

Finding Nuclear Drip Lines and Studying Two Proton Emission

An Undergraduate Thesis

By

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The nuclear landscape, illustrated in figure 1, is visualized by a plot of all known nuclei, with neutron number on the x axis and proton number on the y axis. Each square represents a known nucleus and the black squares represent stable nuclei. As you travel away from this valley of stability, the nuclei begin to get more and more unstable. If you travel far enough away from stability, then some of the nucleons, protons or neutrons, will be completely unbound. The point where this happens is known as a nuclear drip line, because it's as if the extra nucleons drip right off the nucleus.

To find where these drip lines are on the nuclear landscape, the separation energy is needed. Neutron separation energy (S_n) is defined as the amount of energy required to remove a neutron from the nucleus. Proton separation energy (S_p) is defined similarly. In finding both the proton and neutron drip lines, even-even nuclei were used. These are nuclei with both an even proton number and even neutron number. Even-even nuclei are more bound than their odd nucleon neighbors, so the drip lines can be calculated with respect to just even-even nuclei. Since only even-even nuclei were considered, two neutron (S_{2n}) and two proton (S_{2p}) separation energies were used. These are the energies required to remove two neutrons and two protons from a nucleus, respectively. To find these separation energies we can take a look at nuclear binding energy, $B(Z, N)$, where Z represents proton number and N represents neutron number. Nuclear binding energy is the energy required to separate all of the neutrons and protons in a nucleus from each other. By convention, our values for nuclear binding energy are negative for bound nuclei and become more negative for nuclei with more binding. This is because the energy of the system is decreased when the protons and neutrons form a bound nucleus, so the stronger the nucleus is bound, the more the energy of the system is decreased. From the definitions of binding energy and separation energy, we get the relationships $S_{2n}(Z, N) =$

$B(Z, N - 2) - B(Z, N)$ and $S_{2p}(Z, N) = B(Z - 2, N) - B(Z, N)$. Positive separation energy means the nucleus is stable with respect to nucleon emission. Similarly, negative separation energy means the nucleus is unstable with respect to nucleon emission. So the two neutron and two proton drip lines are reached when $S_{2n} \approx 0$ and $S_{2p} \approx 0$, respectively.

In order to calculate these binding energies we used theoretical models known as density functional theory (DFT), specifically Skyrme functionals. We used six different Skyrme DFT models (SkM*, SLy4, SkP, SV-min, UNEDF0, and UNEDF1) to do all the calculations. These computations were done on super computer Jaguar at Oak Ridge National Lab. Using these six models we calculated binding energy and many other properties of all nuclei up to proton number $Z = 130$ and neutron number $N = 300$.

After these calculations were finished, we were able to use these binding energies to calculate S_{2n} and S_{2p} . First, to find the neutron drip line, for each even Z we graphed S_{2n} versus N . For example, the graph for erbium ($Z = 68$) is shown in figure 2. It is easy to see that as the neutron number is increased, S_{2n} approaches zero. The last point where it is still positive is the point on the neutron drip line for erbium. Different models have different predictions for exactly where this point lies. The insert box in the top right of the figure shows that the uncertainty among the models ranges from $N = 154$ to $N = 162$. These uncertainties in the drip line are illustrated on figure 1 in the pink shaded region, and the drip lines shown in figure 1 are the result of averaging the drip lines predicted by each model. The proton drip line was found analogously, that is, by taking each even N and graphing S_{2p} versus Z . For $N = 76$, the point on the drip line happens to lie at erbium, $Z = 68$. This is shown in the insert box at the top left of figure 2. The uncertainty in the proton drip line is much smaller due to us having a good

understanding of the Coulomb interaction, the interaction between charged particles such as protons. Also, the proton drip line is much closer to the valley of stability, so it is much more feasible to reach than the neutron drip line using current experimental methods. This can be seen in figure 1 – the green boxes show nuclei that have been created experimentally, and they go almost all the way to the proton drip line, but are not very close to the neutron drip line.

The calculations done on Jaguar using all these different models can be used for many other things besides finding drip lines. Another nuclear phenomenon that we studied was two proton emissions. We specifically looked at nuclei that are stable with respect to one proton emission, but unstable with respect to two proton emission. This means that these nuclei satisfy the conditions: $Q_p = -S_p < 0$, $Q_{2p} = -S_{2p} > 0$, where Q_p and Q_{2p} represent the decay energies for one proton and two proton emission, respectively. The decay energies represent the amount of excess energy the nucleus will give off when it decays. These decay energies can be used to calculate the decay times, or half-lives. These conditions are illustrated in the insert box at the top left of figure 3.

After identifying the nuclei that satisfy these conditions, we calculated their half-lives, t_{2p} , with respect to two proton emission using two different models. These two models are a direct-decay model and a diproton model. The direct-decay model represents two separate protons being expelled from the nucleus at the same time and is a very complex problem where you have to take into account the three-body problem. The diproton model is much simpler and represents two connected protons, a diproton, being expelled from the nucleus. For the diproton model, the half-life was obtained through the Wentzel-Kramers-Brillouin expression, WKB-approximation. Although both models calculate the half-lives for the isotopes very differently, the results coincided quite well, as explained later in figure 3.

The next step after finding the half-lives of the nuclei was to impose another condition: $10^{-7}s < t_{2p} < 10^{-1}s$. The lower bound of 100 ns on this condition exists to only keep nuclei that can be observed experimentally; if the half-life was shorter, the nuclei would decay too quickly. The upper bound of 100 ms on this condition is in place to ensure that these nuclei's decay modes aren't dominated by beta decay. Additionally, to weed out nuclei that decay quickly by alpha decay, we imposed the condition: $t_{2p} < 10 \cdot t_{\alpha}$. Also, we took a special look at the nuclei that satisfy the condition: $0.1 \cdot t_{2p} < t_{\alpha} < 10 \cdot t_{2p}$. In these nuclei, both two proton decay and alpha decay account for at least 10% of the decay. These special cases are marked as stars in figure 4.

After applying all of these criteria to the calculated half-lives, we averaged the neutron number for all two proton emitters for each even- Z element for both the direct-decay model and the diproton model. These lines were then graphed in figure 3 – the red dotted line represents the results for the direct-decay model and the blue dotted line represents the results for the diproton model. Both models have very little discrepancy as seen in the figure. It is important to note that no candidates for $Z > 82$ remain after all our criteria were applied. These nuclei are way too prone to alpha emission to be likely two proton emitters. The entire picture of all our two proton emission candidates is shown in figure 4. This figure shows a plot of the location of the two proton emitters with proton number on the x-axis and distance from the $2n$ drip line on the y-axis. The colors illustrate the number of different Skyrme DFT models that predict that candidate to be a two proton emitter. The yellow region shows the experimental frontier. It is easy to see that the different Skyrme models don't agree on many cases of two proton emitters. This is due to extreme sensitivity of the half-life, t_{2p} , with respect to the decay energy, Q_{2p} .

However, even though the models don't agree everywhere locally, they still give the same general picture for the entire two proton radioactivity landscape.

High performance computing combined with density functional theory can produce a wide variety of theoretical results in nuclear physics. We were able to calculate the borders of the nuclear landscape (drip lines) with high precision. This led to us also being able to produce a broad view of two proton radioactivity across the nuclear landscape. Other trends and properties across the nuclear landscape can be easily studied with these methods as well. The importance of these results is that as experimental equipment and techniques become more powerful in the future, these nuclei will be able to be tested in the lab. This overview we have provided of the nuclear landscape can serve as a guide for future experimental endeavors.

Figures

Figure 1 [1]: Nuclear even-even landscape as of 2012

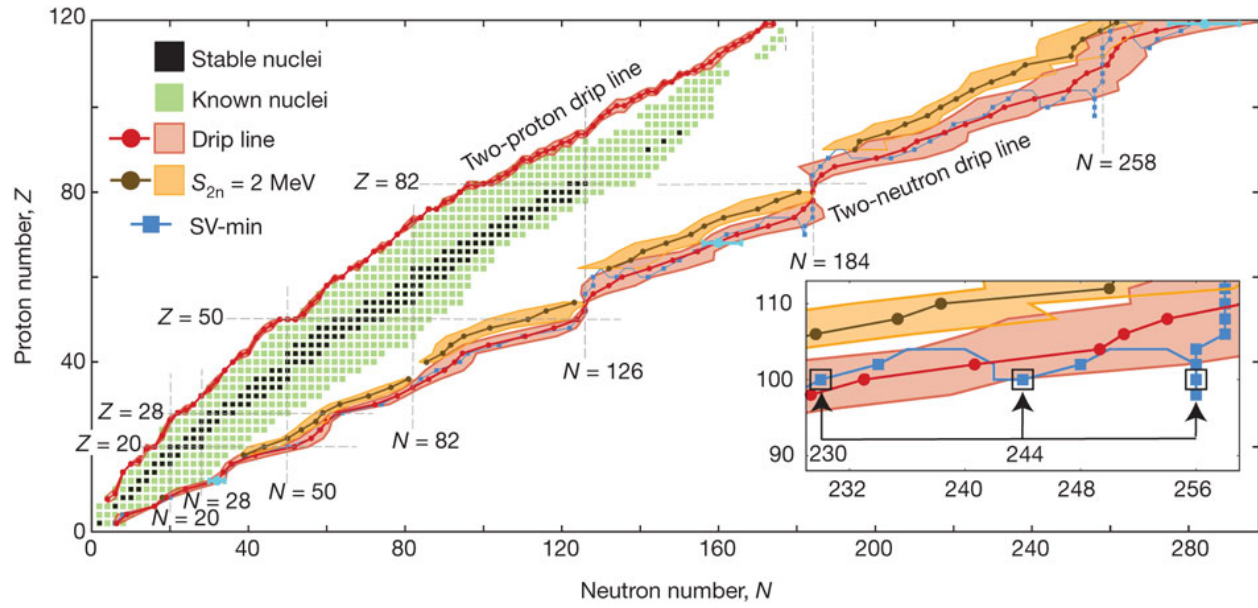


Figure 2 [1]: Calculated and experimental two-neutron separation energies of even-even erbium isotopes

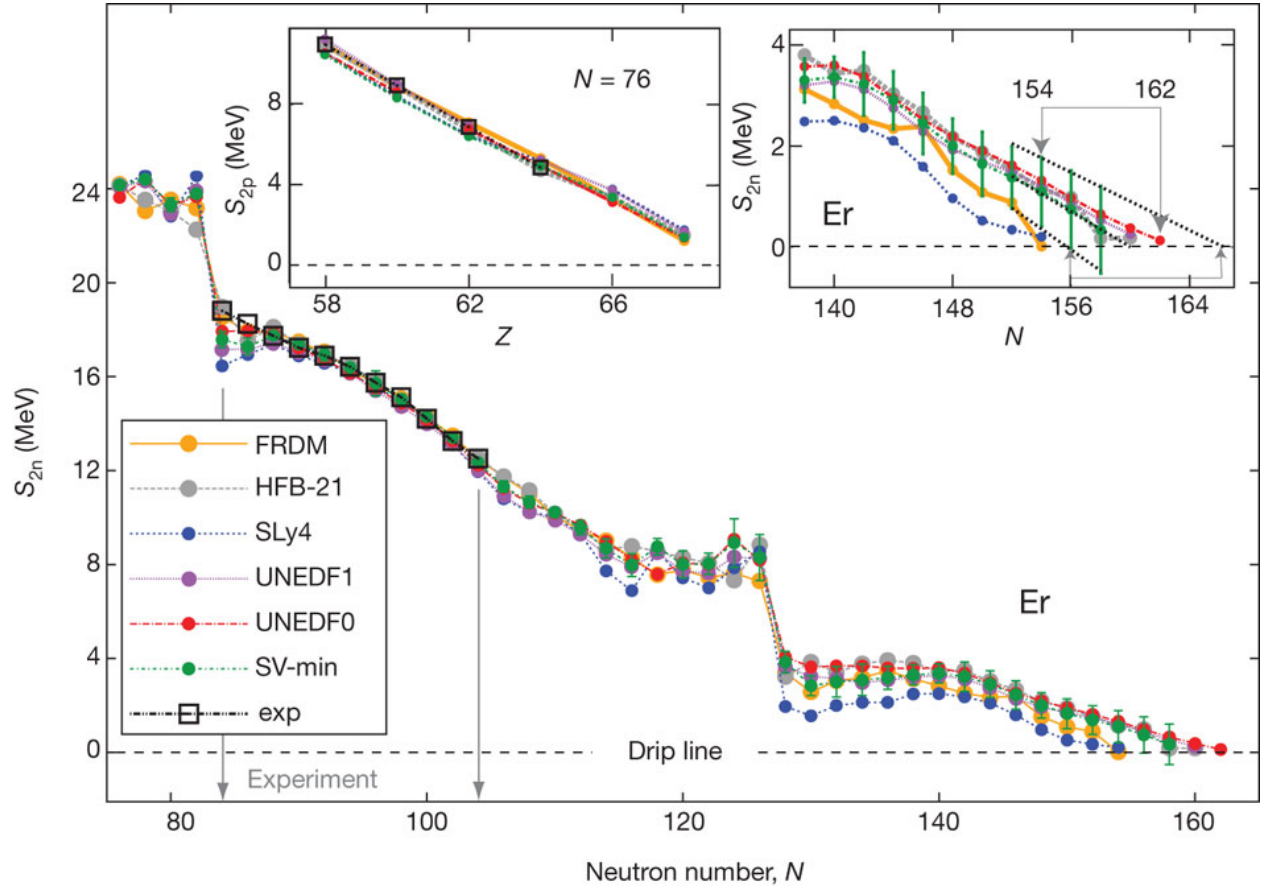


Figure 3 [2]: The landscape of 2p emitters

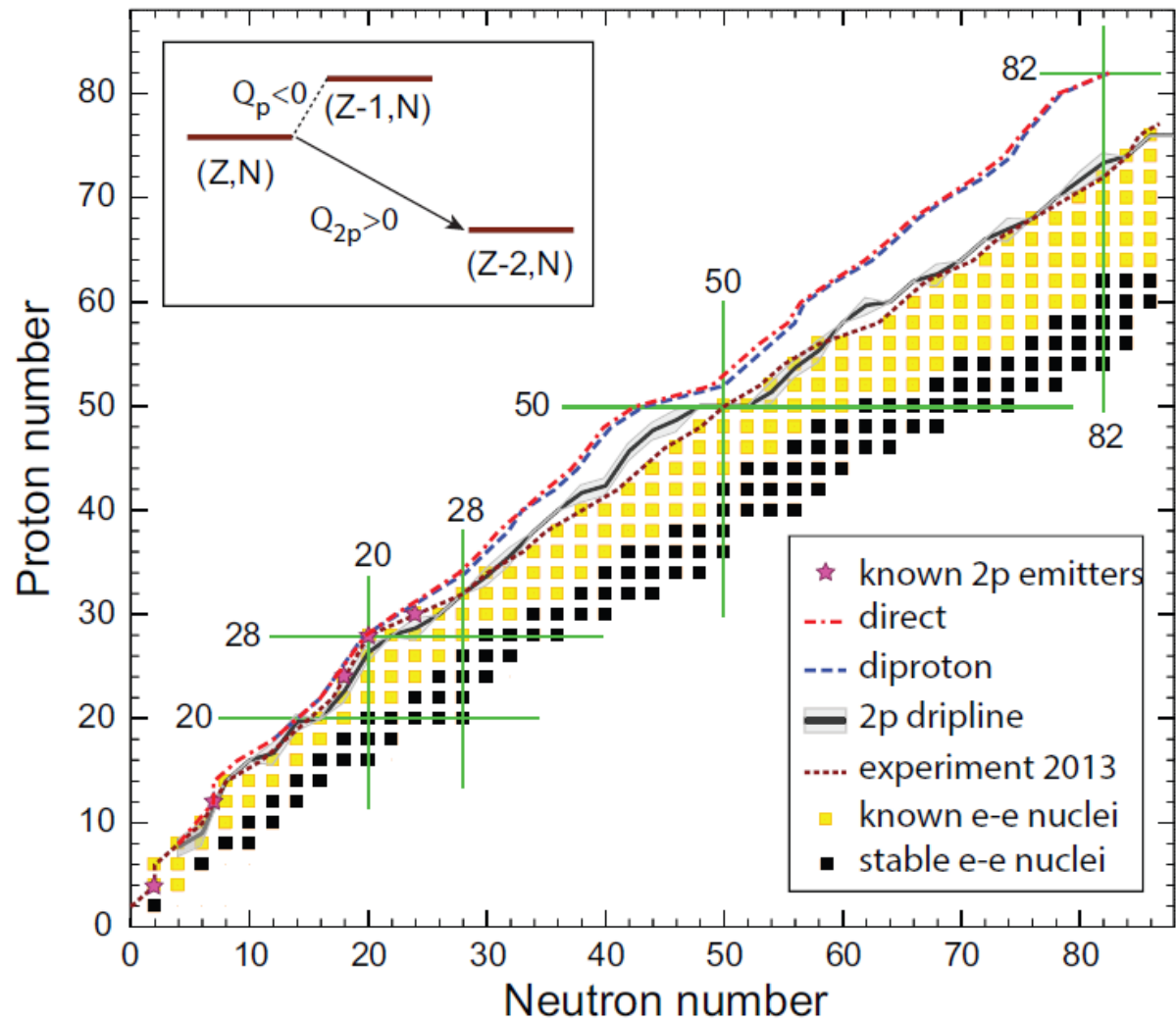
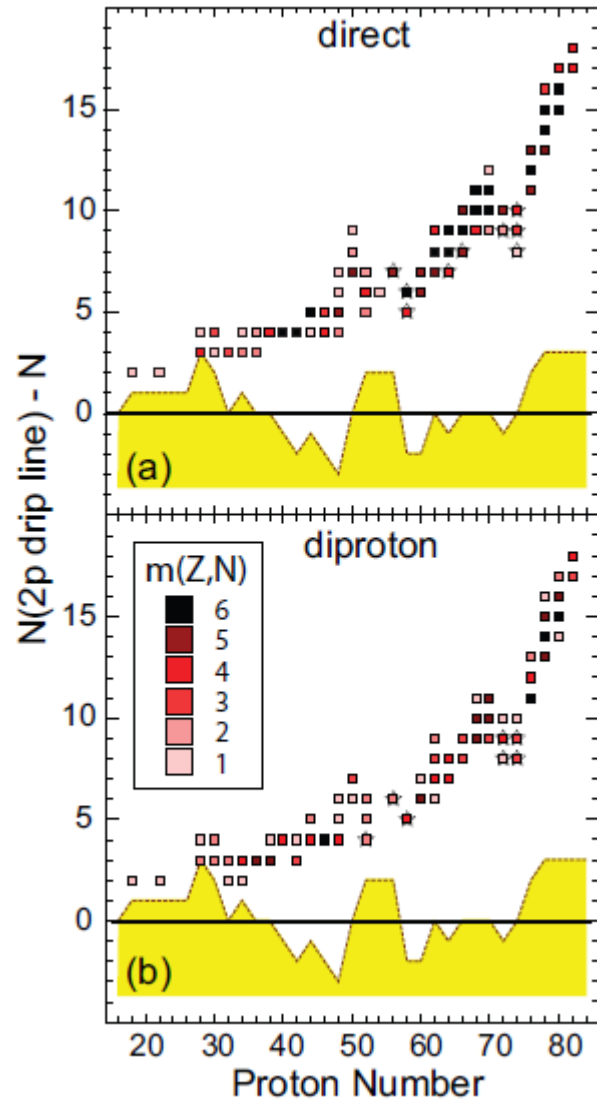


Figure 4 [2]: The predictions of the direct-decay model (a) and diproton model (b) for 2p radioactivity.



References

- [1] J. Erler, N. Birge, M. Kortelainen, W. Nazarewicz, E. Olsen, A. Perhac, and M. Stoitsov, **The Limits of the Nuclear Landscape**, Nature **486**, 509-512 (2012).
- [2] E. Olsen, M. Pfützner, N. Birge, M. Brown, W. Nazarewicz, and A. Perhac, (Publication pending in Phys. Rev. Lett., titled: **The landscape of two-proton radioactivity**).

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