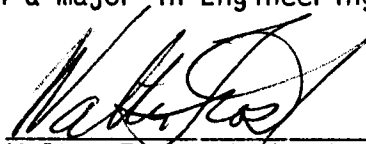


To the Graduate Council:

I am submitting herewith a dissertation written by Ehssanollah F. Arman entitled "Simulation of Atmospheric Boundary Layer Turbulence with Cross-Spectrum and Large-Scale Eddy Structure." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Engineering Science.



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SIMULATION OF ATMOSPHERIC BOUNDARY LAYER TURBULENCE WITH
CROSS-SPECTRUM AND LARGE-SCALE EDDY STRUCTURE

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Ehssanollah F. Arman

December 1986

DEDICATION

This dissertation is dedicated to my parents Mr. and Mrs. Habibollah Arman for their enduring support and encouragement.

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*In the name of God, Most Gracious, Most Merciful.
Praise be to God, The Cherisher and Sustainer of the Worlds;
Most Gracious, Most Merciful; Master of the Day of Judgment.
(Holy Qur'an I:1-4)*

I wish to express my deepest appreciation to the people who have assisted me in this research and have contributed generously to my doctoral program.

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ABSTRACT

The computer-simulated turbulence model plays an important role in the study of diffusive behavior of turbulent flow fields. Diffusion models are used to investigate certain physical phenomena. These models are continuously being developed to include additional features. The model presented in this study is to provide the non-zero cross-spectrum effect and the large-scale eddy structure of the atmospheric boundary layer turbulence.

In simulating turbulence fluctuations, a computer-generated random signal is passed through a digital filter. The filter is designed to produce an output signal having the same energy spectrum as the turbulence observed in the atmospheric boundary layer. Atmospheric turbulence shows that a non-zero cross-spectrum exists in the planetary boundary layer between the longitudinal and the vertical components of turbulence at low frequencies. In this study a technique to account for the non-zero cross-spectrum is considered.

The mechanism for large-scale eddy structure is convection. Thermal plumes exist in the unstable atmosphere. The plumes are effective in carrying warm air to the top of the convective layer. In boundary layers, large eddies are elongated longitudinal vortices, and they are associated with the low-frequency end of the energy spectrum. To consider the effect of large eddies in the turbulence simulation, an energy spectrum expression representative of the low-frequency portion of the spectrum as well as the high-frequency portion is employed.

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LIST OF SYMBOLS

$\hat{A}$	Constant of shaping filter
A_I, B_I	Constant coefficients of large eddy spectra, class I
A_{II}, B_{II}	Constant coefficients of large eddy spectra, class II
A_z	Constant of cross-spectrum expression
$\hat{a}$	Decay parameter
a, b, c	Constant parameters of large eddy equation
$\hat{c}$	Constant $\hat{c} = 1/A_z \times \overline{uw}/\sigma_u\sigma_w$ in cross-spectrum expression
c_m	Constant of lag filter
d_m	Constant $d_m = m\pi/(k\Delta z)_{\max}$
$H(f)$	Filter function in frequency domain
$H(k, z)$	Shaping filter of coherence model
H_m	Discrete filter function
H_u, H_v	Longitudinal and lateral components of filter function
H_{wa}, H_{wb}	Independent vertical components of filter function
$h(\tau)$	Filter function in time domain
i	$i = \sqrt{-1}$
k	Wave number
L	Monin-Obukhov stability length
m, n	Index of discrete quantities
N	Total number of samples per record
$R(\tau)$	Correlation function
S	Constant $S = C_0^2 + 2 \sum_{m=1}^M C_m^2$

T	Total time of record
$U(f), V(f), W(f)$	Fourier transform of turbulence fluctuations
u_*	Friction velocity
$u(t), v(t), w(t)$	Simulated turbulence time histories
$\overline{w}_H$	Horizontal mean wind
w_{mn}	Discrete weight function
x_m	Discrete input signal
$x_m(k)$	Gaussian noise sources of coherence model
$x(t), x'(t), x''(t)$	Independent input signals in time domain
$X(\omega), X'(\omega), X''(\omega)$	Fourier transform of independent input signals
$Y(f)$	Fourier transform of output signal
$Y(k, z)$	Fourier transform of the output signal of coherence model
$Y_C(k, z)$	Fourier transform of composite process output of coherence model
y_m	Discrete output signal
$y(t)$	Output signal time history
z	Height
z_i	Inversion height

Greek Symbols

α	Constant $\alpha = \frac{\sqrt{10}}{2} \frac{\overline{w}_H}{\Lambda_U}$
α_m	Coefficient of Fourier series $\alpha_m = 2C_m^2/S$
β_m	Constant $\beta_m = \hat{A}C_m$
γ	Coherent function
γ_0	Experimental coherent function
Δ	Separation distance between coherent levels
Δt	Time increment

δ	Kronecher delta
$\delta(\omega - \omega')$	Dirac delta function
ϵ	Constant $\epsilon = k\Delta z$
ϵ_0	Constant $\epsilon_0 = (k\Delta z)_{\max}$
η	Reduced frequency, $\eta = fz/\overline{w}_H$
$\hat{\eta}$	Reduced frequency, $\hat{\eta} = f\Lambda_U/\overline{w}_H$
η_i	Reduced frequency, $\eta_i = fz_i/\overline{w}_H$
ξ	Constant $\xi = \epsilon/\epsilon_0$
ξ_0	Constant $\xi_0 = 1/\hat{\alpha}\epsilon_0$
$\sigma_u, \sigma_v, \sigma_w$	Turbulence intensity components
$\sigma_x, \sigma_y, \sigma_z$	Standard deviation of input/output signals
τ	Lag time
$\phi(f)$	Energy spectrum
$\phi_H(f)$	High frequency portion of energy spectrum
$\phi_I(f)$	Energy spectrum of class I
$\phi_{II}(f)$	Energy spectrum of class II
$\phi_L(f)$	Low frequency portion of energy spectrum
$\phi_u(f), \phi_v(f), \phi_w(f)$	One-dimensional energy spectrum components
$\phi_{uw}(f)$	Cross-spectrum

CHAPTER I

LARGE EDDY TURBULENCE SIMULATION MODEL

1. INTRODUCTION

Turbulence fluid motion, although irregular, is generally describable in terms of statistical characteristics. This makes it possible to apply the basic laws of physics to express the turbulent motion in terms of statistical functions of turbulence behavior. These relationships have been long introduced by such giants in the field of turbulence as Taylor, Prandtl, Reynolds, and others. What we are interested in is the application of their relationships, theories, and concepts to engineering problems.

Diffusion action of turbulence is an important characteristic in the study of particle dispersion in such areas as pollution control, agricultural crop dusting, electro-optical weapon system performance, etc. Markov sequence and Monte Carlo techniques are two examples of widely used dispersion models. High-speed computers make Monte Carlo Particle Dispersion Model (MoCaPD) attractive to use. The computer program for the MoCaPD model is basically composed of two models, the turbulent flow model and the particle trajectory model. The turbulence model is computer-simulated and forms the basis of this research.

The technique of computer-generated turbulence simulation widely used today in various atmospheric studies such as diffusion (particle transport) model, wind shear model, etc. has been continuously developed to reflect the latest state of the art. The new emphasis in recent years has been to incorporate additional physics of the

planetary boundary layer such as the coherence property, the non-zero cross-spectrum effect, and the large eddy structure of the atmospheric turbulence into the simulation model.

The Monte Carlo Particle Dispersion Model (MoCaPD) by Arman, et al. (1986) includes the coherence model and the non-zero cross-spectrum feature of the atmospheric turbulence. The knowledge of large eddies in the turbulent flow have existed for some time. However, the study and investigation of large eddy structure is fairly new. This study provides a technique for simulating turbulence which includes the effect of large eddies. The computer code of large eddy simulation is also presented in this study.

2. SIMULATION TECHNIQUE

Large eddies in turbulent shear flows appear as elongated longitudinal vortices with relatively long lifetime, see Figure 1. The somewhat permanent nature of large eddies may provide a deterministic model, treating the large eddies as secondary flows. However, a stochastic approach will be considered in this study in order to expand the existing turbulence simulation models. Large eddies are associated with the low-frequency end of the energy spectrum and atmospheric instabilities. To simulate turbulent fluctuations, a computer-generated Gaussian white noise random signal (see Appendix A) is passed through a digital filter. The filter function, $H(f)$, is designed to produce an output signal which will have the desired energy spectrum, $\phi(f)$, of the atmospheric turbulence being simulated and is given by:

$$|H(f)|^2 = \phi(f) \quad (1)$$

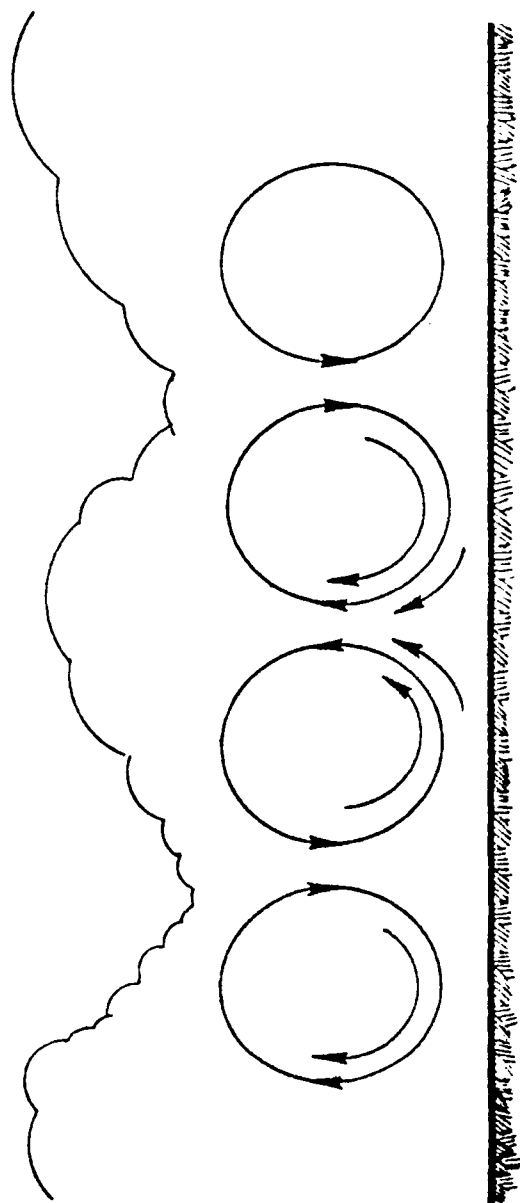


Figure 1. Large-scale eddies in planetary boundary layer.

The energy spectrum $\phi(f)$ is thus required as input to the simulation model. For large eddies, the spectrum is represented by the sum of a low-frequency "convective" portion and a high-frequency "mechanical" portion as given in Panofsky, et al. (1984) by Hojstrup as:

$$\phi(f) = \phi_L(f, z_i) + \phi_H(f) \quad (2)$$

where in terms of reduced frequency ϕ_L depends on $\eta_i = fz_i/\bar{w}_H$ and z_i/L , while ϕ_H depends only on $\eta = fz/\bar{w}_H$; z is the height, z_i is the inversion height, L is Monin-Obukhov stability length, and $\bar{w}_H$ is the horizontal mean wind. The equations suggested by Hojstrup (1981) which add a "convective" function to Kaimal's spectra for $z/L \leq 0$ are:

$$\begin{aligned} \frac{f\phi_u(f)}{u_*^2} &= \frac{0.5\eta_i}{1 + 2.2\eta_i^{5/3}} \left[\frac{z_i}{-L} \right]^{2/3} + \frac{105\eta}{(1 + 33\eta)^{5/3}} \\ \frac{f\phi_v(f)}{u_*^2} &= \frac{0.95\eta_i}{(1 + 2\eta_i)^{5/3}} \left[\frac{z_i}{-L} \right]^{2/3} + \frac{17\eta}{(1 + 9.5\eta)^{5/3}} \\ \frac{f\phi_w(f)}{u_*^2} &= \frac{32\eta}{(1 + 17\eta)^{5/3}} \left[\frac{z}{-L} \right]^{2/3} + \frac{2\eta}{1 + 5.3\eta^{5/3}} \end{aligned} \quad (3)$$

Figure 2 shows the energy spectra for the three components of the turbulent velocity fluctuation vector in unstable air computed from Equation 3. The large eddy effects appear as secondary peaks in the spectrum plots. The above spectra obey two general classes of spectra. These are:

$$\phi_I(f) = \frac{A_I}{B_I + f^{5/3}} \quad (4a)$$

- a: $z/-L = 0.71$
- b: $z/-L = 0.30$
- c: $z/-L = 0.03$
- d: $z/-L = 0.00$

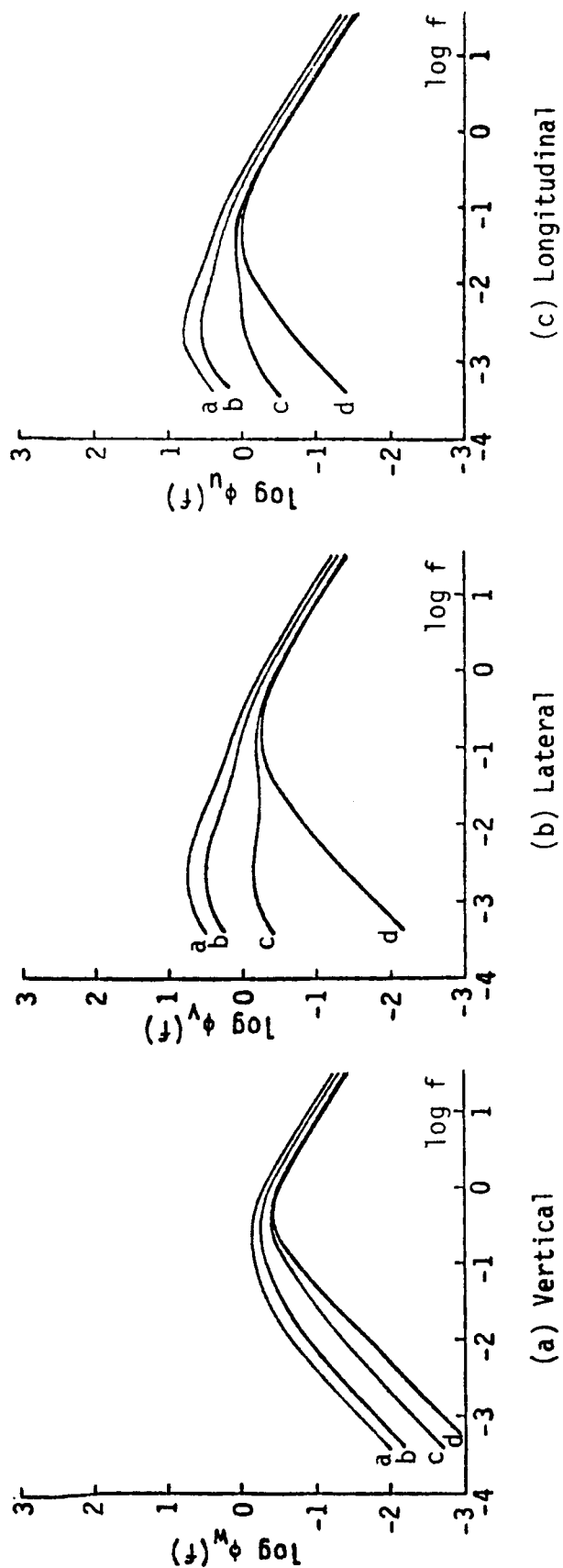


Figure 2. Spectra with large-scale eddy effects.

$$\phi_{II}(f) = \frac{A_{II}}{(B_{II} + f)^{5/3}} \quad (4b)$$

The components of Equation 3 can be adapted to these forms. The results are tabulated in Table 1.

For simple, statistically independent turbulence components, the filter functions are (see Appendix B):

$$\begin{aligned} |H_I(f)|^2 &= \phi_I(f) \\ |H_{II}(f)|^2 &= \phi_{II}(f) \end{aligned} \quad (5)$$

In general, the filter function, $H(f)$, is a complex valued quantity which may be conveniently thought of in terms of a magnitude and an associated phase angle. This can be done by writing $H(f)$ in complex polar notation as:

$$H(f) = |H(f)| e^{-i\theta(f)} \quad (6)$$

The absolute value $|H(f)|$ is called the system gain factor and the associated phase angle $\theta(f)$ is called the system phase factor. In these terms, the filter functions of Equation 5 would be:

$$\begin{aligned} H_I(f) &= \frac{\sqrt{A_I}}{\sqrt{B_I + i(f)^{5/6}}} = \frac{\sqrt{A_I}}{\sqrt{B_I + f^{5/3}}} e^{-i \tan^{-1}(f^{5/6})/\sqrt{B_I}} \\ H_{II}(f) &= \frac{\sqrt{A_{II}}}{(\sqrt{B_{II}} + i\sqrt{f})^{5/3}} = \frac{\sqrt{A_{II}}}{(B_{II} + f)^{5/6}} e^{-i \frac{5}{3} \tan^{-1}\sqrt{f/B_{II}}} \end{aligned} \quad (7)$$

where i is the imaginary number equal to $\sqrt{-1}$.

For isotropic turbulence, the cross-correlation uw , uv , and vw are zero. However, in the atmospheric boundary layer, turbulence is not isotropic and a cross-correlation uw arises due to vertical momentum

TABLE 1. Components of Large Eddy Spectra Equations.

Spectra Component	Class	Coefficients		Parameters		
		A	B	a	b	c
$\phi_{u_L}(f)$	I	$\frac{a}{b} c^{2/3} u_*^2 \left[\frac{z_1}{-L} \right]^{2/3}$	$\frac{c^{5/3}}{b}$	0.50	2.2	$\frac{\bar{w}_H}{z_1}$
$\phi_{u_H}(f)$	II	$\frac{a}{b^{5/3}} c^{2/3} u_*^2$	$\frac{c}{b}$	105.00	33.0	$\frac{\bar{w}_H}{z}$
$\phi_{v_L}(f)$	II	$\frac{a}{b^{5/3}} c^{2/3} u_*^2 \left[\frac{z_1}{-L} \right]^{2/3}$	$\frac{c}{b}$	0.95	2.0	$\frac{\bar{w}_H}{z_1}$
$\phi_{v_H}(f)$	II	$\frac{a}{b^{5/3}} c^{2/3} u_*^2$	$\frac{c}{b}$	17.00	9.5	$\frac{\bar{w}_H}{z}$
$\phi_{w_L}(f)$	II	$\frac{a}{b^{5/3}} c^{2/3} u_*^2 \left[\frac{z}{-L} \right]^{2/3}$	$\frac{c}{b}$	32.00	17.0	$\frac{\bar{w}_H}{z}$
$\phi_{w_H}(f)$	I	$\frac{a}{b} c^{2/3} u_*^2$	$\frac{c^{5/3}}{b}$	2.00	5.3	$\frac{\bar{w}_H}{z}$

transfer. Therefore, a non-zero cross-spectrum exists between u and w components of atmospheric turbulence at low frequencies (Barr, et al., 1974). To include the non-zero cross-correlation effect, which we shall refer to as the u -star turbulence model ($\overline{uw} = -u_*^2$), in the large eddy turbulence simulation model, a similar simulation technique as that used by Perlmutter, et al. (1977) is adopted. Figure 3 shows the control system for the large eddy cross-correlation simulation model. In this figure, $x(t), s$ are the system input functions, $h(\tau), s$ are the response functions, and the system outputs are the u , v , and w turbulence time histories. For this control system, only the cross-correlation effect between u and w components of turbulence is considered. The $\overline{uv}$ and $\overline{vw}$ cross-correlation effects of the atmospheric boundary layer are negligibly small and will not be considered in this study. To model this additional effect ($\overline{uw}$ cross-correlation), an analytical cross-spectrum model is required. Empirical data suggests the following form for the real part (amplitude relationship) of the cross-spectrum expression obtained from Engineering Sciences Data Item No. 74031 (1974):

$$\phi_{uw}(f) = \frac{\hat{c}}{[1 + 0.4 \hat{\eta}^2]^{1/2}} [\phi_u \phi_w]^{1/2} \quad (8)$$

where

$$\hat{c} = \frac{1}{A_z} \frac{\overline{uw}}{\sigma_u \sigma_w}$$

and

$$\hat{\eta} = f \frac{\Lambda_u}{\overline{W}_H}$$

where Λ_u is the longitudinal integral length scale, and A_z is a constant given in Appendix B.

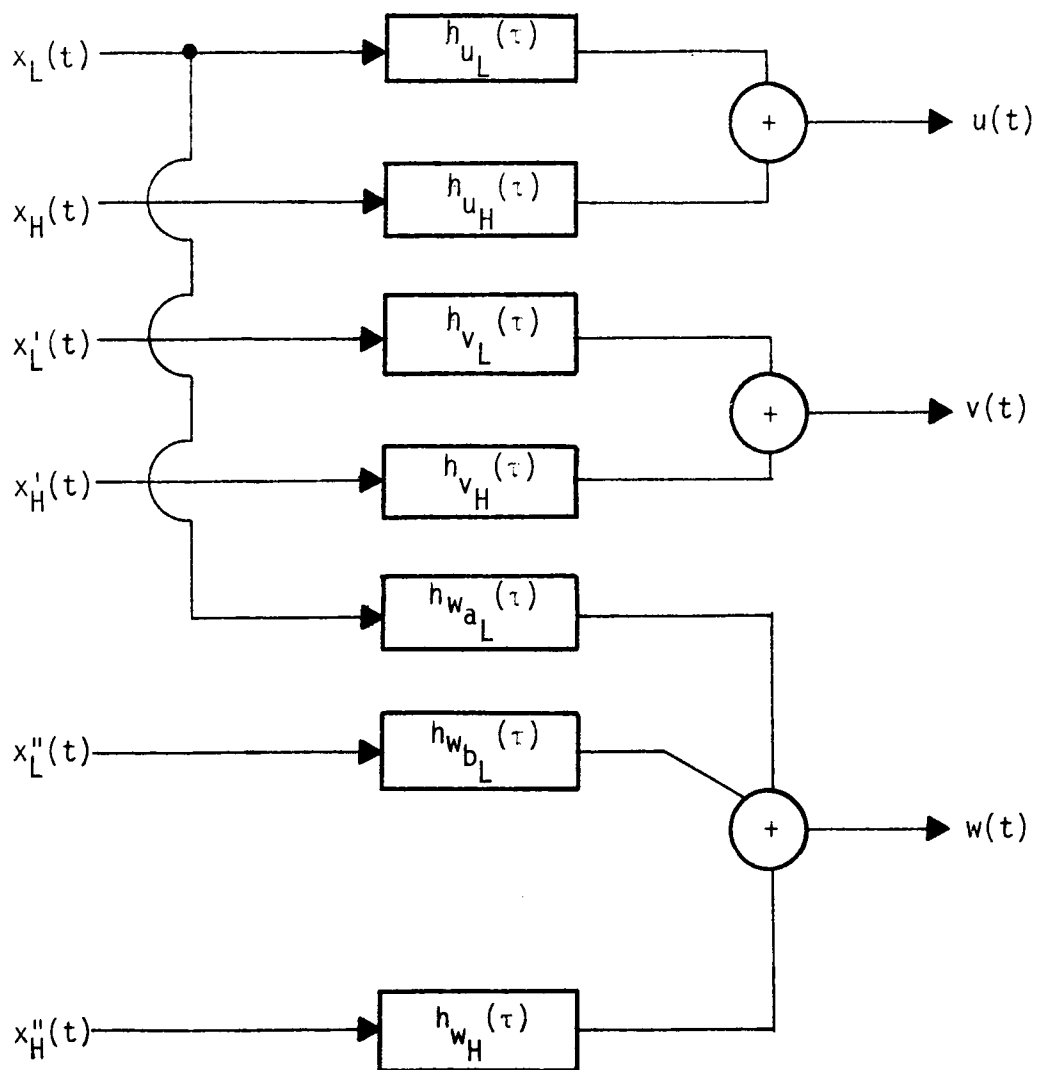


Figure 3. Control system for large eddy cross-correlation simulation model.

With the introduction of Equation 8 into the analysis, the low-frequency filter function for the vertical turbulence component is:

$$H_{w_{aL}}(f) = \left[\frac{\alpha^2 \hat{C}^2 A_{wL}}{\alpha^2 + f^2} \right]^{1/2} (B_{wL} + f)^{-5/6} e^{-i \left[\tan^{-1} \frac{f}{\alpha} + \frac{5}{3} \tan^{-1} \sqrt{\frac{f}{B_{wL}}} \right]}$$

$$H_{w_{bL}}(f) = \left[A_{wL} - \frac{\alpha^2 \hat{C}^2 A_{wL}}{\alpha^2 + f^2} \right]^{1/2} (B_{wL} + f)^{-5/6}$$

$$\times e^{-i \left[\tan^{-1} \frac{f}{\alpha} + \frac{5}{3} \tan^{-1} \sqrt{\frac{f}{B_{wL}}} - \tan^{-1} \frac{f}{\alpha \sqrt{1 - \hat{C}^2}} \right]} \quad (9)$$

where

$$\alpha = \frac{\sqrt{10}}{2} \frac{\bar{W}_H}{\Lambda_u}$$

The high-frequency filter function for the vertical turbulence component follows the same form as the class I filter function of Equation 7. Appendix B provides a detailed derivation of the above equations. Once the filter function is determined, the output signal in the frequency domain, $Y(f)$, for a given input signal, $X(f)$, is obtained by Fourier transformation of the convolution integral as (see Appendix B):

$$Y(f) = H(f) * X(f) \quad (10)$$

Applying a fast Fourier transformation technique (FFT) described in Appendix B, Equation 10 in discrete form becomes:

$$Y_m = H_m X_m \quad (11)$$

where the discrete input signal X_m is:

$$X_m = W_{mn} X_n \quad (12)$$

W_{mn} is the weight function given by:

$$W_{mn} = \exp \left[-i \frac{2\pi mn}{N} \right] \quad (13)$$

and

$$x_n = x(n\Delta t) \quad n = 0, 1, 2, \dots, N-1 \quad (14)$$

is the computer-generated digitized white noise input signal in the time domain. To invert the discrete output signal given by Equation 11 back into the desired time domain, the discrete inverse Fourier transform is applied. The result is:

$$y_n = \frac{1}{N} W_{mn}^* Y_m \quad (15)$$

where

$$W_{mn}^* = \exp\left[i \frac{2\pi mn}{N}\right]$$

A description of the discrete Fourier transformation is given in Appendix B.

This analysis results in a turbulence time history having the given spectra of Equation 3 with the appropriate non-zero cross-correlation effect. To evaluate the simulation model, an example case is computed and compared with existing data.

Figure 4 shows measured time histories of vertical velocity, temperature, and longitudinal velocity which contain large-scale eddies (Wilczak, 1984). The variates, height, mean velocity, stability parameters, and friction velocity are shown on the figure. Wilczak associates the large eddies with the temperature ramps and a height-dependent phase lag of u and w velocity fluctuations where the momentum flux is consistently counter-gradient.

A computer program based on the presented analytical model given in Chapter I of this study is developed. The computer program which is described in Chapter II generates time histories of u , v , and w

Input Conditions: $z = 50 \text{ m}$
 $\bar{w}_H = 11 \text{ ms}^{-1}$
 $L = -70 \text{ m}$
 $z_i = 3000 \text{ m}$
 $u_* = 0.55 \text{ m/s}$
 $\Lambda_u = 500 \text{ m}$

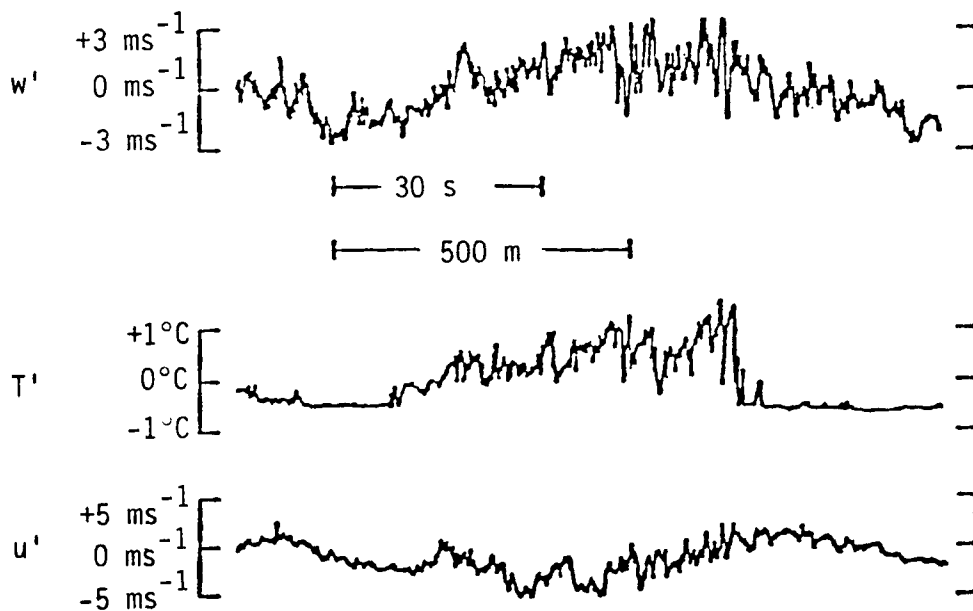


Figure 4. Time series of vertical velocity, temperature, and longitudinal velocity exemplifying the large-scale eddy structure.

components of turbulence for a given input height, wind strength, and stability conditions. Figures 5 and 6 are the simulated time histories of longitudinal and vertical velocity fluctuations generated for the input conditions shown on Figure 4. The cyclic nature of positive and negative regions of vertical velocity is clearly evident both in the actual data and in the simulated data. This cyclic behavior of the velocity fluctuations and the phase lag between u and w components are further reflected as spikes in the uw time histories as shown by data in Figure 7 (Antonia, et al., 1982). These spikes coincide with the temperature ramps. Figure 8 is the uw time history of simulated u and w time histories from Figures 5 and 6. The presence of spikes in uw is clearly evident. Figures 9 through 11 are simulated time histories of u , w , and uw fluctuations for the neutral stability case. The same features of the unstable case are present but with appreciably dampened amplitudes.

One important characteristic of atmospheric turbulence, the interlevel coherence, has not been addressed in previously described simulation. To simulate turbulence which includes the coherence property between vertically separated layers, a coherence function, γ_0 , is introduced (Davenport, 1961; Haugen 1973; Pielke and Panofsky, 1976).

$$\gamma_0(k, \Delta z) = \exp(-\hat{a}|k\Delta z|) \quad (16)$$

where $\hat{a}$ is the decay parameter, k is the wave number, $2\pi f/w_H$, and Δz is the separation distance between vertical levels.

The output transform of a multi-filter system, discussed in Appendix B, is given by:

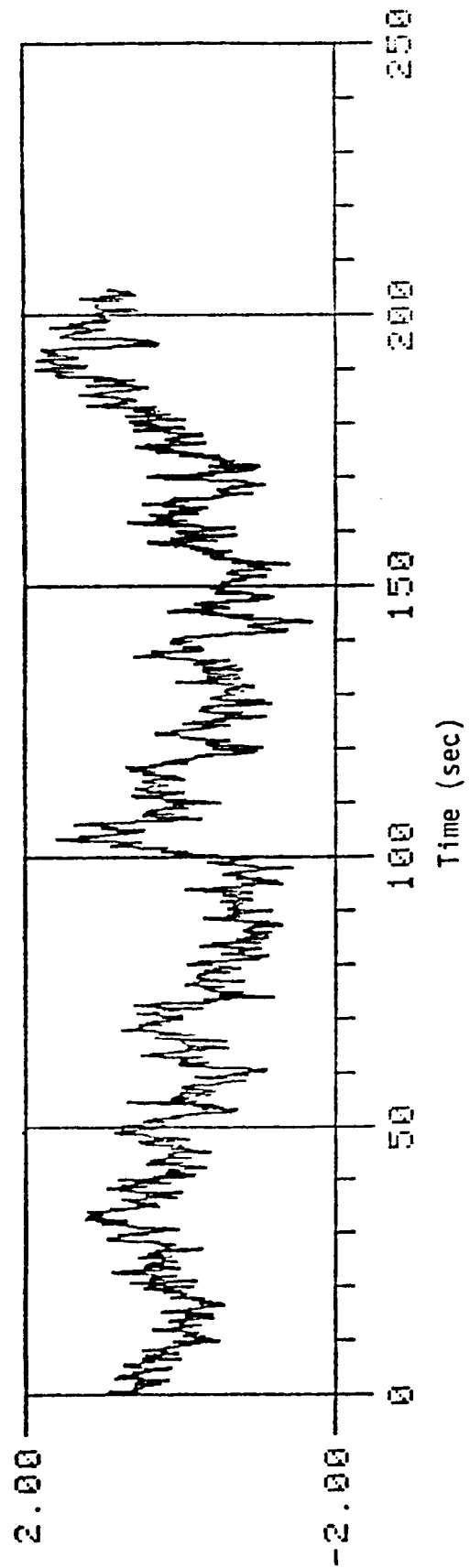


Figure 5. Large-scale eddy turbulence time history (unstable $z/L = -0.71$), longitudinal component ($\sigma_u = 1.27$).

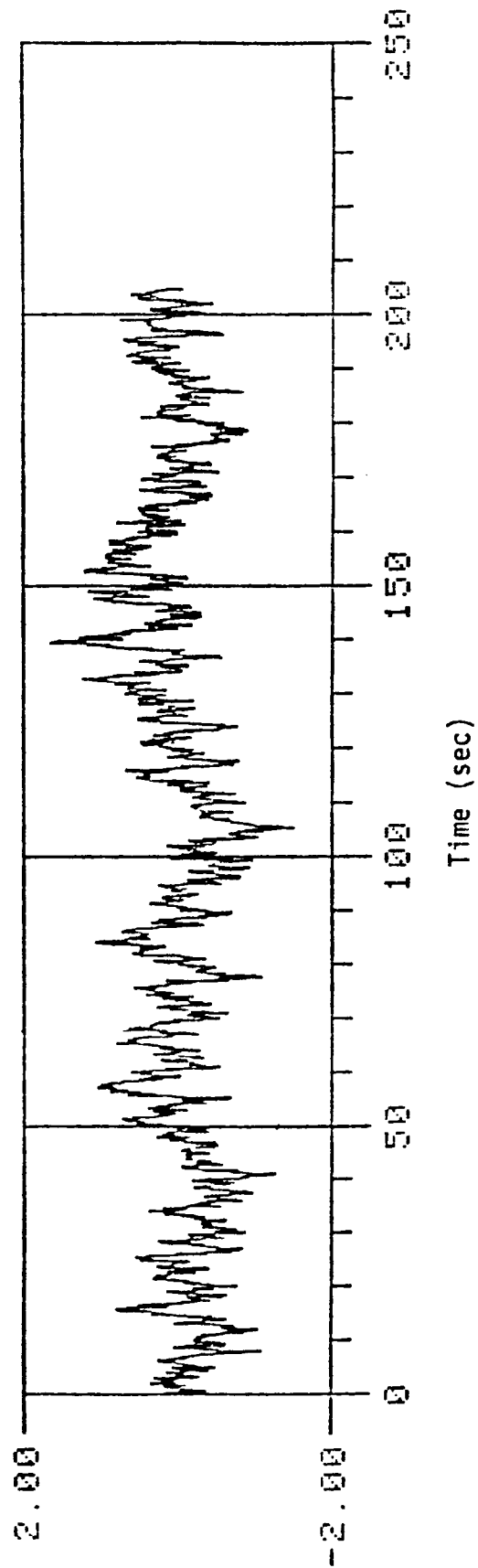


Figure 6. Large-scale eddy turbulence time history (unstable $z/L = -0.71$), vertical component ($\sigma_w = 0.69$).

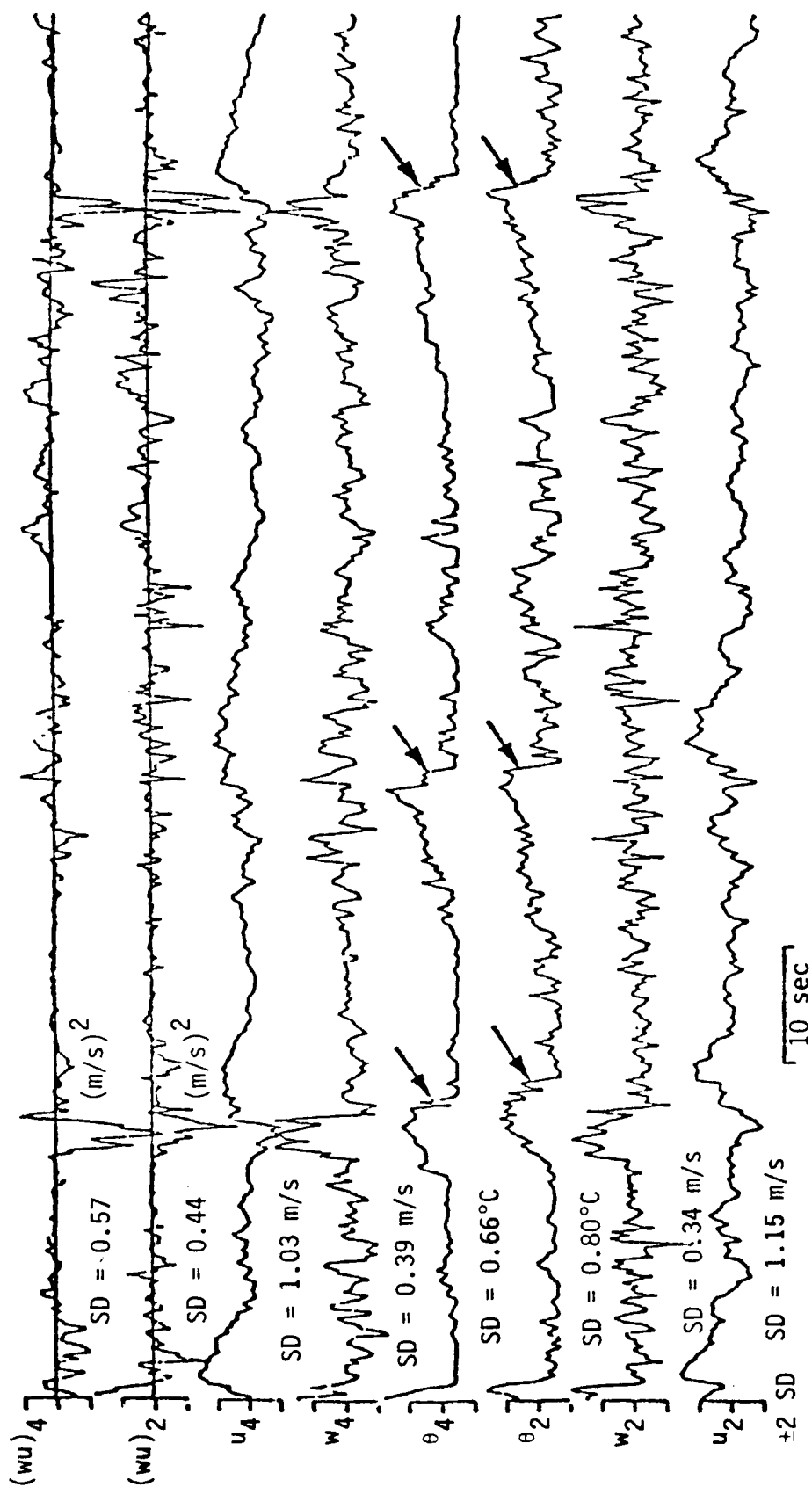


Figure 7. Simultaneous traces of θ , u , w and the product uw at heights of 2 and 4 m. $L = -24.4$ m (run 37). The suffix refers to the particular height. The duration of the record shown in 119 sec. Time progresses left to right. Ramps counted in this record are indicated by arrows. (The heights of the vertical lines at the left of the figures represent a magnitude of ± 2 standard deviations for each variable plotted.)

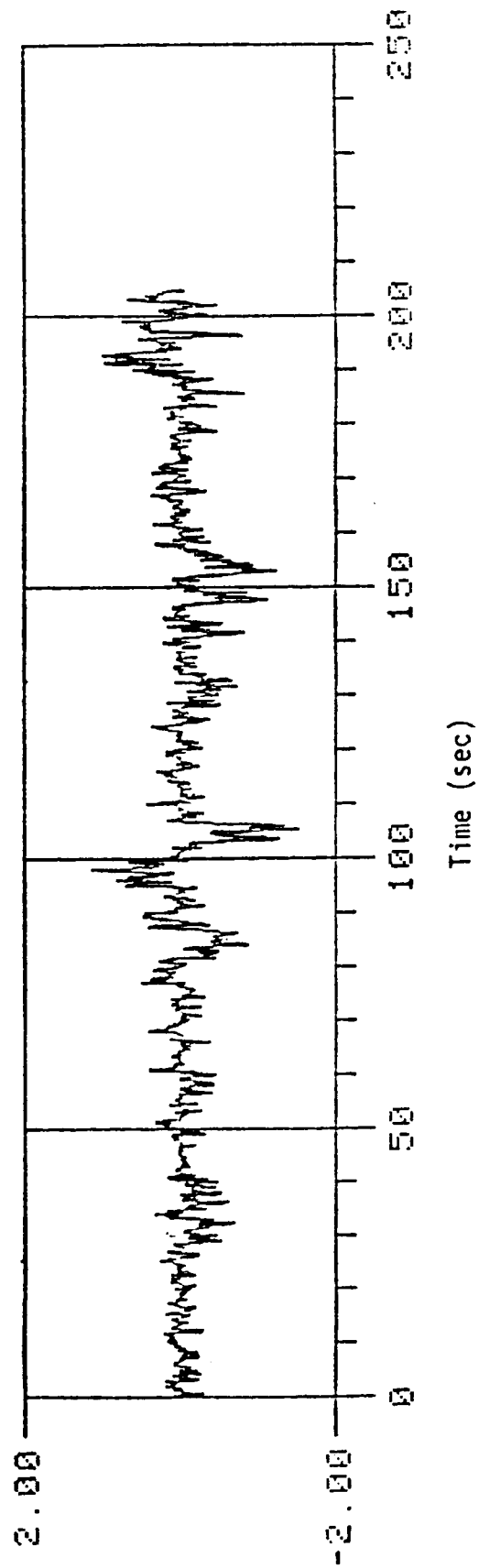


Figure 8. Large-scale eddy turbulence time history (unstable $z/L = -0.71$),
 uw simulation.

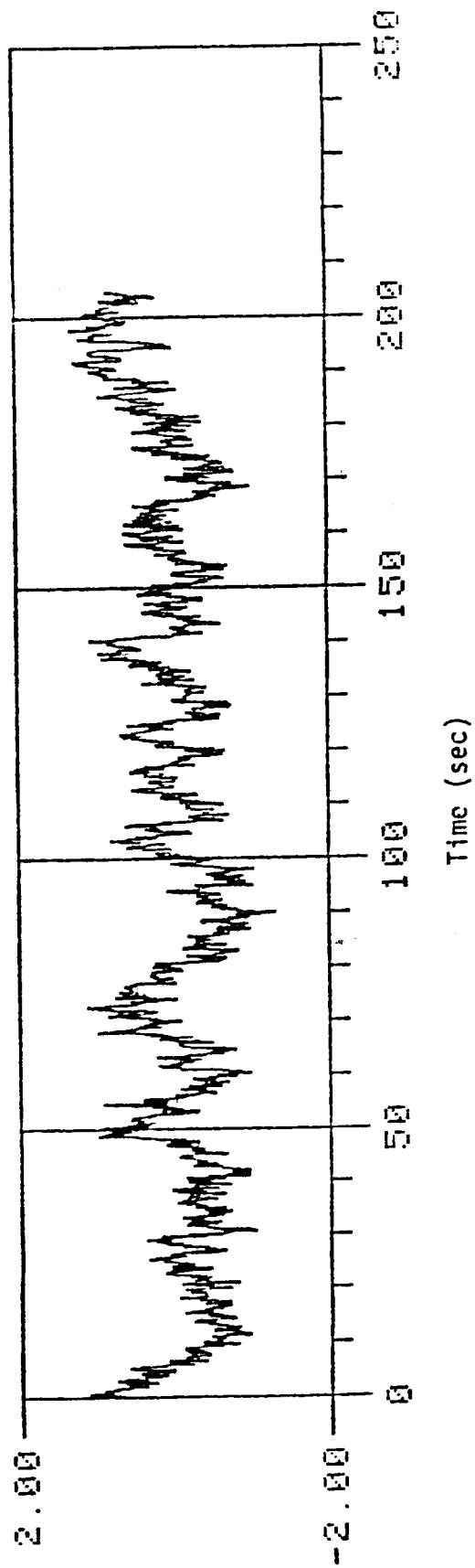


Figure 9. Large-scale eddy turbulence time history (neutral $z/L = 0$), longitudinal component ($\sigma_u = 0.79$).

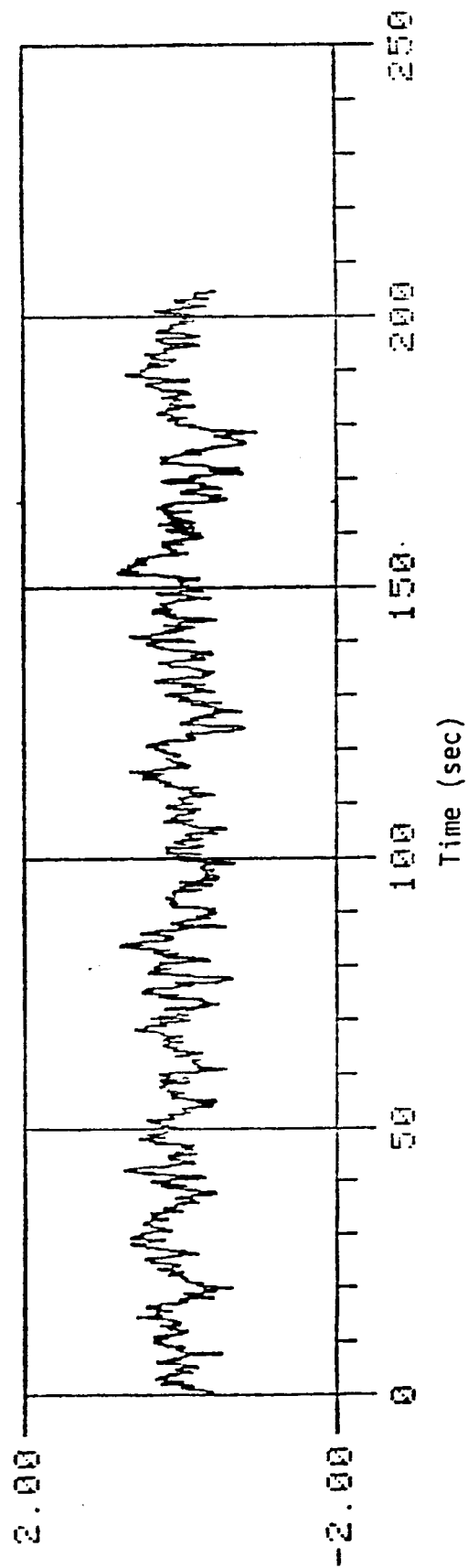


Figure 10. Large-scale eddy turbulence time history (neutral $z/L = 0$), vertical component ($\sigma_w = 0.43$).

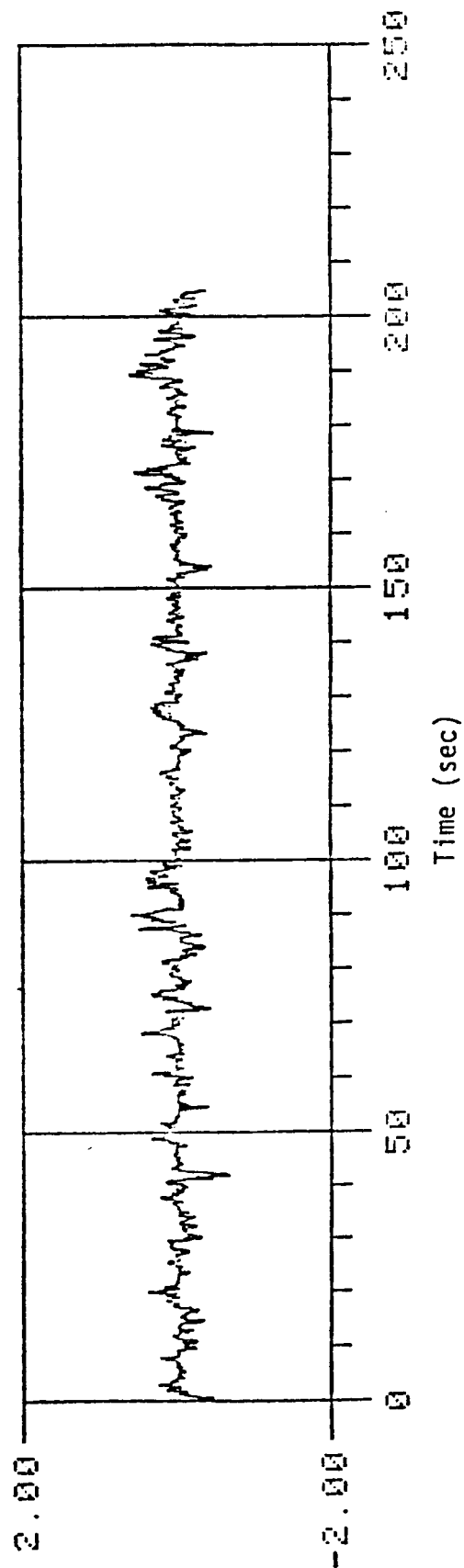


Figure 11. Large-scale eddy turbulence time history (neutral $z/L = 0$),
uw simulation.

$$Y(k,z) = \hat{A}H(k,z) \sum_{m=-M}^M D_m(k,z) X_m(k) \quad (17)$$

where $\hat{A}$ is a constant multiplier, $H(k,z)$ is the frequency response function (filter function), and $D_m(k,z)$ is a level frequency factor (lag filter) given as:

$$D_m(k,z) = C_m \exp(im\pi kz/\epsilon_0) \quad \epsilon_0 = (k\Delta z)_{\max} \quad (18)$$

substituting $D_m(k,z)$ into Equation 17 gives:

$$Y(k,z) = H(k,z) \left[\sum_{m=0}^M (e^{i\theta_m} X_m(k) + e^{-i\theta_m} X_{-m}(k)) \beta_m \right] \quad (19)$$

where

$$\beta_0 = \frac{C_0 \hat{A}}{2}, \quad \beta_m = C_m \hat{A} \quad \text{for } m \neq 0$$

$$\theta_m = m\pi kz/\epsilon_0$$

Equation 19 can be reduced to:

$$Y(k,z) = H(k,z) \hat{X}(k,z) \quad (20)$$

where

$$\hat{X}(k,z) = \sum_{m=0}^M (e^{i\theta_m} X_m(k) + e^{-i\theta_m} X_{-m}(k)) \beta_m$$

Thus, the transform of the output $Y(k,z)$ can be calculated in the same manner as described in Equation 10. Applying the inverse FFT (Equation 15), the simulated output time history is obtained. Figures 12 and 13 show the coherent turbulence time histories of the u and w components, respectively, each at two levels, 47 and 50 m for the unstable case of $z/L = -0.71$. For detailed comparison of the two coherent levels, shorter time histories are plotted in Figures 12 and 13, thus spreading

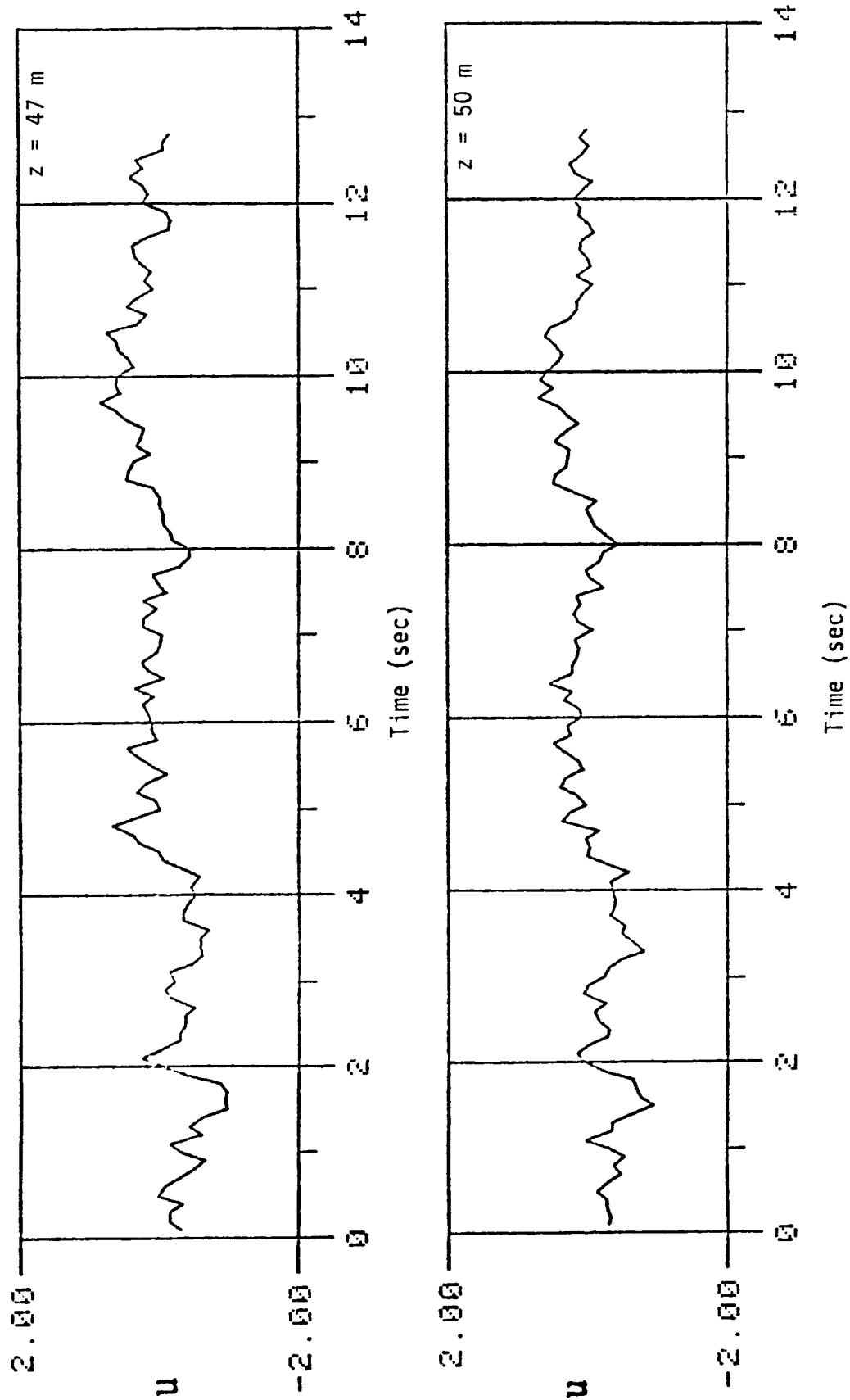


Figure 12. Large-scale eddy turbulence u-component simulation with coherence.

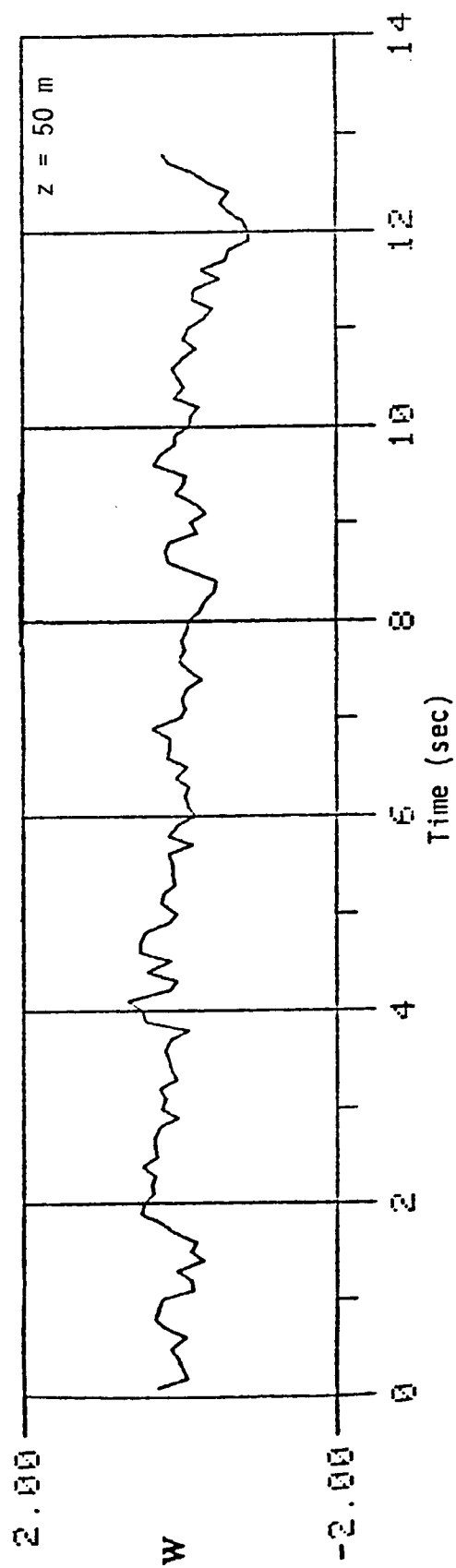
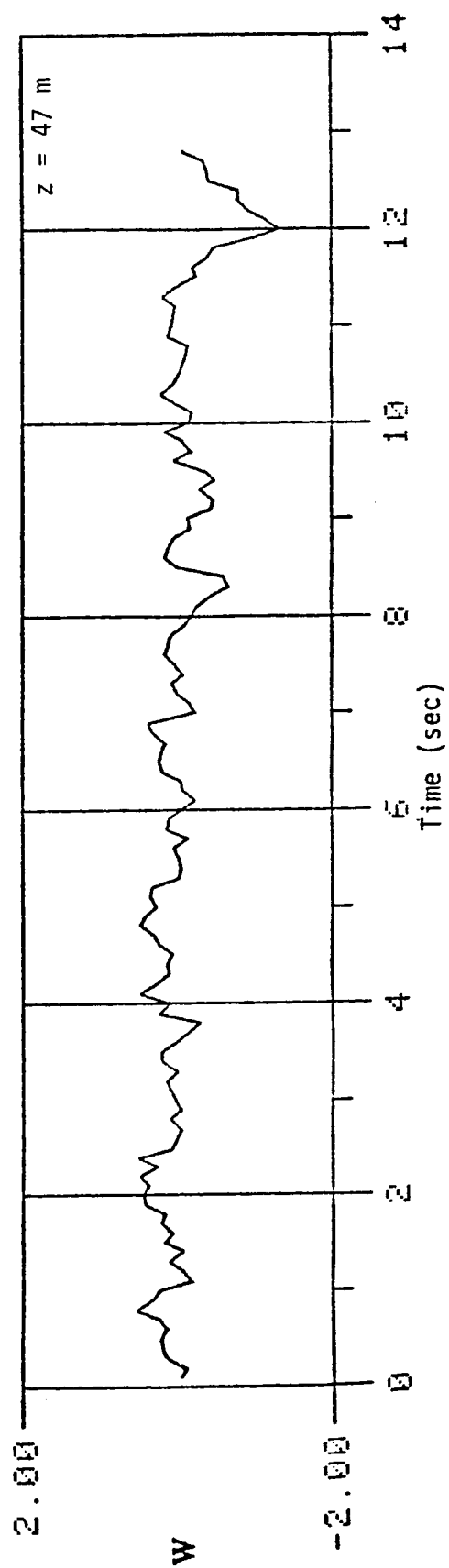
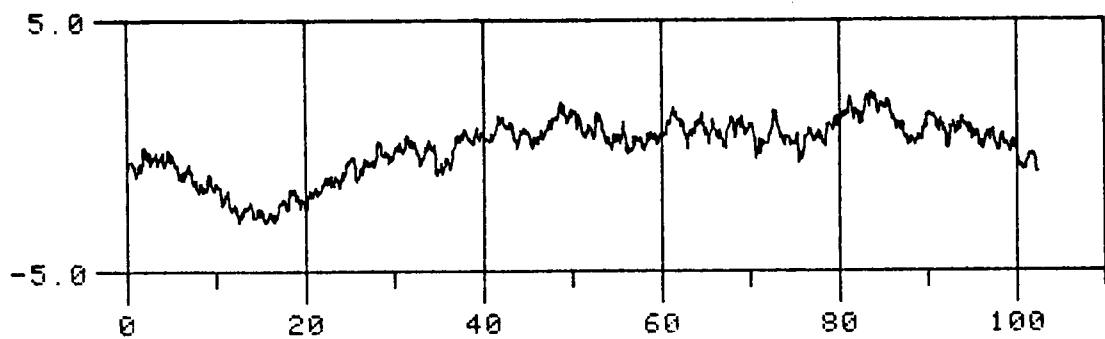


Figure 13. Large-scale eddy turbulence w-component simulation with coherence.

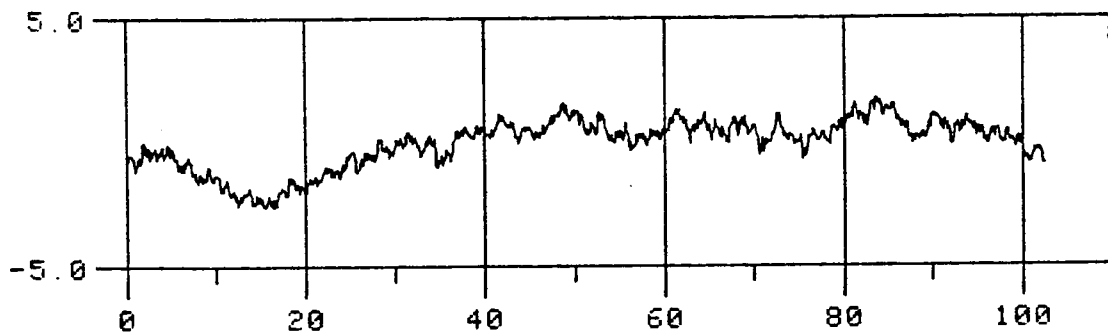
out the peaks and valleys of the time histories (Figures 4 through 11) shown previously. The strong coherence between the two levels is evident in these figures.

To study the effect of stability, several runs (simulated turbulence time histories) are generated. Only stability parameter (z/L) for these runs is varied, while holding all other parameters constant which included using the same random signal as an input to generate the turbulence time histories. Thus, only the effect of varying stability is reflected in the runs. Figures 14, 15, and 16 show the simulated time histories of various stability for u , v , and w components of atmospheric turbulence. The stability values of z/L for the time histories in each figure from top to bottom are -2, -1, -0.3, and 0 which correspond to the Pasquill stability categories A, B, C, and D, respectively. Note that category A is highly unstable while category D is neutral. Other input parameters used in the simulation are the same as given in Figure 4, i.e., $z = 50$ m, $u_* = 0.55$ m/s, $\overline{w}_H = 11$ ms⁻¹, $\Lambda_u = 500$ m, $z_i = 3000$ m. Note that some parameters such as z_i and u_* may vary with stability. However, to study only the effect of varying z/L , those parameters (z_i and u_*) are kept the same as well as other parameters. Study of Figures 14, 15, and 16 shows that in each figure the intensity of turbulence is dampened as the atmosphere becomes more stable.

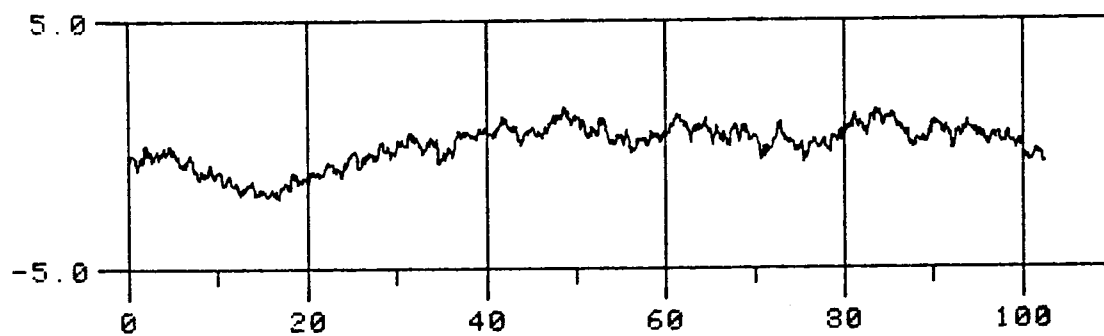
This phenomena, as expected, is associated with the kinetic energy of eddies. An increase in atmospheric stability reflects a decaying of turbulent motion and thus a decrease in the kinetic energy (dynamic action) of the atmosphere. The effect of stability on the energy



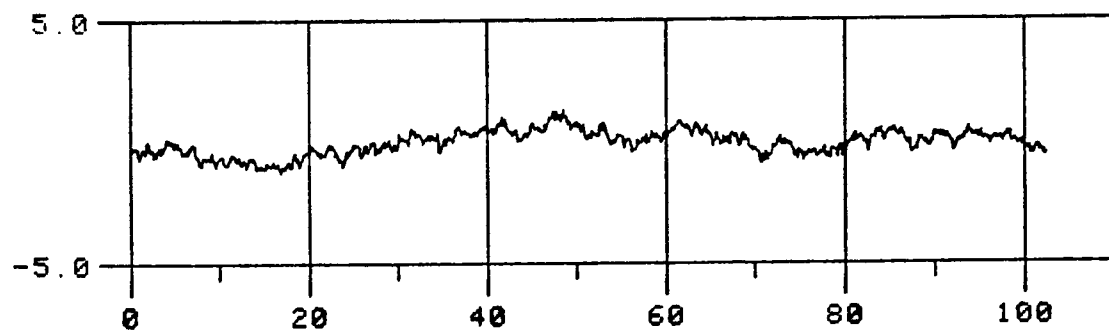
(a) $z/L = -2$



(b) $z/L = -1$

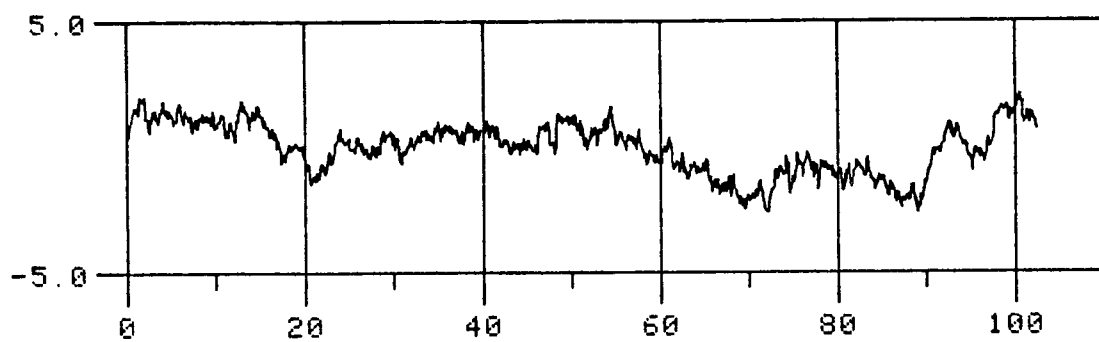


(c) $z/L = -0.3$

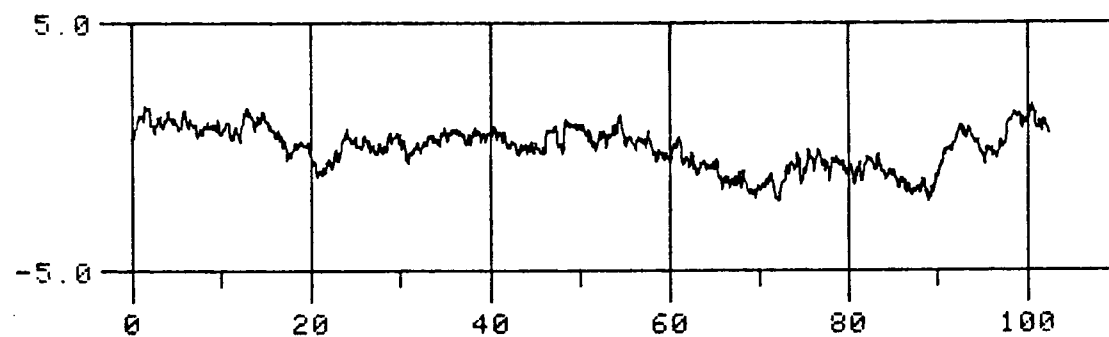


(d) $z/L = 0$

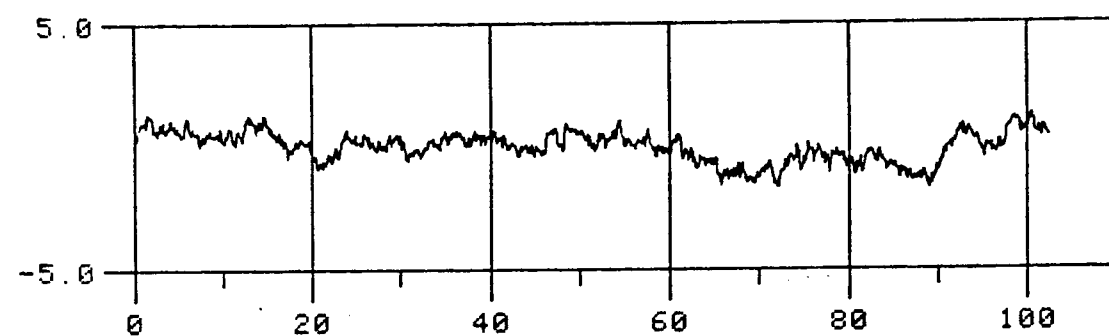
Figure 14. Large-scale eddy simulated u-turbulence component of the Pasquill stability categories A, B, C, and D corresponding to (a), (b), (c), and (d) above.



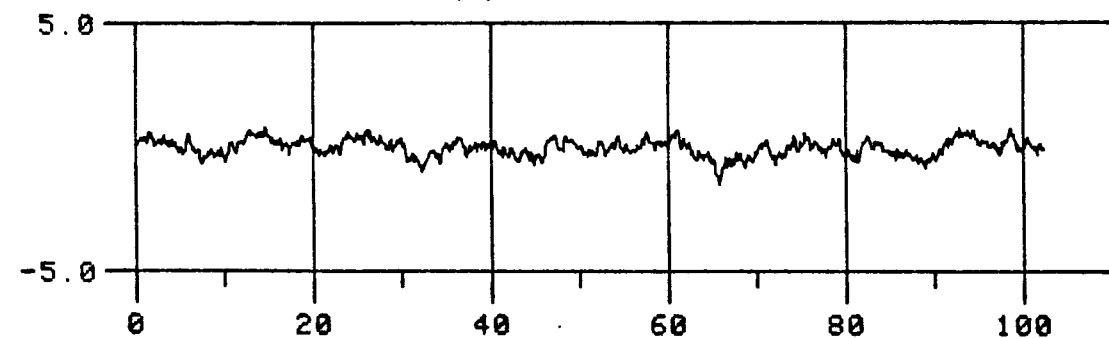
(a) $z/L = -2$



(b) $z/L = -1$

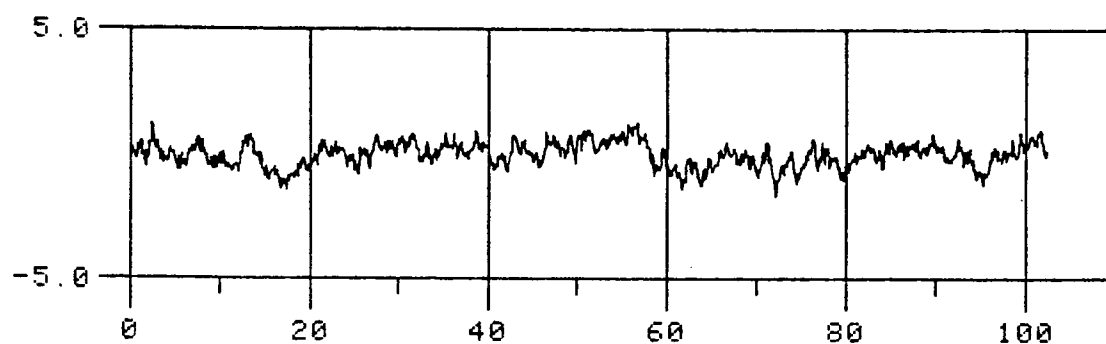


(c) $z/L = -0.3$

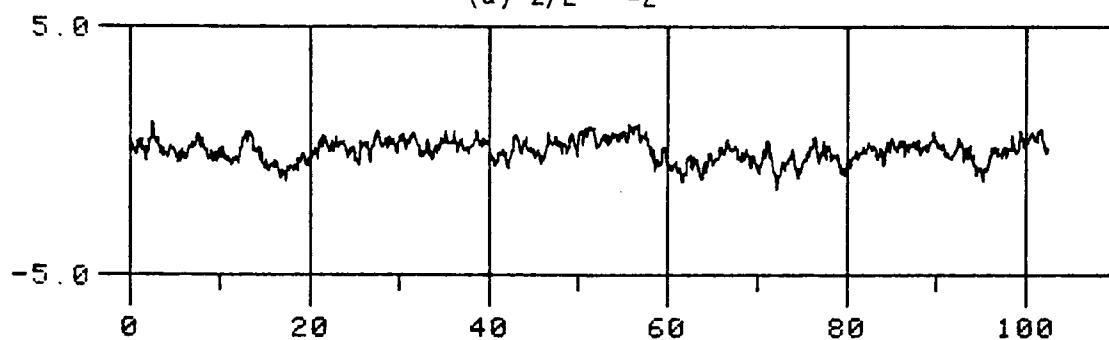


(d) $z/L = 0$

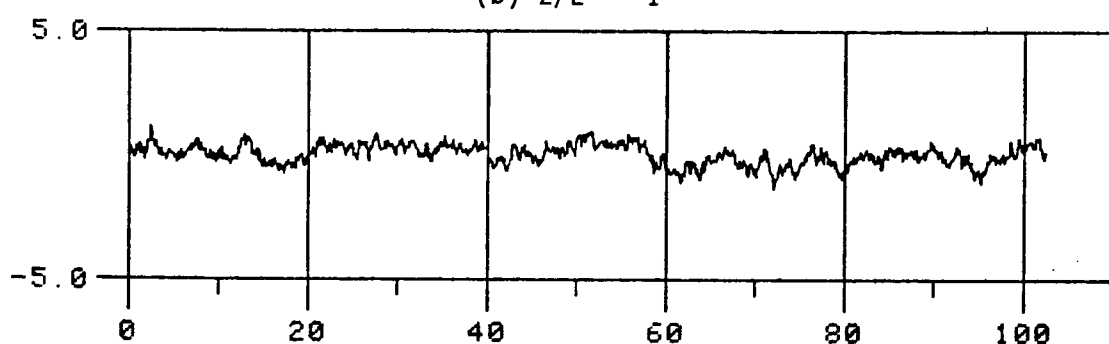
Figure 15. Large-scale eddy simulated v -turbulence component of the Pasquill stability categories A, B, C, and D corresponding to (a), (b), (c), and (d) above.



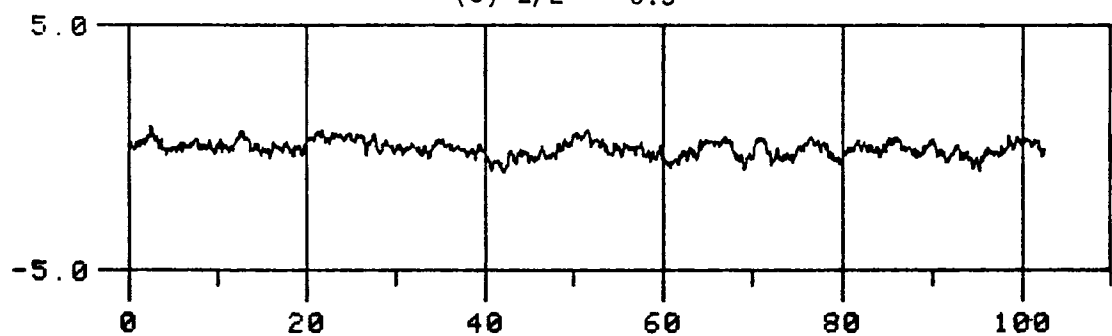
(a) $z/L = -2$



(b) $z/L = -1$



(c) $z/L = -0.3$



(d) $z/L = 0$

Figure 16. Large-scale eddy simulated w-turbulence component of the Pasquill stability categories A, B, C, and D corresponding to (a), (b), (c), and (d) above.

spectrum has been shown in Figure 2, page 5. At low frequencies the kinetic energy of eddies provide the main contribution to the total energy. The long cycles of positive and negative velocity fluctuation regions of Figures 14, 15, and 16 are representative of eddies at low frequencies.

3. CONCLUSIONS

The simulation technique presented in this study provides a turbulence model of the atmospheric boundary layer with new physics included; namely, the non-zero cross-spectrum effect and the large-scale eddy structure which were not incorporated in previous models. The result of simulating specific experimental results presented in the literature illustrates the simulation compares well with the available data.

1. The lag time between large periodic cycles of the longitudinal component and vertical component of turbulence fluctuations are reflected clearly in the simulation, indicating that a non-zero cross-correlation exists.
2. The momentum flux ($-\overline{uw}$) appearing as periodically long spikes in the temporal downwind momentum, i.e., the uw time histories, is present in the simulation. The spikes can be interpreted as specific regions of large-scale motion within the simulated turbulence field.
3. The effect of the atmospheric instability parameter, z/L , is investigated in the simulation. The intensity of the atmospheric turbulence decreases as the atmospheric stability increases.

4. The interlevel coherence property of the atmospheric turbulence generated in the simulation is demonstrated.
5. This study addresses the velocity field characteristic of large-scale eddy structures in the convective atmospheric surface layer. Other characteristics of large-scale eddies such as temperature ramp phenomenon may be considered in future studies.

CHAPTER II

COMPUTER PROGRAM

1. INTRODUCTION

The computer program for the large eddy turbulence simulation model is called EDDYCOHF. This program generates coherent turbulence time histories based on the mathematical model described in Section 2. A flowchart of the main program is shown on Figure 17. A list of subroutines followed by detailed description of input, output, and the computer source code of program EDDYCOHF is given in this section.

2. LIST OF SUBROUTINES

A list of subroutines used in EDDYCOHF are as follows:

FFT	Computes the Fourier transformation (or inverse) of a discrete data set
FILTER	Computes filter function, $H(f)$, for each turbulence component
WYTNOS	Generates a set of Gaussian white noise input signal
SPEC	Computes turbulence intensities by integrating the spectrum functions
BETA	Computes coefficients for multilevel input signal of coherent model
VHAT	Computes the composite input signal of coherent model

3. INPUT REQUIREMENTS

The program EDDYCOHF reads in the input parameters from an input file. The input file is built prior to running the program and has a

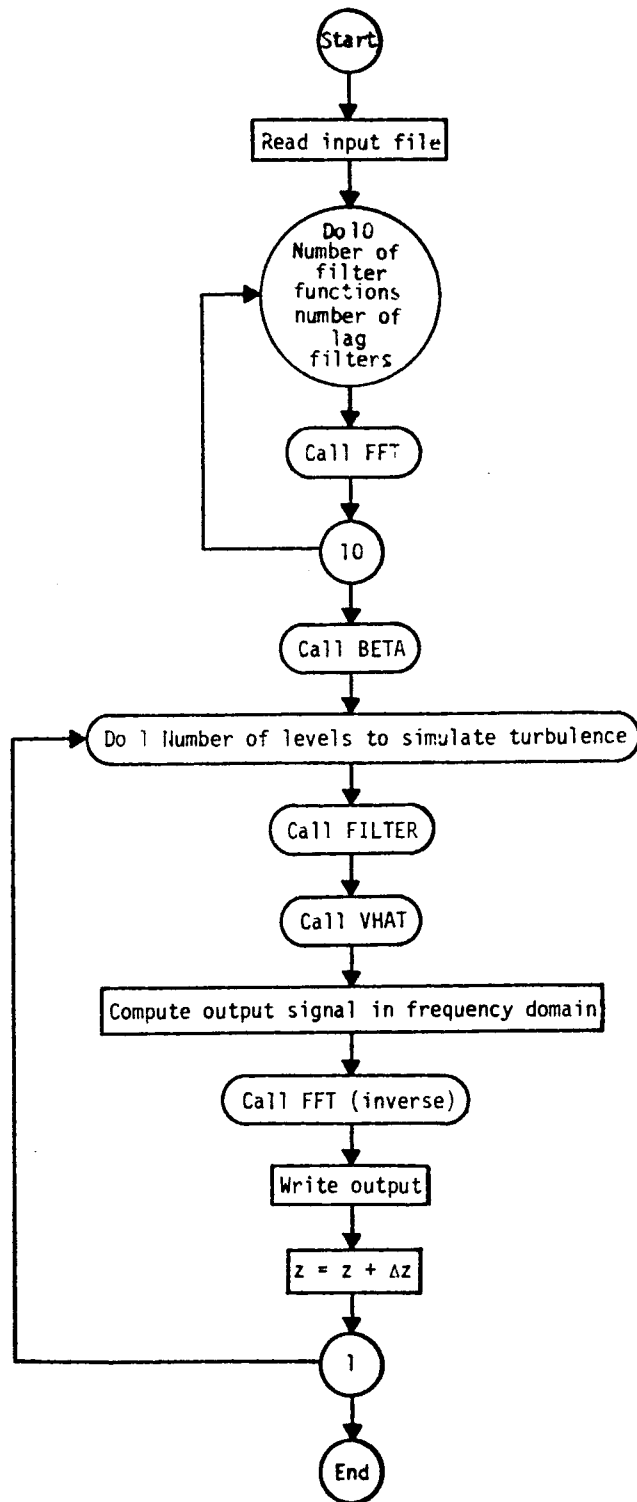


Figure 17. Flowchart of main program EDDYCOHF.

variable name such as EDDY.IN which is assigned by the user. The input file has the following format:

Line 1: ISEED N NP NLF NL IS

Line 2: Z VBAR ZETA ZOZ2 DELZ DT

Line 3: USTAR SLANDA

All variables are separated with a space (free format call). The definition of the input parameters is given below.

Nomenclature

ISEED	Initial seed of random number generator (maximum $2^{31} - 1$)
N	Number of data points (must be power of 2)
NP	Power to which 2 is raised, i.e., $N = 2^{NP}$
NLF	Number of lag filters (an odd number maximum of 11)
NL	Number of levels at which the coherent turbulence is simulated
IS	Stability indicator (1 = unstable, 2 = neutral, 3 = stable)
Z	Height (m)
VBAR	Horizontal mean velocity (m/s)
ZETA	Stability parameter $\zeta = z/L$ where L is the Monin-Obukhov length scale
ZOZI	Stability parameter z/z_i where z_i is the inversion height
DELZ	Δz , the vertical separation distance between coherent turbulence levels
DT	Δt , the time increment of discrete data
USTAR	Friction velocity (m/s)
SLANDA	Integral length scale, Λ

A numerical example of an input file named EDDY.IN is given below. This input file represents a simulation with the following status:

Initial seed of random number generator: ISEED = 0 (Note ISEED of 0 is permissible to use for VAX computer; however, ISEED of zero may not work for some other computers).

Number of data points: $N = 128$ ($128 = 2^7$).

Power to which 2 is raised: $NP = 7$.

Number of lag filters: $NLF = 7$.

Number of coherent turbulence levels: $NL = 2$.

Stability indicator for unstable case: $IS = 1$.

Height of first level: $Z = 50$ m.

Horizontal mean wind velocity: $VBAR = 11$ m/s.

Stability parameter, $\zeta = z/L$: $ZETA = 0.71$. (Note spectrum functions for unstable case are given as a function of $z/-L$; thus, for $Z = 50$ and $L = -70$. $ZETA$ is $50/-(-70) = 0.71$).

Stability parameter $z/z_1 = ZOZI = 0.0167$ ($z_1 = 3000$ m).

The vertical separation distance between coherent turbulence levels $DELZ = 3$.

Time increment of discrete data: $DT = 0.1$.

Friction velocity: $USTAR = 0.55$.

Integral length scale: $SLANDA = 500$.

Listing

```
0 128 7 7 2 1
50. 11. 0.71 0.0167 3. 0.1
0.55 500.
```

4. OUTPUT QUANTITIES

The output of EDDYCOHF program is discrete data representing coherent turbulence time histories of input parameters. The data is

written to an output file. The output file name is generated internally. The output file name designates the turbulence component, stability, and the coherent level for which the turbulence is simulated. An example of an output file name is UTURB.UNS01. This output file contains the u-component values of turbulence for the unstable atmosphere at the level one simulation height. The first line of the output file gives the number of data points in the set. The following lines are the discrete turbulence values in a 10(1X, E10.3) format, i.e., ten discrete values per line, each separated by one space. An example of output file UTURB.UNS01 is listed below.

Listing

```
# OF POINTS      128
-0.265E-01  0.185E+00  0.161E+00  0.739E-01  0.119E+00 -0.388E+00 -0.247E+00 -0.133E+00 -0.165E+00 -0.525E+00
-0.491E+00 -0.592E+00 -0.704E+00 -0.672E+00 -0.587E+00 -0.595E+00 -0.227E+00 -0.459E+00 -0.464E+00 -0.220E+00
-0.431E+00 -0.126E+00  0.184E+00 -0.189E+00 -0.110E+00 -0.248E-01 -0.159E+00  0.170E+00  0.465E+00  0.769E+00
 0.815E+00  0.448E+00  0.384E+00  0.606E+00  0.522E+00  0.678E+00  0.590E+00  0.382E+00  0.292E+00  0.611E-01
 0.123E+00  0.351E-01 -0.312E+00 -0.226E+00  0.240E+00  0.163E+00  0.102E-01  0.923E-01  0.703E-01  0.891E-01
-0.764E-01 -0.652E-01  0.295E+00  0.493E+00  0.369E+00  0.126E+00  0.131E+00  0.299E+00  0.327E+00  0.314E+00
-0.306E-01 -0.518E-01  0.367E-01 -0.583E-01  0.441E+00  0.622E+00  0.432E+00  0.385E+00  0.683E-01  0.989E-01
-0.978E-01  0.451E-01  0.352E-01 -0.276E+00 -0.213E+00 -0.270E+00 -0.104E+00  0.358E-01  0.156E+00 -0.212E-01
 0.474E-01  0.577E-02 -0.693E-01  0.169E+00  0.181E+00  0.163E+00  0.251E+00  0.144E+00  0.970E-01  0.208E+00
-0.915E-01 -0.214E+00 -0.307E-01  0.231E+00  0.156E+00 -0.452E-02  0.601E-01 -0.183E+00 -0.310E-01 -0.166E+00
-0.375E+00 -0.384E+00 -0.445E+00 -0.561E+00 -0.668E+00 -0.504E+00 -0.481E+00 -0.499E+00 -0.341E+00  0.963E-02
 0.180E-01  0.224E+00  0.155E+00  0.193E+00 -0.113E+00 -0.684E-01  0.254E+00 -0.873E-02 -0.184E+00  0.637E-01
-0.500E-01 -0.223E-01  0.126E+00 -0.146E+00 -0.376E+00  0.470E-01  0.691E-01  0.452E-01
```

5. LISTING OF PROGRAM EDDYCOHF

Main Program

The main program reads the input file, calls various subroutines to generate the composite input signal and to compute the filter function, thus computing the output signal, Equation 20. The main program then calls the inverse Fourier transformation to obtain the discrete time histories. Finally, the result, discrete time histories, are written to appropriate output files which were described in Section 4.

Nomenclature.

PI Constant π

DZM $(\Delta z)_{\max}$ of ϵ_0 in Equation 18

INFILE Input file variable name

OFIL Output file variable name

CHTURB() Character array containing turbulence component designation

CHSTAB() Character array containing stability designation

CHLEVEL() Character array containing coherent level designation

YR(),YI() Real and imaginary part of output signal transform, $Y(k,z)$,
Equation 20

Listing.

```
C*****MAIN*****C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SGN/X(2048)
      COMMON/CN1/PI,DT
      COMMON/NDT/ISEED,N
      COMMON/STB/Z,VBAR,ZETA,ZOZI
      COMMON/STAR/USTAR,SLANDA
      COMMON/FIL/HR(7,2048),HI(7,2048)
      COMMON/PAR/A(6),B(6)
      COMMON/T2/DZM,EPO,AA,NP1
      COMMON/T3/NLF,BB(11)
      COMMON/XPRI/RNR(7,11,2048),RNI(7,11,2048)
      DIMENSION XR(2048),XI(2048)
      DIMENSION YR(7,2048),YI(7,2048)
      DIMENSION CHTURB(3),CHSTAB(3),CHLEVEL(10)
      CHARACTER*30 INFILE,OFIL
      CHARACTER*6 CHTURB
      CHARACTER*3 CHSTAB
      CHARACTER*2 CHLEVEL
      DATA CHTURB/'UTURB.','VTURB.','WTURB.'/
      DATA CHSTAB/'UNS','NEU','STB'/
      DATA CHLEVEL/'01','02','03','04','05','06','07','08','09','10'/
      PRINT*,'ENTER INPUT FILE NAME'
      READ99,INFILE
      OPEN(1,FILE=INFILE,STATUS='OLD')
      READ(1,*)ISEED,N,NP,NLF,NL,IS
      READ(1,*)Z,VBAR,ZETA,ZOZI,DELZ,DT
      READ(1,*)USTAR,SLANDA
```

C--

```

DUMMY=1.0
PI=4.*DATAN(DUMMY)
AA=17./(2.*PI)
DZM=3.
EPO=DZM*PI/(VBAR*DT)

C--
DO 10 I=1,7
IF(I.EQ.5) GO TO 10
DO 10 K=1,NLF
CALL WYTNOS
DO 20 J=1,N
XR(J)=X(J)
XI(J)=0.0
20 CONTINUE
CALL FFT(N,NP,1,XR,XI)
DO 25 J=1,N
RNR(I,K,J)=XR(J)
RNI(I,K,J)=XI(J)
25 CONTINUE
10 CONTINUE
DO 5 K=1,NLF
DO 5 J=1,N
RNR(5,K,J)=RNR(1,K,J)
RNI(5,K,J)=RNI(1,K,J)
5 CONTINUE
CALL BETA
ICOD=0
DO 1 IL=1,NL
CALL FILTER
DO 26 I=1,7
CALL VHAT(N,I,XR,XI,Z)
DO 30 J=1,N
YR(I,J)=HR(I,J)*XR(J)-HI(I,J)*XI(J)
YI(I,J)=HR(I,J)*XI(J)+HI(I,J)*XR(J)
30 CONTINUE
26 CONTINUE
C--
DO 40 I=1,3
ICOD=ICOD+1
K=2*I-1
DO 50 J=1,N
XR(J)=YR(K,J)+YR(K+1,J)
XI(J)=YI(K,J)+YI(K+1,J)
IF(I.EQ.3) THEN
XR(J)=XR(J)+YR(K+2,J)
XI(J)=XI(J)+YI(K+2,J)
END IF
50 CONTINUE
C--
CALL FFT(N,NP,2,XR,XI)
IU=ICOD+6
OFILE=CHTURB(I)//CHSTAB(IS)//CHLEVEL(IL)

```

```

      OPEN(IU,FILE=OFILE,STATUS='NEW')
      WRITE(IU,100)N
      WRITE(IU,110)(XR(J),J=1,N)
      CLOSE(IU)
40    CONTINUE
      Z=Z+DELZ
C--
      1    CONTINUE
C--
      99    FORMAT(A)
      100   FORMAT(1X,' # OF POINTS ',I6)
      110   FORMAT(10(1X,E10.3))
      STOP
      END
C--
      BLOCK DATA
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/PAR/A(6),B(6)
      DATA A/0.5,105.,0.95,17.,32.,2./
      DATA B/2.2,33.,2.,9.5,17.,5.3/
      END

```

Subroutine FFT (N, NP, NC, XR, XI)

This subroutine performs the Fourier transformation or the inverse Fourier transformation of a discrete data set (see Equations 12 and 15).

Nomenclature.

N	Number of data points
NP	Power to which 2 is raised, i.e., $N = 2^{NP}$
NC	Control number (1 for Fourier transform, 2 for inverse Fourier transform)
XR,XI	Real and imaginary part of the discrete data set (input and output)

Listing.

```

SUBROUTINE FFT(N,NP,NC,XR,XI)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION XR(1),XI(1)

```

```

        IF(NC.EQ.1) GO TO 120
        DO 110 I=1,N
110     XI(I)=-XI(I)
120     N2=N/2
        NU1=NP-1
        K=0
        DO 150 L=1,NP
130     DO 140 I=1,N2
        P=IBITR(K/2**NU1,NP)
        ARG=6.283185*P/FLOAT(N)
        C=DCOS(ARG)
        S=DSIN(ARG)
        K1=K+1
        K1N2=K1+N2
        TR=XR(K1N2)*C+XI(K1N2)*S
        TI=XI(K1N2)*C-XR(K1N2)*S
        XR(K1N2)=XR(K1)-TR
        XI(K1N2)=XI(K1)-TI
        XR(K1)=XR(K1)+TR
        XI(K1)=XI(K1)+TI
140     K=K+1
        K=K+N2
        IF(K.LT.N) GO TO 130
        K=0
        NU1=NU1-1
150     N2=N2/2
        DO 160 K=1,N
        I=IBITR(K-1,NP)+1
        IF(I.LE.K) GO TO 160
        TR=XR(K)
        TI=XI(K)
        XR(K)=XR(I)
        XI(K)=XI(I)
        XR(I)=TR
        XI(I)=TI
160     CONTINUE
        IF(NC.EQ.1) GO TO 180
        DO 170 I=1,N
        XR(I)=XR(I)/N
        XI(I)=XI(I)/N
170     CONTINUE
180     RETURN
        END
        FUNCTION IBITR(J,NU)
        J1=J
        IBITR=0
        DO 200 I=1,NU
        J2=J1/2
        IBITR=IBITR*2+(J1-2*J2)
200     J1=J2
        RETURN
        END

```

Subroutine FILTER

This subroutine computes the filter functions shown on Figure 3, page 9, given by Equations 7 and 9. The coefficients A and B of Table 1, page 7, are computed from parameters a, b, c, and the input values of the stability parameters. Subroutine SPEC is called to compute the turbulence intensities by integrating the spectrum functions. Finally, the complex valued filter functions are computed in the frequency domain.

Nomenclature.

AC,BC	Coefficients A and B of Table 1
ALFA	Constant α in Equation 9
CHAT2	Constant $\hat{c}^2$ in Equation 9
HR,HI	Real and imaginary part of discrete filter function used in Equation 11

Listing.

```
SUBROUTINE FILTER
IMPLICIT REAL*8(A-H,O-Z)
COMMON/PAR/A(6),B(6)
COMMON/FIL/HR(7,2048),HI(7,2048)
COMMON/STB/Z,VBAR,ZETA,ZOZI
COMMON/STAR/USTAR,SLANDA
COMMON/NDT/ISEED,N
COMMON/CN1/PI,DT
DIMENSION AC(6),BC(6)
ZETAI=ZETA/ZOZI
C1=VBAR/Z
C2=C1*ZOZI
USTAR2=USTAR*USTAR
AC(1)=USTAR2*(A(1)/B(1))*(C2*ZETAI)**(2./3.)
BC(1)=(C2**(5./3.))/B(1)
BC(2)=C1/B(2)
AC(2)=USTAR2*(A(2)/B(2))*BC(2)**(2./3.)
BC(3)=C2/B(3)
AC(3)=USTAR2*(A(3)/B(3))*(BC(3)*ZETAI)**(2./3.)
BC(4)=C1/B(4)
```

```

AC(4)=USTAR2*(A(4)/B(4))*BC(4)**(2./3.)
BC(5)=C1/B(5)
AC(5)=USTAR2*(A(5)/B(5))*(BC(5)*ZETA)**(2./3.)
AC(6)=USTAR2*(A(6)/B(6))*C1**2./3.)
BC(6)=(C1**2./3.)/B(6)
ALFA=1.58114*VBAR/SLANDA
ALFA2=ALFA*ALFA

```

C--

```

CALL SPEC(1,C1,C2,ZETA,ZETA1,SIG1)
CALL SPEC(2,C1,C2,ZETA,ZETA1,SIG2)
CALL SPEC(3,C1,C2,ZETA,ZETA1,SIG3)
SIGU=DSQRT(SIG1*USTAR2)
SIGV=DSQRT(SIG2*USTAR2)
SIGW=DSQRT(SIG3*USTAR2)
PRINT*, 'SIGU =', SIGU
PRINT*, 'SIGV =', SIGV
PRINT*, 'SIGW =', SIGW
RAT=SIG1/SIG3
AZ=.370787+.616271*DEXP(-.17549*RAT)
CHAT2=1./(AZ*AZ*SIG1*SIG3)
IF(CHAT2.GT.1.) CHAT2=1.
SQCS1=DSQRT(1.-CHAT2)

```

C--

```

TN=1./(FLOAT(N)*DT)
DO 10 I=2,4
DO 10 J=1,N
F=(J-1)*TN
D=(DSQRT(AC(I)))/((BC(I)+F)**(5./6.))
TH=(5./3.)*DATAN(DSQRT(F/BC(I)))
HR(I,J)=D*DCOS(TH)
HI(I,J)=-D*DSIN(TH)
10 CONTINUE
DO 20 J=1,N
F=(J-1)*TN
D=DSQRT(AC(1)/(BC(1)+F**2./3.))
TH=DATAN((F**2./3.)/DSQRT(BC(1)))
HR(1,J)=D*DCOS(TH)
HI(1,J)=-D*DSIN(TH)
20 CONTINUE
DO 30 J=1,N
F=(J-1)*TN
F2=F*F
DUMMY1=(ALFA2*CHAT2*AC(5))/(ALFA2+F2)
DUMMY2=(BC(5)+F)**2./6.)
D1=DSQRT(DUMMY1)/DUMMY2
D2=DSQRT(AC(5)-DUMMY1)/DUMMY2
TH1=DATAN(F/ALFA)+(5./3.)*DATAN(DSQRT(F/BC(5)))
IF(SQCS1.EQ.0.) THEN
TH2=TH1-PI/2.
ELSE
TH2=TH1-DATAN(F/(ALFA*SQCS1))
END IF

```

```

        HR(5,J)=D1*DCOS(TH1)
        HI(5,J)=-D1*DSIN(TH1)
        HR(6,J)=D2*DCOS(TH2)
        HI(6,J)=-D2*DSIN(TH2)
30      CONTINUE
        DO 40 J=1,N
        F=(J-1)*TN
        D=DSQRT(AC(6)/(BC(6)+F**(5./3.)))
        TH=DATAN((F**(5./6.))/DSQRT(BC(6)))
        HR(7,J)=D*DCOS(TH)
        HI(7,J)=-D*DSIN(TH)
40      CONTINUE
        RETURN
        END

```

Subroutine WYTNOS

This subroutine generates an array of Gaussian white noise input signal which has zero mean and a standard deviation of unity.

Nomenclature.

X() Array of Gaussian white noise input signal
N Number of data points
ISEED The seed of random number generator
A A uniformly distributed random number between 0 to 1

Listing.

```

SUBROUTINE WYTNOS
IMPLICIT REAL*8(A-H,O-Z)
COMMON/NDT/ISEED,N
COMMON/SGN/X(2048)
COMMON/CN1/PI,DT
SIGMA=DSQRT(0.5/DT)
SUM=0.0
DO 10 J=1,N
A=0.0
DO 20 K=1,12
20  A=A+RAN(ISEED)
X(J)=A-6.0
SUM=SUM+X(J)
10  CONTINUE
DO 30 I=1,8
XMEAN=SUM/(FLOAT(N))

```

```

30    CONTINUE
      SUM=0.0
      DO 40 J=1,N
        X(J)=X(J)-XMEAN
        SUM=SUM+X(J)*X(J)
40    CONTINUE
      SIGX=DSQRT(SUM/(FLOAT(N)))
      DO 60 J=1,N
        X(J)=X(J)*SIGMA/SIGX
60    CONTINUE
      RETURN
      END

```

Subroutine SPEC (IC, C1, C2, ZETA, ZETAI, SIG)

This subroutine computes the turbulence intensities by integrating the spectrum function given by Equation 3. Simpson's rule is the integration technique used.

Nomenclature.

IC	Index code of spectrum component
N	Number of data points
STEP	Integration increment ($d(\ln f)$)
SPN()	Normalized spectrum function given by Equation 3
SIG	Normalized turbulence variance, $(\sigma/u^*)^2$

Listing.

```

SUBROUTINE SPEC(IC,C1,C2,ZETA,ZETAI,SIG)
  IMPLICIT REAL*8(A-H,O-Z)
  COMMON/NDT/ISEED,N
  COMMON/CN1/PI,DT
  DIMENSION SPN(1001)
  STEP=0.01
  NN=1001
  DO 5 I=1,NN
    X=10.**(-8.+STEP*FLOAT(I-1))
    ETAI=X/C2
    ETA=X/C1
10  IF(IC-2) 10,20,30
    D1=(0.5*ETAI*ZETAI**(2./3.))/(1.+2.2*ETAI**(5./3.))
    D2=(105.*ETA)/((1.+33.*ETA)**(5./3.))

```

```

      SPN(I)=D1+D2
      GO TO 200
20    D1=(0.95*ETAI*ZETAI**(2./3.))/((1.+2.*ETAI)**(5./3.))
      D2=(17.*ETA)/((1.+9.5*ETA)**(5./3.))
      SPN(I)=D1+D2
      GO TO 200
30    D1=(32.*ETA*ZETA**(2./3.))/((1.+17.*ETA)**(5./3.))
      D2=(2.*ETA)/(1.+5.3*ETA**(5./3.))
      SPN(I)=D1+D2
200   CONTINUE
      5   CONTINUE
      SM=0.0
      M=NN-1
      DO 40 I=2,M
      ITEST=JMOD(I,2)
      IF(ITEST.EQ.0) GO TO 50
      SM=SM+2.0*SPN(I)
      GO TO 40
50    SM=SM+4.0*SPN(I)
40    CONTINUE
      SIG=STEP*(SPN(1)+SM+SPN(NN))/3.0
      RETURN
      END

```

Subroutine BETA (NP21, BB)

This subroutine computes the coefficients β_m of Equation 19.

Nomenclature.

NLF Number of lag filters (an odd number)
BB Array containing coefficients β_m

Listing.

```

SUBROUTINE BETA
IMPLICIT REAL*8(A-H,O-Z)
COMMON/T3/NLF,BB(11)
COMMON/CN1/PI,DT
COMMON/T2/DZM,EPO,AA,NP1
NP1=(NLF-1)/2+1
D=AA*EPO/2.
BB(NP1)=0.5
SUM=1.
EXEO=DEXP(-D)
PE2=PI/D
ANOM=(1.+EXEO)/(1.-EXEO)
DO 300 I=1,NP1,2

```

```

      K=NP1-I
      BB(K)=ANUM/(1.+(PE2*I)**2)
      IF(K.EQ.1) GO TO 100
      BB(K-1)=1./(1.+(PE2*(I+1))**2)
      SUM=SUM+2.*(BB(K)+BB(K-1))
      BB(K-1)=DSQRT(BB(K-1))
      GO TO 200
100    SUM=SUM+2.*BB(K)
200    BB(K)=DSQRT(BB(K))
300    CONTINUE
      DO 400 I=1, NP1
400    BB(I)=BB(I)/DSQRT(SUM)
      RETURN
      END

```

Subroutine VHAT (N, I, XR, XI, BB, Z)

This subroutine computes the composite input signal, $\hat{X}(k,z)$, given in Equation 20.

Nomenclature.

N	Number of data points
I	Filter index
XR,XI	Real and imaginary part of composite input signal, $\hat{X}(k,z)$, Equation 20
RNR,RNI	Real and imaginary part of multilevel input signal $X_m(k)$, Equation 20
TETA	Part of argument θ_m , see Equation 19
AUG	Argument θ_m , see Equation 19

Listing.

```

SUBROUTINE VHAT(N,I,XR,XI,Z)
  IMPLICIT REAL*8(A-H,O-Z)
  COMMON/XPRI/RNR(7,11,2048),RNI(7,11,2048)
  COMMON/CN1/PI,DT
  COMMON/T2/DZM,EPO,AA,NP1
  COMMON/T3/NLF,BB(11)
  DIMENSION XR(1),XI(1)
  N12=NLF+1

```

```

      PI2=PI*2.
C--
      KK=N/2
      TETA=PI2*Z/(DZM*FLOAT(N))
      DO 200 K=1,N
      K1=K-1
      IF(K1.GT.KK) K1=KK
C--
      XR(K)=0.
      XI(K)=0.
      DO 100 L=1,NP1
      AUG=TETA*FLOAT((NP1-L)*K1)
      AUGN=-AUG
      AUGP=AUG
      CSN=DCOS(AUGN)
      CSP=DCOS(AUGP)
      SNN=DSIN(AUGN)
      SNP=DSIN(AUGP)
      XR(K)=XR(K)+(RNR(I,N12-L,K)*CSN+RNR(I,L,K)*CSP-
1 RNI(I,N12-L,K)*SNN-RNI(I,L,K)*SNP)*BB(L)
      XI(K)=XI(K)+(RNI(I,N12-L,K)*CSN+RNI(I,L,K)*CSP+
1 RNR(I,N12-L,K)*SNN+RNR(I,L,K)*SNP)*BB(L)
100 CONTINUE
200 CONTINUE
      RETURN
      END

```

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BIBLIOGRAPHY

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APPENDICES

APPENDIX A

GENERATION OF A RANDOM SIGNAL

The method used to generate random numbers with a digital computer is based on the formula (Wang and Frost, 1980):

$$x_{n+1} = ax_n + b \quad (\text{mod } m) \quad (\text{A.1})$$

This technique is called the mixed congruential method. By using different values of a and b the process can be repeated for several cycles. Module number m is the largest integer possible with the capacity of the computer used. To generate a random number in the range of 0 to 1, the output data, x_n , is normalized with respect to module number m .

$$x_n' = x_n/m \quad (\text{A.2})$$

where x_n is generated from Equation A.1 since

$$0 \leq x_n \leq m$$

therefore,

$$0 \leq x_n' \leq 1$$

The constants a and b in Equation A.1 are selected to provide speed of computation and good statistical properties, such as a uniform distribution and the maximum period of occurrence. Results presented by Chambers (1977) show that a maximum period can be achieved if a and b are selected as:

$$a = 4 \cdot I + 1 \quad I = 1, 2, 3, \dots$$

$$b = \text{odd} \quad (\text{A.3a})$$

or

$$a = 8 \cdot I \pm 3$$

$$b = 0 \quad (A.3b)$$

As mentioned earlier, Gaussian distributed numbers are required as the white noise input. The Gaussian probability density function is given as:

$$P(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp \left[-\frac{(x - \bar{x})^2}{2\sigma_x^2} \right] \quad (A.4)$$

where σ_x is the standard deviation and $\bar{x}$ is the mean value of the variate.

If x_i' ($i = 1, 2, \dots, N$) is N uniformly distributed numbers over the interval 0 to 1, the central limit theorem yields the following formula:

$$x_j'' = \sigma_x \left(\frac{12}{N} \right)^{1/2} \left[\sum_{i=1}^N x_i' - \frac{N}{2} \right] + \bar{x} \quad (A.5)$$

where x_j'' ($j = 1, 2, \dots, M$) is a set of random numbers having a Gaussian distribution with the mean value equal to $\bar{x}$, and the standard deviation equal to σ_x .

The value of N indicates the number of terms used for each output signal. Selecting N equal to 12 gives a close approximation of the true Gaussian distribution function. Equation A.5 then reduces to:

$$x_j'' = \sigma_x \left[\sum_{i=1}^{N=12} x_i' - 6 \right] + \bar{x} \quad (A.6)$$

To generate a Gaussian distributed white noise signal which will be used to produce a desired output signal with the proper filter, the variance of the input sequence must be adjusted by a factor of $1/2\Delta t$ when using the discrete Fourier transformation given in Appendix B, Section 3. Δt is the time interval (period) at which the signal is

sampled. Hence, a white noise sequence with zero mean and standard deviation of $\sqrt{1/2\Delta t}$ is generated by the following equation.

$$x_j = \frac{\left[\sum_{i=1}^{N=12} x_i - 6 \right]}{\sqrt{2\Delta t}} \quad (\text{A.7})$$

APPENDIX A BIBLIOGRAPHY

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APPENDIX B

DETAILED DERIVATION OF LARGE EDDY TURBULENCE SIMULATION EQUATIONS

1. SIMULATION OF ATMOSPHERIC TURBULENCE

The basic simulation procedure is shown in Figure B.1. Random signals are computer generated and passed through shaping filters to provide the output which is the simulated time history of turbulence. Through the appropriate design of the filter, the turbulence signal output will have specified statistical properties of atmospheric turbulence (i.e., first- and second-moments, etc.). The number of statistical properties simulated is a function of the degree of mathematical complexity of the computational procedure.

Consider the linear system shown in Figure B.2. For the response function $h(t)$ and input $x(t)$, the system output $y(t)$ is given by the convolution integral:

$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t - \tau) d\tau \quad (B.1)$$

Let $H(f)$ be the frequency response function, which is defined as the Fourier transfer of $h(t)$ and $X(f)$ be the Fourier transfer of the input $x(t)$. Then the Fourier transformation of the output given by Equation B.1 is:

$$Y(f) = H(f) X(f) \quad (B.2)$$

The power spectral density function between the input and output can be found from the correlation of the function $y(t)$ as given by:

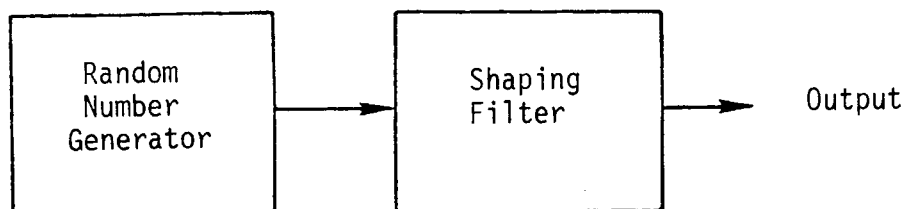


Figure B.1. Turbulence simulation system.

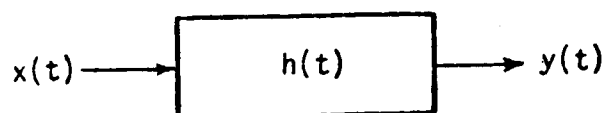


Figure B.2. Simple filter system.

$$R_{lm}(\tau) = \int_{-\infty}^{\infty} y_l(t) y_m(t + \tau) dt \quad (B.3)$$

The Fourier transform of $R_{lm}(\tau)$ yields:

$$\phi_{lm}(f) = \int_{-\infty}^{\infty} R_{lm}(\tau) e^{-i2\pi f\tau} d\tau \quad (B.4)$$

where $i = \sqrt{-1}$. Substituting for R_{lm} in Equation B.4, rearranging, and using a change of variable we have:

$$\phi_{lm}(f) = Y_l^*(f) Y_m(f) \quad (B.5)$$

where

$$Y_m(\omega) = \int_{-\infty}^{\infty} y_m(t) e^{-i2\pi ft} dt$$

and the complex conjugate

$$Y_l^*(\omega) = \int_{-\infty}^{\infty} y_l(t) e^{+i2\pi ft} dt$$

Now the auto-correlation of $y_l(t)$ at $\tau = 0$ is:

$$R_{ll}(0) = \int_{-\infty}^{\infty} y_l^2(t) dt \quad (B.6)$$

and by definition:

$$\phi_{ll}(f) = Y_l^*(f) Y_l(f) = |Y_l(f)|^2 \quad (B.7)$$

is called the energy density spectrum of $y_l(t)$ at $\tau = 0$ and has units of L^2/T when summing over all frequencies (Frost and Moulden, 1977).

Using Equation B.2 in Equation B.7 we can write:

$$\phi_{11}(f) = H^*(f) X^*(f) H(f) X(f) = |H(f)|^2 |X(f)|^2 \quad (B.8)$$

where $|X(f)|^2$ is the energy density spectrum of the input $x(t)$. We can

finally write:

$$\phi_y(f) = |H(f)|^2 \phi_x(f) \quad (B.9)$$

A filter function to obtain the form of the output spectrum $\phi_y(f)$ desired for any given known input $\phi_x(f)$ can now be designed. Introducing the input $x(t)$ as wide-band white noise for which the spectrum $\phi_x(f)$ is a constant adjustable to unity (Bendat and Piersol, 1971) gives:

$$|H(f)|^2 = \phi_y(f) \quad (B.10)$$

Thus, the square of the absolute value of the filter function is equal to the spectrum of the atmospheric turbulence.

2. SIMULATION OF CROSS-SPECTRUM BETWEEN u AND w

From data on atmospheric turbulence, it is seen that a non-zero cross-spectrum particularly in the planetary boundary layer exists between the longitudinal and the vertical components of turbulence at low frequencies. Figure 3, page 9, shows the simulation process used to produce cross-correlated time histories. Referring to Figure 3 and using Equation B.2, we can write the Fourier transform of the system output as:

$$U(f) = H_{uL}(f) X_L(f) + H_{uH}(f) X_H(f) \quad (B.11)$$

$$V(f) = H_{vL}(f) X_L'(f) + H_{vH}(f) X_H'(f) \quad (B.12)$$

$$W(f) = H_{w_{aL}}(f) X_L(f) + H_{w_{bL}}(f) X_L''(f) + H_{wH}(f) X_H''(f) \quad (B.13)$$

(Note: Lower case letters are used to represent the variables in time domain and upper case letters are used to represent the variables in frequency domain. u , v , and w represent the turbulence components in

longitudinal, lateral, and vertical directions, respectively.) Multiplying each component by its complex conjugate gives the energy spectrum defined by Equation B.5. Thus we have:

$$\phi_U(f) = \langle U^*(f) U(f) \rangle = |H_{UL}(f)|^2 + |H_{UH}(f)|^2 \quad (B.14)$$

$$\phi_V(f) = \langle V^*(f) V(f) \rangle = |H_{VL}(f)|^2 + |H_{VH}(f)|^2 \quad (B.15)$$

$$\phi_W(f) = \langle W^*(f) W(f) \rangle = |H_{w_{aL}}(f)|^2 + |H_{w_{bL}}(f)|^2 + |H_{w_H}(f)|^2 \quad (B.16)$$

(Note: Since white noise signals represented here by X are statistically independent, all cross terms are zero.) Recall the energy spectrum of large eddies is represented by the sum of low- and high-frequency portions of the spectra as:

$$\phi_U(f) = \phi_{UL}(f) + \phi_{UH}(f) \quad (B.17)$$

$$\phi_V(f) = \phi_{VL}(f) + \phi_{VH}(f) \quad (B.18)$$

$$\phi_W(f) = \phi_{WL}(f) + \phi_{WH}(f) \quad (B.19)$$

The values of the energy spectrum functions are expressed in Table 1, page 7.

Thus, by comparing Equations B.14, B.15, and B.16 with B.17, B.18, and B.19, the filter functions for longitudinal, lateral, and high-frequency portion of the vertical components of turbulence can be computed directly. However, for the vertical component of turbulence to be cross-correlated with the longitudinal component of turbulence, we have introduced an additional filter function for the low-frequency portion of the spectrum. Thus an additional equation is required. A cross-spectrum relationship between the u and w components of turbulence defined in the Engineering Sciences Data Item No. 74031 (1974) is introduced as:

$$\phi_{uw}(f) = \frac{\hat{c}}{[1 + 0.4\hat{\eta}^2]^{1/2}} [\phi_u \phi_w]^{1/2} \quad (\text{B.20})$$

where

$$\hat{c} = \frac{1}{A_z} \frac{uw}{\sigma_u \sigma_w}$$

and

$$\hat{\eta} = f \frac{\Lambda_u}{W_H}$$

A_z is a constant to be determined such that the relation:

$$uw = \int_0^{\infty} \phi_{uw}(f) df \quad (\text{B.21})$$

is satisfied. Substituting Equation B.20 into Equation B.21 we have:

$$uw = \frac{1}{A_z} uw \int_0^{\infty} \left\{ \left[\frac{\phi_u(f) \phi_w(f)}{\sigma_u^2 \sigma_w^2} \right]^{1/2} / (1 + 0.4\hat{\eta}^2)^{1/2} \right\} df \quad (\text{B.22})$$

or

$$A_z = \int_0^{\infty} \left[\frac{\phi_u(f) \phi_w(f)}{1 + 0.4\hat{\eta}^2} \right]^{1/2} df \quad (\text{B.23})$$

Integrating Equation B.23 numerically, as shown in Figure B.3, and curve fitting the result, the parameter A_z as a function of Λ_u/Λ_w is expressed by the following equation:

$$A_z = 0.370787 + 0.616271 e^{-0.175490 \Lambda_u/\Lambda_w} \quad (\text{B.24})$$

From Equation B.5 the cross-spectrum of the system (see Figure 3, page 9) between u and w component is obtained from multiplying the complex conjugate of Equation B.11 by Equation B.13.

$$\phi_{uw}(f) = \langle U^*(f) W(f) \rangle = H_{uL}^*(f) H_{waL}(f) \quad (\text{B.25})$$

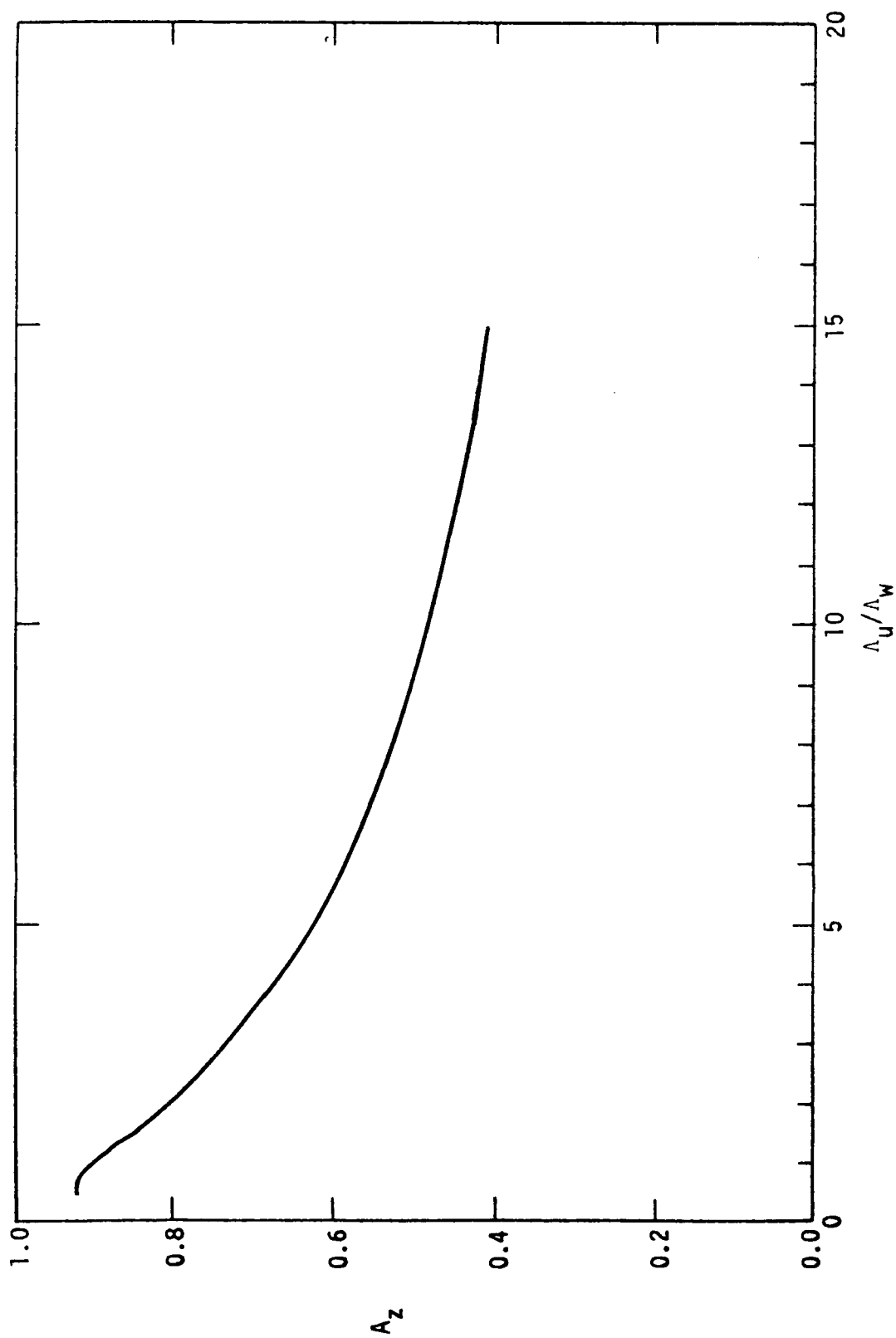


Figure B.3. Values of parameter A_z as a function of Λ_U/Λ_W .

As before, considering the energy spectrum as a sum of the low- and high-frequency portions of spectra we can write the following equation:

$$\phi_{uw}(f) = \phi_{uwl}(f) + \phi_{uwH}(f) \quad (B.26)$$

Noting that a non-zero cross-spectrum exists between u and w components at low frequencies (see Chapter I),

$$\phi_{uwH}(f) = 0$$

To determine the low-frequency portion of the filter functions, the cross-spectrum is multiplied by its complex conjugate (see Equation B.20) giving:

$$\begin{aligned} \phi_{uwl}^* \phi_{uwl} &= \frac{\hat{c}^2}{[1 + 0.4\hat{\eta}^2]} \phi_{uL} \phi_{wL} \\ &= \frac{\hat{c}^2}{[1 + 0.4\hat{\eta}^2]} |H_{uL}|^2 [|H_{waL}|^2 + |H_{wbL}|^2] \end{aligned} \quad (B.27)$$

From the cross-spectrum of the system given by Equation B.25:

$$\phi_{uwl}^* \phi_{uwl} = |H_{uL}|^2 |H_{waL}|^2 \quad (B.28)$$

Equating Equations B.27 and B.28 results in an equation which along with Equation B.16 for the low-frequency portions of spectra can be solved to give the filter functions.

$$\frac{|H_{waL}|^2}{|H_{waL}|^2 + |H_{wbL}|^2} = \frac{\hat{c}^2}{[1 + 0.4\hat{\eta}^2]} \quad (B.29)$$

$$|H_{waL}|^2 + |H_{wbL}|^2 = \phi_{wL} \quad (B.30)$$

The filter functions are:

$$|H_{waL}|^2 = \frac{\hat{c}^2}{1 + 0.4\hat{\eta}^2} \phi_{wL} \quad (B.31)$$

$$|H_{wbL}|^2 = \left[1 - \frac{\hat{c}^2}{1 + 0.4\hat{\eta}^2} \right] \phi_{wL} \quad (B.32)$$

where ϕ_{wL} is given in Chapter I. Let

$$\alpha^2 = \frac{1}{0.4} \left(\frac{\bar{W}_H}{\Lambda_U} \right)^2$$

Then for the low-frequency portion of spectra we have:

$$|H_{waL}|^2 = \frac{\alpha^2 \hat{c}^2}{(\alpha^2 + f^2)} \times \frac{A_{wL}}{(B_{wL} + f)^{5/3}} \quad (B.33)$$

$$|H_{wbL}|^2 = \frac{\alpha^2(1 - \hat{c}^2) + f^2}{(\alpha^2 + f^2)} \times \frac{A_{wL}}{(B_{wL} + f)^{5/3}} \quad (B.34)$$

One solution is:

$$H_{waL} = \frac{\alpha \hat{c} \sqrt{A_{wL}}}{(\alpha + if)(\sqrt{B_{wL}} + i\sqrt{f})^{5/3}} \quad (B.35)$$

$$H_{wbL} = \frac{\sqrt{A_{wL}} (\alpha \sqrt{1 - \hat{c}^2} + if)}{(\alpha + if)(\sqrt{B_{wL}} + i\sqrt{f})^{5/3}} \quad (B.36)$$

The above equations in the complex polar notation form are given in Chapter I by Equation 9.

3. THE DISCRETE FOURIER TRANSFORMATION

For cyclic frequency f , the infinite range Fourier transform of a real-valued or a complex-valued record $x(t)$ is defined by the complex-valued quantity:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-i2\pi ft} dt \quad (B.37)$$

However, since an infinite time record is not practical, by limiting the record $x(t)$ to a range, say 0 to T , then the finite-range Fourier transform will exist as defined by Bendat and Piersol (1971):

$$X(f,T) = \int_0^T x(t) e^{-j2\pi ft} dt \quad (B.38)$$

Now assume the continuous function $x(t)$ is sampled N times at regular intervals of period Δt ($N\Delta t = T$), where Δt has been selected to produce a high cutoff frequency to avoid aliasing. The discrete values of x_n thus become:

$$x_n = x(n\Delta t) \quad n = 0, 1, 2, \dots, N-1 \quad (B.39)$$

and Equation B.38 in discrete form becomes

$$X(f,T) = \Delta t \sum_{n=0}^{N-1} x_n \exp(-j2\pi fn\Delta t) \quad (B.40)$$

A basic frequency f_0 is defined as:

$$f_0 = \frac{1}{T} \quad (B.41)$$

and the discrete frequency value f_m is:

$$f_m = mf_0 = \frac{m}{N\Delta t} \quad m = 0, 1, 2, \dots, N-1 \quad (B.42)$$

At these frequencies, the transformed values give the discrete Fourier components, X_m , defined as:

$$X_m = \frac{X(f_m, T)}{\Delta t} = \sum_{n=0}^{N-1} x_n \exp\left[-j \frac{2\pi mn}{N}\right] \quad (B.43)$$

To simplify the notation, let the discrete weight function, W_{mn} , be:

$$W_{mn} = \exp\left[-j \frac{2\pi mn}{N}\right] \quad (B.44)$$

Thus, Equation B.43 in tensor notation becomes:

$$X_m = W_{mn} x_n \quad (B.45)$$

Equation B.45 is the discrete finite-range Fourier transform. Then the Fourier transform of the output signal generated from the response

(filter) function and the input signal given by Equation B.2 can now be written in discrete form as:

$$Y_m = H_m X_m \quad (B.46)$$

where H_m is the discrete filter function in the frequency domain.

To obtain the output signal in the time domain, the inverse Fourier transform is applied. The inverse Fourier transform of the $X(f)$ in Equation B.37 is:

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{i2\pi ft} df \quad (B.47)$$

From Equation B.43 we have $X(f_m, T) = \Delta t(X_m)$, and noting that $\Delta f = 1/N(\Delta t)$, then Equation B.47 in discrete form becomes:

$$x_n = \frac{1}{N} \sum_{m=0}^{N-1} X_m \exp\left[i \frac{2\pi mn}{N}\right] \quad (B.48)$$

Thus, the discrete inverse Fourier transform of the output signal is:

$$y_n = \frac{1}{N} W_{mn}^* Y_m \quad (B.49)$$

where

$$W_{mn}^* = \exp\left[i \frac{2\pi mn}{N}\right] \quad (B.50)$$

Since the value of W_{mn} repeats for certain combinations of m and n , some additions and multiplications can be eliminated. The fast Fourier transform (FFT) is an algorithm based on this principle which allows computation of the transformation much more rapidly than the direct method by rearranging the matrix W_{mn} to save the calculation of dual node pairs (see subroutine FFT in Chapter II).

When dealing with the discrete Fourier transform instead of the continuous Fourier transform to generate an input signal, the input

variance becomes strongly dependent on the time increment Δt (Oppenheim and Schafer, 1975). Consider the functions $x(t)$ and its Fourier transform $X(f)$, called the Fourier transform pair. To determine the Fourier transform of $x(t)$ by means of digital analysis techniques, it is necessary to sample $x(t)$ at time interval Δt , by multiplying $x(t)$ by a sampling function $\Delta_0(t)$. The sampling at points which are too far apart will lead to confusion between the low- and high-frequency components in the original time history; this is called aliasing. Generally, at least two samples per cycle are required to define a frequency component in the data. Hence, the highest frequency which can be defined by sampling at the rate of $1/\Delta t$ samples per second is $1/2\Delta t$ cps. This is called Nyquist or cutoff frequency:

$$f_c = 1/2\Delta t \quad (B.51)$$

For machine computation, it is necessary to truncate the sampled function $x(t)$ so that only a finite number of points, say N , are considered. Truncation introduces the second modification of the original Fourier transform pair; this effect is to convolve the aliased frequency transform with the Fourier transform of a truncation (rectangular) function $s(t)$. Truncation in the time domain leads to oscillation of the results in the frequency domain termed as leakage. To reduce leakage it is recommended to employ other time domain truncation functions instead of the rectangular function (window).

Thus, the product of the original function with the sampling function and the truncation function yields the finite length time function, $\hat{x}(t)$, given by:

$$\hat{x}(t) = x(t) \Delta_0(t) s(t) \quad (B.52)$$

The Fourier transform of Equation B.52 is given by the convolution of

Fourier transforms $X(f)$ and $S(f)$ as:

$$\hat{X}(f) = X(f) * S(f) \quad (B.53)$$

where $S(f)$, the Fourier transform of the impulse step function, $\Delta_0(t)s(t)$, is:

$$S(f) = \frac{1 - e^{-j2\pi f\Delta t}}{j2\pi f\Delta t} \quad (B.54)$$

The spectrum of the finite length time function $\hat{x}(t)$ is computed by multiplying Equation B.53 by its complex conjugate.

$$\hat{\phi}_X(f) = \phi_X(f) * |S(f)|^2 \quad (B.55)$$

The input variance of a white noise signal for which the spectrum is a constant, can be defined as:

$$\sigma_X^2 = \int_0^{1/2\Delta t} \phi_X(f) df = \phi_X/2\Delta t \quad (B.56)$$

Note that

$$|S(f)|^2 = S(f) S^*(f) = \left[\frac{\sin(\pi f\Delta t)}{\pi f\Delta t} \right]^2 \quad (B.57)$$

Substituting Equations B.56 and B.57 into Equation B.55 results in the input spectrum of the sample data.

$$\hat{\phi}_X(f) = 2\Delta t\sigma_X^2 \left[\frac{\sin(\pi f\Delta t)}{\pi f\Delta t} \right]^2 \quad (B.58)$$

Using this sample data as the input of the turbulence simulation system, the simulated output variance is (see input-output relation given by Equation B.9):

$$\begin{aligned} \hat{\sigma}_Y^2 &= \int_0^{\infty} \hat{\phi}_Y(f) df \\ &= \int_0^{\infty} \hat{\phi}_X(f) |H(f)|^2 df \end{aligned} \quad (B.59)$$

The desired output spectrum $\phi_y(f)$ is defined by Equation B.10 as:

$$\phi_y(f) = |H(f)|^2$$

Thus, Equation B.59 becomes:

$$\hat{\sigma}_y^2 = \int_0^{\infty} 2\Delta t \sigma_x^2 \left[\frac{\sin(\pi f \Delta t)}{\pi f \Delta t} \right]^2 \phi_y(f) df \quad (B.60)$$

Assuming $[\sin(\pi f \Delta t)/\pi f \Delta t]$ to be approximately unity at the lower frequencies where the atmospheric turbulence has the most significant energy density, Equation B.60 reduces to:

$$\hat{\sigma}_y^2 = 2\Delta t \sigma_x^2 \int_0^{\infty} \phi_y(f) df = 2\Delta t \sigma_x^2 \sigma_y^2 \quad (B.61)$$

To have the simulated output variance, $\hat{\sigma}_y^2$, equal to the desired output variance, σ_y^2 , the input variance, σ_x^2 , is selected as:

$$\sigma_x^2 = \frac{1}{2\Delta t} \quad (B.62)$$

Hence, by adjusting the input variance by a factor of $1/2\Delta t$ (see Equation A.7), the discrete system will have the desired output variance.

In the computer program presented in Chapter II, the true output variance is computed by numerically integrating the spectrum function:

$$\sigma_y^2 = \int_0^{\infty} \phi_y(f) df \quad (B.63)$$

The variance of the simulated turbulence is computed by:

$$\hat{\sigma}_y^2 = \frac{\sum_{i=1}^N (y_i - \bar{y})^2}{N} \quad (B.64)$$

where N is the number of data generated. As the value of N increases, the simulated output variance tends to approximate the true value.

4. THE INTERLEVEL COHERENT TURBULENCE GENERATOR

Atmospheric turbulence near the ground has been shown to have strong coherence between vertically separated layers of air. The turbulent wind coherence function, γ_0 , near the ground behaves as:

$$\gamma_0(k, \Delta z) = \exp[-\hat{a}|k\Delta z|] \quad (\text{B.65})$$

where $\hat{a}$ is a non-dimensional coefficient called the decay parameter, k is the wave number, and Δz is the separation distance between the two levels. The technique for generating the two-level fixed-point coherent turbulence is as follows.

Figure B.4 illustrates the interlevel coherent turbulence simulation system. Several independent Gaussian white noise sources, $X_m(k)$, are passed through the lag filters $D_m(k, z)$. The outputs are summed over all the noises to give a composite process $Y_C(k, z)$. The composite process $Y_C(k, z)$ is then passed through the shaping filter $\hat{A}_H(k, z)$ to obtain the output $Y(k, z)$.

Let $D_m(k, z)$ be the Fourier transform of the m th lag filter function, and $X_m(k)$ be the Fourier transform of the m th white noise process, then the Fourier transform of the m th output $Y_m(k, z)$ is given (see Equation B.2) as:

$$Y_m(k, z) = D_m(k, z) X_m(k) \quad (\text{B.66})$$

Summing over all the noises we have:

$$Y_C(k, z) = \sum_{m=-M}^M Y_m(k, z) = \sum_{m=-M}^M D_m(k, z) X_m(k) \quad (\text{B.67})$$

Now invoking Equation B.2 once again with $Y_C(k, z)$ as the Fourier transform of the input and $\hat{A}_H(k, z)$ as the Fourier transform of the response function, the Fourier transform of the output for each filter

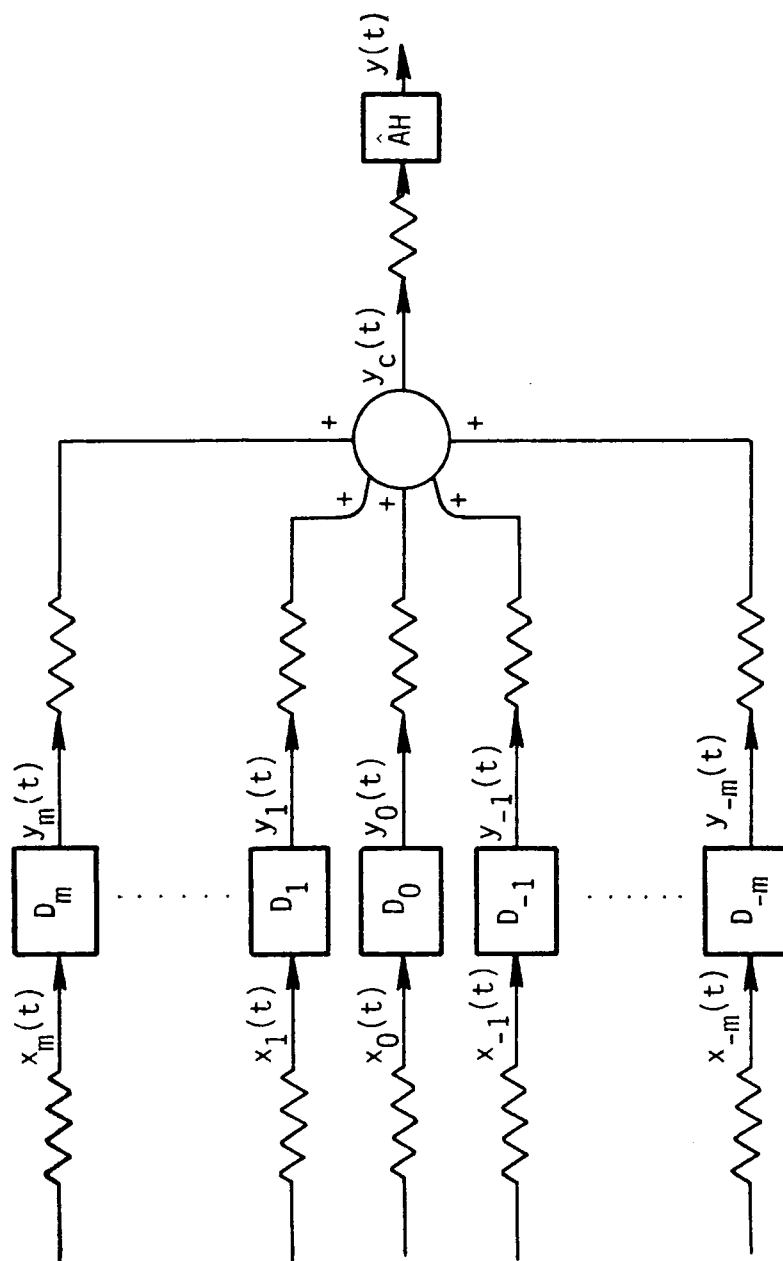


Figure B.4. The interlevel coherence model.

component of the system in Figure 3, page 9, is (see Figure B.5):

$$Y(k,z) = \hat{A}H(k,z) Y_C(k,z)$$

$$= \hat{A}H(k,z) \sum_{m=-M}^M D_m(k,z) X_m(k) \quad (B.68)$$

where $\hat{A}$ is a constant multiplier to be determined later.

Using Equation B.5 to define the two-point spectral function $\phi(k,z_1,z_2)$ at different heights z_1 and z_2 , we multiply Equation B.68 by its complex conjugate evaluated at k' and z_2 to obtain:

$$Y(k,z_1) Y^*(k',z_2) = \hat{A}^2 H(k,z_1) H^*(k',z_2)$$

$$\times \sum_{m=-M}^M \sum_{n=-M}^M D_m(k,z_1) D_n^*(k',z_2) X_m(k) X_n^*(k') \quad (B.69)$$

White noise processes are statistically independent and hence with unit spectral density:

$$X_m(k) X_n^*(k') = \delta(k - k') \delta_{mn} \quad (B.70)$$

where $\delta(k - k')$ is the Dirac delta function and δ_{mn} is the Kronecher delta. Then the two-point auto-spectrum $\phi(k,z_1,z_2)$ can be written as:

$$\phi(k,z_1,z_2) = \hat{A}^2 H(k,z_1) H^*(k,z_2) \sum_{m=-M}^M D_m(k,z_1) D_m^*(k,z_2)$$

$$= \hat{A}^2 \phi_H(k,z_1,z_2) \phi_D(k,z_1,z_2) \quad (B.71)$$

where

$$\phi_H = HH^* \text{ and } \phi_D = \sum_{m=-M}^M D_m D_m^*$$

By definition, the coherence function γ_{ij} is written as:

$$\gamma_{ij}(k,z_1,z_2) = \frac{\phi_{ij}(k,z_1,z_2) \phi_{ij}^*(k,z_1,z_2)}{\phi_{ii}(k,z_1,z_1) \phi_{jj}(k,z_2,z_2)} \quad (B.72)$$

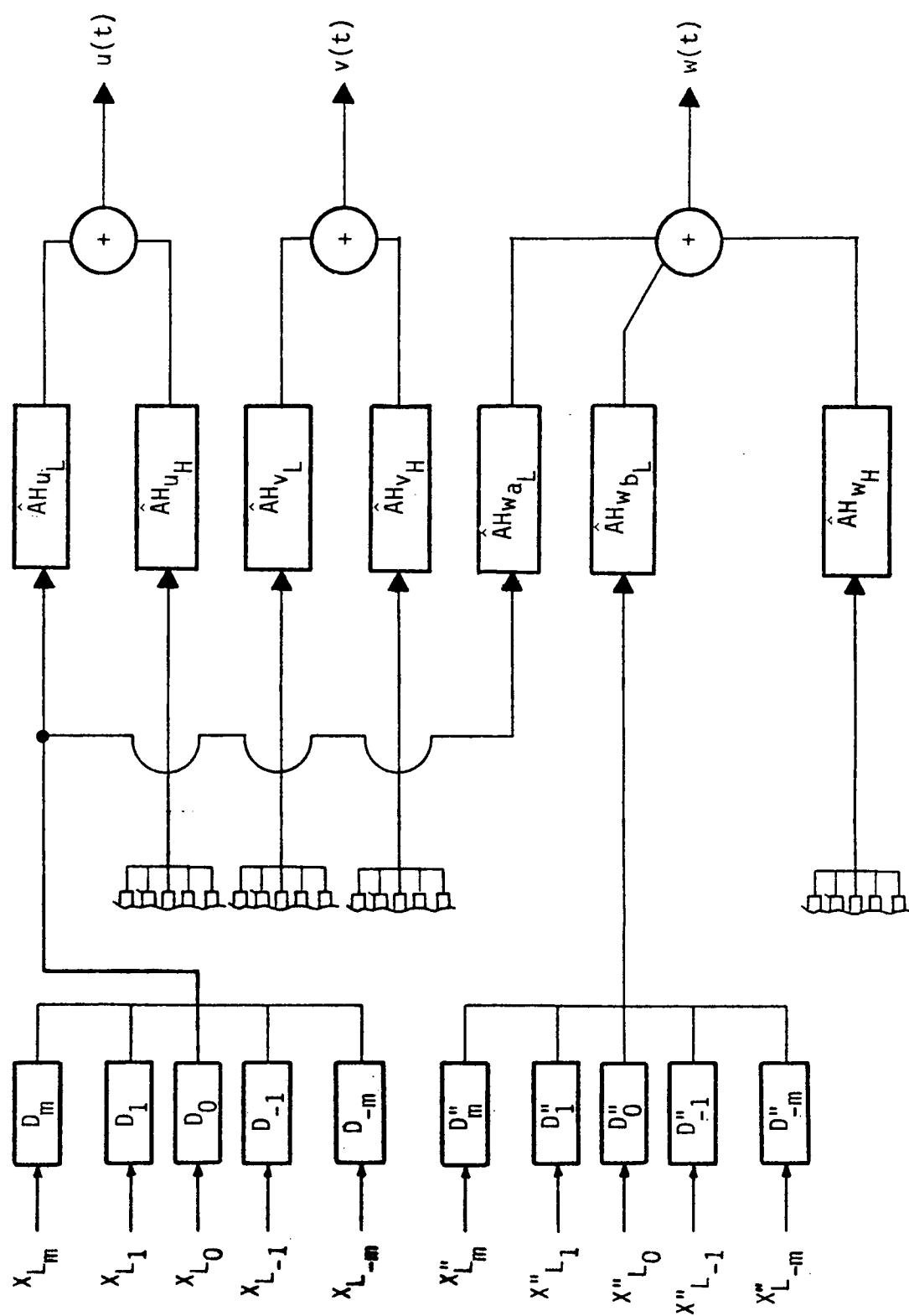


Figure B.5. Control system for large eddy cross-correlation coherent turbulence simulation model.

Then the model coherence function γ is:

$$\gamma(k, z_1, z_2) = \frac{\hat{A}^4 \phi_H(k, z_1, z_2) \phi_D(k, z_1, z_2) \phi_H^*(k, z_1, z_2) \phi_D^*(k, z_1, z_2)}{\hat{A}^4 \phi_H(k, z_1, z_1) \phi_D(k, z_1, z_1) \phi_H(k, z_2, z_2) \phi_D(k, z_2, z_2)} \quad (B.73)$$

Note:

$$\frac{\phi_H(k, z_1, z_2) \phi_H^*(k, z_1, z_2)}{\phi_H(k, z_1, z_1) \phi_H(k, z_2, z_2)} = \frac{H(k, z_1) H^*(k, z_2) H^*(k, z_1) H(k, z_2)}{H(k, z_1) H^*(k, z_1) H(k, z_2) H^*(k, z_2)} = 1 \quad (B.74)$$

Then Equation B.73 reduces to:

$$\gamma(k, z_1, z_2) = \frac{\left[\sum_{m=-M}^M D_m(k, z_1) D_m^*(k, z_2) \right] \left[\sum_{n=-M}^M D_n^*(k, z_1) D_n(k, z_2) \right]}{\left[\sum_{m=-M}^M D_m(k, z_1) D_m^*(k, z_1) \right] \left[\sum_{n=-M}^M D_n^*(k, z_2) D_n(k, z_2) \right]} \quad (B.75)$$

$m = n = -M \dots -1 \ 0 \ 1 \dots M$

Note that the coherence is independent of $\hat{A}H(k, z)$ and is only a function of the lag filters $D_m(k, z)$. The shaping filter $\hat{A}H(k, z)$ will be reserved for auto-spectra and phase angle matching. Thus, the function $D_m(k, z)$ will be formulated so that the model coherence, γ , is equal to the experimental turbulent wind coherence, γ_0 .

Consider a class of processes with coherence function dependent on difference in altitude $\Delta_{12} = z_1 - z_2$. Since the turbulence near the ground ($z < 150$ m) appears to behave as $\gamma_0 = \exp(-\hat{\alpha} k \Delta z)$, a method of satisfying an arbitrary coherence function can be formulated by assuming that $D_m(k, z)$ is a pure lead or lag.

In general, $D_m(k, z)$ in complex polar notation is written as:

$$D_m(k, z) = |D_m(k, z)| e^{i\theta(k, z)} \quad (B.76)$$

For a pure lead or lag, θ is proportional to $\pm m\pi$. Then D_m can be written as:

$$D_m(k, z) = C_m e^{ikz d_m}$$

$$D_0(k, z) = C_0$$

⋮

$$D_{-m}(k, z) = C_m e^{-ikz d_m} \quad (B.77)$$

where C_m is a constant which needs to be determined and d_m can be written as:

$$d_m = \frac{m\pi}{(k\Delta z)_{\max}} \quad (B.78)$$

C_m and d_m are equal for D_m and D_{-m} and $\Delta z = z_1 - z_2$. We can now obtain the result:

$$\begin{aligned} \sum_{m=-M}^M D_m(k, z_1) D_m^*(k, z_2) &= C_0^2 + \sum_{m=1}^M C_m^2 (e^{-ik\Delta z d_m} + e^{+ik\Delta z d_m}) \\ &= C_0^2 + 2 \sum_{m=1}^M C_m^2 \cos(k\Delta z d_m) \end{aligned} \quad (B.79)$$

and

$$\begin{aligned} \sum_{n=-M}^M D_n^*(k, z_1) D_n(k, z_2) &= C_0^2 + \sum_{n=1}^M C_n^2 (e^{+ik\Delta z d_n} + e^{-ik\Delta z d_n}) \\ &= C_0^2 + 2 \sum_{n=1}^M C_n^2 \cos(k\Delta z d_n) \end{aligned} \quad (B.80)$$

Now setting $m = n = 1, 2, \dots, M$ and introducing Equations B.79 and B.80 into Equation B.75, the model coherence function γ is:

$$\gamma(k, \Delta z) = \frac{\left[C_0^2 + 2 \sum_{m=1}^M C_m^2 \cos(k\Delta z d_m) \right]^2}{\left[C_0^2 + 2 \sum_{m=1}^M C_m^2 \right]^2} \quad (B.81)$$

Recall the empirical relationship, Equation B.66, for atmospheric coherence can be expressed as:

$$\gamma_0(k, \Delta z) = e^{-\hat{a}|k\Delta z|} = e^{-\frac{|\xi|}{\xi_0}} \quad (\text{B.82})$$

where

$$\xi = \frac{\epsilon}{\epsilon_0}, \quad \xi_0 = \frac{1}{\hat{a}\epsilon_0} \quad (\text{B.83})$$

and

$$\epsilon = k\Delta z, \quad \epsilon_0 = (k\Delta z)_{\max} \quad (\text{B.84})$$

From Equations B.78, B.83, and B.84, $k\Delta z d_m = m\pi\xi$. Equation B.81 can now be written as:

$$\gamma^{1/2} = \sum_{m=0}^M \alpha_m \cos m\pi\xi \quad (\text{B.85})$$

where

$$\alpha_0 = \frac{C_0^2}{S}, \quad \alpha_m = \frac{2C_m^2}{S} \quad \text{for } m \neq 0$$

and

$$S = C_0^2 + 2 \sum_{m=1}^M C_m^2$$

Coefficients α_m can be found as:

$$\alpha_m = 2 \int_0^1 \gamma^{1/2} \cos m\pi\xi d\xi \quad (\text{B.86})$$

Introducing

$$\gamma = \gamma_0 = e^{-\frac{|\xi|}{2\xi_0}} \quad (\text{B.87})$$

into Equation B.86:

$$\alpha_m = 2 \int_0^1 e^{-\frac{|\xi|}{2\xi_0}} \cos m\pi\xi d\xi \quad (B.88)$$

and carrying out the integration gives:

$$\alpha_m = 4 \frac{\xi_0 \left[1 - (-1)^m e^{-\frac{1}{2\xi_0}} \right]}{[1 + (2m\pi\xi_0)^2]} ; m = 1, 2, \dots \quad (B.89)$$

and

$$\alpha_0 = 2\xi_0(1 - e^{-\frac{1}{2\xi_0}}) ; m = 0 \quad (B.90)$$

Finally,

$$C_0^2 = 2\xi_0(1 - e^{-\frac{1}{2\xi_0}})S, \quad C_m^2 = 2\xi_0 \frac{(1 - (-1)^m e^{-\frac{1}{2\xi_0}})}{[1 + (2m\pi\xi_0)^2]} S \quad (B.91)$$

where

$$\xi_0 = \frac{1}{\hat{a}(k\Delta z)_{\max}}$$

Now rewriting Equation B.71, the two-point auto-spectrum is:

$$\phi(k, z_1, z_2) = \hat{A}^2 H(k, z_1) H^*(k, z_2) \left[C_0^2 + 2 \sum_{m=1}^M C_m^2 \cos(m\pi\xi) \right] \quad (B.92)$$

Thus, the one-point auto-spectrum at z is:

$$\phi(k, z, z) = \hat{A}^2 H(k, z) H^*(k, z) \left[C_0^2 + 2 \sum_{m=1}^M C_m^2 \right] \quad (B.93)$$

Let

$$\hat{A}^2 = \frac{1}{C_0^2 + 2 \sum_{m=1}^M C_m^2} = \frac{1}{S} \quad (B.94)$$

$$\phi(k, z, z) = H(k, z) H^*(k, z) \quad (B.95)$$

where the model one-level auto-spectrum is equated to the one-level auto-spectrum of physical turbulence (see Equation B.10). Thus, using the filter functions of Section 2 of this appendix and the derived constants of this section into Equation B.68 results in:

$$\begin{aligned}
 Y(k,z) &= \hat{A}H(k,z) \sum_{m=-M}^M D_m(k,z) X_m(k) \\
 &= H(k,z) \sum_{m=0}^M \left[e^{im\pi kz/(k\Delta z)_{\max}} X_m(k) \right. \\
 &\quad \left. + e^{-im\pi kz/(k\Delta z)_{\max}} X_{-m}(k) \right] \beta_m
 \end{aligned} \tag{B.96}$$

where

$$\beta_0 = \frac{\hat{A}C_0}{2}, \quad \beta_m = \hat{A}C_m \text{ for } m \neq 0$$

APPENDIX B BIBLIOGRAPHY

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