

To the Graduate Council:

I am submitting herewith a dissertation written by Der-Jhy Shieh entitled "Analysis and Monitoring of In-Core Dynamics in Pressurized Water Reactors." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Nuclear Engineering.

Belle R Upadhyaya

Belle R. Upadhyaya, Major Professor

We have read this dissertation
and recommend its acceptance:

J. W. Kirlin

H. Roland

Charles F. Moore

P. F. Sargua

Accepted for the Council:

Leuminkel

Vice Provost
and Dean of The Graduate School

ANALYSIS AND MONITORING OF IN-CORE DYNAMICS
IN PRESSURIZED WATER REACTORS

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee, Knoxville

Der-Jhy Shieh

March 1985

ii

to Lisa and Frankie

ACKNOWLEDGMENTS

The author wishes to express his gratitude to all who contributed their time and talent for the completion of this work, and in particular to:

Dr. Belle Upadhyaya, for his advice, suggestions and encouragement which made this work possible.

Dr. P.F. Pasqua, head of the Nuclear Engineering Department at UT, and D.N. Fry, head of the Surveillance and Diagnostics Group at ORNL for their continuing interest and support for the research work.

The members of my committee: Dr. T.W. Kerlin, Dr. H.C. Roland and Dr. C.F. Moore, for their time and suggestions.

Dr. F.J. Sweeney of ORNL for all his help, valuable suggestions and discussions.

Special thanks to the staff of the Surveillance and Diagnostics Group: C.M. Smith, J.A. Mullens and W.T. King, and the UT students working there: Jose March-Leuba, Richard Wood and Tommy King for their help in using the computer.

Appreciation is expressed to the Institute of Nuclear Energy Research, Taiwan, for offering scholarship during my entire Ph.D. program, and the support from ORNL for part of the research work.

ABSTRACT

The applicability of the core-exit coolant temperature and the neutron power noise signals for core coolant flow rate estimation and for monitoring the sign of the moderator temperature coefficient of reactivity in pressurized water reactors (PWRs) are studied. Three kinds of models are developed to investigate the mechanisms involved in the noise signals: the phenomenological model, the distributed parameter model and the multi-nodal model. The Loss-of-Fluid Test (LOFT) reactor is selected as the baseline reactor for this study. The phase relationship between the core-exit coolant temperature and the neutron power signals is found to have a linear phase-frequency behavior in a certain frequency region (about 0.1-2 Hz) with lower and upper bounds which are functions of the system dynamics parameters. The slope of this phase is inversely proportional to the core flow rate. The phase at zero frequency extrapolated from this linear phase-frequency region is -180 degrees for a negative moderator temperature coefficient of reactivity and zero degrees for a positive moderator temperature coefficient.

The feasibility of using a stochastic estimator to estimate the flow rate using the core-exit coolant temperature and the neutron noise signals is also studied. The extended Kalman filter is developed and the validity of the method is tested against the 65% and 100% flow rate noise data from the LOFT reactor. The performance of the maximum likelihood function for flow estimation using noise

signals is also investigated using the ambiguity function analysis technique. The sensitivity of the estimation to the variations of other system parameters is evaluated.

CONTENTS

SECTION	PAGE
1. INTRODUCTION	1
1.1 General Description	1
1.2 Previous Work, Motivation and Research Areas	5
1.2.1 Core dynamics study in the frequency domain	5
1.2.1.1 Flow rate estimation using phase slope	5
1.2.1.2 Monitoring moderator temperature coefficient of reactivity using the extrapolated phase at zero frequency	8
1.2.2 Stochastic estimator in the real-time domain	9
1.3 Original Contributions of This Dissertation	12
1.3.1 Investigation of the core dynamics	12
1.3.1.1 Flow rate estimation	12
1.3.1.2 Moderator temperature coefficient of reactivity monitoring.	13
1.3.2 The extended Kalman filter for core mass flow rate estimation	14
1.3.3 The study of the maximum likelihood estimator characteristics using ambiguity function analysis	15
1.4 Organization of the Dissertation	15
2. PHENOMENOLOGICAL MODEL.	17
2.1 Introduction	17
2.2 Heat Source Disturbance	22
2.3 Coolant Inlet Temperature Disturbance	26
2.4 Coolant Velocity Disturbance	29
2.5 Random Heat Transfer Disturbance	33
2.6 Summary and Discussion	36
2.6.1 Summary	36
2.6.2 The case of extremely localized space dependent neutron kinetics	38
2.6.2.1 Heat source disturbance.	38
2.6.2.2 Coolant inlet temperature disturbance.	39
2.6.2.3 Coolant velocity disturbance	40
2.6.2.4 Random heat transfer disturbance	41
2.6.2.5 Concluding remark.	41
3. DISTRIBUTED PARAMETER MODEL	42
3.1 Introduction	42
3.2 The Model	43
3.2.1 Cylindrical fuel region	43
3.2.2 Coolant region	46
3.2.3 Reactor kinetics	49

SECTION

PAGE

3.2.4	Calculation of the transfer function between the core-exit coolant temperature and the neutron power	53
3.3	The Computer Program	54
3.4	Results and Discussion	54
3.4.1	The existence of the linear phase-frequency relationship.	54
3.4.2	The sink frequencies.	55
3.4.3	The phase slope	61
3.4.4	The extrapolated absolute phase at zero frequency	61
4.	MULTI-NODAL MODEL	65
4.1	Introduction	65
4.2	The Model	66
4.3	Results and Discussion	73
4.3.1	Verification of the model	73
4.3.2	The effect of the direct heat to the coolant	78
4.3.3	The effect of neglecting the fuel temperature fluctuation and the feedback interactions between the neutronics and the thermal hydraulics.	81
4.3.4	The effect of heat transfer coefficient fluctuation	81
4.3.5	Consideration of random heat transfer disturbance	86
4.3.6	Monitoring the moderator temperature coefficient of reactivity	89
5.	APPLICATIONS OF THE PHASE RELATIONSHIP BETWEEN THE CORE-EXIT COOLANT TEMPERATURE AND THE NEUTRON POWER NOISE SIGNALS	93
5.1	Introduction	93
5.2	The LOFT Reactor Data Analysis	93
5.2.1	The dominant disturbance source in the LOFT reactor	94
5.2.2	The multi-nodal model simulation	97
5.2.3	The space dependent phenomenon	97
5.3	The Sequoyah Nuclear Power Plant and the Borssele Nuclear Power Plant Data Analysis	99
5.4	Discussion	102
5.4.1	Use of PHASE(TOUT,N) slope to infer the coolant flow rate	102
5.4.2	Monitoring the moderator temperature coefficient of reactivity	106

SECTION	PAGE
6. EXTENDED KALMAN FILTER FOR FLOW ESTIMATION	108
6.1 The Kalman Filtering Algorithm	108
6.2 The Extended Kalman Filter	115
6.3 Modified Extended Kalman Filter Algorithm	116
6.4 The Model for Time Domain Estimation	119
6.5 Test of the Algorithm	124
6.6 The Real Data Analysis	125
6.7 Summary	135
7. AMBIGUITY FUNCTION ANALYSIS	139
7.1 Introduction	139
7.2 The Ambiguity Function Algorithm	141
7.3 Implementation of the Ambiguity Function	145
7.4 Results and Discussion	146
8. SUMMARY, CONCLUSION AND RECOMMENDATIONS	155
8.1 Summary and Conclusion	155
8.1.1 Flow rate estimation using the phase slope.	155
8.1.2 Monitoring the moderator temperature coefficient.	158
8.1.3 Flow rate estimation using stochastic estimator	160
8.2 Recommendations.	161
LIST OF REFERENCES	163
APPENDICES	170
A. ABOUT THE SINK FREQUENCY	171
B. LOFT PARAMETER VALUES	174
C. ONE DIMENSIONAL HOMOGENEOUS TIME DEPENDENT DIFFUSION EQUATION	177
D. PROMPT JUMP APPROXIMATION FOR NOISE DATA ANALYSIS	179
E. ERROR COVARIANCE FOR KALMAN FILTER	180
F. LIST OF COMPUTER PROGRAMS	183
VITA	208

LIST OF FIGURES

FIGURE	PAGE
2-1. Schematic diagram of the mechanisms involved in the neutron power and core-exit coolant temperature fluctuations	18
2-2. The phenomenological model with the heat source disturbance.	23
2-3. The phenomenological model with the coolant inlet temperature disturbance.	27
3-1. Comparison of the phases between the neutron power and reactivity fluctuations with and without including the temperature-reactivity feedback.	56
3-2. The phase angle between the heat generation and the coolant temperature fluctuations at the same axial position.	57
3-3. The gains between the core-exit coolant temperature and neutron power fluctuations for flat neutron flux shape with the heat source, coolant inlet temperature and flow rate disturbance sources.	59
3-4. The phases between the core-exit coolant temperature and neutron power fluctuations for the flat neutron flux shape with the heat source, coolant inlet temperature and flow rate disturbance sources.	60
4-1. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with heat source disturbance for a variety of node numbers with cosine flux shape	74
4-2. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with coolant inlet temperature disturbance for a variety of node numbers with cosine flux shape.	75
4-3. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with coolant flow rate disturbance for a variety of node numbers with cosine flux shape	76

FIGURE

PAGE

- 4-4. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations for the cases without direct heat and with direct heat only with cosine flux shape 79
- 4-5. Comparison of the calculated gains between the core-exit coolant temperature and neutron power fluctuations for the cases without direct heat and with direct heat only with cosine flux shape 80
- 4-6. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with and without including the effect of temperature reactivity feedback for the case with heat source disturbance, cosine flux shape and no direct heat. 82
- 4-7. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with and without feedback for the case with coolant inlet temperature disturbance and cosine flux shape 83
- 4-8. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with and without feedback for the case with coolant flow rate disturbance and cosine flux shape 84
- 4-9. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with and without including the variation of heat transfer coefficient caused by the disturbance of coolant velocity. 85
- 4-10. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with the random heat transfer disturbance for a variety of node numbers. 87
- 4-11. Comparison of the coherences between the core-exit coolant temperature and neutron power fluctuations with the random heat transfer disturbance for a variety of node numbers. 88
- 4-12. The phases for the cases with heat source, coolant inlet temperature, coolant flow rate and random heat transfer disturbances with positive moderator temperature coefficient of reactivity. 91

FIGURE	PAGE
5-1. The experimental phase between the core-exit coolant temperature and neutron power fluctuations of the LOFT reactor at 100% flow condition	95
5-2. The simulated phases between the core-exit coolant temperature and neutron power fluctuations of the LOFT reactor at 100% flow condition with a variety of disturbance sources	98
5-3. The experimental phase between the core-exit coolant temperature and neutron power fluctuations of the LOFT reactor at 100% flow condition with the neutron detectors at different levels A,B and D	100
5-4. Comparison of the phases between the core-exit coolant temperature and neutron power fluctuations with space independent neutron kinetics, and the space dependent neutron kinetics with neutron detectors positions at (a) the fourth coolant node, and (b) the nineteenth coolant node for the case with coolant flow rate disturbance and cosine flux shape	101
5-5. The experimental phase between the core-exit coolant temperature and neutron power fluctuations of the Sequoyah Nuclear Power Plant.	103
5-6. The experimental phase between the core-exit coolant temperature and neutron power fluctuations of the Borssele Nuclear Power Plant.	104
6-1. The EKF estimates of the core flow rates for four sets of simulated data at 100% flow condition.	126
6-2. The EKF estimates of the core flow rates for four sets of simulated data at 50% flow condition with the model linearized at 100% flow	127
6-3. The EKF estimates of the core flow rates for four sets of simulated data at 50% flow condition with the model relinearized at 50% flow.	128
6-4. The noise signal of the normalized neutron power of the LOFT reactor at 100% flow condition	129
6-5. The noise signal of the core-exit coolant temperature of the LOFT reactor at 100% flow condition.	130

FIGURE	PAGE
6-6. The noise signal of the normalized neutron power of the LOFT reactor at 65% flow condition	131
6-7. The noise signal of the core-exit coolant temperature of the LOFT reactor at 65% flow condition.	132
6-8. The EKF estimates of the normalized core flow rates with two sets of 100% flow data and one set of 65 % flow data from the LOFT reactor.	134
6-9. The EKF estimate of the normalized core flow rates with 100% and 65% flow data from the LOFT reactor while the coolant temperature coefficient of reactivity multiplied by 1.5.	136
6-10. The EKF estimates of the normalized core flow rates with 100% and 65% flow data from the LOFT reactor while the break frequency of the temperature sensor is 2 Hz.	137
7-1. The asymptotic ambiguity function of normalized flow rate at 100% flow condition with the design model the same as the true model and $DT=0.078$ sec.	147
7-2. The asymptotic ambiguity function of normalized flow rate at 50% flow condition with the design model the same as the true model and $DT=0.078$ sec.	149
7-3. The asymptotic ambiguity function of normalized flow rate at 100% flow condition with the design model the same as the true model and $DT=0.039$ sec.	150
7-4. The asymptotic ambiguity function of normalized flow rate at 100% flow condition with the design model the same as the true model and $DT=0.156$ sec.	151
7-5. The asymptotic ambiguity function of normalized flow rate at 100% flow condition with the value of the moderator temperature coefficient of the true model 30% higher than that in the design model	153
A-1. Amplitude and phase of a pure delay fluid transport mechanism at the frequency v/L	173
A-2. The time domain sinusoidal fluctuations with frequency being v/L at position (a) 0, and (b) $L/2$	173

LIST OF SYMBOLS

A : ambiguity function, system dynamic matrix

A_{as} : asymptotic ambiguity function

A_c : flow area

APSD_m : auto power spectral density of m

B : deterministic input matrix

C_p : specific heat

C_{pc} : coolant specific heat

C_{pf} : fuel specific heat

CPSD_{nm} : cross power spectral density between n and m

EKF : extended Kalman filter

f_p : peak frequency

f_s : sink frequency

f(z) : neutron flux shape function

F_{fi} : fraction of power deposited in the ith fuel node

F_{ci} : fraction of power deposited in the ith coolant node

G : noise input matrix

GAIN(TOUT,N) : the gain between the neutron flux and the core
exit coolant temperature

H.S. : heat source disturbance

H : measurement matrix

h : film heat transfer coefficient

K : Kalman filter gain

k_f : fuel conductivity

L : active fuel length, likelihood function

m : mass, the dimension of the measurement vector

$\dot{m}$: coolant mass flow rate

MLE : maximum likelihood estimator

n : neutron flux, the dimension of the state variable vector

N : neutron power

p : probability density function

$P(t_i; t_{i-1})$: the conditional covariance at time t_i based on the measurements up to time t_{i-1}

P_e : error covariance matrix

PHASE(TOUT, N) : the phase shift between the neutron flux and the core-exit coolant temperature

PHASE(X, Y; Z) : the phase shift between state variables X and Y with Z as the disturbance source

Q : covariance of the process noise

$QC(\omega)$: heat transfer from fuel to coolant at frequency ω

Q_d : covariance of the process noise for discrete model

Q_m : the total fission energy remaining in the fuel

R : radius of the fuel, covariance of the measurement noise

r : radial position or radius

R.H. : random heat transfer disturbance

S : reactivity disturbance source

T_c : coolant temperature

T_{cl} : cladding temperature

T_f : fuel temperature

TIN : core inlet coolant temperature

TOUT : core-exit coolant temperature

t : time

$\underline{u}$: input vector

v : coolant velocity

$\underline{v}$: white Gaussian measurement noise

$\underline{w}$: white Gaussian process noise

W : mass flow rate

W_{fi} : the weighting of the i th node fuel temperature on the reactivity

W_{ci} : the weighting of the i th node coolant temperature on the reactivity

$\underline{x}$: state variable vector

$\hat{\underline{x}}(t_i; t_{i-1})$: the conditional estimated state at time t_i based on the measurements up to time t_{i-1}

$\underline{z}$: measurement vector

$\underline{Z}(t)$: composite vector comprised of the entire measurement history up to time t

ρ : reactivity, density

$\omega : 2\pi f$, f :frequency

α_c : coolant temperature reactivity coefficient

β : delayed neutron fraction

Λ : prompt neutron generation time

λ : delayed neutron group decay constants

$\phi(t, t_0)$: state transition matrix

1. INTRODUCTION

1.1 General Description

This research work is related to the pressurized water reactor (PWR) core dynamics monitoring by analyzing the neutron power and core-exit coolant temperature signals.

The core of a PWR is essentially a heat source, where the heat generated is due to the fission process; the coolant (i.e., water) is heated up at a high pressure of about 2250 psia without bulk boiling. The core consists of a large number of fuel assemblies, in which the fuel rods are arranged vertically in 15x15 or 17x17 square arrays. The fuel rod consists of uranium oxide pellets. The pellets are contained in a zircaloy tube usually called the cladding. The whole core is submerged in water, which has dual functions as a coolant and as a neutron moderator. As a coolant, the water flows through the core removing the heat. As a moderator, it slows down the fission neutrons so that the neutrons have a better chance to induce fission in the fissile isotopes. Because of these dual functions of the water in the nuclear reactor core, there is intimate interaction between the neutron kinetics and the thermal hydraulics.

Measurements, such as the neutron flux, core-exit coolant temperature and pressure are necessary to monitor the status of the reactor. These measurements are made in the steady state or in the transient state. Even when the system is in steady operation, a closer examination of the measurements will show that superimposed

upon the steady output are some fluctuations about the average. In nuclear reactors, we usually refer to these normal fluctuations about the average value as noise signals. Noise analysis is a technique to retrieve useful dynamic information from the random signals.

The reactor noise could be caused by fluctuating driving sources (hereafter called disturbances) in the process, or could be attributed to the contaminations in the measurements. When the noise is induced by the driving sources in the process, it will contain the process dynamic information and will be useful for the surveillance and diagnostics of system anomalies. Compared with other surveillance and diagnostics techniques, such as the periodic test, noise analysis has the advantage of not interfering with the system operation. Therefore many efforts have been made to investigate the information contained in the noise data.

Until the late 1950's, noise analysis research and development in nuclear reactors was restricted to zero power research reactors [1,2]. However, during the past two decades, significant research has been devoted to the field of power reactor noise, both to improve the theoretical understanding [3] and to develop useful tools for the nuclear industry. The applications include monitoring core barrel motion, instrument tube vibration, fuel motion, stability, steam void velocity, boiling and loose parts [4,5,6]. In the early 1970's, most of the efforts in interpreting the noise data were concentrated on the neutron noise [7,8,9]. Recently, a variety of process

variables, such as temperature [10,11] and pressure [12,13], have been studied. The use of process signals in addition to the neutron noise might improve the usefulness of the noise analysis.

In this research work, we concentrate on analyzing and interpreting the core-exit coolant temperature (TOUT) and the neutron power (N) noises in PWRs at the low frequency region (<2 Hz), and explore the applicability of using these signals to estimate the coolant flow rate and monitor the moderator temperature coefficient of reactivity.

Generally, in PWRs, the in-core instrumentation system consists of thermocouples which measure fuel assembly coolant outlet temperature at preselected locations, and neutron flux sensors that could run along the whole length of selected fuel assemblies. Neutron flux is also measured with the fixed out-of-core neutron detectors. The design calls for approximately one third of the fuel assemblies to have either thermocouples or neutron flux sensors or both [14]. The data obtained from the in-core temperature and flux distribution instrumentation system along with analytical information can be used to determine the core power distribution and estimate the coolant flow distribution. This existing technique uses the average measurement values.

Like most of the measurements, the neutron power and core-exit temperature measurements have some fluctuations around the average values. In addition to the contaminations in the measurement instruments, these noises are believed to be due to the following

possible driving sources: (1) the inlet coolant temperature disturbance, (2) the coolant flow rate (or coolant velocity) disturbance, (3) the random heat transfer processes caused by turbulent flow (hereafter called random heat transfer disturbance), and (4) the reactivity-like driving source, that causes neutron power fluctuation and then affects the coolant temperature (hereafter called heat source disturbance). Since in a PWR, the thermal hydraulics and the neutron kinetics are strongly coupled, the driving sources that cause the core-exit coolant temperature fluctuations might also contribute largely to the neutron power fluctuations. The analysis of both the core-exit temperature and the neutron power noises might be able to get some system dynamic information, such as local flow rate and moderator temperature coefficient of reactivity. This information might be used to monitor anomalies such as blockage, inadequate core cooling and positivity of the moderator temperature coefficient of reactivity.

The main subjects of this dissertation are the analyses of the mechanisms involved in the neutron power and core-exit coolant temperature noises, and the investigation of the algorithms for flow estimation and moderator temperature coefficient monitoring using noise signals. The research work is separated into two areas: one is concerned with the algorithm in the frequency domain, where the usefulness of the phase shift between the core-exit coolant temperature and neutron power noise signals is investigated; the other is for the analysis in the real-time domain (stochastic estimator), where the

feasibility of estimating the coolant flow rate using a stochastic estimator is studied.

1.2 Previous Work, Motivation and Research Areas

The previous works related to the application of the coolant temperature noise analysis are few. Bentley [15,16] studied the properties of coolant temperature noise in fast reactor subassemblies. Ohlmer and Schwalm [17,18] made use of the amplitude probability density function of temperature noise for partial blockage detection in a simulated LMFBR subassembly. Fry and Leavell [11] conducted experimental studies to evaluate the changes in the root-mean-square values and temperature noise amplitude probability density as a function of size and location of blockage in a sodium-cooled heated bundle. Krebs and Weinkotz [19] performed experiments to detect boiling in a sodium-cooled fuel subassembly using thermocouples. The experiences with applications of temperature noise in PWRs is limited. Tsunoda [20] conducted tests using an out-of-pile water loop with an electrically heated section. Sweeney and Upadhyaya [21] and Katona et al.[22] made use of the observed linear phase frequency behavior between the neutron and core-exit coolant temperature noises to estimate the core coolant flow rate in operating PWRs.

1.2.1 Core dynamics study in the frequency domain

1.2.1.1 Flow rate estimation using phase slope

Analysis of the experimental data from operating PWRs shows that the phase shift between the core-exit coolant temperature and

neutron power fluctuations (denoted as $\text{PHASE}(\text{TOUT}, \text{N})$), in the 0.1 to 2 Hz range has a linear phase frequency behavior [10,21,22], and that the slope of $\text{PHASE}(\text{TOUT}, \text{N})$ is related to the coolant velocity or flow rate. In order to use this phase slope as an index for core flow monitoring, we need to know the dynamic mechanisms involved. Several papers [10,21,22,23] have surveyed this problem.

In the paper by Katona et al.[22], a distributed parameter model was developed to relate the coolant velocity with the slope of $\text{PHASE}(\text{TOUT}, \text{N})$ in the Borssele nuclear power plant. In that paper, the neutron power fluctuation due to reactivity driving source was assumed to be the dominant disturbance source causing the core-exit coolant temperature fluctuations. However, the calculated results are not in good agreement with those from the experimental data. Other investigators indicated that power fluctuations and inlet coolant temperature fluctuations are the possible disturbance sources [3,10,21,23]. Canon and Clemo [24] observed a high coherence between the neutron flux and the core flow rate in the 0-2 Hz range in the Loss-of-Fluid Test (LOFT) reactor. Canon and Clemo concluded that the power fluctuation was caused by the fluctuation in the primary coolant flow. The previous results suggest that several disturbance sources will give rise to core-exit coolant temperature fluctuation. Further study is necessary to determine the effect of each disturbance on the noise dynamics.

In this dissertation, the study is aimed at answering the following questions:

- (1) Why is there a linear phase-frequency relationship between the core-exit coolant temperature and the neutron power fluctuations in the 0.1-2 Hz region?
- (2) What is the actual relationship between the slope of phase and coolant velocity (or coolant flow rate)?
- (3) Which fluctuation sources contribute to this relationship?
- (4) What factors affect this relationship?

Because different PWRs may have different dynamic mechanisms, most of our effort focuses on the LOFT reactor, where more complete experimental data are available.

Instead of using only the distributed parameter model as in reference [22], three kinds of physical models are developed to investigate the dynamics: the phenomenological model, the distributed parameter model [25], and the multi-nodal model [25]. The phenomenological model is approximate but gives a good insight into the dynamic mechanisms involved. The nodal model can simulate the real system with fewer assumptions; however, its results in the higher frequency region could be distorted due to the limited number of nodes. This drawback can be compensated using the result from the distributed parameter model. Because of its analytical nature, the distributed parameter model has difficulty in including complicated feedback interaction fully, so the result may be insufficient in the low

frequency range where feedback is important. Hence each model gives some insight into the system and helps to reinforce the results obtained from the other models. The multi-nodal model results are finally compared with those from the LOFT reactor experimental data.

1.2.1.2 Monitoring moderator temperature coefficient of reactivity using the extrapolated phase at zero frequency

The moderator temperature coefficient of reactivity is an important parameter related to the safety of pressurized water reactors (PWRs) and it is required that this parameter be negative for plant operation. For safe reactor operation, an overall negative moderator temperature coefficient is desired [26]. However, due to the boron concentration in the coolant, the moderator temperature coefficient of reactivity may become positive especially at the beginning of the fuel cycle when the boron concentration is high. But it is difficult to measure the moderator temperature coefficient directly [27]. A technique which is able to ensure directly the negativity of this coefficient will be valuable.

The analyses of neutron detector and core-exit temperature noise signals from three different PWRs--the 55 MW(t) LOFT reactor, an 1140 MW(e) commercial PWR [28], and the 477 MW(e) Borssele PWR [29] show that all three reactors exhibit a linear phase-frequency relationship between these two signals in the frequency range 0.1-2 Hz. The phase at zero frequency extrapolated from this linear phase-frequency region is -180 degrees and is an out of phase relationship. This out of phase relationship is speculated to be due to the negative

value of the moderator temperature coefficient of reactivity.

Conversely, if this parameter were to go positive, the extrapolated phase at zero frequency would approach the zero degree phase. The nodal model is used to investigate the viability of monitoring the sign of the moderator temperature coefficient using this extrapolated phase at zero frequency.

1.2.2 Stochastic estimator in the real-time domain

Most of the emphasis concerning physical parameter estimation in a nuclear power plant is on indirect methods [30]: the real-time raw data are first reduced to power spectral density (PSD) [31], correlation function [32] and transfer function [33]. Then for a given model, the parameter values are estimated by least-squares fitting using the reduced data. Using these methods, data averaging is necessary in order to obtain good statistical properties, and therefore the data must be stationary. For an intrinsically stationary type monitoring problem, such as monitoring of structural vibrations, this kind of technique has been shown to be effective and powerful. However, the environment for flow monitoring is quite different. Anomalies in flow could lead to transient conditions; as a tool for flow monitoring, we might prefer an algorithm that can be applied in both steady and transient conditions so that more timely information can be obtained.

The state and parameter estimation technique [34,35,36,37] based on the stochastic model in the real-time domain has the potential to serve this purpose. Among the stochastic estimators for a

linear estimation problem, the Kalman filter is a well known optimal estimator. However, a combined state and parameter estimation is equivalent to a nonlinear estimation problem, even with a linear system model [38]. For a nonlinear estimation problem, the optimal estimator can be formulated with the maximum likelihood technique [39]. But this formulation is numerically cumbersome, and without approximation, it cannot be solved efficiently. Therefore, for the nonlinear problem, the suboptimal estimator with proper approximations is a logical choice. One of the practical suboptimal estimators is the extended Kalman filter (EKF) [37].

The EKF has been widely applied in other engineering disciplines [40,41]. In nuclear engineering, the potential applications in a nuclear power system have been reviewed [42], but the real applications are few [43,44,45]. Both Godbole [43] and Suda, et al. [44] applied the EKF in a zero power reactor for neutron kinetics parameters estimation. Tylee [43] applied the EKF for fuel-clad gap conductance estimation using the transient data from the LOFT reactor.

Some other authors applied the Kalman filter for state estimation in nuclear systems: Fryer and Yarbrough [46] and Griffith [47] applied the Kalman filter for core power estimation; Kitamura [48] applied the technique for analytical redundancy; Clark and Campbell [49] used it for instrument fault detection; and Meek and Doerner [50] showed that use of the Kalman filter to estimate gap conductance is a promising approach.

In this dissertation, we study the feasibility of estimating

the flow rate using the core-exit coolant temperature, and neutron power noise signals using the stochastic estimator. The study is further divided into two parts:

- (1) Develop the EKF for flow rate estimation, and test the algorithm with simulated data and the actual data from the LOFT reactor to justify the capability.
- (2) Investigate the characteristics of the maximum likelihood estimator for flow rate estimation using the ambiguity function analysis technique.

In the first part, a reduced order model was established. Then the EKF was developed with proper approximations to speed up the calculation. Both the simulated data and the real noise data from the LOFT reactor at 65% and 100% flow operation conditions were used to justify the capability of the EKF to monitor the flow rate changes. We noticed that the estimation of flow rate using the EKF is sensitive to other system parameters such as the moderator temperature coefficient of reactivity. In order to have a general understanding of the characteristics of the stochastic estimator for flow rate estimation, the ambiguity function analysis [39] technique is applied. The ambiguity function is a scalar function of the following: (1) the theoretical probability distribution function of the state variables with the assumed true model, and (2) the likelihood function used for estimation. The maximum point in the ambiguity function shows the possible estimation value from the maximum likelihood estimator (an optimal estimator), and the curvature at the maximum point is

inversely proportional to the Cramer-Rao lower bound, which is the minimum achievable error covariance of the estimation. Based on the simplified low order model, the characteristics of the optimal filter for flow monitoring, especially the sensitivity of the estimator to the other parameters in the model, is studied. The result of ambiguity function analysis serves qualitatively as a reference of the attainable goal for all the suboptimal filters including the EKF.

1.3 Original Contributions of This Dissertation

The original contributions of this dissertation are summarized below.

1.3.1 Investigation of the core dynamics

1.3.1.1 Flow rate estimation

Several physical models are developed to investigate the dynamic mechanisms relating the core-exit coolant temperature and neutron power noises, especially the phase shift between them in the low frequency region. The important original results are the following:

- (1) The processes involve the integral of a pure delay mechanism and indicate a linear phase-frequency relationship, but only within certain frequency region with lower and upper bounds.
- (2) The upper bound is the sink or peak frequency (with minimum or maximum gain), as a result of integration. This

upper bound is a function of the driving sources, neutron flux shapes and coolant transport time through the core.

- (3) The phase slope is a function of the driving source, flux shape and the coolant transport time through the core, and not a function of the neutron kinetics parameters, such as the temperature-reactivity coefficient, the neutron generation time, and the delayed neutron fraction.
- (4) With a negative moderator temperature coefficient, for all driving sources considered, the linear phase approaches -180 degrees at zero frequency except for the heat source disturbance.
- (5) If the random heat transfer disturbance is the driving source, the coherence between the core-exit coolant temperature and neutron power fluctuations will be very small under the point reactor kinetics.
- (6) In the LOFT reactor, the coolant flow rate disturbance is the primary driving source for the core-exit coolant temperature and neutron power noises.

1.3.1.2 Moderator temperature coefficient of reactivity monitoring

Comparison of the modeling results with that from the experimental data indicates clearly that the heat source disturbance can be excluded as a possible disturbance source affecting neutron and temperature signal fluctuations in the 0.1 to 2 Hz region. For the other types of disturbance sources, when the moderator temperature

coefficient is negative, $\text{PHASE}(\text{TOUT}, \text{N})$ at zero Hz, extrapolated from the linear phase-frequency region, is -180 degrees. When the moderator temperature coefficient of reactivity is positive, the model predicts that the extrapolated phase at zero Hz is zero degrees. This is a new approach for monitoring the sign of the moderator temperature coefficient of reactivity.

1.3.2 The extended Kalman filter for core mass flow rate estimation

In applying the EKF for core coolant mass flow rate estimation using noise data, the originality lies in the following two aspects:

(1) A new algorithm was developed to avoid the time consuming integration in the basic EKF algorithm for time-continuous models. This new algorithm uses a discretized model, but instead of discretizing the model at each new estimation step (this could be time consuming also), a sensitivity function for the discretized model with respect to the variation of state variables was calculated. Then, based on the first order approximation, the updating of the discretized model can be readily achieved. The bias caused by this first order approximation is corrected by relinearization.

(2) A reduced order model was developed for use in the EKF algorithm for flow monitoring. After proper tuning and subsequent application to the 65% and 100% flow condition noise data from the LOFT reactor, the algorithm was able to monitor the flow rate changes.

1.3.3 The study of the maximum likelihood estimator characteristics using ambiguity function analysis

This is the first work (to the author's knowledge) in surveying the characteristics of the optimal stochastic estimator for nuclear reactor noise analysis, especially for flow estimation, by using the ambiguity function analysis technique.

The result shows that the maximum likelihood estimator will be able to estimate the flow rate. But in comparison to the phase slope method, the estimation of flow rate using the maximum likelihood estimator is very sensitive to the variation of moderator temperature coefficient of reactivity.

1.4 Organization of the Dissertation

In Section 2, the phenomenological models are developed. The results show some simple formulas for $\text{PHASE}(\text{TOUT}, N)$ with different flux shapes, driving sources and neutron kinetics. These models are very helpful in understanding the mechanisms involved in the core-exit coolant temperature and neutron power noises. In Section 3, we further develop a distributed parameter model with point reactor kinetics under some assumptions; this model gives a more precise and complete description of the characteristics of $\text{PHASE}(\text{TOUT}, N)$. The assumptions made in Sections 2 and 3 are verified by using the multi-nodal model developed in Section 4. In the multi-nodal model, a complete neutron kinetics and thermal hydraulics interaction is included. The applicability of the extrapolated $\text{PHASE}(\text{TOUT}, N)$ at 0 Hz

for monitoring the sign of the moderator temperature coefficient is also studied in Section 4. The comparison between the theoretical results and those from the experimental data of the LOFT reactor is shown in Section 5. In this chapter, the PHASE(TOUT,N) data of the Borssele and Sequoyah-1 PWR nuclear power plants are reviewed briefly, and a summary and discussion of the above analyses is presented. Then the study in this dissertation is devoted to the application of the stochastic estimator for flow monitoring. The EKF is developed in Section 6; the algorithm is tested using the simulated data and then applied to analyze the data from the LOFT reactor. Section 7 discusses the characteristics of the maximum likelihood estimator using the ambiguity function analysis technique. Finally, conclusions and recommendations are presented in Section 8.

2. PHENOMENOLOGICAL MODEL

2.1 Introduction

The physics involved in the neutron power and core-exit coolant temperature fluctuations is shown schematically in Figure 2-1, where four kinds of disturbance sources are considered: the heat source disturbance (e.g., reactivity fluctuation) denoted as $\delta\rho'$; the inlet coolant temperature disturbance, denoted as δT_{IN} ; the coolant velocity disturbance, denoted as δv ; and the random heat transfer disturbance, denoted as δh . The entire system forms a closed feedback loop.

To illustrate the mechanism involved, we start from the fluctuations $\delta\rho'$ and $\delta\rho$, where $\delta\rho$ is the reactivity fluctuation caused by temperature reactivity feedback. The fluctuation of reactivity affects neutron power (δN) through core neutron kinetics. Then the heat generated in the fuel transfers through the cladding to the coolant. In this heat transfer process, the heat transfer coefficient could vary due to the fluctuation in the coolant velocity. This coolant velocity fluctuation is considered as combining two components: one is mainly in the axial direction with simultaneous coolant velocity changes along the whole channel (δv), called the coolant velocity disturbance or coolant flow rate disturbance; the other is the local velocity variation in all directions as in turbulence flow, which causes spatially uncorrelated heat transfer coefficient changes (hence called the random heat transfer disturbance

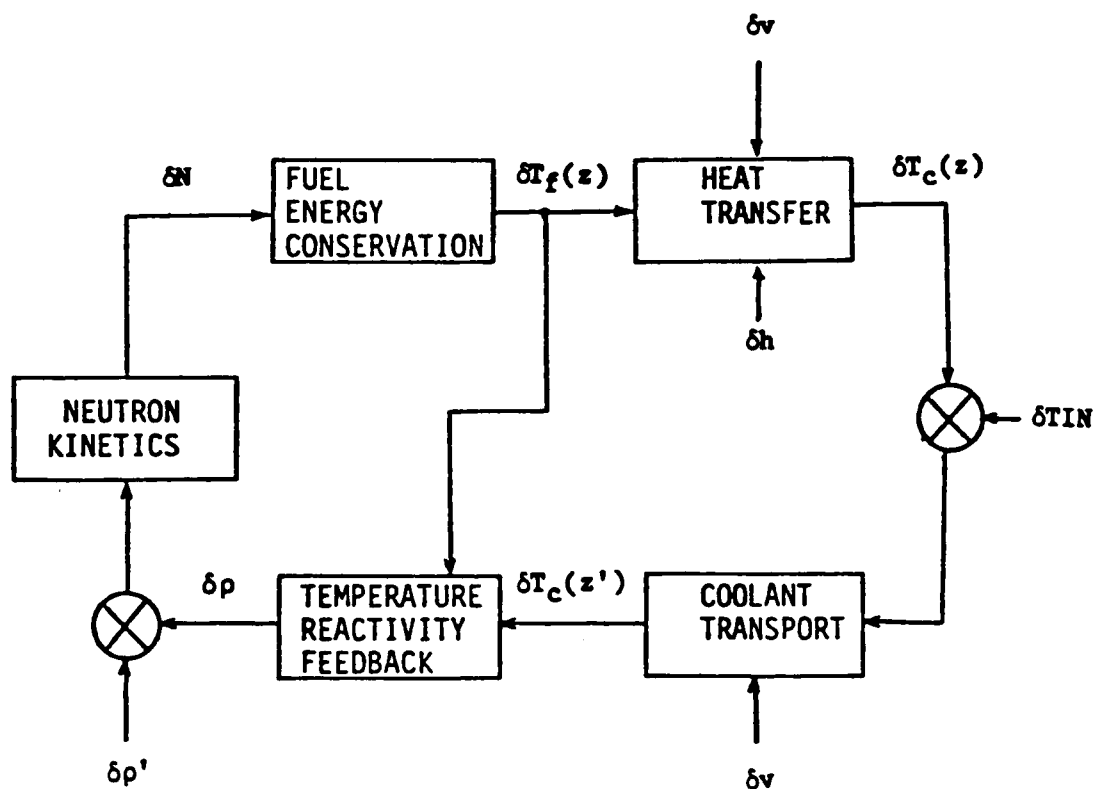


Figure 2-1. Schematic diagram of the mechanisms involved in the neutron power and core-exit coolant temperature fluctuations.

(δh)). The fluctuation of the fuel temperature (δT_f) and the variation of coolant velocity and heat transfer coefficient contribute to the fluctuation of coolant temperature, $\delta T_c(z)$. The other contribution to the coolant temperature fluctuation is the coolant inlet temperature disturbance (δT_{IN}). These coolant temperature fluctuations are transported from upstream (the core inlet) to downstream (the core outlet). The coolant velocity fluctuation causes variation in the coolant transport process and hence the variation of heat removal rate. Finally, both the temperature fluctuations in the fuel and in the coolant induce reactivity fluctuation through temperature-reactivity feedback. This completes a closed feedback loop.

In this dissertation, we are mainly concerned with the noise data taken from the LOFT pressurized water reactor. The LOFT reactor is a 1/55 scale model of a typical PWR. In the LOFT reactor, the neutron noises taken from the neutron detectors at different locations in the core show very little space dependent behavior; hence the point reactor kinetics model is a good approximation for the neutronics.

Before developing more sophisticated models for the whole system, some phenomenological models for each specific disturbance source are described in this chapter.

Phenomenological models under proper assumptions are helpful for understanding the mechanisms involved. The results from these simple models can be used to compare with those from more complicated models, such as a distributed parameter model or a multi-nodal model, and thus avoid inadvertent errors. Most of the assumptions

made in this chapter will be justified using the multi-nodal model in Section 4. The phenomenological models are developed with the following purposes:

- (1) To check the existence of linearity of the phase-frequency relationship between the core-exit temperature and neutron power noise signals.
- (2) To investigate the relationship between the slope of $\text{PHASE}(\text{TOUT}, N)$ and the coolant velocity, and search for simple expressions for the phase slope.
- (3) To examine the sink frequencies, where the gain between the core-exit coolant temperature and the neutron power fluctuations is minimum. (refer to Appendix A)

Four kinds of disturbance sources with cosine and flat flux shapes are investigated.

The general assumptions are as follows:

- (1) The point reactor kinetics is used.
- (2) The heat transfer from fuel to coolant takes place in the radial direction only.
- (3) The coolant temperature fluctuation propagates through the coolant channel with pure delay mechanism, i.e.,

$$\delta T_c(t, z') = \delta T_c(t - (z' - z)/v, z) + n, \quad (2-1)$$

where

z', z : axial position,

$z' > z,$

$\delta T_c(t, z')$: coolant temperature fluctuation at position z'
and time t ,

v : coolant velocity,

n : coolant temperature fluctuation caused by
disturbance between z and z' .

The real coolant temperature fluctuation propagation is a mechanism that is neither pure delay nor well-mixed. But after comparison with the phase shift results from the distributed parameter model and the nodal model, we find that pure delay mechanism is a good approximation in the frequency range where the phase-frequency relationship is linear.

- (4) The feedback process is neglected as described in the following sections for individual disturbance sources.
- (5) The fuel temperature reactivity effect is neglected.
- (6) The coolant temperature reactivity effect is calculated using first order perturbation theory and assuming self-adjointness [51], that is

$$\delta \rho(\omega) = \alpha_c \int_0^L \delta T_c(\omega, z) f(z)^2 dz, \quad (2-2)$$

where

ω : radial frequency,

$\delta \rho(\omega)$: reactivity fluctuation,

α_c : coolant or moderator temperature coefficient
of reactivity,

L : fuel length,

$\delta T_c(\omega, z)$: coolant temperature fluctuation at position z ,

$f(z)$: normalized neutron flux shape with $\int_0^L f(z)^2 dz = 1$.

(7) In the 0.1 to 2 Hz frequency region, the phase between the fluctuations of reactivity and neutron power is zero (the real case may have a deviation of about ten degrees) and the gain is constant and equal to N_0/β , where N_0 is the average neutron power, and β is the delayed neutron fraction. This is to assume that 0.1 to 2 Hz is in the plateau region of the neutron/reactivity transfer function [52].

(8) The direct heat, fission energy directly deposited in the coolant, is negligible.

In the following sections, the phenomenological models for each disturbance source will be discussed separately.

2.2 Heat Source Disturbance

A heat source disturbance includes all the disturbances that affect the neutron flux first, such as reactivity fluctuation, and then through heat transfer processes affect the core-exit coolant temperature, such as reactivity fluctuation. The schematic diagram of the model, Figure 2-2, shows the relationship between neutron flux and core-exit coolant temperature signals in the frequency domain. The reactivity fluctuation, $\delta\rho(\omega)$, affects the neutron power, $\delta N(\omega)$, with the transfer function between them being $G(\omega)$, where ω is the

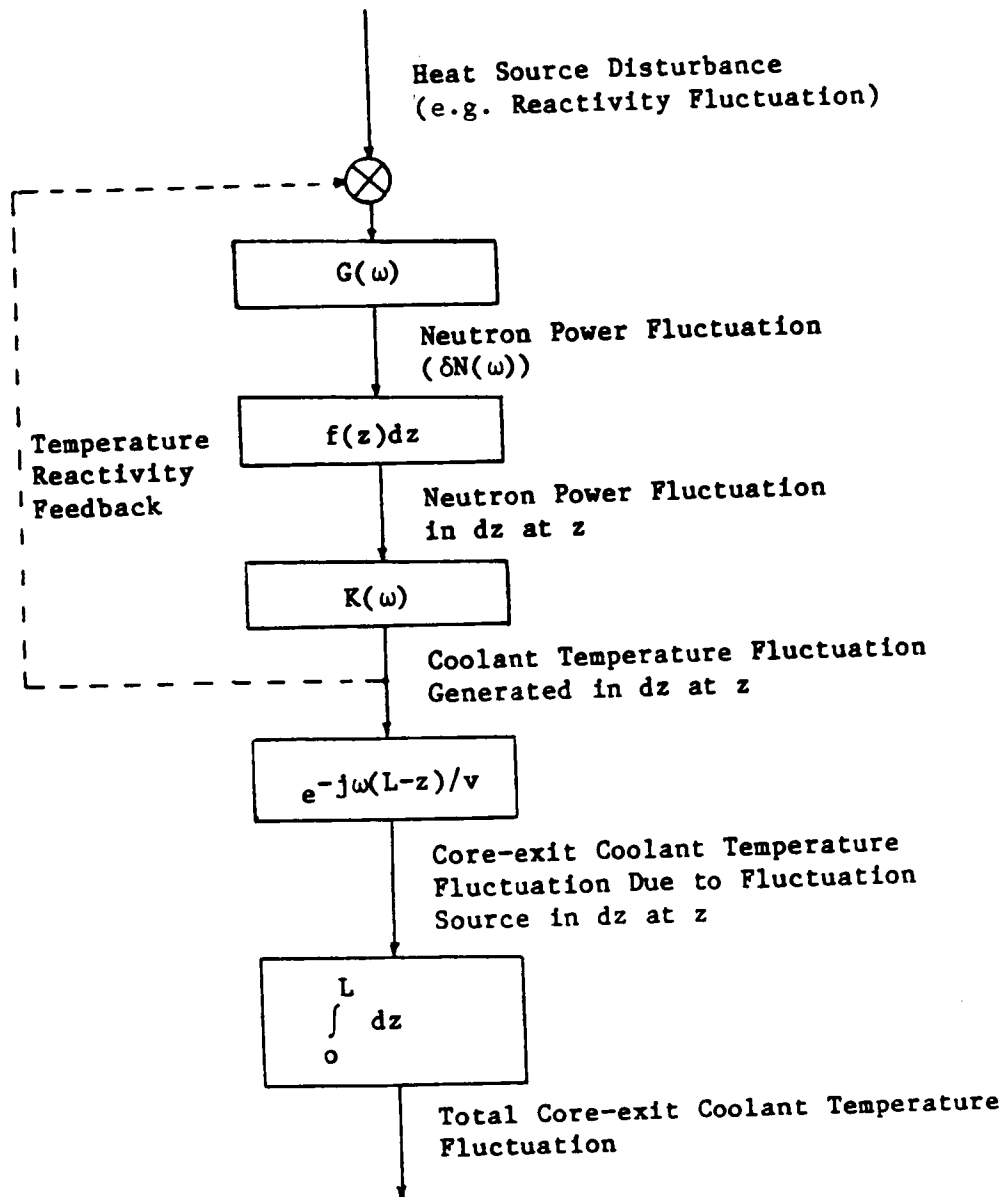


Figure 2-2. The phenomenological model with the heat source disturbance.

frequency. The axial heat generation distribution is $\delta N(\omega)f(z)$, where $f(z)$ is the flux shape. The heat generated in the fuel at position z in dz differential length is $\delta N(\omega)f(z)dz$. This heat is radially transferred from fuel to coolant, with the transfer function between them being $K(\omega)$, and generates a coolant temperature fluctuation source in dz at z . This coolant temperature fluctuation source transports to the core exit through the pure delay mechanism, $\exp(-j\omega(L-z)/v)$, where L is the fuel length and v is the coolant velocity. The contributions from the coolant temperature fluctuation sources at different positions along the whole coolant channel are then summed together by integration.

Because of the long time constant (about five seconds) [52] for the heat transfer process from fuel to coolant, at frequencies above 0.1 Hz, the temperature fluctuation is significantly attenuated. Therefore we assume that the coolant temperature reactivity feedback is negligible above 0.1 Hz.

From Figure 2-1 we get

$$\delta T_c(\omega, L) = \int_0^L \delta N(\omega) K(\omega) f(z) e^{-j\omega(L-z)/v} dz . \quad (2-3)$$

Rearrange this equation to obtain

$$\frac{\delta T_c(\omega, L)}{\delta N(\omega)} = K(\omega) \int_0^L f(z) e^{-j\omega(L-z)/v} dz . \quad (2-4)$$

Eq.(2-4) is then integrated with flat flux shape and cosine flux shape.

(1) Flat flux shape

For the flat flux shape, $f(z)$ is constant. After performing the integration, Eq.(2-4) becomes

$$\frac{\delta T_c(\omega, L)}{\delta N(\omega)} = K(\omega)(2v/\omega)\sin(\omega L/2v)e^{-j(\omega L/2v)} \quad , \quad (2-5)$$

Hence,

$$\text{PHASE}(\text{TOUT}, N) = -(\omega L/2v) + C, \quad (2-6)$$

$$\text{when } \sin(\omega L/2v) \geq 0 \quad ,$$

and,

$$\text{GAIN}(\text{TOUT}, N) = |K(\omega)(2v/\omega)\sin(\omega L/2v)| \quad . \quad (2-7)$$

Because the break frequency of $K(\omega)$ is much less than 0.1 Hz, here we assume that in 0.1 to 2 Hz frequency range the phase of $K(\omega)$ is constant (equal to C).

Letting $\text{GAIN}(\text{TOUT}, N)=0$, we get the sink frequencies

$$f_s = v/L, 2v/L, 3v/L, \dots$$

Notice that before and after the sink frequencies the sign of $\sin(\omega L/2v)$ is altered and $\text{PHASE}(\text{TOUT}, N)$ has 180 degrees jump.

(2) Cosine flux shape

After integration, Eq.(2-4) becomes

$$\frac{\delta T_c(\omega, L)}{\delta N(\omega)} = K(\omega)(L\omega/v)[2/(1-(\omega L/\pi v)^2)]\cos(L\omega/2v)e^{-j(\omega L/2v)} \quad . \quad (2-8)$$

$$\text{PHASE}(\text{TOUT}, N) = -(\omega L/2v) + C \quad , \quad (2-9)$$

$$\text{when } [2/(1-(\omega L/\pi v)^2)]\cos(L\omega/2v) \geq 0 \quad ,$$

and,

$$\text{GAIN}(\text{TOUT}, N) = \left| K(\omega)(L\omega/v) [2/(1-(\omega L/\pi v)^2)] \cos(L\omega/2v) \right| \quad (2-10)$$

Let $\text{GAIN}(\text{TOUT}, N)=0$, we get the sink frequencies

$$f_s = 3v/2L, 5v/2L, 7v/2L, \dots$$

Again before and after the sink frequencies the sign of $[2/(1-(\omega L/\pi v)^2)] \cos(L\omega/2v)$ changes, and $\text{PHASE}(\text{TOUT}, N)$ has 180 degrees jump. The phase always changes sharply around the sink frequencies.

Because the magnitude of $\delta N(\omega)$ is never infinite, the sink frequencies of $\delta T_c(\omega, L)/\delta N(\omega)$ are also the sink frequencies for $\delta T_c(\omega, L)$.

2.3 Coolant Inlet Temperature Disturbance

Figure 2-3 shows the schematic diagram of the model. The inlet coolant temperature disturbance ($\delta \text{TIN}(\omega)$) propagates along the coolant channel with a pure delay mechanism, $\exp(-j\omega z/v)$, and through the coolant temperature-reactivity effect causes the fluctuations of the reactivity and the neutron power. This is calculated using the first order perturbation theory and assuming self-adjointness. Then we assume that the feedback influence of the induced fluctuation of neutron flux on the coolant temperature is negligible based on the same reasoning as in Section 2.2, that above 0.1 Hz the fluctuation is significantly attenuated. Therefore we have

$$\delta T_c(z, \omega) = \delta \text{TIN}(\omega) e^{-j(\omega z/v)} \quad (2-11)$$

According to Eq.(2-2),

$$\delta \rho(\omega) = \alpha_c \int_0^L \delta \text{TIN}(\omega) e^{-j(\omega z/v)} f(z)^2 dz \quad (2-12)$$

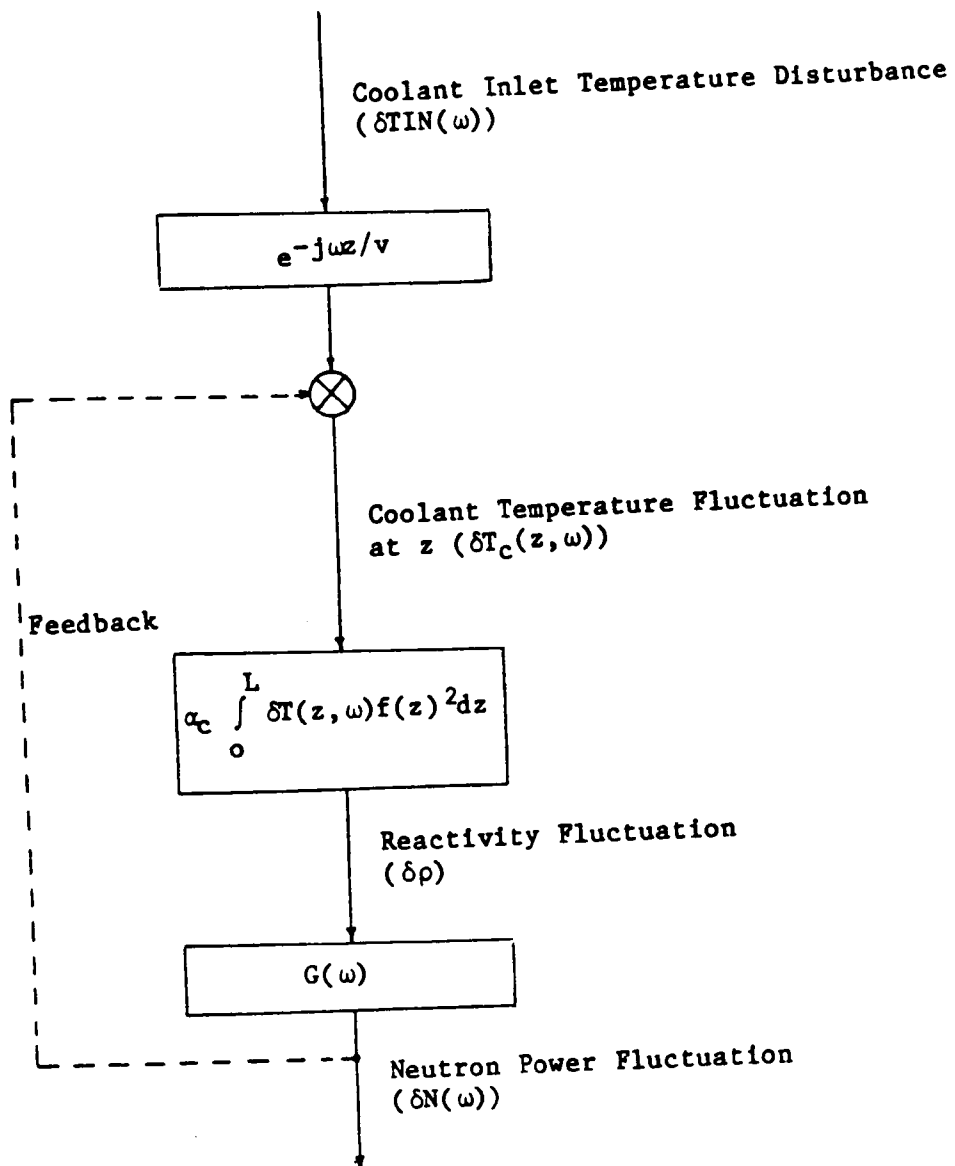


Figure 2-3. The phenomenological model with the coolant inlet temperature disturbance.

And because we assume that in the 0.1 to 2 Hz region,

$\delta N(\omega)/\delta \rho(\omega) = N_0/\beta$, and $\delta N(\omega) = N_0 \delta \rho(\omega)/\beta$, we obtain

$$\frac{\delta T_c(\omega, L)}{\delta N(\omega)} = \beta / [N_0 \alpha_c \int_0^L e^{-j\omega(z-L)/v} f(z)^2 dz] . \quad (2-13)$$

(1) Flat flux shape

Integrate Eq.(2-13) to get

$$\frac{\delta T_c(\omega, L)}{\delta N(\omega)} = e^{-j(\omega L/2v)} \beta / [N_0 \alpha_c (2v/\omega) \sin(\omega L/2v)] . \quad (2-14)$$

$$\text{PHASE}(\text{TOUT}, N) = -(\omega L/2v) - \pi , \quad (2-15)$$

when $\sin(\omega L/2v) > 0$.

Here $-\pi$ is due to the negative value of α_c .

$$\text{GAIN}(\text{TOUT}, N) = \left| \beta / [N_0 \alpha_c (2v/\omega) \sin(\omega L/2v)] \right| . \quad (2-16)$$

From Eq.(2-16) we see that instead of sink frequencies (at which gain is zero), there are peak frequencies (at which gain is infinite). These peak frequencies are

$$f_p = v/L, 2v/L, 3v/L, \dots$$

(2) Cosine flux shape

Integrate Eq.(2-13) to get

$$\frac{\delta T_c(L, \omega)}{\delta N(\omega)} = e^{-j(\omega L/2v)} \beta / [N_0 \alpha_c (1/(4-(\omega L/\pi v)^2)(4v/\omega) \sin(\omega L/2v))], \quad (2-17)$$

and,

$$\text{PHASE}(\text{TOUT}, N) = -(L\omega/2v) - \pi , \quad (2-18)$$

when $\sin(\omega L/2v)/[4-(\omega L/\pi v)^2] > 0$,

$$\text{GAIN}(\text{TOUT}, N) = \left| \beta / [N_0 \alpha_c (1 / (4 - (\omega L / \pi v)^2) (4v / \omega) \sin(\omega L / 2v))] \right| . \quad (2-19)$$

$$f_p = 2v/L, 3v/L, 4v/L, \dots$$

Because $\delta T_c(L, \omega)$ is not infinite, the peak frequencies of $\delta T_c(L, \omega) / \delta N(\omega)$ are also the sink frequencies for $\delta N(\omega)$.

2.4 Coolant Velocity Disturbance

For the coolant velocity disturbance, we consider simultaneous change of the coolant velocity along the whole coolant channel. The effect of a coolant velocity disturbance on coolant temperature is through two processes. First, the coolant velocity disturbance affects the film heat transfer coefficient and hence changes the heat transfer rate from the fuel to the coolant. Second, the coolant velocity disturbance affects heat removal rate. Both processes have the same influence on the changes in the coolant temperature along the whole channel. In order to simplify, here we consider only the second process.

Let QC be the total heat transferred to the coolant, and $QC \cdot f(z)$ be the heat transferred to the coolant per unit length at position z ; the relationship between the heat transferred to the coolant and the increase of coolant temperature is

$$QC \cdot f(z) dz = C_p A_c v \rho \Delta T_c(z) , \quad (2-20)$$

where,

C_p : specific heat capacity of the coolant,

A_c : flow area,

v : coolant velocity,

ρ : coolant density,

$\Delta T_c(z)$: coolant temperature increase in dz at z .

With the coolant velocity disturbance δv , the variation of coolant temperature increase in dz at z is $\delta T_{cs}(\omega, z)$. To avoid confusion, the notations for coolant temperature are repeated here:

$\Delta T_c(z)$ is the coolant temperature increase in dz at z ; $\delta T_{cs}(\omega, z)$ is the variation of the coolant temperature increase in dz at z due to the disturbance, and is the coolant temperature fluctuation generated at z in dz ; $\delta T_c(\omega, z)$ is the accumulated coolant temperature fluctuation at z . Assume that the heat transferred to the coolant per unit length is still the same, then we have

$$C_p A_c (v + \delta v(\omega)) \rho (\Delta T_c(z) + \delta T_{cs}(\omega, z)) = QC \cdot f(z) dz . \quad (2-21)$$

Combine Eqs.(2-20) and (2-21) and neglect the second order perturbation terms to get

$$C_p A_c \delta v(\omega) \rho \Delta T_c(z) = -C_p A_c v \rho \delta T_{cs}(\omega, z) . \quad (2-22)$$

So

$$\delta T_{cs}(\omega, z) / \Delta T_c(z) = -\delta v(\omega) / v . \quad (2-23)$$

Calculate $\Delta T_c(z)$ using Eq.(2-20) and insert into Eq.(2-23) to get the relationship between $\delta T_{cs}(\omega, z)$ and $\delta v(\omega)$. This is

$$\begin{aligned} \delta T_{cs}(\omega, z) &= -(\delta v(\omega) / v) \Delta T_c(z) \\ &= -(\delta v(\omega) / v) [QC \cdot f(z) dz / (C_p A_c v \rho)] . \end{aligned} \quad (2-24)$$

$\delta T_{cs}(\omega, z)$ is the coolant temperature fluctuation generated at

z in dz due to coolant velocity disturbance. This temperature fluctuation will propagate downstream, where the contributions from all the upstream fluctuation sources are summed together. This accumulated coolant temperature fluctuation at axial position x can be calculated as follows:

$$\begin{aligned}
 \delta T_c(\omega, x) &= \int_{z=0}^{z=x} \delta T_{cs}(\omega, z) e^{-j\omega(x-z)/v} \\
 &= - \int_0^x (\delta v(\omega)/v^2) [QC \cdot f(z)/(C_p A_c \rho)] e^{-j\omega(x-z)/v} dz \\
 &= [-\delta v(\omega) \cdot QC/(v^2 C_p A_c \rho)] \int_0^x f(z) e^{-j\omega(x-z)/v} dz .
 \end{aligned}
 \tag{2-25}$$

To calculate $\delta N(\omega)$, use Eq.(2-2) and assumption (7) in Section 2.1 to get

$$\begin{aligned}
 \delta N(\omega) &= (N_0 \alpha_c / \beta) \int_0^L \delta T_c(\omega, x) f(x)^2 dx \\
 &= -[N_0 \alpha_c \delta v(\omega) \cdot QC / (\beta v^2 C_p A_c \rho)] \\
 &\quad \int_0^L f(x)^2 \int_0^x f(z) e^{-j\omega(x-z)/v} dz dx .
 \end{aligned}
 \tag{2-26}$$

The effect of the neutron power fluctuation induced by the primary coolant temperature fluctuation and the subsequent temperature-reactivity feedback are neglected based on the same reasoning as in Section 2.2. Using Eqs.(2-25) and (2-26), we can calculate the transfer function between $\delta T_c(\omega, L)$ and $\delta v(\omega)$, the

transfer function between $\delta N(\omega)$ and $\delta v(\omega)$, and the transfer function between $\delta T_c(\omega, L)$ and $\delta N(\omega)$ for different flux shapes.

(1) Flat flux shape

i. The transfer function between $\delta T_c(\omega, L)$ and $\delta v(\omega)$

After integrating Eq.(2-25), we have

$$\frac{\delta T_c(\omega, L)}{\delta v(\omega)} = -[QC/(v^2 C_p A_c \rho)](2v/\omega) \sin(\omega L/2v) e^{-j(\omega L/2v)} . \quad (2-27)$$

The phase shift is

$$-(\omega L/2v) - \pi , \quad \text{when } \sin(\omega L/2v) > 0 ,$$

and the gain is

$$\left| [QC/(v^2 C_p A_c \rho)](2v/\omega) \sin(\omega L/2v) \right| .$$

The sink frequencies are

$$f_s = v/L, 2v/L, 3v/L, \dots$$

ii. The transfer function between $\delta N(\omega)$ and $\delta v(\omega)$

For $\delta N(\omega)/\delta v(\omega)$, the calculated result shows that when

$\omega L/2v \ll 1$, the phase is $-(\omega L/4v)$ and there are no sink frequencies. The frequencies at which the gain of $\delta N(\omega)/\delta v(\omega)$ is minimum are $v/L, 2v/L, 3v/L, \dots$

iii. The transfer function between $\delta T_c(\omega, L)$ and $\delta N(\omega)$

Because $\delta T_c(\omega, L)/\delta N(\omega) = [\delta T_c(\omega, L)/\delta v(\omega)]/[\delta N(\omega)/\delta v(\omega)]$, the transfer function between $\delta T_c(\omega, L)$ and $\delta N(\omega)$ can be derived directly from the results of i and ii. When $\omega L/2v \ll 1$, the phase shift is $-(\omega L/4v) - \pi$, and there are sink frequencies at $v/L, 2v/L, 3v/L, \dots$

(2) Cosine flux shape

- i. The transfer function between
- $\delta T_c(\omega, L)$
- and
- $\delta v(\omega)$

Again, after integrating Eq.(2-25), we have

$$\begin{aligned} \delta T_c(\omega, L) / \delta v(\omega) \\ = -[QC / (v^2 C_p A_c \rho)] (\omega L / v) [2 / (1 - (\omega L / \pi v)^2)] \cos(\omega L / 2v) e^{-j(\omega L / 2v)} . \end{aligned} \quad (2-28)$$

The phase shift is $-(\omega L / 2v) - \pi$,

$$\text{when } 2\cos(\omega L / 2v) / [1 - (\omega L / \pi v)^2] > 0 .$$

The sink frequencies are at $3v/2L$, $5v/2L$, $7v/2L$,

- ii. The transfer function between
- $\delta N(\omega)$
- and
- $\delta v(\omega)$

After integrating Eq.(2-26), we find that there is no simple expression for the phase shift and there are no sink frequencies.

- iii. The transfer function between
- $\delta T_c(\omega)$
- and
- $\delta N(\omega)$

The sink frequencies are at $3v/2L$, $5v/2L$, $7v/2L$, ..., as those for $\delta T_c(\omega, L) / \delta v(\omega)$. No simple expression is available for the phase shift.

2.5 Random Heat Transfer Disturbance

In this section, we study the phenomenon caused by turbulent flow. Turbulent flow is characterized by the unsteadiness of the flow. The local coolant velocity may fluctuate in every direction. Contrary to the simultaneous coolant velocity changes along the whole channel discussed in Section 2.4, these local fluctuations at different positions are uncorrelated with each other. This local

velocity fluctuation causes the local coolant temperature change through its effect on the heat transfer coefficient and the heat removal rate. Let us denote this coolant temperature fluctuation generated at position z in dz as $\delta T_{cs}(\omega, z)$. Then $\delta T_{cs}(\omega, z)$ and $\delta T_{cs}(\omega, x)$ are uncorrelated if z is not equal to x .

Because in the case of a random heat transfer disturbance there is an infinite number of independent disturbance sources, it is impossible to calculate the relationship between TOUT (or N) and these disturbance sources, and then derive the transfer function between TOUT and N as we have done for the cases with only one disturbance source. Therefore, instead of calculating the transfer function between the core-exit coolant temperature and the neutron power, we calculate the cross power spectral density between TOUT and N (CPSD(TOUT, N)) to get PHASE(TOUT, N). The relationship between the transfer function (TF) and CPSD is as follows:

$$TF(TOUT, N) = CPSD(TOUT, N) / APSD(N)$$

APSD(N) is the auto power spectral density of neutron power. Because APSD(N) is a real number, the phase of TF(TOUT, N) is the same as the phase of CPSD(TOUT, N).

The accumulated coolant temperature fluctuation at position x , $\delta T_c(\omega, x)$ can be calculated using the following equation:

$$\delta T_c(\omega, x) = \int_{z=0}^{z=x} \delta T_{cs}(\omega, z) e^{-j\omega(L-z)/v} . \quad (2-29)$$

This is almost the same as Eq.(2-25) for the coolant velocity disturbance. The difference is that in Eq.(2-25), $\delta T_{cs}(\omega, z)$ and

$\delta T_{cs}(\omega, x)$ are correlated even when z is not equal to x , but now they are uncorrelated. Therefore the CPSD between $\delta T_{cs}(\omega, z)$ and $\delta T_{cs}(\omega, x)$ is zero if z is not equal to x .

According to Eq.(2-2) and assumption (7) in Section 2.1, we have the equation for the neutron power fluctuation with flat flux shape as

$$\delta N(\omega) = (N_0 \alpha_c / \beta) \int_0^L \int_0^x \delta T_{cs}(\omega, z) e^{-j\omega(x-z)/v} dx . \quad (2-30)$$

The cross power spectral density (CPSD) between core-exit coolant temperature and neutron power is calculated as follows:

$$\begin{aligned} \text{CPSD}(T_{OUT}, N) &= \int_{z=0}^{z=L} \delta T_{cs}(\omega, z) e^{-j\omega(L-z)/v} \\ &\quad \cdot [(N_0 \alpha_c / \beta) \int_0^L \int_0^x \delta T_{cs}(\omega, x) e^{-j\omega(y-x)/v} dy]^* \\ &= (N_0 \alpha_c / \beta) \int_0^L \int_0^x \int_0^L \delta T_{cs}(\omega, z) e^{-j\omega(L-z)/v} \\ &\quad \cdot \delta T_{cs}(\omega, x)^* e^{j\omega(y-x)/v} dy , \end{aligned} \quad (2-31)$$

where $*$ means complex conjugate, and

$$\langle \delta T_{cs}(\omega, z) \delta T_{cs}(\omega, x)^* \rangle = C \delta(x-z) dz dx , \quad (2-32)$$

where $\langle \rangle$ means the ensemble average [53], and C is a constant which is a function of the strength of the disturbance.

Hence,

$$\begin{aligned} \text{CPSD}(\text{TOUT}, N) \\ = (N_o \alpha_c / \beta) \int_0^L \int_0^y C e^{-j\omega(L-y)/v} dx dy . \end{aligned} \quad (2-33)$$

Perform the integration to get

$$\begin{aligned} \text{CPSD}(\text{TOUT}, N) = \\ (N_o \alpha_c / \beta) C j [-(vL/\omega) + (2v^2/\omega^2) \sin(\omega L/2v) e^{-j(\omega L/2v)}] . \end{aligned} \quad (2-34)$$

No frequency can make $|\text{CPSD}(\text{TOUT}, N)| = 0$, hence there is no sink frequency. If $\omega L/2v \ll 1$, $\sin(\omega L/2v)$ is approximately equal to $\omega L/2v$, then Eq.(2-34) can be further simplified as

$$\text{CPSD}(\text{TOUT}, N) = (N_o \alpha_c C / \beta) (2vL/\omega) \sin(\omega L/4v) e^{-j(\omega L/4v)} . \quad (2-35)$$

The phase shift is

$$-(\omega L/4v) - \pi .$$

2.6 Summary and Discussion

2.6.1 Summary

From the above analysis, we see that basically the transfer function between the core-exit coolant temperature and the neutron power fluctuations involves the integral and the double integral of the pure delay mechanism. The analysis shows that after integration the linear phase-frequency relationship still could be observed, and the sink or peak frequencies may show up. The results of this chapter are summarized as follows.

Let $\text{PHASE}(\text{TOUT}, \text{N}; \text{X})$ denotes the phase shift between the core-exit coolant temperature and the neutron power with X as the disturbance source. H.S. denotes the heat source disturbance; TIN for the coolant inlet temperature disturbance; VELO for the coolant velocity disturbance (also called flow rate disturbance); and R.H. for the random heat transfer disturbance.

(1) Flat flux shape

i. slope of $\text{PHASE}(\text{TOUT}, \text{N}; \text{H.S.}) = -180L/v \text{ (deg/Hz)}$,

$$f_s = v/L, 2v/L, 3v/L, \dots$$

ii. slope of $\text{PHASE}(\text{TOUT}, \text{N}; \text{TIN}) = -180L/v \text{ (deg/Hz)}$,

$$f_p = v/L, 2v/L, 3v/L, \dots$$

iii. slope of $\text{PHASE}(\text{TOUT}, \text{N}; \text{VELO}) = -90L/v \text{ (deg/Hz)}$,

$$\text{when } \omega L/2v \ll 1 ,$$

$$f_s = v/L, 2v/L, 3v/L, \dots$$

iv. slope of $\text{PHASE}(\text{TOUT}, \text{N}; \text{R.H.}) = -90L/v \text{ (deg/Hz)}$,

$$\text{when } \omega L/2v \ll 1 ,$$

no sink frequencies.

(2) Cosine flux shape

i. slope of $\text{PHASE}(\text{TOUT}, \text{N}; \text{H.S.}) = -180L/v \text{ (deg/Hz)}$,

$$f_s = 3v/2L, 5v/2L, 7v/2L, \dots$$

ii. slope of $\text{PHASE}(\text{TOUT}, \text{N}; \text{TIN}) = -180L/v \text{ (deg/Hz)}$,

$$f_p = 2v/L, 3v/L, 4v/L, \dots$$

iii. slope of $\text{PHASE}(\text{TOUT}, \text{N}; \text{VELO})$ has no simple form ,

$$f_s = 3v/2L, 5v/2L, 7v/2L, \dots$$

The accuracy of these results depends on the validity of the

assumptions. As shown in the following chapters, the assumptions made in this chapter are quite reasonable and the results give a good insight into the problem.

2.6.2 The case of extremely localized space dependent neutron kinetics

In the above analysis, the point reactor kinetics is used. For the reactor with space dependent neutron kinetics, the results will be different. A complete analysis of the cases with space dependent neutron kinetics with neutron-thermal hydraulic interaction needs a large three-dimensional computer code. But under the assumption that the feedback is negligible, the neutron-thermal hydraulic interaction could be decoupled. If we further assume the extremely localized space dependent neutron kinetics so that the local neutron fluctuation at the axial position z can be affected only by the coolant temperature fluctuation at the same position, then the problem is largely simplified and a phenomenological analysis is possible. In the following, we will discuss the extremely localized space dependent neutron kinetics case with different disturbance sources.

2.6.2.1 Heat source disturbance

In this case, a neutron flux fluctuation is treated as a driving force that causes the fluctuations of the heat generation rate and the coolant temperature, and the effect of the coolant temperature-reactivity feedback is assumed to be negligible. Then no

neutron kinetics is involved. PHASE(TOUT,N) is not affected by the neutron kinetics; it is the same as that in Section 2.2.

2.6.2.2 Coolant inlet temperature disturbance

With the extremely localized neutron kinetics, the neutron flux at position z is affected only by the coolant temperature fluctuation at the same position. The coolant temperature fluctuation causes the local reactivity change. In the 0.1-2 Hz frequency region, the neutron flux fluctuation is approximately proportional to the reactivity fluctuation. Hence the neutron flux fluctuation at position z is proportional to the coolant temperature fluctuation at position z . The phase shift between the core-exit coolant temperature and the neutron power fluctuation at position z , $\delta N(z, \omega)$, is the same as that between $\delta T_{OUT}(\omega)$ and $\delta T_c(z, \omega)$. $\delta T_{OUT}(\omega)$ and $\delta T_c(z, \omega)$ have a pure delay relationship with transit distance being $L-z$, i.e.,

$$\frac{\delta T_{OUT}(\omega)}{\delta T_c(z, \omega)} = e^{-j\omega(L-z)/v} \quad , \quad (2-36)$$

hence,

$$\frac{\delta T_{OUT}(\omega)}{\delta N(z, \omega)} = (\text{scalar factor}) e^{-j\omega(L-z)/v} \quad . \quad (2-37)$$

The phase slope is $-360 (L-z)/v$. In Section 2.2, with point reactor kinetics, the slope of PHASE(TOUT,N) is $-180L/v$ deg/Hz.

Because the neutron flux fluctuation is no longer the integral effect (refer to Appendix A) of the coolant temperature fluctuation, there are no sink frequencies in $\delta N(z, \omega)$ and no peak frequencies in $\delta T_{OUT}(\omega)/\delta N(z, \omega)$.

2.6.2.3 Coolant velocity disturbance

As discussed in section 2.6.2.2, $\delta N(z, \omega)$ is proportional to $\delta T_c(z, \omega)$.

According to Eq.(2-25)

$$\delta T_c(\omega, z) = [-\delta v(\omega)QC/(v^2 C_p A_c \rho)] \int_0^z f(x) e^{-j\omega(z-x)/v} dx, \quad (2-38)$$

$$\delta T_{OUT}(\omega) = [-\delta v(\omega)QC/(v^2 C_p A_c \rho)] \int_0^L f(x) e^{-j\omega(L-x)/v} dx, \quad (2-39)$$

and,

$$\begin{aligned} \delta T_{OUT}(\omega) / \delta T_c(z, \omega) \\ = \left[\int_0^L f(x) e^{-j\omega(L-x)/v} dx \right] / \left[\int_0^z f(x) e^{-j\omega(z-x)/v} dx \right]. \end{aligned} \quad (2-40)$$

(1) Flat flux shape

$$\begin{aligned} \frac{\delta T_{OUT}(\omega)}{\delta N(\omega, z)} &\propto \frac{-\delta T_{OUT}(\omega)}{\delta T_c(\omega, z)} \\ &= [-\sin(\omega L/2v) e^{-j\omega L/2v}] / [\sin(\omega z/2v) e^{-j\omega z/2v}] \\ &= [-\sin(\omega L/2v) e^{-j\omega(L-z)/2v}] / \sin(\omega z/2v). \end{aligned} \quad (2-41)$$

The phase slope is $-180(L-z)/v$. In section 2.3, with point reactor kinetics, the slope of $\text{PHASE}(T_{OUT}, N)$ is $-90L/v$.

The gain is $|\sin(\omega L/2v)/\sin(\omega z/2v)|$.

We have sink frequencies at

$$f_s : v/L, 2v/L, 3v/L, \dots,$$

and peak frequencies at

$$f_p : v/z, 2v/z, 3v/z, \dots$$

Because $\delta N(\omega, z)$ is proportional to $\delta T_c(\omega, z)$, sink frequencies of $\delta N(\omega, z)$ are the same as that for $\delta T_c(\omega, z)$, i.e.,

$$f_s : v/z, 2v/z, 3v/z, \dots$$

(2) Cosine flux shape

No simple form for phase slope. The sink frequencies for $\delta T_{OUT}(\omega)/\delta N(\omega, z)$ are

$$f_s : 3v/2L, 5v/2L, 7v/2L, \dots$$

2.6.2.4 Random heat transfer disturbance

With a random heat transfer disturbance, there are independent temperature fluctuation sources at each position z . These fluctuations affect the core-exit coolant temperature separately through pure delay mechanism. Hence in this case, the relationship between $\delta T_{OUT}(\omega)$ and $\delta N(z, \omega)$ is a pure delay mechanism. The phase slope is $-360 (L-z)/v$.

2.6.2.5 Concluding remark

As shown in the above discussions, for the case of extremely localized space dependent neutron kinetics, the slope of the phase between the core-exit coolant temperature and the neutron power fluctuations is a function of the position of the neutron detector. However, with a neutron detector in a specific position, the phase slope is still inversely proportional to the coolant velocity.

3. DISTRIBUTED PARAMETER MODEL

3.1 Introduction

A distributed parameter model is developed using the point reactor kinetics and the energy conservation equations [52]. These energy conservation equations are differential equations in time and space. The equations are first Fourier transformed to the frequency domain in order to get a set of ordinary differential equations with space as the only independent variable. These equations are solved analytically to obtain explicit expressions for the core-exit coolant temperature, the neutron power, and the transfer function between them. Compared with the phenomenological models, the distributed parameter model based on first principles gives more precise results. However, it is very difficult to solve analytically the complete set of system equations with all the interactions between the neutronics and the thermal hydraulics. Hence, here we still use the assumption that in the frequency range of 0.1 to 2 Hz the feedback can be ignored to decouple the interaction as we did with the phenomenological models. The validity of this assumption will be justified using the nodal model in Section 4.

In this chapter, we study the differential equations and the assumptions. The final solutions are implemented in a digital computer to get quantitative results. The values of the parameters in the model are based on the design data of the LOFT reactor [54,55,56]. The characteristics shown in the results are discussed.

3.2 The Model

A single channel geometry with a cylindrical fuel region and a coolant region is used as a representative part of the pressurized water reactor core. Three kinds of disturbance sources, that is, the heat source disturbance, the coolant inlet temperature disturbance and the coolant velocity disturbance, are considered. The reactivity is calculated using Eq.(2-2) and the neutron power fluctuation is calculated using the point reactor kinetics.

3.2.1 Cylindrical fuel region

The heat conduction equation for a solid is used in the analysis of fuel element heat transfer.

$$\nabla \cdot k_f(\underline{r}, t) \nabla T_f(\underline{r}, t) + Q_f(\underline{r}, t) = \rho_f(\underline{r}, t) C_{pf}(\underline{r}, t) \frac{\partial T_f(\underline{r}, t)}{\partial t} \quad , \quad (3-1)$$

where,

∇ : gradient differential operator,

$\underline{r}$: position vector,

$k_f(\underline{r}, t)$: thermal conductivity of the fuel,

$T_f(\underline{r}, t)$: fuel temperature,

$Q_f(\underline{r}, t)$: volumetric heat source in the fuel,

$\rho_f(\underline{r}, t)$: fuel density,

$C_{pf}(\underline{r}, t)$: fuel heat capacity.

The first term in Eq.(3-1) represents the net leakage of heat from a differential volume element, the second term is the heat

generated in the element and the third term the rate of heat accumulated within the element.

Because in the nuclear reactor environment the temperature gradient in the fuel may induce significant variation in thermophysical properties, Eq.(3-1) is usually solved numerically for its complexity. Assumptions are needed to reduce the complexity and make the analytical solution possible.

Generally the dependence of the temperature distribution on the angular position is not important and is ignored. Since the neutron power will not change significantly over a small rod length, conduction along the rod length is assumed negligible. Therefore the temperature distribution for a given axial position is considered one-dimensional. Also the thermal conductivity, density and heat capacity are assumed to be properly averaged constant values. Then Eq.(3-1) will be reduced to the formula

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} T_f(r, z, t) = - \frac{Q_f(t, z)}{k_f} + \frac{\rho_f C_{pf}}{k_f} \frac{d}{dt} T_f(r, z, t) , \quad (3-2)$$

where $Q_f(t, z)$ is the heat generation rate per unit volume in the fuel region. Assume that Q_f is not a function of the radial position r , and its dependence on z and t are separable. That is

$$Q_f(t, z) = Q_m(t)f(z) ,$$

where,

$Q_m(t)$: the total fission energy deposited in the fuel element,

$f(z)$: a shape factor proportional to the neutron flux shape,

$$\int_0^L f(z) dz = 1.$$

Fourier transform Eq.(3-2) and rearrange to get

$$\begin{aligned} \frac{\partial^2}{\partial r^2} T_f(r, z, \omega) + \frac{1}{r} \frac{\partial}{\partial r} T_f(r, z, \omega) - \frac{\rho_f C_{pf} j \omega}{k_f} T_f(r, z, \omega) \\ = - \frac{Q_m(\omega) f(z)}{k_f} \end{aligned} \quad (3-3)$$

The solution of Eq.(3-3) is

$$T_f(r, z, \omega) = A \cdot J_0(ar j^{3/2}) + Q_m(\omega) f(z) / (\rho_f C_{pf} j \omega), \quad (3-4)$$

where,

$$a = \sqrt{\rho_f C_{pf} \omega / k_f},$$

J_0 is the zeroth order Bessel function and A is a constant to be determined from the boundary condition. The boundary condition is

$$k_f \frac{\partial}{\partial r} T_f(r, z, \omega) \Big|_{r=R} = -h [T_f(R, z, \omega) - T_c(z, \omega)], \quad (3-5)$$

where T_c is the bulk coolant temperature at z , and h is the overall heat transfer coefficient that accounts for the heat transfer resistance from the fuel surface through the gap and cladding to the coolant.

Substitute Eq.(3-4) into Eq.(3-5), we get

$$A = G_1(\omega) f(z) + H_1(\omega) T_c(z, \omega), \quad (3-6)$$

where,

$$\begin{aligned} G_1(\omega) \\ = [-h Q_m(\omega) / (\rho_f C_{pf} j \omega)] / [k_f \frac{\partial}{\partial r} T_c(ar j^{3/2}) \Big|_{r=R} + h J_0(aR j^{3/2})], \end{aligned}$$

and,

$$H_1(\omega) = -G_1(\omega) \rho_f C_{pf} j \omega / Q_m(\omega).$$

Insert Eq.(3-6) into Eq.(3-4), we get the explicit expression for the fuel temperature

$$T_f(r, z, \omega) = [G_1(\omega)f(z) + H_1(\omega)T_c(z, \omega)]J_0(\text{arj}^{3/2}) + Q_m(\omega)f(z)/(\rho_f C_{pf} j \omega) . \quad (3-7)$$

3.2.2 Coolant region

The core heat transfer model also requires an energy balance equation for the coolant. Because the coolant density is assumed to be constant, a mass balance equation is not needed. The core pressure is not involved; hence a momentum balance equation is not needed either. In the energy balance equation, the kinetic and potential energy terms are ignored, and the energy balance equation for a single phase, incompressible fluid flow in one dimension is reduced to be

$$\frac{\partial}{\partial t} \rho_c E + \frac{\partial}{\partial z} \rho_c v E + \frac{\partial}{\partial z} [Pv/J] = \frac{2\pi R q}{A_c} + Q , \quad (3-8)$$

where,

E : internal energy per unit mass,

q : wall heat flux,

Q : volumetric heat source in the coolant channel,

J : mechanical equivalent of heat,

A_c : cross section area perpendicular to flow direction,

P : pressure,

v : coolant velocity.

Using the definition of enthalpy, H ,

$$H = E + P/(\rho_c J), \quad (3-9)$$

we have

$$\frac{\partial}{\partial t}(\rho_c E) + \frac{\partial}{\partial z}(\rho_c v H) = \frac{2\pi R q}{A_c} + Q . \quad (3-10)$$

The fluid is single phase and incompressible, hence

$$H = E = C_v T_c = C_p T_c , \quad (3-11)$$

and the wall heat flux, q ,

$$q = h[T_f(R, z, t) - T_c(z, t)] . \quad (3-12)$$

Substitute Eqs.(3-11) and (3-12) into Eq.(3-10) to get the differential equation for coolant temperature:

$$\begin{aligned} \frac{\partial}{\partial t} T_c(z, t) + v \frac{\partial}{\partial z} T_c(z, t) \\ = \frac{2\pi R h}{A_c \rho_c C_{pc}} [T_f(R, z, t) - T_c(z, t)] + \frac{Q(z, t)}{\rho_c C_{pc}} , \end{aligned} \quad (3-13)$$

where $Q(z, t)$ is the direct heat to the coolant per unit volume at position z ; because it is very small, we assume it is negligible.

With the coolant velocity disturbance, δv , Eq.(3-13) becomes

$$\begin{aligned} \frac{\partial}{\partial t} T_c(z, t) + (v + \delta v(t)) \frac{\partial}{\partial z} T_c(z, t) \\ = \frac{2\pi R h}{A_c \rho_c C_{pc}} [T_f(R, z, t) - T_c(z, t)] . \end{aligned} \quad (3-14)$$

Since this is a nonlinear equation, we have to linearize it before Fourier transformation into the frequency domain. Assume that $\delta v(t)$ is small so that its influence on $\partial T_c(z, t)/\partial z$ is negligible, hence

$$\delta v(t) \frac{\partial}{\partial z} T_c(z, t) = \delta v(t) \frac{\partial}{\partial z} T_c(z, 0) , \quad (3-15)$$

then Eq.(3-14) becomes

$$\begin{aligned} \frac{\partial}{\partial t} T_c(z, t) + v \frac{\partial}{\partial z} T_c(z, t) + \delta v(t) \frac{\partial}{\partial z} T_c(z, 0) \\ = \frac{2\pi R h}{A_c \rho_c C_{pc}} [T_f(R, z, t) - T_c(z, t)] . \end{aligned} \quad (3-16)$$

To calculate $\partial T_c(z, 0)/\partial z$, let $t=0$ in Eq.(3-13) to get

$$\frac{\partial}{\partial z} T_c(z, 0) = Q_m(0) \pi R^2 f(z) / (A_c \rho_c C_{pc} v) . \quad (3-17)$$

Insert Eq.(3-17) into Eq.(3-16), we have

$$\begin{aligned} \frac{\partial}{\partial t} T_c(z, t) + v \frac{\partial}{\partial z} T_c(z, t) \\ = \frac{2\pi R h}{A_c \rho_c C_{pc}} [T_f(R, t) - T_c(z, t)] - \frac{Q_m(0) f(z) \pi R^2}{A_c \rho_c C_{pc} v} \delta v(t) . \end{aligned} \quad (3-18)$$

Then Fourier transform Eq.(3-18) to get

$$\begin{aligned} j\omega T_c(z, \omega) + v \frac{\partial}{\partial z} T_c(z, \omega) \\ = b [T_f(R, z, \omega) - T_c(z, \omega)] - \frac{Q_m(0) f(z) \pi R^2 \delta v(\omega)}{A_c \rho_c C_{pc} v} , \end{aligned} \quad (3-19)$$

where,

$$b = 2\pi R h / (A_c \rho_c C_{pc}) .$$

$T_f(R, z, \omega)$ can be calculated using Eq.(3-7). Insert the T_f into Eq.(3-19) and rearrange to obtain

$$\frac{\partial}{\partial z} T_c(z, \omega) + H_2(\omega) T_c(z, \omega) = G_2(\omega) f(z) , \quad (3-20)$$

where,

$$\begin{aligned} H_2(\omega) &= [j\omega - bH_1(\omega)J_0(aRj^{3/2}) + b]/v , \\ G_2(\omega) &= [bG_1(\omega)J_0(aRj^{3/2}) + bQ_m(\omega)/(\rho_f C_{pf} j\omega)]/v \\ &\quad - Q_m(0) \pi R^2 \delta v(\omega) / (A_c \rho_c C_{pc} v^2) . \end{aligned}$$

To solve the first order differential equation, Eq.(3-20), we need one boundary condition. This is

$$T_c(0, \omega) = T_{IN}(\omega) .$$

The solution of Eq.(3-20) depends on $f(z)$. The solutions with the cosine and flat shape are as follows:

(1) Cosine flux shape

Let

$$f(z) = (\pi/2L)\sin(\pi z/L) ,$$

then

$$T_c(z, \omega) = A e^{-H_2(\omega)z} + B \sin(\pi z/L) + C \cos(\pi z/L) , \quad (3-21)$$

where,

$$A = T_{IN}(\omega) - C ,$$

$$B = -H_2(\omega) \cdot C \cdot L / \pi ,$$

$$C = G_2(\omega) / [-2 - 2H_2(\omega)^2 L^2 / \pi^2] .$$

(2) Flat flux shape

Let

$$f(z) = 1/L ,$$

then

$$T_c(z, \omega) = [T_{IN}(\omega)G_2(\omega)/(H_2(\omega)L)]e^{-H_2(\omega)z} + G_2(\omega)/[H_2(\omega) \cdot L] . \quad (3-22)$$

Eqs.(3-21) and (3-22) are used to calculate the coolant temperature fluctuation with the heat source disturbance, the inlet coolant temperature disturbance and the coolant velocity disturbance as the disturbance sources.

3.2.3 Reactor kinetics

The critical condition in a reactor represents a delicate balance between two dynamic processes: the generation rate and the

destruction rate of neutrons. Reactor kinetics is concerned with the changes of the neutron density when the balance condition associated with the critical state is disturbed.

For reactor kinetics, the most detailed analytical description could be formulated in the Boltzmann transport equation, in which the neutron density is considered as a function of time, three position coordinates, the two angles that describe the direction of travel of neutrons, and the energy of the neutron. Although the Boltzmann transport equation is readily derived, it is very difficult to solve. Therefore proper assumptions are necessary to get a practical formula to analyze the reactor kinetics. Because the angular dependence usually is not important, the time-dependent diffusion equations are good approximations. If we further average these equations over the non-temporal variables, then we will get the point kinetics equations

$$\frac{d}{dt}n(t) = \frac{\rho(t) - \beta_T}{\Lambda} n(t) + \sum_{i=1}^I \lambda_i C_i(t) , \quad (3-23)$$

$$\frac{d}{dt}C_i(t) = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) , \quad (3-24)$$

where,

$n(t)$: neutron flux,

β_T : total delayed neutron fraction,

λ_i : decay constant for the i th delayed neutron group,

$C_i(t)$: concentration of the precursor that yields the i th group delayed neutron when it decays,

β_i : delayed neutron fraction for the i th group.

The results obtained from these equations represent the neutron dynamic behavior at any location of the core if the reactor core is small so that space dependent phenomena are not important.

Prior to Fourier transformation of these equations into the frequency domain, we must linearize the product term, ρn . For this purpose, let

$$\rho = \rho_0 + \delta\rho , \quad (3-25)$$

and

$$n = n_0 + \delta n . \quad (3-26)$$

Assume that the reactor is initially critical, $\rho_0=0$.

Substitute Eqs.(3-25) and (3-26) into Eqs.(3-23) and (3-24) and ignore the second order perturbation term to get

$$\frac{d}{dt}\delta n = \frac{n_0 \delta\rho}{\Lambda} - \frac{\beta_T \delta n}{\Lambda} + \sum_{i=1}^I \lambda_i \delta C_i , \quad (3-27)$$

$$\frac{d}{dt}\delta C_i = \frac{\beta_i \delta n}{\Lambda} - \lambda_i \delta C_i . \quad (3-28)$$

Fourier transforming these equations, and after rearrangement, we get

$$\frac{\delta n}{\delta\rho} = n_0 / [j\omega\Lambda + \beta_T - \sum_{i=1}^I \lambda_i \beta_i / (j\omega + \lambda_i)] . \quad (3-29)$$

The reactivity change, $\delta\rho$, in this equation could come from the externally imposed reactivity or from reactivity feedback effects which accompany changes in the nuclear reactor process variables.

Hence the reactivity term in the kinetics equation becomes

$$\delta\rho = \delta\rho_{\text{ex}} + \sum_i \alpha_i \delta x_i, \quad (3-30)$$

where,

$\delta\rho_{\text{ex}}$: externally imposed reactivity,

α_i : coefficient of reactivity = $\delta\rho/\delta x_i$,

δx_i : a process variable such as fuel temperature, coolant temperature, etc.

In this dissertation, we are concerned with the reactivity feedback effects from fuel temperature and coolant (moderator) temperature fluctuations; therefore Eq.(3-30) is reduced to

$$\delta\rho = \delta\rho_{\text{ex}} + \alpha_f \langle \delta T_f \rangle + \alpha_c \langle \delta T_c \rangle, \quad (3-31)$$

where $\langle \delta T_f \rangle$ and $\langle \delta T_c \rangle$ are properly weighted average temperature change of the fuel and the coolant. Based on the perturbation theory and the assumption of self-adjointness, the weighting function is chosen to be the square of the neutron flux as shown in Eq.(2-2), [57]

$$\delta\rho(\omega) = \alpha_c \int_0^L \delta T_c(\omega, z) f(z)^2 dz$$

This formula was used by Kosaly [51,57] to calculate the spectral domain sink frequencies and gave good comparison with the experimental result. However, one should know that self-adjointness comes from the one-group diffusion theory [58], which is not applicable to problems involving changes in the moderating properties or the resonance escape probability. Since the mechanisms for the reactivity coefficients of fuel and coolant temperature are affected by the changes in the neutron energy spectrum, these mechanisms belong to the category where the one-group diffusion equation is not

applicable. Hence the weighting chosen in Eq.(2-2) is experimentally justified rather than having a theoretical basis. A better weighting can be achieved only with a multigroup method which requires a large computer code.

3.2.4 Calculation of the transfer function between the core exit coolant temperature and the neutron power

The transfer function(TF) between the core-exit coolant temperature and the neutron power is

$$TF(TOUT,N) = \delta T_c(L,\omega) / \delta N(\omega) / N_0 .$$

$\delta T_c(L,\omega)$ with the heat source disturbance can be calculated using Eq.(3-21) or Eq.(3-22) by making $TIN(\omega)$ and $\delta v(\omega)$ equal to zero.

Assume that the effect of the induced temperature fluctuation on the neutron power through the temperature-reactivity feedback is negligible, then $\delta N(\omega) / N_0 = Q_m(\omega) / Q_m(0)$.

For the case with the inlet coolant temperature disturbance, $T_c(L,\omega)$ is calculated using Eq.(3-21) or Eq.(3-22) by making $Q_m(\omega)$ and $\delta v(\omega)$ zero. For the coolant velocity disturbance, $T_c(L,\omega)$ is calculated by making $Q_m(\omega)$ and $TIN(\omega)$ zero. Again assume that the coolant temperature fluctuation caused by the feedback of the induced neutron power fluctuation is negligible in the frequency region above 0.1 Hz. Since the normalized neutron power fluctuation $\delta N(\omega) / N_0$ is the same as the normalized neutron flux fluctuation $\delta n(\omega) / n_0$, $\delta N(\omega) / N_0$ can be calculated using Eq.(3-29), in which $\delta \rho$ is calculated by substituting Eq.(3-21) or (3-22) into Eq.(2-2).

3.3 The Computer Program

The explicit expressions derived in the frequency domain are implemented in a PDP11/44 computer with an RSX-11M operating system. For the quantitative results, the parameter values are based on the LOFT reactor design data [54] and References [55,56]. These values are presented in Appendix B. The computer program is listed in Appendix F.

3.4 Results and Discussion

Several results are discussed in this section: (1) the existence of the linear phase-frequency relationship, (2) the sink frequencies, and (3) the phase slope and the extrapolated absolute phase at zero frequency.

3.4.1 The existence of the linear phase-frequency relationship

This linearity is expected for a pure-delay fluid transport mechanism. However, as shown in Chapter 2 (phenomenological model), those processes involving the integral of a pure delay mechanism will also indicate a linear phase-frequency relationship. In general, the results show that this linear relationship does exist for a normal flow rate situation in the low frequency region. But the mechanism in the core-exit coolant temperature and neutron flux involves not just the fluid transport process, but also the heat transfer process and the neutron kinetics. Neither of these processes has a linear phase-frequency relationship for all frequency regions. The phase for

neutron kinetics is not linear with respect to the frequency, but fortunately in the 0.1-2 Hz (especially 0.5 to 2 Hz) frequency region, the phase has only small variations as shown in Figure 3-1. The heat transfer from fuel to coolant is a process with a long time constant. Figure 3-2 shows the phase shift between the coolant temperature fluctuation and the heat generation fluctuation in the fuel. It shows that for the frequency region above 0.1 Hz, the phase shift changes very slowly. Hence, in general, the linear phase-frequency relationship is still observed.

3.4.2 The sink frequencies

Concerning the sink frequencies (or peak frequencies), the results from the distributed parameter model are very close to those predicted using the phenomenological model. Table 3.1 compares those two cases. The deviations are less than 1%.

The results also show that the first sink frequency is the upper limit of the frequency region with linear phase-frequency relationship. Figures 3-3 and 3-4 show some of these results. The sink frequencies are proportional to v/L , where v is the coolant velocity and L is the core height. Hence as the coolant velocity (or coolant flow rate) decreases, the linear region will become narrow or even disappear. This result is also observed using the phenomenological model, as shown in Section 2; the PHASE(TOUT,N) has a 180 degree jump around the sink frequencies.

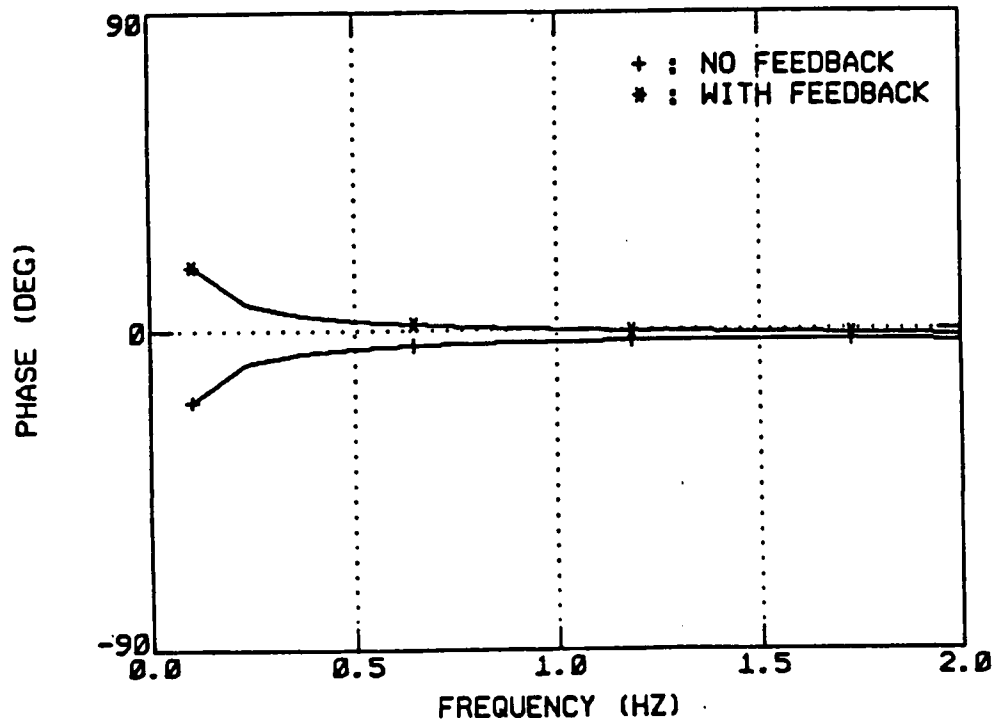


Figure 3-1. Comparison of the phases between the neutron power and reactivity fluctuations with and without including the temperature-reactivity feedback.

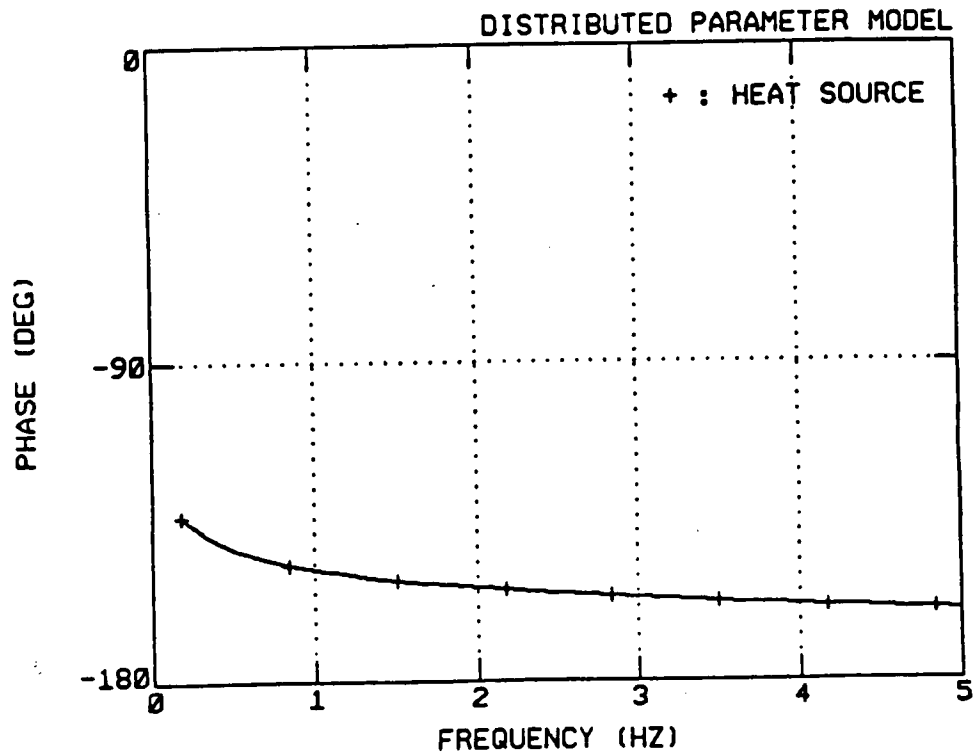


Figure 3-2. The phase angle between the heat generation and the coolant temperature fluctuations at the same axial position.

Table 3.1. The sink frequencies (Hz) of the gain between the core-exit coolant temperature and the neutron flux (coolant velocity=12 ft/sec).

Disturbance Source	Flat flux shape		Cosine flux shape	
	D.P.M.	P.M.	D.P.M.	P.M.
Heat Source	2.16	2.18	3.24	3.27
Flow Rate	2.16	2.18	3.24	3.27
Coolant Inlet Temperature*	2.16	2.18	4.36	4.36

D.P.M. : Distributed Parameter Model

P.M. : Phenomenological Model

* : peak frequency

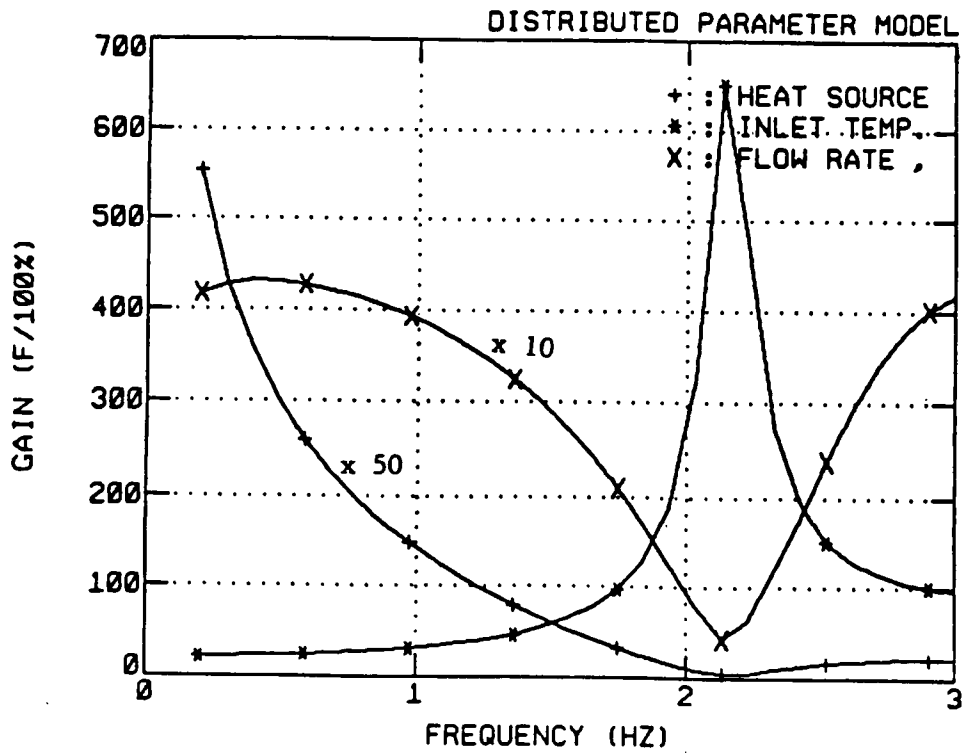


Figure 3-3. The gains between the core-exit coolant temperature and neutron power fluctuations for flat neutron flux shape with the heat source, coolant inlet temperature and flow rate disturbance sources.

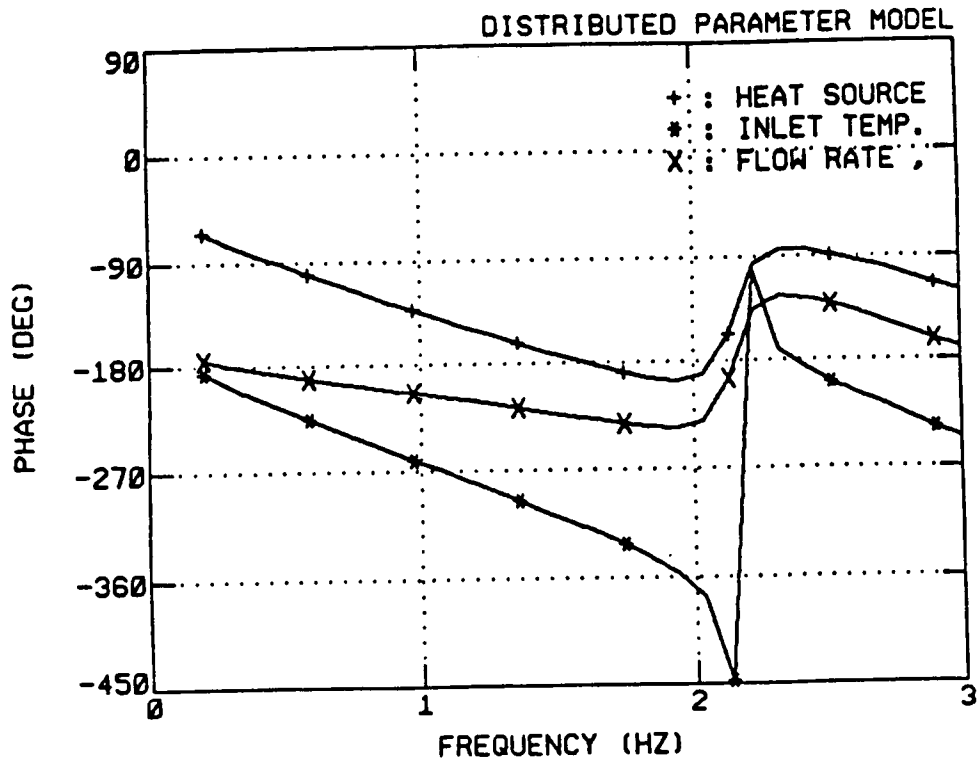


Figure 3-4. The phases between the core-exit coolant temperature and neutron power fluctuations for the flat neutron flux shape with heat source, coolant inlet temperature and flow rate disturbance sources.

3.4.3 The phase slope

The slope of $\text{PHASE}(\text{TOUT}, \text{N})$ for different flux shapes and different disturbance sources are listed in Table 3.2. The phase slopes calculated are based on the phase in the 0.5 to 1 Hz region.

From the phenomenological model, for the flat flux shape, we expect the following relationships:

$$\text{slope of } \text{PHASE}(\text{TOUT}, \text{N}; \text{H.S.}) = \text{slope of } \text{PHASE}(\text{TOUT}, \text{TIN})/2 ,$$

$$\text{slope of } \text{PHASE}(\text{TOUT}, \text{N}; \text{TIN}) = \text{slope of } \text{PHASE}(\text{TOUT}, \text{TIN})/2 ,$$

$$\text{slope of } \text{PHASE}(\text{TOUT}, \text{N}; \text{VELO}) = \text{slope of } \text{PHASE}(\text{TOUT}, \text{TIN})/4 .$$

Where $\text{PHASE}(\text{TOUT}, \text{N}; \text{X})$ is the phase shift between the core-exit coolant temperature and the neutron power with X as the disturbance source, and $\text{PHASE}(\text{TOUT}, \text{TIN})$ is the phase shift between the core-exit coolant temperature and the core inlet coolant temperature with the inlet coolant temperature fluctuation as the disturbance source. In this study, the slope of $\text{PHASE}(\text{TOUT}, \text{TIN})$ is -165 deg/Hz.

Evaluating the above relationships with the results listed in Table 3.2, we find that the simple formulas from the phenomenological models provide a very good guide for the approximate values of the phase slopes with different disturbance sources.

The relationship between phase slope and the coolant velocity is shown in Table 3.3, which shows the inverse proportionality relationships between the coolant velocity and the phase slope.

3.4.4 The extrapolated absolute phase at zero frequency

Figure 3-4 shows that when the coolant inlet temperature fluctuation or the coolant velocity fluctuation is the disturbance

Table 3.2. The slope (deg/Hz) of PHASE(TOUT,N) in 0.5 to 1Hz range for a variety of flux shapes and disturbance sources (coolant velocity=12 ft/sec).

Flux Shape Disturbance	Flat	Cosine	Cosine with upper 10 inches zero
Heat Source	-80.78	-82.41	-95.38
Flow Rate	-34.00	-49.48	-67.35
Coolant Inlet Temperature	-89.67	-87.54	-99.77

Table 3.3. The slope (deg/Hz) of PHASE(TOUT,N) in 0.5 to 1 Hz range for a variety of coolant velocities and disturbance sources with cosine flux shape.

Velocity Disturbance	6 ft/sec	12 ft/sec	24 ft/sec
Heat Source	0.435	0.229	0.119
Flow Rate	0.283	0.137	0.073
Coolant Inlet Temperature	0.479	0.243	0.127

source, the extrapolated absolute phase approaches -180 degrees at zero frequency; this is due to the effect of negative coolant (moderator) temperature coefficient of reactivity. For the case with a heat source disturbance, the phase does not approach -180 degrees. In this case the fluctuation of the neutron power drives the coolant temperature fluctuation, and the negative coolant temperature coefficient of reactivity is not involved; hence the relationship is not out of phase. This is an important observation, and could be used as a new technique to identify the sign of the moderator temperature coefficient of reactivity, which is a safety related parameter. More detailed discussion of the relationship between the extrapolated absolute phase at zero frequency and the sign of the moderator temperature coefficient of reactivity will be described in Sections 4 and 5.

4. MULTI-NODAL MODEL

4.1 Introduction

Although both the phenomenological model and the distributed parameter model give very good insight into the mechanisms involved, the results are based on certain assumptions. The validity of these assumptions will be checked using the multi-nodal model. These assumptions include: (1) neglecting the feedback interactions between the neutronics and the thermal hydraulics, (2) neglecting the variation of heat transfer coefficient due to coolant velocity disturbance, (3) neglecting the feedback effect of the fuel temperature fluctuation, and (4) neglecting the direct heat to the coolant.

The idea of a nodal model is to replace the spatial derivatives with a finite difference approximation. When the number of nodes increases, the result will approach that obtained from the distributed parameter model.

The advantage of the nodal model is its ability to handle the interactive feedback without complicated derivations as in the distributed parameter model. However, due to the limited memory and speed of the digital computer, the number of nodes allowable usually is limited; in the frequency domain analysis, the limited amount of nodes will restrain the accuracy of the result to a certain frequency region.

In addition to verifying the assumptions made in the previous chapters, the nodal model is also used to study the coherence between

the core-exit coolant temperature and the neutron power signals with random heat transfer as the disturbance source. Without measurement noises for the other kind of disturbance sources the coherence is one.

In this chapter, the relationship between the sign of the moderator temperature coefficient of reactivity and PHASE(TOUT,N) is further studied, and a new technique is suggested for monitoring the moderator temperature coefficient of reactivity.

4.2 The Model

A nodal model with the point reactor kinetics with two delayed neutron groups, n fuel nodes, n cladding nodes, and $2n$ coolant nodes is established using the first principle approach. These are shown in Eqs.(4-1) to (4-7). For the coolant, two well-mixed coolant nodes are used with each cladding node. The average temperature of the first node is taken as the coolant temperature to determine the heat transfer driving force to both coolant nodes [25].

The equations are

$$\begin{aligned} \frac{d}{dt} \left(\frac{\delta N}{N_0} \right) = & - \frac{\beta}{\Lambda} \frac{\delta N}{N_0} + \frac{\alpha_f}{\Lambda} \frac{1}{\sum W_{fi}} \sum_{i=1}^n T_{fi} W_{fi} \\ & + \frac{\alpha_c}{\Lambda} \frac{1}{\sum W_{ci}} \sum_{i=1}^{2n} T_{ci} W_{ci} + \sum_{i=1}^2 \lambda_i \frac{\delta C_i}{N_0} + \frac{S(t)}{\Lambda} , \end{aligned} \quad (4-1)$$

$$\frac{d}{dt} \left(\frac{\delta C_i}{N_0} \right) = \frac{\beta_i}{\Lambda} \frac{\delta N}{N_0} - \frac{\lambda_i \delta C_i}{N_0} , \quad i=1,2, \quad (4-2)$$

$$\frac{d}{dt} T_{fi} = \left(\frac{\delta N}{N_0} \right) N_0 F_{fi} \frac{1}{m_{fi} C_{pfi}} - \frac{h_{fcli} A_{fcli}}{m_{fi} C_{pfi}} (T_{fi} - T_{cli}) , \quad (4-3)$$

$$\frac{d}{dt} T_{cli} = \frac{h_{fcli} A_{fcli}}{m_{cli} C_{pcli}} (T_{fi} - T_{cli}) - \frac{h_{clci} A_{clci}}{m_{cli} C_{pcli}} (T_{cli} - T_{c(2i-1)}), \quad (4-4)$$

Coolant node 1,

$$\begin{aligned} \frac{d}{dt} T_{c1} = & \left(\frac{\delta N}{N_0} \right) N_0 F_{c1} \frac{1}{m_{c1} C_{pc1}} + \frac{h_{clc1} A_{clc1}}{m_{c1} C_{pc1}} (T_{c11} - T_{c1}) \\ & + \frac{\dot{m}}{m_{c1}} (T_{IN} - T_{c1}), \end{aligned} \quad (4-5)$$

Coolant nodes $i=2, 4, \dots, 2n$,

$$\begin{aligned} \frac{d}{dt} T_{ci} = & \left(\frac{\delta N}{N_0} \right) N_0 F_{ci} \frac{1}{m_{ci} C_{pci}} + \frac{h_{clc(i/2)} A_{clc(i/2)}}{2m_{ci} C_{pci}} (T_{c1(i/2)} - T_{c(i-1)}) \\ & + \frac{\dot{m}}{m_{ci}} (T_{c(i-1)} - T_{ci}), \end{aligned} \quad (4-6)$$

Coolant nodes $i=3, 5, \dots, 2n-1$,

$$\begin{aligned} \frac{d}{dt} T_{ci} = & \left(\frac{\delta N}{N_0} \right) N_0 F_{ci} \frac{1}{m_{ci} C_{pci}} \\ & + \frac{h_{clc(i+1)/2} A_{clc(i+1)/2}}{2m_{ci} C_{pci}} (T_{c1(i+1)/2} - T_{ci}) \\ & + \frac{\dot{m}}{m_{ci}} (T_{c(i-1)} - T_{ci}) \end{aligned} \quad (4-7)$$

where,

δN : variation of the neutron power,

N_0 : initial neutron power,

F_{fi} : power fraction in i th fuel node,

W_{fi} : weighting of the i th node fuel temperature on the reactivity,
 $S(t)$: reactivity disturbance,
 m_{fi} : i th node fuel mass,
 C_{pfi} : i th node fuel specific heat,
 T_{fi} : i th node fuel temperature,
 T_{cli} : i th node cladding temperature,
 h_{fcli} : i th node fuel to cladding heat transfer coefficient,
 A_{fcli} : i th node fuel to cladding heat transfer area,
 m_{cli} : i th node cladding mass,
 C_{pcli} : i th node cladding specific heat,
 h_{clci} : i th node cladding to coolant heat transfer coefficient,
 A_{clci} : i th node cladding to coolant heat transfer area,
 T_{ci} : i th node coolant temperature,
 m_{ci} : i th node coolant mass,
 C_{pci} : i th node coolant specific heat,
 W_{ci} : weighting of the i th node coolant temperature on reactivity,
 $\dot{m}$: coolant mass flow rate.

In the case with a coolant velocity (or flow rate) disturbance, the heat transfer coefficient will change [3]. For an ordinary cooling medium such as water in the turbulent flow region ($Re > 2000$), the relation for the heat transfer is the Dittus-Boelter equation [59],

$$Nu = 0.023(Re)^{0.8}(Pr)^{0.4}, \quad (4-8)$$

where the Nusselt number, $Nu=hD/k$; the Reynolds number, $Re=Dv\rho/\nu$; and the Prandtl number, $Pr=c\nu/k$ (where D :equivalent diameter of coolant channel, ρ :coolant density, ν :viscosity, h :heat transfer coefficient, A :cooling surface, c :specific heat of coolant, and k :thermal conductivity of the coolant). Based on Eq.(4-8) and the binomial theorem [60], for small variation in h and v , the fluctuation in the heat transfer coefficient, δh , is related to the fluctuation of the velocity, δv , by

$$\delta h/h \approx 0.8 \delta v/v, \quad (4-9)$$

hence we have

$$\delta h_{clci} \approx 0.8 h_{clci} \delta \dot{m}/\dot{m}. \quad (4-10)$$

Including the coolant flow rate fluctuation, after linearization, Eqs.(4-4) to (4-7) become

$$\begin{aligned} \frac{d}{dt} T_{cli} = & \frac{h_{fcli} A_{fcli}}{\dot{m}_{cli} C_{pcli}} (T_{fi} - T_{cli}) - \frac{h_{clci} A_{clci}}{\dot{m}_{cli} C_{pcli}} (T_{cli} - T_{c(2i-1)}) \\ & - \frac{0.8 h_{clci} \delta \dot{m} A_{clci}}{\dot{m} \cdot \dot{m}_{cli} C_{pcli}} (T_{cli0} - T_{c(2i-1)0}), \end{aligned} \quad (4-11)$$

$$\begin{aligned} \frac{d}{dt} T_{cl} = & \left(\frac{\delta N}{N_0} \right) N_0 F_{ci} \frac{1}{\dot{m}_{cl} C_{pcl}} + \frac{h_{clcl} A_{clcl}}{\dot{m}_{cl} C_{pcl}} (T_{cl1} - T_{cl}) \\ & + \frac{\dot{m}}{\dot{m}_{cl}} (T_{IN} - T_{cl}) + \frac{\dot{m}}{\dot{m}_{cl}} (T_{IN} - T_{clo}) \\ & + \frac{0.8 h_{clci} \delta \dot{m} A_{clci}}{\dot{m} \cdot \dot{m}_{ci} C_{pci}} (T_{cli0} - T_{clo}), \end{aligned} \quad (4-12)$$

Coolant nodes $i=2, 4, \dots, 2n$,

$$\begin{aligned}
\frac{d}{dt} T_{ci} = & \left(\frac{\delta N}{N_o} \right) N_o F_{ci} \frac{1}{m_{ci} C_{pci}} + \frac{h_{clc(i/2)} A_{clc(i/2)}}{2 m_{ci} C_{pci}} (T_{cl(i/2)} - T_{c(i-1)}) \\
& + \frac{\dot{m}}{m_{ci}} (T_{c(i-1)} - T_{ci}) + \frac{\delta \dot{m}}{m_{ci}} (T_{c(i-1)o} - T_{cio}) \\
& + \frac{0.8 h_{clc(i/2)} \delta \dot{m} A_{clc(i/2)}}{2 \dot{m} m_{ci} C_{pci}} (T_{cl(i/2)o} - T_{c(i-1)o}) ,
\end{aligned}
\tag{4-13}$$

Coolant nodes $i=3, 5, \dots, 2n-1$,

$$\begin{aligned}
\frac{d}{dt} T_{ci} = & \left(\frac{\delta N}{N_o} \right) N_o F_{ci} \frac{1}{m_{ci} C_{pci}} \\
& + \frac{h_{clc(i+1)/2} A_{clc(i+1)/2}}{2 m_{ci} C_{pci}} (T_{cl(i+1)/2} - T_{ci}) \\
& + \frac{\dot{m}}{m_{ci}} (T_{c(i-1)} - T_{ci}) + \frac{\delta \dot{m}}{m_{ci}} (T_{c(i-1)o} - T_{cio}) \\
& + \frac{0.8 h_{clc(i+1)/2} \delta \dot{m} A_{clc(i+1)/2}}{2 \dot{m} m_{ci} C_{pci}} (T_{cl((i+1)/2)o} - T_{cio}) ,
\end{aligned}
\tag{4-14}$$

where T_{clio} , T_{cio} are the steady state values. The weighting of the i th node fuel temperature on the reactivity, W_{fi} , and the weighting of the i th node coolant temperature on the reactivity, W_{ci} , are proportional to the square of the fraction of the power in the i th nodes, F_{fi} , and F_{ci} .

The above is a set of linear equations, and can be represented in a standard form as

$$\dot{\underline{x}}(t) = \underline{A} \underline{x}(t) + \underline{B} u(t) ,
\tag{4-15}$$

where $\underline{x}$ is the state vector, A is the system matrix, $\underline{u}$ is the input vector, and B is the input matrix.

After Fourier transformation Eq.(4-15) becomes

$$\underline{X}(\omega) = (jI\omega - A)^{-1}B\underline{U}(\omega), \quad (4-16)$$

where $j = \sqrt{-1}$, and I is a unit matrix.

The transfer function between the state variable i and the input n is the matrix element $[(jI\omega - A)^{-1}B]_{in}$; the transfer function between the state variable k and the input n is $[(jI\omega - A)^{-1}B]_{kn}$, and the transfer function between the state variables i and k with n as the input is calculated from $[(jI\omega - A)^{-1}B]_{in} / [(jI\omega - A)^{-1}B]_{kn}$. Hence from $(jI\omega - A)^{-1}B$, we can obtain the transfer function between the state variables (e.g. N and $TOUT$) and the inputs (e.g. the heat source disturbance, the coolant velocity disturbance, the inlet coolant temperature disturbance and the random heat transfer disturbance), and then the transfer function between $TOUT$ and N with any disturbance can be calculated.

For the case with the random heat transfer as the disturbance source, the disturbance is in the fluctuation of the heat transfer coefficient, δh . Each cladding node has an uncorrelated δh fluctuation: δh_{clci} and δh_{clcj} are uncorrelated if $i \neq j$. In other words, only the fluctuation caused by δh variations at the cladding node L , the coolant node $2L-1$ and the coolant node $2L$, which share the same heat transfer coefficient fluctuation, are correlated; L is a dummy variable with the value from 1 to NF , where NF is the total number of fuel (or cladding) nodes. For this multiple input case, the

fluctuation of the state variable i can be calculated as

$$x_i(\omega) = \sum_n [(jI\omega - A)^{-1}B]_{in} \cdot u_n(\omega) . \quad (4-17)$$

And for state variable k , it is

$$x_k(\omega) = \sum_n [(jI\omega - A)^{-1}B]_{kn} \cdot u_n(\omega) , \quad (4-18)$$

where u_n is the disturbance input due to the δh variation at all the related nodes.

Let

$$\langle u_n(\omega) u_m(\omega)^* \rangle = 1, \text{ if } n, m \text{ belong to the nodes} \\ \{(3+NF+L), (3+2NF+2L) \text{ or } (3+2NF+2L-1)\},$$

otherwise,

$$\langle u_n(\omega) u_m(\omega)^* \rangle = 0 .$$

Notice that the total node $(3+NF+L)$ is equivalent to the cladding node L ; the total node $(3+2NF+2L)$ is equivalent to the coolant node $2L$.

Then CPSD_{ik} (i.e. $\langle x_i(\omega) x_k(\omega)^* \rangle$), APSD_i (i.e. $\langle x_i(\omega) x_i(\omega)^* \rangle$), and the APSD_k can be calculated as follows,

$$\begin{aligned} \langle x_i(\omega) x_k(\omega)^* \rangle &= \left\langle \sum_n [(jI\omega - A)^{-1}B]_{in} u_n(\omega) \cdot \left\{ \sum_m [(jI\omega - A)^{-1}B]_{km} u_m(\omega) \right\}^* \right\rangle \\ &= \sum_{L=1}^{NF} \sum_{n'} \{ [(jI\omega - A)^{-1}B]_{in'} \cdot \left[\sum_{m'} [(jI\omega - A)^{-1}B]_{km'} \right]^* \} , \end{aligned} \quad (4-19)$$

n' and m' belong to the nodes

$$\{(3+NF+L), (3+2NF+2L) \text{ or } (3+2NF+2L-1)\} .$$

The transfer function(TF) between variables i and k is

$$TF_{ik} = \text{CPSD}_{ik} / \text{APSD}_k, \quad (4-20)$$

the coherence(COHER) between them is

$$\text{COHER}_{ik} = (\text{CPSD}_{ik} \cdot \text{CPSD}_{ik}^*) / (\text{APSD}_k \cdot \text{APSD}_i). \quad (4-21)$$

Let the i th node be the neutron power and the k th node be the core-exit temperature, then based on the above formulas, we can get the transfer function and the coherence between the neutron power and the core-exit coolant temperature with the random heat transfer disturbance.

4.3 Results and Discussion

4.3.1 Verification of the model

Due to the finite number of nodes used, the nodal model is good only for a certain frequency range. This is checked by increasing the number of nodes to see if the results have significant variations. Figures 4-1, 4-2 and 4-3 show the results with 1, 4, 7, 10 and 13 fuel nodes for different disturbance sources. These figures show that up to 1 Hz, when the number of fuel nodes increases from 10 to 13, the variations are less than 1%. Hence in all the calculations, we use 10 fuel nodes, 10 cladding nodes and 20 coolant nodes, and the results are valid up to 1 Hz.

Under the same assumptions as used in the distributed parameter model, the estimated slope of $\text{PHASE}(\text{TOUT}, N)$ is compared with that from the distributed parameter model and is listed in Table 4.1. The results are in good agreement, thus confirming the validity of the nodal model. The effects of a variety of assumptions on the results will be discussed below.

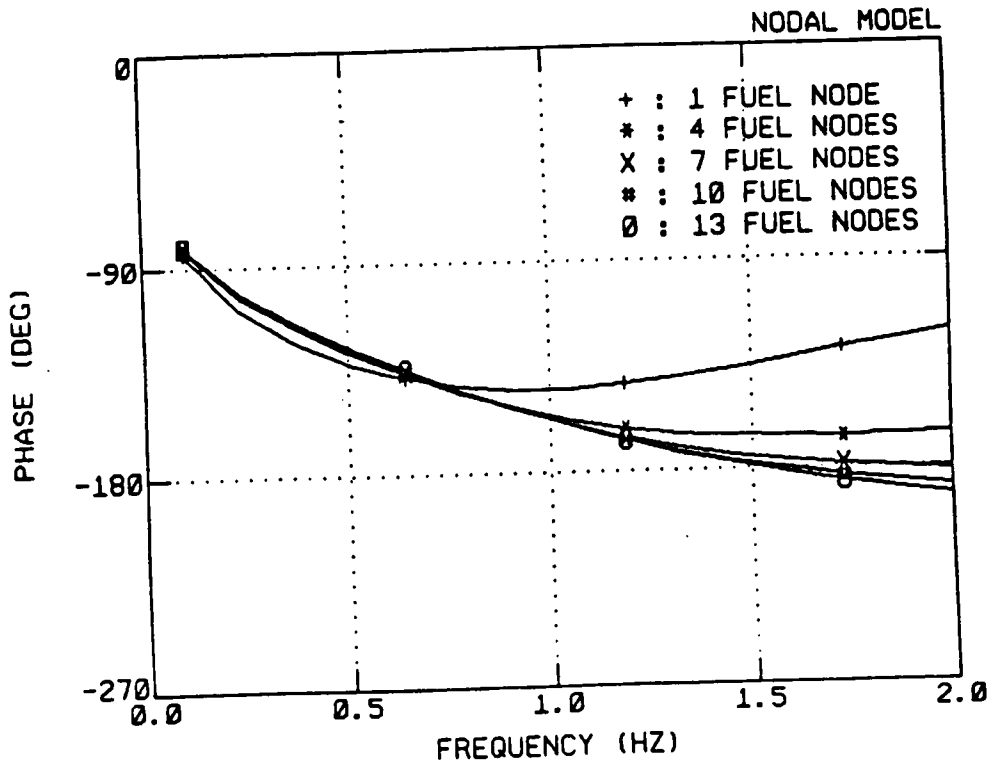


Figure 4-1. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with heat source disturbance for a variety of node numbers with cosine flux shape.

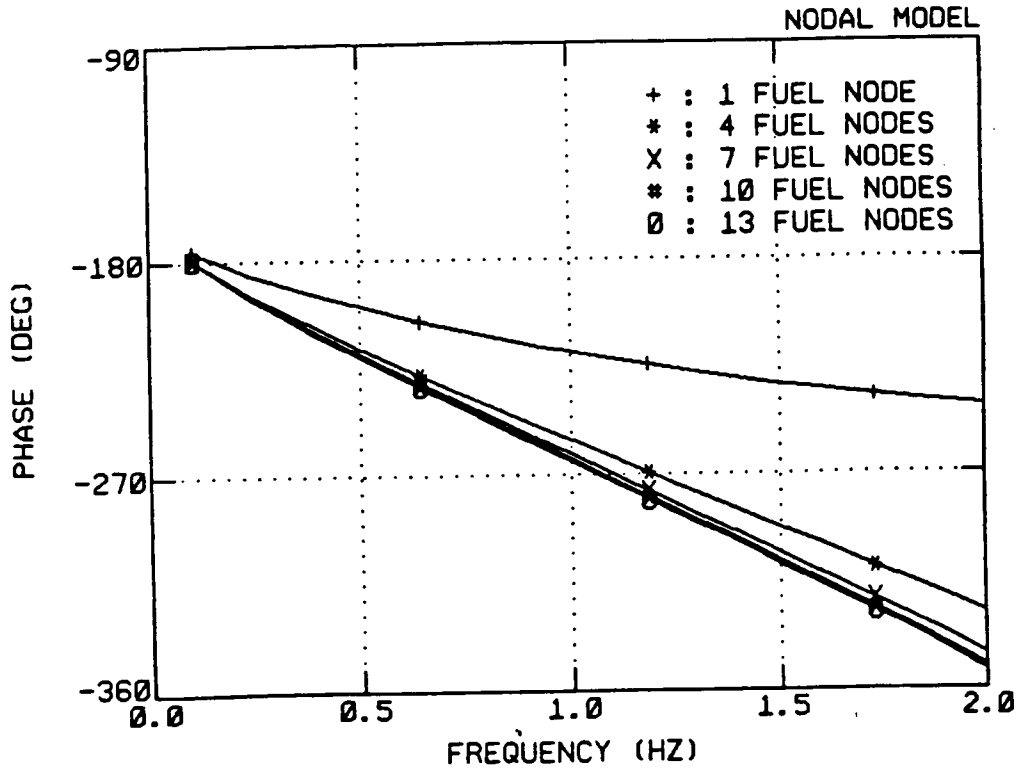


Figure 4-2. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with coolant inlet temperature disturbance for a variety of node numbers with cosine flux shape.

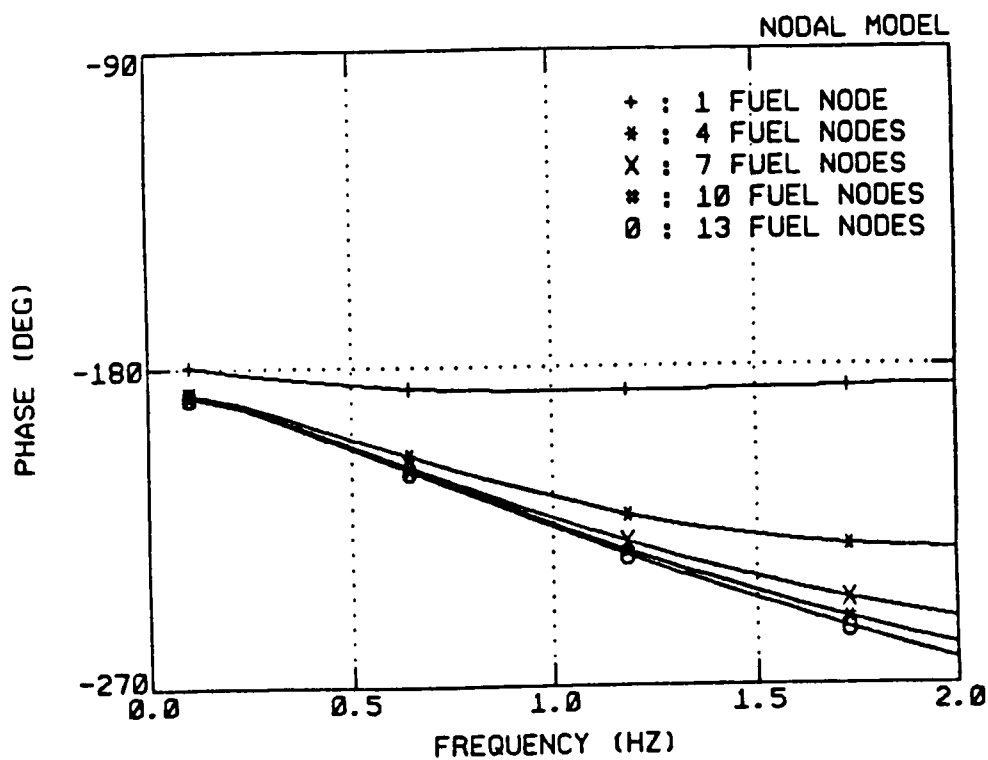


Figure 4-3. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with coolant flow rate disturbance for a variety of node numbers with cosine flux shape.

Table 4.1 Comparison of the calculated slope (deg/Hz) of PHASE(TOUT/N) using distributed parameter model and multi-nodal model (10 fuel nodes) with cosine flux shape.

Model \ Disturbance	Heat Source	Coolant Inlet Temperature	Flow Rate
Distributed Parameter Model	-82.4	-87.5	-49.5
Multi-nodal Model	-81.8	-87.0	-49.1

4.3.2 The effect of the direct heat to the coolant

After fission, the fission products and beta particles which have a short mean free path exchange almost all their kinetic energy into heat energy in the fuel. However, a small portion of energy carried by neutrons and gamma rays which have longer mean free paths can lose their kinetic energy outside the fuel. And about 2% of the total energy is directly deposited in the coolant. The direct heat to the coolant and the heat transferred from the fuel change the coolant temperature through different channels and cause a different PHASE(TOUT,N) for the heat source disturbance case as shown in Figure 4-4. It is seen that the phase between the core-exit temperature and the neutron power through direct heat has less phase shift than that through the heat transfer process. The overall PHASE(TOUT,N) depends on the ratio of the direct heating and heat transferred from the fuel. Although the direct heat is only about 2% of the total heat generated, the heat transferred from the fuel decreases sharply with increasing frequency. Figure 4-5 shows the GAIN(TOUT,N) from the nodal model without direct heat and with direct heat only. This figure shows that at higher frequencies the direct heat could have more contribution to the GAIN(TOUT,N). The combined effect makes PHASE(TOUT,N) nonlinear as a function of frequency. For the other cases in which heat source fluctuation is not the disturbance source, the direct heat does not affect the results because the feedback effect of the induced neutron fluctuation on the coolant temperature is negligible, as will be shown in the next section.

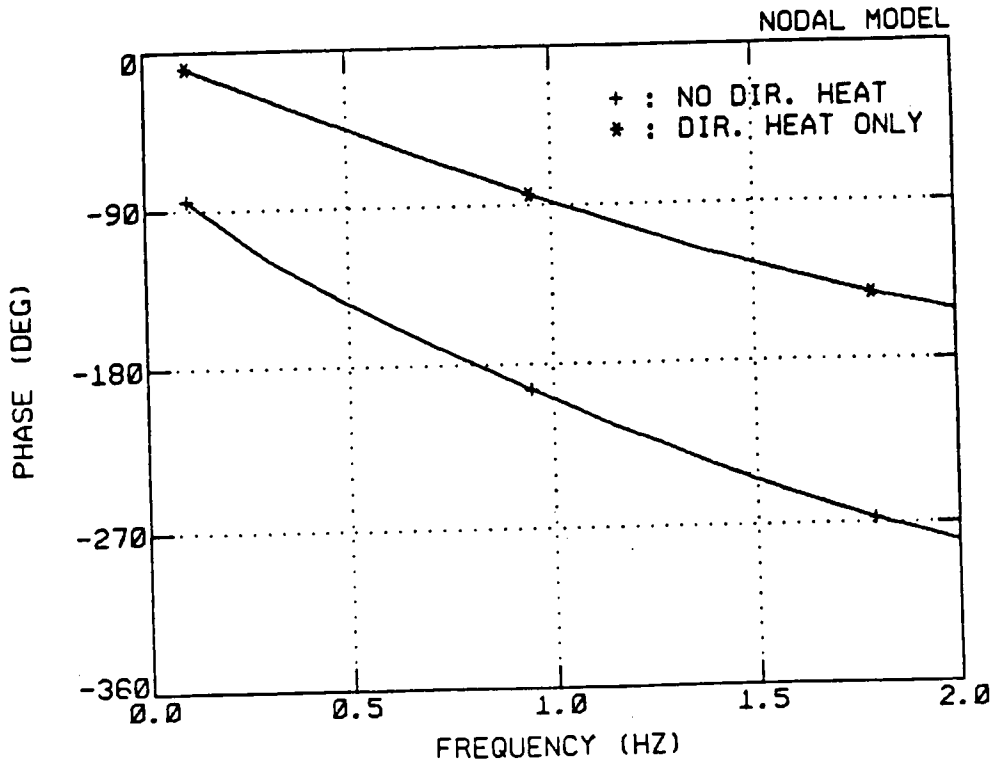


Figure 4-4. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations for the cases without direct heat and with direct heat only with cosine flux shape.

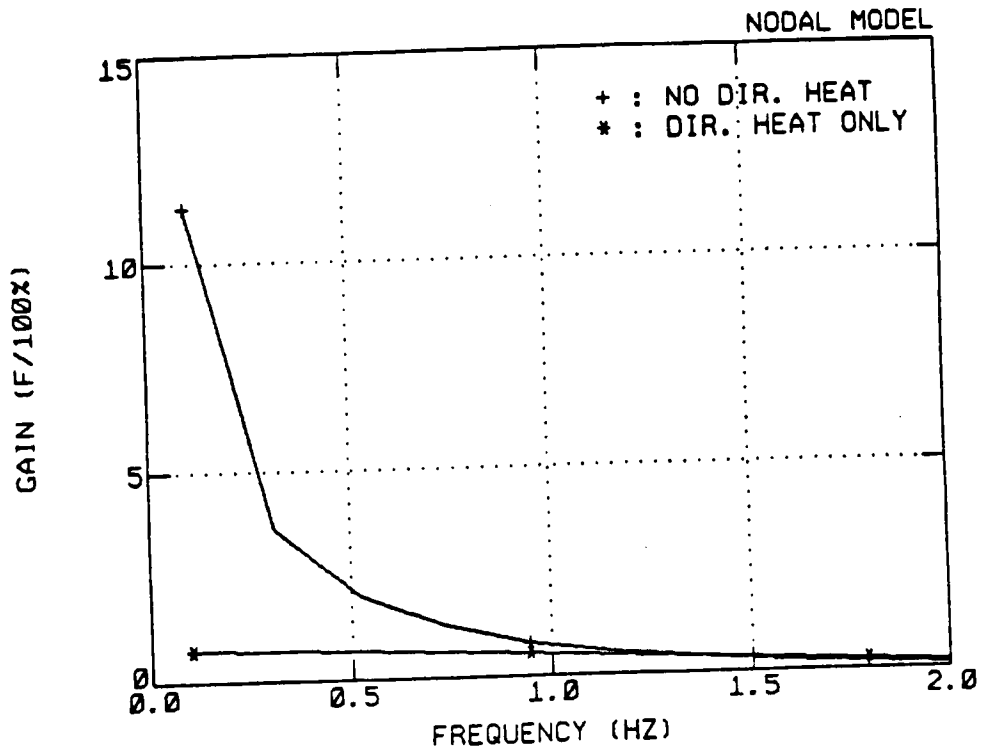


Figure 4-5. Comparison of the calculated gains between the core-exit coolant temperature and neutron power fluctuations for the case without direct heat and with direct heat only with cosine flux shape.

4.3.3 The effect of neglecting the feedback of fuel temperature fluctuation and the feedback interactions between the neutronics and the thermal hydraulics

Figure 4-6 shows $\text{PHASE}(\text{TOUT}, \text{N})$ with and without the assumptions of neglecting the fuel temperature fluctuation and the coolant temperature feedback effect for the heat source disturbance case. As seen from this figure, the assumptions do not affect the result. Figures 4-7 and 4-8 compare the results with and without the assumption made in the distributed parameter model that the effect of the induced neutron power fluctuation on the coolant temperature is negligible for the coolant velocity disturbance and the inlet coolant temperature disturbance cases. Let the slope of $\text{PHASE}(\text{TOUT}, \text{N})$ be estimated using the phase data between 0.5 to 1 Hz. From these figures, we see that the estimated slopes with and without the assumptions have less than 5 deg/Hz difference. Hence the assumption is reasonable.

4.3.4 The effect of heat transfer coefficient fluctuation

As discussed in Section 2.3, the fluctuation of coolant velocity can cause the fluctuation of both the heat transfer coefficient and the heat removal rate. In the distributed parameter model, only the fluctuation of heat removal rate has been considered. Figure 4-9 compares $\text{PHASE}(\text{TOUT}, \text{N})$ for the coolant velocity disturbance case with and without the change of the heat transfer coefficient, and shows that neglecting the fluctuation in the heat transfer coefficient does not affect the result in the frequency range above 0.2 Hz.

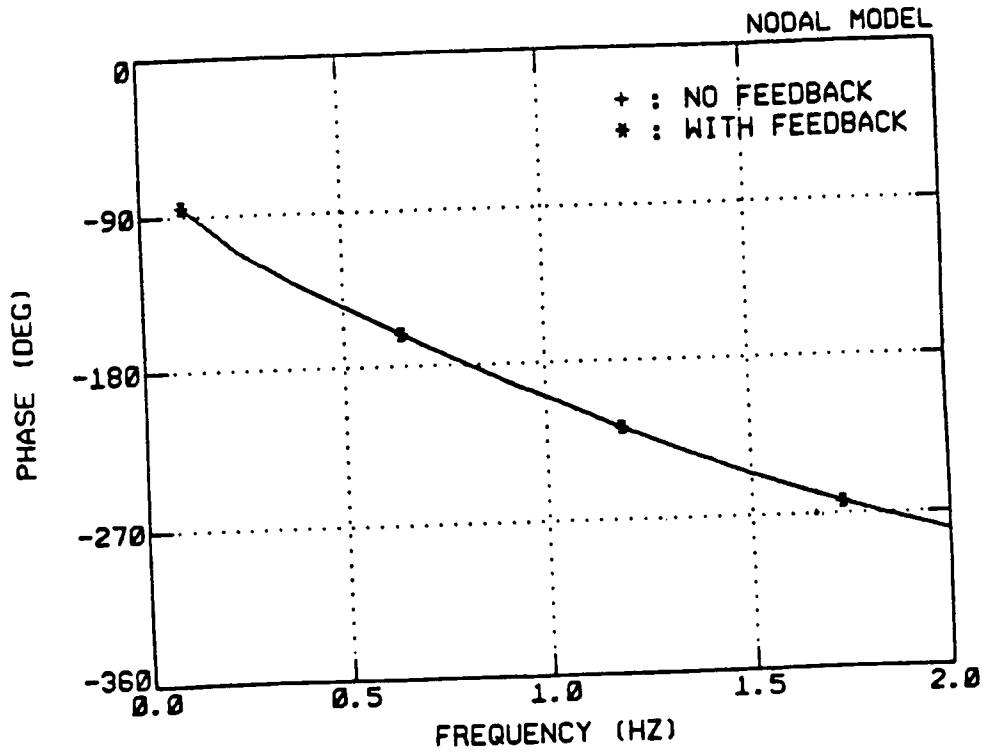


Figure 4-6. Comparison of the calculated phases between core-exit coolant temperature and neutron power fluctuations with and without including the effect of temperature reactivity feedback for the case with heat source disturbance, cosine flux shape and no direct heat.

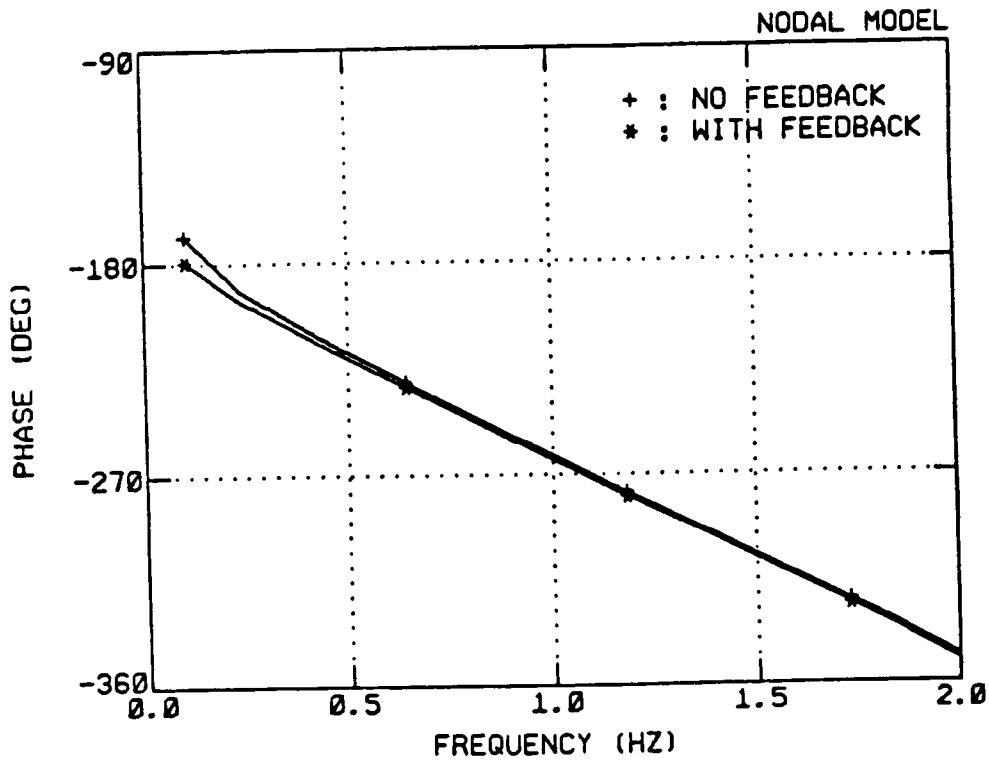


Figure 4-7. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with and without feedback for the case with coolant inlet temperature disturbance and cosine flux shape.

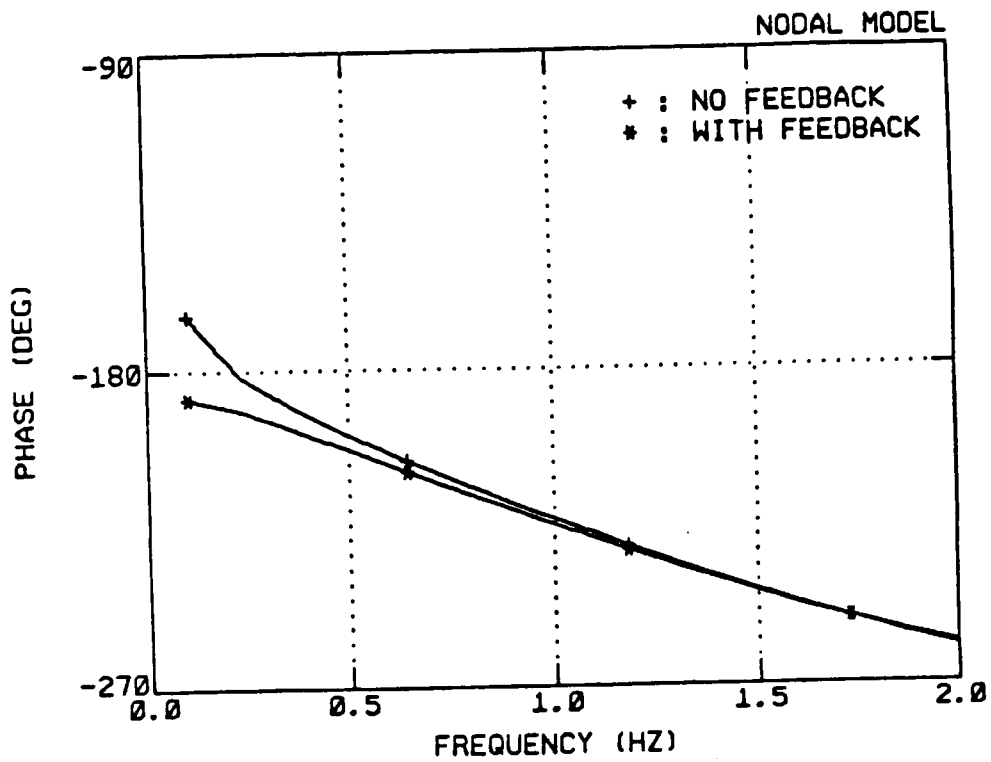


Figure 4-8. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with and without feedback for the case with coolant flow rate disturbance and cosine flux shape.

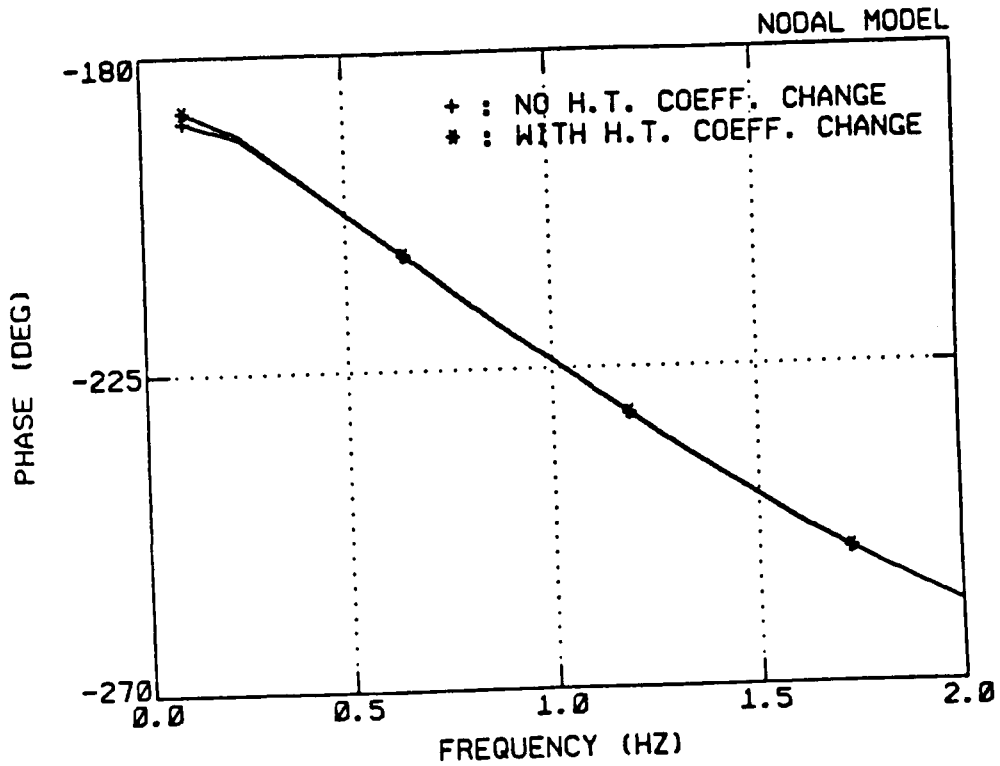


Figure 4-9. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with and without including the variation of heat transfer coefficient caused by the disturbance of coolant velocity.

4.3.5 Consideration of random heat transfer disturbance

The case with random heat transfer disturbance is more complicated than the other cases. In the phenomenological model, we calculated $\text{PHASE}(\text{TOUT}, N)$ for random heat transfer disturbance with the assumption that the disturbance sources at different axial positions are not correlated; hence there is an infinite number of independent random sources caused by the random heat transfer process. This assumption cannot be simulated in the nodal model because the number of nodes must be limited. In the real case, the disturbance sources might be correlated within a certain distance. Using the lumped nodal model is equivalent to assuming that the correlated distance is the length of each fuel node (or cladding node). With more fuel nodes, the correlation distance is shortened.

Figure 4-10 shows $\text{PHASE}(\text{TOUT}, N)$ for the random heat transfer case with different numbers of fuel nodes. With a cosine flux shape and 10 fuel nodes, the phase slope is estimated to be 64.3 deg/Hz. Fig.4-11 shows the coherence between the core-exit coolant temperature and the neutron power with different numbers of fuel nodes; n fuel nodes means that there are n independent disturbance sources. As expected, when $n=1$, the coherence is one. The coherence decreases sharply when the number of nodes increases. This result suggests that if the random heat transfer is the dominant disturbance source, the coherence between the core-exit coolant temperature and the neutron power must be very small with the point reactor kinetics.

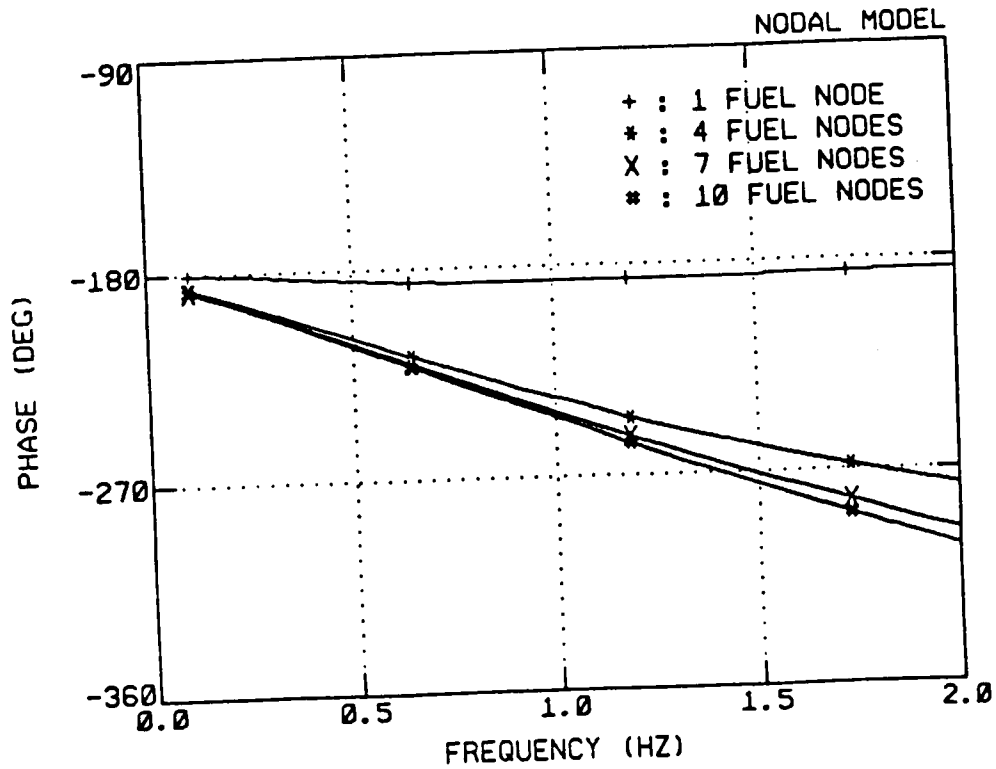


Figure 4-10. Comparison of the calculated phases between the core-exit coolant temperature and neutron power fluctuations with the random heat transfer disturbance for a variety of node numbers.

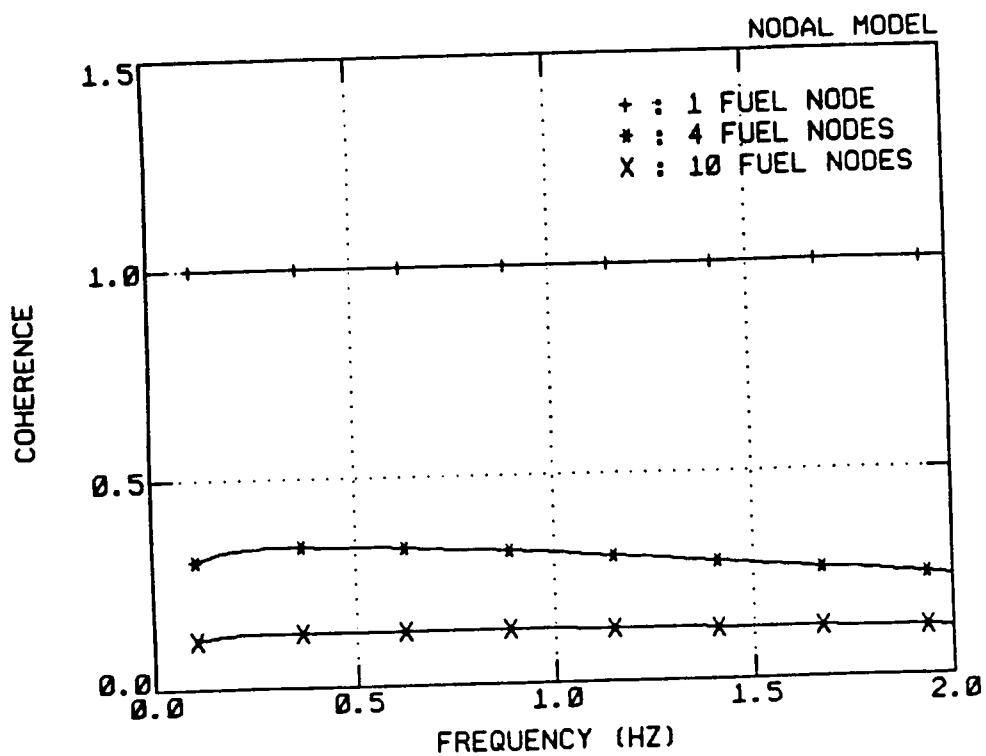


Figure 4-11. Comparison of the coherences between the core-exit coolant temperature and neutron power fluctuations with the random heat transfer disturbance for a variety of node numbers.

4.3.6 Monitoring the moderator temperature coefficient of reactivity

In analyzing the relationship between the phase slope and the coolant flow, another interesting and important phenomenon has been observed. This is the effect of the sign of the moderator temperature coefficient on the PHASE(TOUT,N).

The moderator temperature coefficient of reactivity is an important parameter related to the nuclear reactor safety, and its regulation would be important for reactor availability. For safe reactor operation, a negative moderator temperature coefficient is desired. The Westinghouse Nuclear Energy System Reference Safety Analysis Report [26] states that:

The fuel temperature coefficient will be negative and the moderator temperature coefficient of reactivity will be non-positive for power operating conditions, thereby providing negative reactivity feedback characteristics. Nominal power operation is permitted only in the range of overall negative moderator temperature coefficient.

The Oconee Nuclear Station (a Babcock and Wilcox PWR) Final Safety Analysis Report [72] states that: "The moderator temperature coefficient shall not be positive at power levels above 95% of full power." Among the PWRs from the three PWR vendors in United States, only in the Combustion Engineering PWR the moderator temperature coefficient at full power conditions is allowed to be positive [27].

Due to the boron concentration in the coolant, the moderator temperature coefficient of reactivity may become positive, especially at the beginning of the fuel cycle when the boron concentration is high. But the value of the moderator temperature coefficient cannot

be directly measured. In changing the coolant temperature, the fuel temperature will change too. Hence the directly measurable quantity using the static method is the temperature coefficient that contains the contribution from both the fuel and the coolant temperature changes. Measurements of this temperature coefficient are carried out at zero power and during power ascension. Corrections are applied to these measurements to determine whether the moderator temperature coefficient at full power is within the specified limits. Correction due to measurement uncertainty, Doppler coefficient, dissolved boron concentration, moderator temperature, and the power difference must be made [27].

Considering all the tedious corrections and uncertainties involved in the existing method, a technique that directly monitors and ensures the negativity of the moderator temperature coefficient will be valuable. In our analysis, we find that the $\text{PHASE}(\text{TOUT}, N)$ can be extrapolated from the linear phase-frequency region to -180 degrees at zero frequency with an inlet coolant temperature disturbance, a coolant velocity disturbance or a random heat transfer disturbance as the disturbance sources. This is due to the negative value of the moderator temperature coefficient of reactivity. For comparison, a positive moderator temperature coefficient is used to calculate the $\text{PHASE}(\text{TOUT}, N)$. The results are plotted in Figure 4-12. Figure 4-12 shows that except for the case of a heat source disturbance, the phase can be extrapolated to zero degree at zero frequency. From the analysis we know that in extrapolating $\text{PHASE}(\text{TOUT}, N)$ from the linear phase-

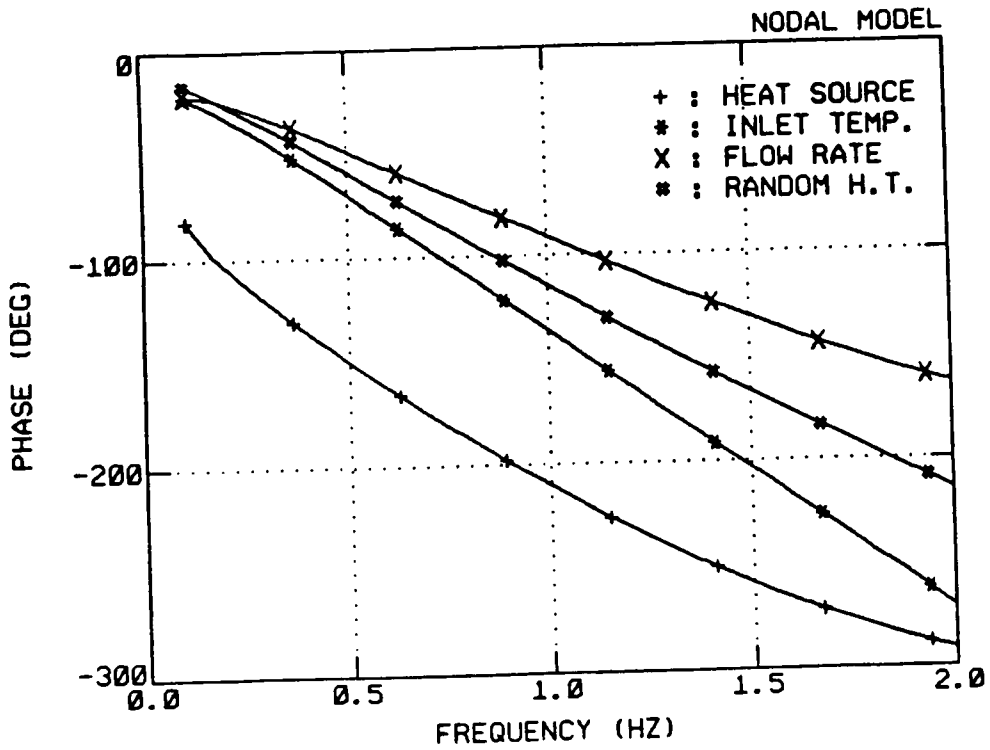


Figure 4-12. The phase for the cases with heat source, coolant inlet temperature, coolant flow rate and random heat transfer disturbances with positive moderator temperature coefficient of reactivity.

frequency region to zero frequency, the phase is zero for a positive moderator temperature coefficient, while for a negative value the phase is -180 degrees. This approach could provide a non-ambiguous technique for monitoring the sign of the moderator temperature coefficient of reactivity.

However, the applicability of this technique is based on the assumption that in the real nuclear power plant, a heat source disturbance is not the dominant disturbance source in the linear phase vs. frequency region. This aspect of the reactor signal property will be discussed in the next chapter.

5. APPLICATIONS OF THE PHASE RELATIONSHIP BETWEEN THE CORE-EXIT COOLANT TEMPERATURE AND THE NEUTRON POWER NOISE SIGNALS

5.1 Introduction

In this chapter, the phase relationship between the core-exit coolant temperature and the neutron flux from the LOFT reactor is interpreted and compared with that from the theoretical model. The data from the Sequoyah nuclear power plant and the Borssele nuclear power plant are also reviewed. Then the feasibility of using the slope of the phase shift between the core-exit thermocouple and the neutron detector noise signals (PHASE(TOUT,N)) to infer flow rate and the applicability of the extrapolated PHASE(TOUT,N) at zero frequency to monitor the moderator temperature coefficient of reactivity are discussed.

5.2 The LOFT Reactor Data Analysis

The LOFT reactor, located at the Idaho National Engineering Laboratory (INEL), is a 55 MW(t) PWR. The core contains five assemblies that have a square cross section with fuel pin and guide tubes in locations typical of those in PWR fuel assemblies, and four assemblies that have a triangular cross section. The height of the core is 5.5 ft. The core contains cobalt self-powered neutron detectors (SPND) at four different axial levels of 11, 27, 44 and 61 inches above the core bottom labeled as D, C, B and A. The core-exit temperature is measured with thermocouples located at the fuel bundle

upper grid. The location of the thermocouple is about one inch above the fuel rod and is approximately five inches above the active fuel region. The thermocouples are stainless steel sheathed K-type chromel-alumel thermocouples with MgO as the insulator. The break frequency of the thermocouple is at 3 Hz. At 100% flow, the coolant velocity measured with a turbine flowmeter at the upper plenum measuring point is 17 ft/sec. The flow area ratio between the upper plenum measuring point and the upper core region is 12 : 17. Hence the coolant velocity in the core region is estimated to be 12 ft/sec.

One of the experimental PHASE(TOUT,N) results [71] for the LOFT reactor at 100% flow is shown in Figure 5-1. The noise experimental data were acquired by the Oak Ridge National Laboratory on July 21, 1983, when the LOFT reactor was at 100% power, 100% flow operation condition. In Figure 5-1 the phase slope is estimated to be 90 deg/Hz. The results of the previous chapters show that the slope of PHASE(TOUT,N) depends on the source of disturbance. Therefore, in order to compare the PHASE(TOUT,N) from the experimental data with that from the theoretical model, we first study the dominant disturbance source that causes the fluctuations of the neutron power and the core-exit coolant temperature in the LOFT reactor. Then the multi-nodal model is used to calculate PHASE(TOUT,N) and to compare with that from the experimental data.

5.2.1 The dominant disturbance source in the LOFT reactor

The coolant flow rate disturbance is found to be the dominant disturbance source for the following reasons:

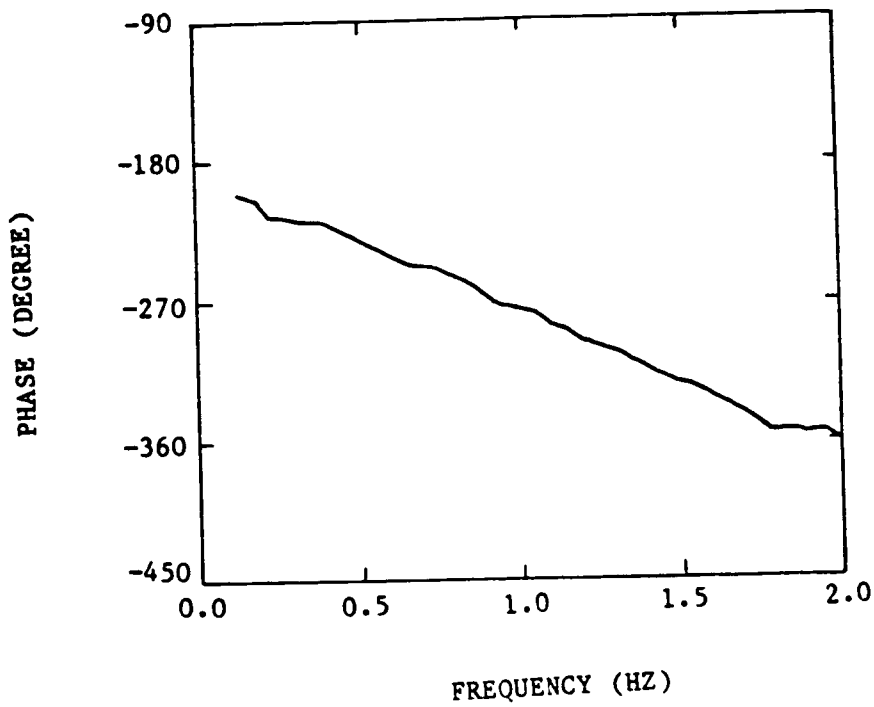


Figure 5-1. The experimental phase between the core-exit coolant temperature and neutron power fluctuations of the LOFT reactor at 100% flow condition.

(1) In an earlier work, Canon and Clemo [24] suggested that fluctuations in the neutron flux in the LOFT reactor is strongly affected by flow fluctuations in the range 0 to 2 Hz. They observed a high coherence between the in-core neutron detector noise and the coolant flow noise as measured by a Venturi flow meter.

(2) The coherence between the coolant inlet temperature and the neutron power noises and the coherence between the coolant inlet and outlet temperatures are very small [71]. This excludes the inlet coolant temperature fluctuation as a dominant disturbance source.

(3) The heat source is not considered as a dominant disturbance source, because the result from the experimental data shows that the extrapolated $\text{PHASE}(\text{TOUT}, \text{N})$ at zero frequency is almost out of phase, i.e., -180 degrees. The theoretical studies show that this kind of relationship exists except for the case with the heat source disturbance.

(4) We exclude random heat transfer as the dominant disturbance source because the coherence between the coolant flow rate and the neutron flux is as high as 0.9 [24]. Hence the contribution from the other disturbance sources must be small. Another reason is that the experimental data shows that the coherence between the core-exit coolant temperature and the neutron power is 0.6 in the 0.1-2 Hz region [71]. However, from the theoretical study in Section 4, we know that if the random heat transfer is the dominant disturbance source, this coherence should be much smaller.

Hence we conclude that in the LOFT reactor, coolant velocity

disturbance is the dominant source giving rise to fluctuations in the neutron power and the core-exit coolant temperature.

5.2.2 The multi-nodal model simulation

In calculating the $\text{PHASE}(\text{TOUT}, \text{N})$ for the real case using the multi-nodal model, some uncertainties are involved and some assumptions are made. These uncertainties and assumptions are as follows: (1) the distance between the top of the active fuel and the temperature sensor is assumed to be 5 inches, and the flow area in this region is assumed to be the same as in the core region; (2) the real neutron flux shape is inferred from the neutron flux measurements at four different levels; and (3) the break frequency of the temperature sensor is assumed to be at 2 Hz instead of 3 Hz. Under these assumptions, the $\text{PHASE}(\text{TOUT}, \text{N})$ calculated is shown in Figure 5-2. As seen from Figure 5-2, if the dominant disturbance source is the coolant velocity fluctuation, the calculated phase slope is 89 deg/Hz. This is very close to the experimental result shown in Figure 5-1, and hence the theoretical result supports the coolant velocity fluctuation as the dominant disturbance source in the LOFT reactor.

5.2.3 The space dependent phenomenon

LOFT is a 1/55 scale model of a commercial PWR. It has a small core with almost point reactor kinetics behavior. In the 0.1-2 Hz frequency range, the neutron noise signals measured from neutron detectors at different positions show very high coherence [71]. The calculated $\text{PHASE}(\text{TOUT}, \text{N})$ with the neutron noises taken from detectors

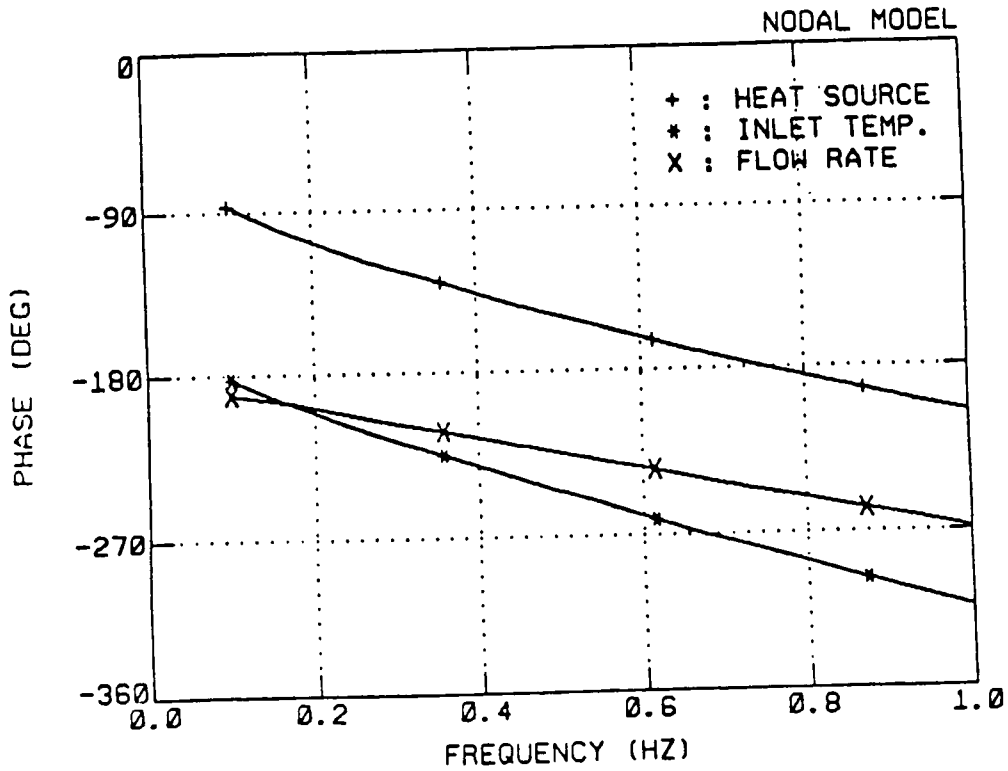


Figure 5-2. The simulated phases between the core-exit coolant temperature and neutron power fluctuations of the LOFT reactor at 100% flow condition with a variety of disturbance sources.

at different positions also are very close to each other. However, if we analyze the data carefully, we still can observe some space dependent behavior as shown in Figure 5-3, in which the locations of neutron detectors are at three different levels from the top to the bottom of the core labeled as A,B and D. These figures show that the slopes decrease slightly as the distance between the core-exit temperature sensor and the neutron detector decreases. This small space dependent phenomenon is also successfully simulated using the same point reactor kinetics, except that the temperature-reactivity weighting is a function of the position of the neutron detector. For the details refer to Appendix C. Figure 5-4 shows the results of the simulation with a coolant velocity disturbance, in which a cosine flux shape is used. The dynamics of the temperature sensor and the effect of the distance between the temperature sensor and the top of the active fuel are not considered. The neutron detectors are assumed to be at the fourth and the nineteenth coolant nodes. A small difference is observed by comparing these space dependent results with those from point kinetics approximation.

5.3 The Sequoyah Nuclear Power Plant and the Borssele Nuclear Power Plant Data Analysis

The Sequoyah-1 nuclear power plant is a 1140 MW(e) Westinghouse PWR. The core height is 3.65 m. The coolant velocity in the core at 100% flow condition is 4.82 m/s. The neutron flux is measured with 1.8 m long ex-core ionization chambers and in-core movable flux

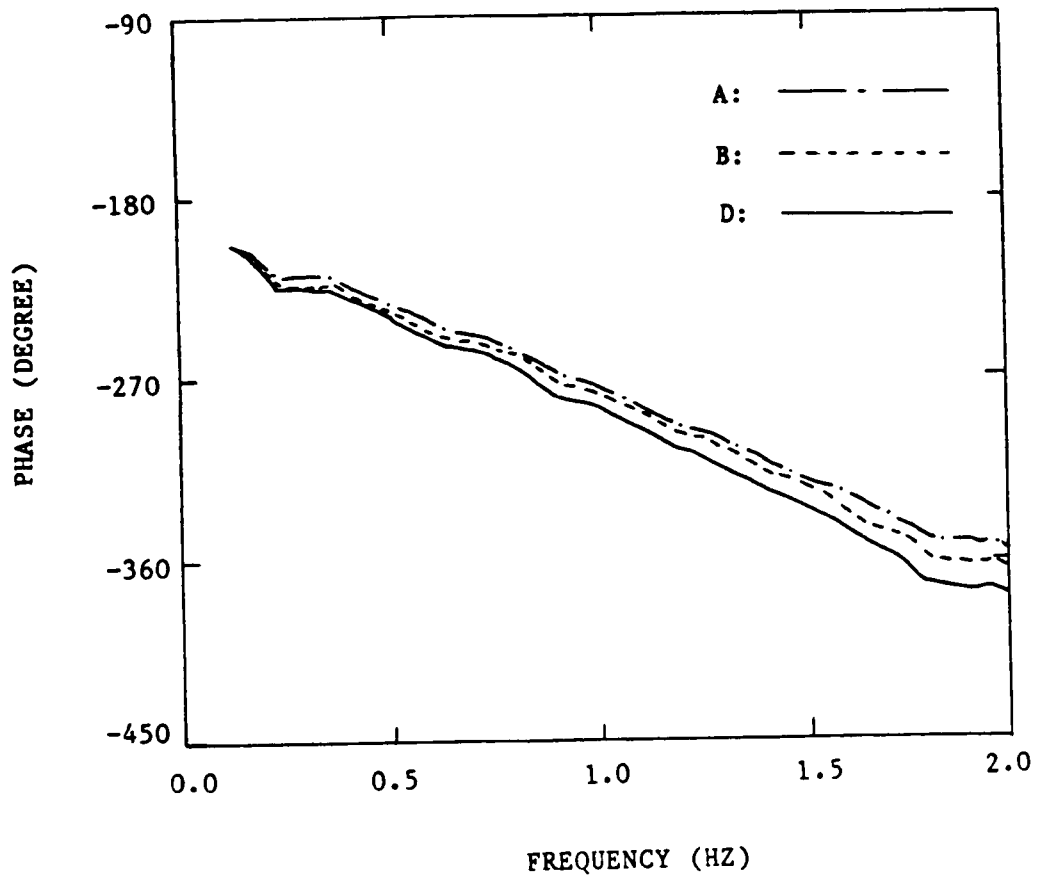


Figure 5-3. The experimental phase between the core-exit coolant temperature and the neutron power fluctuations of the LOFT reactor at 100% flow condition with the neutron detectors at different levels A,B and D.

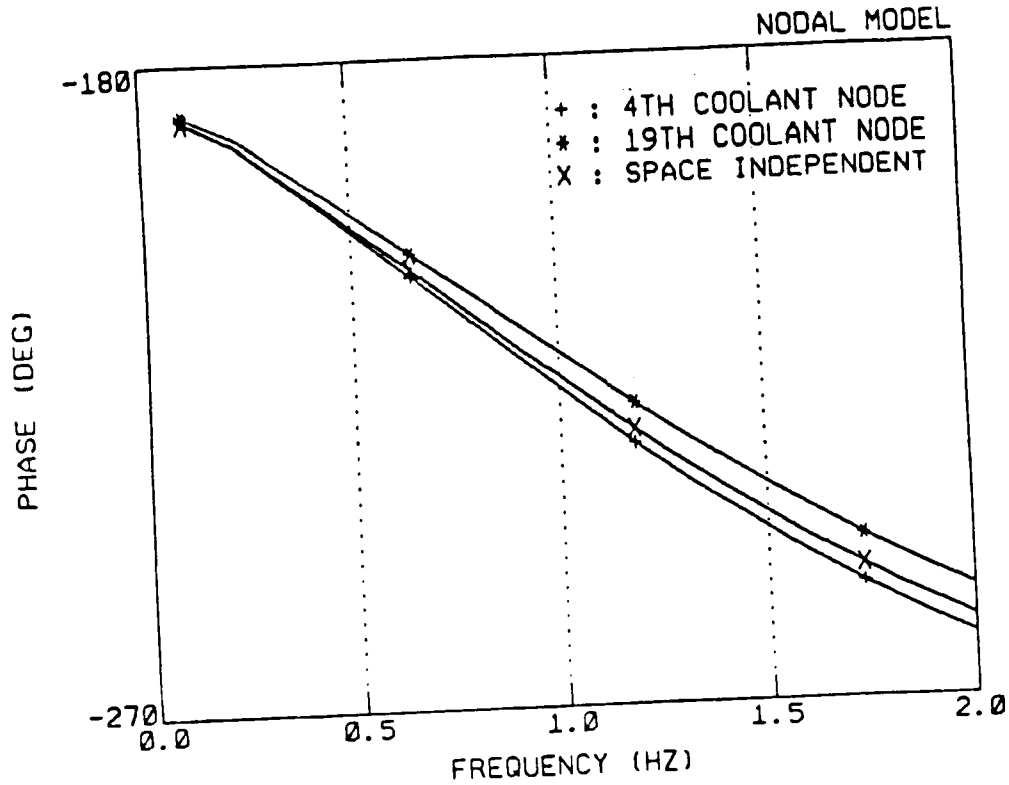


Figure 5-4. Comparison of the phases between the core-exit coolant temperature and neutron power fluctuations with space independent neutron kinetics, and the space dependent neutron kinetics with neutron detectors positions at (a) the fourth coolant node, and (b) the nineteenth coolant node for the case with coolant flow rate disturbance and cosine flux shape.

mapping detectors, and the core-exit coolant temperature is measured using K-type thermocouples located at 10 cm above the top of the core. Figure 5-5 shows one of the PHASE(TOUT,N) results using the out of core neutron detector [10]. The studies of the neutron signals from the neutron flux-mapping detector at different positions show that they are very strongly space dependent. However, a linear phase vs. frequency region is always observed, and the phase at 0 Hz extrapolated from this linear region is nearly -180 degrees.

The Borssele nuclear power plant in the Netherlands is a 477 MW(e) Kraftwerk Union (KWU, West Germany) PWR. The nominal flow velocity is 4.28 m/sec, the core height is 2.55 m. The neutron flux is measured by the ex-core ionization chambers and in-core SPNDs. The core-exit coolant temperature is measured with the thermocouple located at 35 cm above the top of the core. Figure 5-6 shows the phase between an ex-core ionization chamber and the core-exit temperature sensor [29]. The analysis of the neutron power signals from the in-core self-powered neutron detectors shows that the neutron kinetics is space dependent. The linear phase-frequency relationships are observed, and the phase at 0 Hz extrapolated from this linear region is nearly -180 degrees.

5.4 Discussion

5.4.1 Use of PHASE(TOUT,N) slope to infer the coolant flow rate

Summarizing the results of Sections 2 through 5, we have the following conclusions:

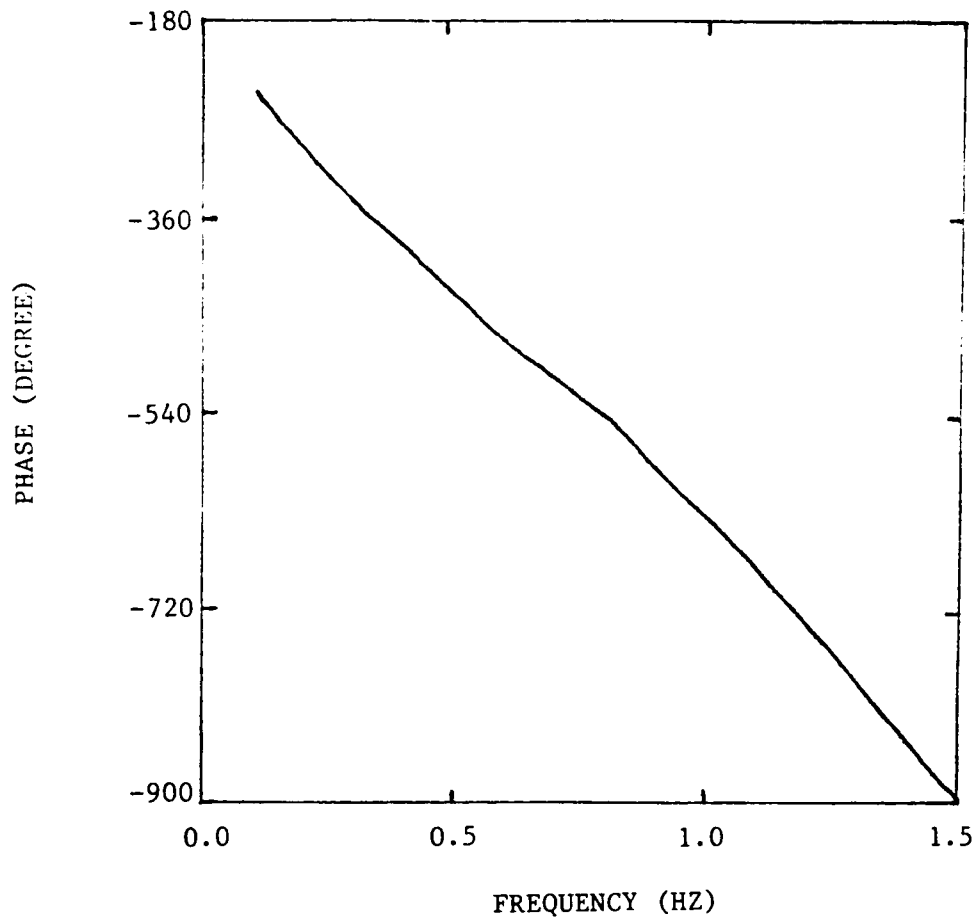


Figure 5-5. The experimental phase between the core-exit coolant temperature and neutron power fluctuations of the Sequoyah Nuclear Power Plant.

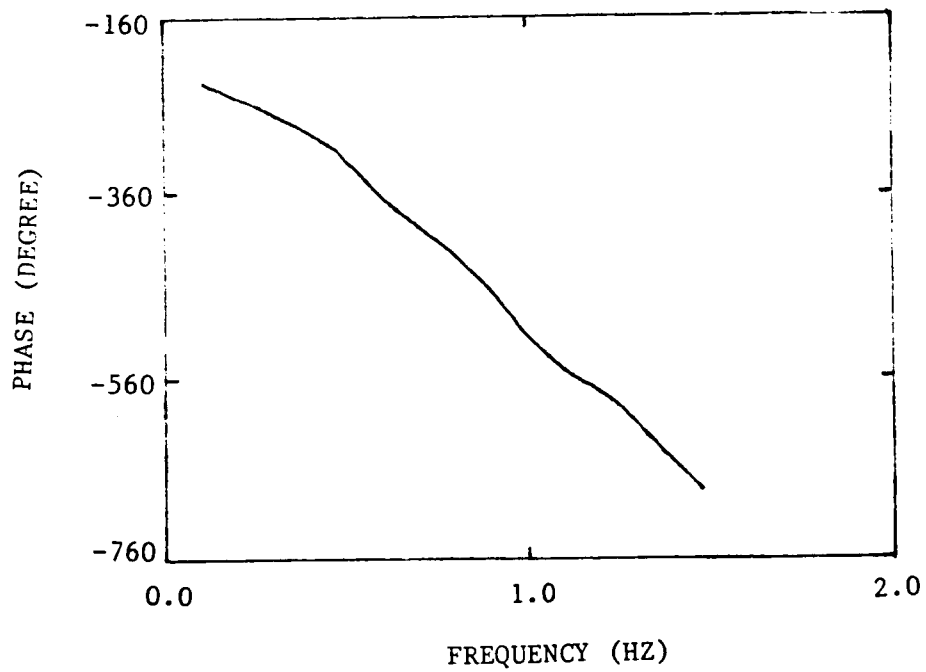


Figure 5-6. The experimental phase between the core-exit coolant temperature and neutron power fluctuations of the Borssele Nuclear Power Plant.

(1) Both the experimental data and the theoretical modeling results show the existence of the linear phase vs. frequency relationship in a certain frequency region (about 0.1 to 2 Hz) with lower and upper bounds. The lower bound is due to the nonlinear phase-frequency behavior of the fuel heat transfer process and the neutron kinetics. The upper bound is the sink or peak frequency obtained as a result of integration. This upper bound is a function of the driving sources, the neutron flux shapes and the coolant transport time through the reactor core.

(2) The theoretical study shows that without the temperature sensor dynamics, the flow rate is inversely proportional to the slope of $\text{PHASE}(\text{TOUT}, \text{N})$ using both the point reactor kinetics and the extremely localized neutron kinetics.

(3) The theoretical study also shows that in this linear phase vs. frequency region, the slope is a function of the disturbance source, the flux shape and the flow rate, but not a function of the neutron kinetics parameters such as the fuel or moderator temperature coefficient of reactivity and the heat transfer coefficient.

It is clear that the slope of $\text{PHASE}(\text{TOUT}, \text{N})$ is related to the coolant velocity or the flow rate. In the reactor, the flow area is not uniform along the whole channel, especially the flow area at the region from the top of the core to the core-exit temperature sensor may be very different from that in the core region. Therefore the coolant velocity varies along the channel, while the flow rate is more likely to be constant. The means of inferring the flow rate from

the slope of $\text{PHASE}(\text{TOUT}, N)$ is our main concern. From the theoretical study, it is safe to say that the flow rate is inversely proportional to the slope of $\text{PHASE}(\text{TOUT}, N)$, but something unrelated to the flow rate also contributes to the slope of $\text{PHASE}(\text{TOUT}, N)$ such as the dynamics of the sensor. Hence the relationship between the flow rate and the slope can be expressed by the following equation:

$$\text{slope}(\text{PHASE}(\text{TOUT}, N)) = A/(\text{flow rate}) + B, \quad (5-1)$$

where A and B are the constants to be determined. From the theoretical study, A is a function of the flux shapes and the disturbance sources and B is a function of the temperature sensor dynamics. Although we would like to calculate A and B based on the theoretical model, due to the uncertainties involved in the model, such as the uncertainties in the space dependent neutron kinetics, the disturbance source and the irregular geometry shape along the coolant channel, it is very difficult to get accurate values theoretically. Theoretical studies can only show the proper range of the values of A and B. The practical approach to obtain the values of the constants A and B is through calibrations based on the real data, similar to the calibrations used for most measurement instruments. By collecting real data for a variety of flow rates, the empirical values for A and B can be obtained, and the formula will be valuable for further flow inference.

5.4.2 Monitoring the moderator temperature coefficient of reactivity

As mentioned in Section 4, a negative moderator temperature coefficient of reactivity is desired for safe reactor operation, and the high boron concentration in the coolant may cause positive

moderator temperature coefficient. The theoretical study shows that except for the case with heat source disturbance, $\text{PHASE}(\text{TOUT}, \text{N})$ at zero frequency extrapolated from the linear phase vs. frequency region will be -180 degrees if the moderator temperature coefficient of reactivity is negative, and zero degrees if it is positive.

All the results of analysis of the real data from the LOFT reactor, the Sequoyah-1, and the Borssele nuclear power plant show that the $\text{PHASE}(\text{TOUT}, \text{N})$ could be extrapolated to -180 degrees at zero frequency. Based on the theoretical study these results exclude the possibility of the heat source disturbance as the dominant disturbance source for the above PWRs. This confirms the applicability of monitoring the sign of the moderator temperature coefficient of reactivity in a commercial PWR by using the extrapolated $\text{PHASE}(\text{TOUT}, \text{N})$ at zero frequency. As a noise analysis technique, this new method is valuable because of its following attractive features:

- (1) no interruption of the reactor operation.
- (2) can be performed in any operating condition.
- (3) because of the big difference between zero degree (for positive coefficient) and -180 degrees (for negative value), the result is non-ambiguous and the technique is robust with regard to the uncertainty and calculational inadequacies.

6. EXTENDED KALMAN FILTER FOR FLOW ESTIMATION

6.1 The Kalman Filtering Algorithm

Data processing schemes which separate desired signals from unwanted disturbances (noises) are referred to as filters. The Kalman filter is a method of combining measurement data, which are contaminated with noises, with the system model to get an optimal estimate of some desired variables.

The Kalman filter algorithm can be derived by various approaches. It can be derived based on the least-squares estimation [62], Bayesian approach [63] or maximum likelihood estimation technique [39]. In order to show the relationship between the Kalman filter and the ambiguity function, a technique that will be used in Chapter 7 for the performance analysis of the maximum likelihood estimator, the Kalman filter derived from the maximum likelihood estimation technique is described briefly here. For details, see reference [39].

A linear system model can be described using the stochastic differential equation

$$\dot{\underline{x}}(t) = A(t)\underline{x}(t) + B(t)\underline{u}(t) + G(t)\underline{w}(t) , \quad (6-1)$$

where,

$\underline{x}$: state variables vector,

A : system dynamic matrix,

B : deterministic input matrix,

$\underline{u}$: input vector,

G : noise input matrix,

$\underline{w}$: white Gaussian process noise with statistics,

$$E[\underline{w}(t)] = \underline{0} ,$$

$$E[\underline{w}(t)\underline{w}(t')^T] = Q(t)\delta(t-t') ,$$

Q is a symmetric and positive semidefinite matrix.

The solution to this stochastic differential equation can be expressed as

$$\begin{aligned} \underline{x}(t) = & \phi(t, t_0)\underline{x}(t_0) + \int_{t_0}^t \phi(t, \tau)B(\tau)\underline{u}(\tau)d\tau \\ & + \int_{t_0}^t \phi(t, \tau)G(\tau)\underline{w}(\tau)d\tau , \end{aligned} \quad (6-2)$$

where,

$\phi(t, t_0)$: state transition matrix.

Based on the assumption that $\underline{u}(t)$ is constant from t_{i-1} to t_i , Eq.(6-2) can be further expressed as a stochastic difference equation

$$\underline{x}(t_i) = \phi(t_i, t_{i-1})\underline{x}(t_{i-1}) + B_d(t_{i-1})\underline{u}(t_{i-1}) + \underline{w}_d(t_{i-1}) , \quad (6-3)$$

where,

$$B_d(t_{i-1}) = \int_{t_{i-1}}^{t_i} \phi(t_i, \tau)B(\tau)d\tau ,$$

$$\underline{w}_d(t_{i-1}) = \int_{t_{i-1}}^{t_i} \phi(t_i, \tau)G(\tau)\underline{w}(\tau)d\tau .$$

The covariance of $\underline{w}_d$ is denoted as Q_d .

$$Q_d(t_{i-1}) = \int_{t_{i-1}}^{t_i} \phi(t_i, \tau)G(\tau)Q(\tau)G(\tau)^T\phi(t_i, \tau)^Td\tau . \quad (6-4)$$

Because the practical implementation of the Kalman filter is through a digital computer, discrete-time measurements are assumed.

The measurement model is

$$\underline{z}(t_i) = H(t_i)\underline{x}(t_i) + \underline{v}(t_i) , \quad (6-5)$$

where,

$\underline{z}$: measurement vector,

H : measurement matrix,

$\underline{v}$: white Gaussian measurement noise with statistics,

$$E[\underline{v}(t_i)] = \underline{0} ,$$

$$E[\underline{v}(t_i)\underline{v}^T(t_j)] = \begin{cases} R(t_i) , & \text{if } t_i = t_j , \\ 0 , & \text{if } t_i \neq t_j . \end{cases}$$

It is also assumed that the process noise $\underline{w}$ and the measurement noise $\underline{v}$ are independent of each other.

Let us define $\underline{Z}(t_i)$ as a composite vector which is comprised of the entire measurement history up to the current time, that is,

$$\underline{Z}(t_i) = [\underline{z}(t_1), \underline{z}(t_2), \dots, \underline{z}(t_i)]^T .$$

Suppose we are at time t_{i-1} and have just taken the measurement $\underline{z}(t_{i-1})$, we are interested in the probability density of $\underline{x}(t_{i-1})$ conditioned on the entire measurement history up to that time, $p[\underline{x}(t_{i-1}) | \underline{Z}(t_{i-1})]$, and the propagation of the density to compute $p[\underline{x}(t_i) | \underline{Z}(t_i)]$ at the next time step t_i . In other words, we want to propagate the conditional density from time t_{i-1} , just after incorporating the measurement $\underline{z}(t_{i-1})$, to time t_i , just after the measurement $\underline{z}(t_i)$. In the derivation, this process is divided into two steps: (1) time propagation from just after the measurement $\underline{z}(t_{i-1})$ to

just before the measurement $\underline{z}(t_i)$, and (2) a measurement update at t_i . By repeated application of Bayes' rule, the density $p[\underline{x}(t_i) | \underline{Z}(t_i)]$ is written in terms of three other densities as shown in Eq.(6-7), each of which can be evaluated rather easily.

$$\begin{aligned}
 p[\underline{x}(t_i) | \underline{Z}(t_i)] &= p[\underline{x}(t_i), \underline{Z}(t_i)] / p[\underline{Z}(t_i)] \\
 &= p[\underline{x}(t_i), \underline{z}(t_i), \underline{Z}(t_{i-1})] / p[\underline{z}(t_i), \underline{Z}(t_{i-1})] \\
 &= p[\underline{z}(t_i) | \underline{x}(t_i), \underline{Z}(t_{i-1})] \cdot p[\underline{x}(t_i), \underline{Z}(t_{i-1})] \\
 &\quad / \{p[\underline{z}(t_i) | \underline{Z}(t_{i-1})] \cdot p[\underline{Z}(t_{i-1})]\} \\
 &= p[\underline{z}(t_i) | \underline{x}(t_i), \underline{Z}(t_{i-1})] \cdot p[\underline{x}(t_i) | \underline{Z}(t_{i-1})] \\
 &\quad / p[\underline{z}(t_i) | \underline{Z}(t_{i-1})].
 \end{aligned}
 \tag{6-6}$$

Once these densities, i.e., $p[\underline{z}(t_i) | \underline{x}(t_i), \underline{Z}(t_{i-1})]$, $p[\underline{x}(t_i) | \underline{Z}(t_{i-1})]$ and $p[\underline{z}(t_i) | \underline{Z}(t_{i-1})]$, are described explicitly, then the optimal estimate at time t_i can be calculated recursively based on the estimate at time t_{i-1} and the measurement at time t_i . The $p[\underline{x}(t_i) | \underline{Z}(t_i)]$ thus obtained has the form

$$p[\underline{x}(t_i) | \underline{Z}(t_i)] = \frac{|H(t_i)P(t_i; t_{i-1})H(t_i)^T + R(t_i)|^{1/2}}{(2\pi)^{n/2} |P(t_i; t_{i-1})R(t_i)|^{1/2}} \exp(-DUM/2)
 \tag{6-7}$$

$$\begin{aligned}
 DUM &= [\underline{z}(t_i) - H(t_i)\hat{\underline{x}}(t_i; t_{i-1})]^T R(t_i)^{-1} [\underline{z}(t_i) - H(t_i)\hat{\underline{x}}(t_i; t_{i-1})] \\
 &\quad + [\underline{x}(t_i) - \hat{\underline{x}}(t_i; t_{i-1})]^T P(t_i; t_{i-1})^{-1} [\underline{x}(t_i) - \hat{\underline{x}}(t_i; t_{i-1})] \\
 &\quad - [\underline{z}(t_i) - H(t_i)\hat{\underline{x}}(t_i; t_{i-1})]^T [H(t_i)P(t_i; t_{i-1})H(t_i)^T + R(t_i)]^{-1} \\
 &\quad \cdot [\underline{z}(t_i) - H(t_i)\hat{\underline{x}}(t_i; t_{i-1})],
 \end{aligned}$$

where,

- $\hat{\underline{x}}(t_i; t_{i-1})$: the conditional estimated mean at time t_i
 based on the measurements up to time t_{i-1} ,
 $P(t_i; t_{i-1})$: the conditional covariance at time t_i based
 on the measurements up to time t_{i-1} ,
 n : the dimension of the state variable vector.

To obtain a maximum likelihood estimate of the system state, an appropriate likelihood function must be defined as a scalar function relating the available measurements, the state variables, and the system parameters. One choice of a likelihood function would be $p[\underline{x}(t_i) | \underline{Z}(t_i)]$. But because we assume that $p[\underline{x}(t_i) | \underline{Z}(t_i)]$ is Gaussian, it is more convenient to define the likelihood function as the natural logarithm of the conditional density

$$L[\underline{x}(t_i), \underline{Z}(t_i)] = \ln [p[\underline{x}(t_i) | \underline{Z}(t_i)]] . \quad (6-8)$$

Maximizing this function yields the same estimate as maximizing the density itself, since for any function p , p and $\ln(p)$ have their maximum at the same point.

To generate the maximum likelihood estimate of $\underline{x}(t_i)$, based on the measurements up to time t_i , denoted as $\hat{\underline{x}}(t_i; t_i)$, we must solve

$$\partial L[\underline{x}(t_i), \underline{Z}(t_i)] / \partial \underline{x}(t_i) = 0 , \quad (6-9)$$

for which the solution is

$$\begin{aligned}\hat{\underline{x}}(t_i; t_i) &= [P(t_i; t_{i-1})^{-1} + H^T(t_i)R^{-1}(t_i)H(t_i)]^{-1} \\ &\quad \cdot [H^T(t_i)R^{-1}(t_i)z(t_i) + P(t_i; t_{i-1})^{-1}\hat{\underline{x}}(t_i; t_{i-1})] .\end{aligned}\quad (6-10)$$

Using the matrix inversion lemma,

$$[P^{-1} + H^T R^{-1} H]^{-1} = P - PH^T [HPH^T + R]^{-1} HP , \quad (6-11)$$

after a lengthy derivation, finally we get that Eq.(6-7) is equivalent to the following

$$p[\underline{x}(t_i) | \underline{Z}(t_i)] = [(2\pi)^{n/2} | P(t_i; t_i) |^{-1/2}]^{-1} \exp(-DUM/2) , \quad (6-12)$$

$$DUM = [\underline{x}(t_i) - \hat{\underline{x}}(t_i; t_i)]^T P(t_i; t_i)^{-1} [\underline{x}(t_i) - \hat{\underline{x}}(t_i; t_i)] ,$$

and

$$\begin{aligned}P(t_i; t_i) &= P(t_i; t_{i-1}) - \\ &\quad P(t_i; t_{i-1})H^T(t_i)[H(t_i)P(t_i; t_{i-1})H^T(t_i) + R(t_i)]^{-1} \\ &\quad \cdot H(t_i)P(t_i; t_{i-1}) .\end{aligned}\quad (6-13)$$

In summary, using the stochastic difference equation, the optimal state estimate Kalman filter is propagated from measurement time t_{i-1} to t_i by the relation,

$$\hat{\underline{x}}(t_i; t_{i-1}) = \phi(t_i, t_{i-1})\hat{\underline{x}}(t_{i-1}; t_{i-1}) + B(t_{i-1})\underline{u}(t_{i-1}) , \quad (6-14)$$

and,

$$P(t_i; t_{i-1}) = \phi(t_i, t_{i-1})P(t_{i-1}; t_{i-1})\phi^T(t_i, t_{i-1}) + Q_d(t_{i-1}) . \quad (6-15)$$

After the measurement $z(t_i)$ the estimate is updated as

$$\hat{\underline{x}}(t_i; t_i) = \hat{\underline{x}}(t_i; t_{i-1}) + K(t_i)[z(t_i) - H(t_i)\hat{\underline{x}}(t_i; t_{i-1})] , \quad (6-16)$$

and,

$$P(t_1; t_1) = P(t_1; t_{1-1}) - K(t_1)H(t_1)P(t_1; t_{1-1}) , \quad (6-17)$$

where $K(t_1)$ is the Kalman filter gain,

$$K(t_1) = P(t_1; t_{1-1})H^T(t_1)[H(t_1)P(t_1; t_{1-1})H^T(t_1) + R(t_1)]^{-1} . \quad (6-18)$$

The initial conditions for the recursion are given by $\underline{x}_0$ and $P(t_0)$.

We see that the algorithm has a predictor-corrector structure: the residual between the measurement and the best predictor, $\underline{z}(t_1) - H(t_1)\hat{\underline{x}}(t_1; t_{1-1})$, is multiplied by the weighting matrix $K(t_1)$ to get the best correction.

For the Kalman filter based on the stochastic differential equation, i.e., Eq.(6-1), the algorithm for the time propagation is modified as follows,

$$\frac{d}{dt} \hat{\underline{x}}(t; t_{1-1}) = A(t)\hat{\underline{x}}(t; t_{1-1}) + B(t)\underline{u}(t) , \quad (6-19)$$

$$\begin{aligned} \frac{d}{dt} P(t; t_{1-1}) \\ = A(t)P(t; t_{1-1}) + P(t; t_{1-1})A^T(t) + G(t)Q(t)G^T(t) . \end{aligned} \quad (6-20)$$

$\hat{\underline{x}}(t_1; t_{1-1})$ and $P(t_1; t_{1-1})$ are obtained by solving these differential equations using algorithms such as the Euler method or the Runge-Kutta method [60].

6.2 The Extended Kalman Filter

The estimation equations in section 6.1 are valid for a linear model. Now consider a nonlinear continuous system with discrete measurements

$$\frac{d}{dt} \underline{x}(t) = f[\underline{x}(t), \underline{u}(t), t] + G(t)\underline{w}(t) , \quad (6-21)$$

and,

$$\underline{z}(t_i) = h[\underline{x}(t_i), t_i] + \underline{v}(t_i) . \quad (6-22)$$

Given such a system model, it is desired to get an approximate state estimate by applying the previous linear estimation algorithm directly. The basic idea of the extended Kalman filter is to achieve this goal by relinearizing about each new estimate. If the deviation in each time step is small, we can apply the first order perturbation technique and get adequate results.

The extended Kalman filter algorithm is as follows: the state estimate propagates from measurement time t_{i-1} to t_i by the relation

$$\frac{d}{dt} \underline{x}(t) = f[\underline{x}(t), \underline{u}(t), t] , \quad (6-23)$$

and the covariance matrix is given by,

$$\begin{aligned} \frac{d}{dt} P(t; t_i) &= F[t, \hat{\underline{x}}(t; t_i)]P(t; t_i) \\ &+ P(t; t_i)F^T[t, \hat{\underline{x}}(t; t_i)] + G(t)Q(t)G^T(t) . \end{aligned} \quad (6-24)$$

The measurement update is

$$K(t_1) = P(t_1; t_{1-1}) H^T [t_1, \hat{x}(t_1; t_{1-1})] \cdot \{H[t_1, \hat{x}(t_1; t_{1-1})] P(t_1; t_{1-1}) H^T [t_1, \hat{x}(t_1; t_{1-1})] + R(t_1)\}^{-1}, \quad (6-25)$$

$$\hat{x}(t_1; t_1) = \hat{x}(t_1; t_{1-1}) + K(t_1) \{z(t_1) - h[\hat{x}(t_1; t_{1-1}), x(t_1; t_{1-1})]\}, \quad (6-26)$$

and,

$$P(t_1; t_1) = P(t_1; t_{1-1}) - K(t_1) H[t_1, \hat{x}(t_1; t_{1-1})] P(t_1; t_{1-1}), \quad (6-27)$$

where,

$$F[t_1, \hat{x}(t_1)] \triangleq \frac{\partial f[x, u, t]}{\partial x} \Big|_{x=\hat{x}(t_1)}, \quad (6-28)$$

$$H[t_1, \hat{x}(t_1)] \triangleq \frac{\partial h[x, t]}{\partial x} \Big|_{x=\hat{x}(t_1)}. \quad (6-29)$$

6.3 Modified Extended Kalman Filter Algorithm

Concerning computational effectiveness, the Kalman filter algorithm based on the difference equation (discrete algorithm) is superior to that based on the differential equation (continuous algorithm). This is clear by comparing Eq.(6-14) and (6-15) for the difference equation representation with Eqs.(6-19) and (6-20) for the differential equation representation. While Eqs.(6-14) and (6-15) involve only a few calculations, the solution of Eq.(6-19) and (6-20) is more complicated. For a linear system, it is easy to discretize differential equations and transform them into difference equations. But for a nonlinear system, we have to first linearize the equations around the initial values. If the deviation is not far from the initial values, this linearized equation will be adequate.

Otherwise we have to relinearize the differential equation with respect to the new state values. To discretize each relinearized equation at every time step is computationally time consuming.

Fortunately for noise analysis, the state variables do not deviate far from the average values, around which the differential equation is linearized. By applying the first order perturbation theory, it will be possible to use the discrete algorithm but avoid discretizing each time.

For a linear time-invariant differential equation

$$\frac{d}{dt} \underline{x}(t) = A\underline{x}(t) + G\underline{w}(t) . \quad (6-30)$$

We can get an equivalent difference equation

$$\underline{x}(t_{i+1}) = \phi(t_{i+1}, t_i) \underline{x}(t_i) + \underline{w}_d , \quad (6-31)$$

where,

$$\phi(t_{i+1}, t_i) = e^{A\Delta t} , \quad (6-32)$$

$$\Delta t = t_{i+1} - t_i ,$$

and the covariance of $\underline{w}_d$, i.e., Q_d ,

$$Q_d = \int_{t_i}^{t_{i+1}} \phi(\tau, t_i) G Q(t) G^T \phi^T(\tau, t_i) d\tau . \quad (6-33)$$

And $e^{A\Delta t}$ can be calculated using the asymptotic series,

$$e^{A\Delta t} = I + A\Delta t + (A\Delta t)^2/2! + (A\Delta t)^3/3! + \dots \quad (6-34)$$

For a linear time-varying continuous system

$$\frac{d}{dt} \underline{x}(t) = F(t) \underline{x}(t) + G(t) \underline{w}(t) , \quad (6-35)$$

where $F(t)$ could be $\partial f / \partial t$ in the EKF algorithm. The equivalent difference equation for Eq.(6-35) is

$$\underline{x}(t_{i+1}) = \phi(t_i)\underline{x}(t_i) + \underline{w}_d, \quad (6-36)$$

where,

$$\phi(t_i) = e^{\int_{t_i}^{t_{i+1}} F(t) dt}$$

$$Q_d = \int_{t_i}^{t_{i+1}} \phi(\tau, t_i) G Q(t) G^T \phi(\tau, t_i)^T d\tau.$$

If we assume $F(t)$ to be constant between t_i and t_{i+1} , then $\phi(t_i)$ becomes $e^{F(t_i)\Delta t}$. But the calculation of $\phi(t_i)$ using Eq.(6-34) for each new $F(t_i)$ will be computationally inefficient. Therefore, instead of doing this, ϕ is expanded in Taylor series (changing the dummy variable from t into x , x is a function of t)

$$\begin{aligned} \phi(\underline{x}) &= \phi(\underline{x}_0) + \frac{d}{dx_1} \phi(\underline{x}) \Big|_{\underline{x}=\underline{x}_0} (x_1 - x_{10}) + \frac{d}{dx_2} \phi(\underline{x}) \Big|_{\underline{x}=\underline{x}_0} (x_2 - x_{20}) \\ &+ \dots + (\text{higher order terms}), \end{aligned} \quad (6-37)$$

where $\underline{x}_0$ is the initial estimate. If $\underline{x}$ is not far from $\underline{x}_0$ as expected in the noise analysis, then we can use the first order approximation

$$\phi(\underline{x}) = \phi(\underline{x}_0) + \sum_i \frac{d}{dx_i} \phi(\underline{x}) \Big|_{\underline{x}=\underline{x}_0} (x_i - x_{i0}), \quad (6-38)$$

where $d\phi(\underline{x}_0)/dx_i$ can be calculated as

$$\begin{aligned} &\frac{d}{dx_1} e^{A\Delta t} \Big|_{\underline{x}=\underline{x}_0} \\ &= \frac{d}{dx_1} A \Big|_{\underline{x}=\underline{x}_0} \Delta t + \frac{d}{dx_1} A \Big|_{\underline{x}=\underline{x}_0} A\Delta t + A \frac{d}{dx_1} A \Big|_{\underline{x}=\underline{x}_0} \Delta t + \dots \end{aligned} \quad (6-39)$$

Because $\phi(\underline{x}_0)$ and $d\phi(\underline{x}_0)/d\underline{x}_1$ can be calculated in advance, with Eq.(6-38), $\phi(\underline{x})$ can be updated with respect to the new estimate easily.

In applying this technique, there could be some bias if $\underline{x}$ is not close to $\underline{x}_0$ (such as the initial estimate of the flow rate is not close to the true value). To correct this error, relinearization around the final estimate values is still needed, but the study shows that usually two or three relinearizations will be sufficient.

Because the estimation usually is not very sensitive to the variation in Q_d , to make this algorithm even more efficient, Q_d will be held constant and modified only during relinearization.

6.4 The Model for Time Domain Estimation

The model developed in this chapter is for the LOFT reactor. In Chapter 4, a 43-node nodal model was used to calculate the phase angle relationship between the core-exit temperature and the neutron power signals. However, due to the capacity of the digital computer, the model used for real time domain estimation has to be a lower order model. Because of the uncertainties in the model, we realize that even with a high order model, for real application, the tuning of the EKF is still inevitable. Hence, here we pursue a model that is simple yet based on the physical laws in order to assure that the model is also applicable to the transient analysis. The uncertainties and errors involved in the model for the analysis of noise data can be

compensated by tuning the noise covariance matrix to the dynamic model by adding a pseudonoise [64].

The simplest model for the reactor core with one delayed neutron group, one fuel node and one coolant node is given by,

$$\frac{d}{dt} \left(\frac{\delta N}{N_0} \right) = \frac{-\beta}{\Lambda} \frac{\delta N}{N_0} + \frac{\lambda \delta C}{N_0} + \frac{\alpha_f}{\Lambda} \delta T_f + \frac{\alpha_c}{\Lambda} \delta T_c + \frac{\delta S(t)}{\Lambda}, \quad (6-40)$$

$$\frac{d}{dt} \left(\frac{\delta N}{N_0} \right) = \frac{\beta}{\Lambda} \frac{\delta N}{N_0} - \frac{\lambda \delta C}{N_0}, \quad (6-41)$$

$$\frac{d}{dt} \delta T_f = \left(\frac{\delta N}{N_0} \right) \frac{N_0 F_f}{m_f C_{pf}} - \frac{h_{fc} A_{fc}}{m_f C_{pf}} (\delta T_f - \delta T_c), \quad (6-42)$$

$$\begin{aligned} \frac{d}{dt} \delta T_c = & \left(\frac{\delta N}{N_0} \right) \frac{N_0 F_c}{m_c C_{pc}} + \frac{h_{fc} A_{fc}}{m_c C_{pc}} (\delta T_f - \delta T_c) \\ & + \frac{\dot{m}}{m_c} (\delta T_{IN} - \delta T_c) + \frac{\delta \dot{m}}{m_c} (T_{IN} - T_{co}). \end{aligned} \quad (6-43)$$

Because the thermal hydraulics is a much slower dynamic process compared with the neutron kinetics, combining these two processes together will get a system with a stiffness problem (i.e. some of the eigenvalues of the system matrix are much larger than others). To discretize these differential equations, the time step must be very small (<0.005 sec) to get good results and avoid computational overflow. In order to increase the time step, some approximations have to be made. One of the most commonly used approximations is the prompt jump approximation [65,66], where $\frac{d}{dt} \left(\frac{\delta N}{N_0} \right)$ in Eq.(6-40) is set equal to zero. As shown in appendix D, this approximation is also proper for noise analysis.

To allow sufficient flexibility to tune the EKF, we use three coolant nodes. There is one more equation to account for the temperature sensor response. The final equations are

$$\frac{d}{dt} \left(\frac{\delta C}{N_o} \right) = \frac{\beta}{\Lambda} \frac{\delta N}{N_o} - \frac{\lambda \delta C}{N_o} , \quad (6-44)$$

$$\frac{d}{dt} \delta T_f = \left(\frac{\delta N}{N_o} \right) \frac{N_o F_f}{m_f C_{pf}} - \frac{X}{m_f C_{pf}} , \quad (6-45)$$

$$\begin{aligned} \frac{d}{dt} \delta T_{c1} = & \left(\frac{\delta N}{N_o} \right) \frac{N_o F_{c1}}{m_{c1} C_{pc}} + \frac{X \cdot f_1}{m_{c1} C_{pc}} \\ & + \frac{\dot{m}}{m_{c1}} (\delta T_{IN} - \delta T_{c1}) + \frac{\delta \dot{m}}{m_{c1}} (T_{IN} - T_{c1o}) , \end{aligned} \quad (6-46)$$

$$\begin{aligned} \frac{d}{dt} \delta T_{c2} = & \left(\frac{\delta N}{N_o} \right) \frac{N_o F_{c2}}{m_{c2} C_{pc}} + \frac{X \cdot f_2}{m_{c2} C_{pc}} \\ & + \frac{\dot{m}}{m_{c2}} (\delta T_{c1} - \delta T_{c2}) + \frac{\delta \dot{m}}{m_{c2}} (T_{c1o} - T_{c2o}) , \end{aligned} \quad (6-47)$$

$$\begin{aligned} \frac{d}{dt} \delta T_{c3} = & \left(\frac{\delta N}{N_o} \right) \frac{N_o F_{c3}}{m_{c3} C_{pc}} + \frac{X \cdot f_3}{m_{c3} C_{pc}} \\ & + \frac{\dot{m}}{m_{c3}} (\delta T_{c2} - \delta T_{c3}) + \frac{\delta \dot{m}}{m_{c3}} (T_{c2o} - T_{c3o}) , \end{aligned} \quad (6-48)$$

$$\frac{d}{dt} \delta T_m = \frac{\delta T_{c3}}{\tau} - \frac{\delta T_m}{\tau} , \quad (6-49)$$

$$\begin{aligned} \frac{\delta N}{N_o} = & \frac{\alpha_f}{\beta} \delta T_f + \frac{\alpha_c}{\beta \Sigma W_{ci}} \sum_{i=1}^3 \delta T_{ci} W_{ci} + \frac{\lambda \Lambda \delta C}{\beta N_o} + \frac{\delta S(t)}{\beta} , \end{aligned} \quad (6-50)$$

where,

$$X = h_{fc} A_{fc} [\delta T_f - (\delta T_{c1} + \delta T_{c2} + \delta T_{c3})/3] ,$$

f_1, f_2 , and f_3 are the flux shape factors; and W_{ci} is the weighting for the coolant temperature reactivity feedback; τ is the time constant of the temperature sensor.

To use the EKF for the estimation of the flow rate, we have to make the flow rate a state variable. We need one more equation

$$\frac{d}{dt} \dot{m} = 0 \quad . \quad (6-51)$$

Here $\dot{m}$ is the average mass flow rate. During steady state operation, the mass flow rate is a constant. By treating flow rate as a state variable, we see that the model becomes nonlinear. Because of the prompt jump approximation, neutron power is no longer treated as a state variable; it is treated as a measurement. So far we have a model for the EKF with seven differential equations, i.e., Eqs.(6-44) to (6-49) and (6-51), where $\delta C/N_0$, δT_f , δT_{c1} , δT_{c2} , δT_{c3} , δT_m , and $\dot{m}$, are the state variables. Two measurements are $\delta N/N_0$ and δT_m , where the relationship between $\delta N/N_0$ and the other state variables is shown in Eq.(6-50).

Some methods have been suggested for the estimation of the noise covariance matrices R and Q_d [67,68]. However, the methods are based on ideal situations, such as the model is perfect and R and Q_d are independent of the state variables. These are not applicable in our case. Hence in this dissertation, tuning is mostly based on physical judgment about the characteristics of the LOFT noise data, in

which the disturbance source for the low frequency range where the PSD is the largest is due to the coolant velocity disturbance. Hence in the model, $\delta S(t)$ and δTIN are set equal to zero. From the analysis in Chapter 4, we also know that the low order model gives a smaller phase angle between TOUT and N. To compensate for this, we have to tune the disturbance artificially to be in the lower part of the coolant channel. Through a trial and error process, we decide to let the coolant velocity disturbance induce the driving random input in the first coolant node only, and the Q matrix has the following form

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

And Q_d is calculated using Eq.(6-4). For the full flow rate, the calculated Q_d matrix is given by

$$Q_d = \begin{bmatrix} 0.00286 & 0.00127 & -0.01234 & -0.00233 & -0.00031 & -0.00008 & 0 \\ & -0.00056 & -0.00547 & -0.00103 & -0.00013 & -0.00003 & 0 \\ & & 0.05882 & 0.01025 & 0.00131 & 0.00024 & 0 \\ & & & 0.00190 & 0.00025 & 0.00006 & 0 \\ & \text{symmetric} & & & 0.00003 & 0.00000 & 0 \\ & & & & & 0.00000 & 0 \\ & & & & & & 0.1 \end{bmatrix}.$$

And the R matrix is chosen to be

$$R = \begin{bmatrix} 0.00001 & 0.00 \\ 0.00 & 0.09 \end{bmatrix}.$$

The validity of this EKF is judged from its ability to estimate both the 100% and 65% flow rate data taken from the LOFT reactor.

In the following sections, the results of the simulated study and of the LOFT reactor data analysis are discussed.

6.5 Test of the Algorithm

Simulated data generated from the reduced order model are used to test the validity of the algorithm. The random signals are generated using the computer library, in which a subroutine to generate the random signal with uniform distribution is available. In order to get Gaussian distributed random data, ten of the uniformly distributed random data are averaged (This is based on the central limit theorem [60]). In order to use the existing digitized real data, the time step is set to be 0.078 sec.

In the EKF estimation, the initial conditions $\underline{x}_0$ and P_0 are chosen as follows:

$$\underline{x}_0 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T ,$$

$$P_0 = \begin{bmatrix} 10. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. \\ & & & & & 1. & 0. & \\ & & & & & & 250000. & \end{bmatrix} .$$

The error covariance of the initial values, P_0 , shows the confidence of the initial guesses. Here, large covariance values are chosen to assume very low confidence on the initial guess. The values of $\underline{x}_0$ and P_0 may affect the convergence rate, but not the final estimation.

Four different random sequences are generated for both 100%

and 50% flow rate. The 100% flow rate estimates using the EKF with 100% flow model are shown in Figure 6-1, which shows that the parameter estimates are approaching the true value. For the 50% flow rate random data, we first use the 100% flow rate model to do the estimation; the result is shown in Figure 6-2. The estimation approaches the 40% flow. The bias error is caused by the large difference between the flow rate used to linearize the model and the true flow rate value. When relinearization of the model is performed with 40% flow rate, the estimates approach the 50% flow rate. Further relinearization of the model with 50% flow rate gives an estimation corresponding to 50% flow rate as shown in Figure 6-3. This study confirms that the algorithm is valid for flow rate estimation using noise data as long as the linearization of the model is performed with an assumed flow rate not too far from the true flow rate, and this can be achieved by relinearizing the model with the final estimated flow when this is very different from the assumed flow rate. Comparing Figure 6-1 with Figure 6-3, we see that the deviation in Figure 6-1 is much larger than that in Figure 6-3. This is due to the larger Q_d value for 50% flow, and hence a larger signal to noise ratio that helps the convergence of estimation.

6.6 The Real Data Analysis

The 100% and 65% flow rate data of the LOFT reactor are shown in Figures 6-4 to 6-7. These noise data were acquired by Oak Ridge National laboratory on Oct. 12, 1981, when the reactor was in 93%

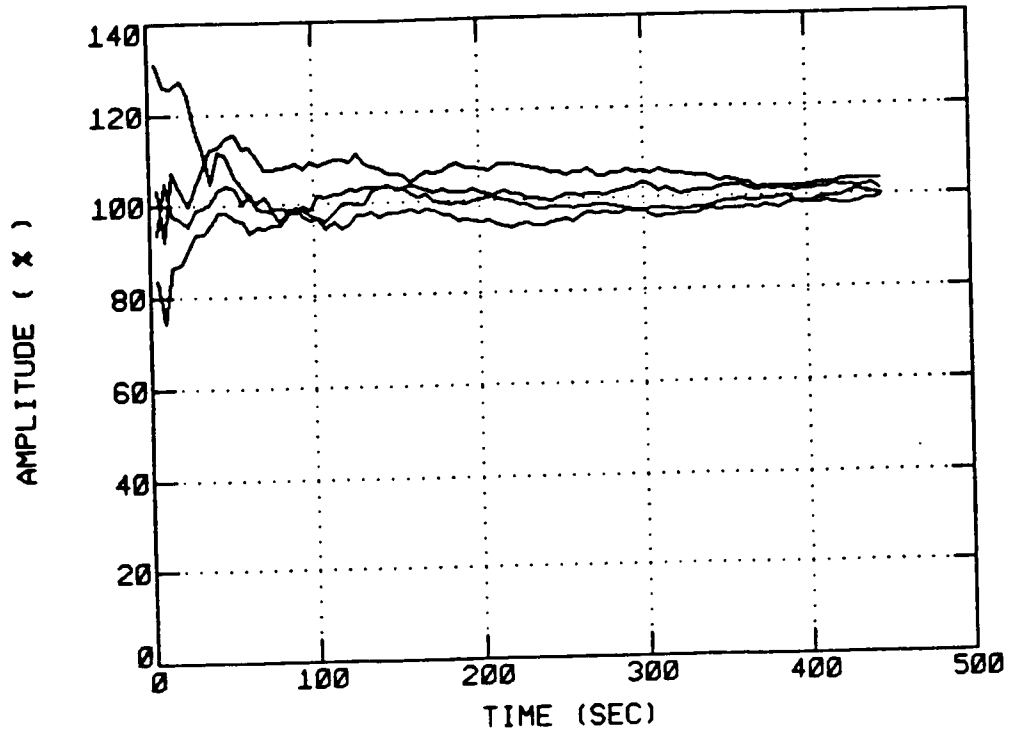


Figure 6-1. The EKF estimates of the core flow rates for four sets of simulated data at 100% flow condition.

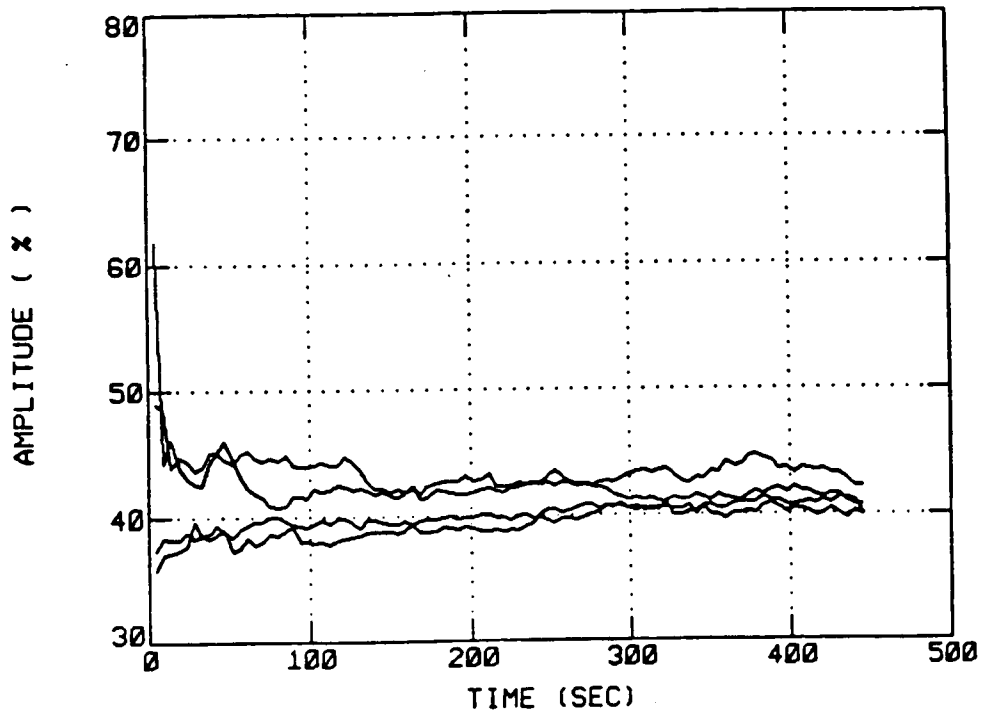


Figure 6-2. The EKF estimates of the core flow rates for four sets of simulated data at 50% flow condition with the model linearized at 100% flow.

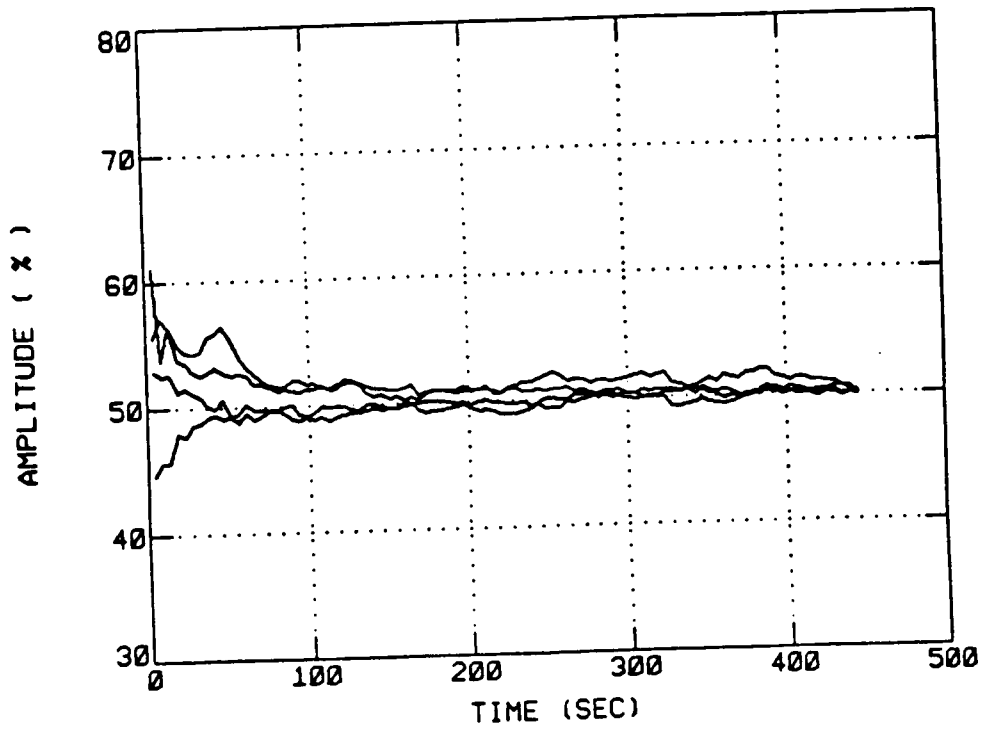


Figure 6-3. The EKF estimates of the core flow rates for four sets of simulated data at 50% flow condition with the model relinearized at 50% flow.

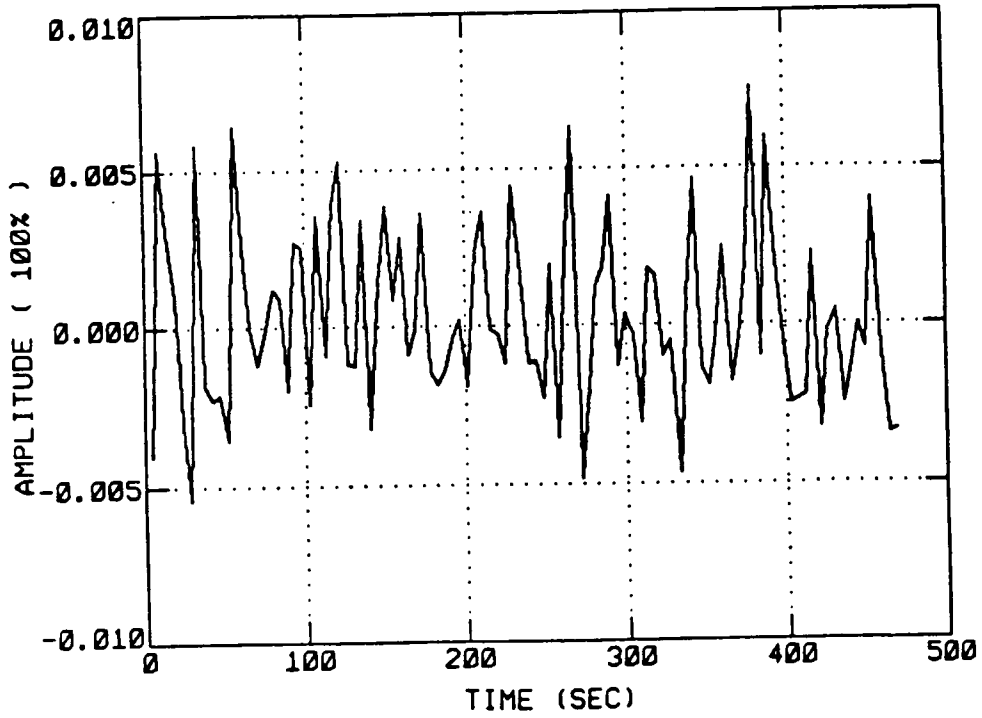


Figure 6-4. The noise signal of the normalized neutron power of the LOFT reactor at 100% flow condition.

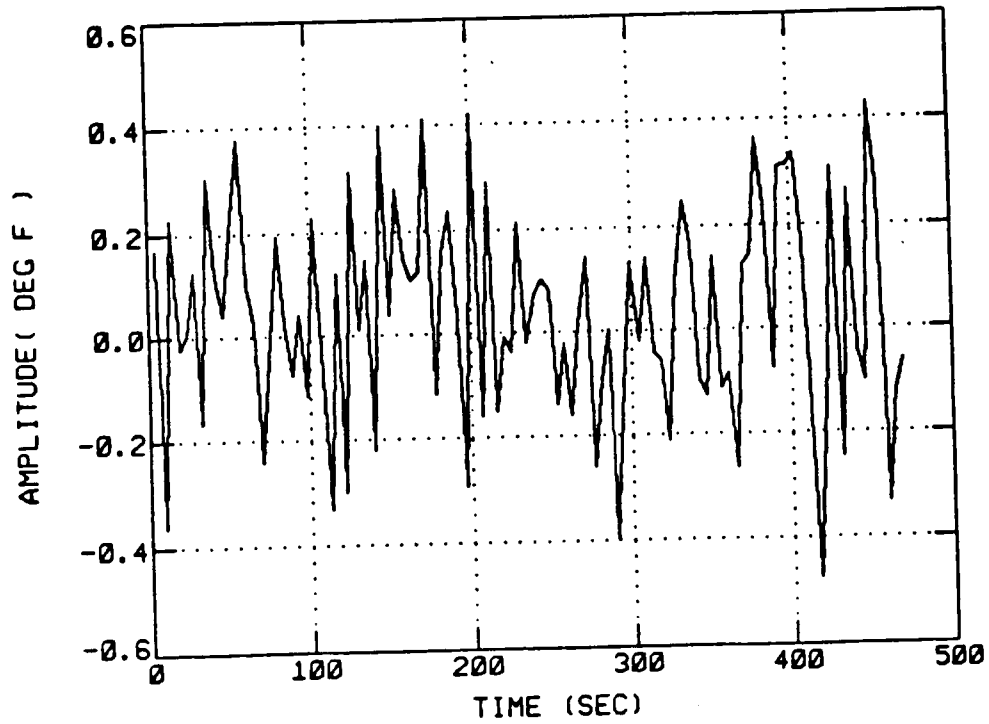


Figure 6-5. The noise signal of the core-exit coolant temperature of the LOFT reactor at 100% flow condition.

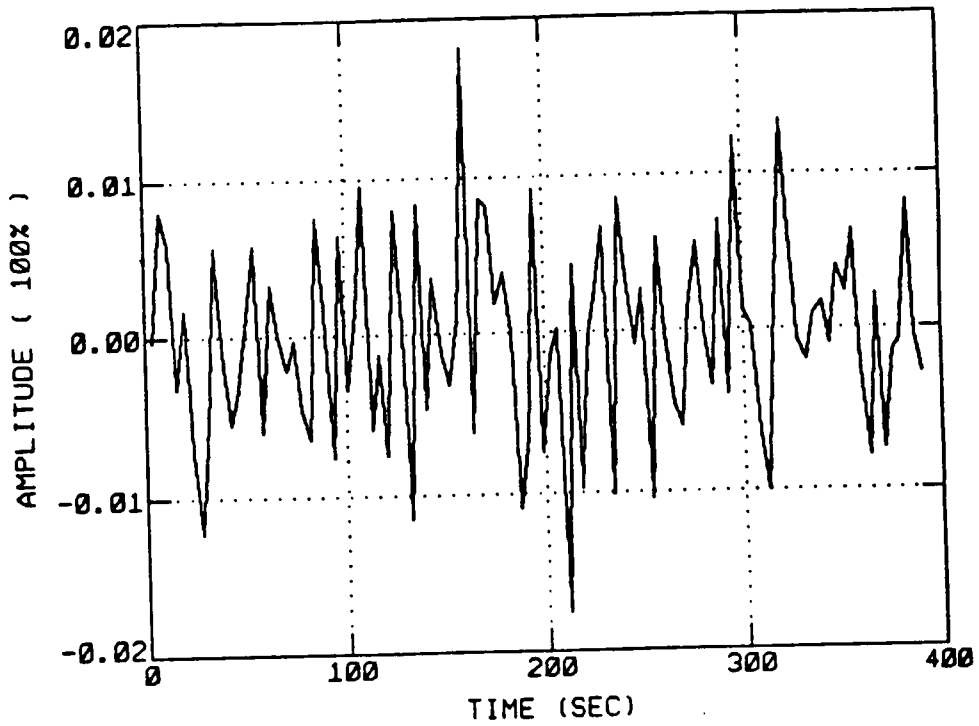


Figure 6-6. The noise signal of the normalized neutron power of the LOFT reactor at 65% flow condition.

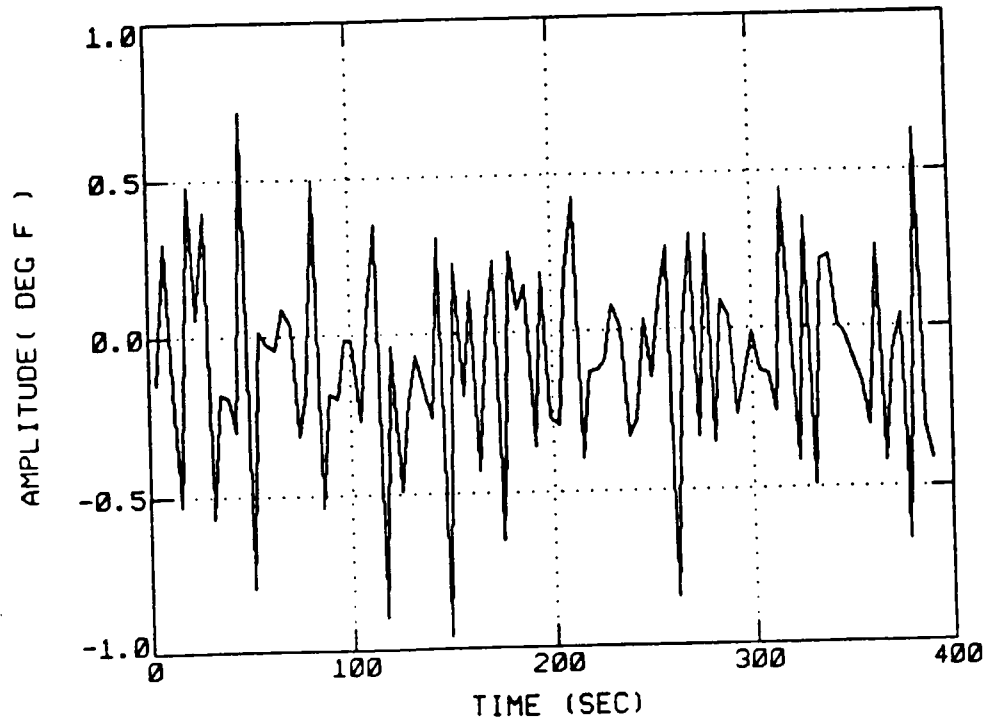


Figure 6-7. The noise signal of the core-exit coolant temperature of the LOFT reactor at 65% flow condition.

power operation condition. These noise signals were digitized with a sampling period of 0.039 sec. In the plot, the time interval between consecutive points is $0.078 \times 60 = 4.68$ sec.

Figure 6-8 shows the mass flow rate estimates for two sets of 100% flow data and one set of 65% flow data. Each plot covers 6000 estimation points. As shown in the figure, the estimated flow rate has fluctuations and does not become constant. The magnitude of the fluctuations depends on the system dynamics and the magnitudes of the Q_d and R covariances. Because the data we used in the estimation is the normal fluctuation part of the measurements, $\underline{z}(t_i)$ in the measurement updating equation of EKF, i.e. Eq.(6-26), is always fluctuating. If we have constant estimate $\hat{\underline{x}}$, then it indicates that the Kalman gain, K , is zero. In such a case the extended Kalman filter is decoupled from the measurements, and the measurement update is no longer effective. Then even in the real world, when the flow rate and the measurements are changing, the EKF will not be able to monitor the changes. Hence it is important that the estimate be able to follow the measurements and fluctuate too. The mean estimation value can be obtained by averaging the fluctuating estimate. If we average the estimates over 2000 points, then for 100% flow data the averaged estimate is 100% flow, while for 65% flow data the averaged estimate is 62% flow. These results show that the EKF using noise data for flow estimation performs very well for changing flow rates.

In the real world, assume that we want to use the EKF to monitor the flow at full power, then because the other parameters in

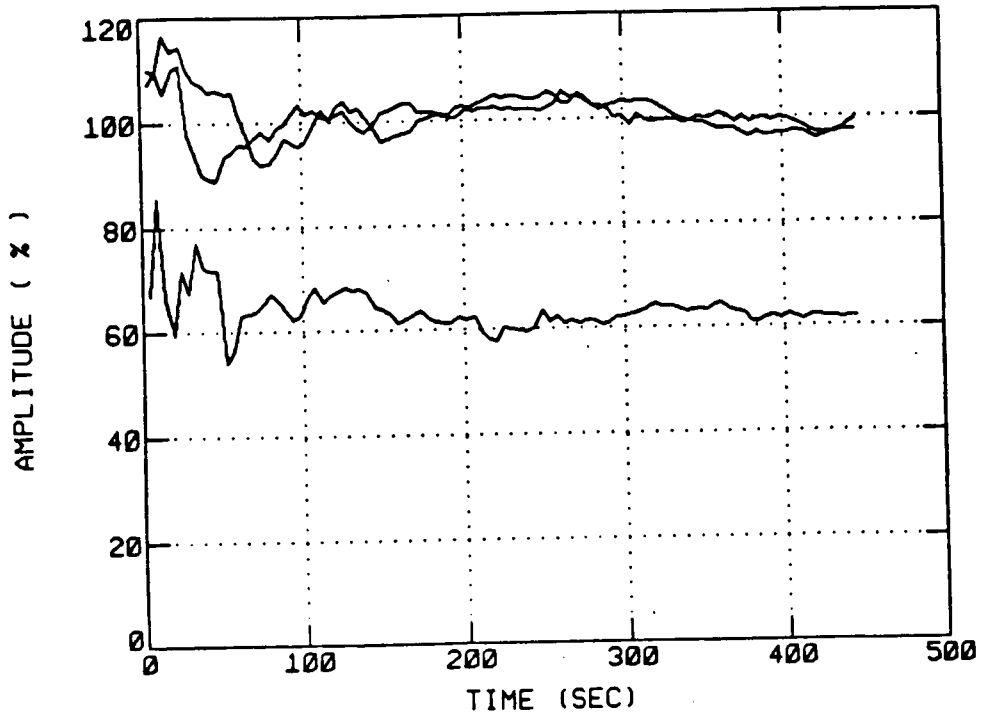


Figure 6-8. The EKF estimates of the normalized core flow rates with two sets of 100% flow data and one set of 65 % flow data from the LOFT reactor.

the system may change too, it is important to know how sensitive the flow estimate is to changes in the other parameters. These parameters include the fuel temperature coefficient of reactivity, the moderator temperature coefficient of reactivity, the fuel-to-coolant heat transfer coefficient and the response of the temperature sensor. The fuel temperature coefficient of reactivity and the heat transfer coefficient generally do not change as much as the moderator temperature coefficient during the operation. The flow estimates with 10% changes in these parameters show that the estimations are not sensitive to these changes. However, the moderator temperature coefficient of reactivity may change significantly from the beginning of the fuel cycle to the end of the fuel cycle due to the change of boron concentration in the moderator. Figure 6-9 shows its effect on the flow estimate, where the coefficient is multiplied by 1.5. This figure shows that the flow rate estimate is sensitive to the changes in the moderator temperature coefficient of reactivity. Figure 6-10 shows the effect of temperature sensor response by changing the break frequency of sensor used in the model from 3 Hz to 2 Hz. The results show that the flow estimates are sensitive to the sensor response too.

6.7 Summary

This chapter has focused on three areas: (1) development of the neutronic thermal-hydraulic model of the system under study; (2) adaptation of the Kalman filter parameter estimation algorithm for

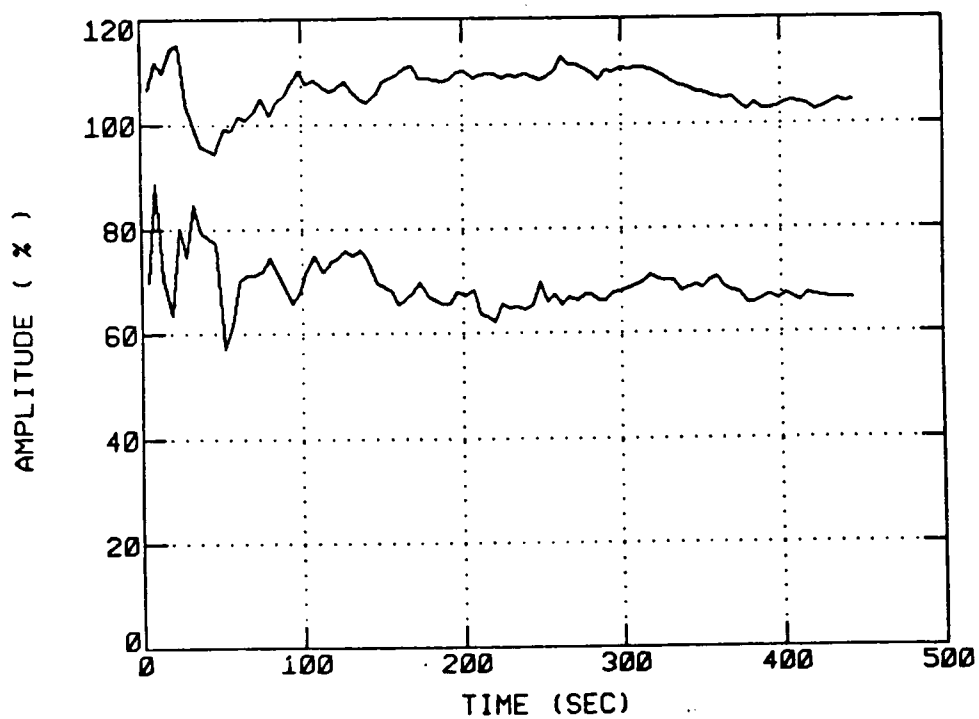


Figure 6-9. The EKF estimate of the normalized core flow rates with 100% and 65% flow data from the LOFT reactor while the coolant temperature coefficient of reactivity multiplied by 1.5.

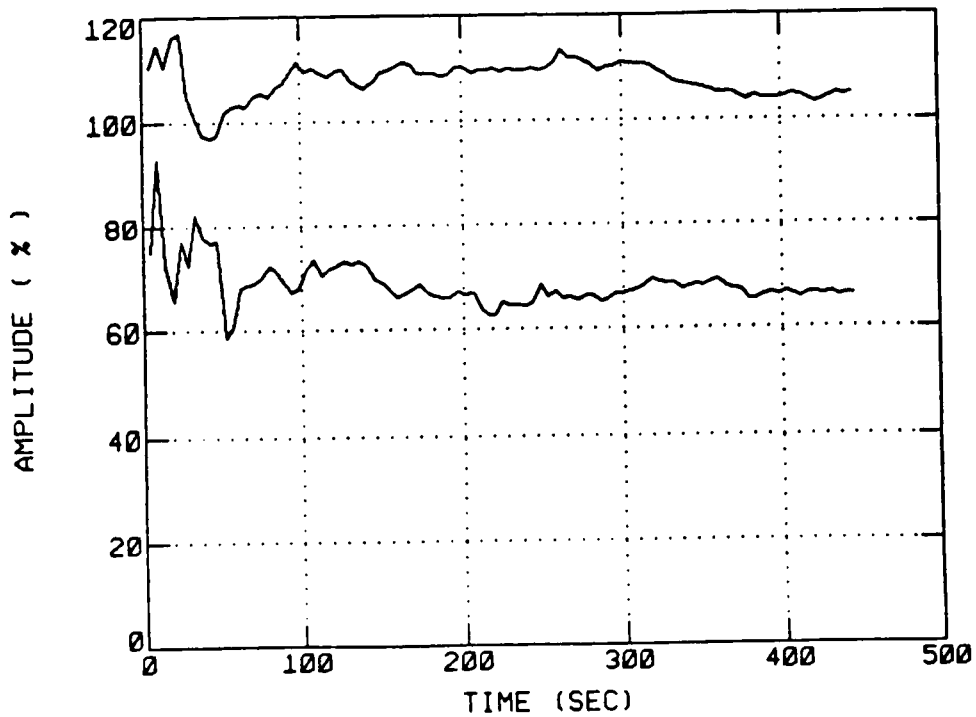


Figure 6-10. The EKF estimates of the normalized core flow rates with 100% and 65% flow data from the LOFT reactor while the break frequency of the temperature sensor is 2 Hz.

the nonlinear problem and for the model uncertainties; and (3) combination of discrete-time model and reactor operational data for flow rate estimation using the extended Kalman filter.

A reduced order discrete-time model is developed to approximate the LOFT reactor core dynamics. The stiffness problem in evaluating the time-domain propagation of the state variables is overcome by making the prompt jump approximation of the neutronic equation. This model and the EKF algorithm is validated against the LOFT data corresponding to 100% and 65% flow conditions. The study shows that a properly tuned extended Kalman filter can be used for flow rate estimation, using the noise signals of the neutron power and the core-exit coolant temperature.

However, we also found that compared with the phase slope method, the flow rate estimation using the EKF algorithm is much more sensitive to the changes in the moderator temperature coefficient of reactivity. This is a disadvantage because the moderator temperature coefficient of reactivity changes significantly during the fuel cycle.

7. AMBIGUITY FUNCTION ANALYSIS

7.1 Introduction

As shown in the previous chapter, the extended Kalman filter (EKF) after proper tuning can estimate very efficiently the change of coolant mass flow rate using random data. But we found that the estimations were sensitive to the variation of the moderator temperature coefficient of reactivity, which may vary significantly during the fuel cycle. Because the EKF is a suboptimal filter, in order to have a better understanding of the applicability of the stochastic estimator for flow monitoring in general, we are interested in investigating the performance that can be achieved with an optimal filter. In this chapter the performance of the maximum likelihood estimator (MLE) is surveyed. The maximum likelihood estimator is an optimal filter in the sense that with an accurate model, the estimation is consistent (it converges to the true value as the number of measurement samples grows without bound), asymptotically unbiased, and asymptotically efficient (as the number of samples grows without bound, it is unbiased and has finite variance; there is no other unbiased estimator whose variance is smaller than that of the MLE estimator).

However, there is no intention to develop a maximum likelihood estimator, because a practical maximum likelihood estimator usually involves approximation. But we would like to know the performance of a maximum likelihood estimator so that it could serve as a reference of the highest attainable goal. The intention is to use

the output from the developed Kalman filter to investigate the performance to be expected from a maximum likelihood estimator based on the ambiguity function analysis technique [39,69]. As shown in the previous chapter, the Kalman filter for the linear system can be derived from the maximum likelihood technique. Hence the Kalman filter is essentially a maximum likelihood estimator, but an extended Kalman filter is not. To investigate the performance of the maximum likelihood estimator for flow rate estimation, we apply the ambiguity function analysis technique which takes full advantage of the Kalman filter developed in the previous chapter. The performance of a filter that includes the estimation bias, error covariance and the sensitivity of the estimation to the modeling uncertainty can be investigated using the Monte Carlo method by generating a large amount of random data. However, this process will be very time consuming. The ambiguity function analysis is an alternative method that is shown to be less time consuming and informative. The ambiguity function is a scalar function of the following: (1) the theoretical probability distribution of the state variables with the assumed true model and (2) the likelihood function used for estimation.

In this chapter we first describe the algorithm of the ambiguity function analysis technique; then based on the simplified low order model, the characteristics of the optimal filter for the flow rate estimation are surveyed. The study includes the following:

- (1) How many estimation points are needed?
- (2) What is a good estimation time interval?

- (3) How sensitive is the estimation to the variations of the other parameters in the model?

7.2 The Ambiguity Function Algorithm

Let $\underline{Z}_1$ be the measurement vector up to time t_1 , and the probability density function of $\underline{Z}_1$ with the true parameter θ_t be defined as $p(\underline{x}_1, \underline{Z}_1 | \theta_t)$. If $L(\theta, \underline{x}_1, \underline{Z}_1)$ is the likelihood function upon which the estimator is based at time t_1 , then the ambiguity function $A(\theta, \theta_t)$ is defined as the scalar function such that

$$A(\theta, \theta_t) \triangleq \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} L(\theta, \underline{x}_1, \underline{Z}_1) p(\underline{x}_1, \underline{Z}_1 | \theta_t) d\underline{x}_1 d\underline{Z}_1. \quad (7-1)$$

In the maximum likelihood technique, the estimate of θ is the one which satisfies the following equation:

$$\left. \frac{\partial}{\partial \theta} L(\theta, \underline{x}_1, \underline{Z}_1) \right|_{\theta=\theta_e} = 0. \quad (7-2)$$

Differentiate Eq.(7-1) to get

$$\frac{\partial}{\partial \theta} A(\theta, \theta_t) = \int_0^{\infty} \dots \int_0^{\infty} \frac{\partial}{\partial \theta} L(\theta, \underline{x}_1, \underline{Z}_1) p(\underline{x}_1, \underline{Z}_1 | \theta_t) d\underline{x}_1 d\underline{Z}_1. \quad (7-3)$$

Hence from Eq.(7-2) and (7-3), we get

$$\left. \frac{\partial}{\partial \theta} A(\theta, \theta_t) \right|_{\theta=\theta_e} = 0. \quad (7-4)$$

Here we see that those estimates which satisfy Eq.(7-2) will also satisfy Eq.(7-4). Hence multiple peaks in the ambiguity function imply that the likelihood function may well have multiple peaks, which can cause convergence to local maxima rather than the global. The

name "ambiguity function" comes from the fact that multiple peaks might cause ambiguity in the estimated quantities.

In Eq.(7-1), for specific θ_t , by changing the value of θ , a curve of the function $A(\theta, \theta_t)$ is obtained, where the peak shows the estimate of θ , and the curvature in the vicinity of the peak conveys the preciseness that a maximum likelihood estimator can achieve. This curvature is inversely related to the Cramer-Rao lower bound [34] on the estimation error covariance matrix. The Cramer-Rao lower bound is the smallest achievable error covariance of the estimation, defined as $-1/\partial^2 A(\theta, \theta_t)/\partial \theta^2$ for an unbiased estimator.

Assume that the system of interest is the linear stochastic difference equation

$$\underline{x}(t_{i+1}) = \phi(t_{i+1}, t_i) \underline{x}(t_i) + B_d(t_i) \underline{u}(t_i) + G_d(t_i) \underline{w}_d(t_i), \quad (7-5)$$

and the discrete time measurements model is

$$\underline{z}(t_i) = H(t_i) \underline{x}(t_i) + \underline{v}(t_i), \quad (7-6)$$

where $\underline{w}_d$ and $\underline{v}$ are independent, zero mean, white Gaussian noise sequences with covariance

$$E \{ \underline{w}_d(t_i) \underline{w}_d(t_i)^T \} = Q_d \delta_{ij},$$

$$E \{ \underline{v}(t_i) \underline{v}(t_i)^T \} = R \delta_{ij},$$

and the initial conditions for the mean and covariance are $\underline{x}_0$ and P_0 .

To get the ambiguity function, we need an explicit expression of the likelihood function. This is achieved by applying the Bayes' rule. Consider $\ln p(\underline{x}_1, Z_1 | \theta)$ as the likelihood function. Bayes' rule can be applied repeatedly to obtain

$$\begin{aligned}
p(\underline{x}_1, \underline{Z}_1 | \theta) &= p(\underline{x}_1 | \underline{Z}_1, \theta) p(\underline{Z}_1 | \theta) \\
&= p(\underline{x}_1 | \underline{Z}_1, \theta) p(\underline{z}_1 | \underline{Z}_{1-1}, \theta) p(\underline{Z}_{1-1} | \theta) \\
&\quad \cdot \\
&\quad \cdot \\
&= p(\underline{x}_1 | \underline{Z}_1, \theta) \prod_{j=1}^1 p(\underline{z}_j | \underline{Z}_{j-1}, \theta) .
\end{aligned}
\tag{7-7}$$

Each of the separate densities in Eq.(7-7) can be written out explicitly as a Gaussian density:

$$\begin{aligned}
p(\underline{x}_1 | \underline{Z}_1, \theta) &= \exp(-DUM/2) / [(2\pi)^{n/2} | P(t_1; t_1) |^{1/2}] , \tag{7-8} \\
DUM &= [\underline{x}_1 - \hat{\underline{x}}(t_1; t_1)]^T P(t_1; t_1)^{-1} [\underline{x}_1 - \hat{\underline{x}}(t_1; t_1)] ,
\end{aligned}$$

where $\underline{x}$ and P are implicitly dependent on the particular parameter values upon which the density is conditioned, and n is the dimension of the vector $\underline{x}$. Also

$$\begin{aligned}
p(\underline{z}_1 | \underline{Z}_{1-1}, \theta) &= \exp(-DUM/2) / [(2\pi)^{m/2} | D(t_1) |^{1/2}] , \tag{7-9} \\
DUM &= [\underline{z}_1 - H\hat{\underline{x}}(t_1; t_{1-1})]^T D(t_1)^{-1} [\underline{z}_1 - H\hat{\underline{x}}(t_1; t_{1-1})] ,
\end{aligned}$$

where,

$$D(t_1) = H(t_1)P(t_1; t_{1-1})H^T(t_1) + R(t_1) .$$

By substituting these expressions into Eq.(7-7), the likelihood function can be written as

$$\begin{aligned}
\ln p(\underline{x}_1, \underline{Z}_1 | \theta) &= -\frac{n+1}{2} \ln(2\pi) - \frac{1}{2} \ln(| P(t_1; t_1) |) \\
&\quad - \frac{1}{2} \sum_{j=1}^1 \ln(| D(t_j) |) - \frac{1}{2} [\underline{x}_1 - \hat{\underline{x}}(t_1; t_1)]^T P(t_1; t_1)^{-1} [\underline{x}_1 - \hat{\underline{x}}(t_1; t_1)]^T
\end{aligned}$$

$$- \frac{1}{2} \sum_{j=1}^i [\underline{z}_j - H(t_j) \hat{\underline{x}}(t_j; t_{j-1})]^T D(t_j)^{-1} [\underline{z}_j - H(t_j) \hat{\underline{x}}(t_j; t_{j-1})] , \quad (7-10)$$

where m is the dimension of vector $\underline{z}$.

Let the true model be identical to the model upon which the likelihood function is based, except that the parameters are evaluated as θ_t in the former, and as θ in the latter. Let the estimation error, e , be defined as the difference between the entire state estimate $\hat{\underline{x}}$ and the true state $\underline{x}_t$. Furthermore, let the covariances of this estimation error before and after measurement incorporation be denoted as $P_e(t_j; t_{j-1} | \theta_t, \theta)$ and $P_e(t_j; t_j | \theta_t, \theta)$. The detailed expression of P_e is shown in Appendix E. By inserting Eq.(7-10) into Eq.(7-1) and performing the integration using the following theorem [70], we will get the formula for the ambiguity function.

Theorem: If R is a positive definite $n \times n$ matrix of constants, A is an $n \times n$ matrix of constants and $\underline{c}$ is an $n \times 1$ vector of constants, then $S = \text{tr}(AR^{-1})$, where

$$S = \frac{|R|}{(2\pi)^{n/2}} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} (\underline{x} - \underline{c})^T A (\underline{x} - \underline{c}) \exp[-(\underline{x} - \underline{c})^T R (\underline{x} - \underline{c})/2] dx_1 dx_2 \dots dx_n$$

The formula obtained is

$$\begin{aligned} A(\theta, \theta_t) = & \sum_{j=1}^i \left\{ -\frac{1}{2} m \ln(2\pi) - \frac{1}{2} \ln[|D(t_j; \theta)|] \right. \\ & - \frac{1}{2} \text{tr} \{ D^{-1}(t_j; \theta) [H(t_j) P_e(t_j; t_{j-1} | \theta, \theta_t) H^T(t_j) + R(t_j)] \} \} \\ & - \frac{n}{2} \ln(2\pi) - \frac{1}{2} \ln[|P(t_1; t_1 | \theta)|] \end{aligned}$$

$$- \frac{1}{2} \text{tr} \{ P^{-1}(t_1; t_1 | \theta) P_e(t_1; t_1 | \theta_t, \theta) \} , \quad (7-11)$$

where $\text{tr}(A) = \sum a_{ii}$ is the trace of matrix A. The explicit expression for P_e is shown in Appendix E.

To investigate the performance of maximum likelihood estimator for flow estimation using noise data, we are more interested in the asymptotic characteristics than the transient behavior. Hence we are interested in the cases for which the value of i in Eq.(7-11) is very large. Once the value of i in Eq.(7-11) is very large, the summation terms in Eq.(7-11) will be dominant and the other terms will be negligible. In addition, for a system in which the matrix ϕ , H , Q_d and R are time independent, P_e will approach a constant value. Then the asymptotic characteristics of the ambiguity function are totally reflected in the single step steady state summation terms, denoted as $A_{as}(\theta, \theta_t)$,

$$\begin{aligned} A_{as}(\theta, \theta_t) = & - \frac{m}{2} \ln(2\pi) - \frac{1}{2} \ln[| D(t_j; \theta) |] \\ & - \frac{1}{2} \text{tr} \{ D^{-1}(t_j; \theta) [H(t_j) P_e(t_j; t_{j-1} | \theta, \theta_t) H^T(t_j) + R(t_j)] \} , \end{aligned} \quad (7-12)$$

where t_j is the time at which the value approaches a constant.

7.3 Implementation of the Ambiguity Function

In implementing the ambiguity function, we need a design model and a true model. The design model is the model used for the

likelihood function $L(\theta, \underline{x}_1, \underline{Z}_1)$ in Eq.(7-1) and the true model is the model used for the probability distribution $p(\underline{x}_1, \underline{Z}_1 | \theta_t)$ in Eq.(7-1). In our analysis, both the design model and the true model have the same structure as shown in Eqs.(6-44) through (6-50), but the values of the parameters are changed.

Both the true model and the design model are first discretized. Then using the Kalman filter program we obtain the steady state Kalman gain, which appears in the formula for P_e as shown in Appendix E. Finally $A_{as}(\theta, \theta_t)$ as in Eq.(7-12) is calculated with $j=0,1,2,3\dots$, until it reaches the steady state. And this steady state value is used to show the characteristics of the ambiguity function.

7.4 Results and Discussion

The first case studies the accuracy of the maximum likelihood estimator for flow estimation, in which the true model has the standard parameter values for 100% flow as in the previous chapter; the design model has the same structure, but the flow rate is changing from 50% to 150 %. As seen from Figure 7-1, the asymptotic ambiguity function is unimodal and the peak value is at 100% flow. Thus the flow rate can be estimated unambiguously and without bias. The curvature of Figure 7-1 is $0.00011(/ \%^2)$; therefore the error covariance based on the Cramer-Rao lower bound formula, which is inversely related to the curvature, for the flow estimate with one estimation step is $9091(\%)^2$. When the number of estimation steps increases,

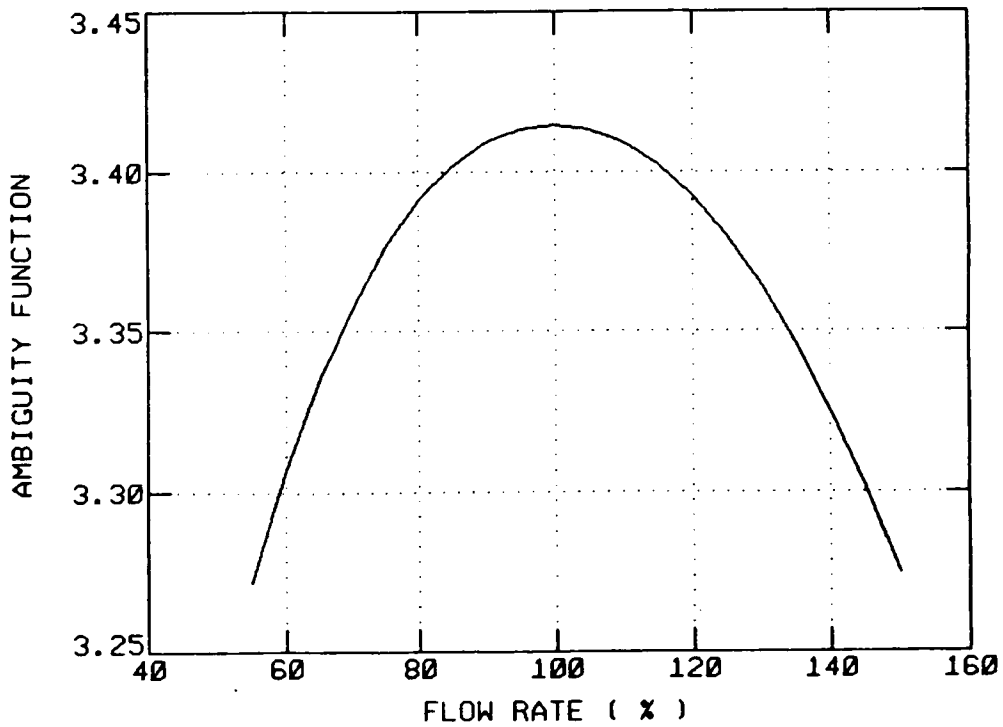


Figure 7-1. The asymptotic ambiguity function of normalized flow rate at 100% flow condition with the design model the same as the true model and $DT=0.078$ sec.

the error covariance will decrease. With 91 estimation steps, the error covariance will be $100(\%)^2$; this is equivalent to a standard deviation of 10%. And to achieve a 1% standard deviation, we need 9091 estimation steps. For the case with 50% flow, the result is shown in Figure 7-2. This figure shows the estimate is unimodal and unbiased as well. The curvature of Figure 7-2 is $0.0007(\%)^2$, which is larger than that of Figure 7-1. This means that estimation for a 50% flow condition will have less error covariance than that for the 100% flow condition. This is because at 50% flow condition, the process noise covariance Q_d is larger. This fact also was revealed in the studies of the test cases in the previous chapter.

In investigating the effect of estimation time interval on the estimation, the ambiguity function for three different time steps of 0.039 sec, 0.078 sec and 0.156 sec are investigated. Figures 7-1, 7-3 and 7-4 show the results. All have unbiased estimates. For the case with the time step 0.039 sec, the curvature is $0.00006(\%)^2$; and the curvature for 0.156 sec time step is $0.00021(\%)^2$. The case with 0.156 sec time step has the largest curvature. Hence, for a fixed number of estimation steps, the filter with 0.156 sec time step will give the best accuracy. But for a fixed total estimation time, the filter with 0.039 time step will give the best accuracy. For example, in 0.156 sec, the filter with 0.039 sec time step will perform four estimations and the curvature becomes $0.00024(\%)^2$. In 0.156 sec, the one with 0.078 sec time step will have two estimations and the curvature becomes $0.00022(\%)^2$. In 0.156 sec, the one with

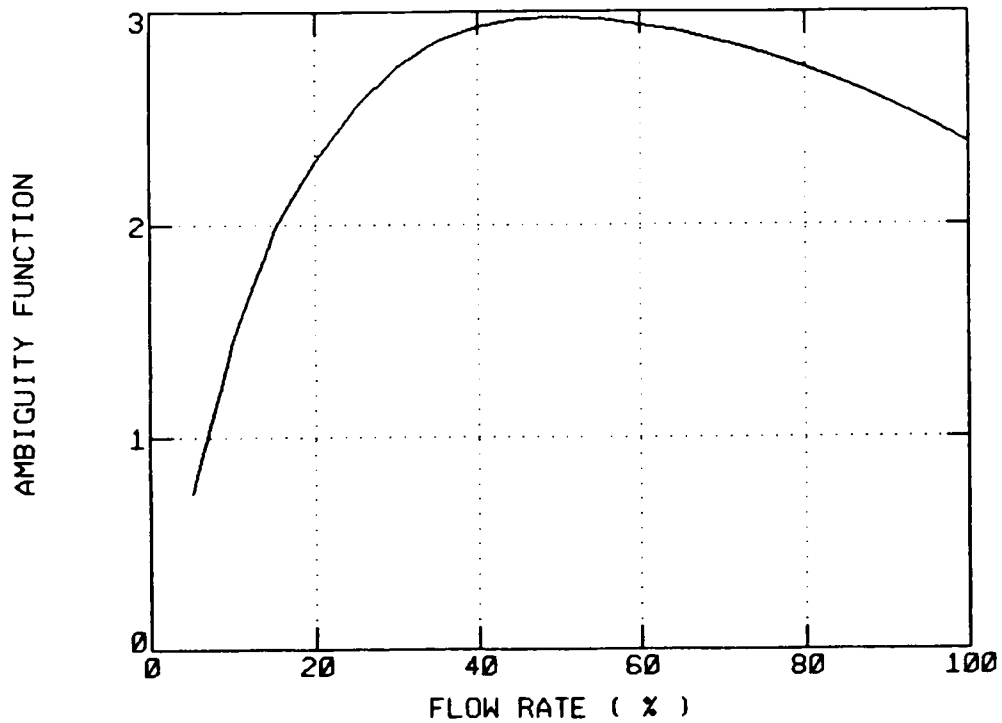


Figure 7-2. The asymptotic ambiguity function of normalized flow rate at 50% flow condition with the design model the same as the true model and $DT=0.078$ sec.

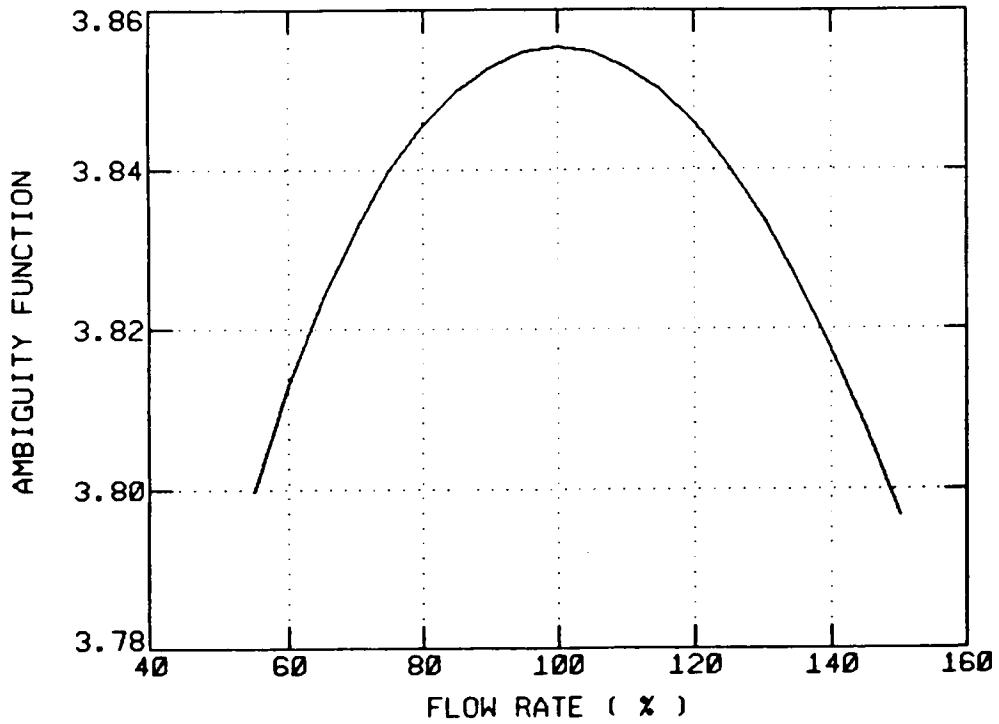


Figure 7-3. The asymptotic ambiguity function of normalized flow rate at 100% flow condition with the design model the same as the true model and $DT=0.039$ sec.

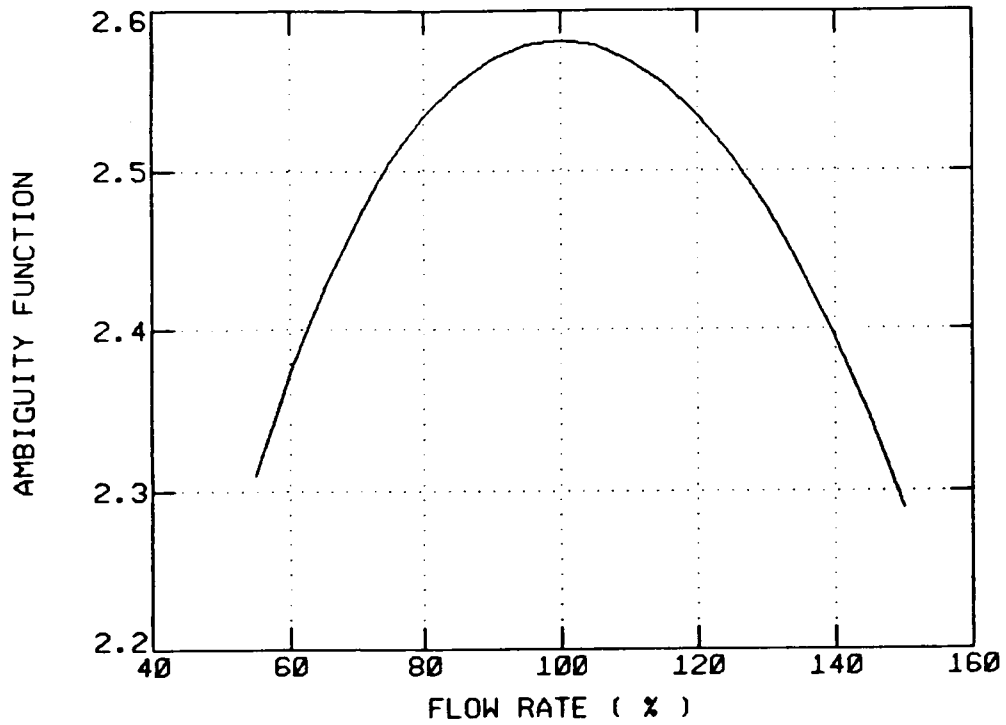


Figure 7-4. The asymptotic ambiguity function of normalized flow rate at 100% flow condition with the design model the same as the true model and $DT=0.156$ sec.

0.156 sec time step has a curvature of $0.00021(\%)^2$. Hence we know that in a fixed total estimation time, the filter with 0.039 sec estimation time interval has the largest curvature and the best accuracy. For on-line application the proper time interval of estimation is a compromise between the speed of computation and the desired accuracy.

The effects of the moderator temperature coefficient on the estimation is investigated by assuming that in the true model and the design model the values of the moderator temperature coefficient are different. Figure 7-5 shows the result of the case in which the true model has the value of the moderator temperature coefficient 30% higher than that in the design model. As seen from the figure, the estimate is biased, the estimation of the flow rate is sensitive to the variation of the moderator temperature coefficient. Several other parameter sensitivities are also studied. By increasing the value of the fuel temperature coefficient or the heat transfer coefficient by 10% in the true model, we get the result that the estimation is not sensitive to the variation of the fuel temperature coefficient or the heat transfer coefficient.

Both the EKF study and the ambiguity function analysis show that the stochastic estimator for flow rate estimation could be sensitive to the variation of the moderator temperature coefficient. We are more concerned with the effect of the moderator temperature coefficient on the estimation because in the normal operation, the moderator temperature coefficient will change significantly due to the burnup of the boron in the coolant. If the estimation of the flow rate

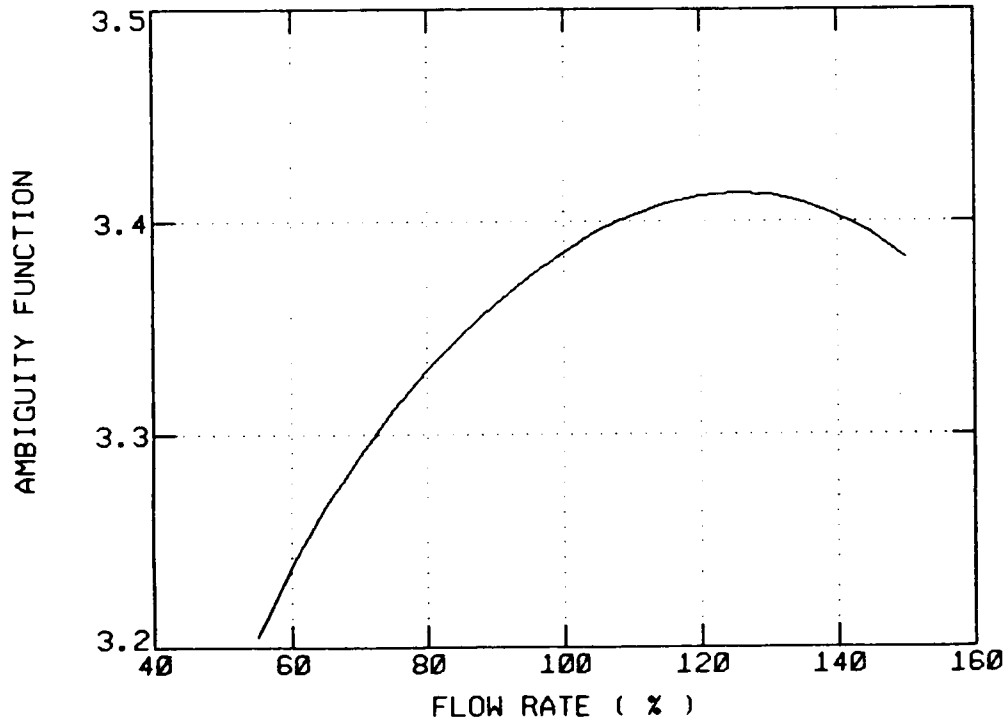


Figure 7-5. The asymptotic ambiguity function of normalized flow rate at 100% flow condition with the value of the moderator temperature coefficient of the true model 30% higher than that in the design model.

is sensitive to the variation of the moderator temperature coefficient, then we have to adjust the model used in the estimator frequently. This is a disadvantage for on-line monitoring. In this aspect, the flow rate estimation based on the phase slope method is superior, because phase slope is not sensitive to the variation of the moderator temperature coefficient. However, the moderator temperature coefficient changes slowly during normal operation. In case of an anomaly that causes a transient, the moderator temperature coefficient could remain constant during the transient period, and the stochastic estimator will still be able to monitor the anomaly. Hence the study shows that the stochastic estimator will be able to monitor the flow rate changes using the steady state noise data. Compared with the phase slope method, the real advantage of the stochastic estimator for flow rate estimation is its potential of processing the transient data, which cannot be done by the phase slope method.

8. SUMMARY, CONCLUSION AND RECOMMENDATIONS

8.1 Summary and Conclusion

8.1.1 Flow rate estimation using the phase slope

The analysis of the experimental data from operating PWRs shows that the phase shift between the core-exit coolant temperature and the neutron power noises in the 0.1 to 2 Hz frequency region has a linear phase-frequency behavior and that the slope of $\text{PHASE}(\text{TOUT}, N)$ is related to the coolant velocity or flow rate. This linear phase-frequency behavior is investigated and interpreted by using three kinds of models: the phenomenological model, the distributed parameter model and the multi-nodal model. Four kinds of disturbance sources are considered: the heat source disturbance, the coolant velocity disturbance, the inlet coolant temperature disturbance and the random heat transfer disturbance.

In the phenomenological model, a set of simple expressions is derived, relating the slope of the phase to the coolant velocity with a variety of disturbance sources for the point reactor kinetics and the extremely localized neutron kinetics. These equations clearly show the inversely proportional relationship between the slope of $\text{PHASE}(\text{TOUT}, N)$ and the coolant velocity. In the distributed parameter model, the linear phase frequency region is found to be limited by lower and upper bounds. The lower bound is due to the fuel heat transfer process and the neutron kinetics. In the frequency range below 0.1 Hz the phase-frequency relationship is not linear. The

upper bound is the sink or peak frequency. This upper bound is a function of the driving sources, the neutron flux shapes and the coolant transport time through the core. The assumptions made in the phenomenological model and the distributed parameter model are validated by the nodal model, in which complete neutronics-thermal hydraulics interactions are included. In the nodal model, the coherence between the core-exit temperature and the neutron power noise signals is shown to be very small if the random heat transfer is the disturbance source. The analysis also reveals that in the LOFT reactor, the coolant velocity disturbance is the dominant disturbance source that gives rise to the fluctuations of the core-exit coolant temperature and the neutron power noises in the linear phase-frequency region.

We conclude that:

- (1) The mechanisms of the core-exit coolant temperature and the neutron power fluctuations involve the integral of a pure delay process. The linear phase-frequency relationship exists within certain frequency region with lower and upper bounds.
- (2) The phase slope is inversely proportional to the coolant velocity.
- (3) The phase slope is a function of the driving sources, the flux shapes and the coolant transport time through the core; and not a function of the neutron kinetics parameters, such as the temperature coefficient of reactivity,

neutron generation time, and delayed neutron fraction.

- (4) For the LOFT reactor, the coolant velocity disturbance is the dominant fluctuation source that gives rise to the fluctuations of the core-exit coolant temperature and the neutron power in the linear phase-frequency region.

It is clear that the slope of $\text{PHASE}(\text{TOUT}, N)$ is related to the coolant velocity or the flow rate. In the reactor, the flow area is not uniform along the whole channel. In particular, the flow area at the region from the top of the core to the core-exit temperature sensor may be very different from that in the core region. Therefore the coolant velocity is varied along the channel, while the mass flow rate must be constant. Our main goal is to infer the flow rate from the slope of $\text{PHASE}(\text{TOUT}, N)$. From the theoretical study, it is shown that the flow rate is inversely proportional to the slope of $\text{PHASE}(\text{TOUT}, N)$, but something unrelated to the flow rate also contributes to the slope of $\text{PHASE}(\text{TOUT}, N)$, such as the dynamics of the sensor. Hence the relationship between the flow rate and the phase slope could be expressed by the following equation:

$$\text{slope}(\text{PHASE}(\text{TOUT}, N)) = A/(\text{flow rate}) + B$$

where A and B are constants to be determined. From the theoretical study, A is a function of the flux shape and the disturbance sources; B is a function of the temperature sensor dynamics. Although we would like to calculate A and B based on the theoretical model, due to the uncertainties involved in the model, such as the uncertainties in the

space dependent neutron kinetics, the disturbance sources and the irregular geometry along the coolant channel, it is very difficult to get accurate values theoretically. The practical approach to estimate the A and B values is through calibration based on operational data.

8.1.2 Monitoring the moderator temperature coefficient

The moderator temperature coefficient of reactivity is an important parameter related to the safety of pressurized water reactors (PWRs). For safe reactor operation, a negative moderator temperature coefficient is desired. But it is difficult to measure the moderator temperature coefficient directly. In light of the complex procedure for establishing this parameter, a direct method to ensure its negativity would be very useful.

The analysis of neutron detector and core-exit thermocouple noise signals from three different PWRs--the 55 MW(t) LOFT reactor, an 1140 MW(e) commercial PWR, and the 477 MW(e) Borssele PWR shows that all three reactors exhibit a linear phase-frequency relationship between the two signals in the frequency range of about 0.1-2 Hz. The phase approaches -180 degrees as the frequency goes to zero. We postulated that this out of phase relationship is due to the negative moderator temperature coefficient. Conversely, if this parameter were to go positive, the extrapolated phase at zero frequency would approach zero degrees. The multi-nodal model was used to establish this postulation. Results of the modeling analysis show the linear phase-frequency behavior. When the moderator temperature coefficient of reactivity is negative, the thermocouple-neutron detector phase

approaches -180 degrees at zero frequency for all the disturbances, except for the case of the heat source disturbance. Comparison of the model results with the experimental data indicates clearly that the heat source disturbance can be excluded as a possible disturbance source affecting neutron and coolant temperature signal fluctuations in this linear phase region. When the moderator temperature coefficient of reactivity is positive, the model predicts a linear phase approaching zero degrees as the frequency goes to zero, except for the case of heat source disturbance. Even though the thermocouple-neutron detector signal phase slope is a function of the coolant transport time through the core, the flux shape and the disturbance sources, the extrapolated phase at zero frequency depends on the sign of the moderator temperature coefficient of reactivity.

This new approach for monitoring the sign of the moderator temperature coefficient of reactivity would be valuable because of its following features: (1) as a noise analysis technique there is no interference with plant operation, and the parameter can be monitored at any operating condition, and (2) the easy distinction between zero frequency phase angles of -180 degrees for a negative coefficient and zero degree for a positive coefficient allows for straightforward tracking of the parameter. Furthermore, this monitoring scheme is robust with respect to measurement uncertainties and calculational inadequacies.

8.1.3 Flow rate estimation using stochastic estimator

The reduced order PWR model and the modified extended Kalman filter algorithm are applied to the LOFT reactor core subsystem to estimate the core coolant mass flow rate. In-core neutron detector and core-exit thermocouple signals are the measurements used for the parameter estimation. These signals were acquired at 100% and 65% flow rate conditions. The performance of the algorithm is tested using the Monte Carlo simulation, which shows the convergence of the coolant flow rate estimation for various random data sequences. The procedure is then applied using operational data at 100% flow. The same model is then used with data from the 65% flow test condition to estimate the flow rate. Since an a priori knowledge of the flow situation is not assumed, a bias in the convergence of the parameter for the latter case is observed. This bias is attributed to the model linearization performed at 100% flow condition. Therefore, for unknown flow situations, the model is relinearized at the end of each estimation sequence, and this procedure is repeated until no more variation in the estimation is observed from one sequence to the next. The study shows that properly tuned and validated PWR models can be used for the estimation of core coolant flow rate.

The ambiguity function analysis technique is applied to study the performance of the maximum likelihood estimator for flow rate estimation. The results show the number of estimation steps needed to achieve a desired accuracy and the effect of the sampling period on the estimation accuracy. The effect of the variation of the other

system parameters on the estimation is also surveyed. Both the ambiguity function analysis and the extended Kalman filter analysis show that the stochastic estimator for flow rate estimation using the core-exit coolant temperature and the neutron power noise signals will be sensitive to the variation of the moderator temperature coefficient of reactivity.

8.2 Recommendations

In this dissertation, based on the available noise data from the LOFT reactor, the theoretical study shows the applicability of the core-exit coolant temperature and the neutron power steady state noise signals for the core flow rate estimation and the negativity of the moderator temperature coefficient of reactivity monitoring. However, for the real application of these techniques, more experiments must be performed and more operational data must be surveyed.

(1) To date, all the available data from the operating PWRs show that the phase at zero frequency extrapolated from the linear phase-frequency region is -180 degrees. Based on the theoretical study, this out of phase phenomenon is due to the negative value of the moderator temperature coefficient. To verify the theoretical result that when the moderator temperature coefficient is positive the extrapolated phase is zero, a low power experiment with a positive moderator temperature coefficient should be performed by increasing the boron concentration in the coolant.

(2) In order to provide needed information about diagnostic approaches in PWRs using the core-exit coolant temperature and the neutron power noises, the experiments that simulate the anomaly conditions, such as boiling or blockage, should be performed, and the techniques developed in this dissertation should be reevaluated using the anomaly condition data.

LIST OF REFERENCES

LIST OF REFERENCES

1. J.A. Thie, Reactor Noise, Rowman and Littlefield, New York (1963).
2. R.E. Uhrig, Random Noise Technique in Nuclear Reactor System, Ronald Press Co., New York (1970).
3. M.M.R. Williams, Random Processes in Nuclear Reactor, Pergamon Press Ltd., Oxford, England (1974).
4. M.M.R. Williams and R. Sher, Eds., Reactor Noise, SMORN-II, Prog. Nucl. Energy 1,2,3,4 (1977).
5. M.M.R. Williams and N.T. McCormick, Eds., Reactor Noise, SMORN-III, Prog. Nucl. Energy 9 (1981).
6. J.A. Thie, Power Reactor Noise, American Nuclear Society, La Grange Park, Illinois (1981).
7. K. Saito, "Source Papers in Reactor Noise," Prog. Nucl. Energy, 3, 157 (1979).
8. N. Pacilio, Reactor Noise Analysis in the Time Domain, TID-24512, U.S. Atomic Energy Commission (1969).
9. Reactor Noise from Critical Assemblies to Power Plants, Ann. Nucl. Energy, 2, 2 to 5 (1975).
10. F.J. Sweeney and B.R. Upadhyaya, "Relationship of Core Exit Temperature Noise to Thermal Hydraulic Conditions in PWRs," The Second International Topical Meeting on Nuclear Reactor Thermal-Hydraulics, Vol.2, pp.1511-1518, Santa Barbara (1983).
11. D.N. Fry and W.H. Leavell, "Temperature Noise Analysis at the Exit of Blocked and Unblocked, 19-Pin Electrically Heated LMFBR Fuel Subassembly Mockups," ORNL/TM-5464 (1976).
12. G. Grunwald, G. Hessel, P. Liewers and W. Schmitt, "Low Frequency Pressure Oscillations in a PWR," Prog. Nucl. Energy, 9, 569 (1982).
13. J.A. Mullens, J.A. Thie, "Understanding Pressure Dynamic Phenomena in PERs for Surveillance and Diagnostic Applications," Proceedings of the Fifth Power Plant Dynamics, Control and Testing Symposium (1983).
14. J.M. Harrer and J.G. Beckerley, Nuclear Power Reactor Instrumentation System Handbook Vol.2, Technical Information Center, USAEC (1974).

15. P.G. Bentley, "Temperature Noise Produced by Turbulent Mixing of Coolant from Fast Reactor Subassemblies," TRG Report 2484(R), VKAEA (1974).
16. P.G. Bentley, "The Problems of Detection of Blockage in the Coolant Passage of a Fast Reactor by Measurement of Coolant Temperature," TRG Report 1197(R), VKAEA (1963).
17. E. Ohlmer and D. Schwalm, "The Evaluation of a One-Dimensional Temperature Profile in a Turbulent Fluid Flow from the Amplitude Distribution of Measured Temperature Fluctuations at a Fixed Detector Position," Int. J. Heat and Mass Transfer, Vol.19, pp.765-770 (1976).
18. E. Ohlmer and D. Schwalm, "The Use of the Amplitude Distribution Function of Temperature Noise for the Malfunction Detection in a Simulated Reactor Subassembly," Atomkernenergie, Vol.25, pp.281-284 (1975).
19. L. Krebs and G. Weinkotz, "Detection of Local Boiling in an LMFBR Subassembly by Temperature Fluctuations Analysis at the Outlet," Progress in Nuclear Energy, Vol.1, pp.507-515 (1977).
20. T. Tsunoda, "Application of Temperature Fluctuation to Anomaly Detection in Fuel Assemblies," J. Nucl. Sci. Tech., Vol.13, pp103-110 (1976).
21. B.R. Upadhyaya and F.J. Sweeney, "Theoretical and Experimental Stochastic Modeling Analysis of PWR Core Heat Transfer," Proceedings of the Fifth Power Plant Dynamics, Control and Testing Symposium, Knoxville, Vol.2, pp.46.01-46.20 (1983).
22. T. Katona, L. Mesko, G. Por and J. Volko, "Some Aspects of the Theory of Neutron Noise Due to Propagating Disturbances," Prog. Nucl. Energy 9 (1981).
23. D.N. Fry, J. March-Leuba and F.J. Sweeney, "Use of Neutron Noise for Diagnosis of In-Vessel Anomalies in Light-Water Reactors," NUREG/CR-3303, ORNL/TM-8774 (1984).
24. J.W. Cannon and T.M. Clemo, "Measurement of Stochastic Flow-Induced Power Fluctuations at LOFT," Trans. Am. Nucl. Soc., 35, 591 (1980).
25. T.W. Kerlin, Dynamic Analysis and Control of Pressurized Water Reactor, Academic Press, Inc. (1978).
26. Reference Safety Analysis Report, Westinghouse Nuclear Energy Systems, Vol.2 (1973).

27. System 80 Safety Analysis Report, Combustion Engineering, Vol.5 (1975).
28. F.J. Sweeney and B.R. Upadhyaya, "Measurement of Core Coolant Flow Velocities in PWRs Using Temperature-Neutron Noise Cross Correlation," *Tran. Am. Nucl. Soc.*, Vol.43, pp.795-797 (1982).
29. B.R. Upadhyaya and E. Turkcan, "Analysis of Neutron Detector and Core-Exit Thermocouple Signals in the Borssele Reactor," Research Report Prepared by the University of Tennessee, Knoxville, Nov. (1984).
30. T.W. Kerlin, G.C. Zwingelstein and B.R. Upadhyaya, "Identification of Nuclear System," *Nuclear Technology*, Vol.36, Nov. (1977).
31. B.R. Upadhyaya and M. Kitamura, "Stability Monitoring of Boiling Water Reactors by Time Series Analysis of Neutron Noise," *Nucl. Sci. Eng.*, Vol.77, p.480 (1981).
32. J.B. Dragt, "Reactor Noise Analysis by Means of Polarity Correlation," *Nukleonik*, 8, 225 (1966).
33. M.N. Moore, "The Determination of Reactor Transfer Functions from Measurements at Steady Operation," *Nucl. Sci. Eng.*, 3, 387 (1958).
34. P. Eykhoff, System Identification, Parameters and State Estimation, John Wiley and Sons, Inc., New York (1974).
35. A.P. Sage and J.L. Melsa, Estimation Theory with Application to Communication and Control, Robert E. Krieger Publishing, Huntington, New York (1979).
36. B.D.O. Anderson and J.B. Moore, Optimal Filtering, Prentice-Hall, Englewood Cliff, New Jersey (1979).
37. P.S. Maybeck, Stochastic Models, Estimation, and Control, Vol.1, Academic Press, New York (1980).
38. D.L. Alspach and A. Abiri, "Optimal Nonlinear System with Unknown Plant and Measurement Noise Covariances," *Proceeding of the Third Symposium on Nonlinear Estimation Theory and Its Application* (1972).
39. P.S. Maybeck, Stochastic Models, Estimation, and Control, Vol.2, Academic Press, New York (1982).
40. Special Issue on Applications of Kalman Filtering, *IEEE Trans. Automat. Contr.*, AC-28, No.3 (1983).

41. L. Ljung, "Asymptotic Behavior of the Extended Kalman Filter as a Parameter Estimator for Linear System," IEEE Trans. on Automatic Control, Vol.AC-24, No. 1, Feb. (1979).
42. B. Frognier, B. Friedlander, and H.S. Rao, "Methods for Identification of Dynamic System," Nucl. Sci. and Eng. 64,644 (1977).
43. S.S. Godbole, "Application of Kalman Filtering Technique to Nuclear Reactors," IEEE Trans. Nucl. Sci. NS-20,661 (1973).
44. N. Suda and S. Shirai, "Estimation of Reactor Parameters by Optimal Filtering Method," J. Nucl. Sci. Technol. 8(8), 438 (1971).
45. J.L. Tylee, "Real Time Nonlinear Estimation of Fuel Clad Gap Conduction," Nucl. Sci. and Eng., 84, 58 (1983).
46. M.O. Fryer and W.M. Yarbrough, "Performance of a Pressured Water Reactor Power Estimation Algorithm," Nuclear Technology, Vol.60, Jan. (1983).
47. M.Griffith, "Power Estimation in the Pressurized Water Reactor," Nuclear Technology, Vol.56, Mar. (1982).
48. M.Kitamura, "Detection of Sensor Failures in Nuclear Plants Using Analytic Redundancy," Tran. Am. Nucl. Soc. 34, 581 (1980).
49. R.N. Clark and B. Campbell, "Instrument Fault Detection in a Pressurized Water Reactor," Nuclear Technology, Vol.56, Jan. (1982).
50. C.C. Meek, R.C. Doerner and W.J. Adams, "Determination of Fuel Clad Thermal Gap Conductance by Use of Kalman Filter Methods," Nucl. Sci. Eng., 81, 560 (1982).
51. G. Kosaly and L. Mesko, "Remarks on the Transfer Function Relating Inlet Temperature Fluctuations to Neutron Noise," Atomkernergie, 20, 33 (1972).
52. T.W. Kerlin, Notes of NE5220: Reactor System Dynamics, the University of Tennessee (1982).
53. B.R. Upadhyaya, Notes of NE6510: Nuclear Reactor Noise Analysis, the University of Tennessee (1984).

54. D.L. Reeder, "LOFT System and Test Description," NUREG/CR-0247, TREE-1208 (1978).
55. J.L. Tylee, "Low-Order Model of the Loss-of-Fluid Test (LOFT) Reactor Plant for Use in Kalman Filter Based Optimal Estimators," EGG-2006 (1980).
56. J.L. Tylee, "Simple Reactor Model Simulation of a LOFT ATWS Event," Nuclear Technology, Vol.61, Apr. (1983).
57. G. Kosaly and M.M.R. Williams, "Point Theory of the Neutron Noise Induced by Inlet Temperature Fluctuations and Random Mechanical Vibrations," Atomkernenergie, 18, 203 (1971).
58. J.R. Lamarsh, Introduction to Nuclear Reactor Theory, Addison-Wesley Pub. Co. Reading, Mass. (1966).
59. C.O. Bennett, J.E. Myers, Momentum, Heat, and Mass Transfer, McGraw-Hill, Inc. (1982).
60. E. Kreyszig, Advanced Engineering Mathematics, John Wiley and Sons, New York (1979).
61. W. Seifritz, "At Power Reactor Noise Induced by Fluctuation in the Coolant Flow," Atomkernenergie, 16, 29 (1970).
62. T.C. Hsia, System Identification Least-Squares Methods, Lexington Books, Lexington (1977).
63. P. Eykhoff, Trends and Progress in System Identification, Pergamon Press, Oxford (1981).
64. A. Gelb, Applied Optimal Estimation, MIT Press, Cambridge, Massachusetts (1974).
65. J. Lewins, Nuclear Reactor Kinetics and Control, Pergamon Press, Oxford (1978).
66. J.J. Duderstadt and L.J. Hamilton, Nuclear Reactor Analysis, John Wiley and Sons, New York (1976).
67. H.W. Brewer, "Identification of the Noise Characteristics in a Kalman Filter," Control and Dynamic Systems, Vol.12 (1976).
68. R.K. Mehra, "Approaches to Adaptive Filtering," IEEE Trans. Automat. Control, AC-17 (5), 693-698 (1972).

69. F.C. Schweppe, Uncertain Dynamic System, Prentice-Hall, Englewood Cliffs, New Jersey (1973).
70. F.A. Graybill, Matrices with Applications in Statistics, Wadsworth International Group, Belmont, California (1983).
71. Private Communication with F.J. Sweeney (1984).
72. Duke Power Company Oconee Nuclear Station Units 1,2,3 Final Safety Analysis Report, Vol.4 (1974).

APPENDICES

APPENDIX A. ABOUT THE SINK FREQUENCY

A sink frequency is a frequency at which the net fluctuation in a signal is zero. This phenomenon is explained as follows:

Assume the coolant inlet temperature fluctuation $T_c(0)$ is the perturbation source. Then the coolant temperature fluctuation at position z is

$$T_c(z) = T_c(0)e^{-j\omega\tau}, \quad (\text{A-1})$$

where $\tau=z/v$, is the delay time, v is the coolant velocity.

The total effect of the coolant temperature fluctuation on the reactivity is calculated with the following equation,

$$\rho = \alpha_c \int_0^L g(z)^2 T_c(z) dz, \quad (\text{A-2})$$

where $g(z)$ is a weighting function proportional to the neutron flux shape, α_c is the coolant temperature-reactivity coefficient, L is the length of the core. To make it simple, assume we have flat flux shape $g(z)=1$. Then Eq.(A-2) becomes

$$\begin{aligned} \rho(f) &= \alpha_c \int_0^L T_c(z) dz \\ &= \alpha_c \int_0^L T_c(0)e^{-j\omega\tau} dz \\ &= \alpha_c T_c(0) \int_0^L e^{-j2\pi f z/v} dz. \end{aligned} \quad (\text{A-3})$$

After performing the integration, the result is zero at $f=v/L$. It means that at $f=v/L$ the total reactivity has no fluctuation. To

explain this result, let us look at the integrand $e^{-j2\pi fz/v}$. At $f=v/L$ the amplitude and phase with respect to z are shown in Figure A-1.

In these figures we see that the amplitude remains the same along the coolant channel, but the phase changes. And the phase at position z and that at position $z+L/2$ has 180 degrees difference, i.e., out of phase. For example at position 0, the temperature fluctuation at $f=v/L$ is as shown in Figure A-2(a) then at position $L/2$, the fluctuation is as shown in Figure A-2(b). If we sum these two fluctuations (Figures A-2(a) and (b)) together, the result will be zero. We see that the effects of the coolant temperature fluctuation at z and $z+L/2$ on the reactivity cancel each other. At this specific frequency, the effect of the coolant temperature fluctuation at position from 0 to $L/2$ cancels that from $L/2$ to L . The total effect is zero. There is no total reactivity fluctuation at this specific frequency. This frequency is called sink frequency. The sink frequency comes from integration. For different $g(z)$ the sink frequency is different.

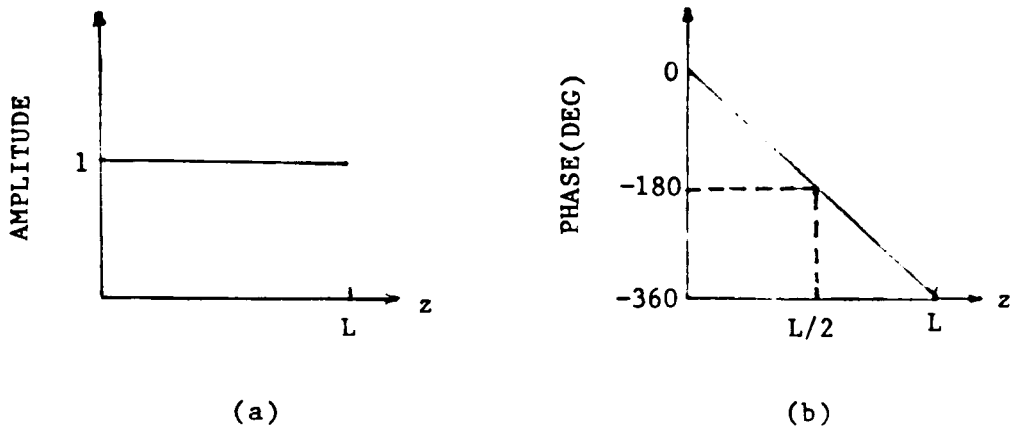


Figure A-1. Amplitude and phase of a pure delay fluid transport mechanism at the frequency v/L .

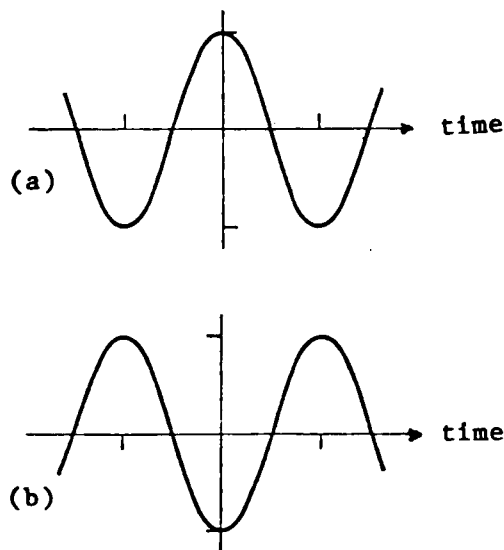


Figure A-2. The time domain sinusoidal fluctuations with frequency being v/L at position (a) 0, and (b) $L/2$.

APPENDIX B. LOFT PARAMETER VALUES

1. Reactor Kinetics

effective fraction of delay neutron: 0.007259

prompt neutron generation time: 0.00001952 s

delayed neutron group fractions: $f_1=0.741$

$f_2=0.259$

delayed neutron group decay constants: $\lambda_1=0.310 \text{ s}^{-1}$

$\lambda_2=0.025 \text{ s}^{-1}$

fuel temperature reactivity coefficient: $-0.002 \text{ } \$/^{\circ}\text{F}$

coolant temperature reactivity coefficient: $-0.0426 \text{ } \$/^{\circ}\text{F}$

2. Core Thermal

thermal power: 55 MWt

total fuel mass: 3256.5 lbm

number of fuel rods: NROD = 1300

length of fuel rods: LROD = 5.5 ft

radius of fuel rods: $r_f = 0.183 \text{ in}$

total clad mass: 668.2 lbm

clad outer radius: $r_{cl} = 0.211 \text{ in}$

clad thickness: $t = 0.0243 \text{ in}$

core flow area: Acore = 1.766 ft^2

fraction of heat remaining in fuel: 0.951

direct heating fraction to coolant: 0.0188

fuel thermal properties: ($T_f: ^{\circ}\text{F}$)

$$C_{pf} = 6.274 \times 10^{-5} + (2.2195 \times 10^{-8})T_f - (1.0554 \times 10^{-11})T_f^2 \\ + (2.1672 \times 10^{-15})T_f^3 \quad (\text{MW-sec/lbm-}^{\circ}\text{F})$$

$$k_f = 1.2558 \times 10^{-6} - (5.9394 \times 10^{-10})T_f + (9.6968 \times 10^{-14})T_f^2$$

$$(\text{MW/ft-}^{\circ}\text{F})$$

cladding thermal properties: (T_{c1} : $^{\circ}\text{F}$)

$$C_{pcl} = 7.103 \times 10^{-5} + (1.606 \times 10^{-8})T_{c1}$$

$$k_{c1} = 2.037 \times 10^{-6} + (1.411 \times 10^{-9})T_{c1} - (4.507 \times 10^{-13})T_{c1}^2$$

$$+ (2.227 \times 10^{-16})T_{c1}^3 \quad (\text{MW/ft-}^{\circ}\text{F})$$

coolant properties: (T_c : $^{\circ}\text{F}$)

$$\rho_c = 51.539 + (5.3628 \times 10^{-2})T_c - (1.1456 \times 10^{-4})T_c^2$$

$$(\text{lbm/ft}^3)$$

$$k_c = 3.1152 \times 10^{-1} + (6.1256 \times 10^{-4})T_c - (1.0447 \times 10^{-6})T_c^2$$

$$(\text{Btu/ft-hr-}^{\circ}\text{F})$$

$$C_{pc} = 3.2442 - (1.0290 \times 10^{-2})T_c + (1.2186 \times 10^{-5})T_c^2$$

$$(\text{Btu/lbm-}^{\circ}\text{F})$$

$$v = 9.4639 \times 10^{-1} - (2.1217 \times 10^{-3})T_c + (1.4648 \times 10^{-6})T_c^2$$

$$(\text{lbm/ft-hr})$$

gap thermal conductivity: (T_g : $^{\circ}\text{F}$)

$$k_g = 2.5586 \times 10^{-8} + (2.4881 \times 10^{-11})T_g - (1.1997 \times 10^{-6})T_g^2$$

$$(\text{MW/ft-}^{\circ}\text{F})$$

the convective coefficient (Dittus-Bolter correlation):

$$h = 0.023 \cdot k \cdot (\text{Re})^{0.8} (\text{Pr})^{0.4} / \text{DEQCOR}$$

$$\text{DEQCOR} = 4 \cdot \text{ACORE} \cdot \text{LROD} / \text{ACLAD}$$

average coolant temperature: 560 $^{\circ}\text{F}$

average cladding temperature: 620 $^{\circ}\text{F}$

average fuel temperature: 1470 $^{\circ}\text{F}$

average gap temperature: 1045 $^{\circ}\text{F}$

coolant velocity: 12 ft/sec

coolant flow rate: 3.54×10^6 lbm/hr

3. The Overall Heat Transfer Coefficient

Let

$$KF = 8 \cdot k_f \cdot NROD \cdot LROD,$$

$$KG = 2 \cdot NROD \cdot LROD \cdot k_g / \ln(r_{2g}/r_{1g}),$$

$$KC1 = 2 \cdot NROD \cdot LROD \cdot k_c / \ln(r_{c1}/r_{2g}),$$

$$KC2 = 2 \cdot NROD \cdot LROD \cdot k_c / \ln(r_{c1}/r_{c1c}),$$

where,

$$r_{1g} = r_f,$$

$$r_{2g} = r_{c1} - t,$$

$$r_{c1c} = r_{c1} - t/2.$$

Then in the multi-node model, the overall heat transfer coefficient from fuel to cladding is:

$$U_{fc1} = 1 / (1/k_f + 1/k_g + 1/k_{c1}),$$

the overall heat transfer coefficient from cladding to coolant:

$$U_{c1p} = 1 / (1/(h \cdot ACLAD) + 1/k_{c2}).$$

In the distributed parameter model, the overall heat transfer coefficient from fuel to coolant is

$$U_{fc} = 1 / (1/k_g + 1/k_{c1} + 1/k_{c2} + 1/(h \cdot ACLAD)).$$

APPENDIX C. ONE DIMENSIONAL HOMOGENEOUS TIME DEPENDENT
DIFFUSION EQUATION

Considering one-dimensional homogeneous critical core with an oscillating absorber located at $x=x_0$, in the frequency domain, the equation satisfied by the neutron fluctuations induced by the oscillating absorber can be written down in the following way*,

$$\begin{aligned} \frac{d^2}{dx^2} \delta\phi(x, \omega) + B(\omega)^2 \delta\phi(x, \omega) \\ = \phi(x) \frac{\Delta\Sigma_a(x, x_0, \omega)}{\Sigma_a M^2} = \frac{\gamma(\omega) \phi(x_0)}{\Sigma_a M^2} \delta(x-x_0) \quad , \quad (C-1) \end{aligned}$$

and,

$$B(\omega)^2 = B^2 - k_\infty / [M^2 G_0(\omega)] \quad , \quad (C-2)$$

where M^2 is the migration area; Σ_a is the absorption cross section; $\gamma(\omega)$ is the strength of the disturbance; B^2 is the geometrical buckling defined by $\pi^2/(2a)^2$, where $2a$ is the length of the core; k_∞ is the infinite multiplication constant defined by $k_\infty = 1 + M^2 B^2$; and $G_0(\omega)$ is the zero power point reactor kinetics transfer function.

The solution of Eq.(C-1) is the following Green's function

$$\begin{aligned} G(\omega, x, x_0) = \frac{\phi(x_0) / (\Sigma_a M^2)}{B(\omega) \sin[2B(\omega)a]} \\ \cdot \begin{cases} \sin[B(\omega)(a+x)] \sin[B(\omega)(a-x_0)], & x < x_0 \\ \sin[B(\omega)(a-x)] \sin[B(\omega)(a+x_0)], & x > x_0 \end{cases} \quad (C-3) \end{aligned}$$

* Refer to G.T. Analytis, "On the Low Frequency Spatial Dependence of the Neutron Response and Noise Field in LWRs," Ann. Nucl. Energy, Vol.10, 5, pp.221-232 (1983).

Then the neutron flux fluctuation at x_0 with a distributed disturbance source $S(\omega, x)$ can be obtained as

$$\begin{aligned}\delta\phi(\omega, x_0) &= \int G(\omega, x, x_0) S(\omega, x) dx \\ &= G_0(\omega) \int [G(\omega, x_0, x)/G_0(\omega)] S(\omega, x) dx .\end{aligned}\quad (C-4)$$

Eq.(C-4) can be written as

$$\delta\phi(\omega, x_0) = G_0(\omega) \rho(\omega, x_0) , \quad (C-5)$$

where

$$\rho(\omega, x_0) = \int [G(\omega, x_0, x)/G_0(\omega)] S(\omega, x) dx . \quad (C-6)$$

Using Eq.(C-5), the space dependent problem is treated with point reactor kinetics, except that the reactivity weighting is dependent on the position of the neutron detector and the frequency. To simplify the calculation, we further assume that in the 0.1 to 2 Hz region G_0 is almost frequency independent and is a constant, and so is $G(\omega, x, x_0)$. Then the reactivity weighting will depend on the detector position only.

APPENDIX D. PROMPT JUMP APPROXIMATION FOR NOISE DATA ANALYSIS

The neutron kinetics equation is

$$\frac{d}{dt} \left(\frac{\delta N}{N_0} \right) = - \frac{\beta}{\Lambda} \frac{\delta N}{N_0} + \lambda \frac{\delta C}{N_0} + \frac{\beta}{\Lambda} \left(\frac{\delta \rho}{\beta} \right) . \quad (D-1)$$

Fourier transform this equation to get

$$j\omega \frac{\delta N(\omega)}{N_0} = - \frac{\beta}{\Lambda} \frac{\delta N}{N_0} + \lambda \frac{\delta C}{N_0} + \frac{\beta}{\Lambda} \left(\frac{\delta \rho}{\beta} \right) . \quad (D-2)$$

In the LOFT reactor

$$\beta = 0.007259,$$

$$\Lambda = 0.00001952,$$

hence,

$$\beta/\Lambda = 372.256 .$$

From the data taken from the reactor, we observe that the bandwidth of the neutron power and the core-exit coolant temperature fluctuations is at most 2 Hz, hence

$$\omega = 2\pi f = 12.5 .$$

The value of ω is only 3% of β/Λ , hence it is reasonable to make the following assumption

$$(j\omega + \frac{\beta}{\Lambda}) \frac{\delta N(\omega)}{N_0} \approx \frac{\beta}{\Lambda} \frac{\delta N(\omega)}{N_0} . \quad (D-3)$$

This is equivalent to setting

$$\frac{d}{dt} \left(\frac{\delta N}{N_0} \right) = 0 , \quad (D-4)$$

as in the prompt jump approximation.

APPENDIX E. ERROR COVARIANCE FOR KALMAN FILTER

Let the truth model be a linear discrete model,
driven by white Gaussian noise w_t .

$$\underline{x}_t(t_i) = \phi_t(t_i, t_{i-1})\underline{x}_t(t_{i-1}) + \underline{w}_t, \quad (E-1)$$

and the measurement be

$$\underline{z}_t(t_i) = H_t(t_i)\underline{x}_t(t_i) + \underline{v}_t. \quad (E-2)$$

And the model for the Kalman filter is

$$\underline{x}(t_i) = \phi(t_i, t_{i-1})\underline{x}(t_{i-1}) + \underline{w}_d, \quad (E-3)$$

$$\underline{z}(t_i) = H(t_i)\underline{x}(t_i) + \underline{v}. \quad (E-4)$$

Being a mathematical representation, the truth model can generate a state process $\underline{x}_t(t_i)$ as well as a measurement process $\underline{z}_t(t_i)$, which is contaminated by the noise v_t . And we can calculate the error committed by the Kalman filter in attempting to estimate the state values at time t_i , before and after measurement incorporation as

$$\underline{e}(t_i; t_{i-1}) = \hat{\underline{x}}(t_i; t_{i-1}) - \underline{x}_t(t_i), \quad (E-5)$$

$$\underline{e}(t_i; t_i) = \hat{\underline{x}}(t_i; t_i) - \underline{x}_t(t_i). \quad (E-6)$$

In the Kalman filter, the time propagation equations are

$$\hat{\underline{x}}(t_i; t_{i-1}) = \phi(t_i, t_{i-1})\hat{\underline{x}}(t_{i-1}; t_{i-1}), \quad (E-7)$$

$$P(t_i; t_{i-1}) = \phi(t_i, t_{i-1})P(t_{i-1}; t_{i-1})\phi^T(t_i, t_{i-1}) + Q_d(t_{i-1}), \quad (E-8)$$

$$Q_d(t_{i-1}) = \int_{t_{i-1}}^{t_i} \phi(t_i, \tau)Q(\tau)\phi^T(t_i, \tau) d\tau, \quad (E-9)$$

and the measurement updates are

$$K(t_i) = P(t_i; t_{i-1})H^T(t_i)[H(t_i)P(t_i; t_{i-1})H^T(t_i) + R(t_i)]^{-1}, \quad (E-10)$$

$$\hat{\underline{x}}(t_1; t_1) = \hat{\underline{x}}(t_1; t_{1-1}) + K(t_1)[\underline{z}_t(t_1) - H(t_1)\hat{\underline{x}}(t_1; t_{1-1})] , \quad (E-11)$$

$$P(t_1; t_1) = P(t_1; t_{1-1}) - K(t_1)H(t_1)P(t_1; t_{1-1}) . \quad (E-12)$$

The filter measurement update Eq.(E-11) can be rewritten as

$$\begin{aligned} \hat{\underline{x}}(t_1; t_1) &= \hat{\underline{x}}(t_1; t_{1-1}) + K(t_1)[\underline{z}_t(t_1) - H(t_1)\hat{\underline{x}}(t_1; t_{1-1})] \\ &= [I - K(t_1)H(t_1)]\hat{\underline{x}}(t_1; t_{1-1}) + K(t_1)\underline{z}_t(t_1) \\ &= [I - K(t_1)H(t_1)]\hat{\underline{x}}(t_1; t_{1-1}) + K(t_1)H_t(t_1)\underline{x}_t(t_1) \\ &\quad + K(t_1)\underline{v}_t(t_1) . \end{aligned} \quad (E-13)$$

The truth model states are not changed with measurement, hence

$$\underline{x}_t(t_1; t_1) = \underline{x}_t(t_1; t_{1-1}) . \quad (E-14)$$

Combining Eqs.(E-13) and (E-14) yields an augmented state vector measurement update as

$$\underline{x}_a(t_1; t_1) = A_a(t_1)\underline{x}_a(t_1; t_{1-1}) + K_a(t_1)\underline{v}_t(t_1) , \quad (E-15)$$

where,

$$\begin{aligned} \underline{x}_a &= [\underline{x}_t, \hat{\underline{x}}]^T , \\ A_a(t_1) &= \begin{bmatrix} I & 0 \\ K(t_1)H_t(t_1) & [I - K(t_1)H(t_1)] \end{bmatrix} , \\ K_a(t_1) &= [0, K(t_1)]^T . \end{aligned}$$

And the augmented state vector time propagation relation is

$$\underline{x}_a(t_1; t_{1-1}) = \phi_a(t_1, t_{1-1})\underline{x}_a(t_{1-1}; t_{1-1}) + \underline{w}_{da}(t_{1-1}) , \quad (E-16)$$

where,

$$\begin{aligned} \phi_a &= [\phi_t, \phi]^T , \\ \underline{w}_{da} &= [\underline{w}_t, \underline{w}_d]^T . \end{aligned}$$

And the error can be generated from the augmented state vector at any time as

$$\underline{e}(t) = C_a(t)\underline{x}_a(t) , \quad (E-17)$$

where

$$C_a(t) = \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix} .$$

The equations for the error covariance analysis are readily obtained from the above equations. Since all processes are assumed to have zero mean, the covariances of the estimation error can be defined as

$$P_e(t) = E[\underline{e}(t)\underline{e}(t)^T] .$$

From Eq.(E-16), propagating between sample times is accomplished by

$$P_a(t_i; t_{i-1}) = \phi_a(t_i, t_{i-1})P_a(t_{i-1}; t_{i-1})\phi_a(t_i, t_{i-1})^T + Q_{da}(t_{i-1}) , \quad (E-18)$$

and the measurement update relation is derived from Eq.(E-15) as

$$P_a(t_i; t_i) = A_a(t_i)P_a(t_i; t_{i-1})A_a(t_i)^T + K_a(t_i)R_t(t_i)K_a(t_i)^T . \quad (E-19)$$

As the state covariance recursion is generated, the error covariance can be obtained at any time as

$$P_e(t) = C_a(t)P_a(t)C_a(t)^T . \quad (E-20)$$

APPENDIX F. LIST OF COMPUTER PROGRAMS

The computer programs are divided into three groups:

(1) Frequency domain analysis

The computer programs used for frequency domain analysis include LOFT.FTN, DISTR.FTN, SYSDAT.FTN and NODAL.FTN.

LOFT.FTN : calculate LOFT system design data, the output data is stored in DESIGN.DAT to be used in the DISTR.FTN and SYSDAT.FTN.

DISTR.FTN : distributed parameter model program.

SYSDAT.FTN : calculate the system matrices for the nodal model program.

NODAL.FTN : nodal model program.

(2) Kalman filter and extended Kalman filter program

The program is in overlay structure with the following parts:

KAL.PAC : root part.

KAL1.PAC : data input subroutine.

KAL2.PAC : Kalman filter main part.

KAL21.PAC, KAL22.PAC : Kalman filter subroutines.

KAL3.PAC : store data for plot.

KAL4.PAC : extended Kalman filter main part.

KAL41.PAC : special subroutines for EKF.

(3) Ambiguity function analysis program : AMBI.FTN


```

C ***** DISTR.FTN *****
C C DISTRIBUTED PARAMETER MODEL WITH FUEL AND COOLANT REGION
C C INCLUDE THE FOLLOWING DISTURBANCE SOURCES
C C 1. HEAT SOURCE 2. VELOCITY 3. INLET TEMP
C C TO CALCULATE THE GAIN AND PHASE BETWEEN NEUTRON POWER (OR CORE
C C EXIT COOLANT TEMPERATURE) AND THE DISTURBANCE SOURCES, AND
C C GAIN AND PHASE BETWEEN NEUTRON POWER AND CORE-EXIT COOLANT
C C TEMPERATURE.
C C THE OUTPUT ARE STORE IN PLOT.DAT TO BE PLOTTED OUT
C *****
C C DIMENSION RGAIN(30),RPHASE(30),TGAIN(30),TPHASE(30)
C C DIMENSION M(30),PHASE(30),GAIN(30)
C C DIMENSION F(2),LAMBDA(2)
C C REAL MCL,KCL,KCL2,KC,NF,LAMBDA,KFI,LROD,KGI,KCL1,KF,KG,KCL
C C COMMON VELD,B,DUM,RCL,GM,OC,DEMSI,CPC,KF,UFC,FDEMSI,CFF
C C COMMON TIN,UNOISE,LROD,ALPHAC
C C LOGICAL%1,HEAD(30)
C C FORMAT(1X,'ENTER 30 CHARACTERS TITLE')
C C FORMAT(30A)
C C FORMAT(1X,'FOR SKEWNESS, 7 INCHES ABOVE SIN SOURCE ASSUMED ZERO?')
C C FORMAT(F10.5)
C C CALL ASSIGN(1,'DESIGN.DAT')
C C READ(1,2)BETA,GAMMA,F(1),F(2),LAMBDA(1),LAMBDA(2),ALPHAF,ALPHAC,PO,
C C > HEATF,HEATC,TFM,XHROD,LROD,RF,TCLM,RCL,CLTHK,ACORE,CFF,KF,
C C > CPCL,KCL,DEMSI,KC,CPC,VISCOB,KG,VELD,H,ACLAB
C C TYPE 3,BETA,GAMMA,F(1),F(2),LAMBDA(1),LAMBDA(2),ALPHAF,ALPHAC,PO,
C C > HEATF,HEATC,TFM,XHROD,LROD,RF,TCLM,RCL,CLTHK,ACORE,CFF,KF,
C C > CPCL,KCL,DEMSI,KC,CPC,VISCOB,KG,VELD,H,ACLAB
C C MODDIMROD
C C FORMATT(2E14.5)
C C COMMON%1,HEAD(30)
C C FLUX,VELOC=ACORE=DEMSI
C C TYPE 19
C C FORMAT(1X,'DEVIATE FLOW RATE BY X.Ns?')
C C READ(9,9)XND
C C FLOWFLOH/XD
C C THE UNIT OF K/MW/TFM, C/MWSEC/LBMW, DEMSILBM/FT=3
C C VISCOILBM/FT=SEC
C C FORMAT(1X,2E12.5)
C C RCL1=RCL-CLTHK
C C TKCL2=9.1416E-06*LROD*KCL/ALDG(RCL/RCL1)
C C TKCL2=9.1416E-06*LROD*KG/ALDG(RCL/RF)
C C UFC1=1./1./((H*ACLAB)+1./TKCL1+1./TKG)
C C UFC/UFC/ACLAB
C C TYPE 41
C C FORMAT(1X,'WHICH TRANSFER FUNCTION? TOUT/GENO, TOUT/TIN1,
C C 1 TOUT/VELO2?')
C C READ(9,4E12)ISOURCE
C C TYPE 49
C C FORMAT(1X,'WHAT SHAPE? FLAT=0, SIN=1')
C C READ(9,4E12)ISHAPE
C C TYPE 9
C C READ(9,9)XNLL

```

```

C C MCLNLL/12.
C C XLL10.
C C IF(1)ISOURCE=9.1100 TO 45
C C IF(1)ISOURCE=9.2100 TO 46
C C FORMAT(15)
C C ASSUME HEAT SOURCE=PO,0.1100/9.1,ULBM,NR
C C OC=HEATC/PO/ACORE TGC,BN IS HEAT/UNIT VOLUME
C C ON=HEATF/PO/(XHROD*9.1416E-06*RCL)
C C TIN=0.
C C UNOISE=0.
C C OC1 TO 44
C C OC10
C C ON10
C C TIN=1.
C C UNOISE=0.
C C GO TO 44
C C OC=0.
C C ON=0.
C C TIN=0.
C C UNOISE=100./((DEMSI+ACORE)
C C FDEMSI=TFM/(XHROD*LROD*9.1416E-06*RF)
C C B=2.*9.1416E-06*RCL*XHROD/UFC/(ACORE+DEMSI+CPC)
C C TYPE 50
C C FORMAT(1X,'PRINT PHANT?')
C C READ(3,70)FMIN,FMAX
C C FORMAT(2F10.5)
C C TYPE 47
C C FORMAT(1X,'HOW MANY FREQUENCY POINTS? MAX=30')
C C READ(3,45)NF
C C DF=(FMAX-FMIN)/NF
C C PRETOR=-3.1416
C C PRETOR=-3.1416
C C PRETOR=-100.
C C DO 80 1,1,NF
C C 1)XN=1.
C C CALL DISTR(XF,GAIN(1),PHASE(1),ISHAPE,KLL,RPHASE(1),TGAIN(1))
C C IF((PHASE(1)-PRETOR).GT.9.1)PHASE(1)=PHASE(1)-9.1416E2
C C IF((PHASE(1)-PRETOR).LT.-9.1)PHASE(1)=PHASE(1)+9.1416E2
C C IF((RPHASE(1)-RRETOR).GT.9.1)RPHASE(1)=RPHASE(1)+9.1416E2
C C IF((RPHASE(1)-RRETOR).LT.-9.1)RPHASE(1)=RPHASE(1)+9.1416E2.
C C PRETOR=RPHASE(1)
C C PRETOR=PHASE(1)
C C CALL KINETC(NF,TPHASE(1),TGAIN(1),F(1),F(2),LAMBDA(1),LAMBDA(2),GAMMA,
C C ) BETA)
C C TPHASE(1)=((PHASE(1)-RPHASE(1))-TPHASE(1))/100./9.1416
C C IF((TPHASE(1)-TRETOR).GT.300.3)TPHASE(1)=TPHASE(1)-360.
C C IF((TPHASE(1)-TRETOR).LT.-300.3)TPHASE(1)=TPHASE(1)+360.
C C TRETOR=TPHASE(1)
C C TGAIN(1)=GAIN(1)/TGAIN(1)/TGAIN(1)
C C RPHASE(1)=RPHASE(1)/100./9.1416
C C PHASE(1)=PHASE(1)/100./9.1416
C C CONTINUE
C C WRITE(9,7)
C C READ(9,8)HEAD
C C CALL ASSIGN(2,'T1')
C C WRITE(9,8)HEAD
C C WRITE(9,9)XNLL,IS1,NF)

```

```

SV:ISTR.PAC      20-DEC-84      14:10:44      PAGE      4

SUBROUTINE DISTRIN,GAIN,PHASE,ISHAPE,XL,RPHASE,ROAIN)
COMPLEX GS,HS,B,AX,BX,CX,THXS
COMPLEX DUM1,DUM2
REAL L,K
COMMON U,B,L,R,QW,QC,RHO,C,K,M,FDEMS1,CPY
COMMON TIN,UNOISE,XLROD,ALPHAC
X=XLROD
S=CMPLX(0.,M)
DUMV=FDEMS1=CPY/M/K
L=SQRT(DUMV)
IF(ISHAPE.EQ.0100 TO 20
  NACT=K
  X=X-XL
  XLROD=XLROD-XL
  BX=GS(B)/(-3.1416/XLROD-HS(B)*HS(B)*XLROD/3.1416)
  BX=BX*3.1416/(2.*XLROD)
  AX=HS(B)*BX*XLROD/3.1416
  CX=TXIN-BX
  TXIN=TXIN*(3.1416/XLROD)+BX*CX*(3.1416/XLROD)
  DUM1=EXP(-HS(B)*HS(B)*XLROD)
  DUM2=DUM1*EXP(-HS(B)*HS(B)*CX)
  THXS=CX*EXP(-HS(B)*HS(B)*K)
  GO TO 30
  AX=XLROD*HS(B)
  BX=GS(B)/AX
  CX=TXIN-BX
  THXS=CX*EXP(-HS(B)*HS(B)*K)/AX
  TYPE 1,THXS
  GAIN=CABS(THXS)
  IF(REAL(THXS).EQ.0.)GO TO 10
  PHASE=ATAN(THXS/REAL(THXS))
  IF(REAL(THXS).LT.0.)PHASE=PHASE-3.1416
  GO TO 40
  PHASE=3.1416/2.
  C CALCULATE REACTIVITY/TEMP
  40 IF(ISHAPE.EQ.0100 TO 50
    DUM1=1.-EXP(-HS(B)*XLROD)
    DUM1=DUM1/HS(B)
    DUM2=1.-4.*B*(3.1416/3.1416/XLROD+XLROD)/(HS(B)*HS(B))
    DUM1=HS(B)*XLROD*(3.1416+CX*DUM1*(1.-1./DUM2)/2.
    DUM2=DUM1*HS(B)*XLROD
    ROAIN=CABS(DUM1)
    RPHASE=ATAN(THXS/DUM1)/REAL(DUM1)
    IF(REAL(DUM1).LT.0.)RPHASE=RPHASE-3.1416
    RETURN
  50 DUM1=1.-EXP(-HS(B)*XLROD)
    DUM1=DUM1/HS(B)
    DUM2=(TXIN-GS(B))/(HS(B)*XLROD)+DUM1*GS(B)/HS(B)
    DUM1=DUM1*ALPHAC/XLROD
    ROAIN=CABS(DUM1)
    RPHASE=ATAN(THXS/DUM1)/REAL(DUM1)
    IF(REAL(DUM1).LT.0.)RPHASE=RPHASE-3.1416
    RETURN
  END

```

```

TYPE 01
WRITE(2,01)
WRITE(2,1)(GAIN(I),I=1,NF)
FORMAT(1X,'TOUT/X,GAIN',THE UNIT ON:LOOK,VELD:100LBM/SEC,TIN:IF')
TYPE 02 01
WRITE(2,02)
WRITE(2,1)(PHASE(I),I=1,NF)
FORMAT(1X,'TOUT/X,PHASE')
TYPE 03 01
WRITE(2,03)
WRITE(2,1)(ROAIN(I),I=1,NF)
FORMAT(1X,'REACTIVITY/TEMP,GAIN')
TYPE 04
WRITE(2,04)
WRITE(2,1)(RPHASE(I),I=1,NF)
FORMAT(1X,'REACTIVITY/TEMP,PHASE')
1234 TYPE 05
WRITE(2,05)
WRITE(2,1)(TGAIN(I),I=1,NF)
FORMAT(1X,'TOUT/NEUTRON,GAIN')
TYPE 06
WRITE(2,06)
WRITE(2,1)(TPHASE(I),I=1,NF)
FORMAT(1X,'TOUT/NEUTRON,PHASE')
CALL ASCII(1,'PLOT.DAT')
WRITE(1,1)(I),I=1,20
FORMAT(1X,01,02,03,04,05,06)
WRITE(1,1)(GAIN(I),I=1,20)
WRITE(1,1)(PHASE(I),I=1,20)
WRITE(1,1)(ROAIN(I),I=1,20)
WRITE(1,1)(RPHASE(I),I=1,20)
IF(ISOURC.NE.0) WRITE(1,1)(TGAIN(I),I=1,20)
IF(ISOURC.NE.0) WRITE(1,1)(TPHASE(I),I=1,20)
IF(ISOURC.EQ.0) WRITE(1,1)(GAIN(I),I=1,20)
IF(ISOURC.EQ.0) WRITE(1,1)(PHASE(I),I=1,20)
CALL CLOSE(1)
STOP
END

```

SVIBSTR.PAC	20-DEC-84	14:10:44	PAGE	5		
<pre> FUNCTION JD(L,R) REAL L,IMAG COMPLEX JO REAL X IMAG=(A/2.)**2 SIGN=-1.*SIGN DO 10 I=1,100 SIGN=-1.*SIGN X1=1. X2=1. I2I=2*I DO 20 I1=1,I2I X1=X1*(A/2.)**I1 I2I=2*I1 DO 30 I2=1,I2I X2=X2*(A/2.)**I2 X=X1*X2*SIGN REAL=REAL*X IMAG=IMAG*X2*X2*SIGN X=ABS(X) IF (X4.LT.0.0000001) GO TO 40 CONTINUE COMPLX(REAL,IMAG) RETURN END </pre>	<pre> FUNCTION MS(S) COMMON U,B,L,R,OM,OC,RMO,C,K,M,FDENB1,CPV REAL L,K COMPLEX JO,MSR,S,MS MS(1:3)=MSR(R,L,K,M)=JO(L,R)*S)/U RETURN END </pre>	SVIBSTR.PAC	20-DEC-84	14:10:44	PAGE	6
<pre> FUNCTION HBR(R,L,K,M) REAL K,L COMPLEX MSR,MSR,GRADJO,JO,S MSR=GRADJO(L,R)*HBJO(L,R) MSR=MSR RETURN END </pre>	<pre> FUNCTION GS(S) COMMON JO,OM,S,OS COMMON U,B,L,R,OM,OC,RMO,C,K,M,FDENB1,CPV COMMON TIM,UNOISE,MLROD,ALPAC,PO REAL L,K OS=(B*OSR(S)=JO(L,R)*S)/OM/(FDENB1*CPV)*OC/(RMO*C))/U OS=OS-UNOISE*PO*(0.0188+0.981)/(1.766*RMO*C*U*U) RETURN END </pre>	SVIBSTR.PAC	20-DEC-84	14:10:44	PAGE	5
<pre> FUNCTION FACTOR(M) M=1. DO 10 I=1,M M=M*I FACTOR=M RETURN END </pre>						

```

FUNCTION GRADJO(L,R)
REAL L, IMAG
COMPLEX GRADJO
REAL X
IMAG=2.0
IMAG=2.0*(A/E-1.0/L-2.)
SIGN=1.
DO 10 I=1,100
  X1=SIGN-1.
  X2=1.
  X21=2.01
  DO 20 J=1,121
    X3=X1*(A/E-2.)/J
    X31=X2*(A/E-2.)/J
    X32=X31*(A/E-2.)/J
    IMAG=IMAG+X2+X2*SIGN=
    X4=ABS(X3)
    IF (X4.LT.0.0000001) GO
      TO 40
    GRADJO=CMPLX(REAL,IMAG)
    RETURN
  END
10
20
30
40

```

```

FUNCTION GSR(S)
  COMPLEX GSR,S,MSR
  REAL L,K
  COMMON U,B,L,R,QA,QC,RHO,C,K,H,PDENSI,CPY
  GSR=ZERO(R,L,K,H)/(PDENSI*CPY)
  RETURN
END

```

```

SV:SYSDAT.PAC 20-DEC-84 14:13:33
4323 FORMAT(1X,'DECIDE UFC BY ?')
      READ(5,4323)DUM
      UFC=UCP/DUM
      DO 30 I=1,NF
        KCL(I)=TCLM/NF
      END DO
      TYPE 301
      FORMAT(1X,'DIVIDE CPC BY ?')
      READ(5,4323)DUM
      CPC=CPC/DUM
      KCL2=93.1416*NRDB*LRDB=KCL/ALOG(RCL/RCL1)
      UCP=1./11./((NRCLAD)+1./KCL2)
      TYPE 4321
      FORMAT(1X,'DIVIDE UCP BY ?')
      READ(5,4322)DUM
      UCP=UCP/DUM
      NF2=2*NF
      TYPE 31
      FORMAT(1X,'DIRECT HEAT TO COOLANT?NO=1')
      READ(5,11) ICHECK
      DO 40 I=1,NF2
        DFC(I)=FC(I)
      END DO
      IF(ICHECK.EQ.1)FC(I)=0.
      MC(I)=ACORE=LRDB*BENSI/(NF*2)
      DO 41 I=1,60
        U(I)=0.
      END DO
      TYPE 42
      FORMAT(1X,'CALCULATE INITIAL VALUETHQ=1')
      READ(5,11) ICHECK
      TYPE 43
      FORMAT(1X,'CHECK
      READ(5,11) ICHECK
      TYPE 44
      FORMAT(1X,'INLET COOLANT TEMPERATURE(I.G. 540 )')
      READ(5,43)TIN
      NT=1+NP+4*NF
      NK=1+NP+2*NF+1
      U(NK)=FLOWMTIN/MC(1)
      TYPE 441
      FORMAT(1X,'USING SPATIAL DEPENDENCE NEUTRON KINETICS?YES=1')
      READ(5,11) ISPACE
      IF(ISPACE.NE.1)GO TO 481
      TYPE 442
      FORMAT(1X,'DETECTOR IN WHICH COOLANT MODE?')
      READ(5,11)ID
      DO 443 I=1,NF2
        CALL SPACEN(I,19,LRDB,WEIGHT(I),NF2)
      END DO
      DO 50 I=1,60
        A(I)=0.
      END DO
      C NEUTRON
      A(1,1)=BETA/GAMMA
      DO 60 I=1,NP
        J=1+I
        A(1,J)=LAMBDA(I)
      END DO
      FF88=0.
      DO 61 I=1,NF
        IF(ISPACE.NE.1)FF88=FF88+FF(I)*FF(I)
      END DO

```

```

C===== SYSDAT.FTN =====
C
C CALCULATE THE SYSTEM MATRICES FOR NODAL MODEL
C WITH POINT NEUTRON KINETIC, NP PRECURSOR GROUPS,
C NF FUEL AND CLAD NODES, 2NF COOLANT MODES
C
C THE OUTPUT IS STORED IN NODAL.DAT TO BE USED FOR NODAL MODEL
C CALCULATION (NODAL.FTN)
C=====
C
      DIMENSION MC(30),FC(20),DFC(30)
      DIMENSION U(60),WEIGHT(30)
      DOUBLE PRECISION MC(60),KCL(19),FF(19),NF(19)
      DIMENSION DFC(19),TCLM(19),KCL1(19),NF1(19),LAMBDA
      REAL KCL,KC,KCL2,KC1,KF,KG,KCL
      CALL NRDB(19,'DESIGN.DAT')
      READ(1,3)BETA,GAMMA,FC(1),FC(2),LAMBDA(1),LAMBDA(2),ALPHAF,ALPHAC,PO,HEATF
      > HEATC,TFM,NRDB,LRDB,NF,TCLM,RCL,CLTHIK,ACORE,CPP,NF,CPCL,KCL,
      > DEMSI,KC,CPC,VISCOS,KG,VELD,N,ACLAD
      TYPE 3,BETA,GAMMA,FC(1),FC(2),LAMBDA(1),LAMBDA(2),ALPHAF,ALPHAC,PO,HEATF,
      > HEATC,TFM,NRDB,LRDB,NF,TCLM,RCL,CLTHIK,ACORE,CPP,NF,CPCL,KCL,
      > DEMSI,KC,CPC,VISCOS,KG,VELD,N,ACLAD
      NRDB=NRDB
      TYPE 3
      FORMAT(1X,6F14.6)
      CALL CLOBE(1)
      NP=2
      TYPE 9
      FORMAT(1X,'NF=?',DEFAULT=10')
      READ(5,11)NF
      IF(NF.EQ.0)NF=10
      TYPE 9
      FORMAT(1X,'DIVIDE ALPHAC BY ?')
      READ(5,4323)DUM
      ALPHAC=ALPHAC/DUM
      TYPE 10
      FORMAT(1X,'INCLUDE TEMPERATURE FEEDBACK? NO=1')
      READ(5,11)IBACK
      IF(IBACK.EQ.1)ALPHAF=0.
      IF(IBACK.EQ.1)ALPHAC=0.
      FLOW=VELD*BENSI*ACORE
      TYPE 12
      FORMAT(1X,'DIVIDE FLOW BY ?')
      READ(5,4323)DUM
      FLOW=FLOW/DUM
      CALL FLRSTR(FF,FC,NF)
      DO 20 I=1,NF
        NF(I)=TFM/NF
      END DO
      KCL1=RCL-CLTHIK/E,
      NF2=NF*(KCL-CLTHIK)/E,
      NRDB=93.1416*NRDB*LRDB,
      KCL2=93.1416*NRDB*LRDB=KCL/ALOG(RCL2/RCL)
      UFC=1./11./((NF)+1./NR2+1./KCL1)
      TYPE 4323

```

```

MFNP=MFNP+DMF
DO 310 I=1,MFNP
  WRITE(8,211)
  WRITE(9,11)(A(I,J),J=1,MFNP)
  FORMAT(1X,I3)
  FORMAT(1X,6E12.5)
  CALL ASSIGN(1,'MODAL.DAT')
  WRITE(11,11A)
  WRITE(11,11A)
  CALL CLOSE(1)
C U11=MFNP*2*H*U1/FLOW*TH/MC(K)
  IF(CHECK.EQ.1)STOP
  CALL INIT(A,U,NT,TIN)
  STOP
END

```

```

61 IF(1SPACE.EQ.1)/FF80/FF80*((HEIGHT(Z=I-1)+HEIGHT(Z=I))/2.)*OFF(I)
  J1=NP+1
  IF(1SPACE.NE.1)(A(I,J)=(ALPHA*F/GAMMA)*OFF(I)/FF80) (HEIGHT*FLUX=Z
70 IF(1SPACE.EQ.1)(A(I,J)=(ALPHA*F/GAMMA)*((HEIGHT(Z=I-1)+HEIGHT(Z=I))/2.)*
  1*OFF(I)/FF80
  FC80=0.
  DO 71 I=1,NF2
    IF(1SPACE.NE.1)FC80=FC80+DFC(I)*DFC(I)
  71 IF(1SPACE.EQ.1)FC80=FC80+HEIGHT(I)*DFC(I)
  DO 80 I=1,NF2
    J1=NP+2*MF+1
    IF(1SPACE.NE.1)(A(I,J)=(ALPHA*F/GAMMA)*DFC(I)*DFC(I)/FC80) (HEIGHT*FLUX=Z
80 IF(1SPACE.EQ.1)(A(I,J)=(ALPHA*F/GAMMA)*HEIGHT(I)*DFC(I)/FC80)
  C PRECURSOR
  DO 90 I=1,NF
    J1=1
    A(I,J)=LAMBDA(I)
  90 A(I,J)=LAMBDA(I)
  C FUEL
  DO 100 I=1,NF
    J1=NP+1
    A(I,J)=POWFF(I)/(MF(I)*CFF)
    K1J=MF
    A(I,K)=UFC/(MF(I)*CFF*MF)
    A(I,J)=UFC/(MF(I)*CFF*MF)
  100 A(I,J)=UFC/(MF(I)*CFF*MF)
  C CLADDING
  DO 130 K=1,NF
    I=1+NP+MF+K
    J1=NP+K
    A(I,J)=UFC/(MCL(K)*CPCL*MF)
    A(I,I)=UFC/(MCL(K)*CPCL*MF)-UCP/(MCL(K)*CPCL*MF)
    L=1+NP+2*MF+2*MK-1
    A(I,L)=UCP/(MCL(K)*CPCL*MF)
  130 CONTINUE
  C COOLANT 2,4,6, ...
  DO 150 K=1,NF2
    I=1+NP+2*MF+K
    J1=NP+K
    A(I,J)=POWFF(K)/(MC(K)*CPC)
    DO 180 K=1,NF
      K2=2*MK
      I=1+NP+2*MF+2*MK
      J1=NP+MF+K
      A(I,J)=UCP/(MC(K2)*CPC*2*MF)+FLOW/MC(K2)
      L=1-1
      A(I,L)=UCP/(MC(K2)*CPC*2*MF)+FLOW/MC(K2)
  180 A(I,I)=FLOW/MC(K2)
  C COOLANT 1,3,5,-----
  DO 200 K=1,NF
    K2=2*MK-1
    I=1+NP+2*MF+2*MK-1
    J1=NP+MF+K
    A(I,J)=UCP/(MC(K2)*CPC*2*MF)
    L=1-1
    IF(K.EQ.1)80 TO 200
    A(I,L)=FLOW/MC(K2)
    A(I,I)=UCP/(MC(K2)*CPC*2*MF)-FLOW/MC(K2)
  200

```

SV18VSDAT.PAC	20-DEC-84	14:13:33	PAGE 5
1	SUBROUTINE FLXDTR(FP,FC,NF)		
2	DIMENSION FP(13),FC(30)		
3	TYPE 1		
4	1N,REAL CASE WITH 10 FUEL NODES ONLY		
5	READ(5,2)ICHECK		
6	FORMAT(13)		
7	NF=2*NF2		
8	IF(ICHECK.NE.0)GO TO 23		
9	DO 21 I=1,NF		
10	FP(I)=0.951/NF		
11	DO 22 I=1,NF2		
12	FC(I)=0.0188/NF2		
13	RETURN		
14	IF(ICHECK.NE.1)GO TO 5		
15	DO 3 I=1,NF		
16	C(I)=1/NF		
17	B=1/2.1416/NF		
18	FP(I)=0.951*(COS(C)-COS(B))/2.		
19	DO 4 I=1,NF2		
20	C(I)=1/NF2		
21	B=1/2.1416/NF2		
22	FC(I)=0.0188*(COS(C)-COS(B))/2.		
23	RETURN		
24	IF(ICHECK.NE.2)TYPE 6		
25	FORMAT(13,'INPUT ERROR')		
26	FF(1)=5.5		
27	FF(2)=10.7		
28	FF(3)=12.5		
29	FF(4)=13.7		
30	FF(5)=14.0		
31	FF(6)=12.5		
32	FF(7)=11.4		
33	FF(8)=9.7		
34	FF(9)=5.5		
35	FF(10)=2.5		
36	FC(1)=1/2		
37	FC(2)=7		
38	FC(3)=9.5		
39	FC(4)=11.5		
40	FC(5)=12.2		
41	FC(6)=13.4		
42	FC(7)=13.4		
43	FC(8)=14.0		
44	FC(9)=14.3		
45	FC(10)=13.6		
46	FC(11)=13.0		
47	FC(12)=12.2		
48	FC(13)=11.7		
49	FC(14)=11.0		
50	FC(15)=9.4		
51	FC(16)=8.0		
52	FC(17)=6.1		
53	FC(18)=4.7		
54	FC(19)=2.3		
55	FC(20)=1.5		
56	TOTAL=0.		

SV18VSDAT.PAC	20-DEC-84	14:13:33	PAGE 6
7	DO 7 I=1,NF		
8	TOTAL=TOTAL+FP(I)		
9	DO 8 I=1,NF		
10	FF(I)=0.951*FF(I)/TOTAL		
11	TOTAL=0.		
12	NF2=NF		
13	DO 81 I=1,NF2		
14	TOTAL=TOTAL+FC(I)		
15	DO 9 I=1,NF2		
16	FC(I)=0.0188*FC(I)/TOTAL		
17	RETURN		
18	END		

SV18VSDAT.PAC	20-DEC-84	14:13:33	PAGE 6
320	SUBROUTINE INIT(A,U,NT,TIN)		
321	DOUBLE PRECISION A(60,60)		
322	DIMENSION U(60),X(60)		
323	LOGICAL SING		
324	TYPE 320		
325	FORMAT(1X,'PREVENT OVERFLOW FACTOR')		
326	READ(5,43)FACTOR		
327	FORMAT(F10.5)		
328	DO 320 I=1,NT		
329	DO 320 J=1,NT		
330	A(I,J)=A(I,J)/FACTOR		
331	CALL INVERT(A,NT,60,SING)		
332	DO 9 I=1,60		
333	X(I)=0.		
334	T=0.		
335	X(1)=1.		
336	DO 91 I=2,NT		
337	T=T+A(1,I)*U(I)/FACTOR		
338	U(I)=(X(I)+T)*FACTOR/A(1,1)		
339	DO 10 I=2,NT		
340	X(I)=0.		
341	DO 10 J=1,NT		
342	X(I,J)=U(I,J)/FACTOR-X(I)		
343	WRITE(5,20)		
344	FORMAT(1X,'INITIAL VALUES')		
345	WRITE(5,30)X		
346	CALL ASSIGN(1,'INIT.DAT')		
347	WRITE(1,20)TIN,X		
348	CALL CLOSE(1)		
349	FORMAT(1X,6E12.5)		
350	RETURN		
351	END		


```

04-DEC-64 15:18:31 PAGE 5
BYIN0001.P0C

      SUBROUTINE MATINV(A,ORDER,DIM A,SING)
      INVERSE OF A REAL MATRIX.
      DIMENSION A(DIM A,1)
      DOUBLE PRECISION Z,PIVOT
      DOUBLE PRECISION S(100),C(100),A
      LOGICAL SING
      INTEGER IP(SO),IS(SO)
      DIMENSION SING(1)
      SING = .FALSE.
      RETURN
      SING = .TRUE.
      RETURN
      END

04-DEC-64 15:18:31 PAGE 6
BYIN0001.P0C

      Z=AL(IPK)
      A(I,I)=A(I,I)+Z
      S02 A(I,K)+Z
      S02 IF(IOM-K)S04,S00,S04
      S04 DO S05 J=1,N
      Z=AL(IOM,J)
      A(IOM,J)=A(I,K,J)
      S06 A(K,J)+Z
      S00 K=K+1
      SING = .FALSE.
      RETURN
      SING = .TRUE.
      RETURN
      END

      SUBROUTINE CORRECT(PHASE,GAIN,FREQ,BUFF)
      C THIS SUBROUTINE CORRECTS THE PHASE AND THE GAIN FOR THE CORRE EXIT
      C TEMPERATURE DUE TO THE RESPONSE OF THE TEMPERATURE SENSOR AND THE
      C DISTANCE BETWEEN THE TOP OF THE FUEL AND THE THERMAL SENSOR
      C ASSUME THE DISTANCE B/W SENSOR AND FUEL IS 5 INCH,THE VELOCITY:
      C 12. FUEL VELOCITY:12.00
      C 12. FUEL VELOCITY:12.00
      DIST=5/12
      Y0=DIST*VELOCITY/BUFF
      ANGLE=1/DUMFREQ*Y0
      AFRQ=2*
      ANGLE=ATAN(A)+180./3.1416
      AMPL=ABS(T1./((A+1.)))
      C
      PHASE=PHASE-ANGLE+ANGLE
      GAIN=GAIN*AMPL
      RETURN
      END

      SUBROUTINE CORRECT(PHASE,GAIN,FREQ,BUFF)
      C THIS SUBROUTINE CORRECTS THE PHASE AND THE GAIN FOR THE CORRE EXIT
      C TEMPERATURE DUE TO THE RESPONSE OF THE TEMPERATURE SENSOR AND THE
      C DISTANCE BETWEEN THE TOP OF THE FUEL AND THE THERMAL SENSOR
      C ASSUME THE DISTANCE B/W SENSOR AND FUEL IS 5 INCH,THE VELOCITY:
      C 12. FUEL VELOCITY:12.00
      C 12. FUEL VELOCITY:12.00
      DIST=5/12
      Y0=DIST*VELOCITY/BUFF
      ANGLE=1/DUMFREQ*Y0
      AFRQ=2*
      ANGLE=ATAN(A)+180./3.1416
      AMPL=ABS(T1./((A+1.)))
      C
      PHASE=PHASE-ANGLE+ANGLE
      GAIN=GAIN*AMPL
      RETURN
      END

      SUBROUTINE CORRECT(PHASE,GAIN,FREQ,BUFF)
      C THIS SUBROUTINE CORRECTS THE PHASE AND THE GAIN FOR THE CORRE EXIT
      C TEMPERATURE DUE TO THE RESPONSE OF THE TEMPERATURE SENSOR AND THE
      C DISTANCE BETWEEN THE TOP OF THE FUEL AND THE THERMAL SENSOR
      C ASSUME THE DISTANCE B/W SENSOR AND FUEL IS 5 INCH,THE VELOCITY:
      C 12. FUEL VELOCITY:12.00
      C 12. FUEL VELOCITY:12.00
      DIST=5/12
      Y0=DIST*VELOCITY/BUFF
      ANGLE=1/DUMFREQ*Y0
      AFRQ=2*
      ANGLE=ATAN(A)+180./3.1416
      AMPL=ABS(T1./((A+1.)))
      C
      PHASE=PHASE-ANGLE+ANGLE
      GAIN=GAIN*AMPL
      RETURN
      END

```

```

C-----
SUBROUTINE ADD(A,M1,M1,B,M2,M2)
  DIMENSION A(12,12),B(12,12),C(12,12)
  IF(M1.M2.NE.12)
    1  FORMAT('X','ERROR')
  IF(M1.M2.NE.12) 1
  DO 10 I=1,M1
    DO 10 J=1,M2
      C(I,J)=A(I,J)+B(I,J)
    10  RETURN
  END

```

SY:hal.pacc	22-DEC-84	18:12:41	PAGE 3
SV:hal.pacc	22-DEC-84	18:11:37	PAGE 1


```

C-----
SUBROUTINE SUB(A,M1,M1,B,M2,M2,C)
  DIMENSION A(12,12),B(12,12),C(12,12)
  IF(M1.ME.M2.OR.M1.ME.M2)TYPE 1
  FORMAT(1X,'ERROR')
  DO 10 I=1,M1
  DO 10 J=1,M1
    C(I,J)=A(I,J)-B(I,J)
  RETURN
END

10
C-----
SUBROUTINE TRANS(A,M,N,B)
  DIMENSION A(12,12),B(12,12)
  DO 10 I=1,N
  DO 10 J=1,M
    B(J,I)=A(I,J)
  RETURN
END

10
C-----
SUBROUTINE INPUT
COMMON A,U,X,Y,R,G,M,B,N,M,MH,DT,P,KK,DEBUG
1  ,MO,NB,IEX,E,FADE,NRK,S1,S2,S3,S4,ISK
  DIMENSION MO(12),E(12)
  DIMENSION A(12,12),U(12),X(12),Y(12),R(12,12),G(12,12),O(12,12),
1 M(12,12),B(12,12),DUM1(12,12),DUM2(12,12)
  DIMENSION S1(12,12),S2(12,12),S3(12,12),S4(12,12)
  DIMENSION P(12,12),MK(12,12)
  LOGICAL S1 IDFIL(27),IFILM(27)
  REAL MK
  INTEGER DEBUG
  FORMAT(3X,S11.7)
  FORMAT(3X,S15)
  FORMAT(7X,I5)
  FORMAT(1X,'ENTER MEASUREMENT DATA FILE NAME',/)
  OPEN(11, 'CORRECTION OF A MATRIX',B=1)
  CALL COMBASS(1,'KSYB.DAT')
  READ(1,1)
  READ(1,1)
  TYPE 2,M
  READ(1,2)M
  TYPE 2,M
  READ(1,1)((A(I,J),J=1,12),I=1,N)
  TYPE 1,((A(I,J),J=1,12),I=1,N)
  READ(1,1)U
  TYPE 1,U
  READ(1,1)X
  TYPE 1,X
  READ(1,1)Y
  TYPE 1,Y
  READ(1,1)((R(I,J),J=1,12),I=1,N)
  TYPE 1,((R(I,J),J=1,12),I=1,N)
  READ(1,1)((G(I,J),J=1,12),I=1,N)
  TYPE 1,((G(I,J),J=1,12),I=1,N)
  READ(1,1)((O(I,J),J=1,12),I=1,N)
  TYPE 1,((O(I,J),J=1,12),I=1,N)
  READ(1,1)((M(I,J),J=1,12),I=1,N)
  TYPE 1,((M(I,J),J=1,12),I=1,N)
  READ(1,1)((B(I,J),J=1,12),I=1,N)
  TYPE 1,((B(I,J),J=1,12),I=1,N)
  READ(1,2)N
  TYPE 2,N
  READ(1,2)M
  TYPE 2,M
  READ(1,2)MH
  TYPE 2,MH
  READ(1,2)IEX
  TYPE 2,IEX
  READ(1,2)MO
  TYPE 2,MO
  READ(1,2)NB
  TYPE 2,NB
  READ(1,3)DEBUG
  TYPE 3,DEBUG
  READ(1,1)DT
  TYPE 1,DT
  READ(1,1)FADE
  TYPE 1,FADE

```



```

SY:kal2.pacc      22-DEC-84      18:13:42      PAGE 1
C KAL2
SUBROUTINE KALMAN
COMMON A,U,X,Y,R,G,H,B,M,M,M,DT,P,KK,DEBUG
1  'POSTERIOR COVARIANCE MATRIX IS',//6E12.4)
DIMENSION A(12,12),U(12),X(12),Y(12),R(12,12),G(12,12),
1  H(12,12),B(12,12)
DIMENSION P(12,12),KK(12,12)
REAL KK
INTEGER DEBUG
1  FORMAT(1X,'THE EXTRAPOLATED STATE VALUES ARE',//6E12.4)
2  FORMAT(1X,'THE PRIORI COVARIANCE MATRIX IS',//6E12.4)
3  FORMAT(1X,'THE PRIORI COVARIANCE MATRIX IS',//6E12.4)
4  FORMAT(1X,'THE GAINS ARE',//6E12.4)
5  FORMAT(1X,'THE POSTERI COVARIANCE MATRIX IS',//6E12.4)
6  FORMAT(1X,'THE UPDATED STATED VALUES ARE',//6E12.4)
CALL EXTRA
IF(DEBUG.EQ.1)TYPE 1, X
C  E(T)=Y(T)-H(T)*X(T)
CALL ERROR
IF(DEBUG.EQ.1)TYPE 2, E
C  P(T)=A(T-1)*G(T-1)+G(T-1)
CALL PRIORI
IF(DEBUG.EQ.1)TYPE 3, ((P(I,J),J=1,12),I=1,N)
C  K(T)=P(T)*X(T)/H(T)*H(T)+X(T)-1
IF(DEBUG.EQ.1)TYPE 4, ((KK(I,J),J=1,12),I=1,N)
C  G(T)=1-(K(T)*H(T))*P(T)/(1-K(T)*H(T))+K(T)*H(T)
C  OR  G(T)=1-(K(T)*H(T))*P(T)
CALL POSTER
IF(DEBUG.EQ.1)TYPE 5, ((G(I,J),J=1,12),I=1,N)
C  X(T)=X(T)+KK(T)*E(T)
CALL UPDATE
IF(DEBUG.EQ.1)TYPE 6, X
RETURN
END

SY:kal3.pacc      22-DEC-84      18:14:09      PAGE 1
C KAL3
SUBROUTINE STORE
COMMON A,U,X,Y,R,G,H,B,M,M,M,DT,P,KK,DEBUG
1  'POSTERIOR COVARIANCE MATRIX IS',//6E12.4)
DIMENSION A(12,12),U(12),X(12),Y(12),R(12,12),G(12,12),
1  H(12,12),B(12,12)
DIMENSION P(12,12),KK(12,12),DUM1(12,12),DUM2(12,12)
REAL KK
INTEGER DEBUG
1  FORMAT(1X,'STATE VALUES ARE',//6E12.5)
2  FORMAT(1X,'STATE VALUES ARE',//6E12.5)
RETURN
END

SUBROUTINE AVE(ICOUNT,MT,XBOT,X)
DIMENSION X(12)
X7=X7+X(7)
XBOT=XBOT+X(7)*X(7)
IF(ICOUNT.LE.1000)GO TO 20
ICOUNT=1
XMTX/1000.
XSGA=XBOT/1000.
SDU=SQRT(XSGA-XMTX)
X7=0.
XBOT=0.
WRITE(2,30)X7,SDU
30  FORMAT(1X,3E12.5)
RETURN
END

```

```

C KAL4
SUBROUTINE EXTKF
COMMON A,U,X,Y,R,G,O,H,B,N,M,MM,DT,P,KK,DEBUG
1 ,NO,NE,EX,E,FADE,NRK,S1,S2,S3,S4
DIMENSION MD(12),E(12)
1 MD(12),B(12),P(12),R(12),Y(12),X(12),U(12),G(12),O(12),H(12),
DIMENSION S1(12),S2(12),S3(12),S4(12),MM(12),MM(12),MM(12),MM(12)
REAL KK
INTEGER DEBUG
1 FORMAT(IX,' THE EXTRAPOLATED STATE VALUES ARE',//6E12.4)
2 FORMAT(IX,' THE ERRORS ARE',//6E12.4)
3 FORMAT(IX,' THE PRIOR COVARIANCE MATRIX IS',//6E12.4)
4 FORMAT(IX,' THE GAINS ARE',//6E12.4)
5 FORMAT(IX,' THE POSTERIOR COVARIANCE MATRIX IS',//6E12.4)
6 FORMAT(IX,' THE POSTERIOR STATE VALUES ARE',//6E12.4)
CALL RESET(AA,M,N)
CALL RESET(HH,M,N)
C AFTER REWIND IN THE FOLLOWING, THE ORIGINAL MATRIX ARE AA,MM, AND
C THE LINEARIZED MATRIX ARE A,H
CALL LINEAR(AA,A,X,DT,N,DEBUG,S1,S2,S3,S4)
C X(T)=A(T-1)*X(T-1)+B(T-1)*U(T-1)
CALL EXTRA(AA,X,B,U,N,MM,DT,NRK,S1,S2,S3,S4)
IF(DEBUG.EQ.1)TYPE 1, (X(I),I=1,N)
CALL LINEAR(HH,M,M,N,X)
C E(T)=Y(T)-H(T)*X(T)
CALL ERROR(Y,M,X,M,N,E)
IF(DEBUG.EQ.1)TYPE 2, (E(I),I=1,M)
C P(T)=A(T-1)*P(T-1)+Q(T-1)
CALL PRIOR
IF(DEBUG.EQ.1)TYPE 3, ((P(I,J),J=1,12),I=1,N)
C K(T)=P(T)*H(T)*(H(T)*P(T)*H(T)+R(T))-1
CALL GAIN
IF(DEBUG.EQ.1)TYPE 4, ((KK(I,J),J=1,12),I=1,N)
C G(T)=E(I-K(T)*H(T))*E(I-K(T)*H(T))-K(T)*R(T)*K(T)
CALL POSTER
IF(DEBUG.EQ.1)TYPE 5, ((G(I,J),J=1,12),I=1,N)
C X(T)=X(T)+K(T)*E(T)
CALL UPDATE
IF(DEBUG.EQ.1)TYPE 6, (X(I),I=1,N)
CALL RESET(AA,M,N)
CALL RESET(HH,M,M,N)
RETURN
END

```

```

C KAL5
SUBROUTINE NOTE
TYPE 1
1 FORMAT(IX,'
1 IEN=0 : KALMAN FILTER'
1 IEN=1 : EXTENDED KALMAN FILTER'
1 IF IEN=0, THE USER MUST SUPPLY THE FOLLOWING INPUTS'
1 A(N,N) TRANSITION MATRIX'
1 U(M,N) INPUT'
TYPE 2
1 FORMAT(IX,' B(N,M,N) CONTROL MATRIX'
1 X(N,S) STATE ESTIMATE'
1 Y(N,S) MEASUREMENT VECTOR'
1 A(N,N) MEASUREMENT ERROR COVARIANCE MATRIX'
1 G(N,N) PROCESS ERROR COVARIANCE MATRIX'
1 G(N,N) STATE ERROR COVARIANCE MATRIX'
TYPE 3
1 FORMAT(IX,' M(N,M,N) OBSERVATION MATRIX'
1 M(N,M,N) OBSERVATION'
1 H(N,M,N) OBSERVATION DIMENSION'
1 M INPUT DIMENSION'
1 IF IEN=1, THE USER MUST SUPPLY THE FOLLOWING
1 MATRICES A,U,B,H ARE NOT NEEDED'
TYPE 4
1 FORMAT(IX,' ADD THE FOLLOWING SUBROUTINE'
1 SUBROUTINE F(X(T-1),U(T-1),X(T)) : THE STATE FUNCTION'
1 SUBROUTINE H(X(T),Y(T)) : THE MEASUREMENT FUNCTION'
1 SUBROUTINE LF(X(T-1),A) : THE GRADIENT OF F'
1 SUBROUTINE LH(X(T-1),M) : THE GRADIENT OF M'
1 THE OUTPUTS INCLUDE
TYPE 5
1 FORMAT(IX,' P(N,N) PRIOR ERROR COVARIANCE MATRIX'
1 K(N,M) GAIN MATRIX'
1 G UPDATED ERROR COVARIANCE MATRIX'
1 X UPDATED STATE ESTIMATE'
1 THE ALGORITHM FOR THE LINEAR KALMAN FILTERS AS FOLLOWING
1 SYSTEM EQUATIONS:
TYPE 6
1 FORMAT(IX,' X(T)=A(T)*X(T)+B(T)*U(T)+C(T)
1 G(T)=G(C(T))
1 Y(T)=H(T)*X(T)+U(T)
1 R(T)=R(U(T))
1 ESTIMATION SCHEME:
1 X(T)=A(T-1)*X(T-1)+B(T-1)*U(T-1)
TYPE 7
1 FORMAT(IX,' P(T)=A(T-1)*P(T-1)+Q(T-1)
1 K(T)=P(T)*H(T)*(H(T)*P(T)*H(T)+R(T))-1
1 X(T)=X(T)+K(T)*(Y(T)-H(T)*X(T))
1 G(T)=E(I-K(T)*H(T))*G(T)
1 INITIAL CONDITION: X(O), G(O)
1
TYPE 8
1 FORMAT(IX,' THE ALGORITHM FOR THE EXTENDED KALMAN FILTER IS
1 SYSTEM EQUATIONS:
1 X(T)=F(X(T),U(T))+C(T)
1 Y(T)=H(X(T))+U(T)
1 R(T)=R(U(T))
1 Y(T)=H(X(T))+U(T)
1 R(T)=R(U(T))

```



```

SYNALS.PACC      22-DEC-84      18:14:54      PAGE 2

      TYPE 9
      FORMAT(IX,' ESTIMATION SCHEME',/,)
      1 K(T)=P(K(T-1),U(T-1)),/
      1 A(T-1) : GRADIENT OF F(X(T-1),U(T-1)) AT X(T-1),/
      1 M(T) : GRADIENT OF H(X(T)) AT X(T),/
      1 P(T)=A(T-1)G(T-1)+A(T-1)*Q(T-1),/
      1 K(T)=P(T)M(T)+CH(T)P(T)M(T)+R(T)-1,
      TYPE 10
      FORMAT(IX,' K(T)=X(T)+K(T)[Y(T)-H(X(T))],/
      1 G(T)=[I-K(T)]H(T)P(T)
      1
      RETURN
      END

SYNALS.PACC      22-DEC-84      18:07:46      PAGE 1

      SUBROUTINE EXTRA
      COMMON A,U,X,Y,R,G,M,B,N,M,MH,DT,P,KK,DEBUG
      1 ,NO,NB,IE,E
      DIMENSION M(12),E(12)
      1 M(12)=R(12),U(12),X(12),Y(12),R(12,12),G(12,12),G(12,12),
      DIMENSION DUM(12,12),DUNE(12,12)
      DIMENSION P(12,12),KK(12,12)
      REAL KK
      INTEGER DEBUG
      CALL HLP(B,N,MH,U,MH,1,DUM)
      CALL HLP(A,N,M,N,1,DUNE)
      CALL ADD(DUM,1,N,1,DUNE,M,1,X)
      RETURN
      END

      SUBROUTINE ERROR
      COMMON A,U,X,Y,R,G,M,B,N,M,MH,DT,P,KK,DEBUG
      1 ,NO,NB,IE,E
      DIMENSION M(12),E(12)
      1 M(12)=R(12),U(12),X(12),Y(12),R(12,12),G(12,12),G(12,12),
      DIMENSION DUM(12,12)
      DIMENSION P(12,12),KK(12,12)
      REAL KK
      INTEGER DEBUG
      CALL HLP(B,N,MH,U,MH,1,DUM)
      CALL SUB(Y,M,1,DUM,M,1,DUM)
      DO 5 I=1,12
      E(I)=0.
      DO 10 I=1,M
      E(I)=DUM(I,1)
      RETURN
      END
5
10

      SUBROUTINE PRIORI
      COMMON A,U,X,Y,R,G,M,B,N,M,MH,DT,P,KK,DEBUG
      1 ,NO,NB,IE,E,FADE
      DIMENSION M(12),E(12)
      1 M(12)=R(12,12),U(12),X(12),Y(12),R(12,12),G(12,12),G(12,12),
      DIMENSION DUM(12,12),P(12,12),KK(12,12)
      REAL KK
      INTEGER DEBUG
      CALL TRANS(A,N,M,N,AT)
      CALL HLP(A,N,M,N,G,N,M,DUM)
      CALL HLP(DUM,N,N,AT,N,N,P)
      NI=N-1
      DO 10 I=1,NI
      DO 10 J=1,NI
      P(I,J)=P(I,J)+FADE
      CALL ADD(P,N,N,G,N,N,P)
      RETURN
      END
10

```

SVIN122.pac 22-DEC-84 18:09:42 PAGE 2

C-----
SUBROUTINE UPDATE
COMMON A,U,X,Y,R,G,H,B,M,M,MH,DT,P,KK,DEBUG
1 ,NO,MB,IE,X,E(12),E(12)
DIMENSION A(12,12),U(12),X(12),Y(12),R(12,12),G(12,12),G(12,12),
1 M(12,12),B(12,12)
DIMENSION DUM(12,12),DUM1(12,12)
DIMENSION P(12,12),KK(12,12)
REAL KK
INTEGER DEBUG
DO 10 I=1,M
DUM(I,1)=E(I)
CALL MLP(KK,M,M,DUM1,M,1,DUM)
CALL ADD(X,M,1,DUM,M,1,X)
RETURN
END

SVIN122.pac 22-DEC-84 18:09:42 PAGE 1

C KAL22 SUBROUTINE GAIN
COMMON A,U,X,Y,R,G,H,B,M,M,MH,DT,P,KK,DEBUG
1 ,NO,MB,IE,X,E(12),E(12)
DIMENSION A(12,12),U(12),X(12),Y(12),R(12,12),G(12,12),G(12,12),
1 M(12,12),B(12,12)
DIMENSION DUM1(12,12),DUM2(12,12),MT(12,12)
DIMENSION P(12,12),KK(12,12)
DOUBLE PRECISION DDUM(12,12)
REAL KK
INTEGER DEBUG
CALL TRANS(M,M,M,MT)
CALL MLP(P,M,M,MT,M,M,DUM1)
CALL MLP(M,M,M,DUM1,M,M,DUM2)
CALL ADD(DUM2,M,M,R,M,M,DUM2)
DO 10 I=1,M
DO 10 J=1,M
DDUM(I,J)=DUM2(I,J)
10 DDUM(I,J)=DUM2(I,J)
C INVERT MPH<R
IF(N.LT.200 TO 40
CALL MATINV(DDUM,M,12,SING)
GO TO 50
DDUM(I,1)=1.000/DDUM(1,1)
DO 45 I=1,12
DO 45 J=1,12
KK(I,J)=0.
DO 60 I=1,M
DO 60 J=1,M
KK(I,J)=0
DO 60 K=1,M
KK(I,J)=KK(I,J)+DUM1(I,K)*DDUM(K,J)
60 KK(I,J)=KK(I,J)+DUM1(I,K)*DDUM(K,J)
RETURN
END

C-----
SUBROUTINE POSTER
COMMON A,U,X,Y,R,G,H,B,M,M,MH,DT,P,KK,DEBUG
1 ,NO,MB,IE,X,E
DIMENSION HQ(12),E(12)
DIMENSION A(12,12),U(12),X(12),Y(12),R(12,12),G(12,12),G(12,12)
1 ,M(12,12),B(12,12)
DIMENSION XI(12,12),DUM1(12,12),DUM2(12,12),DUM3(12,12)
DIMENSION P(12,12),KK(12,12)
REAL KK
INTEGER DEBUG
DO 10 I=1,M
DO 10 J=1,M
XI(I,J)=0.
IF(I.EQ.J)XI(I,J)=1.
CALL MLP(KK,M,M,M,M,DUM1)
CALL SUB(XI,M,M,DUM1,M,M,DUM1)
CALL TRANS(DUM1,M,M,M,M,DUM2)
CALL MLP(DUM2,M,M,P,M,M,DUM3)
CALL MLP(DUM3,M,M,DUM2,M,M,DUM1)
CALL TRANS(KK,M,M,DUM2)
CALL MLP(KK,M,M,R,M,M,DUM3)
CALL MLP(DUM3,M,M,DUM2,M,M,G)
CALL ADD(DUM1,M,M,G,M,M,G)
RETURN
END

```

SV:KAL416.pac      22-DEC-84      18:10:49      PAGE 2

C-----
SUBROUTINE EXTRA(A,B,U,M,MM,DT,NRK,SI,S2,S3,S4)
  DIMENSION DUM(12,12)
  DIMENSION X(12),A(12,12),U(12),B(12,12)
  DIMENSION DUM1(12,12),S1(12,12),S2(12,12),S3(12,12),S4(12,12)
  NI=N-1
  DO 10 I=1,NI
    DO 10 J=1,NI
      DUM1(I,J)=A(I,J)+B(I,J)*X(S1(I,J)+S2(I,J)+S3(I,J)+S4(I,J))
      DUM(I,J)=DUM1(I,J)+S4(I,J)*X(S1(I,J)+S2(I,J)+S3(I,J)+S4(I,J))
      DUM(I,J)=0.
  DO 30 K=1,NI
    DUM1(I,J)=DUM(I,J)+DUM1(I,K)*X(K)
    DO 40 J=1,N
      DUM(I,J)=DUM1(I,J)+DUM(I,J)*X(N)
    DO 40 J=1,N
      DUM1(I,J)=DUM(I,J)+DUM1(I,J)*X(N)
  DO 40 J=1,N
    DUM1(I,J)=DUM1(I,J)+B(J,I)*U(I)
  DO 20 I=1,NI
    X(I)=DUM(I,1)+DUM1(I,1)
  RETURN
END

C-----
SUBROUTINE LINERA(A,B,K,DT,N,DEBUG,S1,S2,S3,S4)
  DIMENSION A(12,12),B(12,12)
  DIMENSION X(12)
  DIMENSION S1(12,12),S2(12,12),S3(12,12),S4(12,12)
  DO 10 I=1,12
    DO 10 J=1,12
      S1(I,J)=A(I,J)+B1(I,J)*X(S1(I,J)+S2(I,J)+S3(I,J)+S4(I,J))
  CONTINUE
  RETURN
END

SV:KAL416.pac      22-DEC-84      18:10:49      PAGE 1

C KAL416: LOFT VELOCITY AS FLUCTUATION SOURCE
SUBROUTINE REIN(A,B,N,M)
  DIMENSION A(12,12),B(12,12)
  DO 10 I=1,M
    DO 10 J=1,M
      B(I,J)=A(I,J)
  RETURN
END

10

C-----
SUBROUTINE RESET(A,B,N,M)
  DIMENSION A(12,12),B(12,12)
  DIMENSION X(N),M(N,N),N(N,1),DUM
  DO 10 I=1,M
    DO 10 J=1,M
      B(I,J)=A(I,J)
  RETURN
END

10

C-----
SUBROUTINE ERROR(Y,N,M,M,X)
  DIMENSION Y(12),X(12),C(12),M(12,12),DUM(12,12)
  CALL HLP(N,M,N,M,X,1,DUM)
  CALL SUB(Y,M,1,DUM,M,1,DUM)
  DO 5 I=1,12
    Y(I)=C(I)
  DO 10 I=1,M
    C(I)=DUM(I,1)
  RETURN
END

5
10

C-----
SUBROUTINE LINERH(M,N,M,X)
  DIMENSION MM(12,12),X(12),M(12,12)
  DO 10 I=1,M
    DO 10 J=1,M
      MM(I,J)=M(I,J)
  RETURN
END

10

```

SVLA001-01NC 22-DEC-04 10:15:31 PAGE 1

```

C FOR AMBIGUITY FUNCTION CALCULATION
1  DIMENSION KK(12,12),M(12,12),NT(12,12),G(12,12),GT(12,12),
1  CT(12,12),R(12,12),AT(12,12),P(12,12),G(12,12),PHI(12,12),
1  PHIT(12,12),PATP(12,12),PETP(12,12),PETH(12,12),
1  DIMENSION C(12,12),GT(12,12)
REAL KK
CALL INPUT(KK,M,NT,G,GT,C,CT,R,AT,NH,ND,NT,P,G,GT,PHI,PHIT)
DO 30 I=1,12
DO 40 J=1,12
DO 50 I=1,12
DO 60 J=1,12
PATP(I,J)=0.
IF(I.EQ.J)PATP(I,J)=1.
C30
DO 40 I=1,NT
I1=I+NT
DO 40 J=1,NT
J1=J+NT
PATP(I,J)=GT(I,J)
C 40
DO 50 I=1,ND
I1=I+NT
DO 50 J=1,ND
J1=J+NT
PATP(I,J)=G(I,J)
C 50
DO 60 I=1,NT
I1=I+NT
DO 60 J=1,NT
J1=J+NT
PATP(I,J)=G(I,J)
C 60
DO 10 I=1,200
DO 10 I=1,SUM
TYPE I,I,SUM
FORMAT(1X,I5,'E14.7')
1  CALL BENIG(KK,M,NT,PATP,GT,PHIT,PHI,CT,C,AT,PETP,PETH,NH,ND,
10  CALL AMBIG(N,R,PETH,P,G,NH,ND,SUM,I,DUM1,DUM2,DUM3,DUM4)
A12=ND*ALOG(2./S)14161/2.
CALL DETRA(G,ND,12,DETA)
A22=ALOG(DETA)/2.
AMBI2=SUM(A1+A2)
CALL ASSIGN(I,'AMBI,DETA')
TYPE 20,AMBI,DETA,DUM1,DUM2,DUM3,DUM4
FORMAT(1X,I5,'E13.6')
1  FORMAT(1X,'AMBI','E14.7',4X)
STOP
END

```

```

1  SUBROUTINE SUB(A,NA,MA,B,NB,MB,C)
   DIMENSION A(12,12),B(12,12),C(12,12)
   IF(MA.NE.NB.OR.MA.NE.MB)TYPE 1
   FORMAT(1X,'DIMENSION ERROR')
   DO 2 I=1,NA
   DO 2 J=1,MA
   C(I,J)=A(I,J)-B(I,J)
   RETURN
   END
2

```

BY: ahl.f.tnc 22-DEC-04 10:15:31 P

```

SUBROUTINE DETMAT(A,NORDER,NDIM,DETA)
  DIMENSION A(NDIM,1)
  N=NORDER-1
  DO 300 I=1,N
    L=I
    IF(A(I,1)-0.)$9.54.53
      IF(L-NORDER)95.90.90
      L=L+1
    DO 60 J=1,NORDER
      A(I,J)=A(I,J)+A(L,J)
    DO 70 J=1,N
      L=L+1
    DO 300 J=L,NORDER
      D=A(J,1)/A(I,1)
      A(J,K)=A(J,K)-D*A(I,K)
    DETA=1.0
    DO 70 J=1,NORDER
      DETA=DETA*A(J,J)
  RETURN
  WRITE(7,105)
  105 CONTINUE THE DETERMINANT IS ZERO !!!
  END

```

```

10
FUNCTION TRACE(A,IORDER)
  DIMENSION A(12,12)
  X=0.
  DO 10 I=1,IORDER
    X=X+A(I,1)
  TRACE=X
  RETURN
END

```

```

SUBROUTINE MULY(A,NA,MB,NB,MC)
  DIMENSION A(1,12),B(12,12),C(12,12)
  IF(MA.NE.NB)TYPE 1
  FORMAT(1X,'DIMENSION ERROR')
  DO 2 I=1,NA
    DO 2 J=1,MB
      C(I,J)=0.
  DO 2 K=1,MA
    C(I,J)=C(I,J)+A(I,K)*B(K,J)
  RETURN
END

```

```

      READ(1,1)((OT(I,J),J=1,12),I=1,N)
      TYPE 1,((OT(I,J),J=1,12),I=1,N)
      READ(1,1)((OT(I,J),J=1,12),I=1,N)
      TYPE 1,((OT(I,J),J=1,12),I=1,N)
      READ(1,1)((MT(I,J),J=1,12),I=1,N)
      TYPE 1,((MT(I,J),J=1,12),I=1,N)
      READ(1,1)((B(I,J),J=1,12),I=1,N)
      TYPE 1,((B(I,J),J=1,12),I=1,N)
      READ(1,2)NT
      TYPE 2,NT
      READ(1,2)NH
      TYPE 2,NH
      CALL CLOSE(1)
      CALL ASSIGN(1,'KAL.DAT')
      READ(1,61)KK,GT,P,AT,OT,MT
      CALL CLOSE(1)
      CALL ASSIGN(1,'KAL.DAT')
      READ(1,61)KK,G,P,A,G,H
      CALL CLOSE(1)
      FORMAT(6E12.5)
      FORMAT(1X,4F17.9)
      FORMAT(1X,3I5,/,10.5)
      DO 80 J=1,12
      DO 80 J1=1,12
      CT(I,J)=0.
      IF(1.EQ.J)CT(I,J)=1.
      IF(1.EQ.J)CT(I,J)=1.
      RETURN
      END

```

61

80

6

80

```

SUBROUTINE UDWMB(A,NA,MA,B,NB,MB,C)
  DIMENSION A(12,12),B(12,12),C(12,12)
  IF(MA.NE.MB)TYPE 1
  FORMAT(1X,'DIMENSION ERROR')
  DO 2 I=1,MA
  DO 2 J=1,MA
  C(I,J)=A(I,J)
  DO 3 J1=1,NB
  INAB=MA-NB
  DO 3 J1=1,NB
  C(INB+J1)=B(I,J1)
  RETURN
  END

```

1

2

3

```

SUBROUTINE INPUT(KK,M,NT,OT,CT,R,RT,NH,ND,NT,P,G,OT,A,AT)
  DIMENSION MO(12),C(12),AT(12,12),MT(12,12),OT(12,12),CT(12,12)
  DIMENSION A(12,12),U(12),N(12),I(12),N1(12),N2(12),N3(12),N4(12),N5(12),
  1 W(12,12),B(12,12),DUM(12,12),DUM2(12,12),C(12,12),R(12,12)
  DIMENSION P(12,12),KK(12,12),OT(12,12)
  REAL KK
  INTEGER DBUG
  FORMAT(5X,4F11.7)
  FORMAT(1X,5I3)
  FORMAT(7X,5I3)
  FORMAT(11X,'ENTER MEASUREMENT DATA FILE NAME',/)
  1 FORMAT(11X,'CORRECTION OF A MATRIXYES=1')
  CALL ASSIGN(1,'KEYS.AMB')
  READ(1,1)
  READ(1,2)H
  TYPE 2,H
  READ(1,2)H
  TYPE 2,H
  READ(1,1)((A(I,J),J=1,12),I=1,N)
  TYPE 1,((A(I,J),J=1,12),I=1,N)
  READ(1,1)U
  TYPE 1,U
  READ(1,1)X
  TYPE 1,X
  READ(1,1)Y
  TYPE 1,Y
  TYPE 1,1,((B(I,J),J=1,12),I=1,N)
  TYPE 1,1,((B(I,J),J=1,12),I=1,N)
  READ(1,1)((O(I,J),J=1,12),I=1,N)
  TYPE 1,((O(I,J),J=1,12),I=1,N)
  READ(1,1)((O(I,J),J=1,12),I=1,N)
  TYPE 1,((O(I,J),J=1,12),I=1,N)
  READ(1,1)((M(I,J),J=1,12),I=1,N)
  TYPE 1,((M(I,J),J=1,12),I=1,N)
  READ(1,1)((B(I,J),J=1,12),I=1,N)
  TYPE 1,((B(I,J),J=1,12),I=1,N)
  READ(1,2)ND
  TYPE 2,ND
  READ(1,2)NH
  TYPE 2,NH
  CALL CLOSE(1)
  CALL ASSIGN(1,'KEYST.AMB')
  READ(1,2)H
  TYPE 2,H
  READ(1,2)H
  TYPE 2,H
  READ(1,1)((AT(I,J),J=1,12),I=1,N)
  TYPE 1,((AT(I,J),J=1,12),I=1,N)
  READ(1,1)U
  TYPE 1,U
  READ(1,1)X
  TYPE 1,X
  READ(1,1)Y
  TYPE 1,Y
  READ(1,1)((OT(I,J),J=1,12),I=1,N)
  TYPE 1,((OT(I,J),J=1,12),I=1,N)

```

1

2

3

4

5


```

SVI0001.finc      22-DEC-84      19:15:31      PAGE 7
SUBROUTINE ANSIG(M,R,PETH,P,O,NM,ND,SUM,IREPE,A1,A2,A3,A123)
C REFER TO MAYBECK VOL 2, P 98
C MPST(A,T),DUM1=-NH*ALOG(2*3.1416)/2-ALOG(DET(A(T,A))) / 2
C --TRACE(INVERSE(A(T,A)*H*PETH*MT+R))/2--ND*ALOG(2*3.1416)/2
C --ALOG(DET(G))/2
C A(T,A)=H*PETH+R
C DIMENSION H(12,12),R(12,12),PETH(12,12),P(12,12),DUM1(12,12),
1 MT(12,12),DUM2(12,12),ATJ(12,12)
A1=-NH*ALOG(2*3.1416)/2.
C GET A(T,A)
CALL MULT(M,NM,ND,P,ND,ND,DUM1)
CALL TRANS(M,NM,ND,MT)
CALL MULT(DUM1,NM,ND,MT,ND,NM,DUM2)
CALL ADD(DUM2,NM,NM,R,NM,NM,ATJ)
CALL DETMAT(ATJ,NM,12,DETA)
A2=-ALOG(DETA)/2.
TYPE 1,A2
FORMAT(1X,'A2=',E12.5)
C GET A3
CALL MULT(M,NM,ND,PETH,ND,ND,DUM1)
CALL MULT(DUM1,NM,ND,MT,ND,NM,DUM2)
CALL ADD(DUM2,NM,NM,R,NM,NM,DUM1)
CALL MULT(M,NM,12,12,ND)
CALL MULT(ATJ,NM,NM,DUM1,NM,NM,DUM2)
A3=TRACE(DUM2,NM)/2.
A123=A1+A2+A3
TYPE 2,A1,A2,A3,A123
FORMAT(1X,'A1=',E12.5,1X,'A2=',E12.5,1X,'A3=',E12.5,'A123=',E12.5)
SUM=SUM+A1+A2+A3
RETURN
END

SUBROUTINE TRANS(A,NA,NA,B)
DIMENSION A(12,12),B(12,12)
DO 10 I=1,NA
DO 10 J=1,NA
B(J,I)=A(I,J)
RETURN
END

SUBROUTINE ADD(A,NA,NA,B,NB,NB,C)
DIMENSION A(12,12),B(12,12),C(12,12)
IF(NA.NE.NB.OR.NA.NE.NB)TYPE 1
FORMAT(1X,'DIMENSION ERROR')
DO 2 I=1,NA
DO 2 J=1,NA
C(I,J)=A(I,J)+B(I,J)
RETURN
END

```

```

SVI0001.finc      22-DEC-84      19:15:31      PAGE 8
SUBROUTINE MINUS(A,NA,NA,B)
DIMENSION A(12,12),B(12,12)
DO 2 I=1,NA
DO 2 J=1,NA
B(I,J)=A(I,J)
RETURN
END

```

VITA

Der-Jhy Shieh was born in Taipei, Taiwan on the eighth of May in 1949. He attended the National Tsing Hua University at Hsin-Chu, where he obtained the B.S. and M.S. degrees in nuclear engineering in 1971 and 1975 respectively. During the period between 1971 and 1973, he was in the military service. After obtaining the M.S. degree, he entered the Institute of Nuclear Energy Research, working at a 40 MWt heavy water research reactor facility for two and a half years as a reactor physicist. And then after training in a commercial BWR nuclear power plant for half a year, he was transferred to the Reactor Engineering Division, simulation, dynamics and control group. In 1981, he was awarded a scholarship from INER to study in the Nuclear Engineering Department of the University of Tennessee in Knoxville, for the Ph.D. degree. He performed part of his research work at the Oak Ridge National Laboratory, Instrumentation and Controls Division in the area of reactor noise analysis.