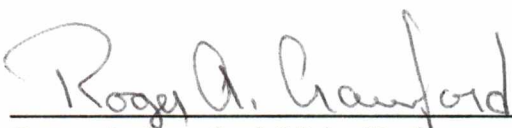
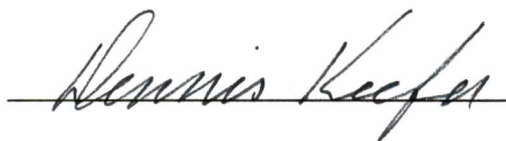


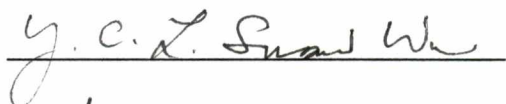
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I am submitting herewith a dissertation written by Bor-Chyuan Lin entitled "A Study of the T-layer MHD Generator/Accelerator Using Nonequilibrium Plasma." I have examined the final copy of this dissertation for form and content and recommended that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Aerospace Engineering.

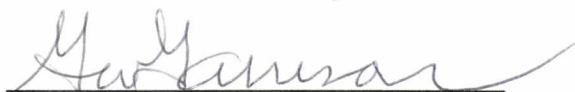

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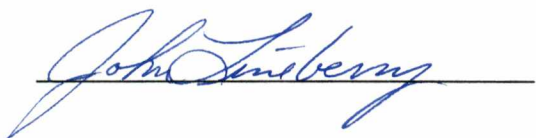
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and recommend its acceptance:











Accepted for the Council:


Associate Vice Chancellor
and Dean of The Graduate School

**A Study of the T-layer MHD Generator/Accelerator
Using Nonequilibrium Plasma**

A Dissertation

Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

Bor-Chyuan Lin

December, 1994

DEDICATION

This dissertation is dedicated to my parents,

Lin, Chung and Lin, Kuo Pao-Mei,

林鍾 林郭寶妹

my wife,

Chen, Yuh-Jiun,

陳昱君

and

other members of my family.

For their constant love and support throughout my life.

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During the years spent on completion of my study, the greatest support was provided by my wife, Yuh-Jiun Chen, and my family. Without their long-term support and encouragement, completion of this thesis would not have been possible.

ABSTRACT

This study concentrates on understanding the gasdynamics of T-layer interaction in the MHD channel, and on predicting the performance potential of MHD devices (generator and accelerator) that use the T-layer principle. The effects of applying a nonequilibrium plasma without seed in the T-layer MHD device is evaluated. Argon is used as the working fluid.

A one-dimensional unsteady model is developed to simulate the T-layer MHD process with a nonequilibrium plasma which is described by a two-temperature model. The unsteady model is coupled with a criterion for determining the onset of nonequilibrium plasma. The derived unsteady governing equations are solved numerically by a second-order high-resolution TVD (total variation diminishing) scheme. A quasi-steady model based on works of Vasilyev et al.^{19,38,44} is included in this study to provide more support to the results obtained.

Sample cases with typical operating conditions of the T-layer MHD generator as well as accelerator are examined. Reasonable consistency is seen between the results calculated by the two models. In the results, significant elevation of the electron temperature over the gas temperature is observed. The unsteady model is also verified by reported experimental results. When including the voltage drop due to the cold wall boundary, acceptable agreement is found between the calculated results and the experimental data with the exception in matching the peak value of current density.

The processes in the MHD channel coupled with a formation section are simulated with single or with multiple T-layer(s) in the channel. The results show that when using multiple T-layers the performance increases only prior to the encounters between the T-layers and the wave perturbations caused by other T-layers. This interaction between T-

layers influences the plasma properties inside the T-layers in an irregular way and subsequently causes fluctuations in the output and degrades performance of the devices.

With a monatomic or molecular gas as the working medium, performance maps based on the quasi-steady model are constructed to provide a general view of the performance potential for the T-layer generator and accelerator.

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LIST OF SYMBOLS†

<i>Symbols</i>	<i>Meaning</i>
A	Cross section of channel
$\mathbf{A}$	Jacobian matrix of the flux vector $\mathbf{F}$ defined in Equation (2.40)
A_{ji}	Einstein A coefficient for the transition from j to i
A_{th}	Cross section of the nozzle throat
$\tilde{A}_{ex}$	Normalized cross section at the nozzle exit
$\tilde{A}_{in}$	Normalized cross section at the nozzle inlet
b_{ij}	Coefficient for collision cross section from state i to j in Equation (3.9)
b_0	Coulomb collision impact parameter for a 90° deflection angle
B	Magnetic induction
$\vec{B}$	Vector of magnetic field
c	Speed of sound
c_p	Specific heat ratio of constant pressure
e	Total energy per unit volume of fluid, electron charge
E	Electrical field measured in laboratory coordinates
E_{cr}	Critical electrical field for onset of nonequilibrium plasma
E_y	Electrical field in y -direction measured in laboratory coordinates
$f(\epsilon_e)$	Electron energy distribution function
$\mathbf{F}$	Flux vector defined in Equation (2.33)
$\tilde{\mathbf{F}}$	Numerical flux vector
g_j	Statistical weight of energy level j

† SI Units are used throughout this work unless other specified.

<i>Symbols</i>	Meaning
G	Normalization factor for tail electron energy distribution in Equation (3.2)
h	Enthalpy per unit mass of fluid, Plank's constant ($= 6.6261 \times 10^{-34}$ Joules/s)
h_g	Enthalpy per unit mass of heavy species
h_λ^o	Heat of formation of ion
H	Total enthalpy per unit mass
I	Total current
$\mathbf{I}$	Unity matrix
J	Current density
$\bar{\mathbf{J}}$	Vector of current density
J_y	Current density in y-direction
k	Boltzmann's constant ($= 1.38047 \times 10^{-23}$ Joules/°K)
k_p	Absorption coefficient at the center of resonance line
K	Load factor of MHD device
K_a	Load factor of accelerator
K_f	Rate coefficient for ionization
K_{ij}	Rate coefficient for transition from state i to j
K_g	Load factor of generator
K_r	Rate coefficient for recombination
l	Channel height
L	A characteristic length, channel length
$\mathbf{L}$	Operator defined in Equations (2.35) and (2.36)
m	Mass flux
$\dot{m}$	Mass flow rate
m_e	Electron mass

<i>Symbols</i>	Meaning
m_j	Mass of particle j
n_{Ar}	Number density of neutrals
n_e	Electron number density
$n_{e,cr}$	Critical electron density for onset of nonequilibrium plasma
n_{e0}	Electron number density found from Saha equilibrium at T_{e0}
$n_{e\infty}$	Electron number density found from Saha equilibrium at $T_{e\infty}$
n_g	Number density of heavy species
n_j	Number density of particles at state j
n_λ	Number density of ions
p	Pressure
p_e	Partial pressure of electrons
p_g	Partial pressure of heavy species
p_i	Pressure of the initial uniform T-layer plasma
p_{t0}	Stagnation pressure
P_{exc}	Power density lost by electron due to excitation collisions
P_{ij}	Power density of electron due to transition from state i to j
P_{inel}	Power density lost by electron due to inelastic collisions
P_{ion}	Power density lost by electron due to ionization
P_J	Power density due to Joule heating
P_{rad}	Power density lost by electron due to radiation
P_{rec}	Power density gained by electron due to recombination collisions
P_{sel}	Power density gained by electron due to de-excitation collisions
q_{ion}	Power density of ionization/recombination
q_{rad}	Total radiative power loss per unit volume

<i>Symbols</i>	Meaning
$\tilde{q}_{rad}$	Total radiative power loss per unit volume per unit solid angle
Q_{ij}	Collision cross section for transition from state i to j
Q_m	Electron-neutral momentum transfer collision cross section
r_p	Half width of the resonance broadening
R	Gas constant
$\mathbf{R}$	Vector of source terms defined in Equation (2.33)
R_{eff}	Effective resistance of the plasma in the MHD channel
R_L	External load of generator
R_h	Ratio of magnetic and gasdynamic pressures
R_m	Magnetic Reynolds number
$R_m R_h$ or R_M	MHD interaction parameter
R_M^*	Critical MHD interaction parameter in Equation (1.2)
R_p	Internal impedance of plasma in the MHD channel
s	Non-dimensional Lagrangian coordinates
S	Saha equilibrium coefficient
t	Time
T	Temperature
T_e	Electron temperature
T_g	Gas temperature, temperature of heavy species
T_{gi}	Gas temperature in the initial uniform T-layer plasma
T_t	Temperature of tail electrons
$T_{\theta 0}$	Stagnation temperature
u	Flow velocity
$\tilde{u}$	Normalized velocity ($= u/u_t$), reduced or enhanced velocity

<i>Symbols</i>	<i>Meaning</i>
$\bar{\mathbf{u}}$	Velocity vector
$\mathbf{U}$	Vector for conservative properties defined in Equation (2.33)
$\mathbf{v}$	Velocity of T-layer in a frame moving with $\mathbf{u}_l$, $v = \mathbf{u} - \mathbf{u}_l $
v_e	Electron thermal velocity
V	Volume
w	Channel width, speed of compression wave
x, y, z	Space coordinates
β_{ji}	Radiative escape factor for transition from state j to i
γ	Specific heat ratio
δ	T-layer thickness
δ_0	Initial T-layer thickness
$\Delta\tilde{p}$	Normalized pressure drop across the T-layer, $\Delta\tilde{p} = (p_2 - p_3)/p_{i0}$
Δt	Magnitude of time step
ΔV_d	Voltage drop across boundary layer
Δx	Size of grid
$\Delta\epsilon$	Lowering of ionization potential
ϵ_{app}	Externally applied voltage
ϵ_e	Electron energy
$\epsilon_{e,mp}$	Electron energy for the most probable distribution
ϵ_i	Effective ionization potential
ϵ_{ij}	Potential difference between energy level i and j
ϵ_0	Permittivity of free space ($= 8.8542 \times 10^{-12}$ farad/m)
$\epsilon_{o\lambda}$	Ionization potential
$\epsilon_{\bar{\mathbf{u}} \times \bar{\mathbf{B}}}$	Electromotive force induced by $\bar{\mathbf{u}} \times \bar{\mathbf{B}}$ effect

<i>Symbols</i>	<i>Meaning</i>
η_a	Enthalpy additional ratio of accelerator
η_N	Enthalpy extraction ratio of generator
κ	A normalized velocity defined in Equation (4.17)
λ_D	Debye length
λ_0	Wave length of the resonance line
Λ	Quantity in the expression for Coulomb collisions in Equation (3.47)
μ_0	Permeability of free space ($= 4\pi \times 10^{-7}$ henry/m)
ν_e	Collision frequency between one electron with neutrals and ions
ν_{e-j}	Collision frequency between one electron with species j
ρ	Mass density
ρ_λ	Mass density of ions
σ	Electrical conductivity
σ_{eff}	Effective conductivity of the MHD channel
τ	$\Delta t / \Delta x$
τ_F	Characteristic time for momentum (force) balance
τ_{ion}	Characteristic time for ionization/recombination
τ_{Te}	Characteristic time for electron energy balance
τ_{Tg}	Characteristic time for heating heavy species

<i>Superscripts</i>	<i>Meaning</i>
b	Bulk electrons
t	Tail electrons
$\dot{x}$	Time rate change of property x

<i>Subscripts</i>	Meaning
Ar	Neutrals
Ar^+	Ion species
e	Electron
g	Heavy species
th	Nozzle throat
$r0$	Stagnation condition
0	Initial value, ground level
1	Unperturbed flow property, first excited level
2	Upstream end of T-layer in generator (downstream end in accelerator)
3	Downstream end of T-layer in generator (upstream end in accelerator)
λ	Ion species, ionized state

CHAPTER 1

INTRODUCTION

1.1 Motivation for T-layer Research

In a magnetohydrodynamic (MHD) generator, a plasma is used as working medium which, when flowing through a magnetic field (externally applied or self-induced), induces an electrical field transverse to both the flow velocity and the magnetic field. The current associated with the induced electrical field interacts with the magnetic field and, due to the Lorentz force effect, a force is exerted on the flowing conducting gas to retard the speed of plasma flow. The net effect of the MHD interaction in a generator is to convert flow kinetic energy into electrical power.

In a conventional MHD generator, the plasma fills the whole volume of the channel. During the operation of the MHD generator, uniform plasma properties are usually desired throughout the channel. This is due to the fact that when current traverses nonuniformities in the plasma, the average internal impedance of the plasma increases, which subsequently degrades the performance of the generator.¹

On the other hand, a T-layer MHD generator operates with intentionally created stratified, high-temperature conducting layers. (The term "T-layer" is an acronym for "high temperature current layer".) This concept was first proposed by Ricateau and Zettwoog in 1963,² and Fraidenraich, Medin, and Thring in 1964.³ Since that time, the T-layer concept has become a branch of the MHD power generation program in the former Soviet Union and now continues in Russia. Extensive T-layer investigations encompassing theoretical, experimental, and numerical studies have been conducted by

Russian scientists. More recently, an international cooperation program on the subject has been established among Russia, Italy, and the Netherlands.⁴

Why is the T-layer concept attracting attention in the MHD community? This answer can be arrived at by considering the recent turndown of traditional MHD research almost worldwide. The development of the conventional MHD generator has been hampered by two major difficulties: the lack of appropriate heat sources and the thermal stress on the channel materials. Both of these limit the availability of high temperature working medium. In the open-cycle MHD system that uses combustion gas seeded with alkali compounds, the electrical conductivity of the plasma is closely related to the gas temperature and is normally limited to 10 Siemens/m due to the combustion flame temperature that is achievable.

On the other hand, high conductivity (10^3 – 10^4 Siemens/m) and high performance (enthalpy extraction ratio ~38%) have been experimentally demonstrated using the closed-cycle MHD concept with inert gas plasmas seeded with alkali metals.⁵ Nevertheless, this high conductivity plasma has only been obtained in a shock tube facility for short duration (a few milliseconds). The lack of technologies to continuously supply a low impurity inert gas with temperature around 2000 °K has long been recognized as the major hurdle for the potential commercial application of the close-cycle MHD power generation.

When using the T-layer concept, only a small portion (~5-15%) of the plasma volume is heated to high temperature (~ 10^4 °K). These hot layers are arranged to be intermittently surrounded by cold gas where the conductivity is extremely low. The high temperature zones provide the necessary high conducting current path required for inducing significant MHD interactions. The channel walls are only heated during the short period when the current layer passes and, as a result, the thermal loads on the walls of the channel are lower and the operating lifetime is longer.

Heating sources for periodically heating small volume of the T-layer to a high temperature are considered to be technically achievable. Some examples of potential choices are applied electrical gas discharges (corona, arc,...), high-intensity (explosive) combustion, microwaves, etc.

Therefore, the T-layer MHD generator is expected to be a device with potential to overcome the two currently insurmountable difficulties met in the traditional scheme of MHD power generation.

Since only a small portion of the gas in the channel is heated, the next question is whether or not a T-layer MHD generator can produce a power level comparable to that of the traditional MHD generator. This question can be roughly answered through an examination of the basic power relation for the MHD power generation, i.e.,

$$Power \propto \int \sigma u^2 B^2 dV \quad (1.1)$$

where integration is taken over the entire volume of channel. For most of the gases used as working medium, the electrical conductivity increases exponentially with temperature in the range of 3000-8000 °K. Thus, for the same amount of heat available, heating only a certain percentage of volume of the working gas to a temperature above 8000 °K and leaving the other portion cold, the integral of the multiplication of the resultant electrical conductivity and the heated volume might be higher than its value in the case of heating the whole volume of gas evenly to a lower temperature. In this situation, according to relation (1.1), the former case can produce a higher power output by raising its effective plasma conductivity if the flow speed and magnetic field strength remain the same in both cases.

That is, if the relationship between the plasma temperature and the electrical conductivity is known, it is possible to increase power output by proper control of the

percentage of gas heated for the same amount of heat available. An estimate by Lyubochko et al.⁶ showed that the power density (per unit volume of channel) of the T-layer generator can be twice the optimum power density of the conventional Faraday MHD generator.

The possibility of providing higher power was the original motivation in examining the T-layer MHD concept in the early 1960's. However, the conclusion obtained above through a simple estimate from the basic power relation is not necessarily true when other factors which influence the properties of T-layer plasma in a non-linear fashion are taken into consideration. Examples of such factors are electrothermal instabilities, which might determine the temperature of the hot plasma; the stabilized size of T-layer, which can only be decided by nonlinear momentum and energy balances in the MHD process; and, the interactions between T-layers when multiple T-layers simultaneously exist in the MHD channel. However, when considering the potential advantages of using the T-layer concept as well as other special features to be discussed in Section 1.3, the T-layer MHD concept is without doubt worthy of further investigation.

1.2 Basic Principles of the T-layer Concept

The operation of a typical MHD generator and/or MHD accelerator with T-layers can be qualitatively explained as shown schematically in Figures 1.1 and 1.2 respectively. The process is initiated by a T-layer formation mechanism, such as the electrical gas discharge shown, by providing a burst of thermal energy which locally heats and ionizes the flow stream. The dominant factors influencing the volume and temperature of the plasma being formed are the heating time, the flow velocity, and the thermal input power. The final volume of the heated plasma is determined by two coupled processes: the initial volume of plasma associated with the heating time, in conjunction with, the

resident time of the fluid as it moves through the heating zone and its subsequent expansion due to a rapidly increasing pressure inside the plasma as a result of intensive heat supplied.

Due to the existence of electrothermal instabilities (ionization or static instability),⁷ the electron temperature in the hot layer is usually in a range around 10^4 °K if no seeding components are added. Whereas when seed is used, the plasma can stabilize in a temperature range close to that of full seed ionization. These stabilized ranges for temperatures correspond to the plateau part on a curve describing the relation between electrical conductivity and electron temperature of a plasma. Through strong ionization of the working medium, the high temperature layer provides a highly conductive zone. This finite volume of high temperature, high conductivity plasma formed in this fashion is referred to as the T-layer.

The T-layer formed is carried by the flow into the downstream MHD channel. Right after it enters the MHD channel, the T-layer plasma will adjust its properties in order to reach a new balance of momentum and energy through MHD interaction. In the flow stream of the MHD channel, there are alternating layers of highly conductive hot plasma surrounded by cold gas. In principle, the effect caused by the existence of the T-layer on the fluid dynamics of the flow can be approximately evaluated by treating the T-layer as a deformable piston that moves with the local speed of the flow.

The MHD processes in a T-layer device are similar to those of a conventional, continuous plasma flow MHD device. That is, an electromotive force is induced by the motion of the T-layer relative to an externally applied transverse magnetic field in accordance with Faraday's law of electromotive induction. In a generator, the induced electric field drives electric current through the T-layer. The current interacts with the magnetic field to produce a $\vec{J} \times \vec{B}$ Lorentz force which retards the T-layer piston within the flow. As a result, a pressure gradient builds up across the T-layer to counter the

Lorentz force. And, relative to the T-layer, a compression wave propagates upstream and a rarefaction (expansion) wave propagates downstream. In the case of an MHD accelerator, a voltage with a magnitude greater than the induced electromotive force is externally applied to induce a current in the direction opposed to the induced electrical field. The Lorentz force is vectored to accelerate the T-layer downstream and an excited compression wave moves downstream while a rarefaction wave expands upstream, both relative to the piston.

In the T-layer MHD device the role of the electrical conductor is played not by the entire volume of the gas as in the conventional MHD device but rather by the small volume of the T-layer. The remaining gas is simply a fluid, working against the electrodynamic force induced by T-layer's MHD interaction. The conversion of energy into different forms is realized by the interaction of flow work. The flow retardation in the generator converts part of the flow work done against the fluid motion into electrical energy and dissipates the other portion into heat, i.e., Joule heating. In the accelerator, the externally input electrical energy is partly converted to flow kinetic energy and the remaining is dissipated as heat. In both devices, Joule heating helps sustain the T-layer. Therefore, the T-layers can remain active inside the channel even though it is surrounded by a cold gas.

The T-layer MHD generator concept was demonstrated as workable in the former Soviet laboratory experiments.^{8,9,10} In these experiments, the validity of considering the T-layer as a piston was verified when using air as the working medium. The existence of the compression wave following the T-layer and the expansion wave preceding it were also experimentally observed.¹¹

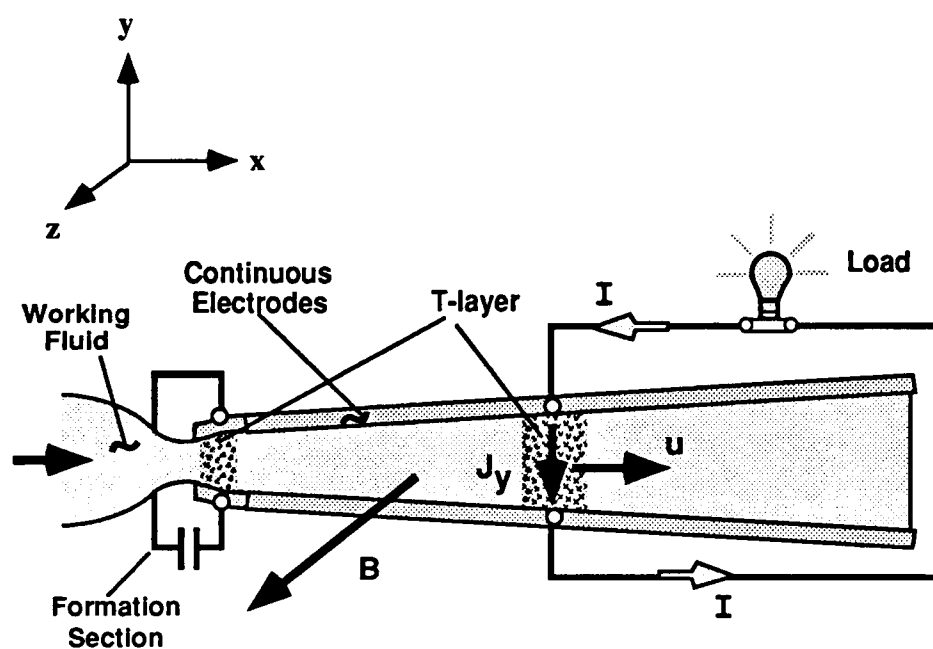


Figure 1.1 Schematic of the T-layer MHD Generator Concept

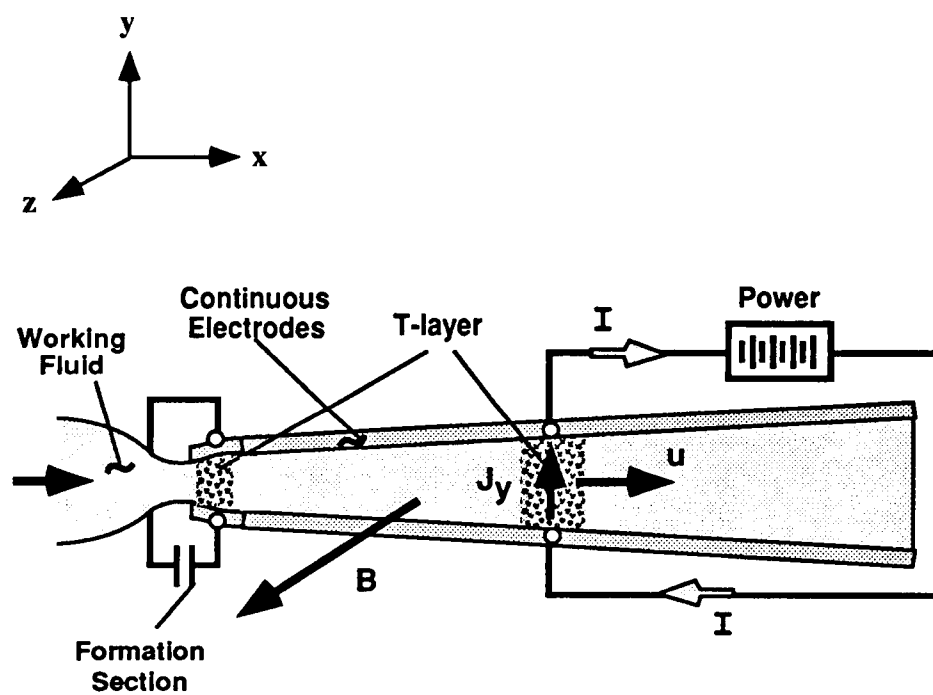


Figure 1.2 Schematic of the T-layer MHD Accelerator Concept

1.3 Features of T-layer Devices

Besides the advantages of the availability of heat sources, low thermal stress, and high power density described in Section 1.1, there are other advantageous features of the T-layer concept which make it more attractive when compared to a conventional continuous plasma flow MHD devices. Some of the more obvious of these are summarized below.

First of all, conventional MHD designs are limited in the degree of ionization that can be achieved. For example, in combustion driven MHD systems plasma conductivity is normally derived by use of seeding of combustion gases with alkali metals (potassium, cesium compounds). The best of the combustion systems, those which used highly energetic fuels such as aluminum, can be expected to deliver a working fluid with conductivity on the order of 10^2 Siemens/m.¹² Whereas, with plasma temperature on the order of 10^4 °K in the T-layer, the ionization of working fluid itself is sufficient to produce a high conductivity in the 10^3 Siemens/m range, which is an order of magnitude greater than the best that seeded combustion driven systems can achieve. Thus, with the T-layer concept, alkali seed addition to promote ionization is not absolutely required. The omission of chemically active seeding can yield longer lifetime of the MHD channel and reduce the tedious seed recovery process which is necessary not only for economic saving from recycling seeding components but also for environmental pollution control.

Secondly, the working gas temperature can be lower in a T-layer device since electrical conductivity in the T-layer is only weakly dependent on this temperature. The relaxation of the necessity of high gas temperature as that required in the conventional MHD device provides the T-layer device more flexibility in thermal cycle analysis when designing a thermal engine with an MHD device as a component.

Thirdly, since the T-layer is a discrete geometric layer, physically bound by the relatively cold gas acting as electrical insulator, Hall currents are ideally eliminated even when continuous electrodes are used. Therefore, the geometry of the electrodes of T-layer devices can be much simplified. The elimination of large scale Hall currents helps limit the scale of Velikhov ionization instability,⁷ which might increase the average internal impedance of the plasma.

Finally, a T-layer MHD generator has the potential to directly produce alternating currents when the electromotive force produced is tapped externally in the fashion shown in Figure 1.3.

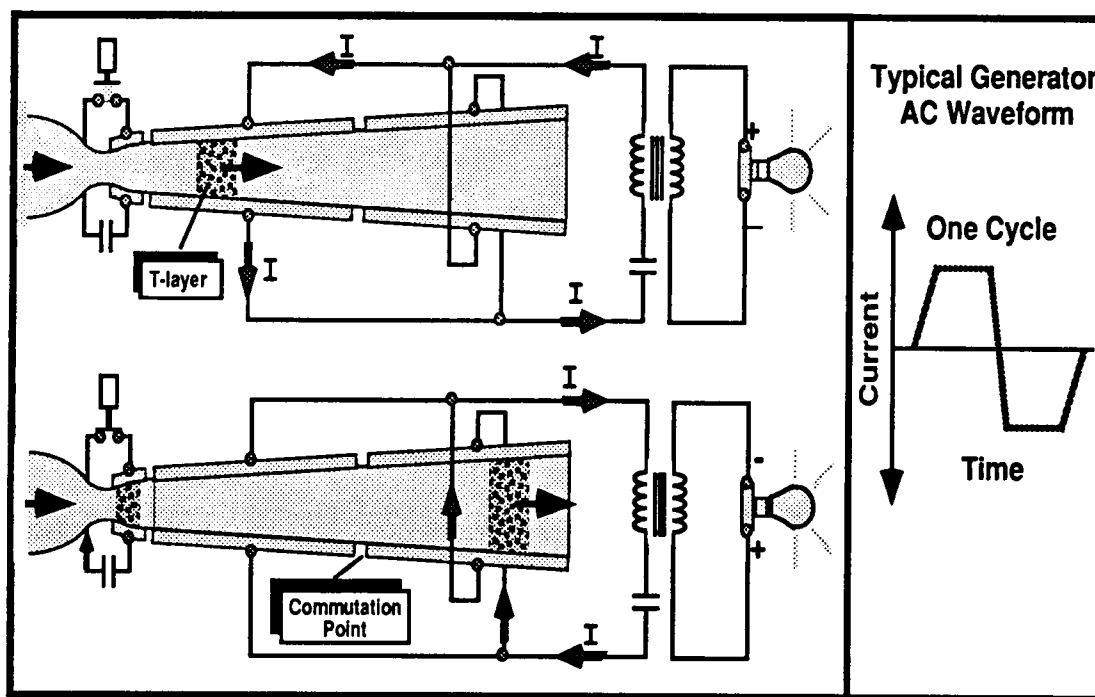


Figure 1.3 Illustration of T-layer MHD AC Power Production

On the other hand, there are some disadvantages to the T-layer generator concept. For example, the T-layer is subject to large radiative losses due to its high temperature. In addition to this, other problems/disadvantages for the T-layer MHD generator can be cited, including,

- a large power supply is needed for the initiation and formation of T-layers;
- a lowering of isentropic efficiency from the irreversible losses associated with compression waves following the T-layers;
- relatively large electrode potential drop (~200-300 Volts) due to cold wall boundary layers can exist;^{9, 10}
- the T-layer is susceptible to hydromagnetic instabilities, e.g., Rayleigh-Taylor type.

The Rayleigh-Taylor instability has been observed experimentally¹¹ and numerically¹³ in a T-layer with an equilibrium plasma. These observations showed that this kind of instability does exist in the T-layer MHD channel but if the T-layer is thick enough (for example, comparable to its width^{22,36,37}), this instability may be stabilized by the existence of some non-linear dissipation mechanisms, such as thermal conduction, Joule dissipation, and radiative loss. However, detailed knowledge of the stabilizing mechanism is still open for further study.

1.4 Literature Review

The T-layer MHD concept has been a subject of MHD research in the USSR for three decades. Although this concept has been theoretically identified as being a viable means for MHD power generation, experimental data has been insufficient to confirm the theoretical results. Up to now, all the research in the field concentrated on T-layer MHD power generation, and no previous study is known to relate to the T-layer application in MHD flow acceleration.

1.4.1 Early T-layer Research

Operating an MHD generator with striated inhomogeneous flow was initially investigated in the early 1960's.^{2,3} An important conclusion of these early works was that high temperature layers could not survive in the MHD channel due to instabilities of Rayleigh-Taylor type. This conclusion was reached after the characteristic time for the growth of instability was examined by a linear analysis in which too many assumptions were made to simplify the problem.

During the late 60's and mid-70's, T-layer research concentrated mainly on theoretical and experimental observations of the processes that occurred in the formation of high temperature, current layers in the MHD channel.^{11,14,15,16} The criteria for development of the T-layer from a thermal/ionization disturbance was identified as¹⁵

$$R_M = R_m R_h \geq R_M^* \quad (1.2)$$

That is, the MHD interaction parameter, R_M , has to be greater than a critical value for the disturbance to grow. Magnetic Reynolds number, $R_m (= \mu_o \sigma_o u_o L_o)$, is the ratio of MHD force to magnetic pressure and $R_h (= B_o^2 / 2\mu_o p_o)$ is the ratio between magnetic and gasdynamic pressures. The critical MHD interaction parameter, R_M^* , is an experimentally determined value which is dependent on the channel geometry and the nature of flow. In summary, if a local disturbance in temperature or in electrical conductivity occurs such that the electrical conductivity σ_o , flow velocity u_o , characteristic length L_o , pressure p_o , and magnetic field B_o satisfy the inequality (1.2), then the disturbance will grow into a T-layer; otherwise, it will be damped.

The above theory was verified by the experimental work of Kerkis et al.¹¹ In the same work, it was also observed that perturbations at the outer boundary of the hot plasma layer appeared which was identified as the Rayleigh-Taylor instability. This instability grew to a certain portion of the layer width but has never been observed in experiment to collapse the current layer - an observation that is consistent with the conclusion obtained by a recent two-dimensional numerical simulation.¹³

1.4.2 T-layer Using Equilibrium Plasma without Seed

The working media observed during the early T-layer research was primarily a plasma of air or combustion products. The T-layer plasma was in thermal equilibrium, i.e., the electron temperature equal to the temperature of heavy species (ions and neutrals). In this type of T-layer plasma, the equilibrium temperature is dominated by counter-balancing of radiative loss and Joule dissipation.

Following the development of extended computer capability, many computational studies on the T-layer concept have been conducted since mid-70's. Among them, two main approaches used to simulate a T-layer with an equilibrium plasma were a local analysis^{17,18,19,44} and an unsteady one-dimensional model.^{18,20,21} The former was developed to model a single stabilized T-layer by treating the T-layer as an impermeable piston. The latter was applied to simulate the unsteady fluid dynamics with interaction of a series of current layers moving with the flow in the MHD generator. The results of both schemes were compared with the available experimental data and reasonable agreement was reported.^{9,10}

The results showed that due to the highly concentrated, highly conductive plasma in the T-layers, the MHD interaction was strongly localized and a strong compression wave formed behind each T-layer. The irreversible losses associated with the shock

wave were large and thus could severely degrade the performance of the generator. Also observed was that only a certain number of current layers could simultaneously exist in the MHD channel owing to the shortage of Joule dissipation to compensate the energy losses (mainly radiative loss). As a consequence, extra current layers decay.

The enthalpy extraction ratio of the T-layer generator using an equilibrium plasma without seed in a typical case observed by Bozhkov et al²¹ (plasma temperature $\approx 10^4$ °K, stagnation gas temperature ≈ 3000 °K, stagnation pressure ≈ 1 MPa, Mach no. ≈ 1.2 , and magnetic field ≈ 2 T) was about 12% and the isentropic efficiency was less than 50%. Both are below the performance required for a competitively efficient MHD generator (enthalpy extraction ratio $\geq 20\%$, isentropic efficiency $\geq 70\%$).

Some other experiments with T-layers using an equilibrium plasma without seed were reported.^{8,9,10,18} In these experiments it was concluded that under the operating conditions for regular MHD power generation with air as working gas, a stabilized T-layer could be formed in the MHD channel after it was initiated upstream. The stabilized T-layer had characteristics qualitatively consistent with theoretical and numerical predictions, i.e., the plasma temperature is not uniformly distributed inside the T-layer and the current density in the T-layer decayed in the compressive region due to a lowering of plasma temperature resulting from a higher radiative loss in the compressive region.

The experiments also verified that radiative loss was the dominant mechanism of energy transfer/loss. Also, it was noted from the experiments that a large voltage drop in the cold boundary layer near the electrodes greatly influenced the performance of the generator. The cold boundary layer effect was critiqued as being the cause for failure in some tests.⁸

Most research works related to T-layer generator using equilibrium plasma without seed were examined from mid 70's to late 80's in the USSR. The final results were

reported by Slavin²², in which this type of T-layer generator was concluded to be inefficient. A simple but more comprehensive parametric analysis regarding the inefficiency of the equilibrium T-layer generator was conducted in a later work of Slavin et al.³⁶. The drawback in efficiency was thought to be mainly related to the relatively high current density ($\geq 10^6$ A/m²) inside the equilibrium T-layer. This effect increased the balanced pressure drop across a single layer and limited the number of current layers that could concurrently exist in the channel; while on the other hand, it enhanced the strength of shock losses and augmented radiative energy escape. The high current density observed was a consequence of the electrothermal instability existing in this type of T-layer plasma.

Based on these studies, it was concluded that alternative types of T-layer generators needed to be researched. Two major alternatives that have been identified are the T-layer that uses an equilibrium plasma with seed and the T-layer with nonequilibrium plasma.

1.4.3 T-layer Using Equilibrium Plasma with Seed

Since the late 1980's, Bityurin et al²³⁻²⁶ of the Institute for High Temperature, the Russian Academy of Sciences, have theoretically demonstrated that high performance is possible in a T-layer MHD generator using an equilibrium plasma with seed. Current layers of this type in the MHD channel were named as Space- and Time-dependent Current Carrying (STCC) nonuniformities in order to show the differences in plasma temperature between the current layers with and without seed. More recently, the same group has cooperated with scientists from Italy and the Netherlands and triggered a new research project in which both numerical and experimental investigations in STCC would be included.^{4,27-32}

The key modification in this new type of stratified current layer in the MHD concept is the addition of seed to the working gas. This provides a possibility to operate the generator using current layers with a plasma temperature at a much lower level (~ 3500 °K) - a temperature level corresponding to a seed fully ionized condition. Under full seed ionization, not only can the required high conductivity be obtained through ionization of the seed atoms but also the electrothermal instability can be eliminated and the plasma in the current layers can be stabilized. By simultaneously operating the channel with multiple current layers of this kind, the MHD interaction is more evenly distributed among various current layers. Consequently, the strength of the shock waves and the degree of irreversible loss are reduced.

Numerical simulations revealed very promising performance of the MHD generator with STCC. The results showed that an enthalpy extraction ratio of 35-40% could be achieved when the stagnation temperature ranged between 1600 to 2000 °K, and an isentropic efficiency of 75-80% could be achieved.²⁵ Based on the high performance of the MHD generator, an overall efficiency of 50-55% was estimated when applied to a coal combustion MHD power plant.

Some experiments of STCC concept using a combustion gas were carried out in a shock tube facility.^{28,30} Significant current increments were measured throughout the channel when the current layers passed through the electrodes. This provided evidence that a current layer of this type could survive in the MHD channel for a long traversing distance. From the experimental results, it was determined that the cold electrode boundary layer was the dominant unfavorable factor for the decrease of current density in the current layers and the collapse of the current layers when the gas temperature was below 2000 °K. Deformation of the current layer along the MHD channel was also observed³⁰ and discussed.³² However, the high performance of the STCC concept

obtained from theoretical and numerical studies is still subject to further experimental verification.

1.4.4 T-layer with Nonequilibrium Plasma

Even though T-layer research using a nonequilibrium plasma existed^{33,34}, this sub-field of the T-layer concept gained more attention after the late 1980's.³⁵⁻³⁹ The current research on this kind of T-layer for MHD power generation in Russia has mainly concentrated on its application to a Faraday-type disk generator.^{22,37,39}

A nonequilibrium plasma is characterized as a plasma in which the electron temperature is different from the temperature of heavy species including neutrals and ions. This is mainly due to the large disparity between the masses of the electrons and heavy particles. In a nonequilibrium plasma, only a very small portion of the mass - the electronic gas - needs to be heated to a high temperature to reach the necessary ionization level of the working gas. Since the electrical conductivity and electron density are principally determined by the electron temperature in a nonequilibrium plasma, the energy input required in the formation of a nonequilibrium T-layer may be substantially reduced.

On the other hand, an additional energy source for the electron gas is needed to balance the energy exchange that occurs from electron-neutral and electron-ion collisions. Thus, besides the fundamental energy losses for both types of plasmas (equilibrium or nonequilibrium) more Joule dissipation is required for a nonequilibrium T-layer to compensate for the collision loss, which in many cases dominates the whole loss mechanism of the nonequilibrium process.

A noble gas is generally accepted as the best working medium for forming a nonequilibrium plasma owing to its small collision cross section for momentum transfer

when colliding with electrons. Also, the radiation from this atomic gas is usually lower in the temperature range observed in the T-layer plasma (≤ 1.5 eV).

In a numerical study of Slavin et al.³⁶, a quasi-one dimensional model was developed to formulate a nonequilibrium plasma with pure argon gas in a linear Faraday type MHD generator. An enthalpy extraction ratio of 25% and an isentropic efficiency of 72% were estimated when steady operation of the generator was reached. In this device, five current layers were driven simultaneously in the channel, the electron temperature in the T-layer was stabilized at around 7000 °K, the electrical conductivity ranged 500 - 700 Siemens/m, and a mean current density about 1×10^5 A/m² when a magnetic field of 4 T was applied. In a more recent work performed by the same group³⁷, it was found that the enthalpy extraction ratio could be increased to 40-50% when a similar type of T-layer was applied in a Faraday type disk generator. In both cases, the flow Mach number was kept low (~ 1.2) to avoid strong shock loss. At the same time, the stagnation pressure was chosen to be relatively small ($\sim 3 \times 10^5$ Pa) to maintain a low collision frequency between electrons and heavy particles in order to get the prominent nonequilibrium effect.

The numerical results that were obtained in the above works implied that a definite potential existed to achieve high performance in a disk MHD generator of Faraday type using the nonequilibrium T-layer concept. Experimental verification of this application is still at a very early stage. Katsnel'son³⁹ reported an experimental observation of the dynamics of forming a current layer in different gas flows including air, argon, helium, and sodium plasma inside a disk shape faraday generator. Both inductive and conductive power to the external load were collected with argon as the working gas in the experiment. However, no specific data associated with generator efficiency was given.

In the another numerical study of Vasilyev et al.³⁸, it was reported that using argon a higher efficiency could be obtained in an MHD process when operating the T-layer

generator with equilibrium plasma than that with nonequilibrium plasma even though the expense for initiating an equilibrium T-layer is higher. This conclusion is questionable. With a pressure equal to or less than the atmospheric pressure, the equilibrium argon plasma can be approached only when the plasma is hot enough to reach a high degree of ionization such that the collisions between electron and heavy species are dominated by Coulomb collisions. With such high temperature and high degree of ionization under a regular T-layer operating condition, the conductivity and the induced current density inside the T-layer must be high, and a strong shock loss can not be avoided. It is believed that the conditions for stabilized plasma, i.e., Equations 10-13 of Reference 38, were inappropriate.

1.5 The Present Work - General Assumptions

Although the T-layer concept has been considered as being a potent alternative in the field of MHD power production for years, there is no precedent for research of this type in the US. After the completion of proof-of concept of open-cycle coal combustion MHD program in this country, this is the moment to enlighten the research interest among the MHD community in this country with other alternative and efficient schemes for the MHD power generation. The T-layer concept is believed to be one of the best potential candidates.

The potential advantages of applying the nonequilibrium plasma in T-layer MHD power generation are studied herein. To achieve this goal, an unsteady quasi-one-dimensional model is formulated and developed to simulate the T-layer forming process and its interaction in the MHD channel. Results from this unsteady model are compared with available experimental data when possible. Also, more verification is made through

the comparison of results obtained from the other model - a quasi-steady model based on the local analysis of Vasilyev and Ovchinnikov et al.^{17,19,38}

A study of the possibility of applying the T-layer concept to flow acceleration is also given herein. The ability to simulate re-entry flight conditions (Mach number ~ 30) in a ground test facility has been a long-term dream among scientists in the field of hypersonic research. Due to the constraints of thermal materials, the most recent wind tunnel technology using arc heaters is only capable of building a wind tunnel with Mach number no greater than 12. Some other means must be sought.

It has been proposed by the MHD community to augment the arc heater tunnel with an MHD accelerator.^{40,41,42,43} When using the traditional MHD concept in flow acceleration, seeding of the plasma with alkali metals is used to obtain the level of electrical conductivity needed for the working medium. The necessity of seeding the plasma results in a major shortcoming for the MHD acceleration as a ground simulation facility, that is, a "true air" condition can never be realized. Only a small amount of seed is needed for a MHD device to accelerate the flow to the desired speed. However, the existence of seed elements, which are extremely chemically active, will result in certain chemical reactions which no similarity rule can theoretically correct for. Or, it is possible for the seeding components to interact with the surface of the flight vehicle, change its property, and thus perturb the flow field.

Here is the application which the T-layer concept can fit into. Since seeding the plasma is unnecessary when using the T-layer concept, it makes a "true air" condition realizable with a T-layer accelerator. Therefore, the T-layer application in flow acceleration is also examined in the present study. Even though the major working medium used in this study is different from air, the results are still helpful in understanding the gasdynamics in the MHD acceleration using T-layers, including the flow field formed by the T-layer MHD effects and by the interaction between different T-

layers. The performance potential of T-layer flow acceleration using air/combustion gases is examined by construction of the performance map in Chapter 5.

Argon is selected as the working gas. This is due to its relatively lower heat capacity and a low electron collision cross section for momentum transfer which makes it susceptible to sustain a nonequilibrium plasma. Also, the argon plasma has been widely studied in arc research and its properties are well known and can easily be found from the literature.

Compared to a conventional MHD process that uses a nonequilibrium plasma, there are more parameters to be considered in the nonequilibrium T-layer concept, such as the size and the number of T-layer that can co-exist in the channel. Although numerical modeling allows parametric studies, finding the optimum operating conditions is not the immediate goal of the present study. Rather, more emphasis is on the inclusion of the important and correct physical effects into the model in order to understand the fundamental physics and the key mechanisms.

Description of the modeling and its limitations are given through presentation of the major assumptions upon which the theory is based. These are summarized here.

- The flow is treated as one-dimensional, inviscid, and compressible.
- Except for electrons, all species are in thermal equilibrium, e.g., two-temperature model is used.
- The electrodes are geometrically continuous and span the width of the channel. A single load is connected to a pair of electrodes in the generator case and a single power supply is applied to the electrode pair in an accelerator and in the T-layer formation section.
- Except for ionization/recombination, the working gas is ideal, monatomic, with constant specific heat ratio ($\gamma = 1.667$) and with negligible atomic excitation energy. The collisions between electrons and heavy particles are assumed to be elastic.

(Electron transition between energy levels will only be considered in the modeling for finding the condition for onset of the nonequilibrium plasma.)

- Only a singly ionized plasma is considered. (For argon, this condition is usually valid when electron temperature is below 1.5 eV.)
- Magnetic Reynolds number is small, i.e., no consideration is given to the induced magnetic field.
- Hall current is negligible in the T-layer due to a relatively small Hall parameter.
- The plasma is assumed to be locally quasi-neutral ($n_e \equiv n_\lambda$).
- The Rayleigh-Taylor type instability is disregarded by maintaining the length of T-layer at least compatible to its width.^{22,36,37}
- The ionization/recombination of particles is dominated by thermal effects. For recombination, only three-body recombination is considered.
- Heat conduction is negligible in the energy balance. Radiation is treated as the major energy loss mechanism.^{33,44}
- T-layers fill the whole cross section of the MHD channel.

Also, due to a lack of appropriate modeling for the complicated gas discharge phenomena existing on the plasma-electrode contact surface, the electrical effect of the cold boundary layer is approximated by a constant voltage drop through the whole channel length in the model.

CHAPTER 2

UNSTEADY MODELING

Motion of the T-layer in the MHD channel is a highly unsteady process. Compression waves as well as rarefaction waves are expected to exist in the channel due to the perturbations of the flow caused by T-layer MHD interaction. To describe these dynamic phenomena in the T-layer flow, any model must be time-dependent.

The unsteady model developed herein includes not only the gasdynamics equations with MHD effects as used in describing a traditional MHD flow but also coupling of the model with a condition for the onset of nonequilibrium plasma as suggested by Slavin.²² This criterion for the onset of nonequilibrium plasma is applied in the present model in a way similar to the "breakdown" concept for the traditional gas discharges. That is, when a nonequilibrium plasma is moving into an adjacent cold gas region, the nonequilibrium plasma can only be effectively triggered if the criterion is locally satisfied, and then the electron energy equation derived from the conventional two-temperature model can properly be used to describe the local nonequilibrium plasma. Otherwise, the nonequilibrium effect will be quenched and differences between the local temperature of electrons and that of heavy species are negligible. Under such circumstances, the ionization mechanism, if there is any, in the relatively cold gas region is no longer dominated by the thermal collisions with high energy electrons. As a result, the ionization rates predicted by the two-temperature model are usually too rapid and the current layer thus calculated may grow fast enough to expand its volume to the entire MHD channel in a very short period, according to Slavin et al.³⁶ The derivation of this criterion will be given in Chapter 3.

In an unsteady model, the properties of the flow are decided at each time step and are advanced in time through an integration of the properties with a certain time scale. The length of the time scale has to be carefully chosen to fit the frame of time for the important unsteady phenomena to be observed. Therefore, prior to modeling, an evaluation of the characteristic times for different processes occurring in the system is necessary.

2.1 Characteristic Times for T-layer Processes

A time scale analysis not only helps in understanding the reaction rate for each process but also helps in simplifying the modeling work by implementing the proper assumptions without losing the exactness in describing the primary phenomena in the system. This type of analysis is especially important in the T-layer modeling process using nonequilibrium plasma since the temperatures of different components scatter over a large range, which usually translates to a large range in time scales.

There are various characteristic times in the T-layer formation and MHD interaction processes. The important characteristic times are those associated with the flow's momentum and energy balances, the production of charged particles, and the resident time.

The momentum and energy balances in a T-layer MHD channel are usually dominated by the contribution from MHD effects due to a high intensity plasma in the T-layer. The characteristic time for momentum balance can be estimated from inertial and Lorentz force terms in the momentum equation as

$$\tau_F = \frac{\rho}{|1 - K|\sigma B^2} \quad (2.1)$$

For a nonequilibrium plasma, the electron energy is decoupled from the energy of heavy species and their associated characteristic times are also different. The characteristic time for heating of heavy species is determined from the gas energy equation (4.4) as

$$\tau_{T_g} = \frac{\gamma}{\gamma-1} \frac{\rho R}{3k m_e n_e \sum_j \frac{v_{e-j}}{m_j}}, \quad j = Ar, Ar^+ \quad (2.2)$$

Similarly, the characteristic time for electron energy balance is calculated from the electron energy equation written as

$$\frac{3}{2} k n_e \frac{\partial T_e}{\partial t} = \frac{J^2}{\sigma} - 3k m_e n_e (T_e - T_g) \sum_j \frac{v_{e-j}}{m_j} - q_{rad} - q_{ion}, \quad j = Ar, Ar^+ \quad (2.3)$$

then its characteristic time can be obtained as

$$\tau_{T_e} = \left[\frac{\partial(\partial T_e / \partial t)}{\partial T_e} \right]^{-1}. \quad (2.4)$$

The characteristic time for the ionization process can be determined based on a method introduced by Hara et al.⁴⁵ From the continuity equation for the electron species, the characteristic time for the production of an electron can be estimated as

$$\tau_{ion} = \int_{n_0}^{n_1} \left(\frac{dn_e}{dt} \right)^{-1} dn_e \quad (2.5)$$

where $\frac{dn_e}{dt}$ is decided from the ionization kinetics (2.28) and $n_{e1} = n_{e\infty} - (n_{e\infty} - n_{e0})/e$ as shown in Figure 2.1 with n_{e0} and $n_{e\infty}$ the electron densities found from Saha equilibrium relation at T_{e0} and $T_{e\infty}$ respectively.

The flow resident time is found as the channel length divided by the average flow speed and usually has a value of 10^{-3} – 10^{-2} second for a channel 1–10 m in length.

The characteristic times are calculated for an argon plasma in some typical cases with different degrees of nonequilibrium effects (represented by ratios of T_g/T_e) and the results are given in Figures 2.2 to 2.4 for pressures equal to (a) 3×10^4 and (b) 1×10^5 Pascal's. In these calculations, the flow speed is assigned as 1000 m/s, the magnetic field as 2.5 T, the load factor is 0.5, and in finding τ_{ion} and τ_{Te} , two T_{e0} 's are chosen for each case, 6000 and 10000 °K. For example, in Fig. 2.2.(a) the two dash-dotted lines are the calculated τ_{ion} for $n_{e0} = 5.06 \times 10^{19}$ ($T_{e0} = 6000$ °K) and $2.49 \times 10^{22} \text{ m}^{-3}$ ($T_{e0} = 10000$ °K) respectively as functions of n_e with T_e constant at $T_{e\infty}$; while the two dash-dot-dotted lines show τ_{Te} for two values of T_{e0} as functions of $T_{e\infty}$ when keeping n_e constant at 5.06×10^{19} and $2.49 \times 10^{22} \text{ m}^{-3}$.

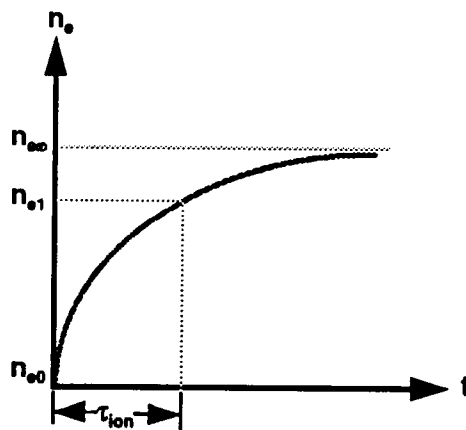


Figure 2.1 Definition of the Relaxation Time for Ionization/Recombination

$$\text{where } n_{e1} = n_{e\infty} - (n_{e\infty} - n_{e0})/e$$

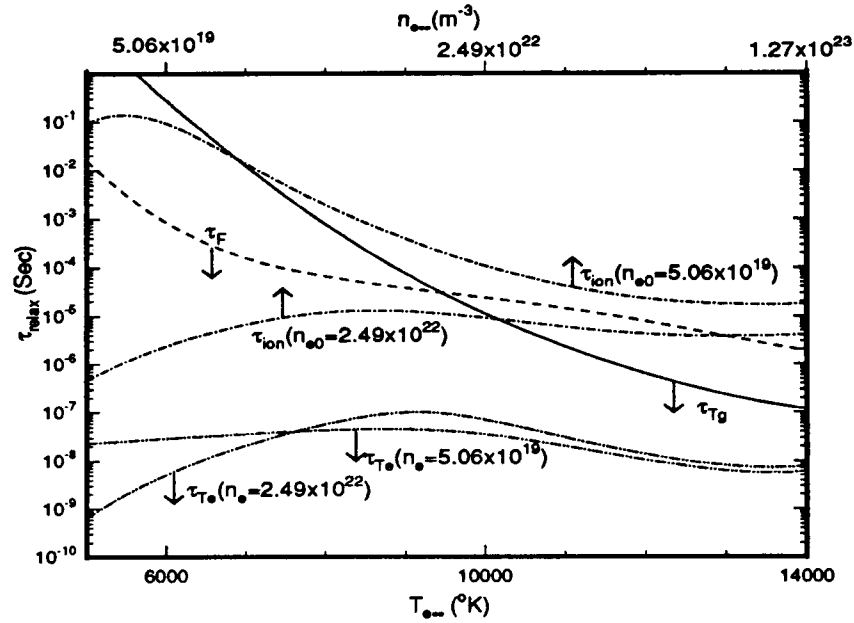
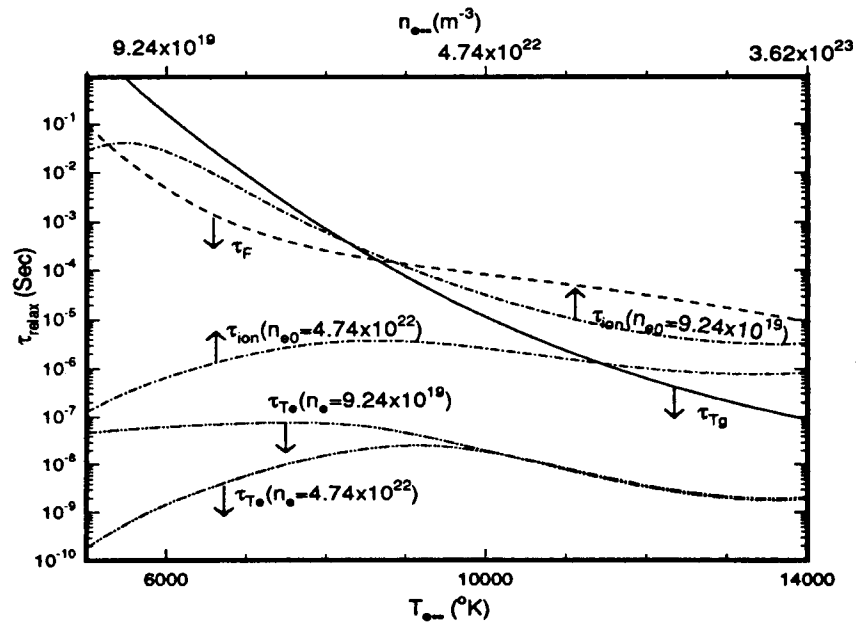
(a) $p = 3 \times 10^4$ pas(b) $p = 1 \times 10^5$ pas

Figure 2.2 The Relaxation Times for Ionization (τ_{ion}), Force Balance (τ_F), Electron Energy (τ_{Te}), and Gas Energy (τ_{Tg}) with $T_g/T_e=0.1$.

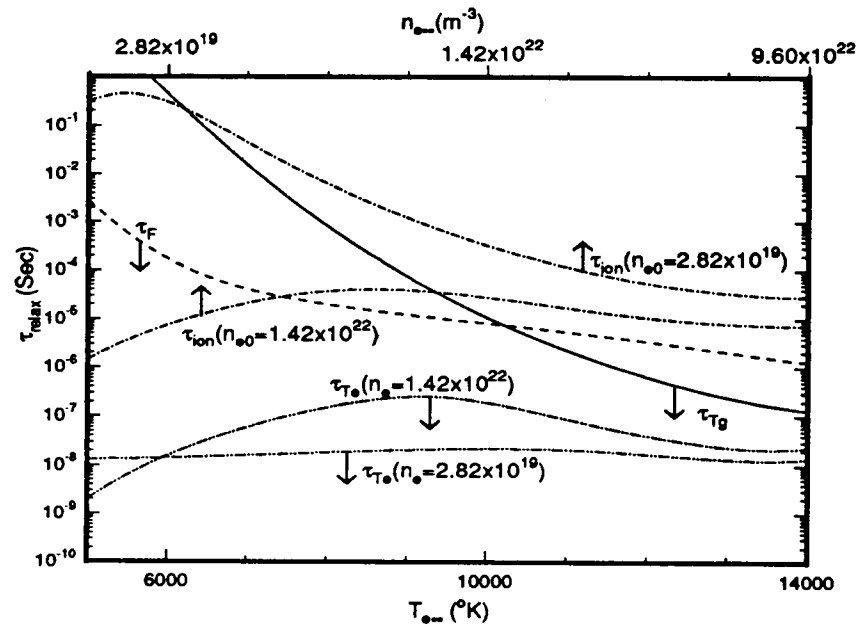
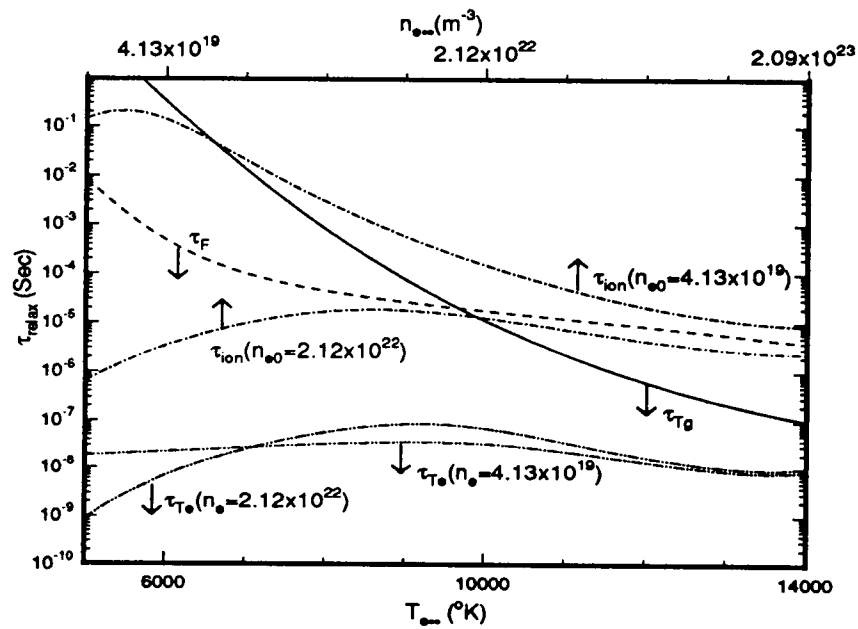
(a) $p = 3 \times 10^4$ pas(b) $p = 1 \times 10^5$ pas

Figure 2.3 The Relaxation Times for Ionization (τ_{ion}), Force Balance (τ_F), Electron Energy (τ_{Te}), and Gas Energy (τ_{Tg}) with $T_g/T_e=0.32$.

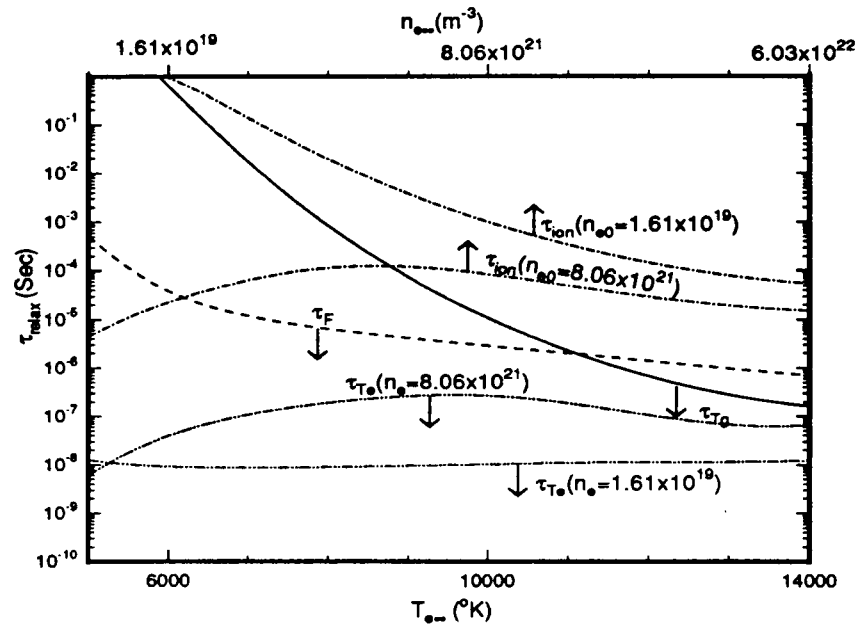
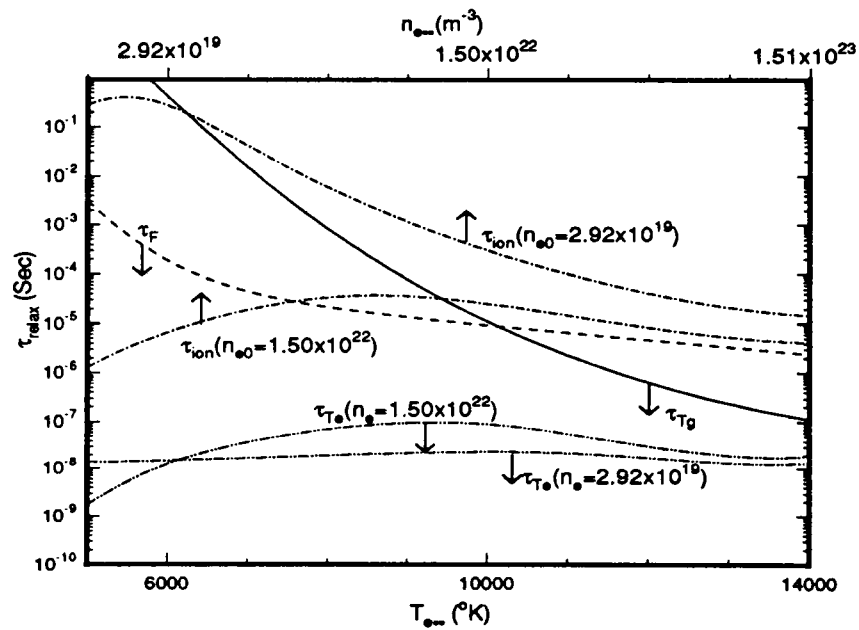
(a) $p = 3 \times 10^4$ pas(b) $p = 1 \times 10^5$ pas

Figure 2.4 The Relaxation Times for Ionization (τ_{ion}), Force Balance (τ_F), Electron Energy (τ_{Te}), and Gas Energy (τ_{Tg}) with $T_g/T_e=0.99$.

From the results shown in Figures 2.2 to 2.4, the following is concluded for a nonequilibrium argon plasma:

- 1) The characteristic time for the relaxation of electron energy is much less than the other characteristic times also observed. With respect to the other processes, an instantaneous relaxation of electron energy can be assumed.
- 2) The Saha equilibrium is only valid for the high pressure, low gas temperature, and high electron density cases.
- 3) The heavy species can be heated at a relatively high rate when the electron temperature is high. This is due to the rapid increase of the Coulomb collisions between electrons and ions with increasing electron temperature.
- 4) The momentum balance can be reached at a faster rate when the density is lower, which corresponds to a small ratio of inertial to Lorentz force when the flow speed is kept constant.

2.2 Mathematical Model

The mathematical formulation for the whole T-layer MHD device is divided into three parts, e.g., gasdynamics with MHD effects, electrodynamics for the MHD channel, and physics for calculation of argon plasma properties. The customary two-temperature model is applied to describe the nonequilibrium plasma.

2.2.1 Gasdynamics with MHD Effects

From the time scale analysis in the preceding Section, it is realized that in simulating the whole process for T-layer motion through an MHD channel, time dependent schemes have to be applied for dominant processes including momentum

balance, relaxation of gas energy, and ionization/recombination of plasma. An infinite rate is only valid for the relaxation of electron energy and the kinetics of ionization/recombination of the argon plasma has to be included in the model.

The formulation for the unsteady quasi-one-dimensional T-layer flow in Eulerian coordinates is listed below.

$$\text{Species continuity} \quad \frac{\partial(\rho_\lambda A)}{\partial t} + \frac{\partial(\rho_\lambda u A)}{\partial x} = \dot{\rho}_\lambda A \quad (2.6)$$

$$\text{Global continuity} \quad \frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho u A)}{\partial x} = 0 \quad (2.7)$$

$$\text{Momentum} \quad \frac{\partial(\rho u A)}{\partial t} + \frac{\partial(p A + \rho u^2 A)}{\partial x} = p \frac{dA}{dx} + J_y B A \quad (2.8)$$

$$\text{Global energy} \quad \frac{\partial(e A)}{\partial t} + \frac{\partial(e + p) u A}{\partial x} = (J_y E_y - q_{rad}) A \quad (2.9)$$

$$\text{Electron energy} \quad \frac{J_y^2}{\sigma} = 3k(T_e - T_g)m_e n_e \sum_j \frac{v_{e-j}}{m_j} + q_{rad} + q_{ion}, \quad j = Ar, Ar^+ \quad (2.10)$$

For an ideal gas, the total energy per unit volume of fluid has the relation

$$e = \frac{p}{\gamma - 1} + \frac{1}{2} \rho u^2 + \rho_\lambda h_\lambda^o \quad (2.11)$$

and the equation of state is also valid as

$$p = p_g + p_e = n_g k T_g + n_e k T_e \quad (2.12)$$

2.2.2 Electrodynamics of MHD Channel

The plasma in the T-layer does not have to be uniform. Therefore, the fluid properties are treated as local values, i.e., $\sigma = \sigma(x)$, $u = u(x)$, $J_y = J_y(x)$, etc. Also, the channel height and width can vary along the channel length, $l = l(x)$, $w = w(x)$.

In an MHD device, the internal impedance of the bulk fluid including T-layer plasma can be calculated as

$$R_p = \left(\int_0^L \frac{\sigma w}{l} dx \right)^{-1} \quad (2.13)$$

The current through the whole channel is found from the integral of current density

$$I = \left| \int_0^L J_y w dx \right| \quad (2.14)$$

Also, the electromotive force induced by the $\bar{u} \times \bar{B}$ effect for the whole channel can be interpreted as the multiplication of the total induced current with the internal impedance, i.e.,

$$\epsilon_{\bar{u} \times \bar{B}} = R_p \int_0^L \sigma u B w dx \quad (2.15)$$

2.2.2.1 Generator Channel

In the present work, a constant resistive load is connected to the generator to extract electrical energy. The local current density is derived from the Ohm's law as

$$J_y = \sigma(E_y - uB + \frac{\Delta V_d}{l}) = \sigma(\frac{I R_L + \Delta V_d}{l} - uB) \quad (2.16)$$

which has a negative value for a generator. By substituting expressions (2.13), (2.15), and (2.16) into (2.14), Kirchhoff's voltage law applied to the circuit of such a generator is then recovered as

$$I R_L + I R_p + \Delta V_d = \epsilon_{\vec{u} \times \vec{B}}, \quad (2.17)$$

The load factor for the generator is defined based on the bulk plasma as

$$K_g = \frac{R_L}{R_L + R_p} = \frac{I R_L}{\epsilon_{\vec{u} \times \vec{B}} - \Delta V_d} \quad (2.18)$$

2.2.2.2 Accelerator Channel and T-layer Formation Section

In an accelerator, an external electrical potential is applied across the channel in direction opposing the MHD induced voltage. A constant load factor greater than unity is assumed in this work. This assures that the voltage applied to the channel always has a magnitude greater than the induced field and the flow will be accelerated all the time. Therefore, the voltage of the power supply, ϵ_{app} , is a time-dependent function under such assumption.

Ohm's law applied to any location in the accelerator channel has the relation

$$J_y = \sigma(E_y - uB - \frac{\Delta V_d}{l}) = \sigma(\frac{\epsilon_{app} - \Delta V_d}{l} - uB), \quad (2.19)$$

By substituting expressions (2.19), (2.13), and (2.15) into (2.14), which now has a positive sign, the relation of Kirchhoff's voltage law applied to the accelerator is thus reached, i.e.,

$$I R_p + \epsilon_{\bar{u} \times \bar{B}} + \Delta V_d = \epsilon_{app} \quad (2.20)$$

The load factor for the accelerator is defined as

$$K_a = \frac{\epsilon_{app}}{\epsilon_{\bar{u} \times \bar{B}} + \Delta V_d} \quad (2.21)$$

treated herein as constant.

Similar to an accelerator, in the T-layer formation section only a single power source is applied on each electrode pair. The equations for Kirchhoff's and Ohm's laws in this situation are similar to relations (2.19)-(2.20) but without contribution from $\bar{u} \times \bar{B}$ effect.

2.2.3 Calculation of Properties of Argon Plasma

The formulation for the MHD flow as given in Equations (2.6-9) can be treated as a combination of the regular governing equations for compressible fluid dynamics and the source terms which represent the contribution from the MHD effects and the production of charged particles. For an MHD device to be efficient enough, generally the influences from these source terms are dominant. Therefore, a careful evaluation of plasma properties is necessary since plasma properties are the key elements needed to evaluate these source terms.

The scalar electrical conductivity of a plasma can be calculated from the expression

$$\sigma = \frac{n_e e^2}{m_e \nu_e} \quad (2.22)$$

where ν_e is the momentum transfer collision frequency between one electron with neutrals and ions, i.e.,

$$\nu_e = \nu_{e-Ar} + \nu_{e-Ar^+} \quad (2.23)$$

The collision frequency between electron and neutrals is found by

$$\nu_{e-Ar} = n_{Ar} Q_m \sqrt{\frac{8kT_e}{\pi m_e}} \quad (2.24)$$

while the Coulomb type collisions between electron and ions have a frequency evaluated by⁴⁶

$$\nu_{e-Ar^+} = 3.64 \times 10^{-6} n_e T_e^{-3/2} \ln(1.24 \times 10^7 \sqrt{\frac{T_e^3}{n_e}}) \quad (2.25)$$

with constant coefficients evaluated in SI units. The average electron-neutral momentum transfer collision cross section can be found from an empirical relation obtained by fitting the data of Milloy⁴⁷ within a temperature range of 3000- 15000 °K; i.e.,

$$Q_m = 1.5 \times 10^{-24} T_e - 4.1 \times 10^{-34} T_e^3 - 3.22 \times 10^{-21} \quad (m^2) \quad (2.26)$$

With the consideration of the lowering of the ionization potential, the resultant effective ionization potential for argon atoms becomes⁴⁸

$$\epsilon_i = \epsilon_{0\lambda} - \Delta\epsilon = 15.76 - 2.086 \times 10^{-11} \sqrt{\frac{2n_e}{T_e}} \quad (eV) \quad (2.27)$$

In the production of charged particles, the ionization kinetics of argon are assumed to be dominated by three-body recombination reactions with electrons as the third body, e.g.,



The rate equation then becomes

$$\dot{n}_e = \frac{dn_e}{dt} = \frac{dn_{\lambda}}{dt} = K_f n_{\text{Ar}} n_e - K_r n_{\lambda} n_e^2 = K_f n_e (n_{\text{Ar}} - \frac{n_e^2}{S}) \quad (2.28)$$

where K_f , K_r , and S are the ionization, recombination, and the Saha equilibrium coefficients respectively. The Saha equilibrium coefficient is calculated as

$$S = \frac{2g_{\lambda}}{g_0} \left(\frac{2\pi m_e k T_e}{h^2} \right)^{3/2} \exp\left(-\frac{\epsilon_i}{k T_e}\right) \quad (2.29)$$

The ionization coefficient is evaluated by an semi-empirical formula given by Beulens et al.⁴⁸ as

$$K_f = \exp(-2.69983 \times 10^{-15} T_e^4 + 1.21373 \times 10^{-10} T_e^3 - 2.1524 \times 10^{-6} T_e^2 + 1.89074 \times 10^{-2} T_e - 114.838) \quad (\text{m}^3/\text{sec}) \quad (2.30)$$

An expression for the radiative energy loss from argon plasma was derived semi-empirically by Wilbers et al.⁴⁹ The formula was concluded after a series of numerical experiments using a analytical model developed by the authors and through comparisons with different sets of laboratory experimental data. In their analytical modeling, the nonequilibrium plasma was described by the two-temperature model as used in the current work. Radiation from argon plasma was assumed to be optically thin.

Contributions due to both continuum emission (free-bound and free-free) and line radiation were included. The expression of Wilbers et al.⁴⁹ is

$$\tilde{q}_{rad} = n_e^2 \left(c_1 + \frac{c_2}{T_e^{1/2}} + \frac{c_3}{T_e} + c_4 T_e + c_5 T_e^{3/2} \right) \exp\left(\frac{c_6}{T_e}\right) \quad (\text{W m}^{-3} \text{ sr}^{-1}) \quad (2.31)$$

with coefficients: $c_1 = -6.932 \times 10^{-41}$, $c_2 = 4.753 \times 10^{-39}$, $c_3 = -5.808 \times 10^{-38}$,
 $c_4 = -2.843 \times 10^{-45}$, $c_5 = 8.629 \times 10^{-47}$, $c_6 = 6.706 \times 10^4$.

This expression for argon plasma radiation is valid for a temperature range less than 15000 °K due to the assumption of a singly ionized plasma, and it was claimed by the original authors that reasonable accuracy could still be maintained even when extrapolated to 20000 °K.

The amount of energy lost per each electron recombination is $(\frac{5}{2} k T_e + \epsilon_i)$. Therefore, the net power density needed for ionization and recombination can be calculated as^{76,77}

$$q_{ion} = \left(\frac{5}{2} k T_e + \epsilon_i \right) \frac{dn_e}{dt}. \quad (2.32)$$

2.2.4 System of Governing Equations

With some extra miscellaneous relations; such as, $\rho_\lambda = m_{Ar} \cdot n_\lambda = m_{Ar} \cdot n_e$, $\rho = m_{Ar} n_g$, $m_{Ar} \equiv m_{Ar}$, and $n_g = n_\lambda + n_{Ar}$, the system of equations is closed once the geometry of the channel, the magnetic field, the voltage drop, and one additional parameter related to the power circuit of each device type are assigned.

In a generator, the additional parameter assigned is the resistive network or load. A single resistive load is similar to ideal operation of T-layer generator with continuous

electrodes. (In a real case, some other components such as inductance might also be connected to the circuit for power consolidation.) Equations (2.6)-(2.12) are employed to find n_e , ρ , u , e , T_e , p , and T_g and relations (2.13), (2.14), (2.16), and (2.22) are applied to calculate R_p , I , J_y , and σ . The load factor is later found from Equation (2.18).

The solution finding process for an accelerator is similar to that for a generator except that, instead of a resistive load, a constant load factor is assigned. The applied electrical field, ϵ_{app} , is then evaluated based on the assigned load factor. Employing a time-varying electrical field to the channel of an accelerator is more reasonable since, when the T-layer traverses the channel, the induced $\bar{u} \times \bar{B}$ field will increase, which might cause some local flow deceleration if the applied field is kept constant. The other possible scheme for reaching uniform acceleration is to segment the electrodes along the channel and apply multiple power supplies with different strengths to various electrode pairs.

In simulating the T-layer formation process, a constant value of ϵ_{app} is initially designated. The current is to be calculated through Equation (2.14) after the other local properties are found.

In the solution scheme, the governing equations are divided into two groups: the time-dependent group, and the electrical group. The time-dependent group includes Equations (2.6)-(2.9) which solve for flow properties as well as charged particle density. In the electrical group, the electron energy equation is coupled with the electrodynamics of the MHD channel in order to find the updated values of electrical properties of plasma other than the electron density. At the same time, the integrals of these electrical properties have to satisfy Kirchhoff's law for the electrical circuit of the channel. The process for solving the system of equations is summarized in a flow chart which is given in Figure 2.5.

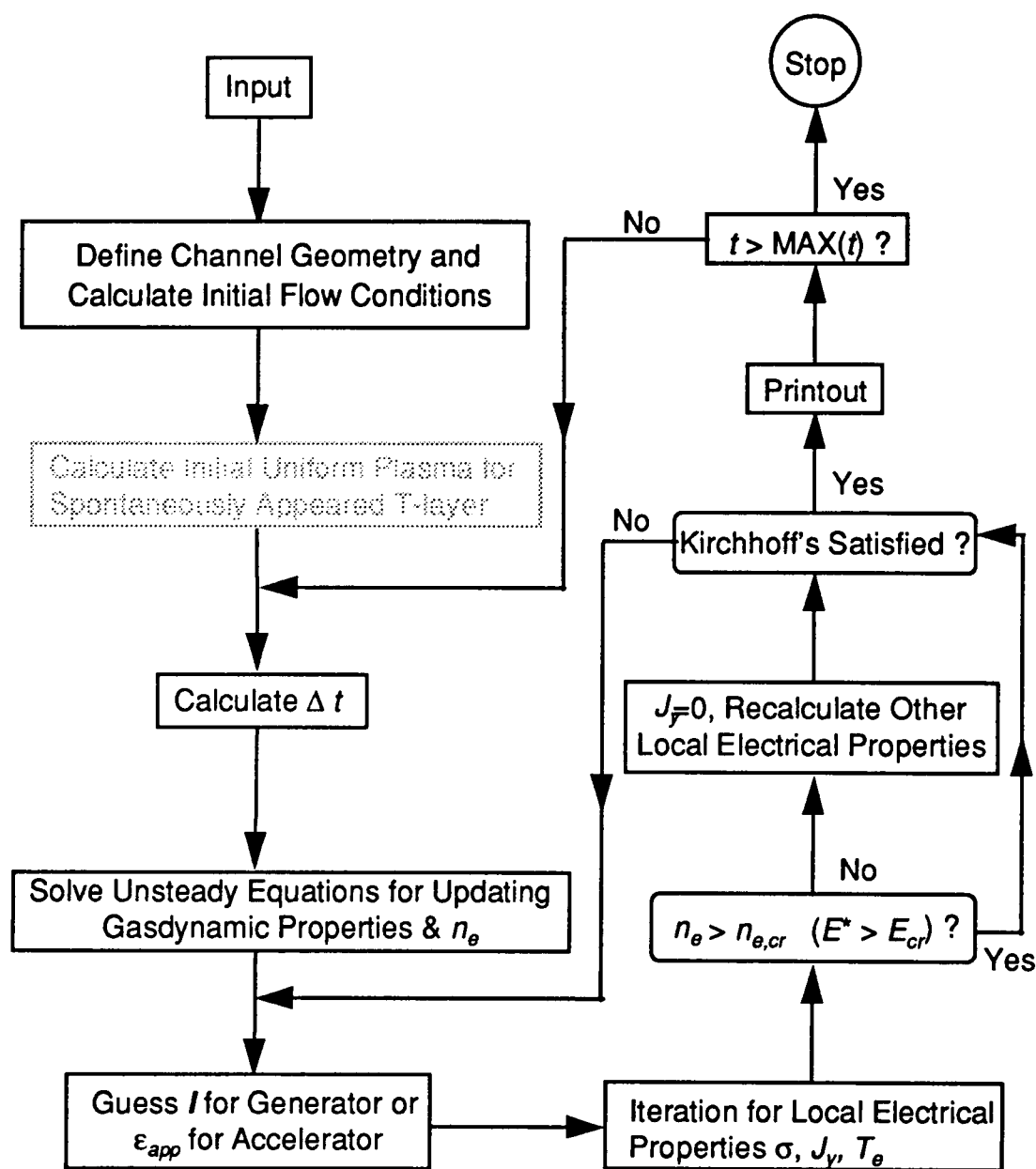


Figure 2.5 Flow Chart for the Calculation Procedure in the Unsteady Model

(The Gray Part is valid only for the Case where T-layer Spontaneously Appears in the MHD Channel.)

2.3 Algorithm for Numerical Solution of Time-dependent Equations

Since the T-layer process is unsteady, the solutions have to be calculated step-by-step, advancing in time after initial conditions are assigned. The time-dependent governing equations, (2.6)-(2.9), have a form as the typical Euler equations with source terms representing the MHD effects and the production of charged particles. Written in conservation form, the equations take the form

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} = \mathbf{R} \quad (2.33)$$

where

$$\mathbf{U} = \begin{bmatrix} \rho A \\ mA \\ eA \\ \rho_\lambda A \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} mA \\ (p + m^2/\rho)A \\ (p + e)mA/\rho \\ \rho_\lambda mA/\rho \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} 0 \\ p \frac{dA}{dx} + J_y BA \\ (J_y E_y - q_{rad})A \\ \dot{\rho}_\lambda A \end{bmatrix},$$

with $m = \rho u$ as the mass flux.

The existence of discontinuities is inherent to a T-layer flow. This is not only due to the intentionally-created alternating cold and hot layers inside the flow, but also owing to the expected strong compression waves that result from the MHD interactions. Therefore, a high-accuracy, high-resolution numerical scheme is needed to appropriately simulate this type of flow field. For this purpose, a finite-difference method with such capability is introduced in this Section.

For the sake of flexibility and reducing the computational burden, operator splitting⁵⁰ between flux vector and source terms is used to solve Equation (2.33). The solution is advanced from $\mathbf{U}^n$ to $\mathbf{U}^{n+1}$ through two partial steps. In the first partial step,

the equations are solved as homogeneous equations, i.e., source terms $R=0$; while flux terms F are taken as zero in the next partial step. A high-resolution second-order TVD (total variation diminishing) scheme originally constructed by Liou⁵¹ is applied to the flux terms on a uniform grid in the first partial step and Euler's predictor-corrector method was used to handle the source terms. By letting the grid spacing be denoted by Δx such that $x = j\Delta x$ and letting the time step be denoted by Δt such that $t = n\Delta t$, the solution procedure from time step n to step $n+2$ can be represented as⁵⁰

$$U_j^{n+2} = L_x(\Delta t)L_t(\Delta t)L_t(\Delta t)L_x(\Delta t)U_j^n \quad (2.34)$$

where $L_x:$ $U_j^* = U_j^n - \tau(\tilde{F}_{j+1/2}^n - \tilde{F}_{j-1/2}^n)$ (2.35)

$$L_R: \begin{cases} \bar{U}_j = U_j^* - \Delta t \cdot R(U_j^*) \\ U_j^{n+1} = \frac{1}{2}[U_j^* + \bar{U}_j - \Delta t \cdot R(\bar{U}_j)] \end{cases} \quad (2.36)$$

with $\tau = \Delta t / \Delta x$. $\tilde{F}$ is referred to as the numerical flux vector which is modified from the first-order numerical flux to make the scheme of higher order accuracy. Using the algorithm given in Equations (2.34)-(2.36), it is possible to extend the model to the multi-dimensional case.

2.3.1 Liou's TVD Scheme

Since the Liou's TVD scheme has successfully been employed to calculate flows with strong shocks and with highly stiff source terms^{52,53,54}, it should be applicable to

simulate the T-layer flow with high accuracy. Only the important results of this numerical scheme are stated below, more details are referred to Liou's original work⁵¹.

When the numerical flux vectors $\tilde{F}$ in Equation (2.35) are expanded, Liou's TVD scheme for the one-dimensional system is written as

$$\begin{aligned} \Delta U_j^n = & -\tau(\Delta^- F_j^+ + \Delta^+ F_j^-)^n - \frac{1}{2}\tau\Delta^-\{\phi_j^+(I - \tau A_{j+1/2})\Delta^+ F_j^+ \\ & - \phi_{j+1}^-(I + \tau A_{j+1/2})\Delta^+ F_j^-\}^n \end{aligned} \quad (2.37)$$

with the notation

$$\Delta^+(\quad)_j = (\quad)_{j+1} - (\quad)_j \quad (2.38)$$

$$\Delta^-(\quad)_j = (\quad)_j - (\quad)_{j-1} \quad (2.39)$$

A is the Jacobian matrix of the flux vector F

$$A(U) = \frac{\partial F}{\partial U} \quad (2.40)$$

The equation system is hyperbolic and can be diagonalized. After diagonalization, the eigenvalues of Jacobian matrix A are obtained as $(u, u+c, u-c, u)$ with $c = \sqrt{\gamma p/\rho}$.

Since the commonly used Roe's average might not be valid for a chemically reacting flow^{55,56}, the components of Jacobian matrix $A_{j+1/2}$, are evaluated at some average state between (U_j, U_{j+1}) similar to the ones defined by Huang⁵⁷ or Carofano⁵⁰ as

$$\rho_{j+1/2} = \frac{(\rho A)_j + (\rho A)_{j+1}}{A_j + A_{j+1}} \quad (2.41a)$$

$$u_{j+1/2} = \frac{(\rho u A)_j + (\rho u A)_{j+1}}{(\rho A)_j + (\rho A)_{j+1}} \quad (2.41b)$$

$$H_{j+1/2} = \frac{(H A)_j + (H A)_{j+1}}{A_j + A_{j+1}} \quad (2.41c)$$

$$(\rho_\lambda)_{j+1/2} = \frac{(\rho_\lambda A)_j + (\rho_\lambda A)_{j+1}}{A_j + A_{j+1}} \quad (2.41d)$$

where $H = (e + p)/\rho$ is the total energy per unit mass. The other average properties are then calculated based on the local values of $\rho_{j+1/2}$, $u_{j+1/2}$, $H_{j+1/2}$, and $(\rho_\lambda)_{j+1/2}$.

Using flux-vector splitting techniques for upwind schemes⁵⁸, the flux vector $\mathbf{F}$ can be split into $\mathbf{F}^+$ and $\mathbf{F}^-$ according to positive and negative signs of the eigenvalues respectively. A Van Leer type flux vector splitting is used in the current study as⁵⁴

$$\mathbf{F}^\pm = \begin{bmatrix} F_1^\pm \\ F_2^\pm \\ F_3^\pm \\ F_4^\pm \end{bmatrix} = \begin{bmatrix} F_1^\pm \\ F_1^\pm \cdot [u - p(u \mp 2c)]/\rho c^2 \\ F_1^\pm \cdot (uc \pm 2h)^2/2(c^2 + 2h) \\ F_1^\pm \cdot \rho_\lambda/\rho \end{bmatrix} \quad (2.42)$$

with

$$F_1^\pm = \frac{1}{4} \frac{\rho A}{c} (u \pm c)^2.$$

The limiter function is chosen as⁵⁹

$$\phi_j^\pm = \phi(r_j^\pm) = \frac{r_j^\pm + |r_j^\pm|}{1 + (r_j^\pm)^2} \quad (2.43)$$

with
$$r_j^+ = \left[\frac{\sum_{i=1}^4 \Delta^- F_i^+}{\sum_{i=1}^4 \Delta^+ F_i^+} \right]_j \quad (2.43a)$$

and
$$r_j^- = \left[\frac{\sum_{i=1}^4 \Delta^+ F_i^-}{\sum_{i=1}^4 \Delta^- F_i^-} \right]_j. \quad (2.43b)$$

2.3.2 Initial and Boundary Conditions

With the unsteady model in the present study, there are two ways of inserting T-layers into the MHD channel: instantaneous appearance and artificial formation. In the first case, the T-layer is assumed to be with the uniform plasma at the beginning of the process (e.g., at $t=0$) and only the MHD channel is involved in the calculations. While in the latter case, a T-layer formation section located upstream of the MHD channel is also included in the calculation and the process starts with initial triggering of a small volume of plasma. The initial triggering is fulfilled by setting a certain voltage across the channel height at the triggering point located at the inlet of the formation section which is a part of the flow nozzle, accelerating flow to the inlet condition needed for the MHD device. Before the appearance of T-layers, the flow conditions in the MHD channel and the formation section (if applicable) are set following isentropic relations.

In the calculations, only a certain length of MHD channel is included which is enough to show the important T-layer interactions inside the channel but not necessarily the length of a practical device designed for producing optimum performance. Therefore, at the channel exit, the boundary conditions are chosen to be of nonreflecting type, to prevent the perturbations from downstream propagating along the incoming characteristics. The model for nonreflecting boundary conditions developed by Thompson⁶⁰ which can handle the passing of strong shock waves is used.

The same nonreflecting boundary condition model is also employed at the inlet of the MHD channel in the case with instantaneous appearance of T-layer. Whenever there is a formation section involved in the calculation, the upstream boundary conditions are set at the subsonic inlet of the nozzle. (The formation section is located at the downstream supersonic portion of nozzle.) Under such circumstances, the local pressure, gas density, and charged particle density are assigned at the nozzle inlet as physical conditions, and the local velocity is found from characteristics extrapolation by including the source term into a model initially formulated by Yee.⁶¹

2.3.3 Stability for Time-dependent Algorithm

The stability of the finite difference method applied for numerically solving the unsteady equations (2.33) requires certain limitations on the length of time step Δt . Without the source terms $\mathbf{R}$, the linearized stability limit is the well-known CFL (Courant-Friedrichs-Lewy) condition, i.e. $\Delta t \cdot (\lambda_F)_{\max} / \Delta x \leq 1$, where $(\lambda_F)_{\max}$ is the maximum value of eigenvalues for the Jacobian matrix $\mathbf{A}$ defined in Equation (2.40) and corresponds to $|u| + c$. Since the CFL condition is obtained from analyzing linearized forms of unsteady equations without source terms, the linearized stability limit is multiplied by a safety factor with a value less than unity to count for the non-linear effect, that is

$$\Delta t \leq f_1 \cdot \frac{\Delta x}{|u| + c} \quad (2.44)$$

Values between 0.2 to 0.75 are chosen for the safety factor f_1 in the calculations of different cases.

Due to the expected strong MHD interaction and the "stiff" term from chemically nonequilibrium ionization, the source terms $\mathbf{R}$ might cause dominant influences. An additional constraint in numerical stability for the existence of source terms is evaluated as

$$\Delta t \leq f_2 \cdot \frac{1}{(\lambda_R)_{\max}} \quad (2.45)$$

where $(\lambda_R)_{\max}$ is the maximum value of the eigenvalues for the Jacobian matrix $\partial \mathbf{R} / \partial \mathbf{U}$ and f_2 is a safety factor smaller than one. In the calculations, the proper range of f_2 is found to be within 0.1 to 0.3.

At each time step, Δt is calculated to satisfy the two criteria (2.44 and 2.45) at each grid point. The minimum value of these local Δt 's is assigned as the length of the time step for each calculation time cycle.

2.4 Solution for Governing Equations of Electrical Group

Right after the electron density and other flow properties are advanced one time step, none of the local electrical properties of plasma including electrical conductivity, current density, etc.; can be directly calculated. This is because all the local electrical properties are coupled to the local electron temperature which is usually found by solving the electron energy equation. Within the electron energy equation, the Joulean dissipation term is strongly coupled to the local current density that is to be obtained through Ohm's law. However, Ohm's law implicitly contains certain bulk electrical properties such as the total current in relation (2.16) for a generator or the applied voltage in (2.19) for an accelerator. The values of the total current or the applied

voltage are decided based on prior knowledge of the distribution of certain local variables when referring to Equations (2.13)-(2.15).

Moreover, every electrical property, no matter whether or not it is a local or a bulk value, cannot be independently solved under the assumed operating configurations for both generator and accelerator. The whole set of electrical properties have to be solved simultaneously in order to satisfy Kirchhoff's law for the MHD circuit. By substituting the temperature of heavy species, T_g , from the equation of state (2.12) into the elastic collision term of electron energy equation (2.10), this temperature is implicitly found with other electrical properties.

Therefore, if there are m local properties and n bulk properties to be found on k grid points, a set of $(m \times k + n)$ equations has to be solved simultaneously. Unfortunately, the commonly used Newton-Raphson method for the nonlinear iteration cannot be applied to the whole set of equations. This is due to the discontinuities in the derivatives of properties created by the judging process for the onset of nonequilibrium plasma to be discussed in the next Chapter.

A method with two iterative loops is developed to overcome the numerical difficulties and still retain acceptable converging rates for iteration. The iterative process begins with a guessed value for a certain bulk property, say, the total current for a generator and the externally applied voltage for an accelerator. At the inner loop, a Newton-Raphson iteration is applied at each grid to find the local properties. After deciding whether the local plasma satisfies the criteria for the onset of nonequilibrium plasma, an outer loop using a conventional bi-section iteration is employed to match the bulk electrical properties (obtained through integrals of local properties) with Kirchhoff's law. Before the next iterative cycle in the outer loop, the iterative process has to go through the inner loop again to evaluate the new values for local properties. The whole process repeats itself until final converged solutions are obtained for both loops.

Only the inner iterative loop is needed in the calculations of the T-layer formation section. This is because the value of applied voltage is given in the formation section. Hence, according to Ohm's law as in (2.19) without the induced field, the local current density can be considered as a function of the local electrical conductivity, which itself is a function of electron temperature. By inserting the relation thus obtained into the electron energy equation, the electron temperature becomes the only unknown in the equation, which can be iteratively solved. After finding the local electron temperature, all the other electrical properties, local and bulk, can be easily calculated.

CHAPTER 3

CONDITIONS FOR THE ONSET OF NONEQUILIBRIUM PLASMA

In a medium-pressure gas discharge with a nonequilibrium plasma, the mean electron energy is usually greater than that of heavy species and the energy transfer between particles are dominated by collisional processes. In this type of plasma, there are three kinds of processes competing with each other for the energy exchange with electrons: elastic collisions between electrons, inelastic collisions with heavy particles for excitation or ionization, and elastic collisions with heavy particles. The colliding rate for the first kind is approximately proportional to n_e^2 ; while the other two are roughly proportional to $(n_e \cdot n_g)$. Due to a stronger dependence on electron density and a higher mean energy (thermal random velocity), the electrons collide and change energy among themselves more frequently when the electron density is high enough. Under such circumstances, the energy transfer in electron-electron collisions dominates the energy exchange in the plasma and the electron energy distribution approaches the most probable distribution in thermodynamic equilibrium - the Maxwellian distribution.

On the other hand, once the electron density decreases, the electron-electron collision frequency will drop at a faster rate compared to collisions of the other two kinds. The rate for energy exchange between electrons decreases dramatically and ultimately becomes comparable to the electron energy loss due to collisions with heavy species. In this situation, electrons tend to be non-Maxwellian in distribution. It was estimated that the Maxwellian distribution might become an appropriate approximation

to describe the energy distribution of electrons in a plasma when the ionization degree increases to 10^{-6} - 10^{-4} .⁶²

In studying the inlet relaxation phenomena existing at the inlet of a closed cycle MHD generator operating with a nonequilibrium plasma, a criterion for the onset of nonequilibrium plasma was identified theoretically by Borghi et al.^{63,64} and the theory was also verified experimentally by Borghi.⁶⁵ The existence of the inlet relaxation region is due to a longer than expected ionization relaxation time. That is, if the electron density is less than a critical value, the electron energy loss due to excitations to the first excited state becomes important with respect to the energy transfer between electrons. These excitations can only be fulfilled by electrons with energies higher than the first excited threshold energy. As a result, the electrons in the high energy tail section are depleted from a Maxwellian distribution. The shortage of these high energy electrons causes a difficulty in triggering the ionization processes which subsequently results in a region of plasma with ionization degrees lower than that predicted by using an assumed Maxwellian distribution. Only if the electron density is greater than the critical electron density, can the nonequilibrium plasma be properly produced in the MHD channel.

To find the critical electron density and to explain the depletion of electrons in the high energy tail, Borghi et al.^{63,64} employed a method originally proposed by Vriens⁶⁶ for noble or noble-alkali gas in gas discharges to describe argon-cesium plasma at operating conditions typical for the MHD generators. The results showed that for electron densities below a critical value, the excitation energy required for free electrons to support excitation collisions from ground to the first excited state in cesium atoms could not be balanced by the corresponding de-excitation energy gain for electrons due to the escape of radiation. In this situation, the high-energy electrons responsible for excitation lose their energy in the excitation collisions and become the low energy electrons with a rate faster than the rate for low-energy electrons transferring to high-energy electron

through gaining energy from de-excitation collisions. The net outcome of this imbalance is the depletion of high-energy electrons.

In the previous studies^{63,64,65}, the argon-cesium plasma was examined with argon being treated as a buffer gas which could not be excited or ionized. In this work, the original method is modified to describe pure argon plasma in order to find its critical electron density which will then be used as a criterion for the onset of a nonequilibrium plasma in the T-layer modeling.

3.1 Two-Electron Group Model (2-EGM)

Vriens⁶⁶ proposed to separate electrons into two groups: bulk electrons with energies below the threshold energy for the first excited state and tail electrons with energies greater than or equal to the same threshold energy. The electron energy distribution function is divided into two parts, each has a form similar to Maxwellian function with a temperature to represent its mean energy, e.g.,

$$\text{Bulk electron: } f^b(\epsilon_e) = 2 n_e \left\{ \frac{\epsilon_e}{\pi(kT_e)^3} \right\}^{1/2} \exp\left(-\frac{\epsilon_e}{kT_e}\right) \text{ for } \epsilon_e < \epsilon_{01} \quad (3.1)$$

$$\text{Tail electron: } f^t(\epsilon_e) = 2 n_e G \left\{ \frac{\epsilon_e}{\pi(kT_t)^3} \right\}^{1/2} \exp\left(-\frac{\epsilon_e}{kT_t}\right) \text{ for } \epsilon_e \geq \epsilon_{01} \quad (3.2)$$

This electron distribution is shown in Figure 3.1. The parameters T_e and T_t are referred to as the "bulk" and "tail" temperatures respectively. Since the mean electron energy is generally much less than the first threshold energy ϵ_{01} most electrons belong to the bulk electrons and the bulk temperature is a good approximation to the mean electron temperature of all the electrons. That is why T_e is used in expression (3.1).

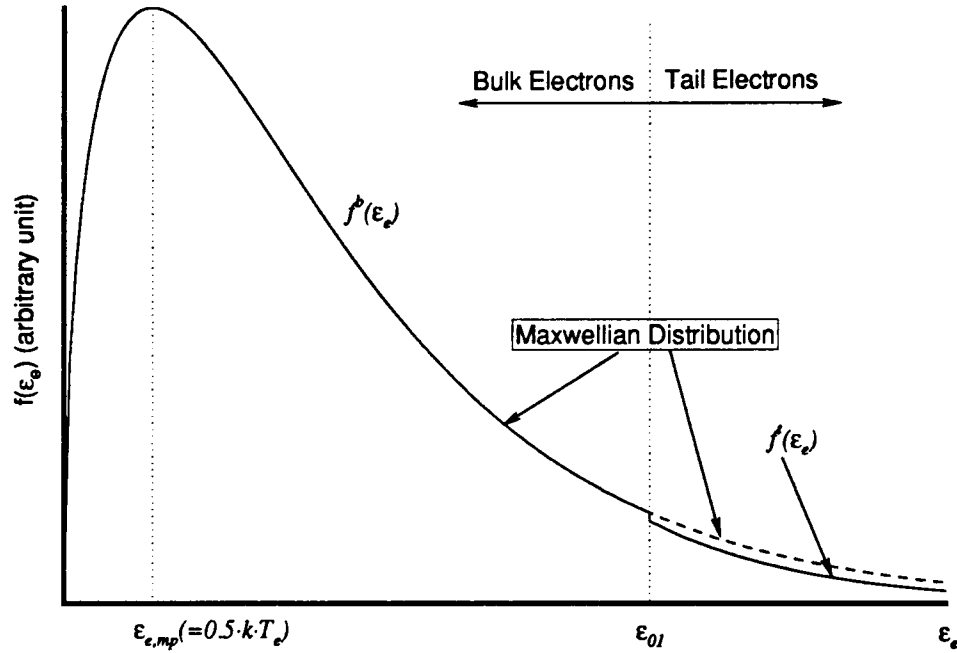


Figure 3.1 Electron Distribution in Two-electron Group Model ($T_i < T_e$)

By the definition given in Equation (3.2), the tail temperature T_i is a measure of the degree of depletion in the tail electrons. The parameter G is the normalization factor. After comparisons between the results from solving the Boltzmann equation and that from 2-EGM, Morgan and Vriens⁶⁷ suggested that, for the argon plasma,

$$G(T_i, T_e) = \left[(T_i / T_e)^{3/2} \exp(-\epsilon_{01} / kT_e + \epsilon_{01} / kT_i) \right]^r \quad (3.3)$$

with

$$r = 1.09 - 0.17(1 - 0.5T_i / T_e)^{-1}.$$

With G given as Equation (3.3) the electron energy has a continuous distribution at $\epsilon_e = \epsilon_{01}$ only if $r = 1$. Maxwellian distribution is arrived at when $T_i = T_e$.

3.2 Three-level Atomic Model⁶⁸

The argon atom is described by a simple approximation, the three-level model including the ground state ($3p^6 \ ^1S_0$), the first excited state ($3d^5 4s$), and the ground state of the singly ionized ion ($3p^5 \ ^2P_{3/2}$), as shown in Figure 3.2. The first excited state is considered as a lumped state including four sub-states which, when expressed in L-S coupling, are $^3P_2$ (93144 cm^{-1}), $^3P_1$ (93751 cm^{-1}), $^3P_0$ (94554 cm^{-1}), and $^1P_1$ (95400 cm^{-1}), with an average excited energy of 11.65 eV. In this case, the energy differences between the three levels are $\epsilon_{0\lambda} = 15.76$ eV, and $\epsilon_{1\lambda} = 4.11$ eV. The statistic weights are $g_0 = 1$, $g_1 = g_1(^3P_2) + g_1(^3P_1) + g_1(^3P_0) + g_1(^1P_1) = 5 + 3 + 1 + 3 = 12$, and $g_\lambda = 6$.

A collisional-radiative model^{69,70} is employed to describe the transitions of electrons between energy levels. The collisional transitions considered are the excitation/ionization and de-excitation/recombination between the levels. The recombination collisions are assumed to be three-body recombination with electrons as the third body. The radiation effects considered include absorption, induced emission and spontaneous emission. It is convenient to treat the combined effects of absorption and induced emission as a reduction in the spontaneous emission transition probability by a parameter known as the radiative-escape factor, β .⁷¹ A detailed definition of the radiative-escape factor can be found in Reference 68. The net rate of radiative transitions, including emission and absorption, from level j to level i ($j > i$) is then expressed as $n_j A_{ji} \beta_{ji}$ with A_{ji} the Einstein A coefficient for the transition j to i . The resultant rate equations for density in the ground state, n_0 , the first excited state, n_1 , and electron density, n_e , ($n_e = n_\lambda$) are

$$\dot{n}_0 = -n_0 n_e (K_{01} + K_{0\lambda}) + n_1 (n_e K_{10} + \beta_{10} A_{10}) + n_e^2 (n_e K_{\lambda 0} + \beta_{\lambda 0} A_{\lambda 0}) \quad (3.4)$$

$$\dot{n}_1 = n_0 n_e K_{01} - n_1 (n_e K_{10} + n_e K_{1\lambda} + \beta_{10} A_{10}) + n_e^2 (n_e K_{\lambda 1} + \beta_{\lambda 1} A_{\lambda 1}) \quad (3.5)$$

$$\dot{n}_e = n_e (n_0 K_{0\lambda} + n_1 K_{1\lambda}) - n_e^3 (K_{\lambda 0} + K_{\lambda 1}) - n_e^2 (\beta_{\lambda 0} A_{\lambda 0} + \beta_{\lambda 1} A_{\lambda 1}) \quad (3.6)$$

K_{ij} 's are the rate coefficients for collisional transitions from state i to j , including excitation, and ionization, while K_{ji} 's are that for de-excitation and recombination.

The rate coefficients K_{ij} are defined in terms of the related transition collision cross sections Q_{ij} . For excitation and ionization,

$$K_{ij} = \int_{\epsilon_{ij}}^{\infty} f(\epsilon_e) Q_{ij}(\epsilon_e) v_e(\epsilon_e) d\epsilon_e$$

$$= \begin{cases} \int_{\epsilon_{ij}}^{\epsilon_{01}} f^b(\epsilon_e) Q_{ij}(\epsilon_e) v_e(\epsilon_e) d\epsilon_e + \int_{\epsilon_{01}}^{\infty} f^i(\epsilon_e) Q_{ij}(\epsilon_e) v_e(\epsilon_e) d\epsilon_e, & \text{if } \epsilon_{ij} < \epsilon_{01}, \\ \int_{\epsilon_{ij}}^{\infty} f^i(\epsilon_e) Q_{ij}(\epsilon_e) v_e(\epsilon_e) d\epsilon_e, & \text{if } \epsilon_{ij} \geq \epsilon_{01}, \end{cases} \quad (3.7)$$

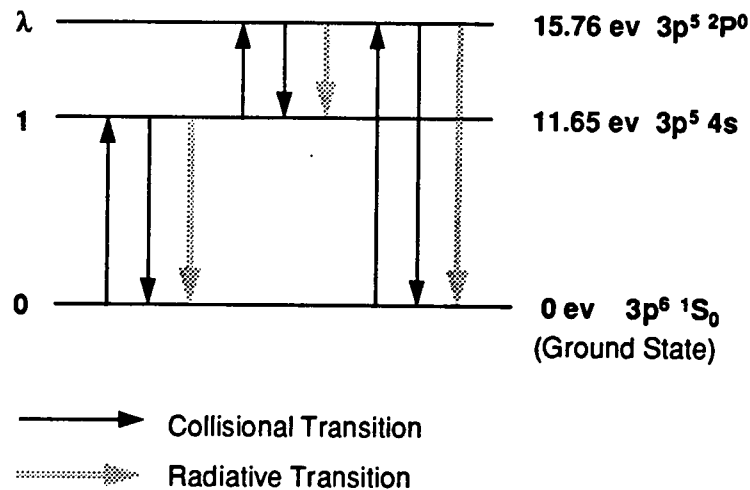


Figure 3.2 Three-level Atomic Model for Argon

where $v_e = \sqrt{2\varepsilon_e/m_e}$ is the electron thermal velocity. For de-excitation or recombination transitions, the threshold energy is zero and thus the transitions are dominated by the bulk electrons. In this case, the rate coefficients become

$$\begin{aligned} K_{ji} &= \int_0^{\infty} f(\varepsilon_e) Q_{ij}(\varepsilon_e) v_e(\varepsilon_e) d\varepsilon_e \\ &\equiv \int_0^{\varepsilon_{0i}} f^b(\varepsilon_e) Q_{ij}(\varepsilon_e) v_e(\varepsilon_e) d\varepsilon_e \equiv \int_0^{\infty} f^b(\varepsilon_e) Q_{ij}(\varepsilon_e) v_e(\varepsilon_e) d\varepsilon_e \end{aligned} \quad (3.8)$$

A detailed discussion for the evaluation of rate coefficients and cross section for electron-impact ionization, excitation, de-excitation, and recombination was given by Vriens and Smeets.⁷² In the current study, a simplified formula is adopted for the calculations of various inelastic electron collision cross sections, that is,⁶⁶

$$Q_{ij}(\varepsilon_e) = \begin{cases} b_{ij}(1 - \varepsilon_{ij}/\varepsilon_e) & \text{if } \varepsilon_e \geq \varepsilon_{ij}; \\ 0 & \text{if } \varepsilon_e < \varepsilon_{ij}. \end{cases} \quad (3.9)$$

This simple formula is expected to provide acceptable accuracy since, in terms of energy, the major contribution of the inelastic collisions comes from the narrow region near the threshold value.

In relation (3.9), b_{ij} has a constant value found from fitting experimental or theoretical data near the threshold energy. For argon, its values for the excitation and ionization were found as $b_{0I} = 0.6 \times 10^{-20}$, $b_{0\lambda} = 2.6 \times 10^{-20}$, and $b_{1\lambda} = 20 \times 10^{-20} \text{ m}^2$.⁶⁶ The corresponding values for de-excitation and recombination can be obtained based on the principle of detailed balancing, which, for transition $i \leftrightarrow j$, has a form as

$$g_i \varepsilon_e' Q_{ij}(\varepsilon_e') = g_j \varepsilon_e Q_{ji}(\varepsilon_e) \quad (3.10)$$

with $\epsilon_e' = \epsilon_e + \epsilon_{ij}$. Then,

$$Q_{ji}(\epsilon_e) = \frac{g_i}{g_j} \cdot \frac{\epsilon_e + \epsilon_{ij}}{\epsilon_e} \cdot Q_{ij}(\epsilon_e + \epsilon_{ij})$$

$$Q_{ji}(\epsilon_e) = \frac{g_i}{g_j} \cdot \frac{\epsilon_e + \epsilon_{ij}}{\epsilon_e} \cdot b_{ij} \cdot \frac{\epsilon_e}{\epsilon_e + \epsilon_{ij}} = \frac{g_i}{g_j} \cdot b_{ij} \quad (3.11)$$

Relation (3.11) is valid for both de-excitation and recombination. However, it is also noted that for three-body recombination, as can be seen from the Saha equilibrium coefficient (2.29); the effective partition function for the ground state of a singly ionized ion is

$$g_\lambda \cdot g_e = g_\lambda \cdot 2 \cdot \left(\frac{2\pi m_e k T_e}{h^2} \right)^{3/2} \quad (3.12)$$

where g_e is considered as the contribution from free electrons with $(2\pi m_e k T_e / h^2)^{3/2}$ as the translational part of partition function and "2" as the contribution from electron spin. When including the influence from this effective partition function into relation (3.11), the collision cross section for recombination is,

$$Q_{\lambda i} = \frac{g_i}{g_\lambda \cdot g_e} \cdot b_{\lambda i} \quad (3.13)$$

The three-body recombination cross section obtained as (3.13) has a dimension of $[L^5]$ while the cross sections for the other processes are of $[L^2]$. The rate coefficients subsequently obtained have dimensions of $[L^6/t]$ and $[L^3/t]$ for recombination and other processes respectively.

The rate coefficients can now be evaluated through integrals in Equations (3.7) and (3.8) with collision cross sections given in (3.9), (3.11), and (3.13). In general,

excitation and ionization are dominated by tail electrons while de-excitation and recombination are mainly due to the bulk electrons. For argon, $\epsilon_{1\lambda}=4.11$ eV < $\epsilon_{01}=11.65$ eV, thus, ionization from the excited state is basically a bulk process. With some approximations, we obtain

$$K_{oj} \equiv b_{oj} G(T_e, T_i) \sqrt{\frac{8kT_i}{\pi m_e}} \cdot \exp\left(-\frac{\epsilon_{01}}{kT_i}\right), \quad j=1, \lambda \quad (3.14)$$

$$K_{1\lambda} \equiv b_{1\lambda} \sqrt{\frac{8kT_e}{\pi m_e}} \cdot \exp\left(-\frac{\epsilon_{1\lambda}}{kT_e}\right) \quad (3.15)$$

$$K_{ji} \equiv b_{ij} \frac{g_i}{g_j} \sqrt{\frac{8kT_e}{\pi m_e}}, \quad ji = \lambda 0, \lambda 1, 10 \quad (3.16)$$

The ionization coefficient given in formula (2.30) was actually obtained as a total rate coefficient for excitation/ionization⁴⁸ based on the partial local thermal equilibrium (PLTE) model of Van der Sijde.⁶² Within the concept of PLTE, the total rate coefficient for excitation/ionization is defined as⁷³

$$K_f = K_{0\lambda} + \sum_{j=1}^N K_{0j} \quad (3.17)$$

where $N=1$ for the upper excited level in the three-level atomic model.

Figure 3.3 compares the ionization coefficient calculated from Equation (3.17) with that found from Equation (2.30). The consistency between both results show the validity of the current three-level model with inelastic collision cross sections as given in Equation (3.9).

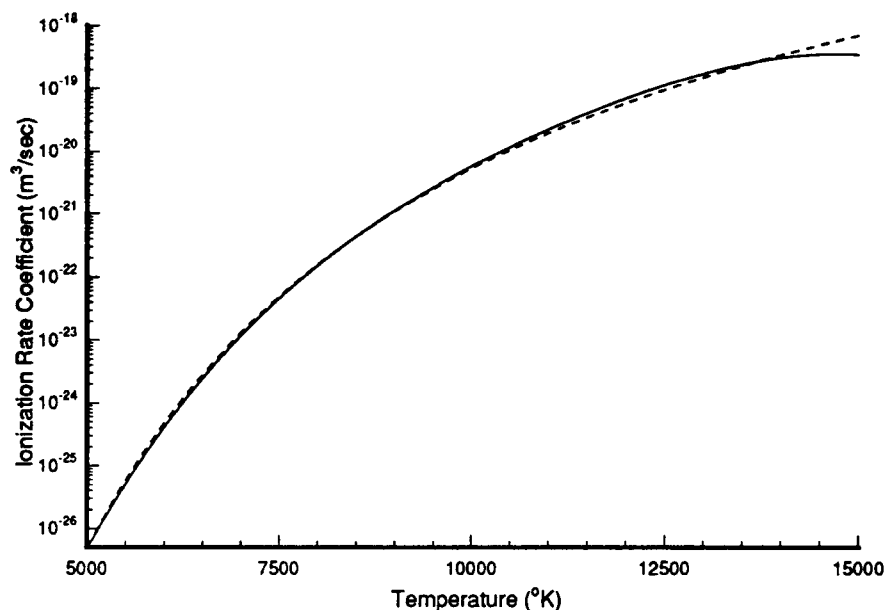


Figure 3.3 Comparison of the Ionization Coefficients of Argon Calculated from Equations (2.30) and (3.17)

The radiation from radiative recombination is usually optically thin⁷⁴, e.g., $\beta_{\lambda 0} \equiv 1, \beta_{\lambda 1} \equiv 1$. However, due to the relatively smaller cross section for radiative recombination (see for example, Chapter 2 of Reference 68) and for the simplicity of the model, the assumption of negligible radiative recombination is applied.

As stated previously, the first excited state of argon in the three-level model consists of four sub-states. Among them, only two sub-states, $^3P_1$ and $^1P_1$, are allowed for radiative transition according to the selection rule. The corresponding two resonance lines are

$$3d^5 4s \ ^3P_1 \rightarrow 3p^6 \ ^1S_0: \lambda_0 = 1066.66 \text{ \AA}, A_{10} = 1.19 \times 10^8 \text{ sec}^{-1}, g_1 = 3;$$

$$3d^5 4s \ ^1P_1 \rightarrow 3p^6 \ ^1S_0: \lambda_0 = 1048.22 \text{ \AA}, A_{10} = 1.19 \times 10^8 \text{ sec}^{-1}, g_1 = 3.$$

where λ_0 is the wavelength at the center of the resonance line. For a Boltzmann distribution, the particle density in each energy level is proportional to $g_j \exp(-\epsilon_j / kT)$.

Due to the small differences between their energies, the population densities in the four sub-states can be approximately assumed to be proportional to their statistical weights:

$n_1(^3P_2) = \frac{5}{12} n_1$, $n_1(^3P_1) = \frac{3}{12} n_1$, $n_1(^3P_0) = \frac{1}{12} n_1$, and $n_1(^1P_1) = \frac{3}{12} n_1$. Thus,

$$n_1 A_{10} \beta_{10} = \frac{1}{4} n_1 \left[A_{10}(1066 \text{ \AA}) \beta_{10}(1066 \text{ \AA}) + A_{10}(1048 \text{ \AA}) \beta_{10}(1048 \text{ \AA}) \right] \quad (3.18)$$

By assuming the geometry of the plasma as a infinite slab, Holstein⁷¹ derived an expression for the radiative-escape factor as

$$\beta_{10} = 1.150 (\pi k_p L)^{-1/2} \quad (3.19)$$

for considering a pressure broadening (resonance broadening in our case due to interactions with same atoms) with a Lorentzian profile. The absorption coefficient, k_p , at the center of the resonance line and can be found as

$$k_p = \frac{\lambda_0^2 n_{Ar} g_1 A_{10}}{2 \pi g_0 \gamma_p} \quad (3.20)$$

with γ_p the half-width of the resonance broadened line. For argon, it was also given that⁶⁶

$$\frac{\gamma_p}{A_{10}} = 0.0144 n_{Ar} \lambda_0^3 \frac{g_1}{g_0} \quad (3.21)$$

In relation (3.19), L is the characteristic length of the infinite slab, i.e., its thickness. In the T-layer calculation, this characteristic length is chosen as the smaller between the thickness of T-layer and the channel height.

The solutions of the rate equations (3.4)-(3.6) can now be found. With the help of conservation of the nuclei of argon atoms

$$n_g = n_0 + n_1 + n_e, \quad (3.22)$$

only two of the rate equations have to be solved to find the three particle densities n_0 , n_1 , and n_e .

3.3 Conservation of Electron Energy

The conservation of electron energy including the effects from inelastic collisions between electrons and atoms can be written in terms of power density as

$$P_J + P_{sel} + P_{rec} = P_{el} + P_{exc} + P_{ion} + P_{rad} \quad (3.23)$$

in which, energy gain terms for electrons consisting of Joule dissipation (J), super-elastic collisions (de-excitation, sel), and recombination (rec) are given in the left-hand-side; while the energy loss terms including elastic collisions with heavy particles (el), excitation (exc), ionization (ion), and radiation (rad) are grouped in the right-hand-side. Comparing Equation (3.23) with (2.10), the differences are from the omission of inelastic excitation and de-excitation contribution and a combination of ionization and recombination into a net term, q_{ion} , in Equation (2.10). The negligence of that portion of inelastic collision effect has been justified in Reference 68 (Chapter 8) when the frequency of inelastic collisions between electrons and neutrals is less than the frequency of electron Coulomb collisions, which is usually true for the argon plasma.

In 2-EGM, the energy conservation equations can be separately written for the tail and bulk electrons as

$$P'_j + P'_{10} + P'_{\lambda 0} + P'_{\lambda 1} + P'_c = P'_{el} + P'_{01} + P'_{0\lambda} + P'_{1\lambda} + P'_{rad} \quad (3.24)$$

$$P_j^b + P_{01}^b + P_{0\lambda}^b + P_{1\lambda}^b + P_c^b = P_{el}^b + P_{10}^b + P_{\lambda 0}^b + P_{\lambda 1}^b + P_{rad}^b \quad (3.25)$$

In which the processes of excitation and ionization are considered to be with energy flow from the tail electrons to the bulk electrons, whereas, the processes of de-excitation and recombination are thought to cause energy flow from the bulk to the tail electrons. Based on this, energy gain terms are then grouped into the left-hand-side and in the right-hand-side are the energy loss terms. P'_c and P_c^b are the energy exchange by Coulomb collisions between two groups of electrons which flows from the group with a higher temperature to the other group.

The power densities from inelastic collisions are formulated as following:

1.) excitation $0 \rightarrow 1$ or ionization $0 \rightarrow \lambda$, $\epsilon_{ij} \geq \epsilon_{01}$, $j=1, \lambda$

$$P'_{0j} = n_e n_0 \left[\epsilon_{0j} \int_{\epsilon_{0j}}^{\infty} f' Q_{0j} v_e d\epsilon_e + \int_{\epsilon_{0j}}^{\epsilon_{01} + \epsilon_{0j}} f' Q_{0j} v_e (\epsilon_e - \epsilon_{0j}) d\epsilon_e \right] \quad (3.26)$$

$$P_{0j}^b = n_e n_0 \int_{\epsilon_{0j}}^{\epsilon_{01} + \epsilon_{0j}} f' Q_{0j} v_e (\epsilon_e - \epsilon_{0j}) d\epsilon_e \quad (3.27)$$

2.) ionization $1 \rightarrow \lambda$, $\epsilon_{ij} < \epsilon_{01}$

$$P'_{1\lambda} = n_e n_1 \left[\epsilon_{1\lambda} \int_{\epsilon_{01}}^{\infty} f' Q_{1\lambda} v_e d\epsilon_e + \int_{\epsilon_{01}}^{\epsilon_{01} + \epsilon_{1\lambda}} f' Q_{1\lambda} v_e (\epsilon_e - \epsilon_{1\lambda}) d\epsilon_e \right] \quad (3.28)$$

$$P_{1\lambda}^b = n_e n_1 \left[-\varepsilon_{1\lambda} \int_{\varepsilon_{1\lambda}}^{\varepsilon_{01}} f^b Q_{1\lambda} v_e d\varepsilon_e + \int_{\varepsilon_{01}}^{\varepsilon_{01} + \varepsilon_{1\lambda}} f^t Q_{1\lambda} v_e (\varepsilon_e - \varepsilon_{1\lambda}) d\varepsilon_e \right] \quad (3.29)$$

3.) de-excitation $1 \rightarrow 0$, $\varepsilon_{ij} \geq \varepsilon_{01}$

$$P_{10}^t = n_e n_1 \left[\int_0^{\varepsilon_{01}} f^b Q_{10} v_e (\varepsilon_e + \varepsilon_{01}) d\varepsilon_e + \varepsilon_{01} \int_{\varepsilon_{01}}^{\infty} f^t Q_{10} v_e d\varepsilon_e \right] \quad (3.30)$$

$$P_{10}^b = n_e n_1 \int_0^{\varepsilon_{01}} f^b Q_{10} v_e \varepsilon_e d\varepsilon_e \quad (3.31)$$

4.) recombination $\lambda \rightarrow 0$, $\varepsilon_{ij} \geq \varepsilon_{01}$

$$P_{\lambda 0}^t = n_e^3 \left[\int_0^{\varepsilon_{01}} f^b Q_{\lambda 0} v_e (\varepsilon_e + \varepsilon_{0\lambda}) d\varepsilon_e + \varepsilon_{0\lambda} \int_{\varepsilon_{01}}^{\infty} f^t Q_{\lambda 0} v_e d\varepsilon_e \right] \quad (3.32)$$

$$P_{\lambda 0}^b = n_e^3 \int_0^{\varepsilon_{01}} f^b Q_{\lambda 0} v_e \varepsilon_e d\varepsilon_e \quad (3.33)$$

5.) recombination $\lambda \rightarrow 1$, $\varepsilon_{ij} < \varepsilon_{01}$

$$P_{\lambda 1}^t = n_e^3 \left[\int_{\varepsilon_{01} - \varepsilon_{1\lambda}}^{\varepsilon_{01}} f^b Q_{\lambda 1} v_e (\varepsilon_e + \varepsilon_{1\lambda}) d\varepsilon_e + \varepsilon_{1\lambda} \int_{\varepsilon_{01}}^{\infty} f^t Q_{\lambda 1} v_e d\varepsilon_e \right] \quad (3.34)$$

$$P_{\lambda 1}^b = n_e^3 \left[\int_{\varepsilon_{01} - \varepsilon_{1\lambda}}^{\varepsilon_{01}} f^b Q_{\lambda 1} v_e \varepsilon_e d\varepsilon_e - \varepsilon_{1\lambda} \int_0^{\varepsilon_{01} - \varepsilon_{1\lambda}} f^b Q_{\lambda 1} v_e d\varepsilon_e \right] \quad (3.35)$$

Substituting these expressions for inelastic power densities into (3.24) and (3.25), adding them up, and applying the relations for the rate coefficients as in Equations (3.7) and (3.8), the electron energy equation can be written in a form as

$$P_j^i + P_j^b = P_{el}^i + P_{el}^b + P_{rad}^i + P_{rad}^b + P_{inel} \quad (3.36)$$

with the net power from inelastic collision, P_{inel} , being defined as

$$P_{inel} = n_e \left[(n_0 K_{01} - n_1 K_{10}) \epsilon_{01} + (n_0 K_{0\lambda} - n_e^2 K_{\lambda 0}) \epsilon_{0\lambda} + (n_1 K_{1\lambda} - n_e^2 K_{\lambda 1}) \epsilon_{1\lambda} \right] \quad (3.37)$$

With regular expressions for the Joule heating, the elastic collision energy, and radiation loss, Equation (3.36) becomes

$$\frac{J_y^2}{\sigma} = 3k(T_e - T_g) m_e n_e \sum_j \frac{v_{e-j}}{m_j} + q_{rad} + P_{inel}, \quad j = Ar, Ar^+ \quad (3.38)$$

Only two of the electron energy equations are needed to solve T_i and T_e and it is usually convenient to solve the final form of the global electron energy equation (3.38) and that for the tail electrons (3.24).

Various power densities in the tail electron energy equation are left to be decided. Contribution from the inelastic electron-atom collisions are obtained through integrals in Equations (3.26), (3.28), (3.30), (3.32), and (3.34). After some approximations, they become

$$\begin{aligned} P_{0j}^i &\equiv n_e n_0 b_{0j} G(T_e, T_i) \sqrt{\frac{8kT_i}{\pi m_e}} \exp\left(-\frac{\epsilon_{0j}}{kT_i}\right) \cdot (2kT_i + \epsilon_{0j}) \\ &= n_e n_0 K_{0j} (2kT_i + \epsilon_{0j}), \quad j = 1, \lambda \end{aligned} \quad (3.39)$$

$$P'_{1\lambda} \equiv n_e n_i b_{1\lambda} G(T_e, T_i) \sqrt{\frac{8kT_i}{\pi m_e}} \exp\left(-\frac{\epsilon_{1\lambda}}{kT_i}\right) \cdot (2kT_i + \epsilon_{1\lambda}) \quad (3.40)$$

Similarly, $P'_{10} \equiv n_e n_i K_{10} (2kT_e + \epsilon_{01}) \quad (3.41)$

$$P'_{\lambda 0} \equiv n_e^3 K_{\lambda 0} (2kT_e + \epsilon_{0\lambda}) \quad (3.42)$$

$$P'_{\lambda 1} \equiv n_e^3 K_{\lambda 1} \left\{ \exp\left(\frac{\epsilon_{1\lambda} - \epsilon_{01}}{kT_e}\right) \left[\epsilon_{1\lambda} \left(1 + \frac{\epsilon_{01} - \epsilon_{1\lambda}}{kT_e}\right) + kT_e \left(1 + \frac{\epsilon_{01} - \epsilon_{1\lambda}}{kT_e}\right)^2 \right] \right. \\ \left. - \exp\left(-\frac{\epsilon_{01}}{kT_e}\right) \left[\epsilon_{1\lambda} \left(1 + \frac{\epsilon_{01}}{kT_e}\right) + kT_e \left(1 + \frac{\epsilon_{01}}{kT_e}\right)^2 \right] + \epsilon_{1\lambda} \sqrt{\frac{T_i}{T_e}} G(T_e, T_i) \exp\left(-\frac{\epsilon_{01}}{kT_i}\right) \right\} \quad (3.43)$$

Expressions for power densities from the electrical field (i.e., Joule dissipation), elastic collisions, and Coulomb relaxation were derived previously^{67,75} and are summarized below:

$$P'_j = n_e f'(\epsilon_e = \epsilon_{01}) \frac{e^2 E^2}{3kT_i n_{Ar} Q_m(\epsilon_e = \epsilon_{01})} \frac{\sqrt{2\epsilon_{01}/m_e}}{(kT_i + \epsilon_{01})} \quad (3.44)$$

$$P'_{el} = n_e f'(\epsilon_e = \epsilon_{01}) \frac{2m_e}{m_{Ar}} \epsilon_{01} n_{Ar} Q_m(\epsilon_e = \epsilon_{01}) \sqrt{\frac{2\epsilon_{01}}{m_e}} (kT_i + \epsilon_{01}) \quad (3.45)$$

$$P'_c = n_e f'(\epsilon_e = \epsilon_{01}) \left(\frac{T_e}{T_i} - 1\right) (kT_i + \epsilon_{01}) n_e \frac{e^4}{8\pi\epsilon_0^2} \sqrt{\frac{2}{m_e \epsilon_{01}}} \ln \Lambda \quad (3.46)$$

where $\ln \Lambda$ is the Coulomb logarithm,

$$\Lambda = \frac{\lambda_D}{b_0} = \frac{\sqrt{\epsilon_0 kT_e / n_e e^2}}{e^2 / 4\pi\epsilon_0 \epsilon_{01}} \quad (3.47)$$

where λ_D is the Debye length, b_0 is the impact parameter for a 90° deflection angle, and ϵ_0 is permittivity of free space. In obtaining Equations (3.44) and (3.45), it is assumed that Q_m has a linear relation with the electron energy and for argon the linear coefficient is $1.4 \times 10^{-20} \text{ m}^2/\text{eV}$ in the energy range above the Ramsauer minimum.

The radiative loss from tail electrons is assumed to be negligible due to its strong dependence on the electron density which, in this case, is the tail electron density and is usually relatively small.

3.4 Results for the Critical Conditions

The critical electron density for the onset of nonequilibrium plasma is decided in the case of a stationary gas discharge. Ohm's law for a regular gas discharge is included, that is

$$J = \sigma E \quad (3.48)$$

By combining Ohm's law, any two of the particle rate equations (3.4)-(3.6), conservation of atomic nuclei (3.22), and electron energy equations for the tail and global electrons, (3.24) and (3.38), a set of six equations is established with seven unknown variables: $n_0, n_1, n_e, T_e, T_i, E$, and current density J .

According to Borghi⁶³, the critical condition corresponds to a situation when $\frac{\partial T_e}{\partial T_i} = 0$ in tail depleted electrons where $T_i < T_e$. Owing to an extremely nonlinear connection between these two temperatures, it is usually very complicated to include this extra condition as the seventh equation into the equation set. Nevertheless, it is much easier to observe the critical condition by assigning one of the variables as a coordinate axis. Then, calculate the corresponding solutions of the other six variables at each point

on the coordinates and locate where $\frac{\partial T_e}{\partial T_i} = 0$ is satisfied. At this point the corresponding electron density is the critical electron density, $n_{e,cr}$.

In the present study, $n_e \geq n_{e,cr}$ is used as the condition for the onset of nonequilibrium plasma only for a recombining plasma. For an ionizing plasma, the Joule dissipation is the only source term in the electron energy equation (2.10). Therefore, an electric field of certain magnitude is needed to support the power consumed in ionization and radiation and to sustain an elevation of electron temperature over gas temperature. The lower limit of the electric field is identified as the electric field corresponding to the critical electron density $n_{e,cr}$, and is symbolized as E_{cr} . That is, the critical conditions are

$$\begin{cases} n_e \geq n_{e,cr} \quad \text{and} \quad E \geq E_{cr}, & \text{if } \frac{dn_e}{dt} \geq 0; \\ n_e \geq n_{e,cr}, & \text{if } \frac{dn_e}{dt} < 0. \end{cases} \quad (3.49)$$

In an MHD device, the electrical field E in the critical condition (3.49) is the absolute value of J_y/σ found from Ohm's law in (2.16) for the generator or (2.19) for the accelerator respectively. It is simply the net applied electrical field $(V_{app} - V_d)/l$ in the T-layer formation section.

Including the extra condition, $\frac{\partial T_e}{\partial T_i} = 0$, there are seven equations for seven unknowns and the system of equations is closed. In other words, the critical conditions can totally be determined once the gas pressure, the gas temperature, and a characteristic length, which decides the radiative-escape factor in Equation (3.19), are assigned. The results of a sample case with argon pressure of 5×10^4 N/m², gas temperature of 1000 °K, and a plasma layer 10 cm in thickness are shown in Figure 3.4. On this plot, the critical electron density and electrical field are marked. When the electron density is

lower than the critical value, the differences between T_i and T_e sharply increase, the tail electrons are strongly depleted.

Figures 3.5 and 3.6 show the maps for $n_{e,cr}$ and E_{cr} with the characteristic length of plasma as 10 cm and 1 cm respectively. Comparison between different figures shows that the values of critical properties decrease with increasing plasma thickness due to a stronger absorption of radiation. In the same map, it is seen that the critical values are closely related to gas density, that is, a higher density (higher pressure or lower gas temperature) has larger critical values for both $n_{e,cr}$ and E_{cr} . The resultant critical electron densities for argon are generally corresponding to ionization degrees between 10^{-5} - 10^{-4} , which fall in the transition domain from non-Maxwellian to Maxwellian electrons.⁶²

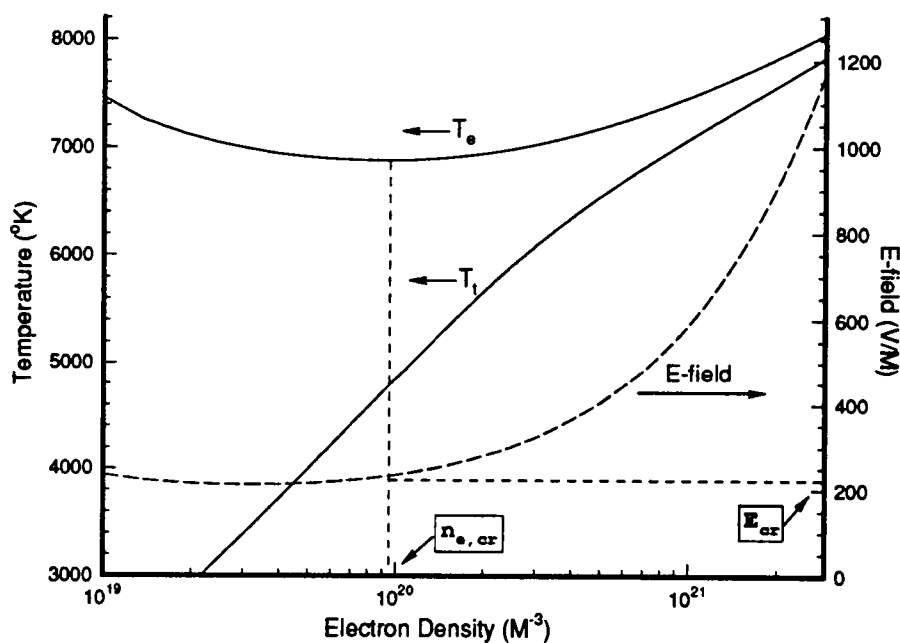


Figure 3.4 Determination of the Critical Conditions for the Onset of Nonequilibrium Plasma ($p=5 \times 10^4$ N/m², $T_g=1000$ °K, Thickness=0.1 m).

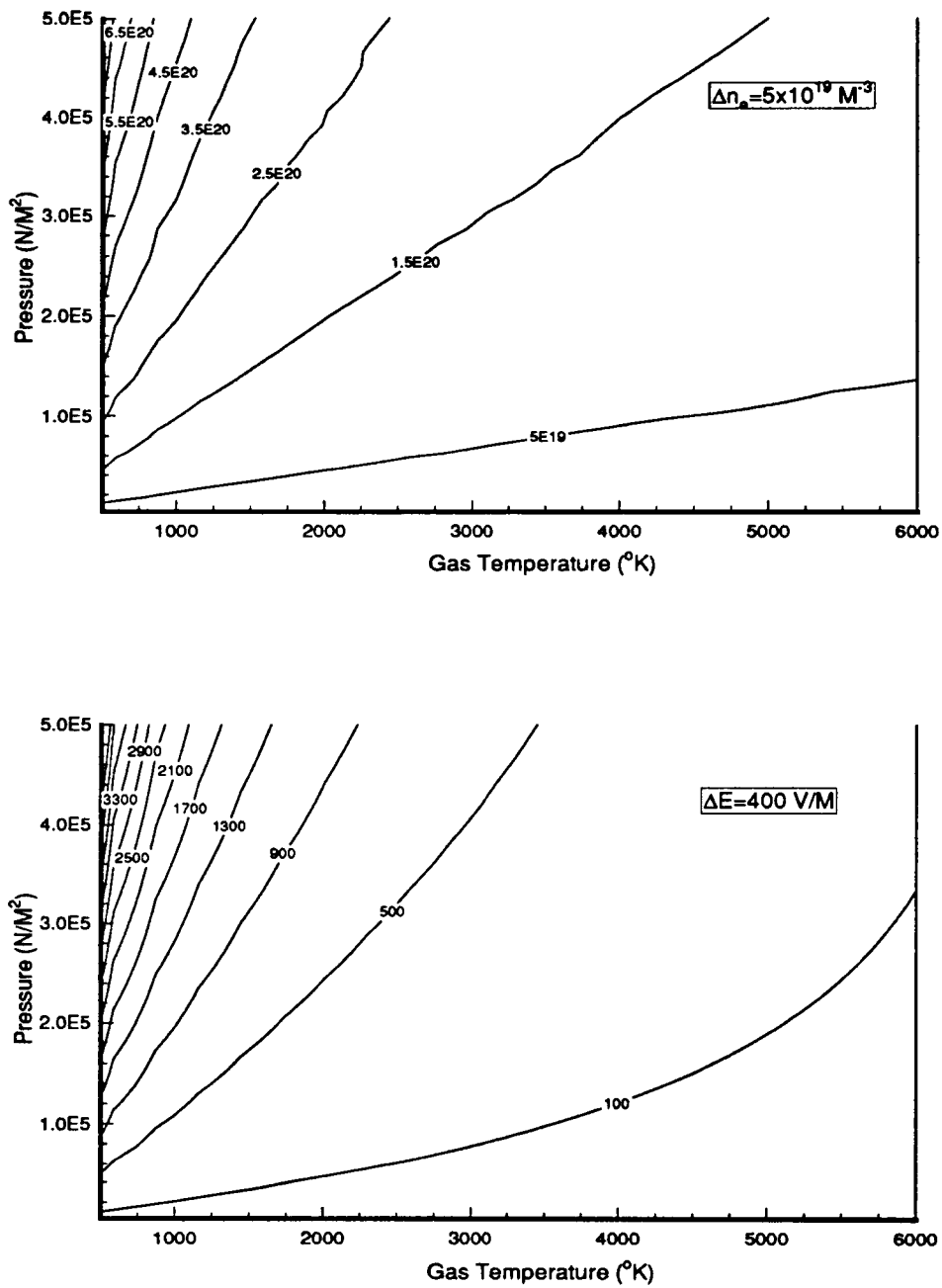


Figure 3.5 Map of the Critical Conditions for the Onset of Nonequilibrium Argon Plasma (Top - Critical Electron Density, Bottom - Critical Electrical Field; Thickness=10 cm)

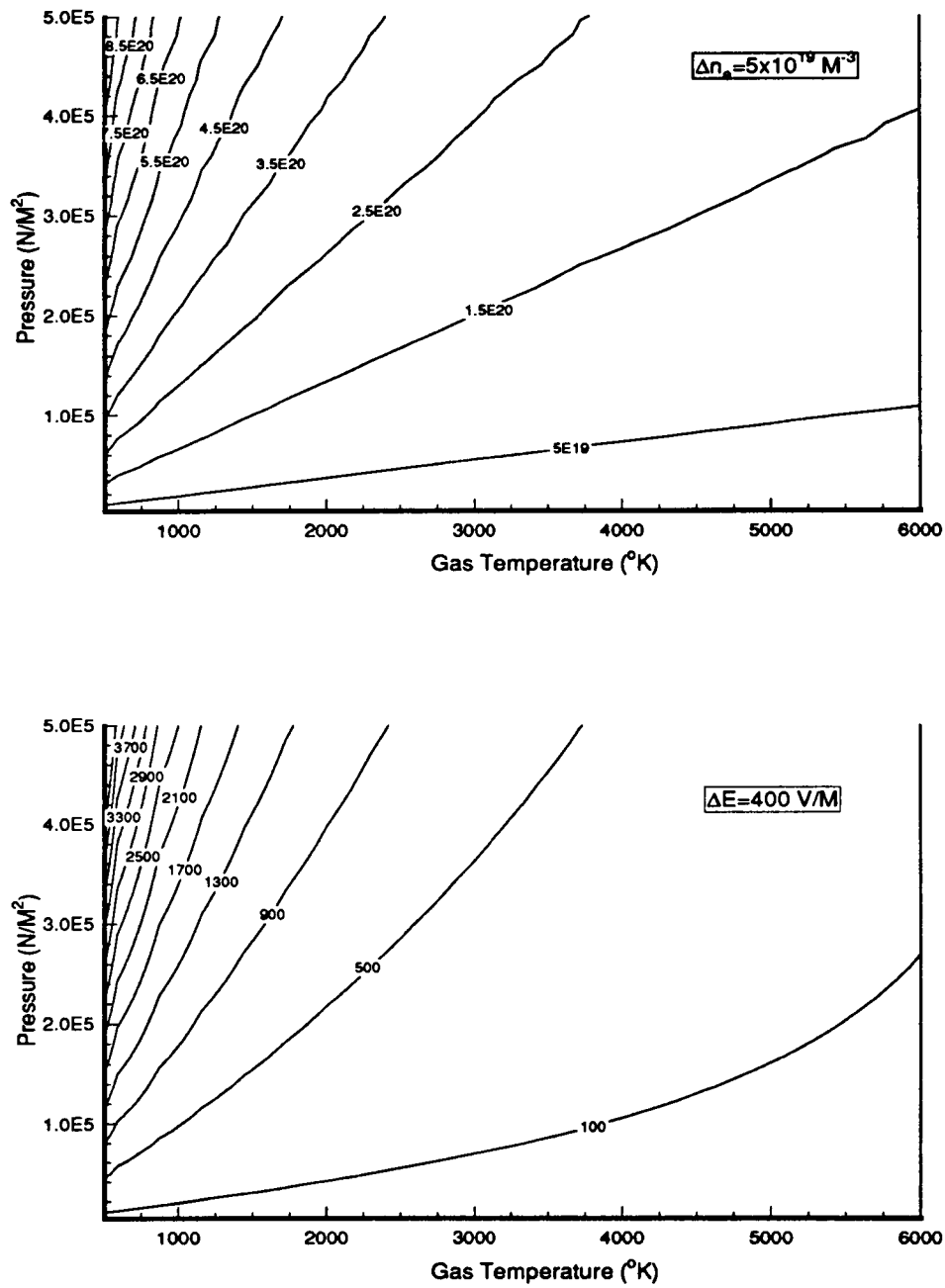


Figure 3.6 Map of the Critical Conditions for the Onset of Nonequilibrium Argon Plasma (Top - Critical Electron Density, Bottom - Critical Electrical Field; Thickness=1 cm)

As pointed out by Morgan and Vriens⁶⁷, even though the 2-EGM was originally developed for low-pressure gas discharge, it is applicable to high pressure discharges with rare gases or metal vapors. The extension of applicable pressure range of 2-EGM was further verified by a consistency between the results obtained from 2-EGM and experimental observation in typical MHD operating conditions.⁶⁵

CHAPTER 4

QUASI-STEADY MODELING

Up to now, the experimental data related to applying the T-layer with a nonequilibrium plasma in MHD devices have been very limited. Thus, it is also desirable to explore other possible means to further validate the results obtained from the unsteady model coupled with the criterion for the onset of nonequilibrium plasma developed in the preceding Chapters. For this purpose, the other T-layer model based on the previous works of Vasilyev et al.^{19,38,44} is also introduced herein.

The model of Vasilyev et al.^{19,44} was originally developed to describe T-layer interaction in an MHD generator and to predict its performance when using an equilibrium plasma. This model was later modified to include the effect of a nonequilibrium plasma by using a two-temperature model for its description. This type of modification is necessary when the plasma pressure is low and when the working gas has a small momentum transfer collision cross section with electrons. Such is the case for inert gases which are of interest for the T-layer process.

The method of Vasilyev is followed to develop a good first order model to study the T-layer power process. However, some modifications are introduced to Vasilyev's model in order to better represent the operating conditions. These modifications include:

- The contribution of the partial pressure of the electron gas to the flow static pressure is not neglected. This portion of the pressure has certain influence when the electron temperature becomes significantly higher than the gas temperature.

- Not only the plasma properties inside the T-layer are calculated, the flow field outside the T-layer perturbed by the MHD interaction is also evaluated for comparison with results found from the unsteady model with the same operating conditions.
- In the generator case, instead of keeping a constant load factor, the external load is assigned and the load factor is later calculated in every time step for the reason cited in Section 2.2.4.
- The model is extended to be applicable in describing the operation of accelerators, in which load factor is kept constant as in the unsteady model.

4.1 Additional Assumptions

In addition to the general assumptions stated in Section 1.5.1 which the unsteady T-layer modeling is based on, more assumptions made in the quasi-steady modeling are listed below.

- The T-layer is treated as an impermeable but deformable piston.
- The whole T-layer has the same speed, i.e., the local flow speed.
- The height of the MHD channel is constant.
- The strength of magnetic field is constant through the channel length.
- Only a single T-layer is considered inside an MHD channel. (No interactions between multiple T-layers are considered.)
- The effect of voltage drop across the plasma-electrode contact layer is neglected.

The assumptions of same speed, constant channel height, and constant magnetic field assure constant loading condition throughout the length of T-layer according to the definition of the load factor in (2.18) and (2.21) for the generator and the accelerator, respectively. These are written as

$$K = \frac{E_y}{uB} = \begin{cases} K_g = I R_L / u B l & \text{for generator} \\ K_a = V_{app} / u B l & \text{for accelerator} \end{cases} \quad (4.1)$$

These additional assumptions further simplify the T-layer modeling and make it amenable to quick analyses. However, to a large extent, the principal features needed to describe the physics in the T-layer interaction processes are still maintained under the assumptions.

From an order-of-magnitude analysis of the relaxation times for the different processes, Vasilyev et al.^{19,38,44} concluded that the relaxation times of ionization/recombination process and of momentum balance were both much shorter than the relaxation time of the energy balance of the heavy species, that is, $\tau_{Tg} \gg \tau_{ion}$, and $\tau_{Tg} \gg \tau_F$. Based on this, Saha equilibrium and the spontaneity of momentum relaxation were assumed to be valid in their model when concentration is on analysis of the slower process for thermal stabilization of T-layer. This is why the model is referred as the quasi-steady model.

However, the results obtained in Section 2.1 from a more detailed time scale analysis show that for argon plasma the relaxation time for heating heavy species, τ_{Tg} , is not necessary greater than both τ_{ion} and τ_F . On the contrary, τ_{Tg} is usually less than the other two relaxation times when the energy exchange between electrons and the heavy species is dominated by collisions of the Coulomb type at higher electron temperatures. Therefore, the properties of T-layer argon plasma obtained in the quasi-steady model might inherently contain certain errors from the assumptions of Saha equilibrium and the spontaneous momentum balance, especially at the high temperature region. Further discussions regarding the possible error caused by these assumptions are to be given in Section 5.1 by examining the results obtained in the sample cases.

4.2 Mathematical Model

In the quasi-steady modeling, properties inside the T-layer and the outside flow field are calculated by two separate groups of formulations. Communications between both parts go through the boundary conditions set on the upstream and downstream ends of the T-layer. That is, all the T-layer MHD interactions occur inside the clearly defined boundaries of high temperature current layer and their influences to the flow field are reflected in the property values on the boundaries. The flow field outside the T-layer is considered as an initially uniform field to be perturbed by the insertion of T-layer.

4.2.1 Local Analysis of T-layer

A normalized Lagrangian coordinate attached to the T-layer is specified in the quasi-steady model to describe the T-layer. The continuity equation can have a simpler form when expressed in this Lagrangian coordinate as

$$\frac{dx}{ds} = \frac{1}{\tilde{\rho}} = \frac{\rho_0 \delta_0}{\rho} \quad (4.2)$$

where s and x are the Lagrangian and Eulerian coordinates, respectively, and, ρ_0 and δ_0 are the gas density and the length of T-layer before MHD interaction begins. By directly integrating Equation (4.2), the T-layer thickness $\delta(t)$ can be found as

$$\delta(t) = \int_0^{\delta} dx = \rho_0 \delta_0 \int_0^1 \frac{ds}{\rho(s)} \quad (4.2a)$$

The other conservation equations are summarized in Lagrangian coordinates below,

$$\text{Momentum} \quad \tilde{\rho} \frac{dp}{ds} = J_y B \quad (4.3)$$

$$\text{Gas energy} \quad \rho \frac{dh_g}{dt} - \frac{dp_g}{dt} = 3k(T_e - T_g) m_e n_e \sum_j \frac{v_{e-j}}{m_j}, \quad j = Ar, Ar^+ \quad (4.4)$$

$$\text{Electron energy} \quad \frac{J_y^2}{\sigma} = 3k(T_e - T_g) m_e n_e \sum_j \frac{v_{e-j}}{m_j} + q_{rad}, \quad j = Ar, Ar^+ \quad (4.5)$$

where $h_g = c_p T_g$ is the enthalpy of heavy species. The continuity equation for charged particles (2.6) in the unsteady model is replaced by the Saha relation as

$$\frac{n_e^2}{n_g - n_e} = 2 \frac{g_\lambda}{g_0} \left(\frac{2\pi m_e k T_e}{h^2} \right)^{3/2} \exp\left(-\frac{\epsilon_i}{k T_e}\right) \quad (4.6)$$

The relations for the electrodynamics in the MHD channel for both the generator and the accelerator are about the same with those in the unsteady model, i.e., Equations (2.13)-(2.21), except that the integrals in relations (2.13)-(2.15) are now integrated through the T-layer thickness only. With constant load factor and constant velocity through the T-layer thickness, Ohm's law can be simplified as

$$J_y = \sigma u B (K - 1) \quad (4.7)$$

and the total current can be integrated as

$$I = \left| \int_0^\delta J_y w dx \right| = |K - 1| u B w \int_0^\delta \sigma dx \quad (4.8)$$

Combining Equations (4.1) to (4.8) with the equation of state (2.12) yields nine equations for nine unknowns, i.e., $\rho, p, T_e, T_g, n_e, \delta, J_y, I$, and K . There still remains one unknown, the flow velocity u . One more relation that is needed to solve for velocity can be obtained by matching the surplus boundary condition on the momentum equation (4.3), which has a form of first-order ordinary differential equation but with two boundary conditions given as

$$\begin{cases} s = 0, & p = p_2, & \text{at upstream boundary of T-layer;} \\ s = 1, & p = p_3, & \text{at downstream boundary of T-layer.} \end{cases}$$

In which both p_2 and p_3 are forced to match the flow gasdynamic relationships as can be easily understood by reference to Figures 4.1 and 4.2.

When treating the T-layer as an impermeable current carrying piston, the interacting body force from Lorentz effect, $\bar{\mathbf{J}} \times \bar{\mathbf{B}}$, applied to the T-layer, perturbs the flow field and a transient flow field develops. The flow structures which are generated by the MHD effects are illustrated in Figures 4.1 and 4.2 for a generator and an accelerator respectively.

Figures 4.1 and 4.2 depicts the situation wherein the T-layer is situated in the channel producing power from electromagnetic induction. The presence of a compression wave moving with velocity, w , is depicted. The traces of temperature and pressure shown in the plots of these two figures indicate the typical flow behavior across the system. Discontinuities in the flow variables are seen as the traces move across the waves and the T-layer.

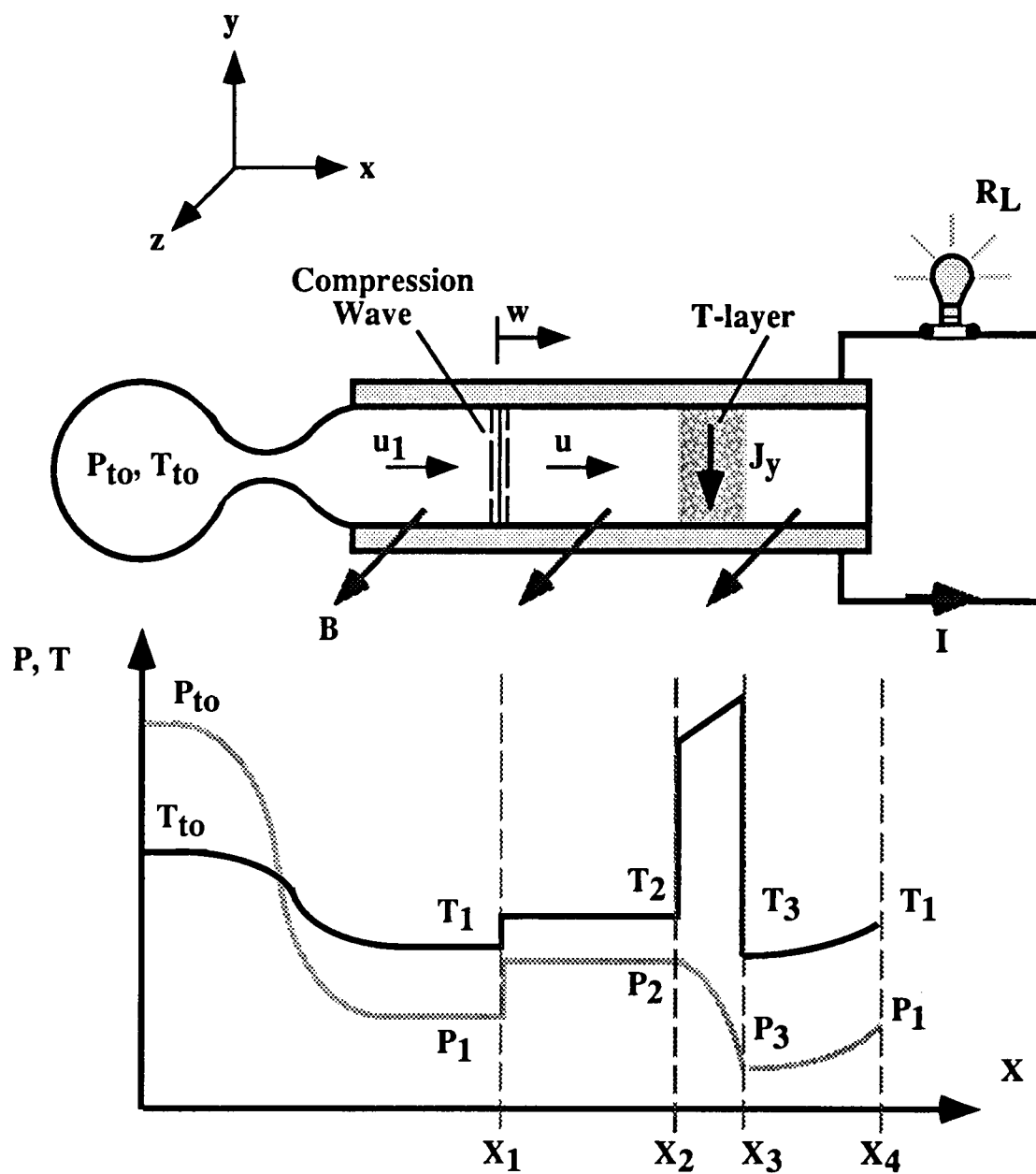


Figure 4.1 Flow Diagram for T-layer Generator

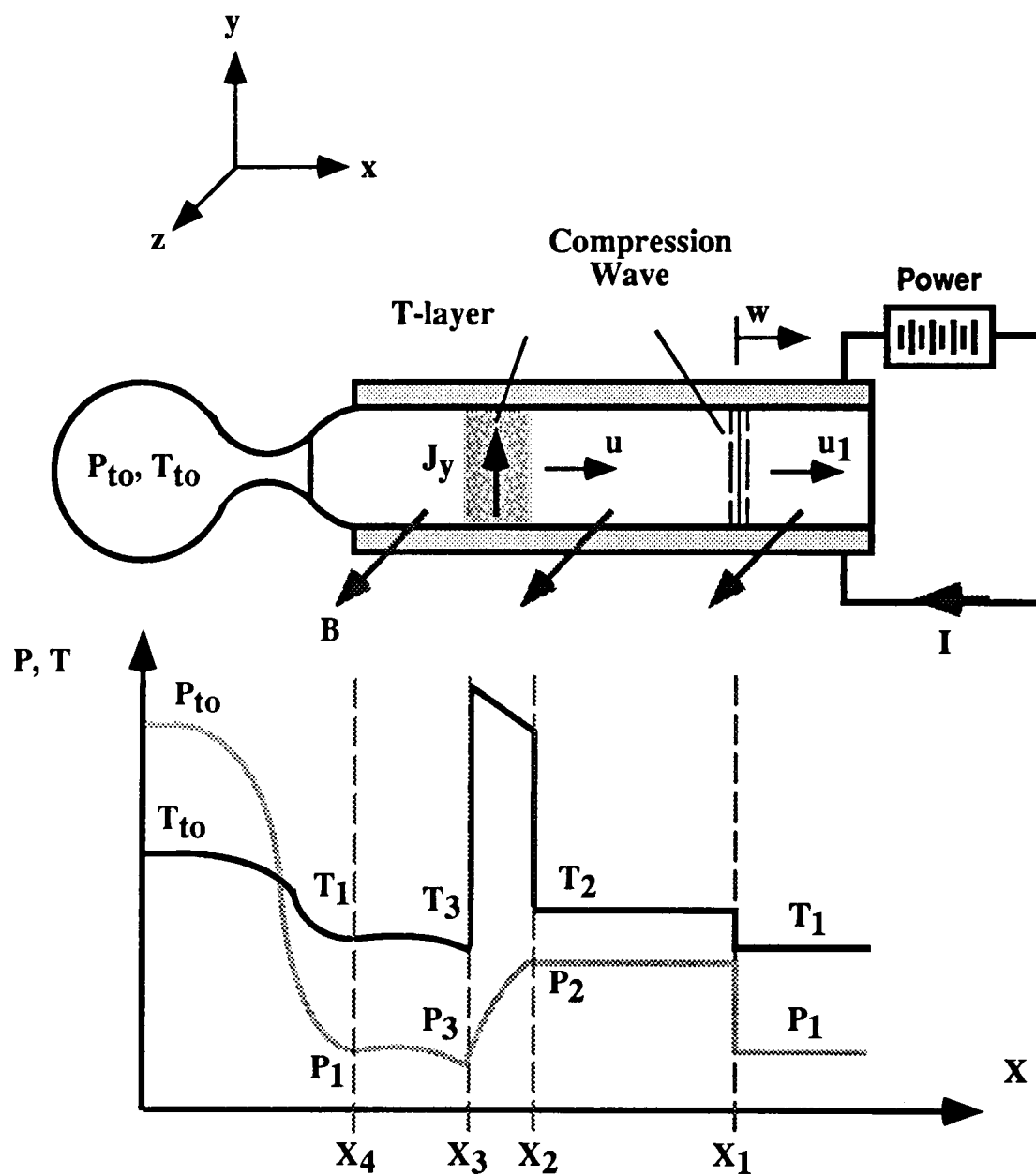


Figure 4.2 Flow Diagram for T-layer Accelerator

In a generator, the Lorentz force is acting to retard the flow. The flow transients in this case, when described in Lagrangian coordinates relative to the T-layer, are a compression wave that propagates upstream and an expansion wave that moves downstream.

Alternatively, the T-layer pushes the flow in the downstream direction in the accelerator. The flow transients then constitute a compression wave that propagates downstream and an expansion wave that moves upstream, both relative to the T-layer.

4.2.2 Perturbed Flow Field Surrounding T-layer

The surrounding flow field perturbed by the T-layer MHD interaction can be treated as analogous phenomena in a shock tube. The well-known shock tube relations can therefore be applied in the formulation of the flow field outside the T-layer. Using the notations in Figures 4.1 and 4.2, the conservation relations across a compression wave can be written as

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1} \left[\left(\frac{u_1 - w}{c_1} \right)^2 - 1 \right] \quad (4.9)$$

$$\frac{u_1 - w}{u - w} = \frac{\frac{\gamma+1}{2} \left(\frac{u_1 - w}{c_1} \right)^2}{1 + \frac{\gamma-1}{2} \left(\frac{u_1 - w}{c_1} \right)^2} \quad (4.10)$$

Also, along a characteristics of the expansion wave (right-running characteristics in a generator or left-running characteristics in an accelerator), the pressure ratio follows its isentropic relation as

$$\frac{p_3}{p_1} = \left[1 - \frac{\gamma-1}{2} \left(\pm \frac{u_1 - u}{c_1} \right) \right]^{2\gamma/(\gamma-1)} \quad (4.11)$$

where "+" of the " $\pm$ " sign is for a generator and "-" is for an accelerator. These expressions, Equations (4.9) through (4.11), contain three of the unknowns, i.e., p_2 , p_3 , and w . The entire set of governing equations describing the T-layer is therefore closed.

After solving the whole set of equations, the strength of the compression wave (p_2/p_1) has been found. The other properties behind the compression wave can be easily evaluated from the widely-known normal shock relations. The locations of the compression wave, T-layer, and expansion wave in the flow field, as those marked in Figures 4.1 and 4.2, are found by the relations summarized in Table 4.1.

Between $x_3(t)$ and $x_4(t)$ is the region of the rarefaction wave where the flow is isentropic. The properties and their locations are evaluated along the characteristics with the Riemann invariants.⁷⁸

Location	Generator	Accelerator
$x_1(t)$	$\int_0^t w(t') dt'$	$\delta_0 + \int_0^t w(t') dt'$
$x_2(t)$	$\int_0^t u(t') dt'$	$\delta_0 + \int_0^t u(t') dt'$
$x_3(t)$	$x_2(t) + \delta(t)$	$x_2(t) - \delta(t)$
$x_4(t)$	$\delta_0 + (u_1 + c_1) \cdot t$	$(u_1 - c_1) \cdot t$

Table 4.1 Equations to Determine the Temporal Locations of Different Waves in the Quasi-steady Model

4.3 Solution Procedure

In the quasi-steady model, the gas energy equation (4.4) is the only governing equation in the time-dependent form. The gas temperature calculated from this equation is advancing with time with a step length restrained from the characteristic time for heating the gas as decided in Equation (2.2). All the other flow properties are to be solved through iterative processes after updating the value of gas temperature.

At each time step, properties other than gas temperature are solved by two iterative loops in an accelerator case while there are three loops involved in a generator case. The extra loop in the generator is to match the load factor found after two loops with the assigned value of the external load.

At the inner loop, after every guess of the T-layer velocity (and the load factor too in the generator), the iteration begins with finding the boundary condition for the momentum equation (4.3) at the upstream end of the T-layer, i.e., p_2 , from solving Equations (4.9) and (4.10) together. The momentum equation, equation of state (2.12), electron energy equation (4.5), Saha relation (4.6), and Ohm's law (4.7) are then combined to iterate for the solutions of the local properties including pressure (p), gas density (ρ), electron temperature (T_e), electron density (n_e), and current density (J) at each grid.

Next is the loop to find the exact value of the T-layer velocity under the given (in an accelerator) or guessed (in a generator) load factor. In a generator, one further iteration is needed to find the correct load factor which satisfies Kirchhoff's law for the circuit.

The iterative processes are summarized in Figure 4.3. Newton-Raphson is the method applied in these iterations. A fourth-order Runge-Kutta scheme is employed to solve the time-dependent gas energy equation.

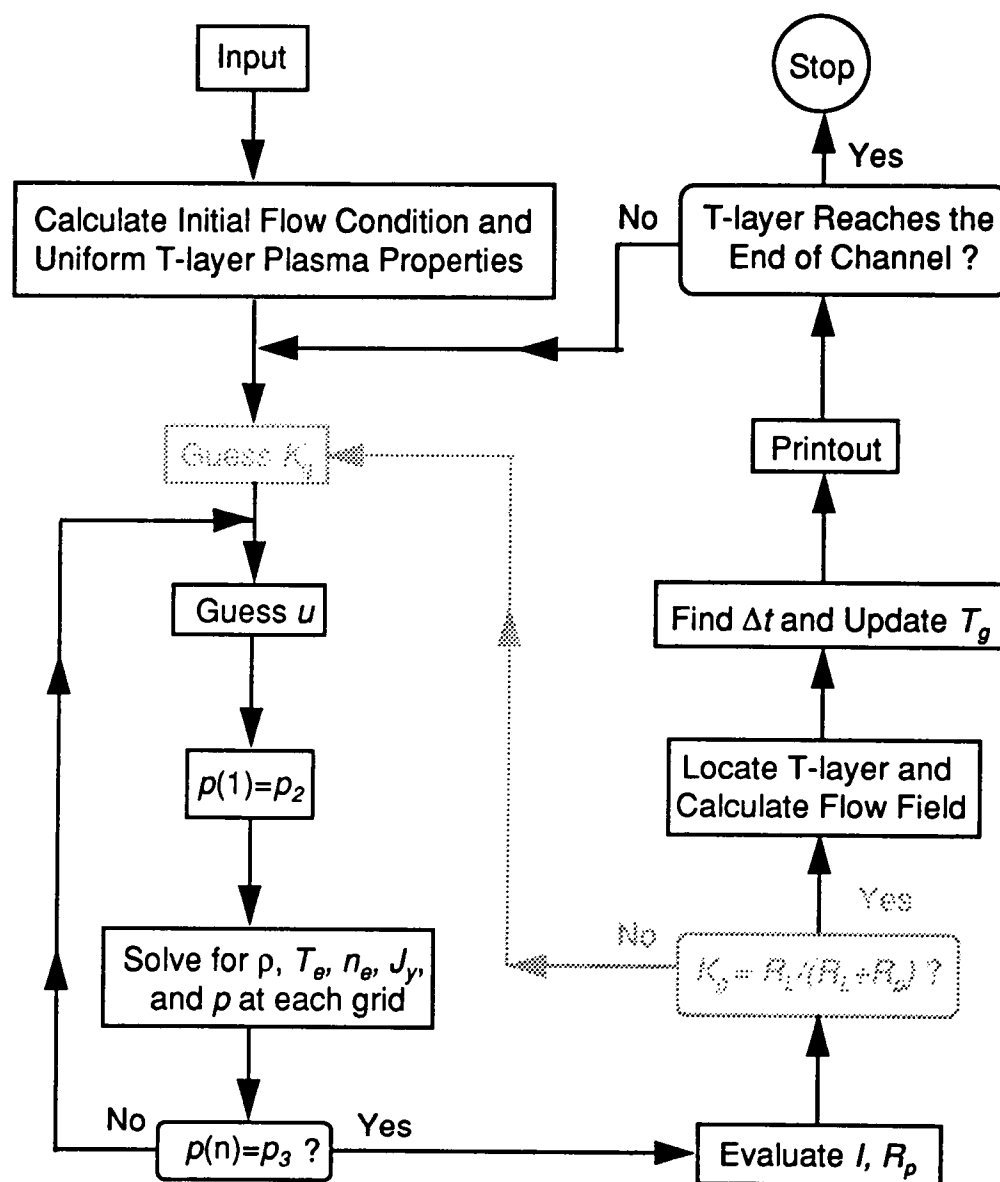


Figure 4.3 Flow Chart for the Calculation Procedure of the Quasi-Steady Model
(The Part Marked with Gray Lines are the Extra Iteration Needed in the Generator Cases.)

4.4 Performance Map

In this Section, the factors which, under the assumptions of quasi-steady model, affect the performance of MHD devices of T-layer type are grouped into a few non-dimensional parameters. The performance of this kind of generator or accelerator is then expressed in terms of these non-dimensional parameters. By doing this, a performance map can be constructed using these parameters as independent variables. After building the performance map, the performance potential of T-layer MHD devices, as well as the influence from a single parameter on the overall performance, can hence be easily seen and predicted.

4.4.1 Generator

To compare the performance of a generator, one important parameter is the enthalpy extraction ratio (E.E.R. or η_N) which is defined as the ratio of the power produced by the generator to the total power content of the T-layer flow at its creation. For a T-layer generator, it can be expressed as

$$\eta_N = \frac{I^2 R_L}{\dot{m} c_p T_{i0}} \quad (4.12)$$

where $\dot{m} = \rho_1 u_1 A$ is the mass flow rate and T_{i0} is the stagnation temperature of the unperturbed flow. Substituting relations (4.1), (4.7), (4.8), and the integrated form of momentum equation (4.3) as

$$p_2 - p_3 = \left| \int_0^\delta J_y B dx \right| = |K - 1| u B^2 \int_0^\delta \sigma dx \quad (4.13)$$

into Equation (4.12), it becomes

$$\eta_N = \frac{K_g u (p_2 - p_3)}{\rho_1 u_1 c_p T_{i0}} = \frac{\gamma - 1}{\gamma} K_g \frac{u}{u_1} \frac{p_2 - p_3}{p_{i0}} \frac{p_{i0}}{p_1} \frac{T_1}{T_{i0}}.$$

By defining a normalized pressure drop across the T-layer as $\Delta\bar{p} = (p_2 - p_3)/p_{i0}$, a normalized velocity ratio as $\bar{u} = u/u_1$ (reduced velocity for the generator or enhanced velocity for the accelerator), and applying isentropic relations to the last two terms in the above expression; the enthalpy extraction ratio can then be written in terms of non-dimensional parameters as

$$\eta_N = \frac{\gamma - 1}{\gamma} K_g \bar{u} \Delta\bar{p} \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{1/(\gamma - 1)} \quad (4.14)$$

with M_1 as Mach number of the unperturbed flow.

An MHD interaction parameter, $R_m R_h$, can be defined as the product of the magnetic Reynolds number, R_m ,

$$R_m = \mu_0 u \int_0^\delta \sigma dx$$

and the magnetic pressure ratio, R_h ,

$$R_h = \frac{B^2}{2\mu_0 p_{i0}}.$$

Using these dimensionless parameters and the integrated form of the momentum equation (4.13), an alternative form of the normalized pressure drop can be derived as

$$\Delta \tilde{p} = \frac{p_2 - p_3}{p_{10}} = \frac{|K-1| u B^2 \int_0^\delta \sigma dx}{p_{10}} = 2|K-1| \mu_0 u \int_0^\delta \sigma dx \cdot \frac{B^2 / 2\mu_0}{p_{10}},$$

that is,

$$\Delta \tilde{p} = 2|1-K| R_m R_h \quad (4.15)$$

Let $v = -(u - u_1)$ as the velocity of T-layer in a frame moving with the unperturbed flow velocity u_1 when choosing the positive direction as that the compression wave propagates relative to the T-layer. The velocity thus defined corresponds to the velocity of the contact surface in a shock tube problem with an initially stationary unperturbed flow. Based on this similarity, we can easily obtain the mathematical connection between the contact surface velocity, v , and the strength of the compression wave from the corresponding relation in a regular shock tube problem, that is

$$\frac{v}{c_1} = \frac{1}{\gamma} \left(\frac{p_2}{p_1} - 1 \right) \left(\frac{2\gamma / (\gamma+1)}{p_2 / p_1 + (\gamma-1) / (\gamma+1)} \right)^{1/2} \quad (4.16)$$

By defining

$$\kappa = v / c_1 = (1 - \tilde{u}) M_1, \quad (4.17)$$

Equation (4.16) can be used to solve for p_2 / p_1 as a function of γ and κ . Similarly, from Equation (4.11), p_3 / p_1 can also be expressed as a function of γ and κ . Therefore, the pressure drop ratio across the T-layer, $\Delta \tilde{p}$, can be derived from the expressions for p_2 / p_1 and p_3 / p_1 as

$$\begin{aligned}
\Delta\tilde{p} &= \frac{p_2 - p_3}{p_{t0}} = \frac{p_2 - p_3}{p_1} \frac{p_1}{p_{t0}} = \frac{p_2 - p_3}{p_1} \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\gamma/(1-\gamma)} \\
&= \left[1 + \gamma\kappa \sqrt{1 + \left(\frac{\gamma+1}{4}\kappa\right)^2} + \frac{\gamma(\gamma+1)}{4}\kappa^2 - \left(1 - \frac{\gamma-1}{2}\kappa\right)^{2\gamma/(\gamma-1)}\right] \\
&\quad \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\gamma/(1-\gamma)}
\end{aligned} \tag{4.18}$$

Thus, $\Delta\tilde{p}$ can be written as a function of M_1 , γ , and κ . Expressing κ in terms of M_1 and $\tilde{u}$ as its definition in (4.17), a relation as

$$\Delta\tilde{p} = f(\gamma, M_1, \tilde{u}) \tag{4.19}$$

can be easily derived.

Combining this relation (4.19) with Equations (4.14) and (4.15), η_N , $\Delta\tilde{p}$, and $\tilde{u}$ can be solved once K_g , $R_m R_h$, M_1 , and γ are assigned. Therefore, by selecting the specific heat ratio of a working fluid, γ , and the operating (unperturbed) Mach number, M_1 , a performance map for the enthalpy extraction ratio, η_N , the corresponding reduced velocity, $\tilde{u}$, and pressure drop ratio, $\Delta\tilde{p}$, can be constructed using K_g and $R_m R_h$ as independent variables.

4.4.2 Accelerator

Similar to the enthalpy extraction ratio for a generator, an enthalpy addition ratio (E.A.R. or η_a) is an important indication for the performance of an accelerator. This enthalpy addition ratio, η_a , is defined as the ratio of the pushing power added to the flow with respect to the total power content of the T-layer flow at its creation, that is,

$$\eta_a = \frac{I u B l}{\dot{m} c_p T_{t0}} \tag{4.20}$$

Similar to the derivation procedure in the generator case, using the load factor in Equations (4.1) with (4.7), (4.8), and (4.13), η_a can be expressed by the dimensionless parameters defined previously as

$$\eta_a = \frac{\gamma-1}{\gamma} \tilde{u} \Delta \tilde{p} \left(1 + \frac{\gamma-1}{2} M_1^2 \right)^{1/(\gamma-1)} \quad (4.21)$$

In an accelerator, the velocity of the contact surface across the compression wave, v , is defined similar to that in the generator case, except that in an accelerator the compression wave is moving downstream with respect to the T-layer, and hence, v has a positive value in the same direction, e.g., $v = u - u_1$. The normalized contact surface velocity κ in (4.17) then turns out to be $\kappa = v / c_1 = (\tilde{u} - 1)M_1$ and exactly the same form of relation (4.18) is applicable in an accelerator.

With the similarities of relations (4.15) and (4.19) in an accelerator and in a generator, it is obvious that, under the assumptions for the quasi-steady model, the performance of a T-layer accelerator can also be totally decided by a group of four dimensionless parameters: K_a , $R_m R_h$, M_1 , and γ . The performance map with K_a and $R_m R_h$ as variables for the enthalpy addition ratio (η_a), the corresponding enhanced velocity ($\tilde{u}$), and the pressure drop ratio ($\Delta \tilde{p}$) of an accelerator can thus be constructed when γ and M_1 are chosen.

CHAPTER 5

RESULTS AND DISCUSSIONS

In this Chapter, the performance of T-layer type MHD devices are evaluated using both the unsteady and the quasi-steady models. The results obtained from the unsteady model are first compared with that of the quasi-steady model under the same operating conditions for some sample cases. With certain modifications in the channel inlet conditions, the unsteady model is next applied to simulate laboratory experiments for a T-layer generator using an argon plasma reported by Gavrilov et al.⁸

Next, a proposed T-layer formation section is included into the numerical simulations of the unsteady model. Here, the channel can operate with multiple T-layers simultaneously. Finally, the performance maps for both T-layer generators and accelerators are constructed under the assumptions of the quasi-steady model. These are discussed to highlight the potential of the T-layer device.

5.1 Comparisons of Results Between Unsteady and Quasi-steady Models

Some cases within the typical MHD operating regime, which also show nonequilibrium effects obtained from using argon gas as working medium, are chosen as cases for calculations using both unsteady and quasi-steady models. Comparisons are made between the results obtained from the two models for the generator as well as the accelerator.

For the purpose of comparison, the initial conditions, the boundary conditions, the channel geometry, and the operating conditions remain the same in both models. The

latter two conditions are easy to fulfill in both models by assigning the same values to variables associated with the geometry and the operating conditions, including: T_{i0} and P_{i0} the stagnation temperature and pressure of the fluid at the channel inlet, M_1 the Mach number of the unperturbed flow at the inlet, B the external magnetic field, A the cross section of the channel, and R_L the external load resistance for a generator or K_a for an accelerator.

The initial conditions of the quasi-steady model as cited in Chapter 4 require an instantaneous appearance of T-layer in the MHD channel. This scenario is assumed to be valid in the unsteady model for comparisons in this Section. The initial T-layer is assumed to be a uniform gas discharge triggered by an externally applied voltage. The plasma properties at the beginning are evaluated by solving the electron energy equation (4.5), equation of state (2.12), and Saha relation (4.6) when the gas temperature, pressure, and the applied voltage are given. Also assigned, as a portion of the initial conditions, is the length of the T-layer which subsequently becomes one of the important factors to determine the initial mass of T-layer plasma.

The T-layer simulation process begins with the insertion of the uniform T-layer into the MHD channel at the moment $t = 0^-$. Right after this, the T-layer plasma adjusts itself instantaneously to fit the requirement of Kirchhoff's law of the MHD channel circuit. Spontaneous relaxation of the momentum equation is at the same time fulfilled in the quasi-steady model. The time after these instantaneous adjustments is marked as $t = 0^+$.

Treating the T-layer MHD interaction as a perturbation to the flow field (as in the quasi-steady model), the boundary conditions of non-reflecting type are applied at the inlet as well as the outlet of the MHD channel while solving the time-dependent gasdynamic equations in the unsteady model, i.e., Equations (2.6)-(2.9). A further

assumption in the comparison exercise is the negligence of voltage drop across the boundary layer.

5.1.1 Generator

A laboratory size generator is chosen as an example to observe the results from both the unsteady and the quasi-steady analyses. The operating conditions are

$$T_{i0} = 2000 \text{ }^{\circ}\text{K},$$

$$P_{i0} = 5 \text{ atm},$$

$$M_1 = 2.5,$$

$$B = 2.5 \text{ T},$$

$$R_L = 0.02 \text{ Ohm},$$

and, $\delta_0 = 0.2 \text{ m}.$

The geometry of the constant cross section channel is chosen to be a height of 1.71 cm, width of 8 cm, and a length of 2 m, of which the channel cross section is derived by matching the chosen flow Mach number with the geometry of experimental nozzle throat in Reference 8 assuming an isentropic expansion.

In the base case, the initial T-layer plasma is obtained with gas temperature equal to five times that of the unperturbed flow gas temperature but pressure is kept at the same value as the surrounding flow, i.e., $T_{gi} = 5 \times T_1$, $p_i = p_1$. The power supplied to initiate the initial uniform plasma is an external source of 15 Volts.

The distributions of various properties in the T-layer flow obtained from the unsteady model and the quasi-steady model for the base sample case are given in Figures 5.1 and 5.2 respectively. In these figures, from top to bottom, distributions of temperatures (T_e and T_g), pressure, velocity, electrical conductivity, and current density

are given at certain time steps; more details at more chosen time steps are shown in the sub-figures (a)-(e) for different properties. From the temperature distributions, the elevation of the electron temperature over the gas temperature, i.e., the nonequilibrium plasma effect is easily seen in the case chosen. The flow fields outside the T-layer, including flow pressure, velocity, and gas temperature, predicted by both models are similar. This means that the differences in the net MHD effects on the surrounding flow between two models are insignificant during the period examined.

The velocity distributions inside the T-layer obtained by the unsteady model are approximately uniform across the T-layer as shown in Figure 5.1.(c), which is consistent with the assumption of the impermeable piston model with the same velocity over the T-layer in the quasi-steady analysis. Inside the T-layer, pressure decays from the upstream end to a value much lower at the downstream end in order to balance the Lorentz force exerted on the T-layer. The magnitude of pressure drop across the T-layer in the unsteady model becomes stable after a duration of $3-5 \times 10^{-4}$ second as given in Figure 5.3. Whereas, the pressure drop obtained in the quasi-steady model converges immediately after the interaction begins due to a spontaneous relaxation of the momentum equation as is assumed in the model.

Also given in Figure 5.3 is the comparison of average local T-layer velocity as calculated by both models for the base case examined. The average velocity evaluated in the unsteady analysis is the mass average velocity of the T-layer. The velocity thus obtained in the unsteady model is shown to reach a condition of stable operation after a duration in the same order as that of pressure. In the stable operation of T-layer generator, the flow kinetic energy exhausted in the MHD interaction by the retarding Lorentz force is mainly reflected in the expansion of volume of the decelerated fluid as shown in Figures 5.1.(c) and 5.2.(c).

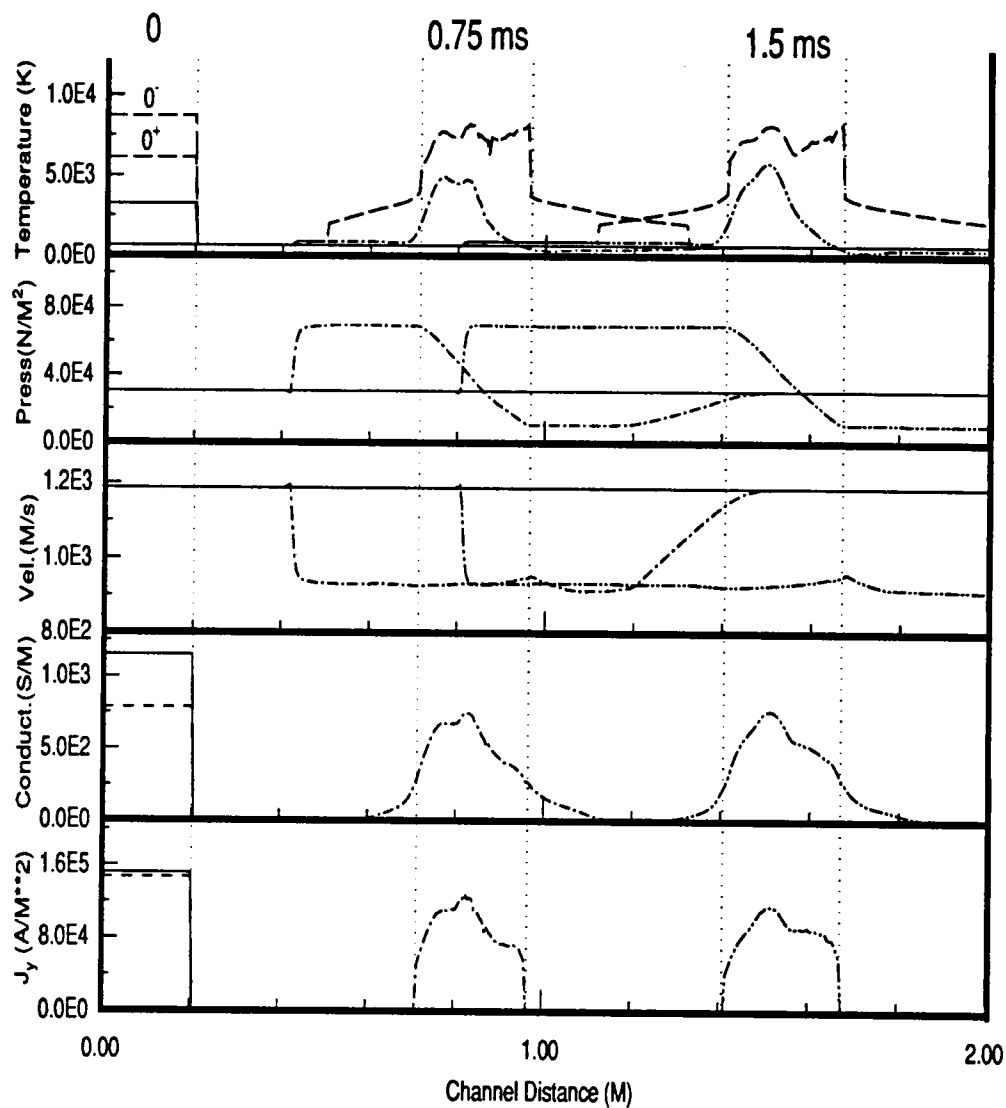
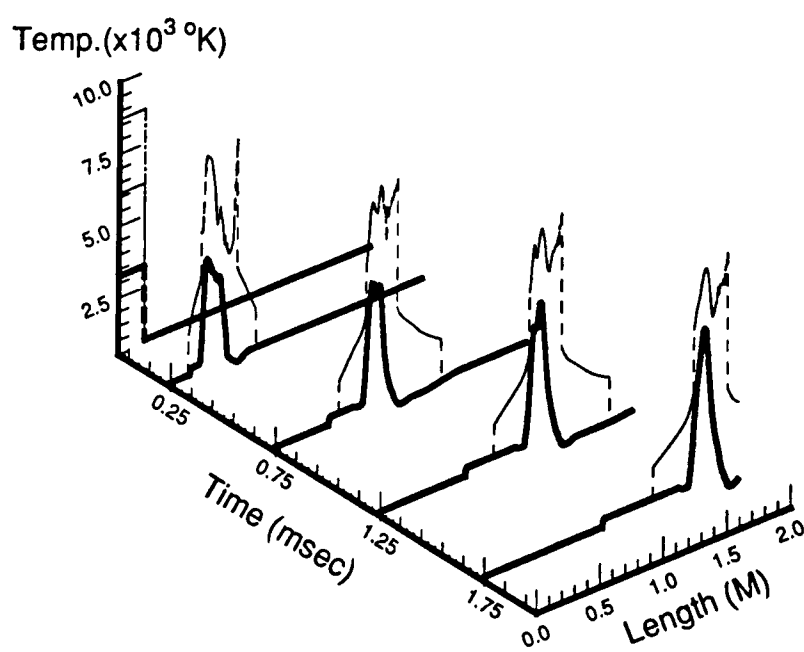
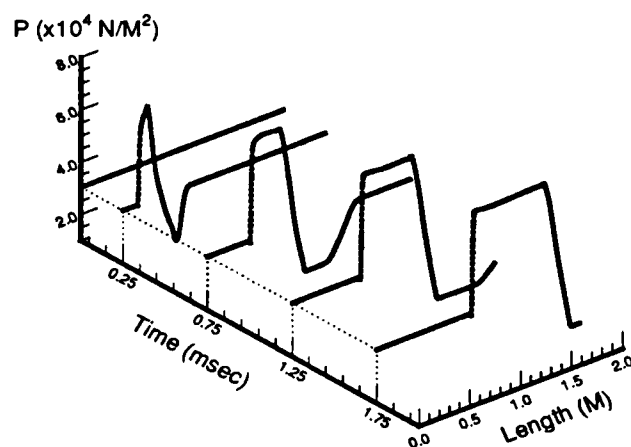


Figure 5.1 Property Distributions of the Base Case T-layer Generator in the Unsteady Model. From Top to Bottom: Electron (---) and Gas Temperatures, Pressure, Velocity, Conductivity, and Current Density.
(—: $t = 0^+$, ---: $t = 0^+$, -.-: $t = 0.75$ ms, - - -: $t = 1.5$ ms.)

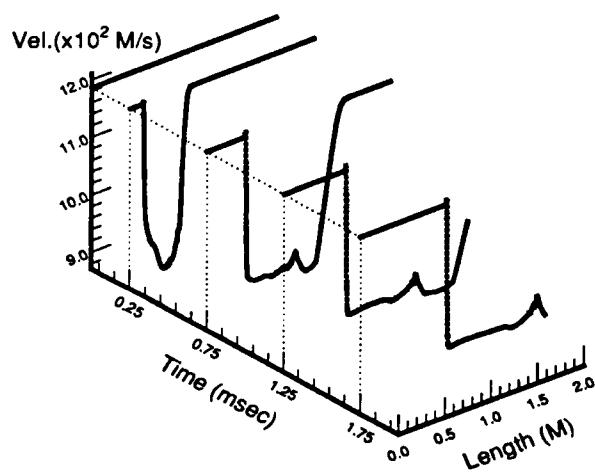


(a) Electron (---) and Gas (—) Temperatures

Figure 5.1 (Continued)

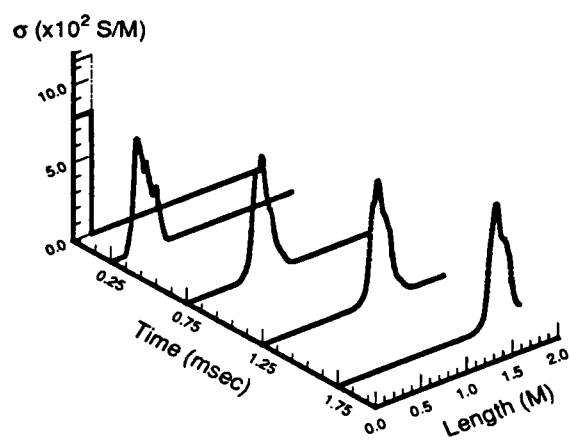


(b) Pressure

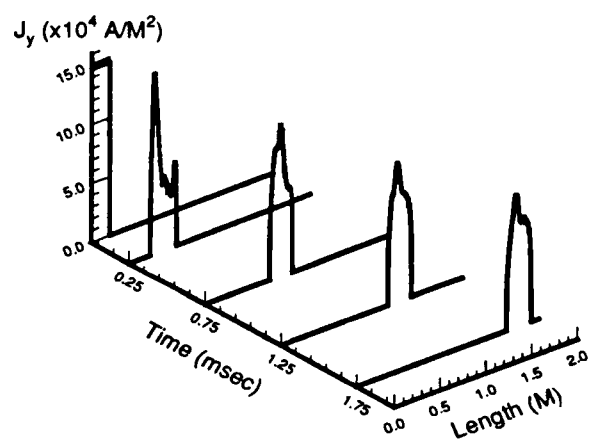


(c) Velocity

Figure 5.1 (Continued)



(d) Electrical Conductivity



(e) Current Density

Figure 5.1 (Concluded)

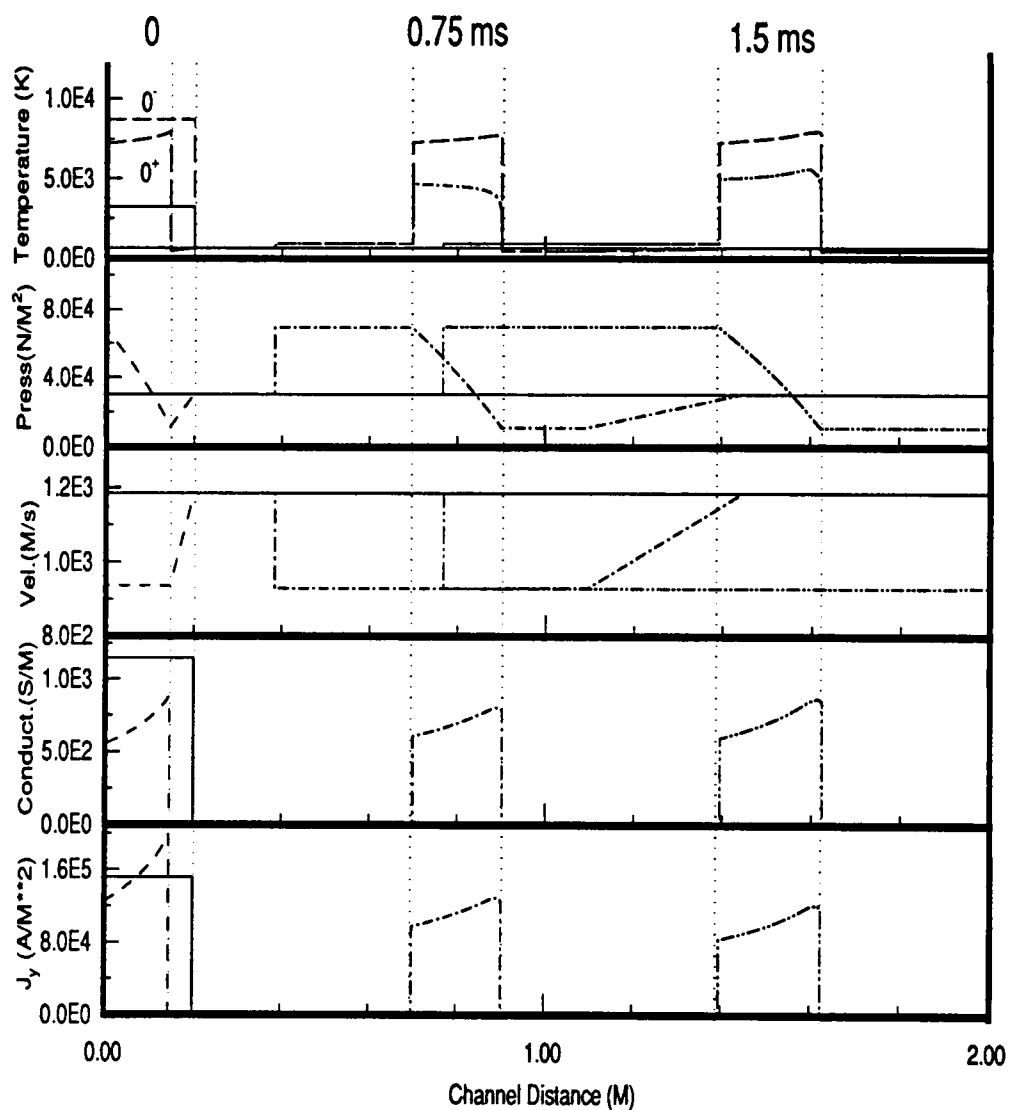
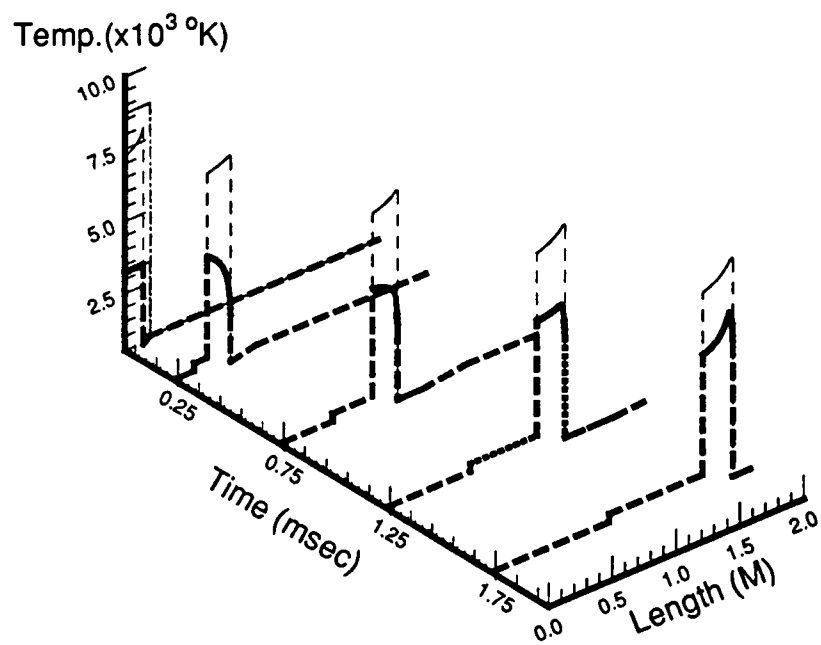
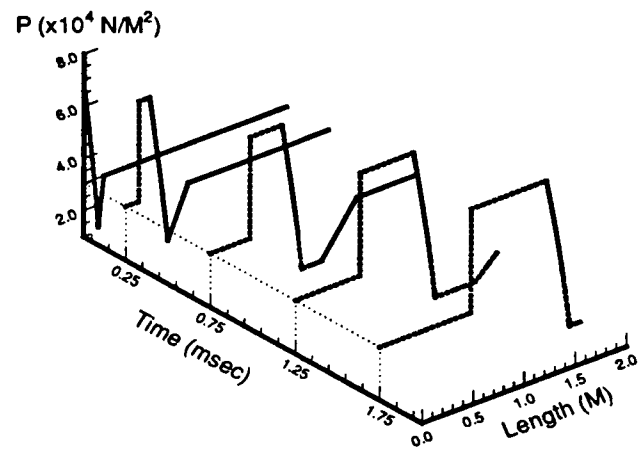


Figure 5.2 Property Distributions of the Base Case T-layer Generator in the Quasi-steady Model. From Top to Bottom: Electron (---) and Gas Temperatures, Pressure, Velocity, Conductivity, and Current Density.
(—: $t = 0^-$, ---: $t = 0^+$, -.-: $t = 0.75$ ms, ...: $t = 1.5$ ms.)

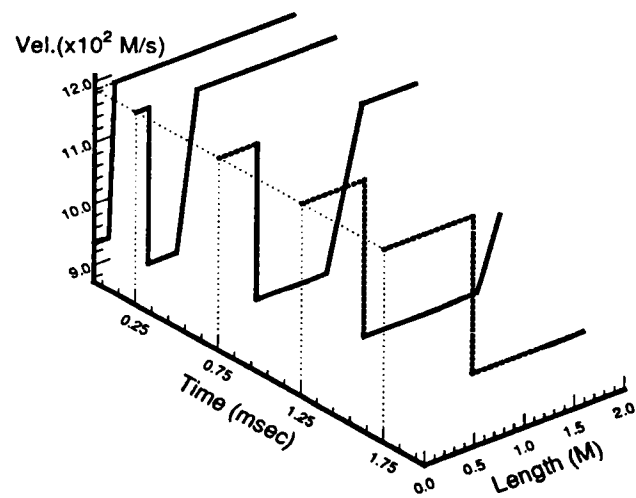


(a) Electron (---) and Gas (—) Temperatures

Figure 5.2 (Continued)

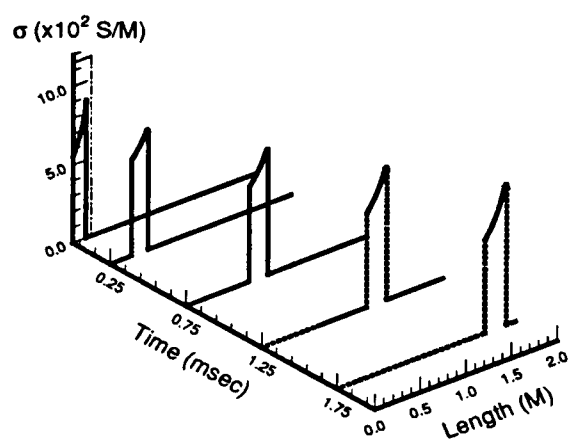


(b) Pressure

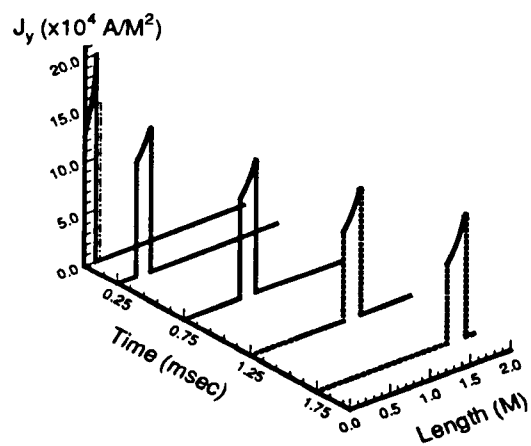


(c) Velocity

Figure 5.2 (Continued)



(d) Electrical Conductivity



(e) Current Density

Figure 5.2 (Concluded)

More evidence regarding the arrival of a stable operating regime a certain period ($3-5 \times 10^{-4}$ second in this example) after the T-layer MHD interaction begins in the unsteady analysis can be seen from the temporal results given in Figures 5.4. Similar to the pressure drop and the average velocity of T-layer examined in Figure 5.3, the results given in Figure 5.4.(b) show an immediate convergence to the stable operation in the quasi-steady model. Nevertheless, after the stable operation is reached, the performance of the T-layer generator including enthalpy extraction ratio, load factor, total current (Figure 5.4), and the energy output (Figure 5.5) predicted by both models are very close to each other in the sample case - a similar conclusion as that obtained in the comparison of the outside flow field disturbed by the T-layer between the two analyses.

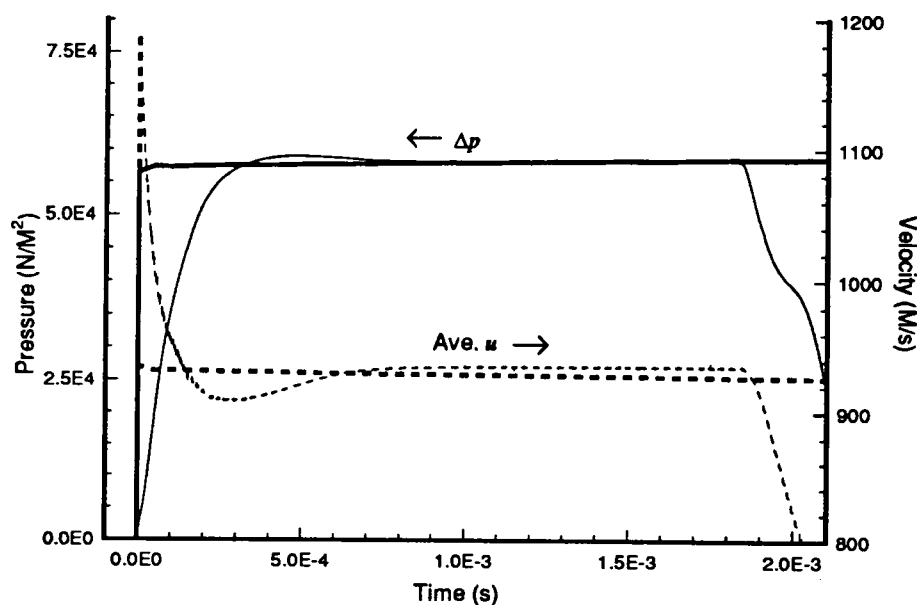


Figure 5.3 Pressure Drop Across the T-layer and Average Velocity of the T-layer in the Generator of Base Case.

(Bold Lines - Quasi-steady Analysis, Thin Lines - Unsteady Analysis)

The continuing increase in the value of interaction parameter shows that the MHD interaction gets stronger and the pressure drop across the T-layer increases. This lowering of pressure at the T-layer's downstream side causes an increase in the T-layer length. Certain, but not serious in this example, discrepancies in the calculated values of MHD interaction parameter between two different models are primarily due to the assumption of an impermeable piston model of T-layer in the quasi-steady analysis. Under this assumption, the plasma mass remains constant. As a result, a higher volume expansion rate, especially at the low pressure end, is expected once the gas is heated. This fact can be easily seen from comparison in the growth rates of T-layer thickness evaluated by the two different models given in Figure 5.6, in which the oscillations in the T-layer length seen in the results of the unsteady model are caused by the finite grid size used in the calculations.

Relative to the impermeable piston model assumed in the quasi-steady analysis, the integrated mass of the T-layer plasma obtained in the unsteady model varies during the MHD process as shown in Figure 5.7. In the example, the mass inside the T-layer increases from the initial value to about a factor of two in the stable operating region. The increase in the mass means a relatively higher gas density distribution in the T-layer plasma compared to that with a constant mass if the T-layer volume is about the same.

From Figures 5.1 and 5.2, inside the current layer, there are some differences in the resultant profiles of electron temperature, gas temperature, electrical conductivity, and current density between the two models. Saha equilibrium assures that the electron density closely relates to the electron temperature, which subsequently results in similar trends in the relation between the electrical conductivity and the electron temperature obtained in the quasi-steady model.

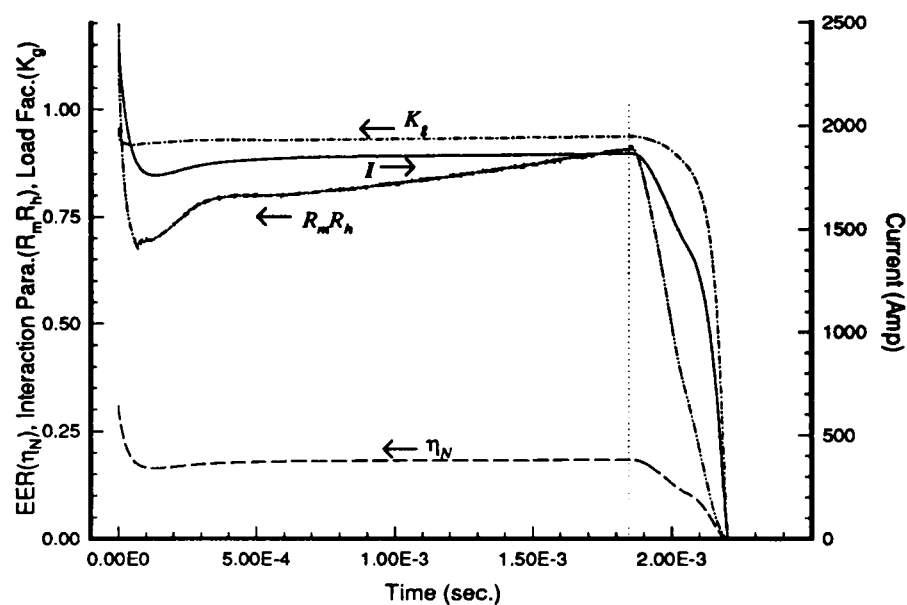
In this model, once the pressure is lower, particle densities are less, which helps reduce the radiative loss as well as promotes the local conductivity through a reduction

in the electron collision frequency. The latter effect passes more current which provides more local Joule heating. Therefore, the electron temperature, electron density, electrical conductivity, and current density all have greater values at the low-pressure downstream end in the quasi-steady model.

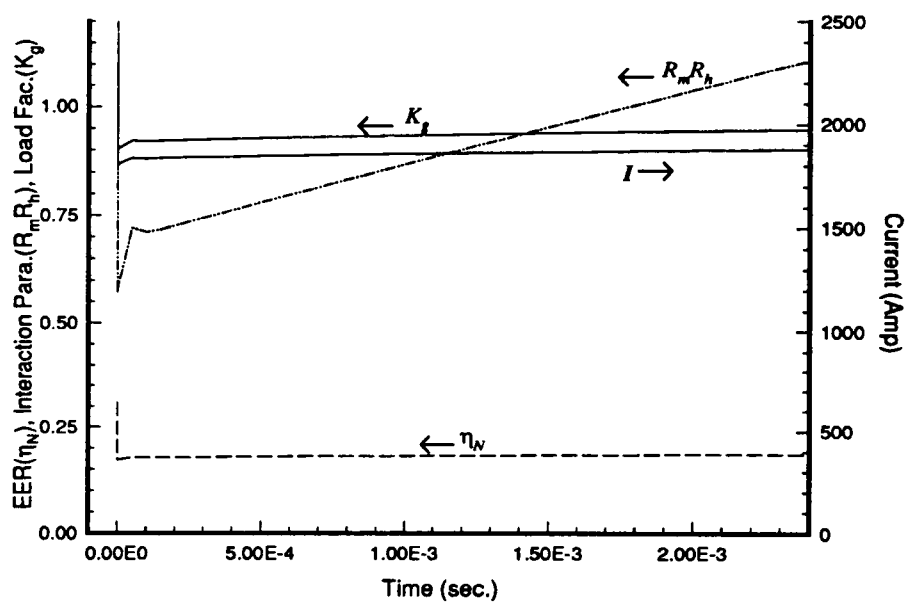
In the unsteady analysis, the finite rate of ionization results in a lower electron density near the downstream end of the T-layer (relative to that obtained in the quasi-steady model), even though the local electron temperature is higher compared to that in the central part of the T-layer plasma. The existence of a lower electron density and a higher particle density obtained in the unsteady model forms a more resistive plasma near the downstream end of the T-layer which subsequently results in a relatively lower current density locally.

A more obvious discrepancy between the results of the two models is in the gas temperature distribution. The much lower gas temperature near the downstream end of the T-layer predicted by the unsteady model is due to a combination of a short heating time, a relatively low local current density, and a low frequency of energy exchange collisions with electrons.

Therefore, we can conclude that, if the operating conditions remain the same, the plasma near the downstream end of T-layer calculated in the quasi-steady analysis usually has a lower pressure but higher plasma intensity when compared to that obtained in the unsteady model. This discrepancy in the results between the two models is primarily due to the assumptions made in the quasi-steady model including Saha equilibrium, the impermeable piston, and the spontaneous relaxation of force balance. The constant mass of plasma inside the T-layer under the impermeable piston model further expands the plasma volume at the downstream side of T-layer due to locally higher heating rates. As a result, the MHD interaction parameter generally increases faster in the quasi-steady model.

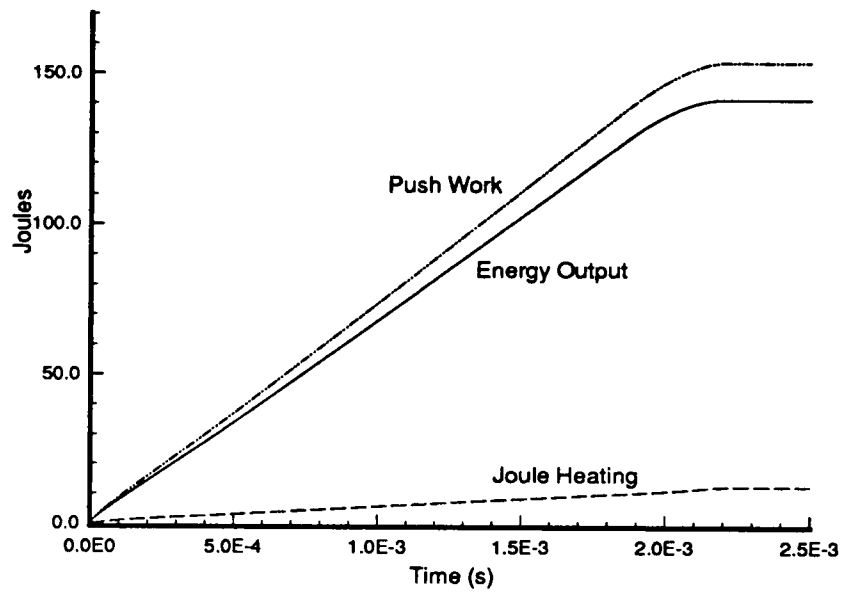


(a) Unsteady Model

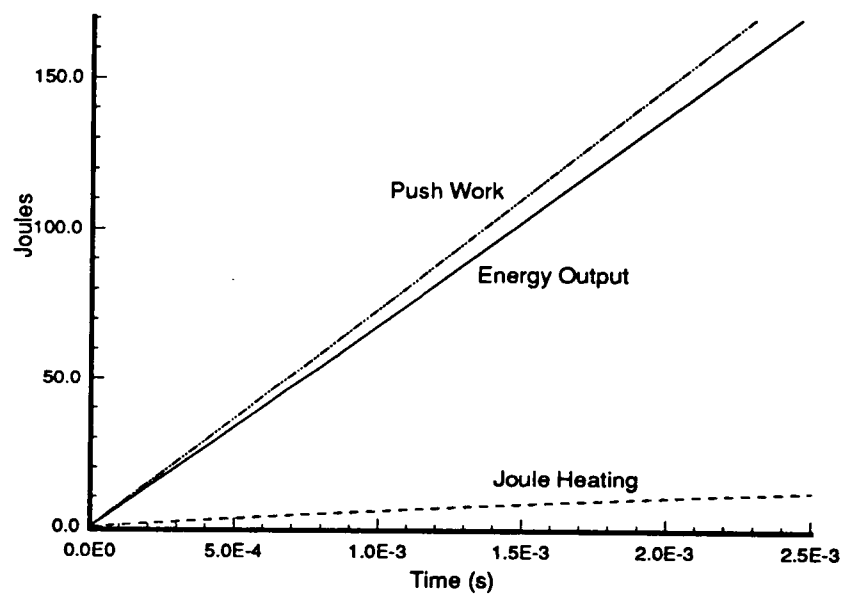


(b) Quasi-steady Model

Figure 5.4 Performance of the Base Case T-layer Generator

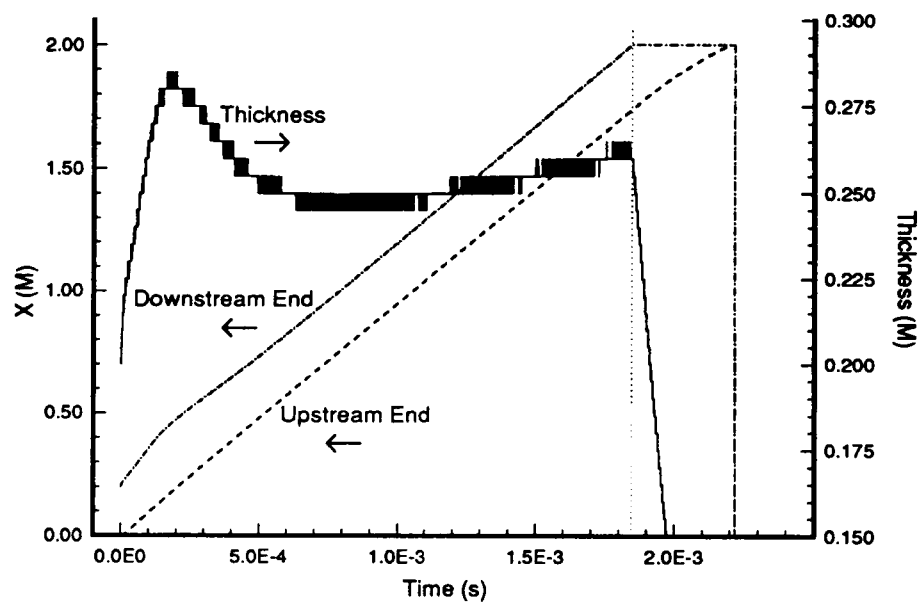


(a) Unsteady Model

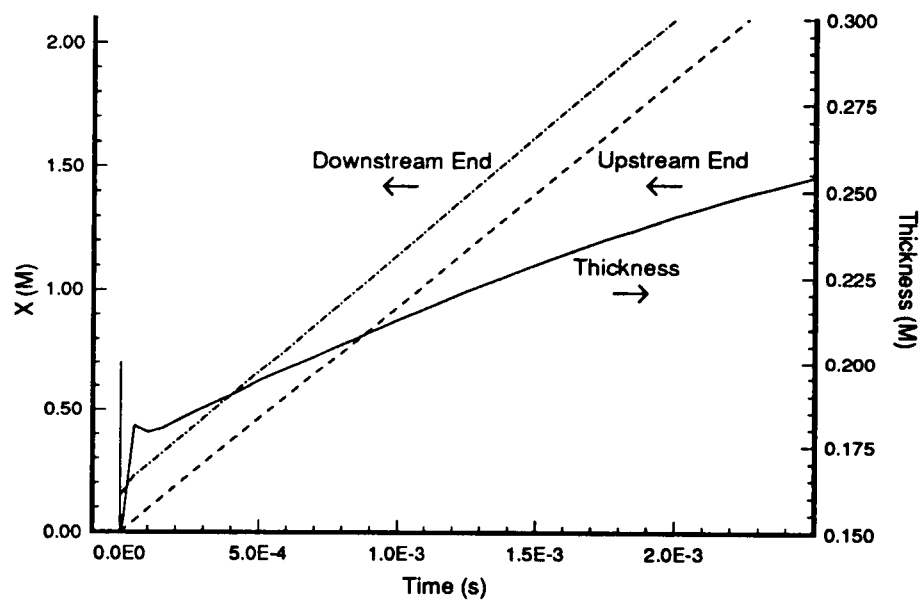


(b) Quasi-steady Model

Figure 5.5 Energy Balance in the Base Case T-layer Generator



(a) Unsteady Model



(b) Quasi-steady Model

Figure 5.6 Thickness and Traces of the T-layer in the Base Case Generator

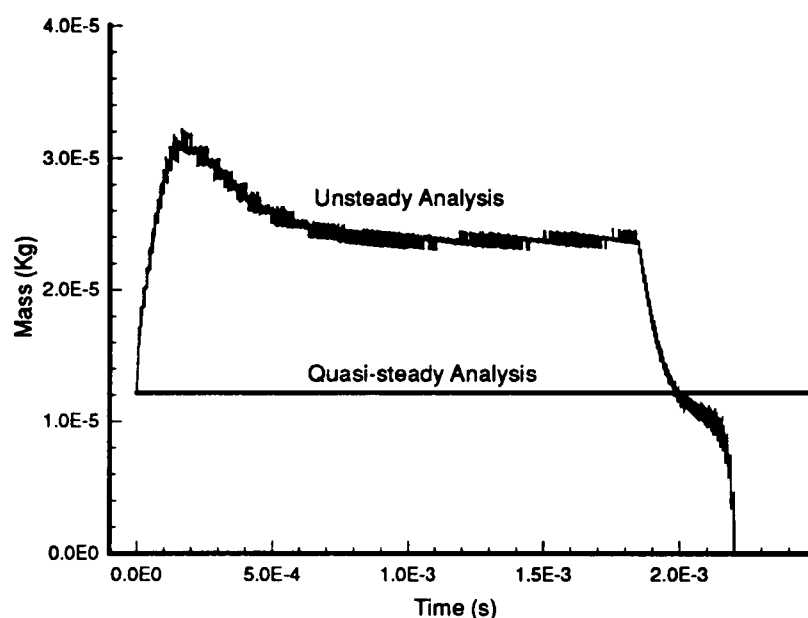


Figure 5.7 Mass of the T-layer in the Base Case Generator

The over-estimate in the MHD interaction rate in the quasi-steady model makes the T-layer plasma more prone to the development of electrothermal instabilities, especially at the low-pressure T-layer's downstream end. As a matter of fact, following the higher rate of increasing MHD interaction, the electrothermal instability will eventually occur in the quasi-steady analysis if calculation of the T-layer MHD process continues for a longer time by setting a longer channel length. This phenomena was previously observed in the example calculated by Vasilyev et al.³⁸ using a model similar to the quasi-steady model described in Chapter 4. With a lower stagnation pressure (~ 3 atm) and a higher magnetic field (3 T) relative to the sample case that we examine, the results of Vasilyev et al.³⁸ showed that electrothermal instabilities could occur at the T-layer's downstream side no later than 1 millisecond after the interaction began.

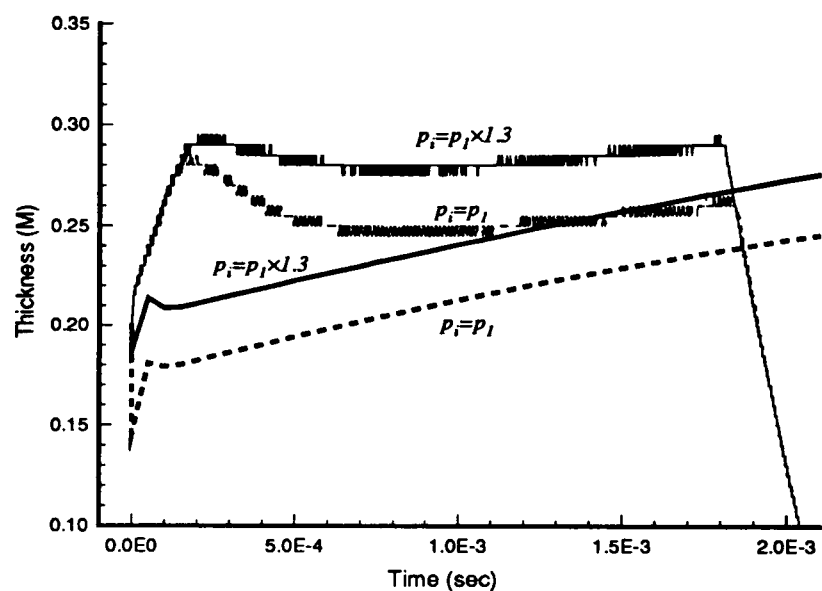


Figure 5.8 Influence of Initial Pressure on the T-layer Thickness in a Generator
(Bold lines - Quasi-steady Analysis, Thin Lines - Unsteady Analysis)

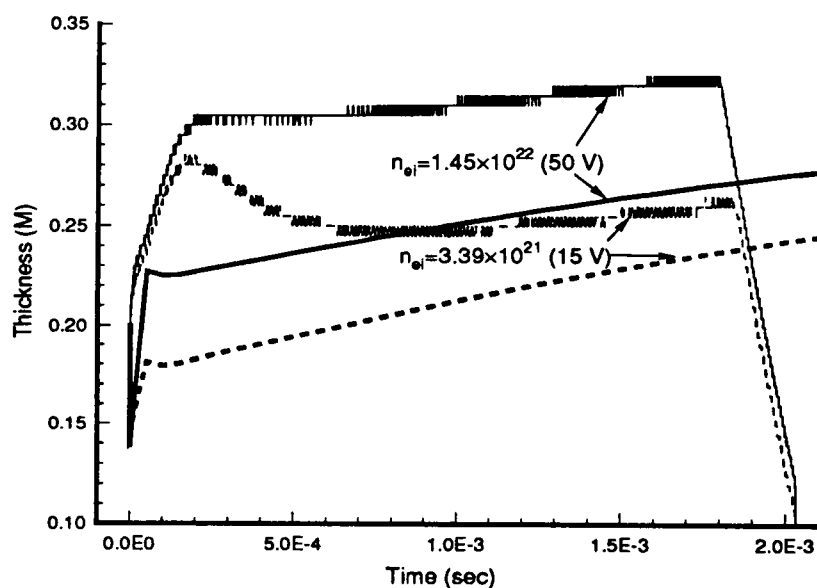


Figure 5.9 Influence of Initial Electron Density on the T-layer Thickness in a Generator
(Bold lines - Quasi-steady Analysis, Thin Lines - Unsteady Analysis)

To understand the factors which may affect the thickness of the T-layer in the region of stable operation, additional calculations are made using both models. After a series of numerical experiments, it was found that the initial pressure and electron density have significant effect on the stable T-layer thickness in the generator. Figure 5.8 shows the increase of the T-layer thickness from the influence of a 30% increase in the initial pressure inside the T-layer compared to the base case. Figure 5.9 then gives the increase of the T-layer thickness due to an initial higher electron density, in which the greater initial electron density is obtained by raising the applied voltage for triggering the initial uniform plasma from 15 to 50 Volts.

5.1.2 Accelerator

A channel with the same cross section as in the generator case is now applied in the calculations of a T-layer accelerator using both the unsteady and the quasi-steady models. The operating conditions are

$$T_{i0} = 2000 \text{ }^{\circ}\text{K},$$

$$P_{i0} = 8 \text{ atm},$$

$$M_1 = 2.5,$$

$$B = 1.0 \text{ T},$$

$$K_a = 1.2,$$

$$\text{and,} \quad \delta_0 = 0.2 \text{ m}.$$

The initial T-layer plasma properties in the base case are obtained with $T_{gi} = 3 \times T_1$, $p_i = p_1$, and a power source of 15 Volts applied on the channel electrodes.

The channel length is elongated to 3 m.

The results for distributions of different properties at certain chosen moments after the appearance of T-layer in the accelerator channel are shown in Figures 5.10 and 5.11 for the unsteady and the quasi-steady models, respectively. The results show that significant nonequilibrium effects in the plasma are obtained in both models for the case chosen. Also, the flow is indeed accelerated by the T-layer MHD interaction and the region being accelerated to higher flow speed spreads in the channel with time before the compression wave exits the channel.

Even though the flow field surrounding the T-layer predicted by both models has similar phenomena, such as the existence of compression and expansion waves and their propagating directions and speeds in the channel etc., velocity distribution inside the T-layer calculated by the unsteady model deviates to a certain degree from a constant value as assumed in the quasi-steady model. When comparing results given in Figure 5.12 for the two models, stronger MHD interaction is observed in the quasi-steady analysis which consequently results in a compression wave with stronger strength and a greater pressure drop across the T-layer, as is seen in Figure 5.13. The pressure drops from the downstream end to the upstream end inside the T-layer in the accelerator to balance the induced Lorentz force. The results obtained in the unsteady analysis show that stable operation of the MHD channel is reached $3\text{--}5 \times 10^{-4}$ second after the MHD interaction begins in the sample accelerator.

Comparing with the generator case examined in the previous sub-section, relatively greater deviations between the results of the unsteady model and that of the quasi-steady model are observed in the sample accelerator case. It is believed that the assumption of using a constant value for the load factor in the accelerator is the primary cause for the greater deviations rather than using a constant load as was done in the generator. The latter loading scheme usually helps stabilizing the MHD operation in the T-layer devices.

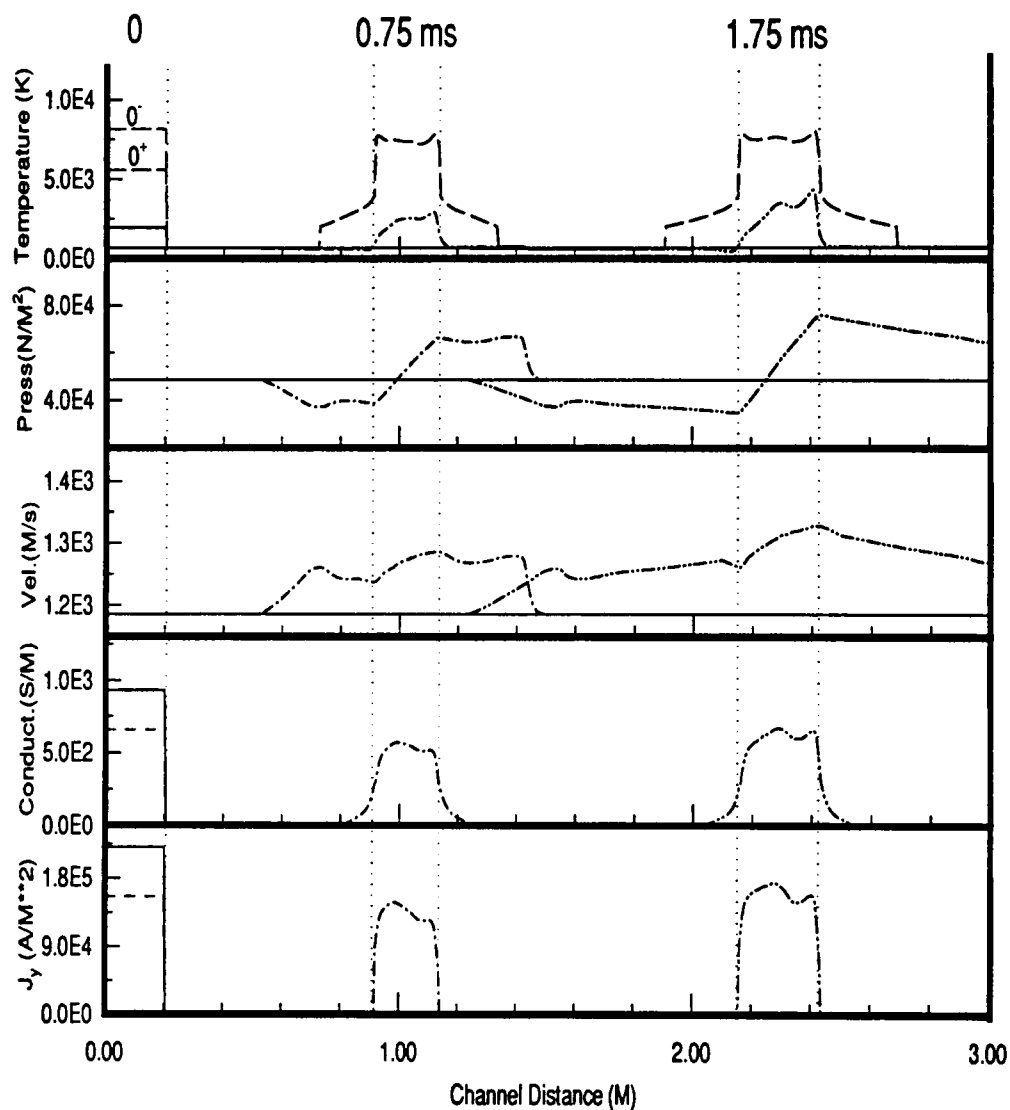
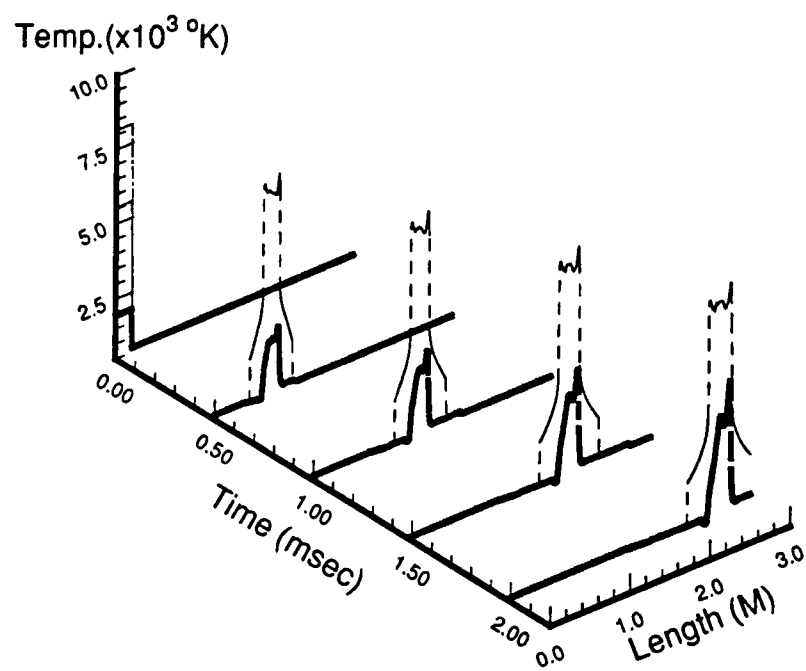
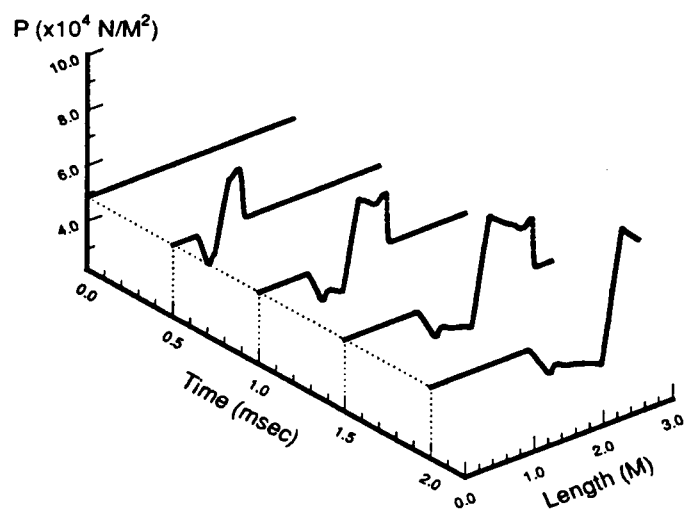


Figure 5.10 Property Distributions of the Sample T-layer Accelerator in the Unsteady Model. From Top to Bottom: Electron (---) and Gas Temperatures, Pressure, Velocity, Conductivity, and Current Density.
(—: $t = 0^-$, ---: $t = 0^+$, -.-: $t = 0.75$ ms, - - - : $t = 1.75$ ms.)

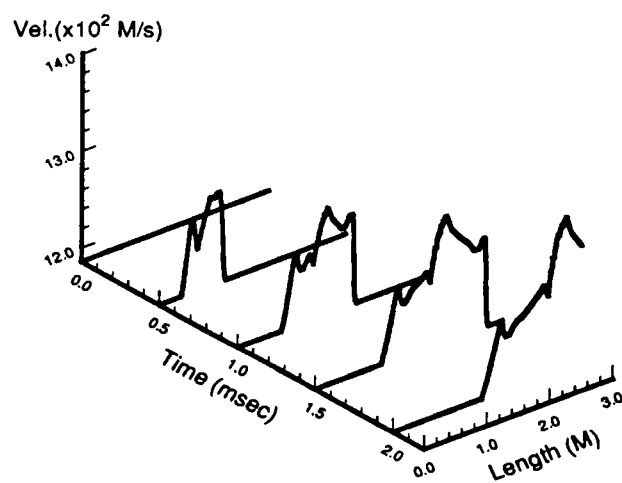


(a) Electron (---) and Gas (—) Temperatures

Figure 5.10 (Continued)

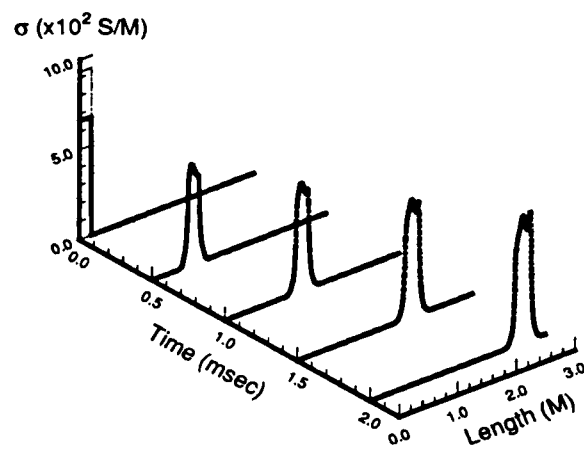


(b) Pressure

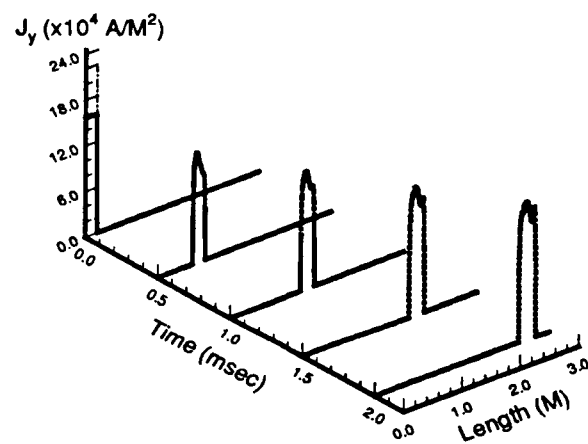


(c) Velocity

Figure 5.10 (Continued)



(d) Electrical Conductivity



(e) Current Density

Figure 5.10 (Concluded)

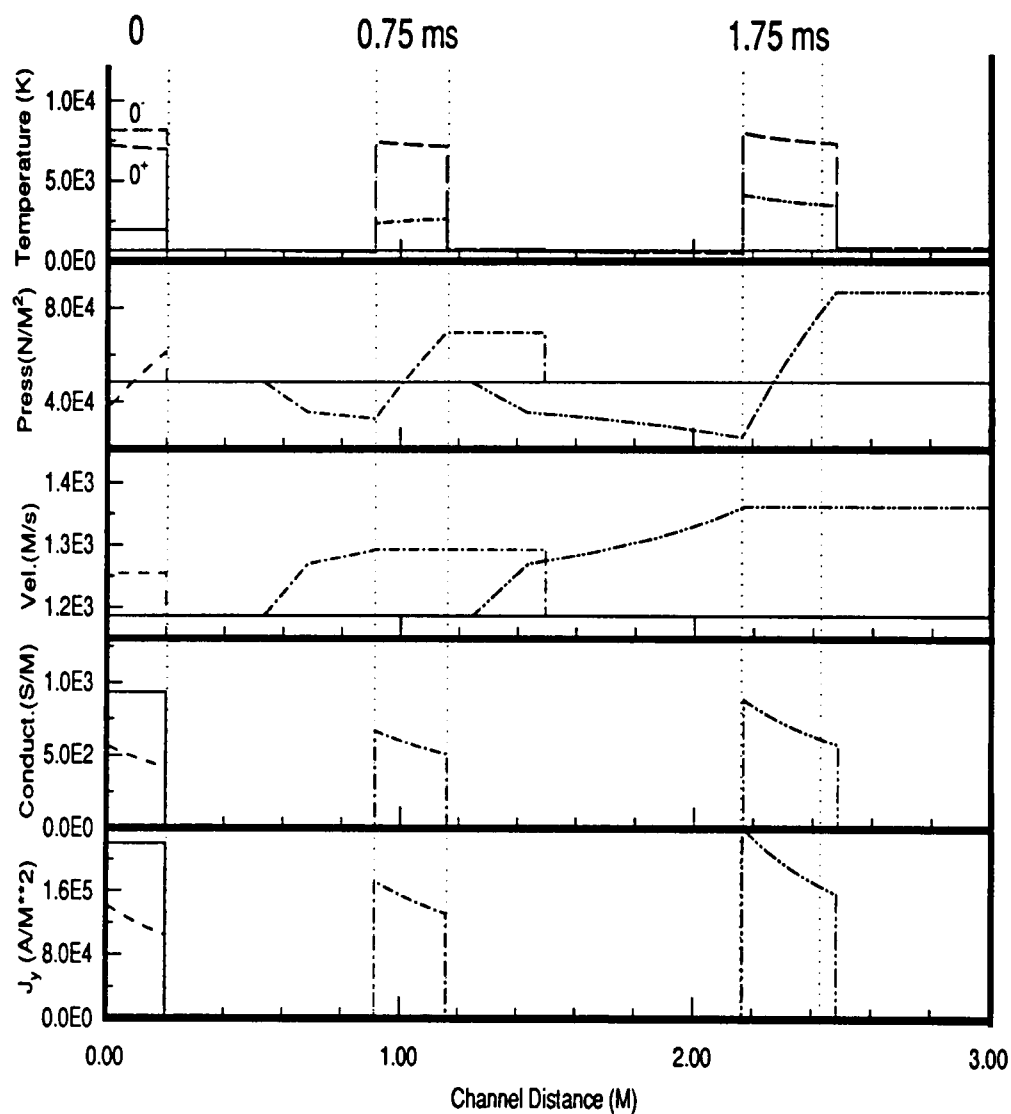
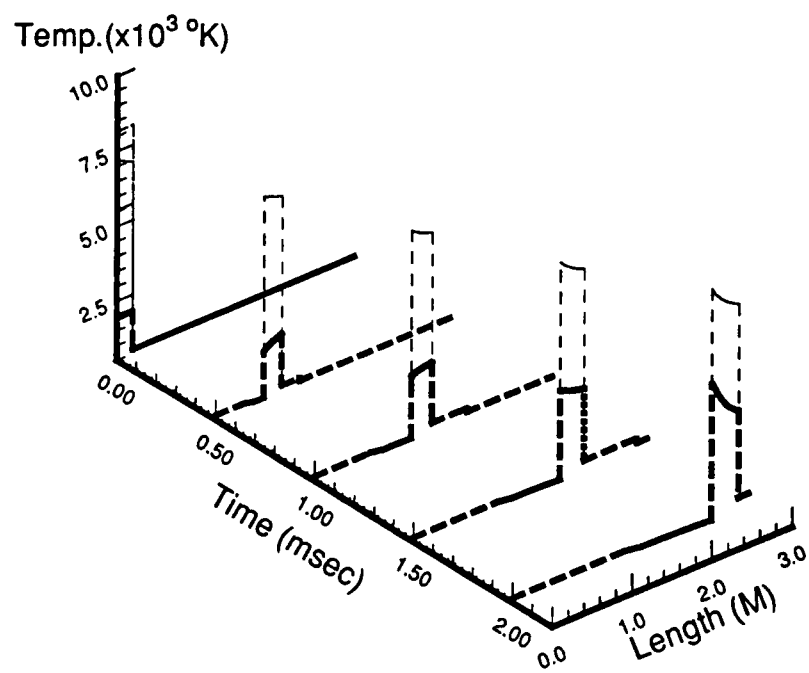
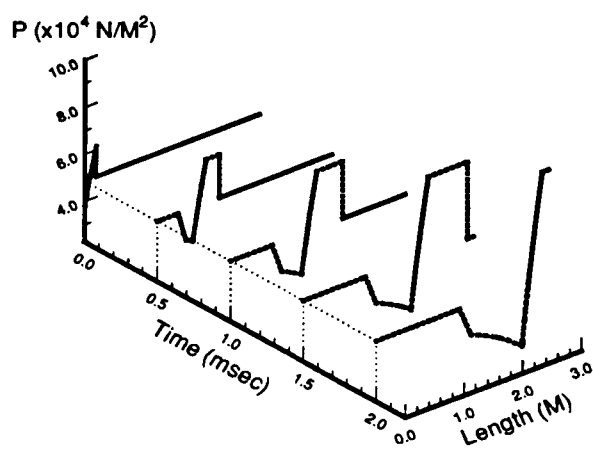


Figure 5.11 Property Distributions of the Sample T-layer Accelerator in the Quasi-steady Model. From Top to Bottom: Electron (— — —) and Gas Temperatures, Pressure, Velocity, Conductivity, and Current Density. (—: $t = 0^-$, ---: $t = 0^+$, -·-·: $t = 0.75$ ms, — — —: $t = 1.75$ ms.)

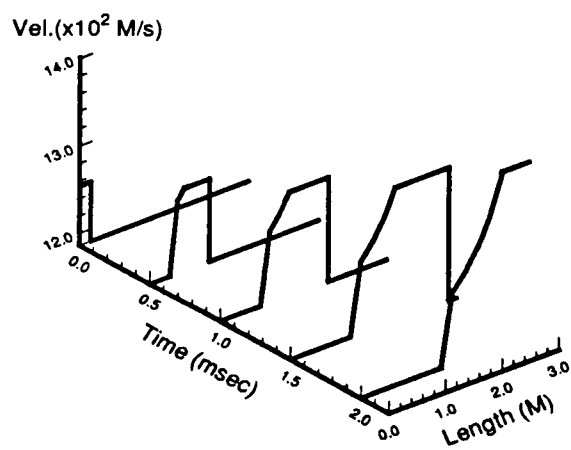


(a) Electron (---) and Gas (—) Temperatures

Figure 5.11 (Continued)

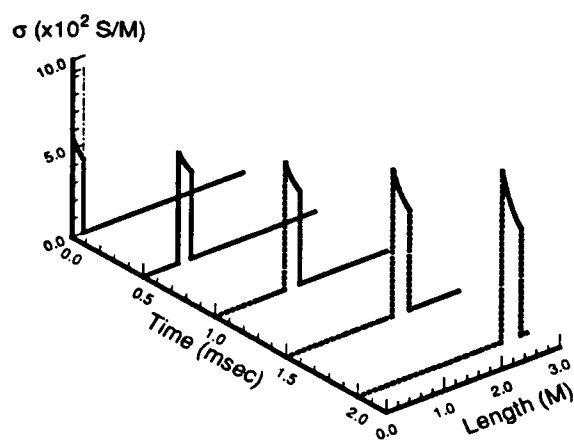


(b) Pressure

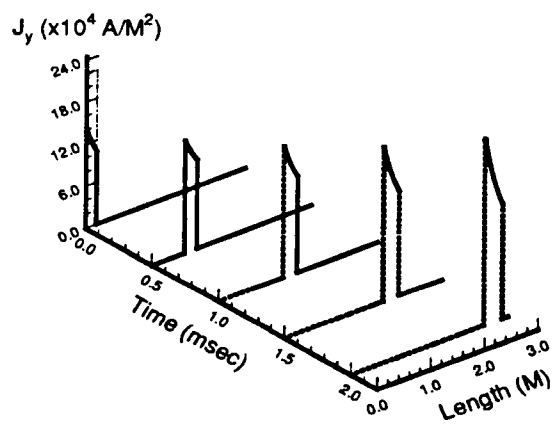


(c) Velocity

Figure 5.11 (Continued)



(d) Electrical Conductivity



(e) Current Density

Figure 5.11 (Concluded)

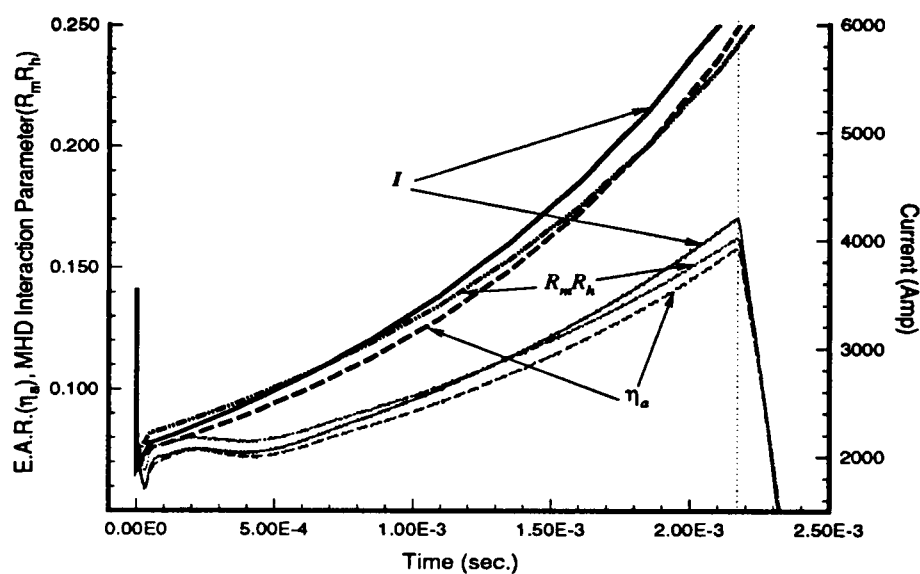


Figure 5.12 Performance of the T-layer Accelerator in the Base Case.

(Bold Lines - Quasi-steady Model, Thin Lines - Unsteady Model)

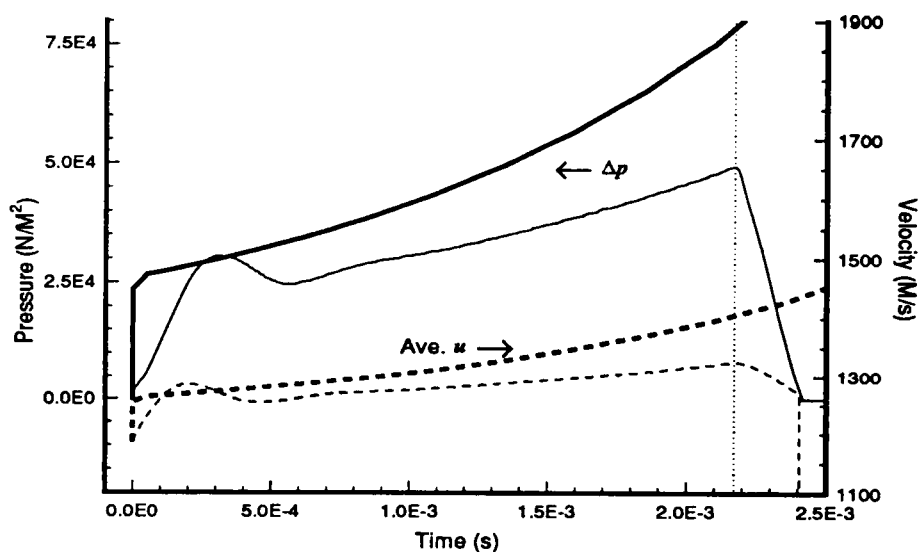


Figure 5.13 Pressure Drop Across the T-layer and Average Velocity of the T-layer in the Base Case Accelerator.

(Bold Lines - Quasi-steady Model, Thin Lines - Unsteady Model)

When the load factor is kept constant, more heat is released into the channel flow which has already had higher MHD interactions, as shown in Figure 5.14 where the heat input predicted by the quasi-steady model is more than that predicted by the unsteady model. This larger amount of heat enhances the push work to accelerate the flow to higher speed. Based on the definition of the load factor as given in Equation (2.21), this operation requires more power. As a result, the channel which originally has the stronger MHD interaction will further raise the strength of the MHD interaction at a higher rate, if the load factor is maintained as constant.

On the other hand, a constant load is assumed in the generator case. Once the MHD interaction is stronger in the channel, more current is driven out from the channel through the electrodes. This tends to increase the voltage drop across the channel and subsequently forces the generator to operate closer to an open-circuit condition. Under such circumstances, the magnitude of current density will drop according to Ohm's law in Equation (2.16) and hence lower the MHD interaction. The operation of the generator is therefore stabilized. This stabilization of the MHD generator with a constant external load applies if no other instabilities exist inside the channel.

As discussed previously in the generator case, the impermeable piston model assumed in the quasi-steady analysis keeps the T-layer mass constant and the T-layer under such assumption is usually subject to stronger local MHD interactions due to expansion to a lower gas density. As shown in Figure 5.15, the quasi-steady model also has less mass in the sample accelerator and is also predicts a stronger local MHD interaction, especially at the low density upstream end of the T-layer. Therefore, in a T-layer accelerator operated with a constant load factor, the quasi-steady model is expected to predict stronger MHD interaction when compared to that predicted by the unsteady model, and the deviation between these two results increases with time.

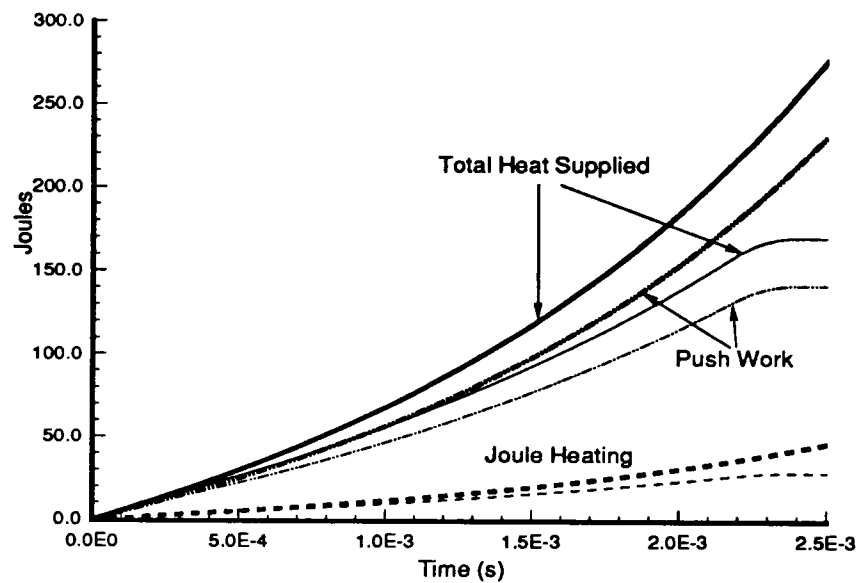


Figure 5.14 Energy Balances in the Base Case T-layer Accelerator
(Bold Lines - Quasi-steady Model, Thin Lines - Unsteady Model)

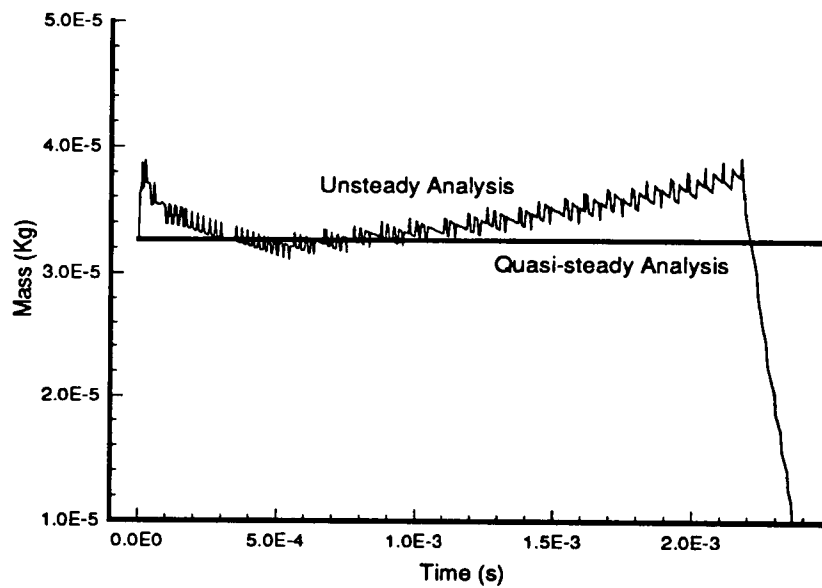
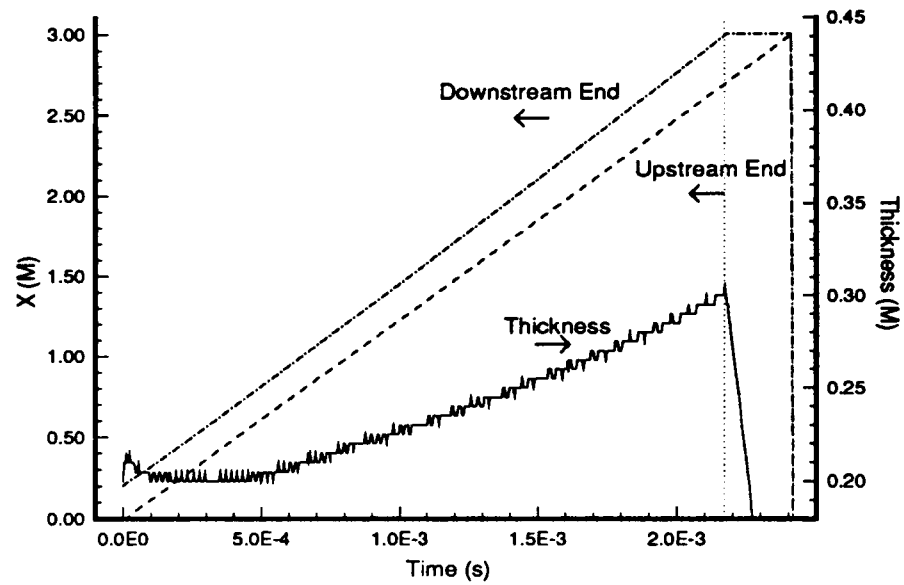
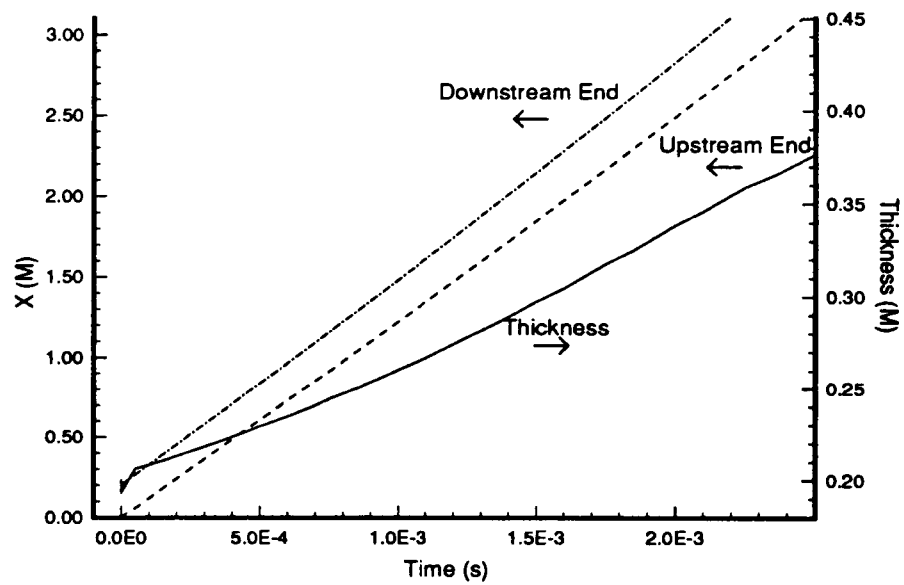


Figure 5.15 Mass of T-layer in the Base Case of T-layer Accelerator
(Bold Lines - Quasi-steady Model, Thin Lines - Unsteady Model)



(a) Unsteady Model



(b) Quasi-steady Model

Figure 5.16 Thickness and Traces of the T-layer in the Base Case Accelerator

It is believed that the differences between the two analyses can be reduced if a constant voltage is applied on the electrodes to accelerate the flow instead of a constant load factor.

It is also noted that the T-layer thickness obtained in the unsteady model is less than that in the quasi-steady model in the base case for the T-layer accelerator as shown in Figure 5.16. That is, more expansion in the volume near the upstream end of the T-layer is needed to compensate the lower pressure region resulting from the force balance with the locally stronger Lorentz force in the quasi-steady analysis.

Similar to the generator, if the initial pressure inside the initially spontaneously inserted T-layer is higher than that of the surrounding flow in the accelerator, the balanced T-layer thickness due to MHD interaction increases compared to the case where the initial pressures inside and outside the T-layer are equivalent. This is true no matter whether the unsteady or the quasi-steady model is employed. For the case shown in Figure 5.17, the pressure of the initial T-layer plasma increases from the unperturbed value to 30% more. This corresponds to an increase in the initial T-layer mass from 3.26×10^{-5} to 4.25×10^{-5} Kg. The same conclusion is obtained if a higher electron density is involved in the initial uniform T-layer plasma. In Figure 5.18, the electron density of the initially uniform T-layer plasma is increased from 2.58×10^{21} to $1.20 \times 10^{22} \text{ m}^{-3}$ by raising the applied voltage of the triggering power source from 15 to 50 Volts.

It is observed that if the initial pressure inside the T-layer is greater than its value in the unperturbed flow, the T-layer will expand its volume right after appearance in the accelerator channel. Compression waves with finite strength then propagate both upstream and downstream relative to the T-layer. However, at the same time the other set of waves, including a compression wave and an expansion wave, are created locally from the T-layer due to the beginning of the MHD interaction. Between them the expansion wave moves upstream with respect to the T-layer. Therefore, upstream of

the T-layer there exists a compression wave owing to the imbalance of inner and outside pressures and an expansion wave produced by MHD effect.

The interaction of these two waves is similar to that described by Shapiro⁷⁹ as "shock overtaking expansion". As a result, the compression and the expansion waves cancel some portion of each others strength and form a finite compressive region. In this region, higher pressure and lower velocity relative to the unperturbed flow values exist. This is shown in Figure 5.19, obtained from the unsteady analysis. Following the increase of the MHD interaction in the accelerator channel, the strength of the expansion wave increases, more cancellation between the expansion and the compression occurs, as can be seen from the comparison of pressure distributions at different time steps in Figure 5.19. However, the existence of the second compression wave moving upstream to the T-layer in the accelerator cannot be seen in the results of the quasi-steady model given in Figure 5.20.

In a T-layer generator, the initial pressure difference across the boundaries of the T-layer also triggers compression waves propagating both upstream and downstream relative to the T-layer. One of the compression waves thus created will interact with the expansion wave caused by the T-layer MHD process at the downstream side of the T-layer. Since the flow is decelerated in the generator channel, the interaction between these two waves becomes the situation known as "rarefaction overtaking shock" as stated by Shapiro.⁷⁹ Here, two waves gradually cancel each other and the compression wave may disappear if the rarefaction wave is stronger than the other. This is what happens in the sample case examined in the last sub-section for the generator where pressure inside the T-layer is 30% higher initially. The resultant flow fields evaluated by the unsteady model are included in Figure 5.21. Therein, it can be seen that the second compression wave is no longer observed at the downstream side of the T-layer.

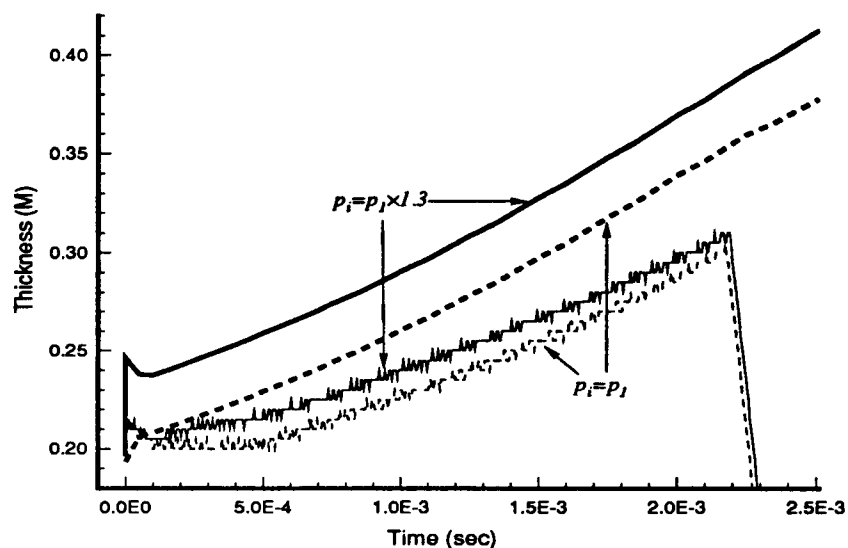


Figure 5.17 Influence of Initial Pressure on the T-layer Thickness in an Accelerator
(Bold Lines - Quasi-steady Model, Thin Lines - Unsteady Model)

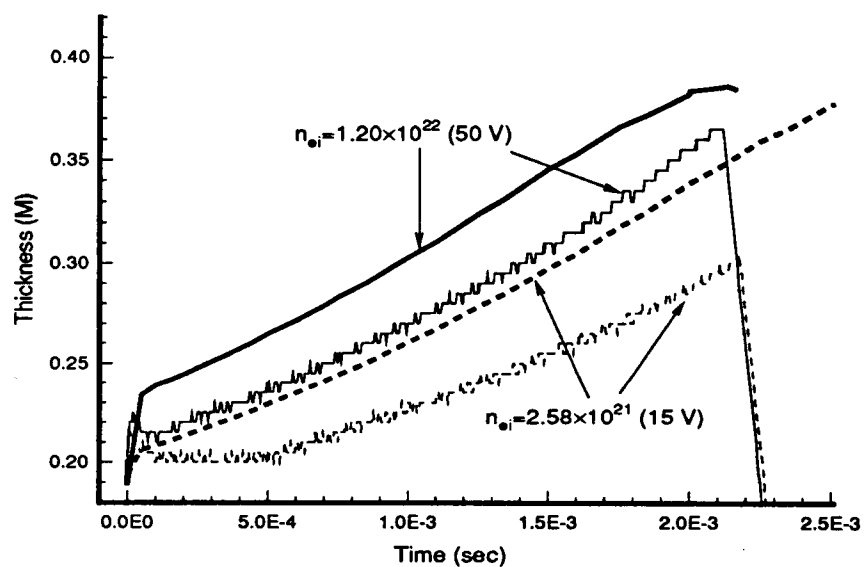


Figure 5.18 Influence of Initial Electron Density on the T-layer Thickness in an Accelerator
(Bold Lines - Quasi-steady Model, Thin Lines - Unsteady Model)

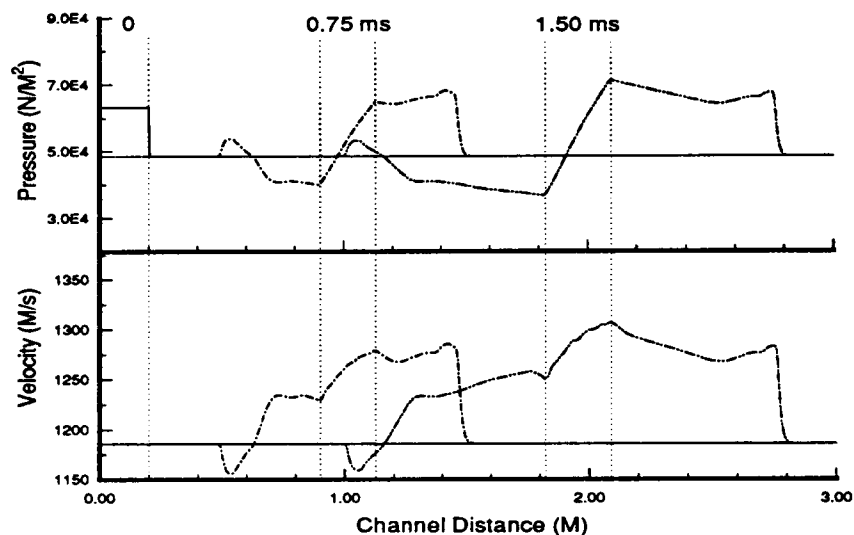


Figure 5.19 Pressure and Velocity Distributions of the T-layer Accelerator

with $p_i = 1.3 \times p_1$ in the Unsteady Model

(—: $t = 0^-$, ---: $t = 0^+$, -·-: $t = 0.75$ ms, - - -: $t = 1.5$ ms)

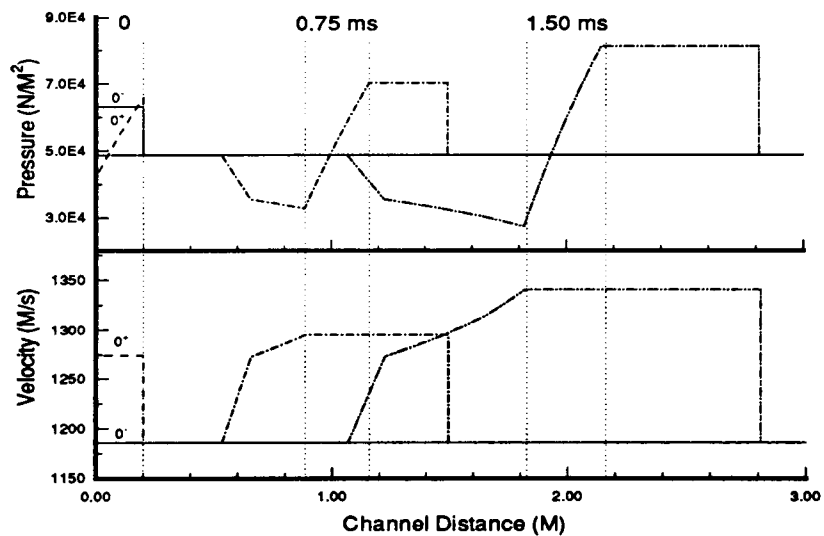


Figure 5.20 Pressure and Velocity Distributions of the T-layer Accelerator

with $p_i = 1.3 \times p_1$ in the Quasi-steady Model

(—: $t = 0^-$, ---: $t = 0^+$, -·-: $t = 0.75$ ms, - - -: $t = 1.5$ ms)

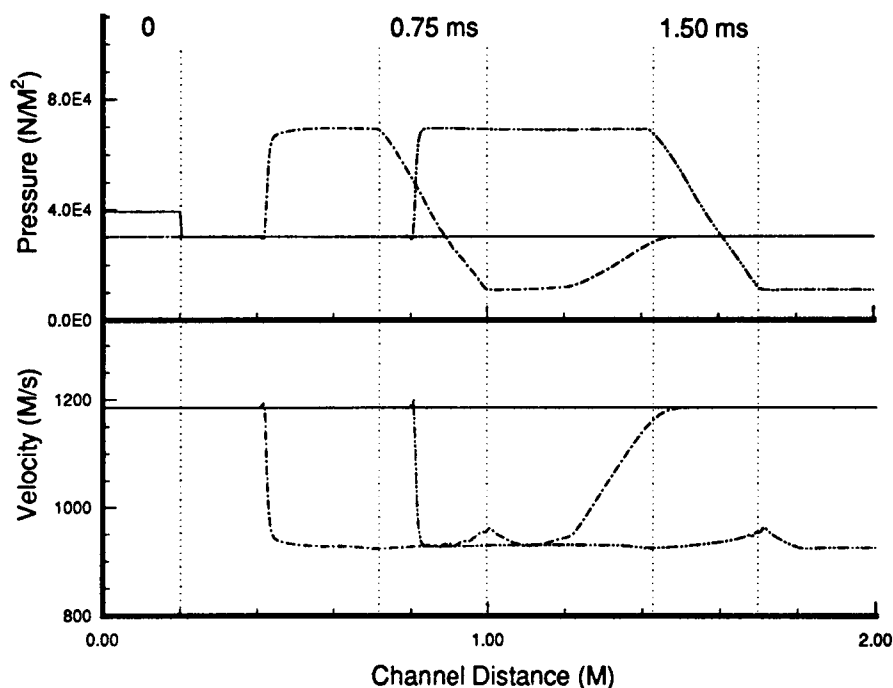


Figure 5.21 Pressure and Velocity Distributions of the T-layer Generator
with $p_i = 1.3 \times p_1$ in the Unsteady Model

(—: $t = 0$, ---: $t = 0.75$ ms, -.-: $t = 1.5$ ms)

5.2 Simulation of Reported Experiment with the Unsteady Model

Information of laboratory work using pure argon as working medium in the T-layer type of MHD device is scarce. To the author's knowledge, the only experimental study of the T-layer generator using argon was reported by Gavrilov et al.⁸ In this Section, the unsteady model is applied to simulate the same experimental conditions of that work. The results thus obtained are compared to the original experimental results.

To match the experiment, the operating conditions are chosen as

$$T_{i0} = 9000 \text{ }^\circ\text{K},$$

$$P_{i0} = 3 \text{ atm},$$

$$M_1 = 1.94,$$

$$B = 0.5 \text{ T},$$

$$R_L = 2.7 \times 10^{-3} \text{ Ohm},$$

and, $\delta_0 = 0.07 \text{ m}.$

The value of Mach number is selected to fit isentropically a gas temperature of 4000 °K at the inlet of the MHD channel. Fixing the nozzle throat of $6.4 \times 10^{-4} \text{ m}^2$, the geometry of the constant cross section MHD channel is decided by the isentropic relation to be a height of 1.18 cm and a width of 8 cm. The channel length was reported as 1 m.

Due to the uncertain energy consumed in the T-layer initiation section in the original experiment when operating with argon gas, the T-layer formed in the initiation section is assumed to be a plasma with uniform properties. These uniform plasma properties are determined by solving the formulation combining the electron energy equation (4.5), equation of state (2.12), Saha equilibrium relation (4.6), with a chosen value for the load factor. The load factor is found as $R_L/(R_L + R_p)$ with the internal impedance calculated as $(\sigma \delta_0 w)/l$ for a uniform T-layer with initial thickness δ_0 . The pressure and gas temperature in the initial uniform T-layer are assigned the same values as that of the surrounding flow field. The differences in the experimental T-layer formed in the initiation section and the initial T-layer plasma properties chosen for the unsteady model are potential sources of error when comparing the results.

The calculation starts with the insertion of the uniform T-layer in front of the inlet to the MHD channel. The upstream boundary conditions for the time-dependent gasdynamic equations (2.6)-(2.9) are chosen to be fixed with time since all the flow

characteristics at the supersonic inlet are propagating towards the flow region. The downstream boundary conditions are taken as the non-reflecting type.

The primary results of the original experiment are summarized in the following:

- the velocity of T-layer was 2.0 km/s;
- the thickness of T-layer along the electrodes was 8-10 cm;
- the peak current density inside T-layer could be as high as 10^6 A/m²;
- the peak current measured was approximately 1000 Amperes;
- the effective conductivity in the channel was 10 Siemens/m.

The calculated T-layer generator performance is shown in Figure 5.22, neglecting the voltage drop across the boundary layer. When compared with the experimental result, the total current obtained in the unsteady model is 3-4 times higher. Numerical experiments were conducted to lower the total current by introducing a boundary layer voltage drop. Once an average voltage drop of 9 volts is implemented into the unsteady model, the current output matches the peak experimental value, i.e., ~1000 Amperes. The performance evaluated with a voltage drop of 9 volts is then given in Figure 5.23. These results show that under such circumstances most of the energy is consumed in the cold boundary layer and both MHD interaction and the enthalpy extraction ratio are very low. This implies that the T-layer generator performance can be severely degraded due to the existence of a cold boundary layer.

More comparisons between the experiments and the numerical solutions obtained in the unsteady model with and without considering the voltage drop are made. In Figures 5.24 the history of the calculated T-layer thickness is shown. Only the length of the current layer inside the MHD channel is accounted for, which is the reason for zero thickness at the beginning of the calculation and for the decreasing thickness after T-layer reaches the exit of channel.

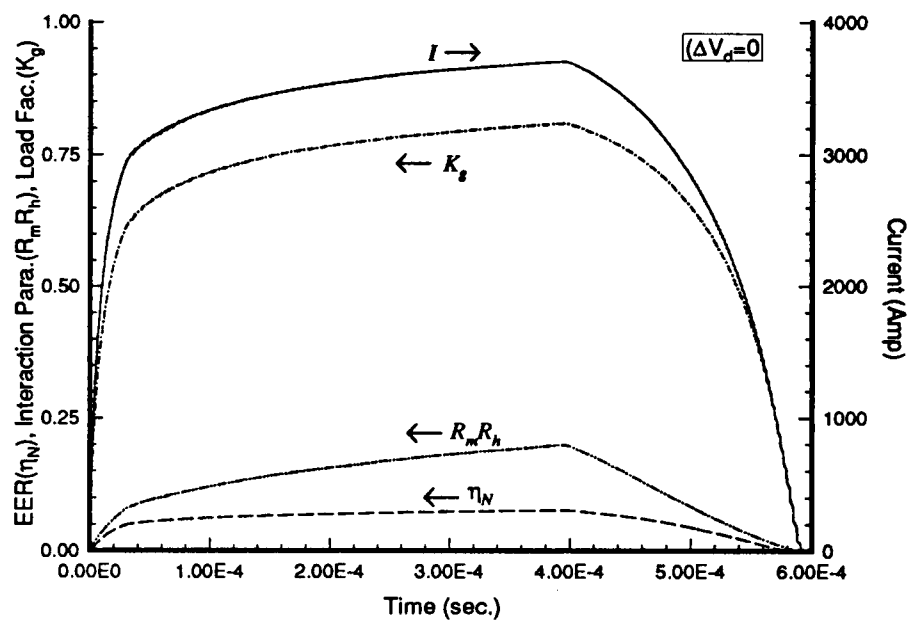


Figure 5.22 Calculated Performance of the Experimental Generator ($\Delta V_d = 0$)

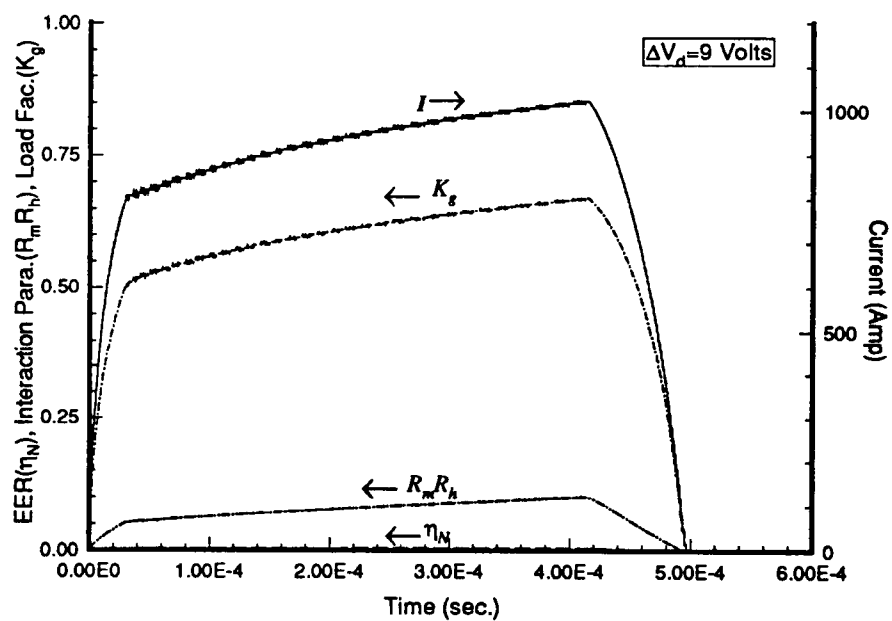


Figure 5.23 Calculated Performance of the Experimental Generator ($\Delta V_d = 9$ Volts)

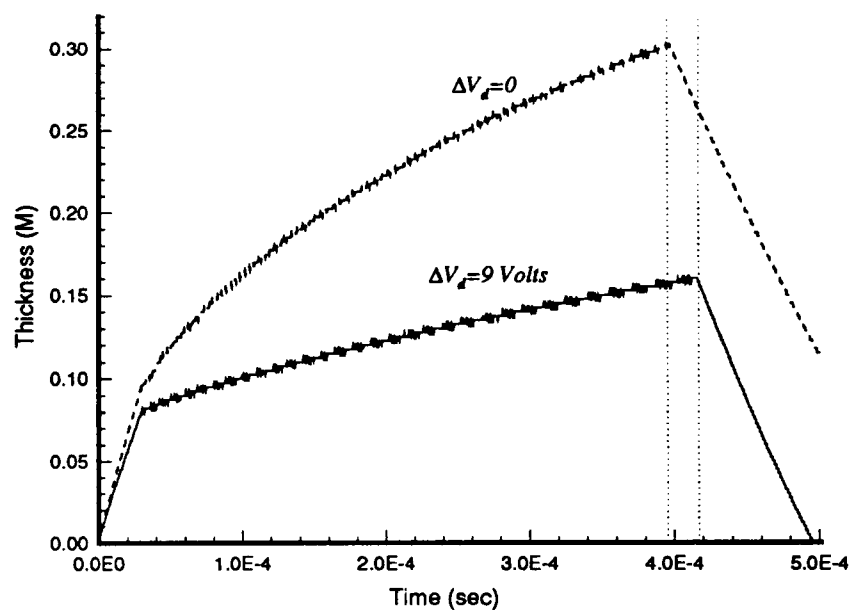


Figure 5.24 Calculated Thickness of T-layer in the Experimental Generator

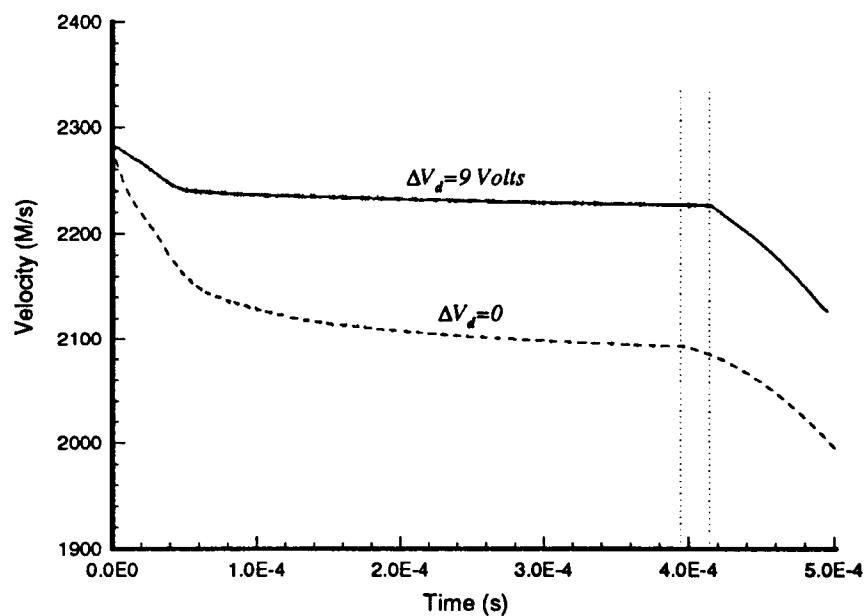


Figure 5.25 Calculated Average Velocity of T-layer in the Experimental Generator

Even though the T-layer thickness with voltage drop of 9 volts is much closer to the experimental value when compared to that without voltage drop, the difference from the experimental thickness (8-10 cm) is still significant. One possible explanation for the difference is due to the fact that regular gas discharges constrict at the contact with the electrodes. The thickness measured in the experiment was the extent of the current layer along the electrode surface. Therefore, the measured value might be less than the real T-layer thickness in the core flow which is the T-layer thickness herein calculated with the one-dimensional model.

Due to the much lower MHD interaction when the voltage drop is included, the resultant T-layer velocity inside the generator channel has higher values (Figure 5.25). Judging from the velocities, it appears that the values obtained without the voltage drop are closer to the experimentally reported T-layer velocity, i.e., ~ 2000 m/s. However, without questioning the accuracy of the measured velocity due to lack of the details of diagnostic techniques, the deviations from the experimental velocity in the calculations are still considered to be in the acceptable accuracy range, even for the case with voltage drop, since viscosity is neglected in the unsteady calculations.

The current densities shown in Figure 5.26 are evaluated at a location in the channel which is reached by the leading edge of T-layer $20 \mu s$ after the interaction begins. Compared to the peak value of 10^6 A/m² measured in the experiment, the calculated values of current density are lower, no matter whether or not the voltage drop is included. The discrepancy in the current density is believed to be related to the constriction of discharge current on the electrode surface in the experiment. The current constriction is a multi-dimensional phenomena and can not be properly described by the one-dimensional unsteady model.

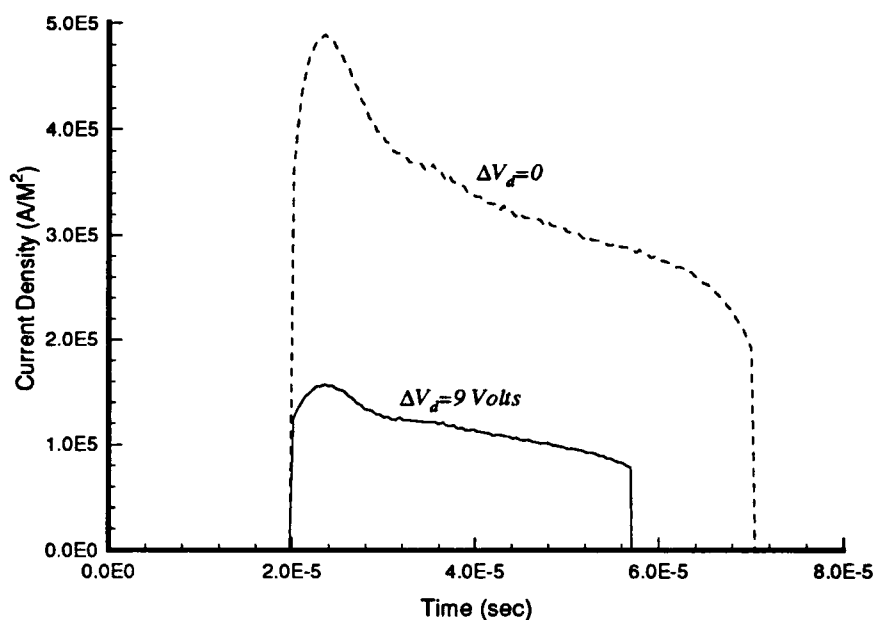


Figure 5.26 Simulation of Current Density in the Experimental Generator

In the experiment, the width of the electrodes was designed to be 4 cm, i.e., half the width of the channel, to avoid potential loss from contact of the T-layer with cold side walls. The shrinkage of the electrode width narrows the path for electrical current flowing between the plasma and the electrodes and thus constricts the current discharge on the electrode surface to an area with cross section less than that in the plasma. By matching the peak values of total current and current density obtained in the experiment, the currents should be confined in an area of 10^{-3} m^2 on the electrode surface. This corresponds to a width of 1 cm if the T-layer has a length as 10 cm. The width thus estimated is much smaller than the electrode width used in the experiment. Therefore, the current must be constricted, especially on the surface of the electrodes.

The heat loss due to boundary layer dissipation is treated as a net energy loss from the MHD channel in the unsteady model. In reality, this energy might be dissipated into

the boundary layer. This would increase the local temperature, reduce the local impedance in the boundary layer, and thus decrease the local value of voltage drop. Based on this, the area on the electrode surface which locally passes a larger amount of electrical current usually has less voltage drop across the boundary layer. This spatial reduction in the impedance is highly localized and further constricts current, which cannot be modeled in the unsteady model assuming a constant voltage drop through the entire channel.

The effective conductivity of the MHD channel is calculated as

$$\sigma_{eff} = \frac{I}{L \cdot w \cdot R_{eff}} \quad (5.1)$$

where L is the channel length. The effective resistance, R_{eff} , is defined as

$$R_{eff} = R_p + \frac{\Delta V_d}{I} \quad (5.2)$$

With a plasma impedance of 1×10^{-3} ohm and an average current of 900 Amperes, the effective conductivity in the case with a voltage drop of 9 volts is evaluated to be 13.4 Siemens/m. Whereas, the effective conductivity corresponding to the case without any voltage drop is about one order of magnitude greater. This provides more evidence of the importance of maintaining a low voltage drop across the boundary layer for the sake of generator performance.

In conclusion, the unsteady model with a voltage drop of 9 volts imposed predicts the total current, the T-layer velocity, the T-layer thickness, and the effective conductivity of the whole channel to a good approximation compared with the experimental results. The possible explanation for the discrepancy in the current density is as discussed in this Section.

5.3 Processes in T-layer Devices with Formation Section

After obtaining acceptable consistency in comparisons with results evaluated by the quasi-steady analysis and with the data reported from previous experimental work in the preceding Sections, the unsteady model is next employed to simulate the processes in T-layer devices including the T-layer formation section. Due to the uncertainty in identifying those complicated factors which influence the voltage drop across the relatively cold boundary layer between the core plasma and the electrode surface, the effect of voltage drop is not considered. However, the voltage drop might cause significant changes in the results, as discussed in the last Section.

This work concentrates on the processes of T-layer interactions in the MHD channel and their associated influence on the performance of the generator or the accelerator. The diffuser is not considered and the downstream boundary conditions of non-reflecting type are imposed at the exit of the MHD channel.

5.3.1 Considerations on the T-layer Formation

The T-layer formation is controlled by operation of a formation section upstream of the channel. The formation section controls the plasma properties and the size of the T-layer by controlling the power supplied to thermally ionize the gas and the time over which energy is added to the stream.

This formation process is sketched in Figure 5.27 for illustration. As seen in Figure 5.27, the whole T-layer formation section is within the nozzle which provides the gas flow to the MHD channel. The formation section is usually considered to be a plasma igniter section and an ionizer section designed to form high-intensity plasma layers. At plasma ignition, some mechanism is needed to produce the necessary initial electron

density of plasma. Examples such as a corona discharge or an arc can be employed in the igniter section to cause the initial breakdown of the working medium by applying a high electrical voltage over the electrodes.

In the ionizer section, an externally applied electrical field with properly chosen power (an arc or diffuse current) is imposed to continually heat and ionize the fluid. One example of the ionizer is the rail heater proposed by Gavrilov et al.⁸ Other approaches such as microwave heating are also possible.

During the heating time in the ionizer section, the plasma absorbs energy from the externally applied power source to increase its temperature and ionize neutrals. At the same time, the plasma suffers an energy loss through heat conduction, radiation, etc. The magnitude of this energy loss strongly depends on the plasma properties (temperature, electron density, etc.) which are continuously changing during the heating process. For a high intensity plasma as exists in a T-layer, these losses generally can not be neglected. This is especially true for the radiative loss. Therefore, for a better estimate of the total energy (including energy absorbed by the plasma and energy loss) consumed in a T-layer formation process, the history of heating process has to be followed in order to integrate the energy needed with time. A time-dependent scheme is required for the calculations in the T-layer formation process.

In the present study, the geometry of the nozzle is given with an area ratio distribution as

$$\begin{cases} \frac{A(x)}{A_{th}} = \frac{(\tilde{A}_{in} + 1)}{2} - \frac{(\tilde{A}_{in} - 1)}{2} \cos\left(\frac{x - x_{th}}{x_{th}} \cdot \pi\right), & \text{if } x \leq x_{th}; \\ \frac{A(x)}{A_{th}} = \frac{(\tilde{A}_{ex} + 1)}{2} - \frac{(\tilde{A}_{ex} - 1)}{2} \cos\left(\frac{x - x_{th}}{L_n - x_{th}} \cdot \pi\right), & \text{if } x > x_{th}. \end{cases} \quad (5.3)$$

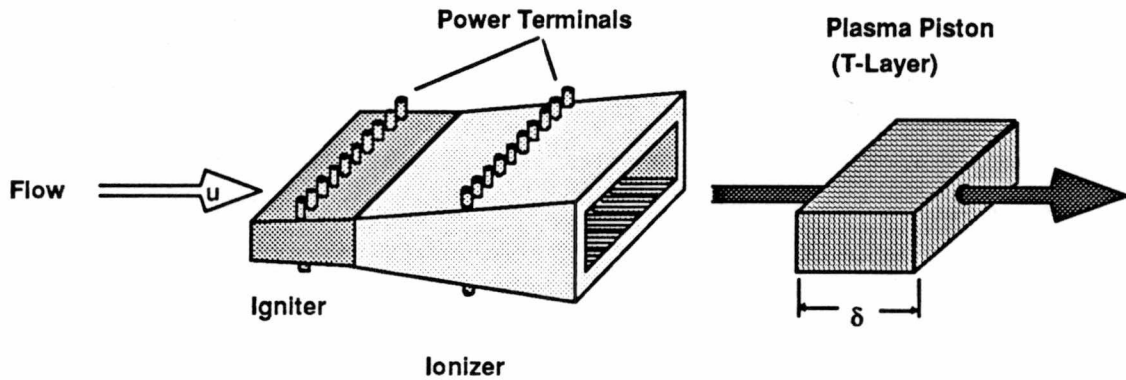


Figure 5.27 T-layer Formation Schematic

where $\tilde{A}_{in}$ and $\tilde{A}_{ex}$ are, respectively, the nozzle inlet and exit cross sections, normalized with that at the throat A_{th} . The length of the nozzle and the location of the throat evaluated from the nozzle inlet are L_n and x_{th} , respectively. In the calculations, L_n is chosen as 50 cm, x_{th} is taken as $0.4L_n$, $\tilde{A}_{in}$ is given a value of 10 to approximate the stagnation conditions, and $\tilde{A}_{ex}$ is decided from the Mach number at the MHD channel inlet according to the isentropic relation. The width of the nozzle is the same as that of the MHD channel, 8 cm, and the throat has a cross section of $6.4 \times 10^{-4} \text{ m}^2$ as given in Reference 8.

The igniting point in the nozzle is selected to be located at 14 cm downstream of the throat, i.e., a local Mach number of 1.98 if the flow is isentropic. The purpose for doing this is to avoid the possibility for thermal choking. If the local Mach number is higher, the supersonic flow can absorb the heat without choking the flow. On the other hand, a greater gap between the electrodes of the igniter is unavoidable for the same geometry of nozzle if the igniting point is installed further downstream in the supersonic expansion part of the nozzle. However, the greater the gap, the higher the applied

voltage required in the igniter section to activate the initial breakdown of the gas. This greater power supply then locally adds more heat into the flowing fluid and increases the chance for choking the flow. Therefore, in practice, careful choices of the location of the igniting point and the proper power supply applied to the T-layer formation section are very important.

The ionizer section begins at 0.5 cm downstream to the igniter in the calculations. The heating mechanism of the formation section is simulated by separately assigning values of the externally applied voltage to the igniter and to the ionizer, respectively. The heating time for the igniter is selected as 10 microseconds while that for the ionizer section is chosen to stop heating at the instant when the downstream end of a current layer arrives at either the inlet of the MHD channel or a pre-selected distance downstream to the inlet.

The operating conditions, including: the stagnation conditions, flow Mach number, magnetic field, and the external load resistance for a generator or load factor for an accelerator, remain the same as that assigned in the generator or the accelerator case in Section 5.1. By doing this, comparisons can be made between cases with spontaneous appearance of the T-layer in the MHD channel and with the T-layer produced by the formation section proposed.

Figure 5.28 show certain steps in the T-layer formation process with power sources of 50 and 25 volts applied to the igniter and the ionizer, respectively. The operating conditions are those for the sample generator examined in Section 5.1. The hot current layer that is formed spreads over the whole formation section in $64.3 \mu\text{s}$ with an average speed around 2400 m/s over that period. This is about a factor of 2 above the local flow velocity. From distributions of the different properties in Figure 5.28, it can be seen that the wave of electrical properties, including electron temperature,

electron density, conductivity, and current density, etc., leads the wave of gasdynamic properties such as gas temperature, pressure, and flow velocity, etc.

The distribution of gasdynamic properties implies that these properties propagate at a speed close to that obtained from superposition of the flow speed with the speed of the compression waves caused by the expansion of the heated gas. This is supported by the observation in the velocity distribution that the local flow speed is about the same with the marching speed of the points moving with the unperturbed flow velocity, i.e., the points marked by circles in the velocity distribution of Figure 5.28. That is, at an instant during the heating process, there is certain point in the heating volume of plasma moving with the unperturbed flow velocity. Upstream to this point, an upstream propagating velocity from the compression wave due to the thermal expansion is superimposed to the unperturbed flow, while a downstream propagating velocity is added to the flow field downstream to the same point.

The compression waves due to the heating expansion spread both upstream and downstream at a speed (relative to the center of the plasma formed) less than the local flow speed in this example, since no disturbance is transmitted upstream beyond the igniting point. The intensity and the spread of the compression wave from gas expansion are influenced by the amount of heat added to the gas and are enhanced to greater values following the heating history.

Owing to the increase of the stagnation pressure in the accelerator case, higher values of voltages are required to create T-layers. Figure 5.29 gives the results wherein 100 and 40 volts are separately applied upon the igniter and the ionizer sections for forming the T-layer in the accelerator case. The heating process continues until the leading edge of the current layer arrives at a certain distance, ~ 2.5 cm in this case, beyond the MHD channel inlet.

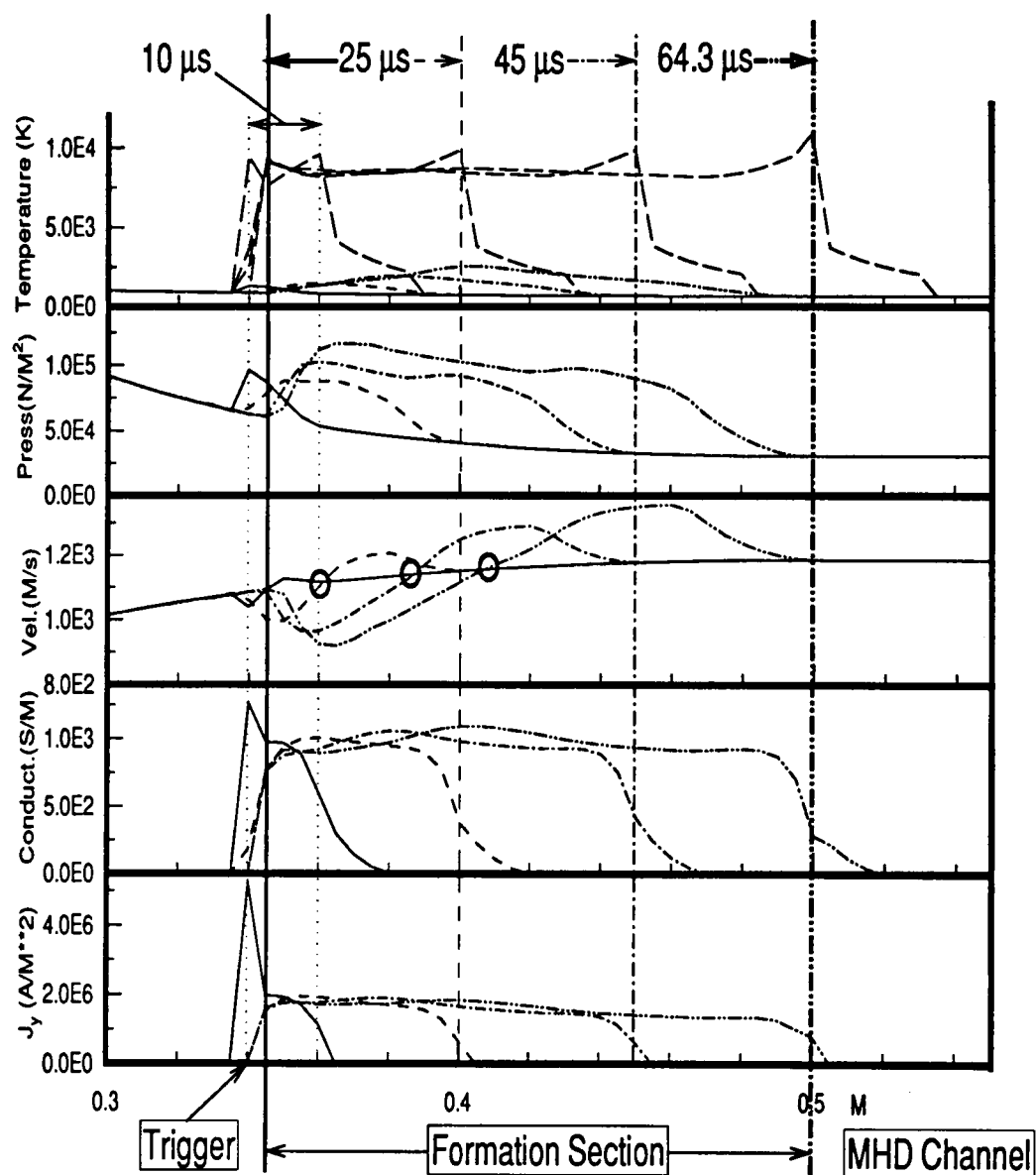


Figure 5.28 Property Distributions in the T-layer Formation Process in the Generator Case. From Top to Bottom: Electron (---) and Gas Temperatures, Pressure, Velocity, Conductivity, and Current Density.
 (—: $t = 10$, ---: $t = 25$, -·-: $t = 45$, ---: $t = 64.3$ μs)

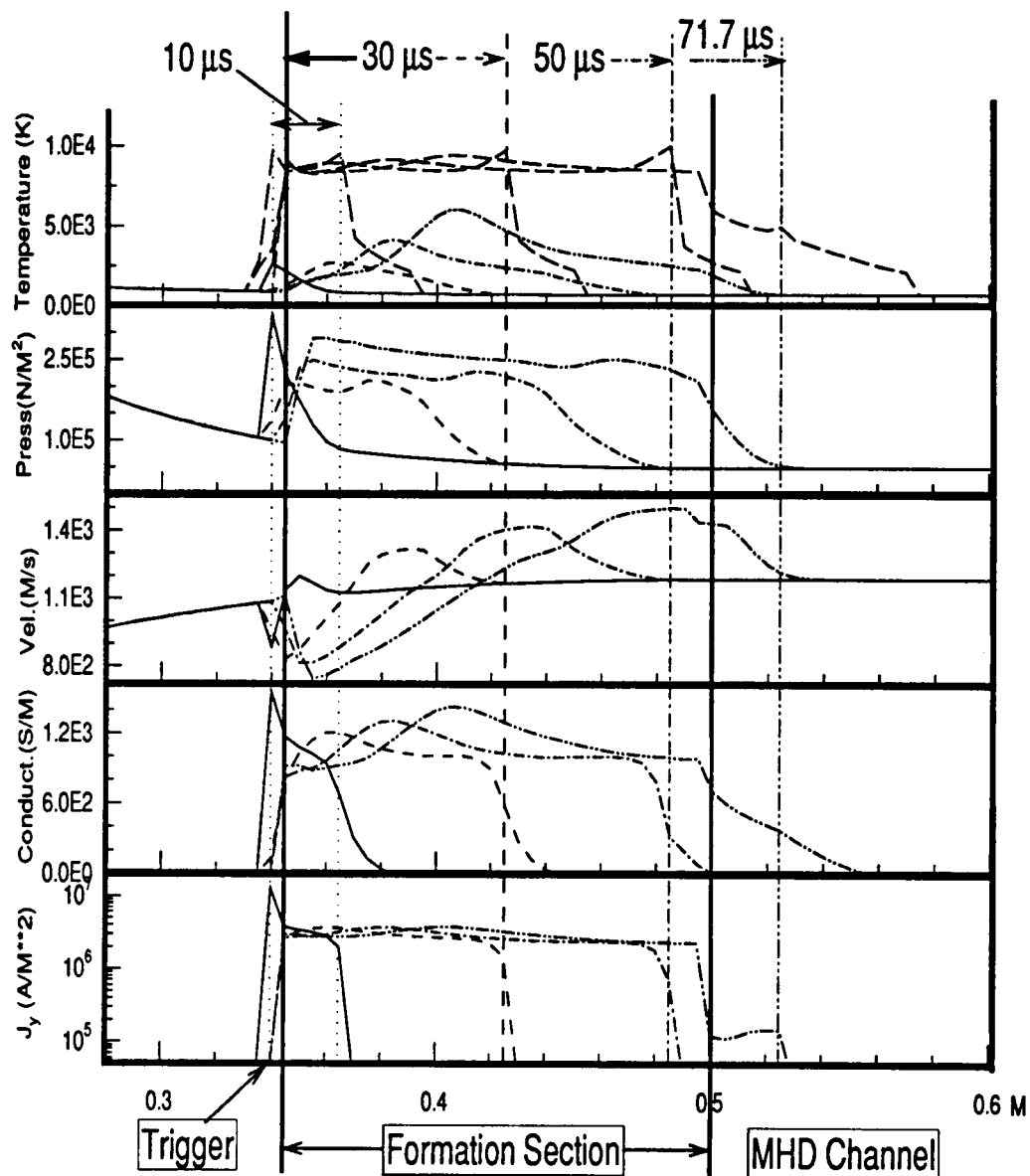


Figure 5.29 Property Distributions in the T-layer Formation Process in the Accelerator Case. From Top to Bottom: Electron (---) and Gas Temperatures, Pressure, Velocity, Conductivity, and Current Density.
 (—: $t = 10$, ---: $t = 30$, -·-: $t = 50$, ---: $t = 71.7 \mu\text{s}$)

It is noted that the gas temperature is generally higher than that given in Figure 5.28 for the generator case even though the electron temperatures in both cases remain at the same level. This higher gas temperature is due to the higher pressure in the channel which, at the same electron temperature, produces a plasma with higher electron density and heats the heavy species with a higher rate through more frequent collisions with electrons.

In both T-layer forming cases examined, the hot plasma eventually fills the entire volume of the formation section even though the power supply at the igniting point is disconnected 10 μ s after the process begins. The local electrical conductivity near the igniter section decays sharply shortly after the power is turned off. The plasma properties formed inside the T-layer are found to be acceptably uniform. Nevertheless, significant disturbances in the gasdynamic properties are observed during the heating process.

5.3.2 Generator

The T-layer formed in the formation section is convected downstream into the MHD channel. The results in Figures 5.30 show the distribution of various properties with a single T-layer in the generator channel. When compared these results to an instantaneously inserted T-layer in the channel as given in Figure 5.1, the flow fields outside the T-layer are similar in both cases except that some disturbances appear near the downstream propagating compression wave in the present case. From traces seen in the pressure and velocity distributions presented relatively in Figures 5.30.(b) and (c), these irregular disturbances are the residual influence from the compression wave that originated from heat expansion in the T-layer formation section.

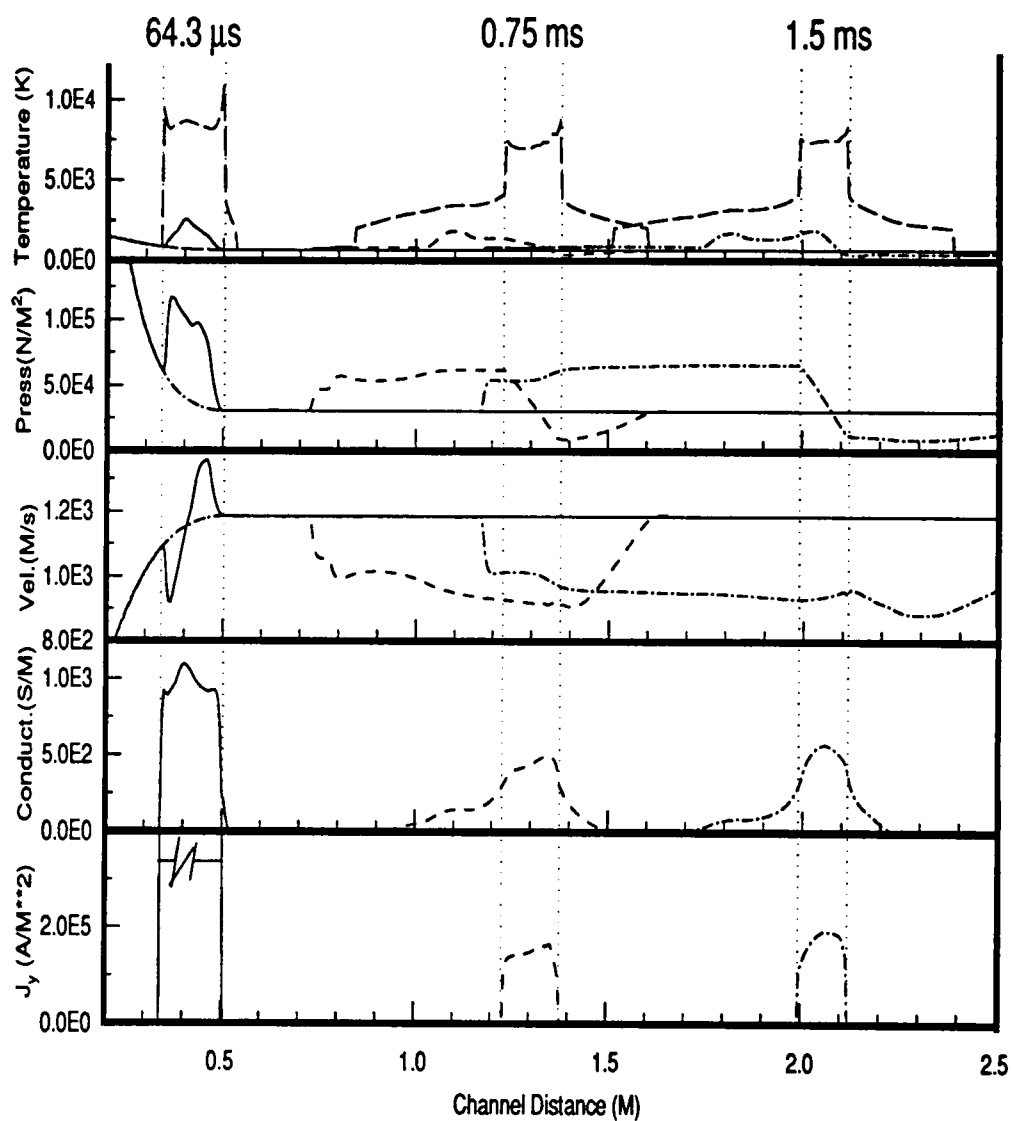
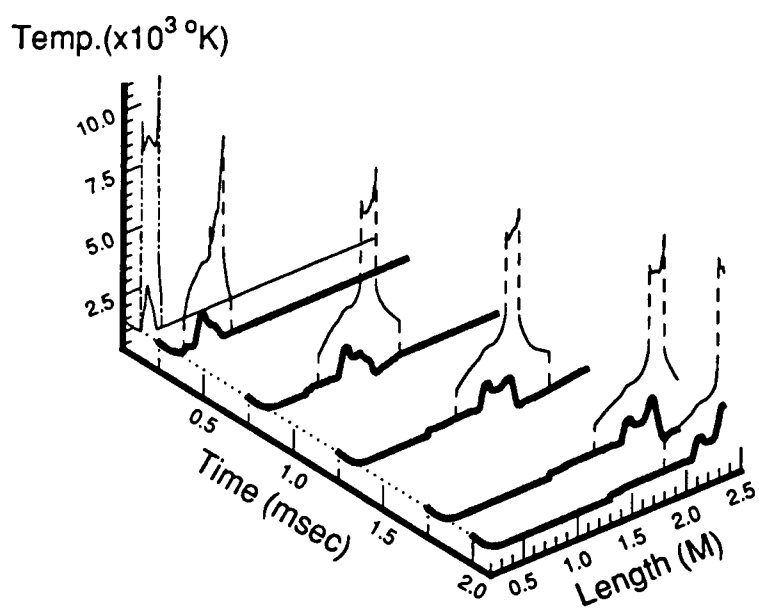
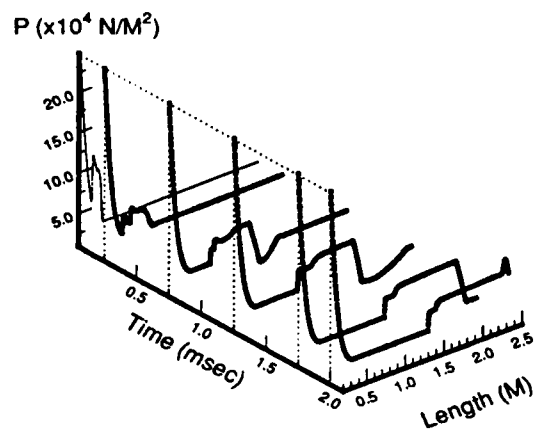


Figure 5.30 Property Distributions in the Single T-layer Generator with the Formation Section. From Top to Bottom: Electron (---) and Gas Temperatures, Pressure, Velocity, Conductivity, and Current Density.
 (—: $t = 64.3 \mu\text{s}$, ---: $t = 0.75 \text{ ms}$, -.-: $t = 1.5 \text{ ms}$)

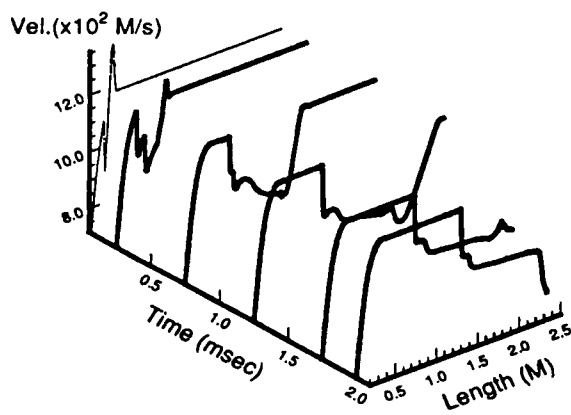


(a) Electron (---) and Gas (—) Temperatures

Figure 5.30 (Continued)

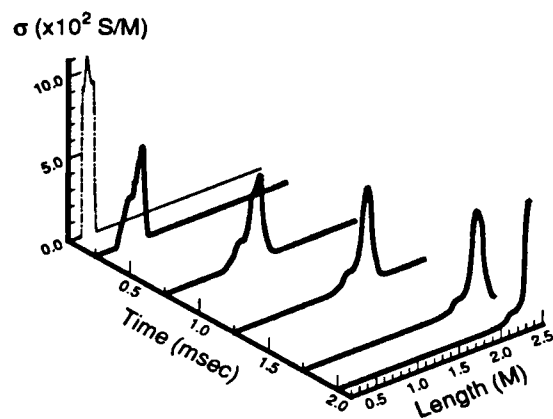


(b) Pressure

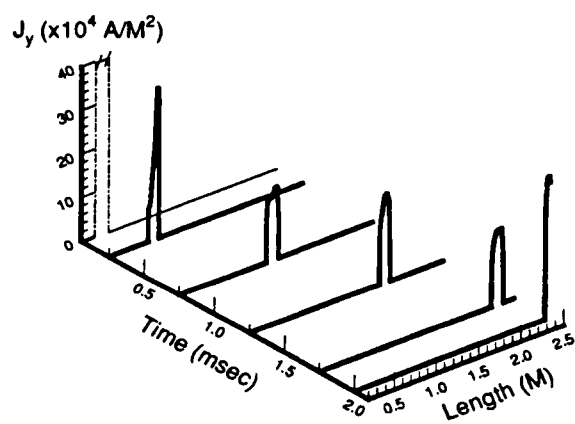


(c) Velocity

Figure 5.30 (Continued)



(d) Electrical Conductivity



(e) Current Density

Figure 5.30 (Concluded)

Although the mass inside the T-layer is about the same in both cases (Figures 5.7 and 5.31), the T-layer thickness in the stable operation region in the present case is only about half of that in the corresponding case with an initial 20 cm of thickness, spontaneously inserted T-layer. The reduction in the T-layer thickness reduces the net MHD interaction and results in lower values of the enthalpy extraction ratio, total current, and load factor, as seen when comparing the results given in Figure 5.32 to those in Figure 5.4.(a).

Attention is also given to the happening of end effects¹⁸. From the distributions of the plasma conductivity and the current density shown in Figures 5.30.(d) and (e), the intensity of plasma is seen to abruptly increase the instant when the T-layer is leaving the channel. The reduction of the cross section for the current path during T-layer's exit increases the effective internal impedance of the plasma inside the channel. Matched to the external load, the load factor decreases its value to respond to the variation in the internal impedance. The channel is then operated closer to the short-circuit condition and more Joule heating is released to heat the residual plasma still inside the channel. Subsequently, the T-layer plasma is heated to higher temperatures, as observed in the temperature distribution of Figure 5.30.(a), and increases its local conductivity. As a result, more current can locally pass through the T-layer.

In order to respond to the suddenly enhanced MHD interaction at the channel exit, flow gasdynamics forms a compression wave immediately in front of the channel exit. This can be easily seen in the pressure and velocity distributions in Figures 5.30.(b) and (c). This final compression wave propagates upstream relative to the T-layer but is still convected downstream by the flow. It finally disappears from the channel following the exit of the T-layer from the channel.

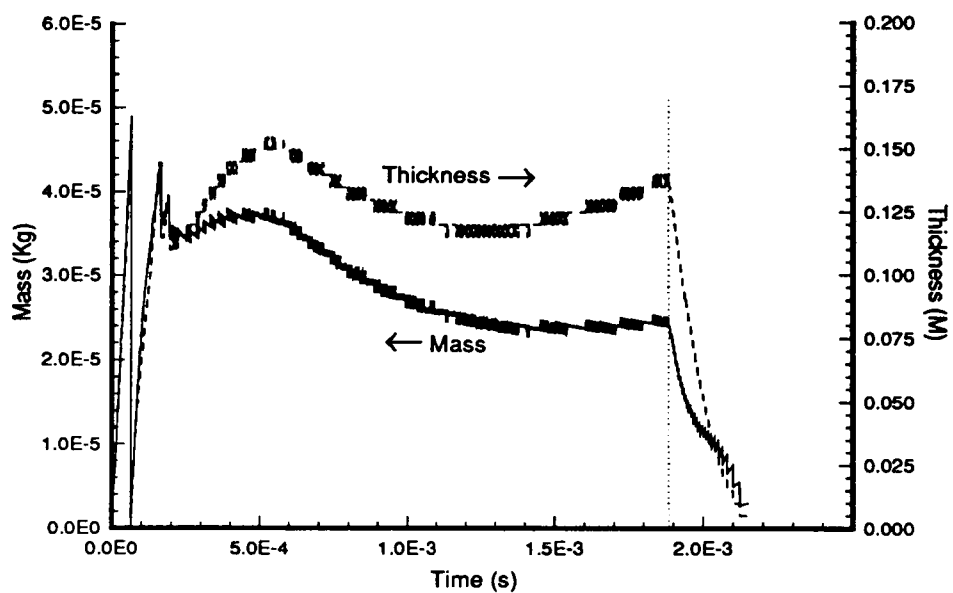


Figure 5.31 Mass and Thickness of T-layer in the Single T-layer Generator with the Formation Section

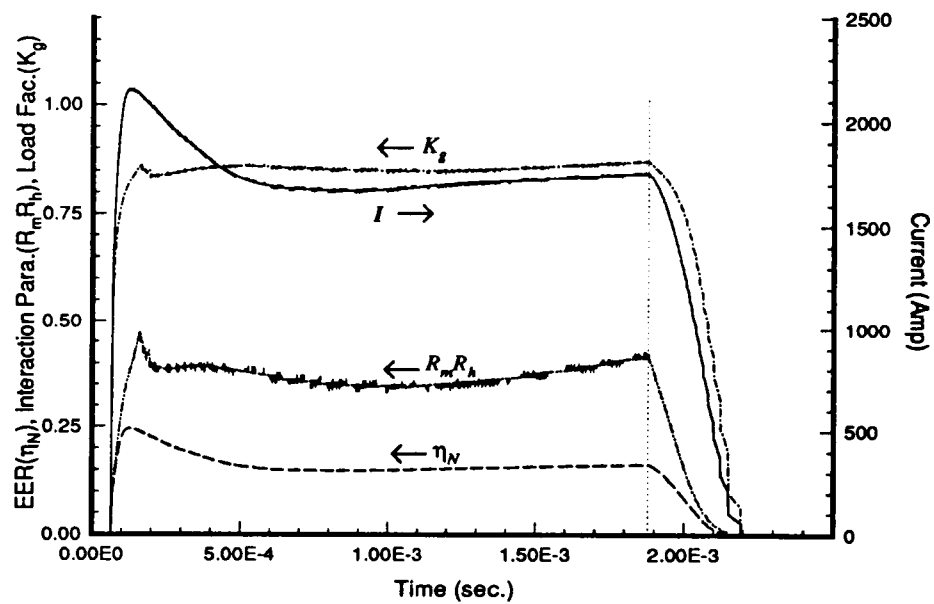
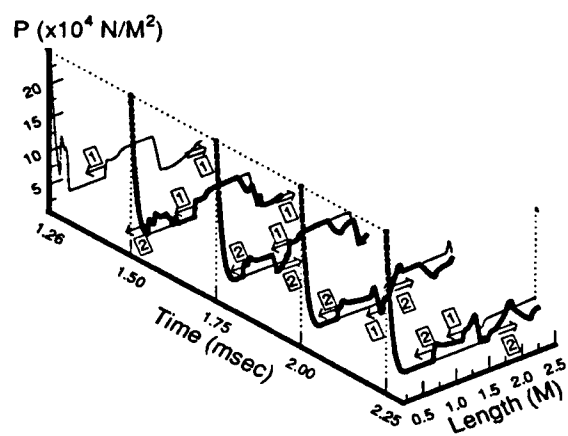


Figure 5.32 Performance of the Single T-layer Generator with the Formation Section

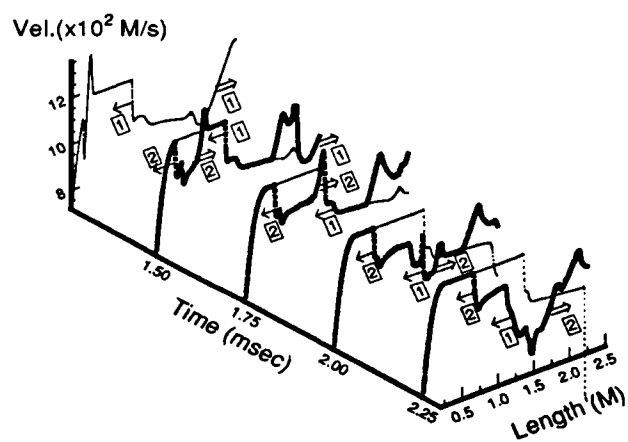
Multiple T-layers are investigated by assigning a cyclic period for powering the formation section. These results are given in Figures 5.33 and are subject to the same operating conditions as used in the single T-layer case. Here, cyclic T-layers are formed every 1.2 ms. In the results, important waves (compression and expansion) existing in the flow field are marked by assigning a number to each of them following the order of their appearance in the channel. Compression and expansion waves are marked by single arrow ($\rightarrow$) and double arrow ($\Rightarrow$) respectively, with arrow heads pointing in the direction of wave propagation. Thin dot-dash lines represent either the results of T-layers prior to entering the MHD channel or, for easy comparison, the results obtained in the corresponding single T-layer case.

The results prior to the entrance of the second T-layer into the MHD channel are the same as that found in the single T-layer case (Figures 5.30) and are therefore omitted in Figures 5.33. In the following, the processes due to the existence of the second T-layer in the MHD channel, the perturbation in the flow field due to the interactions between two T-layers, and their associated influences on the generator performance are observed, following in the time sequence.

Upon entrance of the second T-layer into the channel, the internal impedance of the bulk plasma in the channel decreases due to the continuing increase in the volume of hot plasma. This promotes the MHD interaction in the channel and the generator immediately responds with higher current outputs, as can be seen in Figure 5.34. However, at the same time, the generator is forced to operate closer to the open-circuit condition and the value of the load factor increases. Less Joule heating is then available and the intensities of the plasmas in both T-layers decay, shown in the current density distribution in Figure 5.33.(c) at the time points of 1.5 and 1.75 ms. As a result, the generator current begins to drop after the second T-layer completely enters the channel.

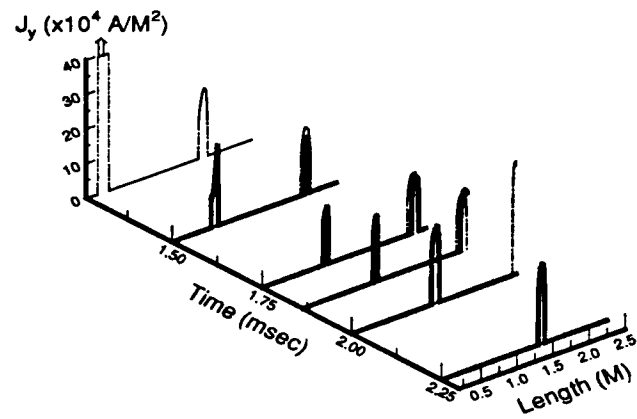


(a) Pressure



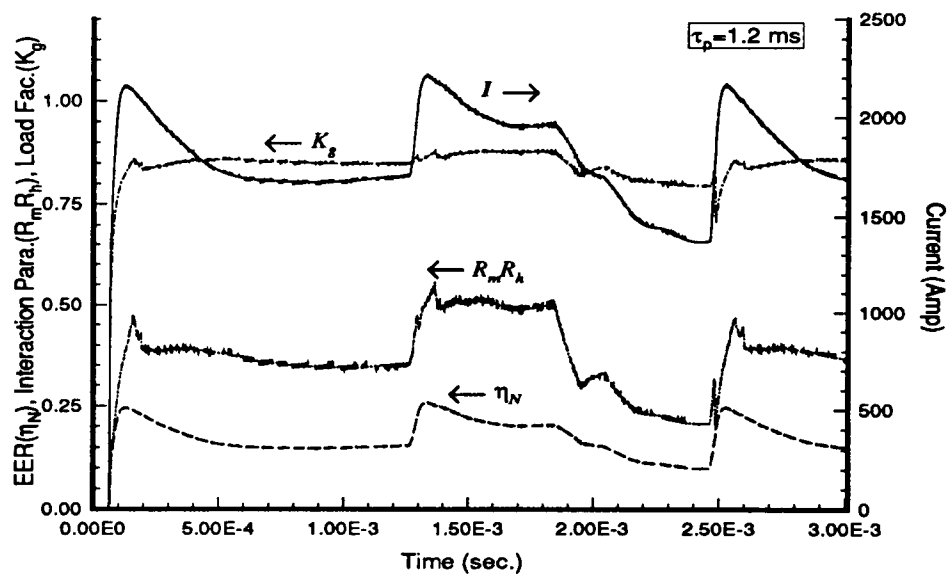
(b) Velocity

Figure 5.33 Property Distributions in the Generator with Multiple T-layers ($\tau_p = 1.2 \text{ ms}$)



(c) Current Density

Figure 5.33 (Concluded)

Figure 5.34 Performance of Generator with Multiple T-layers ($\tau_p = 1.2 \text{ ms}$)

The reduction in the strength of the first T-layer results in the appearance of ripples in the pressure distribution near the downstream end of the T-layer as can be seen at the 1.5 ms time point in Figure 5.33.(a). The reduction in the MHD retarding force makes the first T-layer move downstream with a higher speed compared to that in the single T-layer case.

The variation in the performance of the generator due to multiple T-layers co-existing in the channel is shown in Figure 5.34. The performance jumps immediately after each new T-layer enters the channel at 1.26 or 2.46 ms. Subsequently, the self-adjustment inside the channel required to fit the new loading condition reduces the performance of the generator. The generator then approaches a new stable operation condition.

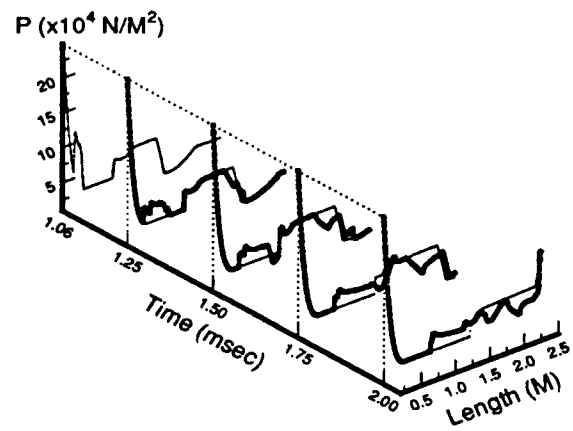
The compression wave created by the MHD interaction of the first T-layer begins to interact with the flow field perturbed by the existence of the second T-layer around 1.75 ms. The resultant pressure disturbances inside the second T-layer make the plasma more resistive and decreases the T-layer's strength. At the same time, the first T-layer regains some part of its strength in order to match the channel loading. During this period, the generator performance decreases but the reduction is not severe.

The first T-layer arrives at the channel exit at 1.85 ms and completely disappears from the channel at 1.96 ms. On its leaving from the channel, the volume of plasma decreases, the internal impedance increases, and the performance of the generator drops at a significant rate. The generator approaches the short-circuit operating condition with a smaller value of the load factor. More Joule heat is then available to heat the second T-layer which picks up strength. Under such circumstances, the more resistive first T-layer cannot obtain enough Joule heat to maintain its plasma intensity as it does in the single T-layer case. Consequently, the end effect due to the first T-layer's exit from the channel is prevented.

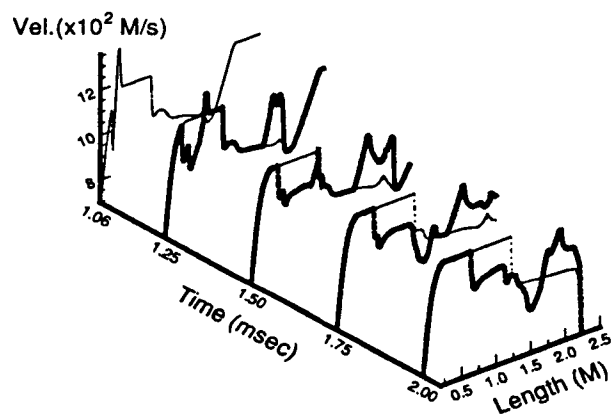
Although the first T-layer has left the channel, the compression wave created by it still remains in the channel 2 ms after the initiation of the first T-layer. The direct interaction between this compression wave and the plasma in the second T-layer further perturbs the plasma properties. As a result, the T-layer shrinks in volume due to the increase in local pressure which increases the plasma impedance and lowers its intensity. The generator at this moment is less efficient than when in stable operation with a single T-layer.

Results with a period of 1 ms for repeated triggering of the T-layers are included in Figures 5.35 and 5.36. By comparing the performance given in Figure 5.36 to that in the case with a period of 1.2 ms (Figure 5.34), the obvious difference is the deep decline of the performance around 2 ms in Figure 5.36. With reference to the history of property distributions given in Figure 5.35, it is understood that the occurrence of the large performance degradation is due to the disappearance of the second T-layer prior to arrival of the first T-layer at the channel exit. In this case, the end effect occurs when the only T-layer in the channel, e.g., the first T-layer, leaves the channel..

With a shorter T-layer forming period, the second T-layer begins to interact with the compression wave created by the first T-layer at an earlier stage. Meanwhile, the first T-layer is under the influence of the expansion wave triggered by the second T-layer. With nonequilibrium plasmas in both T-layers, these pressure disturbances help to reduce the impedance of the first T-layer but at the same time make the second T-layer subject to a lower conductivity. That is, the interaction between the waves resulting from the T-layer MHD interaction in a generator benefits the intensity of the first T-layer but hurts the growth of the second T-layer. As a result, the strength of the second T-layer gets weaker and weaker and eventually vanishes. The generator performance is subsequently seriously degraded when the only T-layer surviving in the channel later arrives at the channel exit.

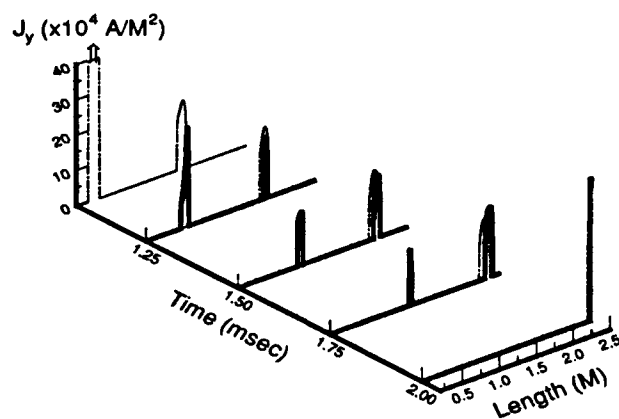


(a) Pressure



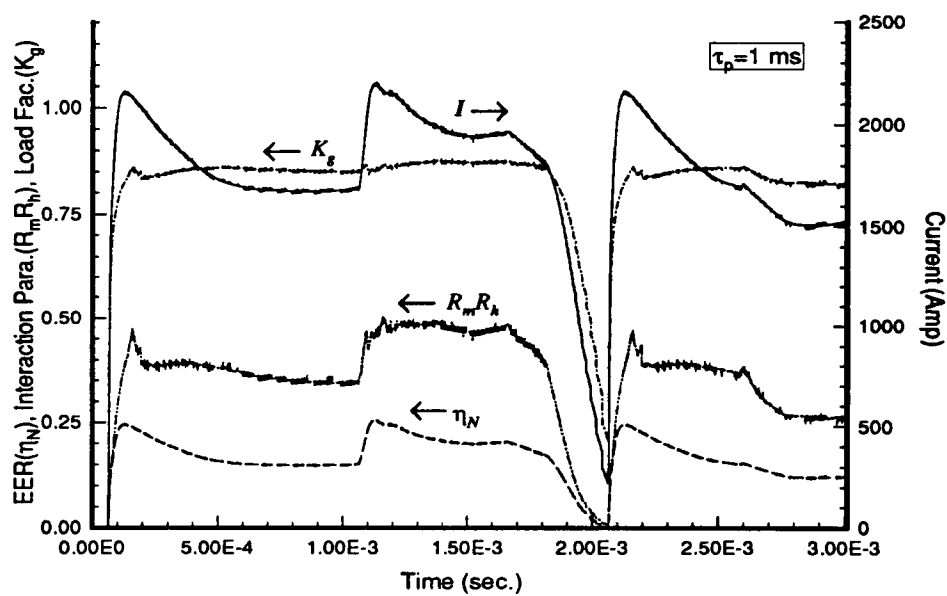
(b) Velocity

Figure 5.35 Property Distributions in the Generator with Multiple T-layers ($\tau_p = 1.0 \text{ ms}$)



(c) Current Density

Figure 5.35 (Concluded)

Figure 5.36 Performance of Generator with Multiple T-layers ($\tau_p = 1 \text{ ms}$)

According to the results given in Figures 5.34 and 5.36, it is concluded that using multiple T-layers in the generator indeed has the potential to improve the generator performance. With such a simple configuration of the generator (a single load) as we have examined, this improvement in performance is valid only in a short duration after the newly created T-layer enters the MHD channel. Fluctuations in the generator outputs are unavoidable unless a more complicated power consolidation mechanism is employed. The interactions between T-layers and their associated perturbations to the flow field are usually harmful in terms of the generator performance.

The comparison between those two examples using multiple T-layers with different cyclic periods ($\tau_p = 1.2$ and 1.0 ms) shows the importance of choosing the proper period for repeatedly creating the T-layers. Possible factors which influence this determination are very non-linear and complicated. These include the strength of the T-layer at its creation, the flow velocity, the channel length, the loading, and the operating conditions, etc.

5.3.3 Accelerator

An accelerator with a single T-layer, initiated by the process shown in Figure 5.29, is simulated with the same operating conditions as that of the accelerator investigated in Section 5.1. The distributions of properties of this single T-layer accelerator are given in Figure 5.37. It is easily seen that the left-running compression wave originated from the thermal expansion in the formation section exists in the MHD channel - the same phenomena observed in Section 5.1 due to an initial imbalance of pressures across the boundaries of the T-layer. The strength of this compression wave decreases following the enhancement of the expansion wave activated by the T-layer MHD interaction.

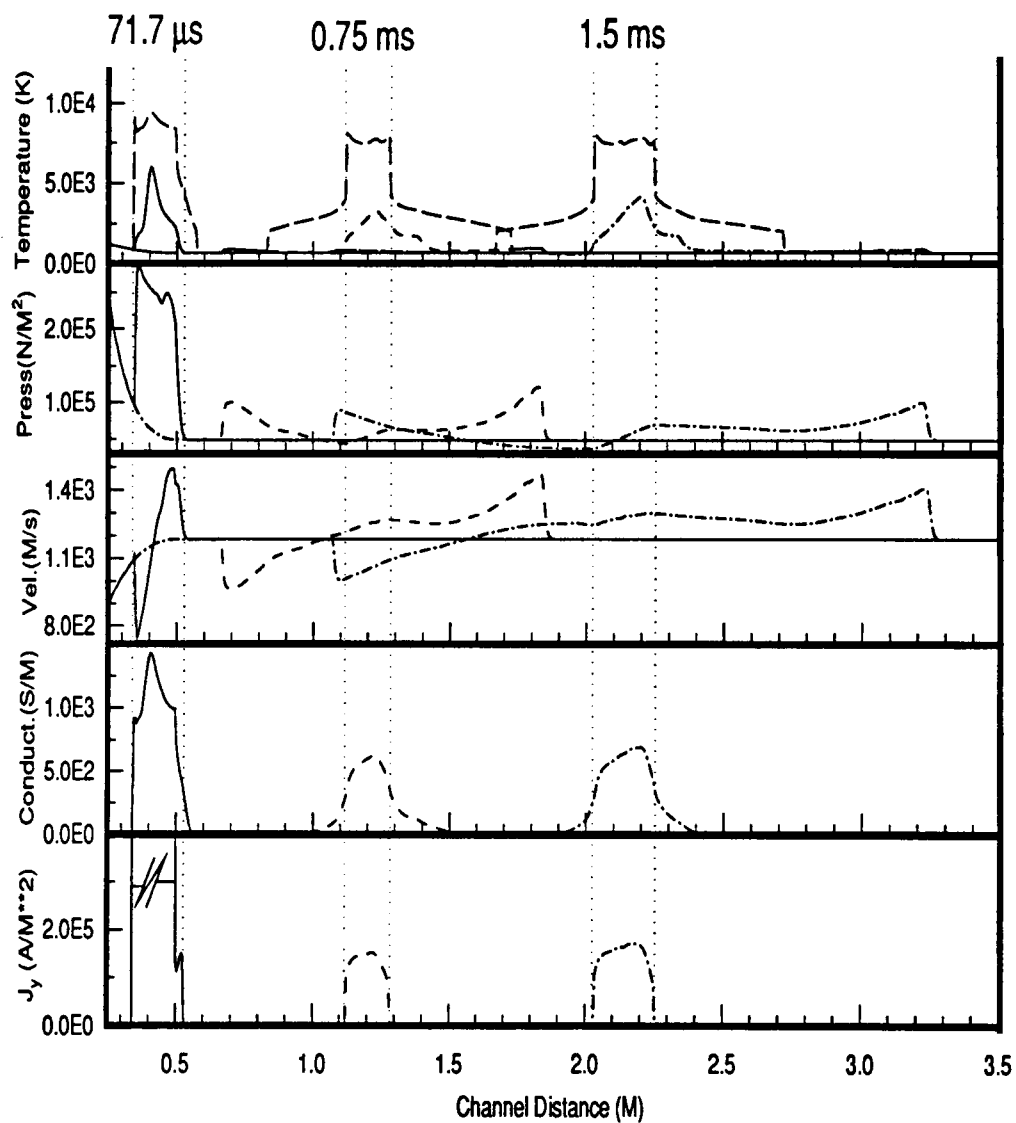
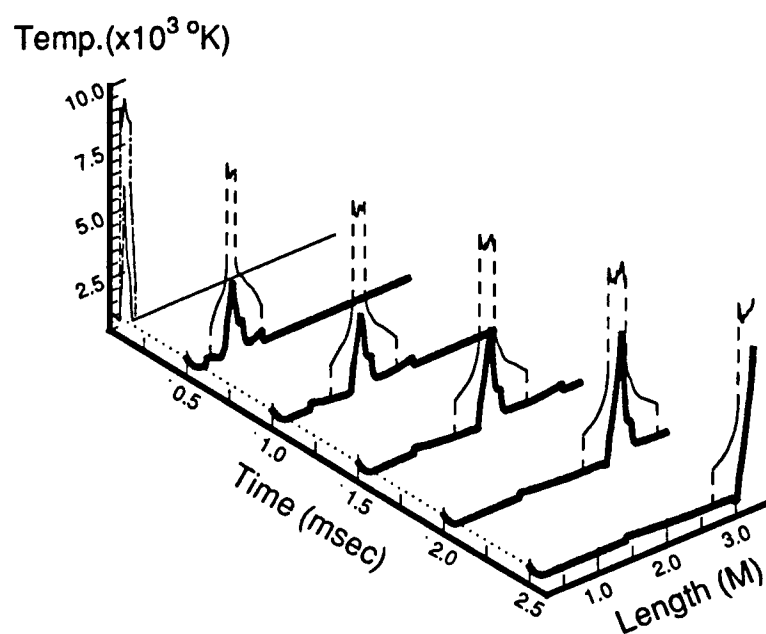
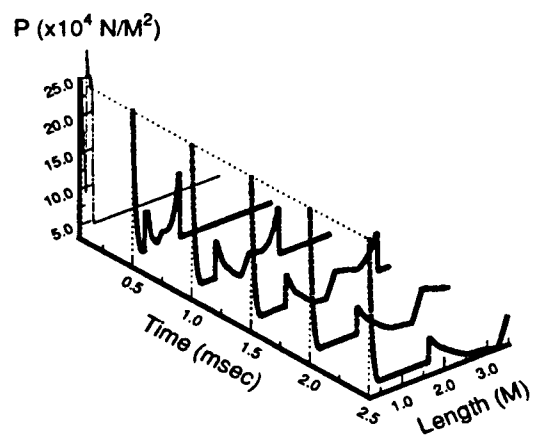


Figure 5.37 Property Distributions in the Single T-layer Accelerator with the Formation Section. From Top to Bottom: Electron (---) and Gas Temperatures, Pressure, Velocity, Conductivity, and Current Density.
 (—: $t = 71.7 \mu\text{s}$, ---: $t = 0.75 \text{ ms}$, -·-: $t = 1.5 \text{ ms}$)

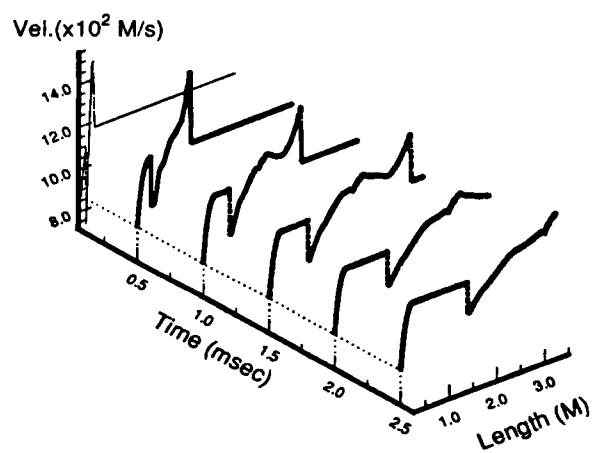


(a) Electron (---) and Gas (—) Temperatures

Figure 5.37 (Continued)

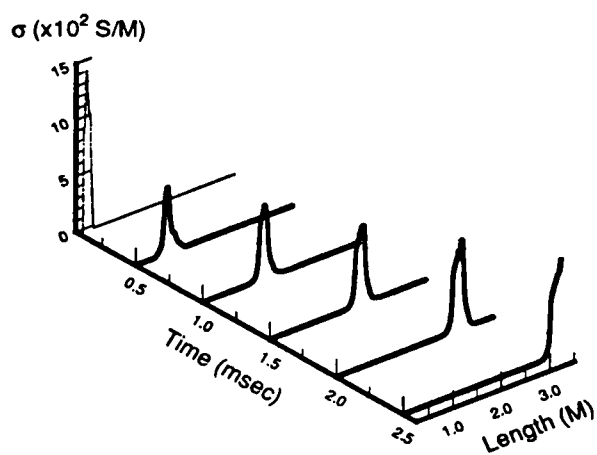


(b) Pressure

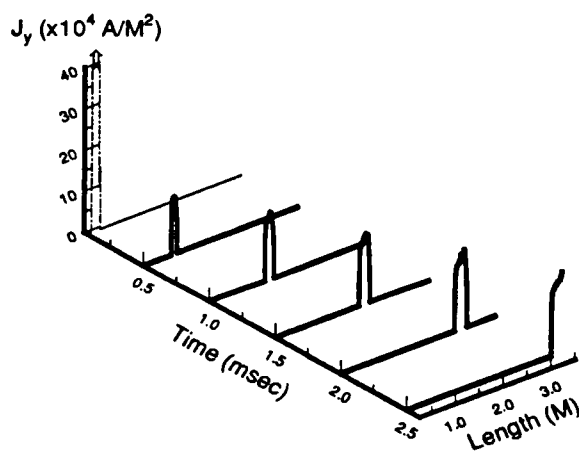


(c) Velocity

Figure 5.37 (Continued)



(d) Electrical Conductivity



(e) Current Density

Figure 5.37 (Concluded)

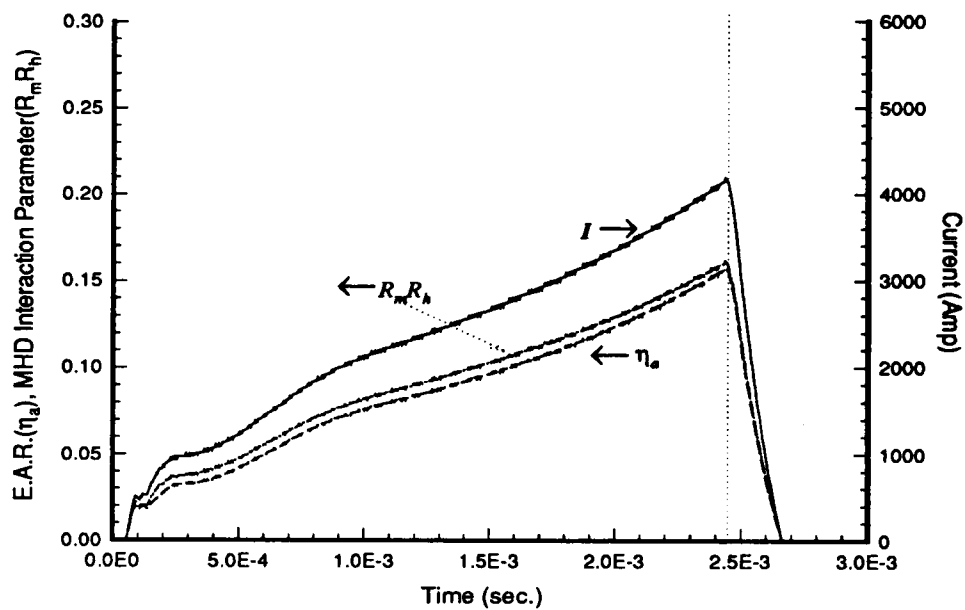


Figure 5.38 Performance of the Single T-layer Accelerator with the Formation Section

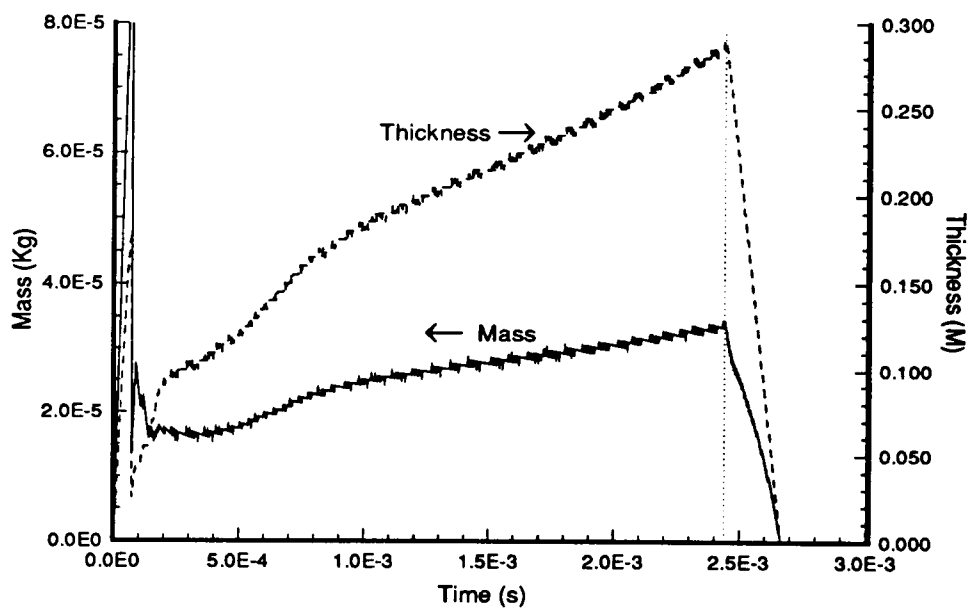
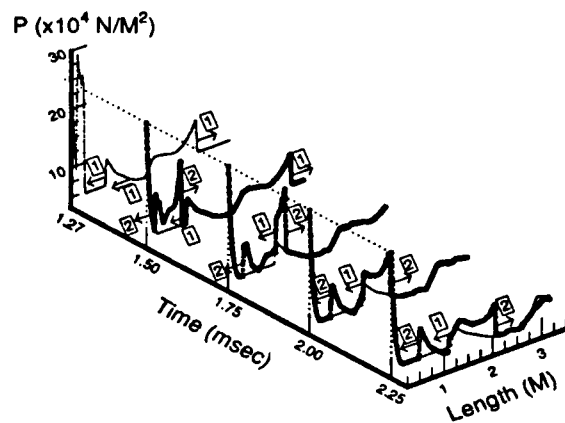


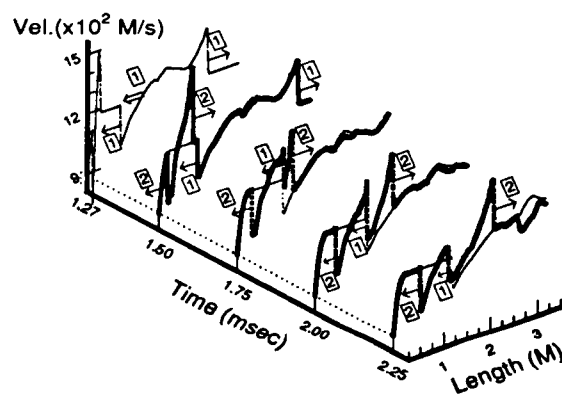
Figure 5.39 Mass and Thickness of the Single T-layer Accelerator with the Formation Section

Comparing to the results obtained with a spontaneously inserted T-layer in Figure 5.10, the flow field perturbed by the T-layer triggered in the formation section is less uniform and more complicated mainly due to the differences in the plasma properties at the MHD channel inlet in both cases. But the electrical properties in the T-layer plasma of both cases approach the same level of values at a certain time after the MHD interaction begins. Even though the T-layer properties at the channel inlet as well as the flow field in the channel during the MHD process are quite different in both cases, the net performance, the mass, and the T-layer thickness of both cases are about the same at the channel exit when relative comparisons are made between Figures 5.38, 5.39, 5.12, 5.15, and 5.16.

An example of operation with multiple T-layers is next examined with a T-layer forming period of 1.2 ms. The results given in Figures 5.40 show that the flow field in the channel thus produced is non-uniform. The flow field simultaneously includes the compression waves caused by the imbalance of the pressure across each T-layer's boundaries, the compression and expansion waves triggered by each T-layer's MHD interaction, and is further complicated by the mutual interactions between these waves. Usually, the interaction between the perturbations created by the MHD effect of two separate T-layers in an accelerator assists the growth in strength of the second (upstream) T-layer. This is because the MHD effects of T-layer create expansion waves propagate upstream and compression waves move downstream both relative to the T-layer in an accelerator. The plasma in the first T-layer suffers a pressure increase from the compression wave triggered by the second T-layer while the expansion wave caused by the first T-layer may decrease the pressure of the second T-layer plasma. This pressure influence in the plasma conductivity is very sensitive, especially when a nonequilibrium plasma is expected to exist.



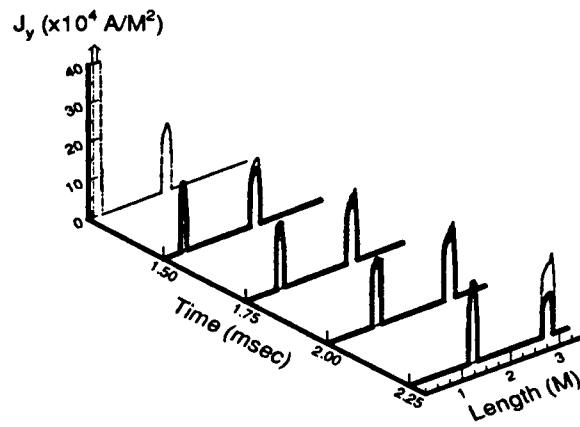
(a) Pressure



(b) Velocity

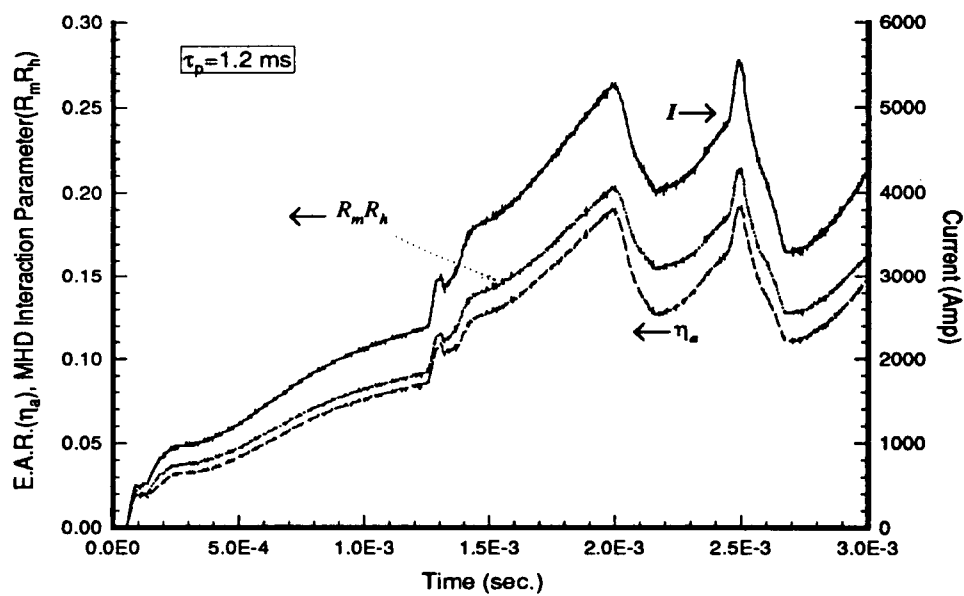
Figure 5.40 Property Distributions in the Accelerator with Multiple T-layers

$$(\tau_p = 1.2 \text{ ms})$$



(c) Current Density

Figure 5.40 (Concluded)

Figure 5.41 Performance of the Accelerator with Multiple T-layers ($\tau_p = 1.2 \text{ ms}$)

The growth in strength of the second T-layer also benefits from the quantity of Joule heat. In this study, the channel height is constant and, a voltage is imposed on the channel to accelerate the flow. The applied electrical field has the same value throughout the channel. When the second T-layer enters the MHD channel, the first T-layer has a relatively higher speed due to preceding acceleration. The higher induced $\bar{\mathbf{u}} \times \bar{\mathbf{B}}$ field on the first T-layer results in a lower Joule heat available to its plasma. Therefore, the strength of the first T-layer decays and that of the second T-layer increases, as can be clearly seen in Figure 5.40.(c).

From the accelerator performance presented in Figure 5.41, it is seen that in comparison with the single T-layer case, a higher enthalpy addition ratio can be obtained during a certain period after the second T-layer enters the accelerator channel. After that (1.95 ms), the effects from the interactions between the T-layers becomes significant and degrades the accelerator performance. In the same results, the second performance drop that begins around 2.5 ms is due to the exit of the first T-layer. Therefore, similar to the generator case, a proper choice of the cyclic period for initiating the T-layers and a better scheme to lessen the effects from the T-layer interaction are crucial for the success of applying multiple T-layers in flow acceleration.

5.4 Results of Performance Map

As described in Section 4.4 for the construction of a performance map, under the assumptions made in the quasi-steady model, the performance of a T-layer MHD channel can be totally determined when four parameters are assigned, i.e., the specific heat ratio γ , the flow Mach number M_1 , the interaction parameter $R_m R_h$, and the load factor K . To describe a T-layer device, the last two parameters entirely decide the effect resulting from the plasma interaction with flow; while the flow gasdynamic effects are decided

mainly by the first two parameters, i.e. γ and M_1 . The dominance of γ and M_1 on gasdynamic effects is a direct consequence of treating the T-layer as an impermeable piston and assuming the fluid is perfect gas and inviscid. Therefore, all gases with the same specific heat ratio will have the same performance map based on $R_m R_h$ and K when the flow Mach number is selected.

When using monatomic or molecular gas in an MHD device, these two types of gases usually need different heating sources to energize them to the required stagnation temperature. The intention herein is to compare the performance potential of a T-layer device which operates using monatomic gas to that of one which uses a molecular gas as the working medium. This purpose can be realized by choosing a proper value of γ for either type of gas when using the model proposed.

In the performance maps that are constructed, contour curves for reduced velocity and enthalpy extraction ratio of a generator (or enhanced velocity and enthalpy addition ratio of an accelerator) are included. The optimum loading curve for T-layer generator operation, which gives a maximum enthalpy extraction ratio at the same MHD interaction parameter, is also shown as a bold dash line. The results calculated in the quasi-steady sample cases in Section 5.1 are also spotted on the performance maps with $\gamma = 1.667$. These appear as bold solid curves at the right lower corner of Figures 5.42 and 5.46 for the generator and the accelerator, relatively. There is consistency between the results from the performance map construction analysis and those from the quasi-steady model.

Even though the performance maps constructed in this work are based on the assumptions made in the quasi-steady model, their predictions regarding the performance of T-layer devices are still in an acceptable range of accuracy as that observed in the sample cases in Section 5.1 when compared to the results from the unsteady model. Therefore, without consideration of the voltage drop across cold boundary layers, the

performance maps can provide a general scope of the performance potential of the T-layer devices and guide its operation toward the right operating conditions.

5.4.1 Generator

Figure 5.42 presents a performance map for T-layer generators using monatomic gases with a specific heat ratio of 1.667 and an unperturbed flow Mach number of 2.5. The map shows that acceptable performance for a generator ($\eta_N \geq 0.2$) can be easily obtained with a medium ranged MHD interaction parameter ($R_m R_h \approx 0.5$) and a loading condition close to the open-circuit condition, ($K_g \rightarrow 1$). The optimum loading curve shifts towards the high loading regime at large values of $R_m R_h$ and points out that a minimum K_g value to provide an acceptable performance is about 0.67. This high value of optimum K_g is contradictory to an engineering judgment widely accepted. A first-order "rule of thumb" for generators of this kind states that an optimum load factor should be around 0.5, which is equivalent to a statement that the maximum power of the generator occurs when the external and internal impedance are matched. This traditionally accepted optimum loading value of a generator does not hold true for a T-layer generator for the following reason.

Mathematically, to get an optimum loading of 0.5 from the relation for the power density produced, an independence between the load factor, the flow velocity, and electrical conductivity must be assumed. This rough estimate is not valid for a T-layer generator. The loading condition influences the magnitude of current through the T-layer and, from the Lorentz effect, influences the force applied on the T-layer, which decelerates the flow. As a result, the flow speed is strongly coupled with the loading situation. The electrical current passes through very narrow regions within high temperature plasma. The local response of this type of plasma structure to any

perturbation to the generator loading condition is more sensitive than that in a regular MHD generator. Wherein, current passes through the bulk volume of plasma and the adjustment to the new loading condition is distributed over a relatively large volume of plasma. Therefore, local plasma properties in T-layer such as electrical conductivity cannot be approximately treated as independent from the load factor. Based on this, the optimum load factor is not necessarily close to 0.5 for the T-layer generator. From observation of the curves for the optimum loading condition in the performance map of Figures 5.42 and 5.43 for different values of γ , it can be seen that the optimum value is close to 0.5 only when the T-layer interaction is weak, a condition where the coupling between flow properties and the loading is weak.

For a certain value of load factor, there also exists an optimum interaction parameter. If the T-layer interaction with the flow is stronger than the optimum value, the flow may be decelerated too much. With a low flow velocity, when moving through the magnetic field, the plasma cannot induce enough electric field and the generator performance degrades. If the interaction parameter increases by either applying a high magnetic field or operating at a lower stagnation pressure, the flow will eventually come to rest. Under such circumstances, there is no power produced which, in the performance map, is designated as a curve for zero enthalpy extraction ratio and zero reduced velocity. This limiting case (zero velocity) is more easily reached at low values of load factor since in that case the generator is subject to a high current, low voltage situation. The Lorentz force will become excessive enough to halt the flow.

It is also noted from these maps that, to provide a higher enthalpy extraction ratio, the optimum value for reduced velocity is between 0.4 - 0.6. This is true over large ranges of load factor and MHD interaction parameter and for both $\gamma = 1.667$ and $\gamma = 1.25$.

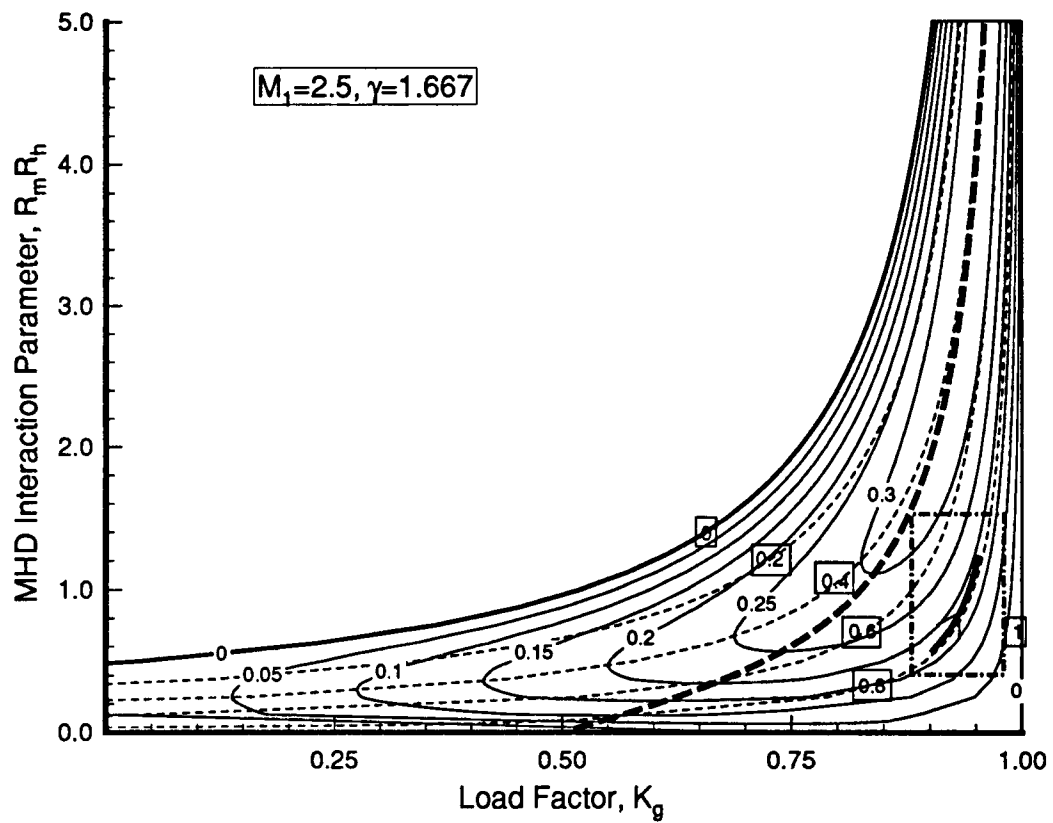


Figure 5.42 Performance Map of Enthalpy Extraction Ratio and Reduced Velocity for $\gamma=1.667$, Mach Number=2.5. Bold Dash Line Is the Optimum Loading Condition. The Operating Curve for the Sample Case Is in the Dot-Dash Box at the Right Corner. Contour Values in the Box Are for Reduced Velocity; Those without Box Are for Enthalpy Extraction Ratio.

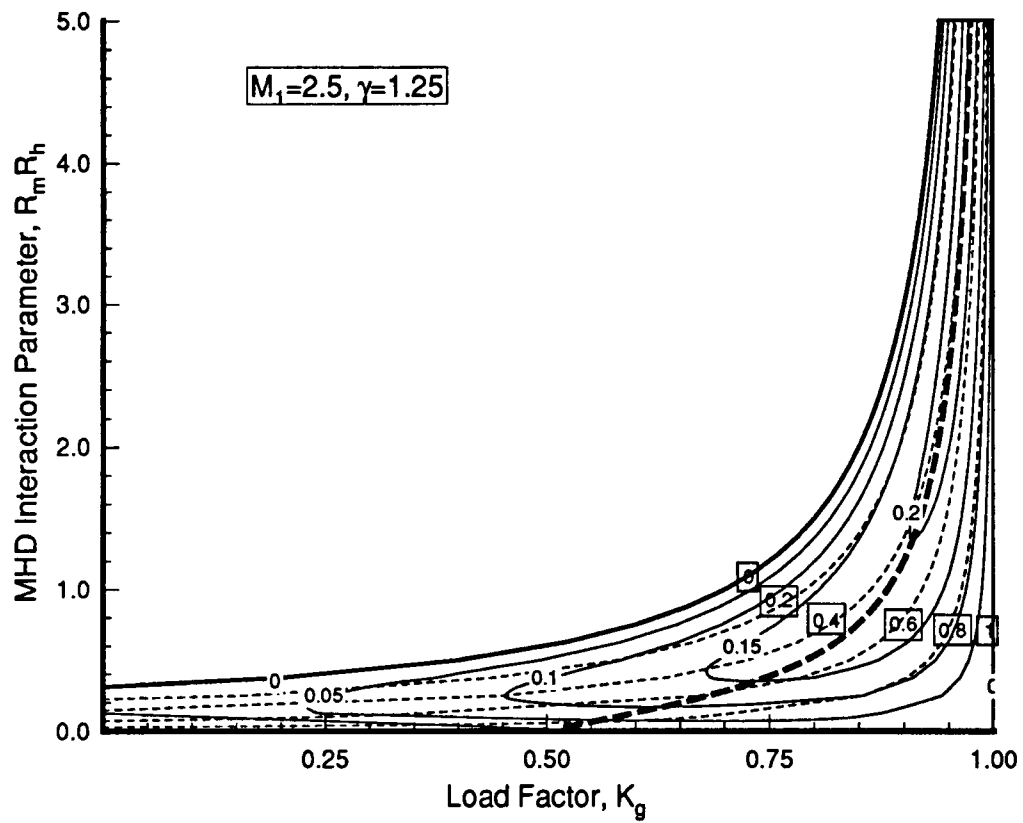


Figure 5.43 Performance Map of Enthalpy Extraction Ratio and Reduced Velocity for $\gamma=1.25$, Mach Number=2.5. Bold Dash Line Is the Optimum Loading Condition. Contour Values in the Box Are for Reduced Velocity; Those without Box Are for Enthalpy Extraction Ratio.

To predict the performance using air/combustion gases as the working medium in a T-layer generator, an average value of specific heat ratio of 1.25 is used. This is a typical value of γ for molecular gases at the working temperature ($\sim 10^4$ °K) of a T-layer device. Figure 5.43 gives the performance map for the case with a specific heat ratio of 1.25 and an unperturbed flow Mach number of 2.5. The results show that an acceptable enthalpy extraction ratio can only become achievable at large values of interaction parameter $R_m R_h$ and within a narrow range of load factor close to an open-circuit condition.

Comparison are made between the results shown in Figure 5.43 and 5.42. With the same load factor and MHD interaction parameter, greater values of enthalpy extraction and of reduced velocity are observed when $\gamma = 1.667$. T-layer generators exhibit better performance potential when using monatomic gases as working medium as compared to using molecular gases. The preferred use of monatomic gas in a generator is basically due to the favorable gasdynamic and thermodynamic effects related to the greater value of specific heat ratio γ .

Also observed is the influence from the flow Mach number, M_1 . Figures 5.44 and 5.45 show the optimum load factor for a certain MHD interaction parameter with its corresponding enthalpy extraction ratio for three different Mach numbers chosen as 1.5, 2.5, and 3.5.

Both cases for monatomic gases ($\gamma = 1.667$, Figure 5.44) and molecular mixtures at high temperature ($\gamma = 1.25$, Figure 5.45) are examined. In both cases, the performance of a T-layer generator increases when a higher Mach number is used. But, the trend of an increase of performance with Mach number might have an upper limit. From the results obtained for each Mach number, the performance is better when a monatomic gas is used.

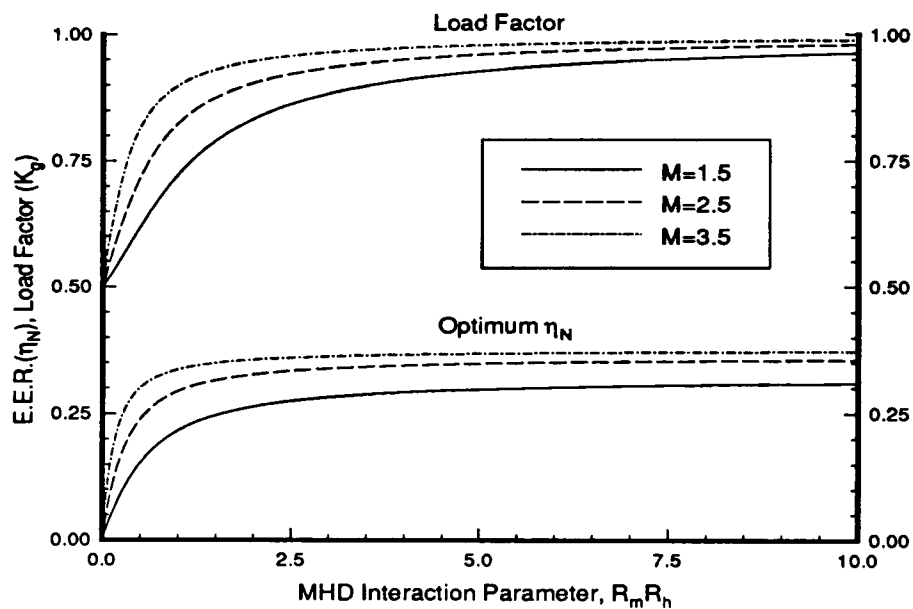


Figure 5.44 Optimum Loading Factor and Its Enthalpy Extraction Ratio for Various Mach Numbers with $\gamma = 1.667$.

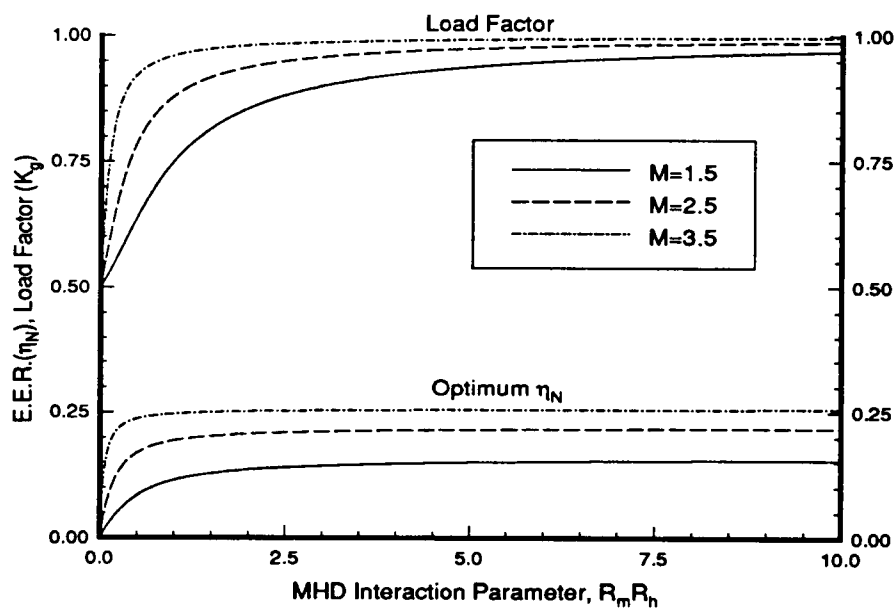


Figure 5.45 Optimum Loading Factor and Its Enthalpy Extraction Ratio for Various Mach Numbers with $\gamma = 1.25$.

5.4.2 Accelerator

The performance maps calculated for the T-layer accelerator are shown in Figures 5.46 and 5.47 for monatomic gases ($\gamma=1.667$) and molecular gases ($\gamma=1.25$), relatively. The flow Mach number is 2.5. It is seen that in each case a better enthalpy addition ratio can be obtained by operating at a higher load factor or at a stronger MHD interaction. A higher load factor corresponds to the use of a greater power supply and a stronger MHD interaction can be achieved by enhancement of plasma intensity, volume; or, by increasing the ratio between the strength of magnetic field and the stagnation pressure.

Comparisons between these two performance maps constructed for the T-layer accelerator show that larger enhanced velocity is found when $\gamma=1.25$ at the same load factor and MHD interaction parameter. The calculated enthalpy addition ratio generally has the same trend as the enhanced velocity, but the increments in its values are very limited when γ changes from 1.667 to 1.25 and Mach number is 2.5. It means that the gasdynamic effects prefer the use of molecular gases in the T-layer type of flow acceleration when flow Mach number is 2.5. This is contradictory to the T-layer generator case operated at the same Mach number in which monatomic gases are favored by providing a better performance.

It is found that in both T-layer generator and accelerator, the normalized velocity $\tilde{u}$ is subject to more significant deviations from unity when a smaller value of γ is used. That is, a greater change in kinetic energy of fluid in these T-layer devices is expected when gas with a smaller specific heat ratio is selected as working medium. The greater deviation from the unperturbed flow velocity lowers the local flow velocity and prevents a generator from providing a higher enthalpy extraction ratio, as seen from the

expression for the enthalpy extraction ratio in Equation (4.14). On the other hand, this greater deviation in velocity promotes the local flow velocity in an accelerator and, according to Equation (4.12), helps the accelerator to reach a higher enthalpy addition ratio.

Even though the lower specific heat ratio of molecular gases is shown to be helpful in flow acceleration and to provide a higher enthalpy addition ratio at Mach number 2.5, other advantages from using monatomic gas should not be neglected. With a larger value of γ , the heat capacity of a monatomic gas is smaller for the same molecular weight. Thus, a monatomic gas requires less energy to be heated to a high temperature. The low heat capacity of monatomic gases has a positive effect in the formation of a T-layer.

Additionally, a monatomic gas usually has a smaller electron-atom collision cross section and is subject to producing a nonequilibrium plasma when it is heated. The advantage of the nonequilibrium plasma effects helps a monatomic gas to save even more energy needed to obtain the high conductivity of a T-layer plasma. That is to say, for the same power available to heat and ionize the working medium, the production of a nonequilibrium plasma usually provides higher conductivity compared to that of an equilibrium plasma. A plasma with higher conductivity can interact more strongly in the MHD channel. This corresponds to a higher value of MHD interaction parameter when the load factor is kept the same. (Joule heating is directly related to the load factor as seen in Ohm's law (4.7).) From the performance map, it is easy to see that a higher value of the MHD interaction parameter provided by using a nonequilibrium plasma always increases performance of the T-layer accelerator and extracts more enthalpy in the generator case when the MHD interaction parameter is below its optimum value at the same load factor.

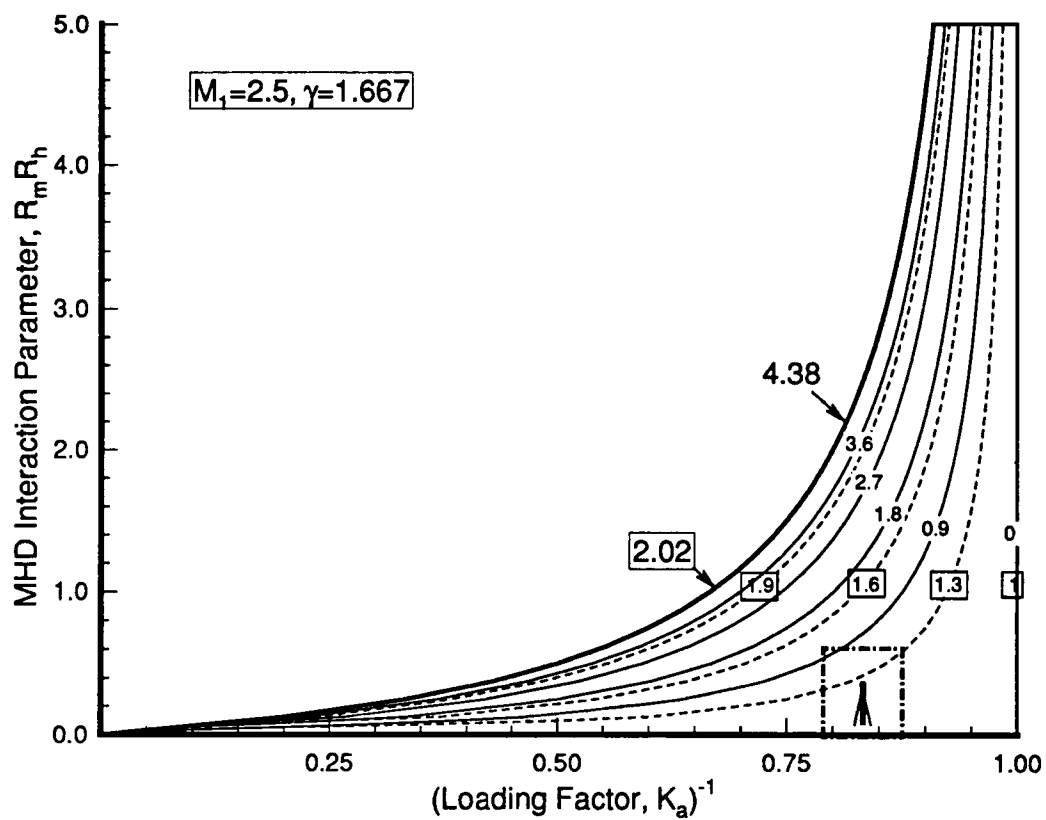


Figure 5.46 Performance Map of Enthalpy Addition Ratio and Enhanced Velocity for $\gamma = 1.667$, Mach Number=2.5. The Operating Curve for the Sample Case Is in the Dot-Dash Box at the Right Corner. Contour Values in the Box Are for Enhanced Velocity; Those without Box Are for Enthalpy Addition Ratio.

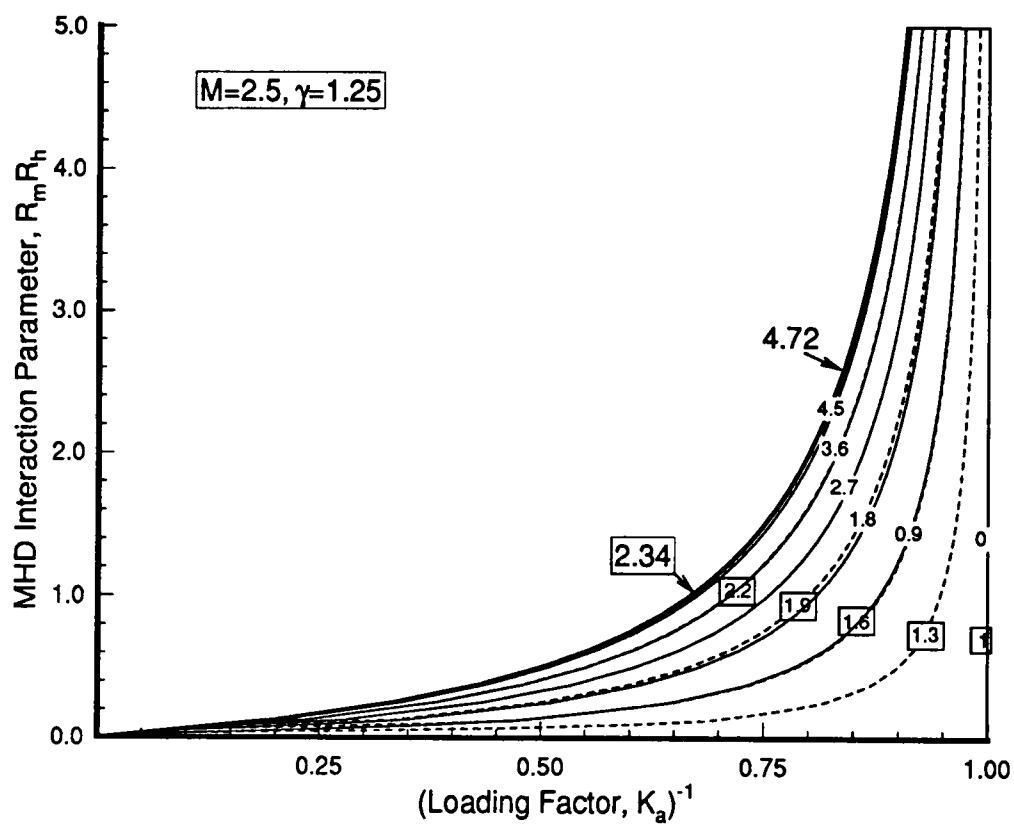


Figure 5.47 Performance Map of Enthalpy Addition Ratio and Enhanced Velocity for $\gamma = 1.25$, Mach Number = 2.5. Contour Values in the Box Are for Enhanced Velocity; Those without Box Are for Enthalpy Addition Ratio.

From observation in the performance map obtained for the generator or for the accelerator, there is an upper limit for the MHD interaction parameter at a chosen load factor. This constraint on the MHD interaction parameter is set by the following conditions:

- 1) The pressure drop across the T-layer has to be smaller than the stagnation pressure, i.e., $\Delta\tilde{p} = (p_2 - p_3) / p_{t0} \leq 1$. Substituting this condition and $\kappa = (1 - \tilde{u})M_1$ for the generator or $\kappa = (\tilde{u} - 1)M_1$ for the accelerator into Equation (4.18), a requirement for normalized velocity $\tilde{u}$ is then obtained.
- 2) The pressure at the upstream end of the T-layer, p_3 , has to be greater than zero. From the expression for p_3/p_1 in Equation (4.11), a limitation in normalized velocity $\tilde{u}$ is derived as

$$\left\{ \begin{array}{ll} \tilde{u} \geq 1 - \frac{2}{(\gamma - 1) M_1}, & \text{for generator;} \\ \tilde{u} \leq 1 + \frac{2}{(\gamma - 1) M_1}, & \text{for accelerator.} \end{array} \right. \quad (5.5)$$

- 3) The lower limit for the flow velocity is zero, i.e., $\tilde{u} \geq 0$.

The results given in Figures 5.42 and 5.43 for the T-layer generator with $\gamma = 1.667$ and $\gamma = 1.25$ show that the upper limit of the MHD interaction parameter for an assigned load factor in both cases is restricted by the condition $\tilde{u} \geq 0$. Once the flow becomes stationary, no further enthalpy can be extracted. On the other hand, in either performance map of Figures 5.46 and 5.47 constructed for the accelerator, the curve for the upper value of MHD interaction parameter is found to be equivalent to $\Delta\tilde{p} = 1$. The corresponding values of the enhanced velocity and enthalpy addition ratio on this curve are also indicated in the same performance map.

CHAPTER 6

SUMMARY AND RECOMMENDATIONS

6.1 Summary

A one-dimensional unsteady model is developed to describe the T-layer process in the MHD channel when using nonequilibrium plasma. The possible benefits from using a nonequilibrium plasma in the T-layer concept are to increase the plasma electrical conductivity to a high level with less power and to reduce the radiative loss. According to the results obtained in the time scale analysis using an argon plasma, it is concluded that certain relaxation processes occurring in the T-layer MHD interaction are necessary to be included as time-dependent processes in this unsteady model. Important time-dependent processes include ionization/recombination of argon plasma, continuity, momentum balance, and global energy balance of working gas. Only electron energy is treated as being instantaneously relaxed. The time-dependent governing equations are solved numerically using a high-resolution second-order TVD scheme to assure proper and accurate simulation of the complicated flow field including the compression and expansion waves that exist in the T-layer MHD channel. The criterion for the onset of a nonequilibrium plasma based on the previous studies of Borghi, et al.^{63,64,65} is also utilized in the unsteady model. It is found that the minimum electron density needed to effectively trigger the nonequilibrium plasma generally corresponds to ionization degrees between 10^{-5} - 10^{-4} for argon plasmas.

For the purpose of verifying the results obtained with the unsteady model, a quasi-steady model modified from former works of Vasilyev et al.^{19,38,44} is also applied to

calculate the sample T-layer processes. This quasi-steady model was intrinsically developed to analyze the slower process for thermal stabilization of T-layer process and thus the energy equation of heavy species is the only equation solved time dependently. The primary differences in the assumptions made in the quasi-steady model from that in the unsteady model are the spontaneous relaxation of momentum balance, Saha equilibrium in ionization/recombination of charged particles, and an impermeable piston model for the T-layer.

In the calculations using both the unsteady and the quasi-steady models, sample T-layer devices operating with a moderate range of plasma intensity (current density $\sim 10^5$ A/m²) were chosen. Significant elevation of the electron temperature above the gas temperature is clearly observed in the results obtained from both models. These results are seen in the temperature distributions in Figures 5.1 and 5.2 for the generator and in Figures 5.10 and 5.11 for the accelerator. Nonequilibrium plasma effects can be significant only in a T-layer with a moderate-intensity plasma. Once the intensity of the plasma is very high; e.g., current density above 10^6 A/m², the electron temperature in the plasma can be more than 12,000 °K. At this high electron temperature, the heavy species (neutrals and ions) in the plasma will be heated at a higher rate due to the more frequent electron-ions collisions. The temperature of the heavy species then approaches the electron temperature and the plasma goes toward thermal equilibrium.

In the results obtained in the sample generator case, good agreement between these two models are observed in the calculated flow fields, levels of electrical properties of plasma (electron temperature, electrical conductivity, and current density), and the generator performance. Deviations in the distribution profiles of plasma electrical properties and gas temperature, plasma mass, and the rate of increase of the MHD interaction parameter are found mainly caused by the additional assumptions made in the

quasi-steady model. The assumptions of spontaneous force balance, Saha equilibrium, and impermeable piston model result in a low pressure, high intensity plasma localized near the downstream end of the T-layer found in the quasi-steady analysis. This subsequently makes the plasma more susceptible to the occurrence of electrothermal instabilities.

The results obtained in the unsteady model for the sample cases show that a condition of stable operation can be reached in a period of $3\text{--}5 \times 10^{-4}$ second after the MHD interaction begins. In the stable operation of a T-layer device, the velocity of the T-layer varies slightly. The effects of the further MHD interaction result in more volume of fluid being decelerated in the generator or accelerated in the accelerator.

Due to the assumption of a constant load factor through the MHD process in the sample T-layer accelerator case, different heating power is added to the plasma in the analyses using the different models. The consistency between the results obtained by the unsteady and the quasi-steady models is not so promising as that seen in the generator case but still remains in an acceptable region. It is noted that the imbalance of the pressure across the boundaries of a T-layer may trigger an additional compression wave that propagates upstream relative to the T-layer during the MHD interaction. The same compression wave is not observed in the generator case.

The unsteady model is applied to simulate the T-layer experimental results using argon gas as reported by Gavrilov et al.⁸ Without including any voltage drop across the cold wall boundary, the calculated peak current density agrees with that measured in the experiment if a factor of 2 correction is applied, due to the effective width of electrodes used in the experiment. However, with the voltage drop neglected the total current evaluated is much higher than the measured value and the T-layer is much thicker than that observed in the experiment. By matching the peak total current, a voltage drop of 9

volts is calculated. With this amount of voltage drop, the calculated flow velocity, the thickness of the T-layer, and the effective conductivity in the channel match the experimental data with reasonable allowance. The current density is then relatively low when compared to the experiment. It is believed that current constriction relates to the discrepancy in the current density. In the unsteady model, flow is treated as one-dimensional and a constant value of voltage drop is assumed through the entire channel. With these two assumptions, the unsteady model cannot properly describe the localized, multidimensional current constriction on the contact surface between the plasma and the electrodes. A large reduction in generator performance due to a voltage drop of 9 volts is observed in the comparison between the results presented in Figures 5.22 and 5.23. The importance of maintaining a low voltage drop across the cold boundary can be easily seen.

The T-layer formation process within a proposed configuration of a T-layer formation section is simulated by the unsteady model. The T-layer formation section consists of an igniter and an ionizer section, upon which power sources are externally imposed. From the results, it is found that the downstream end of the expanded plasma volume moves downstream with a speed as superimposition of the unperturbed flow speed with the speed of compression wave due to thermal expansion of heated gas. The compression waves formed in the formation section are then taken into the MHD channel. The influence from the thermal compression waves in the flow field of the MHD channel can only be clearly seen near the upstream end of the T-layer when merged with the expansion wave created by the T-layer acceleration.

The situation of multiple T-layers co-existing in the MHD channel is also examined. Multiple T-layers are created by intermittent powering of the formation section with an assigned period. The results show that using multiple T-layers indeed

increases the performance of the generator and the accelerator prior to the encounter between T-layer and the wave perturbations caused by other T-layers. Once the interactions between two T-layers take place, the pressures inside the T-layers vary and the plasmas respond with changes in their properties. To match the loading condition of the MHD channel, the strength of one T-layer grows while that of the other decays. The uneven distribution of plasma intensities between T-layers usually increases the internal impedance of the entire channel. As a result, fluctuations are inevitable in the output and the performance of the T-layer device is degraded. To lessen the influence from the interactions between T-layers, a proper selection of the T-layer forming period is crucial. Since the conductivity of a nonequilibrium plasma is very sensitive to the variations in pressure, elimination of the interaction between T-layers is particularly important in case where a nonequilibrium plasma is used.

The performance map for the T-layer device is constructed based on the assumptions made in the quasi-steady model without consideration of the effects from the voltage drop due to the cold boundary layer. The purpose for the performance map is to provide a general scope of the performance potential of T-layer devices. The results show that, relative to molecular gases, monatomic gases are found to provide better performance for the T-layer generator. Using monatomic gases, T-layer generators can easily provide good performance even when operated over a moderate range of flow Mach number. From the performance map obtained an enthalpy extraction ratio as high as 30% is reachable with a Mach number of 2.5 and close to 35% with a Mach number of 3.5. Using air/combustion gases as working medium, the T-layer generator can also provide substantial enthalpy extraction ($\eta_N \sim 0.25$) when operated at high Mach numbers.

In terms of gasdynamics, the T-layer accelerator prefers the use of molecular gases as working medium at Mach number 2.5 by providing a greater enhanced velocity and a higher enthalpy addition ratio. Nevertheless, the differences in enthalpy addition ratio between using molecular and monatomic gases are very limited. At a Mach number of 2.5, the upper limit values of enthalpy addition ratio and enhanced velocity for molecular gases are respectively 4.72 and 2.34; while, those for monatomic gases are relatively 4.38 and 2.02.

On the performance map, the application of a nonequilibrium plasma provides a higher value of MHD interaction parameter at the same load factor. This effect promotes the performance of the T-layer accelerator and increases enthalpy extraction ratio when the generator is operated at an MHD interaction parameter below its optimum value of a certain load factor.

6.2 Recommendations for the Future Work

Compared to traditional MHD devices, there are more important parameters to be considered in the T-layer MHD devices; such as, thickness of T-layer, cyclic period for triggering T-layers, optimum number of T-layers co-existing in the MHD channel, power required to form T-layers, etc. In order to optimize the performance of T-layer devices and to further understand the influence of each parameter on performance, a parametric analysis using the unsteady model developed in this work is necessary to supplement the performance map constructed based on the quasi-steady model. This is especially important for the application of multiple T-layers since the quasi-steady model only deals with single T-layer cases and the interaction between T-layers can significantly degrade the performance.

In practice, a diffuser connected downstream to the MHD channel is required for deceleration of the flow. To comprehensively evaluate the performance of an MHD device, the diffuser should also be considered in the calculations. This is especially true in cycle analysis for the entire power generation system, in which the T-layer MHD generator is involved as a component.

Even though the unsteady model developed in this work contains several primary effects that occur in the T-layer MHD process, modifications to cover more effects are desirable. Viscous effects cause irreversible loss but help smooth the non-uniform flow field created by the T-layer MHD interaction. To accurately estimate the influence from viscous effect, the Navier-Stokes equation coupled with the MHD effect has to be included in the model. Some phenomena that occur in the T-layer concept are multi-dimensional, such as the current constriction, instabilities of Rayleigh-Taylor type, etc. These cannot be properly described by a one-dimensional model. With Liou's TVD scheme to solve the unsteady equations and Euler's predictor-corrector method to handle the source terms, the numerical solver in this work can be extended to multi-dimensional cases.

Besides the viscous effect, the net effects from the flow boundary layer also include the voltage drop across the boundary layer. As discussed in the comparison of results obtained in the unsteady model with the experimental data, the existence of the voltage drop may have very critical influence in the performance of the MHD devices. Therefore, a model to appropriately estimate the voltage drop across the boundary is essential for predicting the performance accurately.

Any theoretical and numerical prediction is subject to the experimental verification. The shortage of the experimental data has been the major hurdle for the development of

the T-layer concept. Therefore, the establishment of an experimental data base is the single aspect most needed for the future T-layer research program.

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VITA

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