

February 24, 1969

To the Graduate Council:

I am submitting herewith a dissertation written by Samuel Benton Roberts, entitled "The Zeroth-Order Current Distribution in Arbitrary Arrays of Thin-Linear Dipoles." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Electrical Engineering.

J. D. Tillman
Major Professor

We have read this dissertation
and recommend its acceptance:

J. F. Bussier
Herbert S. Kopp
W. E. Seeds

Accepted for the Council:

Shirley A. Smith
Vice Chancellor for
Graduate Studies and Research

THE ZERO-ORDER CURRENT DISTRIBUTION IN ARBITRARY ARRAYS
OF THIN-LINEAR DIPOLES

A Dissertation
Presented to
the Graduate Council of
The University of Tennessee

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

by
Samuel Benton Roberts
March 1969

ACKNOWLEDGMENT

The author wishes to express deep gratitude to Dr. J. D. Tillman for his patient guidance and advice throughout this study, and for his unfailing optimism. Appreciation goes also to Dr. H. P. Neff for his counsel and suggestions. Finally, the author wishes to thank his wife for typing the first draft, for her encouragement and moral support, and his daughter, who provided the final incentive for the author to complete the study.

ABSTRACT

The potentials at the terminals of the elements of an array of N thin-linear dipoles is calculated using zeroth-order currents. It is shown that if even currents are present on the elements, the terminal potentials cannot be balanced, and that if the terminal potentials are balanced, odd currents must exist on the elements. A simple method of calculating the complex ratio of odd to even currents on each element is given. The method is applied to several arrays, and the results for tangential circular arrays are compared to an accurate numerical solution. The approximate method has the accuracy of the conventional zeroth-order solution, and is found to be best for half-wave dipoles.

TABLE OF CONTENTS

CHAPTER	PAGE
INTRODUCTION.	1
I. THE ZEROth-ORDER APPROXIMATION TO THE CURRENT DISTRIBUTION.	3
II. DETERMINATION OF THE CURRENTS IN ARBITRARY ARRAYS	17
III. RADIATION ZONE PATTERNS OF AN ARBITRARY ARRAY OF DIPOLES. .	27
IV. APPLICATIONS TO SEVERAL ARRAYS.	35
Tangential Circular Arrays.	35
Two Element Arrays of Dipoles	41
Comments on an Array Consisting of Two Pairs of Crossed Dipoles	54
The Three Element Collinear Array of Half-Wave Dipoles. .	58
V. THE IMPEDANCE OF AN ELEMENT WITH EVEN AND ODD CURRENTS. . .	60
VI. CONCLUSIONS	64
BIBLIOGRAPHY, ...	67
APPENDIX	69
VITA	74

LIST OF FIGURES

FIGURE	PAGE
1. View of Two Elements of an Array of Thin Linear Dipoles. . . .	4
2. Geometry Used in Calculating the Far-Zone Electric Field from the Array	18
3. Equiamplitude and Phase Contours of the Coefficient a_{mn}	23
4. Equiamplitude and Phase Contours of the Coefficient b_{mn}	24
5. View of Single-Ring Tangential Circular Array.	36
6. Theoretical and Zeroth-Order Current Distribution, 3 Element Circular Array, $h = 0.25\lambda$, $kp = 1.27$, 1 Sequence.	42
7. Theoretical and Zeroth-Order Current Distribution, 4 Element Circular Array, $h = 0.25\lambda$, $kp = 2.02$, 1 Sequence	43
8. Theoretical and Zeroth-Order Current Distribution, 3 Element Circular Array, $h = 0.30\lambda$, $kp = 1.45$, 1 Sequence	44
9. Theoretical and Zeroth-Order Current Distribution, 4 Element Circular Array, $h = 0.30\lambda$, $kp = 2.33$, 1 Sequence	45
10. Theoretical and Zeroth-Order Current Distribution, 3 Element Circular Array, $h = 0.50\lambda$, $kp = 2.17$, 1 Sequence	46
11. Theoretical and Zeroth-Order Current Distribution, 4 Element Circular Array, $h = 0.50\lambda$, $kp = 3.50$, 1 Sequence	47
12. Two Element Array of Parallel Dipoles.	48
13. Two Element Planar Array of Dipoles.	50

FIGURE

PAGE

14.	Magnitude of Odd Current Ratio in Two Element Parallel Array, Angle and Spacing Varying.	52
15.	Phase of Odd Current Ratio in Two Element Parallel Array, Angle and Spacing Varying	53
16.	Array of Planar Crossed Dipoles	56

INTRODUCTION

In a recent study, J. D. Tillman (1) demonstrated the fundamental difficulty in feeding an array of asymmetrically coupled elements. This difficulty lies in the fact that the terminal currents and terminal potentials may not necessarily be specified independently. Specification of balanced currents leads, in general, to a non-zero scalar electric potential at the driving point due to the currents in the other elements. Specification of zero potential at the center of the driving terminals of the elements requires, in general, an odd or unbalanced component of current in the element. From a practical standpoint, neither case is desirable, since both lead to radiation from feedlines. There are exceptional cases which do not pose such a problem. The common array of parallel dipoles whose feed-points all lie in the same plane, perpendicular to the plane of the element axes, escapes the problem of non-zero potentials because of geometry.

J. L. Hilburn and C. E. Hickman (2) analyzed tangential and radial circular arrays of antennas, obtaining an exact answer using the Gaussian quadrature technique developed by Hickman for the isolated element. Their results verify the presence of the odd component of current on the elements. As Tillman points out, their assumed driving-point conditions required zero potential at the centers of the elements. Tillman analyzed the same case by an approximate technique assuming balanced currents and showed that non-zero values of scalar potential were present at the centers. In a recent paper

Chang and King (3) analyzed two arbitrarily located parallel dipoles and indicated the presence of the resulting unbalanced current, although they impose an additional boundary condition that the odd currents vanish at the feed points.

The exact solution to the problem of determining the current distribution on an antenna has never been accomplished. The boundary conditions and geometry of the situation do not lend themselves to exact analysis. By making appropriate simplifying assumptions regarding the boundary conditions and method of feeding the antenna, infinite series solutions (4) and purely numerical solutions (2) have been obtained. A determination of the currents in an array of elements is further complicated because of the difficulty in decoupling the integral equations describing the currents. In certain special cases, however, solutions may be obtained. The disadvantage of purely numerical solutions is that they do not lend much physical insight into the problem.

A simple method of estimating the magnitude and phase of the odd component of current on the elements of an arbitrary array of thin dipole antennas is given in the following presentation. The accuracy of the method is approximately that of the zeroth-order solution of conventional antenna theory based on sinusoidal current distributions.

CHAPTER I

THE ZERO-ORDER APPROXIMATION TO THE CURRENT DISTRIBUTION

Two elements of an arbitrary array of N linear antennas are shown in Figure 1. The elements are of length $2h$ and of radius a . They are assumed to be perfectly conducting metal cylinders. The reference element is centered at the origin of the coordinate system and its axis coincides with the z -axis. Each element is fed at the center gap by a difference in scalar electric potential. If the width of the gap, δ , approaches zero, this arrangement is called a slice-generator (5). The difference in potential now becomes a discontinuity in potential.

For the reference element, 1, the potentials on either side of the gap are $\phi_1 + V_1/2$ and $\phi_1 - V_1/2$. Thus, the potential difference across the gap is V_1 . At the center of the gap the scalar potential is ϕ_1 , which is due to the charges on all elements of the array. The value of ϕ_1 depends on how the elements are driven and the geometry of the array.

The currents in the two halves of the antenna are not necessarily equal. For balanced currents $I(\delta)$ is the same as $I(-\delta)$, and as δ approaches zero, $I(\delta)$ approaches the terminal current, I_0 . Specification of either balanced currents or zero scalar potential at the centers is theoretically possible, but not both in general. The most general case possesses both unbalanced currents and non-zero potentials at the feed-points.

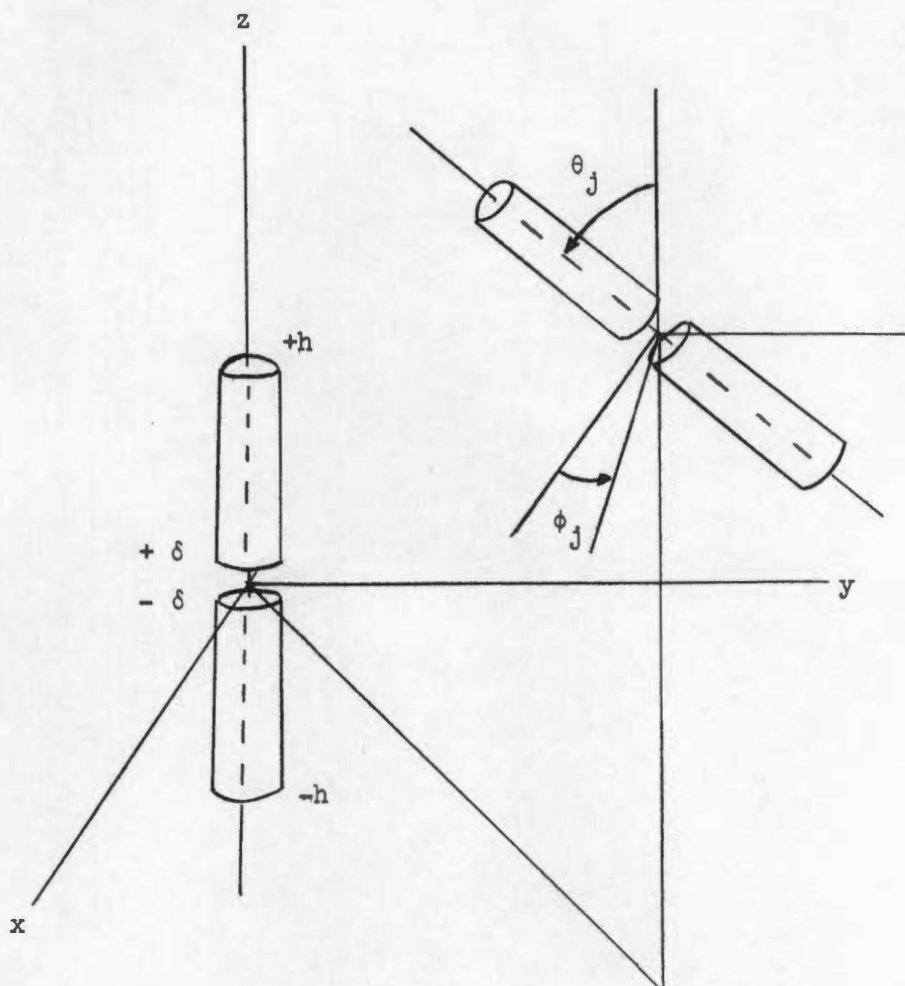


Figure 1. View of Two Elements of an Array of Thin Linear Dipoles.

Other elements of the array are located with centers at (X_m, Y_m, Z_m) and with axes z_m at angles (θ_m, ϕ_m) with respect to the reference coordinate system.

Assume that the radius of the elements is very small, i.e., $ka \ll 1$, where $k = \frac{2\pi}{\lambda}$ is the free space wavenumber. This important simplifying assumption permits the approximation that only axial currents flow in the elements.

A boundary condition for the electric field requires that $\vec{E} \times \hat{n}$ vanish on the surface of the conductors, where $\hat{n}$ is the outward normal to the surface. For the reference antenna this may be expressed as

$$E_z|_{x^2+y^2=a^2} = 0, \quad \delta \leq |z| \leq h. \quad \text{I-1}$$

E_z is the tangential electric field on element 1 due to all currents. This expression neglects the small top and bottom surfaces of the antenna whose areas are small.

The electric field tangential to surface 1 (S_1) is found from the vector magnetic and scalar electric potentials by the dot product

$$\vec{E} \cdot \hat{z} = (-\nabla V - j\omega \vec{A}) \cdot \hat{z}, \quad \text{I-2}$$

and

$$E_z|_{S_1} = (-j\omega A_z + \frac{c^2}{j\omega} (\nabla(\nabla \cdot \vec{A}))_z)|_{S_1}, \quad \text{I-3}$$

where $\vec{A}$ is the vector potential, V the scalar potential, and c is the velocity of wave propagation. I-2 utilizes the continuity equation

between the two potential functions. A sinusoidal ($e^{j\omega t}$) time dependence has been assumed for the fields.

The vector potential on S_1 has only a z component due to element 1, but in general, x , y and z components are present due to the other elements.

Rearranging I-3 and using the boundary condition I-1 yields

$$\frac{d^2 A_z}{dz^2} + k^2 A_z = - \frac{d}{dz} \left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} \right] . \quad \text{I-4}$$

I-4 holds on surface S_1 . Henceforth in this chapter, all equations will hold on S_1 unless otherwise stated. Also in I-4, $\frac{\partial}{\partial z}$ has been replaced by $\frac{d}{dz}$ since the radius a has been assumed to be small.

The complete solution to I-4 is $A_{zc} + A_{zp}$, the complementary and particular solutions. A complementary solution for the top-half of the element 1 is

$$A_{zc} = - \frac{1}{c} [C_1 \cos kz + C_2 \sin kz] , \quad \text{I-5}$$

where C_1 and C_2 are constants determined by the boundary conditions, and c is the free space velocity of light. A particular solution for the equation for the top-half is

$$A_{zp} = - \frac{1}{k} \int_0^z G'(s) \sin k (z - s) ds , \quad \text{I-6}$$

where

$$G'(s) = \frac{d}{ds} \left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} \right] . \quad \text{I-7}$$

Equation I-7 may be verified by the Leibniz rule for differentiating a definite integral. The complete solution for the top-half of the antenna is

$$A_z(z) = -\frac{1}{c} [C_1 \cos kz + C_2 \sin kz] \\ - \frac{1}{k} \int_{\delta}^z G'(s) \sin k(z-s) ds . \quad \text{I-8}$$

By similar arguments the complete solution for the lower half of the reference element is

$$A_z(z) = -\frac{1}{c} [C_3 \cos kz + C_4 \sin kz] \\ - \frac{1}{k} \int_{-\delta}^z G'(s) \sin (z-s) ds . \quad \text{I-9}$$

The integral in I-8 may be simplified slightly using integration by parts so that

$$\int_{\delta}^z \frac{d}{ds} [G(s)] \sin k(z-s) ds = G(s) \sin k(z-s) \Big|_{\delta}^z + k \int_{\delta}^z G(s) \cos k(z-s) ds \quad \text{I-10}$$

$$= -G(\delta) \sin k(z-\delta) + k \int_{\delta}^z G(s) \cos k(z-s) ds . \quad \text{I-11}$$

Hence, I-8 becomes

$$A_z(z) = -\frac{1}{c} [C_1 \cos kz + C_2 \sin kz] + \frac{1}{k} G(\delta) \sin k(z-\delta) - \int_{\delta}^z \left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} \right] \cos k(z-s) ds. \quad \text{I-12}$$

The constant C_2 may be evaluated by considering the scalar potential on the top surface. For this portion

$$\phi(z) = \frac{1}{\omega} \nabla \cdot \bar{A} \quad \text{I-13}$$

$$= \frac{1}{\omega} \left[G(z) + \frac{\partial A_z}{\partial z} \right] \quad \text{I-14}$$

$$= \frac{1}{\omega} G(z) + \frac{1}{\omega} \frac{\partial}{\partial z} \left\{ -\frac{1}{c} [C_1 \cos kz + C_2 \sin kz] + \frac{1}{k} G(\delta) \sin k(z-\delta) - \int_{\delta}^z G(s) \cos k(z-s) ds \right\} \quad \text{I-15}$$

$$= \frac{1}{\omega} G(z) - C_1 \sin kz + C_2 \cos kz + \frac{1}{\omega} \cos k(z-\delta) G(\delta) - \frac{1}{\omega} \frac{d}{dz} \int_{\delta}^z G(s) \cos k(z-s) ds. \quad \text{I-16}$$

The last integral in I-16 is also equal to

$$G(z) + \int_{\delta}^z G(s)(-k) \sin k(z-s) ds, \quad \text{I-17}$$

so I-16 becomes

$$\begin{aligned} \phi(z) = & -C_1 \sin kz + C_2 \cos kz + \frac{1}{\omega} c^2 \cos k(z-\delta) G(\delta) \\ & + jc \int_{\delta}^z G(s) \sin k(z-s) ds. \end{aligned} \quad \text{I-18}$$

Using I-18 and the condition that $\phi(\delta) = \phi_1 + \frac{V_1}{2}$ yields

$$\phi_1 + \frac{V_1}{2} = -C_1 \sin k\delta + C_2 \cos k\delta + \frac{1}{\omega} c^2 G(\delta). \quad \text{I-19}$$

Therefore

$$C_2 = \left[\frac{V_1}{2} + \phi_1 + C_1 \sin k\delta - \frac{1}{\omega} c^2 G(\delta) \right] / \cos k\delta. \quad \text{I-20}$$

Similarly, it may be shown that for the bottom portion of the element,

$$C_4 = \left[-\frac{V_1}{2} + \phi_1 - C_3 \sin k\delta - \frac{1}{\omega} c^2 G(-\delta) \right] / \cos k\delta. \quad \text{I-21}$$

For the slice-generator approximation, δ approaches zero. Hence

$$C_2 = \frac{V_1}{2} + \phi_1 - \frac{1}{\omega} c^2 \left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} \right]_{z=0}, \quad \text{I-22}$$

and

$$C_4 = -\frac{V_1}{2} + \phi_1 - \frac{1}{\omega} c^2 \left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} \right]_{z=0}. \quad \text{I-23}$$

Finally, the solutions for A_z may be written in terms of two constants, C_1 and C_3 , as

$$A_z = -\frac{1}{c} \left[C_1 \cos kz + \left(\frac{V_1}{2} + \phi_1 \right) \sin kz - \frac{1}{\omega} c^2 G(0) \right] - \int_0^z G(s) \cos k(z-s) ds \quad \text{I-24}$$

for the top half, and

$$A_z = -\frac{1}{c} \left[C_3 \cos kz + \left(-\frac{V_1}{2} + \phi_1 \right) \sin kz - \frac{1}{\omega} c^2 G(0) \right] - \int_0^z G(s) \cos k(z-s) ds \quad \text{I-25}$$

for the bottom half.

The left side of equations I-24 and I-25 may be expressed from the integral solution to the Helmholtz wave equation for A_z as

$$A_z = \frac{\mu}{4\pi} \sum_{m=1}^N \int_{-h_m}^{+h_m} I_{mz}(z'_m) \frac{e^{-jkR_{m1}}}{R_{m1}} dz'_m, \quad \text{I-26}$$

where

$$R_{m1} = [(x - x'_m)^2 + (y - y'_m)^2 + (z - z'_m)^2]^{1/2} \quad \text{I-27}$$

and $I_{mz}(z'_m)$ is the current on element m parallel to z .

This equation expresses the total axial component of vector potential on the reference element as a function of the currents in the other elements. A similar equation holds for the top and bottom

surfaces of all N elements of the array. An exact solution for the distribution of currents in the array requires the simultaneous solution of the N integral equations for the currents, using the additional boundary conditions that the currents on all elements vanish at the ends. The equations are further complicated by the presence of the factor ϕ_m in each equation, which is the scalar potential at the center of element m due to the charges on all other elements.

The solution to the general problem has never been accomplished. In special cases, however, the integral equations may be decoupled by the use of special driving sequences (6) in which the terminal voltages and currents are related by complex constants. This procedure requires an identical environment for each element.

The zeroth-order integral equation for the current in the top-portion of element one is obtained by combining and rearranging equations I-25 and I-27 in the following manner:

$$\int_{-h_1}^{h_1} I_{z1}(z') \frac{e^{-jkR_1}}{R_1} dz' = -\frac{j4\pi}{\eta} [C_1 \cos kz + (\frac{V_1}{2} + \phi_1) \sin kz - \frac{j\epsilon^2 4\pi}{\omega\mu} G(0)]$$

$$- \frac{4\pi}{\mu} \int_0^z G(s) \cos k(z-s) ds - \sum_{p=2}^N \int_{-h_p}^{h_p} I_{pz}(z') \frac{e^{-jkR_p}}{R_p} dz' , \quad \text{I-28}$$

where $\eta \equiv \sqrt{\frac{\mu}{\epsilon}}$. The zeroth-order solution is found by approximating the integration on the left and showing that most of the terms on the right are much smaller than the first two.

The integral on the left may be evaluated approximately by noting that $(R)^{-1}$ resembles a delta sequence. This derivation follows Tillman (1). A delta sequence is a sequence of functions $\delta_\epsilon(x)$ which has the properties, for ϵ monotonic and decreasing with limit zero:

$$1. \quad \delta_\epsilon(x) = O(1/\epsilon), \quad |x| < \epsilon \quad \text{I-29}$$

$$2. \quad \int_{-\epsilon}^{+\epsilon} \delta_\epsilon(x) dx = 1 + O(\epsilon) \quad \text{I-30}$$

$$3. \quad \int_{-\infty}^{+\infty} \delta_\epsilon(x) dx = 1 + O(\epsilon). \quad \text{I-31}$$

$$\text{Define } q_a(z) = [z^2 + a^2]^{-1/2}, \quad |z| < h \quad \text{I-32}$$

$$= 0, \quad |z| > h.$$

The function q_a has some of the properties of a delta-sequence. When $z = 0$, $q_a = 1/a$ so property 1 is satisfied. For $-a < z < a$, $[z^2 + a^2]^{1/2} \approx a$, so

$$\int_{-a}^a \frac{dz'}{\sqrt{z'^2 + a^2}} \approx \int_{-a}^a \frac{dz'}{a} = 2, \quad \text{I-33}$$

and property 2 is satisfied with appropriate normalization. Also,

$$\int_{-\infty}^{+\infty} q_a(z') dz' = \int_{-h}^{+h} \frac{dz'}{\sqrt{z'^2 + a^2}} = 2 \ln(2h/a). \quad \text{I-34}$$

Property 2 may again be satisfied with a

different normalization. Using 3 as the most important property,

$$q_a(z) = \Omega \delta(z) \quad \text{I-35}$$

where $\Omega = 2 \ln(2h/a)$.

Additional approximations are necessary for $z \neq 0$, since

$$\begin{aligned} \int_{-h}^{+h} \frac{dz'}{\sqrt{(z-z')^2 + a^2}} &= 2 \ln(2h/a) + \ln(1 - z^2/h^2) \\ &+ \ln \frac{1}{4} \left[1 + \sqrt{1 + \left(\frac{a}{h-z}\right)^2} \right] \left[1 + \sqrt{1 + \left(\frac{a}{h+z}\right)^2} \right], \quad \text{I-36} \end{aligned}$$

which becomes large near the ends of the antenna. However, since the current must vanish at the ends of the conductor, these inaccuracies are no more significant than the approximations already made.

Finally, using the approximate expression for R^{-1} , the left side of I-28 becomes

$$\int_{-h}^{+h} I_z(z') \frac{e^{-jkR}}{R} dz' \approx \Omega \int_{-h}^{+h} I_z(z') \delta(z - z') dz' \quad \text{I-37}$$

$$\approx \Omega I_z(z). \quad \text{I-38}$$

Solving I-28 for $I_z(z)$ yields

$$I_z(z) = -\frac{j4\pi}{\eta\Omega} [C_1 \cos kz + (\frac{V_1}{2} + \phi_1) \sin kz] - \frac{j4\pi}{\eta\Omega} [-\frac{j c^2 4\pi}{\omega\mu} G(0)]$$

$$- \frac{4\pi}{\Omega\mu} \int_0^z G(s) \cos k(z-s) ds - \frac{1}{\Omega} \sum_{p=2}^N \int_{-h_p}^{h_p} I_{pz}(z') \frac{e^{-jkR_p}}{R_p} dz'_p \quad I-39$$

Now note that all terms on the right except the first bracket are at least of order Ω^{-2} , since G and I_p are at least of order Ω^{-1} . Hence, as $\frac{h}{a}$ increases the current approximates more and more closely a sinusoid in form. The conventional zeroth-order approximation to the current is

$$I_z(z) \approx -\frac{j4\pi}{\eta\Omega} [C_1 \cos kz + C_2 \sin kz], \quad 0 < z \leq h. \quad I-40$$

Similarly,

$$I_z(z) \approx -\frac{j4\pi}{\eta\Omega} [C_3 \cos kz + C_4 \sin kz], \quad -h \leq z < 0. \quad I-41$$

The preceding arguments are physically equivalent to assuming that the principal contribution to the vector potential at any point on a thin antenna is due to elemental current sources only in the immediate neighborhood of the point. The accuracy of the assumption improves as the radius decreases, except for points near the ends. Here the current vanishes, so the details of the distribution are unimportant. Since the vector potential is sinusoidal along the wire, the current is also approximately sinusoidal on each half of the antenna.

The current on the antenna may be expressed as the sum of an even part I^e , and an odd part, I^o , where

$$I^e(z) = \frac{I(z) + I(-z)}{2} \quad \text{I-42}$$

and

$$I^o(z) = \frac{I(z) - I(-z)}{2} . \quad \text{I-43}$$

The requirement that the current vanishes at the ends gives

$$C_2 = -C_1 \cos kh / \sin kh , \quad \text{I-44}$$

and

$$C_4 = C_3 \cos kh / \sin kh . \quad \text{I-45}$$

For the top half of the antenna

$$I_T(z) = - \frac{j 4\pi}{\eta \Omega} C_1 \frac{\sin k (h - z)}{\sin kh} , \quad \text{I-46}$$

and for the bottom half

$$I_B(z) = - \frac{j 4\pi}{\eta \Omega} C_3 \frac{\sin k (h + z)}{\sin kh} . \quad \text{I-47}$$

The even part of the current on the antenna is written as

$$I^e(z) = I^e \sin k (h - |z|) , \quad \text{I-48}$$

where I^e is the maximum value of the even part on the wire. Similarly, the odd part may be written as

$$I^o(z) = \pm I^o \sin k (h \mp z) , \quad \text{I-49}$$

where I^o is the maximum value of odd current, and the upper (lower) sign is associated with the top (bottom) portion of the antenna.

The complete zeroth order current has now been expressed as the sum of its even and odd parts. The current may also be written as

$$I_z(z) = (I^e \pm I^o) \sin k (h \mp z) \quad \text{I-50}$$

where the upper (lower) sign is associated with the upper (lower) half of the antenna. I-50 is the final form for the approximate current distribution on the antenna.

The zeroth-order current distribution on elements of an arbitrary array of N thin dipoles has been shown to be sinusoidal in form. The current on each dipole may be expressed as the sum of its even and odd components. The odd current is not necessarily zero at the center, but has a discontinuity there.

CHAPTER II

DETERMINATION OF THE CURRENTS IN ARBITRARY ARRAYS

An array of N arbitrarily located elements is situated in space, element i , the reference element having its center at the origin and lying parallel to the z -axis. Any other element of the array, element j , (see Figure 2) is located by defining its center and angular orientations θ_j , ϕ_j with respect to the reference element.

Each element is assumed to have a sinusoidal current distribution, so that for element n

$$I_n(z_n) = (I_n^e + I_n^0) \sin k(h - z_n), \quad z_n > 0$$

II-1

$$I_n(z_n) = (I_n^e - I_n^0) \sin k(h + z_n), \quad z_n < 0$$

and each element is fed by a slice-generator, so that at the terminal of n ,

$$V_n^T(z_n = \delta) = \phi_n + V_n/2,$$

and

II-2

$$V_n^T(z_n = -\delta) = \phi_n - V_n/2.$$

The potential difference across the terminals of element n is V_n and the terminals are at a common potential ϕ_n ,

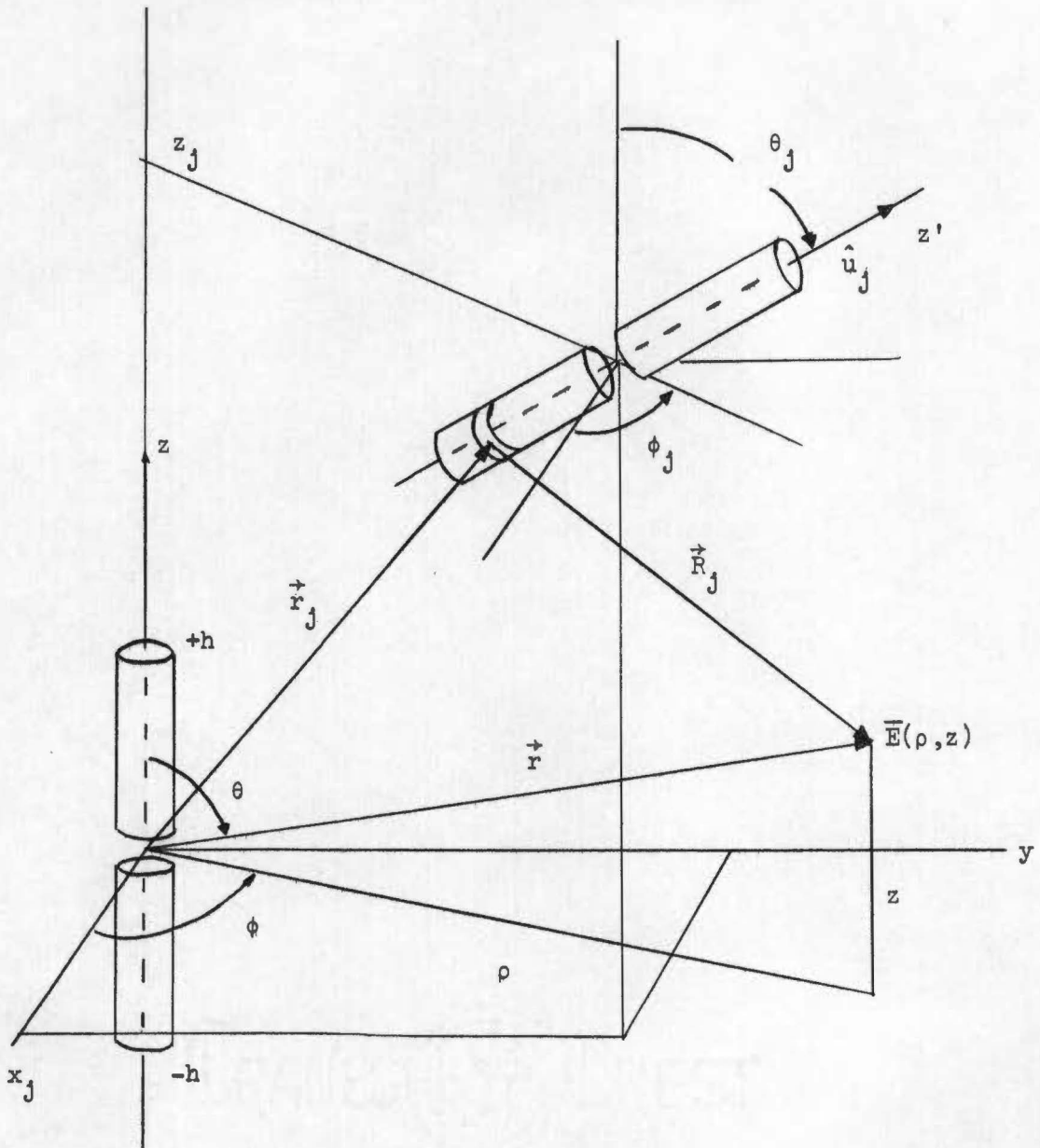


Figure 2. Geometry Used in Calculating the Far-zone Electric Field from the Array.

The charge distribution on n is found to be

$$\rho_n(z_n) = \frac{k}{j\omega} (I_n^e + I_n^0) \cos k(h - z_n), \quad z_n > 0$$

and

II-3

$$\rho_n(z_n) = \frac{-k}{j\omega} (I_n^e - I_n^0) \cos k(h + z_n), \quad z_n < 0.$$

This charge distribution gives rise to a scalar potential whose value at any point in space is

$$\Phi(\rho, z) = \frac{1}{4\pi\epsilon} \int_{-h_n}^{h_n} \rho_n(z') K(\rho, z, z') dz' \quad \text{II-4}$$

where

$$K(\rho, z, z') = \frac{e^{-jk\sqrt{\rho^2 + (z - z'_n)^2}}}{\sqrt{\rho^2 + (z - z'_n)^2}}, \quad \text{II-5}$$

and z and ρ are the right circular cylindrical coordinates of the field point with the center of the n th element as the origin.

Using II-3, II-4 becomes

$$\begin{aligned}
\Phi(\rho, z) = & \frac{-jk}{4\pi\omega\epsilon} I_n^e \left\{ - \int_{-h}^0 \cos k(h + z') K(\rho, z, z') dz' \right. \\
& \left. + \int_0^h \cos k(h - z') K(\rho, z, z') dz' \right\} \\
& \frac{-jk}{4\pi\omega\epsilon} I_n^0 \left\{ \int_{-h}^0 \cos k(h + z') K(\rho, z, z') dz' \right. \\
& \left. + \int_0^h \cos k(h - z') K(\rho, z, z') dz' \right\}
\end{aligned} \quad \text{II-6}$$

and $k/\omega\epsilon \equiv \eta$, the intrinsic impedance of the medium.

If the field point is located at the center of element j , then the potential at this point due to the charges on element i is designated Φ_{ij} . Thus,

$$\Phi_{ij} = I_i^e a_{ij} + I_i^0 b_{ij} \quad \text{II-7}$$

where

$$a_{ij} = \frac{-j\eta}{4\pi} \left\{ - \int_{-h}^0 \cos k(h + z') K(\rho, z, z') dz' + \int_0^h \cos k(h - z') K(\rho, z, z') dz' \right\}, \quad \text{II-8}$$

and

$$b_{ij} = \frac{-j\eta}{4\pi} \left\{ \int_{-h}^0 \cos k(h + z') K(\rho, z, z') dz' + \int_0^h \cos k(h - z') K(\rho, z, z') dz' \right\}. \quad \text{II-9}$$

By changes of variables these integrals may be expressed in terms of the generalized sine and cosine integrals, which appear frequently in antenna theory and have been tabulated by the Harvard Computation Laboratory (7). However, the most general expressions for a_{ij} and b_{ij} require thirty-two terms each, so round-off and interpolation errors resulting from use of the tables may be quite significant. Hence, there is usually very little to be gained from using the tables, as long as a digital computer is available.

It may be shown that for $h = \lambda/4$,

$$\begin{aligned}
 a_{ij}(\rho, z) = & \sin kz \left\{ -\overline{C}_c(k\rho, kz - \frac{\pi}{2}) + j S_c(k\rho, kz - \frac{\pi}{2}) \right. \\
 & \left. + \overline{C}_c(k\rho, kz + \frac{\pi}{2}) - j S_c(k\rho, kz + \frac{\pi}{2}) \right\} \\
 & - \cos kz \left\{ -C_s(k\rho, kz - \frac{\pi}{2}) + j S_s(k\rho, kz - \frac{\pi}{2}) \right. \\
 & \left. + C_s(k\rho, kz + \frac{\pi}{2}) - j S_s(k\rho, kz + \frac{\pi}{2}) \right\}, \quad \text{II-10}
 \end{aligned}$$

and

$$\begin{aligned}
 b_{ij}(\rho, z) = & \sin kz \left\{ -\overline{C}_c(k\rho, kz + \frac{\pi}{2}) + 2\overline{C}_c(k\rho, kz) - \overline{C}_c(k\rho, kz - \frac{\pi}{2}) \right. \\
 & \left. + j S_c(k\rho, kz + \frac{\pi}{2}) - 2j S_c(k\rho, kz) + j S_c(k\rho, kz - \frac{\pi}{2}) \right\} \\
 & + \cos kz \left\{ C_s(k\rho, kz + \frac{\pi}{2}) - 2C_s(k\rho, kz) + C_s(k\rho, kz - \frac{\pi}{2}) \right. \\
 & \left. - j S_c(k\rho, kz + \frac{\pi}{2}) + 2j S_s(k\rho, kz) - j S_s(k\rho, kz - \frac{\pi}{2}) \right\}. \quad \text{II-11}
 \end{aligned}$$

The notation used is that of reference (7). By requiring $\rho = z = 0$, the "self-contributions" to the potential are obtained. These are, for $h = \lambda/4$,

$$a_{11} = 0 + j0, \quad \text{II-12}$$

and

$$\begin{aligned} b_{11} &= -60 \left[S_s \left(0, \frac{\pi}{2} \right) + j S_c \left(0, \frac{\pi}{2} \right) \right] \\ &= -30 \left[\ln 2 + \gamma + \ln \left(\frac{\pi}{2} \right) - \text{Ci}(\pi) + j \text{Si}(\pi) \right] \\ &= -44.45 - j 55.56. \end{aligned} \quad \text{II-13}$$

Here γ is Euler's constant = 0.57772 .

An even current on an element causes no scalar potential at its own center, while an odd current on an element causes a potential proportional to b_{11} .

Although forms such as II-10 and II-11 may be obtained for any ρ , z and h , it is much more accurate to perform the integrations numerically. The coefficient calculations made in this paper were obtained using a 10 point Gaussian quadrature formula on an IBM 7040 computer.

It is interesting to observe the behavior of the complex coefficients a_{ij} , b_{ij} for a half-wave element. Figures 3 and 4 are illustrations of the equiphase and equiamplitude contours of the

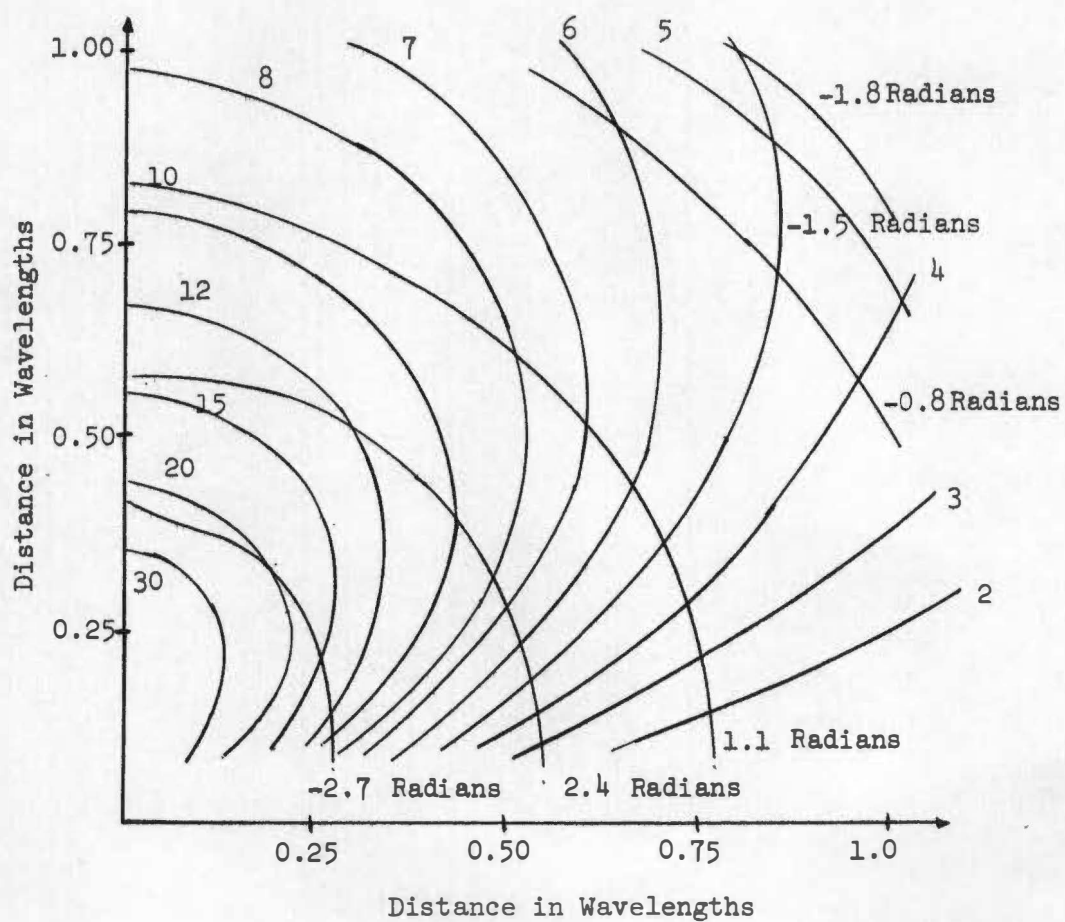


Figure 3. Equiamplitude and Phase Contours of the Coefficient a_{mn} .

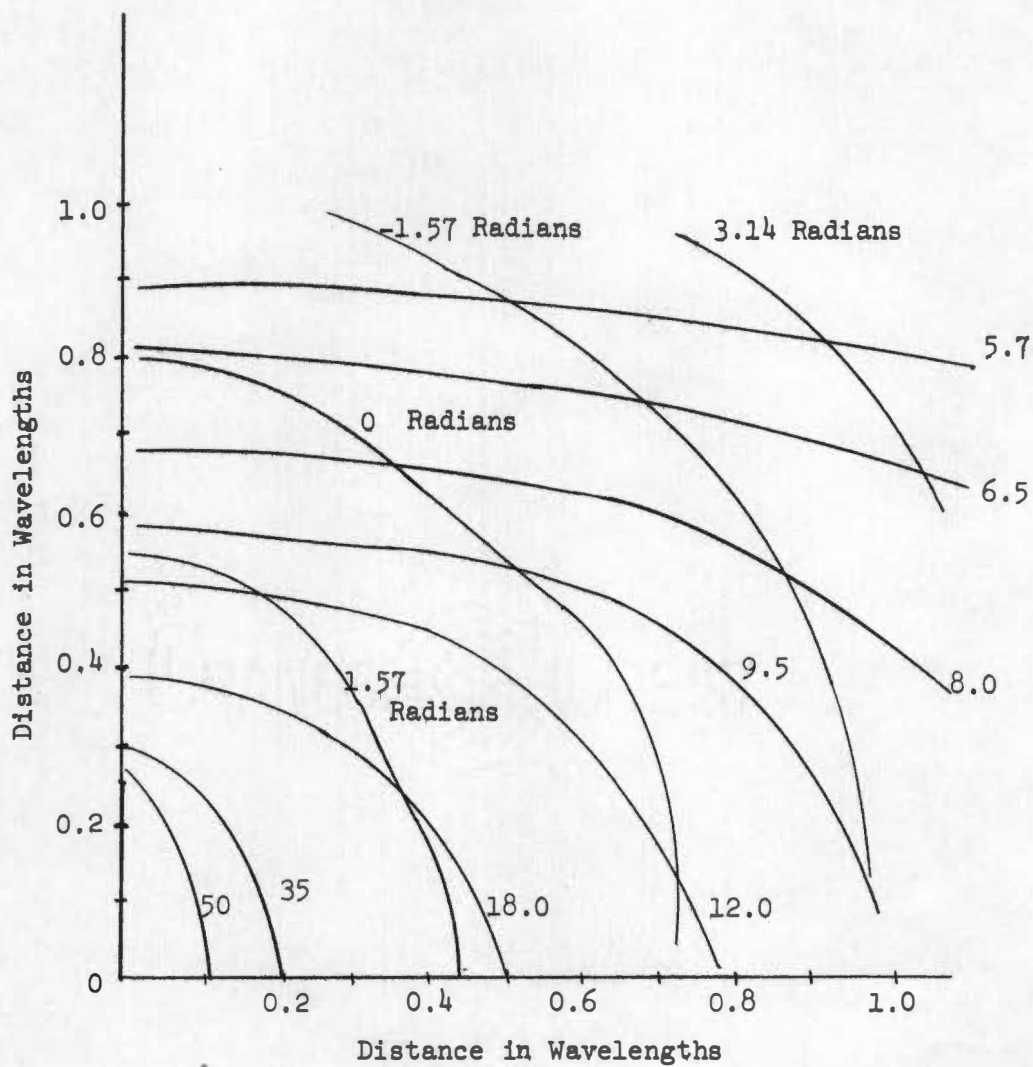


Figure 4. Equiamplitude and Phase Contours of the Coefficient b_{mn} .

coefficients in the region of a few wavelengths. It is necessary only to illustrate one quadrant of the plot. The abscissa is ρ and the ordinate z .

The largest values of a_{1j} and b_{1j} occur along the axis of the element at approximately 0.1750 wavelengths (for $h = \lambda/4$). The approximate values here are $a_{\max} = 337.95 e^{-j1.621}$ and $b_{\max} = 290.32 e^{-j1.715}$. Both coefficients decrease rapidly as the distance from the element increases. The equiphasic curves for a_{1j} indicate to a good degree of approximation expanding spherical waves with a linear phase lag of 2π radians/wavelength. For b_{1j} the curves show the same type behavior. It is expected from physical considerations that this approximation becomes more accurate as the distance from the center increases.

Consideration of these two figures indicates that the maximum odd component of current arises for collinear elements.

In an antenna array, each element makes a contribution to the scalar potential at the center of the other elements, so for an array of N elements, the scalar potential at the center of the p th element is

$$\phi_p = \sum_{q=1}^N \phi_{pq} = \sum_{q=1}^N [I_q^e a_{pq} + I_q^o b_{pq}] \quad \text{II-14}$$

Since a similar equation holds for all N elements, the resulting set of N linear equations is conveniently expressed in matrix form as

$$\underline{\phi} = \underline{A} \underline{I}^e + \underline{B} \underline{I}^o, \quad \text{II-15}$$

where $\underline{\Phi}$ is the $N \times 1$ column vector of scalar potentials at the element feed points, $\underline{A}$ and $\underline{B}$ are the $N \times N$ matrices of coefficients a_{pq} and b_{pq} , (which are dimensionally impedances) and $\underline{I}^e$ and $\underline{I}^0$ are the $N \times 1$ column vectors of amplitudes of currents on the elements.

From equations II-8 and II-9, it is seen that a_{pp} is always zero, and that b_{pp} is never zero, but depends on the element length. The physical reasons for this are intuitively obvious. Also, when all elements are parallel and have centers in the same plane, perpendicular to the axes, $a_{pq} = 0$ for all p and q .

Two special cases of II-15 exist:

1. If $\underline{I}^0 = \underline{0}$, then

$$\underline{\Phi} = \underline{A} \underline{I}^e. \quad \text{II-16}$$

Furthermore, arrays with geometries for which $\det(\underline{A})$ is zero lead to even array currents and zero scalar potentials at the element centers.

2. If $\underline{\Phi} = \underline{0}$, then

$$\underline{I}^0 = -\underline{B}^{-1} \underline{A} \underline{I}^e, \quad \text{II-17}$$

so that $\underline{I}^0$ may be calculated when $\underline{I}^e$ is known.

The general problem would be theoretically solved by forming arrays $\underline{A}$ and $\underline{B}$, determined only by array geometry and choice of directions of reference current, and specifying N elements of $\underline{\Phi}$, $\underline{I}^e$ or a suitable combination thereof.

CHAPTER III

RADIATION ZONE PATTERNS OF AN ARBITRARY ARRAY OF DIPOLES

In this chapter the far-zone electric field patterns for an array of N elements with arbitrary locations and orientations are derived. In Figure 2, page 18, the reference element, 1, is lying along the z -axis of the unprimed coordinate system, its center located at the origin. Element j is located by specifying the location of its center (X_j, Y_j, Z_j) and the angles θ_j, ϕ_j between the reference axis and the axis of element j . Element j has a length $2h_j$.

The vector potential due to element j is

$$\bar{A}_j(r, \theta, \phi) = \frac{\mu}{4\pi} \int_{-h_j}^{h_j} I_j(z') \frac{e^{-jkR_j}}{R_j} \hat{u}_j dz', \quad \text{III-1}$$

where R_j is the distance from the source point to the field point, and $\hat{u}_j$ is a unit vector in the direction of the z -axis of element j .

The electric field is found from

$$\bar{E}_j = -j\omega \bar{A}_j - \{\nabla \cdot \bar{A}_j\} \hat{u}_j. \quad \text{III-2}$$

The radial vector $\bar{r}$ is

$$\bar{r} = x \hat{x} + y \hat{y} + z \hat{z}. \quad \text{III-3}$$

From the geometry of Figure 2, page 18, it is seen that $\bar{r}_j$ is

$$\bar{r}_j = \hat{x}[X_j + z' \sin \theta_j \cos \phi_j] + \hat{y}[Y_j + z' \sin \theta_j \sin \phi_j] + \hat{z}[Z_j + z' \cos \theta_j] . \quad \text{III-4}$$

The scalar $|\bar{R}_j|$, which is needed in equation III-1 is

$$|\bar{R}_j| = |\bar{r} - \bar{r}_j| , \quad \text{III-5}$$

and $\bar{r}_j$ is given in III-4, so

$$\begin{aligned} R_j^2 = & [x^2 - 2x(X_j + z' \sin \theta_j \cos \phi_j) + (X_j + z' \sin \theta_j \cos \phi_j)^2 \\ & + y^2 - 2y(Y_j + z' \sin \theta_j \sin \phi_j) + (Y_j + z' \sin \theta_j \sin \phi_j)^2 \\ & + z^2 - 2z(Z_j + z' \cos \theta_j) + (Z_j + z' \cos \theta_j)^2]^{1/2} . \quad \text{III-6} \end{aligned}$$

In the spherical coordinate system of the reference element

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

III-7

and

$$r^2 = x^2 + y^2 + z^2 .$$

Using III-6 and III-7, R_j may be written as

$$\begin{aligned}
 R_j = & r \left[1 - \frac{2 \sin \theta \cos \phi}{r} (X_j + z' \sin \theta_j \cos \phi_j) + \frac{1}{r^2} (X_j + z' \sin \theta_j \cos \phi_j)^2 \right. \\
 & - \frac{2 \sin \theta \sin \phi}{r} (Y_j + z' \sin \theta_j \sin \phi_j) + \frac{1}{r^2} (Y_j + z' \sin \theta_j \sin \phi_j)^2 \\
 & \left. - \frac{2 \cos \theta}{r} (Z_j + z' \cos \theta_j) + \frac{1}{r^2} (Z_j + z' \cos \theta_j)^2 \right]^{1/2}. \quad \text{III-8}
 \end{aligned}$$

Now in the radiation zone $|\bar{r}| \gg |\bar{r}_j|$, so the terms containing r^{-2} in the brackets are much smaller than the other terms. Neglecting the second and higher order terms in r ,

$$\begin{aligned}
 R_j = & r \left[1 - \frac{2 \sin \theta \cos \phi}{r} (X_j + z' \sin \theta_j \cos \phi_j) \right. \\
 & - \frac{2 \sin \theta \sin \phi}{r} (Y_j + z' \sin \theta_j \sin \phi_j) \\
 & \left. - \frac{2 \cos \theta}{r} (Z_j + z' \cos \theta_j) \right]^{1/2}. \quad \text{III-9}
 \end{aligned}$$

The bracket of equation III-9 is now expanded using the binomial expansion, neglecting terms of order r^{-n} , $n \geq 2$. The reciprocal of the bracket is expanded in the same manner. Finally the approximations are obtained that

$$\begin{aligned}
 R_j \approx & r - [\sin \theta \cos \phi (X_j + z' \sin \theta_j \cos \phi_j) + \\
 & + \sin \theta \sin \phi (Y_j + z' \sin \theta_j \sin \phi_j) + \cos \theta (Z_j + z' \cos \theta_j)], \quad \text{III-10}
 \end{aligned}$$

and

$$R_j^{-1} \approx r^{-1} . \quad \text{III-11}$$

The complete far-zone vector potential is calculated by applying equation III-10 for each element and summing the results.

Write the unit vector $\hat{u}_j$ as

$$\hat{u}_j = \sin \theta_j \cos \phi_j \hat{x} + \sin \theta_j \sin \phi_j \hat{y} + \cos \theta_j \hat{z} . \quad \text{III-12}$$

The vector potential at any point r is

$$\bar{A}(r) = \frac{\mu}{4\pi} \sum_{j=1}^N \int_{-h_j}^{h_j} I_j(z') \frac{e^{-jkR_j}}{R_j} \hat{u}_j dz' , \quad \text{III-13}$$

and using III-10 and III-12

$$\begin{aligned} \bar{A}(r) = \frac{\mu}{4\pi} \frac{e^{-jkr}}{r} \sum_{j=1}^N [\sin \theta_j \cos \phi_j \hat{x} + \sin \theta_j \sin \phi_j \hat{y} \\ + \cos \theta_j \hat{z}] \int_{-h_j}^{h_j} I_j(z') e^{jk[T_j]} dz' , \end{aligned} \quad \text{III-14}$$

where

$$\begin{aligned} T_j = \sin \theta \cos \phi (X_j + z' \sin \theta_j \cos \phi_j) + \sin \theta \sin \phi (Y_j + z' \sin \theta_j \sin \phi_j) \\ + \cos \theta (Z_j + z' \cos \theta_j) . \end{aligned} \quad \text{III-15}$$

The potential $\bar{A}(r)$ can thus be written as

$$\bar{A}(r) = \frac{\nu}{4\pi} \frac{e^{-jkr}}{r} \sum_{j=1}^N \bar{S}_j, \quad \text{III-16}$$

where $\bar{S}_j$ is the argument of the summation sign.

The electric field may now be calculated from III-16. In the far-zone there are only two spherical components of electric field, E_θ and E_ϕ .

From III-2 and the conversion from spherical to cartesian coordinates

$$E_\theta = \cos\theta \cos\phi E_x + \cos\theta \sin\phi E_y - \sin\theta E_z, \quad \text{III-17}$$

and

$$E_\phi = \cos\theta E_y - \sin\theta E_x. \quad \text{III-18}$$

Finally the two components may be expressed as

$$E_\theta = -\frac{j\omega\mu}{4\pi} \frac{e^{-jkr}}{r} \sum_{j=1}^N [\cos\theta \cos\phi S_{jx} + \sin\phi \cos\theta S_{jy} - \sin\theta S_{jz}], \quad \text{III-19}$$

and

$$E_\phi = -\frac{j\omega\mu}{4\pi} \frac{e^{-jkr}}{r} \sum_{j=1}^N [\cos\phi S_{jy} - \sin\phi S_{jx}]. \quad \text{III-20}$$

These last equations give the two far-zone components of electric field at any angle θ and ϕ . Define the principal planes as being the x-y plane, x-z plane and y-z plane.

If III-19 and III-20 are normalized by the factor appearing in front of the summation signs, and F_θ and F_ϕ are called the pattern factors, then in the x-y plane $\theta = \pi/2$, and

$$F_\theta (\theta = \pi/2) = - \sum_{j=1}^N \int_{-h_j}^{h_j} I_j(z') \cos \theta_j e^{jk[T_j]} dz' \quad \text{III-21}$$

$$= - \sum_{j=1}^N S_{jz} . \quad \text{III-22}$$

$$F_\phi (\theta = \pi/2) = \sum_{j=1}^N [\cos \phi S_{jy} - \sin \phi S_{jx}] . \quad \text{III-23}$$

For the x-z plane $\phi = 0$ and the pattern factors are

$$F_\theta (\phi = 0) = \sum_{j=1}^N [\cos \theta S_{jx} - \sin \theta S_{jz}] , \quad \text{III-24}$$

$$F_\phi (\phi = 0) = \sum_{j=1}^N [S_{jz}] . \quad \text{III-25}$$

For the y-z plane $\phi = \frac{\pi}{2}$, and the pattern factors are

$$F_\theta (\phi = \frac{\pi}{2}) = \sum_{j=1}^N [\cos \theta S_{jy} - \sin \theta S_{jz}] , \quad \text{III-26}$$

$$F_\phi (\phi = \frac{\pi}{2}) = \sum_{j=1}^N [S_{jx}] . \quad \text{III-27}$$

Appendix A contains a computer program which will calculate the far-field patterns of an arbitrary array of dipoles with sinusoidal currents in any three planes one chooses to specify.

It is worth while to examine the far-field of a single element with even and odd zeroth-order currents.

Assuming the element to be of length $2h$, centered at the origin and lying along the z -axis, it is easily shown that the far zone electric field is of the form

$$E_{\theta}^e = j60 I^e \frac{e^{-jkr}}{r} \left[\frac{\cos(kh \cos \theta) - \cos kh}{\sin \theta} \right] , \quad \text{III-28}$$

due to the even current, which reduces, for $kh = \frac{\pi}{2}$ to

$$E_{\theta}^e (kh = \frac{\pi}{2}) = j60 I^e \frac{e^{-jkr}}{r} \left[\frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} \right] . \quad \text{III-29}$$

Likewise for the odd current, one obtains

$$E_{\theta}^o = 60 I^o \frac{e^{-jkr}}{r} \left[\frac{\sin kh(\sin(\frac{\pi}{2} \cos \theta) - \cos \theta) - \cos kh \cos \theta \cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} \right] . \quad \text{III-30}$$

For $kh = \pi/2$, this reduces to

$$E_{\theta}^o = 60 I^o \frac{e^{-jkr}}{r} \left[\frac{\sin(\frac{\pi}{2} \cos \theta) - \cos \theta}{\sin \theta} \right] . \quad \text{III-31}$$

Notice that E_{θ}^o is always zero at $\theta = 0$, regardless of kh , and since it is directly proportional to I^o , will usually be much

smaller than E_{θ}^e . The total field, however, is the sum of the two, and since I^0 is complex, the sum cannot be obtained by direct inspection.

CHAPTER IV

APPLICATIONS TO SEVERAL ARRAYS

In the preceding chapters, the solution to the arbitrary array of thin elements was formulated. In this chapter, several specific cases are examined. The results for tangential circular arrays are compared to the exact results obtained by Hilburn and Hickman using the Gaussian Quadrature technique.

A. Tangential Circular Arrays

Figure 5 shows an N element tangential circular array of center-fed dipoles equally spaced around the circumference of a circle of radius ρ . One of the properties of the circular array (8) is that if the terminal voltages are equal in magnitude and the phase differences between successive elements is $2\pi n/N$, where n is any integer from 0 to $N-1$ (called the sequence number) all of the elements will have the same impedance. It is, then, possible to excite the array such that all currents have the same form.

Because of the symmetry of the system the $\underline{A}$ and $\underline{B}$ matrices assume the form

$$\underline{A} = \begin{bmatrix} 0 & a_{12} & a_{13} & \dots & -a_{12} \\ -a_{12} & 0 & a_{12} & a_{13} & \dots & -a_{13} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{12} & a_{13} & \dots & \dots & 0 \end{bmatrix} \quad \text{IV-1}$$

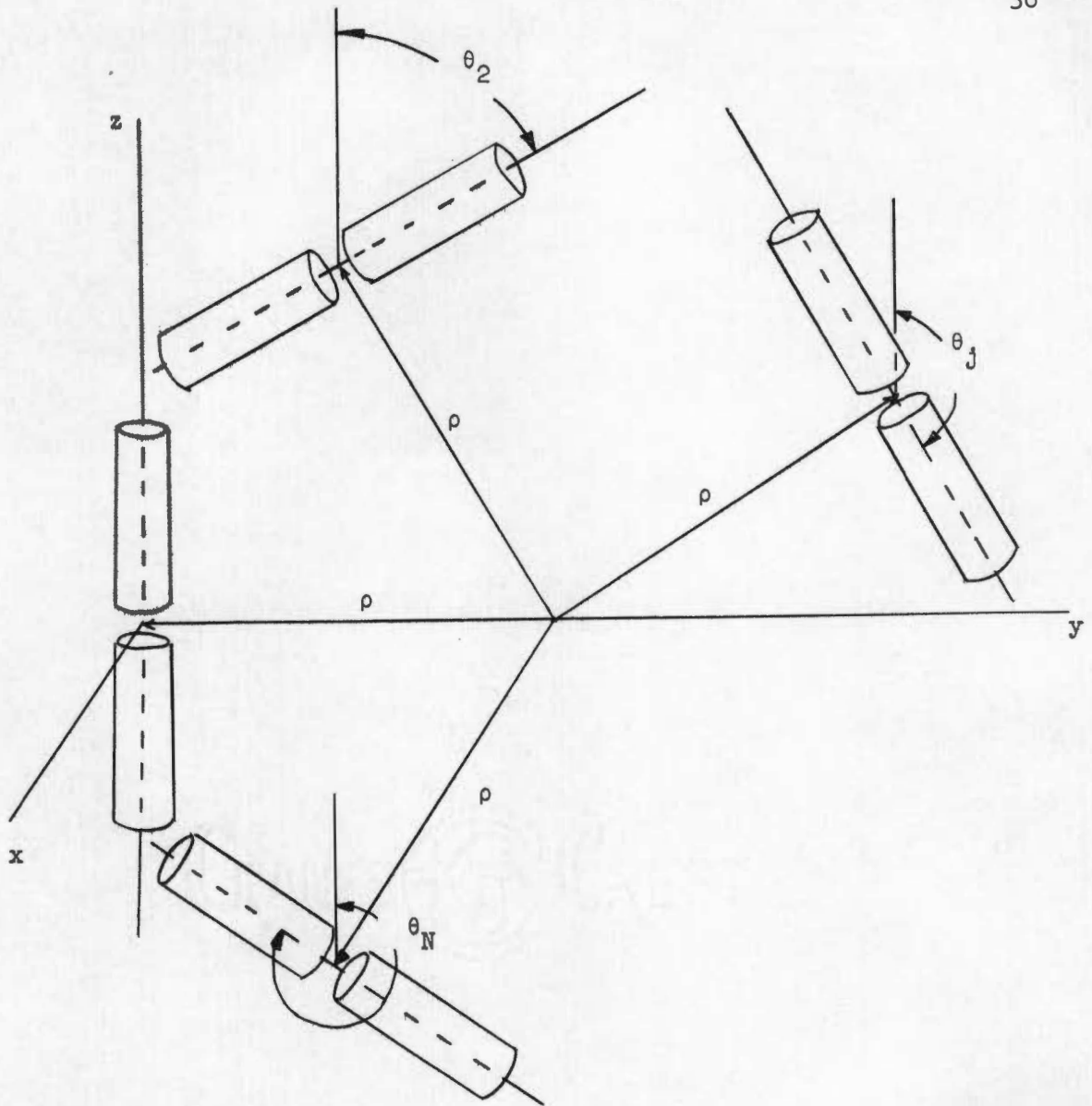


Figure 5. View of Single-Ring Tangential Circular Array.

and

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & \dots & b_{12} \\ b_{12} & b_{11} & b_{12} & b_{13} & \dots & b_{13} \\ & & \dots & & & \\ b_{12} & b_{13} & b_{14} & \dots & b_{11} \end{bmatrix} \quad \text{IV-2}$$

Matrices of this type possess unique properties and have been called "circulant" matrices (9). The eigenvalues and modal matrix may be written almost by inspection.

It is possible to solve the circular array case by considering any general driving sequence as a linear combination of the normal modes of excitation, which may be easily determined. For these cases no matrix inversion is necessary since the odd currents are proportional to the even currents.

The "n" sequence driving current used has the form

$$I^e(n) = I^{(n)} \begin{bmatrix} e^{j2\pi n/N} \\ e^{j2\pi n(2)/N} \\ \vdots \\ e^{j2\pi(N-1)/N} \end{bmatrix} \quad \text{IV-3}$$

If $\lambda_A^{(n)}$ and $\lambda_B^{(n)}$ are the associated eigenvalues of the A and B matrices respectively, then for each sequence (and $\phi = 0$)

$$\lambda_B^{(n)} I^{0(n)} + \lambda_A^{(n)} I^{e(n)} = 0, \quad \text{IV-4}$$

so

$$I^{0(n)} = - \frac{\lambda_A^{(n)}}{\lambda_B^{(n)}} I^{e(n)}. \quad \text{IV-5}$$

Hence it is necessary only to determine the eigenvalues of A and B, which may be easily done.

The three and four element circular arrays will be considered simultaneously. The A and B matrices for these two cases are as follows:

For $N = 3$,

$$A = \begin{bmatrix} 0 & a & -a \\ -a & 0 & a \\ a & -a & 0 \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{12} \\ b_{12} & b_{11} & b_{12} \\ b_{12} & b_{12} & b_{11} \end{bmatrix} \quad \text{IV-6}$$

and for $N = 4$,

$$A = \begin{bmatrix} 0 & a & 0 & -a \\ -a & 0 & a & 0 \\ 0 & -a & 0 & a \\ a & 0 & -a & 0 \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{12} \\ b_{12} & b_{11} & b_{12} & b_{13} \\ b_{13} & b_{12} & b_{11} & b_{12} \\ b_{12} & b_{13} & b_{12} & b_{11} \end{bmatrix}. \quad \text{IV-7}$$

The driving current sequences for the three element array are

$$I^e(0) = I^{(0)} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad I^e(1) = I^{(1)} \begin{bmatrix} 1 \\ e^{i\pi/3} \\ e^{-i\pi/3} \end{bmatrix}, \quad I^e(2) = I^{(2)} \begin{bmatrix} 1 \\ e^{-i\pi/3} \\ e^{+i\pi/3} \end{bmatrix}. \quad \text{IV-8}$$

For the four element array they are

$$I^e(0) = I^{(0)} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad I^e(1) = I^{(1)} \begin{bmatrix} 1 \\ -j \\ -1 \\ +j \end{bmatrix}, \quad I^e(2) = I^{(2)} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad I^e(3) = I^{(3)} \begin{bmatrix} 1 \\ j \\ -1 \\ -j \end{bmatrix}. \quad \text{IV-9}$$

The eigenvalues of A and B for $N = 3$ are respectively, 0 , $ja\sqrt{3}$, $-ja\sqrt{3}$ and $b_{11} + 2b_{12}$, $b_{11} - b_{12}$, $b_{11} - b_{12}$. These yield one solution (the zero sequence) which has no odd current, and two solutions ($n = 1, 2$) which are complex conjugates. For $n = 1$,

$$I^{0(1)} = I^{e(1)} \left(\frac{-ja\sqrt{3}}{b_{11} - b_{12}} \right) \begin{bmatrix} 1 \\ e^{i\pi/3} \\ e^{-i\pi/3} \end{bmatrix}. \quad \text{IV-10}$$

For example, the ratio of odd to even current on each element for $k\rho = 1.27$ is

$$\frac{I^{0(1)}}{I^{e(1)}} = 0.264 \angle -135.5^\circ,$$

so the odd component is about one-fourth that of the even component in magnitude.

The fact that the $N = 0$ sequence yields no odd current is obvious from physical considerations.

For $N = 4$ the eigenvalues of $\underline{A}$ and $\underline{B}$ are respectively 0, 0, $+2ja$, $-2ja$, and $b_{11} + 2b_{12} + b_{13}$, $b_{11} - 2b_{12} + jb_{13}$, $b_{11} - b_{13}$, $b_{11} - b_{13}$. Only two sequences give rise to non-zero odd currents, as is obvious from physical considerations, and the other two sequences are complex conjugates. For $n = 1$,

$$I^{0(1)} = I^{e(1)} \left(\frac{-2ja}{b_{11} - b_{13}} \right) \begin{bmatrix} 1 \\ -j \\ -1 \\ +j \end{bmatrix} \quad \text{IV-11}$$

For $kp = 2.02$, the ratio of odd to even current on each element is

$$\frac{I^{0(1)}}{I^{e(1)}} = 0.312 \angle -140.9^\circ.$$

In order to compare these zeroth-order results with results obtained using a more exact analysis, three cases of $N = 3$ and $N = 4$ were analyzed. Each array had a different element length and array radius. Only the $N = 1$ sequences were calculated. The odd component of current was calculated, plotted as a function of distance along the element, and compared with the corresponding plots using Hilburn and

Hickman's (2) results. The lengths used were $h = 0.25\lambda$, $h = 0.30\lambda$ and $h = 0.50\lambda$. Figures 6 through 11 summarize the results.

The best results are obtained for the $h = 0.25\lambda$ dipoles, where the magnitudes of the currents are about the same. Some of the error may be attributed to the fact that a zeroth-order even current has a maximum at the terminals, while the actual current (for non-zero thickness) has a slight dip near the center. The errors in the angles seem to be significant, but again, most of this error (for $h = 0.25\lambda$, approximately eight degrees) is due to the fact that the Hilburn-Hickman evaluations were made for a thick antenna ($\Omega = 10$, while the zeroth-order calculations assumed very thin antennas, and is exact only as Ω becomes very large. The approximate theory indicates that the phase of the current on either half of the antenna is constant, while the actual current shows some phase variation with distance from the center, as is expected physically.

For $h = 0.3\lambda$, the results are still fairly good, although the phase angle errors are several degrees larger than for $h = 0.25\lambda$. The current distribution is still fairly well approximated by a sinusoid.

For $h = 0.5\lambda$, there are wide variations in both magnitude and phase between the two solutions. This is not surprising, however, since the zeroth-order solution is very poor for a full wave dipole.

B. Two Element Arrays of Dipoles

Figure 12 shows a two element array of parallel dipoles. The spacing between centers is d , and the angle subtended by the line

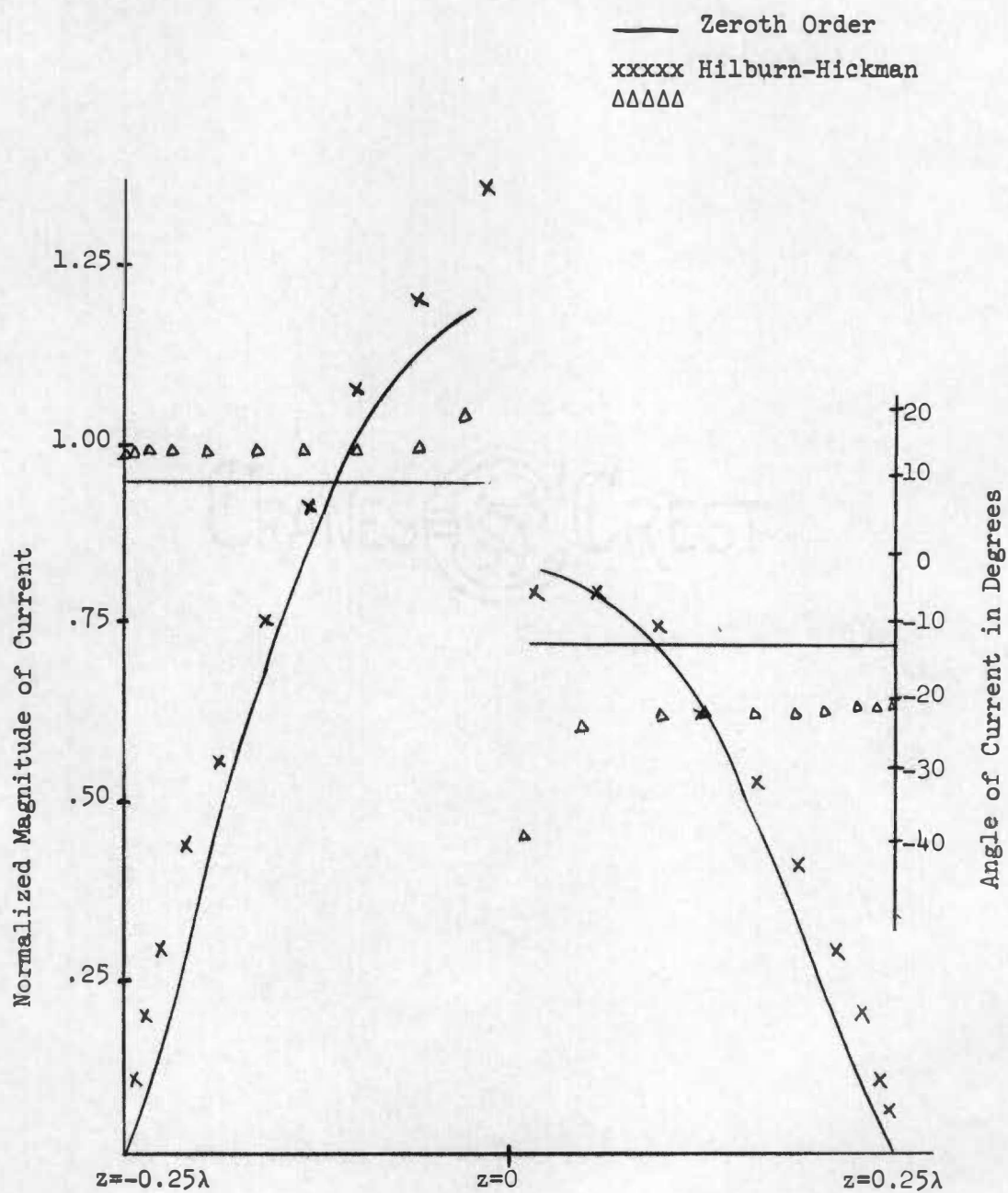


Figure 6. Theoretical and Zeroth-Order Current Distribution, 3 Element Circular Array, $h = 0.25\lambda$, $k\rho = 1.27$, 1 Sequence...

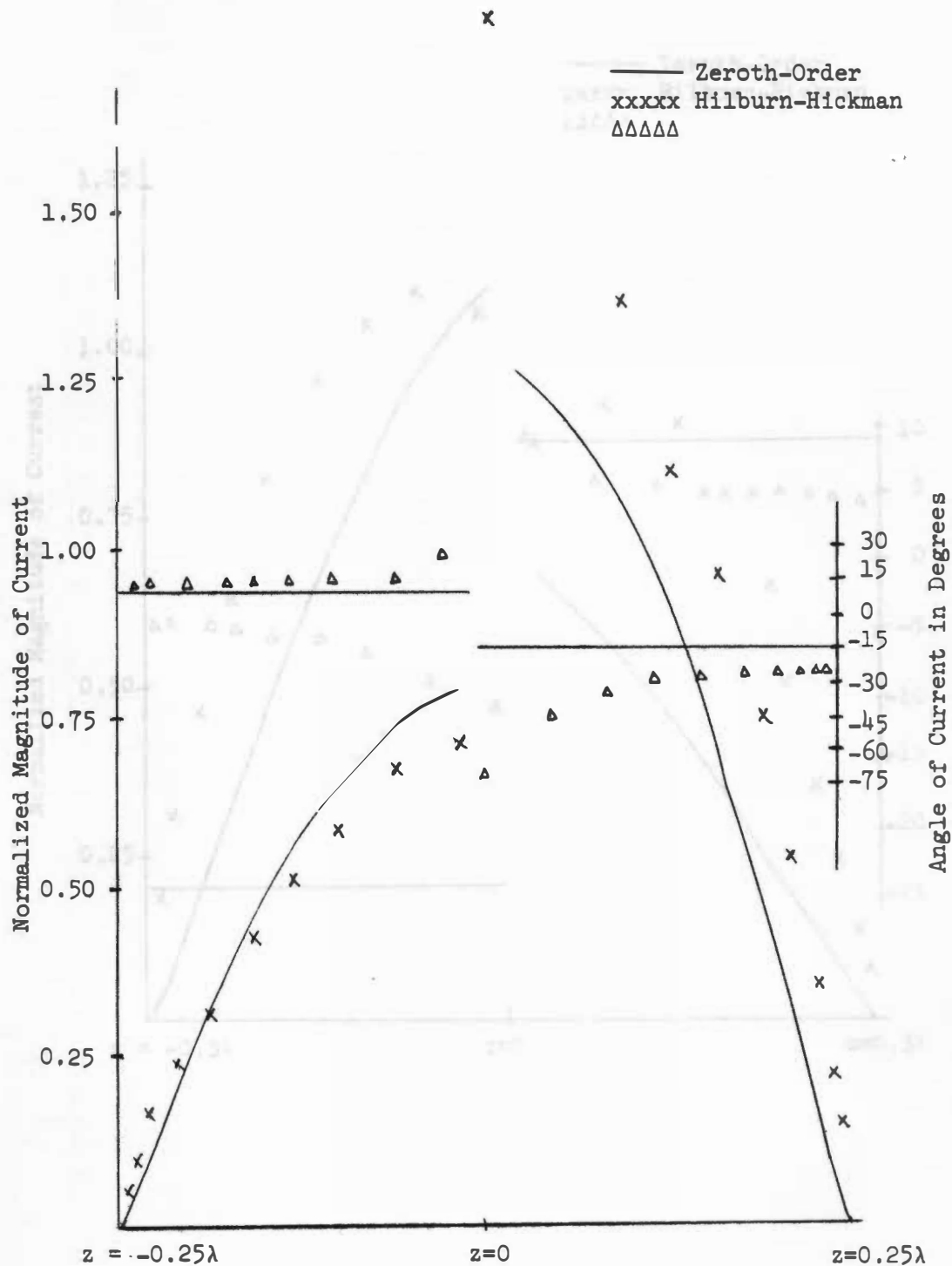


Figure 7. Theoretical and Zeroth-Order Current Distribution, 4 Element Circular Array, $h = 0.25\lambda$, $k\rho = 2.02$, 1 Sequence... .

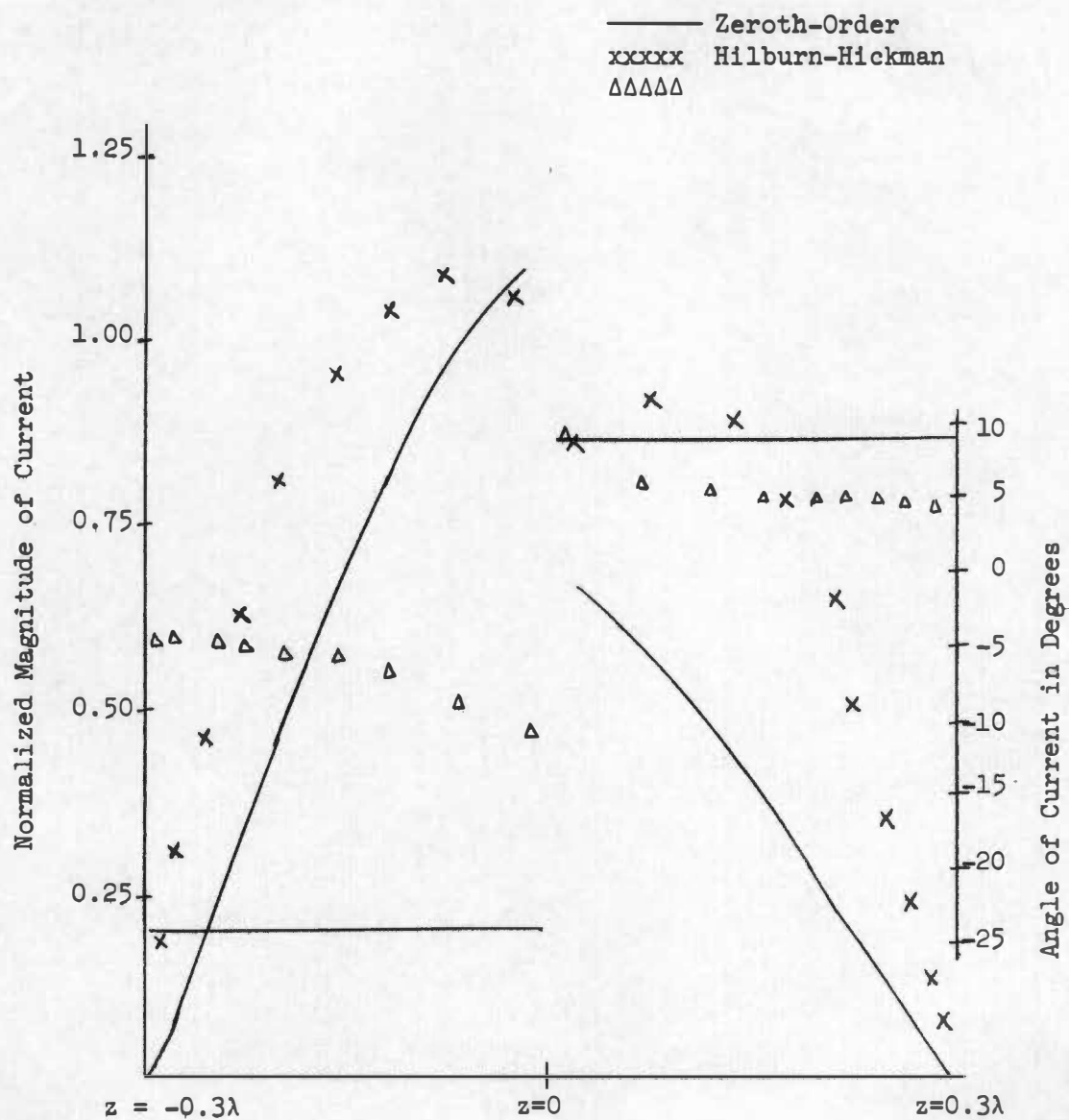


Figure 8. Theoretical and Zeroth-Order Current Distribution, 3 Element Circular Array, $h = 0.30\lambda$, $k_p = 1.45$, 1 Sequence...

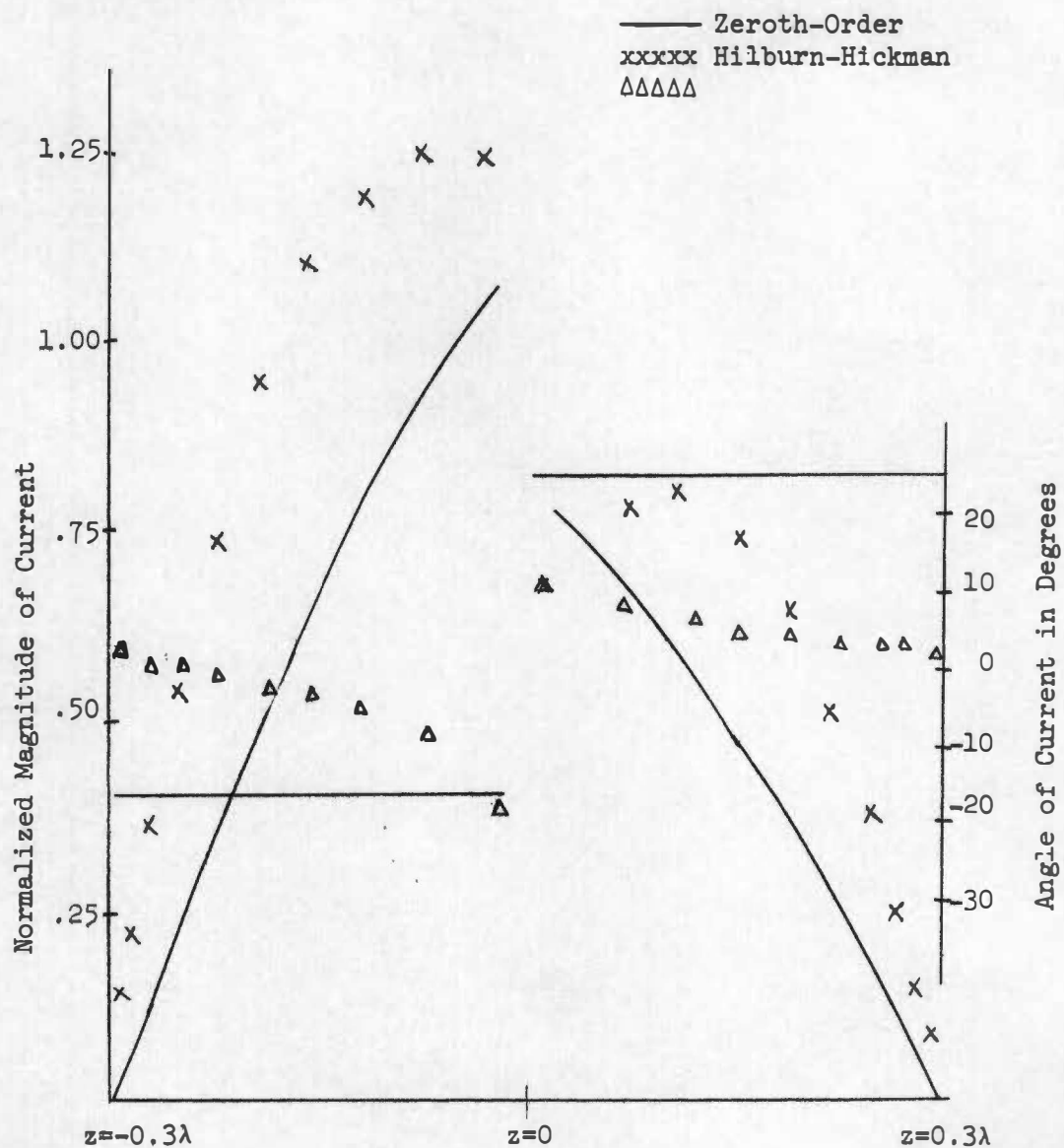


Figure 9. Theoretical and Zeroth-Order Current Distribution, 4 Element Circular Array, $h = 0.30\lambda$, $k_p = 2.33$, 1 Sequence...

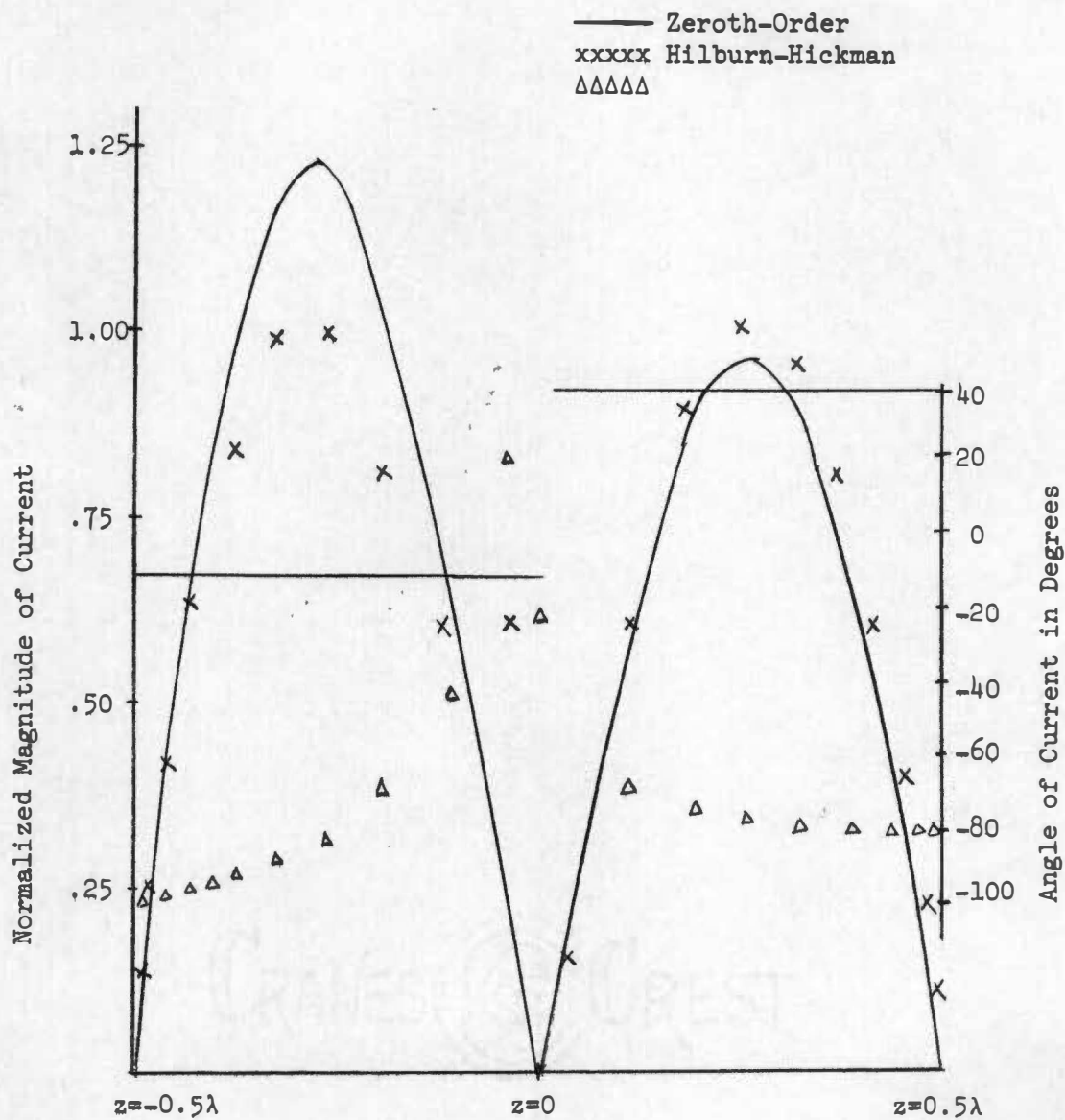


Figure 10. Theoretical and Zeroth-Order Current Distribution, 3 Element Circular Array, $h = 0.50\lambda$, $k\rho = 2.17$, 1 Sequence...

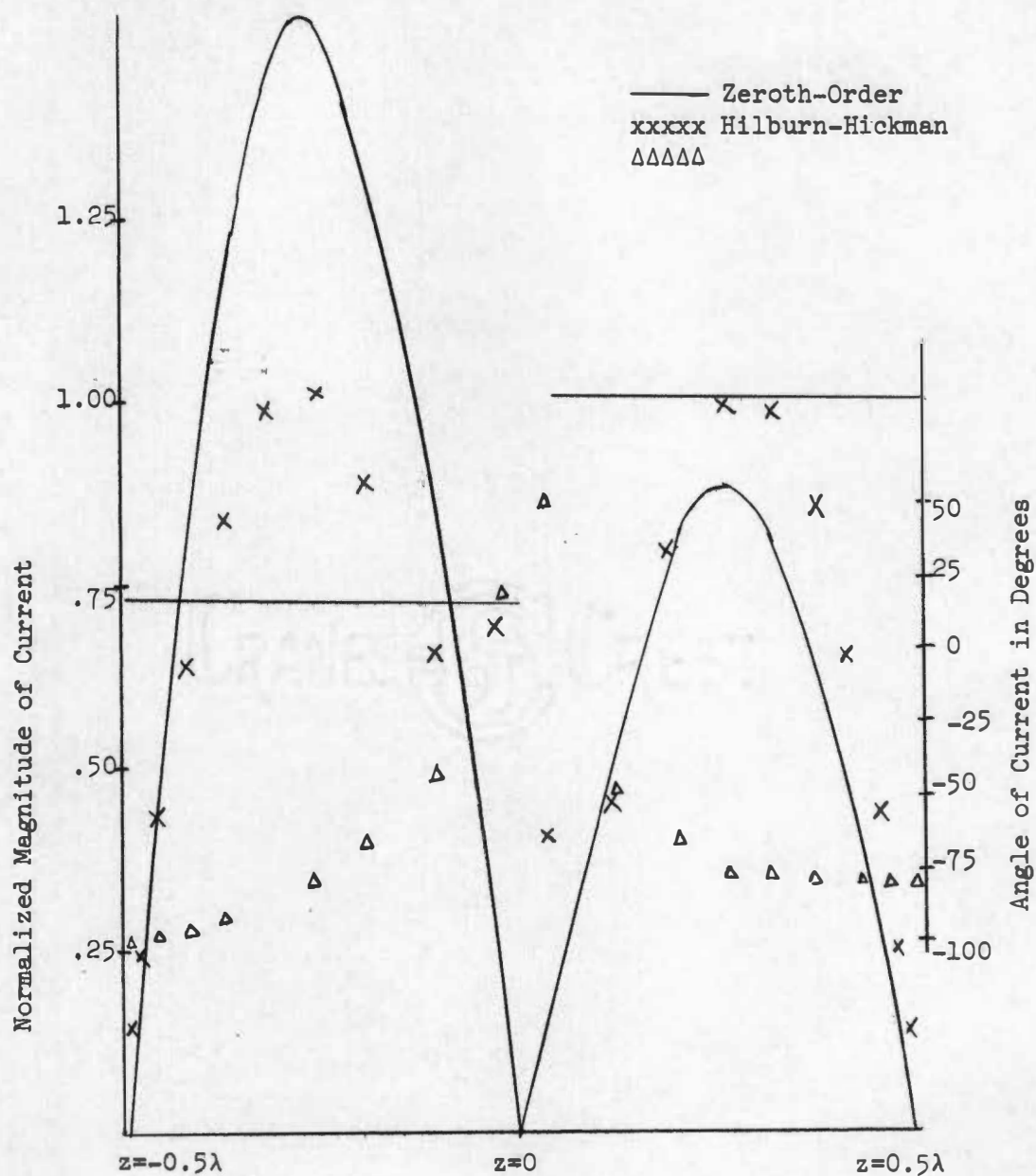


Figure 11. Theoretical and Zeroth-Order Current Distribution, 4 Element Circular Array, $h = 0.50\lambda$, $k\rho \approx 3.50$, 1 Sequence...

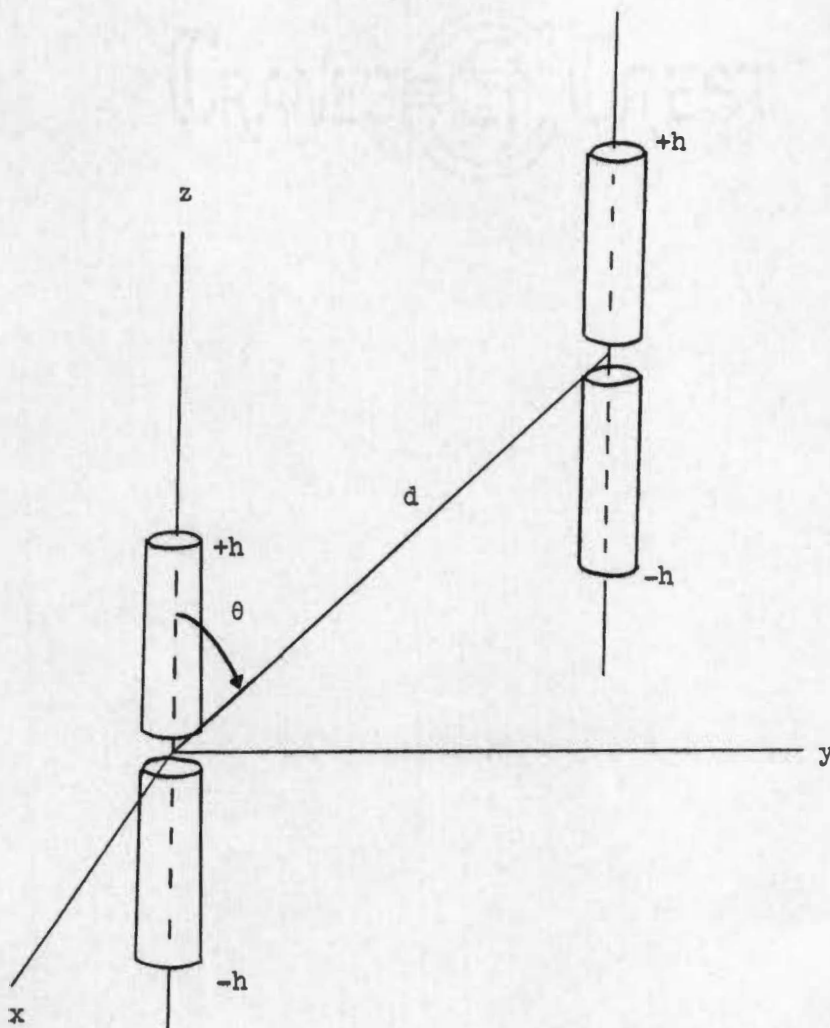


Figure 12. Two Element Array of Parallel Dipoles.

joining the centers and the axis of the reference element at the origin is the polar angle θ . When θ is zero radians the array is collinear, and when θ is $\pi/4$ radians the array is called a 45 degree echelon array. For the $N = 1$ sequence the currents are driven in phase and for the $N = 2$ sequence the currents are driven out of phase. For the $N = 1$ sequence the coefficient matrices assume the form

$$\underline{A}^{(1)} = \begin{bmatrix} 0 & a_{12} \\ a_{12} & 0 \end{bmatrix}, \quad \underline{B}^{(1)} = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{11} \end{bmatrix}. \quad \text{IV-12}$$

From equation II-17, the odd currents may be found by

$$\underline{I}^{0(1)} = - \underline{B}^{-1} \underline{A} \underline{I}^{e(1)}, \quad \text{IV-13}$$

$$\underline{I}^{0(1)} = - \frac{1}{b_{11}^2 - b_{12}^2} \begin{bmatrix} b_{11} & -b_{12} \\ -b_{12} & b_{11} \end{bmatrix} \begin{bmatrix} 0 & a_{12} \\ a_{12} & 0 \end{bmatrix} \begin{bmatrix} I^{e(1)} \\ I^{e(1)} \end{bmatrix}, \quad \text{IV-14}$$

$$\underline{I}^{0(1)} = - \frac{a_{12}}{b_{11} + b_{12}} \underline{I}^{e(1)}. \quad \text{IV-15}$$

Similarly, for the $N = 2$ sequence

$$\underline{I}^{0(2)} = \frac{a_{12}}{b_{11} - b_{12}} \begin{bmatrix} I^{e(2)} \\ -I^{e(2)} \end{bmatrix}. \quad \text{IV-16}$$

Identical results obtained for the two element array whose geometry is pictured in Figure 13. For this case θ is the angle

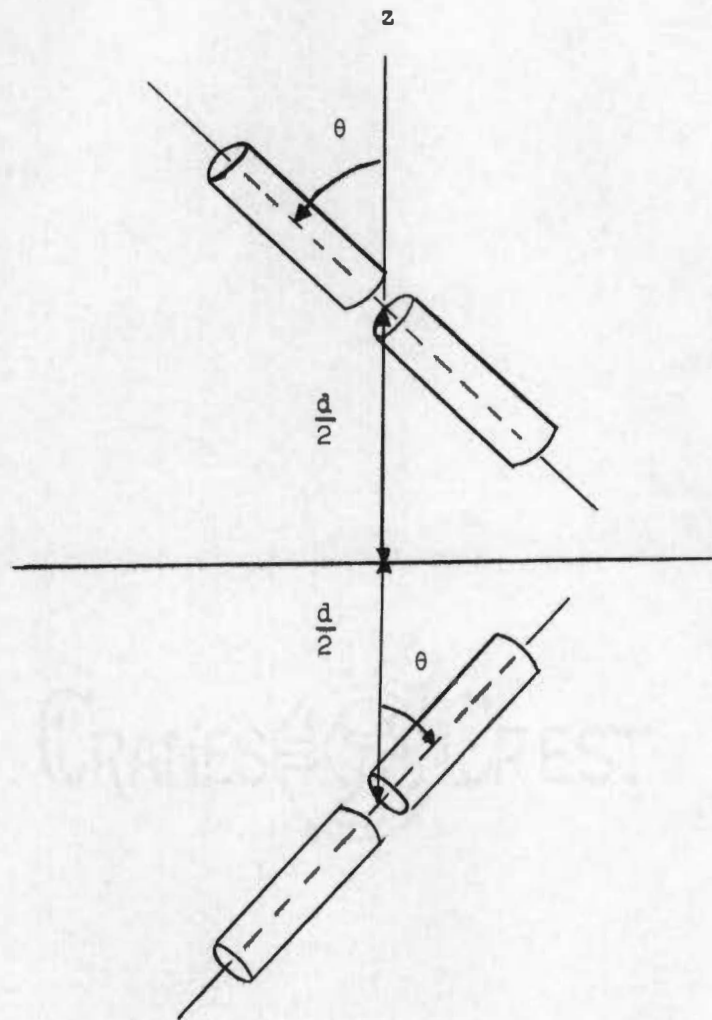


Figure 13. Two Element Planar Array of Dipoles.

between the polar axis and the axes of the two elements, and θ is the same for both elements. A special case of this array is a dipole located at height $d/2$ above a perfectly conducting infinite ground plane, in which case one element is the image of the real element, and only the $N = 2$ sequence is possible.

It is expected from physical considerations of both figures that the odd components of current should be decreasing functions of θ and d . By numerical calculations for the $N = 2$ sequence these expectations are confirmed (for $h = \frac{\lambda}{4}$) as shown in Figure 14, in which the magnitude of the ratio of the odd to even current is plotted as a function of the angle θ for several values of d , ranging from one-half wavelength to four wavelengths. To an excellent degree of approximation (better than 94 per cent at worst) these curves are cosinusoidal in form as a function of θ . An approximate empirical formula which describes the variation for both sequences is

$$\left| \frac{I_o}{I_e} \right| = \frac{0.117\lambda \cos \theta}{d}, \quad \text{IV-17}$$

where λ is the wavelength and d is expressed in the same units as λ . (The $n = 1$ sequence plots are similar in form to the $N = 2$ sequence plots.)

Calculations of the angle of the complex ratio I_o/I_e versus θ and d yield results which are somewhat more complicated. These are shown for the $n = 2$ sequence in Figure 15. The results for $n = 1$

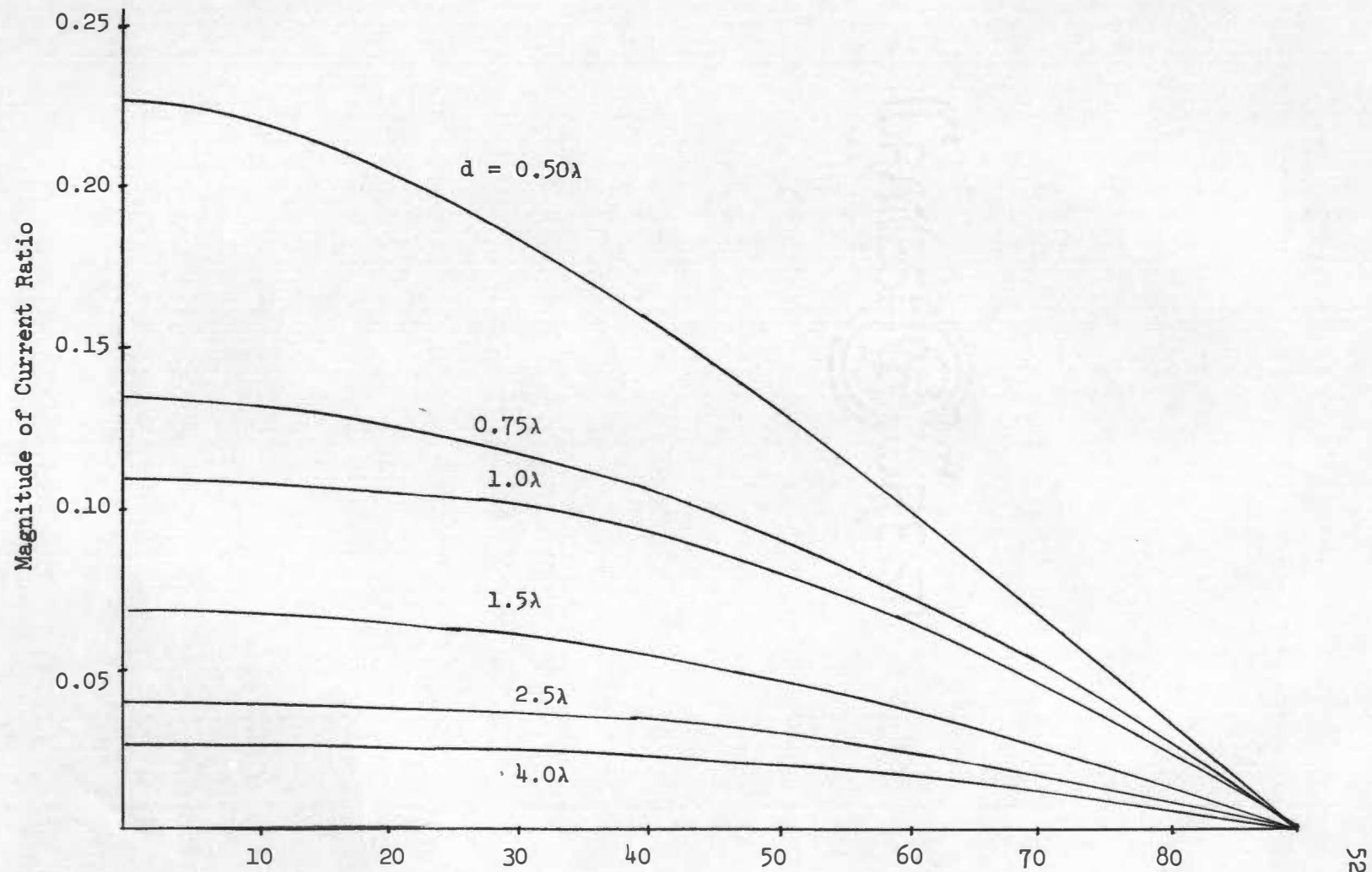


Figure 14. Magnitude of Odd Current Ratio in Two Element Parallel Array, Angle and Spacing Varying.

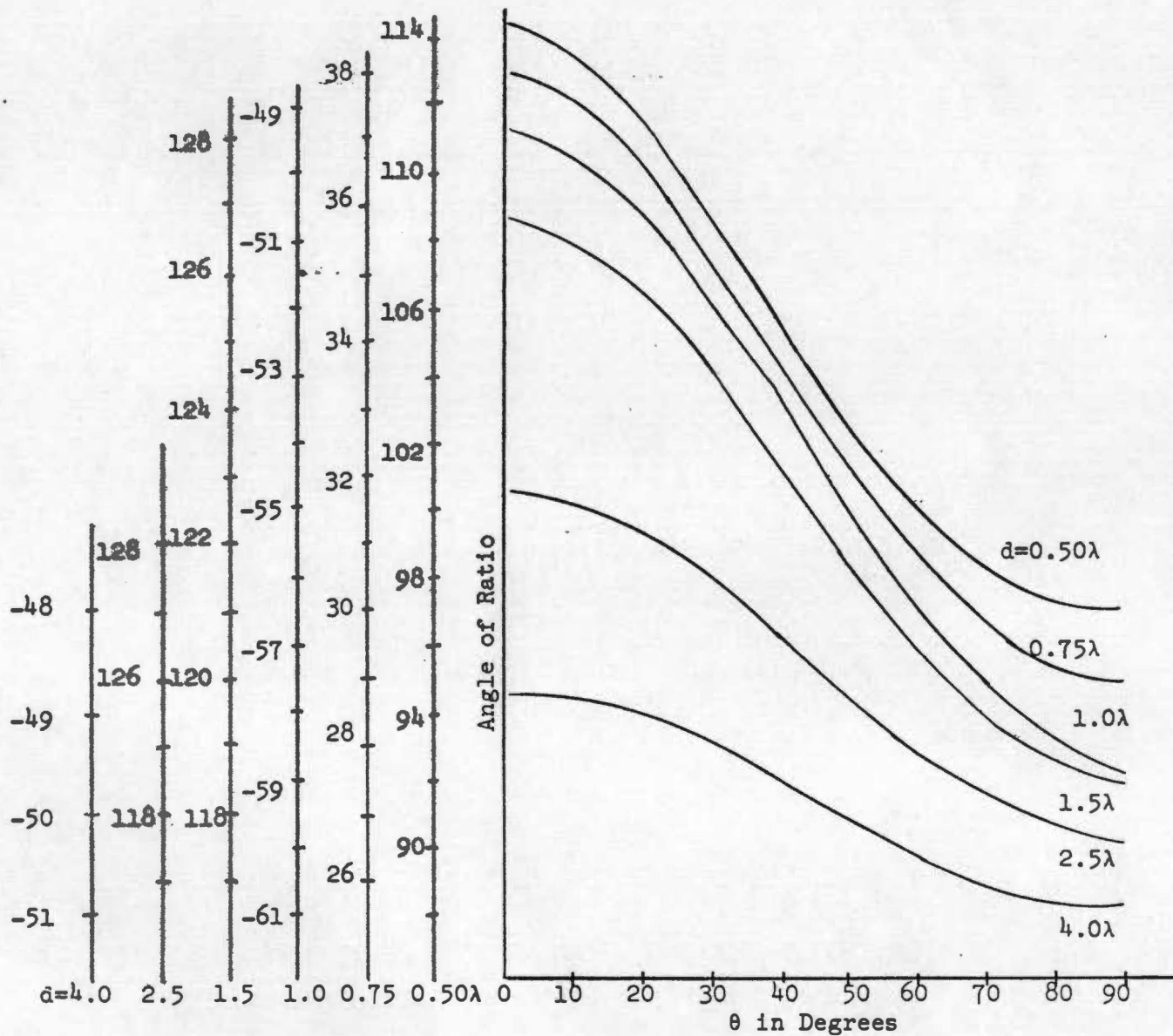


Figure 15. Phase of Odd Current Ratio in Two Element Parallel Array, Angle and Spacing Varying.

are similar in form, but differing in angle by as much as 15 degrees for $d = \frac{\lambda}{2}$ and only 3 degrees for $d = 4\lambda$. The results are plotted as the angle of the ratio I^0/I^e versus θ , for varying d . Since the angular variation of the ratio decreases with d , it is necessary to use a different scale for each spacing. These are labeled in the figure.

An approximate empirical expression for the angular variation of the ratio is

$$\angle \frac{I^0}{I^e} = [(311 - \frac{360d}{\lambda}) - \theta\lambda/9d], \quad \text{IV-18}$$

where θ is expressed in degrees and λ and d are the same as before. There is about 17 degrees error in this expression between the $n = 1$ and $n = 2$ sequences at $d = \lambda/2$ and about 2 degrees at $d = 4\lambda$.

Table I summarizes the results obtained from examination of several cases. The complex ratio of I^0/I^e is given for the $n = 1$ and the $n = 2$ sequences for three collinear arrays with zero tip-to-tip spacing, and three 45° echelon arrays in which the spacing is $\sqrt{2} \lambda/4$. The half-lengths are 0.25, 0.30, and 0.50 wavelengths respectively.

C. Comments on an Array Consisting of

Two Pairs of Crossed Dipoles

Figure 16 shows an array of elements consisting of pairs of crossed dipoles. Crossed dipole arrays are frequently used to obtain

TABLE I
 RATIO OF ODD TO EVEN CURRENTS FOR COLLINEAR
 AND 45° ECHELON TWO-ELEMENT ARRAYS FOR
 THREE DIFFERENT HALF-LENGTHS

Case		n = 1 Sequence	n = 2 Sequence
Collinear	$h = 0.25\lambda$	-0.245 $\angle -47.6^\circ$	-0.224 $\angle -65.4$
	$h = 0.30\lambda$	-0.268 $\angle -73.6^\circ$	-0.255 $\angle -85.5$
	$h = 0.50\lambda$	-0.113 $\angle -3.45^\circ$	-0.661 $\angle -4.7^\circ$
Echelon	$h = 0.25\lambda$	-0.211 $\angle -8.4^\circ$	-0.238 $\angle -41^\circ$
	$h = 0.30\lambda$	0.288 $\angle 4.4^\circ$	0.345 $\angle -35.5^\circ$
	$h = 0.50\lambda$	0.615 $\angle -81.8^\circ$	0.423 $\angle 41.9^\circ$

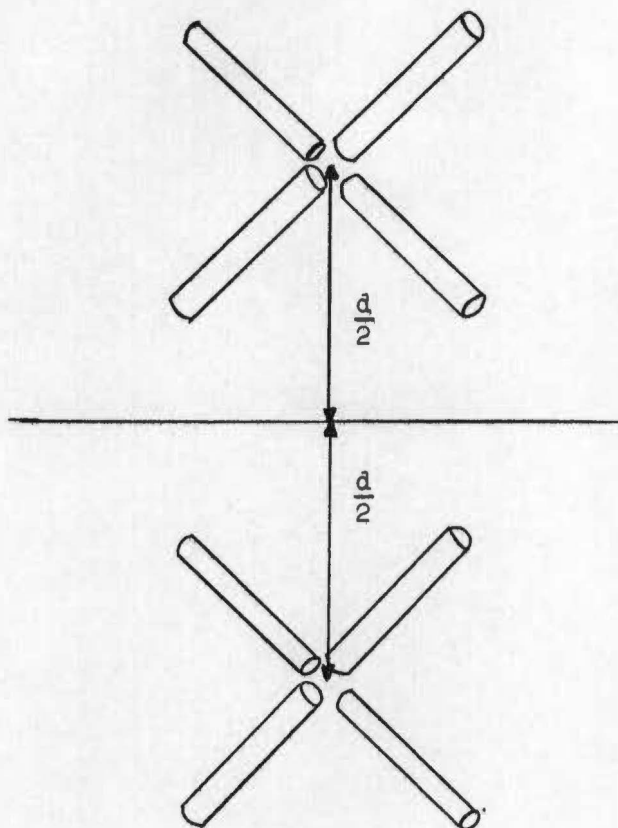


Figure 16. Array of Planar Crossed Dipoles.

circular polarization in a transmitted signal. Equation II-15 assumes the following form when applied to the geometry of Figure 16.

$$\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & a_{13} & a_{14} \\ 0 & 0 & a_{13} & a_{14} \\ -a_{13} & -a_{14} & 0 & 0 \\ -a_{13} & -a_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1^e \\ I_2^e \\ I_3^e \\ I_4^e \end{bmatrix} + \begin{bmatrix} b_{11} & b_{11} & b_{13} & b_{14} \\ b_{11} & b_{11} & b_{13} & b_{14} \\ b_{13} & b_{14} & b_{11} & b_{11} \\ b_{13} & b_{14} & b_{11} & b_{11} \end{bmatrix} \begin{bmatrix} I_1^0 \\ I_2^0 \\ I_3^0 \\ I_4^0 \end{bmatrix}$$

IV-19

Degeneration occurs because the centers of elements 1 and 2, and 3 and 4 are at the same points in space, hence $\phi_1 = \phi_2$, and $\phi_3 = \phi_4$. In other words, the rank of the A and B matrices is 2. If ϕ_1 and ϕ_3 are set equal to zero, and one attempts to obtain circular polarization by setting $I_2^e = j I_1^e$ and $I_3^e = I_1^e$, then it can be shown that $I_2^0 = I_1^0$, and $I_4^0 = I_3^0$, where

$$\begin{bmatrix} I_1^0 \\ I_3^0 \end{bmatrix} = (a_{13} + ja_{14}) I_1^e \begin{bmatrix} \frac{1}{2b_{11} + (b_{13} + b_{14})} \\ \frac{1}{2b_{11} - (b_{13} + b_{14})} \end{bmatrix}$$

IV-20

The important point is that perfect circular polarization of the radiating system is destroyed because of the in-phase odd components of current on 1, 2 and 3,4.

D. The Three Element Collinear Array of Half-Wave Dipoles

Consider an array of three collinear elements lying along the z-axis. Defining the lower element as 1, the center element as 2, and the upper as 3, the coefficient matrices assume the form

$$\underline{A} = \begin{bmatrix} 0 & a_{12} & a_{13} \\ -a_{12} & 0 & a_{12} \\ -a_{13} & -a_{12} & 0 \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{11} & b_{12} \\ b_{13} & b_{12} & b_{11} \end{bmatrix} \quad \text{IV-21}$$

when the reference direction for the positive current is assumed to be upward. If all the elements are driven in phase, Equation II-17 takes the form

$$\underline{I}^0 = -I^e \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{11} & b_{12} \\ b_{13} & b_{12} & b_{11} \end{bmatrix}^{-1} \begin{bmatrix} a_{12} + a_{13} \\ 0 \\ -(a_{12} + a_{13}) \end{bmatrix}. \quad \text{IV-22}$$

Note that the odd currents on the 1 and 3 elements are out of phase, and the odd current on the center element is zero. This is expected from physical considerations, and this situation also conforms to the case of element 3 and the upper half of element 2 lying above an infinite ground plane.

For $h = \lambda/4$ and a spacing of $\lambda/2$ between centers, the odd current matrix is

$$\underline{I}^0 = I^e \begin{bmatrix} -0.101 + j 0.123 \\ 0 + j 0 \\ +0.101 - j 0.123 \end{bmatrix} \quad \text{IV-23}$$

The magnitude of the odd current is less than 16 per cent of the even current.

When the gap width is the limit of the order of the potential difference applied across the gap to the remaining even current that flows, as the gap width becomes very small, and line coupling effects are taken into account. This input impedance is a useful quantity when the line currents are balanced currents. In such cases, antennas are fed from transmission lines. However, an odd current on an element results in unbalanced currents on the transmission line and consequently, non-negligible radiation from the line itself. The overall radiation characteristics of the system, then, depend on the method of feeding, which is a distinct complication in the analysis. Knowledge of the input impedance of the element becomes less important, even somewhat arbitrary, because of the line radiation.

The approximate impedance of an element with even and odd currents may be calculated using the Poynting vector method [1]. The Poynting vector is integrated over a surface that encloses the element to give the complex radiated power. The impedance of the antenna will be defined as the radiated power divided by the square of the even current magnitude:

$$Z_1 = \frac{1}{I_e^2} \int_{\text{surface}} \mathbf{N} \cdot \mathbf{d}\mathbf{S} \quad \text{IV-24}$$

CHAPTER V

THE IMPEDANCE OF AN ELEMENT WITH EVEN AND ODD CURRENTS

If the current distribution on a linear dipole is known precisely, the input impedance may be calculated. From a mathematical viewpoint the input impedance is the limit of the ratio of the potential difference applied across the gap to the resulting even current that flows, as the gap-width becomes very small, and line coupling effects are taken into account. This input impedance is a useful quantity when the line currents are balanced because, in most cases, antennas are driven from transmission lines. However, an odd current on an element results in co-phased currents on the transmission line and consequently, non-negligible radiation from the line itself. The overall radiation characteristics of the system, then, depend on the method of feeding, which is a distinct complication in the analysis. Knowledge of the input impedance of the element becomes less important, even somewhat arbitrary, because of the line radiation.

The approximate impedance of an element with even and odd currents may be calculated using the Poynting vector method (10). The Poynting vector is integrated over a surface just enclosing the element to give the complex radiated power. The impedance of the antenna will be defined as the radiated power divided by the square of the even current magnitude

$$Z_1 = \frac{1}{|I_e|^2} \oint_{\text{Surface}} \langle \vec{S} \rangle_1 \cdot d\vec{s}$$

V-1

where $\langle \vec{S} \rangle_1$ is the time-averaged Poynting vector due to the fields of the antenna. For a thin element lying along the z-axis, and using a cylindrical coordinate system,

$$\langle S \rangle_1 = - E_z H_\phi^* , \quad \text{V-2}$$

and

$$H_\phi^* = \frac{I^*(z)}{2 \pi a} . \quad \text{V-3}$$

The complex radiated power is

$$P = - \int_0^{2\pi} \int_{-h}^{+h} \frac{E_z(z) I^*(z)}{2 \pi a} a \, dz \, d\phi . \quad \text{V-4}$$

$E_z(z)$ is the total electric field parallel to the element due to all sources. Consider only an element along the z-axis with even and odd currents.

The electric field parallel to an even sinusoidal current is well known(10), and may be shown to be

$$E_z^e(\rho, h) = \frac{-j\eta}{4\pi} I_L^e [K(\rho, z, h) + K(\rho, z, -h) - 2 \cos(kh)K(\rho, z, 0)] \quad \text{V-5}$$

where I_L^e is the maximum or loop current on the wire. This result is obtained by calculating the vector potential parallel to the current

and using the relationship between the electric field, vector potential, and scalar potential. A similar analysis performed on an antenna with an odd current yields

$$E_z^0(\rho, h) = \frac{-jn}{4\pi} I_L^0 [K(\rho, z, h) - K(\rho, z, -h) - \frac{2 \sin kh}{k} \frac{\partial K}{\partial z}(\rho, z, 0)] \quad V-6$$

The integral in V-4 contains several parts, some of which do not contribute to the complex radiated power because the integrands are odd, and are integrated over even limits. E_z^e is an even function in z , and E_z^o is an odd function in z . Expanding the integrand yields the non-zero components

$$Z_1 = j30 \{ |I_L^e|^2 \int_{-h}^h \sin k(h-z) [K(\rho, z, h) + K(\rho, z, -h) - 2 \cos kh K(\rho, z, 0)] dz \quad V-7$$

$$+ |I_L^o|^2 \int_{-h}^h \{ \underline{+} \} \sin k(h-z) [K(\rho, z, h) - K(\rho, z, -h) - \frac{2 \sin kh}{k} \frac{\partial K}{\partial z}(\rho, z, 0)] dz \}.$$

Using V-1, and solving for the term $\frac{\partial K}{\partial z}(\rho, z, 0)$ yields

$$Z_1 = j60 \{ \int_0^h \sin k(h-z) [K(\rho, z, h) + K(\rho, z, -h) - 2 \cos kh K(\rho, z, 0)] dz \quad V-8$$

$$+ \frac{|I_L^o|^2}{|I_L^e|^2} \int_0^h \sin k(h-z) [K(\rho, z, h) - K(\rho, z, -h) - \frac{2 \sin kh}{k} F(z, 0)] dz \},$$

where

$$F(z,0) = z e^{-jk\sqrt{\rho^2 + z^2}} \left[\frac{jk}{(\rho^2 + z^2)^2} + \frac{1}{(\rho^2 + z^2)^{3/2}} \right] . \quad \text{V-9}$$

Z_1 is always complex, and reduces to the conventional modified zeroth-order impedance when I_L^0 is equal to zero. The effect of the odd current is to add a correction term to the impedance, whose value is proportional to the square of the ratio of magnitude of the odd to even current.

CHAPTER VI

CONCLUSIONS

The purpose of this paper was to obtain a simple, easy to visualize method of calculating the odd currents in arrays of thin linear dipoles. The scalar potential at the center of each element is found to be proportional to even and odd currents in all elements. These complex constants of proportionality are only functions of the distance from the elements to the point at which the potential is calculated, and are expressible in terms of generalized sine and cosine integrals.

Two sets of driving conditions of interest are the case of balanced currents on each element, or the case of balanced potentials between elements, i.e., the center of each element is at the same scalar potential with respect to the reference. For the case of balanced potentials it is seen that the odd currents in the elements lead to co-phased transmission line currents. An exact analysis of a transmission line with unbalanced currents is not possible in general because the radiated energy depends on the line length, orientation, and proximity to the array elements. When balanced currents are specified, the potentials are unbalanced, and since the scalar potential varies sinusoidally along a thin wire, radiation occurs also in this case. Both cases, then, are undesirable, but physically unavoidable when driving the elements with transmission lines.

One could visualize driving the antennas at each gap with tiny self-contained phase locked transistorized generators so that no transmission lines are necessary. Then the potentials at the centers of the elements could float, and, to the zeroth order approximation, the currents in the elements would be balanced.

The actual physical situation is further complicated by the coupling between the feed lines and the antennas.

The results of applying the method to arrays of antennas seem to be quite reasonable. For tangential circular arrays of half-wave dipoles, the numerical and zeroth-order currents are very similar in magnitude and phase. For longer elements less agreement is found, which is also true in conventional zeroth-order theory. The analysis of the two element parallel staggered array of half-wave dipoles showed that the magnitude of the odd currents in the elements was proportional to the cosine of the angle between them.

By specifying the center location, angular orientation and complex current on all elements of the array, it was possible to derive an expression for the far zone electric field radiated by the N element array. Pattern multiplication is not applicable in this case since the elements have different orientation and element patterns. The element pattern of each element depends on the complex sum of the element patterns due to the even and odd currents.

The input impedance of each element may be defined in terms of its even terminal current, but the impedance loses much of its utility

due to the presence of the unbalanced transmission line currents. It is shown that for an isolated element with even and odd currents, the impedance defined by the E.M.F. method leads to an expression which contains the conventional zeroth-order impedance plus another term dependent on the ratio of the odd to the even current in the element.

It is expected that a more accurate expression for the current distribution will lead to better results. For example, using the first few terms in a Fourier series expansion for the current could yield a much better approximation for the odd currents. Alternatively, the customary iterative method could be used to find a first-order approximation for the odd current.

BIBLIOGRAPHY

CRANE & CO. CREST

BIBLIOGRAPHY

1. J. D. Tillman, Jr., Circular Antenna Arrays Having Tangential Dipoles as Elements, unpublished doctoral dissertation, Auburn University, June, 1968.
2. J. L. Hilburn and C. E. Hickman, Study of Circular Arrays of Tangential and Radial Dipoles, Report Number 1, A. F. 19(628)-4991, Auburn Research Foundation, Auburn University, Auburn, Alabama, March, 1967.
3. V. W. H. Chang and R. W. P. King, "On Two Arbitrarily Located Identical Parallel Antennas," IEEE Transactions on Antennas and Propagation, Vol. AP-16, No. 3, p. 309, May, 1968.
4. R. H. Duncan and F. A. Hinchey, "Cylindrical Antenna Theory," Journal of Research of the National Bureau of Standards, Vol. 64D, No. 5, pp. 569-82, Sept., 1960.
5. R. W. P. King, "The Theory of Linear Antennas," Harvard University Press, Cambridge, Massachusetts, 1956.
6. J. D. Tillman, Jr., "The Theory and Design of Circular Antenna Arrays," The University of Tennessee Engineering Experiment Station, Knoxville, Tennessee, 1966.
7. Harvard Computation Laboratory, "Tables of Generalized Sine and Cosine Integral Functions," Harvard University Press, Cambridge, Massachusetts, 1949, 2 Vols.
8. C. E. Hickman, H. P. Neff and J. D. Tillman, "The Theory of a Single-Ring Circular Antenna Array," Transactions of AIEE, Vol. 80, Pt. 1, p. 110, May, 1961.
9. L. A. Pipes, "Certain Applications of Matrices to Circuit Theory," AIEE Transactions, Vol. 77, Pt. 1, pp. 251-256, May, 1958.
10. E. C. Jordan, "Electromagnetic Waves and Radiating Systems," Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1950.



APPENDIX

\$IBFTC MAIN

C CALCULATION OF THE FARFIELD PATTERNS OF AN ARBITRARY ARRAY
 C OF N IDENTICAL ELEMENTS WITH KNOWN CURRENTS.THIS PROGRAM
 C USES ZEROth ORDER CURRENTS,BUT MAY BE EASILY MODIFIED TO
 C ACCOMODATE ANY SPECIFIED CURRENT DISTRIBUTION.N=NUMBER OF
 C ELEMENTS,H=HALF-LENGTH OF ELEMENTS,TJ AND PJ ARE THE ANG
 C LES WITH RESPECT TO REFERENCE COORDINATE SYSTEM,XJ,YJ,ZJ
 C ARE LOCATIONS OF CENTERS OF JTH ELEMENTS,T=POLAR ANGLE
 C THETA,P=AZIMUTHAL ANGLE PHI,AND M=COMPLEX RATIO OF ODD
 C TO EVEN CURRENT ON EACH ANTENNA.FTHETA AND FPHI ARE THE
 C FAR-FIELD PATTERNS.THE THREE PRINCIPAL PLANES ARE USED
 C IN THIS PROGRAM,BUT OTHER CUTS MAY EASILY BE USED.MODIFI
 C CATIONS OF DIMENSIONS AND FORMATS MUST BE MADE FOR EACH
 C DIFFERENT NUMBER N THAT IS USED.CARDS WITH AN ASTERISK
 C IN COLUMN 60 MUST BE MODIFIED FOR EACH NEW VALUE OF N,H.

```

      INTEGER N,N1,N2,N3,J
      REAL T,TR,P,PR,MFT,MFP,ARGFT,ARGFP,TJ(N),PJ(N),XJ(N) *
      1,YJ(N),ZJ(N) *
      COMPLEX ANS,SUMA,SUMB,SUMC,FTHETA,FPHI,INTEG,M(N) *
      COMMON M,TR,PR,TJ,PJ,XJ,YJ,ZJ,H,J
      EXTERNAL LOWER,UPPER
C      NEXT 2 CARDS SPECIFY N,AND H IN RADIANS
      N = *
      H = 3.14159/2.0 *
      READ (5,100) XJ,YJ,ZJ,TJ,PJ,M
100  FORMAT MUST BE SUPPLIED *
C      PATTERNS FOR X-Y PLANE,THETA = PI/2,PHI VARYS
      WRITE(6,99)
99   FORMAT(1H1)
      T = 90.0
      TR = 90.*3.14159/180.
      DO 1000 N1 = 5,185,5
      P = FLOAT(N1) - 5.0
      PR = P*3.14159/180.0
      SUMA = CMPLX(0.0,0.0)
      SUMB = CMPLX(0.0,0.0)
      SUMC = CMPLX(0.0,0.0)
      DO 900 J = 1,N
      CALL QG10(-H,0.0,LOWER,A1)
      CALL QG10(0.0,H,UPPER,A2)
      ANS = A1 + A2
      SUMA = SUMA + ANS*COS(TJ(J))
      SUMB = SUMB + ANS*SIN(TJ(J))*SIN(PJ(J))
900  SUMC = SUMC + ANS*SIN(TJ(J))*COS(PJ(J))
      FTHETA = -1.0*SUMA
      FPHI = COS(PR)*SUMB - SIN(PR)*SUMC
      MFT = CABS(FTHETA)
      MFP = CABS(FPHI)
      IF ((AIMAG(FTHETA)/REAL(FTHETA)).LE.1.E-6) GO TO 25
      IF ((AIMAG(FTHETA)/REAL(FTHETA)).GE.1.E+3) GO TO 26
      ARGFT = ATAN2(AIMAG(FTHETA),REAL(FTHETA)) *180.0/3.142
  
```

```

      GO TO 27
25    ARGFT = 0.000
      GO TO 27
26    ARGFT = 90.000
27    CONTINUE
      IF ((AIMAG(FPHI )/REAL(FPHI )).LE.1.E-6) GO TO 28
      IF ((AIMAG(FPHI )/REAL(FPHI )).GE.1.E+3) GO TO 29
      ARGFP = ATAN2(AIMAG(FPHI ),REAL(FPHI )) *180.0/3.142
      GO TO 30
28    ARGFP = 0.000
      GO TO 30
29    ARGFP = 90.000
30    CONTINUE
      WRITE(6,102) P,T,FTHETA,MFT,ARGFT,FPHI,MFP,ARGFP
102   FORMAT(1H0,10F13.3)
1000  CONTINUE
      WRITE (6,99)
C     PATTERNS FOR X-Z PLANE,P = 0.0,THETA VARYS
      P = 0.0
      PR = 0.0
      DO 2000 N2 = 5,185,5
      T = FLOAT(N2)-5.0
      TR = T*3.14159/180.0
      SUMA = CMPLX(0.0,0.0)
      SUMB = CMPLX(0.0,0.0)
      SUMC = CMPLX(0.0,0.0)
      DO 1900 J = 1,N
      CALL QG10(-H,0.0,LOWER,A1)
      CALL QG10(0.0,H,UPPER,A2)
      ANS = A1 + A2
1900  SUMA = SUMA + ANS*SIN(TJ(J ))*COS(PJ(J ))
      SUMB = SUMB + ANS*COS(TJ(J ))
      FTHETA = COS(TR)*SUMA - SIN(TR)*SUMB
      FPHI = SUMB
      MFT = -CABS(FTHETA)
      MFP = CABS(FPHI)
      IF ((AIMAG(FTHETA)/REAL(FTHETA)).LE.1.E-6) GO TO 31
      IF ((AIMAG(FTHETA)/REAL(FTHETA)).GE.1.E+3) GO TO 32
      ARGFT = ATAN2(AIMAG(FTHETA),REAL(FTHETA)) *180.0/3.142
      GO TO 33
31    ARGFT = 0.000
      GO TO 33
32    ARGFT = 90.000
33    CONTINUE
      IF ((AIMAG(FPHI )/REAL(FPHI )).LE.1.E-6) GO TO 34
      IF ((AIMAG(FPHI )/REAL(FPHI )).GE.1.E+3) GO TO 35
      ARGFP = ATAN2(AIMAG(FPHI ),REAL(FPHI )) *180.0/3.142
      GO TO 36
34    ARGFP = 0.000
      GO TO 36
35    ARGFP = 90.000

```

```

36      CONTINUE
      WRITE(6,102) P,T,FTHETA,MFT,ARGFT,FPHI,MFP,ARGFP
2000    CONTINUE
      WRITE (6,99)
C      PATTERNS FOR Y-Z PLANE,PHI=PI/2,THETA VARYS
      P = 90.0
      PR = P*3.14159/180.0
      DO 3000 N3 = 5,185,5
      T = FLOAT(N3)-5.0
      TR = T*3.14159/180.0
      SUMA = CMPLX(0.0,0.0)
      SUMB = CMPLX(0.0,0.0)
      SUMC = CMPLX(0.0,0.0)
      DO 2900 J = 1,N
      CALL QG10(-H,0.0,LOWER,A1)
      CALL QG10(0.0,H,UPPER,A2)
      ANS = A1 + A2
      SUMA = SUMA + ANS*SIN(TJ(J))*SIN(PJ(J))
      SUMB = SUMB + ANS*COS(TJ(J))
2900    SUMC = SUMC + ANS*SIN(TJ(J))*COS(PJ(J))
      FTHETA = COS(TR)*SUMA - SIN(TR)*SUMB
      FPHI = SUMC
      MFT = CABS(FTHETA)
      MFP = CABS(FPHI)
      IF ((AIMAG(FTHETA)/REAL(FTHETA)).LE.1.E-6) GO TO 37
      IF ((AIMAG(FTHETA)/REAL(FTHETA)).GE.1.E+3) GO TO 38
      ARGFT = ATAN2(AIMAG(FTHETA),REAL(FTHETA)) *180.0/3.142
      GO TO 39
37      ARGFT = 0.000
      GO TO 39
38      ARGFT = 90.000
39      CONTINUE
      IF ((AIMAG(FPHI)/REAL(FPHI)).LE.1.E-6) GO TO 40
      IF ((AIMAG(FPHI)/REAL(FPHI)).GE.1.E+3) GO TO 41
      ARGFP = ATAN2(AIMAG(FPHI),REAL(FPHI)) *180.0/3.142
      GO TO 42
40      ARGFP = 0.000
      GO TO 42
41      ARGFP = 90.000
42      CONTINUE
      WRITE(6,102) P,T,FTHETA,MFT,ARGFT,FPHI,MFP,ARGFP
3000    CONTINUE
      CALL EXIT
      END
$IBFTC CEX
      COMPLEX FUNCTION CEX(Z)
      CEX = CMPLX(COS(Z),SIN(Z))
      RETURN
      END
$IBFTC LOWE
      COMPLEX FUNCTION LOWER(Z)

```

```

EXTERNAL CEX
REAL T,TR,P,PR,MFT,MFP,ARGFT,ARGFP,TJ(N),PJ(N),XJ(N) *
1,YJ(N),ZJ(N) *
COMPLEX CEX,M(N) *
COMMON M,TR,PR,TJ,PJ,XJ,YJ,ZJ,H,J
TTJ = SIN(TR)*SIN(PR)*(XJ(J)+Z*COS(TJ(J))*SIN(PJ(J)))
1+SIN(TR)*SIN(PR)*(YJ(J)+Z*SIN(TJ(J))*SIN(PJ(J)))
2+COS(TR)*(ZJ(J)+Z*COS(TJ(J)))
LOWER = (1.0-M(J))*SIN(H+Z)*CEX(TTJ)
RETURN
END

$IBFTC UPPE
COMPLEX FUNCTION UPPER(Z)
EXTERNAL CEX
REAL T,TR,P,PR,MFT,MFP,ARGFT,ARGFP,TJ(N),PJ(N),XJ(N) *
1,YJ(N),ZJ(N) *
COMPLEX CEX,M(N) *
COMMON M,TR,PR,TJ,PJ,XJ,YJ,ZJ,H,J
COMPLEX CEX,M(2)
TTJ = SIN(TR)*SIN(PR)*(XJ(J)+Z*COS(TJ(J))*SIN(PJ(J)))
1+SIN(TR)*SIN(PR)*(YJ(J)+Z*SIN(TJ(J))*SIN(PJ(J)))
2+COS(TR)*(ZJ(J)+Z*COS(TJ(J)))
UPPER = (1.0+M(J))*SIN(H-Z)*CEX(TTJ)
RETURN
END

$IBFTC QG10      NODECK
SUBROUTINE QG10(XL,XU,FCT,Y)
COMPLEX FCT,Y
A=.5*(XU+XL)
B=XU - XL
C= .4869533 * B
Y= .03333567 * (FCT(A+C)+FCT(A-C))
C= .4325317 * B
Y=Y + .07472567 * (FCT(A+C)+FCT(A-C))
C= .3397048 * B
Y=Y + .1095432 * (FCT(A+C)+FCT(A-C))
C= .2166977 * B
Y=Y + .1346334 * (FCT(A+C)+FCT(A-C))
C= .07443717 * B
Y= B* (Y+ .1477621 * (FCT(A+C)+FCT(A-C)))
RETURN
END

$ENTRY

```

VITA

Samuel Benton Roberts, son of Harry Mims and Fay (Benton)

Roberts was born in Birmingham, Alabama, on November 8, 1941. He attended the Columbiana, Alabama public school and was graduated from Shelby County High School in 1959. In the fall of that year he entered Alabama College, Montevallo, Alabama, and in 1961 entered Auburn University. Receiving the Bachelor of Electrical Engineering degree in December, 1963, he entered the graduate school of Auburn University, holding a research assistantship. In March, 1965 he received the degree of Master of Science, and remained in school at Auburn, as a teaching assistant. In September, 1966 he began graduate studies at the University of Tennessee, and accepted an Instructorship in the Electrical Engineering Department. He is a member of Tau Beta Pi, Eta Kappa Nu, and Sigma Xi.

He is married to the former Judith Grace Carter of Columbiana, Alabama and has one daughter, Kimberley.