2 The Weak Interaction

Parity is conserved by three fundamental forces: the electromagnetic force, the gravitational force, and the strong force. It was also assumed to be conserved in weak interactions, until attempts to solve the $\theta^+-\tau^+$ puzzle led T. D. Lee and C. N. Yang to question this in 1956 [4]. They realized that conservation of parity in weak interactions had not yet been confirmed experimentally. They noted that observables which were invariant under parity transformations could not be used to test this assumption, and suggested measuring a pseudoscalar quantity. In 1957, C. S. Wu performed an experiment to measure the β -decay of polarized ^{60}Co [5],

$$^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e^- + \bar{\nu}_e, \quad (2.5)$$

a weak process, where the electron emission rates were measured respect to the nuclear spin. The pseudoscalar observable was $(p_e \cdot \sigma_{Co})$. She found an asymmetry in the angular distribution of the emitted electrons, demonstrating that the weak interaction violates parity. Since that time, experimental searches to observe parity violation have been classified into three categories: leptonic, semi-leptonic, and hadronic. The first two of these types have been more precisely measured and understood, since they do not involve the interaction of multiple composite particles (such as nucleons). The theory of weak interactions in the Standard Model can be used to accurately predict interactions between fermions, but not hadronic interactions at low energies. In addition, hadronic weak interactions have proven more difficult to study experimentally due to the added complexity of unavoidable strong or electromagnetic interactions in such reactions. A valuable insight in the study of these interactions is that the maximal parity violation in weak interactions is an indicator of weak physics. The $n^3\text{He}$ experiment is one of a several experiments attempting to measure weak signals in hadronic interactions to more accurately characterize the weak NN interaction.

2.3 Weak NN Interaction

The theory of quantum chromodynamics (QCD) has provided a complete description of couplings between quarks and gluons, and the precision of calculations of weak interactions between hadrons is steadily improving. It is difficult to calculate details of low-energy nuclear interactions, at a comparable precision with experimental results, directly from QCD, as calculations at this scale are non-perturbative. However, we can use effective degrees of freedom to describe the interaction. Although baryons are composite particles, nucleons interact with each other at an energy scale much lower than their constituent quarks, which gives them complex dynamics that are modeled independently of QCD. Because the nucleon-nucleon force is the result of pure QCD interactions at the quark level, it can be thought of as the "residual strong force" at low energies.

The weak interaction between quarks is described by the electroweak model, where the weak force is carried by vector bosons. A rough estimate of the relative interaction strength of the weak and strong nuclear interactions at low energies can be made by comparing two diagrams representing parity-conserving meson exchange and parity-violating meson exchange (Figure 2.1). At nuclear interaction energies, the ratio of the parity-violating nucleon-nucleus potential to the parity-conserving one is proportional to the square of the masses of the mediating particles; here, the pion mass is m_π and the W-boson mass is m_W :

$$\frac{m_\pi^2}{m_W^2} \approx 10^{-6} \quad (2.6)$$

Another estimate of the size of the signal we are looking for was made by Deplanques [6]:

$$\frac{G_F \rho_N \frac{p_F}{M}}{U_0} \approx 10^{-7} \quad (2.7)$$

Here G_F is the Fermi constant, ρ_N is the nuclear density, $\frac{p_F}{M}$ is the velocity of a nucleon at the Fermi level, and U_0 is the strength of the strong nucleon-nucleus interaction. The scale of these estimates presents an obvious difficulty in observing such small differences in the strength of the nucleon-nucleon interaction. However, the weak force does not conserve parity, while the strong force does. This is a unique experimental opportunity to probe the strength of the hadronic weak interaction between nucleons.

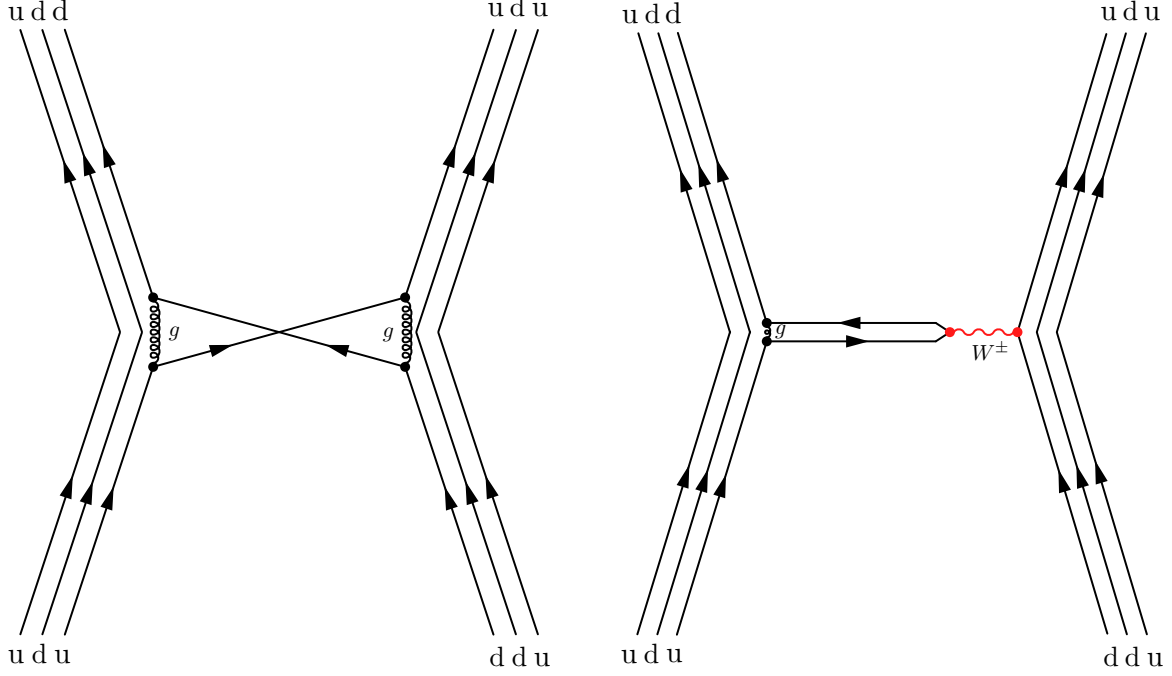


Figure 2.1: Examples of strong-strong and strong-weak meson exchange.

2.3.1 Desplanques, Donoghue, and Holstein meson-exchange model

The Yukawa potential provided an early attempt to describe the interaction between hadrons long before the development of quantum chromodynamics [7]. This nuclear potential models the nucleon-nucleon interaction with a meson as the exchange particle, and assumes that the strength of the potential decays as an exponential:

$$V(\vec{r}) = g \frac{hc}{\vec{r}} e^{-\frac{r}{r_0}} \quad (2.8)$$

Here, g is a dimensionless coupling constant that describes the strength of the interaction, r is the distance between the interacting nucleons, and r_0 is the Compton wavelength of the exchange meson. This early model does reproduce some of the qualitative features of the nuclear potential, since at low energies, the nucleon-nucleon interaction can be effectively modeled by meson exchange. A more complex model can be built from a linear combination of multiple exchange mesons,

$$V(\vec{r}) = \sum_i g_i \frac{hc}{\vec{r}} e^{-\mu_i r}, \quad (2.9)$$

where $\mu_i = \frac{m_i c}{\hbar}$.

A comprehensive model of weak meson exchange was developed by Desplanques, Donoghue, and Holstein (DDH) based on the quark model and SU(6) symmetry [2]. This model describes low-energy nucleon-nucleon interactions in terms of the exchange of light mesons, π , ρ , and ω . The interaction mechanism is constructed from one strong nucleon-meson vertex and one weak interaction vertex (such as the diagram shown on the right side of Figure 2.1). This is interpreted as a parity-conserving meson vertex combined with a parity-violating decay into a weak boson. This model estimates interactions which are at linear order in the weak coupling strength. The size of the interaction is then determined from the DDH Hamiltonian, which can be summarized as a sum of the interaction terms weighted by the model's meson-nucleon coupling parameters, $h_\mu^{\Delta I}$:

$$V_{PV}^{DDH} = h_\pi^1 V_\pi^1 + h_\rho^0 V_\rho^0 + h_\rho^1 V_\rho^1 + h_\pi^2 V_\pi^2 + h_\omega^2 V_\omega^2 + h_\omega^1 V_\omega^1 \quad (2.10)$$

The authors also provided suggested values for the couplings (Table 2.1), although the range of these values and their best estimates have been adjusted as more experimental tests have been performed. In this model, the hadronic weak potential has six nonzero isospin-dependent meson-nucleon couplings: h_π^1 , h_ρ^0 , h_ρ^1 , h_ρ^2 , h_ω^0 , h_ω^1 , corresponding to the exchange of a π , ρ , or ω with a specific isospin transition defined by symmetry. The couplings must be determined through experimental measurement. The values of the reaction-specific terms in the DDH Hamiltonian, $V_\mu^{\Delta I}$ depend on the spin, isospin, and range of the specific interactions, and can be calculated from nucleon-nucleon potentials. However, these values can differ depending in the specific nucleon-nucleon potential used, and the precision can vary depending on the complexity of the system. Several experimental measurements are needed in order to constrain the values of all $h_\mu^{\Delta I}$, and to do this accurately, the choice of model should be consistent among the measurements being combined.

Table 2.1: Coupling values suggested by the authors. Values given in units of $g_\pi = 3.8 \cdot 10^{-8}$.

Coupling	DDH range	DDH best value
h_π^1	0 – 11	4.6
h_ρ^0	-31 – 11	-11
h_ρ^1	-0.4 – 0	-0.2
h_ρ^2	-11 – -7.6	-9.5
h_ω^0	-10.3 – 5.7	-1.9
h_ω^1	-1.9 – -0.8	-1.2

Therefore, it is desirable to conduct a set experiments for which the coefficients can be calculated with the most accurate method. Using few-body systems, where $A \leq 4$, allows these coefficients to be calculated without using mean-field approximations. So by conducting few-body experiments to measure different linear combinations of the coupling constants, we can give the best available estimates of their values. Previous sensitive experimental tests of the hadronic weak interaction in few-body systems include proton-proton scattering at 220 MeV and 45 MeV, the $n + p \rightarrow d + \gamma$ experiment, neutron-alpha scattering $n + \alpha \rightarrow n + \alpha$, and proton-alpha scattering $p + \alpha \rightarrow p + \alpha$. In addition, heavier nuclei experiments measuring the same couplings include the decay of ^{18}F and ^{19}F . The sensitivity of each of these reactions to the relevant meson-nucleon couplings is given in Table 2.2.

The DDH model can be used to perform calculations of the parity-violating obserables in terms of the meson-nucleon couplings for the four-body system [8]. The size of the parity violation in a given experiment where the isospin mixing states are determined then gives a measurement of the weak meson-nucleon couplings for that reaction [8][9][10]. The eventual goal is to have a set of experimental data from few body systems which can be analyzed together to solve most accurately for the values of the couplings, $h_\mu^{\Delta I}$. The analysis of the data from $n + p \rightarrow d + \gamma$ can be found in [11], and there are some existing reviews which consider data from many body systems. For example, in 2013, Haxton and Holstein looked at data from $p + p$, $p + \alpha$, the circular polarization of photons from ^{18}F [12][13][14], and the photon asymmetry in the decay of ^{19}F , and compared the values of the couplings in the isoscalar and isovector dimensions (Figure 2.2). The pattern of experimental coefficients before that time suggested that the isoscalar and isovector interactions dominated most of the NN observables. Based on this experimental data, the best value of h_π^1 is $1.3 \cdot 10^{-7}$. [10]

Table 2.2: The coefficients $V_\mu^{\Delta I}$, from the DDH Hamiltonian, for selected reactions.

Process	Observable	V_π^1	V_ρ^0	V_ρ^1	V_ρ^2	V_ω^0	V_ω^1	
$\vec{n} + {}^3\text{He} \rightarrow p + t$ ¹	A_p	-0.189	-0.036	0.019	-.001	-0.033	.041	
$\vec{n} + p \rightarrow d + \gamma$	A_γ	-0.107	0	-0.001	0	0	0.004	[11]
$\vec{p} + p \rightarrow p + p$ (45 MeV)	A_p	0	0.104	0.104	0.042	0.138	0.138	[15]
$\vec{p} + {}^4\text{He} \rightarrow p + {}^4\text{He}$	A_L	-0.55	0.113	0.048	0	0.091	0.078	[16]
$\vec{n} + d \rightarrow {}^3\text{H} + \gamma$	A_γ	0.69	-0.33	0.99	0.05	-0.22	0.05	[17]
$\vec{n} + p \rightarrow n + p$	P_γ	-3.12	-0.23	0	-0.25	-0.23	0	[14]
$\vec{n} + {}^4\text{He} \rightarrow n + {}^4\text{He}$	ϕ_{pv}	-0.968	-0.199	0.085	0	-.160	0.137	[18]

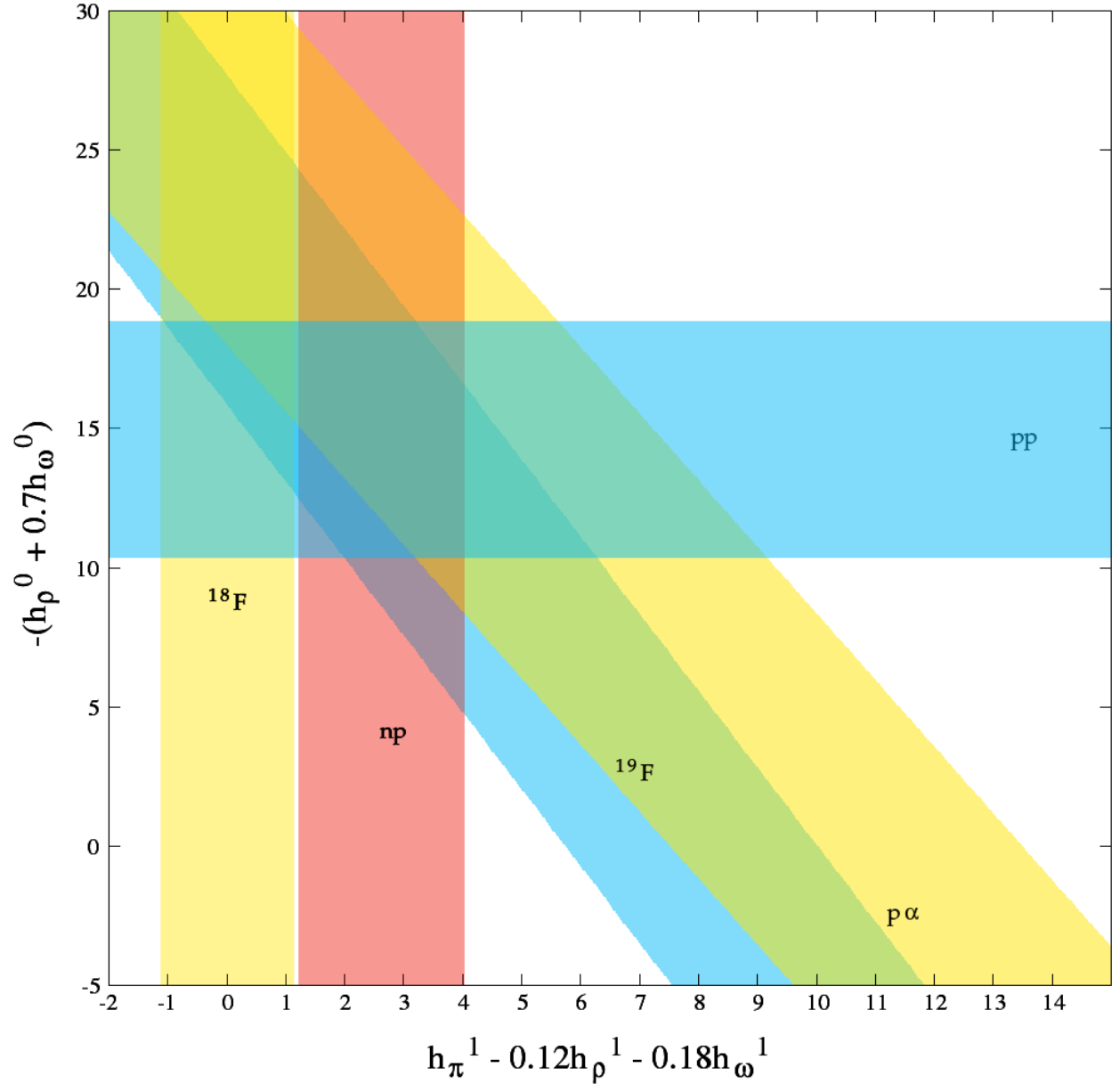


Figure 2.2: Values of the coupling coefficients from selected experiments. [10]

2.3.2 Effective Field Theories

Using the DDH framework to analyze hadronic systems comes with the model limitations of the meson-exchange mechanism [19]. A more modernized approach to treating the NN interaction is through effective field theories (EFT), which expand the interaction terms in the QCD Lagrangian as a function of $\frac{1}{\Lambda}$, where Λ is the momentum scale [20]. This expansion is truncated when the appropriate level of precision is reached for the energy regime of interest, and the coefficients of the different terms are called low-energy constants (LECs). These theories allow for perturbative analysis in parameters other than the strong coupling constant [21]. The LECs, like the DDH couplings, are not known a priori, but must be measured through experiment or calculated by future computations. EFTs may include or exclude the pions as degrees of freedom, depending on where the expansion of Lagrangian is truncated. Theories which "integrate out" pions, and heavier particles, are called "pionless" effective field theories, use contact terms to construct the NN interaction, and can be useful for treating low-energy interactions such as $n+{}^3\text{He}$. At leading order, parity violation in pionless EFT can be described in terms of five operators with five associated low-energy constants: Λ_0^0 , Λ_0^1 , Λ_1^0 , Λ_1^1 , and Λ_2^1 , which can be shown to correspond in linear combination with the DDH couplings. [1]

Recently, another analysis technique has been used to approximate the QCD Lagrangian at low energies, using the expansion of the QCD Lagrangian in terms of the $\frac{1}{N_c}$ parameter, the number of colors allowed in QCD [22]. From a Hartree expansion of the nuclear Hamiltonian in the large- N_c limit [23], twelve operators are derived: two at leading order (LO), four at NLO, and six at NNLO. A study of this technique suggested that this interpretation of the parity-violating NN interaction may better explain observed values of parity violation in reactions which are not dominated by the two leading order terms in the $\frac{1}{N_c}$ expansion [24]. This technique also suggests a future convention for comparing parity-violating measurements, namely, to use the variables Λ_0^+ and $\Lambda_2^{1S_0-3P_0}$ (Table 2.3). The $n^3\text{He}$ measurement is significantly sensitive to one of the two leading terms in this expansion, Λ_0^+ . Given the scarcity of both experimental measurements, and lattice calculations of, these asymmetries, the high-precision nuclear asymmetry measurement made in $n^3\text{He}$ is a valuable addition to the existing data set available to analyze the parity-violating NN interaction.

Table 2.3: Low-energy constants in terms of the DDH variables [1].

Low-energy constant	DDH variable	Large- N_c proportionality
$\Lambda_2^{1S_0-3P_0}$	$-g_\rho h_\rho^2(2 + \chi_\rho)$	$\sim N_c$
$\Lambda_0^+ = \frac{3}{4}\Lambda_0^{3S_1-1P_1} + \frac{1}{4}\Lambda_0^{1S_0-3P_0}$	$-g_\rho h_\rho^0(\frac{1}{2} + \frac{5}{2}\chi_\rho) - g_\omega h_\omega^0(\frac{1}{2} - \frac{1}{2}\chi_\omega)$	$\sim N_c$
$\Lambda_0^- = \frac{1}{4}\Lambda_0^{3S_1-1P_1} - \frac{3}{4}\Lambda_0^{1S_0-3P_0}$	$g_\omega h_\omega^0(\frac{3}{2} + \chi_\omega) + \frac{3}{2}g_\rho h_\rho^0$	$\sim \frac{1}{N_c}$
$\Lambda_1^{1S_0-3P_0}$	$-g_\rho h_\rho^1(2 + \chi_\rho) - g_\omega h_\omega^1(2 + \chi_\omega)$	$\sim \sin^2\theta_W$
$\Lambda_1^{3S_1-3P_1}$	$\frac{1}{\sqrt{2}}g_{\pi NN}h_\pi^1(\frac{m_\rho}{m_\pi})^2 + g_\rho(h_\rho^1 - h_\rho^{1'}) - g_\omega h_\omega^1$	$\sim \sin^2\theta_W$

2.4 Parity Violation in $n+^3\text{He}\rightarrow p+t$

The $n^3\text{He}$ experiment follows other experiments chosen to measure different reactions with alternate combinations of parity-violating couplings. Few-body systems are chosen in order to form a self-consistent description of the weak force between nucleons using exactly-calculable systems. In this experiment, the spin-dependent proton asymmetry in the reaction $n+^3\text{He}\rightarrow p+t$ is measured. The leading terms of the differential cross-section are:

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{4\pi} [1 + \alpha_{pv}(\vec{\sigma} \cdot \vec{k}_p) + \alpha_{pc}(\vec{\sigma} \times \vec{k}_n) \cdot \vec{k}_p] \quad (2.11)$$

Here, $\vec{\sigma}$ is the spin of the neutron. All neutrons in a pulse share the same spin state, up to the polarization efficiency discussed in section 5.3.5. The vectors $\vec{k}_n$ and $\vec{k}_p$ are the momentum of the neutron and the outgoing proton. Let's examine the coefficients of the parity-violating and parity-conserving terms. The parity-violating term, which is pseudoscalar, is proportional to the angle between the neutron spin and proton momentum:

$$\alpha_{pv}(\vec{\sigma} \cdot \vec{k}_p) \propto \alpha_{pv} \cos \theta \quad (2.12)$$

The spin state $\vec{\sigma}$ will be aligned along $\pm\hat{y}$, and the momentum vector of the outgoing proton can be oriented in any direction. So, the angle θ is the angle between the proton momentum and $\pm\hat{y}$. This means that our parity-violating cross-section will have a spherical polar geometry, where reactions more parallel to $\pm\hat{y}$ will have the highest sensitivity to parity violation. Measuring such an effect with a rectangular detector geometry requires a precise alignment of the detector to the magnetic field and accurately calculated geometry factors.

The parity-conserving term can also be analyzed, noting that the cross product $\vec{\sigma} \times \vec{k}_n$ will always point along $\pm \hat{\mathbf{x}}$:

$$\alpha_{pc}(\vec{\sigma} \times \vec{k}_n) \cdot \vec{k}_p \propto \alpha_{pv} \cos \theta' \quad (2.13)$$

Here, θ' is the angle with respect to $\hat{\mathbf{x}}$, not $\hat{\mathbf{y}}$. During the measurement of the parity-conserving asymmetry, the detector is rotated by $\frac{\pi}{2}$, which means it is sensitive to asymmetries in the $\hat{\mathbf{x}}$ -direction. The two detector orientations can be seen in Figure 2.3.

The value of the parity-violating asymmetry in the $n^3\text{He}$ experiment is calculated to have this dependence on the DDH couplings [8]:

$$\alpha_{pv} = -0.189h_\pi^1 - 0.036h_\rho^0 - 0.019h_\rho^1 - 0.001h_\rho^2 - 0.033h_\omega^0 + 0.041h_\omega^1 \quad (2.14)$$

This asymmetry is most sensitive to h_π^1 . The NPDGamma experiment [11] conducted a precise measurement of the value of this coupling:

$$h_\pi^1 = [2.6 \pm 1.2(stat) \pm 0.2(sys)] \cdot 10^{-7} \quad (2.15)$$

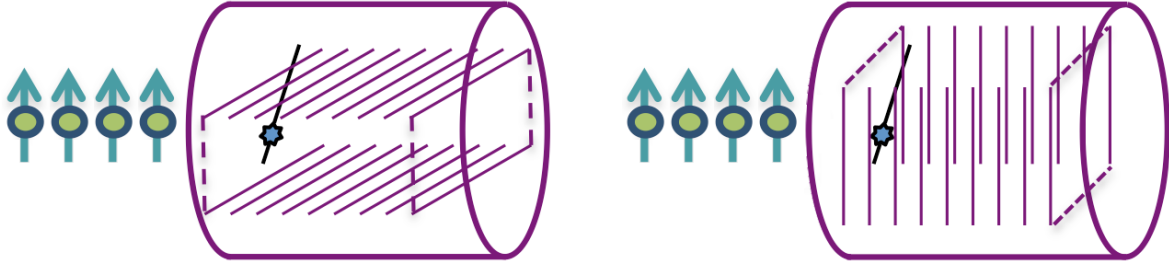


Figure 2.3: Detector orientations sensitive to the $\hat{\mathbf{y}}$ (left) and $\hat{\mathbf{x}}$ (right) directions.

3 EXPERIMENTAL STRUCTURE

The goal of the $n+{}^3\text{He}\rightarrow p+t$ experiment is to measure the size of the proton asymmetry, α_{pv} , with a precision of $1\cdot 10^{-8}$. The asymmetry arises from the parity-violating correlation $(\vec{\sigma}_n\cdot\vec{k}_p)$. A secondary goal is to measure the parity-conserving asymmetry, α_{pc} , which arises from the parity-conserving correlation $(\vec{\sigma}\times\vec{k}_n)\cdot\vec{k}_p$. This is measured in the anisotropic emission distribution of protons in the ionization detector in two perpendicular orientations. This experiment is located at the Spallation Neutron Source at Beamline 13. This facility allows for a high-sensitivity measurement due to the high flux of neutrons provided by the accelerator. The $n{}^3\text{He}$ apparatus allows these observables to be measured with highly polarized and spin-rotated neutrons. Polarized neutrons capture on ${}^3\text{He}$ and the resulting ionization of ${}^3\text{He}$, and correlation with the spin state, is recorded for analysis.

3.1 Spallation Neutron Source

The Spallation Neutron Source (SNS) is a neutron science facility located at Oak Ridge National Laboratory (Figure 3.1). The SNS accelerator complex delivers sub-microsecond pulses of 1 GeV protons at a time-averaged power of up to 1.4MW at 60 Hz. These proton pulses impinge on a circulating liquid mercury target, where each proton produces 25-30 neutrons through the spallation process. Neutrons lose energy in the moderators, made from liquid hydrogen and liquid water, surrounding the target. As the neutrons lose energy, they leak into 18 beam ports serving 24 beamlines. A reflector, made from beryllium, and shielding, made from steel, surround the target and moderator assembly, and redirect neutrons into the moderators for another chance at useful emission. Then, neutrons travel down the guide at Beamline 13 for use in the $n{}^3\text{He}$ experiment.

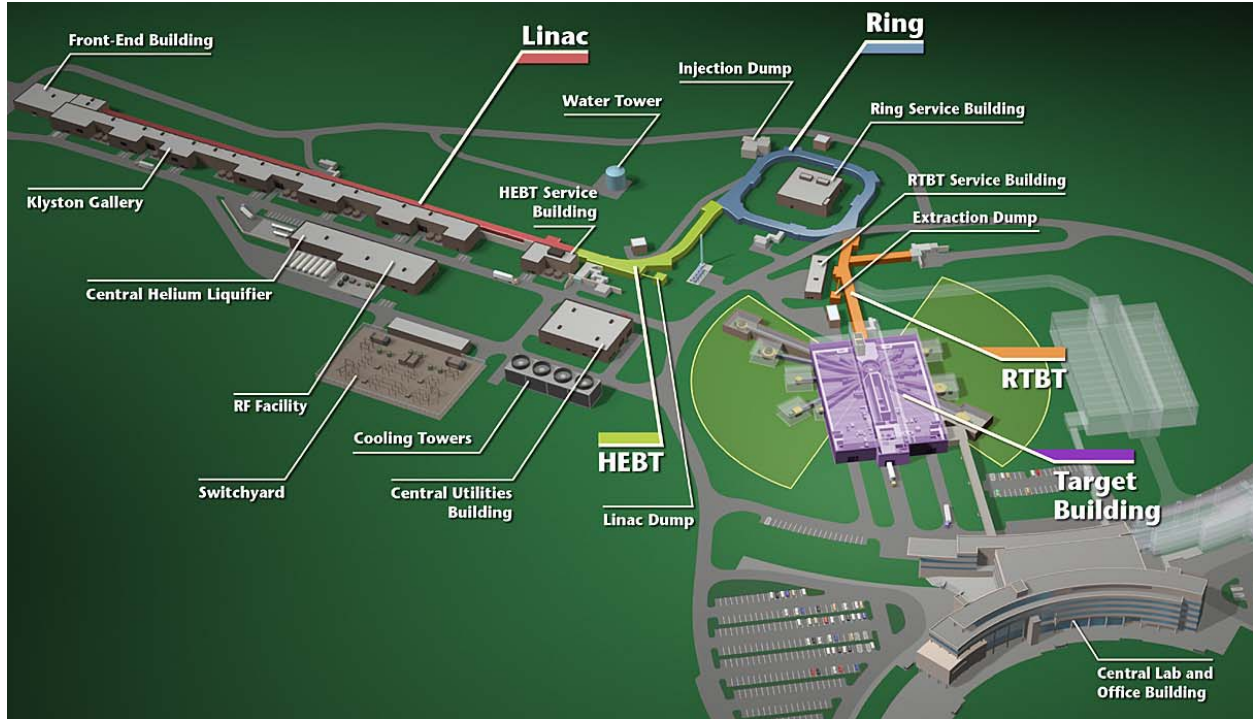


Figure 3.1: SNS site. Figure is from [25].

3.1.1 Proton Accelerator and Accumulator

Neutron production at the SNS begins with H^- ions, which are accelerated through room-temperature and superconducting accelerators to 1 GeV (Table 3.1). After the final linear accelerator stage, the H^- ions are passed through a thin diamond-like carbon foil [26], stripping the electrons. A 1ms series of 700ns bunches is injected into the accumulator ring, which is 248m in diameter, such that the protons traverse the circumference in $1\mu s$ (Figure 3.2). The ring accumulates around 1000 sets of bunches before the entire accumulated pulse is sent to the target as a single 700ns pulse.

At 60Hz, pulses of about $0.9 \cdot 10^{14}$ to $1.5 \cdot 10^{14}$ protons are sent to the spallation target, depending on the operating conditions of the accelerator. The pulse passes through the "Ring to Target Beam Transport" beamline, where the proton current is measured in one of several beam current monitors (BCMs). A plot of this current during 2015, taken from the decimated RTBT record, is shown in Figure 3.3. The average accelerator power during this time period ranged from around 850kW to 1.4MW. For diagnostic purposes, the accelerator regularly "drops" one proton pulse every 10s, or one out of every 600 pulses.

Table 3.1: Energies of the SNS linear accelerator stages [27].

Final Energy	Type	Length
65 keV	Ion Source	0.12m
2.5 MeV	Radio frequency quadrupole	3.76m
86.8 MeV	Drift tube	36.6m
186 MeV	Coupled-cavity	55.1m
387 MeV	Superconducting section 1	68.2m
1.0 GeV	Superconducting section 2	90.6m

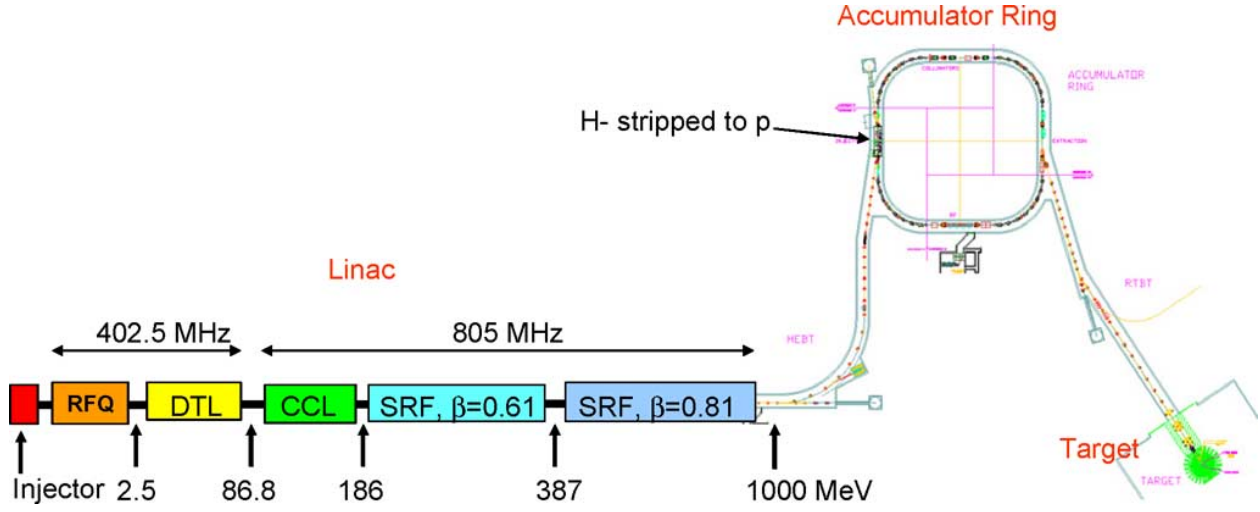


Figure 3.2: Staged linear accelerators and accumulator ring. Figure from [25].

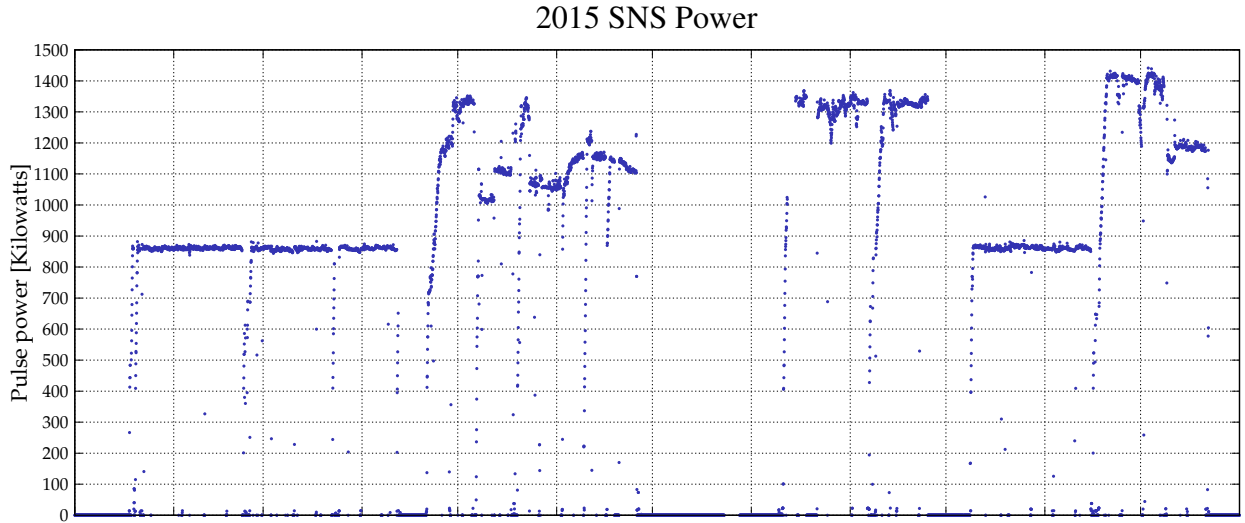


Figure 3.3: SNS average power over the 12 months of the $n+^3\text{He}$ experiment.

3.1.2 Liquid Mercury Spallation Target

The 1GeV protons are directed into a liquid Mercury target, where they spall and produce 20-30 neutrons each. The liquid mercury cycles through a steel containment vessel, dissipating the large amount of heat deposited into it [28]. Water channels around the flowing Mercury provide cooling. During the n^3He experiment, the Mercury target vessel had to be changed. The target designated MTM-002 was removed on October 2, 2015, and the target designated MTX-0095F was installed on October 8th, 2015. Neutron production was resumed at 850kW after the new target was installed.

Reflection and Moderation

Each proton pulse produces $\approx 3 \cdot 10^{15}$ neutrons through spallation, which scatter off of Iron and Beryllium reflectors surrounding the target, and are guided into one of the four H_2 and H_2O moderators [29], and can produce neutrons for up to 18 beamlines at once [30]. The n^3He experiment was performed on Beamline 13, which views a coupled cryogenic Hydrogen moderator at 20K [31] (see Figure 3.4). The time of flight from the moderator face to the n^3He apparatus, given the 2.5Å to 6Å energy range selected by the chopper system [32], covers a ≈ 14 ms time period. Once the neutrons enter the moderator, it takes them approximately 80 μ s to thermalize (Appendix A). Since the thermalization time is much smaller than the time of flight to the 3He target, the arrival time is highly correlated with the energy of the neutron.

3.1.3 Neutron Guide and Choppers

After exiting the bottom, downstream moderator, the neutrons enter the Beamline 13 guide. The neutron guide is composed of straight sections, with one curved section, with a radius of curvature of 117m, to remove a direct line of sight from the moderator to the target. The inner dimensions of the neutron guide are 10cmx12cm. The neutron guide is made out of material with an M-parameter of 3. The wavelength range selected by the choppers is shown in Figure 3.5. The beamline is also shielded by steel and concrete (Figure 3.6).

SNS Target Configuration

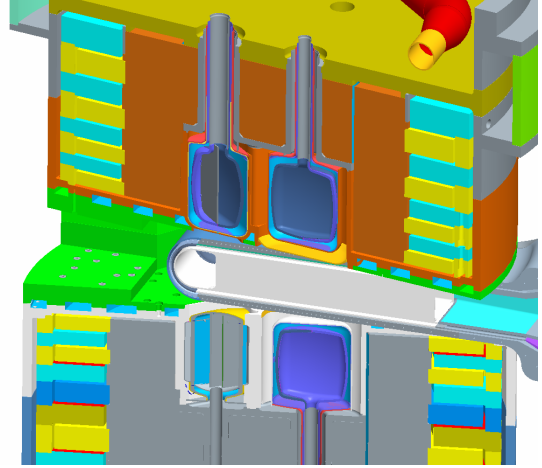
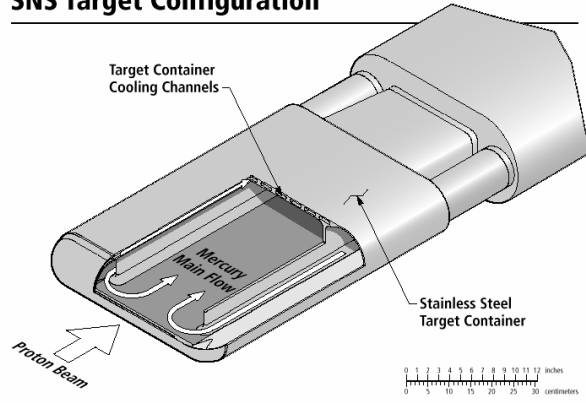


Figure 3.4: The liquid Hg target in its steel vessel, and surrounding moderators. Figures from [28].

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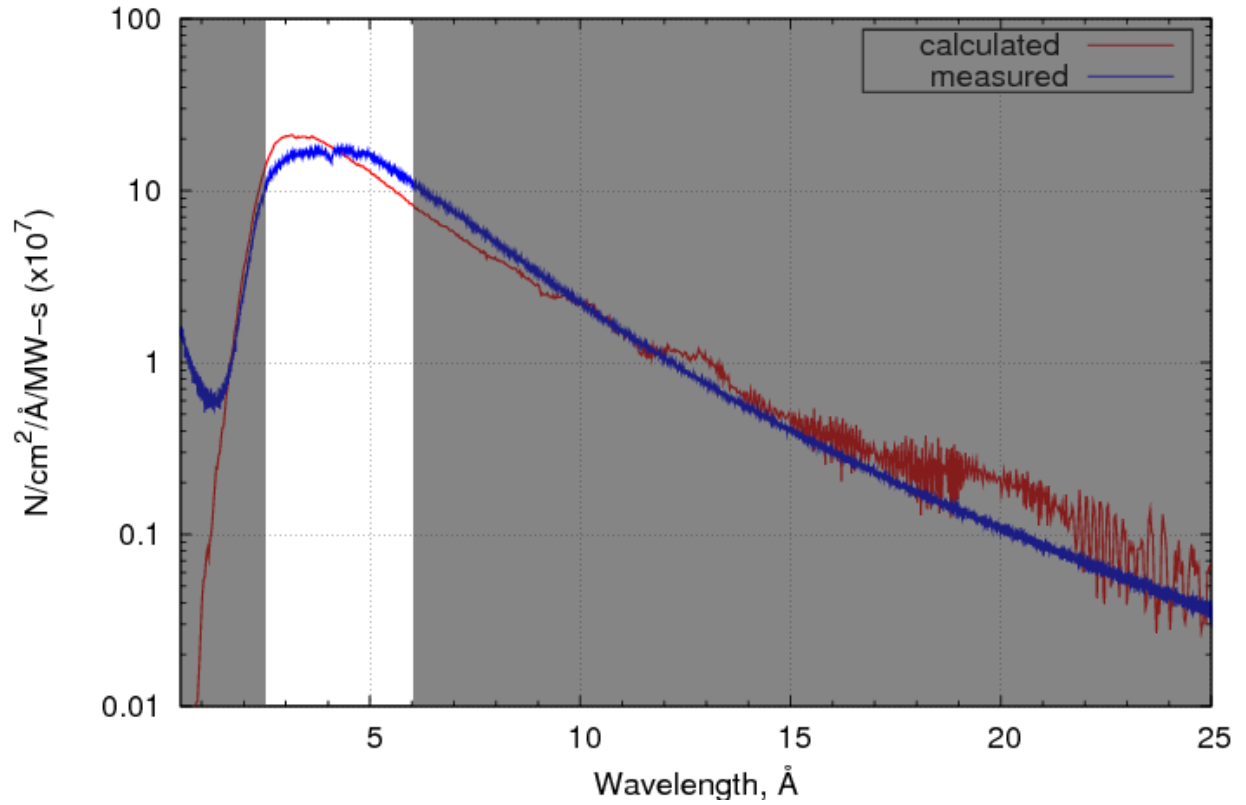


Figure 3.5: Calculated and measured neutron wavelengths at the end of the guide [32].



Figure 3.6: A view of the Beamline 13 radiological shielding.

Beamline 13 also has four choppers installed, two of which were active for the $n^3\text{He}$ experiment, located 5.5m and 7m from the moderator. The choppers are steel disks, rotating at 60Hz, with sectors cut out to permit passage of neutrons with desired timing characteristics. The outsides of each disk are coated in ^{10}B to absorb neutrons which do not arrive in the configured time window. In addition, the flux and wavelength of wrap around neutrons which are admitted through the choppers have to be considered when tuning them. A detailed report of the chopper tuning and configuration can be found in [33].

The $n^3\text{He}$ experiment is located at Beamline 13. Between the target and our experiment, the neutrons travel down a curved supermirror guide with parameter $M=3$ (see section 3.2.2 for the definition of M). Before they reach the experiment, the neutron pulses pass through two chopper disks which spin at 60Hz, and have been configured to select neutrons with energies between 2.3 meV and 13.2 meV in the region of the highest flux. The corresponding wavelength range is approximately 2.5-6Å.

3.2 $n^3\text{He}$ Apparatus

After spallation, neutrons travel down the FNPB beamline, reflect off the inside of the guide surface, pass through adjustable chopper points, and enter the experimental area. During 2015, this space was occupied by the $n^3\text{He}$ equipment (Figure 3.7).

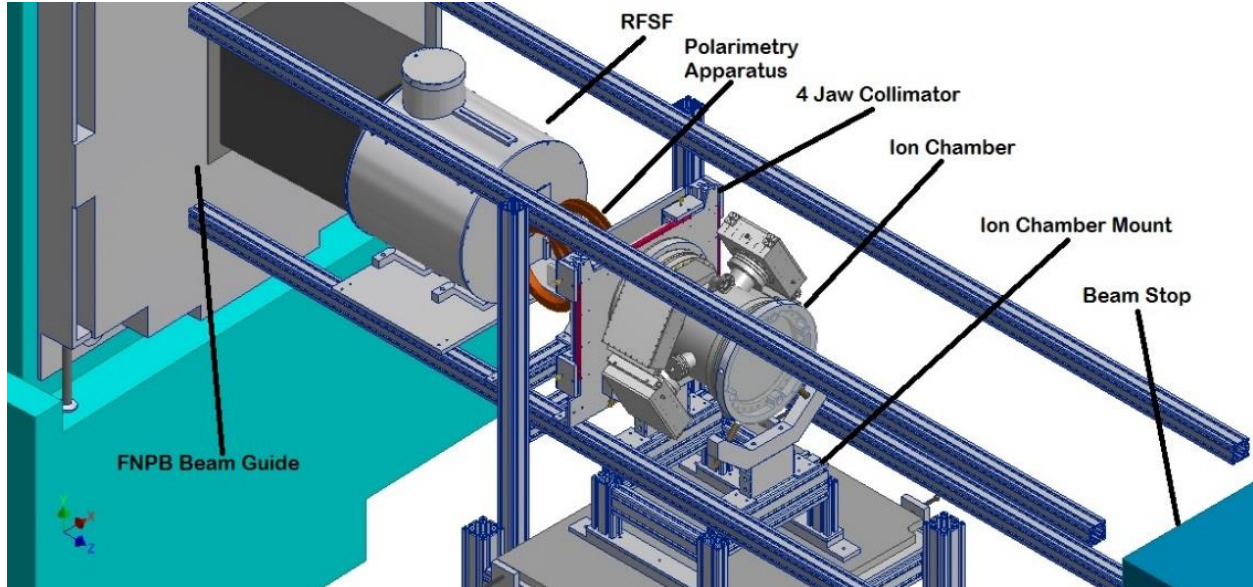


Figure 3.7: The $n^3\text{He}$ apparatus at the end of the neutron guide. Figure from [34].

3.2.1 ^3He Beam Monitor

At the end of the guide, before the neutrons enter the polarizer, they pass through a multiwire ionization chamber, labeled "M1" in other experiments at Beamline 13. This is used to observe the shape, timing, and intensity of the neutron pulses sent to the detector, quantities which are used to make cuts in the analysis of our data. M1 is filled with 98.5% N_2 , and the remaining 1.5% is ^3He [35]. There are three wire layers inside the gas: two outer layers at 1kV and one inner layer of signal wires. The wires are spaced 1.27cm apart. The neutrons enter and exit the gas mixture through 1mm Aluminum windows. This beam monitor is a low efficiency neutron monitor that absorbs only a few percent of the neutron beam. The beam intensity can be determined to a statistical error of 10^{-4} from this monitor.

3.2.2 Magnetic Holding Field

After exiting the polarizer, the collimated beam moves inside of a magnetic field. Four coils around the apparatus create a highly uniform 9.15G holding field which maintains the spin orientation of the neutrons while they travel to the ionization chamber. The uniformity is important because neutrons in different spin states will experience a different force from an inhomogeneous magnetic field. The magnetic field points along $\hat{y}$:

$$\mathbf{B}_0 = B_0 \hat{\mathbf{y}} \quad (3.1)$$

There are two energy eigenstates for a neutron in this magnetic field:

$$\hat{H}_0 = -\boldsymbol{\mu}_n \cdot \mathbf{B}_0 = -\gamma_n \mathbf{S} \cdot \mathbf{B}_0 = \gamma_n B_0 \frac{\hbar}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (3.2)$$

$$E_2 = \frac{\hbar}{2} \gamma_n B_0 \quad E_1 = -\frac{\hbar}{2} \gamma_n B_0 \quad (3.3)$$

Here, $\boldsymbol{\mu}_n = \gamma_n \mathbf{S}$ is the magnetic moment of the neutron. A value for μ_n can be found in [36].

3.2.3 Supermirror Polarizer

After the neutron pulse intensity is measured by M1, it enters the supermirror polarizer [37], where it is spin filtered and passed into the magnetic holding field [38]. The polarizer operates by selectively reflecting neutrons of one spin state inside of a strong magnetic field, due to their effective magnetic scattering length. In the case of this polarizer, the neutrons of the non-reflected spin state pass through the surface coating and are absorbed into the borated glass. The polarizer has a transmission efficiency of around 30%.

The polarizer is made up of 45 vanes of borated glass substrate, with a Ni-Ti supermirror coating, which will be explained below. The spacing between the layers is 0.3mm. The vanes are 40cm long, and have a radius of curvature of 14.8m [39]. The curvature allows the polarizer to selectively adjust neutron momentum. The polarized neutrons are deflected by 2.3° from their initial direction of motion. The whole stack of vanes is placed in a 350G magnetic field. We will first consider neutron reflectivity outside of a magnetic field. In free space with $V = 0$, the energy of the neutron plane wave is

$$-\frac{\hbar^2}{2m} \nabla^2 \psi = E \psi \quad E = \frac{\hbar^2 k^2}{2m} = \frac{2\hbar^2 \pi^2}{m \lambda^2}. \quad (3.4)$$

We can treat neutrons as waves governed by the Schrödinger equation [40]. To solve this equation inside the material, we must choose a form for the neutron-nuclear potential. For a neutron at position $\vec{r}$ inside a material, consider a potential of the form: [41]

$$V(\vec{r}) = \frac{2\pi\hbar^2}{m} \left\langle \sum_i b_i \delta(\vec{r} - \vec{R}_i) \right\rangle, \quad (3.5)$$

where b_i is the scattering length of the nucleus at R_j . Under our conditions, where the neutron wavelength is between 2.5Å and 6Å, and the lattice spacing is 4Å, we can integrate (3.5) over the volume of the crystal [42]:

$$V = \frac{2\pi\hbar^2}{m} b_n \rho, \quad (3.6)$$

where ρ is the number density of the nuclei and b_n is the nuclear scattering length of the material. The effective index of refraction is the ratio of the wavenumbers of the incident neutron plane wave and the spherically scattered wave:

$$n^2 = \left(\frac{k_0}{k} \right)^2 = 1 - \frac{V}{E} \quad (3.7)$$

Using the neutron optical potential (3.7) for V , we have:

$$n^2 = 1 - \frac{\lambda^2 \rho b_n}{\pi} \quad (3.8)$$

Our index of refraction is a function of the scattering length density of the material. For typical materials, $\lambda^2 \rho b_n$ is on the order of 10^{-5} (Table 3.2), so we can expand n^2 into a Taylor series to find n :

$$n = 1 - \frac{1}{2} \frac{\lambda^2 \rho b_n}{\pi} + \frac{1}{4} \left(\frac{\lambda^2 \rho b_n}{\pi} \right)^2 - \dots = 1 - \frac{\lambda^2 \rho b_n}{2\pi} \quad (3.9)$$

See [41] for a derivation of n . The neutron acts as a plane wave when crossing a boundary between two regions with different values of n , and it will undergo reflection. The reflectivity coefficient of a boundary between two regions depends on the values of n in each medium. For neutrons of a given wavelength, reflectivity will be enhanced in systems with a high density of nuclear scattering centers, and where the difference in scattering lengths is large. Consider a boundary between a vacuum, $n_m = 1$, and a region with neutron index $n_m < 1$.

Table 3.2: Densities and scattering lengths of selected materials. [43]

Element	ρ g/cc	b_n fm
Ni	8.8	10.6
Be	1.83	7.75
Fe	7.9	9.7
Pb	11.3	9.6
Al	2.7	3.45
Ti	4.45	-3.34

Similar to light crossing a boundary between media with differing indices of refraction, the neutron will experience refraction according to Snell's law,

$$n_0 \sin \theta_0 = n_m \sin \theta_m, \quad (3.10)$$

where θ_0 and θ_m are measured normal to the boundary. Since $n_m < n_0$, there will be a critical angle below which the neutron will be totally externally reflected from the surface of the material. A convention is define the critical angle as $\phi_c = \frac{\pi}{2} - \theta_m$. We can rewrite the refraction law in terms of the angles perpendicular to the normal, instead:

$$n_0 \cos \phi_0 = n_m \cos \phi_m \quad (3.11)$$

Since $n_0 = 1$, and we are interested in the case of total external reflection, this becomes:

$$\cos \phi_c = 1 - \frac{\lambda^2 \rho b_n}{2\pi} \quad (3.12)$$

The critical angle ϕ_c is small, so we can use the series expansion of cosine to solve for it:

$$1 - \frac{\phi_c^2}{2} \approx \cos \phi_c = 1 - \frac{\lambda^2 \rho b_n}{2\pi} \quad (3.13)$$

$$\phi_c = \lambda \sqrt{\frac{b_n \rho}{\pi}} \quad (3.14)$$

Note that this angle depends on the wavelength of the incident neutron: longer-wavelength neutrons are more easily reflected by a surface.

A useful convention for comparing reflecting surfaces of different types is to define the critical angle of reflective surfaces in units of the critical angle of external reflection between the vacuum and Nickel, which is

$$\phi'_c = \frac{1.73\text{mrad}}{\text{\AA}}\lambda \approx \frac{0.099^\circ}{\text{\AA}}\lambda. \quad (3.15)$$

This critical angle, $\frac{1.73\text{mrad}}{\text{\AA}}$, is used to define the M-parameter mentioned in section 3.13. M=3 means that the reflecting surface will reflect neutrons below the angle

$$3\phi'_c = \frac{5.19\text{mrad}}{\text{\AA}}\lambda. \quad (3.16)$$

The reflectivity coefficient drops below one for neutrons that do not meet the angular, or momentum, requirements for total external reflection. Neutrons which are not scattered by the first boundary will pass through the material. We can take another technique from classical optics and apply it to reflect those neutrons as well. Recall Bragg's law, the relation which describes maximal constructive interference of a wave scattering from periodic boundaries:

$$2d\sin\phi_B = n\lambda \quad (3.17)$$

If we have a stack of layers of alternating values of n , then the neutrons which meet the wavelength and angular criteria of (3.17) will constructively interfere and form a Bragg peak in the scattered neutron spectrum, at the wavelength λ and angle ϕ_B . This may be called a "multilayer mirror," and is useful to provide high reflectivity in a narrow range of angles and wavelengths. The Bragg condition applies for periodically spaced boundaries. A "supermirror" reflective surface is composed of a large number of layers, of alternating values of n , but of gradually decreasing thickness (Figure 3.8). At different depths in the material, the constructive interference condition applies to shorter wavelengths, producing overlapping Bragg peaks in the reflected neutrons. So, instead of a narrow range of wavelengths with a high reflectivity ratio, a supermirror produces constructive interference over a wider range of wavelengths, but with lower overall peak reflectivity.

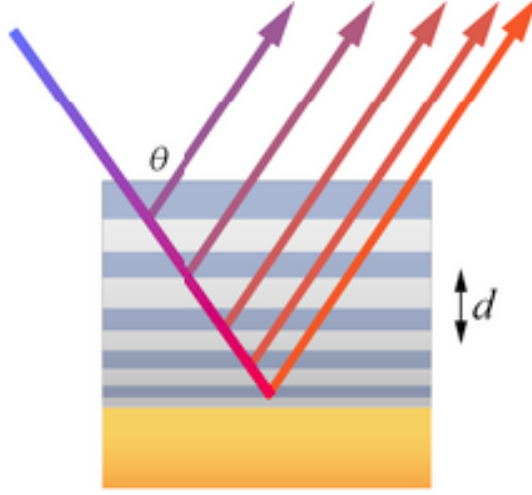


Figure 3.8: Supermirror layering which reflects a range of wavelengths.

Thus, for neutrons with a long enough wavelength relative to the atomic spacing of the medium, a regular supermirror will reflect neutrons over a range of wavelengths. Our supermirror polarizer also applies a strong magnetic field, which causes an energy splitting of the spin states due to its interaction with the magnetic moment of the neutron. Recall from (3.2) that:

$$V_m = -\boldsymbol{\mu}_n \cdot \mathbf{B} = \pm \gamma_n \frac{\hbar}{2} B \quad (3.18)$$

Thus, our Schrödinger equation must be adjusted to include a magnetic potential in addition to the nuclear potential:

$$V_{total} = \frac{2\pi\hbar^2}{m} b_n \rho \pm \gamma_n \frac{\hbar}{2} B \quad (3.19)$$

To illustrate the effect of the supermirror polarizer, let us write the spin-magnetic potential energy in terms of the "magnetic scattering length," b_m :

$$V_m = \pm \gamma_n \frac{\hbar}{2} B = \pm \frac{\pi\hbar^2 \rho}{m} b_m \quad b_m = \frac{\gamma_n m B}{2\pi\hbar\rho} \quad (3.20)$$

Then, our relation (3.9) simplifies to

$$n = 1 - \frac{\lambda^2 \rho}{2\pi} (b_n \pm b_m). \quad (3.21)$$

This expression makes it obvious that the magnetic field causes birefringence in the supermirror. Thus, for one of the coating layers, when the the magnetic field is tuned to

$$|B| = \frac{2\pi\hbar\rho b_n}{\gamma_n m}, \quad (3.22)$$

neutrons in the down state will have a much lower coefficient of reflectivity, they will not experience the same enhanced reflectivity as those in the spin-up state, and they will pass into the borated glass and be absorbed. Under these conditions, only the neutrons in the spin-up state will be reflected off the surfaces and pass through the supermirror polarizer. The polarization efficiency of this device is around 94% (Figure 3.9).

3.2.4 Radio Frequency Spin Rotator

The observation of a spin-dependent asymmetry requires that we observe neutrons in opposite spin states. The magnetic holding field keeps the incoming neutrons polarized in the $+\hat{y}$ direction. To construct asymmetries of the form in (4.38), we want to flip the spins of all neutrons in every other pulse, which means producing neutron beams with spin states alternating between $+\hat{y}$ and $-\hat{y}$. These configurations are shown in Figures 3.10 and 3.11.

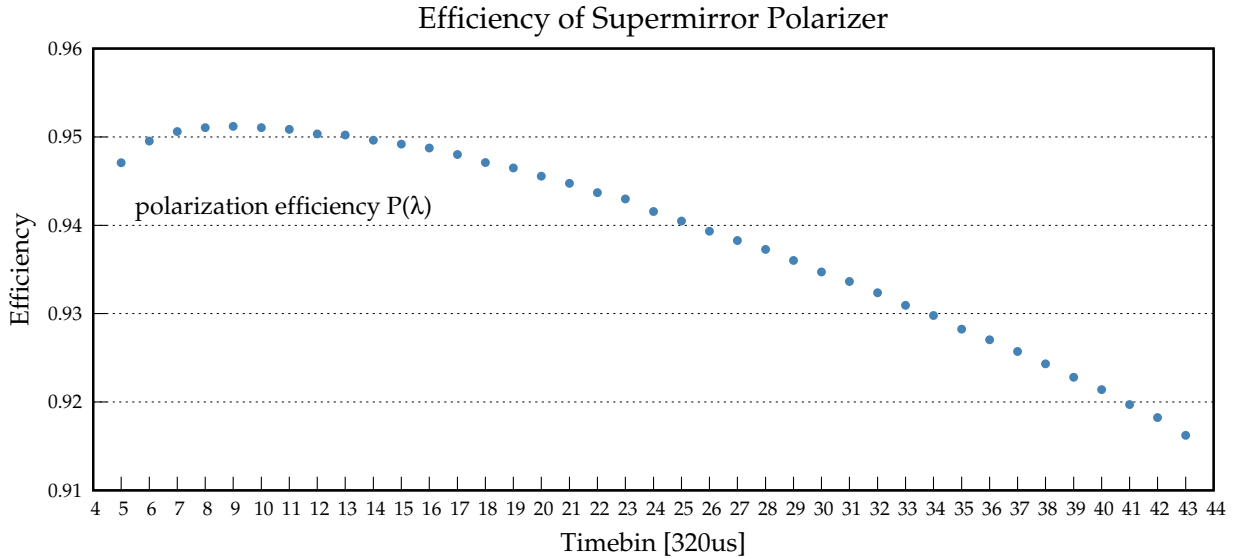


Figure 3.9: Polarization efficiency at the accepted timebins of the detector signals.

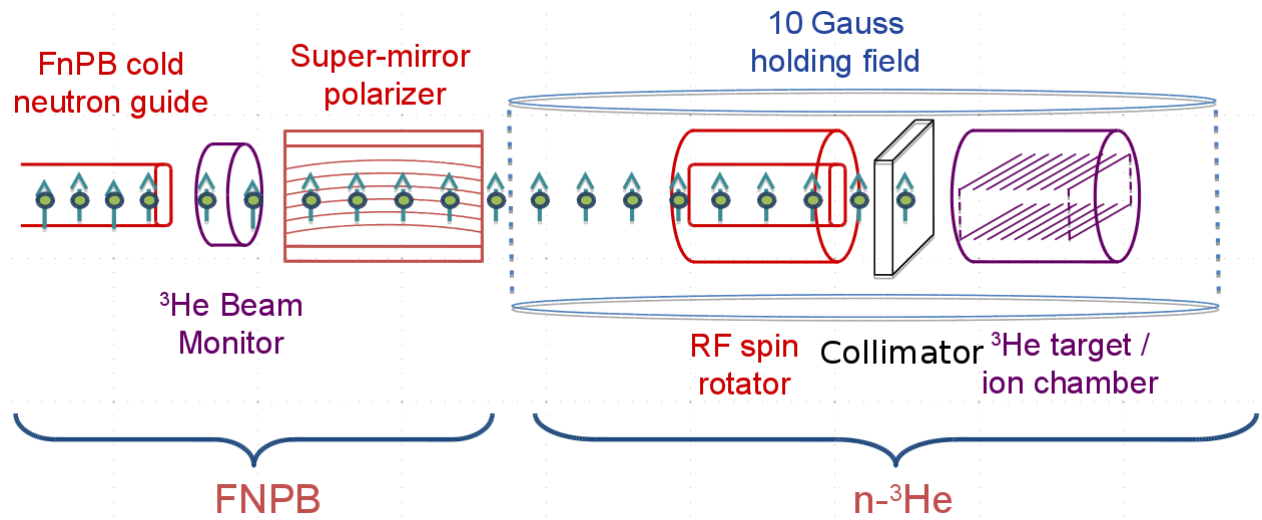


Figure 3.10: Spin configuration with spin rotator off.

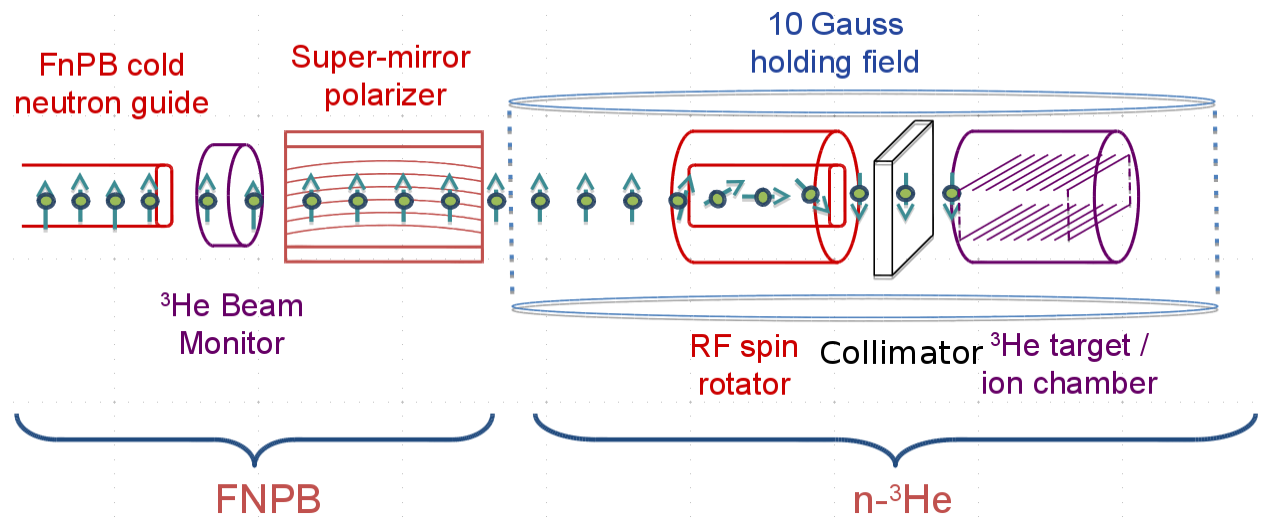


Figure 3.11: Spin configuration with spin rotator on.

3.2.5 Double Cosine-Theta Coil

The design chosen for the spin rotator in the $n^3\text{He}$ experiment is based on a double cosine-theta coil design [45]. Two of the main advantages of this design are a uniform field inside and zero field outside of the coils. The uniform field increases the spin flip efficiency of the device, and the cancelled field outside of the spin rotation volume decreases systematic error. The concept of the coil is based on the dipole field produced by a cosine theta coil, where the name refers to the current density on the surface of a cylinder:

$$k(\phi) = \frac{I}{2\pi r} \sin\phi \hat{\mathbf{z}} \quad (3.23)$$

The interior of such a cylinder has a magnetic field which is uniform in both magnitude and direction, although it may be varied in time. As we will show, this spatially uniform magnetic field can be used to produce the desired spin rotation, with some modifications. Rather than a sheet of current, the field is produced by individual wires placed on the inside and outside surfaces of a solid cylinder approximating the ideal surface current density (Figure 3.12).

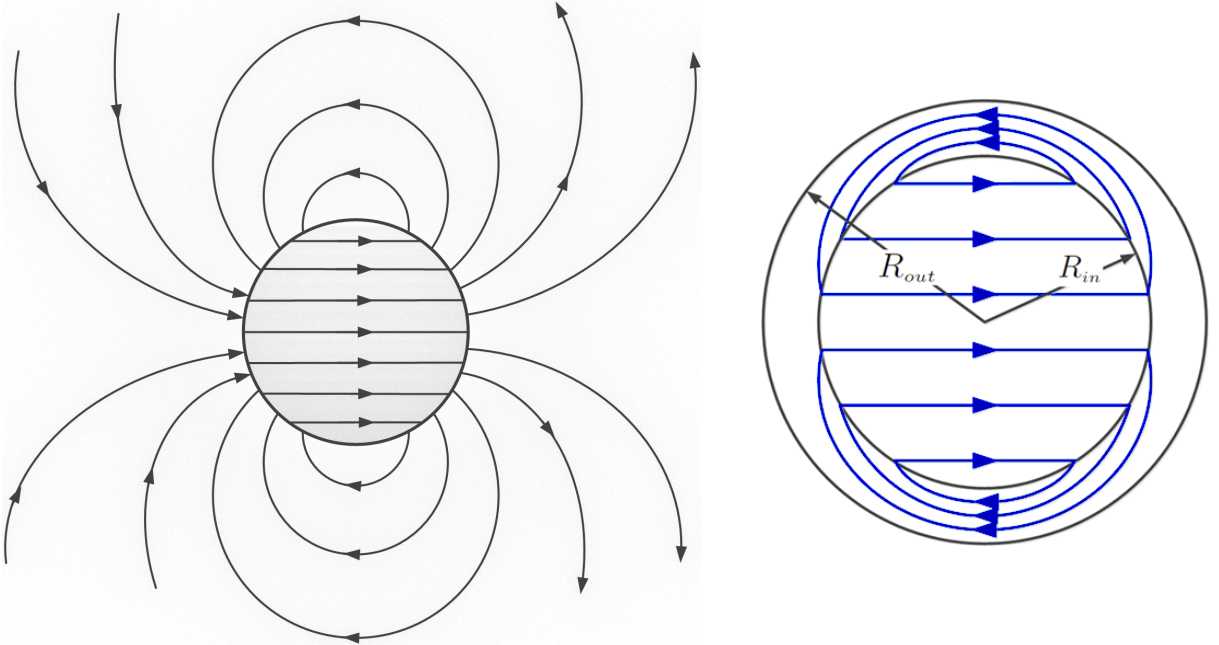


Figure 3.12: Left: Dipole field. Right: Double cosine theta field. Figures modified from [44].

The interior surface has a radius of 16.05cm, and has 320 wires running along it. The exterior surface has a radius of 20.72cm, and has 248 wires running along it. The coil is 39.62cm long.

Spin Precession in a Magnetic Field

This device utilizes the spin precession frequency of neutrons in a static magnetic field to induce controlled Rabi transitions, rotating them between opposite spin states, or spin orientations, in alternating pulses [46]. As discussed in section 3.2.2, after the polarized neutrons exit the supermirror polarizer in the $+\hat{\mathbf{z}}$ direction, their spin orientation is preserved by the magnetic holding field, $\mathbf{B}_0$. By applying the time evolution operator to this Hamiltonian, we observe that the spin orientation of neutrons in this magnetic field will precess about $\mathbf{B}_0$, which points along the y-axis [3], at the frequency ω_L :

$$\omega_L = \frac{\Delta E}{\hbar} = \gamma_n B_0 \quad (3.24)$$

ω_L is called the Larmor frequency. We can rewrite $\hat{H}_0$ as:

$$\hat{H}_0 = \frac{\hbar\omega_L}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (3.25)$$

Spin Rotation

On the left side of Figure 3.13, the two coaxial current cylinders are shown, and on the right side, the Aluminum casing is shown, which also functions as a radio-frequency shield. We want to rotate the spin orientation of the neutrons in alternating pulses from $+\hat{\mathbf{y}}$ to $-\hat{\mathbf{y}}$. We can achieve this by introducing to this system the conditions for Rabi oscillations. The spin rotator produces a second, time-dependent magnetic field, contained in its volume:

$$\mathbf{B}_1 = B_1(t) \hat{\mathbf{x}} \cos \omega t \quad (3.26)$$

Here, $\mathbf{B}_1 \perp \mathbf{B}_0$, and we will have $|\mathbf{B}_1| \ll |\mathbf{B}_0|$.

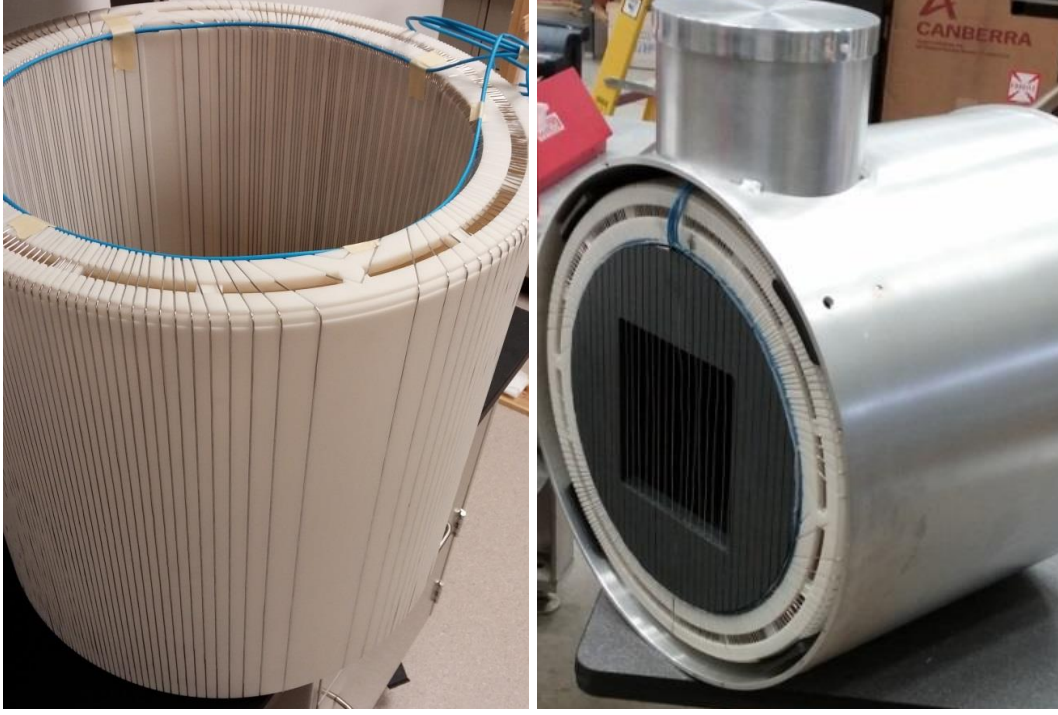


Figure 3.13: Current loops and case of the spin rotator [44].

The neutrons will see two different magnetic field regions:

$$\mathbf{B} = \begin{cases} B_0 \hat{\mathbf{y}} & \text{Exterior} \\ B_0 \hat{\mathbf{y}} + B_1(t) \hat{\mathbf{x}} \cos \omega t & \text{Interior} \end{cases} \quad (3.27)$$

Although $\mathbf{B}_1$ is not a rotating field, we can show that this magnetic field still produces Rabi oscillations in this system. First, rewrite $\mathbf{B}_1$ as the sum of two rotating fields:

$$\mathbf{B}_1 = B_1 \hat{\mathbf{x}} \cos \omega t = \frac{1}{2} [B_1(\hat{\mathbf{x}} \cos \omega t + \hat{\mathbf{z}} \sin \omega t) + B_1(\hat{\mathbf{x}} \cos(+\omega t) - \hat{\mathbf{z}} \sin(+\omega t))] \quad (3.28)$$

This is a superposition of two counter-rotating fields. We can also interpret the field with clockwise rotation as rotating counterclockwise, but with a negative frequency:

$$\mathbf{B}_1 = B_1 \hat{\mathbf{x}} \cos \omega t = \frac{1}{2} [B_1(\hat{\mathbf{x}} \cos \omega t + \hat{\mathbf{z}} \sin \omega t) + B_1(\hat{\mathbf{x}} \cos(-\omega t) + \hat{\mathbf{z}} \sin(-\omega t))] \quad (3.29)$$

Let us first determine the effect of just the field with positive frequency.

Inside the spin rotator volume, the Hamiltonian is:

$$\hat{H} = -\boldsymbol{\mu}_n \cdot \mathbf{B} = -\gamma_n \mathbf{S}_z \cdot \left[B_0 \hat{\mathbf{z}} + \frac{B_1}{2} (\hat{\mathbf{x}} \cos \omega t + \hat{\mathbf{y}} \sin \omega t) \right] = \gamma_n \frac{\hbar}{2} \begin{bmatrix} B_0 & \frac{B_1}{2} e^{i\omega t} \\ \frac{B_1}{2} e^{-i\omega t} & -B_0 \end{bmatrix} \quad (3.30)$$

We can identify a second frequency in the off-diagonal elements, and simplify:

$$\hat{H} = \frac{\hbar}{2} \begin{bmatrix} \omega_L & \omega_R e^{i\omega t} \\ \omega_R e^{-i\omega t} & -\omega_L \end{bmatrix} \quad \omega_R = \frac{\gamma_n B_1}{2} \quad (3.31)$$

In our system, all of the neutrons (up to the polarization efficiency) entering the spin rotator have the same spin orientation, and are thus all in the same energy eigenstate of the magnetic field, although their kinetic energies will vary. For a single neutron,

$$|S\rangle = c_+ |+\rangle + c_- |-\rangle, \quad (3.32)$$

where the state coefficients at $t = 0$ are $c_+ = 1$ and $c_- = 0$.

Under the conditions we have stated, this problem has a known solution. Rabi's formula gives the probability that a neutron will be found in the opposite eigenstate as a function of time, assuming that only one state was populated to begin with:

$$|c_-(t)|^2 = \frac{\omega_R^2}{\omega_R^2 + (\omega - \omega_L)^2} \sin^2 \left\{ \frac{t}{2} \left[\omega_R^2 + (\omega - \omega_L)^2 \right]^{\frac{1}{2}} \right\} \quad (3.33)$$

In order to achieve a total spin rotation, this probability amplitude must be equal to 1. This requires that $\omega = \omega_L$, the resonance condition. For our experiment,

$$\omega = \omega_L = 26.655 \text{ kHz}. \quad (3.34)$$

The spin rotation frequency can be identified from the argument of the sine function:

$$\Omega = \sqrt{\frac{\omega_R^2 + (\omega - \omega_L)^2}{4}} \quad (3.35)$$

At resonance, this reduces to:

$$\Omega = \frac{\omega_R}{2} \quad (3.36)$$

This means that the neutrons will experience a spin rotation of π , i.e., be flipped from being oriented along $+\hat{\mathbf{y}}$ to $-\hat{\mathbf{y}}$, and then back again, at regular timing intervals:

$$\frac{\omega_R}{2}t = (n + \frac{1}{2})\pi \quad (3.37)$$

We want the neutrons to exit $\mathbf{B}_1$ after a rotation of π , so we choose $n = 0$:

$$\omega_R = \frac{\pi}{t} \quad (3.38)$$

Since we require the angular rotation of each neutron to be fixed to π , this relation tells us the necessary magnitude of B_1 , oscillating at ω_L and perpendicular to the holding field, that will induce that rotation in time t . The size of the rotation depends on the velocity of the neutron, so we see that the desired field magnitude is a function of the length of time a neutron spends in the rotator volume:

$$|B_1| = \frac{2\pi}{\gamma_n t} \quad (3.39)$$

The time of flight inside the rotator varies with the neutron wavelength. We can calculate the neutron wavelength as a function of timebin. Then, the desired magnetic field is

$$B_1(t) = \frac{he^{i\omega_L t}}{\gamma_n z_0 m_n \lambda_n}, \quad (3.40)$$

where z_0 is the path length inside the rotator, 39.62cm. B_1 goes as $\frac{1}{t}$ (Figure 3.14). We have calculated the spin-rotation effect that the first rotating field, with a positive frequency, will have on the neutrons. Recall that our magnetic field decomposition (3.29) also included a second field, with a negative oscillation frequency. It is clear that the contribution from this component will be negligible if we calculate the amplitude and frequency of spin rotation arising from this field:

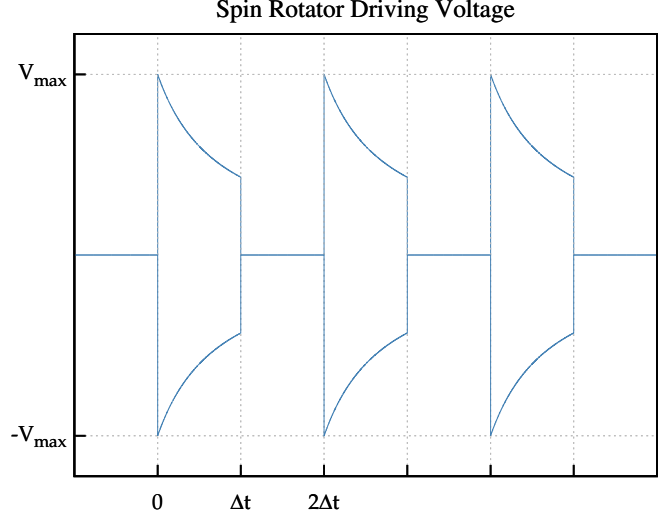
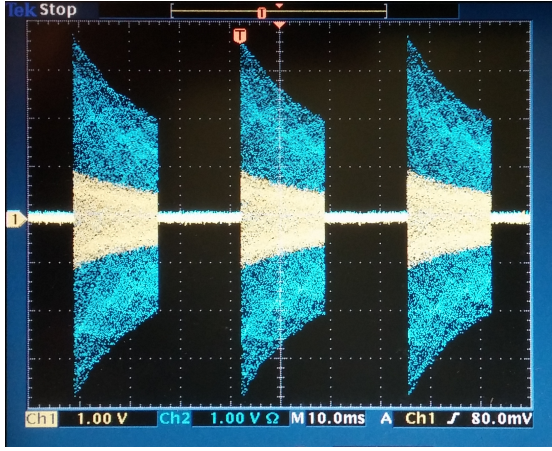


Figure 3.14: Envelope of the magnetic field produced in the spin rotator coil.

$$|c_-(t)|^2 = \frac{\omega_R^2}{\omega_R^2 + (\omega + \omega_L)^2} \sin^2 \left\{ \frac{t}{2} \left[\omega_R^2 + (\omega + \omega_L)^2 \right]^{\frac{1}{2}} \right\} \quad (3.41)$$

Note that a frequency of $-\omega_L$ is far off resonance, giving a transition amplitude of

$$\frac{\omega_R^2}{\omega_R^2 + (\omega + \omega_L)^2} = 1.1 \cdot 10^{-3}, \quad (3.42)$$

compared to unity for the on-resonance field. This transition also has a much higher oscillation frequency, ensuring that it does not interfere with the spin rotation caused by the on-resonance field. Note that the calculation (3.39) assumes that the time a neutron spends inside the rotator volume, t , is small enough that the magnitude of B_1 does not change significantly while the neutron is passing through it. A slower neutron will experience a larger change in the field magnitude, decreasing the precision of the spin rotation, and producing a lower polarization efficiency at that wavelength. However, we can show that this deviation is small and can be neglected. The magnitude of B_1 changes by roughly a factor of two over the pulse width, T . The approximate deviation in magnetic field strength that an average neutron will experience while passing through the rotator volume is:

$$\frac{\Delta B_1}{B_1} \approx \frac{1}{2} \frac{t}{T} \approx 1.2 \cdot 10^{-2} \quad (3.43)$$

3.2.6 Orientation and Precision

There are two measurable asymmetries arising from the capture of polarized neutrons on ^3He (equation 2.11). In order to measure both, the ionization chamber was designed to be rotated by 90° , from the "up-down" mode (which measures the parity-violating asymmetry), to "left-right" mode (which measures the parity-conserving asymmetry). The parity-conserving asymmetry is estimated to be on the order of $5 \cdot 10^{-7}$, and the precision goal of the ^3He experiment is 10^{-8} . To achieve appropriate statistical precision for each, approximately 150 days of data was taken in up-down mode, and 5 days of data was taken in left-right mode. To achieve this level of precision, both the spin rotator and the ionization chamber must be correctly aligned to a precision of 10^{-3} . This was achieved for the spin rotator, as well as for the front of the ionization chamber. However, the signal frames in the ionization chamber (section 3.2) are not all aligned with the front frame, and the variable frame alignment can introduce asymmetry mixing in those elements. This asymmetry mixing due to variable frame alignment is not corrected for in this analysis.

3.2.7 Collimator

After the neutron beam has been rotated to the desired spin state, it is collimated. The precision of the measurement will be increased with higher contrast in signal intensity arising from neutrons in opposite spin states. Collimation increases this contrast at the cost of statistical precision. The collimator is made of Cadmium and ^6Li -coated absorption material (Figure 3.15). It was built to be adjustable, both for diagnostic and optimization reasons. The more the beam is collimated, the lower the total flux, which reduces statistical precision, but increases the value of the off-center geometry factors. A tighter beam collimation leads to greater sensitivity to the asymmetry arising from a greater proportion of the measured yield coming from reactions which occur farther from the cell. The collimation parameter is one of the two major experimental parameters optimized by the simulation. Measurements of detector signals with different collimations were taken on November 30, 2015, and compared with simulated outputs of the detector under the same conditions.

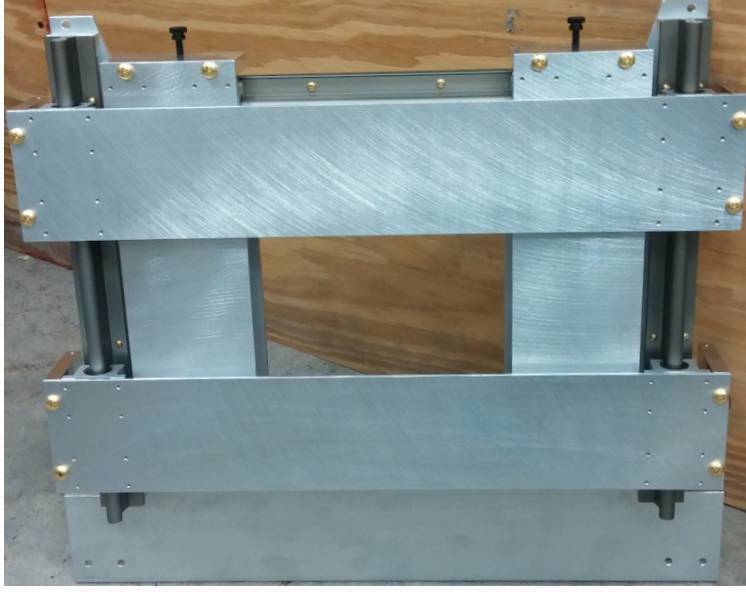


Figure 3.15: Collimator.

3.2.8 ^3He Ionization Chamber

The ionization chamber is filled with ^3He gas at 48.2kPa, which ionizes when the reaction products travel through it, and measures the distribution of ionization energy by the proton and triton emitted from the $n+^3\text{He}$ reaction. There are 16 signal frames with 9 signal wires each, for a total of 144 total signal wires. The ^3He is both the capture target of the experiment and the ionization gas. When the neutron is captured on the ^3He nucleus, it excites and then splits, at rest, into H^+ and $^3\text{H}^+$ [47]:



These emission products travel antiparallel to each other through the gas, for a combined length of approximately 15cm. From conservation of momentum, we know that the proton carries $\frac{3}{4}$ of the energy, or 574keV. The rate of energy loss due from the electronic stopping power of the gas is a function of the charge and velocity of the ions [48][49]. The proton will travel three times as far as the triton into the gas, and it will deposit most of its energy near the end of its track (see section 4.2.1). A view along $\hat{x}$ of an example reaction is shown in Figure 3.16.

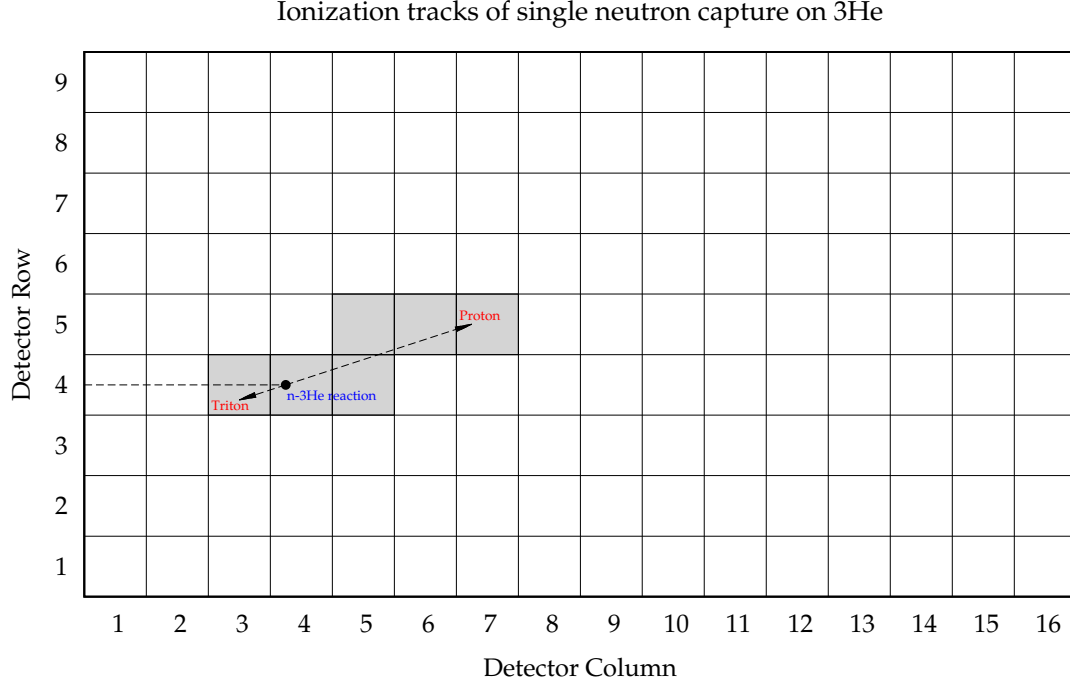


Figure 3.16: Scale of single neutron capture in the ionization chamber.

Both the signal and voltage wires are made of copper, and are 0.51mm in diameter. The vessel has an interior diameter of 25.4cm and is made from Aluminum. The length of the chamber is 33.8cm. The walls are 0.635cm thick, and have ports for power and the outputs, which included current to voltage amplifiers, which are mounted on the sides of the detector vessel [50]. The left side of Figure 3.16 shows the assembled frame stack, and the right side shows the stack enclosed in the Aluminum vessel.

The signal and voltage wires are embedded in alternating frames, which are stacked on top of each other. This "frame stack" is placed inside the vessel (Figure 3.17). There are 16 grounded signal wire frames with 9 signal wires in each of them, for a total of 144 channels. Alternating with signal frames are 17 voltage frames, which place high-voltage wires at -350V around the signal wires when stacked together. When fully assembled during the experiment, each signal wire is surrounded by four high-voltage wires. When the detector is operating to detect the parity-violating asymmetry, the wires are aligned along $\hat{x}$, and when the detector is operating to measure the parity-conserving asymmetry, the detector is rotated by $\frac{\pi}{2}$ and the wires are aligned along $\hat{y}$.

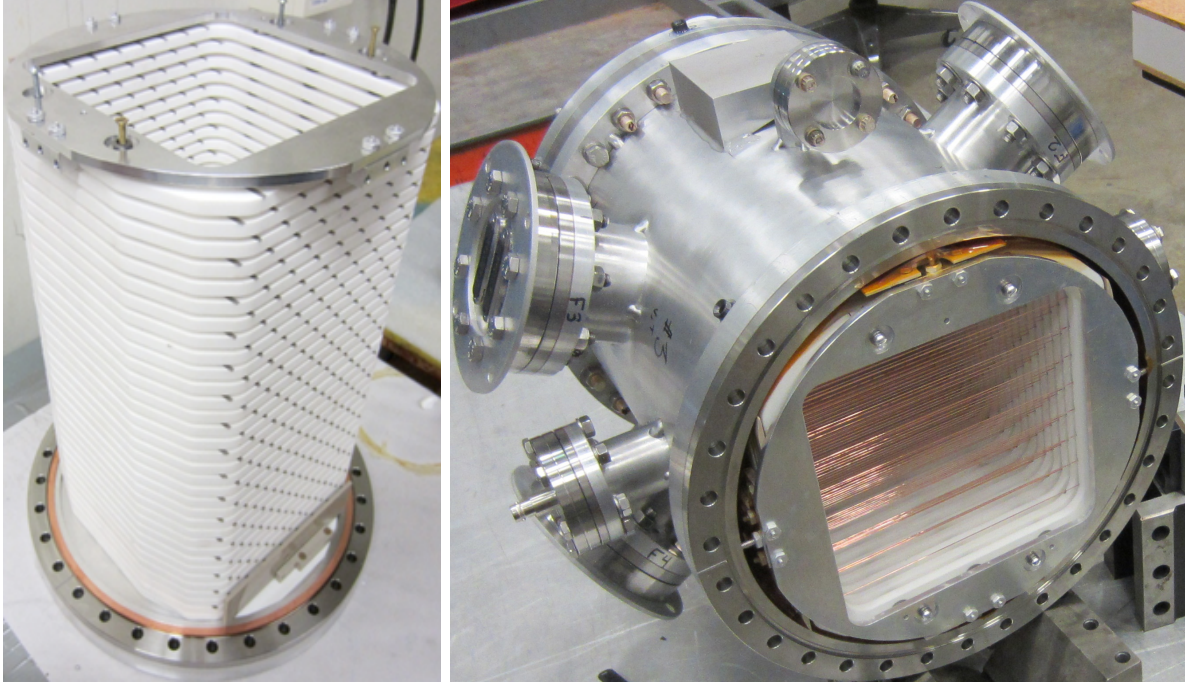


Figure 3.17: Vessel with frame stack in it.

The signal wires are spaced 1.9cm apart, making each signal wire a channel which collects charge from ionization in a 1.9cmx1.9cm "cell" running across the detector volume. Cells along the top and bottom of the chamber are only surrounded by two high-voltage wires; as a result, the simulation of the chamber is corrected with a volume adjustment for those channels. The fill gas pressure is 48.2kPa, a parameter which was optimized through simulation (see section 4.4.3). At this pressure, the mean proton travel length in the gas is six cell widths, which gives us an indication of the size of the necessary covariance correction. When a neutron capture occurs, the H^+ and ${}^3\text{H}^+$ ionize the gas along their trajectory, creating e^- and ${}^3\text{He}^+$ ions as they travel. These ions then travel along the electric field lines between the high-voltage and signal wires, producing a current proportional to the amount of ionization produced in each cell. The ${}^3\text{He}$ in the target vessel captures $\sim 10^8$ neutrons per pulse, so operating in current mode, each element of the detector measures an average yield, not individual capture events (equation 4.36). This current is amplified, converted to a voltage, and sampled by the DAQs and recorded for analysis. The details of the ionization and deposition are discussed in section 4.2. The detector signals are discussed in detail in Chapter 5.

3.2.9 DAQs

The beam monitor signal and the spin rotator current were sampled at 100kHz, and recorded into channels 0 and 1 in DAQ30, giving a resolution of 1624 samples per pulse, or a timebin width of $10\mu\text{s}$. The D-tAcq module type is shown in Figure 3.18. The outputs of the 144 signal wires in the ^3He detector were split up evenly among the remaining four DAQs, labeled 21-24, each recording 36 channels. The sampling rate for these DAQs was set to 50kHz, and they used a 16-point averaging algorithm, giving a resolution of 49 samples per pulse, or a timebin width of $320\mu\text{s}$. The channel map is listed in Table 5.1. All DAQs have a bit depth of 24. The ADC has voltage range of $\pm 10\text{V}$. So, the smallest detectable change in the input signal, or the precision of our detector, is

$$\epsilon = \frac{10\text{V}}{2^{24}} = 1.192\mu\text{V} \quad (3.45)$$

The four DAQs used to record the signals from the detector were shielded from the rest of the electronics of the experiment to reduce electronic noise and decrease any false asymmetry arising from a systematic electronic error. The electronic asymmetry was measured during maintenance days, when neutron production was paused. On those days, the spin rotator was operated normally and the asymmetry in the detector was measured. This is called the "instrumental asymmetry," and was measured to be less than 10^{-9} . The fifth DAQ, DAQ30, which records the beam monitor signals and the spin rotator current, was not shielded, since this DAQ does not record any detector data.



Figure 3.18: Three D-tAcq modules [51].

4 SIMULATION

In experimental physics, simulations have become an increasingly important tool in experimental design, verification of measurements, and analysis. Simulation has a central role in determining the validity and precision of final results. In this chapter, I describe the simulation I wrote to optimize variables in the $n^3\text{He}$ experiment and calculate geometry factors. All parts of this simulation were written in C++. I used the GNU Scientific Library [52] for several mathematical functions such as Fourier transforms, singular value and Cholesky matrix decompositions, Akima interpolation, Lanczos functions, and random number generation. I used ROOT for the histogram function. No other packages or programs (such as GEANT, MCNP, or McStas) were used in this work. The simulation was run on the University of Tennessee cluster Newton, where I used the parallel interface protocol OpenMP to coordinate data flow. The main simulation run was done on 64 nodes, where each instance simulated 2^{25} unique neutron events, for a total of $2.15 \cdot 10^9$ neutron events.

In order to calculate geometry factors, estimate the precision of the experiment, and optimize free parameters in the design, it is necessary to have a precise simulation of the wire chamber signals. The simulation design must be scalable to arbitrary size and parallel in order to take advantage of cluster computation. Scalability requires the creation of accurate event generators, which I derived from tested databases (ENDF and PSTAR) or directly from $n^3\text{He}$ instruments. In order to improve the precision of the results, a weighting system is used which distributes events evenly across space and time but assigns weights proportional to their probability in the generators. Gravitational effects have been neglected in this simulation due to the speed of the particles and small travel distance. Energy deposition times have been tracked up to ionization of the gas.

The basic simulation routine is composed of the following steps:

1. Generate neutron energy (then velocity) and its weight.
2. Generate starting position vector and its weights.
3. Collimate.
4. Calculate cross-section, reaction depth, and its weight.
5. Calculate emission angles of proton and triton.
6. Particle Transport

Simulate ion trajectories and track cell crossings.

Calculate energy depositions and timings.

7. Multiply total weight.

4.1 Generators

To produce the most reliable simulation of the experiment, in order to optimize it and have accurate geometry factors for calculating the asymmetry, the physical parameters of the experiment should be reproduced as closely as possible in the simulation. These details were carefully extracted from various tests and measurements done of the experiment during construction. In addition, computational efficiency was considered with each choice of algorithm, to increase the statistical precision reachable by simulation.

4.1.1 Beam Profile and Divergence

In order to properly align the experiment along the neutron beam, its centroid must be calculated in order to optimize the position of the detector to maximize observed flux and minimize systematic error. For this purpose, two different collimator scans were performed: at the output of the polarizer, 56cm from the forward wall, and the predetermined location of the wire chamber, 342cm from the forward wall.

The upstream profile has a rectangular shape (Figure 4.1). The beam centroid calculation determines the weighted center of the beam in terms of neutron flux, and the experiment is then aligned around it. Generally, the centroid of the beam, $\vec{c}$, is just a weighted average of the flux Φ_i , over the sampling coordinates $\vec{s}_i$:

$$\vec{c} = \frac{\sum \Phi_i \vec{s}_i}{\sum \Phi_i} \quad (4.1)$$

Using this formula, we scan using a grid with constant spacing at the upstream and downstream locations, and calculate the centroids at those two locations, c_{up} and c_{down} . The upstream and downstream "home" locations on the scanning bed were different, so the two centroids are measured in different coordinate systems:

$$c_{up} = (10.042, 12.049)\text{cm}$$

$$c_{down} = (16.867, 11.341)\text{cm}$$

In addition to calculating the centroid for alignment, it is possible to extract the beam shape and divergence from these scans, which were very useful in my simulation.

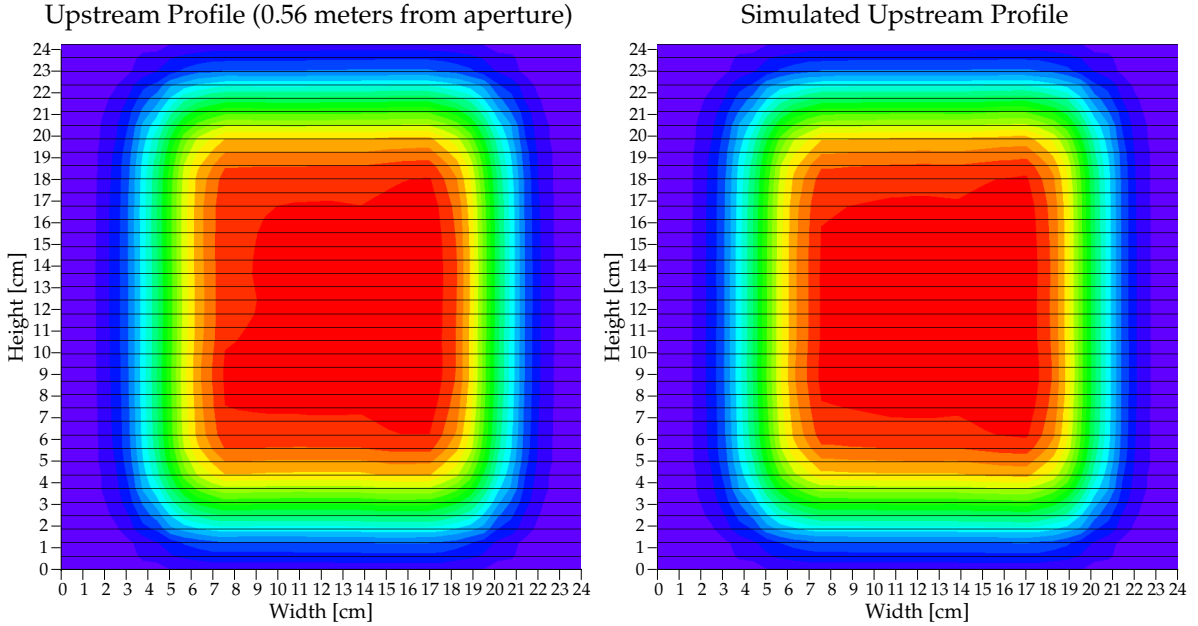


Figure 4.1: Comparison of measured (left) and simulated beam profiles (right).

Two quantities of interest are the physical shapes of the beam profile in two dimensions, and the divergence of the neutron beam in two dimensions. Once this information is extracted from the beam scans, it can be used to accurately model the initial positions of neutrons coming out of the polarizer, and the coordinates where they enter the wire chamber.

We want to reconstruct the profile of the neutron beam and write a generator to randomly fill that shape with an arbitrary number of events. We can take advantage of the relative linear independence of the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ beam profiles to produce an accurate beam profile using two one-dimensional generators. To do this, first we analyze the beam shape to measure how correlated the two profiles are, and then find the best representation of our two-dimensional flux as a product of two one-dimensional profiles. If we treat our flux scan as a two-dimensional matrix, $\mathbf{F}$, we can decompose it into two sets of vectors, $\mathbf{U}$ and $\mathbf{V}$, and determine the eigenvalues, Σ , which indicate how much information each pair of vectors contains. This is also called a singular value decomposition:

$$\mathbf{F}_{\text{up}} \approx \mathbf{U}\Sigma\mathbf{V}^* \quad (4.2)$$

Using this decomposition, we can quantify how well the profile, $\mathbf{F}$, is approximated by the outer product of $\mathbf{U}_{1,j}$ and $\mathbf{V}_{i,1}^*$: The eigenvalues of our products are:

$$\begin{aligned} \Sigma_{1,1} &= 1.634130 \\ \Sigma_{2,2} &= 0.015986 \\ \Sigma_{3,3} &= 0.003993 \\ \Sigma_{4,4} &= 0.002008 \\ \Sigma_{5,5} &= 0.001337 \end{aligned} \quad (4.3)$$

The remaining eigenvalues are less than 10^{-3} . We can see that the first product contains almost all the information about the beam shape. In fact, for this approximation, $\chi^2 < 0.01$. Based on this decomposition, we can create two one-dimensional profiles based on the first vectors in $\mathbf{U}_{1,j}$ and $\mathbf{V}_{i,1}^*$, and multiply them together to generate an accurate reconstruction of our physical beam shape.

We would like to model the divergences of the neutron beam in the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ directions, and include them in the simulation as a refinement of the neutron event generator. To determine the divergences, we need to model the shape of the resultant profile. Since the upstream beam profile is roughly square, we assume a Gaussian form for the divergence, ignoring the energy dependence, and convolve that function with the upstream profile shapes in two dimensions. This gives a good model of the beam evolution, and the resulting shape can be closely approximated as a product of error functions (Figure 4.2).

$$\mathbf{F}_{\text{down}} \approx \frac{A_x A_y}{4} \left[\frac{\text{Erf}(x' - x)}{\sqrt{2}\sigma_x} + \frac{\text{Erf}(x' + x)}{\sqrt{2}\sigma_x} \right] \left[\frac{\text{Erf}(y' - y)}{\sqrt{2}\sigma_y} + \frac{\text{Erf}(y' + y)}{\sqrt{2}\sigma_y} \right] \quad (4.4)$$

This function is the result of the convolution of a Gaussian with a step function, since our upstream profile resembles one. If we fit our scanned downstream profile to this, we can obtain values for the beam divergences in the x and y-directions:

$$\Theta_x = 0.012642$$

$$\Theta_y = 0.015689 \quad (4.5)$$

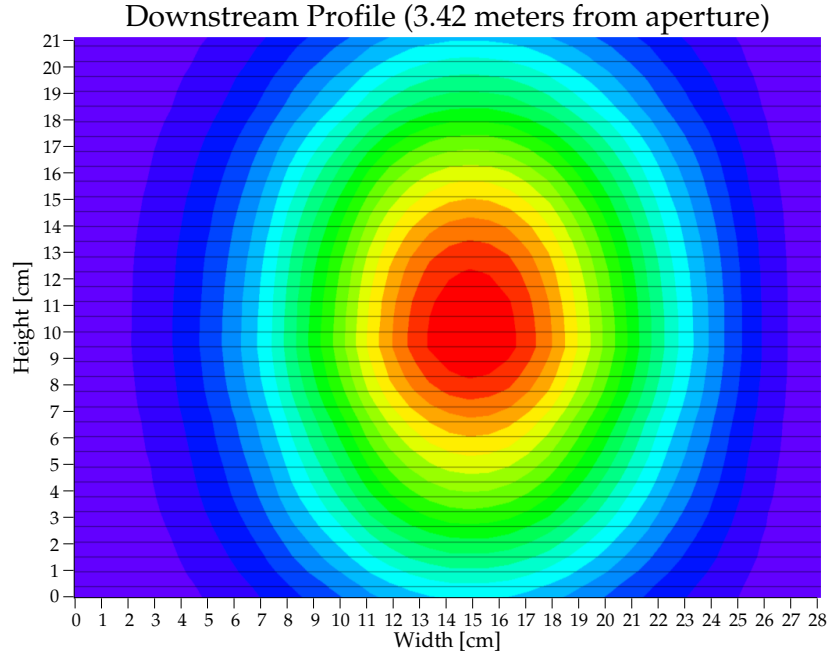


Figure 4.2: Beam profile at the far end of the beamline.

4.1.2 Neutron Spectrum

In order to generate an accurate simulation of n^3He , we need to reproduce the energy spectrum of the neutrons entering the detector. The beam monitor samples the neutron intensity at 100kHz, which we can use to determine this. Since the neutrons come from a pulsed source, the time-of-flight is highly correlated with the energy of the neutron (see Appendix A). If we can make a precise calculation of the time of arrival of each neutron bin, we can determine their relative intensity, and produce a normalized spectrum.

We find the width of the start time at the moderator, $200\mu s$ (A.1), which is much smaller than the time of flight of our fastest neutrons, 15ms. So we can treat the arrival time of the neutron as an indicator of neutron energy, and determine the spectrum from the pulse shape (Figure 4.3). The velocity of a neutron arriving at time t is

$$v(t) = \frac{\Delta x}{\Delta t} \quad (4.6)$$

where Δx is the distance traveled, and Δt is the time taken. To calculate the velocity, we need the path length as well as the exact time-of-flight. Let's first examine the path length.

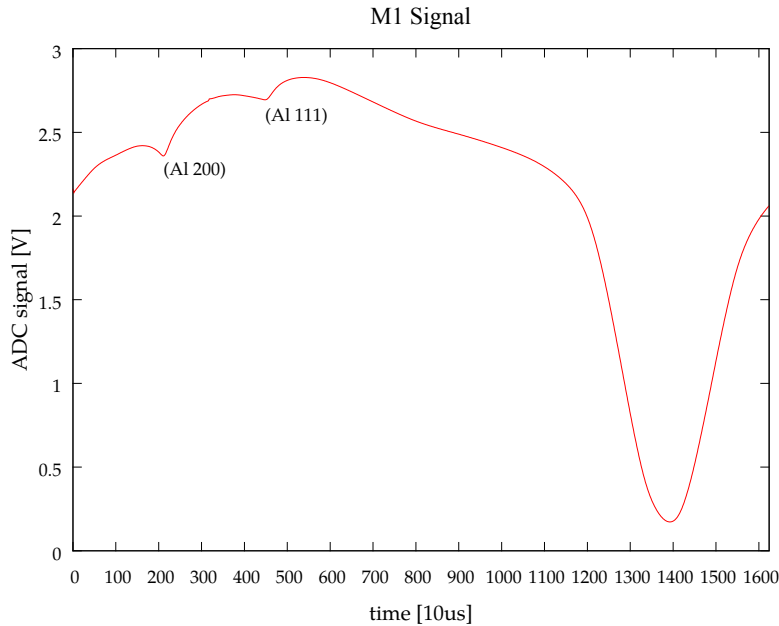


Figure 4.3: Shape of the neutron pulse in the monitor, with the Aluminum Bragg edges indicated.

Path length

We can estimate the path length from the SNS design documents (Appendix B), where it is measured as a linear distance between the moderator and the end of the neutron guide, but we would like to calculate the effective neutron path length. We have precise timing information in the shape of the monitor signal: there are two prominent Bragg edges due to the scattering of neutrons in Aluminum (Figure 4.3). Note that the time signal here is delayed by the time-of-flight to the n^3He apparatus. We know the neutron energy range in the primary pulse window, and which plane spacings in Aluminum these two peaks correspond to. Based on this, we can calculate which neutron wavelengths correspond to the Bragg edges shown in Figure 4.3,

$$\begin{aligned}\frac{\lambda_1}{2} &= \frac{a}{\sqrt{\vec{h}_1^2 + \vec{k}_1^2 + \vec{l}_1^2}} = \frac{4.0495 \cdot 10^{-9}m}{\sqrt{1+1+1}}; & \lambda_1 &= 4.6760\text{\AA} \\ \frac{\lambda_2}{2} &= \frac{a}{\sqrt{\vec{h}_2^2 + \vec{k}_2^2 + \vec{l}_2^2}} = \frac{4.0495 \cdot 10^{-9}m}{\sqrt{4+0+0}}; & \lambda_2 &= 4.0495\text{\AA}\end{aligned}\tag{4.7}$$

where a is the lattice constant of Aluminum, and h , k , and l , are the Miller indices corresponding to the plane spacing which scattered the neutrons out of the beam. The timing measurements at the maximum of these peaks correspond to neutrons of known energies:

$$L_i = \frac{h(t_i - T_E)}{m_n \lambda_i}\tag{4.8}$$

where L is the length traveled, h is Planck's constant, m_n is the mass of the neutron, λ is the neutron wavelength, t is the total time-of-flight, and T_E is defined as the time of peak intensity at the moderator. A simulation of the moderator (A.1) indicates that the delay in peak intensity is approximately $5\mu s$ for neutrons of the two wavelengths (found in equations 4.7 and 4.8) we're using for this calculation. This is significantly less than the FWHM of the neutron pulse, which is around $200\mu s$ for all neutrons in this range, so it is safe to consider both path length solutions to have the same emission time and eliminate this variable in (4.10), although it will introduce a small error in (4.11):

$$L = \frac{h}{m_n} \frac{(t_i - t_j)}{(\lambda_i - \lambda_j)} \quad (4.9)$$

Since we already know the wavelengths, we can find the neutron flight path length by making a precise measurement of the *relative* arrival time of the neutrons. To precisely measure the time between the arrivals, we can isolate the Bragg peaks by subtracting the Bragg edges from a quadratic background signal (Figure 4.4.)

Now we can calculate the path length to the beam monitor. We can identify three sources of error in this calculation. First, half the DAQ resolution is $5\mu s$. Second, the difference in T_E between the two chosen wavelengths is around $5\mu s$. Third, the uncertainty in the time delay from guide reflections, for an M=3 surface, at $\lambda = 4\text{\AA}$, is around $6\mu s$.

$$L = \frac{h}{m_n} \frac{(t_i - t_j)}{(\lambda_i - \lambda_j)} = \frac{h}{m_n} \frac{2.39411 \cdot 10^{-3} s}{6.264598 \cdot 10^{-11} m} = 15.119 m \quad (4.10)$$

$$\sigma_L = \frac{h}{m_n} \frac{\sqrt{2\sigma_{DAQ}^2 + \sigma_{T_E}^2 + \sigma_{ref}^2}}{(\lambda_i - \lambda_j)} = \frac{h}{m_n} \frac{10.520 \cdot 10^{-6} s}{6.264598 \cdot 10^{-11} m} = 0.067 m \quad (4.11)$$

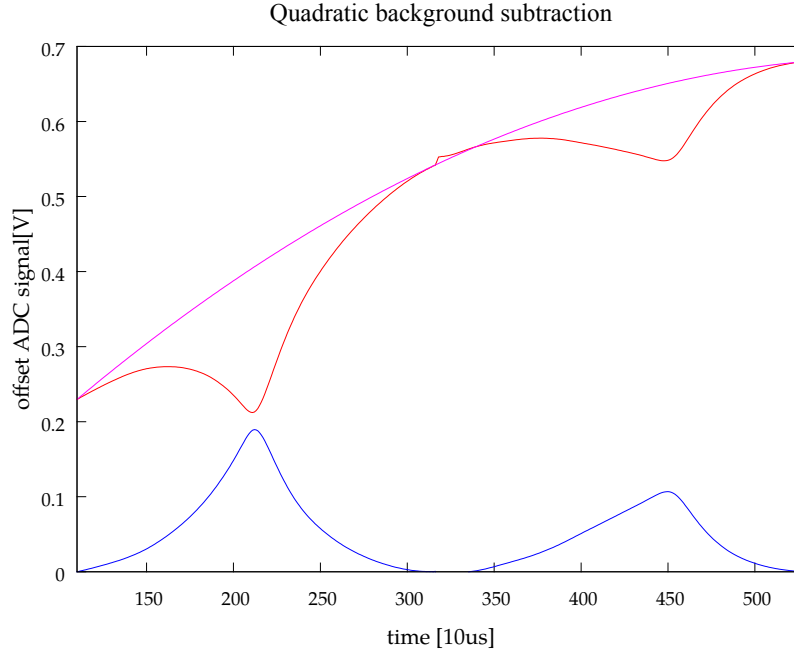


Figure 4.4: Bragg scattering peaks in Aluminum.

Time of Flight

The information contained in the locations of the Bragg peaks can also give an estimate of the absolute emission time of the neutrons. Since $L_i = L_j$, we can calculate T_E :

$$T_E = \frac{\lambda_i t_j - \lambda_j t_i}{\lambda_j - \lambda_i} = 13.424 \text{ms} \quad (4.12)$$

This determines the time of flight to the zeroth timebin of the monitor signal. The recording timings are calibrated to measure a centered pulse in the first frame of the ^3He detector, which is downstream from the monitor. We would like to view an time-ordered velocity spectrum, so we can shift our signal by -2.74ms to get one (Figure 4.5).

$$T_E = 13.424 \text{ms} - 2.74 \text{ms} = 10.684 \text{ms}. \quad (4.13)$$

Now that we have solved for the absolute distance and time, we can construct $E(t)$ from $v(t)$. There is a further correction that must be made before we can create our energy distribution. The beam monitor uses ^3He to detect neutrons. The neutron cross-section of ^3He has a wavelength dependence of the form $\sigma[\lambda] \propto \lambda$, meaning that our observed intensity $S(\lambda)$ is proportional to the true intensity $I(\lambda)$:

$$I(\lambda) \propto S(\lambda) \cdot \lambda$$

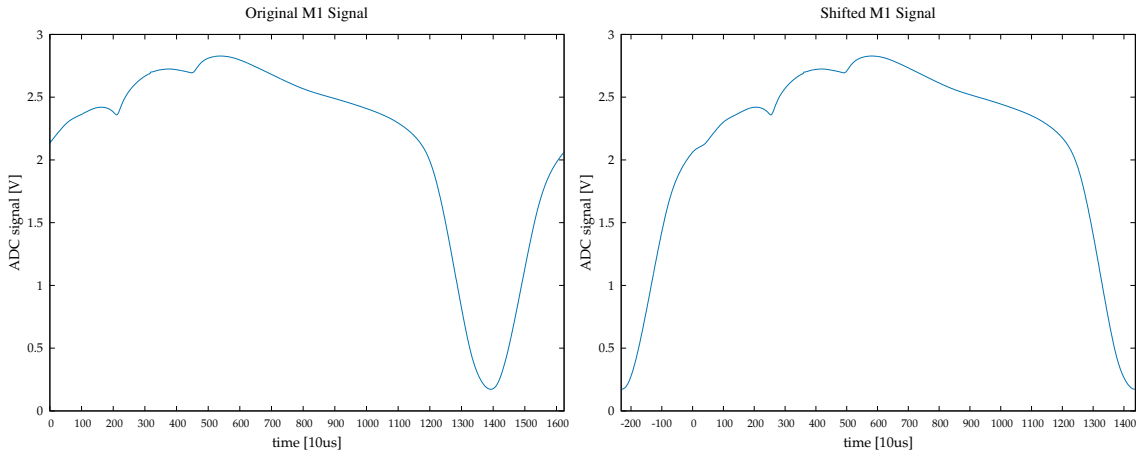


Figure 4.5: Relative time correction of pulse intensity.

Since we need a relative probability distribution, we are only concerned with the relative sensitivity of the monitor to different wavelengths. So, first we take our discrete function $\lambda(t)$, and scale this by $\frac{1}{\lambda}$, and then normalize. Then, we find $E(t)$ from this, and we will produce the correct distribution. Finally, we have the real neutron spectrum incident on the beam monitor (Figure 4.6).

Detector Timing

We may also be interested in the distance to the detector. The locations of the monitor and detector relative to the neutron aperture were measured during commissioning: the polarizer aperture is 0.738m from the monitor, and the detector is 1.492m from the polarizer aperture. Now that we have calculated the flight path distance to the beam monitor, we can calculate the distance to the detector face by adding it to the distance from the monitor to the aperture and the distance from the aperture to the detector face:

$$15.119m + 0.738m + 1.429m = 17.286m \quad (4.14)$$

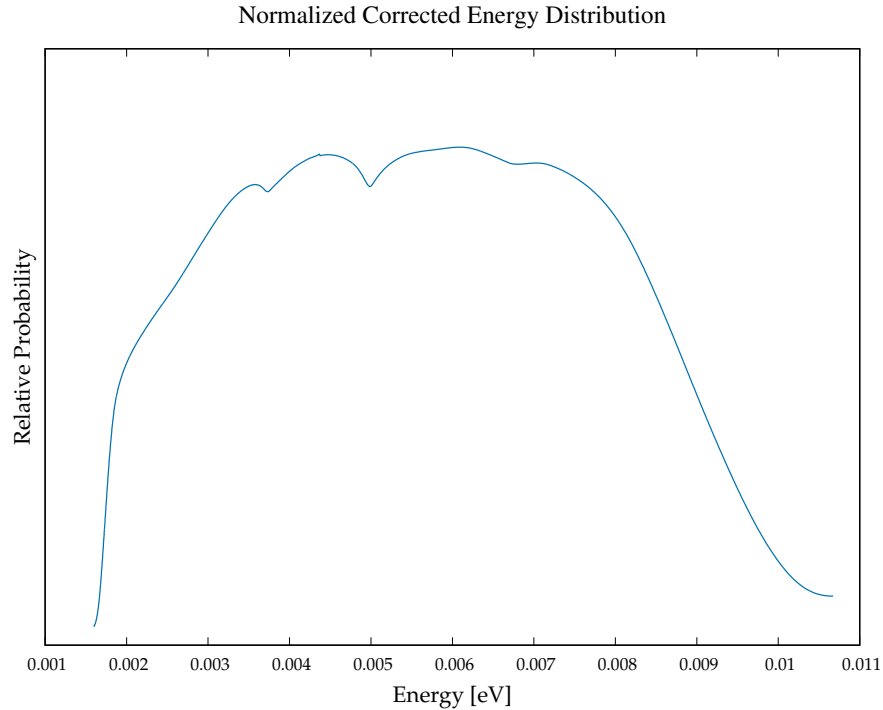


Figure 4.6: Distribution of neutron energies at M1.

The recording timers for monitor and the detector channels are synchronized, so the time offset already calculated in (4.13) can be used to interpret the data in all DAQs. We may also wish to calculate the additional flight time to the beginning of the detector recording frame. We will need a new formula for calculating the flight time between the beam monitor and detector from (4.9), since we don't have a common emission time:

$$L_i = \frac{h(t_i - T_E)}{m_n \lambda_i}$$

We again compare the calculation for each path length, involving two different wavelengths.

$$L_i = \frac{h}{m_n} \frac{[(T_1 + t_i^d) - (T_0 + t_i^m)]}{\lambda_i} \quad L_j = \frac{h}{m_n} \frac{[(t_j^d - T_j^d) - (t_j^m - T_j^m)]}{\lambda_j} \quad (4.15)$$

It is not necessary to know the values of the absolute times T_n to determine the distance between the monitor and detector frames, since we will solve for the relative time difference:

$$T_1 - T_0 = \Delta T = \frac{\lambda_j(t_j^d - t_j^m) - \lambda_i(t_i^d - t_i^m)}{\lambda_j - \lambda_i} \quad (4.16)$$

We can compute this quantity for different frames of the detector. Let's look at the third frame of the detector (index 2) since it has a good pulse shape and a balanced frame window, which will make the interpolation easier.

$$\Delta T_2 = \frac{\lambda_i(t_i^m - t_i^d) - \lambda_j(t_j^m - t_j^d)}{\lambda_j - \lambda_i} = 1.201ms \quad (4.17)$$

4.1.3 Reaction Depth

After generating the neutron wavelength and starting coordinates in the x-y plane, we must calculate the reaction depth, or z-coordinate of the neutron capture. As with the other variables in the simulation, the value of z_0 will be uniformly distributed throughout the depth of the detector in order to improve the stability and convergence behavior of the simulation, and the weights corresponding to those events will correspond to the physical probability of the neutron being captured at that depth.

We can determine a model for generating random events that fits the appropriate probability distribution by starting with the attenuation of a neutron beam in a uniform material:

$$\frac{d\Phi}{dz} = -n\sigma\Phi, \quad (4.18)$$

where Φ is the flux, n is the number density of scattering nuclei, and σ is the total cross-section. Note that absorption cross-section depends on the neutron wavelength, which we will address next. In addition, n depends on the pressure, which is an optimized parameter of the experiment. This gives the flux as a function of depth:

$$\Phi(z) = e^{-n\sigma z} \quad (4.19)$$

The cross-section with ^3He is large, and the neutron beam quickly captures in the first $\approx 6\text{cm}$ of the detector. In order to simulate stable quantities in all cells of the detector, including those where capture events are rare, we use a uniform distribution of depths (Figure 4.7), and weight each event with the probability of it occurring (Figure 4.8). To generate a random event weight, integrate over the depth, d , and normalize:

$$w_z = \frac{e^{-n\sigma z} n\sigma d}{1 - e^{-n\sigma d}} \quad (4.20)$$

To calculate w_z , we need the value of the total cross-section. A simple way to implement this is to use the $\frac{1}{v}$ behavior of the absorption cross-section to produce a function that determines σ from λ every event. Instead of looking up the value for every neutron event, we can take advantage of the very consistent $\sigma \sim \frac{1}{\sqrt{E}}$ behavior of the cross section in our region of interest, which we can observe in the ENDV-VII database [53]. Figure 4.9 shows that $\log(\sigma)$ is a linear function of $\log(E)$:

$$\log(\sigma) = C - \frac{1}{2}\log(E) \quad (4.21)$$

$$C = 2.92709$$

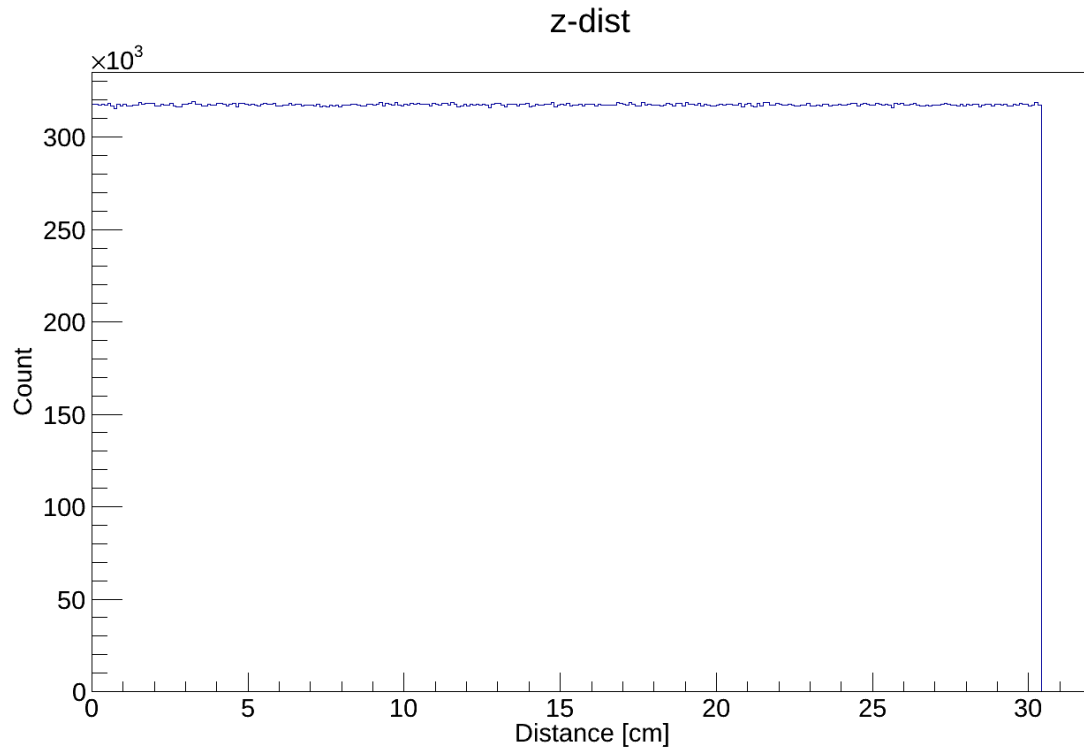


Figure 4.7: Distribution of neutron captures vs. depth into wire chamber.

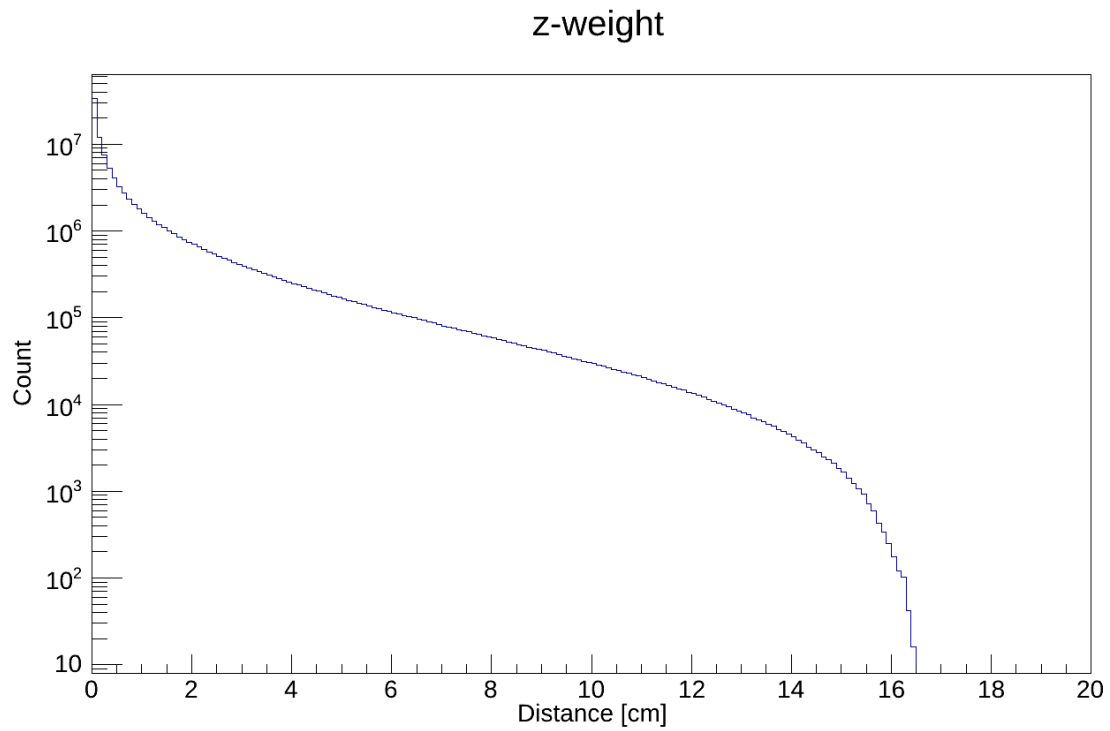


Figure 4.8: Weight of neutron captures at a given neutron depth. Compare with Figure 4.7.

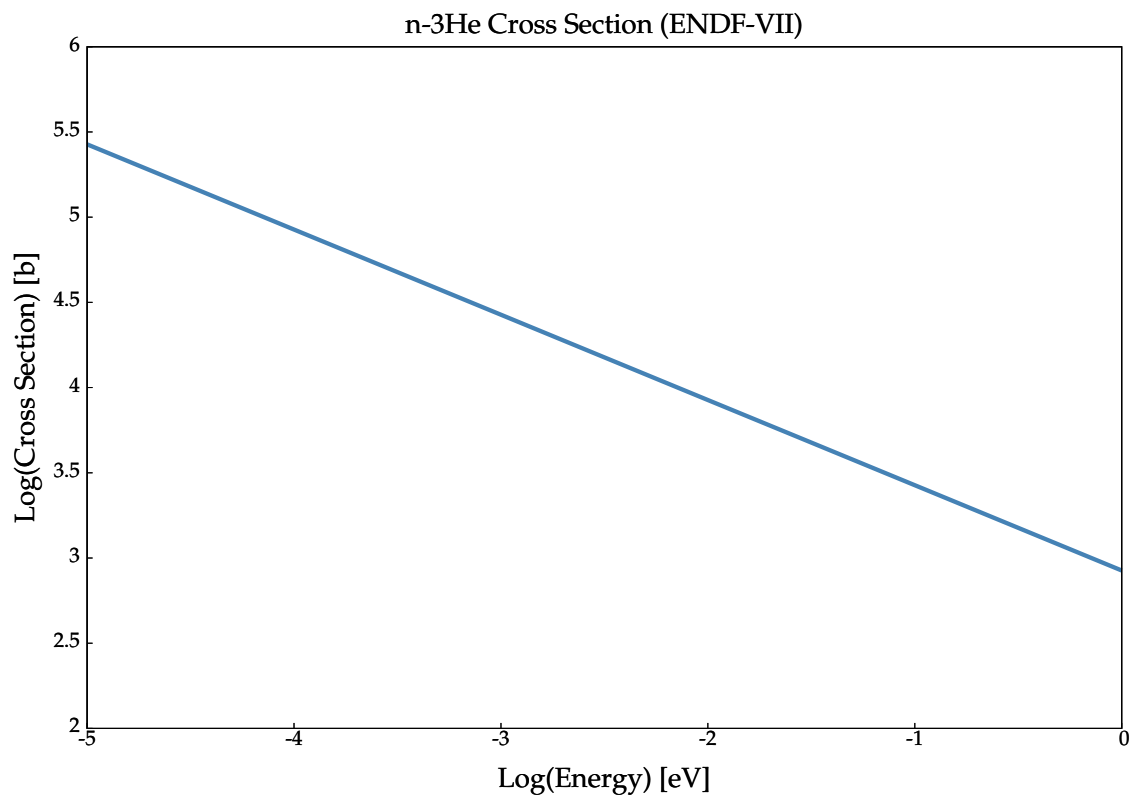


Figure 4.9: $n+{}^3\text{He}$ cross section from ENDF-VII.

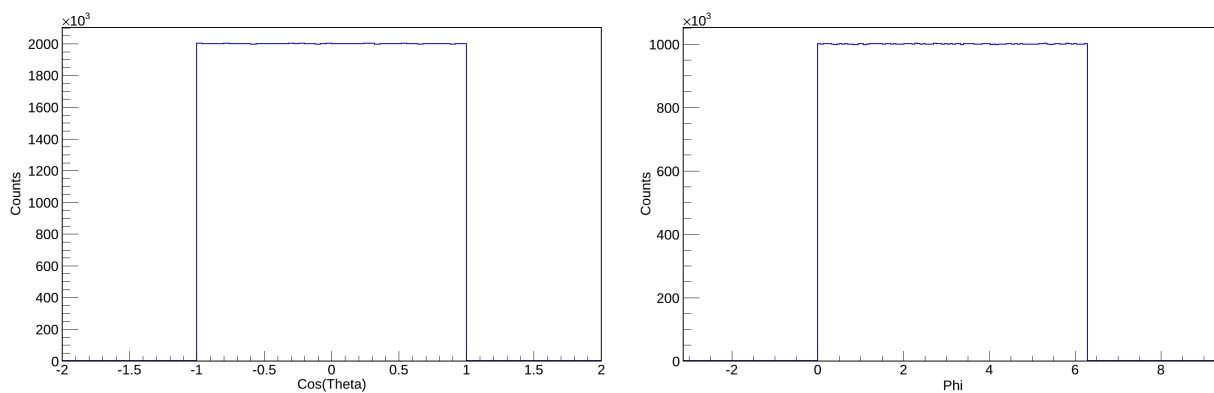


Figure 4.10: Distribution of $\cos(\theta)$ and ϕ .

Once we solve for C , we can then analytically determine the cross section from the energy of each neutron event. When calculating the cross section for each neutron event, we use a constant divided by the square root of the energy:

$$\sigma = \frac{10^C}{\sqrt{E}} \quad (4.22)$$

4.1.4 Emission Angles

After the reaction depth, and weight, are calculated for each neutron, the angles of emission of the reaction products are generated from a uniform spherical distribution (Figure 4.10). A spherical distribution is used because the kinetic energy of the neutron is much smaller than the energy released by nucleon exchange. For the highest-energy neutrons in our pulse,

$$KE_n = \frac{h^2}{2m\lambda^2} = .0131eV \ll 765keV, \quad (4.23)$$

so our reaction products will be emitted in a spherical distribution.

4.2 Particle Transport

Once the proton and triton are emitted into the ^3He gas, the simulation begins tracking the ions, and the energy they deposit in the gas, in three dimensions. This tracking continues until the ions deposit all of their energy into the gas:

$$\begin{aligned} x' &= x + v_x \Delta t \\ y' &= y + v_y \Delta t \\ z' &= z + v_z \Delta t \end{aligned} \quad (4.24)$$

A typical time step for simulations under different experimental parameters is 10^{-9}s . At that size, the ions may move approximately $1\mu\text{m}$ in a single step.

Cell Crossings and Channel Index

The wire chamber has 16 signal frames, each with 9 wires in it. These are placed in between 17 high-voltage frames, also each with 9 wires in them. Based on electric field models [50], every channel collects charge from a rectangular-prism shaped region of space around its signal wire (Figure 4.11).

The simulation uses a cell model to simulate the charge collection observed by these wires. The charge collected in the volumes bounded by the voltage wires are deposited to the signal wire in the center. Edge effects at the top and bottom of the chamber are included with a reduced-volume approximation. Particles are transported through the gas until they exit the active volume or run out of energy.

Every time an ion crosses the boundary of a cell, the distance from the reaction center is recorded, along with the index of the cell being entered or exited. The ionization curve is mapped onto the cell crossings to determine how much energy is deposited into each cell. The signal timings are determined by the entry and exit time of each cell crossing. This produces a time-resolved, three-dimensional simulation of the detector signals.

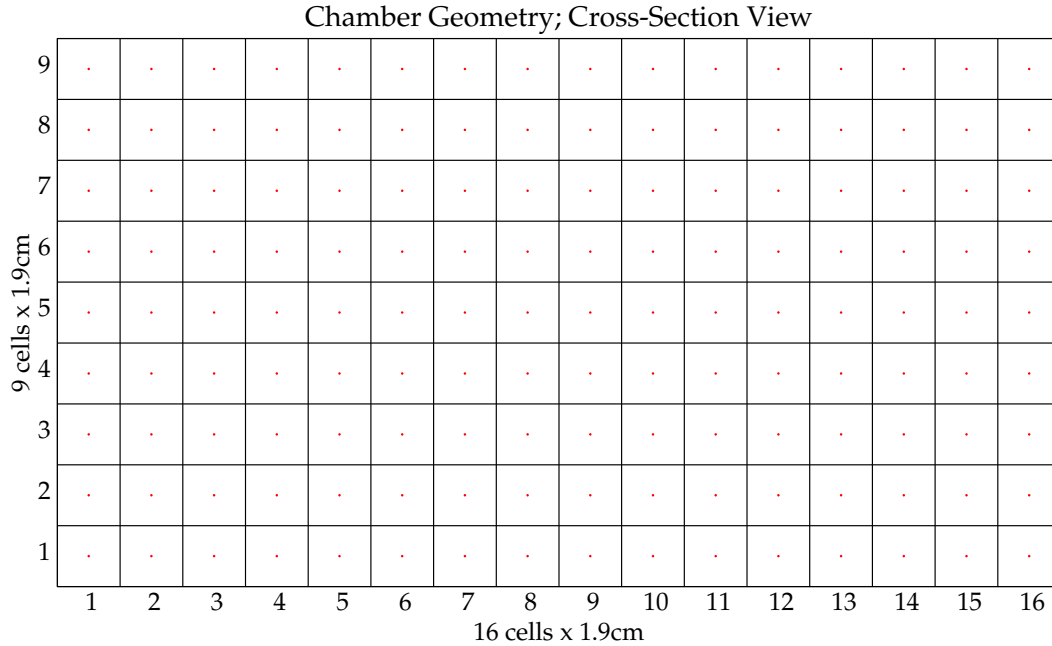


Figure 4.11: The signal wires measure ionization from the cells around them.

4.2.1 Energy Deposition

The energy deposition curves for the proton and triton are both taken from the PSTAR database, which provides the stopping power and ranges of protons in various materials [54]. The kinetic energy of our fastest neutrons is $\sim 10^{-2}$ eV, whereas the kinetic energy of the reaction products is 765keV, so our ionization is dominated by the electronic stopping power of the gas, and we can ignore effects due to nuclear stopping power. To simulate detector signals in $n^3\text{He}$, it is necessary to know the rate of ionization of the gas under different conditions. (This was one of the tasks of the optimization). The rate of energy loss at different energies is sometimes called the stopping power, and is defined as

$$S(E) = -\frac{dE}{dx}, \quad (4.25)$$

the rate of energy loss per unit of length. A familiar reference to this is a plot which has appeared in multiple papers and presentations in the design of $n^3\text{He}$, but at low precision. We would like to have a method to calculate ion energy deposition curves under various conditions. We can derive this from the PSTAR database.

PSTAR data

We are interested in the ionization of ^3He gas by ^1H and ^3H ions. PSTAR provides data for the rate of ionization of ^4He gas by ^1H ions. We can make some analytical corrections to this data, using some additional physics, to get the stopping power data to simulate the ionization rate that we need.

The data is given in units of $\text{MeV cm}^2/\text{g}$. This is the energy loss per unit distance divided by the density of the material. Both the energy loss rate and the density are material-dependent quantities. We can get the linear stopping power for ^4He by multiplying the PSTAR data by the density of ^4He at 101.325kPa and 20°C. Note that the nuclear stopping power is 10^2 - 10^3 times smaller than the electronic stopping power in our region of interest, from 0 to 600 keV (Figure 4.12), so we can ignore energy loss due to nuclear stopping power in our simulation.

Particle Range

An intuitive method to calculate a particle's range is to assume that, since the mean ionization energy of one electron is small compared to the kinetic energy of the proton, and since there is a very large number of collisions, we can treat the energy loss in the medium as being continuous. Then, if we want to calculate the length of the particle's track, we can integrate the inverse stopping power with respect to the particle's energy:

$$\Delta x = \int_0^{E^0} \left[\frac{dE}{dx} \right]^{-1} dE = \int_0^{E^0} \frac{dE}{S(E)} \quad (4.26)$$

With a continuous function of the stopping power, the particle's range in a gas under given conditions can be determined. Let us try this for the most simple system in our case, ^4He at 101.3kPa and 20C (These are the conditions for the data in Figure 4.12). The molar density of ^4He under these conditions is:

$$\frac{1}{V_m} = \frac{N}{V} = \frac{P}{kT} = \frac{101.325 \text{ kPa}}{k \cdot 273.15 \text{ K}} = 4.461 \cdot 10^{-5} \text{ mol/cm}^3 \quad (4.27)$$

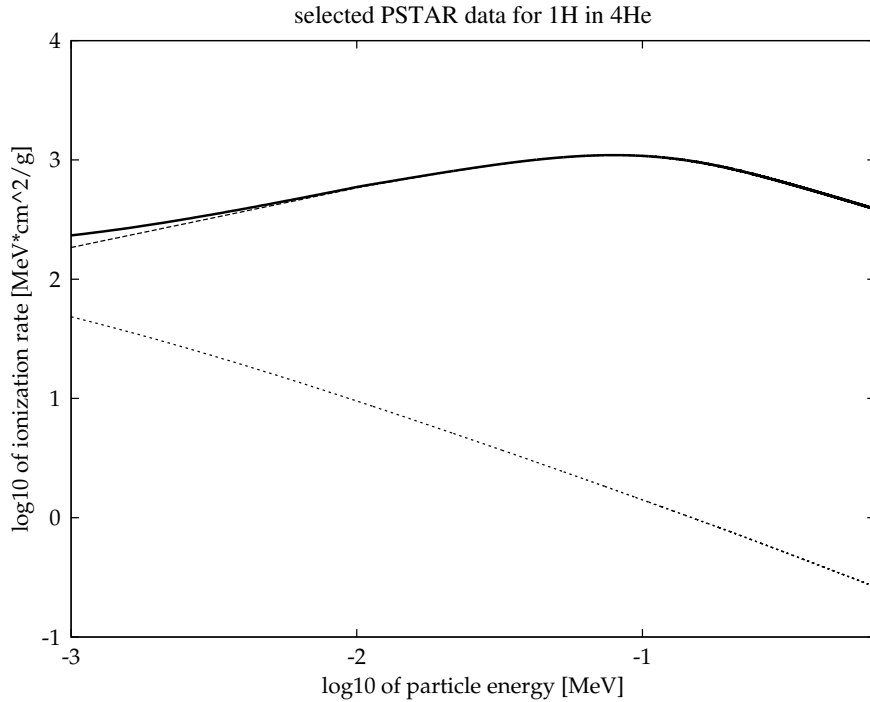


Figure 4.12: Selected PSTAR data for Helium.

The mass density is:

$$\rho = \frac{u}{V_m} = 4.461 \cdot 10^{-5} \text{ mol/cm}^3 \cdot 4.003 \text{ g/mol} = 0.179 \text{ g/cm}^3 \quad (4.28)$$

To get energy loss per unit distance, we multiply the stopping power by mass density:

$$\left[\frac{dE}{dx} \right] = [S] \cdot [\rho] = (\text{MeV cm}^2/\text{g}) \cdot (\text{g/cm}^3) = \text{MeV/cm}; \quad (4.29)$$

Since we do not have an analytic function for the stopping power, we must solve for it numerically. Instead of integrating an analytic function along our particle track, we calculate small, finite energy losses over a short distance (10^{-6}m), and continue adding them until the particle energy goes to zero, and it comes to rest. The results of using this continuous energy loss approximation do not change if the time step is reduced past 10^{-6}m . This method will produce the ionization curve for the proton.

PSTAR is so named because it holds data on the stopping power and range of ^1H . One of our reaction products is ^3H . However, we can still use the PSTAR data to find its ionization rate. This is because the ion-electron interaction depends only on the charge of the ion and its velocity, which we know from the Bethe formula [48]. So, we can find an equivalent energy loss rate from the PSTAR data by converting to the same particle velocity.

As noted above, virtually all of the energy loss the particle experiences traveling through the gas is from the electronic stopping power. Since ^1H and ^3H have the same charge, they will ionize the gas at the same rate as long as their velocity is the same. Then, since the rates of energy loss are given as a function of kinetic energy, we can treat the triton as a proton with its kinetic energy scaled as

$$\frac{E_p}{E_t} = \frac{m_p \mathbf{v}^2}{m_t \mathbf{v}^2} = \frac{u_p}{u_t} = \frac{1.007276466879}{3.01550071632} = 0.334033 \quad (4.30)$$

With this method, we can determine our ionization curve (Figure 4.13). Note that the reaction vertex, in red, indicates where the particles begin their trajectory in the gas, and they travel in opposite directions from conservation of momentum.

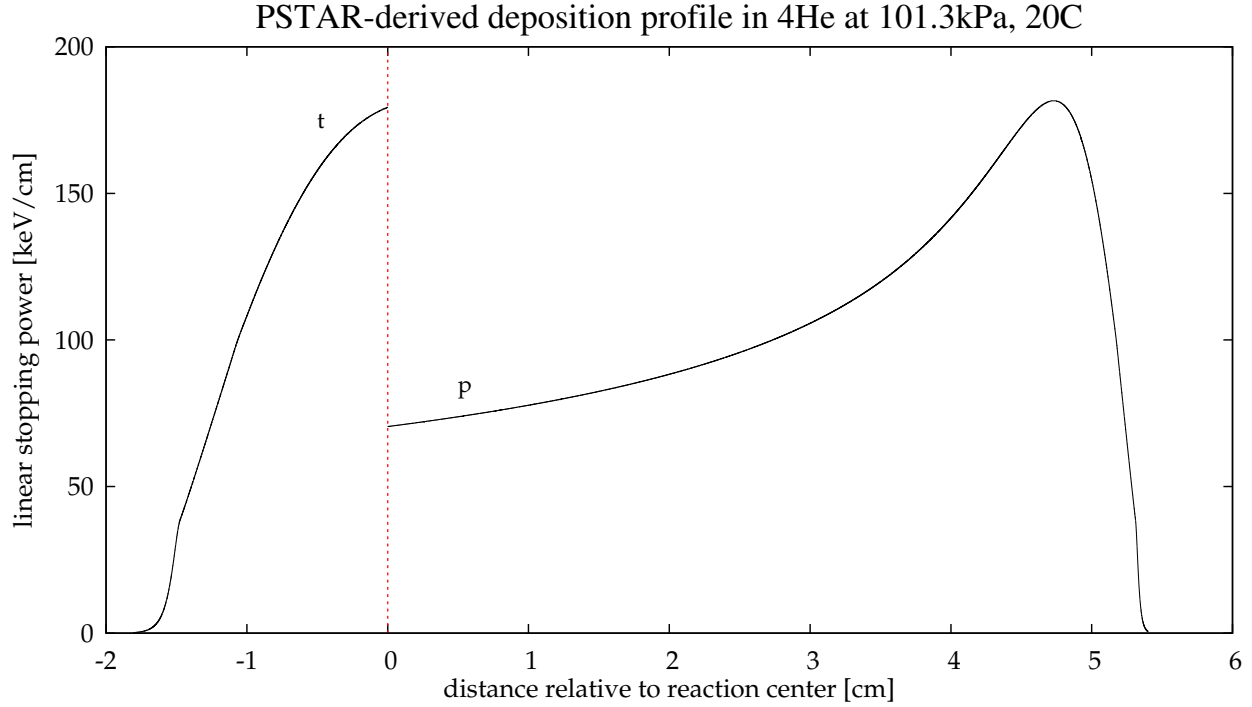


Figure 4.13: The calculated ionization of ^4He .

$n^3\text{He}$ corrections

There are two more differences between this system and our experiment. The first is that the pressure in the wire chamber is not 101.325kPa, but 48.2kPa. Since the gas density is a linear function of the pressure, the rate of energy loss will be scaled by the pressure ratio. To calculate detector sensitivities at different pressures, the energy loss is given a linear scaling according to the pressure ratio. The second difference is that we are using ^3He , not ^4He . PSTAR is given in units of $\text{MeV cm}^2/\text{g}$ for ^4He , but we are not really interested in the mass density of the gas, since the ionization is a function of the *molar* density of the electrons. Once we have converted from $\text{MeV cm}^2/\text{g}$ to MeV/cm , we have the stopping power for ^4He at 1 atm and 20°C . Under these conditions, ^3He has the same molar density of electrons as ^4He , so the stopping power will be the same. This is possible because in our region of interest, the nuclear stopping power is dominated by the electronic stopping power, and can be neglected. We do not need to make a further correction. Now we can produce the chart that corresponds to our experiment: ^3He , at 0.476 atm, 21.3°C , in Figure 4.14.

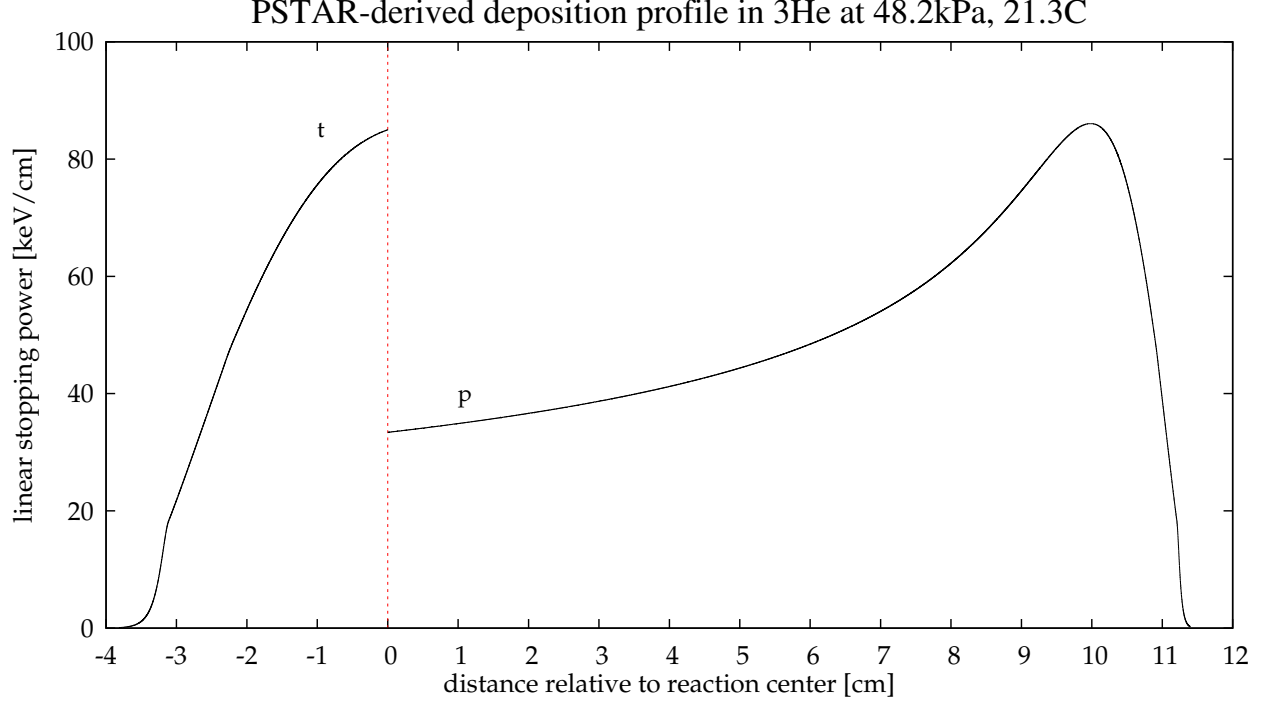


Figure 4.14: ^3H and ^1H ionization in ^3He at 48.2kPa.

Range-Energy Pre-Integration

To determine how much energy is deposited into a cell, the crossing distances are compared with the range-energy curve (Figure 4.14). Since the energy loss is known as a function of ion distance traveled, we can determine the amount of energy deposited into a cell by integrating the amount of energy lost by the ion along that subsection of the particle track,

$$E' = \int_{x_1}^{x_2} \left[\frac{dE}{dx} \right] dx = \int_{x_1}^{x_2} D(x) dx, \quad (4.31)$$

where $D(x)$ is the rate of energy loss per unit distance as a function of distance along the particle track in Figure 4.15. Here, we are integrating a positive quantity since we are calculating energy deposited into a cell, not energy lost by the particle.

Performing a curve integration multiple times every neutron event is resource-inefficient. We can take advantage of the fact that the $D(x)$ is constant for every neutron event and simplify the simulation algorithm. First, rewrite (4.31) as a difference between two integrals:

$$E' = \int_{x_1}^{x_2} D(x)dx = \int_0^{x_2} D(x)dx - \int_0^{x_1} D(x)dx \quad (4.32)$$

Every cell deposition will have this form. So, we only need to integrate $D(x)$ once. Define the cumulative integrated ionization as $CE(x)$:

$$CE(x) = \int_0^x D(x')dx' \quad (4.33)$$

If we pre-integrate $D(x)$ and store the curve, then instead of performing an integration in equation 4.31, we perform a subtraction instead, which is more efficient:

$$E' = \int_{x_1}^{x_2} \left[\frac{dE}{dx} \right] dx = \int_{x_1}^{x_2} D(x)dx = CE(x_2) - CE(x_1) \quad (4.34)$$

4.3 Statistics of Covariances

We want to simulate the uncertainty in the physics asymmetry without knowing the true value of the asymmetry. We can do this by deriving the appropriate statistics for our experiment.

4.3.1 Element Asymmetry

Every neutron event i generates a unique angle θ_i , which is the angle between the proton and neutron momenta. The simulation calculates quantities for K elements, indexed by κ . For each neutron event, the energies deposited into each element, Q_i^κ , are recorded. The total energy received by an element is the sum of all the energies deposited in each event:

$$E^\kappa = \sum_{i=1}^{N_{mc}} Q_i^\kappa \quad (4.35)$$

Let us define the *expectation value* for the energy in cell κ as $\langle E^\kappa \rangle$:

$$\langle E^\kappa \rangle = \left\langle \sum_{i=1}^{N_{mc}} Q_i^\kappa \right\rangle$$

This expectation value is defined over all simulated neutron events. We will analyze how to evaluate this summation further in section 4.3.2. For now, let us define our observable quantities. This expectation value corresponds to our observable, since for a large number of trials, the average simulated value represents the observed energy in an element. For a given spin state, s , the experimentally observed yield for cell κ , Y_s^κ , is

$$Y_s^\kappa = \langle E^\kappa (1 + h\alpha \cos \theta) \rangle, \quad (4.36)$$

where α is the size of the underlying physics asymmetry. From this, we can find a relation between our observable quantities and the simulated ones:

$$\frac{Y_+^\kappa - Y_-^\kappa}{Y_+^\kappa + Y_-^\kappa} = \alpha_\kappa \frac{\langle E^\kappa \cos \theta \rangle}{\langle E^\kappa \rangle}$$

Define the geometry factor for element κ as:

$$G_\kappa = \frac{\langle E^\kappa \cos \theta \rangle}{\langle E^\kappa \rangle} = \frac{\sum_{i=1}^{N_{mc}} Q_i^\kappa \cos \theta_i}{\sum_{i=1}^{N_{mc}} Q_i^\kappa} \quad (4.37)$$

Then the element κ produces an experimental value of the physics asymmetry:

$$\alpha_\kappa = \frac{1}{G_\kappa} \frac{Y_+^\kappa - Y_-^\kappa}{Y_+^\kappa + Y_-^\kappa} \quad (4.38)$$

Equation 4.38 assumes perfect polarization and spin rotation efficiency. Those corrections will be made in Chapter 5.

4.3.2 Statistics

There are 144 physical cells and 49 timebins per pulse, for a total of 7056 signal elements (although we will treat events occurring at different times as relatively independent). The probability that a single neutron event will deposit energy into an element is much smaller than one, even for elements at the front of the chamber. Because of this, the binomial distribution of the number of deposition events n_κ can be approximated with Poisson statistics.

Since the probability of any individual element receiving energy from a given neutron event is much less than one, and each pulse will contain 10^8 neutrons, we can assume that our summation $\sum_n^{N_{mc}} \simeq \sum_n^\infty$. We can introduce the counting statistics by taking advantage of this property. Now we return to evaluating (4.35). The expectation value of the observed energy E^κ is defined as the average value obtained from repeated trials. Let us take advantage of the Poisson nature of the underlying quantities to simplify this procedure. The expectation value of the element signal can be expressed in terms of the average number of depositions in an element and the expectation value of the deposition energy in element κ .

$$E^\kappa = \sum_n^{N_{mc}} \frac{e^{-\lambda_\kappa} \lambda_\kappa^n}{n!} \sum_{i=1}^n Q_i^\kappa \quad (4.39)$$

By Poisson statistics, the quantity λ_κ is equal to the average number of deposition events occurring in element κ . Use the same notation and call this the pulse average of $\bar{n}_\kappa$, where we will assume again that the number of events in a simulation is large enough that this local average converges to the expectation value. The index n is a Poisson-distributed random variable of the number of depositions occurring in element κ , and Q_i^κ is a random variable with a corresponding distribution over the range of possible energies deposited in element κ . Apply the expectation value to this summation:

$$\langle E^\kappa \rangle = \left\langle \sum_n^{N_{mc}} \frac{e^{-\lambda_\kappa} \lambda_\kappa^n}{n!} \sum_{i=1}^n Q_i^\kappa \right\rangle = \sum_n^{N_{mc}} \frac{e^{-\lambda_\kappa} \lambda_\kappa^n}{n!} \left\langle \sum_{i=1}^n Q_i^\kappa \right\rangle \quad (4.40)$$

The expectation operator applies to every term in the summation, so we can move it to just the sum over i . Now, there will be a large number of neutron events in a simulation, so we can assume that the average of the distribution of all possible deposition energies in element κ will be the same as the average of the energies in a single simulation. This will be used to approximate one pulse, so call it the *pulse average*. For clarity, denote the total pulse average with a bar:

$$\left\langle \sum_{i=1}^n Q_i^\kappa \right\rangle = n \bar{Q}^\kappa \quad (4.41)$$

The sum over n gives the result

$$\langle E^\kappa \rangle = \bar{n}_\kappa \overline{Q^\kappa} \quad (4.42)$$

Here we have used the fact that

$$\sum_n \frac{e^{-\lambda_\kappa} \lambda_\kappa^n}{n!} n = \bar{n}_\kappa$$

We have now expressed the expectation value of the observable energy in terms of two averages which we can measure from a single simulation. To calculate the statistics of the running time, as well as optimizing experimental parameters, we can calculate the uncertainty in the physics asymmetry. To do this, consider the covariance of the asymmetries of two elements (as defined in 4.38):

$$\sigma(\alpha_\kappa, \alpha_\beta) = \sigma_{\alpha_\kappa \alpha_\beta} = \sum_{s_1}^{\pm 1} \sum_{s_2}^{\pm 1} \frac{\partial \alpha_\kappa}{\partial Y_{s_1}^\kappa} \frac{\partial \alpha_\beta}{\partial Y_{s_2}^\beta} \sigma_{Y_{s_1}^\kappa Y_{s_2}^\beta} \quad (4.43)$$

The indices s_1 and s_2 refer to the different spin states of the neutron beam, in this case just up and down. Events that occur in different spin states will not be correlated, so the signal covariances are zero unless $s_1 = s_2$. Therefore, we can neglect the implicit derivatives, and rewrite the covariance in the uncertainty as:

$$\sigma_{\alpha_\kappa \alpha_\beta} = \frac{\partial \sigma_\kappa}{\partial Y_+^\kappa} \frac{\partial \sigma_\beta}{\partial Y_+^\beta} \sigma_{Y_+^\kappa Y_+^\beta} + \frac{\partial \sigma_\kappa}{\partial Y_-^\kappa} \frac{\partial \sigma_\beta}{\partial Y_-^\beta} \sigma_{Y_-^\kappa Y_-^\beta}$$

In addition, since the signal yields for different spin states are almost identical, the energy variances for the different spin states are equal: $\sigma_{Y_+^\kappa Y_+^\beta} = \sigma_{Y_-^\kappa Y_-^\beta} = \sigma_{Y^\kappa Y^\beta}$. We can further simplify the expression for the asymmetry covariance in the following way:

$$\sigma_{\alpha_\kappa \alpha_\beta} = \left[\frac{\partial \sigma_\kappa}{\partial Y_+^\kappa} \frac{\partial \sigma_\beta}{\partial Y_+^\beta} + \frac{\partial \sigma_\kappa}{\partial Y_-^\kappa} \frac{\partial \sigma_\beta}{\partial Y_-^\beta} \right] \sigma_{Y^\kappa Y^\beta}$$

Carrying out the partial derivatives and simplifying, we get:

$$\sigma_{\alpha_\kappa \alpha_\beta} = \frac{1}{G_\kappa G_\beta} \left[\frac{4(Y_+^\kappa Y_+^\beta + Y_-^\kappa Y_-^\beta)}{(Y_+^\kappa + Y_-^\kappa)^2 (Y_+^\beta + Y_-^\beta)^2} \right] \sigma_{Y^\kappa Y^\beta}$$

Due to the high statistics in each pulse, we have $Y_+^\kappa = Y_-^\kappa = \langle E^\kappa \rangle = \bar{n}_\kappa \bar{Q}^\kappa$:

$$\sigma_{\alpha_\kappa \alpha_\beta} = \frac{1}{2G_\kappa G_\beta} \frac{\sigma_{Y^\kappa Y^\beta}}{\bar{n}_\kappa \bar{Q}^\kappa \bar{n}_\beta \bar{Q}^\beta} \quad (4.44)$$

Note that the factor of two in the denominator comes from the fact that two pulses are necessary to calculate α_κ . To evaluate this, we need to define the quantity $\sigma_{Y^\kappa Y^\beta}$, which is the covariance of two signal energies. Each observable energy has a term on the order of α (4.36), so the covariance will have terms on the order of α or α^2 :

$$\sigma_{Y^\kappa Y^\beta} = \langle Y^\kappa Y^\beta \rangle - \langle Y^\kappa \rangle \langle Y^\beta \rangle = \langle E^\kappa E^\beta \rangle - \langle E^\kappa \rangle \langle E^\beta \rangle + O(\alpha) + O(\alpha^2) \quad (4.45)$$

For this experiment, the factor α is on the order of 10^{-7} . Any terms in the above expression with one or more coefficients of α will be 10^7 times smaller than the rest, so we can safely neglect them:

$$\sigma_{Y^\kappa Y^\beta} = \langle E^\kappa E^\beta \rangle - \langle E^\kappa \rangle \langle E^\beta \rangle \quad (4.46)$$

The solution in equation 4.41 expresses the product on the right side of equation 4.46 in terms of pulse averages. From (C.8), we have the value of the term on the left:

$$\langle E^\kappa E^\beta \rangle = \widehat{\hat{n}_{\kappa\beta} Q^\kappa Q^\beta} + \bar{n}_\kappa \bar{Q}^\kappa \bar{n}_\beta \bar{Q}^\beta \quad (4.47)$$

Substitute this into (4.46) to get an expression for the signal covariance:

$$\sigma_{Y^\kappa Y^\beta} = \widehat{\hat{n}_{\kappa\beta} Q^\kappa Q^\beta} + \bar{n}_\kappa \bar{Q}^\kappa \bar{n}_\beta \bar{Q}^\beta - \bar{n}_\kappa \bar{Q}^\kappa \bar{n}_\beta \bar{Q}^\beta = \widehat{\hat{n}_{\kappa\beta} Q^\kappa Q^\beta} \quad (4.48)$$

Now we have an expression for the asymmetry covariance:

$$\sigma_{\alpha_\kappa \alpha_\beta} = \frac{\widehat{\hat{n}_{\kappa\beta} Q^\kappa Q^\beta}}{2G_\kappa G_\beta \bar{n}_\kappa \bar{Q}^\kappa \bar{n}_\beta \bar{Q}^\beta} \quad (4.49)$$

Finally, we invert the matrix of uncertainty covariances and sum the elements to calculate the uncertainty in the physics asymmetry:

$$\frac{1}{\sigma_\alpha^2} = \sum_{\kappa}^K \sum_{\beta}^K [\sigma_{\alpha\kappa\alpha\beta}]_{\kappa\beta}^{-1} \quad (4.50)$$

This method is demonstrated in section 5.3.

4.3.3 Diagonal Approximation

We can approximate the uncertainty in the asymmetry by summing only the inverse of the diagonal terms of the covariance matrix. This is equivalent to ignoring covariances and only calculating element variances. This approximation is valid when $l/a < 1$, where l is the length that reaction products travel, and a is the dimension of the cells:

$$\sum_{\kappa}^K \sum_{\beta}^K [\sigma_{\alpha\kappa\alpha\beta}]_{\kappa\beta}^{-1} \approx \sum_{\kappa}^K \frac{1}{\sigma_{\alpha\kappa}^2} \quad (4.51)$$

Consider the variances for the asymmetry of each element:

$$\sigma_{\alpha\kappa}^2 = \frac{\langle E^{\kappa^2} \rangle - \langle E^\kappa \rangle^2}{2G_\kappa^2 \bar{n}_\kappa^2 \overline{Q^{\kappa^2}}} \quad (4.52)$$

To evaluate this, we must calculate the expectation value of the diagonal elements of the signal covariance matrix:

$$\langle E^{\kappa^2} \rangle = \sum_n^{N_{mc}} \frac{e^{-\lambda_\kappa} \lambda_\kappa^n}{n!} \langle \sum_{i=1}^n Q_i^\kappa \rangle^2 = \sum_n^{N_{mc}} \frac{e^{-\lambda_\kappa} \lambda_\kappa^n}{n!} \left[n \overline{Q^{\kappa^2}} + (n^2 - n) \overline{Q^\kappa}^2 \right] \quad (4.53)$$

Now we can simplify the formula for the variance $\sigma_{\alpha\kappa}^2$:

$$\sigma_{\alpha\kappa}^2 = \frac{\bar{n}_\kappa \overline{Q^{\kappa^2}} + (\bar{n}_\kappa^2 + \bar{n}_\kappa) \overline{Q^\kappa}^2 - \bar{n}_\kappa \overline{Q^{\kappa^2}} - \bar{n}_\kappa^2 \overline{Q^\kappa}^2}{2\bar{n}_\kappa^2 G_\kappa^2 \overline{Q^{\kappa^2}}} = \frac{1}{2\bar{n}_\kappa} \frac{\overline{Q^{\kappa^2}}}{G_\kappa^2 \overline{Q^\kappa}^2} \quad (4.54)$$

This solution corresponds to the simplification of equation 4.49 with repeated indices. This approximation can be used as a check on the covariant solution in the appropriate regime.

4.3.4 Additional Computed Quantities

Aside from basic quantities such as (4.42), and (4.47), we will also calculate some additional factors. Define the efficiency of element κ , ϵ_κ , as:

$$\epsilon_\kappa = \frac{1}{N_{mc}} \sum_{i=1}^{N_{mc}} \delta_i^\kappa, \text{ where } \delta_i^\kappa = \begin{cases} 0, & \text{if } Q_i^\kappa = 0 \\ 1, & \text{if } Q_i^\kappa \neq 0 \end{cases} \quad (4.55)$$

The quantity ϵ_κ should converge, so the expected number of deposition events in element κ , n_{exp}^κ , can be predicted by multiplying the element efficiency by the number of experimental neutron events, N_{exp} :

$$n_{exp}^\kappa = \epsilon_\kappa N_{exp}$$

Define the multiplicity, ν , and energy efficiency η , of the chamber as:

$$\nu = \frac{1}{N_{mc}} \sum_i^{N_{mc}} \sum_\kappa^K \delta_i^\kappa = \sum_\kappa^K \epsilon_\kappa; \quad (4.56)$$

$$\eta = \frac{1}{N_{mc} Q^{n(^3He,T)p}} \sum_i^{N_{mc}} \sum_\kappa^K Q_i^\kappa \quad (4.57)$$

These are diagnostic outputs used to verify the simulation.

4.3.5 Simulation Precision

A total of $2.15 \cdot 10^9$ neutron events were calculated, from 64 nodes each generating 2^{25} neutron events. The geometry factors have a mean statistical precision on the order of 10^{-4} .

4.4 Simulation Outputs

4.4.1 Yields

Now that we have matched the beam profile, neutron spectrum, and ionization rates, we can compare the simulated yields to the measured ones to verify that they are similar (Figure 4.15 and Figure 4.16).

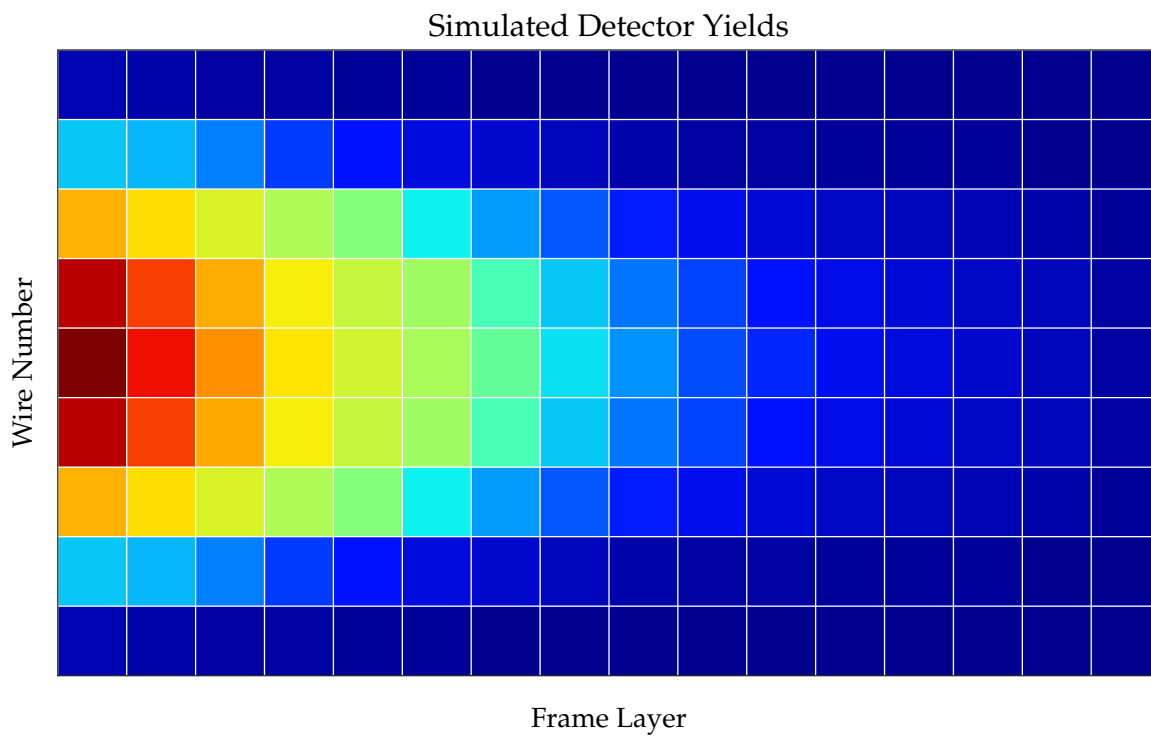


Figure 4.15: Simulated yields in the detector.

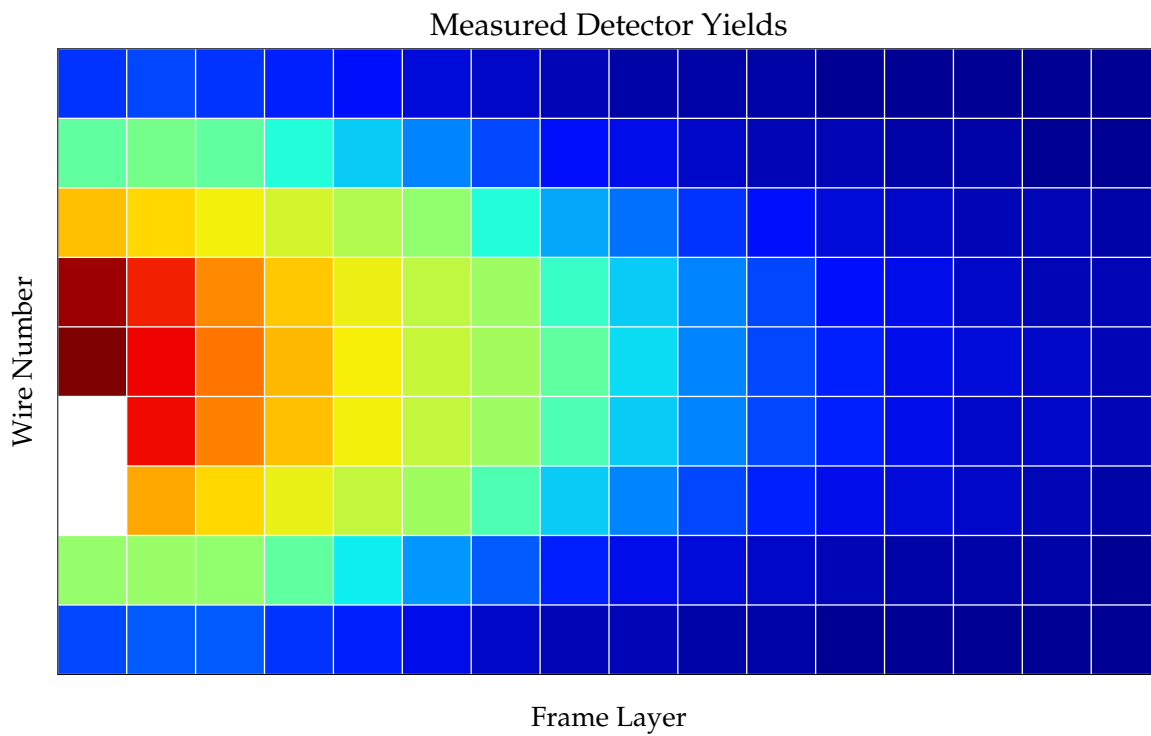


Figure 4.16: Measured yields in the detector.

The main discrepancy between the simulated and measured yields is that the ionization distribution appears wider in the observed signal than in the simulation. The simulation assumes a strict energy deposition according to Figure 4.13, which ignores possible impurities in the fill gas which can alter the mobility of the ions traveling in the gas change their stopping power from the assumed values. The type of the impurity, and the nature of the contamination, is unknown. Channels 5 and 6 are missing due to a short circuit.

We can also compare the yields under tightly collimated conditions. Collimation tests were performed on November 30, 2015, to test the effects of collimation and to compare them with simulated outputs in Figure 4.17 and Figure 4.18. The collimator was set to admit neutrons in rows 3 and 4 to measure the dispersion in the detector.

These measured outputs also show a wider distribution of ionization than in the simulation, but overall are a close approximation to the observed yields under collimation. The aggregate error in comparison between the simulated and measured yields, with and without collimation, is on the order of 10^{-3} .

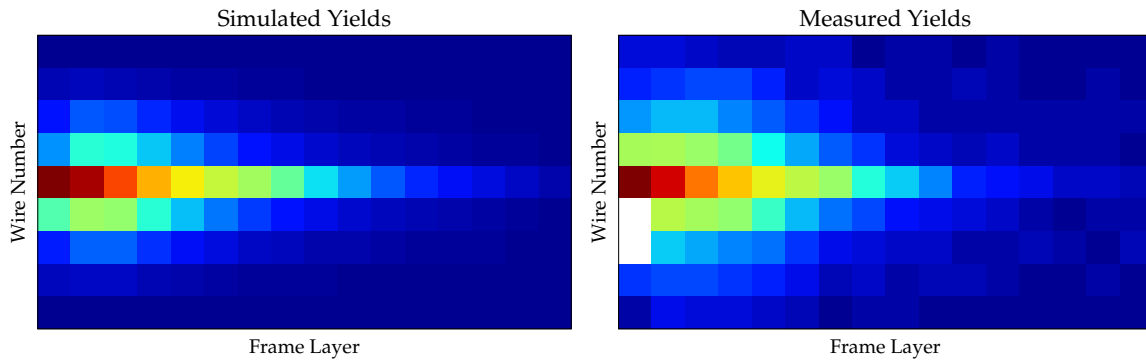


Figure 4.17: Collimation scan in row 4.

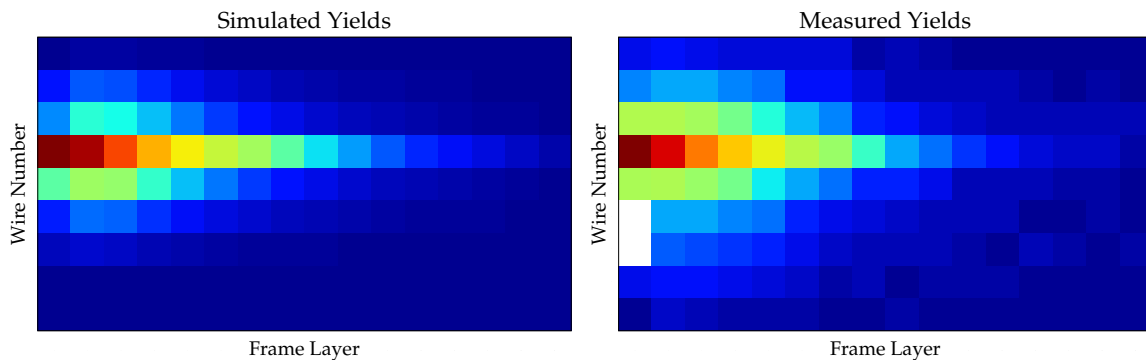


Figure 4.18: Collimation scan in row 3.

4.4.2 Geometry Factors

As detailed in the analysis section, in order to extract the physics asymmetries from each signal wire, the geometry factors for each "active volume" must be calculated. These are the 1.9cmx1.9cm rectangular prisms surrounding each signal wire, bounded by four high-voltage wires. Each spacetime element of the chamber measures a different amount of energy, and a different section of the asymmetry distribution of the reaction products. These two effects must be corrected for before the different measurements can be combined to form one total asymmetry measured for the chamber. The geometry factors are necessary to perform this correction. Calculating these factors as accurately as possible from the physics of the experimental hardware is a very important component of our asymmetry calculation. For most of the analysis, the different time slices of the pulse are summed to provide a more manageable amount of variables, and increased precision when calculating asymmetries. The geometry factors (Figure 4.19) are used throughout Chapter 5 to scale and combine the measured yields in each channel to measure the physical asymmetries.

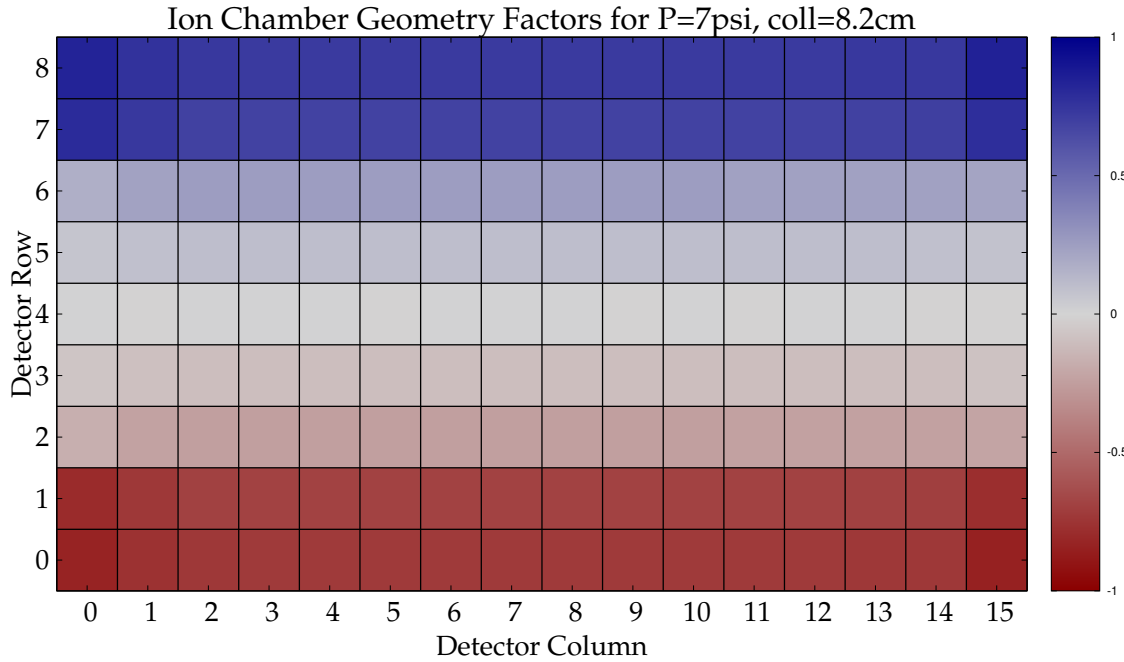


Figure 4.19: Value of the geometry factors across the ion chamber.

4.4.3 Optimizations

An additional objective of simulation is to estimate the precision of the experiment under different configurations, using equation 4.48, which can tell us the statistical contribution of each element of the total result (Figure 4.20). Specifically, the collimation width and gas pressure were optimized based on the simulation (Figure 4.21), but this method could have been used to improve the design of other experimental parameters if more time was available before the start of neutron production. The uncertainty in the extracted asymmetry varies with the configuration of the experiment. The pressure and collimation width both affect the ratio of element sensitivity to statistical precision. By simulating which values of the parameters minimize the uncertainty in the extracted asymmetry, we can optimize the configuration of the experiment. The collimation width and vessel pressure were both modified based on these results. There was insufficient time to reconfigure the vessel pressure after it was set to 48.2kPa, but simulations showed that the experimental precision could have been increased if it was lower. The collimator was adjusted more closely with the optimal values of the simulation. The final collimation was set to 8.2cm.

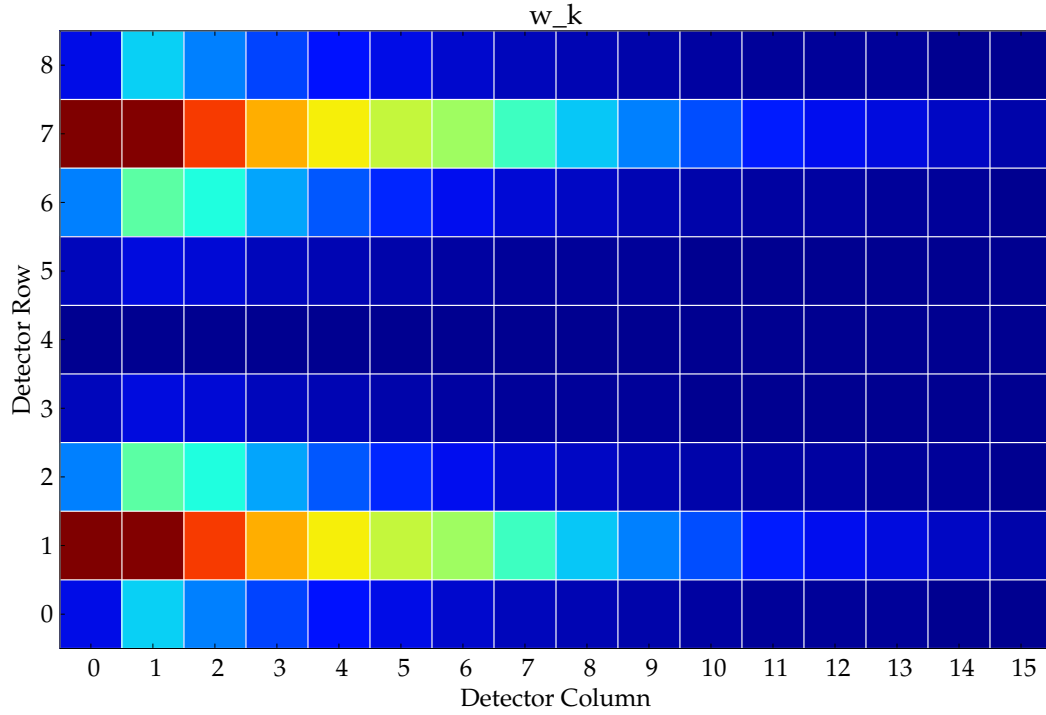


Figure 4.20: Simulated relative weight of each detector element.

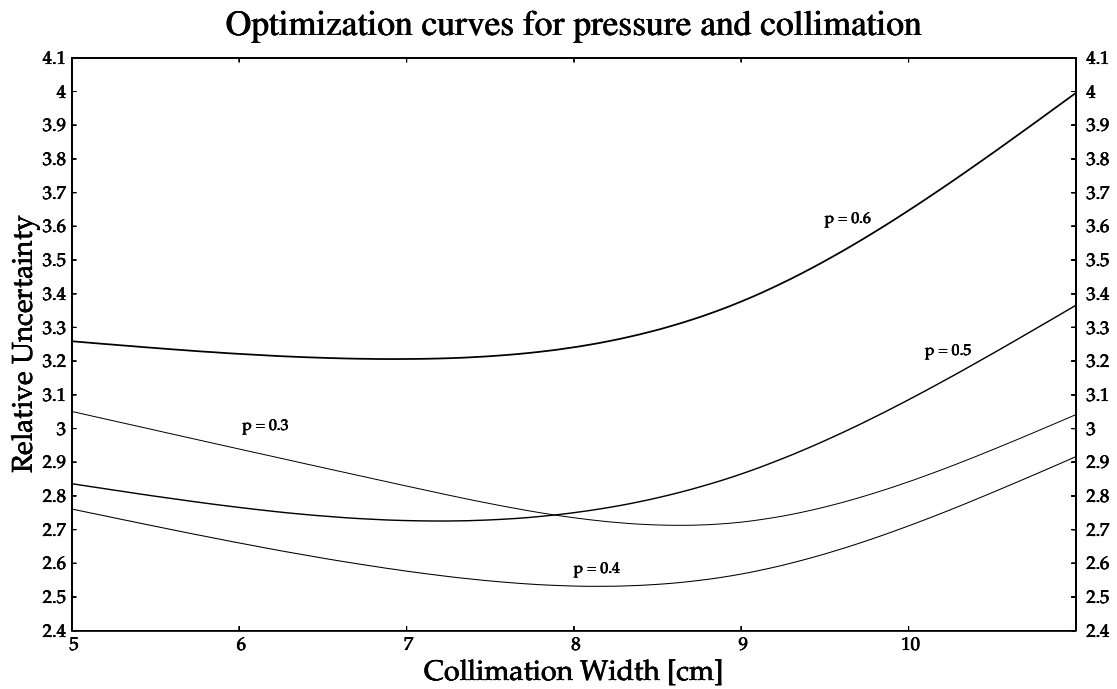


Figure 4.21: Uncertainty curves for different pressures (p) and collimation widths (x-axis).

5 ANALYSIS

The goal of the analysis is to extract the value of the physics asymmetry from a large number of noisy observations. There are 144 detector channels, each of which measured the ionization resulting from 10^9 neutron pulses. Over a twelve month period, significant variations were observed in the expected intensity, pulse shape, and consistency of neutron pulses observed at SNS Beamline 13.

The analysis presented in this chapter is my own work, and is the result of a significant effort to conduct my calculation as independently as possible from any other. I participated in group meetings, and many individual discussions, with other members of both the *npd* γ and $n^3\text{He}$ collaborations, both of which were conducted at SNS Beamline 13, and because of that, there are some analysis techniques that are common among analyses done by members of these groups. Many of the error sources I address here were either also identified by other collaboration members or in common with them. However, I chose my own techniques, devised my own algorithms, and wrote my own code to address these issues. The result I present here is my best determination of the asymmetries using my own tools, and serves as a separate measurement of them.

The analysis procedure is broken up into three parts for ease of computation and to accelerate data transfer. First, the five raw DAQ files are processed and consolidated into a single binary file (sections 5.1 and 5.2). Second, each run's data file is analyzed, and two files are produced: a binary file which contains the main asymmetries and variables, and a diagnostic ROOT file which contains histograms of several physical variables measured in each run (section 5.3). Third, all the analyzed run files are combined at once to determine the final asymmetry and (section 5.4). Results are discussed in (section 5.5).

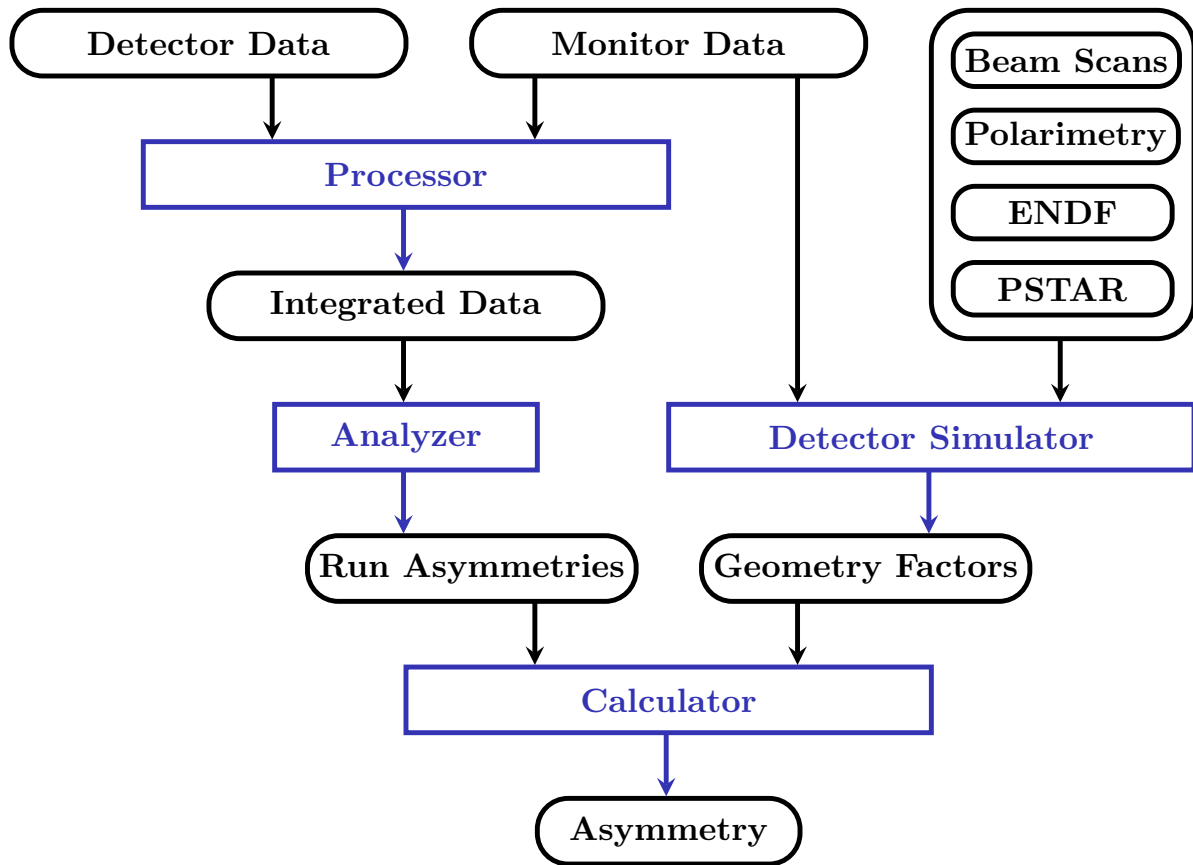


Figure 5.1: Flow of data (black ovals) through numerical operations (blue rectangles) to final result.

The flow of all the sources of data used in the following analysis is outlined in Figure 5.1. Note that no detector data is used in calculating the geometry factors.

5.1 Data Structure and Acquisition

During twelve months of the experimental operation, 55,754 gigabytes of data were collected. Data recording was broken up into small groups indexed by number and time, called runs. The nominal length of a run is 24,996 pulses, which takes approximately six minutes and 56.6 seconds to observe and record. For each run, five files are recorded, one for the diagnostic DAQ, and four which cover all the detector channels. The DAQs all have 24-bit precision, and the samples are stored as single-precision numbers.

The four detectors allocated to record signals from the wire chamber are given the IDs 21, 22, 23, and 24. These four detectors were run at a 50kHz sampling rate, and 16 samples were averaged together to produce one recorded data point. Taking into account the header and write times, each detector channel has 49 samples per pulse. The detector timebins are $320\mu\text{s}$ long. A small recording delay desynchronizes the runs from each other.

We can denote these channels

$$L_{nt}^k, \tag{5.1}$$

where k is the channel number, n is the run number, and t is the timebin index. The DAQ channels are distributed throughout the detector as shown in Table 5.1.

The diagnostic DAQ was run at 100kHz and did not use averaging, giving 1624 samples per pulse. The monitor timebins are $10\mu\text{s}$ long. This DAQ has the ID of 30, and recorded two channels: the output from M1 and the input current to the the spin rotator. Since this is the higher-frequency DAQ, denote its samples as H :

$$\begin{aligned} H_{nt}^0 \\ H_{nt}^1 \end{aligned} \tag{5.2}$$

H^0 is the monitor signal, and H^1 is the spin rotator current.

Table 5.1: Low-frequency DAQ channel map

Frame Number	DAQ ID	DAQ Subchannels
1	21	0 1 2 3 4 5 6 7 8
2	23	0 1 2 3 4 5 6 7 8
3	21	9 10 11 12 13 14 15 16 17
4	23	9 10 11 12 13 14 15 16 17
5	21	24 25 26 27 28 29 30 31 32
6	23	24 25 26 27 28 29 30 31 32
7	21	33 34 35 36 37 38 39 40 41
8	23	33 34 35 36 37 38 39 40 41
9	22	0 1 2 3 4 5 6 7 8
10	24	0 1 2 3 4 5 6 7 8
11	22	9 10 11 12 13 14 15 16 17
12	24	9 10 11 12 13 14 15 16 17
13	22	24 25 26 27 28 29 30 31 32
14	24	24 25 26 27 28 29 30 31 32
15	22	33 34 35 36 37 38 39 40 41
16	24	33 34 35 36 37 38 39 40 41

5.2 Data Processing

Before processing, the integrity of the raw data files is checked through file sizes, checksums, and the synchronization between channels. Analyzing approximately 56Tb of raw experimental data at once over a 1Gbit network connection, under ideal hardware conditions, would take six days just for reading data from memory (without including any logical processing time). This is prohibitive when testing systematic errors or looking for effects which occur over the whole data set. For this reason, the raw data is pre-processed by time-integration and various other methods to extract the quantities needed for analysis. This reduces the total time spent reading data from memory. The monitor and rotator current signals are integrated over the full range:

$$\begin{aligned}
M_n &= \sum_{t=1}^{1624} H_{nt}^0 \\
R_n &= \sum_{t=1}^{1624} H_{nt}^1
\end{aligned} \tag{5.3}$$

The bounds of the detector signals are truncated, as explained in section 5.2.2:

$$Y_n^k = \sum_{t=5}^{43} L_{nt}^k \quad (5.4)$$

Along with the simplifications detailed in the following sections, the total size of the data is reduced by 97.77%, significantly reducing the time necessary to perform analysis.

5.2.1 Pulse Shape

Recall that the neutron burst travels 15.119m (equation 4.10) from the Mercury target to the ^3He target, and also passes through two choppers (section 3.1.3). By the time they reach the ^3He target, the pulse contains some very long-wavelength neutrons, from previous pulses, which pass through both choppers (section 5.3.3). An example of the isolated pulse shape from channel 4 is shown in Figure 5.2. Note the largest component of long-wavelength neutrons visible in the fifth recording frame. These longer wavelength neutrons will have partial or incorrect spin polarization, and this systematic error source must be removed in the analysis, as discussed in section 5.2.3.

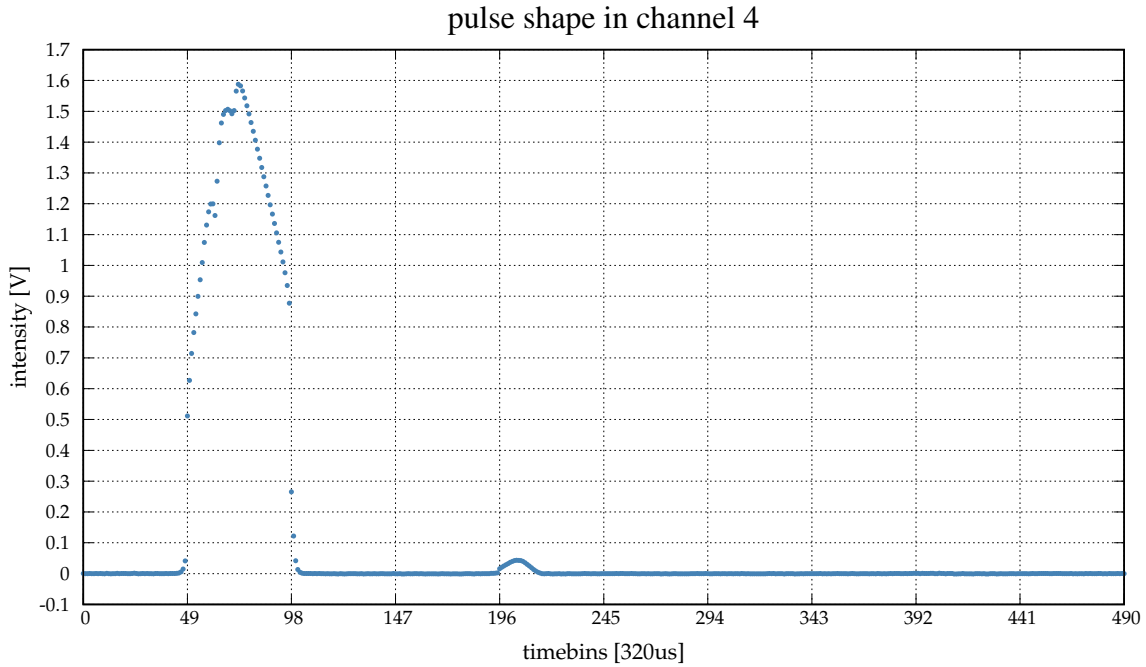


Figure 5.2: Isolated single pulse shape from first wire layer.

5.2.2 Pulse Overlap

Note also in Figure 5.2 that the edges of the 60Hz neutron pulse overlap with adjacent recording windows. This overlap varies slightly in different recording frames, and under different accelerator conditions. Because the spin rotator alternates the spin orientation of successive pulses, adjacent neutron pulses will have opposite parity. To eliminate contamination from opposite polarization neutrons, five timebins have to be cut from the beginning and the end of each pulse before integration to eliminate this effect under all conditions.

Additionally, there is a possibility of overlap ionization signal from ion drift times. A detailed study of these effects was done in [50] by Mark McCrea, who determined that there is no space charge accumulation in the chamber, and the longest charge collection times are 1.1ms, which is the length of three detector timebins (Figure 5.3). The edge samples cut due to the pulse overlap are therefore sufficient to eliminate drift time overlap as well. This cut does not have a significant effect on the statistical precision, since the intensities at the beginning and the end of the pulse are low due to the chopper opening and closing times.

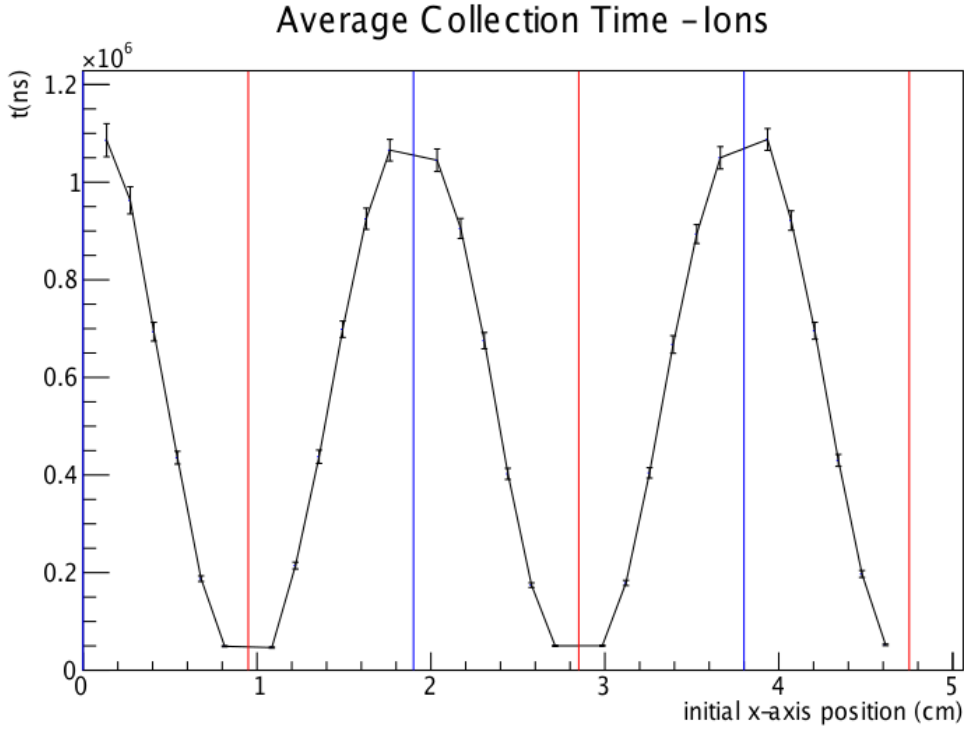


Figure 5.3: Simulation of ion drift times [50].

5.2.3 Dropped Pulses

The SNS intentionally omits one proton pulse every 10 seconds, or every 600th pulse. This means that when all other run conditions are good, every 300th observation of the asymmetry will be missed. As observed in section 5.2.1, the missing long-wavelength neutrons from dropped pulses can produce systematic error in subsequent pulse pairs. The visible error due to these missing neutrons extends for approximately ten total pulses.

In addition to these regularly scheduled omissions, the accelerator randomly fails to deliver a proton pulse from the accumulator ring. In order to ensure the consistency of the sequences used for analysis, the irregularly dropped pulses have to be identified as well as distinguished from the regularly dropped ones. The following algorithm finds the regular dropped pulse set, at unknown locations, in a run with an unknown number of irregular zeroes in it:

1. During normal operation during 2015, the accelerator power never drops below a minimum value of around 600kW. Identify all pulses which are considered "dropped," meaning that their integrated pulse intensity is below the integrated ADC value measured at the lowest normal operating power, around 2500 (see Figure 5.4). For each run, record the time index t_i of every pulse which has an integrated intensity below this value.

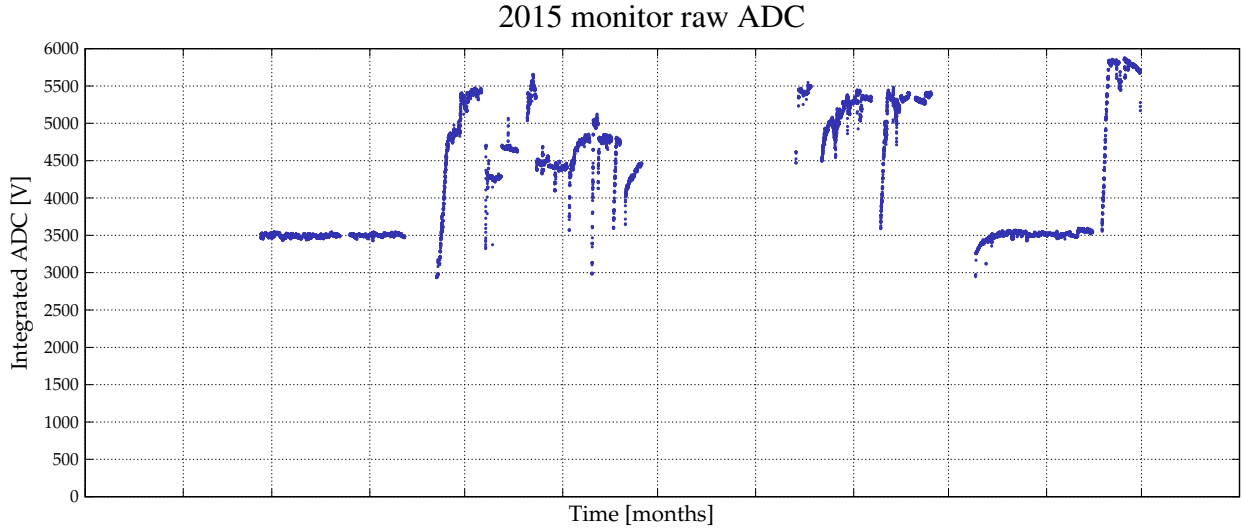


Figure 5.4: Neutron flux measured in beam monitor over the one-year dataset.

2. For every time index t_i recorded in step 1, calculate the distance d_{ij} between t_i and every other index $t_{j \neq i}$. If there are N total dropped pulses in a run, there will be $N-1$ distances for each element t_i . Construct the set $\mathbb{T}_i$ consisting of all the distances d_{ij} between t_i and $t_{j \neq i}$.
3. Count the number of elements in $\mathbb{T}_i$ which are equal to 600. This is the expected distance between regularly dropped pulses. Call this $N(\mathbb{T}_i)$.
4. Construct a set D of all the $N(\mathbb{T}_i)$. Search D for the subset of maximal elements. This is the subset of elements which contain all of the distances from the regularly dropped pulses. Now that these sets are identified, the indices of $\mathbb{T}_i$ are the locations of the all of the regularly dropped pulses in the set. Find the first one of these and begin sequence counting from that point.

Only sequences which have no irregularly dropped pulses are kept for analysis, which means that the regularly dropped pulse set is uniquely determined. The frequency of random pulse failures is shown in Figure 5.5.

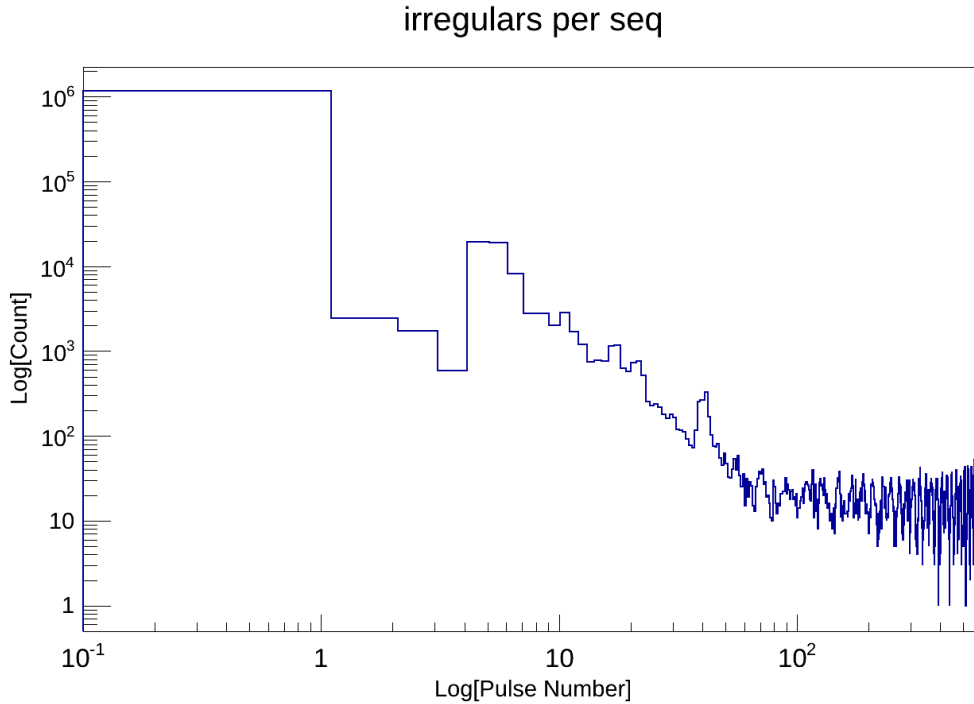


Figure 5.5: Distribution of irregularly dropped pulses per sequence.

5.2.4 Sequence Alignment

The DAQ recording window is not synchronized with the regular dropped pulses, and is in fact randomly aligned (Figure 5.6). This means that the beginning of the run can be located anywhere within a 600-pulse sequence. The algorithm in (5.2.3) identifies the sequence locations. Depending on where the DAQ begins recording, a good run can have 41 or 40 complete sequences in it, which would mean that 396 pulses or 996 pulses must be discarded, since only complete sequences are used. This is the primary source of data cut during processing.

5.2.5 Channel Polarity

Due to imperfections in the electronics wiring, some of the channels have improper detector response. Channels [0,5] and [0,6] create a short circuit, and so data from these two is excluded from the main analysis algorithm. In addition, other channels can show inverted polarity during regular data production (Figure 5.7). Typically, this occurs in channels [2,35], [3,35], and [3,39]. The data processor performs a dynamic polarity check in each run to determine which signals are inverted, and corrects them, before performing time-integration.

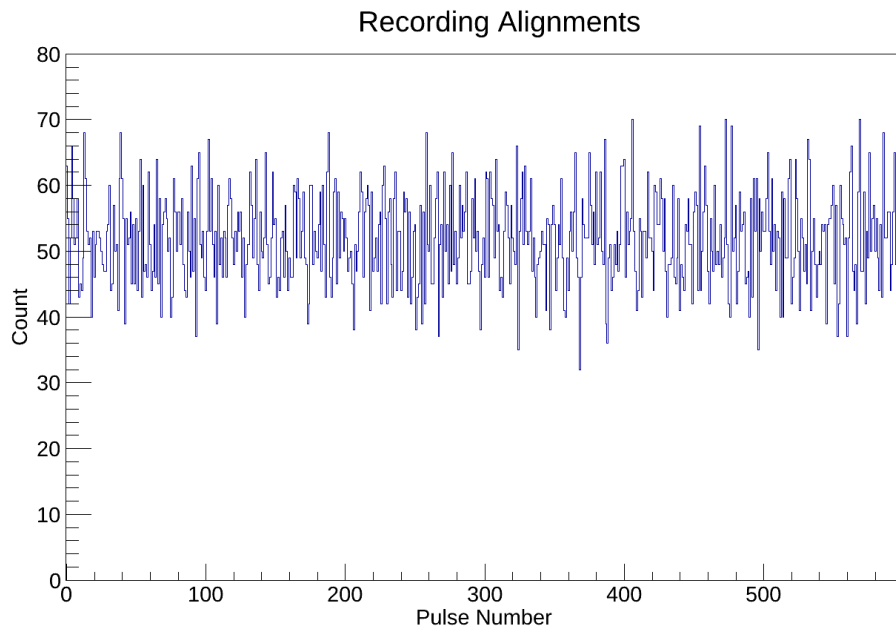


Figure 5.6: Alignment between first sequence and beginning of run record.

5.2.6 Signal Pedestals

Each signal has a background in addition to the current from gas ionization, and a pedestal subtraction method is used to correct the raw signals before they are integrated. In a sequence, there are $600 \cdot 49 = 29,400$ samples, also written as L_{nt}^k . These are the low-frequency DAQ samples before data processing. We search the entire sequence for the minimum value of L_{nt}^k , and subtract that value from every L_{nt}^k in the sequence before they are integrated in section 5.4. This value is typically located in the dropped pulse samples, but may not be, so every sequence has its own pedestal which is dynamically determined. Every pulse in a sequence is given the same pedestal correction. Since these pedestals are re-determined every 600 pulses, they should compensate for slow drifts in the background.

A typical detector signal can be seen in the positive part of Figure 5.7. The background in this signal has contributions from the physics background as well as electronics. Neither the shape nor the intensity of the neutron pulses are consistent enough to use it to attempt to fit the background as part of the pulse shape. The electronics level is measured from beam-off data runs, and contributes to the systematic error of the experiment.

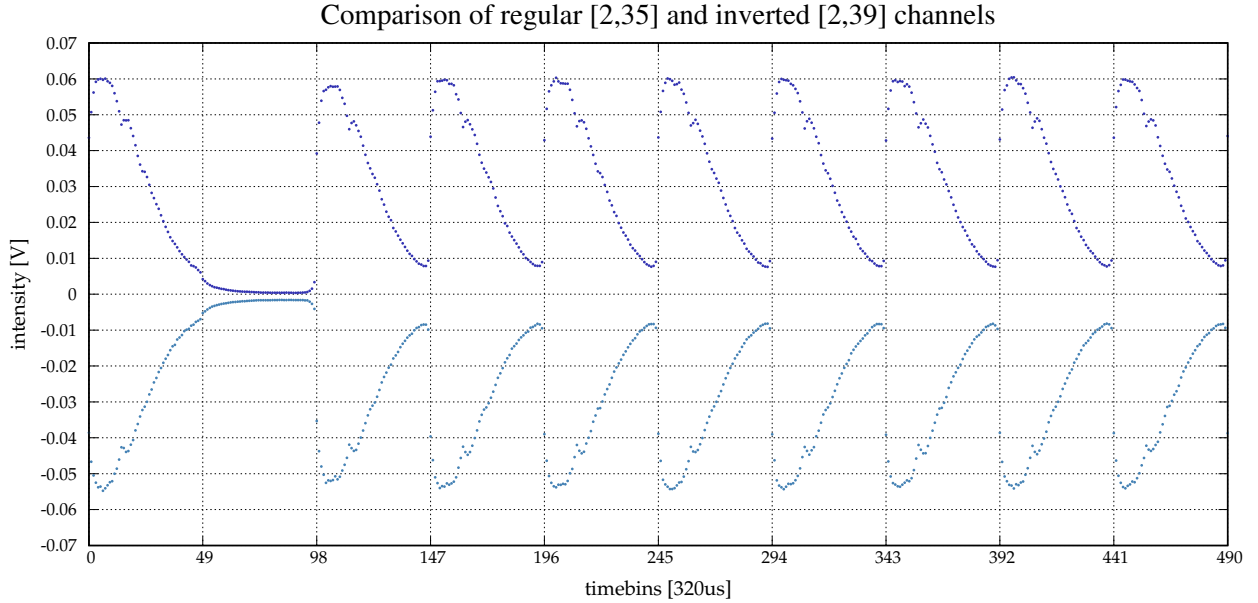


Figure 5.7: Example of inverted channel. The second recording frame is the dropped pulse.

There is no clear way to separate the existing electronics background from the physics background using the same amplifier and DAQ system. The physics background can be partially observed from data taken when the neutron beam is off, but it is not guaranteed that these conditions match those that exist when the beam is on, although they can provide a bound on the electronic asymmetry. Furthermore, these observations are intermittent and would have to be fitted in some way to apply them to the beam-on data.

Instead of assuming we can characterize the background behavior, or introducing an arbitrary fit for it, we can resolve the problem by characterizing the magnitude of the pedestal error, and recording two distinct signal normalizations.

We should first determine what effect that the pedestal magnitude has on the value of the asymmetry. Consider the difference between α with a maximum pedestal, b , and α' with no pedestal:

$$\delta = \alpha - \alpha' = \left(\frac{Y^+ - Y^-}{Y^+ + Y^-} \right) - \left(\frac{Y^+ - Y^-}{Y^+ + Y^- + 2b} \right) = \left(\frac{Y^+ - Y^-}{Y^+ + Y^-} \right) \left(\frac{2b}{Y^+ + Y^- + 2b} \right) \quad (5.5)$$

This difference is characterized by the ratio between the pedestal and the signal:

$$\eta \equiv \frac{2b}{Y^+ + Y^-} \quad (5.6)$$

For typical good pulses, $\eta \ll 1$, so we can expand this quotient:

$$\delta = \left(\frac{Y^+ - Y^-}{Y^+ + Y^-} \right) \left(\eta - \eta^2 + \eta^3 - \dots \right) \quad (5.7)$$

The size of the error introduced into our calculation by the pedestal choice is the ratio of δ to α :

$$\epsilon = \frac{\delta}{\alpha} = \left(\eta - \eta^2 + \eta^3 - \dots \right) \quad (5.8)$$

For runs which pass the other quality checks, ϵ is on the order of 10^{-4} , so the pedestal choice will not introduce any false asymmetry into the result.

5.2.7 Chopper Phase

First, we check the phases of the choppers, and make sure they are operating at the correct phase. The choppers cause the steep leading and falling edges of the neutron pulse, and as such, their operation alters the total flux observed in each pulse. A deviation in the phase of the chopper could possibly alter the total yield in neutron pulses asymmetrically, which could cause a false asymmetry. Therefore, the phase variation must be checked. The chopper controller restores the phases to the correct value over time, but this process is long compared to our basic asymmetry time, $\frac{1}{30}s$. As long as the chopper phase stays relatively constant over the length of a sequence, a false asymmetry will not be introduced, even if it changes from run to run.

We approximate the phase of the chopper by calculating the times at which the pulse intensity is equal to the half of its maximum value. These points occur in the steepest part of the slopes, so they are sensitive to changes in the phase of the choppers. Since we have a discrete set of sampling points, we determine the location of this half-maximum value using a six-point Akima interpolator for the rising and falling edge of every pulse. To detect chopper phases which may create a false asymmetry, we calculate the mean value of the phases over a whole sequence:

$$\bar{\phi}^i = \frac{1}{N_n} \sum_n \phi^i \quad \bar{\phi}^f = \frac{1}{N_n} \sum_n \phi^f \quad (5.9)$$

Then, the difference between the individual pulse's phase and the mean value over the sequence is computed.

$$\Delta_\phi = \phi - \bar{\phi} \quad (5.10)$$

This can be used to measure how much the chopper phase drifts at any timescale. We use the sequence as the relevant timescale, and calculate the chopper drift in that time. A plot of Δ_ϕ values, both before and after being cut, is shown in Figure 5.8.

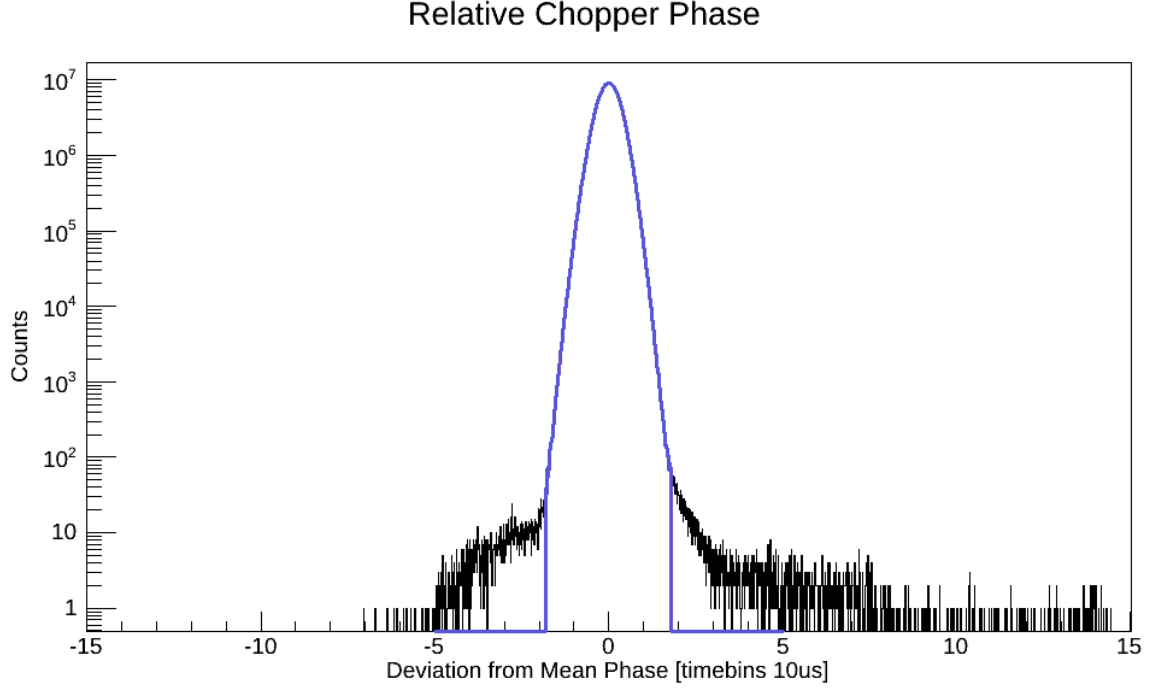


Figure 5.8: Relative chopper phase distribution.

5.2.8 Spin Rotation Current

Second, we check to make sure that every pulse in a sequence is given the correct spin rotation. A malfunction in either the spin rotation or the spin assignment could cause false asymmetries, since any deviation from perfect spin orientation will not measure the spin-dependent asymmetry we are trying to measure. The spin rotation of the pulses is verified by measuring the current sent to the spin rotator, which is very consistent. To check the parity of any neutron pulse, the integrated current to the spin rotator, R_n , is checked to make sure it is within normal bounds.

Once the spin orientation of every pulse is known, we can determine the correct spin orientation of a run in the following way. First, record the spin orientation of every regularly dropped pulse (pulse 1 in every sequence) to determine the parity of the sequence. The spin orientation of the run, as a whole, is determined by the spin orientation of all the sequences in it. A run must have unanimous agreement among all sequences. Then, every pulse in the run is checked to make sure that it has the correct spin orientation relative to the parity of the run. This way, we ensure that all pulses in a run are rotated the correct way.

5.2.9 Pulse Deconvolution

As referenced in section 5.2.2, the neutron pulse arriving at Beamline 13 is "longer" than the 60hz window used for observations in n^3He , and the choppers enable some long-wavelength neutrons to pass through into later pulses. Under ideal operating conditions, the composition of every pulse would be identical, and this leakage would not alter the outcome of the measurement, since the same proportion of every pulse would contain depolarized neutrons, and we would compare pulses of identical polarization. However, when a pulse is dropped, these long wavelength neutrons are missing from later pulses, and they can cause systematic error. We can clearly observe the missing neutrons based on the pulse shape shown in Figure 5.9. (In Figure 5.9, component 1 contains approximately 90% of the intensity. Note that Figure 5.9 is a deconvolution of the beam monitor signal, not the detector signal, so part of the primary pulse is shifted back into pulse 0, and the first wraparound peak is shifted back into pulse 3). To be conservative, we cut the first ten pulses from every sequence to make sure that every remaining pulse has the proper intensity.

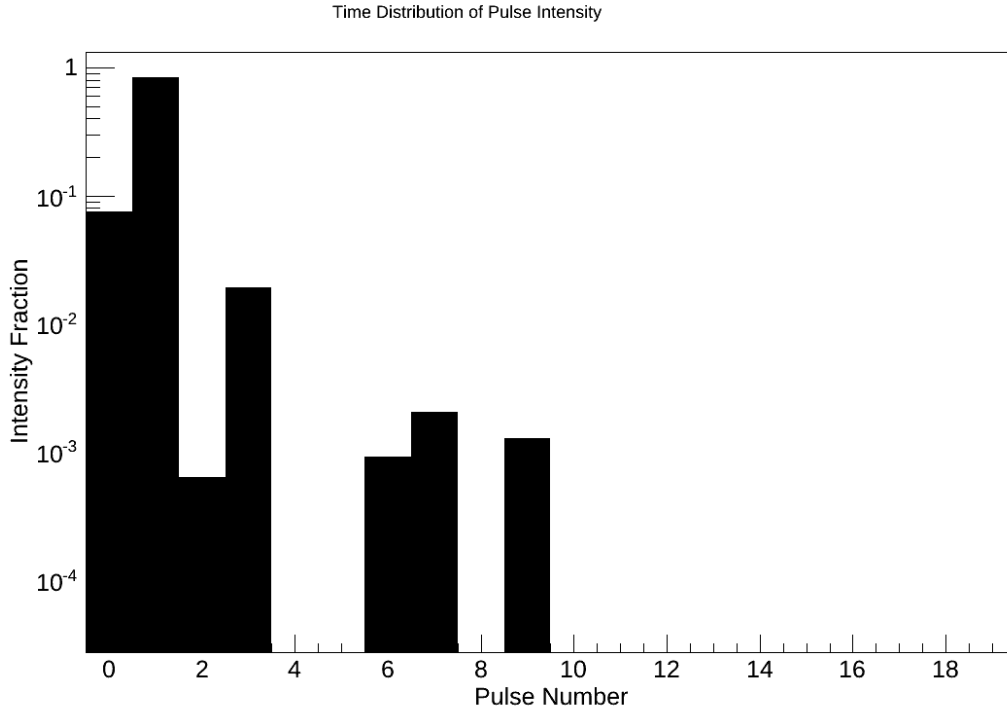


Figure 5.9: Secondary components of pulse intensities.

5.2.10 Run Parity

Based on results found in section 5.6, there is an additional systematic error arising from the different asymmetries observed in "even" and "odd" sequences. If we divide the data set into two subsets, one for each parity, we find that their mean values are split. The physical source of this error was not precisely identified, but we can correct for it based on statistical arguments. The DAQ has a non-zero dead time in between recording runs, and as a consequence of this, the spin orientation of the runs is randomly assigned. In the parity-violating dataset, there are 15529 even runs and 15540 odd runs. An unbiased selector would be expected to produce a difference of 11 runs or fewer 95.5% of the time:

$$2 \sum_{r=N_o}^{N_r} \binom{N_r}{r} \left(\frac{1}{2}\right)^{N_r} = .9548 \quad (5.11)$$

where $N_r = N_e + N_o = 31,069$. This is just a binomial probability. Because of the random assignment of run parity, we can assume that this splitting is due only to the spin orientation of the run, and model the expectation value of the asymmetry as a parity-dependent splitting:

$$\langle \mathbf{A}_s^+ \rangle = A_p + \Delta \quad (5.12)$$

$$\langle \mathbf{A}_s^- \rangle = A_p - \Delta \quad (5.13)$$

where A_p is the physics asymmetry, and Δ is the asymmetry splitting due to the spin ordering of the run. To remove the effect of this splitting from our final result, we analyze the subsets separately and then average the results at the end:

$$\frac{\langle \mathbf{A}_s^+ \rangle + \langle \mathbf{A}_s^- \rangle}{2} = \frac{A_p + \Delta + A_p - \Delta}{2} = A_p \quad (5.14)$$

We can also calculate the estimate of this splitting:

$$\frac{\langle \mathbf{A}_s^+ \rangle - \langle \mathbf{A}_s^- \rangle}{2} = \frac{A_p + \Delta - A_p + \Delta}{2} = \Delta \quad (5.15)$$

5.2.11 Beam Polarization

The neutron pulses are not perfectly polarized by the supermirror polarizer and spin rotator. A fraction of each pulse will be unpolarized, meaning those neutrons have randomly oriented spins, and their net contribution to the observable asymmetry will be zero, but they will dilute the precision of the result. The polarization ratio of each pulse determines the significance each measurement of the asymmetry. The monthly polarimetry studies to measure the polarization and spin rotation efficiency can be used to calculate the weighted mean beam polarization, which we correct the asymmetry by:

$$\mathbf{p} = \frac{\int P(\lambda)S(\lambda)I(\lambda)d\lambda}{\int I(\lambda)d\lambda} = 0.9353 \quad (5.16)$$

Here, $P(\lambda)$ is the polarization efficiency as a function of wavelength, and $S(\lambda)$ is the spin rotation efficiency as a function of wavelength. The polarization efficiency $P(\lambda)$ and spin rotation efficiency $S(\lambda)$ were measured during monthly tests detailed in [44] (Figure 5.10). The product of those efficiencies, weighted by the signal intensity per wavelength, $I(\lambda)$, determines what fraction of each pulse contribute to the asymmetry measurement. The polarization correction, $\mathbf{p}$, is constant, and can be applied at the end of the calculation.

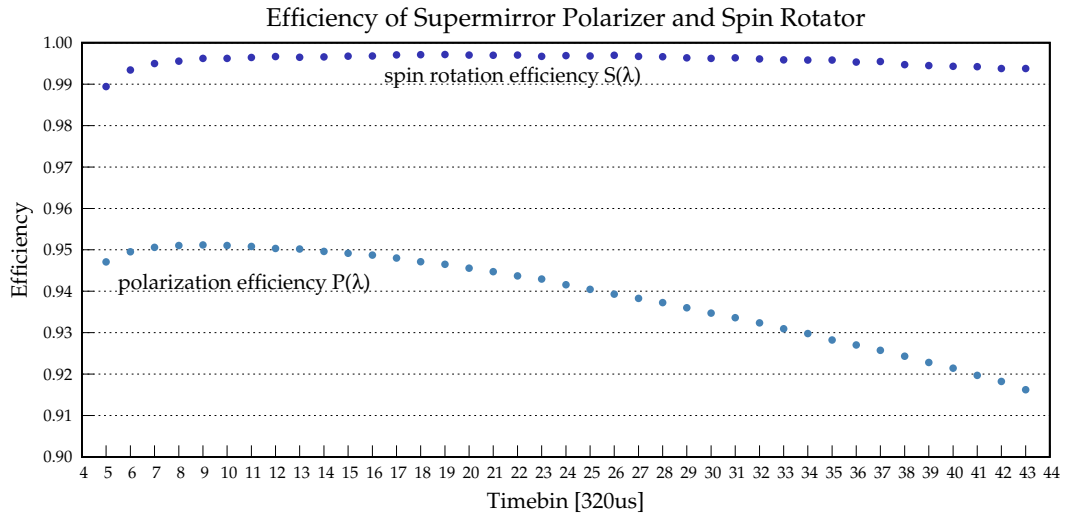


Figure 5.10: Polarization efficiencies in the included time range.

5.2.12 Anomalous Intensity Fluctuations

The accuracy of the measurement is dependent on the intensity of neutron pulses being randomly distributed around a stable mean value. These are not safe assumptions with SNS neutron pulses. Nonrandom behavior may cause systematic error in the result. The pulse intensity measured in the beam monitor can be used to eliminate these events without making any reference to the detector, by measuring the asymmetry in the intensity of neutron pulses before they reach the target. Using the integrated monitor signal from (5.3),

$$M_{rn} = \sum_{t=1}^{1624} H_{rnt}^0$$

we define the beam asymmetry as:

$$\boxed{\frac{M_{r,n} - M_{r,n+1}}{M_{r,n} + M_{r,n+1}}} \quad (5.17)$$

This is the asymmetry that is already present in the pulse intensities, before they reach the target. Pairs with the largest asymmetries are removed. This is plotted in Figure 5.11.

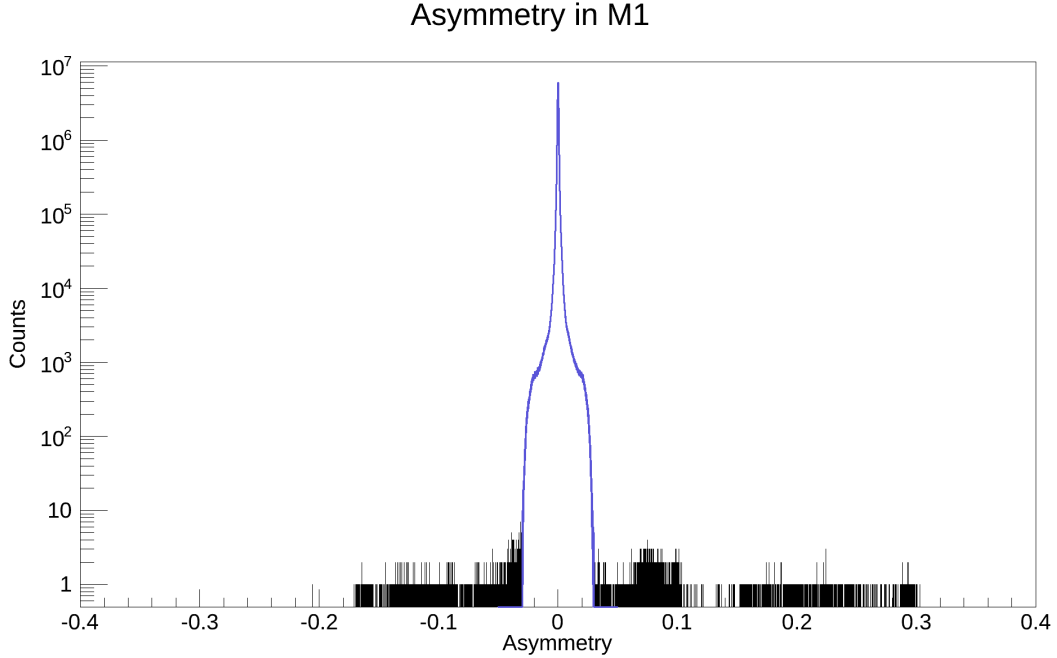


Figure 5.11: Asymmetry in the neutron pulse intensity.

5.3 Asymmetry Calculation

After being processed, the smallest unit of detector data is Y_n^k , which is the yield at pulse number n in detector k , defined in section 4.3 as

$$Y_n^k = \langle E_k(1 + s\alpha_p \cos \theta) \rangle, \quad (5.18)$$

where E_k is the mean value of the yield in channel k , s is the spin orientation of the neutron pulse, and α_p is the value of the physics asymmetry. We wish to combine our observables to extract a value for α_p . Recalling that our geometry factor G_k is defined as

$$G_k = \frac{\langle E_k \cos \theta \rangle}{\langle E_k \rangle}, \quad (5.19)$$

we construct an asymmetry for a single channel

$$\frac{1}{G_k} \frac{Y_+^k - Y_-^k}{Y_+^k + Y_-^k} = \frac{\langle E_k \rangle}{\langle E_k \cos \theta \rangle} \alpha_p \frac{2\langle E_k \cos \theta \rangle}{2\langle E_k \rangle} = \alpha_p. \quad (5.20)$$

Each instance of equation 5.20 is one observation of the physics asymmetry. For ease of computation, the geometry factor scaling will be performed separately during diagonalization. These differences will be combined over each channel and pair to form an asymmetry value measured by whole sequence, α_s . These α_s will then be combined to form one value measured by the entire experiment.

5.3.1 Weight Set

Each pair of pulses represents an independent measurement of the physics asymmetry for each detector. There are 144 channels, 24996 pulses per run, and over 30,000 runs in the good data set. We will combine them in two stages: first, by combining all measurements in a sequence to find a sequence asymmetry, and second, to combine all sequence asymmetries to find a total asymmetry. We would like to combine a large set of independent observations in a way that produces the minimum uncertainty in the combined result.

We can derive this weighting. Let α_p be the weighted average of a set of asymmetries α_i :

$$\alpha_p = \sum_i w_i \alpha_i = \mathbf{w}^\top \mathbf{A} \quad (5.21)$$

Here, w_i are the weights, and $\mathbf{A}$ is the vector of the channel asymmetries, where

$$\sum_i w_i = 1. \quad (5.22)$$

The uncertainty of this measurement is:

$$\sigma_p^2 = \sum_i \sum_j w_i \sigma_{ij} w_j = \mathbf{w}^\top \mathbf{C} \mathbf{w}, \quad (5.23)$$

where σ_{ij} , element ij of the covariance matrix $\mathbf{C}$, is the covariance between α_i and α_j . We would like to minimize the uncertainty with respect to the weights, w_i . We can do this by enforcing the constraint, equation 5.22, using a Lagrange multiplier:

$$\frac{\partial}{\partial w_k} \left[\sum_i \sum_j w_i w_j \sigma_{ij} - \lambda \left(\sum_i w_i - 1 \right) \right] \quad (5.24)$$

The familiar weight solution requires the inverse of $\mathbf{C}$:

$$w_i = \frac{\sum_j \mathbf{C}^{-1}_{ij}}{\sum_{ij} \mathbf{C}^{-1}_{ij}} \quad \mathbf{w} = \frac{\mathbf{C}^{-1} \mathbf{I}}{\mathbf{I}^\top \mathbf{C}^{-1} \mathbf{I}}, \quad (5.25)$$

where $\mathbf{I}$ is the identity vector. The optimally weighted asymmetry and uncertainty are:

$$\alpha_p = \frac{\sum_i \alpha_i \sum_j \mathbf{C}^{-1}_{ij}}{\sum_{ij} \mathbf{C}^{-1}_{ij}} = \frac{\mathbf{I}^\top \mathbf{C}^{-1} \mathbf{A}}{\mathbf{I}^\top \mathbf{C}^{-1} \mathbf{I}} \quad \sigma_p^2 = \frac{1}{\sum_{ij} \mathbf{C}^{-1}_{ij}} = \frac{1}{\mathbf{I}^\top \mathbf{C}^{-1} \mathbf{I}} \quad (5.26)$$

(The right side of equation 5.26 requires the statistical correction $1/\sqrt{N}$.) As a numerical check on our result, we can rewrite and expand equation 5.25 in terms of diagonal modes by finding the eigendecomposition of $\mathbf{C}$. If

$$\mathbf{C} = \mathbf{Q}^{-1} \mathbf{D} \mathbf{Q}, \quad (5.27)$$

where $\mathbf{D}$ is diagonal and $\mathbf{Q}$ is the set of eigenvectors, then

$$\begin{aligned}\alpha_p &= \frac{\mathbf{I}^\top \mathbf{C}^{-1} \mathbf{A}}{\mathbf{I}^\top \mathbf{C}^{-1} \mathbf{I}} = \frac{\mathbf{I}^\top \mathbf{Q}^{-1} \mathbf{Q} \mathbf{C}^{-1} \mathbf{Q}^{-1} \mathbf{Q} \mathbf{A}}{\mathbf{I}^\top \mathbf{Q}^{-1} \mathbf{Q} \mathbf{C}^{-1} \mathbf{Q}^{-1} \mathbf{Q} \mathbf{I}} = \frac{(\mathbf{Q} \mathbf{I})^\top \mathbf{D}^{-1} (\mathbf{Q} \mathbf{A})}{(\mathbf{Q} \mathbf{I})^\top \mathbf{D}^{-1} (\mathbf{Q} \mathbf{I})} \\ \sigma_p^2 &= \frac{1}{\mathbf{I}^\top \mathbf{C}^{-1} \mathbf{I}} = \frac{1}{\mathbf{I}^\top \mathbf{Q}^{-1} \mathbf{Q} \mathbf{C}^{-1} \mathbf{Q}^{-1} \mathbf{Q} \mathbf{I}} = \frac{1}{(\mathbf{Q} \mathbf{I})^\top \mathbf{D}^{-1} (\mathbf{Q} \mathbf{I})}.\end{aligned}\tag{5.28}$$

Note that since $\mathbf{Q}$ is orthogonal, $\mathbf{Q}^\top = \mathbf{Q}^{-1}$. In this new basis, $\mathbf{Q} \mathbf{A}$ are the diagonal asymmetry modes. Since $\mathbf{D}$ is diagonal, the variances are independent. From equation 5.28, we can see that the values are equivalent. Now we are able to calculate the optimally weighted asymmetry using a numerically different method, and we can also calculate the value of the asymmetry for each linearly independent mode of the detector.

5.3.2 Calculation Procedure

The basic asymmetry unit is composed of two pulses of opposite spin, Y_m^k and Y_n^k , where k is the channel index:

$$\alpha^k = \frac{Y_m^k - Y_n^k}{Y_m^k + Y_n^k}\tag{5.29}$$

The $n^3\text{He}$ experiment does not utilize advanced spin sequencing to cancel intensity drifts, so the assumption is that adjacent pulses will always be combined into pairs to form asymmetries. In order to make calculations easier, the indices for these two pulses m and n will be sometimes combined into the pair index p , with the spin up and spin down pulses indicated by p^+ and p^- . As explained in section 5.2.8, the spin state of every individual pulse is checked to ensure that the correct signs are given to Y_m^k and Y_n^k , the summed detector signals as outlined in section 5.1.

Since detector data from two neutron pulses is needed to calculate a single asymmetry observation, we refer to it as a pulse pair asymmetry. In order to determine accurate variances of the asymmetry observations, it is necessary to combine many pair asymmetries together. To minimize the amount of random noise included in this combination, it is desirable to combine pair asymmetries from regions where the experimental conditions are most similar. The most logical unit to use is the 600-pulse sequence, as it is the smallest unit that can be

combined with the same conditions. The pulse pedestal is calculated over whole sequences, so a smaller unit could not be selected. The sequence lasts ten seconds, over which the fluctuations in beam intensity are likely to be much smaller than over the duration of a run, which is seven minutes. In addition, the sequence subunit preserves our ability to measure the splitting in the asymmetry value from the spin parity of the run. In addition, subgroupings which are too small, or which contain observations which are too similar, do not produce numerically stable diagonalizations. To resolve this, the determination of covariances is left to the end of the calculation where the full dataset can be used.

As explained in 5.2.12, typical SNS pulses can vary by as much as 10^{-3} even during the most stable accelerator conditions. This 30Hz neutron pulse modulation can introduce significant false asymmetries into the calculation, unless it is corrected. Scaling by the total detector response gives an equivalent result to using linear regression to correct for this, and is simpler to implement. The scaling algorithm normalizes each pulse pair by the summed intensity over all channels Y_n^k in a given pulse:

$$Y_n = \sum_{k=1}^{144} Y_n^k \quad (5.30)$$

Y_n is the total detector response for a pulse n . Each channel response is scaled by the detector total, eliminating any false asymmetry due to intensity fluctuations:

$$\alpha_p^k = \frac{Y_{p^+}^k - \gamma_p Y_{p^-}^k}{Y_{p^+}^k + \gamma_p Y_{p^-}^k} \quad (5.31)$$

To simplify the numerical calculation, the ratio between two totals, γ , is used instead of performing two divisions:

$$\gamma_p = \frac{Y_{p^+}^k}{Y_{p^-}^k} \quad (5.32)$$

Now, we find the mean value of these corrected pair asymmetries for each channel, A_s^k , over all pairs in a sequence:

$$A_s^k = \frac{\bar{\alpha}_s^k}{G_k} \equiv \frac{1}{G_k} \frac{1}{N_p} \sum_p \alpha_p^k \quad (5.33)$$

Here, N_p is the number of good pairs in a the run, which is typically around 12,000. G_i are the geometry factors calculated in Chapter 4. In addition, we calculate the uncertainty of this value, again per channel per sequence:

$$(\sigma_s^k)^2 = \frac{1}{N_p} \sum_p (A_s^k - \alpha_p^k)^2 \quad (5.34)$$

From the sequence asymmetries per channel, we want to calculate the mean value over all data. There are approximately $1.1 \cdot 10^6$ good sequences, so first we find the mean value of the asymmetry per channel over the whole data set:

$$A^k = \frac{1}{N_s} \sum_s A_s^k \quad (5.35)$$

A^k are the components of $\mathbf{A}$. Next, we determine the covariance relations between channels. Recall, from Figure 4.16, that the emitted proton can travel up to 12cm in the gas before it thermalizes. This means that, for example, ionization from the same reaction event can be measured in up to six channels. As expected, the asymmetries measured in adjacent channels are significantly correlated. In order to correct for this, we first construct the covariance matrix $\mathbf{C}$, between the components of $\mathbf{A}$, by summing over all sequences s :

$$C_{ij} = \frac{1}{G_i G_j} \frac{1}{N_s} \sum_s (A_s^i - A^i)(A_s^j - A^j) \quad (5.36)$$

From the solution in (5.26), we can calculate the physics asymmetry by simply evaluating

$$\boxed{\alpha_p = \frac{\mathbf{I}^T \mathbf{C}^{-1} \mathbf{A}}{\mathbf{I}^T \mathbf{C}^{-1} \mathbf{I}}} \quad s_p^2 = \frac{1}{\mathbf{I}^T \mathbf{C}^{-1} \mathbf{I}} \quad \boxed{\sigma_p^2 = \frac{s_p^2}{N_p}} \quad (5.37)$$

Since our covariance matrix $\mathbf{C}$ is real and symmetric by definition, we can find the uncorrelated modes of the detector by decomposing it:

$$\mathbf{C} = \mathbf{V} \mathbf{D} \mathbf{V}^{-1} \quad (5.38)$$

The eigenvalues of $\mathbf{C}$ are contained in the diagonal matrix $\mathbf{D}$. The eigenvectors, which describe the linearly independent modes of the detector, are in the columns of $\mathbf{V}$. Now that we have the eigenvectors of $\mathbf{C}$, we can change basis from wires, $\mathbf{A}$, to diagonal modes, $\mathbf{B}$:

$$\mathbf{B} = \mathbf{V}^\top \mathbf{A} \quad (5.39)$$

Either weighting set from equation 5.28 can be used to calculate the corrected value of the asymmetry over a set of sequences. As discussed in section 5.3.4, there is an asymmetry splitting due to spin state which we call Δ . In order to evaluate this, we sort the data set into two groups: runs where the even pulses are all "up," and runs where the even pulses are all "down." Then, we calculate the asymmetry for each subset, $\mathbf{A}^+$ and $\mathbf{A}^-$, before finding the value for the entire data set. So, instead of one sum as in equation 5.33 we form two for each channel:

$$A_\pm^k = \frac{1}{N_s^\pm} \sum_s^\pm A_s^k \quad (5.40)$$

Here we sum over two separate subsets to find each mean value, instead of a single mean as in equation 5.33. Then, to combine all channels for each parity subset, we repeat the steps from equations 5.34 to 5.37 for both, including $\mathbf{C}_\pm$. So, we compute the covariance matrix for each subset separately. Once we have a linearly-independent set of mode asymmetries for a subset, we combine them using the weights in equation 5.26 to find a combined asymmetry for each subset:

$$\alpha_\pm = \frac{\mathbf{I}^\top \mathbf{C}_\pm^{-1} \mathbf{A}^\pm}{\mathbf{I}^\top \mathbf{C}_\pm^{-1} \mathbf{I}} \quad s_\pm^2 = \frac{1}{\mathbf{I}^\top \mathbf{C}_\pm^{-1} \mathbf{I}} \quad \sigma_\pm^2 = \frac{s_\pm^2}{N_\pm} \quad (5.41)$$

The two subsets can now be combined to get the physics asymmetry:

$$\boxed{\alpha = \frac{1}{2}(\alpha_+ + \alpha_-)} \quad \boxed{\sigma^2 = \frac{1}{4}(\sigma_+^2 + \sigma_-^2)} \quad (5.42)$$

5.3.3 Parity-Violating Asymmetry

The parity-violating data set has 31,069 good runs, and 1,173,025 good sequences. In total, 8.49% of the data is cut. The sequence alignment, which is part of the data processing, cuts an additional 2.39% of the data which were not part of whole sequences. To illustrate the aggregate effect of our cuts, we calculate all sequence asymmetries with and without them, and compare the distribution. Most of the difference is due to the effect explained in section 5.2.3, which removes sequences with irregularly dropped pulses. Figure 5.12 shows how the precision improves when those sequences are removed. The asymmetries of the remaining sequences are distributed in a highly Gaussian shape, shown in Figure 5.13. After the procedure explained in section 5.4.2, including systematic errors (Table 5.2), the result for the parity-violating detector mode is:

$$\alpha_{pv} = [7.8 \pm 10.3(\text{stat}) \pm 1.6(\text{sys})] \cdot 10^{-9} \quad (5.43)$$

5.3.4 Parity-Conserving Asymmetry

The parity-conserving data set has 996 good runs with 36,224 good sequences. A similar amount of data is cut for this subset of the data: 10.6%. The distribution of sequence asymmetries, before and after all cuts are applied, can be seen in Figure 5.14 and Figure 5.15. The result for the parity-conserving detector mode is:

$$\alpha_{pc} = [-4.2 \pm 0.5(\text{stat})] \cdot 10^{-7} \quad (5.44)$$

Table 5.2: Source and magnitude of systematic errors.

Effect	Parity	Size
Electronic asymmetry	Even	$1 \cdot 10^{-09}$
Stern-Gerlach steering	Even	$1 \cdot 10^{-10}$
Target alignment (non- ϕ)	Even	$5 \cdot 10^{-10}$
Boltzmann polarization	Even	$2 \cdot 10^{-11}$
β -decay	Odd	$1 \cdot 10^{-11}$
Mott-Schwinger scattering	Even	$6 \cdot 10^{-12}$
Induced polarization	Even	$4 \cdot 10^{-13}$
Total		$1.6 \cdot 10^{-9}$

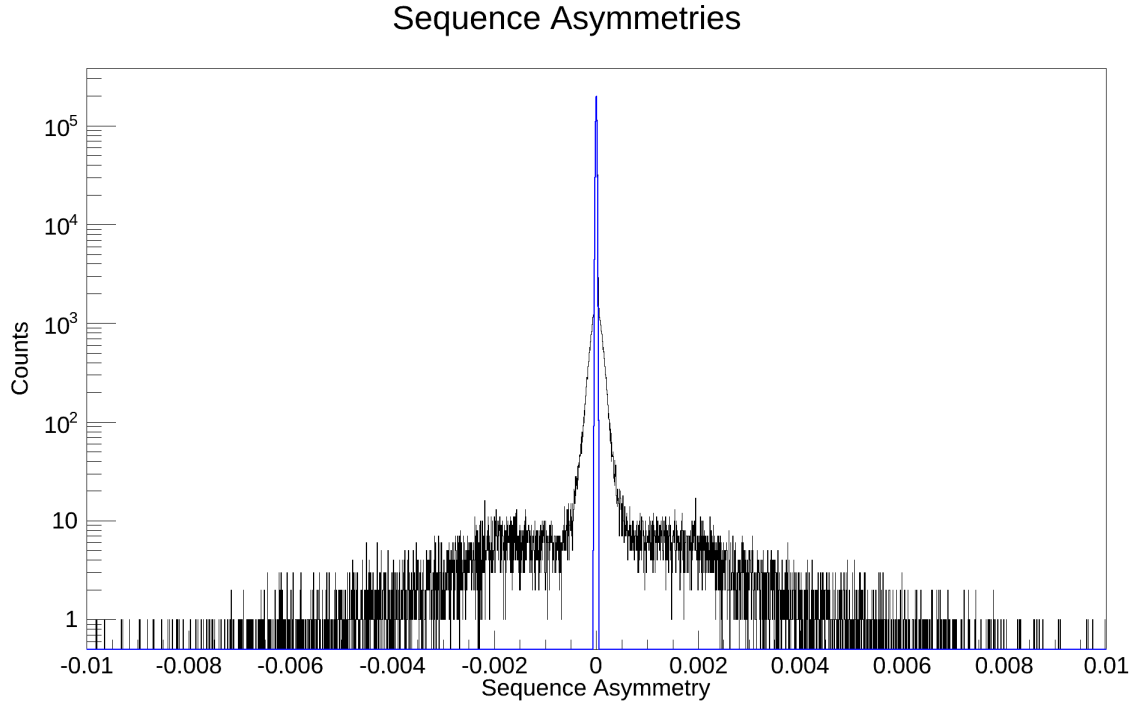


Figure 5.12: Overlay of parity-violating sequence asymmetries before and after cuts are applied.

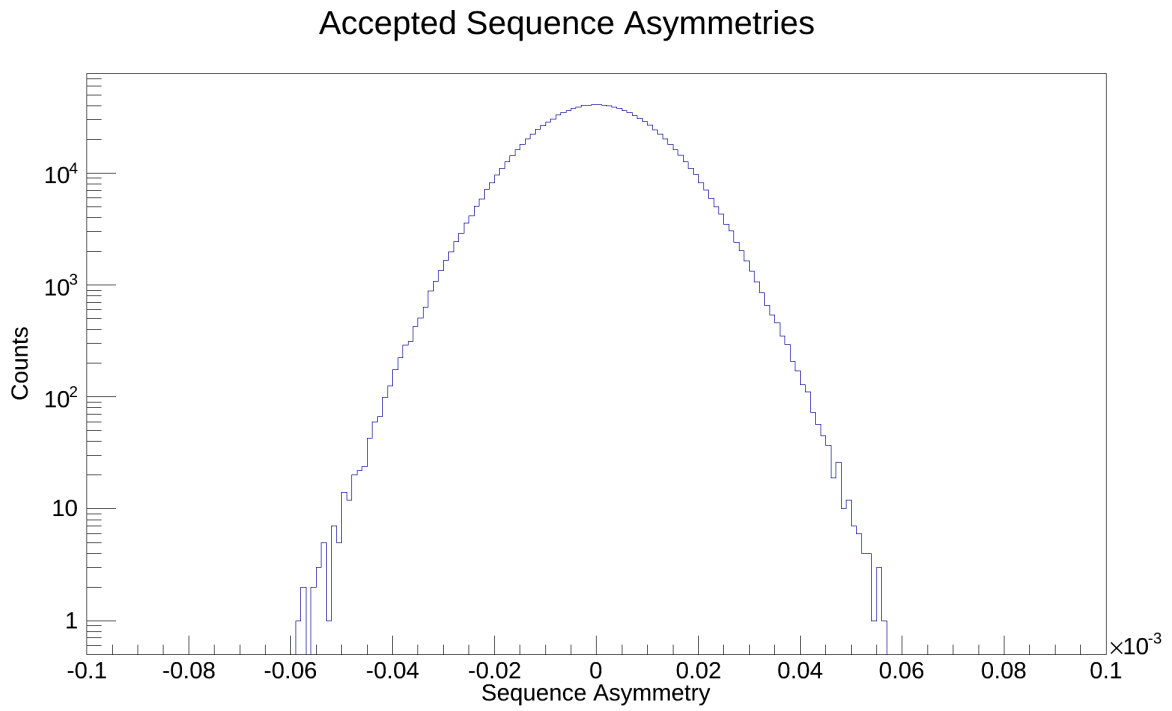


Figure 5.13: Sequence asymmetries which pass all cuts (Zoom of Figure 5.12).

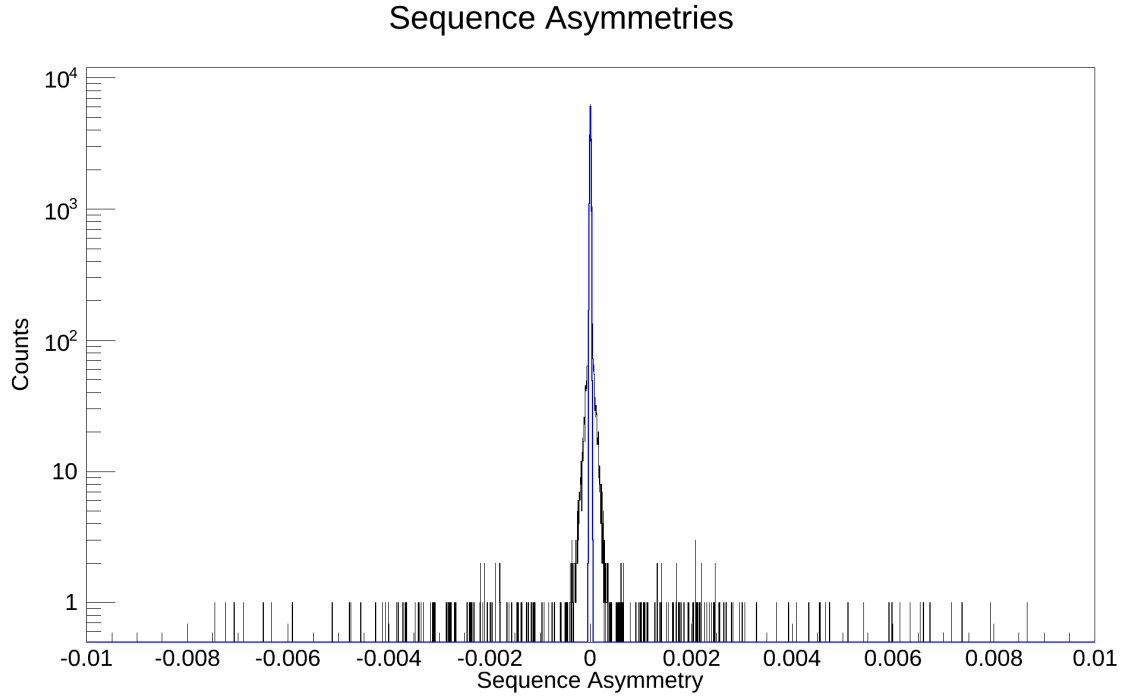


Figure 5.14: Overlay of parity-conserving sequence asymmetries before and after cuts are applied.

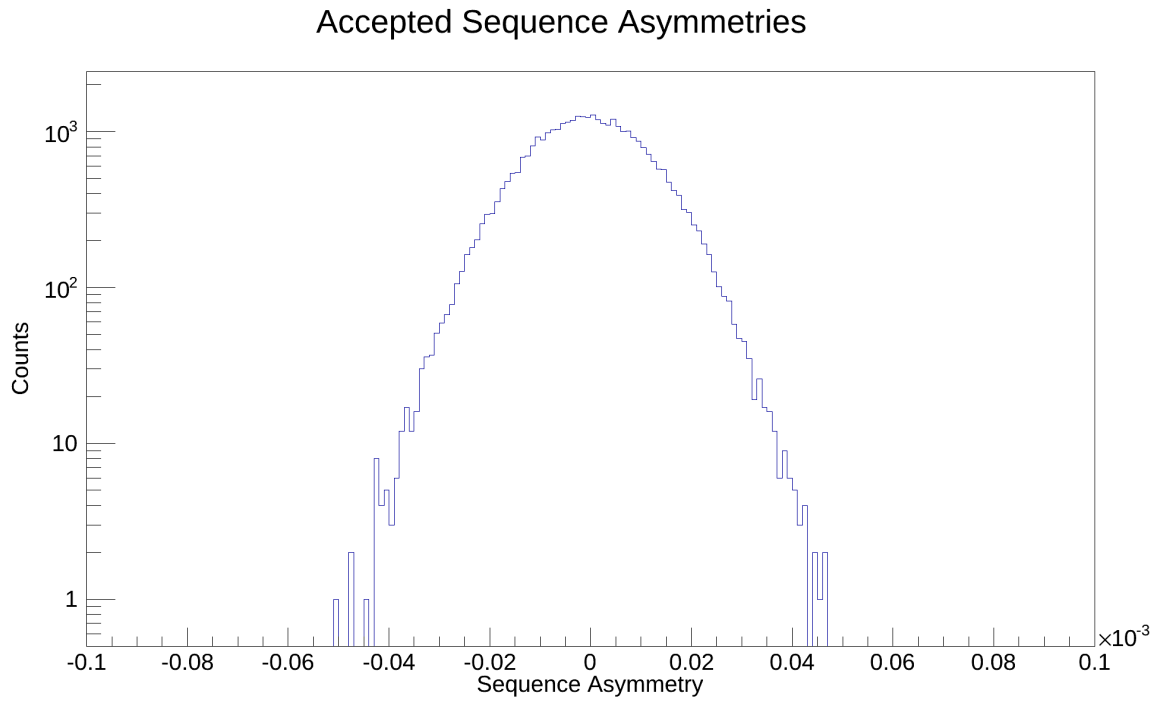


Figure 5.15: Sequence asymmetries which pass all cuts (Zoom of Figure 5.14).

6 CONCLUSION

This measurement of the parity-violating asymmetry in $n+{}^3\text{He}\rightarrow p+t$ is statistically consistent with zero. This is consistent with the results of other few-body experiments, where we do not expect to see amplification of the asymmetry, as found by recent studies of few-body Hadronic Weak Interaction experiments [1]. The statistical precision of the results in equations 5.43 and 5.44 are better than expected by a factor of two.

6.1 Constraints on h_π^1

The asymmetry measured in $n+{}^3\text{He}\rightarrow p+t$ is primarily sensitive to the value of h_π^1 , as the largest coefficient in the potential is $V_\pi^1 = -0.189$, about six times larger than the next largest coefficient, $V_\rho^0 = -0.036$. Each few-body experiment conducted to study the hadronic weak interaction is sensitive to a different linear combination of the coupling coefficients, so aside from the precise measurement of h_π^1 made in the NPDGamma experiment, it is not trivial to compare the results of different measurements.

A previous convention for comparing experimental results was suggested in [55] (see Figure 1.2), and repeated in the literature, using as variables these linear combinations of couplings:

$$\begin{aligned} Y &= -h_\rho^0 - 0.7h_\omega^0 \\ X &= f_\pi - 0.12h_\rho^1 - 0.18h_\omega^1 \end{aligned} \tag{6.1}$$

This approximation allows for bounds on the couplings of the five mesons for which $\Delta I = 0, 1$ to be plotted on two-dimensional exclusion plot. The sixth coupling, h_ρ^2 , has typically

been neglected in this representation, because many observables are either insensitive or only weakly sensitive to it, but the large- N_c treatment of the hadronic weak interaction suggests that this may be incorrect [1].

In lieu of devising a novel solution to the problem of representing a six-dimensional parameter space in two dimensions, we can use the conventional variables in combination with an adjustment of the uncertainty of each measurement based on the DDH "best" values. Recall from equation 2.10 that a measured asymmetry is a weighted sum of the meson-nucleon couplings, where the sum is weighted by a sensitivity to each coupling that is unique for each interaction:

$$A \pm \sigma = \sum h_m^{\Delta I} V_m^{\Delta I} \quad (6.2)$$

The coefficients of each coupling may not match the values in equation 6.1. So let us split each variable term into two parts: one which matches equation 6.1, and the remainders:

$$\begin{aligned} A \pm \sigma &= V_\pi^1 X + V_\pi^1 [Z_\rho^1 h_\rho^1 + Z_\omega^1 h_\omega^1] - V_\rho^0 Y + V_\rho^0 Z_\omega^0 h_\omega^0 + V_\rho^2 h_\rho^2 \\ Z_\rho^1 &= 0.12 + \frac{V_\rho^1}{V_\pi^1} \\ Z_\omega^1 &= 0.18 + \frac{V_\omega^1}{V_\pi^1} \\ Z_\omega^0 &= -0.7 + \frac{V_\omega^0}{V_\rho^0} \end{aligned} \quad (6.3)$$

The linear combinations in equation 6.1 were chosen because V' , the residual magnitudes of the coefficients in equation 6.2, are typically smaller than those in X and Y , and so were historically not as important in comparing experimental results. We can treat the remainders as an additional error term, and evaluate it by using the "reasonable range" of each coupling given in the DDH model. Then, we can rewrite equation 6.3 as:

$$V_\rho^0 Y = V_\pi^1 X + (-A \pm \sigma) + V_\pi^1 [Z_\rho^1 \bar{h}_\rho^1 + Z_\omega^1 \bar{h}_\omega^1] + V_\rho^0 Z_\omega^0 \bar{h}_\omega^0 + V_\rho^2 \bar{h}_\rho^2 \quad (6.4)$$

A plot of this curve reflects the uncertainty in our choice of variables (Figure 6.1).

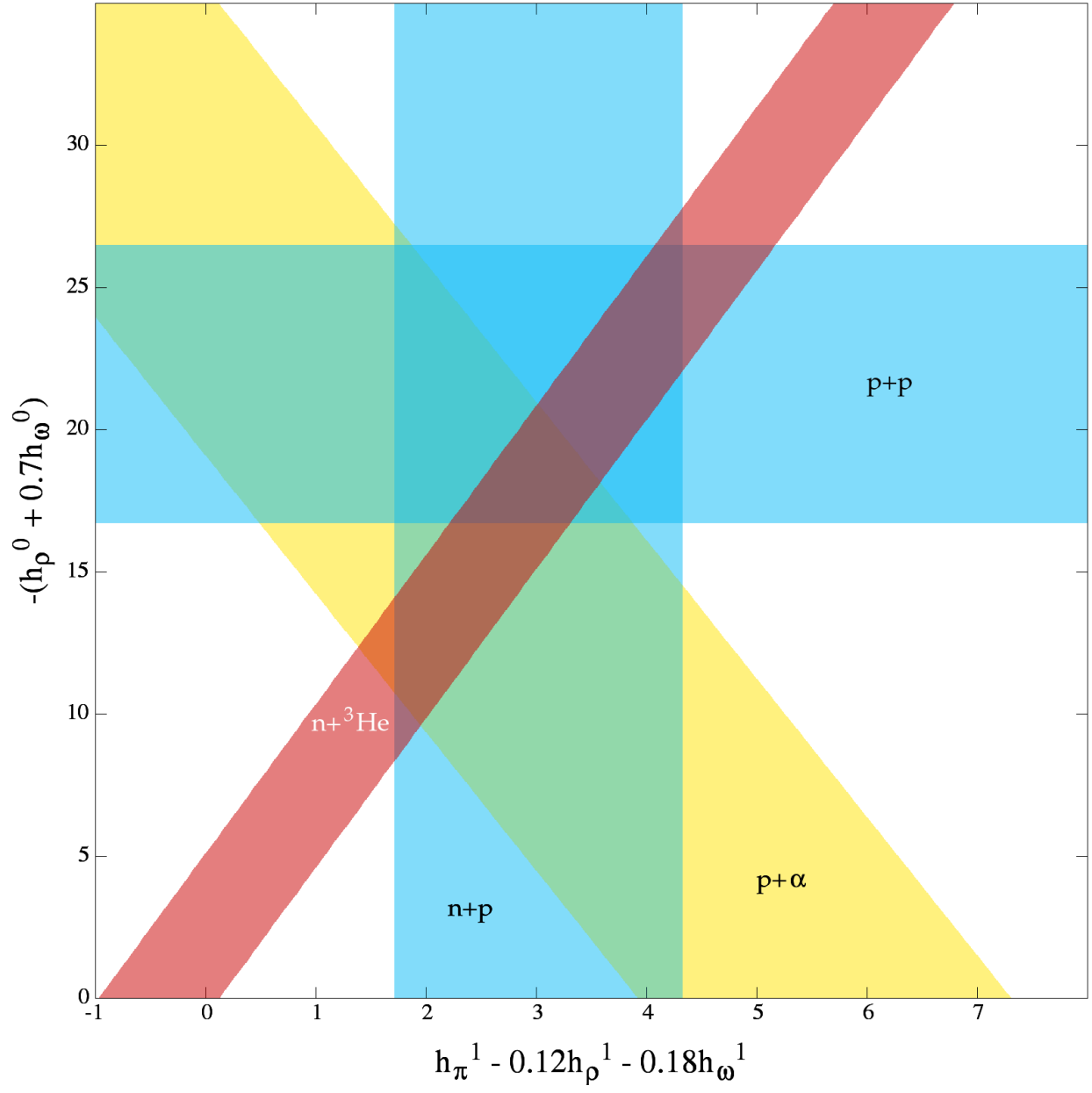


Figure 6.1: Comparison of DDH coupling coefficients from selected few-body experiments.

The program of constraining the values of the couplings $h_m^{\Delta I}$ depends on having a diverse set of accurate experiments with different sensitivities to the coupling set. The $n + {}^3\text{He}$ reaction has a unique linear combination of the meson-nucleon couplings. One representation of this is the positive relationship between X and Y, which is not replicated in any other experiments yet. The $n^3\text{He}$ experiment also has the highest precision of any nuclear asymmetry measurement, and it has unique systematic errors, making it a valuable contribution to the HWI few-body group.

Based on the data taken during this experiment, the calculations of the parity-violating and parity-conserving asymmetry should be consistent with each other to have confidence in them. This work presents an independent analysis of the data recorded the $n + {}^3\text{He} \rightarrow p + t$ experiment. The values of the parity-violating and parity-conserving asymmetries calculated here are statistically equivalent to the other calculations based on the same data. This is an important confirmation of the results, and the next step is to write a publication based on our results and submit it for review. Once they are published, a careful review of the latest results of these measurements is necessary to determine new best values for the meson-nucleon couplings.

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APPENDICES

A Emission time

We would also like to check how good our assumption is about the relationship between arrival time and energy. There is no direct measurement of this we could use in our calculation, but simulations of the emission of neutrons from the moderator are available from the SNS Neutronics group. Figure A.1 shows the intensities as a function of time of four different neutron energies at the coupled hydrogen moderator.

There are two important quantities to note here. The first is that the FWHM of the intensity peaks is about $200\mu s$ for each energy, so the pulse broadening at each energy is significantly larger than the time difference between the peak brightnesses of neutrons at different energies. Second, note that the time difference between the peak brightness of the two energies $4.10\text{\AA}$ and $5.15\text{\AA}$ is around $5\mu s$. The two observed bragg edges of Aluminum occur inside this range.

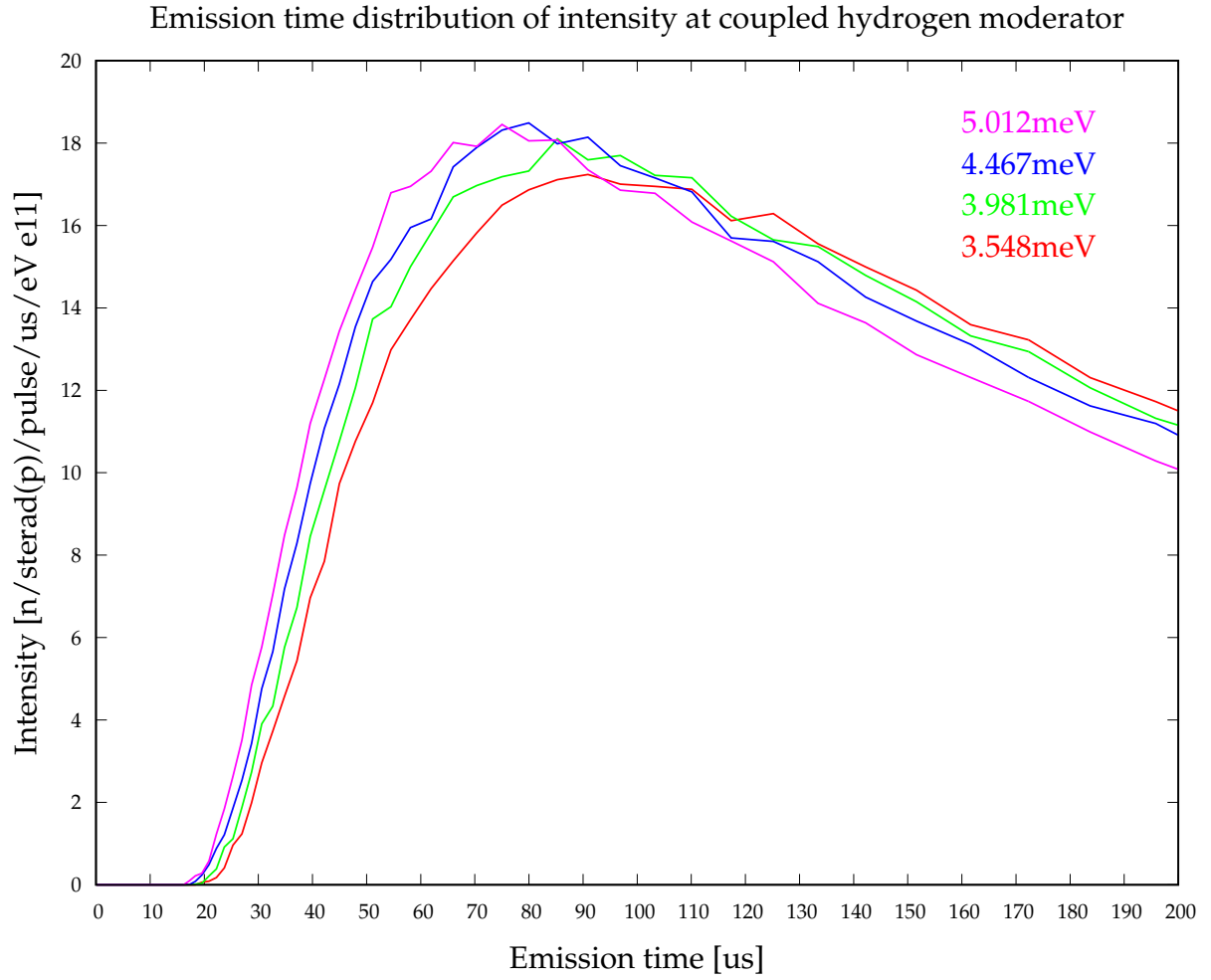


Figure A.1: Emission time distribution of intensity for four different energies.

B Beamline 13 design document

We would like to compare our calculated distance with the physical distance to see how close the values are. The intended path length for beamline 13 is specified in the design document "FUND13NPDG00M8U8713A001." These drawings contain the distances to the end of the guide flange, where M1 is located. Based on this geometry, and the known offset of M1 from the guide flange, we can get an estimate of the ideal direct path length,

$$L_d = \frac{L_g - L_i}{\cos \theta_d} + L_i + L_o = 598.471 in = 15.201 m, \quad (\text{B.1})$$

where L_g is the distance to the guide flange, L_i is the distance to the intersection point, θ is the deflection angle, and L_o is how far the monitor is from the edge of the flange. This differs from our above calculation by 8cm.

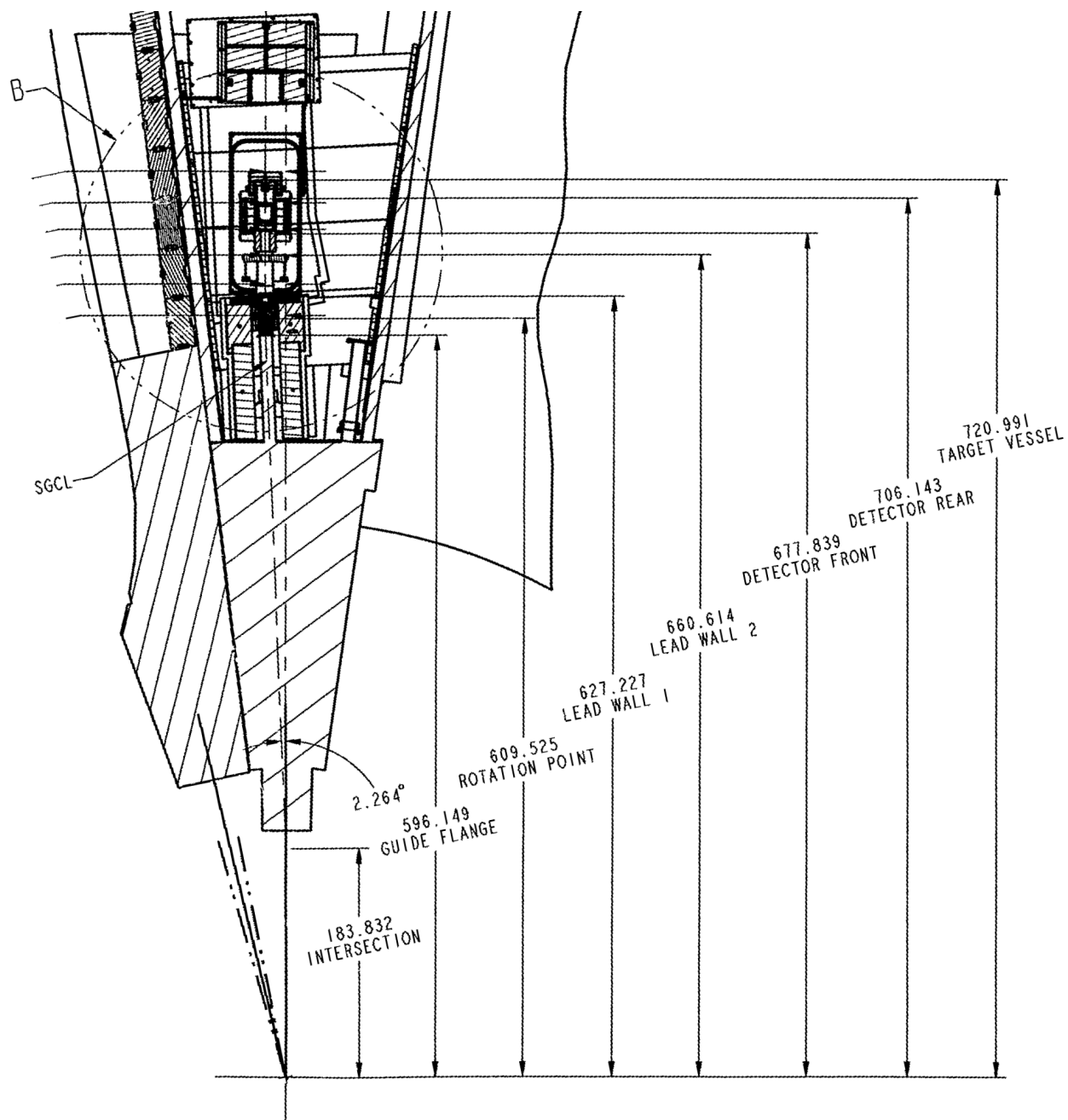


Figure B.1: B-13 design document.

C Derivation of simulated covariances from the Poisson distribution

The solution in equation 4.46 expresses the product on the right in terms of pulse averages. Now we consider the term on the left:

$$\langle E^\kappa E^\beta \rangle = \left\langle \sum_n \frac{e^{-\lambda} \lambda^n}{n!} \sum_i^n Q_i^\kappa \sum_j^n Q_j^\beta \right\rangle \quad (\text{C.1})$$

In order to evaluate this, we must consider the different event subsets in the Poisson sum. Specifically, we must treat the difference between events which deposit energy coincidentally in both elements κ and β (leading to energy correlations) and those which deposit energy only in one. Each of these subsets will have its own separate Poisson parameter. For example, we can rewrite equation C.1 using this decomposition:

$$\begin{aligned} \langle E^\kappa \rangle &= \sum_n \frac{e^{-\lambda_\kappa} \lambda_\kappa^n}{n!} \left\langle \sum_i^n Q_i^\kappa \right\rangle = \sum_n \frac{e^{-\lambda_{\kappa\beta}} \lambda_{\kappa\beta}^n}{n!} \sum_n \frac{e^{-\lambda_{\acute{\kappa}}} \lambda_{\acute{\kappa}}^n}{n!} \left\langle \sum_i^{n_{\kappa\beta}} Q_i^\kappa + \sum_j^{n_{\acute{\kappa}}} Q_j^\kappa \right\rangle \\ &= \sum_n \frac{e^{-\lambda_{\kappa\beta}} \lambda_{\kappa\beta}^n}{n!} \left\langle \sum_i^{n_{\kappa\beta}} Q_i^\kappa \right\rangle + \sum_n \frac{e^{-\lambda_{\acute{\kappa}}} \lambda_{\acute{\kappa}}^n}{n!} \left\langle \sum_j^{n_{\acute{\kappa}}} Q_j^\kappa \right\rangle = \acute{n}_\kappa \acute{Q}^\kappa + \hat{n}_\kappa \widehat{Q}^\kappa = \bar{n}_\kappa \overline{Q}^\kappa \end{aligned} \quad (\text{C.2})$$

The acute denotes events which are unique to the element, and the caret denotes events which are coincident with another cell (unspecified here). This shows that adding the two subsets of depositions gives the total sum of depositions for all events, as expected. Evaluating sums of this form implies the separation of the sum over events into distinct subsets:

$$\sum_i^n Q_i^\kappa = \sum_i^{n_{\kappa\beta}} Q_i^\kappa + \sum_j^{n_{\dot{\kappa}}} Q_j^\kappa + \sum_k^{n_{\dot{\beta}}} Q_k^\kappa = \sum_i^{n_{\kappa\beta}} Q_i^\kappa + \sum_j^{n_{\dot{\kappa}}} Q_j^\kappa \quad (\text{C.3})$$

The third term will be zero, since depositions in element κ will not appear in any sum over events unique to element β . So we see that for every term in the Poisson sum, the number of depositions, n , will be distributed between coincident and unassociated events. Now we can continue evaluating equation C.1, using the decompositions from equation C.2 and equation C.3:

$$\langle E^\kappa E^\beta \rangle = \left\langle \sum_n^{N_{mc}} \frac{e^{-\lambda_{\kappa\beta}} \lambda_{\kappa\beta}^n}{n!} \sum_n^{N_{mc}} \frac{e^{-\lambda_{\dot{\kappa}}} \lambda_{\dot{\kappa}}^n}{n!} \sum_n^{N_{mc}} \frac{e^{-\lambda_{\dot{\beta}}} \lambda_{\dot{\beta}}^n}{n!} \left[\sum_i^{n_{\kappa\beta}} Q_i^\kappa + \sum_j^{n_{\dot{\kappa}}} Q_j^\kappa \right] \left[\sum_i^{n_{\kappa\beta}} Q_i^\beta + \sum_j^{n_{\dot{\beta}}} Q_j^\beta \right] \right\rangle \quad (\text{C.4})$$

The first term of equation C.4 will be the product of sums over coincident events. This is the sum all the elements of a matrix which are of products of the individual depositions. The product of these two sums will include two types of terms - those that correspond to energies all deposited into the same element, and those that contain energies from two separate elements. Taking advantage of the fact that we are calculating the expectation value over an entire pulse (or an arbitrary set of N_{mc} events), which will contain a large number of depositions, we can separate this sum into diagonal and off-diagonal parts and reexpress them in terms of the basic averages:

$$\left\langle \sum_i^{n_{\kappa\beta}} Q_i^\kappa \sum_j^{n_{\kappa\beta}} Q_j^\beta \right\rangle = \left\langle \sum_{i=j}^{n_{\kappa\beta}} Q_i^\kappa Q_j^\beta + \sum_{i \neq j}^{n_{\kappa\beta}} Q_i^\kappa \sum_j^{n_{\kappa\beta}} Q_j^\beta \right\rangle = \left\langle \sum_{i=j}^{n_{\kappa\beta}} Q_i^\kappa Q_j^\beta \right\rangle + \left\langle \sum_{i \neq j}^{n_{\kappa\beta}} \sum_j^{n_{\kappa\beta}} Q_i^\kappa Q_j^\beta \right\rangle$$

We see that there are n diagonal terms, and $n^2 - 1$ off-diagonal ones. Now we take advantage of the expectation value operator and write both parts in terms of the number of events and the pulse average of the basic quantities:

$$\left\langle \sum_i^{n_{\kappa\beta}} Q_i^\kappa \sum_j^{n_{\kappa\beta}} Q_j^\beta \right\rangle = n \widehat{Q^\kappa Q^\beta} + (n^2 - n) \widehat{Q^\kappa} \widehat{Q^\beta} \quad (\text{C.5})$$

In the off-diagonal sum, we can express the average of the products as a product of the averages because events which deposit energy into different elements during different events ($i \neq j$) will not be correlated. We can take the Poisson factors out of the expectation value, and now calculate the deposition sums using our notation for the different averages:

$$\begin{aligned} \langle E^\kappa E^\beta \rangle &= \sum_n^{N_{mc}} \frac{e^{-\lambda_{\kappa\beta}} \lambda_{\kappa\beta}^n}{n!} \left[n \widehat{Q^\kappa Q^\beta} + (n^2 - n) \widehat{Q^\kappa} \widehat{Q^\beta} \right] + \sum_n^{N_{mc}} \frac{e^{-\lambda_\kappa} \lambda_\kappa^n}{n!} n \acute{Q}^\kappa \sum_n^{N_{mc}} \frac{e^{-\lambda_\beta} \lambda_\beta^n}{n!} n \acute{Q}^\beta \\ &+ \sum_n^{N_{mc}} \frac{e^{-\lambda_{\kappa\beta}} \lambda_{\kappa\beta}^n}{n!} n \widehat{Q}^\beta \sum_n^{N_{mc}} \frac{e^{-\lambda_\kappa} \lambda_\kappa^n}{n!} n \acute{Q}^\kappa + \sum_n^{N_{mc}} \frac{e^{-\lambda_{\kappa\beta}} \lambda_{\kappa\beta}^n}{n!} n \widehat{Q}^\kappa \sum_n^{N_{mc}} \frac{e^{-\lambda_\beta} \lambda_\beta^n}{n!} n \acute{Q}^\beta \end{aligned} \quad (\text{C.6})$$

To evaluate equation C.6, we need

$$\sum_n^{N_{mc}} \frac{e^{-\lambda_\kappa} \lambda_\kappa^n n^2}{n!} = \bar{n}_\kappa^2 + \bar{n}_\kappa$$

Note that each element pair produces a unique $\lambda_{\kappa\beta} = \widehat{n}_{\kappa\beta}$. Now we can calculate the Poisson sums:

$$\begin{aligned} &= \widehat{n}_{\kappa\beta} \widehat{Q^\kappa Q^\beta} + \widehat{n}_{\kappa\beta}^2 \widehat{Q^\kappa} \widehat{Q^\beta} + \acute{n}_\kappa \acute{Q}^\kappa \acute{n}_\beta \acute{Q}^\beta + \widehat{n}_{\kappa\beta} \widehat{Q}^\beta \acute{n}_\kappa \acute{Q}^\kappa + \widehat{n}_{\kappa\beta} \widehat{Q}^\kappa \acute{n}_\beta \acute{Q}^\beta \\ &= \widehat{n}_{\kappa\beta} \widehat{Q^\kappa Q^\beta} + \left(\widehat{n}_{\kappa\beta} \widehat{Q^\kappa} + \acute{n}_\kappa \acute{Q}^\kappa \right) \left(\widehat{n}_{\kappa\beta} \widehat{Q^\beta} + \acute{n}_\beta \acute{Q}^\beta \right) \end{aligned} \quad (\text{C.7})$$

Recall from equation C.3 that the sum over coincident and unique events for an element is equal to the total sum:

$$\langle E^\kappa E^\beta \rangle = \widehat{n}_{\kappa\beta} \widehat{Q^\kappa Q^\beta} + \bar{n}_\kappa \overline{Q^\kappa} \bar{n}_\beta \overline{Q^\beta} \quad (\text{C.8})$$

VITA

Christopher Emile Coppola was born in Middletown, New York, and spent time in New York, California, Massachusetts, and North Carolina growing up. After eighth grade, he received a full academic scholarship to, and attended, Asheville School. He received a Bachelors degree in Physics and Mathematics from the University of North Carolina at Asheville where he also conducted computational physics research under Dr. Michael Ruiz. He received a Masters degree in Physics from North Carolina State University. At the University of Tennessee, he founded the Graduate Physics Society, conducted research at Oak Ridge National Laboratory, and received his Ph.D under Dr. Yuri Kamyshkov.