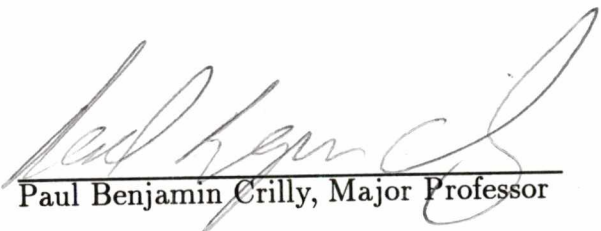
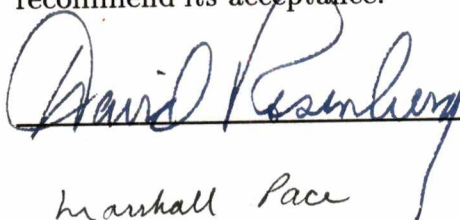


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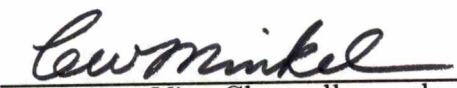
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Edge Enhancement Using Compactly Supported Orthogonal Wavelets

A Thesis

Presented for the Master of Science

Degree

The University of Tennessee, Knoxville

Christopher Carey

August, 1993

ABSTRACT

This thesis examines the use of compactly supported orthogonal wavelets for edge enhancement, within a multiresolution framework. Two methods for generating the modulus of the wavelet transform (*MWT*) image of an input image at the scale of the input image are developed. The first method involves simple linear interpolation of the *MWT* image output from each step of the wavelet pyramid. The second method involves expanding the resolution images output from each step of the wavelet pyramid to four times the size of the input image using an allpass filter. The wavelet transform of the expanded image is then used to generate the *MWT* image. Edge enhancement using the two methods was performed on a set of test images using various order compactly supported orthogonal wavelets and at various resolutions.

Based on the results from the set of test images, it was concluded that the *MWT* methods performed relatively well for images characterized by large contrast between objects within the image and small variation of gray level within each object and poorly for images characterized by small contrast between objects and large variation in gray level within each object. In addition, it was found that, in general, the lower order analyzing wavelets produced better edge enhancement results in the first method and the higher order analyzing wavelets produced better edge enhancement results in the second method.

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1 Introduction

The ultimate goal of many image processing tasks is to extract objects or features from a scene. Once objects or features are extracted from an image, further processing, such as object identification or feature characterization and enhancement can be performed. At some point in the object or feature identification process, image segmentation is used to subdivide an image into regions of similar gray level. Image segmentation techniques are based on identifying regions of the image with similar gray level values (e.g., region growing and region splitting) or with detecting discontinuities in gray level values (e.g., edge and line detection) [1].

In the image segmentation process, objects or features of interest are frequently of very different sizes and the identification of larger structures or objects may aid in the subsequent identification of smaller objects contained in or located near the larger structures. Examining an image at a coarse resolution provides an overall view of the scene and only large objects or structures are evident. At finer resolutions, the details of the image become evident and smaller objects become discernible. Therefore, in some image segmentation problems, it may be optimal to examine an image at different resolutions.

In multiresolution image analysis and processing, resolutions are computed by successive lowpass filtering of the initial image. Methods have been developed for decomposing an image into successive resolutions [2, 3, 4, 5, 6] and pattern

matching algorithms have been developed for identifying objects in the multiresolution framework [7, 8, 9, 10, 11, 12, 13, 14, 15]. In these multiresolution methods, the image segmentation process is done by edge detection. In [2, 3, 4], multiresolution image processing is performed in the Gaussian/Laplacian pyramid. In [2, 3], two options are considered for edge detection. The first involves applying a gradient operator [1] to each resolution. The second involves identifying edges by examining the difference between successive resolutions of an image. In [5, 6] multiresolution image processing is performed by the wavelet pyramid. In the wavelet pyramid, images are generated for each resolution and edges are identified at each resolution by examining the magnitude of the wavelet transform coefficients.

This thesis examines the use of the wavelet transform (*WT*) for edge, enhancement within a multiresolution framework. The *WT* is a decomposition of an image into a particular family of basis functions called wavelets. The wavelets chosen for use in this thesis for edge enhancement are localized in both time and frequency [16, 17]. That is, the wavelet, basis functions, are nonzero over for a limited time range (compact support) and the majority of their frequency content is concentrated within a limited band. The family of wavelets used for multiresolution edge enhancement are dilations and translations of one another. A wavelet similar to the ones used in this thesis have been shown to be useful in the enhancement of multiscale edges [5, 6].

The *WT* of an image results in three images, each one quarter the size of

the original image, containing the *WT* coefficients [5], referred to as the details images. Edges are identified by local maxima in the magnitude of the wavelet transform modulus. The *WT* modulus being the square root of the sum the squares of some combination of the wavelet transform coefficients composing the three details images.

1.1 Statement of Problem

The purpose of this thesis is to enhance multiscale edges in images. The method of performing this edge enhancement is to use the modulus of the wavelet transform of an image. As in the Gaussian/Laplacian pyramid, each application of the two-dimensional wavelet transform results in an image at one-half the resolution and at twice the scale as the input image; the details images are also at twice the scale. A resolution of an image is obtained by filtering the image with a halfband lowpass filter, reducing the frequency or information content of the image by one-half. Since the frequency content of the original image is now reduced by a factor of 2, every other pixel (both horizontally and vertically) may be discarded, resulting in an image one-fourth the size of the original image or at twice the scale of the original image. Here, scale is like the scale on a map: one square inch would show up as one square inch on a map at scale one, as one-fourth a square inch on a map at scale 2 and as one-eighth a square inch on a map at scale 4 (see Section 3.1 for further discussion).

In [5], using the *WT* of an image to detect multiresolution edges was first examined. This thesis continues along this line of investigation, examining methods to enhance multiscale edges of images and by using a different family of analyzing wavelets. In order to examine the enhanced edges at the same scale as the original image, the edge images may be expanded to the scale of the original image by linear interpolation. Alternatively, the resolutions, once computed, may be expanded to one-half the scale of the original image. Details images at the scale of the original image can then be generated for reasonable comparison by taking the (two-dimensional) *WT* of the half-scale image. The resulting output from these methods are a set of resolution images and a set of images showing the enhanced edges at the computed resolutions, all at the scale of the original image. Edge enhancement results from both of these methods are examined in this thesis using a set of the compactly supported, orthogonal wavelets developed in [17]. This set of wavelets was chosen because they are efficiently implemented as a set of finite impulse response filters; the higher order wavelets in this set are also very similar to those used in [5].

1.2 Related Works

In this section, a review of published works which develop or employ wavelet theory for multiresolution edge enhancement and/or image segmentation are reviewed. Several articles which provide insight and develop wavelet theory

are also discussed to provide a general background on the subject. The articles reviewed are generally categorized as being primarily mathematical (theoretical) or concerned with the use of wavelets in signal processing applications.

1.2.1 Wavelet Theory

There are a number of works which examine wavelets from a mathematical perspective [17, 18, 19, 20, 21, 22, 23]. These articles are primarily concerned with the mathematical development of wavelet theory and examine issues, such as, necessary and sufficient conditions for a function to be a wavelet and necessary conditions for the wavelet transform (WT) to have an inverse. That is, the (mathematical) conditions which must be imposed on a wavelet basis function in order for the inverse wavelet transform (IWT) to be able to reconstruct the original signal. Reference [21] is primarily a tutorial. The theory developed in [18, 19, 20, 22] is captured in [17, 23].

The most complete development wavelet theory is in [17]. In [17], the construction of an orthonormal basis of wavelets having compact support and an arbitrarily high degree of regularity is developed. The article reviews the mathematical basis of multiresolution analysis using both the Laplacian pyramid and the wavelet pyramid. From the multiresolution framework, a set of basis functions are developed which are orthogonal and have compact support. Compact support means that the wavelets are of finite duration (length), regularity refers to the

smoothness of the wavelet basis function or (mathematically) how many times a continuous derivative of the function exists. It turns out that the wavelet basis developed in [17] describes a set of optimally flat lowpass filters [16].

In [23], the use of Short Time Fourier Transform (*STFT*) [24] and the *WT* in signal analysis are compared. The main focus of the article is on the formulas for stable and accurate signal reconstruction from the *WT* coefficients. The development is more general than in [17]. Nonorthogonal wavelets are examined and the concept of a frame is developed and discussed. Basically, a frame sets an upper bound on the error that results when an image (signal, function) is reconstructed from its *WT* coefficients. It is shown that decomposition of a function into some wavelet bases result in tight frames (i.e., no error in reconstruction) even though the wavelets are not orthogonal.

1.2.2 Wavelets in Signal Analysis

The articles reviewed in this subsection, [5, 6, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35 and 36] develop and/or employ the *WT* in a signal processing application, primarily in the detection of discontinuities or edges in signals and images, or present a modification or enhancement to the application of wavelets in signal processing. The methods employed or the approach used and the results of each of the above listed articles are summarized individually below. These reviews are intended to provide a perspective on the relationship of this analysis

to previous results using wavelets in signal processing.

Grossman and Morlet [25], proposed to decompose or represent a signal as the sum of a set of functions which are dilations and translations of one another and called this set of functions wavelets. Instead of using the Gabor wavelet [37], which is a set of functions which are translations (spatial) and frequency shifted versions of one another, the wavelets developed in [25] are translations and time dilations of one another. The advantage of using a set of basis functions based on translations and time dilations is that it allows the decomposition to better detect short duration, high frequency events. This, [25], is one of the earliest applications of wavelets to a signal processing problem, where wavelets were used in the processing of seismic data.

In [26], the wavelet transform is applied to the analysis of sounds. The motivation for using wavelets in this application is their property of being increasingly concentrated in time and frequency at higher scales (here scale is interpreted as in music). The article provides criteria for selection of an appropriate wavelet basis function, the most important being a function which is band limited. Basic formulae are given for the computation of the WT and the IWT . It is noted that discontinuities in the input signal are indicated by an increase in the modulus (magnitude of) the WT coefficients. Results are presented graphically in scalograms [16], where the wavelet modulus (on a gray scale) is plotted as a function of time and scale.

In [27], the mathematical properties of a transform based on contributions of functions of constant shape are discussed. The minimal requirements on the prototype function are discussed, that is, the function must be square integrable and have a zero mean. Further discussions in the article provide an intuitive and mathematical basis for the wavelet transform's ability to detect discontinuities in signals.

Mallat [5], develops an efficient method for a multiresolution decomposition of an image using a family of orthogonal wavelets. Starting with six (intuitive) properties that an operator should have to approximate an image at one-half the original image resolution, a suitable smoothing function is developed. This smoothing (or scaling) function is shown to be able to be implemented as a low-pass filter. A wavelet function is then developed from a linear combination of translations of the scaling function. Convolution of the signal with the wavelet is shown to result in a signal which contains the difference in signals at successive resolutions. The results are first developed for a one-dimensional signal, then extended to two-dimensional signals using a separable wavelet.

Image decomposition and reconstruction in [5] is implemented using an efficient pyramid structure of scaling function and wavelet filters. A test image is decomposed into its wavelet coefficients where the sensitivity to edges of the wavelet transform is demonstrated along with its orientational selectivity.

In [28], a physiological basis for the wavelet or constant energy [16] analysis

of signals is discussed and compared to the *STFT* and the Gaussian/Laplacian pyramid method. The major points covered in [28] are essentially those developed and presented in [5].

In [34], edge detection (image segmentation) is performed at a single resolution using an optically created Harr wavelet. It is noted that the size of the analyzing wavelet determines the resolution at which the edges are detected. A large wavelet yields edge detection at a coarse resolution and a small wavelet reveals edges at a fine resolution. Results are presented for a 512×512 binary test image segmented by a 16×16 pixel (Harr) wavelet.

Mallat and Hwang [6], demonstrate how signals can be characterized by the maximum of the modulus of their *WT* and that in two-dimensions, local maxima of the wavelet transform modulus indicate the existence of edges. It is shown how reconstruction of an image from the maxima of the *WT* modulus results in no visible distortion in an image and how modification of modulus maxima can be used in denoising an image. The wavelet transform modulus for a two-dimensional signal is defined as the square root of the sum of the squares of the horizontal and vertical *WT* coefficients (see also [16]). The *WT* phase is defined as the arctangent of the ratio of the vertical to horizontal *WT* coefficients and points in the direction of greatest change.

In [6] it is shown that the local maxima of the *WT* modulus can be used in the detection of sharp variations in a signal (edges) when the analyzing wavelet

is proportional to the first derivative of a smoothing function. Zeros of the WT modulus indicate the existence of a sharp variation (edge) in a signal when the analyzing wavelet is proportional to the second derivative of a smoothing function. Edge detection by the former method has the advantage of keeping the value of the WT coefficients for later image reconstruction. The characterization of a signal by its WT maxima is considered a complete representation of the signal. Reconstructing a signal from the WT modulus maxima result in some errors in the original signal; however, most of these were found to occur in higher frequencies [6].

In [6], it was found that the number of maxima increases linearly with the number of moments of the analyzing wavelet. Therefore, it is best to choose an analyzing wavelet with the minimum number of moments necessary to detect the interesting irregular behavior of a signal. For edge detection, a wavelet with a single vanishing moment is considered adequate.

In [6], the WT modulus maxima of a (one-dimensional) signal are plotted as a function of position (time) and scale. The value of the WT modulus maxima being indicated by its magnitude on a relative scale. It is shown how sharp variations in a signal propagate from one scale to another, forming a maxima line. It is shown that when the WT modulus maxima is mostly due to noise it does not propagate to coarser scales and denoising can be accomplished by removing modulus maxima which do not propagate to coarser scales (i.e., short maxima

lines).

In [33], the reconstruction of an image from the zero crossings of its WT is explored. A procedure is developed for reconstructing an image from the zero crossings of its WT . A stereo matching problem is also solved by computing the distance between matched zero crossings of two images.

When the analyzing wavelet is the second derivative of a smoothing function, then the zero crossings indicate point of variation in the signal. It is shown how to recover the WT of a one-dimensional signal from the position of these zero crossings and a projection operator. The IWT can then be used to recover the image. It is shown by numerical experiment that the reconstruction of an image in this manner is accurate and stable.

Wilson, Calway and Person [30], developed the Multiresolution Fourier Transform (MFT) in this reference. The MFT is considered a superset of the WT and the $STFT$ and has the ability to represent any signal whose support is confined to a compact region without interference from neighboring features. This formulation guarantees that any arbitrary signal feature can be extracted.

In [30], a very general discussion of the problem of identifying an object based on its phase space representation in a multiresolution framework is first presented. Then, the MFT and its inverse are developed and used in an image segmentation application. Maximum likelihood estimation and decision procedures are employed to determine the edges of objects from the MFT coefficients. In

[30], it is concluded that the *MFT* alone is not well suited for image segmentation since a rather complicated line search algorithm needed to be employed.

In [29], the shiftability of the *WT* is discussed. It is noted that a wavelet representation of a signal is based on convolution and subsampling and that translations of the input signal does not result in simple translations of the *WT* coefficients. Instead, a weaker definition of translational invariance is considered, one where the information in a subband (resolution) stays within the subband when the signal is translated. A method is devised where a set of interpolation functions are computed based on the wavelet filter coefficients. The interpolation functions can then be used to select an arbitrary scale, orientation and position of the *WT* of an image based on existing *WT* coefficients. A jointly shiftable *WT* is developed and applied to a stereo matching problem and an image denoising problem.

In [36], techniques are developed for determining orthogonal wavelets of a specified support which results in the best approximation of a signal at a given resolution. In the article, it is shown that the choice of the analyzing wavelet strongly influences the quality of a (limited) reconstruction of a signal from its *WT*. The problem is shown to be one of minimizing the ℓ_2 norm of the signal approximation error at a given resolution. Linear programming methods are used to obtain a solution to the optimization problem.

Rioul and Duhamel [35] developed and compared methods for efficiently

calculating the WT of a signal. Methods that have been applied in speeding up the Fast Fourier Transform are modified as necessary and applied in the computation of the WT . The various algorithms are compared on the basis of the number of multiplications and number of additions required.

Kovacivic and Vetterli [38] have examined the implementation of the WT in multidimensional perfect reconstruction filter banks. Methods are presented for the calculation of the WT of separable (see discussion of [5] above) and non-separable, two-dimensional orthogonal wavelets. It is also shown how wavelet bases can be generated from perfect reconstruction filter banks.

1.2.3 Summary of Related Works

A summary of the survey of published works on the theory of wavelets and their application to various signal processing tasks is presented in this section.

A number of works [17, 18, 19, 20, 21, 22, 23] were found to be primarily concerned with the development of wavelet theory. These articles examined such things as necessary conditions for a function to be a wavelet and necessary conditions for the wavelet transform of a function to have an inverse. The most complete development of wavelet theory was found in [17]. The article by Strang [30] was found to provide a good introduction to wavelet theory.

Additional articles were reviewed [5, 6, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36] which develop and/or employ the wavelet transform in signal processing

applications such as in the detection of discontinuities or for image enhancement. Two articles were found to be informative and provide some of the basis for the edge detection methods developed in Section 3. The original article by Mallat [5], develops an efficient method for multiresolution decomposition of an image using a family of orthogonal wavelets. The article by Mallat and Hwang [6] provides an in-depth discussion and intuitive basis for understanding the use of the wavelet transform in the detection of edges in images. In addition to the above, the tutorial in [16] was found to provide perspective on the relationship of wavelet theory to other signal processing techniques, such as, subband coding, the Short Time Fourier Transform and the multiresolution pyramid.

1.3 Synopsis

The remainder of this thesis is organized as follows. Section 2 presents an overview of wavelet theory, focusing on the use of the *WT* in multiresolution analysis, the generation of compactly supported, orthogonal wavelets and efficient implementation of the *WT* in multidimensional filter banks. In Section 3, two methods for performing multiresolution edge enhancement using the *WT* are presented. The two methods presented in Section 3 are used for multiresolution edge enhancement on a varied set of images in Section 4. Finally, results and conclusions are summarized in Section 5.

2 Wavelet Theory

Wavelet theory is a unified, general description of a particular family of functions which are localized in both time and frequency. The translations and dilations of a prototype wavelet form a family of wavelets which can be used to decompose an arbitrary signal into a linear combination of the members of the wavelet family.

Wavelets are suitable for the analysis of nonstationary signals. Nonstationary signals are signals whose properties (e.g., mean and standard deviation) evolve with time [39]. Unlike the Short Time Fourier Transform which uses a constant (time-width) window function over all frequencies, the wavelet transform (WT) uses a variable (time-width) window function which is small at high frequencies and large at low frequencies. This allows the WT to look closely at small details and globally at coarser details. This is discussed further in Section 2.1.

Wavelet theory has been used in explaining the underlying basis for several signal processing techniques, such as multiresolution analysis [5, 17] and subband coding methods [16, 38]. Multiresolution analysis is used in signal and image processing for extracting information from a signal or image at different resolutions. Information or features present in the signal or image may be obscured at a given resolution and evident at another. Subband coding methods are used for data compression. Coding methods allow for data compression by providing an acceptable approximation to the original signal or image by a fewer point (or reduced

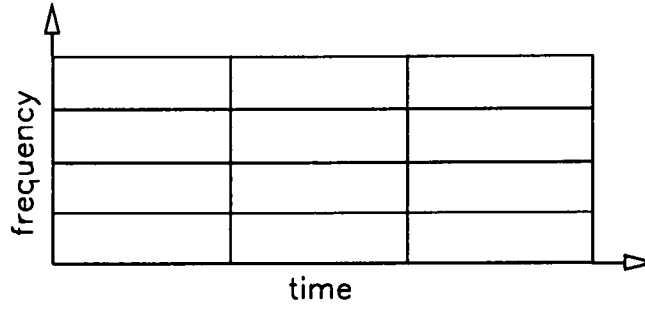
bandwidth) signal. The wavelet theory behind these signal processing techniques is discussed further in Sections 2.2 and 2.3.

As discussed above, wavelets are a family of functions which are dilations and translations of one another. Under certain conditions these families form a basis. A basis is a set of linearly independent functions or vectors which can represent any other function or vector as a unique linear combination of the members of the basis [40]. The implementation of the wavelet transform is presented in Section 2.4 and a method for construction of an orthogonal basis of wavelets having compact support and an arbitrarily high degree of regularity is outlined in Section 2.5. Members of this basis of wavelets is used in the edge enhancement methods proposed in Section 3.

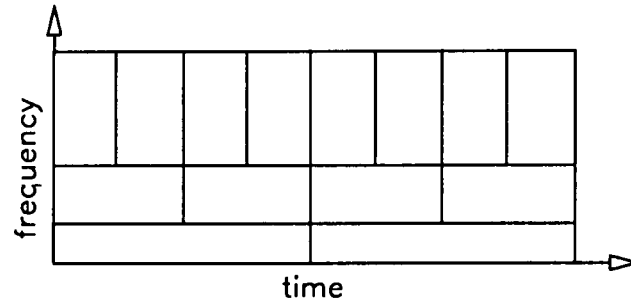
2.1 The *STFT* and the Wavelet Transform

In signal analysis problems, it is often advantageous to examine portions of a signal where there are sharp variations. These locations of sharp variation indicate the start of a note in music [26] or the location of a contour line in images [6]. The Fourier Transform does not allow for the interference free analysis of these locations of sharp variation, but instead provides a global view of the frequency content of the signal.

The Short Time Fourier Transform (*STFT*) is a method of adapting the Fourier Transform to the analysis of locations of sharp variation in a signal. In



(a) Short Time Fourier Transform.



(b) Wavelet Transform.

Figure 2.1: Division of the Time and Frequency Plane for the Short Time Fourier Transform and the Wavelet Transform.

Adapted from: O. Rioul and M. Vetterli, "Wavelets and Signal Processing," *IEEE SP Magazine*, October 1991, pp. 14–38.

the *STFT*, the signal $x(t)$ is first multiplied by a function $w(t)$ which is limited in time and centered about some time point, t_0 . The Fourier Transform of the windowed signal is referred to as the Short Time Fourier Transform:

$$STFT(\tau, f) = \int_{-\infty}^{\infty} x(t)w^*(t - \tau)e^{-2j\pi ft} dt \quad (2.1)$$

where, $w^*(t)$ is the conjugate of $w(t)$. For a given $w(t)$, the window function has a fixed concentration in both time and frequency and the time frequency plane is divided into equal size rectangles as shown in Fig. 2.1(a). When the window function is a Gaussian, the above transform is sometimes referred to as the Gabor

transform [37].

Wavelets are a family of basis functions that are constructed by dilations and translations of a single prototype function, $g(t)$, as shown below [17]:

$$g_{a,b}(t) = \frac{1}{\sqrt{|a|}} g\left(\frac{t-b}{a}\right). \quad (2.2)$$

where, a , is a dilation (or contraction) parameter and, b , is a translation parameter.

The wavelet, $g(t)$, must satisfy the condition:

$$\int_{-\infty}^{\infty} g(t) dt = 0. \quad (2.3)$$

Note that the above condition requires $g(t)$ to have at least some oscillations. The wavelet transform (WT) of a signal, $x(t)$, is defined by the inner product:

$$WT(\tau, a) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t) g^*\left(\frac{t-\tau}{a}\right) dt \quad (2.4)$$

where, $g^*(t)$ is the conjugate of $g(t)$ and $WT(\tau, a)$ is a measure of the correlation between $x(t)$ and $g(t)$ (basis function), that is, the $WT(\tau, a)$ has a large magnitude when $x(t)$ closely matches $g(t)$ (or its negative). When the wavelet, $g(t)$, is thought of as a windowing function, it can be seen that the time width of the window decreases and the frequency extent of window increases as one moves to higher and higher frequency, see Fig. 2.1(b).

The original signal, $x(t)$, is recovered from its wavelet transform by the inverse wavelet transform (IWT), given by the equation:

$$x(t) = c \int_0^a \int_{-\infty}^{\infty} WT(\tau, a) g_{a,\tau}(t) \frac{da d\tau}{a^2} \quad (2.5)$$

where, c is a normalizing constant.

The continuous wavelet transform can be discretized by fixing a dilation step size $a_0 > 1$ and a translation step $b_0 > 0$. The family of analyzing wavelets become:

$$g_{j,k}(t) = a_0^{-j/2} g\left(\frac{t - b_0 k}{a_0^j}\right) \quad (2.6)$$

where j and k are integers greater than or equal to zero. The WT of the discretized signal, $x(n)$, is then calculated by:

$$WT_j(k) = \sum_n x(n) g_{j,k}^*(n) \quad (2.7)$$

and the IWT is calculated by:

$$x(n) = \frac{2}{A+B} \sum_j \sum_k WT_j(k) g_{j,k}(n) \quad (2.8)$$

where A and B are frame bounds. The frame bounds place limits as to the accuracy with which the discretized signal, $x(n)$, can be reconstructed from its WT . That is,

$$A \|x\|^2 < \sum_j \sum_k |WT_j(k)|^2 < B \|x\|^2 \quad (2.9)$$

where,

$$\|x\|^2 = \sum_n |x(n)|^2.$$

When A and B in Eq. 2.9 are equal to 1, then the frame is called a tight frame and exact reconstruction can take place [17]. As might be expected when the discrete dilation parameter a_0 is near 1, $x(n)$ is finely sampled and exact reconstruction

can take place for all $g(n)$, satisfying Eq. 2.3. As a_0 becomes larger, approaching a limit of $a_0 = 2$, the sampling of $x(n)$ becomes sparse and exact reconstruction can take place only for special choices of $g(n)$ [16]. In this limiting case, an arbitrary function can be expressed as a combination of scaling functions, $h(n)$, and a summation of wavelets:

$$x(n) = \sum_k b_{J,k} h_j(n - 2^j k) + \sum_j \sum_k c_{j,k} g_j(n - 2^j k) \quad (2.10)$$

where,

$$b_{J,k} = \sum_n x(n) h_J^*(n - 2^j k).$$

and

$$c_{j,k} = \sum_n x(n) g_j^*(n - 2^j k).$$

Scaling functions and wavelets are closely related, a wavelet is formed by a weighted average of scaling functions. The scaling function is a smoothing function or lowpass filter and the first term in Eq. 2.10 can be considered a resolution of the original signal. The relationship between scaling functions and wavelets is discussed further in Section 2.5.

2.2 Multiresolution Pyramid

The wavelet transform of a signal can be efficiently implemented by pair of highpass and lowpass filters in a pyramid structure similar to the multiresolution pyramid used in image processing [2,3]. The multiresolution pyramid is discussed

in this section to introduce the concepts of resolution and scale and provide a basis for the discussion of the wavelet pyramid in Section 2.4.

In multiresolution analysis, a set of successively blurred images are obtained by repeatedly convolving the original image with a smoothing function creating a set of blurred versions of the original image. The blurred image at each step serves as the input image to the next convolution of the image and the smoothing function. The implementation of one step of a multiresolution pyramid is shown in Fig. 2.2. The purpose in creating this succession of blurred images is that successively larger details of the original image are washed-out after each convolution of the input image allowing for easier detection of the remaining (large scale) objects.

The smoothing function, $h(n)$, is a lowpass filter. After filtering, the blurred image is subsampled (every other sample discarded). If the lowpass filter is a halfband lowpass filter, the highest frequency in the blurred image is now one-half the highest frequency in the original image. therefore, according to the Whitaker Shannon sampling theorem, every other sample may be discarded (vertically and horizontally) and the input image recovered completely from the blurred, subsampled image without aliasing [1].

The blurred subsampled image is now one-quarter the size of the input image. For reasonable comparison and subsequent processing of the set of blurred images, it is desirable to have them all at the same size (scale). In order to expand

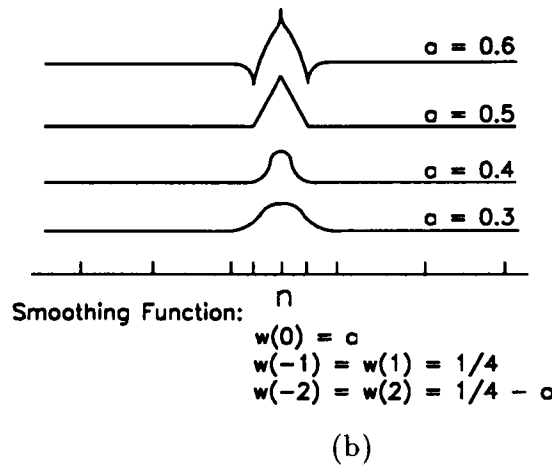
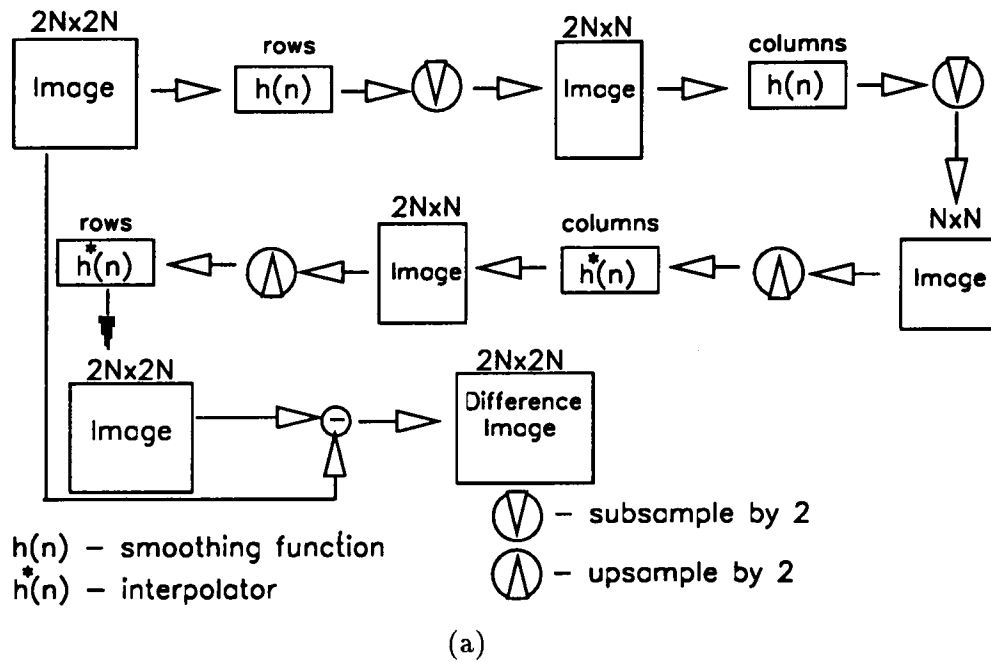


Figure 2.2: Image Decomposition in the Gaussian/Laplacian Pyramid. Schematic of the method is shown in (a). Commonly used smoothing functions are shown in (b).

Adapted from: P. J. Burt and E. H. Adelson, "The Laplacian Pyramid as a Compact Image Code," *IEEE Transactions on Communications*, Vol. COM-31(4), April 1983, pp. 532-540.

the blurred, subsampled images to the size of the original image, an interpolation function, $h^*(n)$, is applied which halves the scale of the input image while maintaining the same resolution. This interpolation operator is the conjugate of the lowpass filter used in blurring the image.

Once the set of blurred images are all returned to the scale of the original image, edge enhancement using a gradient operator or other methods may be applied to the set of blurred images to determine their large and small features. Another method of edge enhancement is obtained by subtracting the n^{th} image from the $n + 1$ image in the pyramid obtaining a difference image. Since the $n + 1$ image is a smoothed version of the n^{th} image, the difference between them will be large at the edges of objects or image contours.

Another way of looking at this is to note that the smoothing function tends to average the pixels in a neighborhood. When there is no variation of the image in the neighborhood, the value of the pixels will be about the same at the n and $n + 1$ resolution and the difference between the n and $n + 1$ resolutions will be near zero. When there is variation in the signal in the neighborhood, such as near the edge of an object, there is a nonzero difference between the n and $n + 1$ resolutions. Note that exact recovery of any image in the set is possible by retaining the last blurred image and all the difference images, described above. This can be done by starting with the last blurred image N of the pyramid and adding the $N - 1$ difference image, creating the $N - 1$ image of the pyramid. This process can

be repeated going backward through each step of the pyramid until the original image is recovered.

2.3 Subband Coding

In the multiresolution pyramid discussed in Section 2.2, resolutions are generated and scaled using a pair of conjugate lowpass filters. The highpass portion of the signal can be recovered from the difference between successive resolutions. The highpass (or bandpass) portion of the original signal can also be determined directly using a pair of highpass and lowpass filters. As is shown in Section 2.4, the wavelet transform and the inverse wavelet transform can be implemented as a set of quadrature mirror filters (QMFs), similar to the type used in subband coding of signals. The subband coding of a signal and properties of QMFs are discussed below to provide a basis for the discussion of an efficient implementation of the *WT* and *IWT* in Section 2.4.

A slightly different procedure used in the subband coding of signals or images than used for the multiresolution pyramid. The one-dimensional case is shown in Fig. 2.3. Here the original signal is convolved with a halfband lowpass filter, $h(n)$, and a halfband highpass filter, $g(n)$. Both the highpass and lowpass signals are now at one-half their original bandwidth and are subsampled by two (every other sample discarded). The original signal can be recovered, as shown in Fig. 2.3, by inserting a zero between each sample and using a lowpass

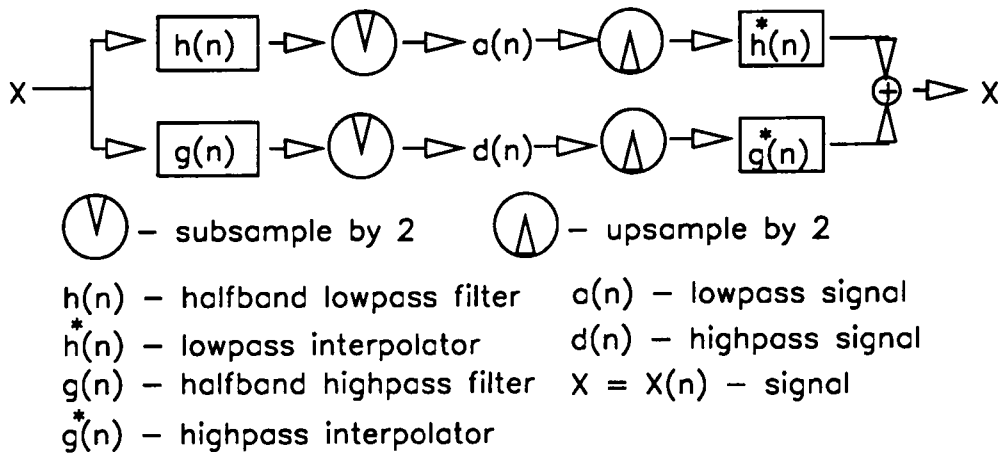


Figure 2.3: Subband Coding of a Signal into a Highpass and Lowpass Signal. Decomposition, transmission and reconstruction of the original signal is shown.

Adapted from: O. Rioul and M. Vetterli, "Wavelets and Signal Processing," *IEEE SP Magazine*, October 1991, pp. 14–38.

and highpass interpolator which are, respectively, conjugates of the lowpass and highpass filters.

Optimally, the lowpass and highpass filters should have a frequency response as shown in Fig. 2.4(a). This frequency response is impractical, so filters with a frequency response as shown in Fig. 2.4(b) are normally constructed [41]. Since the frequency response of the highpass and lowpass filters overlap, there is some aliasing when the highpass and lowpass signals are subsampled. To avoid aliasing in the reconstructed signal, the reconstruction or synthesis filters are chosen as to cancel the aliasing resulting from the use of the (imperfect) decomposition or analysis filters.

In the z -transform space, the entire decomposition and reconstruction pro-

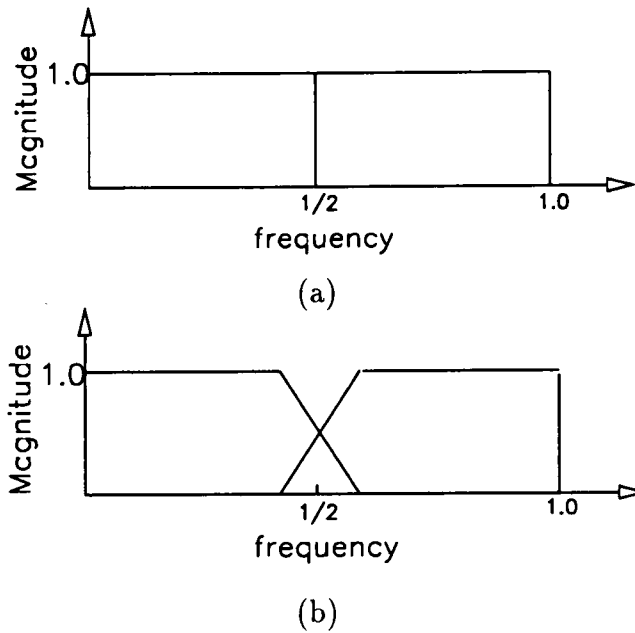


Figure 2.4: Magnitude of the Fourier Transform of Halfband Lowpass and High-pass Filters. The ideal filters are shown in (a). The filters commonly used in perfect reconstruction filter banks are shown in (b).

Adapted from: P. P. Vaidyanathan, "Quadrature Mirror Filter Banks, M-Band Extensions and Perfect Reconstruction Techniques," *IEEE ASSP Magazine*, July 1987, pp. 4-20.

cedure shown in Fig. 2.3 can be expressed as:

$$\hat{x}(z) = \frac{1}{2} \left[h(z)h^*(z) + g(z)g^*(z) \right] x(z) + \frac{1}{2} \left[h(-z)h^*(z) + g(-z)g^*(z) \right] x(-z) \quad (2.11)$$

where,

- x —is the input signal,
- h —is the lowpass decomposition filter,
- g —is the highpass decomposition filter,
- h^* —is the lowpass reconstruction filter and
- g^* —is the highpass reconstruction filter.

Note that the second term in Eq. 2.11 represents the aliasing caused by the decom-

position filters having a frequency response as shown in Fig. 2.4(b). By choosing the reconstruction filters as:

$$h^*(z) = g(-z) \quad (2.12)$$

and

$$g^*(z) = -h(-z), \quad (2.13)$$

the aliasing term in Eq. 2.11 is equal to zero and the transfer function of the system shown in Fig. 2.3 can be written as:

$$T(z) = \frac{\hat{X}(z)}{X(z)} = \frac{1}{2} [h(z)g(-z) - g(z)h(-z)]. \quad (2.14)$$

When the highpass and lowpass filters are chosen such that:

$$g(z) = h(-z), \quad (2.15)$$

the transfer function becomes:

$$T(z) = \frac{1}{2} [h(z)^2 - g(z)^2] = \frac{1}{2} [h(z)^2 - h(-z)^2]. \quad (2.16)$$

The filters described above are referred to as quadrature mirror filters [42, 43, 44, 45, 46, 47, 48] because the lowpass and highpass filters are mirror images of one another in the z -domain (see Eq. 2.15).

Various methods have been developed for designing the lowpass filters so that there is no amplitude distortion [41] and methods have been developed for designing these filters so that there is no amplitude or phase distortion [44, 45, 46, 47], the latter are said to have the perfect reconstruction property.

2.4 Wavelet Transform Implementation

As discussed in Section 2.1, wavelets are a family of functions which are limited in the frequency domain, just as the highpass and lowpass filters in the subband coding scheme described in the previous section. This makes taking the wavelet transform of a signal (as given by Eq. 2.10) adaptable to implementation in a subband coding scheme. In the wavelet decomposition, the lowpass signal gives the first term of Eq. 2.10 and the highpass signal gives the second term of Eq. 2.10. That is, convolving the input signal with a scaling function is the same as lowpass filtering the signal and subsampling by two. Convolving the input signal with the (associated) wavelet is the same as highpass filtering the signal and subsampling by two. The lowpass signal serves as input to the decomposition filter bank and the process is repeated. The resulting lowpass and highpass and subsampled signals correspond to a convolution with a scaling function and its associated wavelet of twice the size of the previous application.

The analyzing wavelets used in this thesis are the compactly supported, orthogonal wavelets developed in [17]. This family of wavelets was chosen because they can be efficiently implemented in a set of finite impulse response filters. In addition, higher order members of this family approach the shape of the wavelet used in [5]. The compactly supported, orthogonal wavelets, developed in [17] are described in detail in Section 2.5. Four members of this family of wavelets are used as analyzing wavelets in this section. These are designated D2, D4, D8 and D16,

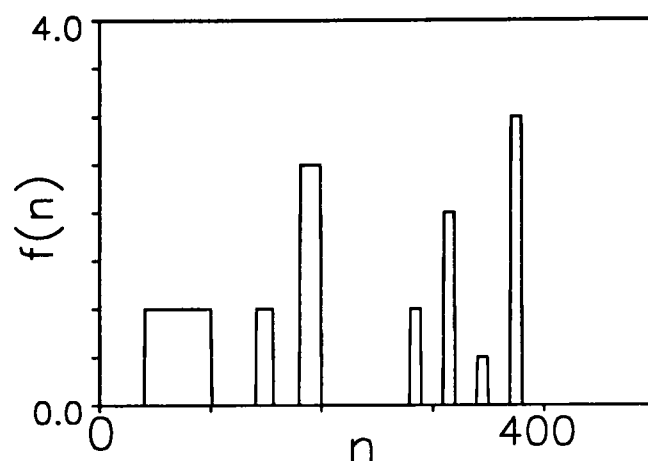


Figure 2.5: Example Signal Having (One-dimensional) Small Scale and Large Scale Features.

where the integers (i.e., 2, 4, 8 and 16) refer to the number of filter coefficients needed in taking the wavelet transform of a signal.

The decomposition of the signal shown in Fig. 2.5 into its wavelet transform is illustrated in Fig. 2.6. The resolutions shown in Fig. 2.6 become successively smoother at lower and lower resolutions. Using the notation of [5], the $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{1}{16}$ resolutions of the signal $f(n)$ are designated $A1f(n)$, $A2f(n)$, $A3f(n)$ and $A4f(n)$, respectively. Since each application of the analysis filter bank results in a signal with half the number of samples as the preceding resolution, linear interpolation is used between the remaining samples to fill out the resolution curves shown in Fig. 2.6. The magnitude of the wavelet transform is also shown in Fig. 2.6. Note that the magnitude of the wavelet transform is large near points of high variation in the signal and near zero when the signal is smooth. The analyzing

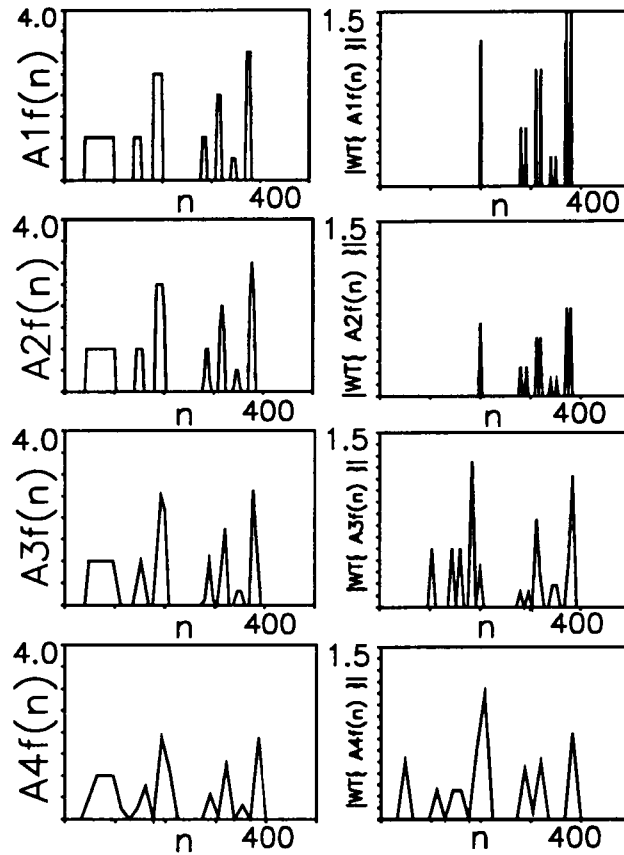


Figure 2.6: Signal Decomposition Using the D2 (or Harr) Wavelet. The signal shown in Fig. 2.5 is shown at resolutions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{1}{16}$ ($A1f(n)$, $A2f(n)$, $A3f(n)$ and $A4f(n)$, respectively) are shown in the lefthand column of plots. The magnitude of the wavelet transform for these resolutions are shown in the righthand column of plots. The analyzing wavelet is the D2 (or Harr) wavelet shown in Fig. 2.7.

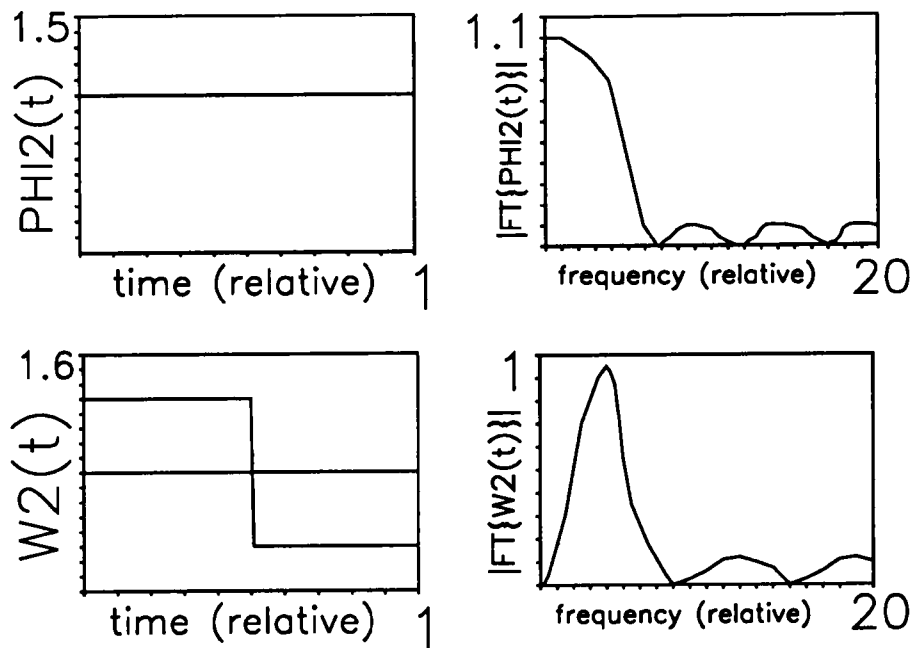


Figure 2.7: The D2 (or Harr) Wavelet. The scaling function, $PHI2(t)$, and the associated wavelet, $W2(t)$, are shown in lefthand column of plots. The magnitude at the Fourier Transform of $PHI2(t)$ and $W2(t)$ are shown in the righthand column of plots.

wavelet is the D2 (or Harr) wavelet, shown in Fig. 2.7. Also, as can be seen from Fig. 2.6, due to the compact support of the D2 wavelet, which is zero outside the interval $(0,1)$ it is possible for some locations of high signal variation to go undetected over several resolutions.

Wavelets D4, D8 and D16, shown in Figs. 2.8, 2.9 and 2.10, respectively, are used to decompose the signal used in Fig. 2.5 into its wavelet transform(s). These are shown in Figs. 2.11, 2.12 and 2.13, respectively. It is interesting to note that the higher order wavelets approach the ideal filter shape of Fig. 2.4(a) and that locations of sharp variations in the input signal are more evident in

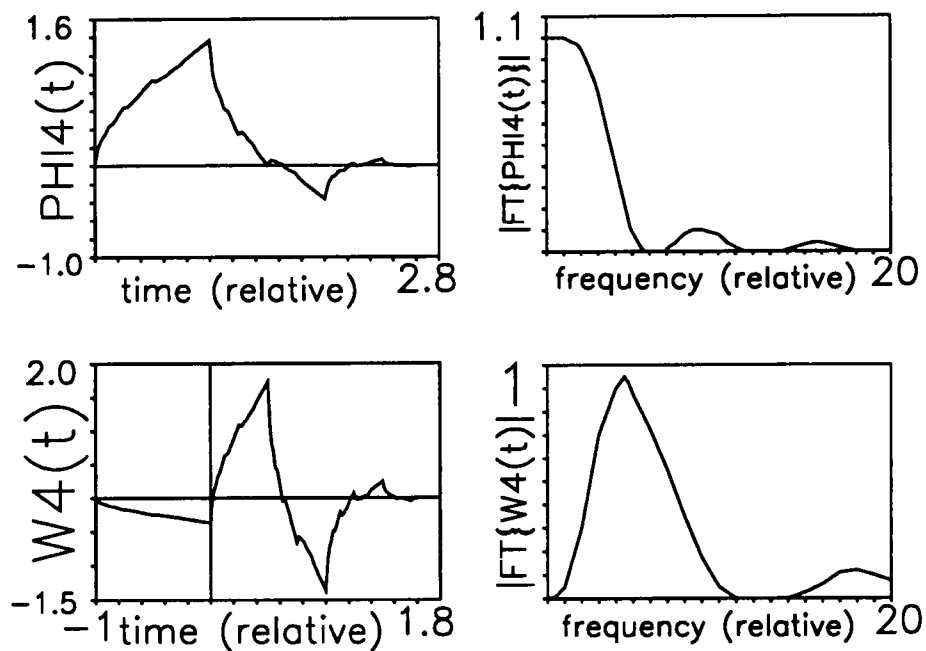


Figure 2.8: The D4 Wavelet. The scaling function, $\text{PHI4}(t)$, and the associated wavelet, $W4(t)$, are shown in the lefthand column of plots. The magnitude at the Fourier Transform of $\text{PHI4}(t)$ and $W4(t)$ are shown in the righthand column of plots.

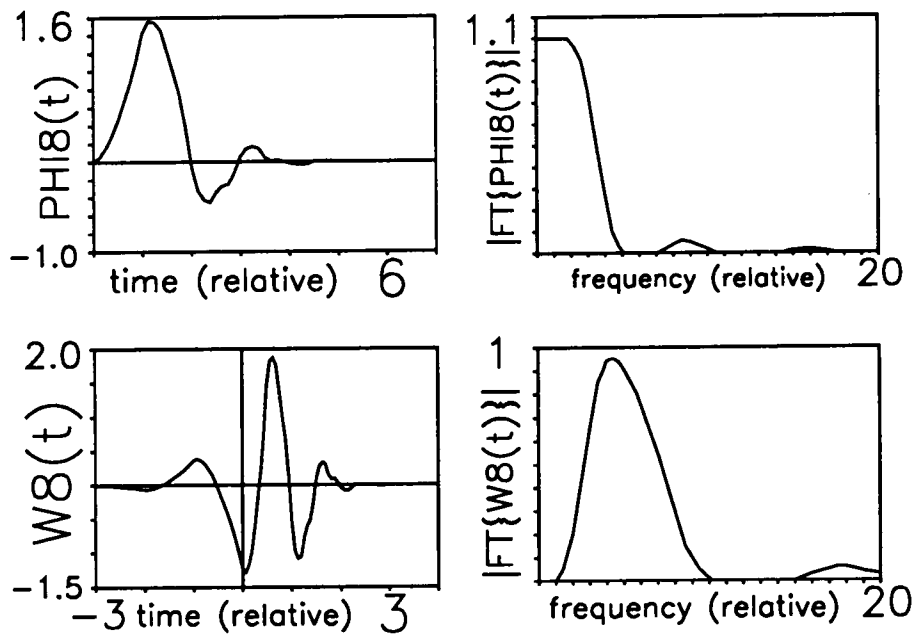


Figure 2.9: The D8 Wavelet. The scaling function, $\text{PHI8}(t)$, and the associated wavelet, $\text{W8}(t)$, are shown in the lefthand column of plots. The magnitude at the Fourier Transform of $\text{PHI8}(t)$ and $\text{W8}(t)$ are shown in the righthand column of plots.

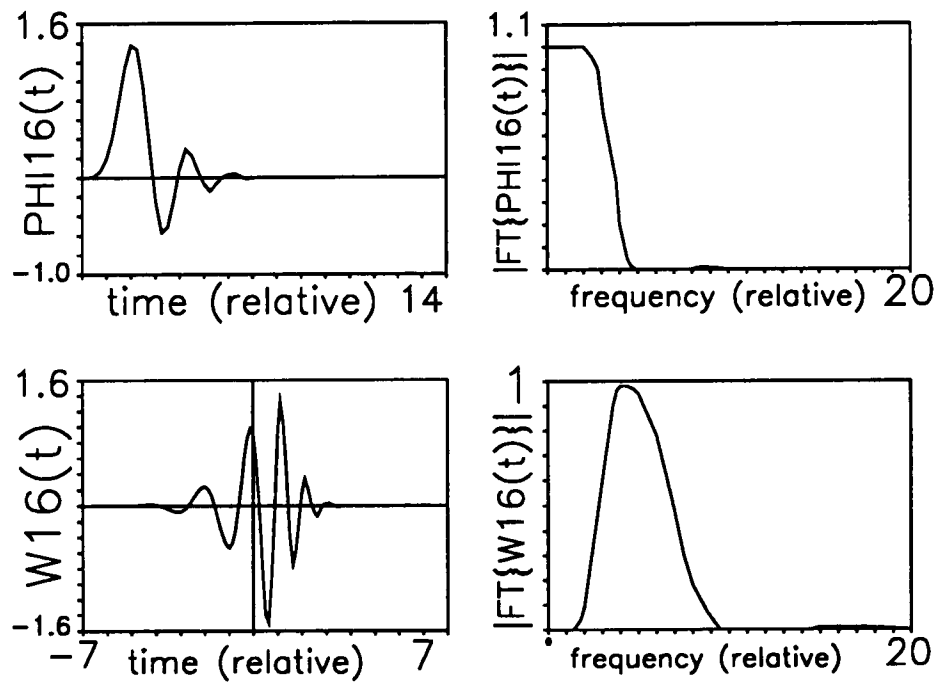


Figure 2.10: The D16 Wavelet. The scaling function, $PHI16(t)$, and the associated wavelet, $W16(t)$, are shown in the lefthand column of plots. The magnitude at the Fourier Transform of $PHI16(t)$ and $W16(t)$ are shown in the righthand column of plots.

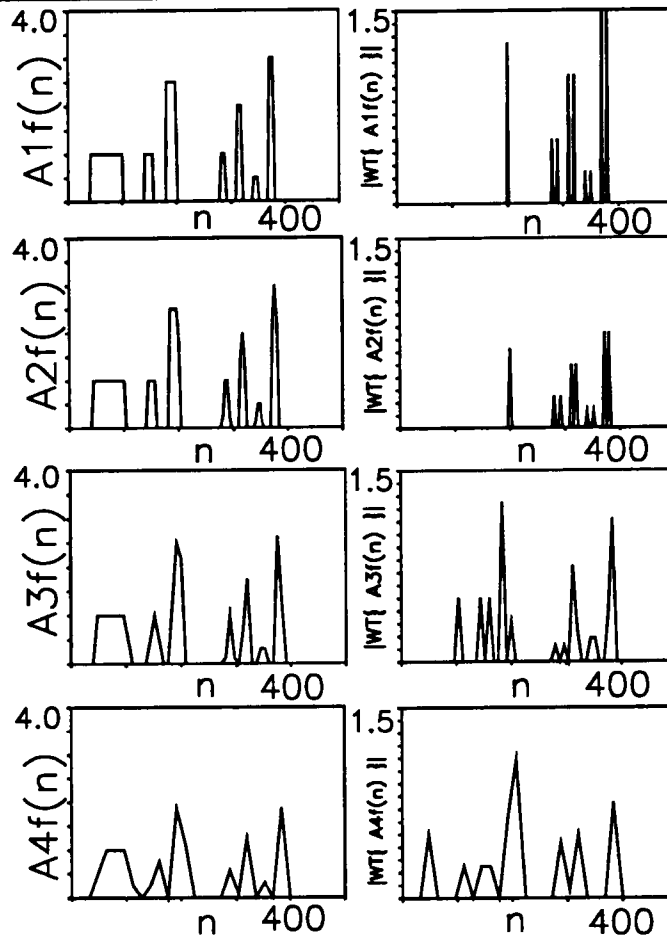


Figure 2.11: Signal Decomposition Using the D4 Wavelet. The signal shown in Fig. 2.5 is shown at resolutions $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ ($A1f(n)$, $A2f(n)$, $A3f(n)$ and $A4f(n)$), respectively in the lefthand column of plots. The magnitude of the wavelet transform for these resolutions are shown in the righthand column of plots. The analyzing wavelet is the D4 wavelet shown in Fig. 2.8.

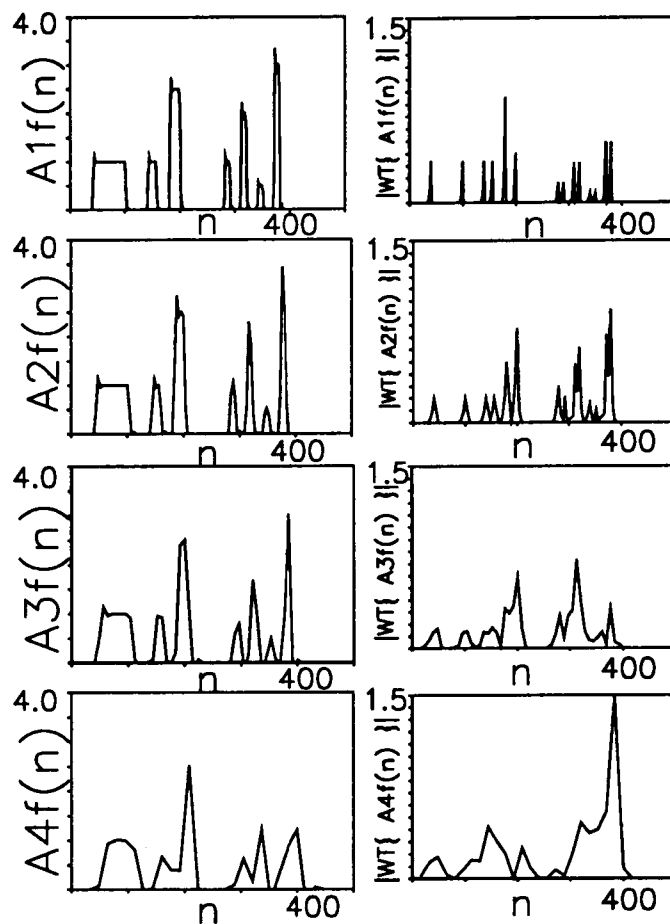


Figure 2.12: Signal Decomposition Using the D8 Wavelet. The signal shown in Fig. 2.5 is shown at resolutions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{1}{16}$ ($A1f(n)$, $A2f(n)$, $A3f(n)$ and $A4f(n)$, respectively) in the lefthand column of plots. The magnitude of the wavelet transform for these resolutions are shown in the righthand column of plots. The analyzing wavelet is the D8 wavelet shown in Fig. 2.9.

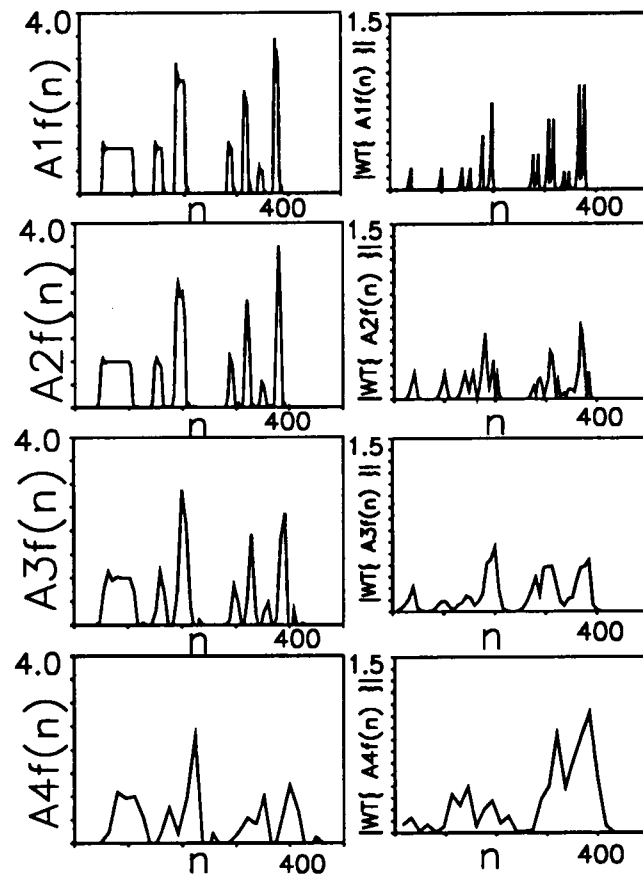


Figure 2.13: Signal Decomposition Using the D16 Wavelet. The signal shown in Fig. 2.5 is shown at resolutions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ and $\frac{1}{16}$ ($A1f(n)$, $A2f(n)$, $A3f(n)$ and $A4f(n)$, respectively) in the lefthand column of plots. The magnitude of the wavelet transform for these resolutions are shown in the righthand column of plots. The analyzing wavelet is the D16 wavelet shown in Fig. 2.10.

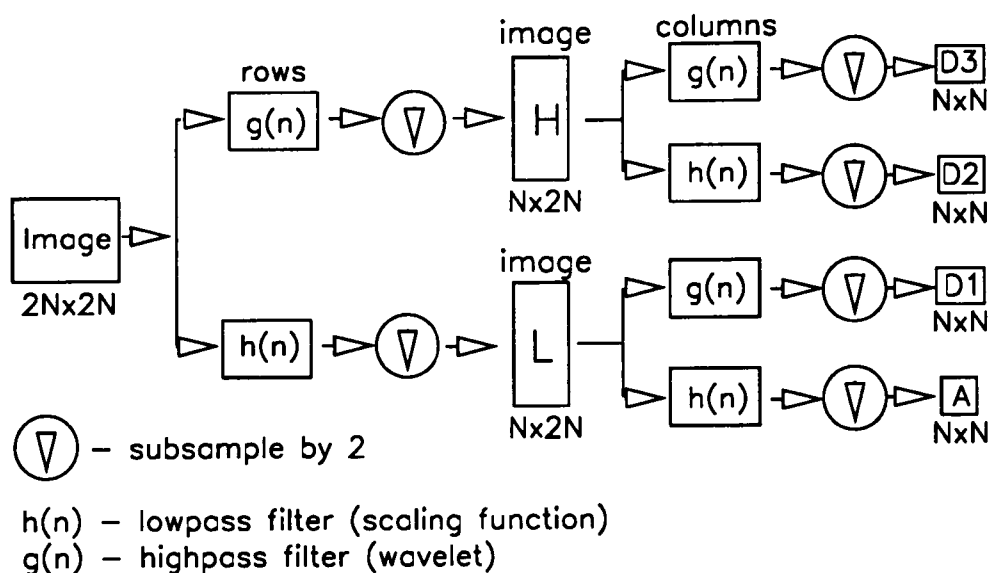


Figure 2.14: Two-dimensional Subband Coding Scheme used to Implement the Wavelet Transform of an Image.

Adapted from: S. G. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 11(7), July 1989, pp. 674-693.

the magnitude of the wavelet transform. However, as the order of the analyzing wavelet increases, the local maxima in the wavelet transform (the peaks) tend to overlap more at each resolution. This is expected since the support of the wavelets (as shown in Figs. 2.7 through 2.10) increase with increasing order.

Now consider the above-described subband coding scheme implemented in two-dimensions as developed in [5] and shown in Fig. 2.14. Lowpass filtering and subsampling (by two) each row of the input image results in an image which is lowpass horizontally, designated image **L**, for the purpose of this discussion. Conversely, highpass filtering and subsampling result in an image which is highpass

horizontally, designated image **H**, for the purpose of this discussion. Next, lowpass filter and subsample of each column of **L**, this results in an image one-quarter the size of the original image which is lowpass horizontally and vertically, this is the resolution image, designated **A** [5]. Highpass filtering and subsampling each column of **L** results in an image which is lowpass horizontally and highpass vertically, designated **D1** [5]. In the **D1** image, vertical edges are indicated by a local maximum in the magnitude of the pixels. Lowpass filtering and subsampling each column of **H** results in an image which is highpass horizontally and lowpass vertically, designated **D2** [5]. In the **D2** image, horizontal edges are indicated by a local maximum in the magnitude of pixels. Highpass filtering and subsampling each column of **H**, results in an image which is highpass horizontally and vertically, designated **D3** [5]. In the **D3** image, corners are indicated by a local maximum in the magnitude of the pixels. The **A** image serves as the input image to the next step of the pyramid. An image can be reconstructed from the **A**, **D1**, **D2**, and **D3** images as shown in Fig. 2.15.

The orientational selectivity of the wavelet pyramid is illustrated using the test image of Fig. 2.16. In Fig. 2.17(a), the $\frac{1}{2}$ resolution **A** image is shown and the **D1**, **D2**, and **D3** images are shown in Fig. 2.17(b). The magnitude of the *WT* is indicated by the darkness of the pixels on a reverse gray scale, that is, the black pixels have a large magnitude and white pixels have a small magnitude. Similar to the definition of the modulus of the wavelet transform given in [6], a

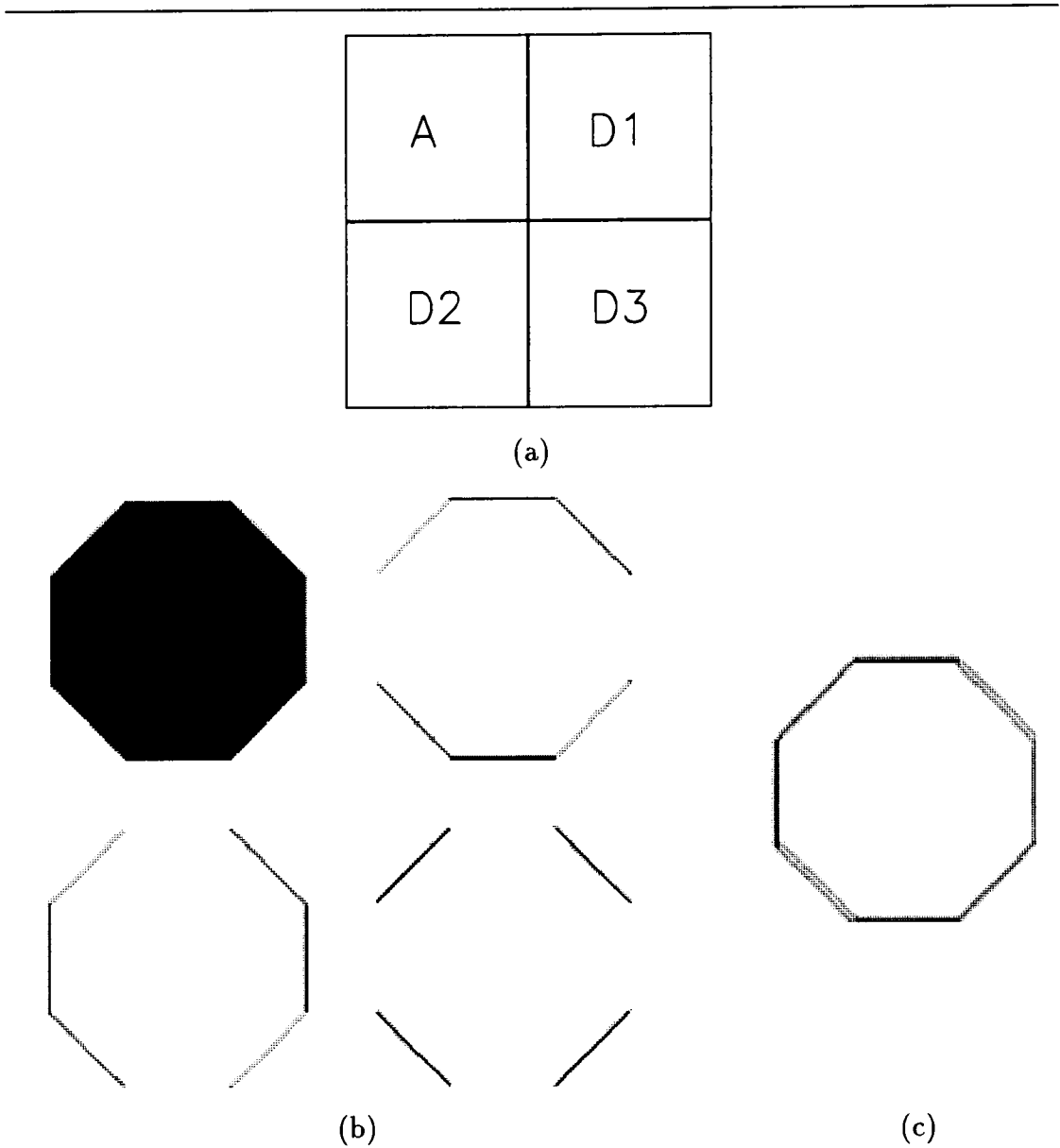


Figure 2.17: Wavelet Decomposition of an Image. The **A**, **D1**, **D2** and **D3** images (see Fig. 2.14) are arranged as shown in (a) and are shown in (b), for test image of Fig. 2.16. The magnitude of the WT modulus given by Eq. 2.17 is shown in (c).

modulus is defined below:

$$MWT_{i,j} = \sqrt{(d1_{i,j})^2 + (d2_{i,j})^2 + (d3_{i,j})^2} \quad (2.17)$$

where,

- $d1_{i,j}$ —is the gray scale value of the i, j pixel of the **D1** image,
- $d2_{i,j}$ —is the gray scale value of the i, j pixel of the **D2** image and
- $d3_{i,j}$ —is the gray scale value of the i, j pixel of the **D3** image.

The above-defined modulus of the wavelet transform is calculated for the test image of Fig. 2.16 and is shown in Fig. 2.17(c). Note that the edges which are not strictly horizontal or vertical are more clearly visible than in the separate **D1**, **D2**, and **D3** images.

As discussed in [16], the main difference between filters used in the decomposition of a signal into its wavelet transform and filters commonly used in subband coding schemes, is that wavelet filters are regular. Regularity refers to the number of times a function is continuously differentiable. If $h(n)$ is the vector of lowpass filter coefficients from a wavelet, its limit function is a continuous and continuously differentiable function, as shown in Figs. 2.7 through 2.10. Not all discrete lowpass filter sequences result in a smooth function, in fact some are very irregular as shown in Fig. 2.18. This is the limit function for the smoothing function given in Fig. 2.2(b) for $a = 0.3$. It was calculated using the procedure described in Section 2.5.1.

The regularity of the limit function, $h_c(n)$, is useful in the reconstruction of a signal from its WT since a small error in one of the WT coefficients results

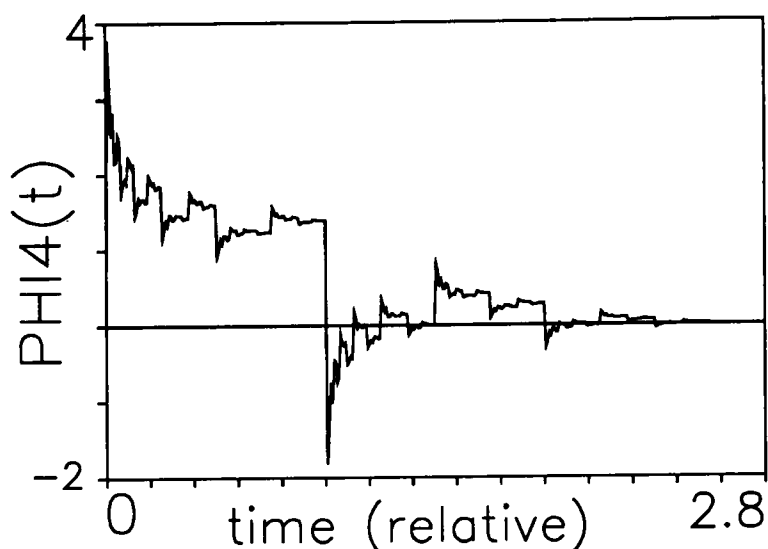


Figure 2.18: Limit Function of an Irregular Lowpass Filter.

in a proportionally small error in the reconstructed signal. In the case of images, the regularity of $h_c(n)$ results in no visible distortion in a reconstructed image.

The scaling functions shown in Figs. 2.7 through 2.10 are the $h_c(n)$ discussed above. A method for constructing $h_c(n)$ with an arbitrarily high degree of regularity is presented in Section 2.5. As is shown in the next section the tradeoff for higher degrees of regularity in the scaling function (and its associated wavelet) is the length of $h(n)$ and correspondingly number of computations.

2.5 Construction of Compact, Orthogonal Wavelets

In the following subsections, the theory and methodology for the determination of the scaling function and wavelet filter coefficients and the construction

of the scaling functions and wavelets are taken from [17 and 21]. The filter coefficients are developed first, in Section 2.5.1 because these coefficients are used in determining the shape and properties of the scaling and wavelets as is shown in Section 2.5.2.

2.5.1 Orthogonal Wavelet Filter Coefficients

In this section, the method for calculating wavelets with an arbitrarily high degree of regularity and compact support is outlined. The conditions for the construction of orthogonal wavelets with an arbitrarily high degree of regularity and with compact support are determined or postulated. From these conditions, the scaling or lowpass filter, $h(n)$, and its associated wavelet or highpass filter, $g(n)$ can be constructed. The development is general and the developed methodology allows for the construction of filters with an arbitrarily high degree of regularity. As can be seen from Figs. 2.6 and 2.11 through 2.13, the support and regularity of the scaling function and its associated wavelet increase with increasing order. Support being the time span over which the wavelet is not always zero. The higher the order of the wavelet, the more coefficients must be used in the lowpass and highpass filters.

The following conditions are imposed on the scaling or lowpass filter, $h(n)$ and the wavelet or highpass filter, $g(n)$:

1. Both filters are bounded:

$$\sum_n |h(n)| < \infty \quad (2.18)$$

$$\sum_n |g(n)| < \infty. \quad (2.19)$$

2. Perfect reconstruction:

$$\mathbf{H}^* \mathbf{H} + \mathbf{G}^* \mathbf{G} = \mathbf{I}. \quad (2.20)$$

where,

$$\mathbf{H} = \begin{bmatrix} h(n) \\ h(n-2) \\ h(n-4) \\ \vdots \end{bmatrix},$$

$$\mathbf{H}^* = \begin{bmatrix} h^*(n) \\ h^*(n-2) \\ h^*(n-4) \\ \vdots \end{bmatrix},$$

$$\mathbf{G} = \begin{bmatrix} g(n) \\ g(n-2) \\ g(n-4) \\ \vdots \end{bmatrix},$$

$$\mathbf{G}^* = \begin{bmatrix} g^*(n) \\ g^*(n-2) \\ g^*(n-4) \\ \vdots \end{bmatrix}$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & \vdots \\ 0 & 0 & \ddots & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & \dots & \dots & 0 & 1 \end{bmatrix}.$$

3. Orthogonal:

$$\mathbf{H} \mathbf{G}^* = \mathbf{0} \quad (2.21)$$

4. $h(n)$ is an averaging weighting function:

$$\sum_n h(n) = \sqrt{2}. \quad (2.22)$$

5. $g(n)$ is a difference function:

$$\sum_n g(n) = 0. \quad (2.23)$$

The above conditions are satisfied by perfect reconstruction filters used in subband coding [41]. Wavelets have the additional constraint of regularity imposed on their associated filters. Since, as shown in the next subsection, the wavelet or highpass filter is constructed from a weighted sum of translations of the scaling function, it is sufficient to impose a regularity condition on the scaling filter only. This regularity condition can be written as (see proposition 3.3 of [17], which is restated below):

$$m_o(\xi) = \left[\frac{1}{2} (1 + e^{i\xi}) \right]^N F(\xi) \quad (2.24)$$

with,

$$F(\xi) = \sum_n f(n) e^{in\xi}$$

such that,

$$\sum_n |f(n)| |n|^\epsilon < \infty \text{ for some } \epsilon > 0 \quad (2.25)$$

and for $\xi \in \mathbb{R}$

$$\sup |F(\xi)| = B < 2^{N-1}. \quad (2.26)$$

In Eq. 2.24, the term,

$$\left[\frac{1}{2} (1 + e^{i\epsilon}) \right]^N,$$

can be considered a smoothing factor which increases the regularity of the scaling functions as N increases. Eq. 2.25 imposes the condition that both the lowpass and highpass filters ($h(n)$ and $g(n)$, respectively) are bounded (see Eqs. 2.18 and 2.19) and Eq. 2.26 imposes the condition that the scaling function has compact support.

The number of coefficients in $h(n)$ determines the order of the wavelet, that is, for a wavelet of order N , there are $2N$ filter coefficients. The coefficients of $h(n)$ are obtained by choosing a sequence $f(n)$ that results in maximally flat perfect reconstruction filters [41]. The coefficients for these filters are obtained for:

$$\begin{aligned} {}_N m_0(u) &= \left[\frac{1}{2} (1 + e^{iu}) \right]^N \sum_{n=0}^{N-1} q_n(n) e^{inu} \\ &= 2^{-1/2} \sum_{n=0}^{2N-1} h_N(n) e^{inu} \end{aligned} \quad (2.27)$$

The polynomial,

$$\sum q_n(n) e^{inu},$$

is determined by retaining all zeros inside the unit circle of $Q_N(e^{iz})$ where,

$$|Q_N(e^{iz})|^2 = \sum_{k=0}^{N-1} \binom{N-1+k}{k} \sin^{2k} \left(\frac{1}{2} z \right) \quad (2.28)$$

and

$$\binom{M}{R} = \frac{M!}{(M-R)!R!}$$

for Q_N of minimal order [17]. The manipulations of Eqs. 2.27 and 2.28 and the numerical procedures used to obtain the values of $h(n)$ are outlined below.

The following expression is substituted to rewrite Eq. 2.28 as a polynomial in $\cos(nz)$:

$$\sin^{2k} \left(\frac{1}{2} z \right) = 4^{-k} \left[\binom{2k}{k} + \sum_{n=0}^{k-1} (-1)^{k-n} 2 \binom{2k}{k} \cos(k-n)z \right]. \quad (2.29)$$

Substituting Eq. 2.29 into Eq. 2.28, and letting $k \rightarrow k-1$ and $n \rightarrow n-1$ (to facilitate programming) yields:

$$\begin{aligned} |Q_N(e^{iz})|^2 &= \sum_{k=1}^N \binom{N+k-2}{k-1} \left(\frac{1}{4} \right)^{k-1} \binom{2k-2}{k-1} \\ &+ \sum_{k=1}^N \left[\binom{N+k-2}{k-1} \left(\frac{1}{4} \right)^{k-1} \sum_{n=1}^{k-1} (-1)^{k-n} 2 \binom{2k-2}{n-1} \cos(k-n)z \right]. \end{aligned} \quad (2.30)$$

For a given N , Eq. 2.10 yields a series of the form:

$$A(z) = \sum_{n=1}^N a_n \cos(n-1)z. \quad (2.31)$$

Once the coefficients of the series given in Eq. 2.31 have been calculated, by summing like terms of $\cos(nz)$ in Eq. 2.30, the associated polynomial, $P_{2N-1}(z)$ is formed:

$$P_{2N-1}(z) = \frac{1}{2} \sum_{n=1}^{N-1} a_{N-n+1} z^{n-1} + a_1 z^{N-1} + \frac{1}{2} \sum_{n=2}^N a_n z^{N+n-2}. \quad (2.32)$$

Note that $P_{2N-1}(z)$ is a symmetric polynomial as typically found in the construction of perfect reconstruction filters [41].

It is now necessary to find the $2N - 2$ roots of the polynomial of Eq. 2.30. The roots of the polynomial of Eq. 2.32 can be found using the Newton-Raphson Algorithm [40]. In general there are K' real roots, denoted r_k and J' pairs of complex roots, denoted $(Z_j, \tilde{Z}_j)$ of $P_{2N-1}(z)$, where $K' + 2J' = 2N - 2$. Choosing only the K real roots and the J pairs of complex roots of $P_{2N-1}(z)$ which lie inside the unit circle of the complex plane, form the polynomial:

$$B(u) = \left[\frac{1}{2} |a_n| \prod_{k=1}^K |r_k|^{-1} \prod_{j=1}^J |z_j|^{-2} \right]^{\frac{1}{2}} \left| \prod_{k=1}^K (e^{iu} - r_k) \prod_{j=1}^J (e^{iu} - z_k) (e^{iu} - \tilde{z}_j) \right|^2. \quad (2.33)$$

From Eq. 2.25:

$$\begin{aligned} {}_N m_0(u) &= \left[\frac{1}{2} (1 + e^{iu}) \right]^N \sum_{n=0}^{N-1} q_n(n) e^{inu} \\ &= \left[\frac{1}{2} (1 + e^{iu}) \right]^N B(u) \\ &= 2^{-1/2} \sum_{n=0}^{2N-1} h_N(n) e^{inu}. \end{aligned} \quad (2.34)$$

The coefficients $h_N(n)$ are determined by equating like powers of e^{inu} in Eq. 2.34.

The values of highpass or wavelet filter, $g(n)$, are determined by

$$g(n) = (-1)^n h(n) \quad (2.35)$$

Where Eq. 2.35 is obtained by imposing the conditions of Eqs. 2.20 and 2.23 on $h(n)$ and $g(n)$. The filter coefficients for the D2, D4, D8 and D16 wavelets determined using the above described procedure are listed in Table 2.1

Table 2.1: Lowpass or Scaling Function Filter Coefficients

n	$h(n)$			
	Wavelet D2	Wavelet D4	Wavelet D8	Wavelet D16
1	0.70710678	0.48296291	0.23037781	0.05441584
2	0.70710678	0.83651630	0.71484657	0.31287159
3		0.22414387	0.63088077	0.67563074
4		-0.12940952	-0.02798377	0.58535468
5			-0.18703481	-0.01582911
6			0.030841382	-0.28401554
7			0.032883012	0.00047248
8			-0.01059740	0.12874743
9				-0.01736930
10				-0.04408825
11				0.01398103
12				0.00874609
13				-0.00487035
14				-0.00039174
15				0.00067544
16				-0.00011748

2.5.2 Determination of the Scaling Function and Wavelet

The scaling function, $\Phi_{2N}(x)$, is determined by the equation [21]:

$$\Phi_{2N}(x) = \prod_{k=0}^{2N-1} c_k \Phi_{2N}(2x - k). \quad (2.36)$$

In Eq. 2.36, $c_{1-k} = c_0, c_1, c_2, \dots, c_{2N-1}$ are recursion coefficients or the coefficients of $h(n)$ which, as shown below, not only are used in the calculation of $\Phi_{2N}(x)$, but are also used in the calculation of the wavelet, $W_{2N}(x)$. Starting with the values of $\Phi_{2N}(x)$ at the integers $x = 1, 2, 3, \dots, (2N - 1)$ and noting that $\Phi_{2N}(x) = 0$, for x outside the interval $(0, 2N - 1)$, the values of $\Phi_{2N}(x)$ at all points $k/2j$ for $k/2j$ in the interval $(0, 2N - 1)$ can be obtained from Eq. 2.36. The values of $\Phi_{2N}(x)$ at integer values of x are determined from the eigenvector of the matrix $\tilde{L}_{2N}$ for eigenvalue $\lambda_1 = 1$ where the elements, $l_{i,j}$ of $\tilde{L}_{2N}$ are given by [21]:

$$l_{i,j} = c_{2i-j}. \quad (2.37)$$

Since $\lambda_1 = 1$ is the dominant eigenvalue (i.e. the eigenvalue with the largest magnitude) the Power Method Algorithm [40] can be used to find the eigenvector associated with λ_1 . The wavelet $W_{2N}(x)$ is constructed from the summation of a scaling function $\Phi_{2N}(x)$ by the equation:

$$W_{2N}(x) = \sum_{k=2-2N}^1 (-1)^k c_{1-k} \Phi_{2N}(2x - k). \quad (2.38)$$

Eqs. 2.34, 2.36, and 2.38 were implemented in a computer program along with the Newton Raphson and Power Method Algorithms to obtain the scaling functions and wavelets shown in Figs. 2.7 through 2.10.

3 Methods for Edge Enhancement Using Wavelets

This section develops and examines two methods for performing edge enhancement using compactly supported orthogonal wavelets in a multiresolution framework. As discussed in Section 2, the modulus of the wavelet transform (*MWT*) is relatively large near an edge in an image and near zero over an area of near constant gray level. The two methods presented in this chapter make use of these properties to detect edges in images.

The first method developed for the enhancement of multiscale edges in a multiresolution framework is presented in Section 3.1. This method is named the direct method because multiscale edges are determined directly from the output of the wavelet pyramid (see Fig. 2.14). The second method is named the underscale method because multiscale edges are determined by taking the wavelet transform of an underscaled resolution of the original image. The underscale method is presented in Section 3.2.

3.1 Direct Method for Multiresolution Edge Enhancement

As discussed in Section 2.2, each step of the multiresolution pyramid halves the resolution and doubles the scale of the input image (reduces the size of the input image by a factor of $\frac{1}{2}$ both horizontally and vertically). This is shown in Fig. 3.1 for an illustration image. In order to reasonably compare or process



(a)



(b)



(c)

Figure 3.1: Illustration Image at Various Resolutions. The illustration image is shown in (a). The illustration image at resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ and scales 2, 4 and 8, respectively is shown in (b). The illustration image at resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ all at scale 1 is shown in (c). The analyzing wavelet is the D4 wavelet.

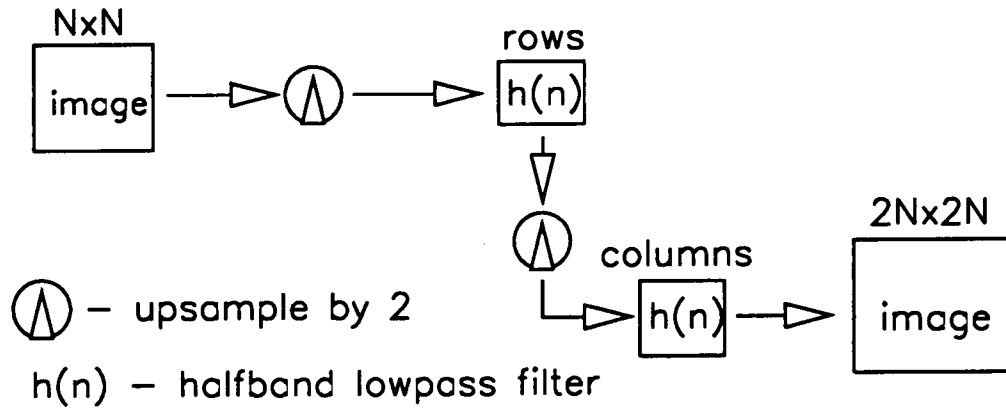


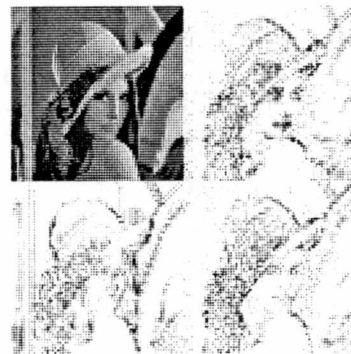
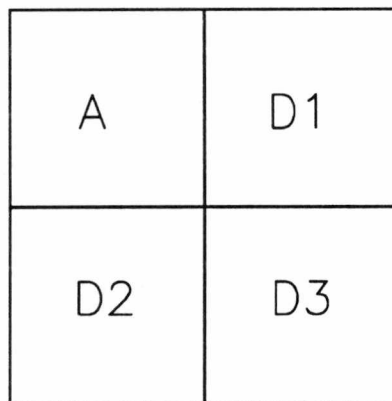
Figure 3.2: Method for Decreasing the Scale of an Image

images at various resolutions (e.g., images with resolutions $1, \frac{1}{2}$ and $\frac{1}{4}$ and scales of 1, 2 and 4, respectively) it is necessary to restore the $\left(\frac{1}{2}\right)^n$ resolution to a scale of 1 (the scale of the original image). An image at the same resolution but at one-half the scale may be obtained by the method shown in Fig. 3.2. This method consists of upsampling the image at the $\frac{1}{2}$ resolution by 2 (inserting a zero between each sample) followed by lowpass filtering with a halfband lowpass filter [16]. An image at $\left(\frac{1}{2}\right)^n$ resolution and a scale of n may be restored to a scale of 1 by n applications of the above method. The images of Fig. 3.1(b) have been restored to a scale of 1 by the above method and are shown in Fig. 3.1(c).

In other edge enhancement methods (see [3] and Section 2.2), the difference between the resolutions shown in Fig. 3.3 would serve in detecting multiscale edges. When an image is decomposed into its wavelet coefficients by the method shown in Fig. 2.14, the edges at each resolution can be detected directly from



(a)



(b)

Figure 3.3: Illustration Image at Resolution $\frac{1}{2}$ and the Magnitude Wavelet Coefficients. The illustration image is shown in (a). The illustration image at resolution $\frac{1}{2}$ and the magnitude of the vertical, horizontal and diagonal wavelet transform coefficients (**A**, **D1**, **D2** and **D3**, respectively) are shown in (b) on a reverse gray scale. The analyzing wavelet is the D4 wavelet.

the magnitude of the wavelet coefficients. The output images from the wavelet decomposition shown in Fig. 2.14 are shown in Fig. 3.3(a) for the image of Fig. 3.1(a). The magnitude of the wavelet coefficients shown in Fig. 3.3(a) are on a normalized reverse gray scale. That is, for the **D1**, **D2** and **D3** images shown in Fig. 3.3(a):

$$dn_{i,j} = g_{max} - \frac{|dn_{i,j}|}{dn_{max}} \quad (3.1)$$

where,

- $dn_{i,j}$ —gray scale value of the i, j pixel for image **DN**, $N = 1, 2$ and 3 ,
- g_{max} —maximum value (white) on the gray scale and
- dn_{max} —maximum of all $dn_{i,j}$.

The images in Fig. 3.3(a) are, of course, also at a scale of 2.

The *MWT* images are shown in Fig. 3.4(b) for resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ for the image of Fig. 3.1(a). This image is also presented using a normalized reverse gray scale as defined by Eq. 3.1. Since the constituent **D1**, **D2** and **D3** images are at scales of 2, 4 and 8, the *MWT* images are also at scales 2, 4 and 8.

To restore the *MWT* images shown in Fig. 3.4(b) to scale 1 (scale of the original image), simple linear interpolation is used as shown in Fig. 3.5.

Expanding the *MWT* images by linear interpolation is computationally efficient and preserves all the edge information in the *MWT* images at each resolution.

Only an additional computations result from expanding the *MWT* images at the same resolution as discussed above for the resolution (**A**) images. The restored *MWT* images at resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ and scale 1 are shown in Fig. 3.4(c). The



(a)



(b)



(c)

Figure 3.4: *MWT* of the Demonstration Image. The original image is shown in (a). The *MWT* images at resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ and scales 2, 4 and 8 are shown in (b). The *MWT* images at resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ and scale 1 are shown in (c). The analyzing wavelet is the D4 wavelet.

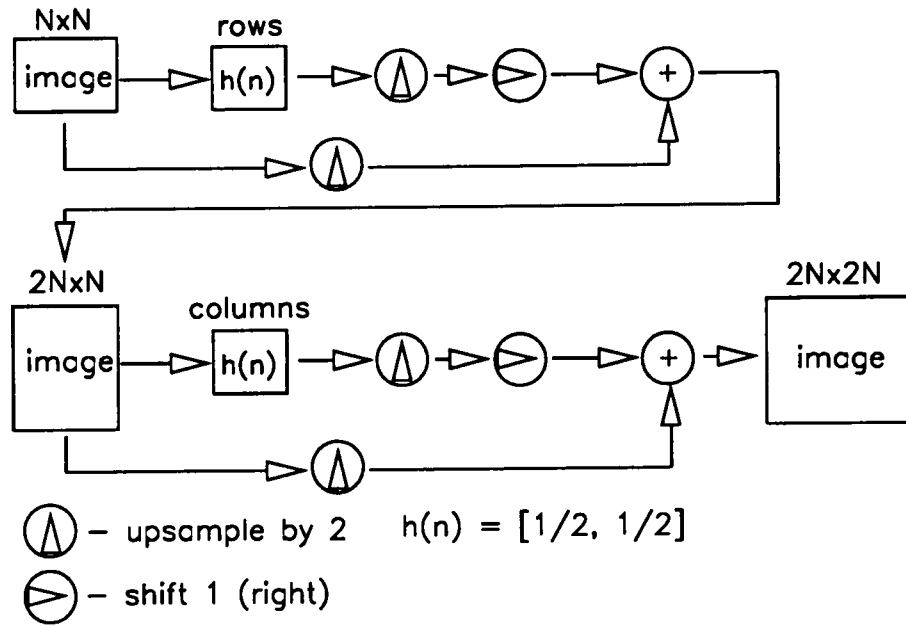


Figure 3.5: Linear Interpolation Method for Returning the *MWT* Images to a Scale of 1.

above discussed method is used for multiscale edge enhancement of a set of test images in Section 4.

Before concluding this section, additional examination of the *MWT* is conducted to assure an efficient definition of the *MWT* has been employed. The *MWT* (as defined by Eq. 2.17), is equal to the square root of the sum of the squares of the horizontal, vertical and diagonal (HVD) wavelet transform coefficients. This definition of the *MWT*, is designated the HVD *MWT*. In reference [6], the *MWT* is defined as:

$$MWT_{i,j} = \sqrt{|d1_{i,j}|^2 + |d2_{i,j}|^2} \quad (3.2)$$

That is, without using the information in the **D3** (diagonal) image. This definition

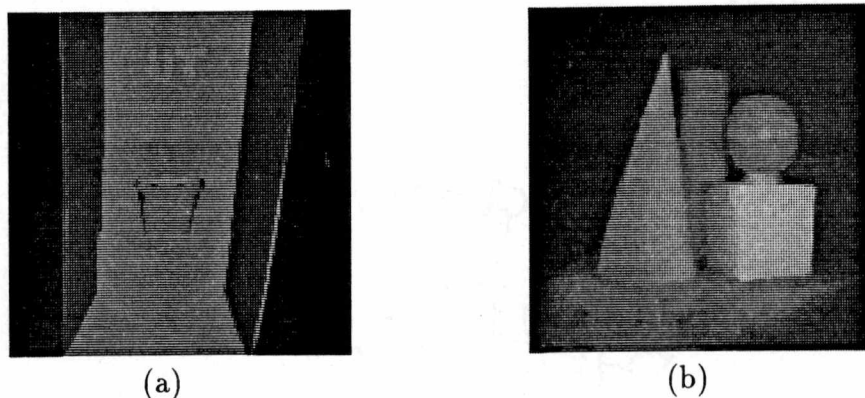
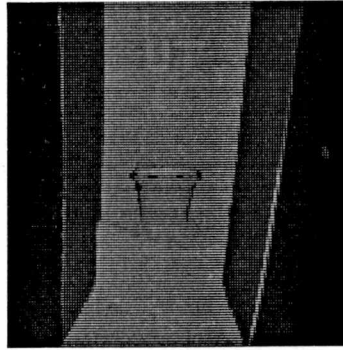


Figure 3.6: *MWT* Test Images. The *MWT* based methods for image segmentation perform relatively well for the image in (a) and relatively poorly for the image in (b).

of the *MWT* is designated the HV *MWT*. As discussed in Section 2.3, the **D3** image is highpass both horizontally and vertically and the magnitude of the wavelet coefficients are large near corners. In order to determine the usefulness of the information contained in the **D3** image, in this multiresolution method, two test images (see Fig. 3.6) are used. The image shown in Fig. 3.6(a) is chosen because the *MWT* of this image yields relatively good edge enhancement. The image shown in Fig. 3.6(b) is chosen because the *MWT* of this image yields relatively poor edge enhancement.

The HVD *MWT* of the image of Fig. 3.6(a) at resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ are shown in Fig. 3.7(b). The HV *MWT* of the image of Fig. 3.6(a) at resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ are shown in Fig. 3.7(c). Similarly, the HVD *MWT*, as of the image of Fig. 3.6(b) at resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ are shown in Fig. 3.8(b). The HV *MWT* of the image of Fig. 3.6(b) at resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ are shown



(a)

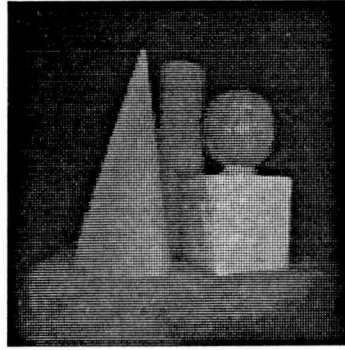


(b)

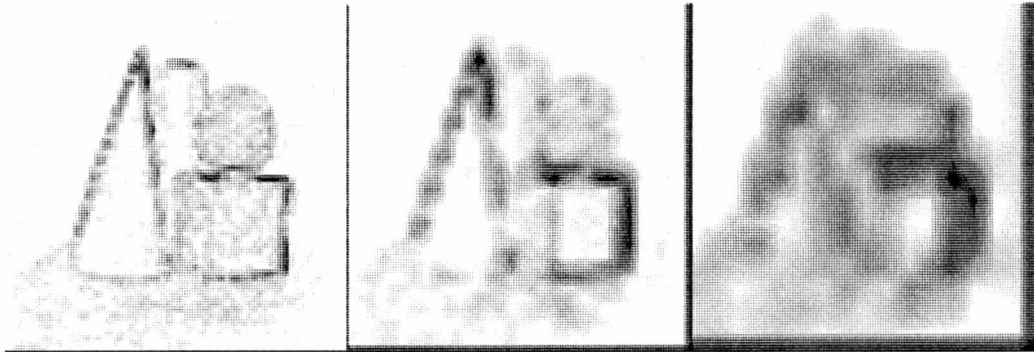


(c)

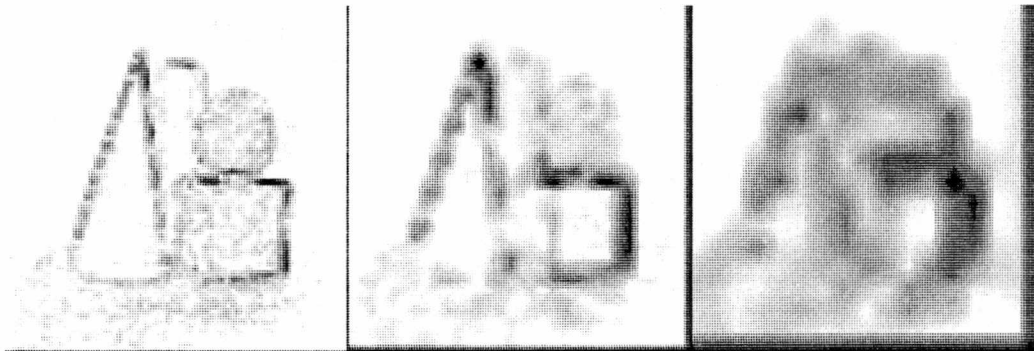
Figure 3.7: Comparison of the HVD and HV *MWT*s for the Image of Fig. 3.6(a), Direct Method. The original image is shown in (a). The HVD *MWT* images at resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ are shown in (b). The HV *MWT* images at resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ are shown in (c). The analyzing wavelet is the D4 wavelet.



(a)



(b)



(c)

Figure 3.8: Comparison of the HVD and HV *MWT*s for the Image of Fig. 3.6(b), Direct Method. The original image is shown in (a). The HVD *MWT* images at resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ are shown in (b). The HV *MWT* images at resolutions $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ are shown in (c). The analyzing wavelet is the D4 wavelet.

in Fig. 3.8(c). In both Figs. 3.7 and 3.8, it can be seen that the HDV and HV *MWTs* produce nearly identical results. This indicates that the diagonal contribution to the *MWT* does not aid in edge enhancement. Note that in all the edge enhancement results for the direct method, the edge lines are at thin in the $\frac{1}{2}$ resolution and become progressively thicker for the lower resolutions. In addition, at the lowest resolutions, object or features run together allowing for only limited edge enhancement. These characteristics are expected due to the multiresolution framework in which the edge enhancement is performed. Characteristics and behaviors of the *MWT* images produced by the direct method are discussed in detail in Section 4.2.

Due to the similarity in the results, for the HVD and HV *MWTs*, it was decided to use the HV *MWT*. The results presented in Section 4.2 are from the HV *MWT*. It should be noted that the analyzing wavelet for the results presented in Figs. 3.7 and 3.8 is the D4 wavelet; however, similar results were obtained with other wavelets (e.g., D2, D8 and D16).

3.2 Underscale Method for Multiresolution Edge Enhancement

The basis for this method comes by noting, again, that each application of the method shown in Fig. 3.2 halves the scale while maintaining the same resolution of an input image. Instead of applying the method of Fig. 3.2 n -times to the $\left(\frac{1}{2}\right)^n$ resolution, apply it $n+1$ times. The resulting image is at the $\left(\frac{1}{2}\right)^n$

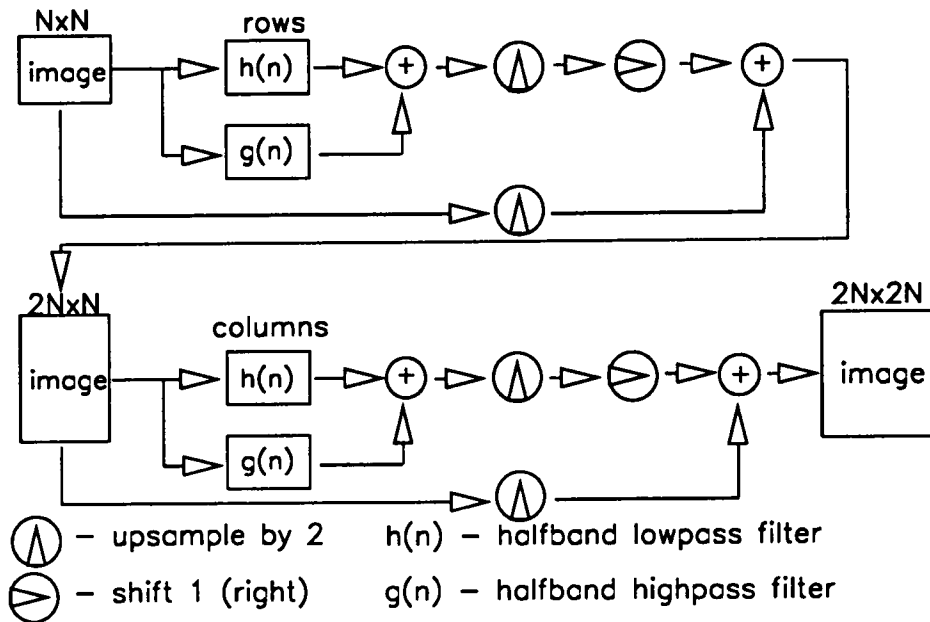


Figure 3.9: Underscale *MWT* Method for Edge Enhancement.

resolution but at $\frac{1}{2}$ the scale of the original image. This means that the resulting image is twice the size of the original image both horizontally and vertically. Applying the *WT* procedure of Fig. 2.14 to the $\frac{1}{2}$ scale image results in **A**, **D1**, **D2** and **D3** images at the scale of the original image.

Now consider an enhancement to this procedure as shown in Fig. 3.9.

In the underscale method, the values of the pixels of the input image are retained and the sum of the highpass and lowpass filtering of the original image are used to provide interpolated values between the (constant values of the) original pixels. The highpass and lowpass filters when used in this manner create an allpass interpolator. The interpolated sequence is upsampled (a zero inserted between

every sample) and displaced to the right by one ($n \rightarrow n + 1$) and then added to the upsampled and unfiltered, original sequence. In this way, the original values of the pixels of the input (resolution) image are unaltered during the interpolation process.

This method was found to result in a resolution image restored to scale which provides better edge enhancement when the *WT* procedure of Fig. 2.14 is used. The motivation for the method of Fig. 3.9 is to build an allpass filter which enhances the edges in the $\left(\frac{1}{2}\right)^n$ resolution as it is scaled (down) to the $\frac{1}{2}$ scale. Images at 1, $\frac{1}{2}$ and $\frac{1}{4}$ resolution are shown in Fig. 3.10(b). These images have been restored to $\frac{1}{2}$ scale by the method shown in Fig. 3.9 followed by a single application of the *WT* procedure shown in Fig. 2.14. Note that the original image shown in Fig. 3.10(a) is not identical to the first image in Fig. 3.10(b), since its edges have been enhanced by the method shown in Fig. 3.9. The *MWT* for the four images shown in Fig. 3.10(b) are shown in Fig. 3.10(c). Fig. 3.11(a) demonstrates the procedure given in Fig. 3.5 for a (one-dimensional) numerical example when the analyzing wavelet is the D2 wavelet. As is shown in Fig. 3.11, the resulting output signal from the method (using the D2 wavelet) is the same as repeating each sample. When the *WT* of the output signal is taken, the resulting coefficients are all zero as shown in Fig. 3.11(b). Therefore, this method does not produce any edge enhancement results when the D2 wavelet is used.

As discussed in Section 3.1, the information contained in the **D3** image



(a)



(b)



(c)

Figure 3.10: Underscale Edge Enhancement Results for the Demonstration Image. The illustration image is shown in (a). The Illustration image at resolutions 1 , $\frac{1}{2}$ and $\frac{1}{4}$ at a scale of 1 are shown in (b). The *MWT* images at resolutions 1 , $\frac{1}{2}$ and $\frac{1}{4}$ and at a scale 1 are shown in (c). The analyzing wavelet is the D4 wavelet.

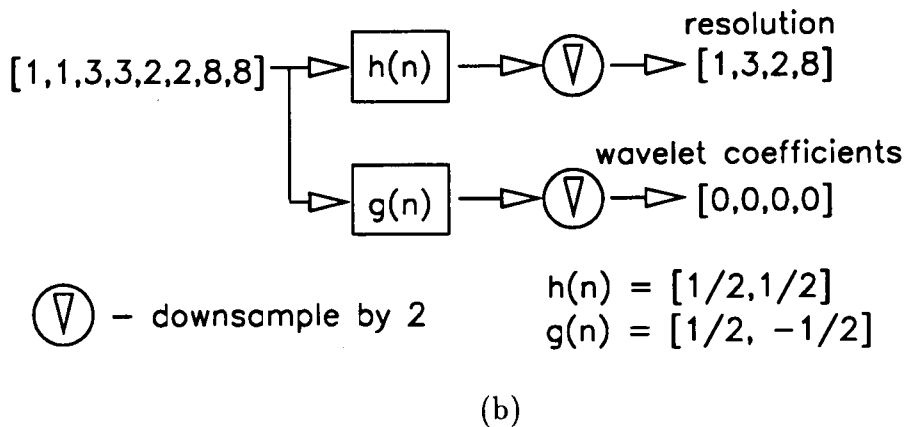
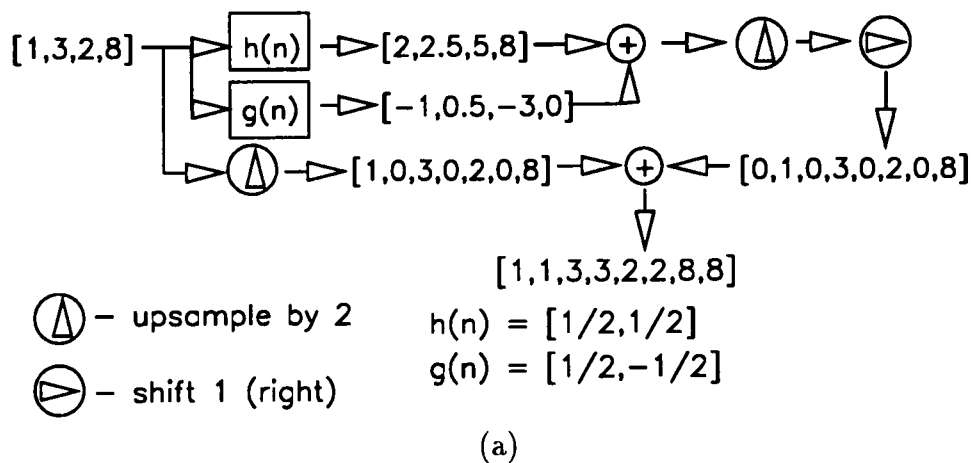


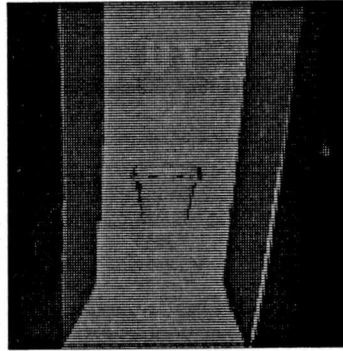
Figure 3.11: Example Results for the Underscale Method when the Analyzing Wavelet is the D2 (or Harr) Wavelet. The results of the method shown in Fig. 3.9 are given in (a) for a one-dimensional example. The wavelet transform of the results are shown in (b).

may not be significant in detecting edges and a simpler definition of the *MWT* given by Eq. 3.2 may be sufficient. The effect on the results of including the information in the **D3** image is also examined for the method discussed above.

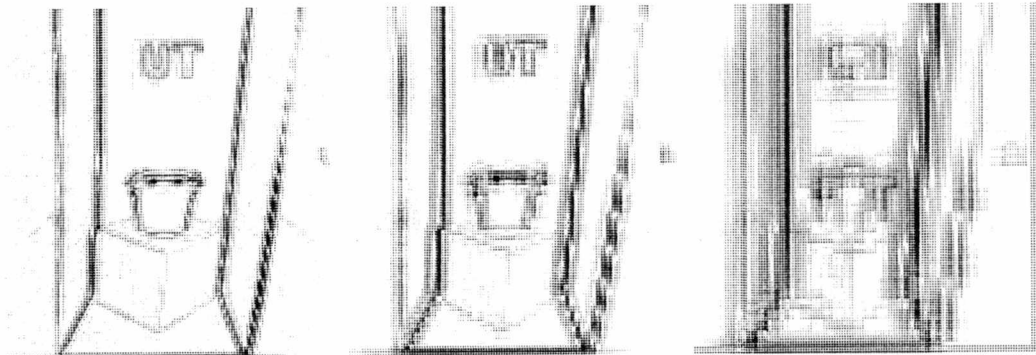
The HVD *MWT* of the image of Fig. 3.6(a) at resolutions $1, \frac{1}{2}$ and $\frac{1}{4}$ are shown in Fig. 3.12(b). The HV *MWT* of the image of Fig. 3.6(a) at resolutions $1, \frac{1}{2}$ and $\frac{1}{4}$ are shown in Fig. 3.12(b). Similarly, the HVD *MWT* of the image of Fig. 3.6(b) at resolutions $1, \frac{1}{2}$ and $\frac{1}{4}$ and are shown in Fig. 3.13(b).

The HV *MWT* of the image of Fig. 3.6(b) at resolutions $1, \frac{1}{2}$ are shown in Fig. 3.13(b). In both Figs. 3.12 and 3.13, it can be seen that the HDV and HV *MWT*s produce nearly identical results. This indicates that the diagonal contribution to the *MWT* does not aid in edge enhancement. Note that in all the edge enhancement results for the underscale method, the edge lines are fairly thin at all resolutions. However, as with the direct method (see Section 3.1), at the lowest resolutions, object or features run together allowing for only limited edge enhancement. Characteristics and behaviors of the *MWT* images produced by the underscale method are discussed in detail in Section 4.3.

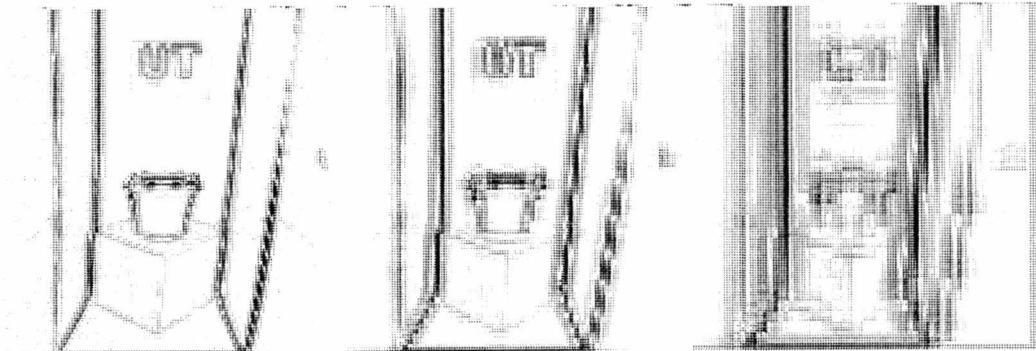
Due to the similarity in the results, for the HVD and HV *MWT*s, it was decided to use the definition of the HV *MWT*. The results presented in Section 4.3 are for the HV *MWT*. It should be noted the analyzing wavelet for the results presented in Figs. 3.12 and 3.13 is the D4 wavelet; however, similar results were obtained with other wavelets (e.g., D2, D8 and D16).



(a)

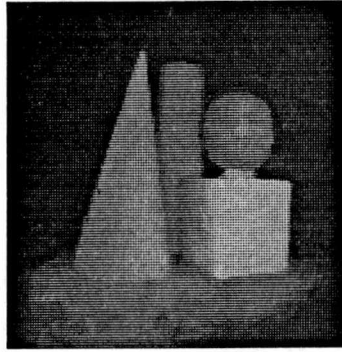


(b)

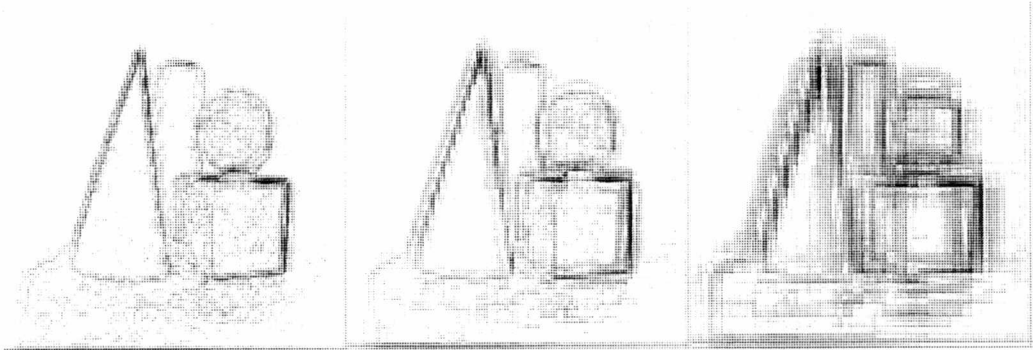


(c)

Figure 3.12: Comparison of the HVD and HV *MWT*s for the Image of Fig. 3.6(a), Underscale Method. The original image is shown in (a). The HVD *MWT* images at resolutions $1, \frac{1}{2}$ and $\frac{1}{4}$ are shown in (b). The HV *MWT* images at resolutions $1, \frac{1}{2}$ and $\frac{1}{4}$ are shown in (c). The analyzing wavelet is the D4 wavelet.



(a)



(b)



(c)

Figure 3.13: Comparison of the HVD and HV *MWT*s for Image of Fig. 3.6(b), Underscale Method. The original image is shown in (a). The HVD *MWT* images at resolutions 1, $\frac{1}{2}$ and $\frac{1}{4}$ are shown in (b). The HV *MWT* images at resolutions 1, $\frac{1}{2}$ and $\frac{1}{4}$ are shown in (c). The analyzing wavelet is the D4 wavelet.

4 Experimental Results

In Section 3, two methods are presented for edge enhancement in a multiresolution framework using compactly supported orthogonal wavelets. Those two methods are applied to the set of test images shown in Fig. 4.1. This set of images was chosen because of the large variation in image texture, edge density and contrast among the images in the set. The images are arranged in order of increasingly poorer edge enhancement results when the two methods are applied. That is, application of the two methods to the first images, that is, Images (a), (b) and (c) of the set results in relatively good edge enhancement results while application of the two methods to the later images, that is, Images (i), (j) and (k) results in relatively poor edge enhancement results.

Section 4 is divided into four parts. Section 4.1 discusses general results and examines the characteristics of images which result in the *MWT* methods producing relatively good or relatively poor edge enhancement results. The results for the direct edge enhancement method described in Section 3.1 are presented in Section 4.2. The results for the underscale edge enhancement method described in Section 3.2 are presented in Section 4.3. The discussions in Sections 4.2 and 4.3 identify characteristic behavior of the resolution and *MWT* images for different order analyzing wavelets and different resolutions. The edge enhancement results obtained by the two methods are compared in Section 4.4.



Image (a)



Image (b)



Image (c)

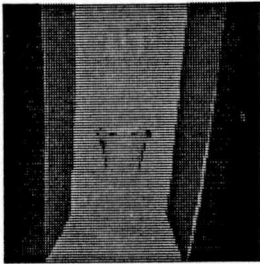


Image (d)

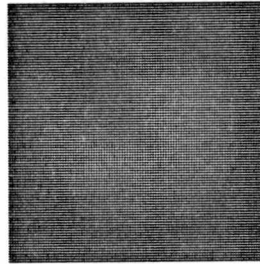


Image (e)

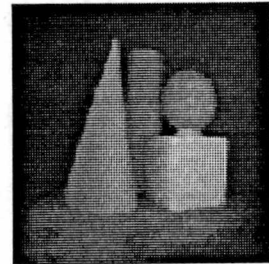


Image (f)

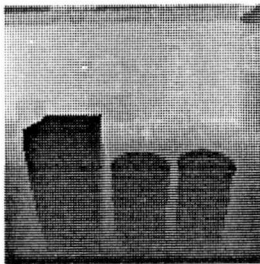


Image (g)

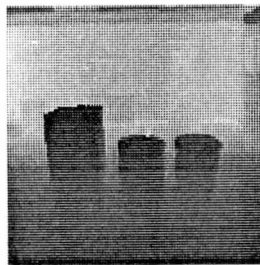


Image (h)

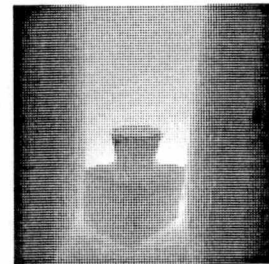


Image (i)

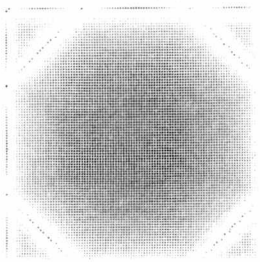


Image (j)

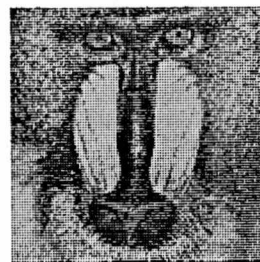


Image (k)

Figure 4.1: The Set of Test Images.

4.1 *MWT* Edge Enhancement Results

4.1.1 Introduction

In this section, the edge enhancement results presented in Sections 4.2 and 4.3 are examined and the characteristics of images which result in relatively good or poor edge enhancement results using the *MWT* methods are discussed. The best edge enhancement results of Sections 4.2 and 4.3 are presented, individually, in this section the majority of the images in Fig. 4.1. This is done to aid in the discussion of the results and present evidence of the trends discussed in the following subsections.

4.1.2 Discussion

The original images shown in Fig. 4.1 are 128×128 pixels in size. The 1 , $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ resolutions are used in each method. The $\frac{1}{8}$ resolution corresponds to a 16×16 pixel image from which little useful information can be extracted (as can be seen from the results in Sections 4.2 and 4.3).

Examining the arrangement of the images in Fig. 4.1 reveals that the *MWT* methods provide relatively good results for images with high contrast, that is, relatively large variation in gray level between the objects in the image and/or between various regions of the image. Another property of the images for which the two *MWT* methods provide relatively good results is small variation of gray level within each region. In contrast, the *MWT* methods provide relatively poor

results for images which have a small variation in gray level between objects or regions and a large variation in gray level within a region. These are expected results since both methods make use of the *MWT* which is a measure of the difference between successively smoothed versions of the input image.

In the following discussions of each image, the *MWT* methods are evaluated. Detailed explanation is provided under the discussion for Image (a); this discussion is applicable to all images.

Image (a): This image and the best edge enhancement results for the *MWT* methods are shown in Fig. 4.2. The *MWT* methods perform well on this image because within each region of the image the gray level is nearly constant. In addition, there are relatively large differences in gray level between the various regions within the image and the gray level changes from one nearly constant value to another over a few pixels. Boundaries between objects or regions like that just described are referred to as strong edges. In contrast, weak edges are characterized by relatively small changes in gray level over many pixels.

Examples of strong edges described above can be seen in Image (a). Scanning Image (a) horizontally from right to left it is seen that the light area of the mirror is surrounded by a dark boarder. The woman's face and shoulder (light areas) are boardered on each side by dark regions (her hair). Similarly, the brim of her hat (a light area) is boardered by the dark boarder of the mirror and the ribbon around the hat. In addition, within the woman's face, the regions around

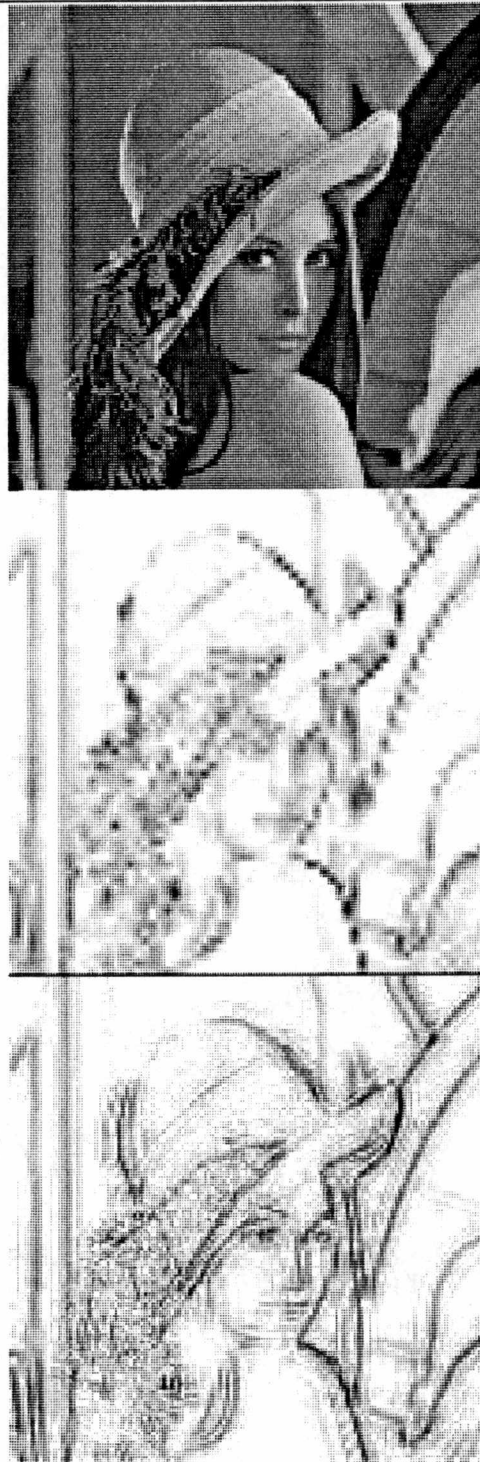


Figure 4.2: Edge Enhancement Results for Image (a). The original image is shown at the top. Edge enhancement results for the direct method using the D2 wavelet and the $1/2$ resolution is shown at the center. Edge enhancement results for the underscale method using the D16 wavelet and the 1 resolution is shown at the bottom.

her eyes are nearly black in contrast to the white areas of her eyes. All the above describe strong edges for which the *MWT* methods perform well.

Note that another reason the edges are easily discernible in the *MWT* image of Image (a) is that most of the edges are of similar difference in gray level. From Eq. 3.1, it can be seen that the simple normalization is biased towards the largest $MWT_{i,j}$. That is, a strong edge will be more easily discernible in a scene than weaker edges in the *MWT* images.

As an example, imagine an image with a black object (i.e., gray level = 0) located in a white region (i.e., gray level = 255), the edge around the black object has an edge strength (difference in gray level) of 255. Also, in the same image there are a great many edges between objects and images that are much weaker, say an average edge strength of only 100. The histogram of the *MWT* image, of this example image, will have one spike in the upper gray levels and many levels centered about gray level = 100. As a result, the strong edge will be very visible in the *MWT* images and the other edges will appear very faint by comparison.

The above example illustrates a characteristic visible in the *MWT* images of Image (a), that is, some edges are very visible while other edges are faint, even though the areas surrounding the edges are bright. For example, the top of the woman's hat in Image (a) is faint in comparison to the adjacent sides.

Images (b) and (c): These images are of the same scene with the camera being closer to the objects in Image (b) and further away in Image (c). The best edge

enhancement results for the *MWT* methods for Image (b) is shown in Fig. 4.3.

As in Image (a), there is not much variation in gray level within the objects in the scenes of Image (b) and Image (c). Scanning the images from bottom to top, it can be seen that the bright barrels are separated from the bright back wall of the room by a dark floor. The doorway on the back wall is also dark compared to the wall. As previously noted, the *MWT* methods perform well in identifying these strong edges. Weaker edges are present at the boundary between the bright floor at the base of the barrels and as expected, the *MWT* images show only a faint edge (line).

Another feature of the *MWT* images of Images (b) and (c) is the nearly constant gray level present in the interior of the objects. For example, in the floor and back wall of the room, the gray level is nearly constant. This is due to the low variation in gray level within the regions within the image. Recall Eq. 2.19 which shows that the sum of the coefficients of the impulse response of the wavelet filter, $g(n)$, is equal to zero. This indicates that across an area of nearly constant gray level, the *MWT* is also nearly zero, regardless of the gray level.

Image (d): This image and the best edge enhancement results for the *MWT* methods are shown in Fig. 4.4. This image consists of a light region in the center of the scene surrounded by two dark regions on each side. The edges between the five panels are very visible due to their strong edges and, in this case, also to the nearly constant gray level within each panel of the image. Note that the

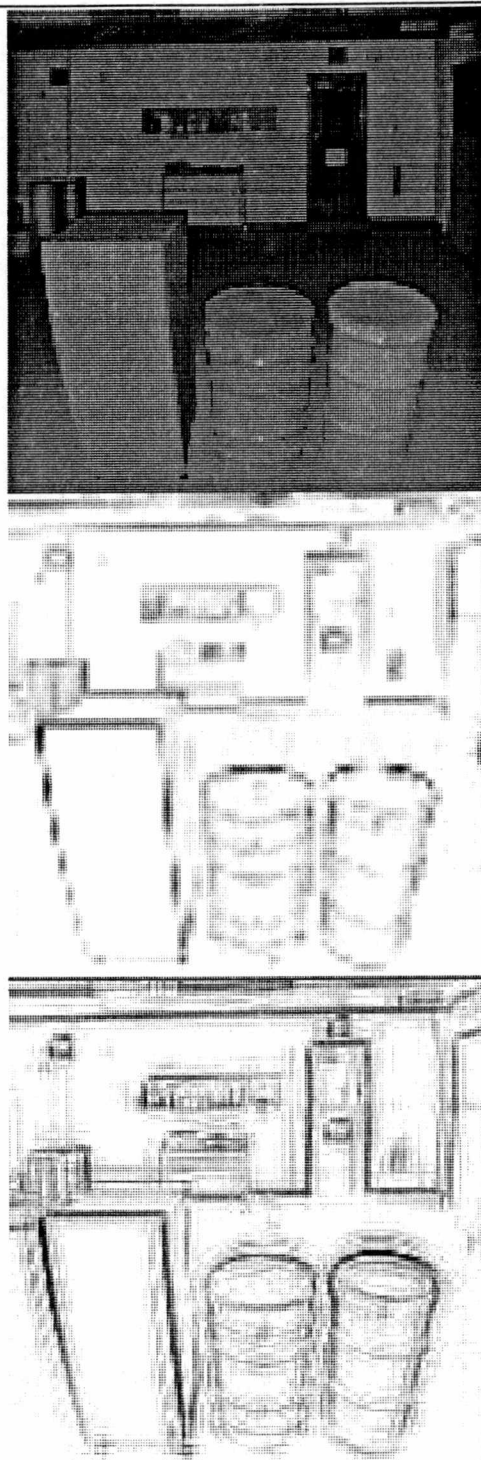


Figure 4.3: Edge Enhancement Results for Image (b). The original image is shown at the top. Edge enhancement results for the direct method using the D2 wavelet and the $\frac{1}{2}$ resolution is shown at the center. Edge enhancement results for the underscale method using the D16 wavelet and the 1 resolution is shown at the bottom.

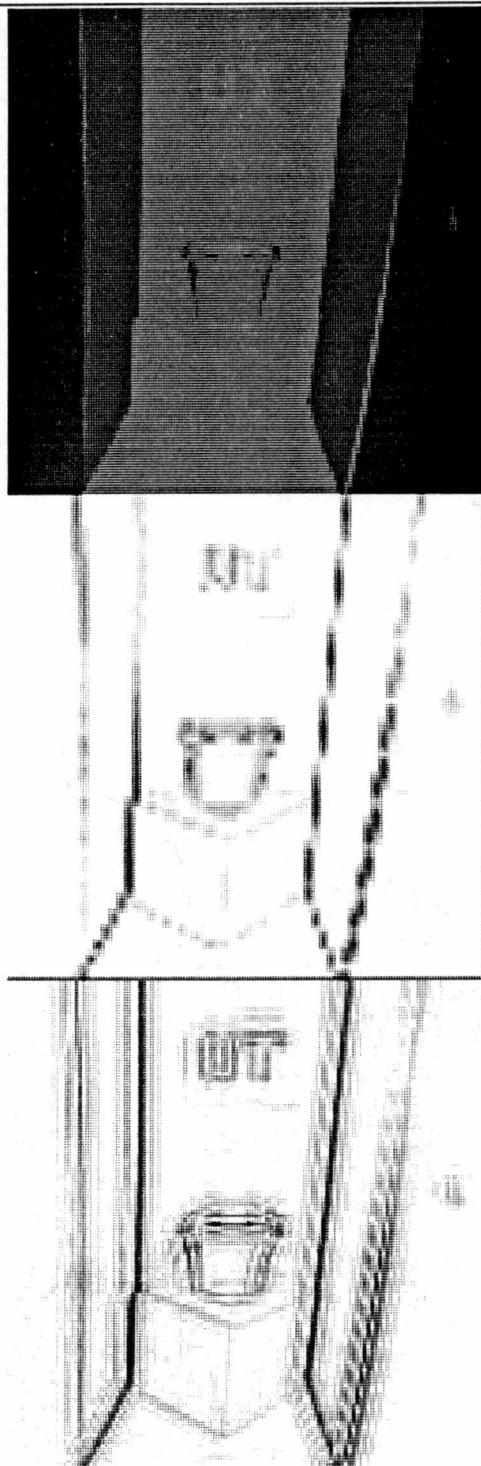


Figure 4.4: Edge Enhancement Results for Image (d). The original image is shown at the top. Edge enhancement results for the direct method using the D2 wavelet and the $\frac{1}{2}$ resolution is shown at the center. Edge enhancement results for the underscale method using the D16 wavelet and the 1 resolution is shown at the bottom.

bucket in the center of the scene has nearly the same gray level as the background. According to earlier results, it would be expected that the *MWT* methods would not detect the edges around the bucket very well, since the small variation in gray level between the background and the bucket result in a weak edge. However, the *MWT* images identify some edges of the bucket. This apparent contradiction is explained by closely examining the region around the bucket, actually what is happening is the the *MWT* detects the dark, thin shadow cast on the back panel by the bucket.

Image (e): This image and the best edge enhancement results for the *MWT* methods are given in Fig. 4.5. In this image there is not much variation in gray level between the sphere and the background. However, the *MWT* methods are still able to detect the edges of the sphere because it is the strongest edge present in the scene. Due to the normalization of gray level in the *MWT* images (see Eq. 3.1 and the discussion for Image (a), above), the edge around the outside of the sphere is the dominant edge and is most visible in the *MWT* images. The quarter slice out of the sphere is a weaker edge and therefore is not very evident, again due to the normalization of the gray levels. The edge on the quarter slice is not evident by a contrasting line, but is evident from the change in gray level in the *MWT* images. Here again, the region of the slice is of nearly constant gray level, which results in a near zero gray level in the *MWT* images.

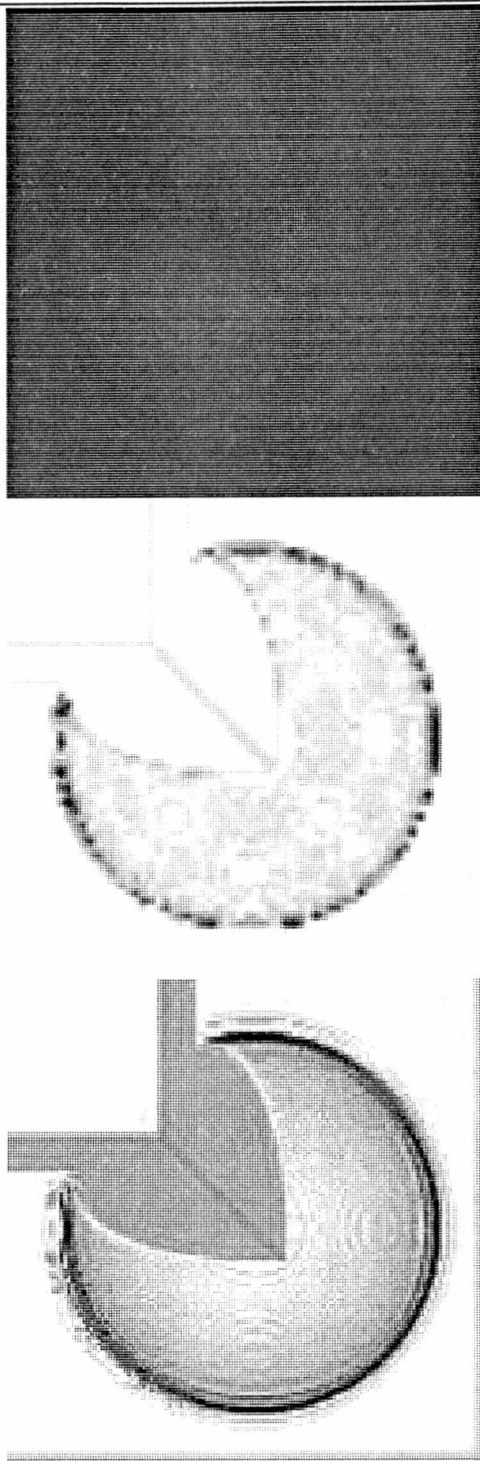


Figure 4.5: Edge Enhancement Results for Image (e). The original image is shown at the top. Edge enhancement results for the direct method using the D4 wavelet and the $\frac{1}{2}$ resolution is shown at the center. Edge enhancement results for the underscale method using the D16 wavelet and the 1 resolution is shown at the bottom.

Image (f): This image and the best edge enhancement results for the *MWT* methods are given in Fig. 4.6. The objects in the center of the scene of this image are bright against a dark background. The edges near the dark background are the strong edges and show up well. The edges in the center of the scene between the cone, square and cylinder are weaker edges. In addition, there is a relatively large variation in gray level within the objects which results in the *MWT* having a nonzero value within the objects so the edges do not stand-out as well in the *MWT* images as the edges in the previous images. The variation in gray level within the objects results in a nonzero *MWT* and consequently to a uniform shading within each object in the image. If the shading is uniform, as it appears for the *MWT* images of Image (f), then simple level filtering should remove much of the shading and allow the edges to be more clearly visible.

Images (g), (h) and (i): These images are the range images of Images (b), (c) and (d), respectively (see Fig. 4.1). The images and the best edge enhancement results for the *MWT* methods are given in Figures 4.7 and 4.8 for Images (g) and (i), respectively. The *MWT* images for Image (g), Image (h) and Image (i) do not as clearly indicate edges as the *MWT* images for Image (b), Image (c) and Image (d), respectively. The primary reason for the poorer edge enhancement results for the range images is due to the reduced contrast or gray level variation among the objects within the range images and the increased variation of gray level within the objects in the range images. In Image (g) and (h), the dark barrels

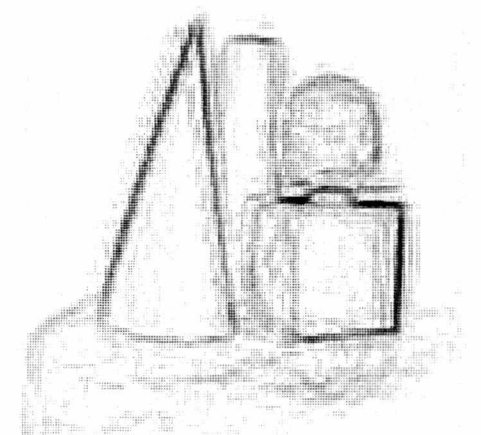
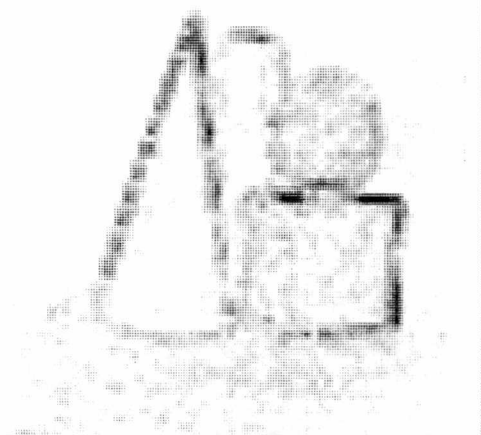
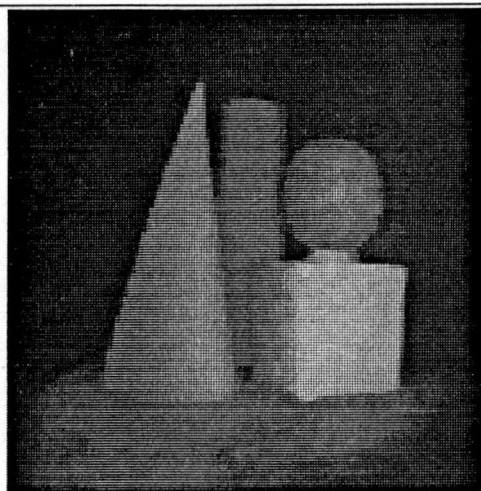


Figure 4.6: Edge Enhancement Results for Image (f). The original image is shown at the top. Edge enhancement results for the direct method using the D4 wavelet and the $\frac{1}{2}$ resolution is shown at the center. Edge enhancement results for the underscale method using the D16 wavelet and the 1 resolution is shown at the bottom.

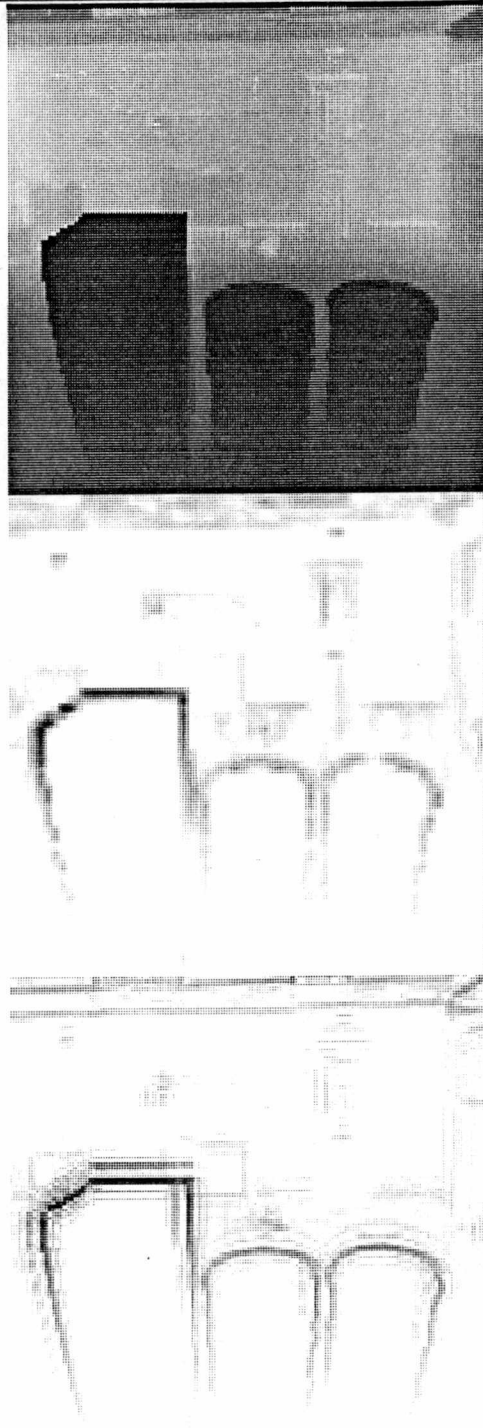


Figure 4.7: Edge Enhancement Results for Image (g). The original image is shown at the top. Edge enhancement results for the direct method using the D4 wavelet and the $\frac{1}{2}$ resolution is shown at the center. Edge enhancement results for the underscale method using the D16 wavelet and the 1 resolution is shown at the bottom.

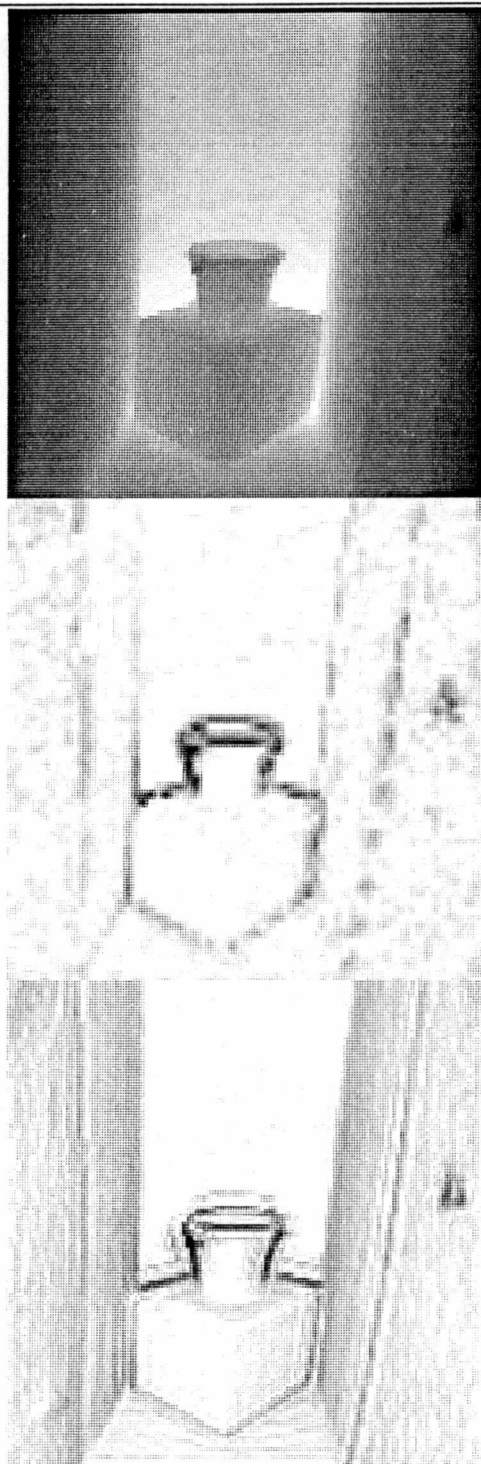


Figure 4.8: Edge Enhancement Results for Image (i). The original image is shown at the top. Edge enhancement results for the direct method using the D4 wavelet and the $\frac{1}{2}$ resolution is shown at the center. Edge enhancement results for the underscale method using the D16 wavelet and the 1 resolution is shown at the bottom.

blend into the dark foreground and the rear wall is gray instead of bright as in Image (b) and Image (c). Also, the gray level in the back wall of the range images varies considerably, making the doorway indistinguishable from the rest of the wall. The above characteristics of these images make for weak edges for which the *MWT* methods do not perform well. In addition, there is a strong edge between the square case and the back wall in Image (g) and Image (h) which dominates the normalization in the *MWT* images, making the remaining edges appear faint. In Image (i), the five panels in the background appear grainy and gray level between the panels changes more slowly than in Image (c). A slow variation in gray level between objects within an image is a characteristic of a weak edge. Thus the edges between the panels are not as evident in the *MWT* images, for Image (i) as they are for Image (c). Also, as previously noted, the variation in gray level within the panels of the image result in a nonzero gray level in the *MWT* images further obscuring the edges.

Image (j): This image and the best edge enhancement results for the *MWT* methods are shown in Fig. 4.9. In this image, gray level varies continuously and at a roughly constant rate of change from the boundaries of the image to the center. Consequently, the *MWT* has nearly the same value throughout the image, varying slightly as the gray level varies slightly. As a result, the normalized *MWT* images look very much like the original image. The results for Image (j) show clearly that the *MWT* methods do not perform well when the smoothed

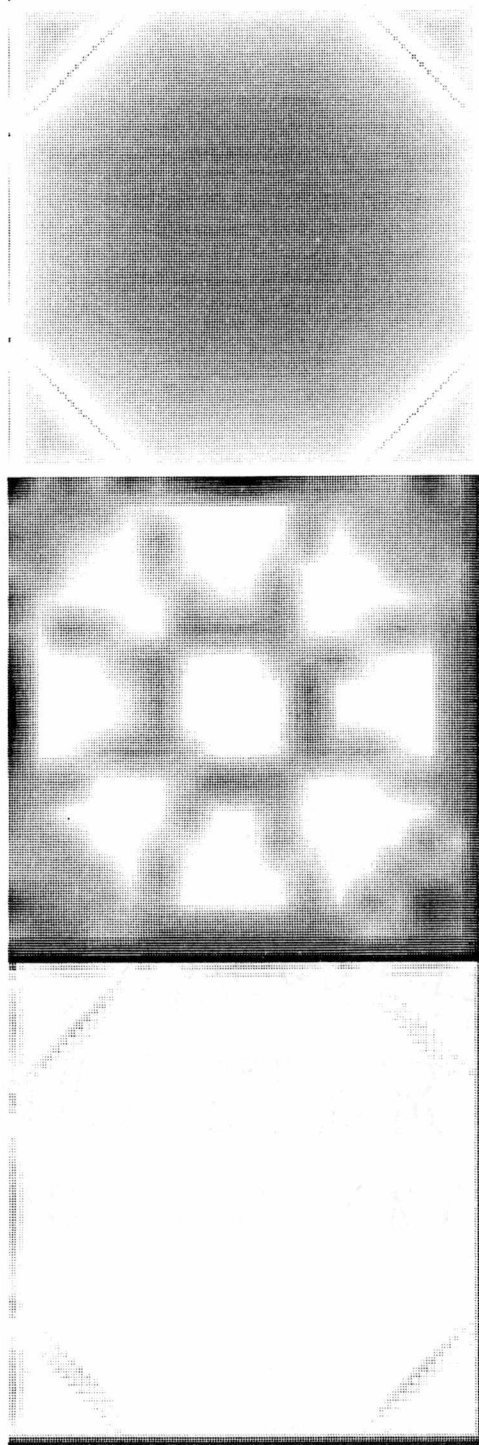


Figure 4.9: Edge Enhancement Results for Image (j). The original image is shown at the top. Edge enhancement results for the direct method using the D4 wavelet and the $\frac{1}{8}$ resolution is shown at the center. Edge enhancement results for the underscale method using the D4 wavelet and the $\frac{1}{2}$ resolution is shown at the bottom.

version of an image is nearly the same as the original image.

Another way of explaining this result is by noting that in the wavelet transform, an image is separated into a highpass and lowpass image, the highpass being the wavelet transform coefficients. If the gray level varies smoothly throughout the image (as in Image (j)) there are no high frequencies present and the wavelet coefficients are near zero and consequently the *MWT* is near zero, resulting in no discernible edges.

Image (k): This image and the best edge enhancement results for the *MWT* methods are shown in Fig. 4.10. The *MWT* methods perform poorly for Image (k) because there are large variations in gray level within each region of the image. As a result, the *MWT* is never near zero and edges are not discernible from the background. As in Image (i), the *MWT* images look very much like the original image.

4.1.3 Conclusions

In the above discussions of each image, the performance of the *MWT* methods have been evaluated for each image shown in Fig. 4.1. The above discussions have been used to identify general characteristics of images for which the *MWT* methods result in relatively good and relatively poor edge enhancement. These characteristics are summarized below:

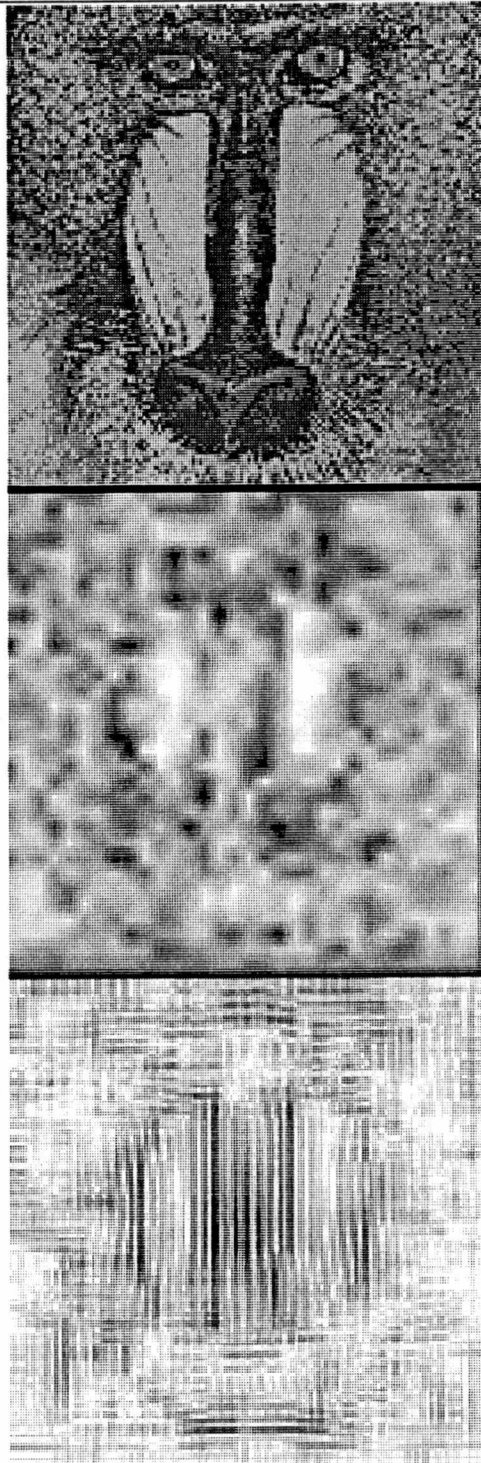


Figure 4.10: Edge Enhancement Results for Image (k). The original image is shown at the top. Edge enhancement results for the direct method using the D8 wavelet and the $\frac{1}{4}$ resolution is shown at the center. Edge enhancement results for the underscale method using the D8 wavelet and the $\frac{1}{2}$ resolution is shown at the bottom.

1. The *MWT* methods perform well for images characterized by large contrast (variation in gray level) between objects within the image and small variation in gray level within each object.
2. The *MWT* methods perform well for images where edges are of nearly the same strength. That is, where the difference in gray level between the objects, at the location of an edge, do not vary greatly from edge to edge throughout the image.
3. The *MWT* methods perform poorly for images having weak edges. That is, boundaries between objects that are characterized by a slow change in gray level over many pixels.

4.2 Results the Direct Edge Enhancement Method

4.2.1 Introduction

The results for the direct method are presented as shown in Fig. 4.11 for each image. The D2, D4, D8 and D16 wavelets are used as the analyzing wavelets. It should be noted that the amount of computation increases by a factor of 8 in going from the lowest order wavelet, D2, to the highest order wavelet, D16, in the set. The edge enhancement results for Images (a) through (k), shown in Fig. 4.1, are presented in this section.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8

(a)

D2 1/2	D2 1/4	D2 1/8
D4 1/2	D4 1/4	D4 1/8
D8 1/2	D8 1/4	D8 1/8
D16 1/2	D16 1/4	D16 1/8

(b)

Figure 4.11: Organization of Results in Figs. 4.12 through 4.22. The resolution images are given in part (a) and the *MWT* images are given in part (b). The top number in each square refers to the analyzing wavelet. The bottom number in each square refers to the resolution.

4.2.2 Discussion

Examining the resolution images in this section, it can be seen that the use of the higher order wavelets results in low resolution images that have smoother variation in gray level. That is, the edges in the lower resolution images are continuous and smooth as opposed to the block like character of the edges in the low resolution images using the low order analyzing wavelets. This is due to the fact that the higher order analyzing wavelets are more localized in frequency, making better lowpass filters (refer to Figs. 2.7 through 2.10). It should be noted that it is computationally more intensive to use the higher order analyzing wavelets.

Examining the *MWT* images in this section, it can be seen that very little useful information about the location of edges is present at the $\frac{1}{8}$ resolution and that the use of the higher order wavelets results in edges which are thicker and more visible than the edges resulting from the use of the lower order wavelets. This result can be explained by noting that the higher order wavelets are less localized, that is, their wavelet filters' impulse responses are longer. This results in edges being less defined in space, but because the higher order wavelets are more localized in frequency, the edges are stronger and better defined. In other words, there is a trade-off, the higher order wavelets are better edge detectors than the lower order wavelets, but the exact location of the edge is less defined when the higher order wavelets are used than when the lower order wavelets are used. Consistent with the above explanation, the higher order wavelets do not

perform as well in images with relatively high edge density since the edges tend to run together. Also, the higher order wavelets are less sensitive to variation in gray level within an object or region because edges are detected over a longer distance (greater number of pixels), so the importance of an individual pixel in detecting a given edge decreases.

The edge enhancement results for the direct method are discussed below for each image.

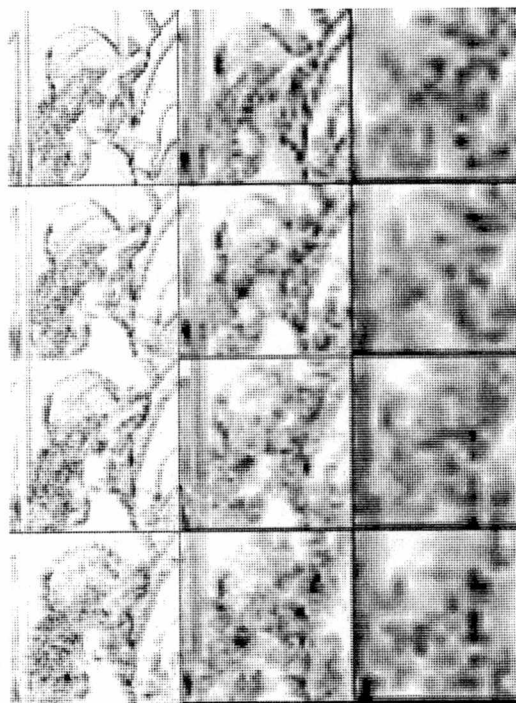
Image (a): The edge enhancement results are shown in Fig. 4.12 for Image (a). The $\frac{1}{2}$ resolution images generate the best edge enhancement. That is, the edge lines are easily discernible and follow the boundaries of the regions of the image completely. For Image (a), the use of the D2 wavelet results in the best edge enhancement. Note that in the $D2-\frac{1}{2}$ image (D2 refers to the analyzing wavelet and $\frac{1}{2}$ refers to the resolution, this convention is used throughout this section to identify a particular image) details such as the edge formed by the reflection of the hat in the mirror is visible along with the wall in the left most portion of the image. As previously discussed, there is very little variation of gray level within the regions of this image and most of the edges in the image are strong edges (see Section 4.1.2). Consequently, the compact spatial support of the D2 analyzing wavelet allows for maximal separation of the enhanced edges. Edges which are close together in the $D2-\frac{1}{2}$ image do not run together as they do when higher order wavelets are used.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

D2 1/2	D2 1/4	D2 1/8
D4 1/2	D4 1/4	D4 1/8
D8 1/2	D8 1/4	D8 1/8
D16 1/2	D16 1/4	D16 1/8



(b)

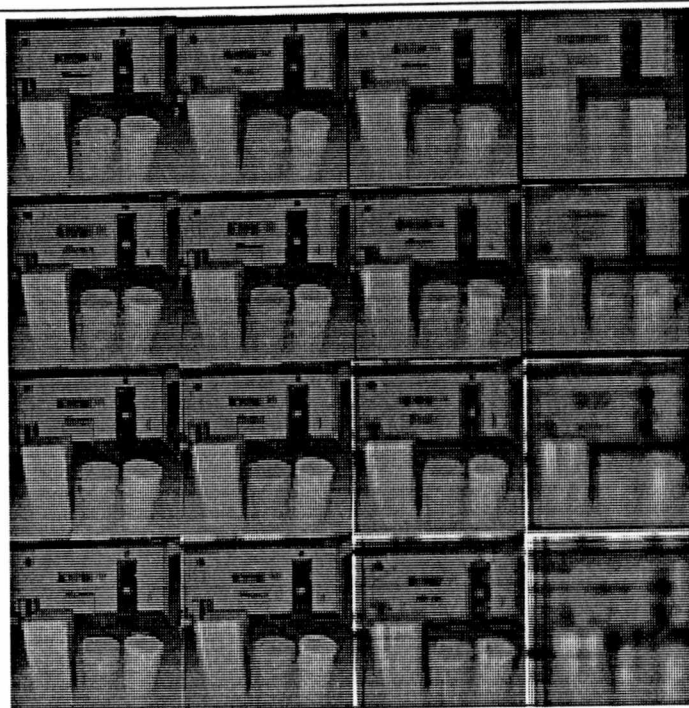
Figure 4.12: Direct Method Results for Image (a). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.11 for key to image arrangement.

Images (b) and (c): The edge enhancement results are shown in Fig. 4.13 and in Fig. 4.14 for Image (b) and Image (c), respectively. As with Image (a), there are strong edges in these Images and very little variation in gray level within the objects in the Images. The $D2-\frac{1}{2}$ and $D4-\frac{1}{2}$ images for both Image (b) and Image (c) provide the best edge enhancement results. Comparing the $D2-\frac{1}{2}$ and $D4-\frac{1}{2}$ images it can be seen that the edges in the $D4-\frac{1}{2}$ images are more visible than in the $D2-\frac{1}{2}$ images for Image (b) and Image (c), especially around the doorway in the back of the room. As previously discussed, this is because the higher order wavelets are better edge detectors, but because of their greater spatial support, the smaller details are not as discernible in the $D4-\frac{1}{2}$ image as in the $D2-\frac{1}{2}$ image.

Image (d): The edge enhancement results are shown in Fig. 4.15 for Image (d). As with Images (a) through (c) there are strong edges and little variation in gray level within each region of Image (d). Again, the $D2-\frac{1}{2}$ image shows the edges most clearly. The UT in the $D2-\frac{1}{2}$ image is almost discernible, in the *MWT* images formed using the higher order wavelets, the UT runs together.

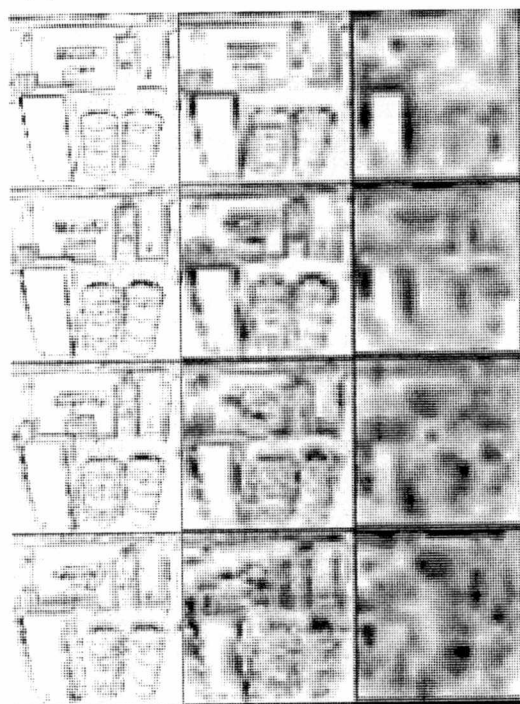
Image (e): The edge enhancement results are shown in Fig. 4.16 for Image (e). In this image, there is a gradual change in gray level over many pixels, that is, this image contains weak edges. The strongest lines around the outside of the sphere in Image (e) are seen in the $D2-\frac{1}{2}$ image. However, the $D4-\frac{1}{4}$ image more clearly shows the edges around the cut-out quarter of the sphere with adequate indication

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

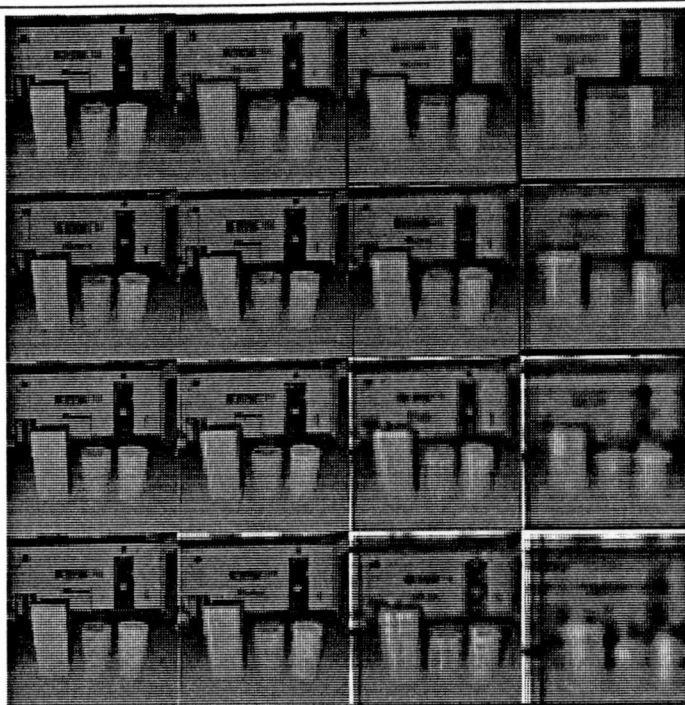
D2 1/2	D2 1/4	D2 1/8
D4 1/2	D4 1/4	D4 1/8
D8 1/2	D8 1/4	D8 1/8
D16 1/2	D16 1/4	D16 1/8



(b)

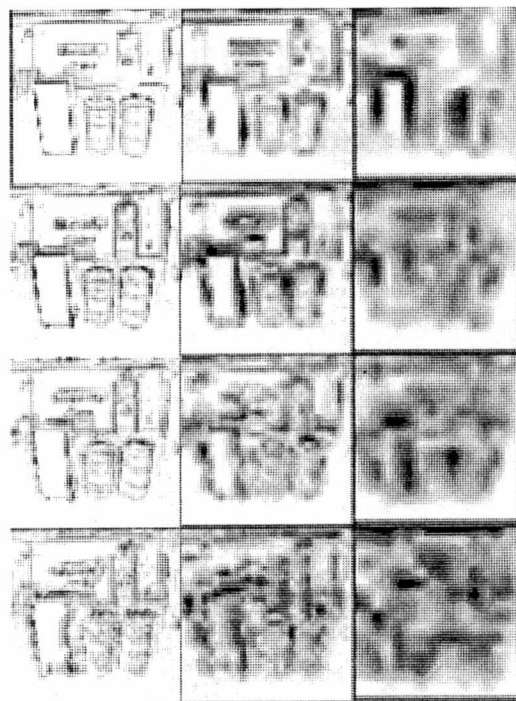
Figure 4.13: Direct Method Results for Image (b). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.11 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

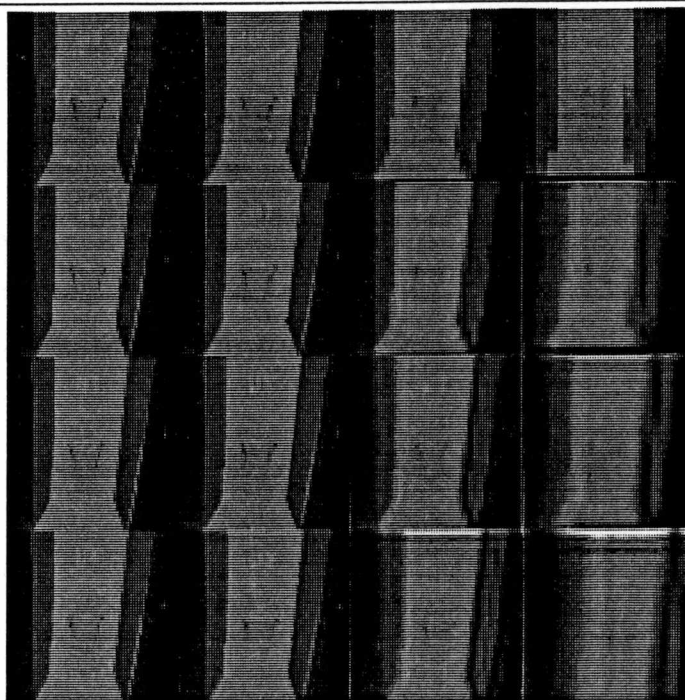
D2 1/2	D2 1/4	D2 1/8
D4 1/2	D4 1/4	D4 1/8
D8 1/2	D8 1/4	D8 1/8
D16 1/2	D16 1/4	D16 1/8



(b)

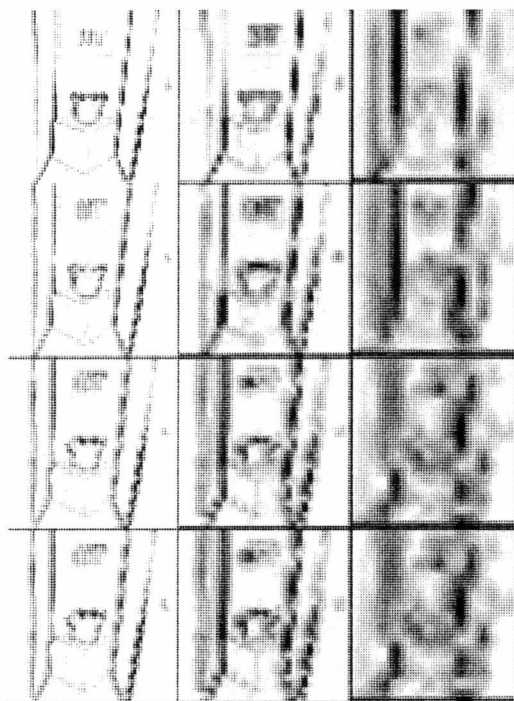
Figure 4.14: Direct Method Results for Image (c). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.11 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

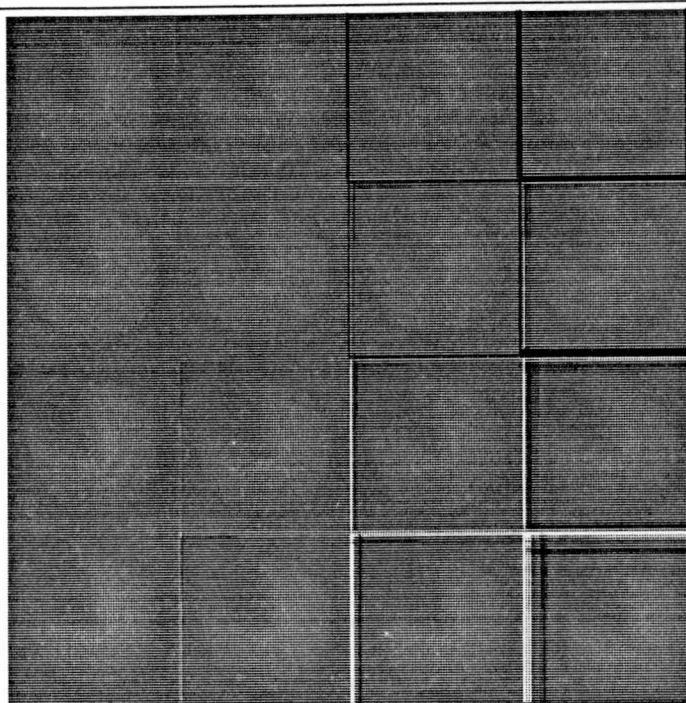
D2 1/2	D2 1/4	D2 1/8
D4 1/2	D4 1/4	D4 1/8
D8 1/2	D8 1/4	D8 1/8
D16 1/2	D16 1/4	D16 1/8



(b)

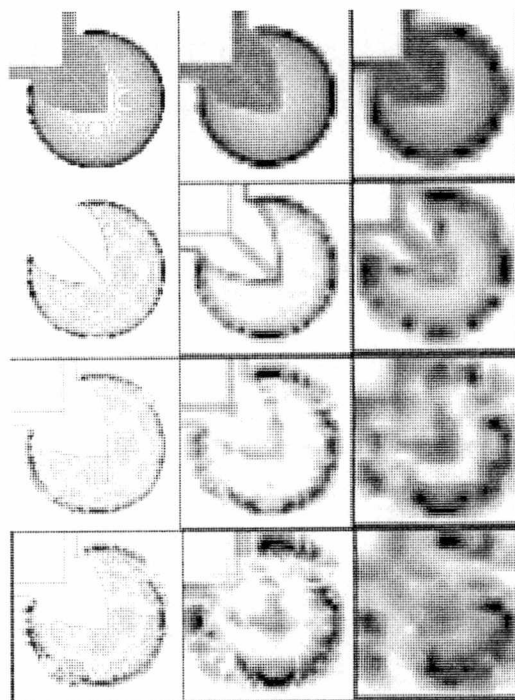
Figure 4.15: Direct Method Results for Image (d). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.11 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

D2 1/2	D2 1/4	D2 1/8
D4 1/2	D4 1/4	D4 1/8
D8 1/2	D8 1/4	D8 1/8
D16 1/2	D16 1/4	D16 1/8



(b)

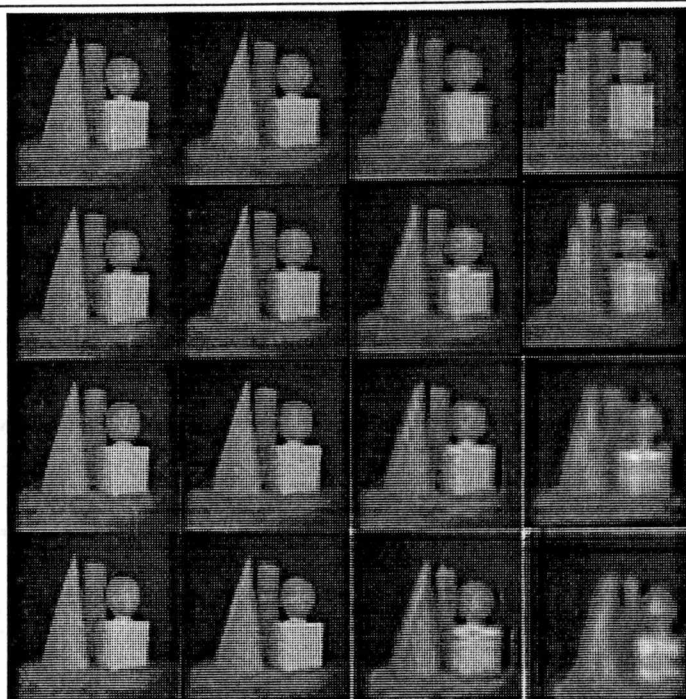
Figure 4.16: Direct Method Results for Image (e). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.11 for key to image arrangement.

of an edge around the outside of the sphere. Since there is a slow variation in gray level within the sphere, the *MWT* has a nonzero value in this region and the sphere is gray in the *MWT* images. The faces of the sphere adjacent to the cut-out quarter are white in the $D4-\frac{1}{4}$ image, indicating a constant gray level.

Image (f): The edge enhancement results are shown in Fig. 4.17 for Image (f). In this image there are strong edges between the objects in the center of the image and the dark background. Also there is a relatively large amount of variation in gray level within the objects in the image. As a result, the outside edges of the objects in the *MWT* images are most visible and the weaker edges in the center of the image between the objects are obscured. Note that the interiors of the objects have a lower gray level (are more white, due to the reverse gray scale) when the higher order wavelets are used. This is due to the higher order wavelets larger support, which tends to average to zero, the variation of gray level within the objects in the image. The best edge enhancement results for Image (f) are shown in the $D4-\frac{1}{2}$ image.

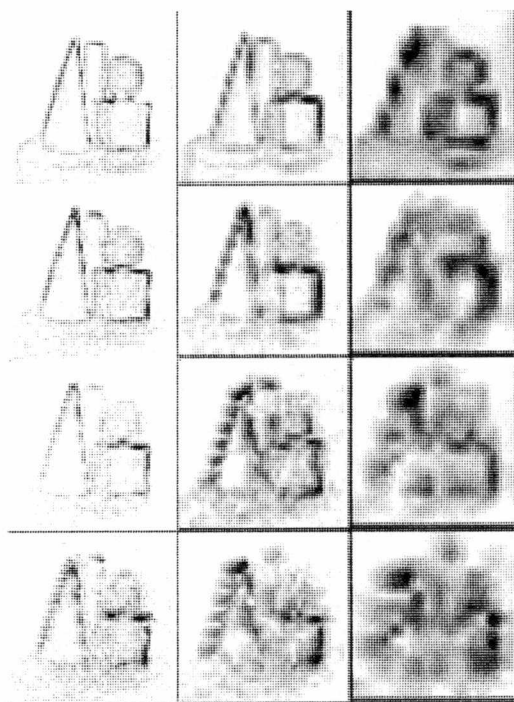
Up until now, the $D2-\frac{1}{2}$ image produced the edge enhancement results for the images in Fig. 4.1. In this case, the $D4-\frac{1}{2}$ image produces the best results because the objects in Image (f) have a greater variation in gray level and the $D4$ wavelet is less susceptible to this variation allowing the edges to be somewhat more discernible from the background in the *MWT* images.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

D2 1/2	D2 1/4	D2 1/8
D4 1/2	D4 1/4	D4 1/8
D8 1/2	D8 1/4	D8 1/8
D16 1/2	D16 1/4	D16 1/8



(b)

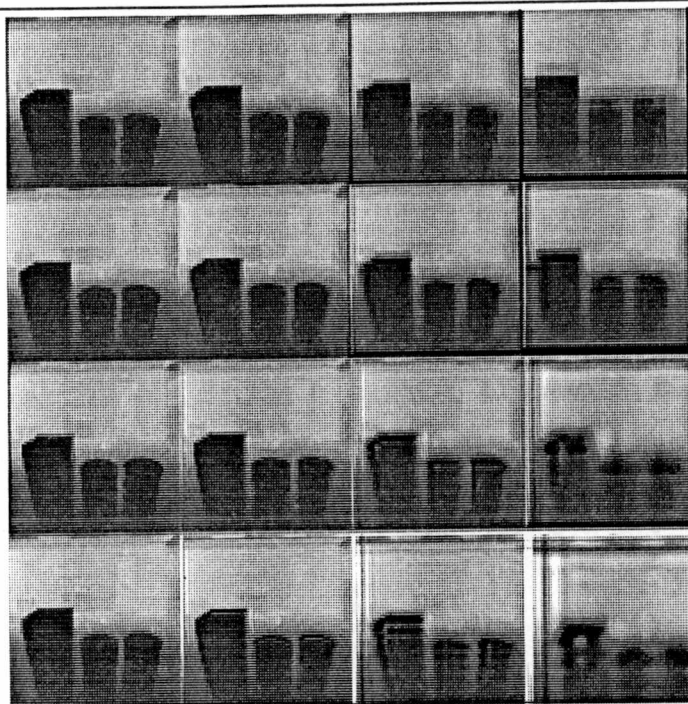
Figure 4.17: Direct Method Results for Image (f). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.11 for key to image arrangement.

Images (g), (h) and (i) These images are the range images of Image (b), Image (c) and Image (d), respectively. The edge enhancement results are shown in Fig. 4.18, 4.19 and 4.20 for Images (g), (h) and (i), respectively. The edge enhancement results are, in general, poorer for the range images due to their reduced contrast. The best edge enhancement is shown in the $D4-\frac{1}{2}$ image for Images (g), (h) and (i). Note that consistent with the results for Image (f), the $D4$ wavelet performs better than the $D2$ because there is less contrast between the objects or regions within the image and greater variation in gray level within the objects or regions.

Image (j): The edge enhancement results are shown in Fig. 4.21 for this image. This image is characterized by slow variation in gray level throughout the image. The best edge enhancement results are shown in the $D4-\frac{1}{8}$ image. The white areas in this image correspond to areas of constant gray level. The gray areas correspond to areas of slow variation in gray level. Because of this slow variation in gray level, no edge enhancement occurs for the $\frac{1}{2}$ resolution images. Note that some edge enhancement is indicated at the boundaries between the white and gray areas.

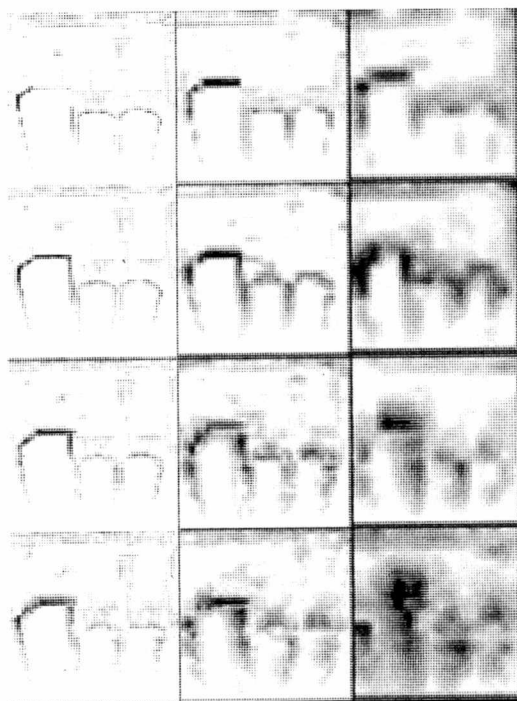
Image (k): The edge enhancement results are shown in Fig. 4.22 for Image (k). This image is characterized by a large variation in gray level within each region. These variations in gray level are detected as edges in the *MWT* methods

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

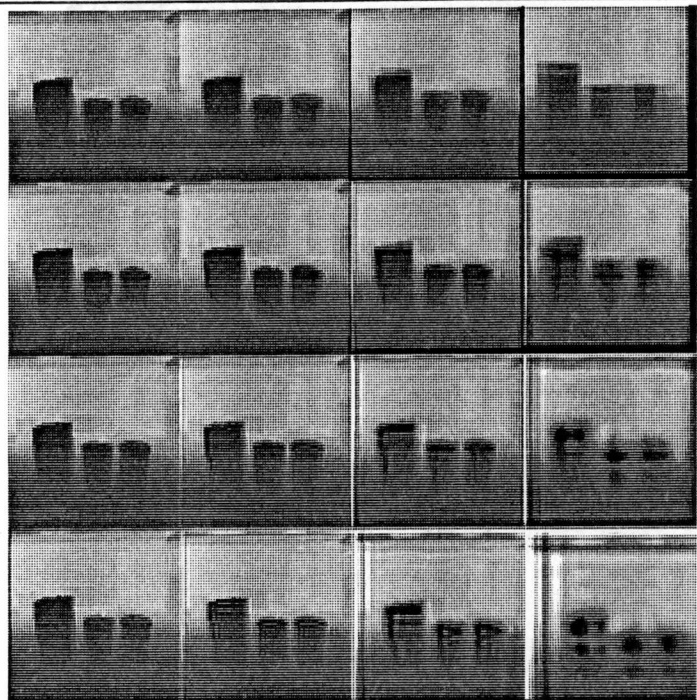
D2 1/2	D2 1/4	D2 1/8
D4 1/2	D4 1/4	D4 1/8
D8 1/2	D8 1/4	D8 1/8
D16 1/2	D16 1/4	D16 1/8



(b)

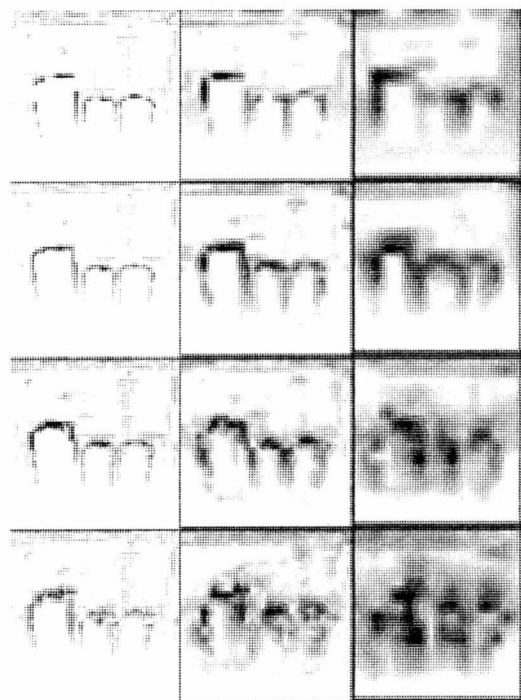
Figure 4.18: Direct Method Results for Image (g). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.11 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

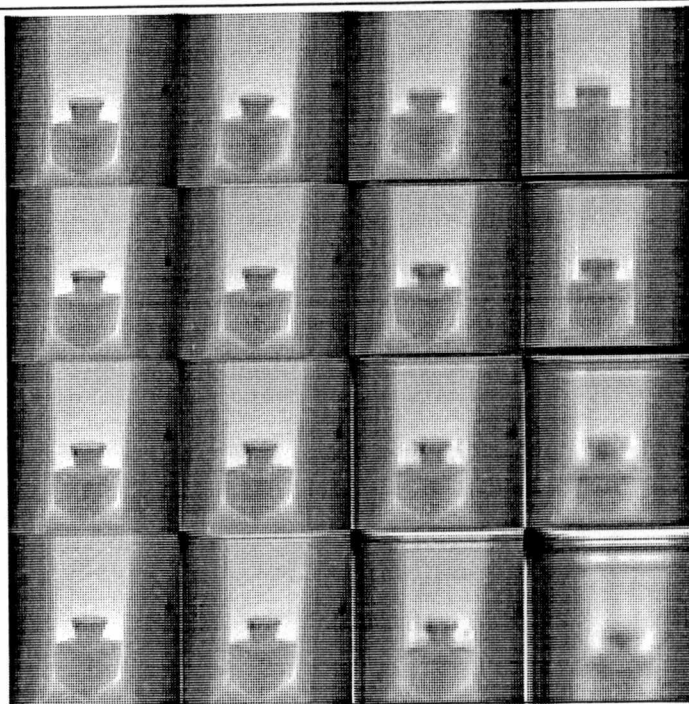
D2 1/2	D2 1/4	D2 1/8
D4 1/2	D4 1/4	D4 1/8
D8 1/2	D8 1/4	D8 1/8
D16 1/2	D16 1/4	D16 1/8



(b)

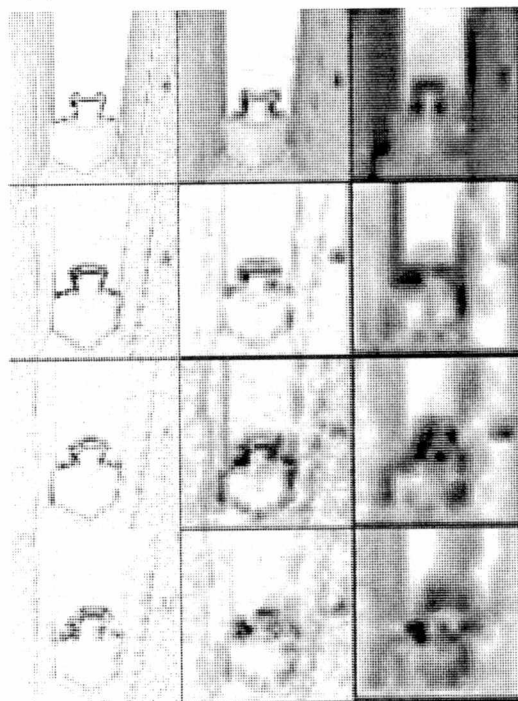
Figure 4.19: Direct Method Results for Image (h). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.11 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

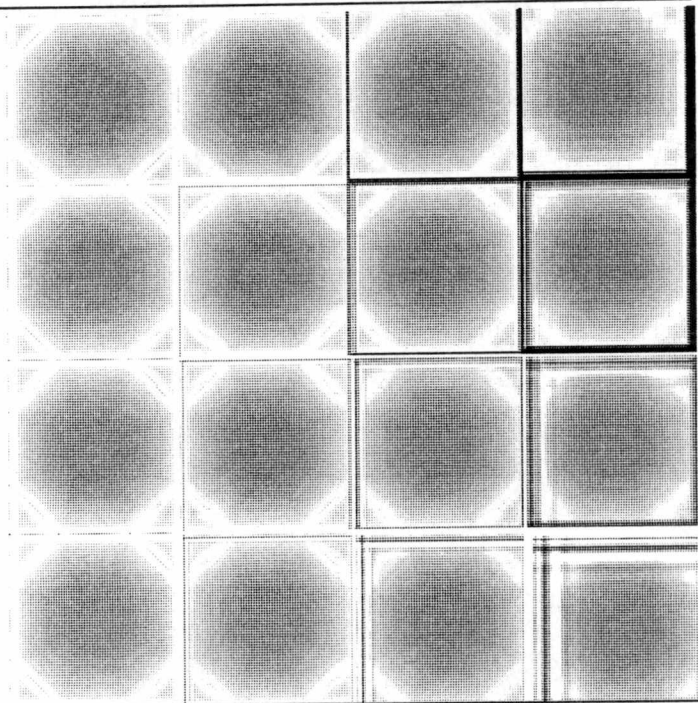
D2 1/2	D2 1/4	D2 1/8
D4 1/2	D4 1/4	D4 1/8
D8 1/2	D8 1/4	D8 1/8
D16 1/2	D16 1/4	D16 1/8



(b)

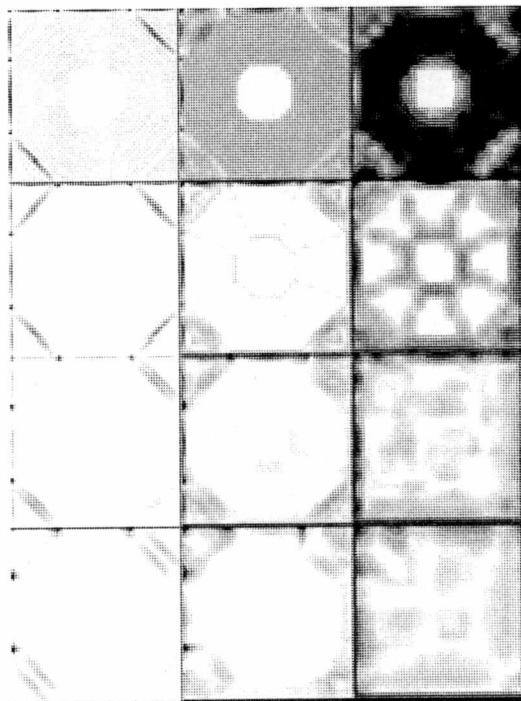
Figure 4.20: Direct Method Results for Image (i). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.11 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

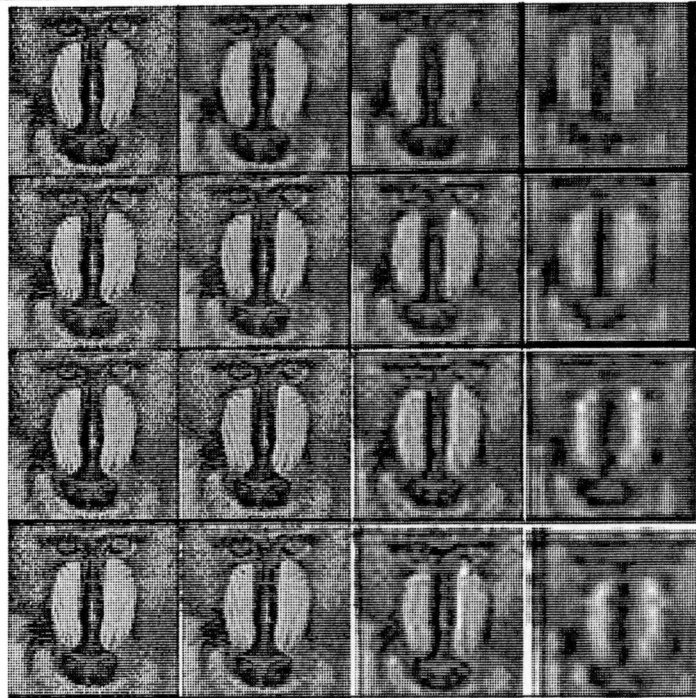
D2 1/2	D2 1/4	D2 1/8
D4 1/2	D4 1/4	D4 1/8
D8 1/2	D8 1/4	D8 1/8
D16 1/2	D16 1/4	D16 1/8



(b)

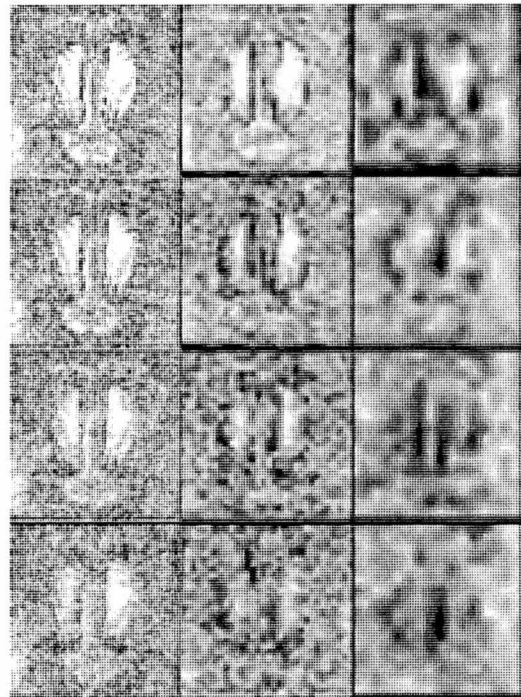
Figure 4.21: Direct Method Results for Image (j). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.11 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

D2 1/2	D2 1/4	D2 1/8
D4 1/2	D4 1/4	D4 1/8
D8 1/2	D8 1/4	D8 1/8
D16 1/2	D16 1/4	D16 1/8



(b)

Figure 4.22: Direct Method Results for Image (k). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.11 for key to image arrangement.

and as a result, the *MWT* images look similar to the original image. The best edge enhancement results are shown in the $D8-\frac{1}{2}$ image, which at least indicates some edge around the orangutang's snout.

4.2.3 Conclusions

In the above discussions of each image, the performance of the direct method has been evaluated for each image shown in Fig. 4.1. Examining the results for different order analyzing wavelet and at various resolutions indicates the higher order wavelets are more localized in frequency and consequently are better edge detectors. That is, a genuine edge detected by a higher order wavelet will, in general, be stronger than the same edge detected by a lower order wavelet. However, the higher order wavelets are less localized in space (have greater support) so the location of the detected edge is smeared over a greater number of pixels. In other words, the better the frequency localization the poorer the spatial localization. Consistent with the above conclusion, it is noted that in Images (a) through (e), the lowest order analyzing wavelet, D2, produces the best edge enhancement results. Whereas in Images (f) through (k), the D4 or higher order analyzing wavelets produce the best edge enhancement results.

The results of this section also indicate that due to the larger support of the higher order wavelets, the importance of each pixel in the final value of the *MWT* is reduced. This makes the *MWT* images produced by higher order wavelets less

susceptible to variations in gray level within an object. For this reason, better edge enhancement results are obtained using a higher order wavelet, D4, for Images (f) through (k).

Finally, the best results of this section indicate that the best edge enhancement results are, in general, obtained at the $\frac{1}{2}$ resolution. Due to the relatively small size of the analyzed images, 128x128, edges tended to run together at the lower resolutions and except for Image (j), no useful edge information is obtained at the $\frac{1}{8}$ resolution for any of the analyzed images. Since the $\frac{1}{8}$ resolution corresponds to only a 16x16 image; this is an expected result.

4.3 Results for the Underscale Edge Enhancement Method

4.3.1 Introduction

The results for the underscale method are presented as shown in Fig. 4.23 for each image. The D2, D4, D8 and D16 wavelets are used as the analyzing wavelets. The D2 wavelet when used in this method results in no edge enhancement as shown in Section 3.2. The edge enhancement results for Images (a) through (k) shown in Fig. 4.1 are presented in this section.

4.3.2 Discussion

Examining the resolution images in the in this section, it can be seen that in this method as the resolution decreases, the images appear more blurred but

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8

(a)

D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8

(b)

Figure 4.23: Organization of Results in Figs. 4.24 through 4.34. The resolution images are given in part (a) and the *MWT* images are given in part (b). The top number in each square refers to the analyzing wavelet. The bottom number in each square refers to the resolution.

also somewhat shinier. This result is expected due to the attempt made in this method to enhance the edges of each resolution as scale is decreased using an "allpass filter". The allpass filter increases the presence of high frequencies in the resolution image (see Section 3.2 for further information) and is expected to enhance the edges within the image and produce better edge detection results than the direct method discussed in Section 3.2. As with the resolution images in the previous section, the use of the higher order wavelets results in low resolution images that have smoother variation in gray level (see Section 4.1).

Examining the *MWT* images in this section, it can be seen that very little useful information about the location of edges is present at the $\frac{1}{4}$ and $\frac{1}{8}$ resolutions. In contrast to the results presented in Section 4.2, the use of the higher order wavelets does not result in edges which are significantly thicker than the edges resulting from the use of the lower order wavelets. This is due to the expansion of each resolution image to $\frac{1}{2}$ the scale of the original image. Resulting in initially greater spatial separation in the input image to the wavelet decomposition. The edge enhancement results for the underscale method are discussed below for each image.

Image (a): The edge enhancement results are shown in Fig. 4.24 for Image (a). The 1 resolution images are the most well segmented. For Image (a), the use of the D8 wavelet results in the best overall edge enhancement, that is, over several resolutions. However, the best results are shown in the D16-1 image (D16

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(b)

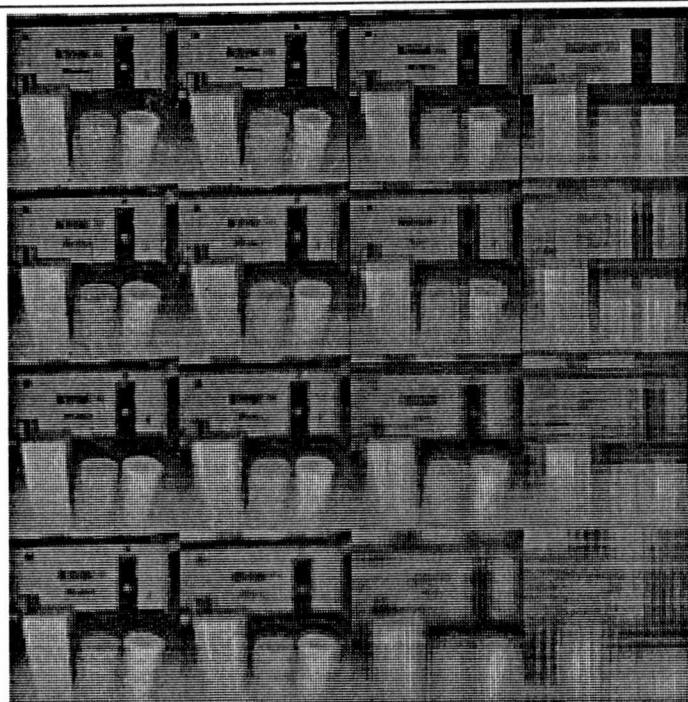
Figure 4.24: Underscale Method Results for Image (a). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.23 for key to image arrangement.

refers to the analyzing wavelet and 1 refers to the resolution, this convention is used throughout this section to identify a particular image) where edges are, in general, stronger and cleaner than in any of the other images. As previously discussed, the higher order wavelets are better at edge enhancement since they are better highpass and lowpass filters; however due to the large support of these higher order wavelets, edges that are close together tend to run together. The use of the underscale method as indicated by these results is a method of preventing close edges from running together by expanding the image to $\frac{1}{2}$ its original scale before the *MWT* is calculated.

Images (b) and (c): The edge enhancement results are shown in Fig. 4.25 and Fig. 4.26 for Image (b) and Image (c), respectively. As expected, the D16-1 image for both Image (b) and Image (c) provides the best edge enhancement results. This is because the higher order wavelets are better edge detectors and in this method edges are maintained separated by expanding the image to $\frac{1}{2}$ its original scale (twice the width and height of the original image).

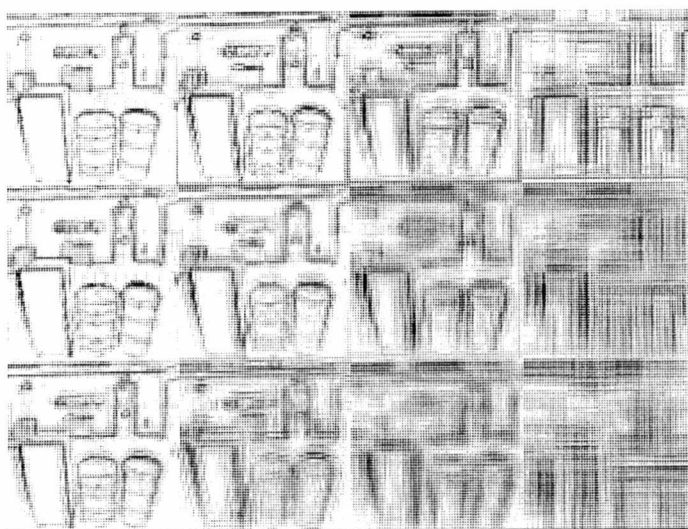
Image (d): The edge enhancement results are shown in Fig. 4.27 for Image (d). Again, the D16-1 image shows the edges most clearly. The UT in the D16- $\frac{1}{2}$ image is almost as discernible in the *MWT* images formed using the higher order wavelets as the UT in the images formed using the D2 wavelet.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

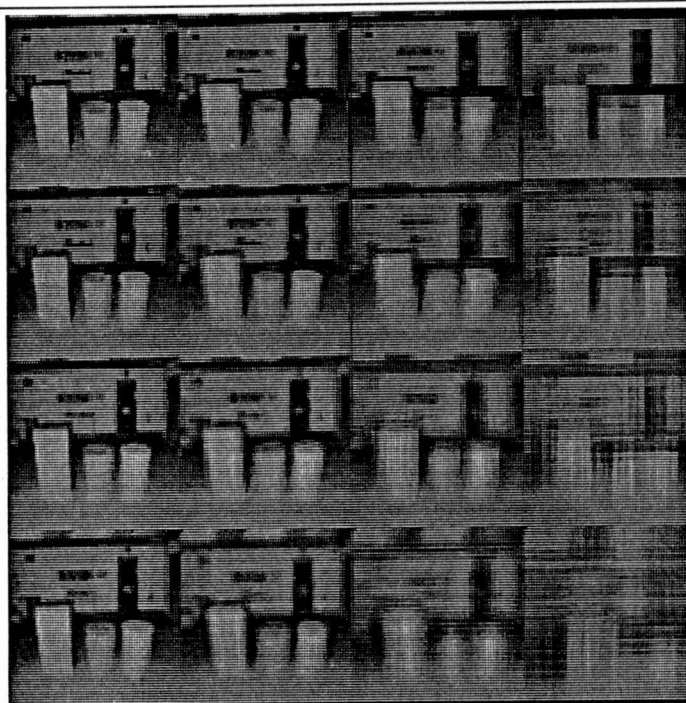
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(b)

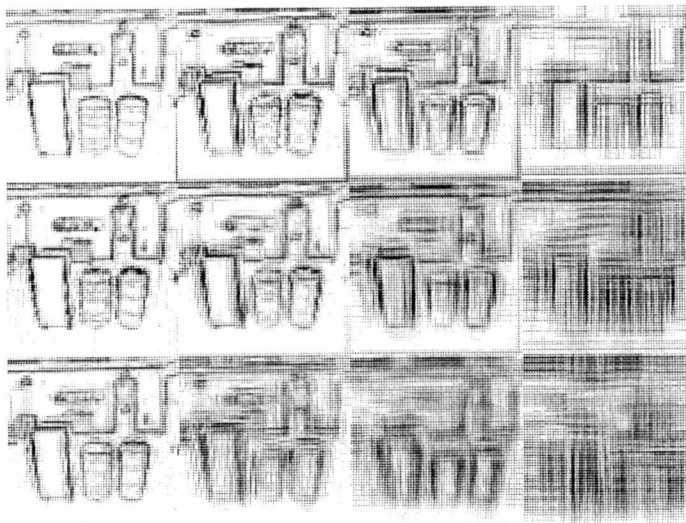
Figure 4.25: Underscale Method Results for Image (b). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.23 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

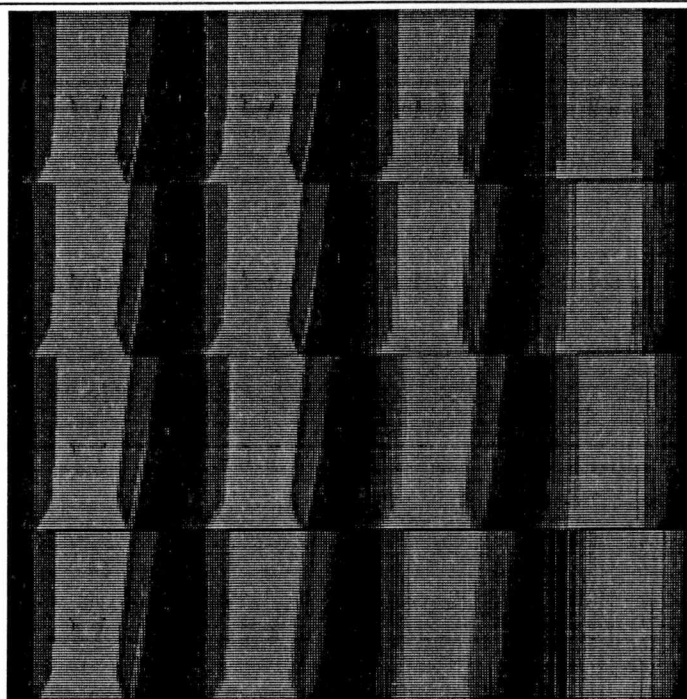
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(b)

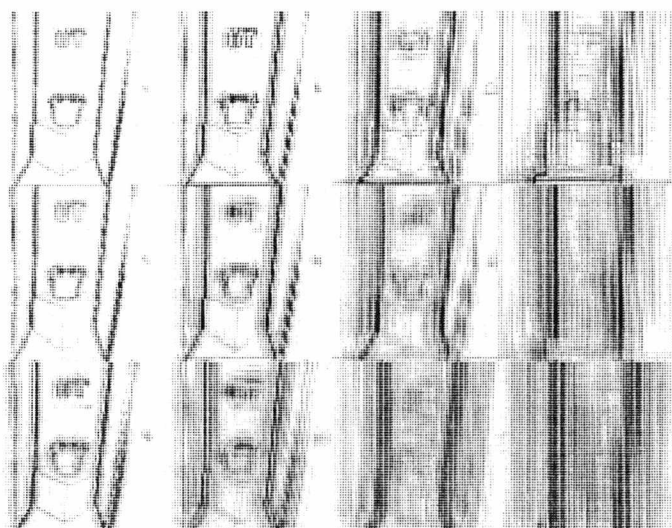
Figure 4.26: Underscale Method Results for Image (c). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.14 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(b)

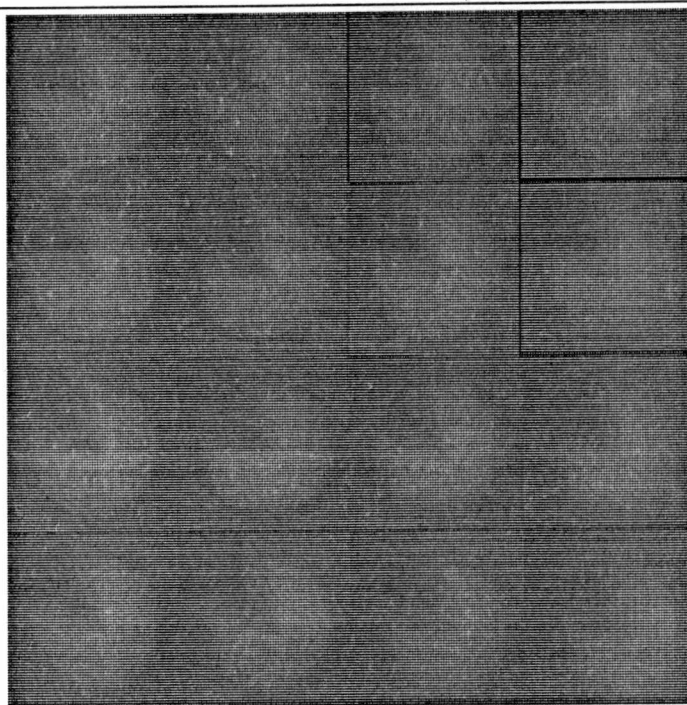
Figure 4.27: Underscale Method Results for Image (d). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.23 for key to image arrangement.

Image (e): The edge enhancement results are shown in Fig. 4.28 for Image (e). The strongest edges around the outside of the sphere in Image (e) are seen in the D16-1 image. However, the $D4-\frac{1}{2}$ image more clearly shows the edges around the cut-out quarter of the sphere with some indication of an edge around the outside of the sphere. Since there is a slow variation in gray level within the sphere, the *MWT* has a nonzero value in this region and the sphere is gray in the *MWT* images.

Image (f): The edge enhancement results are shown in Fig. 4.29 for Image (f). As previously noted, this image is characterized by some variation in gray level within the objects in the image. The interiors of the objects in the *MWT* images have about the same gray level when the higher order wavelets are used. This result is a direct consequence of the underscale method's expansion procedure. Recall, in the direct method, use of the higher order wavelets reduced *MWT* image sensitivity to minor variations in gray level within an object. In this method, sensitivity to gray level variations is about the same, regardless of the order of the analyzing wavelet. The best edge enhancement results are, again, shown in the D16-1 image.

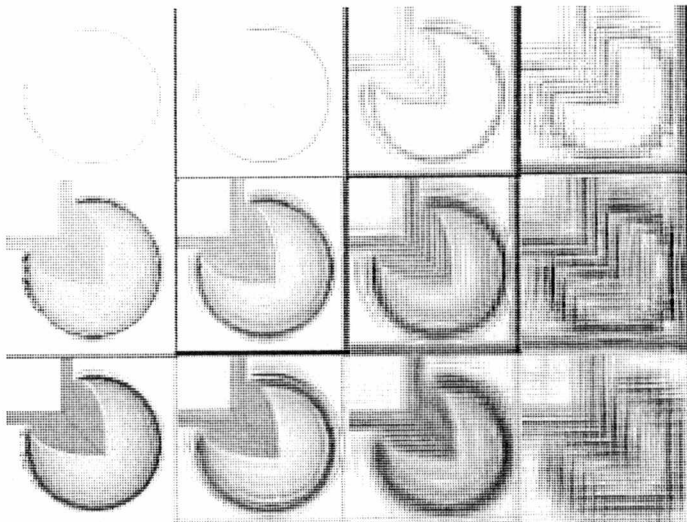
Images (g), (h) and (i): These images are the range images of Images (b), (c) and (d), respectively. The edge enhancement results are shown in Fig. 4.30, 4.31 and 4.32 for Image (g), Image (h) and Image (i), respectively. The

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

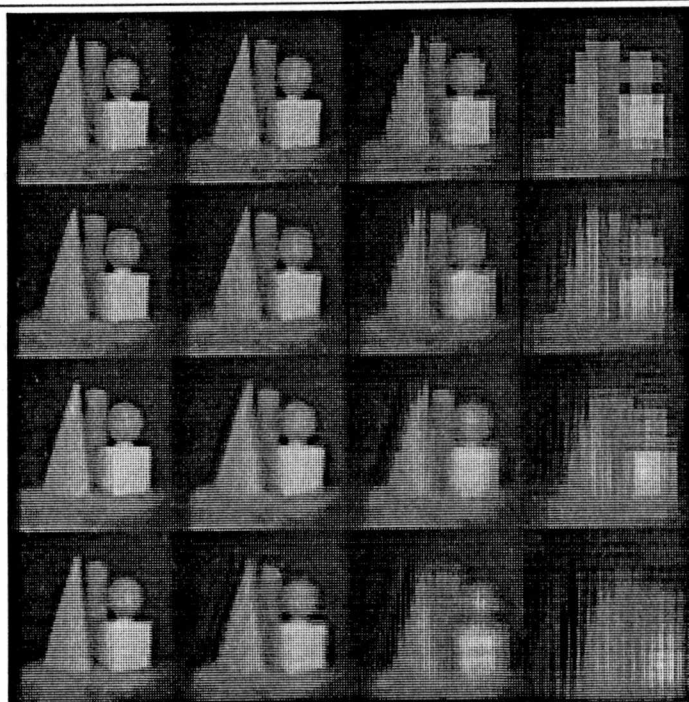
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(b)

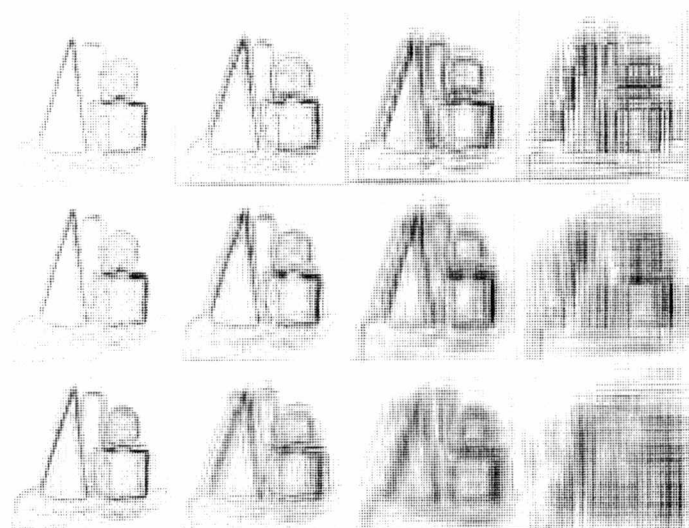
Figure 4.28: Underscale Method Results for Image (e). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.23 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

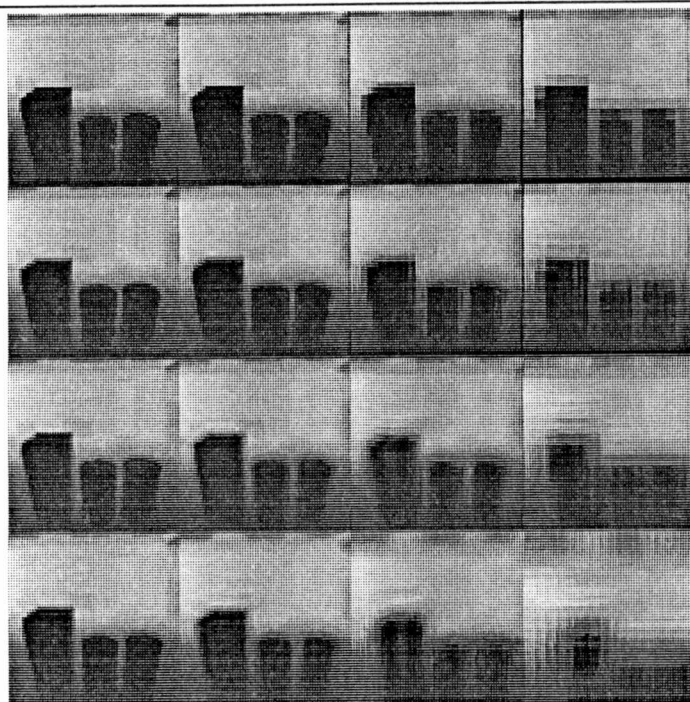
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(b)

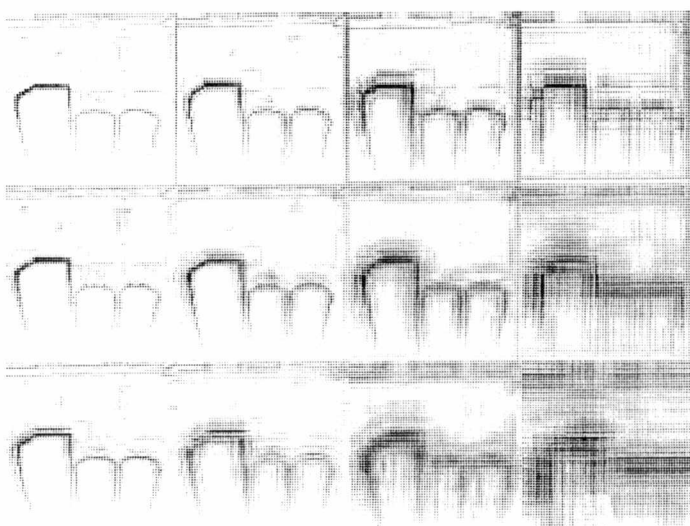
Figure 4.29: Underscale Method Results for Image (f). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.23 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

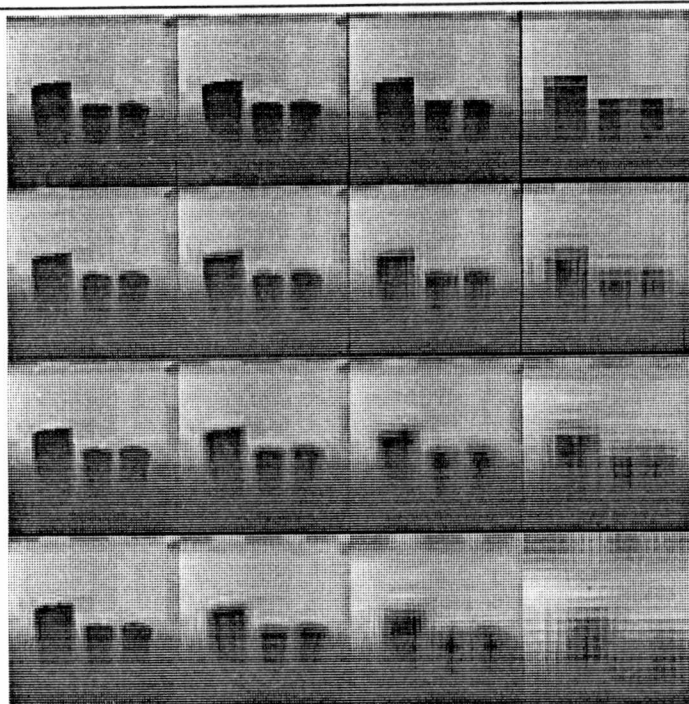
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(b)

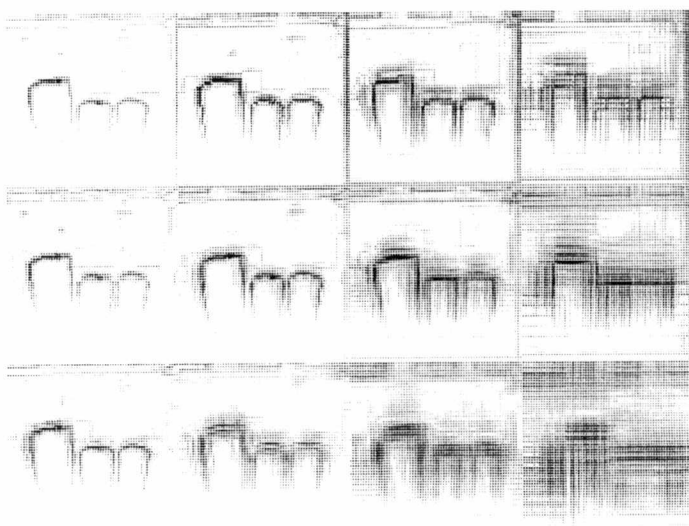
Figure 4.30: Underscale Method Results for Image (g). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.23 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

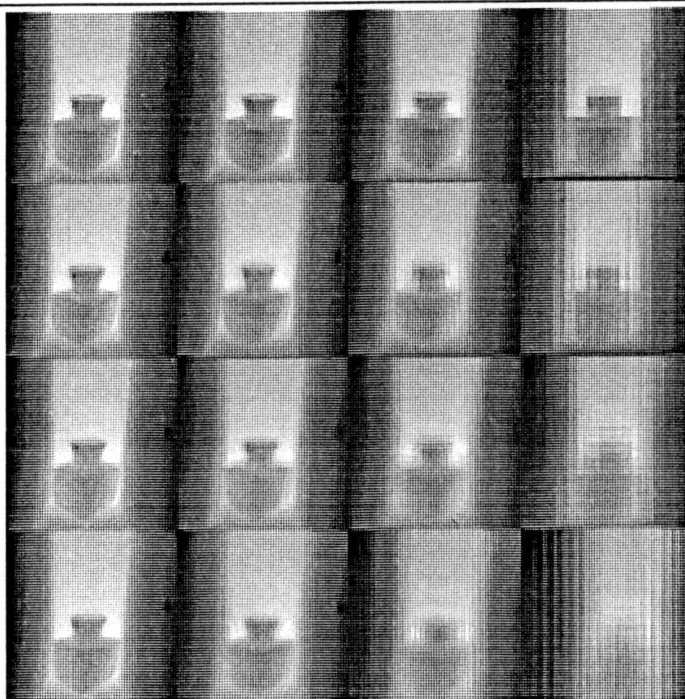
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(b)

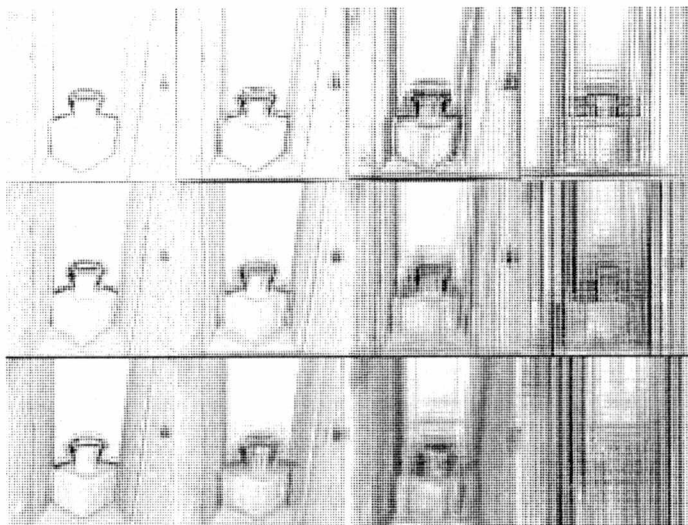
Figure 4.31: Underscale Method Results for Image (h). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.23 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(b)

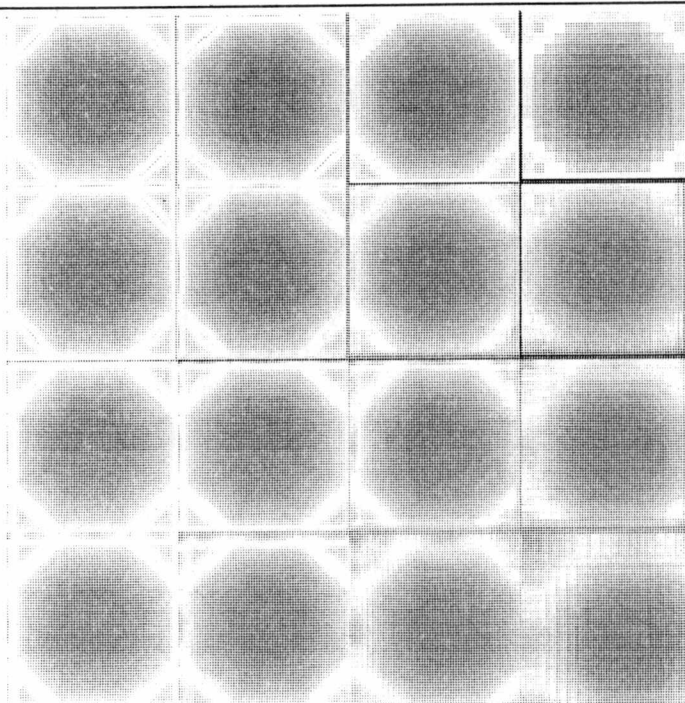
Figure 4.32: Underscale Method Results for Image (i). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.23 for key to image arrangement.

edge enhancement results are, in general, poorer for the range images due to their reduced contrast. The best edge enhancement is shown in the D16-1 image for Image (g), Image (h) and Image (i).

Image (j): The edge enhancement results are shown in Fig. 4.33 for Image (j). This image is characterized by slow variation in gray level throughout the image. The best edge enhancement results are shown in the D4- $\frac{1}{2}$ image. The white areas in this image correspond to areas of constant gray level. The gray areas correspond to areas of slow variation in gray level. Note that no edge enhancement is indicated at the boundaries between the white and gray areas by the edge lines. This image is the only contradiction to the results presented thus far in this section. Up until now, the D16-1 image most clearly indicated edges. For this image, the D4- $\frac{1}{2}$ image gives the best edge enhancement results.

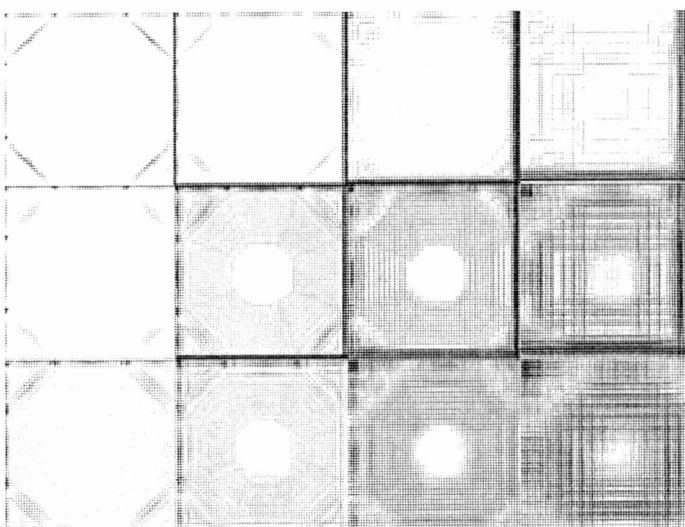
Image (k): The edge enhancement results are shown in Fig. 4.34 for Image (k). This image is characterized by a large variation in gray level within each region. These variations in gray level are detected as edges in the *MWT* methods and as a result, the *MWT* images look similar to the original image. The best edge enhancement results are shown in the D8- $\frac{1}{2}$ image, which at least indicates some edge around the orangutang's snout.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

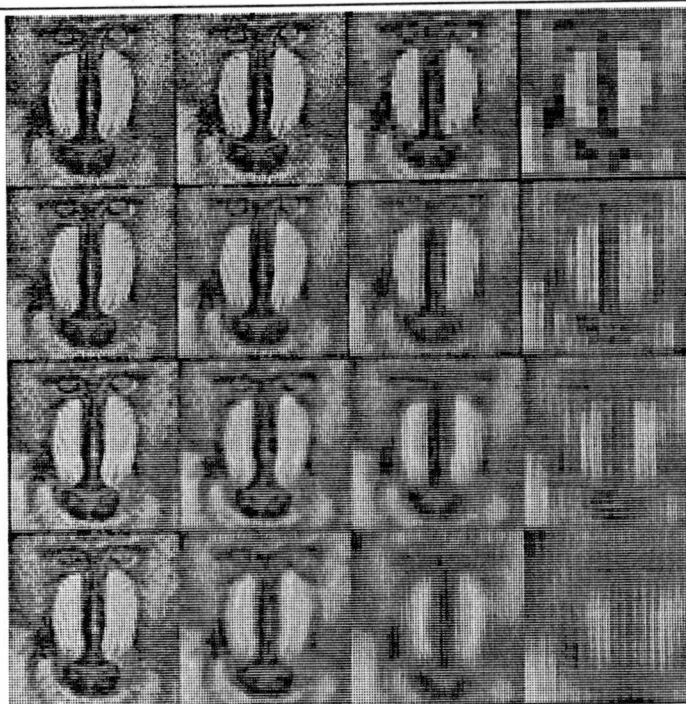
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(b)

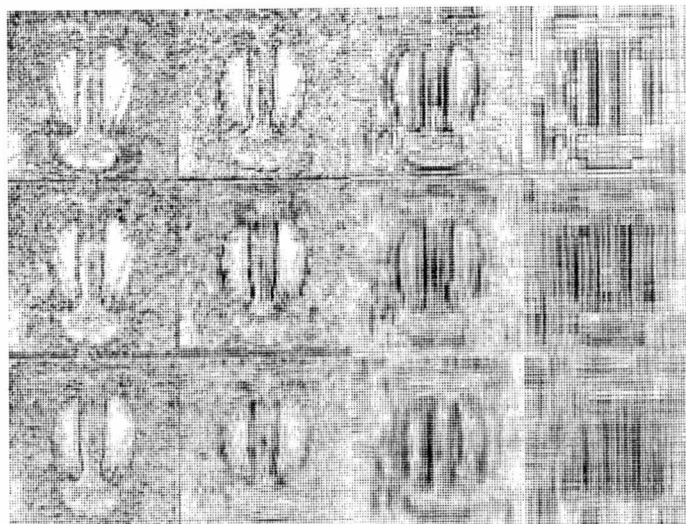
Figure 4.33: Underscale Method Results for Image (j). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.23 for key to image arrangement.

D2 1	D2 1/2	D2 1/4	D2 1/8
D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(a)

D4 1	D4 1/2	D4 1/4	D4 1/8
D8 1	D8 1/2	D8 1/4	D8 1/8
D16 1	D16 1/2	D16 1/4	D16 1/8



(b)

Figure 4.34: Underscale Method Results for Image (k). Resolutions are shown in (a) and *MWT* images in (b), see Fig. 4.23 for key to image arrangement.

4.3.3 Conclusions

In the above discussions of each image, the performance of the underscale method has been evaluated for each image shown in Fig. 4.1.

Examining the results for different order analyzing wavelet and at various resolutions indicates that, with the exception of Image (j) and Image (k), the best results are obtained from the D16-1 image in this method. In addition, the images with the best edge enhancement results are those at the 1 resolution. As previously discussed, the higher order wavelets are better at identifying edges due to their better frequency localization. However, because of their larger support, detected edges are less localized spatially, that is the edge lines are thicker. The thicker edge lines tend to run together in images with high edge density.

In the underscale method, the above problem is avoided by expanding each resolution to $\frac{1}{2}$ the scale of the original image using an allpass filter in the interpolation (see Section 3.2). As a result, the edge lines are roughly the same width regardless of the order of the analyzing wavelet. Another finding of this evaluation of the underscale method is that images segmented using the lower order wavelets are no more sensitive to small variations in gray level within the object than the higher order wavelets. And the use of higher order wavelets does not prevent finer details from being visible (see the UT in Fig. 4.18).

Overall, it can be said that the underscale method allows for relatively good edge enhancement and localization of edges and reduced sensitivity to small variations of gray level within objects.

4.4 Comparison Between the *MWT* Methods

Comparing the edge enhancement results for the direct method in the (b) part of Figs. 4.12 through 4.22 with the corresponding results for the underscale method in the (b) part of Figs. 4.24 through 4.34 several conclusions can be made. First, is that both methods perform well and poorly on the same images in the test set. The second, is that the edge lines in the underscale method are more localized (thinner) than the edge lines for the direct method for the same order analyzing wavelet. Third, both methods produce roughly two resolutions per image, in which useful information about the location of edges can be extracted. Fourth, the underscale method always gave better edge enhancement results than the underscale method for the same image. The exception being Image (j) where the direct method performed better; however, generally poor results are obtained for both methods on Image (j) (and Image (k)). Finally, the underscale method is roughly 4 times as computationally intensive as the direct method for the same order analyzing wavelet.

5 Summary of Results and Conclusions

Two methods have been developed and evaluated for the enhancement of multiscale edges in images in using compactly supported orthogonal wavelets. In this section, the results and findings of the development and evaluation of these methods are summarized. Section 5.1 outlines major points in wavelet theory contained in Section 2. Section 5.2 summarizes the two edge enhancement methods developed in Section 3. Section 5.3 summarizes the results obtained by the two methods when applied to a set of test images in Section 4.

5.1 Summary of Wavelet Theory

In Section 2, an overview of wavelet theory is presented which focuses on the use of the wavelet transform in various signal processing applications, the generation of a family of compactly supported, orthogonal wavelets and efficient implementation of the wavelet transform in multidimensional filter banks.

Wavelet theory is a unified, general description of a particular family of basis functions which are localized in both time and frequency. The translations and dilations of a (prototype) wavelet form a family wavelets. An arbitrary signal can be decomposed into a linear combination of the members of the wavelet family. The variable width of the time window of the wavelet transform allows it to look globally at coarse details and zero-in on small details. This is considered an advantage over the Short Time Fourier Transform in the analysis of nonstation-

ary signals. As with all transform spaces, particular features of signals or images are enhanced in the wavelet transform space, most notably, edges are dominant features in wavelet transform space. Another computationally advantageous property of the wavelet transform is it can be efficiently implemented using a pair of highpass and lowpass filters as shown in Figure 2.14.

5.2 Overview of the Direct and Underscale Methods

In Section 3, two methods are developed for multiscale edge enhancement using compactly supported orthogonal wavelets. Both methods make use of the modulus of the wavelet transform (see Equation 2.17). The *MWT* has a relatively large value near the location of a discontinuity or edge and is near zero when there is little variation in gray level. In the direct method, the *MWT* at each image resolution is calculated. Since with each application of the wavelet pyramid, the output image is at $\frac{1}{2}$ the resolution and twice the scale of the input image, the *MWT* image is expanded back to the scale of the original image by simple linear interpolation. In the underscale method, the resolution image (see Section 2) is expanded to $\frac{1}{2}$ the scale of the original image using an allpass filter. Next, the *MWT* is calculated from the $\frac{1}{2}$ scale image. The underscale method requires approximately four times as much computation as the direct method.

5.3 Summary of Experimental Results

The direct and underscale edge enhancement methods developed in Section 3 are applied to a set of test images in Section 4. Evaluation of the edge enhancement results for the set of test images has provided insight into the characteristics of images for which the (modulus of the) wavelet transform provides relatively good edge enhancement results.

The *MWT* methods were found to perform well for images characterized by large contrast (variation in gray level) between objects within the image and small variation in gray level within each object. This finding is expected, since the *MWT* is a measure of the difference between successively smoothed (or lowpass filtered) versions of an image. In strong edges, where there is a relatively large change in gray level over the span of a few pixels, the edges are indicated by high frequencies. Therefore, the *MWT* has a relatively large magnitude at the location of these, strong edges. Consequently, these edges show up well in the *MWT* images in Section 4. Conversely, in images which are characterized by weak edges, that is, the change in gray level is relatively small and occurs over many pixels, there is reduced high frequencies. As a result the *MWT* has a relatively small magnitude at the location of these weak edges and they show up poorly in the *MWT* images in Section 4. Small variation in the gray level within an object results in a nonzero value of the *MWT* which tends to obscure weaker edges.

In Section 4.2, the results of using the direct method of Section 3.1 is

examined. Edge enhancement results using the D2, D4, D8 and D16 wavelets [17] at the $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ resolutions are compared. The major finding of Section 4.1 is that the use of the higher order wavelets results in edge lines which are thicker and darker than the edges resulting from the use of the lower order wavelets. This result is explained by noting that the higher order wavelets are more localized in frequency and the lowpass and highpass wavelet filters approach perfect lowpass and highpass filters as the order of the wavelet increases (see Figures 2.7 through 2.10). The edges are thicker for the higher order wavelets because they are less localized in space (have a greater support) so the location of the enhanced edge is smeared over a greater number of pixels.

The results of Section 4.2 also indicate that the higher order wavelets are less sensitive to small variations in gray level within an object. This is due to their greater support, the importance of an individual pixel in the value of the *MWT* is reduced, that is, the average is taken over a greater number of pixels. Consistent with the above findings, it was noted in Section 4.2 that use of the higher order wavelets results in better edge enhancement results in noisy images and images characterized by weak edges than the lower order wavelets. In general, the best image segmentation results are obtained at the $\frac{1}{2}$ resolution. Since the analyzed images are 128×128 pixels, edges tended to run together at the lower resolutions.

The best edge enhancement results for the direct method for the Images of Figure 4.1 are summarized below:

1. Images (a)–(d): $D2-\frac{1}{2}$ (where D2 refers to the analyzing wavelet and $\frac{1}{2}$ refers to the resolution).
2. Images (e)–(i): $D4-\frac{1}{2}$
3. Image (j): $D4-\frac{1}{8}$
4. Image (k): $D8-\frac{1}{4}$

In Section 4.3, the results of using the underscale method of Section 3.2 is examined. Edge enhancement results using the D4, D8 and D16 wavelets [17] at the 1, $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ resolutions are compared. The major finding of Section 4.3 is that the use of the higher order wavelets in the underscale method results in edge lines which are darker but not thicker. The edge lines being of approximately the same width, regardless of the order of the analyzing wavelet, is explained by recalling that in the underscale method, the image at each resolution is expanded to $\frac{1}{2}$ the scale of the original image using an allpass filter. The *MWT* images are then formed by taking the wavelet transform of the expanded image. Consequently, the edges have an initially greater spatial separation and the interpolated points, from the expansion procedure (see Section 3.2) prevent a thicker edge line from being formed by keeping the value of the *MWT* near zero away from the edge.

Another finding of the evaluation in Section 4.3, is that the lower order wavelets are no more sensitive to small variations in gray level within an object than the higher order wavelets. Again, this is due to the expansion procedure,

which adds (interpolates) points in the expansion which result in the *MWT* being near zero away from an edge. The results of the underscale method evaluation also indicate that the use of the higher order wavelets does not prevent the finer details of an image from being just as discernible as when the lower order wavelets are used, again due to the expansion procedure used in this method. Consistent with the above findings, the best edge enhancement results were, in general, obtained using the higher order wavelets.

The best edge enhancement results for the underscale method for the Images of Figure 4.1 are summarized below:

1. Images (a)–(i): D_{16-1} (where D_{16} refers to the analyzing wavelet and 1 refers to the resolution).
2. Image (j): $D_{4-\frac{1}{2}}$
3. Image (k): $D_{8-\frac{1}{2}}$

Both methods make use of the orthogonal wavelets developed in [17], where the number of coefficients is twice the order of the wavelet (i.e. the D_4 wavelet is of order 2). These choices of wavelets may not be the optimal family of wavelets for edge enhancement.

Finally, it should be noted that the direct and underscale methods perform relatively good and relatively poorly for the same images, but that overall, the underscale method produced the best edge enhancement results of the two

methods. The advantage the direct method is that it takes only about a fourth as much computation as the underscale method.

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