To the Graduate Council:

I am submitting herewith a thesis written by Solomon Ochola Olebe entitled "Modelling Current Transformers for Abnormal Operating Conditions." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

Frederick W. Symonds

Frederick W. Symonds, Major Professor

We have read this thesis and recommend its acceptance:

Accepted for the Council:

Vice Chancellor
Graduate Studies and Research

MODELLING CURRENT TRANSFORMERS FOR ABNORMAL OPERATING CONDITIONS

A Thesis

Presented for the

Master of Science

Degree

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Solomon Ochola Olebe

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ABSTRACT

Theory and application of a current transformer are first presented. Current transformer performance in relation to various fault conditions is systematically analyzed.

From a current transformer characteristic curve and parameters, "unsaturated" and "saturated" expressions are derived. Mathematical modelling and computer programming are then prepared to predict current transformer performance and design criteria.

Finally, experimental and computer predicted results are discussed and compared.

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CHAPTER 1

INTRODUCTION

With the rapid development in remote control of electric power systems, there has been a continuously growing interest in the study of current transformer performance under numerous fault conditions. This paper involves the development of a mathematical model and computer program to predict the performance and output of current transformer for abnormal operating conditions, which will aid in the design of instrument transformers. The use of numerical methods proves more feasible than the analytical approach, especially when the core saturates and the variables become more complex and nonlinear. In addition, this paper contains a detailed study carried out both theoretically and experimentally.

The heart of this study is the development of "unsaturated" and "saturated" expressions from the characteristic curve and parameters of current transformers. Where the digital computer also proves convenient in this paper is in the determination of the appropriate values of secondary inductances, particularly when a current transformer is subjected to a d.c. offset fault current.

The theoretical and experimental results are found in good agreement with the computer results. Therefore, this study proves that prediction of current transformer performance and output can be done by the use of a digital computer.

Throughout this paper, a current transformer will be referred to as "CT" for brevity.

1.2. Application of Current Transformers: Differential Relays

All forms of electrical equipment such as generators, motors, transformers, transmission lines and buses need protection against faults. One of the methods to protect equipment against faults is the use of differential relays.

There are several types of differential relays, depending on the type of equipment to be protected. As illustrated in Figure 1, the differential relay operates when the difference between the current transformer secondary currents exceeds a set value. Therefore, most operations of differential relay are of the "current-differential kind." The current transformer secondaries are interconnected and the coil of an overcurrent relay is connected parallel to the current transformer secondary circuit. Refer to Figure 2 and assume that the current flows through the primary circuit to an external load or fault located at X. If the ratios of the current transformers are the same, the secondary induced voltages E_a and E_b will be equal. Also the secondary currents of the current transformers will be equal. Consequently, the current transformer secondary currents will only circulate between the two current transformers as illustrated by the arrows. No secondary current will flow through the overcurrent relay.

When a fault occurs anywhere between the two sets of CT's, one of the secondary currents suddenly increases while the other may decrease or suddenly increase and flow in the opposite direction. Accordingly, the fault current flows through the relay and operates

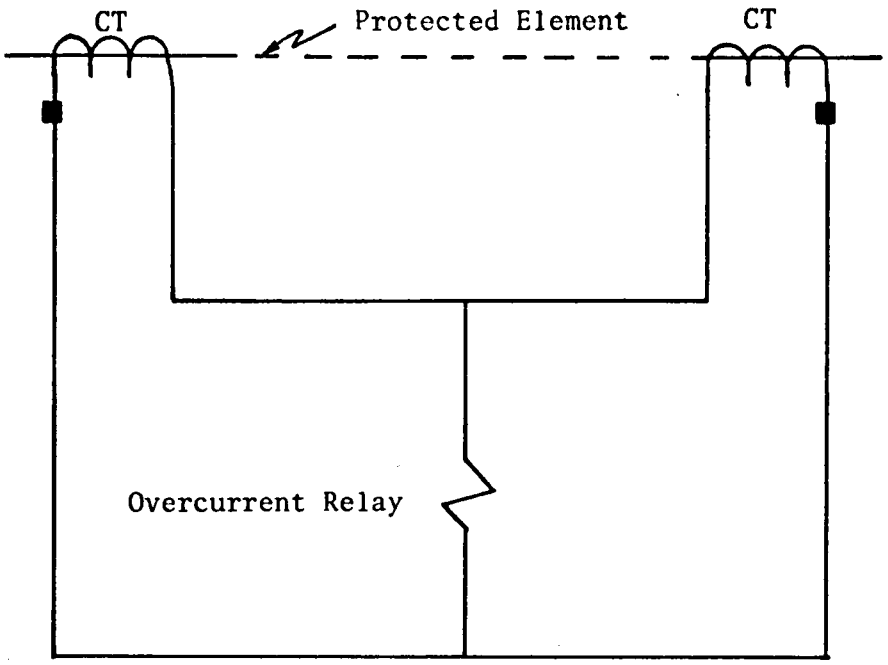


Figure 1. Basic differential relay.

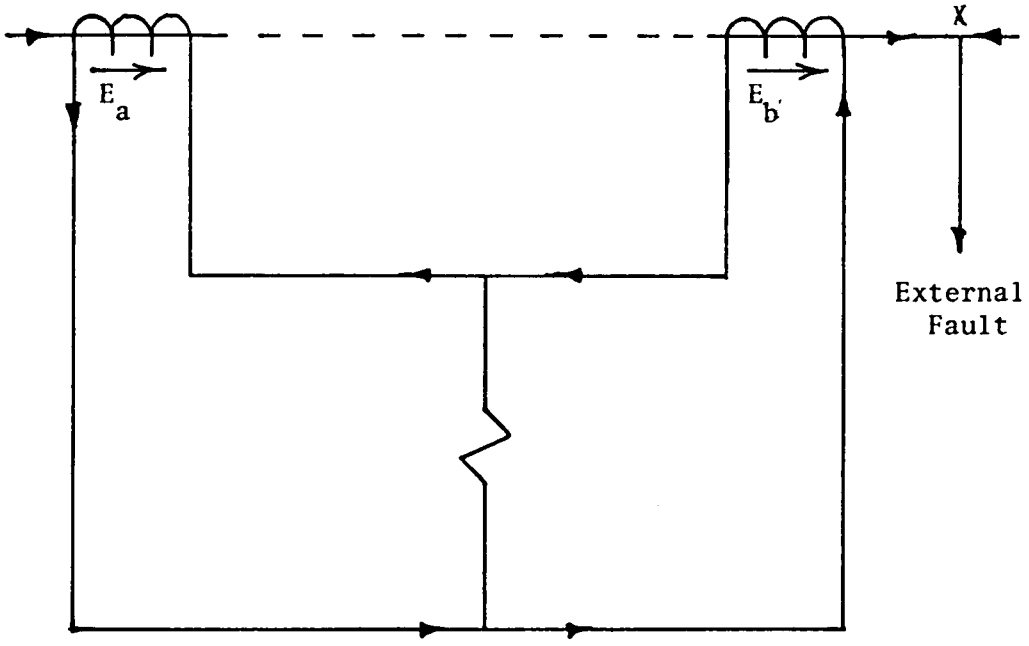


Figure 2. External fault conditions.

it. Refer to Figure 3. It is not necessary for current to flow to the fault from both sides to cause the secondary current to actuate the overcurrent relay. A secondary current flow from one side could be sufficient. Hence, the differential relay current is proportional to the vector difference between the currents entering and leaving the protected circuit. Thus, the overcurrent relay must be set below this maximum error current which can flow through it during fault.

In Figure 4, the CT's are assumed to have the same ratio. The magnetizing currents flow through the shunt impedances Z_{mA} and Z_{mB}. The lead resistance, the CT leakage reactance and resistance are represented by the impedances R_A and R_B. For simplification, all impedances are assumed to be resistances. During an external fault at X, the CT secondary current will merely circulate between the CT secondaries. The only current that can flow through the relay, Z_R, is the vector-difference current in the CT outputs for the same primary current.

Referring again to Figure 4 and assuming that the CT at B is short-circuited because of an internal flaw, the CT at A is delivering its full secondary current which will divide between the shorted CT and the relay. This current division will depend on the values of resistances R_R and R_B. If R_R is large compared to R_B, the relay will not trip unnecessarily since most of the current will go through the shorted CT. This fault will definitely interfere with the output of the CT. In Figure 5, a plot of the rms magnetizing current versus the voltage across the shunt impedance is sketched.

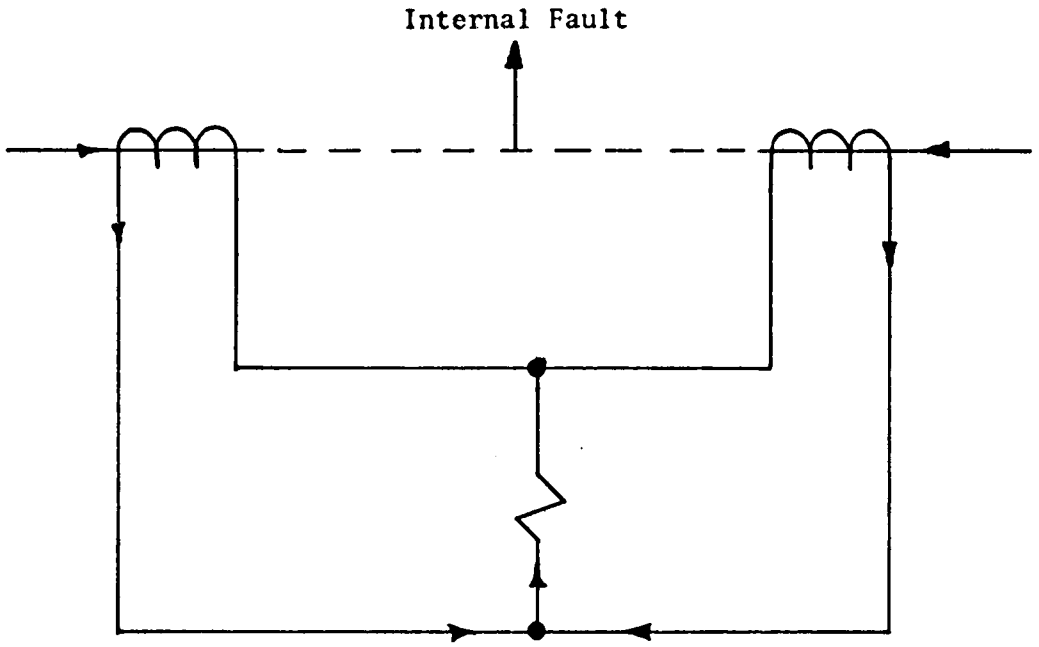


Figure 3. Internal fault conditions.

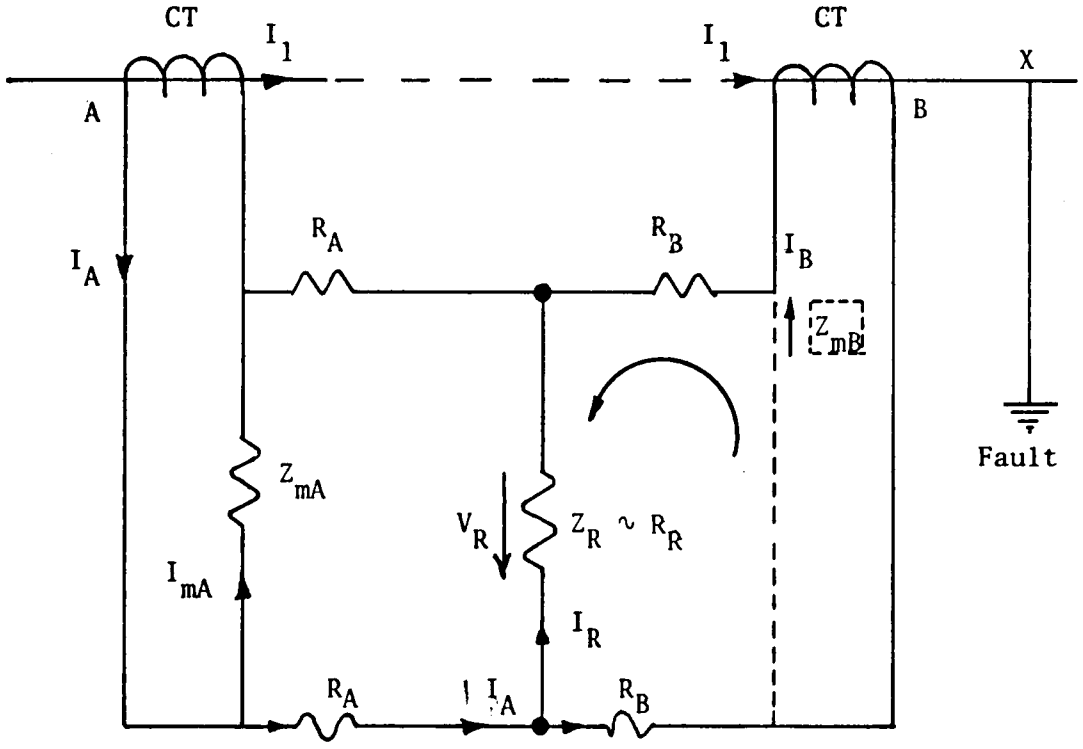


Figure 4. Magnetic saturation of CT.

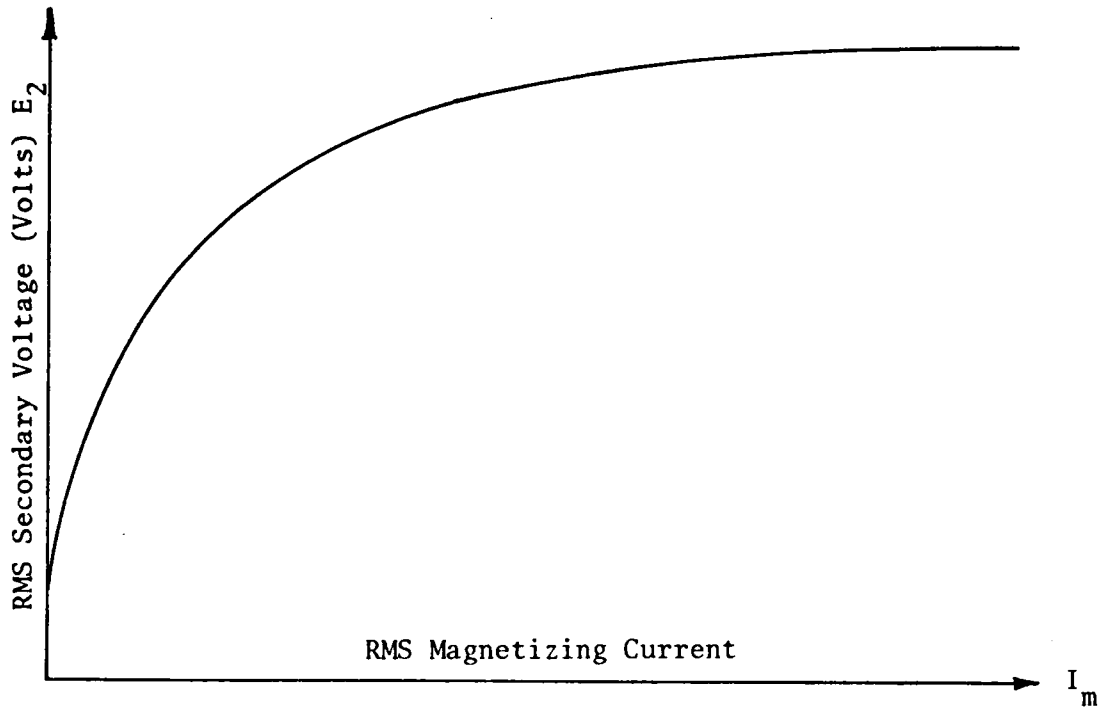


Figure 5. Magnetizing characteristic of CT.

The voltage across the relay is

$$V_R = I_R R_R$$

$$= I_B(R_B + R_B)$$

$$= 2I_B R_B .$$

Therefore, the relay current is $I_R = \frac{2I_B R_B}{R_R}$. The selection of R_R with respect to R_B is made so as to prevent the relay from operating.

$$R_R = \frac{2I_B R_B}{I_R}$$

where I_R is the relay pick-up and R_B is the largest CT lead resistance. Usually, for generators and transformers, R_R is approximately a third of the value expressed above.

$$I_{mA} = I_1 - \frac{N_2}{N_1} I_A \tag{1.1}$$

and

$$I_{mB} = I_1 - \frac{N_2}{N_1} I_B \tag{1.2}$$

where N_1 is the number of turns in the CT's primary winding, and N_2 is the number of turns in the secondary winding. I_{mA} and I_{mB} are the magnetizing components of the primary current I_1. Therefore,

$$I_R = I_A - I_B .$$

Substituting for $\frac{N_2}{N_1} = n$ in equations (1.1) and (1.2),

$$I_R = I_A - I_B$$

$$= \frac{I_1 - I_{mA}}{n} - \frac{I_1 - I_{mB}}{n}$$

$$= \frac{1}{n} (I_{mB} - I_{mA}) \ . \qquad (1.3)$$

From the above equation, it is found that the relay current, I_R, is the difference between the CT magnetizing currents times the inverse of the turns ratio. From the expression for I_R and equation (1.3), as R_R is increased, I_R is reduced; hence I_{mA} and I_{mB} are forced to equality and the CT with the lower magnetizing current will force the balancing current through the other CT secondary. Thus, even if R_R is greatly increased, the relay will still operate during an internal fault because the CT secondary e.m.f. will be additive and will force the current through the relay.

The application of resistance between the relay winding and its series stabilizing resistor depends upon the relay sensitivity and the pick-up setting required for internal faults. The pick-up can be controlled by a series resistance in a very sensitive relay. In a less sensitive relay, the pick-up is controlled by taps on the relay coil. The actual relay pick-up in terms of the primary and magnetizing currents is found by the expression

$$\sum I_m = I_1 - \frac{N_2}{N_1} I_R$$

$$= I_1 - n \cdot I_R$$

$$I_1 = n \cdot I_R + \sum I_m \qquad (1.4)$$

where n is the CT ratio and $\sum I_m$ is the sum of all the CT's magnetizing currents at the relay voltage setting. In other words, the magnetizing currents of all the CT's must be added to the relay current setting to determine the actual relay pick-up.

In order to avoid power failure and production losses, some manufacturing companies have what is called "Dual network system." Such connection is found at the E. I. DuPont De Nemours Nylon Plant at Chattanooga, Tennessee. Refer to Figure 6. The power network is interconnected in such a way that when a fault occurs along the transmission line of relay one, the relay will remote-trip the circuit breaker at Dupont one substation and power is fed from DuPont substation two and vice versa. Relays one and two are approximately one-half mile apart. Such a dual system protects DuPont from tons of waste which can develop in a matter of seconds.

Unfortunately, in the Spring of 1971, lightning struck both of Dupont's 44KV lines simultaneously, faulting both lines and causing the relays to remote-trip the circuit breakers which opened both lines. Although both relays were a half-mile apart, the lightning surge was sufficient to create a tremendous fault to saturate the interconnected

CT's. The three minute power blackout cost DuPont 200,000 pounds of waste.

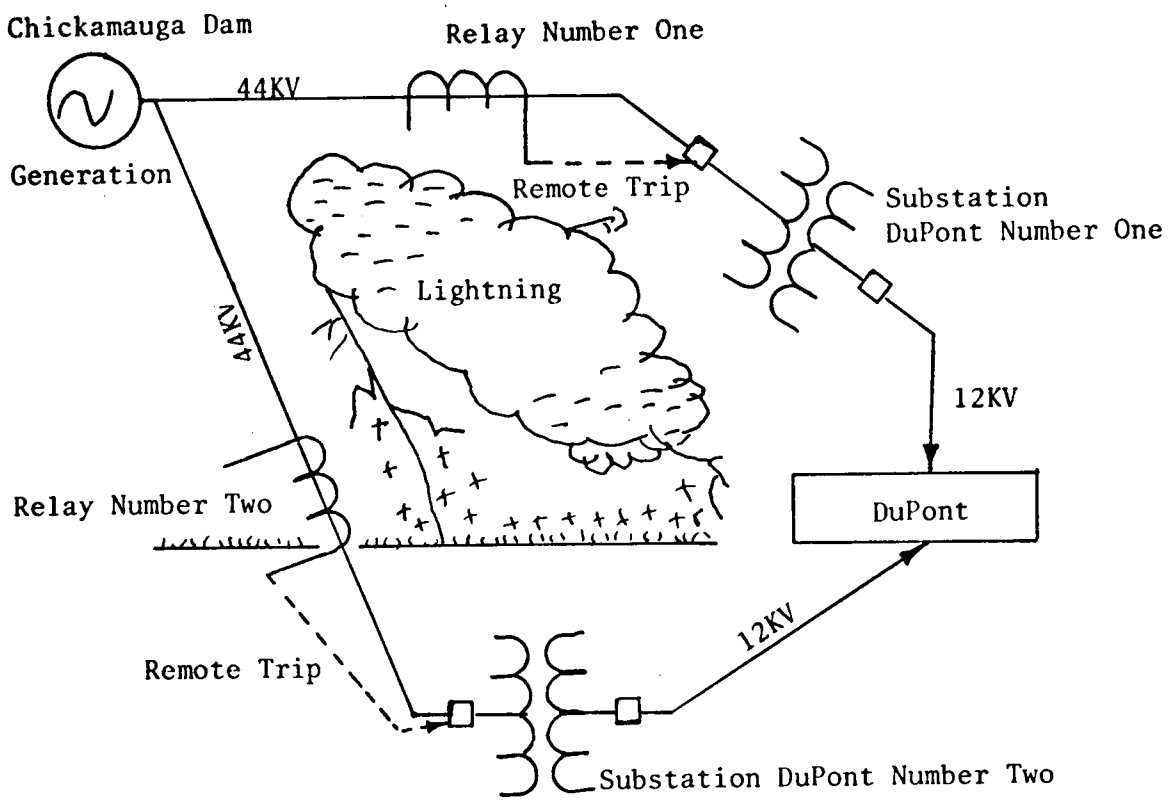


Figure 6. Dual power network and protection of E. I. DuPont Company, Chattanooga, Tennessee.

This example of power failure at DuPont indicates that in the application of CT's in the protection of power systems, there is no guarantee of perfect performance of CT's during a fault.

CHAPTER 2

RATIO ERROR AND PHASE ANGLE ERROR

2.1. Ratio Error

The phasor diagram for a two winding transformer assuming zero power factor on the load side and neglecting impedance of primary and secondary windings is shown in Figure 7. The definition of each component is found in Table 1.

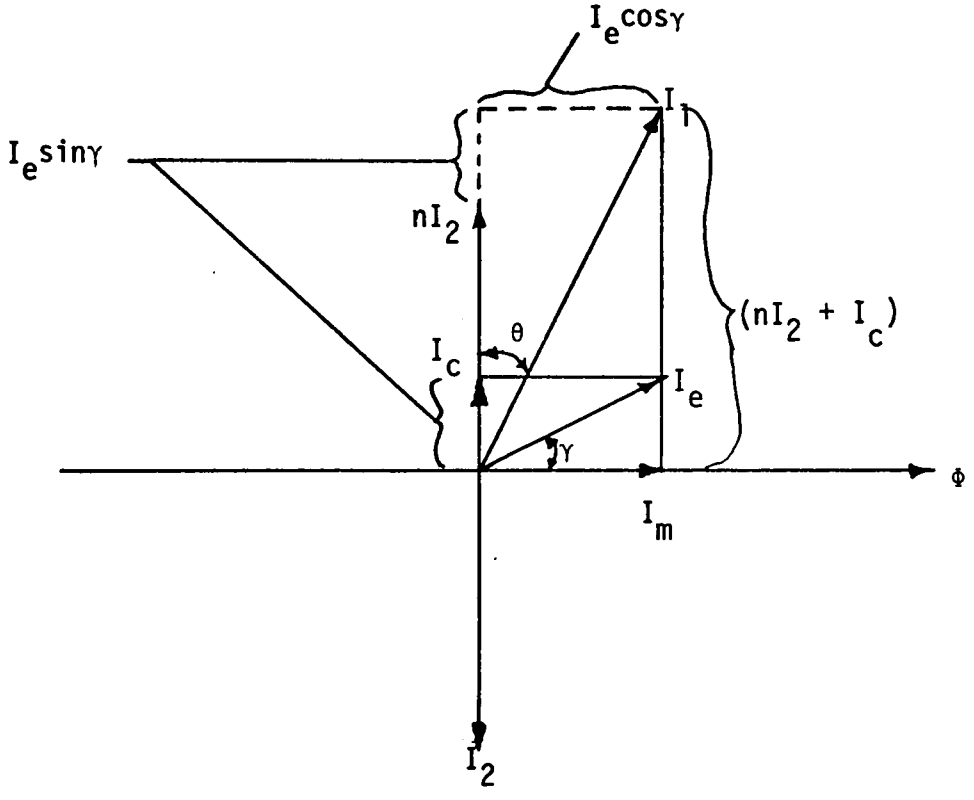


Figure 7. Simple phasor diagram of a two winding transformer.

TABLE 1

SYMBOLS AND DEFINITIONS

Symbol	Definition
I_c	The core loss component of the exciting current
I_m	The magnetizing current
I_1	Primary current
I_2	Secondary current
I_e	Exciting component of I_1 made up of I_c and I_m
nI_2	The primary current components providing the necessary ampere-turns for the secondary current
n	The turns ratio $\frac{N_2}{N_1}$

Assuming zero power factor at the load side of the transformer and neglecting the impedance of both sides of windings,

$$I_1^2 = I_m^2 + (nI_2 + I_c)^2$$

$$= I_m^2 + n^2I_2^2 + 2nI_2I_c + I_c^2 .$$

Since

$$I_e^2 = I_m^2 + I_c^2 ,$$

therefore

$$I_1^2 = I_e^2 + n^2I_2^2 + 2nI_2I_c .$$

Assuming that I_e is negligible,

$$I_1^2 \approx n^2 I_2^2 + 2nI_2 I_c$$

$$\frac{I_1^2}{I_2^2} \approx n^2 + 2n \frac{I_c}{I_2} . \qquad (2.1)$$

Note that

$$(n + \frac{I_c}{I_2})^2 = n^2 + 2n \frac{I_c}{I_2} + (\frac{I_c}{I_2})^2 .$$

Since $(\frac{I_c}{I_2})^2$ will be negligible,

$$(n + \frac{I_c}{I_2})^2 \approx n^2 + 2n \frac{I_c}{I_2} . \qquad (2.2)$$

Comparing equations (2.1) and (2.2),

$$(\frac{I_1}{I_2})^2 \approx (n + \frac{I_c}{I_2})^2$$

therefore

$$\frac{I_1}{I_2} \approx n + \frac{I_c}{I_2} . \qquad (2.3)$$

The $\frac{I_1}{I_2}$ expression in equation (2.3) is called the RATIO ERROR. It is found that the ratio of the primary current to the secondary current is dependent not only on the constant turns ratio, but also on the core loss component of the exciting current and also on the secondary current.

In order to reduce ratio error, a low loss core material such as mumetal (which reduces I_c) must be used. Also increasing the turns ratio, n, will reduce the $\frac{I_c}{I_2}$ term of the ratio error and will increase the secondary current, which further reduces $\frac{I_c}{I_2}$; hence reduces ratio error.

2.2. Phase Angle Error

From Figure 7, page 11

$$\tan \theta = \frac{I_e \cos \gamma}{nI_2 + I_e \sin \gamma}$$

where γ is the angle between I_e and the phasor flux, ϕ. Suppose θ is small and is expressed in radians.

$$\theta \approx \tan \theta$$

then

$$\theta \approx \frac{I_e \cos \gamma}{nI_2 + I_e \sin \gamma} \text{ radians} .$$

Since $I_e \sin \gamma$ is small compared with the nI_2 terms, therefore

$$\theta \approx \frac{I_e \cos \gamma}{nI_2} \text{ radians}$$

but $I_e \cos \gamma = I_m$, so

$$\theta \approx \frac{I_m}{nI_2} . \qquad (2.4)$$

the above is called PHASE ANGLE ERROR.

The phase angle error is important where the current transformer is used in the application of metering. It is required that the wattmeter-coil current be in phase with the load current, I_2. As seen in the phasor diagram, the primary current is slightly out-of-phase with the load current due to the presence of the exciting current, I_e. The phase angle θ is the angle between the reversed load current, along nI_2, and the primary current, I_1.

The phase angle error may be controlled by reducing the magnetizing current and by increasing the turns ratio and the secondary current. Thus, by reducing the ratio error, the phase angle error is also reduced. Ratio and phase angle errors are also a function of power factor of the burden.

2.3. Performance of Current Transformers

The performance of a current transformer is determined by its ratio error and phase angle error. Basically, the error of a current transformer is the vector difference between the primary and secondary ampere turns; therefore, the exciting current of the CT in ampere turns. Hence, it is essential to determine the exciting current.

For a sinusoidal primary current, the exciting current is almost sinusoidal and is a small fraction of the primary current. Any high overcurrent or heavy burden causes the core to saturate, and the exciting current waveform results in large harmonics and the transformer error gets relatively large.

Tremendous improvement has been made in the design of current transformers to minimize errors. Some current transformers are designed so as to override the d.c. transient.

Generally, it is unnecessary to determine the phase angle error of a current transformer used for relaying. One simple reason is that the secondary load of the CT has a very high lagging power factor that the secondary current is almost in phase with the exciting current. Therefore, the effect of the exciting current on the accuracy of the phase-angle is insignificant. More importantly, if the ratio error can be tolerated, the phase angle error can be neglected.

In case there is a leakage flux, calculation of accuracy of CT becomes very difficult. This leakage flux, generally, passes between the primary and secondary windings and completes its path in the core outside the primary winding. It is, thus, difficult to measure the leakage flux.

The data needed to calculate the CT performance at a certain frequency are saturation curves showing exciting current versus secondary voltage, secondary winding resistance and leakage reactance, burden and secondary current.

CHAPTER 3

RESIDUAL FLUX AND SATURATION IN CURRENT TRANSFORMER CORES

Since differential relays are used to protect apparatus from high current, it is essential that the possibility of false operation be investigated. Such a false operation might occur due to a fault outside but close to the protected zone. Such a fault, for example, could be caused by a short circuit.

Current transformers will fail to operate under system short circuit conditions if the heat generated in the windings is sufficient to burn the insulation.

In the generator bushing, CT space is so limited that the core may be easily saturated. Also, with large generator units, the CT's have very high numbers of secondary turns, causing high secondary impedances, and therefore voltages, further aggravating the saturation problem.

Sometimes for a fault outside the protected zone, where the differential relay is not expected to operate, it is possible to obtain differences between the secondary currents of the normally operating CT's. Refer to Figure 8. The relay may operate due to conditions against which it is not required to operate.

Usually, significant errors occur in current transformers during transient and steady-state conditions. The presence of a direct current component in the primary fault current produces oversaturation

of the transformer core, hence maximum error results. The saturation under these kinds of conditions is further worsened if the secondary is overburdened. The magnitude of the error also depends upon the residual magnetism in the transformer core and its polarity with reference to the direct current component. It is unusual for residual magnetism to be present because of demagnetization of the load current. The residual magnetism could be available after the initial fault if the circuit is interrupted before the disappearance of the direct current transient. Then, when the circuit breaker is reclosed against the fault, the residual flux will increase the CT error.

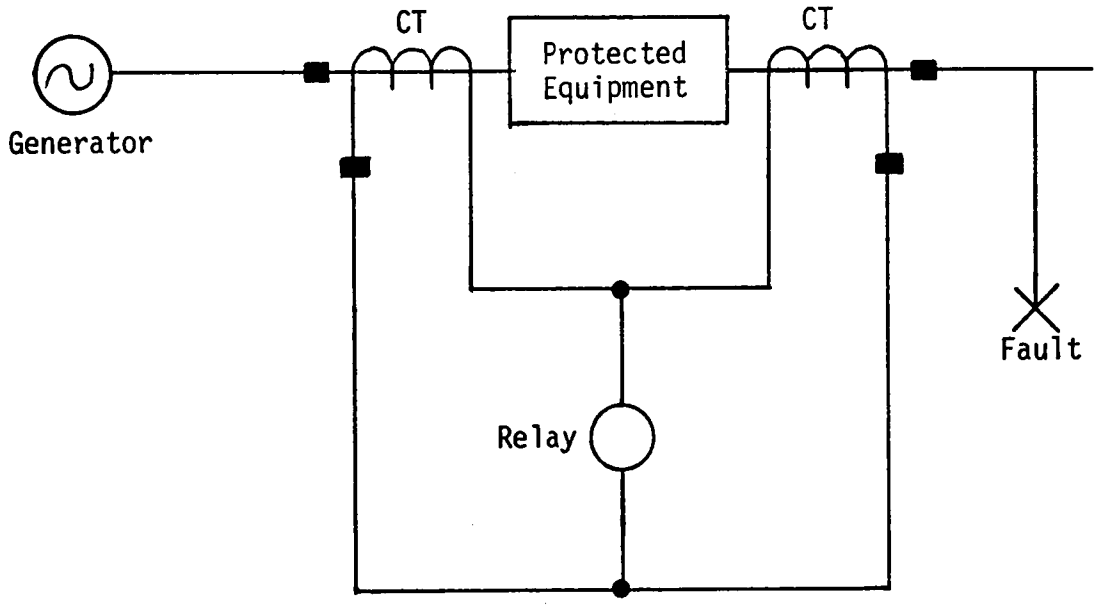


Figure 8. Basic differential connection.

Current transformer cores are usually made of ferromagnetic materials such as steel and cast iron. Referring to Figure 9, the

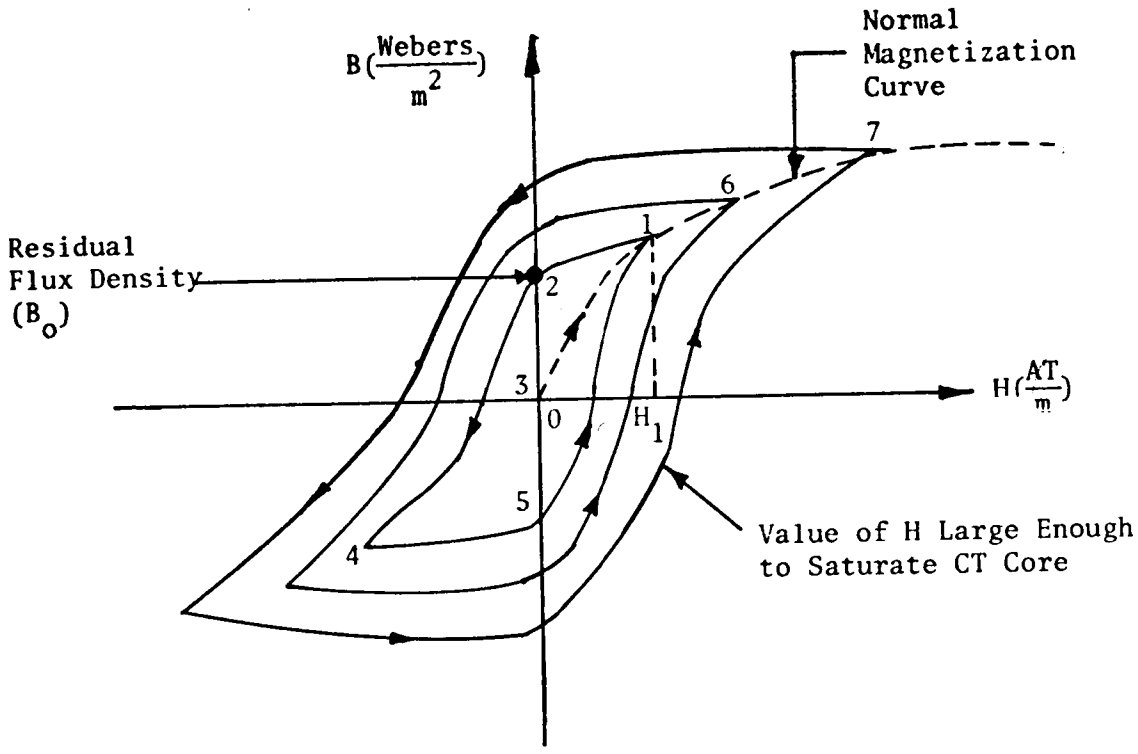


Figure 9. Typical hysteresis loops.

hysteresis loops represent plot of the magnetic flux density (B) versus the magnetic field intensity (H). The usual characteristic of ferromagnetic material is such that once the field intensity is increased to value H_1 and decreased, it is found that demagnetization is opposed. It is found that instead of the demagnetization path retracing the magnetization curve 01, it follows along curve 12, above curve 01. Consequently, when the field intensity is decreased to zero, the flux density does not reduce to zero, but remains at two as indicated by the hysteresis loop. This occurs because of the remnant flux aligned in the direction of the field which was originally applied. The value, B_o, that remains after the field intensity H is removed is known as the

residual flux density. The residual flux density times the current transformer core cross-sectional area is called the residual flux (ϕ_o).

The physical interpretation of CT magnetic saturation is somewhat theoretical. For a CT with a wholly resistive burden, the secondary induced voltage is in phase with the secondary current. Now if the core saturates at ϕ_s, the CT stops generating voltage and the secondary current drops to zero at time t_s. At time t_1, the primary current reverses, requiring a flux change in the opposite direction and the core desaturates. Refer to Figure 10.

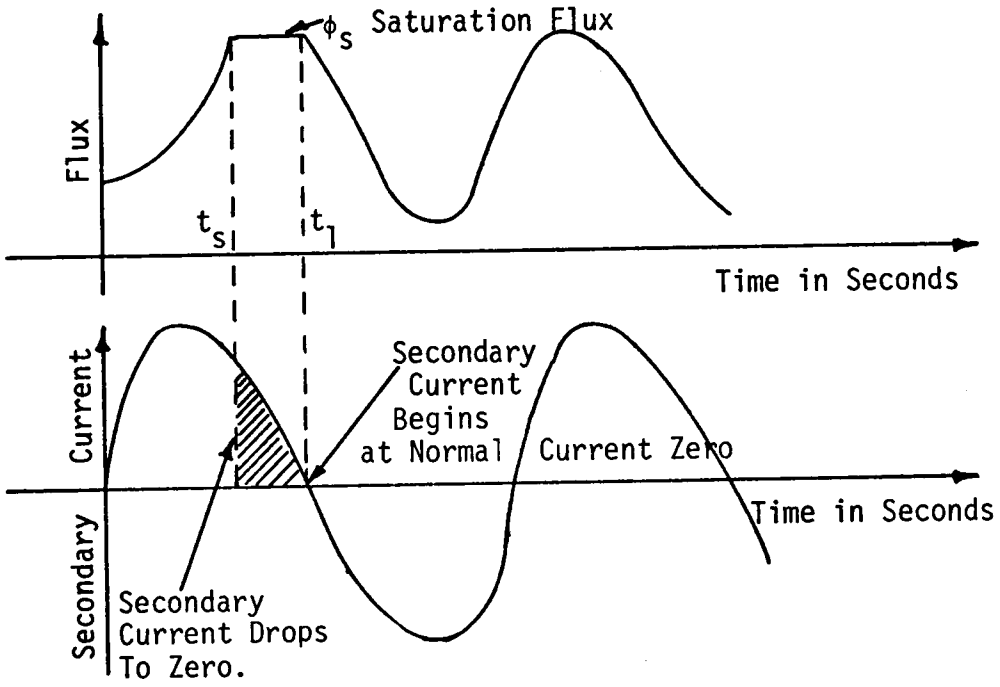


Figure 10. Resistive burden saturation.

A wholly inductive secondary produces flux that follows the primary current until saturation is reached and the flux is flat-topped as in Figure 11. During saturation, the CT ceases to generate voltage and the secondary current is also flat-topped.

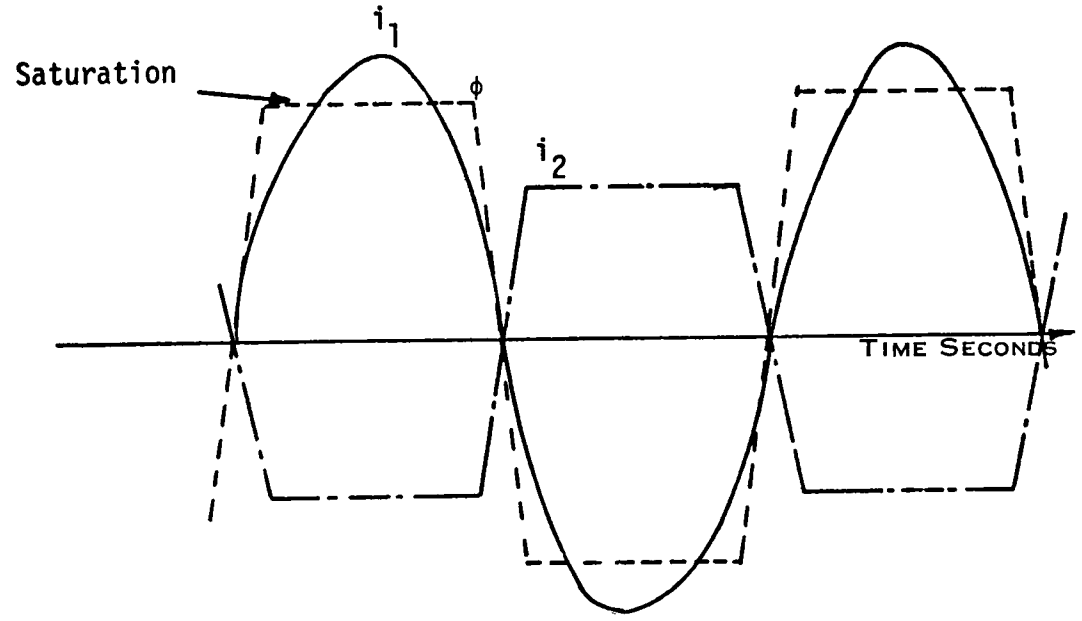


Figure 11. Inductive burden saturation.

The effect of a direct-current component upon CT performance is quite significant. Referring to Figure 12, during a fault, the primary current wave is almost totally offset and a severe saturation results in the relay current transformer. The secondary current is completely suppressed for one cycle, then builds up to a maximum steady state in eleven cycles as the direct-current magnetization disappears. This type of saturation could even slow down the performance of a high-speed differential relay, normally operating in two cycles, to operate in several cycles because of low initial secondary current. The current transformer functions normally after the effects of the direct-current component disappear. The direct-current component appearing in the secondary of CT's tends to reduce the speed of operation of induction-type relays.

The physical and theoretical interpretation of magnetic saturation in current transformers are generally simple. The main objective of

this paper is to develop mathematical models and predict CT performance by the use of digital computer.

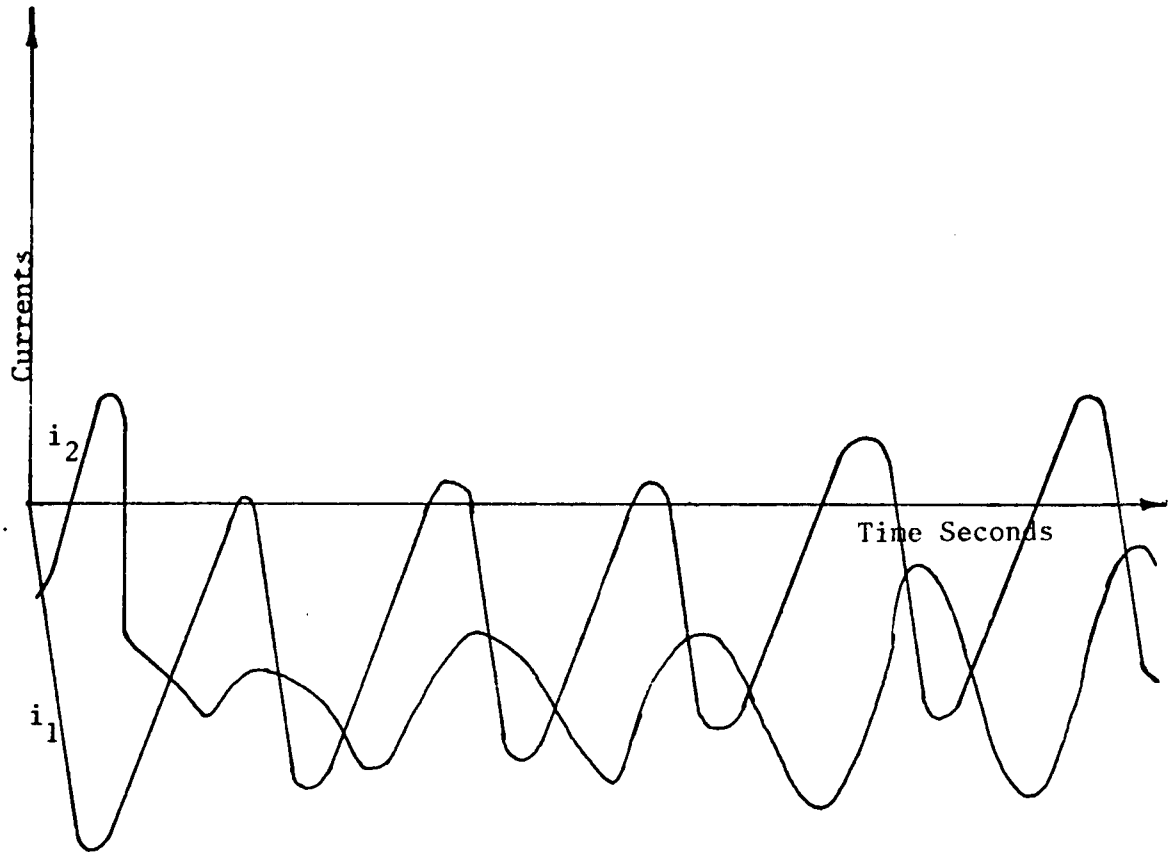


Figure 12. CT saturation due to direct-current component of fault current.

Source: Reference 4.

CHAPTER 4

CALCULATIONS FOR CT PRIMARY FAULT CURRENT

4.1. Introduction

Saturation of a current transformer depends on the type of fault, fault location and most importantly the primary current offset as mentioned in the previous chapters.

The CT primary current, i_1, is also the current in one phase of the generator. It is, therefore, essential to find out which type of fault will cause the greatest amount of fault current. This investigation is particularly intended for a system consisting of a generator and a main transformer connected to a power system. Particular attention is given to the CT currents under conditions of fault on the high voltage side of the main transformer. Therefore, to calculate the fault current, the impedance of this transformer must be taken into account.

Usually, currents having an infinite number of harmonics are produced by single phase faults of either line-to-line or line-to-neutral short circuit. In these calculations, the harmonics have been neglected since their amplitudes are generally very low.

4.2. Line-to-Neutral Fault

Considering the condition of a line-to-neutral fault on the wye side of a neutrally grounded transformer as shown in Figure 13,



Figure 13. A line-to-neutral fault condition.

$$\bar{I}_{a_0} = \bar{I}_{a_1} = \bar{I}_{a_2}$$

$$\bar{I}_a = \bar{I}_{a_0} + \bar{I}_{a_1} + \bar{I}_{a_2} = 3\bar{I}a_1$$

$$\bar{I}_b = 0$$

$$\bar{I}_c = 0$$

$$\bar{V}_a = \bar{V}_{a_0} + \bar{V}_{a_1} + \bar{V}_{a_2} = 0 \ .$$

Therefore, $\bar{V}_{a_1} = -\bar{V}_{a_2} - \bar{V}_{a_0}$

and

$$\bar{V}_{a_1} = \bar{E}_a - \bar{I}_{a_1} \bar{Z}_1$$

$$\bar{V}_{a_2} = - \bar{I}_{a_2} \bar{Z}_2 = - \bar{I}_a Z_a$$

$$\bar{V}_{a_0} = - \bar{I}_{a_0} Z_0 = - \bar{I}_{a_1} Z_0 \ .$$

Therefore,

$$\overline{V}_{a_1} = \overline{I}_{a_1} (Z_2 + Z_0)$$

$$\overline{I}_{a_1} (Z_2 + Z_0) = \overline{E}_a - \overline{I}_{a_1} Z_1$$

$$\overline{I}_a = \frac{\overline{E}_a}{Z_1 + Z_2 + Z_0}$$

$$\overline{I}_a = \frac{\overline{E}_a}{Z_1 + Z_2 + Z_0} . \qquad (4.1)$$

Referring to Figure 14 (A) and (B), a line-to-neutral short circuit on the wye side of the transformer looks to the generator on the delta side like a line-to-line fault. Assuming that the transformer has a reactance X_T,

$$2X_T\overline{I}a = \sqrt{3}\ \overline{E}$$

therefore,

$$\overline{I}_a = \frac{\sqrt{3}\ \overline{E}}{2X_T} . \qquad (4.2)$$

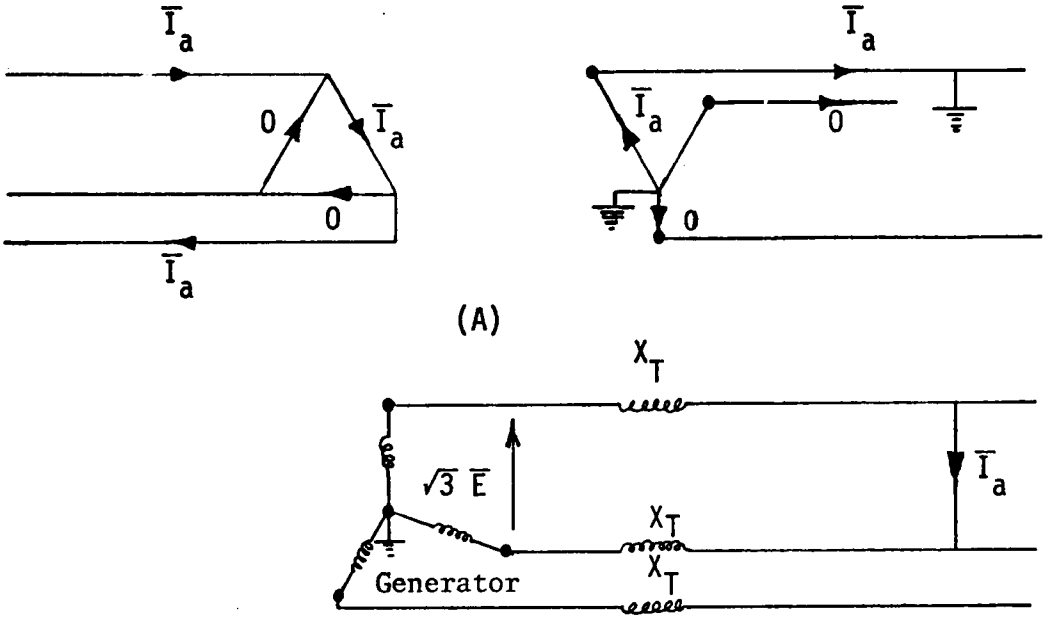


Figure 14. Current conditions of line-neutral-fault. (A) On wye side of transformer; (B) apparent to generator.

4.3. Line-to-Line Fault

For a line-to-line fault as illustrated in Figure 15,

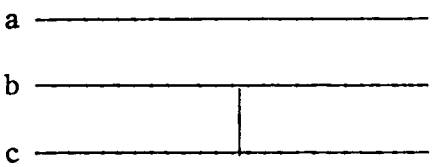


Figure 15. Line-to-line fault condition.

$$\overline{I}_b = -\overline{I}_c$$

$$\overline{I}_a = 0$$

$$\overline{I}_{a0} = \frac{1}{3}(\overline{I}_a + \overline{I}_b + \overline{I}_c) = 0$$

$$\bar{I}_{a1} = \frac{1}{3}\left(\bar{I}_a + a\bar{I}_b + a^2\bar{I}_c\right) = \frac{j\bar{I}_b}{\sqrt{3}}$$

$$\bar{I}_{a2} = \frac{1}{3}\left(I_a + a^2\bar{I}_b + a\bar{I}_c\right) = \frac{-j\bar{I}_b}{\sqrt{3}}$$

therefore,

$$\bar{I}_{a1} = -\bar{I}_{a2}$$

$$\bar{V}_b = \bar{V}_c \ .$$

But

$$\bar{V}_b = a^2\bar{V}_{a_1} + a\bar{V}_{a_2} + \bar{V}_{a0}$$

$$\bar{V}_c = a\bar{V}_{a1} + a^2\bar{V}_{a2} + \bar{V}_{a0}$$

$$\bar{V}_b - \bar{V}_c = (a^2 - a)\,\bar{V}_{a1} + (a - a^2)\,\bar{V}_{a2}$$

$$= 0 \ .$$

Therefore,

$$\bar{V}_{a_1} = \bar{V}_{a_2} \ .$$

Now

$$\bar{V}_{a_1} = \bar{E}_a - \bar{I}_{a_1} Z_1$$

$$\bar{V}_{a_2} = -\bar{I}_{a_2} Z_2 = \bar{I}_{a_1} Z_2 \ .$$

Therefore,

$$\bar{I}_{a_1} = \frac{E_a}{Z_1 + Z_2} = -\bar{I}_{a_2}$$

$$\bar{I}_b = \bar{I}_{a_0} + a^2\bar{I}_{a_1} + a\bar{I}_{a_2}$$

$$= \frac{(a^2 - a)\bar{E}_a}{Z_1 + Z_2}$$

$$= \frac{-j\sqrt{3}\ \bar{E}_a}{Z_1 + Z_2}$$

$$= -\bar{I}_c \ . \qquad (4.3)$$

As shown in Figure 16, a line-to-line fault on the wye side of a delta-wye transformer appears to the generator on the delta side as to double the amount of current in phase B.

Again assuming the transformer to have a reactance X_T and from equation (4.3),

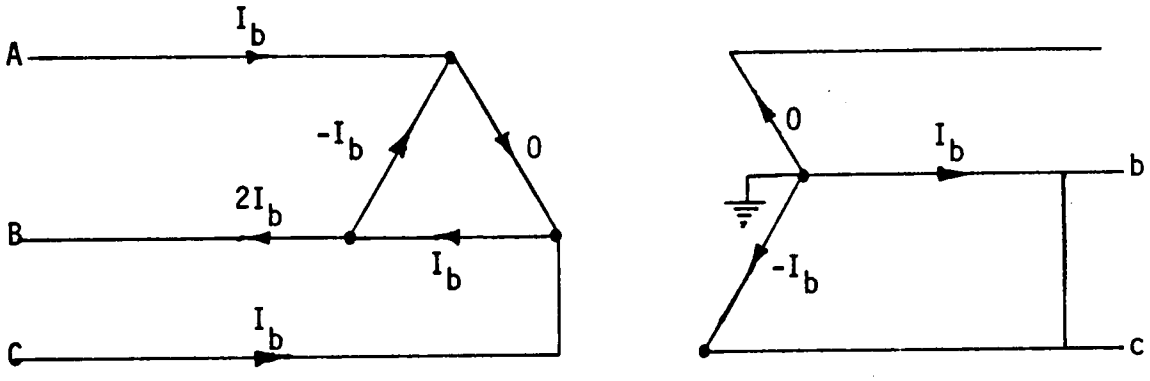


Figure 16. Line-to-line fault current in each phase.

$$\bar{I}_{b\ \text{line-to-line}} = \frac{-2j\sqrt{3}\ \bar{E}_a}{Z_1 + Z_2}$$

$$= \frac{-2j\sqrt{3}\ \bar{E}_a}{2X_T}$$

$$= \frac{-j\sqrt{3}\ \bar{E}_a}{X_T} \ . \qquad (4.4)$$

Comparing equations (4.1) and (4.3) or equations (4.2) and (4.4), it can be concluded that the maximum fault primary current is caused by a line-to-line fault on the high voltage side of the main transformer.

In the design of a CT, consideration of such faults as line-to-line has to be made in order to meet the specifications.

CHAPTER 5

CALCULATIONS FOR CT SECONDARY CURRENT BEFORE AND DURING SATURATION

5.1. Introduction

Given the primary current i_1, it is required to determine the unsaturated and saturated performance expressions of a CT. In the CT equivalent circuit of Figure 17, it is assumed that the CT is wound on a toroidal core resulting in negligible leakage reactance. The secondary leakage reactance is also negligible since a fully distributed winding is assumed, and the core-loss is assumed to be zero. The instantaneous equations are:

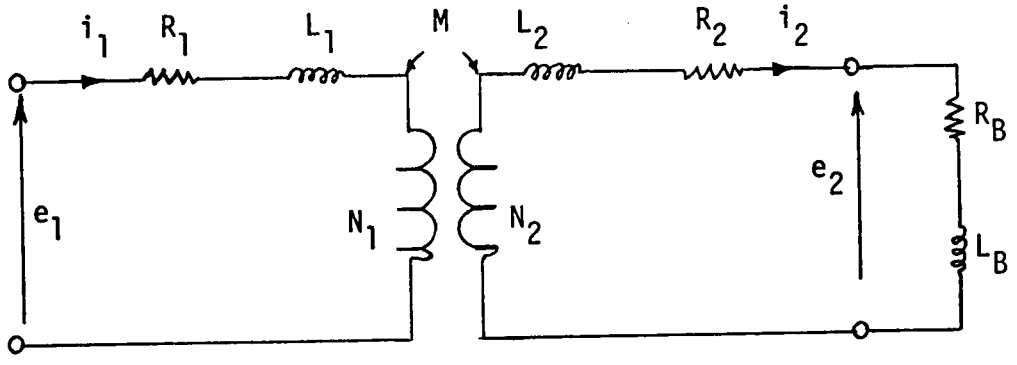


Figure 17. A CT equivalent.

Note: R_1 = primary lead resistance
R_2 = secondary lead resistance
R_B = burden resistance
M = mutual inductance
L_1 = primary lead inductance
L_2 = secondary lead inductance
L_B = burden inductance

$$e_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_m}{dt}$$

$$e_2 = R_B i_2 + L_B \frac{di_2}{dt} = -R_2 i_2 - L_2 \frac{di_2}{dt} + M \frac{di_m}{dt} .$$

We are concerned with the CT output and effects of residual magnetism and saturation.

Substituting for $i_m = i_1 - \frac{N_2}{N_1} i_2$ in the second expression,

$$R_B i_2 + L_B \frac{di_2}{dt} = -R_2 i_2 - L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} - \frac{N_2}{N_1} M \frac{di_2}{dt}$$

$$\left(L_2 + L_B + \frac{N_2}{N_1} M\right) \frac{di_2}{dt} = M \frac{di_1}{dt} - (R_2 + R_B) i_2 .$$

$$\frac{di_2}{dt} = \frac{M \frac{di_1}{dt} - (R_2 + R_B) i_2}{\left(L_2 + L_B + \frac{N_2}{N_1} M\right)} .$$

The above expression for secondary current is similar to equation (5.5) to be solved later.

The mutual inductance $M = \frac{N_1 N_2}{\mathcal{R}}$ where $\mathcal{R} = \frac{\ell}{\mu A}$ = magnetic reluctance.

$\frac{\ell}{A}$ = ratio of CT core primary winding layer length to cross-sectional area;

μ = product of permeability in free space and relative permeability of core material.

Since it is difficult to determine the relative permeability or magnetic reluctance during the unsaturated and saturated states of the

core, Figure 18 shows a simple approximation which will make it easier to determine the mutual inductance. The slope K_1 is designated as a condition of the CT before saturation. As the magnetizing current exceeds the saturation current I_s, slope K_2 for a saturation condition is reached. To allow for residual magnetism, the core conditions are assumed to vary about the point (I_o, ϕ_o). Then the following expressions can be written:

$$\phi = K_1 i_m + \phi_o \qquad -(I_s + I_o) \leq i_m \leq (I_s - I_o)$$

$$= K_2 i_m + \phi_s' \qquad -(I_s + I_o) > i_m > (I_s - I_o)$$

$$\phi_s' = (\phi_s + K_2 I_o) \ .$$

From the above expressions, the value of ϕ for any current i_m can be established. In addition, a known value of ϕ will make it possible to find a time expression for the CT secondary current i_2 for a given primary current i_1.

5.2. Determination of K_1

Referring to the simple two winding transformer of Figure 19 (A), the following equations can be obtained:

$$\phi = K_1 i_m$$

$$V_2 = N_2 \frac{d\phi_2}{dt}$$

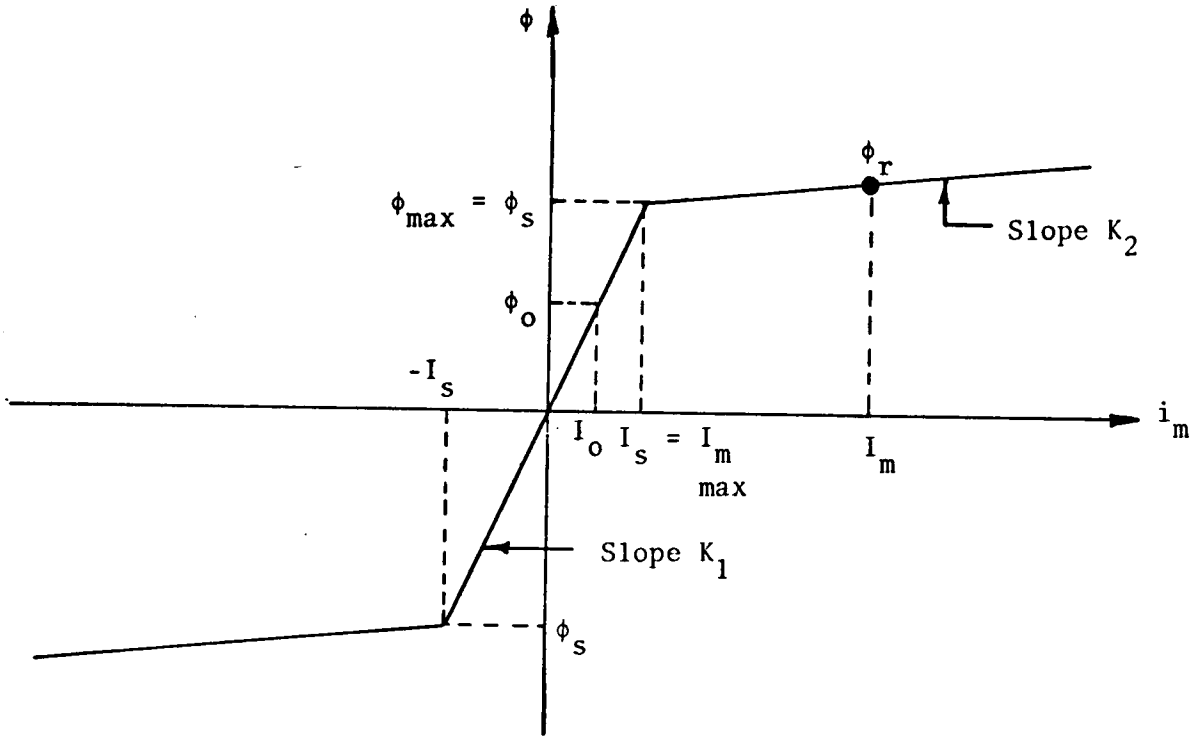
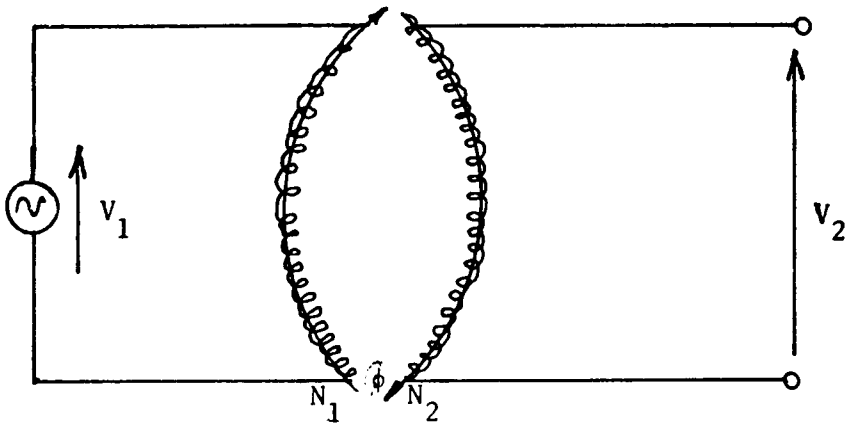
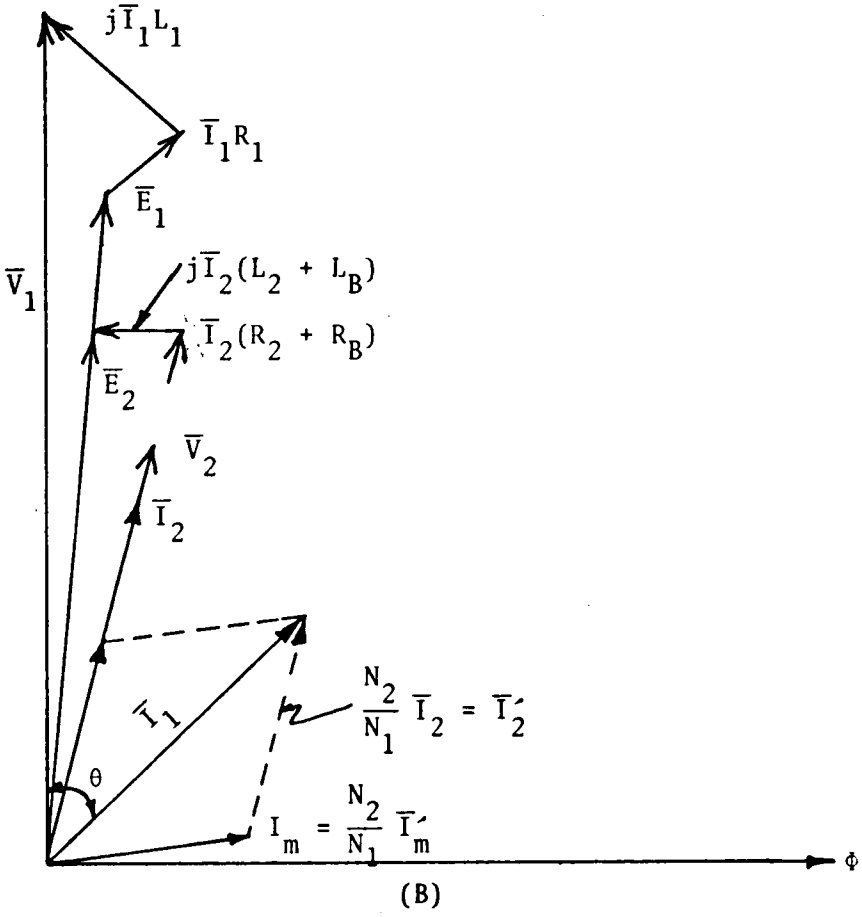


Figure 18. Approximate magnetic characteristic of core material.



(A) A Simple Two Winding Transformer



(B)

Figure 19. Phasor diagram of a CT under burden.

$$V_1 = N_1 \frac{d\phi_1}{dt} .$$

Differentiating both sides of the first equation with respect to time, t,

$$\frac{d\phi}{dt} = K_1 \frac{di_m}{dt} .$$

Substituting the above expression in the second equation,

$$V_2 = N_2 K_1 \frac{di_m}{dt} . \qquad (5.1)$$

But from Figure 19,

$$i_m = \frac{N_2}{N_1} i'_m .$$

Therefore,

$$\frac{di_m}{dt} = \frac{N_2}{N_1} \frac{di'_m}{dt} .$$

Substituting for the above expression in equation (5.1),

$$V_2 = N_2 K_1 \quad \frac{N_2}{N_1} \frac{di'_m}{dt}$$

$$= \frac{N_2^2 K_1}{N_1} \frac{di'_m}{dt} . \qquad (5.2)$$

If i_m is sinusoidal, then

$$i'_m = \sqrt{2}\, I'_m \sin \omega t$$

$$\frac{di'_m}{dt} = \sqrt{2}\, I'_m\, \omega \cos \omega t \ .$$

Substituting for $\frac{di'_m}{dt}$ in equation (5.2),

$$V_2(t) = \frac{N_2^2\, K_1}{N_1} \sqrt{2}\, I'_m\, \omega \cos \omega t \ .$$

Therefore,

$$V_{2_{RMS}} = \frac{N_2^2\, K_1}{N_1} I'_{m_{RMS}}\, \omega$$

$$\frac{N_2^2\, K_1}{N_1}\, \omega = \frac{V_{2_{RMS}}}{I'_{m_{RMS}}} \ .$$

From the magnetization curve (Figure 20) of the RMS secondary voltage versus the RMS secondary magnetizing current (I'_m), for example, the slope

$$\frac{V_{2_{RMS}}}{I'_{m_{RMS}}} = \frac{1,333}{0.004}$$

$$= 333,250 \ .$$

Therefore,

$$K_1 = \frac{333,250 \times N_1}{N_2^2\, \omega}$$

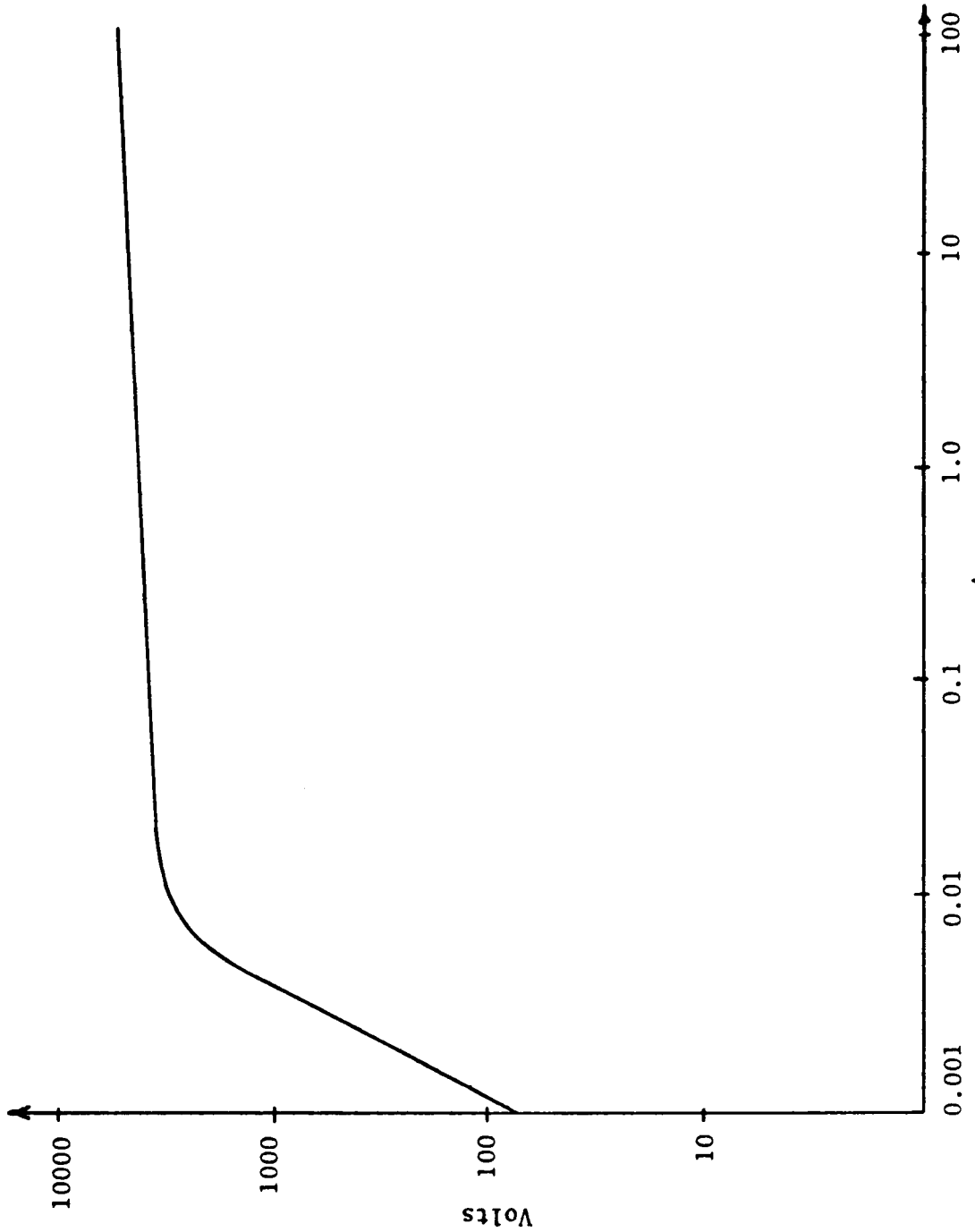


Figure 20. Characteristic curve of a 2.5/22,500 amperes CT.

$$= 1.092 \times 10^{-5} \frac{\text{Webers}}{\text{Ampere}} ,$$

where

$$N_1 = 1 ,$$

$$N_2 = 9{,}000 .$$

It should be remembered that K_1 is the slope of the curve of Figure 18 (page 33) before the point (ϕ_s, I_s) and before saturation. So, given the magnetizing curve of a particular CT, the slope K_1 can be found.

5.3. Determination of K_2

It was easy to determine the slope K_1 as the normal voltage is applied to the CT. However, as the CT core becomes saturated during a fault, a slope represented by K_2 is attained. It will prove very difficult to calculate K_2 analytically due to its dependence on the relative permeability during saturation, the steady state primary current, the maximum flux in the CT core and the maximum secondary magnetizing current. Most of the above named quantities that K_2 depends on will be unknown variables during saturation.

As illustrated in Figure 18, select a point (ϕ_r, I_m) along slope K_2, where ϕ_r and I_m are unknown flux and magnetizing current, respectively. Then

$$K_2 = \frac{\phi_r - \phi_{max}}{I_m - I_{m_{max}}} . \qquad (5.3)$$

Assuming that the maximum magnetizing current and maximum flux can be found from the characteristic curve, the main problem is to determine ϕ_r and I_m. However, it will be most difficult to find these two variables; hence difficult to determine K_2. Therefore, K_2 will have to be represented by a different method.

It is known that $\phi = \mu_o \mu_r N_1 i_m \frac{A}{\ell}$ referred to the primary side where

μ_o = permeability of free space,

μ_r = relative permeability of CT core material,

$\frac{A}{\ell}$ = ratio of CT core cross-sectional area to primary winding layer length,

$$\frac{d\phi}{dt} = \mu_o \mu_r N_1 \frac{A}{\ell} \frac{di_m}{dt},$$

$$\frac{d\phi}{di_m} = \mu_o \mu_r N_1 \frac{A}{\ell} = K.$$

Therefore,

$$\frac{\ell}{A} = \frac{\mu N_2}{K}$$

$$\mathcal{R} = \frac{\ell}{A} \cdot \frac{1}{\mu} = \frac{N_1}{K}$$

$$M = N_1 N_2 \cdot \frac{K}{N_1} = N_2 K .$$

Also $M = N_1 K_1$ referred to the secondary. Therefore, K (referred to primary) $= \frac{N_1 K_1}{N_2}$.

Suppose the relative permeability before saturation is μ_r and during saturation is μ_{rs}. Therefore, before the core saturates,

$$\frac{d\phi}{di_m} = \mu_o \mu_r N_2 \frac{A}{\ell} = K_1$$

and

$$\frac{A}{\ell} = \frac{K_1}{\mu_o \mu_r N_1}$$

then

$$\phi = K_1 i_m \ .$$

If residual flux exists in the core,

$$\phi = K_1 i_m + \phi_o$$

where ϕ_o is the residual flux. If the core saturates, the saturated flux, ϕ_s, is

$$\phi_s = \mu_o \mu_r N_2 \frac{A}{\ell} |i_s| + \mu_o \mu_{rs} N_2 \frac{A}{\ell} (i_m - i_s)$$

$$= K_1 i_s + \frac{\mu_{rs} K_1}{\mu_r} (i_m - i_s)$$

$$= K_1 i_s + \frac{K_1 \mu_{rs}}{\mu_r} (i_m - i_s)$$

$$= K_1 i_s + K_2 (i_m - i_s)$$

where

$$K_2 = \frac{K_1 \mu_{rs}}{\mu_r} \ .$$

The saturated magnetizing current is

$$i_m = \frac{\phi_s - K_1 i_s}{K_2} + i_s \ .$$

5.4. Calculations of CT Output if Core is Unsaturated

From the CT secondary circuit, the following equation can be obtained:

$$N_2 \frac{d\phi}{dt} = (R_2 + R_B) i_2 + (L_2 + L_B) \frac{di_2}{dt} \ . \qquad (5.4)$$

Substituting for

$$\frac{d\phi}{dt} = \mu_r \mu_o N_1 \frac{A}{\ell} \frac{di_m}{dt} = K_1 \frac{di_m}{dt}$$

in equation (5.4),

$$N_2 K_1 \frac{di_m}{dt} = (R_2 + R_B) i_2 + (L_2 + L_B) \frac{di_2}{dt}$$

but

$$i_m = i_1 - \frac{N_2}{N_1} i_2$$

therefore,

$$N_2 K_1 \left(\frac{di_1}{dt} - \frac{N_2}{N_1} \frac{di_2}{dt}\right) = (R_2 + R_B) i_2 + (L_2 + L_B) \frac{di_2}{dt}$$

$$N_2 K_1 \frac{di_1}{dt} - \frac{N_2^2 K_1}{N_1} \frac{di_2}{dt} = (R_2 + R_B) i_2 + (L_2 + L_B) \frac{di_2}{dt}$$

$$\left[L_2 + L_B + \frac{N_2^2 K_1}{N_1}\right] \frac{di_2}{dt} = N_2 K_1 \frac{di_1}{dt} - (R_2 + R_B) i_2$$

$$\frac{di_2}{dt} = \frac{N_2 K_1 \frac{di_1}{dt} - (R_2 + R_B) i_2}{\left(L_2 + L_B + \frac{N_2^2 K_1}{N_1} \right)} \quad . \tag{5.5}$$

The differential equation (5.5) must be solved for the CT secondary current. This is the CT output without saturation. In the later part of this chapter, a digital computer will be used to solve equation 5.5).

5.5. Calculations of CT Output if Core Saturates

When the CT terminal voltage is so high that the core saturates during each half cycle, the performance of the CT may be modeled by equation (5.5) prior to the point where saturation occurs. After saturation is reached, the secondary current is governed by a differential equation containing K_2.

From equation (5.4)

$$\frac{di_2}{dt} = \frac{N_2 \frac{d\phi}{dt} - (R_2 + R_B) i_2}{L_2 + L_B} \quad .$$

Substituting for $\frac{d\phi}{dt} = K_2 \frac{di_m}{dt}$ in the above equation,

$$\frac{di_2}{dt} = \frac{N_2 K_2 \frac{di_m}{dt} - (R_2 + R_B) i_2}{L_2 + L_B} \quad .$$

But $i_m = i_1 - \frac{N_2}{N_1} i_2$. Therefore,

$$\frac{di_2}{dt} = \frac{N_2K_2 \frac{d}{dt}\left(i_1 - \frac{N_2}{N_1} i_2\right) - (R_2 + R_B)i_2}{L_2 + L_B}$$

$$= \frac{N_2K_2 \frac{di_1}{dt} - \frac{N_2^2K_2}{N_1} \frac{di_2}{dt} - (R_2 + R_B)i_2}{L_2 + L_B}$$

$$\left(L_2 + L_B + \frac{N_2^2K_2}{N_1}\right) \frac{di_2}{dt} = N_2K_2 \frac{di_1}{dt} - (R_2 + R_B)i_2$$

$$\frac{di_2}{dt} = \frac{N_2K_2 \frac{di_1}{dt} - (R_2 + R_B)i_2}{\left(L_2 + L_B + \frac{N_2^2K_2}{N_1}\right)} . \qquad (5.6)$$

Equation (5.6) is the nonlinear differential equation to be solved for i_2 when the core is saturated. In solving for i_2 when the core is driven into saturation, the digital computer solution is programmed to test the value of core flux and to decide which of the governing equations for i_2 is the appropriate one at each time interval.

5.6. Core Flux

Calculation of the core flux can be obtained as follows:

$$e_2 = N_2 \frac{d\phi}{dt} = (R_2 + R_B)i_2 + (L_2 + L_B) \frac{di_2}{dt}$$

$$\frac{d\phi}{dt} = \frac{1}{N_2} \left[(R_2 + R_B)i_2 + (L_2 + L_B) \frac{di_2}{dt}\right]$$

$$d\phi = \frac{1}{N_2}\left[(R_2 + R_B)i_2\,dt + (L_2 + L_B)di_2\right]$$

$$\phi = \frac{1}{N_2}\left[(R_2 + R_B)\int i_2\,dt + (L_2 + L_B)i_2\right] \text{ Webers} \quad . \quad (5.7)$$

The equation (5.7) is the general expression for the core flux. The value of this flux is determined by which governing expression for i_2 is calculated at each time interval.

The main expression which will be used for modeling CT's and predicting their performance are equations (5.5), (5.6) and (5.7). An example to solve these equations by the digital computer is given in the Appendix section.

To predict the behavior of a CT, take for example, a CT with ratio 2.5/22,500 amperes and parameters $R_B = 0.128$, $L_B = 0.0001$, $R_2 = 8.0$, $L_2 = 0.1$, $K_1 = 1.092 \times 10^{-5}$ and $K_2 = 0.0546 \times 10^{-5}$. The characteristic curve for this CT was shown in Figure 20, page 37. A current of 20,000 sin ωt amperes is applied to the primary of the CT. The outputs are indicated in Figures 21, 22, 23 and 24.

As observed from the above mentioned figures, the flux, secondary current, and secondary voltage have normal waveforms since the CT is not saturated. These waveforms are obviously symmetrical for any normal voltage.

Now to check for the computer program accuracy analytically, the secondary current of the CT can be found from the expression

$$I_1 = \left(\frac{N_2}{N_1}\right) I_2 \quad .$$

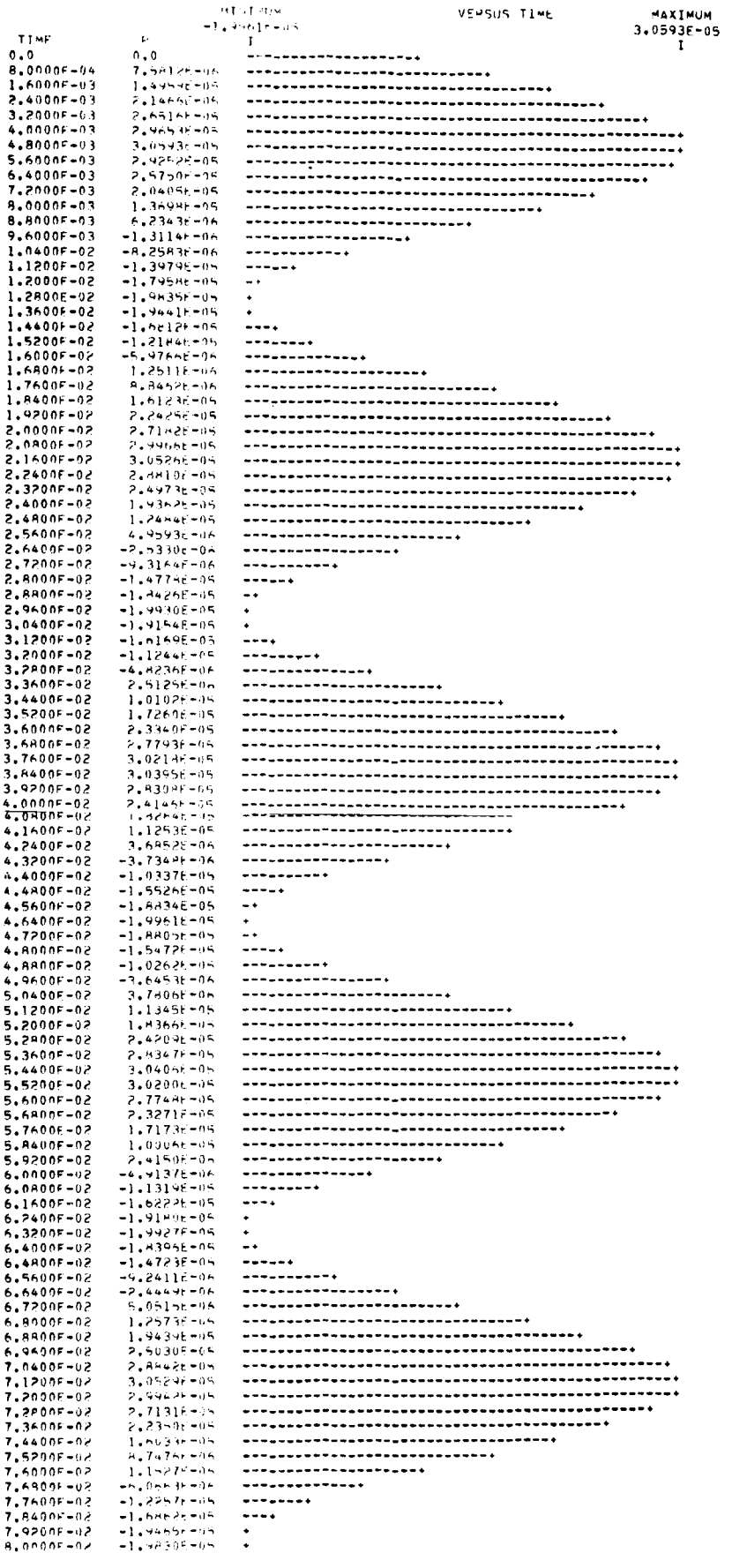


Figure 21. Unsaturated flux.

MINIMUM I2 VERSUS TIME MAXIMUM
-2.2217E 00 2.2217E 00

TIME	I2	I
0.0	0.0	
8.0000E-04	6.6003E-01	
1.6000E-03	1.2605E 00	
2.4000E-03	1.7471E 00	
3.2000E-03	2.0760E 00	
4.0000E-03	2.2175E 00	
4.8000E-03	2.1588E 00	
5.6000E-03	1.9053E 00	
6.4000E-03	1.4797E 00	
7.2000E-03	9.2050E-01	
8.0000E-03	2.7823E-01	
8.8000E-03	-3.8917E-01	
9.6000E-03	-1.0214E 00	
1.0400E-02	-1.5615E 00	
1.1200E-02	-1.9606E 00	
1.2000E-02	-2.1827E 00	
1.2800E-02	-2.2078E 00	
1.3600E-02	-2.0336E 00	
1.4400E-02	-1.6758E 00	
1.5200E-02	-1.1667E 00	
1.6000E-02	-5.5229E-01	
1.6800E-02	1.1196E-01	
1.7600E-02	7.6610E-01	
1.8400E-02	1.3511E 00	
1.9200E-02	1.8141E 00	
2.0000E-02	2.1133E 00	
2.0800E-02	2.2217E 00	
2.1600E-02	2.1296E 00	
2.2400E-02	1.8452E 00	
2.3200E-02	1.3943E 00	
2.4000E-02	8.1743E-01	
2.4800E-02	1.6679E-01	
2.5600E-02	-4.9891E-01	
2.6400E-02	-1.1196E 00	
2.7200E-02	-1.6392E 00	
2.8000E-02	-2.0108E 00	
2.8800E-02	-2.2009E 00	
2.9600E-02	-2.1923E 00	
3.0400E-02	-1.9859E 00	
3.1200E-02	-1.6001E 00	
3.2000E-02	-1.0699E 00	
3.2800E-02	-4.4314E-01	
3.3600E-02	2.2363E-01	
3.4400E-02	8.7022E-01	
3.5200E-02	1.4382E 00	
3.6000E-02	1.8764E 00	
3.6800E-02	2.1452E 00	
3.7600E-02	2.2203E 00	
3.8400E-02	2.0950E 00	
3.9200E-02	1.7805E 00	
4.0000E-02	1.3053E 00	
4.0800E-02	7.1228E-01	
4.1600E-02	5.4935E-02	
4.2400E-02	-6.0738E-01	
4.3200E-02	-1.2149E 00	
4.4000E-02	-1.7127E 00	
4.4800E-02	-2.0559E 00	
4.5600E-02	-2.2135E 00	
4.6400E-02	-2.1713E 00	
4.7200E-02	-1.9331E 00	
4.8000E-02	-1.5204E 00	
4.8800E-02	-9.7046E-01	
4.9600E-02	-3.3290E-01	
5.0400E-02	3.3471E-01	
5.1200E-02	9.7210E-01	
5.2000E-02	1.5217E 00	
5.2800E-02	1.9340E 00	
5.3600E-02	2.1716E 00	
5.4400E-02	2.2132E 00	
5.5200E-02	2.0550E 00	
5.6000E-02	1.7113E 00	
5.6800E-02	1.2131E 00	
5.7600E-02	6.0533E-01	
5.8400E-02	-5.7060E-02	
5.9200E-02	-7.1430E-01	
6.0000E-02	-1.3071E 00	
6.0800E-02	-1.7818E 00	
6.1600E-02	-2.0958E 00	
6.2400E-02	-2.2205E 00	
6.3200E-02	-2.1448E 00	
6.4000E-02	-1.8755E 00	
6.4800E-02	-1.4369E 00	
6.5600E-02	-8.6858E-01	
6.6400E-02	-2.2184E-01	
6.7200E-02	4.4487E-01	
6.8000E-02	1.0715E 00	
6.8800E-02	1.6013E 00	
6.9600E-02	1.9866E 00	
7.0400E-02	2.1925E 00	
7.1200E-02	2.2005E 00	
7.2000E-02	2.0099E 00	
7.2800E-02	1.6377E 00	
7.3600E-02	1.1178E 00	
7.4400E-02	4.9692E-01	
7.5200E-02	-1.6884E-01	
7.6000E-02	-8.1936E-01	
7.6800E-02	-1.3959E 00	
7.7600E-02	-1.8464E 00	
7.8400E-02	-2.1303E 00	
7.9200E-02	-2.221AE 00	
8.0000E-02	-2.112AE 00	

Figure 22. Unsaturated secondary current.

```
                    MINIMUM              IM      VERSUS TIME          MAXIMUM
                  -1.8279E 00                                       2.8015E 00
 TIME          IM                I                                        I
0.0            0.0               ----------+
8.0000E-04     6.9425E-01        ----------+
1.6000E-03     1.3698E 00        ----------+
2.4000E-03     1.9657E 00        ----------+
3.2000E-03     2.4282E 00        ----------+
4.0000E-03     2.7154E 00        ----------+
4.8000E-03     2.8015E 00        ----------+
5.6000E-03     2.6787E 00        ----------+
6.4000E-03     2.3581E 00        ----------+
7.2000E-03     1.8686E 00        ----------+
8.0000E-03     1.2544E 00        ----------+
8.8000E-03     5.7091E-01        ----------+
9.6000E-03    -1.2009E-01        ----------+
1.0400E-02    -7.5625E-01        ----------+
1.1200E-02    -1.2802E 00        ----------+
1.2000E-02    -1.6445E 00        ----------+
1.2800E-02    -1.8164E 00        ----------+
1.3600E-02    -1.7803E 00        ----------+
1.4400E-02    -1.5395E 00        ----------+
1.5200E-02    -1.1158E 00        ----------+
1.6000E-02    -5.4730E-01        ----------+
1.6800E-02     1.1457E-01        ----------+
1.7600E-02     8.1009E-01        ----------+
1.8400E-02     1.4765E 00        ----------+
1.9200E-02     2.0535E 00        ----------+
2.0000E-02     2.4892E 00        ----------+
2.0800E-02     2.7442E 00        ----------+
2.1600E-02     2.7954E 00        ----------+
2.2400E-02     2.6382E 00        ----------+
2.3200E-02     2.2869E 00        ----------+
2.4000E-02     1.7731E 00        ----------+
2.4800E-02     1.1432E 00        ----------+
2.5600E-02     4.5415E-01        ----------+
2.6400E-02    -2.3196E-01        ----------+
2.7200E-02    -8.5315E-01        ----------+
2.8000E-02    -1.3533E 00        ----------+
2.8800E-02    -1.6874E 00        ----------+
2.9600E-02    -1.8251E 00        ----------+
3.0400E-02    -1.7540E 00        ----------+
3.1200E-02    -1.4807E 00        ----------+
3.2000E-02    -1.0297E 00        ----------+
3.2800E-02    -4.4172E-01        ----------+
3.3600E-02     2.3008E-01        ----------+
3.4400E-02     9.2510E-01        ----------+
3.5200E-02     1.5806E 00        ----------+
3.6000E-02     2.1374E 00        ----------+
3.6800E-02     2.5452E 00        ----------+
3.7600E-02     2.7672E 00        ----------+
3.8400E-02     2.7834E 00        ----------+
3.9200E-02     2.5923E 00        ----------+
4.0000E-02     2.2112E 00        ----------+
4.0800E-02     1.6744E 00        ----------+
4.1600E-02     1.0305E 00        ----------+
4.2400E-02     3.3747E-01        ----------+
4.3200E-02    -3.4201E-01        ----------+
4.4000E-02    -9.4664E-01        ----------+
4.4800E-02    -1.4218E 00        ----------+
4.5600E-02    -1.7247E 00        ----------+
4.6400E-02    -1.8279E 00        ----------+
4.7200E-02    -1.7221E 00        ----------+
4.8000E-02    -1.4169E 00        ----------+
4.8800E-02    -9.3974E-01        ----------+
4.9600E-02    -3.3382E-01        ----------+
5.0400E-02     3.4621E-01        ----------+
5.1200E-02     1.0390E 00        ----------+
5.2000E-02     1.6819E 00        ----------+
5.2800E-02     2.2170E 00        ----------+
5.3600E-02     2.5959E 00        ----------+
5.4400E-02     2.7844E 00        ----------+
5.5200E-02     2.7655E 00        ----------+
5.6000E-02     2.5410E 00        ----------+
5.6800E-02     2.1310E 00        ----------+
5.7600E-02     1.5727E 00        ----------+
5.8400E-02     9.1628E-01        ----------+
5.9200E-02     2.2115E-01        ----------+
6.0000E-02    -4.4998E-01        ----------+
6.0800E-02    -1.0365E 00        ----------+
6.1600E-02    -1.4855E 00        ----------+
6.2400E-02    -1.7564E 00        ----------+
6.3200E-02    -1.8249E 00        ----------+
6.4000E-02    -1.6846E 00        ----------+
6.4800E-02    -1.3483E 00        ----------+
6.5600E-02    -8.4626E-01        ----------+
6.6400E-02    -2.2389E-01        ----------+
6.7200E-02     4.6260E-01        ----------+
6.8000E-02     1.1513E 00        ----------+
6.8800E-02     1.7801E 00        ----------+
6.9600E-02     2.2921E 00        ----------+
7.0400E-02     2.6412E 00        ----------+
7.1200E-02     2.7957E 00        ----------+
7.2000E-02     2.7419E 00        ----------+
7.2800E-02     2.4845E 00        ----------+
7.3600E-02     2.0467E 00        ----------+
7.4400E-02     1.4682E 00        ----------+
7.5200E-02     8.0106E-01        ----------+
7.6000E-02     1.0556E-01        ----------+
7.6800E-02    -5.5552E-01        ----------+
7.7600E-02    -1.1224E 00        ----------+
7.8400E-02    -1.5441E 00        ----------+
7.9200E-02    -1.7025E 00        ----------+
8.0000E-02    -1.8159E 00        ----------+
```

Figure 23. Unsaturated secondary magnetizing current.

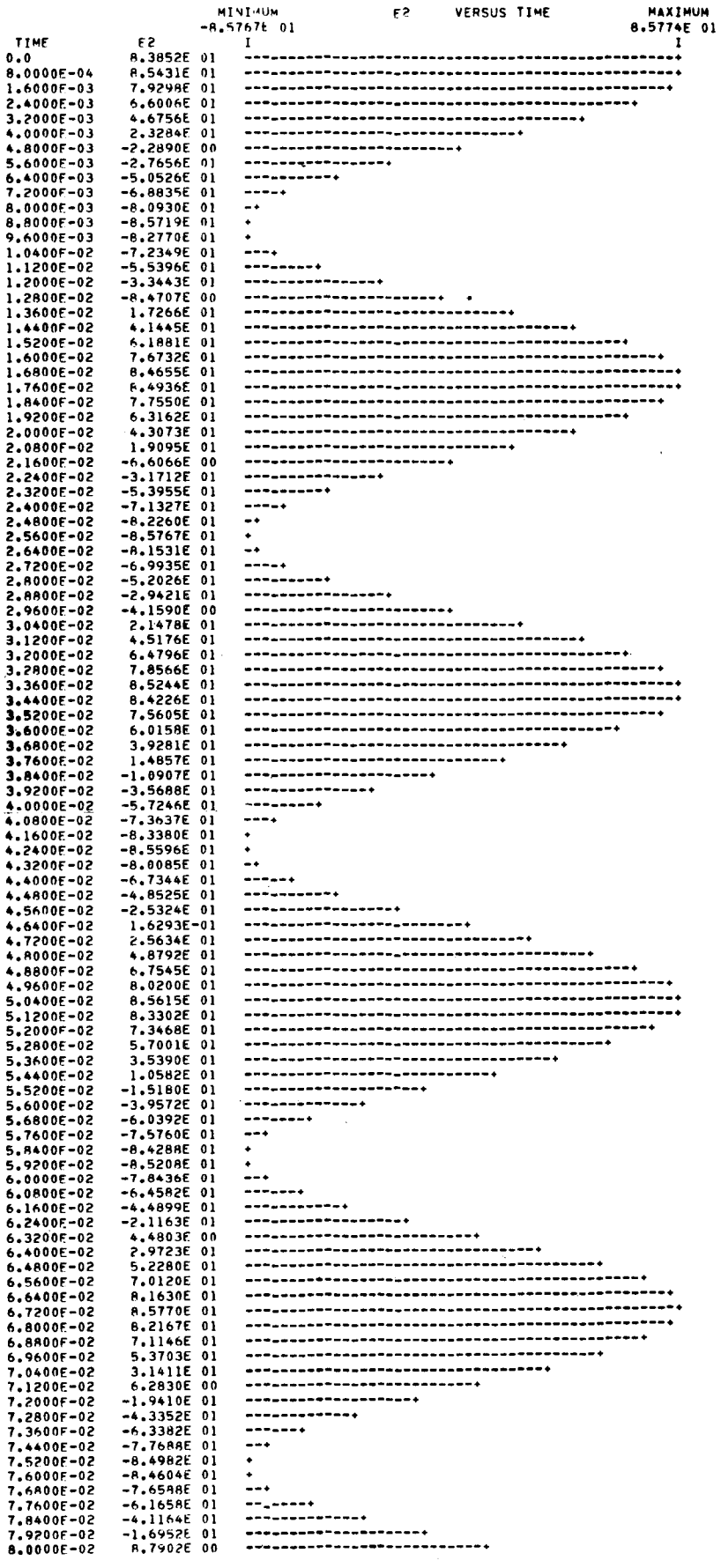


Figure 24. Unsaturated secondary voltage

Therefore,

$$I_2 = \frac{N_1}{N_2} I_1 .$$

For the CT in the last example,

$$\frac{N_1}{N_2} = \frac{1}{9000} = 1.111 \times 10^{-4} .$$

Therefore, for the applied 20,000 ωt input amperes, I_2 should be 2.222 amperes. From Figure 22, the computer result is 2.222 amperes. The computer result is the same as the analytical one.

The CT was then overburdened with R_B = 992 ohms, L_B = 0.5 henry and an abnormally high primary current of 80,000 sin ωt amperes was applied. As a result, the CT was saturated as indicated by the output of Figures 25, 26, 27 and 28. It can be observed that the saturated flux is flat-topped. Also because of the dc component in the flux, the saturating flux starts at a higher value and then goes to steady state when the dc component decays to zero. The secondary current and voltage are more distorted at the top. The magnetizing secondary current experiences the worst distortion as shown in Figure 27. The magnetizing secondary current also has a high dc component which decays to zero.

Thus, given a primary current due to a fault, it is possible to predict a CT output and performance with the aid of a digital computer.

```
                    MINIMUM              P       VERSUS TIME          MAXIMUM
                  -1.6826E-03                                      4.9141E-03
 TIME               P          I                                        I
0.0                 0.0        ----------+
8.0000E-04          2.6615E-04 ----------+                                    
1.6000E-03          7.4429E-04 ----------+                                    
2.4000E-03          1.3883E-03 ----------+                                    
3.2000E-03          2.1273E-03 ----------+                                    
4.0000E-03          2.8945E-03 ----------+                                    
4.8000E-03          3.6185E-03 ----------+                                    
5.6000E-03          4.2318E-03 ----------+                                    
6.4000E-03          4.6772E-03 ----------+                                    
7.2000E-03          4.9123E-03 ----------+                                    
8.0000E-03          4.9141E-03 ----------+                                    
8.8000E-03          4.6803E-03 ----------+                                    
9.6000E-03          4.2303E-03 ----------+                                    
1.0400E-02          3.6028E-03 ----------+                                    
1.1200E-02          2.8526E-03 ----------+                                    
1.2000E-02          2.0457E-03 ----------+                                    
1.2800E-02          1.2532E-03 ----------+                                    
1.3600E-02          5.3294E-04 ----------+                                    
1.4400E-02         -5.0987E-05 ----------+                                    
1.5200E-02         -4.4531E-04 ----------+                                    
1.6000E-02         -6.1459E-04 ----------+                                    
1.6800E-02         -5.4370E-04 ----------+                                    
1.7600E-02         -2.3917E-04 ----------+                                    
1.8400E-02          2.7133E-04 ----------+                                    
1.9200E-02          9.4159E-04 ----------+                                    
2.0000E-02          1.7024E-03 ----------+                                    
2.0800E-02          2.4773E-03 ----------+                                    
2.1600E-02          3.1970E-03 ----------+                                    
2.2400E-02          3.7956E-03 ----------+                                    
2.3200E-02          4.2177E-03 ----------+                                    
2.4000E-02          4.4241E-03 ----------+                                    
2.4800E-02          4.3952E-03 ----------+                                    
2.5600E-02          4.1323E-03 ----------+                                    
2.6400E-02          3.6583E-03 ----------+                                    
2.7200E-02          3.0147E-03 ----------+                                    
2.8000E-02          2.2587E-03 ----------+                                    
2.8800E-02          1.4576E-03 ----------+                                    
2.9600E-02          6.7410E-04 ----------+                                    
3.0400E-02         -2.7794E-05 ----------+                                    
3.1200E-02         -5.8285E-04 ----------+                                    
3.2000E-02         -9.4109E-04 ----------+                                    
3.2800E-02         -1.0703E-03 ----------+                                    
3.3600E-02         -9.5886E-04 ----------+                                    
3.4400E-02         -6.1702E-04 ----------+                                    
3.5200E-02         -7.5724E-05 ----------+                                    
3.6000E-02          6.1605E-04 ----------+                                    
3.6800E-02          1.3932E-03 ----------+                                    
3.7600E-02          2.1717E-03 ----------+                                    
3.8400E-02          2.8833E-03 ----------+                                    
3.9200E-02          3.4635E-03 ----------+                                    
4.0000E-02          3.8593E-03 ----------+                                    
4.0800E-02          4.0346E-03 ----------+                                    
4.1600E-02          3.9731E-03 ----------+                                    
4.2400E-02          3.6800E-03 ----------+                                    
4.3200E-02          3.1812E-03 ----------+                                    
4.4000E-02          2.5214E-03 ----------+                                    
4.4800E-02          1.7598E-03 ----------+                                    
4.5600E-02          9.6183E-04 ----------+                                    
4.6400E-02          1.8622E-04 ----------+                                    
4.7200E-02         -4.9454E-04 ----------+                                    
4.8000E-02         -1.0191E-03 ----------+                                    
4.8800E-02         -1.3375E-03 ----------+                                    
4.9600E-02         -1.4213E-03 ----------+                                    
5.0400E-02         -1.2679E-03 ----------+                                    
5.1200E-02         -8.8997E-04 ----------+                                    
5.2000E-02         -3.1892E-04 ----------+                                    
5.2800E-02          3.9295E-04 ----------+                                    
5.3600E-02          1.1814E-03 ----------+                                    
5.4400E-02          1.9600E-03 ----------+                                    
5.5200E-02          2.6602E-03 ----------+                                    
5.6000E-02          3.2190E-03 ----------+                                    
5.6800E-02          3.5860E-03 ----------+                                    
5.7600E-02          3.7282E-03 ----------+                                    
5.8400E-02          3.6329E-03 ----------+                                    
5.9200E-02          3.3088E-03 ----------+                                    
6.0000E-02          2.7852E-03 ----------+                                    
6.0800E-02          2.1094E-03 ----------+                                    
6.1600E-02          1.3427E-03 ----------+                                    
6.2400E-02          5.4311E-04 ----------+                                    
6.3200E-02         -2.2106E-04 ----------+                                    
6.4000E-02         -8.7922E-04 ----------+                                    
6.4800E-02         -1.3705E-03 ----------+                                    
6.5600E-02         -1.6442E-03 ----------+                                    
6.6400E-02         -1.6826E-03 ----------+                                    
6.7200E-02         -1.4853E-03 ----------+                                    
6.8000E-02         -1.0713E-03 ----------+                                    
6.8800E-02         -4.7163E-04 ----------+                                    
6.9600E-02          2.5887E-04 ----------+                                    
7.0400E-02          1.0543E-03 ----------+                                    
7.1200E-02          1.8324E-03 ----------+                                    
7.2000E-02          2.5184E-03 ----------+                                    
7.2800E-02          3.0535E-03 ----------+                                    
7.3600E-02          3.3898E-03 ----------+                                    
7.4400E-02          3.4977E-03 ----------+                                    
7.5200E-02          3.3680E-03 ----------+                                    
7.6000E-02          3.0128E-03 ----------+                                    
7.6800E-02          2.4648E-03 ----------+                                    
7.7600E-02          1.7741E-03 ----------+                                    
7.8400E-02          1.0003E-03 ----------+                                    
7.9200E-02          2.0136E-04 ----------+                                    
8.0000E-02         -5.4871E-04 ----------+                                    
```

Figure 25. Saturated flux.

```
                    MINIMUM              I2      VERSUS TIME          MAXIMUM
                   -8.9189E 00                                      8.8480E 00
 TIME          I2          I                                            I
0.0           0.0          +
8.0000E-04    2.6377E 00   +----------
1.6000E-03    5.0349E 00   +--------------------
2.4000E-03    6.9359E 00   +----------------------------
3.2000E-03    8.1014E 00   +--------------------------------
4.0000E-03    8.5114E 00   +----------------------------------
4.8000E-03    8.1293E 00   +--------------------------------
5.6000E-03    6.9901E 00   +----------------------------
6.4000E-03    5.1970E 00   +--------------------
7.2000E-03    2.9122E 00   +-----------
8.0000E-03    3.4251E-01   +-
8.8000E-03   -2.2798E 00   +---------
9.6000E-03   -4.7176E 00   +------------------
1.0400E-02   -6.7505E 00   +--------------------------
1.1200E-02   -8.1944E 00   +--------------------------------
1.2000E-02   -8.9189E 00   +
1.2800E-02   -8.8580E 00   +
1.3600E-02   -8.1385E 00   +--------------------------------
1.4400E-02   -6.7012E 00   +--------------------------
1.5200E-02   -4.6607E 00   +------------------
1.6000E-02   -2.2011E 00   +---------
1.6800E-02    4.5543E-01   +-
1.7600E-02    3.0692E 00   +-----------
1.8400E-02    5.4042E 00   +---------------------
1.9200E-02    7.2496E 00   +----------------------------
2.0000E-02    8.3387E 00   +--------------------------------
2.0800E-02    8.6149E 00   +---------------------------------
2.1600E-02    8.0999E 00   +--------------------------------
2.2400E-02    6.8405E 00   +--------------------------
2.3200E-02    4.9506E 00   +-------------------
2.4000E-02    2.6018E 00   +----------
2.4800E-02    4.0998E-03   +-
2.5600E-02   -2.6055E 00   +----------
2.6400E-02   -4.9920E 00   +-------------------
2.7200E-02   -6.9397E 00   +--------------------------
2.8000E-02   -8.2726E 00   +--------------------------------
2.8800E-02   -8.8702E 00   +
2.9600E-02   -8.7748E 00   +
3.0400E-02   -7.9417E 00   +--------------------------------
3.1200E-02   -6.3930E 00   +------------------------
3.2000E-02   -4.2684E 00   +----------------
3.2800E-02   -1.7597E 00   +-
3.3600E-02    9.0658E-01   +-----------
3.4400E-02    3.4898E 00   +-------------
3.5200E-02    5.7566E 00   +---------------------
3.6000E-02    7.5025E 00   +----------------------------
3.6800E-02    8.5308E 00   +--------------------------------
3.7600E-02    8.6729E 00   +---------------------------------
3.8400E-02    8.0268E 00   +--------------------------------
3.9200E-02    6.6508E 00   +-------------------------
4.0000E-02    4.6693E 00   +-----------------
4.0800E-02    2.2613E 00   +---------
4.1600E-02   -3.5590E-01   +-
4.2400E-02   -2.9458E 00   +----------
4.3200E-02   -5.2746E 00   +--------------------
4.4000E-02   -7.1318E 00   +----------------------------
4.4800E-02   -8.3499E 00   +--------------------------------
4.5600E-02   -8.8583E 00   +
4.6400E-02   -8.6817E 00   +
4.7200E-02   -7.7219E 00   +--------------------------------
4.8000E-02   -6.0655E 00   +------------------------
4.8800E-02   -3.8317E 00   +----------
4.9600E-02   -1.2641E 00   +-
5.0400E-02    1.3754E 00   +-----------
5.1200E-02    3.9052E 00   +---------------------
5.2000E-02    6.0982E 00   +----------------------------
5.2800E-02    7.7401E 00   +--------------------------------
5.3600E-02    8.6830E 00   +---------------------------------
5.4400E-02    8.6910E 00   +---------------------------------
5.5200E-02    7.9157E 00   +--------------------------------
5.6000E-02    6.4270E 00   +------------------------
5.6800E-02    4.3592E 00   +-----------------
5.7600E-02    1.8991E 00   +-------------
5.8400E-02   -7.3132E-01   +-
5.9200E-02   -3.2946E 00   +---------
6.0000E-02   -5.5594E 00   +-------------------
6.0800E-02   -7.3213E 00   +----------------------------
6.1600E-02   -8.4212E 00   +--------------------------------
6.2400E-02   -8.8802E 00   +
6.3200E-02   -8.5696E 00   +
6.4000E-02   -7.4857E 00   +--------------------------------
6.4800E-02   -5.6834E 00   +----------------------
6.5600E-02   -3.3542E 00   +---------
6.6400E-02   -7.5915E-01   +-
6.7200E-02    1.8678E 00   +-----------
6.8000E-02    4.3143E 00   +-----------------
6.8800E-02    6.4278E 00   +----------------------------
6.9600E-02    7.9617E 00   +--------------------------------
7.0400E-02    8.7775E 00   +---------------------------------
7.1200E-02    8.6760E 00   +---------------------------------
7.2000E-02    7.7737E 00   +--------------------------------
7.2800E-02    6.1763E 00   +------------------------
7.3600E-02    4.0277E 00   +----------------
7.4400E-02    1.5221E 00   +-----------
7.5200E-02   -1.1148E 00   +-
7.6000E-02   -3.6449E 00   +---------
7.6800E-02   -5.8399E 00   +----------------------
7.7600E-02   -7.5016E 00   +----------------------------
7.8400E-02   -8.5157E 00   +--------------------------------
7.9200E-02   -8.8739E 00   +
8.0000E-02   -8.4302E 00   +--------------------------------
```

Figure 26. Saturated secondary current.

	MINIMUM	IM VERSUS TIME	MAXIMUM
	-1.0296E 03		6.9481E 03
TIME	IM	I	I
0.0	0.0	-------+	
8.0000E-04	2.4373E 01	-------+	
1.6000E-03	6.8159E 01	-------+	
2.4000E-03	4.9070E 02	----------+	
3.2000E-03	1.8442E 03	------------------+	
4.0000E-03	3.2493E 03	----------------------------+	
4.8000E-03	4.5753E 03	----------------------------------+	
5.6000E-03	5.6986E 03	--+	
6.4000E-03	6.5143E 03	--+	
7.2000E-03	6.9449E 03	--+	
8.0000E-03	6.9481E 03	--+	
8.8000E-03	6.5200E 03	--+	
9.6000E-03	5.6958E 03	--+	
1.0400E-02	4.5466E 03	----------------------------------+	
1.1200E-02	3.1726E 03	------------------------+	
1.2000E-02	1.6948E 03	----------+	
1.2800E-02	2.4320E 02	-------+	
1.3600E-02	4.8804E 01	-------+	
1.4400E-02	-4.6691E 00	-------+	
1.5200E-02	-4.0780E 01	-------+	
1.6000E-02	-5.6281E 01	-------+	
1.6800E-02	-4.9789E 01	-------+	
1.7600E-02	-2.1902E 01	-------+	
1.8400E-02	2.4847E 01	-------+	
1.9200E-02	8.6226E 01	-------+	
2.0000E-02	1.0660E 03	----------+	
2.0800E-02	2.4851E 03	----------------------+	
2.1600E-02	3.8034E 03	----------------------------+	
2.2400E-02	4.8996E 03	----------------------------------+	
2.3200E-02	5.6727E 03	--+	
2.4000E-02	6.0508E 03	--+	
2.4800E-02	5.9978E 03	--+	
2.5600E-02	5.5164E 03	--+	
2.6400E-02	4.6481E 03	----------------------------------+	
2.7200E-02	3.4695E 03	------------------------+	
2.8000E-02	2.0849E 03	------------------+	
2.8800E-02	6.1754E 02	----------+	
2.9600E-02	6.1731E 01	-------+	
3.0400E-02	-2.5452E 00	-------+	
3.1200E-02	-5.3375E 01	-------+	
3.2000E-02	-8.6180E 01	------+	
3.2800E-02	-9.8010E 01	------+	
3.3600E-02	-8.7808E 01	------+	
3.4400E-02	-5.6504E 01	-------+	
3.5200E-02	-6.9345E 00	-------+	
3.6000E-02	5.6415E 01	-------+	
3.6800E-02	4.9961E 02	----------+	
3.7600E-02	1.9254E 03	----------------------+	
3.8400E-02	3.2288E 03	----------------------------+	
3.9200E-02	4.2914E 03	----------------------------------+	
4.0000E-02	5.0163E 03	--+	
4.0800E-02	5.3374E 03	--+	
4.1600E-02	5.2248E 03	--+	
4.2400E-02	4.6879E 03	----------------------------------+	
4.3200E-02	3.7744E 03	----------------------------+	
4.4000E-02	2.5660E 03	----------------------+	
4.4800E-02	1.1710E 03	----------+	
4.5600E-02	8.8080E 01	-------+	
4.6400E-02	1.7053E 01	-------+	
4.7200E-02	-4.5288E 01	-------+	
4.8000E-02	-9.3321E 01	------+	
4.8800E-02	-3.9766E 02	---+	
4.9600E-02	-5.5119E 02	--+	
5.0400E-02	-2.7023E 02	----+	
5.1200E-02	-8.1499E 01	------+	
5.2000E-02	-2.9206E 01	-------+	
5.2800E-02	3.5984E 01	-------+	
5.3600E-02	1.1180E 02	--------+	
5.4400E-02	1.5377E 03	----------+	
5.5200E-02	2.8201E 03	----------------------+	
5.6000E-02	3.8435E 03	----------------------------+	
5.6800E-02	4.5157E 03	----------------------------------+	
5.7600E-02	4.7762E 03	----------------------------------+	
5.8400E-02	4.6017E 03	----------------------------------+	
5.9200E-02	4.0080E 03	----------------------------------+	
6.0000E-02	3.0490E 03	------------------------+	
6.0800E-02	1.8114E 03	----------+	
6.1600E-02	4.0714E 02	----------+	
6.2400E-02	4.9736E 01	-------+	
6.3200E-02	-2.0243E 01	-------+	
6.4000E-02	-8.0514E 01	------+	
6.4800E-02	-4.5803E 02	---+	
6.5600E-02	-9.5944E 02	+	
6.6400E-02	-1.0296E 03	+	
6.7200E-02	-6.6825E 02	--+	
6.8000E-02	-9.8105E 01	------+	
6.8800E-02	-4.3190E 01	-------+	
6.9600E-02	2.3706E 01	-------+	
7.0400E-02	9.6549E 01	--------+	
7.1200E-02	1.3040E 03	----------+	
7.2000E-02	2.5605E 03	----------------------+	
7.2800E-02	3.5404E 03	----------------------------+	
7.3600E-02	4.1565E 03	----------------------------------+	
7.4400E-02	4.3541E 03	----------------------------------+	
7.5200E-02	4.1165E 03	----------------------------------+	
7.6000E-02	3.4660E 03	----------------------------+	
7.6800E-02	2.4623E 03	----------------------+	
7.7600E-02	1.1972E 03	----------+	
7.8400E-02	9.1599E 01	--------+	
7.9200E-02	1.8440E 01	-------+	
8.0000E-02	-5.0248E 01	-------+	

Figure 27. Saturated secondary magnetizing current.

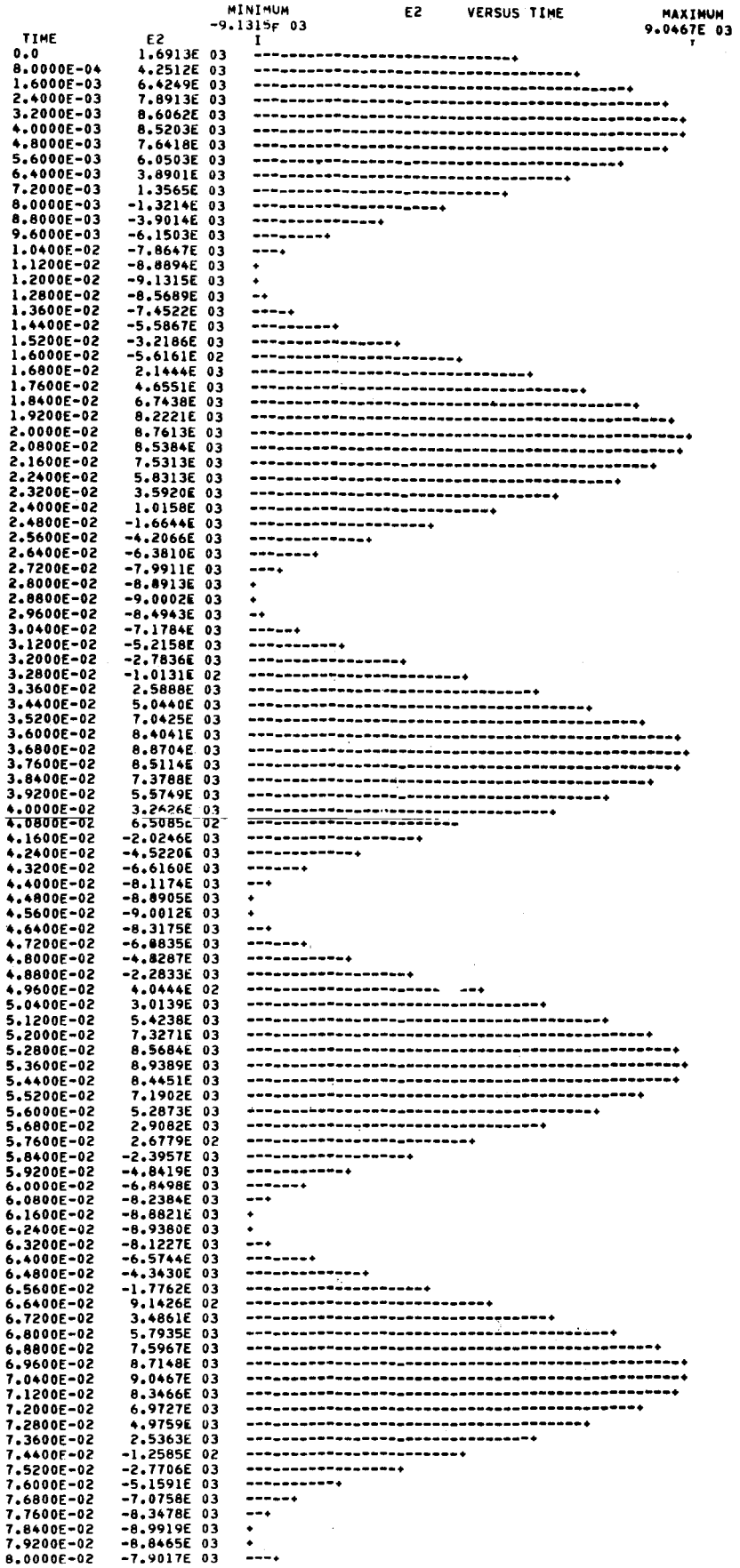


Figure 28. Saturated secondary voltage.

5.7. Experimental Procedures and Results

In order to verify what was mentioned on a theoretical basis in the previous chapters, some tests were conducted in a laboratory using a second CT. The procedures and results are outlined below.

Determination of CT characteristic curve. A simple method to determine a CT characteristic curve is shown in Figure 29. In this case, the primary was excited using a variable transformer for the source voltage. Measurements of the RMS exciting voltage and RMS exciting current were made. The results were plotted as indicated in Figure 30. From Figure 30, the saturating exciting RMS current was found to be 0.8 ampere.

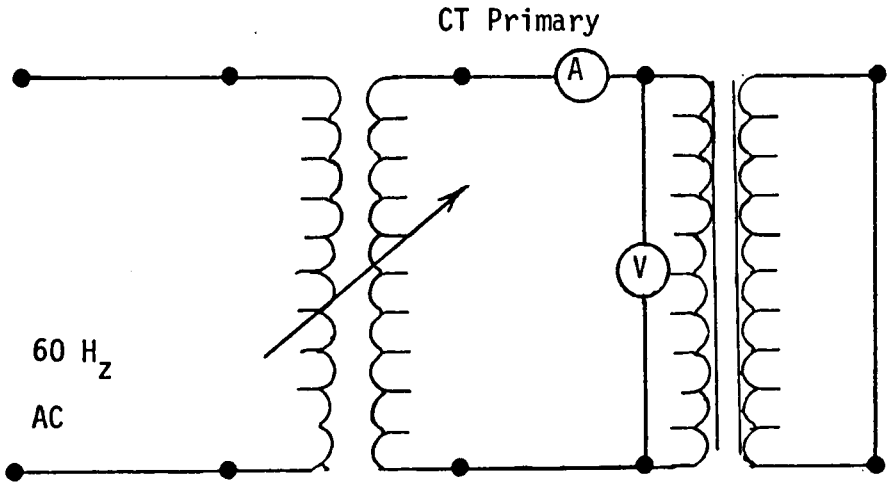


Figure 29. A simple circuit to determine CT characteristic curve.

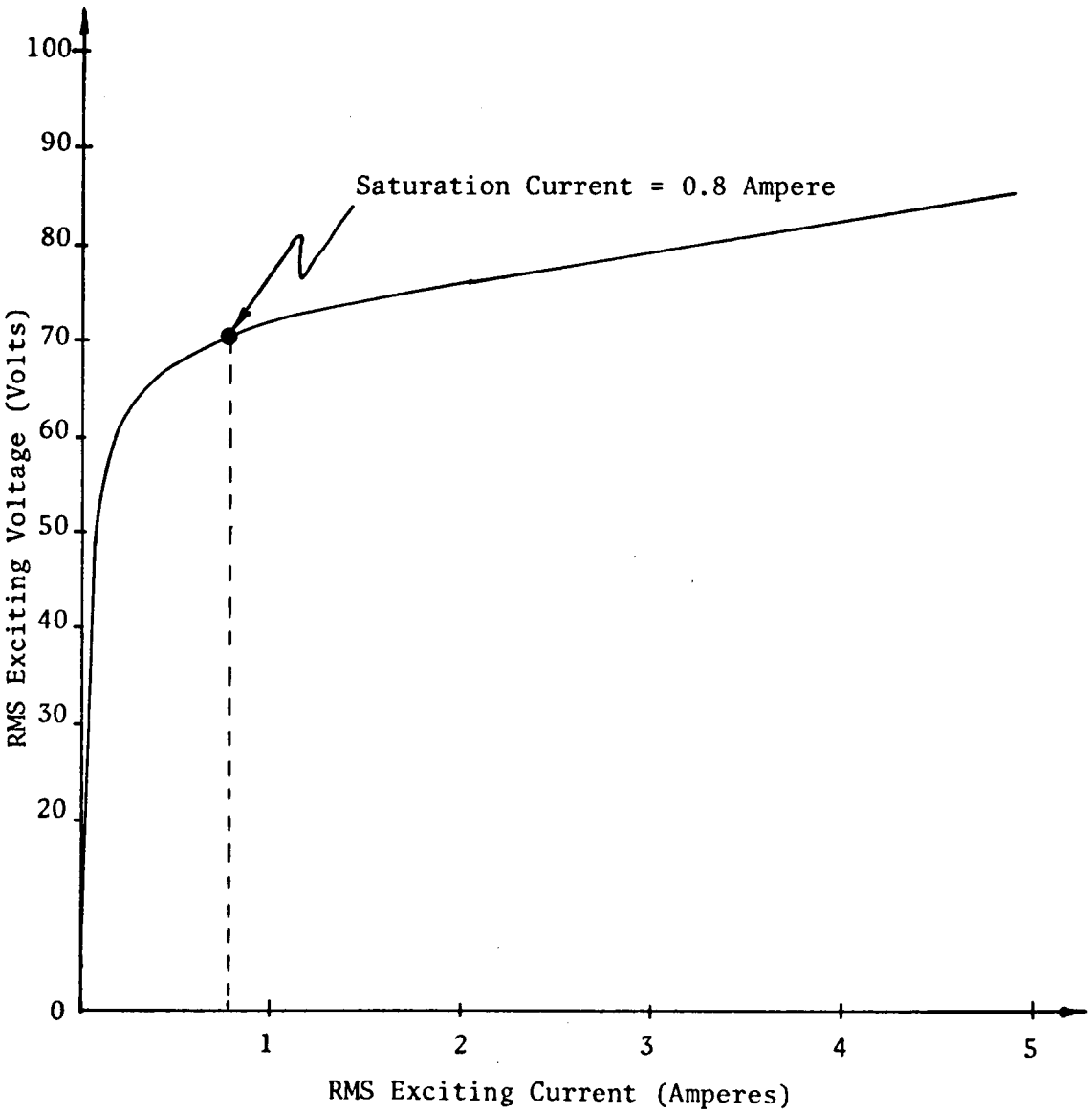


Figure 30. Experimental CT characteristic curve.

Measurement of residual flux with half wave a.c. A method [12] that was more convenient for measuring residual flux is shown in Figure 31. The method involved exciting the CT secondary with half and full wave $60H_z$ voltages. The core flux alternates between residual and maximum values. The half wave was obtained by using a diode as a rectifier. The flux excursion was then calculated from the induction equations (5.8) and (5.9)

The per unit residual flux for half wave excitation can be calculated from (5.8) and (5.9) as follows:

$$\phi_o = \frac{B_r}{B_m} = \frac{E_f - 2E_h}{E_f} = \frac{16.8 - 9}{16.8} = \frac{7.8}{16.8} = 0.464 \text{ WEBERS} .$$

Available p.u. flux excursion (increase) from residual maximum is

$$\phi_\Delta = \frac{B_m - B_r}{B_m} = \frac{2\ E_h}{E_f} = \frac{9}{16.8} = 0.535 \text{ WEBERS} .$$

It can be noticed that the flux increase from minimum to maximum values is approximately equal to the residual flux.

$$\text{FULL WAVE } B_m = \frac{E_f}{4Nf_eA} \tag{5.8}$$

$$\text{HALF WAVE } B_m - B_r = \frac{E_h}{8Nf_eA} . \tag{5.9}$$

Therefore

$$B_r = \frac{E_f}{4Nf_eA} - \frac{E_h}{8Nf_eA} = \frac{2E_f - E_h}{8Nf_eA} .$$

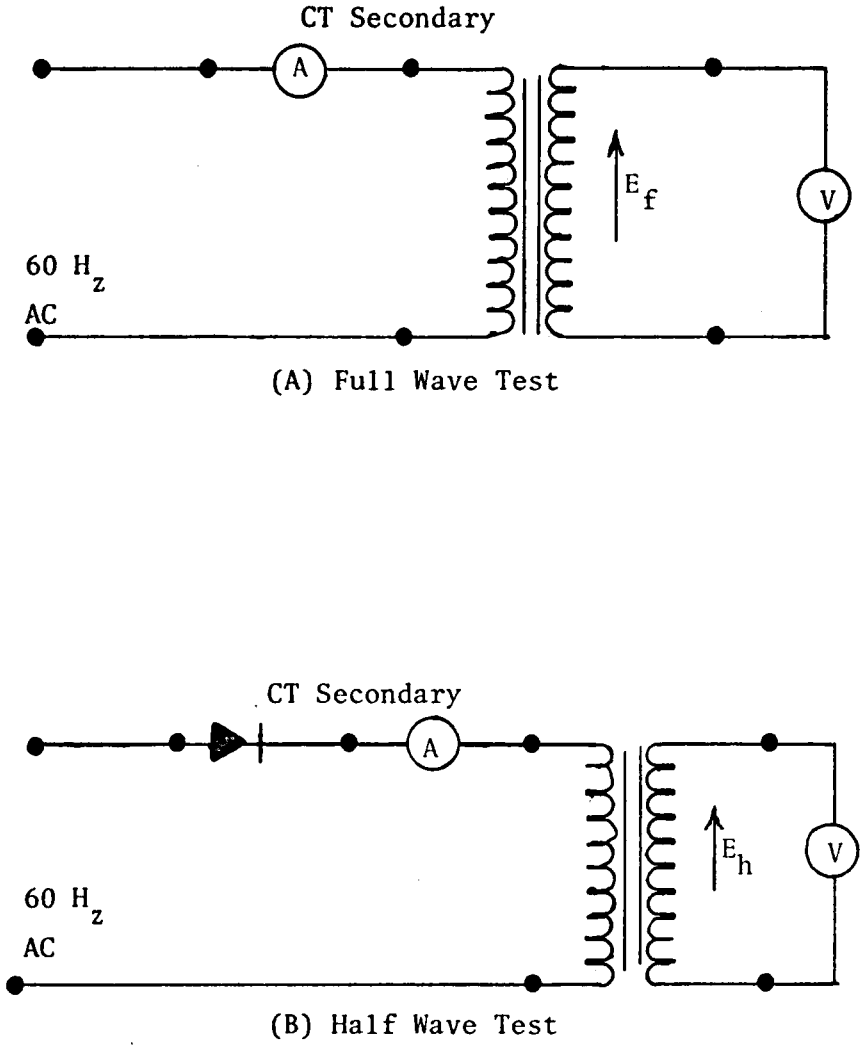


Figure 31. Tests for measuring residual core flux.

B_m = peak induction full wave or half wave in teslas,

B_r = half wave residual induction in teslas,

E_f = full wave a.c. induced average volts = 16.8,

E_h = half wave induced average volts = 4.5,

f_e = excitation frequency in hertz = 60.

Effect of d.c. offset on current transformer performance. The primary impedance $Z_1 = R_1 + j\omega L_1$ was set at different values, while the secondary burden Z_B was left constant at 0.06 ohm. Refer to Figure 32. Switch S was suddenly closed and the CT output was recorded as shown in Figures 33, 34 and 35. As observed in the above resulting output waveforms, the secondary current was offset for several cycles due to the d.c. component in the primary circuit, depending on the time constant $\tau = \frac{L_1}{R_1}$. As shown in Figure 33, the time constant is zero; hence there is no d.c. offset and the output waveform is normal. As the time constant was increased, the d.c. transient also increased. Refer to Figures 34 and 35. For example, in Figure 35, the secondary current was virtually suppressed for seven cycles. The output waveform was normalized after the disappearance of the d.c. component.

A computer program was run with no secondary inductance. The offset primary current was $i_1 = 34.08\ \{\sin(\omega t + \theta) - \sin\theta\varepsilon^{-\tau t}$ amperes. Setting $\tau = 0.0665$ seconds and $\theta = -120°$, the result was obtained as shown in Figure 36. It was found that the secondary inductance has a great influence on the flux d.c. transient. With no secondary

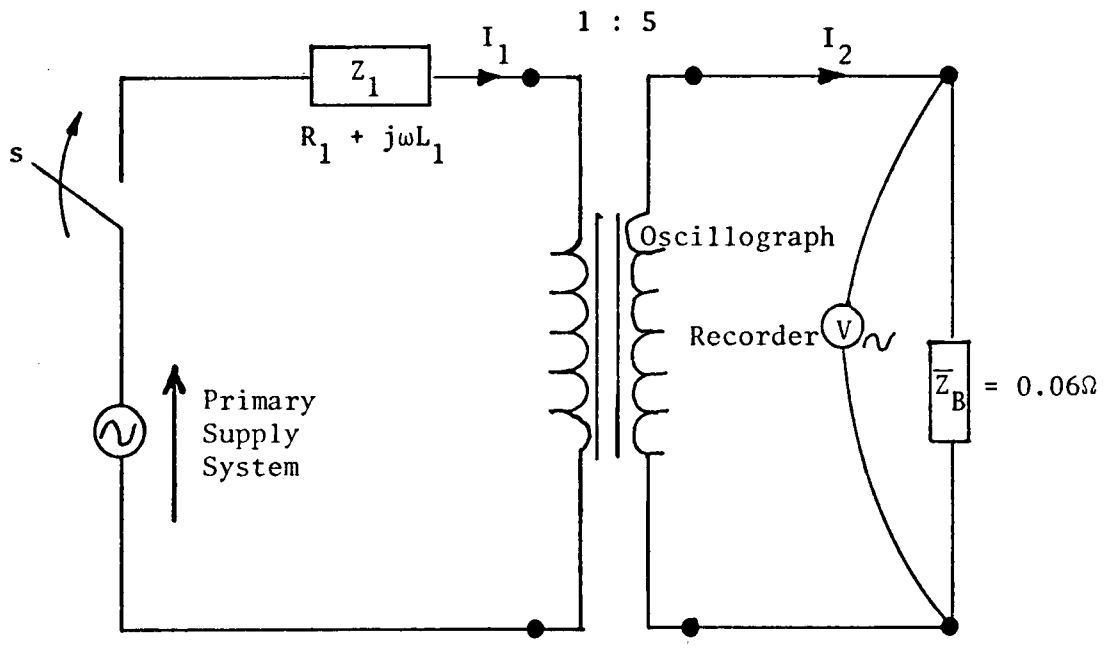


Figure 32. Burdened primary and secondary of a CT.

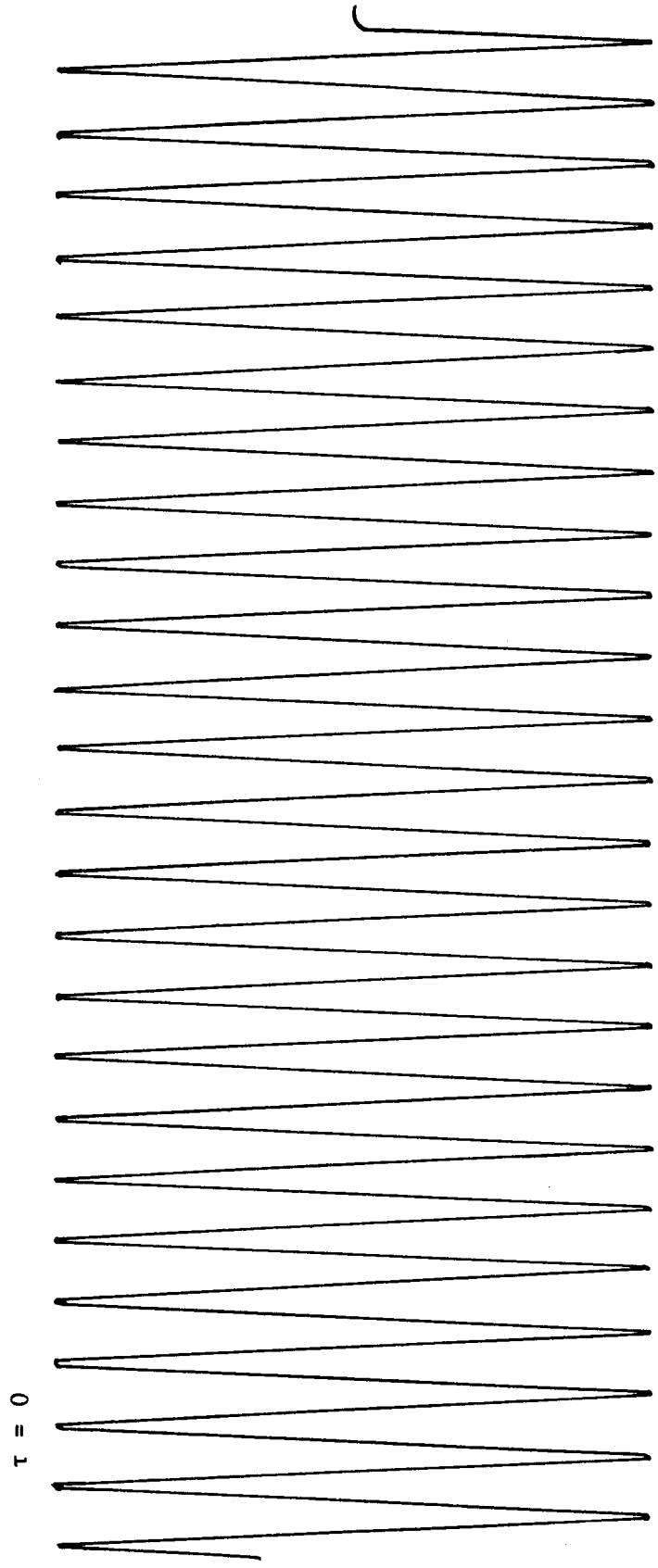


Figure 33. Normal secondary voltage and current waveform.

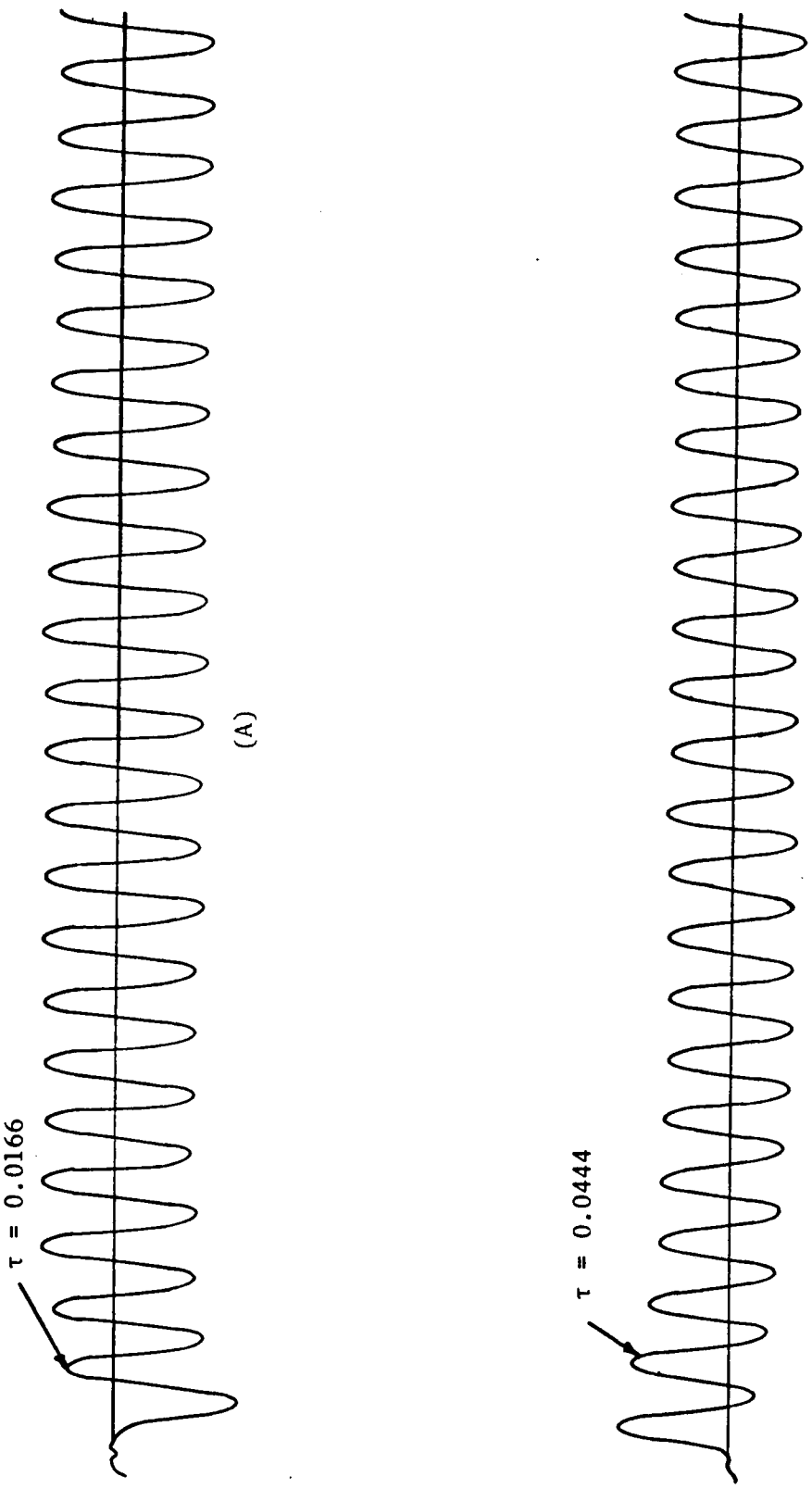


Figure 34. Secondary voltage and current waveforms due to D.C. offset (τ = 0.0166, 0.0444).

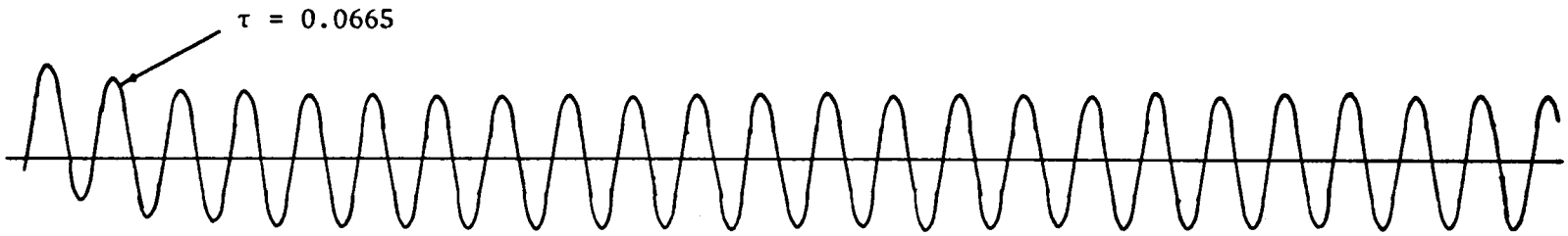


Figure 35. Secondary voltage and current waveforms due to D.C. offset (τ = 0.0665).

MINIMUM P VERSUS TIME MAXIMUM
0.0 3.4696E-03

TIME	P	I
0.0	0.0	+
1.6000E-03	3.4791E-05	+
3.2000E-03	1.4436E-04	--+
4.8000E-03	3.1593E-04	----+
6.4000E-03	5.1422E-04	------+
8.0000E-03	6.9394E-04	----------+
9.6000E-03	8.1574E-04	------------+
1.1200E-02	8.6015E-04	-------------+
1.2800E-02	8.3444E-04	-------------+
1.4400E-02	7.7011E-04	------------+
1.6000E-02	7.1172E-04	----------+
1.7600E-02	7.0124E-04	----------+
1.9200E-02	7.6321E-04	----------+
2.0800E-02	8.9610E-04	-------------+
2.2400E-02	1.0729E-03	----------------+
2.4000E-02	1.2505E-03	-------------------+
2.5600E-02	1.3852E-03	---------------------+
2.7200E-02	1.4479E-03	----------------------+
2.8800E-02	1.4346E-03	----------------------+
3.0400E-02	1.3674E-03	---------------------+
3.2000E-02	1.2874E-03	-------------------+
3.3600E-02	1.2394E-03	------------------+
3.5200E-02	1.2568E-03	-------------------+
3.6800E-02	1.3495E-03	--------------------+
3.8400E-02	1.5003E-03	----------------------+
4.0000E-02	1.6711E-03	------------------------+
4.1600E-02	1.8167E-03	--------------------------+
4.3200E-02	1.9000E-03	---------------------------+
4.4800E-02	1.9058E-03	---------------------------+
4.6400E-02	1.8459E-03	--------------------------+
4.8000E-02	1.7549E-03	-------------------------+
4.9600E-02	1.6781E-03	------------------------+
5.1200E-02	1.6554E-03	------------------------+
5.2800E-02	1.7074E-03	------------------------+
5.4400E-02	1.8279E-03	--------------------------+
5.6000E-02	1.9864E-03	----------------------------+
5.7600E-02	2.1385E-03	------------------------------+
5.9200E-02	2.2420E-03	--------------------------------+
6.0800E-02	2.2715E-03	--------------------------------+
6.2400E-02	2.2272E-03	--------------------------------+
6.4000E-02	2.1356E-03	------------------------------+
6.5600E-02	2.0391E-03	----------------------------+
6.7200E-02	1.9820E-03	---------------------------+
6.8800E-02	1.9942E-03	----------------------------+
7.0400E-02	2.0810E-03	----------------------------+
7.2000E-02	2.2211E-03	------------------------------+
7.3600E-02	2.3743E-03	--------------------------------+
7.5200E-02	2.4953E-03	----------------------------------+
7.6800E-02	2.5502E-03	----------------------------------+
7.8400E-02	2.5280E-03	----------------------------------+
8.0000E-02	2.4449E-03	--------------------------------+
8.1600E-02	2.3384E-03	-------------------------------+
8.3200E-02	2.2538E-03	------------------------------+
8.4800E-02	2.2288E-03	------------------------------+
8.6400E-02	2.2796E-03	------------------------------+
8.8000E-02	2.3958E-03	--------------------------------+
8.9600E-02	2.5434E-03	----------------------------------+
9.1200E-02	2.6773E-03	------------------------------------+
9.2800E-02	2.7571E-03	-------------------------------------+
9.4400E-02	2.7612E-03	-------------------------------------+
9.6000E-02	2.6948E-03	------------------------------------+
9.7600E-02	2.5875E-03	----------------------------------+
9.9200E-02	2.4834E-03	--------------------------------+
1.0080E-01	2.4253E-03	-------------------------------+
1.0240E-01	2.4396E-03	--------------------------------+
1.0400E-01	2.5269E-03	---------------------------------+
1.0560E-01	2.6621E-03	-----------------------------------+
1.0720E-01	2.8028E-03	-------------------------------------+
1.0880E-01	2.9048E-03	---------------------------------------+
1.1040E-01	2.9373E-03	--+
1.1200E-01	2.8938E-03	---------------------------------------+
1.1360E-01	2.7946E-03	--+
1.1520E-01	2.6797E-03	----------------------------------+
1.1680E-01	2.5942E-03	---------------------------------+
1.1840E-01	2.5729E-03	---------------------------------+
1.2000E-01	2.6279E-03	----------------------------------+
1.2160E-01	2.7440E-03	------------------------------------+
1.2320E-01	2.8847E-03	--------------------------------------+
1.2480E-01	3.0043E-03	--+
1.2640E-01	3.0647E-03	---+
1.2800E-01	3.0486E-03	--+
1.2960E-01	2.9655E-03	---------------------------------------+
1.3120E-01	2.8486E-03	--------------------------------------+
1.3280E-01	2.7426E-03	-------------------------------------+
1.3440E-01	2.6887E-03	------------------------------------+
1.3600E-01	2.7094E-03	-------------------------------------+
1.3760E-01	2.8007E-03	------------------------------------+
1.3920E-01	2.9338E-03	--------------------------------------+
1.4080E-01	3.0649E-03	--+
1.4240E-01	3.1510E-03	---+
1.4400E-01	3.1646E-03	---+
1.4560E-01	3.1041E-03	--+
1.4720E-01	2.9937E-03	--------------------------------------+
1.4880E-01	2.8752E-03	-------------------------------------+
1.5040E-01	2.7933E-03	------------------------------------+
1.5200E-01	2.7796E-03	------------------------------------+
1.5360E-01	2.8415E-03	-------------------------------------+
1.5520E-01	2.9598E-03	---------------------------------------+
1.5680E-01	3.0953E-03	---+
1.5840E-01	3.2026E-03	--+
1.6000E-01	3.2462E-03	--+
1.6160E-01	3.2131E-03	--+

Figure 36. Effect of D.C. offset on flux with no secondary inductance.

inductance, the flux increases gradually to a steady state, but the d.c. component does not decay to zero. Furthermore, with no secondary inductance, the secondary current rises to a maximum value and decreases until it reaches steady state. Refer to Figure 37. The secondary current d.c. component decays to zero.

When a very small inductance is added to the secondary, the d.c. flux starts to decay upwards as indicated in Figure 38. The secondary current rises to a maximum and decreases to a steady state. Refer to Figure 39.

When a 0.012 ohm inductance is further added to the secondary, the result reverses in flux waveform. The flux now rises to a maximum and decreases to a steady state. The d.c. flux decays to zero. Refer to Figure 40. The secondary current behaves as in the previous cases and is indicated in Figure 41.

In regards to d.c. transient, we can conclude that with no or very little inductance in the secondary, the d.c. flux will not decay to zero. When adequate inductance is added, the d.c. flux decays to zero. The secondary current d.c. component will generally decay to zero in every case.

CT saturation test. A simple circuit as shown in Figure 42 was connected to run a saturation test on the second CT. A burden of 2.7 + j 0.0265 ohms was first connected to the secondary and primary current of 6.9 amperes was applied. Unsaturated waveform of the secondary voltage and current was observed on oscilloscope and recorded. Refer to Table 2 and Figure 43.

MINIMUM -6.8360E 00 I2 VERSUS TIME MAXIMUM 1.0240E 01

TIME	I2	I
0.0	0.0	
1.6000E-03	3.7524E 00	
3.2000E-03	7.5457E 00	
4.8000E-03	1.0009E 01	
6.4000E-03	1.0240E 01	
8.0000E-03	8.1269E 00	
9.6000E-03	4.3852E 00	
1.1200E-02	3.0590E-01	
1.2800E-02	-2.7002E 00	
1.4400E-02	-3.6003E 00	
1.6000E-02	-2.1041E 00	
1.7600E-02	1.2334E 00	
1.9200E-02	5.2079E 00	
2.0800E-02	8.3911E 00	
2.2400E-02	9.6345E 00	
2.4000E-02	8.4748E 00	
2.5600E-02	5.2974E 00	
2.7200E-02	1.2007E 00	
2.8800E-02	-2.3923E 00	
3.0400E-02	-4.2354E 00	
3.2000E-02	-3.6999E 00	
3.3600E-02	-9.9588E-01	
3.5200E-02	2.9017E 00	
3.6800E-02	6.5969E 00	
3.8400E-02	8.7660E 00	
4.0000E-02	8.6241E 00	
4.1600E-02	6.2026E 00	
4.3200E-02	2.3378E 00	
4.4800E-02	-1.6240E 00	
4.6400E-02	-4.3020E 00	
4.8000E-02	-4.7681E 00	
4.9600E-02	-2.8742E 00	
5.1200E-02	6.9478E-01	
5.2800E-02	4.6635E 00	
5.4400E-02	7.6156E 00	
5.6000E-02	8.4941E 00	
5.7600E-02	6.9742E 00	
5.9200E-02	3.5780E 00	
6.0800E-02	-5.1004E-01	
6.2400E-02	-3.8606E 00	
6.4000E-02	-5.3046E 00	
6.5600E-02	-4.3455E 00	
6.7200E-02	-1.3345E 00	
6.8800E-02	2.6532E 00	
7.0400E-02	6.1980E 00	
7.2000E-02	8.0368E 00	
7.3600E-02	7.5090E 00	
7.5200E-02	4.7898E 00	
7.6800E-02	8.2748E-01	
7.8400E-02	-2.9899E 00	
8.0000E-02	-5.3257E 00	
8.1600E-02	-5.3658E 00	
8.3200E-02	-3.1059E 00	
8.4800E-02	6.4626E-01	
8.6400E-02	4.5571E 00	
8.8000E-02	7.2367E 00	
8.9600E-02	7.7302E 00	
9.1200E-02	5.8548E 00	
9.2800E-02	2.2633E 00	
9.4400E-02	-1.7849E 00	
9.6000E-02	-4.8693E 00	
9.7600E-02	-5.9095E 00	
9.9200E-02	-4.5460E 00	
1.0080E-01	-1.2681E 00	
1.0240E-01	2.7601E 00	
1.0400E-01	6.1092E 00	
1.0560E-01	7.5901E 00	
1.0720E-01	6.6730E 00	
1.0880E-01	3.6749E 00	
1.1040E-01	-3.5282E-01	
1.1200E-01	-3.9950E 00	
1.1360E-01	-5.9726E 00	
1.1520E-01	-5.5938E 00	
1.1680E-01	-2.9983E 00	
1.1840E-01	8.9177E-01	
1.2000E-01	4.6978E 00	
1.2160E-01	7.0706E 00	
1.2320E-01	7.1673E 00	
1.2480E-01	4.9485E 00	
1.2640E-01	1.1918E 00	
1.2800E-01	-2.7818E 00	
1.2960E-01	-5.5748E 00	
1.3120E-01	-6.2065E 00	
1.3280E-01	-4.4585E 00	
1.3440E-01	-9.5259E-01	
1.3600E-01	3.0695E 00	
1.3760E-01	6.1834E 00	
1.3920E-01	7.2860E 00	
1.4080E-01	5.9837E 00	
1.4240E-01	2.7323E 00	
1.4400E-01	-1.3251E 00	
1.4560E-01	-4.7599E 00	
1.4720E-01	-6.3639E 00	
1.4880E-01	-5.5748E 00	
1.5040E-01	-2.6744E 00	
1.5200E-01	1.3096E 00	
1.5360E-01	4.9678E 00	
1.5520E-01	7.0056E 00	
1.5680E-01	6.7003E 00	
1.5840E-01	4.1567E 00	
1.6000E-01	2.6897E-01	
1.6160E-01	-3.5934E 00	

Figure 37. Effect of D.C. offset on secondary current with no secondary inductance.

```
                    MINIMUM              P       VERSUS TIME          MAXIMUM
                   -9.3801E-05                                      4.2313E-03
 TIME          P          I                                             I
 0.0           0.0        -+
 8.0000E-04    4.3482E-04 ------+
 1.6000E-03    9.3396E-04 -----------+
 2.4000E-03    1.4547E-03 ----------------+
 3.2000E-03    1.9524E-03 ----------------------+
 4.0000E-03    2.3844E-03 --------------------------+
 4.8000E-03    2.7139E-03 -----------------------------+
 5.6000E-03    2.9135E-03 -------------------------------+
 6.4000E-03    2.9673E-03 --------------------------------+
 7.2000E-03    2.8727E-03 -------------------------------+
 8.0000E-03    2.6404E-03 ----------------------------+
 8.8000E-03    2.2935E-03 -------------------------+
 9.6000E-03    1.8654E-03 --------------------+
 1.0400E-02    1.3968E-03 ---------------+
 1.1200E-02    9.3217E-04 ----------+
 1.2000E-02    5.1539E-04 -------+
 1.2800E-02    1.8612E-04 ---+
 1.3600E-02   -2.3915E-05 +
 1.4400E-02   -9.3801E-05 +
 1.5200E-02   -1.5293E-05 +
 1.6000E-02    2.0644E-04 ---+
 1.6800E-02    5.5327E-04 -------+
 1.7600E-02    9.9575E-04 ------------+
 1.8400E-02    1.4958E-03 ----------------+
 1.9200E-02    2.0101E-03 ------------------------+
 2.0000E-02    2.4940E-03 ----------------------------+
 2.0800E-02    2.9056E-03 -------------------------------+
 2.1600E-02    3.2095E-03 ----------------------------------+
 2.2400E-02    3.3800E-03 ------------------------------------+
 2.3200E-02    3.4035E-03 ----------------------------------+
 2.4000E-02    3.2795E-03 --------------------------------+
 2.4800E-02    3.020AE-03 -----------------------------+
 2.5600E-02    2.6526E-03 -------------------------+
 2.6400E-02    2.2096E-03 ---------------------+
 2.7200E-02    1.7335E-03 ----------------+
 2.8000E-02    1.2688E-03 ------------+
 2.8800E-02    8.5915E-04 ----------+
 2.9600E-02    5.4298E-04 -------+
 3.0400E-02    3.5040E-04 -----+
 3.1200E-02    3.0032E-04 ----+
 3.2000E-02    3.9877E-04 -----+
 3.2800E-02    6.3834E-04 --------+
 3.3600E-02    9.9887E-04 ------------+
 3.4400E-02    1.4493E-03 ----------------+
 3.5200E-02    1.9503E-03 ------------------+
 3.6000E-02    2.4581E-03 ----------------------+
 3.6800E-02    2.9283E-03 --------------------------+
 3.7600E-02    3.3198E-03 ------------------------------+
 3.8400E-02    3.5987E-03 --------------------------------+
 3.9200E-02    3.7410E-03 ----------------------------------+
 4.0000E-02    3.7353E-03 ----------------------------------+
 4.0800E-02    3.5834E-03 --------------------------------+
 4.1600E-02    3.3003E-03 ------------------------------+
 4.2400E-02    2.9129E-03 --------------------------+
 4.3200E-02    2.4573E-03 ----------------------+
 4.4000E-02    1.9761E-03 ------------------+
 4.4800E-02    1.5138E-03 --------------+
 4.5600E-02    1.1134E-03 ----------+
 4.6400E-02    8.1222E-04 --------+
 4.7200E-02    6.3871E-04 --------+
 4.8000E-02    6.0969E-04 --------+
 4.8800E-02    7.2893E-04 ---------+
 4.9600E-02    9.8681E-04 ------------+
 5.0400E-02    1.3612E-03 ----------------+
 5.1200E-02    1.8194E-03 ------------------+
 5.2000E-02    2.3212E-03 ----------------------+
 5.2800E-02    2.8224E-03 --------------------------+
 5.3600E-02    3.2787E-03 ------------------------------+
 5.4400E-02    3.6501E-03 ----------------------------------+
 5.5200E-02    3.9042E-03 ------------------------------------+
 5.6000E-02    4.0189E-03 -------------------------------------+
 5.6800E-02    3.9850E-03 -------------------------------------+
 5.7600E-02    3.8066E-03 ------------------------------------+
 5.8400E-02    3.5007E-03 ----------------------------------+
 5.9200E-02    3.0961E-03 ------------------------------+
 6.0000E-02    2.6301E-03 --------------------------+
 6.0800E-02    2.1458E-03 ----------------------+
 6.1600E-02    1.6879E-03 ----------------+
 6.2400E-02    1.2987E-03 ------------+
 6.3200E-02    1.0143E-03 ----------+
 6.4000E-02    8.6115E-04 ----------+
 6.4800E-02    8.5410E-04 ----------+
 6.5600E-02    9.9467E-04 ------------+
 6.6400E-02    1.2711E-03 ----------------+
 6.7200E-02    1.6592E-03 ------------------+
 6.8000E-02    2.1249E-03 ----------------------+
 6.8800E-02    2.6270E-03 --------------------------+
 6.9600E-02    3.1210E-03 ----------------------------------+
 7.0400E-02    3.5631E-03 ------------------------------------+
 7.1200E-02    3.9143E-03 --------------------------------------+
 7.2000E-02    4.1437E-03 ----------------------------------------+
 7.2800E-02    4.2313E-03 -----------------------------------------+
 7.3600E-02    4.1701E-03 ----------------------------------------+
 7.4400E-02    3.9664E-03 --------------------------------------+
 7.5200E-02    3.6393E-03 ------------------------------------+
 7.6000E-02    3.2191E-03 ----------------------------------+
 7.6800E-02    2.7446E-03 --------------------------+
 7.7600E-02    2.2593E-03 ------------------------+
 7.8400E-02    1.8077E-03 ----------------------+
 7.9200E-02    1.4314E-03 ----------------+
 8.0000E-02    1.1650E-03 ---------------+
```

Figure 38. Effect of D.C. offset on flux with little secondary inductance.

```
                    MINIMUM          I2      VERSUS TIME          MAXIMUM
                   -5.3177E 00                                   1.0415E 01
TIME                    I                                          I
0.0          0.0        ----------------+
8.0000E-04    1.7771E 00 ----------------------+
1.6000E-03    3.7468E 00 ----------------------------+
2.4000E-03    5.7268E 00 ------------------------------------+
3.2000E-03    7.5344E 00 ----------------------------------------+
4.0000E-03    9.0021E 00 ------------------------------------------+
4.8000E-03    9.9935E 00 --------------------------------------------+
5.6000E-03    1.0415E 01 ----------------------------------------------+
6.4000E-03    1.0224E 01 ----------------------------------------------+
7.2000E-03    9.4352E 00 ------------------------------------------+
8.0000E-03    8.1147E 00 ----------------------------------------+
8.8000E-03    6.3782E 00 ------------------------------------+
9.6000E-03    4.3787E 00 ----------------------------+
1.0400E-02    2.2929E 00 ------------------+
1.1200E-02    3.0546E-01 ----------------+
1.2000E-02   -1.4079E 00 ----------+
1.2800E-02   -2.6961E 00 --------+
1.3600E-02   -3.4465E 00 -----+
1.4400E-02   -3.5949E 00 -----+
1.5200E-02   -3.1312E 00 -------+
1.6000E-02   -2.1009E 00 ----------+
1.6800E-02   -6.0037E-01 ------------+
1.7600E-02    1.2315E 00 ----------------+
1.8400E-02    3.2261E 00 ----------------------+
1.9200E-02    5.2001E 00 ------------------------------+
2.0000E-02    6.9719E 00 ----------------------------------------+
2.0800E-02    8.3785E 00 ------------------------------------------------+
2.1600E-02    9.2896E 00 --------------------------------------------------+
2.2400E-02    9.6200E 00 ----------------------------------------------------+
2.3200E-02    9.3366E 00 --------------------------------------------------+
2.4000E-02    8.4620E 00 ------------------------------------------+
2.4800E-02    7.0721E 00 ----------------------------------+
2.5600E-02    5.2894E 00 ------------------------------+
2.6400E-02    3.2720E 00 ----------------------+
2.7200E-02    1.1989E 00 ----------------+
2.8000E-02   -7.4555E-01 ------------+
2.8800E-02   -2.3886E 00 ----------+
2.9600E-02   -3.5849E 00 ------+
3.0400E-02   -4.2290E 00 ---+
3.1200E-02   -4.2657E 00 ---+
3.2000E-02   -3.6943E 00 -----+
3.2800E-02   -2.5691E 00 --------+
3.3600E-02   -9.9437E-01 ------------+
3.4400E-02    8.8521E-01 ------------------+
3.5200E-02    2.8973E 00 ----------------------+
3.6000E-02    4.8577E 00 ------------------------------+
3.6800E-02    6.5870E 00 ----------------------------------+
3.7600E-02    7.9265E 00 ------------------------------------------+
3.8400E-02    8.7528E 00 --------------------------------------------+
3.9200E-02    8.9889E 00 ----------------------------------------------+
4.0000E-02    8.6111E 00 --------------------------------------------+
4.0800E-02    7.6511E 00 ----------------------------------------+
4.1600E-02    6.1933E 00 ------------------------------------+
4.2400E-02    4.3668E 00 ----------------------------+
4.3200E-02    2.3343E 00 ----------------------+
4.4000E-02    2.7706E-01 ----------------+
4.4800E-02   -1.6215E 00 ----------+
4.5600E-02   -3.1922E 00 -------+
4.6400E-02   -4.2955E 00 ---+
4.7200E-02   -4.8339E 00 --+
4.8000E-02   -4.7608E 00 --+
4.8800E-02   -4.0852E 00 ---+
4.9600E-02   -2.8699E 00 --------+
5.0400E-02   -1.2268E 00 ------------+
5.1200E-02    6.9375E-01 ----------------+
5.2000E-02    2.7164E 00 ----------------------+
5.2800E-02    4.6554E 00 ------------------------------+
5.3600E-02    6.3369E 00 ----------------------------------+
5.4400E-02    7.6041E 00 ------------------------------------------+
5.5200E-02    8.3418E 00 --------------------------------------------+
5.6000E-02    8.4813E 00 ----------------------------------------------+
5.6800E-02    8.0084E 00 --------------------------------------------+
5.7600E-02    6.9637E 00 ----------------------------------+
5.8400E-02    5.4398E 00 ------------------------------+
5.9200E-02    3.5726E 00 ----------------------+
6.0000E-02    1.5286E 00 ----------------+
6.0800E-02   -5.0925E-01 ------------+
6.1600E-02   -2.3588E 00 ----------+
6.2400E-02   -3.8547E 00 ----+
6.3200E-02   -4.8638E 00 --+
6.4000E-02   -5.2966E 00 +
6.4800E-02   -5.1157E 00 +
6.5600E-02   -4.3389E 00 ---+
6.6400E-02   -3.0382E 00 --------+
6.7200E-02   -1.3326E 00 ------------+
6.8000E-02    6.2262E-01 ----------------+
6.8800E-02    2.6492E 00 ----------------------+
6.9600E-02    4.5626E 00 ------------------------------+
7.0400E-02    6.1866E 00 ----------------------------------+
7.1200E-02    7.3789E 00 ------------------------------------------+
7.2000E-02    8.0246E 00 --------------------------------------------+
7.2800E-02    8.0660E 00 --------------------------------------------+
7.3600E-02    7.4977E 00 ------------------------------------------+
7.4400E-02    6.3698E 00 ----------------------------------+
7.5200E-02    4.7825E 00 ------------------------------+
7.6000E-02    2.8778E 00 ----------------------+
7.6800E-02    8.2621E-01 ----------------+
7.7600E-02   -1.1982E 00 ----------+
7.8400E-02   -2.9854E 00 --------+
7.9200E-02   -4.4041E 00 --+
8.0000E-02   -5.3177E 00 +
```

Figure 39. Effect of D.C. offset on secondary current with little secondary inductance.

```
                    MINIMUM                    P    VERSUS  TIME          MAXIMUM
                   -1.2477E-02                                          2.4717E-02
 TIME          P               I                                              I
0.0            0.0             +
1.6000E-03     8.9062E-03      +----------
3.2000E-03     1.7983E-02      +--------------------
4.8000E-03     2.3975E-02      +---------------------------
6.4000E-03     2.4717E-02      +----------------------------
8.0000E-03     1.9898E-02      +----------------------
9.6000E-03     1.1172E-02      +------------
1.1200E-02     1.5708E-03      +-----------------
1.2800E-02    -5.5619E-03      +---------
1.4400E-02    -7.7533E-03      +------
1.6000E-02    -4.2734E-03      +----------
1.7600E-02     3.6071E-03      +-----------------
1.9200E-02     1.3065E-02      +-------------------------
2.0800E-02     2.0722E-02      +-----------------------------
2.2400E-02     2.3836E-02      +-------------------------------
2.4000E-02     2.1269E-02      +-----------------------------
2.5600E-02     1.3890E-02      +-------------------------
2.7200E-02     4.2655E-03      +-----------------
2.8800E-02    -4.2424E-03      +----------
3.0400E-02    -8.6664E-03      +-----
3.2000E-02    -7.4792E-03      +-------
3.3600E-02    -1.1333E-03      +---------------
3.5200E-02     8.0989E-03      +---------------------
3.6800E-02     1.6927E-02      +-------------------------
3.8400E-02     2.2204E-02      +-----------------------------
4.0000E-02     2.2037E-02      +-----------------------------
4.1600E-02     1.6455E-02      +-------------------------
4.3200E-02     7.3995E-03      +--------------------
4.4800E-02    -1.9617E-03      +--------------
4.6400E-02    -8.3524E-03      +-------
4.8000E-02    -9.5439E-03      +----
4.9600E-02    -5.1420E-03      +----------
5.1200E-02     3.2739E-03      +-----------------
5.2800E-02     1.2708E-02      +-----------------------
5.4400E-02     1.9807E-02      +--------------------------
5.6000E-02     2.2040E-02      +----------------------------
5.7600E-02     1.8597E-02      +-------------------------
5.9200E-02     1.0669E-02      +-----------------------
6.0800E-02     1.0324E-03      +-----------------
6.2400E-02    -6.9330E-03      +---------
6.4000E-02    -1.0437E-02      +---
6.5600E-02    -8.2647E-03      +-------
6.7200E-02    -1.2023E-03      +---------------
6.8800E-02     8.2381E-03      +---------------------
7.0400E-02     1.6705E-02      +-----------------------
7.2000E-02     2.1190E-02      +--------------------------
7.3600E-02     2.0093E-02      +-------------------------
7.5200E-02     1.3783E-02      +-----------------------
7.6800E-02     4.4693E-03      +-------------------
7.8400E-02    -4.5780E-03      +-----------
8.0000E-02    -1.0182E-02      +---
8.1600E-02    -1.0382E-02      +---
8.3200E-02    -5.1224E-03      +----------
8.4800E-02     3.7242E-03      +-----------------
8.6400E-02     1.3021E-02      +-----------------------
8.8000E-02     1.9471E-02      +-------------------------
8.9600E-02     2.0783E-02      +--------------------------
9.1200E-02     1.6481E-02      +-----------------------
9.2800E-02     8.0679E-03      +---------------------
9.4400E-02    -1.4990E-03      +---------------
9.6000E-02    -8.8571E-03      +-----
9.7600E-02    -1.1422E-02      +--
9.9200E-02    -8.3010E-03      +-------
1.0080E-01    -6.0822E-04      +---------------
1.0240E-01     8.9297E-03      +---------------------
1.0400E-01     1.6934E-02      +-----------------------
1.0560E-01     2.0569E-02      +--------------------------
1.0720E-01     1.8539E-02      +-----------------------
1.0880E-01     1.1551E-02      +-----------------
1.1040E-01     2.0603E-03      +-----------------
1.1200E-01    -6.5939E-03      +---------
1.1360E-01    -1.1367E-02      +--
1.1520E-01    -1.0585E-02      +---
1.1680E-01    -4.5324E-03      +----------
1.1840E-01     4.6439E-03      +-----------------
1.2000E-01     1.3697E-02      +-----------------------
1.2160E-01     1.9421E-02      +-------------------------
1.2320E-01     1.9788E-02      +-------------------------
1.2480E-01     1.4640E-02      +-----------------------
1.2640E-01     5.8378E-03      +--------------------
1.2800E-01    -3.5727E-03      +-----------
1.2960E-01    -1.0258E-02      +---
1.3120E-01    -1.1867E-02      +-
1.3280E-01    -7.8385E-03      +--
1.3440E-01     3.9742E-04      +---------------
1.3600E-01     9.9271E-03      +---------------------
1.3760E-01     1.7380E-02      +-------------------------
1.3920E-01     2.0118E-02      +--------------------------
1.4080E-01     1.7168E-02      +-------------------------
1.4240E-01     9.5650E-03      +--------------------
1.4400E-01    -1.4289E-05      +---------------
1.4560E-01    -8.1950E-03      +-------
1.4720E-01    -1.2096E-02      +-
1.4880E-01    -1.0347E-02      +---
1.5040E-01    -3.5705E-03      +-----------
1.5200E-01     5.8354E-03      +--------------------
1.5360E-01     1.4546E-02      +-----------------------
1.5520E-01     1.9480E-02      +-------------------------
1.5680E-01     1.8892E-02      +-------------------------
1.5840E-01     1.2984E-02      +--------------------
1.6000E-01     3.8349E-03      +-----------------
1.6160E-01    -5.3296E-03      +----------
```

Figure 40. Effect of D.C. offset on flux with adequate secondary inductance.

MINIMUM -6.7341E 00 I2 VERSUS TIME MAXIMUM 1.0088E 01

TIME	I2	I
0.0	0.0	
1.6000E-03	3.6966E 00	
3.2000E-03	7.4335E 00	
4.8000E-03	9.8598E 00	
6.4000E-03	1.0088E 01	
8.0000E-03	8.0061E 00	
9.6000E-03	4.3201E 00	
1.1200E-02	3.0142E-01	
1.2800E-02	-2.6600E 00	
1.4400E-02	-3.5466E 00	
1.6000E-02	-2.0727E 00	
1.7600E-02	1.2151E 00	
1.9200E-02	5.1305E 00	
2.0800E-02	8.2664E 00	
2.2400E-02	9.4913E 00	
2.4000E-02	8.3489E 00	
2.5600E-02	5.2188E 00	
2.7200E-02	1.1830E 00	
2.8800E-02	-2.3566E 00	
3.0400E-02	-4.1723E 00	
3.2000E-02	-3.6448E 00	
3.3600E-02	-9.8096E-01	
3.5200E-02	2.8586E 00	
3.6800E-02	6.4990E 00	
3.8400E-02	8.6358E 00	
4.0000E-02	8.4960E 00	
4.1600E-02	6.1105E 00	
4.3200E-02	2.3032E 00	
4.4800E-02	-1.5997E 00	
4.6400E-02	-4.2379E 00	
4.8000E-02	-4.6970E 00	
4.9600E-02	-2.8314E 00	
5.1200E-02	6.8459E-01	
5.2800E-02	4.5943E 00	
5.4400E-02	7.5025E 00	
5.6000E-02	8.3680E 00	
5.7600E-02	6.8707E 00	
5.9200E-02	3.5250E 00	
6.0800E-02	-5.0225E-01	
6.2400E-02	-3.8030E 00	
6.4000E-02	-5.2255E 00	
6.5600E-02	-4.2807E 00	
6.7200E-02	-1.3145E 00	
6.8800E-02	2.6139E 00	
7.0400E-02	6.1060E 00	
7.2000E-02	7.9175E 00	
7.3600E-02	7.3976E 00	
7.5200E-02	4.7188E 00	
7.6800E-02	8.1540E-01	
7.8400E-02	-2.9452E 00	
8.0000E-02	-5.2463E 00	
8.1600E-02	-5.2858E 00	
8.3200E-02	-3.0595E 00	
8.4800E-02	6.3685E-01	
8.6400E-02	4.4895E 00	
8.8000E-02	7.1293E 00	
8.9600E-02	7.6155E 00	
9.1200E-02	5.7679E 00	
9.2800E-02	2.2299E 00	
9.4400E-02	-1.7581E 00	
9.6000E-02	-4.7966E 00	
9.7600E-02	-5.8214E 00	
9.9200E-02	-4.4782E 00	
1.0080E-01	-1.2490E 00	
1.0240E-01	2.7192E 00	
1.0400E-01	6.0186E 00	
1.0560E-01	7.4775E 00	
1.0720E-01	6.5740E 00	
1.0880E-01	3.6206E 00	
1.1040E-01	-3.4731E-01	
1.1200E-01	-3.9354E 00	
1.1360E-01	-5.8835E 00	
1.1520E-01	-5.5104E 00	
1.1680E-01	-2.9534E 00	
1.1840E-01	8.7474E-01	
1.2000E-01	4.6282E 00	
1.2160E-01	6.9657E 00	
1.2320E-01	7.0610E 00	
1.2480E-01	4.8752E 00	
1.2640E-01	1.1743E 00	
1.2800E-01	-2.7401E 00	
1.2960E-01	-5.4917E 00	
1.3120E-01	-6.1140E 00	
1.3280E-01	-4.3919E 00	
1.3440E-01	-9.3818E-01	
1.3600E-01	3.0241E 00	
1.3760E-01	6.0917E 00	
1.3920E-01	7.1780E 00	
1.4080E-01	5.8950E 00	
1.4240E-01	2.6919E 00	
1.4400E-01	-1.3051E 00	
1.4560E-01	-4.6889E 00	
1.4720E-01	-6.2690E 00	
1.4880E-01	-5.4916E 00	
1.5040E-01	-2.6344E 00	
1.5200E-01	1.2903E 00	
1.5360E-01	4.8942E 00	
1.5520E-01	6.9017E 00	
1.5680E-01	6.6009E 00	
1.5840E-01	4.0951E 00	
1.6000E-01	2.6525E-01	
1.6160E-01	-3.5397E 00	

Figure 41. Effect of D.C. offset on secondary current with adequate secondary inductance.

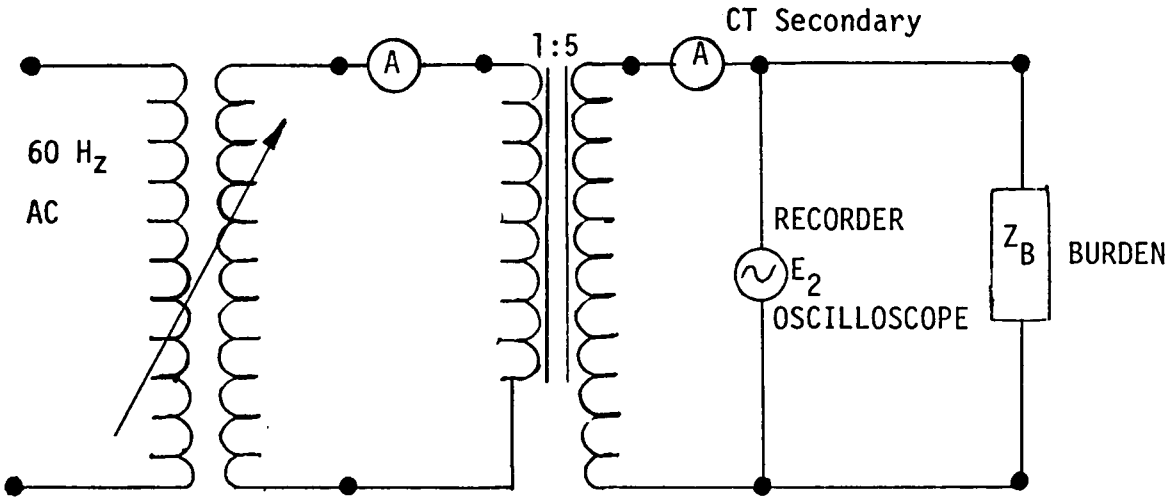


Figure 42. CT saturation test.

TABLE 2

UNSATURATED CONDITIONS

$Z_B = 2.7 + j0.0265$
$E_2 = 63$ Volts
$I_2 = 1.1$ Amperes
$I_1 = 6.9$ Amperes

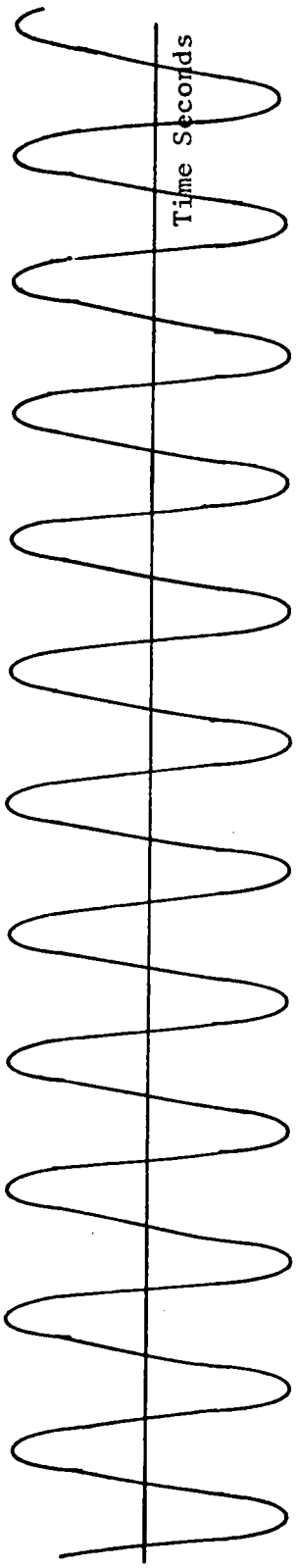


Figure 43. Unsaturated waveform of secondary voltage and current.

With the same burden of 2.7 + j 0.0265 ohms, the CT did not saturate when the primary current was less than 24 amperes as indicated in Table 3. As the burden was increased, less primary current was required to saturate the CT. Refer to Table 3 and Figures 44 and 45. The saturated waveforms are distorted at the top and bottom. These saturated waveforms are similar to those obtained by computer--Figures 26 and 27 (pages 51 and 52).

TABLE 3

SATURATED CONDITIONS

Z_B	I_1 (Amperes)	E_2 (Volts)	I_2 (Amperes)
2.7 + j0.0265	24.	110	2.
5.4 + j0.053	13.3	97	1.66
10.8 + j0.0795	10.8	92	1.5

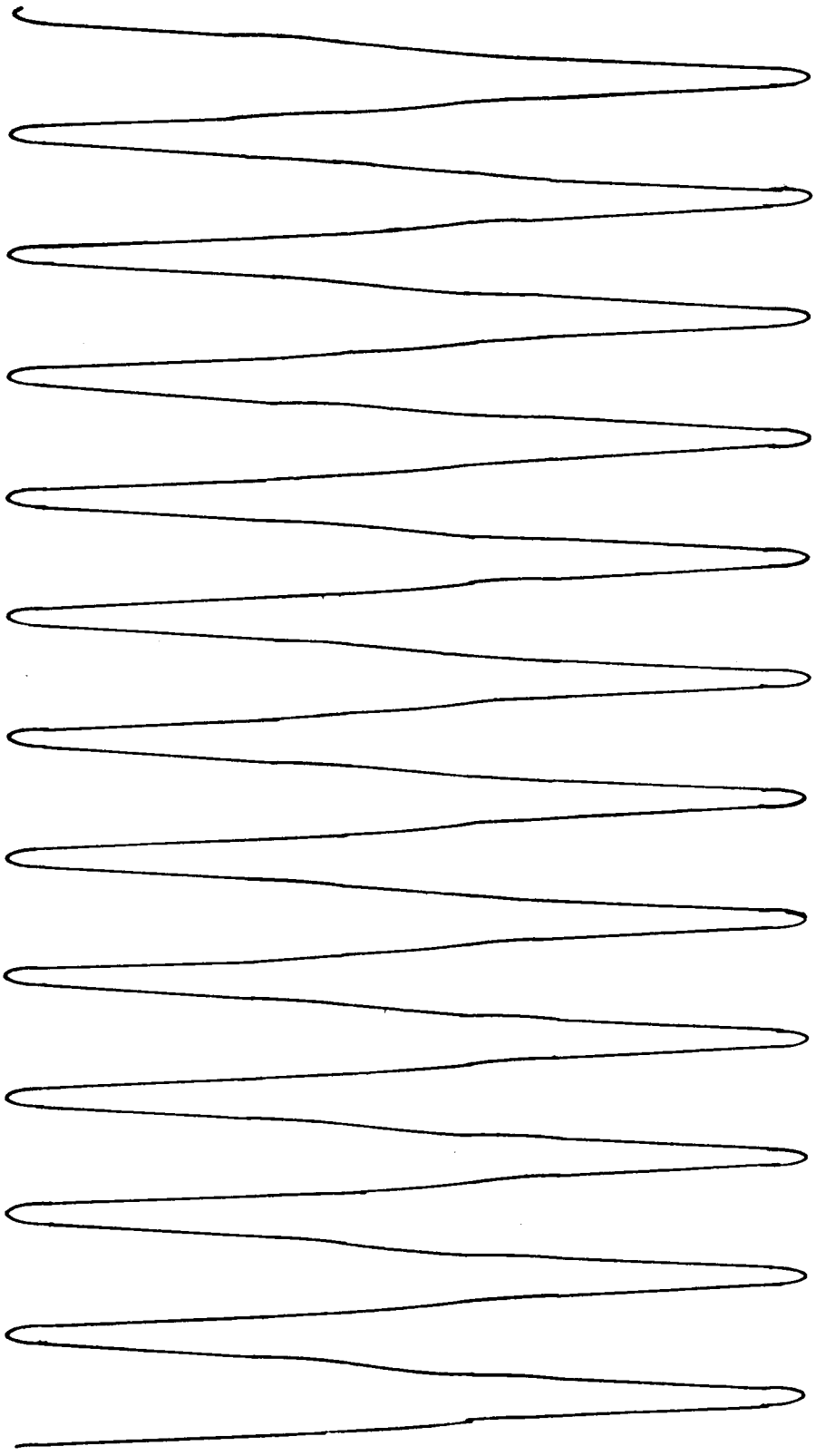


Figure 44. Saturated waveforms of secondary voltage and current with lighter burden.

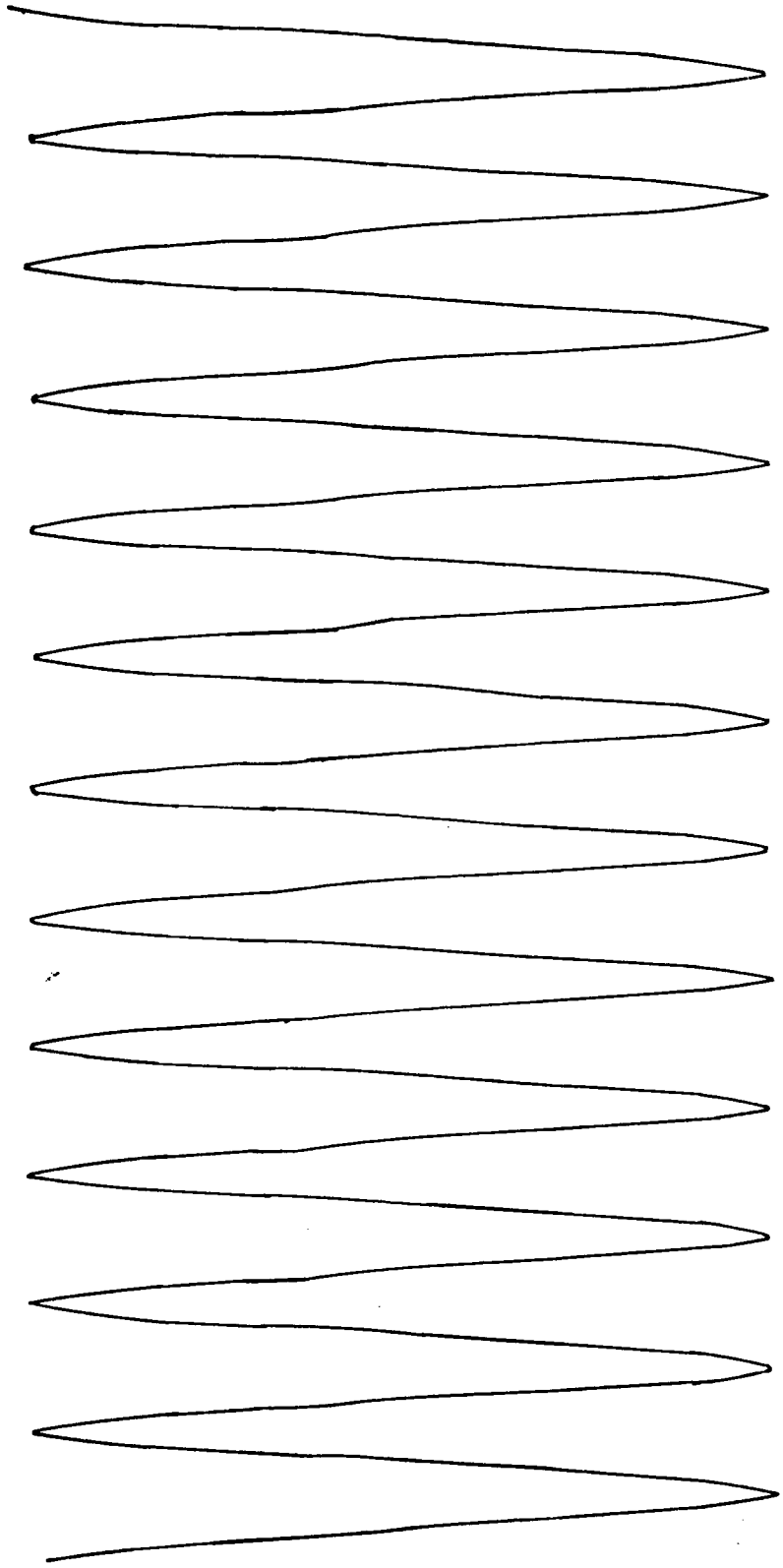


Figure 45. Saturated waveforms of secondary voltage and current with heavier burden.

Ratio and phase angle errors measurement. To measure the ratio and phase angle errors of the second CT, a circuit of Figure 46 was connected. The result was as indicated in Figure 47. Before saturation, the ratio error increases slowly. When the core is saturated, the ratio error almost rises linearly and very sharply. Refer to Figure 47 (A). Conversely, the phase angle error is almost below 0° radian and easy to control before saturation. When the core saturates, the phase angle error rises approximately linearly and also sharply. Refer to Figure 47 (B).

The accomplishment made in this chapter is that a CT performance can be predicted by the use of a digital computer. It is less time consuming to use a computer than to predict analytically.

The accuracy of the prediction has been verified by experimental measurements on a laboratory CT.

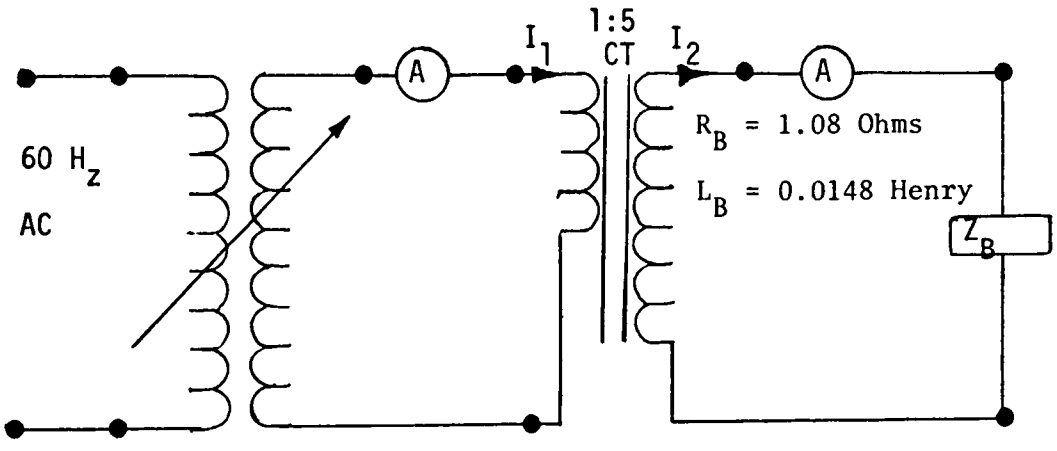


Figure 46. Ratio and phase angle errors measurement circuit.

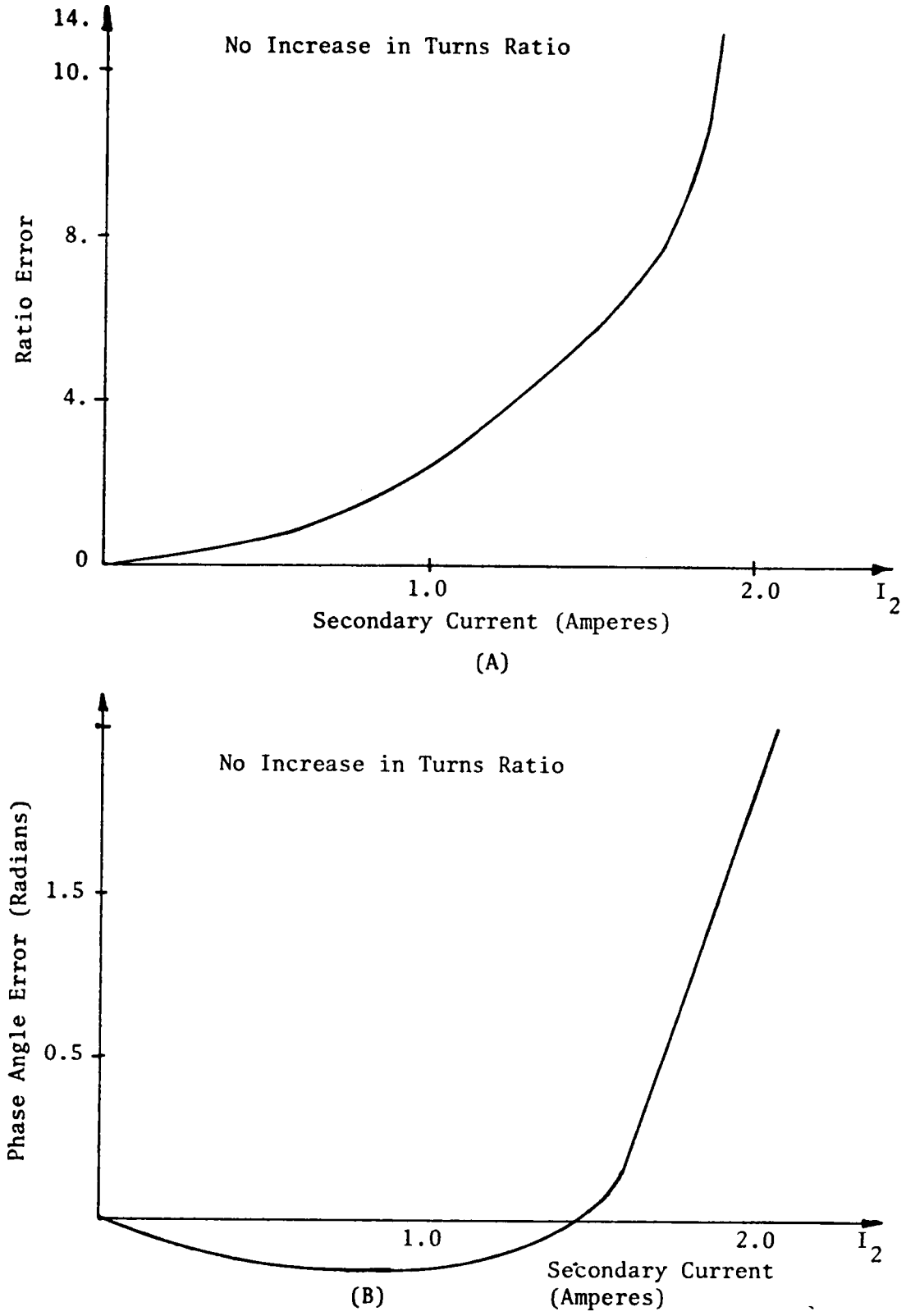


Figure 47. Secondary current versus ratio and phase angle errors.

Note: (A) R_B = 1.08 ohms, L_B = 0.0148 henry; (B) R_B = 1.08 ohms, L_B = 0.0148 henry.

CHAPTER 6

CONCLUSIONS

An accomplishment has been made in this study to develop a mathematical model to predict the performance and output of current transformers. A computer program has been written and applied to various fault conditions. According to this study, the following major conclusions were reached.

1. Both "unsaturated" and "saturated" expressions can be developed from the characteristic curve and parameters of a current transformer.
2. Representation of core inductance, which is nonlinear during saturation, can be developed.
3. Prediction of current transformer performance and output can be made with reasonable accuracy by the use of a mathematical model and computer program for a fault on a power system.
4. The amount of inductance in the secondary of a current transformer has a tremendous influence on the d.c. transient flux. The value of the inductance determines whether the d.c. flux will decay to zero or not. A more accurate method to determine the inductance value is the use of digital computer.
5. The computer and experimental results relative to current transformer performance and output agree with the theoretical analysis.

6. For the purpose of practical application with respect to saturation, since it involves complex nonlinear differential equations, further investigation is required.

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REFERENCES

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APPENDIX

APPENDIX

The computer program below is a model program used in this work to predict the output and performance of a CT. Parameters and the input current may be changed according to the specifications of the CT.

```
INITIAL
  PARAM RB=0.128,R2=8.0,LB=0.001,L2=0.1,N1=1.0,N2=9000.0
  PARAM K1=0.00001092,K2=0.000000546,IS=108.0
  PARAM PS=0.00117936,PT=-0.00117936
DYNAMIC
     NOSORT
     IF(P.GT.PS) GO TO 3
     IF(P.LT.PT) GO TO 5
  SORT
     I1=20000.0*SIN(377.0*TIME)
     JAC=I2*(L2+LB)/N2
     JAS=(R2+RB)/N2
     SIS=INTGRL(0.0,I2)
     SAC=JAS*SIS
     P=JAC+SAC
  I2DOT=(N2*K1*I1DOT-(R2+RB)*I2)/(L2+LB+N2**2.0*K1)
     I1DOT=7540000.0*COS(377.0*TIME)                N1
     I2=INTGRL(0.0,I2DOT)
     E2=(R2+RB)*I2+(L2+LB)*I2DOT
     IM=P/K1
  NOSORT
     GO TO 7
  3 CONTINUE
  SORT
     I1=20000.0*SIN(377.0*TIME)
     JAC=I2*(L2+LB)/N2
     JAS=(R2+RB)/N2
     SIS=INTGRL(0.0,I2)
     SAC=JAS*SIS
     P=JAC+SAC
  I2DOT=(N2*K2*I1DOT-(R2+RB)*I2)/(L2+LB+N2**2.0*K2)
     I1DOT=7540000.0*COS(377.0*TIME)                N1
     I2=INTGRL(0.0,I2DOT)
     E2=(R2+RB)*I2+(L2+LB)*I2DOT
     IM=(P-K1*IS)/K2+IS
  NOSORT
     GO TO 7
  5 CONTINUE
     SORT
```

```
        I1=20000.0*SIN(377.0*TIME)
        JAC=I2*(L2+LB)/N2
        JAS=(R2+RB)/N2
        SIS=INTGRL(0.0,I2)
        SAC=JAS*SIS
        P=JAC+SAC
     I2DOT=(N2*K2*I1DOT-(R2+RB)*I2)/(L2+LB+N2**2.0*K2)
        I1DOT=7540000.0*COS(377.0*TIME)                    N1
        I2=INTGRL(0.0,I2DOT)
        E2=(R2+RB)*I2+(L2+LB)*I2DOT
        IM=(P-PT)/K2-IS
      NOSORT
      7 CONTINUE
   TIMER FINTIM=0.08,OUTDEL=0.0008,DELT=0.00008
   PRTPLT P,I2,IM,E2
   LABEL TRANSIENT RESPONSE FOLLOWING FAULT ON CURRENT XFORM
END
STOP
ENDJOB
```

VITA

The author was born on February 12, 1942, at Paloga Pawaja Aloi, Kitgum, Uganda, East Africa, to Nathaniel O. and Dorothy Ochola. He attended Paloga and Padibe C.M.S. primary schools. He completed his junior secondary education at Kitgum Junior Secondary School and secondary education at Teso College, Aloet, Soroti, Uganda. He entered Morristown College, Morristown, Tennessee, in January 1964, and received an Associate of Arts degree in January 1966.

He entered the College of Engineering, The University of Tennessee, in January 1966 to study electrical engineering on an IIE scholarship, and received the degree of Bachelor of Science in Electrical Engineering in June of 1969. As an undergraduate in electrical engineering, he became a student member of the professional engineering societies of IEEE, Tau Beta Pi and HKN.

On June 14, 1969, he became married to the former Jacqueline Eloise Whiteside of Knoxville, Tennessee.

Also in June 1969, he began his graduate studies in electrical engineering on a teaching assistantship.

In December 1969, he was employed by the E.I. DuPont DeNemours Company, Chattanooga, Tennessee, as an Electrical Design Engineer. Meanwhile he continued his graduate studies part-time at The University of Tennessee. In May 1972, he joined the Chattanooga Electric Power Board as an Engineer.

In March 1974, he took a leave-of-absence from the Power Board to complete his graduate studies. He is currently a member of the IEEE, Tau Beta Pi and HKN.