

Simulation of a Langmuir Probe Sampling a non- Maxwellian Plasma

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ABSTRACT

Langmuir probes are diagnostic tools used to determine electron temperature, number density, and plasma potential. Single, double, and triple Langmuir probe configurations are commonly used in plasma diagnostics because of their relative simplicity. Typical Langmuir probe analysis for determining electron temperature and number density of the plasma (for a single, double, or triple Langmuir probe) includes an assumption that the plasma is in thermal equilibrium. While this assumption may be justified for some applications, it is unlikely that it is fully justifiable for pulsed and time-varying plasmas or for the entire time a plasma device is in use. In this work, Langmuir probe computer models sampled a range of simple equilibrium and non-equilibrium plasmas using fundamental governing equations of probe current collection to compute the current to the probes for a distribution function consisting of two Maxwellian distributions with different temperatures. A variation of this method was also employed, where one of the Maxwellians is offset from zero in velocity space (a drifted Maxwellian or bump-on-the-tail) to add a suprathermal beam of electrons to the tail of the main Maxwellian distribution. For a range of parameters in these non-Maxwellian distributions, the simulation calculates and stores current collection to the probes. Plasma parameters were extracted from the current and voltage curve by applying standard probe theory and compared with the known plasma density and temperature. The collected current from a non-Maxwellian electron distribution illustrates the effect a non-Maxwellian plasma has when interpreted using the equilibrium probe current collection theory, allowing us to

examine the magnitudes of these deviations as a function of the assumed distribution properties. The results of the simulation indicate that all Langmuir probes are ill-suited to report accurate results for non-equilibrium plasmas. Plasmas with bump-on-the-tail electron probability distributions, typical in electric propulsion plasmas, are especially vulnerable to higher inaccuracy in probe measurements and additional investigations into alternative techniques is warranted.

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1 INTRODUCTION

1.1 Plasma Properties

The known universe is primarily composed of plasma, known as the fourth state of matter, which consists of positively or negatively charged freely moving particles that make up an electrically neutral, ionized gas (the net electric charge density is near zero).

Some commonly known examples include fluorescent lightbulbs, lightning, aurorae (polar lights), stars, and the Earth's ionosphere [1]. Understanding plasma lends to furthering our technological capabilities and knowledge of our own environment. Much of the research conducted on plasma is economically motivated due to the large potential energy available from thermonuclear fusion reactions and the semiconductor industry [2].

Diagnostics are necessary in the extreme conditions of a plasma environment where mechanical failure may result from degradation. Plasma diagnostic tools have been used to characterize and quantify plasma since the term plasma was first coined by Irving Langmuir in the 1920's [3]. Early plasma physicists used spectrography to capture the image of a plasma. Specific examples include streak photography for transient plasmas, high speed magneto-optic shutters, and the Schlieren technique. These methods were used to make observations about characteristics such as electron temperature T_e , electron number density, n_e , ionization level, and electron energy distribution [4]. Knowledge of basic plasma physics, such as plasma properties and electromagnetics, is essential to understand the theory behind plasma diagnostics [5].

Until 1820, electricity and magnetism were studied as two separate concepts [5]. In the nineteenth century the two subjects were integrated and the Lorentz force, a result of this integration and named after Hendrik Antoon Lorentz [6], was formalized as

$$\mathbf{j} = \sigma(\vec{\mathbf{E}} + \vec{\mathbf{u}} \times \vec{\mathbf{B}}) \quad 1.1$$

where $\mathbf{j}$ is the current density; with electrical conductivity, σ , and a given velocity $\mathbf{u}$ due to the external electric ($\vec{\mathbf{E}}$ in volts/m), and magnetic induction $\vec{\mathbf{B}}$ (in webers/m²) fields [7]. James Clerk Maxwell, using Gauss's Law and Ampere's Law, formulated four equations used to describe the interaction of the electric and magnetic fields formulated by Heaviside [7].

$$\nabla \times \vec{\mathbf{H}} = \frac{\partial \vec{\mathbf{D}}}{\partial t} + \mathbf{j} \quad 1.2$$

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t} \quad 1.3$$

$$\nabla \cdot \vec{\mathbf{D}} = \bar{q} \quad 1.4$$

$$\nabla \cdot \vec{\mathbf{B}} = 0 \quad 1.5$$

where $\vec{\mathbf{H}}$ is the magnetic field intensity (amp – turn/m), $\vec{\mathbf{D}}$ is the displacement current density (A/m²), and $\bar{q}$ is the electric charge density (C/m³). Eqtns. 1.2 to 1.5 can be used to understand the basis of most electromagnetic phenomenon.

The positively and negatively charged particles in plasma are an assortment of ions and electrons in large enough quantities to develop a quasi-neutral gas that reacts to electromagnetic effects. Plasmas viewed from the “outside”, or macroscopically, are electrically neutral but contain enough charge to interact electromagnetically. This ability

to interact electromagnetically also allows plasma to function as a conductor and carry currents in the form of these charged particles. Maxwell's equations describe the physical effects that occur because of these particles.

With these equations in mind, a broad range of plasma diagnostic devices may be used to quantify physical characteristics such as electron number density and electron temperature. Plasma diagnostic probe types include Langmuir, B-dot [8], Faraday, emissive, heat flux, and neutral particle flux and are all examples of diagnostic devices using plasma dynamics principles to operate and collect information [9]. More examples of plasma diagnostic devices may be found in review papers [10] and textbooks [2]. The methods of collecting information on plasma may be divided into overarching groups. These instruments can be categorized by what plasma frequency they may be used for, as depicted in Figure 1.1 [11].

Oftentimes, these instruments use assumptions in their analyses to make it feasible to calculate plasma characteristics.

1.1.1 Maxwell-Boltzmann Electron Energy Probability Distribution

One assumption that is used in Langmuir probe analysis is the shape of the electron energy probability distribution. Electron energy distributions represent the probability of an electron existing at a certain energy level. This energy function may be expressed in terms of velocity as in a Maxwell-Boltzmann distribution. This distribution describes the

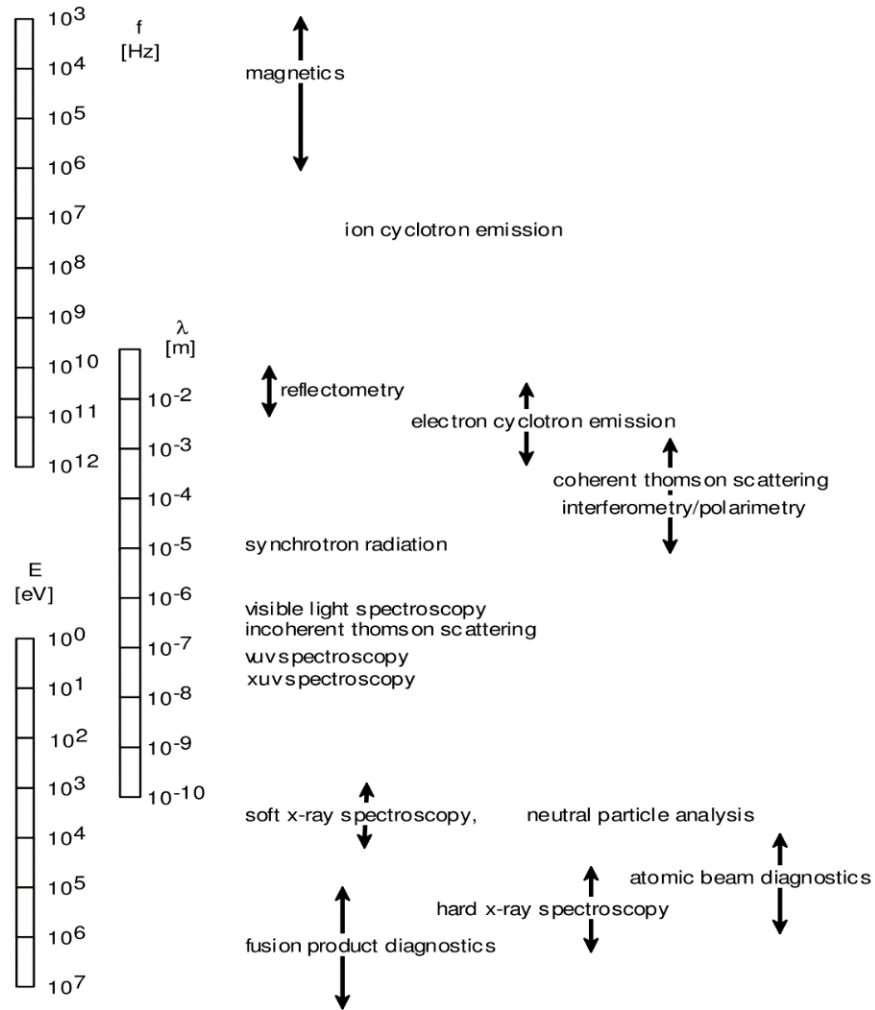


Figure 1.1 Overview of subgroups of plasma diagnostic devices placed against their range [11].

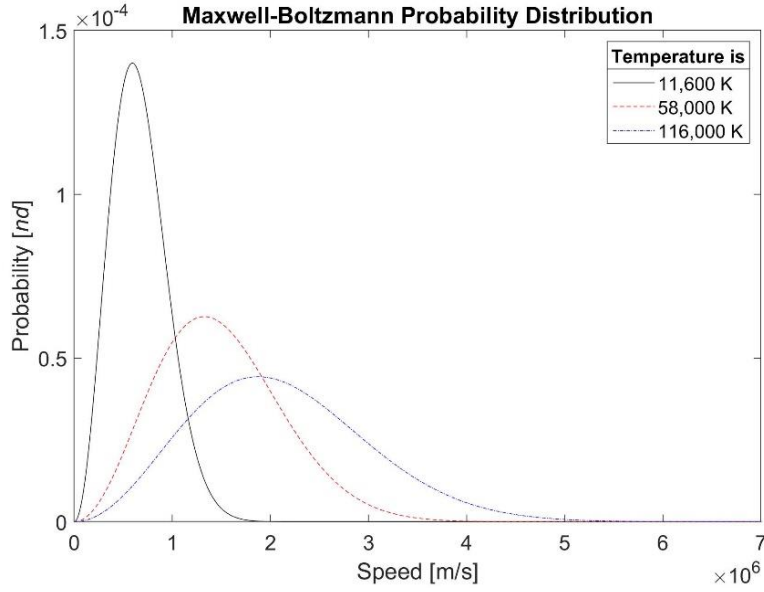


Figure 1.2 Maxwell-Boltzmann Distribution at different energy levels.

velocity and/or the kinetic energy of particles in thermal equilibrium as seen in Figure 1.2 and is a result of kinetic gas theory [12]. It may be referred to mathematically as the chi distribution function representing a particle with three degrees of freedom [13].

The shape of the distribution is primarily a function of the electron temperature since Maxwell-Boltzmann distributions are based on kinetic molecular theory, as a function of the kinetic temperature and the barometric formula.

$$\overline{KE} = \frac{3}{2} k_B T \quad 1.6$$

$$P = P_0 \exp\left(-\frac{mgh}{k_B T}\right) \quad 1.7$$

where the average kinetic energy is $\overline{KE}$, k_B is Boltzmann's constant, T is temperature, the pressure is P , average sea level pressure is P_0 , m is molar mass, g is gravitational acceleration, and h is the height.

With Equations 1.6 and 1.7, an expression for the velocity function for a particle moving in one direction is found. This equation's normalized form (the probability function in one direction or dimension) is shown in Equation 1.9, using Equation 1.6 and 1.7 in combination with 1.8 [14].

$$\int_0^\infty f(\vec{v})d\vec{v} = 1 \quad 1.8$$

$$f(\vec{v}) = \sqrt{\frac{m}{2k_B T_e}} \exp\left(-\frac{m\vec{v}^2}{2k_B T_e}\right) \quad 1.9$$

For some purposes, it may be useful to express this distribution as a differential equation as seen in Equation 1.10.

$$\frac{dN}{N} = \left(\frac{m}{2\pi k_B T}\right)^{D/2} \exp\left(-\frac{m|\vec{v} - u|^2}{2k_B T_e}\right) dv \quad 1.10$$

where N is the molecules moving at velocity $\vec{v}$ to $\vec{v} + d\vec{v}$, particle mass is m , k_B is Boltzmann's constant ($k_B = 1.3806 \times 10^{-23} \frac{\text{m}^2\text{kg}}{\text{s}^2\text{K}}$), T_e is electron temperature, the number of dimensions considered in analysis is seen in the superscript as D , the microscopic velocity is $\vec{v}$, where the macroscopic velocity is $u = \int \vec{v} f d\vec{v}$ is added for this equation. In most cases, u is negligible [15]. Evaluating Equation 1.10 for a one-dimensional model finds Equation 1.11, which is Equation 1.9 with the addition of macroscopic velocity. The result of the same analysis for a particle's one-dimensional

velocity moving in a three-dimensional space is presented in Equation 1.12. For the three-space equations, w represents the particle's speed.

$$f(v) = \sqrt{\frac{m}{2\pi k_B T}} \exp\left(-\frac{m|v - u|^2}{2k_B T}\right) \quad 1.11$$

$$f(w) = 4\pi v^2 \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m|w - u|^2}{2k_B T}\right) \quad 1.12$$

It may be observed in Figure 1.2 that at higher temperatures the particles have more energy and there is a greater probability that they will be found at higher speeds or velocities which results in a wider velocity distribution.

1.1.2 Electron Energy Distribution Instabilities

Two types of non-equilibrium plasmas are two temperature and bump-on-the-tail. These distributions are particularly seen in low-pressure plasmas. The two-temperature plasma consists of two Maxwellian distributions at different temperatures that combine into one electron distribution function. Bump-on-the-tail plasma is a variation of the two-temperature plasma where one of the Maxwellian distributions is shifted in velocity space. It has been shown that modelling two-temperature plasma distributions with an equilibrium Maxwell-Boltzmann distribution model yields higher excitation and ionization rate coefficients in dielectric barrier discharges [16]. Miniaturization of ablative pulsed plasma thrusters has been presented as relying on the emission of secondary electron ejected from a cathode after bombardment that can introduce plasma electron energy distribution instabilities [17].

Examples of two temperature and bump-on-the-tail electron distributions are shown in Figure 1.3 and Figure 1.4 for one- and three-dimensional electron velocity distributions respectively.

Electron temperature and electron number density (n_e) are calculated from the IV characteristic using different methods for each Langmuir probe type. The shape of the electron retarding region of the curve is determined by the electron distribution function that is affected by the plasma temperature(s) [15]. This electron distribution is assumed and fit to the information to extract electron temperature that is required to calculate electron number density [18].

1.2 Langmuir Probes

Langmuir probes are electrostatic plasma diagnostic tools used to determine electron temperature, number density, and plasma potential. Single, double, and triple Langmuir probes are commonly used in plasmas because of their relative simplicity, although they are intrusive diagnostic tools. The probe consists of an electrically isolated piece of conducting material, as shown in Figure 1.5, whose surface is exposed to the plasma. The current generated by particle bombardment from the plasma and collected by the probe is then measured by supporting circuitry as a function of applied probe electric potential. It is assumed that all or most of the charged particle's kinetic energy and charge is transferred to the probe [18].

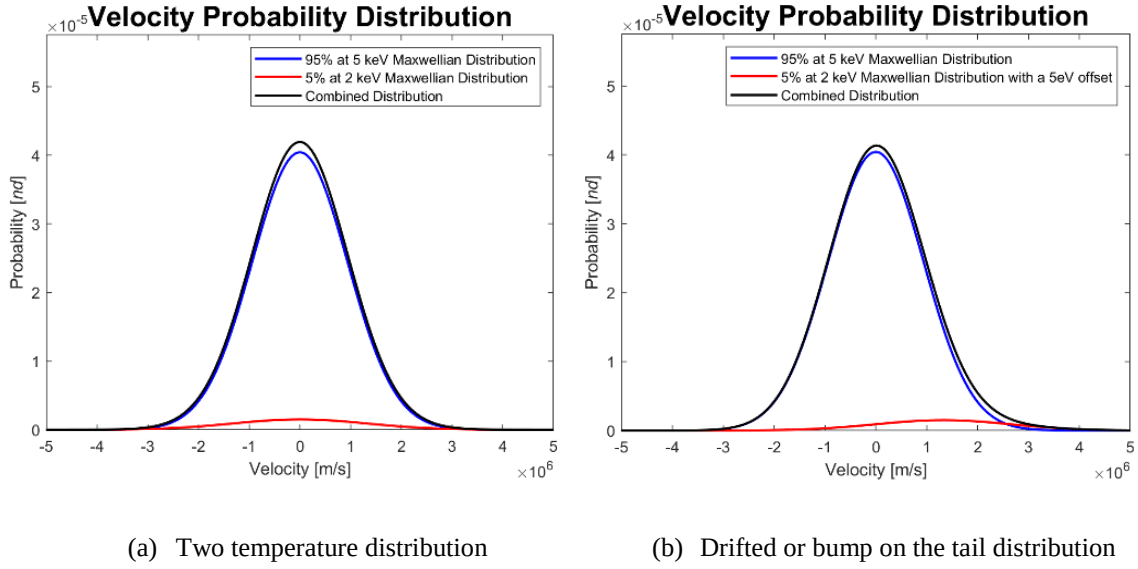


Figure 1.3 Electron velocity probability for a one-dimensional distribution.

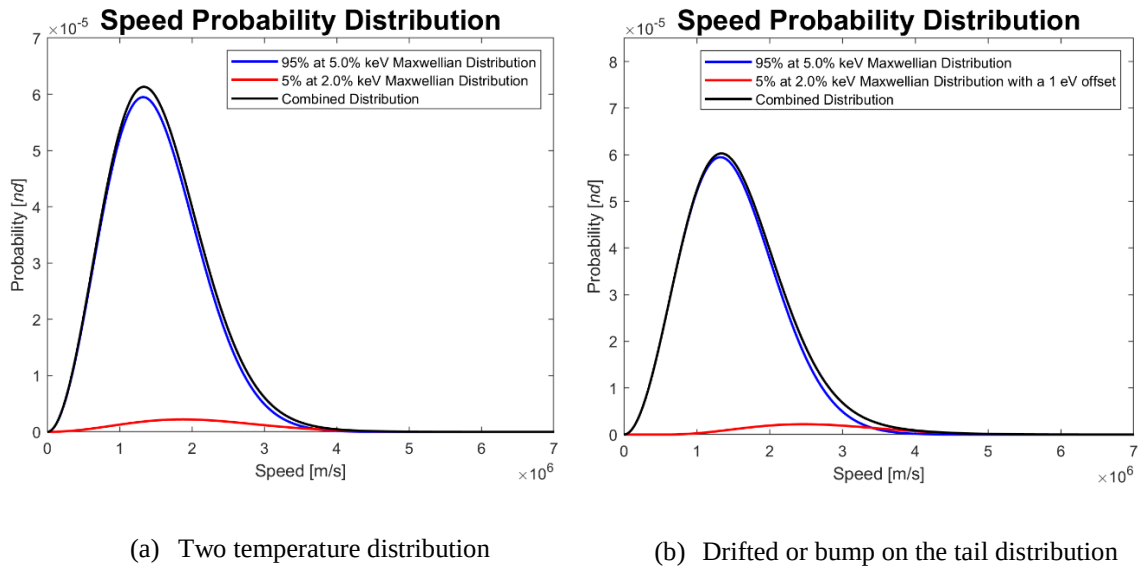


Figure 1.4 Electron speed probability for a three-dimensional distribution.

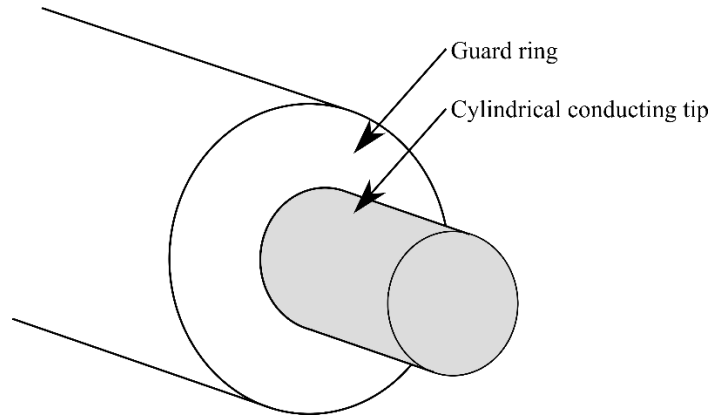


Figure 1.5 Depiction of a Langmuir Probe tip with an insulating guard ring.

The probe tip(s) are immersed in a plasma where current is collected from the ions when the probe is negatively electrically biased and electrons when it is positively biased. For most Langmuir probe applications, it is assumed that the plasma is negligibly magnetized, or that the electron and ion gyro-radii are larger than other lengths of interest, *i.e.* Debye length [18]. The Debye length, λ_D , is a fundamental length defined by the plasma's electrostatic effect and how far into the medium this effect persists. For every Debye length the relative electrical potential decreases by a factor of one over Euler's number.

As the bias is made to be positive with respect to the plasma, electrons are collected. The frequency of the oscillation of the bias must be small enough to neglect transient effects [18]. The magnitude of the current from electrons is generally higher than that of the ions, partially because the electrons have significantly less mass than the ions and because of the role that charge attraction plays.

This difference in bombardment to the probe between ions and electrons generates a current-voltage (IV) characteristic curve based on the probe's bias voltage and how much current is collected (current collected plotted as a function of the applied voltage). A sample IV curve may be seen in Figure 1.6 for single and triple Langmuir probes. Extremely positive or negative voltage biases, the high energy regions shown in the figure, are avoided in part so that the probe is not damaged. These regions are specifically associated with secondary electron emission (the high electron energy region) and neutral ionization (referring to high ion energy). The high energy regions are the secondary emission regions; they are not useful for determining the plasma characteristics of interest. Plasma characteristics (T_e, n_e) are extracted from the IV curve based on electromagnetic theory based on an equilibrium assumption. Electron temperature refers to the energy state of the free electrons of the plasma at equilibrium, and the electron number density is the number of ionized molecules per plasma volume.

There are multiple types of Langmuir probe tips: planar, cylindrical, and spherical. Each probe tip type may be seen in Figure 1.7. The cylindrical probe is the most commonly used in plasma physics diagnostics, followed by the planar probe, then the spherical probe. A planar probe tip is an exposed planar surface with or without a guard ring, sometimes referred to as a faraday probe. The guard rings, coplanar to the probe surface, are used to mitigate edge effects on the end of the coplanar surface; their current is effectively zero and is not collected. This increases the probe tip's apparent planarity [19]. Cylindrical Langmuir probe tips have an exposed cylindrical surface, and their analysis is very similar

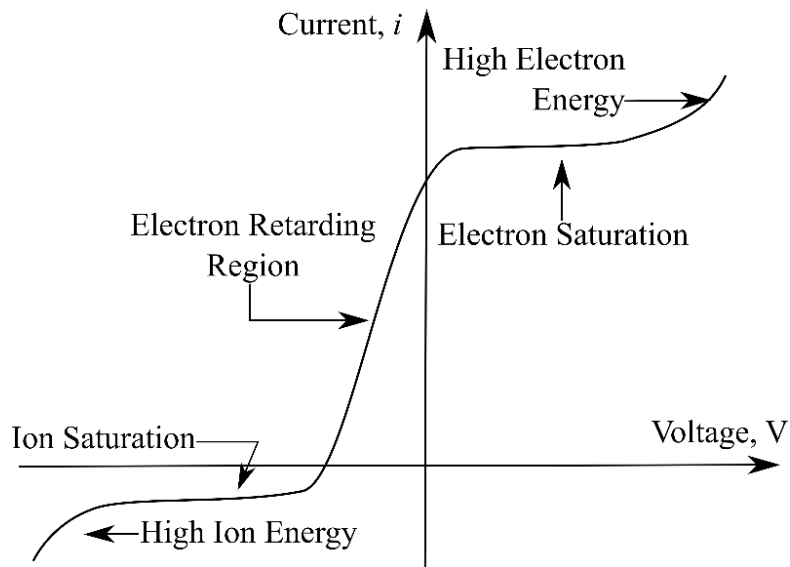


Figure 1.6 Labeled Langmuir probe characteristic curve.

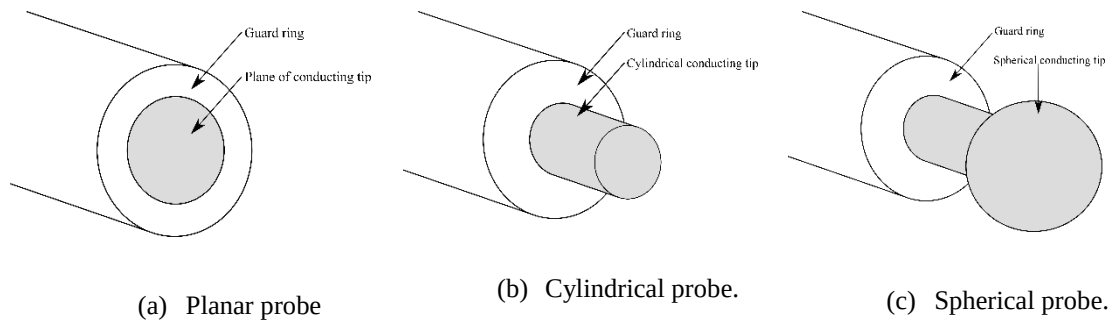


Figure 1.7 Probe tip types [18], [20].

to that using planar probe tips. The spherical Langmuir probe tip is not often used because it is difficult to fabricate and analyzing the ion current is difficult since it is not well defined as the probe tip radius goes to zero [18].

The first two types of Langmuir probes mentioned, single and double Langmuir probes, must have a varying voltage which may also be referred to as a swept or oscillating voltage, applied to record a current value at the corresponding voltage. Figure 1.8 and Figure 1.9 show possible single and double Langmuir probe schematics.

Because of the time associated with varying the voltage across an appropriate range in single and double probes, it is proposed that alternative methods are required for these devices when in a time varying plasma, or plasmas not in thermal equilibrium [21]. Current and voltage mode triple Langmuir probes use static applied voltages and allow for an instantaneous characteristic to be recorded, as well as for T_e and n_e to be determined, if it is assumed that the plasma in question is in thermal equilibrium [22]. Triple Langmuir probe setups are shown in Figure 1.10.

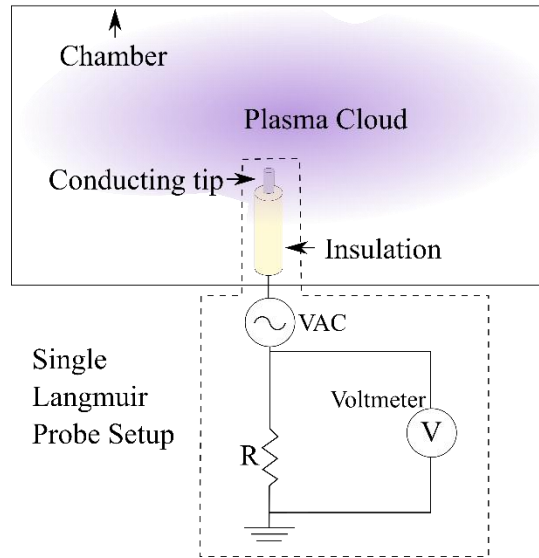


Figure 1.8 Single Langmuir probe schematic.

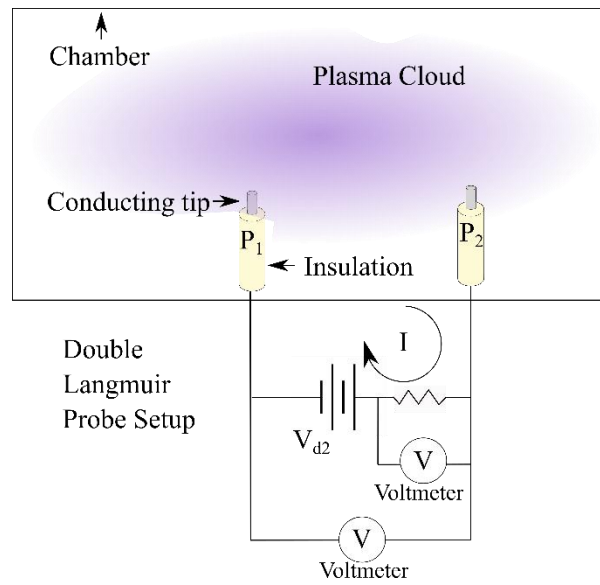


Figure 1.9 Double Langmuir probe schematic.

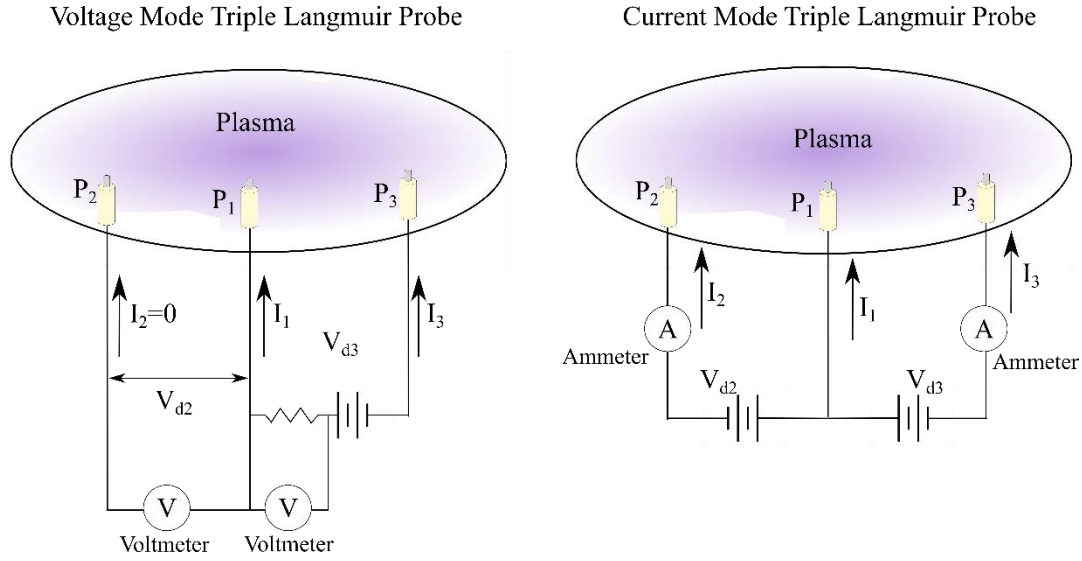


Figure 1.10 Voltage mode (left) and current mode (right) triple Langmuir probe circuits [22].

1.2.1 Common Uses and Practices

Langmuir probes are used in a variety of plasma-based experiments involving glow discharges, radio-frequency, pulsed plasmas, and afterglow [23]. In a glow discharge, a cathode and anode are fixed. An electric potential is applied between them, and electrons accelerate from the cathode to the anode acquire kinetic energy. As they do so, they collide with other particles, creating ions and free electrons, and cause a cascade that will result in a sustained plasma with applied sufficient voltage. Radio-frequency (RF) plasmas are formed by externally applying RF fields to a gas flow, transferring energy to the gas that ionizes into a plasma. Pulsed plasmas, as mentioned earlier in this thesis, are time-varying and often used in thrusters to achieve a higher specific impulse. Plasma afterglow is the period after the source of ionization has been removed either by time (as in a pulsed plasma) or space (distance from the plasma source) [4].

Langmuir probes have been used in a variety of situations: from sampling plasmas on the ground (often radio-frequency plasmas) to in-space. Specific use-cases for radio-frequency discharges include inductively coupled plasma (ICP) discharges [24], characterization of near plume emission from a low-current hollow cathode [25], multi-use diagnostic tool for the Electric Propulsion and Plasma Dynamics Lab in Princeton, NJ [26], and low pressure ICP measurements in a magnetic field [27]. When measuring a radio-frequency plasma with a Langmuir probe, often compensation for the plasma potential oscillation is required. However, if certain discharge conditions are met, this is not required, and the analysis current-voltage characteristic may proceed without compensation [28]. Langmuir probes have also been used to measure plasma characteristics of transient, laser-ablated plasmas [29]. In-space, the upper regions of Earth's atmosphere contains a plasma field which interacts with Earth's magnetic field. Satellites have been launched with Langmuir probes on board to measure current-voltage characteristics that provide plasma parameters [30]. Geomagnetic research is also done with Langmuir probes to monitor the ionosphere's behavior from interplanetary shocks [31].

As previously stated in Section 1.1.2, these plasmas may not have Maxwell-Boltzmann electron probability distributions. In cases where secondary emission is possible, or pulsed power is being used such that distinct electron temperature regimes exist in the same space, the plasma is not in thermal equilibrium [25], [32]. Furthermore, in-space, "auroral regions multicomponent plasmas can be present..." [30]. A Langmuir

probe would be inserted in these plasmas to collect the current generated by charged particles bombarding its surface.

In collecting data from a Langmuir probe, it is useful to be aware of what qualifies as a poor data set for analysis such as low-density curves which do not match the classic shape. For plasmas with low density, the IV characteristic may appear to have no electron saturation or exponential region as shown in Figure 1.11 from *Lecture Notes on Langmuir Probe Diagnostics* by Francis F. Chen [19].

It is common to attempt to fit a drifted electron Maxwellian (bump on the tail) distribution as seen in Figure 1.3 and Figure 1.4 to unusual IV characteristic curves like the one in Figure 1.11. This requires numerically evaluating an integral or expressing the current as an error function [19]. After fitting an equation and plotting the characteristic on a semi-logarithmic scale, the linear portion is fitted. The linear portion of this fitted curve yields unrealistically high electron temperatures when evaluated using equilibrium Langmuir probe theory due to the influence of the secondary electron temperature on the IV characteristic curve slope. A beam current estimated for the drifted function must be subtracted to get a reasonable estimate of the electron temperature [19].

A dirty (non-uniformly coated) probe tip may also report IV characteristics out of line with the true plasma properties [19]. A potential solution to this problem is to make the probe uniformly dirty by coating it in water or alcohol-based carbon coatings [32]. The location of the probe should not interfere with the plasma or be corroded by it (plasmas can

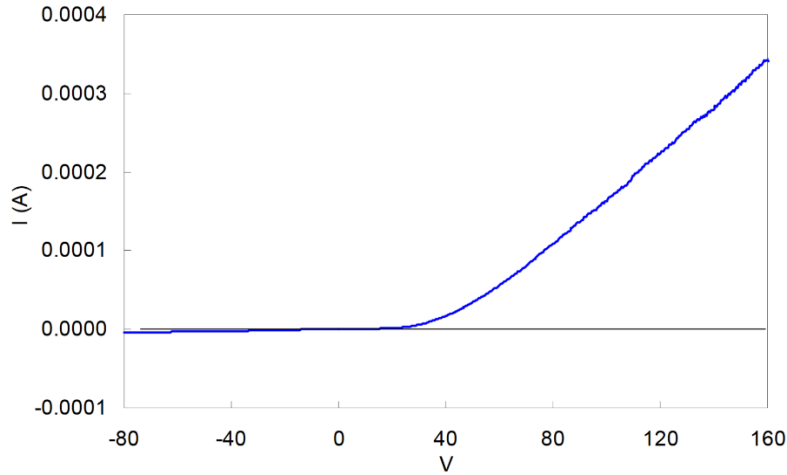


Figure 1.11 I-V characteristic example for a low-density plasma [19].

be damaging to some types of conducting materials, especially if inherently corrosive at lower energy states).

1.3 Statement of Purpose

This thesis will present simulations of a Langmuir probe's response to a plasma using MATLAB R2018a version 9.4.0.81365. The overall objective is to determine the margin of error induced by plasmas not in thermal equilibrium that have non-Maxwellian distributions. A plasma is constructed using chosen parameters for electron and ion energy distributions which are converted from the velocity probability distribution to the amount of current able to overcome the electrostatic sheath and bombard the probe. The Langmuir probe simulation then takes this information and uses Maxwell-Boltzmann Langmuir probe theory to calculate the electron temperature and electron number density. The result of this

calculation and the original plasma parameter values may be compared to illustrate the difference unstable electron distributions make in determining electron temperature and number density. This comparison indicates that as the electron probability distribution is shifted further from the thermal equilibrium assumption, the electron temperature increases and the electron number density decreases. This thesis is organized as follow: Chapter 2 discusses the models used in Langmuir probe analysis, Chapter 3 outlines how the simulation incorporates the models detailed, Chapter 4 depicts key simulation results and Chapter 5 presents conclusions.

2 METHODOLOGY

This chapter describes the theory required to evaluate single, double, current mode triple, and voltage mode triple Langmuir probes. It uses planar probe assumptions and Bohm ion velocity. The electron and ion models common to each probe type are described before discussing individual probe theory.

2.1 Electron Models

To examine a simple non-equilibrium plasma case assuming an electron velocity distribution function, we return to basic governing equations of probe current collection and compute the current to the probes for the combination of two Maxwellian distributions with different temperatures (the two-temperature Maxwellian) in one dimension.

One Maxwellian distribution is constructed as the primary distribution. The second Maxwellian distribution is set to a temperature higher than that of the primary function and assigned a percentage of area taken from the total of its area and the primary Maxwellian. The two distributions are added and normalized to ensure that the result is a probability distribution that integrates to one over all velocities. Equations 2.1 and 2.2 depict the primary and secondary Maxwellians, where p is the percentage of area allocated to the primary function, and m_e is the mass of an electron [33], [15]. An example of this function is found in Figure 1.3 (a), where each distribution is depicted separately and as a combined distribution.

$$f_1(\vec{v}) = p \left(\frac{m_e}{2\pi k_B T_{e_1}} \right)^{1/2} \exp \left(-\frac{m_e \vec{v}^2}{2k_B T_{e_1}} \right) \quad 2.1$$

$$f_2(\vec{v}) = (1 - p) \left(\frac{m_e}{2\pi k_B T_{e_2}} \right)^{1/2} \exp \left(-\frac{m_e \vec{v}^2}{2k_B T_{e_2}} \right) \quad 2.2$$

Similarly, the electron distribution of the three-dimensional movement of a particle moving in one direction are presented as a function of speed in Equations 2.3 and 2.4, and a figure of these distributions may be seen in Figure 1.4 (a).

$$f_1(w) = 4\pi w^2 p \left(\frac{m_e}{2\pi k_B T_{e_1}} \right)^{3/2} \exp \left(-\frac{m_e w^2}{2k_B T_{e_1}} \right) \quad 2.3$$

$$f_2(w) = 4\pi w^2 (1 - p) \left(\frac{m_e}{2\pi k_B T_{e_2}} \right)^{3/2} \exp \left(-\frac{m_e w^2}{2k_B T_{e_2}} \right) \quad 2.4$$

A variation of this method is also employed, where one Maxwellian is offset from zero (in velocity space) to add a suprathermal beam of electrons to the tail of the main Maxwellian distribution as seen in Figure 1.3 and Figure 1.4. This creates a "bump-on-the-tail" distribution function. These electrons affect the sheath potential, ion density, and electron and ion velocity profiles [34]. Equation 2.5 explicitly states the secondary function used for this model where v' represents how much the secondary function is offset [33]. Equation 2.6 similarly states the function for this model in a three-dimensional space. A depiction of function with this distribution type may be seen in Figure 1.3 (b).

$$f_2(\vec{v}) = \frac{1 - p}{N} \left(\frac{m_e}{2\pi k_B T_{e_2}} \right)^{1/2} \exp \left(-\frac{m_e |\vec{v} - v'|^2}{2k_B T_{e_2}} \right) \quad 2.5$$

$$f_2(w) = 4\pi(w - v')^2 \frac{1-p}{N} \left(\frac{m_e}{2\pi k_B T_{e2}} \right)^{1/2} \exp \left(-\frac{m_e |w - v'|^2}{2k_B T_{e2}} \right) \quad 2.6$$

The macroscopic velocity, v' , is integrated from the lower integral bound of v_{req} , which represents the minimum energy an electron must have to overcome the barrier created by the plasma sheath, to infinity. This includes the velocities of particles that may be part of a secondary Maxwell-Boltzmann distribution and at a separate temperature. Both irregular non-equilibrium distributions (two-temperature and bump on the tail) are hypothesized to affect the overall results. Examples of these distribution types, in one and three dimensions, are shown in Figure 1.3 and Figure 1.4.

As the distribution changes, fewer or more (depending on the bulk plasma temperature) electrons overcome the sheath potential to bombard and are collected by the Langmuir Probe. The probe senses the lack or addition of electrons generating current and alters the resulting IV characteristic from which T_e and n_e are calculated using Langmuir probe theory.

We compare the temperatures and number densities associated with the distribution function that were assumed *a priori*, with or without a nonequilibrium component, with the electron plasma density and temperature one would infer when applying standard probe theory as described later in this chapter. The collected electron current is calculated by multiplying the flux, ϕ (where q is the elementary unit charge, and $f_{\vec{v},w}$ is the electron probability distribution function), as seen in Equation 2.7, with the exposed surface area of the probe.

$$\phi = n_e q \int \vec{v} f_{\vec{v},w} d\vec{v} \quad 2.7$$

For a range of parameters in these non-Maxwellian distributions, we compute the current collection to the probes. Total current (I) to the probe is the sum of electron current, I_e , and ion current, I_i . The electron current is the integral of electron flux over velocity [18], so the total current is

$$I = I_i + I_e = I_i + n_e q A \int_{v_{req}}^{\infty} \vec{v} \exp\left(-\frac{m\vec{v}^2}{2k_B T_e}\right) dv. \quad 2.8$$

As stated, the velocity of the electrons must be high enough to overcome the sheath around the probe such that $\vec{v} > v_{req}$, where $v_{req}^2 = \frac{2q}{m}(\phi_P - V_b)$. ϕ_P is the electric potential of the probe, and V_b is the voltage to which the probe is biased.

For equilibrium cases, the current may be calculated by Equation 2.9, which is the analytic solution to the current for an equilibrium electron distribution and Bohm ion velocity [35]. This current may be compared with the current calculated from the integral in Equation 2.6 with Bohm velocity and $V_b \leq \phi_P$ - which includes the assumption that the electron velocity distribution is Maxwellian.

$$I_{eq} = \frac{q n_e A}{4} \sqrt{\frac{8k_B T_e}{m_e \pi}} \exp\left(\frac{q(\phi_P - V_b)}{k_B T_e}\right) \quad 2.9$$

2.2 Ion Models

The ions bombarding the Langmuir probe may be simulated in different configurations, although these methods all fall into two main categories: simulation as a

constant or a distribution. The main two methods explored for simulating ion current velocity or speed (v_i) as a constant are thermal speed and Bohm velocity. Ion current density, j_i , seen in Equation 2.10, is found using the mean ion thermal velocity, or speed, and ion particle density, n_i [33].

$$j_i = -q \frac{n_i \overline{v_{th}}}{4} \quad 2.10$$

where the mean ion thermal speed, $\overline{v_{th}}$, is the integral of the molecular speeds divided by the number of molecules; it is located to the right of the velocity distribution's peak. The mean thermal velocity in one dimension is found using Equation 2.11 and shown in Equation 2.12. The three-dimensional form is shown in Equation 2.13.

$$\frac{1}{2} m \overline{v^2} = \frac{1}{2} k_B T \quad 2.11$$

$$\text{One-dimensional:} \quad \overline{v_{th}} = \sqrt{\frac{k_B T_i}{m_i}} \quad 2.12$$

$$\text{Three-dimensional:} \quad \overline{v_{th}} = \sqrt{\frac{8 k_B T_i}{\pi m_i}} \quad 2.13$$

This method is best used when the electron temperature, T_e , is comparable to the ion temperature, T_i . Thermal velocity is a scalar and so not technically a velocity. It may be thought of as an indicator of the full width at half maximum of the Maxwell-Boltzmann velocity distribution.

In nonequilibrium cases where $T_e \gg T_i$, or the ions are comparatively cold, the Bohm velocity is a more accurate representation of the average value of the ion current density [35].

To obtain the Bohm velocity, first apply Poisson's equation. Following [18]:

$$\frac{\partial^2 \phi}{dx^2} = -4\pi\rho \quad 2.14$$

where ϕ is the electric potential in Volts, and ρ is the charge density in Coulomb per cubic meter defined in Equation 2.15.

$$\rho = q(Z_i n_i(x) - n_e(x)) \quad 2.15$$

where q is the elementary unit charge, Z_i is the ionization level, and n_i , and n_e are the number density of ions and electrons respectively as a function of location along the plasma sheath, x . After the plasma- sheath boundary demarcation, the number density of the electrons and ions become equivalent such that $n_e \cong n_i$. Assuming a Boltzmann distribution and that the electrons are in thermal equilibrium,

$$n_e(x) = n_e \exp\left(\frac{q\phi(x)}{kT_e}\right) \quad 2.16$$

The ion flux is conserved through the plasma, leading to Eq. 2.17.

$$n_i(x) = \frac{n_i V_0}{V_i(x)} \quad 2.17$$

where V_0 is the initial drift velocity, and $V_i(x)$ is the ion velocity at x . Substituting these relations into Eq. 2.14:

$$\frac{\partial^2 \phi}{dx^2} = -4\pi q \left(Z_i \frac{n_i V_0}{V_i(x)} - n_e \exp\left(\frac{q\phi(x)}{kT_e}\right) \right) \quad 2.18$$

Multiplying by e/kT_e and bringing Z_i outside of the parenthetic term:

$$\frac{\partial^2}{dx^2} \left(\frac{\phi q}{kT_e} \right) = -\frac{4\pi q^2 Z_i}{kT_e} \left(\frac{n_i V_0}{V_i(x)} - \frac{n_e}{Z_i} \exp \left(\frac{q\phi(x)}{kT_e} \right) \right) \quad 2.19$$

The Debye length, λ_D , is introduced:

$$\lambda_D^2 = \frac{kT_e}{4\pi n q^2} \quad 2.20$$

The charge density is only dependent on x through the potential ϕ , so that $\rho = \rho(\phi)$.

Expanding ρ about $\phi = 0$, far from the probe, we may use $\rho(\phi) = S(\phi)/\lambda_D^2$ to find

$$\rho(\phi) = \frac{S(0)}{\lambda_D^2} + \frac{\partial S}{\partial \phi}(0) \frac{\phi}{\lambda_D^2} + \dots \quad 2.21$$

Resubstituting this into Poisson's equation:

$$\frac{\partial^2 \phi}{dx^2} = -4\pi \left(\frac{S(\phi)}{\lambda_D^2} + \frac{\partial S}{\partial \phi}(\phi) \frac{\phi}{\lambda_D^2} \right) \quad 2.22$$

$$\frac{\partial^2 \phi}{dx^2} + 4\pi \frac{\partial S}{\partial \phi}(\phi) \frac{\phi}{\lambda_D^2} = -4\pi \frac{S(\phi)}{\lambda_D^2} + \quad 2.23$$

As stated in [18], there are two possible solutions- unstable oscillation or exponential growth or decay. $\partial S/\partial \phi$ provides a stable solution and is chosen so that the general solution is

$$\phi(x) = -S(0) \left(\frac{\partial S}{\partial \phi}(0) \right)^{-1} + C_1 \exp \left(\frac{\alpha x}{\lambda_D} \right) + C_2 \exp \left(-\frac{\alpha x}{\lambda_D} \right) \quad 2.24$$

with initial conditions of $S(0) = 0$ and $\phi(\infty) = 0 \rightarrow C_1 = 0$ where $\alpha^2 = -4\pi \frac{\partial S}{\partial \phi}$.

Therefore,

$$\phi(x) = \phi(x_1) \exp \left(-\frac{\alpha(x - x_1)}{\lambda_D} \right) \quad 2.25$$

From the boundary conditions, $\exp(\alpha x_1/\lambda_D) \sim 1 \rightarrow \frac{\alpha x_1}{\lambda_D} \sim 1 \rightarrow \alpha \sim 0$ because $\frac{x_1}{\lambda_D} \gg 1$ therefore, $S(0) = \partial S(0)/\partial \phi = 0$. Using the conservation of energy (Equation 2.26), $\phi(\infty) = 0$, charge neutrality, and the results of particle velocity (Equation 2.27) and density (Equation 2.28),

$$\frac{1}{2}m_i\vec{v}_0^2 + Z_i q \phi(\infty) = \frac{1}{2}m_i\vec{v}_i^2(x) + Z_i q \phi(x) \quad 2.26$$

where

$$\vec{v}_i^2(x) = \vec{v}_0^2 - \frac{2Z_i q \phi(x)}{m_i} \quad 2.27$$

and

$$\rho(\phi) = q \left[Z_i n_i \vec{v}_0 \left(\vec{v}_0^2 - \frac{2qZ_i}{m_i} \phi \right)^{-\frac{1}{2}} - n_e \exp \frac{q\phi}{kT_e} \right] \quad 2.28$$

we obtain the Bohm velocity condition and the associated ion current density collected by the probe assuming Bohm velocity in Equations 2.29 and 2.30 [16].

$$\vec{v}_0^2(x) = \frac{Z_i k T_e}{m_i} \quad 2.29$$

and

$$j_i = q n_e \vec{v}_0 = q n_e \sqrt{\frac{Z_i k T_e}{m_i}} \quad 2.30$$

2.3 Single Langmuir Probes

Single Langmuir probes are basic electrostatic probes consisting of conducting material exposed to a plasma, where everything except the probe tip (electrode) is electrically isolated so that an appropriate surface area may be estimated. The probe's surface area estimate is used to calculate the theoretical current from the electrons and ions by multiplying the flux to the probe, as found in Equation 2.7, and the exposed surface area of the probe. In practice, the conducting material is biased to a voltage and charged particles bombard the exposed surface where they are collected. This generates a current that is measured. The bias voltage is swept repeatedly from negative to positive limits. The resulting current is plotted against the probe's biased voltage value. A representative plot is shown in Figure 1.6.

In a single Langmuir Probe setup, seen in Figure 1.6, the electron temperature may be found by taking the natural logarithm of the linear portion of the total current (labeled the electron retarding region in Figure 1.6) with respect to the biased voltage:

$$T_e \propto [\ln(I)]^{-1} \quad 2.31$$

where I is the sum of electron and ion current, and T_e is electron temperature in electron-Volts. To convert to Kelvin, multiply by the charge of an electron, $q = 1.6022 \times 10^{-19} \text{C}$, and the Boltzmann constant, $k_B = 1.3806 \times 10^{-23} \text{ m}^2 \text{kg/s}^2 \text{K}$.

$$T_e[\text{K}] = \frac{q}{k_B} T_e[\text{eV}] \quad 2.32$$

From the ion saturation current, i_+ , the electron number density, n_e , of a planar single Langmuir probe may be found by Equation 2.34 [18].

$$i_+ = qn_e A \sqrt{\frac{Z_i k_B T_e}{m_i}} \quad 2.33$$

$$n_e = i_+ \left(qA \sqrt{\frac{Z_i k_B T_e}{m_i}} \right)^{-1} \quad 2.34$$

where A is the exposed surface area of the probe. The number density may also be found by Equation 2.35 where the ion current density, j_i , may be calculated as the ion saturation current divided by the probe tip area [22].

$$n_e = \exp\left(\frac{1}{2}\right) \frac{j_i}{q} \sqrt{\frac{m_i}{k_B T_e}} \quad 2.35$$

The analytic solution for an IV characteristic curve, such as the one seen in Figure 1.6, is a Maxwellian plasma using Bohm velocity, given by Equation 2.36, where V_p is plasma potential and V_B is probe's bias voltage [35].

$$I = \begin{cases} \frac{1}{4} \exp\left(-\frac{q(V_P - V_B)}{k_B T_e}\right) - \exp\left(\frac{1}{2}\right) qn_e A \sqrt{\frac{k_B T_e}{m_i}}, & V_B \leq V_P \\ \frac{1}{4} qn_e v_e A - \exp\left(\frac{1}{2}\right) qn_e A \sqrt{\frac{k_B T_e}{m_i}} \exp\left(\frac{q(V_P - V_B)}{k_B T_i}\right), & V_B > V_P \end{cases} \quad 2.36$$

2.4 Double Langmuir Probes

A double Langmuir probe setup includes an additional probe which is used as a reference instead of ground [18]. This setup is used if the uncertainties around large electron currents need to be avoided, if there is no well-defined ground, or to minimize plasma perturbation. It is difficult to use these probes in practice because of the parasitic capacitance to the ground, distorting the IV curve, and the radio frequencies for sweeping the voltage between the probes must be identical [19]. A differential voltage, $V_{d2} = V_2 - V_1$, is applied between the two probes as shown in Figure 1.9, and the circuit is electrically isolated. The differential voltage is not the voltage of the probes; it is the difference. The double probe characteristic is not plotted against voltage, instead it is plotted against the differential voltage.

The current collected is limited to the ion saturation region of the curve and by current continuity through the two probes; the I-V characteristic is theoretically symmetric about the origin as seen in Figure 2.1.

The electron temperature may be found from the IV characteristic's slope around the origin [18].

$$\frac{kT_e}{q} = i_0 / \left(2 \frac{di}{dV} \Big|_{V=0} \right) \quad 2.37$$

where i_0 is the magnitude of the saturation current. The number density may be found by Equation 2.34, where $i_+ = i_0$ and i_0 is the asymptotic magnitude of the current associated with the increasing voltage magnitude [18].

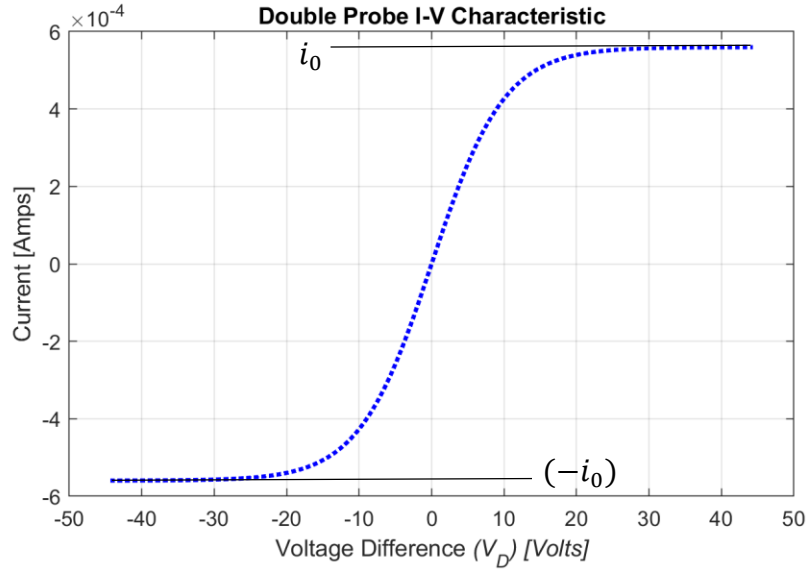


Figure 2.1 Double Langmuir probe characteristic.

The electron temperature and number density may also be found using a two-dimensional grid or step search (a hyperparameter optimization method) although this method is computationally expensive [36]. A parametric mesh of potential electron temperature and number density values is created. The function being evaluated is solved for all combinations of electron temperature and number density which generates a solution mesh. The minimum point of the solution mesh is identified, and the corresponding parameters are the electron temperature and number density [37].

2.5 Triple Langmuir Probes

There are two main modes of operation for a triple Langmuir probe: voltage and current mode. Visual schematics for both modes may be seen in Figure 1.10, and methods for evaluating the two types are described below.

2.5.1 Current Mode

Current mode for a triple Langmuir probe is slightly more complex than voltage mode. It involves two differential probe voltages: one between probes one and two (V_{d2}), and another between probe one and three (V_{d3}) as seen in Figure 1.10. Because the current mode triple Langmuir probe configuration must satisfy current continuity, there is only one configuration where the differential voltages given between the probes are held. In other words, there is only one way the characteristic can look, given the three current values collected by the probe tips.

The current for each probe, as shown in Figure 1.10, for the current mode triple probe, at any instant may be written as:

$$I_1 = Aj_e \exp(-\phi V_1) - Aj_i \exp(-\phi V_1) \quad 2.38$$

$$I_2 = Aj_e \exp(-\phi V_2) - Aj_i \exp(-\phi V_2) \quad 2.39$$

$$I_3 = Aj_e \exp(-\phi V_3) - Aj_i \exp(-\phi V_3) \quad 2.40$$

where $\phi = \frac{q}{kT_e}$, V_i is the voltage of probe i , and $j_e = n_e q \sqrt{kT_e / 2\pi m_e}$. The Bohm velocity is used instead of thermal velocity since the electron temperature is significantly greater

than the ion temperature in nonequilibrium plasmas. To compare our results with those of Chen and Sekiguchi [22], and because the current collected that is generated by ions is negligible compared to that generated by the electrons, we will assume a constant ion flux as a function of voltage. Ion flux, and therefore ion current, can be modeled as a distribution for a more accurate characteristic curve using a Maxwell-Boltzmann velocity probability distribution.

To obtain an electron temperature, we calculate a current ratio as seen in Equation 2.41, assuming that the current density of all three probes is comparable [22]. Numerically, the temperature is found iteratively by using the IV characteristic curve and assigning values of I_1 , I_2 , and I_3 in the current ratio on the left-hand side of Equation 2.41. The right-hand side of Equation 2.41 is subtracted from the current ratio so that the equation is equal to zero. A mesh of T_e values in the electric potential, ϕ , is generated and used in the function. The final value of electron temperature, where Equation 2.41 is satisfied, is accepted as the value of T_e . It should be noted that Chen and Sekiguchi [22] illustrated current to probes two and three in the opposite direction of the schematic shown in this thesis in Figure 1.10; therefore, the current ratio used in their paper is the sum of currents to probes one and two over the sum of current to probes one and three. For the schematic shown in Figure 1.10, the currents are added to obtain:

$$\frac{I_1 - I_2}{I_1 - I_3} = \frac{\exp(-\phi V_1) - \exp(-\phi V_2)}{\exp(-\phi V_1) - \exp(-\phi V_3)} = \frac{1 - \exp(-\phi V_{d2})}{1 - \exp(-\phi V_{d3})} \quad 2.41$$

where V_{d2} is the differential voltage of $V_{d2} = V_2 - V_1$, and $V_{d3} = V_3 - V_1$. The differential voltages are set in the circuit to a constant bias.

After solving for the electron temperature, the plasma number density may be found by resolving the equation for the Bohm or thermal velocity used in calculating the ion current for density. This involves the current density, j_i , which may be found from the current equation to a chosen probe tip, where n_{iBohm} is the ion number density found using Bohm velocity.

$$j_i = \frac{A j_e \exp(-\phi V_1) - I_1}{A \exp(-\phi V_1)} \quad 2.42$$

$$n_{iBohm} = \frac{j_i}{0.6q} \sqrt{\frac{m_i}{kT_e}} \quad 2.43$$

2.5.2 Voltage Mode

In voltage mode, one of the probes of the triple probe is inserted into the plasma without an applied voltage value. The probe is electrically isolated from ground and therefore has no fixed voltage differential, and it will not draw current. Therefore, the voltage value that probe assumes is the floating potential of the plasma. The floating potential may be calculated for an equilibrium plasma using Equation 2.44 or found as the point crossing the x-axis in the IV curve [38].

$$V_f = -\frac{k_B T_e}{q} \ln \left(\frac{m_i}{4\pi m_e} \right)^{3/2} \quad 2.44$$

By current continuity, the other two probe's current values are equal and opposite to maintain a net current to the probe of zero [22]. There is only one current differential element in this configuration, and the value of the differential bias (the difference in voltage

between the two non-floating probes) determines the magnitude of the current drawn by the double probe.

There is only one location along the plasma's generated IV characteristic where the current is both equal and opposite and the corresponding voltage difference is V_{d3} . By using the double probe characteristic that may be generated using probes one and three as depicted in Figure 1.10, the magnitude of the current for those probes (by the assigned voltage differential) is found. For the simulation, the electron temperature, number density, electron distribution and differential voltages were input as a test plasma so that the simulated Langmuir probe function may determine the value of the current flowing through the two non-floating probes. Using Langmuir probe theory, including the equilibrium assumption, electron temperature and number density are calculated from the measured current values.

As in the current mode triple probe, the current ratio is used to find the electron temperature. The main difference in voltage mode analysis lies in the probe that draws no current. In Figure 1.10's schematic of the voltage mode, probe two does not have an assigned voltage. Using this nomenclature, Equation 2.41 becomes Equation 2.45.

$$\frac{I_1 - I_2}{I_1 - I_3} = \frac{I_1}{I_1 - I_3} \quad 2.45$$

Because $I_1 = -I_3$, this expression is always equal to one-half. The number density is calculated as the current mode section, 2.5.1, describes.

In the voltage mode of a triple Langmuir probe, the electron temperature and number density may also be calculated using a grid search as in the double probe [36].

3 LANGMUIR PROBE SIMULATION DESCRIPTION

In this chapter, an overview of the Langmuir probe simulation program is presented.

3.1 Overarching Structure

A simulation of a Langmuir Probe collecting data from a plasma was written using MATLAB software. Separate programs were written for single, double, current mode triple, and voltage mode triple probes; these programs call on subroutines that calculate ion and electron currents collected by each Langmuir Probe. The programs simulate the circuitry involved with each probe as described in Chapter 2: Methodology. To run them, a code called *MainProgram* requests user input to choose the type of probe that is run, the dimensionality (one-dimensional or three-dimensional), the percentages of the main and secondary Maxwellian distributions present in the probability function, and the known electron temperatures and number density. The overall program hierarchy is fairly simple and is depicted in Figure 3.1, although it requires a number of subprograms to make the necessary calculations in a consistent manner.

A chart of possible input options is found in Figure 3.2 showing all options for the *MainProgram* initial inputs and the probe-specific options described below.

Separating the percentages of each Maxwell-Boltzmann distribution was a deliberate choice. By doing so, a multi-energy level plasma component's contribution may be examined in isolation. All electron temperatures input to the program are assumed to be in

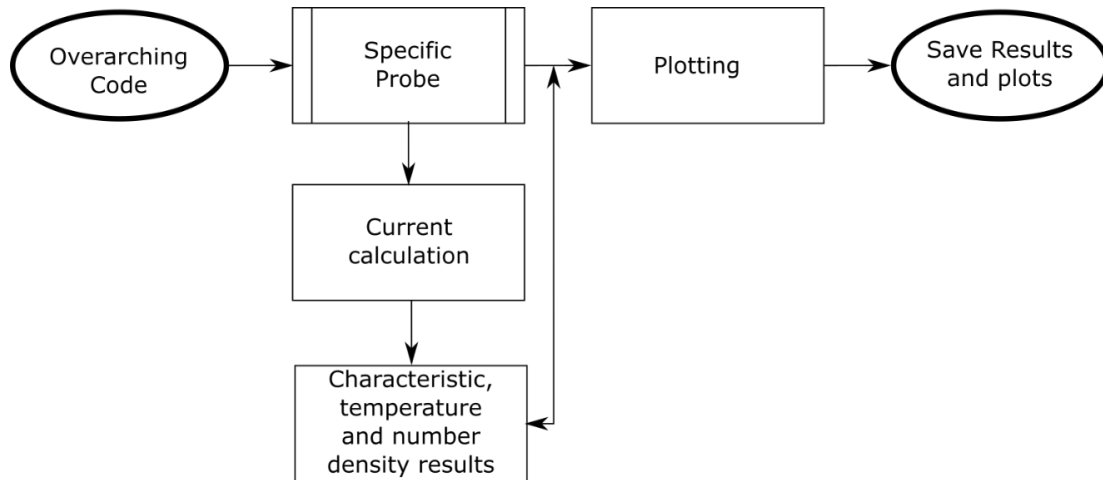


Figure 3.1 Overview of code hierarchy.

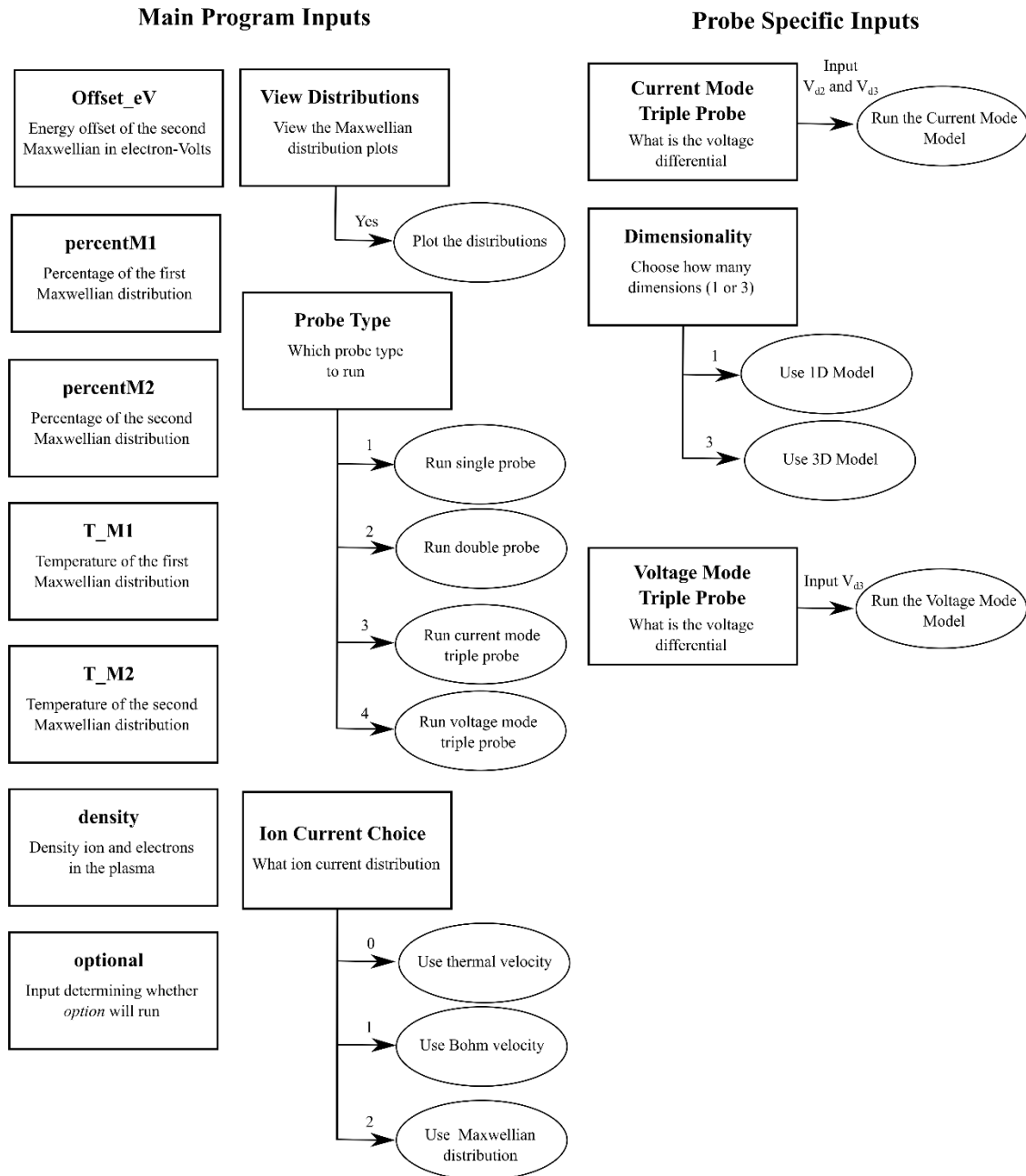


Figure 3.2 Depiction of MATLAB code inputs.

units of electron-Volts, and number density is in particles per cubic meter. A summary of all variables and units used in the program is listed in Table 3.1.

3.2 Subroutines

Subroutine functions are used to calculate values such as electron or ion current across all simulated probe models. They are separate so that the calculations are performed consistently throughout the programs. The individual probe functions do not fall into this category and are covered in the Simulated Probe Models section. The subroutines f , fi , $f_{current}$, *Intersections*, *ionCurrent*, and *order*, are briefly described in the purpose column of Table 3.2. The more unusual subroutines are *option* and *vtoI*; they will be described below.

The subroutine f is used to calculate the electron distribution function as described in Chapter 2 using the inputs from the *MainProgram*. fi and similarly calculates the ion distribution if a Maxwellian distribution is chosen. The $f_{current}$ function is similar to f , but multiplies the distribution function by the velocity so that integration in the *vtoI* function is smoother and provides a mean velocity. The *Intersections* function is fairly self-explanatory: it takes two lines (their values for the x -axis and y -axis) and finds the intersection point(s). *ionCurrent* calculates the ion current to the probe as either the thermal ion velocity associated with the Maxwellian function, fi , or the Bohm ion velocity.

The *option* code, which is executed if any value is in the field, is meant to look specifically at how much charge flux, and therefore current, is contributed by the individual

Table 3.1 Units of common variables in the MATLAB simulation.

Variable	Symbol	Units
Electron Temperature	T_e	eV
Number Density	n_e	particles/m ³
Velocity	v	m/s
Electric Potential	V	V
Current	I	A
Electron Charge	q	C
Boltzmann Constant	k_B	m ² kg/s ² K
Electron Mass	m_e	kg
Probe Area	A	m ²

Table 3.2 List of sub-programs involved in the Langmuir probe simulation.

Function Name	Purpose
f	Calculates the electron Maxwellian distribution function.
fi	Calculates a ion Maxwellian distribution function if input.
fcurrent	Calculates the electron distribution function for flux to be used in the $vtoI$ current calculation.
Intersections	Finds the intersection of two lines. This function was written by Douglas M. Schwarz and obtained from MathWorks Online.
ionCurrent	Calculates the ion current to the probe.
order	Finds the order of a number.
option	Optional area integration.
vtoI	Converts the velocity to current within a probe simulation.

components of a plasma velocity distribution assuming a specific barrier velocity or energy. This is done by integrating over the Maxwellian distribution functions separately. Then, *option* redirects the program to complete the simulation to see the final results as calculated by *MainProgram*.

vtoI calculates the current values of the IV characteristic based on the electron velocity, voltage, and distribution percentages given. It does so in three steps: assigning a barrier voltage, finding the velocity that bombards the probe based on that voltage, and integrating that velocity to find the electron flux and current. The voltage array input is used to assign the minimum velocity, v_{min} , that may bombard the probe by taking every voltage index and iteratively converting them to velocity.

$$v_{min} = \sqrt{\frac{2Vq}{m_e}} \quad 3.1$$

Imaginary values of velocity are not used (values of v_{min} for $V < 0$), and so the IV characteristic is limited to negative voltage values since the electron charge is negative. Positive voltage values have the current assigned as the electron saturation current value. Higher energy states are not calculated or considered. To find the velocities greater than the minimum required, the MATLAB built-in “find” function was used in a logical expression. After this, for every (originally voltage) value, the array of velocities that overcome the minimum velocity requirement are integrated using the built-in trapezoidal integration function to integrate the f function. This produces a flux which is then

multiplied by the probe area and the charge of an electron to get electron current to the probe for every voltage value.

3.3 Simulated Probe Models

As stated previously, the probe models are categorized separately from the subroutines. The subroutines serve as physics-based support for these models so that repetitive calculations may be minimized and to ensure consistent calculations across the probe models. A list of these probe programs may be seen in Table 3.3. Because the single probe code is very simple, it does not require a separate function and is incorporated in *MainProgram*.

Table 3.3 List of first-level programs for the MATLAB code.

Function Name	Purpose
Voltage_mode	Simulates a voltage mode triple Langmuir probe response and analysis.
Current_Probe	Simulates a current mode triple Langmuir probe response and analysis.
doubleprobe	Simulates a double Langmuir probe response and analysis.
doublecheck	Calculates an equilibrium characteristic for a double probe based on Equation 3.1.
Singlecheck	Calculates an equilibrium characteristic for a single probe based on Equation 2.36.

3.3.1 *Single Langmuir Probe*

In this probe model, the electron current, I_e , is calculated using the *vtoI* subroutine. The ion current, I_i , profile is calculated based on user input as the thermal velocity or Bohm velocity, and the total current found by adding I_e and I_i . This results in the numerically-generated IV characteristic.

To validate the model, the analytic solution for an IV characteristic curve for a Maxwellian plasma using Bohm velocity is used. The same parameters are input into the equation, shown as Equation 2.36, and the IV characteristic of a plasma with a distribution in thermal equilibrium is found. The comparison of the numeric simulation IV characteristic and the analytic function's characteristic is discussed in the Results chapter.

3.3.2 *Double Langmuir Probe*

The double probe code first divides the current and associated voltage values of the IV curve calculated for one of the probes (as in the single probe section) into two groups: one for positive current values and one for negative. The magnitude of the negative current values is compared to the magnitude of the positive so that for every instance of $|I_1| = |I_3|$, there is an associated voltage difference, $V_D = V_3 - V_1$ as depicted in Figure 1.9. The double probe IV characteristic is then constructed as the current value (I) plotted against the difference in voltage, V_D as shown in Figure 2.1.

Plasma parameters are found using the slope of the characteristic curve as outlined in Section 2.4. The numeric IV characteristic curve, if Maxwellian, is validated by comparison to Equation 3.1 below [18].

$$I = \frac{1}{4} q n_e A \sqrt{\frac{8ZT_i}{\pi m_i}} \tanh\left(\frac{qV_D}{2k_B T_e}\right) \quad 3.1$$

where Z , the charge state, is unity for this simulation in all cases, m_i is the ion mass, and V_D is the difference in electric potential between the probes.

3.3.3 Current Mode Triple Langmuir Probe

The current mode triple probe program starts with assigning preliminary current and voltage values as I_1, V_1, I_2, V_2, I_3 , and V_3 . These values are governed by the assigned voltage differentials V_{d2} and V_{d3} , shown schematically in Figure 1.10. Starting in the electron saturation region, I_1 and V_1 are assigned as the rightmost curve values. The voltage differential values then dictate the remaining values of current and voltage. For example, a current mode Langmuir probe in a Maxwellian plasma at 5 eV with a number density of 1×10^{19} particles/m³ and values of V_{d2} and V_{d3} of 3 and 6 respectively, the current and voltage values are listed in Table 3.4 for each probe tip.

To find the probe tip's current and voltage values, this construction of differential voltage (and associated current) is “slid” along the IV characteristic to find the minimum value of the sum of the currents to satisfy current continuity (Equation 3.2).

$$I_1 + I_2 + I_3 = 0 (\rightarrow \epsilon) \quad 3.2$$

Table 3.4 Current and voltage value examples for a plasma in thermal equilibrium.

Probe Number	Current Value (A)	Electric Potential (V)
1	2.6298×10^{-5}	-23.518
2	-4.6628×10^{-6}	-26.518
3	-2.1652×10^{-5}	-29.518

The currents are calculated as described in Sections 2.1 and 2.2. To refine the values, the rough current values are taken through an iterative loop until the order of ϵ is significantly (three orders of magnitude) smaller than the order of the smallest current value that has been recalculated. The loop is also terminated if a precision limit ($n \times 10^{-16}$) is reached. It is stated in Section 2.5 that the temperature for triple Langmuir is found Iteratively. To do this, a temperature mesh is generated and Equation 2.41 is solved for each value of the mesh. The temperature value which matches the current ratio found is taken as the electron temperature. The mesh is generated in units of Kelvin and incremented by one from 5,000 K to 6×10^7 K or from roughly 0.5 eV to 5,000 eV. It should be noted that this implies the found electron temperature value's precision is limited by the precision of current values calculated and not by the temperature mesh's increment.

3.3.4 Voltage Mode Triple Langmuir Probe

The voltage mode triple Langmuir probe code is similar to the double Langmuir probe setup in that matching currents must be found to satisfy current continuity ($I_1 = I_3$). The floating probe's current, I_2 , is known to be zero, and the voltage that probe two acquires may be found after the IV curve is constructed by using the *Intersections* code where the second input function is a horizontal line at zero current as seen in Figure 3.3.

The same routine to match current values in the generated IV characteristic is used in this code and the double probe program. To calculate the electron temperature and number density, the procedure used in the current mode probe is implemented to find the value of electron temperature most suited to satisfying the current ratio ($I_1 = I_3$). The same mesh criteria for electron temperature as in the current mode probe program is used in this program.

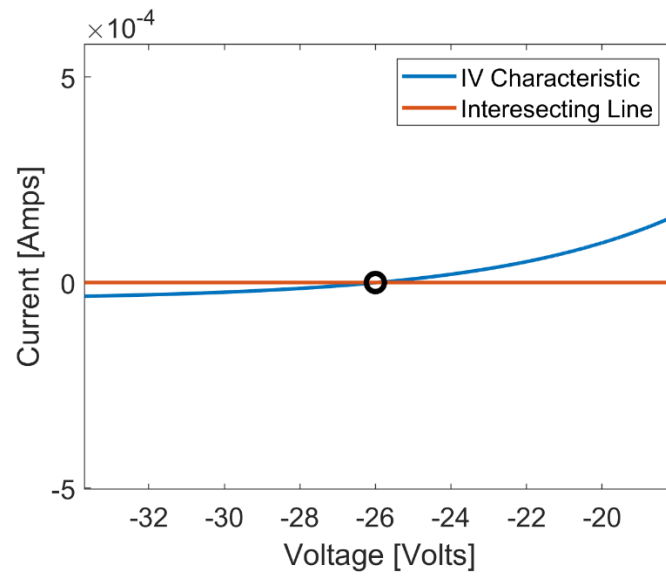
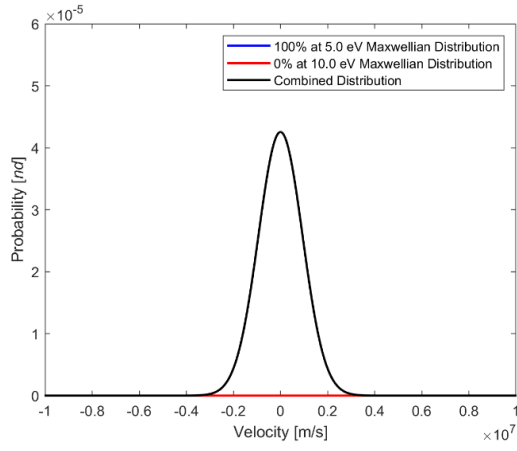


Figure 3.3 Depiction of intersection point.

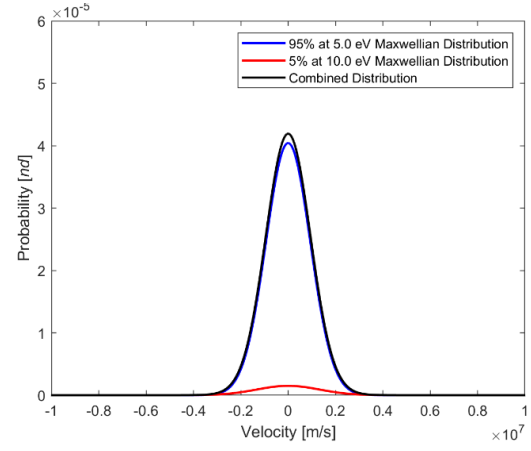
4 RESULTS

The results presented in this chapter are a representative case study of the results from numerous simulations with varying parameters. For all cases presented in this chapter, a one-dimensional model was used with an input number density of $n_e = 1 \times 10^{19}$ particles/m³, and Bohm velocity was assumed for ion velocity. The main Maxwellian curve had an input energy, or temperature, of 5 eV and comprised 95% of the total distribution. For non-equilibrium cases, both two-temperature or bump-on-the-tail, the secondary Maxwellian's energy was set to 10 eV. If an offset energy was included as in the bump-on-the-tail distribution cases, it was set to 1 eV. In the triple Langmuir probe simulations, the differential voltage values were set such that $V_{d2} = 3$ eV and $V_{d3} = 6$ eV, where in voltage mode V_{d2} is the sole differential voltage. Each distribution type described by these parameters may be seen in Figure 4.1.

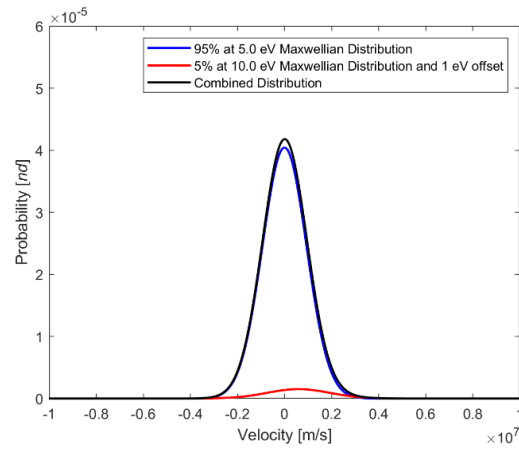
The I-V characteristic for the equilibrium cases, shown in Figure 4.2, were used to validate the simulation by comparing electron temperature and number density from a plasma matching all assumptions used in analysis with the calculated values of these parameters. The curves labeled “Theoretical” were calculated from a single equation based on the chosen plasma parameters, and the “Calculated” curve was generated using the Langmuir probe models. The equilibrium cases run with the one-dimensional model closely follow the theoretical solutions (Equations 2.36 and 2.37) shown in Figure 4.2. Number density



(a) Equilibrium electron distribution.

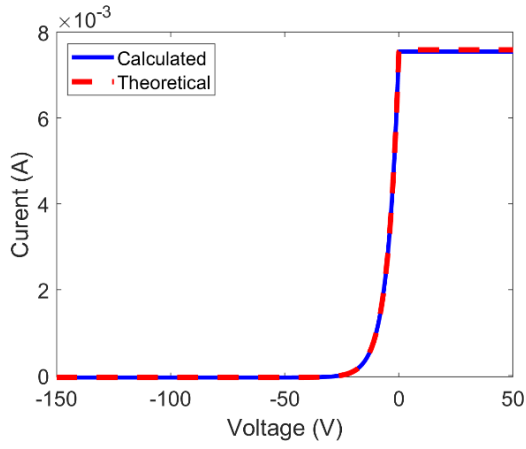


(b) Two-temperature electron distribution.

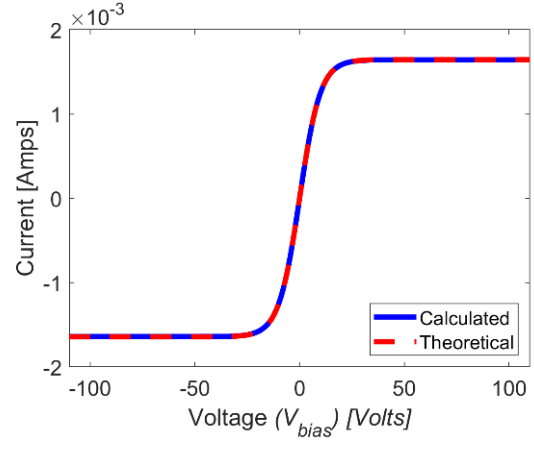


(c) Bump-on-the-tail electron distribution with a 1eV offset.

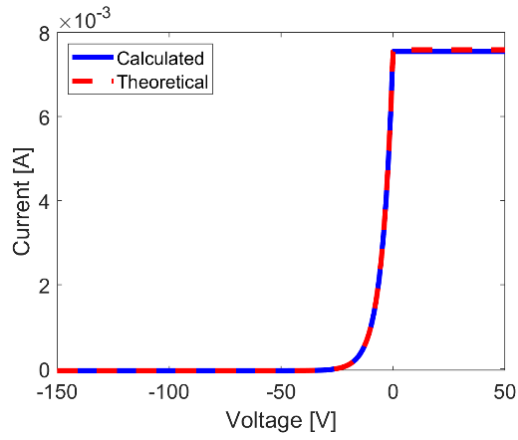
Figure 4.1 Electron distribution depictions.



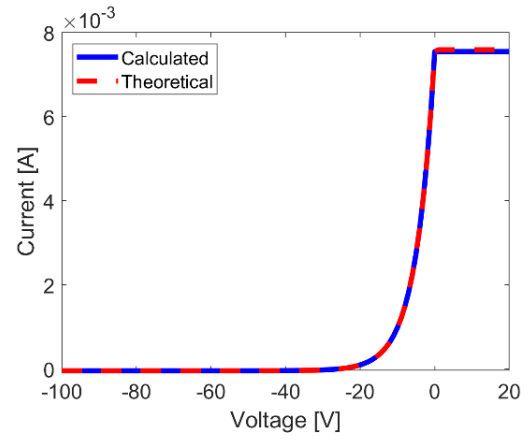
(a) IV characteristic for a single probe.



(b) IV characteristic for a double probe.



(c) IV characteristic for current mode.



(d) IV characteristic for voltage mode.

Figure 4.2 Equilibrium electron probability IV characteristic results.

calculated for all cases with an equilibrium matched well: deviating from the true value by at most $0.141 \times 10^{19} \text{m}^{-3}$ in this case study.

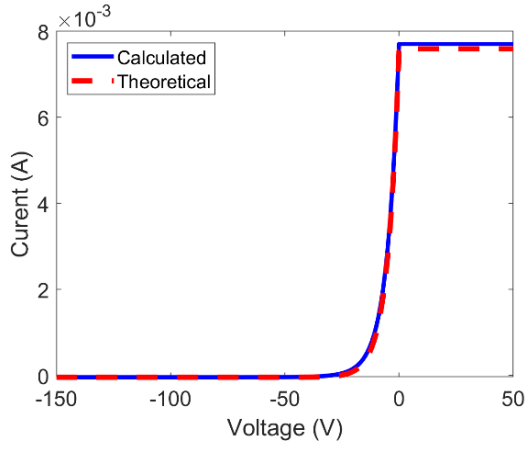
The calculated IV characteristic functions for the current calculated to the probe and the theoretical equation for an equilibrium electron distribution match within 0.15% of the analytic model results using the mean absolute percentage error. The mean absolute percentage error, M , as calculated with Equation 4.1 for each equilibrium curve type is shown in Table 4.1.

$$M = \frac{100\%}{n} \sum_{i=1}^n \left| \frac{I_{t_i} - I_{c_i}}{I_{t_i}} \right| \quad 4.1$$

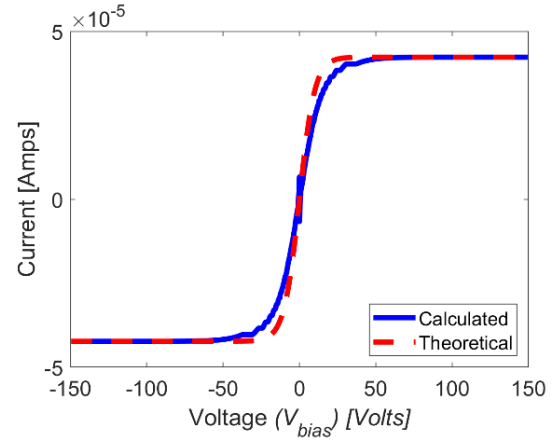
where I_t is the current calculated from the analytic function in Equation 2.36 using solely the bulk temperature of the plasma, and I_c is the current found by the simulation.

All curves were compared to the analytic model which does not account for the secondary (non-equilibrium) electron temperature distribution. The analytic model only uses the plasma bulk temperature to calculate the resulting theoretical IV characteristic and is used as a comparative measure of the curve deviation. For all electron distribution cases the calculated current to the probe is compared to the theoretical curve. With this validation, the results for two-temperature and bump-on-the-tail electron distributions using the described parameters may be seen in Figure 4.3 and Figure 4.4.

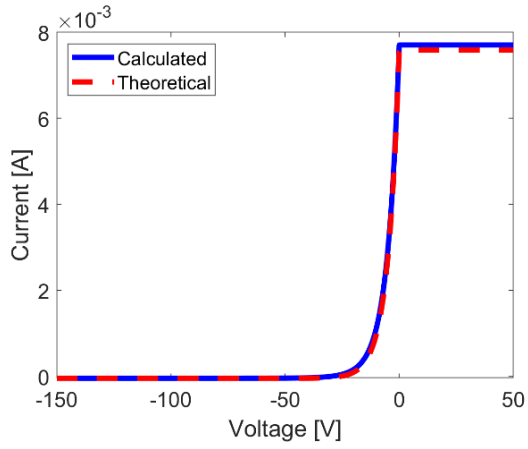
The trend in electron temperature and number density results created by changing the electron velocity probability distribution are consistent as shown in Figure 4.5 and Figure 4.6.



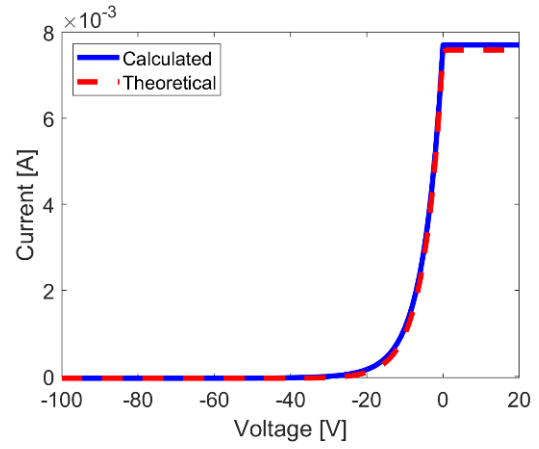
(a) IV characteristic for a single probe.



(b) IV characteristic for a double probe.

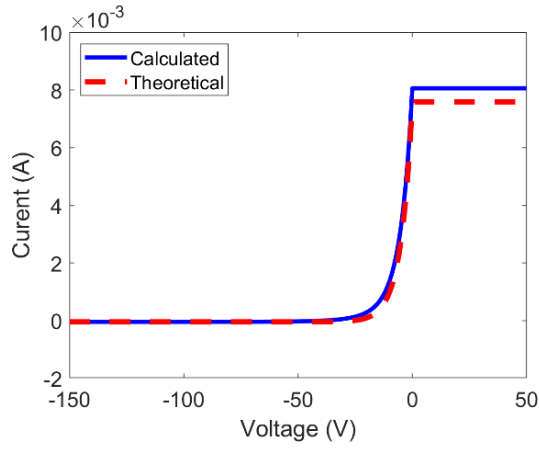


(c) IV characteristic for a current mode probe.

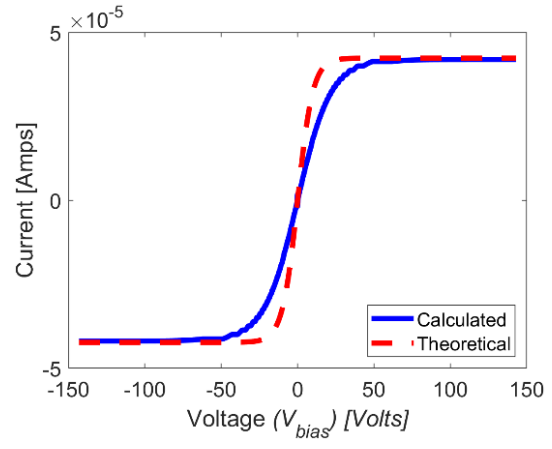


(d) IV characteristic for a voltage mode probe.

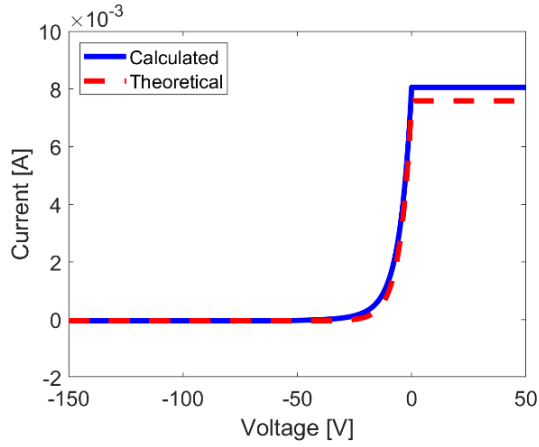
Figure 4.3 Two-temperature electron velocity probability IV characteristic results, one-dimensional.



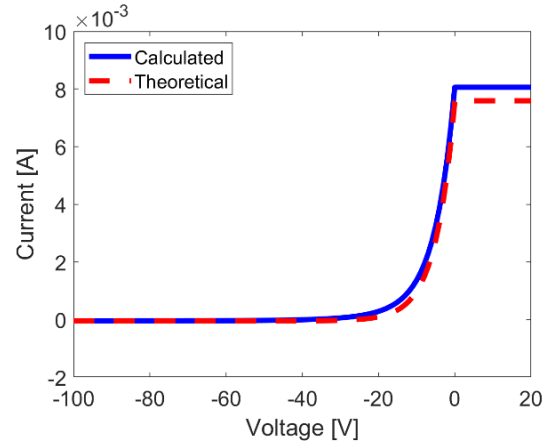
(a) IV characteristic for a single probe.



(b) IV characteristic for a double probe.



(c) IV characteristic for a current mode probe.



(d) IV characteristic for a voltage mode probe.

Figure 4.4 Bump-on-the-tail electron velocity probability IV characteristic results, one-dimensional.

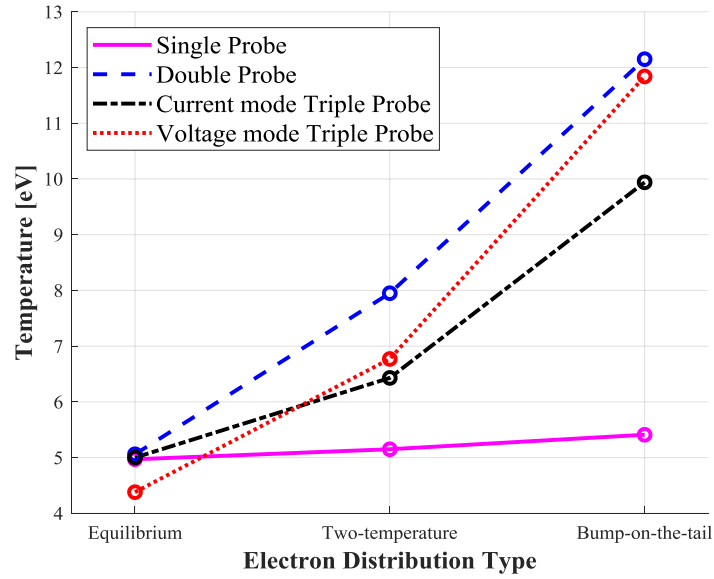


Figure 4.5 Temperature results where electron distribution types one, two, and three indicate an equilibrium, two-temperature, and bump-on-the-tail electron distribution respectively.

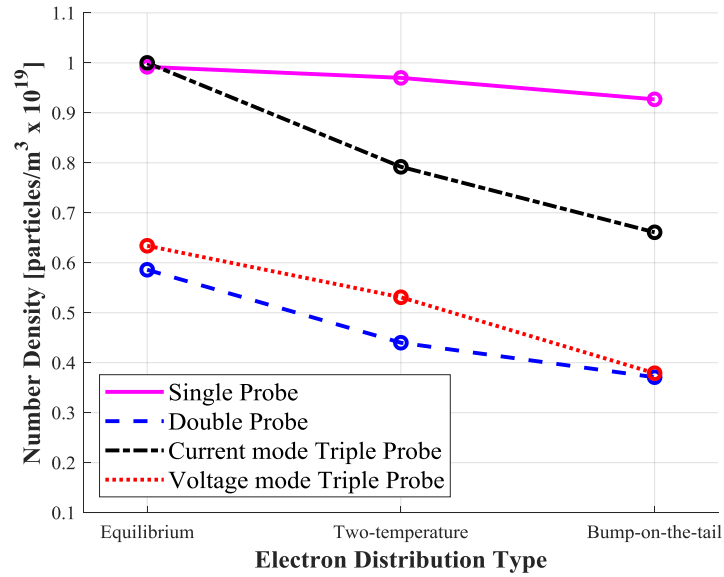


Figure 4.6 Number density results where electron distribution types one, two, and three indicate an equilibrium, two-temperature, and bump-on-the-tail electron distribution respectively.

The results for all Langmuir probe types using the parameters described are enumerated in Table 4.1. The temperature of the primary Maxwellian distribution is set to 5 eV and the electron temperature for the secondary Maxwellian (if present) is set to 10 eV. Electron velocity probability distribution types range further from an equilibrium electron velocity probability distribution as the assigned distribution type number increases. As seen in Figure 4.5 and Figure 4.6, types one through three indicate an equilibrium, two-temperature, or bump-on-the-tail electron probability distribution respectively. As the distribution is taken further from the equilibrium assumption, the electron temperature found by equilibrium Langmuir probe analysis increases and the calculated electron number density decreases.

Inaccuracies in the equilibrium distribution, of up to 41.4%, cases need to be addressed. The simplest way to improve the results for the voltage and current modes is to increase the resolution requirements of the calculated current to the probes. Double Langmuir probe results in the equilibrium case reported 5.06 eV for the electron temperature instead of five electron-Volts. This is not a high margin of error like in the voltage mode, but it is likely caused by taking the line tangent to the slope of the double probe current voltage characteristic at the origin. As the accuracy of the slope calculation improves, the accuracy of the electron temperature improves. The single Langmuir probe has a similar issue: the electron temperature is found from the linear portion of the electron retarding region after the natural logarithm has been applied. Rigorously defining and

automatically finding this linear portion presents a challenge to consistently use the correct number of data points when calculating the slope to find the electron temperature.

Table 4.1 Results for all probe types and all plasma distribution configurations.

	Electron Distribution Type ¹	1-D Electron Temperature (eV)	1-D Number Density ($\times 10^{19}$ particles/m ³)	Mean Absolute Percentage Error (%) (Equation 4.1)
Single Probe	1	4.97	0.992	0.1052
	2	5.15	0.970	68.39
	3	5.41	0.927	224.0
Double Probe	1	5.06	0.586	2.99×10^{-17}
	2	7.95	0.440	5.98×10^{-16}
	3	12.15	0.371	3.48×10^{-17}
Current Mode	1	5.00	1.000	0.1052
	2	6.43	0.792	69.39
	3	9.94	0.661	224.0
Voltage Mode	1	5.14	1.01	0.1052
	2	7.14	0.830	69.39
	3	10.38	0.674	224.0

¹ Electron distribution type is as follows:

- 1 Equilibrium electron probability distribution,
- 2 Two-temperature electron probability distribution, and
- 3 Bump-on-the-tail electron probability distribution

5 CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Single, double, and triple voltage and current Langmuir probe theory was examined and implemented in a program. The probes sampled a variety of plasma types, both equilibrium and non-equilibrium, differing by electron velocity probability distribution shape. To compare the simulation results with the known analytic solution for equilibrium distributions, Bohm velocity for ions was used in the current work. The equilibrium electron probability distribution plasma cases agreed within expected bounds, validating the simulation's ability to model Langmuir probe plasma measurements according to current theory.

The electron velocity probability distributions influenced the electron temperature and number density results. As the distribution was shifted further from the assumed equilibrium distribution shape, the reported electron temperature for all probes rose from bulk equilibrium temperature. The equilibrium cases used to validate the simulation showed a 1.15% deviation from the electron temperature, the two-temperature case resulted in an average deviation of 33.35%, and the bump-on-the-tail distribution an average deviation of 89.4%. From this, the Langmuir probe results for two temperature electron probability distribution plasmas were closer to the bulk plasma temperature than bump-on-the-tail electron distribution plasmas; neither represented the plasma components accurately. Electron number density values fell as the electron velocity distribution shifted

from the equilibrium distribution. Across all probe types (single, double, current mode and voltage mode triple Langmuir probes) the average equilibrium deviation was 10.8%, the two-temperature case was 24.2%, and the deviation for the bump-on-the-tail simulations was 34.18%.

Single Langmuir probe analysis involves first finding the natural logarithm of the electron retarding region, then calculating a slope from the resulting linear portion. How much of the curve is used in this calculation impacts the electron temperature (and the electron number density) calculations. In non-equilibrium cases, the electron retarding region is shown to have a shallower slope, resulting in a higher reported electron temperature than the bulk plasma temperature. The density is dependent on the ion saturation current value, i_+ , taken from the IV characteristic curve as well as the electron temperature; any deviation in calculated electron temperature impacts the number density calculations. Inaccuracy in number density calculations are expected: Langmuir probes are not excellent diagnostic tools for determining number density due to the approximations involved in ion current analysis [18], [19].

All non-equilibrium Langmuir probe simulation results showed significant deviation from the input electron temperature and number density values, especially for the bump-on-the-tail non-equilibrium distributions. Electron temperature results for non-equilibrium plasmas does not represent the bulk temperature of the plasma or the electron temperature of the secondary Maxwellian distribution for cases with non-equilibrium electron velocity probability distributions. The bump-on-the-tail plasma may result in a

temperature higher than the bulk plasma electron temperature or the secondary electron temperature, as seen in the double probe and voltage mode triple probe simulation results. This is due to the extra kinetic energy of the shifted electrons allowing them to overcome the electrostatic barrier around the Langmuir probe(s).

It is apparent from the results that non-Maxwellian plasmas significantly affect the temperature and number density results generated by Langmuir probes. In cases where the plasma is non-equilibrium, Langmuir probes are unacceptable diagnostic tools to garner accurate plasma characteristics. The application of this information is particularly important for pulsed plasma diagnostics, and when there is a known nonequilibrium electron distribution, alternative techniques to garner plasma characteristics should be investigated.

5.2 Future Work

In the future, the fidelity of the model should be improved by rigorously defining how to find the slope of the lines in both single and double Langmuir probe analyses. Furthermore, the ion current can be changed to a Maxwellian distribution to better reflect the ions' behavior. This simulation may also be validated with experimental results by injecting a controlled plasma with a beam of electrons and using both Langmuir probes and other diagnostics such as passive spectroscopy to compare the resulting electron temperature values.

LIST OF REFERENCES

- [1] J. G. Laframboise, "Theory of Spherical and Cylindrical Langmuir Probes in a Collisionless, Maxwellian Plasma at Rest," UTIAS, 1966.
- [2] I. H. Hutchinson, Principles of Plasma Diagnostics, 2 ed., Cambridge: Cambridge University Press, 1987.
- [3] I. Langmuir, "Oscillations in ionized gases," Proceedings of the National Academy of Sciences of the United States of America, Schenectady, NY, 1928.
- [4] R. H. Huddleston and S. L. Leonard, Plasma Diagnostic Techniques, vol. 21, New York, New York: Academic Press Inc., 1965.
- [5] D. Halliday, R. Resnick and J. Walker, Fundamentals of Physics, Hoboken, NJ.: John Wiley & Sons, Inc., 2011.
- [6] R. Fitzpatrick, "The Lorentz Force," University of Texas at Austin, Austin, 2006.
- [7] R. G. Jahn, Physics of Electric Propulsion, New York: McGraw-Hill, 1968.
- [8] S. Bose, M. Kaur, K. K. Barada, J. Ghosh, P. K. Chattopadhyay and R. Pal, "Understanding the working of a B-dot Probe," *European Journal of Physics*, vol. 20, 2019.
- [9] L. B. King, A. D. Gallimore and C. M. Marrese, "Transport-Property Measurements in the Plume of an SPT-100 Hall Thruster," *Journal of Propulsion and Power*, vol. 14, no. 3, pp. 327-335, 1998.
- [10] A. Donne, M. Bock, I. G. J. Classen, M. Von Hellermann, K. Jakubowska, R. Jaspers, C. J. Barth, H. Meiden, T. Oyeveaar, R. E. van de Poll, S. Varshney, G.

- Bertschinger, W. Biel, C. Busch, K. H. Finken, H. R. Koslowski and K, "Overview of Core Diagnostics for TEXTOR," *Fusion Science and Technology*, vol. 47, no. 2, pp. 220-245, 2005.
- [11] A. J. H. Donne, "Introduction to Plasma Diagnostics," *Transactions of Fusion Science and Technology*, vol. 49, p. 349 to 356, 2006.
- [12] J. C. Maxwell, The Scientific Papers of James Clerk Maxwell, W. D. Niven, Ed., New York, NY: Dover Publications, 1965.
- [13] T. Guenault, Statistical Physics, Second ed., Springer, 1995.
- [14] J. W. Rolf, Modern Physics from α to Z, New York, Chichester: Wiley, 1994.
- [15] J. W. Shim and R. Gatignol, "How to obtain higher-order multivariate Hermite expansion of Maxwell–Boltzmann distribution by using Taylor Expansion?," *Zeitschrift für angewandte Mathematik und Physik ZAMP*, vol. 64, pp. 473-482, 2013.
- [16] R. J. Carman and R. P. Mildren, "Electron energy distribution functions for modelling the plasma kinetics in dielectric barrier discharges," *Journal of Physics D: Applied Physics*, vol. 33, no. 19, pp. L99-L103, 2000.
- [17] Z. Peradzyński, K. Makowski and J. Kurzyna, "Early stage of the discharge in ablativepulsed plasma thrusters," *Plasma Sources Sci. Technol.*, vol. 28, no. 024001, 2019.

- [18] D. K. Owens, "Lecture IV: Langmuir Probes," Princeton Plasma Physics Laboratory, Princeton, 2000.
- [19] F. F. Chen, *Lecture Notes on Langmuir Probe Diagnostics*, Jeju, Korea: University of California, Los Angeles, 2003.
- [20] S. Bhattari and L. N. Mishra, "Theoretical Study of Spherical Langmuir Probe in Maxwellian Plasma," *Journal of Physics and Astronomy*, vol. 5, no. 1, 2017.
- [21] W. R. Hoegy and L. H. Brace, "Use of Langmuir probes in non-Maxwellian space plasmas," *Review of Scientific Instruments*, vol. 70, pp. 3015-3024, 1 April 1999.
- [22] S.-L. Chen and T. Sekiguchi, "Instantaneous Direct-Display System of Plasma Parameters by Means of Triple Probe," *Journal of Applied Physics*, vol. 36, no. No. 8, pp. 2363-2375, 1965.
- [23] E. V. Shun'ko, *Langmuir Probe in Theory and Practice*, Boca Raton, Florida: Universal Publishers, 2008.
- [24] S. V. Singh, *Investigation of ICP RF discharges by means of a Langmuir probe*, Varanasi: Ruhr-Universität Bochum, 2004.
- [25] D. J. Asselin, *CHARACTERIZATION OF THE NEAR-PLUME REGION OF A LOW-CURRENT HOLLOW CATHODE*, Worcester Polytechnic Institute, 2011.
- [26] J. Brown, "Design and Implementation of a Double Langmuir Probe for an RF Plasma," Princeton Plasma Physics Laboratory, Princeton, 2017.

- [27] E. Meshcheryakova, M. Zibrov, A. Kaziev, G. Khodachenko and A. Pisarev, "Langmuir probe diagnostics of low-pressure inductively coupled," in *Conference on Plasma-Surface Interactions*, Moscow, 2015.
- [28] L. Oksuz, F. Soberon and A. R. Ellingboe, "Analysis of uncompensated Langmuir probe characteristics in radio-frequency discharges revisited," *Journal of Applied Physics*, vol. 99, 2006.
- [29] S. A. Irimiciuc, S. Gurlui, G. Bulai, P. Nica, M. Agop and C. Focsa, "Langmuir probe investigation of transient plasmas generated by femtosecond laser ablation of several metals: Influence of the target physical properties on the plume dynamics," *Applied Surface Science*, vol. 417, pp. 108-118, 2017.
- [30] S. Ranvier, M. Anciaux, P. Cardoen, E. Gamby, I. S. Bonnewijn, J. D. Keyser, D. Pieroux and J. P. Lebreton, "Use of a Langmuir Probe Instrument on Board a Pico-Satellite," *IEEE Transactions on Plasma Science*, vol. 45, no. 8, pp. 2007-2012, 2017.
- [31] E. Astafyeva, I. Zakharenkova, J. D. Huba, E. Doornbos and J. v. d. IJssel, "Global Ionospheric and Thermospheric Effects of the June 2015 Geomagnetic Disturbances: Multi-Instrumental Observations and Modeling," *Journal of Geophysical Research: Space Physics*, vol. 122, pp. 11,716-11,742, 2017.

- [32] D. A. Lister, *Investigation of Langmuir Probes in Non-Maxwellian Plasma Using Particle-in-Cell (PIC) Modeling*, Knoxville, TN: University of Tennessee-Knoxville, 2016.
- [33] P. Lozano and M. Martinez-Sanchez, "MIT OpenCourseWare," 2015. [Online]. Available: <https://ocw.mit.edu..>
- [34] J. Ou, N. Xiang, C. Gan and J. Yang, "Effect of two-temperature electrons distribution on an electrostatic plasma sheath," *Physics of Plasmas*, vol. 20, no. 063502, 2013.
- [35] R. L. Merlino, "Understanding Langmuir probe current-voltage characteristics," *American Association of Physics Teachers*, vol. 75, no. 12, pp. 1078-1085, 2007.
- [36] J. Bergstra and Y. Bengio, "Random Search for Hyper-Parameter Optimization," *Journal of Machine Learning Research*, vol. 13, pp. 281-305, 2012.
- [37] C. J. Montalvo, *Lecture, Mechanical Engineering Analysis*, Mobile, AL, 2016.
- [38] J. B. Hoag and S. A. Korff, *Electron and Nuclear Physics*, 3rd ed., New York: D. Van Nostrand, 1948.

APPENDIX

MATLAB CODE

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Main Program (Main_Program)

```
function []= MainProgram(offset_eV,percentM1,percentM2,T_M1,T_M2,optional)
density=1E19;
close all;
%%%% The offset_eV describes (in terms of eV) how much the maxwellian of
%%%% the beam is offset. The percentageOfNotShifted is the percent of the
%%%% area of the maxwellians in our density probability space that the not
%%%% shifted Maxwellian has. In other words, how big the 'normal' plasma
%%%% is. The T_M2 describes how hot the suprathermal beam is. It is
%%%% a factor and is multiplied by the temperature of the stable plasma.
% be able to switch ion distribution in the subroutine but keep options in
% one block
% change maxwellian step
% refine battery based on first order delta v (inside a subroutine)

global T2 iT g T offset ne me k mi ni area echarge smallstep Z M A B C
global choice Dim ionmax n % global declarations

%% Assign constants:
Dim = input('One space or three? (1 or 3)\n');
tf1 = isempty(Dim);
while tf1==1
    Dim = input('Please choose. One-space or three? (1 or 3)\n');
    tf1 = isempty(Dim);
end
smallstep = 1;
initialbarrier = -80:smallstep:20;
velocity_limit = 299792458;
g = percentM2;
n = percentM1;
ne = density;
ni = density;
me = 9.11e-31;
mi = 6.6335209e-26;
k = 1.38064852*10^-23;
echarge = 1.60217662e-19;
T = T_M1*11598.58951872581;
T2 = T_M2*11598.58951872581;
iT = T_M1*11598.58951872581;
kinetic = offset_eV*echarge;
offset = sqrt(2*kinetic/me);
% vBohm = sqrt(echarge*T/mi);
```

```

Z          = 1;
M          = 6.6335209e-26;
probe_diameter = 0.000127;
area       = pi*.25*probe_diameter^2;
if Dim == 1
    v       = -velocity_limit:100:velocity_limit;
else
    v       = 0:100:velocity_limit;
end

%% %%%%%%%%%%% Density Probability
%% %%%%%%%%%%%
stablemax= sqrt(me/(2*pi*k*T))*exp((-me*(v).^2)/(2*k*T));
A = trapz(stablemax);
stabilenorm=n*stablemax/A;

supramax = sqrt(me/(2*pi*k*T2))*exp((-me*(v-offset).^2)/(2*k*T2));
B = trapz(supramax);
supranorm= g*supramax/B;

ionmax = sqrt(mi/(2*pi*k*iT))*exp((-mi*(v).^2)/(2*k*iT));
C= trapz(ionmax);

[~,both,first,second] = f(v);
resp = input('See the distribution(s)? (1= Yes, 0=No)\n');
tf2 = isempty(resp);
while tf2==1
    resp = input('Please choose. See the distribution(s)? (1= Yes, 0=No)\n');
    tf2 = isempty(resp);
end
if resp == 1
    fig = figure();
    set(fig,'color','w');
    set(gca,'FontSize',15);
    plot(v,first,'b-','linewidth',1.5); hold on
    plot(v,second,'r-','linewidth',1.5);
    plot(v,both,'k-','linewidth',1.5)
    ylabel('Probability [ {\itnd} ]')
    txt1 = sprintf('%0f%% at %0.1f eV Maxwellian Distribution',...
        percentM1*100,T_M1);
    txt2 = sprintf('%0f%% at %0.1f eV Maxwellian Distribution',...
        percentM2*100,T_M2);

```

```

tinner = ['%.0f%% at %.1f eV Maxwellian Distribution with a %.0f eV offset'];
txt3 = sprintf(tinner,percentM2*100,T_M2,offset_eV);
if Dim ==1
    xlim([-1 1]*10^7)
    ylim([0 8.5]*10^-5)
    xlabel('Velocity [m/s]');
    title('Velocity Probability Distribution','FontSize',18)
    legend(txt1,txt2,'Combined Distribution','Location','northeast')
else
    xlim([0 7]*10^6)
    xlabel('Speed [m/s]');
    title('Speed Probability Distribution','FontSize',18)
end
if offset_eV == 0
    legend(txt1,txt2,'Combined Distribution','Location','northeast')
else
    legend(txt1,txt3,'Combined Distribution','Location','northeast')
end
ylim([0 6]*10^-5)
end

%% Optional Portion
if exist('optional','var')
    option(v,stablenorm,supranorm,offset_eV,percentM1,T_M1,T_M2,density,n)
else
    %% Continuing Inputs
    s1 = ['What type of probe would you like to analyze?\nInpu'...
        't Probe Type\n1   Single Probe\n2   Double Prob'...
        'e\n3   Current Mode Triple Probe\n4   Voltage M'...
        'ode Triple Probe\nChoice: '];
    specification = input(s1);
    tf3 = isempty(specification);
    while tf3==1
        specification = input(strcat('Please choose. \n',s1));
        tf3 = isempty(specification);
    end
    s2 = ['\nIon distribution?\n0   Thermal\n1   Bohm\n2   '...
        'Maxwellian\nChoice: '];
    choice = input(s2);
    tf4 = isempty(choice);
    while tf4==1
        choice = input(strcat('Please choose. \n',s2));

```

```

    tf4 = isempty(choice);
end

if specification == 3
    s3 = ['\nWhat are your battery voltages?\nInput format'...
        ' is [Vd2,Vd3]: '];
    Vds = num2cell(input(s3));
    tf5 = isempty(Vds);
    while tf5==1
        Vds = input(strcat('Please choose. \n',s3));
        tf5 = isempty(Vds);
    end
    [delv2,delv3] = Vds{:};
end
if specification == 4
    delv3 = input('\nWhat is your differential voltage?\nChoice: ');
    tf6 = isempty(delv3);
    while tf6==1
        delv3 = input(strcat('Please choose. \n','\nWhat is your differential voltage?\nChoice:
    '));
        tf6 = isempty(delv3);
    end
end

%% %%%%%%%%%%%%%%% Single Langmuir Probe
%% %%%%%%%%%%%%%%%
eI = vtoI(initialbarrier,v);
iI = ionCurrent(initialbarrier,v);
summation = eI+iI;

%% %%%%%%%%%%%%%%% Single Probe Check
%% %%%%%%%%%%%%%%%
total_I = singlecheck(initialbarrier);

if specification ==1
    digit = 1;
    summation2=summation(summation>0);
    initialbarrier2= initialbarrier(summation>0);
    summation3= log(summation2);
    for iter = 2:length(summation3)
        if abs(summation3(iter)-summation3(iter-1))>1E-4
            summation4(iter-1) = summation3(iter);

```



```

[~,ind] = min(abs(deltavoltage)); val = 5;
% Calculate the slope of the middle portion based on the index of zero
slope = (doublecurrent(ind-val)-doublecurrent(ind+val))/(...
    deltavoltage(ind-val)-deltavoltage(ind+val));
[xout1,~] = intersections(deltavoltage,doublecurrent(end)*ones(1,...
    length(deltavoltage)),deltavoltage,slope*deltavoltage);
[xout2,~] = intersections(deltavoltage,-doublecurrent(end)*ones(1,...
    length(deltavoltage)),deltavoltage,slope*deltavoltage);
graphV = xout1 - xout2;
T_c = graphV*echarge/(4*k*11598.58951872581);
i_0 = doublecurrent(end);
ne_c = i_0/(echarge*area*sqrt(k*11598.58951872581*T_c/M));

%%% Grid Search Method
a1 = 0.25*echarge*area*sqrt(Z*8*k*iT/(pi*M));
a2 = echarge/(2*k);
Ifit = doublecurrent;
cost = 1e20;

ne_guesses = linspace(7e17,20e19,7000);
T_guesses = linspace(0.25*11598,40*11598,7000);

for idx = 1:length(ne_guesses)
    for jdx = 1:length(T_guesses)
        necalc = ne_guesses(idx);
        Tcalc = T_guesses(jdx);
        Igues = a1*necalc*tanh(a2*deltavoltage/Tcalc);
        new_cost = sum((Igues-Ifit).^2);
        if new_cost < cost
            cost = new_cost;
            Tfit = Tcalc;
            ne_fit = necalc;
        end
    end
end
% Tfit/11598.58951872581;
% ne_fit;
strng = ['Original \tElectron Temperature is \t%g \t\tteV\n\t\t'...
    '\tNumber density is \t\t\t%g \t\tcm^-1\nCalculated \tElectron T'...
    '\temperature is \t%g \t\tteV\n\t\t\tNumber density is \t\t\t%.2e'...
    '\tcm^-1\n'];
fprintf(strng,T_M1,ne,round(T_c,2),ne_c)

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Double Probe Check & plot
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
checkcurrent = doublecheck(deltavoltage);
Tempcurrent= log(doublecurrent);
% semilogy(deltavoltage,Tempcurrent)
fig2 = figure();
set(fig2,'color','white');
plot(deltavoltage, doublecurrent,'b-','Linewidth',3.75,'DisplayName',...
    'Calculated');
hold on
plot(deltavoltage,checkcurrent,'r--','Linewidth',3.75,'DisplayName',...
    'Theoretical');
ylabel('Current [Amps]');
xlabel('Voltage \it(V_{bias}) [Volts]');
legend
set(gca,'FontSize',15);
% title('Double Probe I-V Characteristic')
end

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Current Mode Probe
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

if specification ==3
[T_c,ne_c,I1,I2,I3,V1,V2,V3,minval,voltage,current]= Current_Probe(v,...
    delv2,delv3);

```

```

mytext = ['Current\t\t\t\t Voltage \t\t\t\t I1 = %2.8f \t V1 = %2.5f\n' ...
'I2 = %2.8f \t V2 = %2.5f\n I3 = %2.8f \t V3 = %2.5f\n\n I2+I3+I1 = '...
'%2.12f\n\n Temperature Calculated\t = %3.6f eV \n Temperature Input\t\t'...
' = %3.6f eV \n Number Density Found\t = %2.2e m^-3\n Number Density'...
' Input\t = %2.2e m^-3'];

```

```

total_I = singlecheck(initialbarrier);
sprintf(mytext,I1,V1,I2,V2,I3,V3,minval(end),T_c,T_M1,ne_c,density)

```

```

fig = figure();
set(fig,'color','w');
plot(voltage,current,'b-','Linewidth',3.75,'DisplayName','Calculated');
hold on
[~,ind] = find(initialbarrier==voltage(1));
vv = initialbarrier(ind:end);

```

```

cc = total_I(ind:end);
h1=plot(V1,I1,'ro','Markersize',5,'Linewidth',3,'HandleVisibility','off');
h2=plot(V2,I2,'go','Markersize',5,'Linewidth',3,'HandleVisibility','off');
h3=plot(V3,I3,'ko','Markersize',5,'Linewidth',3,'HandleVisibility','off');
% plot(vv,cc,'r--','Linewidth',3.75,'DisplayName','Theoretical');
xlabel('Voltage [V]');ylabel('Current [A]');
% title('Characteristic Curve with Probe Currents');
legend('location','northwest')
set(gca,'FontSize',15)
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Voltage Mode Probe
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if specification ==4
[T_c,ne_c,I1,I2,I3,V1,V2,V3] = Voltage_mode(...
    initialbarrier,summation,delv3,v);

% %%% Grid Search - assuming I have the equivalent of double probe
% % [~,ind] = min(abs(deltavoltage)); val = 5;
% % slope = (doublecurrent(ind-val)-doublecurrent(ind+val))/(...
% %     deltavoltage(ind-val)-deltavoltage(ind+val));
% % [xout1,~] = intersections(deltavoltage,doublecurrent(end)*ones(1,...
% %     length(deltavoltage)),deltavoltage,slope*deltavoltage);
% % [xout2,~] = intersections(deltavoltage,-doublecurrent(end)*ones(1,...
% %     length(deltavoltage)),deltavoltage,slope*deltavoltage);
% % graphV = xout1 - xout2;
% % T_c_op = graphV*echarge/(4*k*11598.58951872581);
% % i_0 = doublecurrent(end);
% % ne_c_op = i_0/(echarge*area*sqrt(k*11598.58951872581*T_c/M));

% %%% Display text
mytext = ['Current\t\t\t Voltage \t\t\nI1 = %2.8f \t V1 = %2.5f\n' ...
'I2 = %2.8f \t V2 = %2.5f\nI3 = %2.8f \t V3 = %2.5f\n\nTemperatur'...
'e Calculated\t = %3.6f eV \nTemperature Input\t\t = %3.6f eV \nNum'...
'ber Density Found\t = %2.2e m^-3\nNumber Density Input\t = %2.2e m^-3'];
sprintf(mytext,I1,V1,I2,V2,I3,V3,T_c,T_M1,ne_c,density)

% %%% Plotting results
% fig = figure();
% set(fig,'color','w');
% plot(initialbarrier,summation,'b-','Linewidth',3.75,'DisplayName',...

```

```

% 'Calculated');
% hold on
% plot(initialbarrier,total_I,'r--','Linewidth',3.75,'DisplayName',...
% 'Theoretical');
% xlabel('Voltage [V]');ylabel('Current [A]');
% legend('location','northwest')
% set(gca,'FontSize',15);
% % title('Characteristic Curve with Probe Currents','FontSize',15);
end
end% the optional if statement
%% % MAPE
if specification == 2
    deltaSignal = abs(checkcurrent - doublecurrent);
    percentageDifference = deltaSignal ./ checkcurrent;
    percentDiff = percentageDifference(~isinf(percentageDifference));
    meanPctDiff = abs(mean(percentDiff))*100
else
    deltaSignal = abs(total_I - summation);
    percentageDifference = deltaSignal ./ total_I;
    meanPctDiff = abs(mean(percentageDifference))*100
end

```

IV Characteristic Equilibrium Case Check (singlecheck)

```

function total_I= singlecheck(initialbarrier)
global ne ni area echarge k T me choice iT mi
Z = 1;
M = 6.6335209e-26;
total_I = zeros(1,length(initialbarrier));

for iter1 =1:length(initialbarrier)
    if initialbarrier(iter1)<=0
        % following equation is messy; it's the check from the lab notes
        % under "the probe characteristic" and uses Bohm Velocity dist
        if choice == 0
            io = -0.25*area*ni*sqrt(8*k*iT/(pi*mi))*echarge;
        elseif choice == 1
            io = -0.6*ni*area*sqrt(k*T/mi)*echarge; %DK Owens
        elseif choice ==2
            io = -fi(v);
        end
        total_I(iter1)= 0.25*ne*area*echarge*sqrt(8*k*T/(me*pi))...
    end
end

```

```

        *exp(echarge*initialbarrier(iter1)/(k*T))+io; %DK Owens
    else
        total_I(iter1)= 0.25*ne*area*echarge*sqrt(8*k*T/(me*pi));
    end
end

```

Double Probe (doubleprobe)

```
function [deltavoltage, doublecurrent]=doubleprobe(summation,initialbarrier,v)
```

```

%% Note, for the double langmuir probe, the increment of the data
%% significantly alters the shape of the characteristic curve.
[~,index]=min(abs(summation));
voltage2= linspace(initialbarrier(1),initialbarrier(index),400);
current2 = vtoI(voltage2,v)+ionCurrent(voltage2,v);
setv1= linspace(initialbarrier(index),initialbarrier(end),length(voltage2));
currentvector1 = vtoI(setv1,v)+ionCurrent(setv1,v);
for iter3 = 1:length(current2)
    difference= abs(currentvector1+current2(iter3));
    [current1diff(iter3), indexcurrent1(iter3)]= min(difference);
    current1(iter3)= current1diff(iter3)+current2(iter3);
    voltage1(iter3)= setv1(indexcurrent1(iter3));
end

```

```

deltav1= voltage2-voltage1;
deltavoltage= [deltav1 fliplr(-deltav1)];
doublecurrent= [current1, fliplr(-current1)];

```

Double Probe IV Characteristic Check (doublecheck)

```
function checkcurrent = doublecheck(deltavoltage)
```

```
global echarge k T ne area iT Z M ni mi
```

```
% deltav1 is V1- Vf
```

```

% constant= 0.25*echarge*ne*area*sqrt(Z*8*k*iT/(pi*M));
constant = 0.6*ni*area*sqrt(k*T/mi)*echarge;
for iteration= 1:length(deltavoltage)
    checkcurrent(iteration) = constant*tanh(echarge*...
        deltavoltage(iteration)/(2*k*T));
end

```

Current Mode Triple Probe (Current_Probe)

```
function
[T_calculated,ne,I1,I2,I3,V1,V2,V3,minum,changedvoltage,characteristic]=Current_Prob
e(v,delv2,delv3)
global area mi choice iT
%%% Iterative constants and changing parameters
timedout      = 0;
counter       = 1;
changedvoltage = -150:.5:150;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Current Mode

%%% Beginning parameters (the initial values of voltage and current that
%%% are later searched to find the first minimum sum)
V1 = changedvoltage(end);
V2 = V1-delv2;
V3 = V1-delv3;

% Location in the matrix is found to match with the current
indexV2 = find(round(changedvoltage,10)==V2);
indexV3 = find(round(changedvoltage,10)==V3);

maxiter    = indexV3;          % for the loop
electrons  = vtoI(changedvoltage,v);
ions       = ionCurrent(changedvoltage,v);
characteristic = electrons + ions;
% voltage   = changedvoltage;
% current   = characteristic;

% Current values to start at
I1         = characteristic(end);
I2         = characteristic(indexV2);
I3         = characteristic(indexV3);
% We are going from right to left on the IV characteristic which requires
% the length of the data
backward   = length(characteristic);

%%% Find the initial minimum value ( $I2+I3-I1=0$  from Chen&S-dawg)
Isum = zeros(length(maxiter),1);
for iteration = 1:maxiter
    I1(iteration) = characteristic(backward-iteration+1);
    V1(iteration) = changedvoltage(backward-iteration+1);
```

```

I2(iteration) = characteristic(indexV2-iteration+1);
V2(iteration) = changedvoltage(indexV2-iteration+1);
I3(iteration) = characteristic(indexV3-iteration+1);
V3(iteration) = changedvoltage(indexV3-iteration+1);
Isum(iteration) = I1(iteration) + I2(iteration) + I3(iteration);
end

[minval(counter),minind] = min(abs(Isum));

% Store the initial guess (the t stands for temporary)
V1t = V1(minind); V2t = V2(minind); V3t = V3(minind);
I1t = I1(minind); I2t = I2(minind); I3t = I3(minind);

clear Isum V1 V2 V3 I1 I2 I3 minind
V1 = V1t; V2 = V2t; V3 = V3t;
I1 = I1t; I2 = I2t; I3 = I3t;

%%% Refinement
while timedout~=1
% Reassign the new minimum values to be refined
if counter~=1
    V1 = V1Range(minind);
    V2 = V2Range(minind);
    V3 = V3Range(minind);
    I1 = I1Range(minind);
    I2 = I2Range(minind);
    I3 = I3Range(minind);
else
    oldstep = 1/10^(counter-1);
    newstep = 1/10^counter;
end

% Assigning the new search ranges
V1Range = (V1-oldstep):newstep:(V1+oldstep);
V2Range = (V2-oldstep):newstep:(V2+oldstep);
V3Range = (V3-oldstep):newstep:(V3+oldstep);
log = isempty([V1Range,V2Range,V3Range]);
if log ==1
    dbstop
end
I1Range = vtoI(V1Range,v)+ionCurrent(V1Range,v);

```

```

I2Range = vtoI(V2Range,v)+ionCurrent(V2Range,v);
I3Range = vtoI(V3Range,v)+ionCurrent(V3Range,v);

% Find the minimum value
for iteration = 1:min([length(V1Range),length(V2Range),length(...
    V3Range)])
    Isum(iteration) = I3Range(iteration) + I2Range(iteration) + ...
    I1Range(iteration);
end
[minval(counter),minind] = min(abs(Isum));
minum= minval(counter);
Order = (10^order(min(abs([I3Range(minind),I2Range(minind),...
    I1Range(minind)]))))\ (10^order(minum));

% Conditions to exit loop
% 1. If the order of the minimum is <<< the smallest current
% 2. If the loop has iterated 16 times it is likely that one of the
% currents is getting smaller along with the minimum value and that the
% order condition will never be fulfilled. Exit the loop.
if Order <=0.001
    timedout = 1;
    thiswasused=0;
end

counter = counter+1;
% Introduce the new step size (delta(V)) and store the old
oldstep = 1/10^(counter-1);
newstep = 1/10^counter;

if newstep == 0
    timedout = 1;
    thiswasused = 1;
end
end
thiswasused
% Stop in debug mode if there is an error
dbstop if error

%%% Temperature
k      = 1.38064852*10^-23;
echarge = 1.60217662e-19;
phi     = echarge/k;

```

```

Searchrange = 5000:1:6E7; % in Kelvin

comparedRatio = (1-exp(-delv2*phi./Searchrange))./(1-exp(delv3*-phi./...
    Searchrange));
% comparedRatio = (exp(-(phi./Searchrange).*delv3)-1)./(exp(-
(phi./Searchrange).*delv3)-1);
currentratio = (I1-I2)/(I1-I3);

 [~,Index] = min(abs(comparedRatio-currentratio));
T_calculated=Searchrange(Index)/phi;

E = exp(-(phi/(T_calculated*phi))*(delv3-delv2));
%%% use Ji at 9 then 14 for ne from Chen
J_i = (1/area)*(I3-I2*E)/(E-1);

if choice == 0
    ne = (4/echarge)*J_i*sqrt(pi*mi/(8*k*iT));
else
    ne = (1.667/echarge)*J_i*sqrt(mi/(k*T_calculated*phi));
end

Voltage Mode Triple Probe (Voltage_mode)
function [T_c,ne,I1,I2,I3,V1,V2,V3]=Voltage_mode(...
    initialbarrier,summation,delv3,v)
global echarge k smallstep area choice mi iT
%%% Note, for the double langmuir probe, the increment of the data
%%% significantly alters the shape of the characteristic curve.
format long
counter= 1; changedvoltage = -150:.5:150;

%%% Split up the current into positive and negative &
%%% Find the voltages associated with those currents

% cutcurrent3= summation(1:length(summation(summation<=0)));
% cutvoltage3= initialbarrier(1:length(summation(summation<=0)));
% cutcurrent1= summation(length(summation(summation<=0))+1:end);
% cutvoltage1= initialbarrier(length(summation(summation<=0))+1:end);

floating_potential = intersections(initialbarrier,summation,...
    initialbarrier,zeros(1,length(summation)));
I2=0;

```

```

V1 = changedvoltage(end);
V2 = floating_potential;
V3 = V1-delv3;
% Location in the matrix is found to match with the current
indexV1 = find(round(changedvoltage,10)==V1);
indexV3 = find(round(changedvoltage,10)==V3);

maxiter      = indexV3;                % for the loop
electrons    = vtoI(changedvoltage,v);
ions         = ionCurrent(changedvoltage,v);
characteristic = electrons + ions;

% Current values to start at
I1           = characteristic(indexV3);
I3           = characteristic(indexV3);
% We are going from right to left on the IV characteristic which requires
% the length of the data
backward     = length(characteristic);

%%% Find the initial minimum value (I2+I3-I1=0 from Chen&S-dawg)
Isum = zeros(length(maxiter),1);
for iteration = 1:maxiter
    I1(iteration) = characteristic(backward-iteration+1);
    V1(iteration) = changedvoltage(backward-iteration+1);
    I3(iteration) = characteristic(indexV3-iteration+1);
    V3(iteration) = changedvoltage(indexV3-iteration+1);
    Isum(iteration) = I1(iteration) + I3(iteration);
end

[minval(counter),minind] = min(abs(Isum));

% Store the initial guess (the t stands for temporary)
V1t = V1(minind); V3t = V3(minind);
I1t = I1(minind); I3t = I3(minind);

clear Isum V1 V3 I1 I3 minind
V1 = V1t; V3 = V3t;
I1 = I1t; I3 = I3t;
nu=12;

timeout = 0;

```

```

while timedout~=1
% Reassign the new minimum values to be refined
if counter~=1
    V1 = V1Range(minind);
    V3 = V3Range(minind);
    I1 = I1Range(minind)/(10^nu);
    I3 = I3Range(minind)/(10^nu);
else
    oldstep = 1/10^(counter-1);
    newstep = 1/10^counter;
end

% Assigning the new search ranges
V1Range = (V1-oldstep):newstep:(V1+oldstep);
V3Range = (V3-oldstep):newstep:(V3+oldstep);
log = isempty([V1Range,V3Range]);
if log ==1
    dbstop
end
I1Range = (10^nu)*(vtoI(V1Range,v)+ionCurrent(V1Range,v));
I3Range = (10^nu)*(vtoI(V3Range,v)+ionCurrent(V3Range,v));

% Find the minimum value
for iteration = 1:min([length(V1Range),length(V3Range)])
    Isum(iteration) = I3Range(iteration) + I1Range(iteration);
end
[minval(counter),minind] = min(abs(Isum));
minum= minval(counter);
Order = (10^order(min(abs([I3Range(minind),I1Range(minind)...
]))))\ (10^order(minum));

% Conditions to exit loop
% 1. If the order of the minimum is <<< the smallest current
% 2. If the loop has iterated 16 times it is likely that one of the
% currents is getting smaller along with the minimum value and that the
% order condition will never be fulfilled. Exit the loop.
if Order <=0.00001
    timedout = 1;
    thiswasused=0;
end

```

```

    counter = counter+1;
% Introduce the new step size (delta(V)) and store the old
    oldstep = 1/10^(counter-1);
    newstep = 1/10^counter;

    if newstep == 0
        timedout = 1;
        thiswasused = 1
        counter
    end
end

%%%% Calculate the plasma parameters
delv2      = V1-V2;
phi        = echarge/k;
Searchrange = 5000:1:6E7; % in Kelvin
comparedRatio = (1-exp(-delv2*phi./Searchrange))./(1-exp(delv3*-phi./...
    Searchrange));
[~,Index] = min(abs(comparedRatio-.5));
T_c = Searchrange(Index)/phi;

E = exp(-(phi/(T_c*phi))*(delv3-delv2));
%%%% use Ji at 9 then 14 for ne from Chen

J_i = (1/area)*(I3)/(E-1);

if choice == 0
    ne = (4/echarge)*J_i*sqrt(pi*mi/(8*k*iT));
else
    ne = (1.667/echarge)*J_i*sqrt(mi/(k*T_c*phi));
end

```

Maxwellian Functions for Electrons (f)

```

function [toint,final_e,first,second] = f(v)
global T T2 offset me k g Dim ne n B A
%%%%% This routine calculates the maxwellian distribution of the stable
%%%%% plasma and the maxwellian of the beam. It assumes the ions do not
%%%%% contribute and normalizes based on that.
% n is the percentage of the stable plasma, g is the remaining area, A
% and B are normalization factors, xx is velocity.
v = v';

```

```

if Dim ==3
    stablemax= 4*pi*(v.^2).*exp(-me*(v.^2)/(2*k*T))*(me/(2*pi*k*T))^1.5;
    first=n*stablemax/A;
    supramax = 4*pi*((v-offset).^2).*exp(-me*((v-offset).^2)/(2*k*T2))*...
        (me/(2*pi*k*T2))^1.5;
    [~,indexo]= max(supramax);  [~,indext]= min(supramax(1:indexo));
    supramax(1:indext)=0;
    second= g*supramax/B;

    if T==0
        final_e = second;
    elseif T2==0
        final_e = first;
    else
        final_e=(first+second);
    end
    toint = final_e.*v*ne;

elseif Dim == 1
    if n==0
        supramax = sqrt(me/(2*pi*k*T2))*exp((-me*(v-offset).^2)/(2*k*T2));
        second= (g/B).*supramax;
        first = zeros(length(second),1);

        final_e = second;
    elseif g==0
        stablemax= sqrt(me/(2*pi*k*T))*exp((-me*(v).^2)/(2*k*T));
        first=(n/A).*stablemax;
        second= zeros(length(first),1);

        final_e = first;
    else
        stablemax= sqrt(me/(2*pi*k*T))*exp((-me*(v).^2)/(2*k*T));
        first= (n/A).*stablemax;
        supramax = sqrt(me/(2*pi*k*T2))*exp((-me*(v-offset).^2)/(2*k*T2));
        second= (g/B).*supramax;

        final_e= first+second;
    end
    toint = ne*v.*final_e;
else
    disp('Dimensionality error.');
```

end

Maxwellian Function for Ions (fi)

```
function final_i = fi(xx)
global iT k ne C mi
%%%%% This routine calculates the maxwellian distribution of the stable
%%%%% plasma and the maxwellian of the beam. It assumes the ions do not
%%%%% contribute and normalizes based on that.

% n is the percentage of the stable plasma, g is the remaining area, A
% and B are normalization factors, xx is velocity.

if Dim ==3
    max= (1/C)*4*pi*(xx.^2).*exp(-me*(xx.^2)/(2*k*iT))*(me/(2*pi*k*iT))^1.5;
    final_i= ne*xx.*(max);
else
    max= (1/C)*sqrt(mi/(2*pi*k*iT))*exp((-mi*(xx).^2)/(2*k*iT));
    final_i= ne*xx.*(max);
end
```

Ion Current Calculation (ionCurrent)

```
function iI= ionCurrent(initialbarrier,v)
global ni k mi echarge iT area me T choice
%%%%% Assuming the ion flux to the probe is a constant:
if choice==0
    ionflux = 0.25*ni*sqrt(8*k*iT/(pi*me))*-echarge;
elseif choice==1
    ionflux = 0.6*ni*sqrt(k*T/mi))*-echarge;
else
    ionflux = fi(v);
end
iI = ones(1,length(initialbarrier))*ionflux*area;
```

Optional Area Function for Probability (option)

```
function entered = option(v,M1,M2,offset_eV,percentageOfNotShifted,T_M1,...
    T_M2,density,n)
global A B mi me k T echarge T2 iT g choice
fig = figure(); set(fig,'color','w')
plot(v,M1,'b-');hold on
```

```

plot(v,M2,'r-'); %xlim([-4e6,4e6]);
Val = trapz(v,M1)*A+trapz(v,M2)*B;
sprintf('The distribution function has area of %3.4f.',Val)
phi_w = sqrt((T+iT)/me);
V_phi_w = sqrt((2*echarge*phi_w)/(me)); %m/s

```

```

M1cut = M1(v>V_phi_w);
M2cut = M2(v>V_phi_w);
plot(v(v>V_phi_w),M1cut,'g-','linewidth',3)
Area_M1 = trapz(M1cut)
Area_M2 = trapz(M2cut)

```

```

MainProgram(offset_eV,percentageOfNotShifted,g,T_M1,T_M2,density)

```

Function to Find the Order of the Number Input (order)

```

function n = order(val)

```

```

n = floor(log(abs(val))./log(10));

```

Convert Velocity to Current (vtoI)

```

function eI = vtoI(V,v)

```

%%% this converts the velocity (v) to current(I) across the voltage (V)

```

global me echarge area

```

```

dbstop if error

```

```

for j = 1:length(V)

```

```

    if V(j)<0

```

```

        barrier(j)= sqrt((-2*echarge*V(j))/(me));

```

```

    else

```

```

        barrier(j)=0;

```

```

    end

```

```

end

```

```

for ind = 1:length(barrier)

```

```

    tempstor = find(v>barrier(ind));

```

```

    indices(ind) = tempstor(1);

```

```

end

```

```

for indexv2 = 1:length(indices)

```

```

    array = v(indices(indexv2):end);

```

% Mean velocity

according to where the electrons get through the plasma potential

```

    integrater = f(array);

```

```

    flux(indexv2) = trapz(integrater');

```

% integrate from upper bound

to Inf b/c these are travelling in the right direction and have enough energy

```

    eI(indexv2) = flux(indexv2)*area*echarge;

```

end

VITA

Alicia C. Sherrod was born in Fairhope, Alabama. She entered the University of South Alabama and graduated with a Bachelor of Science in Mechanical Engineering. During this time, she was a member of the Honor's College as well as the American Institute of Aeronautics and Astronautics, and the Institute of Electrical and Electronics Engineers.