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I am submitting herewith a thesis written by Terry Lynn Dickson entitled "Interface Design Analysis of Cylindrical Vessel with Flat Concentric Head and Nozzle Subject to External Loads." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Engineering Science.

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INTERFACE DESIGN ANALYSIS OF CYLINDRICAL VESSEL WITH FLAT CONCENTRIC HEAD AND NOZZLE SUBJECT TO EXTERNAL LOADS

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Terry Lynn Dickson

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ABSTRACT

Equations are derived for performing an interface design analysis of a cylindrical vessel with a flat concentric head and nozzle which are subject to loads imposed by an attached piping system. Equations for calculating stiffness coefficients derived in Chapter II simulate the translational, flexural, and torsional flexibility of the concentric head and nozzle. Application of these coefficients by the piping analyst will result in a more accurate mathematical model and should produce a more accurate analysis of the piping system. The results of this piping analysis will also give a more realistic prediction of the loads which will be imposed on the head and nozzle by the attached piping system.

Equations for calculating stresses in the shell, head, and nozzle derived in Chapter III may be used to find the maximum allowable loads imposed by the attached piping system.

TABLE OF CONTENTS

CHAPTER		PAGE
I.	INTRODUCTION	1
II.	DEFLECTION ANALYSIS	4
	A. Assumptions	4
	B. Bending Mode	7
	C. Axial Mode	13
	D. Torsional Mode	18
III.	STRESS ANALYSIS	27
	A. Bending Mode	27
	B. Axial Mode	34
	C. Torsional Mode	37
IV.	EFFECT OF PIPE IMPOSED LOADS ON DISCONTINUITY STRESSES IN PRESSURE VESSEL	41
V.	DISCUSSION	44
BIBLIOGRAPHY		45
VITA		47

LIST OF FIGURES

FIGURE		PAGE
1.	Geometry of Shell-Plate-Nozzle System Showing Forces and Moments Applied to Each of the Six Degrees of Freedom	5
2.	Plate and Nozzle Deflections Due to Bending Loads	6
3.	(α) Dimensionless Parameter for Calculating Stiffness Coefficient for Bending Mode	10
4.	Plate Considered as a Torsional Spring	12
5.	Plate Deflection Due to Axial Loads	14
6.	(β) Dimensionless Parameter for Calculating Stiffness Coefficient for Axial Mode	17
7.	Two Dimensional Model for Torsional Mode	19
8.	Differential Element of Circumferential Section Showing Stress and Strain of Plate Subject to Torsional Load	19
9.	Relationship Between Displacement of Plate and Rotation of Nozzle	25
10.	(δ) Dimensionless Parameter for Calculating the Maximum Plate Stress Induced by Bending Loads	31
11.	(ψ) Dimensionless Parameter for Calculating the Maximum Tensile and Compressive Shell Stress Induced by Bending Loads	35
12.	($'$) Dimensionless Parameter Used to Calculate Maximum Tensile and Compressive Plate Stress Induced by Axial Loads	38

NOMENCLATURE

A - Metal Cross Sectional Area, in^2

C_1, C_2 - Constants of Integration

D - Flexural Rigidity of Plate, lb-in

e - Elongation, Inches

En - Modulus of Elasticity of Nozzle Material, lb/in^2

Ep - Modulus of Elasticity of Plate Material, lb/in^2

Es - Modulus of Elasticity of Shell Material, lb/in^2

Fx - Concentrated Force in Axial Mode, lbs

Fy & Fz - Concentrated Force in Bending Mode, lbs

F_R, F_ϕ - Body Forces in Radial and Tangential Directions, lbs

Gn - Modulus of Rigidity of Nozzle Material, lb/in^2

Gp - Modulus of Rigidity of Plate Material, lb/in^2

I - Moment of Inertia of Nozzle Cross Section, in^4

J - Polar Moment of Inertia of Nozzle Cross Section, in^4

K_A - Stiffness Coefficient for Axial Mode, lb/in

K_B - Stiffness Coefficient for Bending Mode, $\frac{\text{in-lb}}{\text{Radian}}$

K_T - Stiffness Coefficient for Torsional Mode $\frac{\text{in-lb}}{\text{Radian}}$

ℓ - Nozzle Length, Inches

ℓn - Natural Logarithim

Mz, My - Externally Applied Bending Moment, in-lb

Mx - Externally Applied Torsional Moment, in-lb

M_R - Internal Radial Bending Moment, $\frac{\text{in-lb}}{\text{in}}$

M_s - Resultant Torsional Moment Acting on Shell, in-lb

M_{ϕ} - Internal Tangential Bending Moment, $\frac{\text{in-lb}}{\text{in}}$

$M_{R\phi}$ - Internal Twisting Moment, $\frac{\text{in-lb}}{\text{in}}$

m - Root of Algebraic Equation

M_{CR} - Critical Moment, in-lb

Q - Shear Force Per Unit Length, lb/in

r - Radial Coordinate, In

Rn - Outside Nozzle Radius, In

Rp - Plate and Shell Outside Radius, In

tp - Plate Thickness, In.

t_s - Shell Thickness, In.

v - Poisson Ratio

V - Tangential Displacement of Plate, in

w - Deflection of Plate in the Lateral Plane, In.

α - Dimensionless Parameter Used to Calculate Stiffness Coefficient for Bending Mode

β - Dimensionless Parameter Used to Calculate Stiffness Coefficient for Axial Mode

γ - Unit Shearing Strain, Radians

ϵ - Tensile Strain

λ_p - Rotation of Plate Due to Torsional Loads, Radians

λ_n - Rotation of Nozzle Due to Torsional Loads, Radians

ζ - Dimensionless Parameter Used to Calculate Maximum Plate Stress Induced by Axial Loads

σ_R - Radial Stress, lb/in^2

σ - Normal Tensile and Compressive Stress, lb/in^2

T - Shearing Stress, lb/in^2

ϕ - Angular Coordinates, Radians

ψ - Dimensionless Parameter Used to Calculate the Maximum Shell Stress Induced by Bending

δ - Dimensionless Parameter Used to Calculate the Maximum Plate Stress Induced by Bending Loads

∇^2 - Laplacian Operator

Θ_P - Rotation of Plate Due to Bending Loads, Radians

Θ_N - Rotation of Nozzle Due to Bending Loads, Radians

Θ_{PEA} - End-Rotation of Plate Due to Axial Loads, Radians

Θ_{PEB} - End-Rotation of Plate Due to Bending Loads, Radians

ΔP - Maximum Deflection of Plate Due to Axial Loads, In.

ΔN - Maximum Deflection of Nozzle Due to Axial Loads, In.

ΔPEA - End Radial Displacement of Plate Due to Axial Loads, In.

ΔPEB - End Radial Displacement of Plate Due to Bending Loads, In.

CHAPTER I

INTRODUCTION

A commonly encountered engineering problem is the interface design of mechanical components and attached piping systems. This problem of ensuring structural compatibility between mechanical components and attached piping systems is generically referred to as the nozzle load problem. The fact that there are so many different components and configurations greatly complicates the nozzle load problem.

There is a considerable amount of literature addressing interface design between cylindrical shells and attached piping systems. Perhaps the best known is the work of Professor P. P. Bijilaard of Cornell University. Bijilaard presented a systematic method of calculating local stresses in cylindrical and spherical shells due to loads transmitted through attached rectangular and cylindrical nozzles [1]. An ASME paper by Billimoria and Hagstrom presented the necessary information required to calculate stiffness coefficients and maximum allowable nozzle loads for a flat bottom cylindrical tank [2]. In general, however, there is a lack of information for the different interface design problems frequently encountered.

The procedure generally followed is to qualify a component to an arbitrary set of nozzle loads and use these loads as design requirements for the attached piping system's reactions acting at the interface. The analysis of the piping system assumes the interface between a floor-mounted component nozzle and the attached piping system is a rigid

anchor point, i.e., restrained in all six degrees of freedom.

This rigidity assumption is in general conservative with regard to static loads such as gravity, thermal expansion, and anchor displacement; however, it could result in overestimating the stiffness of the piping system. This is of particular importance to the nuclear industry where a rigorous dynamic analysis is required for safety related piping systems in order to simulate the effect of the inertial forces produced by seismic events and relief valve blowdown. Overestimation of the system stiffness would generally produce higher natural frequencies and also affect the resulting mode shapes. The stress analysis performed for the dynamic load cases could produce unconservative results.

Simulation of the flexibility of the component and its nozzle would more realistically model the interface and therefore eliminate conservatism for the static load cases and give more accurate results for the dynamic load cases.

This thesis will address the interface design of cylindrical pressure vessels with flat concentric heads and attached piping systems. The first part will be a derivation of stiffness coefficients for each of the six degrees of freedom. These coefficients could easily be incorporated into a piping analysis program to simulate the flexibility of the nozzle-pipe interface.

The second part of this thesis will primarily address qualification of the vessel. Equations for calculating stresses in the vessel, head, and nozzle due to pipe imposed loads will be derived. These stresses

would be combined with the discontinuity stresses and the stresses resulting from pressure and thermal expansion [3].

The effect of the pipe loads on the discontinuity stresses will be considered. The end displacement and rotation of the concentric head due to the pipe loads will be derived. These deformations should be combined with those caused by other loads. The end deformations of the head and vessel would be equated to insure juncture continuity and the redundant discontinuity shear force and moment derived from solving the resulting set of equations simultaneously [3].

CHAPTER II

DEFLECTION ANALYSIS

A. ASSUMPTIONS

The vessel, concentric head, and nozzle as shown in Figure 1 are modeled and analyzed in accordance with the following assumptions:

1. All materials are Hookean and isotropic.
2. The usual assumptions made for small displacement linear elastic analysis, i.e., the deflections curves are fairly flat [4], and the midsurface undergoes no extension.
3. The concentric head is considered to be an annular plate which is simply supported along the outer boundary [3].
4. The vessel is assumed to act as a rigid simple support on the concentric head.
5. The nozzle may be considered as a cylindrical annular beam which is fixed to the concentric head.
6. The total deflection in each degree of freedom of the nozzle-head system can be obtained as the superposition of the deflections due to the flexibilities of the head and the nozzle.
7. The concentric plate may be considered to have a rigid disk located in the annular region. When the plate is subject to an external bending moment, the disk rotates about its center as shown in Figure 2(a) [5]. This implies that the slope of the

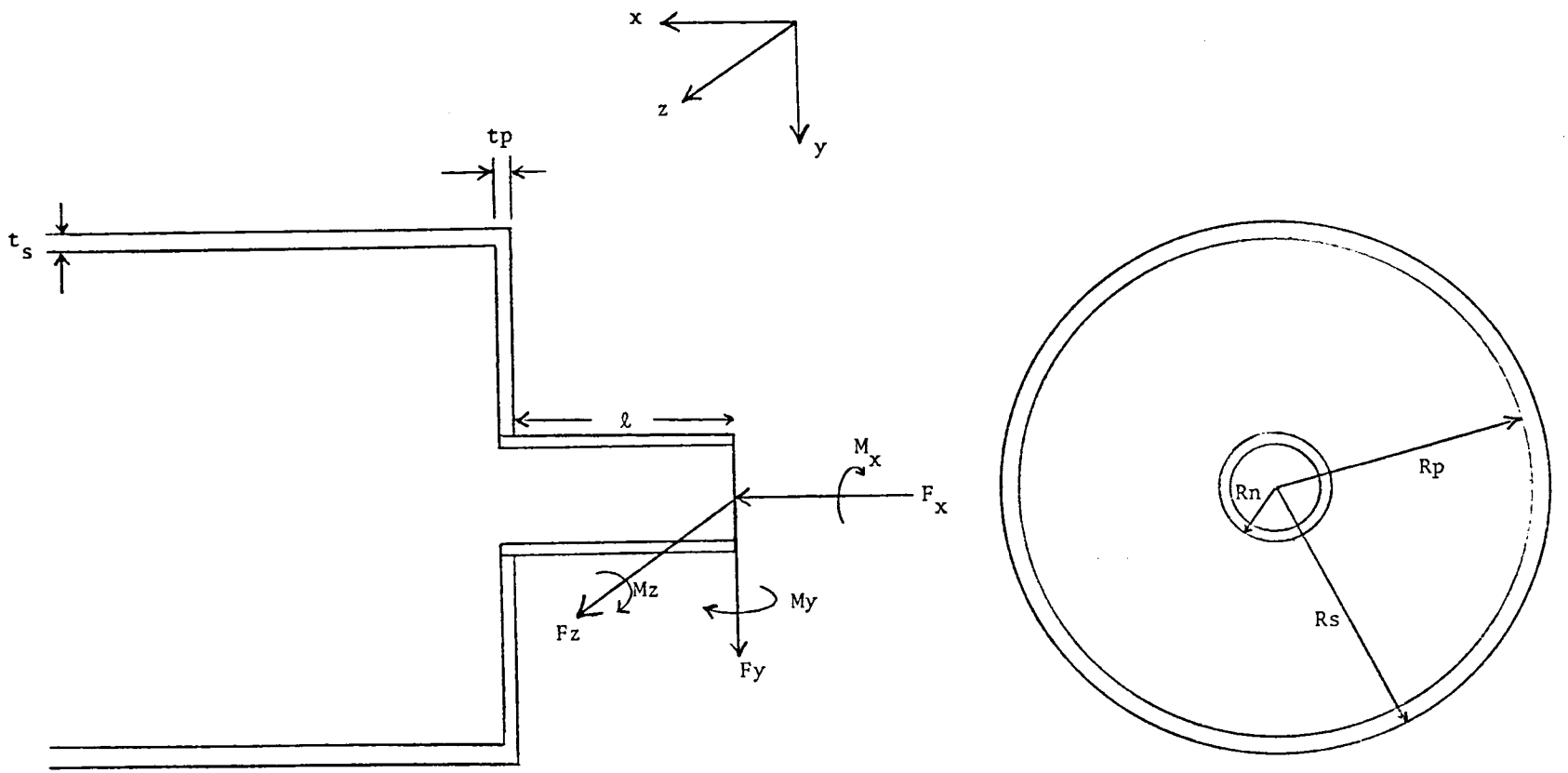


Figure 1. Geometry of shell-plate-nozzle system showing forces and moments applied to each of the six degrees of freedom.

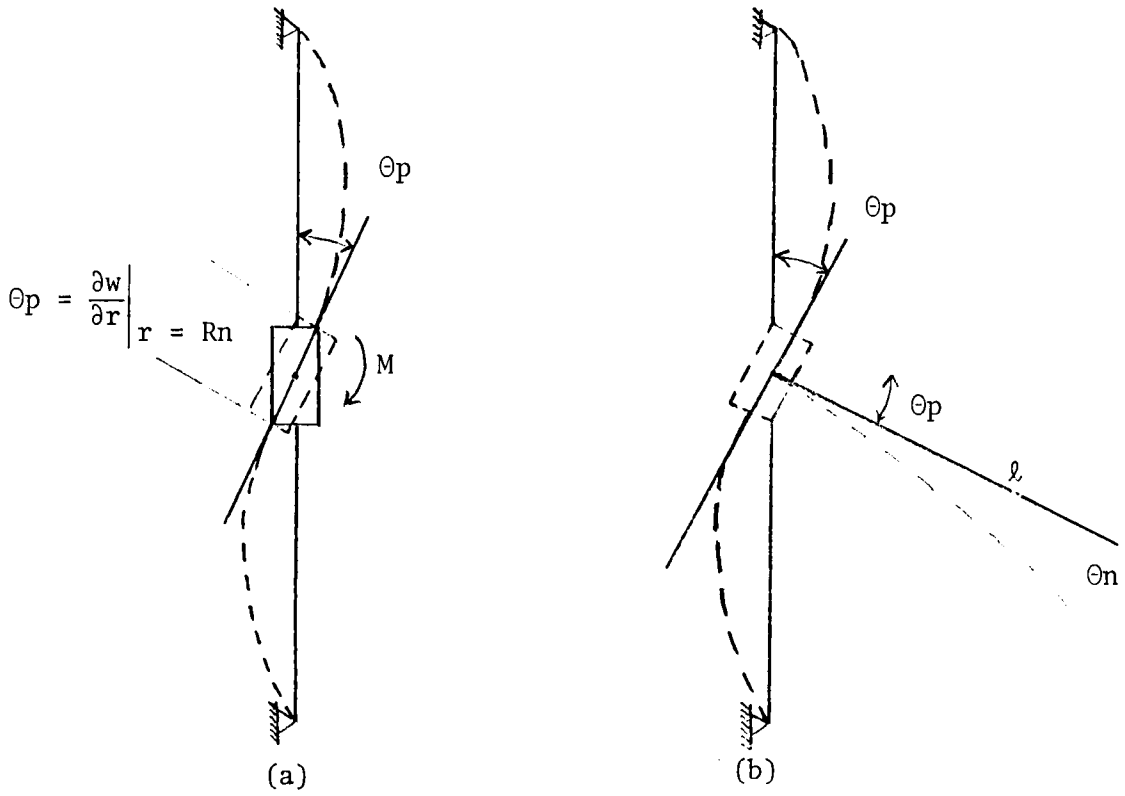


Figure 2. Plate and nozzle deflections due to bending loads.

deflected plate is constant in the annular region as illustrated in Figure 2(a).

8. The nozzle remains perpendicular to the plate during bending. This assumption of zero slope for the nozzle may be used as a boundary condition in the analysis. From the geometry in Figure 2(b), it is seen that the slope of the nozzle at its point of attachment is equal to the slope of the deflected plate in the annular region.

B. BENDING MODE

The concentric plate with a bending moment applied about its center is a nonsymmetric load; therefore, the equation of the deflection surface will be a function of two variables. The equation of the deflected plate surface expressed in polar coordinates is [5]:

$$w(r,\phi) = \frac{MRp}{8\pi D\left[(3+\nu)+(1-\nu)\left(\frac{Rn}{Rp}\right)^4\right]} \left\{ -\left[(1+\nu)+(1-\nu)\left(\frac{Rn}{Rp}\right)^4\right] \frac{r^3}{Rp^3} \right.$$

$$+ (1+\nu)\left[1-\left(\frac{Rn}{Rp}\right)^2\right]^2 \frac{r}{Rp} + 2\left[(3+\nu)+(1-\nu)\left(\frac{Rn}{Rp}\right)^4\right] \frac{r}{Rp} \ln\left(\frac{r}{Rp}\right)$$

$$\left. - \left(\frac{Rn}{Rp}\right)^2 \left[(1+\nu)\left(\frac{Rn}{Rp}\right)^2 - (3+\nu)\right] \frac{Rp}{r} \right\} \cos\phi \qquad (1)$$

The slope of the deflected plate surface at the rigid disk as shown in Figure 2(a) may be calculated as follows:

$$\Theta p = \left.\frac{\partial w}{\partial r}\right|_{r = Rn}$$

From Equation (1):

$$\frac{\partial w}{\partial r} = \frac{MRp}{8\pi D\left[(3+\nu)+(1-\nu)\left(\frac{Rn}{Rp}\right)^4\right]} \left\{ - \left[(1+\nu)+(1-\nu)\left(\frac{Rn}{Rp}\right)^4\right] \frac{3r^2}{Rp^3} \right.$$

$$+ (1+\nu)\left[1-\left(\frac{Rn}{Rp}\right)^2\right]^2 \frac{1}{Rp} + \frac{2}{Rp}\left[(3+\nu)+(1-\nu)\left(\frac{Rn}{Rp}\right)^4\right]\left[1+\ell n\left(\frac{r}{Rp}\right)\right]$$

$$\left. + \left(\frac{Rn}{Rp}\right)^2\left[(1+\nu)\left(\frac{Rn}{Rp}\right)^2 - (3+\nu)\right] \frac{Rp^2}{r^2} \right\} \cos \phi \qquad (1.a)$$

Evaluated at r = Rn:

$$\Theta p = \left.\frac{\partial w}{\partial r}\right|_{r = Rn} = \frac{M \cos \phi}{8\pi D\left[(3+\nu)+(1-\nu)\left(\frac{Rn}{Rp}\right)^4\right]} \left\{-3\left[(1+\nu)+(1-\nu)\left(\frac{Rn}{Rp}\right)^4\right]\right.$$

$$\left(\frac{Rn}{Rp}\right)^2 + (1+\nu)\left[1-\left(\frac{Rn}{Rp}\right)^2\right]^2 + 2\left[(3+\nu)+(1-\nu)\left(\frac{Rn}{Rp}\right)^4\right]\left[1+\ell n\left(\frac{Rn}{Rp}\right)\right]$$

$$\left. + \left[(1+\nu)\left(\frac{Rn}{Rp}\right)^2 - (3+\nu)\right]\right\}$$

Taking ϕ to be the plane shown in Figure 2, i.e., $\phi = 0$ and $\phi = \pi$, and assuming a value of $\nu = 0.3$, the equation for the slope of the deflected plate in the annular region may be written as:

$$\Theta p = \frac{0.43\ M}{Eptp^3\left[3.3+0.7\left(\frac{Rn}{Rp}\right)^4\right]} \left\{ \left[2.7 + 1.4\ \ell n\ \left(\frac{Rn}{Rp}\right)\right]\left(\frac{Rn}{Rp}\right)^4 - 2.1\left(\frac{Rn}{Rp}\right)^6 \right.$$

$$\left. - 5.20\left(\frac{Rn}{Rp}\right)^2 + 6.6\ \ell n\ \left(\frac{Rn}{Rp}\right) + 4.60 \right\} \qquad (2)$$

A more concise way of writing Equation (2) is as follows:

$$\Theta p = \frac{M\alpha}{Eptp^3} \tag{3}$$

where α is a dimensionless parameter dependent on $(\frac{Rn}{Rp})$ and is defined as follows:

$$\alpha = \frac{0.43}{[3.3+0.7(\frac{Rn}{Rp})^4]} \left\{[2.7 + 1.4\ \ell n\ (\frac{Rn}{Rp})]\ (\frac{Rn}{Rp})^4 \right.$$

$$\left. - 5.20\ (\frac{Rn}{Rp})^2 + 6.6\ \ell n\ (\frac{Rn}{Rp}) - 2.1\ (\frac{Rn}{Rp})^6 + 4.60 \right\} \tag{4}$$

Values of α are easily calculated and are plotted as a function of (Rn/Rp) in Figure 3.

The rotation due to the flexibility of the nozzle, Θn, may be calculated from the deflection equation for a cantilever beam subject to an end moment [6]:

$$\Theta n = \frac{M\ell}{EnI} \tag{5}$$

From assumption No. 6, the total rotation at the nozzle end may be considered as the superposition of the rotations due to the plate and nozzle flexibilities. From Equations (3) and (5):

$$\Theta = \Theta p + \Theta n$$

$$\Theta = \frac{M\alpha}{Eptp^3} + \frac{M\ell}{EnI}$$

$$\Theta = M\left[\frac{\alpha}{Eptp^3} + \frac{\ell}{EnI}\right] \tag{6}$$

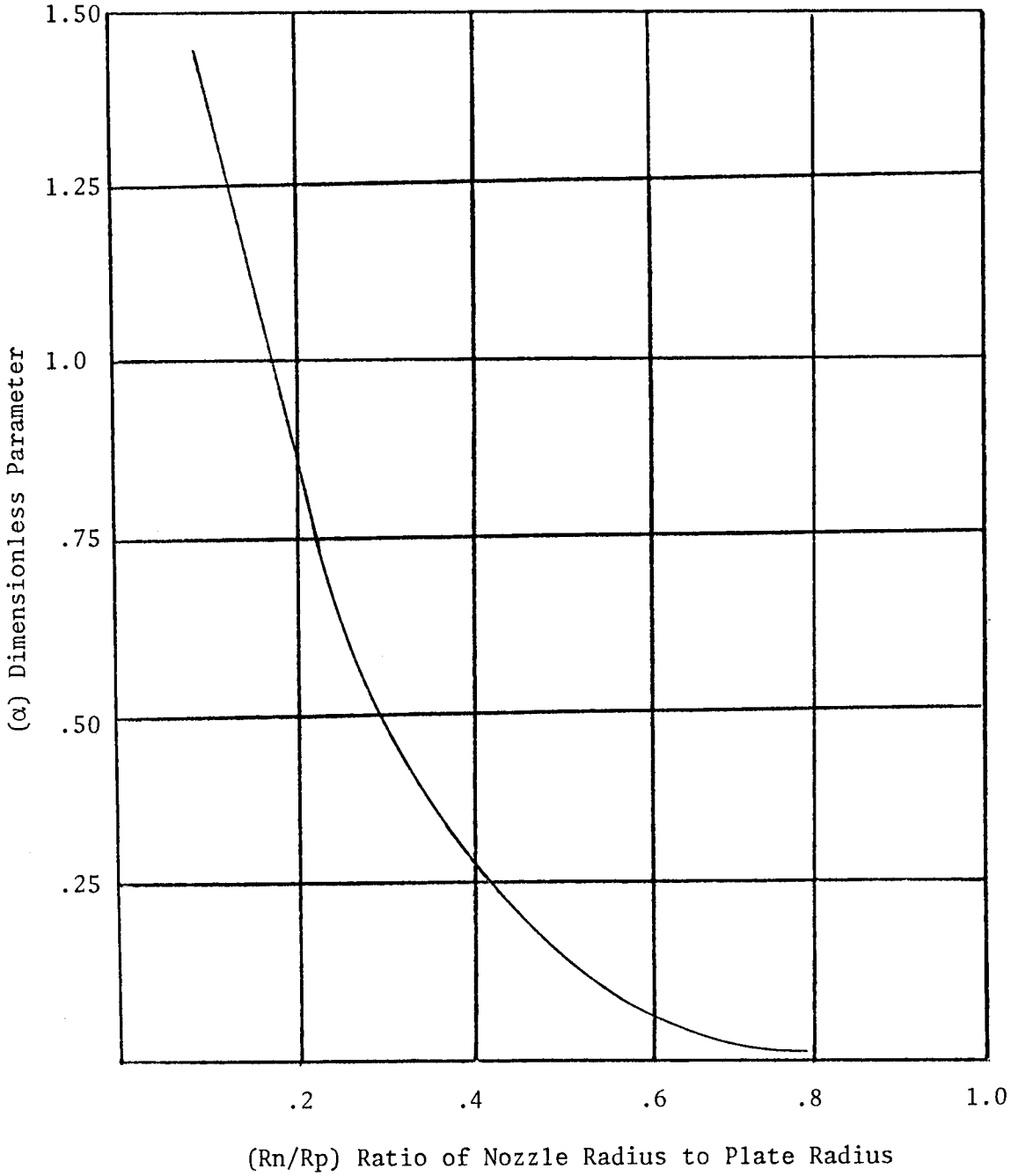


Figure 3. (α) Dimensionless parameter for calculating stiffness coefficient for bending mode.

As a check on the expression for total rotation of the plate and nozzle obtained in Equation (6), an alternate derivation using the classical deflection curve differential equation will be used.

In Figure 4, the plate is modeled as a torsional spring. The classical deflection curve differential equation for the cantilever beam loaded with an end moment is [4]:

$$EnI \frac{d^2y}{dx^2} = M$$

$$\frac{dy}{dx} = \frac{M}{EnI} x + C_1 \tag{7}$$

$$y = \frac{Mx^2}{2\ EnI} + C_1 x + C_2 \tag{8}$$

The following boundary conditions may be used to evaluate the constants of integration:

$$y = 0 \quad \text{at} \quad x = 0 \tag{9.a}$$

From assumption No. 8

$$\frac{dy}{dx} = \Theta p = \frac{M\alpha}{Eptp^3} \quad \text{at} \quad x = 0 \tag{9.b}$$

From Equations (8) and (9.a), $C_2 = 0$. From Equations (7) and (9.b), $C_1 = \Theta p$.

Substitution of C_1 into Equation (7) gives the general expression for the slope of the nozzle.

$$\frac{dy}{dx} = \frac{Mx}{EnI} + \frac{M\alpha}{Eptp^3} \tag{10}$$

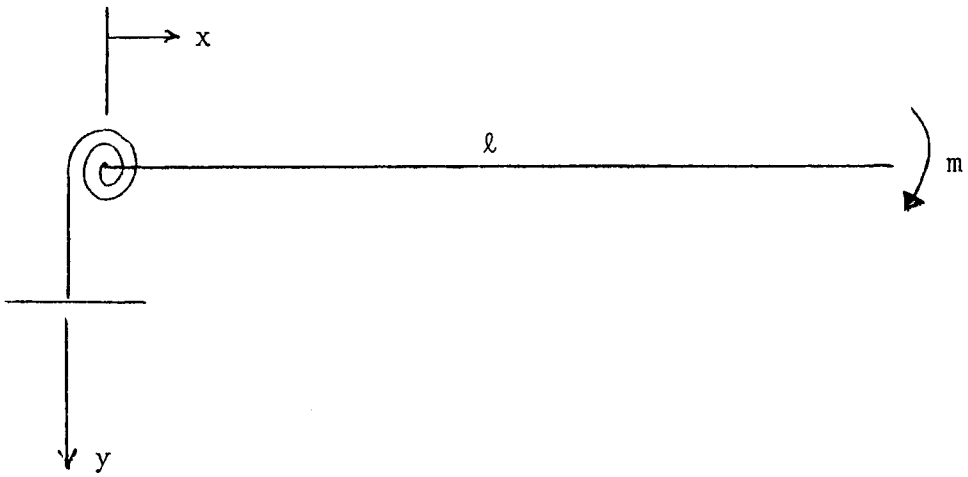


Figure 4. Plate considered as a torsional spring.

To obtain the total rotation at the end of the plate-nozzle system, evaluate Equation (10) at $x = \ell$.

$$\Theta = \left.\frac{dy}{dx}\right|_{x = \ell} = \frac{M\ell}{EnI} + \frac{M\alpha}{Eptp^3}$$

$$\Theta = M\left(\frac{\ell}{EnI} + \frac{\alpha}{Eptp^3}\right)$$

This expression is identical to the one in Equation (6). The stiffness coefficient for the bending mode may be written as:

$$K_B = \frac{M}{\Theta} = \frac{EnEpItp^3}{Eptp^3\ell + EnI\alpha} \qquad (11)$$

A dimensional check verifies that the units for this stiffness coefficient is (lb-in/radian).

It is interesting to note that the same stiffness coefficient could be derived by calculating individual stiffness coefficients for the plate and nozzle and combining them in series.

C. AXIAL MODE

The application of Fx as shown in Figure 1 (page 5) results in lateral deflection of the plate. The plate is considered to be simply supported at the outer radius and fixed at the inner radius and loaded uniformly along the inner radius as shown in Figure 5. The following equation [7] and application of proper boundary conditions may be used to derive the equations of the deflected plate surface:

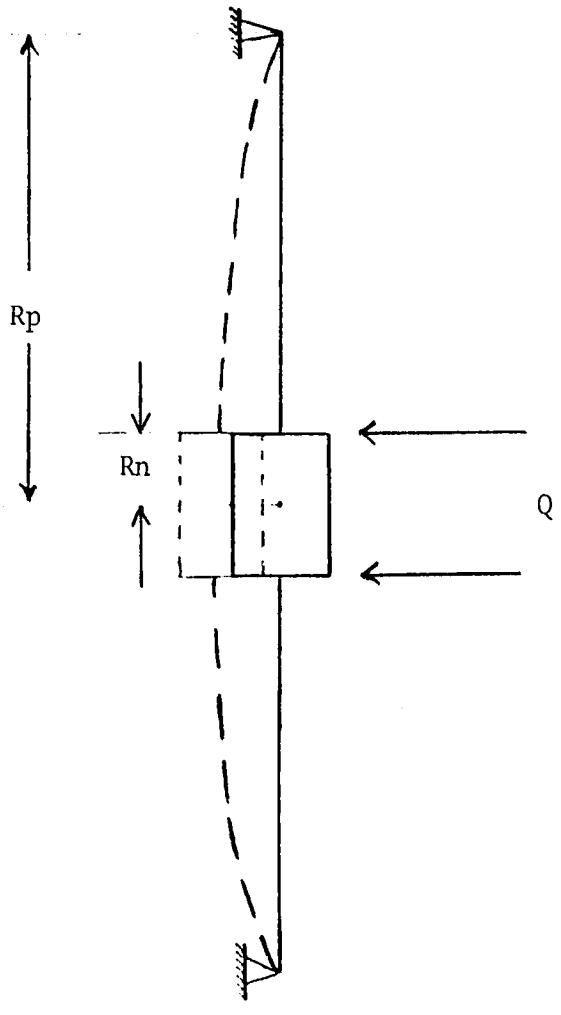


Figure 5. Plate deflection due to axial loads.

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{Q}{D}$$

where $Q = \frac{Fx}{2\pi\ Rn}$

The appropriate boundary conditions for the loading shown in Figure 5 are:

$$w = 0 \quad \text{at} \quad r = Rp$$

$$M_R = 0 \quad \text{at} \quad r = Rp$$

$$\frac{dw}{dr} = 0 \quad \text{at} \quad r = Rn$$

The resulting deflection equation is as follows [8]:

$$w = \frac{Fx\ Rp^2}{8\pi D}\left[\frac{1}{\left(\frac{1+v}{1-v}\right)+\left(\frac{Rn}{Rp}\right)^2}\right]\left\{\frac{1}{2}\left(1-\frac{r^2}{Rp^2}\right)\left(\frac{3+v}{1-v}+\left(\frac{Rn}{Rp}\right)^2\right)\right.$$

$$-\frac{(1+v)}{(1-v)}\left[\left(\frac{2}{1+v}\right)\left(\frac{Rn}{Rp}\right)^2+\frac{r^2}{Rp^2}\right]\ln\left(\frac{Rp}{r}\right)-\left(\frac{Rn}{Rp}\right)^2\ln\left(\frac{Rp}{Rn}\right)$$

$$\left.+\frac{r^2}{Rp^2}\left(\frac{Rn}{Rp}\right)^2\ln\left(\frac{r}{Rn}\right)-2\left(\frac{1+v}{1-v}\right)\left(\frac{Rn}{Rp}\right)^2\ln\left(\frac{Rp}{r}\right)\ln\left(\frac{Rp}{Rn}\right)\right\} \qquad (12.a)$$

The maximum deflection may be obtained by evaluating this expression at r = Rn:

$$\Delta P = w\Big|_{r=Rn} = \frac{FxRp^2}{8\pi D}\left[\frac{1}{\frac{1+v}{1-v}+\left(\frac{Rn}{Rp}\right)^2}\right]$$

$$\left\{\frac{1}{2}\left(1-\left(\frac{Rn}{Rp}\right)^2\right)\left(\frac{3+v}{1-v}+\left(\frac{Rn}{Rp}\right)^2\right)-\left(\frac{4}{1-v}\right)\left(\frac{Rn}{Rp}\right)^2\ln\left(\frac{Rp}{Rn}\right)\right.$$

$$\left.-2\left(\frac{1+v}{1-v}\right)\left(\frac{Rn}{Rp}\right)^2\left[\ln\left(\frac{Rp}{Rn}\right)\right]^2\right\}$$

Assuming a value of v = 0.3:

$$\Delta P = \frac{0.43\ Fx\ Rp^2}{Eptp^3} \left\{ \frac{1}{1.86 + (\frac{Rn}{Rp})^2} \left[(1 - (\frac{Rn}{Rp})^2)\ (2.36 + \frac{1}{2}(\frac{Rn}{Rp})^2 \right.\right.$$

$$\left.\left. - 5.71\ (\frac{Rn}{Rp})^2\ \ell n\ (\frac{Rp}{Rn}) - 3.71\ (\frac{Rn}{Rp})^2\ [\ell n\ (\frac{Rp}{Rn})]^2 \right] \right\}$$

A more concise way of writing the equation of the maximum deflection of the plate when subject to axial loads is:

$$\Delta P = \frac{Fx\ Rp^2}{Eptp^3}\ \beta \qquad (12.b)$$

where β is a dimensionless parameter dependent on (Rn/Rp) and is defined as follows:

$$\beta = \frac{0.43}{[1.86 + (\frac{Rn}{Rp})^2]} \left\{ - (\frac{Rn}{Rp})^2 \left[1.86 + 5.71\ \ell n\ (\frac{Rp}{Rn}) \right.\right.$$

$$\left.\left. + 3.71\ [\ell n\ (\frac{Rp}{Rn})]^2 \right] - 0.5\ (\frac{Rn}{Rp})^4 + 2.36 \right\} \qquad (13)$$

Values of β are easily calculated and are plotted as a function of (Rn/Rp) in Figure 6.

The deflection due to the flexibility of the nozzle ΔN when subject to an axial force is calculated as below:

$$En = \frac{\sigma}{\varepsilon} = \frac{Fx/A}{e/\ell} = \frac{Fx\ell}{Ae}$$

$$e = \Delta N = \frac{Rx\ell}{A\ En} \qquad (14)$$

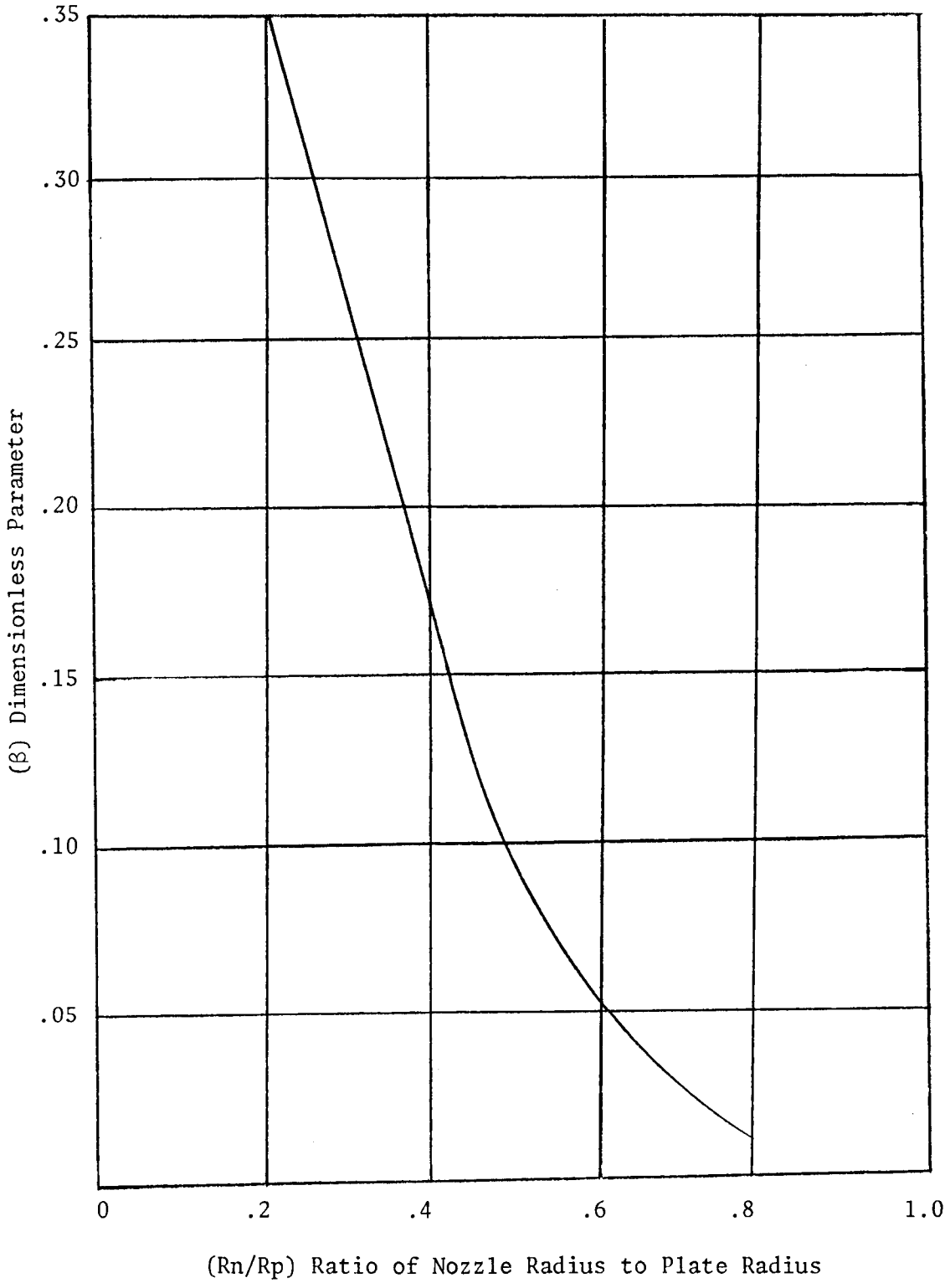


Figure 6. (β) Dimensionless parameter for calculating stiffness coefficient for axial mode.

The total deflection due to axial loads may be written as the sum of Equations (12) and (14).

$$\Delta = \Delta P + \Delta N$$

$$\Delta = \frac{FxRp^2}{Eptp^3}\beta + \frac{Fx\ell}{A\ En}$$

The stiffness coefficient for the axial mode may be written as:

$$K_A = \frac{Fx}{\Delta} = \frac{EpEnAtp^3}{EnRp^2A\beta+Ep\ell tp^3} \qquad (15)$$

A dimensional check verifies that the units are lb/inch.

D. TORSIONAL MODE

The application of M_x as shown in Figure 1 (page 5) results in torsional deformation of the plate and nozzle. It will be assumed that the plate does not deflect in the lateral plane; therefore, a two-dimensional elastic model as shown in Figure 7 will be used. The differential element of the plate as shown in Figure 8 will be assumed to be in a state of pure shear. The two-dimensional equilibrium equations expressed in polar coordinates may be used to relate the stress parameters [10].

$$\frac{\partial \sigma r}{\partial r} + \frac{1}{r}\frac{\partial T}{\partial \phi} + \frac{\sigma r - \sigma \phi}{r} + F_R = 0$$

$$\frac{1}{r}\frac{\partial \sigma \phi}{\partial \phi} + \frac{\partial T}{\partial r} + 2\frac{T}{r} + F_\phi = 0$$

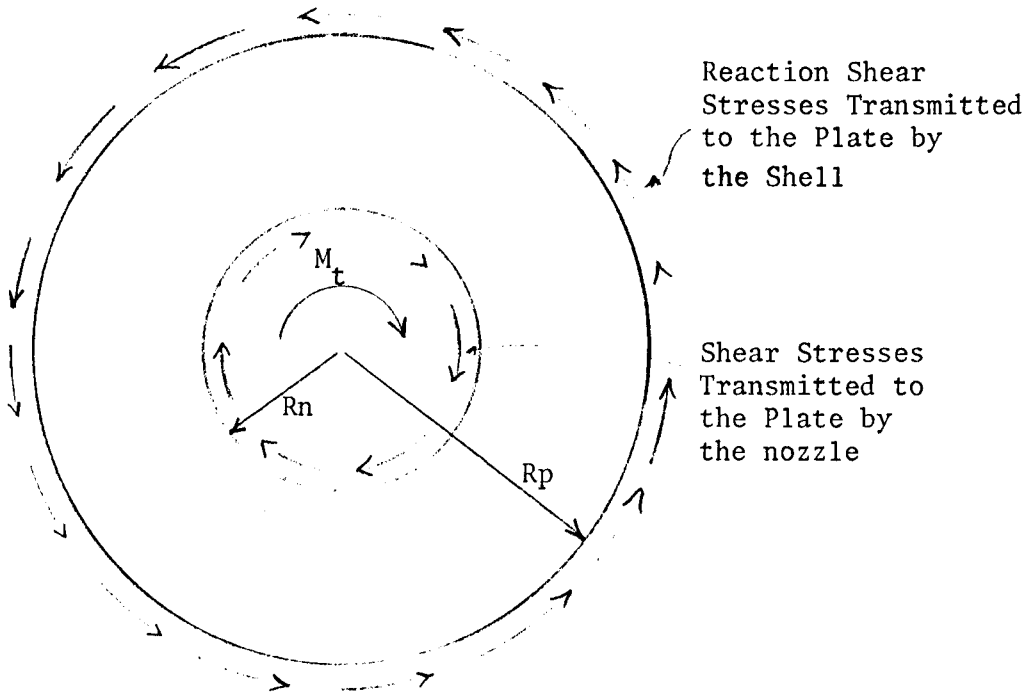


Figure 7. Two dimensional model for torsional mode.

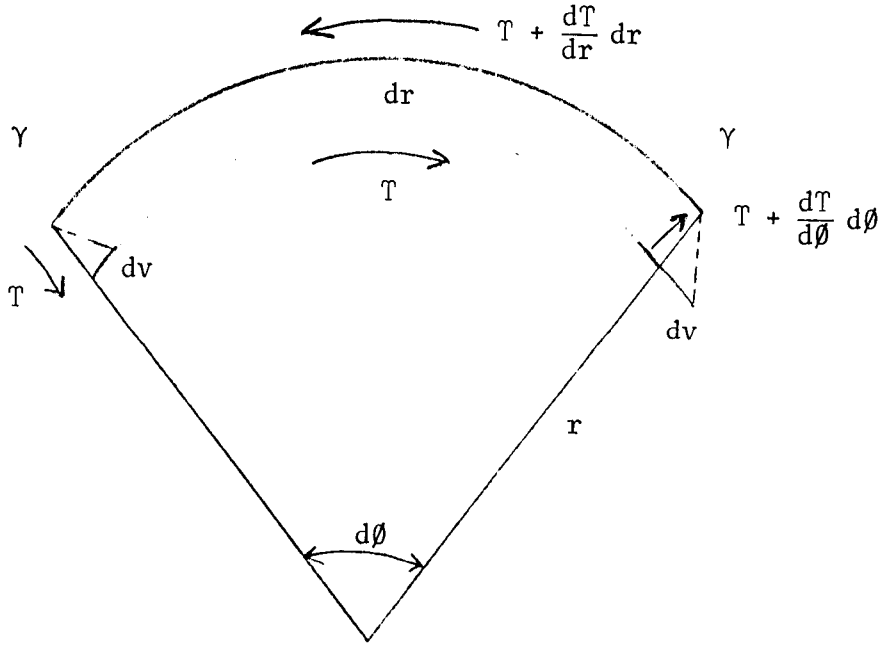


Figure 8. Differential element of circumferential section showing stress and strain of plate subject to torsional load.

These equations reduce to the following expression when applied to Figure 8:

$$\frac{\partial T}{\partial r} + 2\frac{T}{r} = 0 \tag{16}$$

The following relationship may be written between unit shearing strain and tangential displacement [9].

$$\gamma = \frac{T}{Gp} = \frac{\partial V}{\partial r} - \frac{V}{r} + \frac{1}{r}\frac{\partial u}{\partial \emptyset}$$

This equation reduces to the following expression when applied to Figure 8:

$$\gamma = \frac{T}{Gp} = \frac{\partial V}{\partial r} - \frac{V}{r} \tag{17}$$

Solving Equation (17) for T in terms of V and r yields the following expression:

$$T = Gp\ (\frac{\partial V}{\partial r} - \frac{V}{r}) \tag{18}$$

Substitution of Equation (18) into Equation (16) yields:

$$\frac{\partial}{\partial r}\ [Gp\ (\frac{\partial V}{\partial r} - \frac{V}{r})] + \frac{2G}{r}\ (\frac{\partial V}{\partial r} - \frac{V}{r}) = 0$$

The tangential displacements are seen to be a function only of r; therefore, the above partial differential equation may be written as an ordinary differential equation in terms of total differentials as follows:

$$Gp \frac{d^2V}{dr^2} - G \frac{d}{dr} \left(\frac{V}{r}\right) + \frac{2Gp}{r} \frac{dV}{dr} - \frac{2GpV}{r^2}$$

This equation may be simplified as follows:

$$r^2 \frac{d^2V}{dr^2} + r \frac{dV}{dr} - V = 0 \tag{19}$$

Equation (19) is an Euler type of differential equation and may be solved by purely algebraic manipulations [11].

Substitute $V = r^m$ and its derivatives into Equation (19).

$$r^2 m(m-1) r^{m-2} - r^m + r m r^{m-1} = 0$$

Omit that r^m which is not zero when $r \neq 0$

$$m(m-1) - 1 + m = 0$$

$$m^2 - 1 = 0 \tag{20}$$

Equation (20) is the auxiliary equation. Its roots m_1 and m_2 are distinct; therefore, the two functions

$$V_1(r) = r^{m_1} \text{ and } V_2(r) = r^{m_2}$$

Constitute a fundamental system of solutions of Equation (19) for all r as follows:

$$V = C_1 r^{m_1} + C_2 r^{m_2} \tag{21}$$

Solving for the roots of Equation (20), Equation (21) may be written as:

$$V = C_1 r + \frac{C_2}{r} \tag{22}$$

Appropriate boundary conditions required to evaluate the integration constants are:

$$V = 0 \quad \text{at} \quad r = Rp \tag{23.a}$$

$$T = - \frac{M_x}{2\pi Rn^2 tp} \quad \text{at} \quad r = Rn \tag{23.b}$$

Boundary condition (23.b) is a necessary condition for equilibrium at the nozzle-plate interface. The rationale used to arrive at this equation is as follows:

$$M_x = F_t \, R_n \tag{24}$$

where F_t is the resultant tangential force resulting from the shear stress acting uniformly between the plate and nozzle and is calculated as follows:

$$F_t = - (2\pi R_n t_p) \, T \Big| r = R_n$$

Substituting this expression into Equation (24) and solving for $T|r = R_n$ yields:

$$T \Big| r = Rn = - \frac{M_x}{2\pi Rn^2 tp}$$

From boundary condition (23.a):

$$C_1 = - \frac{C_2}{Rp^2} \tag{25}$$

Substitution of boundary condition (23.b) and Equation (22) into Equation (18) yields the following:

$$T\Big|_{r = Rn} = Gp\left[\frac{d}{dr}\left(C_1 r + \frac{C_2}{r}\right) - \frac{1}{r}\left(C_1 r + \frac{C_2}{r}\right)\right]\Bigg|_{r = Rn}$$

$$= -\frac{M_x}{2\pi Rn^2 tp}$$

This equation can be simplified as follows:

$$\frac{2GpC_2}{Rn^2} = \frac{M_x}{2\pi Rn^2 tp}$$

$$C_2 = \frac{M_x}{4\pi Gptp} \tag{26}$$

Substitution of Euation (26) into Equation (25) yields:

$$C_1 = -\frac{M_x}{4\pi GptpRp^2} \tag{27}$$

Substitution of Equations (26) and (27) into Equation (22) yields the general expression for the tangential displacement as a function of r.

$$V = \frac{-M_x r}{4\pi GptpRp^2} + \frac{M_x}{4\pi Gptpr}$$

$$V = \frac{M_x}{4\pi Gptp}\left(\frac{Rp^2 - r^2}{Rp^2 r}\right) \tag{28}$$

A dimensional check verifies that the units for displacement are in inches.

The displacement required for the calculation of torsional stiffness coefficient is at r = Rn. Evaluation of Equation (28) at r = Rn yields:

$$V\Big|_{r = Rn} = \frac{M_x}{4\pi G p t p} \left(\frac{Rp^2 - Rn^2}{Rp^2 Rn} \right) \tag{29}$$

It must be noted that the desired result is an expression for angular deformation in radians. The relationship between the tangential displacement of the plate and the rotation of the nozzle is shown in Figure 9. From Figure 9, the following relationship may be written:

$$V\Big|_{r = Rn} = Rn\lambda p$$

$$\lambda p = \frac{V\Big|_{r = Rn}}{Rn} \tag{30}$$

Substitution of Equation (29) into Equation (30) yields:

$$\lambda p = \frac{M_x}{4\pi G p \ tp} \left(\frac{Rp^2 - Rn^2}{Rp^2 \ Rn^2} \right) \tag{31}$$

The end rotation of the nozzle must be added to the rotation obtained for the plate. The end rotation of the nozzle subjected to a torsional moment is [4]:

$$\lambda_n = \frac{M_x \ell}{J \ Gn} \tag{32}$$

The total rotation of the plate nozzle system may be written as the sum of Equations (31) and (32):

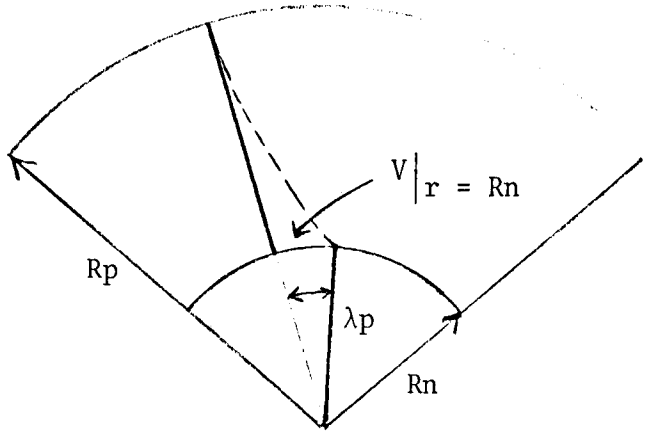


Figure 9. Relationship between displacement of plate and rotation of nozzle.

$$\lambda = \frac{M_x}{4\pi Gptp}\left(\frac{Rp^2 - Rn^2}{Rp^2 Rn^2}\right) + \frac{M_x \ell}{J\ Gn}$$

The stiffness coefficient for the torsional mode may now be written as follows:

$$K_t = \frac{M_x}{\lambda}$$

$$K_t = \frac{4\pi GpGnJtpRp^2Rn^2}{JGn(Rp^2-Rn^2)+4\pi GptpRp^2Rn^2\ell} \qquad (33)$$

A dimensional check verifies that the units are in (lb-in/radian).

CHAPTER III

STRESS ANALYSIS

An investigation of the stresses resulting in the shell-plate-nozzle system from the externally applied loads will be performed. The maximum stress equations will be derived for each component for each mode of loading. These equations can be applied for determining the maximum allowable loads the system is capable of withstanding.

A. BENDING MODE

The governing equations expressed in polar coordinates for the internal bending moments induced in the plate by bending loads are as follows [12]:

$$M_R = -D \left[\frac{\partial^2 w}{\partial r^2} + \nu \left(\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \phi^2} \right) \right] \tag{34.a}$$

$$M_\phi = -D \left[\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \phi^2} + \nu \frac{\partial^2 w}{\partial r^2} \right] \tag{34.b}$$

$$M_{R\phi} = -(1-\nu)D \left[\frac{1}{r} \frac{\partial^2 w}{\partial r \partial \phi} - \frac{1}{r^2} \frac{\partial w}{\partial \phi} \right] \tag{34.c}$$

The equation for the deflection surface of a flat circular plate when loaded nonsymmetrically with a bending moment about its center is expressed in Equation (1). For the convenience of writing this equation in a more compact form in order to perform the manipulations expressed

in Equation (34), Equation (1) may be rewritten as follows:

$$w(r,\phi) = A\left\{Br^3 + Cr + Er(\ell nr - \ell nRp) + \frac{F}{r}\right\}\cos\phi$$

where:

$$A = \frac{MRp}{8\pi D\left[(3+\nu)+(1-\nu)\left(\frac{Rn}{Rp}\right)^4\right]} \tag{35.a}$$

$$B = -\left[(1+\nu)+(1-\nu)\left(\frac{Rn}{Rp}\right)^4\right]\frac{1}{Rp^3} \tag{35.b}$$

$$C = (1+\nu)\left[1-\left(\frac{Rn}{Rp}\right)^2\right]^2\frac{1}{Rp} \tag{35.c}$$

$$E = 2\left[(3+\nu)+(1-\nu)\left(\frac{Rn}{Rp}\right)^4\right]\frac{1}{Rp} \tag{35.d}$$

$$F = -\left(\frac{Rn}{Rp}\right)^2\left[(1+\nu)\left(\frac{Rn}{Rp}\right)^2 - (3+\nu)\right]Rp \tag{35.e}$$

Equations (34) may now be rewritten in terms of the constants expressed in Equations (35);

$$M_R = -D\cos\phi\left\{A\left[6Br + \frac{E}{r} + \frac{2F}{r^3}\right]\right.$$

$$+ \nu\left[\frac{A}{r}\left(3Br^2 + C + E\left(1 + \ell n\, r - \ell n\, Rp\right) - \frac{F}{r^2}\right)\right.$$

$$\left.\left. - \frac{A}{r^2}\left(Br^3 + Cr + Er\left(\ell n\, r - \ell n\, Rp\right) + \frac{F}{r}\right)\right]\right\}$$

$$M_\phi = -D\cos\phi\left\{\frac{A}{r}\left[3r^2 + C + E\left(1 + \ell nr - \ell nRp\right)\right.\right.$$

$$\left. - \frac{F}{r^2}\right] - \frac{A}{r^2}\left[Br^3 + Cr + Er\left(\ell nr - \ell nRp\right)\right.$$

$$\left.\left. + \frac{F}{r}\right] + A\nu\left[6Br + \frac{E}{r} + \frac{2F}{r^3}\right]\right\}$$

$$M_{R\phi} = -(1-v)D\sin\phi \left\{ -\frac{A}{r}\left[3Br^2 + C + E\,(1 + \ell nr - \ell nRp)\right.\right.$$

$$\left.\left. - \frac{F}{r^2} + \frac{A}{r^2}\left[Br^3 + Cr + Er\,(\ell nr - \ell nRp) + \frac{F}{r}\right]\right\}\right.$$

Simplication of these equations yields the following:

$$M_R = -\,DA\cos\phi\,\left[2Br(3+v) + \frac{E}{r}(1+v) + \frac{2F}{r^3}(1-v)\right]$$

$$M_\phi = -\,DA\cos\phi\,\left[2Br(1+3r) + \frac{2F}{r^3}(v-1) + \frac{E}{r}(1+v)\right]$$

$$M_{R\phi} = -(1-v)DA\sin\phi\,\left[-2Br + \frac{2F}{r^3} - \frac{E}{r}\right]$$

Substitution of the constants expressed in Equations (35) and assuming a value of v = 0.3 into Equations (34) yields the following:

$$M_R = \frac{M\cos\phi}{\left[82.94+17.59\,(\frac{Rn}{Rp})^4\right]}\left\{\frac{r}{Rp^2}\left[8.58+4.62\,(\frac{Rn}{Rp})^4\right]\right.$$

$$\left. - \frac{1}{r}\left[8.58+1.82\,(\frac{Rn}{Rp})^4\right] + \frac{Rn^2}{r^3}\left[1.82\,(\frac{Rn}{Rp})^2 - 4.62\right]\right\}$$

$$M_\phi = \frac{M\cos\phi}{\left[82.94+17.59\,(\frac{Rn}{Rp})^4\right]}\left\{\frac{r}{Rp^2}\left[4.94+2.67\,(\frac{Rn}{Rp})^4\right]\right.$$

$$\left. - \frac{Rn^2}{r^3}\left[1.82\,(\frac{Rn}{Rp})^2 - 4.62\right] - \frac{1}{r}\left[8.58+1.82\,(\frac{Rn}{Rp})^4\right]\right\}$$

$$M_{R\phi} = \frac{0.028\,M\sin\phi}{\left[3.3+0.7\,(\frac{Rn}{Rp})^4\right]}\left\{\frac{r}{Rp^2}\left[2.6+1.4\,(\frac{Rn}{Rp})^4\right]\right.$$

$$\left. - \frac{Rn^2}{r^3}\left[2.6\,(\frac{Rn}{Rp})^2 - 6.6\right] - \frac{1}{r}\left[6.6+1.4\,(\frac{Rn}{Rp})^4\right]\right\}$$

Careful observation of these equations verifies that the maximum moment is as follows:

$$M_{max} = M_R\Big|_{r = Rn}$$

This fact is also confirmed by Roark [13]. It should be noted that the maximum values occur at $\phi = 0$ and $\phi = \pi$. The maximum moment on the plate due to bending loads may be written as:

$$M_{max} = M_R\Big|_{r = Rn} = \frac{M}{[82.94+17.59\ (\frac{Rn}{Rp})^4]} \left\{\frac{Rn}{Rp^2}\ [8.58 + 4.62\ (\frac{Rn}{Rp})^4] - \frac{1}{Rn}\ [8.58+1.82\ (\frac{Rn}{Rp})^4] + \frac{1}{Rn}\ [1.82\ (\frac{Rn}{Rp})^2 - 4.62]\right\}$$

Simplification yields:

$$M_{max} = \frac{M}{[82.94+17.59\ (\frac{Rn}{Rp})^4]} \left\{\frac{Rn^2(8.58+4.62\ (\frac{Rn}{Rp})^4)+Rp^2[1.82}{Rp^2Rn} \underline{\{(\frac{Rn}{Rp})^2-(\frac{Rn}{Rp})^4\}-13.2]}\right\}$$

The maximum plate stress for the bending mode is therefore [5]:

$$\sigma_{max} = \sigma_r\Big|_{r = Rn} = \frac{6\ M_{max}}{tp^2}$$

A convenient form of writing this equation is:

$$\sigma_{max} = \frac{M\delta}{Rptp^2}$$

where δ is a dimensionless parameter which is a function of (Rn/Rp) and is defined as below. Values of δ are plotted in Figure 10.

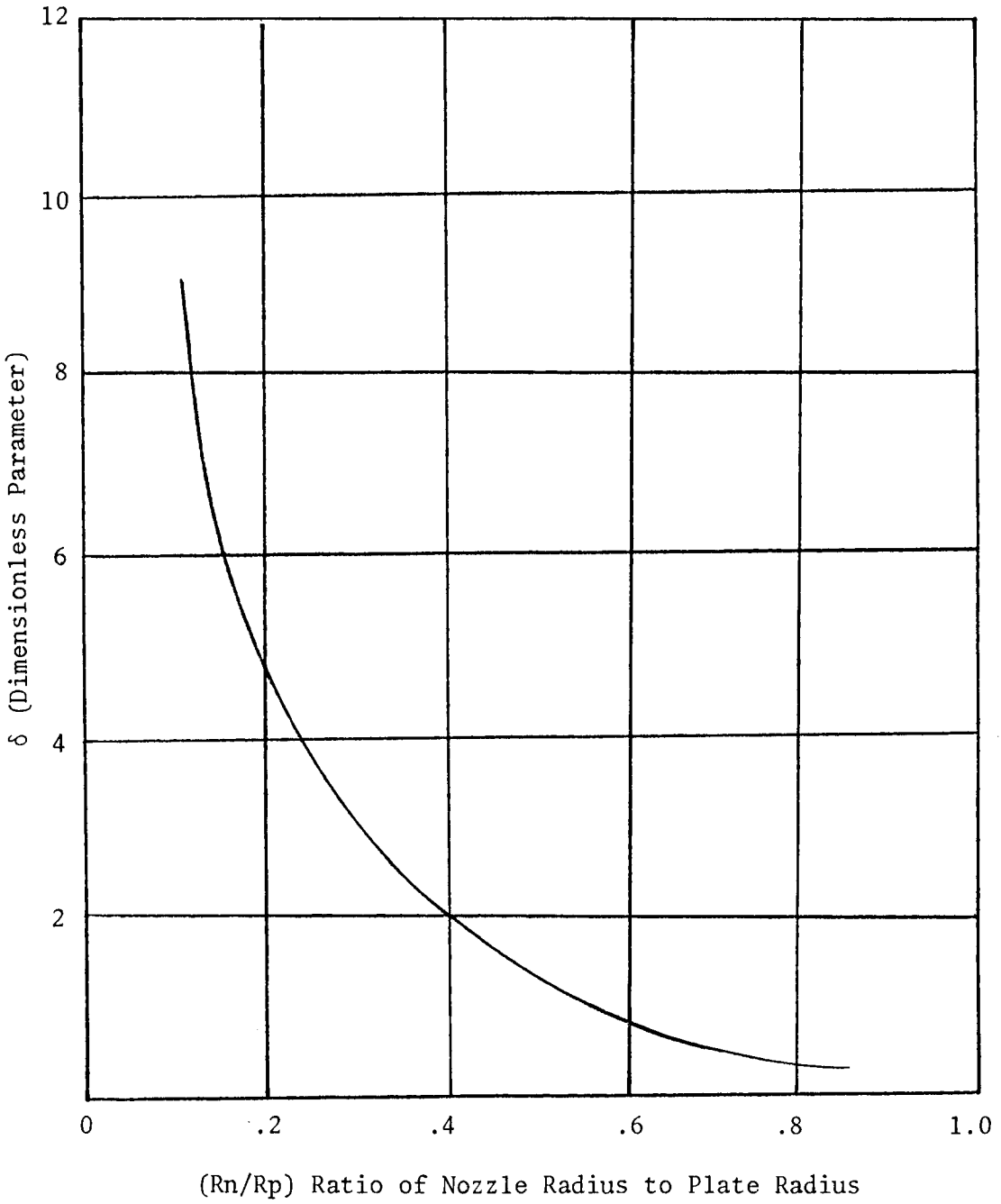


Figure 10. (δ) Dimensionless parameter for calculating the maximum plate stress induced by bending loads.

$$\delta = \frac{5.48\left(\frac{Rn}{Rp}\right)+27.72\left(\frac{Rn}{Rp}\right)^5+\left[10.92\left\{\left(\frac{Rn}{Rp}\right)-\left(\frac{Rn}{Rp}\right)^3\right\}-79.2\left(\frac{Rp}{Rn}\right)\right]}{\left[82.94+17.59\left(\frac{Rn}{Rp}\right)^4\right]}$$

The governing equation for calculating the stress in the nozzle when subject to bending is:

$$\sigma_{max} = \frac{MR_n}{I}$$

The shell was considered to be a rigid simple support in Chapter II and no consideration was given to the elastic deformation. The tensile and compressive forces and corresponding stresses acting on the shell due to the interaction with the plate can be derived by calculating the vertical reaction acting at the outer boundary of the plate. The forces and stresses acting on the shell may be considered to be equal and opposite to the vertical reaction acting at the outer boundary of the plate as required for equilibrium.

The intensity of the vertical reaction of the shell on the plate at the outer boundary is [5]:

$$V = \left[Q_R - \frac{1}{r}\frac{\partial M_{R\phi}}{\partial \phi}\right] \quad r = Rp \tag{36}$$

where: Q_R = Radial Shear = $-D\frac{\partial}{\partial r}\nabla^2 w$

The Laplacian operator ∇^2 expressed in polar coordinates is [11]:

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}$$

After making the proper substitutions, and simplifying, Equation (36) yields

$$V = -D\left[\frac{\partial^3 w}{\partial r^3} + \frac{1}{r}\frac{\partial^2 w}{\partial r^2} - \frac{1}{r^2}\frac{\partial w}{\partial r} + \frac{v}{r^2}\frac{\partial^3 w}{\partial \phi^2 \partial r} - \frac{(1+v)}{r^3}\frac{\partial^2 w}{\partial \phi^2}\right] \tag{37}$$

Written in terms of the constants expressed in Equations (35), Equation (37) yields:

$$V = -DA\cos\phi \left\{(10-2v)B - \frac{(1+v)}{Rp^2}E - \frac{(2-2v)}{Rp^4}F\right\}$$

Substituting the expressions expressed in Equations (35) yields:

$$V = \frac{MRp\cos\phi}{8\pi\left[(3+v)+(1-v)\left(\frac{Rn}{Rp}\right)^4\right]} \left\{-(10-2v)\left[(1+v)+(1-v)\left(\frac{Rn}{Rp}\right)^4\right]\frac{1}{Rp^3}\right.$$

$$\left. - \frac{2(1+v)}{Rp^3}\left[(3+v)+(1-v)\left(\frac{Rn}{Rp}\right)^4\right] + \frac{(2-2v)}{Rp^3}\left(\frac{Rn}{Rp}\right)^2\left[(1+v)\left(\frac{Rn}{Rp}\right)^2 - (3+v)\right]\right\}$$

Assuming a value of v = 0.3, this equation simplifies to:

$$V = \frac{M\cos\phi}{Rp^2}\left[\frac{0.83 + 0.45\left(\frac{Rn}{Rp}\right)^2}{3.3 + 0.70\left(\frac{Rn}{Rp}\right)^4}\right] \tag{38}$$

It should be noted that the reaction intensity expressed in Equation (38) is in lb/in. To express in terms of stress:

$$\sigma = \frac{M\cos\phi}{Rp^2 t_s}\left[\frac{0.83 + 0.45\left(\frac{Rn}{Rp}\right)^2}{3.3 + 0.70\left(\frac{Rn}{Rp}\right)^4}\right]$$

The maximum values would occur at $\phi = 0$ and $\phi = \pi$; therefore, the maximum stresses in the shell induced during bending is:

$$\sigma_{max} = \frac{M\psi}{Rp^2 ts}$$

where ψ is a dimensionless parameter which is a function of (Rn/Rp), is defined as follows and is plotted in Figure 11.

$$\psi = \left[\frac{0.83 + 0.45 \left(\frac{Rn}{Rp}\right)^2}{3.3 + 0.70 \left(\frac{Rn}{Rp}\right)^4} \right]$$

B. AXIAL MODE

The application of F_x as shown in Figure 1 (page 5) results in lateral deflection of the plate. The model and assumptions for the loading shown in Figure 5 (page 14) is applicable to the load case. The governing equations for the internal bending moments are the same as expressed in Equations (34); however, this is a symmetric load and the equations reduce to:

$$M_R = -D \left[\frac{d^2w}{dr^2} + \frac{v}{r} \frac{dw}{dr} \right] \qquad (39.a)$$

$$M_\phi = -D \left[\frac{1}{r} \frac{dw}{dr} + v \frac{d^2w}{dr^2} \right] \qquad (39.b)$$

$$M_{R\phi} = 0 \qquad (39.c)$$

Substitution of Equation (12.a) into Equations (39) yields the following equations for the internal bending moments induced by axial loads [8].

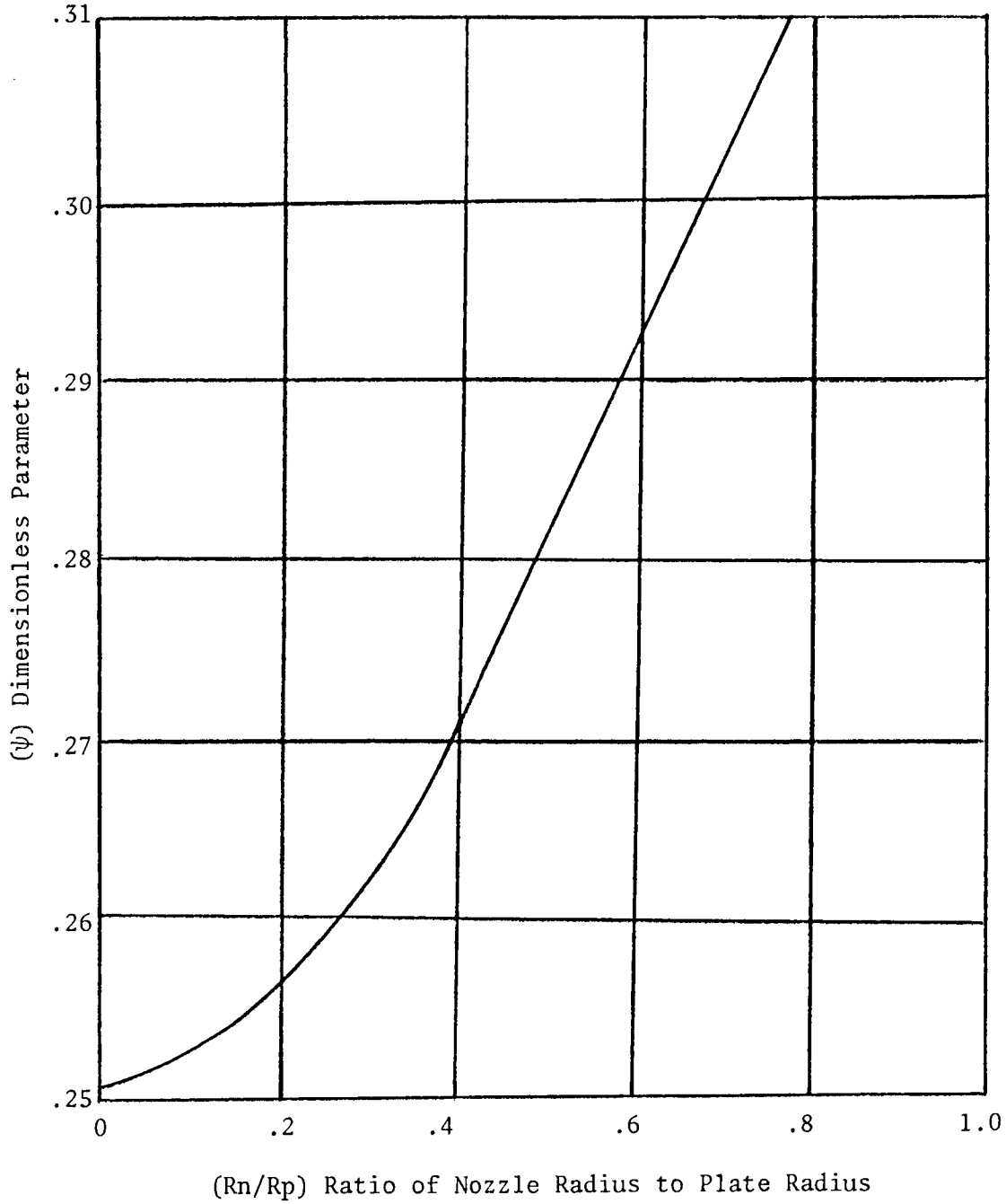


Figure 11. (ψ) Dimensionless parameter for calculating the maximum tensile and compressive shell stress induced by bending loads.

$$M_R = \frac{F_x}{4\pi}\left[\frac{1+v}{\frac{1+v}{1-v} + (\frac{Rn}{Rp})^2}\right]\left[(\frac{1}{1+v})\left(1 - \frac{r^2}{Rp^2}\right)\frac{Rn^2}{r^2}\right.$$

$$\left. + (\frac{Rn}{r})^2 \ln(\frac{Rp}{Rn}) + (\frac{1+v}{1-v})\ln(\frac{Rp}{r}) - (\frac{Rn}{Rp})^2 \ln(\frac{r}{Rn})\right]$$

$$M_\phi = \frac{F_x}{2\pi}\left[\frac{1}{\frac{1+v}{1-v} + (\frac{Rn}{Rp})^2}\right]\left[\frac{1}{2}\left(1 - \frac{Rn^2}{Rp^2}\right) + (\frac{1+v}{1-v})\ln(\frac{Rp}{Rn})\right]$$

The maximum moment is $M_R\Big|_{r = Rn}$ [8] and may be written as follows:

$$M_{max} = \frac{Fx}{2\pi}\left[\frac{1}{\frac{1+v}{1-v} + (\frac{Rn}{Rp})^2}\right]\left[\frac{1}{2}\left(1 - \frac{Rn^2}{Rp^2}\right) + \frac{(1+v)}{(1-v)}\ln(\frac{Rp}{Rn})\right]$$

Assuming a value of v = 0.3, Equation (96) reduces to:

$$M_{max} = \frac{Fx}{1.86 + (\frac{Rn}{Rp})^2}\left[0.08\left(1 - \frac{Rn^2}{Rp^2}\right) + 0.30\ln(\frac{Rp}{Rn})\right]$$

The maximum stress in the plate induced by axial loads may be written as:

$$\sigma_{max} = \frac{6\,M_{max}}{tp^2}$$

$$\sigma_{max} = \frac{6\,Fx}{[1.86 + (\frac{Rn}{Rp})^2]tp^2}\left[0.08\left(1 - \frac{Rn^2}{Rp^2}\right) + 0.30\ln(\frac{Rp}{Rn})\right]$$

This equation may be expressed in terms of a dimensionless parameter as follows:

$$\sigma_{max} = \frac{F_x f}{tp^2}$$

Γ is a dimensionless parameter which is a function of (Rn/Rp), is defined as follows and is plotted in Figure 12.

$$\Gamma = \frac{[0.48(1 - \frac{Rn^2}{Rp^2}) + 1.80 \ln (\frac{Rp}{Rn})]}{[1.86 + (\frac{Rn}{Rp})^2]}$$

The stress in the nozzle is simply:

$$\sigma_{max} = \frac{Fx}{A}$$

C. TORSIONAL MODE

The plate is assumed to be in a state of pure shear when subject to torsional loads. Substitution of Equation (28) into Equation (18) yields the general expression for shear stress as a function of r.

$$T = Gp \left\{ \frac{d}{dr} \left[\frac{Mx}{4\pi Gptp} \left(\frac{Rp^2 - r^2}{Rp^2 r} \right) \right] - \frac{1}{r} \left[\frac{Mx}{4\pi Gptp} \left(\frac{Rp^2 - r^2}{Rp^2 r} \right) \right] \right\}$$

Simplification of this equation yields:

$$T = \frac{- Mx}{2\pi r^2 tp} \tag{40}$$

Careful observation of Equation (40) verifies that the maximum stress induced in the plate by torsional loads occurs at r = Rn.

$$T_{max} = \frac{- Mx}{2\pi Rn^2 tp} \tag{41}$$

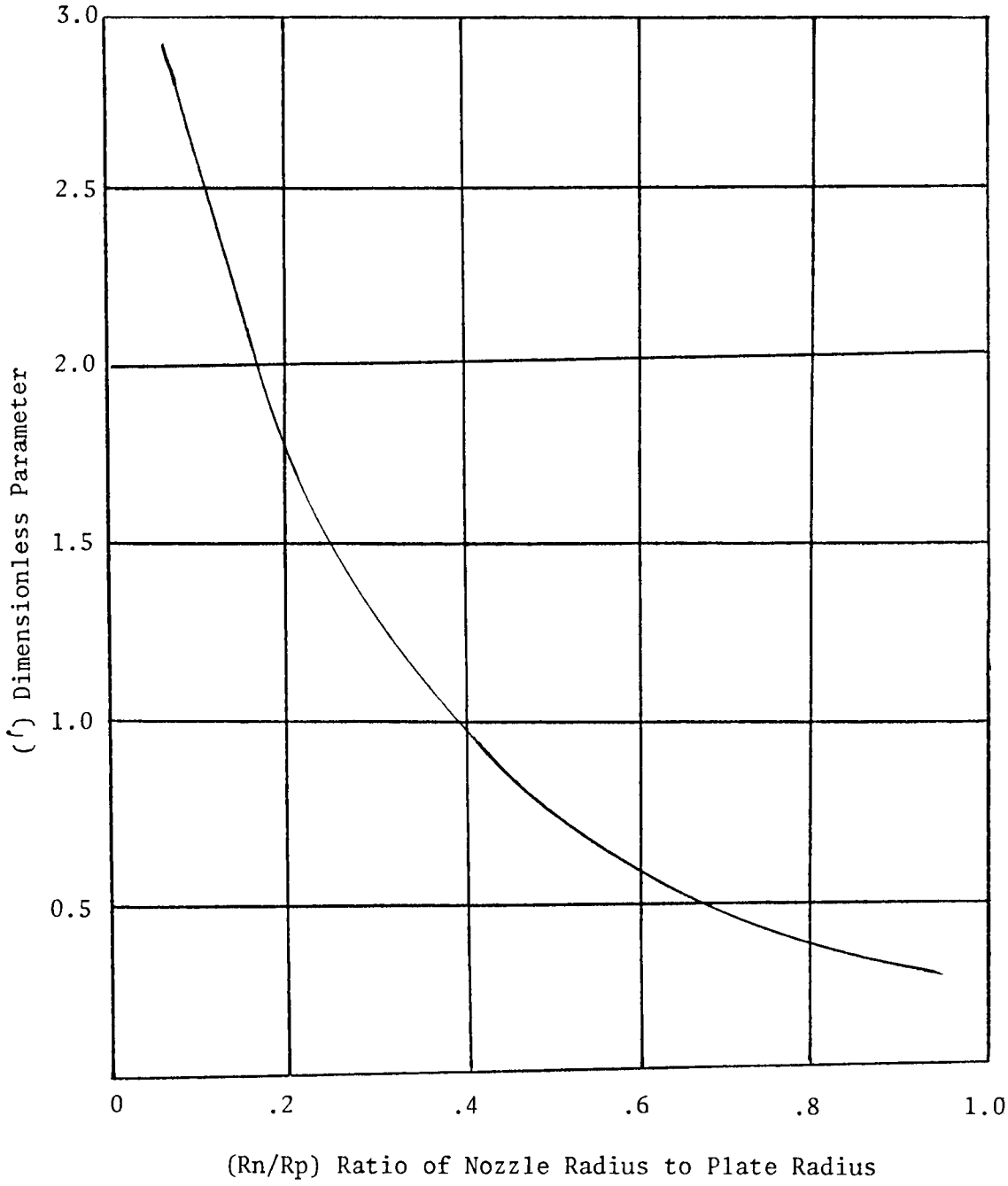


Figure 12. (f') Dimensionless parameter used to calculate maximum tensile and compressive plate stress induced by axial loads.

It should be noted that the expression in Equation (41) is identical to the boundary condition specified in Equation (23.b). The maximum stress induced in the nozzle by a torsional load is [5]:

$$T_{max} = \frac{MxRn}{J}$$

A stress analysis will not be performed for the shell when subjected to torsional loads; however, an equation will be derived which considers the stability of the shell with regard to buckling.

The action-reaction rationale used for calculating stresses in the shell for the bending mode will also be used for the torsional mode. The shear stresses acting on the shell may be considered equal and opposite to the shear stresses acting on the outer boundary of the plate as required for equilibrium. This stress may be obtained by evaluating Equation (40) at the outer boundary of the plate.

$$T\Big|_{r = Rp} = - \frac{Mx}{2\pi Rp^2 tp} \tag{42}$$

The resultant torsional moment acting on the shell may be written as:

$$M_s = \pi \left[Rp^2 - (Rp - t_s)^2\right] Rp \left[T\Big|_{r = Rp}\right] \tag{43}$$

Substitution of Equation (42) into Equation (43) will give an expression for the resultant torsional moment acting on the end of the shell in terms of the externally applied moment M_x.

$$M_s = \frac{\left[Rp^2 - (Rp - t_s)^2\right] Rp\ Mx}{2\ Rp^2 tp}$$

This equation may be simplified to:

$$M_s = \frac{(2Rp^2 t_s - Rpt_s^2)Mx}{2Rp^2 tp} \tag{44}$$

The torsional end moment which would cause buckling of the shell is [14]:

$$M_{CR} = \frac{\pi\sqrt{2}\, E_s}{3(1-v^2)^{3/4}} \left[Rpt_s^5\right]^{1/2} \tag{45}$$

The value of the external moment M_x which would cause the shell to buckle may be derived by equating equations (44) and (45) and solving for M_x.

$$M_x = \frac{2\pi\sqrt{2}\, E_s\, Rp^{5/2}\, t_s^{5/2}\, tp}{3(1-v^2)^{3/4}\,(2Rp^2 t_s - Rpt_s^2)}$$

Assuming a value of v = 0.3, this equation simplifies to:

$$M_x = \frac{3.18\, E_s tp\, Rp^{5/2} ts^{5/2}}{2Rp^2 t_s - Rpt_s^2} \tag{46}$$

CHAPTER IV

EFFECT OF PIPE IMPOSED LOADS ON DISCONTINUITY STRESSES IN PRESSURE VESSEL

Section III of the ASME Boiler and Pressure Vessel Code states that the effect of pipe imposed loads shall be considered in the analysis of the discontinuity stresses in addition to the effects of temperature and pressure. The code provides the equations required to calculate the end deformation of the shell and flat head when subjected to thermal and pressure loads; however, no information is given for end displacements due to pipe imposed loads.

It will again be assumed that the shell is rigid with regard to deforming due to pipe imposed loads. The only information required to evaluate the effect of the pipe imposed loads on the discontinuity stresses is the end deformation of the plate when subjected to bending and axial loads.

The end rotation of the head when subject to bending can be calculated by evaluating the expression for slope of the deflected plate in Equation (1.a) at Rp.

$$\Theta_{PEB} = \frac{MRp}{8\pi D[(3+\nu)+(1-\nu)(\frac{Rn}{Rp})^4]} \left\{ -[(1+\nu)+(1-\nu)(\frac{Rn}{Rp})^4]\frac{3}{Rp} \right.$$

$$+(1+\nu)[1-(\frac{Rn}{Rp})^2]^2\frac{1}{Rp} + 2[(3+\nu)+(1-\nu)(\frac{Rn}{Rp})^4]\frac{1}{Rp}$$

$$\left. +(\frac{Rn}{Rp})^2[(1+\nu)(\frac{Rn}{Rp})^2 - (3+\nu)]\frac{1}{Rp} \right\} \cos\phi$$

Evaluating at $\emptyset = 0$ and $\emptyset = \pi$ and assuming a value of $v = 0.3$, the above equation becomes:

$$\Theta_{PEB} = \frac{0.43\ M}{Eptp^3[3.3+0.7(\frac{Rn}{Rp})^4]}\ [1.90(\frac{Rn}{Rp})^4 - 5.9(\frac{Rn}{Rp})^2 + 4.0] \qquad (47)$$

The value for the end radial displacement may be calculated as follows [3]:

$$\Delta PEB = -\frac{tp}{2}\Theta_{PEB} \qquad (48)$$

Substitution of Equation (47) into (48) yields:

$$\Delta PEB + \frac{0.215\ M}{Eptp^2[3.3+0.7(\frac{Rn}{Rp})^4]}\ [1.90(\frac{Rn}{Rp})^4 - 5.9(\frac{Rn}{Rp})^2 + 4.0]$$

The end rotation of the head when subject to axial loads may be calculated by taking the derivative of Equation (12.a) and evaluating it at Rp. The derivative of Equation (12.a) is:

$$\frac{dw}{dr} = -\frac{Fx\ r}{4\pi D}\left[\frac{1}{(\frac{1+v}{1-v})+(\frac{Rn}{Rp})^2}\right]\left[\frac{1}{(1-v)}\left(1-\frac{Rn^2}{Rp^2}\right)+\frac{(1+v)}{(1-v)}\ln(\frac{Rp}{r})\right.$$
$$\left.-(\frac{1+v}{1-v})(\frac{Rn}{r})^2\ln(\frac{Rp}{Rn})-(\frac{Rn}{Rp})^2\ln(\frac{r}{Rn})\right]$$

Evaluating at $r = Rp$:

$$\Theta_{PEA} = \frac{-F_x Rp}{4\pi D}\left[\frac{1}{(\frac{1+v}{1-v})+(\frac{Rn}{Rp})^2}\right]\left[\frac{1}{(1-v)}\left(1-\frac{Rn^2}{Rp^2}\right)\right.$$
$$\left.-(\frac{1+v}{1-v})(\frac{Rn}{Rp})^2\ln(\frac{Rp}{Rn})-\frac{Rn^2}{Rp^2}\ln(\frac{Rp}{Rn})\right]$$

Assuming a value of v = 0.3:

$$\Theta_{PEA} = - \frac{0.87\ F_x Rp}{Eptp^3} \left[\frac{1}{1.86 + (\frac{Rn}{Rp})^2}\right] [1.43\ (1 - \frac{Rn^2}{Rp^2})$$

$$- 2.86\ (\frac{Rn}{Rp})^2\ \ell n\ \frac{Rp}{Rn}] \qquad (49)$$

The value for the end radial displacement may be calculated as follows [3]:

$$\Delta PEA = - \frac{tp}{2}\ \Theta_{PEA} \qquad (50)$$

Substitution of Equation (49) into Equation (50) yields the following:

$$\Delta PEA = \frac{0.435\ F_x Rp}{Eptp^2} \left[\frac{1}{1.86 + (\frac{Rn}{Rp})^2}\right] [1.43\ (1 - \frac{Rn^2}{Rp^2})$$

$$- 2.86\ (\frac{Rn}{Rp})^2\ \ell n\ (\frac{Rp}{Rn})]$$

The above expressions for end radial displacement and end rotation of the head may be combined with those given in Section III of the ASME Boiler and Pressure Vessel Code for temperature and pressure. Section III also gives the equations required to calculate the end radial displacement and end rotation of the shell due to thermal and pressure loads. The total end radial displacements and rotations for the head and shell are equated and the discontinuity shear force and moment solved from the resulting set of simultaneous equations. The discontinuity stress may then be calculated and combined with stresses due to thermal, pressure, and imposed pipe loads as derived in Chapter III.

CHAPTER V

DISCUSSION

The stiffness coefficients derived in Chapter II and the stress equations derived in Chapter III can be applied to ensure structural compatibility between a cylindrical vessel with a flat concentric head and nozzle and an attached piping system.

The equations for the stiffness coefficients for the bending, axial, and torsional modes are expressed in Equations (11), (15), and (33), respectively. These equations may be used by the piping analyst in order to account for the flexibility of the pipe-tank interface; thereby, producing a more accurate mathematical model, and subsequently, a more accurate analysis.

The stress equations derived in Chapter III may be used by the tank designer for calculating the maximum allowable values for loads imposed by an attached piping system. These values then could be used as design requirements for the attached piping system's reactions acting at the interface.

This method of interface design is more realistic than the procedure generally used as described in Chapter I.

The effect of the pipe imposed loads on the vessel discontinuity stresses in considered in Chapter IV. The equations derived for the plate end rotation due to external loads are supplemental to the equations presented in the ASME code [3].

BIBLIOGRAPHY

BIBLIOGRAPHY

1. Bijilaard, P. P. "Stresses from Radial Loads and External Moments in Cylindrical Pressure Vessel," Welding Journal 33, Research Supplement, 1954, pp. 615-623.

2. Billimoria, H. D. and J. Hagstrom. "Stiffness Coefficients and Allowable Loads for Nozzles in Flat Bottom Storage Tanks," ASME Publication 77-PVP-19, 1977.

3. ASME Boiler and Pressure Vessel Code, Section III, Division I, Nuclear Power Plant Components, Subsection NA, Article A-6000, 1974.

4. Timéshenko, S. P. Strength of Materials. Volume 1, New York: D. Van Nostrand Company, Inc., 1958.

5. Timéshenko, S. P. and S. Woinowsky-Krieger. Theory of Plates and Shells. New York: McGraw-Hill Book Company, Inc., 1959.

6. Gilligan, J. A. and H. M. Priest. Design Manual for High Strength Steels, Pittsburgh: Davis and Warde Printing Company, Inc., 1957.

7. Timéshenko, S. P. Strength of Materials. Volume II, New York: D. Van Nostrand Company, Inc., 1958.

8. Head, J. C. Formulas for Circular Plates Subjected to Symmetrical Loads and Temperatures. United States Atomic Energy Commission, Division of Technical Information, 1966.

9. Timéshenko, S. P. and J. N. Goodier, Theory of Elasticity. New York: McGraw Hill Book Company, Inc., 1970.

10. Gaylord, P. Basic Equations of Engineering Science. New York: Schaum Publishing Company, 1952.

11. Kreyszig, E. Advanced Engineering Mathematics. New York: John Wiley and Sons Inc., 1972.

12. Flugge, W. Handbook of Engineering Mechanics. New York: McGraw Hill Book Company, Inc., 1962.

13. Roark, R. J. Formulas for Stress and Strain, 3rd edition, New York: McGraw Hill Book Company, Inc., 1954.

14. Timeshenko, S. P. and J. M. Gere, Theory of Elastic Stability, New York: McGraw Hill Book Company, Inc., 1961.

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