Ch. 8.

Just like kinematics, other "linear" quantities have similar "angular" cousins.

Force → Torque

~~Force & the result of an~~

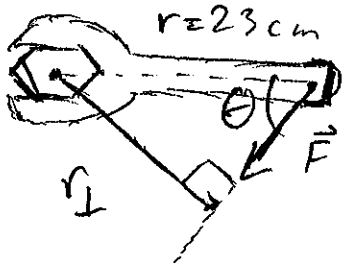
Force results in acceleration.

Torque results in angular acceleration.

Torque is a force applied some distance $r_\perp$ (called "moment arm" or "lever arm") from an axis of rotation, perpendicular to the radial axis.

$$\tau = r_\perp F \quad \text{or} \quad \tau = r \sin\theta \, F$$

Ex: What torque are you applying to the bolt?



$\theta = 60^\circ \quad r = 23\text{cm}$

$$\tau = r_\perp F = (r \sin\theta) F$$

$$\boxed{\tau = 11\ N \cdot m}$$

How can we maximize torque?

- If force is applied perpendicularly, then $\theta \to 90$. $\sin 90 > \sin 60$.

$r = .47\ m$ $\quad \theta_i = 0 \quad \omega_i = 0 \quad \alpha = ?$

$t = 4.3\ s$ $\quad \theta_f = ? \quad \omega_f = 18.8\ rad/s \quad t = 4.3\ s$

$\omega = 18.8\ rad/s$

1.) $\omega = \omega_0 + \alpha t \Rightarrow \alpha = \frac{\omega}{t} = 4.4 \frac{rad}{s^2}$

2.) How many revolutions did the fan-blades travel?

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ then $\frac{\theta}{2\pi} \Rightarrow$ rev. $\boxed{6.4\ rev}$

$\boxed{40.4\ rad}$ (radial)

3.) What was the centripetal acceleration of a point on the tip of the fan blade at $t = 4.3\ s$?

$$a_c = \frac{v_t^2}{r} = \omega^2 r = 170 \frac{m}{s^2}$$

4.) What was tangential acceleration at this time?

$$a_t = \alpha r = 2.1\ m/s^2$$

Torque Problems:

23.) $F = 55\,N$

$r = .74\,m$

a.) $\tau = 40.7\,N\cdot m = 41\,N\cdot m$

b.) $\theta = 45^\circ$

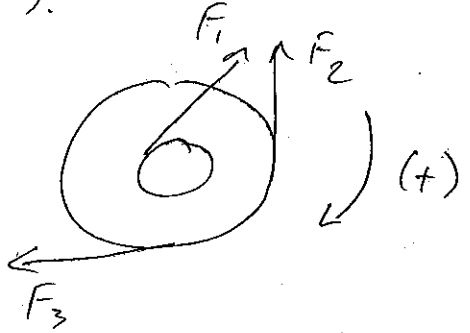
$\tau = rF\sin\theta = 29\,N\cdot m$

$\tau = r_\perp F = r F_\perp$

Explain how we can add torques, recalling Clockwise + Counterclockwise as (+) and (−).

Prob. 24) 4 forces:

~~35 .12 + 18 .24~~



$\sum \tau = r_1 F_{1\perp} + r_2 F_{2\perp} + r_3 F_{3\perp} + \tau_{fr}$

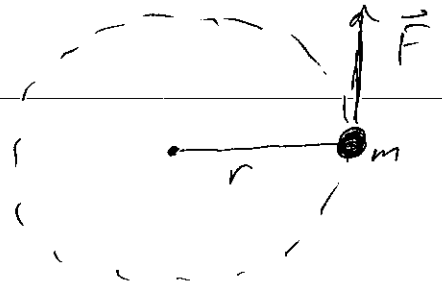
$= .12(35) - .24(28) + .24(18) - .40\,mN = \boxed{1.4\,m\cdot N \ CW}$

~~Maybe 25: What is torque due to left block?~~

Torque + Inertia

Note: Newton's 2nd Law: Imagine a single particle moving in a circle.

$$F = ma$$



recall:

$$a_{tan} = r\alpha$$

$$\Rightarrow F = mr\alpha$$

recall: $\tau = rF$

$$\Rightarrow rF = mr^2\alpha$$

$$\Rightarrow \tau = mr^2\alpha$$

mr^2 → "Moment of Inertia" → I $\left[\frac{kg}{m^2}\right]$

Kind of like an "angular mass."

Inertia: resistance to changes in motion.

$$\boxed{\Sigma \tau = I\alpha}$$ Newton's 2nd Law for rotation.

mr^2 : single particle.

"I" varies for different objects.

page 208, ~~fig~~ fig. 8-21

Ex: 8-11: Heavy Pulley:

~~Shape: 2 Solid Cylinders.~~

~~$I = \frac{1}{2}MR^2$~~

$F_T = 15.0\,N$

$M = 4.00\,kg$

$R = .33\,m$

Find I for unknown shaped pulley.

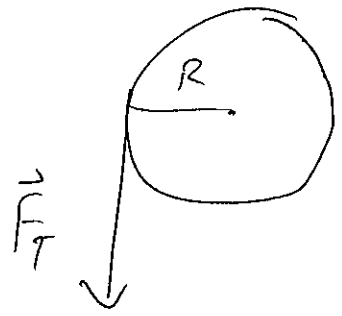
$\omega = 30\,rad/s$

$t = 3.00\,s$

$\tau_{fr} = 1.10\,m \cdot N$



FBD:



$\Sigma\tau = I\alpha$ ~~I_{fr}~~

$\Sigma\tau = RF_T - \tau_{fr} = 3.85\,N \cdot m$

$I\alpha = 3.85\,N \cdot m$

$I = \frac{3.85}{\alpha}$

Note: $\Rightarrow \alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega}{t} = 10\,rad/s^2$

$$\boxed{I = \frac{3.85}{10} = .385\,kg \cdot m^2}$$

Ex. 16.1:

$R_o = 0.065\,m$ $R_t = 0.025\,m$ $M_t = 0.057\,kg$

$M_o = 0.27\,kg$ $L_t = 0.32\,m$

$I_o = \frac{2}{5}MR^2 = 4.6\times10^{-4}\ kg\cdot m^2$

$I_t = MR^2 = 3.6\times10^{-5}\ kg\cdot m^2$

Orange has greater moment of inertia $\Rightarrow$ harder to get rolling.

Q:29: $m = .650\,kg$

$r = 1.2\,m$

a.) I about center of circle.?

$mr^2 = .936\ kg\,m^2 = \boxed{.94\ kg\,m^2}$

b) τ needed to keep ball moving if $F_{air} = 0.020N$

$\alpha = 0$ $\Sigma\tau = I\alpha = 0$

$\Sigma\tau = F_{air}\cdot r - \tau_{app} = 0$

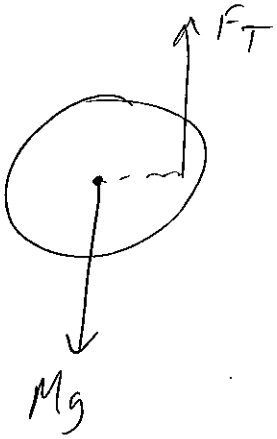
$\tau_{app} = \boxed{.024\ N\cdot m}$

$R = 0.031\ m$

$M = .120\ kg$

$\alpha = 0.78\ rad/s^2$

$L = 1.1\ m$



$$\Sigma F_y = ma_y$$

$$\Sigma F_y = F_T - Mg$$

$$F_T = Ma_y - Mg = M(r\alpha - g)$$

$$\Sigma \tau = I\alpha = RF_T$$

$$I = \frac{RM(r\alpha - g)}{\alpha} = 0.047\ kg\ m^2$$