

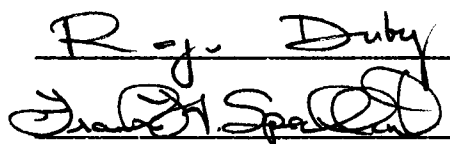
To the Graduate Council:

I am submitting herewith a thesis written by Chenyi Hu entitled "Torsional Control of Thin-Walled Structures by Piezoelectric Actuators." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.



Grzegorz Kawiecki, Major Professor

We have read this thesis
and recommend its acceptance:



Accepted for the Council:



Associate Vice Chancellor
and Dean of The Graduate School

TORSIONAL CONTROL OF THIN-WALLED STRUCTURES

BY

PIEZOELECTRIC ACTUATORS

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Chenyi Hu

August 1994

ACKNOWLEDGMENTS

I would like to thank my advisor Dr. Grzegorz Kawiecki for his invaluable guidance and encouragement throughout the course of this research. I would also like to thank the other committee members, Dr. Frank Speckhart and Dr. Rajiv Dubey, for their comments and assistance in the past two years. I would like to express my thanks to the electrician and the machinists in the Mechanical and Aerospace Engineering Department for their professional help with equipment used in the experiment, especially Mr. Dennis Higdon for his time and numerous valuable suggestions.

I would like to thank my personal friend, Mr. Philip Brantley, for proofreading my manuscript several times.

I am indebted to my family, especially my uncle and aunt, Mr. Y. C. Wu, and Mrs. ShihYing Woo, for their care and support, which made my higher education possible.

Finally, I wish to thank my wife Yichin without whose love and loyal support this work would not have been possible.

ABSTRACT

The topics covered in this thesis are part of the feasibility study in using piezo active materials to modify torsional behavior of thin-walled opened and closed cross-section structural members. Both experimental and analytical methods were used in this investigation. The study began with the investigation of the electro-mechanical characteristics of a single piezo element, in particular the strain transfer mechanism. Then the study was extended to test a composite thin-walled, closed cross-section tube with piezo elements laminated to its walls. The effectiveness of piezo elements in damping torsional vibration was determined. As a part of the fundamental research, Vlasov's theory was used to calculate the amount of twist on a flat thin plate when it was actuated by piezoelectric elements bonded antisymmetrically to both of its surfaces. Also, smart elements based on the ACLD (Active Constrained Layer Damping) was evaluated as an alternative to piezoceramic elements for its potential in controlling structural vibrations.

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NOMENCLATURE

b	width
E	Young's modulus, electrical potential
h	thickness
K_{ij}	stiffness matrix
k	bending curvature, spring constant
G	shear modulus
I	second moment of a cross-sectional area
J	polar moment of inertia of a cross section
J_0	polar mass moment of inertia
l	length
M	bending moment
m	mass density
M_ω	bimoment (or warping moment), referring to beam
T	torsional moment
t	thickness
U, V, W	displacement in x, y, z directions
u, v, w	displacement in n, s, z directions
V	applied voltage
V_x, V_y	shear forces in x, y directions
α	substructure equilibrium parameter

ϵ	strain
ω_n	natural frequency
ϕ	rotation
λ	warping function
ν	Poisson's ratio
σ	stress field

Subscripts

a, s	actuator and substrate, respectively
z, s, n	direction indicator in plate coordinates
x, y, z	direction indicator in Cartesian coordinates
1, 2, 3	index notation for Cartesian coordinates representing x, y, and z, respectively
i, j, k	indices

Superscripts

$()'$	differentiation with respect to z coordinates
$()^{\cdot}$	differentiation with respect to s coordinates
$[]^T$	transposition of a matrix
$[]^{-T}$	transposition and inversion of a matrix

CHAPTER 1

INTRODUCTION

1.1 Objective

Piezo material has the unique capability of producing mechanical strain when subjected to electric potential and of generating electrical charge when subjected to mechanical strain. This capability has been utilized in applications for which conversions between mechanical strains and electric potentials are required. For example, piezo material can be used to pick up sound vibrations in microphones and then convert them into electrical signals; it can also be used to move the cutting tips in precision machining by varying the electrical potential across it. A literature survey shows that research on using piezo elements for shape control and vibration damping has been conducted for at least the last 10 years. The published studies have mainly concentrated on the piezo element's ability to generate pure extension, pure bending, and coupled bending and extension [4], [5], but very little effort has been put into modifying the torsional behavior of a structure [2], [3], [8]. Therefore, it is the purpose of this study to analyze a method of using the piezo actuator for torsional vibration control.

1.2 Significance

The piezo actuator is competing with many other types of mechanical and electrical actuators. Its advantages over other types of actuators are ease of control, short response

time, light weight, simplicity, and ease of installation and maintenance [2]. Its disadvantage is the small magnitude of strain it can generate. The piezo actuator also has some unique attributes, such as a large frequency response bandwidth, a potential for being embedded in the composite material, and the ability to alter the strain in a structure and therefore to modify its displacement or damping characteristics [2].

Although torsional displacements that can be generated by a piezoelectric element are small, its capability of controlling the torsional characteristics of a structure has many potential uses as investigated in this paper. It can be used to change aerodynamic loads on structural elements subjected to air flow, such as controlling helicopter blade vibrations generated by turbulent air flow. This can be achieved by modifying individual blade pitch to reduce the effect of turbulence and by absorbing the blade's vibration energy, both done by piezo actuators.

The proposed concept of using piezo elements for torsional shape and vibration control has been reviewed by several leading authorities in the field. A review done by Boeing Helicopters in Philadelphia states, "This concept could be used to control vibrations created from airflow over aerodynamic surfaces. Primary application would be for helicopter blades that are currently mass balanced during vehicle flight test." A letter from McDonnell Douglas Missile Systems Company comments, "The combination of flexibility and much more violent flight environment has resulted in increased cases of failures due to aero-elastic instabilities and structural fatigue failures due to low-level random vibrations. The push is to find solutions to these problems that will still allow the aircraft to perform at its peak

capability. One of these solutions is the use of active control to reduce the vibration response of critical structural elements on an aircraft or missile."

1.3 Concept Explanation

A cylinder subjected to torsion will have maximum stresses and strains along 45 degree helixes with respect to the cylindrical axis. This phenomenon opens a window for generating twist by creating strains in these directions. Figure 1.1 illustrates the generation of twist by piezo elements to induce strains along helixes of a thin-walled cylinder. For simplicity, only four helix lines are shown. Two of the helix lines are subjected to compressive strains; the other two are subjected to tensile strains. The combination of these two strains will generate a torque about beam's longitudinal axis.

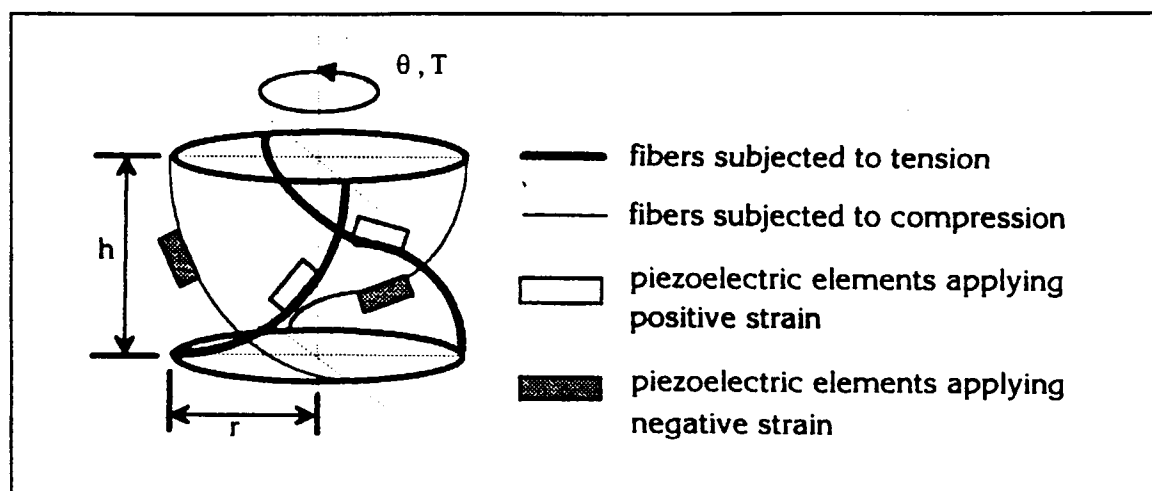


Figure 1.1 Application of twist using piezoelectric actuators [2].

1.4 An Overview of Previous Work

This research project is being conducted by a team led by Dr. Grzegorz Kawiecki. Its goal is to manipulate the torsional motion of a tube, which will ultimately simulate a

helicopter blade, by controlling directionally attached piezoelectric elements. A significant amount of work had already been done by Dr. Kawiecki and his research team before the full involvement of the author. This previous work has been described in reference [3], which includes analytical, numerical and experimental concept validations of a shear plate that resembles an element of the tube and the application of piezo elements to twist a thin-walled closed cross-section beam.

1.4.1 Literature Review

A literature survey on applications and research related to the application of piezoelectric elements for shape and vibration control found that most publications dealt with four categories [3]: pure extension of a beam; pure bending of a beam; coupled bending and extension of a beam, and coupled twisting and extension of a beam. It was also found that most of the research involving piezoelectric materials concerns their application in controlling structural vibrations of flexible beams, plates, etc.. A few publications [1], [13], [32], [33], [34] described the use of piezo elements for torque or twist generation. However, twist investigated in those publications was generated by coupling between bending and axial displacement.

1.4.2 Shear Behavior of an Aluminum Plate

Since a complex thin-walled structure can be finitely subdivided into a number of thin plates, extensive studies using analytical, numerical, and experimental methods were done to

investigate the shape change and dynamic characteristics of a shear plate bonded with piezo elements to both of its sides.

For analytical study, the Classical Laminated Plate Theory (CLPT) was used as the basis for the formulation. The strains transferred to the laminate by the piezoelectric actuator laminae were modeled as thermally induced structural loads in the laminate [3]. The assumptions of the analysis were as follows [14]:

1. The laminate was made up of perfectly bonded laminae.
2. The bonds were infinitesimally thin as well as non-shear deformable (bonds between laminae do not deform).
3. Shear strains perpendicular to the middle surface were zero.
4. The normals did not deform (constant lamina thickness).

For a plate bonded with piezo elements symmetrically to both of its sides, as shown in Fig. 1.2, the piezo elements induced shear in the y -direction and caused no twist in the plate when the elements at $+\alpha$ and elements at $-\alpha$ were activated in the opposite directions.

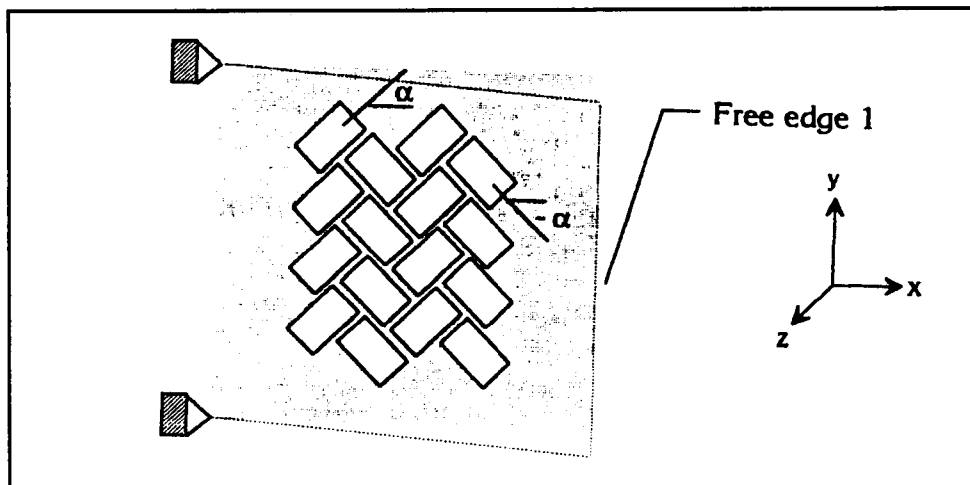


Figure 1.2 Generation of shear displacement in aluminum plate using piezoelectric elements [3].

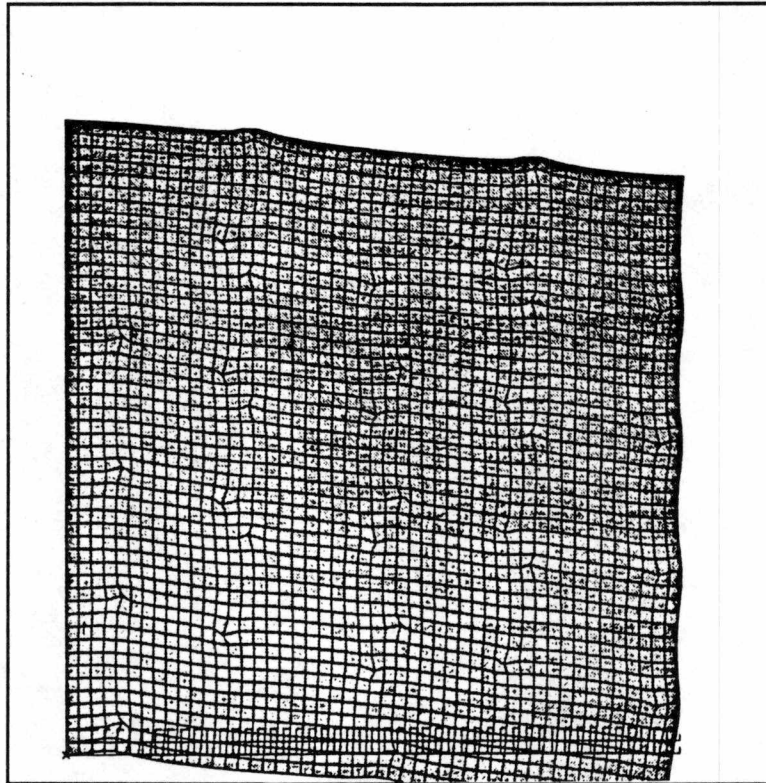


Figure 1.3 Sample displacements of a shear plate [3].

The numerical analysis was done by using a commercially available finite element package, Integrated Design Engineering Analysis Software (I-DEASTM), from Structural Dynamics Research Corporation (SDRC). Several important assumptions were made when modeling the strain transfer from the activated piezo elements to the substrate. First, a perfect bonding between the piezoelectric element and the substrate was assumed; second, the strains generated by the directionally attached piezoelectric elements in the lateral direction were ignored; third, the actual strain transferred to the substrate was modeled by assuming that the strain transfer was due to forces applied on the edges of the activated piezo actuator. A sample output of the simulated shear deformation is shown in Fig. 1.3.

The experimental analysis of a shear plate was performed on a thin aluminum plate with piezo elements bonded symmetrically on both of its sides, as shown in Fig. 1.2. A special testing device was also developed for holding the plate and applying shear loads (in the y-direction) to measure the shear displacements. The plate was clamped at one edge but was free at the other edges. Some additional supports were added to both sides of the plate to prevent it from warping in the z-direction. The shear displacements of the plate along the y-direction were measured on the free edge 1 by a dial indicator.

1.4.3 Composite Tube Testing

In his thesis [3], Mr. W. Smith described bending and twist of a composite tube by piezo actuation, but his results were primitive. Part of this thesis will describe improved testing techniques for more accurate results and a method using piezoelements and feedback control systems for torsional vibration damping.

CHAPTER 2

PIEZO CHARACTERISTICS

2.1 Properties of Piezo Elements

The piezo ceramic elements selected for this research are G-1195 from Piezo Systems Inc., Cambridge, Massachusetts. They were chosen because of high electro-mechanical efficiency, high Curie temperature (the maximum temperature at which piezoelectric material demonstrates the piezoelectric effect) and good mechanical characteristics. The G-1195 met all the requirements of this research and was significantly more efficient in converting electrical energy to mechanical energy than other available materials at the time of purchase. The elements were supplied as 25.4 mm x 6.25 mm x 0.19 mm plates, pre-poled and nickel electroded. The detailed properties of the material are listed in Table 2.1.

In Table 2.1, piezoelectric coefficients are shown with double subscripts, which link electrical and mechanical qualities. The first subscript indicates the direction of the electrical field and the second subscript shows the direction of the corresponding mechanical strain. The directions are shown in Fig. 2.1. The Piezoelectric Charge Coefficient d_{31} is the amount of piezo strain in the longitudinal direction (direction 1) of a piezo element when a voltage potential is applied across the thickness of the crystal (direction 3). The Piezoelectric Voltage Coefficient g_{31} is the amount of voltage potential (in the direction 3) that can be generated by an inplane force (in the direction 1). The Coercive Field is the voltage necessary for changing

the polarity of the crystal; it is equivalent to 228 volts for the selected elements of 0.191 mm thickness.

Table 2.1 Electro-mechanical properties of G-1195 lead zirconate titanate piezoceramic

Property	Symbol	Value	Unit
Density	ρ	7650	Kg/m ³
Curie temperature		360	°C
Piezoelectric charge coefficient	d_{31}	166×10^{-12}	m/V
Piezoelectric voltage coefficient	g_{31}	11×10^{-3}	V-m/N
Young's modulus	E	6.3×10^{10}	N/m ²
Coercive field	E_c	1.2×10^6	V/m ²
Tensile stress--static		7×10^7	N/m ²
Tensile stress--dynamic		2.1×10^7	N/m

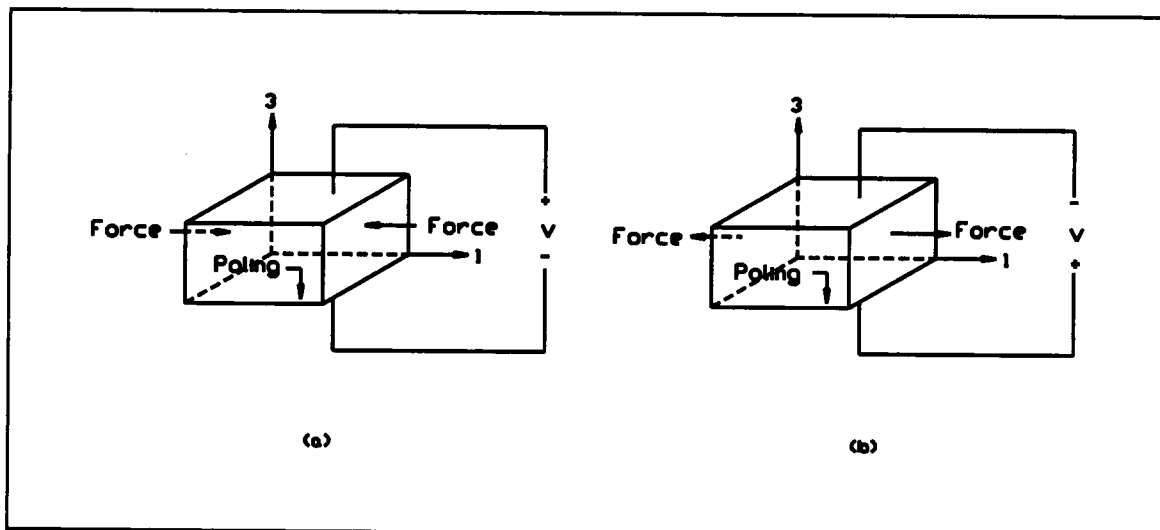


Figure 2.1 Electro-mechanical response of piezo element: a) compression, b) extension.

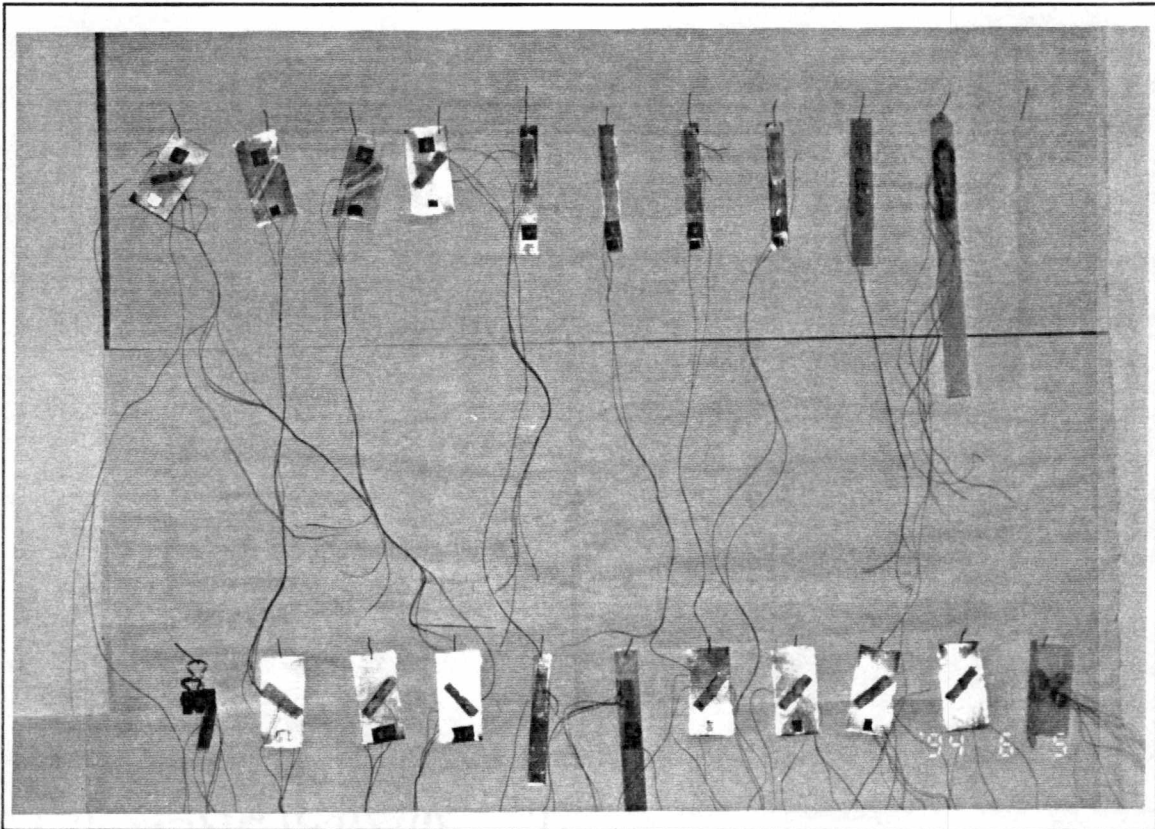


Figure 2.2 Collection of test specimens.

2.2 Experimental Verification of Piezo Element Properties

Experiments were conducted to verify the key data supplied by the manufacturer and to obtain additional information to understand how a piezo element transfers strain to its substrate. Fig. 2.2 is a collection of test specimens that were used in various experiments described in this chapter. As shown in Fig. 2.3, the free strain of a piezo element was tested using two strain gages bonded to the middle and end portion of a piezo element's surface for measuring the strain of the element as well as the strain variation along the longitudinal axis. The adhesive material used to bond the strain gage to the piezo element was Eccobond 45 LV and its catalyst 15 LV, by Emerson & Cuming Inc.. The mixing ratio of Eccobond 45 LV

and catalyst 15 LV is 1:1 for a medium-rigidity bond. During the curing process, pressure was applied through a 5 mm thick silicon pad for even distribution of pressure and minimizing bonding thickness. The pad was separated from the bonding area by a layer of Teflon tape to prevent it from sticking to the test specimen. The temperature was also raised to about 200 degrees Fahrenheit for quick curing. The electric wires leading to the strain indicator were soldered after the curing process. The strain measuring setup is illustrated in Fig. 2.3.

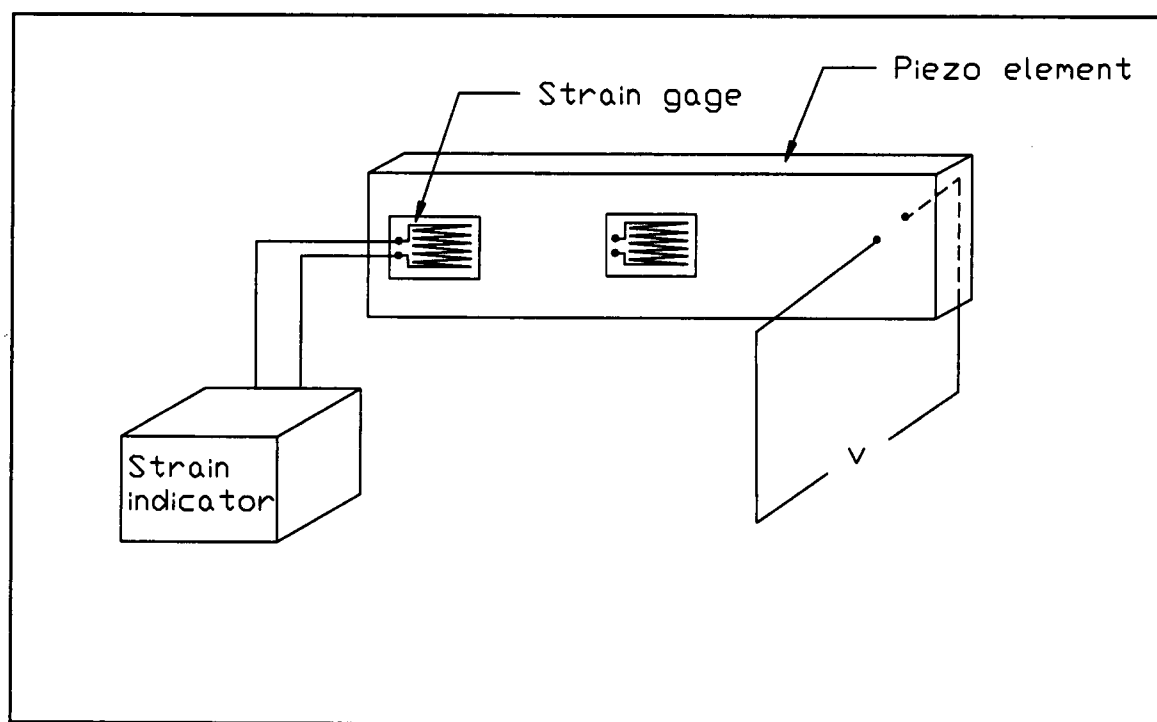


Figure 2.3 Strain gage positions for electro-mechanical property test.

The strain responses of the piezo element were measured by a strain indicator for the applied voltages from 20/-20 volts to 150/-150 volts with a step size of 20/-20 volts. The sequence of applying the selected voltage was as follows: a positive voltage--off--a negative voltage--off--then another cycle starting with a different positive voltage. The measured

strains were plotted in Fig. 2.4. It is noted that strains measured after removal of positive voltages were different than strains after removal of negative voltages. A strain residual was clearly in existence. The strain-voltage relationships have been curve-fitted by third-degree polynomials and are summarized in Table 2.2. Although two strain gages were used, the strains measured by both strain gages were always the same for certain applied voltage, indicating no strain variation along the longitudinal axis of the element.

Table 2.2 Strain-voltage relationship*

	The free strain of a piezo element (E-6)	Constrained strain** (E-6)
Compressive strain	$-0.847 - 0.834 V - 0.0126 V^2 + 0.0000015 V^3$	$101.686 - 4.529 V + 0.0387 V^2 - 0.000132 V^3$
Compressive strain residual	$6.22 - 0.505 V + 0.00177 V^2 - 0.0000376 V^3$	NA
Tensile strain	$25.42 - 0.684 V + 0.038 V^2 - 0.000144 V^3$	$-14.124 + 0.686 V + 0.0112 V^2 - 0.0000574 V^3$
Tensile strain residual	$13.72 - 0.8216 V + 0.01945 V^2 - 0.0000613 V^3$	$-18.979 + 0.3541 V + 0.00224 V^2 - 0.0000145 V^3$

* V is the applied voltage, see Appendix A for details of these polynomials.

** When the thickness of the substrate was 0.292 mm and the thickness of the piezo element was 0.191 mm.

When a piezo element is bonded to the substrate, the actual strain (constrained strain) of the element is smaller than its free strain, and is equal to the induced strain on the substrate if shear lag in the bonding layer is ignored. The induced strain is a function of

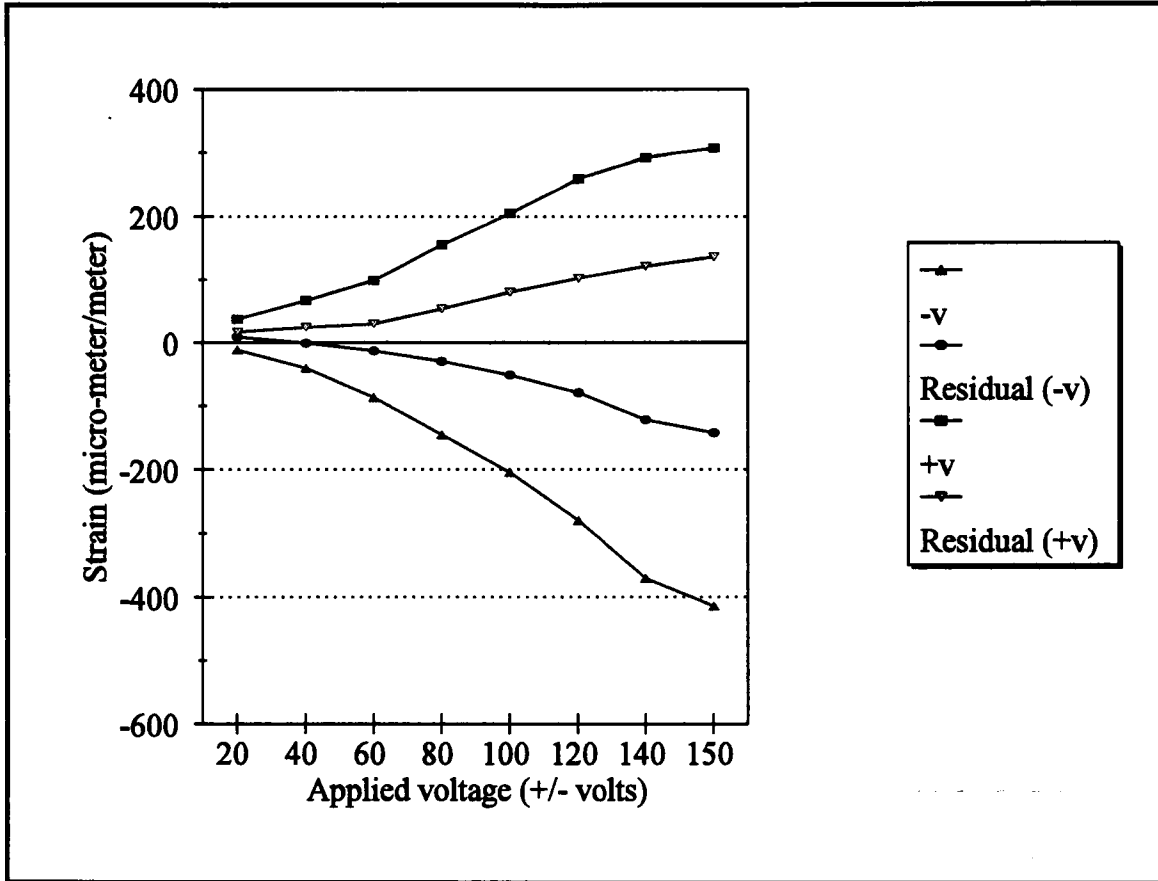


Figure 2.4 Induced free strains versus applied voltages.

applied voltage as well as the thickness and Young's modulus of both piezo element and substrate. The relationship has been formulated by Crawley (Eq. (47) in [14]) as follows:

$$\text{Induced strain} = E_{\max} d_{31} \left[\frac{\alpha}{\alpha + \frac{E_s t_s}{E_a t_a}} \right] \quad (2.1)$$

in which, $\alpha = 6$ for a bending test.

This equation was verified by testing a bending specimen made of a 0.191 mm thick piezo element and its substrate of a 0.291 mm thick aluminum plate. When an electrical potential of 140 volts was applied, the strain measurement indicated 1,905

micrometers/meter; it measured 1,794 micrometers/meter after the electrical potential was turned off. The actual strain was 111 micrometers/meter, which matches well with the calculated value from Eq. (2.1). The detailed data are listed in Appendix A (Table 2).

Both free strain and induced strain have been discussed so far, their physical meaning can be clarified by making an analogy to thermal expansion of a steel bar. The free strain of the piezo element is equivalent to the strain of the bar when its thermal expansion is not restrained; the constrained strain of the piezo element is equivalent to the strain of the bar when its thermal expansion is restrained by a pair of external forces. The restraining forces to the thermal expansion of the bar are proportional to the difference between the unrestrained strain and the restrained strain. Therefore, when a piezo element is bonded to its substrate, the force transmitted to its substrate is also proportional to the difference between the free strain and the constrained strain of the piezo element.

2.3 Effect of Bonding Layer Parameters

A bonding layer between a piezo element and its substrate transmits the strain from the piezo element to the substrate. The mechanical characteristics and the dimensions of the bonding layer significantly affect the amount of strain induced by the piezo element.

2.3.1 Bonding Material Thickness

The thickness of the bonding layer has a major effect on the amount of strain transmitted from the piezo element to the substrate: the thicker the bonding layer, the smaller the strain transferred. A set of bending specimens, which were identical except for bonding

layer thickness, was prepared in order to measure the relation between bonding layer thickness and amount of strain induced in the substrate. Fig. 2.5 is a schematic of the test setup. The dimension t_2 is the thickness of the bond. The mirror attached to the tip of the beam was used to reflect the laser beam, which is omitted in the schematic. The amount of bending was calculated by measuring the span between the boundaries of the laser projection while the beam was being activated by the piezo element. Fig. 2.6 is a plot of the results, which demonstrates the relationship between bonding thickness and its bending effect on a beam.

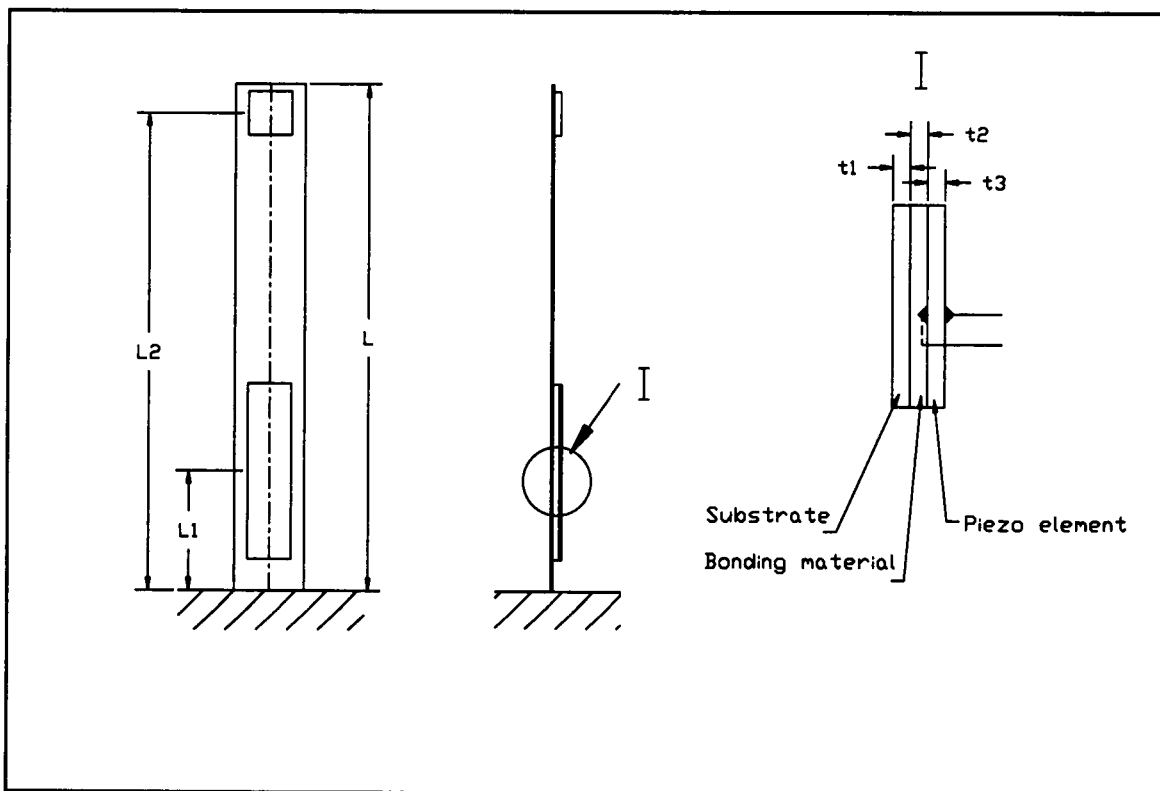


Figure 2.5 Bending test of specimens with different bonding thicknesses.

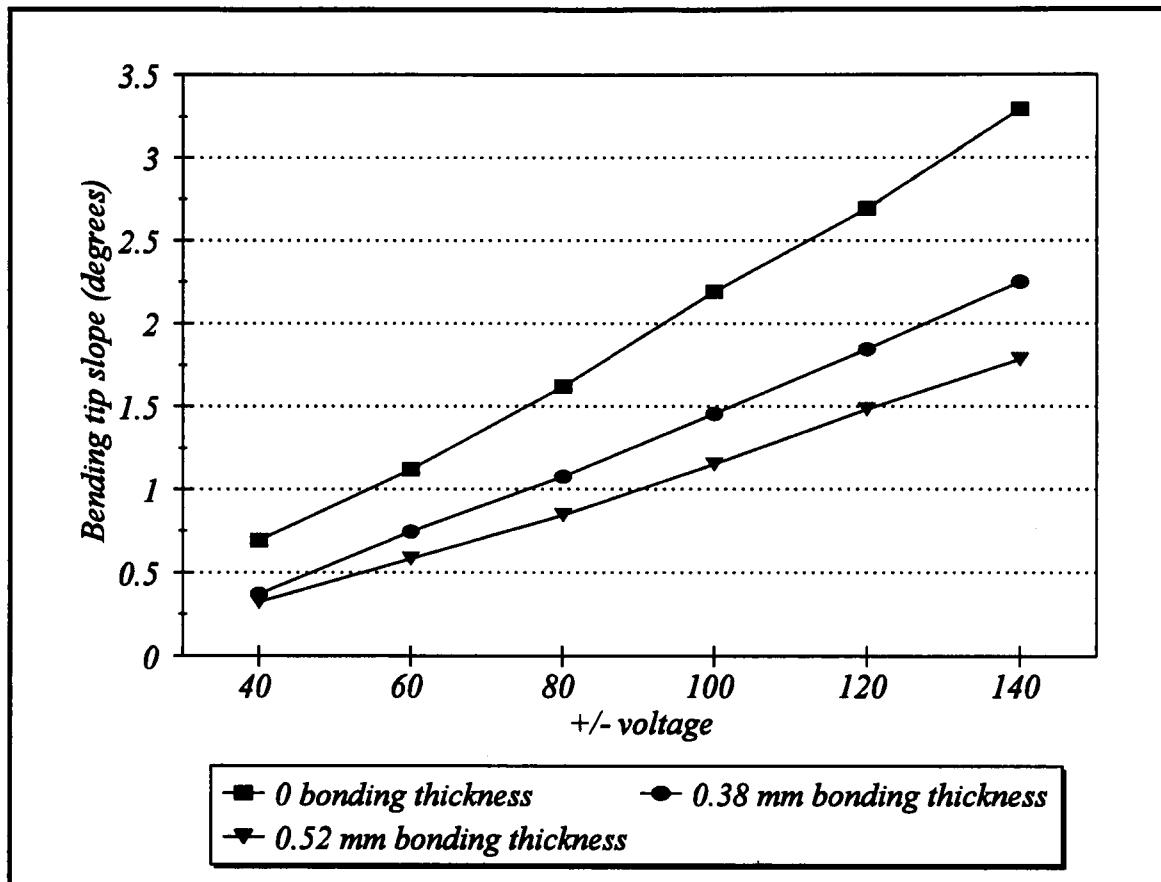


Figure 2.6 Bonding thickness effect on beam bending amplitude (note: slope readings are corresponding to alternating voltages).

It is noted that bonding thickness significantly affects the amount of strain transferred from a piezo element to its substrate. This shows the importance of having minimum bonding thickness in order to achieve the maximum induced strain in the substrate.

2.3.2 Directional Attachment

Directional attachment [1] means that only the central portion of the piezo element is attached, by a longitudinal strip of bonding material. Directional attachment maintains the longitudinal stiffness of the piezo element while reducing its transverse stiffness by allowing

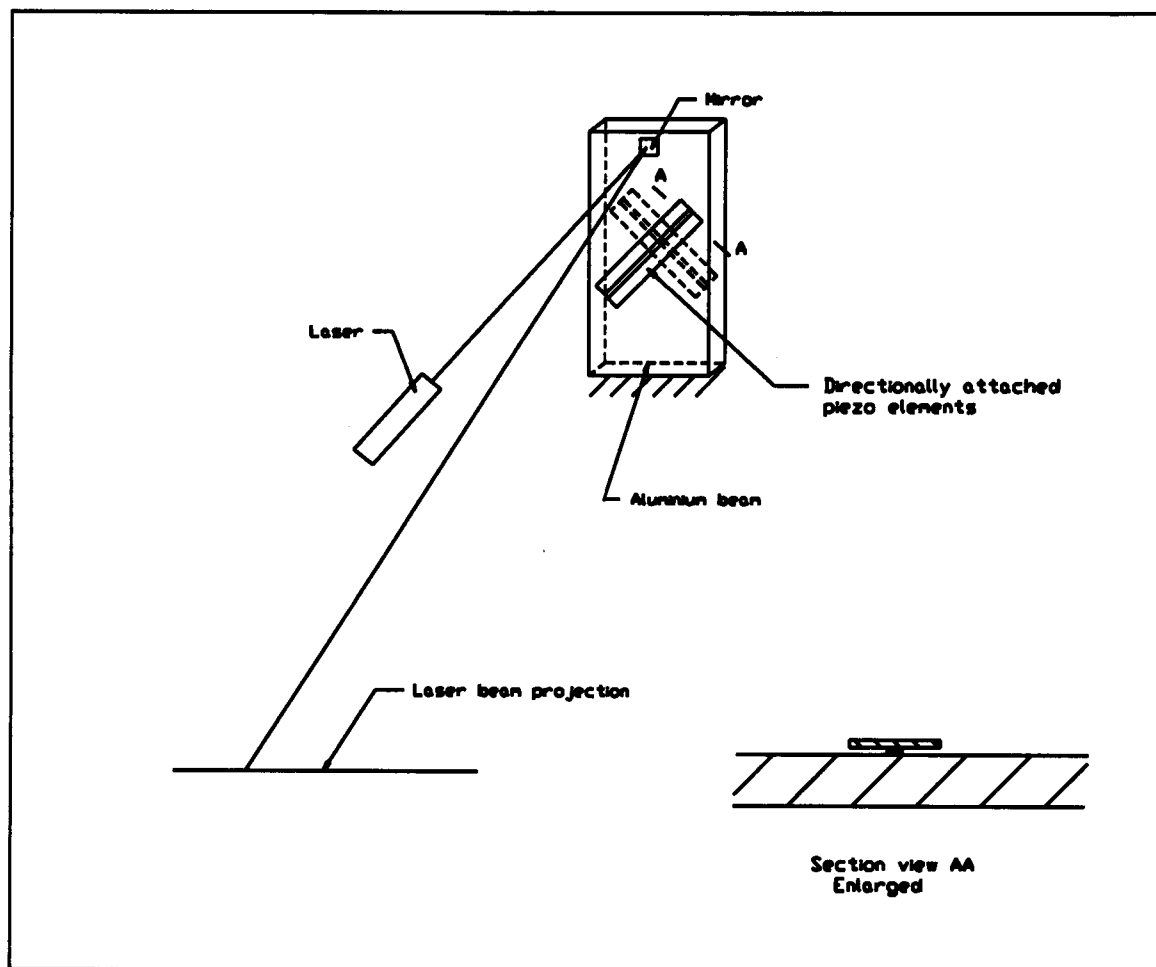


Figure 2.7 Measurement of twist induced in an aluminum plate by directionally attached piezo elements.

the sides to expand and contract with little constraint. This is especially important in generation of twist, where the lateral strain in the element would have a negative effect on the overall twist. A quantitative test was performed to measure the effectiveness of directional attachment, with different bonding strip widths, in generating twist. The test setup is illustrated in Fig. 2.7. A laser was used to measure the twist angle of the specimen.

Experimental results confirmed that the width of the bond does affect the magnitude of twist. Fig. 2.8 shows that the narrower the bonding strip, the bigger the twist angle generated on the specimen.

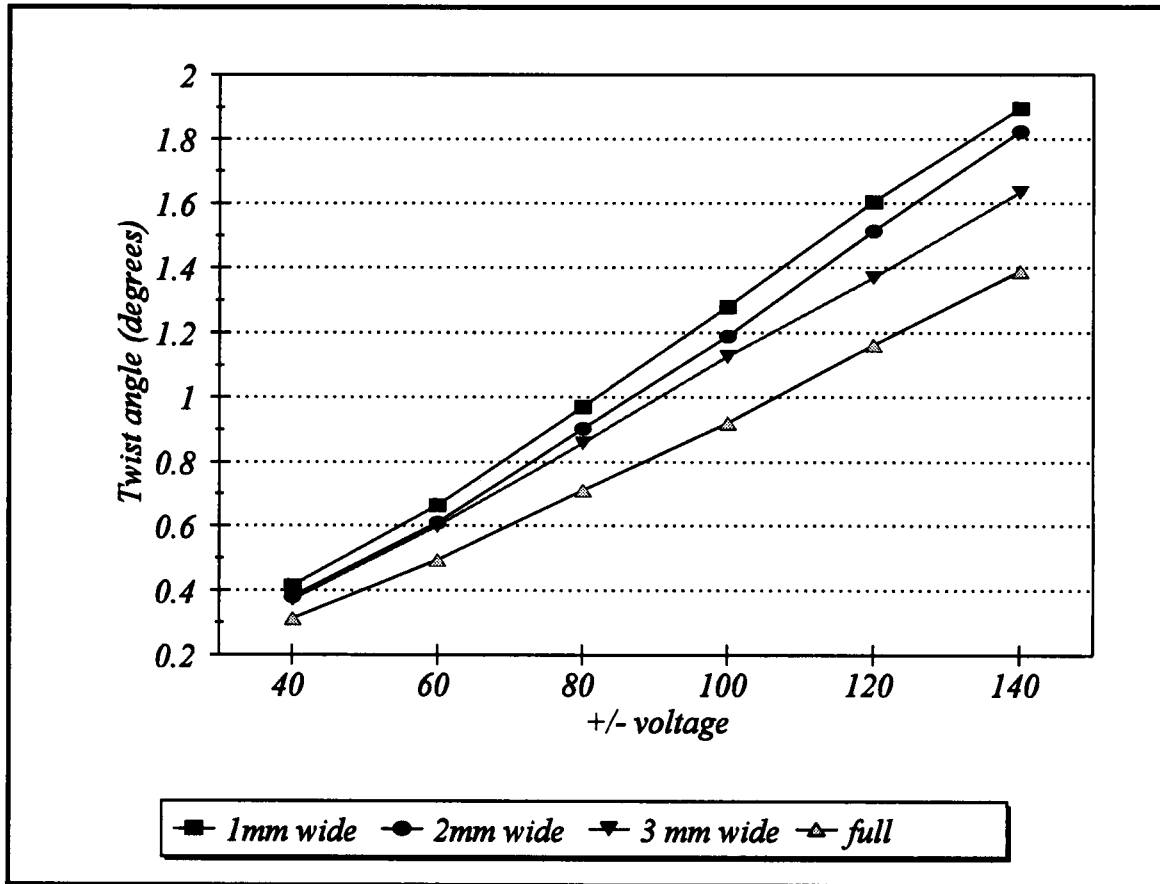


Figure 2.8 Effect of bonding strip width on the magnitude of twist angle.

CHAPTER 3

COMPOSITE TUBE MANUFACTURING

Composite tubes were made of fiberglass and epoxy resin for use in the bending and torsional vibration experiments. The geometry of each tube had to be uniform and precise to prevent possible failures caused by shape imperfections [10]. Furthermore, a high quality tube will more closely match the ideal tube assumed in the analytical model. A procedure was designed for fabricating the high-quality tubes. It includes the following steps:

1. Preparation of square cross-section polystyrene mandrel.
2. Saturation of fiberglass cloth with epoxy resin and fabrication of square cross-section tube.
3. Application of vacuum and pressure to the assembled tube.
4. Removal of the mandrel.

3.1 Mandrel Preparation

In order to use the commercially available flat piezoelectric elements, a square cross-section tube was chosen as the foundation for the piezoelectric elements. A square cross-section mandrel was used to shape the tube. The mandrel had to be removed after the tube was formed. Polystyrene foam material [1], which could be trimmed to the desired size by a hot wire and could be dissolved by acetone after the tube was cured, met the requirements and therefore was chosen as the material for the mandrel.

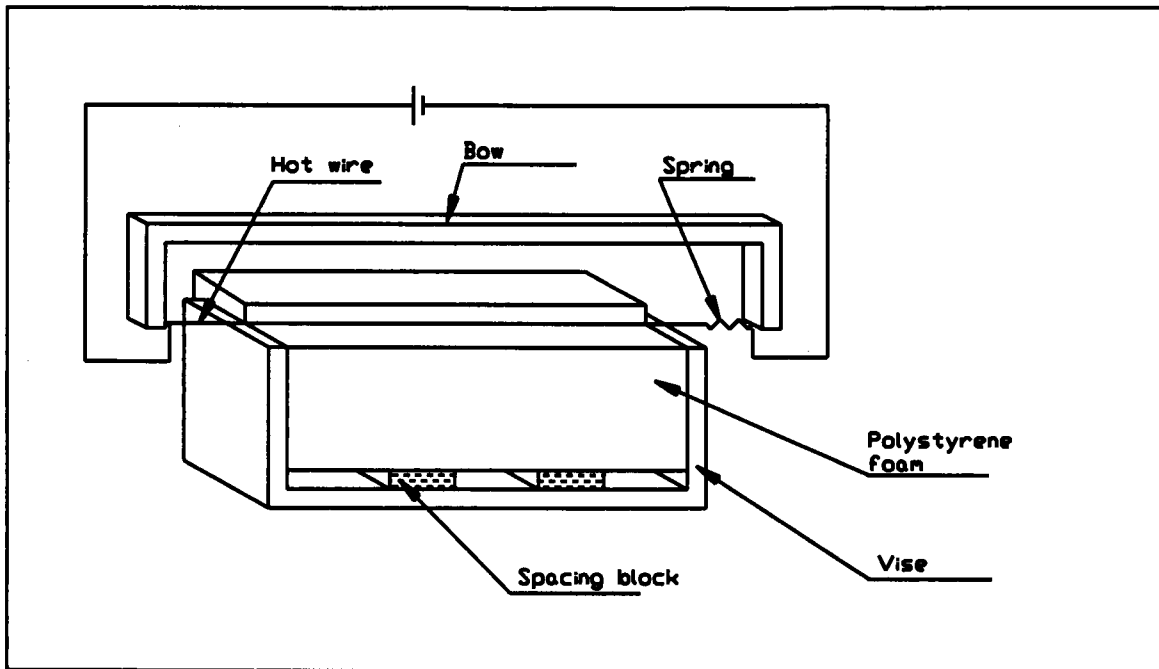


Figure 3.1 Mandrel preparation.

The polystyrene mandrel was made by using a specially designed cutting device. The device consisted of square profiles, spacing blocks, a bow, and a hot wire stretched by a spring, as shown in Fig. 3.1. The spring kept the wire straight by maintaining tension in the wire when it was hot. A block of polystyrene with 20 mm extra thickness material on each side was placed in the device for processing. The hot wire was first moved along the profiles to trim three sides of the block, with the fourth edge being trimmed after the foam block was flipped 180 degrees. Two spacing blocks were placed under the foam so that the distance from the surface of the spacing blocks to the top edge of the vise profile was equal to the desired width for the mandrel. Fig. 3.2 is an actual picture of the cutting device.

Since the device was designed to produce only a few mandrels for the experiments, it was simple and easy to make, but some practice was required before a quality mandrel could be made. The temperature of the hot wire should be kept as low as possible in order

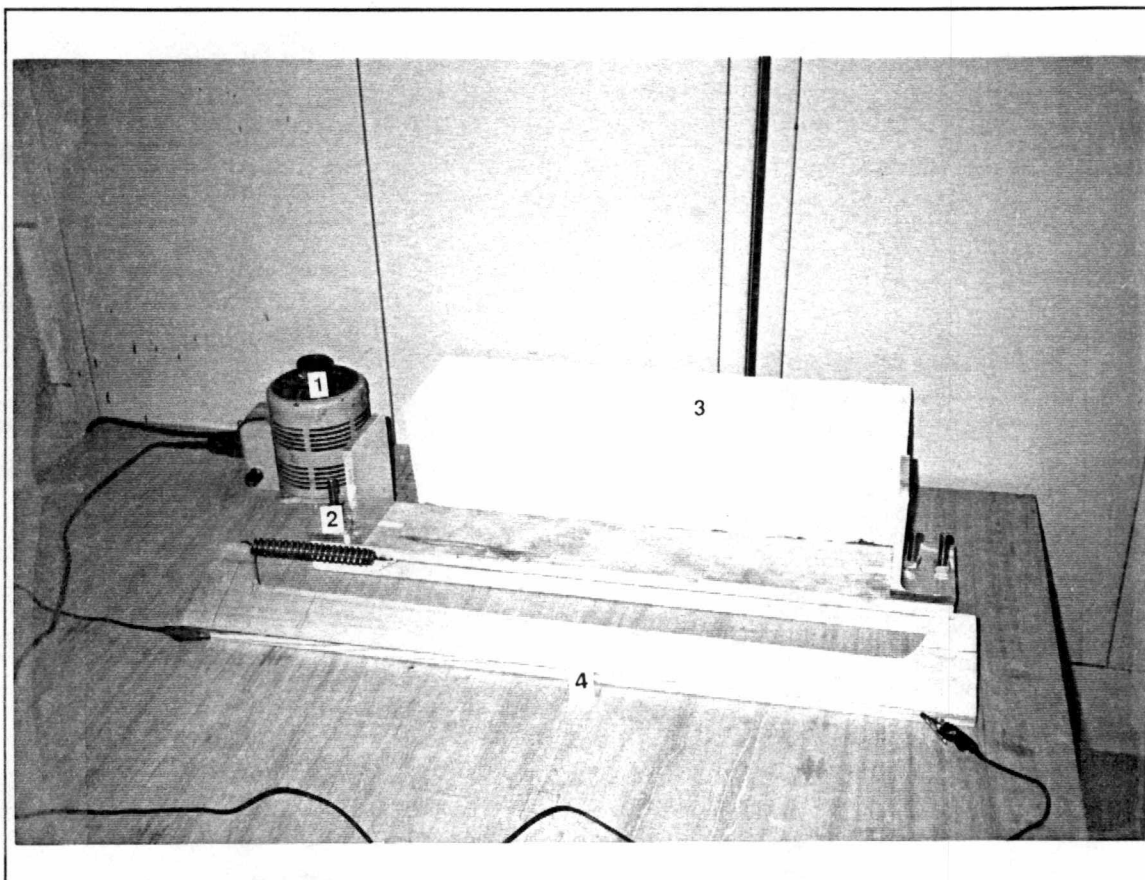


Figure 3.2 Picture of the mandrel cutting device.

to produce a smooth surface and to allow the operator enough time to maintain steady and uniform movement along the profile. The size of the profile should be 1 mm larger than the actual desired size of the mandrel in order to compensate for the heat shrinkage of the polystyrene foam during the process.

Due to flaws in the raw material and speed variations during the cutting process, it was inevitable, even with the most careful operation, that some imperfections were left on the surface of the mandrel. These imperfections were corrected by glossing the surface over with plaster and sanding down. In addition, the core was wrapped longitudinally with plastic

packaging tape to separate the core from the epoxy resin. Otherwise, contacts between epoxy and polystyrene material would have caused a chemical reaction eroding the mandrel surface.

3.2 Resin and Fiberglass Preparation

The fiberglass (9 oz Fabric, 0.008" thick, style 7781 8 Harness Satin weave) and low toxicity lamination epoxy resin (88/87-A from Fibre Glast, Dayton, OH) were chosen for fabrication of the composite tube. The resin was prepared by mixing 5 parts of resin 88 with 1 part of hardener 87 (by volume). One square meter of fiberglass required about 175 ml of mixture for complete saturation. It was noticed that at room temperature the mixture would gel in about three hours and would become very firm in about 60 hours. It usually took a week to complete the curing process. In the final product, the weight ratio of fiberglass to epoxy resin was approximately 3 to 1.

3.3 Specimen Formation

The entire tube fabrication process had to be limited to three hours to avoid the gelling of the epoxy resin. Much care was taken in the process to ensure the quality of the tube. The first challenge encountered was how to stabilize the shape of the fiberglass cloth. Because the harnesses of fiberglass cloth were very smooth and loosely woven, the harnesses slid over each other, making the edges uneven. For the same reason, the overall shape of the fiberglass cloth was also not stable during the operation. This made the operation difficult and affected the quality of the tube. The problem was solved by reinforcing the edges with plastic tape and

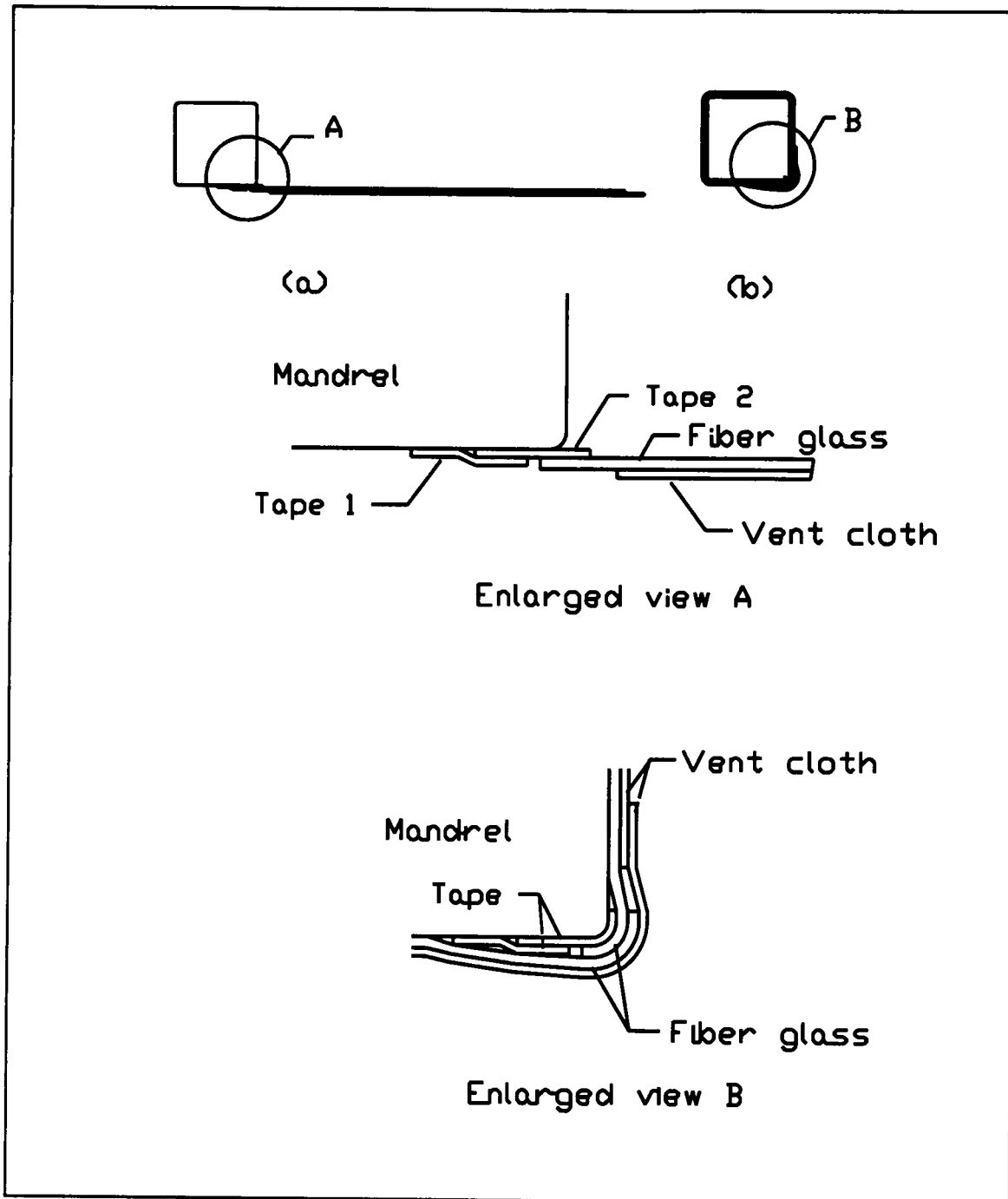


Figure 3.3 Tube fabrication process.

the main fiberglass cloth area with a porous vent cloth. The plastic tape must be an impermeable material; the porous cloth could be perforated or PTFE (polytetrafluoroethylene)-

coated glass cloth. As shown in Fig. 3.3, one edge of the fiberglass cloth was taped to keep the edge even, while the rest of the cloth was reinforced by a piece of porous vent cloth. The resin was sprayed onto the fiberglass and evened out by a pin roller. Taking advantage of the adhesiveness of the uncured resin, the saturated fiber and porous vent cloth were wrapped around the prepared mandrel. The overlap was located at one corner of the tube so that it would not interfere with the piezoelectric elements attached on four sides of the tube. A layer of absorbent cloth (or bleeder cloth) was wrapped around the outside of the vent cloth to absorb the excess epoxy resin.

3.4 Vacuum and Pressure Application During the Curing Process

Pressure and vacuum were applied by using a vacuum bag during the curing process to eliminate bubbles and shape imperfections. Within the first three hours after resin was mixed, the composite tube, along with its mandrel, vent cloth and absorbent cloth, was placed in the vacuum bag from which the air and the volatile gases from the curing process were then vacuumed out. This method eliminated the bubbles produced during the curing process and also applied atmospheric pressure to the tube, to conform the profile of the mandrel. The vacuum was applied for about four hours and then removed. At this point, the resin had gelled but was not completely cured. The removal of vacuum allowed the stressed parts of the tube to bounce back, preventing the formation of wrinkles. The amount of vacuum pressure was also important in controlling the quality of the final product: too much vacuum will cause excessive resin flow, resulting in a resin starved porous laminate. A vacuum pressure of -20 KPa was used during the process.

3.5 Core Removal

After about three days, the curing was completed and the mandrel could be removed. The two ends of the tube were cut with a band saw to obtain the desired length. After the polystyrene was dissolved by acetone, a complete tube was now fabricated. However, a few more days were usually required before it could be used for the experiment.

CHAPTER 4

TORSIONAL ACTUATOR TESTING

The setups of the torsional test include a specimen of thin-walled composite tube with attached piezo elements, a device which holds the specimen, and a set of equipment for measuring, controlling and exciting torsional motions.

4.1 The Torsional Holding Device

A special device was designed to ensure that the composite tube was only subjected to a pure torque in the torsional test. The design criteria for the device were as follows:

1. The device had to accommodate various sizes and shapes of thin-walled tubes, ranging from 20 mm long x 20 mm wide x 100 mm high to 100 mm long x 100 mm wide x 500 mm high [2].
2. The device had to eliminate lateral movement of the tube and only allow rotation about its longitudinal axis of symmetry [2].
3. The device had to allow the introduction of pure torque or twist without coupling with the forces or strains along the lateral or longitudinal directions [2].
4. The device had to be rigid, with the fittings tight enough to prevent free play in the device [2].

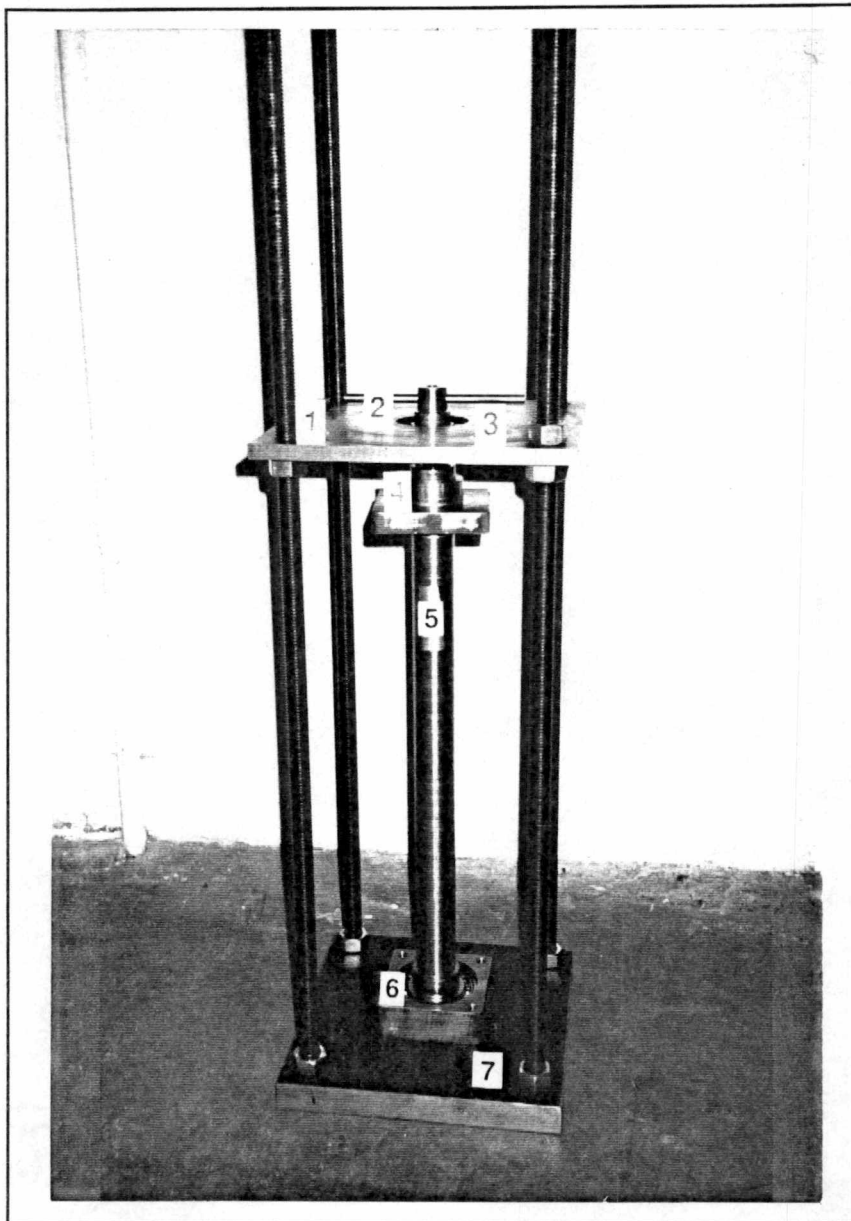


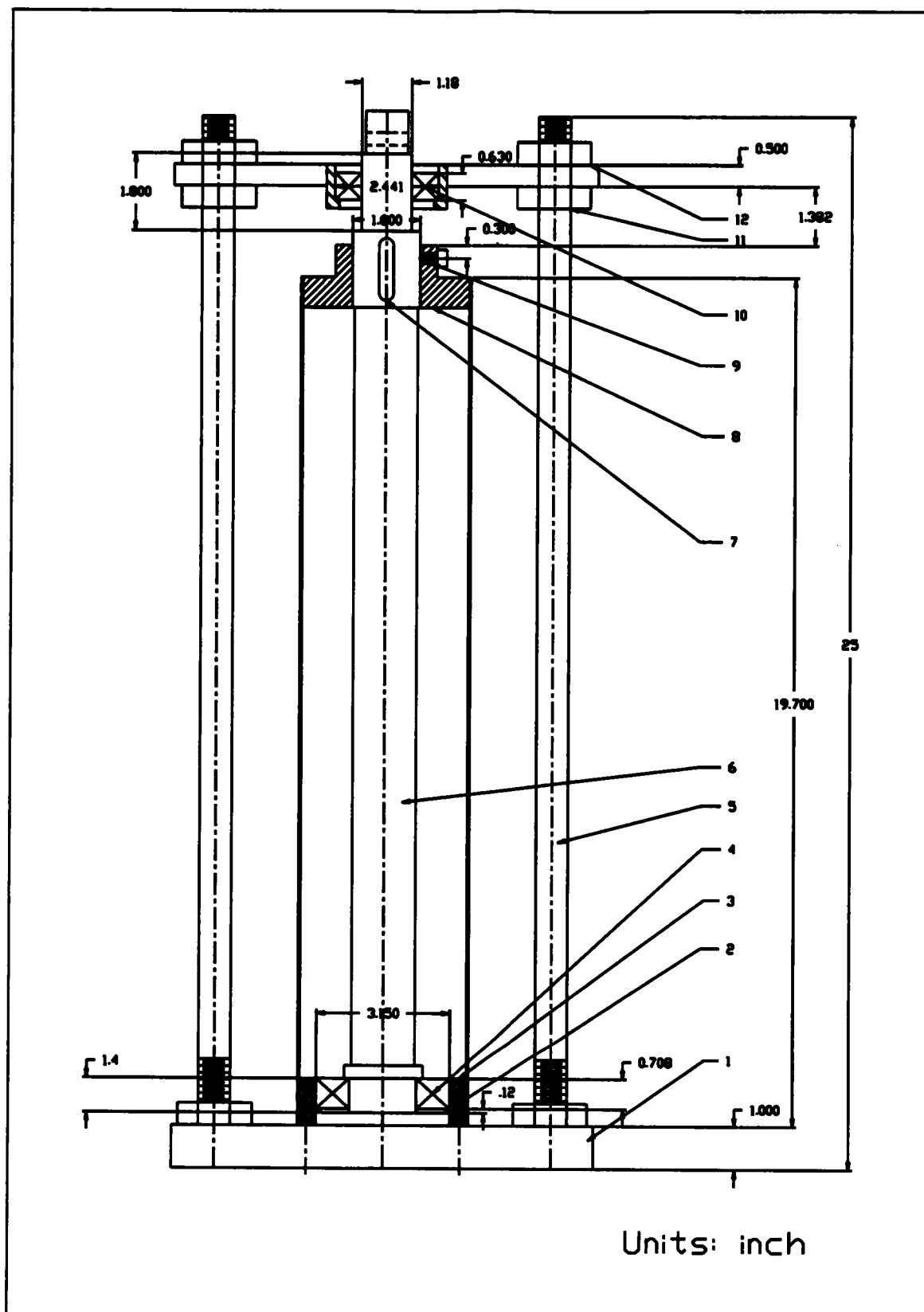
Figure 4.1 Picture of the testing stand.

The testing device is shown in Fig. 4.1. The frame structure of the device consisted of two base plates, 3 and 7, and four threaded rods. With these threaded rods, the height of the device could be adjusted from zero to its maximum value of 500 mm. The threads on the four columns could also be used to attach testing equipment during the experiments. Parts number 4 and 6 serve as bearing housings for the lower and upper bearings,

respectively. The lower bearing was an angular-contact ball bearing which carried the weight of the shaft and other possible longitudinal forces. One end of the test specimen was fitted along the outer profile of the lower bearing case; the other end was fitted on cap 4, which was rigidly connected to the shaft. The fittings at both ends of the tube were also secured by hose clamps. This arrangement left one end of the composite tube to rotate freely with the shaft while the other end was fixed to the device. The shaft could be used to measure torsional displacement as well as to apply a pure torque to the tube. Fig 4.2 is an assembly drawing of the holding device (units are in English system for manufacturing convenience), giving a more detailed configuration of the holding device.

4.2 The Tube

A composite thin-walled tube of 60 mm x 60 mm x 500 mm with wall thickness of 0.4 mm and a fiber orientation 0 x 90 degrees with respect to its longitudinal axis was used as the substrate for the piezo elements. Fig. 4.3 is a front view of the composite tube with attached piezo elements. It shows two banks of vertically aligned piezo elements. The elements in one bank were oriented at +45 degrees from the longitudinal axis of the tube and the elements in the other bank were oriented at -45 degrees. This arrangement gives the most effective generation of torque or twist. In the same configuration, more piezo elements were attached at the other three sides of the external wall and four sides of internal wall. The elements attached inside of the tube were the mirror images of the elements attached outside of the tube about the middle plane of the wall. A total of 64 elements were used, grouped in 16 banks. As shown in the A-A cross-section, holes were drilled in the substrate of the tube to



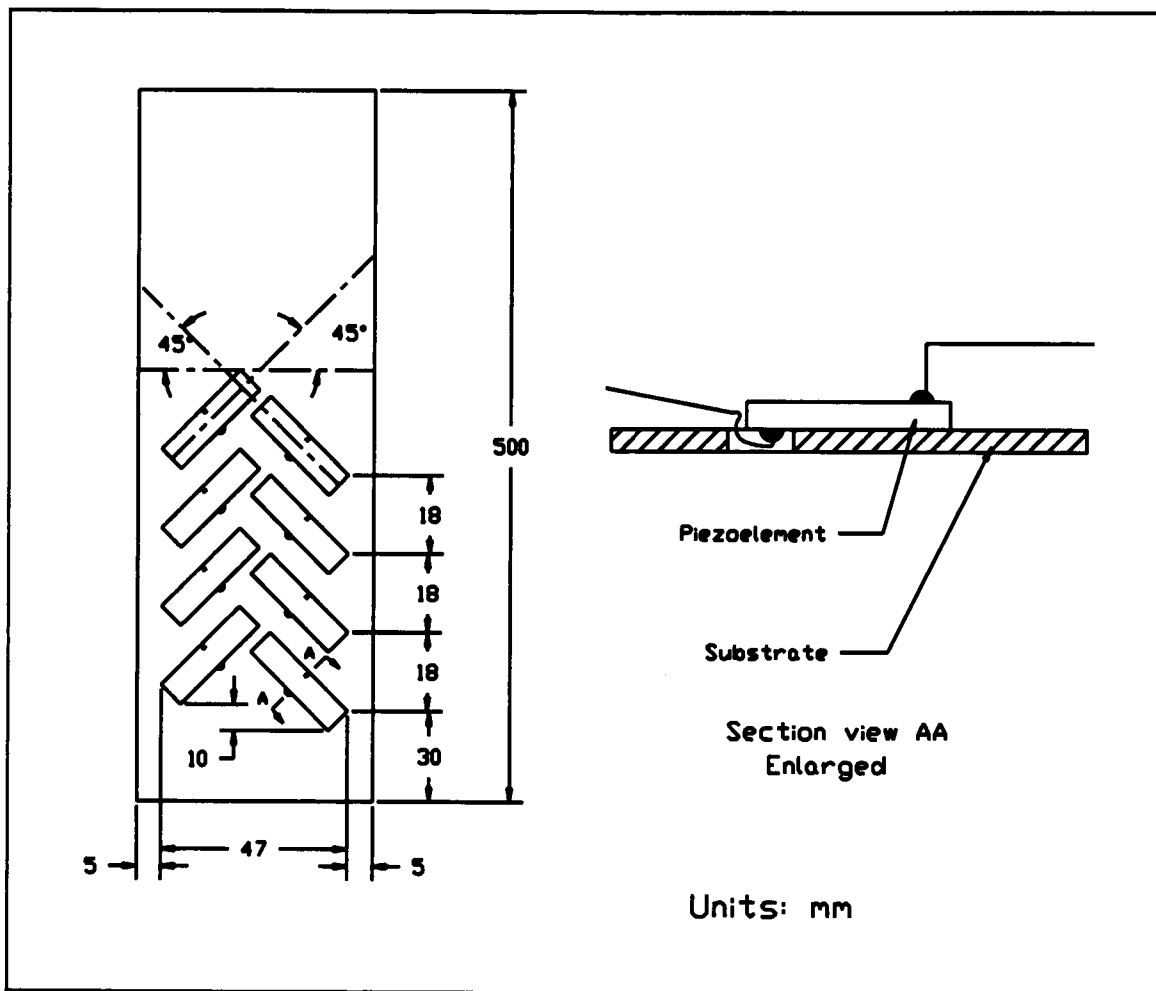


Figure 4.3 A front view of a square cross-section composite tube with piezo elements attached on its surface.

accommodate the soldering spots and to ensure close bonding between the piezo elements and the substrate. The electrical wires were also led out from these holes.

4.3 Tests to Determine Tube's Mechanical Characteristics

The material properties of the composite material, such as Young's modulus, shear modulus, density, etc., are the essential information for the tube analysis.

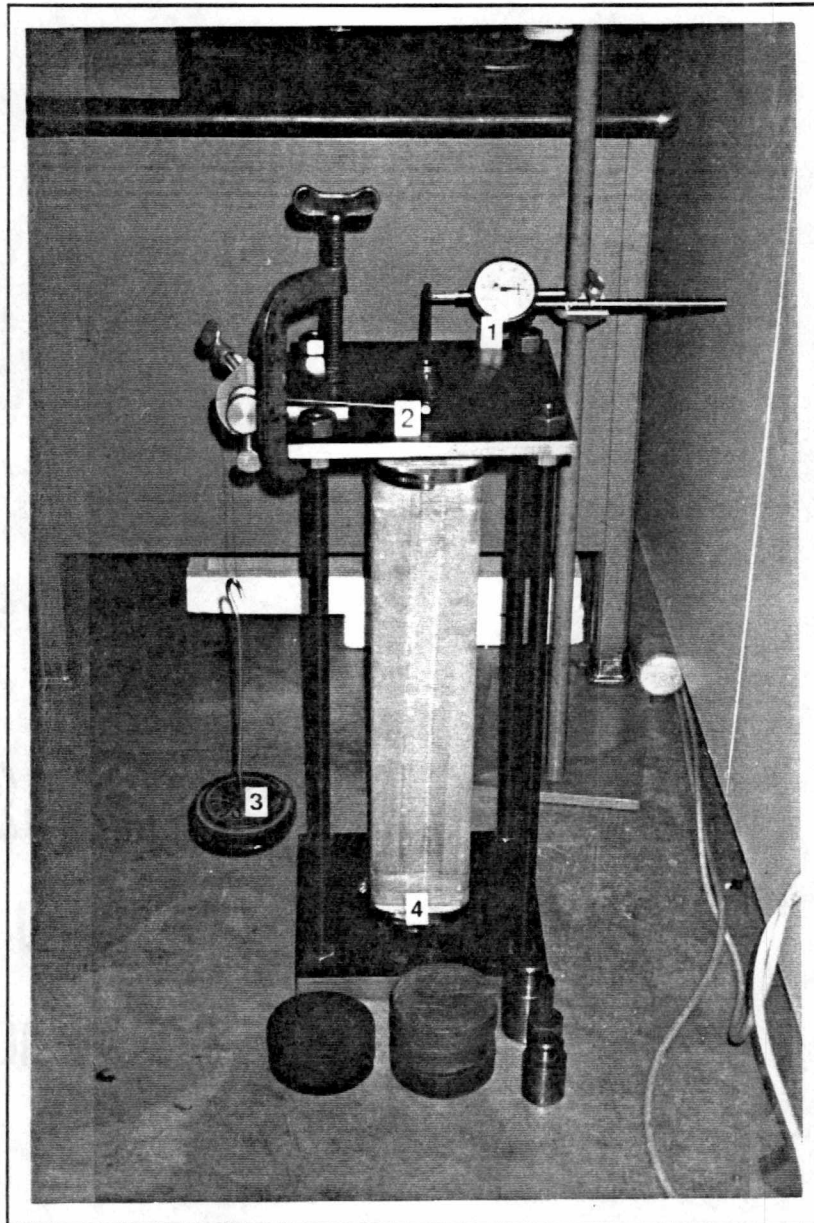


Figure 4.4 Torsional test setup to determine the shear modulus of the composite material.

To determine the shear modulus of the material, a static torsional test was performed on a 60 mm x 60 mm x 500 mm thin-walled tube, with wall thickness of 0.4 mm. The test device described in section 4.1 was utilized as shown in Fig. 4.4. The bar attached to the top of the shaft could be used for torque application and twist measurement. As a force was

applied at one end of the bar, the resulting twist displacement was measured at the other end of the bar by a dial indicator. The actual force was applied by standard weights through a string and pulley assembly. The experimental data showed that rotational displacement and the applied torque had a linear relationship; the shear modulus of the composite material could be easily calculated from:

$$\frac{d\theta}{dz} = \frac{T}{G J} \quad (4.1)$$

which gives $G = 1.047$ GPa for the tube when the fibers of the tube are oriented at 0×90 degrees. The detailed data and calculations are shown in Appendix D.

Although this setup could be used to determine the Young's modulus of the tube, a more direct vibration test was used instead. Fig. 4.5 shows a front view of the test setup. A one-layer composite beam, measuring $75 \text{ mm} \times 29 \text{ mm} \times 0.4 \text{ mm}$, was clamped at one end and free at the other end. Additional mass of 2 grams was attached to the tip of the beam to bring the natural frequency of the beam into a range that could be visually counted. The first step in determining the Young's modulus of the beam was to calculate the beam spring constant from the following formula:

$$\omega_n = \sqrt{\frac{k}{M+m/3}} \quad (4.2)$$

in which ω_n is the measured natural frequency, k is the beam spring constant, M is the attached mass and m is the mass of the beam. The spring constant was the only unknown and could be determined. Then the second formula was used to calculate the Young's modulus:

$$k = \frac{3EI}{l^3} \quad (4.3)$$

in which I and l are related to the geometry of the beam, k being calculated in Eq. 4.2, so that E could be determined. The mechanical properties of the composite material are summarized in Table 4.1.

Table 4.1 Summary of composite material properties

	One Layer [0°/90°]	One Layer [45°/45°]	Two Layer [0°/90°]
Thickness (mm)	0.4	0.4	0.71
Density (g/m ³)	1102	1102	1267
E (GPa)	6.0	5.6	NA
G (GPa)	1.047	NA	NA
G J (60 mm x 60 mm)	88.56 Nm ²	NA	NA

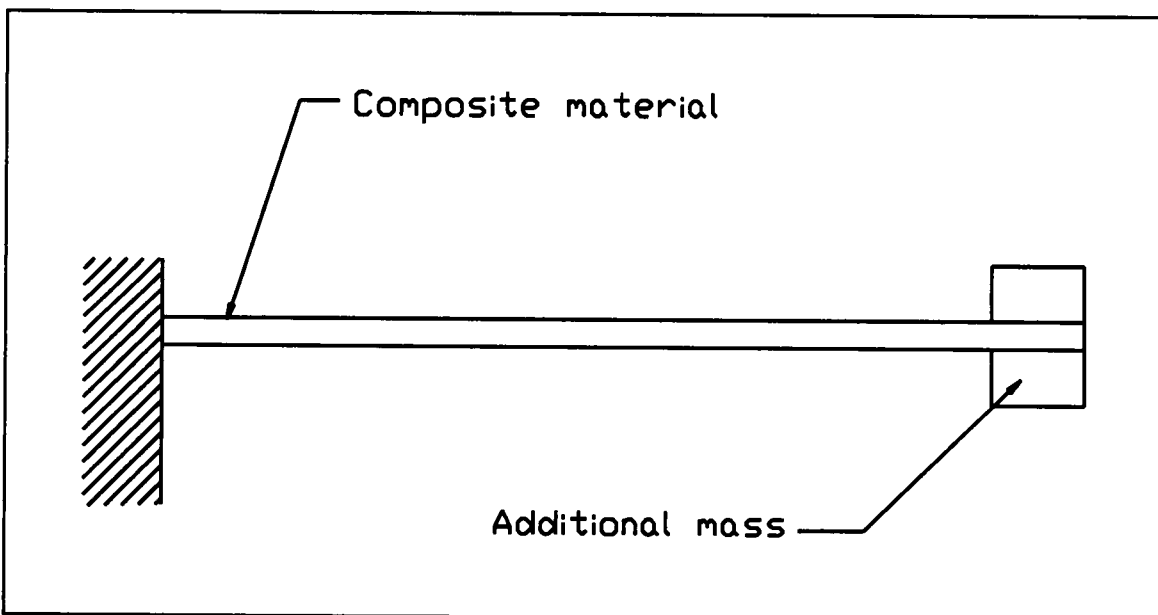


Figure 4.5 Setup for beam vibration test to calculate Young's modulus.

4.4 Dynamic Test of the Tube

The dynamic test of the tube focused on the capability of the piezo actuator to control torsional vibrations. A direct feedback circuit was used for vibration control. A signal generator produced alternating voltage with the maximum amplitude of 0.9 volts. It was amplified by two amplifiers to an alternating voltage with a peak value of 135 volts and was applied to the piezo elements. The two banks of piezo elements in Fig. 4.6 had to extend or contract in opposite directions to generate a torsional vibration on the beam. This was ensured by wiring the front side of the piezo elements in one bank to the backside of the

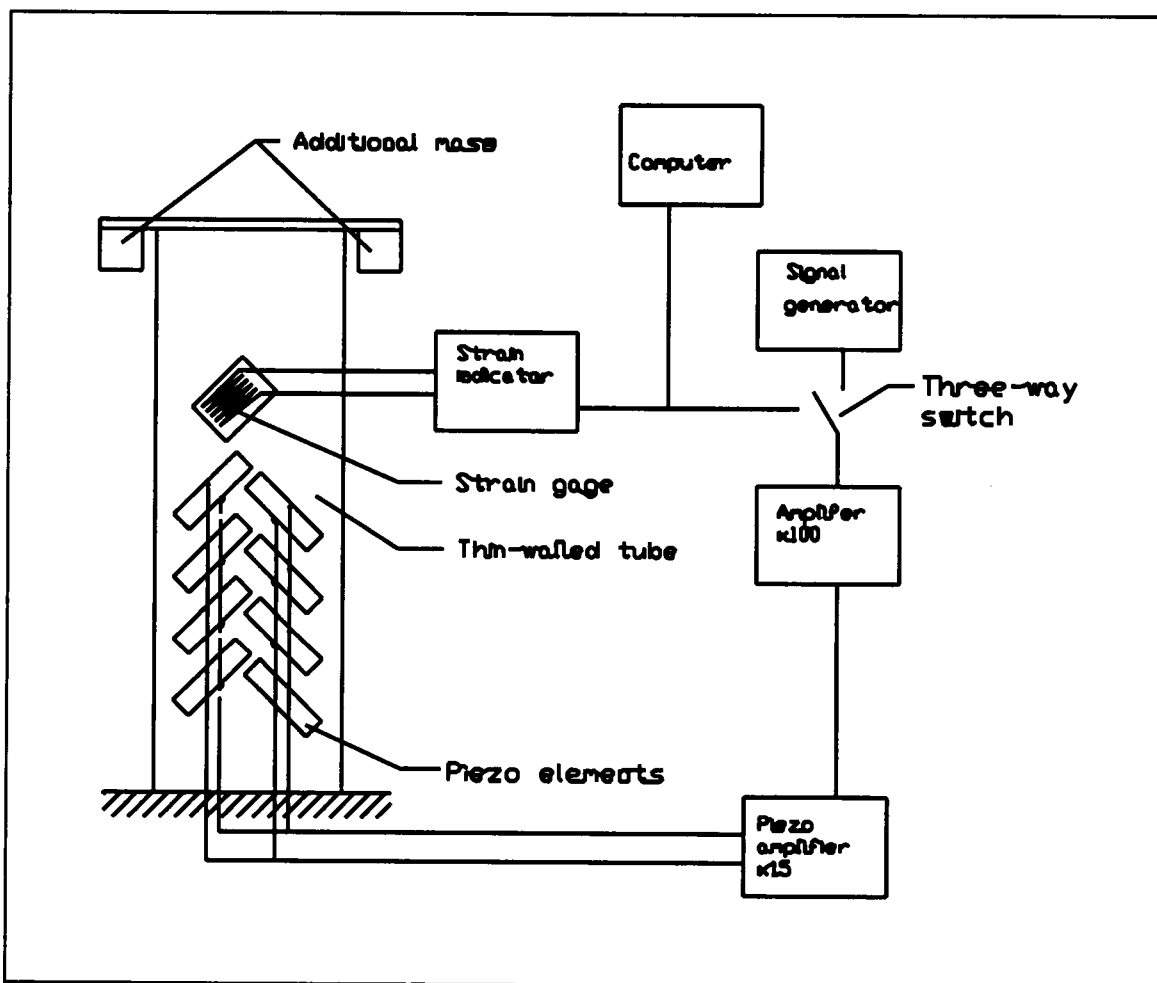


Figure 4.6 Circuit for torsional vibration control.

elements in the other bank. The dashed lines in Fig. 4.5 represent the wires connected to the backside of the piezo elements. The actuator excited the beam to vibrate at its torsional natural frequency. After the beam was excited, the switch shown in Fig. 4.6 was shifted to either the neutral position to allow natural vibration damping or to the position which invokes the direct feedback control circuit. Therefore, one naturally damped vibration time history and one actively damped vibration time history were obtained. These decaying torsional vibrations were represented in Fig. 4.7 with a dashed line and a solid line, respectively. The calculation in Appendix E revealed that when the piezo elements were inactive, the tube had

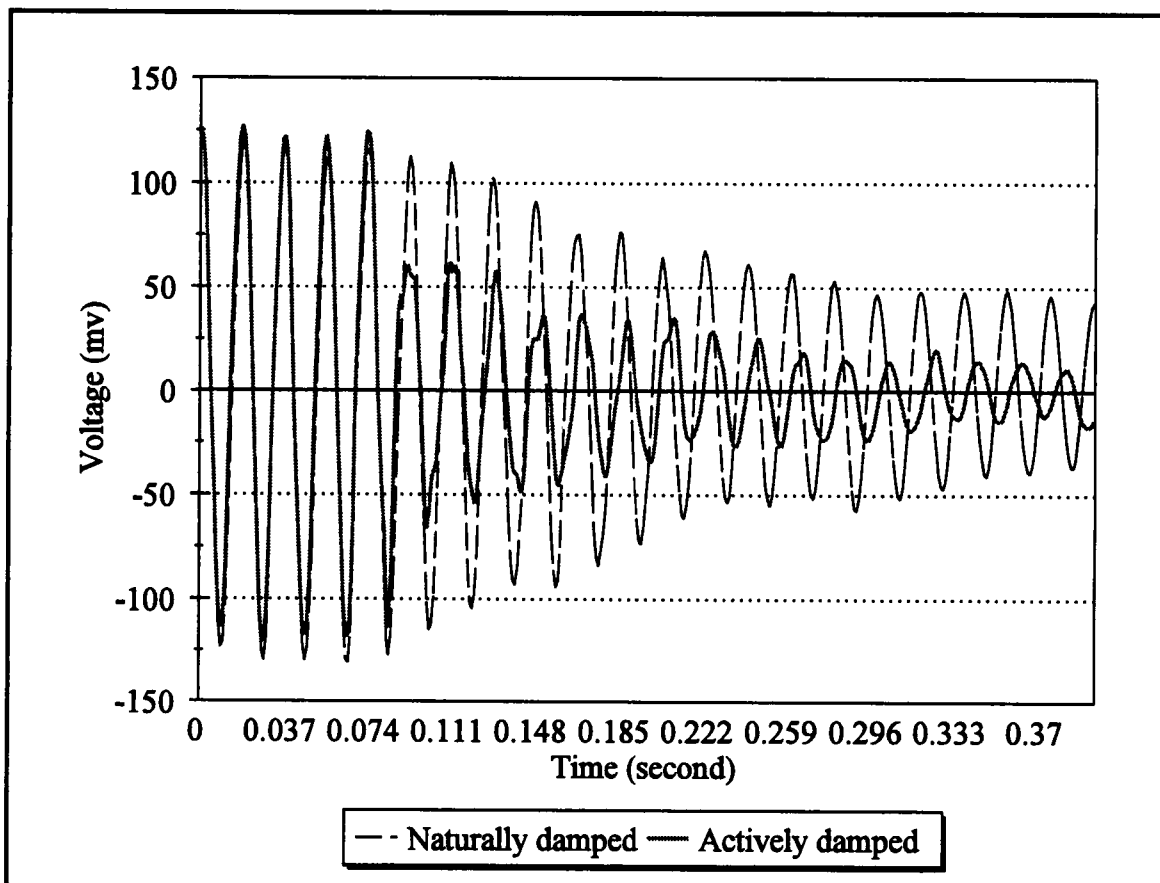


Figure 4.7 Comparison of naturally damped versus actively damped vibrations.

a damping coefficient of 0.0277 Ns/m; when feedback control was invoked and approximately 120 volts were applied to the piezo elements, the overall damping coefficient of the beam was 0.2436 Ns/m. The energy absorbed by the 64 piezo elements was 0.179 joules. The active damping was no longer effective after the vibration amplitudes dropped to about half of maximum amplitude.

Active damping by the feedback control system was ineffective after the amplitude of vibration was significantly reduced. This could be explained by noting that the feedback voltage was proportional to the amplitude of the vibration. It became smaller as the amplitude was reduced. The damping would have been much more effective if a more sophisticated circuit with variable gain were used.

CHAPTER 5

STRUCTURAL MODELING OF A FLAT BEAM WITH ANTISYMMETRICALLY INDUCED STRAIN ACTUATORS

To exploit the potential benefits of smart structure technology, one important approach is to develop an accurate mathematical model which will comprehensively include the substructure and the actuators of the smart structure. This part of the research focused on the modelling of a thin-walled flat beam, which resembles the simplest element of a thin-walled structure. Since fairly accurate mathematical models have already been developed for the bending of thin-walled beams [13] by piezo actuation, this research will describe only the torsional modelling of the beam actuated by antisymmetrically attached piezo elements.

5.1 Approach

This analysis is based on Vlasov's theory for dealing with a thin-walled beam. It uses three sets of coordinate systems, as shown in Fig. 5.1: an orthogonal right-handed Cartesian coordinate system (x, y, z) for the beam; an curvilinear coordinate system (n, s, z) for any plate segment of the beam; and a contour coordinate system (s) . In the curvilinear coordinate system, the n -axis is normal to the mid-surface of any plate segment of the beam, and the s -axis is tangential to the mid-surface along the contour line of the plate, and the z axis is parallel to the longitudinal axis of the beam. In the contour coordinate system, s is measured along the contour line of the cross section from a judiciously selected origin (for ease of

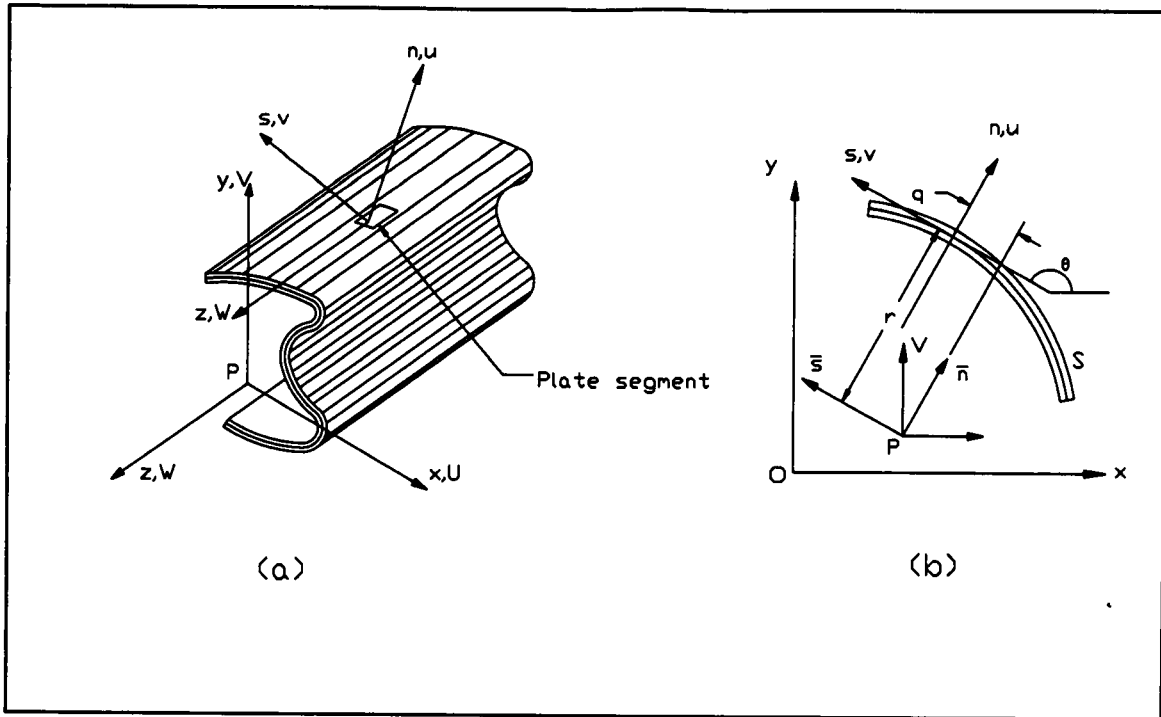


Figure 5.1 Coordinates used in the analysis. a) The coordinates in an open-section beam; b) beam displacements and sign convention.

calculation). Two sets of load systems are used: the generalized beam forces, $V_x, V_y, N_z, M_x, M_y, T, M_\omega$, and M_c ; and the plate loads, $N_z, N_s, N_{zs}, M_z, M_s$, and M_{zs} . The calculation of twist magnitude for a beam subjected to piezo strains can be subdivided into several steps: the first step is to find the stiffness of the beam; the second step is to calculate the plate loads induced by the piezo actuators; the third step is to convert plate loads to beam loads; and the final step is to find the amount of twist from the relation between beam loads, beam stiffness, and strains. Although generalized beam forces include eight terms ($V_x, V_y, V_z, M_x, M_y, T, M_\omega$, and M_c), only M_x, M_y , and T are related to the twist of the beam.

5.2 Kinematics

All the symbols used in the analysis are defined in the Nomenclature at the beginning of the thesis. For quick reference, Table 5.1 gives a list of the variables and their associated coordinate systems.

Table 5.1 Symbols used in the analysis and their associated coordinate system

	Beam coordinates (x, y, z)	Plate coordinates (n, s, z)
Linear displacement	$U(z), V(z), W(z)$	$u(s, z), v(s, z), w(s, z)$
Rotation	ϕ_x, ϕ_y, ϕ_z	θ
Strain	$\epsilon_{zx}, \epsilon_{yz}$	$\epsilon_{zs}, \epsilon_z, \epsilon_s$
Loads	$N, M_y, M_x, T, M_\omega, F_x, F_y, G_x, G_y,$	$N_z, N_s, N_{zs}, M_z, M_s, M_{zs}$

From geometric considerations, as shown in Fig. 5.1, generalized beam displacements are related to the plate displacements as follows [13]:

$$u(s, z) = U(z) \sin \theta(s) - V(z) \cos \theta(s) - q(s) \phi_x(z) + \alpha \psi(z) \quad (5.1)$$

$$v(s, z) = U(z) \cos \theta(s) + V(z) \sin \theta(s) + r(s) \phi_x(z) \quad (5.2)$$

$$w(s, z) = W(z) + x \phi_x(z) + y \phi_y(z) - \lambda \phi_x'(z) \quad (5.3)$$

The expressions for $u(s, z)$ and $v(s, z)$ can be easily verified by observing Fig. 5.1. The expression for $w(s, z)$ is not so visually obvious; however it can be easily derived from the strain-displacement relation--Eq. (5.4), beam transverse shear expression--Eq. (5.5), and plate transverse displacement expression--Eq. (5.2). Eqs. (5.4) and (5.5) are listed as follows:

$$\epsilon_{zz} = \frac{\partial w}{\partial s} + \frac{\partial v}{\partial z} \quad (5.4)$$

$$\epsilon_{zz} = \epsilon_{xz} \cos \theta + \epsilon_{yz} \sin \theta \quad (5.5)$$

In Eq. 5.3, λ , ϕ_x , and ϕ_y , are defined as follows:

$$\lambda = \int_s r ds \quad (5.6)$$

$$\phi(x)_x = \epsilon_{xz} - U' \quad (5.7)$$

$$\phi(x)_y = \epsilon_{yz} - V' \quad (5.8)$$

At this point, the membrane strains and curvatures associated with a generic plate segment can be obtained in terms of bar displacements in beam coordinates. They can be expressed using the following equations:

$$\epsilon_z = w' = W' + x \phi'_x + y \phi'_y - \lambda \phi'_z \quad (5.9)$$

$$\epsilon_x = \dot{v} = (-U \sin \theta + V \cos \theta) \dot{\theta} + \dot{r} \phi_x \quad (5.10)$$

$$\epsilon_{zz} = \epsilon_{xz} \cos \theta + \epsilon_{yz} \sin \theta \quad (5.5)$$

$$k_z = u'' = -\sin \theta \phi'_x + \cos \theta \phi'_y - q \phi''_z + \epsilon'_{xz} \sin \theta - \epsilon'_{yz} \cos \theta - \alpha \psi'' \quad (5.11)$$

$$k_x = \ddot{u} = \ddot{\alpha} \psi \quad (5.12)$$

$$k_{zz} = 2 \dot{u}' = -2(\phi'_z + \dot{\alpha} \psi') \quad (5.13)$$

Eqs. (5.9), (5.10), and (5.5) constitute the strain vector $\epsilon = \{\epsilon_z, \epsilon_x, \epsilon_{zz}\}^T$; and Eqs. (5.11), (5.12), and (5.13) constitute the curvature vector $k = \{k_z, k_x, k_{zz}\}^T$. These vectors can be used to find the plate loads of the thin-walled beam by the classical laminated plate theory as follows:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon \\ k \end{Bmatrix} \quad (5.14)$$

in which, N is the force vector ($N = \{N_z, N_s, N_{zs}\}^T$); M is the moment vector ($M = \{M_z, M_s, M_{zs}\}^T$); A , B , and D are extensional, coupling, and bending matrices determined by the physical sizes and mechanical properties of the substructure. They will be described in section 5.3.

The energy method and virtual work theory are used for further development. The strain energy of the plate segment is:

$$\Pi = \frac{1}{2} \int_s (N_z \epsilon_z + N_s \epsilon_s + N_{zs} \epsilon_{zs} + M_z k_z + M_s k_s + M_{zs} k_{zs}) ds = \frac{1}{2} \int_s (N \cdot \epsilon + M \cdot k) ds \quad (5.15)$$

The full expansion of the above equation by using the definitions in Eqs. (5.9), (5.10), (5.5), (5.11), (5.12), (5.13), and (5.14) will lead to a long expression for Π , which depends on stiffnesses, A , B and D , as well as beam displacements and their first or second order derivatives. It is rather tedious to reorganize the terms in the long expression into a physically meaningful form. This process can be simplified by realizing that the strain energy is equivalent to the virtual work of the bar:

$$\Pi = W_i = NW' + M_y \phi_x' + M_x \phi_y' + T \phi_z' + M_\omega \phi_z'' + F_x \epsilon_{xz}' + F_y \epsilon_{yz}' + G_x \epsilon_{xz} + G_y \epsilon_{yz} + M_c \psi + M_h \psi' + I_\omega \psi'' \quad (5.16)$$

Π can then be symbolically expressed as $\Pi(W', \phi_x', \phi_y', \phi_z', \phi_z'', \epsilon_{xz}', \epsilon_{yz}', \epsilon_{xz}, \epsilon_{yz})$, in which W' , ϕ_x' , ϕ_y' , ϕ_z' , ϕ_z'' , ϵ_{xz}' , ϵ_{yz}' , ϵ_{xz} , and ϵ_{yz} are the degrees of freedom of strain energy. A matrix form that relates beam displacements and beam loads can be established by applying Castigliano's first theorem:

$$\begin{bmatrix} N \\ M_x \\ M_y \\ M_\omega \\ T_s \\ G_x \\ G_y \\ F_x \\ F_y \\ M_c \\ M_h \\ I_\omega \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} & K_{17} & K_{18} & K_{19} & K_{110} & K_{111} & K_{112} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} & K_{27} & K_{28} & K_{29} & K_{210} & K_{211} & K_{212} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} & K_{37} & K_{38} & K_{39} & K_{310} & K_{311} & K_{312} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} & K_{47} & K_{48} & K_{49} & K_{410} & K_{411} & K_{412} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} & K_{57} & K_{58} & K_{59} & K_{510} & K_{511} & K_{512} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} & K_{67} & K_{68} & K_{69} & K_{610} & K_{611} & K_{612} \\ K_{71} & K_{72} & K_{73} & K_{74} & K_{75} & K_{76} & K_{77} & K_{78} & K_{79} & K_{710} & K_{711} & K_{712} \\ K_{81} & K_{82} & K_{83} & K_{84} & K_{85} & K_{86} & K_{87} & K_{88} & K_{89} & K_{810} & K_{811} & K_{812} \\ K_{91} & K_{92} & K_{93} & K_{94} & K_{95} & K_{96} & K_{97} & K_{98} & K_{99} & K_{910} & K_{911} & K_{912} \\ K_{101} & K_{102} & K_{103} & K_{104} & K_{105} & K_{106} & K_{107} & K_{108} & K_{109} & K_{1010} & K_{1011} & K_{1012} \\ K_{111} & K_{112} & K_{113} & K_{114} & K_{115} & K_{116} & K_{117} & K_{118} & K_{119} & K_{1110} & K_{1111} & K_{1112} \\ K_{121} & K_{122} & K_{123} & K_{124} & K_{125} & K_{126} & K_{127} & K_{128} & K_{129} & K_{1210} & K_{1211} & K_{1212} \end{bmatrix} \begin{bmatrix} W' \\ \phi_y' \\ \phi_x' \\ \phi_x'' \\ \phi_z' \\ \epsilon_{xz} \\ \epsilon_{yz} \\ \epsilon_{xz}' \\ \epsilon_{yz}' \\ \psi \\ \psi' \\ \psi'' \end{bmatrix} \quad (5.17)$$

where $K_{ij} = \partial^2 \Pi / \partial D_i \partial D_j$. D_i and D_j are the symbolic expressions for the degrees of freedom of strain energy Π . The K matrix is symmetric; that is, $K_{ij} = K_{ji}$. This is seen by noting that the order of differentiation does not affect the results. K_{ij} is a function of plate stiffness (A , B , and D), mechanical properties of the material, and physical configurations of the beam. Since the study is limited to finding out how much the bar twists, the above expression can be reduced to:

$$\begin{bmatrix} M_x \\ M_y \\ T \end{bmatrix} = \begin{bmatrix} K_{22} & K_{23} & K_{25} \\ K_{32} & K_{33} & K_{35} \\ K_{52} & K_{53} & K_{55} \end{bmatrix} \begin{bmatrix} \phi_y' \\ \phi_x' \\ \phi_x' \end{bmatrix} \quad (5.18)$$

For a solid rectangular section beam, the elements in K matrix and the beam loads are defined as follows:

$$\lambda = 0, \quad q = s, \quad y = 0, \quad \theta = 0, \quad \alpha = s^2/2 \quad (5.19)$$

$$K_{22} = \frac{\partial \Pi}{\partial \phi'_y \partial \phi'_y} = D_{11} s \quad (5.20)$$

$$K_{23} = \frac{\partial \Pi}{\partial \phi'_y \partial \phi'_x} = \frac{B_{11} s^2}{2} \quad (5.21)$$

$$K_{25} = \frac{\partial \Pi}{\partial \phi'_y \partial \phi'_z} = 0 \quad (5.22)$$

$$K_{33} = \frac{\partial \Pi}{\partial \phi'_x \partial \phi'_x} = \frac{A_{11} s^3}{3} \quad (5.23)$$

$$K_{35} = \frac{\partial \Pi}{\partial \phi'_x \partial \phi'_z} = -B_{16} s^2 \quad (5.24)$$

$$K_{55} = \frac{\partial \Pi}{\partial \phi'_z \partial \phi'_z} = 4D_{66} s \quad (5.25)$$

$$M_x = \frac{\partial \Pi}{\partial \phi'_y} \quad (5.26)$$

$$M_y = \frac{\partial \Pi}{\partial \phi'_x} \quad (5.27)$$

$$T = \frac{\partial \Pi}{\partial \phi'_z} \quad (5.28)$$

The expression for above equations are obtained by executing the Mathematica program in Appendix F, Program 1. Knowing the terms defined in the above equations, beam displacements can be readily solved from Eq. (18). The solutions are, of course, still symbolic because stiffness matrices A, B, and D have not yet been actually defined yet.

5.3 Definition of the Plate Stiffness A, B, and D

The three-layer specimen shown in Fig. 5.2 consists of an aluminum plate as a substrate, one piezo layer bonded to the upper surface of the substrate at 45 degrees with

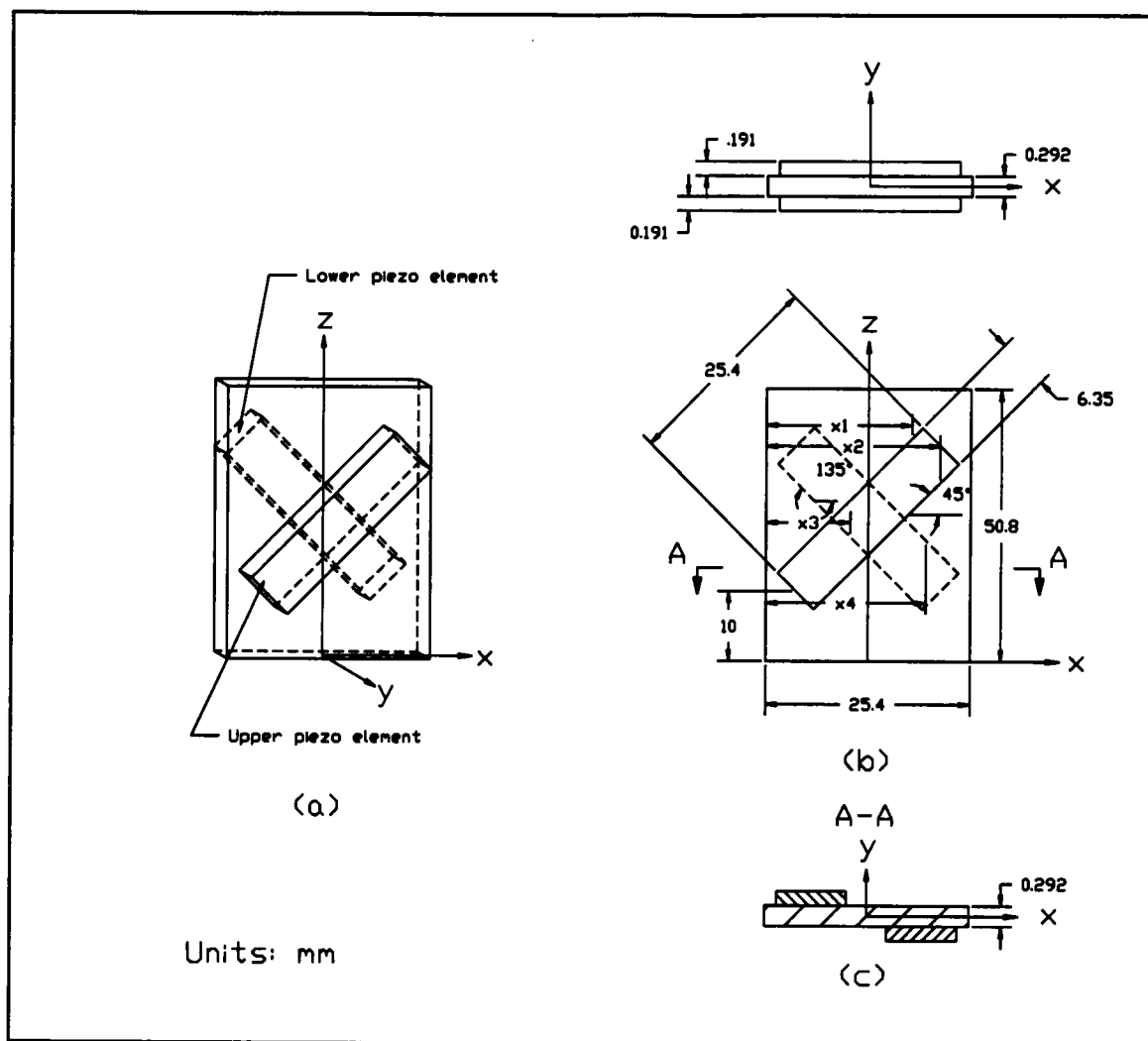


Figure 5.2 Solid beam with antisymmetrically attached piezo elements: a) a three dimensional view of the specimen; b) a two dimensional view of the specimen; c) an arbitrary cross section perpendicular to z-axis.

respect to its longitudinal axis, and another piezo layer bonded to the lower surface of the substrate at - 45 degrees with respect to its longitudinal axis. The adhesive used for bonding was epoxy (ECCOBOND 45 LV/15 LV from Emerson & Cuming, Inc.). The plate stiffness was calculated by using the theories of the mechanics of composite materials. Both piezo elements were subjected to an electrical potential which created a strain on the upper surface

at a 45 degree angle with respect to the z-axis, and a strain on the lower surface at a - 45 degree angle with respect to the z-axis. This causes the beam to twist about its z-axis.

Table 5.2 Specimen specifications

	Substrate	Upper actuator	Lower actuator
Corresponding subscripts	s	u	l
Thickness t (m)	0.000292	0.000191	0.000191
Length L (m)	0.0508	0.0254	0.0254
Width b (m)	0.02694	0.00635	0.00635
Young's modulus E (N/m ²)	6.31×10^{10}	6.31×10^{10}	6.31×10^{10}
Shear modulus G (N/m ²)	2.63×10^{10}	2.63×10^{10}	2.63×10^{10}
Poisson's ratio ν_{12}	0.33	0.33	0.33
Poisson's ratio ν_{21}	0.33	0.33	0.33

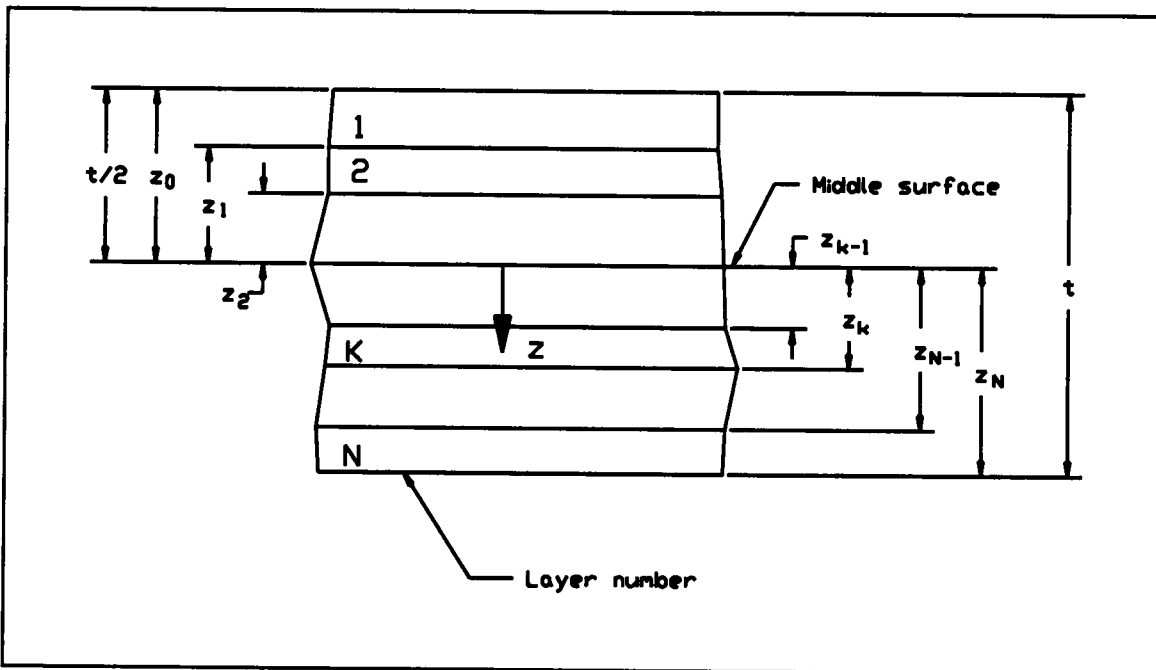


Figure 5.3 Geometry of N-layered laminate.

The generalized formula for one layer of composite material oriented in at angle θ to the x-axis is (Eq. (4.21) in [14]):

$$(A_{ij})_k = (\bar{Q}_{ij})_k (Z_k - Z_{k-1}) \quad (5.29)$$

$$(B_{ij})_k = \frac{1}{2} (\bar{Q}_{ij})_k (Z_k^2 - Z_{k-1}^2) \quad (5.30)$$

$$(D_{ij})_k = \frac{1}{3} (\bar{Q}_{ij})_k (Z_k^3 - Z_{k-1}^3) \quad (5.31)$$

in which, k is the index for the layers, and $\bar{Q}_{ij}$ for each individual branch is defined by the equation (Eqs. (2.78), (2.61), (2.71) in [14]):

$$[\bar{Q}] = [T] \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} [T]^{-T} \quad (5.32)$$

in which:

$$Q_{11} = \frac{E_1}{1 - \nu_{12} \nu_{21}} \quad (5.33)$$

$$Q_{12} = \nu_{12} \frac{E_2}{1 - \nu_{12} \nu_{21}} = \nu_{21} \frac{E_1}{1 - \nu_{12} \nu_{21}} \quad (5.34)$$

$$Q_{22} = \nu_{12} \frac{E_2}{1 - \nu_{12} \nu_{21}} \quad (5.35)$$

$$Q_{66} = G_{12} \quad (5.36)$$

$$[T] = \begin{bmatrix} \cos^2 \theta & \sin^2 \theta & 2 \sin \theta \cos \theta \\ \sin^2 \theta & \cos^2 \theta & -2 \sin \theta \cos \theta \\ -\sin \theta \cos \theta & \sin \theta \cos \theta & \cos^2 \theta - \sin^2 \theta \end{bmatrix} \quad (5.37)$$

Subscript 1 indicates ss direction; subscript 2 indicates nn direction; and subscript 6 indicates sn direction in the local plate coordinate system.

5.4 Assembly of Total Beam Stiffness Matrix

Recalling Eqs. (5.20) to (5.25) for one solid rectangular beam, the total stiffness can be assembled as follows:

$$K_{22} = (D_{11})_{Upper\ actuator}(x_2 - x_1) + (D_{11})_{Lower\ actuator}(x_4 - x_3) + (D_{11})_{Substrate}(x_5 - 0) \quad (5.38)$$

$$K_{23} = \frac{1}{2} \left[(B_{11})_{Upper\ actuator}(x_2^2 - x_1^2) + (B_{11})_{Lower\ actuator}(x_4^2 - x_3^2) + (B_{11})_{Substrate}(x_5^2 - 0) \right] \quad (5.39)$$

$$K_{25} = 0 \quad (5.40)$$

$$K_{33} = \frac{1}{3} \left[(A_{11})_{Upper\ actuator}(x_2^3 - x_1^3) + (A_{11})_{Lower\ actuator}(x_4^3 - x_3^3) + (A_{11})_{Substrate}(x_5^3 - 0) \right] \quad (5.41)$$

$$K_{35} = - \left[(B_{16})_{Upper\ actuator}(x_2^2 - x_1^2) + (B_{16})_{Lower\ actuator}(x_4^2 - x_3^2) + (B_{16})_{Substrate}(x_5^2 - 0) \right] \quad (5.42)$$

$$K_{55} = 4 \left[(D_{66})_{Upper\ actuator}(x_2 - x_1) + (D_{66})_{Lower\ actuator}(x_4 - x_3) + (D_{66})_{Substrate}(x_5 - 0) \right] \quad (5.43)$$

K_{ij} can be solved numerically at this point. By substituting these numerical values into Eq. (5.18), we can find that the angular twist about the z-axis is a function of torque only. The numerical expression is found by the Mathematica program shown in Appendix F, Program 2:

$$\phi'_x = 38.7263 \ T \quad (5.44)$$

where T is the torque induced by the piezo element as defined in the following section.

5.5 Beam Torque Induced by the Piezo Elements

From classical plate theory:

$$\begin{Bmatrix} N^a \\ M^a \end{Bmatrix} = \begin{bmatrix} A^a & B^a \\ B^a & D^a \end{bmatrix} \begin{Bmatrix} \epsilon^a \\ k^a \end{Bmatrix} \quad (5.14)$$

Since the piezo actuator induces strain only, the effect of curvature k is ignored. For the loads induced by one actuator:

$$\{N_x^a, N_s^a, N_{xx}^a\}^T = A^a \cdot \epsilon^a \quad (5.45)$$

$$\{M_x^a, M_s^a, M_{xx}^a\}^T = B^a \cdot \epsilon^a \quad (5.46)$$

where A^a and B^a are the stiffness for the piezo actuator layer and have been defined in Eqs. (5.29) and (5.30). The strain in the piezo layer is $\epsilon^a = \{\Lambda, \Lambda, 0\}^T$, in which Λ is the free strain induced by piezo elements, given in Table 2.2. According to Eq. 32 in [32], the torque of the beam induced by piezo elements is:

$$\begin{aligned} T &= \int_s (N_{xx}r + N_sr - M_{xx} - M_s + Q_s q) ds \\ \text{Since } r &= 0, Q_s = 0 \\ T &= \int_s (-M_{xx} - M_s) ds \end{aligned} \quad (5.47)$$

5.6 Results

Eq. 5.44 was integrated to obtain ϕ_z as a function of induced voltage. The comparison with experimental results is presented in Fig. 5.4. The discrepancy between analytical and

experimental results could be due to the following factors: imperfection in bonding, geometric tolerance of the specimen, minimal variances in voltages applied to the actuators, and shear lag in the bonding layers.

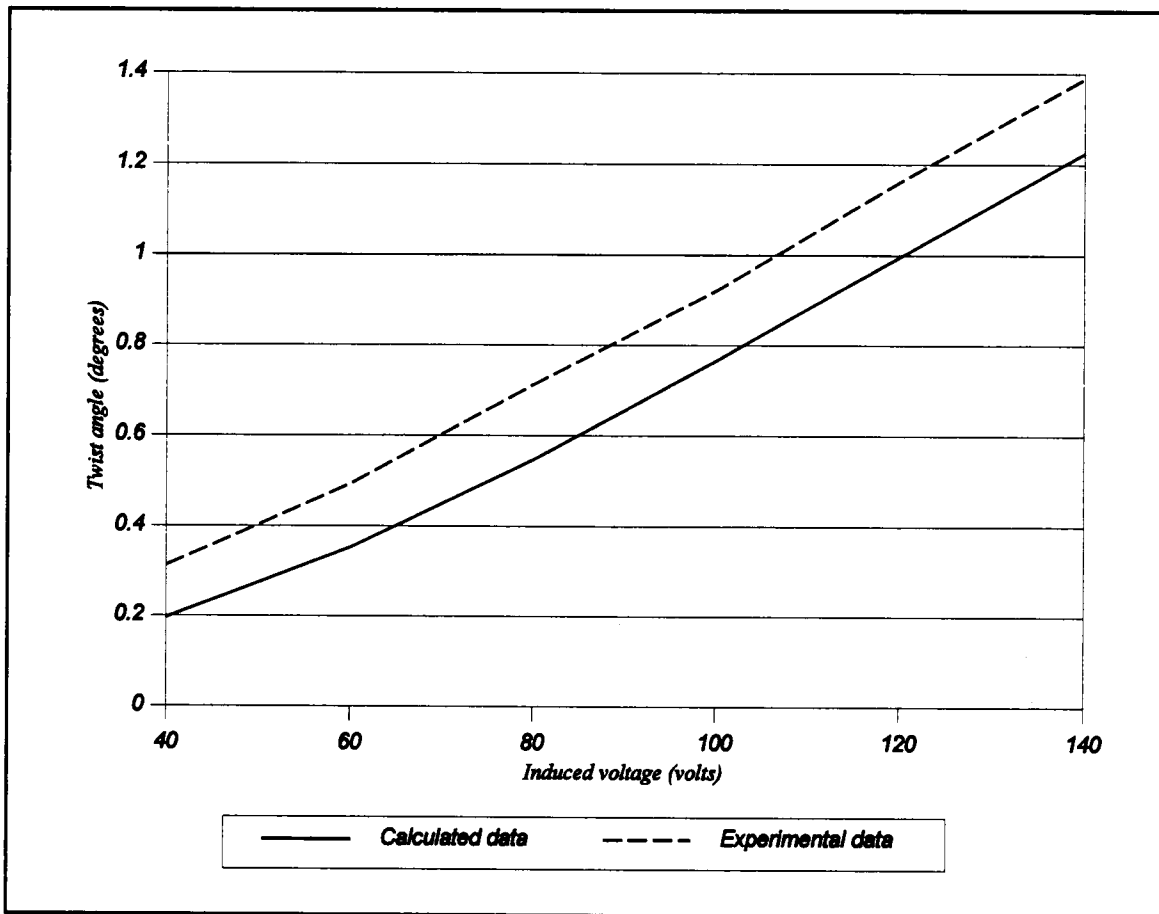


Figure 5.4 Comparison of the calculated results to the experimental results.

CHAPTER 6

ACTIVE CONSTRAINED LAYER DAMPING MODELING

In the effort to find the most effective use of piezo elements for vibration control, another type of smart element is also described here. This part of the thesis presents a feasibility study of using active constrained layer damping (ACLD) to reduce vibrations in non-thin-walled steel structures. ACLD adds the piezo effect to the traditional passive constrained damping (PCLD) to increase damping effectiveness. A quantitative mathematical model of ACLD is developed to find out the amount of damping when a steel cantilever beam (250 mm x 15 mm x 5 mm), with 60 % of its surface covered with ACLD, is subjected to an external harmonic force which has a magnitude of 5 N and a frequency in the vicinity of the beam's first modal natural frequency. The results of the analysis show that the damping by ACLD is 8 times more effective than the traditional passive constrained layer damping (PCLD) for the 10 mm thick beam, and is up to 25 times more effective for a 5 mm thick steel beam.

6.1 Concept Explanation

The damping characteristics of a structural element can be improved by bonding layers of viscoelastic material to its surface. This technique can be more effective if the shear deformation of the viscoelastic material increases. The greater the shear deformation in the viscoelastic layer, the more energy it dissipates. PCLD is a traditional method for increasing

the amount of shear deformation by restraining the free surface of a viscoelastic layer. A recent development combining piezo elements and viscoelastic material has been described by Baz and Ro in their paper [18]: "Active Constrained Layer Damping." They proposed

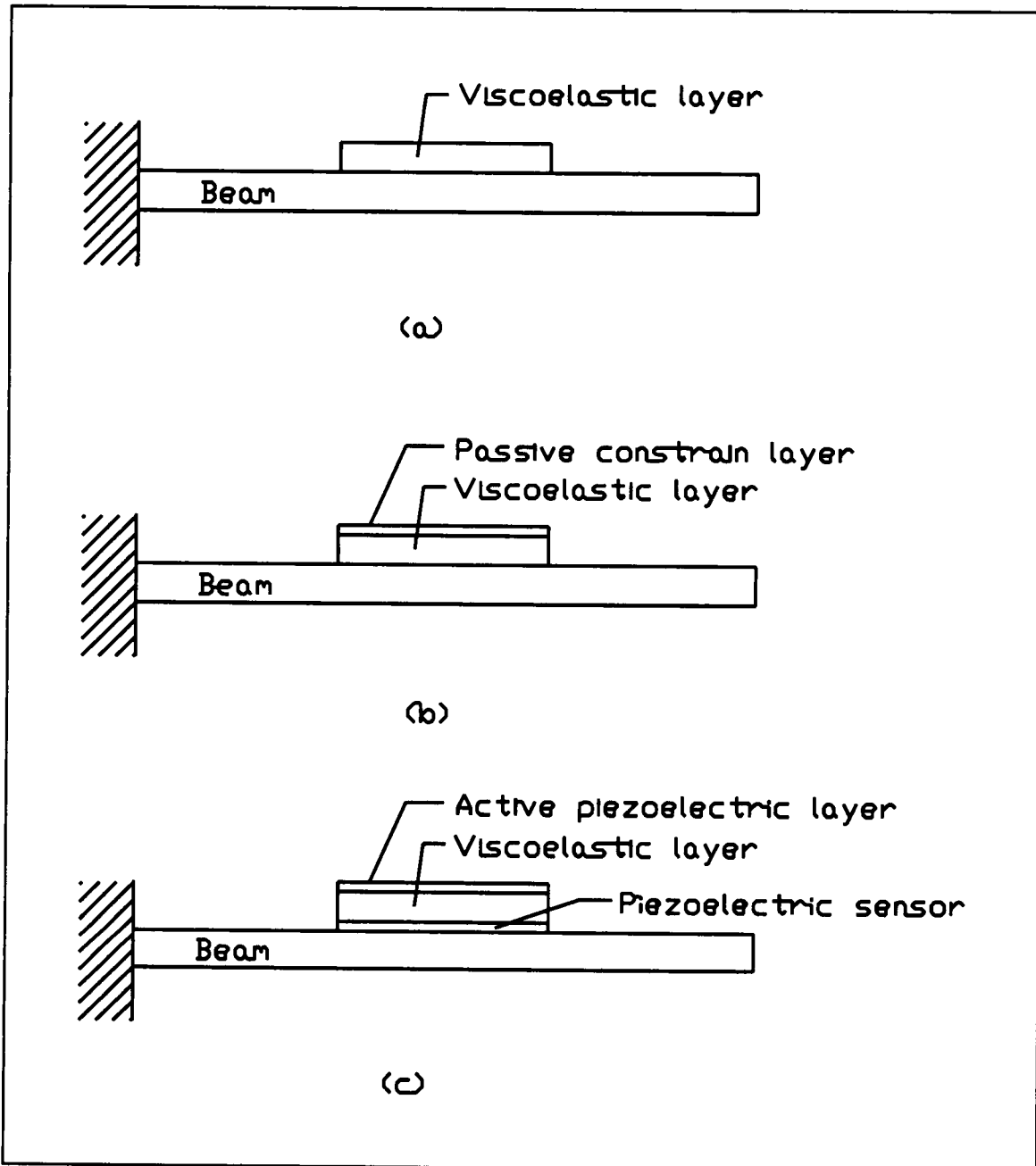


Figure 6.1 Three damping methods for a cantilever beam: a) regular viscoelastic layer damping, b) passive constrained layer damping (PCLD), c) active constrained layer damping (ACLD).

using a layer of piezoelectric material to replace the traditional passive constraining layer on the free surface of the viscoelastic layer. This active layer can be controlled--extended or contracted--in order to maximize the shear deformation in the viscoelastic layer. The ACLD control system for the active piezo layer is normally made of a piezoelectric sensor and an analog amplifier. The ACLD method is attractive for such applications as robotics or aviation, where both the weight and volume of the damping material are of critical importance.

Fig 6.1 illustrates three ways of using viscoelastic material for structural damping: a) is the least effective method of using viscoelastic material for damping; b) is the traditional method (PCLD) using a rigid passive constrained layer to increase damping effectiveness; and c) is the new method of active constrained layer damping (ACLD) with a layer of piezo active material replacing the passive constraint layer. The piezo film induces additional strain on the viscoelastic layer maximizing its damping effectiveness.

The mechanism of ACLD is shown in Fig. 6.2. The viscoelastic layer is sandwiched between two layers of piezoelectric foil--one of the layers acts as an actuator and the other acts as a sensor. The ACLD element is bonded to the surface of a structure. If the structure vibrates, the sensor will generate a charge proportional to the strain on the surface of the structure and will send it to the controller. The controller amplifies the signal (shifting the phase when necessary) and applies appropriate electric potential to the active constrained layer to introduce a piezoelectric strain. This increases the amount of shear deformation in the viscoelastic layer and therefore dissipates more vibration energy. Drawings b) and c) in

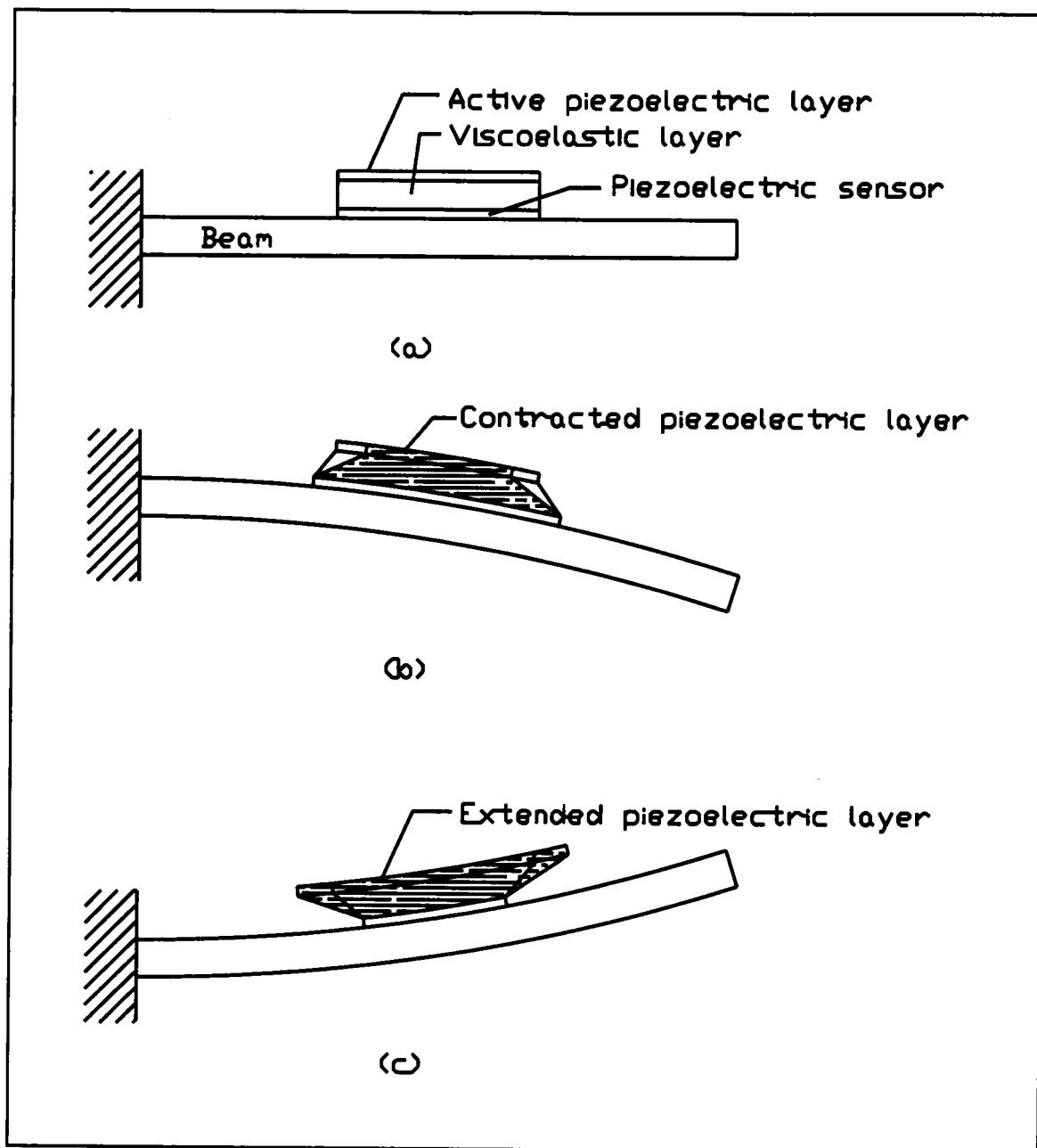


Figure 6.2 Illustration of damping enhancement by an ACLD element.

Fig. 6.2 demonstrate how an active piezoelectric layer increases shear deformation in the viscoelastic layer to improve its damping capability.

6.2 Mathematical Model

A schematic illustration of a beam with an ACLD element is shown in Fig. 6.3. In terms of mathematical modelling, the beam can be divided into three sections: sections 1 and 3 are modeled as ordinary Bernoulli-Euler beams for which mathematical expressions are readily available; section 2 consists of several layers, for which a mathematical equation has been developed by Baz and Ro [18]. These equations are listed as follows [18]:

$$E_4 I_4 \frac{\partial^4 w_1}{\partial x^4} = -m_4 \frac{\partial^2 w_1}{\partial t^2}, \quad 0 \leq x \leq L_1 \quad (6.1)$$

$$\frac{\partial^6 w_2}{\partial x^6} - g(1+Y) \frac{\partial^4 w_2}{\partial x^4} + \frac{m}{D_1} \frac{\partial^4 w_2}{\partial x^2 \partial t^2} = \frac{m g}{D_1} \frac{\partial^2 w_2}{\partial t^2}, \quad L_1 \leq x \leq L_2 \quad (6.2)$$

$$E_4 I_4 \frac{\partial^4 w_3}{\partial x^4} = -m_4 \frac{\partial^2 w_3}{\partial t^2}, \quad L_2 \leq x \leq L \quad (6.3)$$

where

$w_1(x, t)$ = the transverse deflection between $x=0$ and $x=L_1$,

$w_2(x, t)$ = the transverse deflection between $x=L_1$ and $x=L_2$,

$w_3(x, t)$ = the transverse deflection between $x=L_2$ and $x=L$,

$E_4 I_4$ = the flexural rigidity of the plain untreated beam,

D_1 = $(E_1 I_1 + E_3 I_3 + E_4 I_4) / b$,

$E_1 I_1$ = the flexural rigidity of the piezo-actuator,

$E_3 I_3$ = the flexural rigidity of the piezo-sensor,

b = the width of the composite beam,

m_4 = the mass density per unit length of the plain beam,

m = the mass density of the treated beam/unit length and unit width,

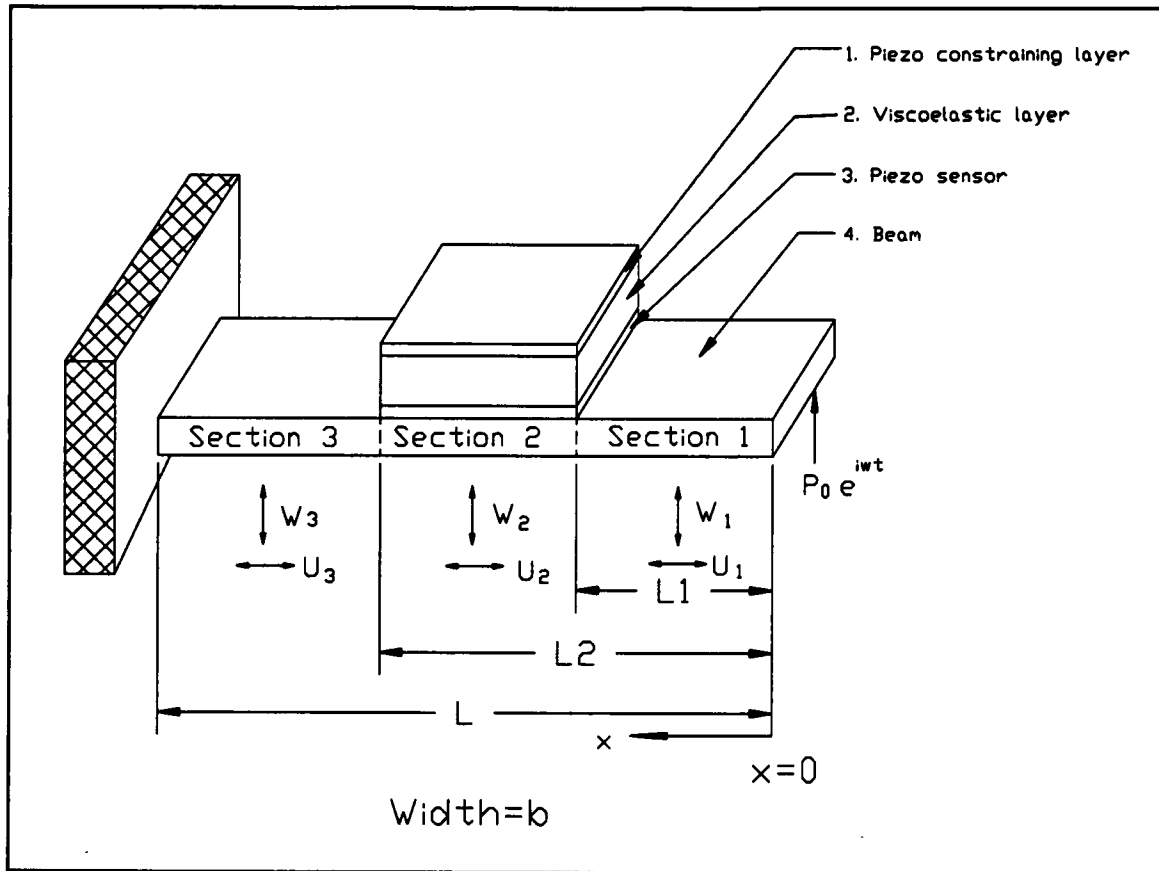


Figure 6.3 Schematic of modelled beam with ACLD treatment.

- $g = G / h_2 [(E_1 h_1 + E_3 h_3 + E_4 h_4) / (E_1 h_1 E_3 h_3 E_4 h_4)],$
 G = the complex shear modulus of the viscoelastic layer,
 $Y = d^2 / D_1 [(E_1 h_1 E_3 h_3 E_4 h_4) / (E_1 h_1 + E_3 h_3 + E_4 h_4)],$
 $d = h_2 + h_1 / 2 + D_2,$
 $D_2 = (E_3 h_3^2 + 2 E_4 h_3 h_4 + E_4 h_4^2) / [2 (E_3 h_3 + E_4 h_4)],$
 h_1 = the thickness of the piezo actuator,
 h_2 = the thickness of the viscoelastic layer,
 h_3 = the thickness of the piezo sensor,
 h_4 = the thickness of the base beam.

Using the classical separation of variables approach, such that the transverse deflection $w_i = W_i(x) T(t)$ and $T''(t)/T(t) = -\omega^2$, the w_i ($i=1, 2$, and 3) can be reduced as a function of x only:

$$E_4 I_4 \frac{d^4 W_1}{dx^4} - m_4 \omega^2 W_1 = 0, \quad 0 \leq x \leq L_1 \quad (6.4)$$

$$\frac{d^6 W_2}{dx^6} - g(1+Y) \frac{d^4 W_2}{dx^4} + m \frac{\omega^2}{D_1} \frac{d^2 W_2}{dx^2} + \frac{m g \omega^2}{D_1} = 0, \quad L_1 \leq x \leq L_2 \quad (6.5)$$

$$E_4 I_4 \frac{d^4 W_3}{dx^4} - m_4 \omega^2 W_3 = 0 \quad L_2 \leq x \leq L \quad (6.6)$$

Since all elements in the equation are known except W_i ($i=1, 2$, and 3), the solution for W_1 can be obtained with 4 unknown constant coefficients, the solution for W_2 can be obtained with 6 unknown constant coefficients, and the solution for W_3 can be solved with another 4 constants coefficients. The total 14 constant coefficients are then determined by solving 14 simultaneous boundary condition equations:

$$\text{When } x = 0: \quad W_1'''(0) = P_0 / E_4 I_4 \quad (6.7)$$

$$W_1''(0) = 0 \quad (6.8)$$

$$\text{For } x = L_1: \quad W_1(L_1) = W_2(L_1) \quad (6.9)$$

$$W_1'(L_1) = W_2'(L_1) \quad (6.10)$$

$$W_1''(L_1) = W_2''(L_1) \quad (6.11)$$

$$W_1'''(L_1) = W_2'''(L_1) \quad (6.12)$$

$$u_1(L_1) = u_2(L_1) \quad (6.13)$$

$$\text{For } x = L_2: \quad W_2(L_2) = W_3(L_2) \quad (6.14)$$

$$W_2'(L_2) = W_3'(L_2) \quad (6.15)$$

$$W_2''(L_2) = W_3''(L_2) \quad (6.16)$$

$$W_2'''(L_2) = W_3'''(L_2) \quad (6.17)$$

$$u_2(L_2) = u_3(L_2) \quad (6.18)$$

$$\text{For } x = L: \quad W_3(L) = 0 \quad (6.19)$$

$$W_3'(L) = 0 \quad (6.20)$$

the Eqs. (6.13) and (6.18) can be fully expanded to include the strain by the piezo layers as follows [18]:

$$\frac{1}{k} \cot(kL_1) \left[\frac{1}{g} \frac{d^4 W_2}{dx^4} - Y \frac{d^2 W_2}{dx^2} - \frac{m \omega^2}{D_1 g} W_2 - \frac{Y}{d} \epsilon_p \right]_{x=L_1} = - \left[\frac{1}{g^2} \frac{d^5 W_2}{dx^3} - \left(Y + \frac{m \omega^2}{D_1 g} \right) \frac{d W_2}{dx} - \frac{Y}{d} x \epsilon_p \right]_{x=L_1} \quad (6.21)$$

$$\frac{1}{k} \tan(k(L_2 - L)) \left[\frac{1}{g} \frac{d^4 W_2}{dx^4} - Y \frac{d^2 W_2}{dx^2} - \frac{m \omega^2}{D_1 g} W_2 - \frac{Y}{d} \epsilon_p \right]_{x=L_2} = - \left[\frac{1}{g^2} \frac{d^5 W_2}{dx^3} - \left(Y + \frac{m \omega^2}{D_1 g} \right) \frac{d W_2}{dx} - \frac{Y}{d} x \epsilon_p \right]_{x=L_2} \quad (6.22)$$

in which ϵ_p is the strain induced by a piezoelectric actuator as defined in Section 6.4.

6.3 The Piezo Actuator

The piezo strain is defined as:

$$\epsilon_p = \left(\frac{d_{31}}{h_1} \right) V_a \quad (6.23)$$

where V_a is the voltage applied across the piezo actuator, and d_{31} is the piezoelectric strain constant which has been described in Chapter 2. The voltage V_a is the amplified signal from the piezo sensor attached to the beam surface, and is defined as [18] [28]:

$$V_a = K_p V_s \quad (6.24)$$

$$V_s = \frac{k_{31}^2 D_d b}{g_{31} C} \int_{L_1}^{L_2} \frac{\partial W_2^2}{\partial x^2} dx \quad (6.35)$$

$$C = 8.854 (10^{-12}) A \frac{k_{3t}}{h_3} \quad (6.26)$$

where V_s is the voltage generated by the piezo sensor as a result of the strain; C is the capacitance of the sensor; A is the sensor surface area; D_d is the distance from the beam neutral axis to the sensor surface; and K_p is the gain by the analog circuit. Finally, k_{31} , k_{3t} , and g_{3t} are related to the piezo properties which are defined in Table 6.1.

Table 6.1 Numerical values used for quantitative evaluation (referring to Fig. 6.3)

	Element 1	Element 2	Element 3	Element 4
Length (L) m	0.15	0.15	0.15	0.25
Thickness (h) m	0.00025	0.000028	0.00025	0.005
Density (ρ) Kg/m ³	2700	2700	2700	7800
Width (b) m	0.015	0.015	0.015	0.015
Young's modulus (E) pa	63.0 E9	63.0 E9	63.0 E9	21.0 E10
Shear modulus (G) pa	28.0 E9	G**	28.0 E9	83.0 E9
External force: $P_0 = 5$ N at the frequency of $\omega = 433$ rad/second				
Properties of piezo sensor and actuator layer:				
d_{31} (m/V)	k_{31}	g_{31} (V m/N)	k_{3t}	
18.6×10^{-11}	0.34	$11.6 \text{ E-}3$	1950	
Results: the tip displacement is 0.016 mm when $K_p = 1000$ Improvement: 2500 % the tip displacement is 1.4964 mm when $K_p = 0$				

** G is defined in [22] as:

$$G = .142 e^{0.494 \ln(\frac{\omega}{2\pi})} (1 + 1.46 i) \quad MN/m^2 \quad (6.27)$$

6.4 Quantitative Evaluation

This investigation is limited to the evaluation of the ACLD's capability of enhancing damping in a clamped cantilever steel beam of 250 mm x 15 mm x 5 mm, with 60% of its surface treated by the ACLD material (increasing the beam's weight by 6-10%), under an oscillating tip force of 5 N at 200 radians per second. The steady state vibration amplitude at the tip of the beam is 0.568938 mm when ACLD is active, compared with 1.68254 mm when only PCLD is engaged. The significant difference between these two numbers is very encouraging and suggests promising outcomes in further investigation.

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

This investigation has comprehensively covered the fundamental issues related to building a piezo actuator for modifying the torsional behavior of a thin-walled, closed cross-section composite tube. Both analytical and experimental approaches have been used for the study. The study has reached conclusions which may find usage far beyond those explored here. However, due the complexity of the matter, future expanded study may be necessary for full exploration.

Piezo actuator with a direct feedback control is effective to damp the torsional vibration of closed cross-section structure. However, the effectiveness of the system is sensitive to the magnitude of torsional vibration. A more sophisticated circuit with the capability of automatically adjusting gain would be more effective in reducing the time constant of the transient vibration.

The ability of the piezo elements to produce twist has been noted. A simple flat beam of 50 mm x 25 mm x 0.292 mm with two antisymmetrically attached piezo elements, measuring 25.4 mm x 6.35 mm x 0.191 mm, can be twisted by induced piezo strain to 1.5 degrees. Analytical results are approximately 15 % smaller than experimental results. This satisfactory agreement shows that Vlasov's theory can also be used for twist analysis. This analytical model can be expanded to incorporate the calculations of beams with different geometries and piezo element arrangements.

Active constrained layer damping is a method investigated as an alternative to pure piezo material damping. This method combines piezo material and viscoelastic material to damp the structural vibration. For a steel cantilever beam measuring 250 mm x 15 mm x 5 mm, with 60% of its surface treated with ACLD, the tip displacement will be reduced from 1.4964 mm to 0.016 mm when it is treated with passive constrained viscoelastic material while being subjected to an external harmonic force of 433 rad / second. This is a 2500 % improvement in beam damping and suggests a promising application of this method in future research.

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- [34] Lazarus, K.B., Crawley, E.F. and Bohlmann, J.D., "Static Aeroelastic Control Using Strain Actuated Adaptive Structures," Proceedings of First Joint U.S./Japan Conference on Adaptive Structures, November, 1990.

APPENDICES

Appendix A Piezo Element Strains

Table 1 Free Strain Measurement Data Sheet

Piezo element strains vs. voltages (micro-meters/meter)

Feb. 10, 1994

Voltage	Compressive strain*					Average	Strain**
volts	test 1	test 2	test 3	test 4	test 5	Reading	10 ⁻⁶
20	8980	8980	8980	8980	8980	8980	-11.3875
40	8950	8952	8954	8948	8948	8950.4	-40.9875
60	8905	8905	8905	8904	8904	8904.6	-86.7875
80	8846	8845	8846	8846	8848	8846.2	-145.1875
100	8786	8788	8786	8788	8784	8786.4	-204.9875
120	8712	8712	8710	8712	8712	8711.6	-279.7875
140	8622	8620	8620	8620	8624	8621.2	-370.1875
150	8560	8566	8580	8590	8590	8577.2	-414.1875

Voltage	Compressive strain residual*						
20 -> 0	8998	8998	9000	9006	9000	9000.4	9.0125
40 -> 0	8992	8990	8992	8992	8988	8990.8	-0.5875
60 -> 0	8980	8980	8978	8976	8977	8978.2	-13.1875
80 -> 0	8967	8960	8960	8960	8960	8961.4	-29.9875
100 -> 0	8946	8940	8940	8940	8932	8939.6	-51.7875
120 -> 0	8924	8912	8912	8912	8900	8912	-79.3875
140 -> 0	8890	8860	8865	8862	8868	8869	-122.3875
150 -> 0	8842	8836	8845	8860	8860	8848.6	-142.7875

Voltage	Tensile strain*						
-20	9030	9030	9028	9026	9028	9028.4	37.0125
-40	9058	9058	9058	9058	9060	9058.4	67.0125
-60	9090	9088	9088	9092	9092	9090	98.6125
-80	9146	9145	9145	9145	9152	9146.6	155.2125
-100	9200	9192	9196	9196	9198	9196.4	205.0125
-120	9250	9250	9250	9250	9252	9250.4	259.0125
-140	9298	9282	9280	9285	9274	9283.8	292.4125
-150	9310	9300	9302	9298	9285	9299	307.6125

Voltage	Tensile strain residual*						
-20 -> 0	9010	9006	9006	9008	9008	9007.6	16.2125
-40 -> 0	9014	9015	9018	9016	9016	9015.8	24.4125
-60 -> 0	9020	9020	9022	9022	9020	9020.8	29.4125
-80 -> 0	9044	9045	9045	9046	9046	9045.2	53.8125
-100 -> 0	9066	9068	9072	9076	9074	9071.2	79.8125
-120 -> 0	9084	9090	9094	9104	9096	9093.6	102.2125
-140 -> 0	9090	9110	9116	9115	9132	9112.6	121.2125
-150 -> 0	9108	9115	9130	9145	9137	9127	135.6125

Approximated origin for the data: 8991.388

* -- Readings from the strain indicator.

** -- Strains with respect to approximated neutral position of 8991.3875.

Table 2 Bonded Strain Measurement Data Sheet

Piezo element strains vs. voltages as the element is bonded to the substrate

Element size: 25.4 mm x 6.35 mm x 0.191 mm

Substrate thickness: 0.292 mm (aluminum)

May 31, 1994

Voltage volts	Tensile strain micro-meters/meter
20	3
40	29
60	55
80	81
100	111
120	129
140	146
150	146

Voltage volts	Tensile residual micro-meters/meter
20 -> 0	-11
40 -> 0	-3
60 -> 0	9
80 -> 0	15
100 -> 0	24
120 -> 0	31
140 -> 0	35
150 -> 0	35

Voltage volts	Compressive strain micro-meters/meter
-20	-33
-40	-43
-60	-54
-80	-71
-100	-97
-120	-117
-140	-139
-150	-147

■ Program 1 Curve Fitting for the Free Strains of A Piezo Element

■ Objective

The experimental data for strains of a piezo element corresponding to several voltages have been collected. This program is to find the generalized strain-voltage relations to approximate the free strain related to an arbitrary voltage.

■ Curve fitting for compressive strain

data={ {20,-23.99}, {40,-53.59}, {60,-99.39}, {80,-157.79},
{100,-217.5}, {120,-292.3}, {140,-382.7}, {150,-426.7}};

These data are fitted by third order polynomial:

eq=Fit[**data**, {1,x,x²,x³},x]

$$-0.78553 - 0.838458 x - 0.0135582 x^2 + 1.23861 \times 10^{-6} x^3$$

■ Curve fitting for compressive strain residual

data={ {20,-3.5975}, {40,-13.1975}, {60,-25.7975}, {80,-42.59},
{100,-64.3975}, {120,-91.9975}, {140,-134.998}, {150,-155.39}}

These data are fitted by third order polynomial:

eq=Fit[**data**, {1,x,x²,x³},x]

$$6.2239 - 0.505082 x + 0.00177744 x^2 - 0.0000375964 x^3$$

■ Curve fitting for tensile strain

data={ {20,24.4}, {40,54.4}, {60,86}, {80,142.6},
{100,192.4}, {120,246.4}, {140,279.8}, {150,295}};

These data are fitted by third order polynomial:

eq=Fit[**data**, {1,x,x²,x³},x]

$$25.4276 - 0.683862 x + 0.03812 x^2 - 0.000144014 x^3$$

■ Curve fitting for tensile strain residual

data={ {20,3.6}, {40,11.8}, {60,16.8}, {80,41.2},
{100,67.2}, {120,89.6}, {140,108.6}, {150,123}};

These data are fitted by third order polynomial:

eq=Fit[**data**, {1,x,x²,x³},x]

$$13.7163 - 0.821643 x + 0.0194514 x^2 - 0.0000613144 x^3$$

■ Program 2 Curve Fitting for the Bonded Strains of A Piezo Element

■ Objective

The experimental data of strains of a piezo element corresponding to several voltages have been collected. This program is to find the generalized strain-voltage relations to approximate the bonded strain in relation to an arbitrary voltage.

■ Curve fitting for compressive strain

```
data={{20,32},{40,-42},{60,-53},{80,-71},  
{100,-97},{120,-117},{140,-139},{150,-147}};
```

These data are fitted by third order polynomial:

```
eq=Fit[data,{1,x,x^2,x^3},x]
```

$$101.686 - 4.52971 x + 0.0387881 x^2 - 0.000132297 x^3$$

■ Curve fitting for tensile strain

```
data={{20,3},{40,29},{60,55},{80,81},  
{100,111},{120,129},{140,146},{150,146}};
```

These data are fitted by third order polynomial:

```
eq=Fit[data,{1,x,x^2,x^3},x]
```

$$-14.124 + 0.686217 x + 0.0112077 x^2 - 0.0000574297 x^3$$

■ Curve fitting for tensile strain residual

```
data={{20,-11},{40,-3},{60,9},{80,15},  
{100,24},{120,31},{140,35},{150,35}};
```

These data are fitted by third order polynomial:

```
eq=Fit[data,{1,x,x^2,x^3},x]
```

$$-18.9791 + 0.35414 x + 0.002243 x^2 - 0.000014583 x^3$$

Appendix B Relationship Between Bonding Thickness and Transferred Strains

Refer to Figure 2.4

Comparason between #1, #2, and #3

1 with 0 bonding thickness

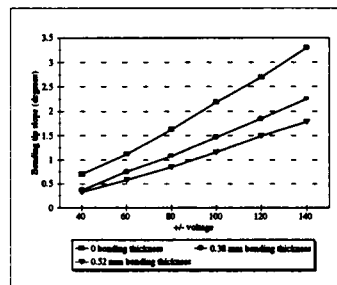
Displacement*

Volt(+/-)	test 1	test 2	test 3	Average	Angle**
	mm	mm	mm	mm	Degree
40	38	36.5	45.6	40.033	0.69224
60	67	64	63.2	64.733	1.11938
80	95	92	94	93.667	1.61981
100	130	124.5	125.5	126.67	2.19073
120	158	154	155	155.67	2.69263
140	195	188	188	190.33	3.29288

#2 with 0.38 mm bonding thicknss

Displacement*

Volt(+/-)	test 1	test 2	test 3	Average	Angle**
	mm	mm	mm	mm	Degree
40	26	25	13	21.333	0.36888
60	42.5	43	43.5	43	0.74611
80	63.5	60	63	62.167	1.07871
100	83.7	82	85.8	83.833	1.45474
120	108	105	105.8	106.27	1.84414
140	130	128.5	130.5	129.67	2.25041



#3 with 0.52 mm thickness

Displacement*

Volt(+/-)	test 1	test 2	test 3	Average	Angle**
	mm	mm	mm	mm	Degree
40	19	19	18	18.667	0.32113
60	35	34	32.4	33.8	0.58148
80	50.4	50.5	46.2	49.033	0.84356
100	68.8	68.2	64	67	1.15269
120	87.5	87	84	86.167	1.4825
140	104	104	103	103.67	1.78368

* The span of laser projection at certain distance when the voltage from - /+volt to -/+volts, the value is only half of listed value if only positive or negative direction potential is applied.

** Calculated from span length and the distance from projection to the specimen.

Appendix C Flat Beam Twist Test with Different Bonding Width

Comparison between #5, #6, #7, and #8

The following displacement refers the laser dot projection displacement:

#5 with 1mm bonding width

Displacement*

Residual***

Volt(+/-)	test 1	test 2	test 3	Average	Angle	test 1	test 2	test 3	Average	Angle
	mm	mm	mm	mm	Degree	mm	mm	mm	mm	Degree
40	23.8	22.8	25	23.8667	0.41439	2.8	4.3	2	3.03333	0.0483
60	40	36.6	38	38.2	0.66326	5	6	4.7	5.23333	0.0833
80	56.8	53.8	57	55.8667	0.97002	10	9	9.8	9.6	0.1528
100	75	72.2	74	73.7333	1.28029	14	11	12.2	12.4	0.1974
120	93	90	94	92.3333	1.60333	16	17	14.9	15.9667	0.2541
140	109.5	109	109	109.167	1.89574	20	19	19	19.3333	0.3077

#6 with 2 mm bonding width

Displacement*

Residual***

Volt(+/-)	test 1	test 2	test 3	Average	Angle**	test 1	test 2	test 3	Average	Angle
	mm	mm	mm	mm	Degree	mm	mm	mm	mm	Degree
40	23.8	20		21.9	0.38024	2	2		2	0.0347
60	36.8	33.5		35.15	0.6103	4	4		4	0.0694
80	53	51		52	0.90288	7.5	7		7.25	0.1259
100	70	67		68.5	1.18941	11.5	11.3		11.4	0.1979
120	88.2	86		87.1	1.51244	16.5	17		16.75	0.2908
140	105	105		105	1.82336	19	18		18.5	0.3212

#7 with 3 mm bonding width

Displacement*

Residual***

Volt(+/-)	test 1	test 2	test 3	Average	Angle**	test 1	test 2	test 3	Average	Angle
	mm	mm	mm	mm	Degree	mm	mm	mm	mm	Degree
40	21.8	21		21.4	0.37156	3	3.2		3.1	0.0538
60	35	34		34.5	0.59901	5.2	6.5		5.85	0.1016
80	50.3	48.5		49.4	0.85773	9	7		8	0.1389
100	66.5	63.2		64.85	1.12602	10.8	8.8		9.8	0.1702
120	77.5	80.2		78.85	1.36915	12.8	13		12.9	0.224
140	95.6	92.5		94.05	1.63315	13.5	13.5		13.5	0.2344

#8 fully bonded

Displacement*

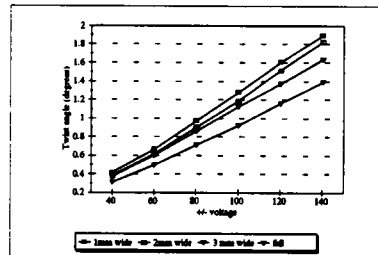
Residual***

Volt(+/-)	test 1	test 2	test 3	Average	Angle**	test 1	test 2	test 3	Average	Angle
	mm	mm	mm	mm	Degree	mm	mm	mm	mm	Degree
40	18	18		18	0.31252	2	2		2	0.0347
60	28.5	28.3		28.4	0.4931	3	4		3.5	0.0608
80	41	41		41	0.71187	5.2	4.2		4.7	0.0816
100	53	53		53	0.92024	8.5	7		7.75	0.1346
120	67.5	66.3		66.9	1.16162	11	10		10.5	0.1823
140	80	80		80	1.38912	11	11.5		11.25	0.1953

* -- The span of laser projection at certain distance when the voltage from -/+ volt to -/+volts, the value is only half of listed value if only positive or negative direction potential is applied.

** -- Calculated from span length and the distance from projection to the specimen.

*** -- Strains after the corresponding electric potential has been removed.



Appendix D Shear Modules of the Composite Tube

Data table of torsional displacement test for one layer tube.

Fiber orientation: 0 x 90 degrees with respect to the longitudinal axis.

Size: 100 mm x 100 mm x 500 mm.

Torque N*m	Amount of twist			Average degrees	G N/m ²
	1st test degrees	2nd test degrees	3rd test degrees		
0.0000	0.0000	0.0000	0.0000	0.0000	
0.9278	0.0373	0.0319	0.0390	0.0361	1.8424E+09
2.0919	0.1242	0.1384	0.1242	0.1290	1.1623E+09
3.4794	0.2130	0.2307	0.2165	0.2201	1.1329E+09
5.2718	0.3372	0.3727	0.3372	0.3490	1.0823E+09
6.3262	0.4259	0.4348	0.4117	0.4242	1.0687E+09
7.3806	0.4969	0.5058	0.4827	0.4952	1.0681E+09
8.4350	0.5857	0.5679	0.5679	0.5738	1.0533E+09
9.4893	0.6567	0.6389	0.6478	0.6478	1.0497E+09
10.5437	0.7632	0.7099	0.7454	0.7395	1.0217E+09
11.5981	0.8341	0.8022	0.8164	0.8176	1.0165E+09
12.6524	0.9406	0.8963	0.8874	0.9081	9.9838E+08
13.7068	1.0471	0.9992	0.9939	1.0134	9.6919E+08
14.7612	1.1536	1.1181	1.1004	1.1240	9.4102E+08
15.8155	1.2778	1.2601	1.2423	1.2601	8.9936E+08
16.8699	1.3843	1.4021	1.3666	1.3843	8.7323E+08
17.9243	1.5086	1.5973	1.5263	1.5441	8.3183E+08
18.9786	1.6683	1.7925	1.6860	1.7156	7.9268E+08
				Average	1.0473E+09

Formulas for Shear modules calculation:

$$J = 4 \times A^2 \times d / s = 4 \times (0.1 \times 0.1)^2 \times 0.0004 / (4 \times 0.1) = 4E-7 \text{ m}^4$$

A: enclosed area

d: wall thickness

s: total length of four sides of the tube

$$G = \text{Torque} \times \text{length} / (\text{angle} \times J)$$

Appendix E Calculation of Torsional Damping Coefficient and Energy Dissipation

Hysteresis damping is caused by the friction between the internal planes that slip or slide as the material deforms. The composite beam torsional damping matches the characteristics, therefore, formulas for hysteresis damping are used for calculation.

Referring to Fig. 4.6, the following parameters are known:

$$J_0 = 3.21 \times 10^{-3} \text{ Kg m}^2$$

$$\omega_n = 52.73 \text{ Hz} = 331.34 \text{ rad/s}$$

$$\text{The magnitude of the vibration is } X = 1.4828 \pi / (2 \times 180) = 0.0129.$$

For natural damping:

The peak values during the natural damping:

Time (s)	Magnitude (mv)
0.242	76.823
0.261	70.303
0.28	65.786
0.299	61.123
0.318	55.890
0.337	51.678
0.356	47.023

Solution:

The logarithmic decrement according to Eq. (2.131) in [31]:

$$\delta = 1/6 \ln(76.823/47.027) = .0818 = \pi \beta$$

$$\beta = 0.026$$

The torsional stiffness of the beam is:

$$k = \omega_n^2 J_0 = 331.34^2 \times 0.00321 = 352.41 \text{ N/m}$$

The equivalent damping constant C_{eq} is given by Eq. (2.132) in [31] as:

$$C_{eq} = \beta \sqrt{J k} = 0.0277$$

Energy loss due to the damping per cycle is:

$$\Delta w = \pi k \beta X^2 = 3.14 \times 352.41 \times 0.026 \times 0.0129^2 = 0.004816 \text{ Joules}$$

For piezo-active damping:

The peak values during the damping:

Time (s)	Magnitude (mv)
0.055	122.58
0.073	124.78
0.091	60.78
0.112	60.58
0.13	58.08
0.147	25.38
0.168	37.38

Solution:

The logarithmic decrement according to Eq. (2.131) in [31]

$$\delta = \ln(124.78/60.78) = 0.7193 = \pi \beta$$

$$\beta = 0.22896$$

The torsional stiffness of the beam is:

$$k = \omega_n^2 J_0 = 331.34^2 \times 0.00321 = 352.41 \text{ N/m}$$

The equivalent damping constant C_{eq} is given by Eq (2.132) in [31] as:

$$C_{eq} = \beta \text{ Sqrt}(J k) = 0.2436$$

Energy loss due to the damping during the first cycle is:

$$\Delta w = 1/2 k (X_1^2 - X_2^2) = 1/2 \times 3.14 \times (0.012938^2 - 0.00622^2) = 0.022675 \text{ Joules.}$$

$$\text{in which } X_1 = 1.4826 \pi / (2 \times 180) = 0.012938 \text{ rad}$$

$$X_2 = X_1 \times 60.78/124.78 = .00622 \text{ rad}$$

For the rest of the cycles, the logarithmic decrement according to Eq. (2.131) in [31]

$$\delta = 1/4 \text{ Ln}(60.78/37.38) = 0.12$$

The number is about 50% larger than the logarithmic decrement of 0.0818 in the natural damping. This means that the active damping using direct feedback control is not very effective after the magnitude of the vibration is reduced.

■ Appendix F Flat Beam Modelling

■ Program 1 Calculation for Elements in K Matrix

■ Objective

This Mathematica program finds beam stiffness constants and beam forces in terms of plate stiffness and beam displacements. The formula used in the program are all described in Chapter 5.

■ Convention

The variable definitions in this program agree with the naming convention used in Chapter 5 everywhere it does not conflict with the software syntax. The Greek characters are typed in their abbreviated English form:

the ... theta

fi ... phi

lam ... lambda

alf ... alpha

eps ... epsilon

A variable name starting with an upper case letter is preceded with a corresponding lower case letter. The letter L is reserved to indicate that the number(s) or letter(s) that follow(s) is/are the subscript(s), while U indicates that the following(s) is/are superscript(s).

The following statements turn off the spelling check for the similiar names used in the program:

```
Off[General::spell];
```

```
Off[General::spell1];
```

■ Define Variables

☐ Define Variables for Rectangular Section Beam

Refer to Fig. 5.1 in Chapter 5:

```
r[s_] := 0;
```

```
lam[s_] := 0;
```

```
q[s_] := s;
```

```
x=s;
```

```
y=0;
```

```
the[s_] := 0;
```

```
alf[s_] := s^2/2;
```

☐ Define the Displacements in Terms of the Shell Coordinates

Refer to Eqs. (5.1), (5.2) and (5.3) in Chapter 5:

```
u[s_, z_] := uU[z]*Sin[the[s]] - vV[z]*Cos[the[s]] - q[s]*fiLz[z]  
alf[s]*psi[z];
```

```

v[s_, z_] := uU[z] * Cos[the[s]] + vV[z] * Sin[the[s]] + r[s] * fiLz[z]
w[s_, z_] := wW[z] + x * fiLx[z] + y * fiLy[z] - lam[s] * D[fiLz[z], z];

```

- Define the extensional stiffness matrix, the coupling matrix and the bending stiffness matrix

```

aA = {{aA11, aA12, 0},
      {aA12, aA22, 0},
      {0, 0, aA66}};

bB = {{bB11, bB12, bB16},
      {bB12, bB22, bB26},
      {bB16, bB26, bB66}};

dD = {{dD11, dD12, 0},
      {dD12, dD22, 0},
      {0, 0, dD66}};

```

- Define the Vectors of Strain and Curvature for a Plate Segment

Refer to Eqs. (5.9), (5.10), and (5.5) in Chapter 5:

```

epsU0 = {D[wW[z], z] + x * D[fiLx[z], z] +
          y * D[fiLy[z], z] - lam[s] * D[fiLz[z], {z, 2}],
          (-uU[z] * Sin[the[s]] + vV[z] *
           Cos[the[s]]) * D[the[s], s] + D[r[s], s] * fiLz[z] / .
          {D[the[s], s] -> 0, D[q[s], s] -> 1},
          epsLxz[z] * Cos[the[s]] + epsLyz[z] * Sin[the[s]]};

```

Refer to Eqs. (5.11), (5.12), and (5.13) in Chapter 5:

```

k = {kLz[s, z], kLs[s, z], kLzs[s, z]};

kLz[s_, z_] := -Sin[the[s]] * D[fiLx[z], z] + Cos[the[s]] *
               D[fiLy[z], z] - q[s] * D[fiLz[z], {z, 2}] +
               D[epsLxz[z], z] * Sin[the[s]] -
               D[epsLyz[z], z] * Cos[the[s]] -
               alf[s] * D[psi[z], {z, 2}]

kLs[s_, z_] := -D[alf[s], {s, 2}] * psi[z];

kLzs[s_, z_] := -2 * (D[fiLz[z], z] + D[alf[s], s] *
                      D[psi[z], z]);

```

- Find the Load Components in the Shell

Force vector:

```
nNSh = aA.epsU0 + bB.k;
```

Moment vector:

```
mMSh = bB.epsU0 + dD.k;
```

- Define the Strain Energy of the Plate Segment

Refer to Eq. (15):

```
energy = 1/2 Integrate[nNSh.epsU0 + mMSh.k, s];
```

■ Calculate the Selected Terms in Stiffness Matrix and Generalized Beam Loads

$$K11=D[\text{energy},\{wW'[z],2\}]$$

$$aA11 \text{ s}$$

$$K12=D[\text{energy},wW'[z],fiLy'[z]]$$

$$bB11 \text{ s}$$

$$k13=D[\text{energy},wW'[z],fiLx'[z]]$$

$$\frac{aA11 \text{ s}^2}{2}$$

$$k15=D[\text{energy},wW'[z],fiLz'[z]]$$

$$-2 \text{ bB16 s}$$

$$K22=D[\text{energy},\{fiLy'[z],2\}]$$

$$dD11 \text{ s}$$

$$k23=D[\text{energy},fiLy'[z],fiLx'[z]]$$

$$\frac{bB11 \text{ s}^2}{2}$$

$$k25=D[\text{energy},fiLy'[z],fiLz'[z]]$$

$$0$$

$$k33=k23=D[\text{energy},fiLx'[z],fiLx'[z]]$$

$$\frac{aA11 \text{ s}^3}{3}$$

$$k35=D[\text{energy},fiLx'[z],fiLz'[z]]$$

$$-(bB16 \text{ s}^2)$$

$$K26=D[\text{energy},fiLy'[z],epsLxz[z]]$$

$$bB16 \text{ s}$$

$$K44=D[\text{energy},\{fiLz''[z],2\}]$$

$$\frac{dD11 \text{ s}^3}{3}$$

$$K45=D[\text{energy},fiLz''[z],fiLz'[z]]$$

$$0$$

```

k55=D[energy,fiLz'[z],fiLz'[z]]
4 dD66 s
tT=Simplify[D[energy,fiLz'[z]]]
s (-2 bB66 epsLxz[z] - bB16 s fiLx'[z] +
      4 dD66 fiLz'[z] + 2 dD66 s psi'[z] - 2 bB16 wW'[z])
mMLx=Simplify[D[energy,fiLy'[z]]]
(s (6 bB16 epsLxz[z] - 6 dD12 psi[z] -
      6 dD11 epsLyz'[z] + 3 bB11 s fiLx'[z] +
      6 dD11 fiLy'[z] + 6 bB11 wW'[z] -
      3 dD11 s fiLz''[z] - dD11 s2 psi''[z])) / 6
mMLy=Simplify[D[energy,fiLx'[z]]]
2
(s2 (-(bB12 psi[z]) - bB11 epsLyz'[z] +
      bB11 fiLy'[z] - 2 bB16 fiLz'[z] + aA11 wW'[z]) \
      + (2 s3 (aA11 fiLx'[z] - 2 bB16 psi'[z] -
      bB11 fiLz''[z])) / 3 -  $\frac{bB11 s^4 psi''[z]}{4}$ ) / 2

```

■ Program 2 Calculation for Amount of Twist

■ Objective

This program computes the displacements of a beam with piezo elements bonded antisymmetrically on both sides of the substrate at the angles of 45 degrees and -45 degrees w.r.t. the longitudinal axis (see Fig. 5.2 in Chapter 5)

■ Convention

The variable definitions in this program agree with the naming convention used in Chapter 5 everywhere it does not conflict with the software syntax. The Greek characters are typed in their abbreviated English form:

the ... theta
fi ... phi
lam ... lambda
alf ... alpha
eps ... epsilon

Any variable name starting with a upper case letter is preceded by a corresponding lower case letter to avoid command names which always start with a upper case letter. The capital letter L is the variable name reserved to indicate that the number(s) or letter(s) that follow(s) is/are the subscript(s), while U indicates the character(s) that follow(s) is/are superscript(s)

The following statements turn off the spelling check for the similiar names used in the program:

```
Off [General::spell];  
Off [General::spell1];
```

■ Define Variables

☐ Define Constant

Define material's engineering constants and the dimensions of the substructure:

```
tUs=2.92*10^-4;  
bUb=0.02694;  
eEUsL1=6.30*10^10;  
eEUsL2=6.30*10^10;  
gGUsL12=2.63*10^10;  
vUsL12=0.33;  
vUsL21=0.33;
```

Define material's engineering constants and the dimensions of the piezo elements:

```
tUa=1.91*10^-4;  
tUc=tUs+tUa;  
lUa=2.54*10^-2;  
bUa=6.35*10^-3;
```

```

eEUaL1=6.3*10^10;
eEUaL2=6.3*10^10;
gGUaL12=2.63*10^10;
vUaL12=0.33;
vUaL21=0.33;

```

Compute the position of the centroid for the composite cross-section:

```

xLc=bUb/2.0;
Solid beam parameters :
the[s_]:=0.0;
r[s_]:=0.0;
q[s_]:=s-bUb/2.0;
x=s-xLc;
y=0.0;
alf[s_]:=s^2/2.0;

```

For a solid, rectangular section the warping function is zero:

```

lam[s_]:=0.0;

```

☐ Define Boundary of Each Branch for An Arbitrary Cross Section

Compute the limits of integration for the composite cross-section. The coordinate system has an origin at the left of the specimen, with the z-axis along the beam. See Fig. 5.3.

```

tau=Pi/4;
xL1=z-.01;
xL2=xL1+bUa/Cos[tau];
xL3=-z+.028;
xL4=xL3+bUa/Cos[tau];
xL5=bUb;
Clear[tau]

```

Compute the position of the centroid for the composite cross-section:

```

xLc=bUb/2.0;
Solid beam parameters :
the[s_]:=0.0;
r[s_]:=0.0;
q[s_]:=s-bUb/2.0;
x=s-xLc;
y=0.0;

```

```
alf[s_] := s^2/2.0;
```

For a solid, rectangular section the warping function is zero:

```
lam[s_] := 0.0;
```

■ Stiffness Matrices

□ Transfer matrix and reduced stiffness matrix

This part of the program defines the matrices needed for calculating the stiffness matrices: extensional: aA, coupling: bB and bending: dD.

The following matrix defines the reduced stiffness matrix for the substrate. It has the information of the material's engineering constant (Young's Modulus, Poisson's Ratio, etc..)

```
qQ := {{qQL11, qQL12, 0.0},
        {qQL12, qQL22, 0.0},
        {0.0, 0.0, qQL66}};
```

The following matrix is the transformation matrix for the substrate (Eq. (5.37)). This matrix is used to transfer matrices from local to global.

```
tm :=
{{(Cos[tau])^2, (Sin[tau])^2,
  2.0*(Sin[tau])*(Cos[tau])},
 {(Sin[tau])^2, (Cos[tau])^2,
  -2.0*(Sin[tau])*(Cos[tau])},
 {-(Sin[tau])*(Cos[tau]), (Sin[tau])*(Cos[tau]),
  (Cos[tau])^2 - (Sin[tau])^2}};
```

The following operation finds the transformed, reduced stiffness matrix for the branch (Eq. (5.32)):

```
qQbar := Inverse[tm] . qQ . Transpose[Inverse[tm]];
```

□ Stiffness matrix for the substrate

```
tau = 0.0;
```

Define the elements of the stiffness matrix for the substrate in terms of engineering constants (Eq. (5.33), (5.34), (5.35), and (5.36)):

```
qQL11 = eEUsL1 / (1.0 - vUsL12*vUsL21);
qQL12 = vUsL12*eEUsL2 / (1.0 - vUsL12*vUsL21);
qQL22 = eEUsL2 / (1.0 - vUsL12*vUsL21);
qQL66 = gGUsL12;
```

Define the vector of ply coordinates for the substrate:

```
dzUs = {-tUs/2.0, tUs/2.0};
```

Define the vector of reduced stiffness multipliers for extensional stiffness matrix calculation for the substrate (Eq. (5.29)):

```
zAUs = dzUs[[2]] - dzUs[[1]];
```

Define the vector of reduced stiffness multipliers for coupling stiffness matrix calculation for the substrate (Eq. (5.30)):

$$\mathbf{zBUS} = (\mathbf{dzUs}[[2]]^2 - \mathbf{dzUs}[[1]]^2) / 2.0;$$

Define the vector of reduced stiffness multipliers for bending stiffness matrix calculation for the substrate (Eq. (5.31)):

$$\mathbf{zDUs} = (\mathbf{dzUs}[[2]]^3 - \mathbf{dzUs}[[1]]^3) / 3.0;$$

Define the coefficients of the extensional stiffness matrix for the substrate:

$$\mathbf{aAUs} = \mathbf{qQbar} * \mathbf{zAUs};$$

Define the coefficients of the coupling stiffness matrix :

$$\mathbf{bBUS} = \mathbf{qQbar} * \mathbf{zBUS};$$

Define the coefficients of the bending stiffness matrix:

$$\mathbf{dDUs} = \mathbf{qQbar} * \mathbf{zDUs};$$

□ Stiffness matrix for the actuator

This part of the program calculates the stiffness matrices: extensional: $\mathbf{aA}$, coupling: $\mathbf{bB}$ and bending: $\mathbf{dD}$ for piezoactuators.

Define actuator reduced stiffness matrix in terms of engineering constants for the actuator (Eq. (5.33), (5.34), (5.35), and (5.36)):

$$\mathbf{qQL11} = \mathbf{eEUaL1} / (1.0 - \mathbf{vUaL12} * \mathbf{vUaL21});$$

$$\mathbf{qQL12} = \mathbf{vUaL12} * \mathbf{eEUaL2} / (1.0 - \mathbf{vUaL12} * \mathbf{vUaL21});$$

$$\mathbf{qQL22} = \mathbf{eEUaL2} / (1.0 - \mathbf{vUaL12} * \mathbf{vUaL21});$$

$$\mathbf{qQU66} = \mathbf{gGUaL12};$$

Define the vector of coordinates for actuators:

$$\mathbf{dzUa} = \{\mathbf{tUs} / 2.0 + \mathbf{tUa}, \mathbf{tUs} / 2.0, -\mathbf{tUs} / 2.0, -(\mathbf{tUs} / 2.0 + \mathbf{tUa})\};$$

Define the vector of reduced stiffness multipliers for extensional stiffness matrix calculation for actuators (Eq. (5.29)):

$$\mathbf{zAUa} = \{\mathbf{dzUa}[[1]] - \mathbf{dzUa}[[2]], \mathbf{dzUa}[[3]] - \mathbf{dzUa}[[4]]\};$$

Define the vector of reduced stiffness multipliers for coupling stiffness matrix calculation for actuators (Eq. (5.30)):

$$\mathbf{zBUa} = \{\mathbf{dzUa}[[1]]^2 - \mathbf{dzUa}[[2]]^2, \mathbf{dzUa}[[3]]^2 - \mathbf{dzUa}[[4]]^2\} / 2.$$

Define the vector of reduced stiffness multipliers for bending stiffness matrix calculation for actuators (Eq. (5.31)):

$$\mathbf{zDUa} = \{\mathbf{dzUa}[[1]]^3 - \mathbf{dzUa}[[2]]^3, \mathbf{dzUa}[[3]]^3 - \mathbf{dzUa}[[4]]^3\} / 3.$$

Define the coefficients of the extensional stiffness matrix for the lower actuator:

$$\mathbf{tau} = -\mathbf{Pi} / 4;$$

$$\mathbf{aAU1} = \mathbf{qQbar} * \mathbf{zAUa}[[2]];$$

bBU1=qQbar*zBUa[[2]];

Define the coefficients of the bending stiffness matrix for the lower actuator:

dDU1=qQbar*zDUa[[2]];

Define the coefficients of the extensional stiffness matrix for the upper actuator:

tau=Pi/4;

aAUu=qQbar*zAUa[[1]];

Define the coefficients of the coupling stiffness matrix for the upper actuator:

bBUu=qQbar*zBUa[[1]];

Define the coefficients of the bending stiffness matrix for the upper actuator:

dDUu=qQbar*zDUa[[1]];

■ Define overall stiffness matrix of the beam

The following equations are derived in the first part of the program.

Define the stiffness coefficients for the substrate - [0, x5]:

kKU1L22 = dDU1[[1,1]]*xL5;
kKU1L23 = bBU1[[1,1]]*xL5^2/2;
kKU1L32 = bBU1[[1,1]]*xL5^2/2;
kKU1L33 = aAU1[[1,1]]*xL5^3/3;
kKU1L35 = -bBU1[[1,3]]*xL5^2;
kKU1L53 = -bBU1[[1,3]]*xL5^2;
kKU1L55 = 4dDU1[[3,3]]*xL5;

Define the stiffness coefficients for the upper piezo element - [x1, x2]:

kKU2L22 = dDUu[[1,1]]*(xL2-xL1);
kKU2L23 = bBUu[[1,1]]*(xL2^2-xL1^2)/2;
kKU2L32 = bBUu[[1,1]]*(xL2^2-xL1^2)/2;
kKU2L33 = aAUu[[1,1]]*(xL2^3-xL1^3)/3;
kKU2L35 = -bBUu[[1,3]]*(xL2^2-xL1^2);
kKU2L53 = -bBUu[[1,3]]*(xL2^2-xL1^2);
kKU2L55 = 4dDUu[[3,3]]*(xL2-xL1);

Define the stiffness coefficients for the lower piezo element - [x3, x4]:

kKU5L22 = dDU1[[1,1]]*(xL4-xL3);
kKU5L23 = -bBU1[[1,1]]*(xL4^2-xL3^2)/2;
kKU5L32 = -bBU1[[1,1]]*(xL4^2-xL3^2)/2;
kKU5L33 = aAU1[[1,1]]*(xL4^3-xL3^3)/3;
kKU5L35 = -bBU1[[1,3]]*(xL4^2-xL3^2);
kKU5L53 = -bBU1[[1,3]]*(xL4^2-xL3^2);
kKU5L55 = 4dDU1[[3,3]]*(xL4-xL3);

Define the stiffness coefficients for the total stiffness matrix:

```

kKL22 = N[kKU1L22+kKU2L22+kKU5L22];
kKL23 = N[kKU1L23+kKU2L23+kKU5L23];
kKL32 = N[kKU1L32+kKU2L32+kKU5L32];
kKL33 = N[kKU1L33+kKU2L33+kKU5L33];
kKL35 = N[kKU1L35+kKU2L35+kKU5L35];
kKL53 = N[kKU1L53+kKU2L53+kKU5L53];
kKL55 = N[kKU1L55+kKU2L55+kKU5L55]

```

```
0.0258222
```

```

kK:={ { kKL22 , kKL23, 0},
      { kKL32 , kKL33, kKL35},
      { 0 , kKL53, kKL55} };

```

■ Define the induced strain vector and piezo-induced loads

```
indstr={lambda,lambda,0.0};
```

The strain defined in the following equation is according to third degree polynomial best fit for a group of experimental data (Curve Fitting for the Bonded Strains of A Piezo Element in Appendix A). The strain is equal to the difference between the free strain and bonded strain, both of which are described in Appendix A.

```

lambda=((25.42-.684 volt +.038 volt^2-
0.000144volt^3)-(-0.78553-.83845 volt-.0135582 volt^2
+0.00000123861 volt^3)) 10^-6

```

```

(26.2055 + 0.15445 volt + 0.0515582 volt^2 -
0.000145239 volt^3) / 1000000

```

■ Solve for displacement in terms of stiffness and loads

Generate the load vector along z-axis (Eq. (5.18)):

```

zaxloa={mMUaLx,mMUaLy,tT}
{mMUaLx, mMUaLy, tT}

```

Solve for the vector of axial and rotation vector derivatives (Eq. (5.18)):

```

disder=Simplify[LinearSolve[kK,zaxloa]];
fiUpLz=Simplify[disder[[3]]]

```

```
38.7263 tT
```

□ Induced force and moment from the lower and upper actuator

according to Eq. (5.45) and (5.46):

```
mMUl=bBUl.indstr;
```

mMUu=Simplify[bBUu.indstr]

$$\{0.11366 + 0.000669891 \text{ volt} + 0.000223622 \text{ volt}^2 - 6.29939 \cdot 10^{-7} \text{ volt}^3,$$

$$0.11366 + 0.000669891 \text{ volt} + 0.000223622 \text{ volt}^2 - 6.29939 \cdot 10^{-7} \text{ volt}^3, 0.\}$$

nNUl=aAUl.indstr;

nNUu=Simplify[aAUu.indstr]

$$\{470.643 + 2.77388 \text{ volt} + 0.92597 \text{ volt}^2 - 0.00260844 \text{ volt}^3, 470.643 + 2.77388 \text{ volt} + 0.92597 \text{ volt}^2 - 0.00260844 \text{ volt}^3, 0.\}$$

□ Induced beam torque

The following formula is defined in Eq. (5.47), the "2" at the end means two elements.

tT=Simplify[(-mMUu[[3]]-mMUu[[2]]) bUa/Cos[tau]*2]

$$0.0127 \text{ Sqrt}[2] (-0.0678511 - 0.0003999 \text{ volt} -$$

$$0.000133494 \text{ volt}^2 + 3.7605 \cdot 10^{-7} \text{ volt}^3)$$

■ Solve for twist angle

The following equation calculates the total amount of twist on the beam when a series of voltages from 20 to 140 volts were applied.

aa=Integrate[Simplify[fiUpLz],{z,.01,.028}]*180/Pi

$$(5.9924 \cdot 10^{-7} \text{ Sqrt}[2] (-180431. - 1063.42 \text{ volt} -$$

$$354.99 \text{ volt}^2 + 1. \text{ volt}^3) / \text{Pi}$$

```

angle=N[Table[-aa,{volt,40,140,20}]]
{0.196097, 0.352351, 0.546368, 0.7652, 0.995899,
1.22552}

```

Experimental data by the author for a flat plate twisted by the piezo elements:

```

dataex={{ 0,0 },
{ 40,.312524},
{ 60,.493097},
{ 80,.711875},
{100,.920245},
{120,1.161621},
{140,1.389125}};

```

Twist angle by this program:

```

dataan={{ 0,0 },
{ 40,angle[[1]]},
{ 60,angle[[2]]},
{ 80,angle[[3]]},
{100,angle[[4]]},
{120,angle[[5]]},
{140,angle[[6]]}};

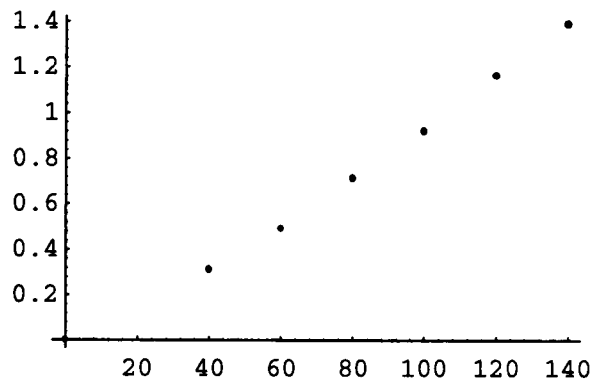
```

Plot of experimental data:

```

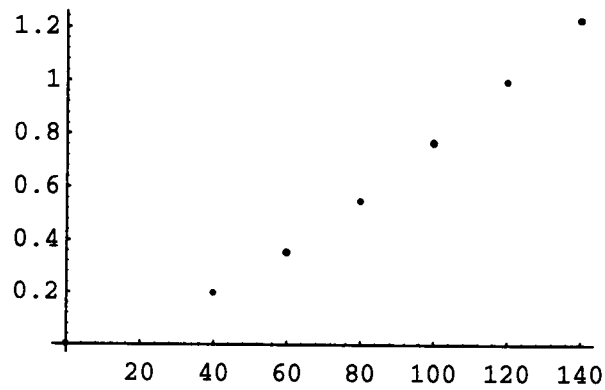
plotex=ListPlot[dataex];

```



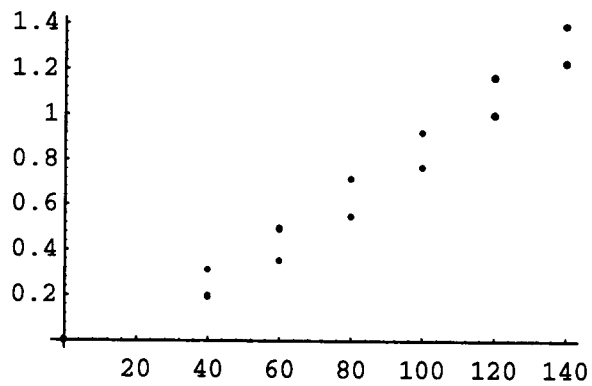
Plot of calculated data:

```
plotan=ListPlot[dataan];
```



Combined plot:

```
show[plotex,plotan];
```



■ Appendix G ACLD Structural Damping Modelling

■ Objective

This Mathematica program numerically evaluates the effectiveness of the Active Constrained Layer Damping (ACLD) method in reducing the tip displacement of a beam when it is subjected to a harmonic external force

The following statement turn off the spelling check for the similar names used in the program:

```
Off[General::spell];  
Off[General::spell1];
```

■ Geometric Definitions and Material Properties

The following constants defines dimensions and mechanical properties of the elements, refer to fig. 6.3.

```
h1=.00025;  
h2=.000028;  
h3=.00025;  
h4=.005;  
L1=0.1;  
L2=0.25;  
L=0.25;  
b=.015;  
p1=2700;  
p2=2700;  
p3=2700;  
p4=7800;  
E1=63.0*10^9;  
E3=63.0*10^9;  
E4=21.0*10^10;  
I1=N[1/12*b*h1^3];  
I2=N[1/12*b*h2^3];  
I3=N[1/12*b*h3^3];  
I4=N[1/12*b*h4^3];  
D1=(E1 I1+E3 I3 +E4 I4)/b;
```

■ External Force

The assumed external force and its frequency are defined as follows:

```
w=433.0;
```

P0=5.0;

■ Piezo-Electric Constraining Layer

The following parameters define the piezo characteristics of the piezo sensor and piezo actuator:

d31=18.6*10[^](-11);

k31=.34;

g31=11.6*10[^](-3);

k3t=1950.0;

Dd=h4/2;

Area=b (L2-L1);

c=8.854*10[^](-12) Area k3t/h3;

Kp=1000.0;

■ Visco-Elastic Layer

The shear modulus of the viscoelastic layer is defined as follows:

G=Simplify[142000.0 Exp[.494 Log[w/2/Pi]](1+1.46I)]/N;

■ Intermediate Variables

The following parameters do not have physical meaning. They are defined to reduce the size of long expressions.

g=G/h2 ((E1 h1 +E3 h3 +E4 h4)/(E1 h1 E3 h3 E4 h4));

D2=(E3 h3[^]2+2 E4 h3 h4 +E4 h4[^]2)/(2(E3 h3+E4 h4));

d=h2+h1/2+D2;

Y=d[^]2/D1 (E1 h1 E3 h3 E4 h4/(E1 h1+E3 h3 +E4 h4));

Dp=E4 I4;

k=Sqrt[p4 w[^]2/E4];

m4=p4 h4 b;

m=(p1 h1+p2 h2+ p3 h3+p4 h4)b;

■ Governing Equations

The following equations are the governing equations for the beam section1, section2, and section 3. The displacement w1 is solved with 4 unknown constants A[1], A[2], A[3] and A[4]; the displacement w2 is solved with 6 unknown constants B[1], B[2], B[3], B[4], B[5] and B[6]; the displacement w3 is solved with 4 unknown constants C[1], C[2], C[3] and C[4].

WW1=DSolve[E4 I4 W1''''[x]-m4 w[^]2 W1[x]==0,W1,x,
DSolveConstants->A];

WW2=DSolve[W2''''''[x]-g (1+Y)W2''''[x]-
m w[^]2/D1 W2''[x]+m g w[^]2/D1 W2[x]==0,W2,x,

```

DSolveConstants->B];
WW3=DSolve[E4 I4 W3''''[x]-m4 w^2 W3[x]==0,W3,x,
DSolveConstants->C];

```

■ Induced Strain

The following equations are related to the voltage generated by the piezo sensor and the voltage applied to the piezo actuator.

```

Vs=-(k31^2 Dd b/(g31 c)) Integrate[(W2''[x]/.WW2),
{x,L1,L2}];
Va=-Kp Vs;
Ep=d31/h1 Va;

```

■ Boundary Conditions

The fourteen boundary conditions of the beam are listed as follows:

```

EQ1=W1'''[0]-P0/Dp /.WW1;
EQ2= W1''[0]/.WW1;
EQ3= W3[L] /.WW3;
EQ4=W3'[L] /.WW3;
EQ5= (W1[L1]/.WW1)-(W2[L1]/.WW2);
EQ6= (W1'[L1]/.WW1)-(W2'[L1]/.WW2);
EQ7= (W1''[L1]/.WW1)-(W2''[L1]/.WW2);
EQ8=( W1'''[L1]/.WW1)-(W2'''[L1]/.WW2);
EQ9=(W3[L2]/.WW3)-(W2[L2] /.WW2);
EQ10=(W3'[L2]/.WW3)-(W2'[L2] /.WW2);
EQ11=(W3''[L2]/.WW3)-(W2''[L2] /.WW2);
EQ12=(W3'''[L2]/.WW3)-(W2'''[L2] /.WW2);
EQ13= (1/k) Cot[k L1] (1/g (W2''''[L1]/.
WW2)-Y (W2'''[L1]/.WW2)-m w^2/(D1 g) (W2[L1]/.WW2)-
Y/d Ep)+ 1/g^2 (W2''''[L1]/.WW2)-Y/g (W2'''[L1]/.
WW2)-(Y+m w^2/(D1 g)) (W2''[L1]/.WW2)-Y/d x Ep/.
x->L1;
EQ14=(1/k) Tan[k(L2-L)] (1/g (W2''''[L2]/.
WW2)- Y (W2'''[L2]/.WW2)-m w^2/(D1 g) (W2[L2]/.
WW2)-Y/d Ep)- (1/g^2 (W2''''[L2]/.WW2)-
Y/g (W2'''[L2]/.WW2)-(Y+m w^2/(D1 g)) (W2'[L2]/.
WW2)-Y/d x Ep )/.x->L2;

```

■ Solution for 14 Coefficients

The 14 unknown constants are solved from 14 simutanuuous equations:

```

SOS=Solve[{EQ1==0,EQ2==0,EQ3==0,EQ4==0,EQ5==0,

```

```
EQ6==0,EQ7==0,EQ8==0,EQ9==0,EQ10==0,EQ11==0,
EQ12==0,EQ13==0,EQ14==0},
{A[1],A[2],A[3],A[4],B[1],B[2],B[3],B[4],B[5],B[6],
C[1],C[2],C[3],C[4]}};
```

■ Displacement at the Tip of the Beam

Find the numerical value for the tip displacement of the beam:

```
Result=Abs[W1[0]/.WW1/.SOS]
{{0.0000610715}}
```

■ Controlling Potential on the Active Piezo-Film (volts)

Find out the maximum electrical potential applied on the actuator:

```
Abs[Va/.SOS]
{{118.445}}
```

VITA

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