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Development and Analysis of Interior Permanent Magnet Synchronous Motor with Field Excitation Structure

A Dissertation

Presented for the degree of

Doctor of Philosophy

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Seong Taek Lee

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DEDICATION

This dissertation is dedicated to my family; my wife Min-young, my daughter Jennifer, and my mother, who are always encouraging me with their love.

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I wish to thank all those who helped me in completing my degree during my graduate studies in the University of Tennessee. Several persons collaborated directly and indirectly with my research and encouraged me to finish the works. That is why I wish to dedicate this section to recognize their support.

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ABSTRACT

Throughout the years Hybrid Electric Vehicles (HEV) require an electric motor which has high power density, high efficiency, and wide constant power operating region as well as low manufacturing cost. For these purposes, a new Interior Permanent Magnet Synchronous Motor (IPMSM) with brushless field excitation (BFE) is designed and analyzed. This unique BFE structure is devised to control the amount of the air-gap flux for the purpose of achieving higher torque by increasing the air-gap flux at low speed and wider operating speed range by weakening the flux at high speed.

On the process of developing the new IPMSM, the following analysis results are presented. Firstly, a new analytical method of output torque calculations for IPMSM is shown. This method works well when using a 2-dimensional magnetic equivalent circuit of a machine by omitting the step of calculating the inductance values which are required for the calculation of the reluctance torque. Secondly, there is a research about the slanted air-gap shape. This structure is intended to maximize the ratio of the back-emf of a machine that is controllable by BFE as well as increase the output torque. The study of various slanted air-gap shapes suggests a new method to increase torque density of IPMSM. Lastly, the conventional two-axis IPMSM model is modified to include the cross saturation effect by adding the cross-coupled inductance terms for calculating the power factor and output torque in comparing different saturated conditions. The results suggest that the effect of cross-coupled inductance is increase when d-axis current is high on the negative direction.

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List of principal symbols

Alphabetic

A_g	air-gap area crossing magnetic flux
A_m	permanent magnet area crossing magnetic flux
B	magnetic flux density
B_g	flux density of the air-gap
$B_{g,1}$	the fundamental component of the flux density in the air-gap
$B_g[n]$	flux density of the air-gap for discrete time function
B_m	flux density of the permanent magnet
B_r	remanence of permanent magnet materials
	magnetic flux-density of the rotor
B_{sat}	saturated flux density
B_{sd}	air-gap flux density caused by d-axis stator current
$B_{sd,1}$	fundamental component of d-axis air-gap flux density caused by d-axis stator current
B_{sq}	air-gap flux density caused by q-axis stator current
$B_{sq,1}$	fundamental component of q-axis air-gap flux density caused by q-axis stator current
$\hat{B}_k$	maximum value of the flux density of the air-gap at k^{th} harmonic
BH_{max}	maximum energy product of permanent magnet materials
D_s	inner diameter of the stator
E	open circuit phase emf voltage, electric field
E_d	d-axis emf voltage at steady-state
E_q	q-axis emf voltage at steady-state
H	magnetic field strength amplitude
H_c	coercivity of permanent magnet materials
H_g	magnetic field strength of the air-gap
H_m	magnetic field strength of the permanent magnet
h_m	height of the permanent magnet
h_s	stator slot height
I	phase current of the stator windings
I_a	current of the armature coils
I_{base}	base phase current
I_{ch}	characteristic current
I_d	d-axis phase current at steady-state

I_q	q-axis phase current at steady-state
$\hat{I}$	peak value of input current
i_a, i_b, i_c	instantaneous phase current of the stator windings
I_{max}	maximum input phase current
J	current density in the elements
K_s	current density of the stator windings
K_{sd}	current density caused by d-axis stator current
K_{sq}	current density caused by q-axis stator current
$\hat{K}_s$	the maximum value of the current density
k_c	Carter coefficient
k_e	coefficient for the calculation end turn leakage inductance
k_w	winding factor
L	phase inductance of the stator windings
L_d	phase inductance of d-axis
L_{es}	windings end turn leakage inductance
L_q	phase inductance of q-axis
L_{rot}	axial length of the rotor
L_{st}	axial length of the stator
L_{sl}	stator slot leakage inductance
l	length of the magnetic flux path
l_b	length of the bridge in the rotor
l_g	length of the air-gap passing magnetic flux
l_m	length of the permanent magnet passing magnetic flux
m	number of phases
mmf_g	magnetomotive force at the air-gap
N	number of turns per phase of the stator windings
N_a	number of turns of armature coils
N_o	total number of data at the air-gap
N_{spp}	number of the slots per pole per phase
n_d	conductor density at the stator
P	real power per phase per pole-pair of the motor
p	number of the pole-pairs
Q	reactive power per phase per pole-pair of the motor
R	resistance of the stator windings
R_g	magnetic reluctance of the air-gap
R_l	leakage reluctance of the bridge in the rotor

$R_{l,top}$	leakage reluctance of the top part of the bridge in the rotor
$R_{l,bot}$	leakage reluctance of the bottom part of the bridge in the rotor
R_{pm}	magnetic reluctance of the permanent magnet
R_{σ}	equivalent magnetic reluctance of the permanent magnet
R_{rot}	outer radius of the rotor
S	complex power per phase per pole-pair of the motor
T	output torque
T_{base}	permanent magnet torque at I_{base}
t_b	thickness of the bridge in the rotor
$t_{b,top}$	thickness of the top part of the bridge in the rotor
$t_{b,bot}$	thickness of the bottom part of the bridge in the rotor
U_r	magnetic scalar potential at the air-gap
U_{sd}	magnetic scalar potential at the stator caused by d-axis stator current
U_{sq}	magnetic scalar potential at the stator caused by q-axis stator current
V	phase voltage of the stator windings, electric potential
V_t	input phase voltage
V_d	d-axis phase voltage at steady-state
V_q	q-axis phase voltage at steady-state
W	total magnetic energy stored in a model of FEA
W_c	total magnetic coenergy stored in a model of FEA
W_d	magnetic energy stored in a machine by only d-axis current
W_q	magnetic energy stored in a machine by only q-axis current
w_s	the average of the stator slot width
X	synchronous phase reactance of the stator windings ($=\omega L$)
X_d	synchronous phase reactance of d-axis
X_q	synchronous phase reactance of q-axis

Greek

α_{cp}	coil pitch angle
γ	slot pitch
δ	voltage angle or torque angle
ϕ	magnetic flux
Φ_r	remanence flux from the permanent magnet
Φ'_r	modified remanence flux from the permanent magnet
Φ_g	magnetic flux of the air-gap
Φ_l	leakage magnetic flux of the bridge in the rotor
$\Phi_{l,top}$	leakage magnetic flux of the top part of the bridge in the rotor
$\Phi_{l,bot}$	leakage magnetic flux of the bottom part of the bridge in the rotor
Φ_{sd}	magnetic flux at the stator caused by d-axis stator current
Φ_{sq}	magnetic flux at the stator caused by q-axis stator current
γ	current angle between current vector and q-axis
Λ	overall flux-linkage per phase of the stator windings
Λ_d	overall d-axis flux-linkage per phase of the stator windings
Λ_q	overall q-axis flux-linkage per phase of the stator windings
λ	flux-linkage at the stator windings
λ_{PM}	flux-linkage produced by the permanent magnet
$\lambda_{d,PM}$	d-axis flux-linkage produced by the permanent magnet
$\lambda_{d,id}$	d-axis flux linkage caused by d-axis current
$\lambda_{q,iq}$	q-axis flux linkage caused by q-axis current
μ_0	magnetic permeability of air
μ_r	relative permeability of permanent magnet
θ	conductor angle from base axis
θ_m	rotor position of d-axis from base axis
θ_r	conductor angle from d-axis
θ_{se}	skew angle
ρ	charge density
σ	conductivity of the material
τ_p	winding pitch
τ_s	slot pitch
ω	synchronous angular speed (electrical speed)
ω_e	electrical angular velocity
ω_{max}	maximum angular speed
ξ	saliency ratio between L_d and L_q

Abbreviations

AC	alternating current
BDCM	brushless DC motor, also called BLDC
BEF	brushless field excitation
CPSR	constant power speed region
DC	direct current
d	direct axis
emf	electromotive force
FEA	finite element analysis
HEV	hybrid electric vehicle
ICE	internal combustion engine
IGBT	insulated gated bipolar transistor
IPMSM	interior permanent magnet synchronous motor
IM	induction motor
LSI	large scale integrated
mmf	magnetomotive force
ORNL	Oak Ridge National Laboratory
PEEMEC	Power Electronics and Electric Machinery Research Center
PM	permanent magnet
PM-RSM	permanent magnet assisted reluctance synchronous motor
PMSM	permanent magnet synchronous motor
q	quadrature axis
RPM	revolutions per minute
SPMSM	surface-mounted permanent magnet synchronous motor
SRM	switched reluctance motor
THD	total harmonic distortion

CHAPTER 1

Introduction

1.1 Overview

Throughout the years hybrid electric vehicles (HEVs) have proved themselves worthy to replace a conventional vehicle with an internal combustion engine (ICE) and are an economical choice considering increasing gas prices. For developing an HEV system, one of the main technologies is designing an electric motor for the traction drive.

The ideal torque-speed and power-speed profiles are indicated in Figure 1.1. The maximum motor torque is determined by acceleration at low speed and hill climbing capability of a vehicle, the maximum motor RPM by the maximum speed of a vehicle, and the constant power by vehicle acceleration from the base speed to the maximum speed of a vehicle. The shaded areas indicate the ranges frequently used for a vehicle; so, high efficiency is required. In general, an electric motor can be operated properly in an urban area to meet the requirement of vehicle performance. But the problem of an electric motor lies in the high-speed region, since increasing motor speed while holding constant power cannot be realized easily. Thus, achieving the wider constant power region in the high-speed range is the key point of the research for the traction motor for HEVs.

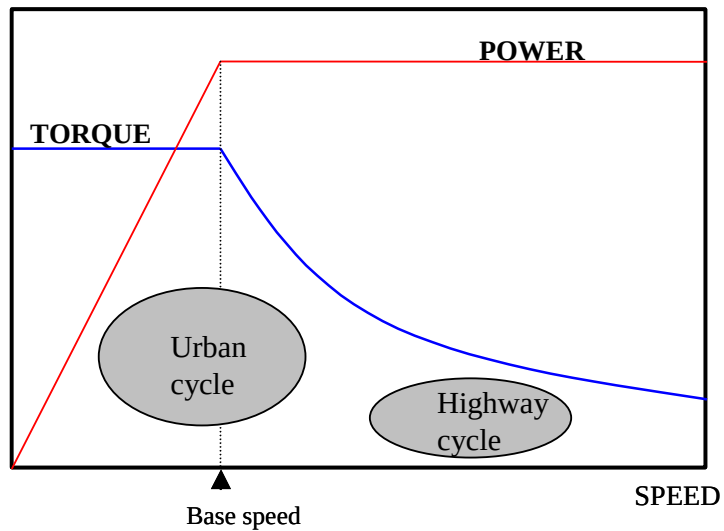


Figure 1.1 Required characteristics of traction motor in a vehicle running mode.

Since DC motor systems have the proper characteristics for the traction application of vehicles, they were popularly used a couple of decades ago [1]. However, in reality, DC motors cannot be attractive for HEV systems anymore because of their low efficiency and frequent need of maintenance caused by their mechanical structures, brushes and commutators. Therefore, thanks to the rapid development of large scale integrated (LSI) circuits and powerful switching devices, such as IGBT (insulated gated bipolar transistor), the Induction Motor (IM), Permanent Magnet Synchronous Motor (PMSM)^{*}, and Switched Reluctance Motor (SRM) have replaced the traction system of most present HEVs. Each type of motor has its own advantages and disadvantages as listed in Table 1.1 which is summarized from references [2–4] with general knowledge about electric machines.

^{*} Sometimes PMSM is also called BDCM (Brushless DC Motor, BLDC). However, in general PMSM has sinusoidal or quasi-sinusoidal distribution of flux in the air-gap and BDCM has rectangular distribution.

Table 1.1 Comparison of the electric motors for HEV application.

Machine Type	Advantages	Disadvantages
DC Motor	• Easy control • Desirable torque-speed characteristics	• Frequent need of maintenance • Low efficiency and reliability • Low speed range
Induction Motor (IM)	• High speed range • High reliability • Low cost • Rigidity in hostile environments	• Low power density and large size • Low efficiency • Thermal problem at high speed
PM Synchronous Motor (PMSM)	• High power density and small size • High efficiency	• Limited speed range • High cost • High stator core loss at high speed
Switched Reluctance Motor (SRM)	• Desirable torque-speed characteristics • High reliability • Low cost • Rigidity in hostile environments	• High torque ripple and noise • Low power density • Low efficiency

A PMSM is the first choice for the HEV system, because a PMSM has the advantages of high torque density and efficiency. The high energy permanent magnets, such as rare earth or samarium cobalt, used for exciting the magnetic field of a PMSM, enable the PMSM to be significantly smaller than the IM and SRM in size and weight.

The PMSM also has better efficiency because of the absence of a rotor winding and the small size of the rotor [4]. Additionally, since the PMSM is efficient at low speed, the HEV using a PMSM is attractive in the city mode in which the vehicle is required to frequently start and stop.

However, the PMSM has some drawbacks caused by its permanent magnets. For operation above its base speed, the permanent magnets produce a significant back electromotive force (emf) that must be reduced for its field weakening capability; a direct

axis demagnetization current produces magnetic flux to oppose the flux from the permanent magnets, and then reduces the flux linked through the stator wires. For this field weakening operation, the fixed huge magnetic flux from the highly energized permanent magnets prevents the direct axis current coming to the stator wires and limits the constant power speed region (CPSR).

There has been much research to increase the CPSR for PMSM. One technique uses an additional field winding of PMSM^{*} to control the field current, which achieves up to 4 times of the base speed, such as the use of conventional phase advance [4]. But the speed ratio is still not enough to meet the vehicle requirement; thus, a booster converter is used for extending the CPSR of the HEV drive system by increasing the input voltage of the motor [5]. This would increase the cost of HEV system. Moreover, the high magnetic flux results in a significant iron loss at the stator, which leads to reduction of the motor efficiency at high-speed operation.

To summarize, the requirements of a PMSM for the HEV system and the main motivations of this research for designing a new PMSM are:

1. improved CPSR capability of PMSM without the use of a boost converter in the HEV system;
2. increased overall motor efficiency at high speed operation;
3. realized higher torque density than conventional PMSM by reducing machine size; and
4. reduced HEV system cost including motor and controller.

^{*} It is called a PM hybrid motor.

To achieve the above purposes, the Power Electronics and Electric Machinery Research Center (PEEMEC) in Oak Ridge National Laboratory (ORNL) has been developing traction motors for HEV systems since 1994. One of the devised machines is ‘16000-rpm Interior Permanent Magnet Reluctance Machine with Brushless Field Excitation’ developed by J. S. Hsu and other engineers at PEEMRC [6].

This dissertation documents the main works in the developing and analyzing process of the 16000-rpm motor focusing on two unique features, axial-side excitation flux structure and slanted air-gap. The purposes of this dissertation are introducing new analysis methods of an electric machine and theoretical approaching to the unique structures of the 16000-rpm motor for the further development and the applications to other types of electric machines in the future.

The brief explanation of two unique features of the 16000-rpm motor is as follows: (1) Since the output torque of an electric machine is proportional to the magnitude of the air-gap flux, this machine can increase its output torque by adding the flux from the side excitation. The controllable excitation flux will be reduced when the machine is operated at high speed, and as a result, the air-gap flux is decreased and the HEV system obtains high CPSR without direct axis demagnetization current. (2) The slanted air-gap is for maximizing the ratio of the controllable magnitude of air-gap flux to increase CPSR capability without losing high torque density. There is no previous research about the slanted air-gap because this structure would have much leakage-flux in the air-gap and cannot avoid losing its output torque caused by its air-gap flux. However, the new research shows that the maximum torque of the machine with the slanted air-gap

is higher than a conventional machine because of the increased reluctance torque caused by the asymmetric air-gap structure.

In the next section of this chapter, permanent magnet materials, the various types of PMSM, and the technical issues for designing a PMSM are presented.

1.2 Permanent Magnet Synchronous Motors

Permanent magnets have not been used for electrical machines for a long time because the development of the permanent magnet materials was not mature until mid 20th century. After the invention of Alnico and Ferrite materials, permanent magnets were widely used for DC machines in small power applications, such as automobile auxiliary motors. Recently, the improvement of the quality of permanent magnet materials and the technical advances of the control methods allow replacing induction machines with permanent magnet machines in many industrial areas.

1.2.1 Permanent magnet materials

Magnetic behavior of permanent magnets is described in terms of the following three major quantities;

1. Remanence ($\mathbf{B}_r$) is the magnetization or flux density remaining in a permanent magnet material after saturation.

2. Coercivity (H_c) is the negative field strength necessary to bring the remanence to zero.
3. Maximum energy product (BH_{max}) indicates the maximum energy that the permanent magnet material can hold.

Figure 1.2 is a typical BH curve of a permanent magnet material. By applying a strong field to a permanent magnet sample, the material is to be initially magnetized. And then, shutting off the field allows the material sample to recoil along the upper curve in Figure 1.2. This curve assumes a fixed and constant slope called permeability. The BH_{max} occurs at the point where BH hyperbola is tangent to the recoil (demagnetizing) line.

Table 1.2 shows the unit of each property. Temperature coefficient (the variance of the remanence in percent per 1°C increase in temperature) is another important property for a design engineer using permanent magnets since some permanent magnet

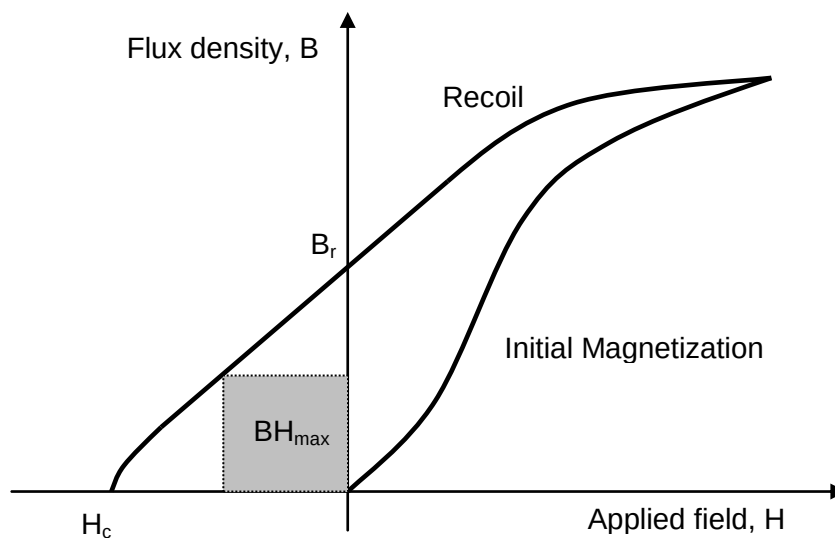


Figure 1.2 Typical B-H loop of a permanent magnet material.

Table 1.2 Major magnetic quantities for permanent magnets and their units.

	SI unit	CGS unit	conversion
B_r	T (tesla)	G (gauss)	1 T = 10 ⁴ G
H_c	A/m	Oe (Oersted)	1 A/m = 4π/10 ³ Oe = 1.257×10 ⁻² Oe
BH_{max}	J/m ³	G·Oe	1 J/m ³ = 1.257×10 ² G·Oe

materials are very sensitive to temperature changes.

Although the manufacturing of permanent magnets started with magnetic carbon steel from the 18th century in London, the actual history of the permanent magnet machines (PM machines) on the industrial basis started with Alnico in the first half of the 20th century. However, Alnico, based on aluminum, nickel, cobalt, and iron, was replaced by Ferrite in the late 1960s, because of Alnico's high price caused by the complex manufacturing process. In spite of its poor maximum energy capability, Ferrite is still widely used in many applications today because of the abundance of the raw materials and the low production cost. Since Ferrite is composed of fine particles made from iron oxide, Fe₂O₃, with barium (Br) or Strontium (Sr), this magnet material is popular for use in complex shapes [7]. However, Ferrite is not suitable for high temperature applications because of its high rate of decrease of **B_r** and **H_c** with increasing temperature [8].

Samarium-Cobalt (Sm-Co) is a rare-earth magnet material developed in the late 1960's. The cost and availability of this type of material limit its commercial success, but the good thermal stability allows its use in applications exposed to high temperature [7], [8].

After General Motors and Sumimoto simultaneously announced the development of Neodymium-Iron-Boron (Nd-Fe-B), this type of magnet became widely used in many industrial areas [7]. For its high energy capability and relatively low cost compared to Samarium-Cobalt, Neodymium-Iron-Boron is the best material in applications requiring a small product size. Samarium-Iron-Nitride (Sm-Fe-N) is the latest permanent magnet material introduced in the mid 1980's. This material has high resistance to demagnetization, corrosion, and temperature changes which are the disadvantage of the Neodymium-Iron-Boron magnet [8].

A historical development of permanent magnet materials throughout the 20th century is illustrated in Figure 1.3 [8], [9]. This graph shows the improvement of the maximum energy in the materials by introducing new types of material except Ferrite. Ferrite does not have desirable values in $\mathbf{BH}_{\max}$ because the increase in coercivity of

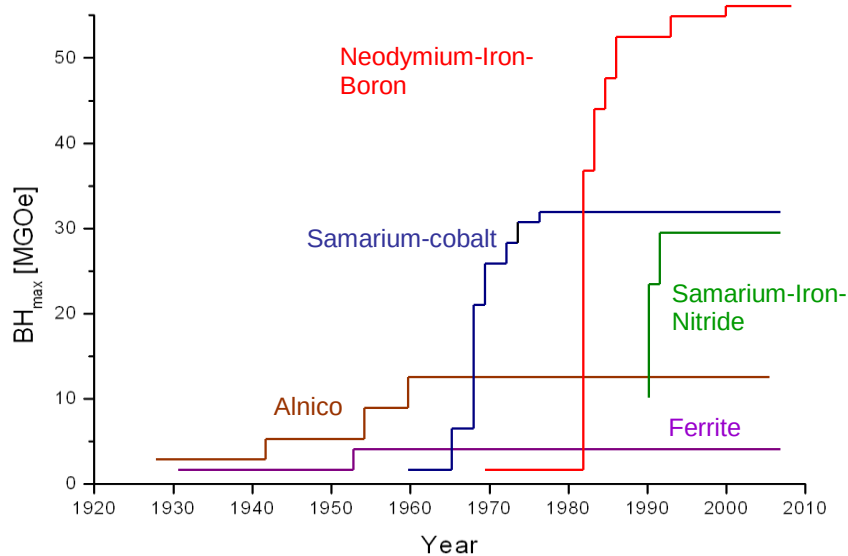


Figure 1.3 The historical development of permanent magnet materials as measured by its $\mathbf{BH}_{\max}$ in the 20th century [8, 9].

Ferrite is accompanied by a decrease in remanence.

1.2.2 Interior permanent magnet synchronous motor

With the development of permanent magnet materials and the techniques of driving an electric machine, the use of PMSMs has rapidly increased in many industrial areas by replacing induction motors because of PMSMs advantages in efficiency and size.

The conventional general type of PMSM has an external stator with conductors and an internal rotor attaching permanent magnets. Among this type of PMSMs, a surface-mounted permanent magnet synchronous motor (SPMSM) is commonly used for its simplicity for manufacturing and assembling. Because SPMSM has permanent magnets that are glued on the surface of its rotor, the rotation speed should be limited in order to keep the permanent magnets at the surface of the rotor from the effect of the centrifugal force. For this reason, most HEV systems use an interior permanent magnet synchronous motor (IPMSM) or a permanent magnet assisted reluctance synchronous motor (PM-RSM)*.

These two types of motors have permanent magnets inside its rotor structure and have almost the same operating principal in using both permanent magnet generated torque and reluctance torque for maximum output torque. The difference is that in PM-RSM the amount of magnet and the magnet flux linkage are small in comparison with

* Reluctance Synchronous Motor without permanent magnet shows similar behavior and characteristic with Switched Reluctance Motor as a traction application.

that of the conventional IPMSM [10], but there is no clear boundary between the two motors.

There are several types of IPMSM, and each type has its own advantages and specific applications. Figure 1.4 shows some examples of IPMSM rotor configurations. The d-axis means the north pole of magnetized direction on which the main magnetic flux from rotor flows to stator through the air-gap. If there are no magnets in each rotor configuration, the motor is to be a pure reluctance synchronous motor. Most IPMSM have some empty spaces, called flux barriers, inside the rotor for increasing its reluctance torque.

Much research has been conducted to determine the PM portion in the flux barriers in the same rotor structure and concluded that more PM increases the torque and efficiency but decreases the constant power region [10 – 11]. Also, the double layer configuration in Figure 1.4 (b) has a higher torque and wider efficiency operating range than the single layer [10], but it cannot avoid the increased PM cost. The arrangement of Figure 1.4 (c) is known as a ‘flux-concentrating’ design because the magnet pole area at the air-gap produces an air-gap flux density higher than that in the magnet [12].

The difference between the asymmetrical flux paths in d-axis and q-axis produces reluctance torque that is not present in a SPMSM. The detailed theory about reluctance torque will be explained in the next chapter. In addition to the merit of high-speed operation, IPMSM has the following useful properties when compared to a traditional SPMSM [10, 12]:

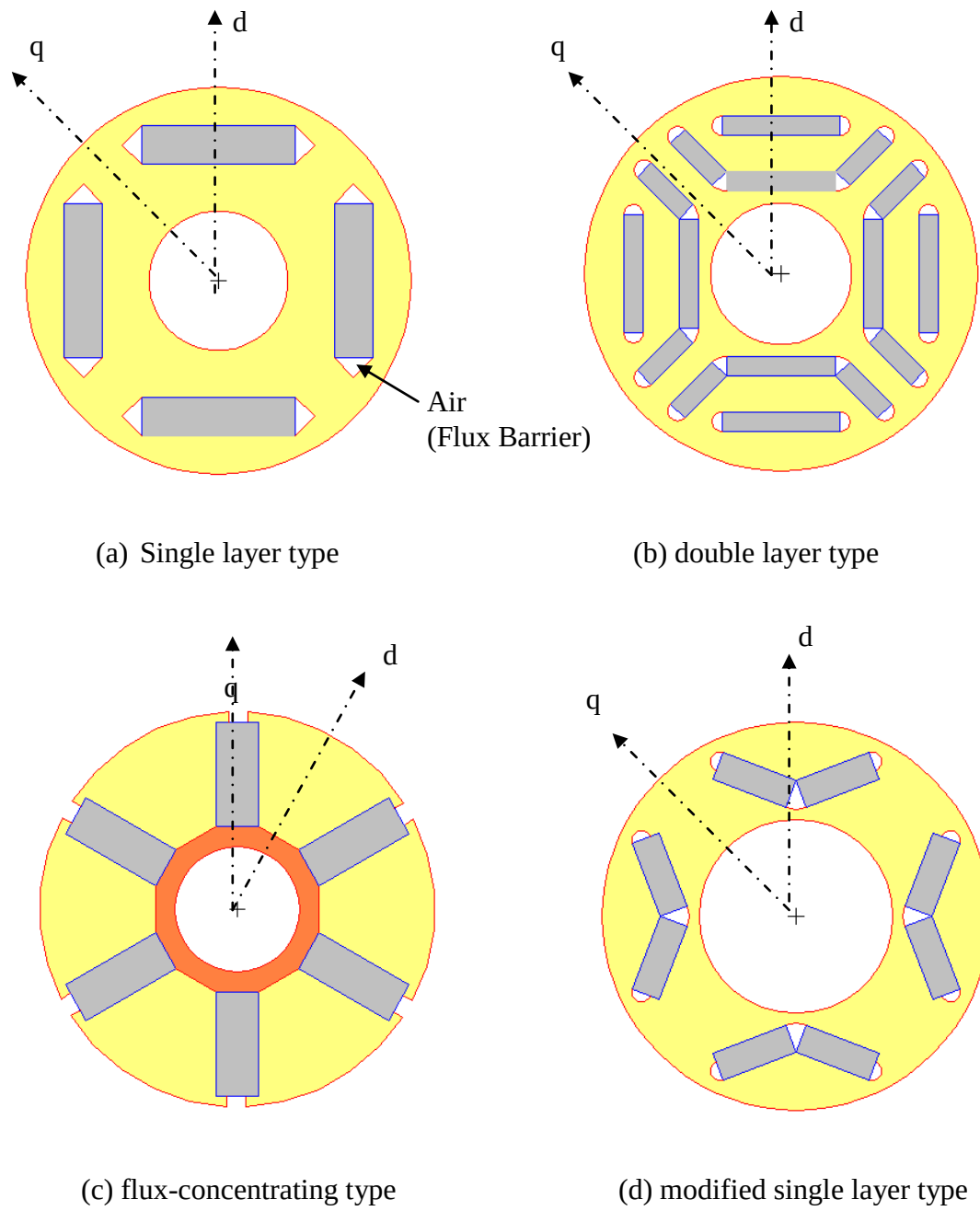


Figure 1.4 Various rotor configurations of IPMSM.

- Field weakening capability with high inductance
- Under-excited operation for most load conditions
- Reducing the risk of demagnetization of permanent magnets
- Increase the resistance against mechanical impacts and corrosion
- High temperature capability

1.3 Research Objective

The importance of high efficiency at low speed and high torque/power density make many vehicle manufacturing companies select a traction motor using permanent magnets, especially IPMSM because it has many advantages compared to the other types of motors in efficiency and power density.

However, the high electro motive force (back-emf) voltage from a motor caused by the permanent magnets limits its maximum constant speed operation, which will be explained in Chapter 2. Further, the high flux from the permanent magnets is also a major factor of the core loss when the machine rotates at high speed.

The primary objective of this research is to introduce and analyze the new concept IPMSM for the application in a HEV propulsion system for the purpose of overcoming the drawbacks of an IPMSM. In order to achieve this objective, this motor has two unique design features, axial-side excitation coils and slanted air-gap.

The concept of the excitation is to control the magnitude of the air-gap flux that determines the peak value of the output torque as well as the maximum speed. The

excitation coils are wound around in the radial direction of the motor; thus, the flux created by the excitation current is in the axial direction, which would come into the rotor and be combined with the PM flux. Therefore, the output torque of the motor can be increased by adding the flux from the side excitation. To achieve the higher CPSR, which is in reverse proportion to the magnitude of the air-gap flux, the excitation current will be reduced. Another advantage of the capability of reducing air-gap flux by controlling the excitation current is that the iron losses will decrease too. The slanted air-gap enables maximizing the ratio of the controllable magnitude of the air-gap flux as well as increasing the maximum torque of the machine.

This dissertation contains the development process of the new IPMSM with new analysis methods of an electric machine; calculation of the output torque of a machine using its equivalent magnetic circuit and consideration of the cross-saturation effect between d- and q- axes to expect the steady-state characteristics of the machine.

The new method of the torque calculation will be useful to expect the output torque of a newly designed motor which uses a reluctance torque. This method is applied to obtain the output torque variation along with the changed slanted air-gap shape. Since there is no previous research about the slanted air-gap, it is necessary to study how the machine characteristics are affected by different depth and width of the slanted air-gap.

The inductance is a very important parameter of PMSM because both the output torque and the maximum operating speed range of the machine are determined by the inductance values. The reason will be explained in Chapter 2. Therefore, calculating inductance values is the one of the main issues in analysis of an IPMSM. Since these inductance values are very sensitive to the magnetic saturation in the flux path of the

motor, it is necessary to develop a new calculation method of the inductance values considering the variations of the magnetic saturation at different load conditions for better prediction of the characteristics of the IPMSM. This dissertation shows how to consider the magnetic saturation when calculating d- and q-axis inductances, especially focusing on the cross-saturation between d- and q-axes.

1.4 Dissertation Organization

This dissertation is organized in the following manner:

- Chapter 2 is a literature review that summarizes the basic theory of IPMSM, to explain the theoretical speed limit of an IPMSM. For this purpose, the steady-state phasor diagram and the circle diagram on the dq-current plane are used. These works show the importance of flux linkage and inductance when analyzing an IPMSM. In a later section, several methods calculating flux linkage and inductance values are introduced including the equivalent magnetic circuit analysis for basic IPMSM models, tangential and radial magnetization, energy method, and using of the values of the flux linkage from finite element analysis (FEA). The last section of this chapter is for the brief explanation about the FEA theory.
- Chapter 3 describes the novel structures of the new IPMSM; side-pole, side-PM, slant-shape rotor, a new method of the expected output torque calculation for a newly designed IPMSM, and the changing pattern of the machine characteristics

with the variation of the slanted air-gap shapes. This chapter shows how these unique structures work and why they are necessary. Also, the new analytical method of the expected output torque calculation will be apply to the slanted air-gap shape to determine the best slant shape.

- Chapter 4 compares FEA simulation and test results focusing on back-emf voltage, output torque, and inductance calculations. Especially, this chapter shows the advantages considering the cross-saturation between d- and q-axes when calculating inductance values along with the varied load current values. The results from simulation and test show the advantages of the novel structures.
- Finally, chapter 5 summarizes conclusions and recommended works for the future research.

CHAPTER 2

Literature Review: Analysis of Interior Permanent Magnet Synchronous Motor (IPMSM)

2.1 Introduction

This chapter provides a summary literature review for the design and analysis of new IPMSM. As shown in the first chapter, an IPMSM is widely used for a high-power and high-speed application such as a HEV system. Since the study of an IPMSM is relatively new technology compared to DC motor and induction motor, there are many research areas which are not still fully developed.

One of the areas to solve is extending the speed limit which is one of the drawbacks of an IPMSM. This chapter provides an explanation of how to decide the speed limit of an IPMSM, which parameters cause the speed limit, and how to calculate these parameters.

The steady-state phasor diagram and the circle diagram on the dq-current plane are used to explain the theoretical speed limit of an IPMSM and show the importance of flux linkage and inductance when analyzing an IPMSM. After that, calculating flux linkage and inductance values are introduced including the equivalent magnetic circuit analysis for basic IPMSM models, tangential and radial magnetization, energy method, and using of the values of the flux linkage from FEA. And the last part of this chapter is for a brief explanation about the FEA theory.

2.2 The Steady-State Modeling of IPMSM

An IPMSM is normally analyzed using rotor reference frame which has a fixed rotor aligned with the direction of permanent magnet flux. In general, the stator winding has three phase quantities, and it can be transformed to rotor reference frame and vice versa by using Park's transformation attached in the Appendices.

At a constant speed, the steady-state phasor diagram of IPMSM can be constructed as shown in Figure 2.1 [13 –15]. The open-circuit phase emf (electromotive force) is

$$E = jE_q = j\omega\lambda_{PM} \quad (2.1)$$

where, ω is synchronous speed (electrical speed) and λ_{PM} is the flux linkage due to the fundamental component of d-axis flux produced by the permanent magnet. Although there exists some d-axis emf associated with the leakage flux [13]*, in most cases it is negligible, especially when the stator winding is sine-distributed. The reason will be shown in section 2.6. All the equations in this section are reorganized from the references [13 –17].

From the phasor diagram shown in Figure 2.1,

$$V_d = -X_q I_q + R I_d \quad (2.2)$$

$$V_q = E_q + X_d I_d + R I_q \quad (2.3)$$

* actually, $E = E_d + jE_q = \omega\lambda_{PM,q1} + j\omega\lambda_{PM,d1}$

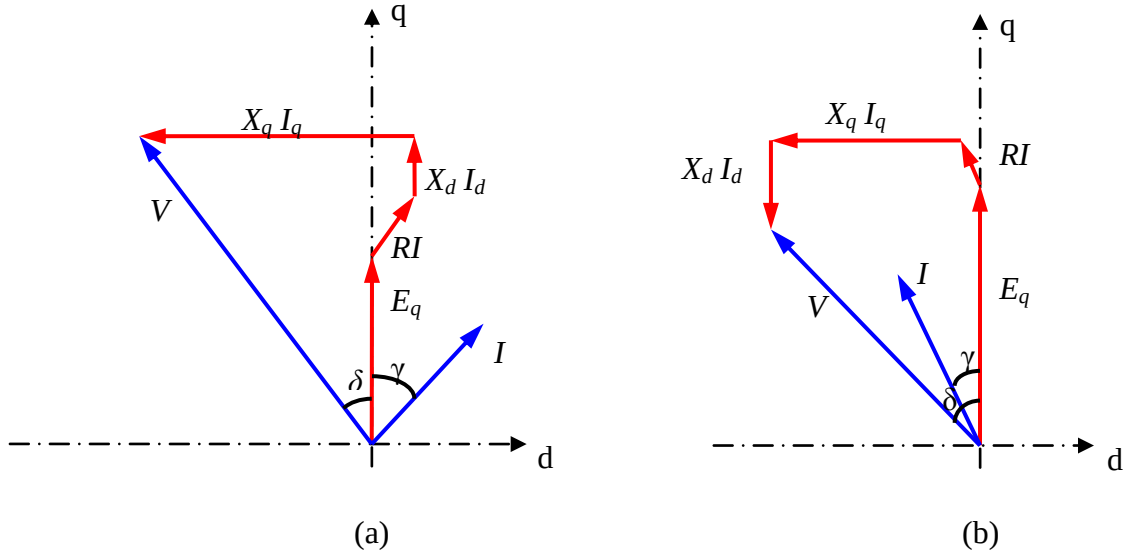


Figure 2.1 Steady-state phasor diagram for IPMSM.

(a) magnetizing armature current in the d-axis.

(b) demagnetizing armature current in the d-axis.

The angles δ and γ are defined as shown in Figure 2.1; then, the voltage and current in the motor are defined by

$$V_d = -V \sin \delta, \quad V_q = V \cos \delta \quad (2.4)$$

$$I_d = \pm I \sin \gamma, \quad I_q = I \cos \gamma \quad (2.5)$$

In a large motor, the stator conductor resistance, R , is negligible, and from (2.2) and (2.3)*,

* including the resistance
$$I_d = \frac{RV_d + X_q(V_q - E_q)}{R^2 + X_d X_q}, \quad I_q = \frac{R(V_q - E_q) - X_d V_d}{R^2 + X_d X_q}$$

$$I_d = \frac{V_q - E_q}{X_d} \quad (2.6)$$

$$I_q = -\frac{V_d}{X_q} \quad (2.7)$$

The complex power per phase per pole-pair into the motor is

$$\begin{aligned} \vec{S} &= \vec{V} \cdot \vec{I}^* \\ &= (V_d + jV_q)(I_d - jI_q) \\ &= V_d I_d + V_q I_q + j(V_q I_d - V_d I_q) \\ &= P + jQ \end{aligned} \quad (2.8)$$

Substitute (2.4), (2.6) and (2.7) into (2.8); the real power is

$$\begin{aligned} P &= V_d I_d + V_q I_q \\ &= V_d \frac{V_q - E_q}{X_d} - V_q \frac{V_d}{X_q} \\ &= \frac{V_d (-E_q)}{X_d} + \frac{(X_q - X_d) V_d V_q}{X_d X_q} \\ &= \frac{V_t E_q}{X_d} \sin \delta + \frac{(X_d - X_q) V_t^2}{2 X_d X_q} \sin(2\delta) \end{aligned} \quad (2.9)$$

where,

$$V_t = \sqrt{V_d^2 + V_q^2}$$

Assume that there is no loss, then the total output torque for three phases with p pole-pairs is

$$T = \frac{3p}{\omega} P = \frac{3p}{\omega} \left[\frac{V_t E_q}{X_d} \sin \delta + \frac{(X_d - X_q) \mathcal{N}_t^2}{2X_d X_q} \sin(2\delta) \right] \quad (2.10)$$

In a pure reluctance synchronous motor,

$$E_q = 0$$

$$T = \frac{3p}{\omega} P = \frac{3p}{\omega} \frac{(X_d - X_q) \mathcal{N}_t^2}{2X_d X_q} \sin(2\delta) \quad (2.11)$$

The first term of (2.10) is the PM generated torque, and the second term is the reluctance torque which is proportional to the difference in stator inductance, $L_d - L_q$. For a general PMSM, L_d is almost the same as L_q , thus the reluctance torque term is canceled. In IPMSM, L_d is lower than L_q because the magnetic flux flowing along the d-axis has to cross through the magnet cavities in addition to the rotor air-gap, while the magnetic flux of the q-axis only crosses the air-gap [11].

Equation (2.10) shows that the period of the reluctance torque is one half of that of the PM generated torque. Figure 2.2 illustrates that the IPMSM can achieve higher torque than a surface mounted PMSM which does not have any reluctance torque component. However, the appearance of the reluctance torque does not mean that the IPMSM can have higher power density than surface mounted PMSM because the magnet flux linkage in IPMSM is not the same as that in the surface mounted PMSM with the same magnet volume. Equation (2.12) is another form of the torque equation (2.10), and (2.13) suggested by Phil Mellor [18] which shows that the total torque of an IPMSM is increased with the saliency of the rotor.

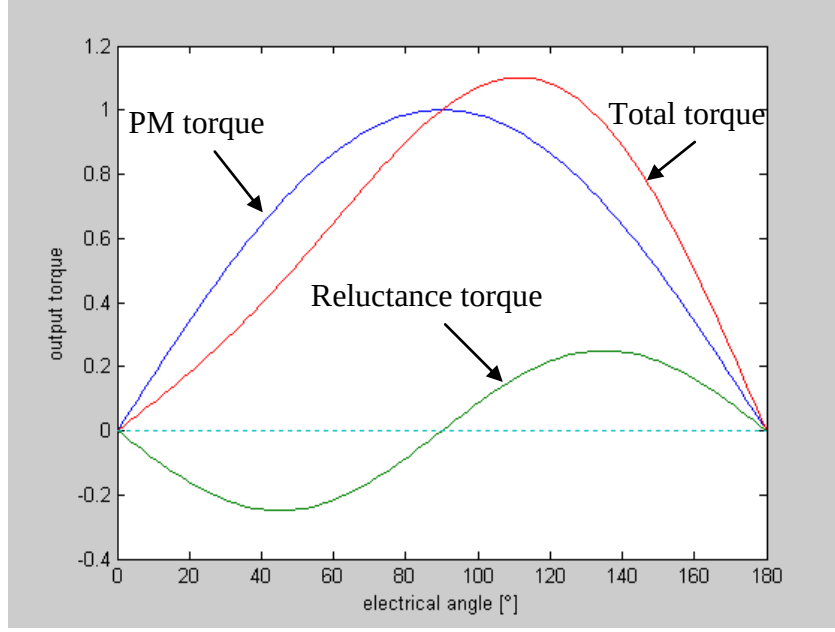


Figure 2.2 Torque-angle characteristic of IPMSM.

$$T = \frac{3p}{2} [\lambda_{PM} I_q + (L_d - L_q) I_d I_q] \quad (2.12)$$

$$\frac{T}{T_{base}} = \cos\delta - \frac{\xi - 1}{2} \frac{I}{I_{base}} \sin(2\delta) \quad (2.13)$$

where,

$$T_{base} = \text{PM torque at } I_{base}$$

$$\xi = \frac{L_q}{L_d} = \text{saliency ratio}$$

$$I = \text{Input current}$$

The ξ term is called the saliency factor and generally cannot be more than 3 [18].

Figure 2.2 also indicates that the optimum commutation angle may have been advanced

from 90° (the optimum commutation angle for PMSM) toward 135° *. Torque equations (2.10) and (2.12) show that the inductance term is important in determining the output torque of an IPMSM.

2.3 Circle Diagram of IPMSM

The circle diagram on the current vector plane is the most convenient method to explain the speed limits of IPMSM [19, 20]. The circle diagram can be constructed by the following equations. The steady-state voltage equations (2.2) and (2.3) can be expressed in terms of corresponding axis currents. Since the resistive voltage drop terms in (2.2) and (2.3) can be negligible for a large capacity motor and at high speeds, these equations are

$$V_d = -X_q I_q \quad (2.14)$$

$$V_q = X_d I_d + E_q \quad (2.15)$$

If the input voltage is V_t , then

$$\begin{aligned} V_t^2 &= V_d^2 + V_q^2 \\ &= (X_q I_q)^2 + (E_q + X_d I_d)^2 \end{aligned} \quad (2.16)$$

* The maximum shifted angle is 135° when the reluctance torque is main source of the total output torque in case of the synchronous permanent magnet assist machine.

Equation (2.16) can be expressed as the current terms below

$$\left(I_d + \frac{E_q}{X_d}\right)^2 + \left(\frac{X_q}{X_d}\right)^2 I_q^2 = \left(\frac{V_t}{X_d}\right)^2 \quad (2.17)$$

using inductance terms,

$$\left(I_d + \frac{\lambda_{PM}}{L_d}\right)^2 + \left(\frac{L_q}{L_d}\right)^2 I_q^2 = \left(\frac{V_t}{L_d}\right)^2 \frac{1}{\omega^2} \quad (2.18)$$

The ellipse equation (2.18) in the d-q current plane is drawn in Figure 2.3. At given constant input voltage V_t , the ellipse shrinks toward the point $(-E_q/X_d, 0)$ as the

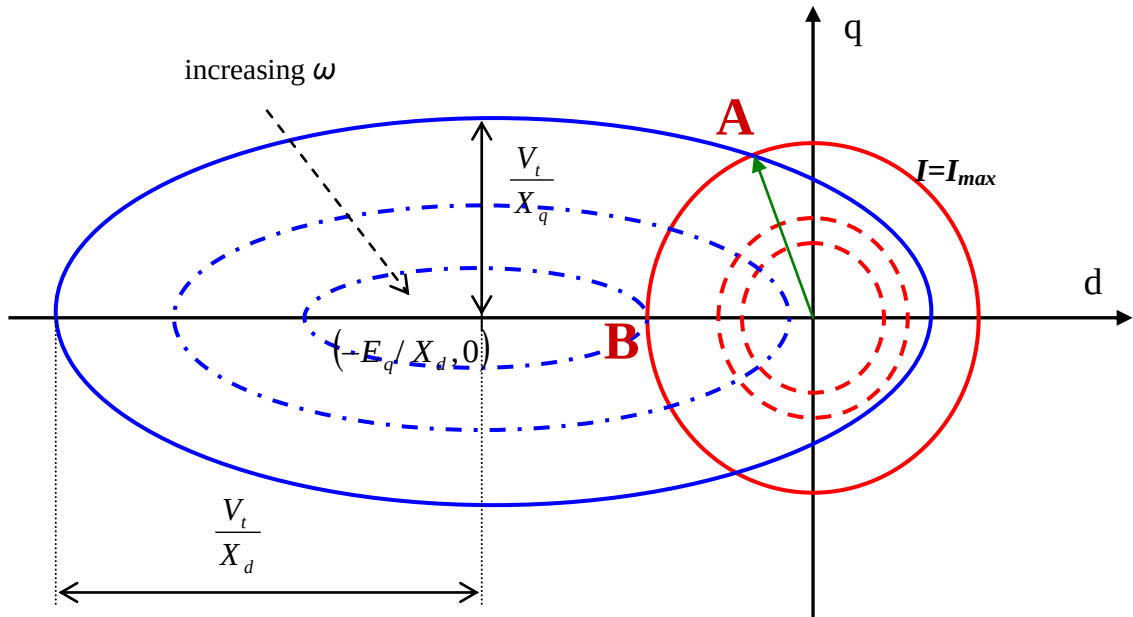


Figure 2.3 The circle diagram of IPMSM on the dq current plane.

speed ω increases. The red circles are the current limit circles of which radius will be increased by enhancing the maximum current capability.

The point A in Figure 2.3 is the current vector position for the maximum torque at a given speed. The point B is the intersection point between the current circle and the negative d-axis. The ellipse can be shrunk until it is tangent to the point B, and the motor has its ideal maximum speed at this point.

R. Schiferl and T. A. Lipo [21] developed the criteria for optimum field-weakening capability of a machine as the center point of the ellipse. This point is also called the characteristic current of an IPMSM as follows [22, 23]

$$I_{ch} \equiv \frac{E_q}{X_d} = \frac{\lambda_{PM}}{L_d} \quad (2.19)$$

At the point B, $I_d = -I_{max}$ and $I_q = 0$. Thus, the ideal maximum speed can be expressed by the characteristic current from (2.18),

$$(-I_{max} + I_{ch})^2 = \left(\frac{V_t}{L_d} \right)^2 \frac{1}{\omega^2} \quad (2.20)$$

$$\omega_{max} = \frac{V_t}{L_d (I_{ch} - I_{max})} \quad (2.21)$$

Equation (2.21) shows that reducing the characteristic current increases the maximum motor speed as shown in Figure 2.3. If $I_{ch} \leq I_{max}$, the machine can have an

infinite speed ideally [19, 22] ^{*}. Although, most IPMSM have the value of I_{ch} higher than the maximum current of the inverters which feed the current into the motor [20, 24], a machine designer will try to reduce the value of I_{ch} as much as possible for high-speed applications.

As expressed in the definition of the characteristic current of (2.19), the lower value of I_{ch} can be achieved by reducing the d-axis flux linkage from the rotor or increasing the d-axis inductance value. J. S. Lawler et al developed a technique that used phase controlled thyristor switches to behave as external inductances for increasing the overall inductance values of a motor [25]. This technique can be applied to not only an IPMSM but also a SPMSM.

However, this is not a solution for a machine itself. The common mechanical technique to a machine is that it has the variable stator windings to reduce the flux linkage through the windings at the high-speed operation. For this purpose, there have been many methods; changing the stator winding [26], using auxiliary stator coils [27], and varying the winding turns [28]. Each method has some favorable features respectively, but the sudden change of the motor winding will cause torque interruption because of the sudden variation in input current when changing stator winding or winding turns. And, the technique of using auxiliary stator coils cannot avoid the increased manufacturing costs caused by the complexity and size of the stator structure.

^{*} The desired value is $I_{ch} = I_{max}$, if $I_{ch} < I_{max}$, then the output power and torque would be reduced at high speed.

2.4 Simple Magnetic Circuit with a Permanent Magnet

In the previous section, it is shown that flux linkage and inductance are the key parameters to determine the high-speed characteristics of an IPMSM. To understand the fundamentals of IPMSM and how to calculate the flux linkage and the inductance values, it is necessary to have a concept of the basic equations of a simple magnetic circuit. This section explains the magnetic flux behavior in a simple magnetic circuit, and the application to an IPMSM will follow in the next section.

Figure 2.4 (a) illustrates a simple magnetic circuit structure with a permanent magnet. Ampere's law states that the line integral of the magnetic field strength or intensity taken around any closed path is proportional to the total current flowing across

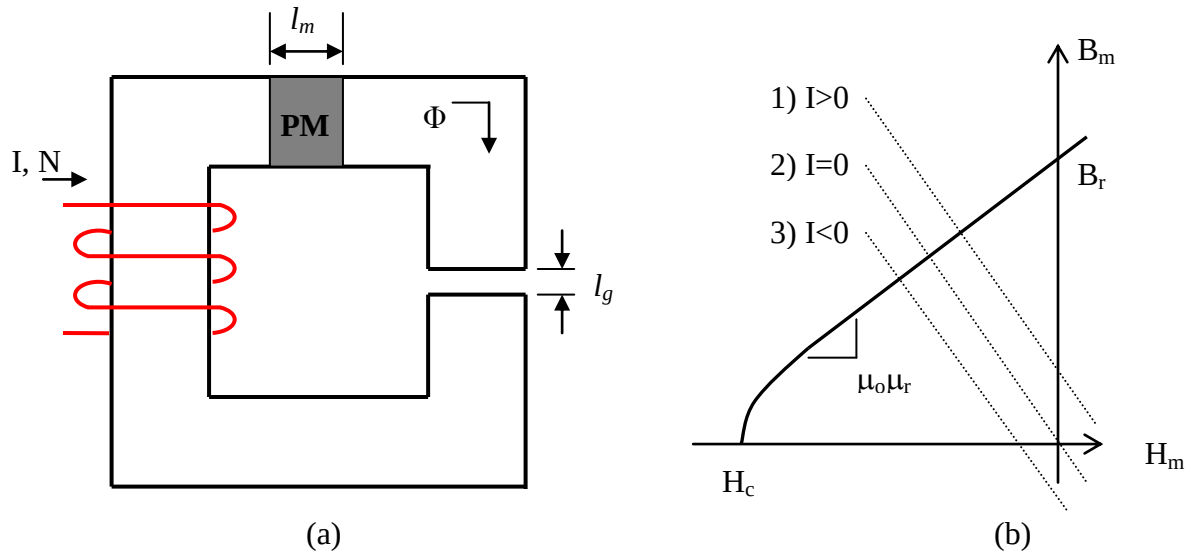


Figure 2.4 A simple magnetic circuit with permanent magnet and demagnetization curve.

any area bounded by that path. From Ampere's law and conservation of flux, the equations for balanced mmf (magneto motive force) are [13 –15, 29]

$$\oint H \cdot dl = \sum I \rightarrow H_m \cdot l_m + H_g \cdot l_g = N_a \cdot I_a \quad (2.22)$$

$$B_m \cdot A_m = B_g \cdot A_g \quad (2.23)$$

In (2.22) and (2.23), B represents flux density; H , magnetic field strength; l , length; A , area; N , the number of coil turns, and I , current. The suffix g stands for the air gap, m for permanent magnet, and a for armature. Under the assumption that the magnetic permeability of the iron is infinity*

$$B_m = \mu_o \frac{A_g}{l_g A_m} (-H_m \cdot l_m + N_a \cdot I_a) \quad (2.24)$$

where μ_o is the permeability of air [13 –15, 29]. This equation (2.24) represents the dashed line in Figure 2.4(b), and this line is called a permeance coefficient line. In Figure 2.4 (b), the linearly approximated equation of demagnetization curve (solid line) is

$$B_m = B_r + \mu_o \mu_r \cdot H_m \quad (2.25)$$

in (2.25), $\mu_o \mu_r$ is the slope of the demagnetization curve in Figure 2.4 (b)[11-13, 30]. The results of combining equations (2.24) and (2.25) are

* The actual value is thousands times of that of air. Air has the value $4\pi \times 10^{-7}$ in SI unit [29].

$$B_m = \frac{1}{\mu_r l_g A_m + l_m A_g} l_m A_g B_r + \frac{\mu_o \mu_r A_g}{\mu_r l_g A_m + l_m A_g} N_a I_a \quad (2.26)$$

This result is the actual flux density from the permanent magnet for the simple magnetic circuit. If $A_m = A_g$, then equation (2.26) will be

$$B_m = \frac{1}{\mu_r l_g + l_m} l_m B_r + \frac{\mu_o \mu_r}{\mu_r l_g + l_m} N_a I_a \quad (2.27)$$

For the axis of H_m side,

$$H_m = \frac{1}{\mu_o \mu_r} \left(\frac{1}{\mu_r l_g + l_m} - 1 \right) B_r + \frac{N_a I_a}{\mu_r l_g + l_m} \quad (2.28)$$

The second term of equation (2.27) indicates the effect of armature reaction. For operating an electric machine, the current is less than zero. As a result, the permeance coefficient line moves to the left and the flux density is reduced. It is called demagnetization. The amount of the demagnetization is determined by the amount of H_m shifted from the origin in Figure 2.4(b) [30] (the second term of equation (2.28)).

If the permeance coefficient line is passed to the nonlinear point (knee point) of the solid line in Figure 2.4(b) by armature reaction, the permanent magnet suffers non-recoverable damage. For this reason, an engineer should be cautious to decide the thickness of the permanent magnets in designing an electric machine.

On the contrary, if there is sufficient current added to this magnetic circuit, then it is possible that the air-gap magnetic flux density increases to above the original point. However, this field enhancement is limited by the saturation of the PM B/H curve because the air-gap flux density can never be higher than the PM flux density [30].

2.5 Magnetic Circuit Analysis of Simple IPMSM

Using a magnetic equivalent circuit is the most common method for designing an electrical machine. Since the equivalent circuit is affected by the shape of the rotor, there are several types of the equivalent circuit. In this section, it is documented how to build the magnetic equivalent circuits for two major rotor shapes, tangential and radial magnetized permanent magnets. Also, 2-dimensional network equivalent circuit of an IPMSM is described in the end of this section. The obtained value of the air-gap flux will be used in the calculation of the flux linkage in section 2.6. The equations in this section are obtained by analogy from the several publications [13 –17, 31, 32] especially the book written by Miller [13] and Hanselman [31].

2.5.1 Tangential magnetization

Figure 2.5 shows a simple structure of a tangential permanent magnet rotor for one pole. By Gauss's law and neglecting leakage, the total flux from the permanent magnet travels through a half part of the air-gap as shown in the equation below.

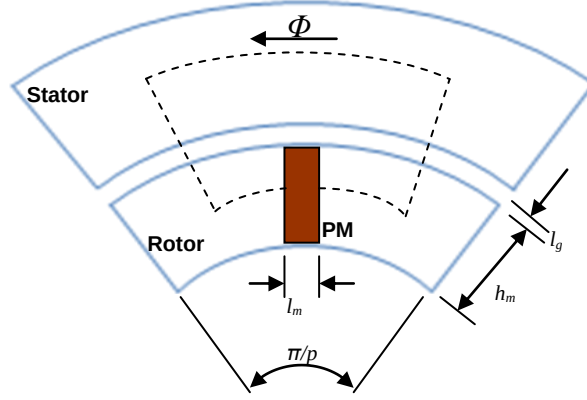


Figure 2.5 A simple structure of IPMSM with tangential magnetization.

$$B_m h_m = B_g \frac{\pi R_{rot}}{2p} \quad (2.29)$$

And using Ampere's law,

$$H_m l_m + 2H_g l_g = 0 \quad (2.30)$$

and applying (2.25) for the permanent magnet, equation (2.30) will be

$$\frac{B_m - B_r}{\mu_o \mu_r} l_m + 2 \frac{B_g}{\mu_o} l_g = 0 \quad (2.31)$$

where μ_r is the relative permeability of the permanent magnet. By combining (2.29) and (2.31), the air-gap flux density is

$$B_g = \frac{B_r}{\frac{\pi R_{rot}}{2p} + \frac{2\mu_r l_g}{l_m}} \quad (2.32)$$

Air-gap flux density equation (2.32) can be obtained from the equivalent magnetic circuit (Figure 2.6). Because the flux from the rotor travels only a half part in the air-gap as shown in Figure 2.5, the air-gap reluctance with one half width of the one pole would be two times of the one pole air-gap reluctance, R_g , which is expressed in (2.33).

$$R_g = \frac{l_g}{\frac{\mu_o \pi R_{rot} L_{rot}}{p}} \quad (2.33)$$

The remanence flux from the permanent magnet and the permanent magnet reluctance are

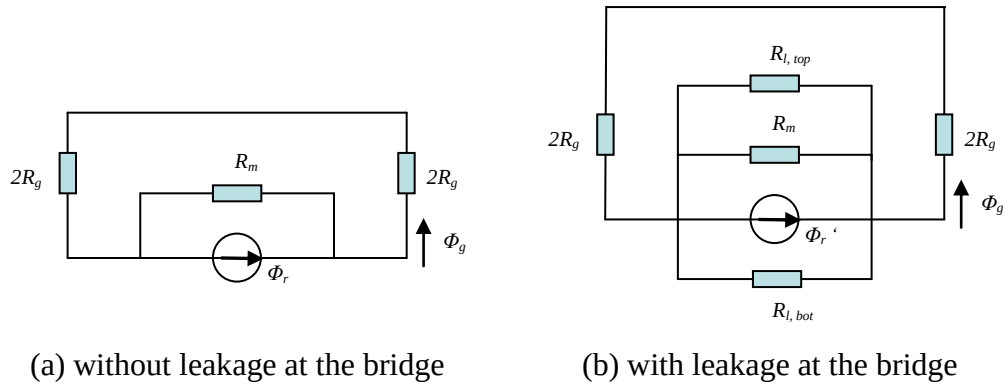


Figure 2.6 The equivalent magnetic circuit of Figure 2.5.

$$\Phi_r = B_r h_m L_{rot} \quad (2.34)$$

$$R_{pm} = \frac{l_m}{\mu_o \mu_r h_m L_{rot}} \quad (2.35)$$

If we can neglect the leakage at the bridge of the rotor as shown in Figure 2.6(a), the air-gap flux density is

$$\Phi_g = \Phi_r \frac{R_m}{R_m + 4R_g} \quad (2.36)$$

Therefore the air-gap flux density can be simply calculated by dividing the air-gap flux by the air-gap surface as

$$B_g = \frac{\Phi_g}{\frac{\pi R_{rot} L_{rot}}{2p}} \quad (2.37)$$

By substituting equations (2.33), (2.34), and (2.35) into equation (2.36), the air-gap flux density equation (2.37) results in the expression (2.32).

The above air-gap flux density equation does not consider the leakage flux in the air-gap, rotor, and stator. Especially, the leakage at the bridges or ribs of the rotor is not negligible as shown in Figure 2.7. These bridges are necessary for mechanical reason to avoid the deformation of the rotor from centrifugal force.

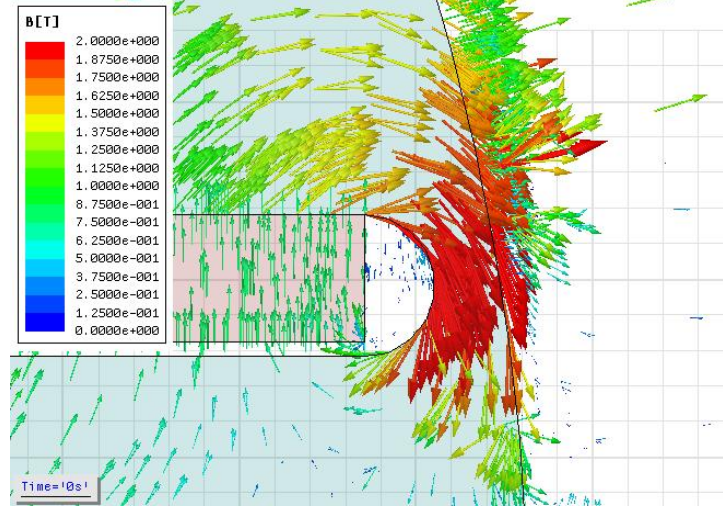


Figure 2.7 An example of leakage flux at a bridge of the rotor.

In general, the bridges are fully saturated under operating situation, and under this condition the bridges behave like air [33]. Therefore, the reluctances at the top and bottom bridges of the rotor in Figure 2.5 are

$$R_{l,top} = \frac{\frac{\pi}{2} l_m}{\mu_o t_{b,top} L_{rot}} \quad (2.38)$$

$$R_{l,bot} = \frac{\frac{\pi}{2} l_m}{\mu_o t_{b,bot} L_{rot}} \quad (2.39)$$

where $\frac{\pi}{2} l_m$ is the approximate distance for traveling leakage flux [33, 34] and $t_{b,top}$ and $t_{b,bot}$ are the thickness of the top and bottom part of the bridges, respectively. Assuming

that the bridges are saturated at a constant flux density B_{sat} , then each leakage flux through the top and bottom bridges is

$$\Phi_{l,top} = B_{sat} t_{b,top} L_{rot} \quad (2.40)$$

$$\Phi_{l,bot} = B_{sat} t_{b,bot} L_{rot} \quad (2.41)$$

The magnitude of B_{sat} is determined by the B-H curve of the material. From the results stated above, the modified flux from the permanent magnet is

$$\Phi_r' = \Phi_r - \Phi_{l,top} - \Phi_{l,bot} \quad (2.42)$$

with a simple circuit analysis in Figure 2.6(b), the air-gap flux is

$$\frac{1}{R_\sigma} = \frac{1}{R_m} + \frac{1}{R_{l,top}} + \frac{1}{R_{l,bot}} \quad (2.43)$$

$$\Phi_g = \Phi_r' \cdot \frac{R_\sigma}{R_\sigma + 4R_g} \quad (2.44)$$

2.5.2 Radial magnetization

A simple example of radial magnetization IPMSM is illustrated in Figure 2.8. In this case, the leakage flux and reluctance at the bridges located at two sides of the permanent magnet have the same value respectively by its symmetry.

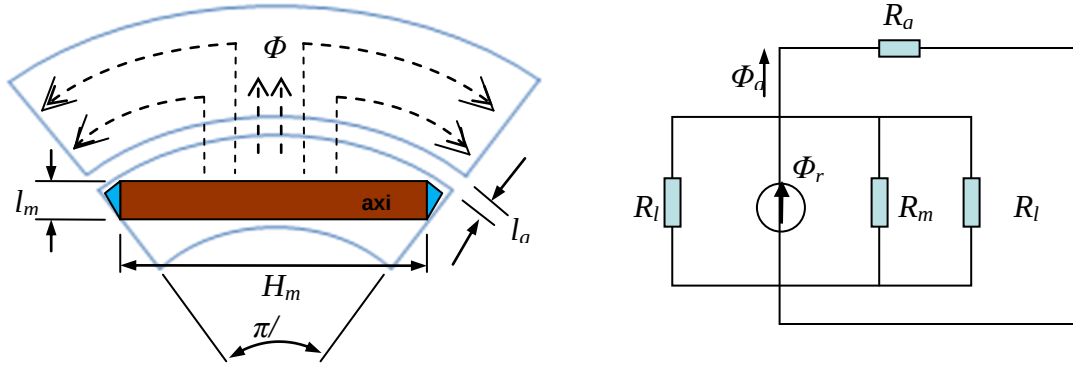


Figure 2.8 A simple structure of IPMSM with radial magnetization and its equivalent circuit.

The equations for the remanence flux from the permanent magnet and the permanent magnet reluctance are the same as those of tangential magnetization as shown in equations (2.34) and (2.35). The reluctance of both bridges and leakage flux are

$$R_l = \frac{l_b}{\mu_o t_b L_{rot}} \quad (2.45)$$

$$\Phi_l = B_{sat} t_b L_{rot} \quad (2.46)$$

where, l_b is the length, and t_b is the thickness of the bridge. Then, modified flux from the permanent magnet is

$$\Phi_r' = \Phi_r - 2\Phi_l \quad (2.47)$$

with a simple circuit analysis in Figure 2.6(b), the air-gap flux is

$$\frac{1}{R_{\sigma}} = \frac{1}{R_m} + \frac{2}{R_l} \quad (2.48)$$

$$\Phi_g = \Phi_r' \cdot \frac{R_{\sigma}}{R_{\sigma} + R_g} \quad (2.49)$$

Therefore, the flux density at the air-gap is

$$B_g = \frac{\Phi_g}{\frac{\pi R_{rot} L_{rot}}{p}} \quad (2.50)$$

This chapter shows how to construct the magnetic equivalent circuit for an IPMSM without the magnetic resistances at rotor and stator under the assumption that the permeability of the iron is infinity. In a practical situation, the magnetic resistances at the rotor and stator should be considered for the accurate analysis as well as the leakage flux and saturation at the stator. Carter's coefficient is also widely used at the analytical design step. This coefficient is a factor considering that the effective air-gap length could be increased by the shape of open slot of the stator [31, 32, 35, 36]. Some well used equations to determine Carter's coefficient are attached in Appendix D.

2.5.3 Two-dimensional network equivalent circuit

The analysis methods of 2.5.1 and 2.5.2 have the assumption that all the flux flow through only d-axis except the leakage flux and the permeability of the rotor and stator is infinite. These assumptions mean that the magnetic flux flows on the 1-dimensional flux

path on the d-axis in the rotor and the stator as shown in Figure 2.6 and 2.8. To get better analysis results, 2-dimensional network equivalent circuit is often used considering non-infinite permeability at rotor and stator [37 – 42].

There are many methods to construct the 2-dimensional equivalent circuit of an electric machine. Figure 2.9 is the example of the network model that is constructed based on the method in references [38], [39], and [40]. The air-gap flux in Figure 2.9 can be calculated by using the nodal analysis similar to an electrical circuit analysis [43].

There are some advantages of a 2-dimensional equivalent circuit: it can be constructed for any kind of permanent magnet configuration in the rotor and the analysis results will be more accurate by increasing the number of nodes in the circuit, although solving the equations might be complicated. One of the methods to solve a 2-dimensional

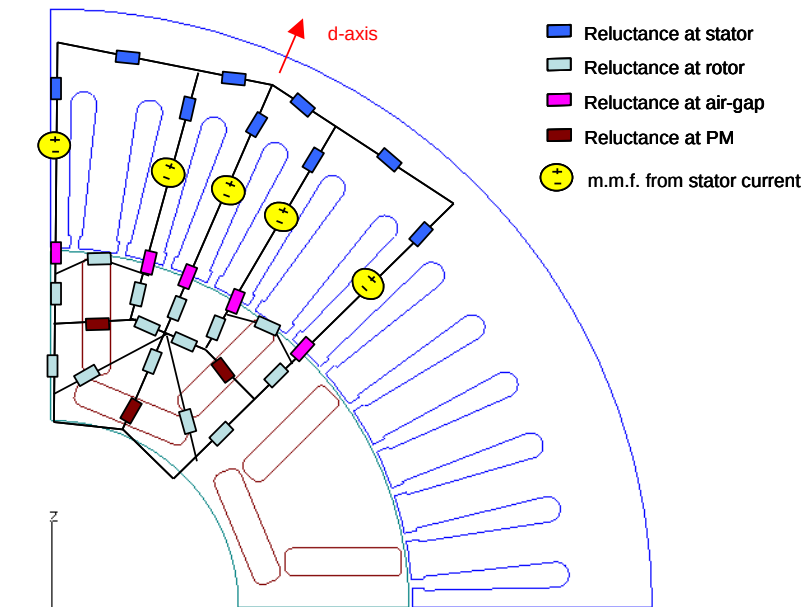


Figure 2.9 Example 2-dimensional equivalent circuit.

equivalent circuit is documented in Chapter 3 for calculating the output torque of an IPMSM.

2.6 Current Density and Flux Linkage in a 3-Phase Winding Distribution

After calculating the air-gap flux from permanent magnets, it is necessary to analyze the magnetic force caused by the stator phase winding. This section is for analyzing how to get sinusoidal distributed mmf (magnetomotive force) from the phase current and calculate the flux linkage for the conductors in the stator by the air-gap flux density.

The simplest winding distribution is a single N-turn winding in a 2-pole machine as shown in Figure 2.10(a). The red dashed line block arc indicates the integration path to obtain mmf in the air-gap for 1 pole. Figure 2.10 and the following equations are obtained by analogy with the book written by Chiasson [29]. Since the permeability of the stator iron is much greater than that of air, Ampere's Law results in

$$\oint H \cdot dl = \sum I \rightarrow 2 \cdot mmf_g(\theta) = 2H_g(\theta) \cdot l_g = Ni \quad (2.51)$$

Then the spatial variation of mmf along θ will be a rectangular shape as illustrated in Figure 2.10(a). This square waveform can be expressed by means of the Fourier series expansion such as

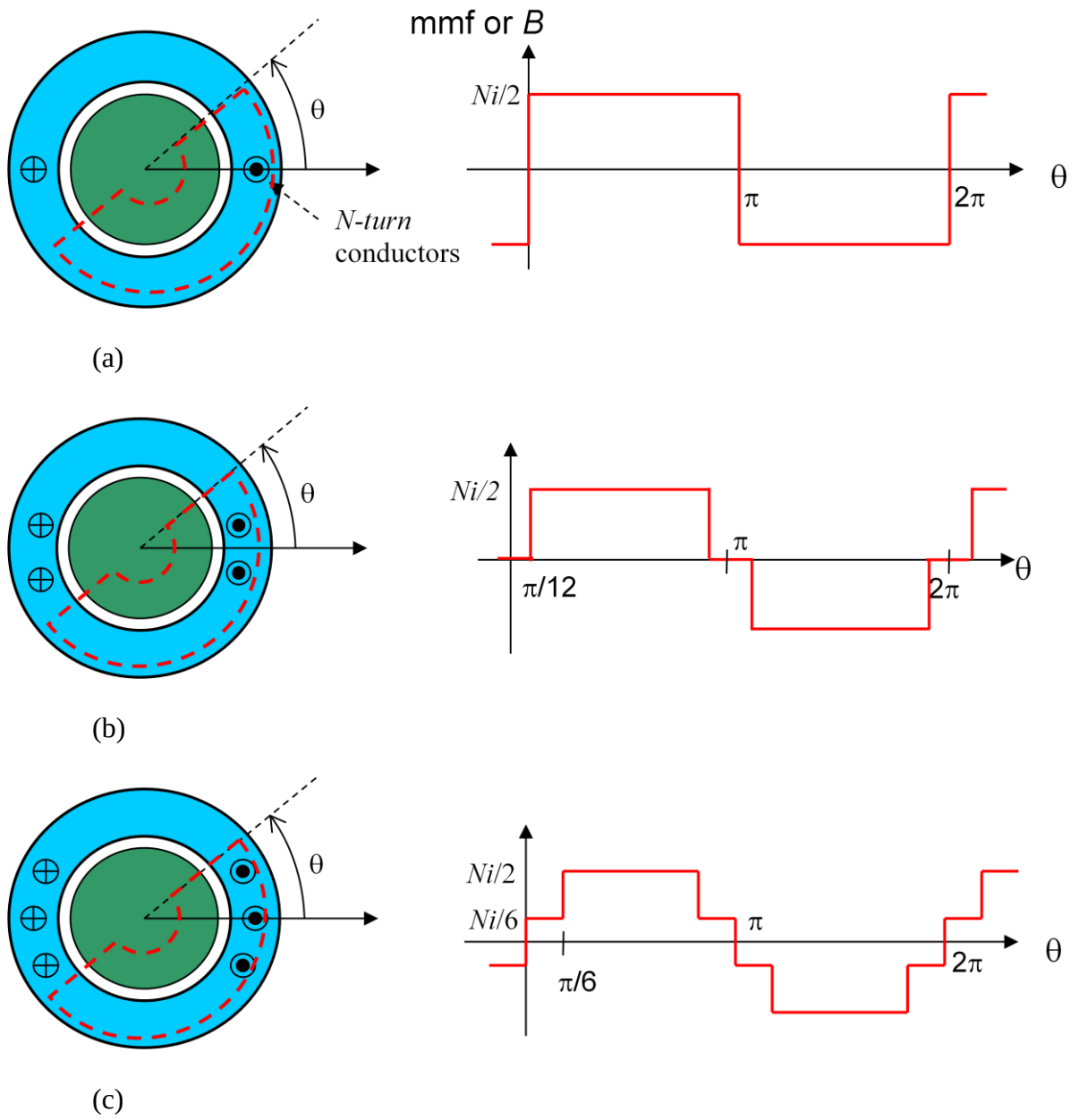


Figure 2.10 A simple structure of IPMSM with radial magnetization and its equivalent.
where i is input current.

$$\begin{aligned}
mmf(\theta) &= \frac{4}{\pi} \frac{Ni}{2} \sum_{k=1,3,5,\dots}^{\infty} \frac{1}{k} \sin(k\theta) \\
&= \frac{4}{\pi} \frac{Ni}{2} [\sin(\theta) + 0.333\sin(3\theta) + 0.2\sin(5\theta) + 0.147\sin(7\theta) + \dots]
\end{aligned} \tag{2.52}$$

If the angle between two neighboring conductors is 30° and the total number of turns is equal to N , the Fourier series expansion is as follows.

For double layered distribution (Figure 2.10(b)),

$$\begin{aligned}
mmf(\theta) &= \frac{4}{\pi} \frac{Ni}{2} \sum_{k=1,3,5,\dots}^{\infty} \frac{1}{k} \cos\left(k \frac{\pi}{12}\right) \sin(k\theta) \\
&= \frac{4}{\pi} \frac{Ni}{2} [0.966\sin(\theta) + 0.236\sin(3\theta) + 0.0518\sin(5\theta) - 0.037\sin(7\theta) + \dots]
\end{aligned} \tag{2.53}$$

For triple layered distribution (Figure 2.10(c)),

$$\begin{aligned}
mmf(\theta) &= \frac{4}{\pi} \frac{Ni}{2} \sum_{k=1,3,5,\dots}^{\infty} \frac{1}{k} \left[\frac{1}{3} + \frac{2}{3} \cos\left(k \frac{\pi}{6}\right) \right] \sin(k\theta) \\
&= \frac{4}{\pi} \frac{Ni}{2} [0.911\sin(\theta) + 0.111\sin(3\theta) - 0.0488\sin(5\theta) - 0.0349\sin(7\theta) + \dots]
\end{aligned} \tag{2.54}$$

By distributing the conductors, the magnitude of high order harmonic terms will be decreased as seen in the equations (2.52) through (2.54). In other words, the mmf waveform will approach the shape of a sine wave as shown in Figure 2.10(b) and (c). The

total harmonic distortion (THD) of each distribution can show how each waveform approaches to the fundamental sine wave mathematically.

The definition of the THD for a function $f(\theta)$ is

$$f(\theta) = \sum_{k=1,2,3,\dots}^{\infty} a_k \sin(k\theta) = \sum_{k=1,2,3,\dots}^{\infty} b_k \quad (2.55)$$

$$THD = \frac{\sqrt{\sum_{k=2,3,\dots}^{\infty} (b_{k,rms})^2}}{b_{1,rms}} = \frac{\sqrt{f_{rms}^2 - b_{1,rms}^2}}{b_{1,rms}} = \sqrt{\frac{f_{rms}^2}{b_{1,rms}^2} - 1} \quad (2.56)$$

Since the rms value of the mmf value of each distribution is $Ni/2$ for Figure

2.9(a), $Ni/2 \times \sqrt{5/6}$ for (b), and $Ni/2 \times \sqrt{\left(\frac{1}{3}\right)^2 \frac{1}{3} + \frac{2}{3}} = Ni/2 \times \sqrt{19/27}$ for (c)

respectively, the total harmonic distortion (THD) of each distribution is

$$THD_{(a)} = \sqrt{\frac{\left(Ni/2\right)^2}{\left(Ni/2 \cdot \frac{4}{\pi} \cdot \frac{1}{\sqrt{2}}\right)^2} - 1} = 0.4834 \quad (2.57)$$

$$THD_{(b)} = \sqrt{\frac{\left(Ni/2 \cdot \sqrt{5/6}\right)^2}{\left(Ni/2 \cdot \frac{4}{\pi} \cdot 0.966 \cdot \frac{1}{\sqrt{2}}\right)^2} - 1} = 0.3189 \quad (2.58)$$

$$THD_{(c)} = \sqrt{\frac{\left(Ni/2 \cdot \sqrt{19/27}\right)^2}{\left(Ni/2 \cdot 4/\pi \cdot 0.911 \cdot 1/\sqrt{2}\right)^2}} - 1 = 0.2147 \quad (2.59)$$

In the equations (2.57) through (2.59), the subscript (a), (b), and (c) represent Figure 2.10 (a), (b), and (c) respectively. Because the pure sine wave with only fundamental component has 0 for its THD value, it is clear that the mmf waveform will approach the shape of a sine wave by distributing the conductors as shown in Figure 2.10(b) and (c).

In practical analysis of a rotating electrical machine, only the fundamental component of mmf is considered. For this reason, the conductor density of a $2p$ -pole machine is

$$n_d(\theta) = \frac{2k_w N}{\pi D_s} \sin(p\theta) \quad (2.60)$$

where D_s is the inner diameter of the stator and k_w is the winding factor which is introduced to take into account the different space positions of the coils by including distribution factor, pitch factor, and skew factor [31–32, 35]. The details about the winding factor are explained in Appendix C. This conductor density means that there are $n_d(\theta)(D_s/2)d\theta$ conductors in a circle arc $(D_s/2)d\theta$ and its unit is $[m^{-1}]$.

If the machine is fed by a 3-phase current, the overall current density is

$$K_s(\theta, t) = \frac{2k_w N}{\pi D_s} \left[\sin(p\theta) i_a(t) + \sin\left(p\theta - \frac{2\pi}{3}\right) i_b(t) + \sin\left(p\theta - \frac{4\pi}{3}\right) i_c(t) \right] \quad (2.61)$$

for

$$\begin{aligned} i_a(t) &= \hat{I} \cos(p\theta_m + \omega_e t) \\ i_b(t) &= \hat{I} \cos\left(p\theta_m + \omega_e t - \frac{2\pi}{3}\right) \\ i_c(t) &= \hat{I} \cos\left(p\theta_m + \omega_e t - \frac{4\pi}{3}\right) \end{aligned}$$

where $\hat{I}$ is the maximum value of input current, ω_e electrical angular velocity, and θ_m rotor position of d-axis as indicated in Figure 2.11.

By inserting each phase current value into (2.61), the resulting current density is

$$K_s(\theta, t) = \frac{3k_w N \hat{I}}{\pi D_s} \sin(p\theta - p\theta_m - \omega_e t) = \frac{3k_w N \hat{I}}{\pi D_s} \sin(p\theta_r - \omega_e t) \quad (2.62)$$

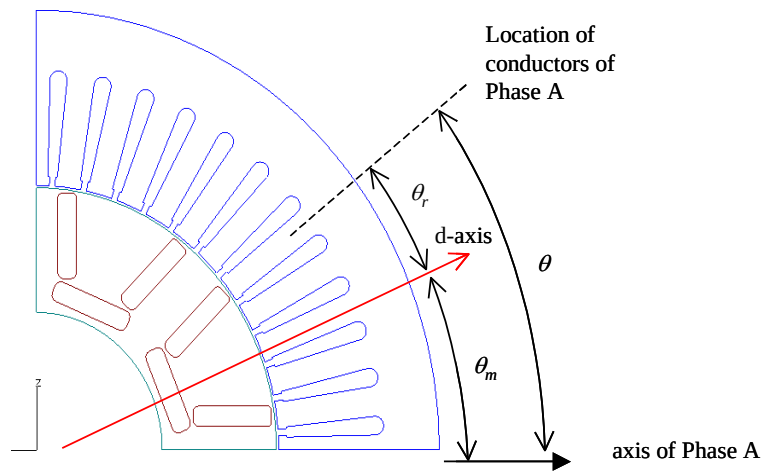


Figure 2.11 Indication of angular position based on phase A.

When $\omega_e t = 0$, the machine receives only d-axis current, and when $\omega_e t = \pi/2$, the machine is loaded by only q-axis current. The equation of the conductor density, (2.60), is used to obtain the flux linkage of the stator winding. Assuming that there is no even harmonic^{*}, then the air-gap flux density distribution will be expressed by

$$B_g(\theta) = \sum_{k=1,3,5,\dots}^{\infty} \hat{B}_k \cos(kp\theta) \quad (2.63)$$

Then, the corresponding magnetic flux across the inner surface defined by the angles between $-\theta$ and θ is

$$\begin{aligned} \phi(\theta) &= \int_{-\theta}^{\theta} \sum_{k=1,3,5,\dots}^{\infty} \hat{B}_k \cos(kp\theta') L_{st} \frac{D_s}{2} d\theta' \\ &= L_{st} D_s \sum_{k=1,3,5,\dots}^{\infty} \frac{\hat{B}_k}{kp} \sin(kp\theta) \end{aligned} \quad (2.64)$$

The infinitesimal flux that is linked by the infinitesimal conductors at θ is the product $\phi(\theta)$ of $n_d(\theta)(D_s/2)d\theta$, yielding

$$\begin{aligned} d\lambda(\theta) &= \phi(\theta) n_d(\theta) \frac{D_s}{2} d\theta \\ &= \frac{L_{st} D_s^2 k_w N}{2\pi} \sin(p\theta) \sum_{k=1,3,5,\dots}^{\infty} \frac{\hat{B}_k}{kp} \sin(kp\theta) d\theta \end{aligned} \quad (2.65)$$

^{*} In general, permanent magnets are magnetized to have only d-axis flux.

The flux linkage corresponding to the overall winding is computed by integrating the above infinitesimal flux over the periphery of stator for one pole-pair, as

$$\begin{aligned}\Lambda &= 2p \int_0^{2\pi/p} d\lambda(\theta) \\ &= \frac{L_{st} D_s^2 k_w N}{\pi} \int_0^{2\pi/p} \sin(p\theta) \sum_{k=1,3,5,\dots}^{\infty} \frac{\hat{B}_n}{k} \sin(kp\theta) d\theta\end{aligned}\quad (2.66)$$

Applying integral methods of sine-function^{*}, (2.66) results

$$\Lambda = \frac{D_s L_{st} k_w N}{2p} \hat{B}_1 \quad (2.67)$$

This result shows that only fundamental components of air-gap flux density affect the sine-distributed stator winding and provide a means to calculate the inductance values that will be shown in the next section.

2.7 Inductances of IPMSM

As shown the previous section 2.2, the large difference of inductances between q and d-axis ($L_q - L_d$) or high ratio of L_q/L_d is necessary for IPMSM to increase its torque or

$$\begin{aligned}^* \int_0^{2\pi/p} \sin(kp\theta) \sin(lp\theta) d\theta &= 0, & \text{if } k \neq l \\ \int_0^{2\pi} \sin(k\theta) \sin(l\theta) d\theta &= \pi, & \text{if } k = l\end{aligned}$$

power density. Also, finding the value of the characteristic current is an important task when designing a new machine. For this reason, obtaining the accurate inductance values of the designed motor is one of the most important issues these days.

There are several methods to calculate the inductance values of IPMSM. One traditional method is to use a magnetic equivalent circuit such as that shown in section 2.6. In this section, the analytical methods are reviewed to obtain the d- and q-axis inductance of IPMSM for tangential and radial magnetization by using a magnetic equivalent circuit. The equations in this section are obtained by analogy from the several publications [13 –17, 31, 32] especially the book written by Miller [13].

Also, there is how to set up the 2-dimensional network equivalent circuit for the inductance calculation. Then, the method using the Finite Element Analysis (FEA) and the issues of calculating inductance values will follow.

2.7.1 Analytical method for tangential magnetization

The basic equation to calculate the value of inductance for electric machines is dividing the value of the magnetic flux linkage by the applied current value such as

$$L = \frac{\lambda}{I} \quad (2.68)$$

Thus, it is important to determine the variation of the flux linkage caused by the phase current of the stator. The current density equation (2.62) from section 2.6 can be used for this analytical calculation. For only d-axis current, the current density will be

$$K_{sd}(\theta_r) = \frac{3k_w N \hat{I}}{\pi D_s} \sin(p\theta_r) = \hat{K}_s \sin(p\theta_r) \quad (2.69)$$

where the subscript “sd” means that the term is caused by d-axis stator current, and $\hat{K}_s$ is the maximum value of the current density. Then, the magnetic scalar potential or mmf at the infinitesimal region on the stator surface by d-axis current is *

$$U_{sd}(\theta_r) = \int \hat{K}_s \sin(p\theta_r) \frac{D_s}{2} d\theta_r = -\frac{\hat{K}_s D_s}{2p} \cos(p\theta_r) \quad (2.70)$$

Equation (2.70) shows that the q-axis line has null magnetic potential ($\theta_r = \pm \pi/2$). Figure 2.12(a) shows the flux line due to d-axis current, and Figure 2.12(b) is its equivalent circuit. Over one pole-pair period, the average value of U_{sd} is

$$U_{sd} = \pm \frac{\hat{K}_s D_s}{\pi p} \quad (2.71)$$

The positive value is for $\pi/2 \leq \theta_r < 3\pi/2$, and the negative value for $0 \leq \theta_r < \pi/2$ and $3\pi/2 \leq \theta_r \leq 2\pi$. The negative sign of (2.71) indicates that the flux caused by this magnetic potential flows from rotor to stator as shown in Figure 2.12. The permanent

* This indefinite integral is possible because both $K_{sd}(\theta_r)$ and $U_{sd}(\theta_r)$ are periodic functions.

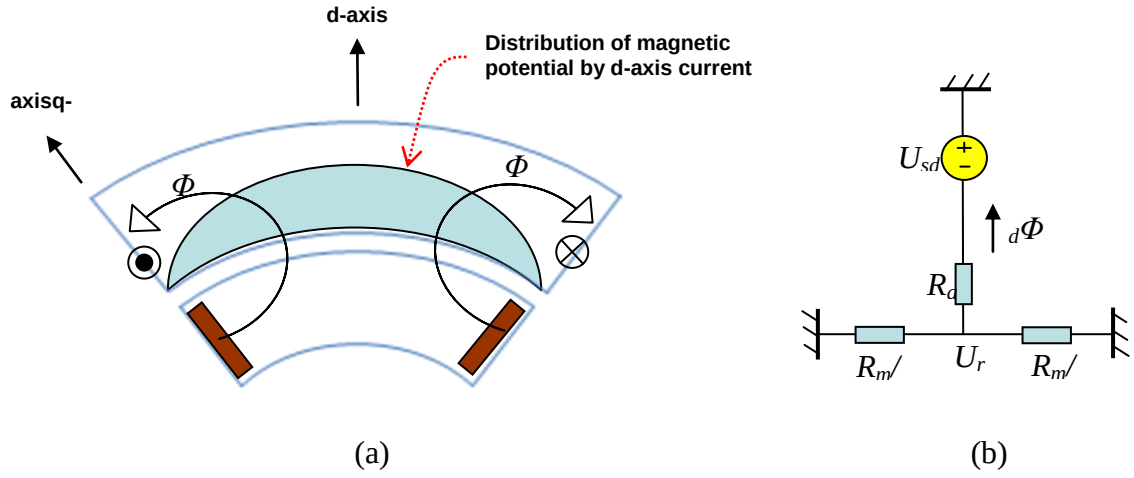


Figure 2.12 Only d-axis current electric loading and its equivalent circuit over the span of $0 \leq \theta_r < \pi/2$ and $3\pi/2 \leq \theta_r \leq 2\pi$ for a tangential magnetized IPMSM.

magnets in Figure 2.12(a) are non-magnetized and only work as reluctances of the equivalent circuit in Figure 2.12(b).

The flux in the stator or air-gap can be obtained from using the equivalent magnetic circuit and the equations of the magnetic reluctances defined in (2.33) and (2.35)

$$\Phi_{sd} = \frac{-U_{sd}}{R_g + \frac{R_{pm}}{4}} = \frac{\hat{K}_s}{p} \frac{4\mu_o\mu_r h_m D_s^2 L_{st}}{8pl_g\mu_r h_m + \pi l_m D_s} \quad (2.72)^*$$

Since the amount of the flux is one half of Φ_{sd} over the span of one half of the pole, the average magnetic potential at the air-gap is

* Under assumption that $D_s \approx 2R_{rot}$ and $L_{st} \approx L_{rot}$

$$U_r = -\frac{\Phi_{sd}}{2} \frac{R_{pm}}{2} = -\frac{\hat{K}_s}{p} \frac{D_s^2 l_m}{8p l_g \mu_r h_m + \pi l_m D_s} \quad (2.73)$$

The air-gap flux density distribution can be obtained from the difference of magnetic potential between stator and rotor by applying Ampere's law

$$B_{sd}(\theta_r) = \frac{\mu_o}{l_g} [U_r(\theta_r) - U_{sd}(\theta_r)] \quad (2.74)$$

Since only fundamental components of air-gap flux density affects the sine-distributed stator winding as shown in section 2.6, it is necessary to determine the fundamental component of (2.74). Using the Fourier series expansion,

$$\begin{aligned} B_{sd1} &= \frac{4p}{\pi} \int_0^{\pi/2p} B_{sd}(\theta_r) \cos(p\theta_r) d\theta_r \\ &= \frac{4p}{\pi} \int_0^{\pi/2p} \frac{\mu_o}{l_g} [U_r - U_{sd}(\theta_r)] \cos(p\theta_r) d\theta_r \\ &= \frac{\mu_o}{l_g} \left[\frac{4}{\pi} U_r + \frac{\hat{K}_s D_s}{2p} \right] \\ &= \frac{\mu_o}{l_g} \frac{\hat{K}_s D_s}{2p} \left[1 - \frac{8D_s l_m}{8\pi p l_g \mu_r h_m + \pi^2 l_m D_s} \right] \end{aligned} \quad (2.75)$$

Recall the flux linkage equation (2.67) from section 2.6 and substitute the definition of $\hat{K}_s$

$$\Lambda_d = \frac{3}{\pi} \frac{\mu_o D_s L_{st} I_d}{l_g} \left(\frac{k_w N}{2p} \right)^2 \left[1 - \frac{8D_s l_m}{8\pi p l_g \mu_r h_m + \pi^2 l_m D_s} \right] \quad (2.76)$$

Since the inductance is computed by dividing flux linkage by the current, the d-axis inductance is

$$L_d = \frac{3}{\pi} \frac{\mu_o D_s L_{st}}{l_g} \left(\frac{k_w N}{2p} \right)^2 \left[1 - \frac{8D_s l_m}{8\pi p l_g \mu_r h_m + \pi^2 l_m D_s} \right] \quad (2.77)$$

The process to determine q-axis inductance is the same as that of d-axis inductance, but it is much easier than the above process. Most IPMSMs have a simple magnetic equivalent circuit to increase q-axis flux caused by q-axis loading as shown in Figure 2.13.

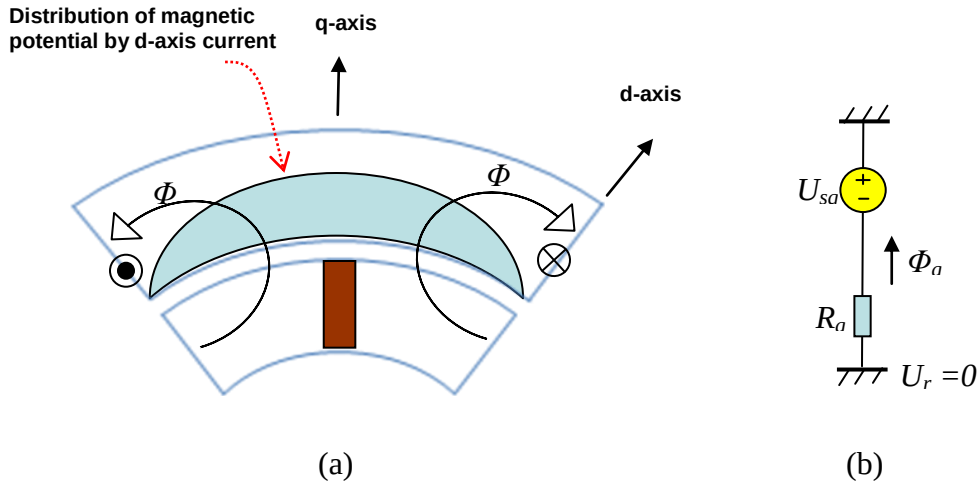


Figure 2.13 Only q-axis current electric loading and its equivalent circuit over the span of $0 \leq \theta_r < \pi$ for a tangential magnetized IPMSM.

The current density equation and the magnetic scalar potential or mmf at the infinitesimal region on the stator surface by q-axis current is respectively

$$K_{sq}(\theta_r) = -\hat{K}_s \cos(p\theta_r) \quad (2.78)$$

$$U_{sq}(\theta_r) = \int K_{sq}(\theta_r) \frac{D_s}{2} d\theta_r = -\frac{\hat{K}_s D_s}{2p} \sin(p\theta_r) \quad (2.79)$$

where the subscript “sq” means that the term is caused by q-axis stator current. Then, the air-gap flux density distribution by applying Ampere’s law is

$$B_{sq}(\theta_r) = \frac{\mu_o}{l_g} [0 - U_{sq}(\theta_r)] = \frac{\mu_o \hat{K}_s D_s}{2pl_g} \sin(p\theta_r) \quad (2.80)$$

Because (2.80) is a simple sinusoidal distribution, the fundamental component of the air-gap flux density is

$$B_{sq1} = \frac{\mu_o \hat{K}_s D_s}{2pl_g} \quad (2.81)$$

The flux linkage after substitution of the definition of $\hat{K}_s$ and q-axis inductance are

$$\Lambda_q = \frac{3}{\pi} \frac{\mu_o D_s L_{st} I_q}{l_g} \left(\frac{k_w N}{2p} \right)^2 \quad (2.82)$$

$$L_q = \frac{3}{\pi} \frac{\mu_o D_s L_{st}}{l_g} \left(\frac{k_w N}{2p} \right)^2 \quad (2.83)$$

The saliency factor ξ defined in section 2.2 is

$$\xi = \frac{L_q}{L_d} = \frac{1}{\left[1 - \frac{8D_s l_m}{8\pi p l_g \mu_r h_m + \pi^2 l_m D_s} \right]} \quad (2.84)$$

2.7.2 Analytical method for radial magnetization

For a radial magnetized IPMSM, the analysis works to calculate the values of inductances are relatively simple compared with a tangential magnetization if the rotor ridges can be neglected.

Figure 2.14 shows how to flow the magnetic flux by only d-axis current loading and its equivalent circuit. The magnetic scalar potential on the stator surface and its average value is the same as those of tangential magnetization in (2.70) and (2.71). Then the air-gap flux and the average magnetic potential at the air-gap from the equivalent circuit are

$$\Phi_{sd} = \frac{-U_{sd}}{R_g + R_{pm}} = \frac{\hat{K}_s}{p} \frac{\mu_o \mu_r h_m D_s^2 L_{st}}{2p l_g \mu_r h_m + \pi l_m D_s} \quad (2.85)$$

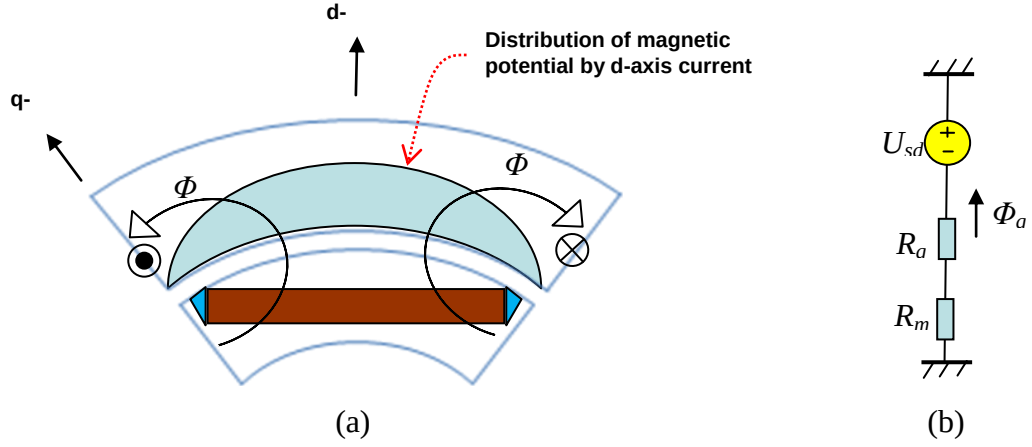


Figure 2.14 Only d-axis current electric loading and its equivalent circuit over the span of $0 \leq \theta_r < \pi/2$ and $3\pi/2 \leq \theta_r \leq 2\pi$ for a radial magnetized IPMSM.

$$U_r = -\Phi_{sd} R_{pm} = -\frac{\hat{K}_s}{p} \frac{D_s^2 l_m}{2 p l_g \mu_r h_m + \pi l_m D_s} \quad (2.86)$$

Therefore, the fundamental components of the air-gap flux density distribution can be obtained as the process of the case of tangential magnetization

$$B_{sd1} = \frac{\mu_o}{l_g} \frac{\hat{K}_s D_s}{2p} \left[1 - \frac{8 D_s l_m}{2 \pi p l_g \mu_r h_m + \pi^2 l_m D_s} \right] \quad (2.87)$$

From the result of (2.86), the d-axis flux linkage and inductance are respectively

$$\Lambda_d = \frac{3}{\pi} \frac{\mu_o D_s L_{st} I_d}{l_g} \left(\frac{k_w N}{2p} \right)^2 \left[1 - \frac{8D_s l_m}{2\pi p l_g \mu_r h_m + \pi^2 l_m D_s} \right] \quad (2.88)$$

$$L_d = \frac{3}{\pi} \frac{\mu_o D_s L_{st}}{l_g} \left(\frac{k_w N}{2p} \right)^2 \left[1 - \frac{8D_s l_m}{2\pi p l_g \mu_r h_m + \pi^2 l_m D_s} \right] \quad (2.89)$$

Since a radial magnetized IPMSM has the same q-axis magnetic equivalent circuit as shown in Figure 2.13(b), it has the same equations for the q-axis flux linkage and inductance respectively. Therefore, the saliency factor ξ is

$$\xi = \frac{L_q}{L_d} = \frac{1}{\left[1 - \frac{8D_s l_m}{2\pi p l_g \mu_r h_m + \pi^2 l_m D_s} \right]} \quad (2.90)$$

Clearly, the difference of the saliency factor between tangential and radial magnetized IPMSMs is in the denominator, and a radial magnetized IPMSM has higher saliency factor than tangential magnetized IPMSM if both are using similar size permanent magnets and rotor. For this reason, a radial magnetized IPMSM has higher reluctance torque than a tangential magnetized IPMSM. However, a tangential magnetized IPMSM is more suitable for the high-speed application such as a HEV system because of the higher d-axis inductance. Also, a tangential magnetized IPMSM has less possibility of the non-recoverable demagnetized damage of the permanent magnets because the armature current does not apply to the permanent magnets directly like a radial magnetized IPMSM [13, 14].

2.7.3 Using 2-dimensional network equivalent circuit

The concept is simple when calculating the inductance values of a machine by its 2-dimensional network equivalent circuit. For d-axis inductance calculation, the method is follows:

- i. Construct the equivalent circuit with the mmf source at the stator and without magnetic flux source caused by permanent magnets.
- ii. Set up the values of each mmf source to have only d-axis current.
- iii. Compute the flux linkage value at each position of the stator conductors by solving the equivalent circuit.
- iv. Divide the flux linkage value by the input d-axis current value.

For q-axis inductance calculation, the values of the mmf sources are just needed to switch with the desired q-axis current and take the same steps above. E. C. Lovelace et al suggested an inductance calculation method that was a different equivalent circuit of the rotor for d-axis and q-axis inductance computation respectively [38], but if the circuit is modeled with sufficient nodes, the differentiating will not be necessary. Figure 2.15 is an example of an equivalent network model for the inductance calculation constructed by the method from references [38 –40].

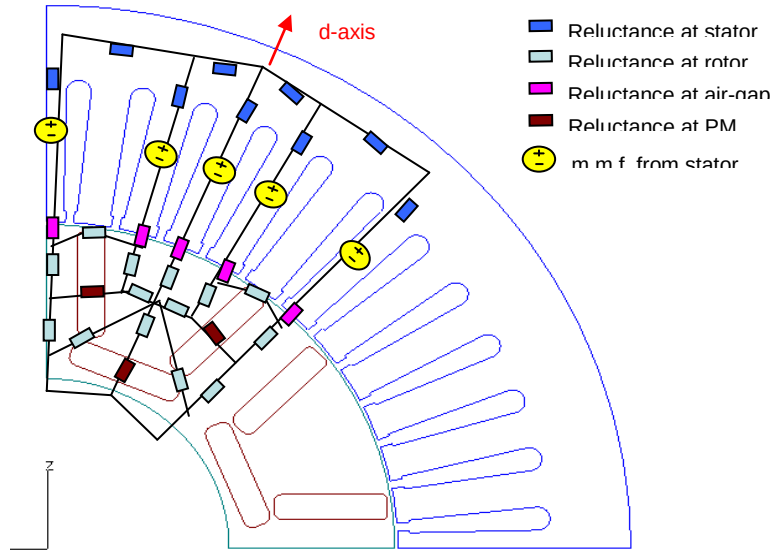


Figure 2.15 Example of an equivalent network model for the inductance calculation.

2.7.4 Energy method for inductance calculation

The above analytical processes to calculate the inductance values are performed under many assumptions, especially neglecting the leakage flux at the bridges of the rotor and the saturation of the flux path in the motor. Actually, it is difficult to consider all the leakage flux at the rotor and the stator because of many possibilities of the variations of motor shape although using a network equivalent circuit.

One of the methods considering the leakage flux is using FEA [44 – 46]. For the computation of the d-axis inductance, the simulations are performed under the conditions of unmagnetized permanent magnets and the stator windings fed by only d-axis current. After obtaining the air-gap flux density distributions from the simulation, the

fundamental components of the air-gap flux density can be obtained by discrete time Fourier series analysis as follows:

$$B_{g1} = \frac{1}{2N_o} \sum_{n=1 \leq N_o} B_g[n] \sin\left(\frac{2\pi}{N_o} n\right) \quad (2.91)$$

where N_o is the total number of data at the air-gap and $B_g[n]$ is the discrete time function at the given data position.

Then, the d-axis inductance is simply calculated by (2.67) and (2.69)

$$\Lambda_d = \frac{D_s L_{st} k_w N}{2p} B_{g1} \quad (2.92)$$

$$L_d = \frac{\Lambda_d}{I_d} \quad (2.93)$$

For the calculation of L_q , the same procedure with I_q is needed. Figure 2.16 and 2.17 are the examples of the simulation results to analyze the traction motor of Toyota Prius [47]. The non-sinusoidal graphs are the real air-gap flux density profiles at the different current values, and sinusoidal graphs are their fundamental profiles. Clearly, the values of q-axis flux density are much higher than those of d-axis flux density under the same conditions. Also, because of the saturation at the stator, the air-gap flux density is not linearly increased by increasing q-axis current. This cause the significant drop of the q-axis inductance value with increasing q-axis current.

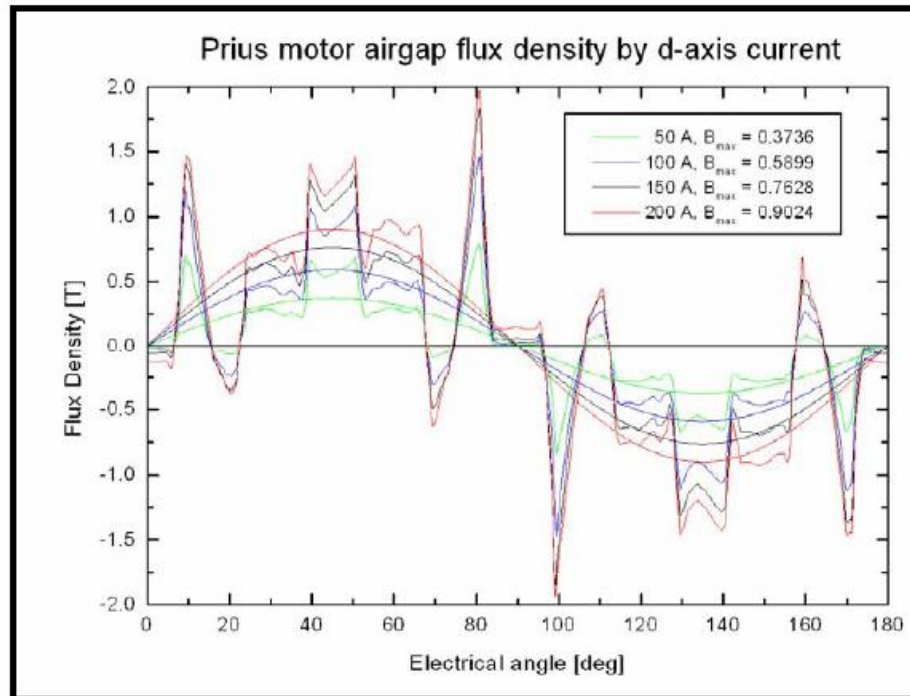


Figure 2.16 Example of d-axis air-gap flux density profiles by d-axis current only.

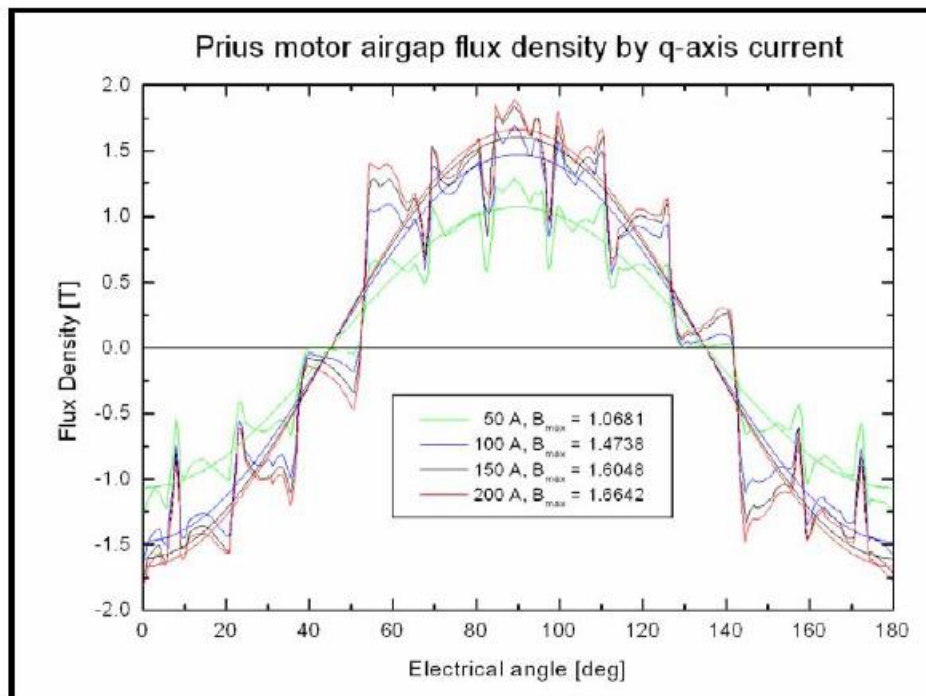


Figure 2.17 Example of q-axis air-gap flux density profiles by q-axis current only.

For accurate results, stator slot leakage inductance and end turn leakage inductance should be added to the results of the FEA and the analytical method [13 – 14, 31, 48]. Although there are many equations defining the slot leakage inductance along with the different slot shape, the simple basic equation is [31]

$$L_{sl} = \frac{\mu_o h_s L_{st} N^2}{3w_s} \quad (2.94)$$

where, h_s is the slot height and w_s is the average of the slot width. Also, the end turn leakage inductance with winding pitch τ_p is [31]

$$L_{es} = \frac{\mu_o \tau_p N^2}{k_e} \ln \left(\frac{\pi \tau_p^2}{4h_s w_s} \right) \quad (2.95)$$

with

$k_e = 8$ for a single layer

$k_e = 16$ for a double layer

Equations (2.94) and (2.95) are just example equations among the many published equations for slot and end turn leakage inductances. Actually, there are no common equations for these leakage terms because of the variance of slot shape.

Pavlik et al [49] developed a method to estimate the inductance values including all the leakage terms by using the magnetic energy obtained from the FEA simulation.

Let W_d be the magnetic energy stored in the IPMSM by only d-axis current. Then, from the relationship between the energy and inductance, W_d is

$$W_d = \frac{3}{2} \left(\frac{1}{2} L_d I_d^2 \right) \quad (2.96)$$

where, the factor 3/2 is from Park's transformation. Therefore, the resulting inductance is

$$L_d = \frac{2}{3} \left(\frac{2W_d}{I_d^2} \right) \quad (2.97)$$

$$L_q = \frac{2}{3} \left(\frac{2W_q}{I_q^2} \right) \quad (2.98)$$

The energy method is a desirable method to calculate the inductance values because this method can consider not only the complex shape of the magnetic path but also the nonlinear magnetic property of the material used in the path. One of the shortcomings of this method is that the output will have a slightly different value by the position of the rotor arrangement because of the difference of harmonic content.

2.7.5 Consideration of the saturation

Although it is very convenient to use the energy method shown in the above section, the major problem is that this method cannot consider the magnetic saturation from the permanent magnets since the energy calculation from (2.96) should be

performed without the presence of the permanent magnets. However, in a real environment an IPMSM is always operated under the influence of the permanent magnets.

Equations (2.99) and (2.100) are the classical form of flux linkage equation in d- and q- axis model [50 –52], which is an easy method considering the saturation from the permanent magnets.

$$\lambda_d(i_d) = \lambda_{d,PM} + L_d i_d \quad (2.99)$$

$$\lambda_q(i_q) = L_q i_q \quad (2.100)$$

The value of the flux linkage can be obtained from FEA using (2.91) and (2.92) as shown in the previous section. Then the inductance values can be obtained from

$$L_d(i_d) = \frac{\lambda_d(i_d) - \lambda_{d,PM}}{i_d} \quad (2.101)$$

$$L_q(i_q) = \frac{\lambda_q(i_q)}{i_q} \quad (2.102)$$

From the above equations, the inductance values of each direction can be determined by the variation of the current applied to it own direction. The drawback of this method is that there is no flux linkage coupling between d- and q-axis due to the saturation from the other axis current applied.

Since El-Serafi et al [53] showed that the cross-magnetizing phenomenon between two axes in saturated synchronous machine in 1988, it has been recognized the

importance of the cross-saturation these days. Especially, when a machine has highly saturated magnetic circuit, the cross-saturation between d- and q- axis is not negligible [46, 54].

One of the methods considering the cross-saturation terms when calculating the inductance values is mapping the values of d- and q-axis flux linkage as the variation of the current in both directions [52, 55] such as

$$\lambda_d = \lambda_d(i_d, i_q), \lambda_q = \lambda_q(i_d, i_q) \quad (2.103)$$

Then, the inductance values of each direction are

$$L_d(i_d, i_q) = \frac{\lambda_d(i_d, i_q) - \lambda_{d,PM}}{i_d} \quad (2.104)$$

$$L_q(i_d, i_q) = \frac{\lambda_q(i_d, i_q)}{i_q} \quad (2.105)$$

If a machine has a complicated shape in its magnetic flux path, it is necessary that the flux linkage (2.103) should take account of angular position variation by averaging over one electrical cycle [52]. To improve the accuracy, there have been many studies to add the mutual inductance terms to (2.104) and (2.105) [53 –55], but it is not mature yet.

2.8 Finite Element Analysis – Maxwell 3D

Obtaining the correct value of flux density and magnetic field remains an important issue for designing an electric machine in verifying the magnetic flux loop and checking the saturation of flux density in each part of the machine. For this purpose, FEA is used extensively for the design and performance prediction of all types of electric machines. This section is for a brief explanation about the FEA.

FEA is a numerical technique based on the determination of the distribution of the electric or magnetic fields inside the machine structure, by means of the solution of Maxwell's equations. FEA can determine the value of torque or force, field distributions, energy variations, etc [56].

To present electric or magnetic fields over a large, irregularly shaped region, each region of the modeled machine is divided into many hexahedral or tetrahedral elements.* The collection of elements is referred to as the finite element mesh. In FEA computation, the value of a vector field at a location within an element is interpolated from its nodal value of the field. For magnetic field calculation, Maxwell 3D solver divides the H-field into a homogeneous and a particular solution. The solver stores a scalar potential at each node for the homogeneous solution of magnetic field density, H , and stores the components of H that are tangential to the element edges [57]. Thus, to obtain a precise description of the field, each element of the model is small enough for the field to be adequately interpolated from the nodal values.

However, there is a trade-off between the number of elements and the amount of computing resources required for the computation. The accuracy of the solution also

* Maxwell 3D uses tetrahedral elements.

depends on how small each of the individual elements is. Thus, a user should choose an adequate mesh size for fitting his computer capability.

The computing procedure is as follows. First, the current density, J , is calculated by input current condition. Using Ohm's law (2.106), the J of all elements in the model is obtained and stored.

$$\vec{J} = \sigma \vec{E} = -\sigma \nabla V \quad (2.106)$$

where,

E is the electric field

σ is the conductivity of the material

V is the electric potential

Under steady state conditions, the charge density, ρ , in any region of the model does not change with time [57]. That is

$$\nabla \cdot \vec{J} = \frac{\partial \rho}{\partial t} = 0 \quad (2.107)$$

Substituting (2.106) into (2.107):

$$\nabla \cdot (\sigma \nabla V) = 0 \quad (2.108)$$

To solve above Laplace's equation, two kinds of boundary conditions can be applied [58, 59]. First one is Dirichlet boundary condition, which means that the nodes at the boundary are singly constrained with a fixed potential value, usually zero for FEA. In other words, the potential value at the outside of the boundary is zero. This Dirichlet boundary condition implies that there is no leakage flux beyond the analysis area. Second condition is Neumann boundary condition, which means that the rate of change of the potential normal to the boundary is specified. The rate is also zero for FEA. Therefore, the flux flows orthogonally on the Neumann boundary condition.

With a boundary condition, the differential equation (2.108) is solved to get the J for all elements. After computing the current density, the FEA solver computes the magnetic field using Ampere's law and Gauss' law in the Maxwell equations describing the continuity of flux,

$$\nabla \times \vec{H} = \vec{J} \quad (2.109)$$

$$\nabla \cdot \vec{B} = 0 \quad (2.110)$$

Next, the energy is calculated. In a linear material, the energy, W , is the same as its coenergy, W_c , as

$$W = \int_v (d\vec{B}) \cdot \vec{H} dv = \frac{1}{2} \int_v \vec{B} \cdot \vec{H} dv \quad (2.111)$$

$$W_c = \int_v \vec{B} \cdot (d\vec{H}) dv = \frac{1}{2} \int_v \vec{B} \cdot \vec{H} dv = W \quad (2.112)$$

However, in a nonlinear material, the coenergy is different from its energy as shown in Figure 2.18. Energy and coenergy in the figure indicate the area above and below the B-H curve.

If the motion of a material has occurred under constant current conditions (steady-state), the mechanical work done can be represented by increasing its coenergy [15]. FEA solver is computing H under constant input current condition, thus, it uses coenergy differentiation over a given angle instead of energy differentiation for torque calculation with nonlinear data of the materials. And the corresponding B vector at any point can be obtained from the inputted B-H curve data.

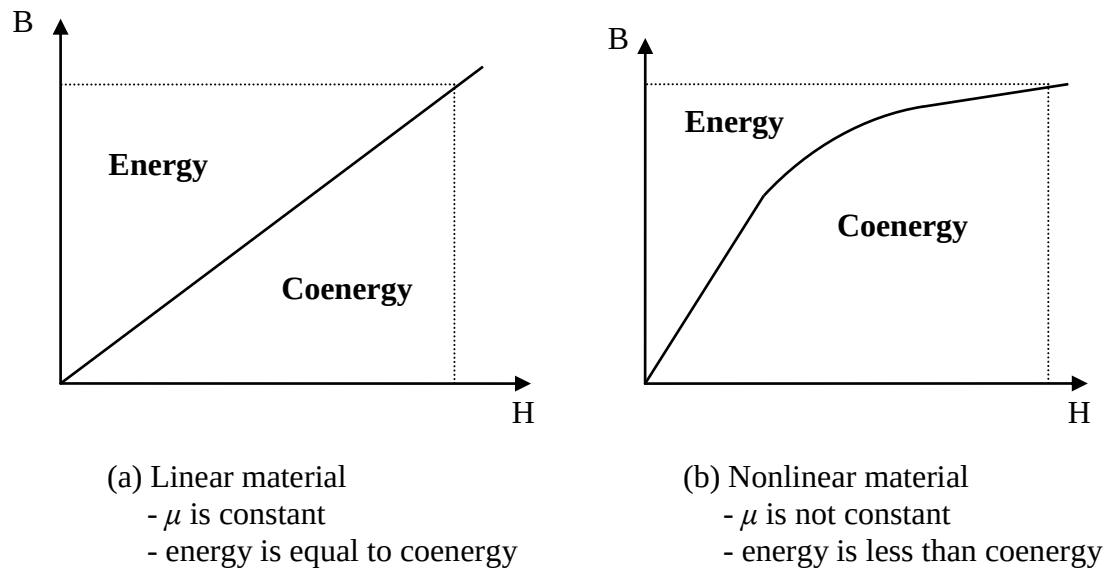


Figure 2.18 Comparing B-H curve between linear and nonlinear material.

2.9 Summary

As a summary, the characteristic current of an IPMSM determines its speed limit and a small value is desirable. To reduce the characteristic current value, it is necessary to reduce the flux from the permanent magnets in the rotor. Ironically, the rotor flux must be high to hold the advantages of the IPMSM, such as high torque density and high efficiency. Therefore, it is necessary to develop a method to increase its high speed capability while still maintaining the ability to provide high torque at low speeds.

This chapter also includes the various equations to calculate the flux linkage and inductance values in d- and q-axis respectively because these parameters are very important factors in analyzing an IPMSM. When calculating the flux linkage and inductance by an analytical method, the theoretical review is explained by two basic models, tangential and radial magnetization IPMSM. The equivalent circuit analysis will be helpful to understand the basic concept of an IPMSM.

In virtue of the progress of computer technology, the inductance values are widely obtained by using FEA these days. Determining the saturation effect between d- and q-axis is the one of the important issues to analyze an IPMSM, and it needs further investigation to solve this issue.

CHAPTER 3

New Features of the IPMSM

3.1 Introduction

It has been widely accepted that an IPMSM is the best type of motor for HEVs since Toyota and Honda adopted the IPMSM as the traction motor of their commercial HEVs in the 1990s^{*}. This is because the IPMSM has higher power density and higher efficiency than other types of motor.

However, an IPMSM has some drawbacks for the application in a HEV, and one of them is the speed limit caused by the high magnetic flux from the permanent magnets inside the rotor which are the main components of the machine to hold high power density and high efficiency ironically. Therefore, the desired goals in designing a new IPMSM for the application in a HEV system are achieving wide operating speed range without losing high power density and high efficiency.

J. S. Hsu at ORNL devised the High-Strength Undiffused Brushless (HSUB) Machine which increases the air-gap flux by the additional excitation current [60]. Based on this machine, the new IPMSM was devised to control the magnitude of the air-gap flux by using three dimensional magnetic rotor flux [6, 61 – 63] with brushless field excitation (BFE) structure.

^{*} The Toyota Prius went on sale as the first mass-produced HEV in Japan in 1997 and Honda introduced the Insight to public in 1999.

Using BFE structure, the air-gap flux of a traditional radial-gap IPMASM can be enhanced by axial-directional flux from the sides of the rotor. As a result, the air-gap flux will be increased and then the motor torque will also increase at a given stator current. Because the intensity of the axial-directional flux is controlled by DC current, the air-gap flux can decrease for the high-speed operation.

Another particular feature of the new IPMSM is the slant shaped rotor [64]. This structure is to increase the operating speed range of the machine by boosting the ratio of the magnitude of the controllable air-gap flux. The irregular shaped air-gap makes a flux barrier only on the d-axis flux path and decreases the d-axis inductance. Therefore, the saliency ratio will be increased and so will the reluctance torque of the IPMSM as shown in the previous chapter.

The contributions of this study are showing the process of the development of the BFE and slanted air-gap structure in terms of electromagnetic analysis including all FEA simulations. The analysis results help J. S. Hsu at ORNL to determine the final shape of the prototype motor. In this chapter, the details are explained about these two novel features; how these structures would function and what the advantages of each are.

3.2 Side Magnetic Flux

Conventional electric machines are constructed from the following parts: a rotor assembly, laminated stator material, phase current-carrying conductors, and a physical structure to support the entire machine. When a current-carrying conductor is placed in a

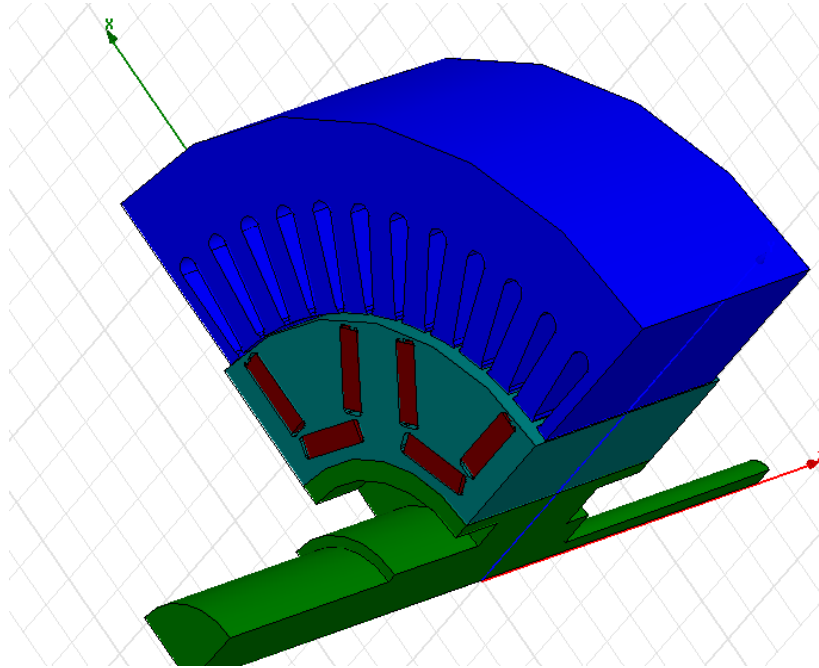
magnetic field from the rotor, the conductor experiences a mechanical force or torque. This force or torque is directly proportional to the intensity of the magnetic field.

Therefore, if the air-gap flux can be increased, then the output power is also increased. However, the enhancement of the air-gap flux requires more or higher quality of permanent magnets. This results in enlargement of the rotor size or raising the manufacturing costs.

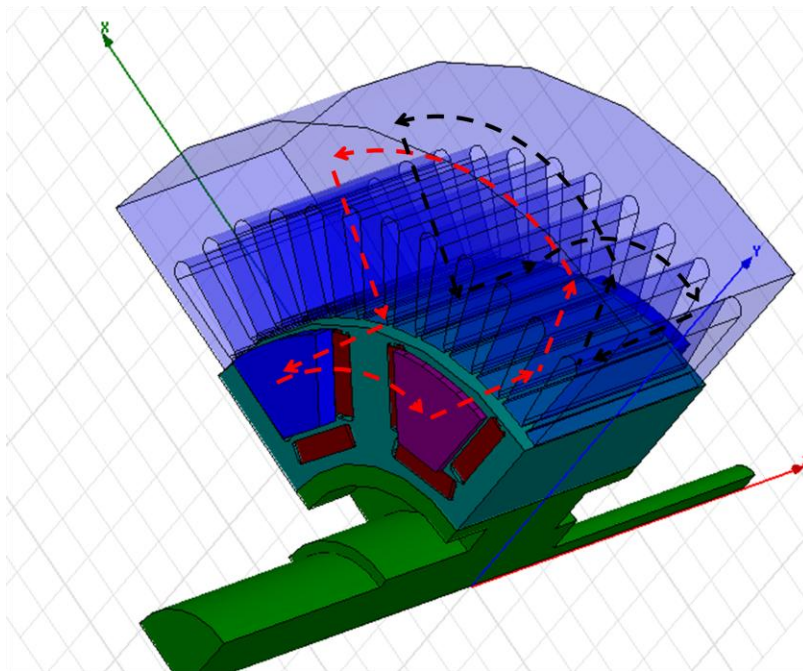
One possible way to increase the air-gap flux density is improving the flux path to break away from the traditional two dimensional approaches for motor designs. This possibility can be accomplished by using the axial flux from both axial sides of the rotor. In a radial IPMSM, additional DC magnetic flux flows from both sides of the rotor axially into the rotor, travels radially through the air-gap to the stator, returns from the adjacent poles of the rotor to complete its magnetic circulation. This flux is added to the magnetic flux from the permanent magnets inside the rotor and interacts with the stator current to produce motor torque.

3.2.1 Structures of the side permanent magnet

Figure 3.1(a) shows various permanent magnet arrangements of an IPMSM that has no additional magnetic field from its sides. The U-shaped permanent magnets are symmetrically arranged for the north and south poles of the rotor, respectively. Figure 3.1(b) shows that in addition to the U-shape permanent magnets, there is either a partial side-permanent magnets (side-PMs) or a full side-PMs arrangement. For the partial side-PMs arrangement, the side-PM and soft magnetic material pieces (side-poles) are



(a) Base IPMSM without side PMs



(b) Tested IPMSM with side PMs

Figure 3.1 Rotor structures for side field excitation for a partial side-PMs arrangement.

alternately located at the sides of the rotor poles. A soft magnetic end ring (not shown) is attached to the sides of the side-PMs to provide a flux return path for the side-PMs.

The arrows in Figure 3.1(b) indicate the directions of the magnetic flux from the side-PMs. Figure 3.2 is the simulation result showing that the flux flows from the sides of the rotor as expected. For the full side-PMs arrangement, permanent magnets replace the side-poles, and boost the magnetic flux from both sides of the rotor.

To confirm the effects of side excitation, the FEA simulation works are performed without side-PMs, partial side-PMs, and full side-PMs respectively. An existing IPMSM of Toyota/Prius is also simulated to obtain the relative result. These simulation models have the same dimensions of outer rotor and inner stator diameter. The stator does not have teeth and conductors to reduce the possibility of interference from the leakage flux at the stator.

Figure 3.3 is the simulation results of the radial air-gap flux density distributions of each model. This figure shows that the air-gap flux can be increased by attaching side-PMs. The flux density of full side-PMs can reach double the value of the Prius model.

The laboratory tests confirmed these simulation results. In case of the Prius model, the air-gap flux density was 0.76 T (simulation result is 0.70 T) and the motor with the partial side PMs has the value of 1.20 T (simulation result is 1.23 T) [61].

An actual prototype motor with partial side-PMs was assembled and tested. Figure 3.4 shows the rotor core of the prototype motor, and Figure 3.5 shows the locations of the side pole and side PM of the prototype motor. The magnetic end rings are not shown in this figure. The prototype uses an aluminum non-magnetic spacer to

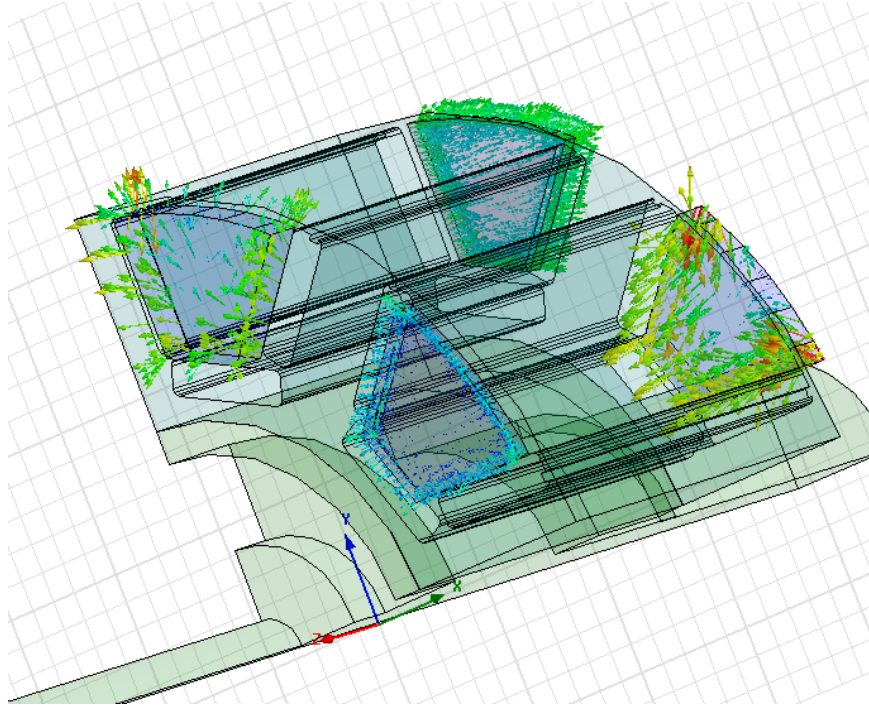


Figure 3.2 Simulation showing the flux from the sides of rotor for a partial side-PMs arrangement.

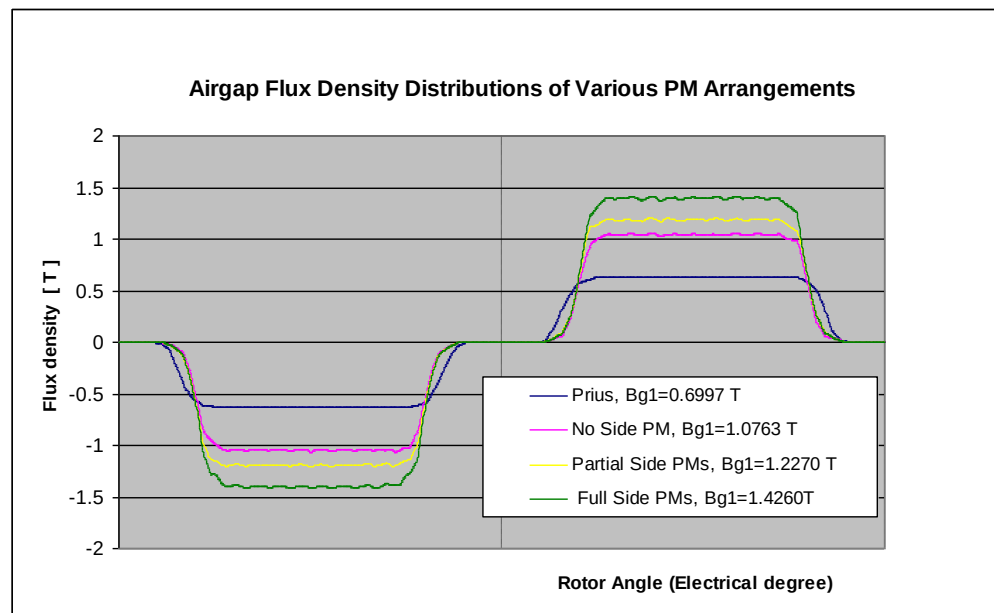


Figure 3.3 Comparison of air-gap flux density distributions.

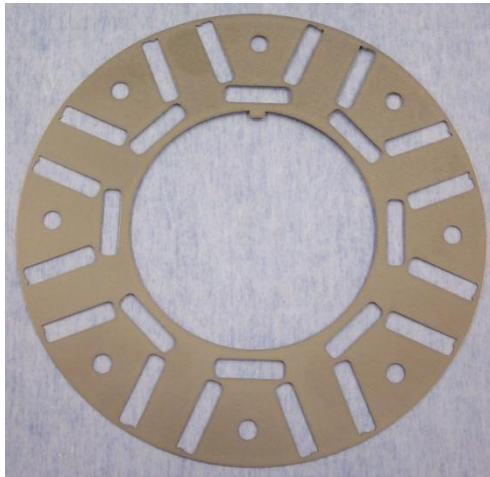


Figure 3.4 Laminated rotor core of the prototype motor.

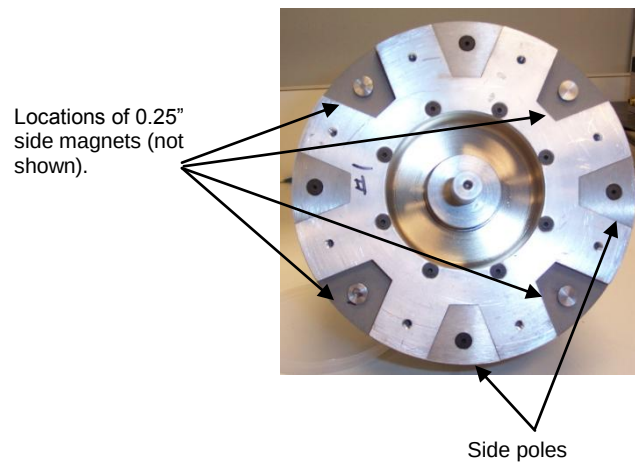


Figure 3.5 Rotor of the prototype motor with the partial side-PMs.

mechanically secure the side poles and side PMs. The laboratory torque tests of the prototype motor with the partial side-PMs agree with those of the simulation (Figure 3.6).

Comparison of the tested efficiency maps of the Prius motor and the prototype motor with partial side PMs are drawn in Figure 3.7 and Figure 3.8. This figure shows that the efficiency yield by the prototype with partial side PMs is comparable (about 1% higher in overall high speed region from 1 complete test run) to that of the Prius motor.

However, the Prius motor is expected to have a relatively higher efficiency at high torque region. These results suggest that the side-excited structure needs further improvement to achieve the higher torque efficiently, such as improving the flux path for reluctance torque production.

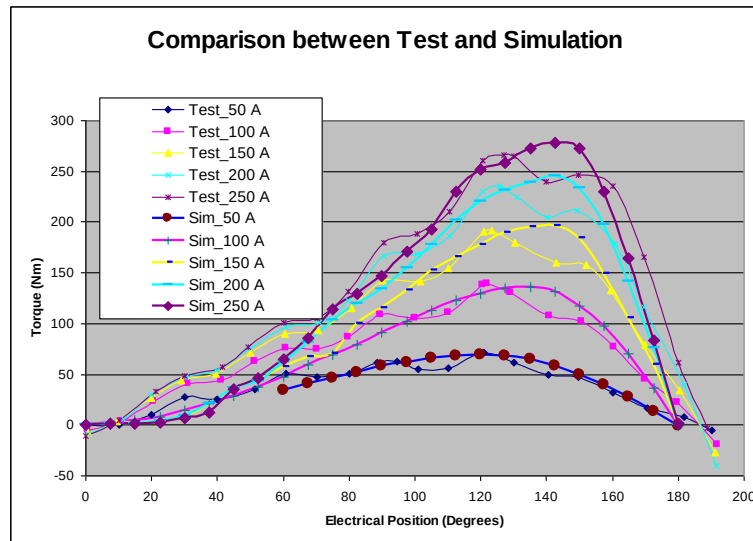


Figure 3.6 Comparison of output torque between simulation and test results of the motor with the partial side-PMs.

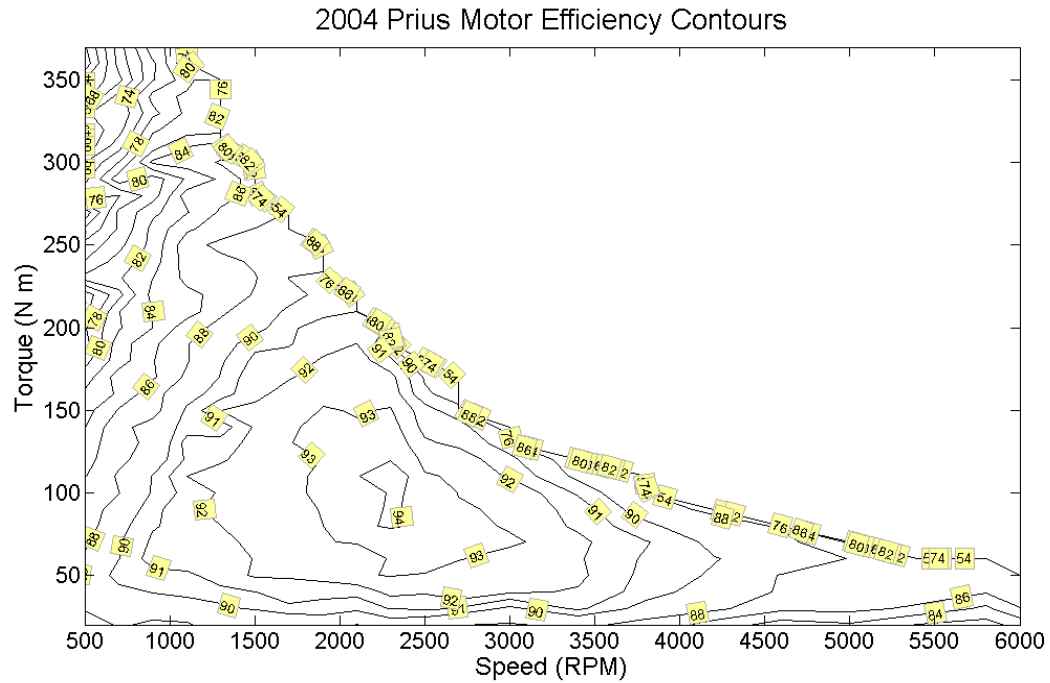


Figure 3.7 Tested efficiency map of 2004 Prius motor.

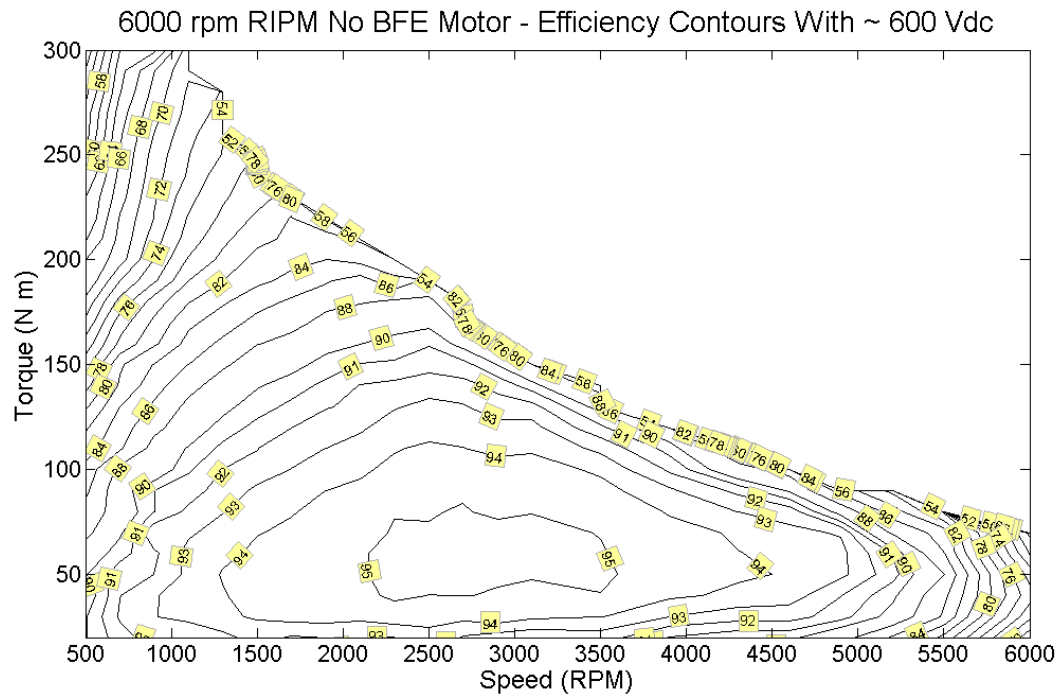


Figure 3.8 Tested efficiency map of the prototype motor with partial side PMs.

3.2.2 Rotor structure with DC current excitation

Although the side-PM structured motor has the advantage for increasing output torque in limited size, the use of high quality permanent magnets on both sides of the rotor is unfavorable for manufacturing because of the high costs of the permanent magnets.

Therefore, the modified IPMSM has DC excitation coils for generating the side magnetic flux. This magnetic flux goes to the rotor through side-poles placed as shown in Figure 3.5. In essence, the magnetic flux generated by DC excitation coils replaces the side permanent magnets.

Figure 3.9 shows the pattern of flow of the excitation flux at d-axis position, and Figure 3.10 is the simulation result showing the magnetic flux movement on the sectional plane of d-axis and q-axis respectively.

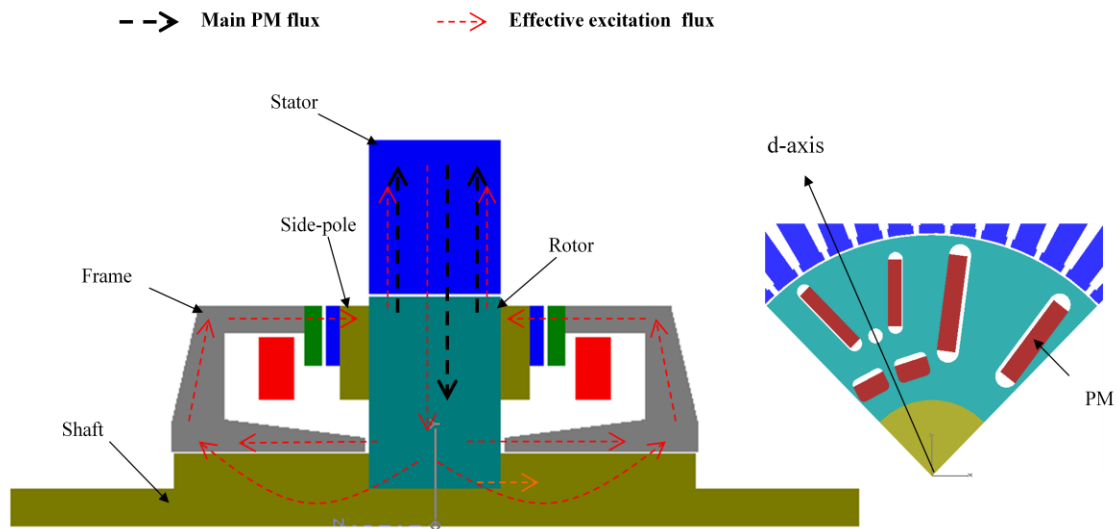
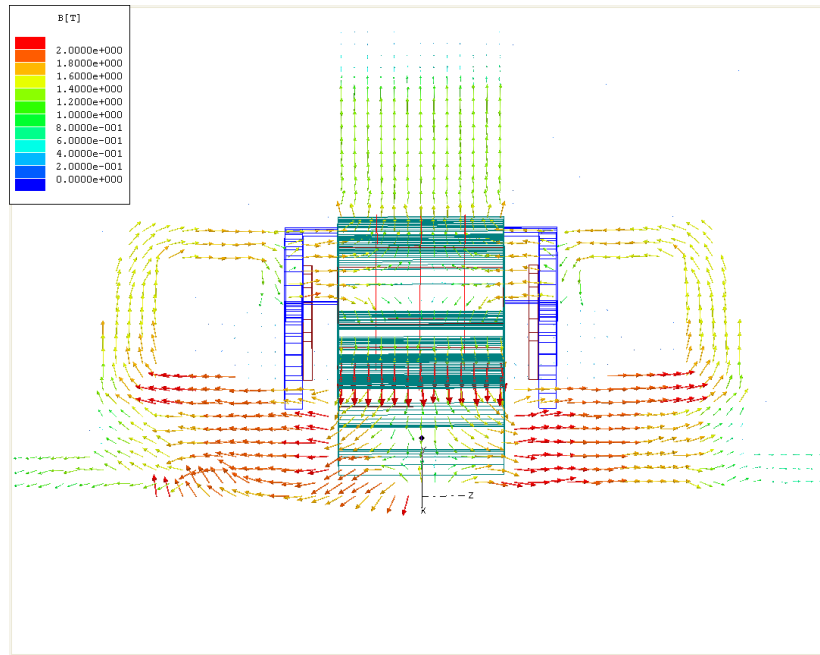
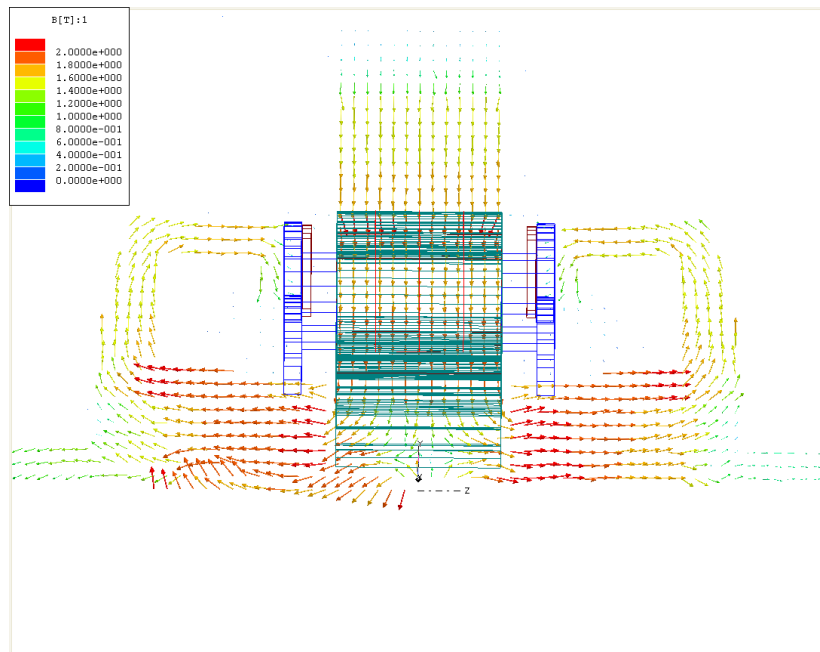


Figure 3.9 Excitation flux concept with DC excitation current.



(a) d-axis plane



(b) q-axis plane

Figure 3.10 Simulation results showing the excitation flux in IPMSM with DC excitation coils.

In addition to reducing the amount of permanent magnets, this structure has another advantage; controlling air-gap flux by DC current. In other words, the amount of the air-gap flux can be changed to meet the requirements of the controller. For an application such as for the traction motor of an HEV, at low speed the motor can achieve higher air-gap flux by increasing the excitation current value and at high speed the motor exhibits lower flux density by reducing the DC current value. Development of this capability requires additional research work to design an improved IPMSM. This is described in the next section.

3.3 Slanted air-gap structure

The newly designed motor has a unique slanted air-gap shape. This structure is designed for increasing output torque by reluctance torque with increasing the controllable back-emf ratio between the lowest and the highest excited condition of the brushless field excitation (BFE) using side-flux [64]. Although the irregularly shaped air-gap reduces the air-gap flux caused by its wide length of the air-gap, it also makes a flux barrier only on the d-axis flux path and decreases the d-axis inductance. Therefore, the reluctance torque of the machine increases to compensate for the decreased PM torque; as a result, the machine achieves a higher ratio of the magnitude of controllable back-emf without losing the high torque capability that results from the BFE.

3.3.1 Theoretical approach to slanted air-gap

Without considering the cross-coupled flux linkage between the d- and q- axes, the reluctance torque equation from (2.12) in Chapter 2 is expressed as

$$T_r = \frac{m}{2} p (L_d - L_q) i_d i_q = \frac{m}{2} p (\lambda_{d,id} i_q - \lambda_{q,iq} i_d) \quad (3.1)$$

In (3.1), m is the number of phase conductors, p is the number of the pole pair, L is inductance, and i is the instantaneous current. The subscripts d and q indicate the d-axis and q-axis, respectively. In the right side of (3.1), $\lambda_{d,id}$ is the d-axis flux linkage caused by d-axis current, and $\lambda_{q,iq}$ is caused by q-axis current. Since the total output torque is determined by mainly the q-axis current, the reluctance torque will be increased when $\lambda_{d,id}$ is small and $\lambda_{q,iq}$ is high. Slanted air-gap is considered for this purpose.

Figure 3.11 shows the example rotor with the slanted air-gap. The solid lines in the figure are the original d- and q-axis when the rotor has a uniform air-gap. To increase the reluctance torque, the new d- and q- axis will be rotated with the angle of α , as shown by the dashed line in Figure 3.11. Then, the new reluctance torque equation will be

$$T_r = \frac{m}{2} p (L_{d'} - L_{q'}) i_{d'} i_{q'} = \frac{m}{2} p (L_{d'} - L_{q'}) i_{d+\alpha} i_{q+\alpha} \quad (3.2)$$

Since the PM flux is sufficiently strong and the rotor has a wide neutral space between the north and south poles, the original d-q axis for the PM flux will be shifted

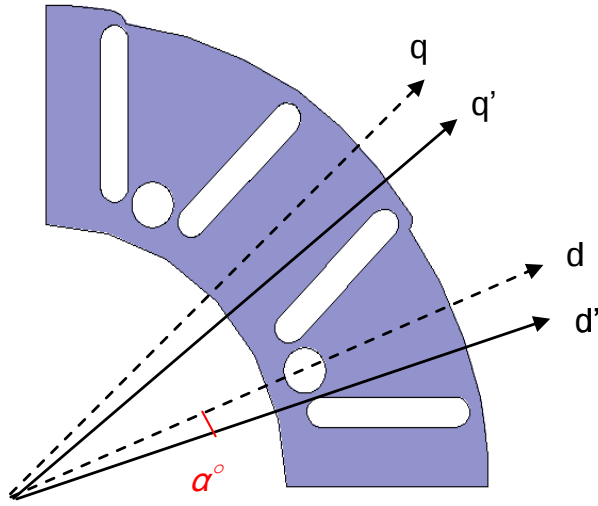


Figure 3.11 Example rotor with slanted air-gap.

slightly. Assuming that the q-axis flux linkage is relatively small in the sine-distributed winding machine, the overall output torque equation is

$$T = \frac{m}{2} p (\lambda_{d,PM} i_q + (L_{d'} - L_{q'}) i_{d+\alpha} i_{q+\alpha}) = \frac{m}{2} p (\lambda_{d,PM} I \sin \theta + (L_{d'} - L_{q'}) I^2 \sin 2(\theta + \alpha)) \quad (3.3)$$

where θ is the current angle on the current d-q plane. Equation (3.3) indicates that the maximum value of the reluctance torque will be shifted from 135° toward 90° with the angle of α as shown in Figure 3.12. Figure 3.12 also shows that the maximum value of the total output torque is little changed, although the increase in the air-gap causes reduced PM torque in the slanted air-gap.

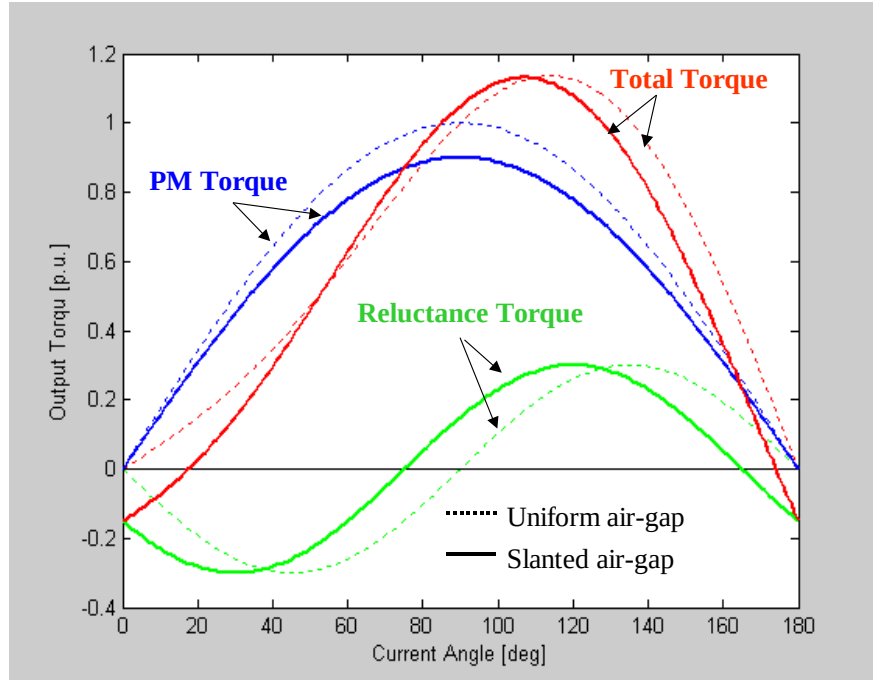


Figure 3.12 Expected output torque of slanted air-gap in Figure 3.11.

3.3.2 FEA simulation results

The theoretical approach to slanted air-gap structure can be ascertained by FEA simulation. Figure 3.13 shows three different rotor models with the same permanent magnet arrangement, stator shape, and excitation structures as described in section 3.2.2. Rotor in Figure 3.13 (a) has an ordinary air-gap width; 0.74 mm (0.029 inches); rotor in (b) has non-uniform air-gap width between the permanent magnets; the deepest air-gap width is 2 mm (0.079 inches); and rotor in (c) has slant air-gap shape; gap width varies from 2.54 mm (0.1 inches) to 0.74 mm (0.029 inches). Each model has the same air-gap length on the q-axis at 0.74 mm but on the d-axis the length is different; 0.74 mm for (a), 2 mm for (b), and variable distance for (c).

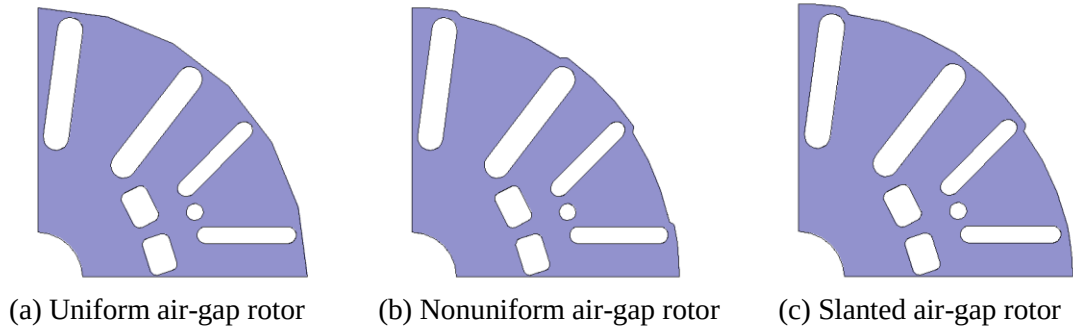


Figure 3.13 Three different basic rotor shapes.

The increased distance of the air-gap aids in blocking the d-axis magnetic flux flowing from stator to rotor or reverse but does not affect significantly the q-axis. This results in higher saliency ratio and increased reluctance torque. Figure 3.14 is the simulation results showing that the reluctance torque is increased for the case of the non-uniform or slant air-gap rotor comparing with the torque of the rotor with uniform air-gap. As expected in theoretical approach, in the case of the slant air-gap rotor, the angle position of maximum torque is shifted toward 90° which is the position at which maximum torque is achieved without reluctance torque.

For this reason, the maximum torque position of the slant air-gap rotor is also shifted toward 90° as shown in Figure 3.15. Therefore, the peak value of the maximum torque of the slant air-gap rotor is almost the same as that of the uniform air-gap rotor in spite of the lack of air-gap flux as shown in Figure 3.16.

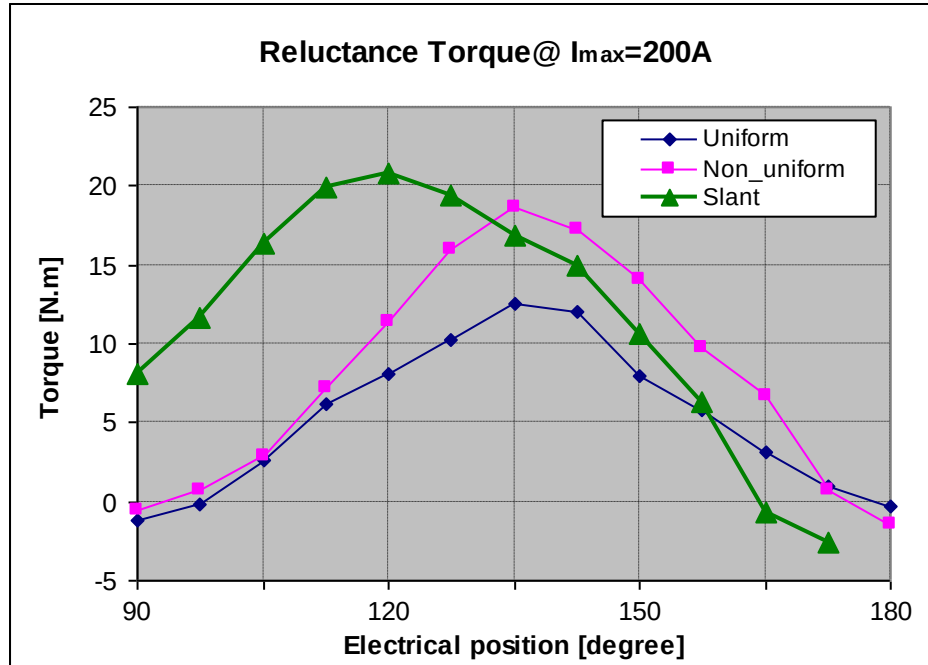


Figure 3.14 FEA simulation results of the reluctance torque of each model.

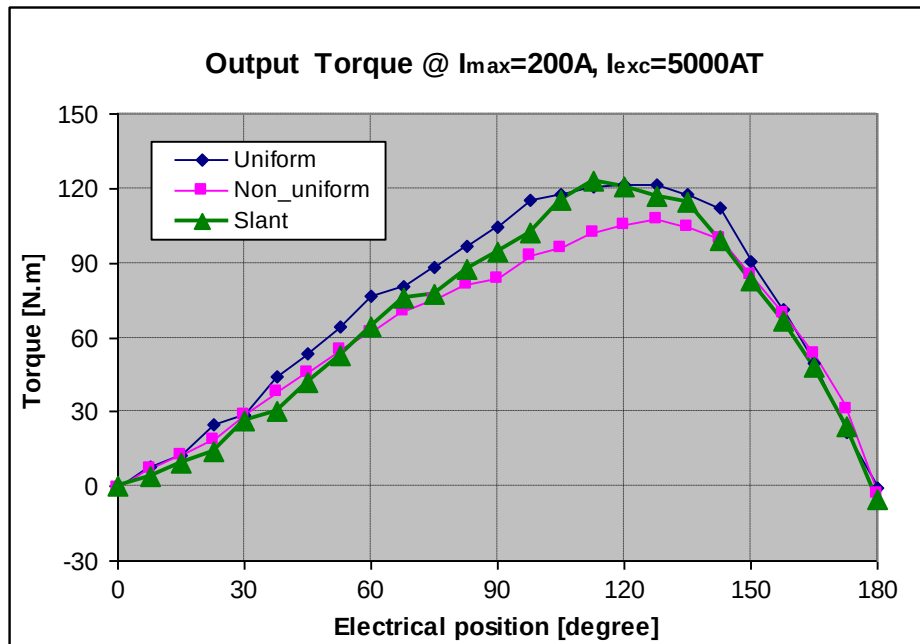
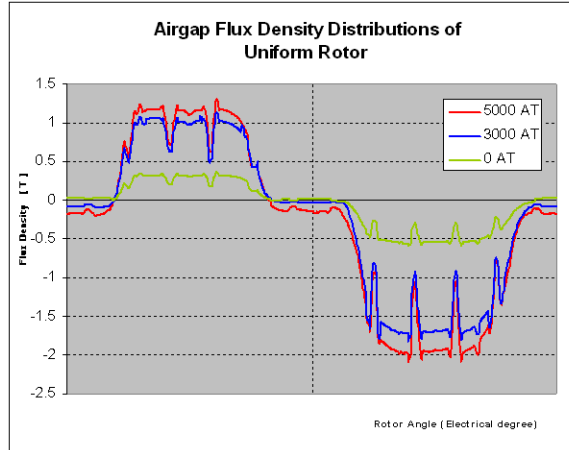
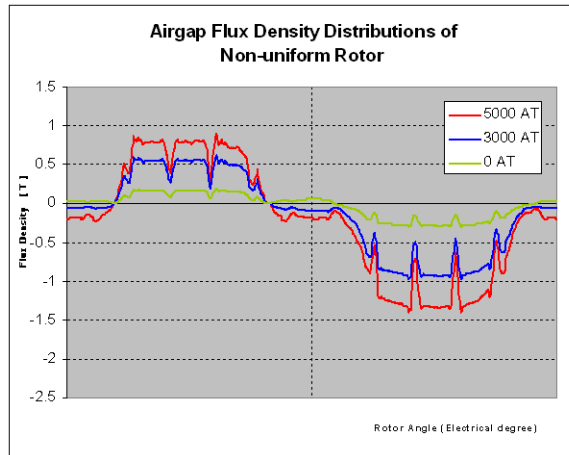


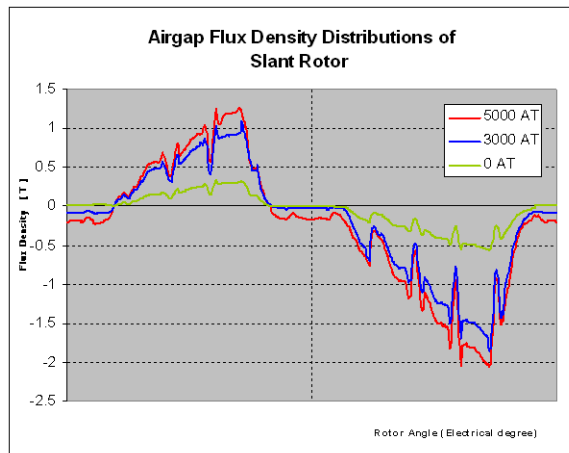
Figure 3.15 FEA simulation results of the output torque of each model.



(a) Uniform air-gap rotor



(b) Non-uniform air-gap rotor



(c) Slant air-gap rotor

Figure 3.16 The air-gap flux density distributions of different rotor types.

This characteristic is one of the advantages of the slanted air-gap IPMSM because the iron loss* will be decreased at the same output torque operation. Figure 3.16 also shows another advantage of the slanted air-gap rotor. By the benefit of the DC excitation structure, the air-gap flux density is altered by changing DC excitation current. Especially, if the air-gap flux can be reduced as much as possible for high speed operation, the iron loss will be significantly decreased. For this reason, a higher value of the back emf ratio between the highest and the lowest excitation condition is desirable because this ratio is related to the value of the controllable air-gap flux directly.

Table 3.1 summarizes simulation results for the output performances of each motor for different excitation conditions; 0 AT, 3000 AT, and 5000 AT. In this table, PM torque means the calculation results of the output torque when the torque angle is 90°.

Table 3.1 Specifications of stator and rotor

Excitation current		Uniform air-gap rotor	Non-uniform air-gap rotor	Slant air-gap rotor
0 AT	PM torque	33.22 Nm	27.97 Nm	29.18 Nm
	Maximum torque	45.19 Nm	37.78 Nm	40.54 Nm
	Air-gap flux density	0.4008 T	0.2047 T	0.2807 T
3000 AT	PM torque	89.60 Nm	64.10 Nm	79.38 Nm
	Maximum torque	103.67 Nm	79.52 Nm	95.63 Nm
	Air-gap flux density	1.2559 T	0.6653 T	0.8545 T
5000 AT	PM torque	104.59 Nm	84.14 Nm	92.22 Nm
	Maximum torque	121.51 Nm	107.31 Nm	123.09 Nm
	Air-gap flux density	1.4263 T	0.9524 T	1.0275 T
Back emf ratio between 0 AT and 5000 AT		2.26	2.74	2.45

* Iron loss is proportional to the square value of the flux density that fluctuates inside the iron.

At this load condition, the reluctance torque should be nearly zero for uniform and non-uniform air-gap rotor models, but in the case of the slant air-gap rotor, the reluctance torque might be included because of its characteristics of shifting reluctance torque as shown in Figure 3.14. The values of the air-gap flux density are obtained from the fundamental components using discrete time Fourier series analysis as described by (2.91) in section 2.7.3.

Table 3.1 clearly shows that the slant air-gap rotor has higher back emf ratio than the uniform air-gap rotor despite the similar output torque performances. Although, the non-uniform air-gap rotor has the highest back emf ratio, this structure cannot overcome its lower air-gap flux density resulting in its lower output torque value. Therefore, the slant air-gap rotor is expected to have higher efficiency and wider speed range holding the capability of high output torque as much as an ordinary rotor model.

3.4 Summary

Although an IPMSM is a suitable motor for the application of an HEV, the limited operating speed is still an obstacle for the utilization in manufacturing field. To solve this problem, the new IPMSM has some unique features. Compared to a traditional electric machine which uses two dimensional magnetic flux paths in its rotor and stator, the new structure of the machine has three dimensional flux way in the rotor by the side-pole and side-PM for increasing its maximum speed as well as improving both the efficiency and power density.

Because the PM torque component is proportional to the product of the PM fundamental air-gap flux density and the current of the motor^{*}, the higher PM fundamental air-gap flux density produces higher torque at a given current of the motor, yielding a higher power density and higher efficiency for the motor.

Additionally, to extend the operating speed range for HEV application by controlling the excitation current, it uses a slant-shape designed rotor. A series of FEA simulation works suggests that this shape is helpful to increase the ratio of air-gap flux density between the highest and the lowest excitation conditions without the loss of the maximum torque. The next chapter will show how to determine the depth and width of the slant shape of the new IPMSM to achieve the maximum output torque with the desired controllable back emf ratio.

Since both the BFE and slanted air-gap structures are newly devised shapes in developing an electric machine, there might be undesired characteristics such as the axial movement at the side air-gap, and increased torque ripple and iron loss caused by unbalanced air-gap flux which is resulted by the slanted air-gap shape. Nevertheless, the achieved results are still attractive in yielding controllable air-gap flux, increased torque density, and high efficiency. Further research can lead to optimization of the machine size and understanding of the machine characteristic for application in various industrial areas.

^{*} $T=K_T \cdot I$, and $K_T \propto \Phi$

CHAPTER 4

New Analytical Method of Output Torque Calculation and Analysis of Various Slanted Air-gap Shapes

4.1 Introduction

Originally, the slanted air-gap shape is devised for increasing the controllable back emf ratio to maximize the effect of BFE. Chapter 3 shows the results of increased back emf ratio of the slanted air-gap design compared to that of the uniformed air-gap design. This chapter determines the best slant shape in terms of the depth and width for developing a new IPMSM with BFE.

Because the slanted air-gap shape causes the shift of the reluctance torque as shown in Figure 3.12, the amount of the shifted angle will affect the maximum output torque. However, a conventional torque calculation method using a magnetic equivalent circuit can show only the magnitude of the reluctance torque not the position. Although the expected output torque shape can be obtained by FEA, it costs the significant amount of time to consider many different slanted air-gap shapes. Therefore, it is necessary to estimate the shifted angle of the reluctance torque along with the different slanted air-gap shapes in a simple analytical method.

For this purpose, a new torque calculation method is considered to find the characteristic of the reluctance torque and the total output torque of a synchronous machine using a 2-dimensional network magnetic equivalent circuit [65].

The new output torque calculation method is applied to the study of finding the characteristics of the new IPMSM dependent on the slanted air-gap structure in comparing the output torque. In addition, a discussion on how the slanted shape affects the back emf ratio and output torque is given.

4.2 New Analytical Method of Output Torque Calculation for an IPMSM

In a conventional analytical method, the calculation of the reluctance torque requires d- and q-axes inductances values, L_d and L_q , because the reluctance torque of an IPMSM is a function of the difference of the d- and q-axes inductances as explained in Chapter 2. In other words, to obtain the expected output torque values of a newly designed IPMSM, the approximated inductance values (L_d and L_q) are necessary [13, 16 – 17, 38 – 40, 66 – 67] using various methods such as solving 1-dimensional magnetic equivalent circuit without permanent magnets as explained in Chapter 2. Then, the overall output torque is the sum of the reluctance torque and the PM torque which is pre-calculated by using the value of the air-gap flux. Below is the overall output torque equation.

$$T = \frac{mp}{2} [\lambda_{PM} I_q + (L_d - L_q) I_d I_q] \quad (4.1)$$

Therefore, the output torque calculation in conventional methods has several steps: 1) calculate the PM torque, 2) calculate d-axis inductances, 3) calculate q-axis inductance, 4) estimate the reluctance torque, 5) add the PM and reluctance torque [13, 38 – 40, 66 – 67].

The above conventional method is not simple and is based on the fundamental component of the air-gap flux. Moreover, the present method of calculating the reluctance torque is under the assumption that all mmf from phase current and the magnetic flux flow to only the two-axes (d- and q- axes) in the machine.

The new method is for obtaining the reluctance and total output torque characteristics with one step by considering the actual distribution of the load current and magnetic flux in 2-dimensional magnetic equivalent circuit. This method is verified by comparing to the results of FEA and experimental test on Prius Motor. Then, it is applied to determine the transition pattern of output torque by changing the slanted air-gap shape.

4.2.1 Concept of new method

The base concept is simple; the output torque is the summation of the difference of the product between the flux linkage and the applied current with the electrical angle difference of 90° all around air-gap. Figure 4.1 is the conceptual equivalent circuit of an IPMSM, which has $4n$ column rods at the stator and rotor respectively over 1 pole-period. The rods indicate the vertical blue lines in Figure 4.1. The column rods at stator and rotor are connected by an air-gap reluctance network to each other, and there are mmf sources

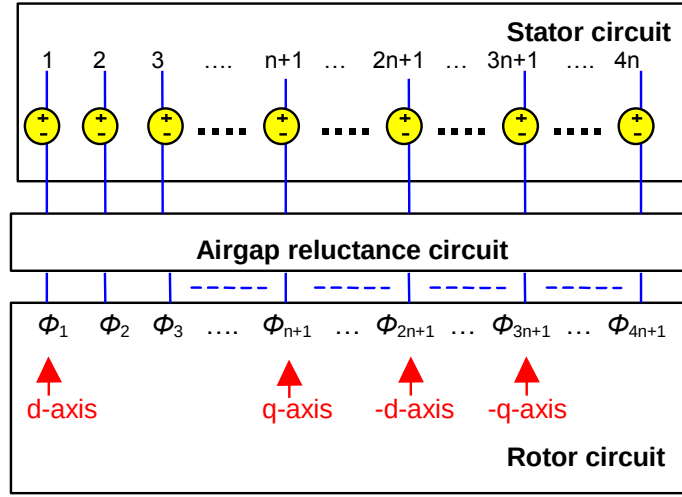


Figure 4.1 Conceptual 2D-equivalent circuit of an IPMSM.

at the stator part, which are modeled for the phase current. The values of the mmf sources are determined by the current density at the stator. Φ_k is the air-gap flux at the k_{th} rod of the air-gap determined by the applied current at the stator. Equation (4.2) is the recalled torque equation of IPMSM from Chapter 2.

$$T = \frac{m}{2} p (\lambda_d i_q - \lambda_q i_d) \quad (4.2)$$

Because (4.2) suggests that the output torque is determined by the difference of the product between the flux linkage and the applied current with the electrical angle difference of 90° , the new torque equation applied to this equivalent circuit is

$$\begin{aligned}
T &= \frac{p}{2} \left[(\lambda_{n+1} i_1 - \lambda_1 i_{n+1}) + (\lambda_{n+2} i_2 - \lambda_2 i_{n+2}) + \dots + (\lambda_{4n} i_{3n} - \lambda_{3n} i_{4n}) \right. \\
&\quad \left. + (\lambda_1 i_{3n+1} - \lambda_{3n+1} i_1) + (\lambda_2 i_{3n+2} - \lambda_{3n+2} i_2) + \dots + (\lambda_n i_{4n} - \lambda_{4n} i_n) \right] \\
&= \frac{p}{2} \left[\sum_{k=1}^{3n} (\lambda_{n+k} i_k - \lambda_k i_{n+k}) + \sum_{k=1}^n (\lambda_k i_{3n+k} - \lambda_{3n+k} i_k) \right] \tag{4.3}
\end{aligned}$$

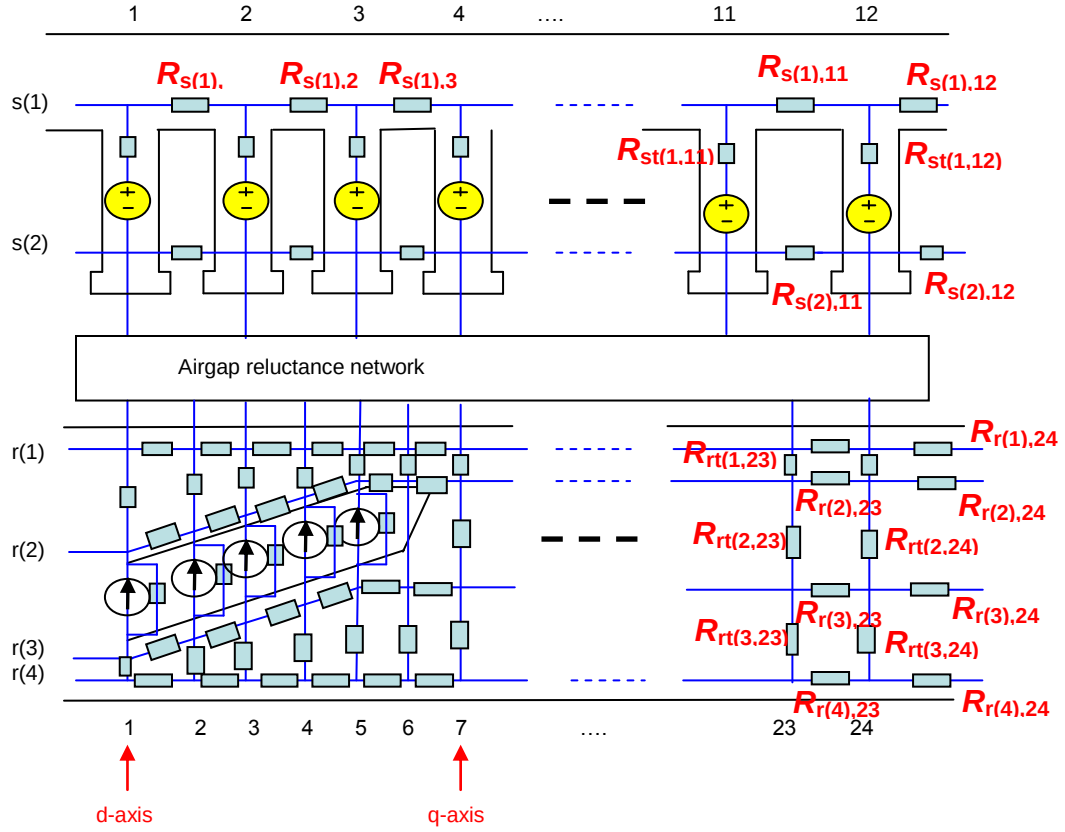
where λ_k is the flux linkage caused by Φ_k . If the machine is symmetric between the d and -d axes, the torque equation (4.3) can be simplified to

$$T = p \sum_{k=1}^{2n} (\lambda_{n+k} i_k - \lambda_k i_{n+k}) \tag{4.4}$$

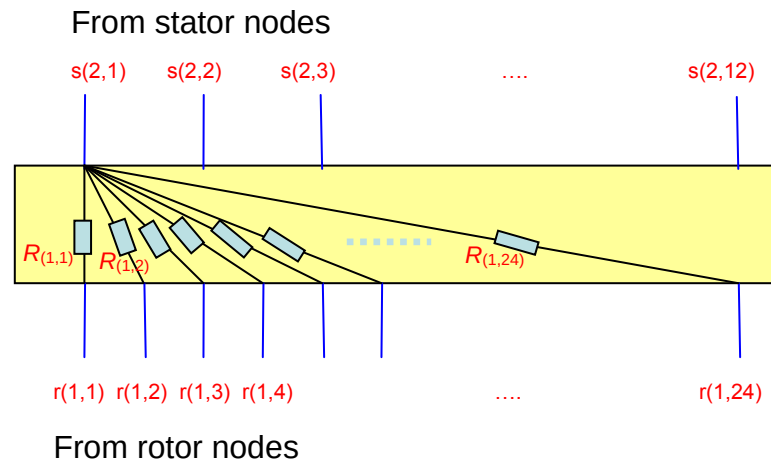
4.2.2 Verifying the method: Analysis of Prius IPMSM

Figure 4.2 (a) is the equivalent circuit of the 2004 Toyota Prius IPMSM used for this test analysis. This circuit has 2 row ($s(1)$ and $s(2)$) and 12 column rods at the stator part and 4 row ($r(1)$ through $r(4)$) and 24 column rods at the rotor part. Every node at stator part is indicated as $s(i,j)$ that means the node intersecting between row $s(i)$ and column j at the stator. Similarly, at rotor part each node is named as $r(i,j)$ between row $r(i)$ and column j . The last column node $s(i,12)$ is connected to the node $s(i,1)$ and $r(i,24)$ to $r(i,1)$ because the machine is periodic with repeated North and South poles.

Every connecting line between two nodes has its reluctance value (small rectangles on the rod in Figure 4.2). At stator part, the reluctance between two nodes, $s(i,j)$ and $s(i,j+1)$, is defined as



(a) Reluctance models



(b) Air-gap reluctance network

Figure 4.2 2D-equivalent circuit of Prius IPMSM.

$$R_{s(i),j} = \frac{l_{s(i),j}}{\mu A_{s(i),j}} \quad (4.5)$$

where μ is the permeability of the material, $A_{s(i),j}$ is the cross-sectional area between the node $s(i,j)$ and $s(i,j+1)$, and $l_{s(i),j}$ is the length between the two nodes. $R_{s(i),12}$ is the reluctance between node $s(i,12)$ and node $s(i,1)$. For the rotor part, the reluctance has the subscript as $r(i),j$ between the nodes of $r(i,j)$ and $r(i,j+1)$ and $R_{r(i),24}$ is the reluctance between node $r(i,24)$ and node $r(i,1)$.

For the vertical connections, the reluctance between $s(1,j)$ and $s(2,j)$ is named as $R_{st(1,j)}$ at the stator part. Similarly, $R_{rt(1,j)}$ is the reluctance between $r(1,j)$ and $r(2,j)$, $R_{rt(2,j)}$ is the reluctance between $r(2,j)$ and $r(3,j)$, and $R_{rt(3,j)}$ is the reluctance between $r(3,j)$ and $r(4,j)$ at the rotor part as shown in some selective names in Figure 4.2 (a).

Every stator node $s(2,i)$ is connected to all nodes on the row $r(1)$ of the rotor, $r(1,1)$ through $r(1,24)$. Figure 4.2 (b) is a sample illustration showing how the air-gap reluctances from stator node $s(2,1)$ is connected to every rotor node on the row $r(1)$. The air-gap reluctance $R_{(i,j)}$ between the nodes of $s(2,i)$ and $r(1,j)$ is defined as

$$R_{(i,j)} = \frac{l_{g(i,j)}}{\mu_o A_{g(i,j)}} \quad (4.6)$$

where μ_o is the permeability of air, $A_{g(i,j)}$ is the average cross-sectional area, and $l_{g(i,j)}$ is the length of two nodes. There are total 288 (12×24) reluctances in the air-gap reluctance network in Figure 4.2.

The magnetic potential at each node is defined as the following vectors.

$$u_{s(1)} = [u_{s(1,1)} \ u_{s(1,2)} \ u_{s(1,3)} \ \cdots \ u_{s(1,12)}]^T \quad (4.7)$$

$$u_{s(2)} = [u_{s(2,1)} \ u_{s(2,2)} \ u_{s(2,3)} \ \cdots \ u_{s(2,12)}]^T \quad (4.8)$$

$$u_{r(1)} = [u_{r(1,1)} \ u_{r(1,2)} \ u_{r(1,3)} \ \cdots \ u_{r(1,24)}]^T \quad (4.9)$$

$$u_{r(2)} = [u_{r(2,1)} \ u_{r(2,2)} \ u_{r(2,3)} \ \cdots \ u_{r(2,24)}]^T \quad (4.10)$$

$$u_{r(3)} = [u_{r(3,1)} \ u_{r(3,2)} \ u_{r(3,3)} \ \cdots \ u_{r(3,24)}]^T \quad (4.11)$$

and

$$u_{r(4)} = [u_{r(4,1)} \ u_{r(4,2)} \ u_{r(4,3)} \ \cdots \ u_{r(4,24)}]^T \quad (4.12)$$

Applying the nodal analysis method of electric circuit based on Kirchoff's current law* [38], the flux $\phi_{st(i)}$ from node s(1,i) to node s(2,i) will be

$$-P_{s(1),12} \cdot u_{s(1,12)} + (P_{s(1),12} + P_{s(1),1}) \cdot u_{s(1,1)} - P_{s(1),1} \cdot u_{s(1,2)} = -\phi_{st(1)} \quad (4.13)$$

$$-P_{s(1),k-1} \cdot u_{s(1,k-1)} + (P_{s(1),k-1} + P_{s(1),k}) \cdot u_{s(1,k)} - P_{s(1),k} \cdot u_{s(1,k+1)} = -\phi_{st(k)}$$

$$\text{for } k = 2 \sim 11 \quad (4.14)$$

$$-P_{s(1),11} \cdot u_{s(1,11)} + (P_{s(1),11} + P_{s(1),12}) \cdot u_{s(1,12)} - P_{s(1),12} \cdot u_{s(1,1)} = -\phi_{st(12)} \quad (4.15)$$

* At a node with n branches, $\sum_{k=1}^n \phi_k = 0$

where $P_{s(1),k}$ is the magnetic permeance at a stator rod, which is defined as the inverse of reluctance of the material such as

$$P_{s(1),k} = \frac{1}{R_{s(1),k}} \quad (4.16)$$

Figure 4.3 is a part of the equivalent circuit showing how to flow the flux $\phi_{st(1)}$ between the nodes $s(1,1)$ to $s(2,1)$. Equations (4.13) through (4.15) can be represented in matrix form as

$$\begin{bmatrix} (P_{s(1),12} + P_{s(1),1}) & -P_{s(1),1} & 0 & 0 & \cdots & 0 & -P_{s(1),12} \\ -P_{s(1),1} & (P_{s(1),1} + P_{s(1),2}) & -P_{s(1),2} & 0 & 0 & \cdots & 0 \\ 0 & -P_{s(1),2} & (P_{s(1),2} + P_{s(1),3}) & -P_{s(1),3} & 0 & \cdots & 0 \\ & & & \ddots & & & \\ -P_{s(1),12} & 0 & & & -P_{s(1),11} & (P_{s(1),11} + P_{s(1),12}) \end{bmatrix} \begin{bmatrix} u_{s(1,1)} \\ u_{s(1,2)} \\ u_{s(1,3)} \\ \vdots \\ u_{s(1,12)} \end{bmatrix} = - \begin{bmatrix} \phi_{st(1)} \\ \phi_{st(2)} \\ \phi_{st(3)} \\ \vdots \\ \phi_{st(12)} \end{bmatrix}$$

$$\rightarrow P_{s(1)} \cdot u_{s(1)} = -\phi_{st} \quad (4.17)$$

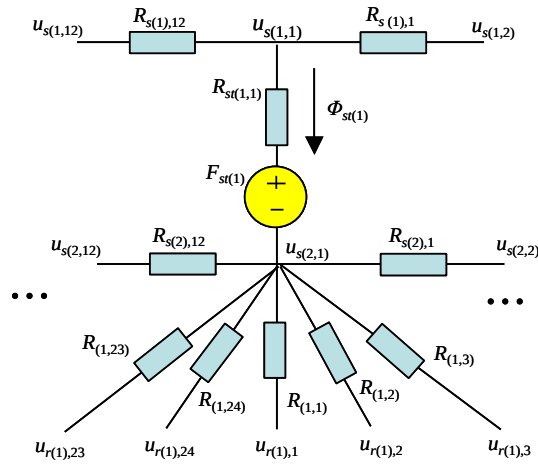


Figure 4.3 A stator part of the equivalent circuit of Prius IPMSM.

Similarly, at the node $s(2,i)$

$$-P_{s(2),12} \cdot u_{s(2),12} + \left(P_{s(2),12} + P_{s(2),1} + \sum_{k=1}^{24} P_{(1,k)} \right) u_{s(2),1} - P_{s(2),1} \cdot u_{s(2),2} - \sum_{k=1}^{24} P_{(1,k)} \cdot u_{r(1,k)} = \phi_{st(1)} \quad (4.18)$$

$$-P_{s(2),1} \cdot u_{s(2),1} + \left(P_{s(2),1} + P_{s(2),2} + \sum_{k=1}^{24} P_{(2,k)} \right) u_{s(2),2} - P_{s(2),2} \cdot u_{s(2),3} - \sum_{k=1}^{24} P_{(2,k)} \cdot u_{r(1,k)} = \phi_{st(2)} \quad (4.19)$$

$\vdots$

$$-P_{s(2),11} \cdot u_{s(2),11} + \left(P_{s(2),11} + P_{s(2),12} + \sum_{k=1}^{24} P_{(12,k)} \right) u_{s(2),12} - P_{s(2),12} \cdot u_{s(2),1} - \sum_{k=1}^{24} P_{(12,k)} \cdot u_{r(1,k)} = \phi_{st(12)} \quad (4.20)$$

In matrix expression,

$$P_{s(2)} \cdot u_{s(2)} - P_g \cdot u_{r(1)} = \phi_{st} \quad (4.21)$$

with

$$P_{s(2)} = \begin{bmatrix} \left(P_{s(2),12} + P_{s(2),1} + \sum_{k=1}^{24} P_{(1,k)} \right) & -P_{s(2),1} & 0 & \cdots & 0 & -P_{s(2),12} \\ -P_{s(2),1} & \left(P_{s(2),1} + P_{s(2),2} + \sum_{k=1}^{24} P_{(2,k)} \right) & -P_{s(2),2} & 0 & \cdots & 0 \\ & & \ddots & & & \\ -P_{s(2),12} & 0 & \cdots & 0 & -P_{s(2),11} & \left(P_{s(2),11} + P_{s(2),12} + \sum_{k=1}^{24} P_{(12,k)} \right) \end{bmatrix} \quad (4.22)$$

and

$$P_g = \begin{bmatrix} P_{(1,1)} & P_{(1,2)} & \cdots & P_{(1,24)} \\ P_{(2,1)} & P_{(2,2)} & \cdots & P_{(2,24)} \\ \vdots & \vdots & \ddots & \vdots \\ P_{(12,1)} & P_{(12,2)} & \cdots & P_{(12,24)} \end{bmatrix} \quad (4.23)$$

Because the voltage source in series with a resistance can be replaced by an equivalent current source in parallel with a resistance as shown in Figure 4.4 [43], the flux $\phi_{st(i)}$ will be

$$\phi_{st(i)} = P_{st(i)} (u_{s(1,i)} - u_{s(2,i)}) - P_{st(i)} \cdot F_{st(i)} \quad (4.24)$$

Therefore, the flux vector ϕ_{st} can be expressed as

$$\phi_{st} = P_{st} \cdot (u_{s(1)} - u_{s(2)}) - P_{st} \cdot F_{st} \quad (4.25)$$

where

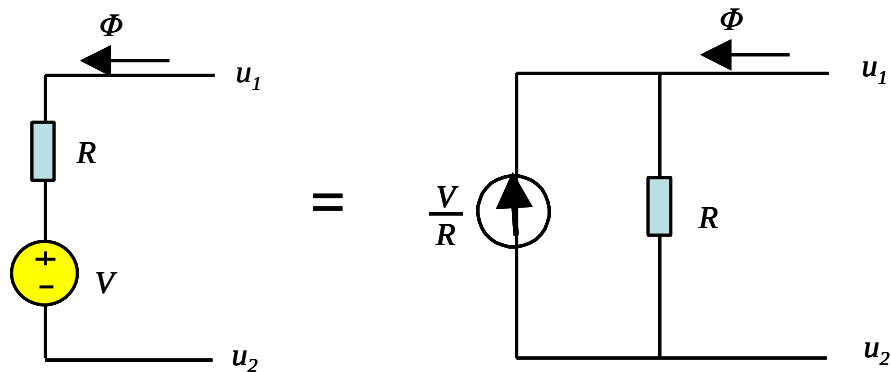


Figure 4.4 Independent source transformation in an electric circuit.

$$P_{st} = \begin{bmatrix} P_{st(1)} & 0 & \cdots & 0 \\ 0 & P_{st(2)} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & P_{st(12)} \end{bmatrix} \quad (4.26)$$

$$F_{st} = \begin{bmatrix} F_{st(1)} & F_{st(2)} & F_{st(3)} & \cdots & F_{st(12)} \end{bmatrix}^T \quad (4.27)$$

For the analysis of the rotor part, let $\phi_{rt(1)}$, $\phi_{rt(2)}$, and $\phi_{rt(3)}$ be defined as the flux vectors between the nodes from $r(1)$ to $r(2)$, from $r(2)$ to $r(3)$, and from $r(3)$ to $r(4)$ of the rows in Figure 4.2 respectively such that

$$\phi_{rt(1)} = \begin{bmatrix} \phi_{rt(1,1)} & \phi_{rt(1,2)} & \phi_{rt(1,3)} & \cdots & \phi_{rt(1,24)} \end{bmatrix}^T \quad (4.28)$$

$$\phi_{rt(2)} = \begin{bmatrix} \phi_{rt(2,1)} & \phi_{rt(2,2)} & \phi_{rt(2,3)} & \cdots & \phi_{rt(2,24)} \end{bmatrix}^T \quad (4.29)$$

and

$$\phi_{rt(3)} = \begin{bmatrix} \phi_{rt(3,1)} & \phi_{rt(3,2)} & \phi_{rt(3,3)} & \cdots & \phi_{rt(3,24)} \end{bmatrix}^T \quad (4.30)$$

In the above equations, $\phi_{rt(i,j)}$ is the flux from node $r(i,j)$ to $r(i+1,j)$ for $i=1,2$, or 3 .

Then, at node $r(1,j)$ the magnetic potential equations will be

$$-P_{r(1),24} \cdot u_{r(1,24)} + \left(P_{r(1),24} + P_{r(1),1} + \sum_{k=1}^{12} P_{(k,1)} \right) u_{r(1,1)} - P_{r(1),1} \cdot u_{r(1,2)} - \sum_{k=1}^{12} P_{(k,1)} \cdot u_{s(2,k)} = -\phi_{rt(1,1)} \quad (4.31)$$

$$-P_{r(1),1} \cdot u_{r(1,1)} + \left(P_{r(1),1} + P_{r(1),2} + \sum_{k=1}^{12} P_{(k,2)} \right) u_{r(1,2)} - P_{r(1),2} \cdot u_{r(1,3)} - \sum_{k=1}^{12} P_{(k,2)} \cdot u_{s(2,k)} = -\phi_{rt(1,2)} \quad (4.32)$$

$\vdots$

$$-P_{r(1),23} \cdot u_{r(1,23)} + \left(P_{r(1),23} + P_{r(1),24} + \sum_{k=1}^{12} P_{(k,24)} \right) u_{r(1,24)} - P_{r(1),24} \cdot u_{r(1,1)} - \sum_{k=1}^{12} P_{(k,24)} \cdot u_{s(2,k)} = -\phi_{rt(1,24)} \quad (4.33)$$

$$\rightarrow P_{r(1)} \cdot u_{r(1)} - P_g^T \cdot u_{s(2)} = -\phi_{rt(1)} \quad (4.34)$$

with

$$P_{r(1)} = \begin{bmatrix} \left(P_{r(1),24} + P_{r(1),1} + \sum_{k=1}^{12} P_{(k,1)} \right) & -P_{r(1),1} & 0 & \cdots & 0 & -P_{r(1),24} \\ -P_{r(1),1} & \left(P_{r(1),1} + P_{r(1),2} + \sum_{k=1}^{12} P_{(k,2)} \right) & -P_{r(1),2} & 0 & \cdots & 0 \\ & & \ddots & & & \\ -P_{r(1),24} & 0 & \cdots & 0 & -P_{r(1),23} & \left(P_{r(1),23} + P_{r(1),24} + \sum_{k=1}^{12} P_{(k,24)} \right) \end{bmatrix} \quad (4.35)$$

A similar process will be applied to nodes $r(2,i)$ including the independent flux sources caused from permanent magnets.

$$-P_{r(2),24} \cdot u_{r(2,24)} + (P_{r(2),24} + P_{r(2),1}) u_{r(2,1)} - P_{r(2),1} \cdot u_{r(2,2)} = \phi_{rt(1,1)} - \phi_{rt(2,1)} + \phi_{PM(1)} \quad (4.36)$$

$$-P_{r(2),1} \cdot u_{r(2,1)} + (P_{r(2),1} + P_{r(2),2}) u_{r(2,2)} - P_{r(2),2} \cdot u_{r(2,3)} = \phi_{rt(1,2)} - \phi_{rt(2,2)} + \phi_{PM(2)} \quad (4.37)$$

$\vdots$

$$-P_{r(2),23} \cdot u_{r(2),23} + (P_{r(2),23} + P_{r(2),24})u_{r(2),24} - P_{r(2),24} \cdot u_{r(2),1} = \phi_{rt(1),24} - \phi_{rt(2),24} + \phi_{PM(24)} \quad (4.38)$$

$$\rightarrow P_{r(2)} \cdot u_{r(2)} = \phi_{rt(1)} - \phi_{rt(2)} + \phi_{PM} \quad (4.39)$$

with

$$P_{r(2)} = \begin{bmatrix} (P_{r(2),24} + P_{r(2),1}) & -P_{r(2),1} & 0 & \cdots & 0 & -P_{r(2),24} \\ -P_{r(2),1} & (P_{r(2),1} + P_{r(2),2}) & -P_{r(2),2} & 0 & \cdots & 0 \\ & & \ddots & & & \\ -P_{r(2),24} & 0 & \cdots & 0 & -P_{r(2),23} & (P_{r(2),23} + P_{r(2),24}) \end{bmatrix} \quad (4.40)$$

and

$$\phi_{PM} = [\phi_{PM(1)} \quad \phi_{PM(2)} \quad \phi_{PM(3)} \quad \cdots \quad \phi_{PM(24)}]^T \quad (4.41)$$

On the nodes $r(3,i)$ and $r(4,i)$

$$P_{r(3)} \cdot u_{r(3)} = \phi_{rt(2)} - \phi_{rt(3)} - \phi_{PM} \quad (4.42)$$

$$P_{r(4)} \cdot u_{r(4)} = \phi_{rt(3)} \quad (4.43)$$

with

$$P_{r(3)} = \begin{bmatrix} (P_{r(3),24} + P_{r(3),1}) & -P_{r(3),1} & 0 & \cdots & 0 & -P_{r(3),24} \\ -P_{r(3),1} & (P_{r(3),1} + P_{r(3),2}) & -P_{r(3),2} & 0 & \cdots & 0 \\ & & \ddots & & & \\ -P_{r(3),24} & 0 & \cdots & 0 & -P_{r(3),23} & (P_{r(3),23} + P_{r(3),24}) \end{bmatrix} \quad (4.44)$$

$$P_{r(4)} = \begin{bmatrix} (P_{r(4),24} + P_{r(4),1}) & -P_{r(4),1} & 0 & \cdots & 0 & -P_{r(4),24} \\ -P_{r(4),1} & (P_{r(4),1} + P_{r(4),2}) & -P_{r(4),2} & 0 & \cdots & 0 \\ & & \ddots & & & \\ -P_{r(4),24} & 0 & \cdots & 0 & -P_{r(4),23} & (P_{r(4),23} + P_{r(4),24}) \end{bmatrix} \quad (4.45)$$

The flux vectors $\phi_{rt(1)}$, $\phi_{rt(2)}$, and $\phi_{rt(3)}$ can be expressed in terms of the potential vectors like

$$\phi_{rt(1)} = P_{rt(1)} \cdot (u_{r(1)} - u_{r(2)}) \quad (4.46)$$

$$\phi_{rt(2)} = P_{rt(2)} \cdot (u_{r(2)} - u_{r(3)}) \quad (4.47)$$

and

$$\phi_{rt(3)} = P_{rt(3)} \cdot (u_{r(3)} - u_{r(4)}) \quad (4.48)$$

with

$$P_{rt(i)} = \begin{bmatrix} P_{rt(i,1)} & 0 & \cdots & 0 \\ 0 & P_{rt(i,2)} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & P_{rt(i,24)} \end{bmatrix}, \quad i = 1, 2, \text{ or } 3 \quad (4.49)$$

Below are assembled all the matrix equations.

$$\begin{bmatrix} P_{s(1)} & 0 & 0 & 0 & 0 & 0 \\ 0 & P_{s(2)} & -P_g & 0 & 0 & 0 \\ 0 & -P_g^T & P_{r(1)} & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{r(2)} & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{r(3)} & 0 \\ 0 & 0 & 0 & 0 & 0 & P_{r(4)} \end{bmatrix} \begin{bmatrix} u_{s(1)} \\ u_{s(2)} \\ u_{r(1)} \\ u_{r(2)} \\ u_{r(3)} \\ u_{r(4)} \end{bmatrix} = \begin{bmatrix} -\phi_{st} \\ \phi_{st} \\ -\phi_{rt(1)} \\ \phi_{rt(1)} - \phi_{rt(2)} + \phi_{PM} \\ \phi_{rt(2)} - \phi_{rt(3)} - \phi_{PM} \\ \phi_{rt(3)} \end{bmatrix} \rightarrow P_{sys} \cdot U = \Phi$$

(4.50)

Using (4.25), (4.46), (4.47) and (4.48), the right side of (4.50) will be

$$\begin{aligned}
\Phi = & \begin{bmatrix} -P_{st} & P_{st} & 0 & 0 & 0 & 0 \\ P_{st} & -P_{st} & 0 & 0 & 0 & 0 \\ 0 & 0 & -P_{rt(1)} & P_{rt(1)} & 0 & 0 \\ 0 & 0 & P_{rt(1)} & -(P_{rt(1)} + P_{rt(2)}) & P_{rt(2)} & 0 \\ 0 & 0 & 0 & P_{rt(2)} & -(P_{rt(2)} + P_{rt(3)}) & P_{rt(3)} \\ 0 & 0 & 0 & 0 & P_{rt(3)} & -P_{rt(3)} \end{bmatrix} \begin{bmatrix} u_{s(1)} \\ u_{s(2)} \\ u_{r(1)} \\ u_{r(2)} \\ u_{r(3)} \\ u_{r(4)} \end{bmatrix} + \begin{bmatrix} P_{st} F_{st} \\ -P_{st} F_{st} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \phi_{PM} \\ -\phi_{PM} \\ 0 \end{bmatrix} \\
= & P_t \cdot U + F_{in} + \Phi_{PM}
\end{aligned}$$

(4.51)

Therefore, we can get the magnetic potential values of all nodes in the equivalent circuit in Figure 4.2 by solving the following matrix equation for U .

$$(P_{sys} - P_t) \cdot U = F_{in} + \Phi_{PM}$$

(4.52)

In (4.52), F_{in} is the phase current input condition and Φ_{PM} is determined by permanent magnet material inside the rotor. The air-gap flux $\Phi_{g(i)}$ at node $r(1,i)$ can be

obtained by summation of all the flux from node $r(1,i)$ to all the nodes on the row of $s(2)$ at the stator such as

$$\Phi_{g(i)} = \sum_{k=1}^{12} P_{(k,i)} (u_{r(1,i)} - u_{s(2,k)}) \quad (4.53)$$

The no-load flux in the air-gap can be calculated simply solving U in (4.52) with $F_{in} = 0$ and then using (4.53). Since this Prius model is symmetric between the d and -d axes, the simple torque equation (4.4) will be used for the output torque calculation. In this equivalent circuit, the number of rods of the rotor is double that of the stator, the torque equation (4.4) is modified as

$$T = 4 \sum_{k=1}^6 (\lambda_{(2k+5)} i_{(k)} - \lambda_{(2k-1)} i_{(k+3)}) \quad (4.54)$$

where $\lambda_{(k)}$ is the flux linkage, which has the relationship with $\Phi_{g(k)}$ like.

$$\lambda_{(k)} = k_w N \cdot \Phi_{g(k)} \quad (4.55)$$

in (4.55), k_w is the winding factor and N is the number of turns of the phase conductors.

Figure 4.5 is the output torque graph obtained from the above equation, which is compared with the FEA and experimental test results respectively. The graphs show that the overall analysis results agree with the experimental test results. Figure 4.6 is the

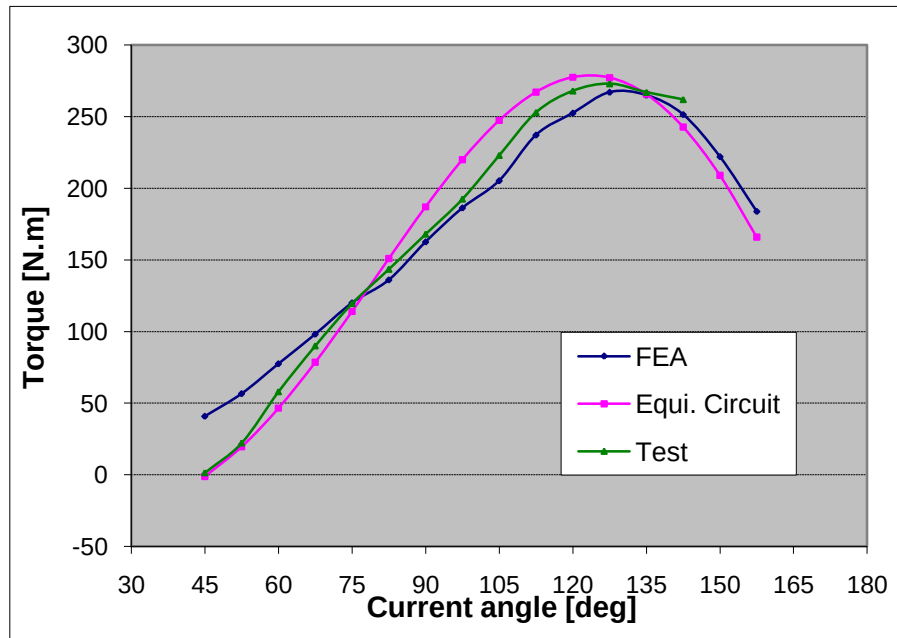


Figure 4.5 Compared results of the output torque when $I=150$ A.

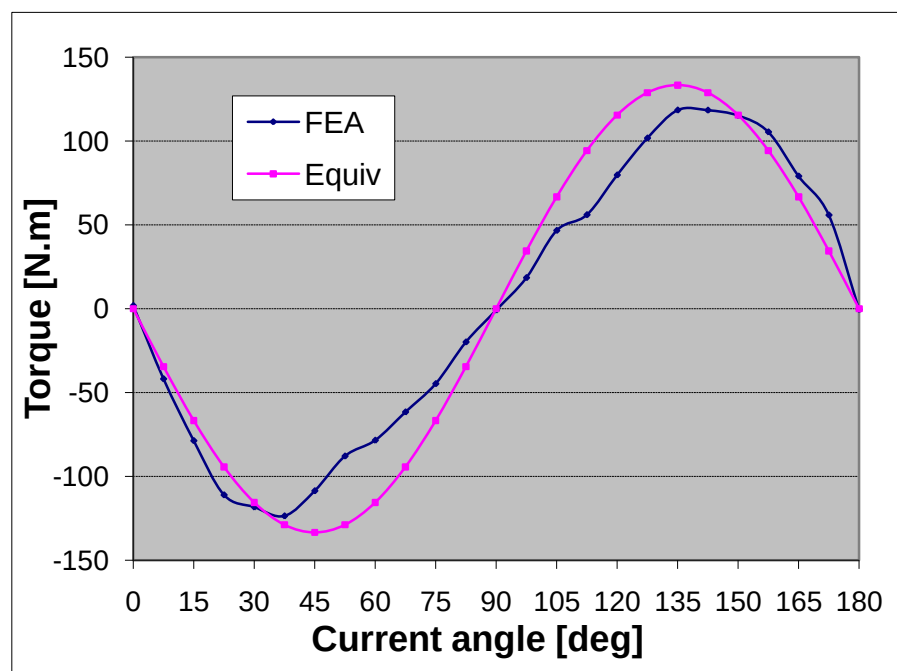


Figure 4.6 Compared results of the reluctance torque when $I=150$ A.

expected reluctance torque graph that is calculated with no-magnetized permanent magnets in the rotor ($\Phi_{PM}=0$ in (4.52)). These two graphs show that the new method of the output torque calculation using 2-dimensional equivalent circuit is very effective in designing a new IPMSM because it has simpler process than a conventional method.

The possibility of the direct calculation of the expected reluctance torque is a major advantage in this process, because a conventional method needs the values of L_d and L_q for calculating reluctance torque. Moreover, using a conventional method a machine designer could determine only the magnitude of the reluctance torque not the position. This means that the conventional method cannot be applied to the analysis of the irregular shaped rotor, such as slant structure. But, this new method can show the shifted position of the reluctance torque that is caused by slanted air-gap.

4.3 Determination of the Slant Air-gap Structure

Since there is no preceding research about the slanted air-gap, it is necessary to understand how the machine characteristics change with the variation of the slanted structure. For this purpose, an equivalent circuit of the new IPMSM with BFE structure is constructed to obtain the expected back emf and maximum output torque values with different slanted air-gap shapes. Then, the computed values are compared to the results from FEA to determine the best-slanted shape in terms of controllable back emf ratio and output torque.

4.3.1 Equivalent circuit with brushless field excitation structure

Comparing to the equivalent circuit of Prius IPMSM in the previous section, the equivalent circuit of the new designed IPMSM should include three special features; U-type permanent magnets arrange inside the rotor, brushless field excitation (BFE) from axial sides of the rotor, and slanted air-gap. Since the slanted shape can be modeled as the length of each air-gap node, the first construction of the equivalent circuit is based on the uniform air-gap with BFE structure.

Since the new designed IPMSM has the same radial stator shape to the Prius IPMSM, its equivalent circuit of the stator part is also the same to that of Prius IPMSM as shown in Figure 4.2; 2 rows ($s(1)$ and $s(2)$) and 12 columns. The rotor part is modeled to have 9 rows ($r(1)$ through $r(9)$) and 24 columns. The connection pattern of the nodes in the rotor part is shown in Figure 4.7(a) for a half section of the equivalent circuit from d-

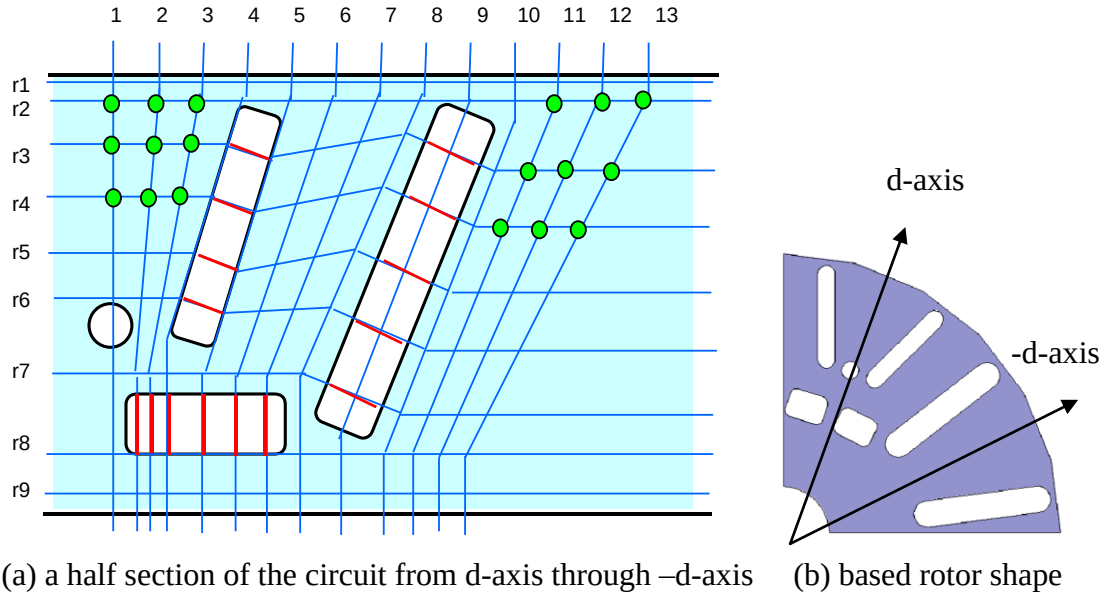


Figure 4.7 Equivalent circuit model in the rotor part.

axis through $-d$ -axis. Considering its symmetry, the modeled properties on column 14 are the same to those of column 12, column 15 to column 11, and so on. The increased number of the rows is to consider the complicated permanent magnets arrangement and the connection with the side-pole and side-PM of the brushless field excitation structure.

Every node is indicated as the same method of Prius equivalent circuit in section 4.2.2. For example, $r(i,j)$ is the node intersecting with the row $r(i)$ and column j . Also, the last column node $s(i,12)$ is connected to the node $s(i,1)$ at the stator part, and $r(i,24)$ is connected to $r(i,1)$ at the rotor part because of its periodic North and South poles.

Every rod between two nodes has its own reluctance value to meet its length and area just like Prius equivalent circuit. $R_{s(i),j}$ is the magnetic reluctance between the node $s(i,j)$ and $s(i,j+1)$ ($R_{s(i),12}$ is the reluctance between node $s(i,12)$ and node $s(i,1)$.) and $R_{r(i),j}$ is the magnetic reluctance between the node $r(i,j)$ and $r(i,j+1)$ ($R_{r(i),24}$ is the reluctance between node $r(i,24)$ and node $r(i,1)$). For the vertical connecting rods, $R_{st(1,j)}$ is the reluctance between $s(1,j)$ and $s(2,j)$, and $R_{rt(i,j)}$ is the reluctance between $r(i,j)$ and $r(i+1,j)$ for $i = 1, 2, \dots, 8$. And, the air-gap reluctance network is the same structure with Prius equivalent circuit as written in section 4.2.2.

There are five more nodes for modeling the excitation structure, which are marked as red circles in Figure 4.8. The node $sd(1)$ means the center point of the side-pole, which is connected to the rotor with the nodes indicated in green circles at the left side in Figure 4.8. These nodes are placed inside of the permanent magnet arrangement on d -axis. Similarly, the node $sd(2)$ (the center point of the side-PM) is connected to the rotor on $-d$ -axis as shown at the right side in Figure 4.8.

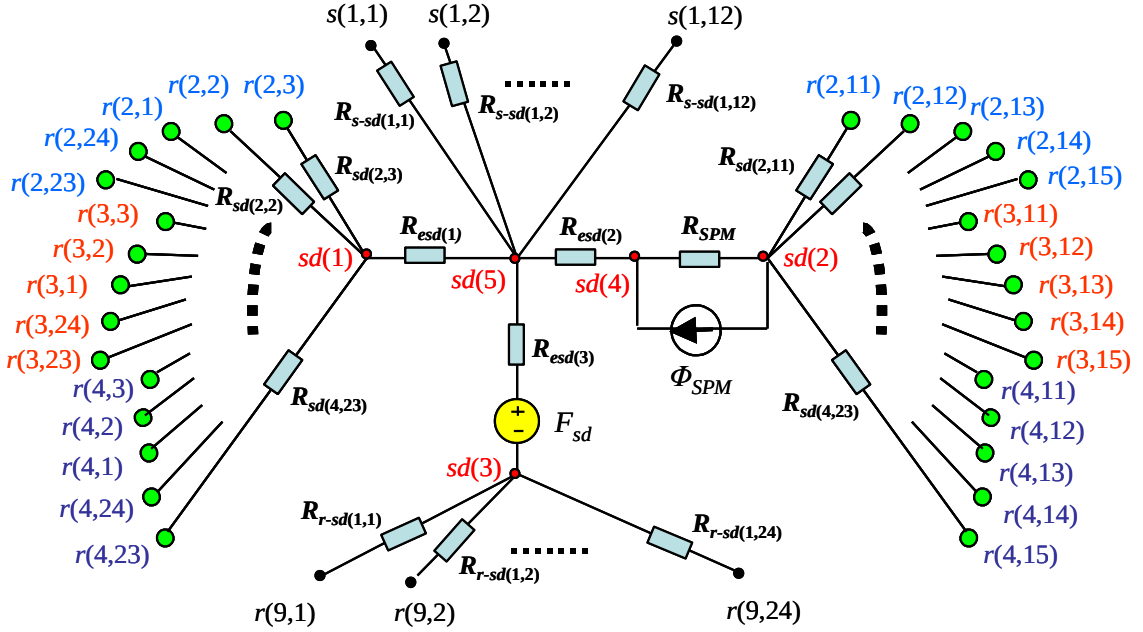


Figure 4.8 Equivalent circuit concept for the BFE structure.

The reluctance between a node on the rotor and $sd(1)$ or $sd(2)$ is defined as $R_{sd(i,j)}$ for $i = 2, 3$, or 4 and $j = 1$ through 24 . In case of $R_{sd(2,j)}$, the reluctance shows the connection between $sd(1)$ and $r(2,j)$ ($j = 1, 2, 3, 23$, or 24) on the d -axis (the left side in Figure 4.8) and between $sd(2)$ and $r(2,j)$ ($j = 11, 12, 13, 14$, or 15) on the $-d$ -axis (the right side in Figure 4.8).

The side-PM places between the nodes $sd(2)$ and $sd(4)$, and the mmf source from the excitation current, F_{sd} , places between the connection of $sd(3)$ and $sd(5)$ which is modeling of the excitation coil housing.

The node $sd(3)$ is the bottom part of the excitation coil housing for the return path of the excitation flux caused by F_{sd} . This node is connected to all nodes of the last low of the rotor ($r(i,9)$) with the reluctances defined as $R_{r-sd(1,j)}$ ($j = 1 \sim 24$). The node $sd(5)$ is the

top part of the excitation coil housing, which has the connection to the stator nodes on the row $s(1)$ for considering the leakage between the stator and the excitation coil housing with the reluctances defined as $R_{s-sd(1,j)}$ ($j = 1 \sim 12$).

There is a total of 12 sets magnetic potential vectors as written below, 2 for the stator, 9 for the rotor, and 1 for the excitation structure as follows

$$u_{s(i)} = [u_{s(i,1)} \ u_{s(i,2)} \ u_{s(i,3)} \ \cdots \ u_{s(i,12)}]^T \quad i = 1, 2 \quad (4.56)$$

$$u_{r(i)} = [u_{r(i,1)} \ u_{r(i,2)} \ u_{r(i,3)} \ \cdots \ u_{r(i,24)}]^T \quad i = 1, 2, \dots, 9 \quad (4.57)$$

$$u_{sd} = [u_{sd(1,1)} \ u_{sd(1,2)} \ u_{sd(1,3)} \ u_{sd(1,4)} \ u_{sd(1,5)}]^T \quad (4.58)$$

The magnetic flux vectors are also defined as

$$\phi_{st} = [\phi_{st(1)} \ \phi_{st(2)} \ \phi_{st(3)} \ \cdots \ \phi_{st(12)}]^T \quad (4.59)$$

$$\phi_{rt(i)} = [\phi_{rt(i,1)} \ \phi_{rt(i,2)} \ \phi_{rt(i,3)} \ \cdots \ \phi_{rt(i,24)}]^T \quad i = 1, 2, \dots, 8 \quad (4.60)$$

In the above equations, $\phi_{st(j)}$ means that the flux flows from the node $s(1,i)$ to $s(2,i)$ and $\phi_{rt(i,j)}$ means that the flux flows from the node $r(i,j)$ to $r(i+1,j)$. Using the same process as shown in section 4.2.2, below are the matrix equations of the magnetic potential vectors on the row $s(1)$, $s(2)$, and $r(1)$ with $P_{s(1)}$, $P_{s(2)}$, $P_{r(1)}$, and P_g that are same magnetic permeance matrices of the equivalent circuit of Prius IPMSM defined in 4.2.2.

$$P_{s(1)} \cdot u_{s(1)} - p_{s-sd} \cdot u_{sd(1,5)} = -\phi_{st} \quad (4.61)$$

$$P_{s(2)} \cdot u_{s(2)} - P_g \cdot u_{r(1)} = \phi_{st} \quad (4.62)$$

$$P_{r(1)} \cdot u_{r(1)} - P_g^T \cdot u_{s(2)} = -\phi_{rt(1)} \quad (4.63)$$

In (4.61), $u_{sd(1,5)}$ is an element of the magnetic potential vector (4.58) and p_{s-sd} is a permeance vector defined as

$$p_{s-sd} = [P_{s-sd(1,1)} \quad P_{s-sd(1,2)} \quad P_{s-sd(1,3)} \quad \cdots \quad P_{s-sd(1,12)}]^T \quad (4.64)$$

The term $p_{s-sd} \cdot u_{sd(1,5)}$ is from the relationship between the stator and the excitation coil housing as shown in Figure 3.24. Actually, $P_{s(1)}$ in (4.61) is slightly different from (4.17) in section 4.2.2 because of the connection between the stator and the excitation coil housing. Below is the modified equation of $P_{s(1)}$.

$$P_{s(1)} = \begin{bmatrix} (P_{s(1),12} + P_{s(1),1} + P_{s-sd(1,1)}) & -P_{s(1),1} & 0 & \cdots & 0 & -P_{s(1),12} \\ -P_{s(1),1} & (P_{s(1),1} + P_{s(1),2} + P_{s-sd(1,2)}) & -P_{s(1),2} & 0 & \cdots & 0 \\ & & \ddots & & & \\ -P_{s(1),12} & 0 & \cdots & 0 & -P_{s(1),11} & (P_{s(1),11} + P_{s(1),12} + P_{s-sd(1,12)}) \end{bmatrix} \quad (4.65)$$

When applying nodal analysis at the rows in rotor part from $r(2)$ to $r(4)$, there are the connections to the excitation structure (the nodes covered in green circle on Figure 3.23) on the node $sd(1)$ or $sd(2)$ as shown in Figure 3.24. Therefore, on the nodes $r(2,i)$

$$P_{r(2)} \cdot u_{r(2)} - p_{r2_sd1} \cdot u_{sd(1,1)} - p_{r2_sd2} \cdot u_{sd(1,2)} = \phi_{rt(1)} - \phi_{rt(2)} \quad (4.66)$$

where

$$p_{r2_sd1} = [P_{sd(2,1)} \ P_{sd(2,2)} \ P_{sd(2,3)} \ 0 \ 0 \ \dots \ 0 \ P_{sd(2,23)} \ P_{sd(2,24)}]^T \quad (4.67)$$

$$p_{r2_sd2} = [0 \ 0 \ \dots \ 0 \ P_{sd(2,11)} \ P_{sd(2,12)} \ P_{sd(2,13)} \ P_{sd(2,14)} \ P_{sd(2,15)} \ 0 \ \dots \ 0]^T \quad (4.68)$$

$$P_{r(2)} = \begin{bmatrix} (P_{r(2),24} + P_{r(2),1} + P_{sd(2,1)}) & -P_{r(2),1} & 0 & \dots & 0 & -P_{r(2),24} \\ -P_{r(2),1} & (P_{r(2),1} + P_{r(2),2} + P_{sd(2,2)}) & -P_{r(2),2} & 0 & \dots & 0 \\ & & \ddots & & & \\ -P_{r(2),24} & 0 & \dots & 0 & -P_{r(2),23} & (P_{r(2),23} + P_{r(2),24} + P_{sd(2,24)}) \end{bmatrix} \quad (4.69)$$

In the definition of $P_{r(2)}$, $P_{sd(2,4)}$ through $P_{sd(2,10)}$ and $P_{sd(2,16)}$ through $P_{sd(2,22)}$ have zero permeance value because there is no connection to the excitation structure. On the nodes $r(3,i)$ and $r(4,i)$, there are connection to the excitation structure and magnetic flux source.

$$P_{r(3)} \cdot u_{r(3)} - p_{r3_sd1} \cdot u_{sd(1,1)} - p_{r3_sd2} \cdot u_{sd(1,2)} = \phi_{rt(2)} - \phi_{rt(3)} + \phi_{PM,r(3)} \quad (4.70)$$

$$P_{r(4)} \cdot u_{r(4)} - p_{r4_sd1} \cdot u_{sd(1,1)} - p_{r4_sd2} \cdot u_{sd(1,2)} = \phi_{rt(3)} - \phi_{rt(4)} + \phi_{PM,r(4)} \quad (4.71)$$

For the above equations, the permeance matrix terms $P_{r(3)}$, $P_{r(4)}$, p_{r3_sd1} , p_{r3_sd2} , p_{r4_sd1} , and p_{r4_sd2} , have the same format with $P_{r(2)}$, p_{r3_sd1} , and p_{r3_sd2} respectively by switching the subscript. In (4.69), the permanent magnet flux source vector $\phi_{PM,r(3)}$ is

$$\phi_{PM,r(3)} = [\phi_{PM,r(3,1)} \quad \phi_{PM,r(3,2)} \quad \cdots \quad \phi_{PM,r(3,24)}]^T \quad (4.72)$$

Figure 4.7 indicates that the permanent magnet sources are placed on the nodes of the 4th, 5th, 8th, 10th, 16th, 18th, 21th, and 22th columns in the equivalent circuit (at 9th and 17th, the flux value is zero because the net amount is null). Therefore, the elements of $\phi_{PM,r(3)}$ are

$$\phi_{PM,r(3,i)} = \phi_{PM_1,r(3)} \quad \text{for } i = 4, 22 \text{ (from short vertical PM in Figure 4.7)}$$

$$\phi_{PM,r(3,i)} = -\phi_{PM_1,r(3)} \quad \text{for } i = 5, 21$$

$$\phi_{PM,r(3,i)} = \phi_{PM_2,r(3)} \quad \text{for } i = 8, 18 \text{ (from long vertical PM in Figure 4.7)}$$

$$\phi_{PM,r(3,i)} = -\phi_{PM_2,r(3)} \quad \text{for } i = 10, 16$$

$$\phi_{PM,r(3,i)} = 0 \quad \text{for the other numbers}$$

The permanent magnet flux sources on the nodes of the row $r(4)$, $r(5)$, and $r(6)$, ($\phi_{PM,r(4)}$, $\phi_{PM,r(5)}$, and $\phi_{PM,r(6)}$) can be defined with the same structure of $\phi_{PM,r(3)}$. Similarly, $\phi_{PM,r(7)}$ and $\phi_{PM,r(8)}$ can be defined as

$$\phi_{PM,r(7)} = [\phi_{PM,r(7,1)} \quad \phi_{PM,r(7,2)} \quad \cdots \quad \phi_{PM,r(7,24)}]^T \quad (4.73)$$

as

$$\phi_{PM,r(7,i)} = \phi_{PM_3,r(7,i)} \quad \text{for } i = 2 \sim 7, 19 \sim 24 \text{ (from bottom PM in Figure 4.7)}$$

$$\phi_{PM,r(7,i)} = \phi_{PM_2,r(7)} \quad \text{for } i = 8, 18 \text{ (from long vertical PM in Figure 4.7)}$$

$$\phi_{PM,r(7,i)} = -\phi_{PM_2,r(7)} \quad \text{for } i = 10, 16$$

$$\phi_{PM,r(7,i)} = 0 \quad \text{for the other numbers}$$

$$\phi_{PM,r(8)} = \left[\phi_{PM,r(8,1)} \quad \phi_{PM,r(8,2)} \quad \cdots \quad \phi_{PM,r(8,24)} \right]^T \quad (4.74)$$

as

$$\phi_{PM,r(8,i)} = -\phi_{PM_3,r(7,i)} \quad \text{for } i = 2 \sim 7, 19 \sim 24 \text{ (from bottom PM in Figure 4.7)}$$

$$\phi_{PM,r(8,i)} = 0 \quad \text{for the other numbers}$$

Using the above flux source definition, the magnetic potential vector equations in the row from $r(5)$ to $r(8)$ are

$$P_{r(i)} \cdot u_{r(i)} = \phi_{rt(i-1)} - \phi_{rt(i)} + \phi_{PM,r(i)} \quad \text{for } i = 5 \sim 8 \quad (4.75)$$

where

$$P_{r(i)} = \begin{bmatrix} (P_{r(i),24} + P_{r(i),1}) & -P_{r(i),1} & 0 & \cdots & 0 & -P_{r(i),24} \\ -P_{r(i),1} & (P_{r(i),1} + P_{r(i),2}) & -P_{r(i),2} & 0 & \cdots & 0 \\ & & \ddots & & & \\ -P_{r(i),24} & 0 & \cdots & 0 & -P_{r(i),23} & (P_{r(i),23} + P_{r(i),24}) \end{bmatrix} \quad (4.76)$$

At the last row $r(9)$, each node is connected to the node $sd(3)$ as shown in Figure 4.8; so, the magnetic potential vector equation is

$$P_{r(9)} \cdot u_{r(9)} - p_{r-sd} \cdot u_{sd(3)} = \phi_{rt(8)} \quad (4.77)$$

where

$$p_{r-sd} = [P_{r-sd(1,1)} \quad P_{r-sd(1,2)} \quad P_{r-sd(1,3)} \quad \cdots \quad P_{r-sd(1,24)}]^T \quad (4.78)$$

$$P_{r(9)} = \begin{bmatrix} (P_{r(9),24} + P_{r(9),1} + P_{r-sd(1,1)}) & -P_{r(9),1} & 0 & \cdots & 0 & -P_{r(9),24} \\ -P_{r(9),1} & (P_{r(9),1} + P_{r(9),2} + P_{r-sd(1,2)}) & -P_{r(9),2} & 0 & \cdots & 0 \\ & & \ddots & & & \\ -P_{r(9),24} & 0 & \cdots & 0 & -P_{r(9),23} & (P_{r(9),23} + P_{r(9),24} + P_{r-sd(1,24)}) \end{bmatrix} \quad (4.79)$$

Since there are 5 nodes of the excitation structure in Figure 4.8, 5 more magnetic potential equations are needed to solve the system. On the node $sd(1)$, the flux from $sd(1)$ to $sd(5)$ is

$$p_{\Sigma_r_sd1} \cdot u_{sd(1)} - p_{r2_sd1}^T \cdot u_{r(2)} - p_{r3_sd1}^T \cdot u_{r(3)} - p_{r4_sd1}^T \cdot u_{r(4)} = -\phi_{sd(1)} \quad (4.80)$$

where

$$p_{\Sigma_r_sd1} = \sum_{k=1}^{24} (p_{r2_sd1}(k) + p_{r3_sd1}(k) + p_{r4_sd1}(k)) \quad (4.81)$$

Similarly, at node $sd(2)$, the flux from $sd(2)$ to $sd(4)$ is

$$p_{\Sigma_r_sd2} \cdot u_{sd(2)} - p_{r2_sd2}^T \cdot u_{r(2)} - p_{r3_sd2}^T \cdot u_{r(3)} - p_{r4_sd2}^T \cdot u_{r(4)} = -\phi_{sd(2)} \quad (4.82)$$

with

$$p_{\Sigma_r_sd2} = \sum_{k=1}^{24} (p_{r2_sd2}(k) + p_{r3_sd2}(k) + p_{r4_sd2}(k)) \quad (4.83)$$

At node $sd(3)$ in Figure 4.8, the flux from $sd(3)$ to $sd(5)$ is

$$\sum_{k=1}^{24} p_{r-sd}(1,k) \cdot u_{sd(3)} - p_{r-sd}^T \cdot u_{r(9)} = -\phi_{sd(3)} \quad (4.84)$$

Between the nodes $sd(2)$ to $sd(4)$,

$$P_{SPM} \cdot u_{sd(4)} - P_{SPM} \cdot u_{sd(2)} = -\phi_{sd(2)} + \phi_{SPM} \quad (4.85)$$

At node $sd(5)$, the last magnetic potential equation at the excitation structure is

$$\left(P_{esd(1)} + P_{esd(2)} + \sum_{k=1}^{12} p_{s-sd}(1,k) \right) \cdot u_{sd(5)} - P_{esd(1)} \cdot u_{sd(1)} - P_{esd(2)} \cdot u_{sd(4)} - p_{s-sd}^T \cdot u_{s(1)} = \phi_{sd(3)} \quad (4.86)$$

Below is the system matrix of the equivalent circuit.

$$P_{sys} = \begin{bmatrix} P_{s(1)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -p_{s_sd} \\ 0 & P_{s(2)} & -P_g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -P_g & P_{r(1)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{r(2)} & 0 & 0 & 0 & 0 & 0 & 0 & -p_{r2_sd1} & -p_{r2_sd2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{r(3)} & 0 & 0 & 0 & 0 & 0 & -p_{r3_sd1} & -p_{r3_sd2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & P_{r(4)} & 0 & 0 & 0 & 0 & -p_{r4_sd1} & -p_{r4_sd2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{r(5)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{r(6)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{r(7)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{r(8)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{r(9)} & 0 & 0 & -p_{r_sd} & 0 & 0 \\ 0 & 0 & 0 & -p_{r2_sd1}^T & -p_{r3_sd1}^T & -p_{r4_sd1}^T & 0 & 0 & 0 & 0 & 0 & p_{\Sigma_r_sd1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -p_{r2_sd2}^T & -p_{r3_sd1}^T & -p_{r4_sd1}^T & 0 & 0 & 0 & 0 & 0 & 0 & p_{\Sigma_r_sd2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -p_{r_sd}^T & 0 & 0 & \sum_{k=1}^{24} p_{r_sd}(k) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -P_{SPM} & 0 & P_{SPM} & 0 \\ -p_{s_sd}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -P_{esd(1)} & 0 & 0 & -P_{esd(2)} & P_{esd(1)} + P_{esd(2)} + \sum_{k=1}^{12} p_{s_sd}(k) \end{bmatrix} \quad (4.87)$$

Then, the system equation is

$$P_{\text{sys}} \cdot U = \Phi \quad (4.88)$$

with

$$U = [u_{s(1)} \ u_{s(2)} \ u_{r(1)} \ u_{r(2)} \ u_{r(3)} \ u_{r(4)} \ u_{r(5)} \ u_{r(6)} \ u_{r(7)} \ u_{r(8)} \ u_{r(9)} \ u_{sd(1,1)} \ u_{sd(1,2)} \ u_{sd(1,3)} \ u_{sd(1,4)} \ u_{sd(1,5)}]^T \quad (4.89)$$

$$\Phi = \begin{bmatrix} -\phi_{st} \\ \phi_{st} \\ -\phi_{rt(1)} \\ \phi_{rt(1)} - \phi_{rt(2)} \\ \phi_{rt(2)} - \phi_{rt(3)} + \phi_{PM_r(3)} \\ \phi_{rt(3)} - \phi_{rt(4)} + \phi_{PM_r(4)} \\ \phi_{rt(4)} - \phi_{rt(5)} + \phi_{PM_r(5)} \\ \phi_{rt(5)} - \phi_{rt(6)} + \phi_{PM_r(6)} \\ \phi_{rt(6)} - \phi_{rt(7)} + \phi_{PM_r(7)} \\ \phi_{rt(7)} - \phi_{rt(8)} + \phi_{PM_r(8)} \\ \phi_{rt(8)} \\ -\phi_{sd(1)} \\ -\phi_{sd(2)} \\ -\phi_{sd(3)} \\ -\phi_{sd(2)} + \phi_{SPM} \\ \phi_{sd(3)} \end{bmatrix} = \begin{bmatrix} -P_{st} \cdot (u_{s(1)} - u_{s(2)}) + P_{st} \cdot F_{st} \\ P_{st} \cdot (u_{s(1)} - u_{s(2)}) - P_{st} \cdot F_{st} \\ -P_{rt(1)} \cdot (u_{r(1)} - u_{r(2)}) \\ P_{rt(1)} \cdot (u_{r(1)} - u_{r(2)}) - P_{rt(2)} \cdot (u_{r(2)} - u_{r(3)}) \\ P_{rt(2)} \cdot (u_{r(2)} - u_{r(3)}) - P_{rt(3)} \cdot (u_{r(3)} - u_{r(4)}) + \phi_{PM_r(3)} \\ P_{rt(3)} \cdot (u_{r(3)} - u_{r(4)}) - P_{rt(4)} \cdot (u_{r(4)} - u_{r(5)}) + \phi_{PM_r(4)} \\ P_{rt(4)} \cdot (u_{r(4)} - u_{r(5)}) - P_{rt(5)} \cdot (u_{r(5)} - u_{r(6)}) + \phi_{PM_r(5)} \\ P_{rt(5)} \cdot (u_{r(5)} - u_{r(6)}) - P_{rt(6)} \cdot (u_{r(6)} - u_{r(7)}) + \phi_{PM_r(6)} \\ P_{rt(6)} \cdot (u_{r(6)} - u_{r(7)}) - P_{rt(7)} \cdot (u_{r(7)} - u_{r(8)}) + \phi_{PM_r(7)} \\ P_{rt(7)} \cdot (u_{r(7)} - u_{r(8)}) - P_{rt(8)} \cdot (u_{r(8)} - u_{r(9)}) + \phi_{PM_r(8)} \\ P_{rt(8)} \cdot (u_{r(8)} - u_{r(9)}) \\ -P_{esd(1)} \cdot (u_{sd(1,1)} - u_{sd(1,5)}) \\ -P_{esd(2)} \cdot (u_{sd(1,2)} - u_{sd(1,5)}) \\ -P_{esd(3)} \cdot (u_{sd(1,3)} - u_{sd(1,5)}) - P_{esd(3)} \cdot F_{sd} \\ -P_{esd(2)} \cdot (u_{sd(1,2)} - u_{sd(1,5)}) + \phi_{SPM} \\ P_{esd(3)} \cdot (u_{sd(1,3)} - u_{sd(1,5)}) + P_{esd(3)} \cdot F_{sd} \end{bmatrix} \quad (4.90)$$

In (4.90),

(4.91)

$$j = 1, 2, \dots, 8 \quad (4.92)$$

(4.93)

The right side of (4.90) can be separated into 4 terms: magnetic potential ($P_t \cdot U$), mmf caused by stator current (F_{in}), permanent magnet flux from rotor (Φ_{PM}), and excited flux (Φ_{EXC}) as shown below.

(4.94)

where

(4.95)

$$F_{in} = [P_{st} \cdot F_{st} - P_{st} \cdot F_{st} \ 0 \ \cdots \ 0]^T \quad (4.96)$$

$$\Phi_{PM} = [0 \ 0 \ 0 \ 0 \ \phi_{PM_r(3)} \ \phi_{PM_r(4)} \ \cdots \ \phi_{PM_r(8)} \ 0 \ \cdots \ 0]^T \quad (4.97)$$

$$\Phi_{EXC} = [0 \ \cdots \ 0 \ -P_{esd(3)} \cdot F_{sd} \ \phi_{SPM} \ P_{esd(3)} \cdot F_{sd}]^T \quad (4.98)$$

Finally, we can determine the magnetic potential values of all nodes in the equivalent circuit of the IPMSM with BFE by solving following matrix equation for U .

$$(P_{sys} - P_t) \cdot U = F_{in} + \Phi_{PM} + \Phi_{EXC} \quad (4.99)$$

One of the notable things to solve (4.99) is that the permeance matrix ($P_{sys} - P_t$) could have very high condition number because the lack of independence between the variables. The high condition number implies that a small change of the values in the input current vector may cause a huge change of the output results of the magnetic potential. Therefore, it is necessary to first determine the effective rank of the permeance matrix ($P_{sys} - P_t$) and reduce its rank down to the effective rank, by projecting the original system matrix to the subspace with the effective rank for instance through the use of singular-value-decomposition.

The analysis results obtained by (4.99) are illustrated in Figure 4.9, 4.10, and 4.11 with comparing the results from FEA simulation. Figure 4.9 shows that the air-gap flux density values are changed by the excitation current values as expected, and the overall values are well matched to the results from FEA simulation.

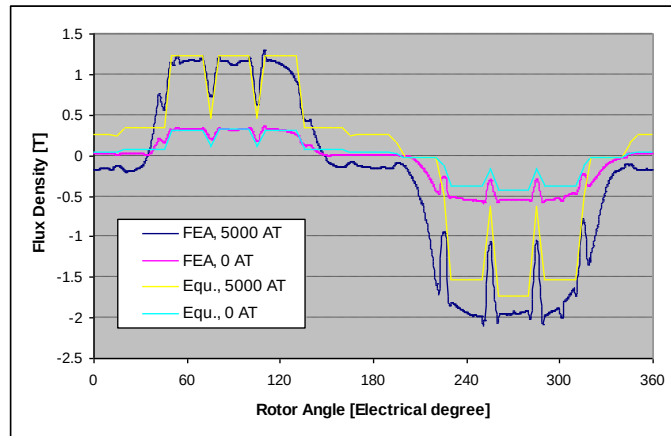


Figure 4.9 Comparison of the air-gap flux density distribution.

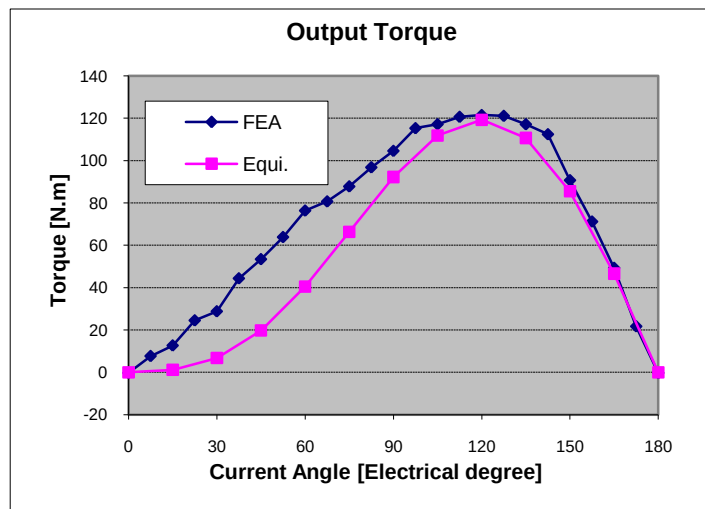


Figure 4.10 Comparison of the calculated output torque.

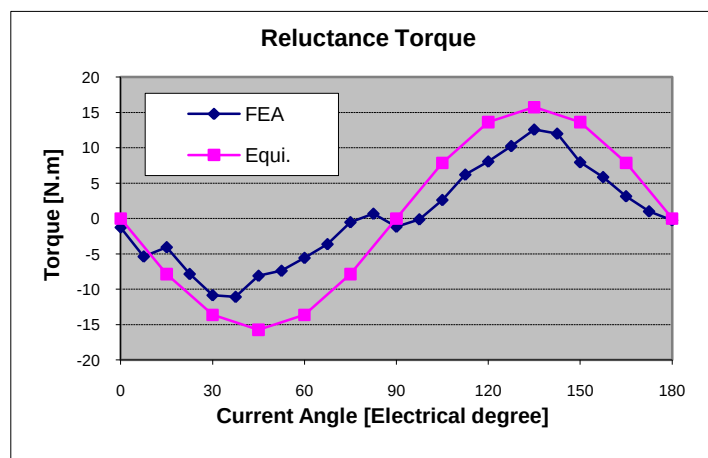


Figure 4.11 Comparison of the calculated reluctance torque.

The magnitude of the maximum torque value is agreeable to the results from FEA simulation as shown in Figure 4.10, but there are some differences of the reluctance torque calculation between the two methods as shown in Figure 4.11. The difference could be caused by the saturation pattern of the stator when phase current is loaded, because the equivalent circuit has too small nodes at the stator to consider the differentiation of the partial saturation at the stator along with the changing input stator current.

4.3.2 Analysis of the effect of different slanted air-gap shapes on machine performance

This research was performed to find the dependence of the characteristics pattern on the slanted air-gap shapes that were indented linearly with different depth (h) and width (l) between the vertical permanent magnets in the rotor as shown in Figure 4.12.

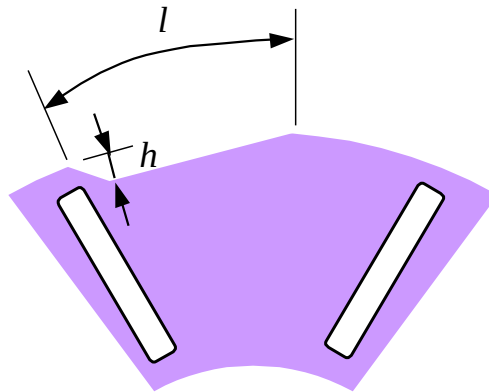


Figure 4.12 An example of the slanted air-gap shape.

The air-gap length of the equivalent circuit is varied to meet the different slanted shapes. Since there is not a sufficient number of nodes on the rotor, the variation of the slanted width has 4 different cases; a quarter span between the vertical permanent magnets, a half, three quarters, and a full span. The depth increases from 0.5 mm to 4.0 mm with a step of 0.5 mm.

The analytical comparison works are focused on three performance measures; reluctance torque, maximum torque, and back emf ratio between 5000 AT and 0 AT of the excitation current. Figure 4.13 is the comparison of the calculated reluctance torque. As expected, the reluctance torque increases with increasing depth at any slant width. And, the reluctance torque can be maximized when the slant width is a half span in any condition of the slant length. Consequently, the expected maximum output torque is highlighted with a half span of the slant width, but the value decreases when the slant length increases more than 2.5 mm as shown in Figure 4.14. The reason presumes that the reduced PM torque could not be compensated by the increased reluctance torque when the slant depth reaches about 2.5 mm.

Figure 4.15 illustrates the expected back emf ratio of different slanted air-gap shapes. The expected value increases significantly as the rotor is slanted in large area in both depth and width. Figure 4.15 also shows that the overall back emf ratio value is not sufficiently high, less than 2.5. Therefore, to achieve a higher ratio, there must be an improvement on the excited flux path, not on the slant shape.

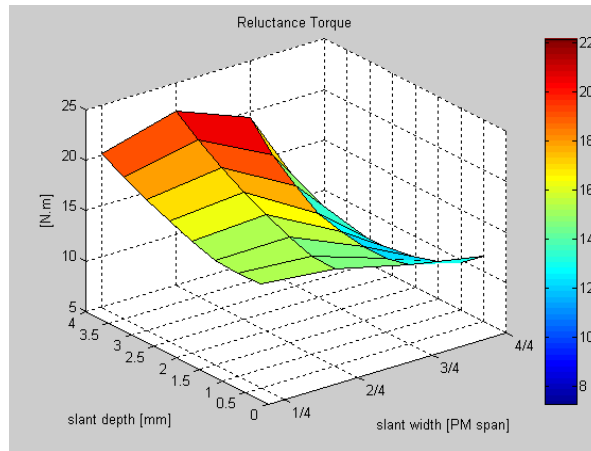


Figure 4.13 Comparison of the reluctance torque.

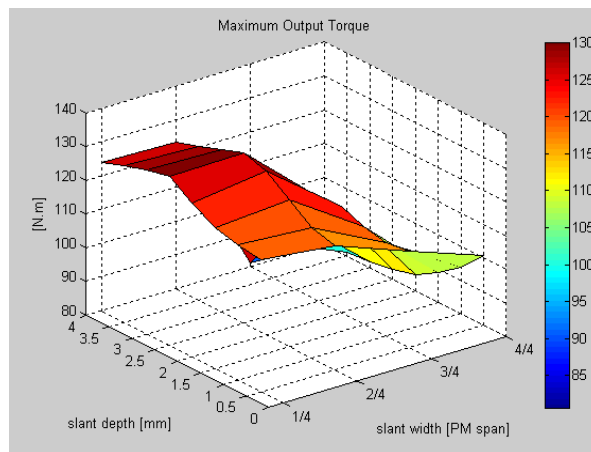


Figure 4.14 Comparison of the maximum torque.

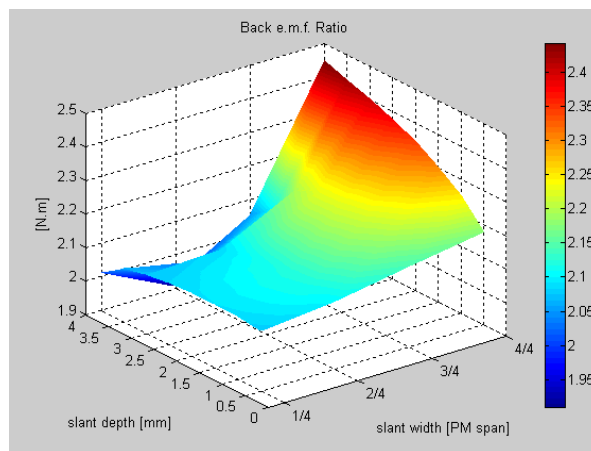


Figure 4.15 Comparison of the back emf ratio.

From the results of the equivalent circuit analysis, the best slant shape is 2.5 mm in depth and a half span in width when a new IPMSM is focused on the maximum output torque. With this shape, the expected machine characteristics are improved 9.5 % in the maximum torque (130.6 Nm from 119.3 Nm with the uniform air-gap) and 2.9 % in the back emf ratio (2.11 from 2.05) compared to the uniform air-gap rotor. If the design is focused on the controllable back emf ratio, the favored slant shape is 2.0 mm in depth and $\frac{3}{4}$ span in width. Then, the expected back emf ratio can be improved 9.8 % (2.25 from 2.05) with a little decreasing in the output torque (111.7 Nm).

To verify the results from the equivalent circuit analysis, some selective slant shapes were simulated by FEA. Figure 4.16 shows the rotor shapes of the simulated models, a $\frac{1}{4}$ span, a $\frac{1}{2}$ span, and a $\frac{3}{4}$ span width with 2.5 mm depth. The reason of choosing 2.5 mm of slant depth is that the output torque can be maximized at this depth with any slant width as shown in Figure 4.14.

Figure 4.17 is the FEA simulation results of the flux density distribution at the air-gap when the excitation current is 5000 AT, which is compared to the results of the

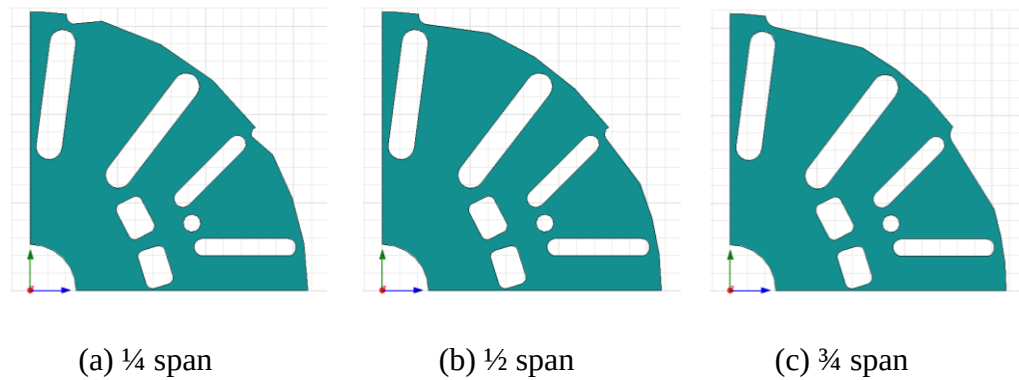


Figure 4.16 Slant-shape rotors with different slant width and 2.5 mm width.

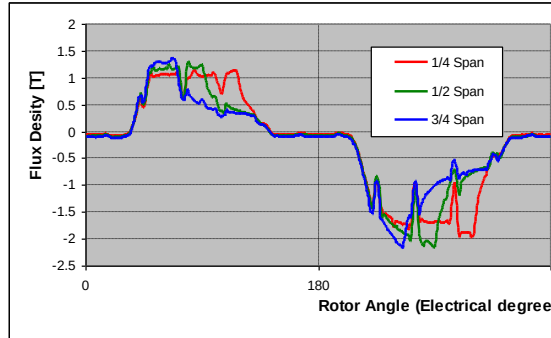
equivalent circuit analysis. The similarity of both graphs proves that the equivalent circuit was constructed well to understand the machine characteristics pattern dependence on slanted air-gap shape.

Figure 4.17 shows that the air-gap flux inclines to the left side of the graph by increased slant portion (from $\frac{1}{4}$ to $\frac{3}{4}$) in both analysis methods. As a result, the maximum value of the flux density is highest in case of $\frac{3}{4}$ slant rotor, but the magnitude of the fundamental component is lowest because of the shrunken width of the effective flux density distributions.

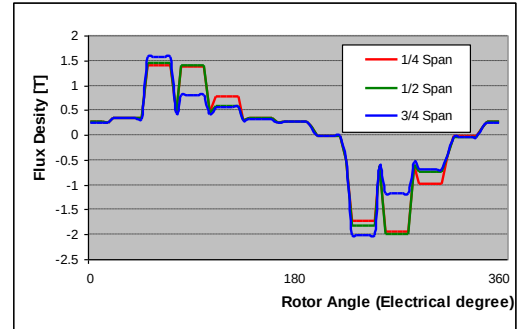
Figure 4.18 is the comparison of the calculated output torque with 5000 AT of the excitation current condition between two methods. Both plots show that the maximum torque position is moved to the left side by increasing the slant width from $\frac{1}{4}$ to $\frac{3}{4}$.

One of the notable things is that the equivalent circuit calculation shows the variation of the output torque is not equidistant by increasing the slant width when compared to the FEA computation (smaller between $\frac{1}{4}$ and $\frac{1}{2}$, and larger between $\frac{1}{2}$ and $\frac{3}{4}$). These results suggest that the equivalent circuit must be improved to obtain more accurate analysis results when considering the slant shape, such as increasing the number of nodes at the air-gap.

Table 4.1 is the comparison of the computation results between the two methods in terms of the output torque and back emf ratio. Although there is some difference in the absolute values of the back emf ratio between the two methods, the overall trend of the variation is similar in both computation results of the maximum torque and back emf ratio. Therefore, the equivalent circuit analysis can be used for determining the machine characteristics dependence on the slant structure.

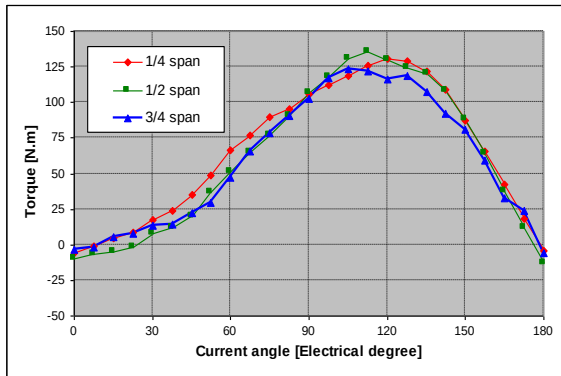


(a) FEA

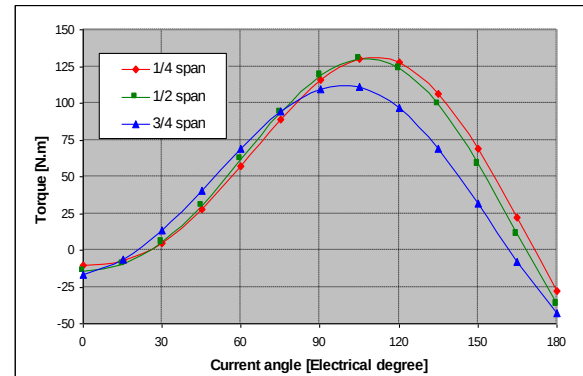


(b) Equivalent circuit

Figure 4.17 Air-gap flux density distributions at 5000AT of the excitation current with different slant width and 2.5 mm depth.



(a) FEA



(b) Equivalent circuit

Figure 4.18 Output torque calculation at 5000AT of the excitation current with different slant width and 2.5 mm depth.

Table 4.1 Comparison of the output torque and back emf ratio between FEA and equivalent circuit analyses.

	Slanted air-gap shape	FEA	Equivalent Circuit
Maximum torque	Uniform	121.51 Nm	119.26 Nm
	$\frac{1}{4}$ span	130.40 Nm (+7.3%)	130.18 Nm (+9.2%)
	$\frac{1}{2}$ span	135.72 Nm (+11.7%)	130.58 Nm (+9.5%)
	$\frac{3}{4}$ span	123.14 Nm (+1.3%)	111.19 Nm (-6.8%)
Back emf ratio between 0 AT and 5000 AT	Uniform	2.26	2.05
	$\frac{1}{4}$ span	2.36 (+4.4%)	2.08 (+1.5%)
	$\frac{1}{2}$ span	2.57 (+13.7%)	2.11 (+2.9 %)
	$\frac{3}{4}$ span	2.82 (+24.8%)	2.23 (+8.8%)

* The percentage number is obtained from the incremental portion to the values of the uniform air-gap.

Although this slanted air-gap structure was devised to increase the controllable back emf ratio through the BFE, the results of this section show that it is possible to achieve higher torque by the selection of the appropriate slant shape. Therefore, it is possible that this slant structure can be applied to the conventional type IPMSM to get high torque density.

4.4 Summary

This chapter shows the process of developing the slanted air-gap shape, which was devised to maximize the effect of the brushless field excitation in terms of the controllable back emf ratio and achievable maximum output torque.

For this purpose, a new torque calculation method is introduced. This method is developed to determine the characteristic of the reluctance and total output torque of a synchronous machine and is an easier procedure than other conventional methods. This method was proved by the analysis of Prius IPMSM and applied to understanding the output torque variation of the new IPMSM by the different shapes of the slanted air-gap structure.

Although it is necessary to improve the equivalent circuit for obtaining better understanding about the slant shape structure, the overall analysis results suggest that the slant shape with about 2.5 mm of the depth and half span of the width is the best when focusing on the output torque performance. Also, the research about the slanted air-gap shape provides a hint about one of the methods to increase the torque density of a conventional IPMSM.

CHAPTER 5

Comparison of Simulated and Test Results of the New IPMSM

5.1 Introduction

Using the unique features shown in the previous chapter, the new IPMSM was developed to run at a speed up to 16,000 rpm. This high speed can be achieved without increasing the input voltage with using a boost converter which is done in the present HEV system [5, 68].

This chapter shows the shape of the prototype motor, analysis results by FEA simulation, and a comparison of those to the test results. The comparison focuses on the expected output torque, back emf, and inductance in various excitation conditions.

5.2 Prototype Motor

Figure 5.1 shows the cross section of the Oak Ridge National Laboratory (ORNL) 16,000-rpm motor design. The dimensions of the stator and rotor are given in Table 5.1. This IPMSM is a three-phase and eight-pole (four-pole pair) machine which means that the number of stator slots per pole per phase is 2. The winding pattern is a single-layer winding with 9 turns, and the winding span is full-pitch in the stator.

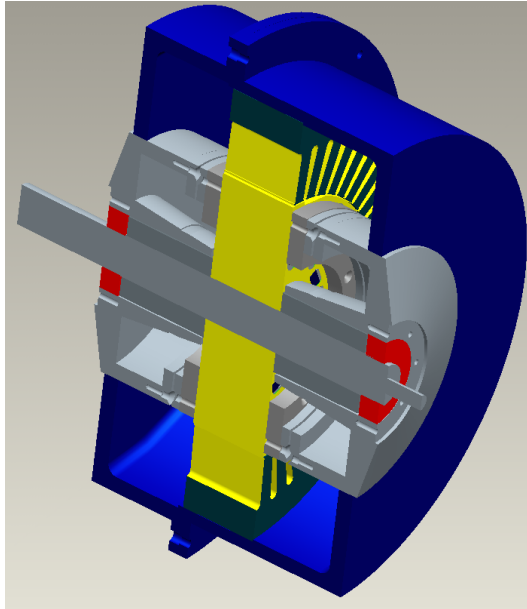


Figure 5.1 Cross sectional view of the ORNL 16,000-rpm motor.

Table 5.1 Specifications of stator and rotor

parameters	value	parameters	value
Number of stator slots	48	Air-gap length	0.74 mm
Outer stator diameter	269.24 mm	Outer rotor diameter	160.45 mm
Inner stator diameter	161.93 mm	Inner rotor diameter	26.67 mm
Stator tooth width	8.86 mm	Bearing distance	189.23 mm
Slot opening width	1.93 mm	PM thickness	5.08/6.1 mm
Stator core length	47.75 mm	Side-PM thickness	2.54 mm

The permanent magnet material used in this motor is NdFeB type of which remanence flux density (B_r) is 1.1 T, coercive field (H_c) is -750000 A/m, and relative permeability (μ_r) is 1.167. Laminated silicon steel (M19_29G) is used for stator and rotor core to reduce hysteresis and eddy current loss, and the shaft is made of mild-steel. These two materials have a non-linear magnetic characteristic, which is shown in Figure 5.2.

Comparing the output characteristics of this motor with those of the Toyota/Prius motor [5, 6] that is selected as a baseline motor, this prototype motor has a higher maximum speed than Toyota/Prius motor (16000 vs. 6000 rpm) with the same power rating (50 kW). By the benefits of a shorter stator core (1.88 vs. 3.3 inches), cost savings can be realized.

Because of its higher speed rating, the HEV system does not require a boost converter for high speed operation. Therefore, this design approach to this prototype motor enables better motor performance as well as the cost saving of the applied HEV system.

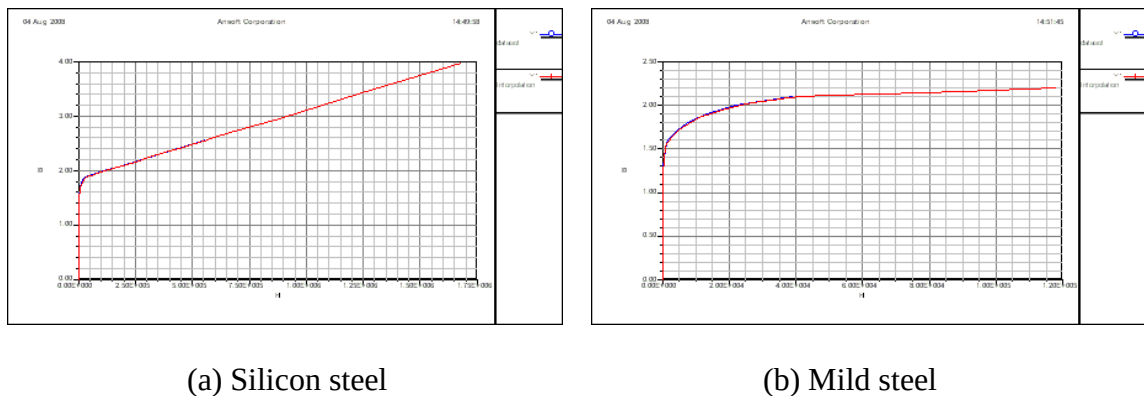
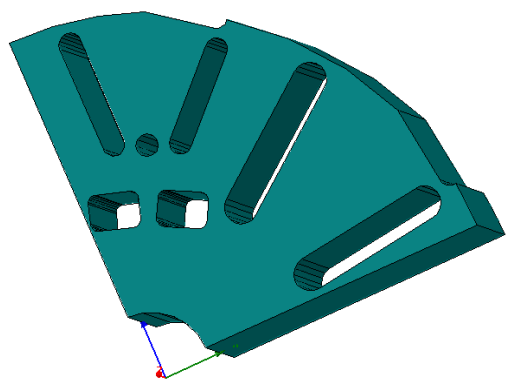


Figure 5.2 B-H curves for non-linear materials

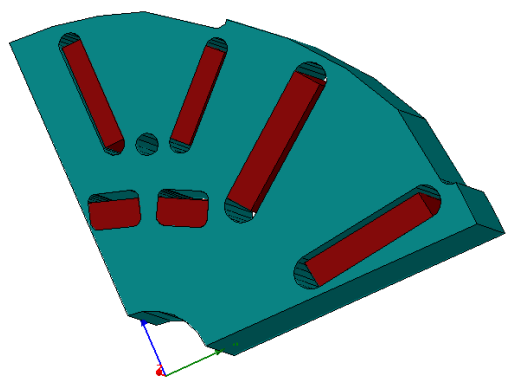
Figure 5.3 is the FEA simulation model drawings illustrating the rotor core, the locations of permanent magnets, the placement of the side-PMs, and the completed rotor. Figure 5.3(a) shows the slanted rotor outer as concluded in the previous chapter. The side-PM in Figure 5.3(c) is held by non-magnetic material such as aluminum. The empty space of the non-magnetic material is in gear with a side-pole to attach the side of rotor stack. `

Figure 5.4 shows several photographs of the actual prototype motor including the wound stator core, a stack of rotor core, a completed rotor after assembled, and the excitation coils inside the housing of the prototype motor. The excitation is wound 865 turns. Lastly, Figure 5.5 is the prototype motor after fully assembled.

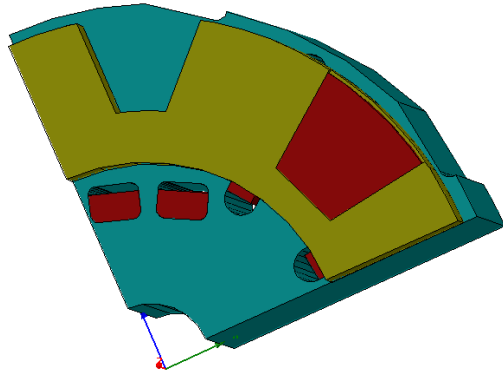
Comparing with the first consideration of DC current excitation structure [6, 61], the mass and volume of the prototype motor can be further reduced by redesigning the interface between the excitation coils and the motor housing. This improvement will reduce the cost to manufacture the motor by using less material and by eliminating some machining steps.



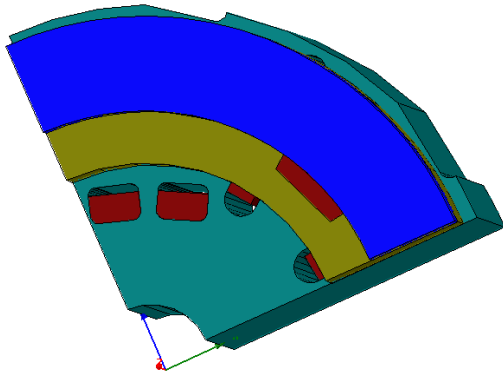
(a) rotor core stack



(b) attached main PMs

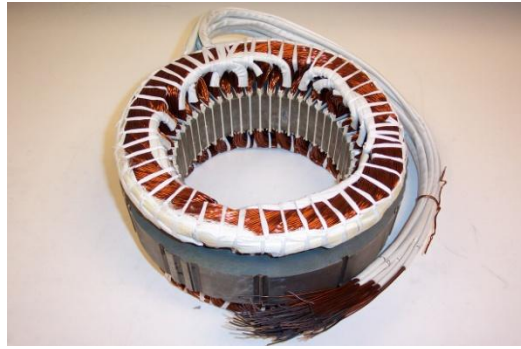


(c) attached side-PMs



(d) attached side-poles

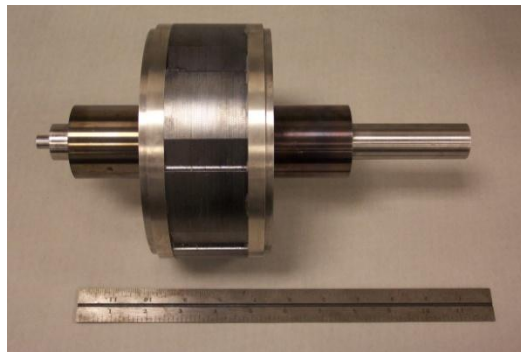
Figure 5.3 The simulation model showing the structure of the rotor.



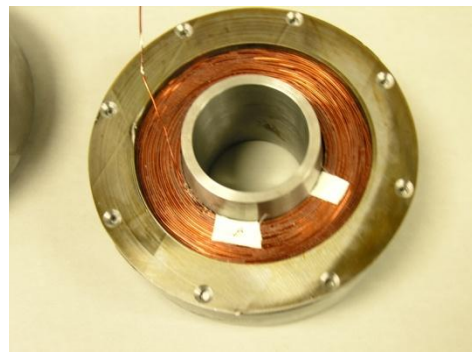
(a) Wound stator core



(b) Rotor punching



(c) Completed rotor



(d) Excitation coils inside the coil housing

Figure 5.4 Photographs of the parts of the prototype motor.

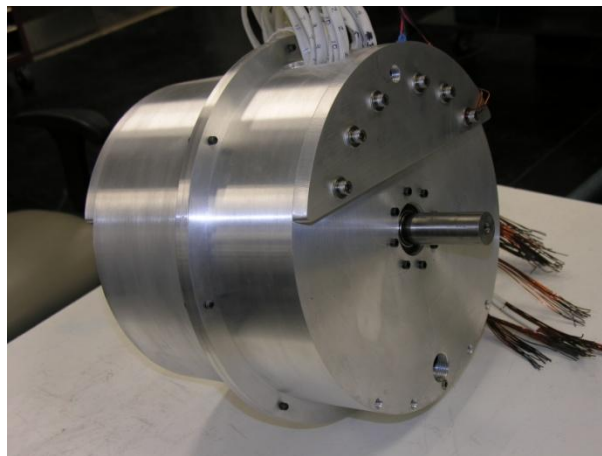


Figure 5.5 Assembled prototype motor.

5.3 Air-gap Flux and Back EMF

5.3.1 Simulation results

To determine the variation of the air-gap flux density by changing the excitation condition, FEA simulations have been conducted under several different excitation current values from 0 AT through 5000 AT with the step of 1000 AT. The simulation results are shown in Figure 5.6.

The simulation results conclude that the air-gap flux density is proportional to the amount of excitation current. Figure 5.6 also shows that the machine would be starting its magnetic saturation from 3000 AT. An excitation current of more than 3000 AT does not help to increase the air-gap flux. This result suggests that the machine requires some optimization for its excitation structures.

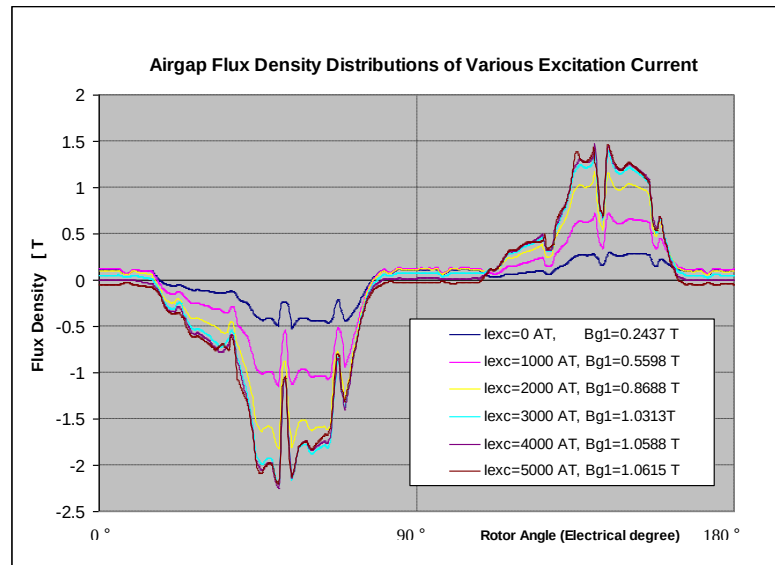


Figure 5.6 Simulation results of air-gap flux density for different excitation conditions.

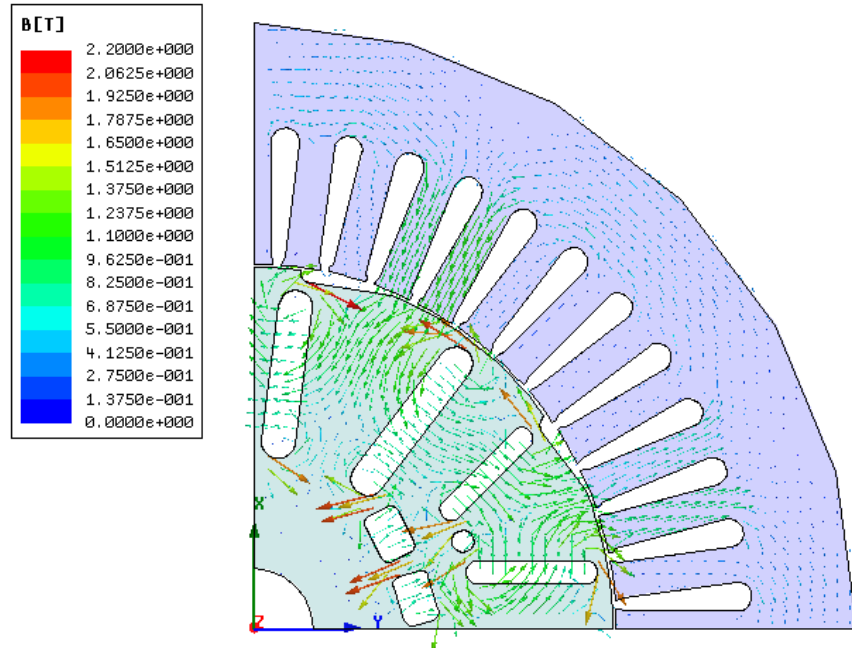
Figure 5.7 is the cut view of the flux density vector distributions in the stator and rotor at no phase current with excitation current of 1000 AT and 5000 AT when the phase current in the stator is zero. Comparing the two figures, the intensity of the vector is increased by strengthening the excitation current, and the results are matched with the graph of Figure 5.6.

The values of flux linkage crossing one of three phase conductors in the stator are also calculated from the simulation. Each value calculated from the simulation is plotted in Figure 5.8 at different rotor positions (from 0° to 90° with the step of 0.5°) and under different excitation conditions. When the excitation current is zero, the calculation results oscillate without any trajectory by some unknown error in the simulation program. Therefore, the case of 0 AT excitation conditions is left out for the calculation of flux linkage and back emf.

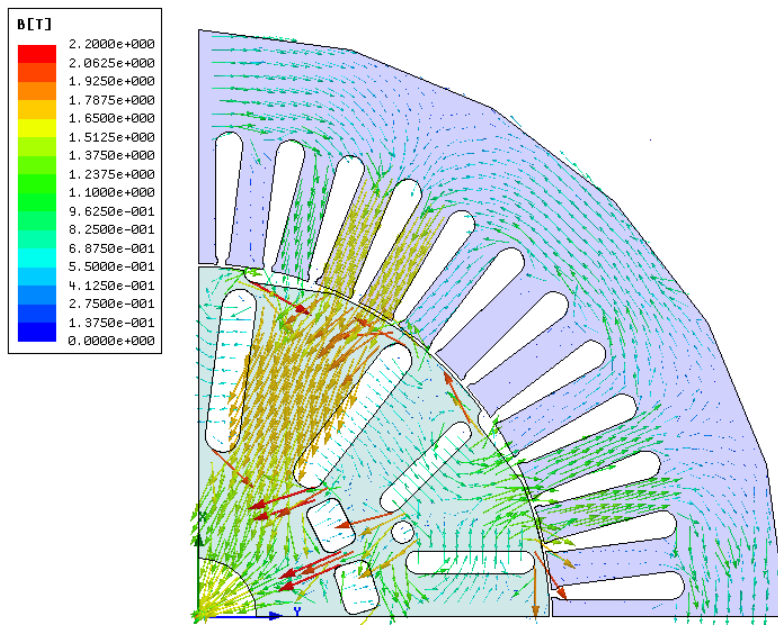
The waveform of the back emf can be obtained from the time derivative of the flux linkage waveform in Figure 5.8 using equation (5.1).

$$E = -\frac{d\psi}{dt} = -\frac{d\psi}{d\theta} \frac{d\theta}{dt} = -\omega \frac{d\psi}{d\theta} \quad (5.1)$$

Figure 5.9 is the resulting back emf waveform and its fundamental waveform with the conditions of 5000 AT and 5000 rpm. Applying the same procedure to the graphs on Figure 5.8, the expected back emf voltages from the fundamental values are plotted versus speed for different excitation field conditions in Figure 5.10.



(a) 1000 AT excitation current



(b) 5000 AT excitation current

Figure 5.7 Flux density vector distributions in stator and rotor at no load conditions.

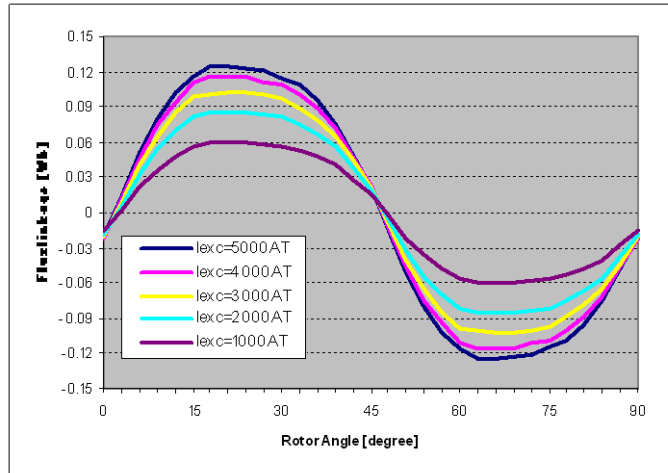


Figure 5.8 Flux linkage crossing a series of conductors in the stator.

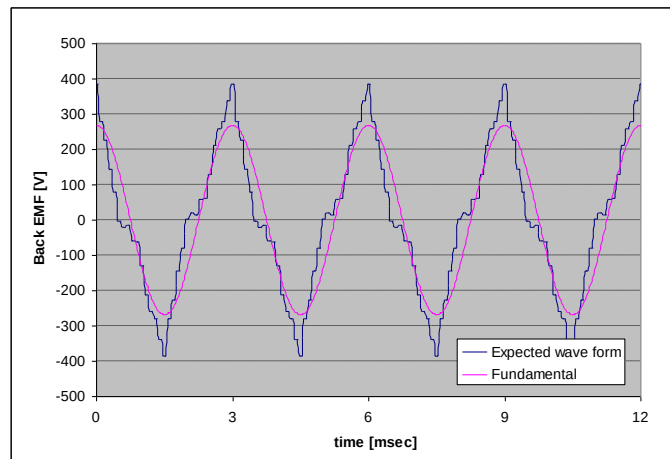


Figure 5.9 A phase back-emf waveform at 5000 AT of excitation current and 5000 rpm.

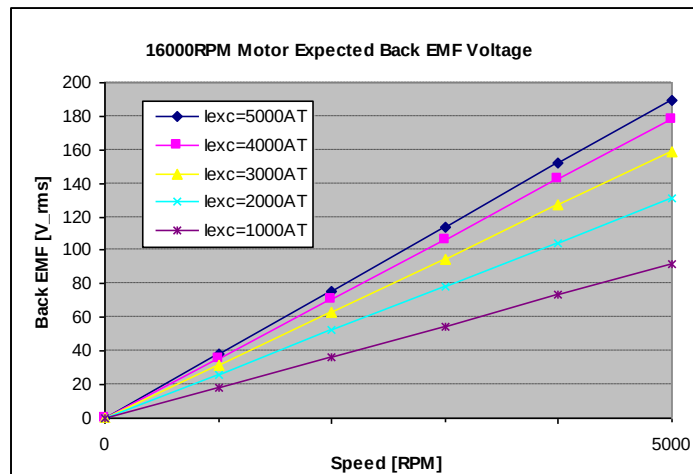


Figure 5.10 Calculated back-emf voltage at various excitation conditions.

5.3.2 Test results

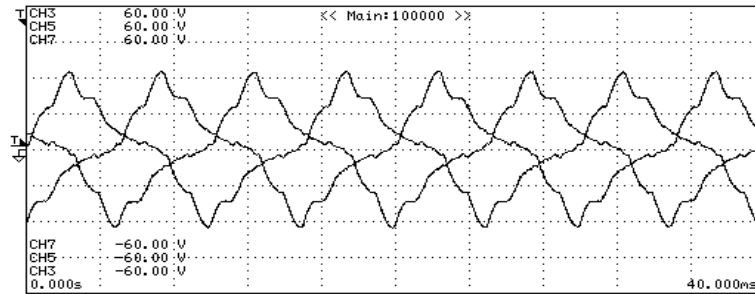
The line-to-neutral RMS voltages were measured as a function of speed for field excitation currents ranging from -5A to +5A in 1A increments. The baseline waveforms obtained at 1,000 rpm with no field excitation and enhanced by a field excitation of +5A are shown in Figure 5.11.

The scale in volts per division has doubled from Figure 5.11 (a) to (b), and both shapes are similar to the calculation results plotted in Figure 5.8. A better representation of the effects of the field excitation current on the magnitude of the back-emf is portrayed in Figure 5.12. The measured RMS voltage is plotted versus speed for each field current.

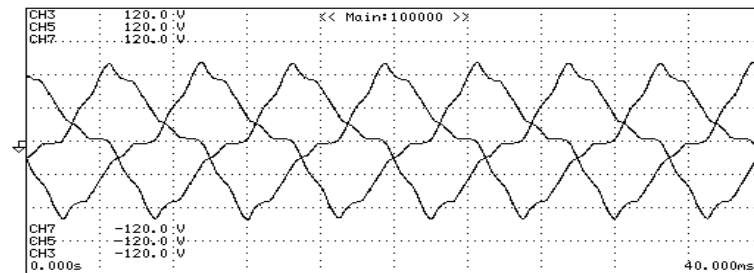
In Figure 5.12 the slope of the -5A to 0A back-emf curves are relatively constant and the overall shape agrees with the simulation results shown in Figure 5.10. The slope increases dramatically as the excitation field is increased from 0 to 3A, yet saturation appears to begin at 3A. The top two lines indicate a large amount of saturation between 4A and 5A of field excitation current as evidenced by a very small change in slope.

From -5A to 0A, there is little change in the back-emf voltage. This result suggests that the negative direction of the excitation flux from both sides of the rotor is not effective to further decrease the air-gap flux.

The effects of saturation by excitation field currents are more obvious if the back-emf is plotted versus field excitation current for each speed as shown in Figure 5.13. This plot substantiates that magnetic saturation becomes effective above 2A where the influence of increased field excitation current on the back-emf curve is significantly reduced. Once again the negative field excitation currents do not help to reduce the back-emf voltage as portrayed in Figure 5.13.



(a) No excitation field current ($16.3 \text{ V}_{\text{rms}}@1,000\text{rpm}$).



(b) 5A excitation field current ($38 \text{ V}_{\text{rms}}@1,000\text{rpm}$).

Figure 5.11 Test results of the back-emf waveform.

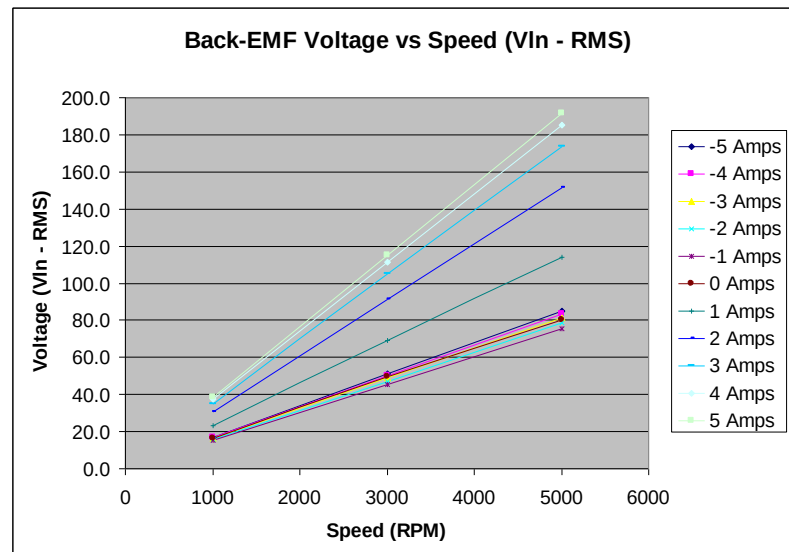


Figure 5.12 Test results of the back-emf curve at various excitation currents.

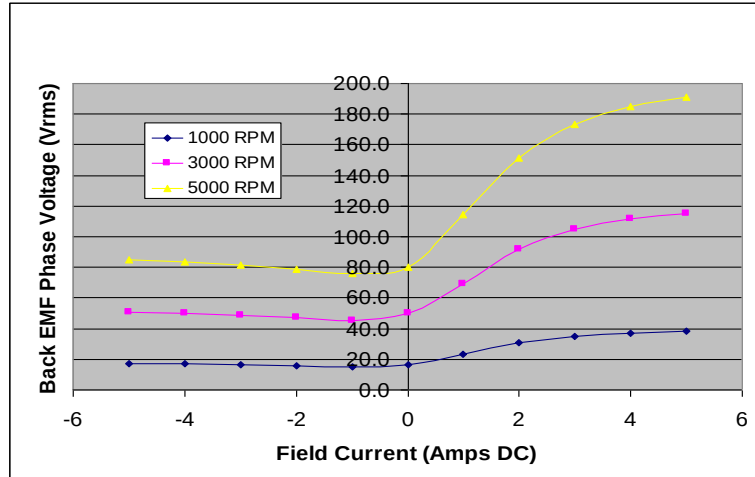


Figure 5.13 Influence of the excitation field current on back-emf voltage.

The results of the back emf for simulation and test are listed in Table 5.2. The simulation results match the test results well at high excitation condition. Both results show that the back emf can be reduced by adjusting the excitation current for extending field weakening operation.

5.4 Output Torque

5.4.1 Simulation results

Figure 5.14 shows the current phase angle versus output torque characteristic at which the maximum phase current is 200 A under various excitation currents. As expected, greater excitation produces stronger motor torque.

Table 5.2 Comparing back emf results between test and simulation at 5000 rpm.

Field current [A]	Test [Vrms]	Simulation [Vrms]
0	80.0	72.4
1	114.2	110.0
2	151.3	140.5
3	173.4	162.6
4	185.1	173.8
5	191.4	182.1

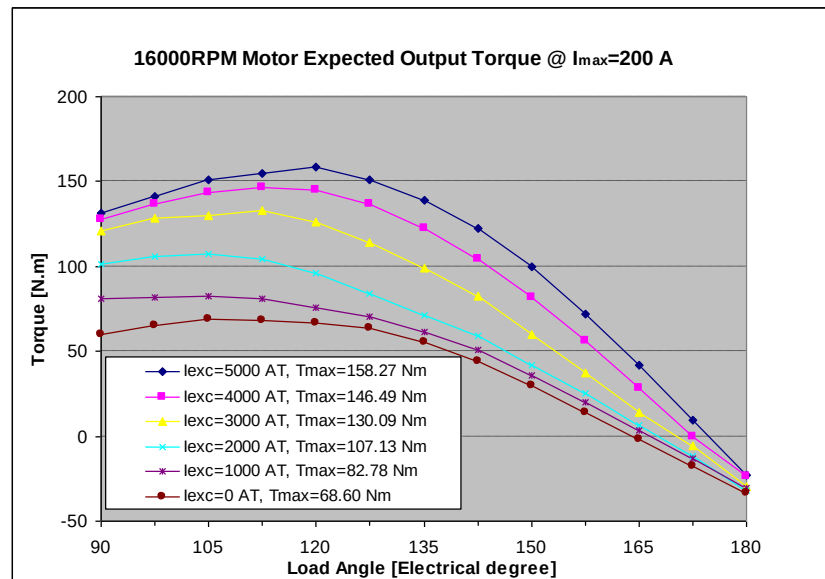


Figure 5.14 Expected output torque at various excitation current values.

Agreeing with the results of air-gap flux density in Figure 5.6, the increasing margin of the PM torque at each excitation current (the values at 90° of load angle in Figure 5.6) is reduced because of the saturation by increasing the excitation current. However, the effects of the saturation on the maximum output torque are less than on the PM torque due to the reluctance torque which also increases by increasing excitation current.

The primary reason is the excitation flux flows mainly through the d-axis; thus, the d-axis would be saturated by increasing excitation current. This saturation would decrease the d-axis inductance; as a result, the saliency ratio of q- and d-axis inductance would increase and so would the reluctance torque.

5.4.2 Test results

The output torque of the prototype motor is measured by the locked rotor tests. Figure 5.15 is the test results waveform when the phase current maintains 50 A. As expected, the peak torque is obtained while using the highest field excitation current of 5A (Since the excitation coils has 875 turns, the total current is 4375 AT.) just like the results of simulation.

Only the peak regions were measured for stator currents other than 50 amps to reduce test time and to avoid the temperature rise that accompanies higher stator currents. Figure 5.16 is the result when excitation current is 5 A.

Contrary to the waveform shape of typical IPMSMs, the torque waveforms in Figure 5.15 and 5.16 are not symmetrical about the horizontal axis. For example, for no

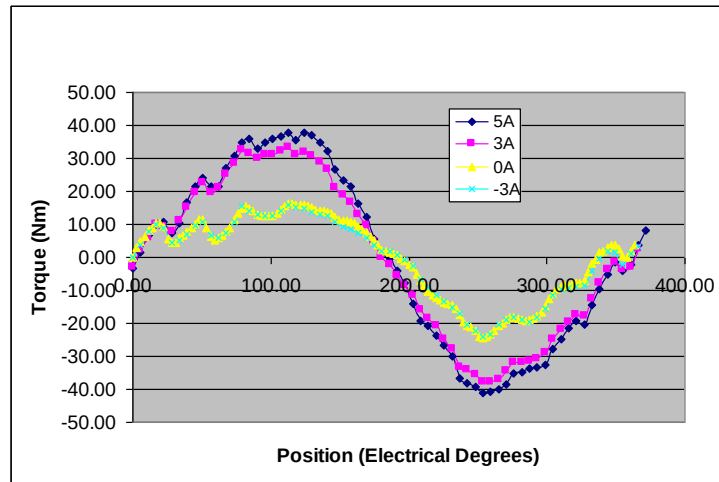


Figure 5.15 Test results of the torque at 50 A of the phase current.

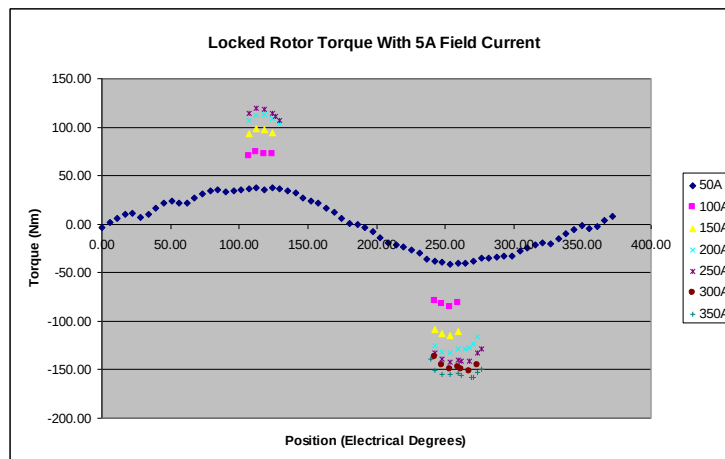


Figure 5.16 Test results of the torque at 5 A of the excitation current.

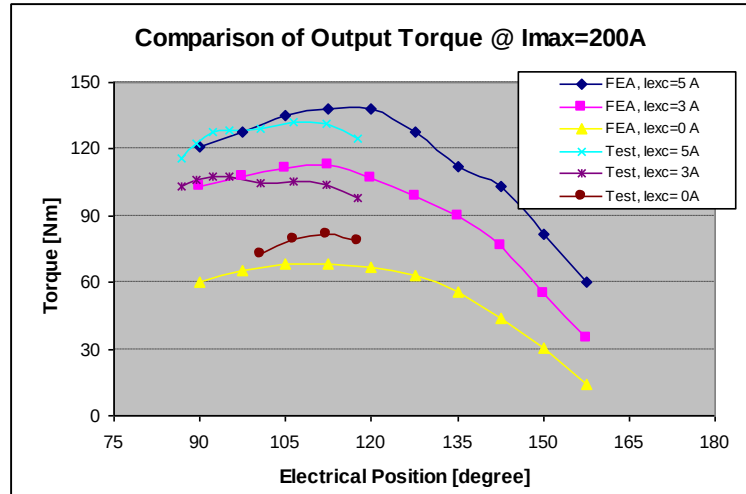


Figure 5.17 Comparison of output torque at 200 A of the phase current.

field excitation current the magnitude of the peak positive torque, 16 Nm, does not equal the magnitude of the peak negative torque, -24 Nm. This feature is for its slant structure; the reluctance torque is not helpful when the motor rotates in reverse.

The comparison with the simulation results is plotted in Figure 5.17 when the phase current is 200 A. Overall the test results follow the simulation results with a similar pattern of the back emf.

5.5 Cross-Coupled and Self Inductances

5.5.1 Development of the equations

As written in Chapter 2, inductance is one of the most important factors that decide the characteristics of a designed machine. Since inductance value is varied by the

operating current condition, the calculation works must be performed for various input currents with considering the saturation by permanent magnets and excitation flux.

Below are the classical flux linkage equations on d- and q- axes with the terms of inductances as shown in Chapter 2.

$$\lambda_d(i_d, i_q) = \lambda_{d,PM} + L_d(i_d, i_q)i_d \quad (5.2)$$

$$\lambda_q(i_d, i_q) = \lambda_{q,PM} + L_q(i_d, i_q)i_q \quad (5.3)$$

In (5.2) and (5.3), λ is the flux linkage, L is the inductance, and i is the instantaneous current. The subscripts d and q indicate the d-axis and q-axis, respectively. And, $\lambda_{d,PM}$ and $\lambda_{q,PM}$ are the d- and q-axis flux linkages by the rotor flux (usually from permanent magnets).

For a conventional permanent magnet motor with uniform air-gap, the term $\lambda_{PM,q}$ is not necessary in general because there is no q-axis flux from its rotor. However, for the new IPMSM of this research the q-axis rotor flux should be considered to calculate L_q because of the slant structure.

Although the classical equations (5.2) and (5.3) are widely used for controlling an IPMSM, recently there are many studies about considering the cross-saturation between d- and q- axes for the development of the sophisticated dynamic model of IPMSM, especially if the machine is operated with high torque [52, 55, 69 – 72]. Below are the nonlinear flux linkage terms with cross-saturation terms [55, 71 – 72]:

$$\lambda_d(i_d, i_q) = \lambda_{d,PM} + L_d(i_d, i_q)i_d + L_{dq}(i_d, i_q)i_q \quad (5.4)$$

$$\lambda_q(i_d, i_q) = \lambda_{q,PM} + L_q(i_d, i_q)i_q + L_{qd}(i_d, i_q)i_d \quad (5.5)$$

In (5.4) and (5.5), the cross coupled inductance L_{dq} means that the amount of changed d-axis flux linkage by the variation of the q-axis current under saturated condition with given both d- and q-axis current, and L_{qd} determines by the same definition of the other axis. Therefore, each value of the cross coupled inductances is determined by linearization of the flux linkage at the applied current condition.

Using (5.4) and (5.5), the output torque can be expressed as

$$T = \frac{3}{2}p(\lambda_d i_q - \lambda_q i_d) = \frac{3}{2}p(\lambda_{d,PM} i_q - \lambda_{q,PM} i_d + (L_d - L_q)i_d i_q + L_{dq}i_q^2 - L_{qd}i_d^2) \quad (5.6)$$

In (5.6), p is the number of pole pairs of the machine. The first two terms in (5.6) relate to the PM torque, the third relates to the reluctance torque, and the rest are from cross saturation. Because d-axis back emf is relatively small in the sine-distributed winding machine, the steady-state voltage equations of the IPMSM at the speed ω are

$$V_d = Ri_d - \omega L_q i_q - \omega L_{qd} i_d \quad (5.7)$$

$$V_q = Ri_q + \omega L_d i_d + \omega L_{dq} i_q + E \quad (5.8)$$

In (5.8), E is the back-electromotive force (back-emf) at the speed ω . Figure 5.18 is the modified phasor diagram for the steady state based on (5.7) and (5.8) from Figure 2.1(b) in Chapter 2. Figure 5.18 suggests that the voltage vector V can be constructed by input current vector I if we know the values of L_d , L_q , L_{dq} , and L_{qd} along with the different current values. Then, the power factor will be determined from the angle between V and I .

5.5.2 Analysis procedure for inductance calculation

First, the calculation of d-axis flux linkage is carried out without current load using the FEA calculation. Then the calculation works are processed with the variation of current loading, and d- and q-axis flux linkages are obtained. When calculating the values of flux linkage, it is necessary to average the values along the angle of the rotor position over one electrical cycle [52], especially if the IPMSM has uneven air-gap flux.

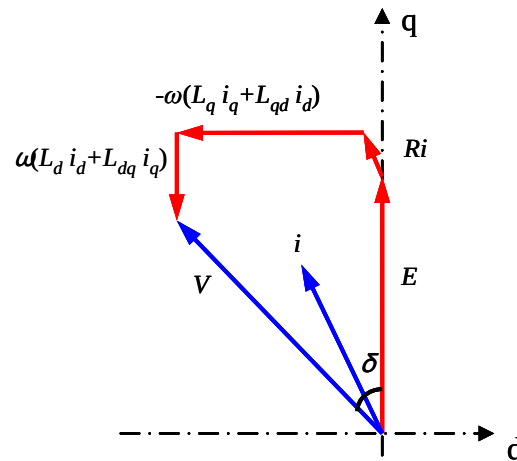


Figure 5.18 Modified steady-state phasor diagram of IPMSM.

After obtaining the whole range of $\lambda_d(i_d, i_q)$ and $\lambda_q(i_d, i_q)$ values, the inductances of cross-saturation terms can be determined by numerical approximation as follows:

$$L_{dq}(i_d, i_q) = \left. \frac{\Delta \lambda_d}{\Delta i_q} \right|_{i_d = \text{constant}} \quad (5.9)$$

$$L_{qd}(i_d, i_q) = \left. \frac{\Delta \lambda_q}{\Delta i_d} \right|_{i_q = \text{constant}} \quad (5.10)$$

To obtain these cross-coupled inductance values, this research uses the flux linkage variation over the current variation with the span of 3 A in each d- and q-axis direction. Using an approximation function of flux linkage as suggested in [69] to reduce the simulation works might be a good method. However, to determine the self-inductance, using the direct derivatives of the approximation function as in [69] could result in very low value of q-axis inductance when the q-axis flux path is saturated.

If the calculated results are then put in (5.4) and (5.5), the values of the self-inductance for the two axes are

$$L_d(i_d, i_q) = \frac{\lambda_d(i_d, i_q) - \lambda_{d,PM} - L_{dq}(i_d, i_q)i_q}{i_d} \quad (5.11)$$

$$L_q(i_d, i_q) = \frac{\lambda_q(i_d, i_q) - \lambda_{q,PM} - L_{qd}(i_d, i_q)i_d}{i_q} \quad (5.12)$$

After calculating the rms value of back emf voltage from (5.1) in section 5.3, the phase voltage can be obtained from (5.7) and (5.8), as can the torque angle δ . Then, the expected power factor is determined by the angle between the given current vector and the calculated voltage vector, and the expected output torque can be obtained from (5.6).

5.5.3 Analysis results of the inductances

In this analysis, there is the comparison between the computed results in two different excited conditions; the highest and the lowest side-field excitation. When the machine is in higher excited condition, it will get higher rotor flux by adding the flux generated from its side-field excitation coils. Therefore, in the condition of the highest excitation (the excitation current is 5 A), the machine could be more saturated than in the lowest excited condition (the excitation current is 0 A), and then, the cross saturation effects were also different.

Figure 5.19 and 5.20 show the FEA results of the flux density distribution on rotor and stator without phase current in two different excited conditions. In the lowest excited condition (Figure 5.20), the flux density is clearly lower than in the highest excited condition, especially on the -d-axis of the rotor. These figures prove that the excited structure works well.

Figure 5.21 shows the variation of the flux linkage in each d- and q-axis direction at different input current conditions, from 0 to 150 A with a step of 30 A in each axis. However, d-axis current has a negative value for motoring operation in the direction of the current vector in Figure 5.18.

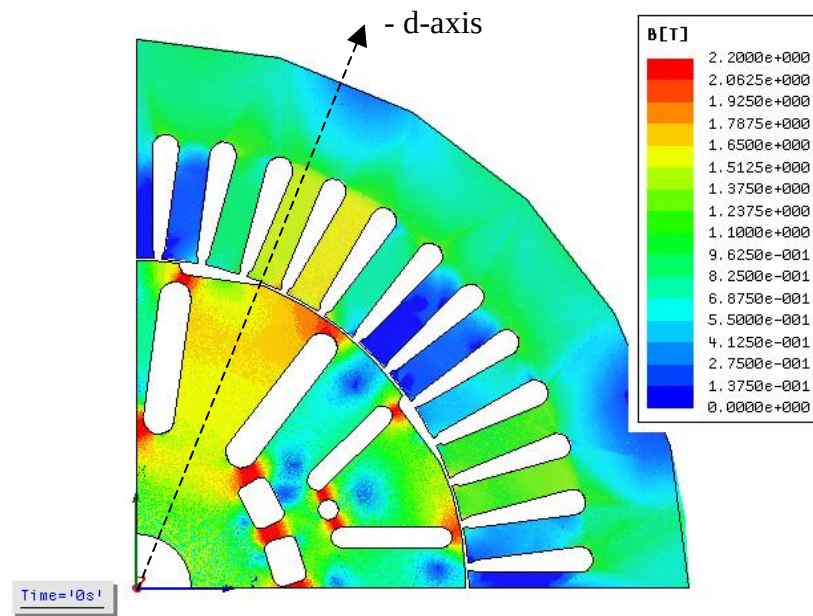


Figure 5.19 Flux density distributions on rotor and stator without phase current in case of the highest excited condition.

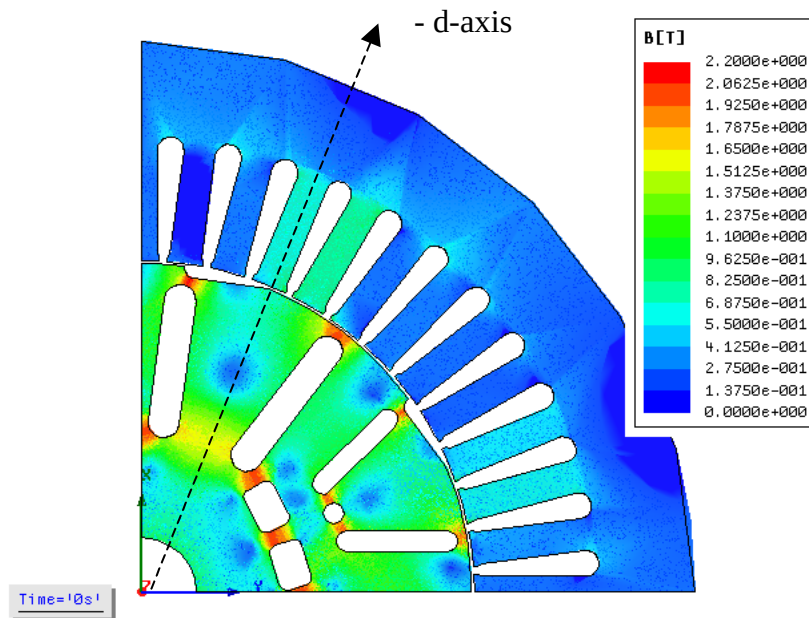


Figure 5.20 Flux density distributions on rotor and stator without phase current in case of the lowest excited condition.

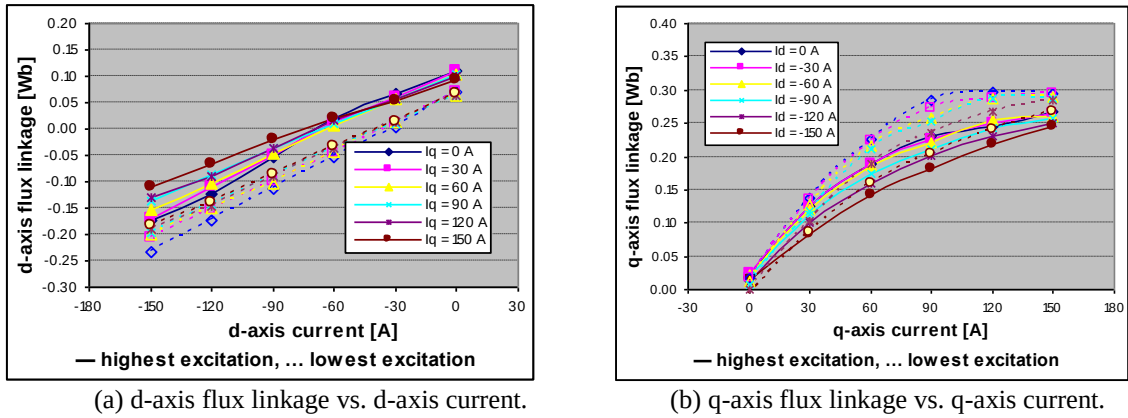


Figure 5.21 Computed flux linkage values.

Figure 5.21 (a) has the shape of a linear function of d-axis current, which suggests that there is no saturation on the d-axis flux path by increasing d-axis current on negative direction because this current acts on demagnetization of the permanent magnet flux. However, Figure 5.21 (b) has the shape of a square root function because the increased q-axis current makes the saturation on the q-axis flux path.

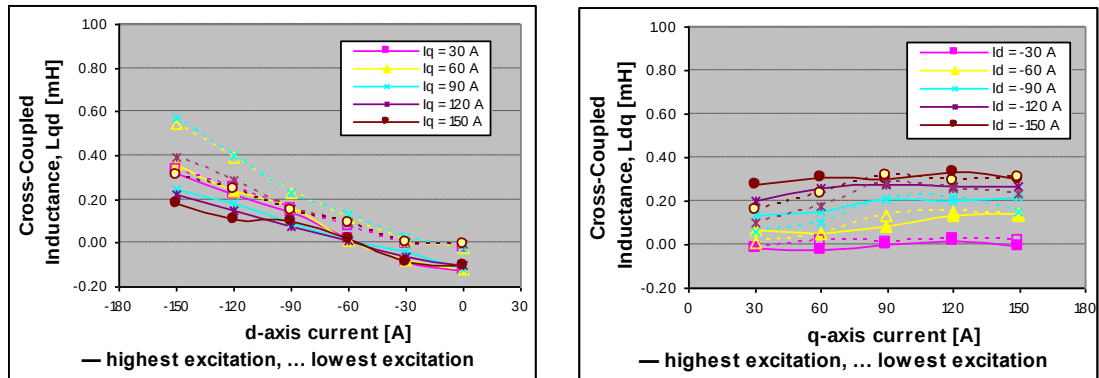
As shown in these figures, the variation of the flux linkage differs by the current change in the other axis, which causes the cross saturation between the two axes. As expected, in the case of the lowest excited condition both d-axis and q-axis flux linkages have wider variation by the change of either d- or q-axis current.

This means that the lowest excited condition has higher self-inductance values on both axes than the highest excited condition. In other words, the flux caused from stator current can travel easily when the machine is operated at lower excited condition.

Therefore, this result suggests that the lowest excited condition can be more affected by the cross-coupled saturation effect, especially in L_{qd} .

Figure 5.22 presents the calculated results of cross-coupled inductance using (5.9) and (5.10). When analyzing Figure 5.21, L_{qd} is easily influenced by d-axis current before saturation, while L_{dq} changes only slightly by the variation of q-axis current but L_{dq} increases by increasing negative d-axis current. One note of interest from Figure 5.22 (a) is that L_{qd} increases by increasing q-axis current until $I_q = 90$ A, then it decreases with increasing q-axis current because of the saturation effect.

Both cross-coupled inductances are determined by mainly d-axis current rather than q-axis current. The overall values of the cross-coupled inductances L_{qd} with the lowest excited condition are much higher than with the highest excited condition, but the values of L_{dq} are a little higher. These results suggest that d- and q-axis have common magnetic flux path through the brushless field excitation structure of the machine, and it causes a small reluctance torque.



(a) Cross-coupled inductance L_{qd} vs. d-axis current. (b) Cross-coupled inductance L_{dq} vs. q-axis current.

Figure 5.22 Computed cross-coupled inductance values.

At the lowest excitation, the maximum values of either L_{qd} or L_{dq} reaches to about 0.6 mH when d-axis current is -150 A. Because this value is more than 30% of the calculated self d-axis inductance, it is not negligible when the machine is operated at a high d-axis current. The difference of the output torque computed with cross saturation terms and without cross saturation terms will show in the next section.

Figure 5.22 also shows that L_{dq} does not have same value with L_{qd} at same current condition as reported in [69], although there are many references assuming $L_{dq} = L_{qd}$ when considering the cross-coupled inductances simply.

Figure 5.23 shows the computed self-inductances in each direction. At low q-axis current, the values of q-axis inductance are increased by negative d-axis current loading as shown in Figure 5.23 (b). These results suggest that the magnetic flux from negative d-axis current is depressed by the rotor flux and goes through the q-axis flux path, especially under low q-axis current, and these results agree with the higher values of L_{qd} at the lowest excited condition.

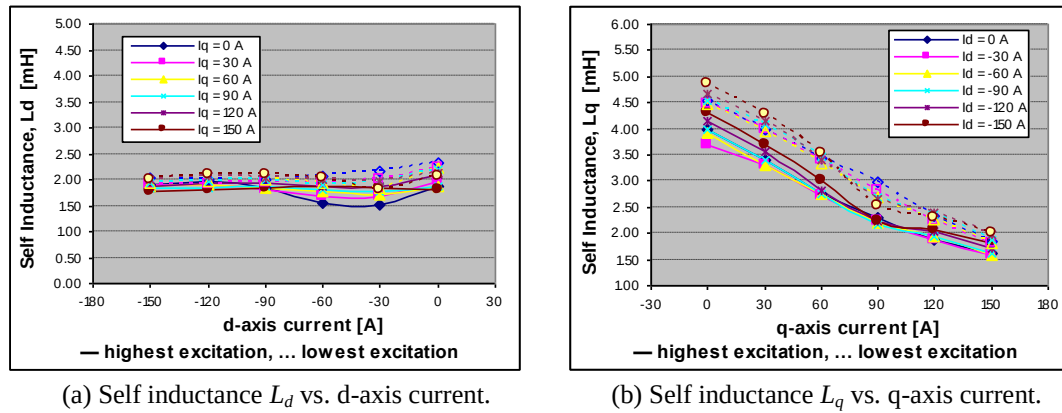


Figure 5.23 Computed self inductance values.

5.5.4 Comparison between the analysis and the test results

To verify the analysis results of the inductance calculation, the computed values of the output torque with the terms of inductance values using (5.6) are compared to the experimental test results. Figure 5.24 shows the contour of input current on the d–q plane in case of the highest excited condition. In case of the lowest excited condition, the input current varied from 0 to 125 A. During the test, the speed is fixed at 3300 rpm in both cases.

Along with the given input current vector, the output torque can be determined from the computed inductance values using (5.6). The results are illustrated in Figure 5.25. To show the advantage of considering the cross-coupled inductances, there are also other plots showing the computed output torque without cross saturation in Figure 5.25.

Figure 5.25 shows that the output torque calculation considering the cross-coupled saturation approach to the experimental torque is more than the calculation without cross saturation. These graphs also suggest that the cross-coupled saturation has

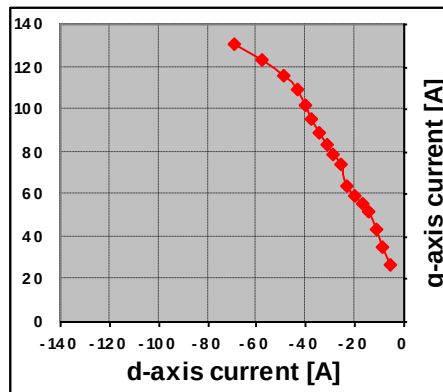
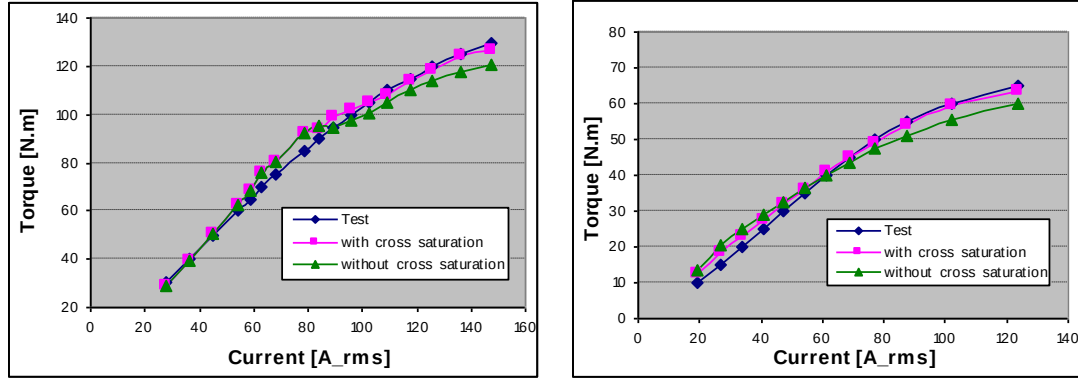


Figure 5.24 Input current contour on d-q current plane in case of the highest excited condition.



(a) In case of the highest excited condition.

(b) In case of the lowest excited condition.

Figure 5.25 Comparison of output torque values vs. input current values.

more influence at higher current conditions which have more negative d-axis current values. Also, the difference between the calculated values with and without the cross-coupled inductance increase in the case of the lowest excited conditions. Therefore, it is possible that a less saturated synchronous machine is more easily affected by the cross-coupled saturation.

5.6 Efficiency Mapping

Data points for each efficiency map were taken from 1,000 rpm to 16,000 in 1,000 rpm increments and from 0 Nm increasing in 10 Nm increments to a final high torque value at each speed. A total of six efficiency maps were generated corresponding to the excitation currents of 0A, 1A, 2A, 3A, 4A, and 5A. Figure 5.26 shows the projected efficiency contours using optimal field current for achieving the highest

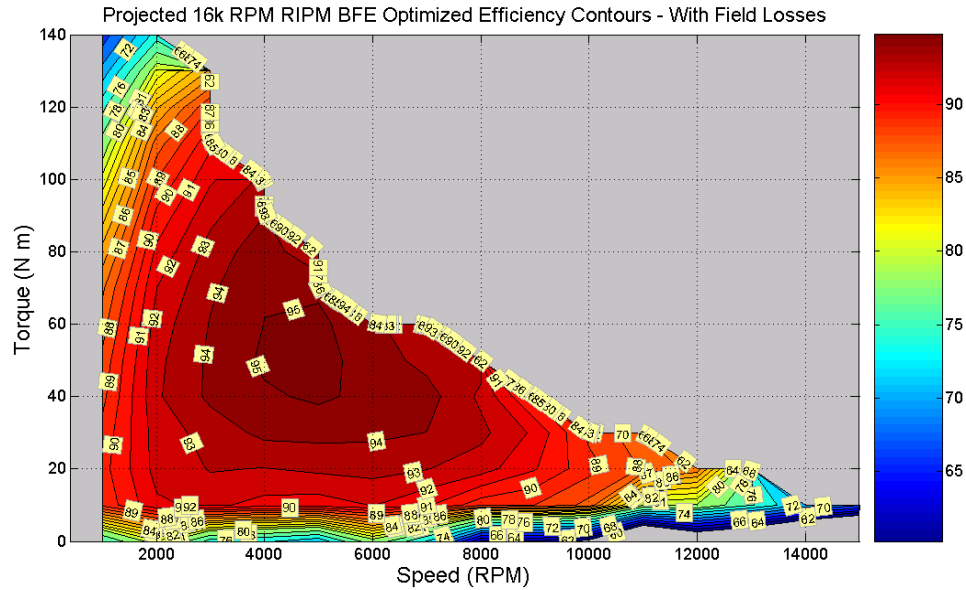


Figure 5.26 Projected efficiency contours using optimal field current.

efficiency by controlling the excitation current. This projected efficiency is substantially superior to the Prius efficiency in Figure 3.7.

5.7 Summary

FEA simulations have been conducted for a newly designed IPMSM that attaches side-PMs and side-poles and has a slant-rotor. Then, the prototype IPMSM was manufactured and tested in various excitation conditions. Good agreement between analytically predicted and experimentally obtained results has been achieved.

Results from both simulations and experimental test clearly show the outstanding characteristics in back-emf and output torque. The air-gap flux and the back emf voltage can be changed by the excitation current value from 0 AT to 5000 AT. So can the output torque. For an IPMSM, the speed is in inversely related to the PM torque or effective flux crossing the phase conductors. Therefore, the new machine can have wider operating speed range than a conventional IPMSM. The back-emf is easily controlled with an external field excitation current in the 0 to 5 A range.

Because the simulation and test results show that the air-gap flux depends mainly on the adjustable excited flux by the excitation current rather than the permanent magnet flux inside the rotor, the permanent magnets should be acting as a flux barrier for the d-axis flux path. The ideal permanent magnet property for the field excitation should be a high coercivity, H_c , and a relatively low remanence, B_r . These would enhance the field adjustment ratio and lower the high speed core losses.

This chapter introduces a new method for calculating the power factor and output torque by considering the cross saturation between d- and q-axes of an IPMSM. The conventional two-axis IPMSM model is modified to include the cross saturation effect by adding the cross-coupled inductance terms. The analysis results are more agreeable to the test results than the computation results that do not consider the cross saturation. Therefore, the modified torque equation (5.6) can be used for the dynamic model of an IPMSM for developing a better control model or control strategy.

CHAPTER 6

Conclusions and Recommendations

Currently the Interior Permanent Magnet Synchronous Motor (IPMSM) is widely used for the application of HEVs by many leading auto manufacturers because the power density and efficiency of this type of motor is high compared with that of induction motors and switched reluctance motors. However, the primary drawback of the IPMSM is caused by its permanent magnets, the fixed PM produces a large back emf or flux linkage that must be reduced for its high speed operation.

In general, the reduction in back-emf is accomplished with a significant d-axis demagnetization current, which reduces the rotor flux by producing the flux to oppose to the rotor flux. This conventional method of field weakening will reduce the efficiency of the motor and there is still limitation caused from its characteristic current.

The objective of this research is to avoid the primary drawbacks of the IPMSM by introducing brushless field excitation (BFE). This offers both high torque density at low speed by using additional flux, which is enhanced by increasing current to a fixed excitation coil at both axial sides, and flux weakening by reducing current to the excitation coil at high speed.

Another expected advantage from BFE is low core loss with lower excitation condition. While the rotor is rotating at high speed with no field current, the core loss is significantly lower than that of conventional IPMSMs. The core and friction loss test results show the benefits of lower air-gap flux density. For example, at 5,000 rpm, the

core and friction loss is 200 W at zero field excitation, compared with 600 W at high field excitation for a high air-gap flux density. This should positively impact the highway fuel efficiency of an HEV.

6.1 Contributions

To maximize the effect of the BFE, the slanted air-gap shape is introduced. The analytical and FEA results show that this unique shape can provide higher output torque and controllable back emf ratio than a uniform air-gap shape. In the process of analyzing the slanted air-gap shape, a new method of output torque calculation using 2-dimensional equivalent circuit is developed. This method enables a machine designer to estimate easily the variation of the output torque without computation of the inductance values. Especially, this method is very effective when the machine has irregular shaped air-gap because the magnitude and position of the reluctance torque cannot be obtained by conventional methods.

There is an investigation for the designed machine characteristic dependence on slant structure using the new method of output torque calculation and FEA. The results suggest that the best slant shape is about 2.5 mm in its depth and a half span between two vertical permanent magnets in its width when focusing on the output torque more than controllable back emf ratio. However, this shape can be changed if the dimensions of the machine and/or the excitation structure are changed. One of the most important results is

that the slanted air-gap shape can be adapted for increasing the output torque of an general typed synchronous machine.

There is a good agreement between analytical prediction and experimental test results of the prototype IPMSM. Both simulation and test works for the prototype prove that the excitation flux from the third dimension (the axial direction) for an interior PM machine is effective and practical in the desirable features of the traction motor for HEV.

When calculating the inductance values, considering the cross saturation between d- and q-axes will give better analysis results. Using these cross coupled inductances L_{dq} and L_{qd} , the conventional two-axis IPMSM model in terms of output torque is modified and compared with the test results. Since the analyzing works were performed in two different conditions, the highest and lowest excited conditions, it is possible to investigate the difference of the cross-saturation effect when a machine has higher magnetic flux from its rotor and lower flux. And, it shows that a less saturated machine could be more affected by the cross-coupled saturation effect.

6.2 Future Research Needs

This prototype motor should not present any manufacturing issues in mass production. Design improvements on the excitation coils to motor housing interface will result in a reduction of both mass and volume of the prototype motor thereby reducing manufacturing costs. Yet, this motor is not optimized since both the BFE and slanted air-gap structures are newly devised shapes in developing an electric machine. There is much

possibility to improve these unique features. Further research can promise the optimization of the machine size and maximize the machine performance for the applications in various industrial areas. The following are the some recommendations and ideas related with the future research subjects for the improvement of the new IPMSM.

Firstly, it is necessary to construct more accurate magnetic equivalent circuit of the machine in order to investigate the slant shaped air-gap in detail. For example, increasing the number of nodes and developing the analytical technique to consider the partial saturation at the stator along with the various input current conditions will be effective to determine the slant shape as well as to examine the designed machine characteristics.

Secondly, the evaluation of the efficiency of the machine can be achieved by the correct computation of the losses. Because the iron loss^{*} is governed by the flux values, the flux values at the stator are calculated with the different excited and input current conditions. Especially, modeling the eddy current loss is a key point in the efficiency analysis because the eddy current loss is greatly influenced by the non-sinusoidal flux variation [35 – 36] and the new IPMSM has many harmonic components of the air-gap flux caused by its slanted shape. Using a frequency spectrum of the air-gap flux density and considering the loss of the flux variation in each d- and q-axis independently will be a possible method.

Lastly, there should be the improvement of the BFE structure. It is possible that the existence of side air-gap could result in axial vibration or unexpected torque ripple.

^{*} Iron loss is usually the summation of the hysteresis loss and the eddy current loss. The hysteresis loss is proportional to the frequency of the flux variation and the eddy current loss is proportional to the square of the frequency.

Another possibility is that the BFE structure has a common magnetic flux path on both d- and q-axis undesirably, and as a result, the reluctance torque of the machine could be reduced by reducing the difference between d- and q-axes inductances. The further study will promise the fully utilization of this unique structure for the application of various industrial areas as well as the application of HEV.

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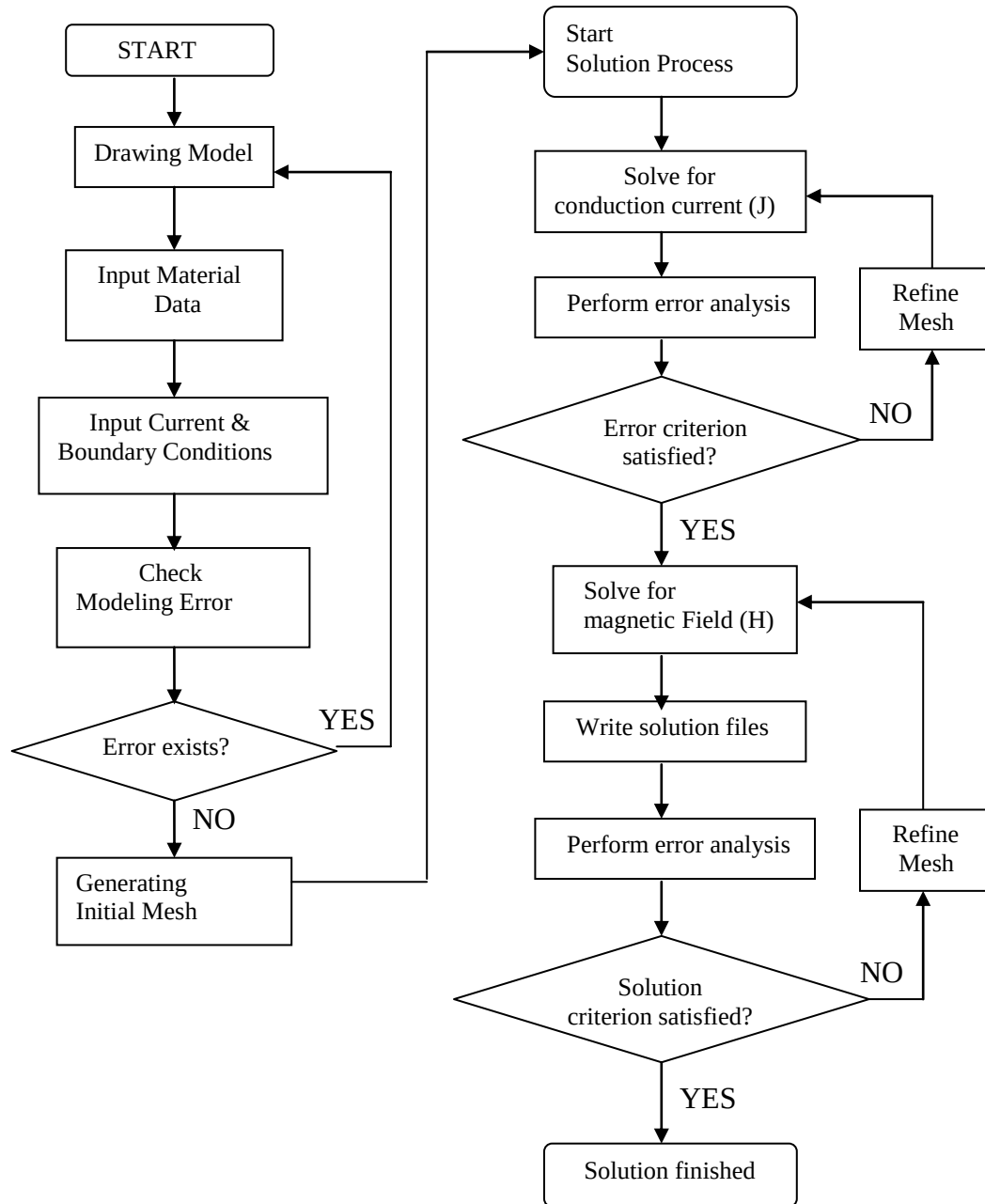
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APPENDICES

Appendix A

Flowchart of Maxwell 3D



Appendix B

Park's transformation and Reference Coordinate systems

The Park's transformation is used to transform three-phase systems into two-axis models, which defines the method of a transformation from the three-phase system to a two-axis model (d and q). For synchronous motors, especially with permanent magnets, a rotor-oriented coordinate system is more convenient. In the dq-system, the d-axis is along the magnetization of the rotor while the q-axis lies electrically perpendicular ahead in the direction of positive rotation.

Figure B.1 shows the structure of the three-phase IPMSM with the reference angles. The a-, b-, and c-axis are fixed on the stator. d- and q-axis are fixed on the rotor, which define the synchronous reference frame. Any set of three-phase quantities, e.g. the flux linkages λ_a , λ_b , and λ_c can be transformed in the synchronous reference frame using Park's transformation method as the following equations:

$$\lambda_d = \frac{2}{3} \left[\lambda_a \cos(p\vartheta_m) + \lambda_b \cos(p\vartheta_m - \frac{2\pi}{3}) + \lambda_c \cos(p\vartheta_m - \frac{4\pi}{3}) \right] \quad (\text{B.1})$$

$$\lambda_q = -\frac{2}{3} \left[\lambda_a \sin(p\vartheta_m) + \lambda_b \sin(p\vartheta_m - \frac{2\pi}{3}) + \lambda_c \sin(p\vartheta_m - \frac{4\pi}{3}) \right] \quad (\text{B.2})$$

where ϑ_m is the mechanical angle between the reference a-phase axis and the d-axis, and p is the number of pole-pair.

The inverse transformation is then;

$$\lambda_a = [\lambda_d \sin(p\vartheta_m) + \lambda_q \cos(p\vartheta_m)] \quad (\text{B.3})$$

$$\lambda_b = \left[\lambda_d \sin\left(p\vartheta_m - \frac{2\pi}{3}\right) + \lambda_q \cos\left(p\vartheta_m - \frac{2\pi}{3}\right) \right] \quad (\text{B.4})$$

$$\lambda_c = \left[\lambda_d \sin\left(p\vartheta_m - \frac{4\pi}{3}\right) + \lambda_q \cos\left(p\vartheta_m - \frac{4\pi}{3}\right) \right] \quad (\text{B.5})$$

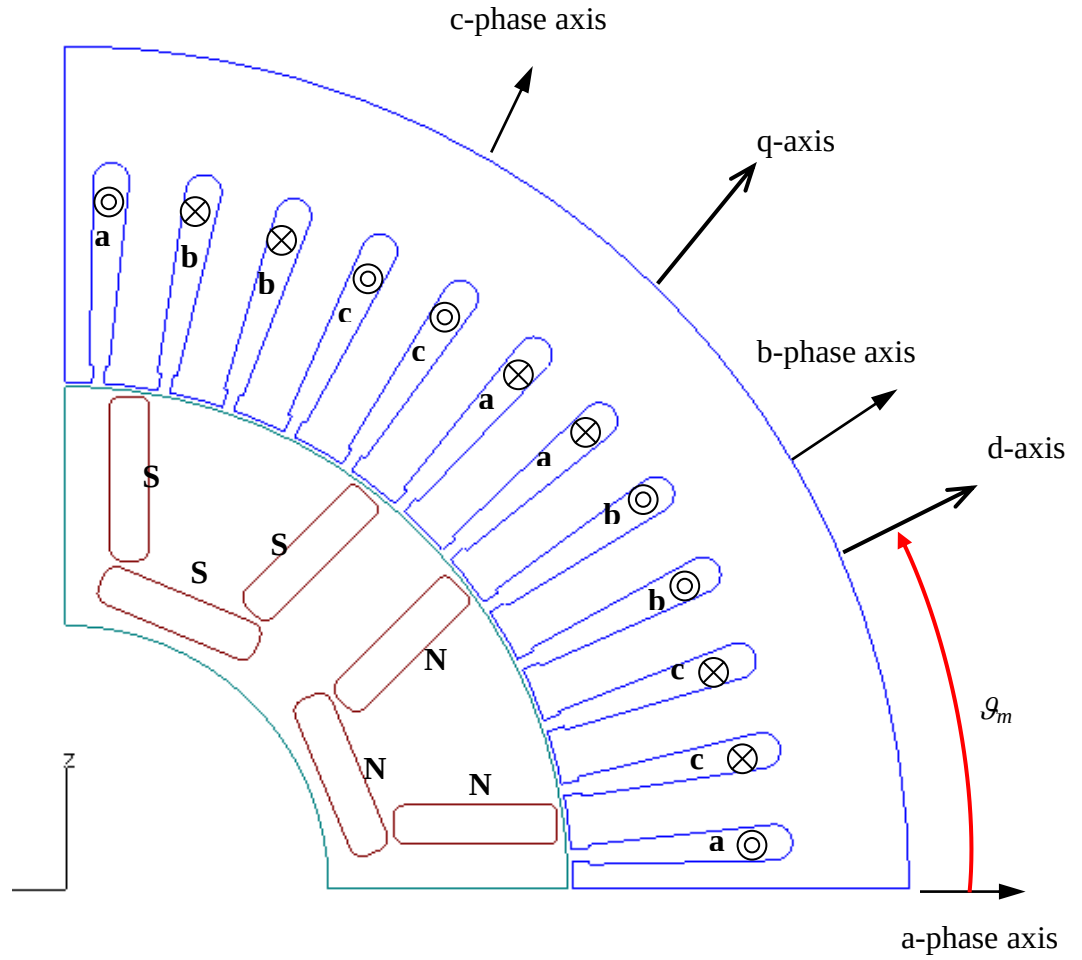


Figure B.1 A quarter IPMSM model for references of the three-phase

Appendix C

Winding factor

When designing an electric machine, it assumes that the stator conductors are wound with a full-pitch (180° electrically) and concentrated at one slot in the stator as shown in Figure C.1 (a) for simple calculation. However, in the real world, most windings are not full pitch nor concentrated to get a smoother sinusoidal mmf wave shape such as Figure A.2 (b). The reason is explained in Chapter 2.

To compensate the difference between the assumed winding and the real winding, a winding factor is usually used. In other words, winding factor is the ratio of the effective mmf produced by the actual windings to the mmf from the full-pitched and concentrated windings with the same number of turns.

This winding factor is the product of distribution factor, pitch factor, and skew factor.

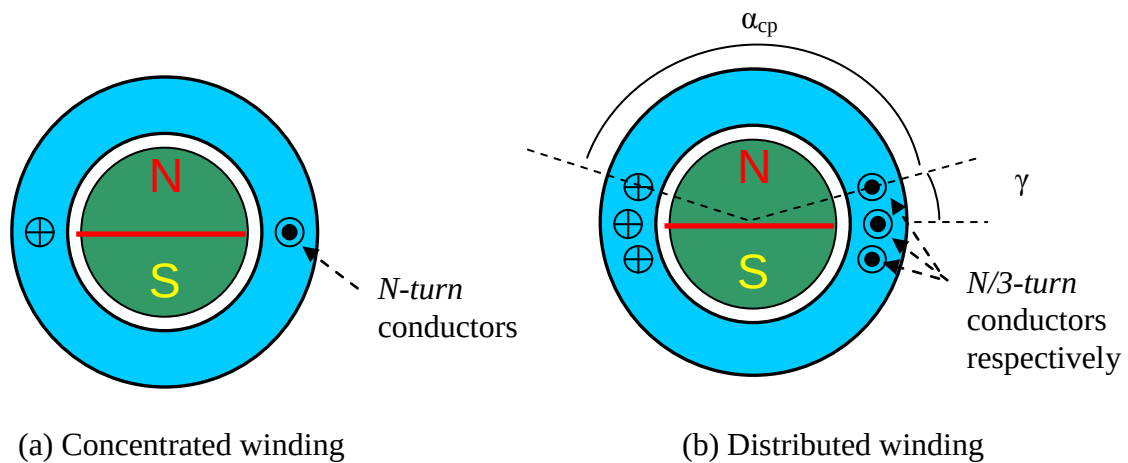


Figure C.1 Comparison of the simple winding patterns.

i. Distribution factor

This factor considers the actual direction of each conductor in different slots. In Figure C.1 (b), the conductors nearby are separated by electrical angle γ , which is also the slot pitch angle. Therefore, the actual mmf is smaller than the mmf produced by concentrated windings as illustrated in Figure C.2.

In Figure C.2 (b), red vectors are the individual directions of each distributed windings, and the blue vector is the summation of all red vectors. Clearly, the magnitude of the added vector is less than the concentrated winding vector (Figure C.2 (a)).

To consider this difference, distribution factor is expressed as [31, 35]

$$k_d = \frac{\sin\left(\frac{N_{spp}\gamma}{2}\right)}{N_{spp} \sin\left(\frac{\gamma}{2}\right)} \quad (D.1)$$

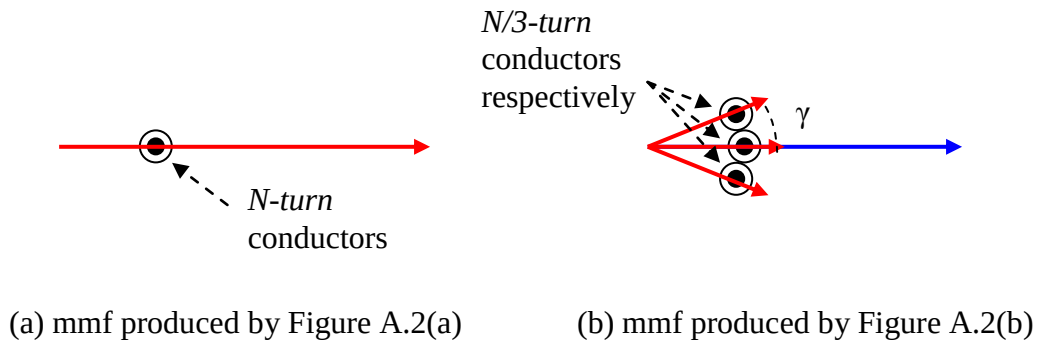


Figure C.2 Comparison of the magnitude of the mmf.

where N_{spp} is the number of the slots per pole per phase and γ is slot pitch in electrical degree. γ can be determined as follow using total number of the slots, N_s .

$$\gamma = \frac{2\pi \cdot p}{N_s} \quad (\text{C.2})$$

ii. Pitch factor

If a machine has distributed windings, the coil pitch will be smaller than 180° (electrical) as shown in Figure C.1 (b). These windings are said to be chorded or short-pitched and have reduced mmf or flux linkage compared to full-pitched windings [31]. The reduction is considered by using pitch factor such as

$$k_p = \sin\left(\frac{\alpha_{cp}}{2}\right) \quad (\text{C.3})$$

where α_{cp} is coil pitch angle.

iii. Skew factor

Skewing the stator slot is a conventional method to reduce cogging torque of an electric machine. A skewed stator has smoother flux linkage and back emf, but it also has

decreased effective slot area and increased coil resistance. Moreover, the flux linkage at the windings from rotor also decreases. The reduction is taken into account by skew factor such as

$$k_s = \frac{\sin\left(\frac{\theta_{se}}{2}\right)}{\frac{\theta_{se}}{2}} \quad (C.4)$$

where θ_{se} is the skew angle in electrical degree. Then, the winding factor is determined by all of the individual factors, such as (C.5).

$$k_w = k_d \cdot k_p \cdot k_s \quad (C.5)$$

Appendix D

Carter's coefficient

Electric machines have slots for current-carrying windings, and these slots are usually open to face the air-gap as shown in Figure D.1. Because of the opening part of the stator, the flux between rotor and stator will travel longer distance than the air-gap at the opening. Therefore, the average magnetic reluctance of the air-gap is higher than the actual value with the actual air-gap distance such as

$$R_g = \frac{k_c \cdot l_g}{\mu_0 A_g} \quad (\text{D.1})$$

where R_g is the average magnetic reluctance, l_g is the length of the air-gap, A_g is the area of the air-gap, μ_0 is the permeability of air ($= 4\pi \times 10^{-7}$), and k_c is the Carter coefficient or Carter factor [31, 36]. Definitely, k_c should be greater than 1 for considering the increased magnetic reluctance of the air-gap.

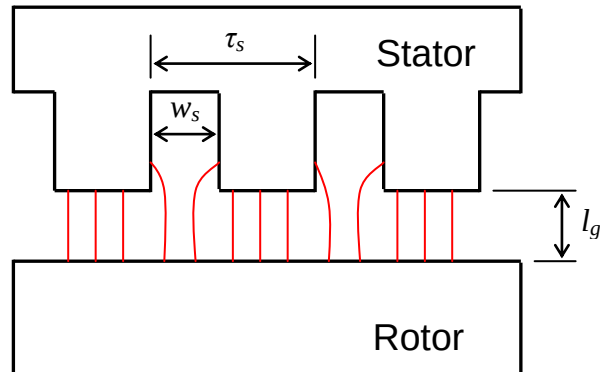


Figure D.1 A simple exmaple of a stator and air-gap model.

There are many expressions for Carter coefficient The simplest one is the (D.2) given by Okawa[32] and Nasar [35].

$$k_c = \left[1 - \frac{1}{\frac{\tau_s}{w_s} \left(\frac{5l_g}{w_s} + 1 \right)} \right]^{-1} \quad (\text{D.2})$$

(D.3) is a little more complicated expression given by Ward and Lawrenson [73].

$$k_c = \left(1 - \frac{2w_s}{\pi\tau_s} \left\{ \tan^{-1} \left(\frac{w_s}{l_g} \right) - \left(\frac{l_g}{2w_s} \right) \ln \left[1 + \left(\frac{w_s}{l_g} \right)^2 \right] \right\} \right)^{-1} \quad (\text{D.3})$$

Hanselman suggested a new expression, (D.4), to have the calculated result placing between the results from (D.2) and (D.3) [31].

$$k_c = \left(1 - \frac{w_s}{t_s} + \frac{4l_g}{\pi\tau_s} \ln \left[1 + \frac{\pi w_s}{4l_g} \right] \right)^{-1} \quad (\text{D.4})$$

Overall, Carter coefficient converges to 1 as l_g/τ_s increases and w_s/τ_s decreases.

VITA

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In 2000, he began attending the University of Tennessee in Knoxville majoring in Electrical Engineering for a Bachelor of Science and a Master of Science degree. He started the Ph. D's program at same school in 2005. While studying for his Master and Ph. D degree, he has worked at the Oak Ridge National Laboratory as a research assistant. His research interests include design, testing, and control of permanent magnet electric machines.