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Date January 7, 1994

Evaluation of Thermoelectric Generators

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Michael Lee Parkison

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ABSTRACT

With increasing emphasis on protecting the environment, many electrical power sources of unconventional nature are being investigated. One such source, which is not a new technology, is thermoelectric generation. The basic principles of thermoelectricity have been known for a hundred years, however, after the introduction of semiconductors in the 1950's thermoelectric generation enjoyed renewed interest and significant advances were made in both generator design and material fabrication. Unfortunately the rate of progress has tapered off recently and the field has reached a plateau where additional gains are becoming painstaking and expensive. This thesis provides a survey of the history and state of the art of thermoelectric generation, describes a computer program for the optimization of a thermoelectric generator (TEG), and illustrates the feasibility of operating a TEG using only waste energy as the heat source (i.e., waste heat, aerodynamic heating, etc.).

Two cases were considered to evaluate the performance of the TEG. The first case evaluates using exhaust gas from an industrial boiler as the heat source for the TEG. The results show that using a TEG (with an efficiency of 9.46

%) 131.9 kW of electric power at 115 volts may be generated simply by recovering a portion of the total energy available in the hot gases. The second case assumes TEG operation near the material limits and shows that for large temperature differences efficiencies as high as 13.5 % are attainable.

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NOMENCLATURE

A	thermoelement cross sectional area, cm^2
Eff_{total}	TEG efficiency based on both heat supplied and internally generated, dimensionless
Eff_{cond}	TEG efficiency based only on the heat supplied, dimensionless
I	electrical current, amps
J	power density, $watts/cm^2$
k	thermal conductivity, $watts/cm-^oK$
K	thermal conductivity, $watts/^oK$
L	length of thermoelement, cm
m	R_L/R, dimensionless
n	charge carrier density, $electrons/cm^3$
N	number of couples or stages
P	electrical power, watts
q	heat flux, $watts/cm^2$
Q	heat rate, watts
r	electrical resistivity, ohms-cm
R	electrical resistance, ohms
R_L	external load resistance, ohms
T	temperature, ^oK
V	electrical potential, volts
x	coordinate direction, cm
Z	figure of merit for a given material, $^oK^{-1}$

α	Seebeck coefficient, volts/°K
β	Thompson coefficient, volts/°K
γ	shape ratio (A/L), cm
ΔT	temperature difference, °K
η	efficiency, dimensionless
π	Peltier coefficient, watts/amp
ρ	electrical resistivity, ohm-cm
χ	generalized material property

subscripts

a	referenced to material a
ab	referenced to the junction of materials a and b
av	average
b	referenced to material b
C	cold junction
cond	conducted into the TEG
gen	generated
H	hot junction
i	indexing integer
in	entering an element
Joule	Joule heat generated
n	n-type (electron movement) semiconductor
N	N^{th} couple or stage
out	leaving an element or TEG

p	p-type (hole movement) semiconductor
Peltier	Peltier heat generated
r	referenced to a heat reservoir
useful	supplied to the load of the TEG
1	referenced to stage or segment one
2	referenced to stage or segment two

<u>superscripts</u>

overbar	average value of parameter

CHAPTER I

INTRODUCTION

The direct conversion of heat into electricity is not a new concept, since the basic principles behind thermoelectricity have been known for over a hundred years. The thermoelectric generator (TEG) has several attractive features. It is quiet, clean, free from vibration, and requires little maintenance. The disadvantages include low thermal efficiencies, vulnerability to increases in contact resistance, and difficulty in fabricating junctions.

It has been estimated that the first TEG constructed would have been capable of converting heat into electricity with an efficiency of about 3 % [1]. In the late 1960's thermoelectric materials with ideal efficiencies of up to 30% were available, but actual efficiencies were still 6-10 % [2,3]. By the late 1970's, with improvements in both material properties and generator design, efficiencies of 10-13 % were reported for some TEG's [4] and unfortunately that is still the range of efficiencies reported in the literature today.

Attempts at developing a TEG for commercial needs have met with varying degrees of success over the years. Several TEG's are available commercially using common fuels (both gaseous and liquid) [5,6,7]. In the 1960's, S.R. Rocklin [8] reported a module using segmented and cascaded elements to improve the efficiency of a TEG used with a radioisotope power source to provide power for space missions. The results show that operating between a hot junction temperature of 1840 °F and a cold junction temperature of 400 °F, the TEG could produce 38.4 Watts at 4.35 volts with an efficiency of 10.8%. Serksnis [9] presented the evaluation of replacing the conventional automobile charging system with a TEG system utilizing the exhaust gas as the heat source. The TEG was designed to operate between hot and cold junction temperatures of 756 °K and 478 °K respectively. The theoretical performance of the TEG resulted in the generation of 2.51 Watts at 68.8 mV with an efficiency of 4.8%. Most recently, Bearden et.al. [10] have reported that for hot junction temperatures as low as approximately 700 °K, TEG's can be designed to produce 30 kW at 230 volts.

Using the basic principles of thermoelectricity, this research develops computer programs to evaluate three different TEG configurations. The three configurations

are the single-stage series TEG, the cascaded TEG, and the segmented TEG; each is described in detail later in this paper. In order to Extend the results of previously reported studies, the programs are then used to compare the performance of the three configurations for the same operating conditions.

Two case studies were were considered to evaluate the TEGs. An arbitrary design requirement to provide a 115-volt power supply was made for both cases. The first case evaluated the specific application of recovering waste heat from the hot exhaust gases of a typical industrial boiler. The exhaust gases were considered a heat reservoir at 525 °F (546 °K) and a heat sink at 80 °F (300 °K) was assumed to be available. Results of this case show that 131.9 kW of electricity at 115 volts could be generated by using a TEG to recover the waste energy.

The second case was evaluated using a heat source near the material limits of the TEG (1000 °K)and the same heat sink (300 °K). This case was considered to establish the potential of the TEG and to substiantiate the comparison of the three types. The results indicate that the cascaded TEG is the most efficient generator, followed by the segmented, and finally by the single-stage series TEG.

The cascaded TEG can generate electricity with efficiencies up to 13.5 % whereas the segmented and single-stage series TEG attain efficiencies of only about 9.3 %. The efficiency of each TEG configuration was also found to be a strong function of the temperature difference at which it was operated. The results of this research agree well with the findings of other studies and show that the TEG is well suited for recovering waste heat.

CHAPTER II

HISTORY

The origins of thermoelectricity can be traced back to 1821 when Thomas Johann Seebeck reported that a small electric current will flow in a closed circuit composed of two dissimilar metals when their junctions are maintained at different temperatures. The TEG Seebeck constructed is estimated to have had an efficiency comparable to the steam engines of that period.

The next significant discovery, by Jean Charles Anthanse Peltier in 1834, was that heat was either liberated or absorbed when a current flows through the junction of dissimilar conductors. The Peltier effect was shown to be reversible, proportional to the current crossing the junction, and independent of the size or shape of the conductors making up the junction. The Peltier effect must be distinguished from the familiar irreversible heating (Joule heating) that occurs in all conductors when electric current is passed through them. Joule heating varies as the square of the current density and directly as the electrical resistance of the conductor (and is therefore a function of the size and shape of the conductor). This means that the Joule heat is always

liberated and is independent of the direction of the current flow.

Realizing a relation between the Seebeck and the Peltier effects should exist, a thermodynamic analysis of the thermoelectric effects was performed by Lord Kelvin (William Thompson) in 1854. Based on an intuitive and arbitrary separation of reversible and irreversible parts of the conduction process [1] Lord Kelvin proceeded to derive the well known Kelvin relations and conclude that there must be a third thermoelectric effect that was not being taken into account. The Thompson effect is the reversible process of heating or cooling that takes place in a single conductor when an electric current is passed through it in the direction of a temperature gradient.

The American chemist Lars Onsager showed in 1931 that the problem of irreversible processes can be treated by means of a general axiom (the Onsager reciprocal relation), which results from the principle of microscopic reversibility. Based on this axiom, H.B. Callen derived relations that supported the original findings of Lord Kelvin.

In 1956, Joffe discovered that PbTe and PbSe were superior thermoelement materials. It was determined by Joffe that

materials with the electron concentrations adjusted to $10^{19}/cm^3$ possessed figures of merit superior to other conducting materials. The figure of merit, Z, is a grouping of the material properties that are most important in the design of a TEG.

With the introduction of better semiconductor materials, much progress has been made in the application of TEGs. Remote weather stations requiring maintenance only about every ten years have been installed by the United States Weather Bureau on the Artic Circle. Similar TEGs power VHF-FM radio relay transmitters located in inaccessible terrain. Small self-contained pumping systems have also been developed to raise productivity in underdeveloped countries [11]. In 1970 the first implant of the radioisotope cardiac pacemaker (RCP) took place in Paris using a miniature nuclear powered thermoelectric source [12]. Other uses include environmental control units, portable generators, sea buoys, and various space vehicles including Voyager I and II [1,2,3,13,14,15].

CHAPTER III

BACKGROUND

Thermoelectric Phenomena:

The science of thermoelectricity is based on three thermodynamic interactions and their relationships. These phenomena are the Seebeck, Peltier, and Thompson effects mentioned previously. Each of the effects is reversible [2] and will be discussed below.

The Seebeck effect, illustrated in Figure 1, is caused by a temperature difference between the two ends of the wire. Electrons at the hot end of the wire will have higher average kinetic energy and speed than at the cold end which will lead to both an energy transfer and a diffusion of electrons towards the cold end [16]. This diffusion of electrons will set up an emf. The open circuit potential developed between terminals 1 and 2 by this emf is the Seebeck voltage. By convention the Seebeck voltage is considered to be positive if the current flows from a to b at the cold junction [17]. The rate of change of the Seebeck voltage with the hot junction temperature is known as the Seebeck coefficient, α_{ab} and is defined in equiation (1).

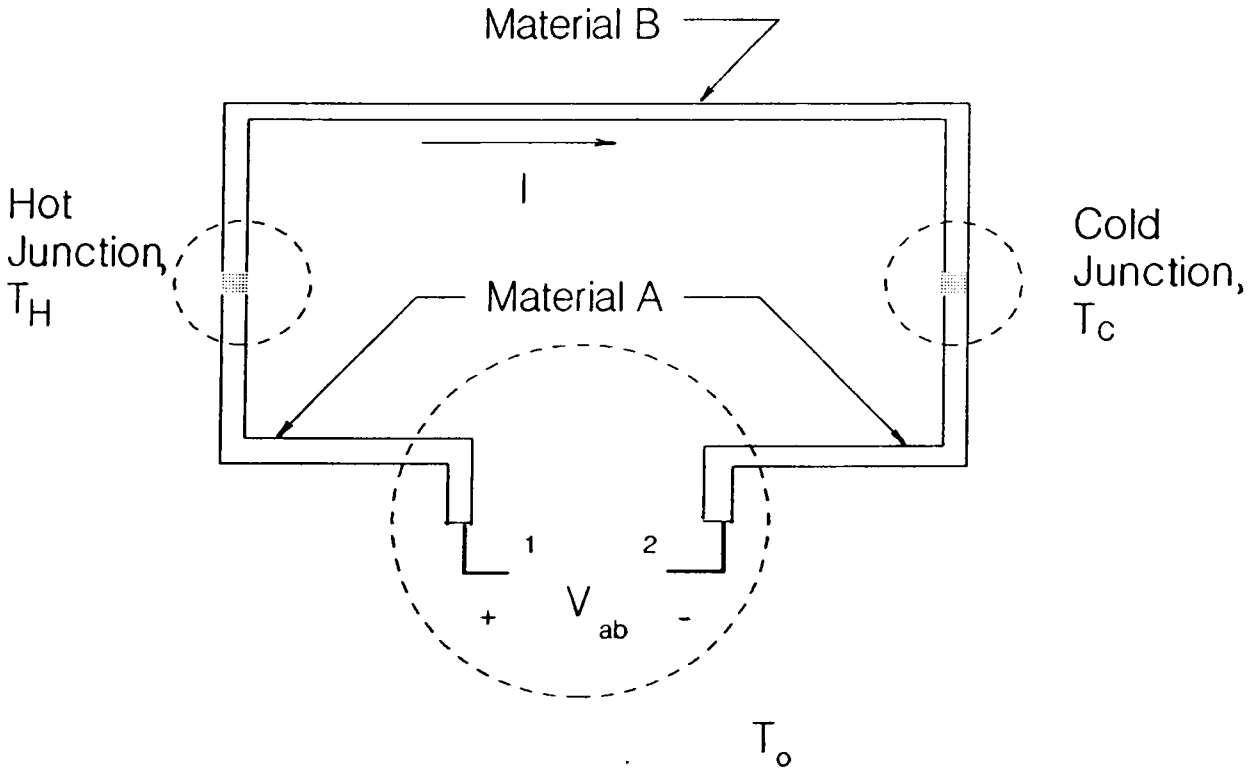


Figure 1. Seebeck Effect

$$\alpha_{ab} = \left(\frac{\partial V_{ab}}{\partial T_H}\right)_{T_C} \qquad (1)$$

The Peltier effect (see Figure 2) arises because the potential energies of the charge carriers on either side of the junction are different. Consequently, when a current, I, flows through the junction, energy must be liberated to or absorbed from the surroundings in order to maintain a conservation of energy and charge. The rate of reversible heat absorption, Q, resulting from passing current I is given by:

$$Q = I\pi_{ab} \qquad (2)$$

Where π_{ab} is the Peltier coefficient of the junction and is considered positive if current flows from a to b and Q is absorbed from the surroundings [12].

The third thermoelectric effect, the Thompson effect (Figure 3) relates to the rate of reversible heat generation resulting from the passage of an electric current, I, along a single conductor over which there is a temperature gradient. The physical explanation of the Thompson effect is the same as for the Peltier effect except that the difference in charge carrier energy is now

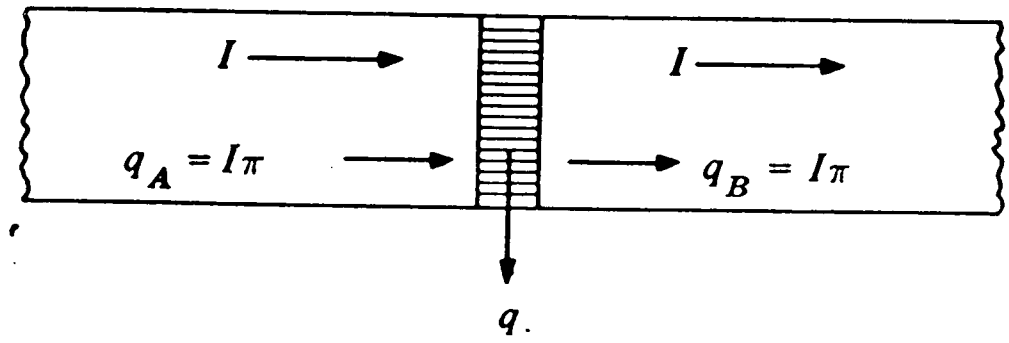


Figure 2. Peltier Effect (Angrist [14])

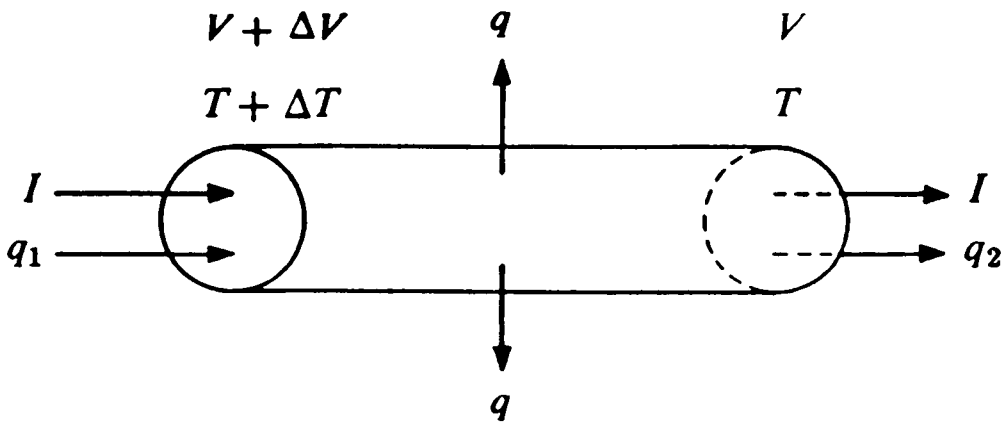


Figure 3. Thompson Effect (Angrist [14])

caused by the thermal gradient along the wire. Providing the temperature differences are small:

$$Q = I\beta\Delta T \tag{3}$$

where β is the Thompson coefficient.

These three thermoelectric coefficients are related by the Kelvin relationships:

$$\alpha_{ab} = \frac{\pi_{ab}}{T} \tag{4}$$

$$\frac{d\alpha_{ab}}{dT} = \frac{\beta_a - \beta_b}{T} \tag{5}$$

These relationships can be derived using the Onsager reciprocal and the standard theory of irreversible thermodynamics as shown by Soo [3].

Two significant relationships can be derived from the Kelvin relationships. One is that the Seebeck coefficient of a junction between two materials is equal to the difference between the absolute Seebeck coefficients of

each material. The other, based on the second Kelvin relationship, allows a definition for the Seebeck coefficient of a single material to be determined and is given by:

$$\alpha = \int_0^T \frac{\beta}{T} dT \qquad (6)$$

Thermoelectric Materials:

Since the practical application of thermoelectric effects is largely a materials problem, a brief discussion of thermoelectric materials, material properties, and material selection will now be presented.

It is very desirable to have a single parameter which characterizes the thermoelectric potential of various materials. This parameter is called the figure of merit, Z, and relates the electrical resistivity, r, the Seebeck coefficient, α, and the thermal conductivity, k, of a material. The figure of merit is defined as:

$$Z = \frac{\alpha^2}{rk} \qquad (7)$$

It has been shown that even small increases in the figure of merit can produce significant changes in the thermal efficiency of a TEG [14]. The problem then becomes simultaneous control of these three macroscopic parameters in the temperature range of interest. Figure 4 shows the interrelationship between these parameters as a function of change in charge carrier density.

Figure 4 has rather arbitrarily been divided into three sections. The insulator portion has electrical resistivity

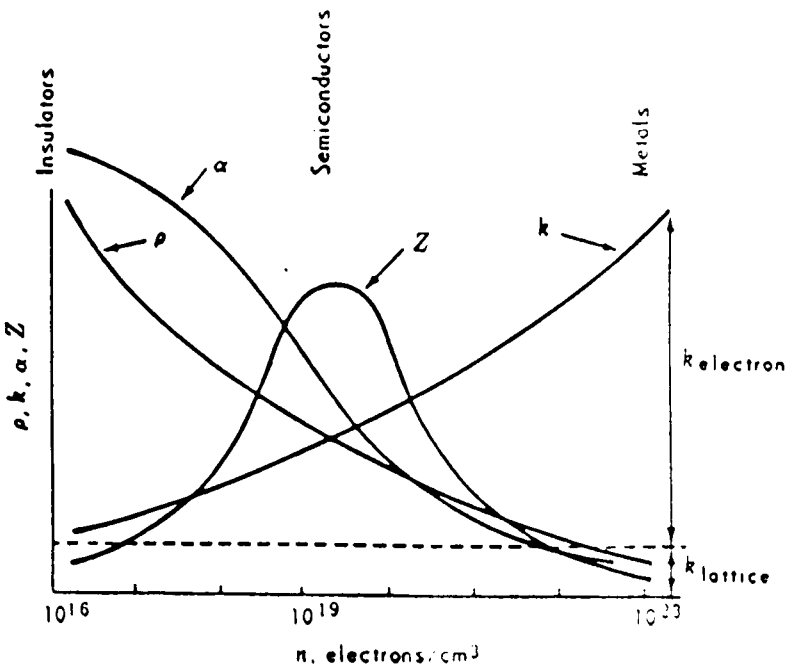


Figure 4. Material Properties as a Function of Charge Carrier Density (Sutton [2])

too large to produce the desired thermoelectric effects. Insulators will therefore not be considered as viable thermoelectric materials. The metals, as shown below in Table 1 [18], typically have small Seebeck coefficients and high thermal conductivity which prevent them from being good thermoelectric materials.

Table 1. Thermoelectric Properties of Selected Metals [18]

Material	r, ohm-cm	k,watt/cm-°K	α, volt/°K
Constantan	48.9×10^{-6}	0.212	32.2×10^{-6}
Alumel	28.1×10^{-6}	0.297	-13.5×10^{-6}
Copper	1.56×10^{-6}	3.770	5.9×10^{-6}
Chromel	70.0×10^{-6}	0.192	25.7×10^{-6}
Iron	8.57×10^{-6}	0.678	17.8×10^{-6}

However, metallic thermoelements could be advantageous when special requirements exist. Demands for ductile materials or high length to area ratios in low current, high voltage generators would require metallic thermoelements. Still, the figures of merit for metals are typically low and the best efficiency (for small temperature differences and maximum power output) as shown

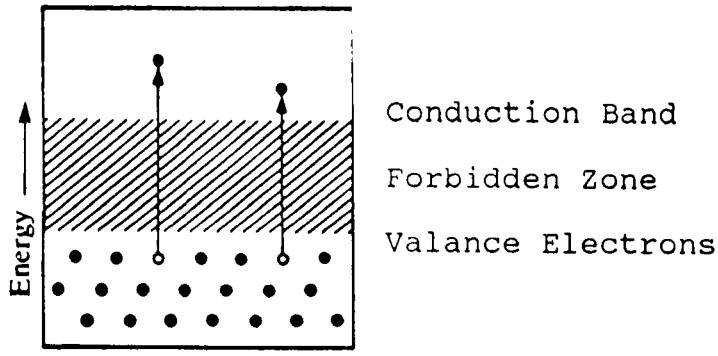
by Sutton [2] can be approximated as:

$$\eta = \frac{(Z\Delta T)}{16} \tag{8}$$

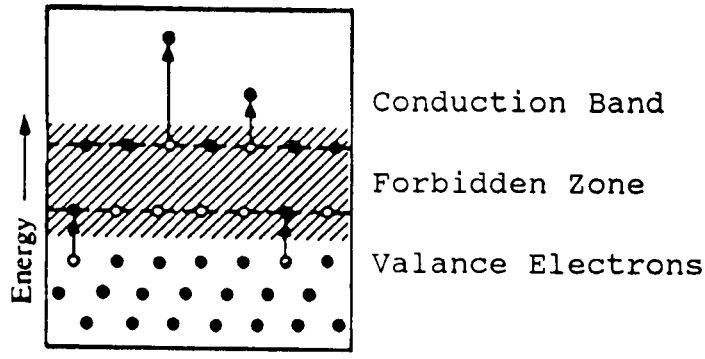
The efficiency of a TEG is defined as the ratio of the power output to the heat input. The power output from the device is easily determined, however, the heat input is somewhat more complex. The efficiency of a TEG and the derivation of the necessary equations are presented with a brief discussion in Chapter V and Appendix I of this paper.

The most promising thermoelectric materials are the semiconductors. The semiconductors have high Seebeck coefficients, low thermal conductivity, and low electrical resistivity as shown by the peaking of the Z curve in the semiconductor region of Figure 4. This indicates that the semiconductors provide the desired thermoelectric characteristics. A detailed explanation of semiconductors is beyond the scope of this paper; however, for completeness a brief discussion is presented below. Several of the references provide a more detailed description of the physics associated with semiconductors [3,14,19].

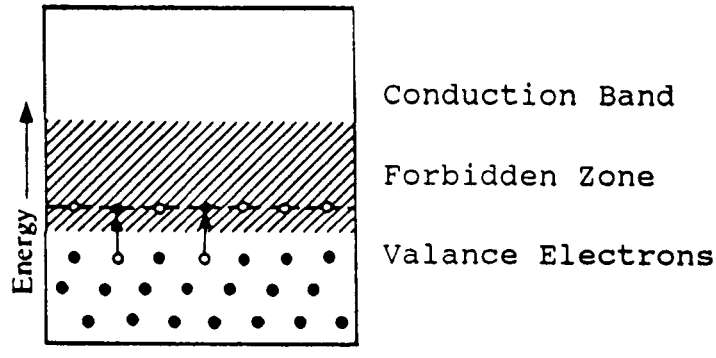
A semiconductor (see Figure 5) is a material in which the



a. Intrinsic Semiconductor



b. Extrinsic n-type Semiconductor



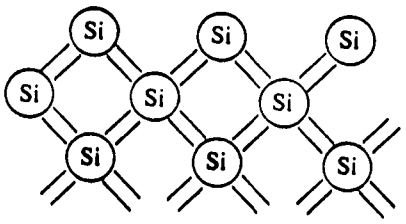
c. Extrinsic p-type Semiconductor

Figure 5. Energy Bands in a Semiconductor (Angrist [14])

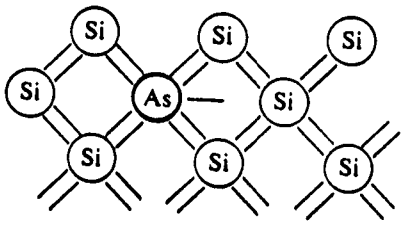
valence band (the upper energy band in an atom occupied by electrons) is separated from the conduction band (the next allowable energy band above the valence) by not too large an energy gap. This allows some valence electrons to be excited either thermally or optically to the lower levels of the conduction band.

There are basically two types of semiconductors, intrinsic and extrinsic. Intrinsic semiconductors are those in which the conduction of current is caused only by those electrons excited up to the conduction band from the valence band. Addition of impurities (dopants) to a semiconductor makes it possible to choose the dominant type of conduction of the material. These materials in which conduction is due to impurities, are called extrinsic semiconductors.

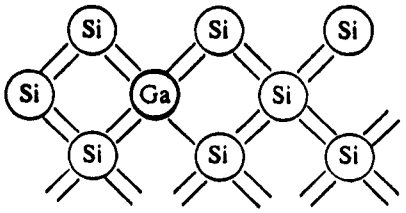
To further illustrate the two types of semiconductors consider Figure 6. Figure 6a represents a silicon atom that is assumed to be intrinsic. Each silicon atom is surrounded by 4 valence electrons. Every silicon atom is then surrounded by 8 valence electrons, 4 of its own and 4 of its neighbors. If an arsenic atom (5 valence electrons) is substituted for a silicon atom, four of the arsenic valence electrons replace the silicon electrons and the fifth is not made part of the valence band



a. Intrinsic Silicon



b. n-type Silicon Doped With Arsenic

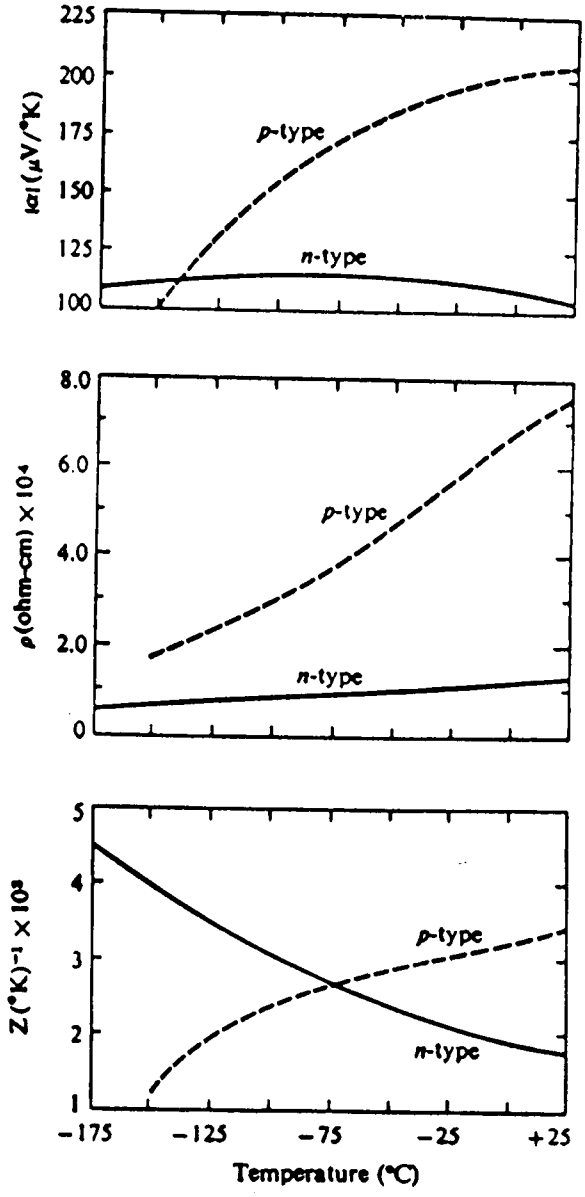


c. p-type Silicon Doped With Gallium

Figure 6. Illustration of Semiconductor Lattice Structures (Angrist [14])

structure. This fifth electron is attracted to the arsenic atom by the positive charge of its nucleus, therefore the energy required to excite this electron into the conduction band is much less than that required to break a valance bond. This is illustrated in Figure 6b. This type of doping results in an n-type extrinsic semiconductor since conduction occurs by means of electrons with a negative charge. If a gallium atom is used as the dopant instead of the arsenic, a hole is left in the valence band. This results because the four valence electrons of the silicon atom are replaced by only three valence electrons in the gallium atom which leaves one empty place in the valence band. This type is illustrated in Figure 6c and is called a p-type extrinsic semiconductor because conduction is by hole movement.

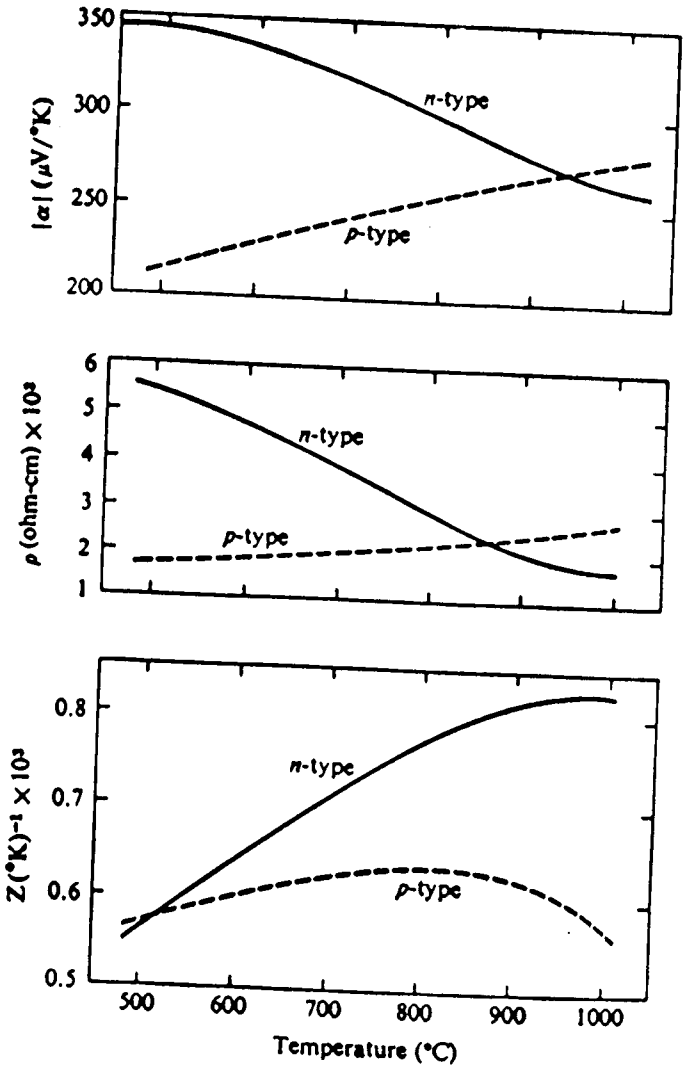
The thermoelectric properties of many materials have been reported in the literature. Often, only some of the properties have been measured and the others were then computed from a theoretical model. Figure 7 presents the thermoelectric properties as a function of temperature [14] and Table 2 presents a summary of some of the more commonly used semiconductor materials [12]. This is only a partial list because with varying concentrations, dopants, and techniques the possibilities for making semiconductors are nearly unlimited.



a. 75% Sb_2Te_3, 25% Bi_2Te_3 p-type alloy
95% Bi, 5% Sb n-type alloy

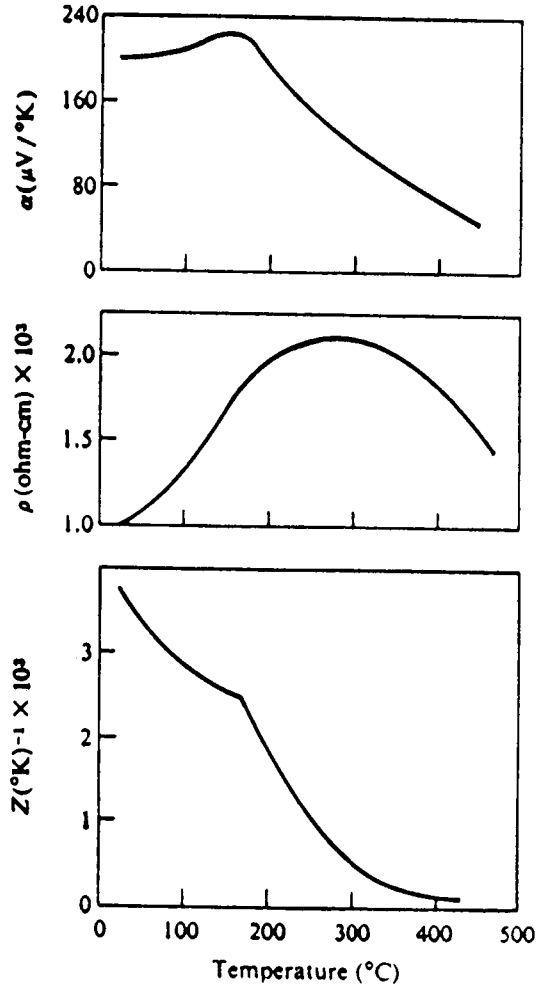
Figure 7. Thermoelectric Properties of Selected Materials

(Angrist [14])



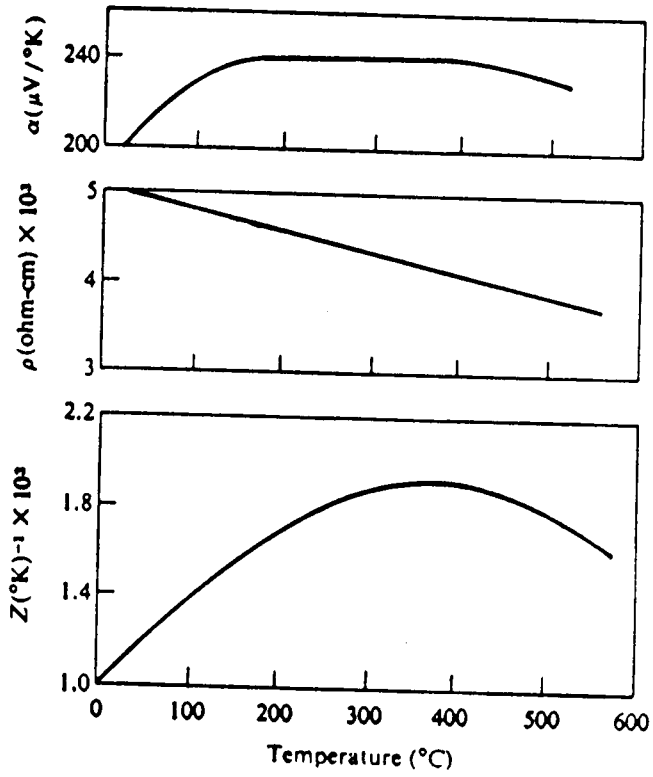
b. p-type and n-type Si Ge

Figure 7. continued



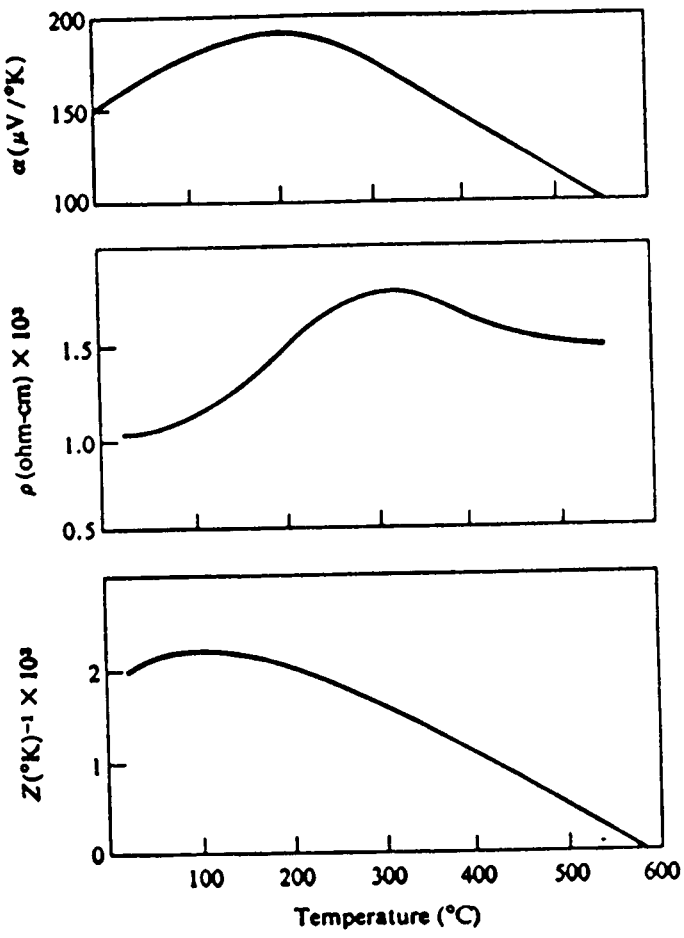
c. 25%Bi_2Te_3, 75% Sb_2Te_3(1.75% Se) p-type alloy

Figure 7. continued



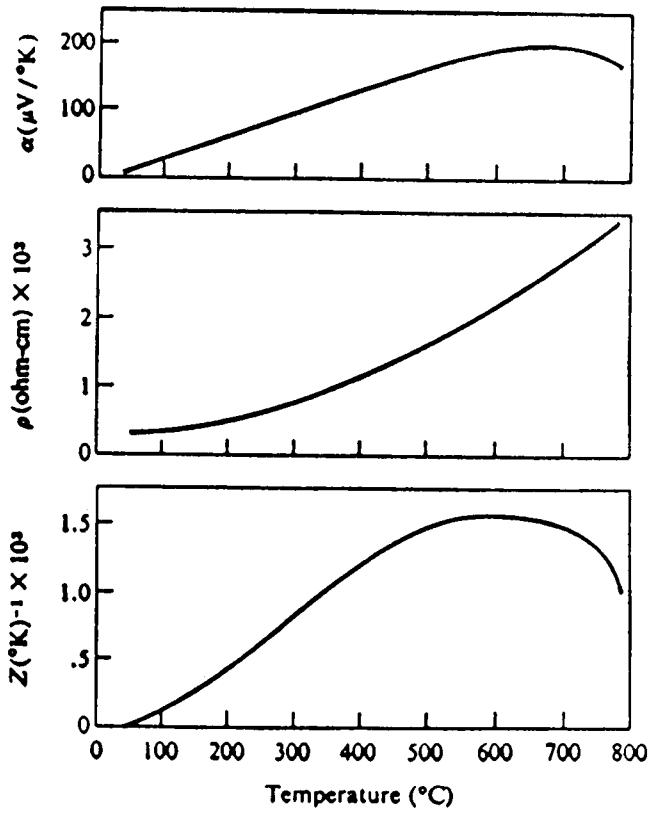
d.$AgSbTe_2$ p-type alloy

Figure 7. continued



e. 75% Bi_2Te_3, 25%Bi_2Se_3 n-type alloy

Figure 7. continued



f. 75% PbTe, 25% SnTe n-type alloy

Figure 7. concluded

Table 2. Thermoelectric Properties of Selected Semiconductors [26]

Material	r, ohm-cm	k, watt/cm-°K	α, volt/°K
$Bi_2Te_{2.4}Se_{0.6}$	1.35×10^{-3}	0.0144	214×10^{-6}
PbTe (cast)	1.86×10^{-3}	0.0152	204.4×10^{-6}
PbTe (sintered)	5.12×10^{-3}	0.0125	240×10^{-6}
$BiSbTe_3$	7.93×10^{-3}	0.0183	164×10^{-6}
ZbSn	2.72×10^{-3}	0.0126	230×10^{-6}

Attempts to increase the figure of merit have been primarily concentrated on optimizing the carrier density by various doping techniques. Still, the discrepancy between the theoretical values of ZT>>1 and the experimental values of ZT=1 seem to indicate that this approach has not developed the full potential of the thermoelectric materials [20]. Recently, the possibility of lowering the thermal conductivity, thereby increasing the figure of merit, has been evaluated and, according to Cahill, a "considerable reduction below the conductivity achieved so far appears to be possible in principle" [13]. Another method of increasing the figure of merit that has

been evaluated is the possibility of mixing together different materials macroscopically as a composite medium. Unfortunately, a theoretical study has led to the conclusion that the effective figure of merit can never be greater than the largest figure of merit of the different materials making up the composite [13]. However, reports indicate that this method allows formation of thermoelectric materials that posses superior thermoelectric properties over the range of 100-300 °C [21]. Therefore it may be an attractive method to tailor thermoelectric materials for specific applications.

Selecting the proper thermoelectric material is a complex topic in itself and is critical to the design of a TEG [22]. Several papers have been written concerning material selection, and a detailed description of the method is beyond the scope of this report. In selecting the material for a single stage TEG the process is fairly straightforward. The materials should be evaluated for the highest figure of merit over the anticipated range of temperatures. However, for a wide temperature range, it becomes necessary to use more than one thermoelectric material. In this case materials with the highest figure of merit for each given temperature range should be selected. It is clear that no one material doped in a particular way will give the best performance, especially

over a range of temperatures. A thorough evaluation of materials which yield the desired results must be made before a material is selected.

CHAPTER IV

GENERATOR DESIGN

Power generation by a single stage thermoelectric element (Figure 8) has inherent output voltage and load limitations. The simplest method to reduce these limitations is to form thermopiles by arranging the thermoelements electrically in series and thermally in parallel. This is known as a single stage series couple and is illustrated in Figure 9. This arrangement is analogous to placing N batteries in series and results in an increase in output voltage, load resistance, and power output by a factor of N times. The current, power density, and thermal efficiency are independent of the number of couples in the element.

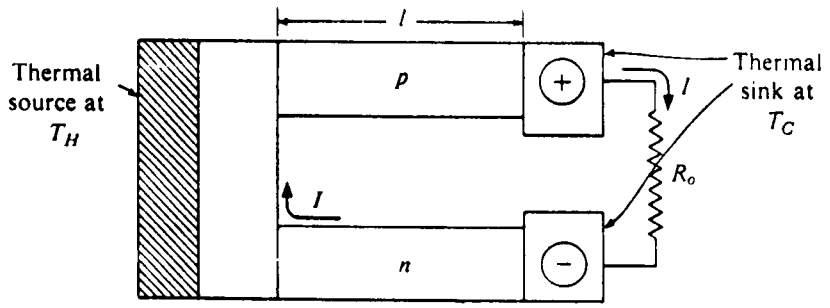


Figure 8. Thermoelectric Element Used for Analysis (Angrist [14])

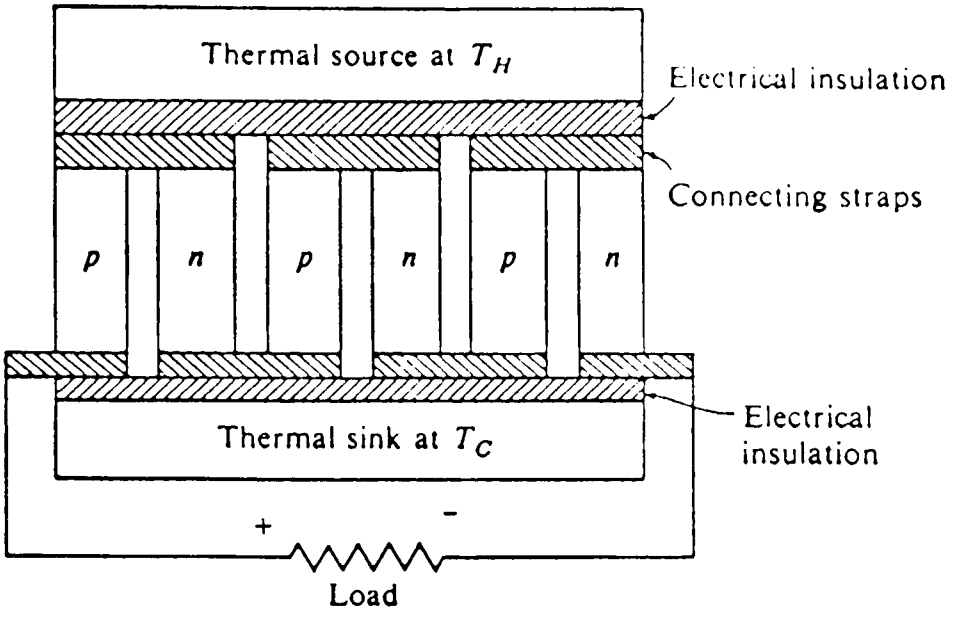


Figure 9. Single-Stage Series TEG (Angrist [14])

As shown in Figure 7, thermoelectric properties vary considerably with temperature. Therefore, to obtain the maximum performance over a large temperature gradient, it would be desirable to use different materials in different temperature regimes. This can be accomplished in two ways, by cascading or by segmenting.

Cascaded generators (see Figure 10) are made by using the heat transferred out of one stage as heat input for the next stage. Each thermoelement material is selected to optimize the output for the temperature range of the given stage. The upper bound of the thermal efficiency for a

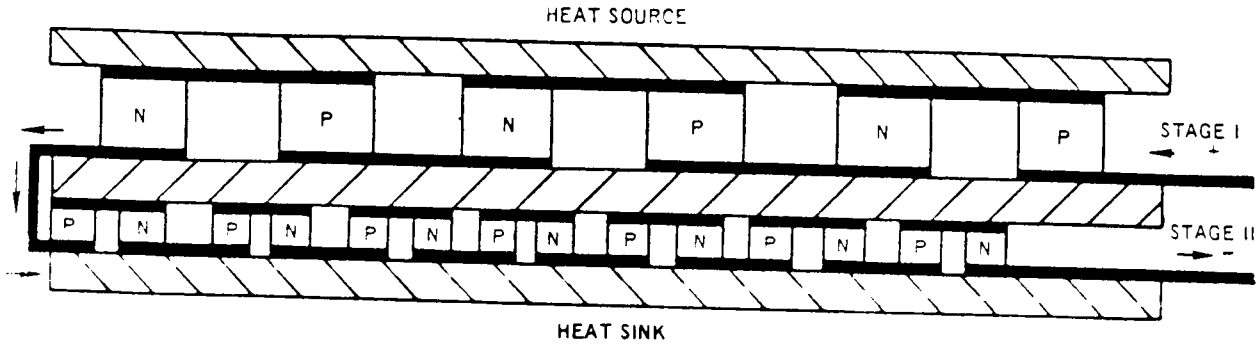


Figure 10. Cascaded TEG (Rocklin [8])

cascaded generator can be determined by considering an infinite stage device as discussed by Angrist [14].

Segmented generators are built from legs of different thermoelectric materials that are connected both thermally and electrically in series as shown in Figure 11. This approach allows each material to operate over the temperature range where its figure of merit is highest. It has been shown that the theoretical efficiency of a segmented generator is always less than or equal to the efficiency of the corresponding multistage generator [14]. Figure 12 illustrates this by comparing the theoretical efficiency of a cascaded and segmented generator[8].

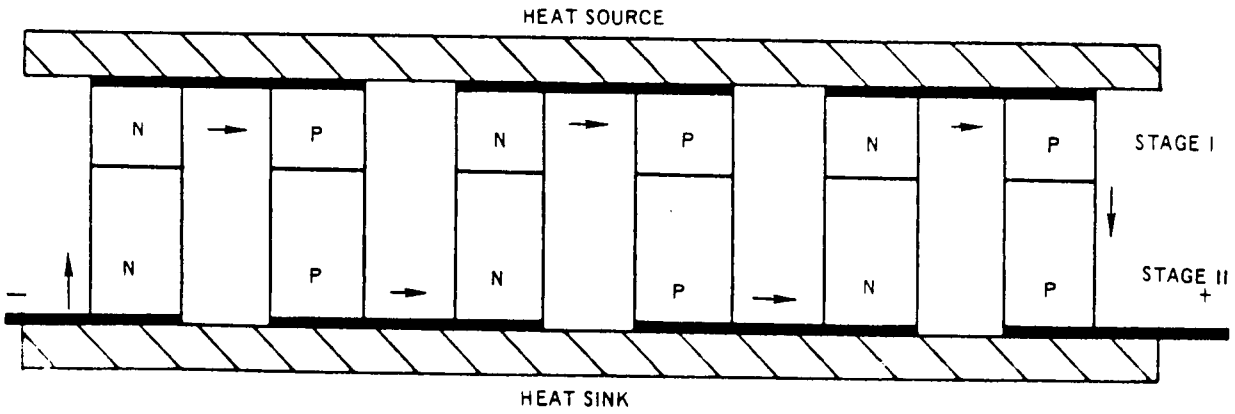


Figure 11. Segmented TEG (Rocklin [8])

The choice of which method to use to obtain the maximum efficiency is largely determined by the thermoelectric materials to be used. When the material properties are similar, the segmented method is usually better since it eliminates the need for interstage electrical connections, many of the interstage joints, and the electrical and thermal losses associated with these joints. If the material properties are significantly different, the maximum efficiency, optimum intermediate temperature, and optimum current may not be met by segmenting since only the geometry can be varied. Cascading allows variation of both the geometry and the number of elements which allows more flexibility to meet the optimum design for materials with significantly different material properties [8].

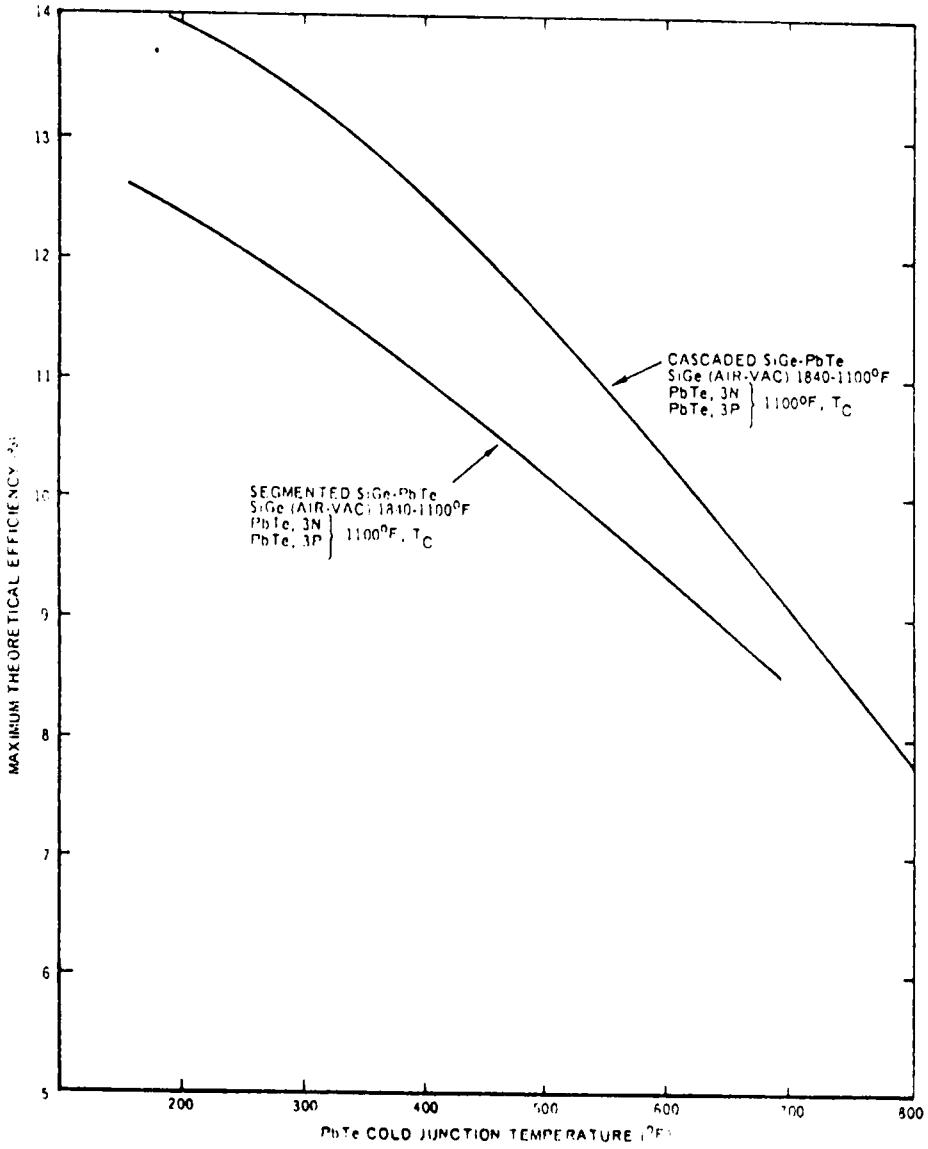


Figure 12. Comparison of Theoretical Efficiency of Cascaded and Segmented TEG Modules (Rocklin [8])

Perhaps the biggest problem with the design of a thermoelectric generator is the design of contacts that provide low resistance to both heat and electricity. Since soldering directly to a semiconductor is not easily accomplished, there are various other techniques in use. At low temperatures, a thin layer of nickel is plated to the ends of the semiconductor and the contacts are then soldered to the nickel. An alternative method is to use a bismuth-antimony alloy as a solder. Contacts are typically made using nickel, iron, or stainless steel for high temperature semiconductors. The semiconductor is heated to just above its melting point and is then brought into contact with the metal connector. While effective, this method is difficult to control. All of the above methods have been shown to give satisfactory results [14].

Another area that must be considered in the design of a TEG is the mechanical properties of the thermoelectric materials. Most semiconductors are relatively weak mechanically. This weakness, coupled with the problems of forming the electrical/thermal contacts for semiconductors, has led to the relatively simple mechanical method of holding the thermoelectric element between the hot and cold junctions illustrated in Figure 13. Variations on compressive loading type arrangements have met with favorable results in several different applications [9,14,23,24,].

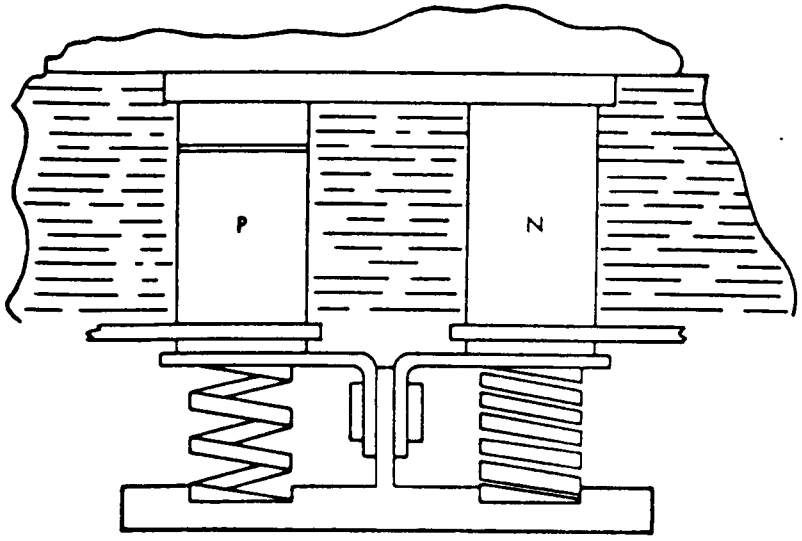


Figure 13. Mechanical Connector Schematic
(Noon and Stapfer [25])

To take full advantage of materials in the design of a TEG and to allow flexibility for uses in varied applications a modular approach which can be used as a building block for TEG's is recommended. The modular approach to design offers several advantages over a system where the thermoelements are constructed as an integral part of the heat source and heat rejection system [4]. The advantages include:

1. A clean,well defined interface between the manufac-

turer of the thermoelectric elements and the generator system integrator.

2. Modules can be integrated with various heat sources or sinks.

3. Improved modules can be easily integrated into existing systems or modules can easily be replaced in the event of failure.

CHAPTER V

DEVICE EVALUATION

Device Performance:

In this section the laws of thermodynamics and the thermoelectric phenomena (discussed in chapter 3) will be used to develop the equations necessary to analyze the performance of a TEG. The analysis will be based on the model shown in Figure 8. Details of the derivation of the equations presented are given in Appendix I. The major problem associated with the evaluation of a TEG with temperature-dependent material properties is the determination of the temperature distribution in the element legs.

Consider a section of the leg of length dx as shown in Figure 14 with current I passing along the leg through a temperature gradient.

From Fourier's law of heat conduction:

$$q_{input} = -k\frac{dT}{dx} \qquad (9)$$

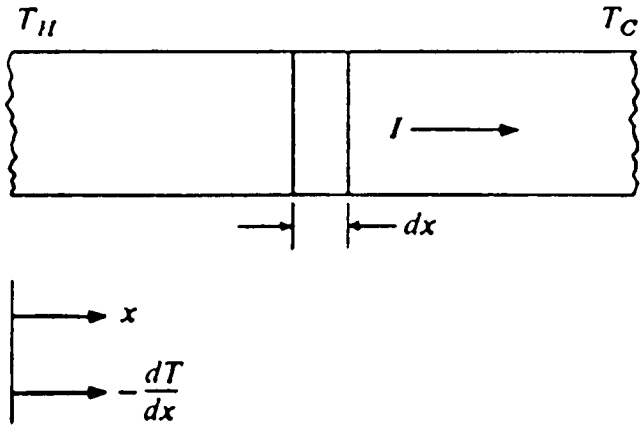


Figure 14. Model Used for Derivation of Equations (Angrist [14])

$$q_{out} = -k\left[\frac{dT}{dx} + \frac{d}{dx}\left(\frac{dT}{dx}\right)dx\right] \quad (10)$$

The heat generated in the rod consists of components from both the Joule heat and the Thompson heat.

$$q_{gen} = J^2 r\,dx - J\beta\frac{dT}{dx}dx \quad (11)$$

Assuming the device is operating under steady-state conditions, the heat in plus any heat generated must equal the heat out. This gives rise to the differential equation for the temperature distribution given below.

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)+J^2r-\beta J\frac{dT}{dx}=0 \qquad \textbf{(12)}$$

The derivation of this equation is straightforward, however, a procedure for a rigorous solution is lengthy and typically requires computer methods. For this reason, approximate methods are commonly used to account for the temperature-dependent material properties. The approximations are usually made by determining a suitable average value for each of the material properties and then treating them as temperature-independent properties [2,14].

The simplest approach is an average of the material property over the temperature range of interest.

$$\overline{\chi}=\frac{1}{\Delta T}\int_{T_c}^{T_H}\chi(T)\,dT \qquad \textbf{(13)}$$

A slight modification to this simple average (shown below)

is recommended by Ure and Heikes [26]. The values of k and α are calculated by equation (13) and then r_{av} can be calculated as:

$$r_{av}=\frac{1}{\bar{k}\Delta T}\int_{T_c}^{T_H} r(T)\,k(T)\,dT \qquad (14)$$

The reasoning for this approximation is that, for I=0, equations (13) and (14) give the correct internal resistance, thermal conductance, and voltage when using the equations that will be developed for constant material properties. Also, for I≠0 the use of equation (13) will result in the correct values for K, R, and V if the temperature is a linear function of x [26]. A comparison between the average property method and an exact solution is presented by Ure and Heikes [26] for three TEG's. The results of this comparison show differences of 7% of the calculated efficiency or less. Sutton [2] reports that by including an approximation for the Thompson heat, the error can be reduced to approximately 2%. The evaluation of TEG's in this research will not require accuracy greater than 10% so the accuracy achieved by the application of average material properties to constant-property equations was sufficient.

In order to develop the required equations, the assumption was made that the material properties were temperature-independent and equation (12) becomes:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)+J^2r=0 \qquad \textbf{(15)}$$

This differential equation may be solved with the boundary conditions: $T=T_H$ at $x=0$

$T=T_C$ at $x=L$

to give the temperature distribution in one leg of the TEG.

$$T(x)=\left[T_H-\frac{\Delta Tx}{L}\right]+\left[\frac{J^2rx(L-x)}{2k}\right] \qquad \textbf{(16)}$$

The first term in equation (16) is due to the conduction through the material caused by a temperature gradient and the second term is the result of a current moving through the element (Joule heating). By differentiating equation

(16) at x=0 and x=L and multiplying by the area and thermal conductivity, the Fourier heat entering the hot and cold junctions respectively may be found.

$$q_H = -KA\frac{dT}{dx}\Big|_{x=0} = K\Delta T - \frac{1}{2}I^2R \qquad \textbf{(17)}$$

$$q_C = -KA\frac{dT}{dx}\Big|_{x=L} = K\Delta T + \frac{1}{2}I^2R \qquad \textbf{(18)}$$

where:

$$R = \frac{r_nL_n}{A_n} + \frac{r_pL_p}{A_p} \qquad K = \frac{k_nA_n}{L_n} + \frac{k_pA_p}{L_p}$$

At the hot and cold junctions the Peltier effect also adds to the energy transfer. The total Peltier effect is the result of heat absorbed between each leg of the generator and each reservoir. The total Peltier effect is defined in equation (19).

$$-(\pi_{pr}+\pi_{rn})\,I_{pn}=-\pi_{pn}I_{pn}=-\alpha T I_{pn} \qquad (19)$$

where $\alpha=|\alpha_n| + |\alpha_p|$

The total thermal power input to the hot junction is given by the sum of the effects just described and is:

$$q_H=K\Delta T-\frac{1}{2}I^2R+\alpha T_H I \qquad (20)$$

Note that the sign of the Peltier term in equation (20) is opposite the definition given by equation (19). This results because the current flow is from the n to the p material at the hot junction (opposite the definition).

The Seebeck voltage can be calculated as

$$V=\alpha\Delta T \qquad (21)$$

From which the current may be calculated.

$$I = \frac{\alpha \Delta T}{R + R_L} \tag{22}$$

where: R = device internal resistance

R_L = external load resistance

The power output is the product of the current (from equation (22)) squared and the load resistance. The load resistance is used since only the useful power is of interest. By considering only the load resistance, the internal voltage losses are accounted for. The power output may then be calculated from equation (23).

$$P = I^2 R_L = \frac{\alpha^2 \Delta T^2 R_L}{(R + R_L)^2} \tag{23}$$

The efficiency of the TEG is defined as the ratio of the electrical power output to the thermal power input to the hot junction.

$$\eta = \frac{P}{q_H} = \frac{I^2 R_L}{(K \Delta T - \frac{1}{2} I^2 R + \alpha T_H I)} \tag{24}$$

The efficiency defined by equation (24) requires some explanation. The heat input used to calculate the efficiency of a TEG was derived using the model shown in Figure 14. The heat input to the hot junction is then defined by equation (20). This heat input to the device includes both the external heat supplied to the device as well as the heat generated within the device. When used to calculate efficiency, this definition of heat input will give conservative values because the heat generated internally is included. This definition of heat input is correct for the multistage devices where heat generated in one stage is input to the next stage and the overall efficiency is based on the efficiency of each stage. Previous research [8,9,10] typically reported efficiencies calculated including heat generated by the TEG, however, to an end user viewing the device as a "black box", the heat provided to the device (what you pay for) and the power provided from the device (what you get) are the two parameters of concern. For completeness, both the external heat input and the total heat input (and the efficiency calculated from each) will be presented for two cases later in this paper.

Device Optimization:

The important design parameters for a TEG are the efficiency and the power output. The two parameters which can be adjusted in the design of a TEG to optimize either the power output or the efficiency are the shape ratio, γ, (defined as the ratio of leg area to length) and the load resistance. The cases of interest in this research are those with maximum power output. The objective in designing for maximum power output is to attain a given power output with minimum size, weight, or use of thermoelectric material. The computer program for the evaluation of a TEG developed in this research (see Appendix II) generates the parameters required for the maximum power output design. The equations required for optimization of the power output are developed below using the equations from the previous sections.

In order to maximize the power output, the optimum load resistance must first be determined. This is accomplished by introducing the ratio of the load resistance to the device internal resistance, m, into equation (23). This results in equation (25).

$$P=\frac{\alpha^2 \Delta T^2 m}{(1+m)^2 R} \qquad (25)$$

where:

$$m=\frac{R_L}{R}$$

Taking the derivative of equation (25) (with respect to m) and setting it equal to zero shows that for maximum power output m must equal 1 (i.e., the load resistance must equal the internal resistance). Substituting this result into equations (21) through (24) gives:

$$V=\alpha\Delta T \tag{26}$$

$$I=\frac{\alpha\Delta T}{2R} \tag{27}$$

$$P=\frac{\alpha^2\Delta T^2}{4R} \tag{28}$$

$$\eta=\frac{I^2R}{(K\Delta T-\frac{1}{2}I^2R+\alpha T_H I)} \tag{29}$$

As can be seen from equations (23),(26),(27), and (28), by using only the load resistance, the power output does not equal the current times the voltage. For the case of maximum power (the load and internal resistances are equal), the power equals one-half the product of current and voltage. As mentioned before, the reason for this is that only the useful power is to be considered.

The geometry of the TEG may now be optimized to provide maximum power output per unit area, J. Substituting m=1 into equation (25) and dividing by the total TEG area yields:

$$J=\frac{\alpha^2 \Delta T^2}{4R(A_n+A_p)} \tag{30}$$

Introducing the definition of R given in equation (18) and rearranging gives:

$$J=\frac{\alpha^2 \Delta T^2}{4\left(r_n L_n+\frac{r_p L_p A_n}{A_p}+\frac{r_n L_n A_p}{A_n}+r_p L_p\right)} \tag{31}$$

With the Seebeck coefficient and ΔT constant, it can be seen that, to maximize J, the denominator of equation (31) must be a minimum. Taking the derivative of the denominator of equation (31) with respect to A_p/A_n and setting it equal to zero yields:

$$\frac{A_n}{A_p} = \left[\frac{L_n r_n}{L_p r_p}\right]^{\frac{1}{2}} \qquad (32)$$

An approach commonly taken is to assume the lengths of the n and p legs are equal[14,26] which then allows the shape ratio of the device to be optimized using equation (32).

The formulas developed thus far have been for a single TEG element. This analysis will now be extended a single-stage series TEG, a cascaded TEG, and a segmented-arm TEG.

The single-stage series TEG is shown in Figure 9. The design parameters (subscripted N) for a TEG with N couples connected thermally in parallel and electrically in series can be derived from equations (26)-(29) and are summarized below.

$$V_N = N\alpha\Delta T = NV \qquad (33)$$

$$I_N = \frac{N\alpha\Delta T}{N2R} = \frac{\alpha\Delta T}{2R} = I \tag{34}$$

$$q_N = NK\Delta T - \frac{1}{2} I_N^2 NR + N\alpha T_H I_N = Nq \tag{35}$$

$$P_N = I_N^2 R_N = NI^2 R = NP \tag{36}$$

$$\eta_N = \frac{P_N}{q_N} = \frac{NP}{Nq} = \frac{P}{q} = \eta \tag{37}$$

The next type of TEG evaluated is the cascaded TEG as shown in Figure 10. Optimizing an ideal, cascaded TEG requires both the appropriate thermoelectric material and the optimum interstage temperature for a given temperature environment. The design procedure is to choose the intermediate temperatures and materials that provide the maxi-

mum performance (i.e., highest Z) over the desired temperature range. This may be seen in the example from Ure & Heikes shown in Figure 15. The curves of Z vs. T make the techniques for material and intermediate temperature selection more obvious. After selecting the appropriate material and temperature range, and assuming the n and p legs have the same length within each stage (but differing lengths in adjacent stages), the area ratio may be determined from equation (32).

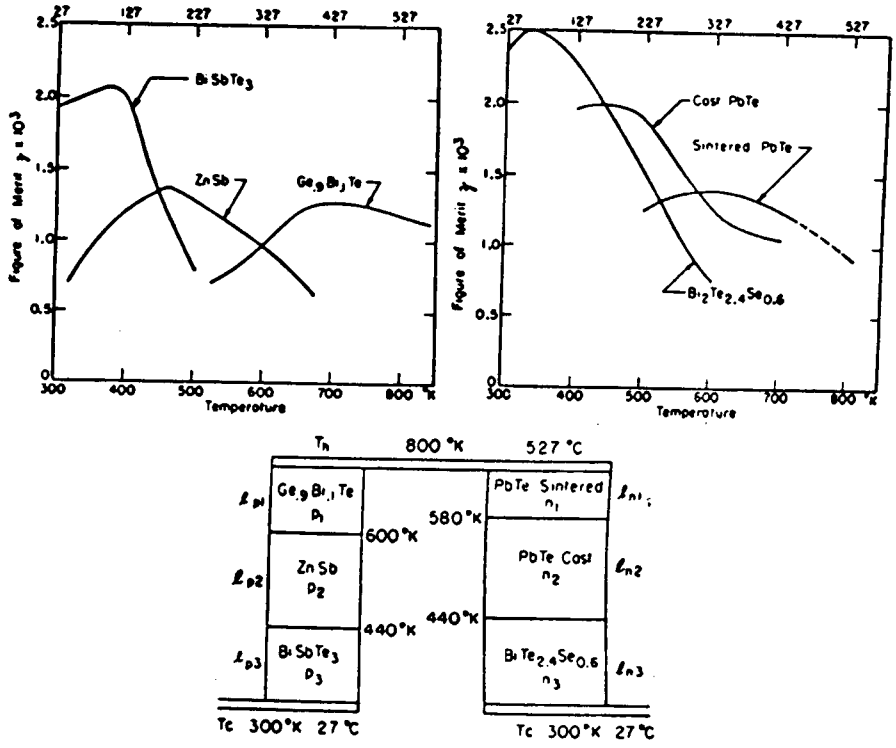


Figure 15. Material Selection for Multistage TEG (Ure and Heikes [26])

The number of couples in each stage and the shape ratio of successive stages must also be determined using the intermediate temperatures previously defined. The method used to determine these parameters is outlined by Ure and Heikes [26]. For the derivation of these equations the first stage will consist of N_1 couples operating between T_1 and T_2. The second stage will have N_2 couples and operate between T_2 and T_3. When the stages are connected electrically in series to provide the maximum output voltage from the TEG, the current which maximizes the efficiency for both stages must be equal. So, setting $I_1=I_2$ (where the subscripts 1 and 2 represent the first and second stages respectively), substituting equation (27) and R (as defined in equation (18)) into equation (32) and making the assumption that $L_p=L_n$, and rearranging yields:

$$\frac{\gamma_1}{\gamma_2} = \left(\frac{\Delta T_2}{\Delta T_1}\right)\left(\frac{\alpha_2}{\alpha_1}\right)\left(\frac{[r_{n1}^{\frac{1}{2}}+r_{p1}^{\frac{1}{2}}]^2}{[r_{n2}^{\frac{1}{2}}+r_{p2}^{\frac{1}{2}}]^2}\right) \qquad \textbf{(38)}$$

This equation defines the relationship of the shape ratio between successive stages of the TEG. Obtaining the relationship between the number of couples in each stage

of the TEG is more complicated. From equations (18) and (36) the heat rejected from the first stage, q_1, and the heat absorbed by the second stage, q_2 may be calculated.

$$q_1 = N_1 (\alpha_1 T_i I_1 + K_1 \Delta T_1 + \frac{1}{2} I_1^2 R_1) \tag{39}$$

$$q_2 = N_2 (\alpha_2 T_i I_2 + K_2 \Delta T_2 - \frac{1}{2} I_2^2 R_2) \tag{40}$$

Under steady-state operation, the heat given off by the first stage must equal the heat absorbed by the second stage. Setting $q_1 = q_2$ and rearranging results in the following relationship between the number of couples in successive stages.

$$\frac{N_2}{N_1} = \left(\frac{\gamma_1}{\gamma_2}\right)\left[\frac{\dfrac{\left(\frac{\alpha_1^2 \Delta T_1 T_i}{2} + \frac{\alpha_1^2 \Delta T_1^2}{8}\right)}{\left(r_{n1}^{\frac{1}{2}} + r_{p1}^{\frac{1}{2}}\right)^2} + \dfrac{\Delta T_1 \left(r_{n1}^{\frac{1}{2}} k_{n1} + r_{p1}^{\frac{1}{2}} k_{p1}\right)}{r_{p1}^{\frac{1}{2}}}}{\dfrac{\left(\frac{\alpha_2^2 \Delta T_2 T_i}{2} + \frac{\alpha_2^2 \Delta T_2^2}{8}\right)}{\left(r_{n2}^{\frac{1}{2}} + r_{p2}^{\frac{1}{2}}\right)^2} + \dfrac{\Delta T_2 \left(r_{n2}^{\frac{1}{2}} k_{n2} + r_{p2}^{\frac{1}{2}} k_{p2}\right)}{r_{p2}^{\frac{1}{2}}}}\right] \tag{41}$$

The efficiency of the cascaded TEG is calculated as:

$$\eta = \frac{\Sigma P_i}{Q_H} \tag{42}$$

The final type of TEG evaluated is the segmented-arm TEG shown in Figure 11. The theoretical efficiency of a segmented-arm TEG is always less than the corresponding multistage generator because the number of parameters that can be adjusted for optimization is fewer for the segmented-arm generator [14]. However, in practice, the elimination of both the temperature drop across the electrical insulation and the electrical- contact resistance between the stages of a cascaded TEG may be enough to offset the losses from the optimization.

There is no simple analytical solution for the segmented TEG. The primary difficulty in the analysis arises because the lengths of the segments are a function of the current and load resistance. The lengths of the segments may be calculated from a heat balance at each interface. The heat flowing out of the interface must equal the heat flowing into the interface. Accounting for the Fourier conduction, Joule heat, and the Peltier effect in this heat balance will give:

$$k_i\left(\frac{\Delta T_i}{L_i}\right)+\frac{1}{2}r_i\frac{I^2}{A^2}Li-k_{i+1}\left(\frac{\Delta T_{i+1}}{L_{i+1}}\right)+\frac{1}{2}r_{i+1}\frac{I^2}{A^2}L_{i+1}$$

$$-(\alpha_{i+1}-\alpha_i)\,T_i\frac{I}{A}=0 \qquad (43)$$

An approximate method developed by Swanson, Sommers, and Heikes has been shown to provide reasonably accurate solutions [26]. A comparison of this approximate method and an exact solution has shown differences of less than 5% [26]. The procedure consists of first defining average material properties for each segment of the generator. using these average properties, the voltage and internal resistance of the TEG may be determined from equations (44) and (45) respectively.

$$V=\left[\alpha_{p1}(T_H-T_{p1i})+\alpha_{p2}(T_{p1i}-T_{p2i})+....\right]+$$

$$\left[\alpha_{n1}(T_H-T_{n1i})+\alpha_{n2}(T_{n1i}-T_{n2i})+....\right] \qquad (44)$$

$$R=\left(\frac{1}{A_p}\right)(r_{p1}l_{p1}+r_{p2}l_{p2}+...)+\left(\frac{1}{A_n}\right)(r_{n1}l_{n1}+r_{n2}l_{n2}+...) \qquad (45)$$

The length of each segment is then calculated from equation (46) (which results from equation (43) being evaluated with the average material properties for each segment for the case of no current flow)[26].

$$\frac{L_{pi}}{L_{pi+1}} = \frac{k_{pi}(T_{pi}-T_{pi+1})}{k_{pi+1}(T_{pi+1}-T_{pi+2})} \qquad (46)$$

Similarly for the n leg:

$$\frac{L_{ni}}{L_{ni+1}} = \frac{k_{ni}(T_{ni}-T_{ni+1})}{k_{ni+1}(T_{ni+1}-T_{ni+2})} \qquad (47)$$

The thermal efficiency of the TEG from equation (24) may then be written as:

$$\frac{P}{q_H} = \left(\frac{V^2 R_L}{R+R_L^2}\right) / \qquad (48)$$

$$\left[\left(k_{p1}(T_H-T_{p1}) + k_{n1}(T_H-T_{n1})\right) + \left(\frac{T_H(\alpha_{p1}+\alpha_{n1})\,V}{(R+R_L)} - \frac{1}{2}\frac{V^2}{(R+R_L^2}(R_{p1}+R_{n1})\right]$$

where:

$$K_{n1} = \frac{\overline{k_{n1}} L_{n1}}{A_n} \qquad K_{p1} = \frac{\overline{k_{p1}} L_{p1}}{A_p}$$

$$R_{p1} = \frac{\overline{r_{p1}} L_{p1}}{A_p} \qquad R_{n1} = \frac{\overline{r_{n1}} L_{n1}}{A_n}$$

As before, for the case of maximum power output, the load resistance is set equal to the internal resistance. Rearranging and introducing the following variables will yield equation (49).

$$K_{p1} = \frac{\overline{k_{p1}} L_{p1}}{A_p}$$

$$k'_{n1} = \frac{\overline{k_{n1}}}{A_n}$$

$$k'_{p1} = \frac{\overline{k_{p1}}}{A_p}$$

$$R'_p = (\overline{r_{p1}} L_{p1} + \overline{r_{p2}} L_{p2} + \ldots\ldots)$$

$$R'_n = (\overline{r_{n1}} L_{n1} + \overline{r_{n2}} L_{n2} + \ldots\ldots)$$

$$\hat{\alpha} = |\alpha_{p1}| + |\alpha_{n1}|$$

$$R = \left(\frac{1}{A_p}\right) R'_p + \left(\frac{1}{A_n}\right) R'_n$$

$$\frac{q_H}{P} = \left[\frac{4\left(\frac{R'_p}{A_p} + \frac{R'_n}{A_n}\right)}{V^2}\right]\left[\overline{k_{p1}}\left(T_H - T_{p1}\right) + \overline{k_{n1}}\left(T_H - T_{n1}\right)\right] +$$

$$\left[\frac{(\alpha 2 T_H)}{V}\right] - \left[\left(\frac{\frac{r_p L_p}{A_p} + \frac{r_n L_n}{A_n}}{2R}\right)\right] \qquad \textbf{(49)}$$

Taking the derivative of equation (49) with respect to A_n/A_p , setting it equal to zero, and rearranging gives equation (50). This is the area ratio of the legs of the TEG that will maximize efficiency.

$$\frac{A_n}{A_p} = \left[\frac{\left(T_H - T_{p1}\right) k'_{p1} R'_n}{\left(T_H - T_{n1}\right) k'_{n1} R'_p}\right] \qquad \textbf{(50)}$$

Equation (50) is then used to calculate the ratio of cross sectional areas of the n and p legs Note that the cross sectional area is constant across all segments for each leg.

CHAPTER VI

PROGRAM DESCRIPTION

General:

The programs developed during this research for the analysis of TEG's are presented in Appendix II. The programs were developed using the equations presented in the previous sections of this report. Three separate programs were developed and are each described below. All three programs were written in the Basic programming language, using the QBasic program contained in MS-DOS, version 5.2. The programs were written to read data from an input file and write the data to an output file (both files are also presented in Appendix II). If desired, the programs could easily be modified to allow interactive input of the data.

The output of each program is basically the same and primarily consists of:

heat input required
output voltage
output current
output power

power density

efficiency of the generator

shape ratio

Some program-peculiar outputs are also presented to aid in evaluation of each stage of the multistage generators. The inputs required, assumptions made, and a few additional comments are presented now for each of the three programs.

Single.Bas:

This program is used to evaluate the single stage series TEG as shown in Figure 9. The program performs the analysis using the equations developed in the previous sections of this report. The materials to be evaluated are selected by the user. The lengths of the p- and n- legs are assumed to be the same and are set equal to 1.0 unit for the evaluation. The cross-sectional area of the p-leg is assumed to be 1 square unit. These assumptions allow the material properties of the couple to be calculated and also permit the area of the n-leg to be calculated. The shape ratio (area/length) of the p- and n- legs may then be established and varied. If necessary,the lengths and areas may be varied as long as the shape ratio is main-

tained constant. The number of couples required to provide the desired voltage is then determined and the performance parameters are calculated for this case.

The required input for the program includes:
hot junction temperature
cold junction temperature
average Seebeck coefficient (for each material)
average thermal conductivity (for each material)
average resistivity (for each material)
desired output voltage

Cascade.Bas:

This program was developed with the equations of the previous sections to evaluate the performance of a multi-stage cascaded TEG (as shown in Figure 10). The number of stages to be evaluated, materials for each stage, and intermediate temperatures are to be determined and provided by the user. The program makes the assumption that the lengths of the p- and n- legs are the same within a given stage but may differ between successive stages. The length of the first stage is assumed to be 1 unit and the cross sectional area of the p-leg in the first stage is assumed to be 1.0 square unit. These assumptions allow

calculation of the heat input and the material properties. The leg length is then adjusted to maintain the desired interstage temperature. The performance parameters are then recalculated based on the assumed area and the calculated length. The total number of couples required to provide the desired power is then calculated.

The required input for the program includes:
hot junction temperature
intermediate temperatures
cold junction temperature
number of couples in the first stage
number of stages to be evaluated
average Seebeck coefficient (for each material)
average thermal conductivity (for each material)
average resistivity (for each material)
desired output voltage

Segment.Bas:

This program is used to evaluate a segmented-leg, multi-stage TEG as shown in Figure 11. The temperatures of each segment (p-and n- legs) must be specified by the user in order to calculate the material properties and performance parameters of the TEG. Again, the lengths of the p- and

n- legs are assumed to be 1 unit for the first segment and the cross-sectional area of the p-leg is assumed to be 1 square unit. This allows calculation of the material properties and heat input. The length of the segment required to maintain the interstage temperature is then determined. The performance parameters are then calculated based on the assumed area and the calculated lengths. The cross-sectional area of each leg (once determined) is constant for all segments of that leg. The total number of couples required to provide the desired output voltage is then determined and the performance of the module is evaluated.

The required input for the program includes:

hot junction temperature
cold junction temperature
intermediate temperatures
number of couples
number of p-segments
number of n-segments
average Seebeck Coefficient (for each material)
average thermal conductivity (for each material)
average resistivity (for each material)
desired voltage

CHAPTER VII

CASE STUDIES

General:

It was the goal of this research to evaluate the feasibility of using a TEG to recover waste energy from existing sources. Large amounts of low grade thermal energy are currently wasted by various industries. This wasted energy could be recovered as economically produced electricity through the use of a TEG. Industries that could potentially benefit from this development include:

Electric Power Utilities

Textile Mills

Paper Mills

Chemical Producers

Steel/ Metal Manufacturers

Food Producers

Two cases were evaluated to verify the performance of the programs and to help establish the relative performance of the three types of TEG's. One case evaluated the use of exhaust gases from a small industrial boiler as the heat source. The expected performance data from the boiler

manufacturer indicates that for a heat input of 67.5 million BTU/hr, approximately 69.2 thousand lbm/hr of flue gas will leave the boiler at a temperature of approximately 525 °F. This assumes that there is no economizer in service. For the purposes of this analysis, the flue gas will be considered to be a heat source (reservoir) at a temperature of 525°F (546 °K). The stack surroundings will be considered a heat sink (reservoir) at a temperature 80 °F (300 °K). The potential energy available from the flue gas is 4.77×10^6 BTU/hr (1,394 kW) based on a simplified analysis (as shown in Appendix II). This is the amount of energy that can be removed before lowering the flue gas temperature below the acid dewpoint of 250 °F.

The second case was run at a high temperature condition (material temperature limit) to establish the potential of the TEG. This case was considered as if the TEG was configured into the wall of a combustion chamber. The maximum hot junction temperature evaluated was 1000 °K with the cold junction temperature maintained at 300 °K. The conditions of both cases are illustrated in Figure 16.

To help evaluate the relative performance of the three types of generators, a design requirement for a 115-volt module was imposed. This module will be considered a

building block which can be combined to meet various power requirements. The 115-volt was an arbitrary requirement and can be changed to meet application specific requirements.

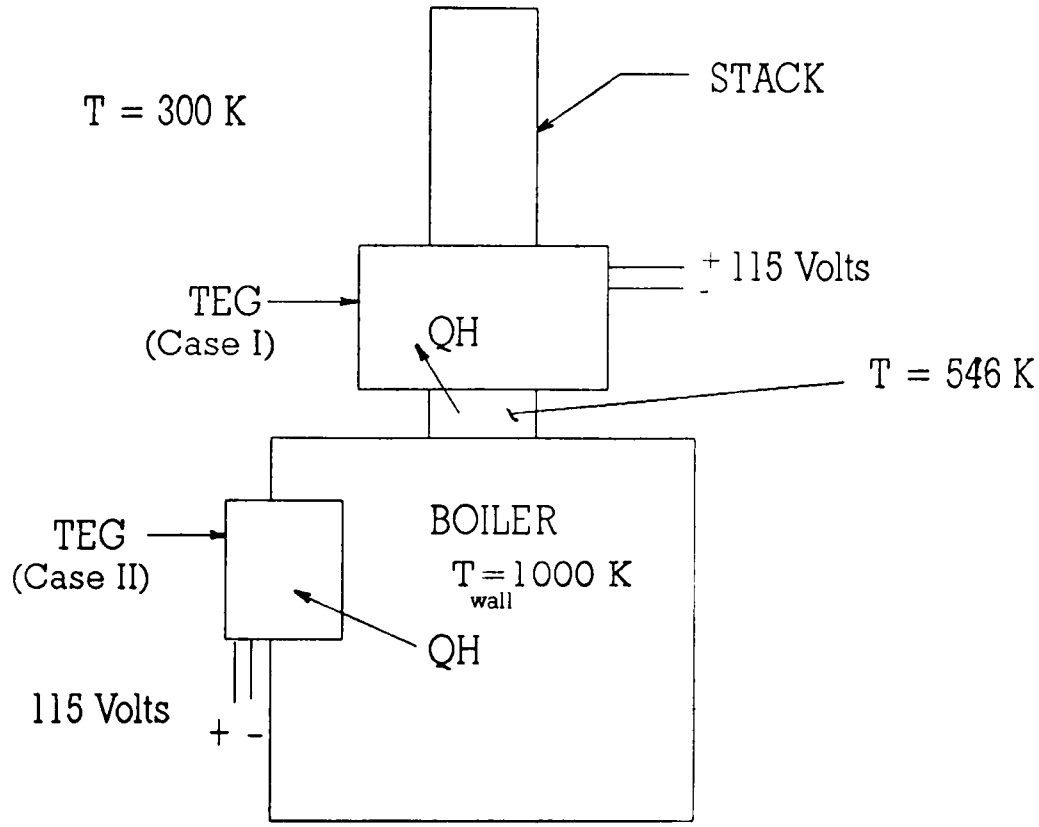


Figure 16. Illustration of Case Studies

The thermoelectric generators were evaluated as if their cold junction temperature was that of the heat sink and their hot junction temperature was that of the heat source. The assumptions made in the development of the equations were also maintained for this study. The ther-

moelectric material properties used for each case were the average values over the temperature range of interest. The input files used to evaluate each TEG and the output files generated are presented along with the program listings in Appendix II. A summary of the results from this evaluation are presented in Table 3-5.

Presentation of Results:

Single-Stage Series TEG: The single-stage TEG was evaluated using PbTe as both the p- and n-type thermoelectric materials. The material properties were average values over the temperature range of 550-300 °K and 1000-300 °K. The material properties used are provided in Appendix II in the program input files. Table 3 presents a summary of the two cases evaluated for the single-stage series TEG. Due to the nature of these elements, they may simply be combined electrically in series and thermally in parallel (as shown in Figure 9) to provide the desired output voltage. The results for the single stage TEG are presented in Appendix II and summarized in Table 3. Cases I and II represent the low and high temperature cases respectively. A discussion of these results will be presented in the next section of the paper.

Table 3. Results for a Single-Stage Series TEG

Parameter	Case I	Case II
T_H, °K	546	1000
T_C, °K	300	300
No. of Elements	2410	710
A_n, cm^2	1.72	0.79
A_p, cm^2	1.00	1.00
L, cm	1.00	1.00
I, amp *	19.52	13.83
R, ohm +	5.89	8.31
V, volt*	115	115
P, Watt*	2245	1591
Q_H, Watt	41889	20729
Efficiency, %	5.36	7.67

+ Internal resistance

* Device output to the load

Cascaded TEG: A two-stage cascaded TEG was also evaluated for both cases described earlier. The materials selected to operate over the ranges of interest were:

Stage 1:	p-leg PbTe	n-leg PbTe
Stage 2:	p-leg $BiSbTe_3$	n-leg $Bi_2Te_{2.4}Se_{0.6}$

The intermediate temperature to maximize performance was determined (using the method described earlier) to be 440 °K. This intermediate temperature was then used as the first stage cold junction temperature and the second stage hot junction temperature. The material properties used were average values determined for the temperature ranges of 550-440 °K and 440-300 °K (for case I) and 1000-440 °K and 440-300 °K (for case II). The output from each stage was considered to be connected electrically in series to increase the output voltage. The length of each stage can be adjusted (as long as the shape ratio is maintained constant within the stage) to maintain the intermediate temperatures constant. The results for the cascaded TEG are presented in Appendix II and summarized in Table 4 below. Again, cases I and II represent the low and high temperature cases respectively. A discussion of these results will be presented in the next section of the paper.

Table 4. Results for a Cascaded TEG

Parameter	Case I		Case II	
Stage No.	1	2	1	2
T_H, °K	546	440	1000	440
T_C, °K	440	300	440	300
No. of Elements	744	3720	243	3159
A_n, cm^2	0.54	0.45	0.80	0.45
A_p, cm^2	1.00	0.36	1.00	0.35
L, cm	0.44	1.49	0.42	0.67
R, ohm +	2.33	28.61	1.44	10.83
V, volt *	18.95	96.18	34.01	81.32
I, amp *	8.12	3.36	23.68	7.51
P, Watt *	154	323	805	610
Q_H, Watt	6730	6253	13224	11808
Efficiency, %	2.29	5.17	6.09	5.17
V, volt ++	115		115	
P, Watt ++	476		1412	
Efficiency, % ++	7.34		10.95	

+ Internal resistance

* Device output to the load

++ Total for device

Segmented TEG: A segmented TEG was also evaluated for both previously defined cases. The materials selected for construction of the TEG were:

p-1	PbTe	p-2	$BiSbTe_3$
n-1	PbTe	n-2	$Bi_2Te_{2.4}Se_{0.6}$

The material properties for each segment were determined using average values over the temperature range of interest and the segment geometry.

The segmented elements (as shown in Figure 11) are connected both thermally and electrically in series to increase the voltage. Each segment length was calculated (using a heat balance at the interface) to provide the desired intermediate temperature to maximize the device output. The results for the segmented TEG are presented in Appendix II and summarized below in Table 5. Again, cases I and II represent the low and high temperature cases respectively. A discussion of these results will be presented in the next section of the paper.

Table 5. Results for a Segmented TEG

Parameter	Case I	Case II
No. of Elements	1872	896
A_n, cm^2	0.18	2.88
A_p, cm^2	1.00	1.00
L_{p1}, cm	0.76	0.80
L_{p2}, cm	0.73	0.73
L_{n1}, cm	0.79	0.89
L_{n2}, cm	0.68	0.68
R, ohm +	38.86	9.21
V, volt *	115	115
I, amp *	2.96	12.50
P, Watt *	340	1438
Q_H, Watt	5641	17575
Efficiency, %	6.03	8.18

+ Internal resistance

* Device output to the load

The heat input and the efficiencies shown in Tables 3-5 were determined including the internal heat generation for the TEG. The programs calculate each component of the heat input (equation (20)) and to show the relative contributions of the external heat supplied (conduction), the Peltier heat, and the Joule heat, each component is presented in Table 6.

Table 6. Comparison of Internal Heat Generation and External Heat Transfer

Parameter	Single-Stage Series TEG		Cascaded TEG		Segmented TEG	
Case	I	II	I	II	I	II
Q_H, W	41889	20729	6730	13224	5640	17575
Q_{cond}, W	33045	16980	5222	10750	4633	15412
$Q_{Peltier}$, W	9967	4544	1586	2876	1060	2515
Q_{Joule}, W	-1123	-795	-77	-403	-66	-391
Eff_{total}, %	5.36	7.67	7.34	10.95	6.03	8.18
Eff_{cond}, %	6.79	9.36	9.46	13.47	7.34	9.32

Viewing the TEG as a "black box", the efficiency will be

increased from 1.3 to 2.5 % above the value when taking the internal heat generation into account. Table 6 also shows that the efficiency of a TEG is a strong function of the temperature difference driving the device. As seen in the two cases presented, increasing the overall temperature difference from 246 °K to 700 °K can increase the device efficiency up to 4 %. This is not suprising since the power output is proportional to the square of the temperature difference and the heat input is directly proportional to the temperature difference. The choice to include or disregard the internal heat generation in the calculation of the efficiency is ultimately of little consequence in the selecting which type of TEG most useful because the relative performance of the three types remains the same.

Discussion of Results:

In this section, the results from the three types of TEGs will be discussed. However, some general discussion concerning the applicability or realism of the cases studied and the TEG module design will be presented first.

The first case study selected has direct application to several industries, as previously mentioned. The boiler

evaluated was a package unit designed to provide 100 psig steam with 65 °F superheat. If a TEG could be designed to recover the wasted thermal energy in the hot exhaust gases (potentially 1394 kW) , it would not only improve the efficiency of the boiler but could possibly generate enough electricity to power the controls for the boiler. While it appears that producing enough power to drive all the associated equipment for the boiler may be questionable, a larger boiler or better TEG may eventually lead to a truly "stand-alone" package boiler.

The use of a TEG in the exhaust stack of a boiler could present various problems. If the modules are physically located inside the stack, the additional pressure drop, as well as mass flow capabilities of the stack, must be evaluated . Another problem would arise if the required heat input lowered the exhaust gases below the acid dew point and acidic condensate began to form on the TEG and stack. Economizer performance could also, obviously, be effected if the stack gas temperature was reduced significantly.

Modular design of the TEG, in addition to the benefits mentioned earlier, would allow various levels of power output to be attained by connecting the modules in series. This would allow generators to be tailored to specific

applications. The complete design of a TEG module is beyond the scope of this study. However, for the case study selected, a compressive loading scheme as shown in Figure 13 is probably the simplest and most economical method available. The design of the heat exchangers for both the hot and cold junctions must be performed. Water-cooled modules would typically be more advantageous since many industrial applications have the requirement for heated process water. In such cases, the TEG would also serve as a preheater. In the case of the boiler stack, the TEG could possibly be integrated into the economizer to perform both tasks.

The performance of the three types of TEGs can be evaluated in a number of ways, however, as was stated previously, this research was concerned with maximum power output. The amount of energy that can be removed from the stack gas before dropping below the acid dewpoint is 1394 kW. Using this energy as the external heat input to the TEG, the maximum power available from each type of TEG can be determined. Using the efficiency based on the external heat only (eff_{cond}) from Table 6 and the 1394 kW available energy from the stack gas, the maximum power possibly provided by each type of TEG may be determined. These values are listed below.

TEG Type	Maximum Power Provided
Single-Stage Series TEG	94.7 kW
Cascaded TEG	131.9 kW
Segmented TEG	102.4 kW

It is obvious from this information that the Cascaded TEG is the best selection for conversion of waste heat into electric energy.

The programs developed in this research are based on theoretical calculations only. The programs do not take into account any parasitic losses in the TEG (i.e., electrical losses across thermal insulation, thermal losses across electrical insulation, temperature drops across insulation, contact resistances, etc.). All three types of TEG's will have losses associated with the hot and cold junction connections and these losses may be assumed to be approximately equal. A qualitative analysis by the author shows that the cascaded TEG will have the largest losses when the additional materials and junctions resulting from the staging of the couples are considered. The segmented TEG will have additional losses only because of the contact resistance between segments. The losses in the single-stage series TEG will be limited to the losses associated with the junctions (common to all three types). If evaluated, the losses identified for each TEG will

likely tend to equalize the performance of the three types.

Thermoelectric generators are commercially available now and should decrease in cost as demand increases in the future or as fabrication techniques are refined. However, the economics of the TEG must also be evaluated as a part of a complete analysis for each application. The use of modular construction would allow the TEG components to be mass produced, which would lower unit cost. The simplest TEG to fabricate is obviously the single stage series TEG since the number of semiconductor junctions is kept to a minimum. The cascaded TEG is simply two or more sets of a single stage series TEG connected thermally in parallel and electrically in series. The segmented TEG poses the additional difficulty of fabricating the intersegment connections (semiconductor to semiconductor) which is difficult and potentially a point of failure. Estimated initial costs of thermoelectric generators are on the order of $30/ Watt [6,7]. This is rather expensive but when no fuel costs and practically no maintenance costs are considered, the payback period would become more attractive.

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

Based on the results of this analysis, the use of thermoelectric generation as a means to recover energy from waste heat sources is a feasible concept. Theoretical calculations show that a cascaded TEG module (with an efficiency of 9.46 %) could provide 0.48 kW of power at 115 volts utilizing flue gas from a boiler as the heat source. Combining these modules could result in power generation of up to 131.9 kW from energy that is currently being wasted throughout industry. Other applications in which thermoelectric generators could take advantage of waste heat are currently being developed [27].

The research done here can provide a tool to facilitate the evaluation of thermoelectric generators for a specific application. Refinement of the programs developed in this research should include the formation of a database of thermoelectric material properties and a routine to calculate the average material properties. Such a database would greatly aid in any future evaluations. Also, in order to improve the realism of these calculations, some consideration of losses within the generator must be made. The theoretical calculations should also be verified

through laboratory or field tests.

The use of TEGs seem to be feasible for many applications and are just beginning to be developed. The performance of TEGs will improve with the continued improvement of generator designs and development of superior materials. The TEG is still not economically competitive with more conventional sources of power; however, with the intent in industry to minimize waste, to use alternative energy sources and to be environmentally conscientious, the use of thermoelectric generators seems an obvious practice in the future.

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APPENDICES

Appendix I

DERIVATION OF EQUATIONS

The equations used for the evaluation of TEG's are presented in the text of this paper. A brief derivation of these equations will now be given.

The analysis of the TEG is based on effective properties for the element (comprised of both the n- and p- leg properties). Below are the definitions of the effective Seebeck coefficient, electrical resistance, and thermal conductivity for a single TEG element.

$$\alpha = |\alpha_n| + |\alpha_p|$$

$$R = \frac{r_n L_n}{A_n} + \frac{r_p L_p}{A_p}$$

$$K = \frac{k_n A_n}{L_n} + \frac{k_p A_p}{L_p}$$

These equations are valid for the single-stage series TEG and for each segment or stage of a multistage device. The overall properties of the segmented TEG may be determined by summing the properties of each segment, while the cascaded TEG is treated as two independent single-stage

series TEG's.

The heat flowing through a TEG element can be calculated from Fourier's law of heat conduction and the temperature distribution given in equation (16). The temperature distribution in equation (16) comes from the steady-state operation of the TEG and includes external heat flowing into the TEG, heat flowing out of the TEG, and any heat generated within the TEG. These three components must be accounted for in the equations used for optimization of the multistage generators. The heat flowing into and out of the element is then seen to be:

$$q_{in} = -kA\frac{dT}{dx}\Big|_{x=0}$$

$$= (-kA)\left(\frac{J^2 r}{2k}L - \frac{\Delta T}{L}\right)$$

$$= k\Delta T\gamma - \frac{I^2 r}{2\gamma}$$

$$= K\Delta T - \frac{1}{2}I^2 R$$

The Peltier effect also contributes to the heat transfer

and will add another term to the equation for heat input to the TEG.

$$q_{in}=K\Delta T-\frac{1}{2}I^2R+\alpha T_H I$$

$$q_{out}=-kA\frac{dT}{dx}\Big|_{x=L}$$

$$=(-kA)\left(-\frac{J^2r}{2k}L-\frac{\Delta T}{L}\right)$$

$$=k\Delta T\gamma+\frac{I^2r}{2\gamma}$$

$$=K\Delta T+\frac{1}{2}I^2R$$

Again, the Peltier effect has its contribution to the heat transfer out of the TEG element.

$$q_{out}=K\Delta T+\frac{1}{2}I^2R+\alpha T_C I$$

The device may be represented as the electrical circuit shown in Figure 17 below. V is the total voltage generated by the device, V_{OUT} is the terminal voltage, R is the internal resistance of the TEG, and R_L is the load resistance.

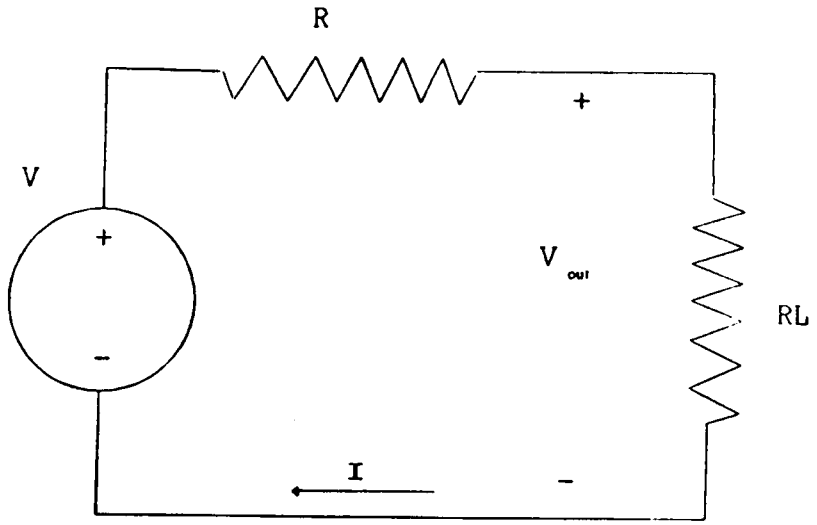


Figure 17. Electric Circuit Representing a TEG

The voltage generated by the device is :

$$V=\alpha\Delta T$$

It was shown in the text that to maximize the power output, the load resistance must equal the internal resistance (i.e., $R_l=R$). This optimum load resistance will be used from this point on without further reference and the total resistance of the circuit twice the internal resistance. The current may be determined from Ohm's Law.

$$I=\frac{V}{R}=\frac{\alpha\Delta T}{2R}$$

Applying Kirchoff's Second Law [28] to the circuit will allow V_{OUT} to be calculated.

$$V-IR-IR_L=0$$

$$V_{OUT}=V-IR=\alpha\Delta T-\frac{\alpha\Delta T}{R+R_L}R_L$$

$$V_{OUT}=\alpha\Delta T-\frac{\alpha\Delta T}{2}=\frac{\alpha\Delta T}{2}$$

The total power produced by the circuit is:

$$P=IV=\frac{\alpha^2\Delta T^2}{2R}$$

The useful power is defined as the power delivered to the load and is calculated below.

$$P_{useful}=IV_{OUT}=\frac{\alpha\Delta T}{2R}\cdot\frac{\alpha\Delta T}{2}=\frac{\alpha^2\Delta T^2}{4R}$$

The efficiency is defined as the useful power divided by the heat input at the hot junction. The heat input (as shown previously) is made up of the external heat supplied

($K\Delta T$), the Joule heat generated (I^2R), and the Peltier heat generated ($\alpha T_H I$). The efficiency is calculated from the following equation.

$$\eta = \frac{P}{q_H} = \frac{IV_{OUT}}{(K\Delta T - \frac{1}{2} I^2 R + \alpha T_H I)}$$

Some additional equations are required for the evaluation and design of a multistage TEG. The number of elements in successive stages of the cascaded TEG is determined by setting the heat out of the first stage, q_1, equal to the heat entering the second stage, q_2.

$$q_1 = [K_1 \Delta T_1 - \frac{1}{2} I_1^2 R_1 + \alpha_1 T_i I_1] N_1$$

$$q_2 = [K_2 \Delta T_2 - \frac{1}{2} I_2^2 R_2 + \alpha_2 T_i I_2] N_2$$

Setting $q_1 = q_2$ gives:

$$q_1 = q_2$$

$$[K_1 \Delta T_1 - \frac{1}{2} I_1^2 R_1 + \alpha_1 T_i I_1] N_1 = [K_2 \Delta T_2 - \frac{1}{2} I_2^2 R_2 + \alpha_2 T_i I_2] N_2$$

$$\frac{N_1}{N_2} = \frac{[K_2 \Delta T_2 - \frac{1}{2} I_2^2 R_2 + \alpha_2 T_i I_2]}{[K_1 \Delta T_1 - \frac{1}{2} I_1^2 R_1 + \alpha_1 T_i I_1]}$$

$$\frac{N_2}{N_1} = (\frac{\gamma_1}{\gamma_2}) \left[\frac{\frac{(\frac{\alpha_1^2 \Delta T_1 T_i}{2} + \frac{\alpha_1^2 \Delta T_1^2}{8})}{(r_{n1}^{\frac{1}{2}} + r_{p1}^{\frac{1}{2}})^2} + \frac{\Delta T_1 (r_{n1}^{\frac{1}{2}} k_{n1} + r_{p1}^{\frac{1}{2}} k_{p1})}{r_{p1}^{\frac{1}{2}}}}{\frac{(\frac{\alpha_2^2 \Delta T_2 T_i}{2} + \frac{\alpha_2^2 \Delta T_2^2}{8})}{(r_{n2}^{\frac{1}{2}} + r_{p2}^{\frac{1}{2}})^2} + \frac{\Delta T_2 (r_{n2}^{\frac{1}{2}} k_{n2} + r_{p2}^{\frac{1}{2}} k_{p2})}{r_{p2}^{\frac{1}{2}}}} \right]$$

The shape ratio for successive stages is determined by assuming that the current from the first stage equals the current from the second stage. First, to evaluate the calculation of the current. As shown in the text:

$$I_i = \frac{\alpha_i \Delta T_i}{2R_i} = \frac{[\frac{\alpha_i \Delta T_i}{2}]}{\frac{r_{in} L_{in}}{A_{in}} + \frac{r_{ip} L_{ip}}{A_{ip}}}$$

Assuming $L_p = L_n$ for each stage and rearranging yields:

$$I_i = \frac{[\frac{\alpha_i \Delta T_i}{2}]}{[\frac{L}{A_{ip}}] [r_{in} \frac{A_{ip}}{A_{in}} + r_{ip}]}$$

Substituting for A_n/A_p from equation (32)

$$I_i = \frac{[\frac{\alpha_i \Delta T_i}{2}]}{[\frac{L}{A_{ip}}]\,[(r_{in}r_{ip})^{\frac{1}{2}} + r_{ip}]}$$

Now,

$$I_1 = I_2$$

$$\frac{[\frac{\alpha_1 \Delta T_1}{2}]}{[\frac{L}{A_{1p}}]\,[(r_{1n}r_{1p})^{\frac{1}{2}} + r_{1p}]} = \frac{[\frac{\alpha_2 \Delta T_2}{2}]}{[\frac{L}{A_{2p}}]\,[(r_{2n}r_{2p})^{\frac{1}{2}} + r_{2p}]}$$

$$\gamma_i = \frac{A_i}{L_i}$$

$$\frac{\gamma_2}{\gamma_1} = \frac{\alpha_1}{\alpha_2}\,\frac{\Delta T_1}{\Delta T_2}\,\frac{[(r_{n2}r_{p2})^{\frac{1}{2}} + r_{p2}]}{[(r_{n1}r_{p1})^{\frac{1}{2}} + r_{p1}]}$$

The efficiency of the cascaded TEG is the total useful power output by the device divided by the heat input to

the hot junction.

$$\eta_{total} = \frac{\Sigma P_{useful}}{Q_H}$$

The segmented TEG is evaluated using the equations for a single element with effective material properties as defined earlier in the paper. The determination of each segment length is accomplished by evaluating equation (43) for the case of no current flow (I=0).

$$k_i\left(\frac{\Delta T_i}{L_i}\right) + \frac{1}{2} r_i \frac{I^2}{A^2} Li - k_{i+1}\left(\frac{\Delta T_{i+1}}{L_{i+1}}\right) + \frac{1}{2} r_{i+1} \frac{I^2}{A^2} L_{i+1} - (\alpha_{i+1} - \alpha_i) T_i \frac{I}{A} = 0$$

For I=0 this reduces to:

$$k_i\left(\frac{\Delta T_i}{L_i}\right) - k_{i+1}\left(\frac{\Delta T_{i+1}}{L_{i+1}}\right) = 0$$

Solving for L_i/L_{i+1} yields:

$$\frac{L_i}{L_{i+1}} = \frac{k_i \Delta T_i}{k_{i+1} \Delta T_{i+1}} = \frac{k_i (T_i - T_{i+1})}{k_{i+1} (T_{i+1} - T_{i+2})}$$

The efficiency of the segmented TEG is calculated from the total useful power from the device and the heat input at the hot junction.

$$\eta = \frac{P}{Q_H}$$

The calculation of potential energy in the exhaust gas from the boiler was done by assuming ideal gas behavior with a constant specific heat. The equation below was used to evaluate the system.

$$\dot{Q} = \dot{m} c_p \Delta T$$

where: $\dot{Q}$ = heat rate, BTU/hr
$\dot{m}$ = mass flow rate, lbm/hr
c_p = specific heat, BTU/lbm-°F*
ΔT = change in gas temperature, °F

* specific heat = 0.25 BTU/lbm-°F for flue gas [29]

Appendix II

Single.Bas

```
1 OPEN "E:SINGLE1.DAT" FOR INPUT AS #1
2 OPEN "E:SINGLE1.OUT" FOR OUTPUT AS #2
5 INPUT #1, NAME$
10 INPUT #1, TH
11 INPUT #1, TC
15 INPUT #1, GP
20 INPUT #1, GN
16 INPUT #1, KP
17 INPUT #1, RP
21 INPUT #1, KN
22 INPUT #1, RN
23 INPUT #1, DV
25 L = 1
30 AP = 1!
35 N = 1
40 FOR ii = 1 TO 2
45 G = (ABS(GN) + ABS(GP)) * N
50 DELT = (TH - TC)
55 R = ((L / AP) * (RP + (RN * RP) ^ .5)) * N
56 RTOT = 2 * R
60 K = ((AP / L) * ((KN * (RN / RP) ^ .5) + KP)) * N
65 AN = AP * ((RN / RP) ^ .5)
70 V = (G * DELT)
71 VOUT = (G * DELT) / 2
75 I = V / RTOT
80 P = I * V
81 PO = I * VOUT
84 QH3 = .5 * I ^ 2 * R
85 QH1 = K * DELT
86 QH2 = G * TH * I
90 QH = (K * DELT) - (.5 * R * (I ^ 2)) + (G * TH * I)
95 EFF = PO / QH
98 PRINT #2, "TH = "; TH; " TC = "; TC
100 PRINT #2, "N = "; N
110 PRINT #2, "L= "; L
120 PRINT #2, "An= "; AN
130 PRINT #2, "Ap= "; AP
140 PRINT #2, "Total Seebeck Coefficient = "; G
150 PRINT #2, "Total Internal Resistance = "; R
160 PRINT #2, "Total Thermal Conductivity = "; K
170 PRINT #2, "Output Voltage = "; VOUT
180 PRINT #2, "Current Output = "; I
190 PRINT #2, "Total Power Output = "; P
191 PRINT #2, "Useful Power Output = "; PO
210 PRINT #2, "Total Heat Input = "; QH
220 PRINT #2, "Total Efficiency of the TEG = "; EFF
```

```
221 PRINT #2, "External Heat Component = "; QH1
222 PRINT #2, "Peltier Heat Component = "; QH2
223 PRINT #2, "Joule Heat Component = "; QH3
224 PRINT #2, "Efficiency based on external heat alone =
"; (PO / QH1)
225 PRINT #2, "Desired Voltage = "; DV
227 PRINT #2,
228 PRINT #2,
230 N = DV / VOUT
240 NEXT ii
245 PRINT #2, NAME$
250 END
260 STOP
```

Cascade.Bas

```
1 OPEN "E:CASCADE.DAT" FOR INPUT AS #1
2 OPEN "E:CASCADE.OUT" FOR OUTPUT AS #2
5 INPUT #1, NAME$
10 INPUT #1, N(1)
15 INPUT #1, NS
20 INPUT #1, DV
25 FOR J = 1 TO NS
35 INPUT #1, T(J)
40 INPUT #1, GP(J)
45 INPUT #1, KP(J)
50 INPUT #1, RP(J)
55 INPUT #1, GN(J)
60 INPUT #1, KN(J)
65 INPUT #1, RN(J)
66 AP(J) = 1!
70 NEXT J
75 INPUT #1, T(NS + 1)
80 L(1) = 1!
90 EFFSUM = 1!
105 FOR JJ = 1 TO NS
109 G(JJ) = (ABS(GN(JJ)) + ABS(GP(JJ)))
115 DELT(JJ) = (T(JJ) - T(JJ + 1))
120 AN(JJ) = ((RN(JJ) / RP(JJ)) ^ .5) * AP(JJ)
125 NEXT JJ
130 FOR JJJ = 1 TO NS
131 XXX = 1
132 GAMP(1) = AP(1) / L(JJJ)
135 AP(JJJ) = GAMP(JJJ) * L(JJJ)
138 AN(JJJ) = ((RN(JJJ) / RP(JJJ)) ^ .5) * AP(JJJ)
142 GAMN(JJJ) = AN(JJJ) / L(JJJ)
144 R(JJJ) = ((RP(JJJ) + (RN(JJJ) * RP(JJJ)) ^ .5) /
GAMP(JJJ))
145 RLOAD = R(JJJ)
146 RTOT(JJJ) = RLOAD + R(JJJ)
```

```
150 K(JJJ) = (GAMP(JJJ) * ((KN(JJJ) * (RN(JJJ) / RP(JJJ))
^ .5) + KP(JJJ)))
155 V(JJJ) = G(JJJ) * DELT(JJJ) * N(JJJ)
156 VOUT(JJJ) = ((G(JJJ) * DELT(JJJ)) / 2) * N(JJJ)
160 I(JJJ) = V(JJJ) / (RTOT(JJJ) * N(JJJ))
162 P(JJJ) = I(JJJ) * V(JJJ)
163 PO(JJJ) = I(JJJ) * VOUT(JJJ)
164 QH(JJJ) = ((K(JJJ) * DELT(JJJ)) - (.5 * I(JJJ) ^ 2 *
R(JJJ)) + (G(JJJ) * T(JJJ) * I(JJJ))) * N(JJJ)
165 Q1(JJJ) = K(JJJ) * DELT(JJJ)
166 Q2(JJJ) = G(JJJ) * T(JJJ) * I(JJJ)
167 Q3(JJJ) = .5 * I(JJJ) ^ 2 * R(JJJ)
172 IF XXX = 1 THEN
173 L(JJJ) = (KP(JJJ) * DELT(JJJ) * AP(JJJ)) / (QH(JJJ) /
N(JJJ))
176 XXX = XXX + 1
177 GOTO 132
178 ELSE
179 END IF
180 EFF(JJJ) = PO(JJJ) / QH(JJJ)
181 IF JJJ = NS GOTO 192
182 GAMP(JJJ + 1) = GAMP(JJJ) * (G(JJJ) / G(JJJ + 1)) *
(DELT(JJJ) / DELT(JJJ + 1)) * ((RP(JJJ + 1) + (RP(JJJ + 1)
* RN(JJJ + 1)) ^ .5) / (RP(JJJ) + (RP(JJJ) * RN(JJJ)) ^
.5)) * ((RP(JJJ + 1) / RP(JJJ)) ^ .5)
183 L(JJJ + 1) = AP(JJJ) / GAMP(JJJ + 1)
185 N(JJJ + 1) = N(JJJ) * (GAMP(JJJ) / GAMP(JJJ + 1)) *
(DELT(JJJ) / DELT(JJJ + 1)) * ((RP(JJJ + 1) / RP(JJJ)) ^
.5) * (((G(JJJ) ^ 2 / (RN(JJJ) ^ .5 + RP(JJJ) ^ .5)) *
(DELT(JJJ) / 8 + T(JJJ + 1) / 2) + (KN(JJJ) * (RN(JJJ) ^
.5) + KP(JJJ) * (RP(JJJ) ^ .5))) / (((G(JJJ + 1) ^ 2 /
(RN(JJJ + 1) ^ .5 + RP(JJJ + 1) ^ .5)) * (DELT(JJJ + 1) /
8 + T(JJJ + 1) / 2) + (KN(JJJ + 1) * (RN(JJJ + 1) ^ .5) +
KP(JJJ + 1) * (RP(JJJ + 1) ^ .5)))))
192 EFFX(JJJ) = 1 - EFF(JJJ)
200 EFFSUM = EFFSUM * EFFX(JJJ)
300 PRINT #2, "FOR STAGE "; JJJ
301 PRINT #2, "TH = "; T(JJJ); "  TC = "; T(JJJ + 1)
305 PRINT #2, "NUMBER OF COUPLES = "; N(JJJ)
307 PRINT #2, "AREA OF THE n-LEG = "; AN(JJJ)
309 PRINT #2, "AREA OF THE p-LEG = "; AP(JJJ)
311 PRINT #2, "LENGTH OF THE STAGE ="; L(JJJ)
313 PRINT #2, "SHAPE FACTOR p-LEG = "; GAMP(JJJ)
314 PRINT #2, "SHAPE FACTOR n-LEG = "; GAMN(JJJ)
315 PRINT #2, "TOTAL STAGE SEEBECK COEFFICIENT = "; G(JJJ)
* N(JJJ)
316 PRINT #2, "TOTAL STAGE INTERNAL RESISTANCE = "; R(JJJ)
* N(JJJ)
317 PRINT #2, "TOTAL STAGE THERMAL CONDUCTIVITY = ";
K(JJJ) * N(JJJ)
318 PRINT #2, "STAGE OUTPUT VOLTAGE = "; VOUT(JJJ)
319 PRINT #2, "STAGE OUTPUT CURRENT = "; I(JJJ)
```

```
320 PRINT #2, "STAGE OUTPUT POWER (useful) ="; PO(JJJ)
322 PRINT #2, "HEAT INPUT TO STAGE = "; QH(JJJ)
323 PRINT #2, "STAGE EFFICIENCY = "; EFF(JJJ)
324 PRINT #2, "EXTERNAL HEAT COMPONENT = "; Q1(JJJ)
325 PRINT #2, "PELTIER HEAT COMPONENT  = "; Q2(JJJ)
326 PRINT #2, "JOULE HEAT COMPONENT = "; Q3(JJJ)
327 PRINT #2, "NUMBER OF COUPLES REQUIRED FOR NEXT STAGE =
"; N(JJJ + 1)
328 PRINT #2,
329 PRINT #2,
330 VTOT1 = VTOT1 + VOUT(JJJ)
331 ITOT1 = ITOT1 + I(JJJ)
332 PTOT1 = PTOT1 + PO(JJJ)
333 RTOT1 = RTOT1 + R(JJJ)
334 NEXT JJJ
335 EFFTOT = 1 - EFFSUM
337 PRINT #2,
340 PRINT #2, "TOTAL TEG EFFICIENCY "; EFFTOT
344 PRINT #2,
350 NREQ = (DV / VTOT1)
351 PRINT #2, "NUMBER OF COUPLES REQUIRED = "; NREQ
355 VMOD = NREQ * VTOT1
356 IMOD = VTOT1 / (2 * RTOT1)
357 PMOD = PTOT1 * NREQ
358 EFFMOD = EFFTOT
359 PRINT #2, "MODULE TOTAL RESISTANCE = "; RTOT1
360 PRINT #2, "MODULE VOLTAGE OUTPUT = "; VMOD
361 PRINT #2, "MODULE CURRENT OUTPUT = "; IMOD
362 PRINT #2, "MODULE POWER OUTPUT = "; PMOD
363 PRINT #2, "MODULE EFFICIENCY = "; EFFMOD
385 PRINT #2,
390 PRINT #2, NAME$
395 END
400 STOP
```

Segment.Bas

```
1 OPEN "E:SEGMENT.DAT" FOR INPUT AS #1
2 OPEN "E:SEGMENT.OUT" FOR OUTPUT AS #2
5 INPUT #1, N$
10 INPUT #1, DV
11 N = 1
15 INPUT #1, NSP
20 INPUT #1, NSN
25 FOR J = 1 TO NSP
27 INPUT #1, TP(J)
30 INPUT #1, GP(J)
35 INPUT #1, KP(J)
38 INPUT #1, RP(J)
39 AN(J) = 1!
```

```
41 NEXT J
42 INPUT #1, TP(NSP + 1)
44 FOR JJ = 1 TO NSN
46 INPUT #1, TN(JJ)
48 INPUT #1, GN(JJ)
50 INPUT #1, KN(JJ)
53 INPUT #1, RN(JJ)
56 NEXT JJ
60 INPUT #1, TN(NSN + 1)
80 RPTOT = 0!
85 RNTOT = 0!
90 VPTOT = 0!
95 VNTOT = 0!
96 FOR IJ = 1 TO NSP
97 XXX = 1!
98 AP(IJ) = 1!
99 LP(IJ) = 1!
100 TRP(IJ) = (RP(IJ) * LP(IJ)) / AP(IJ)
101 TGP(IJ) = GP(IJ)
102 TKP(IJ) = (KP(IJ) * AP(IJ)) / LP(IJ)
103 DELTP(IJ) = TP(IJ) - TP(IJ + 1)
104 VP(IJ) = (TGP(IJ) * DELTP(IJ))
105 IP(IJ) = VP(IJ) / (2 * TRP(IJ))
106 QP(IJ) = (TKP(IJ) * DELTP(IJ)) - (.5 * (VP(IJ) ^ 2) /
(4 * TRP(IJ))) + (TGP(IJ) * IP(IJ) * TP(IJ))
107 QP1(IJ) = (TKP(IJ) * DELTP(IJ))
108 QP2(IJ) = (.5 * (VP(IJ) ^ 2) / (4 * TRP(IJ)))
109 QP3(IJ) = (TGP(IJ) * IP(IJ) * TP(IJ))
111 IF XXX = 1 THEN
112 LP(IJ) = (KP(IJ) * AP(IJ) * DELTP(IJ)) / QP(IJ)
113 XXX = XXX + 1
114 GOTO 104
115 ELSE
116 END IF
117 VPTOT = VPTOT + VP(IJ)
118 RPTOT = RPTOT + (RP(IJ) * LP(IJ))
119 LP(IJ + 1) = LP(IJ) * ((TKP(IJ + 1) * (TP(IJ + 1) -
TP(IJ + 2))) / (TKP(IJ) * (TP(IJ) - TP(IJ + 1))))
120 NEXT IJ
121 YYY = 1!
122 FOR IJJ = 1 TO NSN
123 XXX = 1!
124 LN(IJJ) = 1!
125 TRN(IJJ) = (RN(IJJ) * LN(IJJ)) / AN(IJJ)
126 TGN(IJJ) = GN(IJJ)
127 TKN(IJJ) = (KN(IJJ) * AN(IJJ)) / LN(IJJ)
128 DELTN(IJJ) = TN(IJJ) - TN(IJJ + 1)
129 VN(IJJ) = (TGN(IJJ) * DELTN(IJJ))
130 IN(IJJ) = VN(IJJ) / (2 * TRN(IJJ))
131 QN(IJJ) = (TKN(IJJ) * DELTN(IJJ)) - (.5 * (VN(IJJ) ^
2) / (4 * TRN(IJJ))) + (TGN(IJJ) * IN(IJJ) * TN(IJJ))
135 QN1(IJJ) = (TKN(IJJ) * DELTN(IJJ))
```

```
136 QN2(IJJ) = (.5 * (VN(IJJ) ^ 2) / (4 * TRN(IJJ)))
137 QN3(IJJ) = (TGN(IJJ) * IN(IJJ) * TN(JIJ))
140 IF XXX = 1 THEN
141 LN(IJJ) = (KN(IJJ) * AN(IJJ) * DELTN(IJJ)) / QN(IJJ)
142 XXX = XXX + 1
143 GOTO 125
144 ELSE
145 END IF
146 VNTOT = VNTOT + VN(IJJ)
147 RNTOT = RNTOT + (RN(IJJ) * LN(IJJ))
148 LN(IJJ + 1) = LN(IJJ) * ((TKN(IJJ + 1) * (TN(IJJ + 1)
- TN(IJJ + 2))) / (TKN(IJJ) * (TN(IJJ) - TN(IJJ + 1))))
149 IF YYY = 1! THEN
150 AN(IJJ) = (KP(1) * (RNTOT) * (TP(1) - TP(2))) / (KN(1)
* RPTOT * (TN(1) - TN(2))) * AP(IJJ)
155 YYY = YYY + 1
156 GOTO 122
160 END IF
165 AN(IJJ) = AN(1)
175 NEXT IJJ
245 R = (RPTOT / AP(IJJ - 1)) + (RNTOT / AN(IJJ - 1))
250 VTOT = VNTOT + VPTOT
251 VOUT = VTOT / 2
255 I = VTOT / (2 * R)
260 P = (I ^ 2) * R
270 QH = (QP(1) + QN(1))
275 EFF = P / QH
290 FOR K = 1 TO NSP
291 PRINT #2, "FOR p-LEG SEGMENT  "; K
292 PRINT #2, "AP = "; AP(K)
293 PRINT #2, "LP("; K; ") = "; LP(K)
294 PRINT #2, "KP("; K; ") = "; TKP(K)
295 PRINT #2, "RP("; K; ") = "; TRP(K)
296 PRINT #2, "SEEBECK COEFFICIENT ("; K; ") = "; TGP(K)
297 PRINT #2, "DELTA TP("; K; ") ="; DELTP(K)
298 PRINT #2, "VP("; K; ") ="; VP(K)
299 PRINT #2, "IP("; K; ") ="; IP(K)
300 PRINT #2, "QP("; K; ") ="; QP(K)
301 PRINT #2, "LP("; (K + 1); ") = "; LP(K + 1)
302 PRINT #2,
303 NEXT K
320 FOR KK = 1 TO NSN
321 PRINT #2, "FOR n-LEG SEGMENT  "; KK
322 PRINT #2, "AN = "; AN(KK)
323 PRINT #2, "LN("; KK; ") = "; LN(KK)
324 PRINT #2, "KN("; KK; ") = "; TKN(KK)
325 PRINT #2, "RN("; KK; ") = "; TRN(KK)
326 PRINT #2, "SEEBECK COEFFICIENT ("; KK; ") = "; TGN(KK)
327 PRINT #2, "DELTA TN("; KK; ") ="; DELTN(KK)
328 PRINT #2, "VN("; KK; ") ="; VN(KK)
329 PRINT #2, "IN("; KK; ") ="; IN(KK)
330 PRINT #2, "QN("; KK; ") ="; QN(KK)
```

```
331 PRINT #2, "LN("; (KK + 1); ") = "; LN(KK + 1)
332 PRINT #2,
333 NEXT KK
335 PRINT #2, "FOR THE ENTIRE MODULE"
337 PRINT #2, "NUMBER OF COUPLES = "; N
340 PRINT #2, "An = "; AN(1)
345 PRINT #2, "Ap = "; AP(1)
350 PRINT #2, "TOTAL INTERNAL RESISTANCE = "; R
355 PRINT #2, "TOTAL VOLTAGE OUTPUT = "; VOUT
360 PRINT #2, "TOTAL CURRENT OUTPUT = "; I
365 PRINT #2, "TOTAL POWER OUTPUT = "; P
370 PRINT #2, "GENERATOR EFFICIENCY = "; EFF
380 NC = DV / VOUT
390 VMOD = NC * VOUT
392 IMOD = I
394 QMOD = QH * NC
396 PMOD = P * NC
398 EFFMOD = EFF
400 PRINT #2,
402 PRINT #2, "NUMBER OF COUPLES TO PROVIDE "; DV; " VOLTS
IS "; NC
403 PRINT #2, "MODULE OUTPUT VOLTAGE = "; VMOD
404 PRINT #2, "MODULE CURRENT OUTPUT = "; IMOD
405 PRINT #2, "MODULE POWER OUTPUT = "; PMOD
406 PRINT #2, "MODULE HEAT INPUT REQUIRED = "; QMOD
407 PRINT #2, "MODULE EFFICIENCY = "; EFFMOD
410 PRINT #2, "EXTERNAL HEAT COMPONENT = "; ((QP1(1) +
QN(1)) * NC)
411 PRINT #2, "JOULE HEAT COMPONENT = "; ((QP2(1) +
QN2(1)) * NC)
412 PRINT #2, "PELTIER HEAT COMPONENT = "; ((QP3(1) +
QN3(1)) * NC)
439 PRINT #2,
440 PRINT #2, N$
445 END
450 STOP
```

PROGRAM INPUT

Single1.Dat

```
Case I: TEG in exhaust gas: Single1.dat: PbTe used for
both n- and p- legs
546
300
0.000163
0.000225
0.0240
0.00090
0.0185
0.00265
115
```

Single2.Dat

```
Case II: TEG in combustion gas: Single2.dat: PbTe used for
both n- and p- legs
1000
300
0.000258
0.000205
0.0175
0.00653
0.0210
0.00412
115
```

Cascade1.Dat

```
Case I: TEG in exhaust gas: Cascade1.dat:
PbTe/Bi2Te2.4Se0.6 & PbTe/BiSbTe3
1
2
115
546
0.000301
0.0165
0.00463
0.0001795
0.02336
0.00135
440
```

0.000164
0.0183
0.000793
0.000214
0.0144
0.00135
300

Cascade2.Dat

Case II: TEG in combustion gas: Cascade2.dat:
PbTe/Bi2Te2.4Se0.6 & PbTe/BiSbTe3
1
2
115
1000
0.000281
0.01688
0.00789
0.0002188
0.02013
0.005013
440
0.000164
0.0183
0.000793
0.000214
0.0144
0.00135
300

Segment1.Dat

Case I: TEG in exhaust gas: Segment1.dat:
PbTe/Bi2Te2.4Se0.6 & PbTe/BiSbTe3
115
2
2
546
0.000301
0.0165
0.00463
440
0.000164
0.0183
0.000793
300
546

0.0001795
0.02336
0.00135
440
0.000214
0.0144
0.00135
300

Segment2.Dat

Case II: TEG in combustion gas: Segment2.dat:
PbTe/Bi2Te2.4Se0.6 &PbTe/BiSbTe3
115
2
2
1000
0.0002813
0.01688
0.00789
440
0.000164
0.0183
0.000793
300
546
0.0002188
0.02013
0.005013
440
0.000214
0.0144
0.00135
300

PROGRAM OUTPUT

Single1.Out

```
TH =  546  TC =  300
N =  1
L=  1
An=  1.715938
Ap=  1
Total Seebeck Coefficient =  .000388
Total Internal Resistance =  2.444345E-03
Total Thermal Conductivity =  5.574486E-02
Output Voltage =  .047724
Current Output =  19.52425
Total Power Output =  1.863551
Useful Power Output =  .9317753
Total Heat Input =  17.38352
Total Efficiency of the TEG =  5.360107E-02
External Heat Component =  13.71324
Peltier Heat Component =  4.136174
Joule Heat Component =  .4658877
Efficiency based on external heat alone =  6.794715E-02
Desired Voltage =  115


TH =  546  TC =  300
N =  2409.689
L=  1
An=  1.715938
Ap=  1
Total Seebeck Coefficient =  .9349594
Total Internal Resistance =  5.89011
Total Thermal Conductivity =  134.3278
Output Voltage =  115
Current Output =  19.52425
Total Power Output =  4490.579
Useful Power Output =  2245.289
Total Heat Input =  41888.89
Total Efficiency of the TEG =  5.360107E-02
External Heat Component =  33044.64
Peltier Heat Component =  9966.895
Joule Heat Component =  1122.645
Efficiency based on external heat alone =  6.794716E-02
Desired Voltage =  115


Case I: TEG in exhaust gas: Single1.dat: PbTe used for
both n- and p- legs
```

```
TH =  1000  TC =  300
N =  1
L=  1
An=  .7943137
Ap=  1
Total Seebeck Coefficient =  .000463
Total Internal Resistance =  1.171687E-02
Total Thermal Conductivity =  3.418059E-02
Output Voltage =  .16205
Current Output =  13.83049
Total Power Output =  4.482461
Useful Power Output =  2.24123
Total Heat Input =  29.20931
Total Efficiency of the TEG =  7.673001E-02
External Heat Component =  23.92641
Peltier Heat Component =  6.403515
Joule Heat Component =  1.120615
Efficiency based on external heat alone =  9.367183E-02
Desired Voltage =  115


TH =  1000  TC =  300
N =  709.6575
L=  1
An=  .7943137
Ap=  1
Total Seebeck Coefficient =  .3285714
Total Internal Resistance =  8.314963
Total Thermal Conductivity =  24.25651
Output Voltage =  115
Current Output =  13.83049
Total Power Output =  3181.012
Useful Power Output =  1590.506
Total Heat Input =  20728.61
Total Efficiency of the TEG =  .07673
External Heat Component =  16979.56
Peltier Heat Component =  4544.303
Joule Heat Component =  795.2531
Efficiency based on external heat alone =  9.367181E-02
Desired Voltage =  115
```

Case II: TEG in combustion gas: Single2.dat: PbTe used for both n- and p- legs

```
FOR STAGE  1
TH =  546   TC =  440
NUMBER OF COUPLES =  1
AREA OF THE n-LEG =  .5399784
AREA OF THE p-LEG =  1
LENGTH OF THE STAGE = .4397083
SHAPE FACTOR p-LEG =  2.274235
SHAPE FACTOR n-LEG =  1.228038
TOTAL STAGE SEEBECK COEFFICIENT =  .0004805
TOTAL STAGE INTERNAL RESISTANCE =  3.135164E-03
TOTAL STAGE THERMAL CONDUCTIVITY =  6.621184E-02
STAGE OUTPUT VOLTAGE =  .0254665
STAGE OUTPUT CURRENT =  8.12286
STAGE OUTPUT POWER (useful) = .2068608
HEAT INPUT TO STAGE =  9.046082
STAGE EFFICIENCY =  2.286745E-02
EXTERNAL HEAT COMPONENT =  7.018455
PELTIER HEAT COMPONENT  =  2.131057
JOULE HEAT COMPONENT =  .1034304
NUMBER OF COUPLES REQUIRED FOR NEXT STAGE =  4.885527


FOR STAGE  2
TH =  440   TC =  300
NUMBER OF COUPLES =  4.885527
AREA OF THE n-LEG =  .4511957
AREA OF THE p-LEG =  .3458077
LENGTH OF THE STAGE = 1.489261
SHAPE FACTOR p-LEG =  .2322009
SHAPE FACTOR n-LEG =  .3029662
TOTAL STAGE SEEBECK COEFFICIENT =  1.846729E-03
TOTAL STAGE INTERNAL RESISTANCE =  3.845441E-02
TOTAL STAGE THERMAL CONDUCTIVITY =  4.207412E-02
STAGE OUTPUT VOLTAGE =  .129271
STAGE OUTPUT CURRENT =  3.36167
STAGE OUTPUT POWER (useful) = .4345666
HEAT INPUT TO STAGE =  8.404655
STAGE EFFICIENCY =  5.170547E-02
EXTERNAL HEAT COMPONENT =  1.205679
PELTIER HEAT COMPONENT  =  .559113
JOULE HEAT COMPONENT =  4.447489E-02
NUMBER OF COUPLES REQUIRED FOR NEXT STAGE =  0



TOTAL TEG EFFICIENCY  7.339054E-02

NUMBER OF COUPLES REQUIRED =  743.1939
MODULE TOTAL RESISTANCE =  1.100625E-02
```

```
MODULE VOLTAGE OUTPUT =  115
MODULE CURRENT OUTPUT =  7.02953
MODULE POWER OUTPUT =  476.7049
MODULE EFFICIENCY =  7.339054E-02

Case I: TEG in exhaust gas: Cascade1.dat:
PbTe/Bi2Te2.4Se0.6 & PbTe/BiSbTe3
```

Cascade2.Out

```
FOR STAGE  1
TH =  1000   TC =  440
NUMBER OF COUPLES =  1
AREA OF THE n-LEG =  .7970955
AREA OF THE p-LEG =  .9999999
LENGTH OF THE STAGE = .4167792
SHAPE FACTOR p-LEG =  2.399352
SHAPE FACTOR n-LEG =  1.912512
TOTAL STAGE SEEBECK COEFFICIENT =  .0004998
TOTAL STAGE INTERNAL RESISTANCE =  5.909548E-03
TOTAL STAGE THERMAL CONDUCTIVITY =  7.899993E-02
STAGE OUTPUT VOLTAGE =  .139944
STAGE OUTPUT CURRENT =  23.681
STAGE OUTPUT POWER (useful) = 3.314013
HEAT INPUT TO STAGE =  54.41872
STAGE EFFICIENCY =  .0608984
EXTERNAL HEAT COMPONENT =  44.23996
PELTIER HEAT COMPONENT  =  11.83576
JOULE HEAT COMPONENT =  1.657007
NUMBER OF COUPLES REQUIRED FOR NEXT STAGE =  12.64781


FOR STAGE  2
TH =  440   TC =  300
NUMBER OF COUPLES =  12.64781
AREA OF THE n-LEG =  .4511957
AREA OF THE p-LEG =  .3458077
LENGTH OF THE STAGE = .6668493
SHAPE FACTOR p-LEG =  .5185695
SHAPE FACTOR n-LEG =  .6766081
TOTAL STAGE SEEBECK COEFFICIENT =  4.780872E-03
TOTAL STAGE INTERNAL RESISTANCE =  4.457661E-02
TOTAL STAGE THERMAL CONDUCTIVITY =  .243255
STAGE OUTPUT VOLTAGE =  .334661
STAGE OUTPUT CURRENT =  7.507548
STAGE OUTPUT POWER (useful) = 2.512484
HEAT INPUT TO STAGE =  48.59222
STAGE EFFICIENCY =  5.170548E-02
EXTERNAL HEAT COMPONENT =  2.692617
PELTIER HEAT COMPONENT  =  1.248655
```

```
JOULE HEAT COMPONENT =  9.932486E-02
NUMBER OF COUPLES REQUIRED FOR NEXT STAGE =  0


TOTAL TEG EFFICIENCY  .1094551

NUMBER OF COUPLES REQUIRED =  242.3067
MODULE TOTAL RESISTANCE =  9.434002E-03
MODULE VOLTAGE OUTPUT =  115
MODULE CURRENT OUTPUT =  25.15396
MODULE POWER OUTPUT =  1411.8
MODULE EFFICIENCY =  .1094551

Case II: TEG in combustion gas: Cascade2.dat:
PbTe/Bi2Te2.4Se0.6 & PbTe/BiSbTe3


                        Segment1.Out


FOR p-LEG SEGMENT   1
AP =  1
LP( 1 ) =  .7644958
KP( 1 ) =  .0165
RP( 1 ) =  .00463
SEEBECK COEFFICIENT ( 1 ) =  .000301
DELTA TP( 1 ) = 106
VP( 1 ) = .031906
IP( 1 ) = 3.445572
QP( 1 ) = 2.287782
LP( 2 ) =  .7271096

FOR p-LEG SEGMENT   2
AP =  1
LP( 2 ) =  .7271096
KP( 2 ) =  .0183
RP( 2 ) =  .000793
SEEBECK COEFFICIENT ( 2 ) =  .000164
DELTA TP( 2 ) = 140
VP( 2 ) = .02296
IP( 2 ) = 14.47667
QP( 2 ) = 3.52354
LP( 3 ) =  0

FOR n-LEG SEGMENT   1
AN =  .1830729
LN( 1 ) =  .7902727
KN( 1 ) =  5.411527E-03
RN( 1 ) =  5.827561E-03
SEEBECK COEFFICIENT ( 1 ) =  .0001795
```

```
DELTA TN( 1 ) = 106
VN( 1 ) = .019027
IN( 1 ) = 1.632501
QN( 1 ) = .725853
LN( 2 ) =  .677029

FOR n-LEG SEGMENT   2
AN =  .1830729
LN( 2 ) =  .677029
KN( 2 ) =  .0212694
RN( 2 ) =  9.139891E-04
SEEBECK COEFFICIENT ( 2 ) =  .000214
DELTA TN( 2 ) = 140
VN( 2 ) = .02996
IN( 2 ) = 16.38969
QN( 2 ) = 4.398211
LN( 3 ) =  0

FOR THE ENTIRE MODULE
NUMBER OF COUPLES =  1
An =  .1830729
Ap =  1
TOTAL INTERNAL RESISTANCE =  2.076382E-02
TOTAL VOLTAGE OUTPUT =  .06144
TOTAL CURRENT OUTPUT =  2.958993
TOTAL POWER OUTPUT =  .1818005
GENERATOR EFFICIENCY =  6.032598E-02

NUMBER OF COUPLES TO PROVIDE  115  VOLTS IS  1871.745
MODULE OUTPUT VOLTAGE =  115
MODULE CURRENT OUTPUT =  2.958993
MODULE POWER OUTPUT =  340.2841
MODULE HEAT INPUT REQUIRED =  5640.757
MODULE EFFICIENCY =  6.032598E-02
EXTERNAL HEAT COMPONENT =  4632.293
JOULE HEAT COMPONENT =  65.97714
PELTIER HEAT COMPONENT =  1059.906

Case I: TEG in exhaust gas: Segment1.dat:
PbTe/Bi2Te2.4Se0.6 & PbTe/BiSbTe3
```

Segment2.Out

```
FOR p-LEG SEGMENT   1
AP =  1
LP( 1 ) =  .7965075
KP( 1 ) =  .01688
RP( 1 ) =  .00789
SEEBECK COEFFICIENT ( 1 ) =  .0002813
```

```
DELTA TP( 1 ) = 560
VP( 1 ) = .157528
IP( 1 ) = 9.982763
QP( 1 ) = 11.86781
LP( 2 ) =  .7271096

FOR p-LEG SEGMENT   2
AP =  1
LP( 2 ) =  .7271096
KP( 2 ) =  .0183
RP( 2 ) =  .000793
SEEBECK COEFFICIENT ( 2 ) =  .000164
DELTA TP( 2 ) = 140
VP( 2 ) = .02296
IP( 2 ) = 14.47667
QP( 2 ) = 3.52354
LP( 3 ) =  0

FOR n-LEG SEGMENT   1
AN =  2.881712
LN( 1 ) =  .8902913
KN( 1 ) =  6.515718E-02
RN( 1 ) =  1.548743E-03
SEEBECK COEFFICIENT ( 1 ) =  .0002188
DELTA TN( 1 ) = 106
VN( 1 ) = .0231928
IN( 1 ) = 7.487623
QN( 1 ) = 7.757753
LN( 2 ) =  .677029

FOR n-LEG SEGMENT   2
AN =  2.881712
LN( 2 ) =  .677029
KN( 2 ) =  .0212694
RN( 2 ) =  9.139891E-04
SEEBECK COEFFICIENT ( 2 ) =  .000214
DELTA TN( 2 ) = 140
VN( 2 ) = .02996
IN( 2 ) = 16.38969
QN( 2 ) = 4.398211
LN( 3 ) =  0

FOR THE ENTIRE MODULE
NUMBER OF COUPLES =  1
An =  2.881712
Ap =  1
TOTAL INTERNAL RESISTANCE =  .0102757
TOTAL VOLTAGE OUTPUT =  .1284168
TOTAL CURRENT OUTPUT =  12.49714
TOTAL POWER OUTPUT =  1.604842
GENERATOR EFFICIENCY =  8.177306E-02
```

NUMBER OF COUPLES TO PROVIDE 115 VOLTS IS 895.5215
MODULE OUTPUT VOLTAGE = 115
MODULE CURRENT OUTPUT = 12.49714
MODULE POWER OUTPUT = 1437.171
MODULE HEAT INPUT REQUIRED = 17575.12
MODULE EFFICIENCY = 8.177306E-02
EXTERNAL HEAT COMPONENT = 15412.42
JOULE HEAT COMPONENT = 390.9452
PELTIER HEAT COMPONENT = 2514.76

Case II: TEG in combustion gas: Segment2.dat:
PbTe/Bi2Te2.4Se0.6 &PbTe/BiSbTe3

VITA

Michael Lee Parkison was born in Knoxville,Tn on December 13, 1962. He lived in Kingston where he graduated from Roane County High School in June, 1981. He entered Tennessee Technological University the following September. While attending Tennessee Tech, he participated in the Cooperative Education program and worked for NASA in the Materials Testing Branch at the Kennedy Space Center. In March 1986, he received the degree of Bachelor of Science in Mechanical Engineering. Upon graduation, he accepted employment with the Calspan Corporation at the Arnold Engineering Development Center in Tullahoma Tn. In 1987 he married Karen A. Grosch and they have three children, Austin, Rachel, and Jacob. In 1986 he enrolled in the University of Tennessee Space Institute at Tullahoma Tn and received a Masters of Science degree in Mechanical Engineering in December 1993. He returned to Roane county in 1992 and is currently employed as a systems engineer with Martin Marietta Energy Systems at the K-25 Plant in Oak Ridge, Tn.