To the Graduate Council:

I am submitting herewith a thesis written by Cuong Van Nguyen entitled "The Use of Trigonometric Series to Formulate the Solution for Cylindrical Dipole Current." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

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THE USE OF TRIGONOMETRIC SERIES TO FORMULATE THE SOLUTION FOR CYLINDRICAL DIPOLE CURRENT

A Thesis

Presented for the

Master of Science

Degree

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To my parents for their utmost mental support, and to my
brother Dung whose advice and encouragement were
vital during my graduate study

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ABSTRACT

The goal of this work is to investigate the current distribution on cylindrical dipole antennas. The basis for the analytic work is the antenna integral equation involving the tangential electric field intensity and the antenna surface current density. The current is initially expanded into a trigonometric series, making radiation patterns very simple to obtain once the coefficients of this series have been determined.

Since there are several approaches to the problem via the trigonometric method, the solution obtained in this work is compared with others to verify its validity.

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CHAPTER I

INTRODUCTION AND GENERAL CONSIDERATIONS

A. INTRODUCTION

The practical design of antennas requires an accurate expression of the current distribution along the antennas. Since 1937 when L. V. King [8] had presented his work in solving the problem, several methods were invented to obtain theoretical results which show the current on the lateral surface of a dipole antenna. It is the purpose of this paper to solve the important problem stated above and also, at the same time, show that the trigonometric theory, among others, is the most dependable and appropriate in this case.

Before designing the antennas, one must first develop basic field concepts and obtain equations for calculating the fields in order to examine their principal implications. The famous Hallén integral equation is used in this investigation. The method of moments, as described in detail by R. Harrington [3], A. Barrero [4], and R. Montiel [5], is then applied to obtain a trigonometric series representation of the current. Lastly, the coefficients are solved with the aid of a high-speed digital computer.

The remaining part of the work is dedicated to the investigation of the correlations among the results obtained by A. Barrero, R. Montiel, R. Mack [1] and by the author of this work. Thorough considerations are given to the formulation of the solutions from A. Barrero and R. Montiel because of the similarity of their work and this work.

B. METHODS OF SOLUTION

There are three methods for solving the antenna integral equation: iterative solutions, numerical integration, and series representation of the current distribution.

There are many disadvantages in the iterative procedure. Firstly, high-order iterations are not feasible; secondly, only in thin antennas can approximations made in the kernel be relevant; and finally, for long dipoles, the theory is not very appropriate [9, p. 431].

The dependency of numerical procedures in the method used for numerical integrations makes this solution seem somewhat limited by matrix inversion capabilities. In addition to this drawback, the form of the solution is not proper for field calculations as the set of point-values usually does not give much insight into the problem.

Consequently, series representation of the current distribution is the most attractive method. By applying the method of moments, this technique can be carried out by analytic work, and eventually the current can be found by computer programming.

Duncan and Hinchey [7] were pioneers in this line of investigation. In a revised version in 1960, they presented the Fourier series representation of the kernel. This was done to avoid the complications encountered by Zuhrt. The formulation, however, seemed to have met other difficulties, and the indication is that there may have been an undetected error in their calculations.

In 1972 and 1979, A. Barrero and R. Montiel, respectively, proceeded to solve the antenna current problem using the series representation method and the kernel of Duncan and Hinchey. In A. Barrero's approach, the integral equation is in terms of the electric field, while it is in terms of vector potential in R. Montiel's work. Both obtained good results as compared with the measured data of Mack and with each other.

CHAPTER II

SUMMARY OF OTHER TRIGONOMETRIC APPROACHES

A. BARRERO'S APPROACH

In this approach, the current distribution is expressed in the series

$$I(z) = \sum_{n=0}^{N} In \cos(n\pi z/h) \ , \tag{II.1}$$

where the coefficients are determined by the method of moments. Notice that the current does not vanish identically at the ends of the dipole ($z = \pm h$). It should also be pointed out that this is the same form of current as used in the original work by Duncan and Hinchey. Figure 1 shows the dipole in a right-handed Cartesian system of coordinates.

The kernel of the integral equation for the electric field is given as below

$$K_E(z-z') = (d^2/dz^2 - K^2)k(z-z') \ , \tag{II.2}$$

$$K(z-z') = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp(-jkR)/R \ d\phi \ , \tag{II.3}$$

where

$$R = [4a^2 \sin^2\phi/2 - (z-z')^2]^{1/2} \ . \tag{II.4}$$

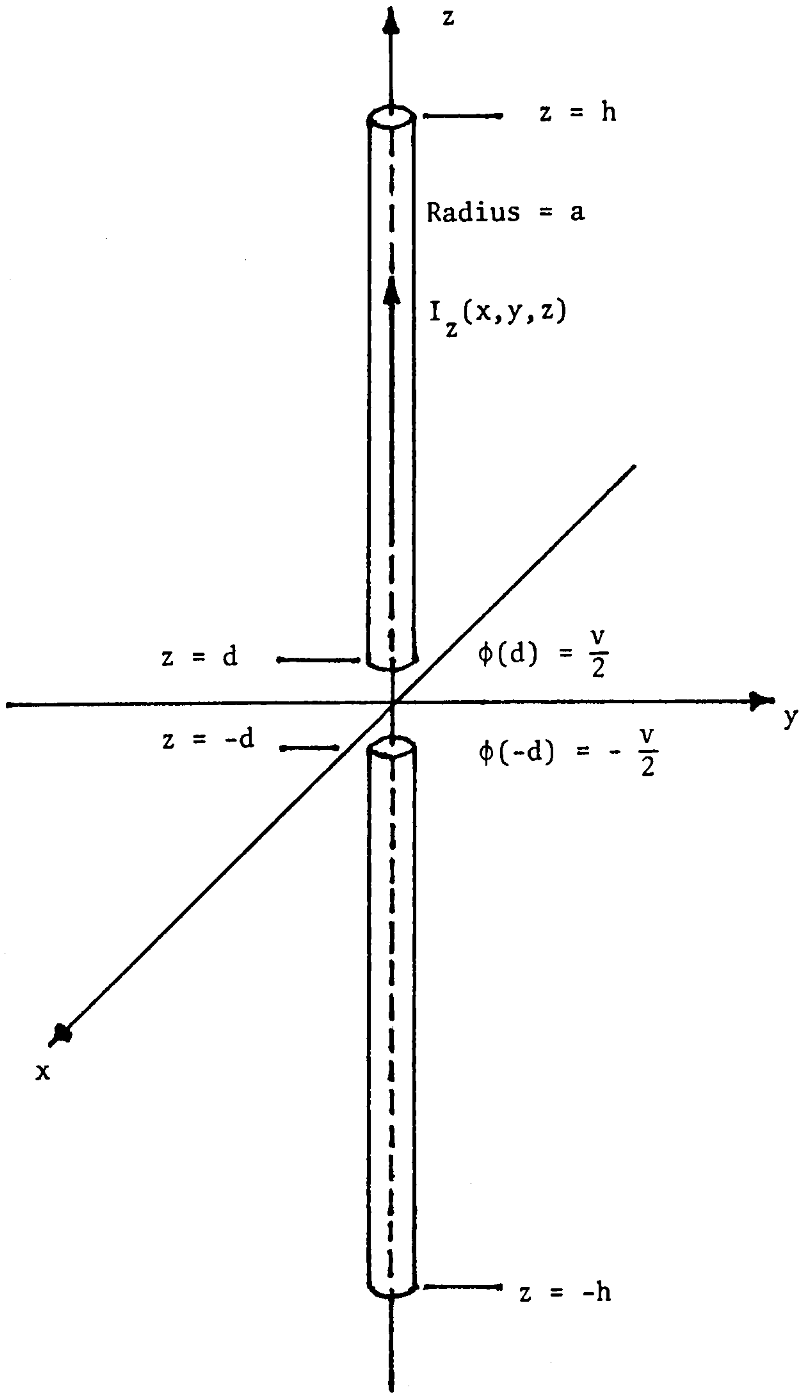


Figure 1. Center-Fed Isolated Dipole Antenna.

Duncan and Hinchey showed that the above kernel can be expanded into a Fourier series

$$K_E(z-z') = \sum_{m=0}^{\infty} k_m \cos(m\pi z/2h)\cos(m\pi z'/2h)$$

$$+ \sin(m\pi z/2h)\sin(m\pi z'/2h) \ . \tag{II.5}$$

By setting V = 1, the integral equation for the z-component of the electric field along the antenna surface may be normalized. Thus A. Barrero obtained

$$\int_{-h}^{h} I(z')K_E(z-z')dz' = \delta(z), \quad |z| \le h \ . \tag{II.6}$$

Since $I(z')$ must be an even function, the kernel $K_E(z-z')$ may be reduced to

$$K_M(z,z') = \sum_{m=0}^{\infty} k_m \cos(m\pi z/2h)\cos(m\pi z'/2h) \ . \tag{II.7}$$

We are then led to consider

$$\int_{-h}^{h} I(z')K_M(z,z')dz' = \delta(z) \ . \tag{II.8}$$

Following the method of moments, we multiply both sides by $\cos(2p+1)\pi z/2h$ and integrate on z from -h to h; the result is

$$\sum_{n=0}^{N} T_{pn} I_n = 1, \quad p = 0,1, \ldots N . \tag{II.9}$$

Substituting in Eq. II.7 yields

$$T_{pn} = \sum_{m=0}^{2N} k_m A_{nm} B_{pm} , \tag{II.10}$$

where

$$A_{nm} = 2 \int_0^h \cos(m\pi z'/2h)\cos(n\pi z'/h)dz' , \tag{II.11}$$

and

$$B_{pm} = 2 \int_0^h \cos(m\pi z/2h)\cos[(2p+1)\pi z/2h]dz . \tag{II.12}$$

Due to orthogonality, there results:

$$T_{pn} = h(k_{2n}B_{p,2n} + k_{2p+1}A_{n,2p+1}) . \tag{II.13}$$

The final result is

$$T_{pn} = hB_{p,2n}(k_{2n}+k_{2p+1}) , \tag{II.14}$$

where

$$B_{p,2n} = h(-1)^{n+p}[1/(2n-2p+1) + 1/(2n+2p+1)]2/\pi . \tag{II.15}$$

The set of equations generated by Eq. II.9 may be cast in matrix form as

$$TI = B \ , \tag{II.16}$$

where I is the column vector containing the coefficients of $I(z')$, and B is a column vector with the first element unity. The formal solution is simply

$$I = T^{-1} B \ . \tag{II.17}$$

Notice that in Barrero's method, the end current $I_z(\pm h)$ assumes whatever value is required to satisfy the boundary value problem in Nth order. The "unknown constant" in Hallén's integral equation is usually evaluated by forcing the dipole current to vanish at $z = \pm h$. There is no "unknown constant" in A. Barrero's integral equation!

B. MONTIEL'S APPROACH

In the formulation of the solution for the antenna current distribution, R. Montiel utilized the equation

$$\int_{-h}^{h} f(z')K(z-z')dz' = C_1 \cos kz + \frac{V}{2} \sin k|z| \ , \tag{II.18}$$

where

$$K(z-z') = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp(-jK|\bar{r}-\bar{r}'|)/|\bar{r}-\bar{r}'| d\phi' \ , \tag{II.19}$$

and

$$f(z') = \sum_{0}^{\infty} F_n \cos(n\pi z'/h) \tag{II.20}$$

is the trigonometric form for the current. This formulation is identical to that of Duncan and Hinchey, and the series for the current is also identical to that of A. Barrero. The kernel can be obtained in a similar fashion as in A. Barrero's work

$$\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_n D_m A_{nm} B_{pm} = C_1 R_p + V_p \ , \tag{II.21}$$

where

$$B_{pm} = 2 \int_0^h \cos[(2p+1)\pi z/2h] \cos[m\pi z/2h] dz \ , \tag{II.22}$$

$$A_{nm} = 2 \int_0^h \cos(n\pi z'/h) \cos(m\pi z'/2h) dz' \ , \tag{II.23}$$

$$R_p = 2 \int_0^h \cos kz \cos[(2p+1)\pi z/2h] dz \ , \tag{II.24}$$

and

$$V_p = 2 \int_0^h \sin kz \cos[(2p+1)\pi z/2h] dz \ . \tag{II.25}$$

By using the table of integrals and by the condition of orthogonality, vectors R_p and V_p become:

$$R_p = \frac{\sin\alpha}{\alpha} + \frac{\sin\beta}{\beta} \qquad \text{(II.26)}$$

$$V_p = -\frac{\cos\beta}{\beta} - \frac{\cos\alpha}{\alpha} + C\ , \qquad \text{(II.27)}$$

where

$$\alpha = (2p+1) \qquad \text{(II.28)}$$

$$\beta = k + (2p+1)\ \pi/2\ , \qquad \text{(II.29)}$$

and

$$C = \frac{2k}{k^2 - [(2p+1)\ \pi/2]^2}\ . \qquad \text{(II.30)}$$

The impedance matrix is

$$T_{pn} = h\ B_{p,2n}(D_{2n}+D_{2p+1})\ , \qquad \text{(II.31)}$$

where

$$B_{p,2n} = h(-1)^{n+p}[1/(2p+1-2n) + 1/(2p+1+2n)]\ 2/\pi\ . \qquad \text{(II.32)}$$

The formal solution of the current is

$$F = C_1 T^{-1} R + T^{-1} V\ , \qquad \text{(II.33)}$$

where C_1 is the "unknown constant" mentioned earlier in this chapter.

C. JUSTIFICATION FOR THE FORMULATIONS

A. Barrero took the approach of solving the integral equation of the electric field along the antenna. Since this approach avoids the constant C_1 in Eq. II.18, it gives a satisfactory result with less difficulty in formulating the solution. In addition to that, there is no need to solve for other column vectors R, V, as in R. Montiel's work, once the impedance matrix is inverted. Thus, the process of solving the antenna current is shortened, and the analytic work is simpler. A notion about this approach is that only the very first column of the admittance matrix is used to calculate for the current distribution. This is due to the fact that the end point of the dipole is treated as the center of an interval, and, therefore, the starting point is at one-half subsection in from the tips, namely, the center of the dipole. Since the antenna investigated in this work is center-fed, the first element of the column vector B in Eq. II.16 is of the value of the generating function. The inversion of the whole impedance matrix thus seems to be wasteful. Furthermore, a convergence factor has to be included in order to be able to invert the impedance matrix.

The author of this thesis recognizes that even though it is more lengthy to use the vector potential in the integral equation to obtain the solution as R. Montiel did, the results are very satisfactory. Granted that extra work is needed to solve the problem, but on the other hand, the approach is easier to understand and is somewhat self-explanatory. A significant advantage of this technique is that it allows the user to obtain the solution for either the case of a nonzero end current or that of an identically zero current at the tips of the antenna. The author of this work utilized this and went about solving the problem with R. Montiel's approach, even though the analytic work resembles more of A. Barrero's formulation.

D. EVALUATION OF THE CONSTANT C_1

In most cases, when the solutions of the antenna integral equation are called for, a zero end current is assumed in order to allow evaluation of the constant C_1. Infield [2] stated that the current does not necessarily vanish at the tips of a dipole antenna. It is mostly a matter of convenience to assume that $I_z(\pm h)$ is identically zero. Previous work done by R. B. Mack, A. Barrero, and R. Montiel also indicated that the nonzero end current provided a better solution for the problem. In this paper the author has verified that an identically zero end current is not necessary, even though it may be assumed to solve for the constant C_1.

CHAPTER III

THE FORMULATION OF THE SOLUTION

A. INITIAL CONSIDERATION

Although the general formulation here is similar to that of A. Barrero's, i.e., the electric field approach, the initial assumption is different, and as a result, leads to a similar—though not exactly the same—result. Actually the impedance matrix of this work turns out to be the transpose of the one in A. Barrero's solution. As a result, further investigations are made, and the twenty-fifth order solution in this work is as satisfactory when compared with Mack's experimental results as Barrero's. However, in order to verify that the zero end current may or may not be needed in the formulation of the solution, the computer program written takes the form of R. Montiel's approach, i.e., vector potential. As a result, there is a need to understand all of the formulations above, and thus, a summary of the above work is undeniably a necessity.

B. STATEMENT OF THE PROBLEM

Initiated by Dr. H. P. Neff's idea, the author merely proceeded to formulate the solution for the dipole antenna current. Since the integral equation in terms of the electric field has already been investigated by A. Barrero, the first part of this formulation is very much the same as his. The remaining part of the investigation, from the point where the current is expressed as a trigonometric series and the method of moments is applied to the integral equaticn to the final solution, is the work of the author.

Consider a center-driven, cylindrical dipole symmetrically on the z-axis (as shown in Figure 1, p. 5). The conductor is of infinite conductivity. Assume that there are no ϕ variations and that the surface current depends on z alone. Thus, ka must be much less than unity so that there is no radial current, and we can obtain Hallén's integral equation

$$j \frac{V_0}{2C} \sin k \left|z\right| + C_1 \cos kz = \frac{\mu}{4\pi} \int_{-h}^{h} I_z(z') K(z-z') dz' \ ,$$

$$-h \le z \le h \qquad \text{(III.1)}$$

where

$$c = (\mu\varepsilon)^{-1/2} \ , \qquad \text{(III.2)}$$

$$k = \omega(\mu\varepsilon)^{1/2} \ , \qquad \text{(III.3)}$$

V_0 = the voltage across the gap such that the electric field at the gap is $-V_0\delta(z)$,

$I_z(z')$ = the unknown dipole current distribution,

$$K(z-z') = \frac{1}{2\pi} \int_0^2 e^{-jkR}/R \ d\phi \qquad \text{(III.4)}$$

and

$$R = [4a^2 \sin^2\phi'/2 + (z-z')^2]^{1/2} \; . \tag{III.5}$$

C_1 is a constant, usually evaluated by assuming that $I_z(\pm h)$ is zero. In this work the constant C_1 is avoided entirely.

The z-component of the electric field is related to the vector potential by

$$E_z(\rho,z) = \frac{1}{j\omega\mu\varepsilon}(k^2 + \partial^2/\partial z^2)A_z(\rho,z) \tag{III.6}$$

for the dipole described in the preceding paragraph. In particular, for $\rho = a$, Eq. III.1 and Eq. III.6 give

$$-j4\pi\omega\varepsilon V_0\delta(z) = (k^2 + \partial^2/\partial z^2) \int_{-h}^{h} I_z(z')K(z-z')dz' \; ,$$

$$-h \leq z \leq h \; . \tag{III.7}$$

The current $I_z(z')$ remains the unknown to be found, but the constant C_1 has disappeared in Eq. III.7.

Solving Maxwell's equations simultaneously and applying the continuity of current equation results in

$$\nabla^2 E_z + k^2 E_z = j\omega\mu[J_z + \frac{1}{k^2}\partial^2 J_z/\partial z^2] \tag{III.8}$$

which yields the solution

$$E_z = -j\omega\mu/4\pi \iiint_{\text{all space}} [I_z(z') + \frac{1}{k^2}\partial^2/\partial z^2 J_z(z')] \times \frac{e^{-jkR}}{R} dz' \; . \tag{III.9}$$

which results (for $\partial I_z/\partial z' \Big|_{z'=0} = 0$)

$$j4\pi\omega\varepsilon E_z = - I_z(h)\,[K'(z+h) - K'(z-h)]$$
$$- I_z'(h)\,[K(z+h) + K(z-h)]$$
$$+ \int_{-h}^{h} (k^2 + \partial^2/\partial z'^2) I_z(z') \times K(z-z')dz' \, . \qquad \text{(III.10)}$$

On the other hand, if we write:

$$j4\pi\omega\varepsilon E_z = \int_{-h}^{h} I_z(z')(k^2 + \partial^2/\partial z'^2)K(z-z')dz' \qquad \text{(III.11)}$$

and integrate by parts twice, we obtain the previous result identically. Moreover, since

$$\frac{\partial^2}{\partial z'^2} K(z-z') = \frac{\partial^2}{\partial z^2} K(z-z') \, , \qquad \text{(III.12)}$$

the final result can be written

$$j4\pi\mu\varepsilon E_z = \int_{-h}^{h} I_z(z')(k^2 + \partial^2/\partial z^2)K(z-z')dz' \, , \qquad \text{(III.13)}$$

and on the dipole surface

$$-j4\pi\omega\varepsilon \; V_0\delta(z) = \int_{-h}^{h} I_z(z')(k^2 + \partial^2/\partial z^2)K(z-z')dz \qquad \text{(III.14)}$$

valid for $\rho = a$ and $-h \le z \le h$.

This integral was derived without the use of vector potential and contains no unknown constants.

The kernel of the integral equation of the electric field is given as

$$K_E(z-z') = (\partial^2/\partial z^2 - k^2)K(z-z') \,, \tag{III.15}$$

with $K(z-z')$ as in Eq. III.4.

As in the work of Duncan and Hinchey, $K(z-z')$ is a function of the variable $x = (z-z')$. Since z and z' are in the range $-h \leq z, z' \leq h$, x is in the range $-2h \leq x \leq 2h$. Furthermore, since $K(x)$ contains only x^2, a cosine series is sufficient. Thus, the kernel is given as

$$K(x) = \frac{D_0}{2} + \sum_{m=1}^{\infty} D_m \cos(m\pi x/2h) \,, \tag{III.16}$$

where

$$\begin{aligned} D_m &= \frac{1}{2h} \int_{-2h} K(x)\cos(m\pi x/2h)\,dx \\ &= \frac{1}{2h} \int_{-\infty}^{\infty} K(x)\cos(m\pi x/2h)\,dx \\ &\quad - \frac{1}{h} \int_{2h}^{\infty} K(x)\cos(m\pi x/2h)\,dx \,. \end{aligned} \tag{III.17}$$

The first integral in Eq. III.17 can be found immediately as it merely represents known samples of Fourier Transform of $K(x)$ [7]. The second integral is evaluated by expanding K(x) in a series of negative power of x, performing the ϕ' integration, and finally performing the x integration. Thus, the D_m can be found with as much accuracy as desired. In terms of z and z', Eq. III.16 is

$$K(z-z') = \frac{D_0}{2} + \sum_{m=1}^{\infty} \cos(m\pi z/2h)\cos(m\pi z'/2h)$$

$$+ \sum_{m=1}^{\infty} \sin(m\pi z/2h)\sin(m\pi z'/2h) \ . \qquad \text{(III.18)}$$

In the present work, the second sum in Eq. III.18 will disappear since $I_z(z')$ is an even function for an isolated and center-driven dipole. This merely simplifies the problem, and the formulation being represented is not limited to this special case. Equation III.7 can now be written

$$-j4\pi\omega\varepsilon\ V_0\delta(z) = (k^2 + \partial^2/\partial z^2) \int_{-h}^{h} I_z(z') \sum_{m=0}^{\infty} \frac{\varepsilon_m}{2} D_m$$

$$\times \cos(m\pi z/2h) \cdot \cos(m\pi z'/2h)dz',$$

$$-\ h \leq z \leq h \qquad \text{(III.19)}$$

or

$$-j4\pi\omega\varepsilon\ V_0\delta(z) = (k^2 + \partial^2/\partial z^2) \sum_{m=0}^{\infty} \frac{\varepsilon_m}{2} D_m \cos(m\pi z/2h)$$

$$\times \int_{-h}^{h} I_z(z')\cos(m\pi z'/2h)dz' \ , \qquad \text{(III.20)}$$

where

$$\varepsilon_m = \begin{cases} 1, & m = 0 \\ 2, & m = 1,2,3, \ldots \end{cases} \tag{III.21}$$

then

$$-j4\pi\omega\varepsilon\, V_0 \delta(z) = \sum_{m=0}^{\infty} \frac{\varepsilon_m}{2} D_m \, [k^2 - (\frac{m\pi}{2h})^2] \cos(m\pi z/2h)$$

$$\times \int_{-h}^{h} I_z(z') \cos(m\pi z'/2h)\, dz' \; . \tag{III.22}$$

To apply the method of moments, let

$$I_z(z') = \sum_{n=0}^{N} I_n \cos(n\pi z'/h) \;, \quad n=0,1,2, \ldots \tag{III.23}$$

which is not a truncated Fourier series since the coefficients I_n will be determined by the method of moments rather than by the Euler formulas. Defining the following quantities:

$$V_0 = (-j4\pi\omega\varepsilon)^{-1} \text{ volts} \;, \tag{III.24}$$

and

$$K_m(z,z') = \sum_{m=0}^{\infty} \frac{\varepsilon_m}{2} D_m [k^2 - (\frac{m\pi}{2h})^2]$$

$$\times \cos(m\pi z/2h) \cos(m\pi z'/2h) \;, \tag{III.25}$$

Eqs. III.22 and III.23 give

$$\delta(z) = \sum_{n=0}^{N} I_n \int_{-h}^{h} K_m(z,z')\cos(n\pi z'/h)dz' \ . \tag{III.26}$$

Now the current can be expressed as:

$$I_z(z') = \sum_{n=0}^{N} I_n \cos(2n+1)\pi z'/2h \ , \quad n = 0,1,2, \ldots \tag{III.27}$$

which can never be a Fourier series (even in infinite order) because the set in Eq. III.27 is not complete. Notice that $I_z(\pm h) = 0$. By defining V_0 and $K_m(z,z')$ exactly as in the preceding paragraph, an equation results:

$$\delta(z) = \sum_{n=0}^{N} I_n \int_{-h}^{h} K_m(z,z')\cos(2n+1)\pi z'/2h \ dz' \ . \tag{III.28}$$

Multiply both sides of Eq. III.28 by cos $p\pi z/h$, $p = 0,1,2, \ldots N$, and integrate on z from -h to h. Then a set of N+1 simultaneous equations can be obtained.

$$1 = \sum_{n=0}^{N} I_n \int_{-h}^{h} \left\{\int_{-h}^{h} K_m(z,z')\cos p\pi z/h \ dz\right\} \times \cos(2n+1)\pi z'/2h \ dz' \ . \tag{III.29}$$

or

$$1 = \sum_{n=0}^{N} T_{pn}' I_n \ , \tag{III.30}$$

where

$$T_{pn}' = \int_{-h}^{h} \left\{ \int_{-h}^{h} K_m(z,z')\cos p\pi z'/h \, dz \right\} \times \cos(2n+1)\pi z'/2h \, dz' \quad \text{(III.31)}$$

or

$$T_{pn}' = \sum_{m=0}^{\infty} \varepsilon_m/2 \, D_m \, [k^2 - (m\pi/2h)^2] \times A_{n,m} B_{p,m} \, , \quad \text{(III.32)}$$

where

$$A_{n,m} = 2 \int_0^h \cos(2n+1) \, z \, /2h \, \cos(m\pi z'/2h) dz \quad , \quad \text{(III.33)}$$

and

$$B_{n,p} = 2 \int_0^h \cos(p\pi z/h)\cos(m\pi z/2h) dz \, . \quad \text{(III.34)}$$

Due to the orthogonality between the trigonometric functions,

$$A_{n,m} = \begin{cases} 0, \ m \text{ odd}, \ m \neq 2n+1 \\ h, \ m = 2n+1 \\ 4h/\pi[(-1)^n(2n+1)\cos(m\pi/2)]/ \\ \quad [(2n+1)^2 - (m)^2], \text{ otherwise} \end{cases} \quad \text{(III.35)}$$

and

$$B_{p,m} = \begin{cases} 0, \text{ m even, } m \neq 2p \\ h, \ m = 2p \neq 0 \\ 2h, \ m = 2p = 0 \\ 4h/\pi[(-1)^{p}m \sin(m\pi/2)]/ \\ \quad [(m)^2-(2p)^2], \text{ otherwise .} \end{cases} \tag{III.36}$$

After separating the sum in Eq. III.32 into two parts; one for m even and one for m odd, there results,

$$T'_{pn} = h[D_{2p}[k^2 - (p\pi/h)^2]A_{n,2p} + D_{2n+1}[k^2 - ((2n+1)\pi/2h)^2]B_{p,2n+1}] \tag{III.37}$$

But

$$A_{n,2p} = B_{p,2n+1} = 4h/\pi(-1)^{n+p}2n+1/[(2n+1)^2 - (2p)^2] . \tag{III.38}$$

So,

$$T'_{pn} = hB_{p,2n+1}[D_{2p}[k^2 - (p\pi/h)^2] + D_{2n+1}[k^2 - ((2n+1)\pi/2h)^2]] . \tag{III.39}$$

By comparing Eq. III.39 with Eq. II.14, it is obvious that the impedance matrix T'_{pn} derived in this work is the transpose of the matrix T_{pn} from A. Barrero's formulation.

The set of equations, III.30, may be written in the matrix form

$$B = T\ I , \tag{III.40}$$

where T is the matrix T'_{pn}, I is a column vector containing I_n, and B is a column vector with the first element unity. The final solution is simply

$$I = T^{-1} B . \tag{III.41}$$

This method of solution has been used to determine approximately the current distribution for isolated cylindrical dipole antennas. The calculation of the matrix elements, and the solutions of the simultaneous equations were accomplished by the IBM-360 computer.

The results obtained in this paper for the half-wave and full-wave dipoles are in good agreement with the measured values of Mack [1], especially at the tips of the antennas where the current vanishes. The results are presented and compared with those obtained by A. Barrero and R. Montiel in Chapter IV.

C. THE ANTENNA RADIATION FIELD

The very task of an antenna is to radiate electromagnetic energy efficiently. As a result, this calls for the calculation of radiation patterns. It is known from elementary theory that the radiation field for a thin cylindrical dipole is

$$E_\theta = j \frac{\omega}{4\pi} \frac{e^{-jkr}}{r} \sin\theta \int_{-h}^{h} I_z(z') e^{jkz'\cos\theta} \, dz' , \tag{III.42}$$

$$H_\phi = \frac{E_\theta}{\eta} . \tag{III.43}$$

Retaining only that part which depends on θ, the "radiation pattern" is given by

$$F(\theta) = \sin\theta \int_{-h}^{h} I_z(z') e^{jkz'\cos\theta} \, dz' . \tag{III.44}$$

Substituting Eq. III.23 gives

$$F(\theta) = \sin\theta \sum_{n=0}^{N} I_n \int_{-h}^{h} \cos(n\pi z'/h) \times e^{jkz'\cos\theta} \, dz' . \tag{III.45}$$

Performing the indicated integration

$$F(\theta) = kh^2 \sin 2\theta \, \sin(kh\cos\theta) \times \sum_{n=0}^{N} I_n \frac{(-1)^n}{(kh\cos\theta)^2 - (n\pi)^2} \tag{III.46}$$

which is easy to evaluate once the I_n's have been found.

CHAPTER IV

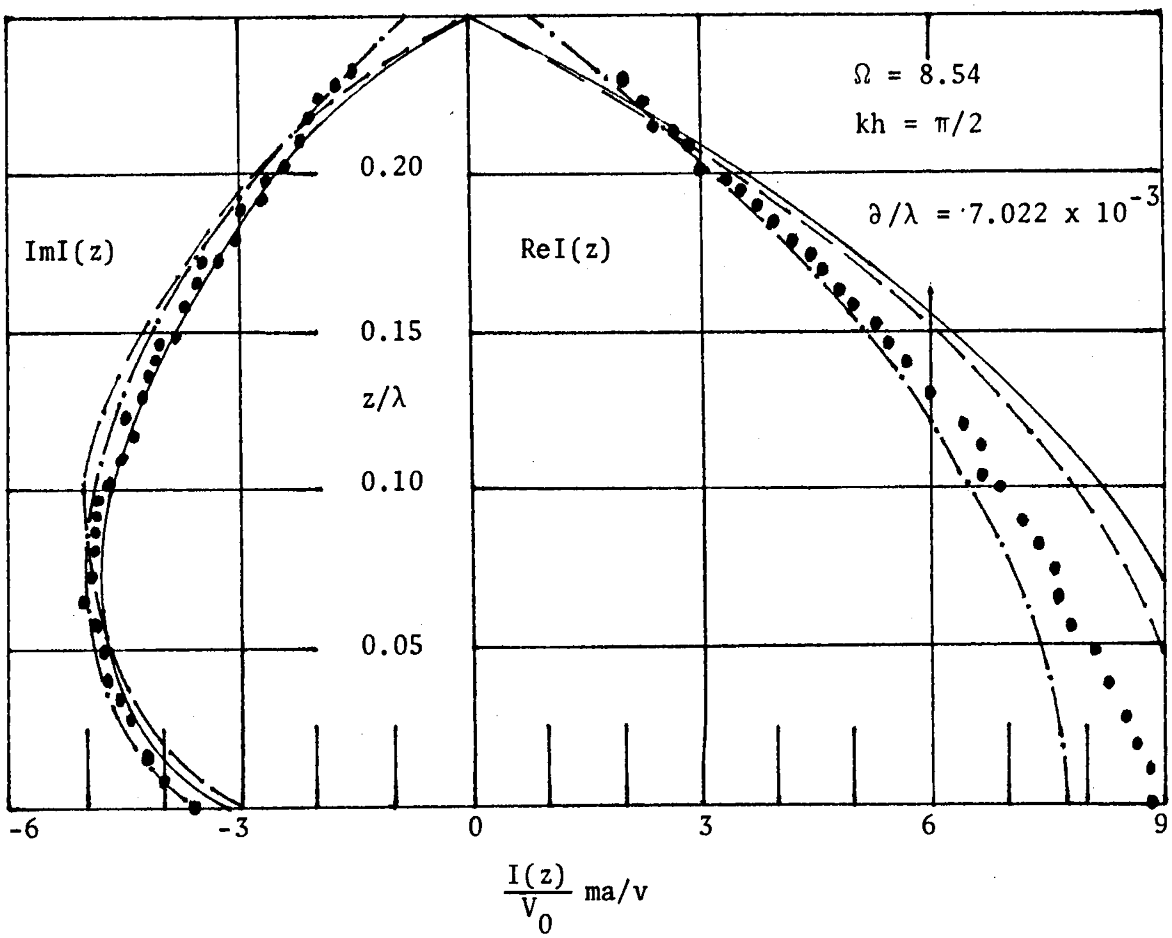
RESULTS

A. RESULTS FOR HALF- AND FULL-WAVE ANTENNAS

As indicated previously, the method used to solve for the antenna current distribution in this thesis is very satisfactory. As this work is a combination of A. Barrero's and R. Montiel's work, its results resemble both of the solutions.

The results of applying my solution to antennas with lengths of half wave and full wave were obtained. The reason for this is that the half-wave dipole represents a very practical case, while the full-wave dipole represents a good test of any theory because of the rapid change of current near the gap.

The twenty-fifth order solutions are in good agreement with Mack's measured results. Figures 2 to 5 show the current distribution on the half-wave dipole. The first two are for identically zero end current, i.e., FH = 0, while the latter ones are for a small end current, i.e., FH $\neq$ 0. The values of the end current were obtained by A. Barrero in his formulation. As one observes, the convergence of the different orders of solutions towards the experimental results can be verified by Tables I and II. Similarly, for the full-wave dipole, the solutions correspond very well with those of A. Barrero, R. Montiel, and Mack. Figures 6 to 9 illustrate this fact. Tables III and IV show the convergence of the solutions and show conclusively that the formulation should be considered satisfactory.



. . . Mack (Measured)

— • — A. Barrero

——— R. Montiel

— — — Nguyen

Figure 2. Current in Half-Wave Dipole (20th Order, FH = 0).

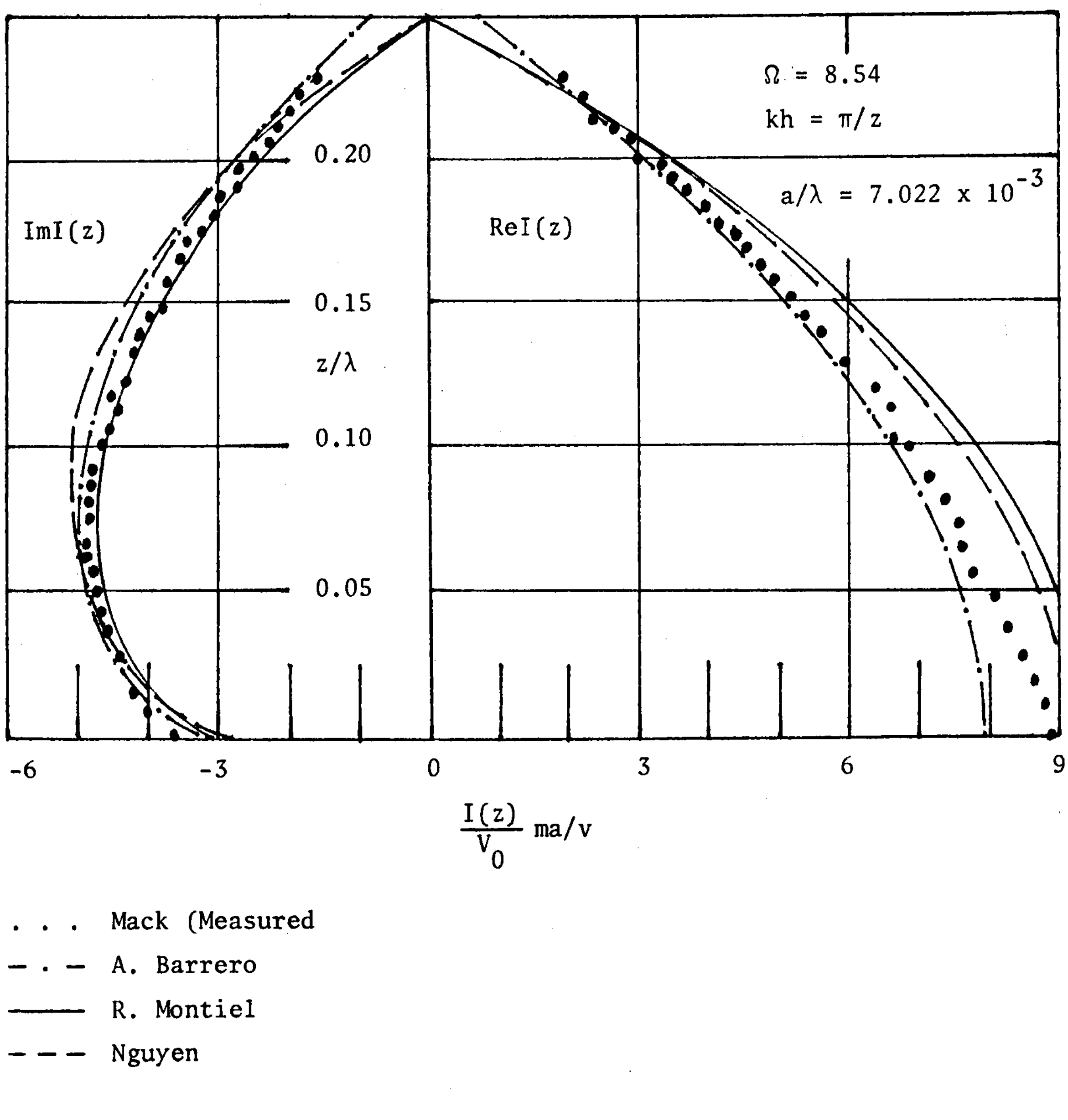


Figure 3. Current in Half-Wave Dipole (25th Order, FH = 0).

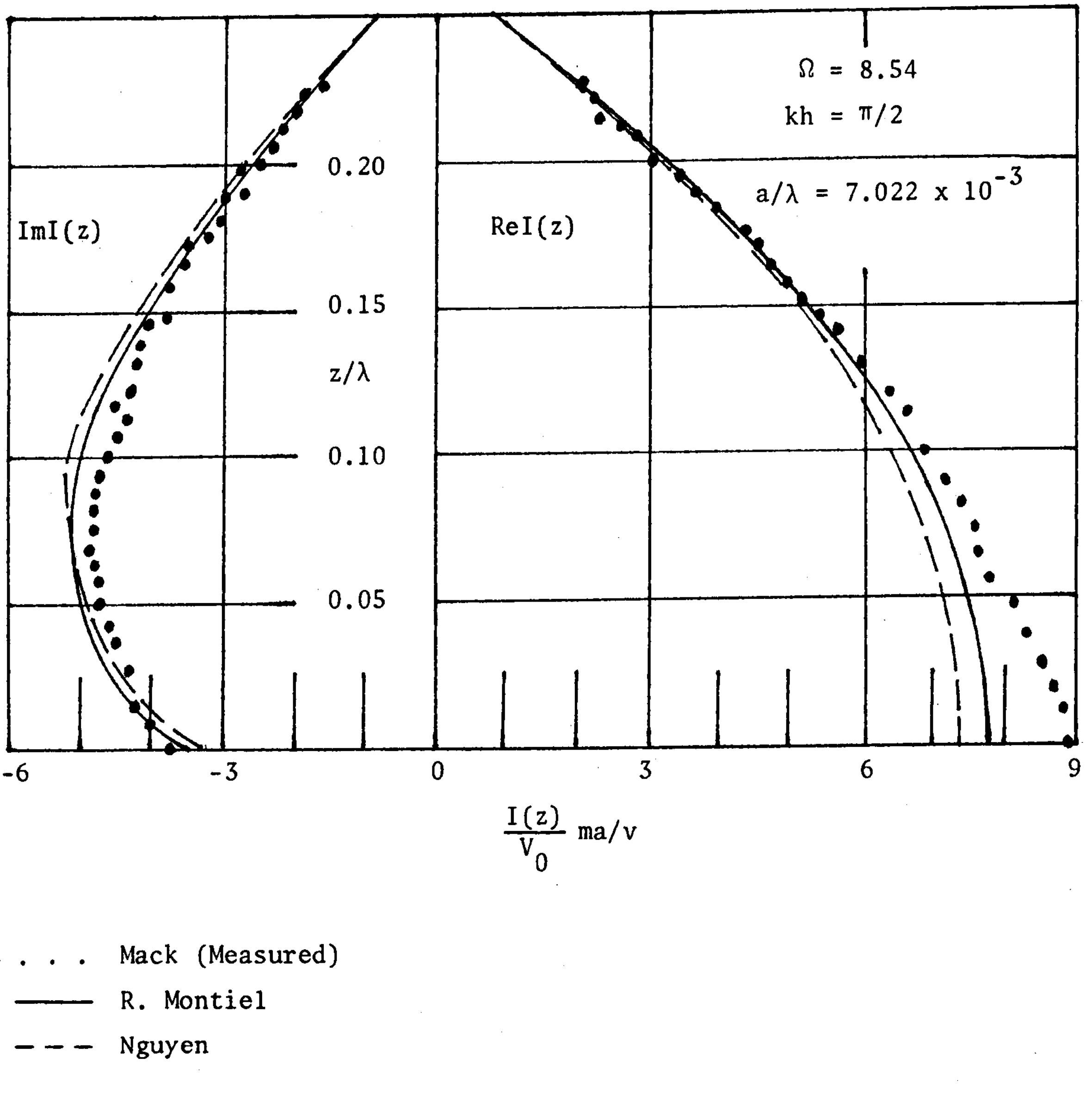


Figure 4. Current in Half-Wave Dipole (20th Order, FH $\neq$ 0).

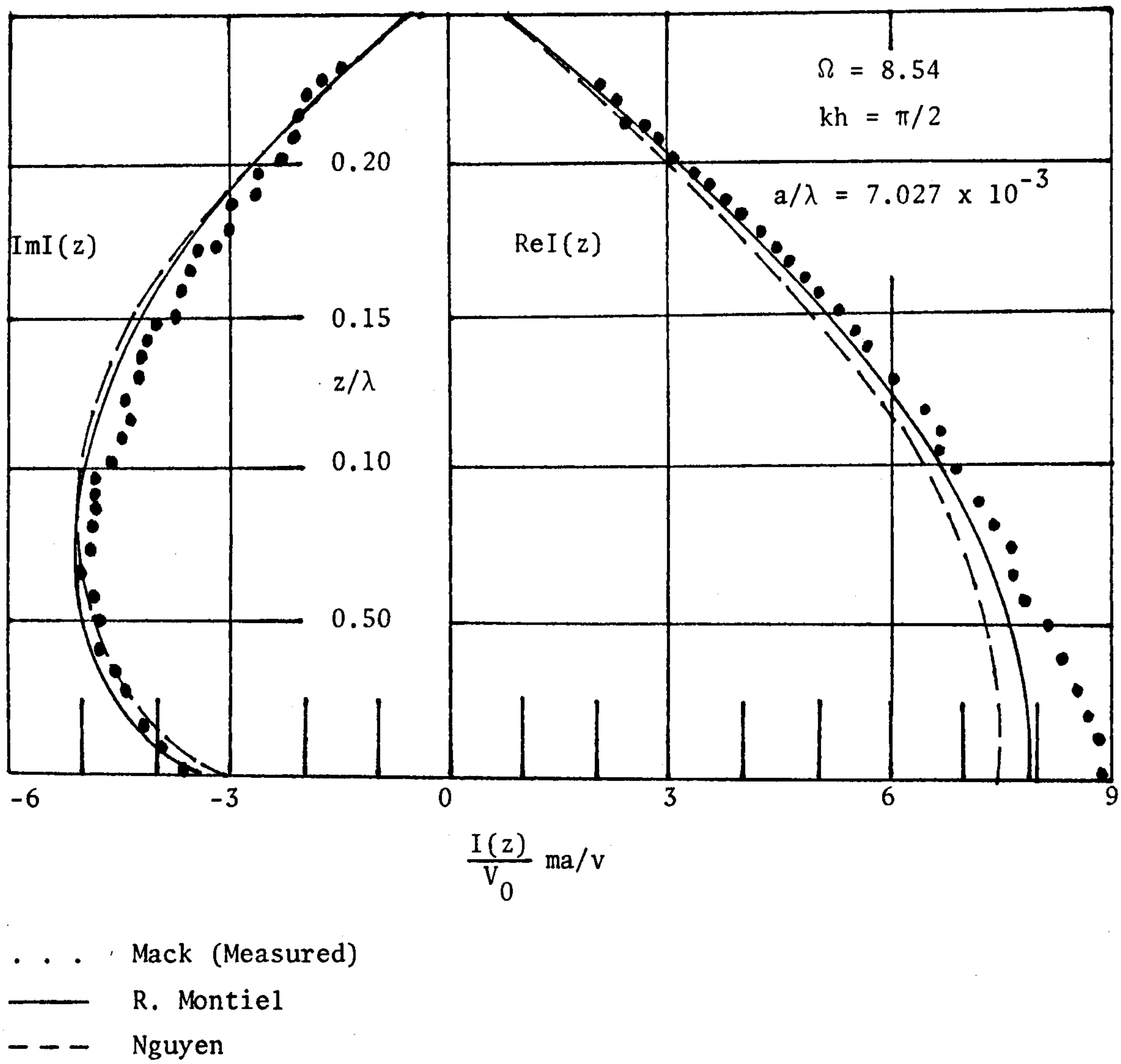


Figure 5. Current in Half-Wave Dipole (25th Order, FH ≠ 0).

TABLE I

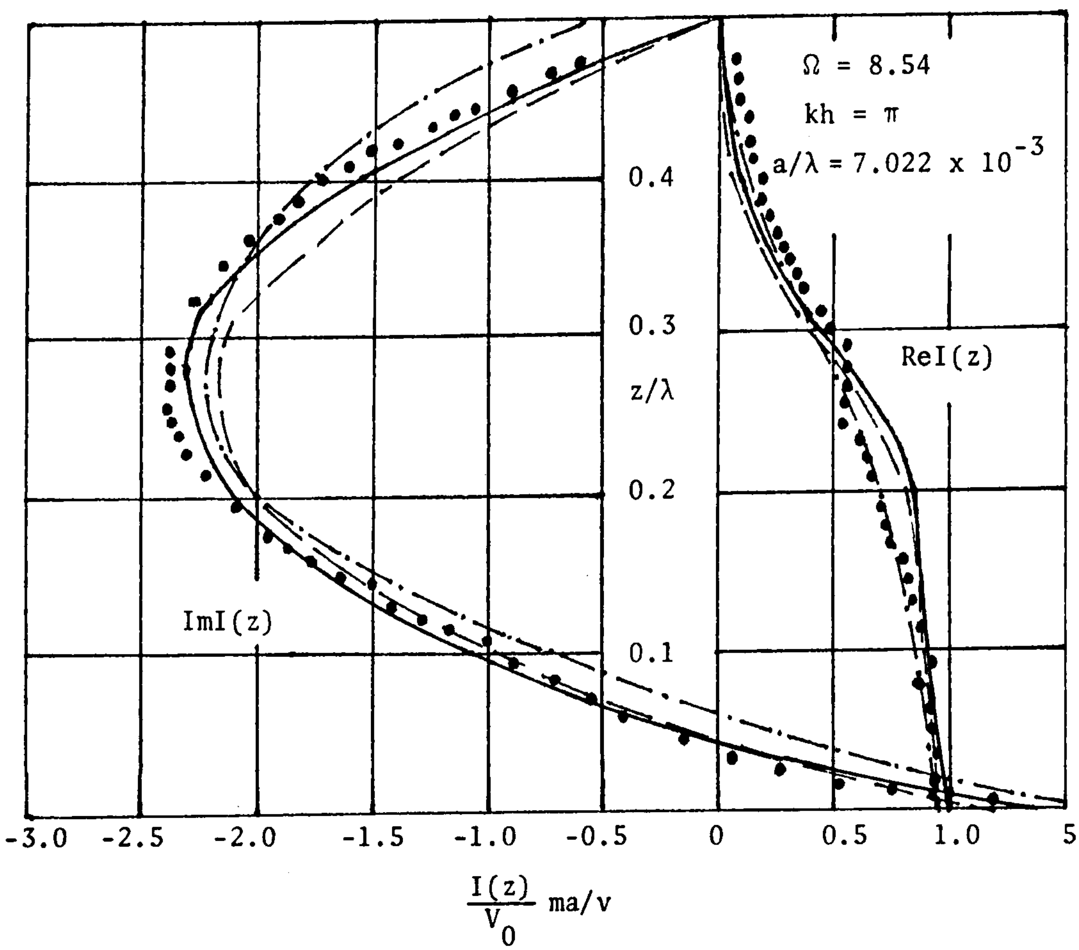
INPUT ADMITTANCE FOR A HALF-WAVE DIPOLE AT 20th ORDER
(ma/v)

Nguyen	FH = 0	9.408 - j 3.010
	FH $\neq$ 0	7.299 - j 3.210
R. Montiel	FH = 0	9.854 - j 3.228
	FH $\neq$ 0	7.845 - j 3.494
A. Barrero		7.865 - j 3.539
Mack (Experimental)		8.981 - j 3.736

TABLE II

INPUT ADMITTANCE FOR A HALF-WAVE DIPOLE AT 25th ORDER
(ma/v)

Nguyen	FH = 0	9.350 - j 2.821
	FH $\neq$ 0	7.423 - j 3.092
R. Montiel	FH = 0	9.377 - j 3.103
	FH $\neq$ 0	7.974 - j 3.460
A. Barrero		8.225 - j 3.499
Mack (Experimental)		8.981 - j 3.736



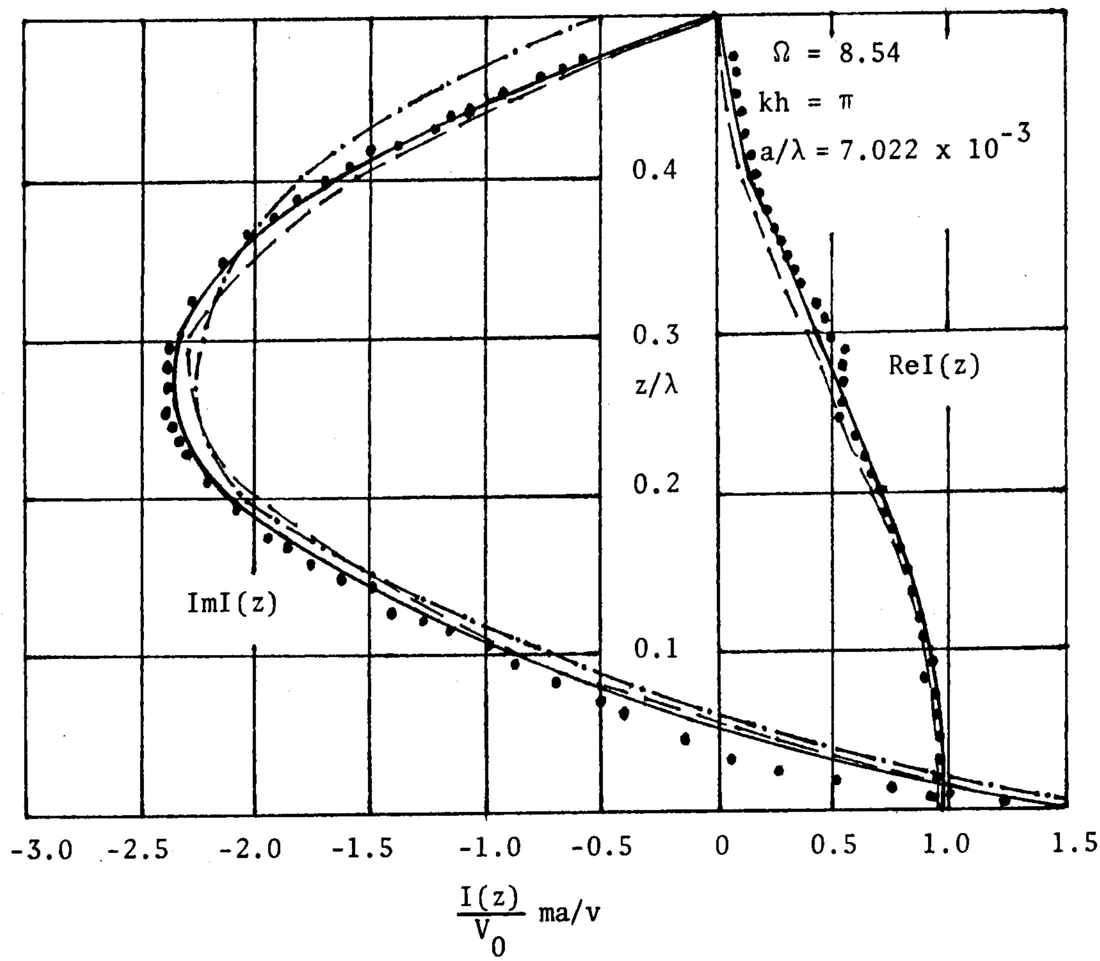
. . . Mack (Measured)

— · — A. Barrero

——— R. Montiel

— — — Nguyen

Figure 6. Current in Full-Wave Dipole (20th Order, FH = 0).



. . . Mack (Measured)

— · — A. Barrero

——— R. Montiel

— — — Nguyen

Figure 7. Current in Full-Wave Dipole (25th Order, FH = 0).

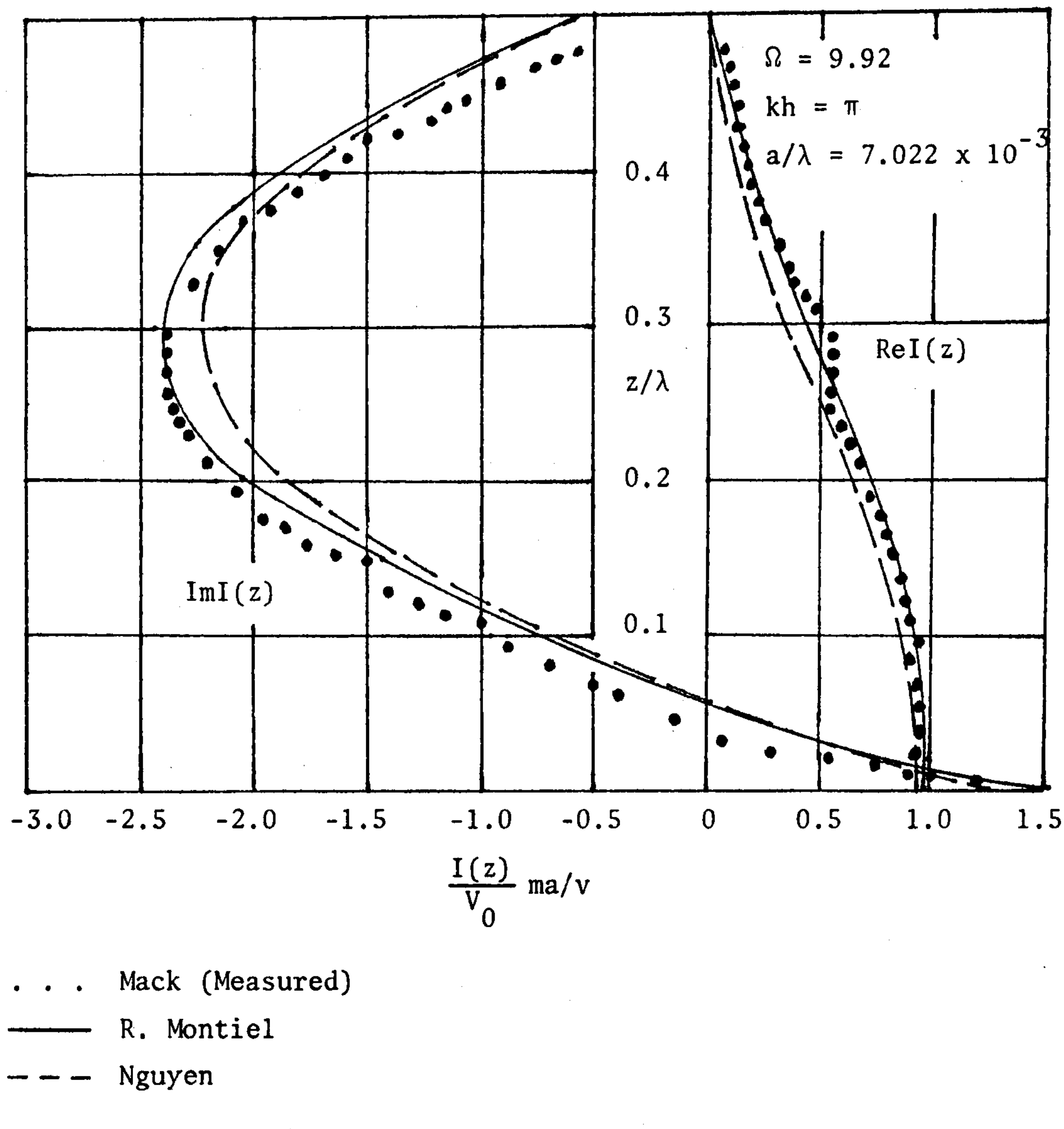
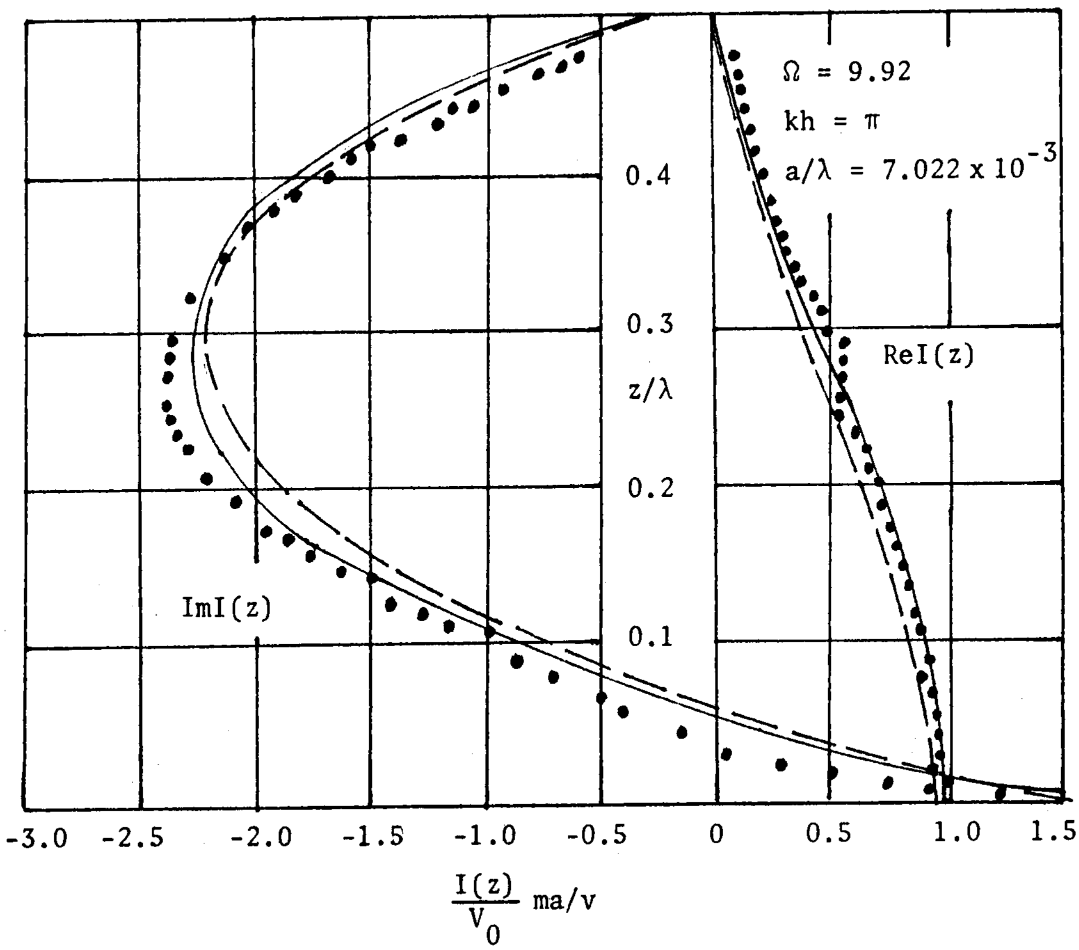


Figure 8. Current in Full-Wave Dipole (20th Order, FH ≠ 0).



. . . Mack (Measured)

——— R. Montiel

– – – Nguyen

Figure 9. Current in Full-Wave Dipole (25th Order, FH ≠ 0).

TABLE III

INPUT ADMITTANCE FOR A FULL-WAVE DIPOLE AT 20th ORDER
(ma/v)

Nguyen	FH = 0	0.975 + j 1.213
	FH $\neq$ 0	0.902 + j 1.264
R. Montiel	FH = 0	1.005 + j 1.371
	FH $\neq$ 0	0.977 + j 1.677
A. Barrero		0.976 + j 1.665
Mack (Experimental)		1.023 + j 1.394

TABLE IV

INPUT ADMITTANCE FOR A FULL-WAVE DIPOLE AT 25th ORDER
(ma/v)

Nguyen	FH = 0	0.953 + j 1.425
	FH $\neq$ 0	0.922 + j 1.589
R. Montiel	FH = 0	0.977 + j 1.568
	FH $\neq$ 0	0.981 + j 1.697
A. Barrero		0.980 + j 1.768
Mack (Experimental)		1.023 + j 1.394

It should be mentioned again at this point that even though the formulation in this work is an integral equation for E_z which does not have the constant C_1 in it, the program written utilized the vector potential approach and, therefore, has to evaluate C_1.

A closer look at the methods of formulating the solutions reveals that while A. Barrero's work yielded results as good as those of R. Montiel and those in this thesis, it did not provide the option to choose whether or not to force the end current be identically zero. According to Dr. H. P. Neff and Dr. J. D. Tillman of The University of Tennessee, the end current may take any form to produce a better solution to the original boundary value problem. Due to the flexibility mentioned above, the formulation in this work is undoubtedly dependable, and what is more important, it strengthens the fact that the trigonometric method is very appropriate to solving the cylindrical dipole antenna current problem.

B. ADMITTANCE FOR ISOLATED DIPOLES

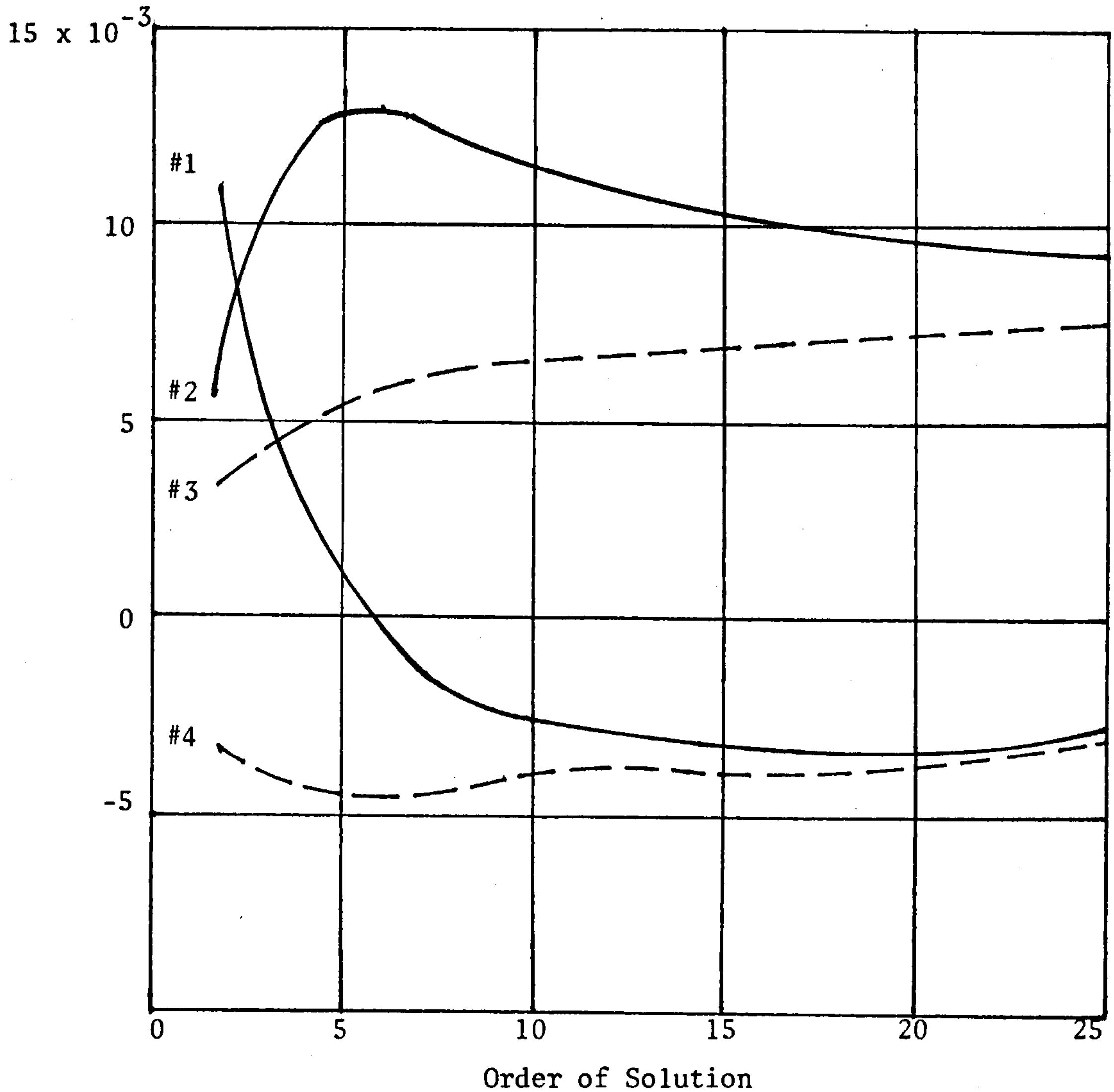
Normally the knowledge of antenna admittance is considered important due to the fact that it is an essential part of antenna design. The input admittances are, in the treatment of the antenna in this work, the diagonal elements in the admittance matrix. Tables I to IV list the input admittances for half- and full-wave dipoles with $\Omega = 8.54$ and $\Omega = 9.92$, respectively. The parameter Ω describes the height-to-thickness ratio of the antennas based on the assumption that $h >> a$, and the ratio h/a equals the ratio of the total length to the diameter of the cylindrical antenna. Thus,

$$\frac{\text{Total length}}{\text{Diameter}} = \frac{2h}{2a}$$

and Ω is given by

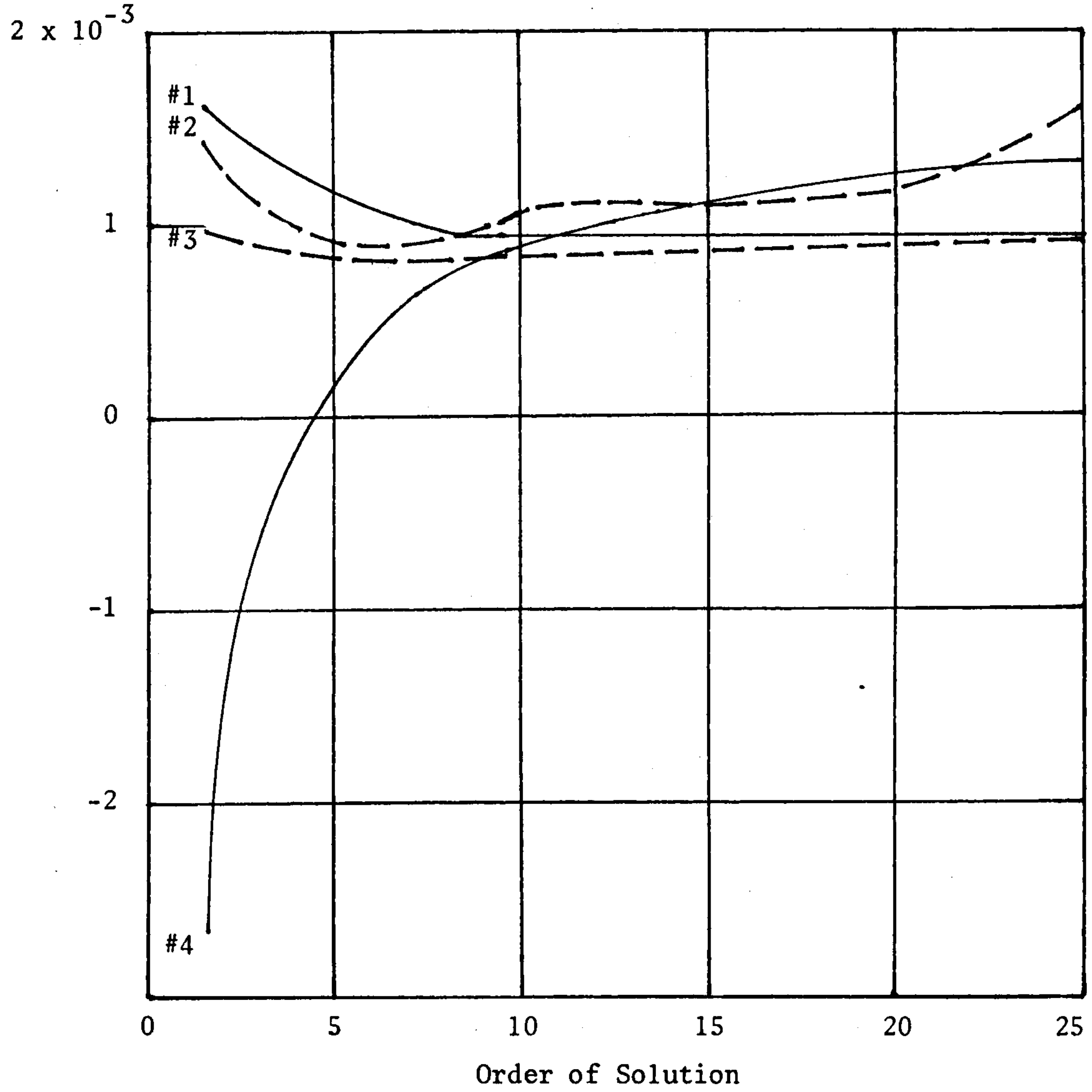
$$\Omega = 2 \ln \frac{2h}{a} .$$

Figures 10 and 11 show the conductance and susceptance components of antenna admittance as a function of the order of solution.



#1 Susceptance (FH = 0)
#2 Conductance (FH = 0)
#3 Conductance (FH $\neq$ 0)
#4 Susceptance (FH $\neq$ 0)

Figure 10. Admittance versus Order of Solution for Half-Wave Dipole.



#1 Conductance (FH = 0)
#2 Susceptance (FH ≠ 0)
#3 Conductance (FH ≠ 0)
#4 Susceptance (FH = 0)

Figure 11. Admittance versus Order of Solution for Full-Wave Dipole.

CHAPTER V

CONCLUSION

The goal of this thesis was to formulate and obtain an approximate solution for the dipole antenna current distribution. The results, as investigated thoroughly in the previous chapter, show that the technique is quite satisfactory.

The trigonometric theory developed utilized the fact that the current is expanded into a series of cosine terms.

It should be noted that this work is a following step of two previous theses. The similarities among the approaches of the author of this paper with A. Barrero and R. Montiel contribute to the fact that there are several ways one can solve the antenna current problem—with the trigonometric method as a framework.

It is also important to point out that as a result of this investigation, the end current does not necessarily have to be identically zero, but on the contrary, may take on any form that would result in a satisfactory solution—a satisfactory solution being one which agrees well with measured results.

Lastly this work concludes that the original work of Duncan and Hinchey should have given results more satisfactory than what they actually obtained. This would indicate that there must have been an error somewhere in their procedure, but not in their intention.

LIST OF REFERENCES

LIST OF REFERENCES

1. R. P. King, R. B. Mack and S. Sandler, Arrays of Cylindrical Dipole. London: Cambridge University Press, 1968.

2. L. Infeld, "The Influence of the Width of the Gap upon the Theory of Antennas," Quart. Appl. Math., Vol. 30, pp. 74-76, January 1959.

3. R. F. Harrington, Field Computation by Moment Methods. New York: Macmillan Co., 1968.

4. Alejandro Barrero, "A Trigonometric Solution to the Problem of Cylindrical Dipole," M.S. Thesis, The University of Tennessee, Knoxville, Tennessee, 1972.

5. Rafael S. Montiel, "Trigonometric Theory to Approximate the Current Distribution on Cylindrical Dipoles," M.S. Thesis, The University of Tennessee, Knoxville, Tennessee, 1979.

6. Robert E. Collin, Foundation for Microwave Engineering. New York: McGraw-Hill Book Co., 1966.

7. R. H. Duncan and F. A. Hinchey, "Cylindrical Antenna Theory," J. Res. NBS, Vol. 64D, pp. 569-584, September-October 1960.

8. L. V. King, "On the Radiation Field of a Perfectly Conducting Base Insulated Cylindrical Antenna Over a Perfectly Conducting Plane Earth, and the Calculation of a Radiation Resistance and Reactance," Phil. Tran. Royal Society, Series A, Vol. 236, pp. 381-422, November 1937.

9. T. T. Wu, "Theory of Long Antennas," in Antenna Theory Part I. R. E. Collin and F. J. Zucker, Eds. New York: McGraw-Hill Book Co., 1969.

10. R. W. P. King and C. W. Harrison, Antenna and Waves: A Modern Approach. Cambridge: The MIT Press, 1969.

11. E. Hallên, "Theoretical Investigation into the Transmitting and Receiving Qualities of Antennas," Nova Acta Regiae Soc. Sci. Upsalienisis, Ser. 4, Vol. 11, pp. 1-44, 1968.

12. R. W. P. King, Electromagnetic Engineering. New York: McGraw-Hill Book Co., 1945.

13. M. I. Skolnik, "Large Antenna System," in Antenna Theory Part II. R. E. Collin and F. J. Zucker, Eds. New York: McGraw-Hill Book Co., 1969.

14. T. T. Wu and R. W. P. King, The Scattering and Diffraction of Waves. Cambridge: Harvard University Press, 1959.

15. B. J. Ley, Computer Aided Analysis and Design for Electrical Engineers. New York: Holt, Rinehart and Winston, Inc., 1970.

APPENDIX

```
C       CYLINDRICAL ANTENNA CURRENT DISTRIBUTION          C
C                                                         C
C*********************************************************C
C
C
C IN THIS PROGRAM,THE CURRENT DISTRIBUTION OF A
C CYLINDRICAL DIPOLE ANTENNA IS SOLVED BY THE
C TRIGONOMETRIC METHOD.
C
C THE ANTENNA SIMULATED IS 2 METERS LONG,AND THE
C CONDUCTOR IS OF INFINITE CONDUCTIVITY.THE GENERATOR
C IS AT THE CENTER OF THE DIPOLE AND IS AN IMPULSE
C FUNCTION.
C
C ALL NUMERICAL VALUES IN THE PROGRAM ARE CALCULATED
C IN DOUBLE PRECISION.
C
      REAL*8 ZZ(11)
      COMPLEX*16 O(625)
      LOGICAL SING
      REAL*8 FN,FI,GI,HPI,I00,J00,API,PI,THETA,Z
      REAL*8 AL,ARG,OMEGA,RA
      REAL*8 J0,H0,SI,CI,K0,I0,F,G
      REAL*8 BE
      REAL*8 INCZ
      REAL*8 E,ALPHA,BETA
      COMPLEX*16 CK(6),C(50),K1,K2,CONS,J
      COMPLEX*16 TPN(25,25),TPNI(25,25),B(11)
      COMPLEX*16 EXSIN(25),EXCOS(25),PEXCO(25),PEXSI(25)
      COMPLEX*16 A(25)
      COMPLEX*16 C1,NUM,DEN,FH,AI
      COMMON J,RA,FI,GI,FN,HPI,I00,J00,API,PI,
     $THETA,Z,AL,ARG,CK
      INTEGER ORDER,DIM A
C
C PARAMETERS - CHANGEABLE IF NECESSARY
C
C NPN=ORDER,THE SIZE OF THE SQUARE MATRIX.
C OMEGA=IMPEDANCE,AL=ANTENNA LENGTH,
C RA=RADIUS OF THE DIPOLE.
C
      NPN=25
      OMEGA=8.54D0
      AL=1.D0
      RA=2.D0*DEXP(-OMEGA/2.D0)
C CONSTANTS - GENERAL
      PI=3.1415926500
      HPI=PI/2.D0
      J=(0.D0,1.D0)
```

```
      API=PI*AL
      NP2=2*NPN
      CONS=-J*376.73D0/(4.D0*PI*API)
C
C BY USING THE METHOD OF MOMENTS,THE CURRENT
C DISTRIBUTION,WHICH IS EXPRESSED IN TRIGONOMETRIC
C SERIES,IS FOUND.THE PRIMARY TASKS WOULD BE:
C
C 1/ TO EVALUATE THE KERNEL OF THE ELECTRIC FIELD.
C 2/ TO DETERMINE THE ELEMENTS OF THE IMPEDANCE MATRIX.
C 3/ TO SOLVE FOR THE CONSTANT C1 AND THE ADMITTANCE MATRIX.
C 4/ TO SOLVE THE SIMULTANEOUS EQUATIONS IN ORDER TO
C OBTAIN THE CURRENT DISTRIBUTION.
C
C CONSTANTS FOR FUNCTION K1
      CK(1)=(1.D0,0.D0)
      CK(2)=-J*API*(RA**2)
      CK(3)=-(RA**2)-3.D0*(API**2)*(RA**4)/4.D0
      CK(4)=9.D0*J*API*(RA**4)/4.D0-5.D0*J*(API**3)
     $*(RA**6)
      CK(5)=9.D0*(RA**4)/4.D0+5.D0*(API**2)*(RA**6)/
     $2.D0+53.D0*(API**4)*(RA**8)/192.D0
      CK(6)=-5.D0*J*API*(RA**6)-175.D0*J*
     $(API**3)*(RA**8)/96.D0
C CALCULATION OF FOURIER COEFFICIENTS OF THE KERNEL
C
C K2 IS A DEFINITE INTEGRAL WHICH CAN BE EVALUATED BY
C APPROXIMATION METHOD USING BESSEL'S FUNCTIONS.
C K1 IS APPROXIMATED BY INTEGRATING THE POWER EXPANSION
C OF THE KERNEL.
```

```
      DO 10  MM=1,NP2
      M=MM-1
      ARG=(AL*2.D0-M)*(AL*2.D0+M)/4.D0
      C(MM)=(K2(M)-K1(M))/2.D0
      C(MM)=CONS*C(MM)
  10  CONTINUE
C COEFFICIENTS OF MATRIX TPNI:
C
C MATRIX TPNI IS AN IMPEDANCE MATRIX.ITS DIMENSION IS
C CHANGEABLE FROM 1X1 TO 25X25.LARGER MATRICES WILL
C RESULT MORE ACCURATE SOLUTIONS.
C
      DO 20 N=1,NPN
      KP=N-1
      DO 20 NN=1,NPN
      NP=NN-1
      TPNI(NN,N)=(1.D0/(2*KP+1-2*NP)+1.D0/(2*KP+1+2*NP))
     1*(C(2*NP+1)+C(2*KP+2))
     2*(2.D0/PI)*(-1.D0)**(N+NN)
  20  CONTINUE
C INVERSE MATRIX TPNI
C
C THE ADMITTANCE MATRIX IS USED TO EVALUATE THE
C ANTENNA CURRENT.UTILIZING THE APPROXIMATIONS FROM
C MATH HANDBOOK,THE ELEMENTS CAN BE FOUND BY
C TRIGONOMETRIC MANIPULATIONS.
```

```
      ORDER=25
      DIM A=25
      CALL INVERS(TPNI,ORDER,DIM A,SING)
C DETERMINATION OF EXCOS AND EXSIN
      E=0.000000100
      DO 50 I=1,NPN
      N=I-1
      ALPHA=API-((2*N)+1)*HPI
      BETA=API+((2*N)+1)*HPI
      EXCOS(I)=(1.D0,0.D0)
      EXSIN(I)=-(1.D0,0.D0)/(2.D0*API*API)
      IF(DABS(ALPHA).LT.E)GO TO 50
      EXCOS(I)=(DSIN(ALPHA)/ALPHA)+(DSIN(BETA)/BETA)
      EXSIN(I)=-(DCOS(BETA)/BETA)-(DCOS(ALPHA)/ALPHA)
     1+1.D0/BETA+1.D0/ALPHA
      EXSIN(I)=-EXSIN(I)/(2.D0*API)
   50 CONTINUE
C DETERMINATION OF THE PRODUCT OF TPNI AND EXCOS
C DETERMINATION OF THE PRODUCT OF TPNI AND EXSIN
      DO 60 I=1,NPN
      PEXCO(I)=(0.D0,0.D0)
      PEXSI(I)=(0.D0,0.D0)
      DO 60 N=1,NPN
      PEXCO(I)=PEXCO(I)+(TPNI(I,N)*EXCOS(N))
      PEXSI(I)=PEXSI(I)+(TPNI(I,N)*EXSIN(N))
   60 CONTINUE
```

```
C EVALUATION OF THE CONSTANT C1
C
C CONSTANT C1 IS NEEDED TO DETERMINE THE CURRENT IN
C THE DIPOLE,ESPECIALLY AT THE TIPS OF THE ANTENNA
C WHERE THE CURRENT VANISHES.
C
C IN THIS COMPUTER PROGRAM,IF FH IS IDENTICALLY ZERO,
C C1 WILL FORCE THE END CURRENT TO BE ZERO.OTHERWISE,
C THE CURRENT WILL HAVE CERTAIN VALUE AT Z=+-H
C
      FH=(0.19223092D-4,-0.27651576D-3)
      NUM=(0.D0,0.D0)
      DEN=-FH
      DO 70 I=1,NPN
      NUM=NUM+PEXCO(I)*(-1)**(I+1)
      DEN=DEN+PEXSI(I)*(-1)**(I+1)
   70 CONTINUE
      C1=-(DEN/NUM)
      DO 80 I=1,NPN
      A(I)=(C1*PEXCO(I))+PEXSI(I)
   80 CONTINUE
C CALCULATION OF CURRENT DISTRIBUTION
C
C THE CURRENT DISTRIBUTION ALONG THE ANTENNA IS IN
C MILLIAMPERES.
C
C THE LENGTH OF THE DIPOLE IS BROKEN DOWN TO 10
C EQUAL SEGMENTS WHERE THE CURRENT IS SHOWN.
C
      WRITE(6,85)
```

```
 85   FORMAT(1H ,21X,'Z',5X,'REAL PART',13X,'IMAGINARY PART')
      WRITE(6,86)
 86   FORMAT(1H0)
      Z=0.D0
      INCZ=0.1D0
      DO 110 I=1,11
      AI=(0.D0,0.D0)
      DO 90 N=1,NPN
      NN=N-1
 90   AI=AI+A(N)*DCOS(NN*PI*Z)
      WRITE(6,100)Z,AI
 100  FORMAT(/1H ,19X,F5.2,2(D18.8,4X))
      Z=Z+INCZ
 110  CONTINUE
      STOP
      END
```

```
      COMPLEX FUNCTION K1*16(M)
      REAL*8 Y1,Y2,F1,F2,F3,F4,F5,F6,SI,CI
      REAL*8 FN,FI,GI,HPI,I00,J00,API,PI,THETA,Z
      REAL*8 AL,ARG,OMEGA,RA
      COMPLEX*16 CK(6),J,I
      LOGICAL PLUS
      COMMON J,RA,FI,GI,FN,HPI,I00,J00,API,PI,
     $THETA,Z,AL,ARG,CK
C CALCULATION OF K1(M)
      Y1=2.D0*(API+M*HPI)
      F1=-CI(Y1)
      F3=HPI-SI(Y1)
      Y2=(AL*2.D0-M)*PI
      PLUS=.FALSE.
      IF(Y2.NE.0.D0)GO TO 10
      F2=.5772156649D0
      F4=HPI
      GO TO 30
   10 IF(Y2.GT.0.D0)GO TO 20
      Y2=-Y2
      PLUS=.TRUE.
   20 F2=-CI(Y2)
      F4=HPI-SI(Y2)
      K1=(0.D0,0.D0)
   30 DO 60 L=1,6
      I=F1+F2-J*F3
      IF(PLUS)GO TO 40
      I=I-J*F4
      GO TO 50
   40 I=I+J*F4
   50 K1=K1+CK(L)*I
      IF(L.EQ.6)RETURN
      F5=F1
      F6=F2
      F1=(DCOS(Y1)-(Y1*F3/2.D0)*2.D0**L)/(L*2.D0**L)
      F2=(DCOS(Y2)-(Y2*F4/2.D0)*2.D0**L)/(L*2.D0**L)
      F3=(DSIN(Y1)+(Y1*F5/2.D0)*2.D0**L)/(L*2.D0**L)
      F4=(DSIN(Y2)+(Y2*F6/2.D0)*2.D0**L)/(L*2.D0**L)
   60 CONTINUE
      RETURN
      END
```

```
      COMPLEX FUNCTION K2*16(M)
      REAL*8 Y
      REAL*8 FN,FI,GI,HPI,I00,J00,API,PI,THETA,Z
      REAL*8 AL,ARG,OMEGA,RA
      REAL*8 J0,H0,SI,CI,K0,I0,F,G
      COMPLEX*16 CK(6),J
      COMMON J,RA,FI,GI,FN,HPI,I00,J00,API,PI,
     $THETA,Z,AL,ARG,CK
C CALCULATION FOR K2(KZ)
      Y=(ARG*PI)**2
      IF(Y.GE.0.D0)GO TO 10
      Y=-Y
      Y=RA*DSQRT(Y)
      I00=I0(Y)
      K2=2.D0*I00*K0(Y)
      RETURN
   10 IF(Y.NE.0.D0)GO TO 20
      K2=-DLOG((RA**2)*API/2.D0)-1.1544313298D0-J*PI
      RETURN
   20 Y=RA*DSQRT(Y)
      J00=J0(Y)
      K2=-J*PI*J00*(J00-J*H0(Y))
      RETURN
      END
```

```
      REAL FUNCTION CI*8(X)
      REAL*8 X,F,G
      REAL*8 FN,FI,GI,HPI,I00,J00,API,PI,THETA,Z
      REAL*8 AL,ARG,OMEGA,RA
      COMPLEX*16 CK(6),J
      COMMON J,RA,FI,GI,FN,HPI,I00,J00,API,PI,
     $THETA,Z,AL,ARG,CK
C APPROXIMATION OF COSINE INTEGRAL
      IF(X.GE.1.D0)GO TO 20
      WRITE(6,10)
   10 FORMAT('1','NO APPROXIMATION IS PROVIDED
     * FOR THIS CASE')
   20 FI=F(X)
      GI=G(X)
      CI=FI*DSIN(X)-GI*DCOS(X)
      RETURN
      END
```

```
      REAL FUNCTION SI*8(X)
      REAL*8 X
      REAL*8 FN,FI,GI,HPI,I00,J00,API,PI,THETA,Z
      REAL*8 AL,ARG,OMEGA,RA
      COMPLEX*16 CK(6),J
      COMMON J,RA,FI,GI,FN,HPI,I00,J00,API,PI,
     $THETA,Z,AL,ARG,CK
C APPROXIMATION OF SINE INTEGRAL
      IF(X.GE.1.D0)GO TO 20
      WRITE(6,10)
   10 FORMAT('1','NO APPROXIMATION IS PROVIDED FOR
     * THIS CASE')
   20  SI=HPI-FI*DCOS(X)-GI*DSIN(X)
      RETURN
      END
```

```
      REAL FUNCTION G*8(X)
      REAL*8 X
C CALCULATION FOR G(X)
      G=(
     121.821899D0+
     2352.018498D0*X**2+
     3302.757865D0*X**4+
     442.24855D0*X**6+
     5X**8)
      G=G/(
     6499.690326D0*X**2+
     71114.978885D0*X**4+
     8482.485984D0*X**6+
     948.196927D0*X**8+
     1X**10)
      RETURN
      END
```

```
      REAL FUNCTION F*8(X)
      REAL*8 X
C CALCULATION FOR F(X)
      F=(
     1 38.102495D0+
     2335.677320D0*X**2+
     3265.187033D0*X**4+
     438.027264D0*X**6+
     5X**8)
      F=F/(
     6157.105423D0*X+
     7570.236280D0*X**3+
     8322.624911D0*X**5+
     940.021433D0*X**7+
     1X**9)
      RETURN
      END
```

```
C
C J0,H0,K0,I0 ARE BESSEL'S FUNCTIONS USED TO
C APPROXIMATE K2,WHICH IS NEEDED TO EVALUATE THE KERNEL.
C
C HERE THE ARGUMENT SWAP IS PASSED TO Y WHICH IS THE
C ACTUAL VALUE.
C
      REAL FUNCTION J0*8(SWAP)
      REAL*8 Y,SWAP
      REAL*8 FN,FI,GI,HPI,I00,J00,API,PI,THETA,Z
      REAL*8 AL,ARG,OMEGA,RA
      COMPLEX*16 CK(6),J
      COMMON J,RA,FI,GI,FN,HPI,I00,J00,API,PI,
     $THETA,Z,AL,ARG,CK
C CALCULATION FOR J0
      Y=SWAP
      IF(Y.GT.3.D0)GO TO 10
      Y=Y/3.D0
      J0=1.D0-
     12.2499997D0*Y**2+
     21.2656208D0*Y**4-
     30.3163866D0*Y**6+
     40.0444479D0*Y**8-
     50.0039444D0*Y**10+
     60.0002100D0*Y**12
      RETURN
 10   Z=Y
      Y=3.D0/Y
      THETA=Z-
     1.78539816D0-
     2.04166397D0*Y-
     3.00003954D0*Y**2+
     4.00262573D0*Y**3-
     5.00054125D0*Y**4-
     6.00029333D0*Y**5+
     7.00013558D0*Y**6
      FN=.79788456D0-
     1.00000077D0*Y-
     2.00552740D0*Y**2-
     3.00009512D0*Y**3+
     4.00137237D0*Y**4-
     5.00072805D0*Y**5+
     6.00014476D0*Y**6
      J0=FN*DCOS(THETA)/DSQRT(Z)
      RETURN
      END
```

```
      REAL FUNCTION H0*8(SWAP)
      REAL*8 Y,SWAP
      REAL*8 FN,FI,GI,HPI,I00,J00,API,PI,THETA,Z
      REAL*8 AL,ARG,OMEGA,RA
      COMPLEX*16 CK(6),J
      COMMON J,RA,FI,GI,FN,HPI,I00,J00,API,PI,
     $THETA,Z,AL,ARG,CK
C CALCULATION FOR H0
      Y=SWAP
      IF(Y.GT.3.D0)GO TO 10
      Y=Y/3.D0
      H0=DLOG(1.5D0*Y)*J00/HPI+
     1.36746691D0+
     2.60559366D0*Y**2-
     3.74350384D0*Y**4+
     4.25300117D0*Y**6-
     5.04261214D0*Y**8+
     6.00427916D0*Y**10-
     7.00024846D0*Y**12
      RETURN
   10 H0=FN*DSIN(THETA)/DSQRT(Z)
      RETURN
      END
```

```
      REAL FUNCTION K0*8(SWAP)
      REAL*8 Y,SWAP
      REAL*8 FN,FI,GI,HPI,I00,J00,API,PI,THETA,Z
      REAL*8 AL,ARG,OMEGA,RA
      COMPLEX*16 CK(6),J
      COMMON J,RA,FI,GI,FN,HPI,I00,J00,API,PI,
     $THETA,Z,AL,ARG,CK
C CALCULATION FOR K0
      Y=SWAP
      IF(Y.GT.2.D0)GO TO 10
      Y=Y/2.D0
      K0=-DLOG(Y)*I00-0.57721566D0+
     2.42278420D0*Y**2+
     3.23069756D0*Y**4+
     4.03488590D0*Y**6+
     5.00262698D0*Y**8+
     6.00010750D0*Y**10+
     7.00000740D0*Y**12
      RETURN
   10 Z=Y
      Y=2.D0/Y
      K0=DEXP(-Z)*(
     1+1.25331414D0
     2-.07832358D0*Y
     3+.02189568D0*Y**2
     4-.01062446D0*Y**3
     5+.00587872D0*Y**4
     6-.00251540D0*Y**5
     7+.00053208D0*Y**6)/DSQRT(Z)
      RETURN
      END
```

```
      REAL FUNCTION I0*8(SWAP)
      REAL*8 Y,SWAP
      REAL*8 FN,FI,GI,HPI,I00,J00,API,PI,THETA,Z
      REAL*8 AL,ARG,OMEGA,RA
      COMPLEX*16 CK(6),J
      COMMON J,RA,FI,GI,FN,HPI,I00,J00,API,PI,
     $THETA,Z,AL,ARG,CK
C CALCULATION FOR I0
      Y=SWAP
      IF(Y.GT.3.75D0)GO TO 10
      Y=Y/3.75D0
      I0=1.D0
     1+3.5156229D0*Y**2
     2+3.0899424D0*Y**4
     3+1.2067492D0*Y**6
     4+0.2659732D0*Y**8
     5+0.0360768D0*Y**10
     6+0.0045813D0*Y**12
      RETURN
   10 Z=Y
      Y=3.75D0/Y
      I0=DEXP(Z)*(
     1+.39894228D0
     2+.01328592D0*Y
     3+.00225319D0*Y**2
     4-.00157565D0*Y**3
     5+.00916281D0*Y**4
     6-.02057706D0*Y**5
     7+.02635537D0*Y**6
     8-.01647633D0*Y**7
     9+.00392377D0*Y**8)/DSQRT(Z)
      RETURN
      END
```

```
C
      SUBROUTINE INVERS(TPNI,ORDER,DIM A,SING)
C
C THE INVERSION SUBROUTINE INVERTS A MATRIX IN ITS
C OWN SPACE USING THE GAUSS-JORDAN METHOD AS ARRANGED
C BY H. RUTIHAUSER WITH FULL PIVOTING.
C
C TPNI=THE MATRIX TO BE INVERTED.THE INVERSE OF TPNI
C IS RETURNED IN TPNI.
C
C ORDER=AN INTEGER VARIABLE.THE ORDER OF THE MATRIX
C TO BE INVERTED.
C
C DIM A=AN INTEGER VARIABLE CONTAINING THE FIRST
C DIMENSION OF THE VARIABLE TPNI IN THE CALLING PROGRAM.
C DIM A MAY BE GREATER THAN ORDER.
C
C SING=A LOGICAL VARIABLE.WHEN THE ROUTINE RETURNS,
C SING IS 9FALSE. IF THE MATRIX WAS NON-SINGULAR AND
C .TRUE. IF THE MATRIX WAS SINGULAR.
C IF SING IS .TRUE.,THE MATRIX TPNI CONTAINS
C ARBITRARY NUMBERS.
C
      INTEGER ORDER,DIM A
      COMPLEX*16 TPNI(DIM A,DIM A),U(100),W(100)
      COMPLEX*16 PIVOT,V
      LOGICAL SING
      INTEGER IP(100),IQ(100)
      REAL*8 CDABS
      N=ORDER
      DO 1 K=1,N
C DETERMINATION OF PIVOT ELEMENT
      PIVOT=DCMPLX(0.D0,0.D0)
      DO 100 I=K,N
      DO 100 J=K,N
      IF(CDABS(TPNI(I,J)).LE.CDABS(PIVOT))GO TO 100
      PIVOT=TPNI(I,J)
      IP(K)=I
      IQ(K)=J
  100 CONTINUE
      IF(CDABS(PIVOT)) 102,900,102
  102 IPK=IP(K)
      IQK=IQ(K)
C EXCHANGE OF THE PIVOTAL ROW WITH THE KTH ROW
      IF(IPK-K) 200,299,200
  200 DO 201 J=1,N
      V=TPNI(IPK,J)
      TPNI(IPK,J)=TPNI(K,J)
  201 TPNI(K,J)=V
  299 CONTINUE
C EXCHANGE OF THE PIVOTAL COLUMN WITH THE KTH COLUMN
      IF(IQK-K)300,399,300
  300 DO 301 I=1,N
      V=TPNI(I,IQK)
```

```
      TPNI(I,IQK)=TPNI(I,K)
 301  TPNI(I,K)=V
 399  CONTINUE
C JORDAN ELIMINATION STEP
      DO 400 J=1,N
      IF(J-K)403,402,403
 402  U(J)=1.D0/PIVOT
      W(J)=1.D0
      GO TO 404
 403  U(J)=-TPNI(K,J)/PIVOT
      W(J)=TPNI(J,K)
 404  TPNI(K,J)=DCMPLX(0.D0,0.D0)
 400  TPNI(J,K)=DCMPLX(0.D0,0.D0)
      DO 405 I=1,N
      DO 405 J=1,N
 405  TPNI(I,J)=TPNI(I,J)+W(I)*U(J)
   1  CONTINUE
C REORDERING THE MATRIX
      K=N
      DO 500 KDUM=1,N
      IPK=IP(K)
      IQK=IQ(K)
      IF(IPK-K)501,502,501
 501  DO 503 I=1,N
      V=TPNI(I,IPK)
      TPNI(I,IPK)=TPNI(I,K)
 503  TPNI(I,K)=V
 502  IF(IQK-K)504,500,504
 504  DO 506 J=1,N
      V=TPNI(IQK,J)
      TPNI(IQK,J)=TPNI(K,J)
 506  TPNI(K,J)=V
 500  K=K-1
      SING=.FALSE.
      RETURN
 900  SING=.TRUE.
      RETURN
      END
```

VITA

Cuong Van Nguyen was born in Long-Xuyen, Vietnam, on April 21, 1956. He later moved to Saigon and attended the Tran-Qui-Cap elementary school. After spending three years at the Vo-Truong-Toan high school, he transferred to the Chu-Van-An (Bưởi) high school. Early in 1975 he was admitted to the Middle Tennessee State University as an undergraduate student of the Overseas Study Program by the Vietnamese government in Saigon. In the fall of 1976,he transferred to The University of Tennessee where he became a recipient of a University of Tennessee General Scholarship. He graduated with a Bachelor of Science in Electrical Engineering in Fall 1978 and in the Winter Quarter of the following year, was awarded a Graduate Assistantship by the Department of Electrical Engineering at the same university. In December, 1979, he will receive the Master of Science degree in Electrical Engineering. His major interest lies in the computer option, and the minor is electromagnetic theory.

He is a member of the Institute of Electrical and Electronics Engineers, the Eta Kappa Nu Society, and the Tau Beta Pi Engineering Honor Society.