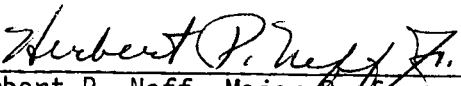
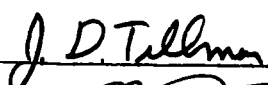
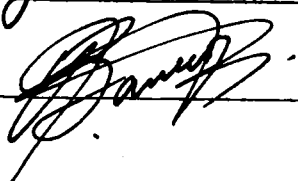


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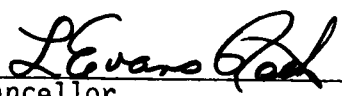
I am submitting herewith a thesis written by Rafael Segundo Montiel entitled "Trigonometric Theory to Approximate the Current Distribution on Cylindrical Dipoles." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.


Herbert P. Neff, Major Professor

We have read this thesis
and recommend its acceptance:

Accepted for the Council:


Vice Chancellor
Graduate Studies and Research

TRIGONOMETRIC THEORY TO APPROXIMATE THE
CURRENT DISTRIBUTION ON
CYLINDRICAL DIPOLES

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Rafael Segundo Montiel

March 1979

181538

To my mother Olga de Montiel
and my father Rafael A. Montiel
for cheerful endurance of time
which have been spent during
my education.

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Last but certainly not least, the author conveys his deepest appreciation to Flor Negrin for her spiritual support during this study.

ABSTRACT

This investigation involves the analysis of the cylindrical dipole antenna. The purpose of the analysis is to accurately determine the current distribution on thin cylindrical, center-driven dipole antennas. The basis for the theoretical analysis was the antenna integral equation first developed by E. Hallén. Chapter II considers this equation in detail.

Expanding the kernel of the integral equation in a Fourier cosine series, and with the use of the concept of the Delta function generator, the approximate current distribution can be expanded in the following series, which satisfies the boundary conditions on the antenna current at the ends of the dipole.

$$I_z(z) = \sum_{n=0}^N F_n \cos\left(\frac{n\pi z}{h}\right)$$

Chapter III develops a solution for this current approximation. The coefficients were computed using the method of moments.

The results of this trigonometric theory are compared with results obtained from various theoretical techniques of King-Middleton and the experimental measurements of Mack. The trigonometric theory is demonstrated to provide accurate current distributions and terminal impedances in twenty-fifth-order solutions for half and full wave length antennas. Some results are presented for different order solutions in agreement with the Barrero [22] solutions.

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CHAPTER I

INTRODUCTION AND GENERAL CONSIDERATIONS

I. THEORY OF ANTENNA

The antenna name was probably suggested by the resemblance of the thin arms of the probe to the feelers or antennae of an insect. At present, however, this name is applied to all circuits designed for effective long-distance coupling.

An antenna is a device designed to radiate an electromagnetic wave into free space when excited by a wide range of alternating current or, alternatively, to absorb energy from an electromagnetic wave and produce a wide range of alternating currents [1]. In the case of transmission, the impedance presented to the transmission line or wave-guide by the antenna should match the characteristic impedance of the line or wave-guide, while in the case of reception the line should be matched to the antenna. An antenna may be capable of functioning over a wide band of frequencies or, alternatively, at one specific frequency. In the first case the antenna is called aperiodic, and in the second case tuned.

There are several problems to be considered before designing a practical antenna. Some of them are:

- a. to develop basic field concepts.
- b. to obtain equations for calculating fields and to examine their principal implications.
- c. to derive the expressions for the electric and magnetic intensity of the wave excited by an element of electric current in free space.

- d. to examine directive effects of spatial arrangements of current elements.
- e. to obtain formulas for transmission of power between antennas.
- f. to derive current distributions in antennas with given connections to generators.
- g. to examine the reciprocity between transmitting and receiving antennas.
- h. to evaluate antenna impedance.

Finally the general principles have to be applied to order to select examples of practical antennas.

II. BASIC ANTENNA PRINCIPLES

The antenna functions as a transducer, coupling power from waveguides or coaxial lines to free space, and viceversa. The basic principles of antennas are better understood by the radiation intensity pattern. Antennas do not radiate power equally well in all directions in free space.

If the radiation intensity pattern is scaled such that the maximum intensity is unity the result is commonly called simply the radiation pattern. Figure 1 shows a possible pattern, the angular separation between points on the radiation pattern that are $3d_B$ down from the maximum is called the beam width (θ_B).

An important antenna parameter is its gain. There are two types of gain, one is directive gain, and the other is the power gain. Other antenna parameters are the aperture efficiency, signal reception and the polarization.

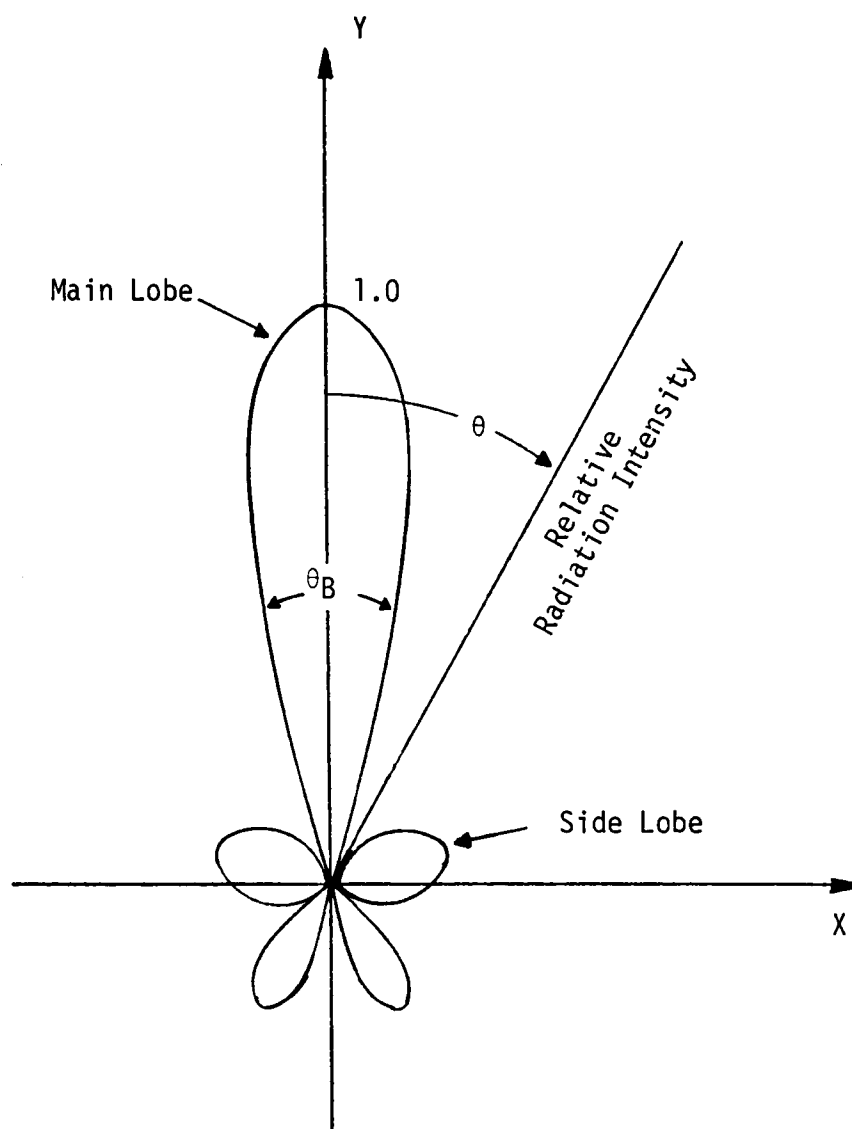


Figure 1. Radiation pattern in polar coordinates.

III. CYLINDRICAL ANTENNA

The first antenna investigated was the Hertizian dipole, consisting essentially of an oscillating filament of current of very short length between two metal spheres. The analysis of cylindrical wires of finite length rather than infinitesimal dipoles was begun with the determination by Abraham in 1896 of the natural modes of a thin ellipsoid.

Figure 2 illustrates a Hertizian dipole for which, the length $2h$ of the dipole is restricted to values such that $kh \ll 1$. The ends of the antenna are assumed to be capable of storing a charge.

In 1937 L. V. King [2] presented work based on an integral equation, but the significant advances in linear dipole analysis use Hallen's formulation. Modern dipole antenna theory began with Hallen's studies in 1938 [3].

A dipole antenna consists of two equal colinear rods separated axially by a small gap. In general the antenna may be of any length.

The shape of the ends of the antenna has been a subject of such discussion. Some authors using accurate analysis of the problem specify that the ends of the antenna must be shaped so as to prevent the accumulation of electric charge there, and thus make them hemispherical [4]. But in any case, the consensus seems to be that unless the radius of the antenna approaches 1/10th of its half length, the shaping of the ends is not critical, and for antennas of more practical proportions, half-wave length and full wave length, the ends effects are small.

Then a dipole as described above is connected to a balanced source of pulsating electrical energy, current flows in the two rods [5]. This current must decay in some manner along the length of the antenna, for no

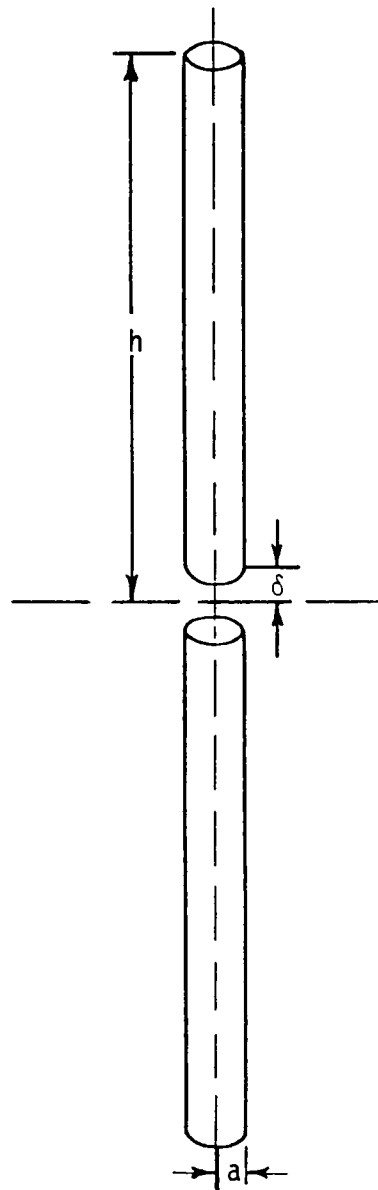


Figure 2. Dimensions of the Hertzian dipole.

current can leave the ends and only a very small amount of charge can accumulate there.

The current at any point in the antenna will certainly depend on the distance of that point from the base gap, and if a right-handed cartesian system of coordinates is used for the problem (Figure 3), the current at any point will be a function of z .

Thus, the problem of the dipole becomes that of finding a solution to Maxwell's equations with a particular set of boundary conditions corresponding to the above description of the dipole. The voltage across the base gap must be equal to the voltage applied to the antenna.

The current at the point, $z = +h$ and $z = -h$, must be zero. Also the field and current within the antenna must everywhere satisfy Ohm's law, as the antenna must be made of a conducting material, in order to satisfy the following boundary conditions:

- a. $V_{z=\delta} = V/2$ $V_{z=-\delta} = -\frac{V}{2}$
- b. $I_{z=h} = 0$ $I_{z=-h} = 0$
- c. $\tilde{E} = -\rho\tilde{J}$ for all points

For the most difficult part is that of finding the values of current that satisfy completely all the above boundary conditions.

There are many reasons to study the current distribution in a dipole. Some of them are listed below.

- a. It makes possible accurate calculation of the impedance of the antenna, which is very important in practical consideration of matching for maximum power transfer.

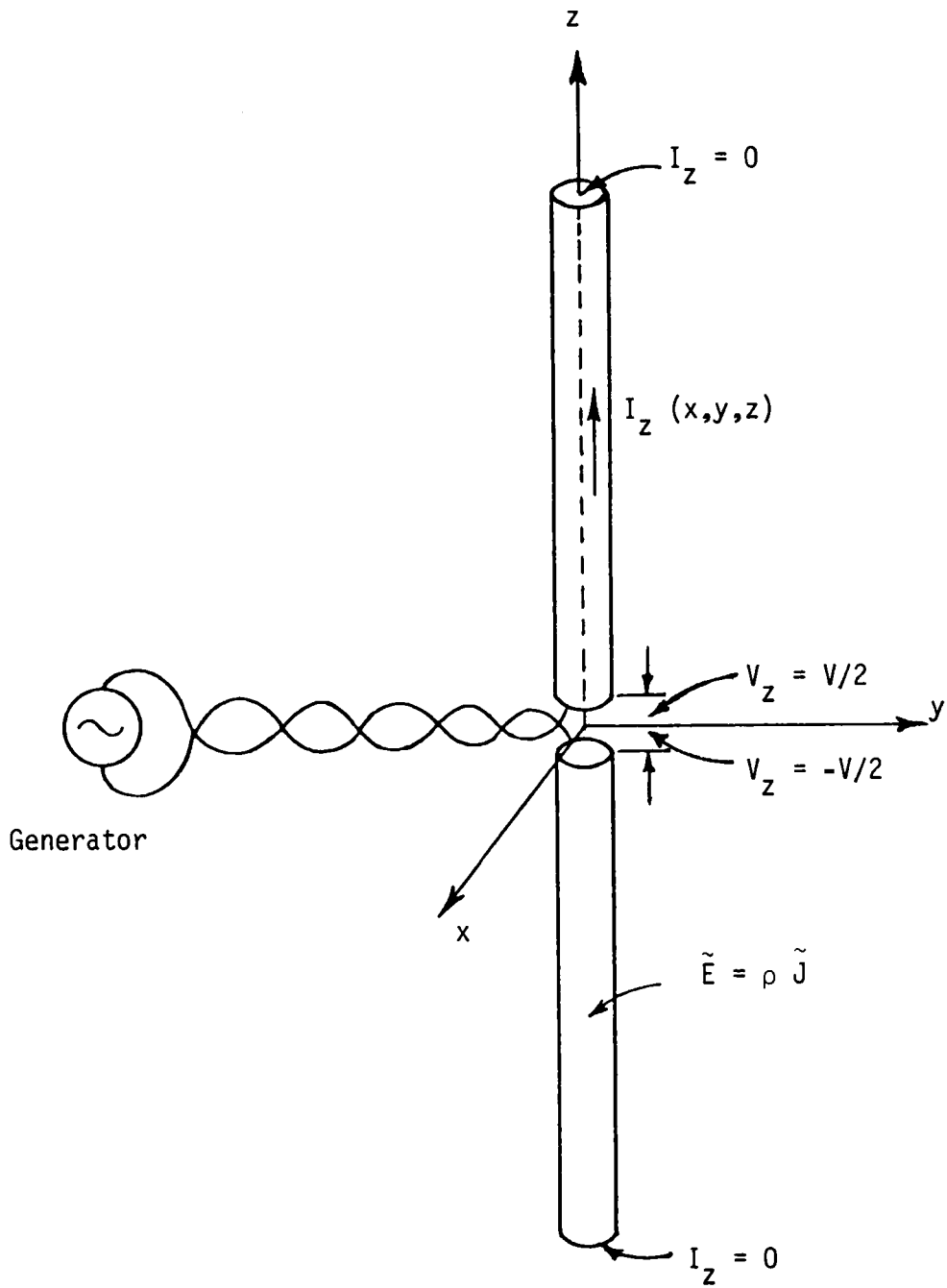


Figure 3. The dipole with coordinates axes and boundary conditions.

- b. It can be used to accurately determine the mutual coupling between adjacent antennas. This is important due to the increasing use of arrays consisting of large numbers of closely spaced dipoles.
- c. It could possibly shed more light on the actual manner in which antennas radiate power, and thus extend out theoretical knowledge of antennas in general.

IV. TERMINAL-ZONE PHENOMENA

One important consideration in a cylindrical dipole model is the terminal-zone phenomena and the treatment of the gap. Several investigators, have found that the terminal-zone phenomena and coupling between the line and antenna greatly affects the apparent impedance of an antenna as a load terminating a transmission line. In simple transmission line analysis the conventional line equations are not valid. Some considerations are [6]

- a. the absence of a conductor in the gap of the antenna.
- b. deviation of capacitance per unit length and inductance per unit length at the end of the line since a finite line behaves differently than an infinite line.
- c. electromagnetic coupling of the antenna and transmission line R. King [7] has shown that the nonuniform line near the antenna junction may be replaced by a fictitious section of line which contains only values of impedance and admittance.

Figure 4 illustrates an actual transmission line coupled to an antenna and an ideal line, with King's terminal-zone correction network

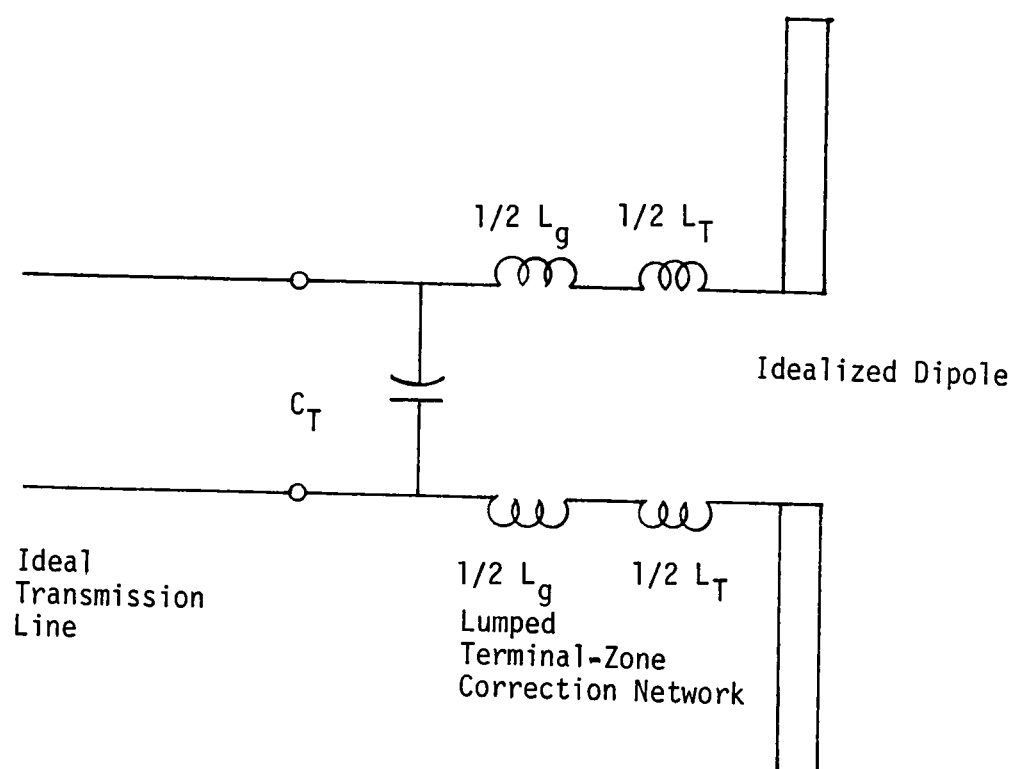


Figure 4. Equivalent circuit for Terminal-zone phenomena.

connected to the idealized dipole. C_T is the lumped capacitance which compensates for electromagnetic coupling and the deviation of the line's characteristic impedance near the junction. L_g is the lumped inductance necessary to correct for the gap. L_T is the lumped inductance necessary to compensate for changes of the characteristic impedance toward the end of the line. The apparent impedance Z_a is the measured quantity, while the idealized impedance Z_b , results from a theoretical analysis of the excitation of the cylindrical dipole model by a delta function generator.

CHAPTER II

DERIVATION OF INTEGRAL EQUATIONS

I. PRELIMINARY CONSIDERATIONS

Maxwell's Equations

In the most general sense, antenna theory deals with the interactions between charges and currents in a configurations of conductors immersed in free space or some other medium. Moving charges in the elements of a transmitting antenna interact to produce motion in the charges present in the elements of a receiving antenna.

The quantitative description of the aforementioned interactions may be facilitated by the introduction of the electric field vector $\tilde{E}$ and the magnetic flux density vector $\tilde{B}$. In general $\tilde{E}$ and $\tilde{B}$ are interrelated. The relationship between them is specified by a set of partial differential equations known as Maxwell's equations. Maxwell's equations also relate $\tilde{E}$ and $\tilde{B}$ to their associated charge and current density functions, so that Maxwell's equations, in conjunction with appropriate boundary conditions, may be regarded as the definitions of the electric field and magnetic flux density vectors. For an arbitrary hemogeneous isotropic medium, Maxwell's equations may be stated as follows:

$$\tilde{\nabla} \cdot \tilde{E} = \rho/\epsilon, \quad \text{II-1}$$

$$\tilde{\nabla} \times \tilde{E} = - \frac{\partial \tilde{B}}{\partial t}, \quad \text{II-2}$$

$$\tilde{\nabla} \times \tilde{\mathbf{B}} = \mu(\tilde{\mathbf{J}} + \epsilon \frac{\partial \tilde{\mathbf{E}}}{\partial t}), \quad \text{II-3}$$

$$\tilde{\nabla} \cdot \tilde{\mathbf{B}} = 0. \quad \text{II-4}$$

Where $\tilde{\mathbf{E}}$ is the electric field intensity vector in volts per meter, $\tilde{\mathbf{B}}$ is the magnetic flux density vector in webers per square meter, ϵ and μ are the permittivity and permeability constants respectively, whose free-space values are

$$\epsilon_0 = \frac{1}{36\pi} \times 10^{-9} \text{ farads/meter}, \quad \text{II-5}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ henries/meter}. \quad \text{II-6}$$

The volume charges density ρ in coulombs per cubic meter and the current density $\tilde{\mathbf{J}}$ in amperes per square meter are related to each other by the continuity equation

$$\tilde{\nabla} \cdot \tilde{\mathbf{J}} = - \frac{\partial \rho}{\partial t}. \quad \text{II-7}$$

Which may be obtained by taking the divergence of Eq. II-3 and combining the resulting equation with Eq. II-1.

The frequency domain representation of Maxwell's equation is [1, page 123]

$$\tilde{\nabla} \times \tilde{\mathbf{E}} = - j\omega \tilde{\mathbf{B}}, \quad \text{II-8}$$

$$\tilde{\nabla} \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega \tilde{\mathbf{D}}, \quad \text{II-9}$$

$$\tilde{\nabla} \cdot \tilde{D} = \rho , \quad \text{II-10}$$

$$\tilde{\nabla} \cdot \tilde{B} = 0 . \quad \text{II-11}$$

The determination of the current distribution is a boundary value problem involving the solution of Eqs. II-1 and through II-4 subject to the boundary conditions imposed by the physics of a particular problem

Boundary Conditions

In order to simplify the specification of the boundary conditions it is customary to define the following functions [8, page 34-36]

$$\tilde{H} = \frac{\tilde{B}}{\mu} , \quad \text{II-12}$$

$$\tilde{D} = \epsilon \tilde{E} . \quad \text{II-13}$$

These functions, known as the magnetic field intensity and electric flux density respectively, allow Maxwell's equations to be written as

$$\tilde{\nabla} \cdot \tilde{D} = \rho , \quad \text{II-14}$$

$$\tilde{\nabla} \times \tilde{E} = - \frac{\partial \tilde{B}}{\partial t} , \quad \text{II-15}$$

$$\tilde{\nabla} \times \tilde{H} = \tilde{J} + \frac{\partial \tilde{D}}{\partial t} , \quad \text{II-16}$$

$$\tilde{\nabla} \cdot \tilde{B} = 0 . \quad \text{II-17}$$

For these equations the following statements can be made: (a) The tangential component of $\tilde{E}$ is continuous at an interface between two media, (b) The tangential component of $\tilde{H}$ is continuous at an interface except at the surface of a perfect conductor, where it is discontinuous by an amount equal to the surface current per unit width

(c) The normal component of $\tilde{B}$ is continuous at the surface of discontinuity, (d) The normal component of $\tilde{D}$ is continuous if there is no surface charge density, the discontinuity is by an amount equal to the surface charge density (if present).

In terms of the variables of Eqs. II-14 through II-17, the boundary conditions on the normal and tangential field components at the interface between two media may be summarized as follows:

$$E_{t1} = E_{t2} , \quad \text{II-18}$$

$$H_{t1} - H_{t2} = K, \quad \text{II-19}$$

$$D_{n1} - D_{n2} = \rho_s, \quad \text{II-20}$$

$$B_{n1} = B_{n2} . \quad \text{II-21}$$

Where K represents surface current density and ρ_s represents a surface charge density at the boundary between the two media. If medium 2 is a perfect conductor in which $\tilde{E}_1 = \tilde{B}_1 = 0$, then Eqs. II-18 through II-21 can be simplified to

$$E_{t1} = E_{t2} = 0 , \quad \text{II-22}$$

$$H_{t1} = K , \quad \text{II-23}$$

$$D_{n1} = \rho_s , \quad \text{II-24}$$

$$B_{n1} = B_{n2} = 0 . \quad \text{II-25}$$

Development of Potential Functions

Since the divergence of $\tilde{B}$ is zero, the magnetic flux density may be represented as

$$\tilde{B} = \nabla \times \tilde{A}' . \quad \text{II-26}$$

This is due to the vector identity

$$\nabla \cdot (\nabla \times \tilde{A}') = 0 .$$

This complete definition of $\tilde{A}'$ may be given as

$$\tilde{A}' = \tilde{A} + \nabla \psi \quad \text{II-27}$$

Since $\nabla \times \nabla \psi = 0$, the curl of $\tilde{A}'$ yields

$$\nabla \times \tilde{A}' = \nabla \times \tilde{A} + \nabla \times \nabla \psi = \nabla \times \tilde{A} = \tilde{B} . \quad \text{II-28}$$

Where $\tilde{A}$ is called a vector potential and ψ is called a gauge function.

Substituting Eq. II-26 into Eq. II-8 yields

$$\nabla \times (\tilde{E} + j \omega \tilde{A}) = 0 . \quad \text{II-29}$$

In words, $(\tilde{E} + j \omega \tilde{A})$ is a vector with zero curl, then a scalar potential, V , exists, such that

$$-\tilde{\nabla} V = \tilde{E} + j \omega \tilde{A} . \quad \text{II-30}$$

The electric field may be eliminated, with the combination of Eqs. II-9, II-12, II-13, and II-30 to obtain

$$\nabla(\nabla \cdot \tilde{\mathbf{A}}) - \nabla^2 \tilde{\mathbf{A}} = \omega^2 \mu \epsilon \tilde{\mathbf{A}} - j\omega \mu \epsilon \nabla V + \mu \tilde{\mathbf{J}} . \quad \text{II-31}$$

Substituting Eq. II-30 into Eq. II-1 yields

$$\nabla^2 V + j\omega \nabla \cdot \tilde{\mathbf{A}} = -\frac{\rho}{\epsilon} . \quad \text{II-32}$$

A convenient choice of the potential for a radiation problem which satisfies the invariance of $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{E}}$ is

$$\nabla \cdot \tilde{\mathbf{A}} = -j\omega \mu \epsilon V . \quad \text{II-33}$$

This particular assumption is called the Lorentz condition, and the resulting potentials are said to be in the Lorentz gauge. Now the Eqs. II-31 and II-32 can be written in a more convenient form

$$\nabla^2 \tilde{\mathbf{A}} + \omega^2 \mu \epsilon \tilde{\mathbf{A}} = -\mu \tilde{\mathbf{J}} , \quad \text{II-34}$$

$$\nabla^2 V + \omega^2 \mu \epsilon V = -\frac{\rho}{\epsilon} . \quad \text{II-35}$$

These are called the inhomogeneous vector and scalar wave equation, also known as inhomogeneous Helmholtz equations. The cartesian components of the vector quantities are,

$$\nabla^2 A_x + k^2 A_x = -\mu J_x , \quad \text{II-36}$$

$$\nabla^2 A_y + k^2 A_y = -\mu J_y , \quad \text{II-37}$$

$$\nabla^2 A_z + k^2 A_z = -\mu J_z , \quad \text{II-38}$$

$$\nabla^2 V + k^2 V = -\frac{\rho}{\epsilon} . \quad \text{II-39}$$

Where $k^2 = \omega^2 \mu \epsilon = (2\pi/\lambda)^2$.

By applying δ -function excitation and taking the Fourier transform, the impulse response may be found. The inverse Fourier transform of the impulse response is the Green's function in free space and is defined by King and Harrison [9, page 23].

$$\nabla^2 G(\tilde{r}, \tilde{r}') + k^2 G(\tilde{r}, \tilde{r}') = \delta(\tilde{r}, \tilde{r}'). \quad \text{II-40}$$

A form of convolution of the forcing function and the Green's function expresses the vector potential as

$$\tilde{A}(\tilde{r}) = \frac{\mu}{4\pi} \iiint \frac{\tilde{J}(\tilde{r}') e^{-jk|\tilde{r}-\tilde{r}'|}}{|\tilde{r}-\tilde{r}'|} dV_0. \quad \text{II-41}$$

II. CLASSICAL FORMULATION OF THE PROBLEM

The classical solution to the dipole antenna problem is included in any text containing linear dipole theory. The model used is a thin, cylindrical dipole of radius a with the axis of the dipole lying along the z axis of the cylindrical coordinate system (ρ, ϕ, z) . The dipole extends from $z = -\delta$ to $+\delta$ and the elements extend from $+\delta$ to h and $-\delta$ to $-h$. The geometry of the problem is shown in Figure 2 (Chapter I, page 5).

The associated boundary value problem is for a conductor of radius a extending from h to $+h$ with an infinitesimally small gap at $z = 0$. For most practical purpose infinite conductivity may be assumed,

however Hickman and Tillman [10], and King, Harrison and Arosen [11] have carried out solutions to the antenna problem by retaining the finite conductivity of the dipole. The other assumption considers the source as a slice generator such that the electric field in the gap is azimuthally symmetric. R. King [12, page 87], Duncan and Hinchey [13, page 571] and T. Wu [14, page 333] have studied this problem. The voltage across the gap is defined by

$$V = - \int_{\text{gap}} E_s dz . \quad \text{II-42}$$

Where E_s is the z - component of electric field at $\rho = a$. Therefore, the assigned boundary values are

$$E_z = 0 \quad \text{for} \quad \begin{array}{c} \rho=a \\ \delta \leq |z| \leq h , \\ \text{All } \phi \end{array} \quad \text{II-43}$$

$$E_z = - V \delta (z) \quad \text{for} \quad |z| \leq h . \quad \text{II-44}$$

Where V , the amplitude of the time-harmonic input voltage, is held constant and $\delta (z)$ is the usual Dirac delta function. With these conditions, the combination of Eqs. II-30 and II-33 gives

$$\begin{aligned}
E_z &= \frac{1}{j\omega\mu\epsilon} \left[\tilde{\nabla} \cdot (\tilde{\nabla} \tilde{A}) \right]_z - j\omega A_z \\
&= -\frac{j}{\omega\mu\epsilon} \left[(\tilde{\nabla}^2 \tilde{A})_z + k^2 A_z \right] \\
&= -\frac{j}{\omega\mu\epsilon} \left(\frac{\partial^2 A_z}{\partial z^2} + k^2 A_z \right)
\end{aligned}
\tag{II-45}$$

From Eqs. II-44 and Eqs. II-43, II-45 we obtain

$$\frac{\partial^2 A_z}{\partial z^2} + k^2 A_z = -j\omega\mu\epsilon V\delta(z), \quad |z| \leq h
\tag{II-46}$$

$$\frac{\partial^2 A_z}{\partial z^2} + k^2 A_z = 0 \quad \cdot \quad \begin{array}{l} \rho = a \\ \delta \leq |z| \leq h \\ \text{All } \phi \end{array}
\tag{II-47}$$

In general the vector potential A_z is independent of the azimuth angle ϕ , and also A_z is a function of both z and ρ . We will ultimately take ρ at the antenna surface.

Solution of the homogeneous equation in the upper half of the antenna and on the lower half may be expressed as

$$A_z = C_1 \cos kz + C_2 \sin kz, \quad \begin{array}{l} \rho = a \\ \delta \leq z \leq h \\ \text{All } \phi \end{array}
\tag{II-48}$$

$$A_z = C_3 \cos kz + C_4 \sin kz \quad \begin{array}{l} \rho = a \\ -h \leq z \leq h \\ \text{All } \phi \end{array} \quad \text{II-49}$$

From the symmetry of the antenna A_z must be an even function of z ; therefore it is expected that $C_1 = C_2$ and $C_3 = -C_4 = 0$. The corresponding particular solution of Eq II-46 is

$$A_z = - \frac{j\omega\mu\epsilon}{2k} \sin k|z| . \quad \text{II-50}$$

then a single equation valid over the full length of the antenna is

$$A_z = C_1 \cos kz - \frac{j\omega\mu\epsilon}{k} \frac{V}{2} \sin k|z| . \quad \begin{array}{l} \rho = a \\ -h \leq z \leq h \\ \text{All } \phi \end{array} \quad \text{II-51}$$

The result is the familiar equation originally derived by Hallén [12, page 79]. The equation is

$$\frac{\mu}{4\pi} \int_{-h}^h I_z(z') K(z-z') dz' = C_1 \cos kz - \frac{j\omega\mu\epsilon}{k} \frac{V}{2} \sin k|z| . \quad \text{II-52}$$

Where $K(z-z')$ is called the kernel and is given by

$$K(z-z') = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\exp(-jk|\tilde{r}-\tilde{r}'|)}{|\tilde{r}-\tilde{r}'|} d\phi' \quad \text{II-53}$$

Delta Function Generator

In the derivation of Eq. II-52 the base gap of the antenna is made to approach zero so that the antenna is driven by a Dirac delta function of electric field. Since the gap approaches zero width, the capacitance of the gap approaches infinity producing a singularity in the current at the gap [9, page 147]. This singular charging current has no physical significance and must be subtracted out. The remaining finite current that is characteristic of the antenna without the gap may then be related to the corresponding current in an identical antenna driven by a practical feed system. Since the singularity is of very short range, it will only begin to show up in current approximations which contain a large numbers of terms. The singularity is not reproduced by approximations made of continuous functions and containing a relatively small number of terms.

The Kernel Function

A principal difficulty in the solution of the integral equation II-52 is the complicated Kernel II-53. Considerable simplification is brought about if the current in the dipole is approximated by a filamentary current lying along the axis of the antenna. Such an approximation to the Kernel is given by

$$K(z - z') = \exp(-jkR)/R . \quad \text{II-54}$$

Where

$$R = \sqrt{(z-z')^2 + a^2} .$$

II-55

Strictly speaking Eq. II-52 has no exact solution. Duncan and Hinchey [13, page 575] pointed out that high-order solutions cannot converge to the correct solution of the antenna integral equation. However, T. Wu [15, page 329] developed a theory showing that low-order solutions are very similar, within the range of validity of the approximation.

Neff's Derivation

A different method to approach the antenna integral equation has been developed by H. P. Neff of the University of Tennessee. He considered the vector potential first formulated in terms of the tangential electric field over the cylindrical antenna surface, and then considered the tangential magnetic field over the same surface. In this method Neff noted that it was more informative to carry out the exact analysis in terms of the cylindrical wave function. Neff found, as did Duncan and Hincey [13]

$$K(z-z') = \frac{1}{2\pi} \int_{-\infty}^{\infty} J_0 \left[a \sqrt{k^2 - k_z^2} \right] H_0^{(2)} \left[a \sqrt{k^2 - k_z^2} \right]$$

$$\times \exp [j(z-z')k_z] dk_z .$$

II-56

Working with this equation Neff supposed that $J_0 \left[a \sqrt{k^2 - k_z^2} \right]$ may be approximated by one, valid for $k_z < k$, but approaches infinity for $k_z > k$. This is precisely what is done to approximate the Kernel. Also Neff found no relationship between the value of the current density at the end of the dipole and the constant C_1 , and its analytic expression is

$$C_1 = - \int_h^{\infty} E_z(z') \operatorname{sinc} k z' dz' . \quad \text{II-57}$$

This equation can be interpreted using the Lorentz condition.

$$\Phi = - \frac{\nabla \cdot \mathbf{A}}{j\omega\mu\epsilon} = - \frac{V_0}{2} \cos kz + C_1 \sin kz \quad \text{II-58}$$

C_1 is just the negative of the potential evaluated on the antenna at $z = \lambda/4$. Siller [16, page 26] presents a complete discription of this technique.

CHAPTER III

TRIGONOMETRIC SOLUTION APPROXIMATION TO THE CURRENT

I. CURRENT APPROXIMATION

Solutions to the integral equation II-52 have taken several forms. The zero-order solution presented by King, Mack and Sandler [4] is very simple. Siller [16] also developed a simple trigonometric approximation to the current on a cylindrical dipole antenna. His current distribution along the length of the element is the trigonometric approximation given by

$$I_{z_n}(z_n) = \sum_{p=1}^P A_{np} \sin \frac{p\pi}{2h_n} (h_n - |z_n|). \quad \text{III-1}$$

Where A_{np} are complex coefficient which may be determined by fitting the solution at P points along the length of each antenna element. In this approximation the number of terms used is dependent upon the degree of accuracy required by the investigator; however the approximation begins to diverge if more than five terms are used.

Eq. III-1 can be expanded using simple trigonometric identities and considering the case of "p odd" and "p even," the equivalent is

$$I_z(z) = \sum_{\substack{p=1 \\ p \text{ odd}}}^N C_p \cos \frac{p\pi z}{2h} + \sum_{\substack{p=2 \\ p \text{ even}}}^N D_p \sin \frac{p\pi |z|}{2h}. \quad \text{III-2}$$

where

$$C_p = (-1)^{\frac{p+3}{2}} A_p , \quad \text{III-3}$$

$$D_p = (-1)^{\frac{p+6}{2}} A_p , \quad \text{III-4}$$

Unlike the work of Duncan and Hinchey [13], the functional form chosen to depict the approximate current distribution cannot be a Fourier series.

II. METHOD OF MOMENTS

Solutions to the integral equations which arise in electromagnetic field problems can rarely be achieved through analytical means. Several techniques have been developed, but we will focus on a single broad class which is referred to as the Method of Moments. References for this method are discussed using the notation of Harrington [17, 18].

Using this technique, the integral equation may be written in the following operator notation

$$L \{ D \} = B \quad \text{III-5}$$

Where L is a linear operator and B is the response to the excitation D .

The essence of the Method of Moments is:

- a. Expand the unknown function using a set of basis functions $\{\bar{f}_n; n = 1, 2, \dots, N\}$ in the domain of the operator

$$D = \sum_{n=1}^N A_n f_n . \quad \text{III-6}$$

- b. Define a suitable inner product of the tangential vector function over the surface S as

$$\langle \bar{P} \cdot \bar{Q} \rangle = \int_S \bar{p} \cdot \bar{q} \, da . \quad \text{III-7}$$

- c. Define a set of weight functions in the range of the operator L

$$\{ W_m; m = 1, 2, \dots, M \} . \quad \text{III-8}$$

- d. Take the inner product of the weight functions with the operator equation in which the expansion has been substituted

$$\sum_{n=1}^N A_n \langle W_m \cdot L(f_n) \rangle = \langle W_m \cdot B \rangle \quad m = 1, 2, \dots, M \quad \text{III-9}$$

This equation defines M relationships for the N unknown values a_n . Since it is a linear system of equations, it is ideally suited for numerical computation. Eq. III-9 may be equivalently written in matrix form as

$$[C_{m n}] \cdot [A_n] = [B_m] . \quad \text{III-10}$$

where

$$C_{m n} = \langle W_m, L(\bar{f}_n(x)) \rangle . \quad \text{III-11}$$

and

$$B_m = \langle W_m, B \rangle . \quad \text{III-12}$$

If the matrix C_{mn} is nonsingular, its inverse exists. Then the solution of Eq. III-10 is

$$[A_n] = [C_{mn}]^{-1} \cdot [B_m] . \quad \text{III-13}$$

and the response A is approximated by

$$D = [f_n] \cdot [A_n] = [f_n] \cdot [C_{mn}]^{-1} \cdot [B_m] , \quad \text{III-14}$$

where

$$[f_n] = [f_1, f_2, f_3 \dots]^\top . \quad \text{III-15}$$

III. DEVELOPMENT OF FOURIER SERIES SOLUTIONS

Duncan and Hinchey [13] developed a FOURIER SERIES for the Kernel of Hallén's equation,

$$K(z-z') = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\exp(-j k R)}{R} d\phi , \quad \text{III-16}$$

where

$$R = [4a^2 \sin^2 \phi/2 + (z-z')^2]^{1/2} . \quad \text{III-17}$$

The integral equation is valid only on the range $-h \leq z \leq z+h$.

To simplify the calculation it is possible to avoid a double Fourier Series using the fact that z and z' enter only as the square of their difference. So let $y = (z-z')$, then $K(y)$ contains only y^2 , so a cosine series in the range $-h$ to $+h$ will be sufficient. Now an expansion of the Kernel is

$$K(y) = \frac{D_0}{2} + \sum_{m=1}^{m=\infty} D_m \cos \left(\frac{m\pi y}{2h} \right) \text{ for } -h \leq y \leq +h \quad \text{III-18}$$

And the value of any coefficient of D_m is given by

$$D_m = \frac{1}{2h} \int_{-2h}^{2h} K(y) \cos \left(\frac{m\pi y}{2h} \right) dy \quad \text{III-19}$$

This equation can be simplified using the cosine transformation.

$$D_m = \frac{1}{h} \sqrt{\frac{\pi}{2}} \left[G \left(\frac{m\pi}{2h} \right) - G_2 \left(\frac{m\pi}{2h} \right) \right] . \quad \text{III-20}$$

Where $G(\alpha)$ can be defined as

$$G(\alpha) = -J \sqrt{\frac{\pi}{2}} J_0(\beta a) H_0^{(2)}(\beta a) . \quad \text{III-21}$$

But it is necessary to rewrite $G(\alpha)$ considering the cases for $\alpha < k$, $\beta = b$ and $\alpha > k$, $\beta = -jb$.

$$G(\alpha < k) = -j \sqrt{\frac{\pi}{2}} J_0(ba) H_0^{(2)}(ba), \quad \text{III-22}$$

and

$$G(\alpha > k) = \sqrt{\frac{2}{\pi}} I_0(ba) K_0(ba). \quad \text{III-23}$$

Also $G_2(\alpha)$ can be defined as

$$G_2(\alpha) = \sqrt{\frac{2}{\pi}} \int_{2h}^{\infty} K(y) \cos \alpha y \, dy. \quad \text{III-24}$$

For a high degree of convergence the Kernel can be expanded in a negative power series of y , and for convenience set

$$\mu^2 = (4a^2 \sin^2 \phi/2)/y^2. \quad \text{III-25}$$

And then Eq. III-16 becomes

$$K(y) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\exp(-jky \sqrt{1+\mu^2})}{y} (1+\mu^2)^{-1/2} d\phi. \quad \text{III-26}$$

Using the table of functions [19] appropriate expansion of the integrand are made and ϕ - integration is performed, resulting in

$$K(y) = \frac{e^{-jky}}{y} \left[1 - \frac{C_1}{y} - \frac{C_2}{y^2} - \frac{C_3}{y^3} - \dots \right] \quad \text{III-27}$$

where the coefficients are

$$C_1 = -jka^2, \quad \text{III-28}$$

$$C_2 = -\frac{a^2}{2} - \frac{3k^2a^4}{4}, \quad \text{III-29}$$

$$C_3 = \frac{9jka^4}{4} - \frac{5jk^3a^6}{12}, \quad \text{III-30}$$

$$C_4 = \frac{9a^4}{4} + \frac{5k^2a^6}{2} + \frac{35k^2a^8}{192}, \quad \text{III-31}$$

$$C_5 = -\frac{5jka^6}{4} - \frac{175jk^3a^8}{96}, \quad \text{III-32}$$

etc.

With a few trigonometric identities, a few elementary changes of variables, depending on whether $\alpha < k$ or $\alpha > k$, $G_2(\alpha)$ becomes [13, page 579]

$$G_2(\alpha < k) = -\sqrt{\frac{1}{2\pi}} \left\{ Ci[(k + \alpha)2h] + Ci[(k - \alpha)2h] \right. \\ \left. + j\pi - j Si[(k + \alpha)2h] - j Si[(k - \alpha)2h] \right\}. \quad \text{III-33}$$

$$G_2(\alpha > k) = -\sqrt{\frac{1}{2\pi}} \left\{ \text{Ci} [(\alpha + k)2h] + \text{Ci} [(\alpha - k)2h] \right. \\ \left. -j\text{Si} [(\alpha + k)2h] + j\text{Si} [(\alpha - k)2h] \right\} . \quad \text{III-34}$$

The special case for $\alpha = k$ is given by the singularity of the functions so the logarithmic singularities subtract off in $G(\alpha) - G_2(\alpha)$ and one obtains

$$G_1(\alpha = k) = \frac{1}{\sqrt{2\pi}} \left\{ \text{Ln} \frac{4h}{\delta Ka^2} + \text{Ci}(4kh) - j\text{Si}(4kh) \right\} \quad \text{III-35}$$

Using the tables of functions [19, pages 232-369] it is possible to compute the values of sine integral (Si), cosine integral (Ci), and polynomial approximations to the modified Bessel functions, J_0 , Y_0 , Y_1 , J_1 , $H_0^{(2)}$ and K_0 .

IV. TRIGONOMETRIC SOLUTION

The equation to be solved is

$$\int_{-h}^h f(z') K(z-z') dz' = C \cos kz + \frac{V}{2} \sin k|z| , \quad \text{III-36}$$

where

$$K(z-z') = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\exp(-jk|\tilde{r} - \tilde{r}'|)}{|\tilde{r} - \tilde{r}'|} d\phi' \quad \text{III-37}$$

and

$$f(z') = \sum_{n=0}^{\infty} F_n \cos \left(\frac{n \pi z'}{h} \right) . \quad \text{III-38}$$

is the trigonometric solution.

The Kernel can be expanded to

$$K(z - z') = \sum_{m=0}^{\infty} D_m \left[\cos \left(\frac{m \pi z}{2h} \right) \cos \left(\frac{m \pi z'}{2h} \right) + \sin \left(\frac{m \pi z}{2h} \right) \sin \left(\frac{m \pi z'}{2h} \right) \right] , \quad \text{III-39}$$

The sine terms are not needed in the treatment of a center-fed antenna, then, the Kernel reduces to

$$K_E(z - z') = \sum_{m=0}^{\infty} D_m \cos \left(\frac{m \pi z}{2h} \right) \cos \left(\frac{m \pi z'}{2h} \right) \quad \text{III-40}$$

Now expanding $f(z')$ as

$$f(z') = \sum_{n=0}^{\infty} F_n \cos \left(\frac{n \pi z'}{h} \right) , \quad \text{III-41}$$

and substituting into Eq. III-36 the result is

$$\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_n D_n A_{nm} \cos \left(\frac{m \pi z}{2h} \right) = C \cos kz + \frac{1}{2} \sin k |z| . \quad \text{III-42}$$

An infinite set of linear simultaneous equations is obtained by multiplying both sides by $\cos (2p + 1) \pi z/2h$ and integrating on z from $-h$ to $+h$. The result is

$$\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} F_n D_m A_{nm} B_{pm} = CR_p + V_p, \quad \text{III-43}$$

where

$$B_{pm} = 2 \int_0^h \cos [(2p + 1)\pi z/2h] \cos [m\pi z/2h] dz, \quad \text{III-44}$$

$$A_{nm} = 2 \int_0^h \cos (n\pi z'/h) \cos (m\pi z'/2h) dz', \quad \text{III-45}$$

$$R_p = 2 \int_0^h \cos kz \cos [(2p + 1) \pi z/2h] dz, \quad \text{III-46}$$

and

$$V_p = 2 \int_0^h \sin kz \cos [(2p + 1) \pi z/2h] dz, \quad \text{III-47}$$

Let $h = lm$. Using tables of integrals and by the condition of orthogonality, $A_{nm} = 0$. For m even, $m \neq 2n$; $B_{pm} = 0$ for m odd,

$m \neq 2p + 1$. Thus the vector R_p and V_p become

$$R_p = \frac{\sin \alpha}{\alpha} + \frac{\sin \beta}{\beta} , \quad \text{III-48}$$

$$V_p = - \frac{\cos \beta}{\beta} - \frac{\cos \alpha}{\alpha} + C . \quad \text{III-49}$$

where

$$\alpha = (2p + 1) , \quad \text{III-50}$$

$$\beta = k + (2p + 1)\pi/2 , \quad \text{III-51}$$

and

$$C = \frac{2k}{k^2 - [(2p + 1)\pi/2]^2} . \quad \text{III-52}$$

Now for the case of $k = 2(p + 1)\pi/2$. Eqs. III-46 and III-47 result in

$$R_p = 2 \int_0^1 \cos^2 kz \, dz = 1 , \quad \text{III-54}$$

and

$$V_p = 2 \int_0^1 \sin kz \cos kz \, dz = \frac{1}{k} . \quad \text{III-55}$$

Equation III-43 can be written as

$$\sum_{m=0}^{\infty} T_{pn} F_n = C R_p + V_p , \quad \text{III-56}$$

where

$$T_{pn} = \sum_{m=0}^{2N} D_m A_{nm} B_{pm} . \quad \text{III-57}$$

Since m is either even or odd, this basic set of functions can be broken into two parts, and for the case of $m = 2n$ and $m = 2p + 1$

$A_{n,2n} = B_{p,2p+1} = h$, and T_{pn} reduces to

$$T_{pn} = h (D_{2n} B_{p,2n} + D_{2p+1} A_{n,2p+1}) . \quad \text{III-58}$$

There D_0 has been redefined by a factor of two, and is included in the computation of A_{00} . For the condition $A_{n,2+1} = B_{p,2n}$, the final result for the matrix coefficients T_{pn} is given by

$$T_{pn} = h B_{p,2n} (D_{2n} + D_{2p+1}) , \quad \text{III-59}$$

Where

$$B_{p,2n} = h (-1)^{n+p} \left[\frac{1}{2p+1-2n} + \frac{1}{2p+1+2n} \right] \frac{2}{\pi} \quad \text{III-60}$$

The set of equations generated by Eq. III-56 as p ranges from zero to infinity can be cast in matrix form as

$$TF = CR + V \quad \text{III-61}$$

The formal solution of the current given in terms of the inverse matrix, where T must of course be nonsingular, is

$$F = CT^{-1}R + T^{-1}V \quad \text{III-62}$$

The evaluation of the constant, C , is obtained by setting Eq. III-41 to an arbitrarily small value, that may be zero.

$$FH = \sum_{n=0}^{Nth} F_n \cos(n\pi z/h), \quad \text{III-63}$$

for $z = \pm h$. Thus,

$$\cos(n\pi) = (-1)^n \quad \text{for} \quad z = h = 1. \quad \text{III-64}$$

Then

$$\sum_{n=0}^N (C M_n + N_n) (-1)^n = FH, \quad \text{III-65}$$

$$c = - \frac{\sum_{n=0}^{Nth} N_n (-1)^n + FH}{\sum_{n=0}^{Nth} M_n (-1)^n} . \quad \text{III-66}$$

Where the vectors M_n and N_n are given by

$$M = T^{-1} R , \quad \text{III-67}$$

and

$$N = T^{-1} V . \quad \text{III-68}$$

The final solution for the approximate current distribution for isolated thin cylindrical dipole elements is obtained in the expansion form as

$$F_n(z) = \sum_{n=0}^{Nth} F_n(C) \cos(n\pi z/h) . \quad \text{III-69}$$

The evaluation of the inverse matrix elements was accomplished on a high-speed digital computer, using a subroutine furnished by A. Barrero.

V. IMPEDANCE AND ADMITTANCE IN LINEAR ANTENNAS

A knowledge of the impedance of an antenna is necessary in order to match the antenna to a generator or to a transmission line, and to compute the gain of the antenna.

The input impedance Z_T of a center-fed cylindrical antenna is found by taking the ratio of the input or terminal voltage V_T and the current I_T at the input terminals. That is,

$$Z_T = \frac{V_T}{I_T} = R_T + j X_T . \quad \text{III-70}$$

Where

$$I_T = I_z (0) ,$$

R_T = terminal resistance,

X_T = terminal reactance .

and the admittance is given by

$$Y_T = \frac{I_T}{V_T} = G_T + j B_T , \quad \text{III-71}$$

where

G_T = terminal conductance,

B_T = terminal susceptance.

The subject of antenna impedance has been studied very thoroughly and exact solutions have not yet been found for the cylindrical dipole.

The EMF Method

The most common way of obtaining the impedance of a linear antenna is called the conservation of complex power or Induced EMF method. The

impedance of the antenna may be written as

$$Z = - \frac{1}{|I_z(0)|^2} \int_{-h}^h E_z(a, z) I_z^*(z) dz . \quad \text{III-72}$$

This is the form in which the impedance is usually expressed [1, p. 538; 12, pp. 268-260]. This method is frequently referred to as the "Poynting vector method." This name is usually attached to the impedance computation when the radiation resistance is computed using the far-zone fields of the antenna [1, p. 536]. King, Mack, and Sandler [4, p. 23] note that Eq. III-72 can be reduced to an identity if the electric field is a delta-function generator. That is,

$$Z = \frac{1}{|I_z(0)|^2} \int_{-h}^h V_0 I_z^*(z) \delta(z) dz ,$$

$$= \frac{1}{I_z(0) I_z^*(0)} V_0 I_z^*(0) ,$$

$$Z = \frac{V_0}{I_z(0)} .$$

III-73

CHAPTER IV

RESULTS

I. JUSTIFICATION FOR NONZERO END CURRENTS

The concept of nonzero end currents is not new, but it is usually set aside by forcing zero current at $|z| = h$ in the evaluation of C_1 in Hallén's integral equation. In general, this is not a necessity, but it is done as a matter of convenience. King discusses the possibility of nonzero end current [12, page 70]. He concludes that the antenna surface could appear longer than it actually is. Infield [20, page 911] points out that the current is not necessarily zero at the tips of a linear dipole.

Experimental data by R. B. Mack [21] also indicate that the current is not necessarily zero. It has also been called to the attention of the writer by Dr. H. P. Neff, Dr. J. D. Tillman and Dr. A. Barrero, all with the University of Tennessee, that a nonzero end current could produce a better solution to the original boundary value problem.

In 1972 A. Barrero [22] solved the linear dipole problem (approximately) and found a nonzero current at the end of each element. His solution was in good agreement with measured data, and it also yielded a good solution to the original boundary value problem.

II. RESULTS FOR HALF AND FULL-WAVE ANTENNAS

The antenna current distribution under study is calculated using Eq. III-69, derived in Chapter III. The current distribution is solved by the computer program given in the Appendix.

The comparison of the result with other theories and experimental measurements, is based on two kinds of dipole antennas which are the most important ones. R. King [23] presented the same results to compare with Mack's measurements.

Figure 5 and 6 show the current distribution on half and full-wave dipoles at twentieth-fifth order, for the case where FH in Eq. III-63 is forcing zero current at the end of the dipole $z = \pm h$, and Figures 5b and 6b show the current distribution on half and full-wave dipole, for the case where FH in Eq. III-63 is forcing the current to a very small value.

All the results presented in this chapter are in good agreement with measured data by Mack and the King-Middleton third-order solution.

Admittance of Isolated Dipole

As noted earlier, a knowledge of antenna admittance is important in the development of antenna systems. Fortunately almost all theoretical treatments of isolated cylindrical antennas provide valuable information in this area. Table I lists values of antenna input admittance for the half-wave dipole with $\Omega = 8.54$; the results are given for orders 15, 17, 19, 21, 23, 24, 25. For the case of $FH \neq 0$, the nonzero current of Barrero's computer program is used for each

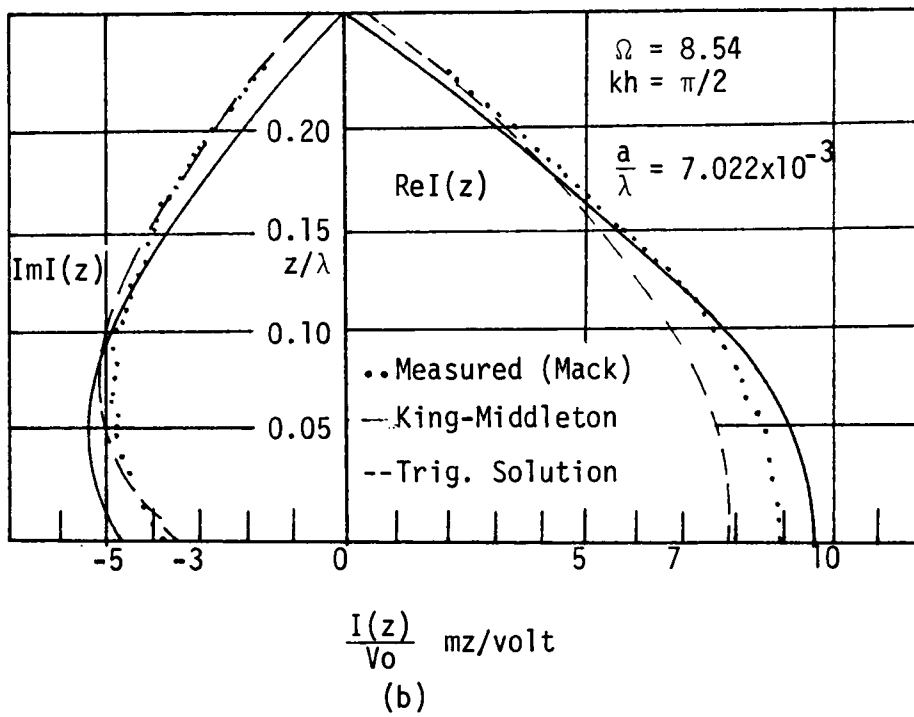
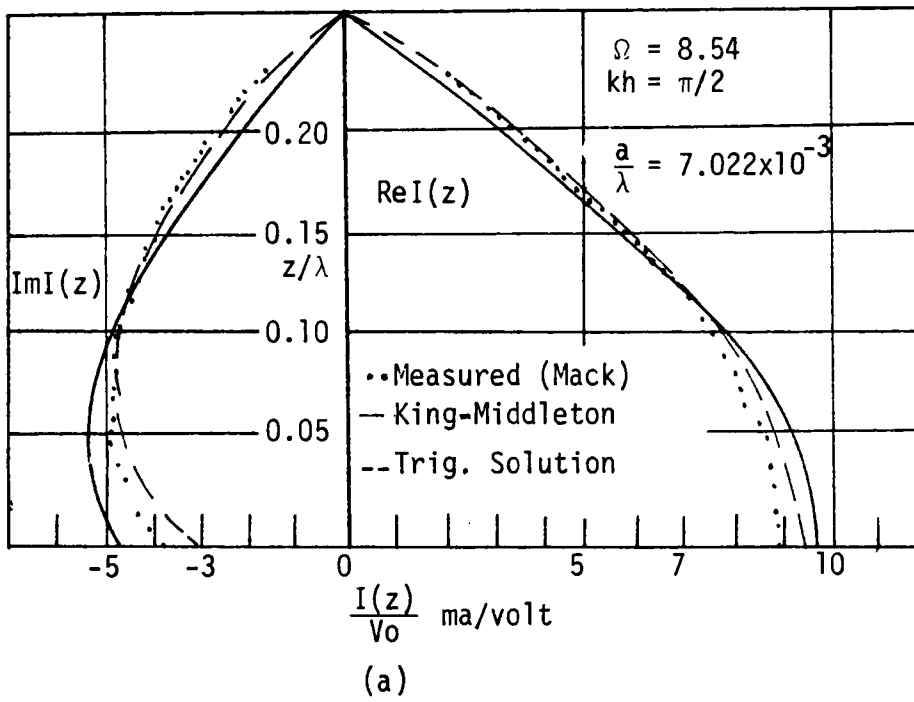


Figure 5. Current in half-wave dipole for
(a) $f(\pm h) = 0$ and (b) $f(\pm h) \neq 0$.

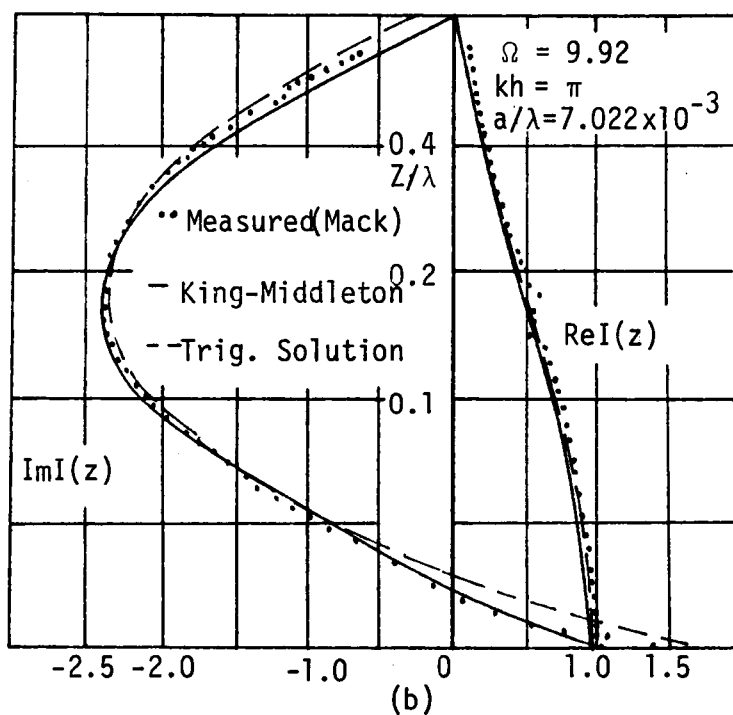
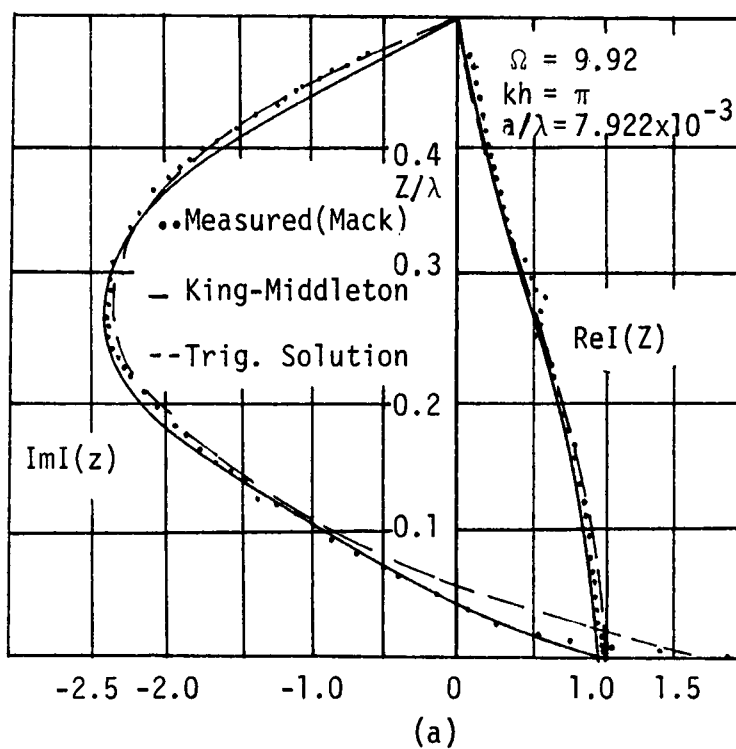


Figure 6. Current in full-wave dipole for
 (a) $f(\pm h) = 0$ and (b) $f(\pm h) \neq 0$.

TABLE I
INPUT ADMITTANCE FOR A HALF-WAVE DIPOLE

Order	Input Admittance $f(\pm h) = 0$	Input Admittance $f(\pm h) \neq 0$
15	10.152-j2.995	7.854-j3.779
17	9.921-j3.053	7.907-j3.713
19	9.741-j3.084	7.952-j3.651
21	9.595-j3.099	7.991-j3.595
23	9.476-j3.105	7.944-j3.505
24	9.577-j3.217	7.832-j3.366
25	9.377-j3.103	7.974-j3.460
MEASURED (MACK) 8.981-j3.736		

respective order. Since the results converge to the experimental value of $8.981 - j3.736 \text{ mA/v}$, the theory must be considered satisfactory.

The parameter Ω describes the height-to-thickness ratio of the antennas based on the assumption that $h \gg a$, and the ratio h/a equals the ratio of the total length of the cylindrical antenna to the diameter.

Thus,

$$\frac{\text{Total length}}{\text{Diameter}} = \frac{2h}{2a} \quad \text{IV-1}$$

and Ω is given by

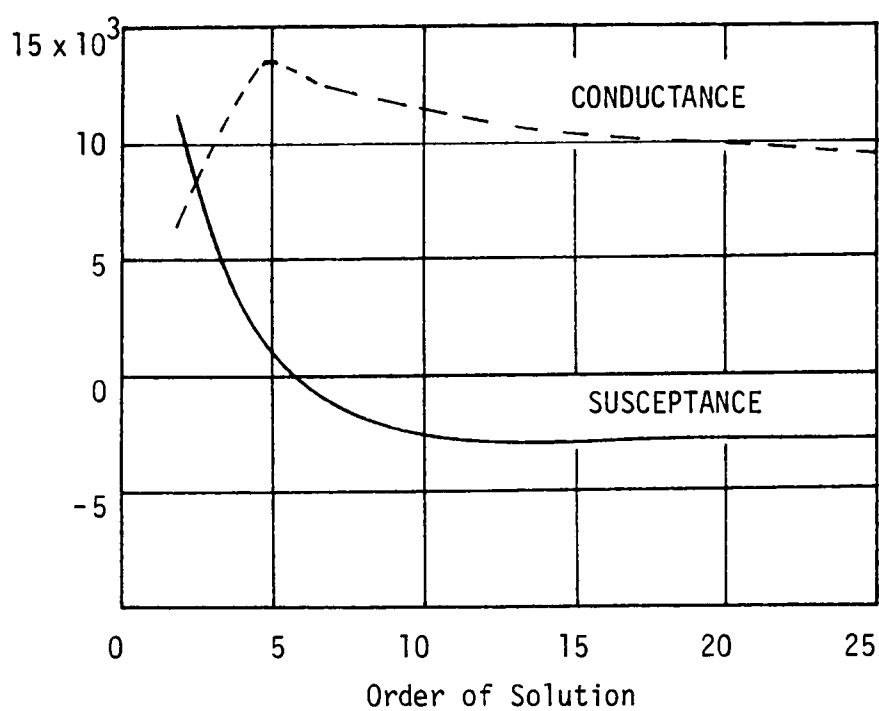
$$\Omega = 2 \ln \frac{2h}{a} \quad \text{IV-2}$$

Table II lists values of input admittance for a full-wave dipole with $\Omega = 9.92$; the results are given under the same condition of Table I, and the results converge to the experimental value of $1.023 + j1.394 \text{ mA/v}$.

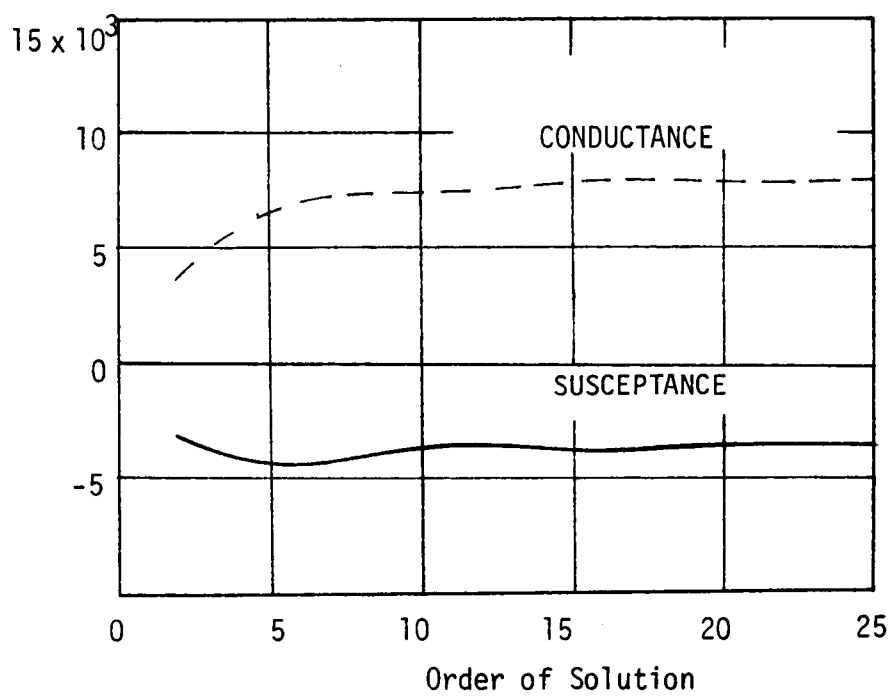
Figure 7 and Figure 8 show the conductance and susceptance components of antennas admittance as a function of the order of the solution. The results are obtained using the boundary condition $f(\pm h) = 0$ and $f(\pm h)$ not zero but equal to a very small value obtained from Barrero's solution.

TABLE II
INPUT ADMITTANCE FOR A FULL-WAVE DIPOLE

Order	Input Admittance $f(\pm h) = 0$	Input Admittance $f(\pm h) \neq 0$
15	1.006+j1.276	0.979+j1.477
17	1.003+j1.351	0.978+j1.540
19	1.001+j1.4156	0.981+j1.574
21	0.999+j1.472	0.980+j1.626
23	0.998+j1.522	0.981+j1.658
24	1.001+j1.485	0.979+j1.738
25	0.977+j1.568	0.981+j1.697
MEASURED (MACK) 1.023+j1.394		



(a)



(b)

Figure 7. Admittance versus order of solution for half-wave with (a) $f(\pm h) = 0$ and (b) $f(\pm h) \neq 0$.

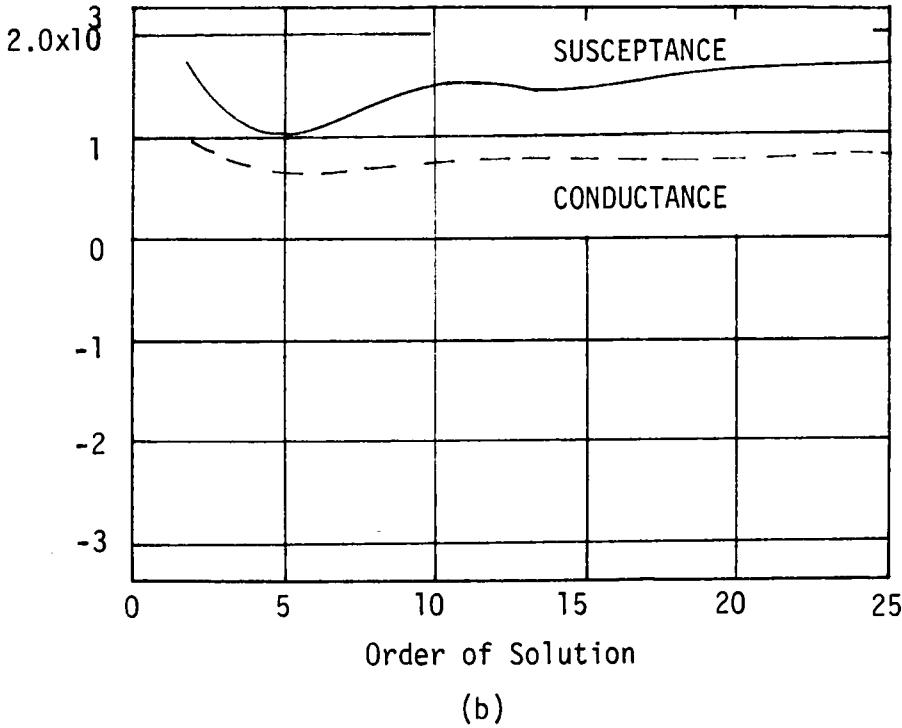
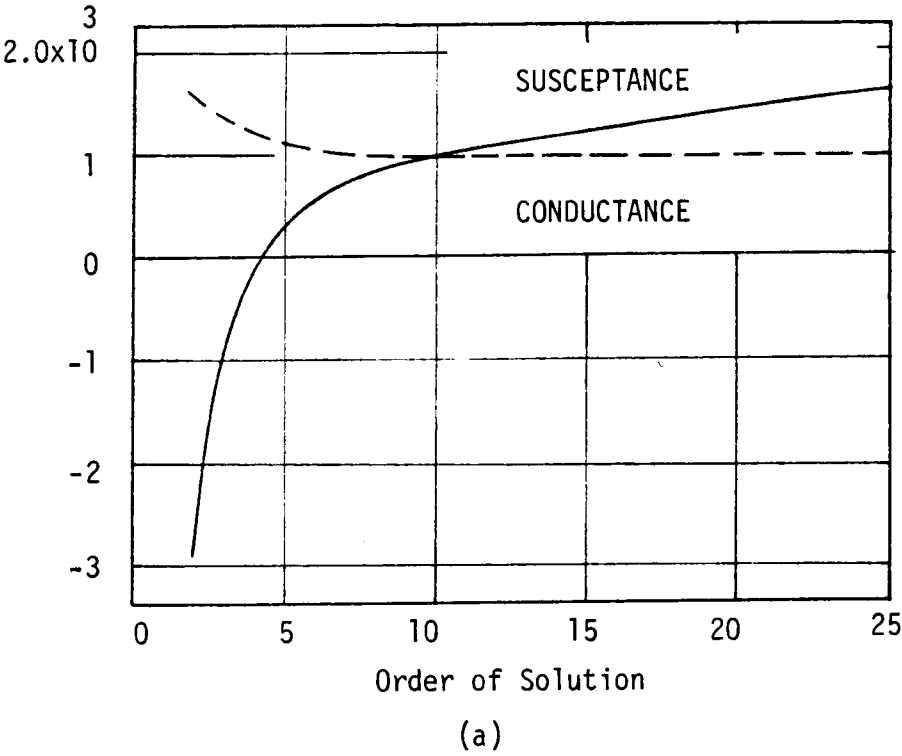


Figure 8. Admittance versus order of solution for full-wave with (a) $f(\pm h) = 0$ and (b) $f(\pm h) \neq 0$.

CHAPTER V

CONCLUSIONS AND SUGGESTIONS FOR FUTURE STUDY

I. CONCLUSIONS

This work has presented a theory to approximate the current distribution on center-fed thin cylindrical dipole antenna. The current distribution function is shown to yield very accurate impedances for half and full-wave length dipoles compared with the King-Middleton third order iteration and the finest experimental measurements by Mack.

The method of solution of the trigonometric theory is sufficiently simple that the antenna engineer may easily generate his own data. For this reason a copy of the computer program is included in the Appendix.

The trigonometric theory developed in this work expresses the current in a series of cosine terms. The work presented by this thesis justified the possibility of nonzero currents at the tips of a linear dipole antenna. Figures 5 and 6 pages 42-43 and Tables I and II page 44-46 compare the current approximations and admittances respectively for the cases when the current at the tip is forced to be zero and when these same current is set equal to an arbitrary small value. Both results are in good agreement with each other and also with other theories. For the case shown in Figure 5a, the current at the end of the dipole does not converge to zero, however for the case shown in Figure 5b the current converges uniformly to the given value. As a conclusion it must be pointed out that the results from the

trigonometric theory give an overall agreement as good as the more sophisticated iterative solution. All these results for currents and admittance were computed using the Eq. III-69 which is in disagreement with the paper presented by Duncan and Hinchey [13].

It can be seen from the graphs of admittance versus order that a great deal of margin for the study of thicker antennas is inherent in the twentieth-fifth order solutions. Also studies of full wave length antennas with $h/a < 60$ will not require more than 25 terms for fast convergence.

Finally, it should be pointed out that since the matrix cannot be inverted in finite order, the trigonometric series for the current is not strictly a Fourier series. However, it does converge to a Fourier series as the order is increased.

II. SUGGESTIONS FOR FUTURE STUDY

The analysis presented in this work may be extended, with only slight modification, to general linear dipole arrays with arbitrary excitation. The most direct application would probably be to Yagi-Uda antennas.

Other possibilities are the study of the receiving antenna, and the study of scattering of electromagnetic waves by cylindrical wires.

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APPENDIX

```

C THIS PROGRAM COMPUTES THE CURRENT DISTRIBUTION ON A
C CENTER-DRIVEN CYLINDRICAL DIPOLE BY SOLVING AN INTEGRAL
C EQUATION FOR THE ELECTRIC FIELD.
C
C THE CURRENT IS EVALUATED AS A TRIGONOMETRIC SERIES.
C THE COEFFICIENTS ARE COMPUTED BY THE METHOD OF MOMENTS.
C TO SIMPLIFY THE COMPUTATION OF THE MARRIX COEFFICIENTS
C THE KERNEL IS EXPANDED IN A FOURIER SERIES.
C
C THE PARAMETERS ARE THE ANTENNA LENGTH IN WAVLENGTHS
C (ANTL) AND THE RADIUS IN METERS (A).
C THE ANTENNA DIPOLE HAS LENGTH OF 2 METERS.
C
C FOR ANTENNAS VERY CLOSE TO BUT NOT EXACTLY AT
C AT RESONANCE LARGE ERRORS MAY OCCUR. AWARNING
C MESSAGE IS PROVIDED FOR THIS CASE.
C*****
C*****
      REAL*8 OMEGA,ALPHA,BETA,VAL,VALI,THRESH,A,FF
      REAL*8 FNOT,GG,HPIE,I00,J00,K,PIE,THETA,Z,ANTL,ARG
      REAL*8 J0,Y0,SI,CI,K0,I0,F,G
      COMPLEX*16 T1,T2,T3,CCC,S,S1,FH,J,K1,K2,CONS,FI
      COMPLEX*16 T(25,25),B(25),CC(6),RPP(25),VPP(25)
      COMPLEX*16 TI(25,25),F1(25),C(50),AA(25),D(25)
      INTEGER P,PP,P1
      COMMON C,J,A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,
      $THETA,Z,ANTL,ARG
C*****
C*****
C PARAMETERS
C PARAMETERS CAN BE VARIED TO OBTAIN SOLUTIONS FOR DI-
C POLES OF DIFFERENT DIMENSIONS AND AT DIFFERENT FREQUENCIES.
C NP CAN CHANGE THE NUMBER OF TERMS IN THE SOLUTION.
      NP=25
      OMEGA=8.5400
      ANTL=0.500
      A=2.00*DEXP(-OMEGA/2.00)
C CONSTANTS.
C FH CAN BE ZERO OR A VERY SMALL VALUE.
      FH=(0.00,0.00)
      NO=2*NP
      THRESH=1.0-3
      PIE=3.1415926500
      HPIE=PIE/2.00
      K=PIE*ANTL
      J=(0.00,1.00)
      CONS=-J*376.7300/(4.00*PIE*K)
C*****
C*****

```

```

C GENERATE ARRAY OF COEFFICIENTS CC FOR USE BY FUNCTION K1
  CC(1)=(1.00,0.00)
  CC(2)=-J*K*(A**2)
  CC(3)=-A**2+3.00*(K**2)*(A**2)/4
  CC(4)=9.00*J*K*(A**4)/4.00-5.00*J*(K**3)*(A**6)
  CC(5)=9.00*(A**4)/4.00+5.00*(K**2)*(A**6)/2.00
  *+53.00*(K**4)*(A**8)/192.00
  CC(6)=-5.00*J*K*(A**6)-175.00*J*(K**3)*(A**8)/96.00
C*****
C COMPUTE FOURIER COEFFICIENTS OF KERNEL C.
  DO 40 MM=1,NO
    M=MM-1
    ARG=(ANTL*2.00-M)*(ANTL*2.00+M)/4.00
    IF((DABS(ARG).EQ.0.00).OR.(DABS(ARG).GT.
    *THRESH)) GO TO 30
    WRITE(6,20)
  20  FORMAT(1H1,20X,'***WARNING***ANTENNA VERY CLOSE TO
    * RESONANCE. LARGE ERRORS MAY OCCUR')
  30  C(MM)=(K2(M)-K1(M))/2.00
    C(MM)=CONS*C(MM)
  40  CONTINUE
C*****
C COMPUTE MATRIX COEFFICIENTS T.
  DO 50 NN=1,NP
    N=NN-1
    DO 50 PP=1,NP
      P=PP-1
      T(PP,NN)=(1.00/(2*P+1-2*N)+1.00/(2*P+1+2*N))
      1*(C(2*N+1)+C(2*P+2))
      2*(2.00/PIE)*(-1.00)**(NN+PP)
  50  CONTINUE
C*****
C GENERATE FOURIER COEFFICIENTS OF ELECTRIC FIELD
C THE ELECTRIC FIELD USED HERE IS AN IMPULE FUNCTION
C SOLVE SIMULTANEOUS EQUATIONS TO COMPUTE THE INVERSE MATRIX
C VECTOR B(N) WILL CONTAIN THE SOLUTION
C AND THE INVERSE MATRIX IS ASSIGN TO TI.
  WW=1.0
  LIM=10000
  E=0.00000001
  DO 52 L=1,NP
    DO 54 I=1,NP
      B(I)=(0.00,0.00)
  54  CONTINUE
      B(L)=(1.00,0.00)
      CALL SOLVE(T,B,NP,WW,LIM,E)
      DO 56 I=1,NP
        TI(I,L)=B(I)
  56  CONTINUE
  52  CONTINUE

```

C COMPUTE THE VALUES OF VECTORS RPP AND VPP

```
DO 58 I=1,NP
  P1=I-1
  VAL=K-((2*P1)+1)*HP1E
  VALI=K+((2*P1)+1)*HP1E
  RPP(I)=(1.00,0.00)
  VPP(I)=-(1.00,0.00)/(2.00*K*K)
  IF(DABS(VAL).LT.0.000000100) GO TO 58
  RPP(I)=(DSIN(VAL)/VAL)+(DSIN(VALI)/VALI)
  VPP(I)=-(DCOS(VALI)/VALI)-(DCOS(VAL)/VAL)
  *+1.00/VALI+1.00/VAL
  VPP(I)=-VPP(I)/(2.00*K)
```

58 CONTINUE

C COMPUTE VALUES OF VECTOR VPP BY THE INVERSE MATRIX

C COMPUTE VALUES OF VECTOR RPP BY THE INVERSE MATRIX.

```
DO 60 I=1,NP
  AA(I)=(0.00,0.00)
  D(I)=(0.00,0.00)
  DO 60 INDICE=1,NP
    AA(I)=AA(I)+(TI(I,INDICE)*RPP(INDICE))
    D(I)=D(I)+(TI(I,INDICE)*VPP(INDICE))
```

60 CONTINUE

C COMPUTE VALUES OF CONSTANT CCC

```
S=(0.00,0.00)
S1=-FH
DO 62 I=1,NP
  S=S+AA(I)*(-1)**(I+1)
  S1=S1+D(I)*(-1)**(I+1)
```

62 CONTINUE

CCC=-(S1/S)

DO 64 I=1,NP

64 F1(I)=(CCC*AA(I))+D(I)

C COMPUTE VALUES OF CURRENT DISTRIBUTION.

WRITE (6,70) ANTL,A

70 FORMAT (1H1,20X,'CURRENT DISTRIBUTION.

* ANTENNA LENGTH = ',F7.3/',' ',20X,'ANTENNA

* LENGTH = 2 MTS, RADIUS = ',D15.8,'MTS')

WRITE (6,74)

74 FORMAT(1H0)

WRITE (6,78)

```
78  FORMAT(1H ,21X,'Z',5X,'REAL PART',13X,'IMAGINARY PART')
    WRITE (6,74)
    Z=0.00
    DELZ=0.0100
    DO 100 L=1,101
    FI=(0.00,0.00)
    DO 80 NN=1,NP
    N=NN-1
80  FI=FI+FI(NN)*DCOS(N*PIE*Z)
    WRITE (6,90) Z,FI
90  FORMAT(1H ,19X,F5.2,2(D18.8,4X))
    Z=Z+DELZ
100 CONTINUE
    STOP
    END
```

```

COMPLEX FUNCTION K1*16(M)
  RFAL*8 Y1,Y2,F1,F2,F3,F4,F5,F6,SI,CI,
  $A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,THETA,Z,ANTL,ARG
COMPLEX*16 C(6),J,I
LOGICAL PLUS
COMMON C,J,A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,
  $THETA,Z,ANTL,ARG
C*****
C CALCULATE K1(M) USING RECURSIVNESS OF FM,FM'.
  Y1=2.D0*(K+M*HPIE)
  F1=-CI(Y1)
  F3=HPIE-SI(Y1)
  Y2=(ANTL*2.D0-M)*PIE
  PLUS=.FALSE.
  IF(Y2.NE.0.D0) GO TO 5
  F2=.5772156649D0
  F4=HPIE
  GO TO 15
5  IF(Y2.GT.0.D0) GO TO 10
  Y2=-Y2
  PLUS=.TRUE.
10  F2=-CI(Y2)
  F4=HPIE-SI(Y2)
  K1=(0.D0,J.D0)
15  DO 40 L=1,6
  I=F1+F2-J*F3
  IF(PLUS) GO TO 20
  I=I-J*F4
  GO TO 30
20  I=I+J*F4
30  K1=K1+C(L)*I
  IF(L.EQ.6) RETURN
  F5=F1
  F6=F2
  F1=(DCOS(Y1)-(Y1*F3/2.D0)*2.D0**L)/(L*2.D0**L)
  F2=(DCOS(Y2)-(Y2*F4/2.D0)*2.D0**L)/(L*2.D0**L)
  F3=(DSIN(Y1)+(Y1*F5/2.D0)*2.D0**L)/(L*2.D0**L)
  F4=(DSIN(Y2)+(Y2*F6/2.D0)*2.D0**L)/(L*2.D0**L)
40  CONTINUE
  RETURN
END

```

```

      COMPLEX FUNCTION K2*16(M)
      REAL*8 Y,I0,K0,J0,Y0,
      *A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,THETA,Z,ANTL,ARG
      COMPLEX*16 C(6),J
      COMMON C,J,A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,
      $THETA,Z,ANTL,ARG
      Y=ARG*PIE**2
      IF(Y.GE.0.D0) GO TO 10
      Y=-Y
      Y=A*DSQRT(Y)
      I00=I0(Y)
      K2=2.D0*I00*K0(Y)
      RETURN
10   IF(Y.NE.0.D0) GO TO 20
      K2=-DLOG((A**2)*K/2.D0)-1.1544313298D0-J*PIE
      RETURN
20   Y=A*DSQRT(Y)
      J00=J0(Y)
      K2=-J*PIE*J00*(J00-J*Y0(Y))
      RETURN
      END

```

```

      REAL FUNCTION J0*8(SAVE)
      REAL*8 Y,SAVE.
      $A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,THETA,Z,ANTL,ARG
      COMPLEX*16 C(6),J
      COMMON C,J,A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,
      $THETA,Z,ANTL,ARG
      Y=SAVE
      IF(Y.GT.3.D0) GO TO 10
      Y=Y/3.D0
      J0=1.D0
      1-2.249999700*Y**2
      2+1.265620800*Y**4
      3-0.316386600*Y**6
      4+0.044447900*Y**8
      5-0.003944400*Y**10
      6+0.000210000*Y**12
      RETURN
10    Z=Y
      Y=3.D0/Y
      THETA=Z
      1-.7853981600
      2-.0416639700*Y
      3-.0000395400*Y**2
      4+.0026257300*Y**3
      5-.0005412500*Y**4
      6-.0002933300*Y**5
      7+.0001355800*Y**6
C      FNOT=.7978845600
      1-.0000007700*Y
      2-.0055274000*Y**2
      3-.0000951200*Y**3
      4+.0013723700*Y**4
      5-.0007280500*Y**5
      6+.0001447600*Y**6
C      J0=FNOT*DCOS(THETA)/DSQRT(Z)
      RETURN
      END

```

```

      REAL FUNCTION Y0*8(SAVE)
      REAL*8 Y,SAVE.
      $A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,THETA,Z,ANTL,ARG
      COMPLEX*16 C(6),J
      COMMON C,J,A,FF,FNOT,GG,HPIE,I00,J00,K,PIE.
      $THETA,Z,ANTL,ARG
      Y=SAVE
      IF(Y.GT.3.00) GO TO 10
      Y=Y/3.00
      Y0=DLOG(1.500*Y)*J00/HPIE+
      1.3674669100+
      2.6055936600*Y**2-
      3.7435038400*Y**4+
      4.2530011700*Y**6-
      5.7426121400*Y**8+
      6.0042791600*Y**10-
      7.0002484600*Y**12
      RETURN
10    Y0=FNOT*DSIN(THETA)/DSQRT(Z)
      RETURN
      END

```

```

      REAL FUNCTION CI*8(X)
      REAL*8 X,F,G,
      $A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,THETA,Z,ANTL,ARG
      COMPLEX*16 C(6),J
      COMMON C,J,A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,
      $THETA,Z,ANTL,ARG
      IF(X.GE.1.00) GO TO 10
      WRITE(6,5)
5     FORMAT('1',*ARGUMENT OF CI < 1.0, THE
      * APPROXIMATION IS NOT VALID*)
10    FF=F(X)
      GG=G(X)
      CI=FF*DSIN(X)-GG*DCOS(X)
      RETURN
      END

```

```

      REAL FUNCTION SI*8(X)
      REAL*8 X,
      $A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,THETA,Z,ANTL,ARG
      COMPLEX*16 C(6),J
      COMMON C,J,A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,
      $THETA,Z,ANTL,ARG
      IF(X.GE.1.00) GO TO 10
      WRITE(6,5)
5     FORMAT('1','ARGUMENT OF SI < 1.0, THE
      * APROXIMATION IS NOT VALID')
10    SI=HPIE-FF*DCOS(X)-GG*DSIN(X)
      RETURN
      END

```

```

      REAL FUNCTION K0*8(SAVE)
      REAL*8 Y,SAVE.
      *A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,THETA,Z,ANTL,ARG
      COMPLEX*16 C(6),J
      COMMON C,J,A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,
      *THETA,Z,ANTL,ARG
      Y=SAVE
      IF(Y.GT.2.D0) GO TO 10
      Y=Y/2.D0
      K0=-DLOG(Y)*I00
      1-.57721566D0
      2+.42278420D0*Y**2
      3+.23069756D0*Y**4
      4+.03488590D0*Y**6
      5+.00262698D0*Y**8
      6+.00010750D0*Y**10
      7+.00000740D0*Y**12
      RETURN
10   Z=Y
      Y=2.D0/Y
      K0=DEXP(-Z)*
      1+1.25331414D0
      2- .07832358D0*Y
      3+ .02189568D0*Y**2
      4- .01062446D0*Y**3
      5+ .00587872D0*Y**4
      6- .00251540D0*Y**5
      7+ .00053208D0*Y**6)/DSQRT(Z)
      RETURN
      END

```

```

      REAL FUNCTION IO*(SAVE)
      REAL*8 Y,SAVE,
      *A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,THETA,Z,ANTL,ARG
      COMPLEX*16 C(K).J
      COMMON C,J,A,FF,FNOT,GG,HPIE,I00,J00,K,PIE,
      *THETA,Z,ANTL,ARG
      Y=SAVE
      IF(Y.GT.3.7500) GO TO 10
      Y=Y/3.7500
      I0=1.00
      1+.3515622900*Y**2
      2+.3.084942400*Y**4
      3+.1.206749200*Y**6
      4+.0.265473200*Y**8
      5+.0.036076800*Y**10
      6+.0.004531300*Y**12
      RETURN
10    Z=Y
      Y=3.7500/Y
      I0=DEXP(Z)*(
      1+.3989422800
      2+.0132859200*Y
      3+.0022531900*Y**2
      4+-.0015756500*Y**3
      5+.0091628100*Y**4
      6+-.0205779600*Y**5
      7+.0263553700*Y**6
      8+-.0164763300*Y**7
      9+.0039237700*Y**8)/DSQRT(Z)
      RETURN
      END

```

```

REAL FUNCTION G*(X)
REAL*8 X
G=(
1 21.82189900
2+352.01849800*X**2
3+302.75786500*X**4
4+ 42.24285500*X**6
5+          X**8)
G=G/((
6 499.69032600+
71114.97888500*X**2
8+482.48598400*X**4
9+ 48.19692700*X**6
1+          X**8)*X**2)
RETURN
END

```

```
REAL FUNCTION F*(X)
REAL*8 X
F=(
1 38.10249500
2+335.67732000*X**2
3+265.18703300*X**4
4+ 38.02726400*X**6
5+                               X**8)
  F=F/((
6+157.10542300
7+570.23628000*X**2
8+322.62491100*X**4
9+ 40.02143300*X**6
1+                               X**8)*X)
  RETURN
END
```

```

SUBROUTINE SOLVE(T,B,NP,W,LIM,E)
COMPLEX*16 T(25,25),B(25),X(25),XI
REAL*8 CDABS
DO 10 I=1,NP
IF(CDABS(T(I,I)).NE.0.D0) GO TO 10
WRITE(6,5)
5  FORMAT(1H1,20X,'SUBROUTINE SOLVE: 0 IN THE DIAGONAL')
RETURN
10 CONTINUE
DO 20 I=1,NP
20  X(I)=(0.D0,0.D0)
DO 50 N=1,LIM
E2=0.0
DO 40 I=1,NP
XI=B(I)
DO 30 J=1,NP
30  XT=XI-T(I,J)*X(J)
XI=X(I)+W*XI/T(I,I)
E1=CDABS(XI-X(I))
X(I)=XI
IF(E1.GT.E2) E2=E1
40 CONTINUE
IF(E2.LE.E) GO TO 70
50 CONTINUE
WRITE(6,60) E2
60  FORMAT(1H1,20X,'TOO MANY ITERATIONS. ERROR=',E13.6)
70  DO 80 I=1,NP
80  B(I)=X(I)
RETURN
END

```

VITA

Rafael Segundo Montiel was born in San Carlos del Zulia, Venezuela, on May 28, 1946. He received his primary and secondary education in the public schools of Maracaibo, Zulia, where he graduated from Liceo Rafael Ma. Baralt in June 1965.

The following year he was employed by Telephone Company of Venezuela. In September 1967, he entered the schools of Technician, Centro de Entrenamiento en Telecomunicaciones, John Deket, where he held several scholarships. In September 1970 he received the degrees of Técnico en Telecomunicaciones and Técnico en Radiocomunicaciones.

From October 1970 to October 1974 he was employed again by Telephone Company of Venezuela. In January 1975 Mr. Montiel entered the Institute of Language, Louisiana State University learning the English Language, in the following September he transferred to the University of Tennessee as an undergraduate student under the sponsorship of the Fundación Mariscal de Ayacucho. In December 1977, he received the degree of Bachelor of Science in Electrical Engineering. The following January he entered the graduate school and in March 1979, he received the Master of Science degree in Electrical Engineering. His major area of interest is electromagnetic theory.