

**Scaling and Dynamics of
Cylinder–Induced Transitional and
Turbulent Shockwave /
Boundary-Layer Interactions in a
Mach 4 Freestream**

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Doctor of Philosophy
Degree

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I dedicate this dissertation to my family:

To my parents, Sam and Cara Lester, who gave me life and my foundation.

To my brother, Grant Lester, for the laughter and a listening ear.

To my cats, Gibbous and Elvira, for their company in the midnight hours.

*And everlastingly, to Michael Shook, for bringing me happiness and staying by my side
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Abstract

Shockwave / boundary-layer interactions (SBLI) are a prevalent, high-risk fluid phenomena on supersonic and hypersonic aircraft that have significant implications for a vehicle's overall flight performance and safety. Their unsteady nature can generate large gradients in shear layer properties and localized thermal loads, and due to their complexity. With the use of newly developed diagnostic capabilities and analysis methods, it is possible to begin a systematic and efficient study of the driving behavior in transitional and turbulent shockwave / boundary-layer interactions. This investigation aimed to quantitatively measure characteristic SBLI features, capture general SBLI dynamics as a product of the incoming boundary layer state, understand how frequency content associated with characteristic SBLI unsteadiness changes during the transition process, and assess what driving mechanisms promote the growth and collapse of the separation bubble.

To complete this assessment a cylindrical shock generator was installed on a 6-degree axisymmetric cone to study both transitional and turbulent shockwave / boundary-layer interactions in the UTSI Mach 4 Ludwig tube. High-speed schlieren imaging and pressure-sensitive paint were utilized. Data analysis included power spectral densities, spectral heat maps, POD, DMD, a custom feature-tracking algorithm, and analysis between the separation shock, upstream influence, and a distinct boundary-layer thickening feature.

The high-speed schlieren imaging time series images display a shock collapse from an upstream disturbance in the incoming boundary layer. The upstream influence is seen to bow in response to the turbulent packet as it is pushed downstream. Following this collapse, the upstream influence appears to have a linear restoring behavior upstream towards its originating position. Local frequency probes generally see the most prominent frequencies at the mean location of these ranges of influence. As the SBLI shock generator is moved

downstream, this frequency peak shifts to a higher frequency and is suspected to scale with the incoming boundary layer height. The dominant modes in the POD analysis revealed a shift in the SBLI structure between the first two and the final two-cylinder shock generator locations. In the DMD analysis, modes associated with rapid growth mechanisms were identified. A relationship between the UI presence and boundary layer thickening behavior was observed.

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Nomenclature

σ	Standard Deviation
σ^2	Variance
d	Cylinder Diameter
f	Frequency
G_{xx}	Power Spectral Density Magnitude
M_∞	Freestream Mach Number
M_e	Boundary Layer Edge Mach Number
P_0	Stagnation Pressure
P_∞	Freestream Static Pressure
Re	Reynolds Number
Re/x	Unit Reynolds Number
St	Strouhal Number
T_0	Stagnation Temperature
T_∞	Freestream Static Temperature
U_∞	Freestream Velocity
UI	Upstream Influence Shock

x Distance from the cylinder face
 X_{cylinder} Distance along surface from leading edge to cylinder face
DMD Dynamic Mode Decomposition
HORIZON High-speed Original Research and Innovation Zone
PDF Probability Density Function
POD Proper Orthogonal Decomposition
PSD Power Spectral Density
PSP Pressure Sensitive Paint
SBLI Shockwave / Boundary-Layer Interaction
SPOD Spectral Proper Orthogonal Decomposition
STBLI Turbulent Shockwave / Boundary-Layer Interaction
UTSI University of Tennessee Space Institute
XSBLI Transitional Shockwave / Boundary-Layer Interaction

Chapter 1

Introduction

1.1 Motivation

Shockwave / boundary layer interactions (SBLI) are a pervasive feature in both the internal and external flow paths for flight in the supersonic and hypersonic regimes that can affect performance, safety, and design considerations [38]. These interactions can cause boundary-layer separation, generating significant local thermal and mechanical loads at separation and reattachment locations [31, 37, 39, 89, 99, 111], and thus, structural damage can occur. Furthermore, this aforementioned boundary-layer separation, due to the adverse pressure gradient associated with the shockwave, can lead to a reduction in control authority or a decreased lift-to-drag ratio. For internal flows, the shock-induced boundary-layer separation could restrict the mass flow, resulting in inlet degradation and unstart [60, 61]. Moreover, the unsteady dynamic behavior associated with these interactions can manifest as high-amplitude, low-frequency oscillations that can harmonize with the aircraft. Due to these ramifications, SBLI has been the focus of extensive research since its first appearance in the work of Ferri in 1939 [28].

One of the most notable examples of risk associated with shockwave / boundary-layer interactions was demonstrated when the X-15 research craft experienced catastrophic damage to a pylon due to shock-shock and shockwave / boundary-layer interactions. These fluid phenomena appeared again on the Apollo capsule and prompted numerous studies to understand stability issues associated with the shockwave / boundary-layer interactions on

the aft side of the body. The U.S. space shuttle program also highlighted the importance of understanding shockwave / boundary-layer interactions in two distinct ways.

The first example from the U.S. space shuttle program occurred during Space Transportation System (STS) 1 and showcases how SBLI, in combination with real-gas effects, impacts flight vehicle stability. An investigation, prompted by the narrow success of the program's maiden flight, found discrepancies between preflight measurements in ground-testing facilities where low-enthalpy flow fields were significantly different from measurements recorded during the maiden flight, subject to the Earth's atmosphere. This discrepancy is often referred to as the Shuttle Flap Anomaly. It was found that viscous-interaction effects notably reduced surface pressure to the aft-windward expansion surfaces in the presence of real-gas effects.

More specifically, a shuttle model was tested in the Large Energy National Shock Tunnel (LENS) I facility at Calspan University of Buffalo Research Center (CUBRC) and found that the real-gas effects decreased the size of the shockwave / boundary-layer interaction regions, generated larger pressures, and increased the area of flow attachment over the flap. Oxygen dissociation levels were then varied, and overall pressure levels were found to be relatively small; however, over a large area, a significant decrease in the pitching moment was observed as the total enthalpy was increased. This indicated that reduced stability in relatively higher enthalpy flows was a result of a decrease in the pressures on the rearward part of the curved wing. These results were evident from in-flight data from the STS-1 mission when the shuttle flaps had to be fully deflected to maintain stability at a much larger angle than initially predicted. Moreover, this reasoning was later validated by Direct Numerical Simulation (DNS) [4, 71] under limited conditions.

The second example from the U.S. space shuttle program is the unfortunate disaster of the space shuttle Columbia. The Columbia disaster was a result of structural damage caused by a large piece of foam insulation that had fallen and impacted the reinforced carbon-carbon panels on the leading edge of the shuttle's left wing just after liftoff. The foam insulation originated from the left bipod ramp holding the external tank to the nose of the shuttle, and it was separated through dynamic loading caused by a shockwave / boundary-layer interaction. NASA engineers at the time had concluded the impact had no entry-critical damage implications; however, the disaster became evident when Columbia experienced a

sudden loss of data first from the protective thermal tiles and then promptly followed by the pressure sensors associated with the landing gear system on the vehicle’s left side. Leroy Cain, the entry flight director, disclosed at the disaster briefing that “when the failure occurred, the shuttle was experiencing maximum heat loads of nearly 3,000 degrees Fahrenheit.” He went on to state, “If we did have a structural problem or a thermal problem, you would expect to get it at the peak heating... The most extreme thermal environment was right at Mach 18, and that’s where we lost the vehicle.” The situation clearly displayed how significant the effects of SBLI can be, especially in terms of the extreme thermal loading that they can produce and how destructive this can be to the structural integrity of the vehicle. Fig. 1.1 shows photographs of the incident, including notes on the debris path, impact, and damage as it struck the left wing [109].

Typically, shockwave / boundary-layer interactions are categorized by the state of the incoming boundary layer [36, 50], referred to here as laminar SBLI, transitional SBLI, and turbulent SBLI for incoming laminar, transitional, and turbulent boundary layers, respectively [61]. As originally discussed in Lester, et al. [61], Figure 1.2 shows the general structure and major features of an SBLI induced by a cylindrical shock generator for the cases of turbulent and transitional incoming boundary layers. For the turbulent case, displayed in Fig. 1.2 on the left-hand side, the strong inviscid shock created by the cylinder in the freestream flow results in an adverse pressure gradient that causes the boundary layer to separate upstream of the cylinder shock generator, thereby creating the separation shock feature. This separation shock is unsteady due to both the dynamics of the separation bubble downstream of the shock [25, 31] and the turbulent fluctuations of the incoming boundary layer [6, 110]; however, the precise unsteadiness mechanism is still unknown [17, 24, 31]. The transitional interaction shown in Fig. 1.2(b) has many of the same features as the turbulent case, though an additional intermittent shock feature is typically present. This feature is known as an upstream influence shock and is one of the defining characteristics of transitional interactions. Much like the turbulent case, the separation shock foot is unsteady, though the precise extent and dynamics of its motion are not as well-characterized as that of the turbulent interaction [50]. This type of interaction also has a wider range of streamwise motion [20, 61].

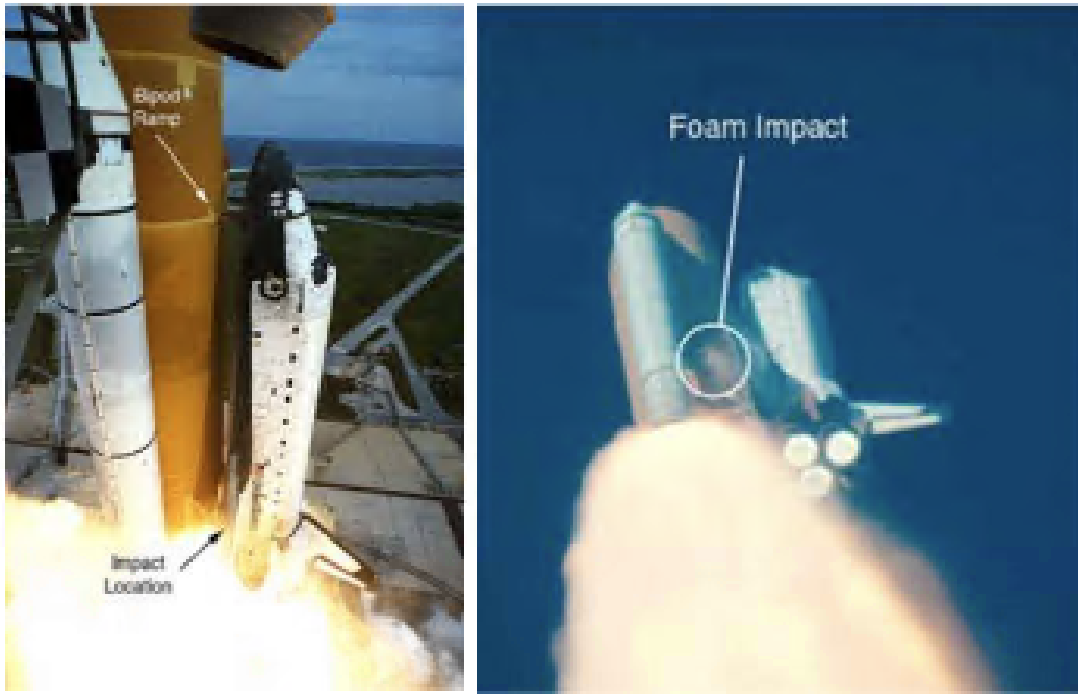


Figure 1.1: Space Shuttle Columbia just after takeoff. Visual of foam breaking off and impacting the left wing [109].

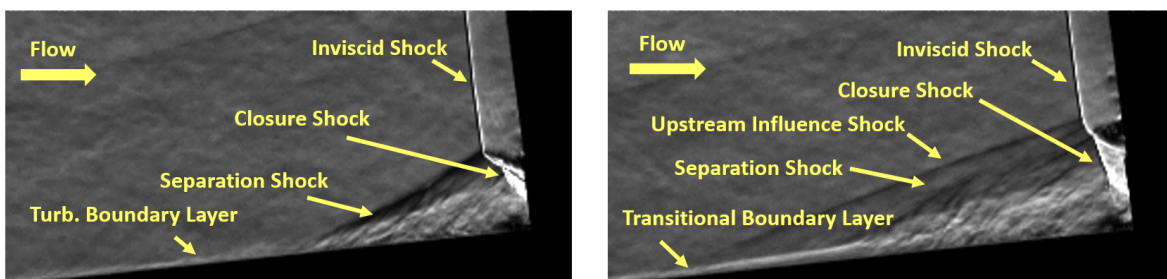


Figure 1.2: Flow features of turbulent (left) and transitional (right) SBLI. The transitional case exhibits an extra, intermittent shock feature known as an upstream influence. However, the separation shock is unsteady for both cases.

Laminar shockwave / boundary-layer interactions have generally been considered by the aerospace community to be predicted quite well and easily solvable for most flow regimes through computational analysis [52]. In light of emerging data, high-speed vehicles are expected to undergo extended flight times in the transitional and turbulent boundary-layer regime than previously anticipated. As such, the aerospace community has shifted its SBLI interrogation to transitional and turbulent interaction types. Work to date has yielded a limited amount of nonintrusive turbulent interaction data, and experimental studies have revealed characteristic features of transitional SBLI, including their statistically probable regions of influence and dynamics, to be more complex than their turbulent counterparts [14]. Transitional SBLI are presently known to have a larger separation region in comparison to turbulent interactions but possess the same elevated thermal and mechanical loads at equivalent or even greater magnitudes. Due to the uncertainty in transition prediction, it is common practice to overdesign supersonic vehicles. Not only is this practice cost-prohibitive, but vehicle performance is also limited [16]. Understanding the driving mechanics behind SBLI would provide better transition prediction methods and improve the supersonic vehicle design approach.

1.2 Research Objectives

In an effort to understand the underlying driving mechanics of SBLI, this research will look to leverage new diagnostic capabilities and data analysis techniques and begin a systematic approach to understanding how SBLI dynamics evolve as they undergo transition from a laminar to turbulent interaction. A cylinder-induced shockwave / boundary-layer interaction (SBLI) occurring in Mach 4 flow over a 6° axisymmetric cone model was studied with high-speed schlieren imaging at the University of Tennessee Space Institute. By adjusting the streamwise location of the shock generator, the current study examines both transitional and turbulent interactions. The following research aim is proposed: To identify the evolution of scaling and dynamics parameters of a transitional and turbulent cylinder-induced shockwave / boundary-layer interactions and the underlying mechanisms responsible for unsteady

characteristics. The proposed research effort aims to address the following open questions on this subject:

1. How do SBLI characteristics and the interaction dynamics evolve when the incoming boundary-layer state changes from transitional to turbulent?
2. How does the unsteadiness associated with characteristic SBLI features evolve during the transition process?
3. What promotes the growth of the separation bubble, and what causes its subsequent collapse?

1.2.1 Objective 1

An investigation into key SBLI features and behaviors was conducted to give insight into the driving underlying physics of both transitional and turbulent interactions. Quantitative measurements of averaged data sets, such as the boundary layer and separation length scale, are key length scales for general SBLI structures based on prior work in the community. Standard methodologies of evaluating SBLI based on high-speed schlieren and pressure sensitive paint data are completed, and a quick look at time series data reveals interesting dynamics that may be further investigated or presented as spurious, potentially upstream driving behaviors.

1.2.2 Objective 2

Using high-speed schlieren data and pressure sensitive paint, the frequency content of SBLI is evaluated in the form of traditional power spectral densities and remapped back to the image domain in the form of a spectral heat map. These experiments investigate changes in the frequency content, specifically assessing differences in relative dominance and spatial extent of unsteady features as the interaction's incoming boundary layer progressed from transitional to turbulent. Pressure sensitive paint data was utilized to validate frequency information in the schlieren data and briefly look at spanwise variations.

1.2.3 Objective 3

Utilizing computational learning techniques such as (e.g., POD and DMD), dominating structures for each case were reviewed, and modes associated with characteristic high frequency content were evaluated based on their modal growth rates. Identified high frequency content and rapid growth modes may prove insightful in identifying SBLI driving mechanisms and the key features necessary for changes in the global SBLI structures. This data will provide insight into why specific systems may manifest as traditionally labeled “oscillator” or “amplifier” responses and identify key variables that dictate the system’s behavior. Further analysis with the use of a shock tracker of high-speed schlieren imaging allowed for the upstream influence and separation shock locations to be tracked. Moreover, a cross-correlated boundary-layer thickening behavior was completed to investigate the origin of driving mechanisms and explore how these dominating physics may shift throughout the boundary-layer breakdown process.

Chapter 2

Overview

2.1 Laminar Interactions

Laminar shockwave / boundary-layer interactions in hypersonic flows are known to have a large area of influence but relatively lower thermal and pressure loads than their transitional and turbulent counterparts. Laminar shockwave / boundary-layer interactions may develop in one of two ways. The first development path may be “internally” from a compression surface due to “free interactions” between the viscous and inviscid flow and may be considered as an attached or a separated interaction. These shockwave / boundary-layer interactions can be quasi-two-dimensional or three-dimensional in nature. Quasi-two-dimensional interactions, like those seen with compression ramps, are traditionally used as a simplified case where it is easier for researchers to evaluate the boundary layer and interpret results. The second development pathway may be induced via the impingement of a shockwave onto a boundary layer, otherwise known as shock-induced separation.

Separated boundary layers for both free interactions and impingement-type shockwave / boundary-layers are known to influence the location of the interaction, moving it upstream from its attached counterpart. This movement upstream results in a more extensive separation. The reattachment-compression process is a result of a significant increase in the flow curvature. In turn, this process results in an increased strength in the normal pressure gradient. As such, the more complex and stronger cases with separation were made the focus of research for many years, and it was critical for the aerospace community to

develop simulations that could account for this change in pressure to adequately simulate and predict the subsequent flow field.

Experimental results have found that incipient separation, the minimal amount of a pressure ratio needed to cause separation in free interactions, trends well with the overall pressure rise necessary to induce this type of separation for both a ramp- and shock-induced interaction. When a viscous interaction parameter seen in Eqn. 2.1 is used, both data sets collapse well [4]. Moreover, incipient separation is congruous with the hypersonic similitude parameters. These relationships suggest in high-Mach number flows, flow separation is dictated predominately between viscous and inviscid flows by free interaction. Additionally, the observed pressure rise through the interaction was found to be related to the reattachment region’s peak heat-transfer rate [43].

$$\bar{X} = \frac{M_\infty^3}{\sqrt{(Re_x)}} \sqrt{(C_w)} \quad (2.1)$$

2.1.1 Triple-deck Theory

Before the advanced Navier-Stokes and DSMS prediction methods of today, the underlying characteristics of shockwave / boundary-layer interactions were focused on creating numerical approximations for the first- and second-order boundary-layer equations. These early models, developed in roughly 1941 through 1953, investigated the influence of the upstream influence through the work of Lighthill [62] and Oswatitsch and Weighardt [78]. Initially, the upstream influence was thought to be related to upstream propagation through the subsonic region of the boundary layer. In low-Mach number flows, it was found that the separation regions have a more considerable upstream extent, prompting a new theory for understanding the underlying driving mechanics behind this rapid boundary-layer growth.

In the early works of Lighthill and Landau [62], boundary-layer separation was characterized by different turbulence scales and substantial boundary-layer displacement. At the time, researchers found that Prandtl’s boundary-layer model, characterized by dividing the problem into two layers; an outer region governed by inviscid flow and an inner region that is dominated by viscous flow (the boundary layer), was insufficient for evaluating

flows with separation. In separated flows, the magnitude of retained terms from this two-layer model matched asymptotic expansion, becoming equal in magnitude to one another. This realization prompted the works of Stewartson and Williams [102], Messiter [72], and Neiland [76] to propose a new theory. The triple-deck theory describes the boundary-layer behavior best by subdividing the problem into three layers; the upper deck, lower deck, and main deck, when significant disturbances exist in the boundary layer. Triple-deck theory reserves the same streamwise order as the boundary layer, $\mathcal{O}(Re^{-3/8})$; however, it has three different transverse-size orders, $\mathcal{O}(Re^{-3/8})$, $\mathcal{O}(Re^{-1/2})$, and $\mathcal{O}(Re^{-5/8})$, respectively. The relative layer thicknesses may be seen with their respective layer more clearly in Fig. 2.1. The upper deck is outside of the boundary layer and encapsulates inviscid and irrotational flow governed by the Euler equations. Meanwhile, the main deck and lower deck are within the boundary layer. The main deck has an inviscid but rotational flow, while the lower deck is both viscous and rotational. Each of these layers is responsible for passing information to the next. As velocity decreases upon proceeding closer to the wall, the sensitivity of the system to pressure variations increases as well. In turn, should these variations in pressure decelerate fluid particles, this method will detect regions of subsequent detachment, rapid streamline displacement, and reverse flow downstream of separation. More importantly, this algorithm does not encounter the Goldstein singularity, where Prandtl’s boundary-layer method arrives at a single solution close to the separation point [57, 35], and therefore is essential for boundary-layer separation analysis [1, 56].

Laminar separation in supersonic flows was then postulated to be a byproduct of rapid boundary-layer growth through the mutual interaction of the viscous layer and inviscid flow. This development process was coined a “free interaction” and may be extended to supersonic flows over adiabatic walls.

The models for these free interactions utilized momentum integral techniques adding the moment of momentum equation by Glick [34], Honda [44], and Lees and Reeves [59] in 1957 through 1964. These momentum-integral techniques allowed for the prediction of the flow structure in and above the reverse flow regions of shock-induced separation regions from separation to reattachment. Nielson’s findings showed agreeable velocity distribution for quasi-two-dimensional flows [77]. Glick later modified these equations using the Crocco-Lees

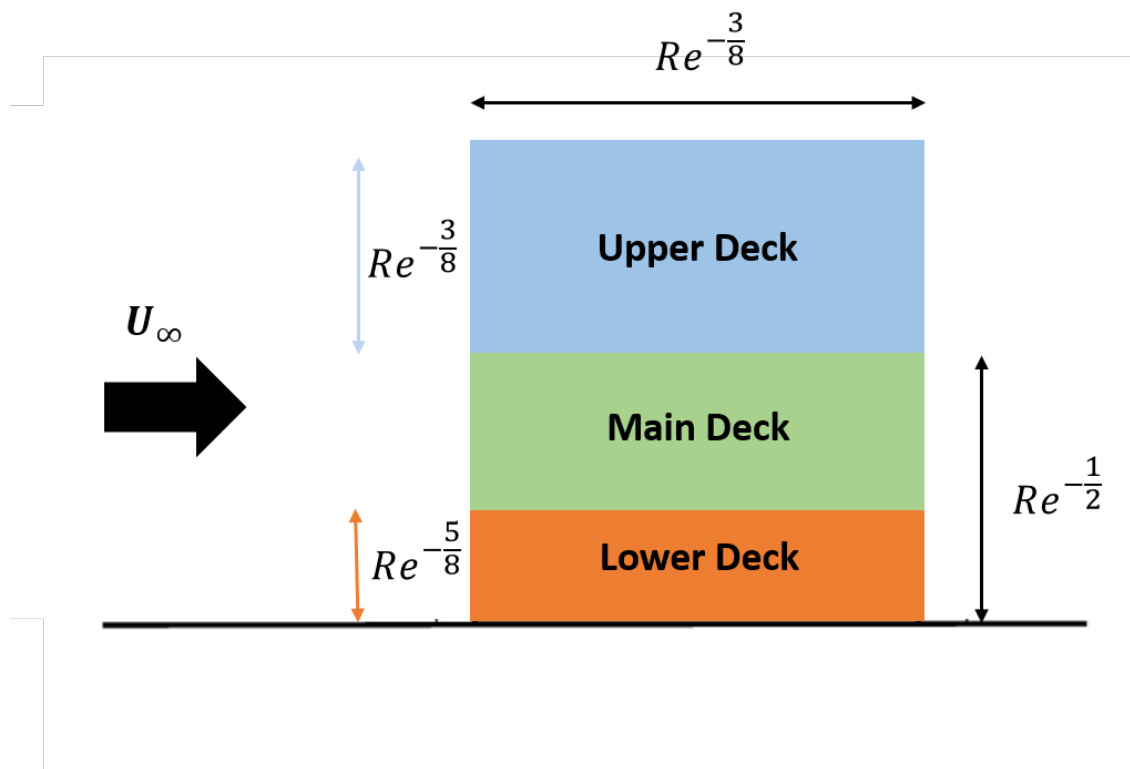


Figure 2.1: Triple-deck structure. Image adapted from Aljohani [1].

mixing theory and could then predict the pressure rise and boundary-layer growth leading to separation [34].

Work by Holden in 1966 [41] utilized an integral analysis that included the energy equation and a standard pressure rise for approximating the first- and second-order boundary layer equations. These were done to predict the skin friction distribution, pressure, and heat transfer of shockwave / boundary-layer interactions over cold wall conditions. While the first two parameters could be approximated with reasonable accuracy, the methodology would overpredict the heat transfer in the separation region.

The momentum-integral analysis will break down in regions of viscous-inviscid interactions over small boundary-layer thicknesses in hypersonic flows. As such, these integral analysis techniques had high Mach number limitations, prompting the work of MacCormack and his accurate numerical solution to the Navier-Stokes in 1975 [68]. These Navier-Stokes solutions were capable of determining the characteristics of both attached and separated regions of corner flows. These results compared well with skin friction and heat transfer measurements for attached and small separated regions, but they underpredicted the separation region. These discrepancies were later remedied in 1989 by Rudy et al. [86] with the introduction of three-dimensional effects.

Currently, even complex laminar shockwave / boundary-layer cases can be predicted up to velocities above 4.25 km/s (14,000 ft/s) in the air in which the shock layer and separated regions are in a state of chemical non-equilibrium [42].

2.2 Turbulent Interactions

As originally discussed in Smith, et al. [98], Unlike laminar shockwave / boundary-layer interactions, modeling for their turbulent counterparts has been met with limited success. Partially, this is due to the nature of turbulent interactions being fundamentally different from their laminar counterpart. A turbulent boundary-layer profile has much stronger velocity gradients. In turn, this ensures that the sonic point is located lower in the boundary layer (i.e., closer to the wall) and allows for turbulent interactions to resist separation. This means that the embedded separation regions seen in this interaction cannot be assumed

to be induced by the turbulent boundary-layer flow field and cannot be described by mutual interaction between the rapid boundary-layer growth as a function of an adverse pressure gradient, as is the case for laminar flow. Turbulent interactions have limited interaction regions on the order of 2-3 times the diameter of a cylinder-shock generator for shock impingement SBLI cases [92]. Moreover, they have much stronger loading than laminar interactions. However, a notable feature of turbulent interactions is the inherent low frequency unsteadiness that exists several orders of magnitude below the incoming boundary layer’s characteristic frequency [24]. The probability of this unsteadiness creating acoustic resonance with aerospace vehicles has made it a captivating scientific area of study [98, 98].

2.2.1 Unsteadiness

Unsteadiness was believed to be first encountered in experiments by Holden in 1991 [40]. Measurements on a cone cylinder flare model found large-scale instabilities that were thought to be susceptible to upstream disturbances and not simply a product of transition. This meant that if the observed shockwave / boundary-layer interaction was not sufficiently downstream of flow instabilities, these disturbances would perturb and generate instabilities within the interaction itself. This study went on to look at cylinder flare configurations for the ground testing phase of the HIFiRE flight experiments that featured a blunted nose cone. It should be noted that the HIFiRE flight experiments were the first flight test to confirm the existence of shockwave / boundary-layer interaction unsteadiness [49]. These flight test experiments confirmed that the measured unsteadiness does not result as a consequence of conducting ground testing. Results showed that the surface-pressure and heat-transfer measurements displayed the importance of their computational counterparts in including compressibility effects in their models, as they significantly affected the output heating levels. The shear stress transport turbulence model used in the simulations modeling these experiments was able to match pressure and heat transfer rates down in the interaction, but overpredicted the separated region length on the cone-flare junction. This disagreement between experiments and computational models was seen over several additional cases, including work by Gaitonde et al. [30]. Ultimately, it was stated that future computations

must include turbulence models that can model/predict the separation, recirculation, and reattachment regions reliably.

Past conceptual debates on SBLI have focused on rectifying contradictory data regarding where SBLI dynamics originate and how these systems impact the global system [17, 24, 25]. More explicitly, the competing theories surrounded whether the dominating mechanisms emerged from an upstream or downstream origin and if their dynamics were best described as an amplifier or oscillator behavior, respectively. As recently as 2021 [107], a new model has been widely accepted in which the high-risk unsteadiness in SBLI may be considered a function of both upstream and downstream mechanisms such that the dominant driver and overall system behavior may shift based on the shockwave / boundary-layer interaction strength [17]. This model is a first-order dynamic system, first proposed by Plotkin [80], and it operates as a low-pass filter of external forcing [82]. In Eqn. 2.2, Plotkin relates turbulent boundary-layer fluctuations to the shock motion’s statistical properties, assuming that $x(0) = 0$. For the following equations, time is t , a time average is indicated by an overbar, and τ_R is the shock’s recovery time from a perturbation.

$$x = \int_0^t u(\xi) \exp\left(\frac{\xi - t}{\tau_R}\right) d\xi \quad (2.2)$$

Eqn. 2.3 uses the integral of the autocorrelation time scale to quantify the relative time scale for the shock to respond to the incoming turbulence. Here, the subscript i may be replaced by u , x , or $[hp!]$ to denote the autocorrelation of the turbulent boundary-layer velocity, shock position, or wall pressure fluctuations, respectively. Once the integral is taken for the autocorrelations, the time constant of the restoring velocity (τ_R) is equivalent to the integral time scale of the shock position (x), explicitly ($\tau_R = \tau_x$)

$$\tau_i = \int_0^\infty R_i(\tau) d\tau \quad (2.3)$$

The fundamental results of Plotkin’s theory are encapsulated in Eqns. 2.4, 2.5, and 2.6. Equation 2.4 can approximate the mean square shock excursion and is later simplified for the shock position’s autocorrelation ($R_x(t)$) and power spectrum ($G_x(f)$) to Eqns. 2.5 and 2.6.

These approximations require the turbulent fluctuations to be much faster than the shock response ($\tau_R \gg \tau_u$), operate over a considerable time scale ($t \gg \tau_u$) and ($f \ll 1/(s\pi\tau_u)$).

$$\overline{x^2} = \overline{u^2}\tau_u\tau_R \quad (2.4)$$

$$R_x(t) = e^{-t/\tau_R} \quad (2.5)$$

$$\frac{G_x(f)}{\overline{x^2}\tau_R} = \frac{4}{1 + (2\pi f\tau_R)^2} \quad (2.6)$$

Touber and Sandham [108] proposed a very similar closed quantitative model capable of capturing the low frequency response of the shock foot motion utilizing only upstream flow properties and the impinging shockwave. Their derivation of shock foot unsteadiness from the momentum integral equation provides both the mathematical and physical basis for Plotkin's model. Turbulent spectra studies by Poggie and Smits [83, 81] further validated this stochastic model choice. Work by Threadgill [106] compared unsteady SBLI features across a variety of two-dimensional SBLI configurations and found good agreement with Plotkin's model.

More recent work by Jenquine et al. [45] in 2023 provides a possible physical basis for the model wherein mass imbalance in the separation bubble via shear layer entrainment is suspected to be the driving mechanism of SBLI. The study utilizes fast response pressure sensitive paint (PSP) on an axisymmetric compression ramp with an incoming turbulent boundary layer to isolate maximum and minimal pressure fluctuations. Cross-correlation and cross-coherence mapping of spanwise distributions of pressure fluctuations revealed similar time scales between the duration of the separation bubble pulsations and variations in the inflow momentum. In general, the separation bubble pulsations showed a gradual response to variations in the global inflow momentum and subsequent mass imbalance in the system. However, rapid upstream/downstream separation shock motion was observed in response to acute variations of the mass imbalance in the separation bubble and the precursory global momentum fluctuations only when they approached zero. Hence, it gives physical meaning to

flow visualization of SBLI separation bubble motion, most commonly displaying separation bubble pulsations or a breathing motion with intermittent rapid separation shock motions.

The selective dynamical system model has given insight into legacy data and prompted researchers to carefully consider experimental parameters and analysis methods when comparing data sets as the dominating mechanisms and their impacts, especially the low frequency unsteadiness associated with the separation bubble, are still not properly discernible.

2.3 Transitional Interactions

As shock strength and the local Reynolds number increase (using the boundary-layer thickness at the separation location as the scaling in the Reynolds number), laminar interactions are more likely to become transitional. Shockwave / boundary-layer interactions in transitional flow are more scarcely discussed in the literature than turbulent shockwave / boundary-layer interactions, with early work beginning in the late 1950s. The focus of early transitional studies was to understand the heat-transfer characteristics of laminar-separated flows over nose tips for reentry vehicles [13]. The literature covering transitional interactions up to 1993 was encapsulated in a review by Knight [52] and again in 2018 [51]. Transitional SBLI research was later resumed due to their characteristic intense pressure and thermal loads, which are much larger than their turbulent counterparts, making them a point of interest for predicting maximum loading. For cylinder shock generated SBLI, transitional interactions also operate over relatively more prominent areas of influence with length scales between that of laminar and turbulent interactions, making their overall influence a severe design challenge to engineers [75]. The accurate prediction of thermal and pressure fluctuation magnitudes, as well as an approximation of transition location and interaction range, are of vital importance to future hypersonic technologies.

In order to create an accurate model for transition that is capable of approximating these variables, one must first understand the transition process and how energy is created, stored, and dispersed in hypersonic systems. From this understanding, the transition process can be seen to occur through transition waves, where low-amplitude boundary-layer disturbances

can feed the growth of instabilities in the flow field, or spot-type transition, where high-amplitude freestream disturbances cause bypass transition and their formulation of turbulent spots can superimpose on the SBLI itself.

It should be noted at this time that a new categorization of transitional interactions has been established by Threadgill [107] based on the presence of intermittency in the incoming boundary layer to rectify discrepancies in transitional SBLI behavior. Intermittency is defined differently for boundary layers and SBLI. Intermittency in the boundary layer refers to the percentage of time a flow is turbulent flow [92]. In SBLI, intermittency is related to the separation and reattachment of flow. Transitional SBLI are considered to be upstream-transitional when intermittency is detected in the incoming boundary layer or interaction-transitional if intermittency is detected only in the separation bubble. This categorization method is based on a review of turbulent interactions and their control from 1985 by Delery [22]. Delery noted two categories of turbulent interactions in his review, the first type acting on incoming boundary-layer properties before the shock region and those with only a local response to the shock foot itself.

2.3.1 Turbulent Intermittency

A key variable in the transition from a laminar to a turbulent boundary-layer state is turbulence intermittency. Intermittency is defined as the discontinuous generation of large velocity changes and other gradients in turbulent fluids. More simply, it represents the probability that the flow is turbulent at some given location. Initial transition modeling efforts excluding intermittency resulted in approximations that only provided the transition onset location but not its length. Such limitations drove the development and implementation of the intermittency energy transport equations in turbulence models [70]. This highlighted the importance of understanding how large-scale quantities, such as energy, are transported via turbulence to the smallest eddies, where dissipation can occur.

Initial work by Spiegel [100] in 1972 instituted the first steps in developing a model capable of insight into the “qualitative nature of intermittency” and investigated the origins of intermittency that would lead to the understanding of energy modes. At this time, turbulence intermittency was seen as a two-fluid model comprised of turbulence packets

surrounded by a laminar flow field that would dampen inhomogeneities. Spiegel mentioned a notable “separation in wavenumber space between the large and small scales” whose “gap [in order of magnitude] often plays a decisive role in these kinds of models.” [100]

Hence, the study of intermittency also requires differentiation between large-scale, small-scale, and edge intermittency due to the multiscale phenomenon of turbulence. Large-scale intermittency, such as those visualized in the freestream, are the main flow eddies and, rather obviously, have no need to account for viscosity effects. They are much weaker than their small-scale counterparts and operate on a global scale. On the other hand, small-scale or even microscale intermittency is, according to Mahrt [69], “where the spatial variation of the fine-scale velocity gradients and dissipation are organized by individual main turbulent eddies.” Here, the eddies are viewed as impulses of activity when turbulent signals are filtered at high frequencies. Often, the strongest small-scale velocity gradients are concentrated at the sharp edges of the main eddies, like that of the outer fluctuating boundary layer of transitional and turbulent flows. This type of turbulent intermittency is referred to as edge intermittency or microfronts and is believed to be responsible for a large portion of the total dissipation of energy despite the minute total volume of the turbulent fluid it resides in [5]. Moreover, Mahrt [69] reports that “the eddy fronts may [also] dominate the total transport” of the system.

In general, the model of choice for energy transport in the boundary layer is the turbulent cascade, in which eddies become stronger as they decay to smaller sizes. In 2007, Jimenez [47] noted, “when the physical consequences of the cascade were first explored by Kolmogorov, he assumed that the process was complex enough for the eddies to lose all the memory of their previous histories and that their properties after each cascade step could be described by purely random distributions.” Therefore, eddies of such a small scale will not be affected by large-scale inhomogeneities and eddies that are too large for viscosity to be of significant consequence. In more simplified terms, eddies of a specific size only interact with other eddies of a relatively similar magnitude, and all interactions between eddies with a significant difference in size occur through a series of small energy cascade steps.

It is worth bearing in mind that the turbulent cascade, though generally accepted for most energy dissipation models, is not always the best representation of the turbulent

intermittency on certain scales nor for certain geometric configurations where the flow may be driven by other variables. One such example is the log region of a transitional boundary layer. Jimenez [47] noted that in the case of a decaying two-dimensional model, turbulent structures would block the classic cascade process by the consolidation of the vortical forces in stable vortex cores, resulting in a highly intermittent entropy distribution that is not well modeled via a cascade or by a multifractal. Moreover, in a three-dimensional case, which tends to be milder due to the three-dimensional relief effect, data fits favor a mixed multiplicative-stochastic cascade model [47]. This means that specific data sets can be collapsed and compared across facilities should the underlying intermittency driving mechanism be the same.

2.3.2 Transition Process

The naturally occurring boundary-layer transition process begins with the receptivity process wherein the acoustic, entropic, and vortical disturbances in the freestream may be allowed into the boundary layer and excite existing instabilities. Primary instabilities are superimposed onto the stable laminar base flow. These instabilities grow linearly until their amplitude is sufficient to influence the overall mean flow field and possibly establish a new steady-state. The amplification process via small disturbances either feeds the exponential growth of eigenmodes or the nonmodal growth of optimal disturbances, or it creates a nonmodal response to forcing. The new steady-state or quasi-steady-state behavior is then acted upon by secondary instabilities whose nonlinear growth rapidly accelerates the transition process. Secondary instabilities, three-dimensional waves, and vortex structures are then superimposed on the flow and lead to the formation of turbulent spots. The breakdown of the instabilities is the final stage of the transition process. Further downstream, these turbulent spots coalesce, and a fully turbulent boundary-layer state results [93, 95].

In the boundary-layer transition from laminar to turbulent states, the final stages can be associated with the inception and spreading of turbulent spots, and the frequency of their passage at a given streamwise location in the flow field is defined as intermittency. Furthermore, the appearance of these turbulent spots is both temporally and spatially irregular and is theorized to be the byproduct of linear instabilities after the breakdown

process or a result of a bypass mechanism [46]. It should be noted here that bypass transition is characterized by a distinct bypass of the linear and secondary instability stages of disturbance growth. More specifically, transition does not emanate from exponential instabilities; instead, it is considered to emanate from nonmodal growth mechanisms. Should bypass transition occur, the boundary layer becomes receptive to only low frequency disturbances in the freestream that amplifies high-amplitude streaks. Bypass transition concludes similarly to natural transition with the production of turbulent spots just before a fully turbulent boundary layer is established [66].

2.3.3 Theory of Spots

In 1985, Narashima [74] completed early work on what would be later coined the “theory of spots.” These turbulent spots are localized patches of turbulence with vortical structures, boundary-layer parameters, and energy transport properties equivalent to fully-developed turbulent boundary layers [48, 91, 116]. The geometry of these turbulent spots is arrowhead in shape, and their spanwise spreading rate is predominantly controlled by compressibility. Computational studies by Sandham [90] observed an increased lateral growth as turbulent spots pass through a separation bubble.

Narashima’s early hypothesis was held for depicting the distribution of intermittency in flow fields with one- to three-dimensional geometries, incompressible flow fields up to hypersonic speeds, and in both constant and varying pressure conditions. Moreover, he concluded that turbulent spot geometry was predominantly affected by favorable pressure gradients in the boundary layer or sub-transitions within the transition region. Narashima, at this time, also noted a link to spot production rate and the local boundary-layer thickness and viscosity [74].

These same principles were observed again in a study by Marxen and Zaki [70], who investigated the turbulent characteristics associated with the early intermittent region of a transitional boundary layer. The two studied these characteristics from direct numerical simulations completed for the boundary layer over a flat-plate model that had experienced bypass transition. In this experiment, the skin friction coefficient was found to be proportional to the intermittency variable in the area occupied by turbulent spots.

Unconditional averages and turbulent ensemble averages found a high-intensity skin friction outline around the turbulent spots. The noticeable fluctuation in skin friction values was much lower than the approximated background coefficient of skin friction. Correlation analysis of the turbulence production rate shows that these values “at the end of the transition region [...] decay towards the expected value downstream as the turbulence production rate approaches that of a fully turbulent flow.” [70]

The enhanced turbulence production rate can be attributed to higher shear stress in that region, based on the wall shear stress relation to the overall power input for accelerating the mean flow field and the dissipation rate of the mean-flow profile. Once the generated turbulent spots are of ample size, they are seen to share the same physical behaviors of a fully turbulent boundary layer and are best described as a localized packet of turbulence. A diagram of the laminar boundary layer breakdown into a fully turbulent flow, including the coalescing of turbulent spots in the final stage of the transition process, is depicted in Fig. 2.2. Note at the downstream and lateral edges, the constrained turbulence statistics dramatically change. Elevated values of the root-mean-squared streamwise perturbation velocity and turbulent kinetic energy production, as well as high pressure and turbulent fluctuations, are observed. Downstream of the spot, there is also a low-pressure region that increases the overall pressure change felt by the advancing structure [70]. A schematic and general characteristics of a turbulent spot are displayed in Fig. 2.3.

The study of spot propagation and its relation to intermittency continued in a series of experiments completed by Casper et al. [10, 11] using a 7° cone model tested at Mach numbers of 5, 6, 8, and 14 at Sandia National Laboratories’ Hypersonic Wind Tunnel and the Boeing/AFOSR Mach-6 Quiet Tunnel at Purdue University. Initial experiments utilized high frequency pressure sensors and high-speed schlieren [10], as well as thermocouples [11], to study the growth and breakdown of boundary-layer disturbances into turbulent spots. Observed boundary-layer intermittency was directly tied to the appearance of these turbulent spots. In successive experiments, the system was artificially forced using an electrical discharge perturber [79]. Results showed the turbulent spot convected along edge streamlines and an increase in intermittency due to the forcing of the artificial disturbances. High frequency pressure fluctuations peaked just before the appearance of the turbulent

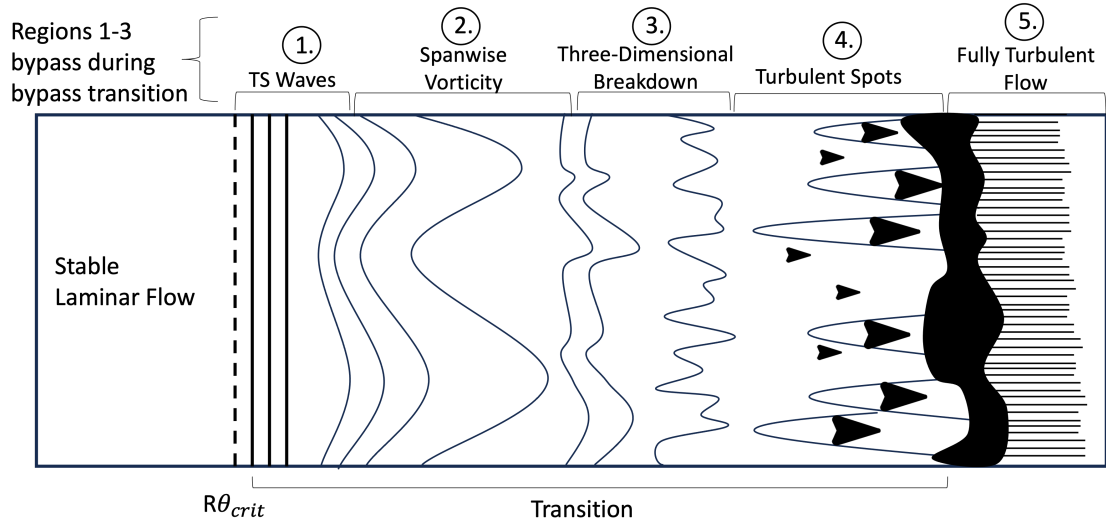


Figure 2.2: Transition to turbulent flow diagram. Adapted from Gen et al. [33].

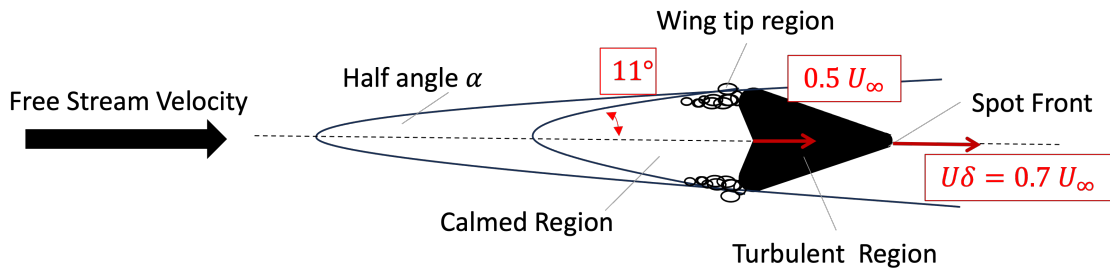


Figure 2.3: Turbulent spot schematic. Adapted from Redford et al. [85].

spots due to the growth of second-mode instabilities, and a rise in low frequency pressure measurements was observed after the transition to a fully turbulent boundary layer.

Experiments in 2019 by Casper et al. [12] included a composite panel insert and accelerometers on the cone configuration. The system was artificially forced using an electrical discharge perturber at both periodic and randomly distributed repetition rates of 2.11-12.7 kHz. Elevated vibrational response in a naturally occurring transitional boundary layer was directly linked to the intermittence forced by turbulent spots, and their distribution determined which natural structural frequencies were excited. They found that the vibrational response was dependent on both the forcing frequency as well as the flow direction in the periodically forced case. As such, the wall-normal and streamwise responses were easily excited; moreover, the spanwise forcing occurred at high frequencies due to the turbulence within the spots. For the case of random spot generation, an average burst rate frequency was calculated, and the recorded oscillation frequencies of the panel at or above this average burst rate were excited by the transitional flow [12].

Chapter 3

Methodology

3.1 Wind Tunnel Facilities and Conditions

3.1.1 UTSI Mach 4 Ludwieg Tube

The UTSI Mach 4 Ludwieg tube hosts a large $609 \text{ mm} \times 609 \text{ mm}$ (24 in $\times$ 24 in) test section with a variable per length freestream Reynolds number range from $3.48\text{-}52.48 \times 10^6 \text{ m}^{-1}$ ($1.0\text{-}16.5 \times 10^6 \text{ ft}^{-1}$). The facility has an approximate run time of 105-120 ms at steady flow conditions, and a schematic of the Mach 4 Ludwieg Tube is shown in Fig. 3.1. Additional facility details may be found in the tunnel characterization conference dissertation [53]. Nominal run conditions for the facility are tabulated in Table 3.1 below for reference. Note that the nominal pressures were adjusted per run to augment the freestream Reynolds number, and these values are tabulated on a run-to-run basis in the next section and displayed in Table 3.2.

3.1.2 UTSI 6° Cone Model

As originally discussed in Lester, et al. [61], the present work builds upon the results of Lash et al. [58] and Combs et al. [20, 19] by studying the dynamics of cylinder-induced transitional and turbulent SBLI on a 6°-half angle cone in Mach 4 flow. The details of the model configuration and a CAD rendering are shown in Fig. 3.2. The model has an aluminum body and a steel nose with a tip diameter of less than $250 \mu\text{m}$ to ensure sufficient sharpness

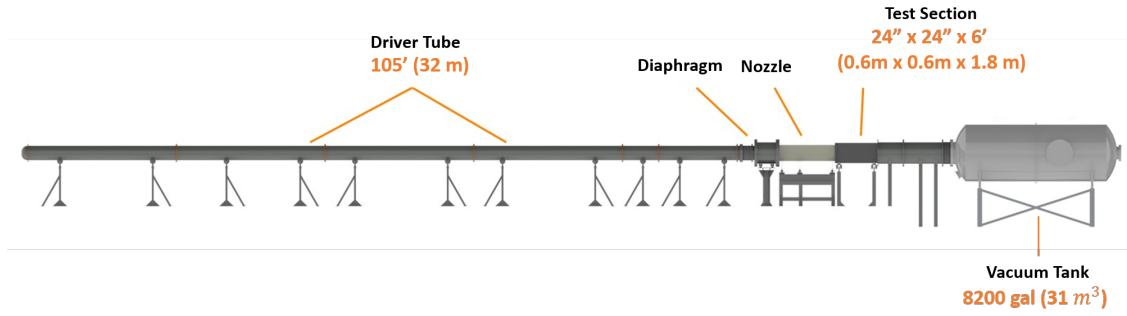


Figure 3.1: Diagram of the UTSI Mach 4 Ludwieg tube.

Table 3.1: Summary of nominal tunnel conditions used in the experiments. Freestream values were calculated using the isentropic relations.

Facility	T_0 [K]	M_∞	T_∞ [K]	U_∞ [m/s]
UTSI Mach 4	287	$4.00 \pm 0.5\%$	68.5	663.4

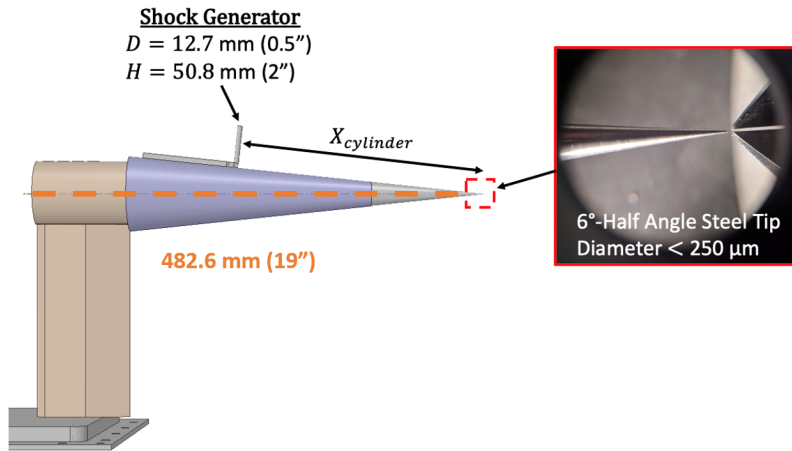


Figure 3.2: CAD model and dimensions of a 6° cone.

of the leading edge. The edge Mach number is calculated to be $M_C = 3.76$ using a Taylor-Maccoll analysis. Having an axisymmetric model is favorable for schlieren imaging, as it helps remove or avoid edge effects that are present with a flat plate model due to schlieren's path-integrated nature. Additionally, since the prior work of Lash et al. [58] demonstrates that the dynamic boundary-layer thickening and upstream influence shock are related on a 10° half-angle cone in a Mach 2 freestream, there were no *a priori* concerns about using such a model in the current work. The shock generator installed on the cone is a cylinder with a diameter of 12.7 mm (0.5") and a height of 50.8 mm (2"). The height of the boundary layer on the model without the shock generator was found to be less than 2 mm. The cone is installed with a 0° angle of attack, and by relocating the shock generator to various positions downstream of the leading edge, the boundary-layer state (and the interaction dynamics) can be varied [61].

3.2 Experimental Techniques

3.2.1 High-Speed Schlieren

Schlieren imaging is a standard diagnostic tool in ground testing providing qualitative flow visualization and, more recently, with the rise of high-speed cameras, the technique is capable of extracting quantitative results for fluid dynamics research [97, 18]. The origins of this diagnostic technique date back to the work of Robert Hooke in the mid-1600s from whence it derives its name from the visualized veins (streaks, *schliere*) in glass. In the early 1900s, Toepler advanced the ease of use and sensitivity of the system, catching the attention of Ernst Mach, who first introduced it to the aerospace industry, where it has since been a staple [97]. The path-integrated diagnostic technique is based on the Gladstone-Date relation, displayed below in Eqn. 3.1. Here, n is the index of refraction, $n - 1$ denotes the refractivity, k is the Gladstone-Dale coefficient, and ρ is the local fluid density in compressible flows [2].

$$n - 1 = k\rho \tag{3.1}$$

UTSI's TALon Mach 4 Ludwig tube is instrumented with an ultra-high-speed schlieren/shadowgraph system that is capable of acquiring high-resolution schlieren images at substantially high framerates. The system is aligned into a modified Z-type schlieren setup, which includes a flat plate mirror to redirect the received image into the camera's sensor due to spatial constraints around the tunnel. A schematic of the setup can be viewed in Fig. 3.3. A Photron FASTCAM SA-Z high-speed camera is synchronized to the pulsed illumination provided by a high-powered LED built in-house. In this work, the Photron FASTCAM SA-Z is equipped with a 300 mm zoom lens that acquires images at a frame rate of 200 kHz in the Mach 4 wind tunnel. A total of 10,240 frames during the steady-state run time of the Mach 4 Ludwig tube were captured at each streamwise cylinder shock generator station. Table 3.2 summarizes the individual test cases and their conditions. The custom-built LED light source was designed after the work of Willert et al. [115] and can provide short-lived pulses of light at rates up to 400 kHz. The highly light-sensitive CMOS imaging sensor captures images with LED pulse widths as short as 1 μs or less, which is short enough to effectively freeze the high-speed flow features in instantaneous images. The two primary mirrors in this Z-type configuration have a diameter of 12.5" and a focal length of 100" ($f/8$). A horizontal knife-edge cutoff ($\partial n / \partial y$) was utilized to ensure boundary-layer visualization, where ($\rho \frac{\partial p}{\partial y} = 0$), for data analysis purposes. With the high-speed camera and long focal length mirrors, this system provides sufficient resolution and sensitivity to capture the dynamics of the shockwave / boundary-layer interactions.

The different cylinder shock generator stations will be denoted as X_{cylinder}/D throughout the dissertation. Note that the nomenclature chosen has the same cylinder shock generator diameter for all cases, so variation in the overall value is only a byproduct of the cylinder shock generator's position from the cone's leading edge. The nomenclature chosen does not include several other useful parameters, such as the Reynolds number or other length scales that are known to collapse the SBLI information. This choice provides a nondimensionalized number that captures test cases between different diagnostics and includes repeat tests where the freestream conditions may slightly change. If too many similarity parameters were included, a different reference value would be seen for each individual run, requiring the

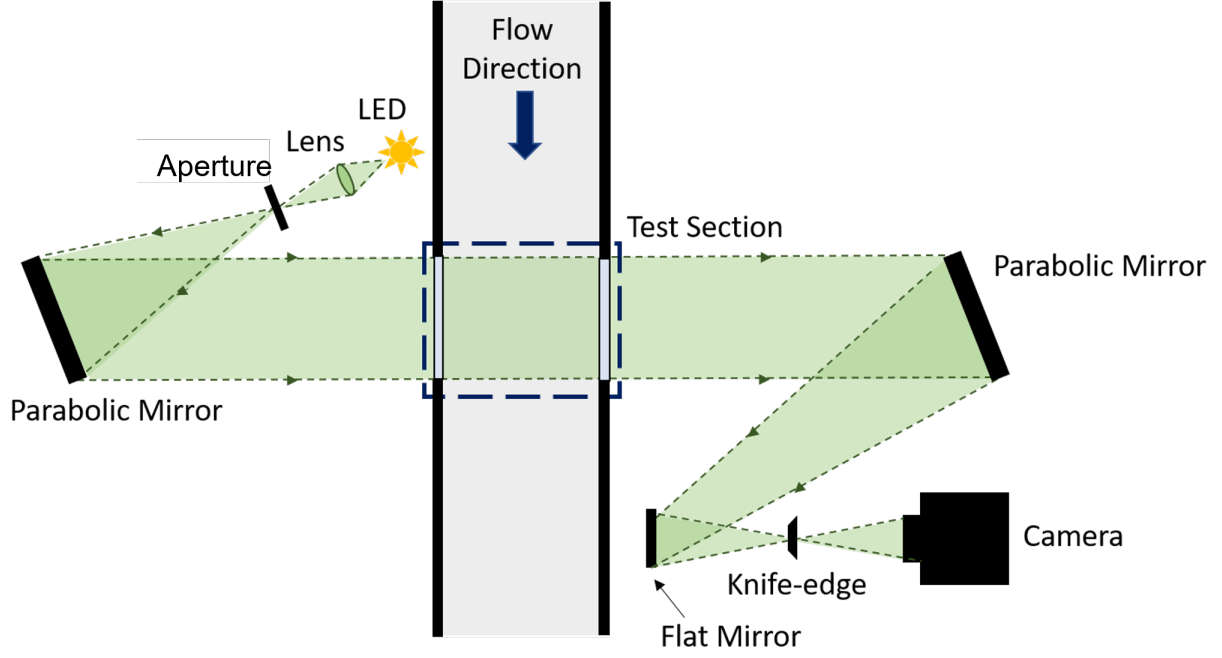


Figure 3.3: Schematic of the modified Z-type schlieren configuration used in this study.

Table 3.2: Summary of wind tunnel conditions used in the schlieren experiments. X_{cylinder} refers to the location of the cylinder relative to the leading edge of the 6° cone (along the cone surface).

Run Number	P_0 [psi]	Re_∞/x [$\times 10^6 \text{ m}^{-1}$]	X_{cylinder}/D	Interaction
15	57	19.03	12	Transitional
16	56.64	18.91	12	Transitional
17	58.10	19.39	13	Transitional
18	58.19	19.42	13	Transitional
19	57.27	19.12	18	Turbulent
20	55.54	18.54	18	Turbulent
23	57.27	19.12	11	Transitional
24	56.82	18.97	11	Transitional

reader to continuously reference the experimental conditions table. The fundamental point of the parameter is to provide clarity during the discussion portion of this dissertation.

3.2.2 Pressure-Sensitive Paint

Fast-response, Unsteady Pressure-Sensitive Paint, or uPSP, is used to verify the dynamic SBLI information found in high-speed schlieren imaging and its subsequent processing methods. This diagnostic technique operates based on the oxygen quenching rate of specific luminescent molecules embedded in the oxygen-permeable paint. PSP has the ability to offer better spatial-resolution and cost advantages compared to point-sensor measurements like pressure probes on a model surface [64]. The technique’s fast-response rate allows for a finer frequency resolution necessary to properly resolve low frequency content. Moreover, it has been used to extract spectral content from SBLI on numerous occasions [27, 29, 21, 26, 87, 3, 98]. In these experiments, a polymer ceramic paint developed by Innovative Scientific Solutions Inc. (ISSI) is used to achieve an estimated 20 kHz response rate that results in frequencies up to 10 kHz that may be reliably resolved from this diagnostic technique based on the Nyquist theorem. However, trusted and experimentally verified frequency measurement ranges in pressure sensitive paint data are known to be variable based on the applied base and luminophore paint layer thickness. The paint thickness and associated response rate across the model are difficult to quantify but recent research has noted a decay in the paint’s coherence at higher frequencies. Frequency content and the associated coherence were noted to be between 90% and 100% for frequency content between 1 and 4 kHz. At 5 kHz this amplitude response drops to about 80%, a coherence rate of 68% was noted at 7 kHz, and drops below 40% at 10 kHz [23]. Most of the frequency content of interest in this study is expected to fall in the 1-5 kHz range as is typically documented for characteristic SBLI frequency content.

The ISSI paint consists of an oxygen-permeable binder as the base coat, the porosity of which allows for the rapid diffusion of oxygen and an oxygen-sensitive top coat with platinum (pentafluorophenyl) porphyrin (PtTFPP) luminophore molecules [96, 98]. During the steady-state run time, a high-powered UV light at wavelengths of $\sim 390\text{-}450$ nm emits photons that are absorbed by the luminescent molecules, transitioning them to an excited

singlet energy state. The molecules then emit red-shifted photons in an effort to return to ground state, resulting in the observed bright pink luminescence at about 650 nm. During this interaction, a process known as oxygen quenching is observed; this is the physical method by which PSP is exploited to correlate its fluorescence rate to the local concentration of oxygen and, by extension, pressure. As oxygen interacts with the luminescent molecules, the transition to the ground state becomes non-radiative, and the rate at which this process is conducted is dependent on the local partial pressure of oxygen [96].

Therefore, higher oxygen quenching rates result in a lower observed intensity and vice versa. This change in intensity is recorded via a scientific high-speed camera after passing through a long-pass filter that eliminates the excitation light over UV wavelengths. The approximated change in pressure is based on the intensity ratio between a wind-off, or ambient set of images, and a wind-on state. A schematic of the process is displayed below in Fig. 3.4, and a setup image of the camera, LED array, and painted model installed in the Mach 4 Ludwig tube is shown in Fig. 3.5. Each of the PSP test conditions are documented in Table 3.3. For these experiments, a Photron AX-200 high-speed camera is used in combination with a Nikon AF zoom lens featuring a 50 mm focal length. Using a larger depth of field allows the surface of the 6° cone model that is not parallel to the camera lens and LEDs to retain a higher focus quality over a larger extent of the model’s surface. It should also be noted that the camera’s view of the cylinder shock generator is slightly lower than the centerline of the model. With the sidewall geometry constraints and relative model angle, a shadow is cast at the base of the cylinder’s front face, where dynamic content from the SBLI is expected to be seen. Adjusting the camera’s angle to be just below centerline removes this shadow from the region of interest, as can be seen in the unaltered image from the AX camera below in Fig. 3.6, with the slight angle in the cylinder shock face that is obscured in the processing phase.

The conversion from an intensity captured by a camera to a calibrated pressure is completed by using an intensity ratio between the wind-on images of the wind tunnel run and wind-off images taken at a vacuum pressure close to the expected static pressure of the run. These intensity ratios are used to create a pressure calibration curve derived by collecting the paint’s emitted response at several pressures during the pre-run tunnel evacuation, where a

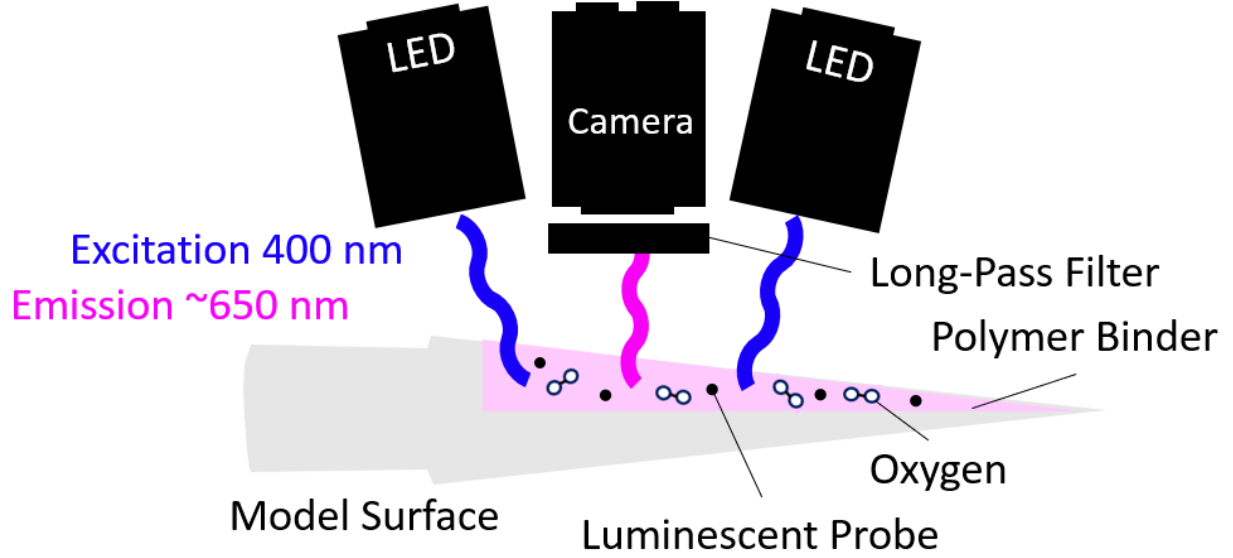


Figure 3.4: Schematic of a typical PSP using ISSI paint, not to scale.

Table 3.3: Summary of wind tunnel conditions used in the PSP experiments. X_{cylinder} refers to the location of the cylinder relative to the leading edge of the 6° cone (along the cone surface).

Run Number	P_0 [psi]	P_c	Re_c/x [$\times 10^6 \text{ m}^{-1}$]	X_{cylinder}/D	Interaction
1	57.02	0.52	25.42	11	Transitional
2	57.79	0.53	25.77	12	Transitional
3	55.81	0.51	24.88	13	Transitional
4	56.65	0.52	25.25	18	Turbulent

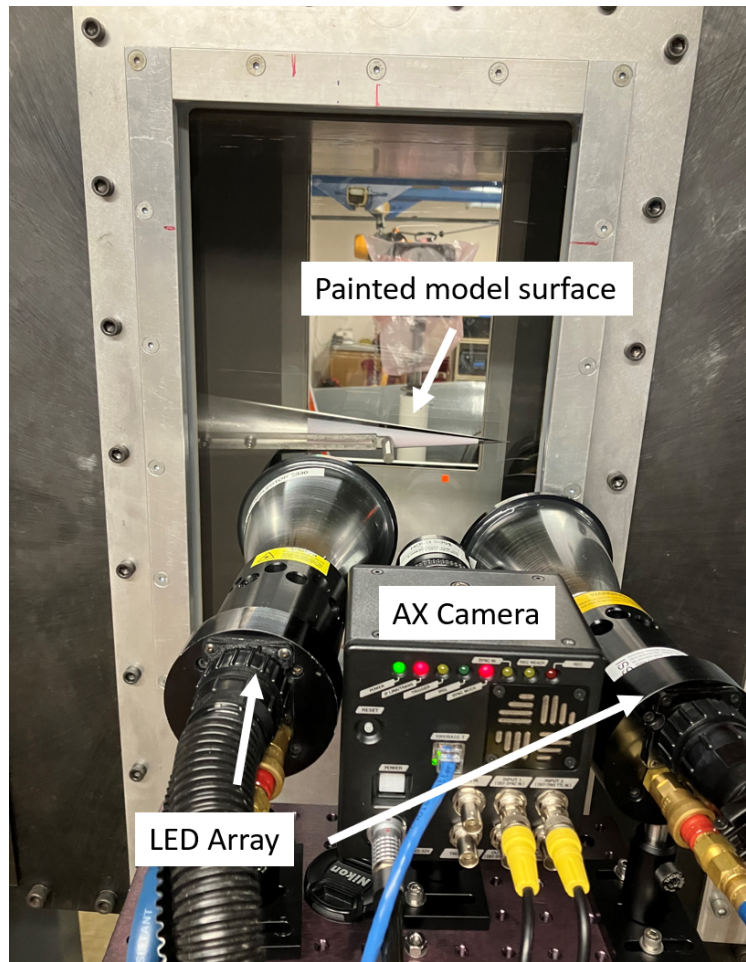


Figure 3.5: PSP setup in the Mach 4 Ludwig Tube at UTSI with the cone model mounted in the test section.

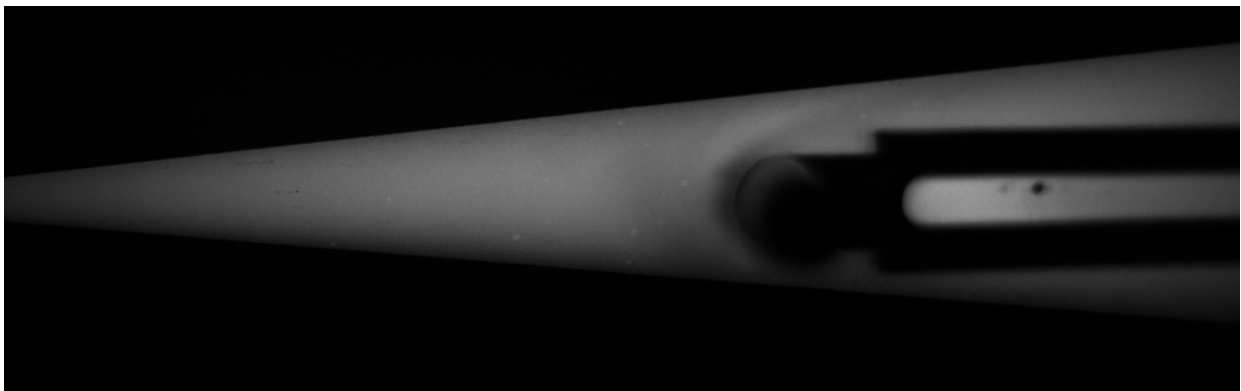


Figure 3.6: Raw image from the AX Camera of the 6° cone painted with pressure-sensitive paint.

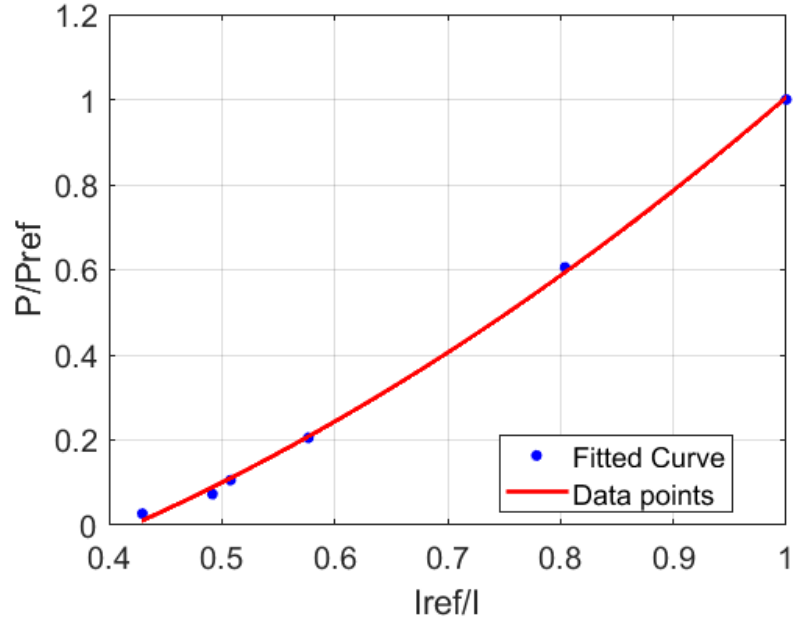
Druck pressure calibrator is used as the measurement standard. A Druck DPI 612 Flex Series Pressure Calibrator was utilized in the experimental setup. The calibration curves created for this set of experiments are shown in Fig. 3.7. Two calibration curves were used in this analysis, where a second calibration curve had to be created for the final shock generator station due to a repositioning of the camera. Approximately 2,400 frames were captured during the steady-state flow events at an acquisition rate of 20 kHz. All data are presented in the form of their local surface pressure divided by the experimental cone pressure P_C upstream of the interaction’s footprint. The experimental cone pressure P_C is calculated using an average of 100x20 pixel range centered on the cone at the most upstream available location in the frame for the entire time series. Theoretical conical flow relations predict a P_1/P_c ratio of 0.72 for a Mach 4 freestream.

It is worth bearing in mind that the ISSI pressure-sensitive paint does exhibit some temperature sensitivity. However, due to the limited run time of the Ludwig Tube facility, there is no significant change in temperature throughout the steady-state run time, and the stagnation temperature is assumed to be 298 K or roughly room temperature. For these experiments, only the change in pressure, based on the oxygen quenching rate and luminescence intensity, will be reported. The direct, instantaneous pressure values themselves are not reported, so as to bypass issues associated with measurement accuracy typically seen in pressure-sensitive paint experiments.

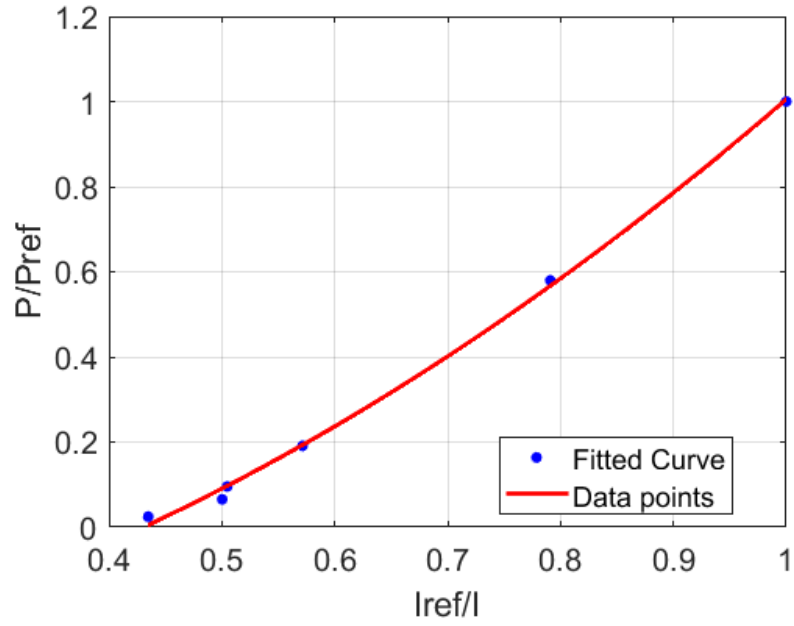
3.3 Pressure-sensitive Paint Uncertainty

This section details an uncertainty analysis of the pressure-sensitive paint. Pressure-sensitive paint errors are generally expected to be a function of several factors, including: photo-degradation, camera noise, illumination distribution, model movement, calibration error, and temperature effects, to name a few. This analysis will be segmented to evaluate systematic and random uncertainties before calculating and tabulating the setup’s total uncertainty.

Within the random uncertainty analysis, specific aspects of the experiment must be considered, such as the inherent camera sensor noise, luminophore concentration, paint thickness, optical distortions via flow phenomena, tunnel movement / vibrations during



(a) Calibration curve for $X_{\text{cylinder}}/D = 11, 12, \text{ and } 13$



(b) Calibration curve for $X_{\text{cylinder}}/D = 18$

Figure 3.7: Pressure-sensitive paint calibration curve

the run, and LED fluctuations. The equations and background information related to this uncertainty analysis are reviewed in detail by Wheeler and Gangi [113]. Moreover, portions of this methodology are commonly utilized in PIV uncertainty analysis [114] in which positional jitter, out-of-plane motion, and camera pixel noise are considered appropriately captured. For the captured pressure-sensitive paint data, the pixel resolution was 0.0171 cm/pixel for the $X_{\text{cylinder}}/D = 11, 12,$ and 13 cases and a 0.0166 cm/pixel resolution for the $X_{\text{cylinder}}/D = 18$ case.

To calculate the random uncertainty, the student's t -distribution is utilized and evaluated at a 95% confidence level, as is recommended generally for uncertainty analysis [113]. Here, the random uncertainty, denoted as R , may be calculated as seen in Eqn. 3.2 below. In this equation, t represents the student's t -distribution at the chosen confidence level and sample size, x is the variable being measured, n denotes the number of times x is sampled, and σ_x is the standard deviation in the variable x measured over the full sample size.

$$R_{\bar{x}} = \pm t \frac{\sigma_x}{\sqrt{n}} \quad (3.2)$$

The standard deviation component of the equation is captured from the wind-off images during tunnel pull-down when the test section and vacuum tank are being evacuated; therefore, this captures the random uncertainties in the experimental configuration without any flow field component influence. In Fig. 3.8, the calculated pressures for one reference pixel in the wind-off images are plotted. Several additional lines at constant y -values are shown as well. One of the lines showcases the mean pressure while the others illustrate statistical and uncertainty bounds with respect to this mean. The outer bounds are plotted two standard deviations from the mean value, encapsulating an estimated 96% of the data. The narrower bounds represent the uncertainty in the average value, $R_{\bar{x}}$, with a 95% confidence level. This same data is displayed in the form of a histogram to visualize the data from a different perspective in Fig. 3.9. Here, it is clear that approximately 1/3 of the data are grouped within the same bin as the mean, with a bin width of 0.002 psi. The uncertainty in the mean scales to a maximum of 36% of the mean value, or about 0.19 psi.

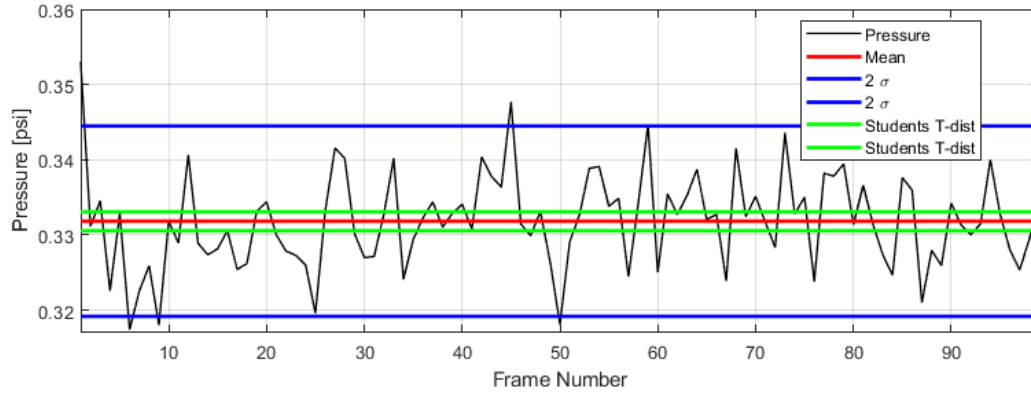


Figure 3.8: Pressure trace measurements and statistical uncertainty for the $X_{\text{cylinder}}/D = 18$ case over a single pixel for wind-off data at $x/d = -3.5$ and $y/d = 0$.

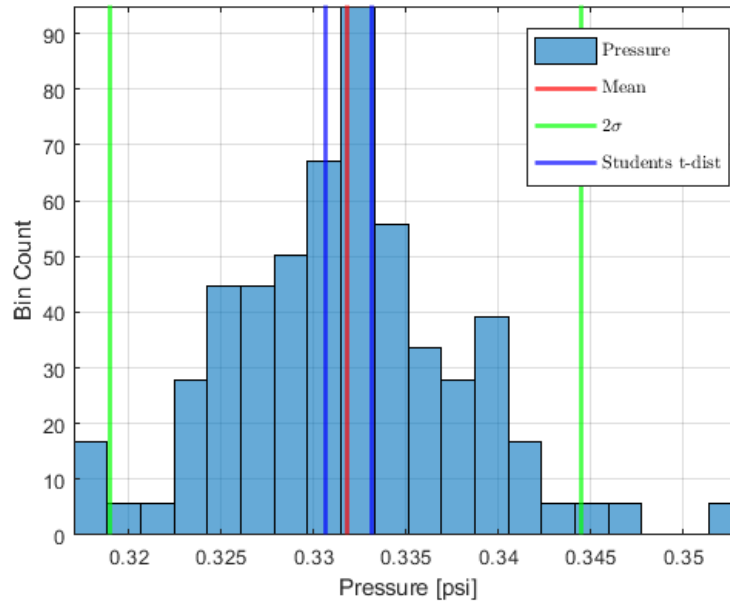


Figure 3.9: Histogram of pressure measurements and statistical uncertainty for $X_{\text{cylinder}}/D = 18$ case over a single pixel for wind-off data at $x/d = -3.5$ and $y/d = 0$.

One source of random errors is related to the tunnel’s movement during the run. Over the run time, the PSP images shift in the field of view by approximately 25 to 35 pixels based upon the results of applying an image stabilization algorithm. Using the camera resolution, this results in a maximum displacement of 0.6 cm in the data. For reference, the pressure-sensitive paint is evaluated typically from an x/d location of 0 to -3.5, or 4.45 cm. This motion results in variations in the illumination (excitation) of the PSP, thereby potentially altering the emission levels in proportion. Overall, however, the change in intensity from the tunnel movement is much less than the intensity variation across the model.

Systematic uncertainty for pressure-sensitive paint can best be approximated by understanding and quantifying errors in the calibration process, using the curve fit to convert the intensity data to pressure, and comparing these values with the pressures measured by the calibration standard. This process could also be performed with data acquired during a test, so long as there is a well-calibrated sensor nearby and no significant fluctuations exist in the area of interest. The reduction of pressure sensitive paint data is performed with an intensity ratio between the data point, a wind on image, and a reference point, a wind off image. The paint’s emission intensity is captured via a high-speed camera, and this intensity ratio is able to negate the need to determine an excitation light intensity [88]. This relationship is achieved through the Stern-Volmer coefficients, and the uncertainty in those coefficients can be prominent. Often, prior calibration that may be done in a vacuum results in more significant bias errors when compared to *in situ* calibration via pressure taps on the model surface. This is expected to be a product of reducing bias error associated with temperature effects [65]. In this experiment, the temperature dependence of the ISSI pressure-sensitive paint is expected to be captured in the calibration measurements. Changes to the calibration curve based on the temperature variation are expected to be negligible based on the limited run time during which little to no heat transfer is expected, as the Mach 4 Ludwig Tube operates over a short duration (approximately 160 ms) with unheated test gas.

The systematic uncertainties expected during this experiment would include errors in the calibration curve relating intensities to pressure. The experiment’s systematic (bias) uncertainty is calculated via Eqn. 3.3 with the “true” value measurement taken utilizing the Druck pressure calibrator for the wind-off images. Pressure values of 0.323, 0.320, 0.355, and

0.330 psi were recorded for the referenced wind-off images for each cylinder shock generator station at $X_{\text{cylinder}}/D = 11, 12, 13,$ and 18 , respectively. Note that no pressure sensors were installed on the cone to provide a “true” pressure measurement *in situ*. Wind-on images are compared to theoretical surface pressures on a cone model.

$$B_x = |(P_{\text{true}} - P_{\bar{x}})| \quad (3.3)$$

To contrast the uncertainties relative to one another, the data at several pixel locations were evaluated for both the wind-on and wind-off data sets for all four cylinder shock generator stations. Two streamwise locations and three spanwise locations were analyzed, and their uncertainties are displayed in Table 3.4 as a percentage of their relative “true” pressure value captured from the Druck for the reference images. The wind-off images show less than a 1% difference in the total uncertainty compared to the “true” pressure at all centerline stations, $y/d = 0$. A maximum total uncertainty of 3.35% was observed for the $X_{\text{cylinder}}/D = 11$ case at $y/d = -0.5$ and $x/d = 0.5$. The total uncertainty at the downstream probe station of $x/d = 1.1$ for the wind-off case is not greater than 1.74%. These are notably much smaller than their wind-on counterparts, which is to be expected. Resultant uncertainties calculated in the wind-off images would be a product of geometry, camera noise, and lighting only.

The random uncertainty results for the wind-on runs at several probe locations are displayed in Table 3.5. There is a significant increase in the uncertainties in the upstream wind-on cases compared to those calculated in the wind-off images. For the three most upstream cylinder shock generator stations, a significant amount of the pressure data is expected to be a byproduct of the presence of the upstream influence. It is not a measure of the paint uncertainty, but of the unsteadiness of the UI. However, this larger uncertainty is still present in the turbulent case where only a limited number of frames were seen to be transitional in nature. Further investigation shows a significant climb in the pressure data between frames 1300 and 1450 and again between frames 1900 and 2150. This rise in uncertainty is likely associated with flow physics as well as illumination changes from the facility’s translation with respect to the stationary camera and lighting setup during the run.

Table 3.4: PSP random uncertainty at different probe locations for wind-off frames.

X_{cylinder}/D	y/d	Mean [psi] $x/d = -3.5$	σ [psi] $x/d = -3.5$	$R_{\bar{x}}$ $x/d = -3.5$	Mean [psi] $x/d = -1.1$	σ [psi] $x/d = -1.1$	$R_{\bar{x}}$ $x/d = -1.1$
11	0.5	0.3266	0.0076	0.0015	0.3243	0.0052	0.0010
11	0	0.3228	0.0064	0.0013	0.3232	0.0062	0.0012
11	-0.5	0.3123	0.0083	0.0016	0.3284	0.0083	0.0017
12	0.5	0.3189	0.007	0.0014	0.3176	0.007	0.0014
12	0	0.3173	0.007	0.0014	0.3191	0.0062	0.0012
12	-0.5	0.3102	0.0092	0.0018	0.3194	0.0084	0.0017
13	0.5	0.3573	0.0071	0.0014	0.357	0.0075	0.0015
13	0	0.3549	0.0074	0.0015	0.3543	0.0088	0.0017
13	-0.5	0.3496	0.0091	0.0018	0.3515	0.0094	0.0019
18	0.5	0.3316	0.0062	0.0012	0.3315	0.0056	0.0011
18	0	0.3318	0.0063	0.0013	0.3318	0.007	0.0014
18	-0.5	0.3315	0.0069	0.0014	0.3322	0.0076	0.0015

Table 3.5: PSP random uncertainty at different probe locations for wind-on frames.

X_{cylinder}/D	y/d	Mean [psi] $x/d = -3.5$	σ [psi] $x/d = -3.5$	$R_{\bar{x}}$ $x/d = -3.5$	Mean [psi] $x/d = -1.1$	σ [psi] $x/d = -1.1$	$R_{\bar{x}}$ $x/d = -1.1$
11	0.5	0.3503	0.0119	0.0005	0.4111	0.0415	0.0017
11	0	0.3482	0.0106	0.0004	0.4362	0.0564	0.0023
11	-0.5	0.3525	0.0187	0.0007	0.4289	0.0506	0.0020
12	0.5	0.3402	0.0184	0.0007	0.4804	0.0532	0.0021
12	0	0.3363	0.0159	0.0006	0.5178	0.0645	0.0026
12	-0.5	0.3333	0.0196	0.0008	0.4702	0.0512	0.0020
13	0.5	0.3719	0.0122	0.0005	0.5857	0.0599	0.0024
13	0	0.3801	0.0145	0.0006	0.6189	0.0707	0.0028
13	-0.5	0.3745	0.0142	0.0006	0.6089	0.0637	0.0025
18	0.5	0.3509	0.0132	0.0005	0.5355	0.0319	0.0013
18	0	0.3412	0.0152	0.0006	0.5459	0.0335	0.0013
18	-0.5	0.3541	0.014	0.0006	0.5252	0.0316	0.0013

This pressure trace is included for the centerline pixel probed in the turbulent wind-on case at $x/d = 3.5$ in Fig. 3.10.

Though the total uncertainties in these cases is greater than that in the wind-off reference conditions, it is clear that this change is significantly smaller than those associated with the physics of this flow. The total uncertainty of the measurement, W , is then calculated with the root-sum-square of the systematic and random uncertainties. Note that the total uncertainty inherits the 95% confidence level in which the values B_x and $R_{\bar{x}}$ are calculated.

$$W_{\bar{x}} = (B_x^2 + R_{\bar{x}}^2)^{1/2} \quad (3.4)$$

The calculated mean, standard deviation, random error, bias error, and total uncertainty for three different probe conditions for all four test cases are recorded in Table 3.6. These metrics are provided with dimensions of [psi]. The probe conditions cover both the wind-on and wind-off image sets at an upstream station of $x/d = -3.5$ or a downstream station of $x/d = -1.1$. The true pressure reference for the bias errors for both wind-on and wind-off conditions have been included and are denoted as P_{true} . For the wind-off images, the true pressure reference is the measured pressure value from the Druck pressure calibrator. For the wind-on images, a theoretical pressure was calculated on the cone surface based on freestream conditions.

One area of potential uncertainties not evaluated in the aforementioned analyses is associated with suspected vertical gaps that may exist at the cylinder shock generator location between the model surface and the baseline of the cylinder shock generator. The cylinder shock generator is mounted to the cone model as a cantilever beam that is secured via a pair of screws at the aft end and zip ties upstream. The cylinder is secured and inspected to be flush mounted before the experiment; however, the cylinder may bend during flow on conditions. Potential effects of the SBLI pressure distribution may be seen below by comparing Figs. 3.11 and 3.12 for a cylinder shock generator station at $X_{cylinder}/D = 13$ where later analysis found a relatively larger amount of movement in the separation shock position warranting visual inspection. These images featured unprocessed images from the acquisition at timestamps of 31.35 ms and 35.25 ms, respectively. An overlay of the images

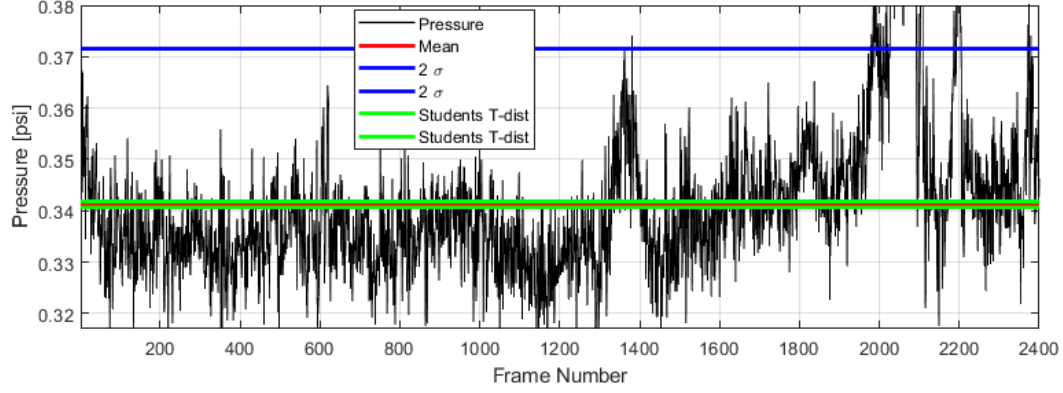


Figure 3.10: Pressure trace measurements and statistical uncertainty for the $X_{\text{cylinder}}/D = 18$ case over a single pixel for wind-on data at $x/d = -3.5$ and $y/d = 0$.

Table 3.6: PSP uncertainty metrics

Probe Condition	X_{cylinder}/D	Mean [psi]	σ [psi]	$R_{\bar{x}}$ [psi]	B_x [psi]	$W_{\bar{x}}$ [psi]	P_{true} [psi]	$W_{\bar{x}}$ [%]
Wind-On, $x/d = -3.5$	11	0.3482	0.0106	0.0004	0.1703	0.1703	0.5185	32.85
	12	0.3363	0.0159	0.0006	0.1892	0.1892	0.5255	36.00
	13	0.3801	0.0145	0.0006	0.1274	0.1274	0.5075	25.10
	18	0.3412	0.0152	0.0006	0.1739	0.1739	0.5151	33.77
Wind-Off, $x/d = -3.5$	11	0.3228	0.0064	0.0013	0.0002	0.0013	0.323	0.40
	12	0.3173	0.0070	0.0014	0.0027	0.0030	0.32	0.95
	13	0.3549	0.0074	0.0015	0.0001	0.0015	0.355	0.42
	18	0.3318	0.0063	0.0013	0.0018	0.0022	0.33	0.67
Wind-Off, $x/d = -1.1$	11	0.3232	0.0062	0.0012	0.0002	0.0013	0.323	0.39
	12	0.3191	0.0062	0.0012	0.0009	0.0015	0.32	0.48
	13	0.3543	0.0088	0.0017	0.0007	0.0019	0.355	0.53
	18	0.3318	0.0070	0.0014	0.0018	0.0022	0.33	0.68

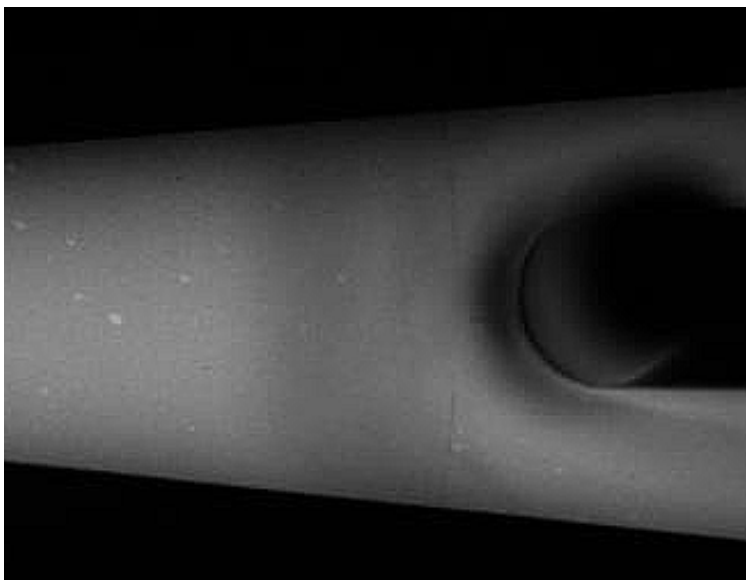


Figure 3.11: Raw image collected of the PSP, frame 1427.

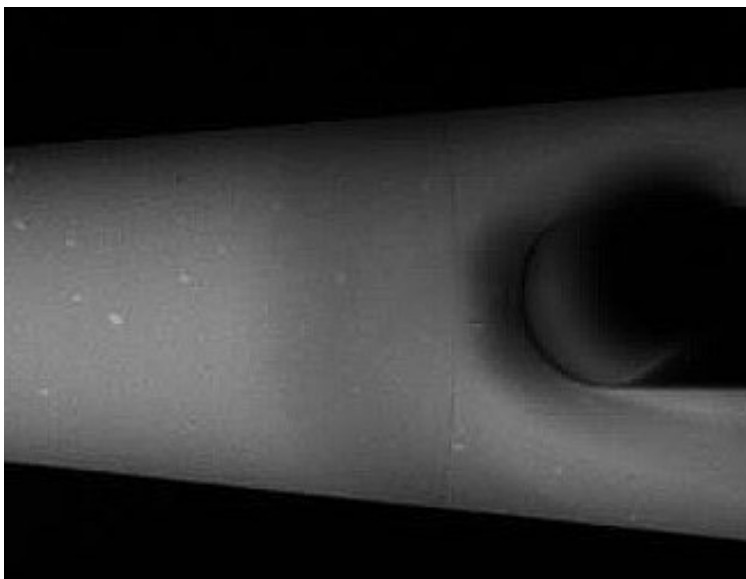


Figure 3.12: Raw image collected of the PSP, frame 1505.

and comparison of the images does not reveal any insight into whether these changes are based on SBLI characteristics or are a byproduct of the cylinder shock generator bending during low on conditions that may result in a small gap at the upstream base of the shock generator. The gap heights may affect relative pressure data but are expected to be washed out in the average and standard deviation images. The instantaneous image time series were set to avoid this timestamp. Moreover, previous experimental data sets on the variable gap heights between a cylinder and model surface for a turbulent SBLI interaction showed no influence on the captured frequency content [32].

3.4 Analytical Methodology

3.4.1 Preprocessing Stage

The Ludwig Tube is an impulse facility that translates as a response to high pressure differentials breaking Mylar diaphragms to initiate the test. As such, the facility moves slightly, causing a delayed shift in the camera’s field of view that must be corrected. The schlieren and PSP images captured in these experiments were stabilized using the `imregister` command in MATLAB with a ‘translation’ transformation type and a ‘monomodal’ optimizer. The high-speed schlieren image data sets were then background subtracted using an average of 100 images of the model before the tunnel was fired to improve the resultant image signal-to-noise ratio. The background subtracted images were then contrast adjusted, and the overall intensity was equalized across the entire run duration and with respect to each individual test. This step was completed by taking the standard deviation of a region of interest from an initial image that is then divided by the following image and so on to correct for changes in the image contrast. The intensity is adjusted by simple addition and subtraction to the desired brightness. This step was included due to facility vibrations that can change the amount of light cutoff by the knife edge before entering the camera in the schlieren system. Where possible, images used for analysis were chosen early in the steady-state run time to avoid these changes in the schlieren system’s sensitivity. Sharp dips in the standard deviation and intensity were evaluated

for disturbances and found to be associated with the passages of diaphragm fragments. The individual frames were stabilized utilizing the `dftregistration` command in MATLAB to achieve sub-pixel alignment. The row and column shifts of the high-speed schlieren data for Run 15 can be seen below in Fig. 3.13. The results of the sub-pixel stabilization exhibit similar trends as the intensity and standard deviation variations, affirming their correlation with one another. The row and column shifts for the pressure sensitive paint data for Run 3 is shown in Fig. 3.14. The pressure sensitive paint data, acquired at 20,000 fps, captures more tunnel recoil behavior by time the 2,400 frames used in the analysis are acquired at an end time of 160 ms after the system is triggered. In comparison, the schlieren data, for the case shown, has a maximum shift of just over 5 pixels compared to the 38 pixel shift for the psp data across the entire run duration. Note that the high-speed schlieren data acquired at 200,000 fps is able to acquire the desired 10,240 frames used in this analysis 91 ms after the tunnel trigger where the recoil behavior of the Ludwig tube is more benign.

3.4.2 Spectral Analysis

As originally discussed in Smith, et al. [98], spectral analysis of the schlieren image sequences was completed to interrogate the unsteadiness in the fluid dynamics of the observed SBLI. In this analysis, some pertinent regions of interest were selected, and a power spectra was created based on the pixel intensity as a function of time. Each of the power spectra computed within the area of interest can be used to identify the locations where SBLI unsteadiness is concentrated. The power spectra are obtained by applying the fast Fourier transform (FFT) on a pixel-by-pixel basis using the `pwelch` command in MATLAB based on Welch’s method [112]. In this manner, each pixel’s time history is treated as a unique measurement that is proportional to the density gradient across the tunnel at that location. [98]

For each case presented, a $N_{fft} = 2^{10} = 1024$ was chosen along with a $N_{ovlp} = N_{fft}/2 = 512$ (or 50% overlap) to correct for some substantial leakage of harmonic frequencies into nearby frequency bins, a common practice dating back to Welch’s original work [101, 98]. A Hann window was also applied to the dataset. The power spectral density (PSD) for each region of interest is then plotted in reference to one another in normalized units of $G_{xx} \cdot df / \sigma^2$.

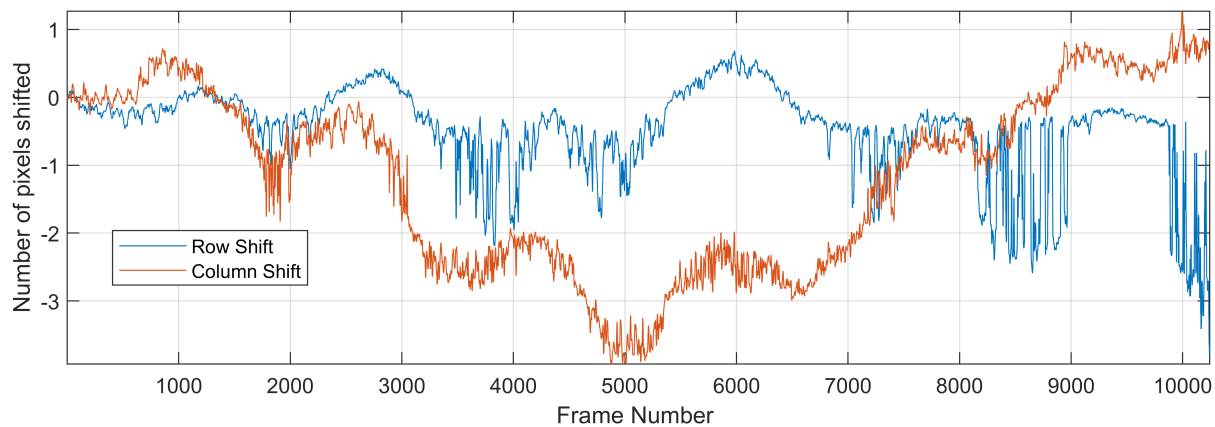


Figure 3.13: Results of preprocessing stage on schlieren data.

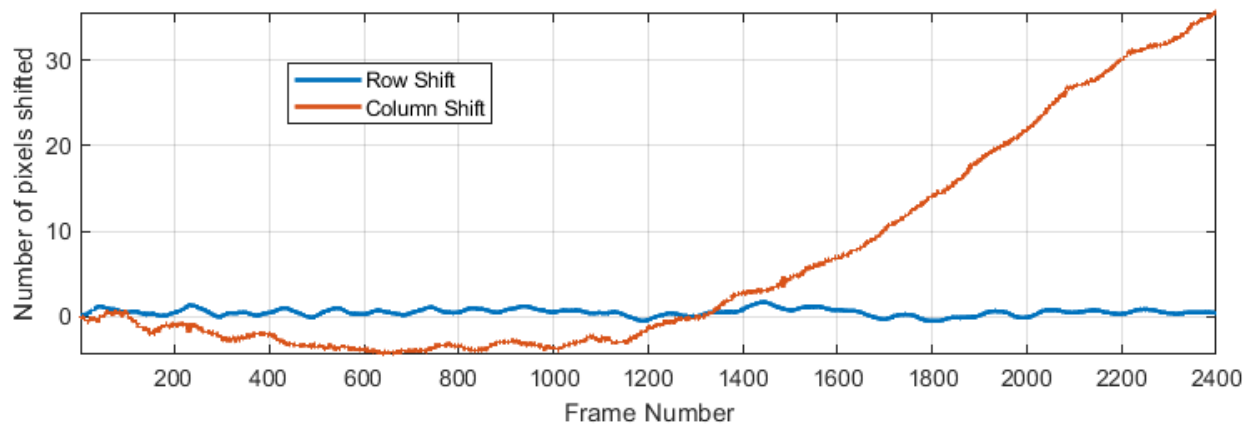


Figure 3.14: Results of preprocessing stage on pressure sensitive paint data.

Each test case utilized 10,240 points to remove augmentation of the spectral content due to zero padding and $f_s = 200$ kHz. Based on the Nyquist theorem, this allows frequencies up to 100 kHz to be resolved ($f_{max} = 100$ kHz). The resulting frequency resolution, or bin size, is $df = 195.31$ Hz. The number of total blocks used for each case was 19. The spectra have been integrated across particular frequency ranges to highlight the relative power contained within those bands. These values are then mapped onto the original image domain to generate a heat map corresponding to the locations where these frequencies are most dominant.

Power spectral densities were taken of the PSP data set in order to evaluate frequency content in their spanwise direction and further verify spectral content seen in the high-speed schlieren imaging. Power spectral densities displayed here were created using the same methodology as the schlieren images by use of MATLAB's `pwelch` command. A window size of $nfft = 2^8 = 256$ was utilized based on the limited sample length of the data set, which is a total of 2304 frames for each case, to ensure no zero padding was included in the data set. The data set has a limited sampling frequency of 20 kHz based on the pressure-sensitive paint response rate, and the resulting frequency resolution is 86.80 Hz. A 50% overlap was used, resulting in a total of 17 windows that were averaged. The maximum frequency resolved without aliasing in this case is 10 kHz via the Nyquist theorem; however, significant spectral attenuation is expected above 6 kHz as reviewed in the methodology section for this diagnostic technique.

3.4.3 Proper Orthogonal Decomposition

Modal decomposition is a mathematical technique for extracting the essential features of a flow field based on perceived energy and dynamics. For an overview of all modal decomposition methods, the reader is encouraged to review Taira et al. [103]. Proper Orthogonal Decomposition (POD) is a form of modal decomposition based on the single-value decomposition (SVD) of a data matrix. POD is also known as principle component analysis (PCA) or Karhunen-Loève decomposition, and much of the current framework for modern POD was introduced by Lumley [67] to extract coherent structures from turbulent

flows. Introductory POD information is summarized well by Chatterjee [15] and by Brunton and Kutz [8].

POD operates by decomposing, via a Fourier series, a matrix into modes with descending order of energy. However, POD has a limited ability to extract fluid dynamic behavior in the temporal domain. In other words, POD does not provide information on how these modes evolve in time.

In this study, the SVD method was used to conduct POD. This method is used to obtain a low-rank matrix approximation and generalize the eigendecomposition to rectangular matrices, unlike PCA, which is limited to a squared matrix. The SVD can be seen in its equation form below in Eqn. 3.5.

$$A = U\Sigma V^* \quad (3.5)$$

Here, A is the resulting rectangular matrix, U and V are the left and right singular vectors, respectively, $*$ denotes the conjugate transpose of a matrix, and Σ is a diagonal matrix with the unit vectors' magnitudes in decreasing order along its diagonal.

A custom code was created based on the work of Brunton and Kutz [8] as well as Price [84], and the code `POD.m` is displayed in Appendix B.1. Data processing of the schlieren images began with mean subtraction before reshaping the data to loop through each pixel's intensity change in time. The SVD is then taken using MATLAB's `svd` command in "econ" mode for the sake of limiting the computational expense. The left singular matrix, U , contains the "power" of the modes which are then shaped back into the original dimensions of the schlieren data set for view. Individual modes above 2% of the cumulative energy were examined, and an image reconstruction was completed based on these dominant modes and the cumulative energy at 90% and 99% of the total energy captured. A spectral analysis of these dominant modes was also examined.

3.4.4 Dynamic Mode Decomposition

Dynamic Mode Decomposition, or DMD, represents a similar data analysis methodology that is similar to the POD modes, though DMD also extracts dynamic information regarding how

the modes evolve in time. DMD approximates the time-evolution of a possibly nonlinear system with a linear operator, as shown in Eqn. 3.6. DMD methods, alterations, and mathematical principles are well documented in the archival literature [55, 94, 103].

$$X' \approx AX \quad (3.6)$$

For a given data set, the data is reshaped from matrix to vector form to construct X . A temporally shifted vector, X' , is then constructed for the calculation such that:

$$X = X(:, 1:\text{end}-1) \quad (3.7)$$

$$X' = X(:, 2:\text{end}) \quad (3.8)$$

POD analysis is then completed on variable X , and the data set is truncated so only the dominant modes are kept. Additional parameters, $\tilde{A}$ and ϕ , are then calculated based on Eqns. 3.9 and 3.10, respectively. Here, Γ is the eigenvectors matrix from $\tilde{A}$ and may be found using the `eig` command in MATLAB. The eigenvectors matrix is denoted by the variable ϕ , where its real component consists of oscillation frequency information and its imaginary component contains information regarding the exponential growth or decay of that particular mode as a function of time. The variable ϕ can then be reshaped back into the original frame dimensions to construct a heat map with the energy distribution for a chosen frequency or exponential growth component.

$$UX'V\Sigma^{-1} = U^*AU = \tilde{A} \quad (3.9)$$

$$\phi = X'V\Sigma^{-1}\Gamma \quad (3.10)$$

Note that DMD information is organized in order of importance based on dynamic information, not by energy. As such, one of the main limitations of DMD is the number of modes to keep. Currently, DMD modes are evaluated based on a researcher's interpretation of output modes based on the physically known characteristics of the fluid of interest. To

this end, a custom code was created based on the work of Kutz et al. [54] and is displayed in Appendix B.2.

3.4.5 Shock Tracker

As originally discussed in Lester, et al. [61], a feature tracking algorithm originally developed at UTSI was modified and utilized to identify and locate the various features of the shock interactions on a frame-by-frame basis. Initially, the MATLAB-based algorithm identifies the bow shock and triple point to define an interrogation region automatically. The image input to the algorithm undergoes a contrast adjustment before intensity thresholds are used to binarize the image. This process isolates the lambda shock structure and applies a linear fit to the pixels that are successfully identified as being located along the separation shock and the upstream influence. The linear fit is used to approximate the intersection point of these features with the model surface. The resulting data are “filtered” based upon the following statistical analyses wherein interactions with shock foot locations determined to be less than 2.7 diameters from the cylinder face are categorized as turbulent interactions. In Fig. 3.15 below, an instantaneous image of the shock tracker probes and subsequent linear fit of those points is shown [61].

3.4.6 Shock Tracker Uncertainty

Shock tracker uncertainty calculations involve quantifying uncertainty in both the diagnostic technique employed (i.e., high-speed schlieren imaging) and the data analysis methodology by which this information is extracted. These parameters and others are tabulated below in Table 3.7. The former of these two areas of concern may be approximated by quantification of the image resolution and captures errors associated with sub-pixel inaccuracy of tracked SBLI features. The second portion is focused on the designed shock tracker algorithm. The shock tracker code has several assumptions that balance the accuracy of the results with the ease of code employment on a given data set. The algorithm looks to find the leading edge of a substantial density gradient, focusing on locating a darker region in the schlieren data, within a predefined area of expected appearance as specified by the operator. Identified

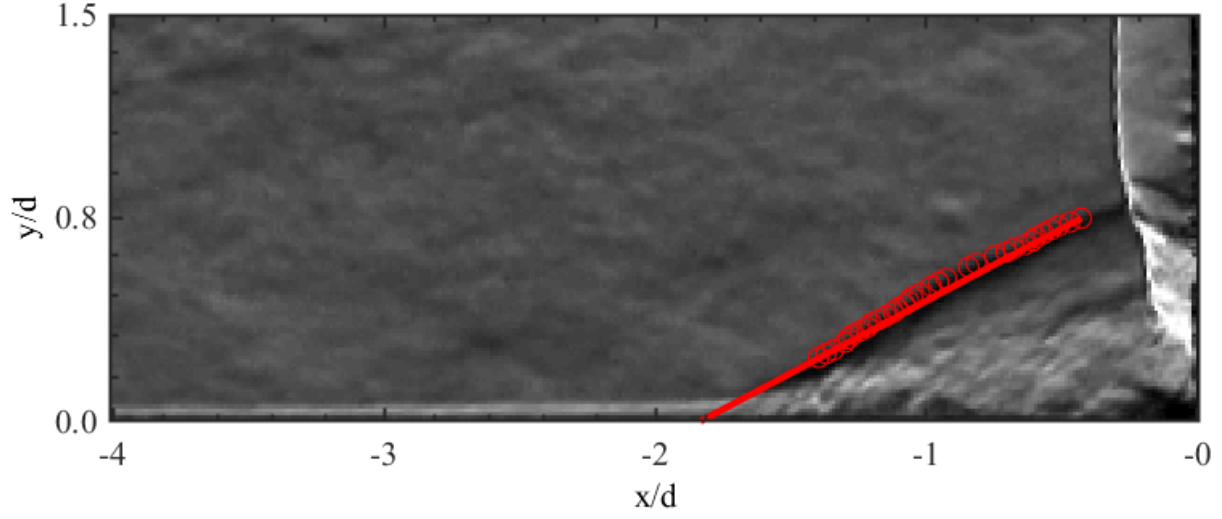


Figure 3.15: Instantaneous schlieren image of SBLI with shock tracker results including found data points and their subsequent linear fit.

Table 3.7: Summary of Shock Tracker Errors

X_{cylinder}/D Case	Pixel Res. [mm/px]	R-Squared [$R^2 < 0.95$]	Slope Outlier	Two Sigma	Manual Check [N, Frames]	Uncertainty [%]
11	0.1751	712	273	94	16	2.61
12	0.1751	802	506	823	30	5.15
13	0.1751	197	63	3396	23	8.76
18	0.1751	51	63	312	39	1.11

points along the shock are run through a linear regression and an outlier rejection process. Matches outside of a two-sigma range are removed before the remaining data are passed through a second linear regression.

One of the first parameters included in this table is the R-squared value, or the square of the linear correlation coefficient for the data identified as an SBLI feature. The shock tracker assumes that the upstream influence and separation shock are best fit by a straight line, although this is not a universally valid assumption, especially in instances of turbulent packets causing the SBLI structure to collapse back towards the cylinder shock generator. As a result, the number of frames with an R-squared value less than 0.95 are tabulated. Second, a parameter known as the slope outlier was created to identify cases where negative slopes resulted from the linear regression. In this case, a positive slope is always expected, and only in outlying situations does such a situation occur (e.g., where identified points have potentially been split between two or more strong features such as a dark separation shock foot and a stronger gradient at the top of the upstream influence location). The number of occurrences of negative slopes is calculated and tabulated below as the slope outlier parameter.

Finally, given the amount of data being reduced here, it would be a laborious task to evaluate the credibility of each individual frame. Instead, the author has reviewed the first 500 images of each case in the fashion of typical shock tracker uncertainty evaluation [18]. Note that this manual review of the first 500 frames showed a significant increase in incorrectly tracked frames, where the shock tracker output did not match the SBLI features upon visual inspection. More incorrectly tracked frames were identified in the later transitional and fully turbulent cases. The transitional cases more frequently have separation shocks that have increased curvature such that a linear fit of the data can result in errors associated with the extracted shock foot location. The case for the turbulent interaction had larger errors than were expected. In particular, this case showed small upstream extensions of the separation shock foot that did not follow the linear behavior of the rest of the forward lambda shock.

A total of 42,000 frames were evaluated for each cylinder shock generator station, where two test cases were concatenated in the time dimension to increase the total amount of data

collected. For clarity, 21,000 frames were taken from each run. The percentage of frames associated with these uncertainty metrics is summed directly to predict the final uncertainty in the shock tracker data for each shock generator station.

Chapter 4

Results and Discussion

4.1 High-Speed Schlieren Imaging

In this section, high-speed schlieren imaging results are reviewed. Shockwave / boundary-layer interaction statistics and dynamics are investigated as a function of the incoming boundary-layer state that is adjusted by changing the cylinder shock generator streamwise station on the model. Instantaneous, average, and standard deviation images are presented for each cylinder shock generator location, or X_{cylinder}/D case, and the test conditions are reported below in Table 4.1 for each run. Note that X_{cylinder} refers to the location of the cylinder relative to the leading edge of the model along the cone's surface. Note that Re/x is calculated for the conditions experienced on the cone surface. For improved clarity in printed form, each of these images has had slight brightness and contrast adjustments to better distinguish the various interaction features from one another. Approximately 10,240 images (corresponding to a 51.2 ms duration of steady flow) were used to compute the average and standard deviation in each run, and similar results were seen for each repeated case. For parity with current SBLI literature, the description given to an SBLI that is between fully laminar and fully turbulent flow will be that it is a transitional SBLI.

Representative instantaneous images are presented below in Fig. 4.1, where these images were selected to distinguish the two distinct interaction types observed in this body of work. The first three cases of $X_{\text{cylinder}}/D = 11, 12, \text{ and } 13$ exhibit characteristics typical of transitional SBLI, specifically the presence of the upstream influence. Meanwhile, the

Table 4.1: Summary of wind tunnel conditions. X_{cylinder} refers to the location of the cylinder relative to the leading edge along the cone surface.

Run Number	P_0 [psi]	Re_C/x [$\times 10^6 \text{ m}^{-1}$]	X_{cylinder}/D	Interaction
15	57.0	21.3	12	Transitional
16	56.6	21.2	12	Transitional
17	58.1	21.7	13	Transitional
18	58.2	21.7	13	Transitional
19	57.3	21.4	18	Turbulent
20	55.5	20.8	18	Turbulent
23	57.3	21.4	11	Transitional
24	56.8	21.2	11	Transitional

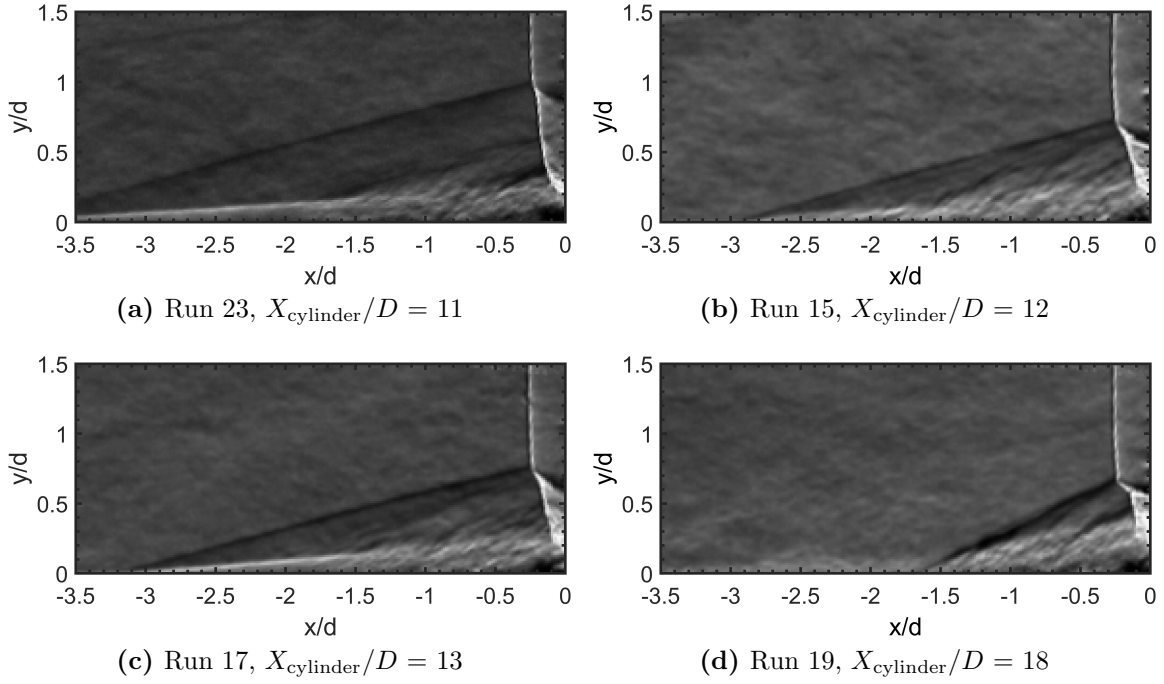


Figure 4.1: Instantaneous schlieren images for the 6° cone at $X_{\text{cylinder}}/D = 11, 12, 13$, and 18 respectively.

last case at $X_{\text{cylinder}}/D = 18$ has the typical structure of a turbulent SBLI. That is to say, the forward shock foot of the lambda structure, or separation shock, is pronounced, and the upstream influence is absent. A detailed study of the entire image set from the $X_{\text{cylinder}}/D = 13$ case (run 17 as shown in Fig. 4.1) and a repeated case (run 18, not shown) reveals that this cylinder location does correspond with transitional SBLI behavior. However, as a significant portion of these data appear to have a turbulent SBLI, this cylinder position can be further classified as one that produces a late-stage transitional SBLI. This behavior is also observable in the averaged images.

Looking at the average images in Fig. 4.2, the interaction scale and incoming boundary-layer thickness can be easily interpreted. At $X_{\text{cylinder}}/D = 18$, the separation shock is well defined and has an interaction scale of about 1.7 diameters, or 22.06 mm. As the cylinder shock generator is moved upstream towards the leading edge of the cone model, the separation shock diminishes in the average. The mean location also moves closer to the cylinder: the separation region length varies from roughly 1.4 diameters at $X_{\text{cylinder}}/D = 13$, to 1.3 and 1.2 diameters for $X_{\text{cylinder}}/D = 12$ and 11, respectively. Simultaneously, the upstream influence appears (although it is more prominent in the instantaneous images), and its location can be interpreted. The overall interaction scale also grows as the shock generator is moved upstream towards the cone’s leading edge. When considering the $X_{\text{cylinder}}/D = 18$ case, these averaged images suggest that nearly all data capture the behavior of a turbulent interaction. Meanwhile, as the cylinder’s location approaches the leading edge, transitional interactions are observed with increasing prevalence.

In the standard deviation images of Fig. 4.3, the black regions represent areas with low standard deviations, while the white regions characterize high standard deviations. Each of these SBLI shows a large standard deviation in the region near the boundary-layer separation, and these areas are concentrated underneath the separation shock foot (e.g., between $-2 < x/d < -1$). This unsteadiness is smallest in the $X_{\text{cylinder}}/D = 11$ case but has roughly the same scale for the other three cases. However, the $X_{\text{cylinder}}/D = 18$ case appears to have less variation suggesting the turbulent interaction features to be more stationary.

Further forward, the three cases with transitional SBLI behavior ($X_{\text{cylinder}}/D \leq 13$) have high standard deviation over a longer extent, which corresponds to the unsteadiness of the

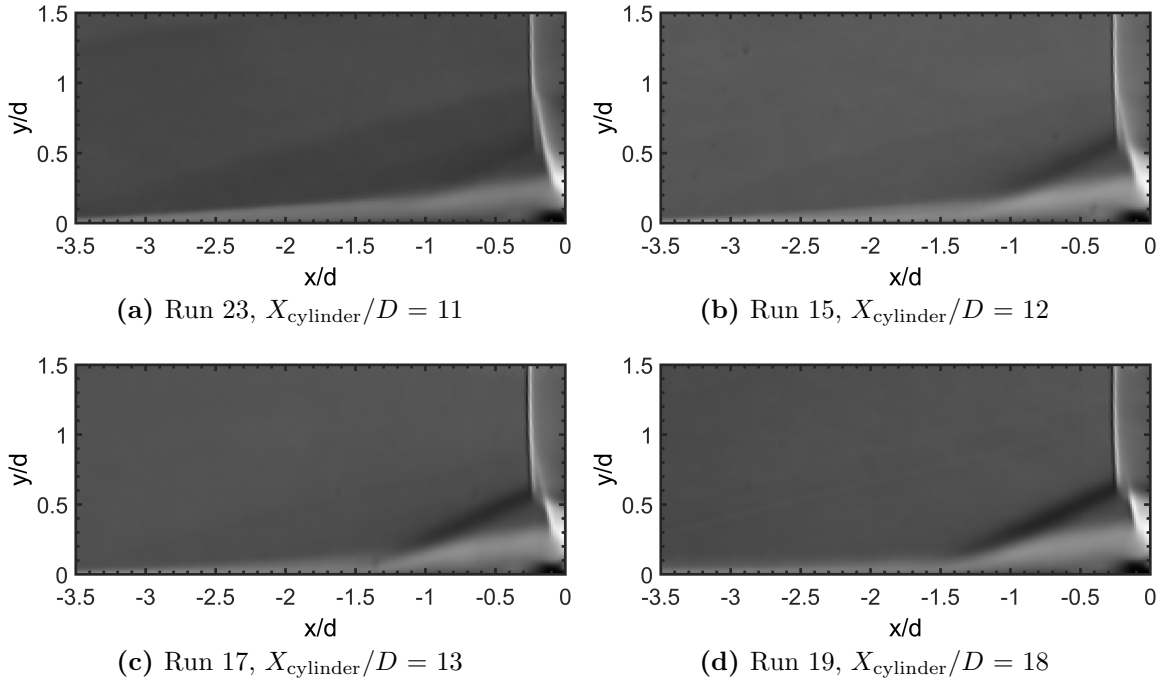


Figure 4.2: Average images for the 6° cone at $X_{\text{cylinder}}/D = 11, 12, 13$, and 18 respectively.

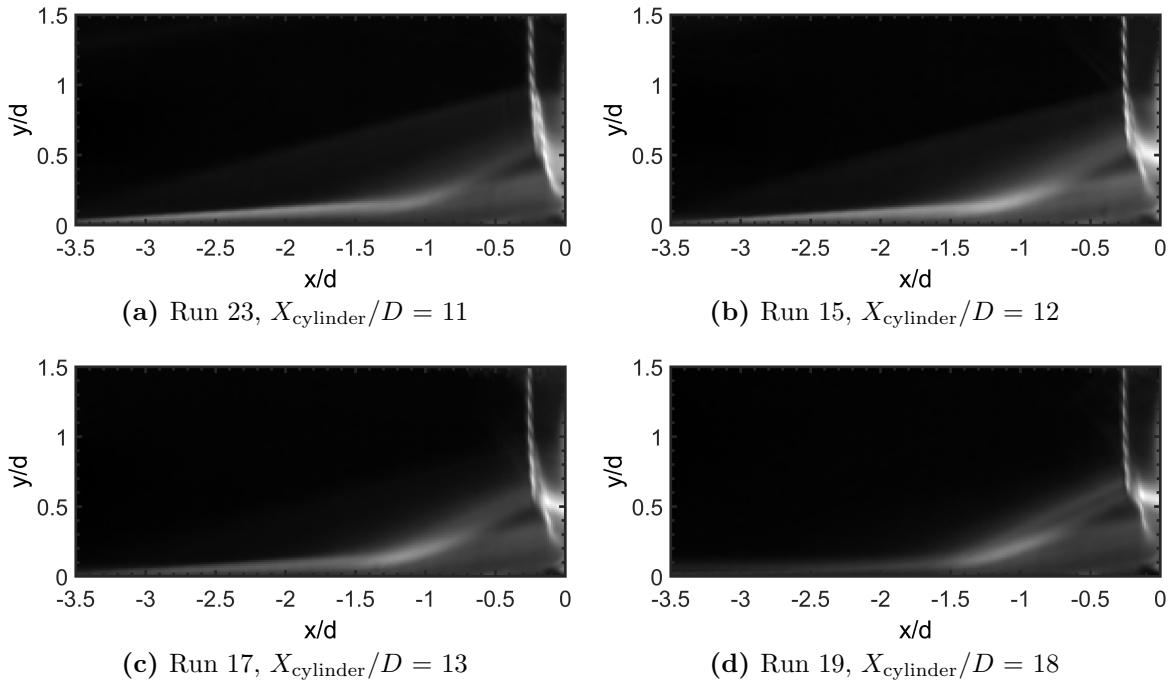


Figure 4.3: Standard deviation images for the 6° cone at $X_{\text{cylinder}}/D = 11, 12, 13$, and 18 respectively.

upstream boundary layer and provides a rough estimate for the extent of the upstream influence motion. The rapid growth of the boundary layer behind the upstream influence may account for the large change in standard deviation. This is also observed in the instantaneous images and is discussed later. Contrasting these cases against the turbulent SBLI at $X_{\text{cylinder}}/D = 18$, it is clear that the upstream unsteadiness disappears for the turbulent case, which is to be expected since this interaction tends to collapse to having only a separation shock and no upstream influence. Although the upstream influence tends to dissipate in the time-averaged images, its presence is highlighted here by the elevated unsteadiness within the upstream boundary layer for $X_{\text{cylinder}}/D \leq 13$ [61].

The intermittency of the boundary layer on the cone surface is plotted in Fig. 4.4 without the presence of a cylinder shock generator. The boundary layer is seen to transition between ~ 2.5 inches to 6.5 inches from the leading edge of the cone. Note that while the cylinder shock generator was placed at a location of 5.5, 6, 6.5 and 9 inches from the leading edge (LE), the SBLI footprint extends upstream of the shock generator face and responds to the incoming boundary layer state.

The incoming conditions for each case are tabulated in Table 4.1 while the boundary-layer thickness just upstream of the separation shock foot location, separation shock location, and separation scale (L_{sep}) for each case are presented in Table 4.2. The boundary-layer thickness and interaction length (L_{sep}) are reported here in units of mm. Measurements of the incoming boundary layer are taken ahead of the upstream influence location, typically at approximately $x/d \approx -4$. Theoretical laminar boundary-layer thicknesses were calculated based on the local Reynolds Number. The transitional interactions were found to have an incoming boundary layer similar to what would be expected from the theoretical laminar boundary-layer thickness calculation and were within about 1.5-32%. The difference between the theoretically predicted height and measured height may be a product of the camera resolution or the measurement method. The incoming boundary-layer thickness was measured via an average image that included 1,000 frames, and the camera resolution, approximately ~ 0.17 mm per pixel, would suggest an uncertainty of 0.34 mm in the boundary-layer measurement. The rapid growth of the boundary layer is visible downstream of the upstream influence (UI). The transitional cases see rapid growth of the boundary layer to about 1.75 mm at $x/d = -2$,

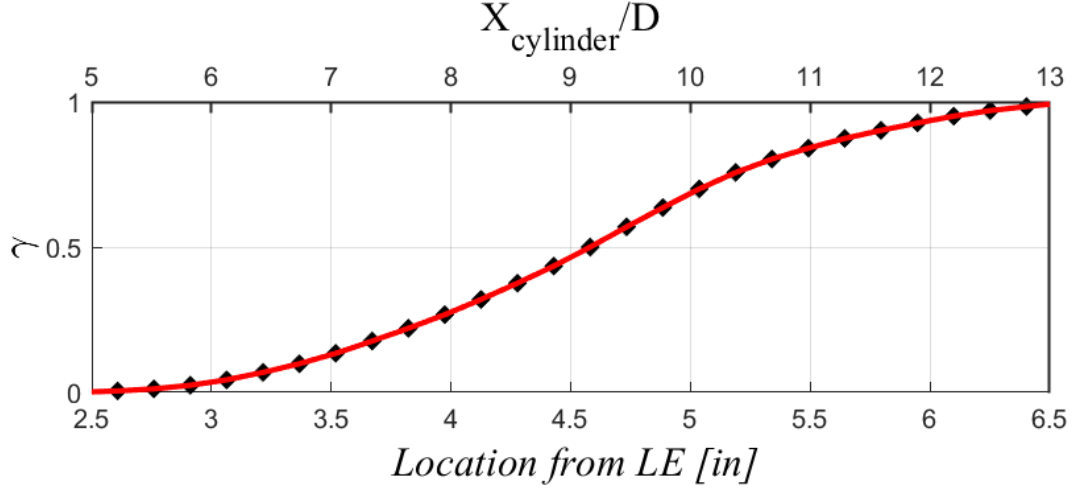


Figure 4.4: Intermittency of boundary layer on the 6° cone model without a cylinder shock generator

Table 4.2: Summary of SBLI statistics from the schlieren data.

Run Number	$X_{cylinder}/D$	Re_C/x [$\times 10^6$] m^{-1}	BL Thickness [mm]	λ_1 [x/d]	λ_2 [x/d]	L_{sep} [mm]
15	12	21.30	0.53	-1.4	-0.1	16.51
16	12	21.17	0.53	-1.45	-0.15	16.51
17	13	21.71	0.53	-1.6	-0.1	19.05
18	13	21.74	0.53	-1.6	-0.1	19.05
19	18	21.40	1.05	-1.55	-0.15	17.78
20	18	20.40	1.23	-1.7	-0.15	19.68
23	11	21.40	0.53	-1.3	-0.1	15.24
24	11	21.23	0.70	-1.3	-0.1	15.24

consistently, and the boundary-layer thickness reaches 1.92 mm for the turbulent case. The locations of the front and rear leg of the lambda shock foot are extracted from the average images and denoted as λ_1 and λ_2 , respectively. The separation length is also calculated, which is simply the length between the front and rear legs of the lambda shock. Note that the boundary-layer thickness increases as expected with regards to the increase in Reynolds number as the cylinder shock generator is shifted downstream. The leading lambda shock foot location collapses back towards the cylinder shock generator as the incoming boundary-layer state changes from transitional to turbulent. This follows the expected trend, where the front shock foot location remains between $-2 < x/d < -1.5$ [20]. The rear leg of the lambda shock does not appear to change when evaluating the averaged images, and it remains at a location of about $x/d \approx -1$ as expected [63]. The closure shock location does move slightly upstream in the turbulent case to about $x/d = -0.15$. Note that while the boundary layer thickness does increase as the cylinder shock generator is moved downstream, it only increases by a small amount compared to the significant changes in the interaction locations and scales.

From the high-speed schlieren imaging, some recurring behavior in the interaction dynamics was identified. A time series is shown in Fig. 4.5 depicts the distinct features and sequence of events involved in the collapse of the shockwave / boundary-layer interaction in response to a disturbance in the incoming boundary layer. The first frame shown is 8621 which is 43.1 ms into the steady state run time, or 93.1 ms after the camera is triggered to begin recording. The upstream influence appears to respond first to the presence of an upstream disturbance in the incoming boundary layer. The UI is pushed downstream and begins to shock bows inward on itself. This rapid collapse in the interaction structure is followed by the return of the separation shock back upstream. This restoration occurs at a comparatively slower rate than the initial collapse, and the slope of the separation shock is linear in nature, as is typically characteristic of SBLI. The separation shock appears to diminish in strength as it moves upstream until it reaches a maximum separation length, just as the upstream influence becomes apparent. The upstream influence then continues to develop further upstream, with it eventually diminishing in strength as it approaches a maximum outboard position.

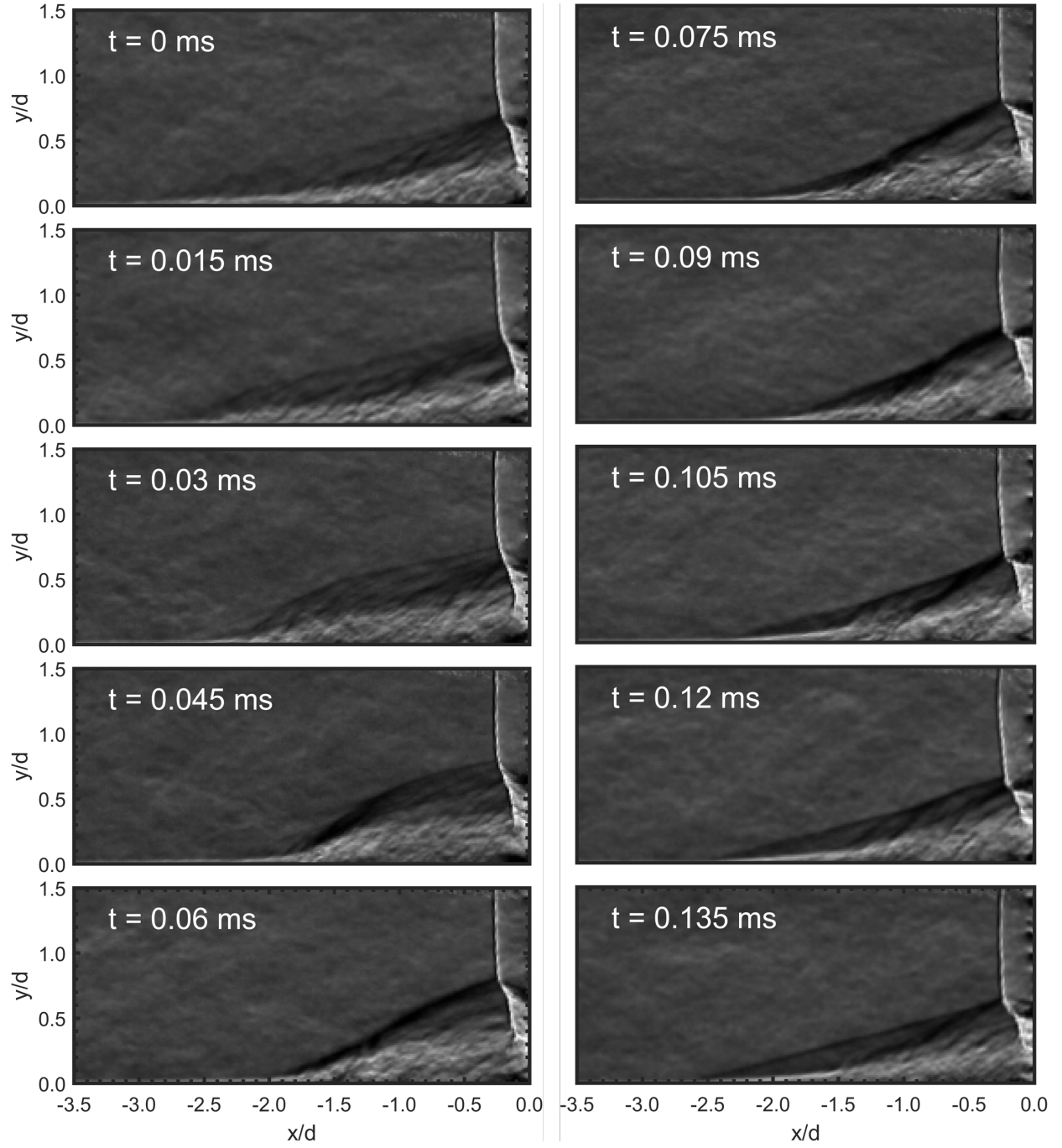


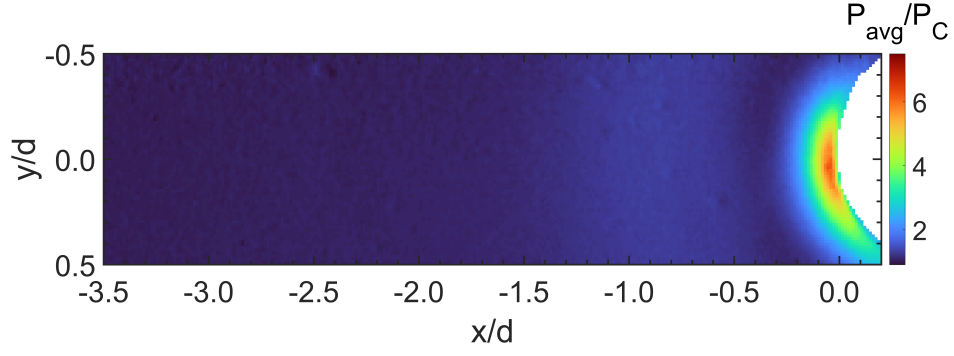
Figure 4.5: Time series of high-speed schlieren images of transitional SBLI dynamics from Run 17 at $X_{\text{cylinder}}/D = 13$.

4.2 Pressure Sensitive Paint Results

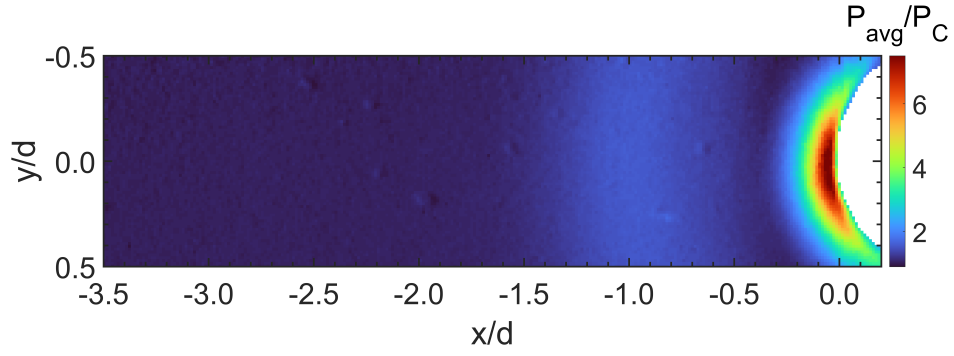
Pressure-sensitive paint was applied to the 6° cone model to characterize the SBLI based on its pressure footprint. Average and standard deviation images, pressure traces along the cone’s centerline, time series images, localized power spectral densities, and centerline pressure distributions in time are evaluated in this section to discern the interaction statistics and dynamics.

Average PSP images are provided for each X_{cylinder}/D station in Fig. 4.6. Here, the average pressure has been normalized by the theoretical post-shock pressure, which is denoted as P_{avg}/P_C . A total of 2,400 frames are used to construct the averaged image. In the average images, the flow direction is from left to right. The cylinder shock generator’s geometry is marked in white on the right-hand side of the image where its most upstream edge is at an x/d location of 0. The pressure rise from the forward lambda shock foot and closure shock structure in the separation bubble is visible in all average images. However, the presence of the upstream influence is not. Instantaneous images indicate significant movement in the upstream influence, which would reduce its signature in the average. The upstream x/d locations in the averages have higher pressure ratio values in the transitional cases compared to the turbulent case, indicating the UI’s range of influence is seen in the average images. The chosen colormap and scaling may also contribute to the UI’s limited appearance.

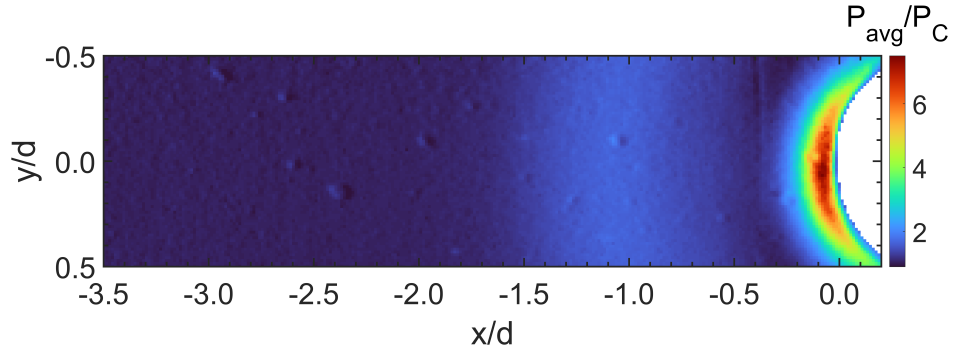
As a reminder, the first two cylinder shock generator stations are often referred to in this section as early transitional cases; meanwhile, the third station at $X_{\text{cylinder}}/D = 13$ is referred to as late transitional. The $X_{\text{cylinder}}/D = 18$ case is a turbulent interaction case. This description is based on each run’s intermittency or interaction type throughout the evaluated run times. Transitional interactions are traditionally defined as anything between an all laminar and all turbulent interactions. A transitional case may oscillate between these two boundaries. The “early” transitional cases show typical transitional structures for the majority of the run duration on a frame-by-frame basis. This is quantified during the shock tracker analysis section. Specific methods of evaluation, such as an average, are expected to result in an output image that displays transitional SBLI structures, which includes the presence of the upstream influence. The late-stage transitional case shows



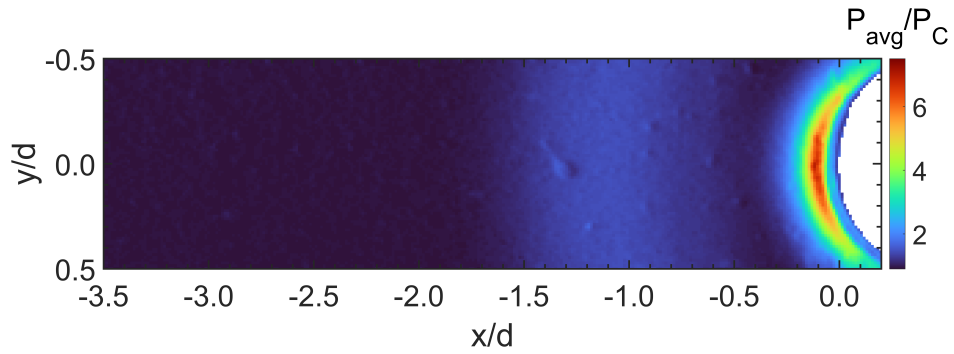
(a) $X_{\text{cylinder}}/D = 11$



(b) $X_{\text{cylinder}}/D = 12$



(c) $X_{\text{cylinder}}/D = 13$



(d) $X_{\text{cylinder}}/D = 18$

Figure 4.6: Average PSP images for the 6° cone configuration at $X_{\text{cylinder}}/D = 11, 12, 13$, and 18.

a majority of its frames have a “turbulent” SBLI structure while the rest are typically transitional interactions. The “early” and “late-stage” transitional interactions are pointed out here due to the known changes in the incoming boundary-layer state to include more turbulent spots. The “late-stage” transitional cases also have changes to the “global” results that mimic their fully turbulent counterparts even though their averages display elements of transitional interactions including their upstream extent and presence of the upstream influence.

Note that the resolution of a single pixel to physical units results in a change of 0.0171 cm for $X_{\text{cylinder}}/D = 11, 12$, and 13 cases and 0.0166 cm for the $X_{\text{cylinder}}/D = 18$ case. These correspond to an x/d shift of 0.135 and 0.0131, respectively. Shifts below these thresholds are not reported as they do not correspond to anything meaningful.

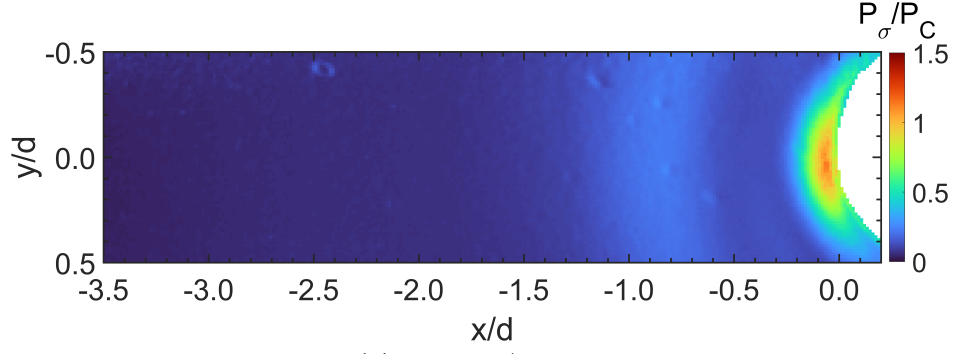
The interaction’s dominating feature is associated with the closure shock. The $X_{\text{cylinder}}/D = 11$ case spans upstream to about $x/d = -0.2$ with a centered peak pressure above $P_{\text{avg}}/P_c = 5$. The high pressure region has a width of 0.1. This higher pressure region expands and strengthens in the $X_{\text{cylinder}}/D = 12$ station. The entire region spans from $x/d = 0$ to -0.15. This case has a closure shock region with the highest P_{avg}/P_c measurements of all four cylinder shock generator locations investigated. For $X_{\text{cylinder}}/D = 13$ and 18 cases, the closure shock region remains consistent and spans from 0 to -0.3. However, the peak pressure decreases at more downstream cylinder shock generator stations. In the late transitional case, the most elevated pressure values operate over the same x/d centerline span; however, the interaction at a $X_{\text{cylinder}}/D = 18$ decreases to a width of $x/d = 0.1$, that is, from -0.05 to -0.15, and shifts off the surface of the cylinder by about $x/d = 0.05$.

At an X_{cylinder}/D location of 11, From the average images the separation shock tends to oscillate between $x/d = -0.5$ and -1.3 along the centerline for cylinder shock generator locations of $X_{\text{cylinder}}/D = 11, 12$, and 13. The peak pressure in these regions increases as the interaction becomes more turbulent. In the fully turbulent interaction, the high-pressure region appears to operate from about $x/d = -0.8$ to -1.4, and the local pressure is reduced when compared to the previous station.

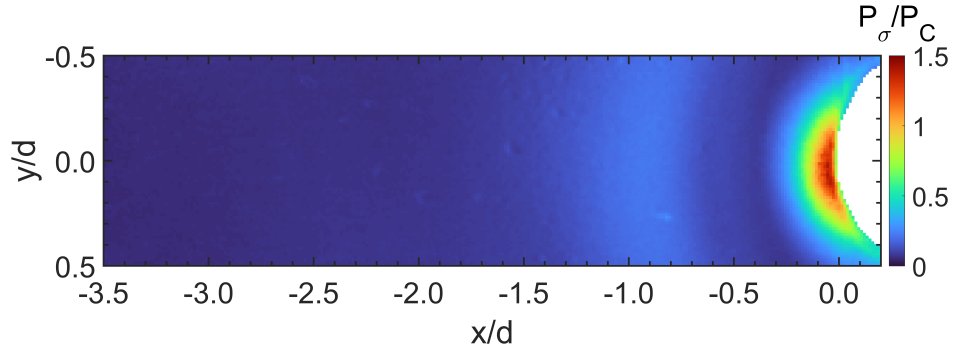
The interaction’s closure shock is the dominant P_{avg}/P_c feature for all four cylinder shock generator cases. The closure shock in the $X_{\text{cylinder}}/D = 11$ case extends upstream

to about -0.2 with a centered maximum P_{avg}/P_c reading of 6.95 centralized at x/d -0.12. The closure shock's upstream extent appears consistent across all cylinder shock generator stations. Explicitly, the elevated P_{avg}/P_c values begin at about $x/d = -0.2$ for all evaluated cases; however, the peak pressure within these regions shifts upstream as a function of downstream shock generator location. The peak pressure for the $X_{cylinder}/D = 12$ case is visible at $x/d = -0.04$ and has a value of $P_{avg}/P_c = 6.18$. At $X_{cylinder}/D = 13$ the max pressure is located at $x/d = -0.05$ and has a value of $P_{avg}/P_c = 8.06$. The peak pressure decreases to 7.47 at $x/d = -0.08$ for $X_{cylinder}/D = 18$, and the high pressure region shifts off the surface of the cylinder by about $x/d = -0.05$. High pressure areas result in more local oxygen quenching and decreases the intensity of the PSP luminophore picked up by the camera. While local shadows may be present at the cylinder shock generator base, they would result in a darker region in the raw data and register as a higher pressure than are physically present. Ideally shadows are expected to be removed by the wind on to wind off ratio. Regardless, this localized higher pressure due to a shadow does not match the results seen here and would suggest local shadows have minimal effects in the averaged images.

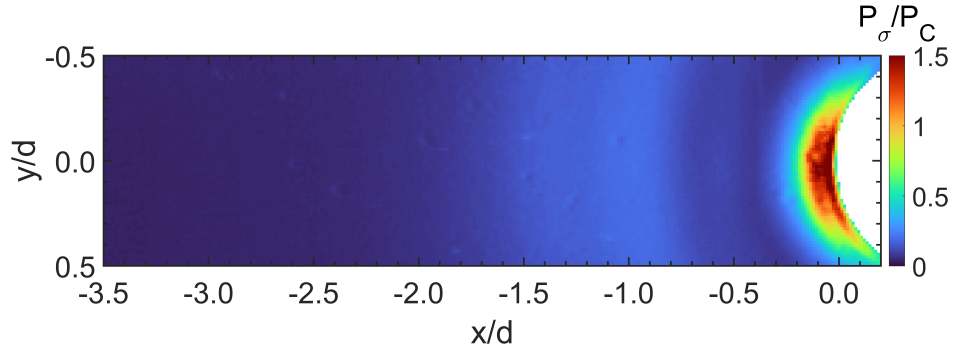
Standard deviation images of the PSP data of the cone with a cylinder shock generator are shown in Fig. 4.7. At a $X_{cylinder}/D$ station of 11, some unsteadiness is seen in the range of -0.3 to -1.1 with a locally elevated standard deviation value around -0.3 to -0.4. Most of the unsteadiness overlaps with the separation shock location from about -0.6 to -1, as evident from the averaged images. This region of unsteadiness appears to shift upstream from $x/d = -0.4$ to -1.7 and the maximum standard deviation spans from $x/d = -0.7$ to about -1.2. The standard deviation has maximum $x/d = -0.9$, again significantly overlapping with the average image's elevated P_{avg}/P_c area. The later transitional case shows elevated standard deviation values from about -0.4 to -1.8, with the largest P_σ/P_c values between $x/d = -0.75$ to -1.15. Note there is an uniform decrease in P_σ/P_c as the separation shock moves upstream. This gradient is visible from -1.15 to -1.55. Areas of maximum P_σ/P_c overlap much of the same areas of high P_{avg}/P_c values. This may indicate an increase in unsteadiness at upstream locations and lower occurrence at these same upstream locations. The turbulent case shows two narrow and distinct higher standard deviation regions centralized at a centerline x/d location of -1.45 and -0.9 with a width of about $x/d = 0.4$ for the upstream station and



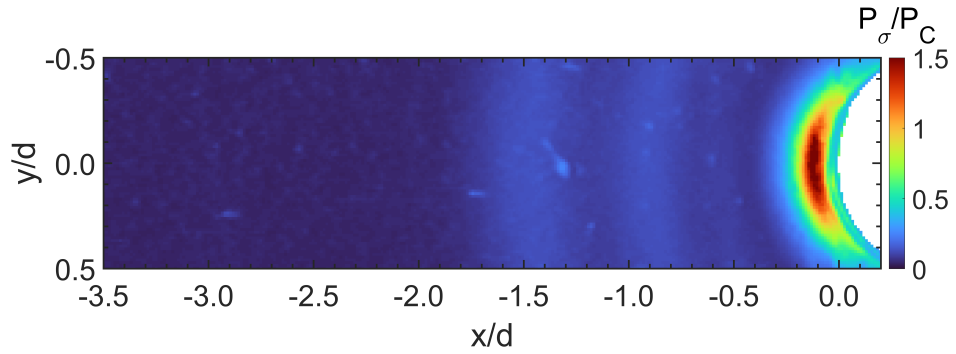
(a) $X_{\text{cylinder}}/D = 11$



(b) $X_{\text{cylinder}}/D = 12$



(c) $X_{\text{cylinder}}/D = 13$



(d) $X_{\text{cylinder}}/D = 18$

Figure 4.7: Standard deviation PSP images for the 6° cone configuration at $X_{\text{cylinder}}/D = 11, 12, 13$, and 18.

0.3 width for the downstream station. These locations are at either edge of the higher value average pressure ratio range spanning from about -0.75 to -1.6. In this case, the data indicates that the interaction typically stays within a range of about -0.75 to -1.6, but reaches these extreme extents on occasion.

The standard deviation closure shock contours show very similar trends in their structure when compared to the average image's closure shock contours. The $X_{\text{cylinder}}/D = 11$ case has nearly the same size area of its elevated pressure ratio values when compared to the average. For the $X_{\text{cylinder}}/D = 12$ case, the width of the elevated P_σ/P_c shows no significant change in its extent compared to the previous case. Both upstream shock generator stations have significant unsteadiness decreases at more outboard positions. For the $X_{\text{cylinder}}/D = 13$ and 18 cases, significantly elevated P_σ/P_c ratios are observed. Notably, the turbulent case has a centerline x/d extent of about 0.1 compared to its average extent of about 0.05, indicating significant fluctuations in the closure shock's relative size or upstream extent in a fully turbulent case.

Typical parameters extracted from SBLI, such as the separation location and length scale of the separation, are tabulated below in Table 4.3. The separation location, λ_1 , is reported in physical units (i.e., [mm]) which can be seen to move upstream as the cylinder shock generator is moved downstream as the incoming boundary-layer state undergoes transition. The separation length scale, L_{sep} also follows the trend. The separation location, λ_1 , appears to stay constant, but the separation length increases at each subsequent station.

A time series of instantaneous images is displayed in Fig. 4.8 for the $X_{\text{cylinder}}/D = 13$ case. The time series shown starts at frame 842. Every third frame is shown; which, corresponds to a 0.15 ms iteration between each step shown in the time series. Frame 842 is 42.1 ms in the steady state run time and 82.1 ms after the camera's is triggered to start based on the starting shock passage in the upstream driver tube. The separation shock fluctuates forward and aft, with the most downstream edge of the pressure plateau oscillating from about $x/d = -1.3$ to $x/d = -1.5$. The geometry on the back side of the pressure plateau appears to mirror the geometry of the closure shock. This may be indicative of fluctuations originating at the end of the plateau region, driving unsteadiness in both the upstream and downstream separated flows as proposed by Murphree [73]. The elevated pressures of the closure shock

Table 4.3: Summary of measured SBLI parameters from the pressure sensitive paint data. X_{cylinder} refers to the location of the cylinder relative to the leading edge of the 6° cone (along the cone surface).

X_{cylinder}/D	Interaction	$\lambda_1 [x/d]$	$\lambda_2 [x/d]$	$L_{\text{sep}} [mm]$
11	Transitional	-1.3	-0.35	12.07
12	Transitional	-1.5	-0.35	14.61
13	Transitional	-1.7	-0.4	16.51
18	Turbulent	-1.8	-0.4	17.78

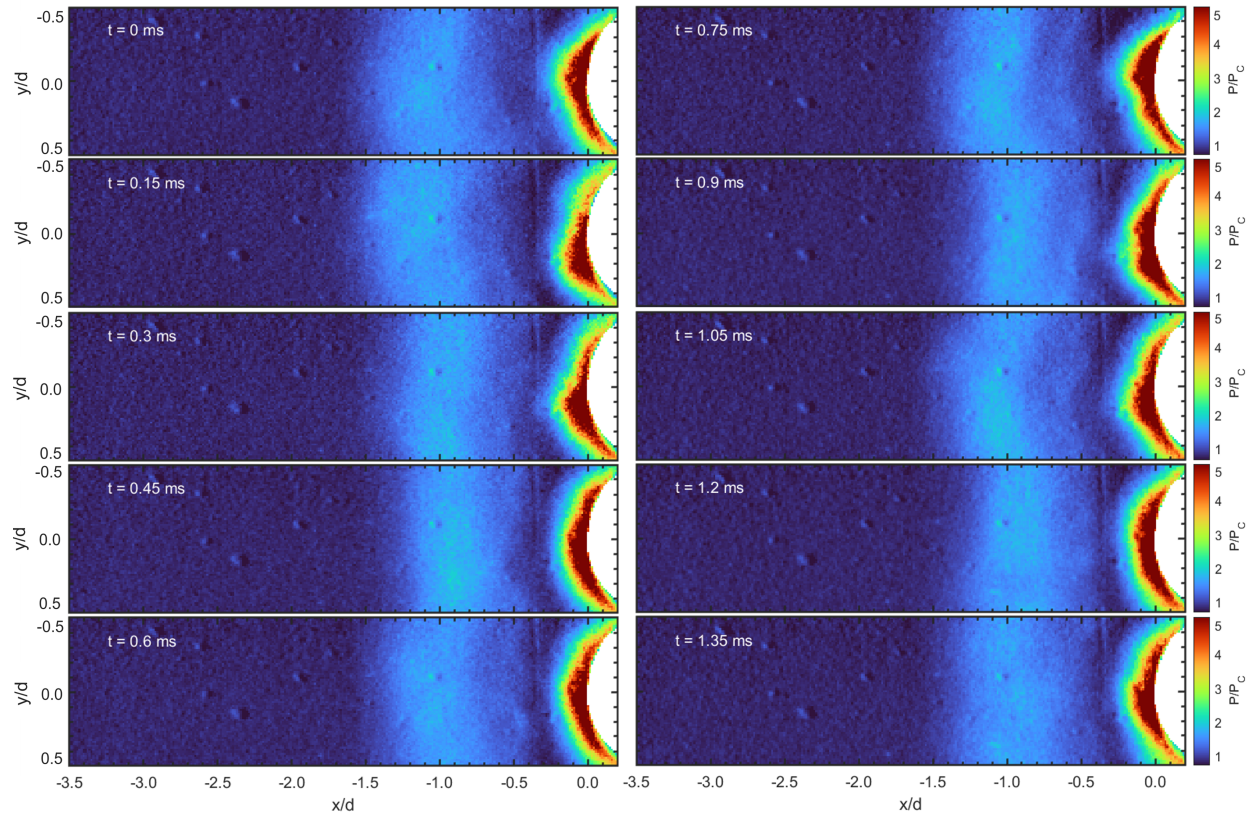
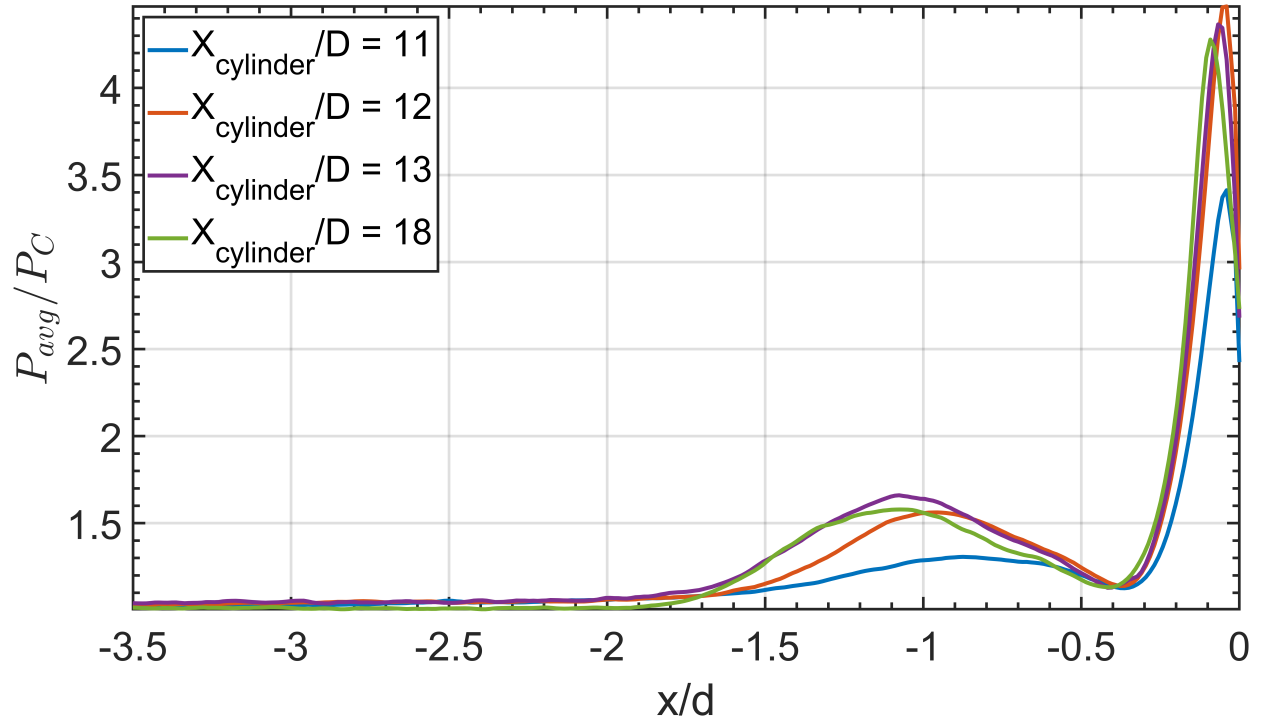


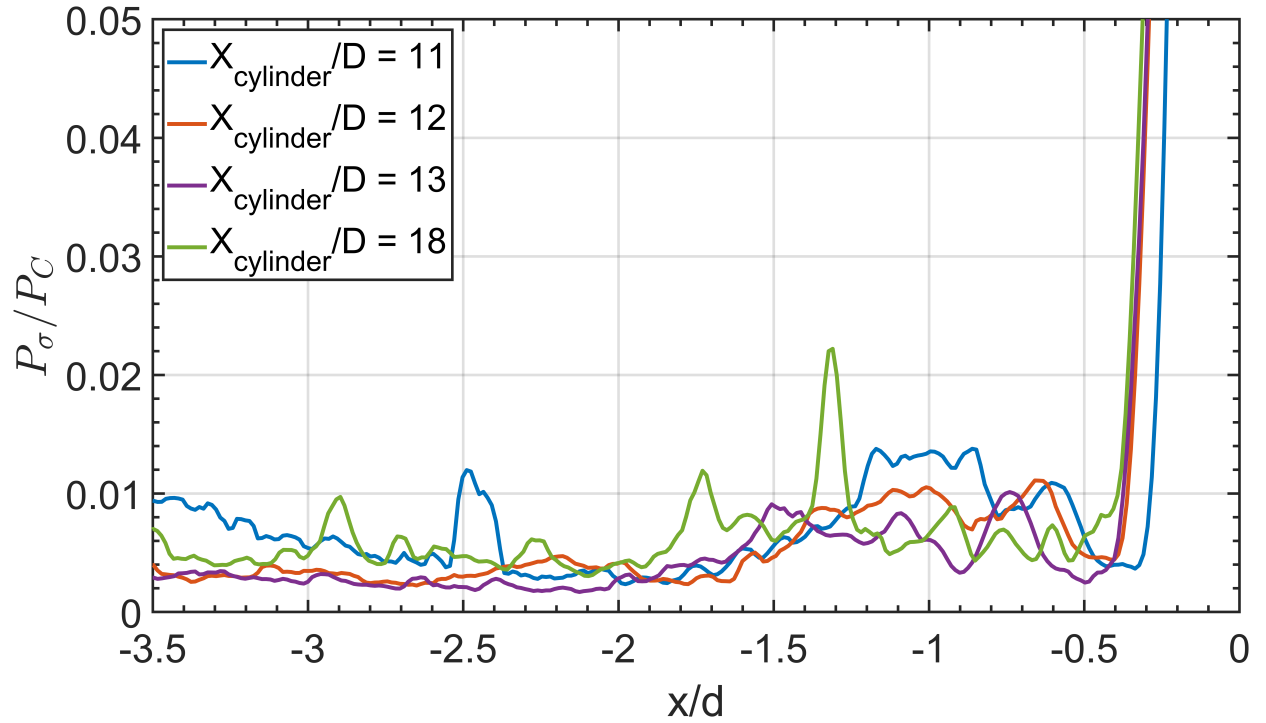
Figure 4.8: Time series of pressure sensitive paint data.

change shape rapidly and are not symmetric. This asymmetry may be due to the SBLI dynamics or the introduction of cross flow. Note that some of this asymmetry is expected to be a byproduct of possible bending of the cylinder shock generator being mounted via a cantilever beam. No clear driver of this behavior is seen between the separation shock foot position and closure shock. Notably, even in the instantaneous images, there is no evident pressure rise as a result of the upstream influence, as previously seen in the average and standard deviation contours over the entirety of the run time.

Traces of the average and standard deviation pressures from the PSP images are plotted below in Fig. 4.9. To create the traces 40 pixels above and 40 pixels below the cone centerline (total of 80 pixels) were utilized to extract the average or standard deviation. Note this spatial averaging process will decrease the maximum P_{avg}/P_c and P_σ/P_c values observed in Fig. 4.6 and Fig. 4.7. The spatial averaging in the traces was completed to smooth the curves and remove spurious pixel data. The average and standard deviation traces should provide better spatial accuracy in key features than the average and standard deviation images at the compromise of amplitude data. The full 2,400 frames were used to compute these statistics for each case. The computed traces allow trends in average pressure values to be compared more easily and accurately than those plotted in Fig. 4.9(a). The initial pressure rise is known from literature to be delayed for increasing values of d/δ and corresponds to higher pressure plateau rates [63]. As the constant-diameter cylinder shock generator is moved downstream, the boundary-layer thickness is expected to increase; therefore, d/δ would decrease as $X_{cylinder}/D$ increases. The centerline average pressure ratios follow this trend for the initial rise in pressure, but the trend inverts when it comes to the plateaus' local maxima. The pressure trough is seen to be constant, only moving upstream as the interaction becomes more turbulent. The trough locations are expected to be consistent. This is because the trough is suspected to correspond to the separation line [63]. The cylinder shock generator stations decrease in P_{avg}/P_c values between the plateau region and the pressure trough. There is a decrease in P_{avg}/P_c values 13.8%, 27%, 32%, and 28.3% at $X_{cylinder}/D$ stations of 11, 12, 13, and 18, respectively. Finally, the peak pressures, which are a function of inviscid features such as the bowshock, are expected to increase with increasing shock strength before experiencing some pressure relief and dropping in strength at the



(a) Average pressure ratio traces



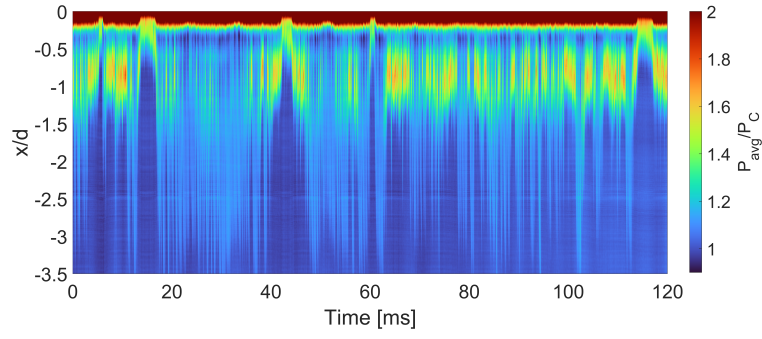
(b) Standard deviation of pressure ratio traces

Figure 4.9: Average and standard deviation pressure values along the centerline of the cone divided by the upstream cone pressure.

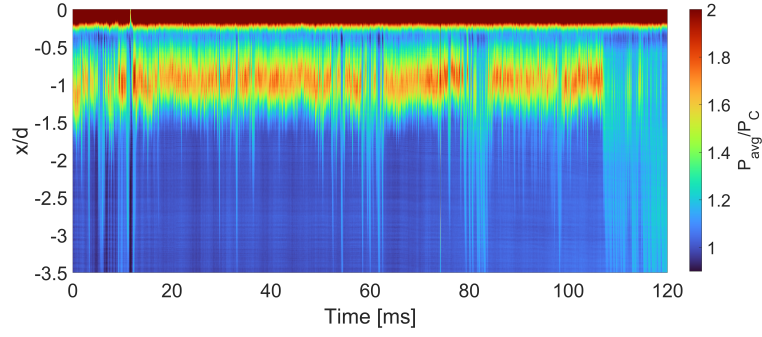
attachment line [63]. The second transitional station at $X_{\text{cylinder}}/D = 12$ has the dominant peak pressure of all four cases, as previously seen in the average PSP images.

The standard deviation traces are displayed in Fig. 4.9(b). All transitional interaction cases have an increased standard deviation from about $x/d = -0.5$ to -1.7 with a larger streamwise range of elevated unsteadiness as the interaction is moved downstream. This behavior would indicate that the separation shock oscillates over an upstream extent range of movement as the interaction becomes more turbulent. For the $X_{\text{cylinder}}/D = 11$ case, two localized peaks at $x/d \approx -2.5$ and -1 mark the more persistent presence of the upstream influence and separation shock. The fully turbulent case also contains several localized peaks at -1.35 , -1.75 , and -2.9 . At each subsequent location P_σ/P_c values decrease from about 0.023, to 0.013, and finally to 0.01. These three peaks indicate areas of higher unsteadiness, and are associated with the upstream extent of the upstream influence, upstream extent of the separation shock, and downstream extent of the separation shock. These higher unsteadiness levels were also seen in the schlieren data.

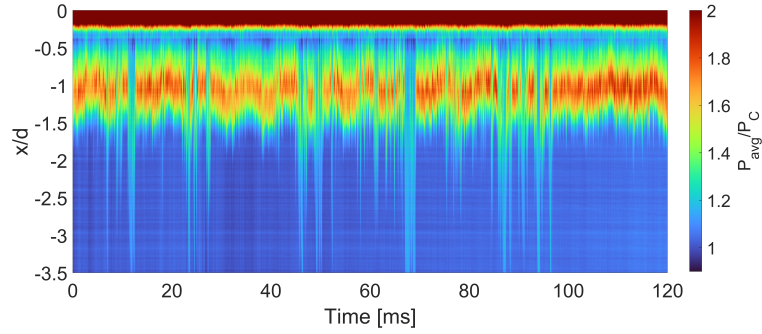
Below is Fig. 4.10, a centerline average of the pressure sensitive paint from an x/d position of 0 to -3.5 is plotted vs. time to evaluate SBLI dynamics. The centerline average is spatially averaged in the same methodology as the centerline traces previously discussed. The early transition cases, at $X_{\text{cylinder}}/D = 11$ and 12 , show significantly different trends when compared to one another. The $X_{\text{cylinder}}/D = 11$ case shows elevated pressure ratios typically from $x/d = -0.5$ to about $x/d = -1.3$. This elevated pressure, at a $P_{\text{avg}}/P_c > 1.2$ at an x/d location of ~ 1.3 , was measured to be present for $\sim 36\%$ of the test duration. This is much shorter compared to the 79%, 98% and 99% of the test duration at these conditions for the other three cylinder shock generator positions at $X_{\text{cylinder}}/D = 12, 13$, and 18 , respectively. For the $X_{\text{cylinder}}/D = 11$ case five distinct instances of a collapse towards the cylinder face. Three are larger in duration at ~ 18 ms, 43 ms, and 116 ms. The final two instances of collapse are much shorter in duration at ~ 8 ms and at ~ 60 ms. For the $X_{\text{cylinder}}/D = 12$ case, the pressure rise from the separation shock foot is the maximum of all four cylinder shock generator stations. The plateau region spans from about -0.7 to about $x/d = -1.3$. There are shorter deviations from the typical behavior in this case occurring at the beginning and end of the run. This behavior likely corresponds with higher noise in



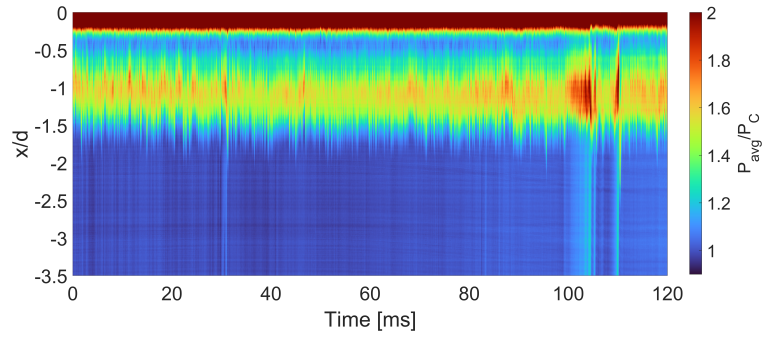
(a) $X_{\text{cylinder}}/D = 11$



(b) $X_{\text{cylinder}}/D = 12$



(c) $X_{\text{cylinder}}/D = 13$



(d) $X_{\text{cylinder}}/D = 18$

Figure 4.10: Centerline pressure distribution in time for $X_{\text{cylinder}}/D = 11, 12, 13$, and 18 .

the test section pressure during startup and shutdown behavior of the Ludwig tube. At a X_{cylinder}/D position of 13, there is some sinusoidal behavior of the separation shock foot between an upstream extent of $x/d = -1.4$ and a downstream extent of $x/d = -0.8$. Finally, the turbulent SBLI case has a consistent lower pressure ratio of $P_{\text{avg}}/P_c = 1.4$ that varies around an x/d position of -1. This plateau region's upstream extent appears reduced during the initial 20 ms of run time and has elevated P_{avg}/P_c starting at about 100 ms and 108 ms.

4.3 Spectral Analysis

Spectral analysis was performed on both the high-speed schlieren and pressure sensitive paint data to provide insights into the overall SBLI dynamics. The power spectral density, or PSD, of the mean subtracted data was computed via Welch's method, and detailed information regarding this process is included at length in Chapter 3. In this section, spectral content for transitional and turbulent interactions are examined, and comparisons are made between the two diagnostics. The spectral content of the data is displayed in the form of spectral heat maps and in traditional PSD plots for the frequency content of individual probe stations.

4.3.1 High-Speed Schlieren Data

In this section, the spectral content of the high-speed schlieren data is reviewed in several different forms. To begin, the global spectral content from each test case across the steady-state flow duration is calculated by averaging PSDs computed from each pixel in all frames. The results of this are shown below in Fig. 4.11. Overall, no global frequency content is present in these data; however, given the highly unsteady and varying behavior known to be present at certain conditions, additional methods that do not spatially average the data were used, including the creation of spectral heat maps and localized PSDs.

Individual power spectral densities were analyzed at specific stations to verify the dynamic content seen in the spectral heat maps and to understand (i) how the content is distributed for specific features as a function of position and (ii) how the content changes as the interaction evolves from a transitional to a turbulent SBLI. In Table 4.4, the various probe stations (x/d

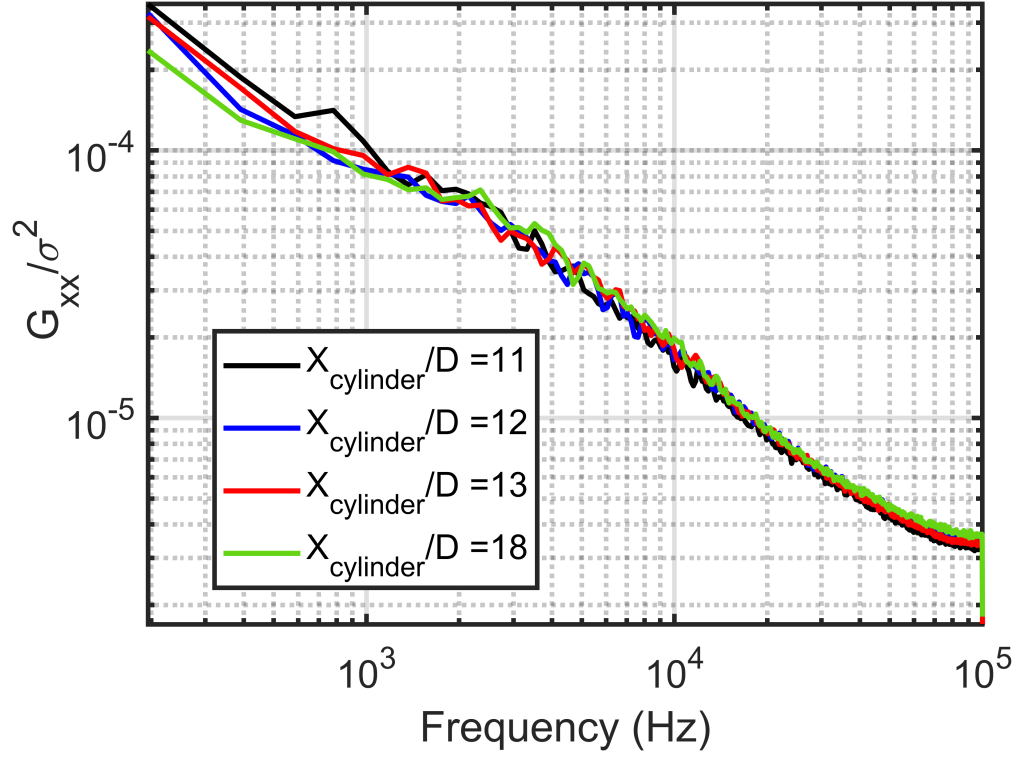


Figure 4.11: Global power spectral density averaged across all schlieren data in all steady state frames for each cylinder shock generator position.

Table 4.4: PSD Probe Stations

X_{cylinder}/D	x/d Range	y/d Range	Step Size
Upstream Influence	-2.5 to -1.5	0.5	0.2
Separation Shock	-1.5 to -0.5	0.3	0.2
Boundary Layer	-3 to -1.6	0.05	0.2

and y/d) for this analysis are tabulated. The same data extraction methodology and probe stations are applied to all four cylinder shock generator positions.

For this analysis, the parameter G_{xx}/σ^2 was used. The output of MATLAB's `pwelch` command, G_{xx} , denotes the power spectral density in squared magnitude units of the time series data per Hertz. For the schlieren data, the output is in squared intensity per Hertz, where the intensity is proportional to density. The division of the PSD by the variance of the original signal in the time domain normalizes the output such that its integral over all frequencies is 1. This scaling effectively represents a probability density function of the power distribution (per Hertz) having normalized by the total power. Experimentally, the PSD output from a single pixel in time is divided by the variance of the pixel intensity in time. The parameter G_{xx}/σ^2 is then integrated with respect to frequency such that the resulting normalized and non-dimensional power spectrum describes the percentage of the overall power contribution for each frequency bin. Each pixel in the image has a PSD computed based on its temporal information. For the global spectra, the PSDs are summed and divided by the number of signals, or pixels, that act as individual sensors. This ensures the global PSD remains on the same scale as individual PSDs and operates as different measurements of the same fluid features. This would also report the information in terms of the percentage of unsteadiness contained in the selected frequency band.

Before averaging the power spectral density over the entire high-speed schlieren frame, the resulting amplitudes can be remapped back to the original image domain to create a spectral heat map. Since the PSD of each pixel in the images is a function of frequency, these data are best plotted in regards to a specified frequency band to analyze the associated SBLI features and their relative magnitude. Data viewed using this method may allow for a better understanding of the dynamics associated with characteristic SBLI features by viewing the spatial distribution of the spectral content.

Four specified frequency bands were investigated. These frequency bands are from 0-3 kHz, 3-6 kHz, 6-30 kHz, and 30-60 kHz. The first spectral heat map range uses the relatively low frequency band from 0 to 3 kHz and is shown in Fig. 4.12 for all $X_{cylinder}/D$ locations. Within this band, the contour map shows a notable increase in spectral content associated with the upstream influence, in contrast to the incoming freestream. There is also high

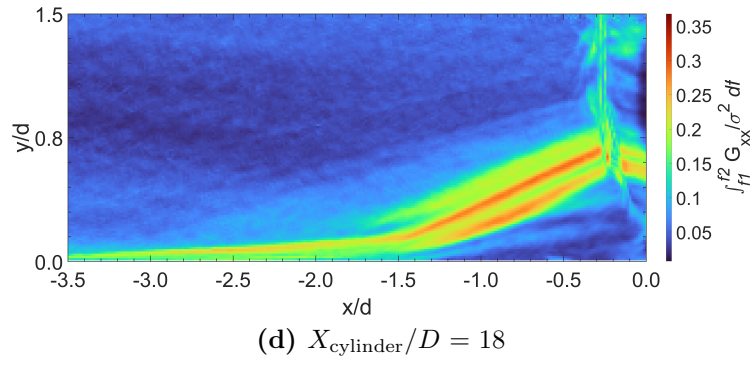
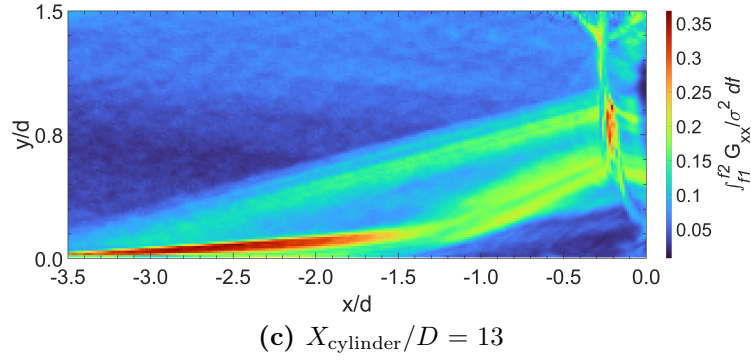
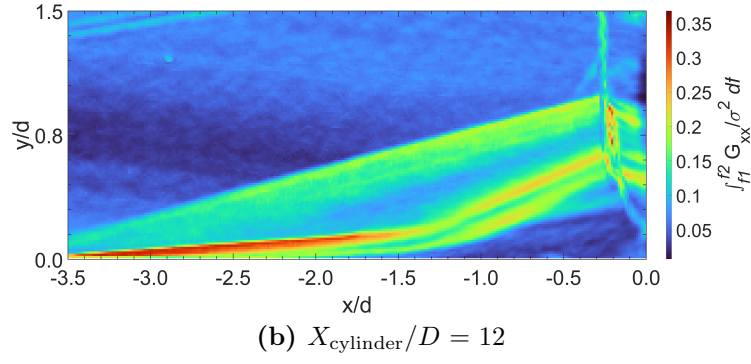
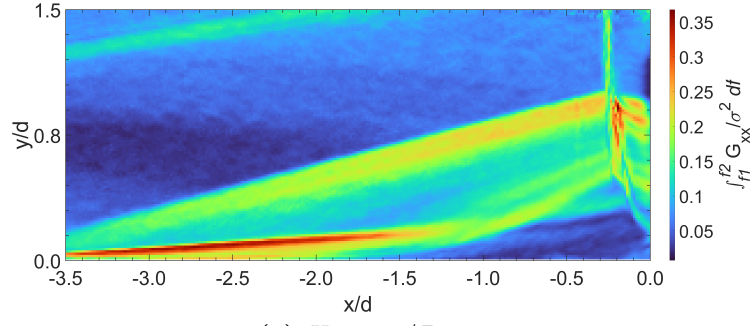


Figure 4.12: Spectral heat map for all cases in the 0-3 kHz band.

amplitude content in the upper portion of the boundary layer, which is likely related to the UI's shock foot movement. Finally, there are two distinct portions of high amplitude unsteadiness in the vicinity of where the separation shock is located.

In the transitional cases, there is high amplitude, low frequency content associated with the small-scale movement of the UI that intersects the cone surface at $x/d = -3.7$ to -3.5 . This relative amplitude abates as the cylinder shock generator is moved further downstream on the cone from $X_{\text{cylinder}}/D = 11$ to 12 ; however, as the cylinder is moved from 12 to 13 diameters from the leading edge, the region of unsteadiness surrounding the UI narrows while its relative amplitude in this band increases. Further downstream at $X_{\text{cylinder}}/D = 18$, the SBLI becomes turbulent, and one would expect for the UI to diminish (with only spurious appearance) as can be seen here. The higher relative amplitudes of unsteadiness in this band of frequencies suggest the UI exhibits distinct low frequency behavior under certain conditions (e.g., a transitional boundary layer) for this type of SBLI. Global PSDs that show high relative amplitude peaks may be dependent on the duration of the run spent at these x/d positions and the mitigation of freestream disturbances that would force a shock foot position change. It is possible freestream fluctuations may cause deviations from this behavior of the UI via a frequency shift if the shock foot is just upstream and downstream.

The three transitional cases show high relative amplitude content present within the boundary layer, which is likely correlated with the UI shock foot position. Generally, the UI shock thickens and strengthens as it is pushed downstream until the shock foot is expected to correspond to an x/d location where the features are traditionally labeled as a leading lambda shock foot. Tester et al. [104] found a boundary-layer thickening precursor based on the movement of the UI, and this region in the spectral heat maps may be indicative of an increase in the boundary-layer thickness occurring. Clemens and Narayanaswamy [17] have theorized that the boundary-layer thickness and the shock foot position may both shift in an attempt to control changes in momentum.

Another notable feature in the band spanning $0\text{-}3$ kHz can be found in the vicinity of the forward lambda shock foot associated with the SBLI. It is clear from observing the raw data that this shock foot oscillates over a finite region. The unsteady motion of the shock foot has clearly been captured in these spectral heat maps as there are two local maxima

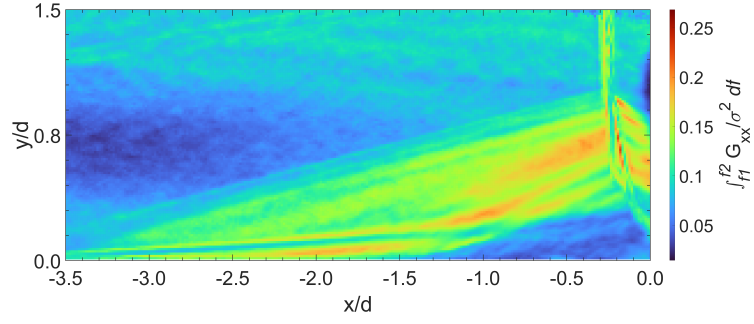
in the integrated PSDs which surround the extreme positions (i.e., the most upstream and downstream locations) of this shock. It was possible that these locations are, in fact, the upstream influence and separation shock with the UI's collapse near the separation shock, as previously suggested in the literature by Leidy et al. [60]. This behavior was visualized in the high-speed schlieren imaging. The amplitude of the high-amplitude separation shock positions increases as a function of the downstream cylinder shock position.

Evaluating the spectral heat maps in a slightly higher frequency band of 3-6 kHz, typical dynamic features of the upstream influence and separation shock are viewed over all four observed cases in Fig. 4.13. Higher amplitudes are now associated with the movement of the interaction shock features in the inboard x/d locations between a typical separation shock location and the most upstream UI x/d positions. This frequency band may provide insight into the sweeping behaviors of SBLI. It should be noted that the spectral content shown does not differentiate between upstream and downstream movement, and future work may find separating this content favorable.

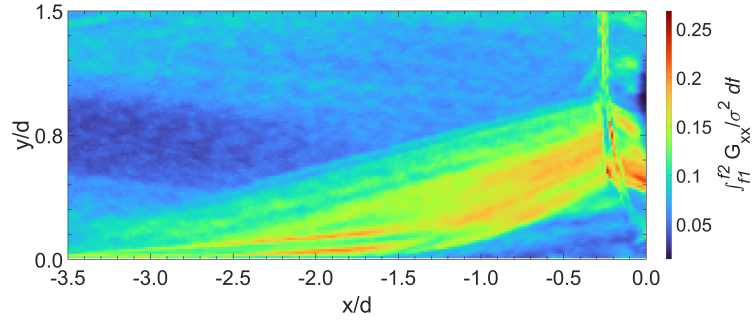
In this frequency band from 3-6 kHz, the late transitional case and turbulent case have much higher amplitudes associated with the separation shock location and upstream in the incoming boundary layer. As the interaction becomes more turbulent, the upstream extent decreases and the strength of this region increases.

In review, the upstream influence of most outboard positions is more strongly connected to low frequency content in the 0-3 kHz band. The separation shock also has most of its dynamic content occupied in the low frequency band of 0-3 kHz. However, frequency content associated with the separation shock appear at similar magnitudes across both regions, just at different spatial positions. This data emphasizes how sensitive traditional instrumentation such as pressure transducers may be to their relative position; the unsteady content could be perceived at different frequencies, and the peaks in the spectra may appear in different forms (e.g., sharp or broadband peaks in the PSD).

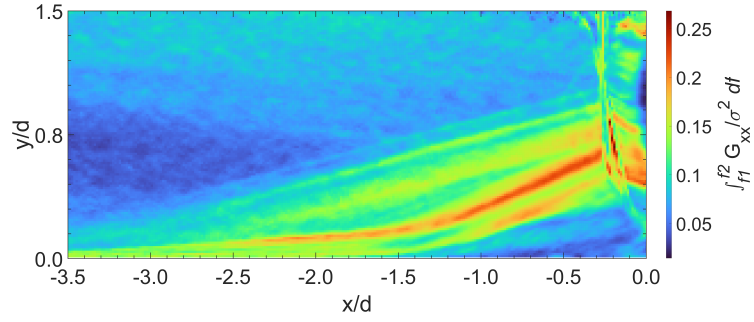
Spectral heat maps of frequency content in the 6 to 30 kHz band are displayed in Fig. 4.14. For all four cylinder shock generator cases, the dominant feature appears to be associated with the vortices on the shock generator face. Specifically, this high amplitude region appears just above the jet impingement location on the cylinder shock generator for the upstream



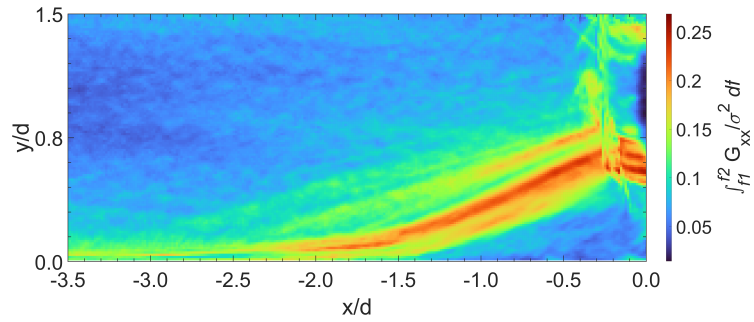
(a) $X_{\text{cylinder}}/D = 11$



(b) $X_{\text{cylinder}}/D = 12$



(c) $X_{\text{cylinder}}/D = 13$



(d) $X_{\text{cylinder}}/D = 18$

Figure 4.13: Spectral heat map for all cases in the 3-6 kHz band.

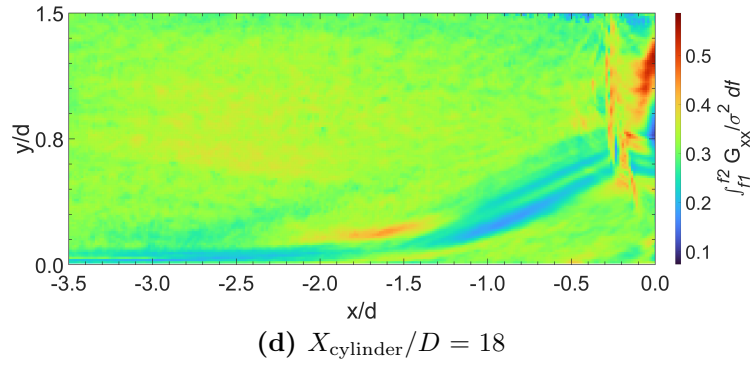
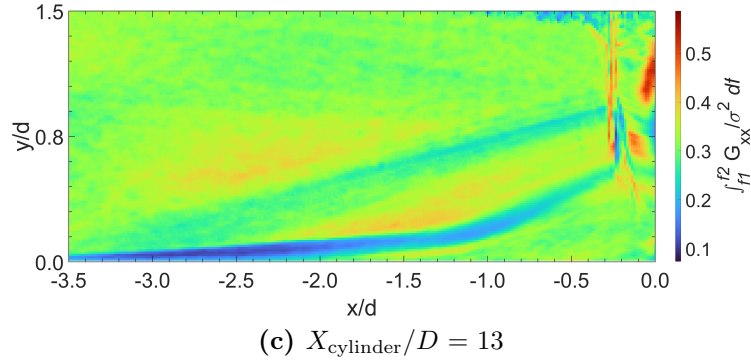
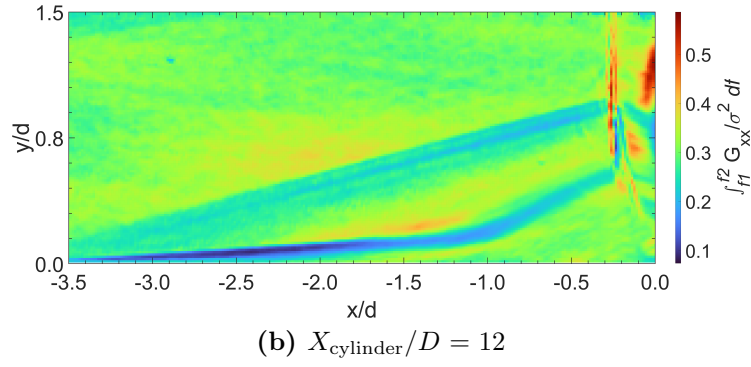
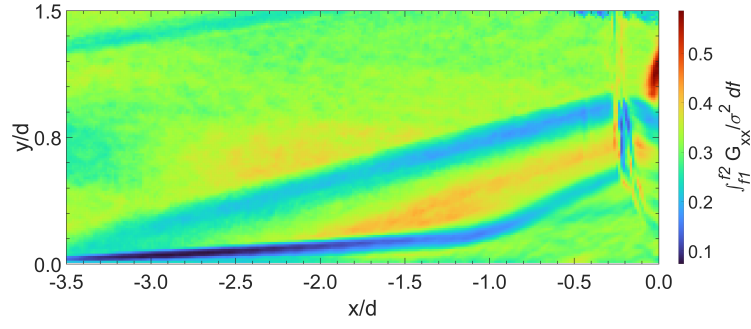
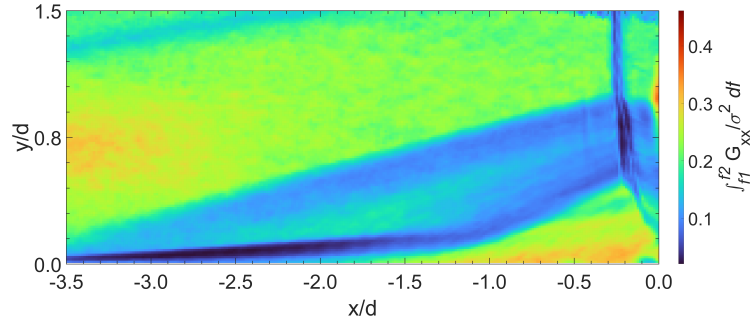


Figure 4.14: Spectral heat map for all cases in the 6-30 kHz band.

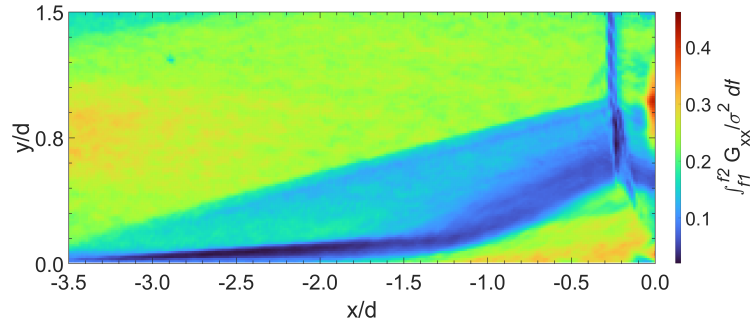
influence in all three transitional cases and for the separation shock for the turbulent case. This vortex region exists between $y/d = \sim 0.97$ to 1.25 at $X_{\text{cylinder}}/D = 11$. This range expands slightly to $y/d = 0.92$ to 1.25 for the other two transitional cases. The vortex region for the turbulent case, at $X_{\text{cylinder}}/D = 18$, shifts upward from $y/d = 0.1$ to 1.3 . This vortex region maintains its spectral amplitude across all shock generator locations even though the separation shock is known to increase in strength as the interaction becomes more turbulent. These vortices originate with the jet impingement and triple point location that drop towards the model surface as the interaction becomes more turbulent. This information suggests these vertical structures share a relationship between their relative size, location, and perceived velocity as they travel up the face of the cylinder shock generator. There is some residual frequency content located between the UI and the separation shock that decay as the cylinder shock generator is shifted downstream.

Another spectral heat map of the shockwave / boundary-layer interaction at all four downstream positions of the cylinder shock generator is displayed in Fig. 4.15. This figure displays the high frequency turbulent content at 30-60 kHz. Notably, the vortex as a result of the triple point's jet impingement on the surface of the cylinder is the dominant feature within these data. SBLI unsteadiness is not expected to be visible in this frequency band. However, associated SBLI features, such as the vortex on the cylinder shock generator face above the triple point and freestream turbulence, have frequency content at this higher frequency band that can help to infer what the system is doing as a whole. At $X_{\text{cylinder}}/D = 11$, the region encompassing this vortex is visible at a y/d height of 0.9 to 1.1 , though the region is very thin in the streamwise direction. At $X_{\text{cylinder}}/D = 12$, the region associated with this vortex is larger and spans a y/d extent from about 0.7 to 1.15 , suggesting that the overall SBLI features including the triple point, UI and separation shock are more often collapsed than in the previous case and experience a much larger range of travel. This region remains nearly the same size at $X_{\text{cylinder}}/D = 13$, though for this case, the region is more evenly distributed, possibly indicating a more consistent travel velocity in the region.

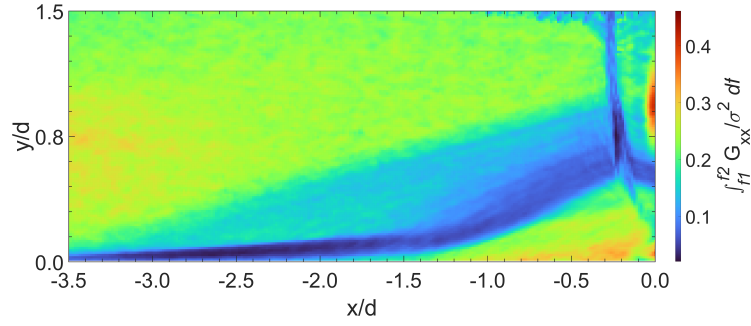
The vortex region on the cylinder face seems to align more with UI and separation features. The previous case suggests a bimodal distribution with a clear upstream and downstream location wherein the UI or separation shock may remain at longer instances of



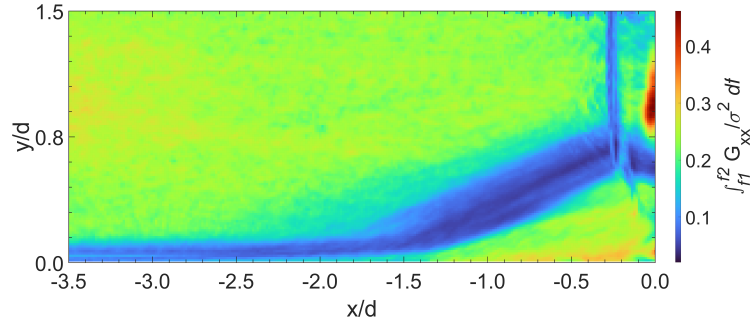
(a) $X_{\text{cylinder}}/D = 11$



(b) $X_{\text{cylinder}}/D = 12$



(c) $X_{\text{cylinder}}/D = 13$

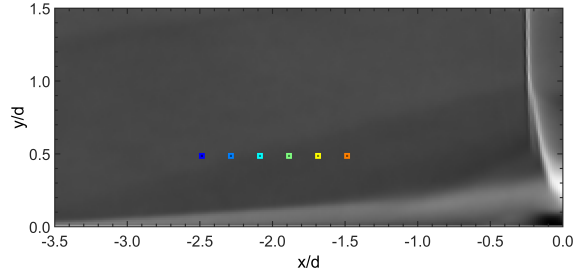


(d) $X_{\text{cylinder}}/D = 18$

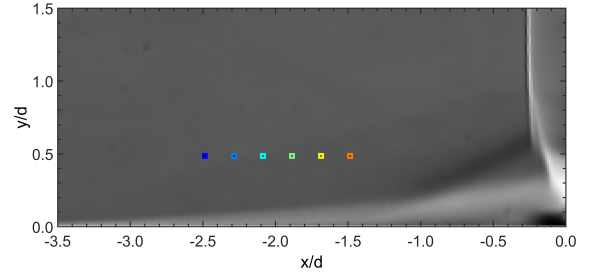
Figure 4.15: Spectral heat map for all cases in the 30-60 kHz band.

time. Finally, at the most downstream location, $X_{\text{cylinder}}/D = 18$, where the interaction is fully turbulent, the vortex region exists at $y/d = 0.75$ to 1.2 with a significantly larger area in the streamwise direction. This suggests that a larger vortex exists between $y/d = 0.8$ to 1 and that it is associated with the separation shock triple point and the consistent presence of the impinging jet at the lower height. At $X_{\text{cylinder}}/D = 12$, a bimodal distribution is apparent and the interaction still has the same range of travel but is more likely to be collapsed with the typical location of the separation shock and absence of the UI for most time instances. These same vortex structures were visualized earlier in the 10-30 kHz band, although they had nearly double the magnitude associated with them, and they were located higher on the cylinder for the transitional cases due to the jet impingement from the upstream influence. The higher amplitude, lower frequency content may indicate that the vortices reduce in frequency and increase in strength as they travel up the surface of the cylinder shock generator. This may be a product of their relative size and velocity decreasing. Therefore, frequency is a function of the triple point impingement strength or relative height on the cylinder shock generator at its time of generation. In this band, there are some higher amplitudes in the spectral content associated with the freestream, which are likely an artifact of the LED light's location indicated by the intensity drop off from the centralized location. In this schlieren based system, the intensity variations in the LED would be proportionally related to the system's sensitivity to variations in density. The gradient would not be indicative of variations in the unsteadiness physics but of the system's ability to measure those variations consistently with equal amplitude across every pixel. As the cylinder shock generator is moved downstream, the most forward-reaching part of the separation region also shifts downstream. The region is seen to start at roughly $x/d \approx -1.5$ and translate downstream to $x/d \approx -1.3$, for $X_{\text{cylinder}}/D = 12$ and 13 and, then to $x/d = -1.2$ for the turbulent case for $X_{\text{cylinder}}/D = 18$.

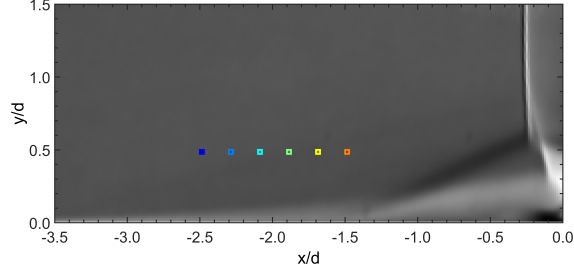
The first individual probe series is displayed in Fig. 4.16, where the placement of the probes is intended to investigate the spectral content of the upstream influence. The PSDs were taken at x/d locations spanning from -2.5 to -1.5 , at interval of $x/d = 0.2$, and at a constant y/d location of 0.5 . The power spectral densities were averaged over a 2-pixel by 2-pixel area. The color map of the pixel stations matches their corresponding PSDs. For



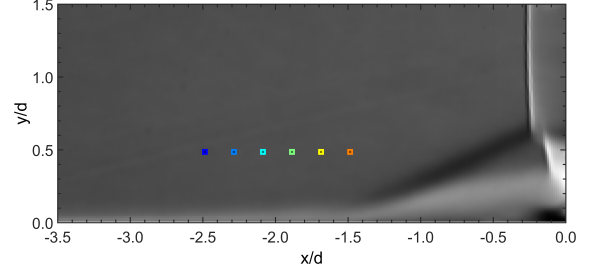
(a) $X_{\text{cylinder}}/D = 11$



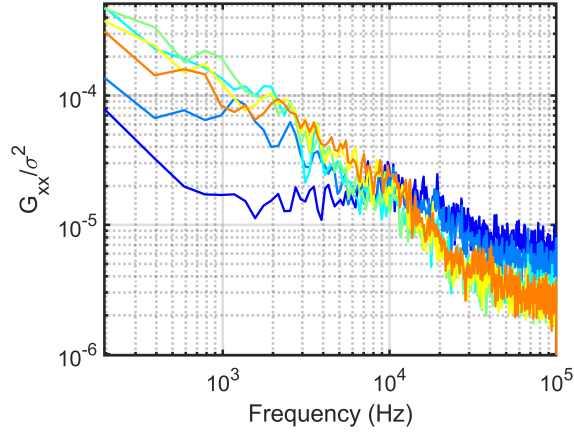
(b) $X_{\text{cylinder}}/D = 12$



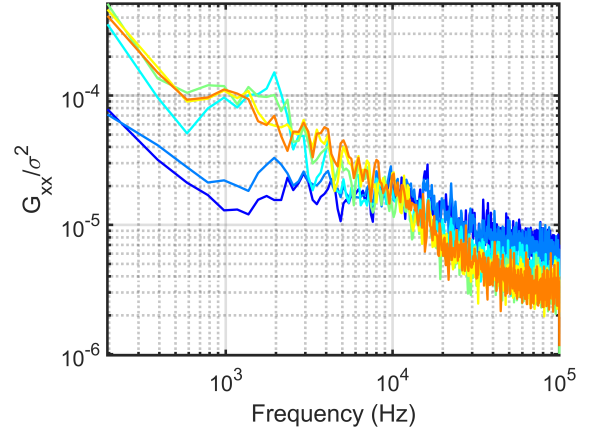
(c) $X_{\text{cylinder}}/D = 13$



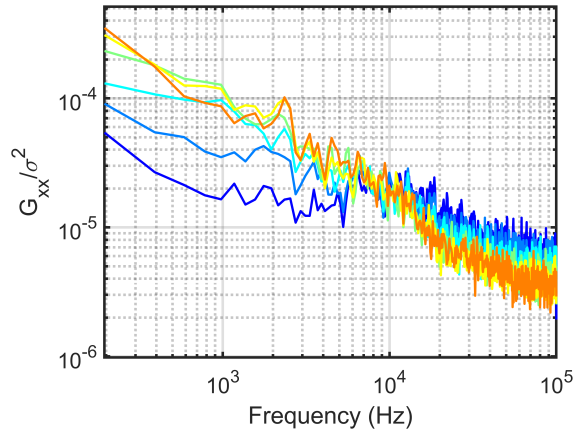
(d) $X_{\text{cylinder}}/D = 18$



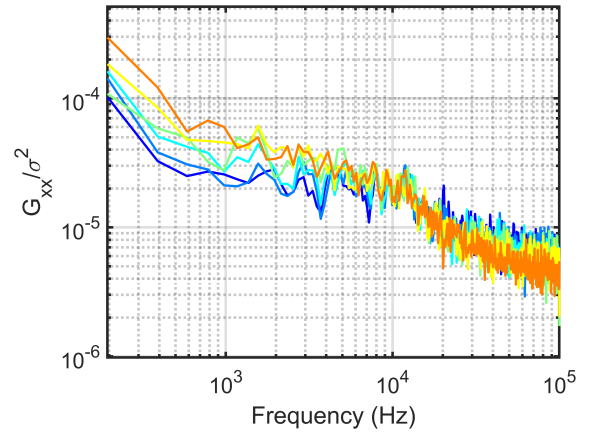
(e) $X_{\text{cylinder}}/D = 11$



(f) $X_{\text{cylinder}}/D = 12$



(g) $X_{\text{cylinder}}/D = 13$



(h) $X_{\text{cylinder}}/D = 18$

Figure 4.16: Power spectral density analysis for the upstream influence.

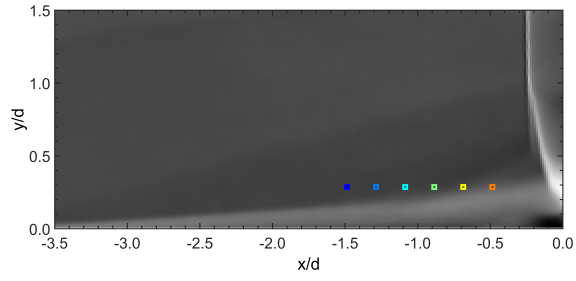
this case, horizontally spaced probes were staggered to examine changes in the frequency content associated with the upstream influence.

At $X_{\text{cylinder}}/D = 12$, a distinct local peak is seen at 2000 Hz for the probe at $x/d = -2.1$ (cyan). The upstream station at $x/d = -2.3$ (light blue) shows a broader peak also centered at a frequency of 2000 Hz at this cylinder shock generator location. At an X_{cylinder}/D position of 13, a localized peak at 2300 Hz is also visible over PSD stations from -2.1 (cyan) to -1.5 (orange). At the upstream stations, there is a clear decay in both the amplitude and shape of the PSD in the vicinity of this frequency. The amplitudes of this peak are slightly higher than the similar peak for the previous cylinder shock generator position.

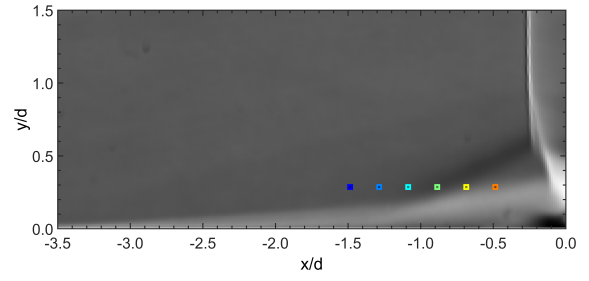
For all four SBLI cases, the most upstream station (dark blue) shows similarity in the overall trends, indicating its position clearly in the freestream and outside of the upstream influence. The turbulent case further corroborates this as all spectra begin to fall within a similar band and exhibit a similar distribution of the content in comparison to the most upstream probe station at $x/d = -2.5$ (dark blue). For all X_{cylinder}/D positions, the PSD probe located at a position of $x/d = -2.3$ (light blue) appears the energy distribution is more closely similar to the upstream behavior (dark blue line) but shares some of the localized peaks of the closest downstream PSD station (cyan). The overall energy in the spectral domain is between that of the freestream and spectral probes that are known to be in areas where the UI is a prominent feature.

A second power spectral density probe series was set to capture frequency content associated with the separation shock foot and can be seen in Fig. 4.17. The probe stations were adjusted to span from $x/d = -1.5$ to -0.5 (spaced by $x/d = 0.2$) along a constant y/d location of 0.3. The higher y/d probe station was chosen to examine the unsteadiness of this SBLI structure at all cylinder locations without requiring the reference probe stations to be moved, as the separation shock slightly adjusted its position. This higher y/d location also allows for an interrogation of the collapsed upstream influence locations that appear just slightly upstream of the separation shock in the 0 to 3 kHz band from the spectral heat maps that have weaker spectral content closer to the boundary layer.

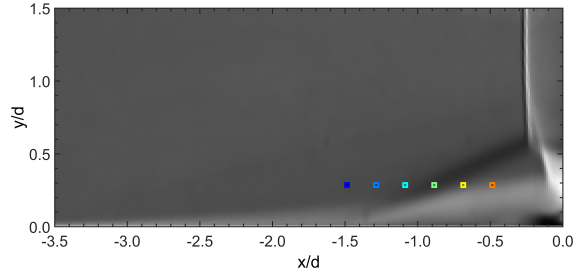
At $X_{\text{cylinder}}/D = 11$, the probe stations at $x/d = -1.3$ and -1.1 (i.e., the light blue and cyan points, respectively) have a similar energy distribution around 1 kHz that seem



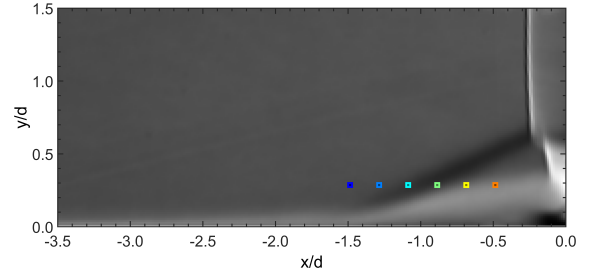
(a) $X_{\text{cylinder}}/D = 11$



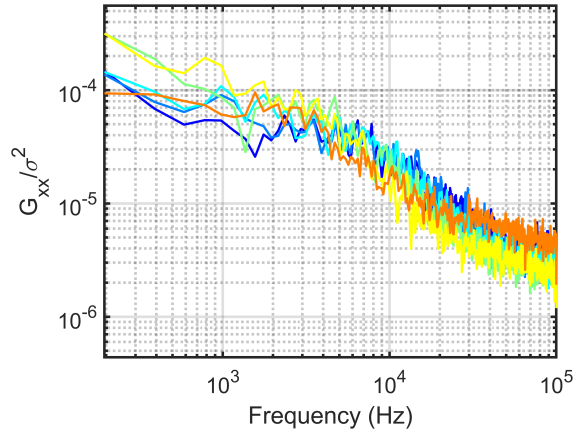
(b) $X_{\text{cylinder}}/D = 12$



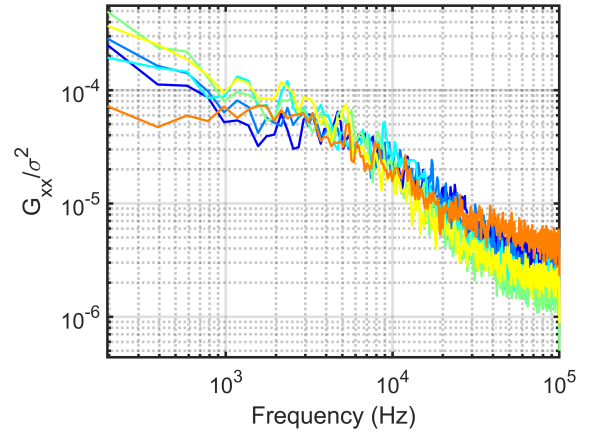
(c) $X_{\text{cylinder}}/D = 13$



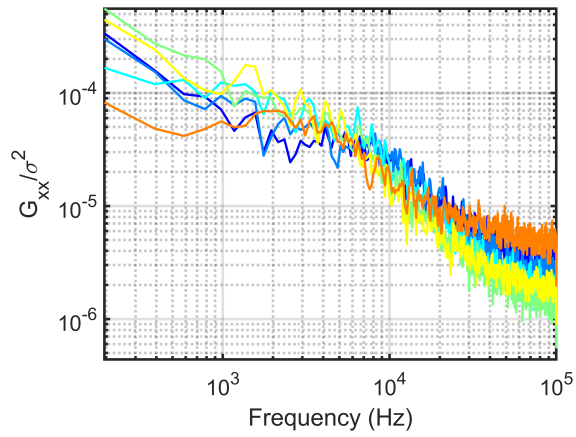
(d) $X_{\text{cylinder}}/D = 18$



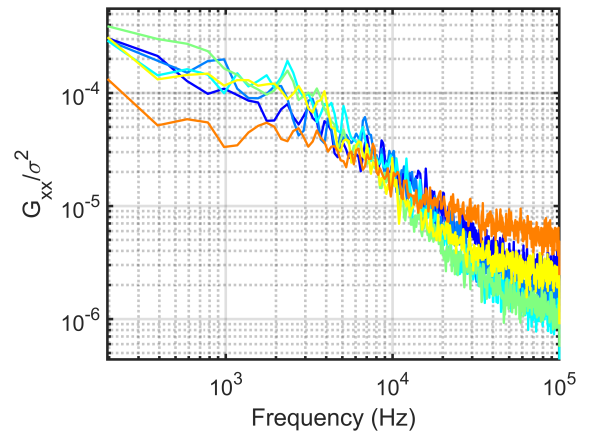
(e) $X_{\text{cylinder}}/D = 11$



(f) $X_{\text{cylinder}}/D = 12$



(g) $X_{\text{cylinder}}/D = 13$



(h) $X_{\text{cylinder}}/D = 18$

Figure 4.17: Power spectral density analysis near the separation shock.

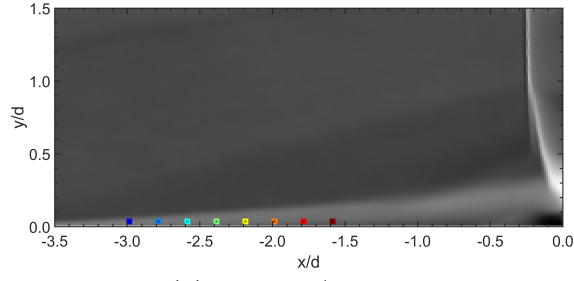
to be indicative of the upstream influence's presence based upon the previous spectra in Fig. 4.16. The cyan station shows significant random fluctuations in the amplitude in the spectrum, obscuring any notable spectral content at this probe station, likely a byproduct of the dynamic UI and the separation shock behavior at this cylinder shock generator location. However, further downstream, the green spectrum ($x/d = -0.9$) exhibits increased amplitudes over a broadband band, ranging from approximately 2 to 6 kHz, before undergoing a more rapid decay in the high frequencies. The broadband content represents a shift in the unsteadiness to higher frequencies. At $X_{\text{cylinder}}/D = 12$, the cyan and yellow spectra ($x/d = -1.1$ and -0.7 , respectively) show higher peaks with matching amplitudes at frequencies of 1.2 kHz and about 2.2 kHz. Between these two probe stations is the green spectrum ($x/d = -0.9$), which has no distinct characteristics compared to its neighboring probes. At an X_{cylinder}/D location of 13, a localized peak is seen at the x/d probe station of $x/d = -0.7$ at 1.5 kHz (the yellow line). Particularly, the cyan and light blue spectra ($x/d = -1.1$ and -1.3 , respectively) display a localized rise also in this frequency band that spans from 1 to 1.5 kHz before devolving into random variations in the amplitude at higher frequencies. The dark blue ($x/d = -1.5$), show characteristics similar to freestream turbulence spectra as were seen in the other cases; meanwhile, the green spectrum ($x/d = -0.9$) seems to be picking up both the UI and separation shock in this case and averaging out any other notable features. The turbulent case at $X_{\text{cylinder}}/D = 18$ displays a distinct peak around 2.5 kHz for probes at x/d locations of -0.9 and -1.1 (cyan and green), which are located on either side of the average separation shock position. The lighter blue spectrum is extracted from $x/d = -1.3$, a location where a behavior associated with a collapsed upstream influence would be expected to reappear, and in this case, it has a peak associated with a slightly lower frequency of around 1 kHz. Both the dark blue and orange spectra ($x/d = -1.5$ and -0.5 , respectively) show no distinctive features other than broadband random variations in the amplitude, indicative of turbulence.

Murphree et al. [73] conducted experiments on a cylinder shock generator mounted to a flat plate in a Mach 4.95 freestream. Their frequency analysis of a series of pressure probes revealed that high amplitude unsteadiness exists at ~ 2000 Hz at locations of $x/d = -5.83$ and -3.83 , and this is likely associated with the upstream influence. Moreover, frequency content

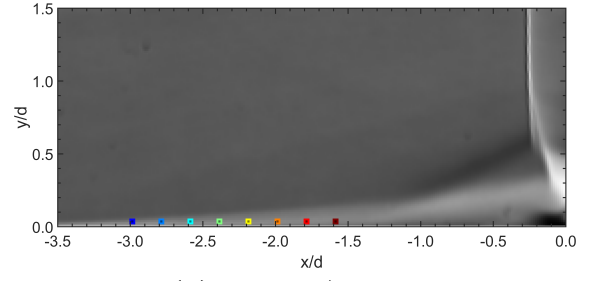
of around 1200 Hz was identified at an x/d location of -2.17, shifting to approximately 3000 Hz at $x/d = -1.83$. The results from Murphree et al. [73] match the data seen here well.

Power spectral densities were extracted from the boundary layer to evaluate if any upstream frequency content could be identified that would possibly feed the unsteadiness in the shockwave / boundary layer interaction. Probes were placed at x/d locations from -3 to -1.5 at intervals of $x/d = 0.2$, and each has a constant y/d location of 0.05. These data can be seen in Fig. 4.18. For these probe stations, the upstream stations would have a higher probability of separation shock foot unsteadiness influencing the frequency measurements in the boundary layer due to the boundary-layer thickness.

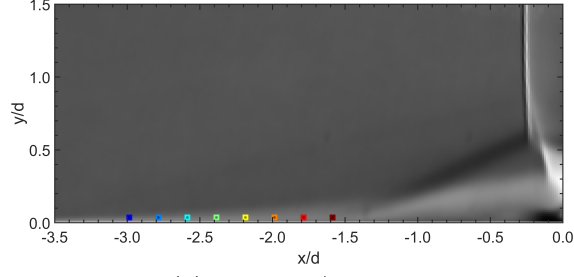
For each of the shock generator positions, there is a gradual change in the spectral content that depends primarily upon the probe positions. At a $X_{cylinder}/D = 11$ position, the spectra seem to overlap at 3.5 kHz. At lower frequencies, the amplitudes decrease as the probe station is moved downstream. However, at higher frequencies, this trend reverses and the amplitudes increase as the probe station moves downstream. This suggests that the interaction's frequency data is related to the interaction's incoming boundary layer state, relative length scale in time, and sub 3 kHz data is more likely to be emphasized at upstream positions and high frequency information at downstream locations. At the next cylinder shock generator location, $X_{cylinder}/D = 12$, the upstream PSD station shows higher amplitude unsteadiness centered about 2 kHz. Recall the spectral heat maps in the 0-3 kHz band, where known dynamic content associated with the UI and separation shock was seen to be present. The individual probe stations previously examined have also shown a 2 kHz peak exists associated with the UI. The forward lambda shock foot also had some frequency content around 2 kHz, but not in a distinct manner. This probe is likely tied to the UI due to the distinct 2 kHz frequency. At $X_{cylinder}/D = 13$, all spectra show an amplitude increase in the 1 to 2 kHz band. Around 2.3 kHz the amplitudes generally increase in magnitude until the second and third PSD probe positions (i.e., the blue and cyan lines, respectively) before decreasing in magnitude at each subsequent downstream position. The three most downstream positions show a localized peak at a frequency around 1.5 kHz with nearly equivalent magnitude. The turbulent SBLI case shows a significant amount of random fluctuations in the amplitude of the spectral content. Overall, the spectra appear



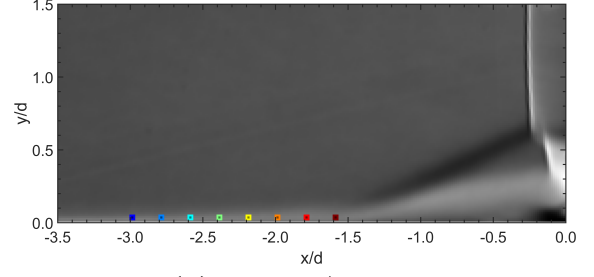
(a) $X_{\text{cylinder}}/D = 11$



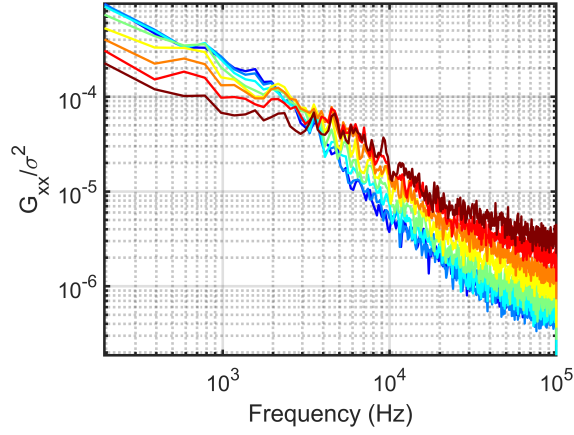
(b) $X_{\text{cylinder}}/D = 12$



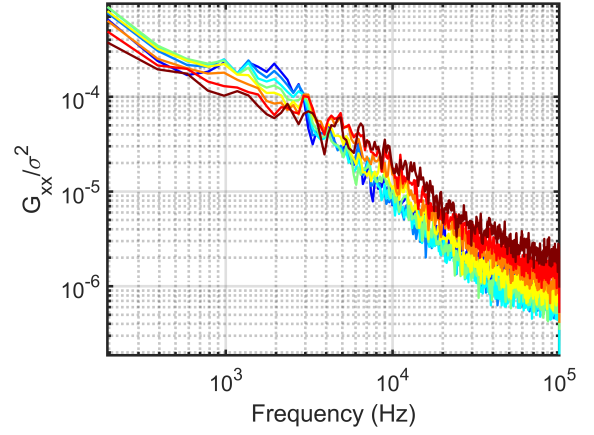
(c) $X_{\text{cylinder}}/D = 13$



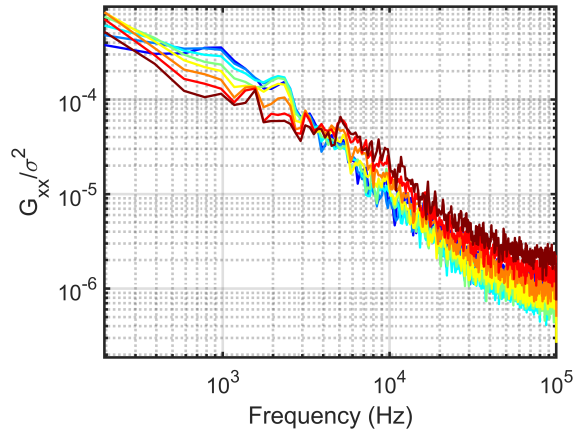
(d) $X_{\text{cylinder}}/D = 18$



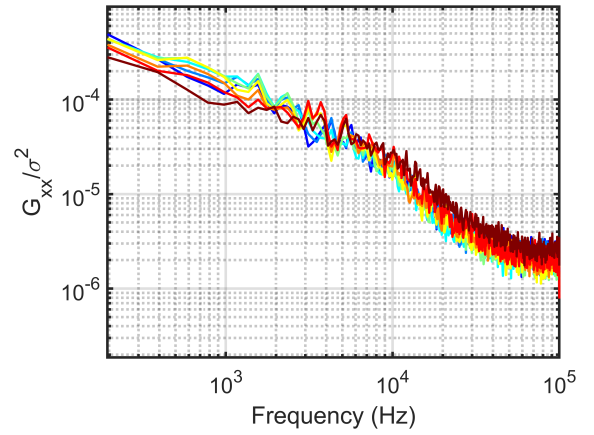
(e) $X_{\text{cylinder}}/D = 11$



(f) $X_{\text{cylinder}}/D = 12$



(g) $X_{\text{cylinder}}/D = 13$



(h) $X_{\text{cylinder}}/D = 18$

Figure 4.18: Power spectral density analysis in the boundary layer.

consistent in their overall amplitude and slope across the multiple probe stations with no notable features. These features and the increased variations in amplitude for each probe station simply indicate the turbulent content for this case. The high frequency content of the boundary layer is shown to collapse as a function of the downstream cylinder shock generator location.

The prominent identified peaks are tabulated with their respective frequencies and Strouhal numbers in Table 4.5. The Strouhal number is calculated based on Eqn. 4.1.

$$St = fL/U \quad (4.1)$$

Here, U represents the fluid velocity, and the length scale used in these calculations is the separation length, which facilitates comparison with archival data in the literature. In this experiment, the freestream velocity is 663.47 m/s and U_c is 623.8 m/s, which is used in the Strouhal calculation below. Strouhal numbers of approximately 0.01 to 0.2 are indicative of “shock breathing” motion. Strouhal numbers of 1 are expected for phenomena associated with incoming freestream turbulence and would be primarily identifiable upstream of the interaction. Vortices and other large-scale turbulent structures like Kelvin-Helmholtz instabilities in the separation region are expected to have Strouhal numbers of 0.1-0.5 [17, 20]. All Strouhal numbers calculated for the interactions investigated throughout this study correspond to shock breathing motion.

Notably, there are patterns in the spectral content that appear to resemble extraneous shockwaves above the interaction. These can be seen in all four cases above the triple point upstream of the bow shock. These “features” are expected to be a byproduct of diaphragm particles impacting the cylinder shock generator face and disrupting the bow shock’s structure. An instantaneous image of one such event is displayed in Fig. 4.19. Their area of influence on the SBLI is well seen in the spectral heat maps, and while it is expected that these rare occurrences will contribute to the data, their impacts were found to be localized. Additionally, the frequencies identified within the primary interaction area match the literature and correspond to shock breathing motion. Therefore, the impacts of these particle strikes are not expected to disrupt the local PSDs. Further processing identified

Table 4.5: Summary of SBLI frequency content and conditions.

Probe station	Run Number	X_{cylinder}/D	L_{sep} [m]	Frequency [Hz]	Strouhal
Upstream Influence	23	11	15.24	1171.88	0.0286
	15	12	16.51	1953.12	0.0517
	17	13	19.05	2343.75	0.0716
Separation Shock	15	12	16.51	1171.88	0.0310
	15	12	16.51	2343.75	0.0620
	17	13	19.05	1367.19	0.0418
	19	18	17.78	2343.75	0.0668
Boundary Layer	15	12	16.51	1953.12	0.0517
	17	13	19.05	1562.5	0.0477
	19	13	17.78	2148.44	0.0612

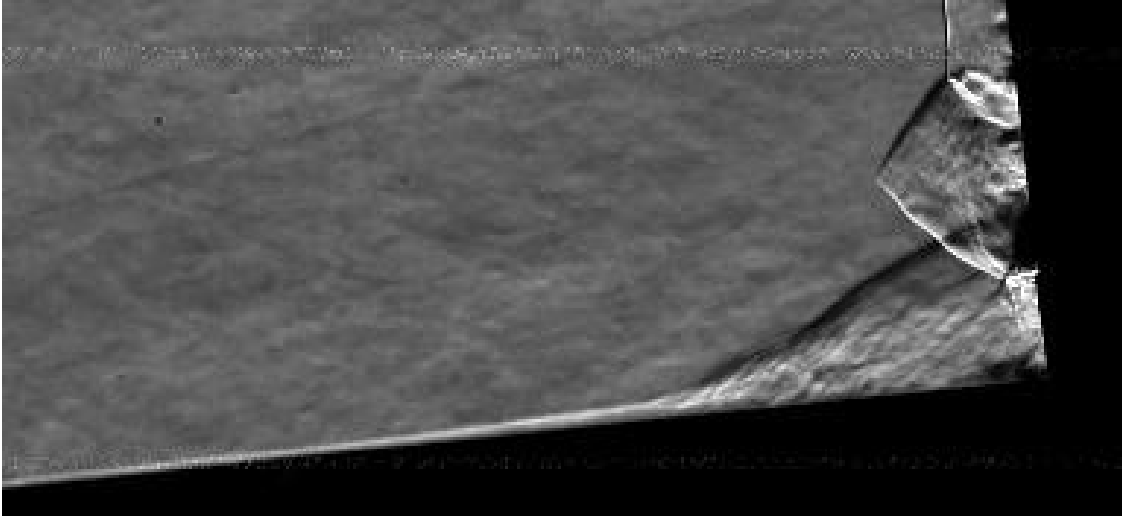


Figure 4.19: Diaphragm particle strike on the cylinder shock generator in Run 15 that disrupts the SBLI structure.

particle strikes during the steady state run time and removed the spectra calculated within the windows corresponding to these time instances. This additional processing step removed nearly all of the anomalous behavior seen in these data and confirmed that any impacts on the spectra of localized probes were minimal. Evidence of the particle impact effects were identified to be present between 1.9% and 7.6% of the total number of frames.

4.3.2 Pressure-Sensitive Paint Data

Power spectral densities were taken of the PSP data set in order to evaluate frequency content in their spanwise direction and further verify spectral content seen in the high-speed schlieren imaging. Power spectral densities displayed here were created using the same methodology as the schlieren images by use of MATLAB's `pwelch` command. A window size of $nfft = 2^8 = 256$ was utilized based on the limited sample length of the data set, which is a total of 2304 frames for each case, to ensure no zero padding was included in the data set. The data set has a limited sampling frequency of 20 kHz based on the pressure-sensitive paint response rate, and the resulting frequency resolution is 86.80 Hz. A 50% overlap was used, resulting in a total of 17 windows that were averaged. The maximum frequency resolved without aliasing in this case is 10 kHz via the Nyquist theorem; however, significant spectral attenuation is expected above 6 kHz as reviewed in the methodology section for this diagnostic technique.

In this section, the spectral content for both transitional and turbulent interaction types for the pressure sensitive paint. The power spectral density values for a frequency band of 0 to 3 kHz and 3 to 6 kHz were remapped back to the original image domain in the form of a spectral heat map. The first frequency band spanning from 0 to 3 kHz is shown in Fig. 4.20. The cylinder shock generator at a downstream position of $X_{cylinder}/D = 11$ has a relatively high amplitude where $\int_0^3 G_{xx}/\sigma^2 = 0.9$. This region begins at a centerline $x/d = -0.6$ and spans to $x/d = -1.1$, where this unsteadiness is associated with forward lambda shock foot location. From $x/d = -1.1$ to -3.5 , there is an elevated amplitude of $\int_0^3 G_{xx}/\sigma^2 = 0.75$ to 0.8, picking up the movement of the upstream influence that rapidly drops off at $x/d = -3.3$. A lower amplitude region associated with the reattachment shock or vortex at the base of the shock generator has an amplitude of 0.5 from $x/d = 0$ to -0.3 . The high amplitude, dark red, regions at the base and upper side of the cylinder are effects of the cylinder's

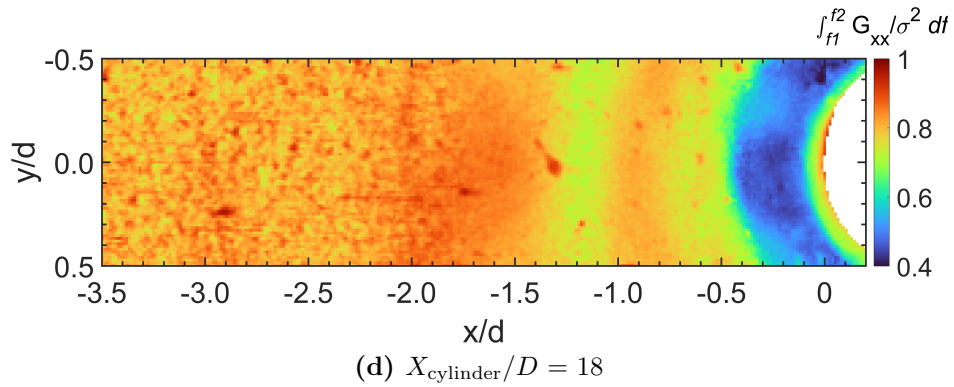
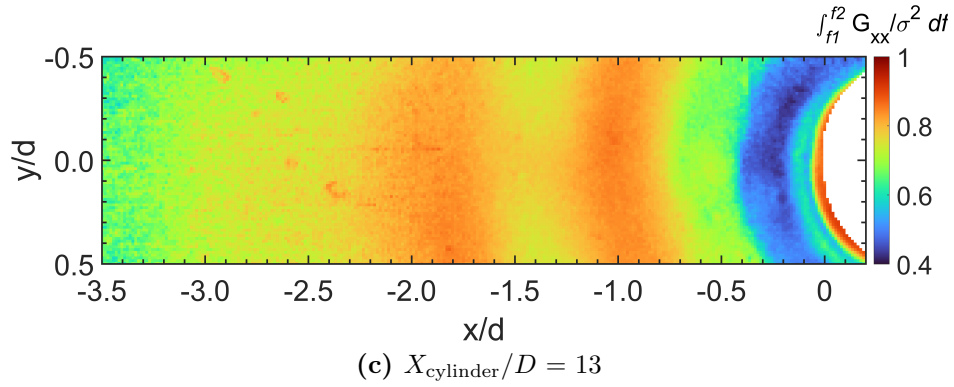
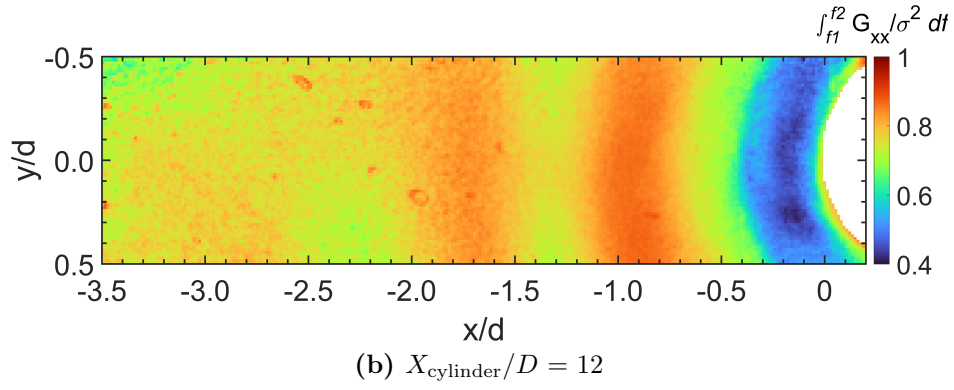
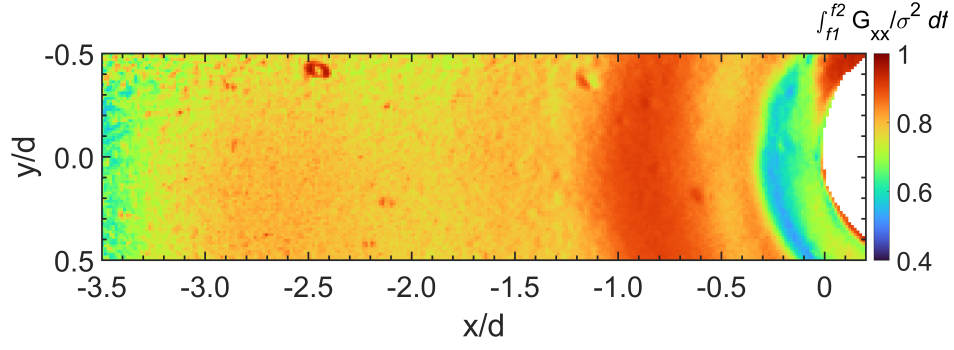


Figure 4.20: Spectral heat map of pressure sensitive paint data over a frequency band of 0-3 kHz.

shadow and do not represent any real physics that are present. For the $X_{\text{cylinder}}/D = 12$ position, a region of higher amplitude unsteadiness spans from $x/d = -0.7$ to -1.1 , and there is a secondary high amplitude location from $x/d = -1.6$ to -2.0 . The secondary, upstream, higher amplitude spectral content location is associated with a collapsed upstream influence location. Meanwhile, the closure shock's lower amplitude region covers a streamwise area from $x/d = 0$ to -0.4 . For the late transitional case at $X_{\text{cylinder}}/D = 13$, the lower amplitude region is seen to span from $x/d = -0.4$ to -0.8 , with high amplitude regions existing between $x/d = -0.8$ to -1.3 and $x/d = -1.6$ to -2.3 . This late-stage transitional case displays the highest amplitude content of all four cases with a similar spectral heat map to the $X_{\text{cylinder}}/D = 12$ case. The fully turbulent case at $X_{\text{cylinder}}/D = 18$ shows dominant amplitudes for this frequency band at two PSD probe stations similar to what was seen in the standard deviation images. The high amplitude range from about $x/d = -0.8$ to -1.1 and from $x/d = -1.4$ to about -2.0 before dissipating upstream. There is a significant amount of upsteadiness in the upstream boundary layer from $x/d = -3.5$ to -2.0 indicative of a turbulent incoming boundary-layer state. Similar to the PSD of the schlieren data, significant spectral content associated with the upstream influence is visible in this frequency band from 0 to 3 kHz.

Spectral heat maps in the 3 to 6 kHz band show very similar pressure distributions of the separation shock and closure shock or vortex region between all cases and can be seen in Fig. 4.21. This aligns well with spectral data seen in the schlieren imaging that displayed a dominance of spectral content associated with the separation shock in the 3 to 6 kHz band. At the most upstream shock generator position, $X_{\text{cylinder}}/D = 11$, relatively higher amplitudes at $\int_3^6 G_{xx}/\sigma^2 \approx 0.25$ to 0.28 in the 3 to 6 kHz band are isolated to the vortex produced by the cylinder shock generator at x/d locations of -0.2 to -0.3 ; meanwhile, the forward lambda shock foot's impression on the surface of the model starts at $x/d = -1.15$. For the $X_{\text{cylinder}}/D = 12$ case, the region around the vortex is much more expanded from the previous transitional case. The high amplitude region associated with the vortex spans from $x/d = -0.05$ to -0.4 and continues out to $x/d = -0.7$ at a lower amplitude. For this case, the pressure rise from the separation shock foot's presence is seen from $x/d = -1.2$ to -1.5 in this frequency band with low relative amplitudes of $\int_3^6 G_{xx}/\sigma^2 \approx 0.22$. The $X_{\text{cylinder}}/D = 13$ case has higher amplitudes from the vortex from $x/d = -0.1$ to -0.5 , and the separation shock

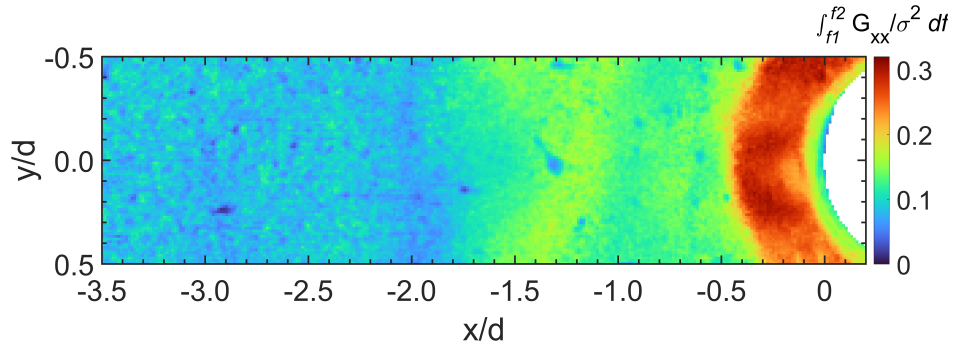
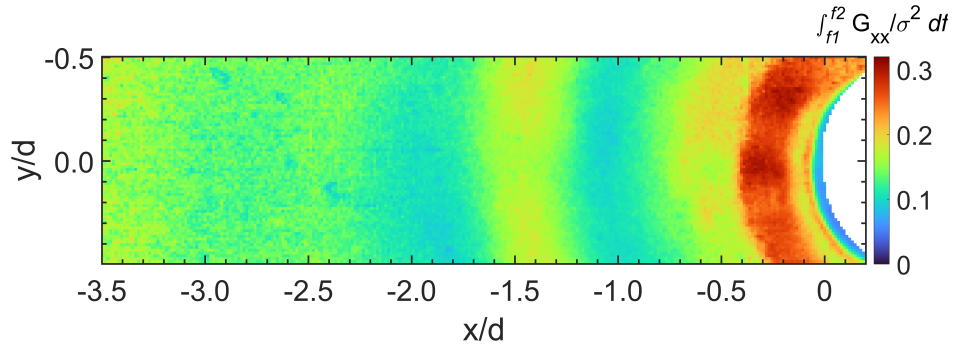
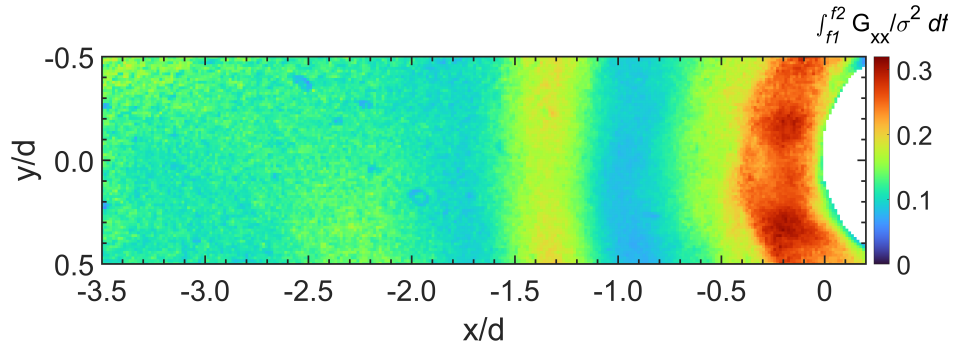
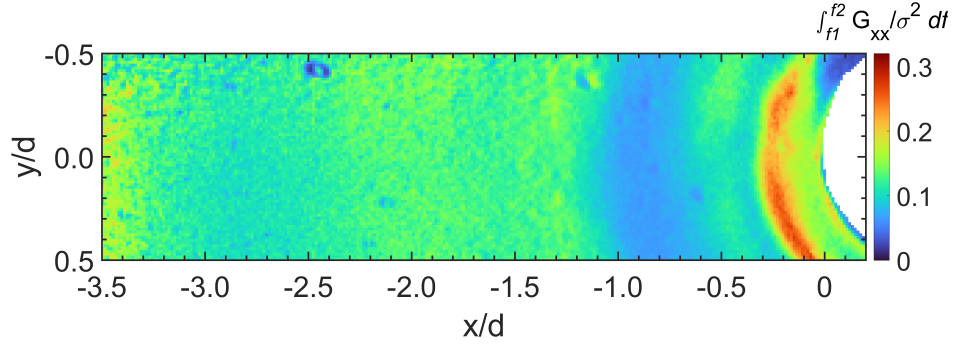
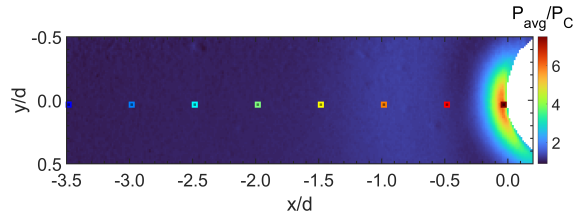


Figure 4.21: Spectral heat map of pressure sensitive paint data over a frequency band of 3-6 kHz.

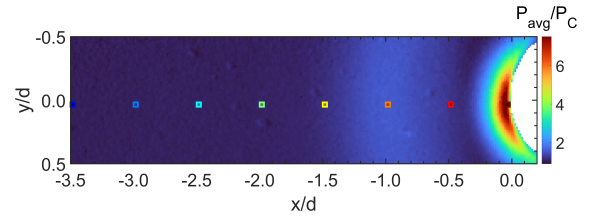
foot varies from $x/d = -1.2$ to -1.6 . Finally, for the turbulent interaction case at $X_{\text{cylinder}}/D = 18$, higher amplitudes associated with the shock-generated vortex span between $x/d = -0.2$ to -0.5 and are seen to have a maximum value of about 0.3, which are higher and with more occurrence than in the transitional interaction. The separation shock foot, in this case, appears to vary from about $x/d = -1.1$ to -1.2 , and there is a significant drop in spectral content in the 3-6 kHz band upstream of $x/d = -1.8$.

Power spectral densities were evaluated over eight streamwise probe stations, centered on the cone. The data have been extracted along the centerline, in increments of $x/d = 0.5$, except for the last station at the base of the cylinder face, which is centered at a position of $x/d = -0.05$. The extracted data are the average of the spectral content over a 2-pixel by 2-pixel area, or an area of 0.0342 by 0.0342 when expressed in physical coordinates (x/d area). These probe stations are plotted over the average images for each shock generator position in parts (a) through (d) of Fig. 4.22. The spectral content for each station is plotted together in parts (e) through (h) in the color corresponding to their stations. Recall, that the pressure sensitive paint significantly attenuates the amplitudes of frequency content above 6 kHz.

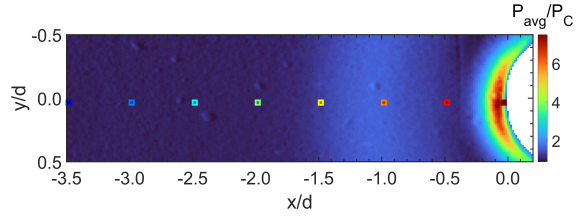
At $X_{\text{cylinder}}/D = 11$, the spectral content associated with this early transitional case is very benign with no significant features outside of a 1.3 kHz localized peak in the two most upstream stations, $x/d = -3.5$ and -3 (dark blue and light blue, respectively), that would be associated with the pressure fluctuations produced by the upstream influence. For the $X_{\text{cylinder}}/D = 12$ case, the spectral content dramatically changes. Spectral probes positioned at $x/d = -0.05$ and -1 (brown and orange, respectively) show elevated amplitude at a frequency of 300 Hz. Spectral heat maps between 0 and 3 kHz displayed also had higher amplitudes in these regions. The closure shock region's geometry and the downstream side of the pressure plateau associated with the lambda shock foot in the standard deviation images mirror one another. This may suggest the behavior in these regions may be correlated at very low-frequencies. Beginning at the PSD probe station of $x/d = -1.5$ (the yellow line) there is increased unsteadiness centered around 400 Hz. This unsteadiness is then seen to be centered around 500 Hz at $x/d = -2.0$ (the green line) and then at 600 Hz for PSD probe stations at $x/d = -2.5$, -3.0 , and -3.5 (cyan, light blue, and dark blue, respectively). This suggests



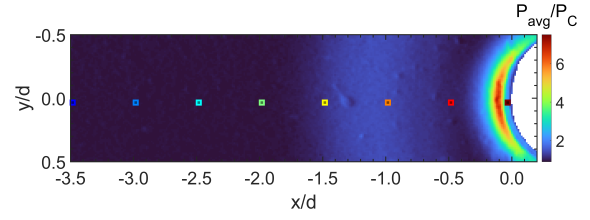
(a) $X_{\text{cylinder}}/D = 11$



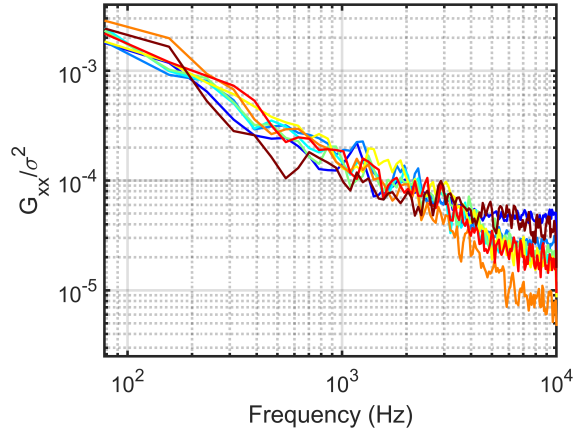
(b) $X_{\text{cylinder}}/D = 12$



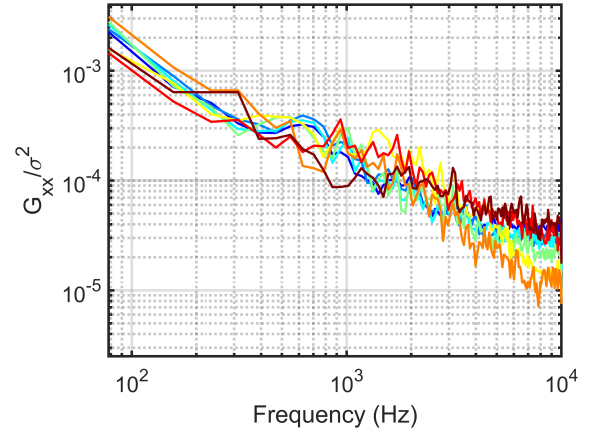
(c) $X_{\text{cylinder}}/D = 13$



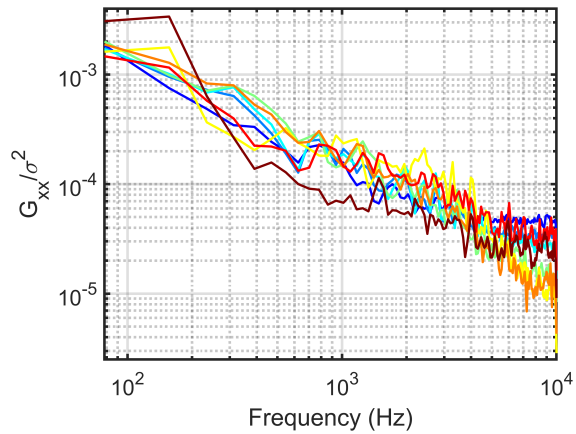
(d) $X_{\text{cylinder}}/D = 18$



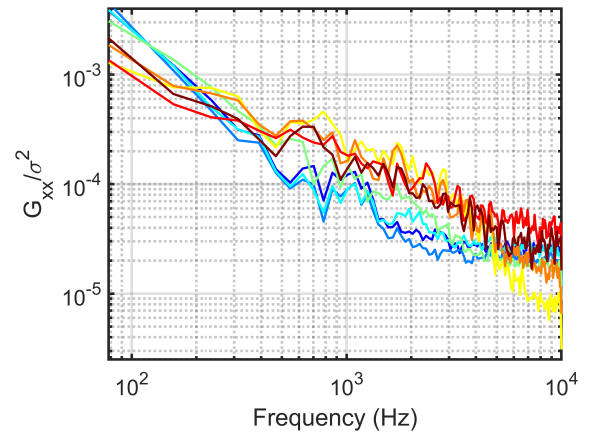
(e) $X_{\text{cylinder}}/D = 11$



(f) $X_{\text{cylinder}}/D = 12$



(g) $X_{\text{cylinder}}/D = 13$



(h) $X_{\text{cylinder}}/D = 18$

Figure 4.22: Power spectral densities extracted at increments of $x/d = 0.5$ over the PSP data set at each shock generator position.

SBLI shock foot behaviors are being attenuated between $x/d = -1.5$ and -2.5 . Finally, strong localized peaks at $x/d = -1.5$ are seen at 1.5 kHz, 2.2 kHz, and 2.7 kHz, and localized peaks for the $x/d = -0.5$ station at 1.4 kHz and 1.8 kHz. When $X_{cylinder}/D = 13$, the spectral content for each case appears to have more amplitude variation, and it becomes harder to distinguish between the content that dominates each probe station. This may be potentially due to the heavy mixture of transitional and turbulent behavior. There appears to be some elevated frequency content in the 700 to 800 Hz band for most of the probe stations, except for the most downstream case at $x/d = 0.05$ (brown). The $x/d = 0.05$ probe station (brown) shows dominating low frequency content at about 160 Hz, and another distinct peak occurs at 1.5 kHz, similar to the upstream influence behavior when the shock generator is located at $X_{cylinder}/D = 11$. For the fully turbulent case at $X_{cylinder}/D = 18$, spectral probes at $x/d = -2.5$, -3.0 , and -3.5 again share similar trends, indicating an incoming boundary layer with elevated broadband behavior centralized at 700 Hz and 1.1 kHz. The downstream probe positions show significant random fluctuations in the amplitude, as seen in the prior shock generator position. However, there are broad peaks for $x/d = -0.05$, -1.0 , and -1.5 centered at 600, 650, and 800 Hz, respectively.

Overall, the PSD stations along the cone centerline yielded inconclusive results. Large variations in the spectral amplitude results and several elevated peaks in particular spectra suggest the frequency resolution may not be appropriate to resolve the dynamic behavior of this transitional SBLI. It is essential to consider that SBLI behaviors are dependent on the Reynolds number, Mach number, incoming instabilities, which may also be influenced by freestream tunnel noise, the incoming boundary layer state, model geometry, shock generator geometry, three-dimensional relief, and several other factors. Initial schlieren results that took the PSD of the entire image domain did not capture some of the frequency trends seen. This suggests the interaction's dynamics are more obscured than seen in other experimental data, and the same would be expected in the pressure sensitive paint results

It is important to note that image stabilization during the preprocessing phase shows small scale (about 2-3 pixels) fluctuations in the streamwise direction and even smaller oscillations in the spanwise direction for the first three shock generator positions. The oscillations occur every 100 frames for the $X_{cylinder}/D = 11$ and 12 cases, and every 150

frames for the $X_{\text{cylinder}}/D = 13$ case. This results in artificial unsteady content that is visible from about 100 to 133 Hz for the previously mentioned cases; therefore, the frequency content in this band should generally be neglected, since it could be a byproduct of tunnel movement and/or the image stabilization routine.

4.4 POD Results

Proper Orthogonal Decomposition (POD) was conducted on the schlieren data set for each case, and one set of results for each case of $X_{\text{cylinder}}/D = 11, 12, 13$, and 18 will be reviewed and discussed in this section. The singular value spectrum and cumulative energy content for run 16 ($X_{\text{cylinder}}/D = 12$) is shown in Fig. 4.23. Both plots are normalized by the total energy of the system and are displayed as a total percentage. For this analysis type, 10,240 images were supplied, resulting in a mode index of the same size (i.e., there are 10,240 resultant modes). The singular value spectrum denotes the percentage of energy found in a given mode, or eigenvalue, compared to the total. The first mode contains about 17% of the total energy, with the following modes having about 6% and 3% for modes two and three. As expected, there is a rapid drop in the percentage of energy contained in each mode. This percentage drops to under 2% after six modes. Two gray dotted lines denote the modes at which 90% and 99% of the total energy has been captured. For this case, the 90% and 99% cumulative energies occur at modes 947 and 4166, respectively. This information is also readily seen by observing the cumulative energy plot with its expected log plot approaching these values rapidly before slowly converging towards 100%. Individual modes with energy content above 2% and the mode at which the cumulative energy content has reached 90% and 99% of the total energy are tabulated in Table 4.6 for each case. Most cases see only significant energy contribution within the first six modes.

To ensure the POD outputs are correct and capture the desired information, it is useful to reconstruct a typical frame via POD reconstruction. Fig. 4.24 shows the POD reconstruction process for run 16. The first image displayed is an instantaneous image taken at 50 ms into the steady-state run time, specifically frame 1000. From here, it is clear how the summation of each additional mode captures smaller details to reconstruct the image. The first few

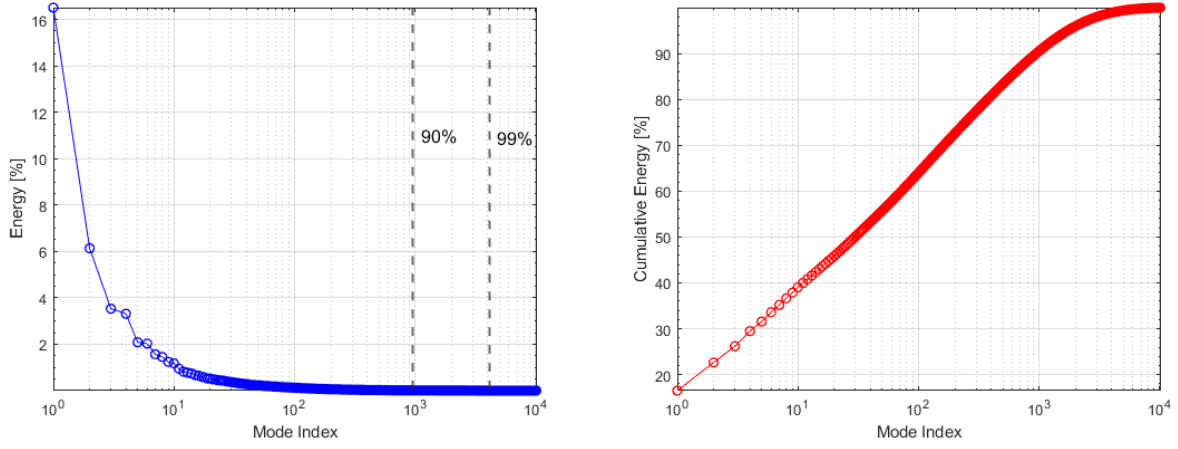


Figure 4.23: Singular value spectrum (a) and cumulative energy content (b) from POD of the schlieren data for run 16.

Table 4.6: Summary of POD singular value energy of the schlieren data.

Run Number	X_{cylinder}/D	Mode $\epsilon(\%) > 2\%$	Mode $\Sigma\epsilon(\%) = 90\%$	Mode $\Sigma\epsilon(\%) = 99\%$
15	12	6	1014	4328
16	12	6	947	4166
17	13	6	967	4187
18	13	5	1025	4529
19	18	6	1126	4856
20	18	5	1192	4856
23	11	6	890	4126
24	11	5	1077	4626

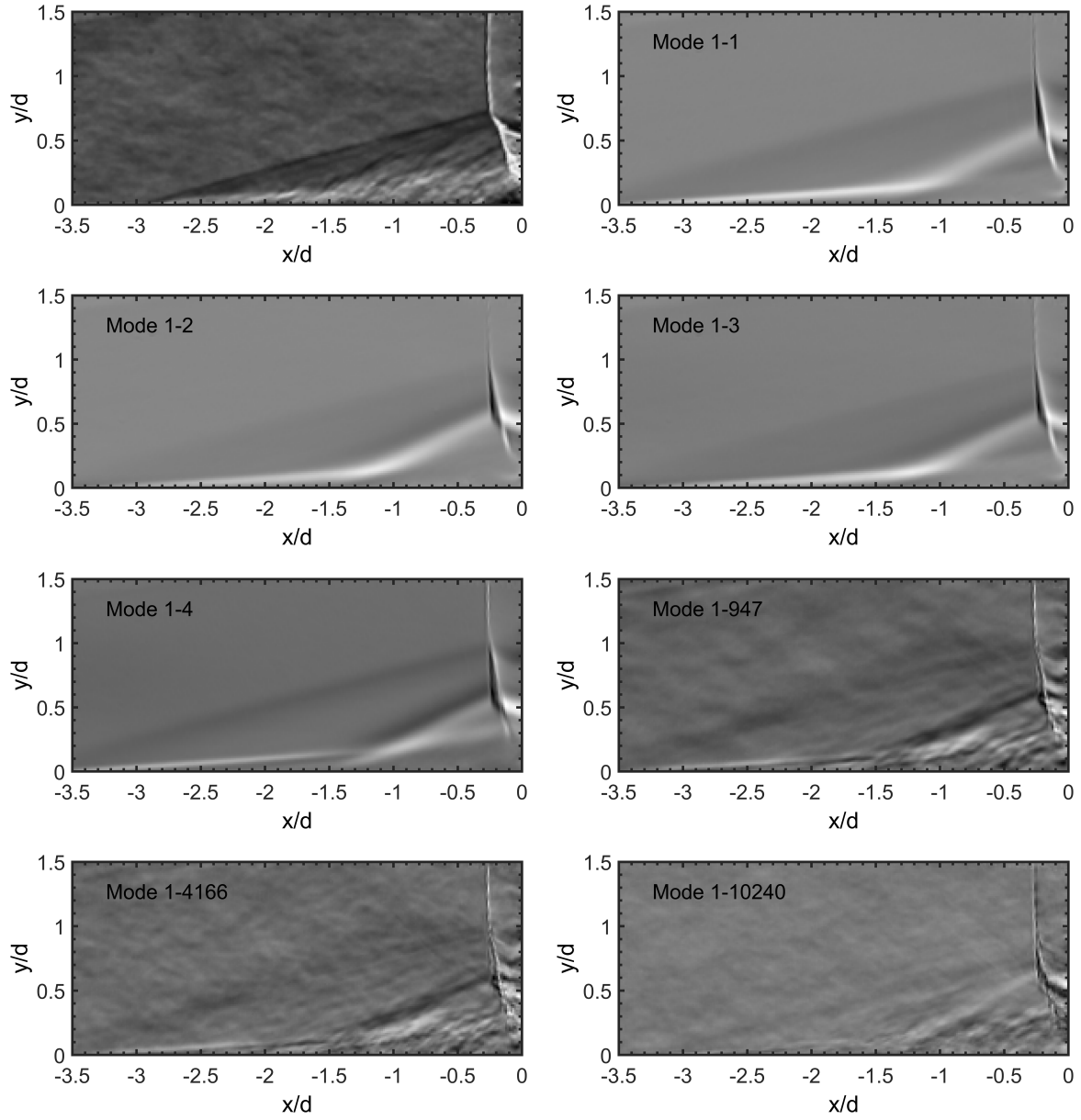


Figure 4.24: Reconstruction of original images by summation of POD modes in order of cumulative energy.

modes observed show that most of the deviations from the mean information is encapsulated in the first few modes and desired smaller-scale turbulent structures are only present in later modes like those seen in the summation of modes 1 through 947.

Fig. 4.25 shows the first five POD modes for run 16 as well as modes 947, 4166, and 10240, which correspond to 90%, 99%, and 100% of the cumulative energy. Note the color scale, which has the spatial mode information non-dimensionalized by the maximum value of the spatial mode referenced. All colormaps for the POD modes range from +1 to -1 denoted as red and blue, respectively. As ϕ , or the spatial modes, approach zero the colormap will turn from a bright red or blue to white, effectively highlighting only relatively stronger spatially orthogonality. It should be noted in this context that a negative value may represent a part of a coherent structure that is moving in the opposite direction as the positive value of the coherent structure, or opposing phase shift. Only a handful of initial modes will be evaluated as they are more likely to be associated with a real coherent structure. Run 16, is considered transitional and is at an X_{cylinder}/D location of 12. The transitional nature of this SBLI is apparent in modes 1-5, which clearly identify the separation shock and upstream influence. Mode 1 clearly shows the upstream influence and the separation shock with the locations of both being typical of a transitional interaction. In mode 2, a fluctuating component with the opposite sign is shown to be just upstream of the separation shock. Modes 3, 4, and 5 display stronger fluctuations associated with the boundary layer in the separation region, and their strongest components correspond to the separation shock. Modes 947, 4166, and 10240 display varying scales of turbulent packets in both the freestream and separation regions.

The first four POD modes for a $X_{\text{cylinder}}/D = 11$, test 24, case mirror the results of the shock generator location at an X_{cylinder}/D position of 12. These results are shown in Fig. 4.26. Again, the upstream influence is clearly observed far upstream with a strong separation shock. Mode 2 displays collapsed fluctuating components at the separation shock's typical location and may indicate the collapse of the upstream influence against the separation shock. Modes 3 and 4 highlight the separation shock and emphasize the larger-scale boundary-layer fluctuations in the separation region that appear to enclose the region.

The first four POD modes for the $X_{\text{cylinder}}/D = 13$ case, test 18, distinctly differ from the first two cylinder shock generator locations. These results are shown in Fig. 4.27. Evidence

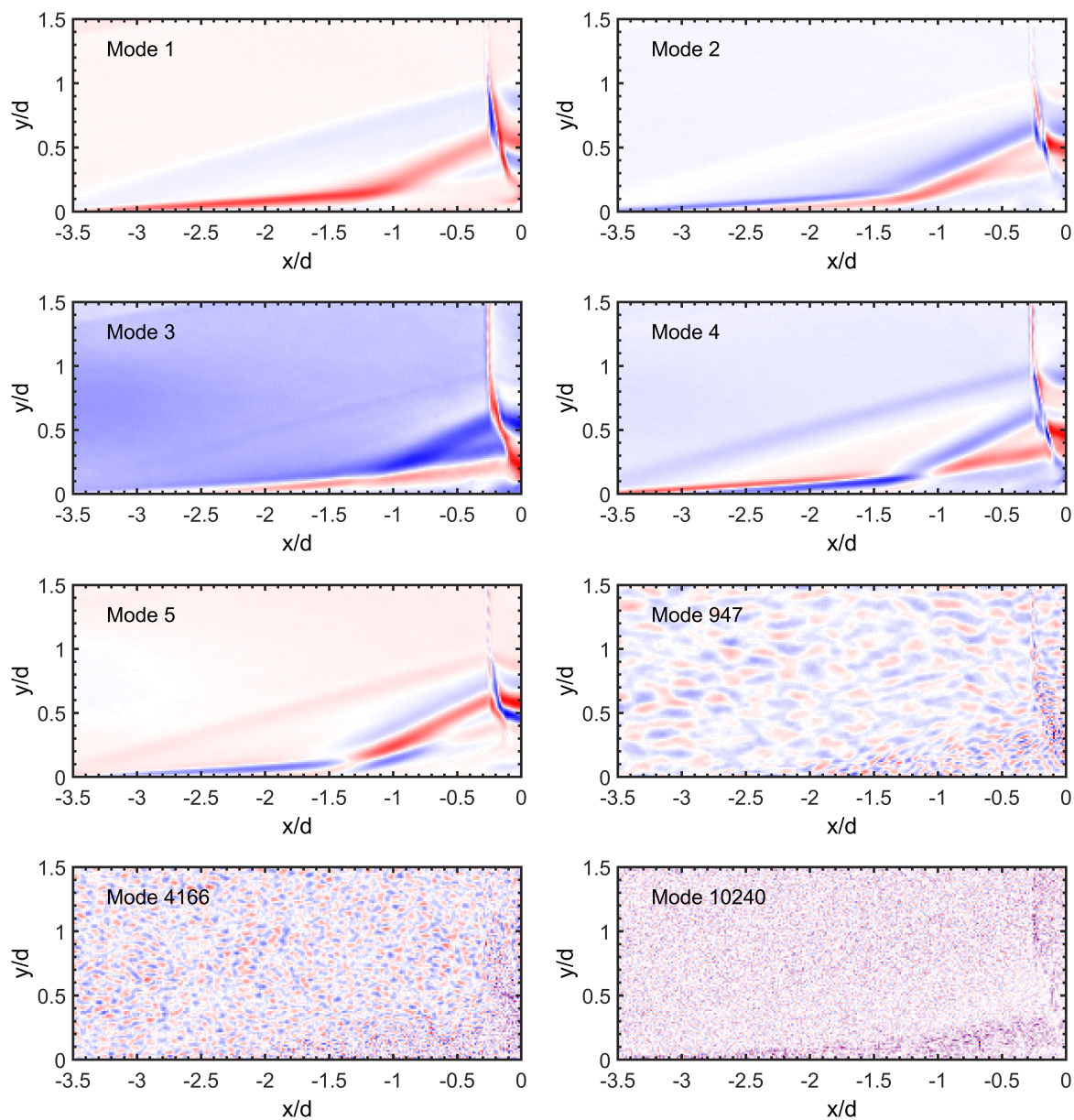


Figure 4.25: Visualization of first five POD modes and three other modes encapsulating noise from test 16, a $X_{\text{cylinder}}/D = 12$ case.

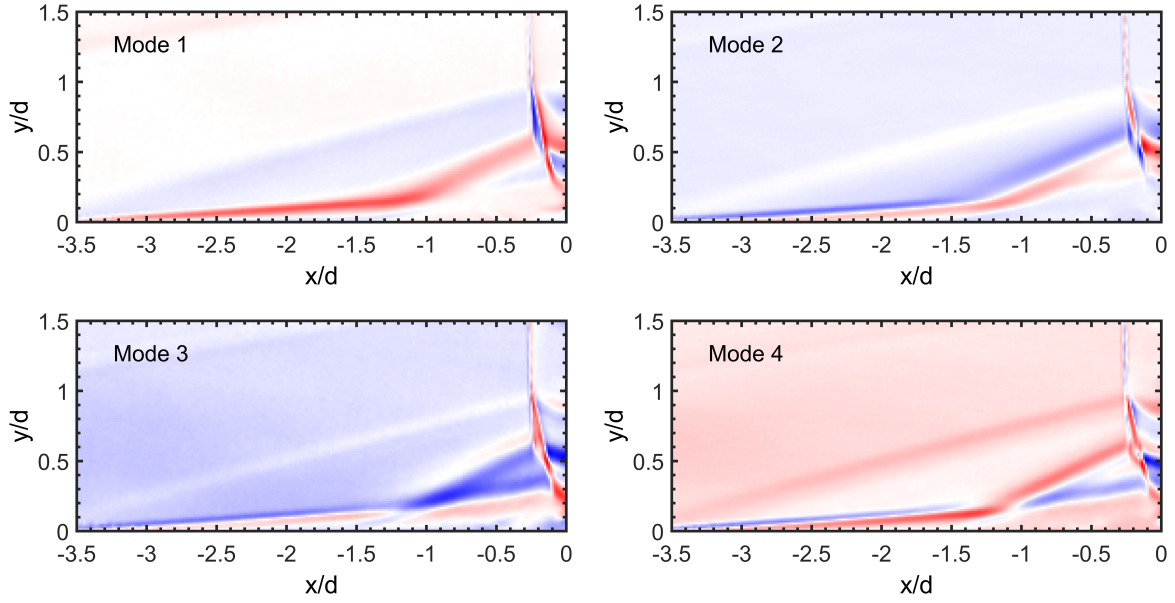


Figure 4.26: Visualization of first four POD modes from test 24, a $X_{\text{cylinder}}/D = 11$ case.

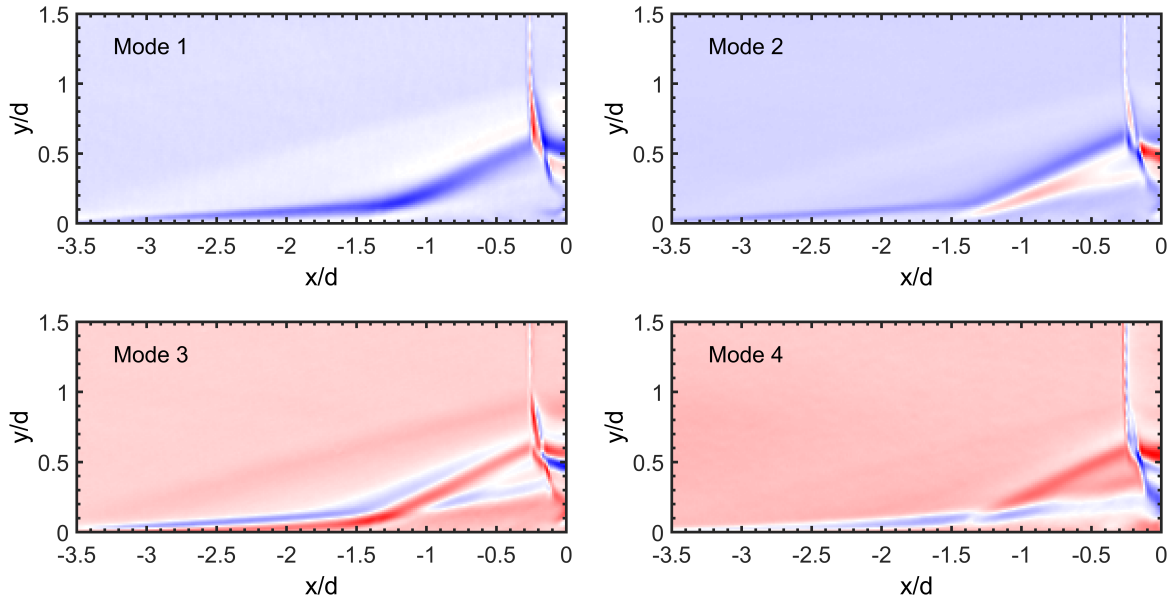


Figure 4.27: Visualization of first four POD modes from test 18, a $X_{\text{cylinder}}/D = 13$ case.

of the upstream influence is only visible by the lack of a strong fluctuating component in the region. In the first mode, the separation shock is dominant (dark blue), and the triple point is associated with a positive value (dark red). Mode 2 again distinguishes the separation shock; however, the impinging jet and region just below the separation shock are the dominating features with an opposite sign of the spatial mode. Mode 3 and 4 highlight a region of the boundary layer in the separated region as previously seen in the early transitional cases, and Mode 3 has strong positive and negative spatial model values collapsed against one another in the separation shock region.

POD analysis of run 20, which has the shock generator located at $X_{\text{cylinder}}/D = 18$, shows the distinct structure of a turbulent interaction. These results are shown in Fig. 4.28. Mode 1 highlights only the separation shock and its adjoined impinging jet with strongly positive and negative spatial mode values. Mode 2 highlights the separation shock with both an upstream and downstream opposing fluctuation component. Mode 3 highlights that the vast majority of the content in this mode is for negative fluctuations with respect to the mean, apart from the rearward-facing lambda shock structure, which has positive fluctuations within this mode. Finally, mode 4 shows two parallel structures that surround the separation shock region.

4.5 DMD Results

Dynamic Mode Decomposition (DMD) allows for the returned eigenvalues of a singular value decomposition to be evaluated in terms of energy modes, frequency, and growth rate. This analysis allows characteristic features of shockwave / boundary-layer interactions to be further investigated, similarly to POD, but results are anchored to the frequency domain and provide a more intuitive, physical understanding of the returned modes. The returned eigenvalues found in the singular value decomposition MATLAB command are complex numbers, where the real component returns frequency information and the imaginary component returns the growth rate. One method of interpreting the eigenvalues is the use of a unit circle. Using a unit circle, the total component of the eigenvalue can be evaluated and various modes selected based on specified metrics like those parsed out in Fig. 4.7. A mode

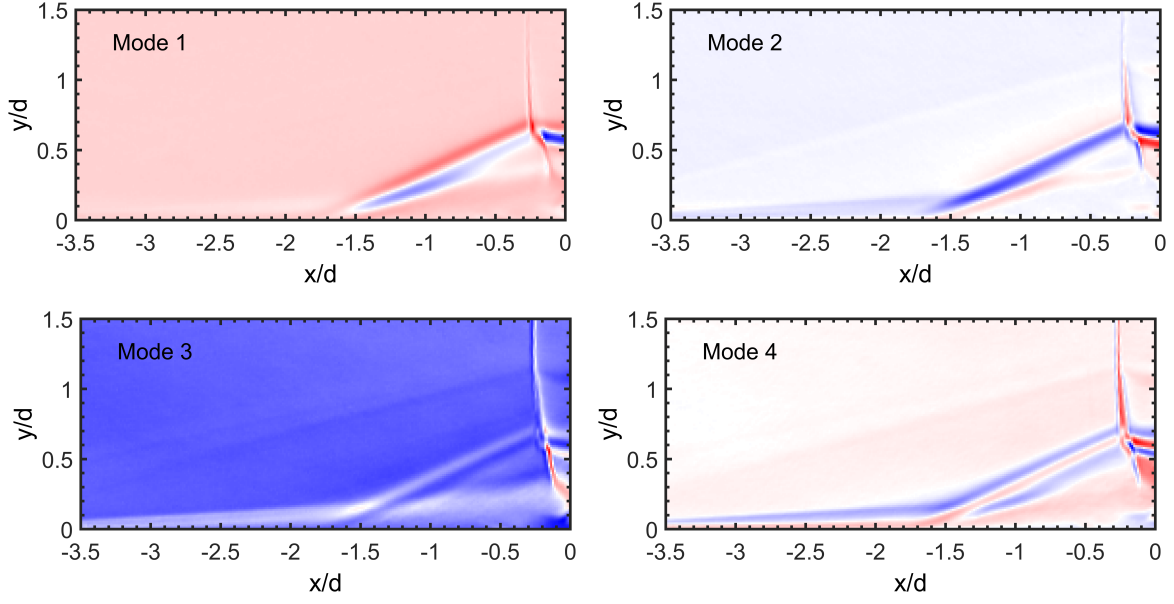


Figure 4.28: Visualization of first four POD modes from test 20, a $X_{\text{cylinder}}/D = 18$ case.

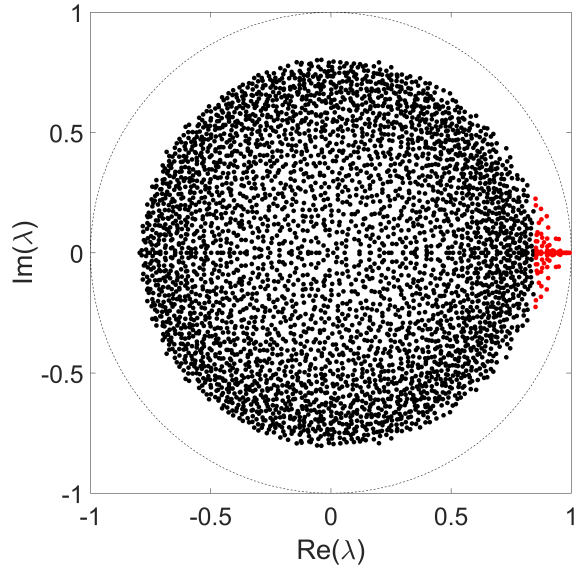
Table 4.7: Summary of DMD cumulative energy at evaluated frequencies for modes with a total eigenvalue magnitude greater than 85%.

Run Number	23	24	15	16	17	18	19	20
X_{cylinder}/D	11	11	12	12	13	13	18	18
Number of Modes (#)	78	90	78	73	73	76	68	76
Number of Modes (%)	0.76	0.88	0.76	0.71	0.71	0.74	0.66	0.74
All frequencies (%)	37.6	46.1	40.8	39.4	41.5	41.7	46.8	55.5
$f = 0 \text{ kHz}$ (%)	8.1	10.8	11.0	7.4	12.3	9.2	11.5	12.3
$0 < f < 1.5 \text{ kHz}$ (%)	13.6	15.4	12.6	15.2	13.8	18.6	17.9	20.6
$1.5 < f < 3 \text{ kHz}$ (%)	8.5	7.0	8.2	6.4	6.7	6.5	5.3	8.6
$3 < f < 6 \text{ kHz}$ (%)	4.7	8.9	6.1	7.3	5.5	5.3	8.0	7.1
$f > 6 \text{ kHz}$ (%)	2.8	3.9	3.0	3.1	3.2	2.1	4.0	6.9

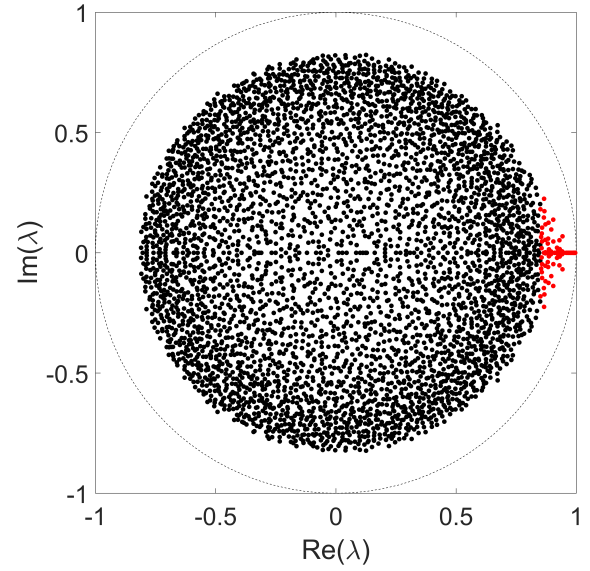
with a radius greater than one is considered unstable and may provide isolated unstable mechanisms in the phenomena evaluated. Unit circle plots were created for each cylinder shock generator station using the high-speed schlieren images and are shown in Fig. 4.29. All cases are stable; however, at low imaginary numbers and high real numbers, the unit circle values climb and approach unity for all cases [54]. Modes with an eigenvector magnitude greater than 0.85 are plotted in red and extracted for further evaluation. All extracted modes are associated with low frequency information and strong growth or decay rates.

For each of the runs, the total number of modes that were extracted based on a total eigenvalue magnitude greater than 0.85 and their cumulative energy has been reported in Table 4.7. Recall that a total of 10,240 modes were generated, and then truncated such that only the modes with the largest amount of energy contribution were kept. The number of modes used in this analysis were roughly half the total amount of generated modes and had a cumulative energy of 99%. In the table shown, the total number of modes in each run were reported along with the percentage of the total 10,240 modes originally generated. From this information, it is clear that less than 1% of the total modes generated contribute between 37.6% and 55.5% of the total energy found in the entire system and will be analyzed in this section. The cumulative energy is then broken down further based on the real value input for each mode. This gives an approximate percentage of the cumulative energy of the entire system that is contained in each frequency range. All of the tests show that the cumulative energy contribution is most significant in the 0 to 1.5 kHz range as would be expected for SBLI. A substantial amount of the cumulative energy is also associated with modes that register a frequency of zero and have significant exponential growth. These purely real eigenvalues imply the presence of a non-oscillatory instability that has persistent growth [7]. In transition, this could be associated with the growth of disturbances, amplification of small perturbations, or the onset of secondary instabilities [9, 103, 94].

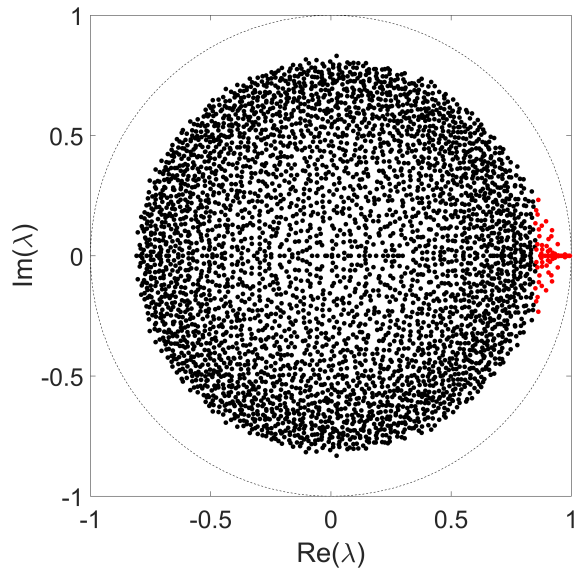
The selected modes with a total eigenvalue magnitude greater than 0.85 (85%) have been parsed based on a frequency range of $f = 0$ kHz, $0 < f < 1.5$ kHz, $1.5 < f < 3$ kHz, and $3 < f < 6$ kHz, and all frequencies with a radius greater than 0.85. These modes have been remapped back onto the original imaging domain for inspection. The associated colormap indicates the relative magnitude of growth rates, defined as σ . The parameter



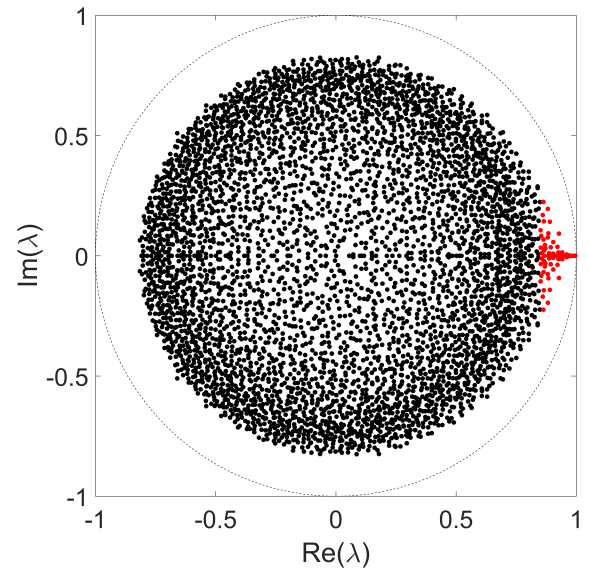
(a) $X_{\text{cylinder}}/D = 11$



(b) $X_{\text{cylinder}}/D = 12$



(c) $X_{\text{cylinder}}/D = 13$

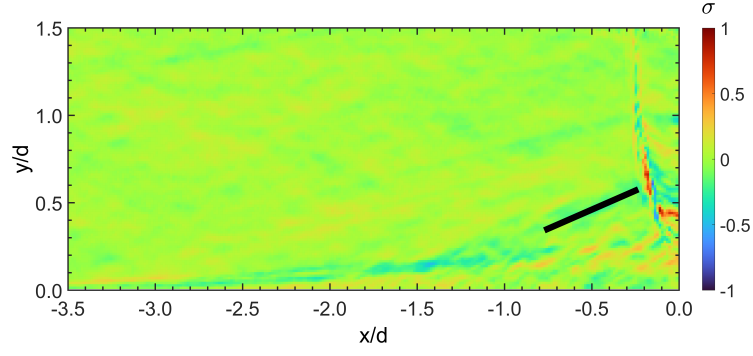


(d) $X_{\text{cylinder}}/D = 18$

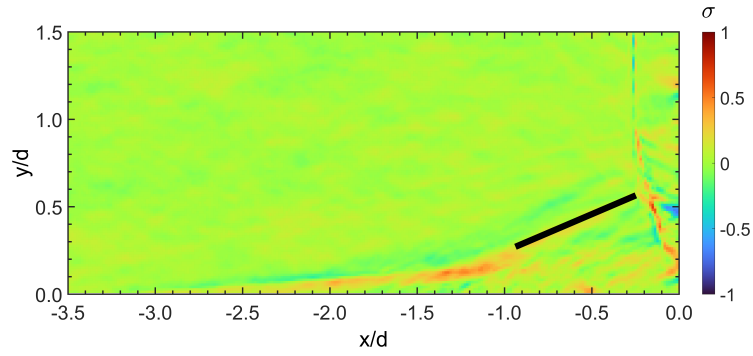
Figure 4.29: Unit Circle of DMD data

σ represents the real part of the eigenvector indicating the growth or decay rate, and it has units of (1/s). In DMD, the growth and decay rate are exponential in nature and evolve as a function of $e^{\sigma t}$ where t represents the total time over which the growth or decay occurs [9]. In these experiments, t is the input data duration [94] which is equal to 51.2 ms here. The overall impact and significance of features and their associated value of sigma can vary based on this associated time scale [9]. The dark red regions indicate large positive growth rates and may designate areas where the dynamic content at these frequencies would exhibit amplifier behaviors. Blue regions indicate areas of decreasing growth rates and may highlight a dampening behavior. Features colored in green have growth rates near zero. The presence of a strong amplification and strong decay within the same mode may suggest competing instability growth and dissipation. However, the overall dominant negative or positive growth would indicate global amplification and global dampening occurring in the SBLI system. Note that a different colormap is used for the DMD data to distinguish these plots visually from the POD data during the comparison process. The addition of a color, green, at values of $\sigma = 0$ also helps emphasize gradients to give the reader more spatial context of the SBLI system as a whole. The black line represents the extracted mean location of the SBLI from the average schlieren images. Spectral data suggests that the interaction sees an increase in the characteristic frequencies associated with the UI and separation shock as the cylinder shock generator is moved downstream. Boundary layer probes in the PSD sections also see a reversal of spectral amplitude as a function of x/d position based at 3 kHz. The data here is parsed into finer bins here to discern if DMD can identify different driving mechanisms associated with frequency changes seen in the PSDs.

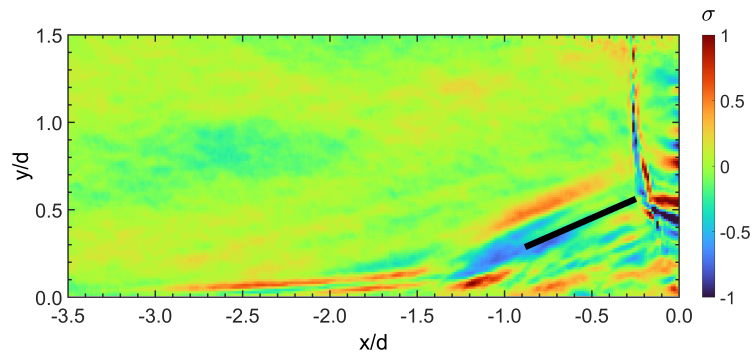
DMD data for modes with a frequency of 0 kHz were analyzed first and are presented in Fig. 4.30. Growth rates for the most upstream cylinder shock generator stations with a frequency content of 0 Hz are quite benign. The most dominant features for $X_{\text{cylinder}}/D = 11$ and $X_{\text{cylinder}}/D = 12$ are associated with the closure shock. The $X_{\text{cylinder}}/D = 12$ case does appear to have a small amount of growth at the forward lambda shock foot that seems in line with the mean location. The downstream cylinder shock generator stations however, have significantly larger growth and decay rates present together in the modes presented. Both $X_{\text{cylinder}}/D = 13$ and $X_{\text{cylinder}}/D = 18$ have strong growth and decay rates associated



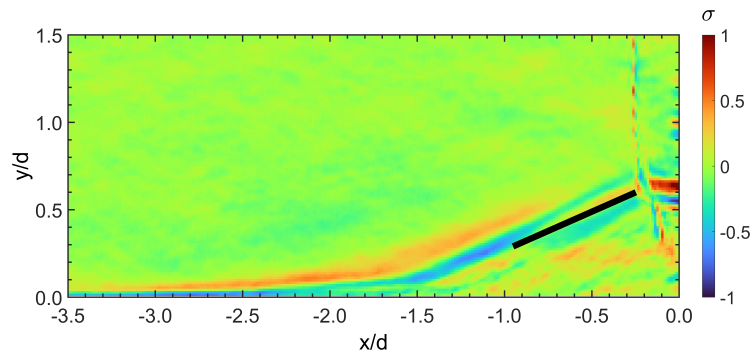
(a) $X_{\text{cylinder}}/D = 11$



(b) $X_{\text{cylinder}}/D = 12$



(c) $X_{\text{cylinder}}/D = 13$



(d) $X_{\text{cylinder}}/D = 18$

Figure 4.30: DMD results for each X_{cylinder}/D location of 11, 12, 13, and 18 for frequencies at 0 kHz, and with a radius greater than 0.85.

with the jet impingement originating from the triple point and the separation shock. Note that later shock generator stations have areas of positive and negative values of σ that run parallel to one another and are upstream of the mean SBLI station.

DMD data in the $0 < f < 1.5$ kHz range with a total eigenvalue magnitude greater than 0.85 are remapped back to their original image form and displayed in Fig. 4.31. For the $X_{\text{cylinder}}/D = 11$ case, there are regions of positive and negative growth rates that appear to be dominant for this case that run parallel to one another. The interaction is dominated by positive values of σ and appears to straddle the mean separation shock foot location for this case. The more upstream blue component is weaker in dominance and extends far upstream in the boundary up to about $x/d = -2.6$. The red component strengthens as the boundary layer grows, extends upstream to about $x/d = -2.4$, and reaches a peak in amplitude at the base of the separation shock foot. The $X_{\text{cylinder}}/D = 12$ case shows similar features as the previous cylinder shock generator station; however, the relative dominance has shifted. First the positive and negative values of σ in the separation shock appears to have moved slightly upstream in respect to the mean location. The dominating growth rate in this case is decaying and is located in the incoming boundary layer from $x/d = -3.2$ to -1.8 . The positive growth rate in the boundary layer extends upstream to $x/d = -3.2$ as the negative growth rate now only extends up to about $x/d = -2.2$. This may indicate a change in driving mechanisms or the extent of the separation shock to be influenced by the incoming boundary layer. The two most downstream cylinder shock stations have significantly stronger growth rates as indicated by the colormap's scale changes. The late transitional case, at $X_{\text{cylinder}}/D = 13$, has the most dominant growth of all cases in the 0 to 1.5 kHz range. Both positive and negative growth rates appear to be associated with the separation shock that is upstream of the mean separation shock location this case. Strong amplification and decay are located at the separation shock foot and in the shear layers and compression waves seem to be present. Unlike the previous cases, the strong growth rate associated with the separation shock is much more significant than the growth rate in the incoming boundary layer. Note the different scaling such that the growth rate in the boundary layer is comparable to what was seen in the early transitional cases. Incoming boundary-layer turbulence in this frequency range is mainly associated with the weaker dampening component. Fine-scale

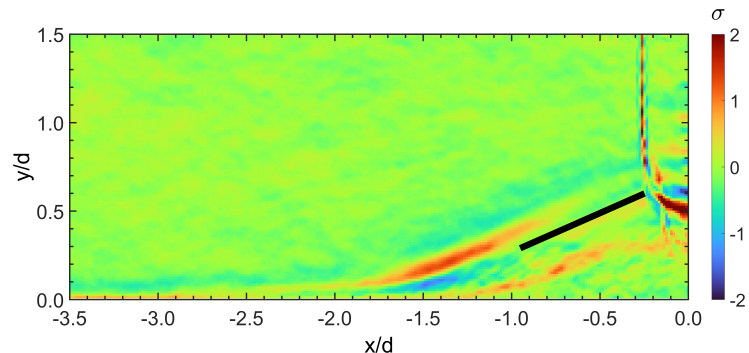
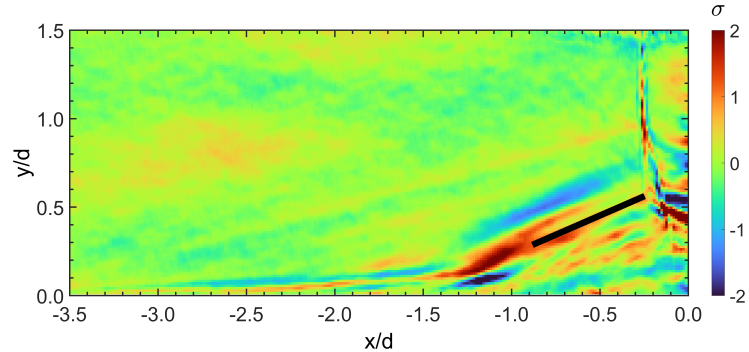
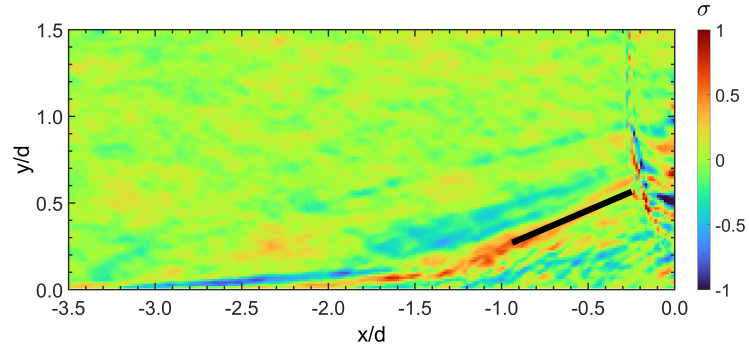
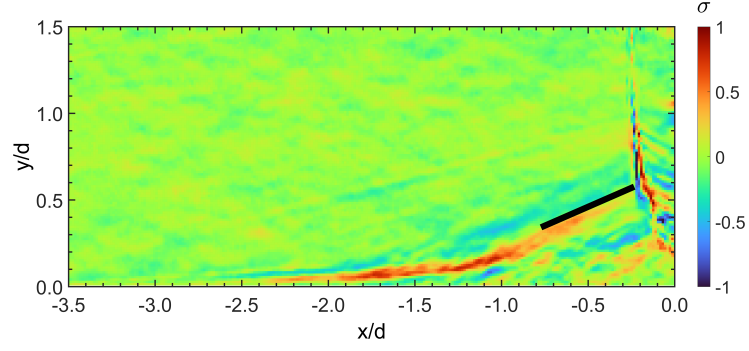
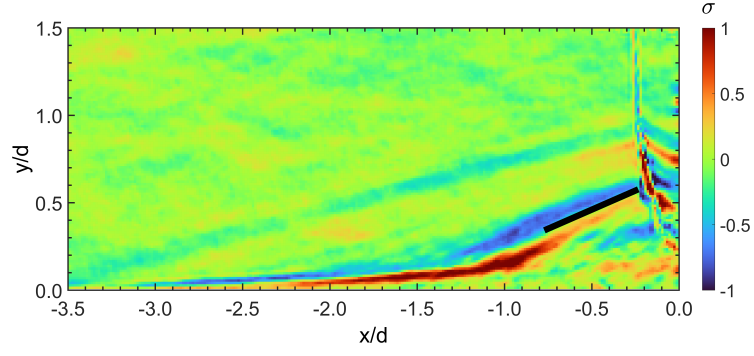


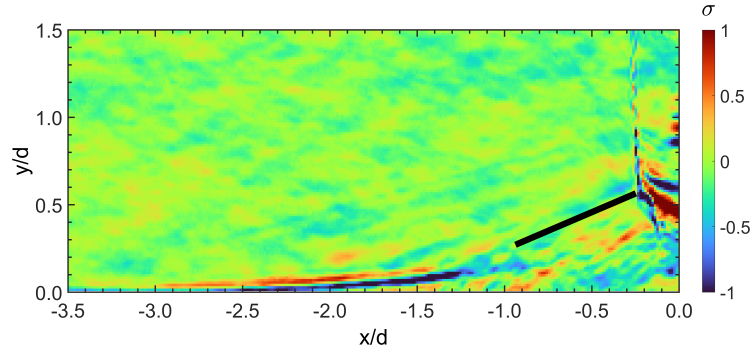
Figure 4.31: DMD results for each X_{cylinder}/D location of 11, 12, 13, and 18 for frequencies between 0-1.5 kHz, and with a radius greater than 0.85.

turbulence would be expected at much higher frequencies that are not examined in the DMD information here. Given the low frequency range of this information, information seen in the boundary layer is likely associated with larger structures or the occasional large-scale turbulent spots like those seen in the schlieren time series information. This may indicate that the late transitional case does not have a strong correlation in the separation shock foot motion with the incoming boundary layer for this frequency range. For the turbulent case ($X_{\text{cylinder}}/D = 18$), there is a clear oscillation at the base of the separation shock foot that has strong opposing growth rates. The positive growth rate component appears upstream of the mean separation shock foot locations, while the negative growth rate component appears to be downstream of the mean separation location. This may indicate a region where the incoming boundary-layer state and separation bubble balance with one another to create a stabilized region where the leading lambda shock foot remains. Moreover, the strong oscillating components are not seen to have counterparts in the incoming boundary layer. This may occur as a result of the turbulent boundary layer lowering the sonic line thereby preventing upstream propagation of information in the thinner subsonic portion of the boundary layer. Moreover, the strong amplification component of the separation region may prevent further downstream motion of the leading lambda shock foot.

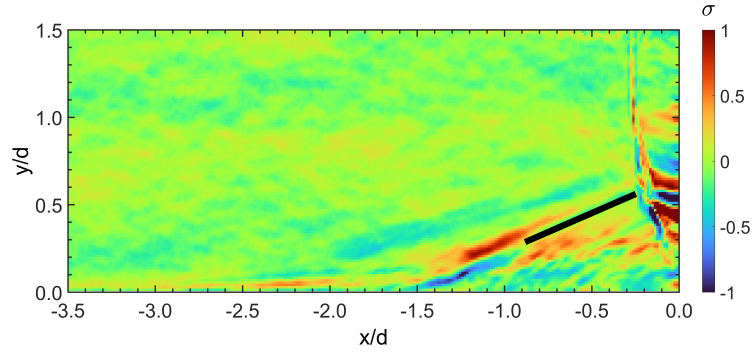
DMD data in the 1.5 to 3 kHz range are displayed in Fig. 4.32. At $X_{\text{cylinder}}/D = 11$ the separation shock and its upstream extent in the incoming boundary layer are prominent. Overall, stronger negative growth rates are visible in the separation shock and the incoming boundary layer than positive growth rates. Regions of high decay rate appear downstream of the mean location as regions of relatively high growth rate appear upstream of the mean location. The red and blue regions are seen to extend upstream to about $x/d = -2.3$ and -3.1 , respectively. Though most of the energy for this test is located in the $0 < f < 1.5$ kHz particularly in the closure shock and at the forward lambda shock foot, most of the typical SBLI structure is visible in the $1.5 < f < 3$ kHz range. In the $X_{\text{cylinder}}/D = 12$ case, again, the dominant behavior in a transitional interaction for the 1.5 to 3 kHz range appears in the incoming boundary layer and the impingement jet. In this case, the dominant location in dark blue appears to stretch upstream in the lower portion of the boundary layer to about $x/d = -2.6$. Note that in most of the modes examined the upper portion of the boundary



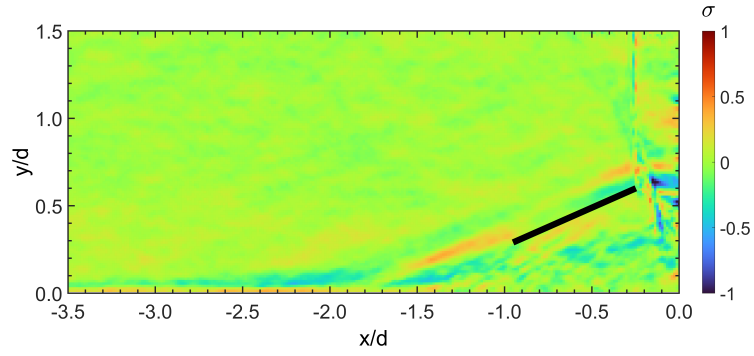
(a) $X_{\text{cylinder}}/D = 11$



(b) $X_{\text{cylinder}}/D = 12$



(c) $X_{\text{cylinder}}/D = 13$



(d) $X_{\text{cylinder}}/D = 18$

Figure 4.32: DMD results for each X_{cylinder}/D location of 11, 12, 13, and 18 for frequencies between 1.5-3 kHz, and with a radius greater than 0.85.

layer appears to be a dampening feature and the lower portion appears to have a positive growth rate associated with it. This is opposite for $X_{\text{cylinder}}/D = 12$ case in the 1.5 to 3 kHz range. For an X_{cylinder}/D position of 13, the separation shock foot area has several features that align with compression waves that are seen in typical instantaneous schlieren images. The most dominant behaviors are associated with the closure shock and several visible impingement jets near the triple point. There are some more benign positive and negative growth rates associated with features at the separation shock foot locations from $x/d = -1.5$ to -0.9 . As before, the turbulent case shows the regions of positive and negative growth rates side-by-side associated with the separation shock at the mean location; however, this region is stronger in the 0 to 1.5 kHz range than in the 1.5 to 3 kHz range.

DMD data in the 3 to 6 kHz range are displayed in Fig. 4.33. The three most upstream cylinder shock generator locations at $X_{\text{cylinder}}/D = 11, 12$, and 13 at have lower values of σ than their fully turbulent counterpart and for frequency ranges from 0 to 3 kHz. These cases do show elevated growth modes in the incoming boundary layer and at the mean separation shock location. The $X_{\text{cylinder}}/D = 12$ case does have elevated levels of positive σ connecting the separation shock to a lower portion of the incoming boundary layer. In the fully turbulent case, the separation shock and incoming boundary layer are visible. The boundary layer appears to stretch as far forward as $x/d = -3.5$ and two layers including a decaying growth rate in the upper portion of the boundary layer and a positive growth rate in the lower portion of the boundary layer that is clearly connected to the separation shock. The separation shock has two stronger parallel layers. A positive value of σ is upstream of the mean shock location and a negative growth rate just downstream of the mean location. The most prominent features is the impingement shock that displays both positive and negative growth components.

4.6 Statistical Analyses of Feature-Tracking Data

The feature-tracking algorithm implemented in MATLAB was used to locate the contact point of the leading lambda shock foot along the model surface on a frame-by-frame basis of the high-speed schlieren images. The identified shock foot locations are summarized in

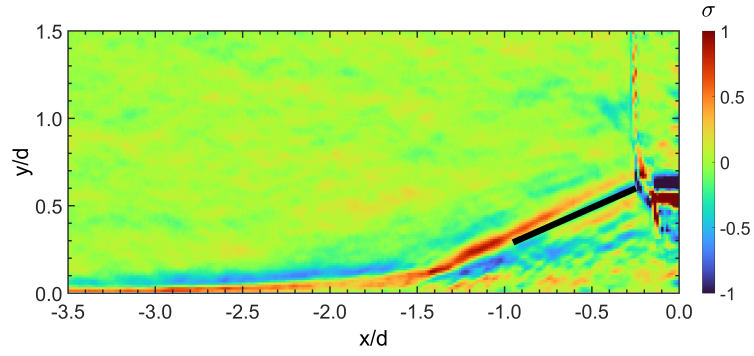
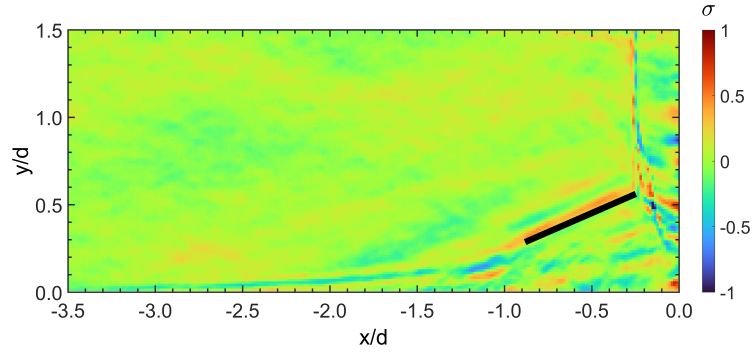
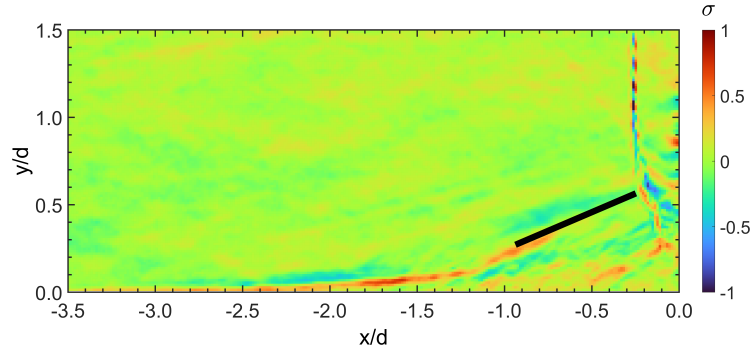
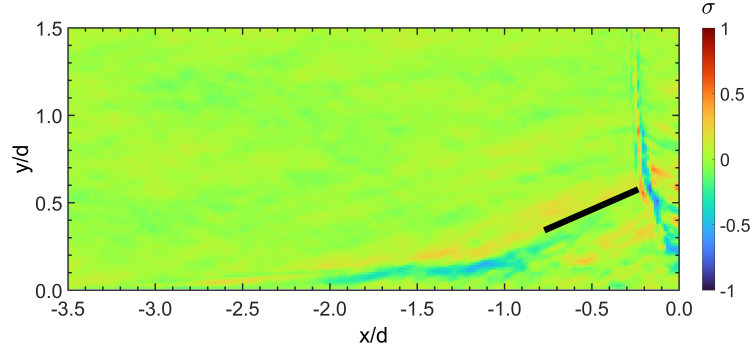


Figure 4.33: DMD results for each X_{cylinder}/D location of 11, 12, 13, and 18 for frequencies between 3-6 kHz, and with a radius greater than 0.85.

histograms as shown in Fig. 4.34. Shock foot locations identified to be within 2.7 diameters of the cylinder shock generator are categorized as a turbulent interaction. For the case of $X_{\text{cylinder}}/D = 11$, many frames indicate that the interaction is likely transitional as the shock foot location was identified very far upstream—only 46% of the data, in this case, were categorized as turbulent. Considering the same metric for the remaining cases, this percentage increases as the shock generator is translated downstream. For the locations of $X_{\text{cylinder}}/D = 12, 13$, and 18, turbulent interactions were found to be present 64%, 77%, and 96% of the time, respectively. These data are presented in Table 4.8 for the individual runs processed; these data were averaged in determining the overall likelihood of the interaction state. While the cylinder shock generator position of $X_{\text{cylinder}}/D = 18$ contains nearly all turbulent interactions, the particular frames categorized as transitional exhibit shifts in the incoming boundary-layer state with seemingly random intermittency. Although further analysis is needed, the best practice may be to describe these shifts as an evolution from early to late stages of boundary-layer transition.

4.7 Boundary-layer Thickening Relation

Much of the material contained in this section was originally presented in Lester et al. [61], though it has been copied here for clarity, consistency, and completeness of this dissertation.

Prior work conducted by Lash et al. [58] focused on analyzing SBLI that occur with both cylindrical and blunt fin shock generators on a flat plate as well as a cylindrical shock generator on a cone, with each subjected to a Mach 2 freestream. These experiments were conducted at UTSI, and one of the goals of the current work is to build upon those results with additional data collected for SBLI in a Mach 4 freestream. In their analyses, Lash et al. [58] postulate that there is a relationship between the upstream influence (UI) and a distinct growth or thickening of the incoming boundary layer (they termed this the boundary-layer precursor). The work carried out was meant to explore this hypothesis further, and the data collected within this work provides additional support for this theory.

The feature-tracking algorithm can also identify a sudden thickening of the boundary layer. This feature's position can be compared to the upstream influence, as shown in

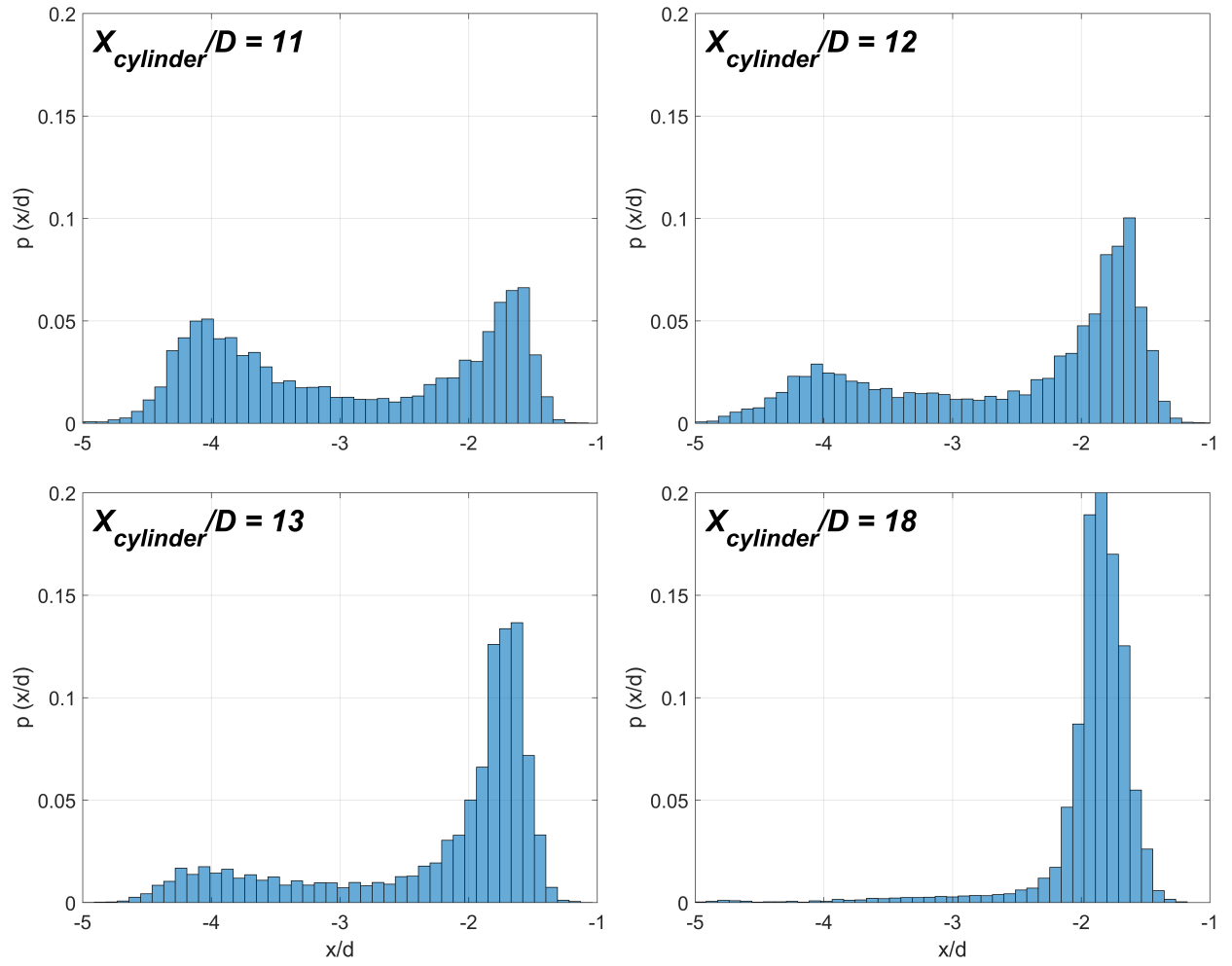


Figure 4.34: Histograms of MATLAB-identified shock foot locations along the cone's surface.

Table 4.8: Percentage of frames identified as turbulent.

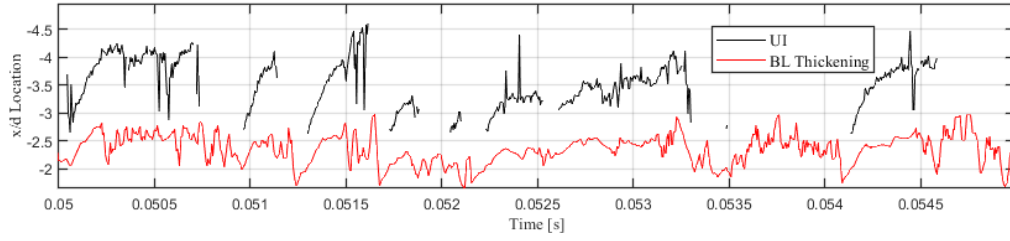
Run Number	$X_{cylinder}/D$	Percentage Turbulent [%]
23	11	51.27
24	11	39.88
15	12	64.91
16	12	61.99
17	13	71.09
18	13	82.09
19	18	95.98
20	18	95.68

Fig. 4.35. In these data, the x-axis corresponds to a length of 1,000 images, and comparisons are made for $X_{\text{cylinder}}/D = 11$ and 12. Comparing the boundary-layer thickening (or precursor) with the upstream influence, it is clear that the two move in concert with one another for these cases, which corroborates the results seen in the works of Lash et al. [58] and Tester et al. [104]. In those instances, however, the two features were observed to have a sinusoidal behavior, which is not apparent here. In the present data, the sharp declines in the upstream influence position appear to correspond to periods where the SBLI displays behavior similar to turbulent interactions.

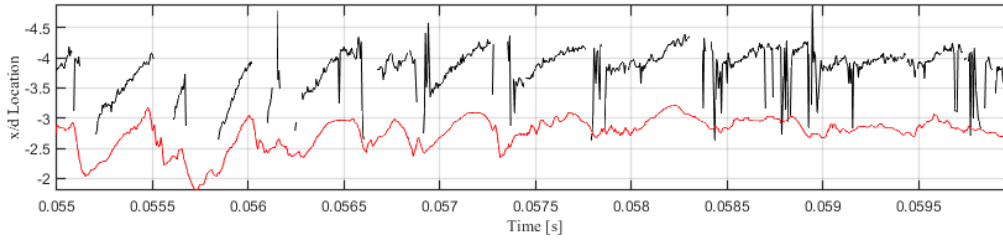
The behavior of the boundary-layer thickening location was further examined as a function of its derivative, which indicates the direction in which it travels. These data are first categorized into two bins: upstream movement and downstream movement. Next, the categorized data are analyzed by computing their probability density functions and probabilities. For a direct comparison of these data, the nonzero minimum of the number of counts and the maximum number of counts for both the PDF and the probability of the upstream influence appearance regarding the detected boundary-layer thickening precursor are displayed in Table 4.9.

The probability density function (PDFs), is shown in the top half of Fig. 4.36. For nearly all cases examined in this study, the mean location of the boundary-layer thickening is between $x/d = -3$ and -2.6 , and the points of highest probability (i.e., the maxima) are seen near $x/d \approx 2.8$. When considering each of these cases (which include transitional and turbulent interactions), it is clear that the PDFs tend to decrease in magnitude and broaden as the cylindrical shock generator is moved downstream from $X_{\text{cylinder}}/D = 11$ to $X_{\text{cylinder}}/D = 18$. Interestingly, however, these changes are not as substantial for the data categorized as having downstream movement of the boundary-layer thickening feature. This indicates that these interactions may collapse to similar scales as the boundary-layer precursor moves closer to the separation shock.

In the bottom half of Fig. 4.36, the probability of an upstream influence (UI) shock is determined as a function of the categorized boundary-layer precursor position. Upon initial inspection, it is clear that there is a stark contrast between the cases of $X_{\text{cylinder}}/D = 11$ & 12 and those of $X_{\text{cylinder}}/D = 13$ & 18. This helps to clarify that the cases of



(a) $X_{\text{cylinder}}/D = 11$



(b) $X_{\text{cylinder}}/D = 12$

Figure 4.35: Typical time-history (over 1,000 images) of the boundary-layer precursor and the upstream influence location.

Table 4.9: Minimum and maximum number of points evaluated for each X_{cylinder}/D plot in the PDF and probability graphs.

X_{cylinder}/D	Precursor Movement	Percentage of Frames	PDF min	PDF max	p min	p max
11	Upstream	43.2	40	1597	5	1583
11	Downstream	49.5	19	1438	5	822
12	Upstream	43.8	3	1915	49	1883
12	Downstream	48.1	5	1245	29	686
13	Upstream	43.9	6	2926	50	2960
13	Downstream	47.6	6	658	28	1235
18	Upstream	41.4	2	5758	28	5803
18	Downstream	42.4	1	662	42	1150

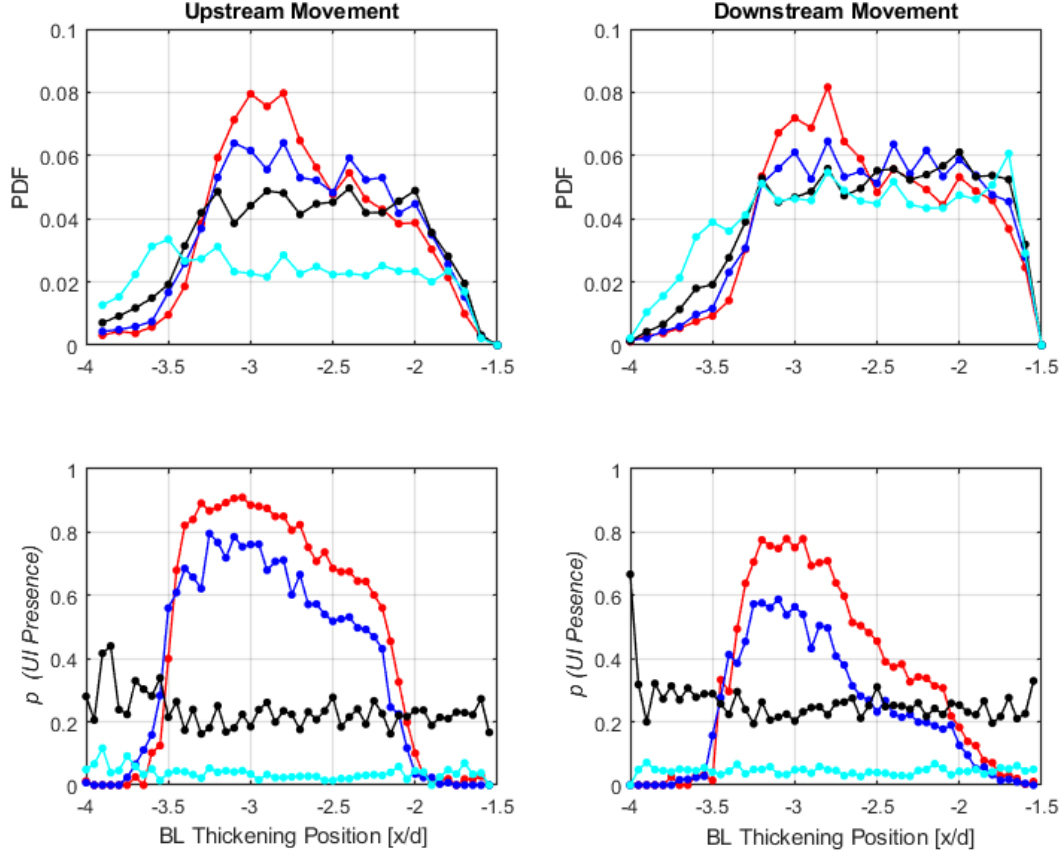


Figure 4.36: Probability density functions of the boundary-layer precursor (top) and the probability for the existence of the upstream influence as a function of the boundary-layer precursor position (bottom). The figure illustrates the following cases: — $X_{\text{cylinder}}/D = 11$ — $X_{\text{cylinder}}/D = 12$ — $X_{\text{cylinder}}/D = 13$ — $X_{\text{cylinder}}/D = 18$

$11 \leq X_{\text{cylinder}}/D \leq 13$ correspond with the SBLI being transitional, although the location of $X_{\text{cylinder}}/D = 13$ has a much lower probability for an upstream influence to be present. This noteworthy characteristic indicates that this location is on the verge of having a fully turbulent interaction. For the most downstream location of $X_{\text{cylinder}}/D = 18$, it is evident that with such a low probability for an upstream influence to be present, this case can be considered fully turbulent. These trends are also supported by the decreasing probability and broadening of these data as the cylinder is moved downstream. In contrasting the two categorized plots (upstream vs. downstream movement), one can see that in the transitional cases of $X_{\text{cylinder}}/D = 11$ and 12 , there is a higher probability for the UI presence when the boundary-layer precursor is moving upstream, or away from the separation shock. This indicates an increasing interaction scale, as evidenced by the standard deviation of the schlieren images in Fig. 4.3.

Given the visualized dynamics of the upstream influence displayed in the high-speed schlieren time series displayed in Fig. 4.5. The discrepancy between the upstream and downstream movement of the boundary-layer thickening precursor represented in the PDF may be a function of the rapid collapse of the shockwave / boundary-layer interaction as a result of upstream forcing. Recall the quick collapse of the upstream influence and separation shock foot downstream and the bow outward bow of the structure. The PDFs associated with the downstream movement show a relatively consistent behavior across $X_{\text{cylinder}}/D = 12, 13$, and 18 . The most upstream location of the cylinder shock generator at $X_{\text{cylinder}}/D = 11$ shows a slightly higher probability at a boundary-layer thickening position of $x/d = -2.75$ to ~ -3.2 , with similar amplitudes when compared to the upstream movement PDF. This suggests that an early transition or partially laminar interaction may resist some of this forcing behavior.

Chapter 5

Conclusions

5.1 Summary and Conclusions

A cylindrical shock generator was installed on a 6° axisymmetric cone to study both transitional and turbulent shockwave / boundary-layer interactions in the UTSI Mach 4 Ludwieg tube. This investigation aimed to answer the following research questions:

1. How do SBLI characteristics and the interaction dynamics evolve when the incoming boundary layer state changes from transitional to turbulent?
2. How does the unsteadiness associated with characteristic SBLI features evolve during the transition process?
3. What promotes the growth of the separation bubble, and what causes its subsequent collapse?

High-speed schlieren imaging and pressure-sensitive paint were utilized as the primary diagnostic techniques while conducting research to address these questions. Data analysis included investigations into power spectral densities, spectral heat maps, POD, DMD, a custom shock-tracking algorithm, and cross-correlation analysis between the separation shock, upstream influence, and a distinct boundary-layer thickening feature. The time-resolved images were acquired at 200 kHz and 20 kHz for the schlieren and PSP, respectively. Four shock generator locations downstream of the cone leading edge ($X_{\text{cylinder}}/D = 11, 12,$

and 13) had a significant portion of their data display structures which are characteristic of transitional interactions. Specifically, images with a turbulent structure were identified 46%, 63%, and 77% of the time for the positions of $X_{\text{cylinder}}/D = 11, 12,$ and 13, respectively. For the case of $X_{\text{cylinder}}/D = 13$, the high percentage indicates that this case should be described as an interaction in the late stages of transition. In the fully turbulent case, $X_{\text{cylinder}}/D = 18$, the data are predominantly characterized by features of a turbulent SBLI with approximately 96% of the images exhibiting these characteristics [61].

The high-speed schlieren imaging shows that the transitional SBLI has a much larger extent of motion in the streamwise direction compared to a fully turbulent SBLI. The forward lambda shock foot collapses towards the cylinder face as the interaction becomes more turbulent, aligning with known interaction behaviors. The incoming boundary-layer thicknesses measured in averaged images are in good agreement for the transitional cases, and rapid growth of the boundary layer occurs downstream of the upstream influence location. A slight increase in the boundary-layer thickness is expected, but similar measurements across the four cylinder shock generator cases are expected to be a limitation of the camera resolution. Time series images display a shock collapse from an upstream disturbance in the incoming boundary layer. The upstream influence is observed to bend in response to turbulent packets as it is pushed downstream next to the separation shock while retaining its structure. Following this collapse, the upstream influence appears to have a linear restoration behavior upstream towards its originating position.

Pressure-sensitive paint data from the instantaneous images show that the aft side of the pressure plateau mirrors the corrugated structure of the high-pressure separation region at the base of the cylinder shock generator. Average pressure traces along the centerline show that the initial rise of the pressure plateau region corresponds well with known scaling relating the cylinder shock generator diameter and boundary-layer thickness as described in Lindorfer et al. [63]. As the constant-diameter cylinder shock generator is moved downstream, the incoming boundary-layer thickness increases, and the initial rise in pressure in the plateau region moves upstream. The pressure trough is fairly constant over all four cases, as expected, since the region is known to correspond to the separation line [63]. Peak pressures, which are a function of inviscid features, are expected to increase with increasing shock strength before

experiencing some pressure relief and subsequent reduction at the attachment line [63]. The second transitional case at $X_{\text{cylinder}}/D = 12$ has the dominant average peak pressure of all four cases and much lower variations in the region than the $X_{\text{cylinder}}/D = 13$ case, which displays similar magnitudes in the average images.

Spectral analyses of the data were completed for both diagnostics, and results were displayed in the form of traditional PSDs and spectral heat maps. PSD data extracted from the entirety of the schlieren images and the resultant shock foot position from the shock tracker did not appear to have any prominent unsteadiness associated with particular frequencies. Spectral heat maps and localized PSDs of the high-speed schlieren and PSP data identified frequency content associated with the upstream influence and separation shock foot. Notable characteristic changes were observed as the cylinder shock generator was translated downstream and the incoming boundary-layer state shifted.

The upstream influence and separation shock displayed ranges of oscillation for each feature within the spectral heat maps. The upstream influence, for each test case, appears to account for approximately 20% of the unsteadiness in the 0 to 3 kHz frequency band of the schlieren data based on the spectral heat maps. Localized peaks at ~ 1200 , 2000, and ~ 2400 Hz were seen in PSD probes extracted near where the UI is known to be. In the PSP spectral heat maps, about 60 to 80% of the uncertainty in the same frequency range was associated with the UI's area of influence for all of the transitional cases.

The separation shock's primary streamwise location appears to be correlated with peaks in the 3 to 6 kHz band, and this accounts for 20-25% of the unsteadiness in this frequency range for the schlieren data and between 10-20% of the unsteadiness in the PSP data. Localized PSD probes near the separation shock region of the schlieren data saw a localized peak at 2300 Hz in the fully turbulent case and in a transitional case ($X_{\text{cylinder}}/D = 12$). This peak is not seen in $X_{\text{cylinder}}/D = 13$ cases, but it may be obscured by mixing behaviors as the interaction becomes more turbulent. Spectral analyses from the pressure-sensitive paint show $\sim 80\%$ of the unsteadiness in the 0-3 kHz range is associated with the separation shock just upstream and downstream of its mean position. In the 3-6 kHz range, the vortex at the cylinder shock generator base dominates with $\sim 30\%$ of the unsteadiness.

PSDs of the schlieren image intensities in the boundary layer generally show peaks matching frequency content in the localized probe analysis for the upstream influence. A decrease in amplitude of the spectral content is seen below 3000 Hz as a function of the localized PSD probe's downstream location and vice versa for frequency content above 3000 Hz. As the shock generator is moved downstream, the boundary-layer probe's spectral content collapses as a result of increased levels of turbulence. Spectral heat maps of the schlieren data show about 30-35% of the unsteadiness in the 0 to 3 kHz range is in the upper portion of the boundary layer between the upstream influence and the separation shock location. At higher frequencies, such as the 3 to 6 kHz range, approximately 25% of the unsteadiness within this band was associated with the separation shock foot and its upstream extent in the boundary layer. This low frequency content may be a strong indicator of the boundary-layer thickening behavior modulating the boundary layer momentum as first seen by Tester et al. [105] and reviewed by Clemens and Narayanaswamy [17]. The frequency content can dramatically change based on the probe location, interaction strength, and tunnel unsteadiness levels.

PSD data of the pressure sensitive paint identified the presence of the separation shock in the spectral heat maps, but yielded inconclusive results when looking at several PSD stations along the cone centerline. Large variations in the spectral amplitude results and several elevated peaks in particular spectra suggest the frequency resolution may not be appropriate to resolve the dynamic behavior of this transitional SBLI. It is essential to consider that SBLI behaviors are dependent on the Reynolds number, Mach number, incoming instabilities, which may also be influenced by freestream tunnel noise, the incoming boundary layer state, model geometry, shock generator geometry, three-dimensional relief, and several other factors. Initial schlieren results that took the PSD of the entire image domain did not capture some of the frequency trends seen. This suggests the interaction's dynamics are more obscured than seen in other experimental data and would be expected in the pressure sensitive paint results for this experiment.

POD and DMD were utilized to identify large amplitude, low frequency content with rapid growth modes that may be associated with SBLI driving mechanisms and the key features necessary for changes in the global SBLI structures. The dominant modes in the

POD analysis revealed a shift in the SBLI structure between the first two and the final two-cylinder shock generator locations. In the DMD analysis, modes associated with rapid growth mechanisms were identified through a unit circle analysis and isolating modes with a radius exceeding 0.85. Based on this metric, less than 1% of the 10,240 total modes generated were used in the analysis. These specific modes contributed between 37.6% and 55.5% of the total energy found in the entire system.

In several instances, DMD data displays regions with positive and negative growth rates parallel to the shear layer near the separation shock with varying extents upstream. The presence of a strong amplification and strong decay within the same mode may suggest the contending growth and dissipation of instabilities. All of the tests show that the cumulative energy contribution is most significant in the 0 to 1.5 kHz band. The late stage transitional case at $X_{\text{cylinder}}/D = 13$ and the fully turbulent case at $X_{\text{cylinder}}/D = 18$ have the strongest growth rates, which appear to be associated with the separation shock. The largest growth and decay rates are at the intersection of the separation shock foot locations with the boundary layer and in the impingement shock where it originates from the triple point.

In the 1.5 to 3 kHz range, the early transitional cases appear to have stronger positive and negative growth rates associated with the interaction than what was observed in the 0 to 1.5 kHz band. Most notably, the separation shock and the incoming boundary layer in the $X_{\text{cylinder}}/D = 11$ case are highlighted; meanwhile, the $X_{\text{cylinder}}/D = 12$ case has strong growth and decay rates in the impingement jet and the upstream boundary layer. The upstream amplification and decay behavior in the boundary layer for the early transition cases may be indicative of a higher sonic line, allowing more information to be propagated upstream. This upstream extent would be limited as the interaction becomes more turbulent and as the sonic line moves lower in the boundary layer.

An image-based feature-tracking algorithm developed in-house permits data quantification and statistical analyses of the schlieren images. The identified features of the interaction include the separation shock, the upstream influence (for transitional cases), and a distinct boundary-layer thickening indicator. A cross-correlation of the UI presence and boundary-layer thickening behavior was completed to investigate the origin of driving mechanisms and explore how the dominant physics may be altered throughout the boundary-layer breakdown

process. Statistical analyses of the feature locations were completed, and the data were categorized based on the upstream or downstream movement of the separation shock (i.e., its time derivative). By comparing the boundary-layer precursor with the upstream influence location, it was clear that the two features move in concert with one another, corroborating the results of Lash et al. [58] and Tester et al. [104] with Mach 2 freestream conditions.

The categorized data were inspected in terms of various probabilities, and the resulting analysis indicates a higher probability that the upstream influence will be present when the boundary-layer thickening precursor is moving upstream. Moreover, it appears from the probability density functions that downstream movement of the separation shock based on the position of boundary-layer precursor for all cases is similar. Meanwhile, the probability densities for upstream movement of the separation shock for a given boundary-layer precursor position decrease as the interaction becomes more turbulent. This suggests that the boundary layer may help modulate upstream and downstream movement for the transitional cases. The results indicate that the late transitional phase SBLI tends to exhibit similar probability density functions as the fully turbulent SBLI case.

5.2 Recommendations for Future Research

Future efforts building upon this research may benefit from a few lessons learned. First, in reference to the configuration analyzed, the pressure-sensitive paint data displayed some unsteadiness in the results as a consequence of the mounting methodology. Although this artifact was identified and verified in the raw data, it became noticeable only after several processing steps. Future configurations should utilize a fixed mount for the cylinder shock generator, as opposed to one that can be traversed. This becomes difficult when the main objective is to probe several downstream stations. Therefore, future configurations may benefit from using a cone model that can be rotated and has different mounting stations at various angles, thereby avoiding upstream effects or disturbances caused by an upstream threaded hole. Note that this would result in a limited number of cylinder shock generator stations that could be tested on a single manufactured cone. Furthermore, pressure taps along the centerline of the cone model would help validate some of the frequency content

and pressure loads seen by the SBLI in each interaction case. This would require an array of sensors at each station, which could become cost-prohibitive due to the total number of sensors. A computational comparison of this experimental data set may help link the frequency content of the shock foot movement to the relative momentum flux in the incoming boundary-layer state. The division of SBLI data based on identifiable unique behaviors, shock foot velocity, and the relative direction of the shock foot's movement before completing POD, DMD, and shock tracker analysis may yield more insightful correlations and spectral responses by effectively dividing up possible driving mechanisms. However, the difficulty in this task lies in identifying and parsing large datasets while maintaining timing, so that the information can be driven back to the frequency domain. Shock trackers that may assume nonlinear behavior may help parse out SBLI behaviors. Note that the success of such codes is highly based on the SBLI strength, advanced outlier detection algorithms, and how dynamic the interaction is in the experiment. Additional advancements in these codes will likely require the development of AI-based tracking algorithms.

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Appendix A

Use of Figures from Published Works

A.1 NASA Media Usage Guidelines

NASA media usage as it pertains to Still Images, Audio Recordings, Video, and Related Computer Files for Non-Commercial Use is as follows.

NASA content - images, audio, video, and computer files used in the rendition of 3-dimensional models, such as texture maps and polygon data in any format - generally are not subject to copyright in the United States. You may use this material for educational or informational purposes, including photo collections, textbooks, public exhibits, computer graphical simulations and Internet Web pages. This general permission extends to personal Web pages.

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For additional information on distribution policies visit:

<https://www.nasa.gov/multimedia/guidelines/index.html>

Appendix B

Selected Scripts

B.1 POD MATLAB Code

```
function [PODoutput]=POD(data)
%=====Inputs:=====
%data – Schlieren frames or time series to be analyzed
%=====Outputs:=====
%phi – Spatial modes, mode time-series or snapshot coefficients
%lambda – Mode eigenvalues
%ai – Mode matrix or time coefficients
%size – size of input matrix supplied
%locs – Index of modes above 2% of the cumulative energy captured in a single
%x90 – Mode number where 90% of the energy has been captured
%x99 – Mode number where 99% of the energy has been captured

%% Universal variables (for figures and saving data to folder)
global saveoutpath outputfilename scale height diameter mask xadjust yadjust

%% Prepare data sets
[r,c,d] = size(data); %grab size of input matrix
```

```

Field=(data)-(mean(data,3)); %mean subtract
Field2 = reshape(Field,[r*c],d); %reshape

%% POD processing – covariance method (or direct POD)
% R = (Field2' * Field2) ./ (d-1); % covariance matrix (also called snapshot matrix)
% [phi lambda] = eig(R, 'vector'); %solve eigvals R*a = lambda*a
% [lambda, indexs] = sort(lambda, 'descend'); %lambda is eigenvalues
% phi = phi(:, indexs); %spatial modes
% ai=Field2*phi; %time coefficients
% % Utilde=ai*phi'; %reconstruction

%% POD Processing – svd method (faster using "econ")– without econ will be ab
[U,S,V] = svd(Field2/sqrt(d-1), 'econ');
    phi=V; %phi is the spatial modes, mode time-series or snapshot coefficients
%     ai=Field2*phi; %A is the time coefficients
    ai=U; %U Mode matrix and should be the same as ai
    lambda= diag(S).^2; %mode eigenvalues

%% Error Approximation
    singular_values = lambda; % Extract singular values from the eigenvalues
    importantmodes=sum((singular_values./sum(singular_values).*100)>2); % count
    [pks, locs]=maxk(lambda, importantmodes); % index of important modes

% Cumulative Energy:
    hf = figure; %Open figure and keep handle
    hf=colordef(hf, 'white'); %Set color scheme
    hf.Color='w'; %Set background color of figure window
    cumsumvalues=((cumsum(singular_values)./sum(singular_values)).*100);
    plot(cumsumvalues, 'ro-', 'LineWidth', 1);
    grid on

```

```

set(gca,'XMinorTick','on')
set(gca,'YMinorTick','on')
xlabel('Mode Index');
ylabel('Cumulative Energy [%]');
% title('Cumulative Energy Content');
exportgraphics(gcf, strcat(saveoutpath, outputfilename, '_PODCumulativeEnergyContent'), 'png');
saveas(hf, strcat(saveoutpath, outputfilename, '_PODCumulativeEnergyContent'), 'png');

% Singular Value Spectrum:
hf = figure; %Open figure and keep handle
hf=colordef(hf,'white'); %Set color scheme
hf.Color='w'; %Set background color of figure window
sumsingularvalues=((singular_values./sum(singular_values)).*100);
plot(sumsingularvalues,'bo-', 'LineWidth', 1);
grid on
set(gca,'XMinorTick','on')
set(gca,'YMinorTick','on')
xlabel('Mode Index');
ylabel('Energy [%]');
[~,x90]=min(abs(cumsumvalues-90));
[~,x99]=min(abs(cumsumvalues-99));
xline(x90,'--','LineWidth',1.5) % cumsumvalue of 90%
ylim_value = ylim;
txt = texlabel('90%');
text(x90-10,ylim_value(2)*2/3,txt,'FontSize', 12)
xline(x99,'--','LineWidth',1.5) % cumsumvalue of 99%
txt =texlabel('99%');
text(x99-10,ylim_value(2)*2/3,txt,'FontSize', 12)
% title('Singular Value Spectrum');
exportgraphics(gcf, strcat(saveoutpath, outputfilename, '_PODSingularValueSpectrum'), 'png');

```

```

saveas(hf, strcat(saveoutpath, outputfilename, '_PODSingularValueSpectrum', '

%% PLOT mode number
for ii = 1:length(locs) %mode of interest
    modeshape = reshape(ai(:, locs(ii)), [r, c]); %reshape back into normal compone
    modeshape = modeshape/max(max(abs(modeshape))); %normalize colormap
    txt =(strcat('Mode', { ' ' }, num2str(locs(ii))));
    hf=easyfigure(modeshape, strcat('_PODMode_', num2str(locs(ii))), 'redblue'
end
for ii = [x90, x99, 100, d] %mode of interest
    modeshape = reshape(ai(:, ii), [r, c]); %reshape back into normal compone
    modeshape = modeshape/max(max(abs(modeshape))); %normalize colormap
    txt =(strcat('Mode', { ' ' }, num2str(ii)));
    easyfigure(modeshape, strcat('_PODMode_', num2str(ii)), 'redblue', [], [], [
end

%% Reconstruction based on modenumber range ( $X=U*S*V$ )
for ii = 1:length(locs) %mode of interest
    Utilde =sum((ai(:, 1:locs(ii))*phi(:, 1:locs(ii)))', 2)'; % Reconstruction
    Utilde_new = reshape(Utilde, [r, c]); %reshape back into normal compone
    txt =(strcat('Mode 1-', num2str(locs(ii))));
    hf=easyfigure(Utilde_new, strcat('_PODReconstructionMode_', num2str(locs(ii))
end
for ii = [x90, x99, 100, d] %mode of interest
    Utilde =sum((ai(:, 1:ii)*phi(:, 1:ii)))', 2)'; % Reconstruction on mode k
    Utilde_new = reshape(Utilde, [r, c]); %reshape back into normal compone
    txt =(strcat('Mode 1-', num2str(ii)));
    easyfigure(Utilde_new, strcat('_PODReconstructionMode_', num2str(ii)), '
end

```



```

%% Plot PSD of POD modes with additional Strouhal Number Axis
hf = figure; %Open figure and keep handle
hf=colormap(hf,'white'); %Set color scheme
hf.Color='w'; %Set background color of figure window
hold on
modes=[1,2,3,x90,x99,100];
for ii=1:length(modes)
[pxd, pxd_f]=pwelch(ai(:,modes(ii)), hann(nfft), round(.5*nfft), nfft,fs)
npsd=pxd/trapz(pxd_f(2),pxd); %not sure this is pxd*f/sigma^2
plot(pxd_f*IntermittentL/Velocity,pxd);
Legend{ii}=string(strcat('Mode ',{ ' ' },num2str(modes(ii))));
end
legend(Legend,'Location','northeast')
grid on
box on
set(gca,'XMinorTick','on')
set(gca,'YMinorTick','on')
xlabel('St');
set(gca,'Xscale','log')
set(gca,'Yscale','log')
ylabel('G_{xx} df/\sigma^2');
exportgraphics(gcf,strcat(saveoutpath,outputfilename,'_PODPSDs','.png'),'');
saveas(hf,strcat(saveoutpath,outputfilename,'_PODPSDs','.m'))

%% Save out data
PODoutput.phi= phi;
PODoutput.lambda=lambda;
PODoutput.ai=ai; %size of Region
PODoutput.size=[r,c,d]; %size of Region
PODoutput.locs=locs; %size of Region

```

```
    PODoutput . x90=x90 ;  
    PODoutput . x99=x99 ;  
end
```

B.2 DMD MATLAB Code

```
function [DMDoutput]= dmd(data,fs,DMDrank)
%=====Inputs:=====
%data – Schlieren frames or time series to be analyzed
%fs – sampling frequency of data set
%DMDrank – manually truncate the data set
%=====Outputs:=====
% phi– Dynamic or DMD modes
% lambda – Discrete–time eigenvalues
% omega – Continuous–time eigenvalues
% W – Right eigenvector of Atilde
% DMDrank – rank
% locs – Index of modes above 2% of the cumulative energy captured in a single
% Xdmd – Reconstruction matrix

%% Universal variables (for figures and saving data to folder)
    global saveoutpath outputfilename scale height diameter mask xadjust yadj

%% Prepare data sets
    [r,c,d] = size(data);
    Field=(data)-(mean(data,3)); %mean subtract
    Field2 = reshape(Field,r*c,d); %reshape

%% Perform DMD
    %Constructing shift–stacked data matrices
    X = Field2(:, 1:end-1); % Shift the data by one column to create X1 and X2
    Xprime = Field2(:, 2:end);

    %SVD and truncate to first r modes
```

```

[U, S, V] = svd(X, "econ"); % Perform Singular Value Decomposition

if exist('DMDrank', 'var') == 1 && ~isempty(DMDrank)==1
else
DMDrank=rank(X);
end

Ur = U(:, 1:DMDrank); % Truncate matrices U, S, and V based on the desired rank
Sr = S(1:DMDrank, 1:DMDrank);
Vr = V(:, 1:DMDrank);

%DMD modes
Atilde = Ur' * Xprime * Vr / Sr; % Compute the approximate low-rank reconstruction
[W, D] = eig(Atilde); % Perform eigenvalue decomposition on Atilde
phi = Xprime * Vr / (Sr * W); %dynamic or DMD modes

%DMD eigenvalues
lambda=diag(D); %discrete-time eigenvalues
omega=log(lambda)*fs/(2*pi); %continuous-time eigenvalues

%% Calculate Growth Rates (Real Part of Eigenvalue):
growth_rates = log(real(lambda))* fs; %log of eigenvalues and also known as growth rates

%% Calculate Oscillation Frequencies (Imaginary Part of Eigenvalue):
% oscillation_freqs = abs(imag(log(lambda) / (2*pi *(1/ fs))));
oscillation_freqs = abs(imag(omega));

%% Find local max in amplitude
b=phi./X(:,1); % Normalize by first mode
b=abs(diag(b));

```

```

freqtest= oscillation_freqs;
freqtest(freqtest>0)=1; % Do not select frequencies at zero
b2=b.*freqtest;
[v,ix] = sort(oscillation_freqs);
b2 = b2(ix);
[locs,P] = islocalmax(b2,'MaxNumExtrema',8,'MinSeparation',1000);
locs=ix(locs); %index of important modes
[~,locsGR]=max(growth_rates);

%% Visualizing discrete time eigenvalues (lambda) relative to a unit circle (
hf = figure; %Open figure and keep handle
hf=colorddef(hf,'white'); %Set color scheme
hf.Color='w'; %Set background color of figure window
hold on
plot(lambda, 'b.', 'MarkerSize',15);
rectangle('Position',[-1 -1 2 2 ],'Curvature',1,'EdgeColor','k','LineStyle'
plot(lambda(locs), 'r.', 'MarkerSize',15);
axis(1.2*[-1 1 -1 1]);
axis square;
axis tight
xlim([-1 1])
ylim([-1 1])
box on
ylabel('Im(\lambda)')
xlabel('Re(\lambda)')
% title('Eigenvalues Relative to Unit Circle');
exportgraphics(gcf, strcat(saveoutpath,outputfilename,'_DMDUnitCircle','p
saveas(hf, strcat(saveoutpath,outputfilename,'_DMDUnitCircle','m'))

```

```

%% FFT of data
    xhat=fftn(data);
    N=size(data,3); %number of samples
    xpower = abs(xhat(1:N/2+1)*2/N);
    freqs=fs*(0:(N/2))/N;

% Plot Single-Sided Amplitude Spectrum
    hf = figure; %Open figure and keep handle
    hf=colordef(hf,'white'); %Set color scheme
    hf.Color='w'; %Set background color of figure window
    hold on
    plot(freqs,xpower)
%     title("Single-Sided Amplitude Spectrum of X(t)")
    xlabel("Frequency [Hz]")
    ylabel("|P1(f)|")
    set(gca,'XScale','log')
    set(gca,'YScale','log')
    box on
    axis tight
    grid on
    set(gca,'XMinorTick','on')
    exportgraphics(gcf,strcat(saveoutpath,outputfilename,'_FFT','_png'),'Reso
    saveas(hf,strcat(saveoutpath,outputfilename,'_FFT','_m'))

%% Plot Growth Rate vs Oscillation Frequency and St
    hf = figure('Units','centimeters','Position',[5, 3, 25, 15],'PaperPosi
    hf=colordef(hf,'white'); %Set color scheme
    hf.Color='w'; %Set background color of figure window
    stem(oscillation_freqs,real(growth_rates),'k','LineWidth',1.2);
    hold on

```

```

stem( oscillation_freqs( locs), real( growth_rates( locs) ), 'r', 'LineWidth', 1.2);
xlabel( 'Oscillation Frequency' );
ylabel( 'Growth Rate' );
grid on
set( gca, 'XMinorTick', 'on' )
    ax1 = gca; % current axes
    ax1_pos = ax1.Position; % position of first axes
    ax2 = axes( 'Position', ax1_pos, ...
        'XAxisLocation', 'top', ...
        'YAxisLocation', 'right', ...
        'Color', 'none' );
    ax2.YAxis.Visible = 'off'; % remove y-axis
    ticks = ax1.XTick;
    ticks=ticks.*IntermittentL/Velocity;
    set( gca, 'xticklabel', num2str( ticks, '%.1f' ) )
    xlabel( 'St' )
%      title( 'Growth Rate vs Oscillation Frequency' );
exportgraphics((gcf, strcat( saveoutpath, outputfilename, '_DMDGrowthRateVsOscillationF'
saveas( hf, strcat( saveoutpath, outputfilename, '_DMDGrowthRateVsOscillationF'

```

```

%% Oscillation frequency vs Amplitude (Units of Frequency)
hf = figure %('Units', 'centimeters', 'Position', [5, 3, 25, 15], 'PaperPosDef', 'landscape');
hf=colordef(hf, 'white'); %Set color scheme
hf.Color='w'; %Set background color of figure window
stem( oscillation_freqs, b, 'k', 'LineWidth', 1.2); %oscillation freq vs power
hold on
stem( oscillation_freqs( locs), b( locs), 'r', 'LineWidth', 1.2); %oscillation frequency vs power
xlabel( 'Frequency [Hz]' );
ylabel( 'a_m (a.u.)' );
axis tight

```

```

grid on
set(gca,'XMinorTick','on')
set(gca,'XScale','log')
set(gca,'YScale','log')
exportgraphics(gcf,strcat(saveoutpath,outputfilename,'_DMDFreqVsAmplitude
saveas(hf,strcat(saveoutpath,outputfilename,'_DMDFreqVsAmplitude','.m'))

%% Oscillation frequency vs Amplitude (Units of Strouhal)
hf = figure%('Units','centimeters','Position',[5,3,25,15],'PaperPos
hf=colordef(hf,'white'); %Set color scheme
hf.Color='w'; %Set background color of figure window
stem(oscillation_freqs*IntermittentL/Velocity,b,'k','LineWidth',1.2); %os
hold on
stem(oscillation_freqs(locs)*IntermittentL/Velocity,b(locs),'r','LineWidt
% legend('FFT','DMD')
xlabel('St');
ylabel('a_m (a.u.)');
axis tight
grid on
set(gca,'XMinorTick','on')
set(gca,'XScale','log')
set(gca,'YScale','log')
exportgraphics(gcf,strcat(saveoutpath,outputfilename,'_DMDFreqVsAmplitud
saveas(hf,strcat(saveoutpath,outputfilename,'_DMDFreqVsAmplitudeStrouhal'

%% Mode Number vs Amplitude
hf = figure; %Open figure and keep handle
hf=colordef(hf,'white'); %Set color scheme
hf.Color='w'; %Set background color of figure window
hold on

```



```

%      plot(freqs ,xpower , 'k' , 'LineWidth' ,1.2) %freq vs power/amplitude
stem(b, 'k' , 'LineWidth' ,1.2); %oscillation freq vs power
hold on
stem(locs ,b(locs) , 'r' , 'LineWidth' ,1.2); %oscillation freq vs power
xlabel('Mode Index ');
ylabel('a_m (a.u.) ');
set(gca , 'YScale' , 'log ')
axis tight
box on
grid on
set(gca , 'XMinorTick' , 'on ')
exportgraphics(gcf ,strcat(saveoutpath ,outputfilename , '_DMDModeVsAmplitude
saveas(hf ,strcat(saveoutpath ,outputfilename , '_DMDModeVsAmplitude' , '.m'))

%% Mode Number vs Growthrate
hf = figure; %Open figure and keep handle
hf=colordef(hf , 'white '); %Set color scheme
hf.Color='w'; %Set background color of figure window
hold on
%      plot(freqs ,xpower , 'k' , 'LineWidth' ,1.2) %freq vs power/amplitude
stem(growth_rates , 'k' , 'LineWidth' ,1.2); %oscillation freq vs power
hold on
stem(locs ,growth_rates(locs) , 'r' , 'LineWidth' ,1.2); %oscillation freq vs p
xlabel('Mode Index ');
ylabel('GrowthRates ');
set(gca , 'YScale' , 'log ')
axis tight
box on
grid on
set(gca , 'XMinorTick' , 'on ')

```

```

exportgraphics(gcf, strcat(saveoutpath, outputfilename, '_DMDModeVsGrowthRate', '.m'))
saveas(hf, strcat(saveoutpath, outputfilename, '_DMDModeVsGrowthRate', '.m'))

%% Visualize dynamic modes (phi) in St
    phi1=real(phi);
    phi2=imag(phi);
for ii = 1:length(locs) %size(modes, 2)
    jj=locs(ii);
    mode_image1 = reshape(phi1(:, jj)', size(data, 1), size(data, 2)); %r
    mode_image2 = reshape(phi2(:, jj)', size(data, 1), size(data, 2)); %i
    txt =textlabel(strcat('St= ', num2str(oscillation_freqs(jj)*Intermittency), ' '), 'St= ', num2str(oscillation_freqs(jj)*Intermittency));
    easyfigure(mode_image1, strcat('_DMDModeReal_', num2str(jj)), 'redblue', 'r');
    easyfigure(mode_image2, strcat('_DMDModeImag_', num2str(jj)), 'redblue', 'i');
end

%% DMD reconstruction matrix
mm1=size(X,2); %mm1 = m-1;
time_dynamics=zeros(DMDrank,mm1);
t=(0:mm1-1)*(1/fs); %time vector
for iter=1:mm1
    time_dynamics(:, iter)=(b.*exp(omega*t(iter)));
end
Xdmd=phi*time_dynamics;

%% Save out Data
DMDoutput.phi = phi; %dynamic or DMD modes
DMDoutput.lambda =lambda;%discrete-time eigenvalues
DMDoutput.omega =omega; %continuous-time eigenvalues
DMDoutput.W=W;
DMDoutput.DMDrank =DMDrank; %rank

```

```
DMDoutput.locs =locs; %location of  
DMDoutput.Xdmd=Xdmd; %reconstruction  
end
```

Vita

Lauren Elise Lester was born in Memphis, TN, on April 2nd, 1996. She was raised in Millington, TN and graduated from Millington Central High School in May 2014. She completed her Bachelor of Science in Mechanical Engineering at the University of Tennessee in Knoxville in December of 2018. Upon graduation she joined the HORIZON Research Group at the University of Tennessee Space Institute in pursuit of her PhD in Aerospace Engineering. She obtained her Master of Science in Aerospace Engineering in December of 2022.