

Coulomb Corrections for Inclusive Electron Scattering Data

A Thesis Presented for the
Master of Science
Degree

The University of Tennessee, Knoxville

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May 2019

Abstract

Experiment E89-008 was performed using the Continuous Electron Beam Accelerator Facility (CEBAF) at the Thomas Jefferson National Accelerator Facility in 1996. It measured inclusive electron scattering for D [deuterium], ^{12}C [carbon 12], ^{56}Fe [iron 56], and ^{197}Au [gold 197]. Of those, the ^{12}C , ^{56}Fe , and ^{197}Au data was analyzed in terms of $F(y)$, which is a tool that was used to understand the momentum distribution of nucleons in the nucleus. However, the processing on this data was incomplete, and Coulomb corrections were never applied. These corrections are essential in finalizing and completing the processing of the E89-008 data. To apply these corrections, a suite of code that simulates cross sections using the XEM model was restored from a specific experimental version to fit the conditions imposed by the E89-008 data. Using this simulation, Coulomb corrections were finally applied to the E89-008 data.

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Chapter 1

Introduction

Scattering experiments are ubiquitous in physics. Starting with Hans Geiger and Ernest Marsden's gold foil experiment for Ernest Rutherford, measuring the energy and angle of scattered particles off a target has long been an essential method to study nuclear and nucleon structure.

Thankfully, scattering experiments have improved in both scope and sophistication since the gold foil experiment. Instead of emitting particles from a glass tube, modern experiments accelerate electrons through a sophisticated track of magnets, with the energy of these accelerated particles increasing as time has gone on. With the recent 12 GeV upgrade to the Thomas Jefferson National Accelerator Facility, modern scattering experiments are a far cry from a glass tube and gold foil.

1.1 Theory

While alpha particle scattering was the tool of choice for the gold foil experiment, electron scattering is also an effective tool for studying nuclear structure. Given the lack of significant electromagnetic interaction between an electron and a target nucleus, the interaction between the two can be modeled as the exchange of a photon between the electron and a single particle in the nucleus. The ambiguity in the use of the word 'particle' is intentional. The scale of the particle probed is dependent on experimental kinematics. Specifically, the resolution of the

probe is inversely proportional to the wavelength of the photon, $\lambda = \frac{h}{\sqrt{Q^2}}$, where Q^2 is the square of the 4-momentum transfer. Q^2 can be further defined as $Q^2 = -q^2 = \vec{q}^2 - (\nu)^2 = 4 \cdot E_0 \cdot E' \cdot \sin^2(\frac{\theta}{2})$ where ν is the energy transfer $\nu = E_0 - E'$, the beam energy minus the scattered energy, q is the three momentum, and θ is the scattering angle [1]. At the lowest Q^2 settings, the exchanged photon interacts with the nucleus as a whole, scattering elastically. As Q^2 increases, the photon interacts on a more focused scale. At higher Q^2 the initial elastic scattering will instead become quasielastic (QE) scattering, where the photon interacts with a single nucleon instead of the nucleus as a whole. Increasing Q^2 further allows the photon to probe deeper than a single nucleon and instead scatter off a single quark [1].

Another way to look at the scattering is the Bjorken x , $x = \frac{Q^2}{2m\nu}$, where m is the nucleon mass. Bjorken x will be referred to simply as x in this thesis. x will range from 0 to 1 for scattering off a free nucleon, with $x = 1$ representing elastic scattering off the nucleon, and $x < 1$ representing inelastic scattering [5]. In a nucleus with A nucleons, x can vary from 0 to A because the nucleons can share momentum. The higher the x , the larger the fraction of nuclear momentum carried by the struck particle. At high ν and Q^2 , x can be viewed specifically as the fraction of the nucleon's momentum carried by the struck quark [6]. All of this combines to make an exploration of scattering $x > 1$ quite interesting as it provides a look at a regime forbidden to the free nucleon. Consequently, one can explore the transition from quasielastic to deep inelastic scattering in a region of momentum sharing.

1.2 Scaling

For scattering, where the method of interaction is well understood, it can be useful to look for scaling behavior. Doing so allows one to use scaling to study the underlying nuclear structure. Scaling is the regime where the measured cross section, ordinarily dependent on Q^2 and ν , can be expressed by a single variable that, in turn, is a function of Q^2 and ν . Scaling expresses itself differently between the deep inelastic and quasielastic regimes, which reflects the main process in each zone [7].

1.2.1 Quasielastic Scattering

Quasielastic scattering has, for a long time, been analyzed in terms of y -scaling because quasielastic scattering can be expressed as a function of y , the nucleon's momentum along the direction of the exchanged photon.

The use of y is applicable because quasielastic scattering involves the scattering off one nucleon. Thinking in this way, the quasielastic cross section can be expressed as the summation of all the bound nucleon cross sections:

$$\frac{d^5\sigma}{dE'd\Omega d^3\vec{p}} = \sum_{nucleons} \sigma_{eN} \cdot S'_N(E_0, \vec{p}_0) \quad (1.1)$$

With E' being the energy of the scattered electron, E_0 and $\vec{p}_0$ being the energy and momentum of the hit nucleon, and $\vec{p}$ being the final momentum of that nucleon. σ_{eN} is the electron-nucleon cross section for scattering off of a bound nucleon and $S'_N(E_0, \vec{p}_0)$ is the spectral function that represents the probability of finding a nucleon with those kinematics in the nucleus. An inclusive cross section will be an integral over this spectral function, though the quasielastic portion of that can be simplified to yield:

$$\frac{d\sigma}{d\Omega dE'} = \sigma_{eN} \cdot F(y) \quad (1.2)$$

In doing this, the y dependence is revealed as a part of $F(y)$, a scaling function that is related to the momentum and energy distribution of the nucleons. Conveniently, this allows one to view the quasielastic cross section as two separate terms. The electron-nucleon cross section represents the interaction piece, where $F(y)$, being a related to the momentum and energy distribution of the nucleons, represents the underlying nuclear structure of the target [5].

1.2.2 The Limitations of Scaling

While the above treatment of y provides a general understanding of y as and $F(y)$ as a scaling variable and scaling function, more specific conditions can demonstrate different scaling relationships. For example, in the case of quasielastic scattering at high- Q^2 , with no

final-state interactions (FSIs), where a nucleon interacts and overlaps with another nucleon after the scattering event, the cross section can be reduced to the following:

$$\nu + M_A - E_s^{min} = \sqrt{(M_N^2 + (q + y)^2)} + \sqrt{(M_{A-1}^2 + y^2)} \quad (1.3)$$

where M_A and M_{A-1} are the masses of the target and spectator (A-1) system respectively. In the case of quasielastic scattering, M_A is the nucleon mass, and $A - 1$ is the mass of the remaining nucleons. E_s^{min} is the minimum separation energy, the minimum energy to separate a nucleon from its nucleus. From this point, $F(y)$ can be written as:

$$F(y, Q^2) = \frac{d^2\sigma}{d\Omega d\nu} [Z\sigma_p + N\sigma_n]^{-1} \frac{q}{\sqrt{(M_N^2 + (y + q)^2)}} \quad (1.4)$$

$F(y, Q^2)$ will only be dependent on y at large Q^2 . This holds true over a wide range of nuclei and momenta. Additionally, if the spectator (A-1) nucleus is unexcited, $F(y)$ can be directly related to the nucleon momentum distribution, $n(k)$: $\frac{F(k)}{dk} \approx -2\pi k n(k)$ [2]. This is an extremely useful relationship because the momentum distribution is not an observable. Since $F(y)$ can be related to the measured cross section, it provides a clear path to understanding the momentum distribution. However, the ability to directly relate to the momentum distribution is only applicable for the deuteron (^2H) because the spectator nucleus is only one nucleon.

Figure 1.1 shows such an extraction using the deuteron. The points represent extracted momentum distributions from a range of experimental θ 's and Q^2 's. Higher Q^2 's were not included in the above plot because they do not contain sufficient precision in the desired high momentum region. However, with the remaining data, it is clear that extracted momentum distributions are Q^2 independent. Additionally, the extracted distributions match the modeled distributions, which further demonstrates that such a relationship between $F(y)$ and $n(k)$ is valid for ^2H [7].

However, the requirements for such a relationship are not fulfilled in nuclei heavier than the deuteron. The spectator (A-1) for the deuteron is a single nucleon, which is much easier to understand and quantify. For heavier nuclei, where the spectator (A-1) system can involve

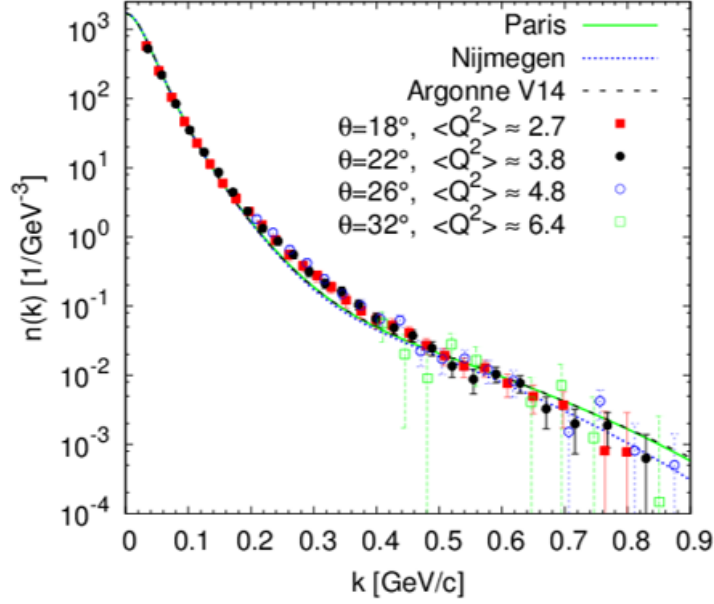


Figure 1.1: Plot of k vs extracted $n(k)$ from E02-019 data for $A = 2$ for various angle settings. [2]

many many nucleons, there can be a break up or excitation of this system, which leads to an uncertainty when dealing with the spectator system.

While attempts have been made to account for these shortcomings in the scaling function [8], the solutions were never fully model independent with regards to $n(k)$.

However, it is possible to achieve a more meaningful physical result by comparing nuclei in a kinematic regime where the scattering is k , the momentum, is greater than k_F . In Figure 1.2, the momentum distribution given by the black dotted line roughly represents the distribution given by mean field theory. The solid line represents the calculated momentum distribution, with its noticeable high momentum tail. The high momentum region is interesting because the tail is thought to be caused by short range hard interactions between nucleons. These short range correlations (SRCs) allow for the study of short-distance structure through looking at interactions of high-momentum nucleons [9].

By analyzing the tails in this high momentum region, it allows for information to be extracted in a unique way. Looking at the regime where $k > k_f$, the observed nucleons are ones that, in the SRC picture, gained momentum from short range interactions before

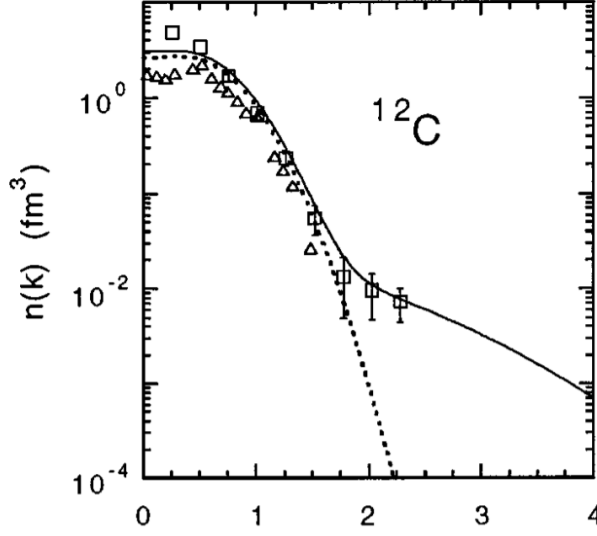


Figure 1.2: Plot of k (x -axis. Not labled) vs $n(k)$ for ^{12}C . The dotted line represents n_0 , the 0^{th} order mean field term. The full theoretical multi-body nucleon momentum distribution, $n(k)$, is given by the solid line. The square points were obtained using y scaling analysis of inclusive scattering data and the triangular points were momenta extractions from exclusive data [3]

scattering. This is compared to FSIs, where the nucleon interacts with another nucleon after scattering. This region provides a different way to probe the high momentum tail for various nuclei. The compared nuclei should have a similar high momentum tail if the high momentum components are related to two-nucleon short range correlations (2N-SRCs). That is when the nucleons have a large relative momentum, but a small momentum for the center of mass because of their hard two-body interaction [2]. This holds true for heavy nuclei down through the deuteron and can be seen in Figure 1.3. However, while the momentum distributions in Figure 1.3 are not identical to one another, the high momentum tails for $A > 2$ should be scaled versions of the deuteron tail.

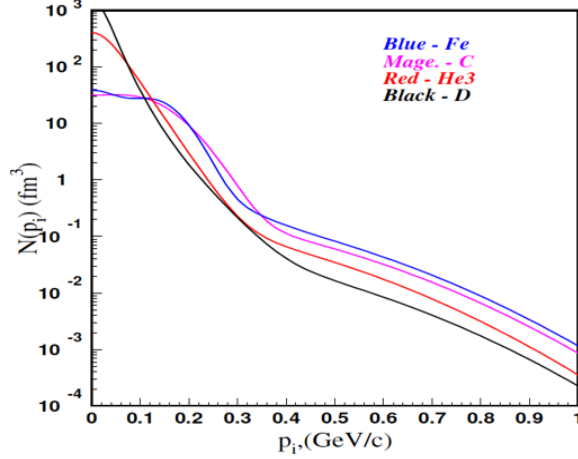


Figure 1.3: k vs calculated $n(k)$, labeled as p_i vs $N(p_i)$, for Fe (blue), C (magenta), ^3He (red), and ^2D (black). [3]

If the high momentum tails of various nuclei are related to the deuteron, then evaluating these nuclei using ratios should yield consistent results. However, since momentum distributions aren't directly measured, cross sections are used instead. If the segment of the cross section that is used is in the $x > 1$ region then that region contains nucleons with $k > k_f$, with k_f being the fermi momentum. The main ratio used is a per nucleon cross section ratio. Specifically, in comparing $\frac{(\sigma_A/A)}{(\sigma_D/2)}$, with D standing for the deuteron and A is an arbitrary nucleus greater than 2, a plateau is expected to appear as x approaches 2 [2]. However, the visibility of this plateau is dependent on kinematics. As is obvious by Figure 1.4, data at lower Q^2 is not able to reach as far into the high momentum region. This is especially true for heavier nuclei. If Q^2 is too low, the SRC plateau is far less defined. This is shown clearly by Figure 1.5.

The appearance of a plateau is significant because this means that a per nucleon cross section ratio is a valid way of studying SRCs. The data from JLab experiment E89-008, the data this thesis is based on, was able to explore higher Q^2 than the Stanford Linear Accelerator Center (SLAC) or CEBAF Large Acceptance Spectrometer (CLAS), but was limited by its deuterium statistics. As a result, while it is able to provide evidence of SRCs, it, and most of the data taken to this point, cannot provide precise A/D ratios for numerous nuclei over the desired x and Q^2 range [2].

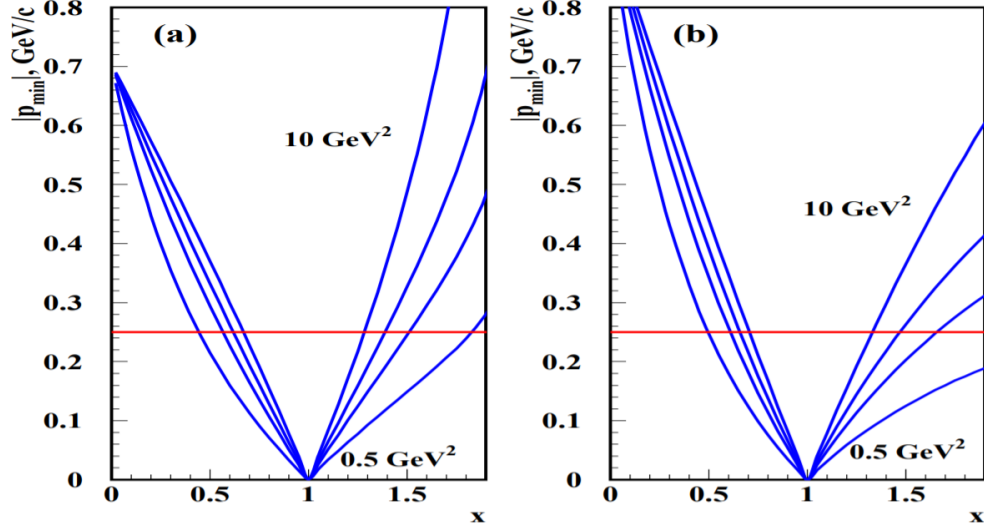


Figure 1.4: Plot of the minimum momentum required to scatter a nucleon in deuterium (left) and gold (right) as a function of x and Q^2 . Specifically, this corresponds to quasielastic scattering for Q^2 of 0.5, 1.5, 3, and 10 GeV^2 . For heavier nuclei, which have larger Fermi momenta, x or Q^2 must be higher than that of a lighter nucleus in order to reach the $k > k_f$ SRC region. [8]

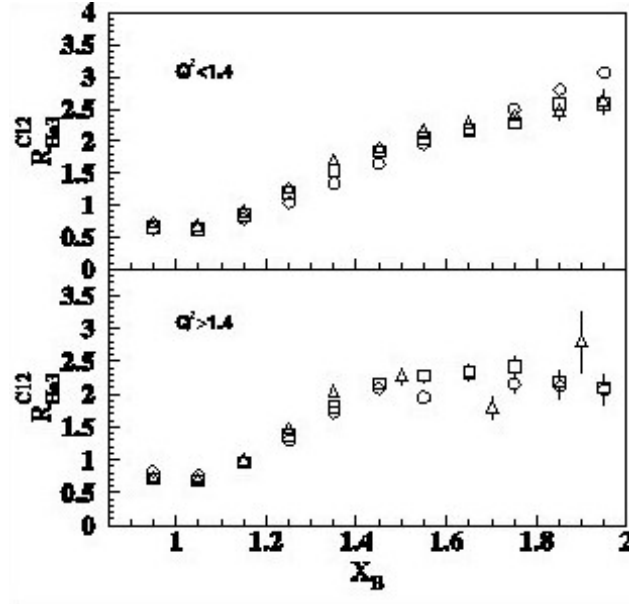


Figure 1.5: Plot of per nucleon cross section ratio for ^{12}C in different Q^2 regimes. The upper plot shows results for $Q^2 < 1.4$ whereas the bottom plot shows results for $Q^2 > 1.4$. The SRC plateau is better defined in the bottom plot because the higher Q^2 allows for a more effective probe of the high momentum region. In the upper plot, results from mean field nucleons combine with high momentum nucleons to muddle the picture [10]

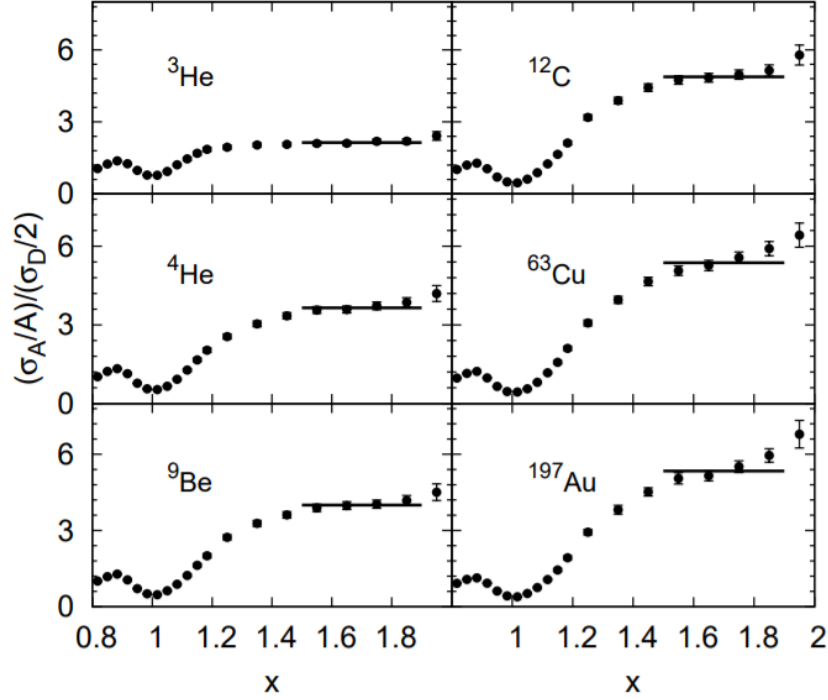


Figure 1.6: $\frac{(\sigma_A/A)}{(\sigma_D/2)}$ for E02-019 targets at 18 degrees. The expected location of the plateau is represented by the solid line. [2]

As demonstrated in 1.6, the ratio of $\frac{(\sigma_A/A)}{(\sigma_D/2)}$ has a similar shape regardless of A . Though the onset of the expected plateau rises slightly as A increases, such a change does not introduce much of a variation into the shape of the ratio. The reason the plateau increases with A is because larger nuclei will have more correlations. While it might appear that the end of the ratio near $x = 2$ begins to vary noticeably with A , the spike is not as important as it might seem. As x approaches 2, σ_D approaches 0. This effect is magnified as A increases, but σ_D behaves the same as x approaches 2 in every situation. Additionally, in heavier nuclei, the correlated pair is affected to a much greater degree by the surrounding nucleons. In the helium nuclei, there are only one or two remaining nucleons to affect the correlated pair. In gold, on the other hand, there are dozens of nucleons that impact the correlated pair.

Another reason the ratio of $\frac{(\sigma_A/A)}{(\sigma_D/2)}$ is useful is because it provides a way to account for final state interactions (FSIs). If one wants to extract $F(y, |q|)$ from experimental data, not

only must the spectator nucleus be in its ground state, but there must be no contribution from FSIs. It is possible to work around FSIs if very specific values of the scaling function can be determined. However, analyzing SRCs using the ratio of $\frac{(\sigma_A/A)}{(\sigma_D/2)}$ is a much more effective method for dealing with FSI contributions because these contributions are the same for the deuteron and the heavier nuclei. This means that FSIs cancel when using a ratio [2].

Without FSIs, the per nucleon cross section ratio represents the relative strength of the A nucleus' high momentum tail. If the high momentum tails are produced entirely by quasielastic scattering from a neutron-proton (n-p) SRC [11], then the plateau in the per nucleon cross section ratio represents the 2N-SRC contribution to the nuclear wave function relative to the deuteron.

The quasielastic cross section can be broken up into contributions from single nucleon scattering all the way through, in theory, A nucleon scattering:

$$\sigma(x, Q^2) = \sum_{j=1}^A A \frac{1}{j} a_j(A) \sigma_j(x, Q^2) \quad (1.5)$$

Using this expression, $\sigma(x, Q^2) = 0$ when $x > j$ and $a_j(A)$ is proportional to the likelihood that a nucleon is in a j -nucleon correlation. For the deuteron, where the highest j correlation is a 2N correlation, a_2 will be dominated by such correlations when $x > 1.4$. In this regime, k_N , the nucleon momentum, is much larger than k_F . Referencing Figure 1.2, the mean field contribution has also died out at this point and SRCs dominate [4].

Substituting the above expression for $\sigma(x, Q^2)$ into the per nucleon cross section ratio, the scaling relationship between heavy nuclei and the deuteron is further reinforced:

$$\frac{\sigma_A(x, Q^2)/A}{\sigma_D(x, Q^2)/2} = a_2(A) \quad (1.6)$$

The scaling of this ratio has been demonstrated by both SLAC [12] and Jefferson Lab [10] [13]. Where a_2 is the raw per nucleon cross section ratio, there is a value R_{2N} that is this same ratio, corrected for center of mass motion. As the raw ratio, a_2 indicates the relative strength of the high momentum tail. The higher a_2 is for a given nucleus, the more high momentum nucleons there are in the tail of the nucleus' momentum distribution. R_{2N} , having removed the center of mass motion from a_2 , is now the relative likelihood that a

nucleon is in an SRC. If gold has an R_{2N} value of 4, that means that a gold nucleon is 4 times as likely to be in an SRC than a deuterium nucleon. Using these quantities, it is possible to thoroughly examine the A dependence of SRCs.

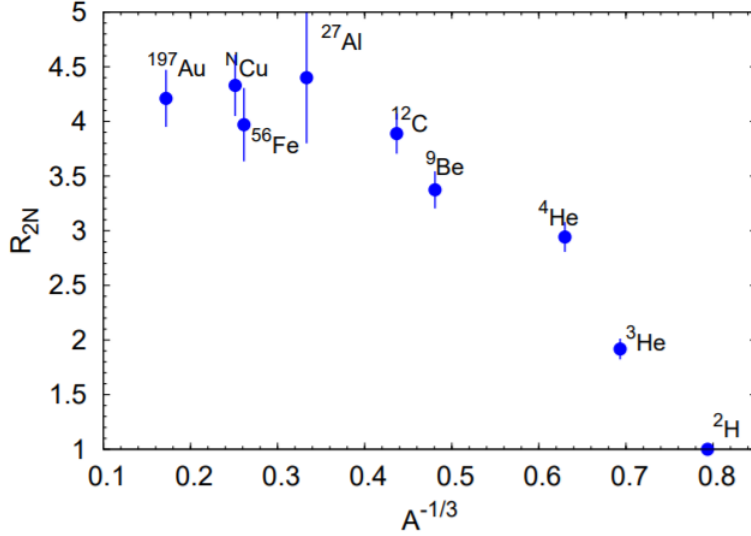


Figure 1.7: R_{2N} vs $A^{-1/3}$ [4]

The first method of R_{2N} comparison is by using $A^{-1/3}$. The surface of a nucleus is almost universally modeled by $\rho(r - R)$. R , in this case, is the half-density radius $R = r_o A^{1/3}$. Using this radius, contributions from the nuclear surface increase as R^2 or $A^{2/3}$. Using this, the per nucleon cross section should scale by $A^{-1/3}$ [14]. A plot of R_{2N} vs $A^{-1/3}$ in Figure 1.7 yields a smooth distribution, but not a linear relationship. There is a disconnect in the linearity between the lighter nuclei and the heavier nuclei. While the heavier nuclei have a clear linear relationship, the lighter nuclei do not follow the same relationship, instead having a linear relationship of their own. This deviation is expected, however, because the smaller the nucleus, the more nuclear response is dominated by surface effects. With larger nuclei, the constant density region dominates [4].

Linear SRC scaling was thought to be more likely with respect to nuclear density because, the more tightly packed a nucleus is, the closer the nucleons are and the higher the probability two nucleons will interact at short distances. Nonetheless, as in Figure 1.7 vs $A^{-1/3}$, the relationship in Figure 1.8 between the heavy nuclei differs from that of the lighter nuclei [4].

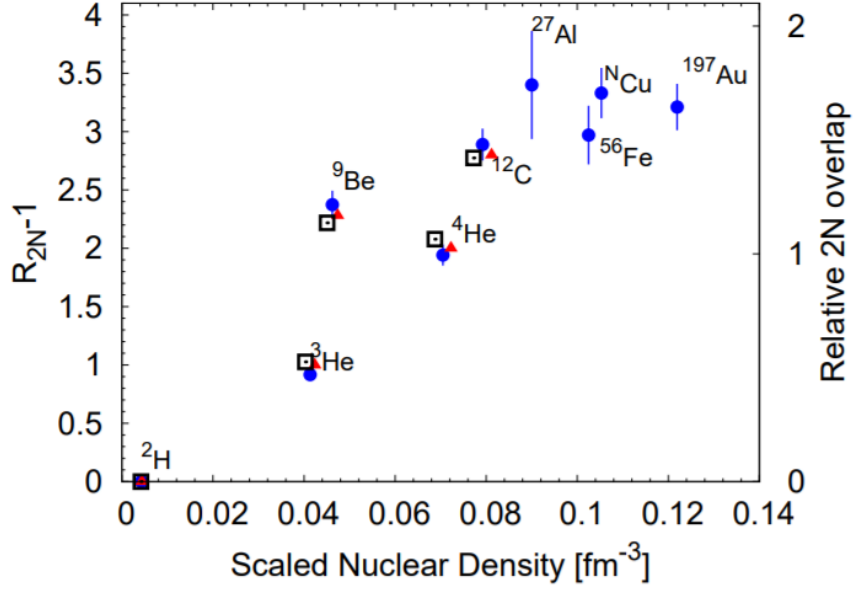


Figure 1.8: R_{2N} vs scaled nuclear density. The triangles and squares represent calculated R_{2N} overlaps, with the deuteron value subtracted. [4]

While $F(y)$ is an interesting tool, it is limited. It can only be extracted with and certainty for ^2H . Beyond that, it cannot provide a meaningful characterization of the momentum distribution for heavier nuclei. On the other hand, the ratio of $\frac{(\sigma_A/A)}{(\sigma_D/2)}$ provides a more universal approach. Coupled with examination of high-momentum tails, target ratios provide an effective method for understanding the A and Q^2 dependence of 2N-SRCs.

Chapter 2

Analysis

In inclusive cross-section experiments, where only the scattered electrons are measured, every possible final state is included. To extract cross sections from electrons detected in an inclusive scattering experiment, there are a number of processes that alter the energy of the scattered electron, which must be accounted for. For example, radiative corrections account for radiative processes like bremsstrahlung, where an electron can lose energy before or after the interaction of interest. However, the main correction in question for this thesis is the Coulomb correction. This alteration of the electron's wave function by the electrostatic field of the target nucleus is not accounted for in the JLab E89-008 analysis. It is therefore important to find a way to retroactively include this correction to complete the analysis of the E89-008 data.

In order to guarantee the accuracy of Coulomb corrections, there must be a high degree of confidence in the cross section model used to determine these corrections. If the model is able to sufficiently replicate the uncorrected results, then that allows the subsequent corrections to be well grounded in the existing data. The model is described below.

2.1 Coulomb Correction

After passing through the atomic electron cloud around the target, an incoming electron will encounter the nucleus. The bare nucleus will accelerate the electron towards it, thereby increasing its energy. However, while the energy of the incoming electron is increased,

the energy of the scattered electron is decreased. This change in energy, caused by the electrostatic field of the nucleus, leads to a difference between the incoming and scattered electron energies in this regime. A measured beam energy is expected to produce a scattered electron at a specific momentum. If a 10 GeV electron from a beam does is not actually a 10 GeV electron when it scatters, it can have a meaningful effect on the measured cross section. Additionally, the electrostatic field of the nucleus can deflect incoming electrons if the electrons scatter near the edge of the nucleus. Such a deflection can allow for electrons to hit the detector that may not have otherwise, or vice versa.

To understand the effect of the above energy shift, it can be helpful to look at the shift in momentum because of the Coulomb acceleration: $k'_f = k_f + \Delta k$ for the final momentum and $k'_i = k_i + \Delta k$ for the initial momentum. In this case, $\Delta k = -.775 \frac{V_0}{c}$ where V_0 is the potential energy of the electron in the center of the nucleus, or the lowest order of an electrostatic potential in a charged sphere [1]

$$V_0 = \frac{-3\alpha(Z-1)}{2R} \quad (2.1)$$

For the Equation 2.1, R is the radius of the nucleus, α is the fine structure constant, and $Z-1$ is used instead of Z because the acceleration is being calculated for the $A-1$ spectator nucleus. R for heavier nuclei is calculated by using, $R = 1.1A^{1/3} + 0.86A^{-1/3}$ [4] and relevant R are listed in Table 2.1. Additionally, the scattering is distributed over the whole volume of the nucleus, so the potential must resemble the average potential inside a homogeneously charged sphere. This is where the factor of .775 originates [15].

Table 2.1: Table of R values in Coulomb corrections.

Target	Radius (fm)
C	2.89
Fe/Cu	4.60
Au	6.55

Another effect of the nucleus on the electron is that the attractive force focuses the electron wave function. This focusing is accounted for by an aptly named "focusing" factor. This factor is implemented quadratically. Enhancing the phase space in the shifted cross

section cancels the outgoing factor for the outgoing electron, so the remaining focusing factor is $(\frac{k'_i}{k_i})^2$ [1].

Accounting for the above factors, the Coulomb correction can be expressed as follows:

$$C_{coulomb} = \frac{\sigma_{born}(k_i, k_f)}{\sigma_{born}(k'_i, k'_f)} \frac{1}{(k'_i/k_i)^2} \quad (2.2)$$

With σ_{born} representing the full, modeled cross section. The Coulomb correction is a ratio of the cross section model with experimental kinematics to the cross section model with shifted kinematics multiplied by a focusing factor. The Coulomb correction varies from target to target, as is clearly exhibited by Figure 2.1, a plot of the Coulomb correction factor for various targets at 18 degrees from the E02-019 analysis. Where the Coulomb correction for ^{12}C reaches a maximum between 1-2%, ^{197}Au 's correction can reach a maximum of 10%. This is due to the fact that the larger the nucleus, the more protons there are to affect incoming electrons. For larger angles, the Coulomb correction is also larger. This is because, as angle increases, the electron is probing deeper into the nucleus, allowing for the nuclear protons to have a greater effect on it [1].

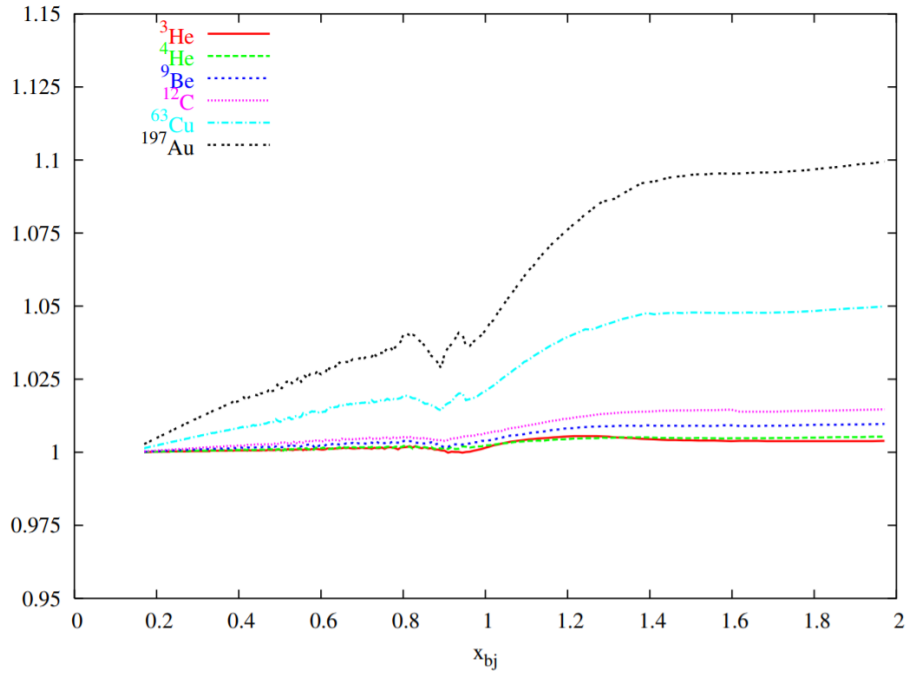


Figure 2.1: Coulomb correction factor for E02-019 targets at 18 degrees [1].

2.2 Cross Section Model

The below cross section model was originally developed for JLab experiment E02-019. Dubbed the XEM model, a combination of cross section and EMC effect, it is made up of two kinematic components. The first is a quasielastic scattering piece and the second is an inelastic scattering piece. Originally laid out in Fortran, the XEM model was translated into ROOT/C++ for JLab experiment E08-014. While the model's framework was accurately copied from Fortran to ROOT/C++, parameters were refit to facilitate analysis for E08-014. These discrepancies will be noted as they appear.

The quasielastic cross section is made up of contributions from $F(y)$, a kinematic factor K , and the electron-nucleon cross sections, σ_p and σ_n . The full form of which is given here [1]

$$\frac{d\sigma}{d\Omega d\nu} = F(y) \cdot (Z \cdot \sigma_p + N \cdot \sigma_n) \cdot K \quad (2.3)$$

For the purposes of this paper, however, much more attention will be paid to $F(y)$, which is given for ^2H as:

$$F(y) = (f_0 - B) \cdot \frac{\alpha^2 e^{-\alpha y^2}}{\alpha^2 + y^2} + B e^{-b|y|} \quad (2.4)$$

For heavier elements, the trailing exponential of $F(y)$ is modified to give:

$$F(y) = (f_0 - B) \cdot \frac{\alpha^2 e^{(-\alpha y)^2}}{\alpha^2 + y^2} + B e^{-(by)^2} \quad (2.5)$$

It is important to remember the underlying significance of $F(y)$. Though described above using a set of parameters, $F(y)$ is related to the momentum distribution of the nucleons in a target. In the forms given above, none of the parameters f_0 , B , a , b , α , are measured values. Instead, $F(y)$ is extracted from experimental data, and the parameters are determined from fits to this experimental data [1].

Additionally, there was a discrepancy between E08-014's minimum separation energies, E_s^{min} and E02-019's separation energies. As the minimum energy required to separate a nucleon from its nucleus, the separation energies in the E08-014 code were given as raw

averages of the proton and neutron separation energies. However, at $x > 1$, the electron-proton cross section, is equal to approximately three times the electron-neutron cross section. Therefore, the proton separation energy is used in lieu of an averaged separation energy.

Table 2.2: Table of $F(y)$ fit values from the JLab experiment E02-019 [1].

Name	E_s^{min} (MeV)	F0	B	a	b	alpha
C	16.0000	3.1882	1.3591	3.0265	7.0505	137.2846
Fe	10.0000	2.8900	1.4016	3.1802	7.2635	165.7000
Au	5.8000	2.6424	0.7632	3.0654	6.7678	132.4517

To produce an accurate quasielastic simulation, additional corrections must be applied to the modeled cross section. Without these corrections, the raw simulation would not match low Q^2 data:

$$\sigma_{QEfinal} = \sigma_{QEinitial} \cdot DQAF \cdot T \quad (2.6)$$

Where $DQAF$ is the deep inelastic-quasielastic asymmetry factor, and T is the tail correction, which will be discussed later in more detail. $DQAF$ is a factor that is introduced to account for underlying asymmetry in the calculation of the cross section for $x < 1$. $DQAF$ brings the simulation in line with expected results and is given by:

$$DQAF = 1 + y \cdot 1.4 \cdot c_j \quad (2.7)$$

$$y = \frac{-(4 \cdot Q \cdot (Q^2 + M_p^2 - M_{A-1}^2 - (M_T + \nu)^2))}{8 \cdot (Q^2 - (M_T + \nu)^2)} + \frac{[(4 \cdot Q \cdot (Q^2 + M_p^2 - M_{A-1}^2 - (M_T + \nu)^2))^2 - 16 \cdot (Q^2 - (M_T + \nu)^2) \cdot ((Q^2 + M_p^2 - M_{A-1}^2 - (M_T + \nu)^2))^2 - 4 \cdot M_{A-1}^2 \cdot (M_T + \nu)^2]^{1/2}}{[8 \cdot (Q^2 - (M_T + \nu)^2)]} \quad (2.8)$$

With c_j being a target specific constant, M_p being the proton mass, M_{A-1} being the mass of the A-1 system, and M_T being the target mass. DQAF was originally removed for E08-014's code, and was reintroduced to match E02-019's original simulation [1].

Unlike the quasielastic cross section, which required the fitting of multiple variables to existing data to supply an accurate model, the deep inelastic cross section is first built on nuclear structure functions, F_1^A and F_2^A . These structure functions are functions of ν and Q^2 , the energy loss and momentum transfer evaluated analytically in the lab frame.

These functions combine to give the unpolarized cross section for electron scattering off a nuclear target in the inelastic regime [16] [1]:

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{d\sigma}{d\Omega_{Mott}} [F_2^A(\nu, Q^2) + 2F_1^A(\nu, Q^2) \tan^2 \frac{\theta}{2}] \quad (2.9)$$

The structure functions are described like so:

$$F_1^A(Q^2, \nu) = \int d^4p_0 S'(E_0, p_0) \left(\frac{M}{p_0} \right) [F_1^N + \frac{F_2^N}{2M^2} \frac{|\mathbf{p}_0 \times \mathbf{q}|^2}{|\mathbf{p}_0|^2}] \quad (2.10)$$

$$F_2^A(Q^2, \nu) = \int d^4p_0 S'(E_0, p_0) \times \left\{ F_1^N \frac{q^2}{|\mathbf{q}|^2} \left(\frac{q^2}{\bar{q}^2} - 1 \right) + \frac{F_2^N}{M^2} \left[\frac{q^4}{|\mathbf{q}^4|} (\bar{E}_0 - \bar{\nu} \frac{\bar{\mathbf{q}} \cdot \bar{\mathbf{p}}_0}{\bar{q}^2}) - \frac{q^2}{|\mathbf{q}|^2} \frac{|\mathbf{p}_0 \times \mathbf{q}|^2}{|\mathbf{p}_0|^2} \right] \right\} \quad (2.11)$$

Having defined these structure functions and how they relate to the inelastic cross section, one can analyze the electron-nucleon cross section using the F_2^A structure function, given here:

$$F_2^A = \frac{d^2\sigma}{dE'd\Omega} \cdot \frac{\nu}{\sigma_{mott} [1 + 2 \tan^2(\frac{\theta}{2}) \frac{1 + \nu^2/Q^2}{1 + R}]} \quad (2.12)$$

The F_2^p and F_2^n , proton and neutron, structure functions are used to calculate the deep inelastic cross section for $x < 0.8$ like so:

$$\sigma_{DIS} = (F_2^p + F_2^n) \cdot n(k) \quad (2.13)$$

Where $n(k)$ is a target specific momentum distribution that is calculated using the derivative of $F(y)$. The nucleon structure functions are smeared in this way to avoid discontinuities in the transition from the deep inelastic regime to the quasielastic regime. This smearing is also a more realistic representation for bound nucleons [1].

In the original XEM model, the F_1^A and F_2^A integrals are iterated thirty times in order to maximize the accuracy of the deep inelastic cross section. However, in the case of the E08-014 version, these integrals were run six times in order to shorten the calculation time. E08-014 made such adjustments as the focus of their experiment was $x > 2$ which is well outside

the deep inelastic regime. Nonetheless, such measures made a substantial difference when comparing E02-019 results to the E08-014 simulations, as shown in Figure 2.2. Reverting the iterations to the original thirty erased this deviation.

Fixing both the quasielastic fit and deep inelastic iteration numbers to match the E02-019 values allowed the ROOT/C++ code for E08-014 to match its original Fortran counterpart.

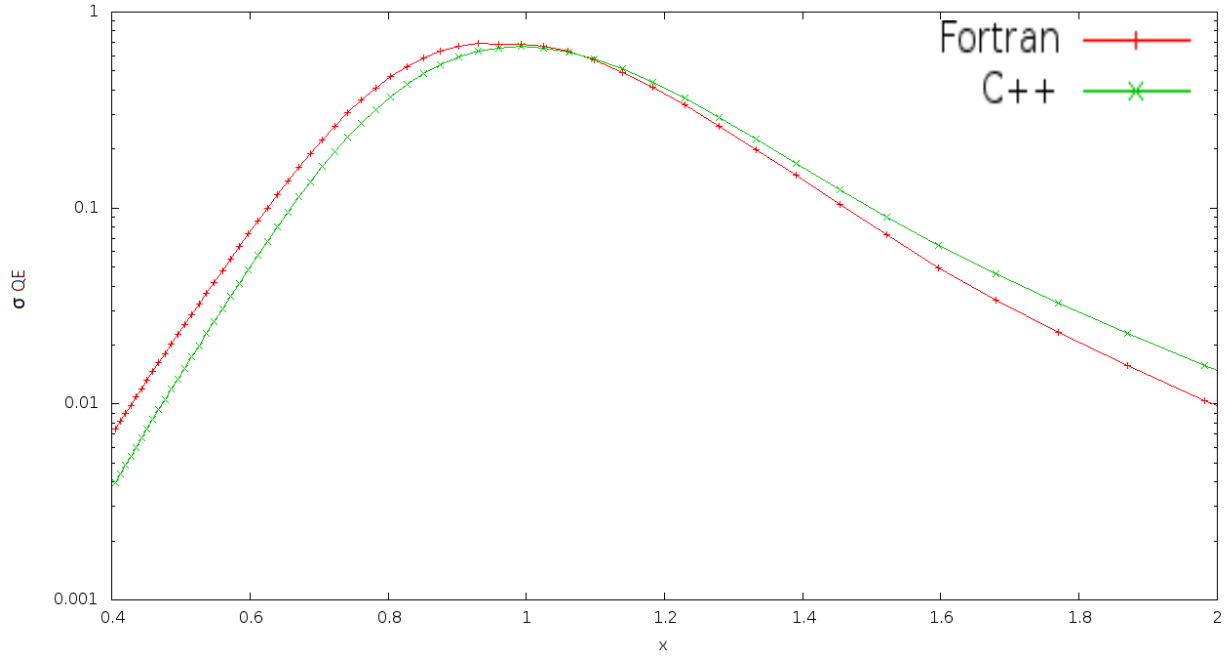


Figure 2.2: Plot of the quasielastic cross section using the original Fortran XEM simulation and the original translated C++ XEM simulation. The disparity is caused by the difference in $F(y)$ parameters used. Using the same parameter values brought the two σ_{QE} in line.

2.3 Tail Correction

Having removed the assumptions made by the E03-103 translation of the XEM model, the simulation was able to be used with the E89-008 data. E89-008 was focused on a region similar to E02-019, for which the XEM code was originally written.

Comparing the XEM model to the E89-008 data in Figure 2.3, there was sufficient agreement at $x < 1$, especially for higher Q^2 , to be confident in the deep inelastic piece of the XEM model. However, for $x > 1$, especially for low Q^2 , the XEM cross section was lower than the given E89-008 cross section. For $x \sim 2$, the XEM model was less than 75 percent

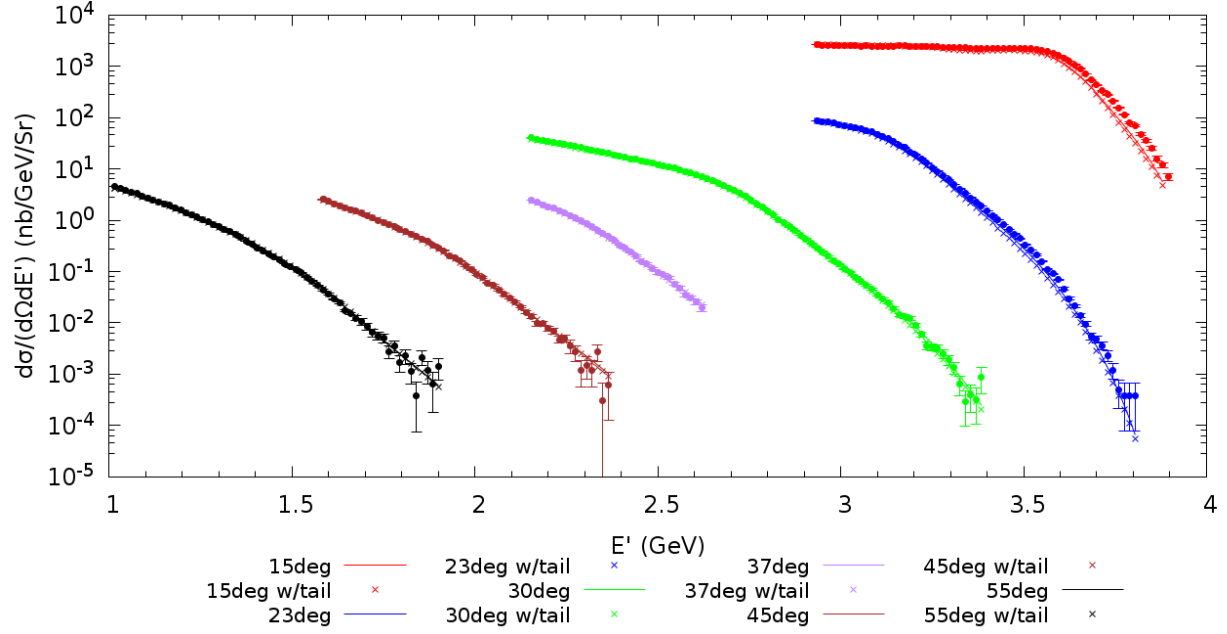


Figure 2.3: Comparison between E89-008 data, the XEM Model with unshifted base energy with the original tail correction, and the XEM model with unshifted energy and no original tail correction for ^{12}C .

the value of the given data. With poor agreement in the $x > 1$ regime, it was clear there was work to be done in that region. Though the older data was suspect for a few of the settings, the data could not be changed. Instead, changes to the simulated tail correction could remove disagreement between the simulation and the E89-008 data. The tail correction is implemented into the model because there exists a residual Q^2 dependence in taken data.

Instead of being a correction that is abruptly turned on, the tail correction is slowly turned on and blended into the cross section over an x range. Implementation of the tail correction depends on the target. For deuterium, the activation range is narrow, from $x = 1.4$ to $x = 1.45$. For targets like ^4He , ^{56}Fe , and ^{197}Au , the tail correction slowly turned on from $x = 1.2$ to $x = 1.4$. For anything else, the range is $x = 1.4$ to $x = 1.6$.

The original high- x tail correction was presented as:

$$T = a \cdot \exp(b \cdot x_{local}) + c \cdot x^6 + d \cdot x^4 + e \cdot x^2 + f \quad (2.14)$$

As given, each target had its own set of variables for each of the parameters, though f was always 0, and e was non-zero for only ^2H . However, the disagreement between the XEM

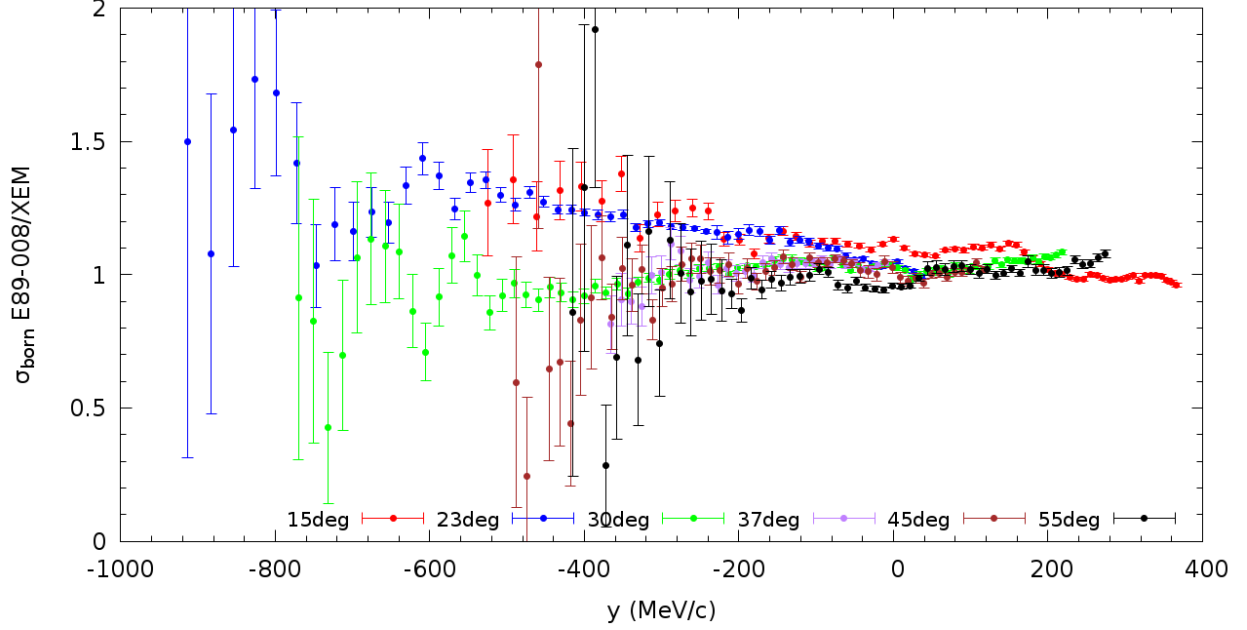


Figure 2.4: Ratio of E89-008 ^{12}C data to XEM Model with unshifted base energy with no linear tail correction.

model and the E89-008 data was only present in the region where the tail correction was active, which is why the original tail correction was altered. This tail correction was replaced by an exponential tail correction. The reason for this was two-fold. Firstly, as demonstrated by Figure 2.4, the ratio between E89-008 and the XEM model appeared mostly, but not entirely linear. An exponential fit provided a flexible way to account for any abnormalities in the ratio.

Each angle setting was fit using a basic $a \cdot e^{b \cdot x}$ form. The a 's and b 's from each angle fit were saved and then plotted. Each a and b entry was plotted with respect to the Q^2 value that corresponded to $x = 1$ for that angle setting. The plot of the a 's and the plot of the b 's were then fit using more complicated exponential forms. In fitting the a 's and b 's, Q^2 dependence was incorporated into the existing y dependence.

The basic form of the tail correction is listed:

$$T = A \cdot e^B \cdot y \quad (2.15)$$

While this fit may seem simplistic, the parameter A was fit using a more involved exponential, and B was fit using a combined exponential and polynomial form:

$$A = a \cdot e^{(b \cdot Q^2)} + c \quad (2.16)$$

$$B = d \cdot \exp^{e \cdot Q^2} + f + g \cdot (Q^2)^2 \quad (2.17)$$

Which gives the final tail correction form, combining both Q^2 and y dependence as:

$$T = [a \cdot \exp^{(b \cdot Q^2)} + c] \cdot \exp^{d \cdot \exp^{e \cdot Q^2} + f + g \cdot (Q^2)^2} \cdot y \quad (2.18)$$

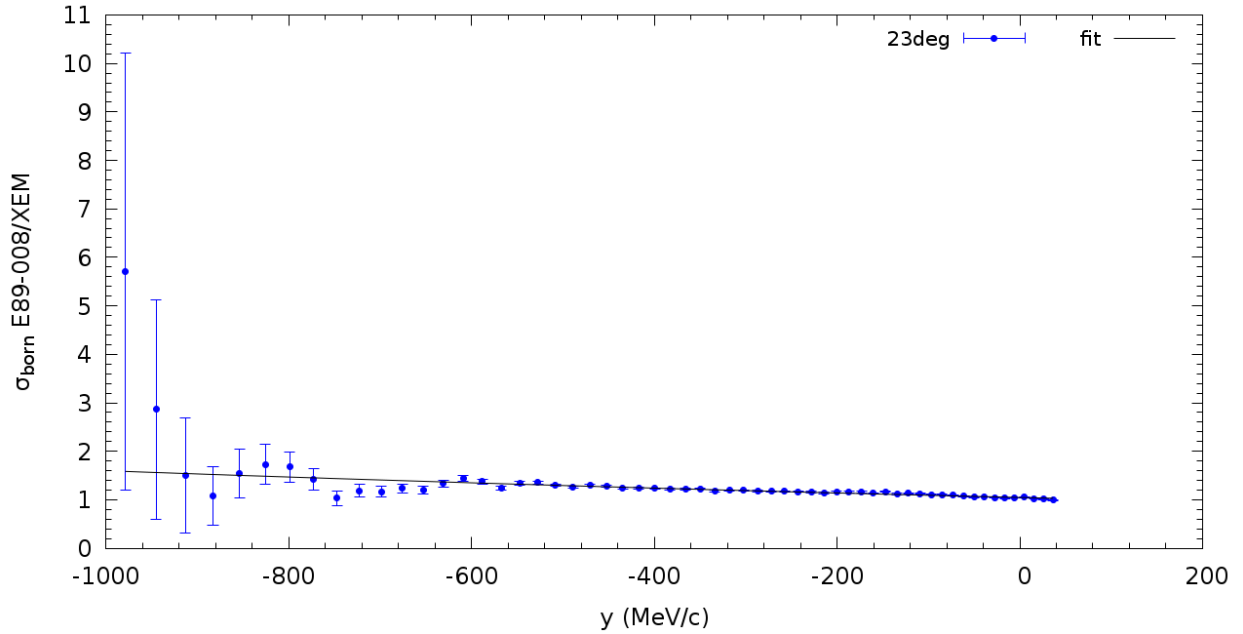


Figure 2.5: Plot of the ratio of E89-008 data to the XEM simulation with respect to y at 23° for ^{12}C , done using an exponential fit.

The goal of the tail correction is to act as a scaling factor in the high- x (low- y) region that will bring the XEM simulation in line with the E89-008 data. Therefore, the ratio of the E89-008 to the simulation was fit using the exponential fit in the case of ^{12}C . As is obvious by Figures 2.5 and 2.6, the fit is good in the high- y (low- x) region, where the data has small error bars. The error bars increase dramatically as y decreases (x increases), but the fit does an adequate job of accounting for the uncertainty in that region. The 23° and 45° fits are

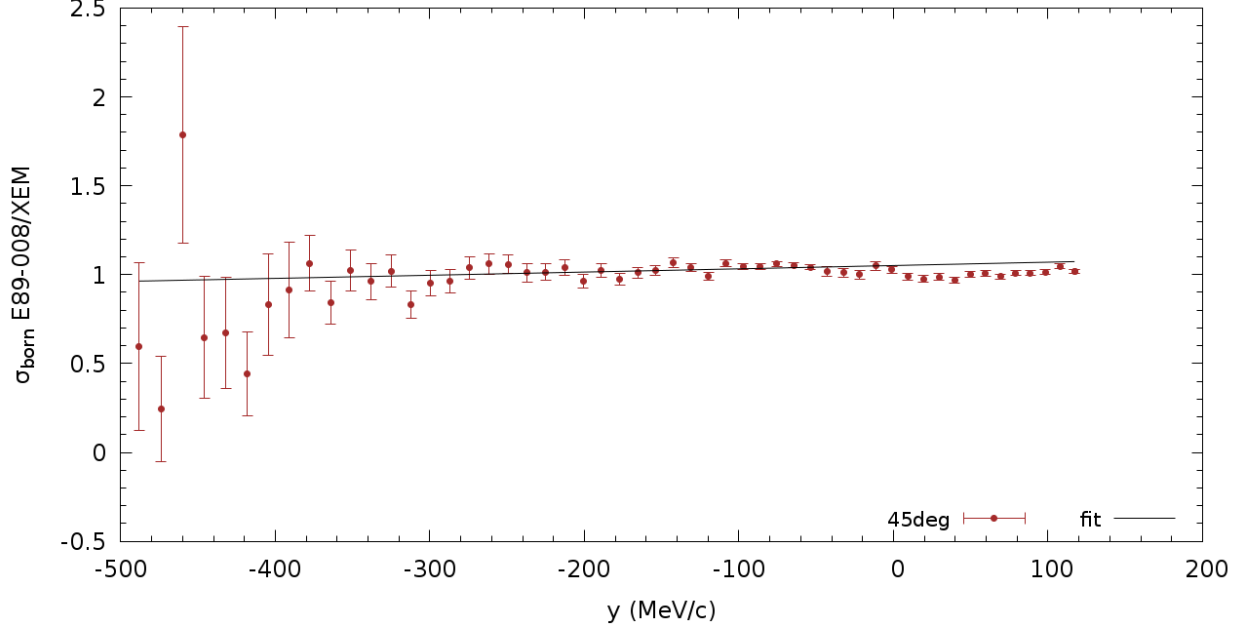


Figure 2.6: Plot of the ratio of E89-008 data to the XEM simulation with respect to y at 45° , ^{12}C , done using an exponential fit.

representative of the fits as a whole. Using the fits from each angular setting, a further fit of the tail correction variables is shown in Figures 2.7 and 2.8.

This exponential fit was appropriate for the ^{12}C data and, when applied to the XEM model, brought the model cross section values in line with the E89-008 data, as demonstrated by Figures 2.4 and 2.9. The ratio between the two cross sections is very close to 1, indicating that the simulation has sufficiently matched the data in the $y < 0$ region.

Both ^{56}Fe and ^{197}Au behaved similarly when fit. Figures 2.10 and 2.13 show the exponential fit for the ratio at both 23° and 45° for ^{197}Au . The fitting of ^{197}Au is representative of ^{56}Fe . These angles were chosen to remain consistent with the ^{12}C figures.

Though most of the ratios between the E89-008 data and the XEM model maintain similar shapes between angles and targets, the 15° ratio for ^{197}Au stands out as an oddity. The ratio's erratic shape is due to a known issue with the low Q^2 data. While the remaining erraticism in the high- y part of the ratio remains after the tail correction, it is a purely cosmetic issue. The tail correction was still effectively implemented for the 15° ratio.

The tail corrections that were produced by the fits were applied to the ^{56}Fe and ^{197}Au ratios. In both the ^{56}Fe and ^{197}Au cases, the ratio between the E89-008 data and the XEM

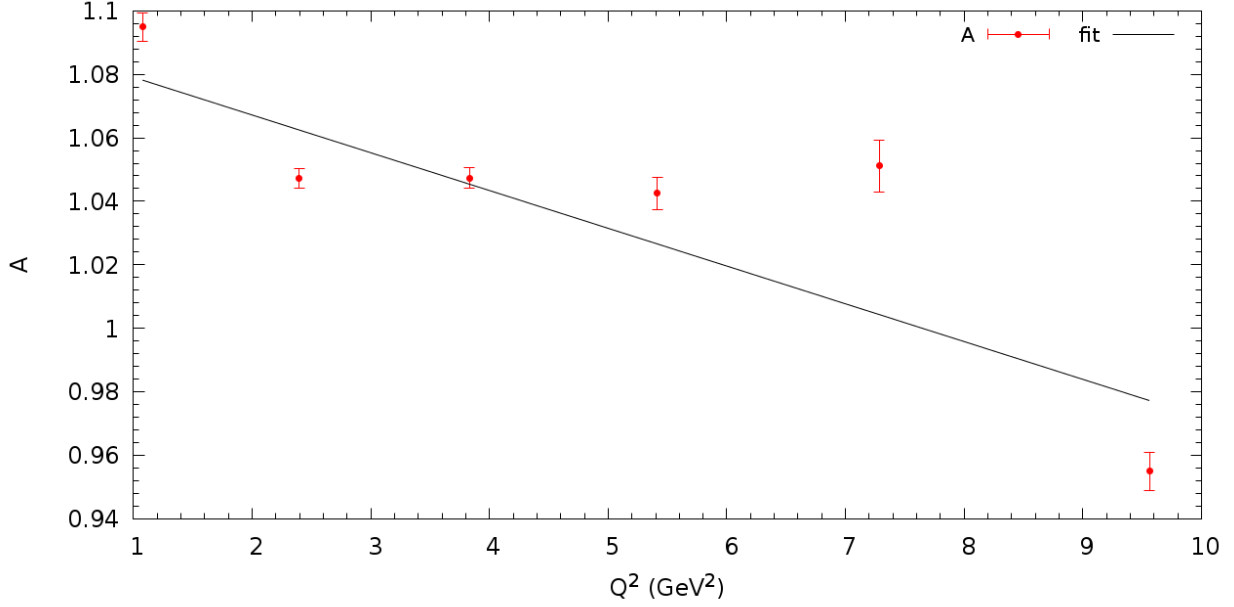


Figure 2.7: Plot of the fit applied to the A 's used to fit every angle of E89-008 ^{12}C data vs Q^2 .

model was brought close to 1. With the new tail corrections allowing the simulation to closely mirror the data, it was time to generate coulomb corrections for the data.

As given in Equation 2.2, the coulomb correction involves the ratio of a cross section at given kinematics to a cross section at shifted kinematics. Multiplying this ratio by the focusing factor yields the coulomb correction. In the case of the E89-008 data, the coulomb correction had to be determined using the XEM simulation. This is why it was essential to match the simulation output with the data, as shown in Figures 2.9, 2.17, and 2.15. The kinematics of the E89-008 data were run through the XEM simulation for every angle setting for ^{12}C , ^{56}Fe , and ^{197}Au . The simulations were run again at the same kinematics, but this time the code shifted the kinematics as prescribed in 2.1. However, instead of using Δk , the shifts were applied as ΔE 's for ease of use in the code. These shifts are listed in Table 2.3.

Additionally, the focusing factor, outlined in 2.1, is applied by the code automatically if the kinematics are shifted for coulomb corrections. The two sets of simulations give files with $\sigma_{\text{born}}(k_i, k_f)$ and $\sigma_{\text{born}}(k'_i, k'_f) \cdot (k'_i/k_i)^2$, the two components necessary in calculating coulomb

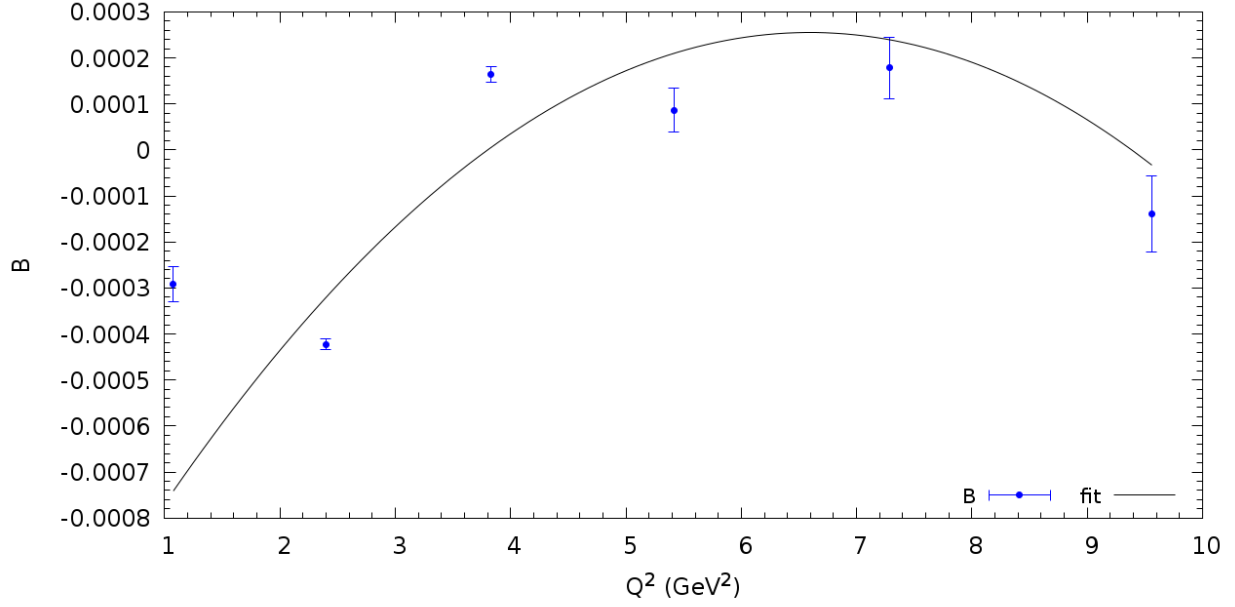


Figure 2.8: Plot of the fit applied to the B 's used to fit every angle of E89-008 ^{12}C data vs Q^2 .

corrections. The ratios of the unshifted cross sections to the shifted cross sections were taken and plotted for each target in Figures 2.18, 2.19, and 2.20.

Systematic error in the Coulomb correction is calculated by applying a 10 percent shift to Equation 2.1 and then taking the ratio of the subsequent cross section to the original Coulomb shifted cross section. The systematic errors for each target at all angle settings are presented in Figures 2.21, 2.22, and 2.23.

Table 2.3: Table of ΔE values for Coulomb corrections.

Target	ΔE (GeV)
C	0.00292
Fe	0.01020
Au	0.02400

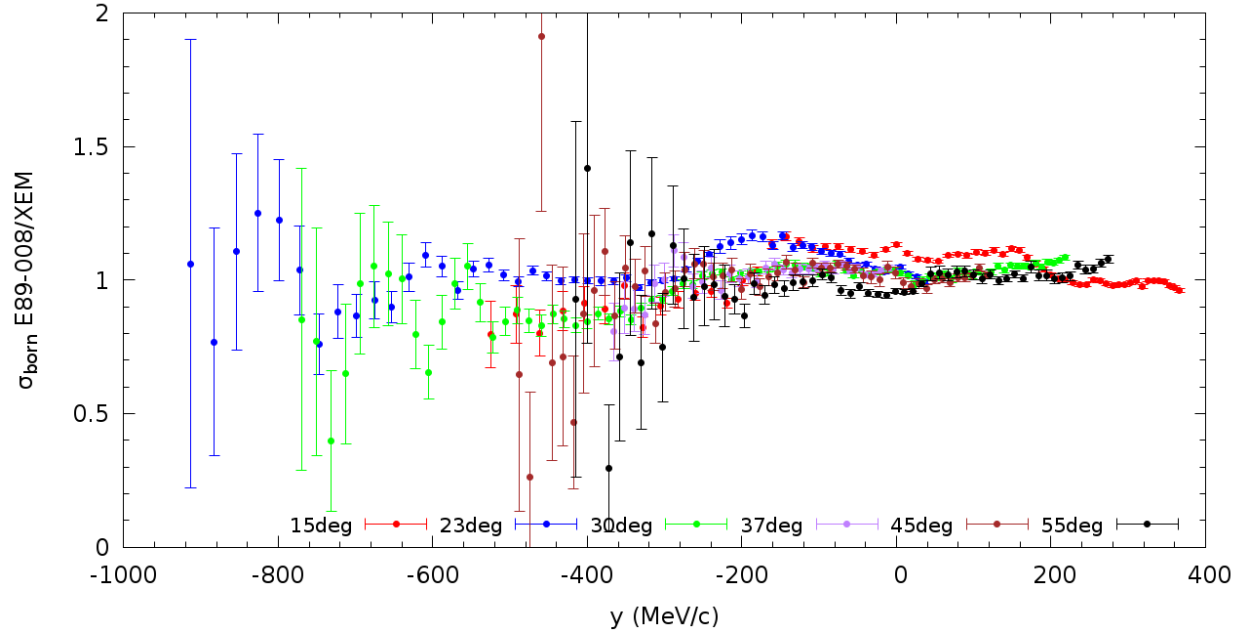


Figure 2.9: Ratio of E89-008 ^{12}C data to XEM Model with unshifted base energy with the new linear tail correction.

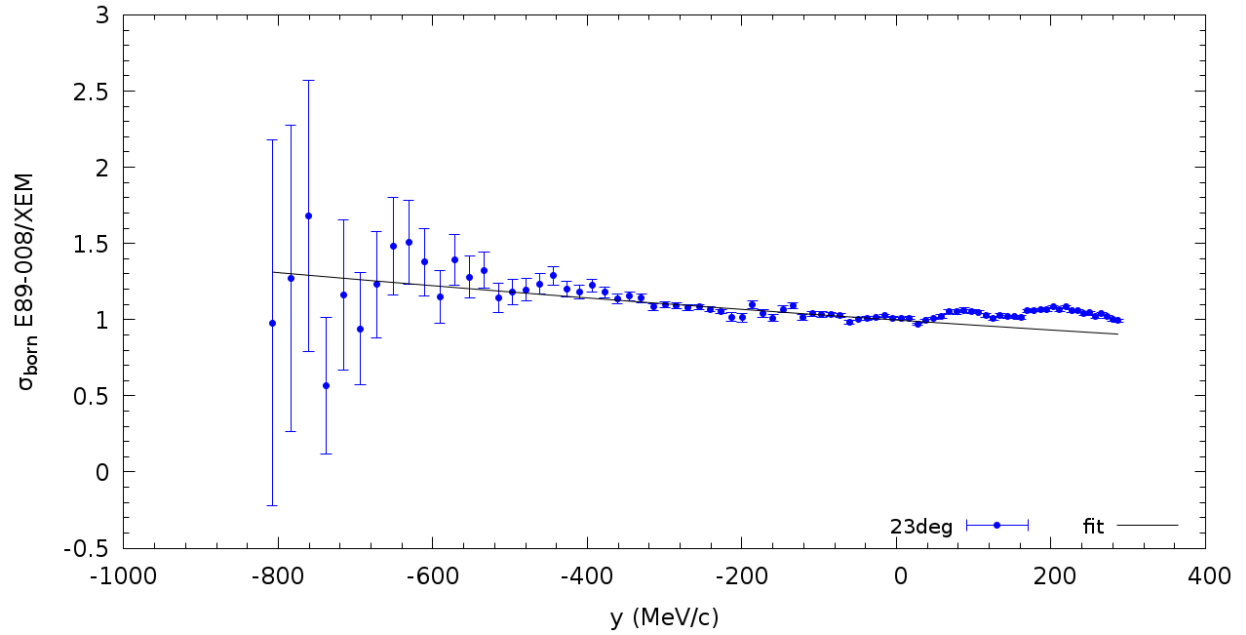


Figure 2.10: Plot of the ratio of E89-008 data to the XEM simulation of ^{197}Au with respect to y at 23° , fit using a linear fit.

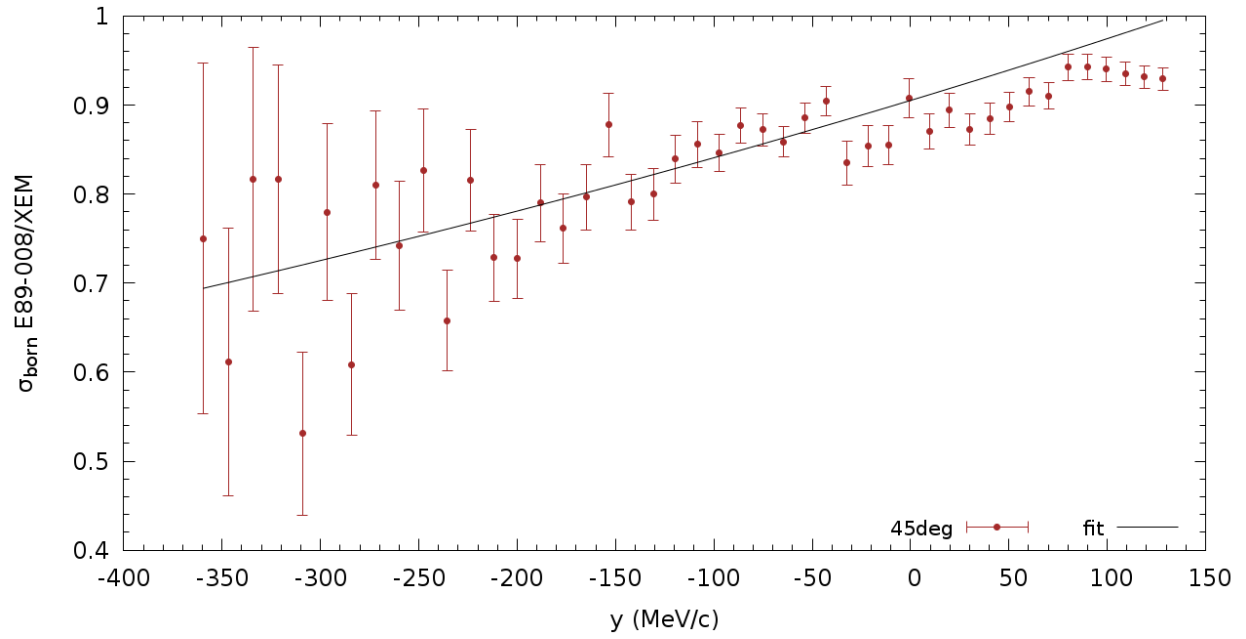


Figure 2.11: Plot of the ratio of E89-008 data to the XEM simulation of ^{197}Au with respect to y at 45° , fit using an exponential fit.

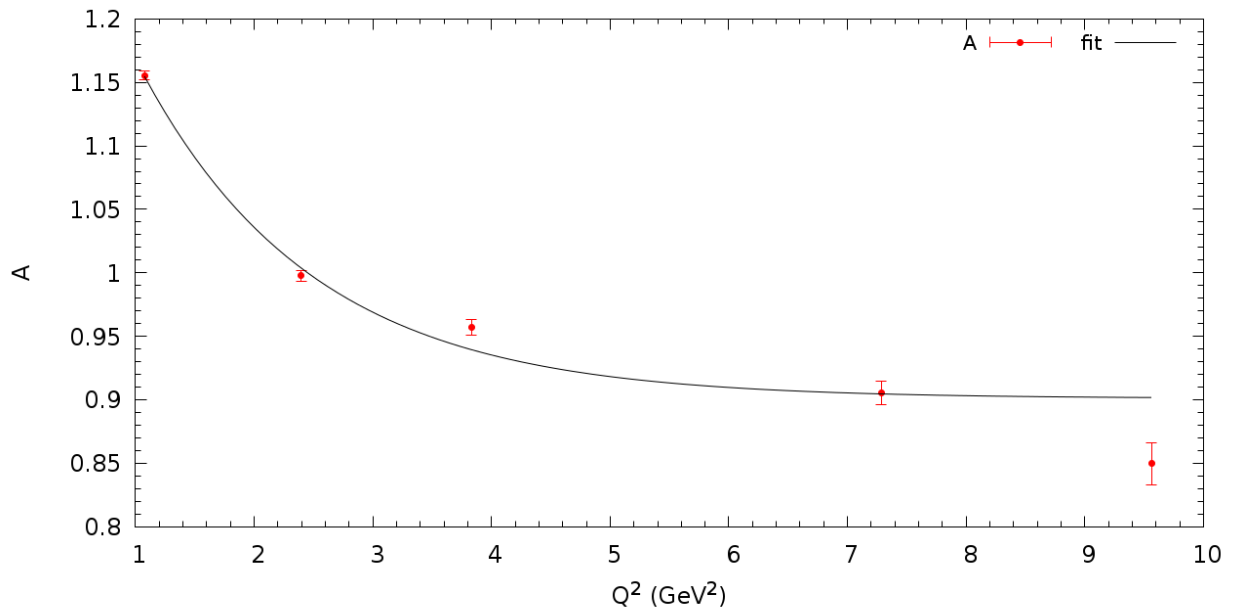


Figure 2.12: Plot of the fit applied to the A 's used to fit every angle of E89-008 ^{197}Au data vs Q^2 .

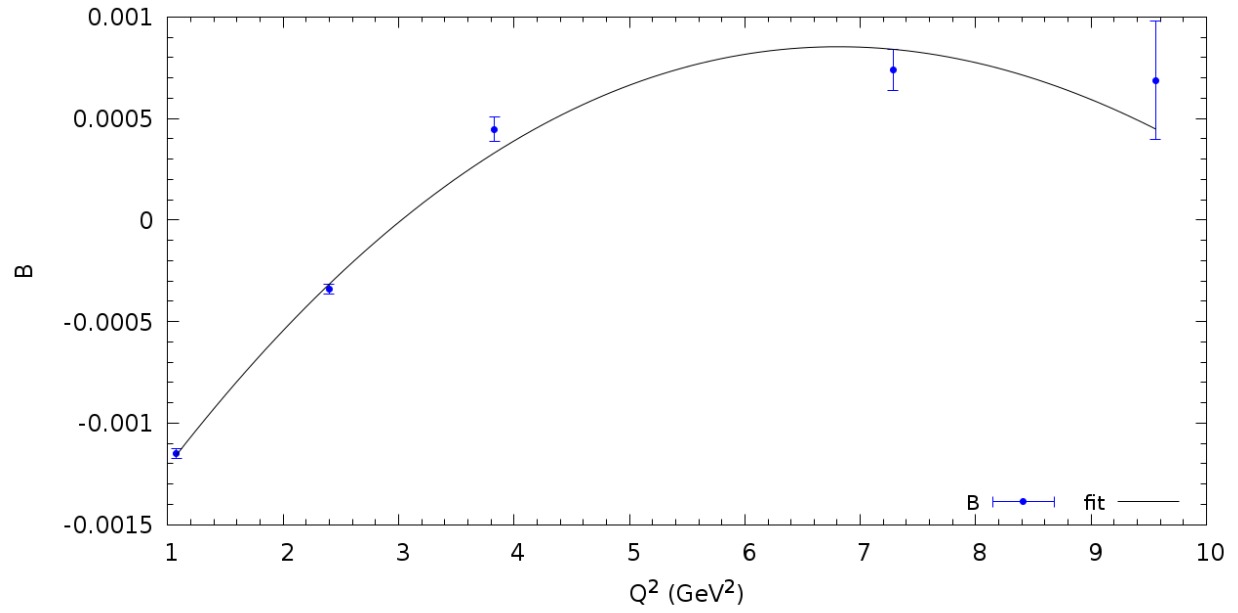


Figure 2.13: Plot of the fit applied to the B 's used to fit every angle of E89-008 ^{197}Au data vs Q^2 .

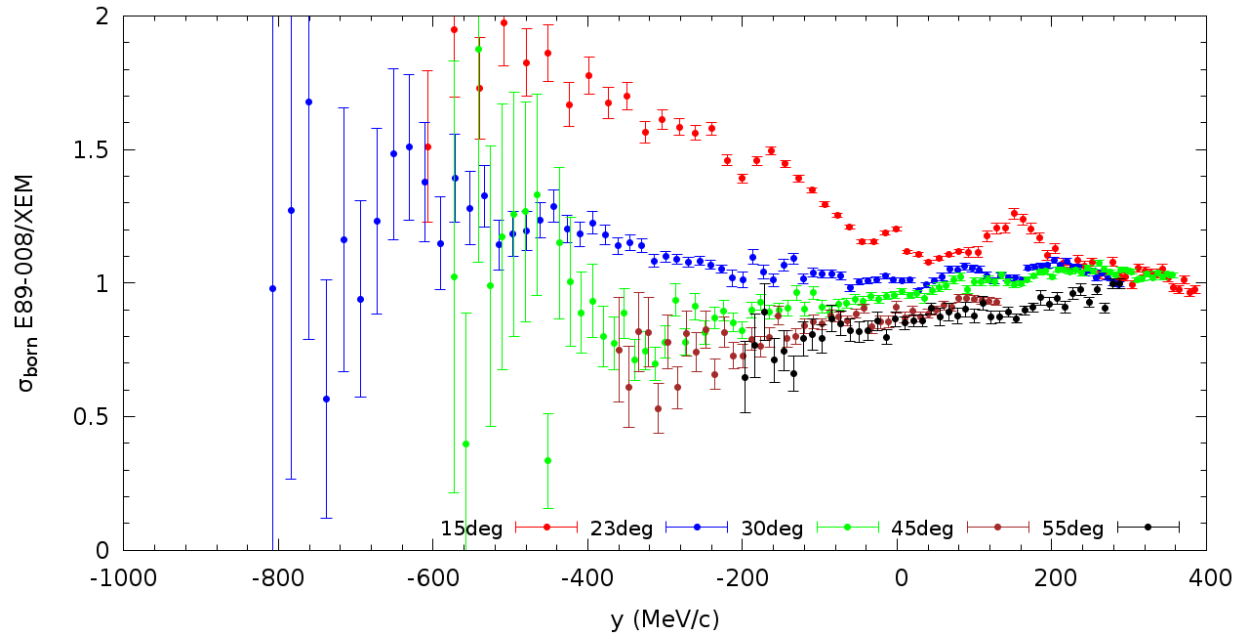


Figure 2.14: Ratio of E89-008 ^{197}Au data to XEM Model with unshifted base energy without any tail correction.

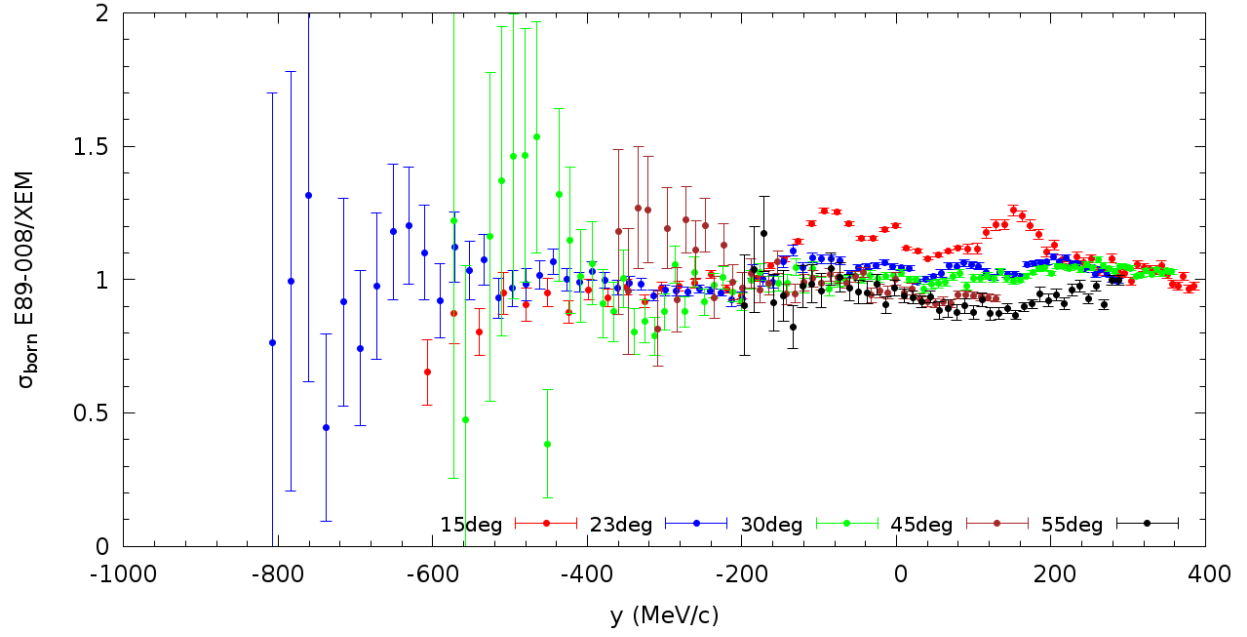


Figure 2.15: Ratio of E89-008 ^{197}Au data to XEM Model with unshifted base energy with the new exponential tail correction.

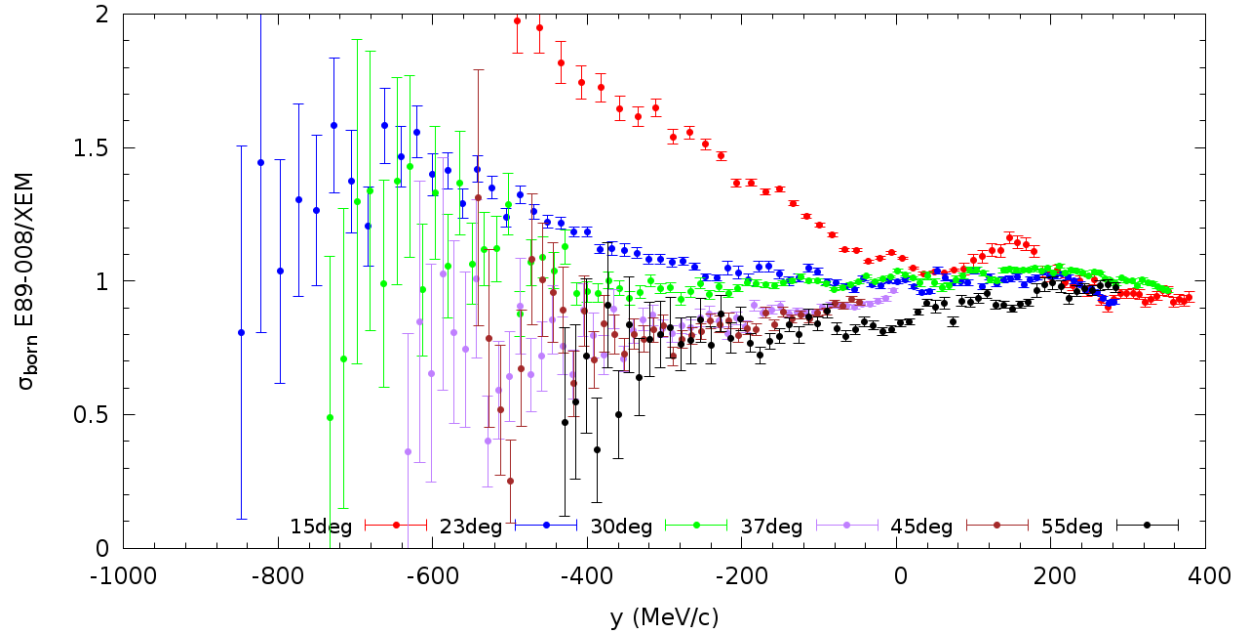


Figure 2.16: Ratio of E89-008 ^{56}Fe data to XEM Model with unshifted base energy without any tail correction.

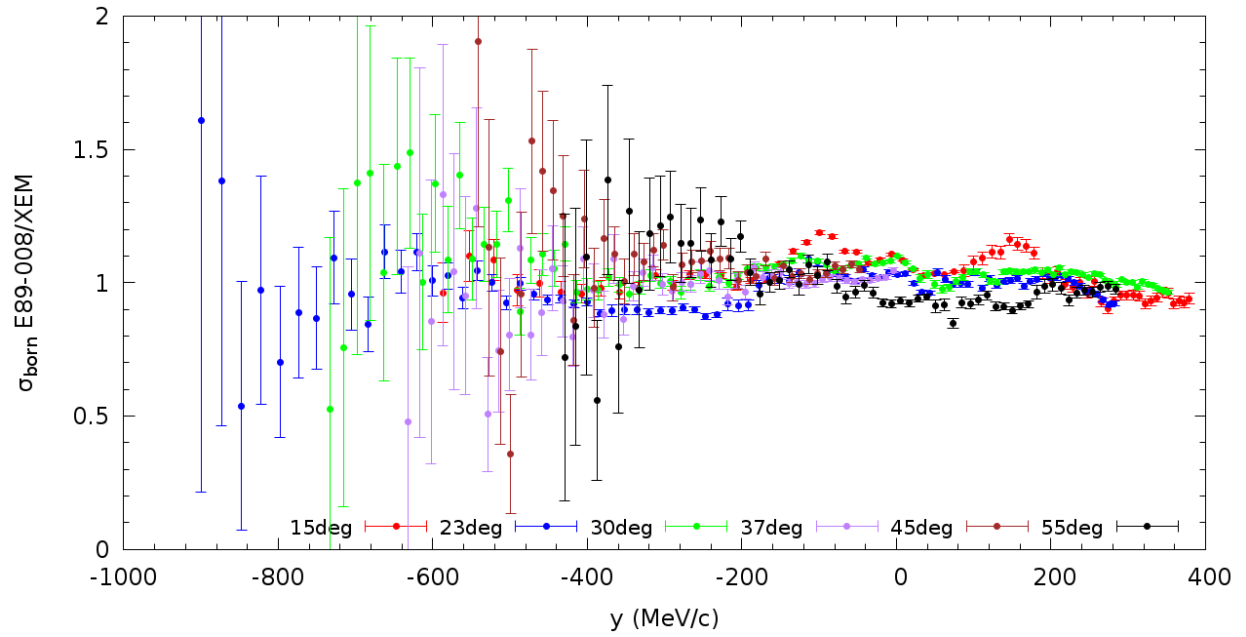


Figure 2.17: Ratio of E89-008 ^{56}Fe data to XEM Model with unshifted base energy with the new exponential tail correction.

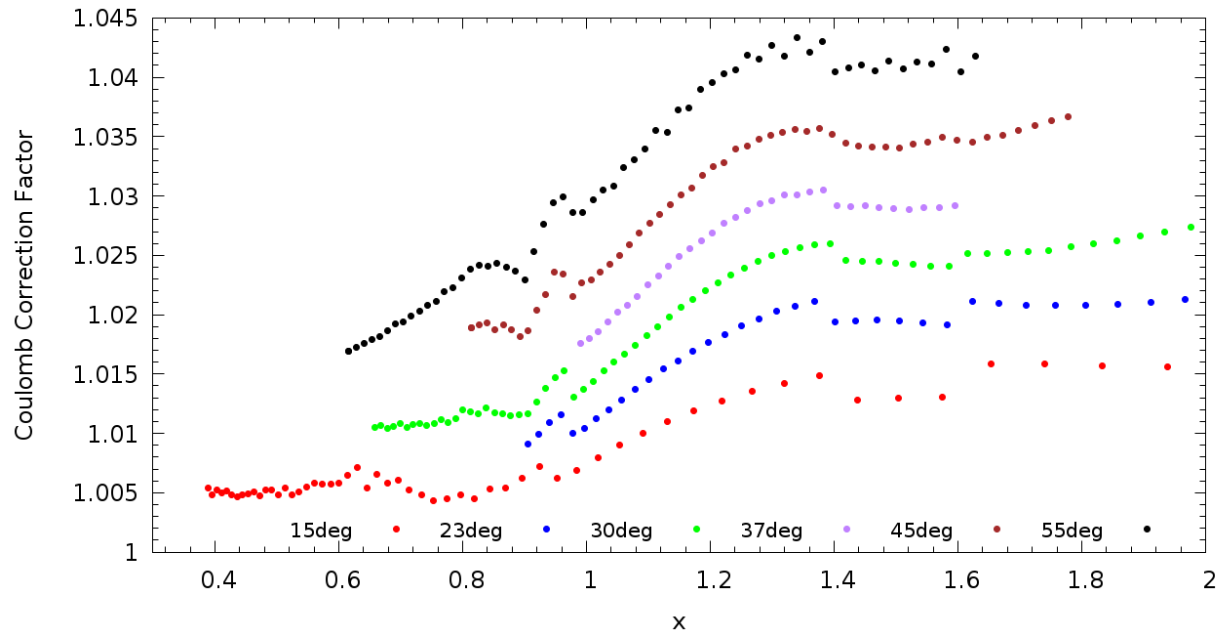


Figure 2.18: Plot of the ratio of unshifted cross section to shifted cross section for ^{12}C . This ratio represents the coulomb correction.

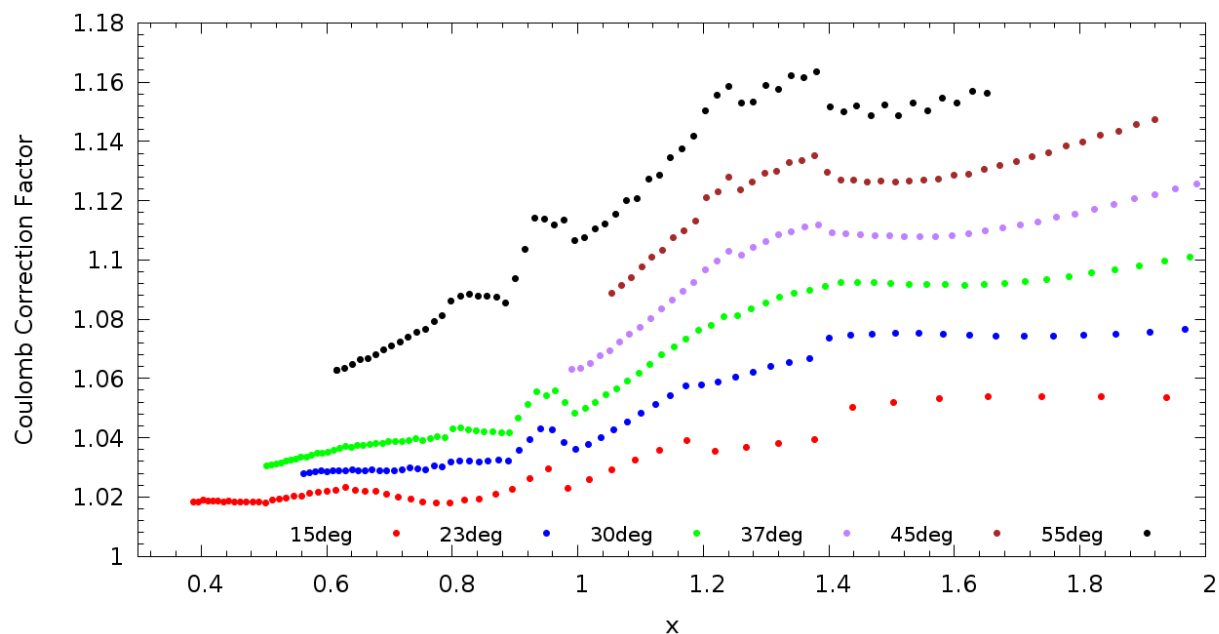


Figure 2.19: Plot of the ratio of unshifted cross section to shifted cross section for ^{56}Fe . This ratio represents the coulomb correction.

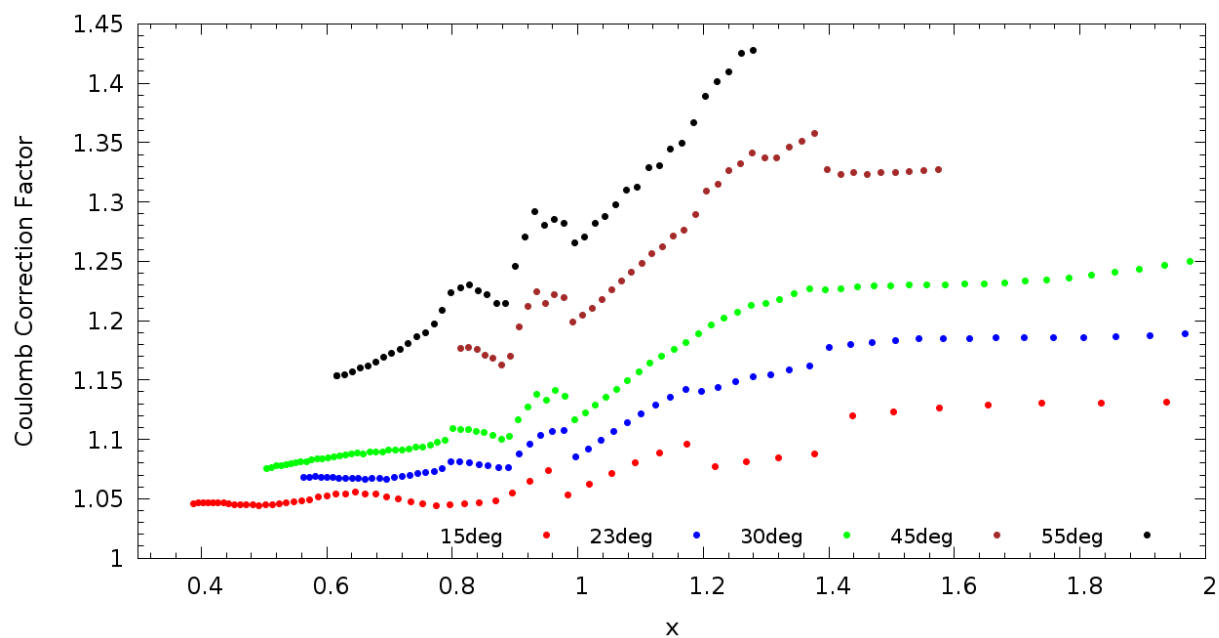


Figure 2.20: Plot of the ratio of unshifted cross section to shifted cross section for ^{197}Au . This ratio represents the coulomb correction.

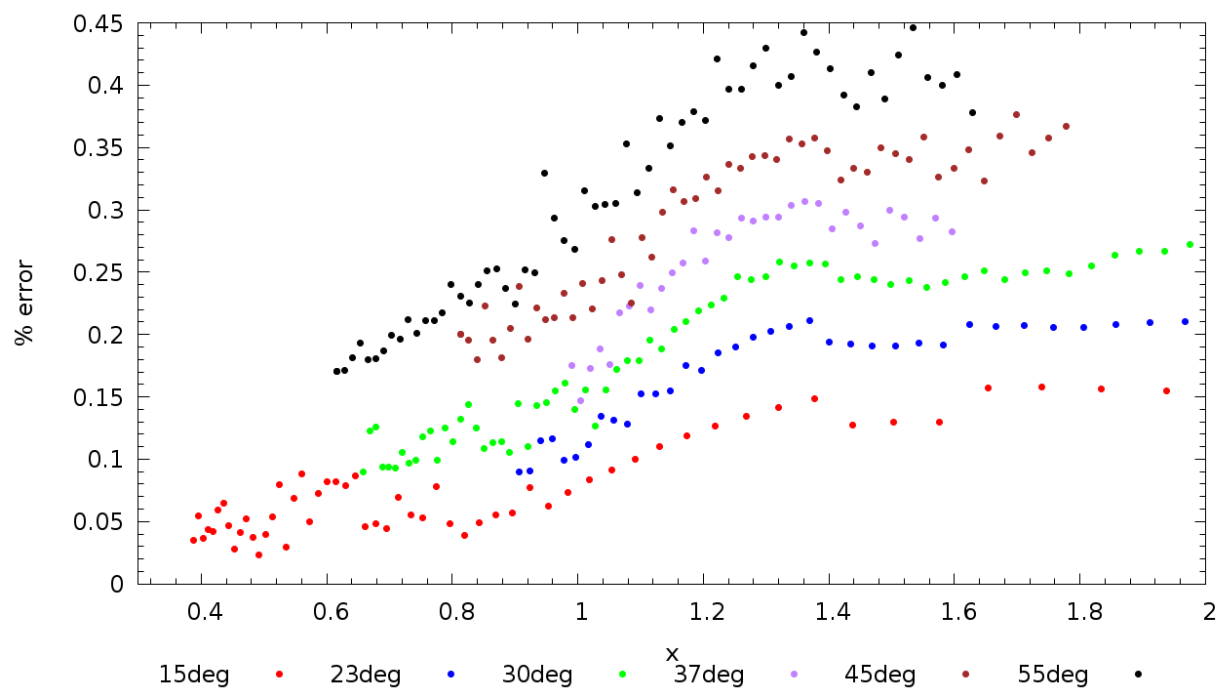


Figure 2.21: Plot of systematic error in the Coulomb correction for ^{12}C at all angle settings.

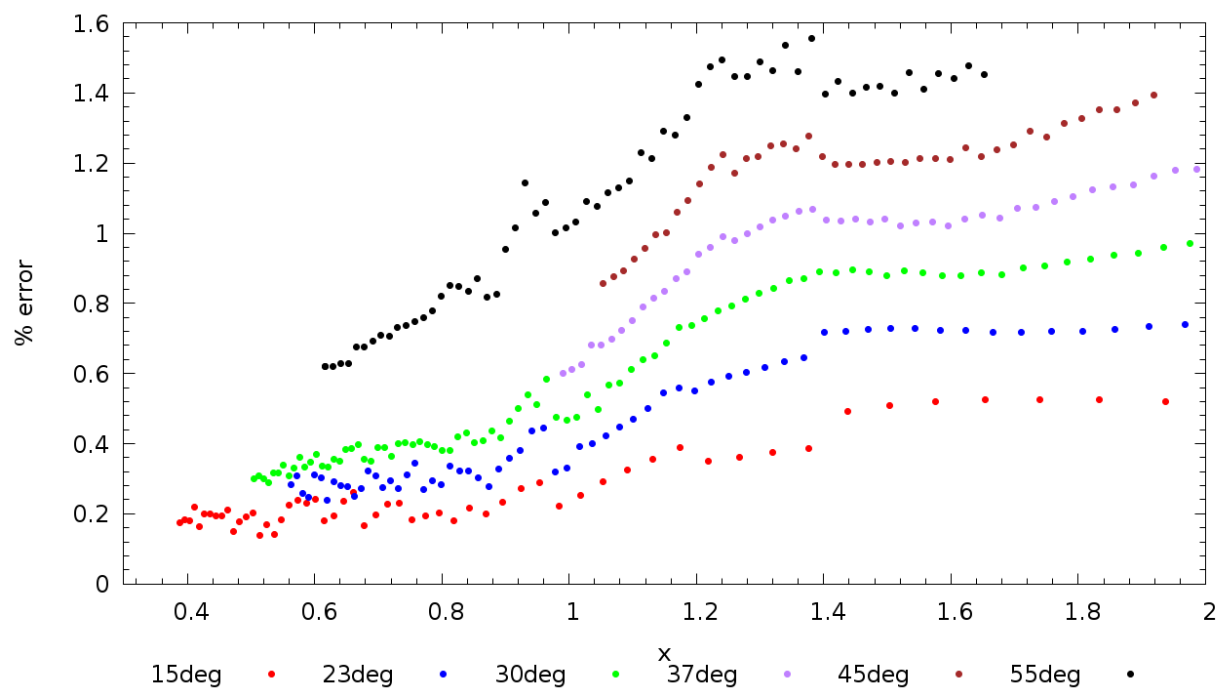


Figure 2.22: Plot of systematic error in the Coulomb correction for ^{56}Fe at all angle settings.

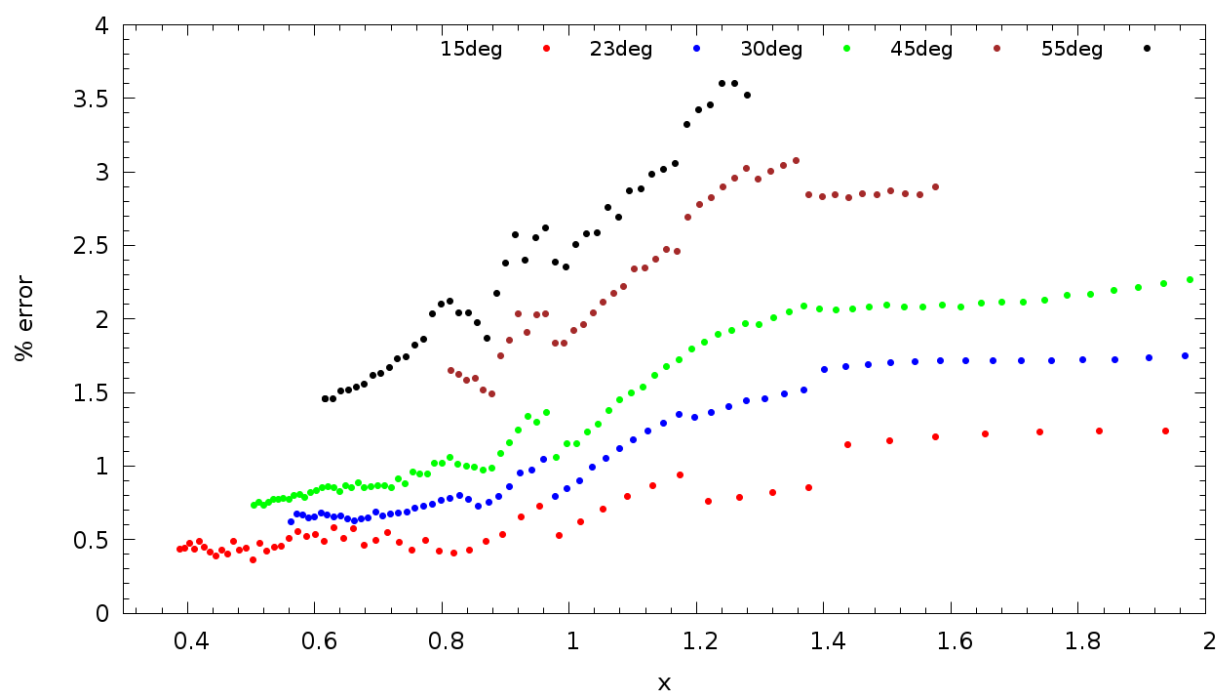


Figure 2.23: Plot of systematic error in the Coulomb correction for ^{197}Au for all angle settings.

Chapter 3

Conclusion and Outlook

Applying Coulomb corrections to the E89-008 data served two purposes. With the newly added corrections, shown in Figures 3.1, 3.2, and 3.3, the data analysis is completed as all corrections and associated uncertainties are finalized. This will allow the data to be published. Additionally, calculating Coulomb corrections allowed for a rigorous testing, application, and documentation of the XEM model code. While the simulation has undergone changes and has been adapted to suit various experiments, this paper should serve as a reference for the fundamental pieces of the model. From the core of the model, the simulation can be adapted to suit conditions of future experiments.

There are many ways forward for inclusive scattering experiments. In experiment E12-11-112 in Jefferson Lab Hall-A, measurements of tritium and ^3He , $A = 3$ nuclei, are being performed to look at possible isospin dependence. A simple SRC model views nucleon interactions as isospin independent, but E12-11-112 is looking to extract cross-section ratios of ^3He to tritium with an uncertainty of less than 4 %. In doing so, the goal is to clearly see possible isospin dependence in 2N-SRCs. Isospin independent and isosinglet dominated cross sections could have an up to 40 percent difference. Additionally, E12-11-112 is using inclusive scattering to test isospin structure in 3N-SRCs. Since inclusive scattering includes all final states, it is possible to differentiate between scattering configurations.

Additionally, Jefferson Lab Hall-C are moving forward with E12-06-105, another inclusive scattering experiment. Figure 3.4 shows the diversity in targets and kinematics that E12-06-105 will explore. Unlike E12-11-112, where the focus is on tritium and ^3He , E12-06-105 is

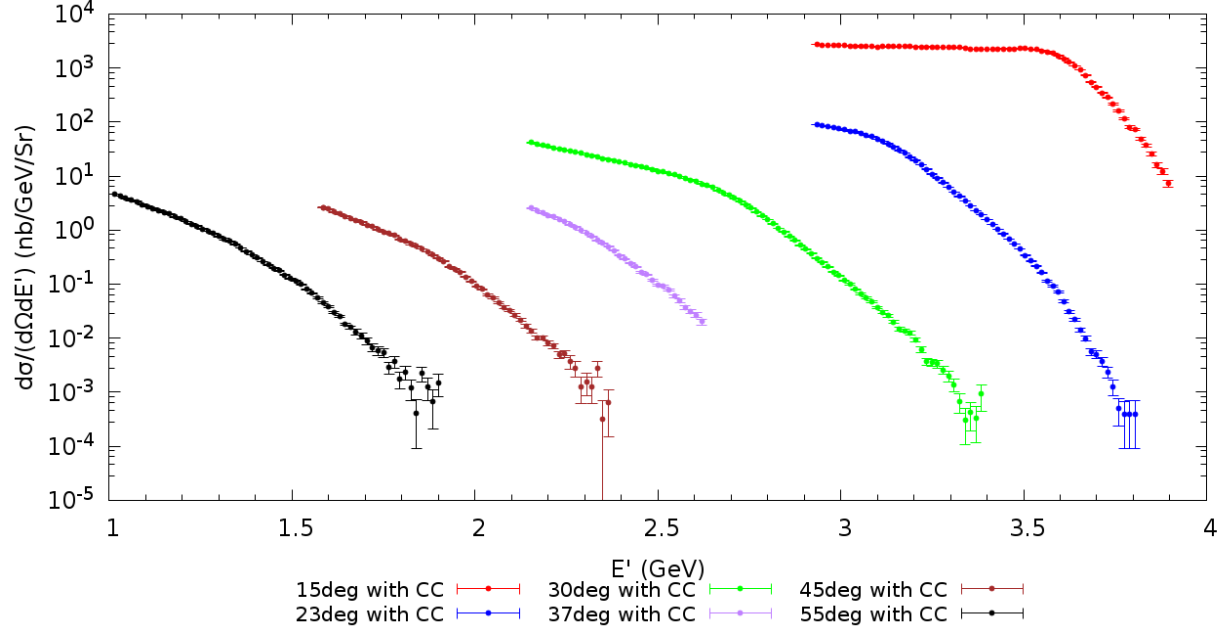


Figure 3.1: Plot of E89-008 σ' born with respect to E' for ^{12}C with Coulomb correction applied.

looking at a wide range of nuclei. With the 12 GeV upgrade to the accelerator at Jefferson Lab coupled with detector upgrades, the measurements proposed by E12-06-105 will provide much higher precision the region where $2.25 < x < 3$, for example. Experiments will probe the highest Q^2 ever in this region, which is the best chance of seeing 3N-SRCs. Added precision in this region allows for a detailed study of scaling for $A/{}^4\text{He}$ as well as much clearer cross section measurements in 3N and 4N SRC regions.

The way forward for inclusive scattering experiments is diverse and exciting. With new experimental results from Jefferson Lab, the SRC picture becomes clearer and clearer. While E89-008 helped set the groundwork in extracting momentum distributions of scattered nuclei, the approach to analyzing scattering experiments has evolved and will continue to evolve as new insights reveal themselves.

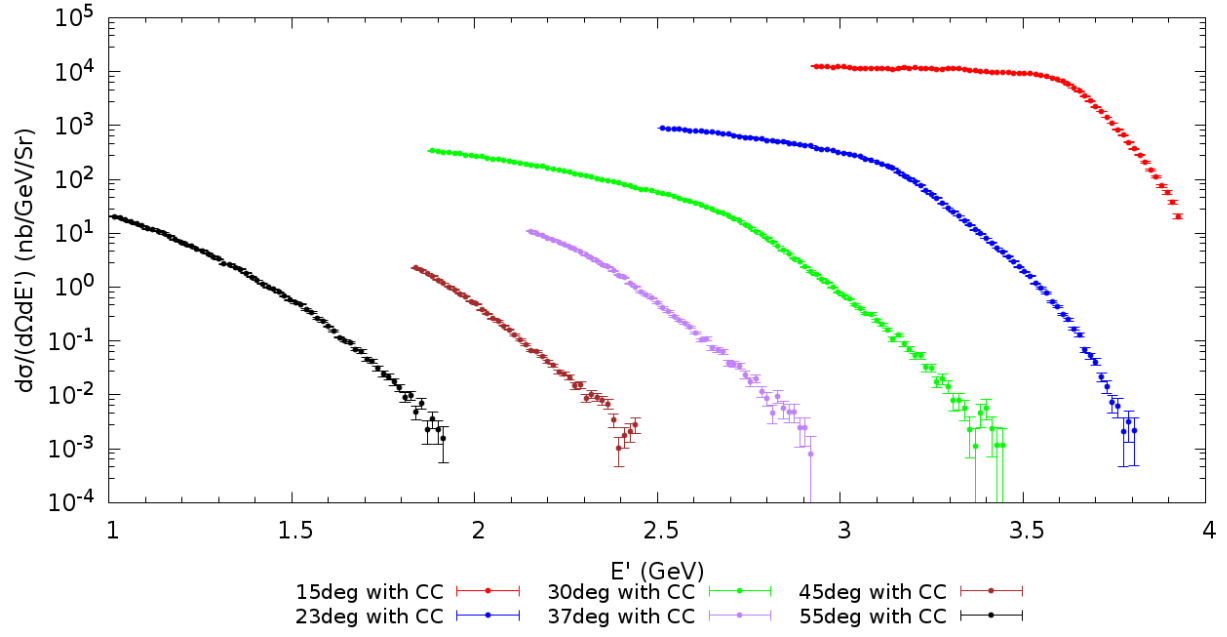


Figure 3.2: Plot of E89-008 σ' born with respect to E' for ^{56}Fe with Coulomb correction applied.

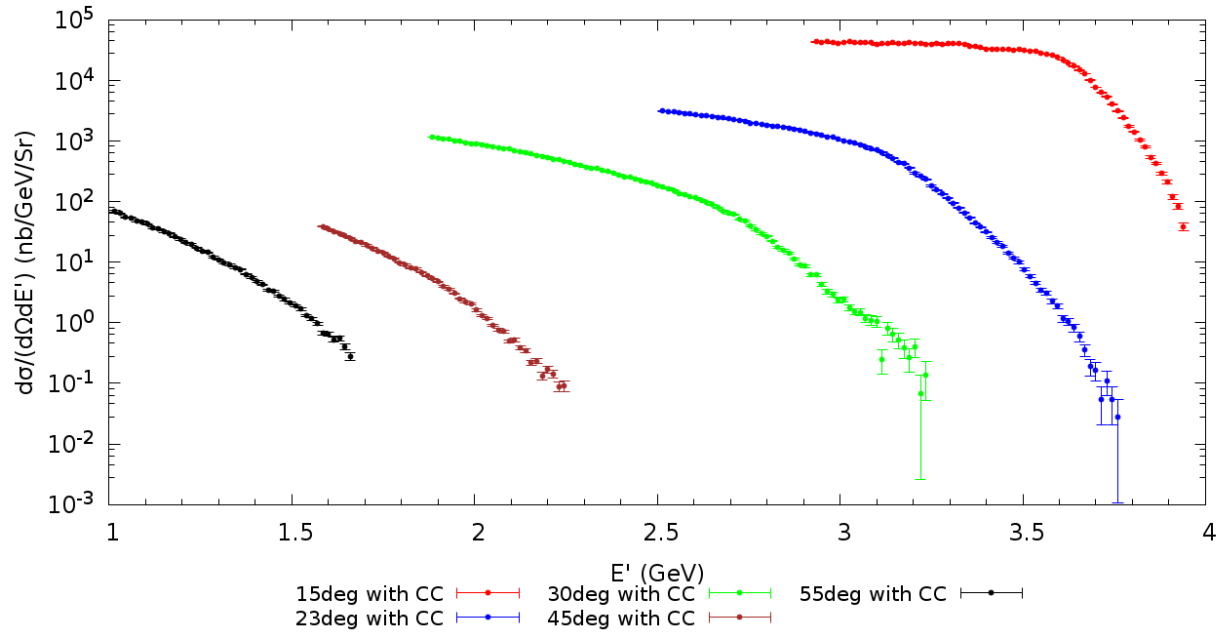


Figure 3.3: Plot of E89-008 σ' born with respect to E' for ^{197}Au with Coulomb correction applied.

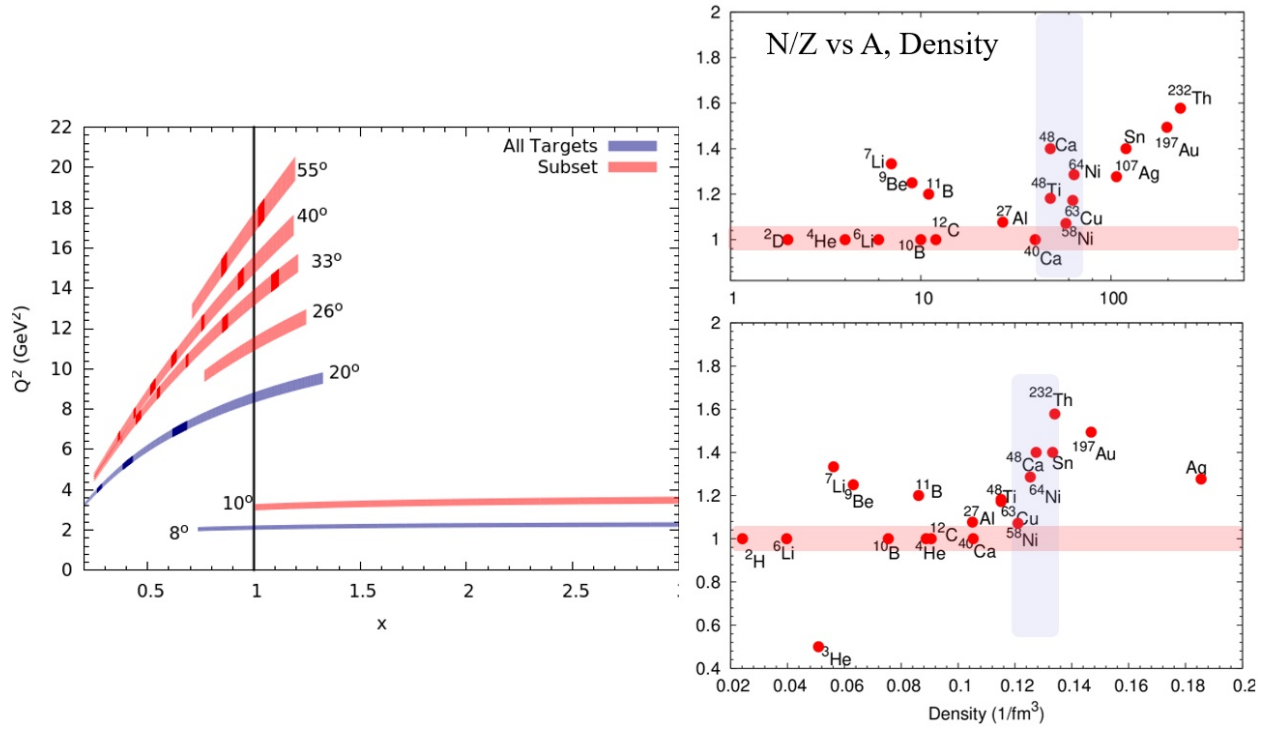


Figure 3.4: Plot of targets and angle settings that will be measured in the upcoming Jefferson Lab Hall-C experiment E12-06-105.

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Vita

Jay Carroll was born in Cincinnati, Ohio to Bob and Janel Carroll. He is the older brother to Mary Carroll. He attended school in the Indian Hill Village school system until high school, where he attended St. Xavier high school. After St. Xavier high school, Jay attended the University of Notre Dame, graduating in 2016. At the University of Notre Dame, Jay majored in Physics, with concentrations in Advanced Physics and Astrophysics. It was also at the University of Notre Dame where Jay began working with Dr. Justin Crepp. Jay worked under Dr. Crepp for four years and was given responsibilities and challenged in ways that he never expected. The research was an opportunity that he will always be thankful for. At Dr. Crepp's suggestion, Jay applied to the University of Tennessee - Knoxville, and was accepted into the Physics PhD program. Jay later switched to the Masters program and will graduate in May 2019. As a graduate teaching assistant, Jay had the pleasure of working for Dr. Sean Lindsay. Jay also did work for, and completed his thesis under, the attentive and encouraging Dr. Nadia Fomin. Jay will be moving back to Cincinnati to apply for jobs in earnest.