

To the Graduate Council:

I am submitting herewith a Thesis written by Richard Henry Swan Jr., entitled "Surface Location Error in Robotic Milling: Modeling and Experiments". I have examined the final electronic copy of this Thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.

Dr. Tony Schmitz, Major Professor

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(Original signatures are on file with official student records.)

Surface Location Error in Robotic Milling: Modeling and Experiments

A Thesis Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Richard Henry Swan Jr.
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Abstract

Robotic milling offers new opportunities for discrete part manufacturing as an alternative to milling using large conventional machine tools. The advantage of industrial robots is their large work volume, configurability, and comparatively low cost. However, robots are significantly less stiff than conventional machine tools, which can lead to poor surface finish, low machining accuracy, and low material removal rates. The purpose of this research is to predict the geometric errors, or surface location errors, that occur in a robotic milling tool path, validate these predictions with machining tests, and compensate these errors by tool path modification. Compared with conventional machine tools, robots possess low stiffness, low frequency vibration modes and the presence of these modes causes surface location errors that are nearly independent of spindle speed in the range typically used for machining. Additionally, the robot often exhibits errors relative to the commanded tool path. By developing an understanding of both the dynamics of the robot and its tool path accuracy, predictions were made of the surface location error for a machined part and a compensation algorithm was developed. The accuracy of the predictions and compensation algorithm were verified with a series of experiments. Through this research it was determined that robotic milling is prone to large surface location errors, but it is possible to reduce these through offline compensation.

Table of Contents

Chapter 1: Introduction	1
Chapter 2: Background and Literature Review	2
Milling Description	2
Radial Depth of Cut	4
Cutting Forces	6
Tap Testing and Modal Fitting	6
Predicting Chatter	12
Average Tooth Angle Approach	14
Fourier Series Approach	15
Surface Location Error	18
Cutting Force Coefficients	24
Robotic Milling	26
Low Stiffness Compensation	26
Low Frequency Modes and Chatter	27
Chapter 3: Experimental Description	29
Chapter 4: Robot Orientation and Dynamics	31
Spindle Orientation	31
Dynamic Testing	31
Tap Testing Across Linear Track	31
Tap Testing While Moving	34
Tap Testing at Different Locations Over the Plate	44
Tool Change Repeatability	44
Tap Testing Repeatability	54
Low Frequency Robot Modes	54
Final System Dynamics	63
Robot Dynamics	63
Workpiece Dynamics	63
Chapter 5: Robot Motions without Cutting	69
Dimensional Accuracy of Robot Tool Paths	75
Chapter 6: Post-Process Measurement	81

Chapter 7: Parameter Selection and Error Prediction	84
Dynamic Model	84
Cutting Force Coefficients	84
Stability Analysis	90
Machining Parameter Selection	90
Spindle Speed Selection	90
Selecting Axial Depth and Feed Per Tooth	91
Robot Path Error Prediction	91
SLE Predictions and Compensation	91
Chapter 8: Conventional Machine Tool Comparison	103
Chapter 9: Results	108
Chapter 10: Conclusions	112
Chapter 11: Future Work	113
References	114
Vita	117

List of Tables

1	Table showing modal parameters of moving versus stationary low frequency FRFs, and stiffness scaling factors	43
2	Table showing the modal fit data of the plate	51
3	Table showing the average dimension and standard deviation of the squares of a sample plate	83
4	Average x direction modal fits of tool-tip robot-spindle modes	87
5	Average y direction modal fits of tool-tip robot-spindle modes	88
6	Workpiece modal fits in x and y	88
7	Cutting conditions for each square of the workpiece	96

List of Figures

1	Example of a face mill (left) and an end mill (right)	3
2	Schematics of up and down milling for radial immersion of 50%	5
3	Diagram showing the cutting force components in milling	5
4	Diagram showing a spring mass damper system	8
5	The modal hammer used in the experiment	9
6	The low mass low sensitivity accelerometer used to measure the tool-tip FRFs . . .	10
7	The large mass high sensitivity accelerometer used to measure low frequency robot modes	10
8	The laser Doppler vibrometer used to measure the low frequency tool-tip FRF . . .	11
9	Cutting tool deflections on a work piece in-phase (left) and out-of-phase (right) . .	13
10	Average cutting angle and cutting force for up milling (left) and down milling (right)	16
11	Example stability map	16
12	SLE: (a) shows intended geometry, (b) shows undercut, and (c) shows overcut . . .	19
13	Plot of stability lobes and SLE at same spindle speeds	19
14	Cutting force predictions using time domain simulation (solid red), Fourier series (n=5)(dotted red), and Fourier series (n=50)(blue)	21
15	Layout of the squares on the work-piece	30
16	Spindle in the parallel configuration (left), and the perpendicular configuration (right)	32
17	Locations of the testing points along the linear track	33
18	Results of the testing in all 10 locations for the x direction	35
19	Results of the testing in all 10 locations for the y direction	36
20	Graph showing the changing dynamics of the robot in the y direction	37
21	Graph showing the changing dynamics of the robot in the x direction	38
22	Graph showing the changing dynamics at the extremes in the x direction	39
23	Graph showing the changing dynamics at the extremes in the y direction	40
24	Real and imaginary low frequency FRF in the x direction	41
25	Real and imaginary low frequency FRF in the y direction	42
26	Graph showing the FRFs of squares 1-3 in the x direction	45
27	Graph showing the FRFs of squares 1-3 in the y direction	46
28	Graph showing the FRFs of squares 4-6 in the x direction	47
29	Graph showing the FRFs of squares 4-6 in the y direction	48
30	Graph showing the FRFs of squares 7-9 in the x direction	49
31	Graph showing the FRFs of squares 7-9 in the y direction	50
32	Graph of the tool change repeatability at the high frequency mode	52
33	Graph of the tool change repeatability at the low frequency modes	53
34	Graph showing the repeatability of x-direction FRF	55
35	Graph showing the repeatability of y-direction FRF	56
36	Graph showing the repeatability of x-direction FRF zoomed in around large mode .	57

37	Graph showing the repeatability of x-direction FRF zoomed in around large mode .	58
38	Impact forces of the hammer during tap testing procedure	59
39	The vibrometer setup with the laser source and target	60
40	Real and imaginary parts of the x direction tool-tip FRF for the vibrometer and accelerometer	61
41	Real and imaginary parts of the y direction tool-tip FRF for the vibrometer and accelerometer	62
42	X Direction FRF of average fit compared with FRFs for each side at low frequency	64
43	X Direction FRF of average fit compared with FRFs for each side at high frequency	65
44	Y Direction FRF of average fit compared with FRFs for each side at low frequency	66
45	Y Direction FRF of average fit compared with FRFs for each side at high frequency	67
46	X and y direction dynamics of the workpiece	68
47	Image of the indicator setup in the spindle	70
48	Comparison of indicator and robot results for all five tests along the front surface .	71
49	Comparison of indicator and robot results for all five tests along the back surface .	72
50	Comparison of indicator and robot results for all five tests along the left surface . .	73
51	Comparison of indicator and robot results for all five tests along the right surface .	74
52	Example of the indicator test run on a conventional machine tool	76
53	Comparison of the commanded and actual robot positions along the front surface .	77
54	Comparison of the commanded and actual robot positions along the back surface .	78
55	Comparison of the commanded and actual robot positions along the left surface . .	79
56	Comparison of the commanded and actual robot positions along the right surface .	80
57	Diagram showing the tool path of the probe	82
58	The dynamics of the robot-spindle-workpiece combination for the high frequency mode	85
59	The dynamics of the robot-spindle-workpiece combination for the low frequency modes	86
60	Average forces of cutting tests in x,y, and z directions across f_t range	89
61	Stability map for x direction feed	92
62	Stability map for y direction feed	93
63	Variation in surface location error near the 5,400 Hz natural frequency	94
64	Variation in surface location error near the tenth subharmonic of the 5,400 Hz natural frequency	95
65	SLE resulting from the errors in the accuracy of the robot paths	97
66	SLE predictions for x axis dimension	98
67	SLE predictions for y axis dimension	99
68	Commanded x direction radial depths to compensate for SLE and robot motion errors	101
69	Commanded y direction radial depths to compensate for SLE and robot motion errors	102
70	Tap testing results of the CNC machine tool compared with the robot in the x direction	104
71	Tap testing results of the CNC machine tool compared with the robot in the y direction	105

72	SLE predictions and measured values for the CNC machine tool x direction	106
73	SLE predictions and measured values for the CNC machine tool y direction	107
74	Measured SLE and predicted SLE in the x direction	109
75	Measured SLE and predicted SLE in the y direction	110
76	Error in the SLE compensation in the x and y directions	111

Chapter 1: Introduction

This thesis describes an implementation of a robotic milling system, a modelling procedure to characterize the dynamics of the system, and a surface location error prediction and compensation algorithm. Robotic milling is a technology that has inspired great interest in recent years. There are many benefits to the successful implementation of robotic milling. For example, when machining large parts, robotic milling can offer a lower cost alternative to large and expensive machine tools, which must be larger than the part to be produced in general. The serial structure of the robot allows it to reach a much larger work volume than a conventional CNC machine with a similar footprint and cost. Also, the robot is capable of greater part complexity due to its six degrees of freedom and ability to reach nearly any area on the part from nearly any direction. When performing five-axis milling, the six-axis robot provides a redundant degree of freedom that can be used to prevent the robot body from colliding with the part, or can be used for stiffness optimization. Additionally, it is possible to combine robotic milling with additive systems to iteratively produce parts with machined interior and exterior surfaces. However, a primary limitation to robotic milling is its low, pose-dependent stiffness. The serial nature of the robot kinematics is the main reason for the low stiffness. The deflections (and corresponding vibrations) of one joint are experienced by every joint that follows it in the kinematic chain, creating a propagation of errors. This issue is well documented in the literature, which is reviewed in Chapter 2, but the potential advantages of robotic milling justify additional research in this area despite its known limitations. The purpose of this research is to develop a robotic milling system, model the dynamics of this system, use this model to predict the corresponding surface location errors, and compensate for these errors to improve the geometric accuracy of machined parts.

Chapter 2: Background and Literature Review

Understanding the topic of surface location error (SLE) in robotic milling requires a background knowledge of two well understood and researched fields: milling and robotics. This chapter describes milling, implications for adding a milling spindle to a robot, milling stability, and the prediction of surface location errors in milling. An understanding of all these topics is necessary to implement a successful robotic milling system.

Milling Description

The process of milling involves a rotating tool with one or more cutting edges and removing material from a non-rotating workpiece using these cutting edges. In a typical three-axis milling machine the tool is mounted to a spindle that is capable of controlled motion in the z axis direction, while the workpiece is mounted to a table that is able to move in the x and y axis directions. There are many different types of tools that can be used for milling, and each will behave differently when cutting. Some examples of tools are face mills, end mills, ball-nose end mills, and chamfer mills. A face mill and end mill are shown in Figure 1. While different tool geometries are available, most share common characteristics that are used when modeling the behavior of the tools. These include the number of teeth (N_t), diameter (d), radius (r), and helix angle (γ). All equations in this thesis refer to milling using a square end mill. However, the techniques can be extended to different tool geometries.

For square end mills, the number of teeth (N_t) refers to the number of unique cutting edges that come into contact with the workpiece. These cutting edges are often oriented at a helix angle (γ), that is inclined relative to the axis of the tool. This allows cutting to occur at different instantaneous locations along the edge, which reduces local pressure on the cutting edge and decreases tool wear. In milling, the tool rotates at a rate referred to as the spindle speed (Ω) typically in units of revolutions per minute (rpm). As the tool rotates, each tooth comes into contact with the workpiece, and removes material producing a "chip". The frequency that the tool comes into contact with the workpiece is referred to as the tooth passing frequency (f_{tooth}) and is calculated in Equation 1.1, where the division by 60 converts revolutions per minute to revolutions per second and provides the final units of teeth per second, or Hz.

$$f_{tooth} = \frac{\Omega N_t}{60} (Hz) \quad (1.1)$$

Some other process parameters that are important are radial depth of cut (a), axial depth of cut (b), feed per tooth (f_t), and cutting forces.

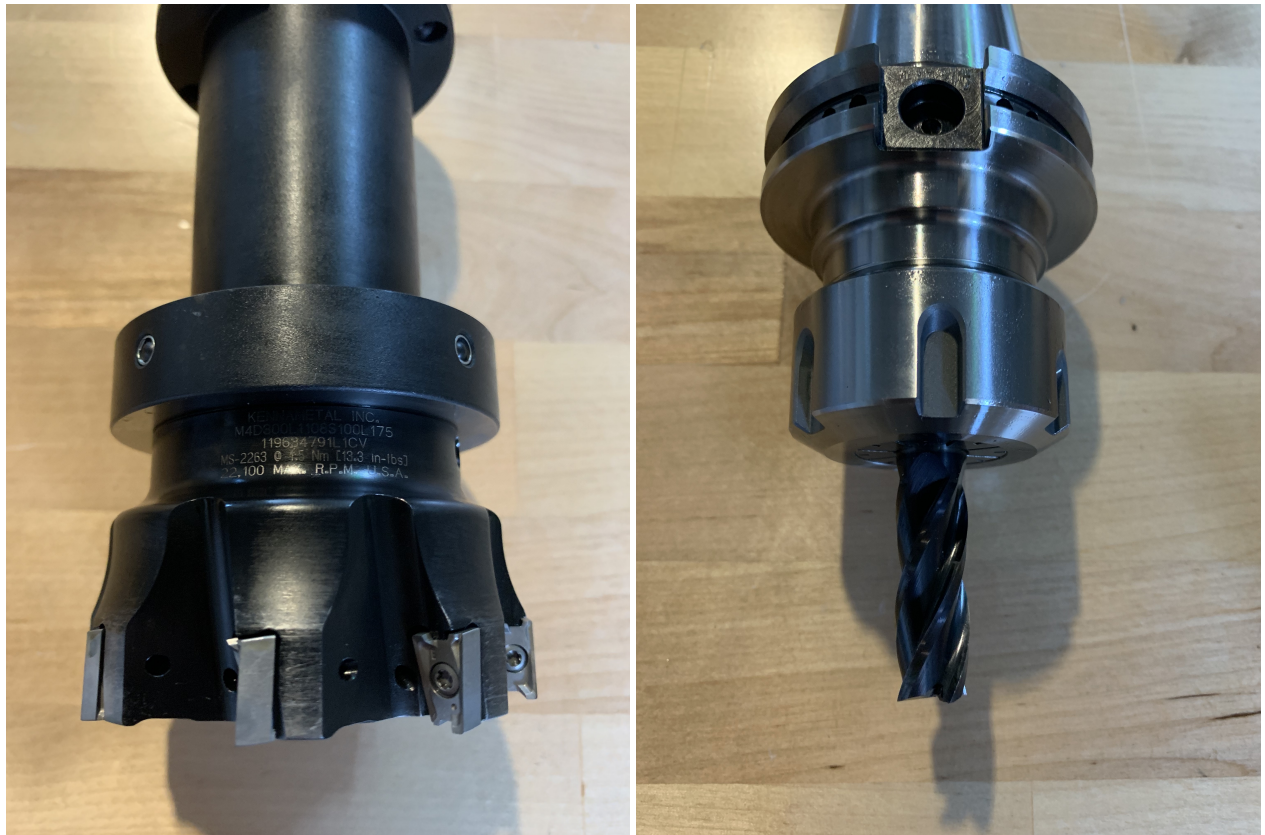


Figure 1: Example of a face mill (left) and an end mill (right)

Radial Depth of Cut

The radial depth of cut is the the depth of cut in the radial direction (perpendicular to the tool axis). This is often reported as a percentage of the tool diameter, or percent radial immersion. For example, 100% radial immersion or "slotting" refers to a cut where the entire diameter of the tool is engaged in the cut. When using a radial immersion less than than 100%, there are two different types of milling that can occur: conventional (up) milling and climb (down) milling. For each of these types, there is a starting angle (ϕ_s), referring to the angle of rotation of the tool when entering the cut, and an exit angle (ϕ_e) referring to the angle of rotation of the tool at the exit of the cut. Schematics of these two different styles of milling are shown in Figure 2.

For up milling $\phi_s = 0^\circ$ by convention because the tool will always enter the cut at the 0° position, while ϕ_e can be calculated using Equation 1.2

$$\phi_e = \cos^{-1}\left(\frac{r-a}{r}\right) \quad (1.2)$$

For down milling $\phi_e = 180^\circ$ by convention because the tool will always exit the cut at the 180° position, while ϕ_s can be calculated using Equation 1.3

$$\phi_s = 180 - \cos^{-1}\left(\frac{r-a}{r}\right) \quad (1.3)$$

In milling the thickness of the chip being created differs depending on the angle (ϕ) of the tool in the cut. This relationship is calculated using Equation 1.4. In up milling the chip thickness increases as the tool progresses through the workpiece, and in down milling the chip thickness decreases.

$$h = f_t \sin(\phi) \quad (1.4)$$

Axial Depth of Cut

The axial depth of cut (b) refers to the length of cut along the axial direction of the tool. The axial depth is used to determine the chip width ($\tilde{b}$) using Equation 1.5 where γ is the helix angle.

$$\tilde{b} = \frac{b}{\cos(\gamma)} \quad (1.5)$$

Feed Per Tooth

Feed per tooth (f_t) refers to the advance of the tool in the feed direction per tooth period. This is dependent on the linear feedrate (f), the spindle speed (Ω) and the number of teeth on the cutting tool (N_t). The calculation is shown in Equation 1.6

$$f_t = \frac{f}{\Omega N_t} \quad (1.6)$$

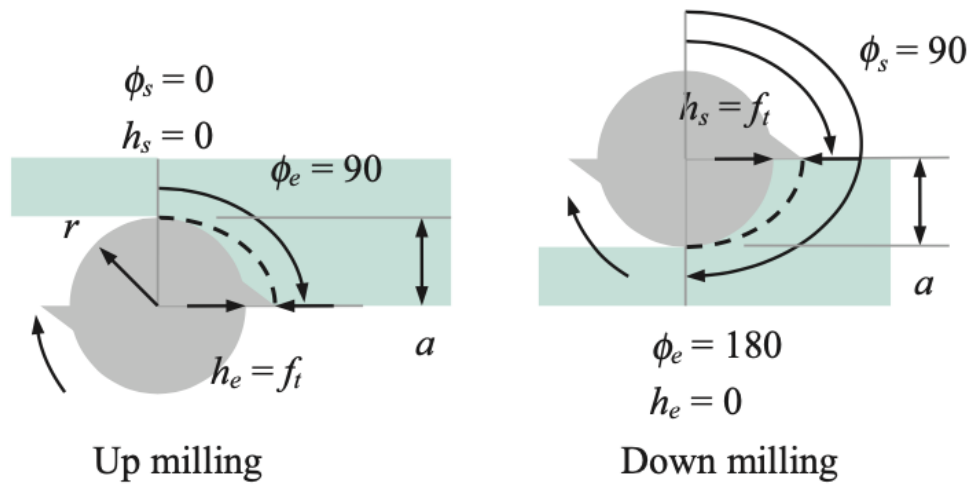


Figure 2: Schematics of up and down milling for radial immersion of 50%

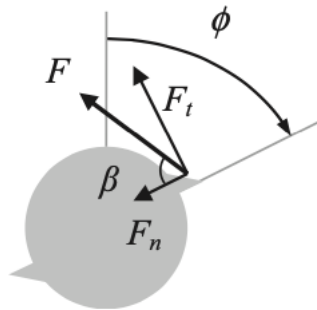


Figure 3: Diagram showing the cutting force components in milling

Cutting Forces

As the tool is fed through the workpiece, a force is required for each tooth to create a chip. This is referred to as the cutting force. The formula for cutting force is shown in equation 1.7, where K_s is the specific cutting force coefficient for the material, b is the axial depth of cut and h is the instantaneous chip thickness.

$$F = K_s b h \quad (1.7)$$

This cutting force can be broken down into two components, one in the normal direction to the cutting edge (F_n), shown in Equation 1.8, and one in the tangent direction to the cutting edge (F_t), shown in Equation 1.9. These values are related by a force angle (β). Figure 3 provides an illustration of these values.

$$F_n = \cos(\beta)F = \cos(\beta)K_s b h = k_n b h \quad (1.8)$$

$$F_t = \sin(\beta)F = \sin(\beta)K_s b h = k_t b h \quad (1.9)$$

These cutting forces are oriented in the tangent and normal directions of the cutting edge of the tool, but the tool is constantly rotating so this coordinate system moves with the tool. It can be helpful to express the cutting forces in an x,y coordinate frame, which does not rotate with the tool but is fixed to the workpiece. The equations for the x direction force component (Equation 1.10) and the y direction force component (Equation 1.11) are shown below.

$$F_x = F_t \cos(\phi) + F_n \sin(\phi) \quad (1.10)$$

$$F_y = F_t \sin(\phi) - F_n \cos(\phi) \quad (1.11)$$

Substituting the equations for F_t and F_n and the equation for instantaneous chip thickness (h) into the equations for F_x and F_y results in the following equations for F_x and F_y .

$$F_x = k_t b f_t \sin(\phi) \cos(\phi) + k_n b f_t \sin(\phi)^2 \quad (1.12)$$

$$F_y = k_t b f_t \sin(\phi)^2 + k_n b f_t \sin(\phi) \cos(\phi) \quad (1.13)$$

When milling with either high radial depths of cut or with cutters with many cutting teeth, it is possible for multiple teeth to be engaged in the cut at the same time. In this case, the equations for F_x and F_y are simply the summation of the forces for each of the teeth engaged in cutting.

Tap Testing and Modal Fitting

In milling, understanding the dynamics of the machine being used for the milling process is crucially important to being able to predict how a cutting process will behave. The dynamics of a system are described by its frequency response function (FRF). For a simple system, such as the spring, mass, damper system shown in Figure 4, the single mode FRF can be calculated using Equations 1.14 and 1.15 for the real and imaginary parts respectively. In the equations, k refers to

the stiffness of the mode, ζ refers to the damping ratio, and r is the ratio of the excitation frequency and the natural frequency (ω/ω_n).

$$FRF_{real} = \frac{1}{k} \frac{1 - r^2}{(1 - r^2)^2 + (2\zeta r)^2} \quad (1.14)$$

$$FRF_{imag} = \frac{1}{k} \frac{-2\zeta r}{(1 - r^2)^2 + (2\zeta r)^2} \quad (1.15)$$

However, machine tools are not simple systems, and contain many vibration modes. The FRFs are typically measured and then fit with multiple modes to fully describe the system dynamics. The FRF measurements are performed using a modal tap test. In this test, a modal hammer and vibration sensor are applied. The modal hammer measured the force when tapping the tool and the sensor records the corresponding tool vibration. The sensor can be an accelerometer, capacitance probe, or a laser Doppler vibrometer. A hammer is shown in Figure 5, two different accelerometers are shown in Figures 6 and 7. The first accelerometer is low mass and low sensitivity, and is used on the tool-tip to measure the higher frequency FRFs. The second accelerometer has a larger mass and a higher sensitivity and is used to measure the low frequency vibration modes. The vibrometer is used to measure low frequency vibration modes at the tool-tip and is shown in Figure 8. The accelerometer is mounted to the tool using modal wax, and then the tool is hit with the hammer. The hammer records the impact force it creates on the tool and the accelerometer measures its motions after the impact. The accelerometer data is double integrated in the frequency domain to yield the displacement, and the displacement and force data are used to determine the real and imaginary parts of the tool-tip FRF in (m/N) across a range of frequencies. This impact is repeated multiple times in a single test and the results are averaged to reduce noise and increase accuracy. Similarly, the laser beam from the vibrometer is focused on the tool, and it measures the motion of the tool when the hammer impacts it.

This process results in a FRF with the selected frequency measurement range, usually 0-10,000 Hz. To model the system, the most flexible modes are identified and the system dynamics are represented only for those modes. This is done using both the real and imaginary parts of the FRF, through a process called peak picking. The first step is to find the natural frequency of a mode (ω_n). This is done by finding the local minimum of the imaginary portion of the FRF. The frequency where this local minimum occurs is the natural frequency of the mode. Next, the dimensionless damping ratio is calculated using Equation 1.16. In this equation ω_1 refers to the frequency of the local maximum of the real part of the FRF, and ω_2 refers to the frequency of the local minimum of the real FRF.

$$\zeta = \frac{\omega_2 - \omega_1}{2\omega_n} \quad (1.16)$$

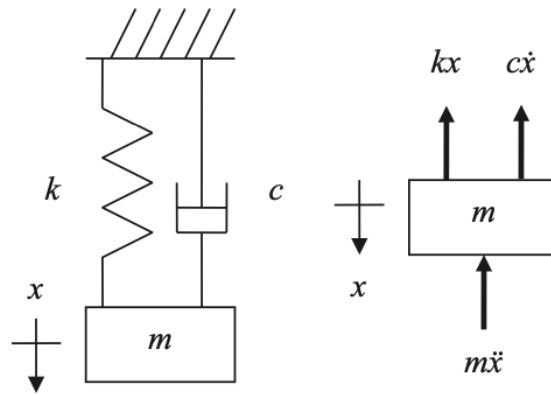


Figure 4: Diagram showing a spring mass damper system



Figure 5: The modal hammer used in the experiment

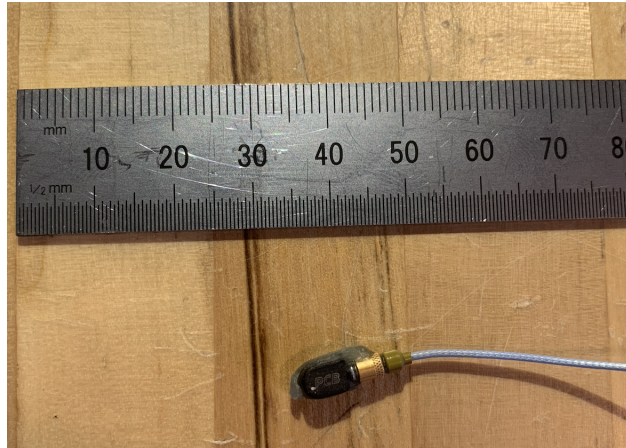


Figure 6: The low mass low sensitivity accelerometer used to measure the tool-tip FRFs

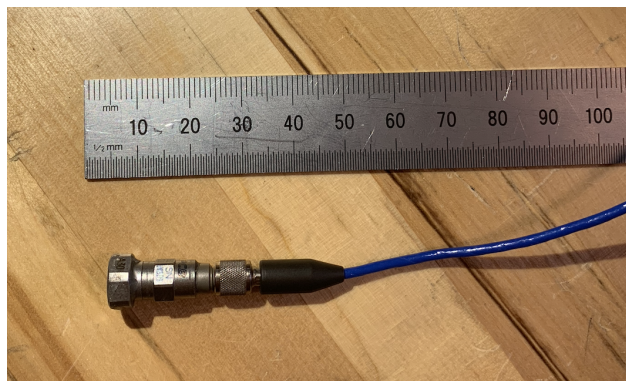


Figure 7: The large mass high sensitivity accelerometer used to measure low frequency robot modes



Figure 8: The laser Doppler vibrometer used to measure the low frequency tool-tip FRF

Next the stiffness (k) of the system is calculated using Equation 1.17. In this equation (A) refers to the local minimum value of the imaginary FRF where the natural frequency was identified.

$$k = \frac{-1}{2\zeta A} \quad (1.17)$$

From these values it is possible to calculate the modal mass (m) for the selected mode using Equation 1.18, and the modal damping coefficient (c) using Equation 1.19.

$$m = \frac{k}{\omega_n^2} \quad (1.18)$$

$$c = 2\zeta\sqrt{km} \quad (1.19)$$

This process is repeated for as many modes as are desired to obtain an accurate fit of the FRF.

Predicting Chatter

During the milling process, as cutting forces are applied to the tool it will deflect. These deflections will affect the surface of the part, and each subsequent tooth will cut away the uneven surface the tooth before it left behind. If the deflection of the previous pass is in phase with the deflection of the next pass as shown on the left in Figure 9, then the chip thickness will remain roughly the same as it would if the tool was not deflecting, and the cutting force will remain stable. However if the deflections are out of phase, as shown on the right in Figure 9, the chip thickness experienced by the tool will vary considerably from what it would be if the tool was not deflecting.

In the areas where the chip thickness is higher, the cutting force will also be higher, causing higher deflections which will be encountered by the next tooth. This causes the cutting forces to grow significantly. This phenomenon is called "regenerative chatter". In unstable cuts (i.e., cuts that experience chatter), the cutting forces grow significantly from those experienced in stable cutting, and the final machined surface will have a wavy look, from where the tool deflected in and out of the surface. This is a condition that should be avoided in milling, and this can be done by modeling the cutting process and selecting cutting conditions that are stable.

There are two different frequency domain solutions used to model chatter, the average tooth angle approach, developed by Tlustý [25, 26, 27, 28], and the Fourier series approach developed by Altintas [29]. While both provide a closed-form, frequency domain solution for milling stability, they use different simplifying assumptions. Therefore, both are used in order to predict and avoid chatter, in this thesis.

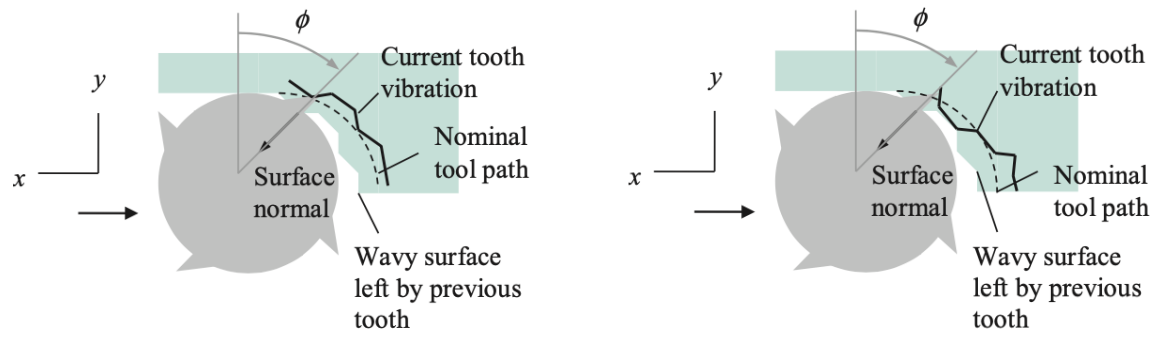


Figure 9: Cutting tool deflections on a work piece in-phase (left) and out-of-phase (right)

Average Tooth Angle Approach

For the average tooth angle approach, the milling process is simplified by assuming an average tooth angle in the cut and an average cutting force. Using directional orientation factors, this cutting force is projected into the x and y directions of the system, and then projected into the surface normal at the average tooth angle.

The first step of modeling chatter using this method is to determine the frequency response function (FRF), in both the x and y directions, of the system being used. This can be done by performing a modal tap test of the tool mounted in the machine. In this case the x direction is defined as the feed direction of the tool. Once the FRFs have been determined for the x and y directions, the directional orientation factors must be calculated. Figure 10 presents a diagram illustrating the significant characteristics in this calculation for up and down milling respectively. These orientation factors are dependent on the average angle of the tooth in the cut (ϕ_{avg}), and on the angle between the surface normal direction and the cutting force (β), which is determined experimentally. ϕ_{avg} is the average of the start angle (ϕ_s), and the exit angle (ϕ_e), as shown in equation 1.20.

$$\phi_{avg} = \frac{\phi_s + \phi_e}{2} \quad (1.20)$$

The directional orientation factors are used to orient the FRFs of both the x and y directions into the surface normal direction. As a result there is a directional orientation factor for both the x (μ_x) and y (μ_y) directions, and these are calculated differently for up and down milling. For up milling (μ_x) and (μ_y) are calculated using Equations 1.21, and 1.22 respectively, and for down milling Equations 1.23 and 1.24 are used.

$$\mu_x = \cos(\beta - (90 - \phi_{avg}))\cos(90 - \phi_{avg}) \quad (1.21)$$

$$\mu_y = \cos(180 - \phi_{avg} - \beta)\cos(180 - \phi_{avg}) \quad (1.22)$$

$$\mu_x = \cos(\phi_{avg} - 90 + \beta)\cos(\phi_{avg} - 90) \quad (1.23)$$

$$\mu_y = \cos(\beta - (180 - \phi_{avg}))\cos(180 - \phi_{avg}) \quad (1.24)$$

After the directional orientation factors are calculated, the oriented FRF is calculated using Equation 1.25.

$$FRF_{orient} = \mu_x FRF_x + \mu_y FRF_y \quad (1.25)$$

After the oriented FRF is calculated, the next step is to determine the maximum axial depth of cut that can be performed without experiencing chatter. The oriented FRF and its corresponding frequency range are then trimmed to only include the valid chatter frequency range. This includes

only the frequencies where the real part of the oriented FRF is negative. The maximum (or limiting) stable axial depth is referred to as (b_{lim}), and is calculated using Equation 1.26.

$$b_{lim} = \frac{-1}{2K_s \text{Real}[FRF_{orient}]N_t^*} \quad (1.26)$$

The variable N_t^* in Equation 1.26 represents the average number of teeth in the cut, and is calculated using Equation 1.27.

$$N_t^* = \frac{\phi_e - \phi_s}{\frac{360}{N_t}} \quad (1.27)$$

Additionally, it is needed to calculate the difference in phase (ϵ) between the current tooth and the next tooth over the valid chatter frequency range. This is done with Equation 1.28.

$$\epsilon = 2\pi - 2\tan^{-1}\left(\frac{\text{Real}[FRF_{orient}]}{\text{Imaginary}[FRF_{orient}]}\right) \quad (1.28)$$

Once b_{lim} and ϵ have been calculated, it is possible to create the stability lobe diagram, which maps b_{lim} to the spindle speeds of the machine. This is done using Equation 1.29. In this equation ω refers to the range of frequencies where the real FRF is negative. Since the stability lobes repeat for each integer fraction of the tooth passing frequency of the system, the variable $N = 0, 1, 2, \dots$ defines a separate stability limit for each instance. The final stability map plot incorporates these individual stability limits for a range of N values. An example is provided in Figure 11.

$$\Omega_N = \frac{(\omega/N_t)}{N + \frac{\epsilon}{2\pi}} \quad (1.29)$$

Fourier Series Approach

The Fourier series approach is used to generate a stability map to predict spindle speed-axial depth combinations where stable cutting can occur. For this approach, the time varying coefficients of the dynamic milling equations are expanded into a Fourier series, then the series is truncated to only include the average component [1]. The cutting force coefficients K_n and K_t are defined using Equations 1.30 and 1.31 respectively.

$$K_n = \frac{1}{\tan(\beta)} \quad (1.30)$$

$$K_t = \frac{K_s}{\sqrt{1 + K_n^2}} \quad (1.31)$$

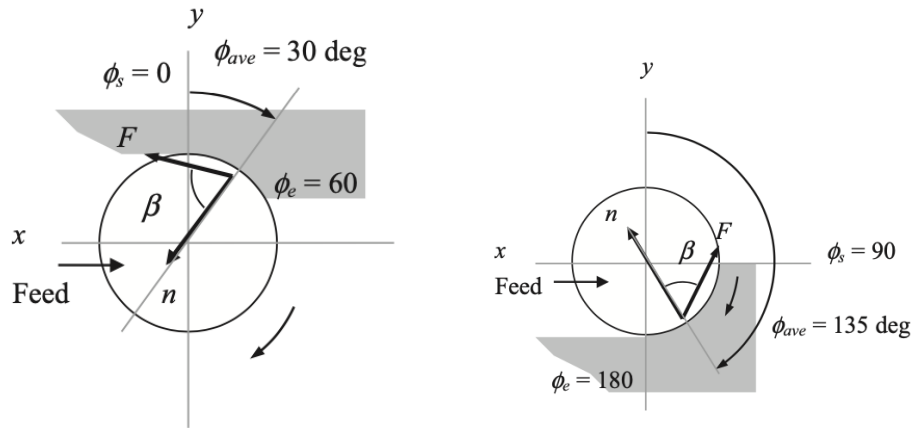


Figure 10: Average cutting angle and cutting force for up milling (left) and down milling (right)

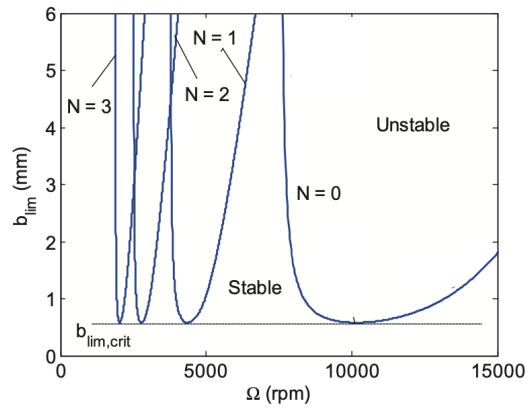


Figure 11: Example stability map

In the average tooth angle approach, the positive x direction is defined as the opposite of the feed direction, while in the Fourier series approach the positive x direction is the same as the feed direction. The FRFs in the x and y directions are determined by modal testing, and an oriented FRF is determined. In this approach the directional dynamic force coefficients α_{xx} , α_{xy} , α_{yx} , and α_{yy} , are determined using Equations 1.32, 1.33, 1.34, and 1.35 respectively. The oriented FRF is represented as the matrix shown in equation 1.36.

$$\alpha_{xx} = \frac{1}{2}((\cos(2\phi) - 2K_n\phi + K_n\sin(2\phi)) \Big|_{\phi_s}^{\phi_e}) \quad (1.32)$$

$$\alpha_{xy} = \frac{1}{2}((- \sin(2\phi) - 2\phi + K_n\cos(2\phi)) \Big|_{\phi_s}^{\phi_e}) \quad (1.33)$$

$$\alpha_{yx} = \frac{1}{2}((- \sin(2\phi) + 2\phi + K_n\cos(2\phi)) \Big|_{\phi_s}^{\phi_e}) \quad (1.34)$$

$$\alpha_{yy} = \frac{1}{2}((- \cos(2\phi) - 2K_n\phi - K_n\sin(2\phi)) \Big|_{\phi_s}^{\phi_e}) \quad (1.35)$$

$$FRF_{or} = \begin{bmatrix} \alpha_{xx}FRF_x & \alpha_{xy}FRF_y \\ \alpha_{yx}FRF_x & \alpha_{yy}FRF_y \end{bmatrix} \quad (1.36)$$

From the oriented FRF matrix, the eigenvalues (λ_1 , λ_2) are determined. These are computed for every value of the oriented FRF across the entire frequency range. This differs from the average tooth angle approach which only considers the frequencies where the real part of the oriented FRF is negative. Across the frequency spectrum, a limiting axial depth value is calculated for each eigenvalue, b_{lim1} and b_{lim2} . These are calculated using Equations 1.37 and 1.38 respectively.

$$b_{lim1} = \frac{\frac{2\pi}{N_t K_t}}{real(\lambda_1)^2 + imag(\lambda_1)^2} real(\lambda_1) (1 + (\frac{imag(\lambda_1)}{real(\lambda_1)})^2) \quad (1.37)$$

$$b_{lim2} = \frac{\frac{2\pi}{N_t K_t}}{real(\lambda_2)^2 + imag(\lambda_2)^2} real(\lambda_2) (1 + (\frac{imag(\lambda_2)}{real(\lambda_2)})^2) \quad (1.38)$$

Once these b_{lim} values have been calculated across the entire frequency range, they are truncated to only the frequencies with positive b_{lim} values. Next the inverse tangents, ψ_1 and ψ_2 , of the ratio of the imaginary and real parts of each eigenvalue are calculated, and this value is used to calculate the phase difference, ϵ , as in the average tooth angle approach. This is shown in Equations 1.39

and 1.40.

$$\epsilon_1 = \pi - 2\psi_1 \quad (1.39)$$

$$\epsilon_2 = \pi - 2\psi_2 \quad (1.40)$$

Finally, the range of spindle speeds (Ω) corresponding to the b_{lim} values is calculated using Equations 1.41 and 1.42, across the range of frequencies where b_{lim} is positive (ω). As with the average tooth angle approach, the stability lobes repeat with each integer fraction (N).

$$\Omega_1 = \frac{60}{N_t} \frac{w}{\epsilon_1 + 2N\pi} \quad (1.41)$$

$$\Omega_2 = \frac{60}{N_t} \frac{w}{\epsilon_2 + 2N\pi} \quad (1.42)$$

Surface Location Error

Chatter is not the only factor to consider when attempting to achieve accurate milling dimensions. Even when using stable cutting conditions, it is possible for forced vibrations to negatively affect part geometric accuracy. This phenomenon is called surface location error (SLE), and is illustrated in Figure 12. There have been many studies performed to characterize the phenomenon of surface location error [19, 20, 21, 22, 23, 24]. The work from these studies and others was used to develop the model for SLE presented by Schmitz and Smith in the Machining Dynamics textbook [1], which was used for the surface location error predictions in this research. In general, surface location error results from deflections of the tool. These deflections are based on both the flexibility of the tool and the cutting forces of the operation. Higher cutting forces relative to the flexibility of the tool will generally result in higher magnitudes of SLE. However, SLE is also highly dependent on the natural frequencies of the system, and the tooth passing frequency of the tool. This is explored in a paper by Schmitz et al. [18]. In the paper the authors examined how surface location error changes with spindle speed. They noted that when holding other process parameters constant (f_t , axial depth, radial depth) changing the spindle speed resulted in large changes in SLE around the natural frequency corresponding to the most flexible mode of the system, as well as its integer fractions. These large variations in SLE occur at the same spindle speeds as the best speeds for avoiding chatter, so it is often difficult to achieve both high material removal rates and accurate surfaces. To illustrate this point, a plot of the stability lobes of a system across a range of spindle speeds, and the corresponding surface location error at each speed is shown in Figure 13.

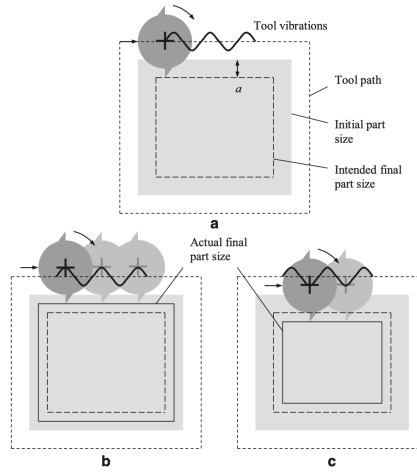


Figure 12: SLE: (a) shows intended geometry, (b) shows undercut, and (c) shows overcut

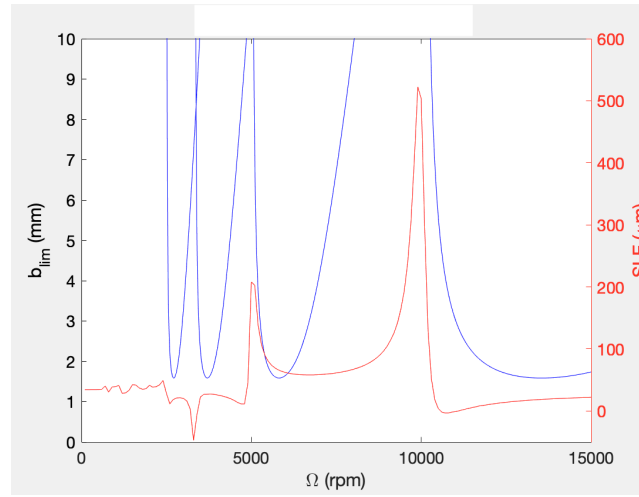


Figure 13: Plot of stability lobes and SLE at same spindle speeds

To select machining parameters that will achieve the desired part accuracy, it is important to model the SLE over a range of possible spindle speeds. For this thesis, a frequency domain solution for SLE was used [1, 30]. In this method two basic assumptions are made. The first is that while vibrations occur in both the feed direction (x) and the direction normal to the finished surface (y), the y direction vibrations dominate the final SLE. The second is that the effects of regenerative chatter can be neglected because stable cutting conditions are used. Given these assumptions, the SLE model is computed by expressing the y direction force in the frequency domain, using a Fourier series, determining the frequency domain y displacement by multiplying the Fourier series by the y direction FRF of the machining system, then performing an inverse Fourier transform and sampling this deflection at the tool rotation angle which will leave the final surface (cut entry for up milling, and cut exit for down milling).

The first step is to compute the Fourier series coefficients. Any number of coefficients (terms) can be used. However, the more terms are used the better the prediction will be. Figure 14 shows an example of Fourier series approximations using five and 50 terms, to compare with the results from a time domain simulation. Because of the improved agreement with 50 terms, this was the number used for this thesis. This frequency domain solution considers the tangential k_t and normal k_n cutting force coefficients, as well as their corresponding edge coefficients k_{te} and k_{ne} . The instantaneous tangential and normal cutting force equations are shown in Equations 1.43 and 1.44 respectively. In this equation $h(\phi)$ refers to Equation 1.4 for instantaneous chip width.

$$F_t(\phi) = k_t b h(\phi) + k_{te} b \quad (1.43)$$

$$F_n(\phi) = k_n b h(\phi) + k_{ne} b \quad (1.44)$$

The complete equation for the force in the y direction, perpendicular to the feed direction, is shown in Equation 1.45, with $g(\phi_i)$ being a switching function that equals 1 when the tooth is engaged in the cut and 0 otherwise.

$$F_y(\phi) = -b \left[\frac{-k_t f_t}{2} \sum_{i=1}^{N_t} g(\phi_i) (1 - \cos(2\phi_i)) + \frac{k_n f_t}{2} \sum_{i=1}^{N_t} g(\phi_i) \sin(2\phi_i) \right] - k_{te} \sum_{i=1}^{N_t} g(\phi_i) \sin(\phi_i) + k_{ne} \sum_{i=1}^{N_t} g(\phi_i) \cos(\phi_i) \quad (1.45)$$

In terms of the Fourier series coefficients F_y can be written as shown in Equation 1.46.

$$F_y(\phi) = \sum_{i=1}^{N_t} \left(a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\phi_i) + b_n \sin(n\phi_i)) \right) \quad (1.46)$$

The first coefficient to be calculated is a_0 . This is done using Equation 1.47.

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} F_y(\phi) d\phi \quad (1.47)$$

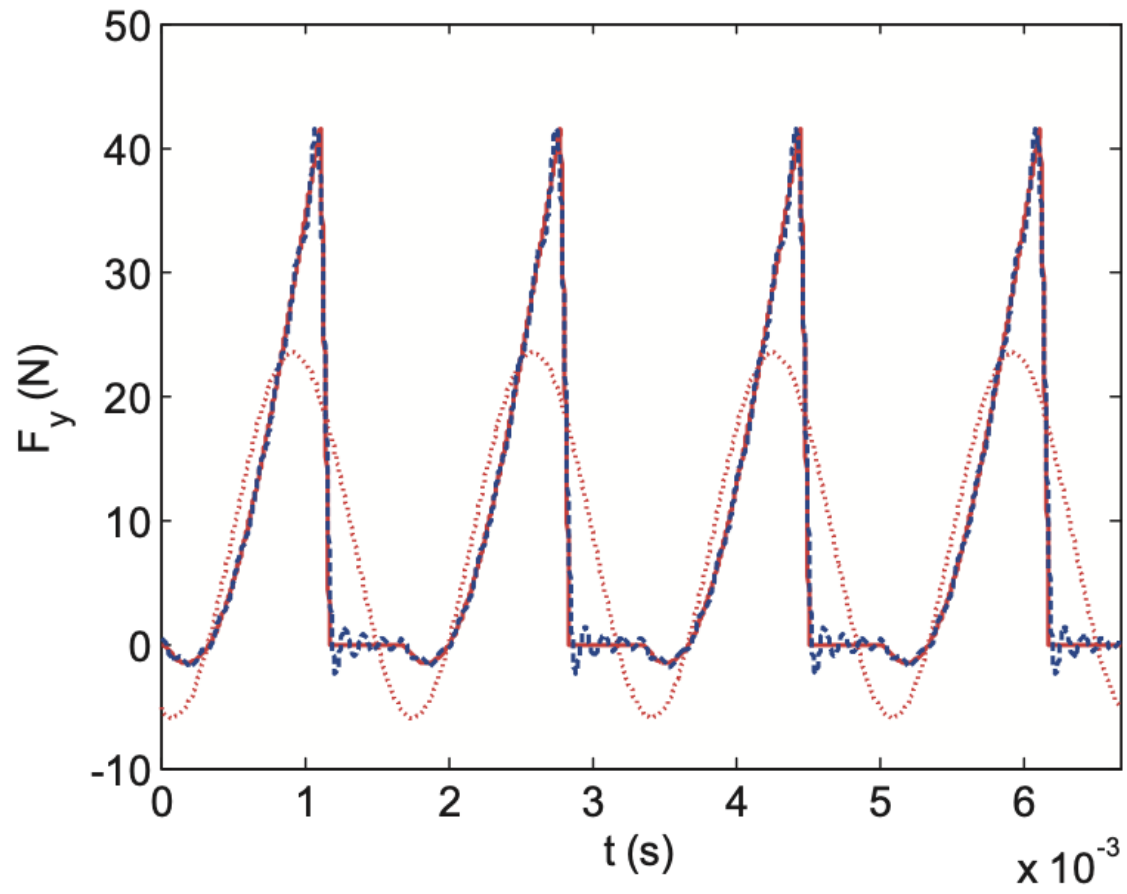


Figure 14: Cutting force predictions using time domain simulation (solid red), Fourier series (n=5)(dotted red), and Fourier series (n=50)(blue)

However, for examining an individual tooth the integration from 0 to 2π can be broken up into smaller sub integrals. During one rotation of the tool a single cutting tooth is only engaged in the cut from ϕ_s to ϕ_e . The maximum cutting angle possible is 180° or π radians. Because of this the π to 2π section of the integral must be 0 for this tooth. To account for both up and down milling the angle ϕ_1 is defined between 0 and π , and for up milling this angle refers to the exit angle of the cut and for down milling it refers to the start angle. Equation 1.48 shows the equation for a_0 divided into multiple integrals.

$$a_0 = \frac{1}{2\pi} \left[\int_0^{\phi_1} F_y(\phi) d\phi + \int_{\phi_1}^{\pi} F_y(\phi) d\phi + \int_{\pi}^{2\pi} F_y(\phi) d\phi \right] \quad (1.48)$$

Since the tooth is only engaged in the cut between ϕ_s and ϕ_e the choice of either up or down milling results in a single integral as shown in Equation 1.49.

$$a_0 = \frac{1}{2\pi} \left[\int_{\phi_s}^{\phi_e} F_y(\phi) d\phi \right] \quad (1.49)$$

Expanding F_y to include all of its components yields the final form of a_0 as shown in Equation 1.50.

$$a_0 = -\frac{bN_t}{2\pi} \left[-\frac{k_t f_t \phi}{2} + \frac{k_t f_t}{4} \sin 2\phi - \frac{k_n f_t}{4} \cos 2\phi + k_{te} \cos \phi + k_{ne} \sin \phi \right]_{\phi_s}^{\phi_e} \quad (1.50)$$

The same procedure is completed for the other Fourier series coefficients a_n and b_n shown in Equations 1.51 and 1.52.

$$a_n = \frac{1}{\pi} \int_0^{2\pi} F_y(\phi) \cos(n\phi) d\phi \quad (1.51)$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} F_y(\phi) \sin(n\phi) d\phi \quad (1.52)$$

The equations for a_1, a_2, b_1 , and, b_2 are shown in Equations 1.53, 1.54, 1.55, 1.56 respectively. Starting with the $n = 3$ term for both a and b, a recursive pattern develops, and the a_n and b_n terms can be calculated using Equations 1.57, and 1.58.

$$a_1 = -\frac{bN_t}{\pi} \left[k_t c \left(-\frac{1}{4} \sin \phi + \frac{1}{12} \sin 3\phi \right) + k_n c \left(-\frac{1}{4} \cos \phi - \frac{1}{12} \cos 3\phi \right) \right. \\ \left. + k_{te} \left(\frac{1}{4} \cos 2\phi \right) + k_{ne} \left(\frac{1}{2} \phi + \frac{1}{4} \sin 2\phi \right) \right]_{\phi_s}^{\phi_e} \quad (1.53)$$

$$a_2 = -\frac{bN_t}{\pi} \left[k_t c \left(\frac{1}{4} \phi - \frac{1}{4} \sin 2\phi + \frac{1}{16} \sin 4\phi \right) + k_n c \left(-\frac{1}{16} \cos 4\phi \right) \right. \\ \left. + k_{te} \left(-\frac{1}{2} \cos \phi + \frac{1}{6} \cos 3\phi \right) + k_{ne} \left(\frac{1}{2} \sin \phi + \frac{1}{6} \sin 3\phi \right) \right]_{\phi_s}^{\phi_e} \quad (1.54)$$

$$b_1 = -\frac{bN_t}{\pi} \left[\begin{array}{l} k_t c \left(\frac{3}{4} \cos \phi - \frac{1}{12} \cos 3\phi \right) + k_n c \left(\frac{1}{4} \sin \phi - \frac{1}{12} \sin 3\phi \right) \\ + k_{te} \left(-\frac{1}{2} \phi + \frac{1}{4} \sin 2\phi \right) + k_{ne} \left(-\frac{1}{4} \cos 2\phi \right) \end{array} \right]_{\phi_s}^{\phi_e} \quad (1.55)$$

$$b_2 = -\frac{bN_t}{\pi} \left[\begin{array}{l} k_t c \left(\frac{1}{4} \cos 2\phi - \frac{1}{16} \cos 4\phi \right) + k_n c \left(\frac{1}{4} \phi - \frac{1}{16} \sin 4\phi \right) \\ + k_{te} \left(-\frac{1}{2} \sin \phi + \frac{1}{6} \sin 3\phi \right) + k_{ne} \left(-\frac{1}{2} \cos \phi - \frac{1}{6} \cos 3\phi \right) \end{array} \right]_{\phi_s}^{\phi_e} \quad (1.56)$$

$$a_n = -\frac{bN_t}{\pi} \left[\begin{array}{l} k_t c \left(-\frac{1}{2n} \sin n\phi + \frac{1}{4(n-2)} \sin(n-2)\phi + \frac{1}{4(n+2)} \sin(n+2)\phi \right) \\ + k_n c \left(\frac{1}{4(n-2)} \cos(n-2)\phi - \frac{1}{4(n+2)} \cos(n+2)\phi \right) \\ + k_{te} \left(-\frac{1}{2(n-1)} \cos(n-1)\phi + \frac{1}{2(n+1)} \cos(n+1)\phi \right) \\ + k_{ne} \left(\frac{1}{2(n-1)} \sin(n-1)\phi + \frac{1}{2(n+1)} \sin(n+1)\phi \right) \end{array} \right]_{\phi_s}^{\phi_e} \quad (1.57)$$

$$b_n = -\frac{bN_t}{\pi} \left[\begin{array}{l} k_t c \left(\frac{1}{2n} \cos n\phi - \frac{1}{4(n-2)} \cos(n-2)\phi - \frac{1}{4(n+2)} \cos(n+2)\phi \right) \\ + k_n c \left(\frac{1}{4(n-2)} \sin(n-2)\phi - \frac{1}{4(n+2)} \sin(n+2)\phi \right) \\ + k_{te} c \left(-\frac{1}{2(n-1)} \sin(n-1)\phi + \frac{1}{2(n+1)} \sin(n+1)\phi \right) \\ + k_{ne} c \left(-\frac{1}{2(n-1)} \cos(n-1)\phi - \frac{1}{2(n+1)} \cos(n+1)\phi \right) \end{array} \right]_{\phi_s}^{\phi_e} \quad (1.58)$$

Once the Fourier series coefficients have been calculated, they are used to simulate the cutting force. The cutting force model F_y already includes all the terms of the Fourier series and each of the cutting teeth, but to simulate tools with a non-zero helix angle, the model must also divide the cut into slices along the axial direction, where each slice is assumed to have a 0° helix angle (γ). The axial direction is divided into A slices with each slice represented by j. The final representation of F_y is shown in Equation 1.59.

$$F_y(\phi) = \sum_{j=1}^A \sum_{i=1}^{N_t} \left(a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\phi_i) + b_n \sin(n\phi_i)) \right) \quad (1.59)$$

In this equation ϕ_i is calculated using Equation 1.60. This includes the spindle speed in rad/s, ω , and the angular rotation of each of the axial slices χ , calculated in Equation 1.61. In this equation db is the height of each slice, and d is the diameter of the tool.

$$\phi_i = \omega t + \frac{2\pi}{N_t} (i - 1) - \chi(j - 1) \quad (1.60)$$

$$\chi = \frac{2db \tan(\gamma)}{d} \quad (1.61)$$

The frequency domain y direction force is then multiplied by the frequency response function for the milling system in the y direction. The result is the frequency domain y direction vibration. Then the inverse Fourier transform is performed to transform the y displacement back into the time domain, and the displacement is sampled at the cut entry, for up milling, or the cut exit for down milling.

Cutting Force Coefficients

Both the algorithms for the stability maps and for the surface location error require cutting force coefficients. These values are unique to each tool-workpiece combination and depend on the geometry of the tool and the physical properties of the material being cut. For rough estimates of SLE or stability generic coefficients based on the workpiece material can be used, but for more accurate predictions, the coefficients should be determined experimentally using cutting tests.

The procedure used in this research to determine cutting force coefficients is based on work by Schmitz et al. [1]. In the procedure a series of cutting tests are performed with all cutting parameters held constant except feed per tooth, which is varied over the range of interest. The force is measured and the known cutting conditions are used to determine the cutting force coefficients. This cutting force in the x, y, and z directions is measured with a dynamometer, and a linear fit is applied. The slope of this fit corresponds with the cutting force coefficients, k_n and k_t , while the y intercept of the fit corresponds with the cutting force edge coefficients k_{ne} and k_{te} .

The equations for the normal and tangential cutting forces are shown in Equations 1.62 and 1.63 respectively. In these equations the cutting force coefficient multiplies both the axial depth and the chip thickness which depends on feed per tooth, while the edge coefficient only multiplies axial depth.

$$F_n = k_n b h + k_{ne} b \quad (1.62)$$

$$F_t = k_t b h + k_{te} b \quad (1.63)$$

The dynamometer measures and reports forces in the x and y directions not the normal and tangential so these cutting forces must be projected into the x and y directions to represent the values that would be measured. For the cutting angle ϕ the forces in the x and y directions are shown in Equations 1.64 and 1.65 respectively.

$$F_x = k_t b f_t \frac{\sin(2\phi)}{2} + k_{te} b \cos(\phi) + k_n b f_t \frac{(1 - \cos(2\phi))}{2} + k_{ne} b \sin(\phi) \quad (1.64)$$

$$F_y = k_t b f_t \frac{(1 - \cos(2\phi))}{2} + k_{te} b \sin(\phi) - k_n b f_t \frac{\sin(2\phi)}{2} - k_{ne} b \cos(\phi) \quad (1.65)$$

Since the only variable changing between tests is feed per tooth it is only required to consider the average cutting force in each direction during the cut. This is done using Equations 1.66 and 1.67

for the x and y directions respectively.

$$\bar{F}_x = \left[\frac{N_t b f_t}{8\pi} (-k_t \cos(2\phi) + k_n(2\phi - \sin(2\phi))) + \frac{N_t b}{2\pi} (k_{te} \sin(\phi) - k_{ne} \cos(\phi)) \right]_{\phi_s}^{\phi_e} \quad (1.66)$$

$$\bar{F}_y = \left[\frac{N_t b f_t}{8\pi} (k_t(2\phi - \sin(2\phi)) + k_n \cos(2\phi)) - \frac{N_t b}{2\pi} (k_{te} \cos(\phi) + k_{ne} \sin(\phi)) \right]_{\phi_s}^{\phi_e} \quad (1.67)$$

In these equations, the first term depends on f_t and corresponds to the slope of x and y forces, while the second term is constant for constant axial depths and corresponds to the y intercept of the x and y forces. To solve for the cutting force coefficients k_t , and k_n , the first term of $\bar{F}_x$ is set equal to the slope of the x cutting force, and the first term of $\bar{F}_y$ is set equal to the slope of the y cutting force. This is shown in matrix form in equation 1.68. With (C) equaling $\frac{N_t b}{8\pi}$ and cs, sn, and df, resulting from the integration limits of the $\bar{F}_x$ and $\bar{F}_y$ equations. These calculations are shown in Equations 1.69, 1.70, and 1.71 respectively.

$$\begin{bmatrix} -C * cs & C * (df - sn) \\ C * (df - sn) & C * cs \end{bmatrix} * \begin{bmatrix} k_t \\ k_n \end{bmatrix} = \begin{bmatrix} slope_x \\ slope_y \end{bmatrix} \quad (1.68)$$

$$cs = \cos(2\phi_e) - \cos(2\phi_s) \quad (1.69)$$

$$sn = \sin(2\phi_e) - \sin(2\phi_s) \quad (1.70)$$

$$df = 2\phi_e - 2\phi_s \quad (1.71)$$

The same process can be done for the edge coefficients, but with using the y intercepts from the linear regression instead of the slopes. This is shown in Equations 1.72, 1.73, and 1.74.

$$\begin{bmatrix} C/4 * sne & -C/4 * cse \\ -C/4 * cse & -C/4 * sne \end{bmatrix} * \begin{bmatrix} k_{te} \\ k_{ne} \end{bmatrix} = \begin{bmatrix} intercept_x \\ intercept_y \end{bmatrix} \quad (1.72)$$

$$cse = \cos(\phi_e) - \cos(\phi_s) \quad (1.73)$$

$$sne = \sin(\phi_e) - \sin(\phi_s) \quad (1.74)$$

Robotic Milling

Now that the concepts of traditional milling have been discussed, the next topic is milling when using an industrial robot with a milling spindle attached at the end of the six degree of freedom kinematic chain. Many of the basic concepts are the same, such as up/down milling, depths of cut, feed rates and cutting force coefficients. The main difference in robotic milling is the reduction in stiffness between an industrial robot and a conventional machine tool, as well as its variation with the industrial robot orientation or pose. In a conventional machine tool the axes are often aligned in parallel, allowing individual axes to move independently of each other. For example in a three-axis mill, the z axis holds the spindle and is rigidly mounted to the body of the machine, while the large table holds the workpiece and is constrained to move in the x and y directions. This allows each axis to have high structural stiffness, and good positional accuracy. Five-axis conventional milling machines work in much the same way, but with the addition of rotary axes to allow extra degrees of freedom. However, industrial robots work differently. These machines are arranged in a serial configuration, with each additional axis mounted to the one preceding it. This allows robots to have great flexibility of motion, but it does not offer the stiffness advantages of conventional machine tools because the stiffness of each axis depends not only on itself but on the stiffness of each axis preceding it. This leads to significant stiffness reductions for industrial robots relative to conventional machine tools, potentially by a factor of 50x or more [5]. Machine tools are often assumed to have the same structural dynamics throughout their entire work volume. This assumption cannot be made for robots because their dynamics can change dramatically with end-effector location and robot pose.

Low Stiffness Compensation

It is well documented that industrial robots have much lower stiffness than conventional machine tools. Nearly every paper on robotic milling begins with this statement. To address this challenge, there have been many studies conducted attempting to model the stiffness of the robot across its working range and find optimal machining conditions. A study done by Xiong et al. [4] takes advantage of the redundant degree of freedom introduced by performing five-axis milling with a six-axis robot to orient the robot in such a way to optimize the cutting stiffness of individual machining passes. They developed a stiffness model for each joint and use it to select orientations with optimal stiffness. Using this approach, they were able to achieve great improvements in part accuracy. Similarly, Celikag et al. [6] explored using the same redundant degree of freedom to optimize stiffness. They measured the stiffness of the robot across a grid of points in various orientations of the redundant degree of freedom to find the optimal machining conditions. They concluded that the stiffness is typically higher when the force direction is aligned with the joints of the robot and is lower when higher torques are placed on the joints. In a different paper, Celikag et al. [7] exploited the redundant tool axis degree of freedom to suppress chatter during the machining process. To do this, they dynamically changed the orientation of the robot during the cut, thus changing the dynamics of the system. This inhibited the formation of regenerative chatter, allowing for more stable cutting.

The pose of the robot is not the only factor influencing the stiffness of the machining process. Another study by Lin et al. [5] explored how the configuration of the spindle can play a role in end effector positional accuracy. They proposed a method to optimize the spindle placement on the end effector to increase part accuracy. Another study by Schnoes et al. [9] developed an offline model for robot path planning, which optimized workpiece placement within the work area of the robot and attempted to align the cutting forces in the stiffest direction of the robot by altering the radial depth of cut. Robotic milling is similar in many ways to conventional milling, but due to the low structural stiffness different considerations may be needed for accurate modeling. A study by Riviere-Lorpevre et al. [8] developed a model for robotic milling to better predict cutting forces and resulting surfaces. This model considered more complex toolpaths, and the possibility of tool axis tilt, due to the low stiffness of the robot. In addition to the lack of stiffness of the robots, there are also occasions when the workpiece has low stiffness such as when machining thin-walled parts. A study by Ozturk et al. [3] discusses a robotic milling system for machining thin-walled aerospace parts. These parts are difficult to machine because the cutting forces often cause significant vibrations in the workpiece. In the study this was compensated by using a robot to machine the part, and supporting the workpiece using either stationary supports, or using mobile supports positioned opposite the cutting surface. However, the use of supports can introduce form errors in the final machined surface. This is explored by Sun et al. [15] who further confirmed the pose-dependent stiffness of industrial robots.

Low Frequency Modes and Chatter

As a result of being less stiff than conventional machine tools, industrial robots are more prone to experiencing chatter. Multiple studies have been conducted to understand the role of chatter in robotic milling and attempt to minimize it. For example, in a paper by Gonul et al. [10] the researchers focused on using the previously discussed redundancy of robots to orient the robot in a way that maintains stable cutting conditions over large work areas, and long toolpaths. Another study by Chen et al. [11] developed a method for predicting stability at any pose of the robot. This was done by positioning the tooltip in a variety of different locations and poses, and conducting impact tests to determine the modal parameters. From the dynamics of the robot at these measured locations an inverse distance weighted model is used to predict the modal parameters for any location and pose to ensure stable cutting conditions at all tool locations. A study by Wu et al. [14] also predicted pose dependent cutting stability. They concluded that the stability lobes are highly pose dependent and dominated by low frequency modes. A study by Cordes et al. [2] further expands on the low frequency modes. In the study, the researchers conducted modal tests of the dynamics of the robot and used the resulting frequency response function (FRF) to predict chatter at a range of spindle speeds, then performed cutting tests to verify the predictions. From this study, they concluded that the low frequency modes of the robot cause significant pose dependent chatter at low spindle speeds, which they attribute to the mode coupling effect, while the high frequency modes cause the typical regenerative chatter at higher spindle speeds. The mode coupling effect is further considered in a few studies. One by Celikag et al. [12] questioned whether mode coupling chatter is even possible in milling operations. In the study, they apply the mode coupling chatter

principles to the milling process and predict stability boundaries. Their experimental tests did not align with the mode coupling predictions and they concluded that mode coupling chatter is not present in robotic milling. However, Cen et al. [13] claim the presence of mode coupling chatter and present a method for reducing its effect.

In addition to low frequency robot modes being pose dependent, they also can change based on whether the robot is in motion or stationary, due to the effect of the gears in each joint being engaged. In a paper by Tunc et al. [17] the differing dynamics between the robot while stationary and in motion are considered. Modal testing was conducted both while the robot was stationary and when moving at low speed. The study showed that the low frequency modal parameters of the robot were significantly different under motion, as opposed to at rest. As a result, to obtain accurate representations of the low frequency modes, they recommended that testing should be conducted while the robot is in motion. To add further to the complication of modeling the dynamics of robotic milling systems, a paper by Mohammadi et al. [16] claims that the dynamics of the robot behave non-linearly. This means that the modal parameters of the system depend on the applied force and can change dramatically over a range of potential cutting forces.

Chapter 3: Experimental Description

The purpose of this thesis was to gain an understanding of vibrations during robotic milling, and use this understanding to predict the surface location error when milling aluminum samples using various milling parameters. The experimental setup included a PushCorp STC1515 milling spindle attached to the end effector of a KUKA KR250 six-axis industrial robot. See Figure 16. A series of 6061-T6 aluminum workpieces were created consisting of nine squares, of known dimensions, arranged in a grid pattern, as shown in Figure 15. After performing dynamic analysis of the robot spindle system, stable cutting conditions were selected to perform machining operations with a constant radial depth around the perimeter of each square. This experimental setup is similar to the one used by Schmitz et al. [18]. At this constant radial depth two different axial depths, feed per tooth values, and spindle speeds were chosen and each combination was used to machine a single square on the plate (eight total tests). The test grid was repeated three times to establish the repeatability of the process. After the machining was completed the part was measured using a Zeiss coordinate measurement machine (CMM). These measurements were then compared to the predicted value of surface location error (SLE) obtained using the frequency domain SLE prediction discussed in Chapter 2. Once the repeatability of the process was established, and the SLE predictions were validated, a compensation algorithm was developed to enable milling parts with increased geometric accuracy. This compensation algorithm was then used to machine an additional plate with compensated radial depths of cut to reduce the error. Finally, the initial testing was repeated on a conventional three-axis machine tool using the same cutting tool geometry, and this result was compared to the results from robotic milling tests.

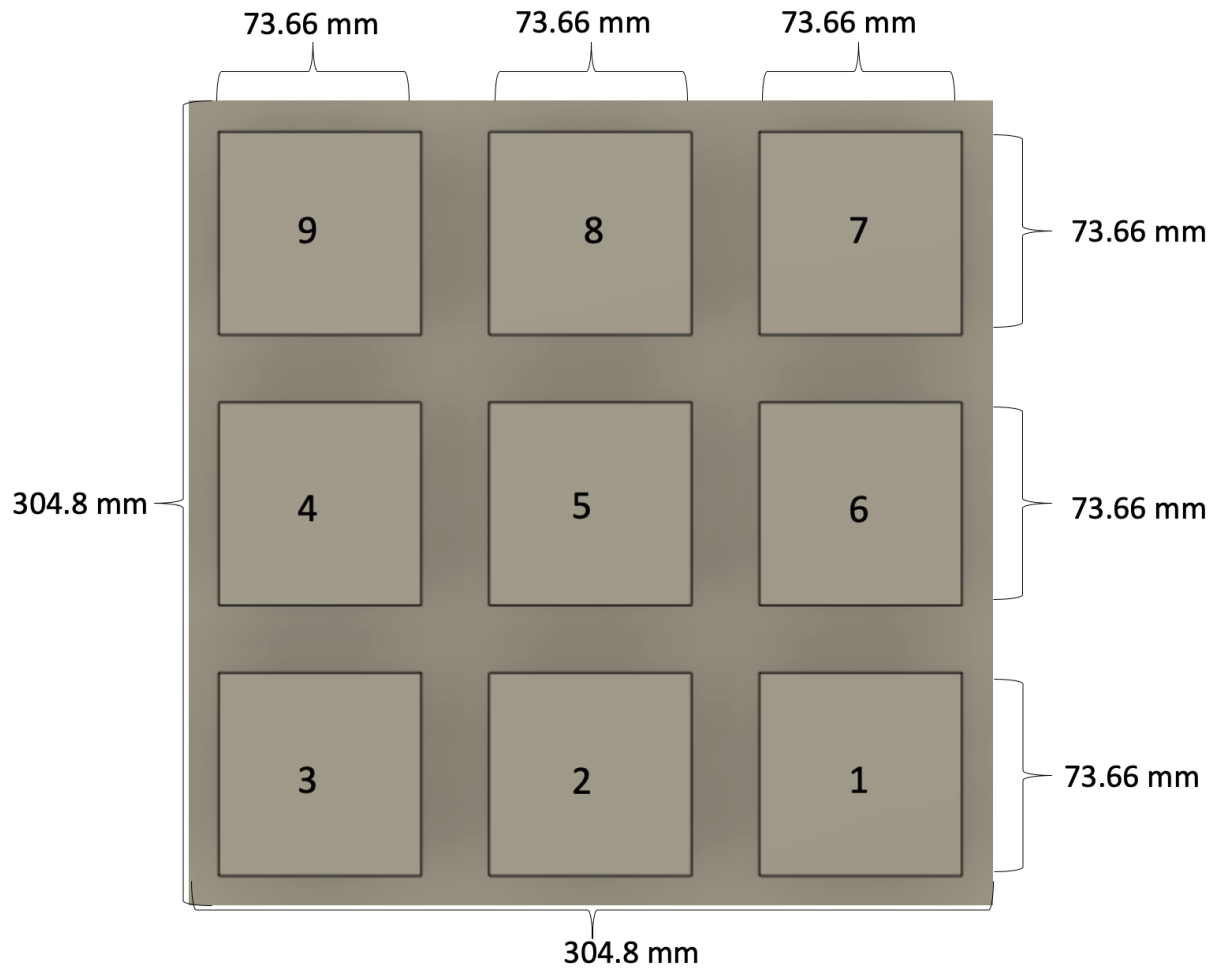


Figure 15: Layout of the squares on the work-piece

Chapter 4: Robot Orientation and Dynamics

From the literature it is known that the dynamic performance of the robot is heavily dependent on its pose. A nominal orientation for the robot was selected to perform the machining activities near the center of its work volume. To evaluate the variation in dynamics within the area required to machine the aluminum plate, the end effector frequency response functions (FRFs) were measured to determine the variation in dynamic response.

Spindle Orientation

The first consideration when setting up the robotic machining cell was the configuration of the spindle on the robot end effector [5]. The spindle was capable of mounting to the robot in two different configurations, parallel and perpendicular to the end effector plate. These configurations are shown in Figure 16. Both configurations were implemented and tested. The perpendicular mounting configuration was selected to conduct the experiments because the parallel configuration aligns the spindle axis with the final robot rotary axis. This limits the usefulness of the final robot axis and robot performance. In addition, for the perpendicular configuration the center of mass of the spindle, and the location of the tool are closer to the robot end effector, which reduces the moment exerted on the robot by the weight of the spindle and the cutting forces. Finally, the perpendicular orientation allowed for a mounting fixture with increased stiffness to connect the spindle to the robot.

Dynamic Testing

Tap Testing Across Linear Track

From the literature it is widely understood that industrial robots have different structural dynamics depending on their configuration. However, each robot is constructed differently and, as a result, it can be difficult to predict the best machining locations, and the best robot poses in these locations. To develop an understanding of the varying structural dynamics of the KUKA KR250 used in this research, it was necessary to test the structural dynamics in a variety of possible machining locations.

To obtain this information, a test was conducted along the 2 meter span of the linear track system where the workpiece is mounted. Ten equally spaced locations were chosen and the robot was moved from point to point along the track. The stationary dynamics of the robot at the end effector location were measured using tap testing at each location. The x-direction refers to the direction along the linear track and the y-direction is perpendicular to the x-axis such that the positive y-direction points away from the robot. An illustration of the experimental setup and the locations of the positions along the linear track is shown in Figure 17.

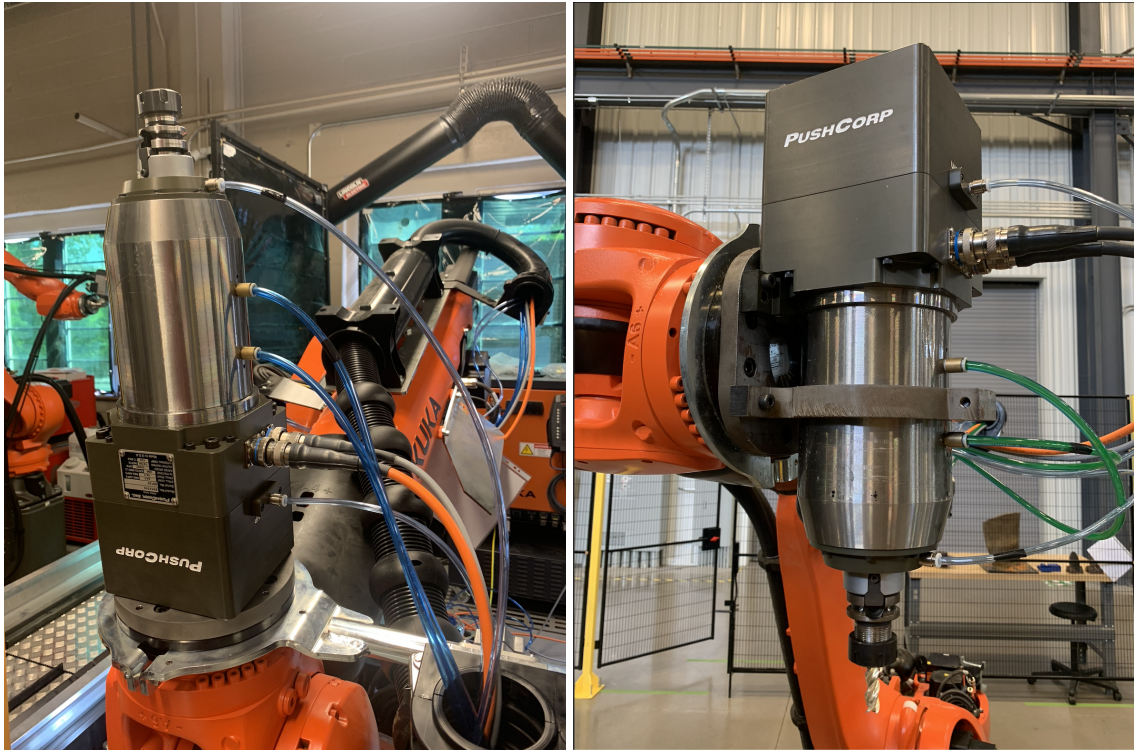


Figure 16: Spindle in the parallel configuration (left), and the perpendicular configuration (right)

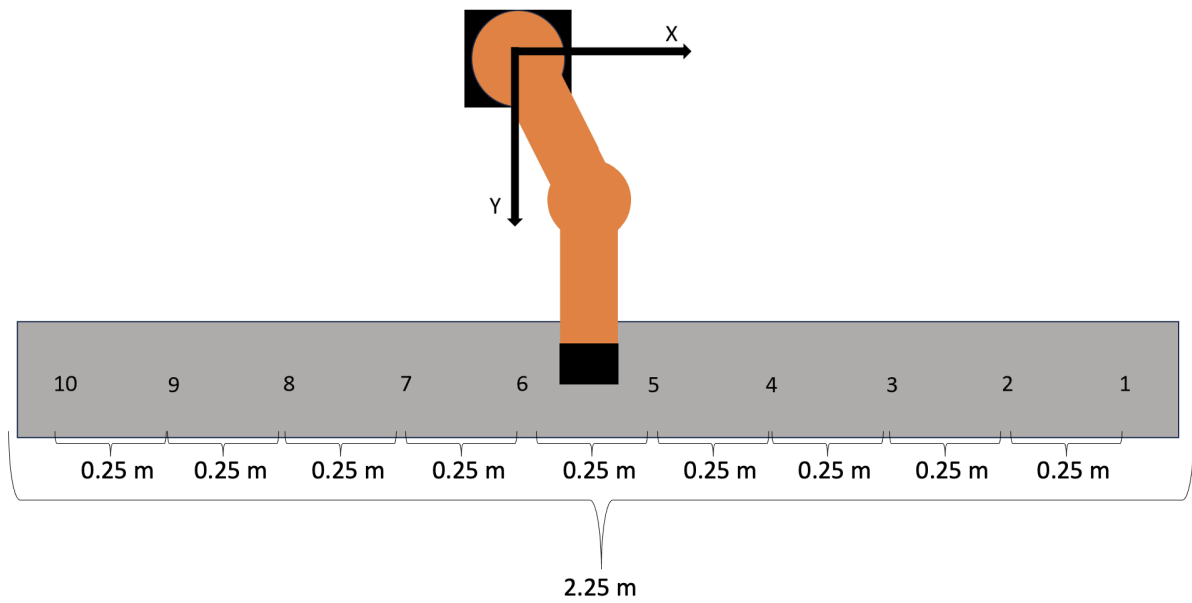


Figure 17: Locations of the testing points along the linear track

From this illustration it can be seen that locations (5-7) are closest to the base of the robot so it would be expected that the dynamics of these location might be more favorable than that of other locations further from the base. Figures 18 and 19 show the robot FRFs for each of the 10 locations in the x and y directions respectively. These graphs show that the dynamics of the robot at each location are similar, however there are some significant changes over this range. Figure 20 shows a dramatic change in the two most prominent modes in the y-direction. The mode occurring around 385 Hz is the more prominent mode in the lower numbered locations, while the mode occurring around 345 Hz is less significant. Toward the later locations the 345 Hz mode is more prominent and the 385 Hz mode has become less significant. A similar but opposite shift occurs in the x-direction as shown in Figure 21. From these tests it was determined that the dynamics of the system are dependent on location and there was a correlation between distance from the base of the robot and dynamic performance, with locations 1 and 10 performing worse than locations 5-7 specifically in the x direction, as can be seen in Figure 22 for the x direction and Figure 23 for the y direction.

Tap Testing While Moving

When the robot is at rest for an extended period of time a braking system is activated and the drive motors are turned off. The tap testing in the previous section was completed under this condition. From the literature it has been reported that the dynamics of the robot when in motion may differ from the dynamics when the robot is stationary [17], which has the potential to influence the milling results because milling is performed with the robot in motion. To investigate this potential difference, tap testing was conducted with a high sensitivity accelerometer mounted to the spindle housing. The robot was commanded to move along a line at a speed of 2 mm/s (KUKA automatic mode), and the tap testing was conducted in the location perpendicular to the direction of motion. This direction was chosen to measure the FRF that would most strongly influence SLE. The results of the tap testing are shown in Figures 24, and 25 for the x and y directions, respectively. The modal parameters for the x and y directions are also shown in Table 1. From these tests it was determined that for the y direction, while feeding in x, the stiffness was significantly different when moving versus when stationary. The 8 Hz mode became more stiff while the other two low frequency modes became more compliant. The x direction, while feeding in y, also experienced changes but not at the same level as the y direction.

Given the results of this testing, the in motion dynamics of the robot were used for the SLE model and prediction. However, the measurements completed here did not include the tool-tip dynamics. The high sensitivity accelerometer was too large to attach to the end mill and would cause significant mass loading for the end mill. To address these challenges, the following steps were followed: 1) the robot FRFs were measured at on the spindle housing using the high sensitivity (larger mass) accelerometer for both the stationary and in motion conditions; 2) the tool-tip FRFs were measured using a lower sensitivity (lower mass) accelerometer in the stationary condition; 3) the scaling between the stationary measurements for the low frequency modes was used to scale the moving FRF modes measured at the end effector to the tool-tip. These scaling factors are shown in Table 1.

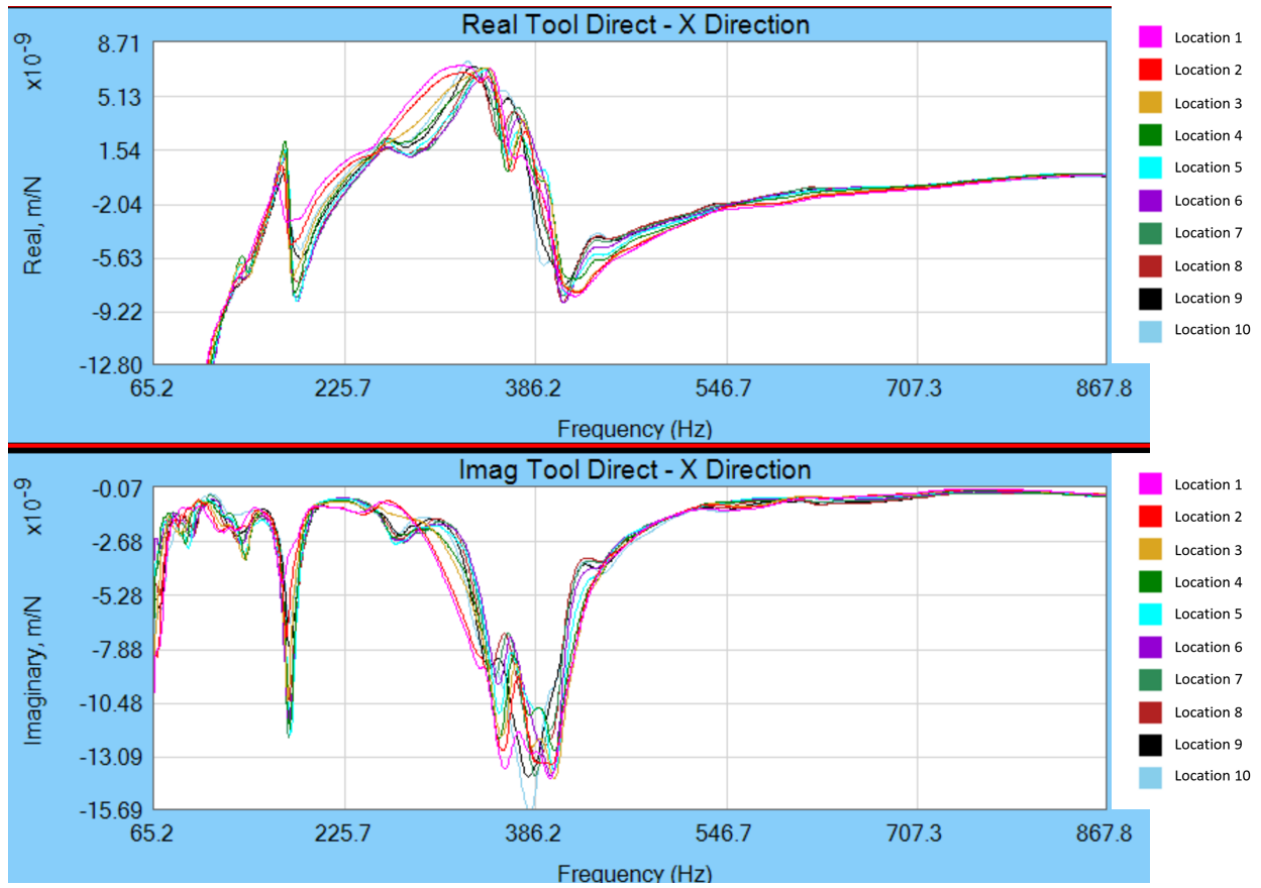


Figure 18: Results of the testing in all 10 locations for the x direction

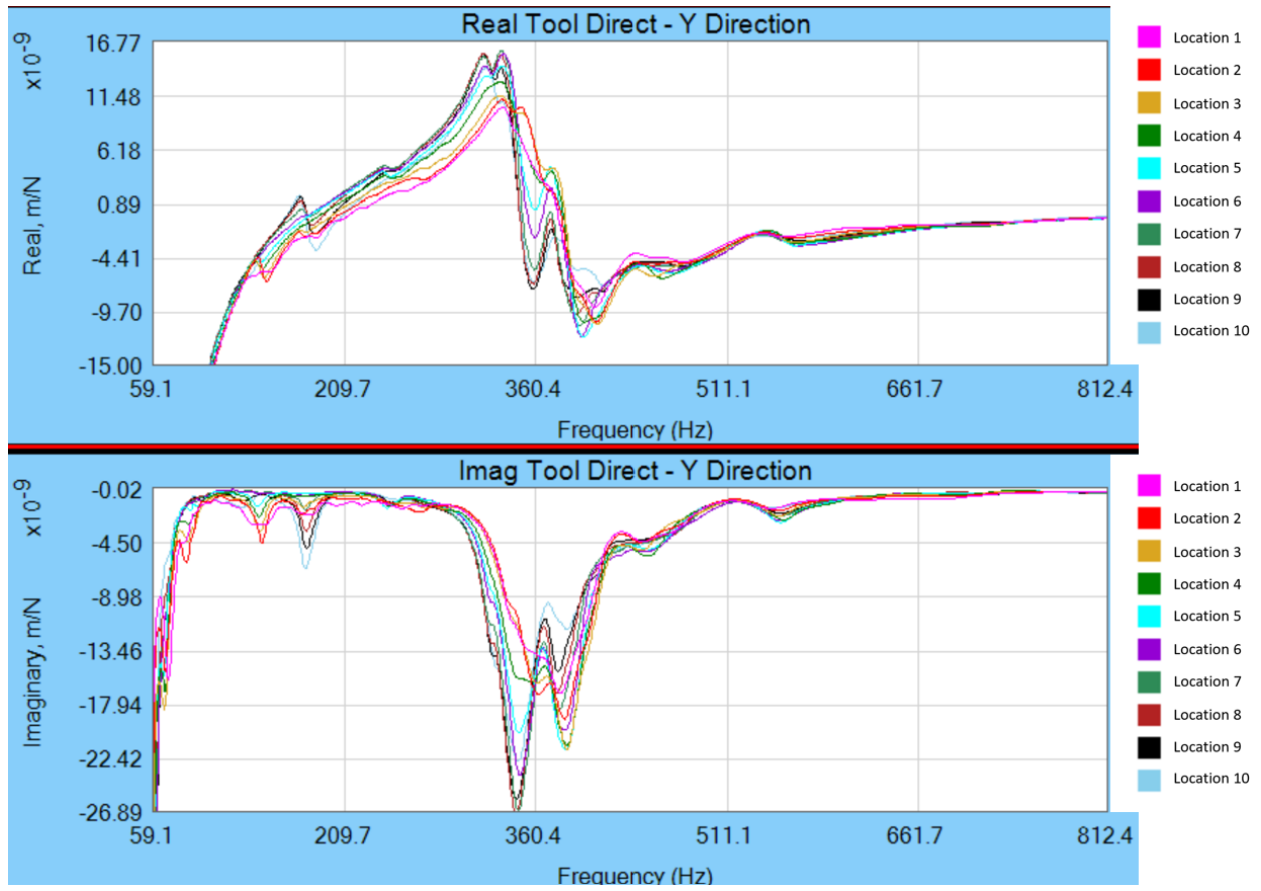


Figure 19: Results of the testing in all 10 locations for the y direction

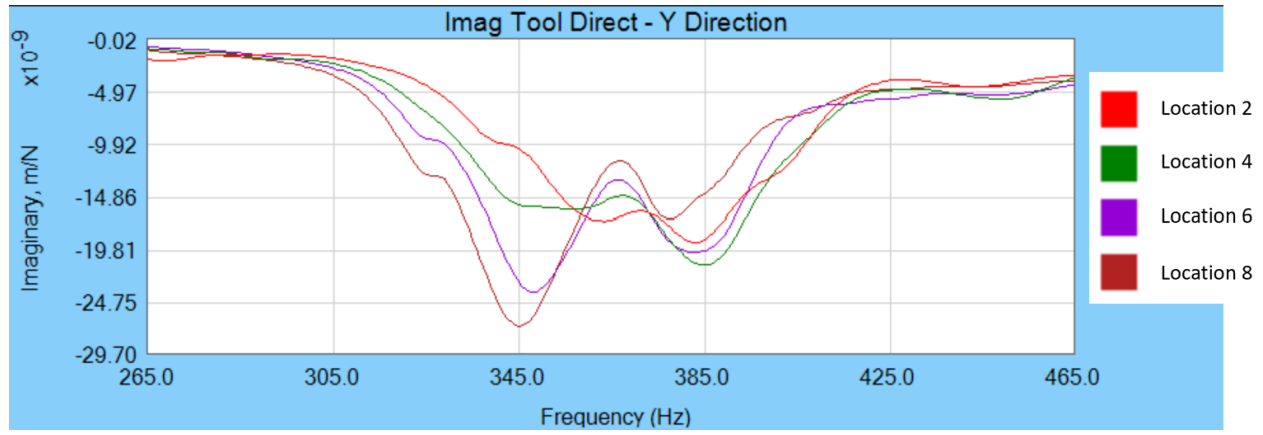


Figure 20: Graph showing the changing dynamics of the robot in the y direction

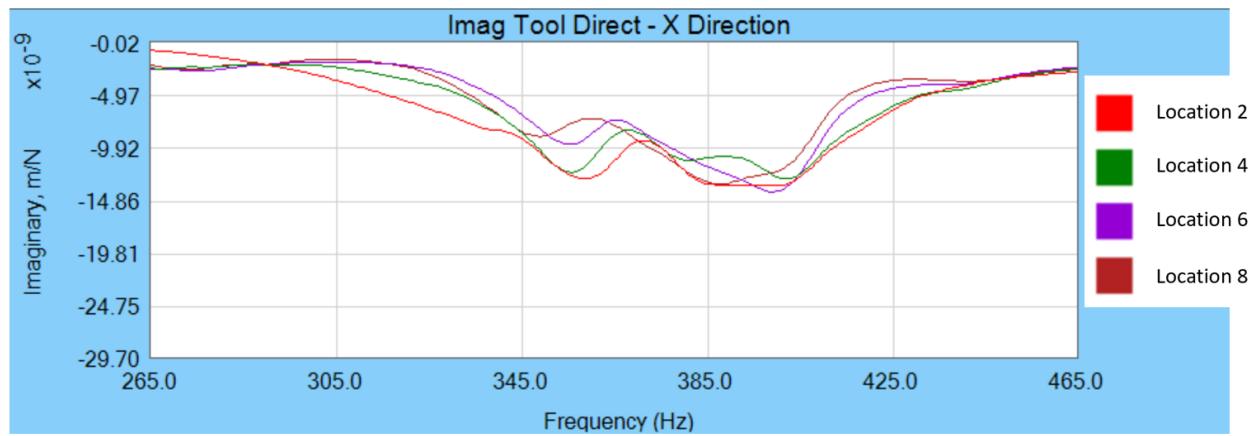


Figure 21: Graph showing the changing dynamics of the robot in the x direction

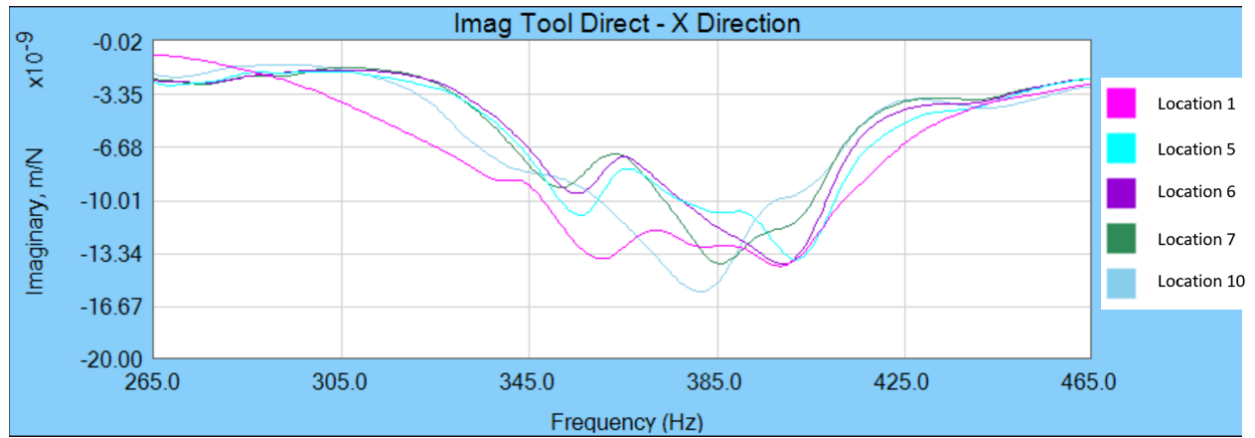


Figure 22: Graph showing the changing dynamics at the extremes in the x direction

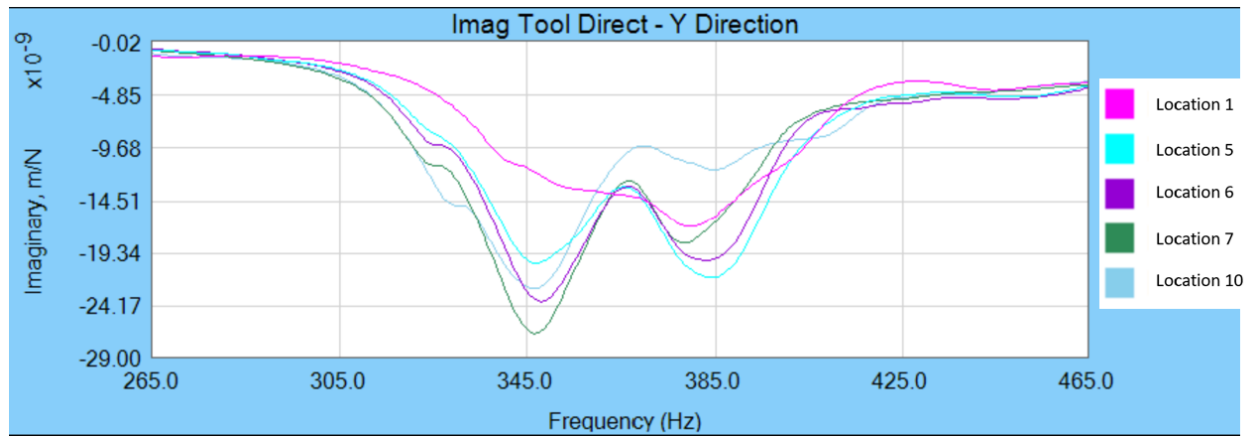


Figure 23: Graph showing the changing dynamics at the extremes in the y direction

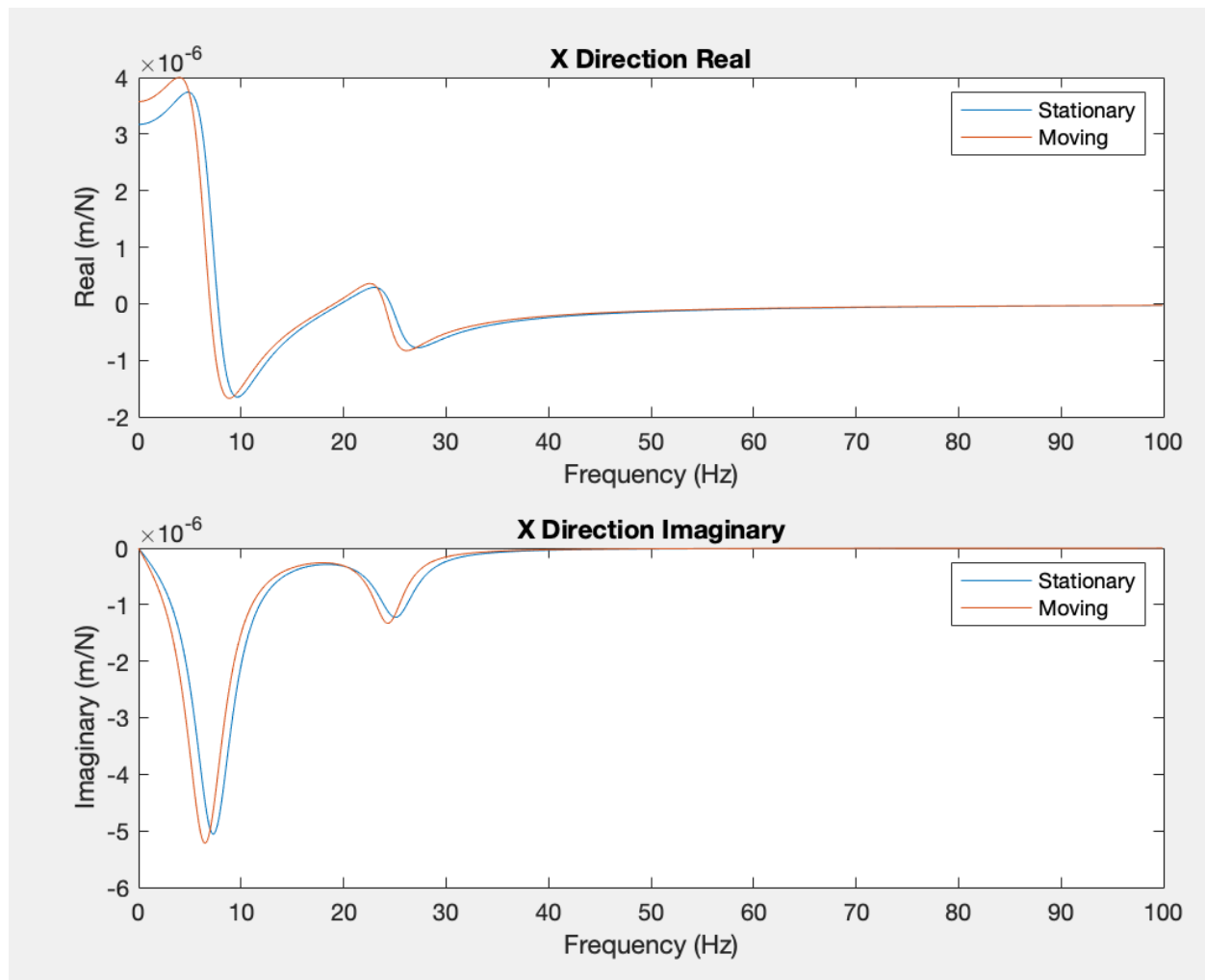


Figure 24: Real and imaginary low frequency FRF in the x direction

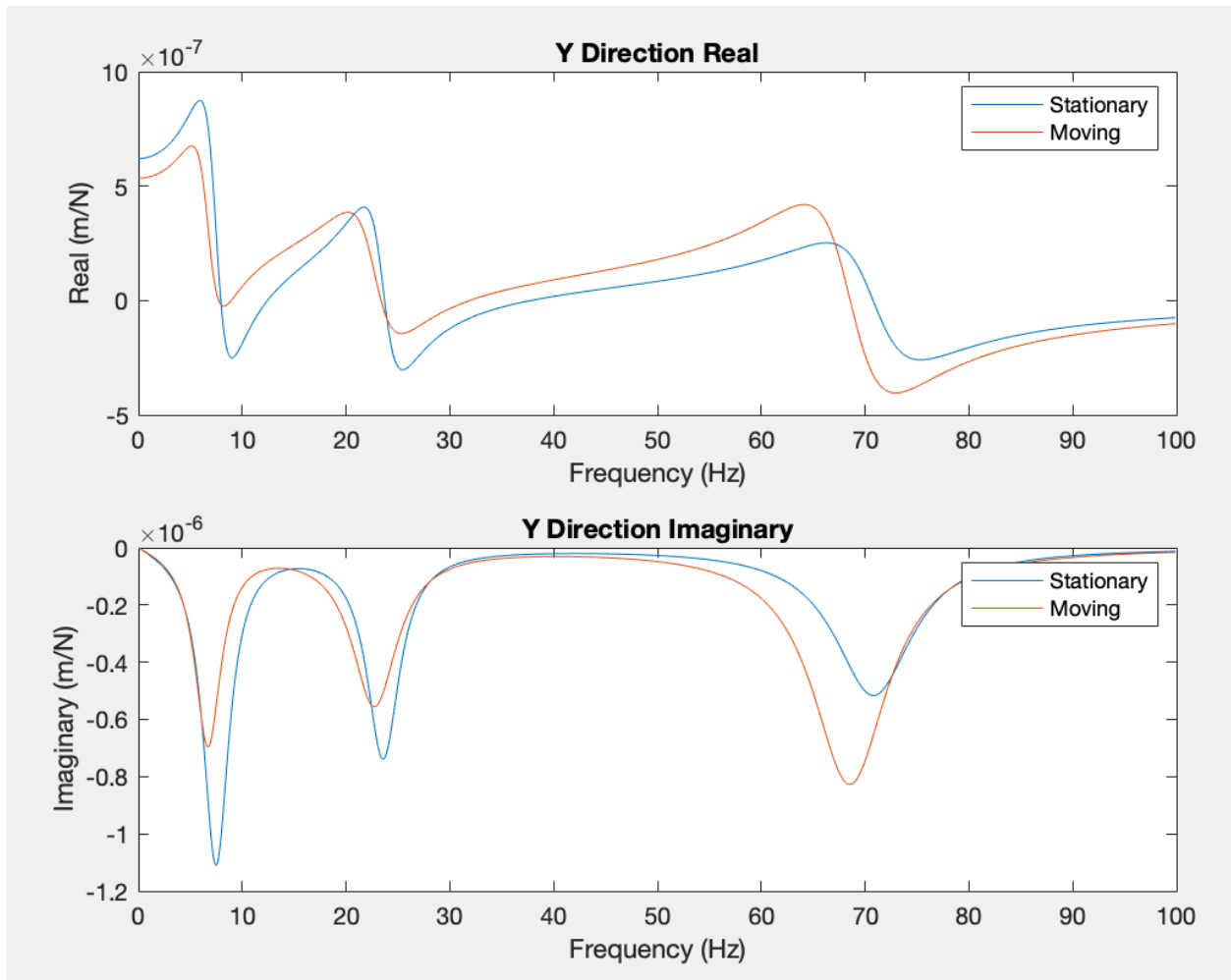


Figure 25: Real and imaginary low frequency FRF in the y direction

Table 1: Table showing modal parameters of moving versus stationary low frequency FRFs, and stiffness scaling factors

X Direction			
Frequency (Hz)	7	25	
Damping (%)	32	8.5	
Stiffness Stationary (N/m)	3.38×10^5	4.72×10^6	
Stiffness Moving (N/m)	2.96×10^5	5.04×10^6	
Stiffness Scaling	0.88	1.07	
Y Direction			
Frequency (Hz)	7	23	70
Damping (%)	21	9	6.5
Stiffness Stationary (N/m)	2.29×10^6	8.52×10^6	1.51×10^7
Stiffness Moving (N/m)	3.32×10^6	7.87×10^6	9.40×10^6
Stiffness Scaling	1.45	0.92	0.62

Tap Testing at Different Locations Over the Plate

To determine the variation in FRFs at different robot poses while machining the plate, tap testing at the tool-tip was completed at a grid of locations relative to the plate. Because the high frequency tool modes are less sensitive to robot motion [17], tap testing was conducted while the robot was stationary. Once a location close to the base of the robot had been chosen to mount the workpiece, dynamics of the system across the surface of the plate were mapped.

Similar to the measurements at locations along the linear track, it was also expected that the structural dynamics would be different over each of the squares on the plate. To characterize these differences, another series of tap tests were performed over each of the squares on the plate. Modal fitting was then performed for the FRF of each of the locations in both the x and y directions, and these modal fits were compared to determine their variation over the surface of the plate, shown in Figure 15. The tool-tip FRFs are shown in the following figures: Figures 26 and 27 show the FRFs for locations 1-3 in the x and y directions respectively, Figures 28 and 29 show the same for locations 4-6, and Figures 30 and 31 show locations 7-9. All the modal fit results are shown in Table 2. The lowest frequency modes used the high sensitivity (larger mass) accelerometer, while all other modes used the lower sensitivity (smaller mass) accelerometer. From the figures and the table, it is seen that the dynamics are similar at different locations across the plate, but not identical. For this experiment it was desired to develop as accurate a prediction of surface location error as possible, so it was decided this unnecessary source of error should be eliminated. This was done by using the degrees of freedom of the linear track system to rotate and translate the plate so that each square was machined in approximately the same location, minimizing the effect of changing the robot location. The location selected most closely aligns with the location of square 2.

Tool Change Repeatability

An assumption often made in machining is that the tool changing process is dynamically repeatable, meaning removing the tool from the spindle then replacing it will not affect the dynamics of the system. In conventional machining, this is a crucial assumption because it would be impractical for any industrial machining process to have to perform a new dynamic test every time a tool change occurred. The validity of this assumption was tested for the robotic machining setup used in this thesis. A series of tap tests were performed on the same tool five times after removing and replacing the tool-holder in the spindle between each test. The results of this experiment are shown for the high and low frequencies in Figures 32, and 33, respectively. From the graphs it is seen that, while there are some differences in the dynamics between tests, these changes are small and the errors they would cause in the predictions are small compared to other errors in the machining process. This is crucial because a tool changing operation is required to accurately probe the part.

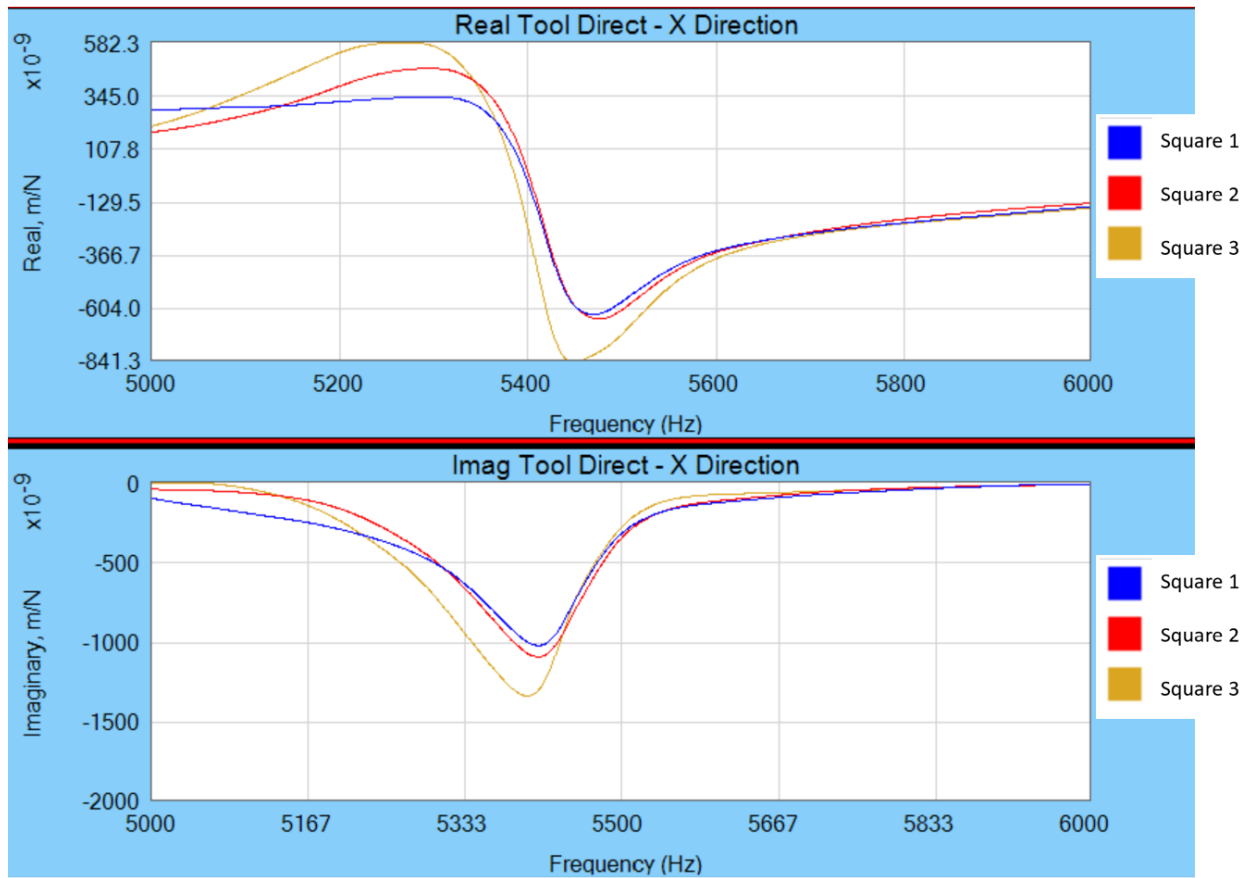


Figure 26: Graph showing the FRFs of squares 1-3 in the x direction

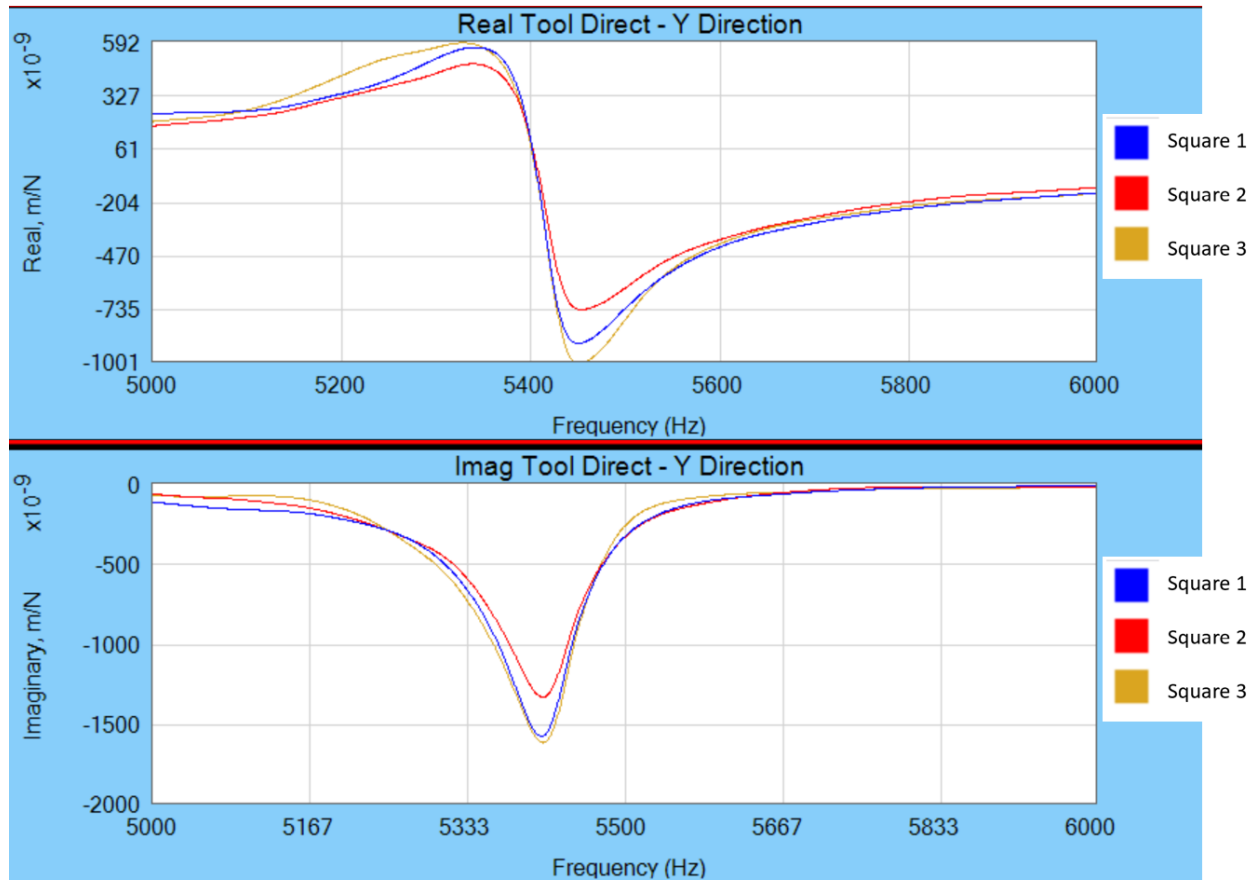


Figure 27: Graph showing the FRFs of squares 1-3 in the y direction

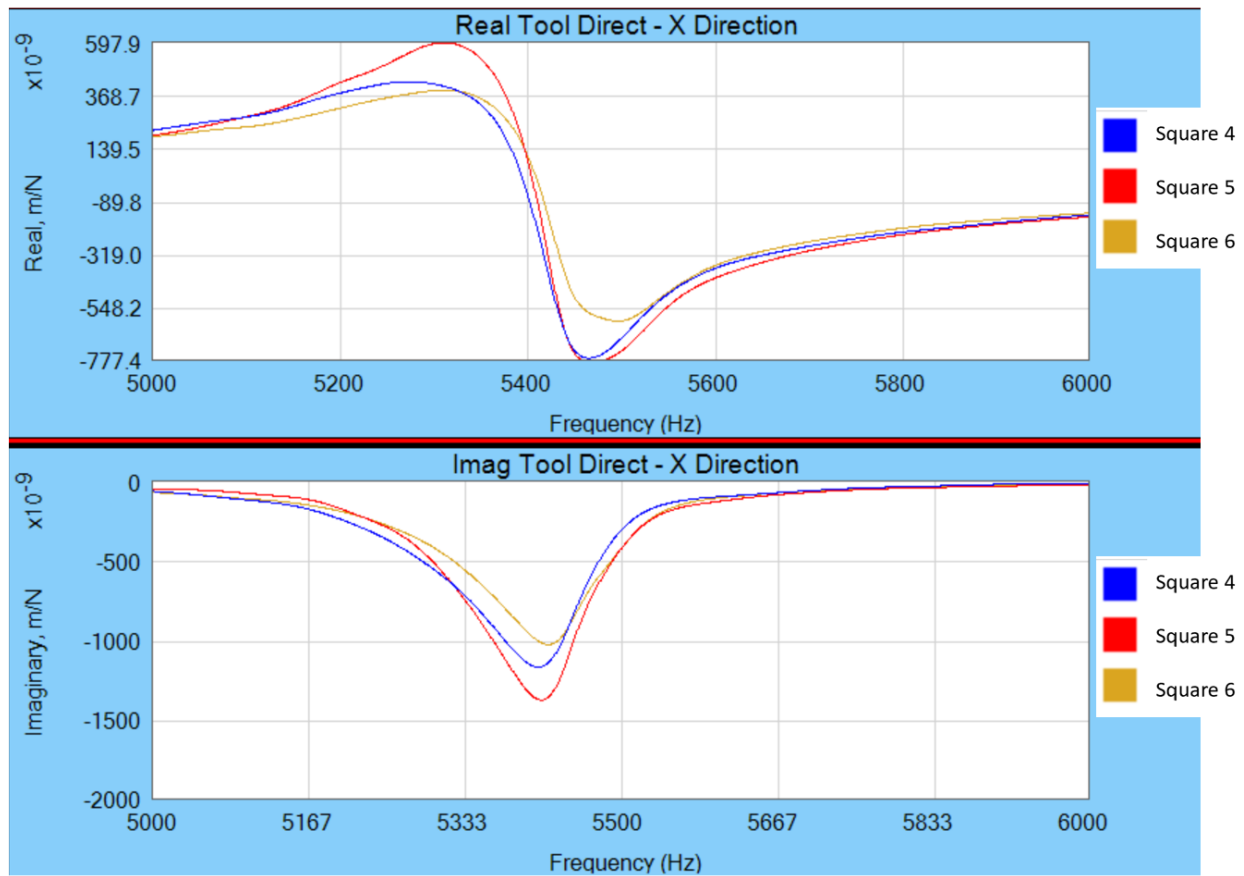


Figure 28: Graph showing the FRFs of squares 4-6 in the x direction

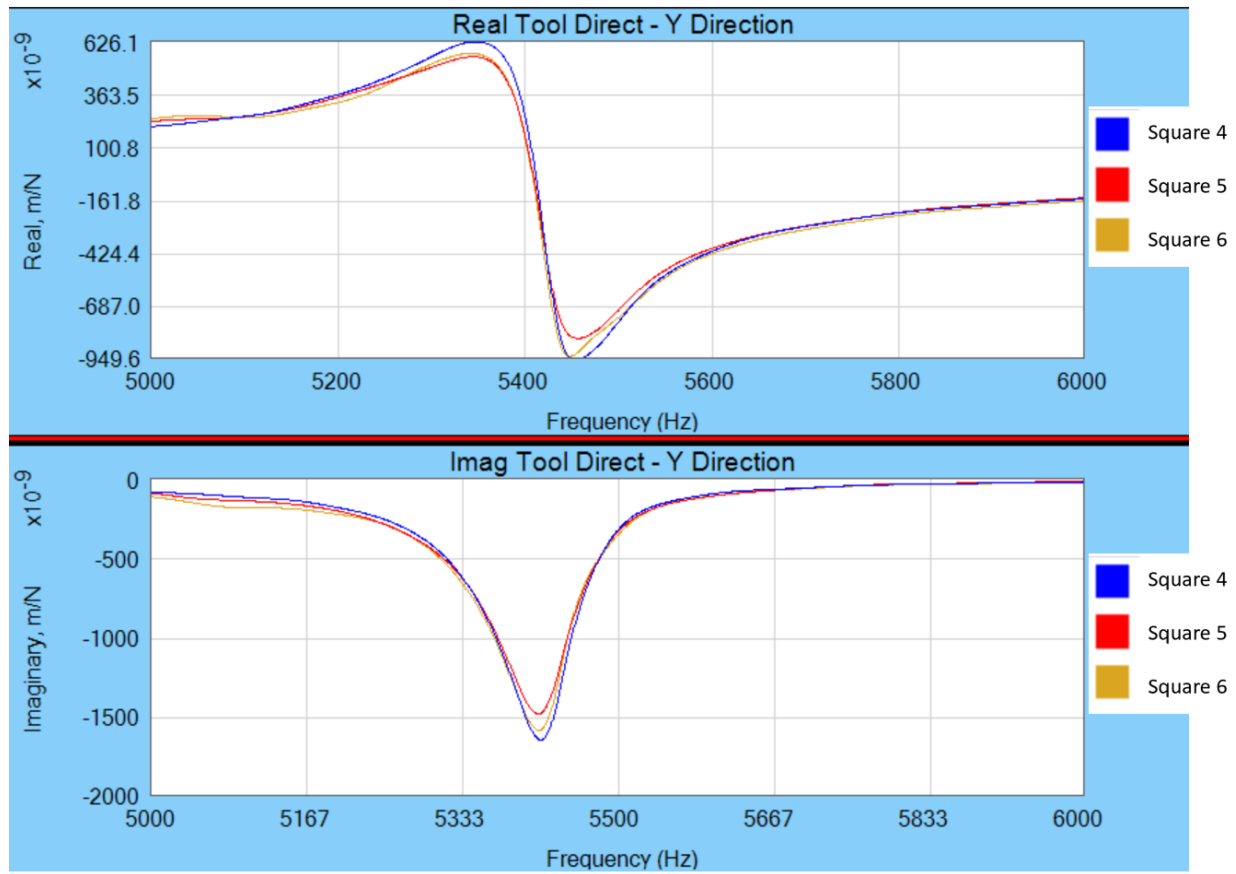


Figure 29: Graph showing the FRFs of squares 4-6 in the y direction

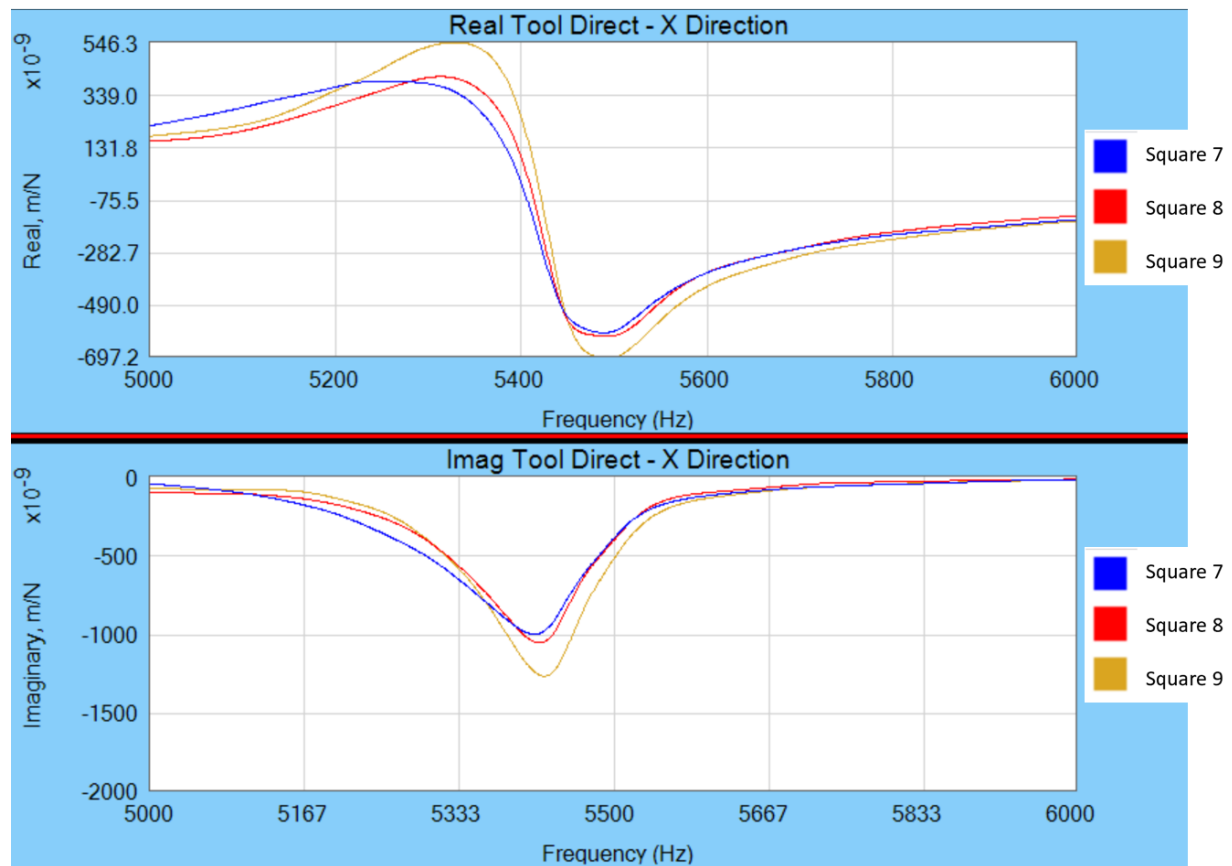


Figure 30: Graph showing the FRFs of squares 7-9 in the x direction

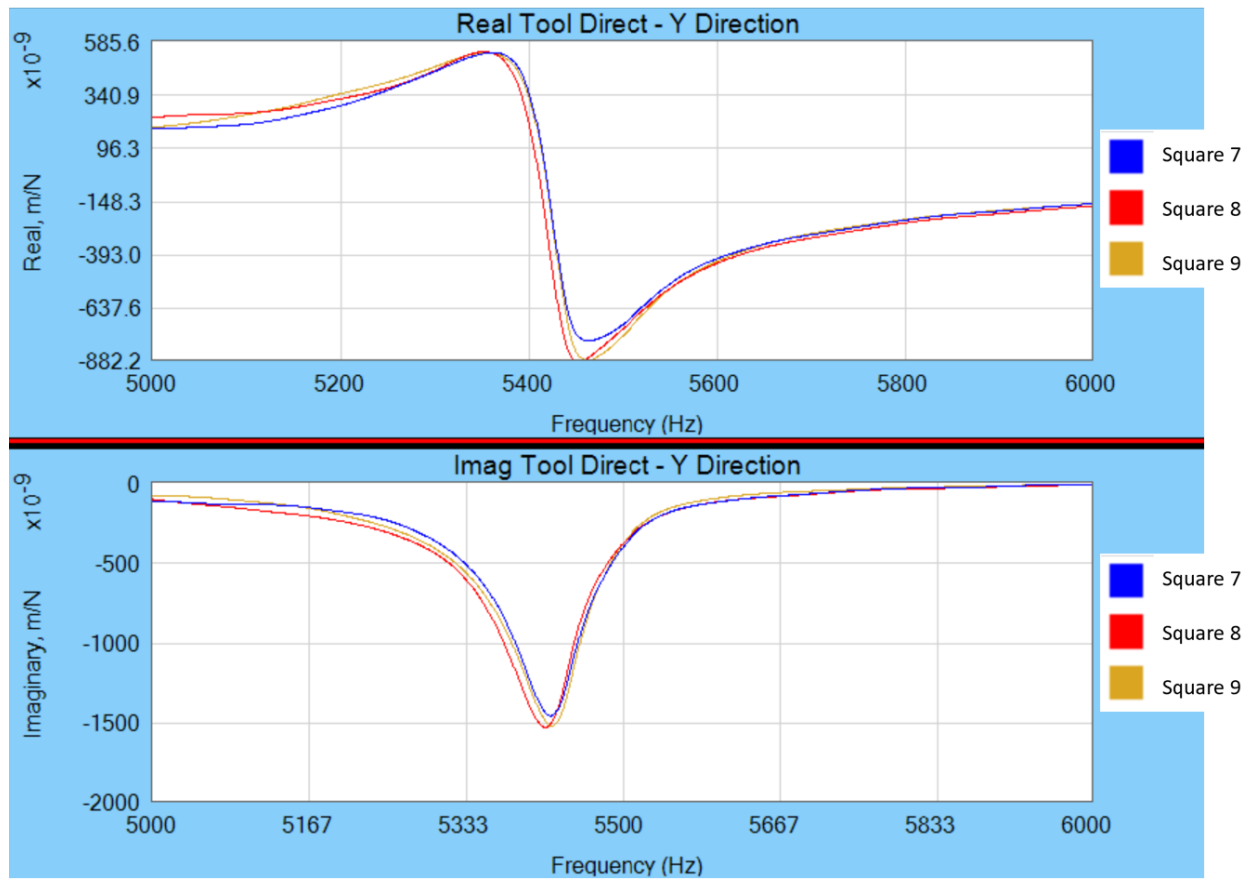


Figure 31: Graph showing the FRFs of squares 7-9 in the y direction

Table 2: Table showing the modal fit data of the plate

X								
Frequency (Hz)			Stiffness (N/m)			Damping (%)		
Mean	STD	% Error	Mean	STD	% Error	Mean	STD	% Error
8.1	0.40	4.97%	5.155×10^5	4.96×10^4	9.62%	18.99	0.96	5.05%
25.6	1.71	6.69%	4.813×10^6	5.94×10^5	12.35%	9.78	0.69	7.08%
63.8	1.92	3.01%	5.398×10^6	1.21×10^6	22.43%	6.07	3.20	52.66%
123.1	1.36	1.11%	6.857×10^7	3.37×10^7	49.15%	4.31	0.67	15.59%
186.8	6.51	3.49%	4.476×10^7	5.88×10^6	13.13%	12.48	4.92	39.44%
278.6	3.50	1.26%	3.022×10^7	6.85×10^6	22.67%	10.71	3.55	33.11%
657.8	3.56	0.54%	2.200×10^7	4.07×10^6	18.50%	9.54	1.48	15.49%
854.6	14.28	1.67%	2.673×10^7	4.69×10^6	17.53%	14.90	2.70	18.09%
1317.0	9.23	0.70%	3.118×10^7	2.05×10^6	6.58%	9.66	0.85	8.83%
2231.6	39.58	1.77%	6.991×10^7	1.35×10^7	19.32%	5.64	0.73	12.95%
2421.2	17.39	0.72%	2.715×10^8	1.38×10^8	50.88%	2.00	0.59	29.60%
2774.0	37.21	1.34%	8.962×10^7	1.55×10^7	17.26%	5.12	0.63	12.29%
3560.3	65.25	1.83%	1.496×10^8	3.48×10^7	23.27%	4.86	0.80	16.46%
5301.1	22.64	0.43%	1.389×10^8	5.64×10^7	40.61%	1.67	0.55	32.70%
5410.7	7.47	0.14%	4.058×10^7	3.93×10^6	9.69%	1.18	0.10	8.33%
6634.2	10.08	0.15%	1.111×10^9	1.82×10^8	16.40%	1.28	0.24	18.43%

Y								
Frequency (Hz)			Stiffness (N/m)			Damping (%)		
Mean	STD	% Error	Mean	STD	% Error	Mean	STD	% Error
7.5	0.25	3.36%	2.340×10^6	3.48×10^5	14.87%	21.48	4.45	20.70%
24.4	0.66	2.71%	1.099×10^7	2.02×10^6	18.33%	6.78	0.80	11.88%
71.3	2.60	3.64%	1.383×10^7	9.48×10^5	6.85%	8.05	1.87	23.23%
120.9	1.27	1.05%	1.148×10^7	1.25×10^6	10.90%	7.57	0.63	8.28%
216.3	0.87	0.40%	1.619×10^7	5.08×10^5	3.13%	10.20	0.44	4.30%
672.8	8.01	1.19%	3.016×10^7	8.59×10^6	28.49%	10.87	1.09	10.05%
806.1	7.49	0.93%	3.778×10^7	1.38×10^7	36.60%	11.37	2.03	17.86%
1129.7	20.11	1.78%	2.287×10^7	3.85×10^6	16.82%	21.01	2.13	10.15%
1456.1	15.69	1.08%	9.954×10^7	3.21×10^7	32.23%	4.76	0.95	19.95%
2211.7	24.44	1.10%	9.978×10^7	2.17×10^7	21.79%	5.79	1.01	17.49%
2372.9	11.26	0.47%	1.982×10^8	7.52×10^7	37.96%	1.95	0.42	21.38%
2814.2	27.48	0.98%	8.545×10^7	7.20×10^6	8.43%	3.75	0.60	15.88%
3400.4	58.41	1.72%	1.136×10^8	1.83×10^7	16.08%	6.89	1.75	25.44%
5299.9	22.67	0.43%	9.593×10^7	2.32×10^7	24.15%	2.46	0.73	29.57%
5412.2	4.38	0.08%	4.212×10^7	1.55×10^6	3.69%	0.86	0.07	8.18%
6632.4	8.32	0.13%	1.330×10^9	2.71×10^8	20.34%	0.98	0.10	10.63%

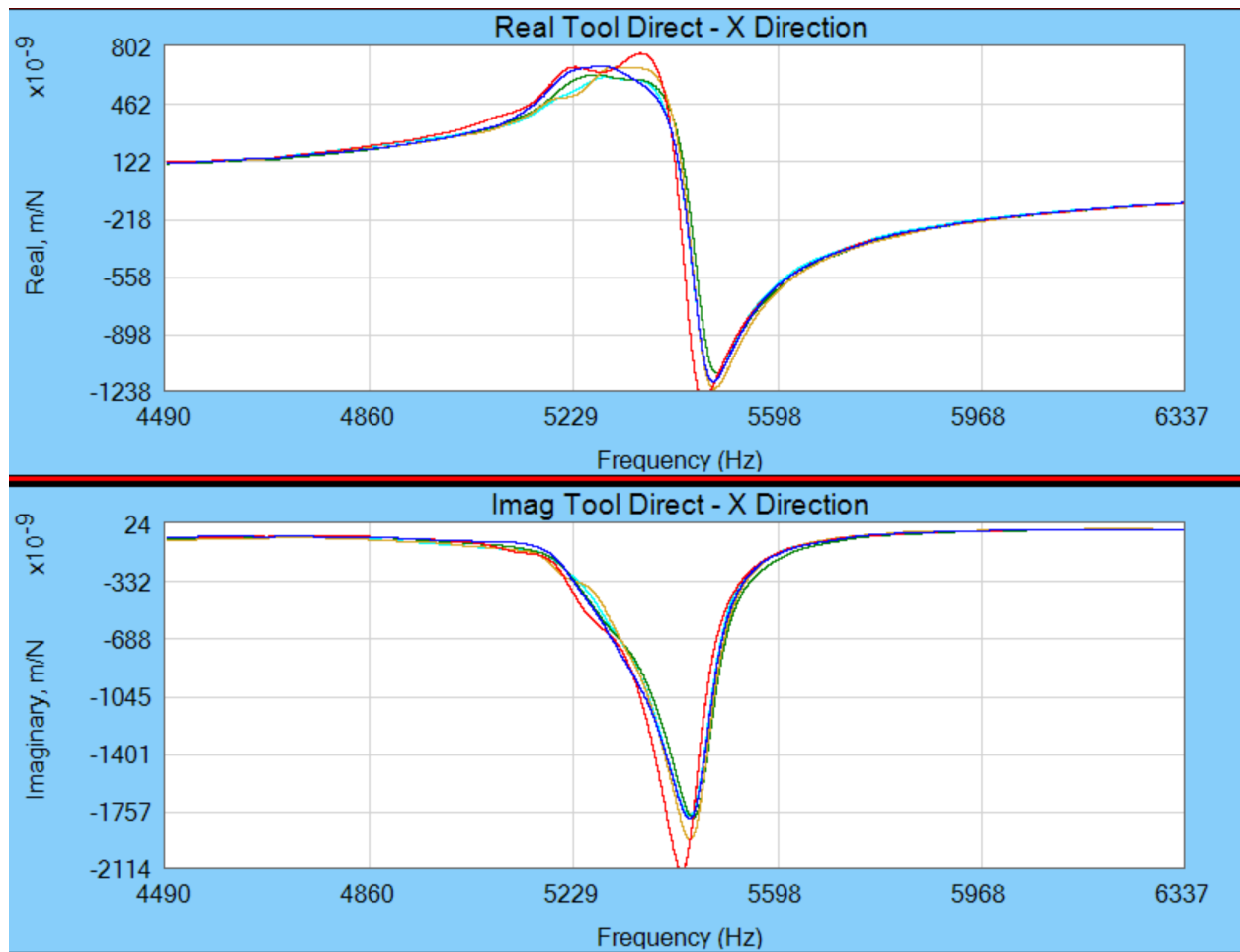


Figure 32: Graph of the tool change repeatability at the high frequency mode

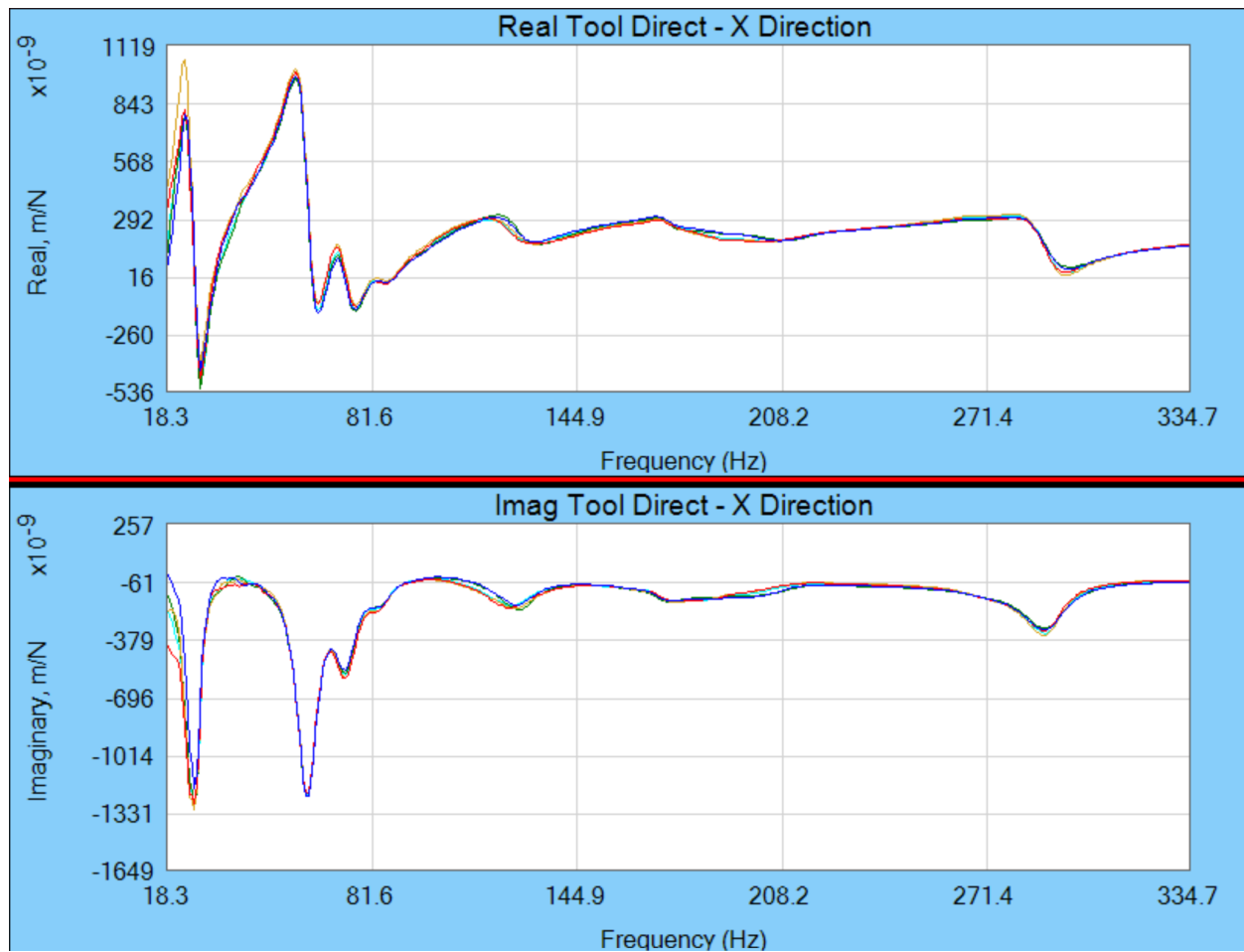


Figure 33: Graph of the tool change repeatability at the low frequency modes

Tap Testing Repeatability

Because the system dynamics are key inputs to the SLE predictions, the repeatability of the tap testing measurements used to identify the systems dynamics was evaluated. Additionally from the literature [16] it has been reported that the robot dynamics are nonlinear. This means that the vibration output is not linearly variable with the force input. Since a tap test consists of several unique impacts averaged together unless every impact is done with identical force levels, which is nearly impossible for a manual process, there could be differences between tests. Also, given that the dynamics of the robot are pose dependent, it is reasonable to assume that the dynamics of the robot may not be the same every time it returns to the same position. Considering all these factors, it was desired to understand how repeatable the tap testing results were when the robot was moved away from a point and returned to it. To study this, a point was chosen in the middle of the plate, over square 5, and five modal tap tests were performed in the same location with the robot being moved and returned between tests. The results of this test formed the baseline repeatability level for the tap tests at different locations, and were used to verify that the dynamics were actually changing with location, and the differences were not measurement non-repeatability. The results of the five repeatability tests are shown in Figures 34 and 35 for the x and y directions, respectively. Figures 36 and 37 show the same FRF for the x and y direction, respectively, but are resized to show in detail lower stiffness high frequency mode. Figure 38 displays sample impact forces to observe the variation between manual taps. These force levels represent typical cutting forces in aluminum.

Low Frequency Robot Modes

Due to their different kinematics, industrial robots exhibit low frequency, low stiffness vibration modes relative to a conventional stacked-axis machine tool. In conventional machining, the tool modes are typically the most flexible modes for the tool-holder-spindle-machine assembly and are, therefore, the most important for predicting the dynamics of the cutting process. These modes typically occur at high frequencies from 1000 Hz to 6000 Hz. To measure these modes, a low mass accelerometer is attached to the tool-tip to measure the vibration response during tap testing. This low mass accelerometer is able to measure the high frequency modes accurately but is less effective for low frequency modes (less than 100 Hz). In robotic milling, these low frequency modes are important because they can be as flexible, or even more flexible, than the tool modes. They therefore influence the surface location error and milling stability. To measure the low frequency modes (less than 100 Hz) modes a laser Doppler vibrometer was used. The vibrometer setup is shown in Figure 39, and a comparison of the vibrometer and accelerometer FRFs at low frequencies is shown for the x and y directions in Figures 40 and 41, respectively. The final FRF of the robot along each side of the square is the result of the combination of the low frequency modes from the vibrometer and the higher frequency modes from the accelerometer. The resulting FRF was used for predicting stability and surface location error.

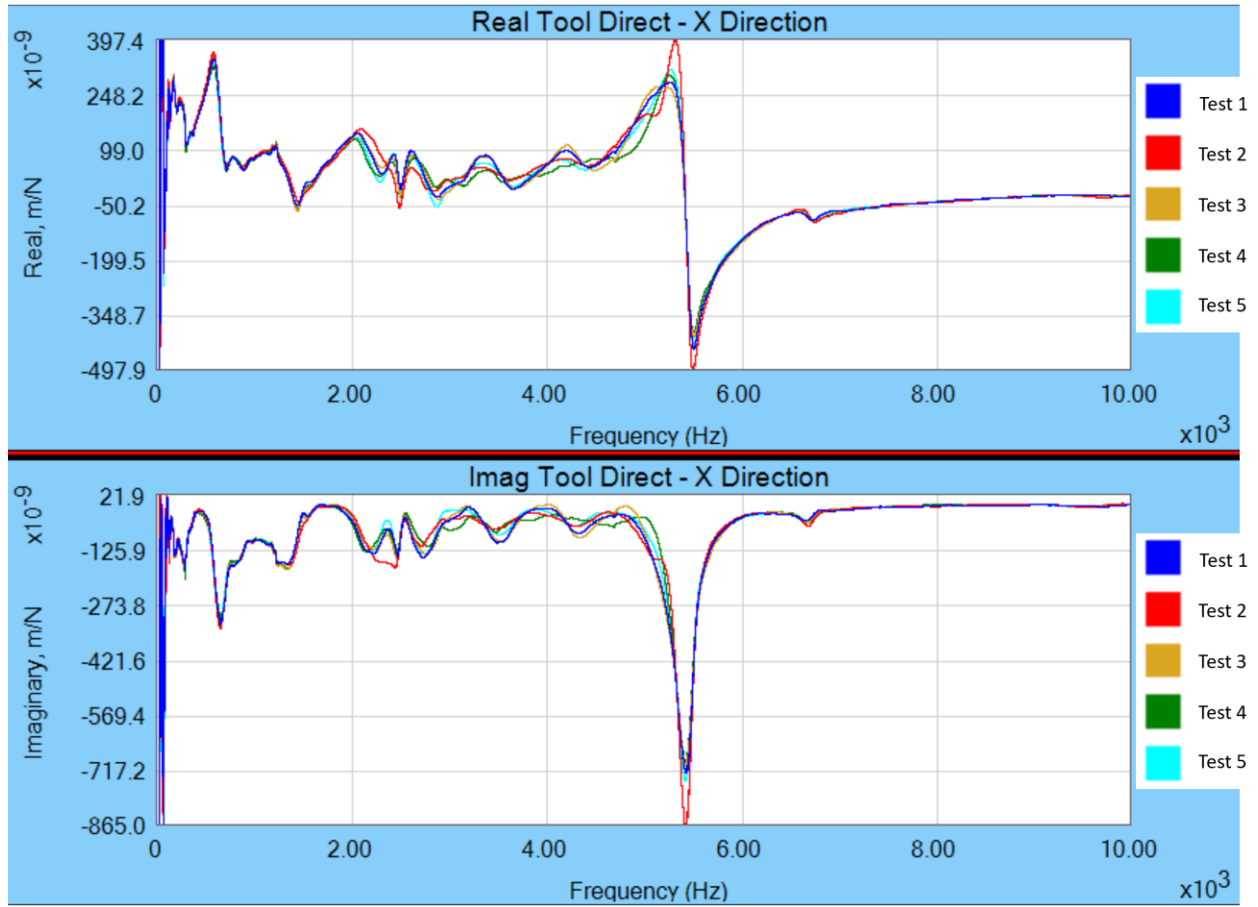


Figure 34: Graph showing the repeatability of x-direction FRF

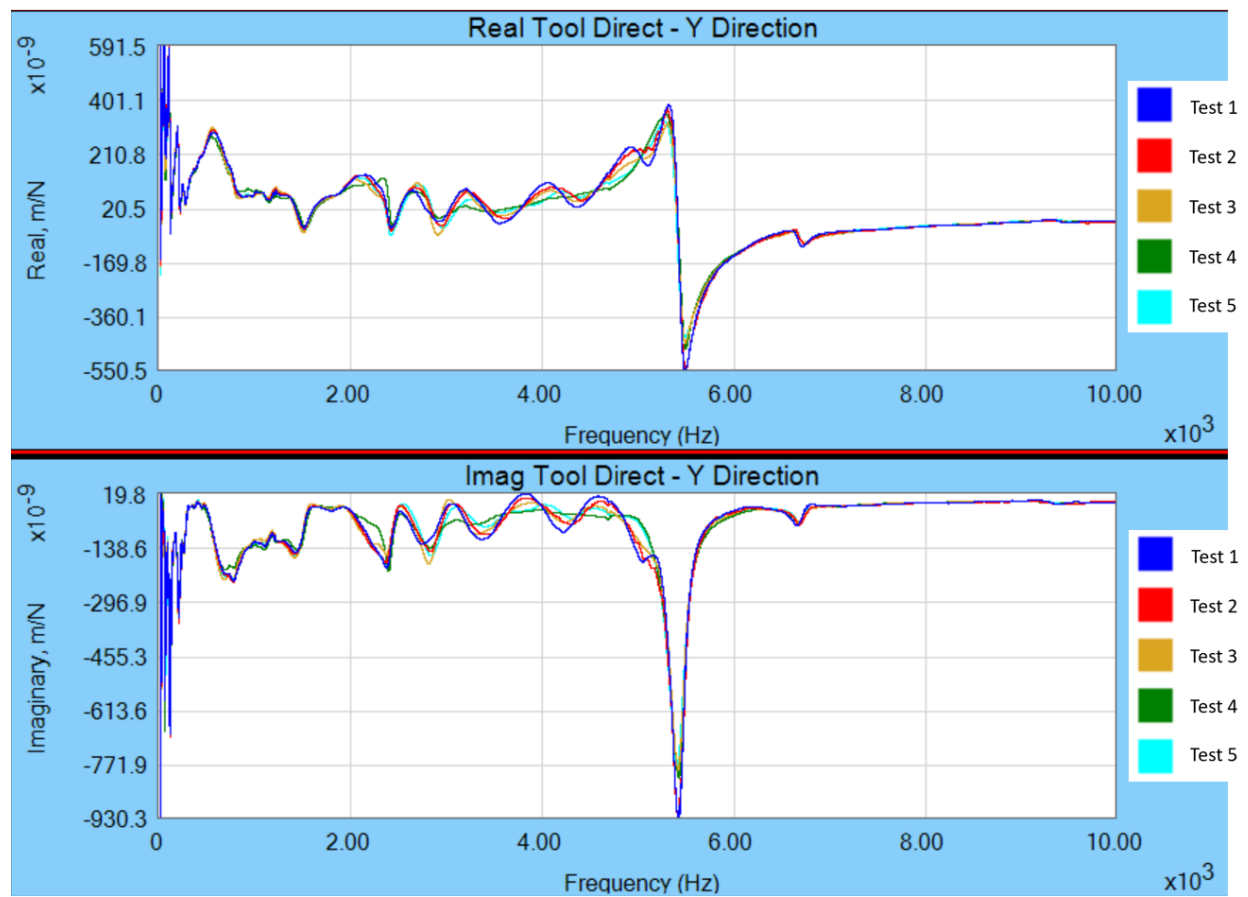


Figure 35: Graph showing the repeatability of y-direction FRF

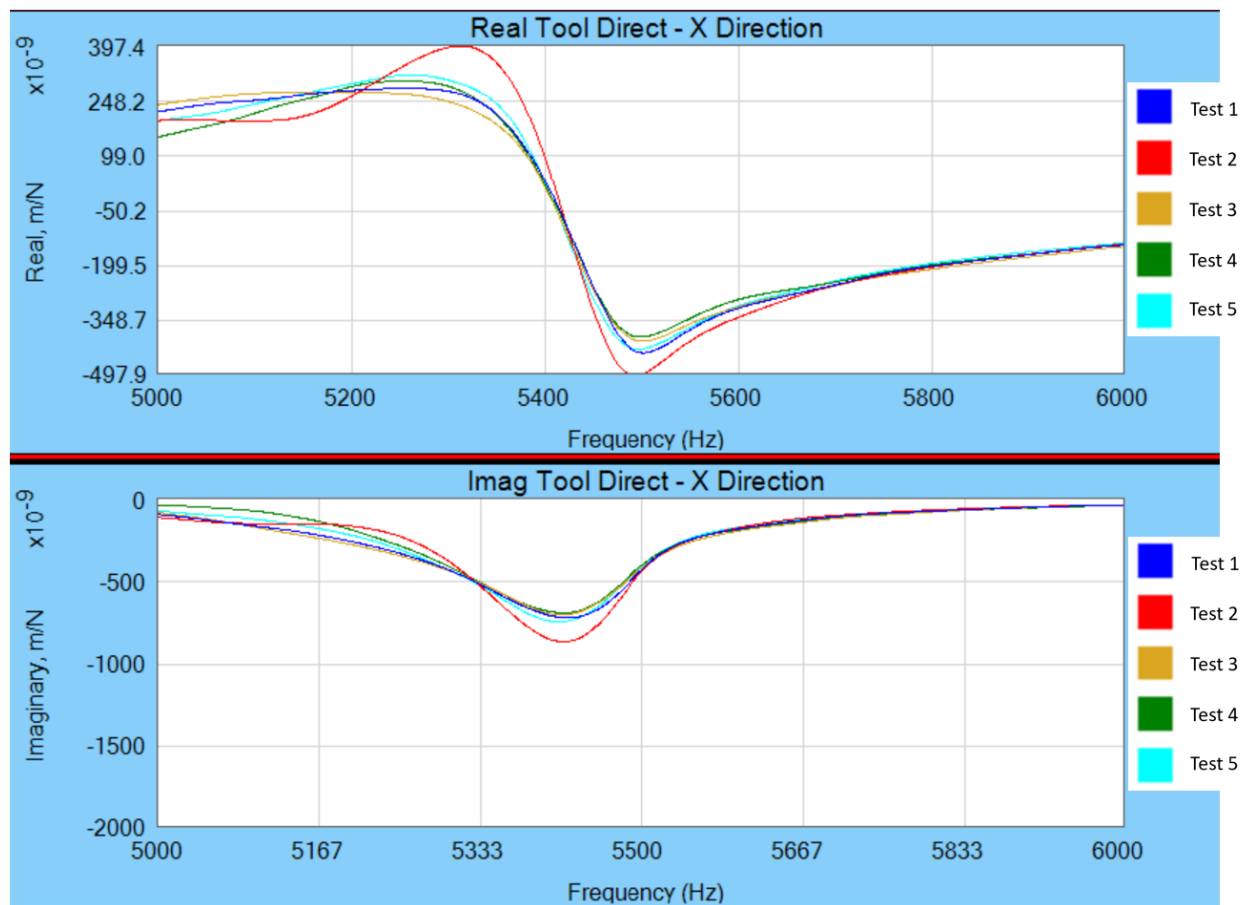


Figure 36: Graph showing the repeatability of x-direction FRF zoomed in around large mode

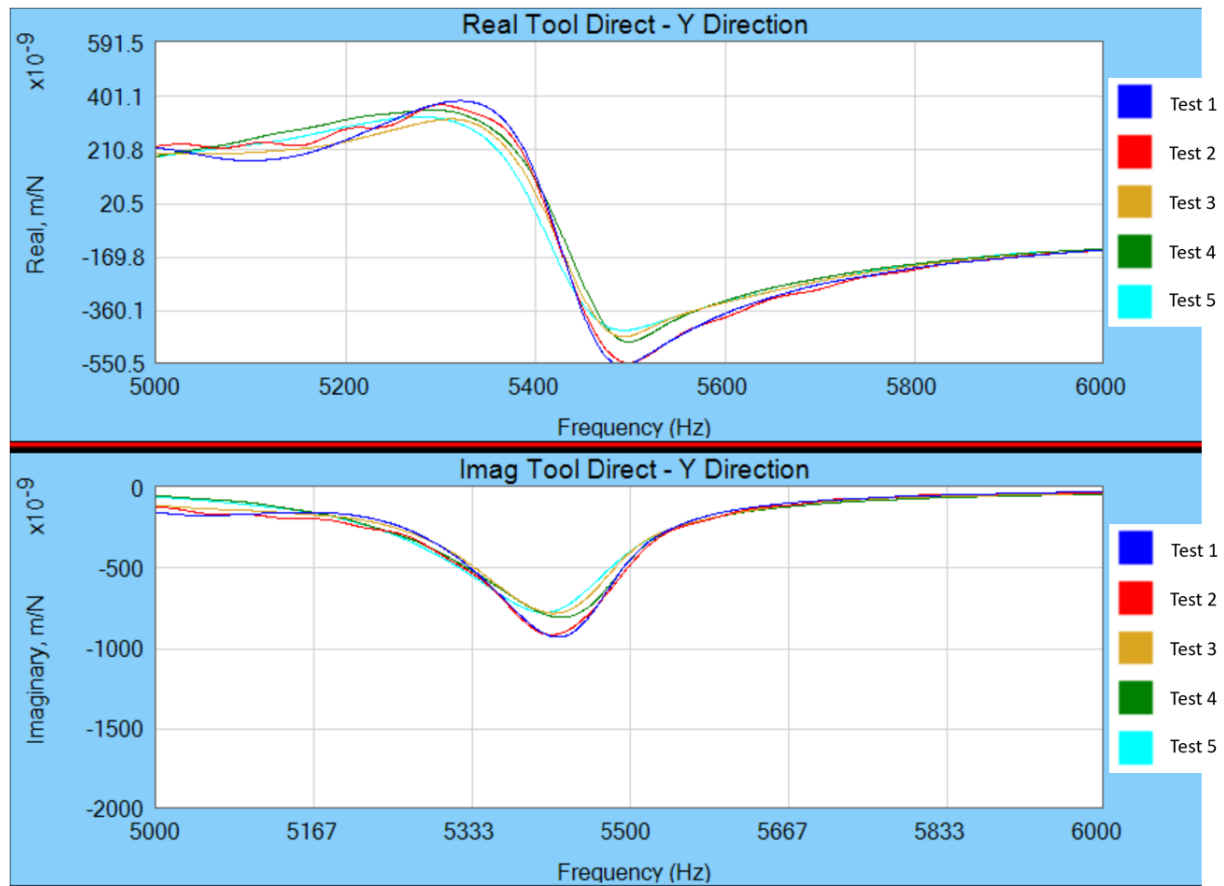


Figure 37: Graph showing the repeatability of x-direction FRF zoomed in around large mode

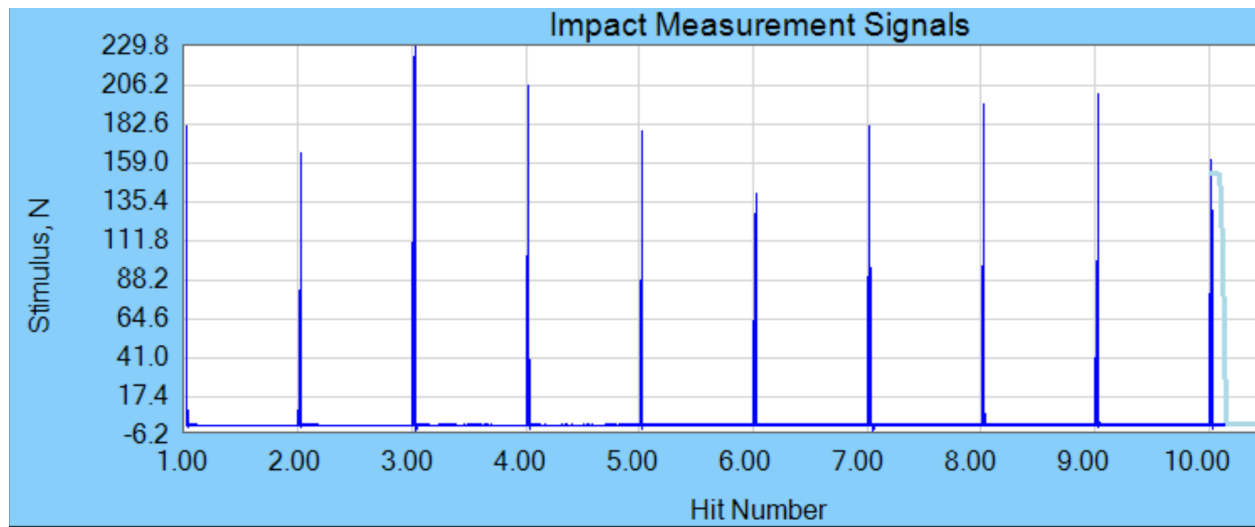


Figure 38: Impact forces of the hammer during tap testing procedure

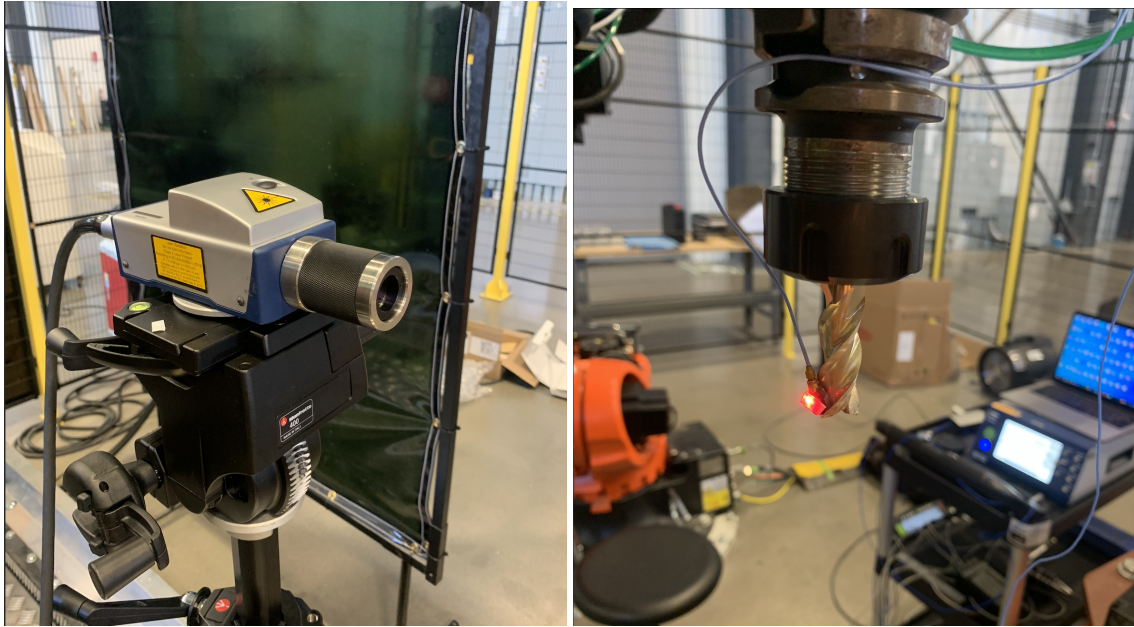


Figure 39: The vibrometer setup with the laser source and target

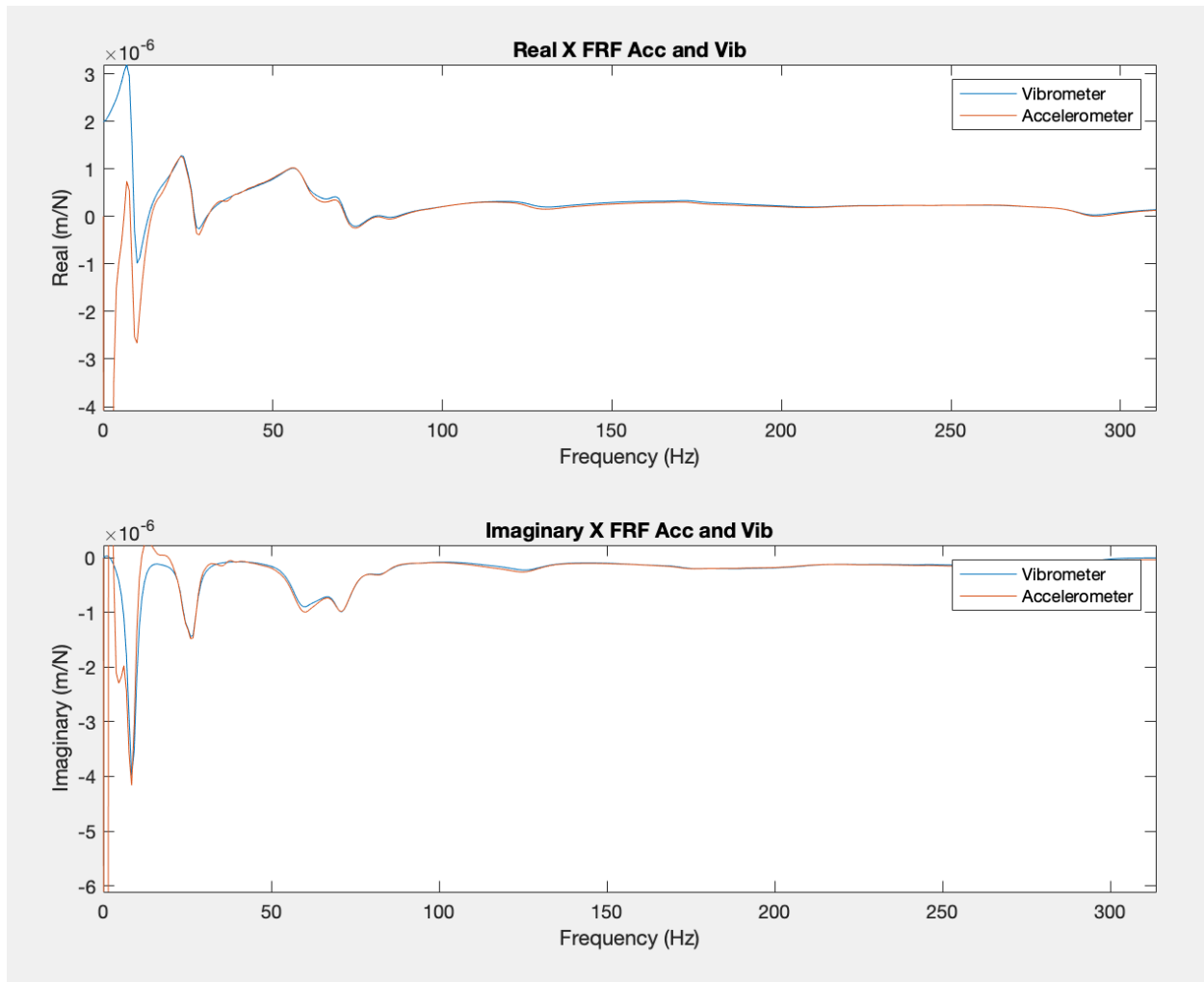


Figure 40: Real and imaginary parts of the x direction tool-tip FRF for the vibrometer and accelerometer

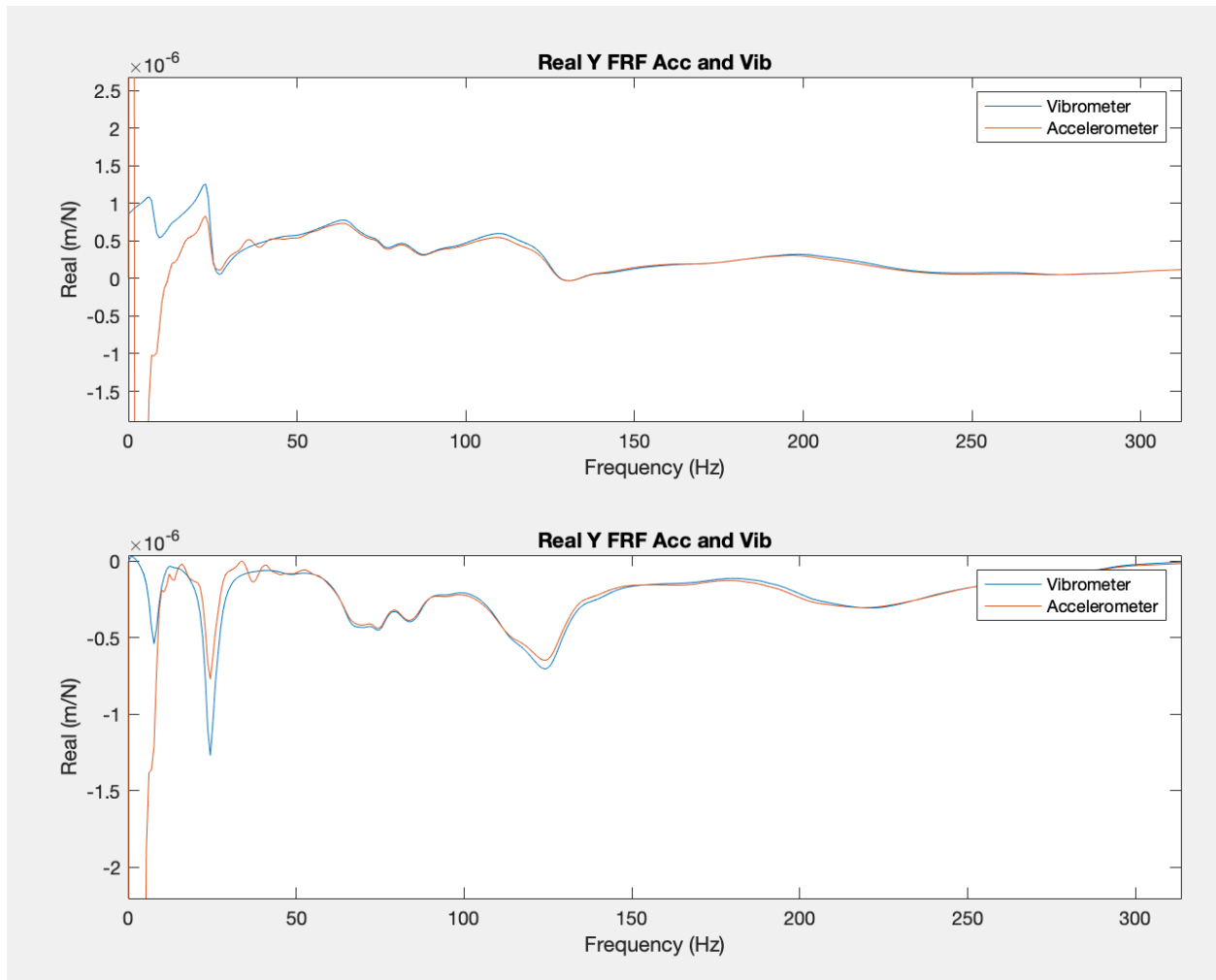


Figure 41: Real and imaginary parts of the y direction tool-tip FRF for the vibrometer and accelerometer

Final System Dynamics

Robot Dynamics

A single region of the workspace was used for machining each square on the plate. The plate was translated and rotated in order to present each of the eight perimeter squares to the robot at this location. The center square was left uncut to serve as a datum for post machining dimensional measurements. A single location was chosen to minimize errors resulting from changing dynamics of the robot at different locations across the plate. However, even over the area of a single square, the dynamics were not assumed to be constant. To explore this potential variation, tap testing was completed at the tool-tip for each side of the square to be machined. These results were then compared and the modal fits were averaged together to form the final modeled dynamics of the robot in both the x and y directions. These tests, along with the average fit at low and high frequencies, are shown in Figures 42, 43, 44, and 45, for the x direction and y directions, respectively. From these figures it is seen that the average modal fit is a good approximation of the dynamics of the robot across each of the sides of the square to be machined. Additionally, from these graphs it is seen that the most compliant modes of the system occur near 5400 Hz, due to a tool bending mode, and near 8 Hz and 24 Hz, due to the robot flexibility. The stiffness of the 8 Hz mode in the x direction is significantly lower than that of the y direction. This discrepancy is likely due to the fact that force in the x direction is primarily resisted by only the first joint in the kinematic chain while force in the y direction is resisted by most of the other joints and the entire arm of the robot. This directional discrepancy causes larger oscillations of the tool in the x direction. The final step in modeling the robot dynamics was to account for the changing dynamics of the robot during motion versus stationary. To do this, the scaling factors calculated in the previous section were applied to the stiffnesses of each of the low frequency modes, in both the x and y directions.

Workpiece Dynamics

In this system the robot is the least stiff component, but the flexibility of the workpiece cannot be neglected. The workpiece is mounted to a robotic linear track with two stacked rotary axes and this system also has significant vibration modes. To investigate these modes, a similar tap testing experiment was performed. The high sensitivity/larger mass accelerometer was used for this experiment since significant low frequency modes were expected. The dynamics of the x and y directions of the workpiece are shown in Figure 46. From the figure it is seen that the x and y directions both have several prominent low frequency modes, but the y direction is significantly less stiff. This is likely because the y axis of the workpiece is tangent to one of the rotational axes stacked on the linear track. As a result, force in the y direction causes a large moment around this axis. To incorporate the workpiece dynamics in to the dynamic model for stability and SLE prediction the FRFs of the robot and workpiece were summed (the real and imaginary parts were added frequency by frequency).

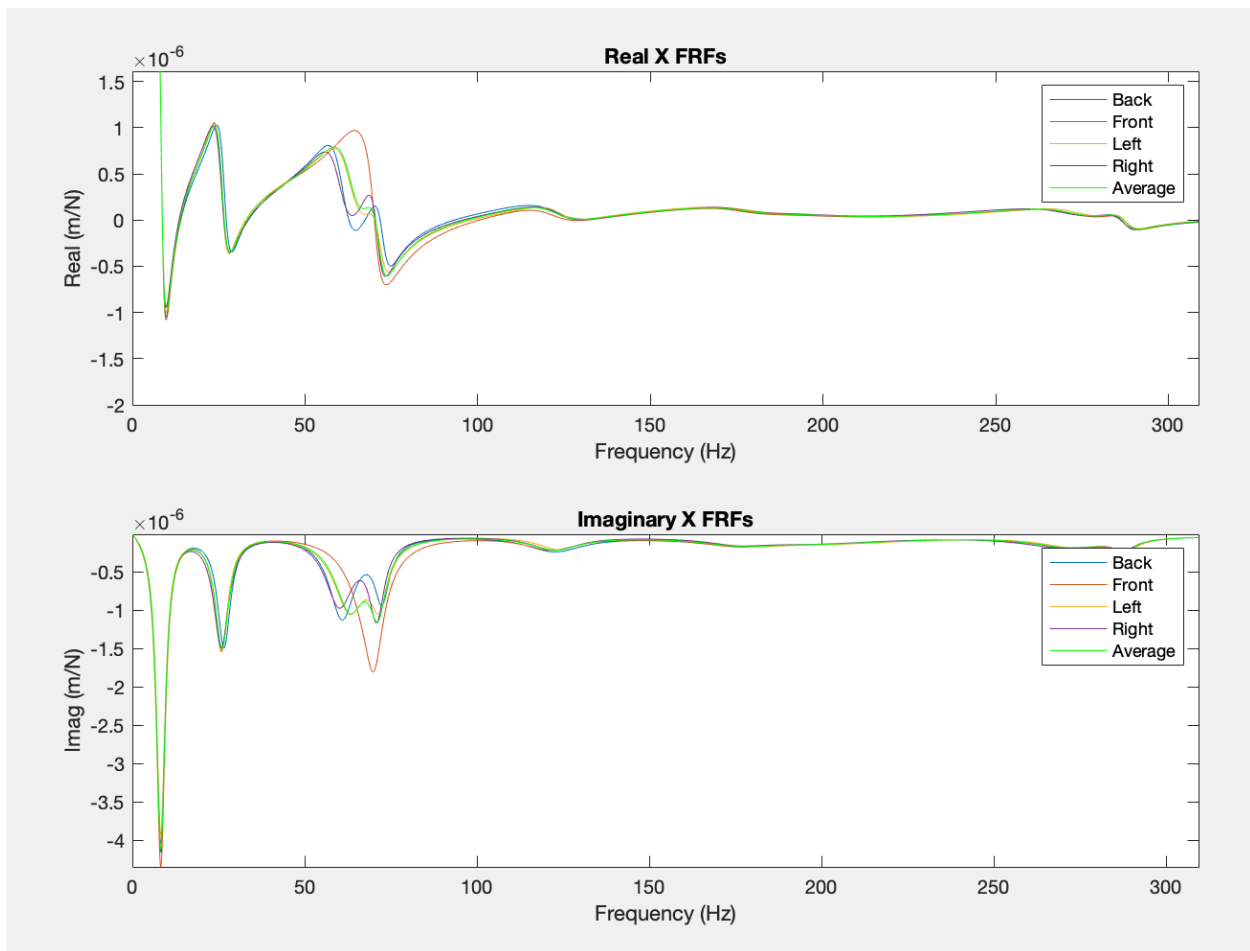


Figure 42: X Direction FRF of average fit compared with FRFs for each side at low frequency

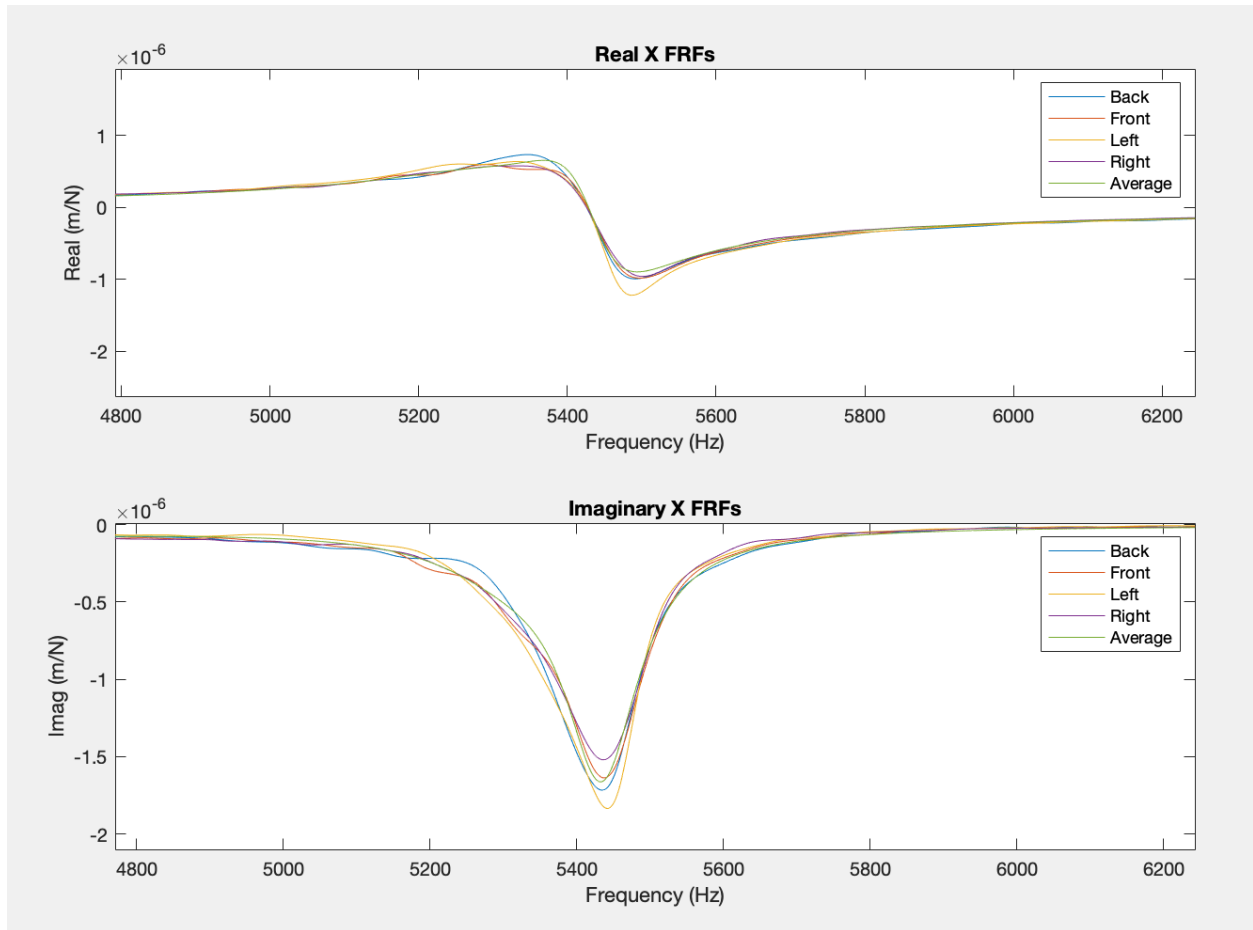


Figure 43: X Direction FRF of average fit compared with FRFs for each side at high frequency

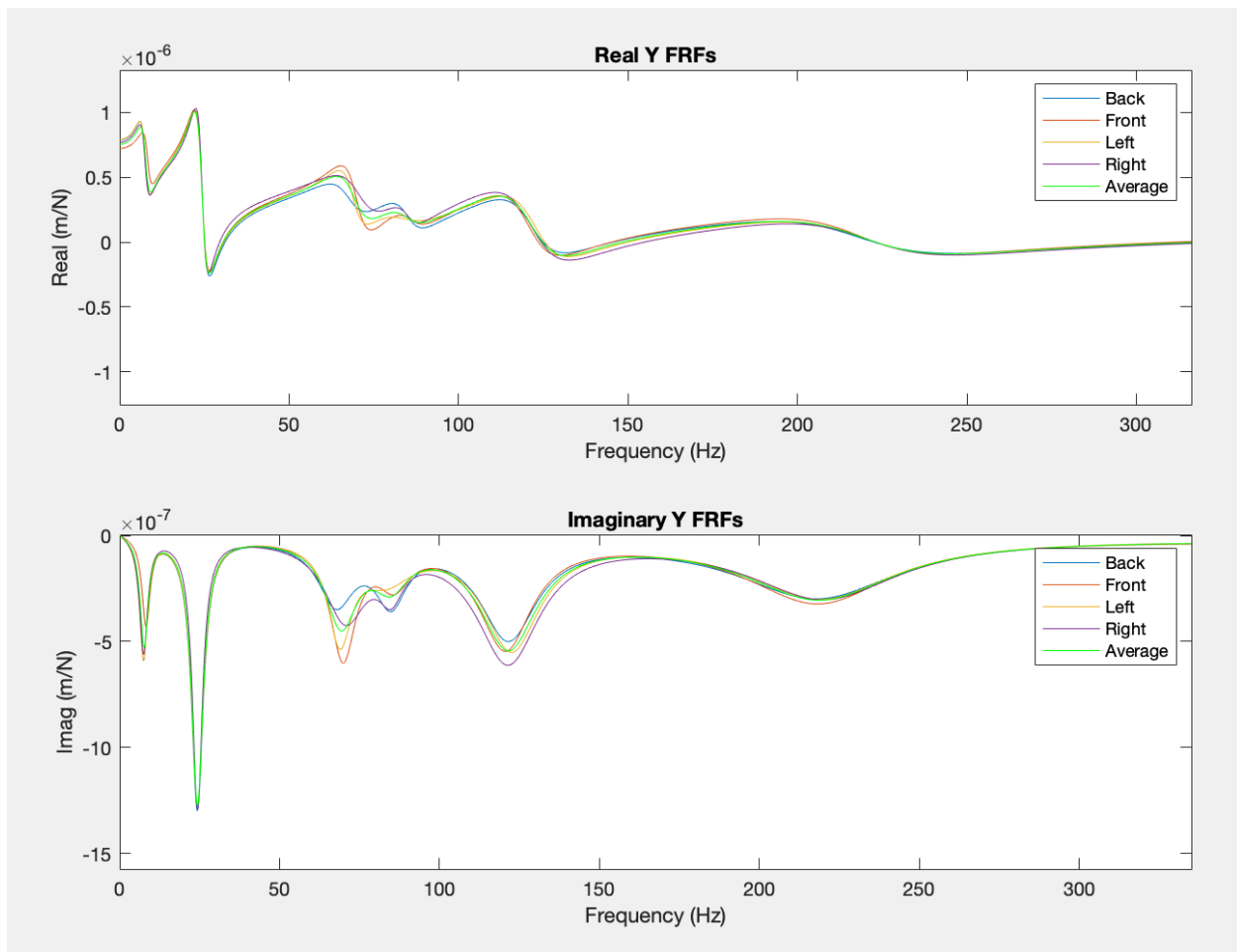


Figure 44: Y Direction FRF of average fit compared with FRFs for each side at low frequency

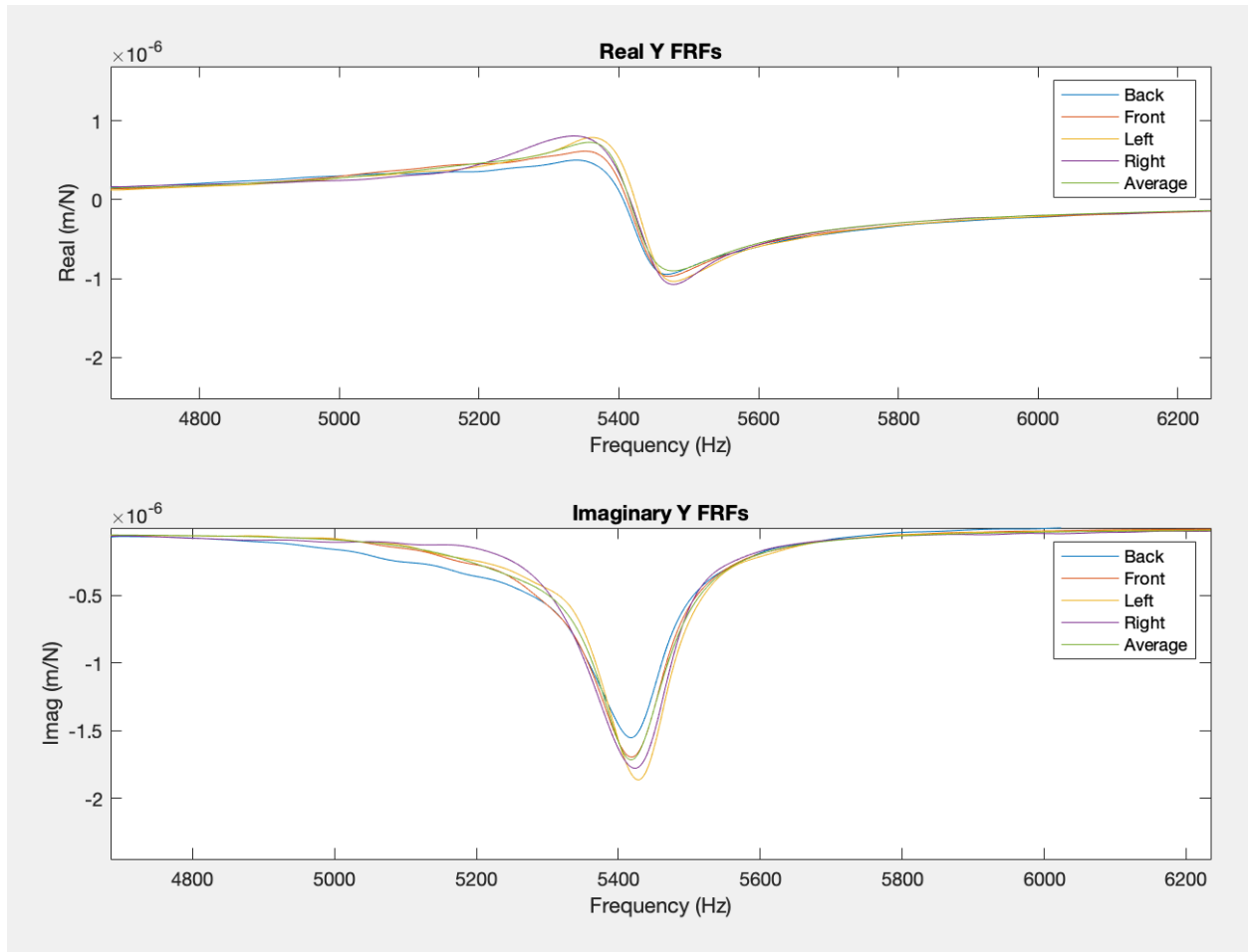


Figure 45: Y Direction FRF of average fit compared with FRFs for each side at high frequency

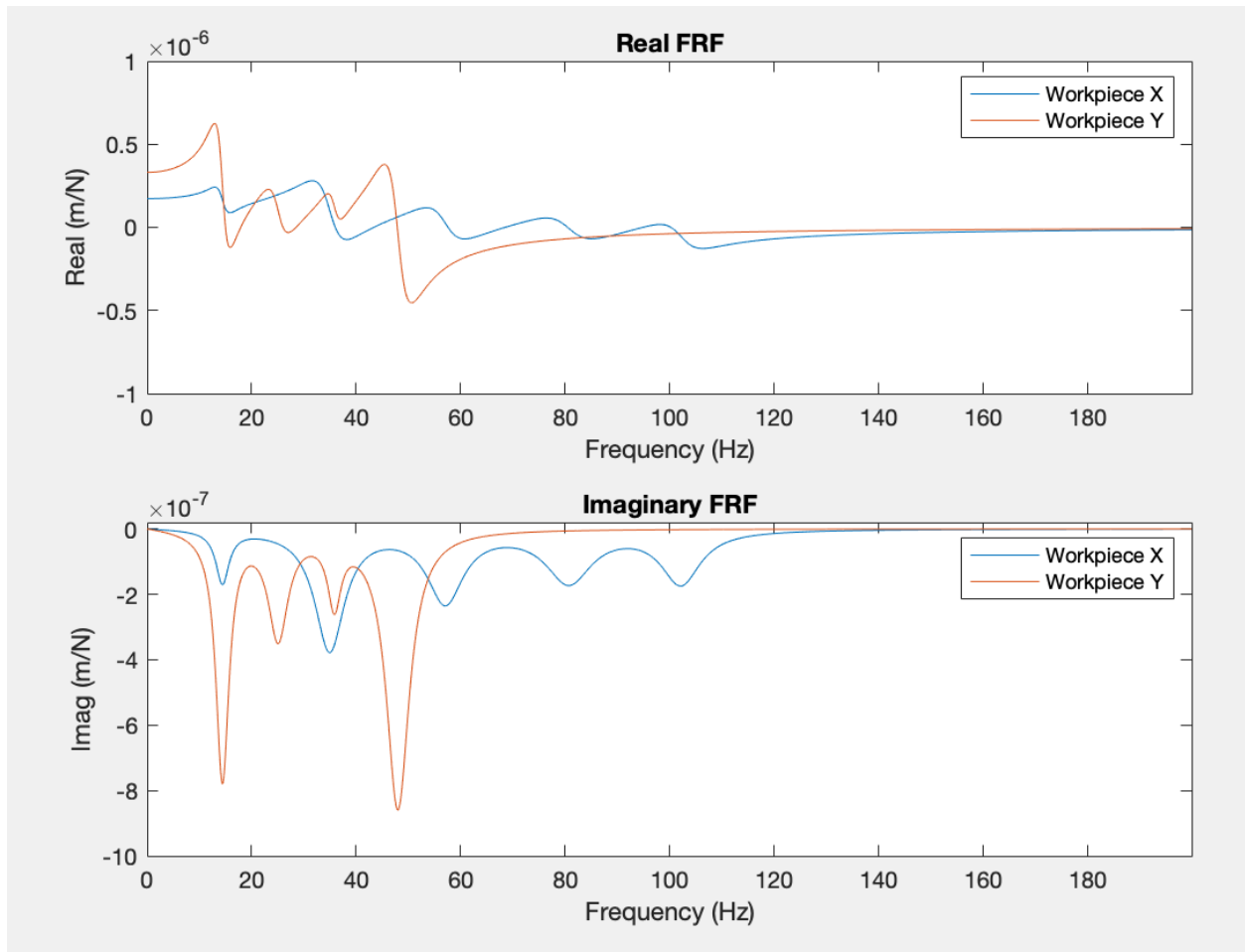


Figure 46: X and y direction dynamics of the workpiece

Chapter 5: Robot Motions without Cutting

Many industrial robots are designed to be used in pick and place applications, where location repeatability is far more important than path accuracy. This was evident during initial testing, where the ability of the robot to follow a straight line was less than the performance of conventional machine tools. It was desired to understand the magnitude of these deviations because they affect the machined surface of the workpiece. The robot represents an open loop system, where rotary axis angles are commanded to position the tool-tip at the desired location in space given the link lengths that connect the joints. While the robot controller does track its joint positional data, it does not use this data to compensate the actual end effector trajectory in real time. This data can be extracted from the robot after executing a tool path and used to characterize the actual position of the robot end effector compared to its commanded position.

To study deviations from the commanded path, testing was performed using a linear variable differential transformer (LVDT) sensor mounted in the spindle. During tool path execution, the LVDT tip was placed against the surface of a flat block. The robot was commanded to move in a straight line parallel to the block surface. If no deviations occurred, the indicator would read a constant value. If deviations occurred, on the other hand, the indicator would record these departures from the intended straight line path. The results from the data recorded by the robot controller were compared to the results from the indicator. The test was repeated five times across four different flat surfaces, each representing a side of the square to be machined. This resulted in 20 total tests. The experimental setup is shown below in Figure 47, with (a) referring to the direction of travel of the spindle during a test, and (b) referring to the direction of deflections measured during the test. For example, if the robot was traversing in the y direction, the indicator recorded deflections in the x direction. The results of this test are shown in Figures 48, 49, 50, and 51, representing the front, back, left, and right surfaces of the part respectively. Each figure displays the results of all five tests on each surface for both the robot and the indicator. From the figures it is seen that the tests display repeatability for both the robot and indicator, but the agreement between the two measurement sources is not perfect. There are a few reasons for this disagreement. First the robot has a lower spatial sampling frequency than the indicator, so it is possible the actual deflections aliased in the robot controller data. Second, the robot controller only measures the motions in each of its joint axes, so its estimation of the tool-tip position is based only on the position of its end effector, neglecting any deflections that occur due to the addition of the spindle-holder-sensor to the system. The agreement is also worse along the left and right surfaces, than it is on the front and back surfaces. This is likely due to the reduced stiffness of the robot in the x axis compared to the y axis. However, even with these limitations it was determined that the robot was capable of measuring the average value of its position compared to its commanded position. These tests formed a baseline for the path accuracy of the robot and any differences between the actual and commanded tool path measured in this test would be present in machining operations, in addition to any vibrations resulting from the machining process.

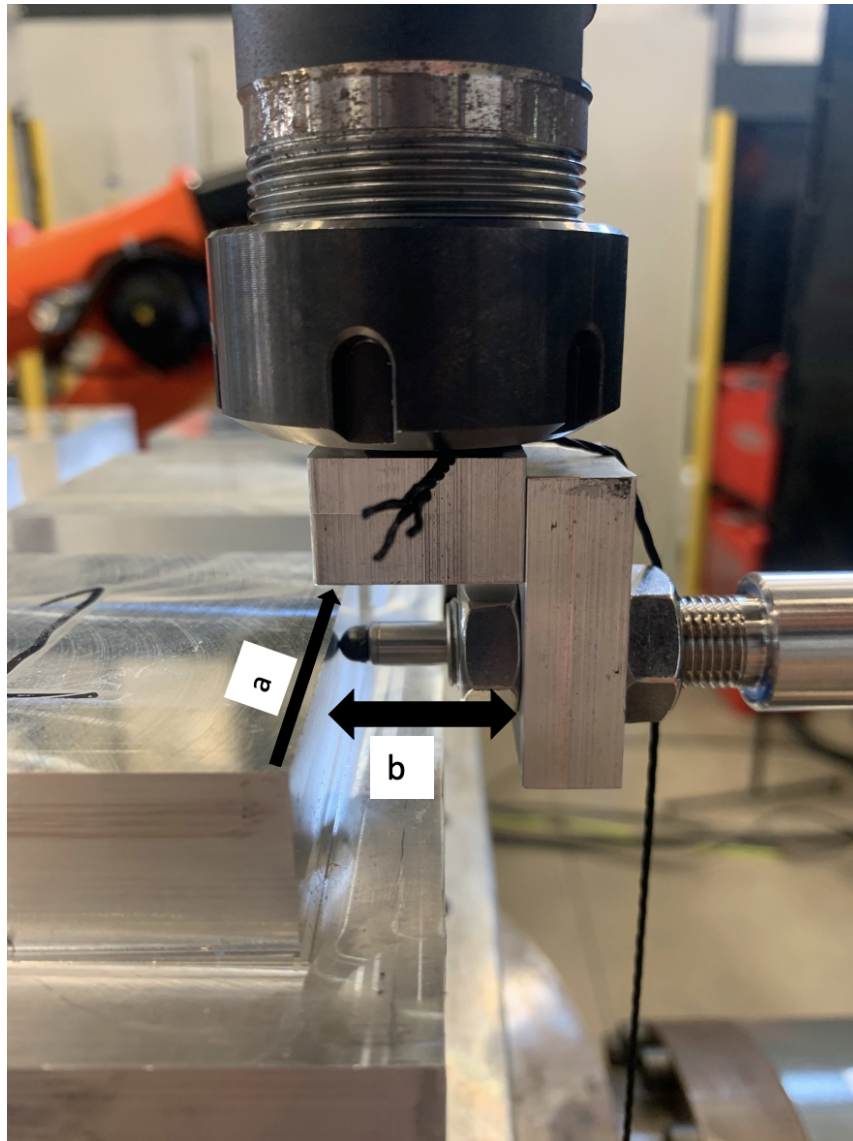


Figure 47: Image of the indicator setup in the spindle

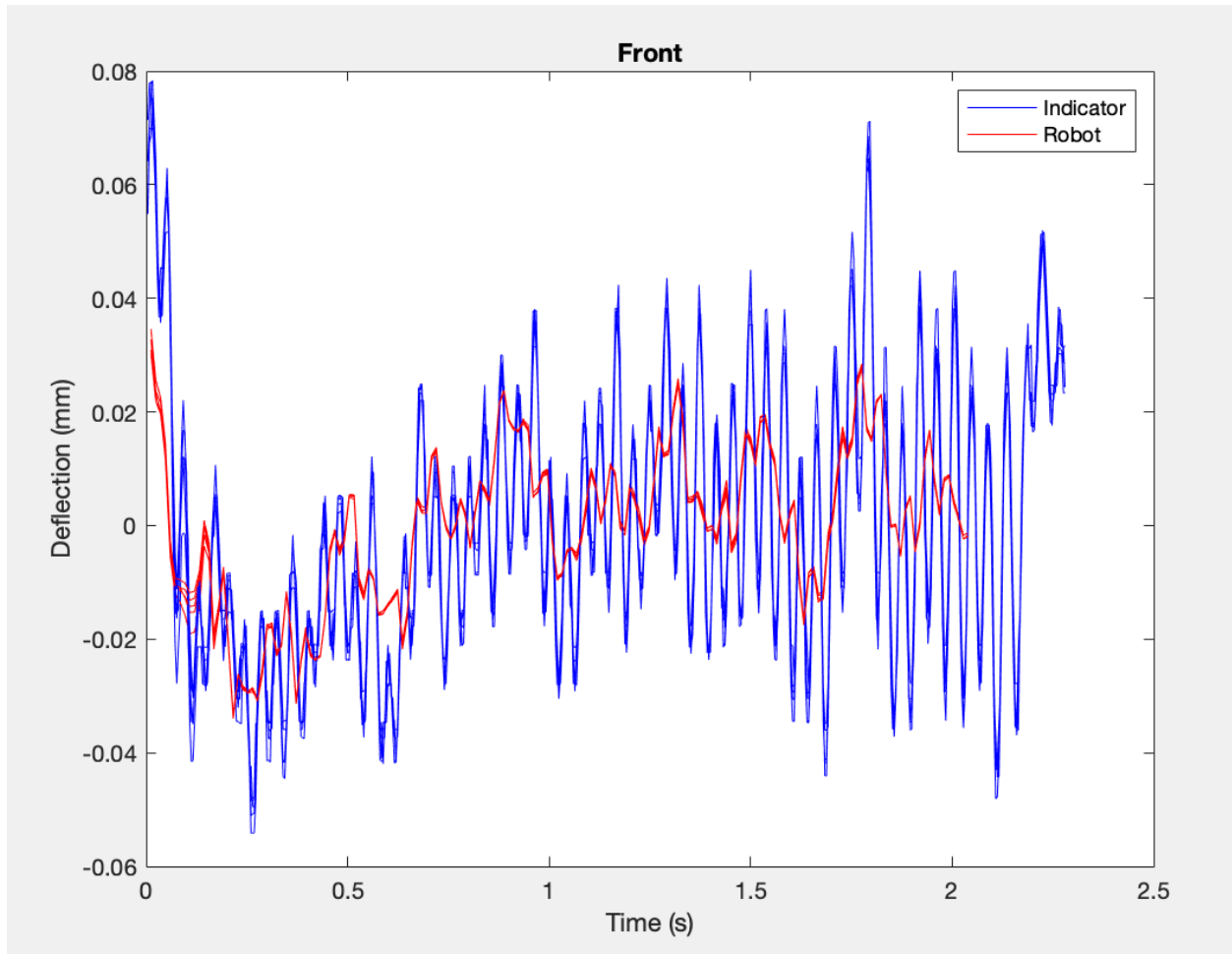


Figure 48: Comparison of indicator and robot results for all five tests along the front surface

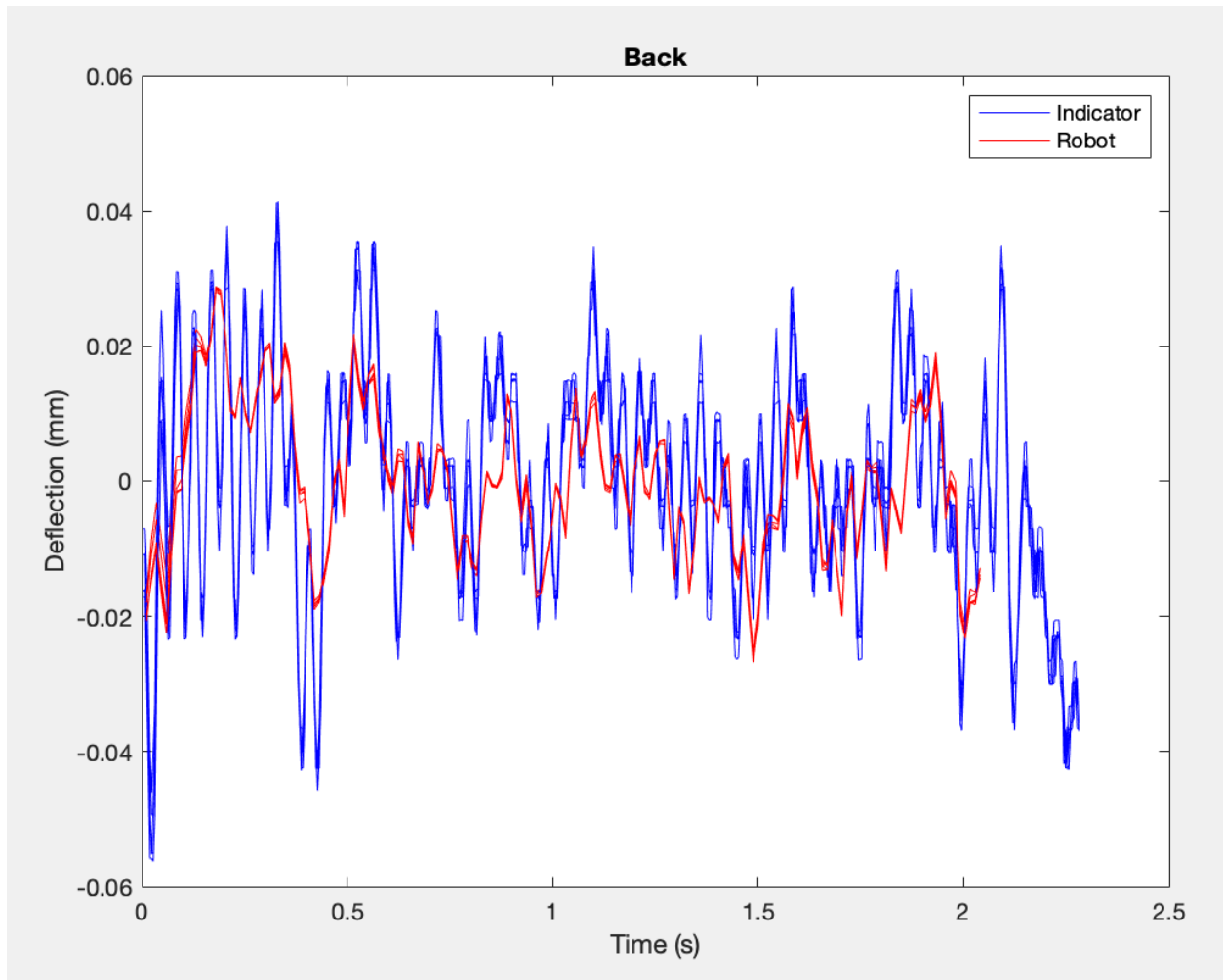


Figure 49: Comparison of indicator and robot results for all five tests along the back surface

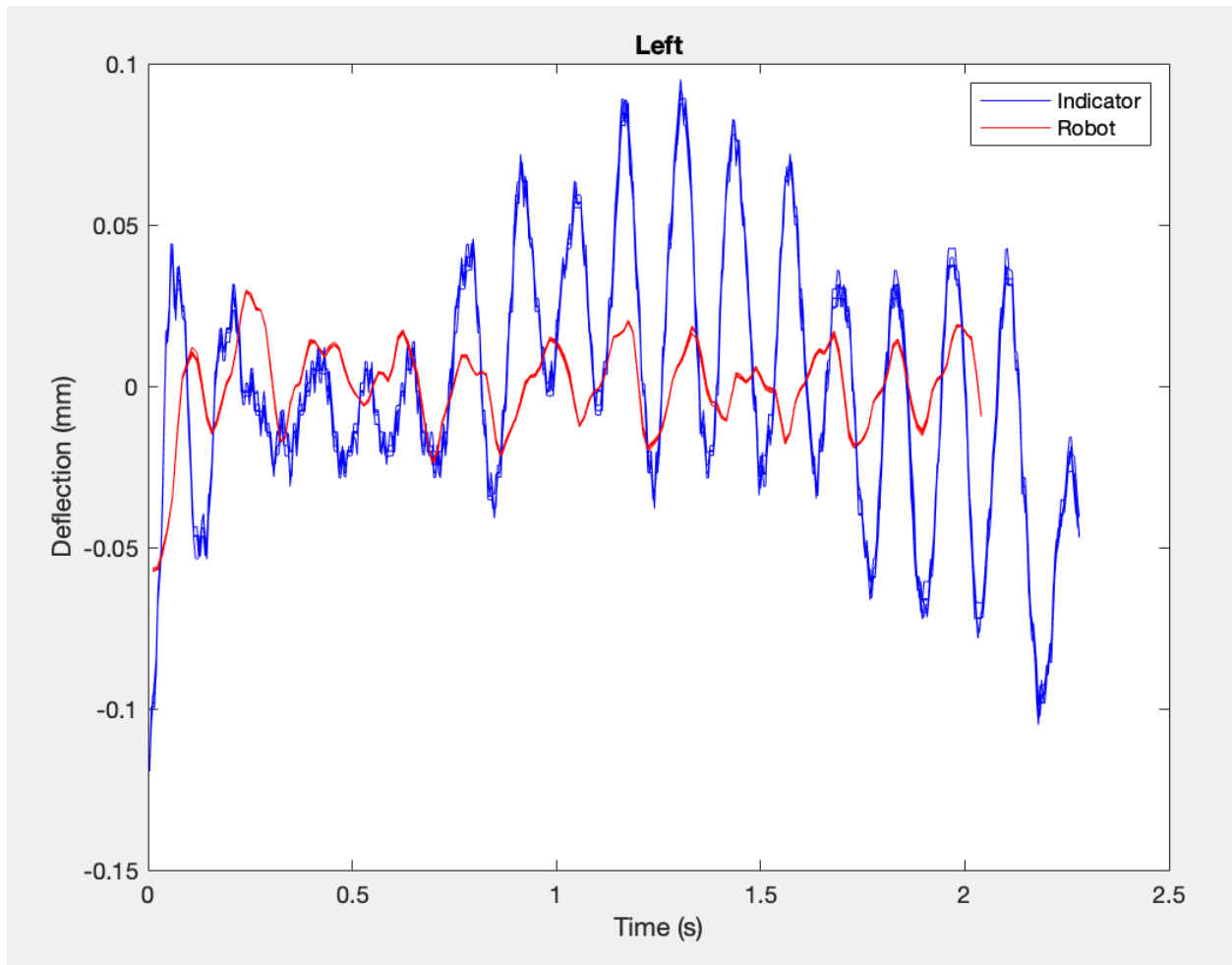


Figure 50: Comparison of indicator and robot results for all five tests along the left surface

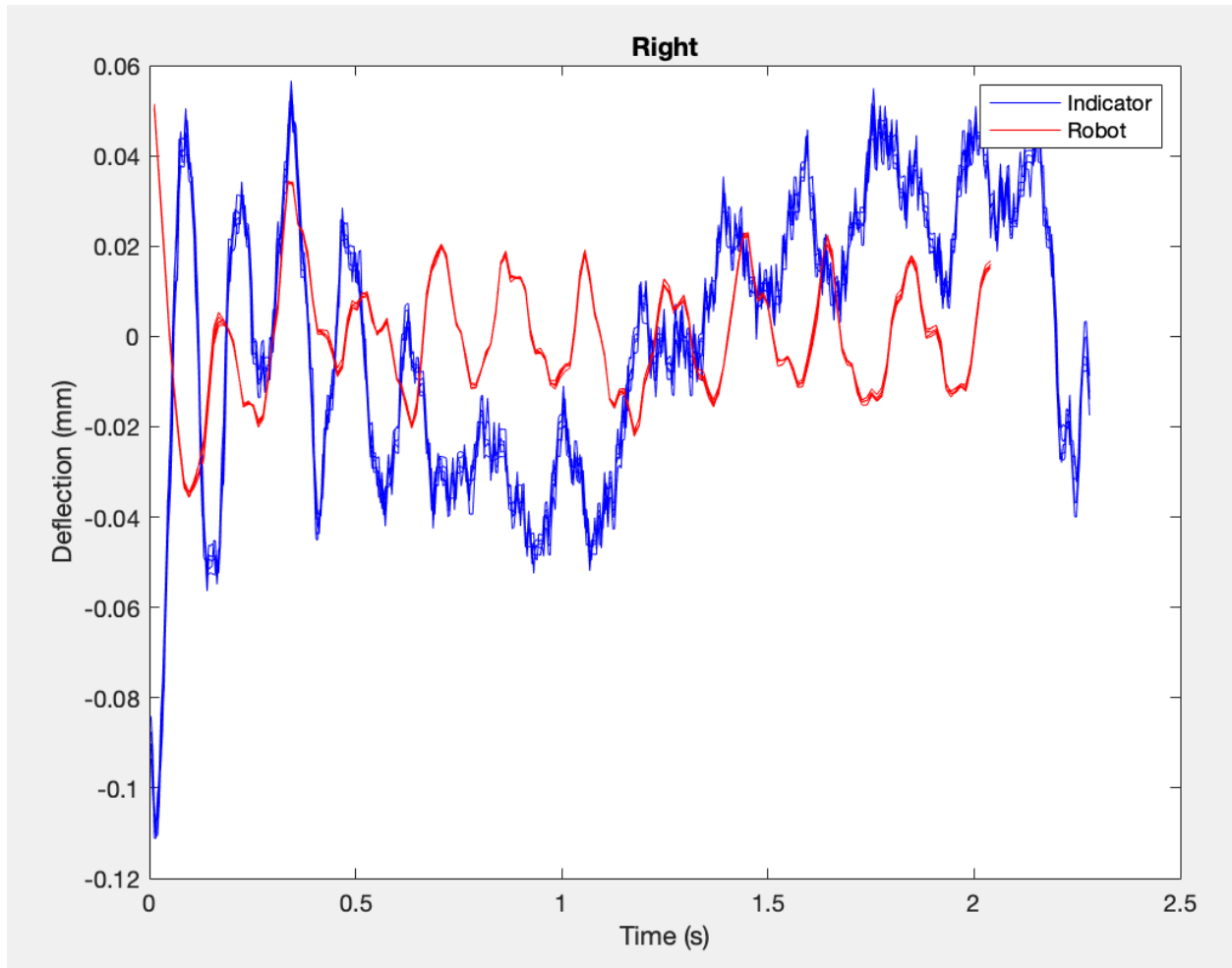


Figure 51: Comparison of indicator and robot results for all five tests along the right surface

Dimensional Accuracy of Robot Tool Paths

Based on the results from the previous section, it is observed that the robot path accuracy is inferior to conventional machine tools. To provide a comparison, the same experiment reported in the previous section was conducted on a conventional machine tool. The results are shown in Figure 52. The deflections while the machine is in motion are within the resolution provided by the LVDT. To further evaluate the robot path accuracy, its positional accuracy during tool paths was tested by programming the robot to move in a square tool path with defined side lengths, which represent the square to be machined during the follow-on machining trials. To simulate the machining tool paths the measurements were performed in the same location and with a traversal speed corresponding to one of the machining tests. The square tool path was approximately 74 mm in length and width, and was repeated five times with a step down in the z direction between each repetition. The results of the test along each side of the square are shown in Figures 53, 54, 55, and 56, for the front, back, left, and right sides, respectively. The figures include arrows that point towards the center of the square. From these figures it is seen that, for this part geometry, the robot path is consistently offset from the commanded path towards the exterior of the part. This would result in less material being removed than commanded (i.e. an undercut surface) if a part was machined using this tool path. It is possible to calculate the average value of the actual robot path along each of the sides, and compare it to the average value of the commanded robot path, to determine the average error along each side. From the figures it is seen that the robot paths are repeatable with the oscillations in each path matching closely between layers. Therefore, compensation for the observed offsets is possible.

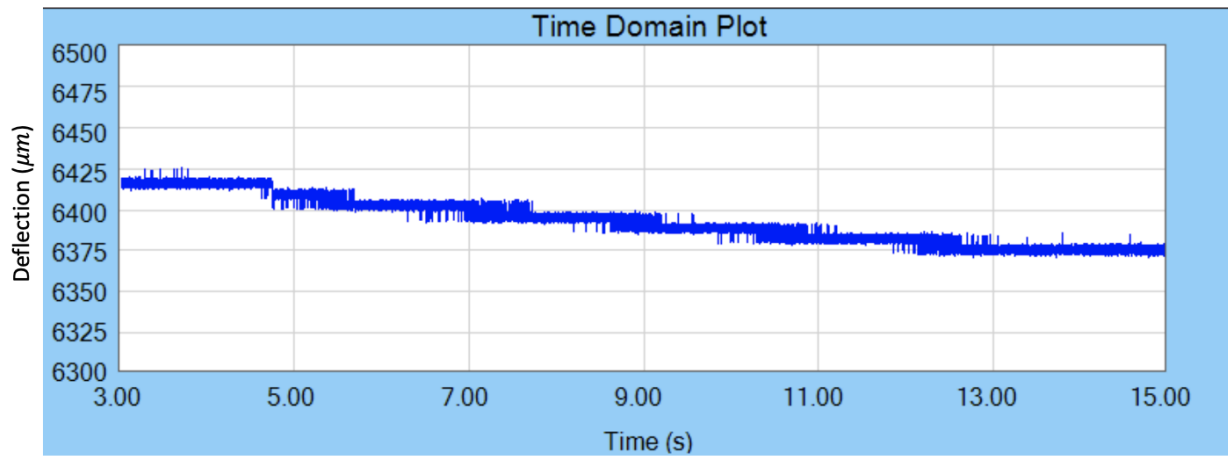


Figure 52: Example of the indicator test run on a conventional machine tool

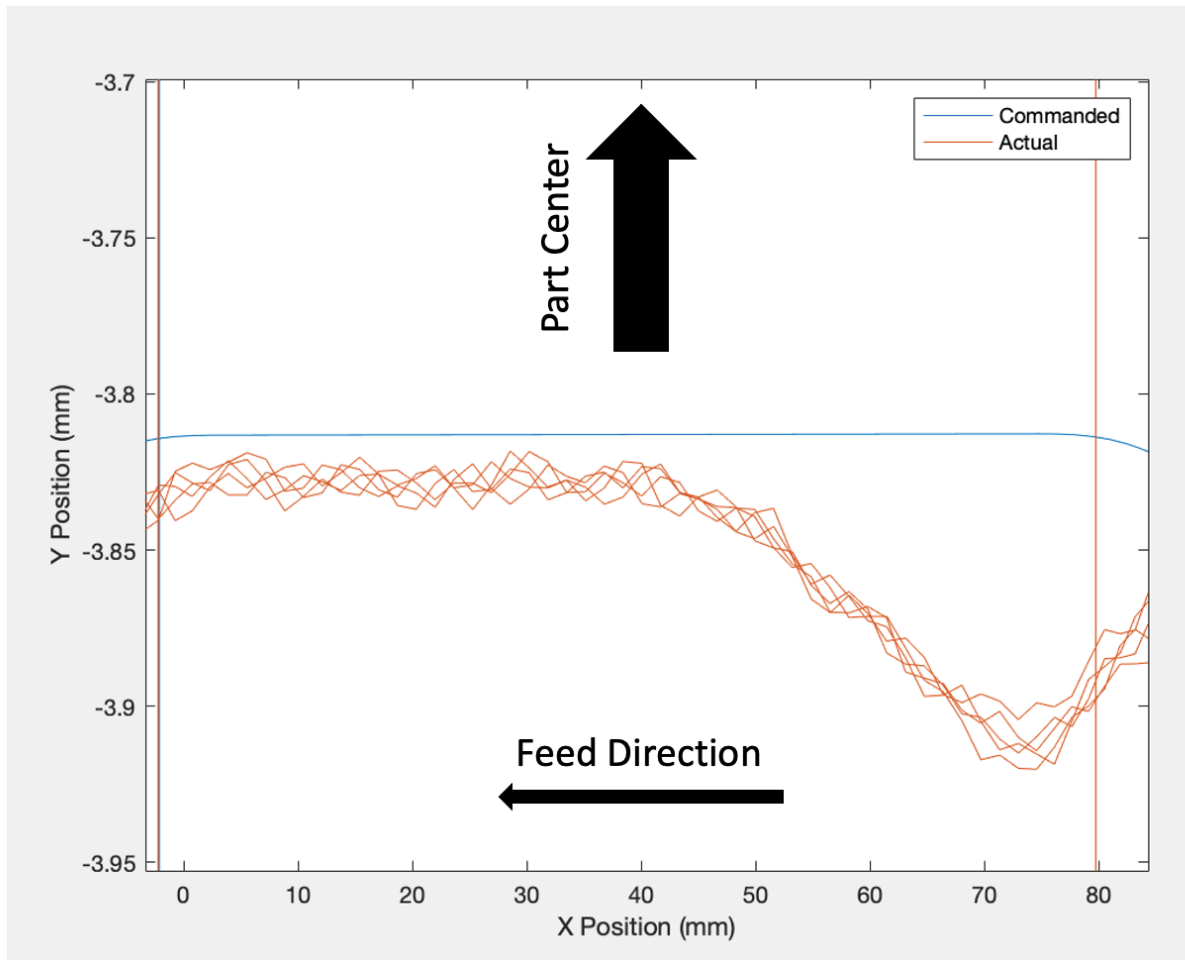


Figure 53: Comparison of the commanded and actual robot positions along the front surface

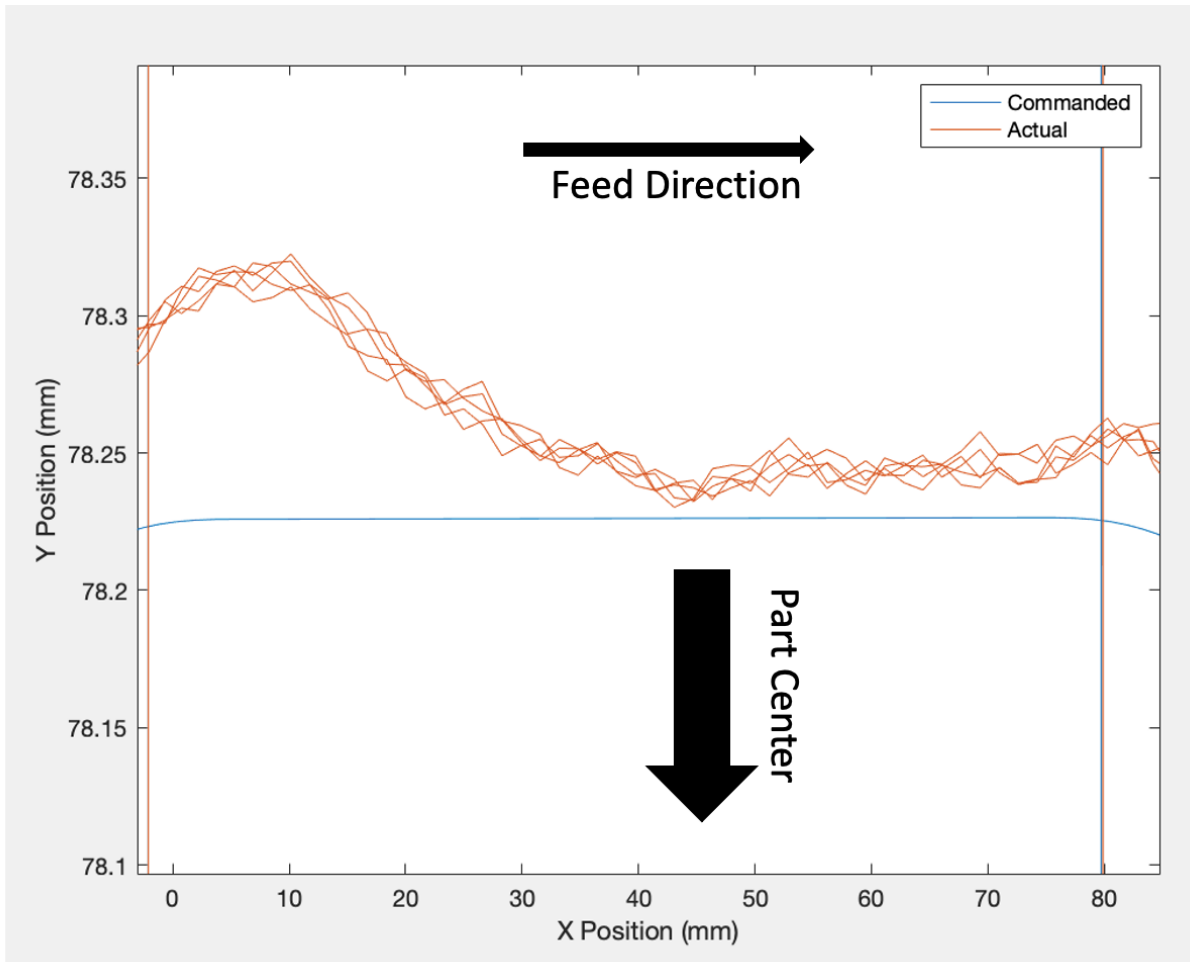


Figure 54: Comparison of the commanded and actual robot positions along the back surface

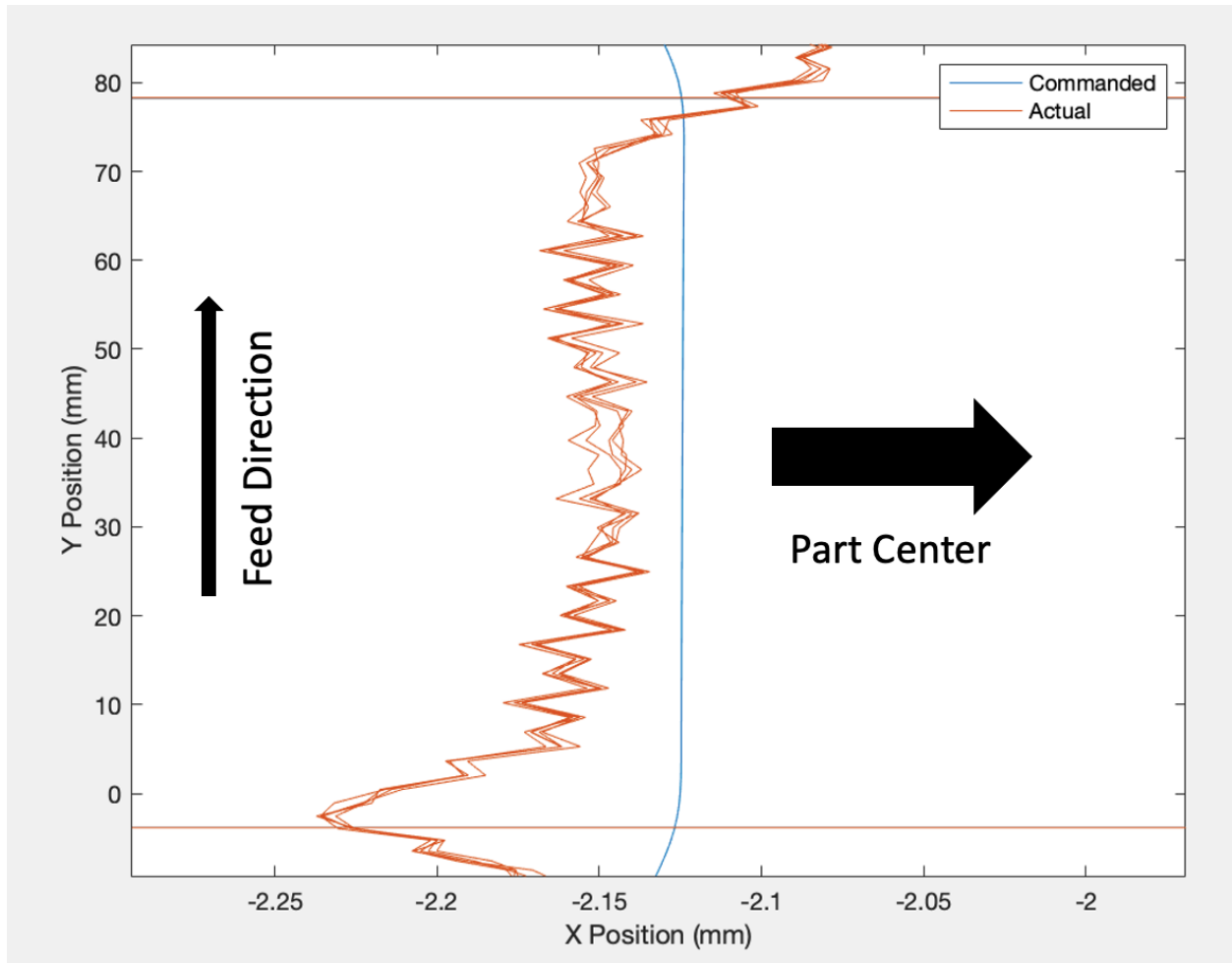


Figure 55: Comparison of the commanded and actual robot positions along the left surface

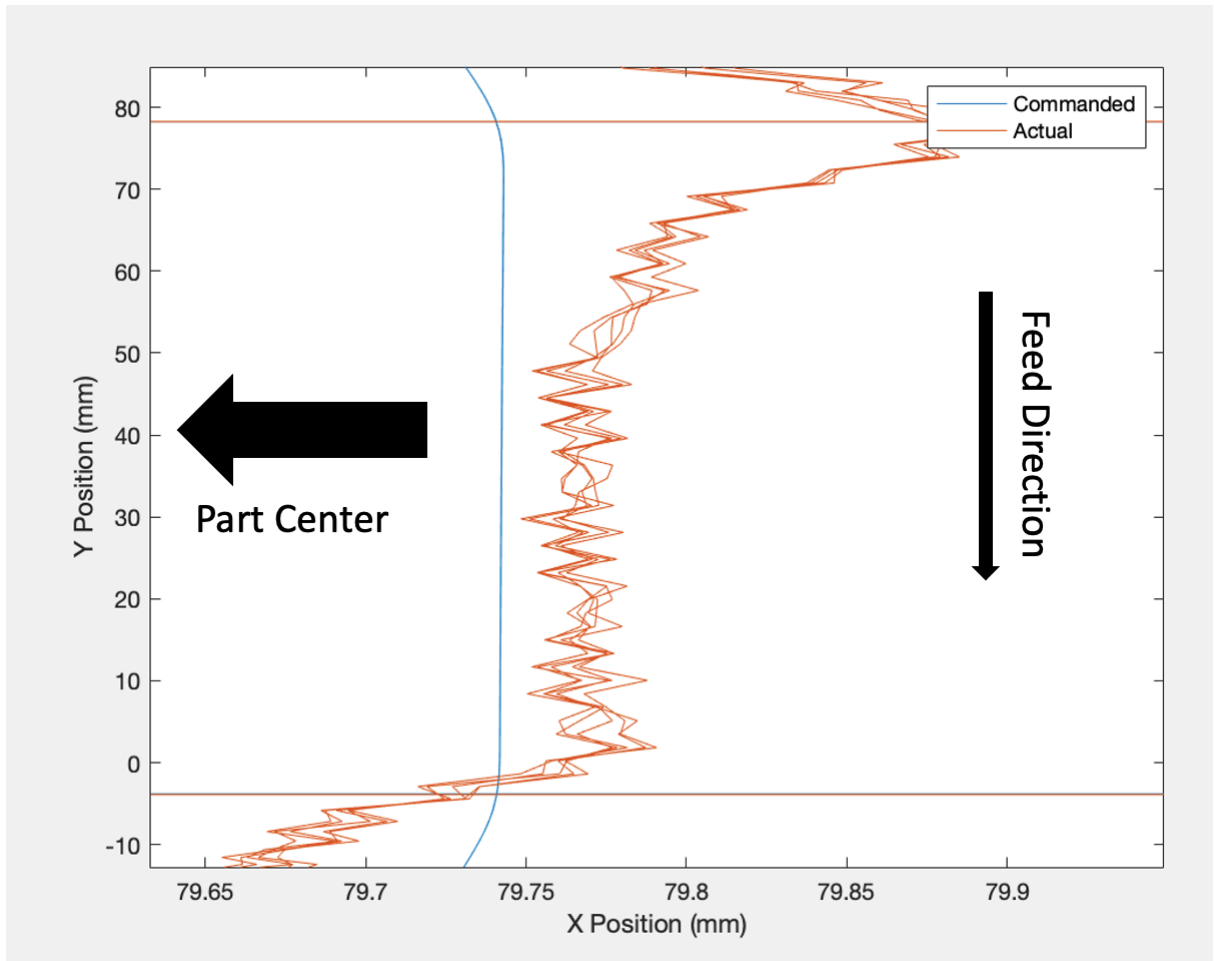


Figure 56: Comparison of the commanded and actual robot positions along the right surface

Chapter 6: Post-Process Measurement

To validate the surface location error predictions, it was necessary to accurately measure the machined workpieces for comparison to the commanded dimensions. The initial geometry of each workpiece was machined on a Haas VF-4, and while low radial and axial depth finishing passes were performed to produce the final surface, it could still not be assumed that the dimensions of the part were equal to the commanded dimensions. The squares on each plate were measured on a Zeiss Duramax coordinate measuring machine (CMM) before and after the cutting tests were performed, using an identical measurement routine to minimize errors. To measure the dimensions of the squares a 3 mm diameter probe was moved along the surface of each of the sides of each square near the top surface while the probe position was sampled at a constant rate of once per 0.1 mm of travel distance. The probing tool path was a rectangle with a width of 25.4 mm (along the tool path direction) centered in the middle of the tool path, and a height of 3 mm (along the axial direction of the tool) with the top edge 3 mm from the surface of the part. This tool path is shown in Figure 57. Only the center portions of the square sides were sampled to avoid the transient effects introduced at the cut start when the tool first engaged with the material and at the cut end when the tool ended the material engagement. The measurement results would therefore represent the steady-state cutting response, which corresponded to the surface location error predictions. The differences in the dimensions of each square before and after cutting were calculated. The measured radial cut width for each pass was half of this difference for each dimension because the cutting tool removed material along the entire perimeter of the square. The resulting surface location error was the difference between the measured radial depth and the commanded radial depth.

Because the CMM measurement uncertainty affects the surface location error algorithm validation, a repeatability study was conducted on the CMM to ensure that the measurement results were consistent. In this study, one plate (with 9 squares) was measured using the same computer controlled measurement path five times. The averages and standard deviations of the dimensions for each square were then calculated. The results of this study for one of the plates are given in Table 3. From the table it is seen that the measurement process is repeatable with a maximum standard deviation of less than 1 (μm). This value is well below the path accuracy of the robot, so it was determined that the CMM results were sufficient for the robotic milling tests.

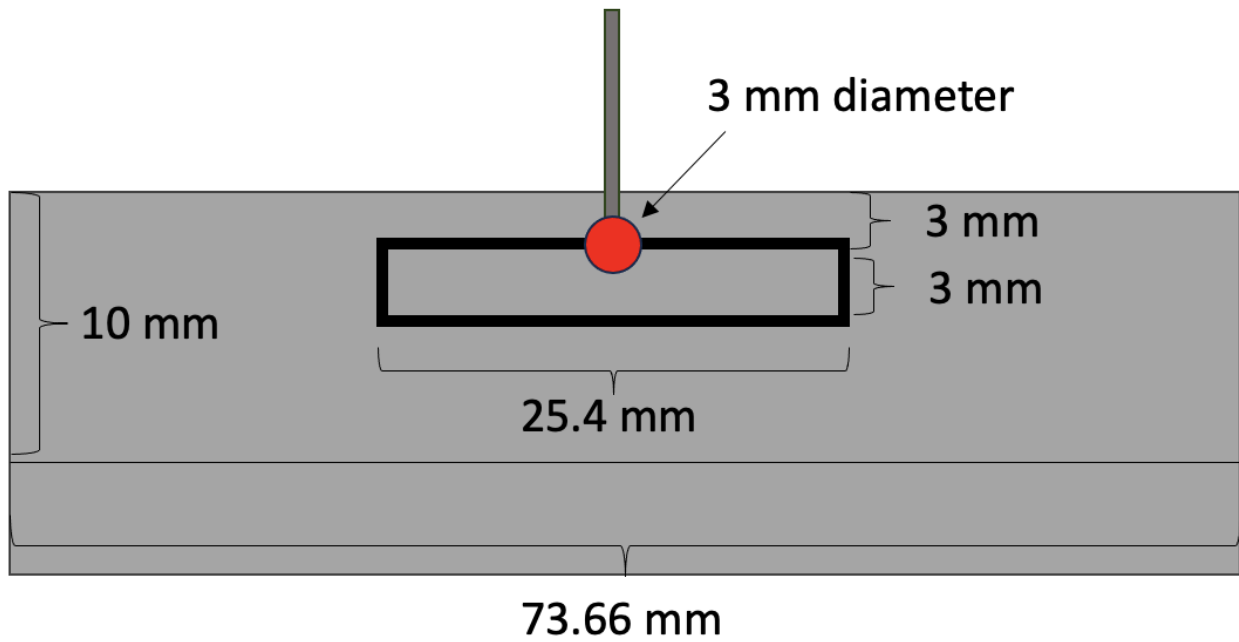


Figure 57: Diagram showing the tool path of the probe

Table 3: Table showing the average dimension and standard deviation of the squares of a sample plate

Square Location	Dimension	Average (mm)	Standard Deviation
1	X	75.957	0.15
1	Y	75.953	0.23
2	X	75.953	0.28
2	Y	75.955	0.28
3	X	75.903	0.13
3	Y	75.907	0.24
4	X	75.902	0.19
4	Y	75.912	0.19
5	X	75.953	0.21
5	Y	75.957	0.63
6	X	75.956	0.41
6	Y	75.960	0.28
7	X	75.954	0.16
7	Y	75.957	0.44
8	X	75.947	0.38
8	Y	75.956	0.26
9	X	75.944	0.42
9	Y	75.954	0.42

Chapter 7: Parameter Selection and Error Prediction

The goal of this research is to advance knowledge and modeling capability for the robotic milling process to make it a viable option for discrete part manufacturing. This requires the robot to be capable of accurately machining parts with high material removal rates. It is therefore necessary to select stable cutting parameters, predict geometric errors due to forced vibrations (surface location error, SLE), and compensate for these errors. The dynamic models developed in this research are presented and used in this chapter to identify the cutting parameters for the experiment plan. The SLE is then predicted with the goal of performing error compensation to produce accurate parts.

Dynamic Model

The dynamic model for the machining system includes the frequency response functions, FRFs, for both the robot-spindle and workpiece. For the robot-spindle, a vibrometer was used to measure the low frequency modes and an accelerometer was used to measure the high frequency modes. The low frequency stiffnesses were also compensated to accommodate the changing stiffness when the robot is in motion. For the workpiece, all modes were measured using an accelerometer. The final dynamic model of the system is shown in Figure 58 for the high frequencies, and in Figure 59 for the low frequencies. The full list of modal parameters of the robot-spindle, in the x and y directions, and the workpiece are shown in Tables 4, 5, and 6. The x and y directions differ so the compensation algorithm was implemented independently for each direction.

Cutting Force Coefficients

After the dynamic model was established the next step was to determine the cutting force coefficients for the tool-workpiece combination. This was completed using the procedure detailed in Chapter 2. Five cutting tests were performed at evenly spaced feed per tooth (f_t) values ranging from 0.1 mm/tooth to 0.3 mm/tooth. The tool was a three flute endmill, the spindle speed was 10,700 rpm, and both the radial and axial depths of cut were 2mm. The average forces of each test for each direction are shown in Figure 60. The slope of the average forces in both the x and y directions are used to calculate the cutting force coefficients k_t and k_n . From this test, $k_t = 597.4$ N/mm² and $k_n = 59.8$ N/mm², resulting in $K_s = 600.4$ N/mm², and $\beta = 84.28^\circ$. These cutting force coefficients were used in the models for stability and surface location error.

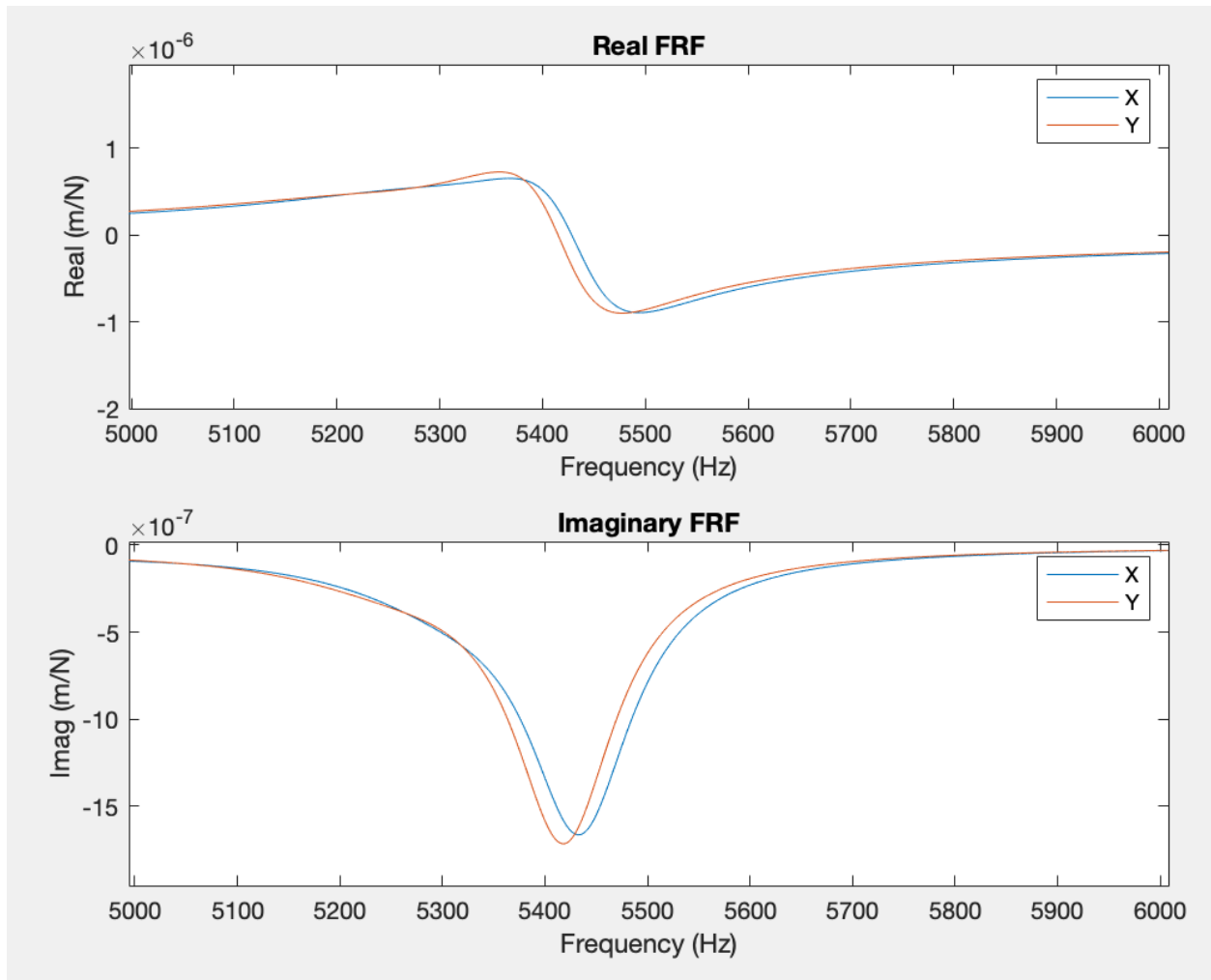


Figure 58: The dynamics of the robot-spindle-workpiece combination for the high frequency mode

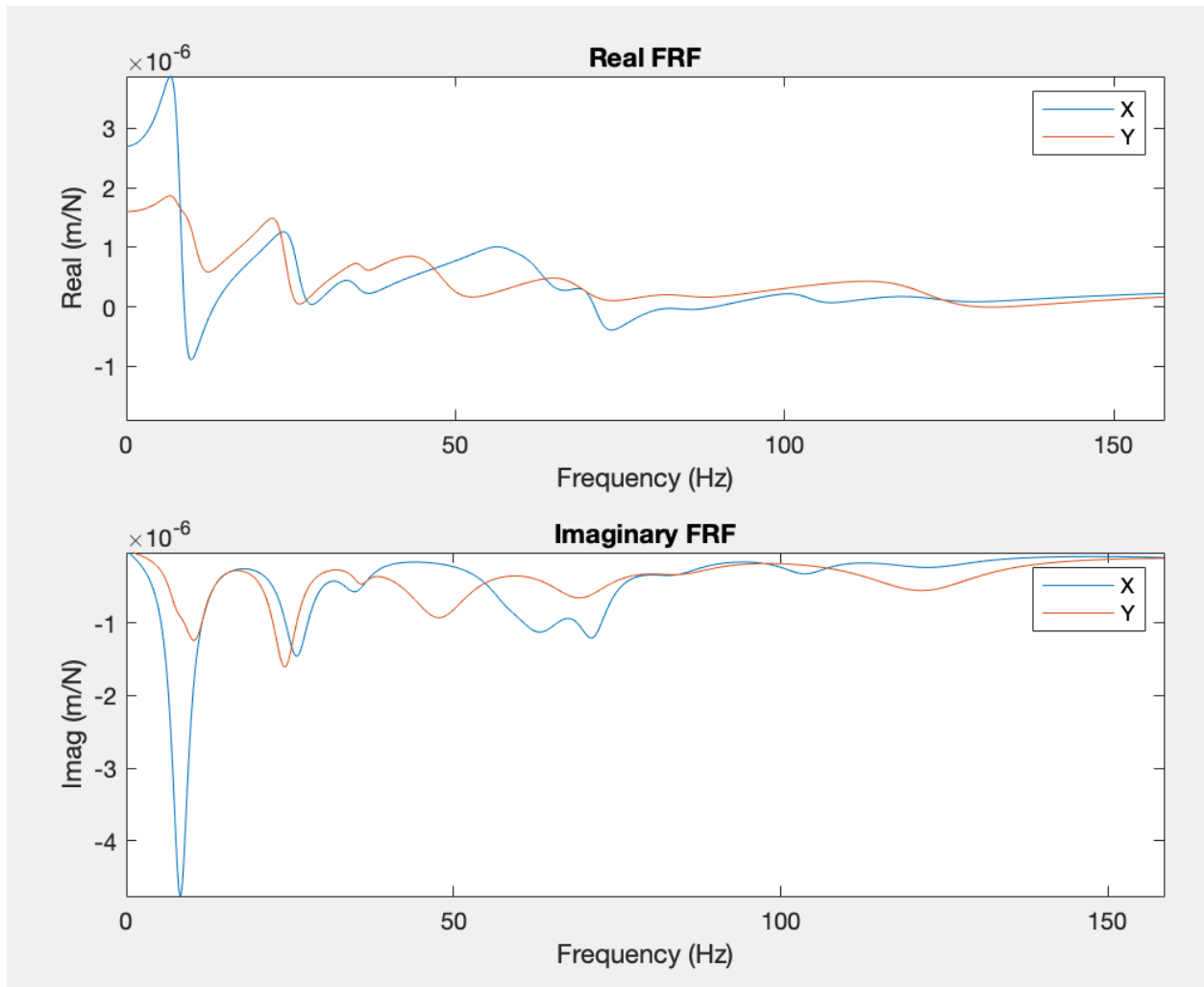


Figure 59: The dynamics of the robot-spindle-workpiece combination for the low frequency modes

Table 4: Average x direction modal fits of tool-tip robot-spindle modes

Robot-spindle x		
Frequency (Hz)	Stiffness (N/m)	Damping (%)
8.4	6.78×10^5	18.18
26.1	3.77×10^6	9.13
63.3	6.27×10^6	8.55
71.3	1.47×10^7	4.01
123.0	3.83×10^7	7.35
176.5	9.71×10^7	7.21
200.8	2.72×10^7	17.25
272.5	7.26×10^7	4.92
287.5	2.22×10^8	1.39
657.8	1.23×10^7	15.21
1014.5	6.36×10^7	17.50
1443.5	3.30×10^7	6.88
2223.8	1.10×10^8	5.96
2439.3	1.48×10^8	2.22
2786.5	2.80×10^8	3.17
3114.3	5.63×10^7	9.59
4519.5	6.87×10^7	11.53
5311.3	1.01×10^8	2.25
5434.3	2.88×10^7	1.13
6629.3	9.10×10^8	0.76

Table 5: Average y direction modal fits of tool-tip robot-spindle modes

Robot-spindle y		
Frequency (Hz)	Stiffness (N/m)	Damping (%)
7.8	5.07×10^6	19.55
24.4	4.43×10^6	8.99
69.5	1.48×10^7	8.73
85.0	3.38×10^7	8.09
122.0	1.10×10^7	8.98
220.8	1.32×10^7	13.20
676.5	1.45×10^7	18.52
1184.0	2.30×10^7	22.80
1480.0	6.47×10^7	4.78
2197.0	5.99×10^8	2.54
2404.0	2.23×10^8	1.32
2672.0	5.18×10^8	3.10
2933.3	5.89×10^7	7.92
4362.3	6.28×10^7	20.95
5257.8	1.33×10^8	2.38
5418.8	2.76×10^7	1.11
6622.8	9.03×10^8	0.75

Table 6: Workpiece modal fits in x and y

Workpiece x		
Frequency (Hz)	Stiffness (N/m)	Damping (%)
14.5	2.97×10^7	10.53
35.1	1.39×10^7	9.78
57.2	3.13×10^7	7.33
80.9	4.79×10^7	6.60
102.2	6.84×10^7	4.48
Workpiece y		
14.5	6.25×10^6	10.53
25.2	1.71×10^7	9.09
35.9	5.68×10^7	4.26
48.1	1.06×10^7	5.56

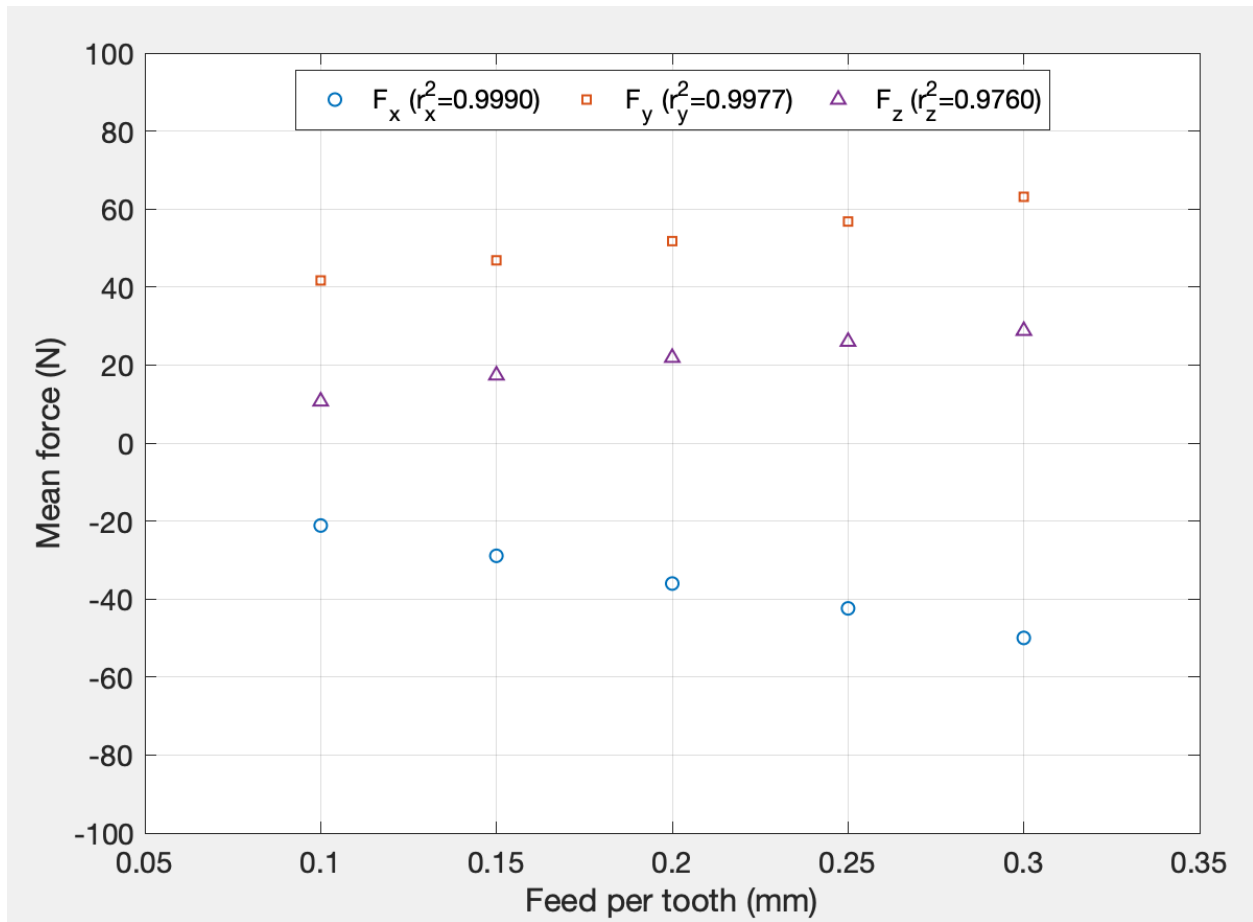


Figure 60: Average forces of cutting tests in x,y, and z directions across f_t range

Stability Analysis

After identifying the dynamic model and the cutting force coefficients for the system the next step was to identify stable cutting conditions. Both the average tooth angle and Fourier series stability maps were used, and the parameters were selected to be stable for both methods. These algorithms have several input parameters and output a stability map across a range of spindle speeds. These input parameters are the tool-tip and workpiece FRFs in the x and y directions, the characteristics of the cutting tool (number of teeth, and diameter), the cutting force coefficients, and the machining type (up or down milling). For these experiments, a three flute, 12.7 mm diameter tool was chosen, for down milling operations with a radial depth of cut of 2 mm. At these conditions the Fourier series approach resulted in the lower limiting axial depths. The stability maps are shown in Figures 61 and 62, for the x and y feed directions respectively.

Machining Parameter Selection

Spindle Speed Selection

When predicting SLE the largest variation in the error occurs when the tooth passing frequency of the tool is near the natural frequency corresponding to the most flexible vibration mode of the system, and integer fractions of these frequencies. For the robot system these natural frequencies are 8, 24, and 5400 Hz. The corresponding tooth passing frequencies to match these natural frequencies are 160, 480, and 108,000 rpm respectively. Interestingly these spindle speeds are well below and well above those traditionally selected for milling.

To select machining parameters that represent traditional robotic milling speeds, it was decided to limit the spindle speed range to be above 5,000 rpm and below 12,000 rpm. Given these spindle speed constraints the large SLE variations due to the low frequency modes are not present because the selected tooth passing frequencies are well above the associated spindle speed range. However, these modes still contribute to the SLE. Because these modes have very low stiffness, they produce a constant offset in machined surface, which results in spindle speed independent SLE. For the high frequency mode the variation in SLE occurs at each integer fraction of the modes natural frequency, but the magnitude of variation decreases as the integer (divisor) increases. For the selected spindle speed range, the integer is 10 and the associated spindle speed range is near 10,800 rpm. The SLE change in this spindle speed range is significantly less than the change when the tooth passing frequency is near the natural frequency, as shown by Figures 63 and 64, for the natural frequency, and tenth subharmonic respectively. These figures show the results for the x direction, but the y direction demonstrates similar behavior. To validate the model, spindle speeds of 10,700 and 10,950 rpm were chosen. These spindle speeds occur near the largest variation in SLE resulting from the tenth subharmonic of the 5,400 Hz mode. In addition to this contribution to the SLE, the low frequency modes cause a large constant offset in SLE. The result is SLE is not that sensitive to spindle speed.

Selecting Axial Depth and Feed Per Tooth

Due to the low stiffness robot modes, the SLE within the selected spindle speed range, is large and has low sensitivity to spindle speed. Because the error is caused by dynamic deflections of the cutting tool due to the cutting force, the SLE should scale linearly with cutting force (within the limits of linear robot dynamics). While keeping the radial depth constant at 2 mm, there are two ways to change the cutting force: 1) axial depth; and 2) feed per tooth. Changing either value should cause a linear change in cutting force and, thus, a linear change in surface location error. To test this, two stable axial depths were chosen, 1.5 mm and 2.5 mm, and two feed per tooth values were chosen, 0.125 mm/tooth and 0.25 mm/tooth. Each combination of these axial depths and feed per tooth values were tested at each of the two spindle speeds for a total of eight tests. The parameters for each test and the square of the workpiece where each test was performed are shown in Table 7.

Robot Path Error Prediction

When the robot is commanded to move along a tool path, there are errors associated with this commanded motion. The robot operates in an open loop mode, where the end effector position is obtained by the commanded angles for each rotary axis. However, there is no feedback about the actual position to change its motion profile. It is possible to record the end effector position data while the robot is in motion, though, to compare the actual robot motion to the commanded robot motion. This information can then be used to inform the SLE prediction, because the actual position of the tool center point serves as the baseline error for the machining process. To obtain this data, the tool paths were executed without cutting and the robot path data was recorded. The deviations from the commanded tool path were extracted and a baseline SLE resulting from the robot motions only was established for each set of machining parameters. These results are shown in Figure 65, where a positive SLE denotes radial depths of cut lower than commanded, or under-cut. In this case, the part is larger than commanded. From the figure it is seen that the robot path errors are dependent on the robot speed, with higher errors at the 0.25 mm/tooth condition when the robot is moving faster, and are the same regardless of the travel direction of the robot. These baseline errors were added to the machining dependent SLE prediction for each set of machining parameters.

SLE Predictions and Compensation

After the cutting parameters were selected (see Table 7, the next step was to predict the SLE for each set of cutting parameters. This was completed using the frequency domain SLE algorithm with the addition of the errors from the robot path. The results are shown in Figures 66 and 67 for the x and y directions, respectively. From these predictions it is seen that there is a clear increase in SLE as the axial depth and feed per tooth are increased, while there is a lower dependence on spindle speed. Also, the y direction SLE predictions are lower than the x direction. This is because the low frequency modes are stiffer in the y direction.

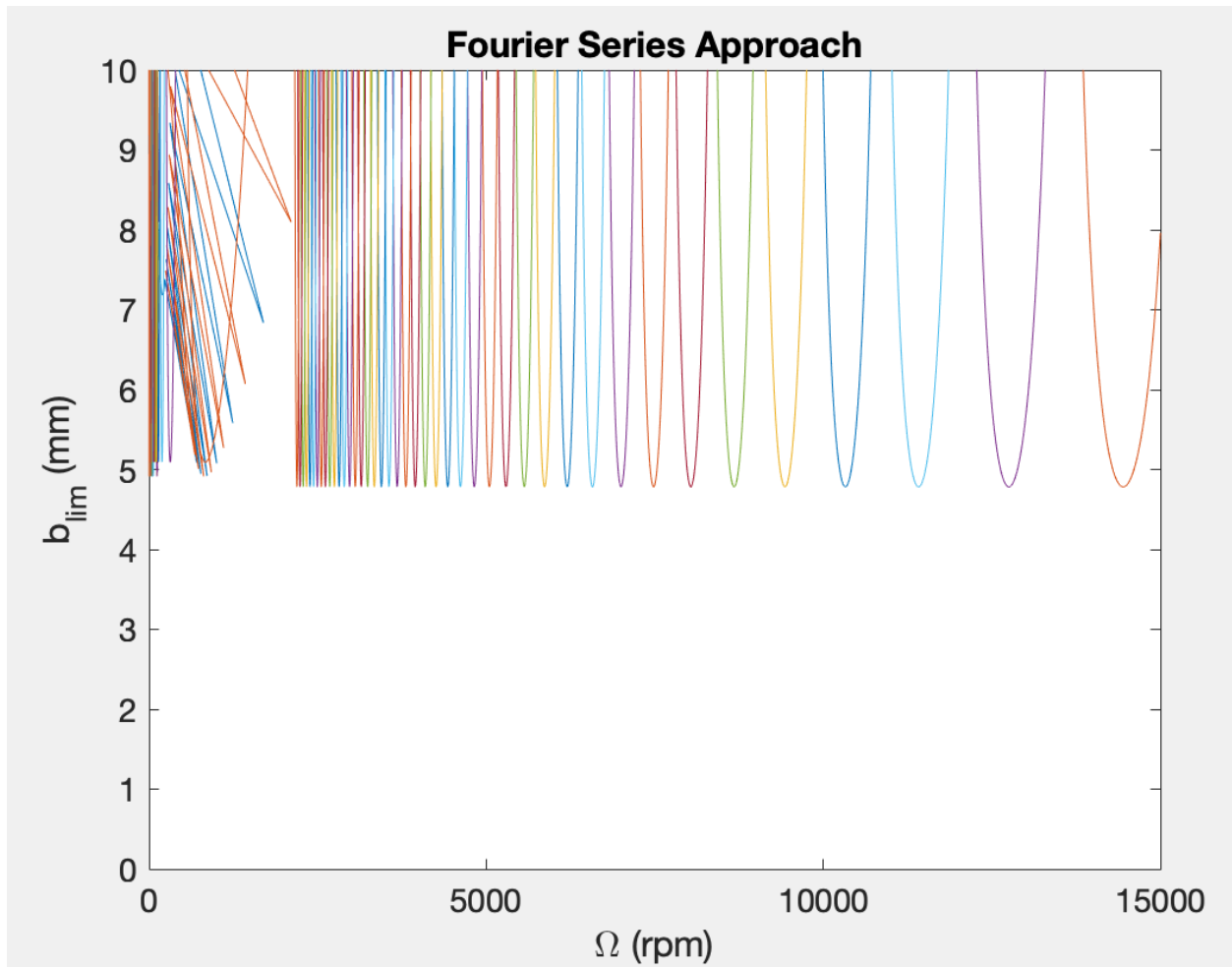


Figure 61: Stability map for x direction feed

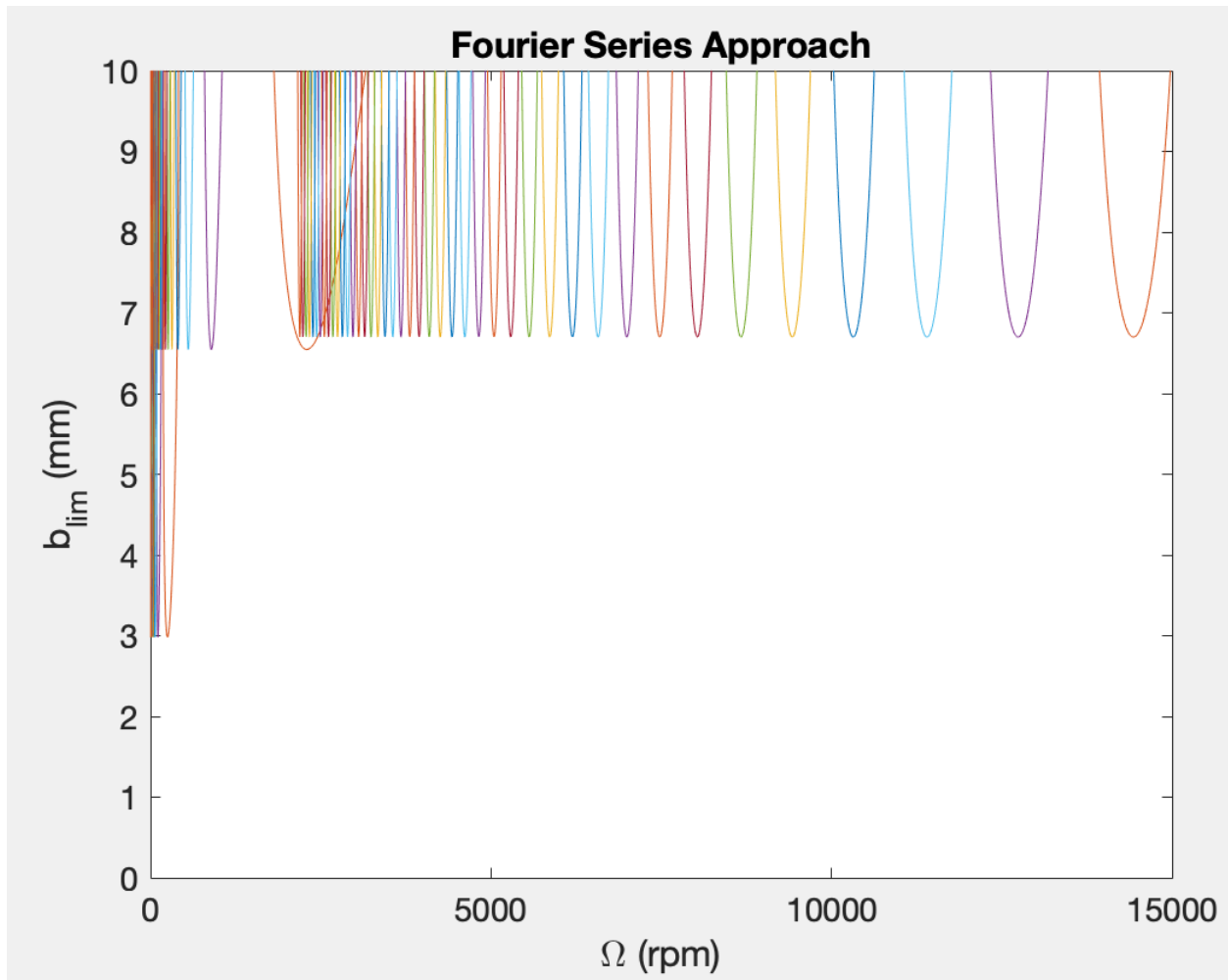


Figure 62: Stability map for y direction feed

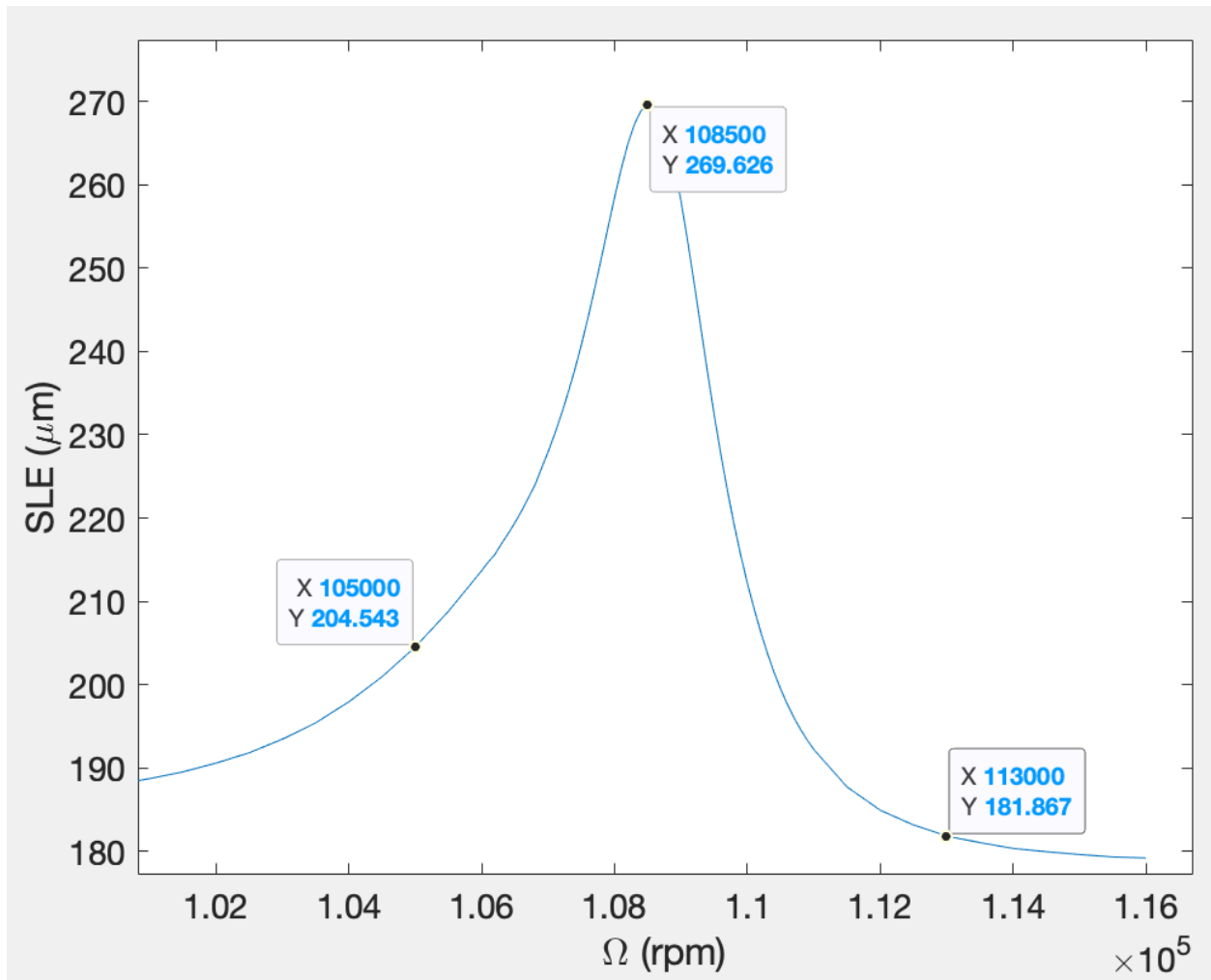


Figure 63: Variation in surface location error near the 5,400 Hz natural frequency

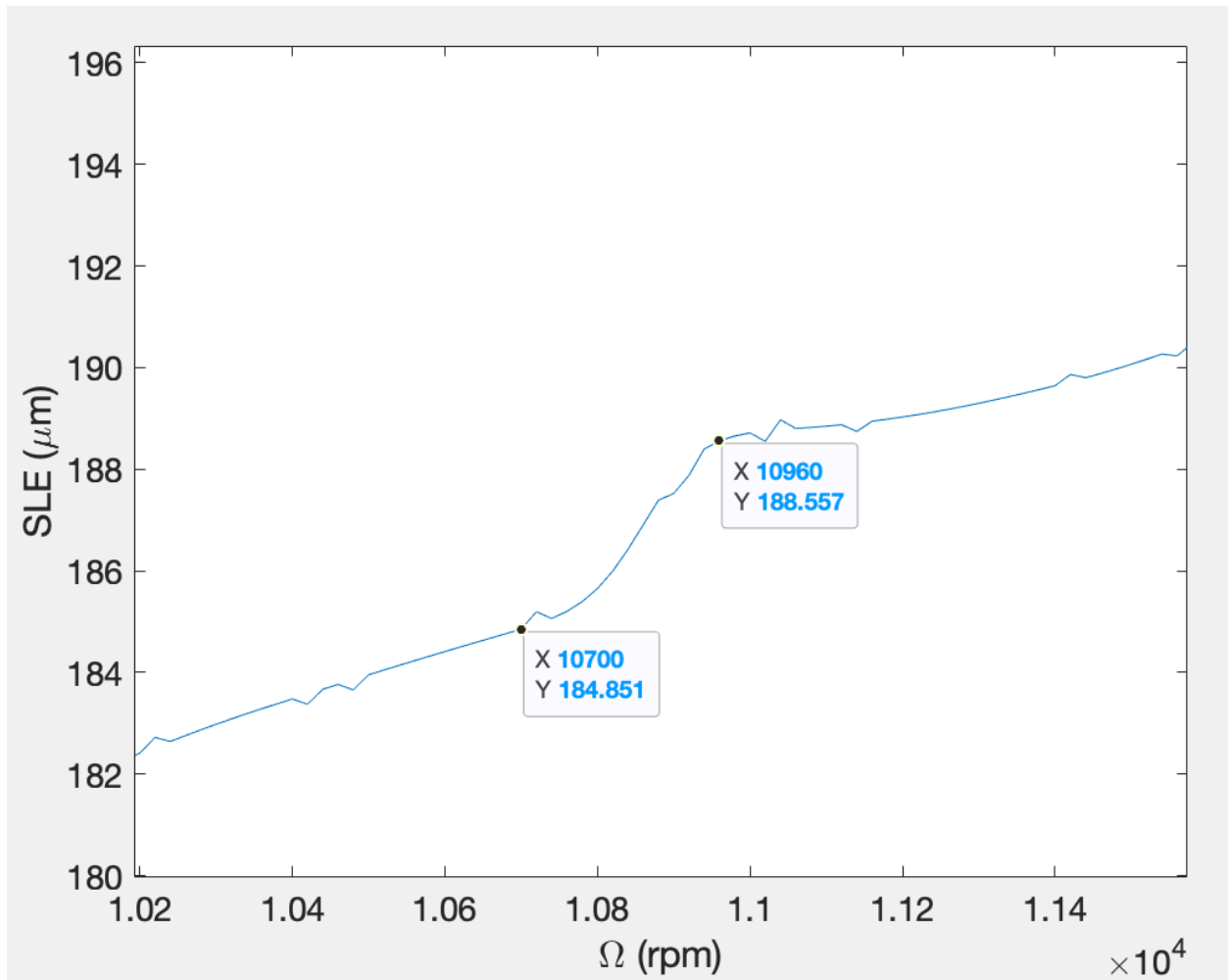


Figure 64: Variation in surface location error near the tenth subharmonic of the 5,400 Hz natural frequency

Table 7: Cutting conditions for each square of the workpiece

Square number	Axial depth (mm)	Feed per tooth (mm/tooth)	Spindle speed (rpm)
2	1.5	0.125	10,700
3	1.5	0.125	10,950
4	1.5	0.25	10,700
9	1.5	0.25	10,950
8	2.5	0.125	10,700
7	2.5	0.125	10,950
6	2.5	0.25	10,700
1	2.5	0.25	10,950

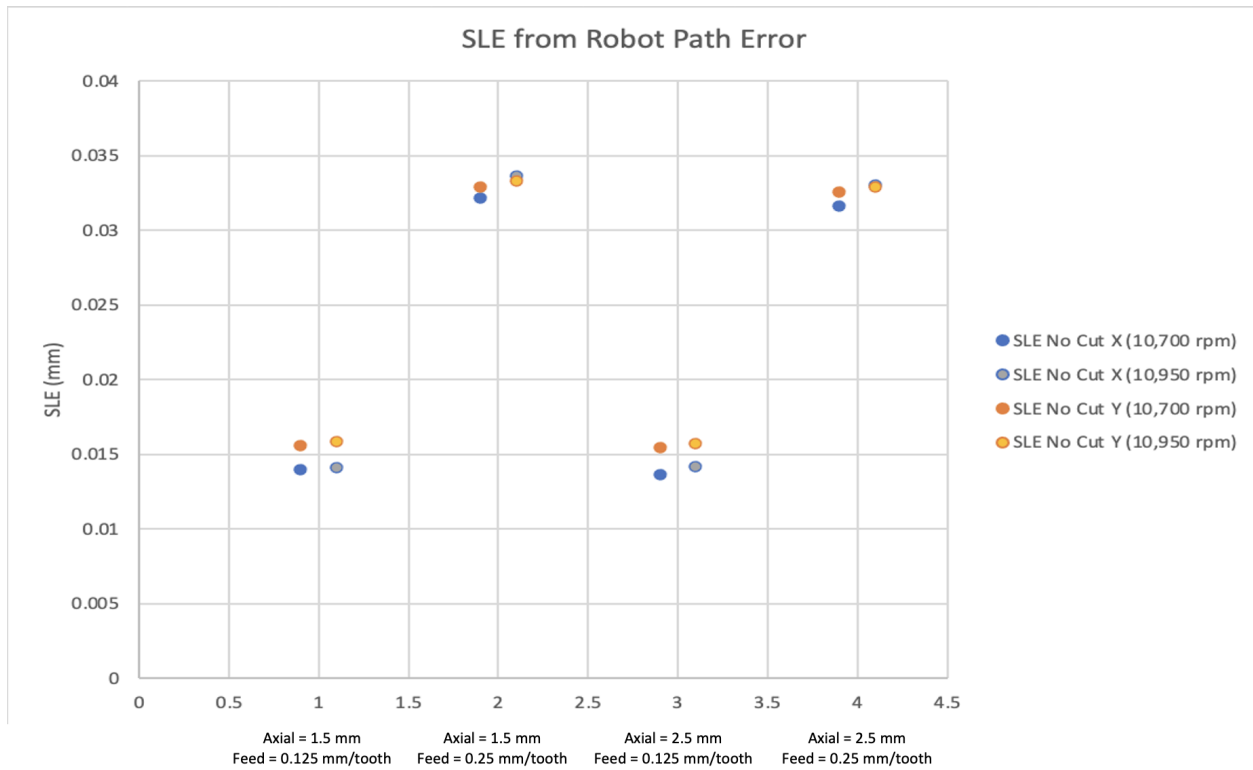


Figure 65: SLE resulting from the errors in the accuracy of the robot paths

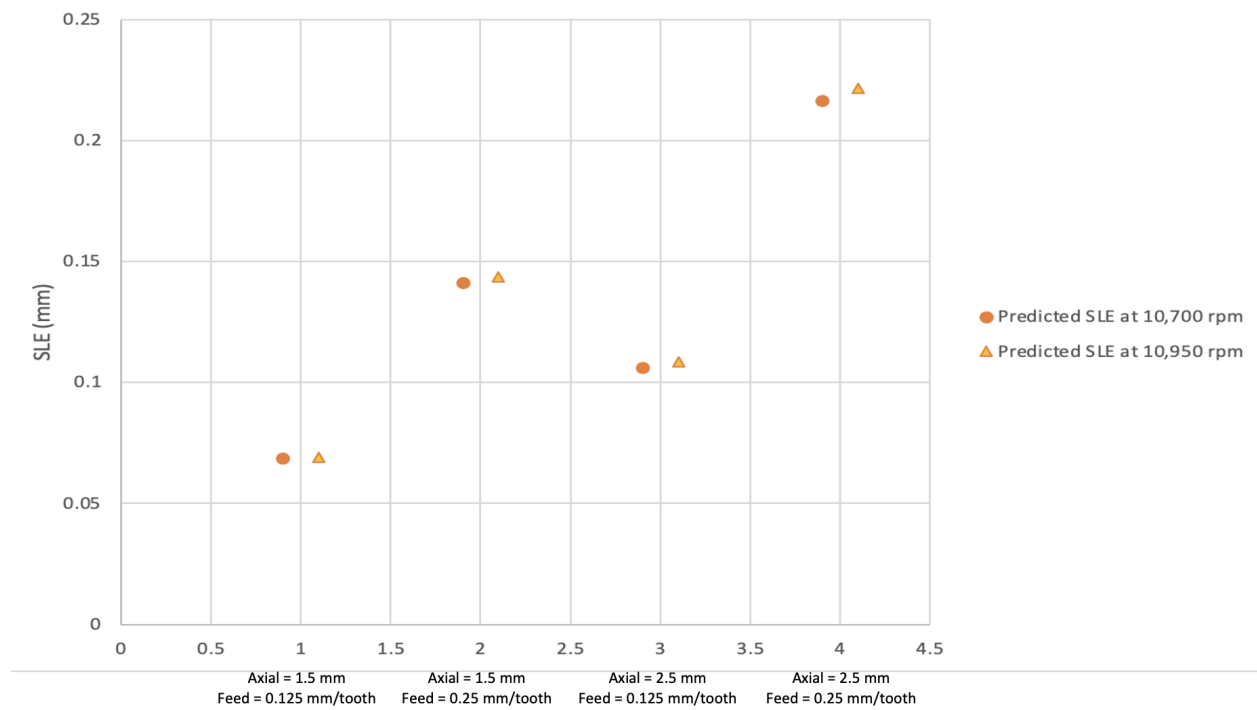


Figure 66: SLE predictions for x axis dimension

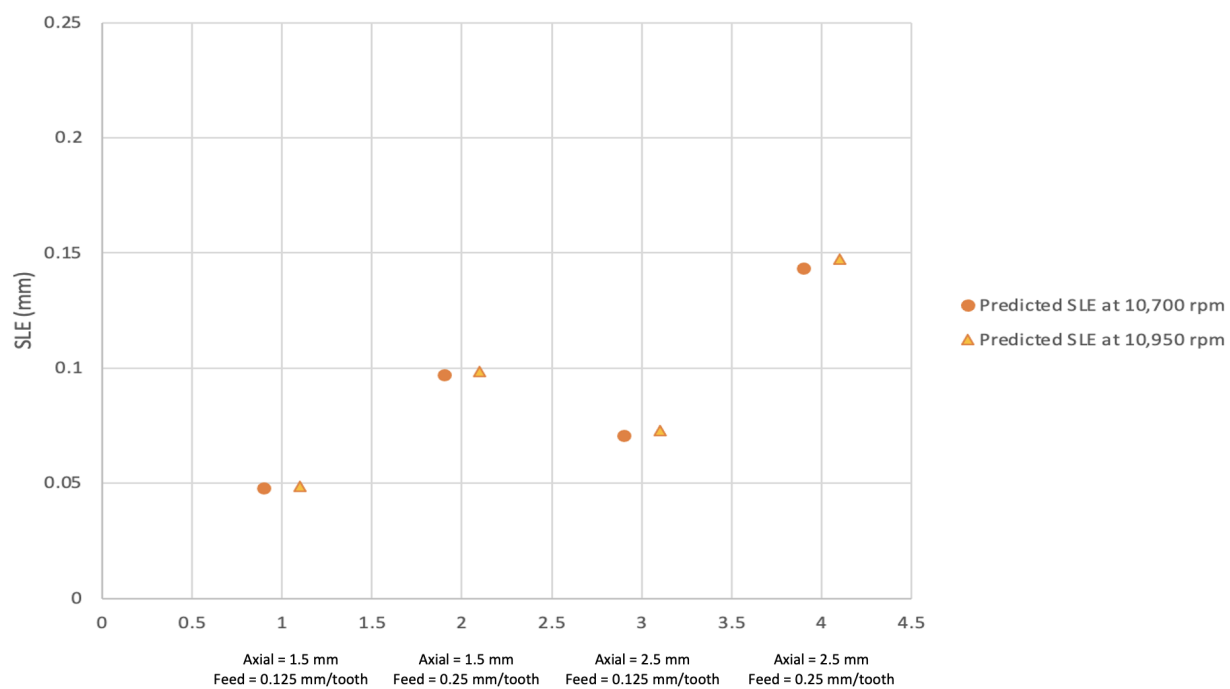


Figure 67: SLE predictions for y axis dimension

Once the errors were predicted, it was possible to compensate for them. This is done by increasing the radial depth to compensate for the undercut resulting from the SLE. The dynamics and, thus, the errors are different in the x and y directions, so each direction must be compensated separately. The first step was to add the initial predicted SLE for each case to the radial depth. This should result in a final tool path that achieves the original desired radial depth of cut. However, at this compensated radial depth, the SLE prediction will increase, because the cutting force is now higher, and there will still be error between the desired surface and the resulting surface. The process was repeated by adding the new error to the radial depth and using it to recalculate the SLE until increasing the radial depth caused a change in SLE of less than $2\text{ }\mu\text{m}$. After this process was completed, the SLE resulting from the robot path errors was added to the radial depths in both the x and y directions. The compensated radial depths are shown in Figures 68 and 69 for the x and y directions, respectively.

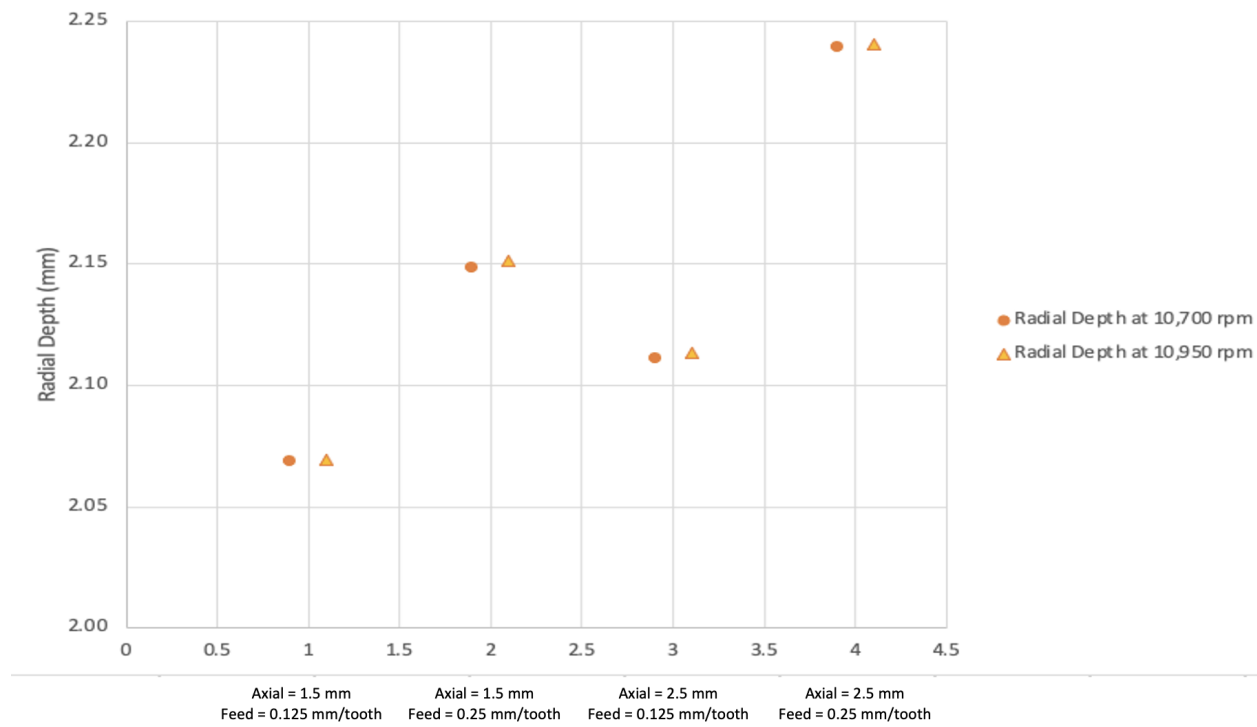


Figure 68: Commanded x direction radial depths to compensate for SLE and robot motion errors

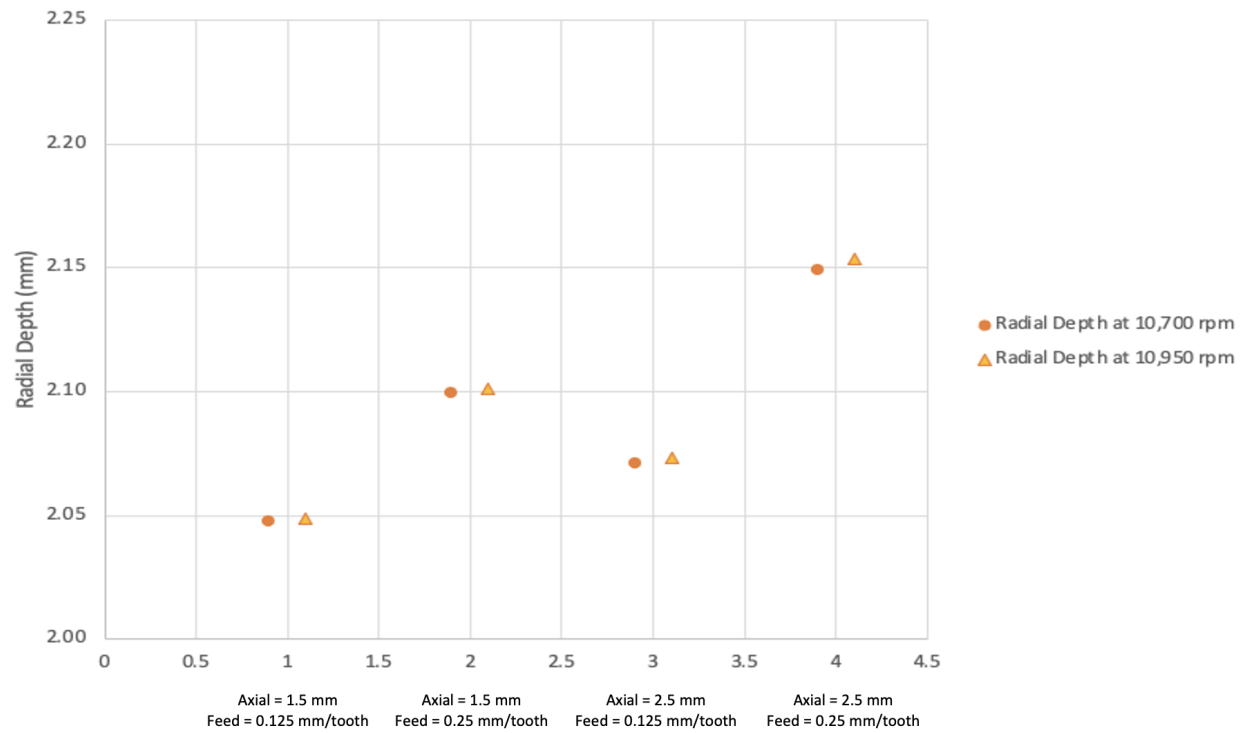


Figure 69: Commanded y direction radial depths to compensate for SLE and robot motion errors

Chapter 8: Conventional Machine Tool Comparison

When evaluating the success of a robotic milling system, its performance relative to a conventional machine tool must be considered. Conventional machine tools offer stiffness and accuracy advantages over robotic milling systems and, as a result, generally produce parts with improved part accuracy and surface finish, at higher material removal rates. For this research a direct comparison of the robotic milling system to a conventional machine tool was desired. For this comparison, the same experiment was conducted on the robot and a Haas VF-4 three-axis CNC machine tool. The same tool geometry, radial and axial depths of cut, spindle speeds, and feed per tooth values were used. The first step was to tap test the tool and workpiece mounted in the machine tool. The results of this tap test in the x and y directions are shown in Figures 70 and 71 in x and y respectively, compared with the tap testing results for the robot. From the figures it is seen that the machine tool has a significantly higher stiffness, and does not have the presence of the low stiffness low frequency modes as seen for the robot. After the dynamic model of the CNC was established, surface location error predictions were made and the cutting tests were performed. It was assumed that the tool-tip dynamics did not change over the size of the machined squares. After the squares were machined, they were measured using the same probing routine as described for the testing and the measured SLE was compared to the predictions. The results are shown in Figures 72 and 73 for the x and y directions, respectively. From the figures it is seen that the errors from the CNC machining process were all less than 10 μm , which is significantly less than the path accuracy of the robot. From this test it was confirmed that the machine tool is able to produce more accurate parts than the robot under the same conditions.

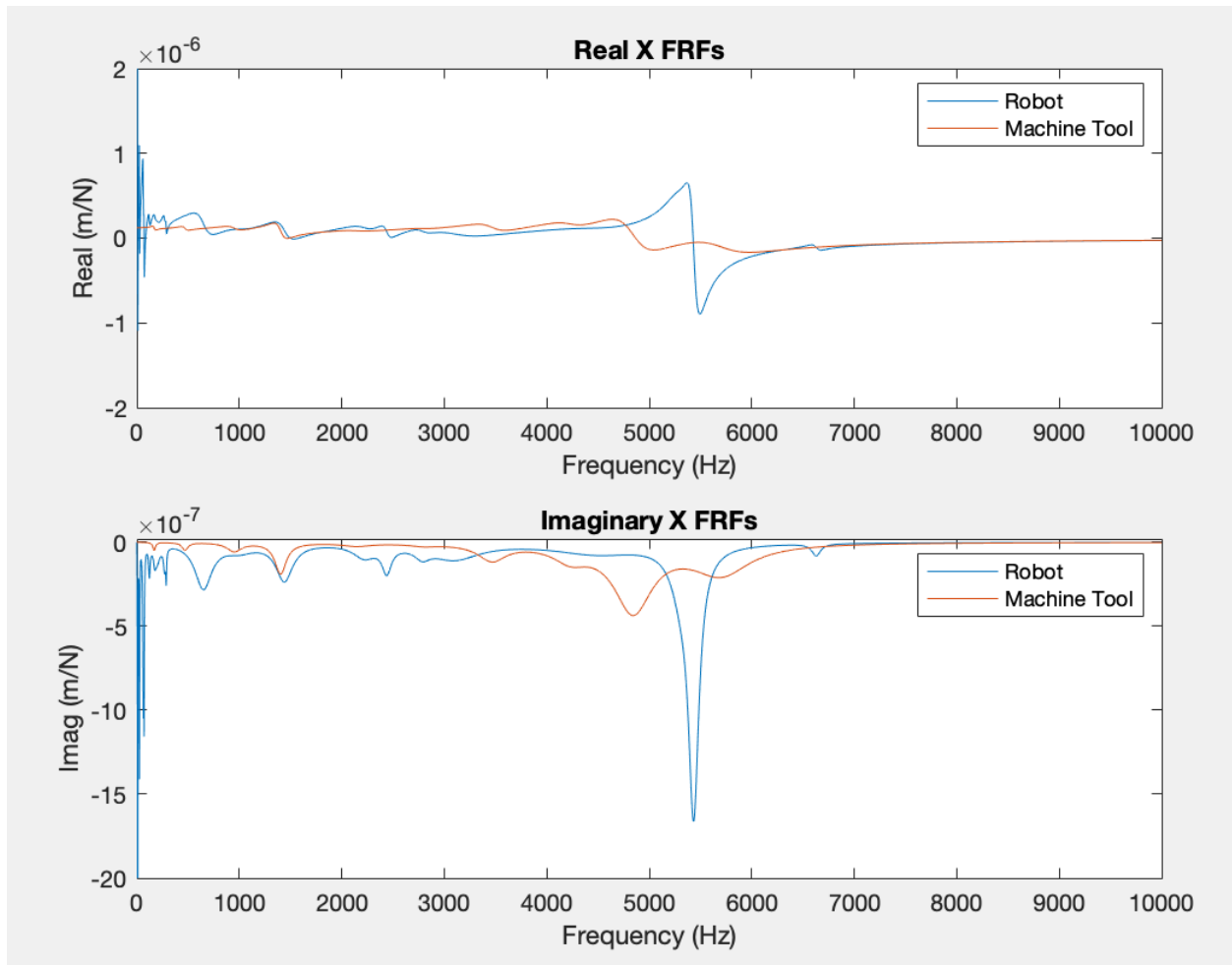


Figure 70: Tap testing results of the CNC machine tool compared with the robot in the x direction

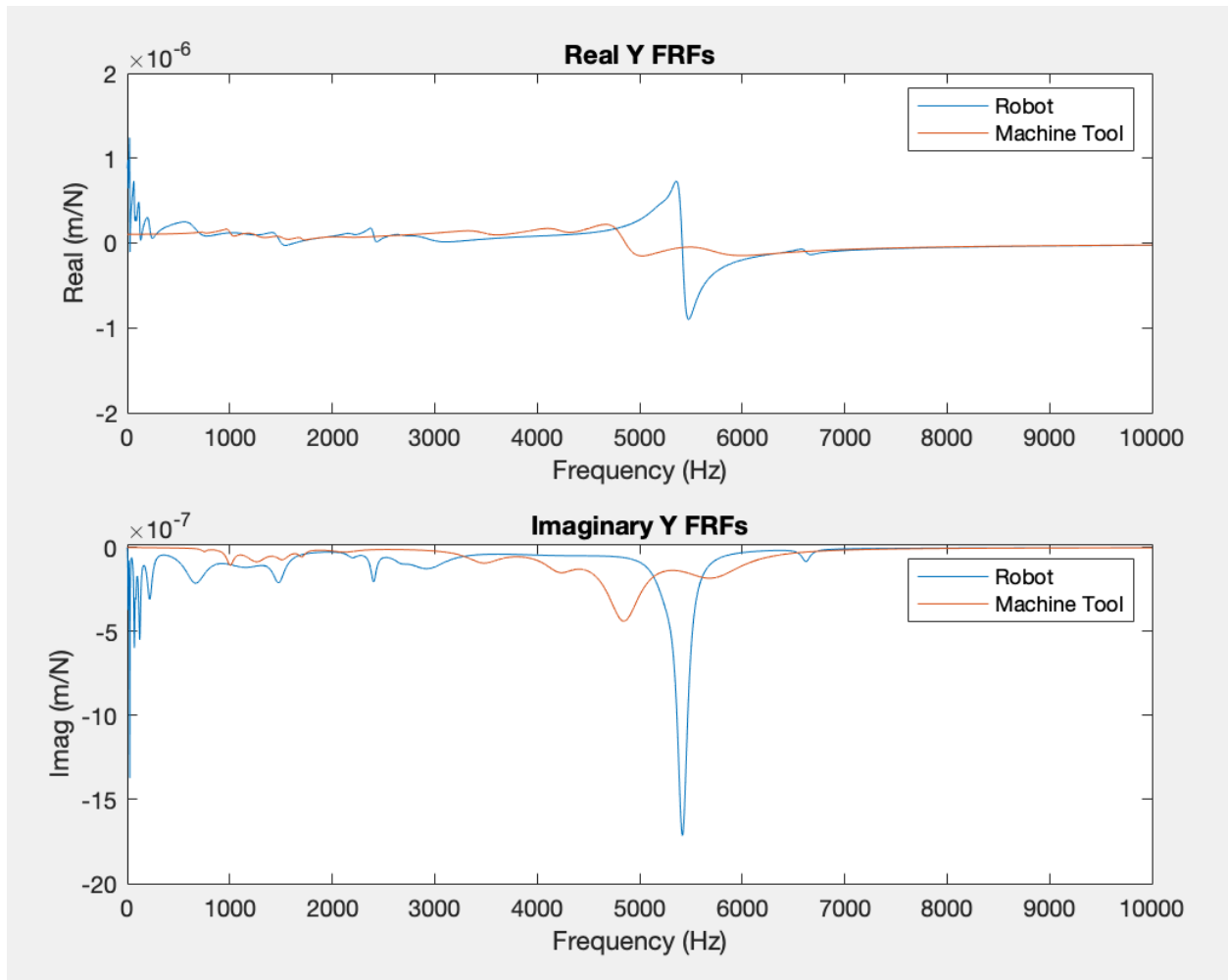


Figure 71: Tap testing results of the CNC machine tool compared with the robot in the y direction

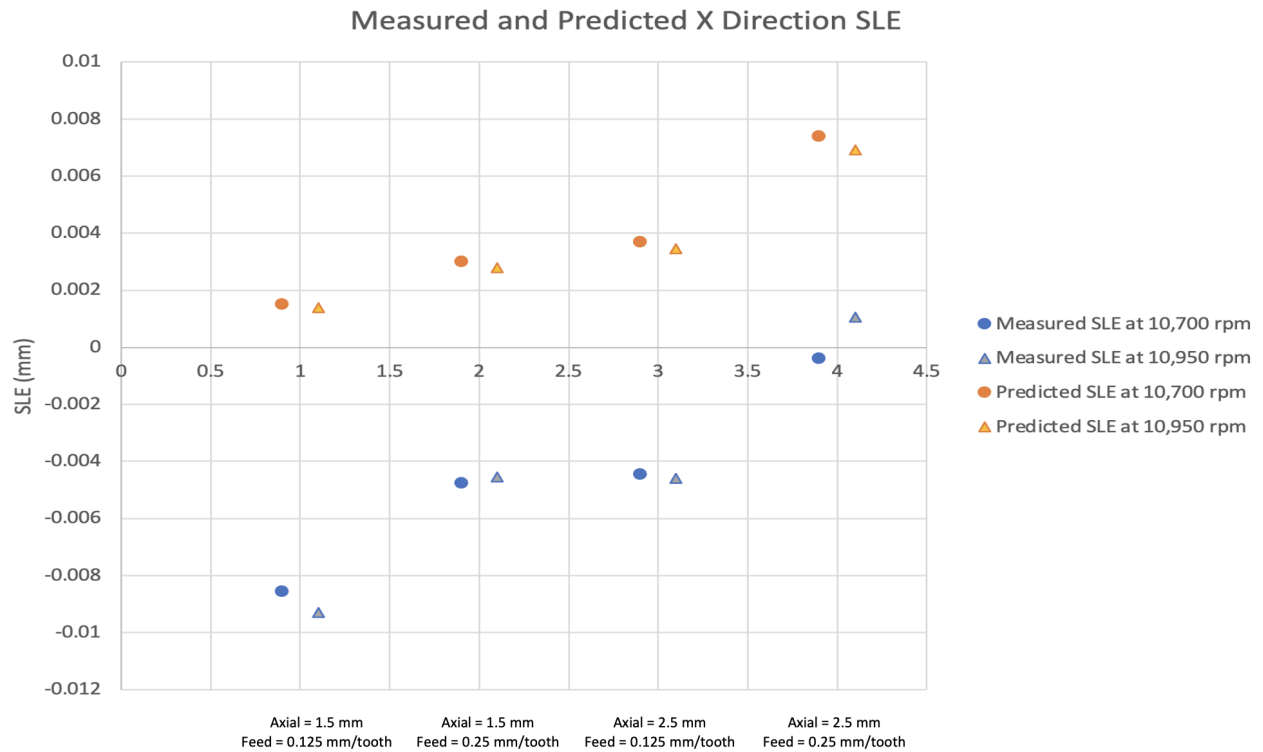


Figure 72: SLE predictions and measured values for the CNC machine tool x direction

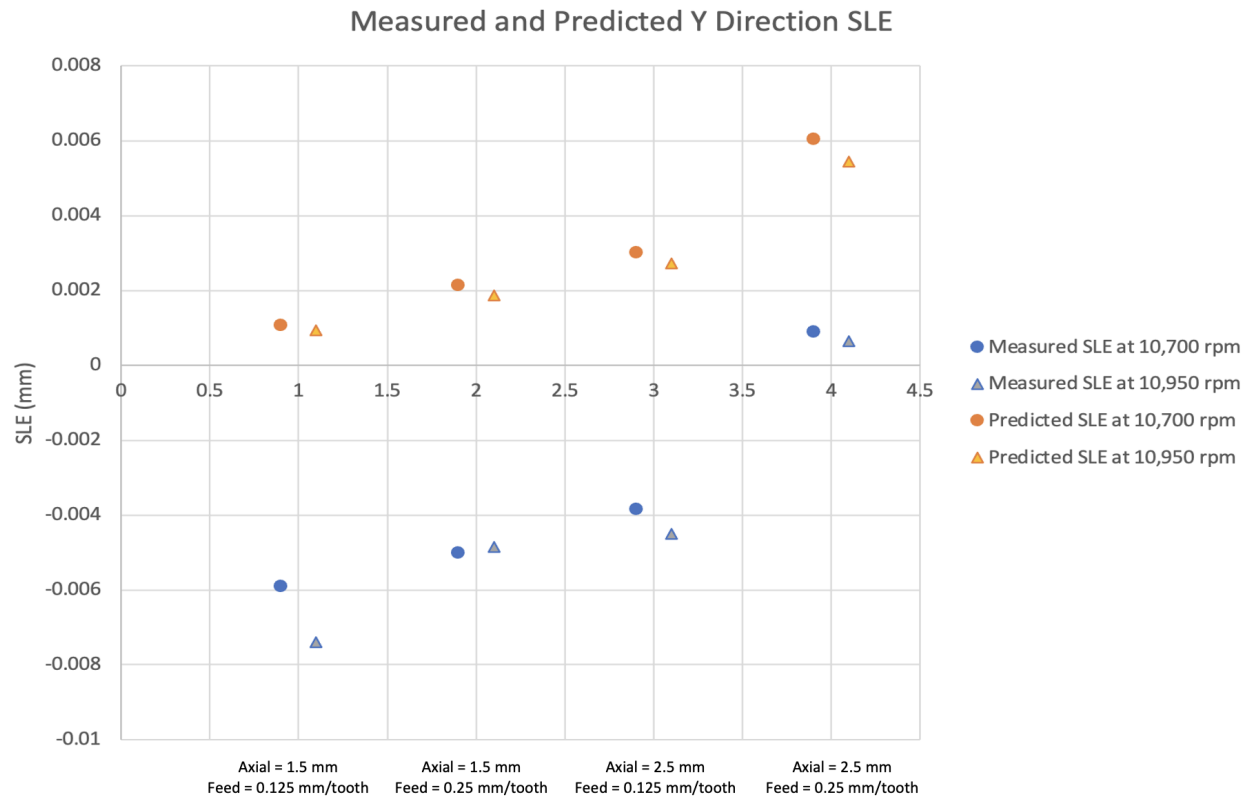


Figure 73: SLE predictions and measured values for the CNC machine tool y direction

Chapter 9: Results

As shown in Table 7 eight sets of machining parameters were tested to include two spindle speeds, 10,700 rpm and 10,950 rpm, two axial depths, 1.5 mm and 2.5 mm, and two feed per tooth values, 0.125 mm/tooth and 0.25 mm/tooth. These tests were repeated three times to establish the repeatability of the process. A comparison between the experiments and predictions are shown in Figures 74 and 75 for the x and y directions, respectively. From the figures it is seen that there was good repeatability in the results from the different tests. Additionally, the results showed good agreement between the prediction and the experimental results, with the x direction showing better agreement than the y direction.

The SLE compensation experiment used the same spindle speeds, axial depths, and feed per tooth values as the initial tests, but increased the radial depth of cut in the x and y directions to achieve a final part with the originally commanded dimensions. These results for the x and y directions is shown in Figure 76, where a zero error value indicates agreement with the commanded dimension, positive errors denoting undercut (larger than commanded), and negative errors denoting overcut (smaller than commanded). The measured error values reported in this section were obtained from the average surface of each of the sides of the part as reported by the CMM. The effects of surface finish on the accuracy of the finished part are not considered.

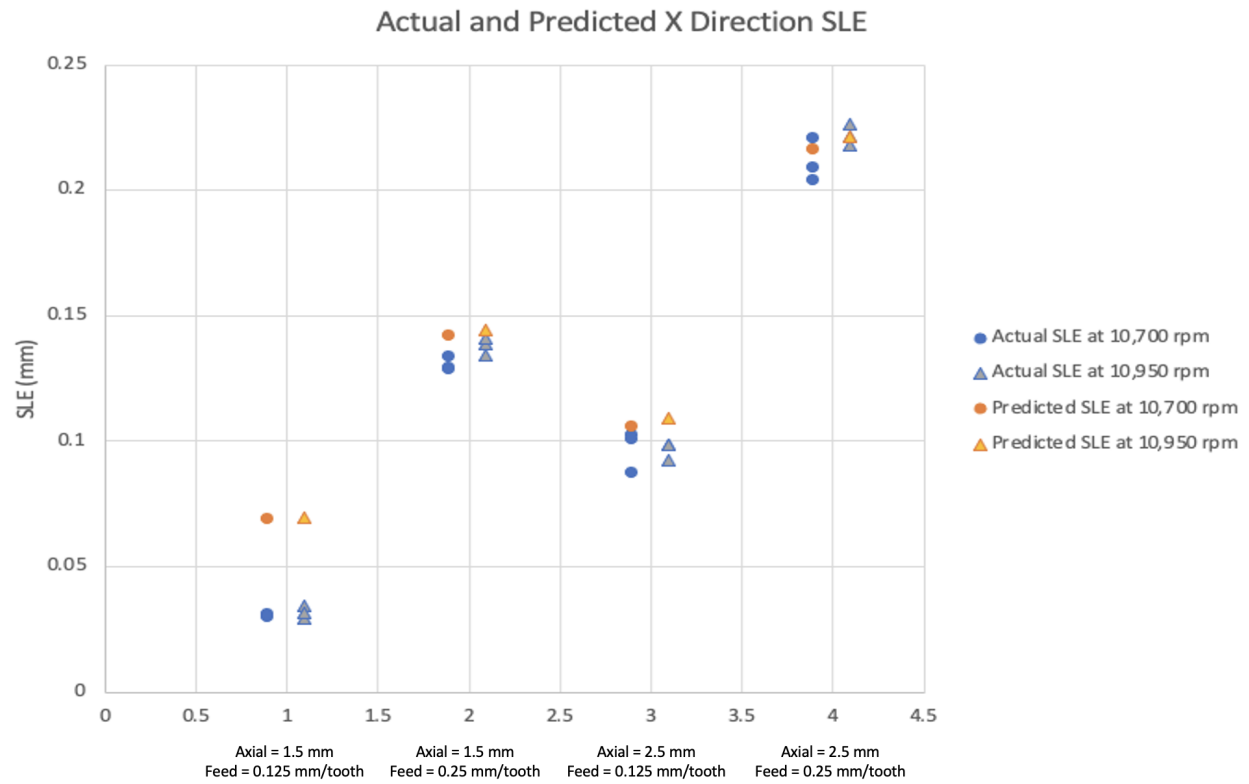


Figure 74: Measured SLE and predicted SLE in the x direction

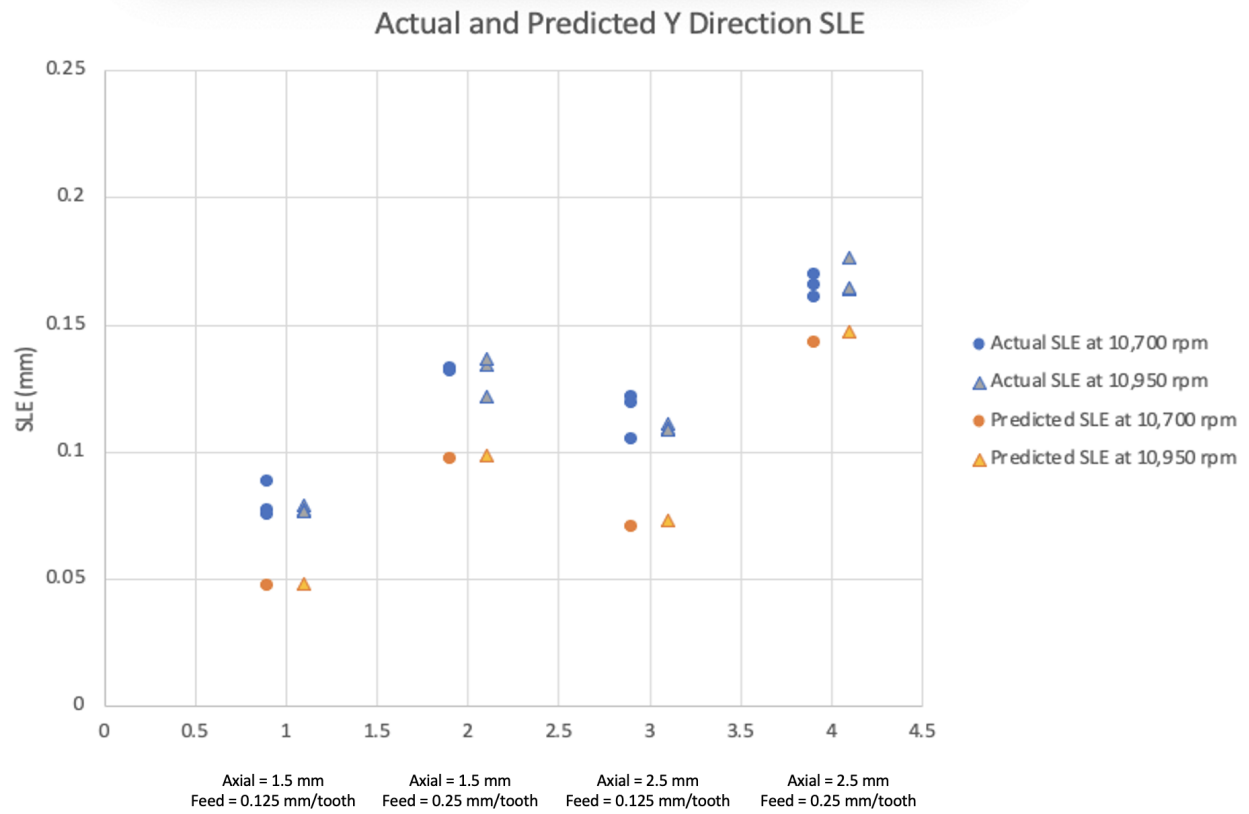


Figure 75: Measured SLE and predicted SLE in the y direction

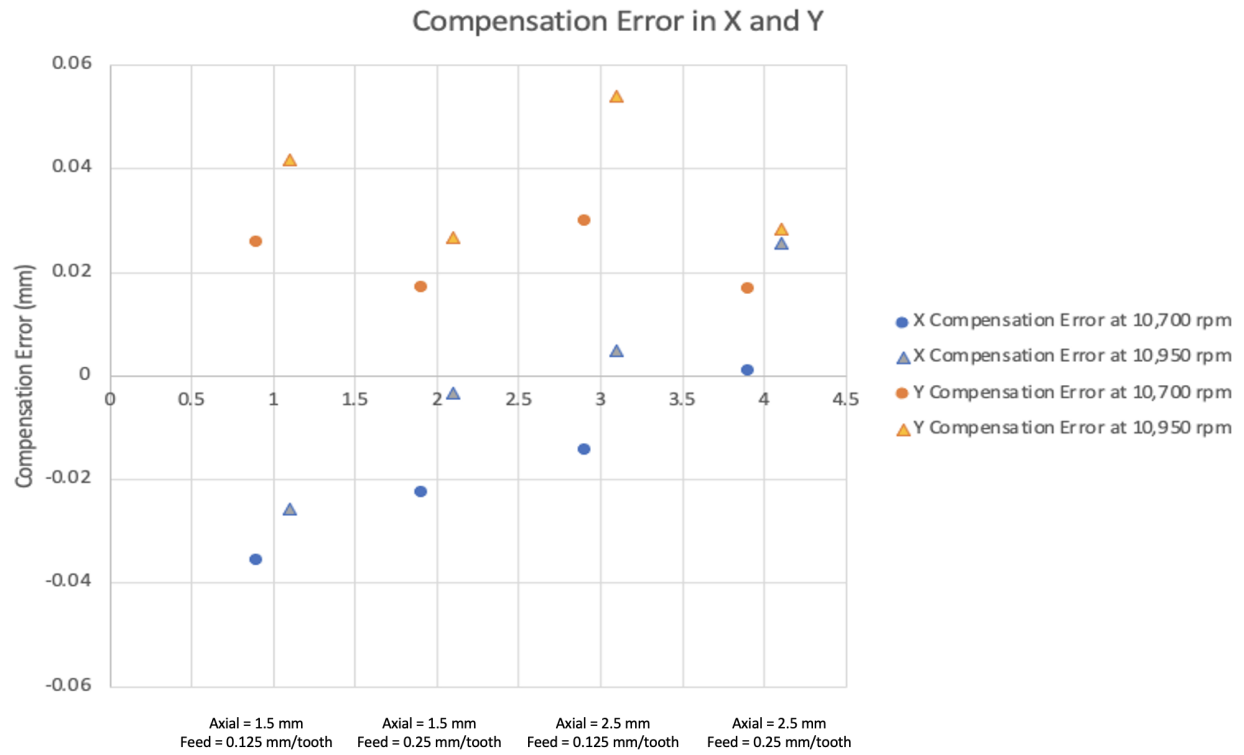


Figure 76: Error in the SLE compensation in the x and y directions

Chapter 10: Conclusions

In this research the dynamic behavior of a milling spindle attached to a six-axis robot was investigated and a machining performance model was developed. It was confirmed that the robotic milling setup had significantly lower stiffness compared to a conventional machine tool, which affects stability and surface location error when machining. The dynamic model developed through this research was used to predict surface location errors at a variety of milling process parameters. The presence of low frequency modes with low stiffness caused significant surface location error, which did not depend on spindle speed. Through experiments it was determined that robot tool paths generated repeatable machining processes and the surface location model was able to accurately predict the resulting final part geometry. The model was then used to compensate for the surface location error by increasing the radial depth. This aspect of the model was also verified experimentally, showing that the model enables significant improvements in robotic milling accuracy.

Chapter 11: Future Work

Because the model developed in this research can be used to accurately predict surface location error in robotic milling, there is opportunity for future work in this area. In this research, only tool paths in the x and y coordinate directions were explored. To take full advantage of the six degrees of freedom the robot possesses, tool paths and robot dynamics in any direction or orientation in 3D space should be explored. Also, from the research it is known that the dynamics of the robot change with position of the tool, and pose of the robot. Developing an algorithm to predict the dynamics of a location and pose within the workspace, based on interpolation of the dynamics of known configurations, would enable more complex tool paths and the creation of larger parts.

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Vita

Buddy Swan was born in Knoxville, Tennessee and has been a volunteer all his life. He has always wanted to attend The University of Tennessee but it wasn't until high school that he figured out what he wanted to study there. While taking several math and science classes in high school he realized engineering might be a good fit for him, but it wasn't until he took some engineering fundamentals and programming classes did he realize that he wanted to become an engineer.

Once in college he selected Mechanical Engineering with a minor in Computer Science. Both fields require strong problem solving skills and this is an area that brings Buddy great enjoyment. Throughout his undergraduate studies, Buddy was debating which major he wanted to pursue, mechanical engineering or computer science. In the end he decided not to choose and instead did both. He completed the mechanical engineering degree first and then went on to pursue a masters degree.

Beginning the Masters degree led him to Dr. Tony Schmitz and Buddy began a project working on robotic milling. This project allowed him to use both his skills as a mechanical engineer in addition to plenty of programming work. Thanks to the guidance of Dr. Schmitz, as well as help from his fellow graduate students, Buddy was able to complete this thesis and make a contribution to the scientific community. After graduation Buddy hopes to get a job with a company working to develop green energy technology, or in some other field where he can use his skills to make a positive impact in the world.