ELECTRICAL CHARACTERISTICS

OF

TRANSMISSION LINES

by

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This work is, of necessity, a compilation of material from various sources. The authors are indebited to Prof. J. G. Tarboux, Professer of Electrical Engineering at the Univerity of Tennessee, for his kind and invaluable assistance in the gathering together and in the correlation of this material. He also aided in the solution of some of the important formulae.

University of Tennessee
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May 29, 1930

Signed:-
L. L. Wilson
L. R. Crittendon

INTRODUCTION

For the benefit of those who are interested in electrical transmission of energy, this work, dealing with the theoretical electrical considerations involved in planning transmission lines, is presented.

On the market today are treatises of a similar nature, which are more complete than this, but, wherein this presentation is brief, an effort has been made to develop the theories in a manner which may be understood by the average electrical engineer, and to apply them in such a way as to make possible the solution of actual circuits by the engineer without recourse to tedious mathematical analysis of each problem encountered.

The history of power development shows that up until about the year 1890, power-generating stations were almost exclusively located at, or near the point where the power was to be used. With the advent of electric transmission, however, centralization of power-generation was made possible, and then there was only a short step ahead to the inter-connection of the lines of the large power companies. This inter-connection has had the effect of creating a better load factor, of making available power from large and remote, low-cost developments, of decreasing the standby, or reserve equipment, of increasing the radius of service from a given plant, and of allowing the division of load between stations for most economical operation. Thus the growth in power development during the past thirty years, or more is due primarily to progress made in electrical transmission.

In the following pages a few fundamental mathematical considerations have been reviewed; equations derived for the constants of various types of lines; equations given for voltage regulation, line efficiency, most economical size of conductor, line losses, power circles for the generator end of the line, and for the receiver end, and for synchronous condenser capacity.

An effort has been made to develop simple and general equations for transmission characteristics which, in their final form, will render possible the relatively easy solution of most transmission problems.

ELECTRICAL CHARACTERISTICS

OF

TRANSMISSION LINES

SIMPLE EQUATIONS

Direct Current Lines --

Although direct current transmission over long distances is not very practical and is very expensive, any treatise on transmission line calculations would not be complete without mentioning it.

The three important factors governing direct current design are: (1) voltage drop in line, (2) power loss in line, and (3) safe current capacity of conductors.

The only loss in the transmission of direct current is that due to resistance, so considering a system as shown in Figure 1:

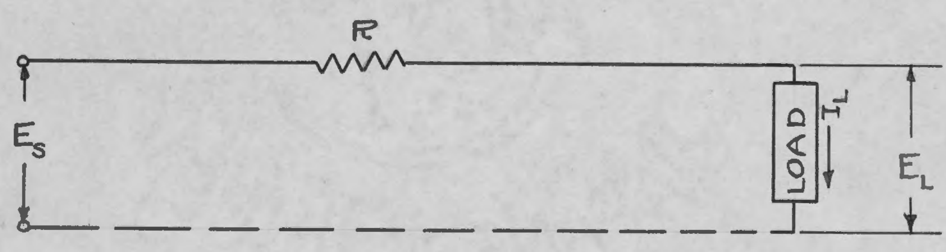


Figure 1

$$E_s = E_L + I_L R \qquad (1)$$

$$I_s = I_L \qquad (2)$$

E_s , I_s - sending voltage and cirrent respectively

E_L , I_L = load voltage and current respectively

The IR drop should be kept within two to three percent of load voltage for lighting loads, and within five per cent of load voltage for industrial power loads. The power loss, I^2R, should of course be kept as low as possible. On outdoor lines the carrying capacity of the conductors will be taken care of by the above considerations, but on indoor installations the capacity limits are given by the National Electric Code.

Voltage Regulation and Efficiency --

The voltage regulation of a line is the ratio of the volts drop in the line between the reciever end and supply end, to the receiver voltage, expressed in per cent, or

$$V. R. = \frac{E_s - E_L}{E_L} \times 100 \qquad (3)$$

The efficiency of the line is the ratio of the output of the line to the input.

$$E = \frac{\text{output}}{\text{input}} \times 100 = \frac{\text{output}}{\text{output} + \text{losses}} \times 100 \qquad (4)$$

Economical Size of Conductor --

The power loss in a line, the current remaining constant, depends upon the resistance of the line which is in inverse proportion to the conductor area. The cost of the conductor varies directly as the area of cross-section of the conductor. There is some conductor area that is most economical for any given set of load conditions. Consider the three items entering into the cost of a conductor, namely: the cost which is independent of conductor size, the cost which is dependent upon the size of the conductor, and the energy loss in the conductor. Then the total cost is

$$C = K + C_1 + C_2 \qquad (5)$$

C = total annual cost
K = constant annual cost
C_1 = annual cost dependent on conductor size
C_2 = annual cost of power loss

$$C_1 = W\,A\,L\,C_c F \qquad (6)$$

$$C_2 = \frac{I_e^2 R}{1000} \cdot t\,C_e = \frac{I_e^2 p\,L\,t\,C_e}{1000\,A} \qquad (7)$$

W = weight of conductor per circular mil-foot = 3.03×10^{-6} for copper
A = area of conductor in circular mils
L = total length of conductor in feet
C_c = that cost per lb. of conductor which is proportional to conductor size
F = annual fixed charges, taxes, insurance, interest and depreciation = 15 per cent (good average)
I_e = equivalent constant current in amperes
R = total resistance of conductor in ohms
t = total hours per year that current flows over line
C_e = cost of energy per kilowatt hour
p = ohms resistance per circular mil-foot = 10.6 per annealed copper.

Therefore

$$C = K + W\,A\,L\,C_c\,F + \frac{I_e^2\,p\,L\,t\,C_e}{1000\,A} \qquad (8)$$

Differentiating with respect to A and setting the first derivative equal to zero

$$\frac{dC}{dA} = 0 + W\,L\,Cc\,F - \frac{I_e^2\,p\,L\,t\,C_e}{1000\,A^2} = 0$$

or for the most economical size of conductor

$$A = \sqrt{\frac{I_e^2\,p\,t\,C_e}{1000\,W\,C_c\,F}} \qquad (9)$$

For annealed copper this becomes

$$A = 59\,I_e\sqrt{\frac{C_e t}{C_c F}} \qquad (10)$$

I_e as stated is the equivalent constant current, and is determined by finding the average of the squares of the currents in the line, and extracting its square root, just as the effective current of A. C. is the square root of the average of the squares of the instantaneous values, or

$$I_e = \sqrt{\text{average } I^2}$$

The determination of the most economical conductor size for A. C. lines is not so easy since it involves other items, namely: corona, mechanical strength, and voltage regulation. The above conclusions, though partially applicable to all transmission circuits, may be considered as final for low voltage lines alone. In the usual high voltage line the corona effect is the determining factor. This effect is treated in a later section.

Alternating Current Lines --

For alternating currents the transmission line problem is much more complex. It involves three circuits which are distinctly different, yet very closely related to each other. These are: (1) the electric circuit, (2) the magnetic circuit, and (3) the dielectric circuit. The presence of the magnetic and dielectric circuits introduce effects upon the line which are measured as inductance L and capacity C, respectively. The voltage and current values along the line depend not only on their component parts, but also on their time-phase and space-phase relations. The voltage at the load may be less than, equal to, or even greater than that at the generator. Instead of dealing only with resistance, three basic factors must be considered, namely, resistance (R), inductive reactance (X_L), and capacity susceptance (B). In many cases, due to faulty insulation a fourth factor must be considered, namely, leakage (G). These factors are known as line constants. Then the total line impedance is

$$Z = R + j\,X_L \qquad (11)$$

and the total shunt admittance is

$$Y = G + j\,B \qquad (12)$$

Resistance. -- Due to the tendency of alternating current to concentrate at the surface of a conductor, which tendency is known as "skin effect", the effective resistance is more than the direct current resistance.

$$R^1 = K R \text{ *} \qquad (13)$$

R^1 = effective resistance, where

$$K = \frac{1 + \sqrt{1 + F^2}}{2}$$

for copper

$$F = 0.0105 \, d^2 \, f$$

for aluminum

$$F = 0.0063 \, d^2 \, f$$

d = diameter of conductor in inches
f = frequency of current in cycles per second

Reactance. -- The reactance of transmission lines for each conductor per mile of length is

$$X_L = 2 \pi f (80 + 741.1 \log_{10} \frac{S}{r}) 10^{-6} \text{ ohms per mile!} \qquad (14)$$

S = distance between conductor in inches
r = radius of conductor in inches

S must be an average or equivalent spacing and in three phase lines

$$S = \sqrt[3]{S_1 \, S_2 \, S_3} \qquad (15)$$

S_1, S_2, S_3 = distances between the three conductors

Capacity Susceptance. -- The capacity susceptance

$$B = 2 \pi f C = 2 \pi f \left(\frac{0.03883}{\log_{10} \frac{S}{r}} \right) 10^{-6} \text{ mhosper mi.} \qquad (16)$$

* Formula by Alfred Still (Electric Power Transmission)

! Standard Handbook for Electrical Engineers, Sec. 11-42.

where S is greater than 20 r; if S is less than 20 r, a more exact equation must be used, namely,

$$B = 2 \pi f \left(\frac{0.03883}{\log_{10} (a + \sqrt{a^2 - 1})} \right) 10^{-6} \quad * \qquad (17)$$

where B is mhos per mile.

$$A = \frac{S}{2 r}$$

Here again S is an equivalent spacing.

The leakage G depends upon so very many variables, such as temperature, humidity, motion of air, dust particles in air, etc., that it can not easily be pre-determined; however, it is usually so small that it can be neglected without disagreeable error.

Conductor Spacing. -- The spacing between conductors and between conductors and tower depends upon many factors, the most important of which are: span, material and diameter of conductors, voltage, temperature, and climatic conditions, type of insulators and type of tower. However, through practice empirical formulae have been developed. D. D. Ewing has developed one which can safely be used for practically all cases. It is,

$$S = 10 + 1.28 E \qquad (18)$$

S = horizontal spacing in inches between conductors
E = line kilovolts

Approximate Equations for Solving Alternating Current Lines --

For short transmission lines up to twenty-five miles in length, and for any voltage up to 50,000 volts the capacity is so small as to be negligible, and the equations for voltage and current relations are:

$$\dot{I}_s = \dot{I}_L \qquad (19)$$

$$\dot{E}_s = \dot{E}_L + \dot{I}_L \dot{Z} \qquad (20)$$

Letters shown with a dot above denote a vector quantity.

These conditions are shown diagrammatically in Figure 2

* Standard Handbook for Electrical Engineers, Sec. 11-42

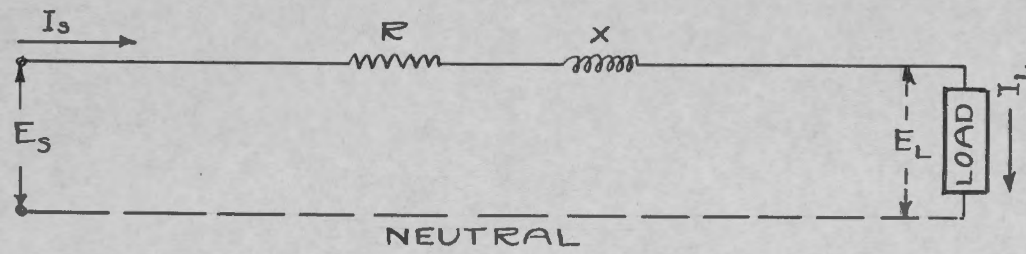


Figure 2

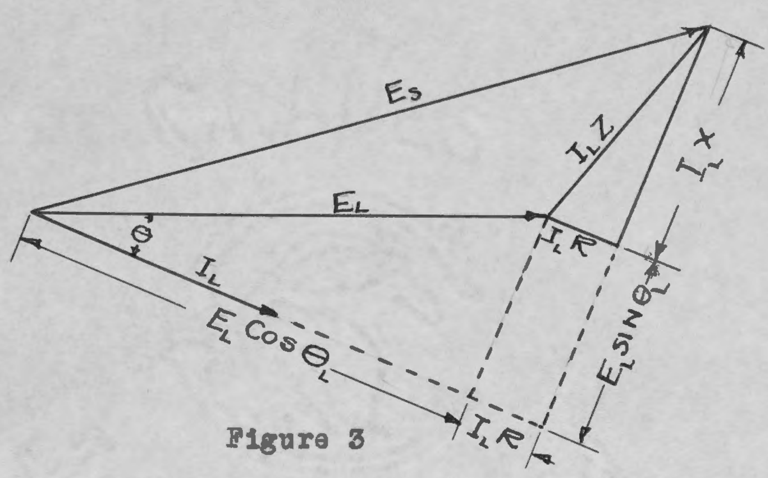


Figure 3

From Figure 3

$$E_s = \sqrt{(E_L \cos \theta_L + I_L R)^2 + (E_L \sin \theta_L \pm I_L X)^2} \quad (21)$$

For lines longer than twenty-five miles the capacity effect becomes great enough that it must be considered. There are two approximate methods for considering capacity which are sufficiently accurate for lines up to fifty miles in length under not more than 50,000 volts pressure. These are known as: (1) the T line method, and (2) the π line method.

Nominal " T " Line. --

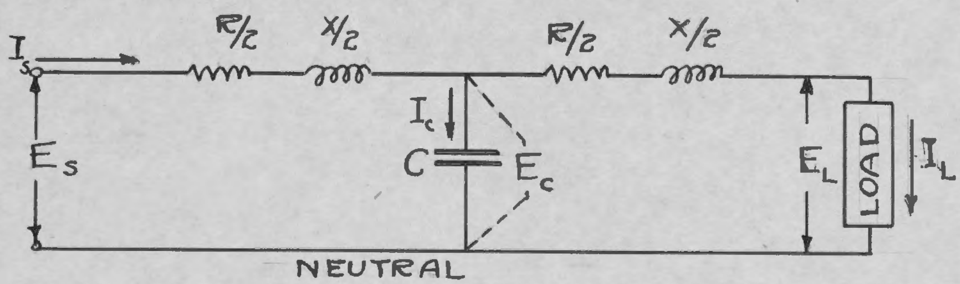


Figure 4

In Figure 4 the resistance and inductance are shown divided into two equal parts, and the capacity is considered as concentrated in the middle of the line. Then

$$\dot{E}_c = \dot{E}_L + \dot{I}_L \frac{Z}{2}$$

$$\dot{E}_s = \dot{E}_c + \dot{I}_s \frac{Z}{2}$$

but

$$\dot{I}_s = \dot{I}_L + \dot{I}_c$$

and

$$\dot{I}_c = \dot{E}_c Y$$

therefore

$$\dot{E}_s = \dot{E}_L + \dot{I}_L \frac{Z}{2} + \dot{I}_L \frac{Z}{2} + \dot{E}_L Y \frac{Z}{2} + \dot{I}_L Y \frac{Z}{2} x \frac{Z}{2}$$

$$\dot{E}_s = (1 + \frac{Z}{2} Y) \dot{E}_L + (Z + \frac{Z^2Y}{4}) \dot{I}_L \qquad (22)$$

$$\dot{I}_s = \dot{I}_L + Y \dot{E}_L + Y \frac{Z}{2} \dot{I}_L$$

$$= Y \dot{E}_L + (1 + Y \frac{Z}{2}) \dot{I}_L \qquad (23)$$

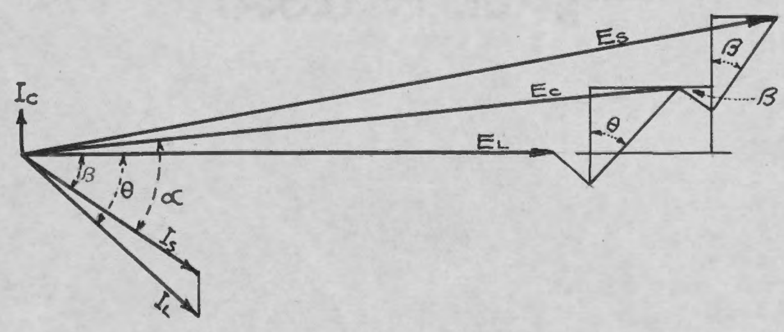


Figure 5

Nominal " π " Line. -- Another approximate equivalent circuit is shown in Figure 6. Here the capacity is considered as divided equally between the two ends of the line with all the resistance and reactance between the lumped capacities.

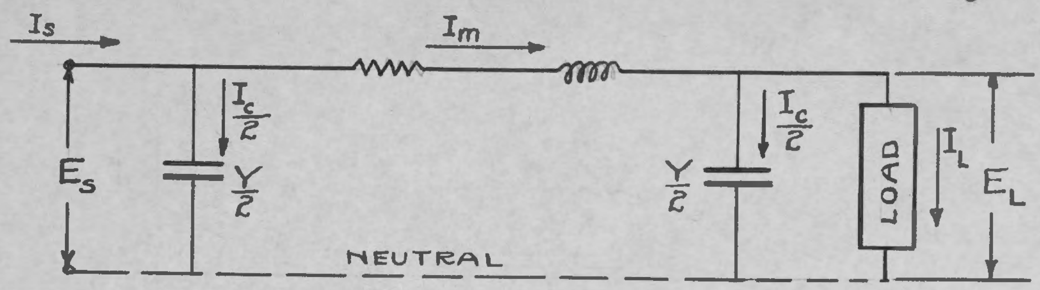


Figure 6

In this case

$$\dot{I}_m = \dot{I}_L + \dot{E}_L \frac{Y}{2}$$

and

$$\dot{I}_s = \dot{I}_m + \dot{E}_s \frac{Y}{2}$$

but

$$\dot{E}_s = \dot{E}_L + \dot{I}_m Z = \dot{E}_L + \dot{I}_L Z + \dot{E}_L Z \frac{Y}{2}$$

collecting

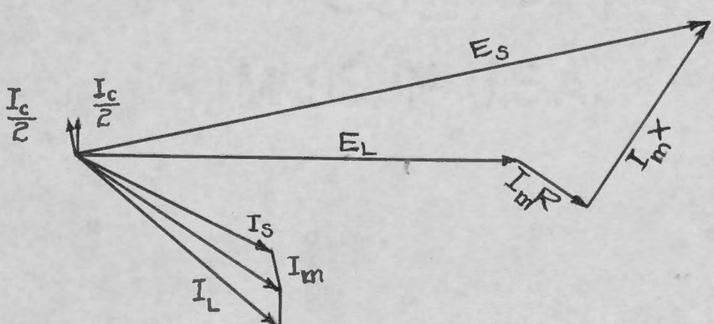
$$\dot{E}_s = (1 + Z \frac{Y}{2}) \dot{E}_L + Z \dot{I}_L \qquad (24)$$

Therefore

$$\dot{I}_s = \dot{I}_L + \dot{E}_L \frac{Y}{2} + \dot{E}_L (1 + Z \frac{Y}{2}) \frac{Y}{2} + Z \dot{I}_L \frac{Y}{2}$$

or collecting

$$\dot{I}_s = \dot{E}_L (Y + Z \frac{Y^2}{4}) + \dot{I}_L (1 + Z \frac{Y}{2}) \qquad (25)$$



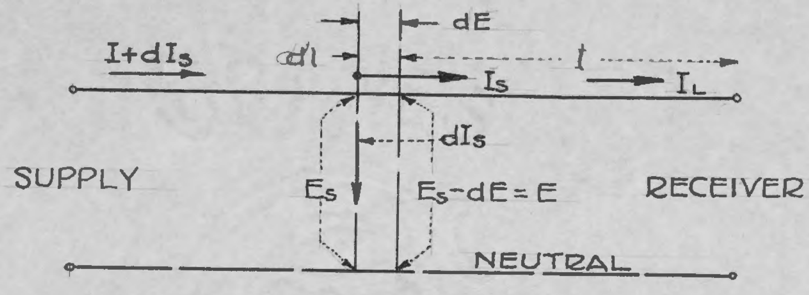
Vector Diagram for " π " Line

Figure 7

GENERAL EQUATIONS FOR A. C. LINES

Exact Solution --

In as much as the constants of the line are not concentrated either at the center or on each end, but are distributed along the line, there will be some error introduced in considering them as concentrated. If the line is long this error becomes great so that it is better to deduce the exact equations for lines over fifty miles in length under any voltage.



Transmission Line With Distributed Constants

Figure 8

Consider a differential length of line d l at a distance l from the receiver end as in Figure 8. The current flowing to the load is I, the leakage current in the element d l is dI, and therefore the supply current is $I_s = I + dI_s$ The voltage on the receiver end is $E = E_s - dE$, on the supply end E_s, and the voltage drop along the element dl is dE. Let $Z = r + jX$ be the series impedance per mile; $Y = g + jb$ be the shunt admittance of the line per mile. The mean current in the element dl is $\dfrac{I + dI_s + I}{2}$,

then

$$dE = \frac{(I + dI_s + I)}{2} Zdl$$

and

$$dE = (I + \frac{DI_s}{2}) Zdl \qquad (26)$$

As $dI_s = 0$ as a linit, $I + \dfrac{dI_s}{2} = I$

then

$$dE = IZdl, \text{or} \quad IZ = \frac{dE}{dl} \qquad (27)$$

in a similar way

$$dI = EYdl, \quad \text{or} \quad EY = \frac{dI}{dl} \qquad (28)$$

Differentiating Equation (27) and substituting Equation (28) into the result

$$\frac{d^2E}{dl^2} = Z\frac{dI}{dl} = ZYE = \alpha^2 E \qquad (29)$$

$$= \sqrt{ZY}$$

this last term is called the propagation factor.

In a like manner

$$\frac{d^2I}{dl^2} = Y\frac{dE}{dl} = ZYI = \alpha^2 I \qquad (30)$$

Re-arranging

$$I = \frac{d^2I}{\alpha^2 dl^2} \qquad (31)$$

and

$$E = \frac{d^2E}{\alpha^2 dl^2} \qquad (32)$$

Solving (See Cohen A., "Differential Equations", Art.43 & 45)

$$I = C \sinh \alpha l + D \cosh \alpha l \qquad (33)$$

$$E = A \sinh \alpha l + B \cosh \alpha l \qquad (34)$$

From Figure 8

when $l = 0$, $I = I_L$, and $E = E_L$

Substituting these values in Equations (33) and (34)

$$I_L = C \sinh 0 + D \cosh 0$$

$$I_L = D$$

$$E_L = A \sinh 0 + B \cosh 0$$

$$E_L = B$$

therefore

$$I = C \sinh \alpha l + I_L \cosh \alpha l \qquad (35)$$

$$E = A \sinh \alpha l + E_L \cosh \alpha l \qquad (36)$$

Differentiating Equation (35) with respect to l and substituting Equation (28)

$$\frac{dI}{dl} = \alpha C \cosh \alpha l + \alpha I_L \sinh \alpha l = EY$$

Substituting Equation (36) for E

$$Ay \sinh \alpha l + E_L Y \cosh \alpha l = \alpha C \cosh \alpha l + \alpha I_L \sinh \alpha l$$

As an identity

$$AY \sinh \alpha l = \alpha I_L \sinh \alpha l$$

$$E_L Y \cosh \alpha l = \alpha C \cosh \alpha l$$

and

$$AY = \alpha I_L$$

$$E_L Y = \alpha C$$

Therefore

$$A = \frac{\alpha I_L}{Y} = \sqrt{\frac{ZY I_L^2}{Y}} = \sqrt{\frac{Z I_L^2}{Y}} = Z_0 I_L \qquad (37)$$

and

$$C = \frac{E_L Y}{\alpha} = \frac{E_L Y}{\sqrt{ZY}} = \sqrt{\frac{Y E_L^2}{Z}} = \frac{E_L}{Z_0} \qquad (38)$$

where

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

Substituting Equations (37) and (38) in Equations (36) and (35) respectively

$$E = Z_0 I_L \sinh \alpha l + E_L \cosh \alpha l \qquad (39)$$

$$I = E_L \sinh \alpha l + I_L \cosh \alpha l \qquad (40)$$

Where $l = L$ the length of the line, $E = E_s$, and $I = I_s$
Then

$$E_s = Z_0 I_L \sinh \alpha L + E_L \cosh \alpha L \qquad (41)$$

$$I_s = \frac{E_L}{Z_0} \sinh \alpha L + I_L \cosh \alpha L \qquad (42)$$

These may be written

$$E_s = AE_L + BI_L \qquad (43)$$

$$I_s = CE_L + DI_L \qquad (44)$$

where

$$A = \cosh \alpha L \qquad C = \frac{\sinh \alpha L}{Z_0}$$

$$B = Z_0 \sinh \alpha L \qquad D = A = \cosh \alpha L$$

These equations represent the general fundamental transmission line equations. As has been noted these equations deal with complex numbers, and with hyperbolic functions of complex numbers. It has been found that these hyperbolic functions are given by an un-ending series of algebraic terms.

$$\cosh \alpha L = \left(1 + \frac{\alpha^2 L^2}{2!} + \frac{\alpha^4 L^4}{4!} + \text{etc.}\right)$$

$$\sinh \alpha L = \left(\alpha L + \frac{\alpha^3 L^3}{3!} + \frac{\alpha^5 L^5}{5!} + \text{etc.}\right)$$

So the line constants may be written

$$A = \left(1 + \frac{\alpha^2 L^2}{2!} + \frac{\alpha^4 L^4}{4!} + \text{ETC.}\right)$$

$$B = Z_0 \left(\alpha L + \frac{\alpha^3 L^3}{3!} + \frac{\alpha^5 L^5}{5!} + \text{etc.}\right)$$

$$C = \frac{1}{Z_0}\left(\alpha L + \frac{\alpha^3 L^3}{3!} + \frac{\alpha^5 L^5}{5!} + \text{etc.}\right)$$

$$D = A$$

Only the first few terms of the series are important as the successive terms decrease in value very rapidly, and can be neglected without appreciable error.

However, if an exact solution is desired the values of the functions themselves must be determined exactly, and substituted in the equation.

Since α is a complex quantity then the functions may be written

$$A = \cosh (u + jv) \qquad B = Z_0 \sinh (u + jv)$$

$$C = \frac{1}{Z_0} \sinh (u + jv) \qquad D = \cosh (u + jv)$$

but

$$\sinh (u + jv) = \sinh u \cos v + j \cosh u \sin v \quad (45)\ *$$

$$\cosh (u + jv) = \cosh u \cos v + j \sinh u \sin v \quad (46)\ *$$

and the constants may be converted into complex quantities with the aid of tables of hyperbolic and trigonometric functions.

Summary. --

The equations so far given may be summarized thus:

$$E_s = AE_L + BI_L \qquad (43)$$

$$I_s = CE_L + DI_L \qquad (44)$$

where the constants are as shown as in Table 1--

* See Appendix

Table 1.

TYPE LINE	MAX. LEN. MILES	MAX. VOLTS	A	B	C	D
D. C.			1	R	0	1
A. C. IMPED.	10	25,000	1	Z	0	1
NOM. "T"	50	50,000	$1 + \frac{ZY}{2}$	$2 + \frac{YZ^2}{4}$	Y	$1 + \frac{ZY}{2}$
NOM. "π"	50	50,000	$1 + \frac{ZY}{2}$	Z	$Y + \frac{ZY^2}{4}$	$1 + \frac{ZY}{2}$
EXACT METH.	any	any	$\cosh \alpha L$	$Z_0 \sinh \alpha L$	$\frac{\sinh \alpha L}{Z_0}$	$\cosh \alpha L$

If the characteristics of the line are known the proper constants may be substituted in the general equations, and the relations between the generator and receiver ends calculated.

Combination of Networks --

Frequently, especially in long transmission lines, transformers are put in on the supply end to raise the voltage, and on the receiver end to lower the voltage. This is done to decrease the line losses, for with a given amount of power transmitted the losses vary as an inverse function of the voltage. The addition of these transformers to the transmission system, changes the system characteristics to such an extent that they can not be neglected.

The usual problem involves a step-up transformer, the transmission line and a step-down transformer. It may be considered as one involving three networks in series as in Figure 9.

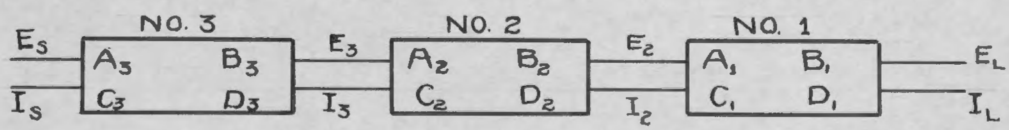


Figure 9.

The networks 1, 2, 3, have constants A, B, C & D, suitably distinguished by subscripts. These constants are defined by the following equations:

$$E_2 = A_1E_L + B_1I_2 \quad (a)$$

$$I_2 = C_1E_L + D_1I_2 \quad (b)$$

$$E_3 = A_2E_2 + B_2I_2 \quad (c)$$

$$I_3 = C_2E_2 + D_2I_2 \quad (d)$$

$$E_s = A_3E_3 + B_3I_3 \qquad (e($$

$$I_s = C_3E_3 + D_3I_3 \qquad (f)$$

Substituting (a) and (b) in (c) and (d) respectively

$$E_3 = A_2\,(A_1E_L + B_1I_L) + B_2\,(C_1E_L + D_1I_L)$$

$$E_3 = (A_2A_1 + B_2C_1)\,E_L + (A_2B_1 + B_2D_1)\,I_L \qquad (g)$$

$$I_3 = C_2\,(A_1E_L + B_1I_L) + D_2\,(C_1E_L + D_1I_L)$$

$$I_3 = (C_2A_1 + D_2C_1)\,E_L + (C_2B_1 + D_2D_1)\,I_L \qquad (h)$$

In a like manner by substituting equation (g) and (h) in (e) and (f) respectively, it is found that the resultant constants A_o, B_o, C_o, and D_o, including the three networks are:

$$A_o = A_3\,(A_1A_2 + C_1B_2) + B_3\,(A_1C_2 + C_1D_2) \qquad (47)$$

$$B_o = A_3\,(B_1A_2 + D_1B_2) + B_3\,(B_1C_2 + D_1D_2) \qquad (48)$$

$$C_o = C_3\,(A_1A_2 + B_2C_1) + D_3\,(A_1C_2 + C_1D_2) \qquad (49)$$

$$D_o = C_3\,(B_1A_2 + D_1B_2) + D_3\,(B_1C_2 + D_1D_2) \qquad (50)$$

It only remains now to determine the proper values for the various constants, and to substitute them in the above equations for A_o, B_o, C_o, and D_o, then substitute these values in the general transmission line equations.

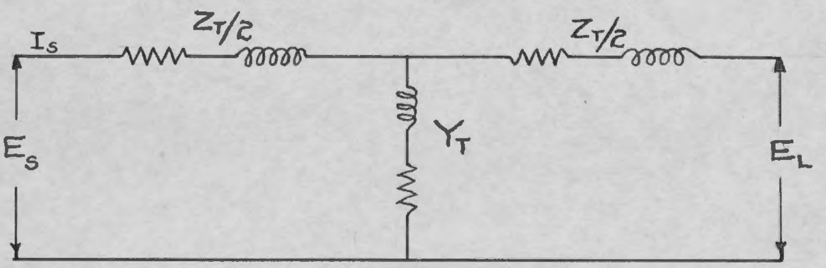


Figure 10.

Atransformer may be accurately represented by the diagram (Figure 10.), in which Z_T REPRESENTS THE TRANSFORMER impedance and Y_T represents the shunt admittance. Z_T and Y_T are complex quantities in which the real parts represent the copper and iron loss, and the imaginary parts represent the transformer reactive, and magnètizing Kva. The current constants would then be:

TABLE II

Item	Type of Network	Circuit Constants A_0	B_0	C_0	D_0	Comments
(a)	E_s T_r E_r Series Impedance	1	T_r	0	1	
(b)	E_s Y_r E_r Shunt Admittance	1	0	Y_r	1	
(c)	E_s $T_r/2$ Y_r $T_r/2$ E_r Transformer	$1+\frac{T_r Y_r}{2}$	$T_r\left(1+\frac{T_r Y_r}{4}\right)$	Y_r	$1+\frac{T_r Y_r}{2}$	Transformer, Exact
(d)	E_s $\frac{Y_r}{2}$ T_r $\frac{Y_r}{2}$ E_r	$1+\frac{T_r Y_r}{2}$	T_r	$Y_r\left(1+\frac{T_r Y_r}{4}\right)$	$1+\frac{T_r Y_r}{2}$	Transformer, Approximate Error
(e)	E_s T_r Y_r E_r	$1+T_r Y_r$	T_r	Y_r	1	Transformer, Approximate Error
(f)	E_s Y_r T_r E_r	1	T_r	Y_r	$1+T_r Y_r$	Transformer, Approximate Error
(g)	Transformer Approximate Network	1	T_r	Y_r	1	Transformer, Approximate Error
(h)	E_s A. B. C. A E_r Transmission Line (T.L.)	$A^* = \cosh\sqrt{ZY}$	$B^* = \sqrt{\frac{Z}{Y}}\sinh\sqrt{ZY}$	$C^* = \sqrt{\frac{Y}{Z}}\sinh\sqrt{ZY}$	$A^* = \cosh\sqrt{ZY}$	Transmission Line, Exact
(i)	E_s A. B. C. A T_r E_r T.L. and Series Impedance at Receiver	A	$B+AT_r$	C	$A+CT_r$	Exact
(j)	E_s T_s A. B. C. A E_r T.L. and Series Impedance at Supply	$A+CT_s$	$B+AT_s$	C	A	Exact
(k)	E_s A. B. C. A E_r T.L. and Series Impedance at Both Receiver and Supply	$A+CT_s$	$B+A(T_r+T_s)+CT_rT_s$	C	$A+CT_r$	Exact
(l)	E_s A. B. C. A E_r Items (c) Plus (h)	$A\left(1+\frac{T_r Y_r}{2}\right)+BY_r$	$B\left(1+\frac{T_r Y_r}{2}\right)+AT_r\left(1+\frac{T_r Y_r}{4}\right)$	$C\left(1+\frac{T_r Y_r}{2}\right)+AY_r$	$A\left(1+\frac{T_r Y_r}{2}\right)+CT_r\left(1+\frac{T_r Y_r}{4}\right)$	Transmission Line and Receiver Transformer, Exact
(m)	E_s A. B. C. A E_r Items (d) Plus (h)	$A\left(1+\frac{T_r Y_r}{2}\right)+BY_r\left(1+\frac{T_r Y_r}{4}\right)$	$B\left(1+\frac{T_r Y_r}{2}\right)+AT_r$	$C\left(1+\frac{T_r Y_r}{2}\right)+AY_r\left(1+\frac{T_r Y_r}{4}\right)$	$A\left(1+\frac{T_r Y_r}{2}\right)+CT_r$	Transmission Line and Receiver Transformer, Approximate Error
(n)	E_s A B. C. A E_r Items (e) Plus (h)	$A(1+T_r Y_r)+BY_r$	$B+AT_r$	$C(1+T_r Y_r)+AY_r$	$A+CT_r$	Transmission Line and Receiver Transformer, Approximate Error
(o)	Transmission Line and Receiver Transformer Approximate Network Error .5%	$A+BY_r$	$B+AT_r$	$C+AY_r$	$A+CT_r$	Transmission Line and Receiver Transformer, Approximate Error
(p)	E_s A. B. C. A E_r Items (h) Plus (c)	$A\left(1+\frac{T_s Y_s}{2}\right)+CT_s\left(1+\frac{T_s Y_s}{4}\right)$	$B\left(1+\frac{T_s Y_s}{2}\right)+AT_s\left(1+\frac{T_s Y_s}{4}\right)$	$C\left(1+\frac{T_s Y_s}{2}\right)+AY_s$	$A\left(1+\frac{T_s Y_s}{2}\right)+BY_s$	Transmission Line and Supply Transformer, Exact
(q)	E_s A. B. C. A E_r Items (h) Plus (d)	$A\left(1+\frac{T_s Y_s}{2}\right)+CT_s$	$B\left(1+\frac{T_s Y_s}{2}\right)+AT_s$	$C\left(1+\frac{T_s Y_s}{2}\right)+AY_s\left(1+\frac{T_s Y_s}{4}\right)$	$A\left(1+\frac{T_s Y_s}{2}\right)+BY_s\left(1+\frac{T_s Y_s}{4}\right)$	Transmission Line and Supply Transformer, Approximate Error
(r)	E_s A. B. C. A E_r Items (h) Plus (f)	$A+CT_s$	$B+AT_s$	$C(1+T_s Y_s)+AY_s$	$A(1+T_s Y_s)+BY_s$	Transmission Line and Supply Transformer, Approximate Error
(s)	Transmission Line and Supply Transformer, Approximate Error 0.5%	$A+CT_s$	$B+AT_s$	$C+AY_s$	$A+BY_s$	Transmission Line and Supply Transformer, Approximate Error
(t)	E_s A B C A E_r Transmission Line and Both Transformers	$A_0=A\left[\left(1+\frac{T_r Y_r}{2}\right)\left(1+\frac{T_s Y_s}{2}\right)+T_sY_r\left(1+\frac{T_s Y_s}{4}\right)\right]+BY_r\left(1+\frac{T_s Y_s}{2}\right)+CT_s\left(1+\frac{T_r Y_r}{2}\right)\left(1+\frac{T_s Y_s}{4}\right)$ $B_0=B\left(1+\frac{T_r Y_r}{2}\right)\left(1+\frac{T_s Y_s}{2}\right)+A\left[T_r\left(1+\frac{T_r Y_r}{4}\right)\left(1+\frac{T_s Y_s}{2}\right)+T_s\left(1+\frac{T_r Y_r}{2}\right)\left(1+\frac{T_s Y_s}{4}\right)\right]+CT_rT_s\left(1+\frac{T_r Y_r}{4}\right)\left(1+\frac{T_s Y_s}{4}\right)$ $C_0=C\left(1+\frac{T_r Y_r}{2}\right)\left(1+\frac{T_s Y_s}{2}\right)+A\left[Y_r\left(1+\frac{T_s Y_s}{2}\right)+Y_s\left(1+\frac{T_r Y_r}{2}\right)\right]+BY_rY_s$ $D_0=A\left[\left(1+\frac{T_r Y_r}{2}\right)\left(1+\frac{T_s Y_s}{2}\right)+T_rY_s\left(1+\frac{T_r Y_r}{4}\right)\right]+BY_s\left(1+\frac{T_r Y_r}{2}\right)+CT_r\left(1+\frac{T_r Y_r}{4}\right)\left(1+\frac{T_s Y_s}{2}\right)$				Transmission Line and Supply and Receiver Transformer, Exact
(u)	E_s A B C A E_r	$A_0=A\left[\left(1+\frac{T_r Y_r}{2}\right)\left(1+\frac{T_s Y_s}{2}\right)+T_sY_r\left(1+\frac{T_r Y_r}{4}\right)\right]+BY_r\left(1+\frac{T_r Y_r}{4}\right)\left(1+\frac{T_s Y_s}{2}\right)+CT_s\left(1+\frac{T_r Y_r}{2}\right)$ $B_0=B\left(1+\frac{T_r Y_r}{2}\right)\left(1+\frac{T_s Y_s}{2}\right)+A\left[T_r\left(1+\frac{T_r Y_r}{2}\right)+T_s\left(1+\frac{T_s Y_s}{2}\right)\right]+CT_rT_s$ $C_0=C\left(1+\frac{T_r Y_r}{2}\right)\left(1+\frac{T_s Y_s}{2}\right)+A\left[Y_r\left(1+\frac{T_r Y_r}{4}\right)\left(1+\frac{T_s Y_s}{2}\right)+Y_s\left(1+\frac{T_r Y_r}{2}\right)\left(1+\frac{T_s Y_s}{4}\right)\right]+BY_rY_s\left(1+\frac{T_r Y_r}{4}\right)\left(1+\frac{T_s Y_s}{4}\right)$ $D_0=A\left[\left(1+\frac{T_r Y_r}{2}\right)\left(1+\frac{T_s Y_s}{2}\right)+T_rY_s\left(1+\frac{T_s Y_s}{4}\right)\right]+BY_s\left(1+\frac{T_r Y_r}{2}\right)\left(1+\frac{T_s Y_s}{4}\right)+CT_r\left(1+\frac{T_s Y_s}{2}\right)$				Transmission Line and Supply and Receiver Transformer Approximate Error 0.25
(v)	E_s A, B, C, A E_r	$A_0=A\left[(1+T_rY_r)+Y_rT_s\right]+BY_r+CT_s(1+T_rY_r)$ $B_0=B+A(T_r+T_s)+CT_rT_s$ $C_0=C(1+T_rY_r)(1+T_sY_s)+A\left[Y_r(1+T_sY_s)+Y_s(1+T_rY_r)\right]+BY_rY_s$ $D_0=A\left[(1+T_sY_s)+Y_sT_r\right]+BY_s+CT_r(1+T_sY_s)$				Transmission Line. Supply and Receiver Transformer Approximate Error 0.5
(w)	Transmission Line, Supply and Receiver Transformer Approximate Network	$A_0=A+CT_s+Y_r(AT_s+B)$ $B_0=B+A(T_r+T_s)+CT_rT_s$ $C_0=C+BY_rY_s+A(Y_r+Y_s)$ $D_0=A+CT_r+Y_s(B+AT_r)$				Transmission Line. Supply and Receiver Transformer Approximate Error 1%
(x)	E_s T_s A_2 B_2 C_2 A_2 T_m A_1 B_1 C_1 A_1 T_r E_r Two T.L. and Three Trans. Solution is approximate. Item (g) Represents Middle Trans.	$A_0=(A_2+C_2T_s)(A_1+C_1T_m)+(B_2+A_2T_s)(A_1Y_m+C_1)$ $B_0=(A_2+C_2T_s)(B_1+A_1T_r+A_1T_m+C_1T_mT_r)+(B_2+A_2T_s)(A_1+C_1T_r+Y_mB_1+Y_mA_1T_r)$ $C_0=C_2(A_1+C_1T_m)+A_2(A_1Y_m+C_1)$ $D_0=(A_1+C_1T_r)(A_2+C_2T_m)+(B_1+A_1T_r)(A_2Y_m+C_2)$				Two Transmission Lines in Series and Three Transformers, Approximate Error 1% Note Receiver and Supply Transformer Exciting Kv-a Considered as Part of the Loads
(y)	Transmission Systems In Parallel	$A_0=\frac{A_1B_2+A_2B_1}{B_1+B_2}$	$B_0=\frac{B_1B_2}{B_1+B_2}$	$C_0=C_1+C_2+\frac{(A_1-A_2)(D_2-D_1)}{B_1+B_2}$	$D_0=\frac{B_1D_2+B_2D_1}{B_1+B_2}$	Two Transmission Systems In Parallel, Exact

$E_s = A_0E_r + B_0I_r$, $I_s = C_0E_r + D_0I_r$ $\qquad$ $E_r = D_0E_s - B_0I_s$, $I_r = -C_0E_s + A_0I_s$ $\qquad$ $^*A = \left(1+\frac{ZY}{1\cdot2}+\frac{Z^2Y^2}{1\cdot2\cdot3\cdot4}\cdots\right)$ $\qquad$ $^*B = Z\left(1+\frac{ZY}{2\cdot3}+\frac{Z^2Y^2}{2\cdot3\cdot4\cdot5}\cdots\right)$ $\qquad$ $^*C = Y\left(1+\frac{ZY}{2\cdot3}+\frac{Z^2Y^2}{2\cdot3\cdot4\cdot5}\cdots\right)$

Z = Impedance Per Mile × Miles
Y = Admittance Per Mile × Miles

$$A_T = \left(1 + \frac{Z_T Y_T}{2}\right) \tag{51}$$

$$B_T = Z_T \tag{52}$$

$$C_T = Y_T \left(1 + \frac{Z_T Y_T}{4}\right) \tag{53}$$

$$D_T = \left(1 + \frac{Z_T Y_T}{2}\right) \tag{54}$$

The constants for the whole system would then take the following forms for various combinations: (See Table 2).

Where the highest degree of accuracy is required the exact solutions are employed, but for the usual case approximately solutions are adequate. It is advisable to use general constants applicable to the transmission system as a whole, since their use simplifies the calculation of different load conditions, and also enables diagrams to be drawn whereby the transmission problems may be solved graphically.

OTHER CHARACTERISTICS OF A. C. LINES

Corona --

The above considerations do not tell the whole story of the transmission line. In some cases the dielectric circuit becomes very important, especially in high voltage systems. When the voltage of a conductor in air is raised to a point above the dielectric strength of the surrounding air the air will break down and become ionized. This ionized air is a conductor, and hence an additional source of power loss. As the conductivity of the air increases, a faint purplish light may be seen near the conductor (visible in the dark), and a hissing sound is made. This phenomenon, of which the light and hissing are indications, is called corona.

In high voltage transmission lines it is corona which determines to a great extent the size and kind of conductor used.

The best formulae for finding the disruptive critical voltage, the visual critical voltage, and the power loss due to corona effect have been developed by F. W. Peek, Jr. They are:

Disruptive critical volts, fair weather;

$$e_o = 2.303\, m_o g_o\, a r \log_{10} \frac{S}{r} \tag{55}$$

effective kilovolts to neutral,

Visual critical voltage, fair weather;

$$e_v = 2.303\, m_v g_o\, a r \left(1 + \frac{0.189}{\sqrt{ra}}\right) \log_{10} \frac{S}{r} \tag{56}$$

effective kilovolts to neutral,

Power loss, fair weather,

$$P = \frac{390}{a}(f + 25)\sqrt{\frac{r}{S}}\,(e - e_o)^2 10^{-5} \qquad (57)$$

Where

e = effective applied voltage in kilovolts to neutral

e_o = effective disruptive critical voltage in kilovolts to neutral

e_v = effective visual critical voltage in kilovolts to neutral

P = power loss in fair weather in kilowatts per mile of each conductor

a = air density factor $= \frac{17.9\,b}{459 + t}$ = 1 at 77° F. and 29.92 inches barometric pressure.

g_o = 53.6 kilovolts per inch (effective disruptive gradient of air)

b = barometric pressure in inches of mercury

t = maximum temperature in degrees F.

f = frequency in cycles per second

m_o = irregularity factor, which is 1 for smooth polished wires, 0.98 to 0.93 for roughened or weathered wire, 0.87 to 0.83 for seven strand cables, concentric lay, and 0.85 to 0.80 for nineteen, thirty-seven and sixty-one strand

m_v = irregularity factor, which is 1 for polished wires, 0.72 for local corona all along cables, and 0.82 for decided corona all along cables

r = radius of conductor in inches

S = spacing between conductor centers in inches

During stormy or bad weather the power loss is somewhat higher, and may be estimated by taking the value of e_o as 0.8

times the value obtained for good weather.

These equations apply only for two-wire single-phase, and three-wire equilateral three-phase circuits. The solution of the corona loss for unsymmetrical and double circuit three-phase lines necessitates a determination of the potential gradient at the surface of each conductor.* For instance, if three conductors are placed symetrically in a plane, the critical voltage for the center conductor will be approximately four per cent lower, and for the two outside conductors six per cent higher than the value for the same minimum spacing in the equilateral arrangement.

Transposition of Conductors --

By transposition of the conductors of a transmission system is meant the inter-changing of the relative positions of the conductors. This is necessary in order to

(1) Eliminate electrostatic and electromagnetic unbalancing of the phases.
(2) To eliminate mutual induction between parallel lines.
(3) To prevent disturbances in neighboring telephone and telegraph work. As a general rule this transposition should be made every mile to prevent disturbance troubles, while for the balancing effect a transposition every two to forty miles is sufficient, depending on the type and length of the line.

Transmission Line Steady State Stability --

The characteristics of a transmission line, as an electric circuit are closely similar to those of a synchronous machine, the chief difference being in the effect of the magnetic saturation on the latter, and the distributed capacity effect on the former. When a synchronous motor is loaded it takes more current from the line, and also its rotor drops back in phase position by an angle governed by the synchronous impedance of the machine. As the rotor drops back in phase, the torque is increased up to a certain point, known as the pull-out point, after which any further increase in load will cause the phase angle to be so great that the torque will begin to decrease, and instability will occur. Similarly, as the load on a transmission line is increased there will be an increase in the phase angle between the sending and receiving voltages. (See Figure 11)

* Woodruff, L. F., Electric Power Transmission and Distribution

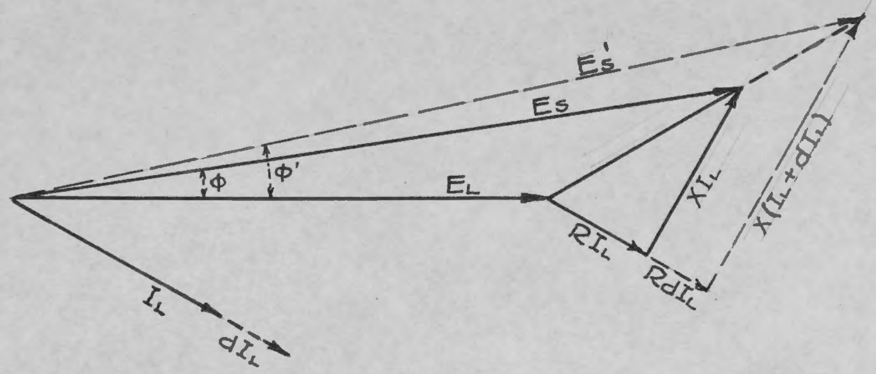


Figure 11.

Since the resistance, inductance and capacity of the line are distributed all along the line, therefore the time-phase position of the current and voltage shifts continually along the line. Such a change may be represented as in Figure 12.

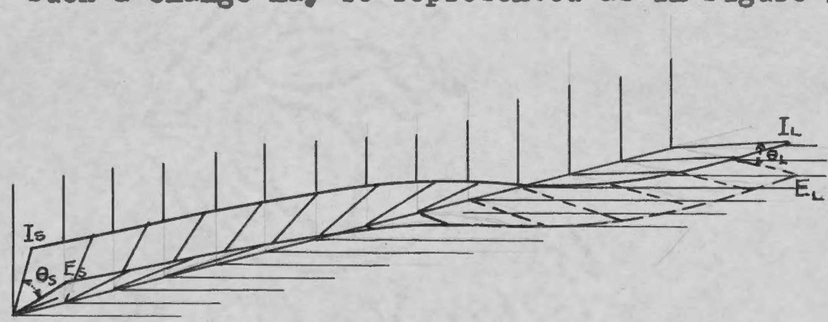


Figure 12.

However the load can not be increased indefinitely for a point is reached after which any further increase in phase angle will decrease the amount of power delivered, and beyond this pull-out point the line will be unstable.

Assume a single-phase line, capacity neglected, and the receiver and sending voltages constant. The current and voltage relations are as shown in Figure 13.

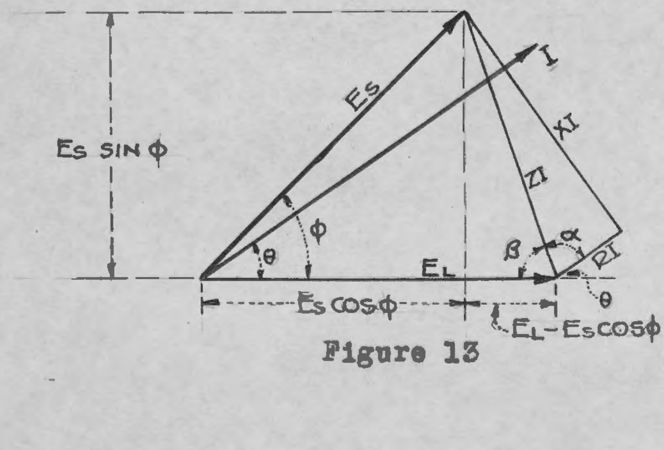


Figure 13

The power delivered to the load is

$$P = E_L I \cos\theta = \frac{E_L}{Z} IZ\,(-\cos(B + a))$$

$$= \frac{E_L}{Z} IZ\,(\sin B \sin a - \cos B \cos a) \qquad \text{(a)}$$

and from Figure 13

$$\sin B = \frac{E_s \sin \phi}{IZ} \qquad \sin a = \frac{X}{Z}$$

$$\cos B = \frac{E_L - E_s \cos \phi}{IZ} \qquad \cos a = \frac{R}{Z}$$

substituting in Equation (a)

$$P = \frac{E_L}{Z^2}(XE_s \sin \phi - RE_L + RE_s \cos \phi) \qquad (58)$$

differentiating P with respect to ϕ, and setting the derivative equal to 0

$$\frac{dP}{d\phi} = \frac{E_L}{Z^2}(XE_s \cos \phi - RE_s \sin \phi) = 0$$

Factoring and collecting it is found that the maximum power is delivered when the phase angle is such that

$$\frac{\sin \phi}{\cos \phi} = \tan \phi = \frac{X}{R}$$

or

$$\phi = \tan^{-1} \frac{X}{R}$$

In a transmission line of negligible resistance

$$\phi = \tan^{-1} \infty = 90^\circ$$

Thus the power limit of a transmission line can be increased by

(a) decreasing the line impedance,
(b) using lower frequencies,
(c) using generators and transformers of low impedance, and
(d) using generators equipped with quick acting voltage regulators, so that the generator voltage may be increased as the load is increased.

CIRCULAR DIAGRAMS

In the pages preceding several possible combinations of net-works have been shown, and it has been demonstrated that any of these combinations can be replaced by an equivalent network represented by a single set of general circuit constants, which can be applied in the usual manner. The operations have been almost purely mathematical. A method will now be presented for a graphical solution of transmission problems, and, although all types of conditions may not be covered herein, the possibilities for solution of problems in all forms will be indicated.

The primary object of an approximate graphical solution is not one of accuracy, although this should be within a few per cent of that obtained by an exact method of calculation, but one that gives, in as simple and general way as possible, a maximum set of solutions with a mimimum of calculation. When such a diagram has been finished the most desirable conditions of operation can be readily selected, and the rigid mathematical solution may be applied to the particular case with any further degree of accuracy that may be desired. Usually this unnecessary for the majority of problems, as a good graphical method is limited in accuracy, only in the drawing and reading of the scalar quantities.

In order for the reader to clearly understand the derivation of this method it is necessary that he keep in mind the idea of complex quantities, and know something of conjugate functions as explained in the appendix of this treatise. For convenience of usuage a few relations of quantities will be repeated.

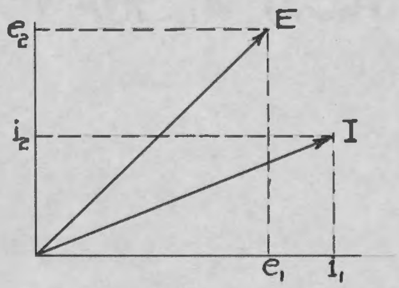


Figure 14.

See Figure 14.

$$\dot{E}\dot{I} = (e_1 + je_2)\,(i_1 + ji_2)$$

$$= (e_1 i_1 - e_2 i_2) + j\,(e_1 i_2 + e_2 i_1)$$

$(e_1 i_1 - e_2 i_2)$ represents real power

$j\,(e_1 i_2 + e_2 i_1)$ represents reactive power

$$\dot{E}I = (e_1 - je_2)\,(i_1 + ji_2)$$

$$= (e_1 i_1 + e_2 i_2) + j\,(e_1 i_2 - e_2 i_1)$$

This result we shall define as equivalent to

$$P + jQ$$

that is,

$$\bar{E}\dot{I} = P + jQ$$

and similarly

$$\dot{E}\bar{I} = P - jQ$$

$$\dot{E}\bar{E} = (e_1 + je_2)(e_1 - je_2)$$

$$= e_1^2 + e_2^2$$

$$= E^2$$

Similarly

$$\dot{A}\bar{A} = A^2$$

$$\dot{B}\bar{B} = B^2$$

etc.

These relations will be used much in the following pages.

Power Circles for Receiver End --

In order to draw circles which will represent conditions at receiver end of circuit, the problem is essentially one of developing an equation involving P_L and Q_L (real and reactive power), and of showing that it is the equation of a circle. Starting with a fundamental equation and its conjugate:

$$\dot{E}_s = \dot{A}\dot{E}_L + \dot{B}\dot{I}_L$$

$$\bar{E}_s = \overline{AE}_L + \overline{BI}_L$$

the product of the two is obtained

$$\dot{E}_s\bar{E}_s = (\dot{A}\dot{E}_L + \dot{B}\dot{I}_L)(\overline{AE}_L + \overline{BI}_L)$$

$$E_s^2 = \dot{A}\bar{A}\dot{E}_L\bar{E}_L + \dot{B}\bar{B}\dot{I}_L\bar{I}_L + \dot{A}\dot{E}_L\overline{BI}_L + \overline{AE}_L\dot{B}\dot{I}_L$$

$$= A^2E_L^2 + B^2I_L^2 + \dot{A}\dot{E}_L\overline{BI}_L + \overline{AE}\dot{B}\dot{I}_L$$

multiply thru by $\dot{E}_L\bar{E}_L$

$$E_s^2E_L^2 = A^2E_L^4 + B^2E_L^2I_L^2 + \dot{A}\bar{B}E_L^2(\dot{E}_L\bar{I}_L) + \bar{A}\dot{B}E_L^2(\bar{E}_L\dot{I}_L)$$

$$= A^2E_L^4 + B^2(P_L^2 + Q_L^2) + \dot{A}\bar{B}E_L^2(P_L - jQ_L) + \bar{A}\dot{B}E_L^2(P_L + jQ_L)$$

Resolving above equation into a form with P_L and Q_L as co-ordinates

$$E_s^2E_L^2 - A^2E_L^4 = B^2P_L^2 + (\dot{A}\bar{B} + \bar{A}\dot{B})P_LE_L^2 + B^2Q_L^2 - j(\dot{A}\bar{B} - \bar{A}\dot{B})Q_LE_L^2$$

Divide by B^2

$$\frac{E_s^2E_L^2}{B^2} - \frac{A^2E_L^4}{B^2} = P_L^2 + \frac{(\dot{A}\bar{B} + \bar{A}\dot{B})}{B^2}P_LE_L^2 + Q_L^2 - J\frac{(\dot{A}\bar{B} - \bar{A}\dot{B})}{B^2}Q_LE_L^2$$

Completing the squares,

$$\frac{E_s^2E_L^2}{B^2} - \frac{A^2E_L^4}{B^2} + \left(\frac{\dot{A}\bar{B} + \bar{A}\dot{B}}{2B^2} E_L^2\right)^2 + \left(j\frac{\dot{A}\bar{B} - \bar{A}\dot{B}}{2B^2} E_L^2\right)^2 =$$

$$P_L^2 + \frac{\dot{A}\bar{B} + \bar{A}\dot{B}}{B^2} P_LE_L^2 + \left(\frac{\dot{A}\bar{B} + \bar{A}\dot{B}}{2B^2} E_L^2\right)^2 + Q_L^2 - j\frac{\dot{A}\bar{B} - \bar{A}\dot{B}}{B^2} E_L^2Q_L$$

$$+ \left(j\frac{\dot{A}\bar{B} - \bar{A}\dot{B}}{2B^2} E_L^2\right)^2$$

Simplifying

$$\frac{E_s^2E_L^2}{B^2} = \left(P_L + \frac{\dot{A}\bar{B} + \bar{A}\dot{B}}{2B^2} E_L^2\right)^2 + \left(Q_L - j\frac{\dot{A}\bar{B} - \bar{A}\dot{B}}{2B^2} E_L^2\right)^2$$

Let

$$l = \frac{\dot{A}\bar{B} + \bar{A}\dot{B}}{2B^2} = \frac{a_1b_1 + a_2b_2}{B^2}$$

and

$$m = j\frac{\dot{A}\bar{B} - \bar{A}\dot{B}}{2B^2} = \frac{a_1b_2 - a_2b_1}{B^2}$$

$$n = \frac{1}{B}$$

Then

$$(P_L + lE_L^2)^2 + (Q_L - mE_L^2)^2 = n^2E_s^2E_L^2$$

Power Circles for Sending End --

By a similar reasoning, beginning with

$$\dot{E}_L = \dot{D}\dot{E}_s - \dot{B}\dot{I}_s$$

$$\bar{E}_L = \overline{DE}_s - \overline{BI}_s$$

$$(P_s - l^1E_s^2)^2 + (Q_s + m^1E_s^2)^2 = n^2E_s^2E_L^2$$

where

$$l^1 = \frac{\dot{B}\bar{D} + \bar{B}\dot{D}}{2B^2} = \frac{b_1d_1 + b_2d_2}{B^2}$$

and

$$m^1 = j\frac{BD - BD}{2B^2} = \frac{b_2d_1 - b_1d_2}{B^2}$$

Thus the equation for the receiver, or load end is

$$(P_L + lE_L^2)^2 + (Q_L - mE_L^2)^2 = n^2E_s^2E_L^2$$

and for the sending end is

$$(P_s - l^1E_s^2)^2 + (Q_s + m^1E_s^2)^2 = n^2E_s^2E_L^2$$

These are evidently equations of circles. (See Figure 15)

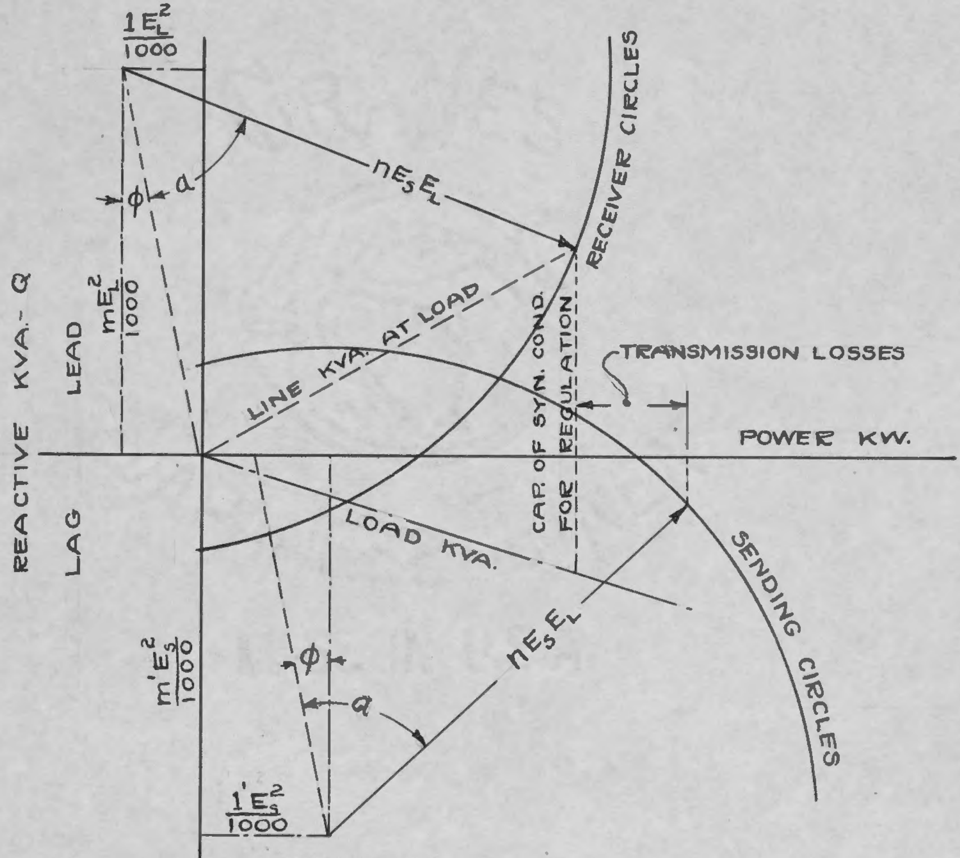


Figure 15.

Figure 15 shows that the center of the receiver circle depends upon receiver voltage, and not sending voltage, and the center of sending circle depends on sending voltage. The radius of each circle however depends both on receiver and sending voltages.

Should conditions be assumed whereby the receiver, or load voltage is to be maintained as constant, the center of all receiver circles will be the same point. With a change in sending voltage, however, the center of sending circles, in addition to the radius, will change.

Reference lines may be drawn through the centers of the receiver and sending circles, which lines may be defined as the position of the radius vector when the voltages at the load and at sending end are in phase. The direction of the lines is determined by the angle ϕ which they make with the

Q, or reactive power, axis.

$$\phi = \tan^{-1} \frac{b_1}{b_2}$$

The efficiency of a transmission line may be determined from the diagram with the aid of the reference lines. A point on the power axis, representing power being consumed at load, is chosen, and the point projected along a vertical line until it intersects the receiver power circle. A radius vector is drawn to the intersection from the center of the receiver circle, and the angle a, between the radius vector and the reference line, is measured. The angle a is then transferred to the sending circle, a radius vector being drawn through its center at an angle a with the reference line. From the intersection of this sending radius vector and the sending circle a line is projected to the power axis. The point intersected on power axis represents sending power.

$$\text{Line efficiency} = \frac{\text{Receiver power}}{\text{Sending power}} \times 100$$

As the power is increased, with constant sending and load voltages, a point is reached where, for a very small increase of power, the current increases greatly, or probably excessively, causing the line to pull out. This pull-out point of the power is called the Static Stability Limit, and is represented in Figure 15 by the point on power axis which is intersected by a vertical line tangent to receiver circle.

In general practice the power at which a line is normally operated is considerably less than the Static Stability Limit. Any limit set up for normal loading conditions is called a Transient Stability Limit. Ordinarily speaking for this limit, the angle a should not exceed thirty degrees.

ILLUSTRATIVE EXAMPLE

In order that the principles given may be more thoroughly understood a representative problem will be worked out in detail.

Assume a line with the following data given:

length = 500 miles

line voltage = 250,000 volts

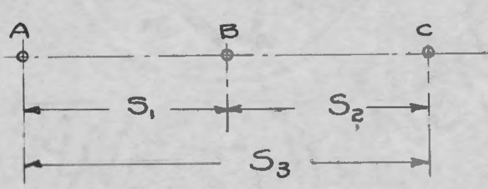
maximum load at receiver end = 150,000 kilowatts

auto-transformers at each end of line = 110,000 to 250,000

x = 5 per cent

R = 0.35 per cent

conductors located in horizontal line, thus:



frequency = 60 cycles

exciting current neglected, for transformers.

The solution will be made in the following order:

(1) Computation of conductor spacing

(2) Computation of conductor diameter

(3) Resistance, reactance and capacity computation

(4) Determination of line constants

(5) Determination of overall constants

(6) Circle diagrams

(1) Conductor Spacing. --

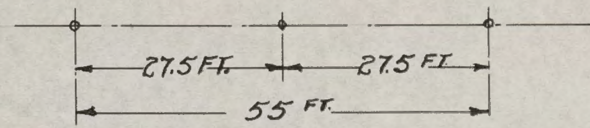
From Equation (18)

$$S = 10 + 1.28 E$$

$$S = 10 + 1.28 \times 250$$

$$S = 330 \text{ inches}$$

$$S = 27 \text{ feet and } 6 \text{ inches}$$



(2) The diameter of the conductors will be determined by the corona effect.

$$e = \frac{250,000}{\sqrt{3}} = 144,500 \text{ volts effective applied}$$

voltage to neutral

$m_o = .85$ for hard drawn copper stranded cable

$g_o = 53.6$ kilovolts to the inch

$a = 1$ at ordinary atmosphere conditions

$S = 330$ inches

It is desirable that there shall be no corona in fair weather, and a very little in stormy weather. Since the disruptive critical voltage in stormy weather is about 0.8 that in fair weather, a critical voltage somewhere between 144,500 volts and 180,000 volts should be assumed, depending upon the prevalent weather conditions. If the air is generally dry, and free from any particles in suspension a low value may be assumed. In this case 160,000 volts is assumed as the disruptive critical volts.

By Equation (55)

$$e_o = 2.303\, m_o g_o\, a r \log \frac{S}{r}$$

$$160 = 2.303 \times 0.85 \times 53.6 \times 1 \times r \log \frac{330}{r}$$

$$r = 0.550$$

The commercial cable that has a radius nearest this is the 950,000 cm., the diameter of which is 1.123 inches. This cable gives a disruptive critical voltage of 163 kilovolts. This value is reduced to 157 kilovolts due to the fact that the conductors are in a horizontal row. However 157 kilovolts is well above the applied voltage.

(3) Resistance ---

Assuming an average temperature of seventy-five degrees Farenheit the resistance as given in tables for hard drawn copper stranded cable is 0.0176 ohms per 1000 feet, or 5.28 x 0.0176, or .0930 ohms per mile.

$$R^1 = KR$$

$$K = \frac{1 + \sqrt{1 + 0.0105\ d^4 f^2}}{2}$$

$$R^1 = 1.59 \times .0930$$

$R^1 = 0.148$ ohms effective resistance per mile.

Reactance--

From Equation (15)

$$S = 415$$

From Equation (14)

$$X_L = 2\pi \times 60\ (80 + 741.1 \log_{10} \frac{415}{.561})\ 10^{-6}$$

$X_L = .832$ ohms per mile.

Capacity Susceptance --

From Equation (16)

$$B = 2\pi \times 60\ \left(\frac{0.03883}{\log_{10} \frac{415}{.561}} \right) 10^{-6}$$

$B = 5.11 \times 10^{-6}$ per mile

Therefore the impedance

$Z = 0.148 + j\ 0.832$ per mile

and the shunt admittance

$$Y = j \times 5.11 \times 10^{-6}$$

(4) Line Constants. --

$$a^2 = ZY$$

$$Z = 0.148 + j\ 0.832$$

$$= .845\ \underline{\lfloor 79^{o} - 55'}$$

$$Y = 5.11 \times 10^{-6}\ \underline{\lfloor 90^{o}}$$

$$ZY = 4.31 \times 10^{-6}\ \underline{\lfloor 169^{o} - 55'}$$

$a = \sqrt{ZY} = 2.08 \times 10^{-3} \underline{|84^\circ - 57'}$

$L = 500$

$aL = 1.040 \underline{|84^\circ - 57'}$

$= .0915 + j\,1.036$

Also

$\sqrt{\frac{Z}{Y}} = Z_0 = 406.6 \overline{|5^\circ\ 2.5'}$

$\sqrt{\frac{Y}{Z}} = \frac{1}{Z_0} = 2.46 \times 10^{-3} \underline{|5^\circ\ 2.5'}$

Let

A_2, B_2, C_2, and D_2 = the line constants

then

$A_2 = D_2 = \cosh aL$

$B_2 = Z_0 \sinh aL$

$C_2 = \frac{\sinh aL}{Z_0}$

But

$\cosh aL = \cosh u \cos v + j \sinh u \sin v$

and

$\sinh aL = \sinh u \cos v + j \cosh u \sin v$

$u = .0915$

$v = 1.036$

Finding the values in tables of trigonometric and hyperbolic functions, substituting and simplifying:

$\cosh aL = .518 \underline{|8^\circ\ 45'}$

$\sinh aL = .866 \underline{|86^\circ\ 55'}$

therefore

$A_2 = D_2 = .518 \underline{|8^\circ\ 45'}$

$B_2 = 352 \underline{|81^\circ\ 52.5'}$

$C_2 = 2.13 \times 10^{-3} \underline{|91^\circ\ 5715'}$

In order that the power loss in the line may be cared for, assume the transformers to have a capacity of 175,000 kilowatts. With a power factor of 80 per cent this means that the current flowing in the line is 506 amperes, and since the transformers are connected in Y, the transormer current is equal the line current, which is 506 amperes. Then

$$X_T = \frac{\frac{5}{100} \times \frac{250,000}{\sqrt{3}}}{506} = 14.3 \text{ ohms}$$

also

$$R_T = \frac{\frac{0.35}{100} \times \frac{250,000}{\sqrt{3}}}{506} = 1.00 \text{ ohms}$$

As only one phase, or one line to neutral, is considered only one transformer at each end will be considered. Where

$$Z_{RT} = Z_{ST} = 1 + j\,14.3 = 14.34 \underline{\left|\, 86^\circ\ 0.5' \right.}$$

$$Y_{RT} = Y_{ST} = 0$$

The equivalent network is similar to item (v) in Table 2.

$$A_1 = A_3 = D_1 = D_3 = 1$$

$$B_1 = B_3 = Z_T = 14.34 \underline{\left|\, 86^\circ\ 0.5' \right.}$$

$$C_1 = C_3 = 0$$

Then from Equations (47) to (50)

$$A_o = D_o = .487 \underline{\left|\, 9^\circ\ 27' \right.}$$

$$B_o = 365.58 \underline{\left|\, 82^\circ\ 24.5' \right.}$$

$$C_o = 2.13 \times 10^{-3} \underline{\left|\, 91^\circ\ 57.5' \right.}$$

(5) A_o, B_o, and C_o being the overall constants.

Substituting these values in the general Equations (43) and (44)

$$E_s = .487 \underline{\left|\, 9^\circ\ 27' \right.} \times E_L + 365.58 \underline{\left|\, 82^\circ\ 24.5' \right.} \times I_L \quad (a)$$

$$I_s = 2.13 \times 10^{-3} \underline{\left|\, 91^\circ\ 57.5' \right.} \times E_L + .487 \underline{\left|\, 9^\circ 27' \right.} \times I_L \quad (b)$$

By way of illustration, suppose the voltage to neutral at the load end is 144,000 volts, and the phase load is 50,000 kilowatts at 80 per cent power factor. Then with E_L as a reference vector

$$I_L = 435 \overline{\left|\, 36^\circ\ 50' \right.}$$

Substituting in (a) and (b)

$$E_s = 219{,}640 \angle 34^\circ\ 41'$$

$$I_s = 321.0 \angle 56^\circ\ 28'$$

These conditions are represented in Figure 16.

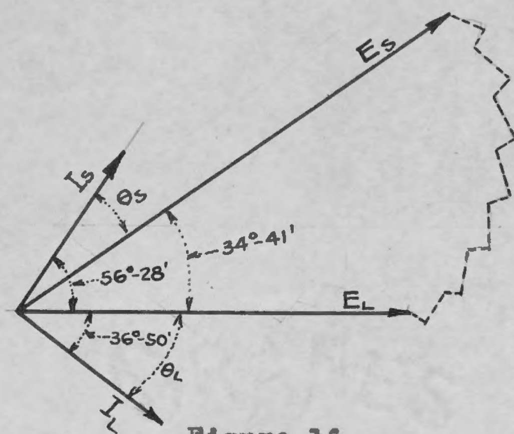


Figure 16.

Then the power input is

$$E_s I_s \cos \theta_s = 65{,}200 \text{ kilowatts}$$

and the power output is

$$E_L I_L \cos \theta_L = 50{,}000 \text{ kilowatts}$$

Thus the line efficiency is

$$\frac{50{,}000}{65{,}200} \times 100 = 76 \text{ per cent}$$

(6) The Circle Diagram --

The use of the circle diagram aids in determining the conditions which will give the most economical operation. Consider the equations for the power circles at the load and supply ends,

$$(P_L + lE_L^2)^2 + (Q_L - mE_L^2)^2 = \frac{E_s^2 E_L^2}{B^2}$$

$$(P_s - l^1 E_s^2)^2 + (Q_L + m^1 E_s^2)^2 = \frac{E_s^2 E_L^2}{B^2}$$

From the line constants given in the derivation of the above equations

$$l = l' = .00038$$

$$m = m' = .00127$$

$$B^2 = (365.58)^2$$

To find the most economical operating conditions with the load voltage given, assume different sending conditions, assume a constant load voltage of 144 kilovolts. Then assume

(1) E_s = 160 kilovolts

(2) E_s = 144 kilovolts

(3) E_s = 130 kilovolts

Then (1)

$$(P_L + 7.88 \times 10^6)^2 + (Q_L - 2.433 \times 10^7)^2 = (6.3 \times 10^7)^2$$

$$(P_s - 9.73 \times 10^6)^2 + (Q_s + 3.251 \times 10^7)^2 = (6.3 \times 10^7)^2$$

and (2)

$$(P_L + 7.88 \times 10^6)^2 + (Q_L - 2.433 \times 10^7)^2 = (5.67 \times 10^7)^2$$

$$(P_s - 7.88 \times 10^6)^2 + (Q_s + 2.433 \times 10^7)^2 = (5.67 \times 10^7)^2$$

lastly (3)

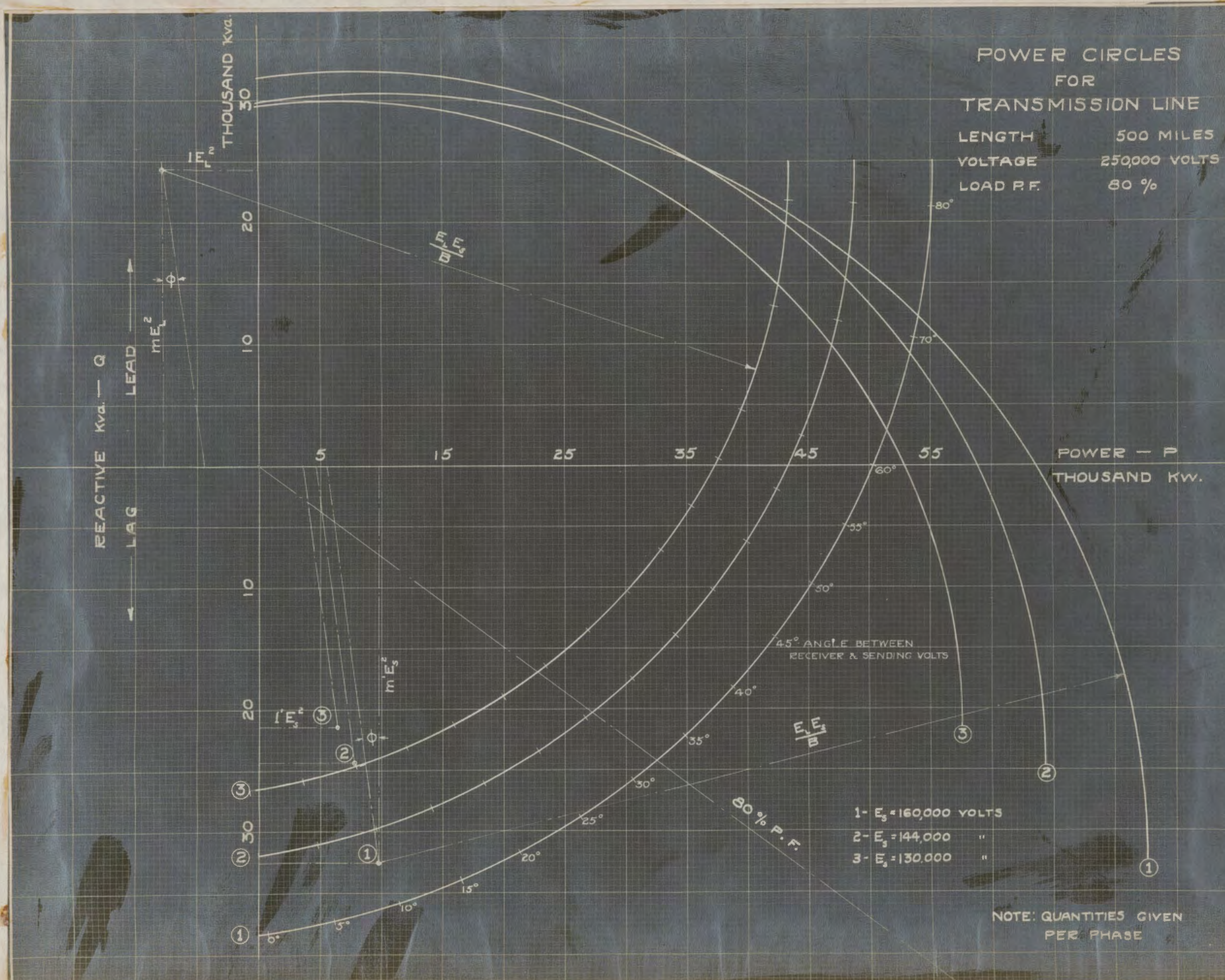
$$(P_L + 7.88 \times 10^6)^2 + (Q_L - 2.433 \times 10^7)^2 = (5.12 \times 10^7)^2$$

$$(P_s - 6.42 \times 10^6)^2 + (Q_s + 2.146 \times 10^7)^2 = (5.12 \times 10^7)^2$$

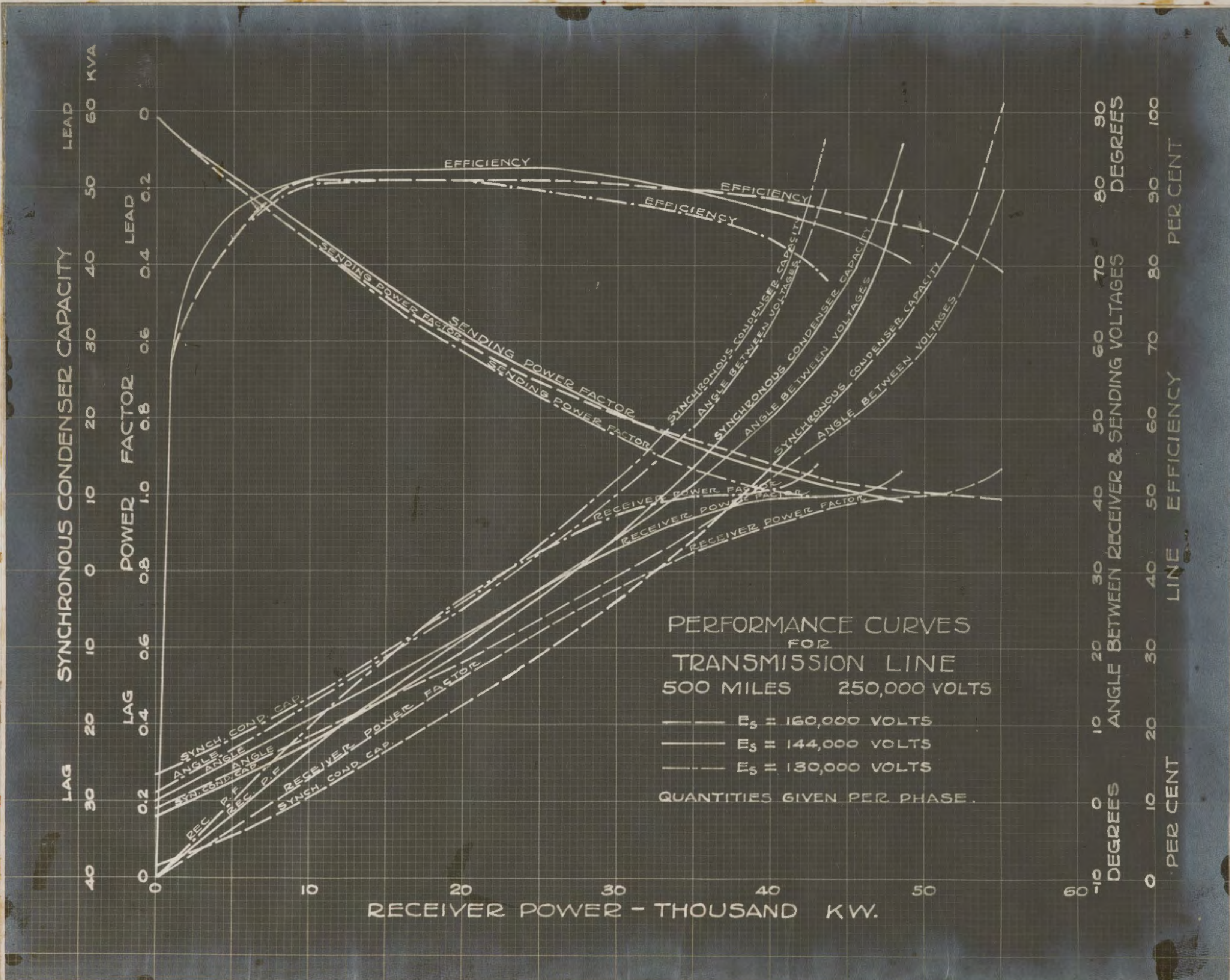
By drawing the circles represented by the above formulae, the performance for various load conditions may be studied. For instance, with the power factor of the load at .80, and a load of 30,000 kilowatts, with sending voltage equal to receiver voltage, it is found that the sending voltage leads the receiver voltage by 34.5°, a synchronous condenser of 4,500 Kva. capacity is required, the overall receiver power factor is 86 per cent, the sending power factor is .79 and the line efficiency is 91.5 per cent.

Curves may be drawn as illustrated, showing the performance of the line at all load conditions.

In the case of this line, assuming the sending voltage equal to the receiver voltage, the static stability limit is 48,800 kilowatts, and the transient stability limit is 26,600 kilowatts. In other words this line can not be safely used for the load conditions assumed in the problem if the sending voltage is only 144,000 volts per phase.



POWER CIRCLES
FOR
TRANSMISSION LINE
LENGTH 500 MILES
VOLTAGE 250,000 VOLTS
LOAD P.F. 80 %
POWER — P
THOUSAND KW.
REACTIVE Kva. — Q
LEAD
LAG
THOUSAND Kva.
45° ANGLE BETWEEN
RECEIVER & SENDING VOLTS
80 % P. F.
1- E_s = 160,000 VOLTS
2- E_s = 144,000 "
3- E_s = 130,000 "
NOTE: QUANTITIES GIVEN
PER PHASE


PERFORMANCE CURVES
FOR
TRANSMISSION LINE
500 MILES 250,000 VOLTS
E_S = 160,000 VOLTS
E_S = 144,000 VOLTS
E_S = 130,000 VOLTS
QUANTITIES GIVEN PER PHASE.
RECEIVER POWER - THOUSAND KW.
SYNCHRONOUS CONDENSER CAPACITY
LAG
LEAD
KVA
POWER FACTOR
ANGLE BETWEEN RECEIVER & SENDING VOLTAGES
DEGREES
LINE EFFICIENCY
PER CENT
EFFICIENCY
SENDING POWER FACTOR
RECEIVER POWER FACTOR
SYNCHRONOUS CONDENSER CAPACITY
ANGLE BETWEEN VOLTAGES
SYNCH. COND. CAP.
ANGLE
REC. P.F.

APPENDIX

Hyperbolic Functions --

In order to get an idea of the hyperbolic functions let us study the common trigonometric functions in a different light. Assume a circle of unit radius as in figure 1.

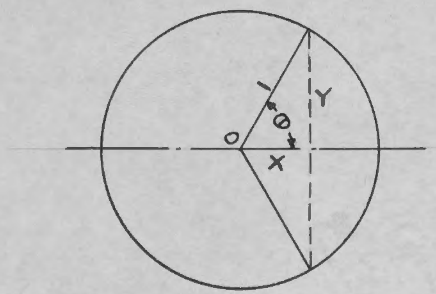


Figure 1.

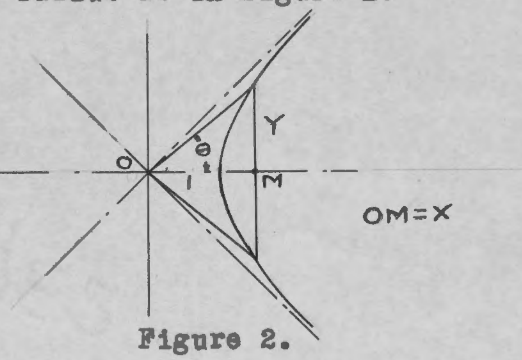


Figure 2.

Let A be the area of a sector included between two radii, whose included angle is 2θ, then

$$A = \frac{1}{2}(r^2)\, 2\theta) = \theta$$

But the half chord y and its distance x from the center are

$$x = r \cos \theta = \cos \theta$$

$$y = r \sin \theta = \sin \theta$$

Substituting A for its equal θ

$$x = \cos A$$

$$y = \sin A$$

Then x and y are trigonometric, also called circular functions of the area A.

Now consider the corresponding area A, and lines x and y for a rectangular hyperbola, as Figure 2.

$$x^2 - y^2 = 1 \text{ , or}$$

$$y^2 = x^2 - 1$$

By analogy x and y are called hyperbolic functions of the area A.

$$x = \cosh A$$

$$y = \sinh A$$

Many higher mathematical functions are defined and studied by means of infinite series, so chosen as to facilitate the solution of some problem. It has been proved that

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \text{ ------------etc. to infinity}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \text{ ------------etc. to infinity}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} \text{ -----------etc. to infinity}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} \text{ -----------etc. to infinity}$$

$$E^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \text{ -----------etc. to infinity}$$

$$E^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} \text{-----------etc. to infinity}$$

$$E^{jx} = 1 + jx - \frac{x^2}{2!} - \frac{jx^3}{3!} + \frac{x^4}{4!} + \frac{jx^5}{5!} \text{---etc. to infinity}$$

$$E^{-jx} = 1 - jx - \frac{x^2}{2!} + \frac{jx^3}{3!} + \frac{x^4}{4!} - \frac{jx^5}{5!} \text{--etc to infinity}$$

Then combining the above series,

$$\frac{E^x + E^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \text{etc.} = \cosh x$$

$$\frac{E^x - E^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \text{etc.} = \sinh x$$

$$\frac{e^{jx} + e^{-jx}}{2} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \text{etc.} = \cos x$$

$$\frac{e^{jx} - e^{-jx}}{2} = jx - \frac{jx^3}{3!} + \frac{jx^5}{5!} + \text{etc.} = j \sin x$$

These relations may then be used in the solution of complex quantities and functions. For instance

$$\cosh (u + jv) = \frac{E^{u + jv} + E^{-u - jv}}{2}$$

If $E^{-u + jv}$ and $E^{u - jv}$ are added and subtracted from the right hand side of the equation, the value of the equation will not be changed, but will take the form:

$$\cosh (u + jv) = \tfrac{1}{4}(E^{u+jv} + E^{-u+jv} + E^{u-jv} + E^{-u-jv}) +$$
$$\tfrac{1}{4}(E^{u+jv} - E^{-u+jv} - E^{u-jv} - E^{-u-jv})$$

factoring

$$\cosh (u + jv) = \left(\frac{E^u + E^{-u}}{2} \times \frac{E^{jv} + E^{-jv}}{2}\right) +$$
$$\left(\frac{E^u - E^{-u}}{2} \times \frac{E^{jv} - E^{-jv}}{2}\right)$$

$$\cosh (u + jv) = \cosh u \times \cos v + j \sinh u \times \sin v$$

In a like manner

$$\sinh (u + jv) = \sinh u \times \cos v + j \cosh u \times \sin v$$

Polar Expressions for Complex Quantities --

In this treatise, as in most techinal papers dealing with electrical phenomena, the roots of minus one are used as operators indicating time and space-phase displacements, or a rotation of similar vectors.

The operator

$j = \sqrt{-1}$ indicates a rotation of 90^0

also

$j^2 = -1$ indicates a rotation of $\pm 180^0$

$j^3 = -\sqrt{-1}$ indicates a rotation of $+ 270^0$ or -90^0

$j^4 = +1$ indicates a rotation of $+ 360^0$ or -0^0

or it is noted that the product of two operators indicates a rotation equal to the sum of the separate rotations and the quotient indicates the difference of these rotations. When dealing with products, quotients, powers and roots of complex quantities it is convienient to keep the expressions in polar form, indicating the scalar length and the angle of rotation. Let

$$\begin{aligned} \dot{A} &= a_1 + ja_2 \\ &= A(\cos a + j \sin x) \\ &= A(\cos \tan^{-1} a_2/a_1 + j \sin \tan^{-1} a_2/a_1) \\ &= A \underline{\left| \tan^{-1} a_2/a_1 \right.} \\ &= A \underline{\left| a \right.} \end{aligned}$$

where A is the scalar value and a or $\tan^{-1} a_2/a_1$ is the angle of rotation. Likewise

$$\begin{aligned} \dot{B} &= b_1 + jb_2 \\ &= B(\cos B + j \sin B) \\ &= B(\cos \tan^{-1} b_2/b_1 + j \sin \tan^{-1} b_2/b_1) \\ &= B \underline{\left| \tan^{-1} b_2/b_1 \right.} \\ &= B \underline{\left| B \right.} \end{aligned}$$

The following are some of the operations easily performed with complex quantities in this form:

$$\dot{A}\dot{B} = AB\,\underline{\big|\, a + B}$$

$$\dot{A}/\dot{B} = A/B\,\underline{\big|\, a - B}$$

$$\dot{A}^2 = A^2\,\underline{\big|\, 2a}$$

$$\sqrt{\dot{A}} = \sqrt{A}\,\underline{\big|\, \frac{a}{2}}$$

$$\dot{A}^n = A^n\,\underline{\big|\, na}$$

$$1/\dot{A} = 1/A\,\underline{\big|\, -a}$$

$$\sqrt[n]{\dot{A}/\dot{B}} = \sqrt[n]{A/B}\,\underline{\big|\, (a - B) / n}$$

$$(\dot{A}\dot{B})^n = (AB)^n \quad \underline{(a + B)\, n}$$

Conjugate Functions --

In complex quantities, if the sign of the imaginary term is changed, the resultant expression is called a conjugate function. For example

$$\dot{A} = (a_1 + ja_2)$$

$$\bar{A} = (a_1 - ja_2)$$

$\bar{A}$ being the conjugate, as may be readily seen

$$\dot{A}\bar{A} = a_1^2 - j^2 a_2^2 = a_1^2 + a_2^2$$

$$= A^2$$

A = scalar value

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The foregoing thesis is hereby approved as a creditable study of an engineering subject, carried out and presented in a manner sufficiently satisfactory to warrant its acceptance as a prerequisite to the degree for which it has been submitted. It is to be understood that by this approval the undersigned does not necessarily endorse or approve any statement made, opinions expressed, or conclusions drawn therein, but approves the thesis only for the purpose for which it is submitted.

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