To the Graduate Council:

I am submitting herewith a thesis written by Larry Edward Williams entitled "Measurements of the Electrical Resistivity of Aluminum by Electromagnetic Induction." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Physics.

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MEASUREMENTS OF THE ELECTRICAL RESISTIVITY OF ALUMINUM BY ELECTROMAGNETIC INDUCTION

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Larry Edward Williams

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ABSTRACT

A through transmission apparatus is designed and built to measure resistivities. It is used to measure the resistivity of each of five samples of aluminum 7075 alloy and the result is compared to a previously made calibrated measurement for each sample. Both the magnitude and phase of the EMF induced in the pick-up coil are dependent upon the resistivity. These are compared with computed values determined for several resistivities and linearly interpolated to find two values of the resistivity of each sample--one for the magnitude and one for the phase--for at least three different readings. Also the error analysis is made, including edge effects, and is compared with the errors from the previous measurements.

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INTRODUCTION

The electrical resistivity of a material is usually measured by passing a direct current through a very precisely machined cylindrical sample with enlarged ends, as shown in Figure 1, measuring the potential between two carefully measured points, and applying Ohm's Law. This method gives very accurate measurements,[1] but it requires samples machined to the experimenter's exact specifications, which may not be the material's final form. If the material's resistivity is measured, and the material is then machined, rolled, or otherwise changed in form, the disturbance changes the resistivity--possibly far beyond the error of the measurement. It is clearly preferable to find some way to measure the resistivity of a material already in its final form.

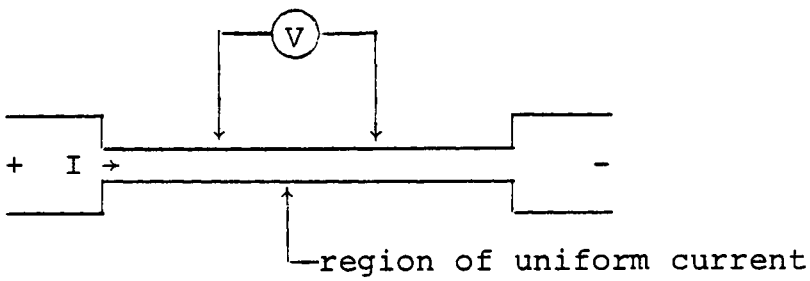


Figure 1. DC resistivity apparatus

Electromagnetic induction is useful in a number of applications becuase the EMF induced in one coil by a

current in another coil is dependent upon many parameters. One of these parameters is the resistivity of any nearby conductor. If all the other parameters are accurately known, and only one conductor is present, then the induced EMF gives the resistivity of that conductor. Such measurements may be made on materials in several forms. If the sample is cylindrical, then two coils coaxial to the sample may be used. For blocks or plates with flat surfaces, two coaxial coils on the same side of the material--called reflection coils--may be used. This thesis focuses on the design and use of a through transmission apparatus--two coaxial coils on opposite sides of a thin plate of material.

Any induction instrument measures the magnitude and phase of an EMF induced in one coil, called the pick-up coil, due to a current in a coaxial coil, called the driver coil. In this case, a thin plate of aluminum 7075 alloy is between the coils. The AC voltage on the driver coil is of constant magnitude and a single frequency. With the coil dimensions and all of the electrical parameters of the coils, the driver source, and the amplifier connected to the pick-up coil precisely measured, the resistivity of the sample is the only variable, and it determines the magnitude and phase of the pick-up voltage. Thus, knowing the relation between the resistivity and and magnitude and phase, we may find the resistivity from

the measured magnitude and from the measured phase independently.

Program MULTRU,[2] used extensively here, is a FORTRAN program designed to simulate the operation of an ideal through transmission instrument. MULTRU has far more capabilities than will be used here, since it was designed for use with multiple layers of conductors, and only one layer is used. The user inputs the physical dimensions and electrical parameters of the coils, the electrical parameters for the driver source and the pick-up amplifier, and the thickness, relative permeability (1.0 for the case of aluminum), and resistivity of the sample. The output includes the magnitude and phase of the output voltage of the pick-up amplifier.

The program MULTRU is used to design a through transmission apparatus to measure resistivities as accurately as possible. The apparatus and the program are then used together to measure the resistivities of several samples of aluminum 7075 alloy. These measurements are then compared with calibrated measurements made previously.

CHAPTER I

THEORY

In a paper by Dodd, Deeds, and Luquire,[3] integral representations for the vector potential produced by a coil carrying a sinusoidal current in the presence of conductors are found for several cases. Once found, the vector potential gives the EMF induced in a coaxial coil through the relation

$$V = j2\pi\omega \int_{\text{coil cross section}} \int rA(r,z)N\,dr\,dz \tag{1}$$

or, for the EMF induced in a coil of rectangular cross section with a constant coil density of N turns per unit area of cross section, as shown in Figure 2,[3]

$$V = \frac{j2\pi\omega n'}{(l_2'-l_1')(r_2'-r_1')} \int_{l_1'}^{l_2'} \int_{r_1'}^{r_2'} rA(r,z)\,dr\,dz \tag{2}$$

where ω is the angular frequency of the sinusoidal current, n' is the total number of turns in the rectangular pick-up coil, and $A(r,z)$ is the spatial part of the vector potential.

For the case of a driver coil above a semi-infinite block of one material with a layer of thickness c of another material as shown in Figure 3,[3] the vector potential in the block is found to be

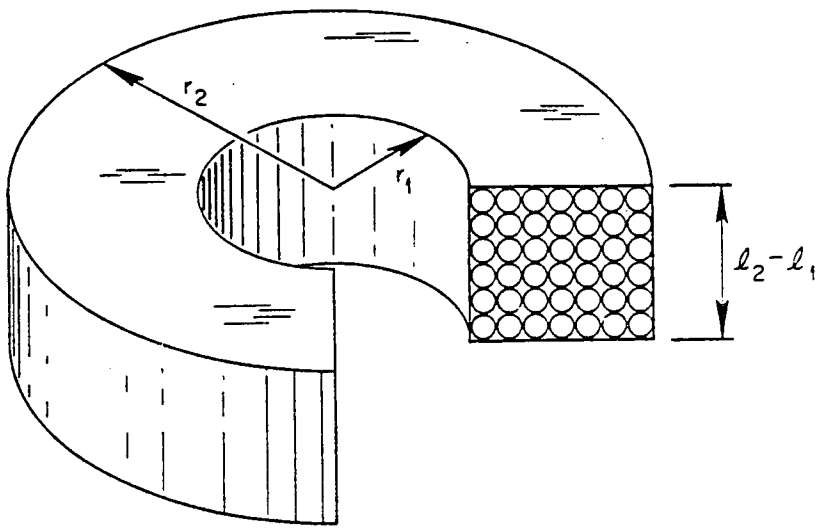


Figure 2. Coil with rectangular cross section.

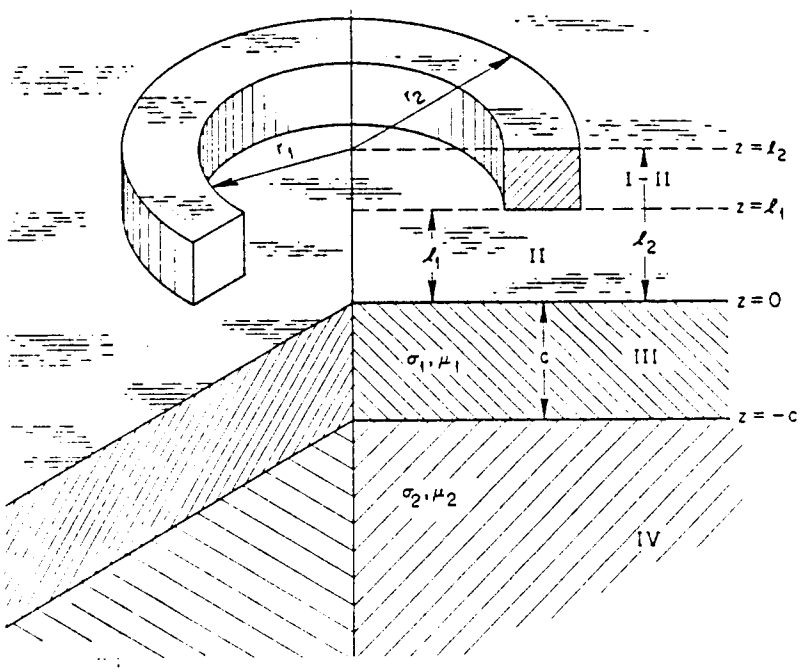


Figure 3. Coil above a semi-infinite block of one material with a layer of another.

$$A(r,z)=\mu IN\int_0^\infty \frac{1}{\alpha^2\alpha_0} J(r_2,r_1)J_1(\alpha r)(e^{-\alpha_0 l_1}-e^{-\alpha_0 l_2})$$

$$X\frac{2\alpha\beta_1 e^{(\alpha_2+\alpha_1)c}\, e^{\alpha_2 z}}{(\alpha_0-\beta_1)(\beta_1-\beta_2)+(\alpha_0+\beta_1)(\beta_1+\beta_2)e^{2\alpha_1 c}} \qquad (3)$$

where $\alpha_0=(\alpha^2-\omega^2\mu_0\varepsilon_0)^{\frac{1}{2}}$

$\alpha_i=(\alpha^2-\omega^2\mu_i\varepsilon_i+j\omega\mu_i\sigma_i)^{\frac{1}{2}}$

$J(r_2,r_1)=\alpha^2\int_{r_1}^{r_2} r_0 J_1(\alpha r_0)dr_0$

$\beta_i=(\mu_0/\mu_i)\alpha_i$

i=1 in the top layer, i=2 in the semi-infinite block

σ_i is the conductivity (inverse of the resistivity, ρ) in the region corresponding to i.

Now, let the semi-infinite block be air ($\sigma_2=0$, $\mu_2=\mu_0$, and $\varepsilon_2=\varepsilon_0$) and put a coil coaxial to the first in that region as shown in Figure 4.[3] Substituting (3) in (2), we have

$$V=\frac{j4\pi\omega nn'I\mu}{(r_2-r_1)(r_2'-r_1')TT'}\int_0^\infty \frac{1}{\alpha^5} J(r_2,r_1)J(r_2',r_1')$$

$$X(1-e^{-\alpha T})(1-e^{-\alpha T'})\frac{e^{-\alpha(l+l')}\beta_1 e^{\alpha_1 c}}{(\alpha+\beta_1)^2 e^{2\alpha_1 c}-(\alpha-\beta_1)^2}\,d\alpha \qquad (4)$$

where $T=l_2-l_1$ and $T'=l_1'-l_2'$, simply the lengths of the driver and pick-up coils, respectively, l and l' are the "lift-offs," or distances from the sample to each coil, and μ with no subscript is μ_1.

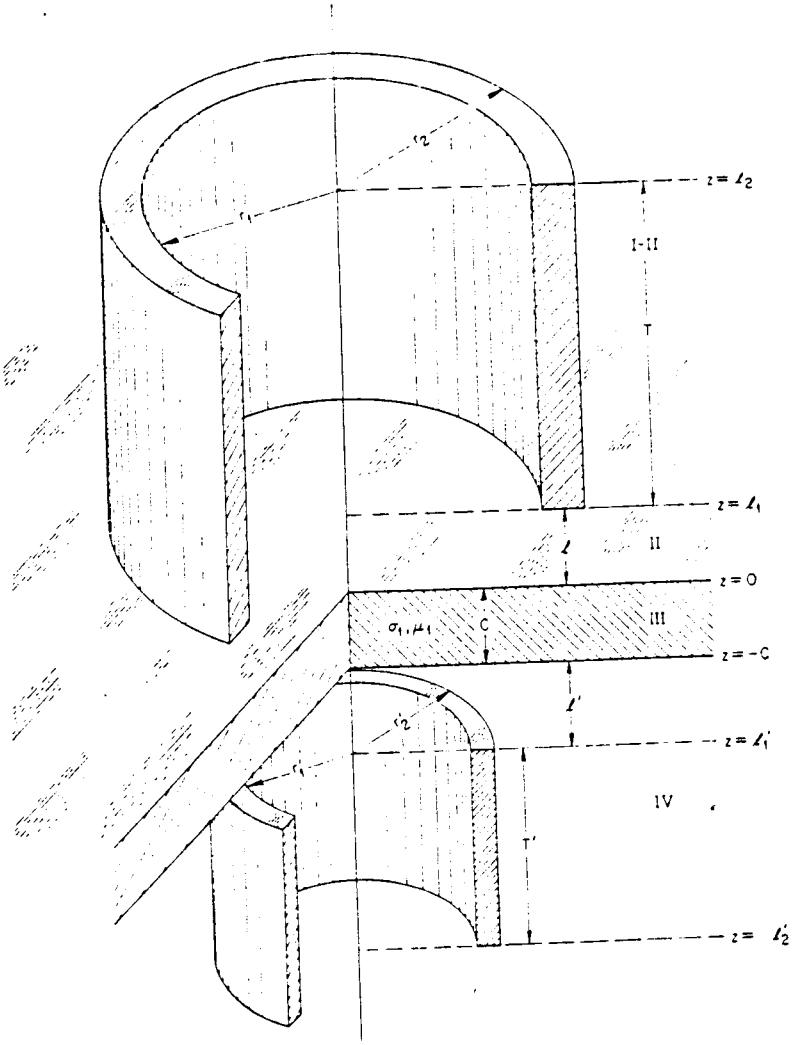


Figure 4. Through transmission apparatus.

Equation (4) may be expressed in any system of units. For convenience, l, r_1, r_2, T, the corresponding primed dimensions, and c are often expressed normalized by dividing by the mean radius of the driver coil, $\bar{r}$. When this is done, α, α_1, and β_1 must be multiplied by $\bar{r}$ to make them dimensionless (they have dimensions of inverse length), and to preserve the form of V. This changes the form of α_1 to

$$\alpha_1=(\alpha^2-\bar{r}^2\omega^2\mu\varepsilon+j\bar{r}^2\omega\mu\sigma)^{\frac{1}{2}}$$

where the subscripts of μ_1, ε_1, and σ_1 have been dropped. The resistivity is to be determined by experiment. The conductivity occurs in terms of $\bar{r}^2\omega\mu\sigma$, which is dimensionless. Thus, in the design phase and in the error analysis, $\bar{r}^2\omega\mu\sigma$ is studied instead of the resistivity.

Because V is complex, evaluating (4) gives the magnitude and the phase of the pick-up voltage. The integrations are performed by MULTRU. This relates V and ϕ (phase) to the resistivity.

CHAPTER II

APPARATUS

The experimental apparatus is basically very simple. It consists of two coaxial coils facing each other, as shown in Figure 4, with the sample plate between them perpendicular to the coil axes. The driver coil is driven by a single frequency sine-wave generator. The pick-up coil is connected to the input of a bandpass amplifier tuned to the same frequency as the driver source, thus eliminating noise and pick-up from other sources. The output of this amplifier is then connected to either an AC voltmeter, or to one input of a phase meter, the other input of which is connected to the driver source. Each coil has the following parameters:

Inner diameter=.296" ± .001"

Outer diameter=.504" ± .004"

Length=.100" ± .001"

DC resistance=5.80 Ω ± .01 Ω

Each coil has 194 turns of #34 copper wire, and an epoxy coating over the face 5.3 to 5.5 mils thick.

The driver coil is driven at about 1 kHz, 3.535 V RMS. The pick-up bandpass amplifier is tuned to 1 kHz. Magnitude measurements are taken from the amplifier output using a Hewlett Packard 3450A Multifunction Meter. Phase

measurements are made with a Tektronix 7904 oscilloscope with a 7D15 225 MHz Universal Counter Timer. The pick-up coil is mounted on a barrel micrometer-driven support. The micrometer gives the total separation of the coils. The pick-up coil is placed in contact with the sample, so the epoxy thickness is taken as the coil-to-conductor spacing. The sample is mounted between the coils on a screw-driven micrometer support. It is important for the center of the sample to be as close as possible to the coil axes to minimize error due to edge effects, since the theory assumes an infinite plate.

CHAPTER III

DESIGN

In the design of the apparatus, dimensionless normalized parameters are used, as mentioned in CHAPTER I. All parameters with units of length (r_1, r_2, l, T, r_1', r_2', l', T', and c) are expressed as $d/\bar{r}$, where d is the dimension being normalized, and $\bar{r}$ is the mean radius of the driver coil. The quantities α, α_1, and β_1 all have dimensions of the reciprocal of length and are multiplied by $\bar{r}$ to make them dimensionless and to preserve the form of equation (4), changing the last term of α_1 and β_1 to $\bar{r}^2\omega\mu\sigma$, which is dimensionless, and may be used instead of ρ for design calculations. We try to maximize the dependence of V and ϕ on $\bar{r}^2\omega\mu\sigma$ and minimize their dependence on the normalized values of the coil dimensions, lift-offs, and the plate thickness. We see immediately that the errors in these normalized values are minimized by maximizing $\bar{r}$ and the unnormalized values of those dimensions, and minimizing the corresponding errors. For example,

$$\Delta(T/\bar{r}) = [(\Delta T/T)^2 + (\Delta\bar{r}/\bar{r})^2]^{\frac{1}{2}} , \qquad (6)$$

which is very small when T and $\bar{r}$ are very large and ΔT and $\Delta\bar{r}$ are very small. When the errors in the parameters themselves are minimized, the error in $\bar{r}^2\omega\mu\sigma$ must also be

minimized. Thus, it is seen that the measurement of $\bar{r}^2_{\omega\mu\sigma}$ will be the most accurately made with coils whose dimensions are large, and of course, precisely known. By computing and plotting $\bar{r}_{\omega\mu\sigma}$ versus $c/\bar{r}$ for several magnitudes and several phases of the pick-up voltage, it is found that minimum sensitivity to the sample's thickness occurs for very thick material, as can be verified by maximizing $\Delta(c/\bar{r})$ as before. It is also found that maximum sensitivity to $\bar{r}_{\omega\mu\sigma}$ occurs for very thick material. The thickness must be limited, however, to that thickness for which the amplified pick-up voltage is much greater than circuit noise (say $|V|/|V|_{air}$ is approximately equal to ½).

In a paper by Smith and Dodd,[4] it is found that flat "pancake" coils, where T is very small, are very sensitive to lift-off. However, equation (4) shows that $V \propto 1/T$, so for a long coil, pick-up voltage magnitude is lost. We therefore compromise by using coils of square cross-section.

It has been mentioned that error is reduced by maximizing $\bar{r}$. This is only true if the sample is large enough to minimize the effects of the edges of the plate. No theory exists yet to calculate edge effects--the theory used assumes an infinite sheet--so we must rely upon empirical measurements to determine the behavior of V and ϕ as the distance from the edge of the sample is varied.

Then, knowing the dimensions of the available samples (which are 2-7/8"x2-7/8"), we may estimate the error due to edge effects and the maximum $\bar{r}$ for which edge effects are within acceptable limits. The measurements of edge effects are found in CHAPTER V. For the purposes of design, it is found that if $\bar{r}$=.200", it is small enough for the edge of the coil to be 13.375$\bar{r}$ from the edges of the sample, for which edge effects should be tolerably small.

CHAPTER IV

DATA ANALYSIS

Program MULTRU predicts that the phase shift in air of the pick-up coil voltage is 90°. This is in agreement with the relation of mutual induction,

$$V=-M\frac{dI}{dt}$$

where M, the mutual inductance, is a real function of geometry only. For a sinusoidal driven current I, the derivative changes the current's phase by 90°.

However, other components in the circuit also change the phase of the output voltage by an amount given by $\tan^{-1}(X/R)$ (X=Im(Z), R=Re(Z), Z=impedance), which is dependent only on the circuit parameters, and ω, which is constant. Therefore, the change in phase through the pick-up circuit (mainly the bandpass amplifier) is constant. We therefore subtract the phase of the pick-up voltage with a sample from that in air, and this is the same difference as that of MULTRU's phase from 90°.

The driving potential, V_0, is not always 3.535V RMS. However, $|V_{PU}| \propto |I_{DR}|$ and $I_{DR}=|V_0|/|Z_{DR}|$, so $|V_{PU}| \propto V_0$. The constant of proportionality between $|V_{PU}|$ and $|V_0|$ is dependent only on circuit parameters, the frequency of the drive source, and the geometry of the system. For each measurement, $|V_{PU}|$ is taken for both air and a

sample, varying nothing else. Therefore, for air $|V_{PU}|/|V_0|=C_1$, and for a sample, $|V_{PU}|/|V_0|= C_2$, where C_1 and C_2 are constants. Thus, $|V_{PU}|_{sample}/|V_{PU}|_{air}=C_3$, a constant independent of V_0. We may thus let the computer give $|V_{PU}|$ for various resistivities within a range about the expected value, then tabulate $|V_{PU}|/|V_{PU}|_{air}$ and interpolate for the resistivity. The same kind of linear interpolations is also used to derive the resistivity from the phase measurements, $\phi-\phi_{air}$.

Magnitude measurements are taken simply by connecting the output of the bandpass amplifier to the Hewlett Packard 3450A Multifunction Meter and reading the RMS voltage. The phase measurements are made with a Tektronix 7904 Oscilloscope with a 7D15 225MHz Universal Counter Timer. The driver source is connected to one input of the Timer, and the output of the bandpass amplifier is connected to another. The timer then gives the time delay between the two inputs. Using this, and the period of the driver source, measured by the same timer, the phase is found since

$$\phi=\frac{\Delta t}{T}X360^\circ . \qquad (7)$$

For both magnitude and phase measurements, at least 3 readings are taken and averaged for each sample and coil separation, and the corresponding resistivities found from MULTRU. The final averages are tabulated in TABLE I.

TABLE I

MEASUREMENTS

c (inches)	$\phi-\phi_{air}$ (degrees)	$\lvert V\rvert/\lvert V\rvert_{air}$	ρ_{ϕ} ($\mu\Omega$-cm)	ρ_{M} ($\mu\Omega$-cm)	Given ρ ($\mu\Omega$-cm)
.17044	78.776	.5124	3.678	3.917	3.7483
.17044	79.147	-----	3.660	-----	3.7483
.17044	78.940	.5068	3.676	3.869	3.7483
.17044	78.372	.5136	3.693	3.902	3.7483
.17044	78.758	.5085	3.685	3.880	3.7483
.17044	78.862	.5085	3.682	3.890	3.7483
.15009	69.568	.5589	3.750	3.890	3.7473
.15009	70.671	.5510	3.717	3.910	3.7473
.15009	69.783	.5582	3.753	3.929	3.7473
.15009	70.700	.5526	3.714	3.930	3.7473
.15009	70.558	.5522	3.718	3.910	3.7473
.15009	70.087	.5563	3.727	3.907	3.7473
.14616	69.100	.5590	3.726	3.910	3.7607
.14616	69.059	.5585	3.735	3.912	3.7607
.14616	69.319	.5587	3.723	3.934	3.7607
.14616	68.964	.5592	3.734	3.906	3.7607
.12587	63.245	.5882	3.674	3.909	3.7527
.12587	63.779	.5821	3.664	3.900	3.7527
.12587	63.778	.5836	3.655	3.903	3.7527
.11569	59.065	-----	3.685	-----	3.7583
.11569	58.907	.6137	3.687	3.870	3.7583
.11569	59.060	.6138	3.690	3.901	3.7583

CHAPTER V

ERROR ANALYSIS

Now we ask how much error we may reasonably expect in our measurements. Resistivity is related to a number of parameters, each of which has some error associated with it. We may write, then,[5]

$$\rho = f(y_1, y_2, \ldots y_n) \tag{8}$$

from which follows

$$\sigma_\rho{}^2 = \sum_i (\partial f/\partial y_i)^2 \sigma_i{}^2 \tag{9}$$

where y_i is any one of the independent parameters, σ_ρ is the standard deviation of the resistivity, and σ_i is the standard deviation of y_i. If we assume that there is no error in any parameter except y_k, then

$$\sigma_{\rho k}{}^2 = (\partial f/\partial y_k)^2 \sigma_k{}^2, \tag{10}$$

where the subscript k of $\sigma_{\rho k}$ indicates that it is the error in the resistivity due to the parameter y_k only. This may be calculated by MULTRU by varying the desired parameter, then using the relation between the resistivity and the phase and magnitude to determine the error in the resistivity. Then the total error may be found by simply summing the calculated $\sigma_{\rho k}{}^2$'s and taking the square root. TABLE II gives the errors in each parameter, and the associated error in the resistivity measured by the magnitude and by the phase.

TABLE II

ERRORS ASSOCIATED WITH CERTAIN PARAMETERS

Parameter	Δ parameter	$\Delta\rho_\phi$ ($\mu\Omega$-cm)	$\Delta\rho_M$ ($\mu\Omega$-cm)
r_1, r_1'	.0005"	.015	.030
r_2, r_2'	.002"	.13	.18
T, T'	.001"	.035	.10
epoxy thickness	.0001"	0	0
DC resistance	1.0 Ω	0	0
gap between pick-up and sample	0	0	0
frequency	.12 Hz	.00045	.000035
Edge effects (Distance of coil axes off center of sample)	.2"	.0041	.019
coil spacing	.002"	.0006	.0285
c			
for .17044"	.000055"	.03535	.02767
for .15009"	.0001134	.08643	.05727
for .14616"	.0001644"	.10604	.08302
for .12587"	.0000846"	.05457	.04272
for .11569"	.0000891"	.05747	.04500
Mag/Mag_{air}	.00064		.00821
$\phi-\phi_{air}$	.032°	.00302	

All errors may be calculated by MULTRU except those due to edge effects, since the theory assumes an infinite sheet. It is necessary to measure empirically the deviation from the theory due to the finite dimensions of the sample, and the deviation from the best possible measurements due to the coil axes not intersecting the sample at its exact center. The latter is found by measuring the magnitude and phase with the coil axes at varying distances from one edge of a sample. At a distance of $4\bar{r}$, we find the resistivity varies, on the average, with a slope of .004134 $\mu\Omega$-cm/$\bar{r}$ when measured by magnitude. Assuming the coils may be a maximum of 0.2", or $1\bar{r}$, off center, the resistivity may vary by .019 $\mu\Omega$-cm for magnitude measurements or .0041 $\mu\Omega$-cm for phase measurements.

For each thickness of aluminum, several resistivity measurements were taken for both the magnitude and phase. This allows us to find an average and standard deviation for each thickness for both magnitude and phase measurement. The standard deviation is translated to a percent error and tabulated in TABLE III. Also tabulated is the percent error of the average measured resistivities with respect to the previous measurements. These data show that the through transmission apparatus is probably useable for measuring resistivities.

TABLE III

AVERAGES AND COMPARISON OF ERRORS

c (inches)	avg. ρ_ϕ ($\mu\Omega$-cm)	Standard Deviation in ρ_ϕ%	% error from prev.	expected % error from phase	avg. ρ_M ($\mu\Omega$-cm)	Standard Deviation in ρ_M%	% error from prev.	expected % error from magnitude
.17044	3.679	.29	1.847	5.21	3.892	.51	3.687	7.95
.15009	3.730	.46	.471	5.62	3.914	.33	4.267	8.06
.14616	3.729	.16	.842	5.83	3.915	.34	3.953	8.19
.12587	3.664	.27	2.415	5.34	3.904	.12	3.882	7.99
.11569	3.686	.10	1.973	5.31	3.894	.55	3.473	7.97

CHAPTER VI

SUMMARY

The standard deviations and the percent errors from the previous measurements are seen from the tables to be well within the estimated errors. Thus, the through transmission apparatus gives accurate measurements of resistivity. In fact, the measurements are more accurate than expected.

The largest contributions to the estimated errors are made by the outer radii of the coils. Error could be reduced by measuring these more accurately than they have been. It would also be useful to have a theoretical means of estimating the edge effects. Some possible sources of error are not taken into account because there are no ways to measure them with the available equipment. Among these are the alignment of the coil axes with each other and the alignment of the sample perpendicular to the coil axes. These alignments are made as well as possible by sight, and the coils and sample are held very securely in place. Therefore, errors due to alignment are minimized as well as they can be, and are assumed negligible. Although some improvements could be made, through transmission measurements of resistivity are promising.

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APPENDIX

LIST OF SYMBOLS

Symbol	Name	Units
$A(r,z)$	Spatial component of Vector Potential	$\frac{\text{weber}}{\text{meter}}$
c	Thickness of Sample	meter
I or I_{DR}	Applied Current	ampere
$J(r_2,r_1)$	$\alpha^2 \int_{r_1}^{r_2} r_0 J_1(\alpha r_0) dr_0$	
l_2	Distance from metal to top of driver coil	meter
l_1	Distance from metal to bottom of driver coil--lift-off	meter
N	Turns per unit area	$\frac{\text{turns}}{\text{meter}^2}$
n	Number of turns	turn
r_1	Driver coil inner radius	meter
r_2	Driver coil outer radius	meter
$\bar{r}$	Mean driver coil radius	meter
Δt	Time difference between driver and pick-up	second
T	period or coil length	second meter
V or V_{PU}	Induced EMF	volt
V_0	Driver input voltage	volt
Z	Impedance	ohm
α	Separation constant	meter^{-1}

Symbol	Name	Units
α_0	$(\alpha^2-\omega^2\mu_0\varepsilon_0)^{\frac{1}{2}}$	meter^{-1}
ϕ	Phase	degrees
ε	Dielectric Constant	$\frac{\text{farad}}{\text{meter}}$
μ	Permeability	$\frac{\text{henry}}{\text{meter}}$
σ	Conductivity	$\frac{\text{mho}}{\text{meter}}$
ρ	Resistivity	ohm-meter
ρ_ϕ	Resistivity taken from phase	ohm-meter
ρ_M	Resistivity taken from magnitude	ohm-meter
ω	Angular Frequency	$\frac{\text{radian}}{\text{second}}$
Symbol′	Symbol pertaining to pick-up coil	

VITA

Larry Edward Williams was born in Knoxville, Tennessee on November 26, 1955. He attended Sterchi Elementary School and Central High School in that city, graduating in June of 1973. That month he entered the University of Tennessee, Knoxville, and in June 1977 he received a Bachelor of Science degree in Engineering Physics with Highest Honors.

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