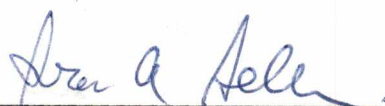


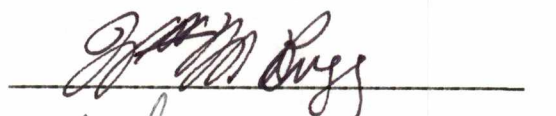
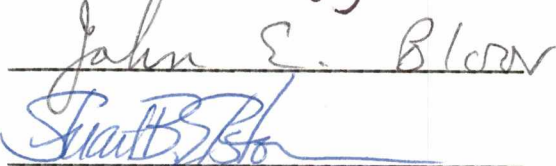
To the Graduate Council:

I am submitting herewith a dissertation written by Scott Donald Berry entitled "Multipole Expansion of the Shape of the Electron Loss to the Continuum Cross-Section at Asymptotically High Velocities." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Physics.



Ivan A. Sellin, Major Professor

We have read this dissertation  
and recommend its acceptance:

Accepted for the Council:



Vice Provost  
and Dean of the Graduate School

MULTIPOLE CHARACTERIZATION OF THE ELECTRON LOSS TO THE CONTINUUM  
CROSS SECTION AT ASYMPTOTICALLY HIGH VELOCITIES

A Dissertation  
Presented for the  
Doctor of Philosophy  
Degree  
The University of Tennessee, Knoxville

Scott Donald Berry

August 1985

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## ABSTRACT

A powerful new technique, using a two-dimensional position sensitive multichannel plate detector coupled with a spherical sector electron analyzer, allows rapid data collection of doubly- and triply-differential electron spectra. This device is used to collect spectra, doubly differential in electron energy and emission angle, for the electron loss to the continuum cross section, where an electron originally bound to the incoming projectile is excited just over the ionization limit into the continuum of the ion, for  $O^{5+}$  projectiles incident on helium and argon gas targets at energies of 41, 82, and 105 MeV at the Holifield Heavy Ion Research Facility at Oak Ridge National Laboratory. By making a multipole expansion of the cross-section in the projectile frame, and appropriately including the effects of the electron analyzer, a general characterization of the electron loss to the continuum cusp (ELC) shape is made. These results are directly compared with the recent theory of J. Burgdörfer, which derives the multipole content for the ELC cross-section, and excellent agreement is found for the case of argon targets at the highest two energies, where the theory is expected to be more reliable. The lack of good agreement with helium target data suggests the need for better treatment of electron-electron interactions within the theoretical framework, which are expected to be proportionately more significant for helium targets. Also, the importance of this approach in obtaining higher multipole information and some possible future investigations are discussed.

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## CHAPTER 1

### INTRODUCTION

The velocity spectrum of free electrons ejected into the forward direction in swift ion-atom collisions contains a cusp-shaped peak when the emergent electron velocity  $\vec{v}_e$  is very close to the outgoing projectile velocity in both direction and speed. For a bare incident projectile, the electrons must arise from transfer of a target electron into a low-lying projectile-centered continuum state (which is inherently a high-momentum transfer process analogous to bound state electron capture); this process is termed electron capture to the continuum (ECC). If there are several electrons available from the incident projectile, it is much more favorable for one of these electrons to be excited via a low-momentum transfer process into the projectile continuum (analogous to excitation to high Rydberg states) and this method of electron production into the forward direction is termed electron loss to the continuum (ELC). The case of one or a few electrons initially attached to the incoming ion leads to a mixture of these two processes, with one-electron ions producing roughly comparable cross sections for ECC and ELC. If instead of atomic or molecular targets, a thin foil target is introduced, a similar cusp-shaped feature is found in the electron spectra. These electrons are commonly known as convoy or beam-foil convoy electrons.

Since the discovery<sup>1,2</sup> of these processes in 1970, collectively known as electron transfer to the continuum (ETC), there has been increased interest in both the experimental and theoretical aspects of this problem. Research progress has been steadily made, and there are several excellent reviews of previous work in this field<sup>3,4,5</sup>.

In this work, we will be concerned chiefly with the ELC process at high velocities, and its implications pertaining to excitation of high Rydberg states in a high velocity ion-atom collision. First, we will briefly review the history of investigations into ETC processes and discuss the major theoretical approaches to describing ELC. Secondly, we will detail the experimental arrangement used in collecting the doubly-differential electron spectra. We will then describe the mathematical expansion used to describe the collected data, and the fitting procedure employed. Lastly, we will present the results of the data analysis, and discuss its implications to the current state of ELC theory and to future experiments.

### Historical Background

Before the first conclusive measurements were made demonstrating the nature of an ETC process in 1970<sup>1</sup>, investigations of the doubly differential cross sections,  $d^2\sigma/dE d\Omega$  for electrons ejected with energy  $E \pm \Delta E/2$  into solid angle  $\Omega$  by protons colliding with various gas targets were initiated by Kuyatt and Jorgensen<sup>6</sup> about a decade earlier, and shortly after by Rudd, et al.<sup>7</sup> A comparison of those results with a first Born calculation of the differential cross sec-

tion for target ionization of Oldham<sup>8</sup> found that the experimental values were greater than the expected theoretical ones by up to several orders of magnitude. The largest discrepancies were for forward emission angles, and for energies where the ejected electron and projectile velocities were nearly equal. Oldham<sup>8</sup> suggested that at velocity matching (in both magnitude and direction) the cross section calculations would show the greatest difference as compared to the experimentally measured values, possibly due to the final state interaction between the ejected electron and the outgoing projectile ion (which would be minimally deflected in such a collision). The Born approximation used, while taking into account some of the influence of the projectile after the collision, neglected higher order terms of this interaction, treating the electron as moving in the field of the ionized target atom.

### **Initial Discovery**

Early in 1970, Macek<sup>9</sup> proposed a theory to account for the observed forward peak based on an expansion of the three-body Fadeev equations describing the wave function for three interacting asymptotically free particles. His approach, while not in good numerical agreement with the measured doubly differential cross sections, qualitatively matched the variation with energy and angle including the forward angle peak. Soon after, Crooks and Rudd<sup>1</sup> performed the first direct measurement of the forward peak, and compared the experimental results with that of Macek<sup>9</sup>, as shown in Figure 1.1, which is

reproduced from Ref. 1. This figure also contains a calculation using the first Born approximation treating the ejected electron state as a plane wave, and its obvious lack of any peak indicates the

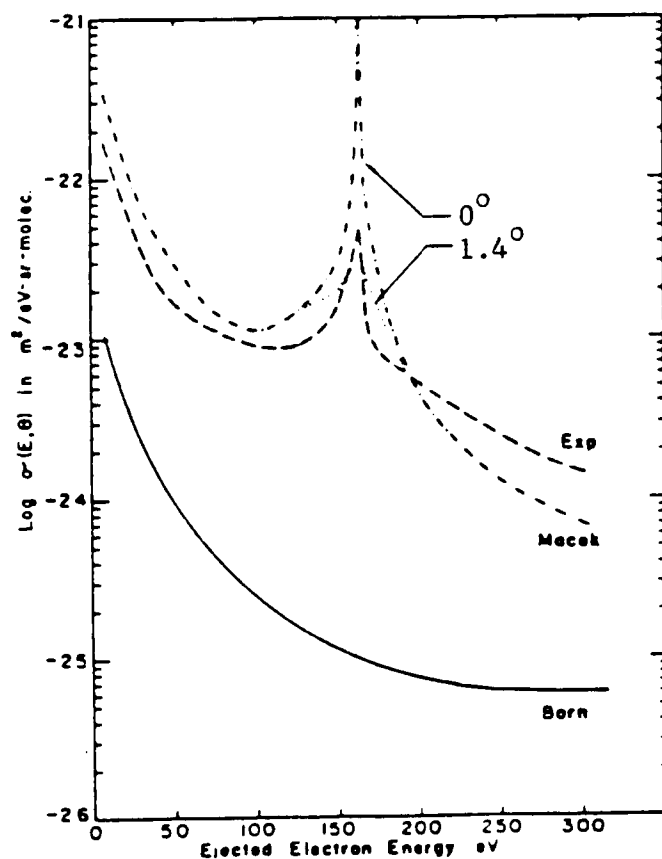


Figure 1.1. Initial direct observance of the forward capture peak by Crooks and Rudd. Reproduced from Ref. 1; abscissa is laboratory frame electron energy, ordinate is the logarithm of the cross section. Theoretical calculations are from Ref. 9 for the energy spectrum of electron transfer to the projectile continuum for 0 degrees and 1.4 degrees. The solid line indicates a first Born target ionization calculation.

necessity of including the electron-projectile ion interaction to a greater degree. Note also the divergence of the theoretical calculation for emission at exactly zero degrees: this arises from the description of the electron wave function when  $\vec{v}_e = \vec{v}_p$  as a projectile centered Coulomb wave. The singularity, when integrated over the range of energies and angles of collection in an electron spectrometer, is removed, leaving a sharp peak.

An important point is made in Reference 9 in a discussion of the connections between target excitation, charge exchange, and ionization. Macek notes that target excitation and ionization producing low energy electrons with respect to the target differ only in the amount of energy received in the collision: the electron final state is most nearly in a target eigenstate and the preferred reference frame of the electron is in terms of the target. However, where the electron and projectiles are almost coincident in velocity, this is no longer valid, and the natural coordinates for these electrons refer to the outgoing ion. Here the electron can be considered to be "captured" by the passing particle, but with just enough energy such that the capture is into the continuum (and therefore ECC), rather than into a bound state of the projectile.

### Early Investigations

Shortly after the pioneering efforts of Crooks, Rudd, and Macek a similar peak was found by Harrison and Lucas<sup>1</sup> for fast protons emerging from thin carbon foil targets. This also was attributed to

an ECC mechanism, but further investigation into the case of a foil target led to the labelling of this process as convoy electron production. Within a few years, investigation of the forward peak produced in collisions with both gas and solid targets was being carried out through several theoretical and experimental efforts. (A more complete account of the early work on the ETC process appears in Refs. 2, 3, and 4).

Two important early theories were developed by Salin<sup>10</sup> and by Dettmann, Harrison, and Lucas<sup>11</sup>. Within the framework of the first Born approximation, Salin introduced a scheme for including the projectile distortion to higher orders than had been previously incorporated into target ionization theories. The final state of the electron may be thought of as a two-center continuum molecular orbital, as pointed out by Ponce and Meckbach<sup>12</sup>, and Salin made use of a distorted-wave approximation which leads to one-center continuum atomic orbitals, for electrons travelling slow with respect to one of the nuclei, but electrons travelling fast with respect to both heavy particles are described by the combined-atom continuum. As Rodbrø and Andersen<sup>13</sup> have discussed, this theory predicts an asymmetric peak in the velocity spectrum of ejected electrons for lower energy particles, tending toward symmetry for higher projectile velocities (reflecting the underlying shape of the Born cross section).

The theory of Dettmann, et al., is the best known of the early theories, because it develops a simple analytical form for the velocity spectrum of ejected electrons collected into a cone centered

about the projectile direction. Here, charge transfer to the continuum is considered to be the dominant process in the production of the forward peak. The development of the T matrix describing the scattering of the electrons follows that for bound state capture, but the final bound-state wave function is replaced by a continuum-state wave function.

This normalization factor for the Coulomb wave,  $e^{i\pi/2}\Gamma(1+i\eta)$  (where  $\eta \equiv Z_p/(a_0 k)$ , and  $k = |\vec{k}| = (m/\hbar)(\vec{v}_e - \vec{v}_p)$ ,  $Z_p$  is the projectile charge, and  $a_0$  the Bohr radius) contains a singularity which give rise to the divergence of the cross section at exact velocity matching. Dettmann et al. made the approximation  $\eta \gg 1$ , or

$$e^{i\pi} |\Gamma(1 + i\eta)|^2 = \frac{2\pi\eta}{1 - e^{-2\pi\eta}} \approx 2\pi\eta \equiv f(k). \quad (1.1)$$

The form of the cross section derived is

$$\frac{d\sigma}{d\vec{v}_e} = f(k) F(v_p), \quad (1.2)$$

with  $F(v_p)$  a smoothly varying function at the cusp peak. This explicitly introduces a  $1/v_e'$  dependence into the cross section, where  $v_e' \equiv |\vec{v}_e - \vec{v}_p|$ , which produces the cusp singularity.

Integrating over a collection cone of half angle  $\theta_0$ , the form of the cross section becomes

$$\frac{d\sigma}{d\vec{v}_e} \approx 4\pi^2 Z_p^2 F_{BK}(v_p) (e^2/\hbar) \times \quad (1.3) \\ \{[(v_e - v_p)^2 + (v\theta_0)^2]^{1/2} - |v_e - v_p|\},$$

where

$$F_{BK} = \frac{2^{16} z_p^2 z_t^5}{10 \pi^2 (e^2/n)^3} \left( \frac{e^2}{n v_e} \right)^{12} \pi a_0^2 .$$

where  $F_{BK}$  is the so-called Brinkman-Kramers cross section (which neglects the internucleus interaction during the scattering). Note that the form of the cusp in this prediction is symmetric about  $v_e = v_p$ , and a simple derivation leads to a full-width-at-half-maximum for the cusp  $\Gamma = 3/2(v_p \theta_0)$ . Since this approach is a first Born approximation, the validity of the calculation is limited to projectile velocities much greater than the initial velocity of the electron to be transferred; often this regime is termed the asymptotically high velocity region.

Experimentally, there were many investigations into the process of ETC during the 1970's. Important differences arose between theoretical predictions and experiment, and among experimental groups over methods of interpreting the collected electron spectra in the forward direction. In ECC, especially for heavy projectiles such as C and O, an asymmetry was noted in the velocity spectrum favoring velocities in the laboratory lower than the projectile velocity<sup>14</sup>, even though the full widths of the cusp seemed to scale roughly as  $\Gamma = 3/2(v_p \theta_0)$ . In 1978, and later in a review article, Shakeshaft and Spruch<sup>15</sup> suggested that the inclusion of the second order terms in a Born series expansion would explain the asymmetry, even for velocities appreciably below those where this term would dominate the total cross section (this term has a leading  $v^{-10}$  dependence of the

cross section as opposed to the first Born  $v^{-1}$  dependence). Chan and Eichler<sup>16</sup> pointed out that further refinements within the first Born approximation would produce a similar asymmetry. Several experimental investigators<sup>17,18,19,20</sup> have favored the former approach, and it is now generally agreed that the inclusion of higher order Born terms in a Born series expansion is important in describing ECC.

For convoy electron production, there have been two differing theoretical approaches. The first is to treat the production of electrons within a solid target as a direct combination of binary ETC processes<sup>21</sup>. The alternative has been a solid state wake-riding model, where a potential wake which can trap electrons just behind the projectile travelling through the solid is formed<sup>22</sup>. Current experimental results disagree with the predictions of both approaches, although the binary ETC models have gained some recent favor<sup>23</sup>.

For the case of electron loss to the continuum, experimental investigations have been somewhat less extensive. Drepper and Briggs<sup>24</sup> published a theory concerning the ELC peak in 1978. The resulting peak shape closely resembles that of the ECC theories of Macek<sup>8</sup>, Salin<sup>10</sup>, and Dettmann et al.<sup>11</sup>. This is because the same Coulomb factor  $\sim 2\pi\eta$  arises in the differential cross section for both ECC and ELC.

Menendez and coworkers<sup>25</sup> performed a charge-state-variation experiment with  $\text{He}^+$  (mixture of ECC and ELC) and  $\text{He}^{++}$  incident on argon, and found no significant difference in the cusp shapes. In

similar experiments, Lucas et al.<sup>26</sup> also found highly similar shapes. In contrast, Meckbach and coworkers<sup>27</sup> at Bariloche, Argentina have found symmetric and narrower cusps for  $\text{He}^+$  projectiles.

For heavy projectiles, investigations<sup>28</sup> have uniformly found cusp shapes to be both approximately symmetric and narrower than the corresponding ECC shapes. Figure 1.2 is a comparison of ECC and ELC reproduced from Suter, et al.<sup>28</sup>. A progression to a narrower and more symmetric cusp is seen as the incident projectile brings into the collision more electrons. In addition, especially for the curves corresponding to incident  $\text{O}^{4+}$  and  $\text{Si}^{10+}$ , structure in the wings of the cusp is present, and is due to projectile autoionization lines of very low energy in the projectile frame<sup>4</sup>. In Figure 1.3 another spectrum, taken with improved resolution in energy and a smaller collection cone clearly indicates these lines. In fact, the frame transformation and the corresponding scale change that is inherent in going from the projectile frame from which these electrons are emitted to the laboratory frame where the spectra are measured, allows measurement of cross sections for very low energy processes<sup>29</sup>.

Systematic investigations into the ELC cusp shape as a function of the number of electrons carried into the collision by the ion have been carried out<sup>30</sup>. It was found that there was minimum width and maximum symmetry when there were 2p electrons available for loss in the collision, and that when these electrons are present the yields of the forward peak rise sharply. This minimum width is found to be

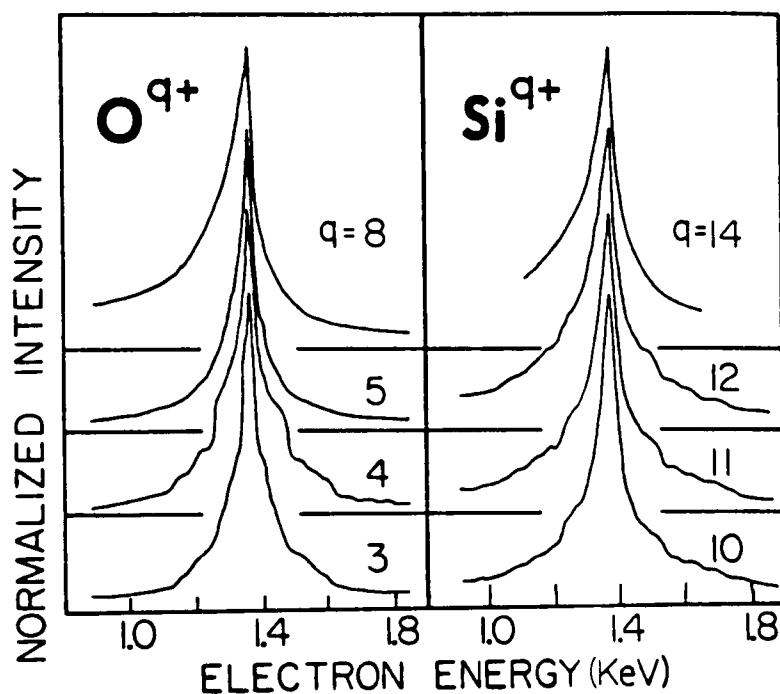


Figure 1.2. Normalized spectra for electron capture and loss to continuum states in the forward direction for 2.5 MeV/u  $O^{q+}$  and  $Si^{q+}$  on Ar. Zero-count base lines are shown for each spectrum. Reproduced from Ref. 28.

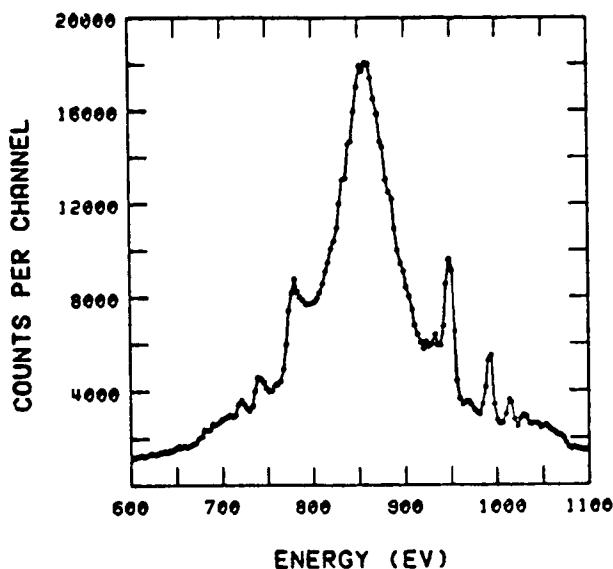


Figure 1.3. Energy spectrum of electrons emitted in the forward direction in collisions of 25 MeV  $O^{4+}$  on Ar.

$\Gamma = 0.26 \pm 0.04$  au (atomic units of velocity) for Si, O, and C projectiles, He, Ne, and Ar targets, over the velocity range 7 - 12 au. Laubert et al.<sup>31</sup> find similar ( $\Gamma = 0.25 \pm 0.02$  au) widths for convoy electron production, suggesting a close relationship between the ELC and convoy production processes.

The narrow widths for heavy particle ELC have even been found for loss from an initial 1s electron state<sup>4</sup>. Using 20 MeV  $O^{7+}$  projectiles and Ar targets, and detecting the exiting charge state of the projectile in coincidence with the cusp electrons, separation of the processes of ECC and ELC from a one-electron ionic projectile was

possible. In Figure 1.4, reproduced from Ref. 4, the coincidence spectrum for exiting bare ions, indicating ELC only, is shown in comparison with all non-coincident electrons, indicating ECC only. The ELC cusp has a width that agrees with the findings of Ref. 30 and the characteristic ELC shape, while the ECC portion is clearly broader and asymmetric as expected.

More recently, theories have been developed by Day<sup>32</sup>, Briggs and Day<sup>33</sup>, and Burgdörfer et al.<sup>34</sup> to explain the discrepancies between the early theoretical and experimental results. These theoretical approaches all contain anisotropic terms in the calculated cross section (the shape of the cusp is no longer dependent only on the magnitude  $|\vec{v}_e - \vec{v}_p|$  but on a vectorial function  $f(\vec{v}_e - \vec{v}_p)$  which permits angular deviation from spherical symmetry), and are discussed in more detail in the next section.

### Theoretical Characterization of Multipole Content

Unlike the case of ECC, which is difficult to treat theoretically by simple approximations due to the high momentum transfer required to move the involved electron into a rapidly moving frame of reference, the low momentum transfer necessary in an ELC process simplifies the theoretical treatment considerably<sup>35,36</sup>. Specifically, use of the first Born term only in an asymptotic expansion of the scattering cross section is acceptable when the incident projectile velocity is much greater than the velocity of the electron to be

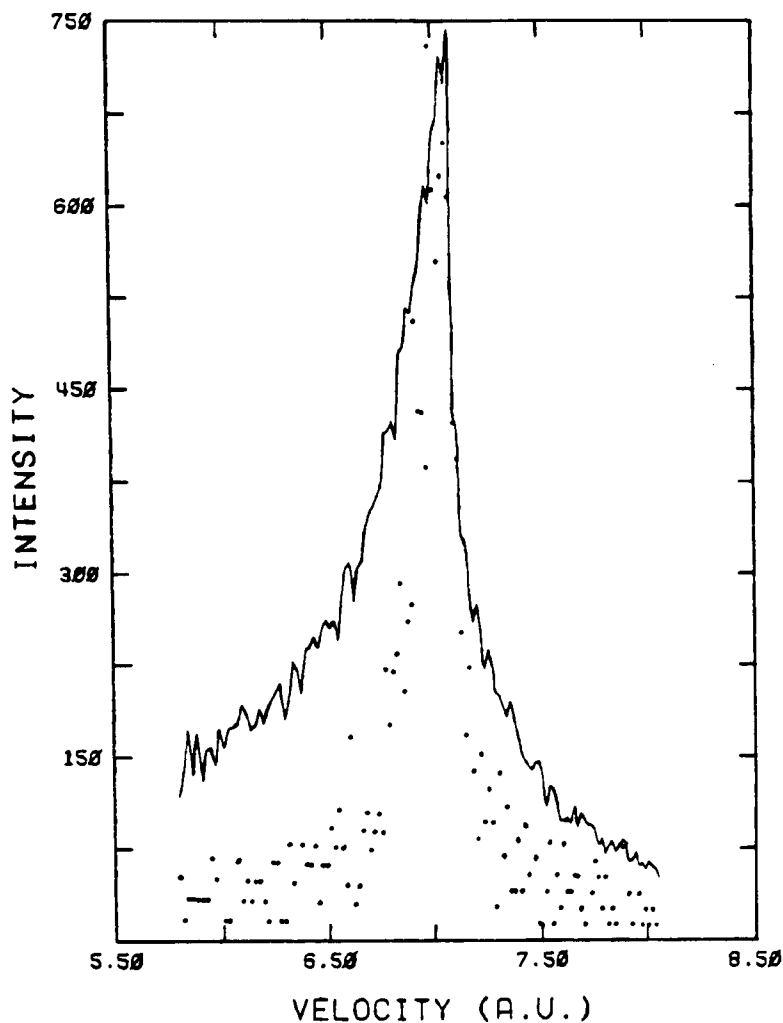


Figure 1.4. Cusp for 20 MeV  $O^{7+}$  projectiles on Ar observed in coincidence with  $O^{8+}$  (dots), overlaid with cusp containing all electrons not detected in coincidence with  $O^{8+}$  (solid line). The former is an ELC spectrum and the latter represents ECC:

lost in the collision, which for ELC processes involving outer electrons is often satisfied. More precisely, if  $v_{eo}$  is the initial orbital velocity of the electron attached to the projectile ion,

$$\frac{v_p}{Z_p} \gg v_{eo} \quad (1.4)$$

is the condition to be satisfied for the validity of the Born approximation. Although this Born validity criteria is only marginally satisfied in the case of several experiments in which faster inner electrons are lost, the first Born approximation has been used in the theories reviewed here.

### **Drepper, Briggs, and Day**

The original theoretical description of the electron loss to the continuum process was formulated by Drepper and Briggs<sup>24</sup> in 1976 and 1978. These papers dealt with the Born approximation applied to the case of projectile ionization as seen in the laboratory frame through a simple frame transformation. In the small velocity limit of the projectile frame electron velocity, they showed that the cross section for ELC is dominated by the square of the Coulomb wave function normalization factor ( $\sim 2\pi\eta$  from Eq. 1.1) as in the case of an ECC process. However, there was no investigation into any angular dependence of the cross section in the neighborhood of the cusp peak, as pointed out by Day<sup>32</sup>. Day used the original formulation of Drepper and Briggs, and by inclusion of an angle-dependent term in investigating the transition form factor

$$T(n, l, m \rightarrow \vec{v}, \vec{Q}) = \langle \vec{v}^- | e^{i\vec{Q} \cdot \vec{r}} | n, l, m \rangle \quad (1.5)$$

in its zero velocity projectile frame limit (here  $n$ ,  $l$ , and  $m$  are the initial electron state quantum numbers,  $\vec{v}$  represents the final unbound state of the electron, and  $\vec{Q}$  the momentum transfer), was able to separate the cusp cross section into two parts:

$$\frac{d\sigma}{dv_e} = \frac{A}{v_e} [ 1 + \beta P_2(\cos \alpha) ] , \quad -1 \leq \beta \leq 2, \quad (1.6)$$

where  $\alpha$  is the projectile frame emission angle, relative to the projectile direction and the limits on  $\beta$  are to insure that the cross section is positive definite. The leading term of this expression is just the result of Drepper and Briggs and has the same form of the cross section as that of Dettmann, Harrison, and Lucas<sup>11</sup> for ECC.

The angle dependent term of Eq. 1.6, when viewed after integration over a typical electron analyzer resolution value and a narrow angular collection cone will either widen or narrow the observed cusp depending on the sign of  $\beta$ . Day also derived a series expansion for the width of the cusp giving:

$$\Gamma = \frac{3}{2} v_p \theta_0 ( 1 + 3/2\beta + \dots ) . \quad (1.7)$$

In a following paper Briggs and Day<sup>33</sup> expanded upon the ideas of Day's previous letter by incorporating the anisotropy into a numerical calculation for He<sup>+</sup> incident on H, using a closure approximation to treat the effect of target electrons. Two different methods were

used in the calculation of the value of  $\beta$ , and approximately the same results for each were obtained. In general, their results predict a positive value for  $\beta$  when the incident projectile velocity is less than about 5 au and a negative value asymptotically approaching -0.5.

### Burgdörfer

The calculation of Briggs and Day for ELC from an initial  $1s$  state determined the  $v_e' \rightarrow 0$  for the full transition form factor listed in Eq. 1.5. To extend this approach to higher initial  $n$  states, the calculation would become increasingly complicated. Additionally, the approximate equality of ECC and ELC cross sections for one-electron projectiles<sup>4</sup> requires performing a coincident experiment which detects the outgoing projectile charge state simultaneously with the cusp electrons to make a valid comparison with their results. To systematically study the theoretical properties of ELC as a function of initial released electron state, its binding energy, velocity of the projectile, and the structure of the target, Burgdörfer et al.<sup>34</sup> approached the problem of ELC using a group-theoretical method.

By assuming the initial state of the electron attached to the projectile can be treated as hydrogenic, previous algebraic approaches<sup>37,38</sup> to calculating the matrix elements for transitions between hydrogenic bound states were extended by exploiting the continuity of electron wave functions over the ionization limit, and replacing the final electron bound states by low lying continuum states. This procedure, extending a continuity argument for total

cross section for electron excitation and charge transfer by Rudd and Macek<sup>36</sup>, allowed the  $v_e' \rightarrow 0$  limit of the bound-free transition form factor to be reduced to relatively simple expressions, even for higher  $n$  states.

The form of the doubly differential cross section within this calculation proved quite simple:

$$\frac{d\sigma}{d\vec{v}_e} = \frac{1}{v_e} \sum_{\ell=0, \text{even}}^{2n} a_{\ell}(n, \ell, |m|) P_{\ell}(\cos \theta_e) . \quad (1.8)$$

The appearance of only even terms in this multipole expansion, and the limit to twice the initial quantum number  $n$  value, is an important result. This selection rule (derived by considering the symmetries of the final electron distribution) applied to an initial  $1s$  state recovers the form previously derived by Briggs and Day<sup>33</sup>, in Equation 1.6, and it explains why their result contained only two terms of the expansion.

The Born approximation used in Ref. 34, is only valid under two conditions. The first is that Equation 1.4 must be satisfied, with  $v_{e0}$  given by  $1/n$  in atomic units. Also, the initial state must be well approximated by a hydrogenic wave function to take advantage of the group algebraic approach. In order to compare directly with experiment, a third condition is required, that the ECC process be either negligible or excluded by a coincident requirement.

One experimental case that did satisfy these conditions was for  $O^{5+}$  projectiles at velocities  $7 \leq v_p \leq 12 \text{ au}^{34}$ . The outer  $2s$

electron (Li-like configuration) is reasonably well approximated by a hydrogenic wave function using an effective charge of 6.3. Furthermore, the large ELC cross sections at these energies (about an order of magnitude above that for the roughly equal ECC and ELC cross sections for the 1s electrons<sup>4</sup>) provided a case in which the ELC contribution would dominate the ECC contribution. The theoretical results for this case contained a large negative value for the coefficient with  $l = 2$  ( $a_2/a_0 \sim -.6$ ). This changes the character of the cross section to exhibit an anisotropy transverse to the projectile direction) Excellent agreement of the widths expected by this theory to the measured widths suggested that the predicted large transverse nature of the ELC cusp cross section is the source of the ELC cusp narrowing. The agreement of the theoretical approach was not as close, deviating from the calculated values by about a factor of three.

To investigate the proposed nature of the ELC cross section in greater detail, it was decided to investigate the ELC cusp shape using a newly developed position sensitive detector, which in combination with a suitable electron spectrometer could measure the doubly differential cross section for electrons directly, for  $O^{5+}$  ions incident on helium and argon gas targets. The measurements presented in this work are in contrast to traditional singly differential measurements involving collection into a cone centered about the projectile direction. This is the first (to my knowledge) direct doubly differential measurement of the ELC cross section for three

electron projectiles. Meckbach, et al.<sup>39</sup> performed a direct measurement for  $H^0$  projectiles and helium targets using a simple (but slower) angular scanning procedure, but analysis of the results are complicated by the use of a one-electron projectile and an extended target region<sup>39</sup>. In order to insure that the asymptotic velocity condition, Eq. 1.4 was satisfied, application was made for experimental beam time at the Holifield Heavy Ion Research Facility Tandem Accelerator, where projectile energies meeting this condition were easily obtainable, and approval for this proposal was made allowing the collection of the spectra desired in April 1984.

## CHAPTER 2

### EXPERIMENTAL ARRANGEMENT

#### Spectrometer and Beamline

The apparatus used in collecting the doubly differential electron spectra was located at the Holifield Heavy Ion Research Facility (HHIRF) 25 MV Tandem Van de Graaff Accelerator at Oak Ridge National Laboratory, where  $O^{5+}$  ion beams at the desired energies of 41, 82, and 105 MeV were available. The ion beam provided by the accelerator was magnetically charge state and energy selected, transported through an ultra-high vacuum (pressure  $\leq 10^{-7}$  Torr) beamline containing focusing, steering, and collimation elements, into a vacuum chamber, where the gas target, electron spectrometer, and position sensitive detector were located; a schematic layout of the experiment is shown in Figure 2.1. The ion beam was essentially undeflected by the collision, and an exit hole was provided in the spectrometer to allow the outgoing beam to leave the experimental chamber via another beamline where it was transported to a distant Faraday cup used for ion collection and normalization.

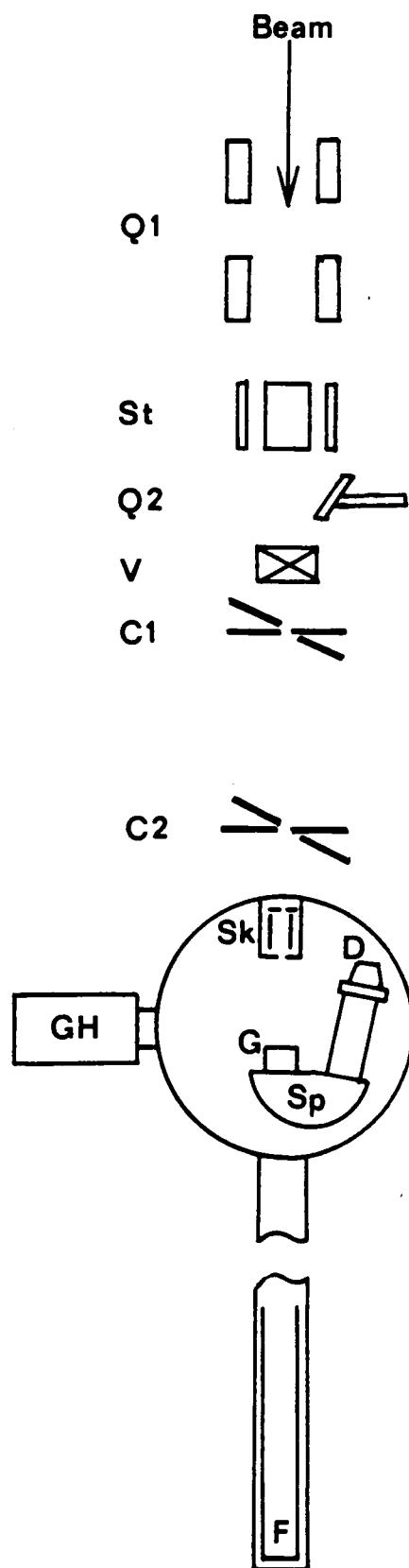
#### Ion Beam Steering and Collimation

Because the ELC cusp is sharply defined in angle (>95% of the cusp intensity is found within  $\sim 3$  degrees of the projectile ion direction) it is necessary to provide a well-collimated, nearly parallel ion beam at the gas target where the ion-atom collision is

Figure 2.1. Schematic diagram of the beamline and experimental apparatus.

From top to bottom (projectile beam direction):

- (Q1) Magnetic quadrupole doublet lens used to focus ion beam.
- (St) Magnetic steering elements (vertical and horizontal).
- (Q2) Quartz plate which can be moved into the path of the ion beam to view focusing conditions.
- (V) Valve which separates accelerator beamline from experimental station.
- (C1) First four-jaw collimator unit.
- (C2) Second four-jaw collimator unit.
- (Sk) Ion beam halo skimmer, field plate and vacuum separation aperture assembly.
- (D) Position sensitive detector assembly.
- (GH) Gas handling and control system to regulate target gas cell.
- (G) Target gas cell.
- (Sp) Electrostatic spherical sector spectrometer.
- (F) Faraday cup to monitor ion beam current.



to occur. Failure to restrict the angular spread of the ion beam leads to a mixing of the angular information desired in measuring the variation of the cross section with angle, so two collimator housings containing four movable carbon jaws each forming an adjustable (via micrometer drives) rectangular aperture (typically 1.4 mm square), were installed with a separation of 91 cm between them. With appropriate adjustment of both aperture sizes, ion beam angular divergence was limited to  $\leq 0.3$  degrees and more typically  $\sim 0.1$  degrees.

In addition, these collimators were used to limit the ion beam spot size at the target gas cell located at the entrance focus of the spectrometer. This was done for two reasons: the ion beam diameter at this point defines an object size for the electron source which will be energy selected by the spectrometer and partially determines the energy resolution (discussed below), and collisions between the ion beam and metal parts of the analyzer would provide a copious supply of electrons in the energy and angular range of the spectrometric apparatus which could drastically change the measured cross section amplitude and shape.

For oxygen nuclei at all three of the energies used here, nuclear reactions induced by collisions with components of the apparatus are within the realm of possibility for many materials used, and for oxygen beams exceeding  $\sim 100$  MeV, the Coulomb barrier is exceeded for almost all possible nuclei<sup>40</sup>, as shown in Figure 2.2. These nuclear reactions can provide particles, such as protons,

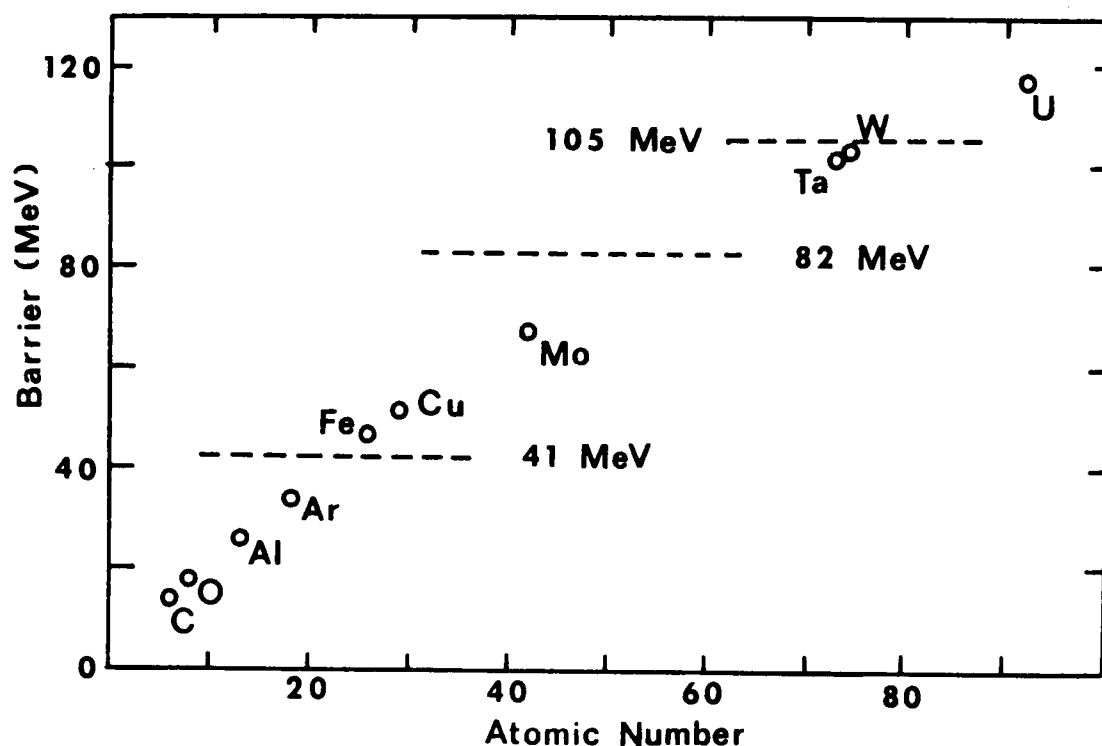


Figure 2.2. Plot of Coulomb barrier height versus atomic number for oxygen projectiles incident on selected nuclei. Values shown are calculated from the formula presented in Ref. 40 for the Coulomb barrier height as a function of nuclei atomic number and weight (the heaviest stable isotope of a given target nucleus was used to form an upper limit on the barrier height).

neutrons, alphas, and electrons, as well as gamma rays from nuclear excitations within the level structure of the target nucleus<sup>41</sup>. All of these products can produce electrons in the energy range collected by the apparatus via collisions with any solid surfaces in the vacuum system. In addition, especially for heavy atoms, high energy x rays from inner shell transitions<sup>42</sup> will provide an additional photon flux originating from collisions with collimator materials. Because of the relatively simple structure of the carbon nucleus, especially relative to heavy nuclei such as tantalum<sup>41</sup>, and the lack of atomic transitions with photon energies  $>1$  keV<sup>43</sup>, carbon was chosen as the material for all collimating materials instead of the usual choice of tantalum<sup>44</sup>.

To remove any particles produced at the collimation units, and to remove beam ions deflected in collision with slit edges, a skimmer assembly (labelled "Sk" in Figure 2.1), consisting of a 2.5 mm carbon aperture followed by a pair of plates to which a high potential difference could be applied, was inserted 28 cm downstream from the second collimator assembly (labelled "C2" in Figure 2.1). The electric field established,  $\sim 2$  kV/cm, was sufficient to deflect any electrons still travelling with the ion beam (while affecting the ions insignificantly) before a separation aperture at the entrance to the vacuum chamber containing the electron analyzer.

In addition to the above efforts to carefully define the projectile beam at the target location, skillful operation of the final steering and focusing elements (the units labelled "Q1" and

"St" in Figure 2.1) was required in order to minimize collisions between the ions and the target gas cell apertures, while maintaining a sufficient particle intensity for the measurement. By electrically isolating the gas cell and measuring any induced current, and by operating the detector during all beam focus and steering changes, two "noise" signals were available to the accelerator operators, who were instructed to maximize transmitted beam current (as measured in the Faraday cup, "F" in Figure 2.1) while minimizing the noise signals. It was found that current induced on the target cell could be reduced to unmeasurable levels, but that there was still a measurable electron noise count rate which could not be removed. However, this rate was small compared to the actual ELC signal collected, and could be appropriately measured, and removed from the electron signal, as discussed later.

The beam current was measured by transporting the outgoing ions into a Faraday cup located about 3.5 meters (as far as possible within the experimental area to reduce the effect of Bremsstrahlung radiation from stopping the ions) from the electron spectrometer. The cup was constructed of a 1 meter long, 10 cm diameter aluminum tube, with a back surface constructed of carbon, where the ions were stopped. This choice of material was made for precisely the same reason as listed above in making the collimators. The large length-to-diameter ratio of the Faraday cup was found sufficient in previous experiments to contain any secondary electrons produced in stopping the ions.

## Vacuum System and Target Gas Cell

The experimental vacuum system which contained the collimation elements, the spectrometer chamber, and the Faraday cup is shown schematically in Figure 2.3. The basic requirements were as follows: the design of the system must be capable of handling large gas loads resulting from the gas target cell, and prevent any significant gas flux (especially helium) into the tandem accelerator vacuum system. The first requirement was necessary to allow the gas target effective length to remain short, and so present no significant variation of focusing of electrons arising from different regions of the target area, and to reduce charge changing collisions within the vacuum system which would no longer preserve a single charge state of the projectile from participating in the collision. This basic requirement was met by the use of a Varian oil diffusion pump and liquid nitrogen trap connected to the spectrometer chamber, with an estimated pumping speed of  $\sim 1000$  liters/second. This pump/trap combination was capable of keeping the vacuum system pressure near the analyzer to  $\leq 3 \times 10^{-7}$  Torr when there was no gas load present from the gas target.

The second condition required that an additional pumping station with a  $\sim 300$  l/s turbomolecular pump be installed between the experimental chamber and the accelerator beamline proper. Two apertures were placed between the experimental chamber and the turbo pump, and between the turbo pump and the accelerator beam line to provide as

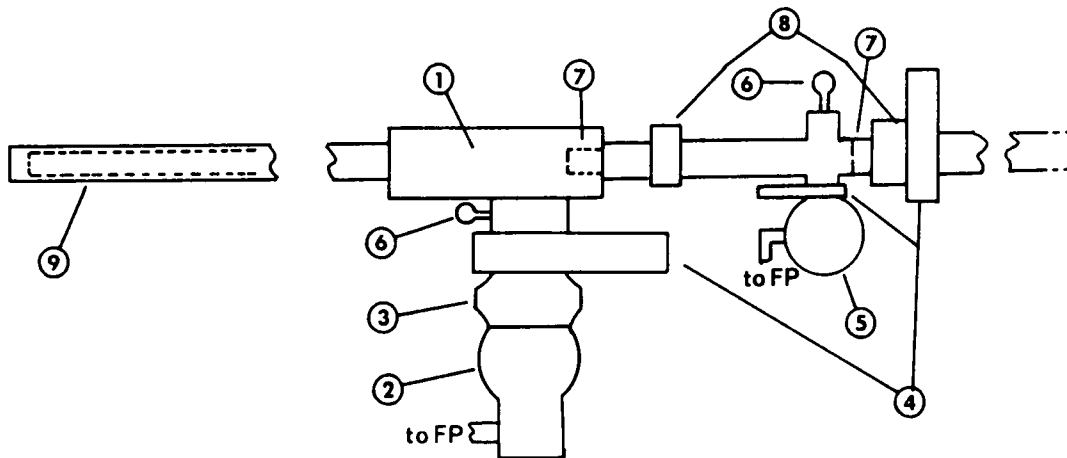


Figure 2.3. Vacuum system schematic layout.

- (1) Vacuum chamber housing target cell and spectrometer.
- (2) Varian model VHS-4 diffusion pump.
- (3) Liquid nitrogen trap (to keep oil from the diffusion pump from entering the experimental system).
- (4) Automatic valves.
- (5) Balzers model TP-270 turbomolecular pump.
- (6) Ionization pressure gauges.
- (7) Vacuum separation apertures (to limit flow of target gases into accelerator vacuum system).
- (8) Collimation housings.
- (9) Beam line extension and Faraday cup.

much restriction against gas flow into the accelerator beamline as possible, consistent with preventing ion beam-aperture collisions. Under typical operating conditions with a 10 mTorr gas pressure in the target cell, spectrometer chamber pressure was  $\sim 1.5 \times 10^{-5}$  Torr, pressure at the turbo pump was  $\sim 6 \times 10^{-7}$  Torr, and the accelerator beamline pressure was  $\leq 3 \times 10^{-8}$  Torr.

The target gas cell, centered about the entrance focus of a spherical sector electron analyzer described below, consisted of a cylindrical region 8 mm long terminated by apertures with 2.5 mm holes, through which the interacting projectiles could pass. The actual effective length of the target region was unknown, but was estimated to be approximately 1.3 mm<sup>45</sup>. This cell was connected to two lines (0.25 inch tubing), one connected to a regulated gas supply, and the other to one side of a differential capacitive manometer (MKS Baratron model 315), which had an accuracy of 0.1 mT (millitorr)<sup>46</sup>. The other side of the manometer led to the vacuum chamber housing the analyzer apparatus, and therefore it measured the pressure difference between the interior of the gas cell region and the system (gas loaded) background pressure. A voltage proportional to the differential pressure was fed into a MKS model 250B pressure controller, which operated a Veeco model PV-10 fast piezoelectric metering valve located in the gas supply line. This system was capable of maintaining the pressure variation within the target cell (at a desired pressure of 5 mT for argon, and 10 mT for helium) to <1 %.

In addition to the capability of keeping the gas pressure within the target cell to a constant value over the background system pressure, it was necessary to account for "background" events occurring during the collection of experimental data. The first type of background events arose from collisions between projectile ions and collimation materials as discussed above, and the second from charge changing collisions with background gas atoms before the ion beam entered the gas cell. While the rate of charge changing was expected to be insignificant in, for example, affecting the Faraday cup beam normalization, adding one or more electrons to a projectile (thereby increasing the ELC cross section) could affect the measured multipole content of the ELC cross section. To account for these affects, electron spectra were collected in a "gas dump" mode where the gas flow was kept at the same rate, but instead allowed directly into the analyzer chamber instead of the target cell, by using a signal from the chamber ion gauge to replace the signal from the capacitive manometer and reproducing the same chamber pressure as when the gas was directed into the target cell. These resulting spectra were later subtracted from the normally collected data, as detailed below in the chapter on data analysis.

### **Spherical Sector Analyzer**

The electron spectrometer used was a spherical sector electrostatic analyzer as originally introduced by Purcell<sup>47</sup>. A simple schematic layout of the analyzer is presented in Figure 2.4. The spectrometer consists of two concentric sectors of spherical copper

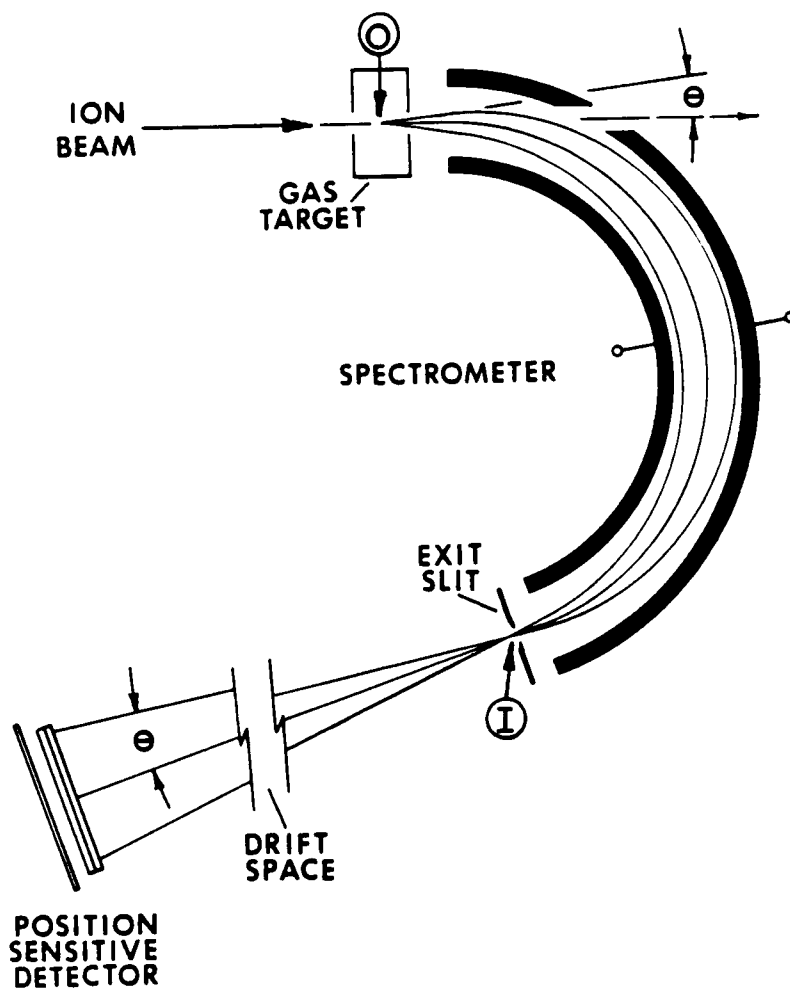


Figure 2.4. Schematic layout of the spherical sector spectrometer. The paths shown include the central ray trajectory and two typical paths at non-zero angles with respect to the ion beam direction, and O and I respectively label the object and image foci respectively.

surfaces, to which a potential difference is applied. The resulting electric field,  $\propto 1/r^2$ , where  $r$  is radial position relative to the spherical surfaces' common center, will deflect and focus electrons entering the ends of the sector close to the central ray as shown in Figure 2.4, and only those electrons having a certain energy range will exit the analyzer at the other sector boundary. If the sector subtends an angle of 180 degrees, the analyzer has several useful properties<sup>48</sup>: it is stigmatic and double-focusing, meaning electron paths emanating from a common object point differing in "launch" angle from the central ray path either parallel or perpendicular to the bending plane will be focused to a common image point (in the 180 degree analyzer case the object and image points lie at the boundaries to the sector field region); in the plane perpendicular to the bending plane the focusing is perfect, meaning that all angles from the central ray will be focused to the image point; and in the bending plane the focus is to first order, where the deviation  $x$  from the image point has a zero first (but not higher order) derivative with respect to the emission angle.

Unlike the original instrument of Purcell, which used a sector angle of 90 degrees, the analyzer used here (identical in size and characteristics to that used by Glass<sup>49</sup>) has a sector angle which subtended 160 degrees. This is very close to the ideal case, preserving to a good degree<sup>50</sup> the properties of the 180 degree spectrometer, while having the added advantage of removing the focal points from the sector boundaries. This, for example, allowed the

placement of the gas target cell as mentioned above, where the central point of the cell coincided with the analyzer object focal point. If a 180 degree analyzer had been used, the placing of the gas cell at the focal point of the analyzer would have seriously perturbed the radial electric field at the sector boundary, introducing severe aberrations in the focal properties of the analyzer. In the 160 degree electron spectrometer there are small effects due to field fringing at the sector boundary, but the analyzer is constructed to minimize them. Magnetic fields also affect the analyzer focal properties, so three mutually orthogonal pairs of coils surrounded the vacuum chamber containing the spectrometric apparatus which nulled the magnetic field in the chamber to  $\leq 100$  milligauss.

Since there could be no defining aperture at the object point of the spectrometer (to avoid production of electron via collision between the incoming ions and the aperture), the size of the projectile beam, along with the size of the exit aperture of the analyzer determined the range of energies  $\Delta E$  allowed through the exit slit. As a general rule for spectrometers of this type<sup>48</sup>, the transmission function  $T(E)$ , which expresses the probability of transmission of an electron of a given energy  $E$  through the spectrometer, is approximated by a trapezoidal function characterized by a top width (TW), and a full width at half maximum (FWHM, normally associated with  $\Delta E$ ), where

$$\Delta E = E [\max(x_0, x_i)/(2\bar{r}) + \alpha^2]. \quad (2.1)$$

Here  $x_o$  and  $x_i$  are the object and image sizes respectively,  $\bar{r} = 54.7$  mm is the radius of the central ray, and  $\alpha$  is the emission angle relative to the central path in the bending plane (and there is only first-order focusing). This angular aberration term can be quite significant for collection over a few degrees of angle, but here the angle is resolved to about 0.35 degree (see next section), and this term can be neglected, giving

$$\Delta E = E [\max(x_o, x_i) / (2\bar{r})], \quad (2.2)$$

(the formula for TW is that for  $\Delta E$  with  $\max(x_o, x_i)$  replaced by  $|x_o - x_i|$ ) and the resolution<sup>51</sup> (normally quoted as a percentage)

$$R \equiv \Delta E / E = \max(x_o, x_i) / (2\bar{r}) = \text{constant}. \quad (2.3)$$

For the ion beam size at the center of the gas cell (the object focus), estimated to be ~1.0 mm, and the exit aperture size of 1.0 mm (located at the image focus) the resolution should therefore be 0.91 % (FWHM).

### Position Sensitive Detector

In focusing the electrons emitted from the object point, the initial angular relationship between the electron trajectory and that of the central ray is reestablished, and the same angular distribution of electrons, now selected in energy, is found at the image focus. Figure 2.5 graphically illustrates, for typical electron paths through the analyzer, the preservation of these angles. By allowing a drift distance after the electrons pass by the

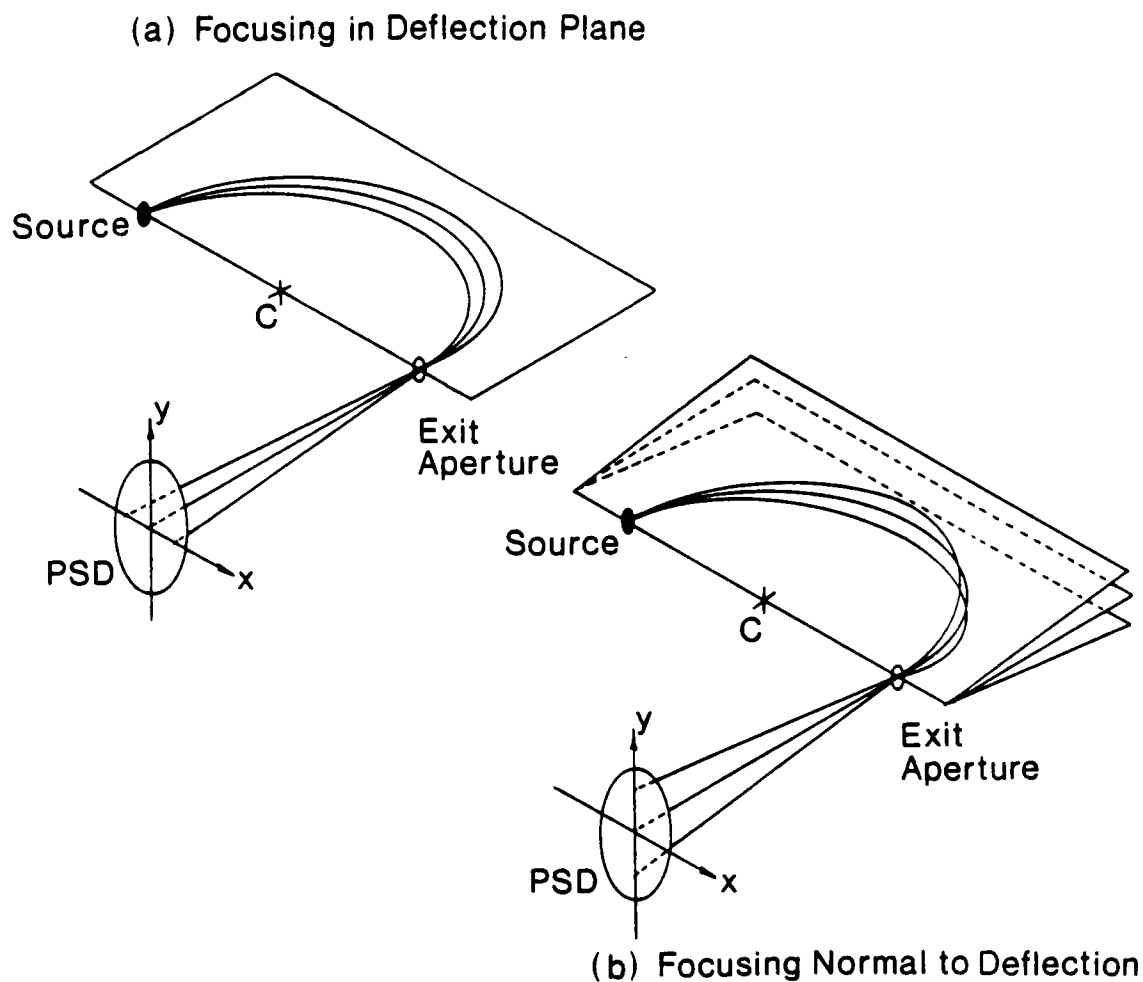


Figure 2.5. Typical trajectories through the electron spectrometer. Top portion of figure shows focusing action within the deflection plane, and bottom half indicates focusing perpendicular to the deflection plane.

spectrometer exit aperture, those electrons with initial angles near zero degrees will remain close to a straight line extension of the central ray, and those electrons with constant angular divergence from the central ray direction will be found at a constant distance  $d$  from this line, given by the simple formula  $d = (\ell)\tan^{-1}(\theta_0)$ , where  $\theta_0$  is the initial launch angle, and  $\ell$  is the drift distance. By aligning the analyzer so that the ion beam direction coincides with the tangent of the central ray at the object focus, and placing a planar detector after such a suitable drift length, the angular relationship (near zero degrees) between the electron and the ion that created it can be measured, if the planar detector is capable of resolving the position of the detected electron in two dimensions. Additionally, because the ETC cusp is strongly peaked in angle, alignment of the spectrometer is less critical due to the lack of apertures restricting the collection cone; small misalignments will shift the angular peak position on the detector, but this effect can be cancelled during later data reduction, since the zero degree position will have the highest count rate and is easily found.

To accomplish this, a position sensitive detector was developed within the University of Tennessee Atomic Physics Group, based on a microchannel plate electron multiplier - resistive anode design as first described by Lampton and Carlson<sup>52</sup>. The complete details of this detector are beyond the scope of this work, so only a simple description of the detector will be presented.

The microchannel plate detector, consisting of a pair of chevronned (channel direction alternating between plates) microchannel plate electron multipliers, amplifies a single electron striking the front face of the detector into a pulse of  $\sim 10^7$  electrons which can be counted by standard electronic modules, while maintaining the original position information of where the electron first struck (the centroid of the charge pulse coincides with that of the original position very closely). This multiplier is followed by a circular arc terminated resistive anode, as detailed in Ref. 52, and shown in Figure 2.6, which contains four corner electrodes where output pulses are detected by standard charge sensitive pulse detection techniques to produce charge pulse amplitudes  $A_0$  through  $A_3$  shown in Figure 2.6 from the four corner pickups. By forming the sums  $\Sigma_x = A_1 + A_2$ ,  $\Sigma_y = A_2 + A_3$ , and  $\Sigma_t = A_0 + A_1 + A_2 + A_3$ , the ratios  $\Sigma_x/\Sigma_t$  and  $\Sigma_y/\Sigma_t$  give the x and y positions of the pulse arrival at the anode. These sum pulses are formed by standard analog techniques using linear summer/inverter NIM modules, and connected to the input of a custom-built electronics module, which stretches the sum pulse peak values, performs the analog divisions given, and finally generates two 8-bit digital values which correspond to the two coordinates of the event position. Additional analog position pulses are provided (for display on an oscilloscope screen) in order to allow real-time monitoring of the incoming signals during data acquisition. The detector system is capable of a resolution between 100 and 150  $\mu\text{m}$ <sup>53</sup>.

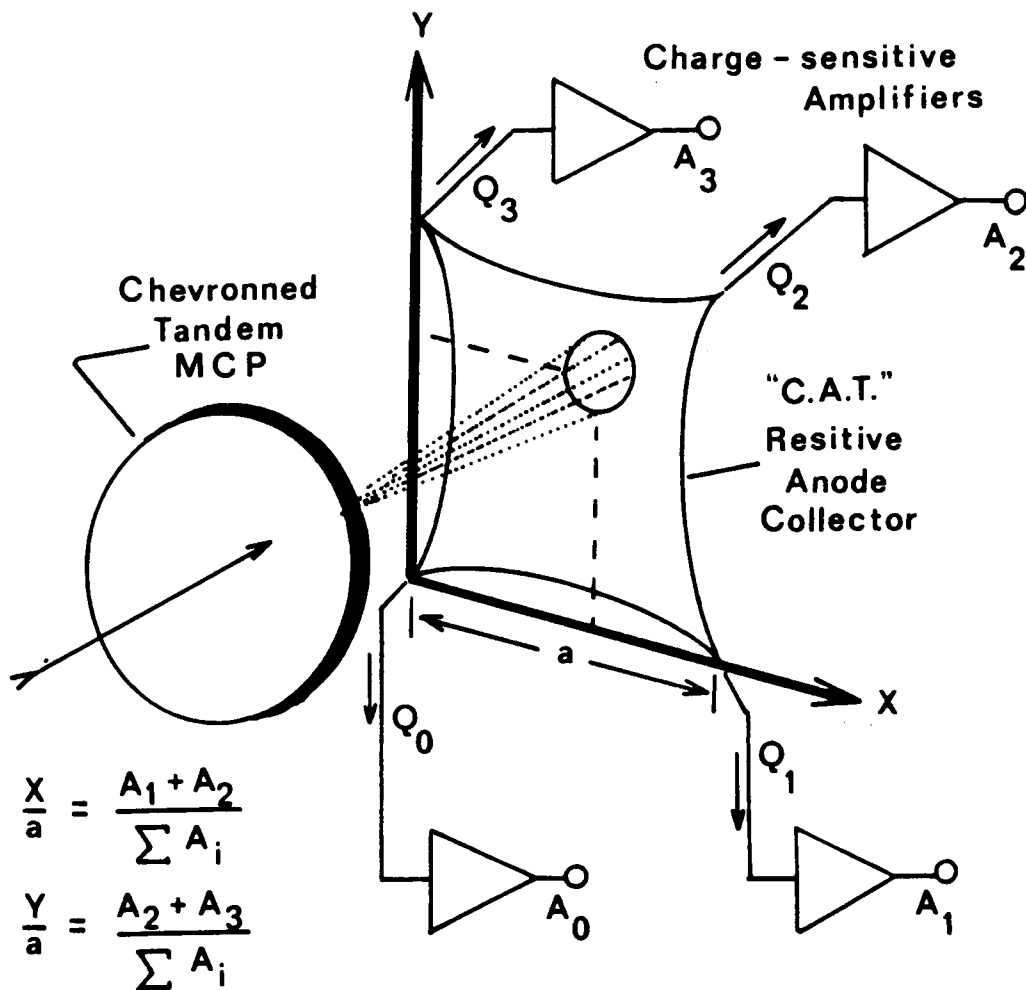


Figure 2.6. Schematic of the resistive anode collector. The charge cloud, from the electron multiplication process occurring in the microchannel plates, divides between the four corner pickups labelled 0 through 4. The position of the charge centroid is then calculated by forming the indicated ratios, as discussed in the text.

Additionally, the front surface of the detector is preceded by a pair of 80 % transparent mesh grids separated by 3 mm, between which a voltage ( $\sim 400$  V) is applied to prevent any low energy electrons from triggering the microchannel plates. These electrons were detected without this field applied; their probable origin was from secondary electrons produced from other electrons colliding with the analyzer sector plates. A simple numerical calculation was performed to assess the effect this retarding field had on the arrival position of the electron, and this was found to be negligible.

The angular resolution of the detector, as mentioned above, was electronically less than 0.08 degrees (based on better than 200  $\mu\text{m}$  linear position resolution and a 150 mm drift distance). However, because of the limited scheduled machine time, the beam spot size and the exit aperture were 1.0 mm, to allow for a reasonable electron count rate. This effectively determined the angular resolution, assuming negligible spectrometer aberrations: all electrons passing through the exit aperture of the analyzer with the same angle of emission, but traversing different points in the aperture plane will form an image on the detector equal to this size. This image size then dominates, leading to an angular resolution value for this work of .354 degrees.

#### Data Collection and Storage

The amount of data that this position sensitive detector can collect in a short period of time is substantial: at a peak rate of

~3 kHz, a megabyte of information (three bytes - two positional and one byte of energy information) can be collected in less than two hours. Due to both speed and storage limitations (especially the latter) imposed by the use of standard multichannel data acquisition microcomputers available at the time, it was decided to make use of the considerably more powerful<sup>54</sup> HHIRF user facility data collection system.

This system is comprised of two parts: a CAMAC (Computer Automated Measurement and Control<sup>55</sup>) based ORNL custom built microcomputer controller, and a complementary histogramming and data collection system resident on the HHIRF Perkin-Elmer 8/32 computer system.

The CAMAC controller, entitled the "Event Handler<sup>56</sup>," is a user programmable module which can control electronic devices connected to the CAMAC system, and store information collected when a user specified "event" occurs, and provides a means by which this data can be transferred to the HHIRF computer system. In the experiment here, for example, an event consisted of either a detected electron or an accumulation of a particular amount of charge in the Faraday cup. The former event required the storage of the two bytes of position information<sup>57</sup> and one byte which specified the electron spectrometer energy setting (also controlled by the Event Handler). The latter event type was used in controlling the spectrometer voltage setting by advancing a counter (in a cyclical or modulo fashion) through 256 values, and using this value via a digital-to-analog (DAC) converter to control the electron analyzer power supply. This allowed the

collection of data at 256 closely spaced electron energy values, for each of which an equal number of ions passed through the target gas cell (as measured by the charge collected in the Faraday cup). Repeated scans of the 256 channels of energy collected were performed in order to reduce the sensitivity of the spectrum to projectile ion beam variations.

The Event Handler, through a fast data buffer, sent the collected data stream to the HHIRF computer system, where a resident data collection system program performed three tasks: (1) overall system control of the Event Handler via user terminals, (2) dumping of the data in a fast event-by-event "list" mode to magnetic tapes for later, complex data reduction and analysis, and (3) (nearly) live-time extraction of one and two dimensional histograms (reduction of the data into spectra where, for example, a given channel of the spectra contains all counts at a particular energy or angle), and their display on user color graphics terminals.

With such a flexible system of data collection and analysis, rational decisions as to the progress of an experiment is possible within the normal constraints of limited accelerator operation time (the entire amount of data collected here represents about 53 hours of running time with the 25 MV Tandem). Figure 2.7 summarizes the entire data collection system in a block diagram form, and the important numerical parameters discussed above are presented in Table 2.1.

Figure 2.7. Block diagram of computer control and data collection system. (A) Current signal from Faraday cup is sent to electrometer/amplifier which multiplies current signal by typically 1000. That signal is connected to a current digitizer, which sends a pulse to a counter after collecting 100 picoCoulombs of charge. When counter reaches a predetermined value, a dose event signal is sent to the Event Handler. (B) Upon receiving a dose event, the Event Handler increments a register (modulo 256) which controls a digital-to-analog converter (DAC). The DAC in turn controls a programmable power supply which provides a given voltage to the spectrometer sectors. (C) Simultaneously, anode signals from the position sensitive detector are amplified and added to be sent to an analog-to-digital converter (ADC) for collection by the Perkin-Elmer computer. The computer stores position and energy information on the magnetic tape, calculates histograms stored on disk, displays histograms on color graphics terminals, and responds to user control via user terminals.

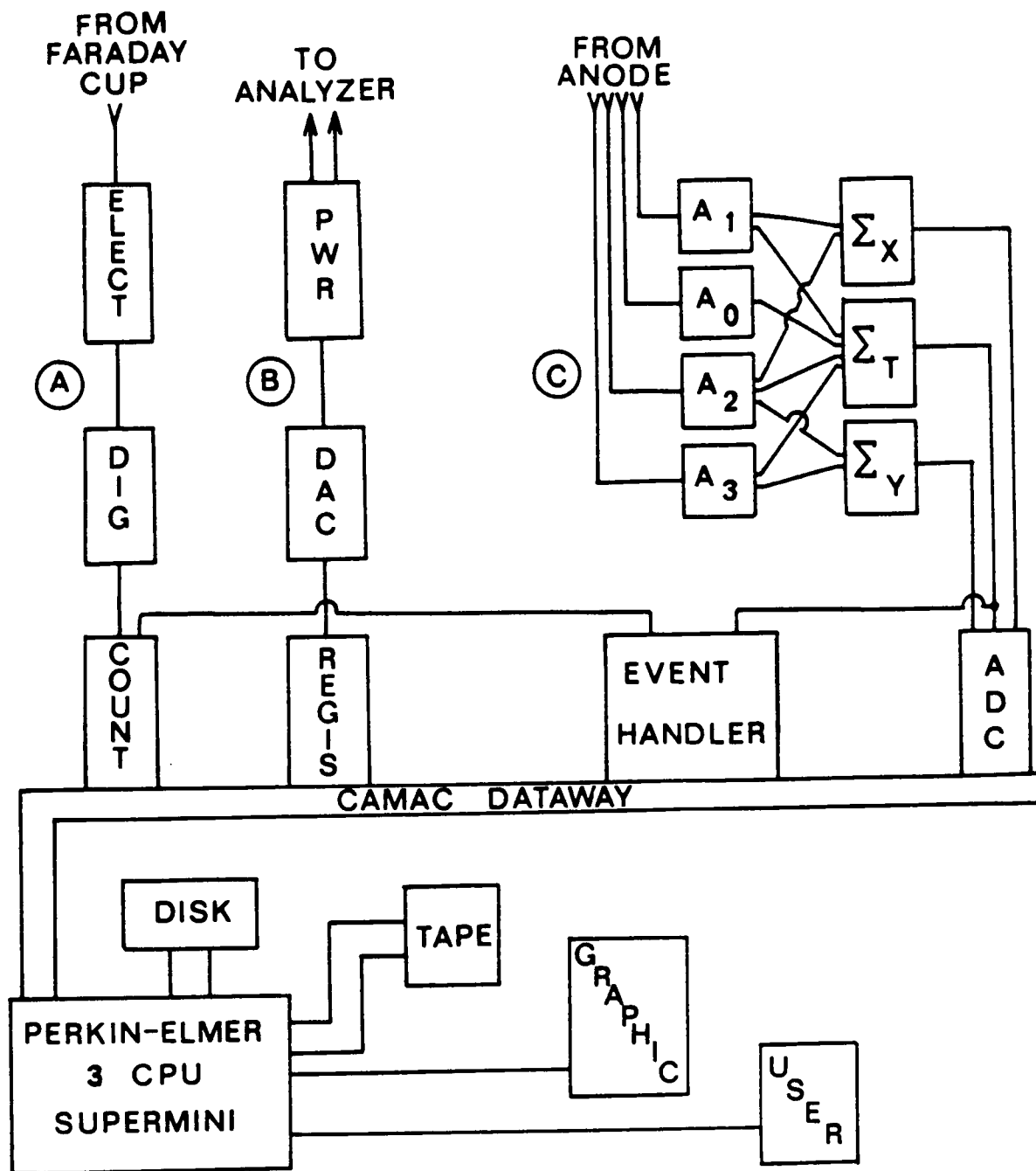


Table 2.1. Summary of important numerical parameters characterizing the experimental conditions.

| Parameter                        | Value                              |
|----------------------------------|------------------------------------|
| Ion beam angular divergence      | $\leq 0.3$ deg (typically 0.1 deg) |
| Ion beam spot size at gas target | $\sim 1.0$ mm                      |
| Analyzer FWHM                    | 0.91 %                             |
| Analyzer TW                      | 0.00 %                             |
| Detector angular resolution      | 0.354 deg                          |
| Maximum electron count rate      | 3 kHz                              |

## CHAPTER 3

### DATA ANALYSIS

#### Extraction and Reduction of Spectra

In order to perform a more complete analysis of the data than was available through the limited live-time histogramming features of the HHIRF data collection system, the collected data needed to be reformulated into a form that would be amenable to data analysis, especially the characterization of the data by an extension of a fitting procedure developed earlier for singly differential spectra<sup>20</sup>.

The first step in that procedure involved scanning through the original list mode data tapes (>1 gigabyte), condensing the information stored into a more compact format, and ignoring known bad data sets and, for example, ignoring the edge regions of the detector where the angle of collection relative to the ion beam direction was outside of the desired total collection cone. This prescanning procedure reduced the original number of tapes by 30 %, and the following tape scans used to extract histogram information were increased markedly in speed.

Then, a tape scanning procedure was followed for the remaining data, in which the list mode format was histogrammed into a three dimensional parameter space of 64 by 64 by 64 channels with one dimension representing the electron energy, and the other two the

position components of the event at the detector. This form of the spectra is shown diagrammatically in Figure 3.1.

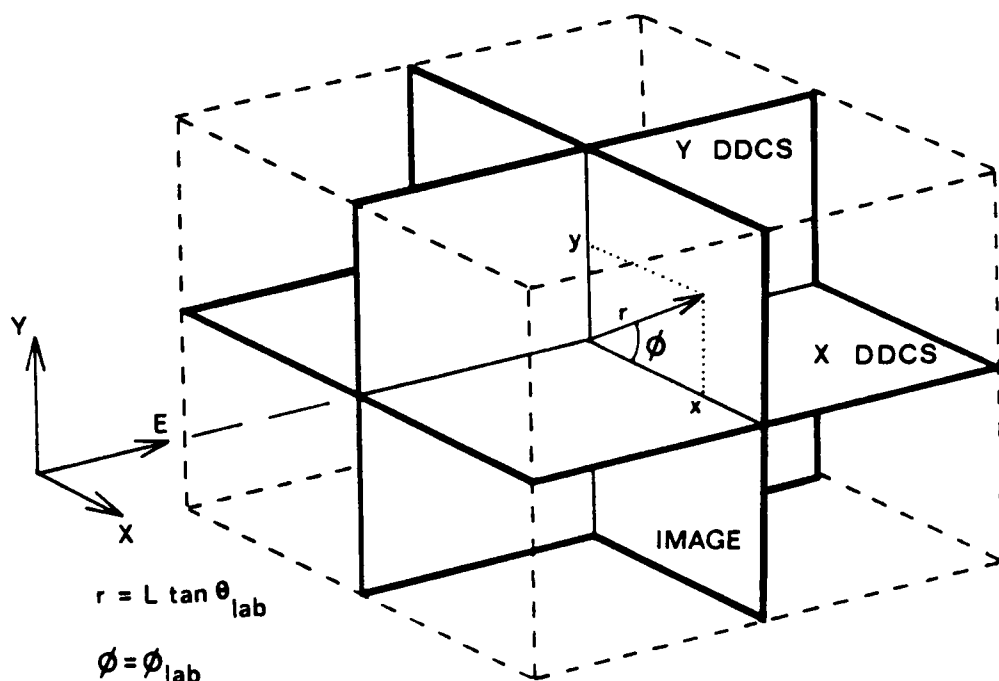


Figure 3.1. Three dimensional histogram representation of the collected data (extent marked by dashed lines). Three possible planes for two dimensional spectra extraction are labelled and bordered with thick solid lines. The two planes labeled DDCS are differential in energy and angle  $\theta_e$  (holding  $\phi = \text{constant}$  or  $\phi = \text{constant} + 180$  degrees). The image plane with electron energy such that  $v_e = v_p$  (maximum differential cross section) is used to find the  $\theta_e = 0$  position, then the radial component from that point is directly proportional to  $\theta_e$ .

By considering only a two dimensional slice of this data, either a representation of a detector image (for a small range of energies) or a doubly differential form of the cross section (differential both

in energy and ejection angle) can be formed. Because of the nature of the focusing of the spherical sector spectrometer, as discussed in Chapter 2, we chose to consider a plane of extraction that corresponded with the direction of optimum focal conditions (perpendicular to the deflection plane of the analyzer). This representation of the data was transferred to the University of Tennessee Atomic Physics Group's NOVA 4/X minicomputer for final data analysis -- utilizing a fitting procedure to recover detailed shape information for direct comparison with the recent theory of Burgdörfer et al.<sup>34</sup>, and extracting cross section information for both experimental and theoretical comparison.

### **Multipole Expansion Characterization of the Cross Section**

In order to characterize the collected two dimensional distributions, and make a meaningful comparison with theoretical results, it was decided to extend a previously developed fitting procedure<sup>20</sup> that made use of a double expansion of the cross section in the projectile frame. By incorporating an appropriate approximation to the electron spectrometer transmission function and carrying out the appropriate integrations (described below), measured singly differential ECC cross sections ( $d\sigma/dE$  or  $d\sigma/dv$  depending on the use of a magnetic or electrostatic analyzer) could be quantified via a fitting process in a manner highly independent of specific choices of experimental parameters, such as energy resolution or cone-of-collection angle. Here the nature of this approach and its extension to accommodate the

fitting of doubly differential continuum electron spectra are reviewed.

The simple results of Dettmann, Harrison, and Lucas<sup>11</sup> give the form of the cross section as

$$\frac{d\sigma}{d\vec{v}} \propto \frac{1}{|\vec{v}_e - \vec{v}_p|}, \quad (3.1)$$

where  $\vec{v}_e$  and  $\vec{v}_p$  represent the laboratory frame electron and projectile velocities, for an ETC process. Due to the generally accepted fact<sup>4</sup> that both ECC and ELC measured distributions exhibit anisotropies which disagree with this result, the cross section can be written more generally<sup>19,58</sup> as

$$\frac{d\sigma}{d\vec{v}} = \frac{f(v_e, v_p, \theta_e)}{|\vec{v}_e - \vec{v}_p|} = \frac{f(v_e, v_p, \theta_e)}{v'_e}, \quad (3.2)$$

in accordance with Figure 3.2.

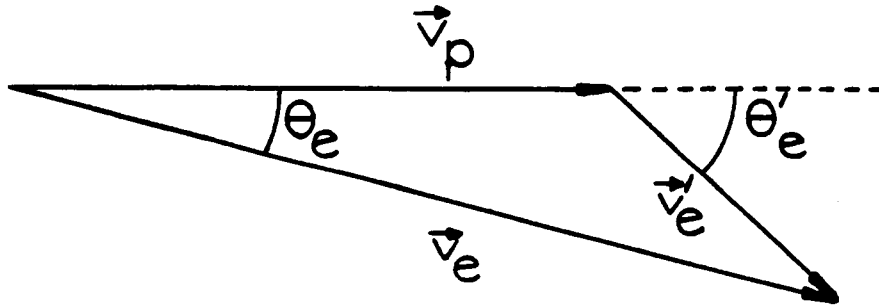


Figure 3.2. Vector diagram indicating the relationship between the projectile frame variables (primed) and the laboratory frame variables (unprimed). Subscript p refers to the projectile ion and subscript e to the unbound ELC electron.

Any asymmetry or anisotropy can be incorporated into the function  $f(v_e, v_p, \theta_e)$ , and so following the treatment of Refs. 19, 20, and 58 we expand it in the projectile frame as

$$f(v_e, v_p, \theta_e) = \sum_k a_k P_k(\cos \theta'_e) . \quad (3.3)$$

This can be viewed as either a "partial wave" expansion (but here an expansion of the cross section, not a wave function as is more commonly the case) or as a projectile frame spherical harmonic expansion of the electron "emission" pattern into its  $2^k$  multipoles (dipole, quadrupole, octupole, hexadecapole, . . . for  $k = 1, 2, 3, 4, \dots$ ). The coefficients  $a_k$  of Eq. 3.3 are now functions only of  $v_e$  and  $v_p$ , for a given target  $Z$  and ionic projectile, and to completely specify them we expand the  $a_k$  in a Taylor's series in  $v'_e$ , giving

$$\frac{d\sigma}{d\vec{\sigma}} = \frac{C}{v'_e} \sum_{n,k} \beta_{n,k}(v_p) (v'_e)^n P_k(\cos \theta'_e) , \quad (3.4)$$

where  $\beta_{0,0} \equiv 1$ .

The measured distributions  $Q(E_e, \theta_e)$  collected by the spectrometer/detector combination are integrals of the product of the spectrometer acceptance function  $S(E_e, \Omega_e)$  (describing the range of electron energies and angles associated with a given measured electron energy  $E_e$  and solid angle  $\Omega_e$ ) and the differential cross section  $d^2\sigma/dE d\Omega$  over the range of  $S(E_e, \Omega_e)$  with non-zero values:

$$Q(E_e, \Omega_e) = \int_{E_e} \int_{\Omega_e} S(E_e, \Omega_e) \frac{d^2\sigma}{dE_e d\Omega_e} dE_e d\Omega_e, \quad (3.5)$$

By relating the cross section differential in velocity to that differential in energy

$$\frac{d^2\sigma}{dE_e d\Omega_e} = \left( \frac{v_e}{\mu_e} \right) \left( \frac{d\sigma}{d\vec{v}_e} \right), \quad (3.6)$$

where  $\mu_e$  is the reduced mass of the ejected electron - target nucleus system, the measured distribution becomes

$$Q(E_e, \theta_e) = C \sum_{n,k} \beta_{n,k}(v_p) \times \quad (3.7)$$

$$\int_{E_e} \int_{\Omega_e} \left( \frac{v_e}{\mu_e} \right) (v_e')^{n-1} P_k(\cos \theta_e) S(E_e, \theta_e) dE_e d\Omega_e.$$

Finally, a representation for the function  $S(E_e, \theta_e)$  must be developed. We make the assumption for the small energy and angular resolutions used in this work that the transmission function is separable:

$$S(E_e, \Omega_e) = R(E_e) G(\theta_e, \phi_e). \quad (3.8)$$

For the case of singly differential spectra, Meckbach, et al.<sup>58</sup> have determined that equation 3.8 is an excellent approximation, and this form will be assumed acceptable here also. As mentioned in Chapter 2, the function  $R(E_e)$  is well approximated by a triangular or

trapezoidal line shape specified by a FWHM and a TW. Because the exit aperture of the spectrometer limits the range of angles seen by any point on the detector, the form of the function chosen for  $G(\theta_e, \phi_e)$  is a "pillbox" function unity within an angle  $\delta\gamma = (\text{angular resolution})/2$  of the central direction  $(\theta_e, \phi_e)$ . This is a simple two dimensional extension of the functional form used in Ref. 20, and can be visualized graphically in Figure 3.2. However, this function now depends on both  $\theta_e$  and  $\phi_e$ , and integration limits of the integral over these variables are interdependent. Unlike the case of Ref. 20, where these integrals became elementary, numerical techniques must now be employed to perform the integrations. A more detailed description of the method used to calculate the integrals of Eq. 3.7 is found in the Appendix.

### Fitting Procedure

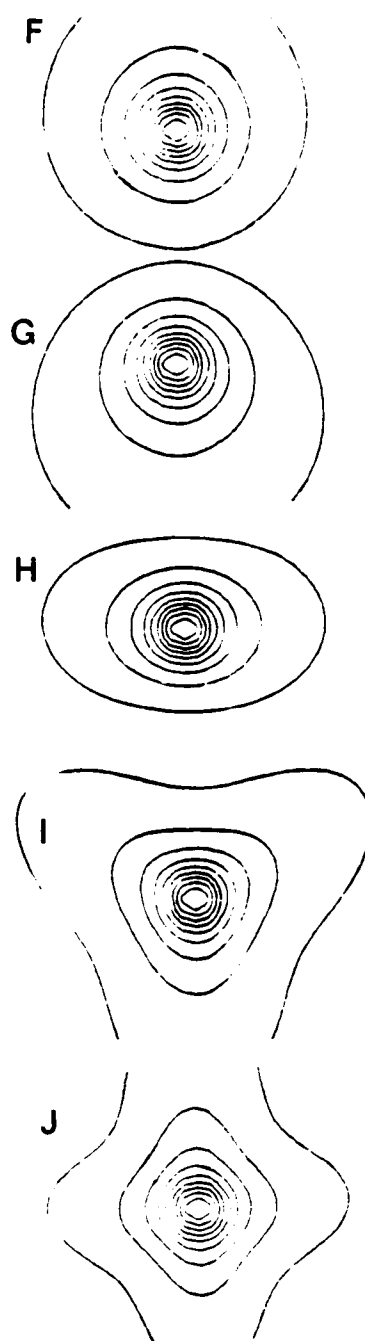
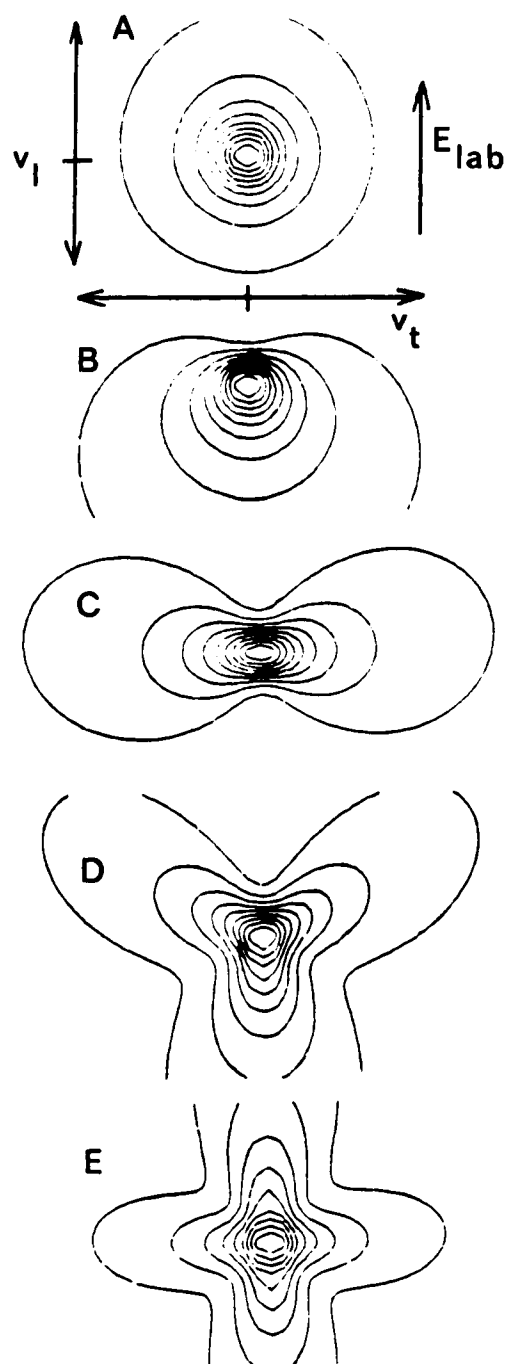
By using the method outlined in the Appendix for calculating the functional form of Eq. 3.7, which reduced the two angular integrations to a series of elementary integrals, a FORTRAN computer program was written which could calculate a test spectrum (up to 64 by 64 energy and angular values in size) given a set of  $\beta_{n,k}$  values. The infinite expansion developed in the previous section was truncated to  $0 \leq k \leq 4$  and for  $n = 0, 1$ . The limit of  $k$  values was chosen in consideration of the theory of Ref. 34, which predicts specific non-zero values of  $\beta_{0,2}$  and  $\beta_{0,4}$ . Only two terms in the Taylor series expansion were retained; the contribution of even the  $n = 1$  terms is

significant for only larger values of the projectile frame velocity magnitude  $v'_e$ , and for small values of  $v'_e$ , the contribution becomes vanishing small. Figure 3.3 displays contour plots of sample two dimensional spectra for various combinations of terms in the cusp expansion. Note that, except for the outer few contours, the terms with  $n = 1$  differ insignificantly from the single Dettmann cusp term (only  $\beta_{0,0}$ ).

Additionally, there were several other parameters necessary: the energy and angular center values, the acceptance function parameters FWHM, TW, and the angular resolution value, and an additional term allowing for a linearly varying efficiency of the position sensitive detector over the range of angular values within a two dimensional distribution. This last parameter was found to be empirically necessary for good fit results and will be discussed in Chapter 4.

This program was incorporated into a general purpose fitting system based on the method of linearizing the test function in the chi-square space formed by the variable parameters, combined with a gradient-least-squares procedure as formulated in Bevington<sup>59</sup>. The system allowed any arbitrary subset of the parameters listed in the previous paragraph to be variable, while the remaining parameters were held fixed. In this manner, various combinations of fitted parameters could be tried, and, for example, testing of the significance of adding particular terms could easily be accomplished.

Figure 3.3. Contour plots displaying combinations of all possible expansion terms with the Dettmann cusp term ( $\beta_{0,0}$ ). The magnitudes of the  $\beta$  values are  $\pm 1.0$  (recall that  $\beta_{0,0} \equiv 1$ ), with the negative sign for  $k = 1$  to  $3$  and the positive sign for  $k = 0$  and  $k = 4$  chosen in accordance with usual signs found for each term. The contours shown represent multiples of 10 %, with the maximum values occurring at the center of each plot, corresponding to  $v'_e = 0$ . A scale (arbitrary units, but equal velocity scaling in the projectile frame for each direction) indicating directions specifying energy and angle is found in the upper right corner of the figure.



The fitting system also incorporated background subtraction capability, appropriately incorporating the corrections necessary to the statistical weights used to calculate the reduced chi square (as defined in Ref. 59). Additionally, the region within one resolution volume of the cusp peak could be excluded from the fit, to avoid problems associated with the approximations used in describing the analyzer acceptance function (the error associated with the approximations will be greatest here since the cross section is singular for  $\vec{v}_e = \vec{v}_p$ , and the integrations of the approximate resolution functions with a very rapidly changing  $1/v_e'$  functional form will depend heavily on the exact nature of the resolution functions).

Additional routines were written to accomplish extraction of fitted values, fitted functions, and other related tasks. Interrogation routines were written to report the current status of the fit in progress: the program once started was automatic until a minimum value of  $\chi^2$  was found, and a typical single fit using 12 to 16 variable terms required about three to six hours of computer time to fit around 2000 data points.

## CHAPTER 4

### RESULTS AND DISCUSSION

#### Results and Comparison with Theory

Figure 4.1 displays sample data concerning the distribution observed for electrons ejected by  $v_p = 10.1, 14.3$  and  $16.2$  au  $O^{5+}$  ions undergoing single collisions in He and Ar targets. As expected from Refs. 4 and 34 the distribution is dominated by ELC from the loosely bound  $n = 2$  levels by an order of magnitude compared to combined contributions from ECC and  $n = 1$  ELC. These data exhibit the first direct confirmation of Burgdörfer's explanation<sup>34</sup> of the narrow ELC widths seen in earlier experiments: preferential transverse electron transmission. There can be no doubt concerning the strongly transverse character of the ELC angular distributions; the presence of appreciable  $\beta_{0,2}$  and  $\beta_{0,4}$  multipole strengths is confirmed; and a striking contrast with the strong dipole character ( $\beta_{0,1}$ ) of the ECC distribution illustrated for  $15.4$  au  $O^{8+}$  in Ne is seen. Also plotted is the theoretical angular distribution calculated by Burgdörfer for ELC in Ar, after convolution with the spectrometer acceptance function using the calculated values for the FWHM and angular acceptance as listed in Table 2.1 (page 45).

Table 4.1 presents a brief summary comparison of fitted experimental values with theoretical values for  $\beta_{0,k}$ . (Values for  $\beta_{1,k}$  and other terms are deferred until a later, more comprehensive table.)

Figure 4.1. Sample data comparing ELC measured and theoretical distributions for 82 MeV projectiles. Horizontal scales shown indicate the electron laboratory frame energy and the vertical scales indicate the laboratory frame electron ejection angle  $\theta_e$ . The data is shown by indicating contour levels of multiples of 12.5 %, with the highest level occurring near the center of each distribution shown. Fit results (b) and (e) are plotted using the same contour levels as in the corresponding data.

- (a) Measured spectrum for Ar target.
- (b) Corresponding fit results for (a).
- (c) Theoretical distribution expected from Ref. 34 (essentially the same for both targets).
- (d) Measured spectrum for He target.
- (e) Corresponding fit results for (d).
- (f) ECC spectrum from 94 MeV bare O incident on Ne for comparison. Note the low/high energy asymmetry not present for ELC.

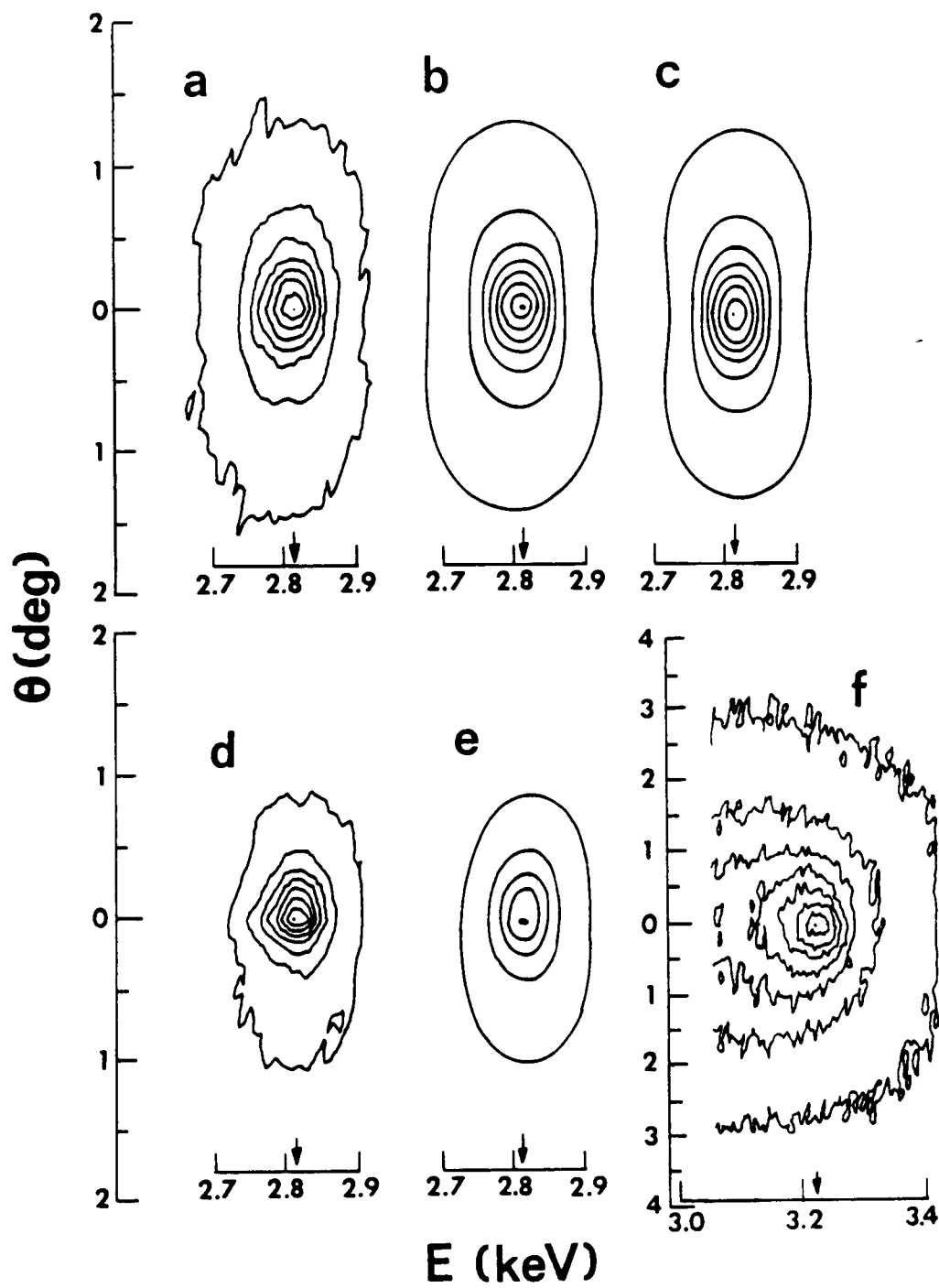


Table 4.1. Comparison of experimental fit results with theoretical results of Burgdörfer (Ref. 34). Where multiple data sets were available, the fitted values represent the mean, and the statistical standard deviation of these averages appears beneath the values in parentheses.

| $v_p$<br>(au)     | Target | $\beta_{0,2}$   |       | Th. | $\beta_{0,4}$  |      | Th. | $\beta_{0,1}$   |     | $\beta_{0,3}$   |     | $\chi^2$ b     |
|-------------------|--------|-----------------|-------|-----|----------------|------|-----|-----------------|-----|-----------------|-----|----------------|
|                   |        | Exp.            | Th.   |     | Exp.           | Th.  |     | Exp.a           | Th. | Exp.a           | Th. |                |
| 16.2<br>(105 MeV) | Ar     | -0.68<br>(0.00) | -0.70 |     | 0.16<br>(0.00) | 0.18 |     | -0.02<br>(0.00) |     | -0.05<br>(0.00) |     | 1.74<br>(0.14) |
| 14.3<br>(82 MeV)  | Ar     | -0.62<br>(0.02) | -0.67 |     | 0.12<br>(0.02) | 0.16 |     | -0.08<br>(0.03) |     | -0.01<br>(0.01) |     | 1.64<br>(0.11) |
| 10.1<br>(41 MeV)  | Ar     | -0.26           | -0.56 |     | 0.05           | 0.10 |     | -0.09           |     | -0.04           |     | 1.55           |
| 16.2              | He     | -0.23           | -0.68 |     | -0.16          | 0.16 |     | 0.28            |     | -0.16           |     | 2.43           |
| 14.3              | He     | -0.43<br>(0.03) | -0.64 |     | 0.06<br>(0.02) | 0.14 |     | 0.18            |     | -0.14           |     | 1.83<br>(0.12) |
| 10.1              | He     | -0.23           | -0.52 |     | -0.04          | 0.07 |     | 0.03            |     | 0.00            |     | 1.86           |

aTheoretical values calculated in Ref. 34 are zero for  $\beta_{0,1}$  and  $\beta_{0,3}$ .  
bReduced  $\chi^2$  as defined in Reference 59.

The small values found for  $\beta_{0,1}$  and  $\beta_{0,3}$  in Ar are attributed to small ECC contributions, seen to have the anticipated sign and to converge toward 0 along with  $\sigma_{\text{ECC}}$  as  $v_p$  increases. For Ar excellent agreement between theoretical and experimental values of  $\beta_{0,k}$  prevails for the two higher  $v_p$  values; the lesser agreement at 10.1 au is not of serious concern, because of the suspect accuracy<sup>34,60</sup> of the Born approximation at low  $v_p$ .

The worse agreement for He is of more concern, since its simpler structure and the use of more accurate scattering functions<sup>34</sup> to calculate the interaction than were available for Ar would suggest otherwise. Experimental error is contraindicated for two reasons. First, all possible experimental conditions were held fixed during the collection of data at a single energy for which both Ar and He target spectra were acquired. The accelerator operators were instructed to make no changes during the acquisition of cusp distributions at a single energy that would affect the steering and focusing of the ion beam, and possibly affect the resolution parameters characterizing the function  $S(v_e, \theta_e)$ . If it was necessary to change the ion beam intensity when switching gas targets, due to the approximately 30:1 ratio of the Ar to He ELC cross sections, the changes were made by adjusting the stripper gas pressure in the terminal of the accelerator. (The terminal stripper is a necessary part of any tandem accelerator, changing the injected negative ions to positive ions for further acceleration; changing the pressure here will have a strong effect on the ion beam intensity with negligible effect on

the focusing properties.) Second, although channel-by-channel gas dump background subtraction represented a greater source of statistical and systematic error for He because of smaller ELC yields, in the worst case--for  $v_p = 16.2$  au, where  $\gamma$ -ray/x-ray induced photoelectron flux at the detector was most serious--the ELC to background ratio at the peak of the cusp was  $\sim 11:1$ , and better still for the integrated yields.

Unable to find a source of error adversely affecting the He data selectively, we seek reasons why the theoretical results might better describe the multipole distribution for Ar. Since doubly inelastic collisions between the active projectile electron and target electrons are taken into account using the closure approximation<sup>34</sup> and this contribution is fractionally more important for target  $Z_t = 2$  than for  $Z_t = 18$  (the strong variation of the nucleus-electron interaction as a function of  $Z_t$  makes this contribution dominant when Ar is the target), reinvestigation of the adequacy of the approximation is suggested.

Alternatively, some other interaction process contributing to the measured distributions could be responsible for altering the cusp shape. It is possible, for example, that for He targets (with a much lower cross section within the ELC cusp) autoionization processes similar to that pictured in Figure 1.3 (page 11) add to the collected shape; these contributions, if present, would add "rings" around the two dimensional cusp distribution. This is not likely, however: for autoionization to occur within the  $O5^+$  projectile, a double excita-

tion is necessary requiring a K shell vacancy to be created; any transition involving the repopulation of that vacancy would produce an electron with an energy in the projectile frame too high to be seen in the ELC cusp. Therefore, it would be necessary to have either simultaneous capture and excitation to form  $O^{4+}$  or a Coster-Kronig transition accompanying a doubly excited state to contribute to the ELC cusp in this way<sup>61</sup>. This is considered very unlikely.

The values presented in Table 4.1 represent results from fits containing all ten multipole terms available through the fitting program. In addition, there were two variable parameters describing the center of the cusp distribution (in  $E, \theta$  space), three variables related to the spectrometer resolution (FWHM, TW, and angular resolution), and a "tilt" parameter (to be discussed later), totalling 16 variable terms allowed in these fits. The region within one FWHM in energy and one angular resolution width from the cusp center was excluded from these fit results also, for all cases except 41 MeV projectiles on Ar targets, where the results were not improved.

The angular resolution parameters were allowed to vary within the fitting procedure for the simple reason that good  $\chi^2$  values ( $\leq 3$ ) were not in general possible for the collected spectra without this variation. However, as was the case in Reference 20, the values achieved by the fitting process for these resolution parameters were reasonably close to the expected values. This was especially the case when the small center region around the cusp peak was excluded from participation in the fit (discussed later).

It was also found necessary to apply a linear "tilt" to the angular variation direction of the cusp distributions. The spectra displayed a pronounced apparent variation in collection efficiency, presumably due to variation of detection efficiency of the microchannel plate electron multiplier. Unfortunately, the short time interval of data collection did not allow for detailed testing of the detector efficiency, and the magnitude of this effect ( $\sim 12\%$  per degree of angular change) was not noticed until the complete tape scanning procedures had been performed several weeks later. There is some indication within the literature<sup>62</sup> of an efficiency change with differing entrance angles to the channels, but other sources<sup>63</sup> disagree with the magnitude of those results.

The exclusion of the central region near the cusp peak was not necessary to obtain reasonable fit results, but problems with characterizing the He target data led to its exclusion. The reason to not use that portion of the spectra in determining the best values within the fit is that errors introduced by the simplifying assumptions used to evaluate the resolution functions and the errors associated with the numerical evaluations of the integrals would become more important in this area. This is especially true when the resolution volume edge is located near the cusp singularity; the rapidly varying  $1/v_e'$  dependence multiplied by an approximation to a complete resolution description will enhance the error made in these approximations, while far away from the singularity the effect is fractionally much less important. Exclusion of this region was done

only after determining the best values with the cusp peak included within the fit, and those values were used as starting points, in order to insure that the computed fit function would not seriously differ from the cusp peak when the exclusion was made.

Finally, there was an additional measure made to help determine the absence of systematic variations between the measured data and the calculated fit spectra. Spectra were formulated in which each channel contained the contribution to  $\chi^2$  for that channel multiplied by the sign of that difference. Systematic errors in the fitted spectrum would then be evident within this spectrum. Also, this procedure was found very useful in investigating the conditions necessary for the setting of initial values of the fitting parameters in a manner that would insure that the fitting method converged to an absolute, as opposed to a local, minimum by the fitting procedure.

The better Ar  $\chi^2$  values reflect difficulty in understanding the origin of the more sharply spiked He cusps. Even the modest He  $\chi^2$  values required inclusion of unstable and highly  $v_p$ -dependent values of  $\beta_{0,1}$  and  $\beta_{0,3}$  of equal size but opposite sign, whose cancellation accounts for the approximate forward-backward symmetry obvious in the data (the cancellation was not complete: these terms alone were approximately zero away from the cusp peak, and formed a bipolar spike at the cusp peak which had the effect, through a small shift in the energy channel center of narrowing the cusp peak for the helium data). Inability of the routine to produce a sufficiently sharp spike using the expansion form chosen, even with  $\beta_{0,1}$ ,  $\beta_{0,3}$  available

--unfortunately the only form suggested to date by theorists--is at fault. Because of these fitting difficulties and because odd order multipoles are inconsistent with theoretical expectations, these offsetting  $\beta_{0,1}, \beta_{0,3}$  values are regarded as unphysical fitting artifacts and are not believed to reflect an ECC contribution but rather an attempt by the fitting routine to account for the variation from the expected functional form. For He the fitted values for  $\beta_{0,2}, \beta_{0,4}$  are therefore also considered to be less accurately determined. Fitting the helium cusps with only even terms in the expansion showed that for the lower two projectile energies, there were only small changes ( $\sim .05$  or less) in the values for  $\beta_{0,2}$  and  $\beta_{0,4}$ , but that for 105 MeV ions, the presence of the offsetting odd multipole terms was too large to obtain a good fit result with only even terms; even term only fits had much larger increases of  $\chi^2$ . The values of  $\beta_{0,2}, \beta_{0,4}$  for Ar are believed accurate.

Table 4.2 presents a complete compilation of the final results of the fitting procedure, including the values of all  $\beta_{n,k}$  terms used. Errors quoted are based on standard deviations of mean values obtained when multiple data sets were available, and represent only that error. The relatively small values indicate the lack of substantial changes in experimental conditions during the measurement of the ELC distributions for a single target and energy. Additionally, Figures 4.2, 4.3 and 4.4 present data for 41 and 105 MeV projectiles not shown in Figure 4.1 (page 58).

Table 4.2. Complete summary of fitting parameter results. Listed are values for all ten term coefficients, the analyzer parameters FWHM, TW, and the angular resolution, the reduced chi square as defined in Ref. 59, the value of the empirical tilt parameter, and the number of degrees of freedom (number of data points less number of variable parameters in the fit). When multiple data sets were available for a single combination of ion energy and target, mean values are listed, with standard deviations below in parentheses.

| Term          | Projectile<br>Energy: |        | 105 MeV         |                 | 82 MeV |        | 41 MeV |        |
|---------------|-----------------------|--------|-----------------|-----------------|--------|--------|--------|--------|
|               | Target:               |        |                 |                 |        |        |        |        |
|               | Argon                 | Helium | Argon           | Helium          | Argon  | Helium | Argon  | Helium |
| $\chi^2$      | 1.74<br>(0.14)        | 2.43   | 1.64<br>(0.11)  | 1.83<br>(0.12)  | 1.55   | 1.85   |        |        |
| $\beta_{0,1}$ | -0.02<br>(0.00)       | 0.28   | -0.08<br>(0.03) | 0.18<br>(0.06)  | -0.08  | 0.03   |        |        |
| $\beta_{0,2}$ | -0.68<br>(0.00)       | -0.23  | -0.62<br>(0.02) | -0.43<br>(0.02) | -0.26  | -0.23  |        |        |
| $\beta_{0,3}$ | -0.05<br>(0.00)       | -0.16  | -0.01<br>(0.01) | -0.14<br>(0.03) | -0.04  | 0.00   |        |        |
| $\beta_{0,4}$ | 0.16<br>(0.00)        | -0.16  | 0.12<br>(0.02)  | 0.06<br>(0.02)  | 0.05   | -0.04  |        |        |
| $\beta_{1,0}$ | -0.10<br>(0.00)       | 1.6    | -0.21<br>(0.02) | 0.26<br>(0.10)  | 0.80   | 0.56   |        |        |
| $\beta_{1,1}$ | -0.44<br>(0.00)       | -0.87  | -0.30<br>(0.09) | -0.47<br>(0.11) | -0.69  | -0.09  |        |        |
| $\beta_{1,2}$ | 0.28<br>(0.00)        | -0.52  | 0.31<br>(0.05)  | -0.51<br>(0.13) | -0.41  | -0.45  |        |        |
| $\beta_{1,3}$ | 0.28<br>(0.00)        | 0.26   | 0.18<br>(0.03)  | 0.34<br>(0.04)  | 0.31   | 0.00   |        |        |
| $\beta_{1,4}$ | -0.23<br>(0.00)       | 0.01   | 0.06<br>(0.02)  | 0.24<br>(0.08)  | -0.31  | 0.22   |        |        |

Table 4.2 (continued)

| Term                    | Projectile<br>Energy:<br>Target: | 105 MeV        |        | 82 MeV         |                | 41 MeV |        |
|-------------------------|----------------------------------|----------------|--------|----------------|----------------|--------|--------|
|                         |                                  | Argon          | Helium | Argon          | Helium         | Argon  | Helium |
| FWHM                    |                                  | 0.92<br>(0.00) | 0.62   | 0.97<br>(0.03) | 0.96<br>(0.14) | 1.56   | 0.99   |
| TW                      |                                  | 0.44<br>(0.00) | 0.11   | 0.03<br>(0.04) | 0.00<br>(0.00) | 0.32   | 0.00   |
| Angular<br>resolution   |                                  | 0.28<br>(0.00) | 0.32   | 0.30<br>(0.01) | 0.35<br>(0.02) | 0.40   | 0.35   |
| Tilt (% per<br>channel) |                                  | 1.3<br>(0.01)  | 2.3    | 0.82<br>(0.02) | 2.3<br>(0.17)  | 1.6    | 2.4    |
| Degrees of<br>Freedom   |                                  | 2718           | 1646   | 2726           | 958            | 973    | 958    |

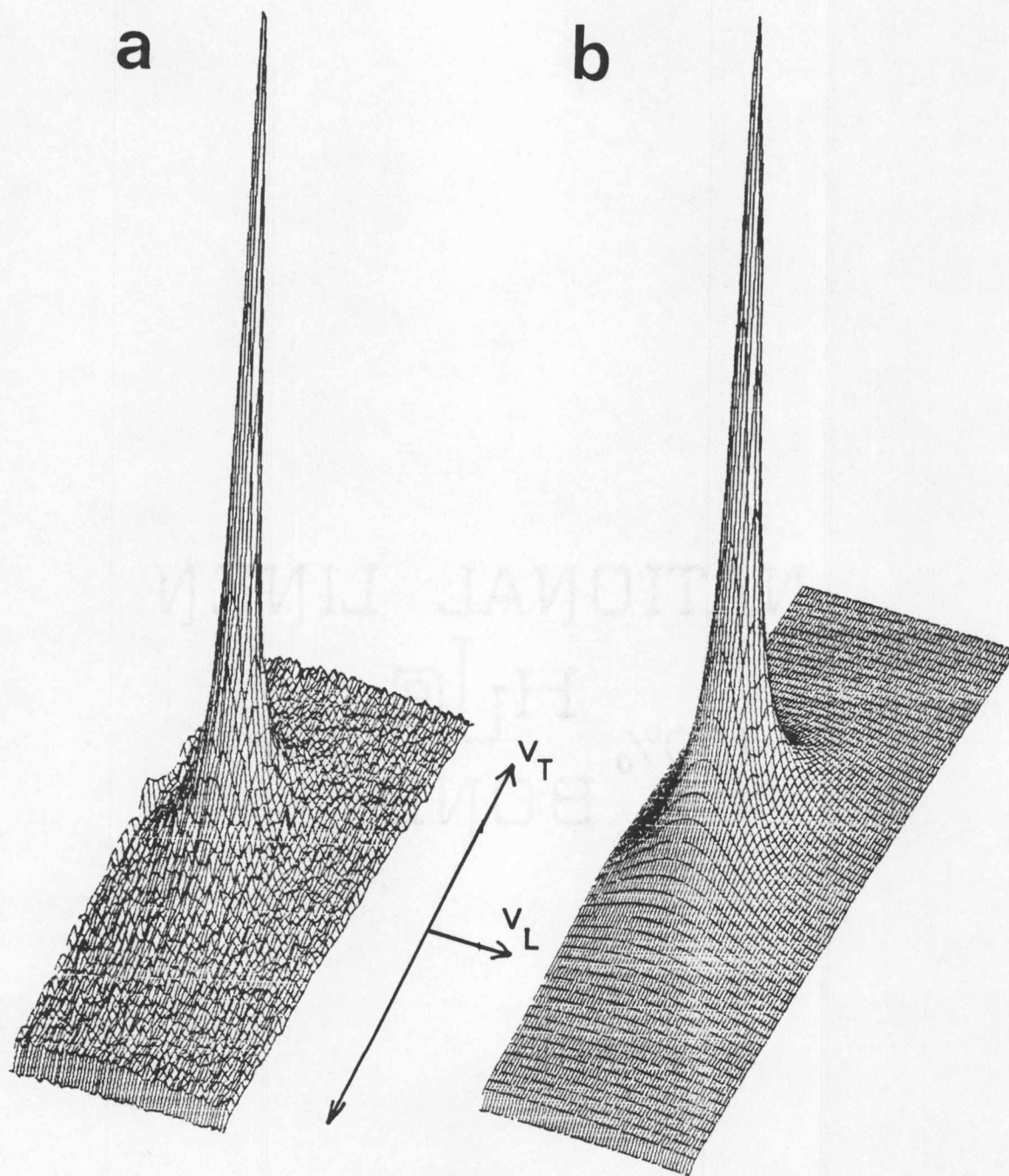


Figure 4.2. Isometric view display of data for 105 MeV projectiles comparing measured distributions and fit results. Vector diagram of longitudinal and perpendicular velocity directions (projectile frame). (a) Data for 105 MeV projectiles and Ar targets. (b) Corresponding fit to (a).

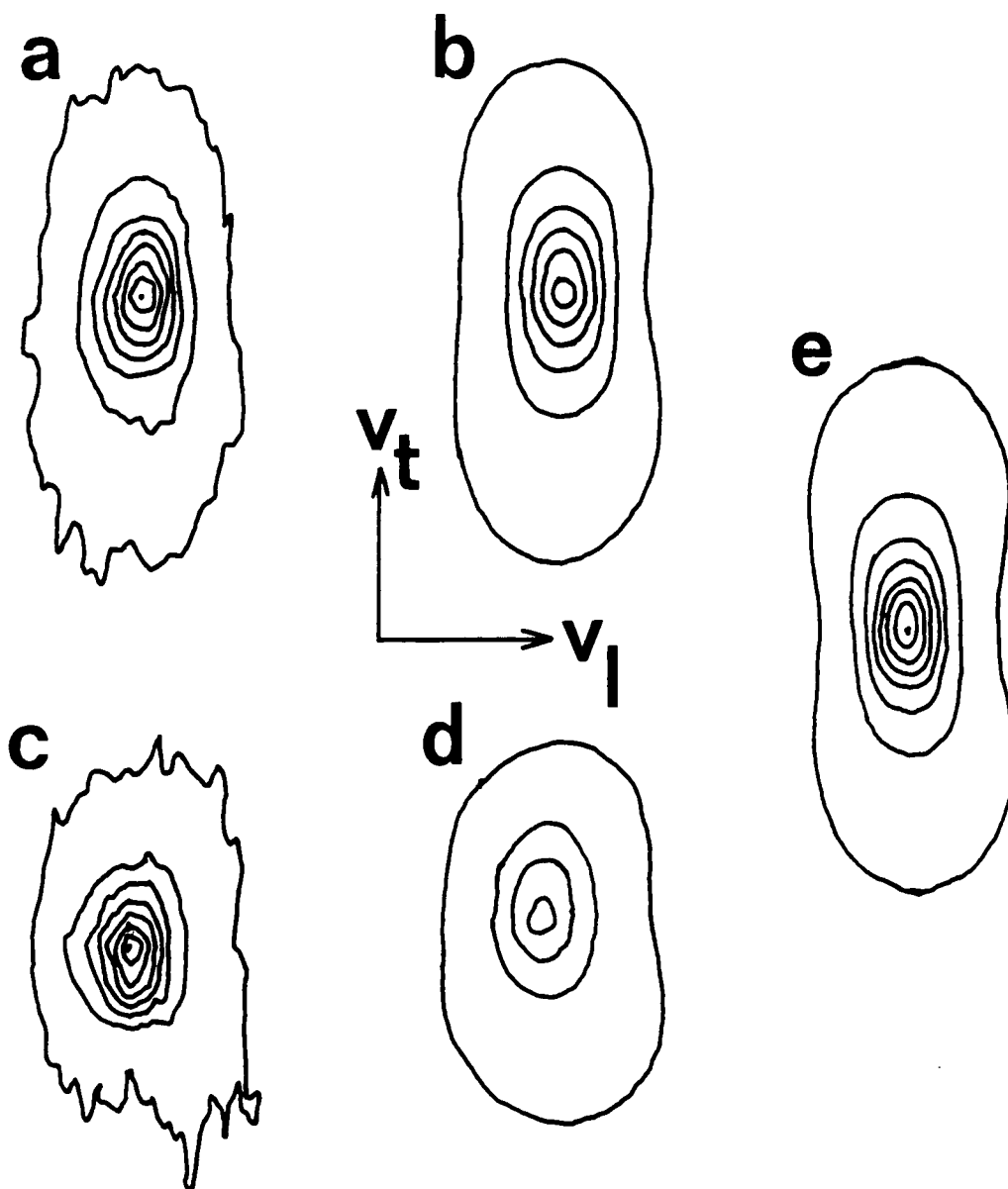


Figure 4.3. Summary of data for 105 MeV projectiles. Display is the same as in Figure 4.1 (page 58). (a) Data for 105 MeV projectiles and Ar targets. (b) Corresponding fit to (a). (c) Data for He targets. (d) Corresponding fit to (c). (e) Theoretical distribution expected, Ref. 34.

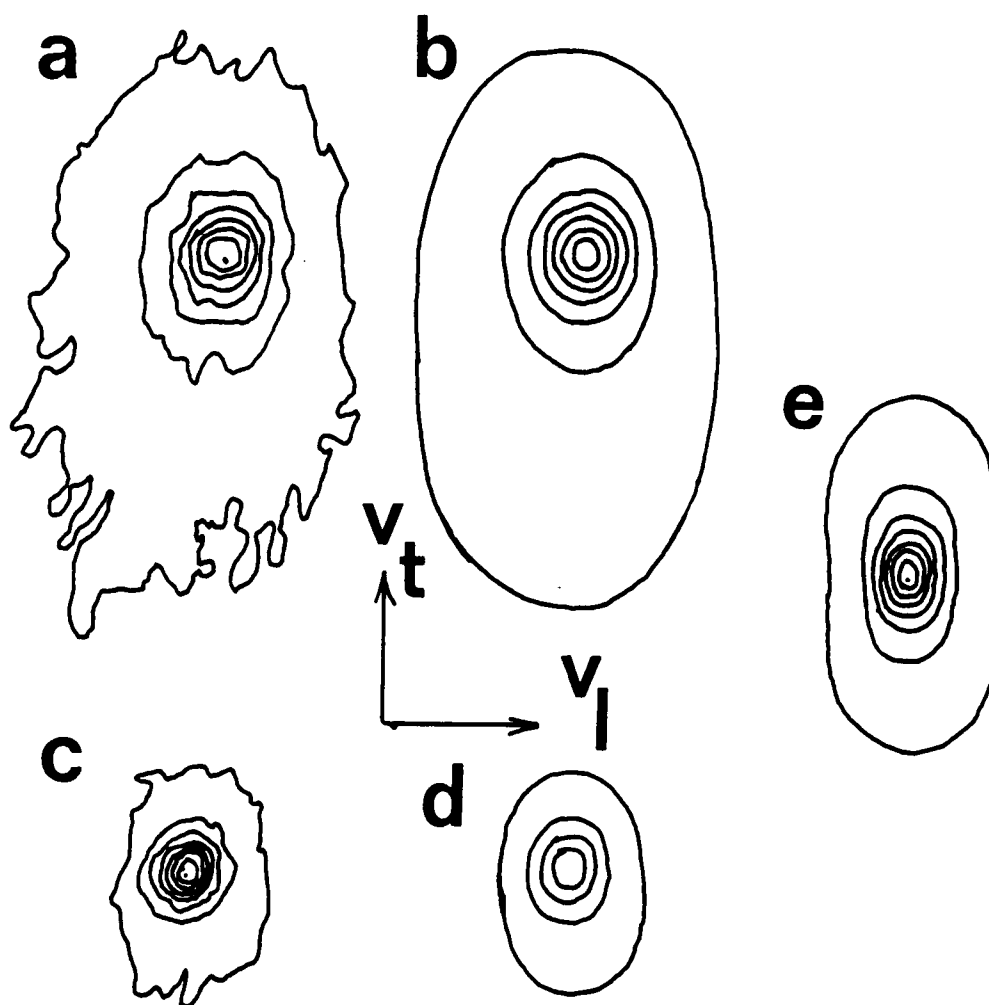


Figure 4.4. Summary of data for 41 MeV projectiles. Display is the same as in Figure 4.1 (page 56). (a) Data for 41 MeV projectiles and Ar targets. (b) Corresponding fit to (a). (c) Data for He targets. (d) Corresponding fit to (c). (e) Theoretical distribution expected, Ref. 34.

## Summary

The above results demonstrate the power and speed with which the use of the position sensitive detector and double expansion fitting method can determine high order multipole content of ejected electron distributions in ion-atom collisions. The close agreement with the theoretical calculations of Reference 34 for Ar targets is an indication of the correctness of the approach taken there; however the lack of good agreement for He targets merits more investigation.

By making use of the extension over the ionization limit used in Ref. 34 to extend previous bound state theory for high Rydberg states into the continuum, but in a reverse manner, the methods of this experiment have an important application in understanding the properties of these bound states, which are notoriously inaccessible to direct experimental investigation<sup>64</sup>.

In conclusion, three examples of experiments using the method here which can settle important issues raised in other work are discussed. For example, in a singly differential study of coincident double electron capture by 20 MeV  $\text{Au}^{15+}$  ions in He, one by ECC and one to a bound state<sup>65</sup>, Andersen et al. find that the measured shape is much more reminiscent of ELC in its forward-backward symmetry than of the characteristic dipolar shape for ECC alone. They conjecture that a double capture event involving a tightly space-correlated, two-electron wave packet is quickly followed by rapid autoionization. This is supposed to lead to a two-electron final state in

which one electron is bound and the other has a continuous energy spectrum like that of ELC owing to a short autoionization lifetime.

When combined with coincident detection of emergent ion charge state  $q_e^4$ , this method for determining multipole content is ideally suited to testing their conjecture that ECC accompanied by bound state capture transforms a cusp with large dipole content into one dominated by the monopole term. Quantitative determination of possible higher multipoles would reveal significant new information concerning the conjectured two-electron spatial correlation.

A coincidence experiment recently set up at Oak Ridge National Laboratory<sup>66</sup> also indicates it will be possible to study the phase space sharing in double capture events in which one electron has a positive energy just above the ionization limit while the other occupies a high Rydberg state just beneath. Thus far coincidences between convoy electrons and those freed by subsequent field stripping of ions in high Rydberg states have been detected downstream of a C foil. When converted to gas targets, these experiments will be cousins of experiments on  $e^-$  excitation of doubly excited Rydberg states of the  $He^-$  ion carried out at Manchester<sup>67</sup>, and related to recent theoretical work on the subject by Feagin and Macek<sup>68</sup>.

In still another experiment<sup>69</sup>, the multipole content of the convoy electron distributions emergent from thin foils of sub-equilibrium thickness in the company of emergent  $Oq^+$  projectiles at  $v_p = 10.1, 14.3, \text{ and } 16.2 \text{ au}$  have been compared and contrasted with the ECC and ELC multipole distributions discussed above. The results

clearly delineate the primary role of ELC in the bulk material as distinct from proposed mechanisms of formation at or near the exit surface<sup>70</sup>. The presence of still higher  $\beta_k$  values - up to  $\sim 10!$  - in these data indicates the likely importance of steady state excitation of surprisingly high  $n$  and  $l$  states as projectile ions traverse the bulk. It also offers the exciting possibility that this method can provide a novel and unique means of measuring mean excitations of swift projectiles while immersed in bulk material, as a function of  $Z_p$ ,  $Z_t$ ,  $v_p$ ,  $q_e$ , and other dynamical variables, for channeled as well as unchanneled ions.

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## APPENDIX

## APPENDIX

To calculate the measured ELC cusp distributions  $Q(E_e, \theta_e)$  it is necessary to evaluate the integrals over energy and angle given in Equation 3.7 (page 51), with the spectrometer transmission function  $S(E_e, \theta_e)$  as defined in Equation 3.8 (page 51), using the functional forms for  $R(E_e)$  and  $G(\theta_e, \phi_e)$  discussed in Chapter 2 (page 35) and Chapter 3 (page 52) respectively. Because the speed of the fit will be enhanced greatly, it is desired to perform as much of the integration as possible using analytical techniques.

The trapezoidal energy acceptance function  $R(E_e)$ , which better characterizes the analyzer properties than the often used rectangular acceptance functions<sup>58</sup> (unity for a small range of energies centered about  $E_e$  and zero elsewhere) are complex enough in an analytical form to make integration of the energy integral difficult by elementary techniques. Also, integration over  $E_e$  alone does not remove the cusp singularity, and removal of this singularity by partial calculation of the integrals using analytical techniques is useful in avoiding the special care required to treat divergences within numerical calculations.

The integrations over  $\theta_e$  and  $\phi_e$ , however, when both performed over a range of angles which includes  $\theta_e = 0$ , results in a functional form without a divergence for the velocity matching condition. If the angular integration is performed for a cone centered about  $\theta_e = 0$ , (and the angular acceptance function  $G(\theta_e, \phi_e)$  takes a very

simple form) as would be the case for a singly differential measured cross section<sup>20</sup>, the integrals become separable and elementary, if not a bit tedious for large  $k$  values. (Separable is used here to denote no dependence of one integral's limits on the other variable.) It should be noted that this property also holds true for integration over  $\theta_e$  between any two such cones, and any arbitrary "cone sector" defined by a simple limit on the range of  $\phi_e$ , i. e.  $\phi_1 \leq \phi \leq \phi_2$ .

For the doubly differential measurements of concern in this work, when the angular value of  $\theta_e$  is nonzero separability does not hold, because in general the limits of one angular integral depends on the other. However, the integrals can be divided into "cone sectors", each of which is evaluable by elementary techniques. To evaluate the integrals exactly, this division must be into an infinite number of thin such "cone sectors". This is impractical, but an approximate division into a smaller number of subregions can be performed to calculate a value of the integration with a reasonably small error.

Given the velocity and angular variables as defined by Figure 3.2 (page 49), the integrals over  $\theta_e$  and  $\phi_e$  take the form

$$I_{nk} = \int_{\phi_1}^{\phi_2} d\phi_e \int_{\cos \theta_2}^{\cos \theta_1} d(\cos \theta_e) (v')^{n-1} P_k(\cos \theta'_e) \quad (A.1)$$

for each of the terms in the multipole expansion defined in Equation 3.4 (page 50). The integral over  $\phi_e$  is trivial, and by using the projectile frame-laboratory frame relationships  $v_e'^2 = v_e^2 + v_p^2 - 2v_e v_p \cos \theta_e$  and  $v_e' \cos \theta_e' = v_e \cos \theta_e - v_p$ , and making the

variable change  $z = z(\cos\theta_e) = v_e'/v_p$  ( $z$  has the functional form  $[A + B\cos\theta_e]^{1/2}$ ) the integrals become

$$I_{nk} = (-1)^k \frac{2 \Delta\phi (v_p)^{n-1}}{\rho} \int_{z_2}^{z_1} dz (z)^{n-1} \times P_k\left(\frac{z^2 - (\rho^2 - 1)}{2z}\right), \quad (A.2)$$

where  $\rho \equiv v_e/v_p$ ,  $z_{1,2}$  are evaluations of  $z = v_e'/v_p$  at  $\theta_{1,2}$ . By expanding the Legendre polynomials to get a polynomial in  $z$  (giving powers from  $-k$  to  $k$ ), the integral over  $z$  is reduced to a simple sum of elementary integrals over a single power of  $z$ .

This expansion is increasingly tedious for larger  $k$ , and for the range  $0 \leq k \leq 4$  the results are:

$$\begin{aligned} P_0 &= 1. \\ P_1 &= z. \\ P_2 &= [3z^2 - 2(u-2) + 3u^2z^{-2}]/8, \quad u = \rho^2 - 1. \\ P_3 &= [5z^3 - az + auz^{-1} - 5u^3z^{-3}]/16, \quad a = 15u + 12. \\ P_4 &= [35z^4 - bz^2 + c - bu^2z^{-2} + 35u^4z^{-4}]/128, \\ &\quad b = 20(7u + 6), \quad c = 6(35u^2 + 40u + 8). \end{aligned} \quad (A.3)$$

These relationships are substituted into Equation A.2 and evaluation of that integral is straightforward (taking care to evaluate integrands of the form  $u^n/z^n$  carefully in the limit of  $z \rightarrow 0$  and therefore  $v_e' \rightarrow 0$ ).

After these angular integrations have been performed for each "cone sector" and the results totalled for a given angular resolution

volume centered about the desired direction ( $\theta_e, \phi_e$ ), the integral over energy (including the function  $S$ ) is handled by simple numerical methods. To save computation time, the energy acceptance function  $R(E_e)$  is computed for a number of closely spaced points, the energy values computed over the entire desired range of energies plus the (base) width of the acceptance function and a sum of their product is formed at each desired energy value over the range of  $R$ .

## VITA

Scott Donald Berry was born in Ann Arbor, Michigan on May 27, 1957. He attended elementary school in Flint, Michigan and secondary schools in Flushing, Michigan, graduating from Flushing Senior High School in June 1975. In September of that year he entered Albion College, Albion, Michigan and graduated Summa Cum Laude in December 1978 with a Bachelor of Arts degree with majors in Mathematics and Physics.

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