Development of an Optimal Replenishment Policy for Human Capital Inventory

A Dissertation Presented for

the Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

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May 2011

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Dedication

This dissertation is dedicated in memory of my loving father and mother, Kenneth and Anne Holt, whose unwavering support and words of encouragement throughout my lifetime has given me the strength to continue my academic endeavors during the last 40 years.

This work is also dedicated to my loving wife, Carol, and my daughter, Jordan, who have been pillars of support for me and have made personal sacrifices for me to achieve my academic goals.

Acknowledgements

I would like to express my deepest gratitude to my graduate advisor, Dr. Denise F. Jackson, for her guidance, caring and patience during the course of my academic studies, research and dissertation. Without her inspiration and encouragement, the achievements of this research and the completion of this dissertation would not have been possible.

I would also like to thank my graduate committee, Dr. Charles H. Aikens, Dr. Xueping Li and Dr. Charles E. Noon, for their time and participation in this academic endeavor, both in the classroom and participating in my dissertation proposal reviews and finally my dissertation defense.

Abstract

A unique approach is developed for evaluating Human Capital (workforce) requirements. With this approach, new ways of measuring personnel availability are proposed and available to ensure that an organization remains ready to provide timely, relevant, and accurate products and services in support of its strategic objectives over its planning horizon. The development of this analysis and methodology was established as an alternative approach to existing studies for determining appropriate hiring and attrition rates and to maintain appropriate personnel levels of effectiveness to support existing and future missions.

The contribution of this research is a prescribed method for the strategic analyst to incorporate a personnel and cost simulation model within the framework of Human Resources Human Capital forecasting which can be used to project personnel requirements and evaluate workforce sustainment, at least cost, through time. This will allow various personnel managers to evaluate multiple resource strategies, present and future, maintaining near “perfect” hiring and attrition policies to support its future Human Capital assets.

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Chapter 1 - Introduction

In order for organizations to meet their goals, they must begin thoughtful and deliberate planning for the talent needed to enable them to do so. The retirements of Baby Boomers and changing demographics are creating skill gaps and, potentially, a workforce supply-demand crisis. The risk (probability of occurrence and the impact of an occurrence) of such a crisis may differ across industries, but the consequences of not being able to fulfill the organizational mission indicate a questionable future for most. Hence organizations in both the private and public sectors are realizing that there is an immediate and critical need for improving the management of their workforce – not just in terms of the quantity of people, but also the quality. This, then, calls for a new perspective in managing the workforce. It calls for the recognition that the workforce is a special type of working capital – Human Capital. In fact, according to Spirgi (Spirigi, 2009) "As the manufacturing economy continues to give way to the knowledge economy, talent—rather than capital (hard goods) and headcount (bodies)—has become the essential asset of ***every*** organization."

The term "Human Capital" is being used here to mean the stock of ability, knowledge and skills that reflects the accumulated value of information and other forms of education possessed by an employee (www.economics.noaa.gov/). It is the "value" of what the individual brings to the job that sustains competitiveness in the organization and it is the Human Capital that favorably impacts performance and productivity. Hence, it is imperative that an organization have available the necessary type and amount of knowledge/skills/abilities to do the

work required. This requires a strategic approach to a strategic problem. It requires planning and managing to ensure its success.

Strategic Workforce Planning requires an ongoing analysis in order to evaluate future Human Capital needs and also for providing a path to achieve optimal placement of personnel. In addition to identifying the proper size, location, and specialties of Human Capital, the real underlying challenge is to minimize personnel costs while maintaining balanced levels of Human Capital despite the constantly changing personnel environment.

Traditionally, the workforce has been thought of in terms of the physical sum of people employed. However, the Human Capital investment is defined as the total amount and quality of talent, knowledge, expertise, and training that these workers possess. As executives and investors begin to view the workforce in these terms instead of as a general cost of business, they are beginning to realize the opportunities exist for capitalizing on this asset. Resultantly, the necessity to determine the current value of Human Capital and achieve workforce optimization is increasing (Barrette, 2004). Recognizing that this is not just an "HR" issue, organizations are instituting workforce planning.

Many organizations perform very specialized work that requires highly qualified individuals who not only have critical levels of education, but extensive experience for which there is no substitute. Today's commercial-off-the-shelf Enterprise Resource Planning (ERP) software packages, such as SAS, PEOPLESOFT or SYTELINE, provide Human Resource Management System (HRMS) capabilities. However, these systems fall short in analyzing the present and future workforce levels and associated capital expenditures as they change through time. Workforce levels constantly change due to changes in promotions, retirements, and other

departures. Without a system to provide real-time forecasts, organizations may not respond quickly enough to changes so they may avert a workforce crisis.

What is needed, then, is a system that incorporates quantitative techniques to develop forecasts that may be used to propose various solution sets and management alternatives to the decision-maker by integrating system dynamics and simulation. The result is a more suitable methodology to adequately predict and control a proper balance between competing future requirements and identifying personnel and monetary shortfalls. This assists in planning and support of the personnel base and all of the assets needed for future operations.

Workforce planning involves the translation of business strategy into Human Capital needs. It is a systematic process that includes:

- the analysis of an organization's current workforce situation relative to the requirements of meeting its strategic goals
- the assessment of its current people practices
- identification of risks in getting and keeping critical talent
- a recommendation of solutions to close the gaps and to protect its intellectual capital
- prioritization of solutions in terms of economic return.

These activities are needed to ensure an available pool is on hand when needed and to optimize Human Capital investments. To keep the organization "right-sized", i.e., minimizing peaks and valleys, an organization must have the ability to manage the inventory of this capital asset. Critical to performance improvements is minimizing negative consequences due to

shortages in number or inadequacies in type and mix. Hence, the management of the Human Capital must include:

- A classification system that is aligned with work requirements;
- A retainment and replenishment strategy; and
- A mechanism to measure and minimize the costs of Human Capital inventories.

This is not different from the management of any other asset, and yet the asset is radically different from any other asset. In general, an asset is a physical entity used in the production of wealth. So is the worker. The worker not only brings him/herself to the workplace, but also its experience, skills, and knowledge. Managing capital goods inventory involves specifying the quantity, size and placement of stocked goods to protect against the random disturbance of running out of them. This requires the development of policies based on replenishment lead time, carrying costs, asset care, inventory demand forecasting, inventory valuation, future inventory price forecasting, availability of inventory, quality management, replenishment, and handling of returns and defective goods. Balancing these competing requirements leads to optimal inventory levels, which is an on-going process as the business needs shift and react to external influences, especially budgetary. This includes the monitoring of movement into and out of storage locations and the reconciling of the inventory balances.

Managing human inventories also has as its primary objective the task of determining and controlling stock levels to balance the need for meeting the demand while minimizing holding and handling costs. This too requires the development of policies based on recruitment lead time, salaries, professional development, staffing demand forecasting, pay plans, future salary

forecasting, number on the payroll, quality management, hiring, retiring, and firing. . Such policies are necessary for managing the impact of intangible and opportunity costs associated with Human Capital needed for meeting performance requirements.

Given that most organizations view their human resources as the primary area for cutting costs (e.g., downsizing with budget cuts); there is a need to know that such decisions are truly improving performance through increased productivity and financial stewardship. Can an organization be both efficient and effective if it has not "right-sized" its workforce and put measures in place to ensure that it remains at the "right" size? Perhaps it can, if it recognizes that the workforce is indeed capital, an investment, that is kept sufficiently lean to meet its goals. Carrying too much is costly, and so is carrying too little.

1.1 Background

Based on the Bureau of Labor Statistics estimate (Hennessey, 2006), between 2006 and 2020, approximately one-third, 25 million, of the 76 million of those born in the United States between 1946 and 1964 (the "Baby Boomers") will retire. Not only will this create a shortage of workers to fill jobs, but will also create a "boomer brain drain" that will last for decades. The loss of the baby boomers will hit some industries harder than others such as manufacturing, utilities, retail and health care. This loss can be viewed as a crisis. As many organizations have realized, the loss is not only a loss of bodies, but also a significant loss of skills. Although some are taking actions to protect against the "brain drain", others are slow to react. Those who are reacting are initiating processes to focus on staffing analysis of current jobs, forecasting future needs and determining who has what skills.

During the George W. Bush administration, the number one management initiative for the federal government was the Strategic Management of Human Capital (Liebowitz, 2003). According to a Joint Hearing on Federal Human Capital, by 2005 half of the 1.8 million non-postal civilian workers would be eligible for early or regular retirement. An even greater percentage of the Senior Executive Service, the government's core managers, would be eligible to leave. Because of the potential high level of possible attrition, all government agencies were required to develop a Human Capital strategy by 2005. Attrition is defined (Mifflin, 2009), in the context of Human Capital, as a gradual, natural reduction in membership or personnel, as through retirement, resignation, or death. Use of the term attrition in this document also includes any other reduction in personnel in an organization, whether voluntary or involuntary.

The need for understanding the demand and supply of the Human Capital workforce is not unique to the United States. A report developed by the Economic Research Institute of Northern Ireland (McGuinness & Bennett, March 2008) assessed the trend in occupational forecasting and skills research among fifteen countries from North America, Europe, Australia and Asia. The areas of comparison included the data sources utilized, who does and pays for the forecast, who uses the forecast, and who is responsible for implementation and the policy impact. The conclusion of this study is that although the macro-economic modeling approach is a good proxy for general demand and movements of the workforce within an economy, it lacks the ability to identify specific Human Capital requirements for volatile industries or unfilled vacancies occurring in industries. It also found that the classification of workforce skills tend to be highly generic and does not reflect key occupations and qualifications within an industry.

Although significant studies have been directed or commissioned by public and private entities, these studies have not yielded the desired results. Studies that result in demographic studies of classes within a population do not address the needs within organizations at the functional level of planning. Current literature reviews reveal that the macro-economic or the multi-sectored approach of Human Capital inventory forecasting is adequate for forecasting within the general economy. However, it lacks the forecasting methodologies that are necessary at an organizational level.

1.2 Problem Statement and Research Question

Although workforce planning is one of the most important issues for organizations today, the task of actually implementing workforce planning is discouraging because it is so difficult to define and to quantify (Sullivan, 2002). Research has shown that when companies are more efficient and plan ahead, managers can be provided with the right number of people, with the right skills, in the right place, and at the right time, avoiding layoffs or panic hiring (Sullivan, 2002). Studies show that developing workforce plans ensure that an organization has the resources to deliver promised services to its customers in a timely and quality manner. Workforce plans facilitate change management and contribute to improved people management. Workforce planning also allows for smarter decisions and better long-term investments (Delahoussaye, Ellis, & Bolch, 2002).

A well know quotation, *"If you can't measure it, you can't manager it"*, often attributed to Peter Ferdinand Drucker, emphasizes the fact that management cannot manage what is not measured. Davenport's and Harris' (Davenport & Harris, 2007) research established that there is a positive correlation between business intelligence and the level of analytic evaluation that

businesses conduct. The extent of most businesses and organizations involve only the analysis of business and organization demographics, particularly Human Capital demographics for the organization's workforce (Figure 1.1). Although this information provides a description of the workforce, it does not provide adequate business knowledge or intelligence critical for competitive advantage. Moving beyond descriptive reports and into detailed analyses to explain, forecast, and optimize the workforce increases business intelligence, and, hence, competitive advantage. This relationship is depicted in Figure 1.2.

The problem then, is the lack of analytics to provide business intelligence relevant to the workforce so that competitive advantage can be obtained and sustained. To address this problem, the following research question is formed:

Can the Human Capital workforce that is necessary to accomplish work in an organization be quantified? If so, then how?

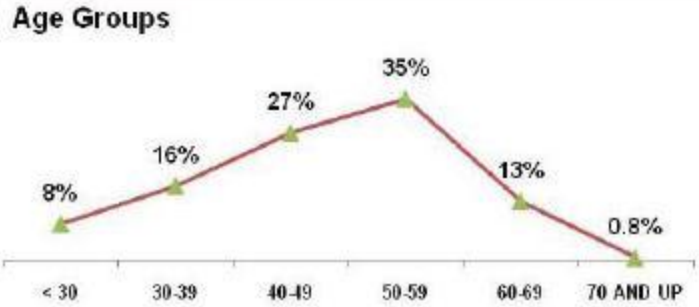
Hypothesis*: The Human Capital workforce that is necessary to accomplish work can be viewed as an inventoried asset and can be planned and managed as any other significantly valued asset utilizing techniques and methodologies applied to traditional inventory asset management.*

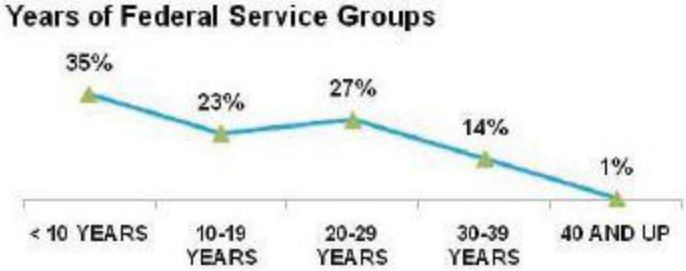
This hypothesis will be tested by utilizing datasets obtained from two large organizations and performing analyses to demonstrate the applicability of techniques and methodologies of physical inventory models to Human Capital inventories. The expected outcome, utilizing existing predictive modeling and minimum cost optimization techniques, will demonstrate that physical inventory management techniques and methodologies can be applied effectively to Human Capital inventories. This outcome will include a forecast of the Human Capital

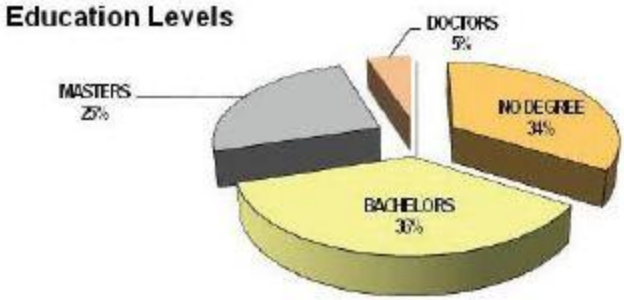
Department of Energy (Complex Wide)

As of SEPTEMBER 30 2010

TOTAL EMPLOYEES	14945	
RETIREMENT	**Employees**	**% of Total**
ELIGIBLE TO RETIRE IN 2010	2691	18%
ELIGIBLE TO RETIRE BY END OF 2014	4936	33%
PROBABLE RETIRE BY END OF2014	3100	21%
GENDER	**Employees**	**% of Total**
MALES	9287	62%
FEMALES	5658	38%
AGE GROUPS	**Employees**	**% of Total**
< 30	1204	8%
30-39	2413	16%
40-49	4103	27%
50-59	5192	35%
60-69	1916	13%
70 AND UP	117	0.8%
AVERAGE AGE	48.1 YRS	
FEDERAL SERVICE	**Employees**	**% of Total**
< 10 YEARS	5264	35%
10-19 YEARS	3364	23%
20-29 YEARS	4003	27%
30-39 YEARS	2129	14%
40 AND UP	185	1%
AVERAGE LENGTH	17.0 YRS	
EDUCATION LEVELS	**Employees**	**% of Total**
NO DEGREE	5085	34%
BACHELORS	5388	36%
MASTERS	3717	25%
DOCTORS	755	5%
ETHNIC GROUPS	**Employees**	**% of Total**
NATIVE AMERICAN	244	2%
ASIAN	694	5%
AFRICAN AMERICAN	1564	10%
HISPANIC	1003	7%
CAUCASIAN	10760	72%
NOT SELF-IDENTIFIED	680	5%
SELF-IDENTIFIED	**Employees**	**% of Total**
DISABILITY	972	7%
VETERANS PREF.	2901	19%







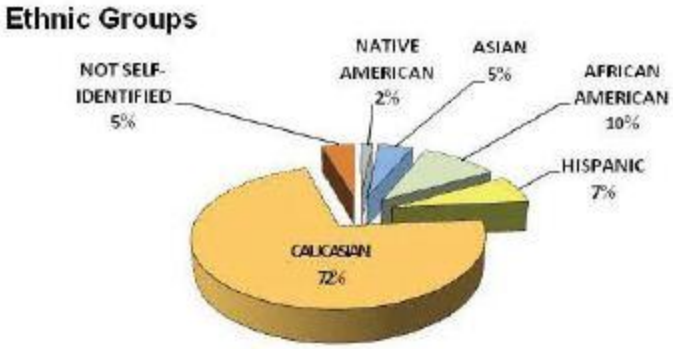


Figure 1-1 DOE Complex Wide Workforce Demographics (Energy U. S., 2011)

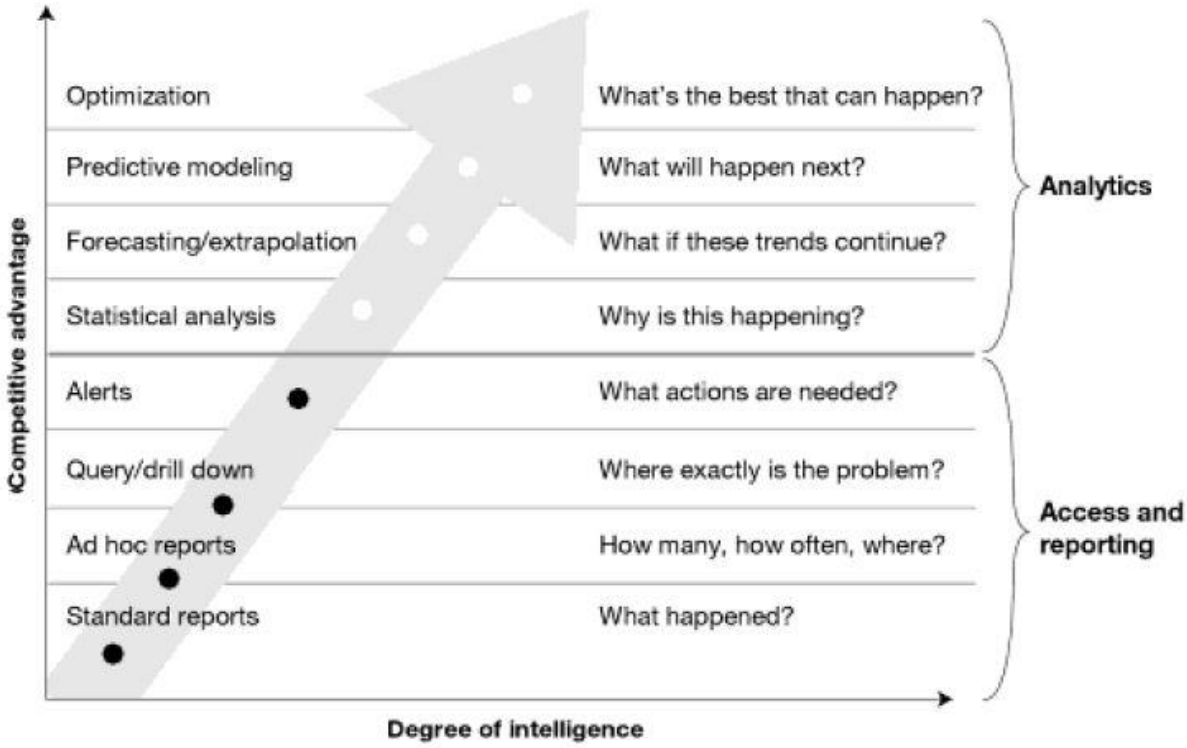


Figure 1-2 Business Intelligence and Competitive Advantage (Davenport & Harris, 2007)

availability, service levels, and costs associated with this availability. It will then be used to establish a Human Capital inventory policy.

1.3 Assumptions

The development of this inventory policy is built upon certain assumptions. These assumptions capture characteristics of Human Capital availability and forecasting of this availability. Specifically, they are:

- Human Capital is, and therefore can be managed, like other physical assets, with recognition of its specific characteristics.
- Demand of Human Capital in this study is deterministic and is constant in each

demonstration scenario. For this study, ten (10) years of personnel data from two large organizations was obtained. This data items utilized from these datasets are used 'as is' without modifications of manipulation.

- The Human Capital inventory availability is derived by empirical analysis however there is no *a priori* assumptions for the distribution of the availability.
- Lead-time is considered constant and varies from no lead time to a two period lead time in the demonstration scenarios in this study.

1.4 Limitations and Constraints

The process of developing this information system is bound by certain limitations and constraints. The limitations are primarily due to availability of hardware and software while the constraints are relative to the nature of the items, the inventory policies and procedures, and the forecasting methods themselves. Specifically, they are:

- Only quantitative forecasting methods will be considered.
- Only Type A Inventories, as defined by the ABC Inventory Classification System (Hillier & Lieberman, 2005), are considered for applicability in this study.
- The forecasting horizon will be short to medium term (Minimum of 1 to a maximum of 10 years)
- No negative demands or stock-outs are to be considered.
- This study is limited to the study of inventory methodologies that answer the questions of how much to order and when to order and the associated inventory costs. Other aspects of inventory management such as inspections, location, and monitoring of material are not going to be considered. Since this is a general study, it will focus on answering only those two general questions that can be generalized to several types of industry and products.

- Partial satisfaction of demand is not allowed for replenishment events.
- Organizational capacity and availability limitations are ignored. For example, it is not going to be considered whether replacements come from the organization or the environment.
- This study is going to be limited to simple and basic forecasting models. The type of forecasting required to provide useful application for inventory control must be simple enough that it will be possible to create repetitive forecasts at a minimum cost. Hence, not only does simplicity positively affect effectiveness, but it also may increase economic efficiency.

1.5 Expected Significance of This Study to the Body of Knowledge

The need for this research has been stated in the literature by several authors (Chiles A. H., March 1, 1999), (Cohen & Richardson, November 3, 2000), (D'Agostino, July 29, 2009) and (Dychtwald, Erickson, & Morison, 2006). The model presented in this study is designed to address these needs and to complement the current body of knowledge in the area of industrial engineering by providing an efficient, step by step procedure to help management produce effective forecasts and inventory models of Human Capital, thereby expanding the application of proven inventory management techniques.

(Cappelli, 2009) and (Huang, Lee, song, & Eck, 2009) discusses the current need for quantitative methods and modeling for Human Capital workforce planning. The quantitative Human Capital (workforce planning) methods and models utilized during the 1970's and 1980's and subsequently abandoned, are being resurrected due to the recognition that the Human Capital

(workforce) is in a major transitional state for the next 20 years because of the effects that the "baby boomers" will have on Human Capital attrition.

The decision systems and guidelines found in the literature review of inventory systems pertained to physical assets and do not help management efficiently calculate accurate forecasts and effectively manage Human Capital inventories. There is software on the market, such as SAP, PeopleSoft or Oracle, that is designed to help management plan the application of workforce resources but they do not provide for quantitative analyses to determine the demand and supply gaps of the Human Capital inventories. Although this software offers limited advanced planning functions, these systems are costly and not affordable except for the largest of corporations.

1.6 Research Organization

This research paper is organized in the following manner.

Chapter 1 is the Introduction chapter. This chapter discusses the need for Human Capital planning with references for such needs with the public sector.

Chapter 2 is a literature review of the history of public sector Human Capital planning models and traditional inventory models. Chapter 2 also discusses the data analysis and methods associated with organizational attrition data, survival analysis associated with a methodology for attrition and replenishment modeling and relevant simulation and optimization methodologies and techniques for minimum cost optimization.

Chapter 3 presents the detailed methodologies associated with the data analysis and modeling presented in Chapter 2.

Chapter 4 presents the analysis and models that are utilized in this research: Cluster Analysis, Survival Analysis and Minimum Cost Optimization modeling.

Chapter 5 includes the conclusions of the study based on the analysis and results of Chapter 4, the operationalization of the methodologies presented in this dissertation, future research, conclusions and other areas of applicability.

Chapter 2 - Literature Review

2.1 Overview

This chapter is divided into 4 major sections. The first section addresses the history and development of Human Capital planning in general. The second section addresses the history and development of staffing approaches of Human Capital. Section 3 addresses the prior Human Capital Planning approaches. Section 4 discusses the types of Human capital Inventory models approaches. Section 5 discusses the possible statistical approaches to clustering and survival analysis for consideration of determining Human capital attrition. Section 6 reviews optimization techniques utilized within inventory systems.

2.2 Human Resources Planning Models

The human resources planning model is a method companies use to make sure it has enough employees and the right employees at the right time to successfully carry out the functions of the company. As Collins found in his research (Collins, 2001), the best companies "*first got the right people on the bus and then figured out where to drive it*". The executives found that you must get the right people on the bus, the right people in the right seats and the wrong people off the bus in order to take an organization to greatness. From a workforce planning perspective, human resource workforce planning is having "the right number of people, with the right skills sets, in the right jobs, at the right time" (Fisher, 2010). The three key elements of a human resources planning model include predicting the employees the company needs, analyzing if the supply of employees meets the demand and learning to balance the supply and demand of employees (Lorette, 2011). Lorette discusses the need within a human resource

planning model for analytical processes for assessing employee data, forecasting and predicting attrition, assessing employee demand and identifying the time frames of the attrition and availability of the workforce. However, Lorette identifies no specific analytical, statistical or simulation models and methodologies.

(Bohlander & Snell, 2010) define Human Resource Planning as the process of anticipating and making provisions for the movement of people into, within and out of an organization. Their strategic analysis includes the identification of what resources are needed and what are available and balancing the supply and demand considerations. The analytical techniques presented by (Bohlander & Snell, 2010) only include a Markov Analysis of the transition of employees into and out of positions. No methodologies are presented in their paper that encompasses the determination of optimum staffing levels based on Human Capital demand, attrition or the minimization of costs associated with the availability of employees (Human Capital inventory).

When considering Human Capital inventory management in the framework of traditional physical inventory management, it is obvious that the expectations and objectives for the management of a Human Capital inventory are the same as that of a traditional physical inventory.

A Workforce Planning (Human Capital Planning) model that utilizes optimized computer simulation was developed by Huang, Lee, Song and Eck (Huang, Lee, song, & Eck, 2009). The objective of their model is to determine a desired workforce staffing level based on a desired service level (Safety Stock) entering a subsequent period. Stochastic workforce demands and attrition are utilized in the model to determine end-of-period workforce availability. Although

this model uses advanced workforce (Human Capital) simulation and attempts to determine optimum staffing levels based on an inventory service level, this model does not utilize Human Capital inventory costs in determining the optimum workforce staffing levels nor does it support a multiple planning period horizon.

In the late 1970's, thirty percent (30%) of employers used elaborate statistical regression models to forecast their talent needs (Human Capital) which fell to nine percent (9%) by 1984 (Cappelli, 2009). Cappelli also found that Markov Chain analysis models fell from twenty-two percent (22%) to six percent in the same period and operations research tools declined in general from twenty-three percent (23%) to four and one half percent (4.5%). By the 1990's only 19% of the companies surveyed maintained planning of any kind compared to 96% that had a dedicated workforce planning department in the decades prior to the 1980's.The inadequacy of the models of the 1960's, 1970's and 1980's due to the incorrect assumptions of growth, rather than economic decline, lead to incredibly wrong estimates which ultimately lead many companies to abandon the previously used modeling techniques. With many companies again realizing the needs for workforce planning tools, Cappelli has identified the following in order to develop adequate workforce plans:

- availability of the workforce (Human Capital inventory)
- the attrition associated with a workforce
- the demand for the workforce that meets the organizations objectives
- simulation modeling for evaluating alternate planning scenarios
- costs associated with alternate planning scenarios.

These components, as Cappelli identified, are the same components that are typically associated with traditional inventory models (demand, supply, cost and simulation)

2.3 Human Capital Planning Approaches In Industry

The International Personnel Management Association (IPMA, 2002) describes workforce planning as

> *' the strategic alignment of an organization's Human Capital with its business direction. It is a methodical process of analyzing the current workforce, identifying future workforce needs, establishing the gap between the present and future, and implementing solutions so the organization can accomplish its mission, goals, and objectives.'*

The International Professional Management Association states that the *IPMA Workforce Planning Resource Guide for Public Sector Human Resource Professionals* represents the best thinking of a diverse task force of IPMA members. Because IPMA represents public sector HR professionals from all levels of government, their *Guide* presents state-of-the-art information on this important and timely topic (IPMA, 2002). The IPMA states that a workforce plan is not a static document that tries to predict the future or describe the past. Instead, a workforce plan focuses on developing information that can help an organization make decisions for the short term and the long term based on changing organization strategies. The workforce plan is intended to help solve staffing problems related to the management of employee movement into, around, and out of an organization.

The IPMA Workforce Planning Guide (IPMA, 2002) identifies four steps that should be used to develop a workforce planning model. These four steps are:

- Step 1: Scan
- Step 2: Supply and Demand
- Step 3: Gap Analysis
- Step 4: Action

The activities that need to be addressed in each step follows:

Step 1: Scan - Environmental scanning examines internal and external **S**trengths,

Weaknesses, **O**pportunities, and **T**hreats (SWOT analysis). Budget must also be a major factor in this review.

Step 2: Supply and Demand - The supply and demand analysis should lead one to assess whether there is balance in the organization, or whether imbalances among the workload, workforce, and competencies.

Step 3: Gap Analysis - Gap analysis compares information from the supply and demand analysis to identify the differences between the current and the future organizational workload, workforce, and competencies. This analysis must use comparable workload and workforce elements and the competency sets developed in Step 2, the supply and demand analysis.

Step 4: Action - Action plans should be developed to address the most critical gaps facing the organization so *Human Capital can support organizational strategy.*

The IPMA has identified numerous government entities as well as private sector entities that have used the IPMA Guide as the foundation of their workforce planning model. These entities include the federal government (Health and Human Services (HHS, 1999) and the U.S. Department of Transportation (DOT, 1999) , the state of Georgia, Minnesota Department of Transportation, Booz-Allen and Hamilton, Inc., the state of Washington, Duke Power, the city of Minneapolis, and the city of Los Angeles.

The steps defined in the IPMA Workforce Planning Guide, particularly step 2 (supply and demand) and step 3 (gap analysis), is the focus of this research and the application of traditional physical inventory models to the management of Human Capital inventories. Although the IPMA guide details the steps that are necessary in creating a workforce planning model, it does not include any references, guidelines or analytical methods that should be utilized in the supply and demand analysis or the gap analysis in the planning.

2.4 Inventory Models and Policies

This study is being performed to test the hypothesis that the management of Human Capital inventory can be accomplished with the same tools and techniques used to manage physical inventories. Hence, a review of literature on different types of models in practice and the inventory policies associated with these models for managing physical assets is vital.

Typically, the major objectives of traditional inventory models consists of minimizing total inventory costs and balancing the economics of large orders or large production runs against the cost of holding inventory and the cost of having a shortage or surplus of an inventory item. (Zappone, 2010) states that "*the purpose of an inventory system is to determine rules that management can use to minimize the costs associated with maintaining and meeting customer*

demand". Zappone also states that inventory models should answer the questions: when should an order be placed for an inventory item and how large should each order be and how often should the inventory status be determined (reviewed). The answers to these questions determine the *inventory policy*.

A successful inventory policy is vital for customer satisfaction. And, without customer satisfaction it is impossible to sustain profit; this is because without enough inventories in stock to satisfy demand an organization cannot survive in the long term. Success in terms of inventory policy is defined here as having the right levels of inventory to satisfy the demand at a minimum cost. Excess inventory is considered a liability and can be very expensive. On the other hand, not enough inventory can mean the loss of market share or competitive advantage. An inventory policy must be able to provide two types of information: when to order, and how much to order to satisfy a minimum level of demand. This information is found based on the type of inventory model employed.

2.4.1 Types of Workforce Planning Models

In a review of operations research applications in workforce planning, Wang (Wang, 2005) identified four analytical models that are utilized for workforce planning. These models include Markov Chain Models, Simulation Models, Optimization Models and System Dynamic Models. The purpose of these models is to ensure that effective workforce planning is performed in order "to ensure that the right people are available at the right places and at the right times to execute corporate plans with the highest quality (Khoong, 1996)" The framework for each one of the four types of models is discussed below. The details of the statistical methodologies will not

be presented in this paper. Instead, the application of how the statistical technique was utilized is presented.

The Markov Chain model is applied in a discrete-time stochastic process environment. This Markov Chain model is used to statistically move from one state in the system to another state. The population of the workforce is divided into classes according to characteristics and attributes such as age, experience, rank, trade. The classes are exhaustive and mutually exclusive so that an individual only belongs to one class. It is assumed that the there is no differences between the members of the population in the same class, with each class having its own probability of transition to any other state. The number of members transitioning from one state to another is the product of the transition probability and the number of members in the state prior to the transition. Attrition rates are assigned to each class with the sum of the transition probability and the attrition probably is equal to one. A replenishment probability is also assigned to each class and the sum of the replenishment rates for all classes equals one. The transition, attrition and replenishment probabilities are based on historical data. The results of this model are the prediction of the number of members in each state at some point in time.

Although the Markov model is an acceptable statistical method of forecasting workforce movement and availability form one state to another state, it does not provide for the forecasting of costs associated with manpower planning such as cost of maintaining the workforce, attrition costs, replenishment costs, workforce shortage costs or excess workforce cost that exceeds demand. The Markov model also does not provide for an optimum solution based on workforce planning objectives. Typically, Markov models would require classes that must be large enough

that would provide stability for the transition probabilities. Class sizes of 100 or more would be necessary to establish stable transition probabilities among classes.

Simulation modeling is a technique of mimicking real-world situations, establishing relationships among the parts of the system. It is advantageous to use Simulation models when systems evolve over time, when model variables are either stochastic or deterministic and when time can be either discrete or continuous. The Rand Corporation developed a simulation model for the U.S. Air Force for their manpower planning for use in the Air Force's pilot training program (Mooz, 1970). This model forecasts the availability of pilots over time through the training program based on the demand within the Air force for pilots. This model also calculates the costs of the personnel required for the training program. The attrition rate used in this model is based on historical or estimated data and is a deterministic variable in this model. This model did not incorporate optimization techniques in order to minimize Human Capital costs.

Although simulation modeling is a good methodology when systems are too complex for analytical models, simulation models can be time consuming and costly to develop. Until the recent advancements in capabilities of simulation software, optimization within simulation models was not possible. Optimization models are a good methodology when needing to answer "what if" questions for multiple planning scenarios.

Optimization models are utilized in workforce planning when it is desired to determine the "best" solution to a planning model objective when constraints in the models must be considered. Objectives in optimization models typically include either the minimization of costs associated with workforce staffing or the minimization of staffing levels to meet service level objectives. Constraints in a manpower planning optimization model typically include limits on

staffing costs, limits on the availability of replenishment staff or maintaining a particular staffing level within the manpower planning time horizon.

Optimization techniques used in workforce planning include Linear Programming, Integer Programming, Goal Programming or Dynamic Programming. Linear Programming solutions involve problems where the objective functions and the constraints are linear functions of the decision variables. Integer Programming solutions involve problems where the objective function and the constraints are either linear or non-linear and the decision variables are integer. Pure Integer Programming includes decision variables that are all integer. Mixed-Integer Programming includes some integer decision variables and zero-one or binary programming includes decision variables if all variables are binary.

Goal Programming is an optimization methodology when multiple objective functions exist. Goal programming is an extension of Linear programming and is utilized when there is a conflict among multiple objectives.

Dynamic Programming (DP) is a method of solving multistage decision problems in which a series of decisions need to be made at each stage to reach an optimal solution. Dynamic Programming problems are typically large and or complex. DP breaks these complex problems into a series of single stages, finding an optimal solution at each stage based on the objective function for the stage. The optimal solution at each stage may be obtained by using either Linear, Non-Linear, Integer or analytical solutions.

The application of traditional physical inventory planning and analysis and the appropriate application of these methodologies and techniques to workforce planning (Human Capital) is the focus of this research.

2.4.2 Traditional Physical Inventory Models and Policies

Typically, the major objectives of traditional inventory models consists of minimizing total inventory costs and balancing the economics of large orders or large production runs against the cost of holding inventory and the cost of having a shortage or surplus of an inventory item. (Zappone, 2010) states that "*the purpose of an inventory system is to determine rules that management can use to minimize the costs associated with maintaining and meeting customer demand".* Zappone also states that inventory models should answer the questions: when should an order be placed for an inventory item and how large should each order be and how often should the inventory status be determined (reviewed). The answers to these questions determine the *inventory policy*.

The literature review of traditional inventory models resulted in the identification of two basic, and most well-known, models: the Economic Order Quantity model (EOQ) and the Economic Production Quantity (EPQ), also known as the Production Order Quantity model. Although these models are based on rather simple assumptions, they are both important for both practical and theoretical reasons. These models describe the tradeoff between the different types of inventory costs and form the basis for the mathematical development and analysis of more complex systems. Details of the EOQ and EQP can be found in an abundance of Operations Research and Manufacturing Planning literature (Hillier & Lieberman, 2005), (Hadley & Whitin, 1963) and (Johnson & Montgomery, 1974). Each of the two models includes assumptions or characteristics that determine the fundamental nature of the application of the models. These characteristics include whether there is a deterministic or stochastic demand, whether the inventory model is for one period or multiple inventory periods, the cost components of the

models and whether the inventory availability is allowed to fall to zero, is allowed to fall below zero or whether a certain level of inventory is desired to be maintained that is greater than zero. The assumptions and characteristics of these models are discussed in detail in the following sections in Chapter 2.

Traditional physical inventory models identified by (Aggarwal, 1974) can be grouped based on six categories:

- models for determining optimum inventory policies,
- lot size optimization,
- optimization of various specific management objectives,
- models for optimizing highly specialized inventory situations;
- application of advanced theories to inventory problems
- models bridging the gap between theory and practice.

However, difficulties arise in applying these models to routine practical inventory situations. The complexity of a model when all the situations are simultaneously considered overwhelms those who are responsible for the management of inventories. Another difficulty in developing comprehensive inventory models is the need for broad organizational support for such efforts. It is not very typical for a Human Capital organization to have the broad range of knowledge and skills to implement sophisticated manpower planning models. Contributions would be required from a diverse group of organizations such as Production Planning for forecasting demands, Information Technology or Industrial Engineering for computer simulation modeling, Human Capital Planning for employment demographics and Strategic Planning for providing long range forecasts and business strategies. The assemblage of such a group and the

coordination of these groups is typically not a priority item within a Human Resource planning organization.

The use of Operations Research is one methodology for organizations to improve their inventory policy to determine when and how much to replenish their inventory. As defined by (Hillier & Lieberman, 2005), Operations research uses scientific inventory management, comprised of the following steps:

1. *formulate a mathematical model describing the behavior of the inventory system.*
2. *Seek an optimal inventory policy with respect to this model.*
3. *Use a computerized information processing system to maintain a record of the current inventory levels.*
4. *Using this record of current inventory levels, apply the optimal inventory policy to signal when and how much to replenish inventory.*

2.4.2.1 Deterministic and Stochastic Inventory Models

Mathematical inventory models can be divided into two broad categories (*deterministic models* and *stochastic models)* according to the predictability of the demand involved.

The demand for a product in a traditional inventory system is the number of units, whether they are parts, materials or labor hours that will be withdrawn or utilized from an inventory for some use during a specific time. The Human Capital Inventory Cost Model is characterized by the demand of Human Capital labor hours. The hours are provided by human resources that are available during the period of demand. The Human Capital inventory is not

consumed as in the traditional inventory system, but is utilized to service the demand and is available from period to period with the exception of attrition within the Human Capital inventory. The demand in the deterministic model is a known quantity. In a stochastic model, the demand in any inventory period is a random variable rather than a known quantity.

2.4.2.2 Continuous and Periodic Inventory Review

Two methods are utilized to monitor an inventory: a Continuous Review and a Periodic Review (Hillier & Lieberman, 2005). The continuous review places a request for inventory replenishment when an inventory falls below a prescribed reorder point, the minimum level of inventory at which a new order must be placed. In a periodic review, the inventory level is checked at discrete intervals even if the inventory levels fall below a reorder point between review periods.

2.4.2.3 Single Period and Multiple Period Inventory Models

In some situations, there is a relatively short consumption period and inventory products are of no value to satisfy demand in the next period. This multiple period situation exists when inventory products cannot be utilized to meet the demand for the inventory product in future periods. This inventory situation is characterized with one-time inventory buys. An example of this inventory situation is the "Newsboy Model". These vendors must place a one-time order for the next day's papers with no chance of reordering. Since demand for the next day is unknown, the newspaper vendor may order "too much" or "not enough". If this vendor orders too much, the vendor will face the cost of being overstocked. Likewise, if the vendor orders too few papers, then the vendor will incur the cost of being under stocked. The vendor is interested in balancing these two costs so that expected profit is maximized. There are many similar situations under

which managers face this same kind of dilemma. When a specialized piece of equipment is purchased, a one-time spare parts order may be placed at the same time. The local grocery store buys bread once a day and many other perishable products on a regular basis. Airlines must decide how many reservations to take for a given flight – they may overbook since they don't know how many people will actually show up and use their reservation.

Multi-period inventory control addresses one form of the inventory control problem. During a certain time period, a certain number of items in inventory exist. During an inventory period a certain number of items are consumed and also a certain number of items are received. There are costs involved in receiving, retaining and consuming the items. These costs are realized over many similar time periods, creating a control problem. Mathematically, we can represent the multi-period model as:

$$y_t = y_{t-1} + \mu_t - \varepsilon_t$$

where y_t is the inventory level at the beginning of time *t*, y_{t-1} is the inventory level at the end of *t-1* period, μ_t is the order quantity received at the beginning of period *t*, ε_t is the demand for the resource in period *t*.

2.4.2.4 Traditional Physical Inventory Components

Components that are typically used in traditional inventory model costing include:

Ordering Costs: The ordering cost includes all costs related to replenishing the inventory item. There are two components of the order cost. The first component is the fixed order cost which is the amount of money paid when you place an order regardless of the number of units ordered. The other component is the variable order cost which is a cost per unit of order.

Holding Costs: Holding cost is money spent to keep and maintain a stock of goods in storage. The most obvious holding costs include rent for the required space; equipment, materials, and labor to operate the space; insurance; security; interest on money invested in the inventory and space, and other direct expenses. Holding cost also includes the opportunity cost of reduced responsiveness to customers' changing requirements, slowed introduction of improved items, and the inventory's value and direct expenses, since that money could be used for other purposes. While it is possible to measure the components of holding cost, it is common to estimate them as at least one-third the value of the stored goods per year. If opportunity cost is included, it is reasonable to use one-half the value of stored goods as their holding cost.

Shortage Costs: This penalty cost is the cost per unit of not satisfying the order when it is received. Shortage cost or stock-out cost is the total of all costs associated with shortage units. We use penalty cost in inventory planning. The penalty cost should not be something you pay actually. It can be a chance of profit missed, which is called opportunity cost. However, there is a case when you should pay a penalty for the shortage. This happens when you have an agreement with a customer to satisfy the demand by a certain date with the right quantities, or you will pay a penalty for the breach of contract.

Surplus Costs: The item cost and the residual cost associated with the left over inventory items that have little or no value from one inventory period to another.

Lead Time: Lead-time is defined as the time period from initiating of an activity to its completion. For inventory management, *lead time* is the time interval between placing an order and receiving delivery of the items.

Another component of inventory management concerns backlogging. *Backlogging* is defined as not having sufficient inventory of an item to meet current demand. Typically, when demand cannot be met, two inventory policy options can be established. One option is to replenish the quantity of the inventory item using a priority option to obtain more inventory items. The other option is to not meet the current demand and the demand for the inventory item is eliminated.

2.4.2.5 The Continuous-Review Deterministic Model

The Continuous-Review Deterministic model is one of the most common inventory models used within industries and organizations. This model is also referred to as the Economic Order Quantity Model (EOQ model). In this model, units of inventory are withdrawn from the inventory at a constant rate, i.e. the demand rate is at a constant rate. In this model, the replenishment of the inventory is immediate when the inventory quantity drops to zero and is replenished by a constant fixed batch size and all of the units arrive simultaneously. Planned shortages or surpluses are not allowed. The unit price per item, the holding cost rate, and the order cost per order is constant. The lead time is fixed. Only one inventory type item is considered. The objective of this model is to determine when and by how much to replenish inventory so as to minimize the sum of costs per unit time.

2.4.2.5.1 The Economic Order Quantity Model without Shortages

A graphical representation of the Continuous-Review EOQ Deterministic model, without shortages, taken from (Zappone, 2010) is represented in Figure 2-1 (Zappone, 2010). In the EOQ model, the inventory is depleted to a position where the inventory on hand is zero. The inventory is immediately replenished with a quantity of Q units. The replenishment can be seen to occur

every Q/α time periods where α is the demand rate per unit time. This time period is known as the cycle time. As seen in Figure 2-1, at time 0, the inventory is replenished with batch size Q.

2.4.2.5.2 The EOQ Model Costs

The primary cost associated with any physical inventory management system is the *Total Cost Per Cycle*. The *Total Cost Per Cycle* is determined by summing the total production costs per cycle plus and the cost of holding the current inventory:

$$Total\ Cost\ per\ Cycle = K + cQ + \frac{hQ^2}{2a}$$

where K = ordering cost per cycle, c = item unit cost, Q = the replenishment quantity, h = per unit holding cost, a = the demand rate per unit time.

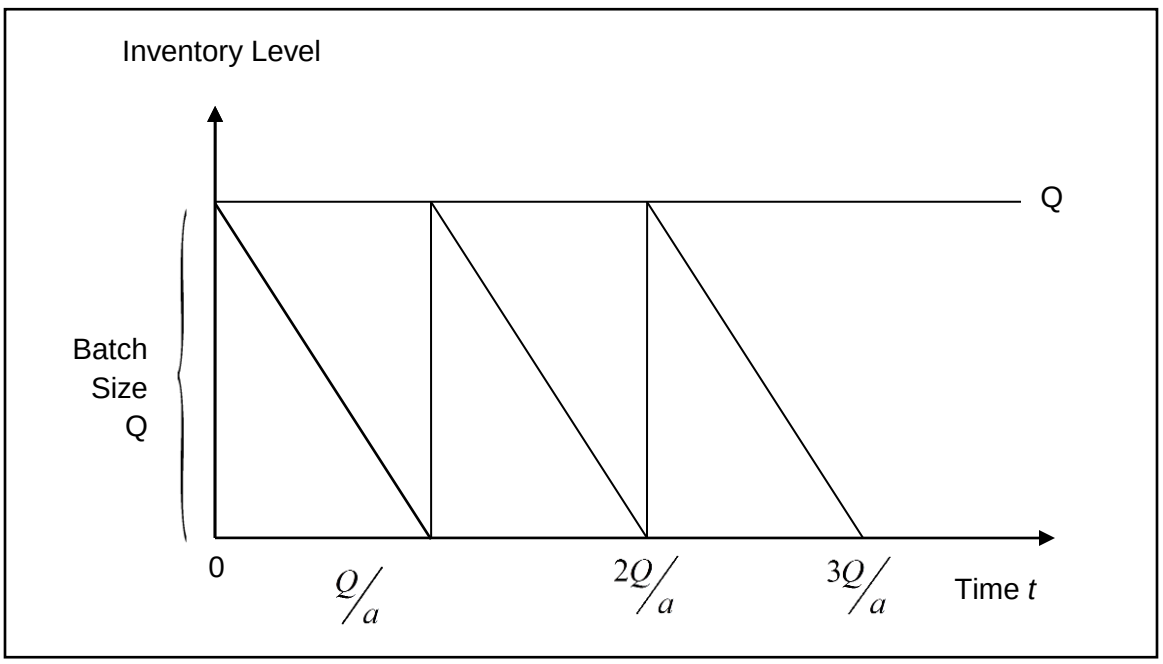


Figure 2-1 EOQ Inventory Level With No Shortages

The value of Q^* that minimizes the total cost is found by taking the derivative of the total cost per unit time with respect to Q and setting the derivative to zero and solving for Q.Thus Q is determined by the following: $Q = \sqrt{\frac{2aK}{h}}$.

Figure 2-2 depicts the holding, ordering and total cost curves for an EOQ model. It can be seen that the minimum total cost occurs at the intersection of the holding and ordering cost curves.

2.4.2.5.3 The EOQ Model with Planned Shortages

The occurrence of an inventory shortage, also referred to as a stock-out, leads to either an acceptable delay in satisfying the demand for the out-of-stock inventory items or the demand may be lost. This is a change from the basic EOQ model. The demand for the out-of-stock inventory items is delayed until the inventory item is again available. Backorders are filled immediately when the order quantity arrives to replenish the inventory.

This model, with shortages, taken from (Hillier & Lieberman, 2005), is similar to Continuous-Review Deterministic model with planned shortages:

Figure 2-3 depicts the effects on the inventory when shortages are allowed. The total cycle cost for the EOQ Model with Planned Shortages is calculated by

$$S^* = \sqrt{\frac{2aK}{h}}\sqrt{\frac{p}{p+h}} \; and \; Q^* = \sqrt{\frac{2aK}{h}}\sqrt{\frac{p+h}{p}}.$$

If it is assumed that the unit cost of an item is constant throughout the time horizon and is independent of the batch size, the unit cost does not appear in the optimal solution for the batch

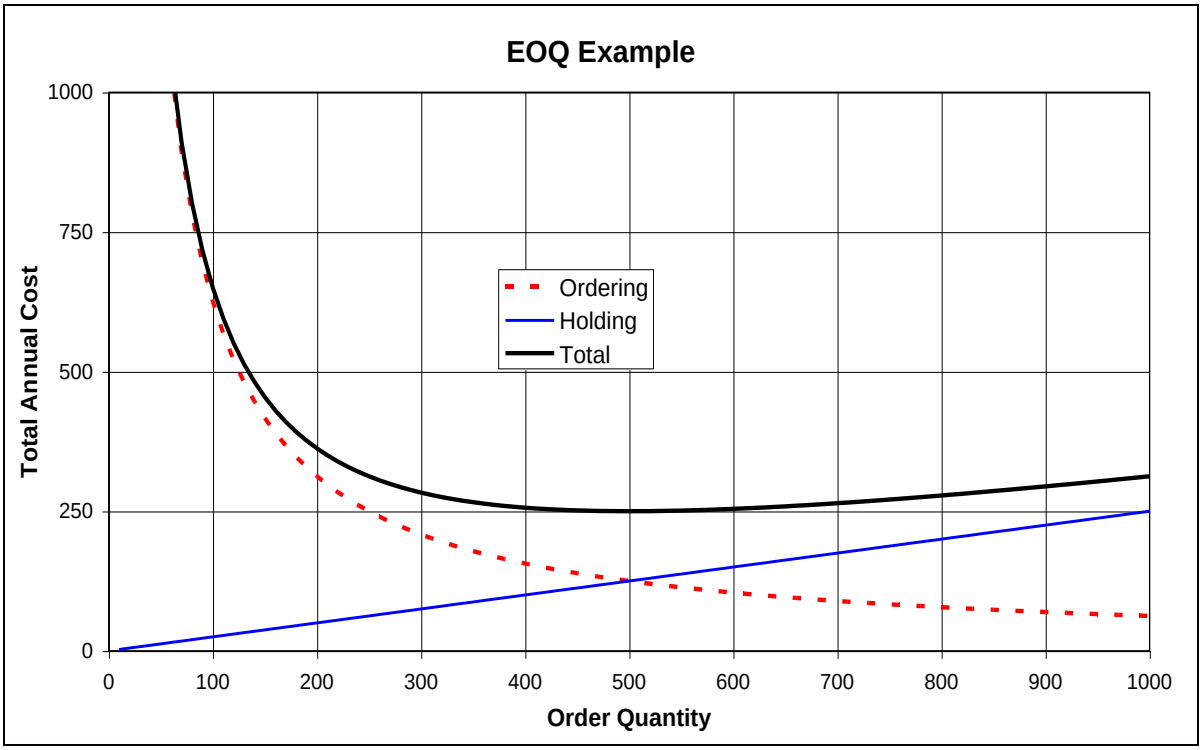


Figure 2-2 Optimized Order Quantity (Aquilano, 2005)

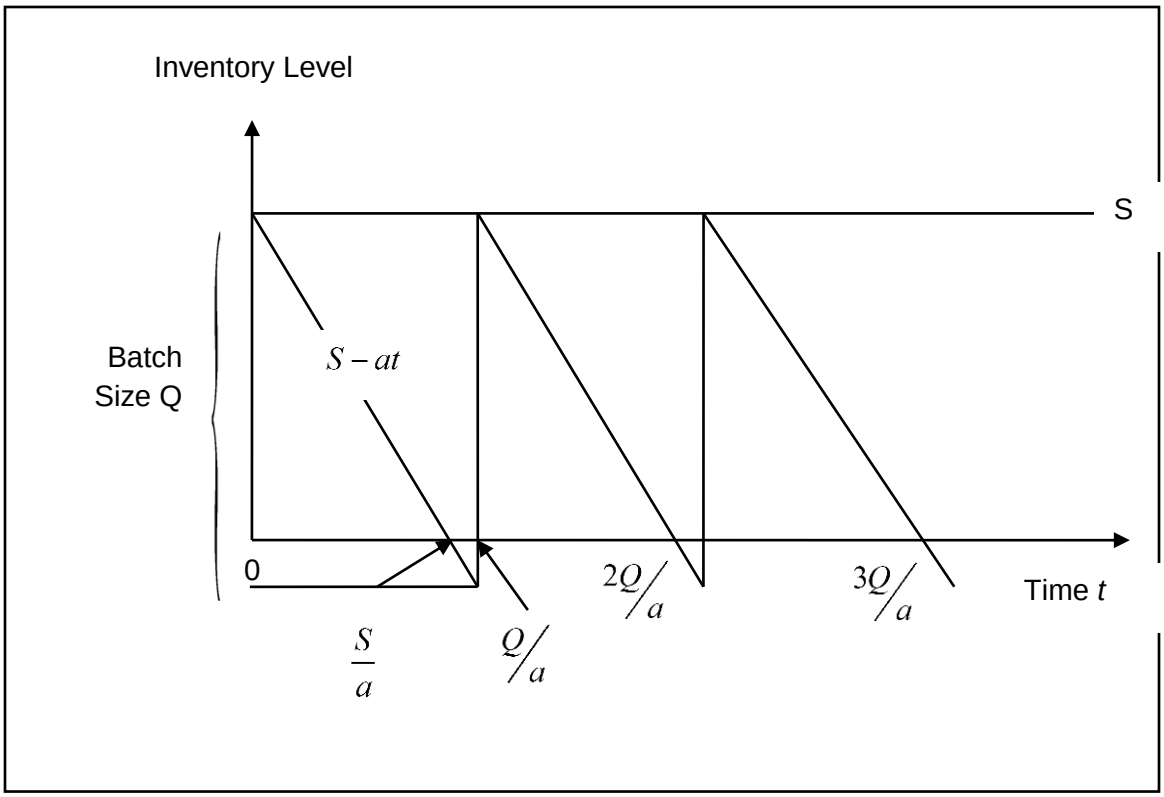


Figure 2-3 EOQ With Inventory Shortages (Zappone, 2010)

$$Total\ Cost\ per\ Cycle = K + cQ + \frac{hS^2}{2a} + \frac{p(Q-S)^2}{2a}$$

where *K*, *c*, *Q* and *a* are the same as in the EOQ model without shortages. *p* is defined as the shortage cost per unit time and *S* is defined as the inventory level after replenishing the inventory with Q units.

Optimizing for the two decision variables by setting the partial derivatives $\partial T/\partial S$ and $\partial T/\partial Q$ to zero and solving, we have

2.4.2.6 Observations about EOQ Models

If it is assumed that the unit cost of an item is constant throughout the time horizon and is independent of the batch size, the unit cost does not appear in the optimal solution for the batch size. This result occurs because no matter what inventory policy is used, the same number of units is required per unit time, so this cost per unit time is fixed.

The analysis of the EOQ models assumed that the batch size Q is constant from cycle to cycle. The resulting optimal batch size Q^* actually minimizes the total cost per unit for any cycle, so the analysis shows that this constant batch size should be used from cycle to cycle even if a constant batch size is not assumed.

2.4.2.7 A Deterministic Periodic-Review Model

An assumption for the Basic EOQ model is a constant demand rate. When the demand on an inventory item is allowed to vary from period to period, the EOQ formula no longer ensures a minimum-cost solution. In a deterministic periodic-review model, planning is done for the next n periods with regards to the quantities to replenish the inventory at the beginning of each of the periods. The demands for each period are known but not necessarily the same. The costs incurred in the Deterministic Periodic-Review model are the same as the EOQ model. The objective for this model is also to minimize the total cost over n periods. However, the optimal solution for this model does not have a closed-form solution as the EOQ model. By using a backward recursive simulation modeling approach as defined in (Hillier & Lieberman, 2005), minimum period costs can be determined.

2.4.2.8 The Stochastic Continuous-Review Model

A stochastic inventory model is designed for analyzing inventory systems where there is considerable uncertainty about future demands. In a stochastic continuous review model, the demands are uncertain and the inventory is being reviewed continuously so that an order for inventory replenishment can be made as soon as the inventory level drops below a predefined reorder point. The continuous-review inventory system for an inventory item will be based on two critical numbers:

$$R = \text{reorder point}$$
$$Q = \text{order quantity}$$

The inventory policy based on this model is to place an order for Q units to replenish the inventory when the inventory level drops to R units. The assumptions of this model as defined by (Hillier & Lieberman, 2005):

- Only a single product is modeled
- The inventory level is under continuous review so that its current value is always known
- An (R,Q) policy is to be used so that the only decisions to be made are to choose R and Q
- There is a lead time between the time the order is placed and when the order quantity is received. This lead time can be either fixed or variable.

- The demand for withdrawing units from the inventory during the lead time *m* is uncertain. However, the probability distribution of demand is known (or at least estimated)
- If a stock-out occurs before the order is received, the excess demand is backlogged, so that the backorders are filled once the order arrives.
- A fixed set-up cost (denoted by *K*) is incurred each time an order is placed
- The cost of the order is proportional to the order quantity *Q*.
- A certain holding cost (denoted by *h*) is incurred for each unit in inventory per unit time.
- When a stock-out occurs, a certain shortage cost (denoted by *p*) is incurred for each unit backordered per unit time until the backorder is filled.

2.4.2.8.1 Determining the Order Quantity *Q*

The assumptions are similar to the EOQ model with planned shortages except for the fact that the EOQ model assumed a known fixed demand. Because of uncertain demand, safety stock needs to be added when setting the reorder point to provide some cushion for having well-above-average demand during the lead time. To approximate the Order Quantity Q, the formula is

$$Q = \sqrt{\frac{2AK}{h}}\sqrt{\frac{p+h}{p}}$$

where A is the average demand per unit of time and K, h and p as defined in the EOQ model. This Q is only an approximation since no closed-form formula is available for the exact value of the optimum order quantity.

2.4.2.8.2 Determining the Reorder Point R

Choosing the reorder point R is based on a desired level of service. Therefore, a service level must be chosen in order to determine the reorder point. One alternative service level that is commonly used (Hillier & Lieberman, 2005) is the probability that a stock-out will not occur between the time an order is placed and the order quantity is received.

If D is the demand per period and R is the desired reorder point, then R is calculated as $R = \bar{d}L + z\sigma$ where $\bar{d}$ is the average demand during lead time, L is the lead time, z is the number of deviations for the desired service level and σ is the standard deviation of the demand during the lead time (Hillier & Lieberman, 2005).

The behavior of the stochastic demand model is characterized by the graph in Figure 2-4. The effect of the stochastic demand is apparent by the various changes in the inventory levels from period to period.

Continuous review systems are not typically utilized when the inventory item is not very expensive or very critical or when a customer doesn't mind waiting for a backorder.

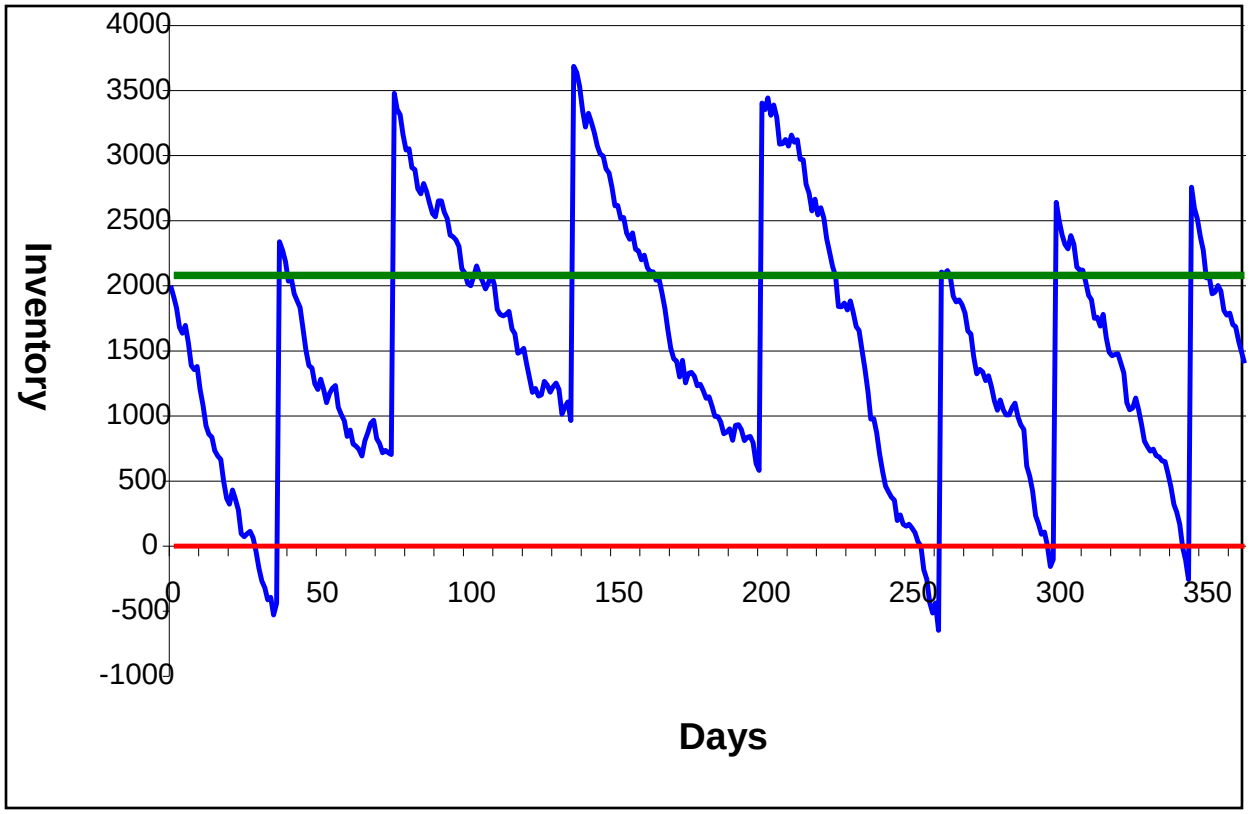


Figure 2-4 Behavior of Stochastic Demand Model (Aquilano, 2005)

2.4.2.9 Stochastic Periodic Review System

The stochastic periodic-review model assumes (Hillier & Lieberman, 2005) that the demand is uncertain (random) and has a defined probability distribution function. The periodic-review model reviews the inventory level at predetermined time intervals. At the end of each period, a decision is made on how much to order in order to replenish inventory for the next period. When considering more than two periods, this model becomes *increasingly difficult.* A determination for a two-period model is provided by (Hillier & Lieberman, 2005). Because of the complexity of the multi-period model, the analysis is suited for recursive algorithm

techniques. The details of a recursive programming approach is developed in Hillier and Lieberman. The inventory review policy for the multiple period stochastic demand model is:

Every T time periods, check the inventory level I , and order enough to bring the inventory back up to predetermined level. This "order-up-to" level should be enough to cover expected demand during the lead time, plus the time that will elapse before the next periodic review. The order quantity is defined as

$$Q = \bar{d}(T + L) - I$$

With Safety Stock

$$Q = \bar{d}(T + L) - I + z\sigma_{T+L}$$

$\bar{d}, z \; and \; \sigma$ are previously defined above.

2.4.2.10 The Economic Production Quantity (EPQ) Model

The EPQ, first developed by E.W. Taft in 1918, is an extension of the EOQ model (Taft, 1918). The difference between the these two models is that the EPQ model assumes that a firm will either produce its own quantity of inventory or the inventory will be procured and shipped to the firm as they are being produced. In either case, the inventory is replenished ***incrementally*** over time while consumption is occurring. In contrast, the basic EOQ model assumes that a complete order arrives ***instantaneously*** at a single point in time. The EPQ model and the EOQ model are single product lot scheduling models.

Similar assumptions for both the EOQ model and the EPQ model include:

- Only on type item is involved in the inventory

- The demand is known and is constant over the planning horizon
- The usage rate (demand) is constant
- Lead times are constant
- There is a fixed ordering/setup cost incurred by the appropriate model regardless of the number of units requested
- There is a holding or storage costs associated with each inventory item held in storage
- The unit acquisition cost is the same for all items acquired, i.e., no quantity discounts

The assumptions of the EPQ model that differ from the EOQ model are:

- While demand is continuous from period to period, the production occurs periodically.
- The production rate is constant
- ***Incremental*** replenishment begins when the inventory level reaches zero, i.e., during a production run, the production of items is continuous and at a constant rate

As can be seen in Figure 2-5, there is a *production and usage* phase and a *usage only* phase in the EPQ model. During the production and usage phase, inventory builds up at a rate equal to the difference between the production and usage rates with the assumption that the production rate is greater than the consumption rate. As long as production occurs, the inventory will continue to build. When production ceases, the inventory level will begin to decrease. The maximum inventory level is achieved at the point when production ceases. When the amount of

inventory on hand is exhausted, production is resumed and the cycle repeats itself. If a company is able to produce the desired inventory, there are no ordering costs as in the EOQ model. However, there are production setup costs associated with a production cycle. The EPQ inventory cycle is defined as the time of the start of one production run until the time of the start of the next production run as can be seen in Figure 2-5, encompassing both the Production and Usage periods and the Usage Only period. The production cycle begins when the inventory reaches zero and ends when the run size, Q_0 is reached. These setup costs would typically include equipment preparation such as cleaning, adjusting, changing tools and fixtures. These setup costs are analogous to the ordering costs in the EOQ model and are also independent of the production lot size.

Just as in the EOQ model, the economic production quantity is based on the determination of an optimum production lot size that minimizes the total inventory cycle costs.

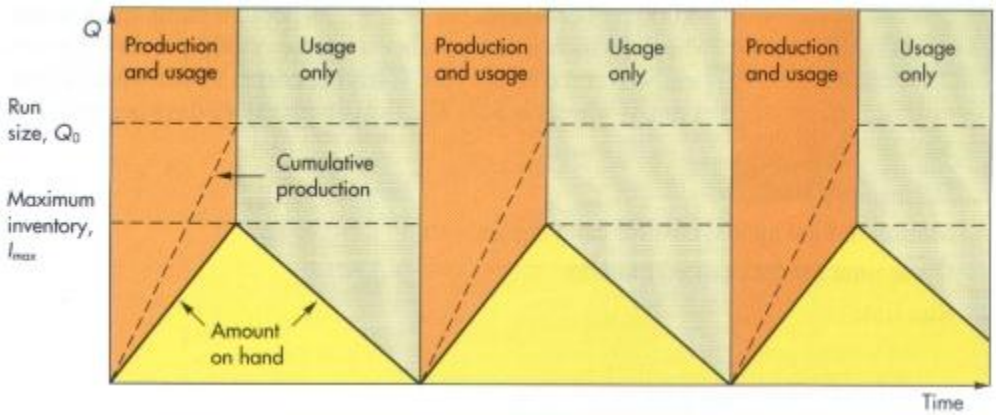


Figure 2-5 The Economic Production Quantity Model (Chen, 2004)

For the EPQ model, the total cycle cost is calculated as the sum of the carrying cost plus the setup costs:

$$Total\ Cycle\ Cost = \frac{I_{max}}{2}H + \frac{D}{Q_0}S$$

where I_{Max} is the maximum inventory, *H* is the holding cost per unit, *D* is the period demand, *S* is the setup/ordering costs and Q_0 is the economic run quantity that minimizes the Total Cycle Cost.

The economic run quantity is calculated as

$$Q_0 = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}}$$

where *p* is the production or delivery rate, and *u* is the usage rate per period, typically determined by dividing the demand (*D*) by the production rate *p*.

Other relevant EPQ estimates include:

The inventory cycle time: $\frac{Q_0}{u}$;

The production run time: $\frac{Q_0}{p}$;

The maximum inventory level per inventory cycle : $I_{max} = \frac{Q_0}{p}(p-u)$;

The average inventory per cycle: $I_{average} = \frac{I_{max}}{2}$.

As can be seen in Figure 2-2 for the EOQ model, the analogous economic run quantity for the EPQ model is determined by the intersection of the carrying (holding) cost curve and the order (production) quantity curve, resulting in the minimum for the total cost curve.

The EOQ and the EPQ models are based on a single inventory cycle. If the assumptions are maintained in each inventory period, then each cycle will be identical to every other cycle. Just as with the EOQ model, when constant demand assumptions are violated, the closed-form estimates of the model variables are not appropriate. In inventory theory, the Dynamic Lot Size model is one generalization of EPQ model. This model, just as in the material presented for the EOQ in the previous sections where the demand varies over time, must resort to solutions that do not have a closed form. Wagner and Whitin (Wagner & Whitin, 1958) provide an algorithm for finding the optimal solution by dynamic programming. Just as with the EOQ model, algorithms, heuristics and optimization methods must be applied for determining optimum economic run quantities and minimum total cycle cost for the EPQ model when period demand varies over time.

2.4.3 The EOQ, EPQ and the Human Capital Inventory Models

As a result of the literature search conducted for this study relating to traditional inventory models, it has been observed that the EOQ and EPQ model are the earliest and most well-known models in inventory theory. Since the development of the EOQ model (Harris, 1915) and the EPQ model (Taft, 1918), numerous research efforts have been undertaken to extend the basic EOQ and EPQ model by relaxing various assumptions so that these new models perform within a desired environment. Just as new inventory models have evolved over time by adjusting the basic assumptions of these two models, this study continues with the evolutionary

development of a Human Capital inventory model by modifying the basic assumptions of these models.

In the basic form of the EOQ and EPQ models, a primary assumption is that the demand is constant over time and that the inventory is consumed over time based on the demand. For the Human Capital inventory, this assumption is not true. In the Human Capital inventory model, the inventory is reduced through attrition, independent of demand. Since attrition reduces the Human capital inventory, the attrition assumption is that the attrition rate varies from period to period. The inventory replenishment within the Human Capital inventory model is assumed to be similar to the EOQ model, instantaneous, not incremental, and at the beginning of the first period within an inventory cycle. The instantaneous assumption is based on the fact that when the replenishment event is triggered, a procurement action for the optimum replenishment quantity is initiated for Human Capital that meets the Human Capital requirements. As observed in practice, only individuals that immediately meet the Human Capital requirements are considered. There is no gradual or incremental fulfillment of the replenishment quantity for Human Capital once a replenishment event and quantity has been identified. This instantaneous replenishment is assumed for any fixed replenishment lead time.

A summary comparing the basic EOQ, EPQ and Human Capital inventory models is found in Table 2-1. This table is not an exhaustive comparison of all potential model variations but it is a representation of the fundamental differences between the models. This summary identifies the similarities and differences between the basic EOQ/EPQ model and the Human Capital inventory model, with the most significant cells highlighted in ‘green’. Necessary extensions are made to the EOQ/EPQ model in the development of the Human Capital inventory

Table 2-1 EOQ, EPQ and Human Capital Model Comparison

Assumptions/Model Type	EOQ Model	EPQ Model	Human Capital Inventory Model
Objective	Optimum number of units of inventory to order with minimum total cost associated with inventory (Single item inventory).	Optimum number of units of inventory to produce/procure with minimum total cost associated with inventory (Single item inventory).	Optimum number of units of inventory to obtain with minimum total cost associated with inventory (Single item inventory.
Economic Quantity Solution – Minimum Cycle cost	Closed Form for single inventory Cycle	Closed Form for single inventory Cycle	No closed form exist, period demand varies
Demand	Deterministic – Constant rate per period during an inventory cycle	Deterministic – Constant rate per period during an inventory cycle	Deterministic – Rate can vary by period in inventory cycle.
Inventory Availability/Position	Consumed by Demand	Consumed by Demand	Independent of demand. Reduced by attrition

Table 2-1 EOQ, EPQ and Human Capital Model Comparison (continued)

Ordering/Setup/Procurement	Fixed one-time ordering cost for each inventory order, not based on quantity in order	Fixed one-time production setup or ordering cost for each inventory order, not based on quantity in order	Fixed one-time procurement cost for each inventory replenishment, not based on quantity in replenishment
Holding Cost	Applied to average inventory during inventory cycle	Applied to average inventory during inventory cycle	Applied to each unit of available inventory during each period
Shortage Costs	No Shortage costs in this model	No Shortage costs in this model	When inventory availability is less than period demand
Reorder Order point (ROP)	Inventory reaches zero	Inventory reaches zero	Reorder point based on desired service level
Inventory Cycle Time (T)	Closed form based on Optimum cycle quantity and period demand	Closed form based on Optimum cycle quantity and usage demand	No Closed Form Method of calculation due to varying attrition rate per period
Quantity discounts	None	None	None

Table 2-1 EOQ, EPQ and Human Capital Model Comparison (continued)

Economic Order/Production Quantity	Single Product – Lot scheduling	Single Product – Lot scheduling	Single Product – Lot scheduling
Reorder/Replenishment Method	Complete and immediate after specified lead time	Finite Replenishment (Constant Production rate). Production rate > Demand Rate. Incremental Replenishment during production cycle	Complete and immediate after specified lead time
Determination of Economic Order Quantity/Minimum Total Cycle Cost	First and second order derivatives	First and second order derivatives	No Closed form method. algorithms, heuristics, optimization
Lead Time	Fixed	Fixed	Fixed

model presented in this research. The full capabilities and the more complex assumptions of the Human Capital inventory model are discussed in Chapter 3, Methodology and Chapter 4, Model Development and Application.

2.5 Data Analysis

Based on the research question and objectives, data analysis associated with the Human Capital Inventory Cost Model requires a methodology and model for the attrition rate of the Human Capital inventory within an organization. Attrition is defined (Mifflin, 2009), in the context of Human Capital, as a gradual, natural reduction in membership or personnel, as through retirement, resignation, or death. Use of the term attrition in this document also includes any other reduction in personnel in an organization, whether voluntary or involuntary.

In the traditional inventory model, demand of the inventory resource consumes the available physical inventory, therefore reducing the number of units of inventory from period to period. However, in a Human Capital inventory, demand for resources do not consume available inventory, i.e., deplete, the resource from period to period. The reduction in the Human Capital inventory occurs due to attrition of the available resources. Because of this difference between the traditional inventory and the Human Capital inventory, estimation of attrition within the Human Capital inventory is necessary. Among the many statistical techniques for prediction and having knowledge of the nature of Human Capital inventories and the nature of attrition within a population, cluster analysis is a method to identify and group attrition related natural groups of Human Capital inventory personnel associated with attrition. After such grouping is accomplished, the techniques and statistical methods for applying survival analysis are utilized

for determining the attrition rates. Survival analysis is appropriate since attrition in a Human Capital inventory is typically event based, i.e., someone leaves and organization and there is a duration associated with the time to event, i.e., how long and individual was a member of the Human Capital inventory.

2.5.1 Cluster Analysis

The term cluster analysis, first used by (Tryon, 1939), encompasses a number of different algorithms and methods for grouping objects of similar kind into respective categories. A general question facing researchers in many areas of inquiry is how to organize observed data into meaningful structures, that is, to develop taxonomies, groups or clusters based on independent variables. In contract, discriminant analysis classifies observed data items into already known groups. Also, cluster analysis makes no distinction between dependent and independent variables. Cluster analysis is the obverse of factor analysis. Factor analysis reduces the number of variables by grouping items into a smaller set of factors. Whereas, cluster analysis reduces the number of observations or cases by grouping the data items into a smaller set of clusters. In other words, cluster analysis is an exploratory data analysis tool which aims at sorting different objects into groups in a way that the degree of association between two objects is maximal if they belong to the same group and minimal otherwise. Cluster analysis does not require that there is any prior knowledge about which elements belong to which clusters. Given the above, cluster analysis can be used to discover structures in data without providing an explanation/interpretation. In other words, cluster analysis simply discovers structures in data (Wasilewska, 2010).

2.5.1.1 What is Cluster Analysis?

Cluster analysis identifies and classifies objects, individuals or variables on the basis of the similarity of the characteristics they possess. It seeks to minimize within-group variance and maximize between-group variance. The result of cluster analysis is a number of heterogeneous groups with homogeneous contents. There are substantial differences between the groups, but the individuals within a single group are similar.

Data may be thought of as points in a space where the axes correspond to the variables. Cluster analysis divides the space into regions characteristic of groups that it finds in the data. The data may need to be preprocessed by outlier detection and standardization. Cluster Analysis can be used for outlier detection. Outliers may emerge as singletons or as small clusters far removed from the others. To do outlier detection at the same time as clustering the main body of the data, enough clusters should be developed to represent both the main body of the data and the outliers. Cluster Analysis is dependent upon the determination of 'distance' from one entity in a data set to another entity within the dataset.

2.5.1.2 Cluster Analysis Techniques

Cluster analysis follows one of two approaches: hierarchical and non-hierarchical (Wasilewska, 2010). Partitioning methods include Hierarchical methods, Density-based methods, Grid-based methods and Model-based methods. These partitioning methods divide the objects into a set of partitions based on some criteria. The partitioning process improves the partitions by shifting objects between them for higher intra-class similarity, interclass dissimilarity and other such criteria.

The choice of the algorithm depends on the type of data available and the nature and purpose of the application.

2.5.1.3 Hierarchical Clustering

Hierarchical clustering follows one of two approaches: Agglomerative methods or divisive methods. Agglomerative methods start with each observation as a cluster and with each step combine observations to form clusters until there is only one large cluster. Divisive methods begin with one large cluster and proceed to split into smaller clusters items that are most dissimilar. The use of inter-cluster distance measures determines whether a data item is associated with a particular cluster. There are five ways of defining inter-cluster distance:

1) single linkage (based on the shortest distance between objects);
2) complete linkage (based on the largest distance between objects);
3) average linkage (based on the average distance between objects);
4) Ward's method (based on the sum of squares between the two clusters, summed over all variables), and
5) centroid method (based on the distance between cluster centroids).

2.5.1.4 Density-based Methods

The density-based clustering approach is capable of finding arbitrarily shaped clusters, where clusters are defined as dense regions separated by low-density regions. Usually, density-based clustering algorithms are not suitable for high-dimensional data sets since data points are sparse in high-dimensional spaces. Density-based approaches apply a local cluster criterion. Clusters are regarded as regions in the data space in which the objects are dense, and which are

separated by regions of low object density (noise). These regions may have an arbitrary shape and the points inside a region may be arbitrarily distributed.

2.5.1.5 Grid-Based Methods

In general, a grid-based clustering algorithm consists of the following five basic steps:

1) partitioning the data space into a finite number of cells (or creating grid structure)
2) estimating the cell density for each cell,
3) sorting the cells according to their densities,
4) identifying cluster centres,
5) traversal of neighbor cells.

A major advantage of grid-based clustering is that it significantly reduces the computational complexity.

2.5.1.6 Model-Based Methods

In the framework of model based clustering algorithms, the data are assumed to come from a mixture of probability distributions, each of which represents a different cluster. There is a huge number of published works related to model-based clustering algorithms.

2.5.1.7 Nonhierarchical Clustering

Nonhierarchical clustering is a partitioning of the sample data set. Each cluster has a seed point and all objects within a prescribed distance are included in that cluster.

Another way of nonhierarchical clustering is to loop through the sample, assigning each case to the seed point to which it is closest. Nonhierarchical clustering has three approaches:

1) the sequential threshold (based on one cluster seed at a time and membership in that cluster fulfilled before another seed is selected, i.e., looping through all n points before updating the seeds, as in the K-MEANS procedure),

2) parallel threshold (based on simultaneous cluster seed selection and membership threshold distance adjusted to include more or fewer objects in the clusters, i.e., updating the seeds as you go along, as in the ISODATA procedure), and

3) optimizing (same as the others except it allows for reassignment of objects to another cluster based on some optimizing criterion).

The process when using nonhierarchical methods involves an initial selection of seed points for clusters. If we let *k* denote the number of clusters to be formed, we assign an initial "seed" for each *k*. The result of the initial clustering depends upon the seed points. Therefore, there is an advantage to perform the clustering using different seed points. The selection of the *k* initial seeds can arbitrarily be

- the first *k* cases
- a randomly chosen *k* cases
- *k* specified cases
- or chosen from a *k*-cluster hierarchical solution.

2.5.1.8 Hierarchical versus Nonhierarchical Methods

While there is no definite rule as to which type of clustering to use, it is suggested that both be used. It is recommended to start with hierarchical to generate and profile the clusters and

then use nonhierarchical to fine tune the cluster membership with its switching ability. In this case, the centroids from hierarchical clustering are taken as the seeds for nonhierarchical clustering.

2.5.1.9 How Many Clusters Should Be Formed?

There is no generally accepted procedure for determining the number of clusters. This decision should be guided by theory and practicality of the results, along with use of the inter-cluster distances at successive steps. When using a criterion such as between-groups sum of squares or likelihood, this can be plotted against the number *k* of clusters. Also, the likelihood can be used in model selection criteria such as AIC (Akaike's Information Criterion) or BIC (Bayesian Information Criterion) to estimate *k*.

1) AIC = - 2 log likelihood + 2*number of parameters
2) BIC = -2 log likelihood + log(n)*number-of-parameters

Most statisticians who use model-selection criteria are leaning toward BIC instead of AIC.

2.5.1.10 Interpretation of the Clusters

This is a creative process. Examination of the cluster profiles will provide the researcher with insight as to what the clusters mean.

2.5.1.11 Cluster Validation

Statistical tests can be utilized to determine the degree and effectiveness of the clustering process. In order to test the hypothesis that there is no clustering (i.e., that the population is homogeneous) among the clustering process that yields *k* clusters, compute the mean vector and

covariance matrix of the whole sample. Draw pseudorandom samples of from each of the *k* clusters from the corresponding multinomial distribution and compute a measure of spread of the clusters. This generates a sampling distribution for the measure of spread. If the value for the actual sample is among the highest, you've got statistical significance.

Another validation methodology is to split the original sample into a training set and a test set. The centroids from the clustering of the training cases can be used to cluster the test cases to see if comparable results are obtained.

Another method of validating and determining the effectiveness of clustering is the use of *silhouettes* (Rousseeuw, 1987) . Assuming a dataset has been clustered via any technique, such as K-Means, into *k* clusters, a S*ilhouette Index* is calculated based on a measure of within cluster similarity and a measure of among cluster dissimilarity. The method of calculating the *Silhouette Index* is follows.

For each data item *i* in the dataset let *a*(*i*) be the average dissimilarity of *i* with all other data within the same cluster. Any measure of dissimilarity can be used but distance measures are the most common. The interpretation of *a*(*i*) is how well matched *i* is to the cluster it is assigned (the smaller the value, the better the matching). The average dissimilarity of *i* with the data of other clusters must also be determined. The similarity and dissimilarity measure is performed for each data item in each cluster. The lowest average dissimilarity to *i* is denoted by *b*(*i*). The *Silhouette Index* is defined as

$$s(i) = \frac{b(i) - a(i)}{\max\{a(i), b(i)\}}$$

Which can be written as:

$$s(i) = \begin{cases} 1 - a(i)/b(i), & \text{if } a(i) < b(i) \\ 0, & \text{if } a(i) = b(i) \\ b(i)/a(i) - 1, & \text{if } a(i) > b(i) \end{cases}$$

From the above definition it is clear that

$$-1 \leq s(i) \leq 1$$

For $s(i)$ to be close to 1. It is required $a(i) < b(i)$. As $a(i)$ is a measure of how dissimilar i is to its own cluster, a small value means it is well matched. Furthermore, a large $b(i)$ implies that i is badly matched with other clusters. Thus an $s(i)$ close to one means that the data items in a cluster is appropriately clustered. If $s(i)$ is close to negative one, then by the same logic, i would be more appropriate if it was clustered in another cluster. An $s(i)$ nears zero, data items are on the border of two natural clusters.

The average $s(i)$ of a cluster is a measure of how tightly grouped all the data in the cluster are. Thus the average $s(i)$ of the entire dataset is a measure of how appropriately the data has been clustered. If there are too many or too few clusters, as may occur when a poor choice of k is used in the k-means algorithm, some of the clusters will typically display much narrower silhouettes than the rest. Thus silhouette plots and averages may be used to determine the natural number of clusters within a dataset.

2.5.2 Survival Analysis

The U.S. Bureau of Labor Statistics (Dohm, 2000) estimates that about 76 million Baby Boomers, or 43 percent of the current work force, will be eligible to retire within the next 10 to

15 years. The boomer generation, those born post World War II from 1948 to 1964, will hit retirement age anywhere from 2010 to 2026. As the oldest baby-boomers begin retiring in the next several years, the implications for the workforce could be enormous. In many organizations and industries, the attrition of the “baby boomers” is causing what has been referred to as a "huge knowledge gap". The current tight labor market situation could also be exacerbated, hindering prospects for economic growth and putting a greater burden on those remaining in the workforce, perhaps forcing them to work longer hours, especially in occupations with functions less conducive to technology-driven productivity innovations. Many of the jobs in health services and educational services, for example—service may suffer.

Attrition is a reduction in the number of employees in an organization. This attrition could be caused by retirement, resignation, reassignment, transfers, layoffs or terminations. Understanding the nature of the attrition in a workforce is critical in evaluating the future staffing of organizations. Attrition analysis is necessary to identify the relationship between a subject’s attrition and the variables that affect attrition. The goal of attrition analysis is to understand the most important variables that contribute to attrition and what the likelihood of attrition is for each subject. A powerful method for analyzing attrition and forecasting the probability of subject attrition is *Survival Analysis* (Hosmer & Lemeshow, 2008).

Survival analysis encompasses a wide variety of methods for analyzing the timing of events. Survival Analysis is concerned with the studying the time between an entry point in a study and the subsequent event of interest. The typical event that is utilized in Survival Analysis is death. Other examples of the application of Survival Analysis include applications in the medical as well as non-medical fields. Medical examples include the times to relapse after drug

application, times from drug treatments to death. Non-medical examples include time from making a loan to full repayment (Banking), time from graduation until first job (Economy) or time from being single to getting married (social). In the application of Human Capital inventory attrition analysis, the event to be utilized is the point of time an individual leaves the organization and the duration of the employee in an organization.

Survival Analysis has been applied in different disciplines where the terminology varies from discipline to discipline. The term “Survival Analysis” is used in the biostatistics field. “Failure Time Analysis” is used in the Engineering field and “Event History Analysis” is used in the sociology field.

Survival Analysis, an appropriate statistical method, is utilized to investigate, analyze and apply survival regression models to forecast attrition in a public sector Human Capital inventories. The event of interest is the event of leaving the organization.

2.5.2.1 Why Use Survival Analysis

Unlike linear regression that yields linear changes, survival analysis has a dichotomous (binary) outcome. Unlike logistic regression that results in odds ratios, survival analysis analyzes the time to an event. The advantages of Survival Analysis include (Gage, 2004):

1. Ability to account for censoring.
2. Comparing survival between 2+ groups
3. Assessing the relationship between covariates and survival time

In a comparison of regression analysis with survival analysis, survival analysis is a more appropriate method for analyzing time-to-event data. The advantages of survival analysis over

linear regression includes the ability of survival analysis to have independent time, categorical or continuous variables where linear regression only allows categorical or continuous independent variables. The dependent variable in linear regression is assumed to be normally distributed where the dependent variable in event analysis is binary. Linear regression does not allow censoring of data where censoring is necessary in survival analysis. Logistic regression would seem to be an appropriate statistical analysis to use in event analysis. However, logistic regression does not allow the censoring of data. Logistic regression does allow categorical or continuous independent variables but does have a binary dependent variable except in the case of multinomial logistic regression (more than two dependent variables).

More generally, survival analysis involves the modeling of time to event data. Many concepts in Survival analysis have been explained by the Counting Process Theory (Chang & Hsiung, 2005) which has emerged more recently. The flexibility of a counting process is that it allows modeling multiple (or recurrent) events. This type of modeling fits very well in many situations (e.g. people can go to jail multiple times, alcoholics can start and stop drinking multiple times, and people can get married and divorced multiple times).

Survival analysis attempts to answer questions such as: what is the fraction of a population which will survive (or experience an event) past a certain time? Of those that survive (experience the event), at what rate will the event of interest occur? Survival Analysis assumes well-defined events at specific times. The theory of survival presented here also assumes that the attrition event *happens just once for each subject.*

2.5.2.2 Significant Survival Analysis Functions

The object of primary interest is the survival function, also called survivorship function, and is conventionally denoted as S, which is defined as

$$S(t) = \Pr(T > t)$$

where t is some time, T is a random variable denoting the time to the event of interest, and "Pr" stands for probability. That is, the survival function is the probability that the time of the event is later than some specified time. The survival function is also called the survivorship function in problems of biological survival, and the reliability function in mechanical survival problems. In the latter case, the reliability function is denoted R(t). Usually one assumes S(0) = 1, although it could be less than 1 if there is the possibility of immediate death or failure.

The survival function must be non-increasing: $S(u) \leq S(t)$ if $u > t$. This property follows directly from $F(t) = 1 - S(t)$ being the integral of a non-negative function. This reflects the notion that survival at a later age is only possible if surviving all younger ages. Given this property, the lifetime distribution function and event density (F and f below) are well-defined.

The survival function is usually assumed to approach zero as age increases without bound, i.e., $S(t) \to 0$ as $t \to \infty$, although the limit could be greater than zero if eternal life is possible.

2.5.2.3 Lifetime Distribution Function and Event Density

Related quantities are defined in terms of the survival function. *The lifetime distribution function*, conventionally denoted *F*, is defined as the complement of the survival function,

$$F(t) = \Pr(T \leq t) = 1 - S(t)$$

and the derivative of *F* (i.e., the density function of the lifetime distribution) is conventionally denoted *f*,

$$f(t) = F'(t) = \frac{d}{dt}F(t)$$

f is sometimes called the event density; it is the rate of the event per unit time.

The survival function is often defined in terms of distribution and density functions

$$S(t) = \Pr(T > t) = \int_t^\infty f(u)\,du = 1 - F(t).$$

Similarly, a survival event density function can be defined as

$$s(t) = S'(t) = \frac{d}{dt}S(t) = \frac{d}{dt}\int_t^\infty f(u)\,du = \frac{d}{dt}[1 - F(t)] = -f(t)$$

2.5.2.4 Hazard Function and Cumulative Hazard Function

The hazard function, conventionally denoted λ, is defined as the event rate at time t conditional on survival until time t or later (that is, T ≥ t),

$$\lambda(t)\,dt = \Pr(t \leq T < t + dt \,|\, T \geq t) = \frac{f(t)\,dt}{S(t)} = -\frac{S'(t)\,dt}{S(t)}$$

The hazard function must be non-negative, λ(t) ≥ 0, and its integral over [0, ∞] must be infinite, but is not otherwise constrained; the hazard function may be increasing or decreasing, non-monotonic, or discontinuous. An example is the bathtub curve hazard function, which is large for small values of t, decreasing to some minimum, and thereafter increasing again; this can

model the property of some mechanical systems to either failure soon after operation, or much later, as the system ages.

The hazard function can alternatively be represented in terms of the cumulative hazard function, conventionally denoted Λ:

$$\Lambda(t) = -\log S(t)$$

solving for *S(t)* yields:

$$S(t) = exp(-\Lambda(t)) ,$$

or differentiating (with the chain rule):

$$\frac{d}{dt}\Lambda(t) = -\frac{S'(t)}{S(t)} = \lambda(t)$$

Λ is called the cumulative hazard function because the preceding definitions together imply

$$\Lambda(t) = \int_0^t \lambda(u)\, du ,$$

which is the "accumulation" of the hazard over time.

From Λ(t) = − logS(t) we see that Λ(t) increases without bound as t tends to infinity (assuming S(t) tends to zero). This implies that λ(t) must not decrease too quickly, since, by definition, the cumulative hazard has to diverge. For example, exp(− t) is not the hazard function of any survival distribution, because its integral converges to 1.

2.5.2.5 Quantities Derived From the Survival Distribution

Future lifetime at a given time t_0 is denoted by the time remaining until the occurrence of the event, thus future lifetime is $T - t_0$ in the present notation. The expected future lifetime is

the expected value of future lifetime. The probability of an event at or before $T + t_0$, given survival until t_0, is just

$$P(T \leq t_0 + t | T > t_0) = \frac{P(t_0 < T \leq t_0 + t)}{P(T > t_0)} = \frac{F(t_0 + t) - F(t_0)}{S(t_0)}$$

Therefore the probability density of future lifetime is

$$\frac{d}{dt} \frac{F(t_0 + t) - F(t_0)}{S(t_0)} = \frac{f(t_0 + t)}{S(t_0)}$$

and the expected future lifetime is

$$\frac{1}{S(t_0)} \int_0^\infty t\, f(t + t_0)\, dt = \frac{1}{S(t_0)} \int_{t_0}^\infty S(t)\, dt$$

For $t_0 = 0$, i.e., at birth, this reduces to the expected lifetime. In reliability problems, the expected lifetime is called the mean time to failure, and the expected future lifetime is called the mean residual lifetime. The probability of individual survival until *t* or later is *S(t)*, by definition. The expected number of survivors, in a population of n individuals, is $n \times S(t)$, assuming the same survival function for all. Thus the expected proportion of survivors is *S(t)*, and the variance of the proportion of survivors is $S(t) \times (1-S(t))/n$.

The age at which a specified proportion of survivors remain can be found by solving the equation $S(t) = q$ for *t*, where *q* is the quartile in question. Typically one is interested in the median lifetime, for which q = 1/2, or other quartiles such as q = 0.90 or q = 0.99.

2.5.2.6 Survival Regression Analysis

Cox Regression is a well-recognized statistical technique for analyzing survival data (Cox D. , 1984). Cox regression is a statistical technique for exploring the relationship between

the survival of a subject and several explanatory variables. A Cox model is a multivariate technique for analyzing the effects of two or more covariates (metric or non-metric) on survival. It also estimates the hazard (or risk) of the event of interest for an individual given their covariates. Unlike other models (Life Tables, Kaplan-Meier or Event History models) that are utilized in event modeling, the Cox regression model is semi-parametric (Cox D. R., 1972). No baseline hazard rate is required to be identified a priori for the hazard function. Survival-time data have two important special characteristics:

1. Survival times are non-negative and are typically positively skewed.
2. Typically, subjects in the study have censored survival times. Censored survival times of subjects are not observed because the event of interest does not take place in the time frame of the study. Failure to take censoring into account can produce serious bias in estimates of the distribution of survival time and related quantities.

Subject censoring takes the following forms (Figure 2-6):

1. Uncensored – a subject enters and leaves within the period of study.
2. Right Censoring – a subject enters the study period but does not leave the study during the study period.
3. Left censoring – a subject leaves during the study period but the beginning of the subject is not known.

Survival time is the object of study in Survival Analysis. Survival time should not be confused with calendar time. Survival time is measured relative to a time origin such as the beginning of employment. However, subjects may be entering and leaving the study at different calendar time.

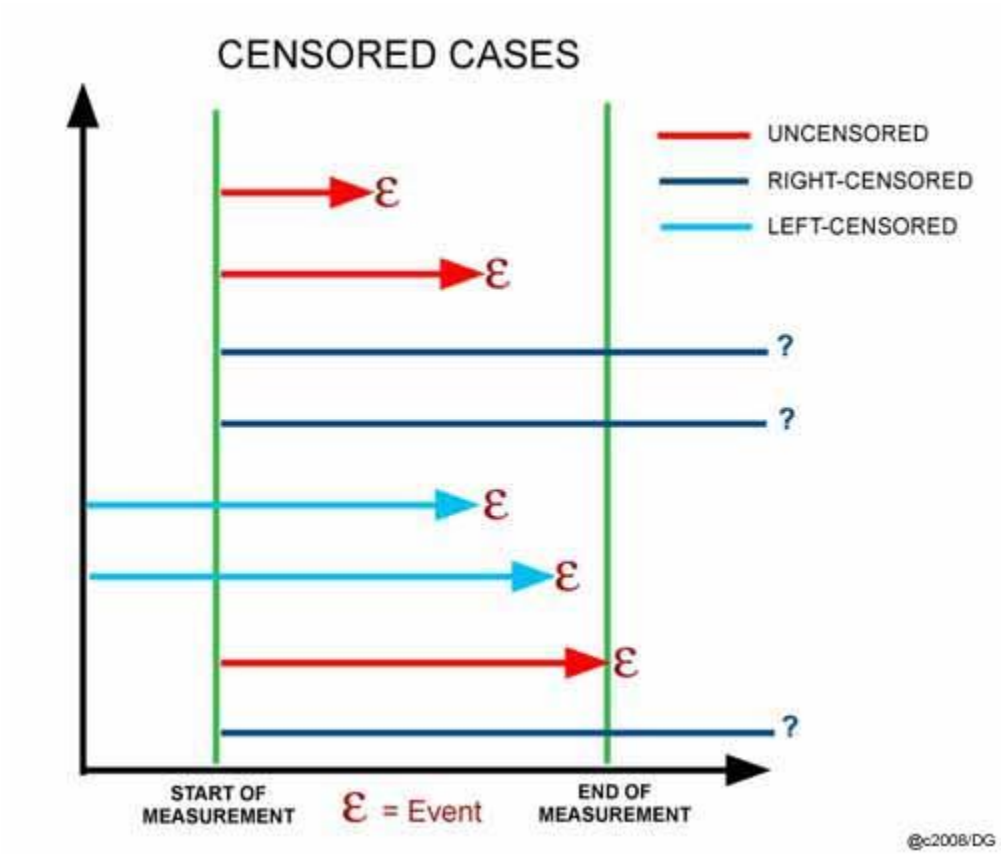


Figure 2-6 Censored and Uncensored Data Visualization

The techniques developed in studying survival data include these basic concepts:

1. Right censored survival data
2. The survival time of each subject or the time that the observation of the subject is censored.
3. Whether or not the subject's survival time is censored.

4. The value of one or more explanatory variables (covariates) that may influence survival times.

2.5.2.7 The Cox Hazard Model

Let x_{ij} be the jth covariate for the ith individual with $j = (1,2, \dots N)$. The basic model:

$$H_i(t) = \Lambda_o(t)e^{b_i x_{i1} + \cdots + b_n x_{in}}$$

where $\Lambda_0(t)$ is the baseline hazard function and is a linear combination of the *n* covariates which is exponential. The model (Cox D. R., 1972) is called the proportional hazards model because the hazard of any individual is a fixed proportion of the hazard of any other individual:

$$\frac{H_i(t)}{h_k(t)} = e^{\{b_1(x_{i1} - x_{k1}) + \cdots + b_n(x_{in} - x_{kn})\}}$$

The model (Log-relative Hazard) can also be expressed as:

$$ln\left\{\frac{h(t)}{h_o(t)}\right\} = b_1 x_{1+\cdots+} b_k x_k$$

Where *h(t)* is the Hazard function at time *t*, $h_0(t)$ is the baseline hazard or hazard for an individual when the value of all covariates are zero.

2.5.2.7.1 Cox Model Assumptions

The hazard function is an estimate of the relative risk of the terminal event. The potential of the terminal event in the hazard function is a per-unit-time rate for an individual that has survived up to that point. As a rate, it can vary:$0 \leq h(t)$. The hazard rate is not a probability of

the terminal event but the rate of the occurrence of an event at time t. The greater the value of $h(t)$, the greater the rate of the event. The Cox Model assumptions include:

1. Independent variables are identified where one independent variable is the duration of the event, another is a dichotomous variable for the event, typically 1 if the event occurred, 0 (censored) if the event has not occurred and other covariates may be metric or non-metric.
2. There is no assumption about the distribution of the hazard function
3. Changes in the levels of the independent variables will produce proportional changes in the hazard function, independent of time.
   - $ln\left\{\frac{h(t)}{h_o(t)}\right\} = b_1x_{1+\cdots+}b_kx_k$
   - A log-linear relationship exists between the hazard function and the independent covariates: $ln\left\{\frac{h(t)}{h0(t)}\right\} = b_1x_1 + \cdots + b_kx_{k1}$

The relationship between the Cumulative Hazard function and the Cumulative Survival function is

$$H(t) = -\ln(S(t))$$

when $S(t)$is the Cumulative Survival Function.

2.5.2.7.2 Cox Statistical Tests

In statistics, *a likelihood ratio test* is used to compare the fit of two models, one of which is nested within the other. This often occurs when testing whether a simplifying assumption for a model is valid, as when two or more model parameters are assumed to be related. Both models

are fitted to the data and their log-likelihood recorded. The test statistic (usually denoted D) is twice the difference in these log-likelihoods:

$$D = -2(\ln(likelihood\ for\ null\ model) - \ln(likelihood\ for\ alternative\ model))$$

or

$$D = -2\ln\left(\frac{likelihood\ for\ null\ model}{likelihood\ for\ alternative\ model}\right)$$

The model with more parameters will always fit at least as well (have a greater log-likelihood). Whether it fits significantly better and should thus be preferred can be determined by deriving the probability or p-value of the obtained difference D. In many cases, the probability distribution of the test statistic can be approximated by a chi-square distribution with $(df1 - df2)$ degrees of freedom, where $df1\ and\ df2$ are the degrees of freedom of models 1 and 2 respectively. The test requires nested models, that is, models in which the more complex one can be transformed into the simpler model by imposing a set of linear constraints on the parameters.

2.5.2.7.3 Significant Survival Analysis Statistical Tests

A Wald test is used to test the statistical significance of each coefficient in the Cox regression model. *A Wald* test calculates a Z statistic, which is:

$$z = \frac{\hat{B}}{SE}$$

This z value is then squared, yielding a Wald statistic with a chi-square distribution. However, several authors have identified problems with the use of the Wald statistic. For large coefficients, the standard error is inflated, lowering the Wald statistic (chi-square) value. The likelihood-ratio test is more reliable for small sample sizes than the Wald test.

Likelihood-Ratio Test: The likelihood-ratio test uses the ratio of the maximized value of the likelihood function for the full model (L_1) over the maximized value of the likelihood function for the simpler model (L_0). The likelihood-ratio test statistic equals:

$$-2\log\left(\frac{L_0}{L_1}\right) = -2[\log(L_0) - \log(L_1)] = -2(L_0 - L_1)$$

This log transformation of the likelihood functions yields a chi-squared statistic. This is the recommended test statistic to use when building a model through backward stepwise elimination.

2.6 Optimization Methods

Based on the literature review of traditional inventory models, a review of only relevant optimization techniques was performed. Rather than a comprehensive review of the field of optimization techniques and methodologies, this section will focus only on the relevant optimization techniques and methods applicable to this study. Application of the researched optimization techniques and methods are applied in Chapter 3, Methodologies.

In order to determine the appropriate optimization methods to consider for this research, it is advantageous to understand the underlying closed form calculation of the independent variables associated with traditional inventory modeling. A discussed in the traditional Economic Order Quantity model, the order quantity is calculated as

$$Q^* = \sqrt{\frac{2aK}{h}}\sqrt{\frac{p+h}{p}}.$$

Although this is a closed form for calculating the order quantity for a single period EOQ inventory model, it is obvious that this is a non-linear independent variable solution for the EOQ.

Based on this non-linear nature of this solution, only non-linear optimization techniques and methods will be considered for this research. Also, it is obvious that the only constraints associated with the solution for the EOQ model is that *Q* be positive. The objective function for the traditional EOQ model is of the form

$$\text{Total Cost per unit time} = \frac{K + cQ + \frac{hQ^2}{2a}}{\frac{Q}{a}} = \frac{aK}{Q} + ac + \frac{hQ}{2}.$$

Although the Human Capital inventory optimization model will not have this exact form for its cost per unit time, the calculation will be similar in nature, resulting in a non-linear objective function. Based on the non-linear nature of the order quantity and the object function, research will be limited to unconstrained, non-linear techniques and methods.

2.6.1 NonLinear Optimization

The general non-linear optimization problem has the form (Bazaraa, Hanif, & Shetty, 2006):

$$\min_{x \in R^d} f(x) \quad where\ x = (x_1, ..., x_d)$$

subject to

$$\begin{aligned} &g_i(x) = 0 && i = 1, ..., m_e \\ &g_i(x) \le 0 && i = m_{e+1}, ..., m \\ &x_l \le x \le x_u \end{aligned}$$

where $f(x)$ and $g(x)$ are given functions of the *n* decision variables.

In particular, if $m = 0$, the problem is called an unconstrained optimization problem. In our research, there are no constraints. Therefore $m = 0$ in our study. Again, depending on the

characteristics of *f(x)* and *g(x)*, different algorithms are used. Because of the many types and the many algorithms, we will not attempt to survey the non-linear subject in its entirety. We will address the specific problem as defined in this paper, i.e., an unconstrained non-linear optimization with non-linear objective function and non-linear independent variables.

2.6.2 Local and Global Solutions

The necessary conditions that a particular solution $x = (x_1, x_2, \dots,)$ be optimal is a differentiable function is.$\frac{df}{dx_j} = 0\ at\ x = \ x^* for\ j = 1,2, \dots, n$. However, when solving the system of *n* equations obtained by setting the *n* partial derivatives equal to 0, there is no guarantee that these equations would be linear which would enable one to solve these equations without extreme difficulty. Thus, algorithmic search procedures for finding x^* are utilized for finding optimum solutions (Hillier & Lieberman, 2005).

Linearly Constrained Optimization problems are characterized by linear constraints that fit the linear programming method with a non-linear objective function. Special algorithms that have extended the simplex method have been developed for the linearly Constrained Optimization problem. Quadratic Programming also has linear constraints but the objective function is quadratic, i.e., terms in the objective function is squared or there is a product of two variables. Many algorithms have been developed for this case with the additional assumption that *f(x)* is a concave function. Convex programming is characterized by a concave objective function, *f(x)* with each constraint, *g(x)* is also convex. This situation ensures that a local maximum is a global maximum. Separable Programming is a special case of convex programming with one additional assumption that *f(x)* and *g(x)* functions are separable. A separable function is a function where each term involves just a single variable so that the

function is separable into a sum of functions of individual variables. Thus if *f(x)* is separable, $F(x) = \sum f_j(x_j)\ where f_j(x_j) * F(x)$ only includes the terms involving just x_j. Nonconvex Programming encompasses all nonlinear programming problems that do not satisfy the assumptions of convex programming. In this model, if a local optimum solution is found, it does not guarantee that the local solution is a global solution. Other such nonlinear programming problems exist such as Geometric Programming and Fractional Programming.

In reference to the EOQ inventory model, since the Total Cycle Cost, $K + cQ + \frac{hS^2}{2a} + \frac{p(Q-S)^2}{2a}$. as well as the Cycle Order Quantity, $Q^* = \sqrt{\frac{2aK}{h}}$ do have first order derivatives, they are not linear. Therefore, the methods described above do not adequately provide optimum solutions for the non-linear optimization problem. However, methods and algorithms have been developed to address the unconstrained nonlinear estimation problem that rely on the comparison of each trial solution with the best obtained up to the last trial. One such method is referred to as 'direct search' (Hooke & Jeeves, 1961).

Direct search methods assume that even if the objective function *f(x)* is differentiable, the gradient of *f(x)* is either unavailable or unreliable (Gill, Murray, & Wright, 1986). The most famous of all direct search methods is the simplex algorithm of Nelder and Mead (Nelder & Mead, 1965). A popular and effective search algorithm, the Nelder-Mead algorithm is designed to solve the classical unconstrained optimization problem of minimizing a given nonlinear function $f : \mathbb{R}^n \to \mathbb{R}$. The method uses only function values at some points in $\mathbb{R}^n$, and does not try to form an approximate gradient at any of these points. Hence it belongs to the general class of direct search methods (Wright, 1966).

In many practical problems, like parameter estimation and process control, the function values are uncertain or subject to noise. Therefore, a highly accurate solution is not necessary, and may be impossible to compute. All that is desired is an improvement in function value, rather than full optimization.

The Nelder-Mead method frequently gives significant improvements in the first few iterations and quickly produces quite satisfactory results. Also, the method typically requires only one or two function evaluations per iteration, except in shrink transformations, which are extremely rare in practice. This is very important in applications where each function evaluation is very expensive or time-consuming. For such problems, the method is often faster than other methods, especially those that require the least function evaluations per iteration. In many numerical tests, the Nelder-Mead method succeeds in obtaining a good reduction in the function value using a relatively small number of function evaluations. Apart from being simple to understand and use, this is the main reason for its popularity in practice.

Due to the complexity of the computation of the objective function, analytical expressions for the first and second derivatives are not available. Since these derivatives are not available. Any optimization or search techniques that rely on these derivatives are not an option for use in this research.

Since a function for computing the Hessian used in computing the direction is rarely available, attention has focused on computing it numerically (Schoenberg, 2001). The calculation of the Hessian is very expensive computationally, however, and efforts were made to find a way to produce the Hessian more cheaply. The critical insight from which came the current quasi-

Newton methods was made by (Broyden, 1969). Further work has developed other types of secant updates, the most important of which are the DFP. DFP, the first quasi-Newton method, is named after Davidson, who discovered it in 1959, and Fletcher and Powell, who explored its mathematical properties over the next few years. Instead of computing the true Hessian as in Newton's method, an approximation is made based on the change in gradient between iterations. The primary advantage is that there is not a need to compute the exact Hessian at each point, which may be computationally expensive. The BFGS, named after (Broyden, 1969, Fletcher, 1970, Goldfarb, 1970, and Shanno, 1970), is generally regarded as the best performing method. This quasi-Newton optimization method has been successful primarily because its method of generating an approximation to the Hessian encourages better conditioning. Nearly all implementations of the Newton method involve a numerical calculation of the Hessian.

The use of the quasi-newton method is advantageous when the evaluation of the Hessian is impractical or costly. The central idea underlying quasi-Newton methods is to use an approximation of the inverse Hessian. The form of approximation differs among methods.

The quasi-Newton methods that build up an approximation of the inverse Hessian are often regarded as the most sophisticated for solving unconstrained problems. The following two formulas have received wide acceptance:

- Davidon -Fletcher-Powell (DFP) formula
- Broyden-Fletcher-Goldfarb-Shanno (BFGS) formula.

- Earliest (and one of the most clever) schemes for constructing the inverse Hessian. Originally proposed by Davidon (1959) and later developed by Fletcher and Powell (1963).
- It has the interesting property that, for a quadratic objective, it simultaneously generates the directions of the conjugate gradient method while constructing the inverse Hessian.
- The method is also referred to as the variable metric method (originally suggested by Davidon).
- Broyden–Fletcher–Goldfarb–Shanno formula is more complicated than DFP, but straightforward to apply
- BFGS update formula can be used exactly like DFP formula.

Both DFP and BFGS methods have theoretical properties that guarantee superlinear (fast) convergence rate and global convergence under certain conditions. However, both methods could fail for general nonlinear problems. Specifically, the DFP is highly sensitive to inaccuracies in line searches. Both methods can get stuck on a saddle-point. In Newton's method, a saddle-point can be detected during modifications of the (true) Hessian. Therefore, searches are made around the final point when using quasi-Newton methods. The update of the Hessian becomes "corrupted" by round-off and other inaccuracies. All kind of "tricks" such as scaling and preconditioning exist to boost the performance of the methods.

2.7 Summary

The literature research performed in Chapter 2 has focused on the need for developing appropriate methods of an attrition model for the members of a population. Clustering techniques

for data analysis is explored due to the natural clustering of the Human Capital data that is used in this research. Survival analysis techniques are utilized to determine the survival function for attrition of members of the population in the data sets. After appropriate clustering techniques have been applied and after the utilization of survival analysis has been performed to create an attrition model, non-linear optimization techniques are utilized to determine optimum Human Capital inventory variables associated with Human Capital availability.

Chapter 3 - Methodology

3.1 Introduction

The Human Capital planning process is a phased approach that reflects the strategic business goals of and organization (Figure 3-1). This research will focus on Phase II, conducting the workforce analysis. Phase II should answer the question whether the Human Capital will meet the strategic goals and how ready is the workforce and organizational structure to execute these goals? The analysis also includes an analysis of the current supply and forecasting the expected demand for Human Capital to successfully execute the agency's business plan. Phase II activities should also include the analysis of the attrition factors and the current supply or availability of the Human Capital workforce. The ability to identify the gap between the demand and supply is also performed in Phase II.

This chapter presents the methodologies and techniques utilized for cluster and survival analysis and the application of these techniques within the application of an optimized Human Capital Inventory Cost Model. The techniques and analysis applied in this research and the relationship and flow of the analysis is depicted in Figure 3-2. The two datasets that are utilized in this research are transformed into the format that is most effective for use in this research. Utilizing the dataset records of employees that have separated from the organization during the study, clustering techniques are utilized for creating appropriate groupings of individual employees into 'homogeneous' groups based on the age and years-of-company-service. Silhouette analysis is utilized to determine and select the 'best' grouping (clusters) of the separated employees. Once the clusters are determined from the separated employees, the set of

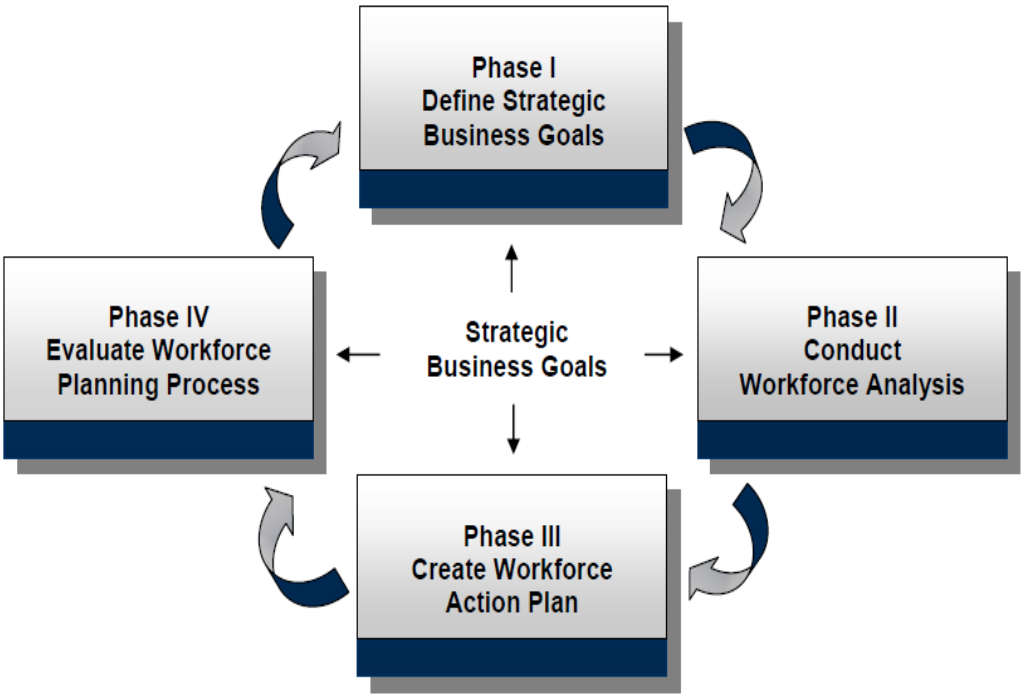


Figure 3-1 Human Capital Phased Planning Process (OSP, 2008)

Data Analysis, Modeling and Optimization

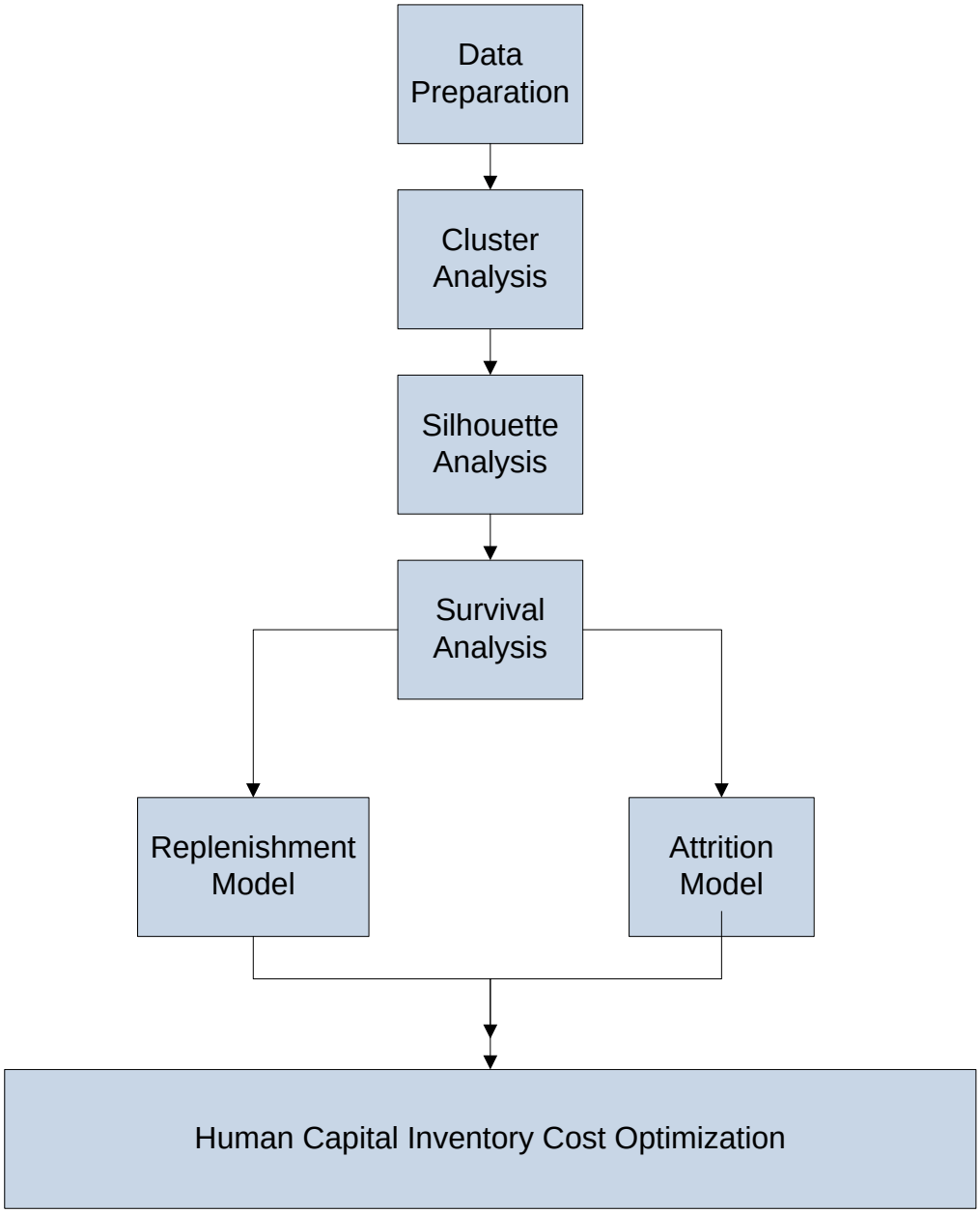


Figure 3-2 Data Analysis, Modeling and Optimization

employees that are a part of the current population of employees at the end of the study is assigned to a cluster based on the 'nearest neighbor' technique. Employee attrition distributions are created for each cluster. These attrition distributions are utilized in the inventory cost model for the purpose of reducing the available Human Capital inventory from one period to the next. The New Hire Distribution and the Employee Attrition distribution are utilized in the Human Capital inventory model to determine inventory availability, inventory replenishment and inventory period costs. The Human Capital Inventory Cost Model is 'wrapped' within the Optimization function to determine minimum inventory cycle costs. Details of each of the functions are discussed in the following sections.

3.1.1 Cluster Analysis

Cluster analysis is utilized for the assignment of a set of observations into subsets (called clusters) so that observations in the same cluster are similar, or clustered in such a manner so that a homogeneous grouping of the observations is achieved. The results achieved by the cluster analysis is a statistical significant grouping of the observations associated with the age and years of company service of individuals in the observed observations. This clustering provides a more accurate method of determining attrition rates within the observed population, thus reducing the variance in the measurements for multiple attrition rates. Once statistical significant clusters are determined, survival analysis is utilized to determine attrition rates for each cluster.

3.1.2 Survival Analysis

Survival analysis involves the application of statistical techniques such as Life Tables, Kaplan-Meier or Cox Regression to determine the probability of survival over time. The Kaplan-

Meier procedure uses a method of calculating life tables that estimates the survival or hazard function at the time of each event. The Life Tables procedure uses an actuarial approach to survival analysis that relies on partitioning the observation period into smaller time intervals and are useful for dealing with large samples. If there are variables that are related to survival time or variables that you want to control for (covariates), the Cox Regression procedure is a more appropriate statistical methodology to apply. If covariates have different values at different points in time for the same case, Cox Regression with Time-Dependent Covariates should be utilized. Since the data that is utilized in this research has only one observed time for each observation and clusters will be determined for each age and years-of-company service observation, the most appropriate method for survival analysis is the Kaplan-Meier Survival Analysis.

3.1.3 Simulation Optimization Model

The Simulation Optimization Model determines the minimum cost of maintaining a Human Capital inventory, based on a time phased demand for this inventory. Inventory replenishment quantities and inventory replenishment times are determined. The reduction in the inventory is determined by the attrition rates associated with the aging of the Human Capital inventory. The Human Capital cost in a time period is determined by applying a unit cost rate per Human Capital inventory item for inventory utilization, surplus inventory, shortage inventory and human inventory replenishment cost. The minimum cost optimization model is an appropriate method for determining optimum inventory levels based on the assumption that traditional minimum cost optimization models is appropriate for Human Capital Inventory Cost Models.

3.2 Clustering Analysis

As discussed in Chapter 2, Clustering is a type of unsupervised machine learning. It is distinguished from supervised learning by the fact that there is not an *a priori* classification for the clustering of the data. The objective is to learn the classification/grouping from the data. As defined in Chapter 2, a cluster is a collection of objects which are *similar* in some way. Clustering is the process of grouping *similar* objects into groups. An example of Clustering would be to group people based on their height and weight. Other possible applications of clustering include data reduction (reduce data that are homogeneous (similar)), finding "natural clusters" and describing their unknown properties, finding useful and suitable groupings, finding unusual data objects (i.e. outlier detection). Typically, clusters are created using distance measures. i.e., two or more objects belong to the same cluster if they are "close" according to a given distance (geometrical distance like Euclidean or Manhattan). Another measure used for clustering is conceptual. Conceptual clustering would assign two or more objects to the same cluster if there is the ability to define a concept *common* to all objects. In other words, objects are grouped according to their fit to descriptive concepts, not according to simple similarity measures.

There are several common clustering approaches. *Exclusive Partitioning* groups data in a way that one data item can only belong to one cluster. The most widely used means for exclusive clustering is by using K-Means clustering. *Agglomerative Clustering* initially assigns each data point in its own cluster. This method iteratively reduces the number of clusters through the unions between the nearest clusters. *Hierarchical Clustering* uses this technique. Overlapping approaches do not assign each data point to a unique cluster. *Overlapping Clustering* creates

fuzzy sets of cluster data. Overlapping techniques assign data points to clusters so that each point may belong to two or more clusters with different degrees of membership. Fuzzy C-Means clustering is an example using this approach. *Probabilistic Clustering* uses probability distribution measures to create the clusters. An example of this technique is the Gaussian mixture model clustering, which is a variant of *K-M*eans.

With prior knowledge of the data sets used in this research, Exclusive Clustering using K-Means clustering is used in performing the clustering analysis. The Overlapping, Hierarchical and the Probabilistic approaches and techniques will not be utilized in this research.

3.2.1 Exclusive clustering: *K*-means Algorithm

K-means clustering is one of the more commonly used clustering algorithms (MacQueen, 1967). The objective in clustering is to construct a partition of a dataset *D* of *N* objects into a set of *K* clusters. When using K-Means clustering, an initial choice of *K*, the number of desirable partitions of the dataset *D*, must be chosen. K-Means Clustering is a heuristic method where each cluster is represented by the centre of the cluster (i.e. the centroid). The two datasets that are used in this research is two dimensional. However, clustering algorithms can be used in much higher dimensions.

3.2.1.1 The K-Means Algorithm

Given *K*, the *K-means* algorithm is implemented in four steps:

1. Choose *K* points at random as cluster centres (centroids)
2. Assign each datapoint in a dataset to its closest cluster centre using a distance measure (usually Euclidean or Manhattan)

3 Calculate the centroid (mean) of each cluster, use it as the new cluster centre (one measure of centroid is mean)

4 Go back to Step 2, stop when cluster centres do not change any more

3.2.1.2 Strengths

The K-Means algorithm is relatively efficient. When there are N objects in the dataset and a selection of K clusters is chosen, *K* is always less that than *N*. The K-Means algorithm always terminates successfully.

3.2.1.3 Weaknesses

The K-Means algorithm does not necessarily find the most optimal configuration. The algorithm is also significantly sensitive to the initial randomly selected cluster centres. The mean of each cluster must be definable (i.e., can be computed). Another weakness is that *K*, the number of clusters, must be specified in advance.

3.2.2 Agglomerative Clustering: Hierarchical Algorithm

The K-means algorithm starts out with a fixed number of clusters and allocates all data into the exact number of clusters. However, the agglomeration algorithm does not require the number of clusters *K* to be predetermined. The Agglomeration starts out by forming a cluster for each data item, i.e. there will initially be only one data item per cluster. Initially, with a dataset of N items, there will be N clusters. By using a distance measure (or similarity), the Agglomerative algorithm reduces the number of clusters by one in each iteration, merging the two closest (or most similar) clusters. This continues until all data items are in a single cluster.

3.2.2.1 Strengths

The Agglomerative clustering algorithm is preferable for detailed data analysis. The technique provides more information than exclusive clustering. When using the agglomerative clustering technique, a decision can be made on any number of clusters without the need to redo the algorithm which would be required in exclusive clustering. Changes in the number of clusters in exclusive clustering would require that the exclusive algorithm would need to be re-performed.

3.2.2.2 Weaknesses

Agglomerative clustering is less efficient than exclusive clustering. There is also no backtracking in the process. Once clusters are merged, there is no ability to reassign data points to other clusters other than when merging one cluster to another.

3.2.3 Cluster Validation

The goal of clustering is to determine the intrinsic grouping in a set of *unlabeled* data. It can be shown that there is no absolute "best" criterion which would be independent of the final aim of the clustering. The user must supply the criterion to suit the application. Questions and concerns, as well as validation, of the application of clustering techniques are:

- How to decide the optimal number of clusters that fits a data set
- How do we know which clustering method is better/suitable since various clustering algorithms behave in a different ways depending on
  - the features of the data set (geometry and density distribution of clusters)

- the input parameters values (e.g.: for *K*-Means, initial cluster choices influence the result)

In general, clusters, should have high intra–cluster similarity, i.e. low variance among intra-cluster members The variance of a cluster is a typical measure of intra-cluster similarity where the variance for *x* is defined by

$$\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2 \qquad \text{with } \bar{x} \text{ as the mean of } x.$$

As an example of intra-cluster similarity, consider two clusters with the following data values: Cluster 1 = [3,7,10], Cluster 2 = [17, 18, 20]. The variance of Cluster 1 is 2.33. The variance of Cluster 2 is 12.33. Using the variance as a measure of cluster similarity, Cluster 1 is a better cluster than Cluster 2.

Although the variance is a good measure of intra-cluster similarity, it does not measure how good the overall data clustering is or if there is a suitable number of clusters.

In conjunction with intra-cluster similarity, it is also needed to determine how well the clustering technique has performed related to inter-cluster dissimilarity or inter-cluster variance. Using the concepts of intra-cluster variance and inter-cluster variance, good clustering should result in small intra-cluster variances and large inter-cluster variance. Good clusters will also have low inter–cluster similarity, i.e. high variance among inter-cluster members in addition to high intra–cluster similarity, i.e. low variance among intra-cluster members. There are numerous measures of clustering quality that have been developed. Three of the more popular and frequently used measures are the Davies-Bouldin index (Davies & Bouldin, 1976), the Silhouette method (Rousseeuw, 1987) and the Dunn's Validity Index (Dunn, 1974). In a study by (Saitta,

Raphael, & Smith, 2007), several scenarios were analysed to compare various clustering indices for effectiveness. An evaluation by (Saitta, Raphael, & Smith, 2007) identified the Davies-Bouldin and the Silhouette indices as the two better means for measuring cluster validity when using data that is represented in Figure 3-3.

3.2.3.1 The Davies-Bouldin Index

The Davies-Bouldin index is a function of the ratio of the sum of within-cluster (i.e. intra-cluster) scatter to the between cluster (i.e. inter-cluster) separation. Because a low scatter and a high distance between clusters lead to low values of the Davies-Bouldin index, a minimization of the Davies-Bouldin (DB) index is desired.

Let C={C_1,….., C_k} be a clustering of a set of *N* objects, then the *DB* Index is

$$\mathrm{DB} = \frac{1}{\mathrm{k}}\sum_{\mathrm{i}=1}^{\mathrm{k}} \mathrm{R_i}$$

where with $\mathrm{R_{ij}} = \frac{\mathrm{var(C_i)+var(C_j)}}{||\mathrm{c_i-c_j}||}$, $\mathrm{R_i} = \max_{\mathrm{j=1,..,k,i\neq j}} \mathrm{R_{ij}}$, and where C_i is the ith cluster and c_i is the centroid for cluster *i* . The numerator of R_{ij} is a measure of intra-cluster similarity while the denominator is a measure of inter-cluster separation.

3.2.3.2 The Silhouette Index

The Silhouette index (SI) computes for each point a width depending on its membership in any cluster. This silhouette width is then an average over all observations. The Silhouette Index is calculated as:

$$\mathrm{SI_k} = \frac{1}{\mathrm{n}}\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{(\mathrm{b_i} - \mathrm{a_i})}{\max(\mathrm{a_i}, \mathrm{b_i})}$$

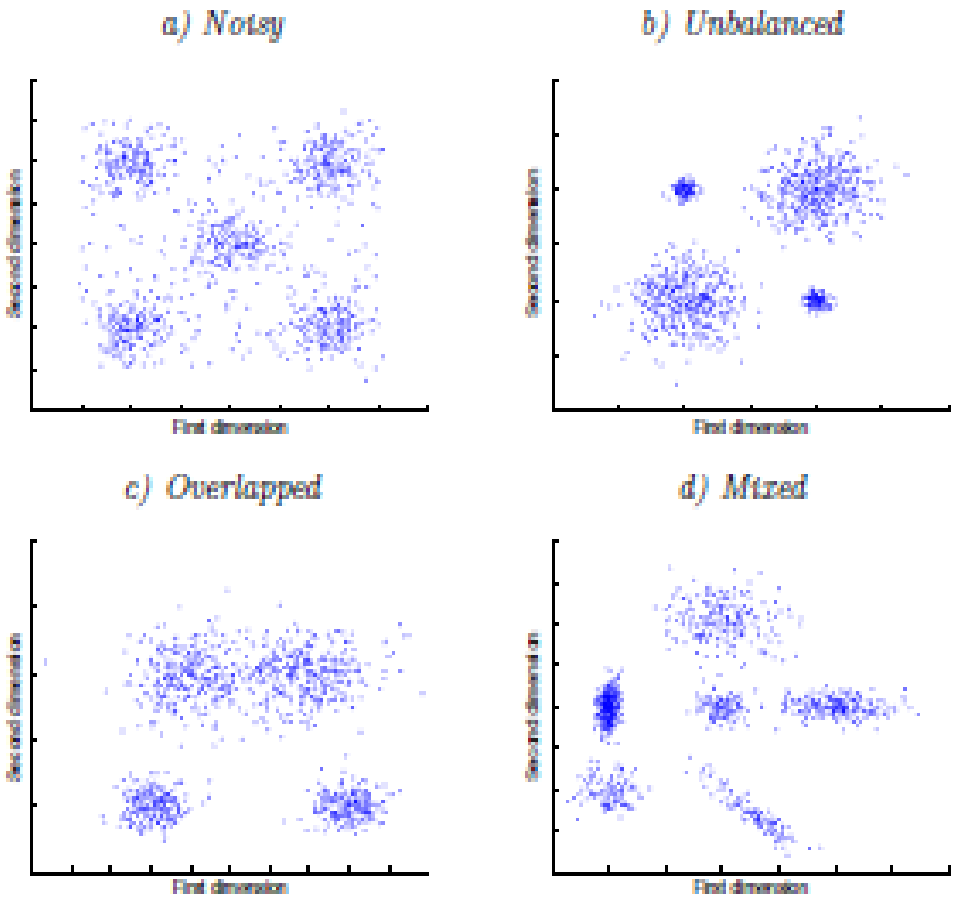


Figure 3-3 Visualization of Various Clustering Data Scenarios

where *n* is the total number of points, a_i, is the average distance between point *i* and all other points in its own cluster and b_i is the minimum of the average dissimilarities between i and points in other clusters. Finally, the partition with the highest Silhouette Index (SI) is taken to be optimal.

3.3 Survival Analysis

As discussed previously, the popular methods used in survivor analysis includes Life Tables, Kaplan-Meier analysis and Cox Regression.

3.3.1 Life Tables

The most straightforward way to describe the survival in a sample is to compute the *Life Table*. The life table technique is one of the oldest methods for analyzing survival (failure time, event) data ((Berkson & Gage, 1950), (Cutler & Ederer, 1958), (Gehan & Thomas, 1969)). A Life Table can be thought of as an "enhanced" frequency distribution table. The distribution of survival times is divided into a certain number of intervals. For each interval computations of the number and proportion of cases or objects that entered the respective interval "alive," the number and proportion of cases that failed in the respective interval (i.e., number of terminal events, or number of cases that "died"), and the number of cases that were lost or censored in the respective interval are made. Life tables involve the use of fixed-length survival time intervals instead of variable length survival times. When using Life tables, information is lost when survival times are categorized in fixed intervals of time. The wider the time intervals, the more information that is lost. The following description for the development of Life tables is taken from (Statsoft, 2010).

Based on those numbers and proportions, several statistics can be computed:

- Number of Cases at Risk
- Proportion Failing
- Proportion surviving
- Cumulative Proportion Surviving (Survival Function)
- Probability Density
- Hazard rate
- Median survival time
- Required sample sizes

Number of Cases at Risk. This is the number of cases that enter the respective interval alive, minus half of the number of cases lost or censored in the respective interval.

Proportion Failing. This proportion is computed as the ratio of the number of cases failing in the respective interval, divided by the number of cases at risk in the interval.

Proportion Surviving. This proportion is computed as 1 minus the proportion failing.

Cumulative Proportion Surviving (Survival Function). This is the cumulative proportion of cases surviving up to the respective interval. Since the probabilities of survival are assumed to be independent across the intervals, this probability is computed by multiplying out the probabilities of survival across all previous intervals. The resulting function is also called the *survivorship* or *survival function*.

Probability Density. This is the estimated probability of failure in the respective interval, computed per unit of time, that is:

$$F_i = (P_i - P_{i-1})/h_i$$

In this formula, F_i is the respective probability density in the ith interval, P_i is the estimated cumulative proportion surviving at the beginning of the ith interval (at the end of interval $(i - 1)$), P_{i+1} is the cumulative proportion surviving at the end of the ith interval, and h_i is the width of the respective interval.

Hazard Rate. The hazard rate (the term was first used by (Barlow, Marshall, & Proschan, 1963) is defined as the probability per time unit that a case that has survived to the beginning of the respective interval will fail in that interval. Specifically, it is computed as the number of failures per time units in the respective interval, divided by the average number of surviving cases at the mid-point of the interval.

Median Survival Time. This is the survival time at which the cumulative survival function is equal to *0.5*. Other percentiles (25th and 75th percentile) of the cumulative survival function can be computed accordingly. Note that the 50th percentile (median) for the cumulative survival function is usually not the same as the point in time up to which 50% of the sample survived. (This would only be the case if there were no censored observations prior to this time).

Required Sample Sizes. In order to arrive at reliable estimates of the three major functions (survival, probability density, and hazard) and their standard errors at each time interval the minimum recommended sample size is 30.

The basic algebra used in life tables is as follows.

- q_x**:** the probability that someone aged exactly x will die before reaching age $(x+1)$.
- p_x**:** the probability that someone aged exactly x will survive to age $(x+1)$.

  $$p_x = 1 - q_x$$

- l_x**:** the number of people who survive to age x

  Note: this is based on a starting point of l_0 lives

  $$l_{x+1} = l_x \cdot (1 - q_x) = l_x \cdot p_x$$

  $$\frac{l_{x+1}}{l_x} = p_x$$

- d_x**:** the number of people who die aged x last birthday

  $$d_x = l_x - l_{x+1} = l_x \cdot (1 - p_x) = l_x \cdot q_x$$

- ${}_tp_x$**:** the probability that someone aged exactly x will survive for t more years, i.e. live up to at least age $x + t$ years

  $${}_tp_x = \frac{l_{x+t}}{l_x}$$

- ${}_{t|k}q_x$**:** the probability that someone aged exactly x will survive for t more years, then die within the following *k* years

$$_{t|k}q_x = {}_tp_x \cdot {}_kq_{x+t} = \frac{l_{x+t} - l_{x+t+k}}{l_x}$$

- μ_x : the *force of mortality*, ie the instantaneous mortality rate at age *x*, ie the number of people dying in a short interval starting at age *x*, divided by l_x and also divided by the length of the interval. Unlike q_x, the instantaneous mortality rate, μ_x, may exceed 1.

3.3.2 Kaplan-Meier Method

The Kaplan-Meier method is a method for estimating survival times using non-parametric survival times and allows for variable length measures of survival time. The length of time is considered an independent variable. The Kaplan-Meier method relies on a dependent variable. This dependent variable is typically coded as a zero or a one. The dependent variable represents whether or not an event has occurred. The code that is selected for the case where the event has not occurred is considered *censored*. Otherwise, the occurrence of the event is considered *uncensored*. The Kaplan-Meier is useful when the number of cases is small and exact survival times are known.

The estimated survival function at time t is calculated as:

$$S(t) = \prod_{i=1}^{t} \left[\frac{(n-i)}{(n-i+1)}\right]^{C_i}$$

where $\prod_{i=1}^{t}()$ denotes the multiplication of the survival times across all cases less than or equal to t (the geometric mean); *t* = time, e.g. days, weeks, months, etc., *n* is the total number of cases

in the sample, i is the number of cases surviving up to time t, C_i is a constant such that C_i is 0 for the uncensored or terminal case and 1 for the censored case.

The Kaplan-Meier use of censored cases in survival analysis, i.e. not considering this data as missing, avoids the situation where survival probabilities will be too low, underestimating the probability of survival. The Kaplan-Meier method involves the following steps in determining the estimated survival function:

1. List the time when the pre-established event occurs based on the subject's involvement in the survey (participation time);
2. For every participation time, identify the number of subjects that continue to participate in the survey (those who did not achieve the pre-established event (employment in our case);
3. determine the number of subjects who achieved the pre-established event within each time interval;
4. calculate the probability of occurrence of the pre-established event, for each participation interval;
5. repeat this process until all time periods have been covered;

3.3.3 Cox Regression

Cox regression (Cox D. , Regression Models and Life Tables, 1972) , or proportional hazards regression, is method for investigating the effect of several variables upon the time a specified event takes to happen. In the context of an outcome such as death, this is known as Cox regression for survival analysis. The method does not assume any particular "survival model" but

it is not truly non-parametric because it does assume that the effects of the predictor variables upon survival are constant over time. Provided that the assumptions of Cox regression are met, this function will provide better estimates of survival probabilities and cumulative hazard than those provided by the Kaplan-Meier function. The model can be written as:

$$h\{(t), z_1, \dots, z_{m)}\} = h_0(t)e^{(b_1 z_1 + \cdots + b_m z_m)}$$

where *h(t,...)* denotes the resultant hazard, given the values of the *m* covariates for the respective case (z_1, z_2, ..., z_m) and the respective survival time (t). The term $h_0(t)$ is called the *baseline hazard*; it is the hazard for the respective individual when all independent variable values are equal to zero. This model can be linearized by dividing both sides of the equation by $h_0(t)$ and then taking the natural logarithm of both sides:

$$log\left[\frac{h\{(t), (z \dots)\}}{h_0(t)} = b_1 z_1 + \cdots + b_m z_m\right]$$

We now have a fairly "simple" linear model that can be readily estimated.

While no assumptions are made about the shape of the underlying hazard function, the model equations shown above do imply two assumptions. First, they specify a multiplicative relationship between the underlying hazard function and the log-linear function of the covariates. This assumption is also called the *proportionality assumption*. In practical terms, it is assumed that, given two observations with different values for the independent variables, the ratio of the hazard functions for those two observations does not depend on time. The second assumption of course, is that there is a log-linear relationship between the independent variables and the underlying hazard function.

3.3.3.1 Hazard and Hazard-Ratios

Cumulative hazard at a time *t* is the risk of dying between time 0 and time *t*, and the survivor function at time *t* is the probability of surviving to time *t*. The coefficients in a Cox regression relate to hazard; a positive coefficient indicates a worse prognosis and a negative coefficient indicates a positive effect of the variable with which it is associated. Cox regression is suitable when there are time dependent and fixed covariates. When individuals are followed over time, the values of covariates may change with time. Covariates can thus be divided into fixed and time-dependent. A covariate is time dependent if the difference between its values for two different subjects changes with time. A covariate is fixed if its values cannot change with time, e.g. sex or race. Lifestyle factors and physiological measurements such as blood pressure are usually time-dependent. Cumulative exposures such as smoking are also time-dependent but are often forced into an imprecise dichotomy, i.e. "exposed" vs. "not-exposed" instead of the more meaningful "time of exposure". There are no hard and fast rules about the handling of time dependent covariates.

3.3.3.2 Model Analysis and Deviance

A test of the overall statistical significance of the model is accomplished by utilizing the likelihood chi-square statistic by calculating and comparing the deviance (- 2 * log likelihood) of the model, with all of the covariates you have specified, against the model with all covariates dropped. The individual contribution of covariates to the model can be assessed from the significance test given with each coefficient.

The deviance is minus twice the log of the likelihood ratio for models fitted by maximum likelihood (Hosmer & Lemeshow, 1989) and is tested by subtracting the deviance of the model with the new parameter from the deviance of the model without the new parameter. The difference is then tested against a chi-square distribution with degrees of freedom equal to the difference between the degrees of freedom of the old and new models.

3.3.3.3 Survival and Cumulative Hazard Rates

The survival/survivorship function and the cumulative hazard function are calculated relative to the baseline (lowest value of covariates) at each time point. Cox regression provides a better estimate of these functions than the Kaplan-Meier method when the assumptions of the Cox model are met and the fit of the model is strong.

3.3.3.4 Data Preparation

The two datasets that are part of this research include the following data items:

- Age at time of event
- Years of company service at time event
- Event (0 if event not occurred, 1 if event occurred)
- Strata (determined as a result of the data clustering)

3.4 The Human Capital Inventory Cost Optimization

As stated in Chapter 1, the purpose of this research is to develop an effective method of Human Capital inventory planning utilizing a minimum cost optimization Human Capital

inventory planning model. This Human Capital Inventory Cost Model is developed utilizing the techniques and policies for traditional inventory models. The Human Capital Inventory Model is composed on two components: (1) a Human Capital Inventory Cost Model to calculate the costs associated with maintaining a Human Capital inventory and (2) a Human Capital inventory cost minimization function that determines the appropriate staffing level based on the demand for the Human Capital inventory and minimizes the cost that is associated with providing a Human Capital inventory in an inventory cycle.

3.4.1 The Human Capital Inventory Cost Model

As with any inventory management system, parameters and assumptions need to defined and established. The parameters and assumptions of the Human Capital Inventory Cost Model include:

***t* (time period):** The model is a discrete time model. Time is measured in periods. A period in this model will be annual. However, any time period could be utilized as long as all model variables can be associated on a per period basis.

***d(t)*:** Demand per time period. This is the demand for the resource. In this model, the demand is deterministic rather than stochastic (random). The purpose of the deterministic demand is to demonstrate the applicability of the model. Random demands could be applied if desired and appropriate statistical analysis utilized for stochastic estimates of Human Capital availability.

C_i: Inventory Cycle. An inventory cycle is defined as a grouping of periods from the period that an inventory replenishment made up to the period that the next replenishment quantity is made to the inventory. The length of this cycle must be greater than or equal to one.

***I(t)*:** Available Inventory level in a period. This is the staffing level that is available to meet the period demands. Inventory level is computed differently in the Human Capital Inventory Cost Model than in the traditional inventory models. Traditional inventory models calculate the inventory level as $I(t+1) = I(t) - d(t) + Q(t)$ where *d(t)* is the demand in period *t*. In the Human Capital Inventory Cost Model, the inventory level is calculated as $i(t+1) = i(t) - a(t) + Q(t)$ where *a(t)* is the attrition of the Human Capital inventory in period *t*. *a(t)* is independent of the demand *d(t).*

***a(i)*:** This is the attrition per period. This is determined from the survival analysis that is discussed in Section 3.4.

***Q(i)*:** This is the replenishment quantity that is determined by the inventory policy.

3.4.1.1 Human Capital Inventory Costs

Human Capital inventories are necessary for any organization to provide goods and/or services. These inventories, just as with traditional non-labor inventories, require companies to invest in their Human Capital inventories and incur costs associated with maintaining and replacing these Human Capital inventories. Just as with traditional non-labor inventories, organizations incur costs when surplus or excess inventories exist, i.e. when more or less resources exist to meet customer demands or services. Also, a shortage or lack of adequate Human Capital inventories results in incurring costs for the replacement of these inventories or lost opportunity costs due to the shortage of resources. Human Capital inventory costs include direct and indirect cost components. Direct Human Capital inventory costs typically refer to the costs of wages, salaries and fringe benefits associated with work that can be directly associated

and applied to products or services. Indirect costs are incurred for common or joint objectives and therefore cannot be readily and specifically identified with a product or service.

The accumulation of the indirect costs associated specifically with labor inventories is commonly referred to as labor burden. Labor burden is the cost to a company to carry their labor force aside from salary actually paid to them. Labor Burden typically includes, but is not limited to, costs related to labor inventories such as federal and state payroll taxes, life, health and disability insurance, vacation pay, holiday pay, sick pay, and pension and retirement.

Costs that are incurred that only benefit an identifiable unit or activity of an organization are considered overhead costs. It is common to find separate overhead pools for engineering, manufacturing, material handling, and for certain off-site activities Examples of overhead pool costs are:

- Department supervision
- Depreciation of department buildings and equipment
- Training of department employees
- Fringe benefits of department employees

Overhead rates are typically developed by dividing the overhead pool costs by the selected allocation base, e.g., direct labor dollars or direct labor hours.

General and Administrative (G&A) expenses represent the cost of activities that are necessary to the overall operation of the business as a whole, but for which a direct relationship to any particular cost objective cannot be shown. G&A includes the top management functions

for executive control and direction over all personnel, departments, facilities, and activities of the contractor. Typically, it includes human resources, accounting, finance, public relations, contract administration, legal, and an expense allocation from the corporate home office.

The General and Administrative (G&A) rate is typically developed by dividing total general and administrative expenses by the selected allocation base, e.g., total cost input (i.e., total direct and indirect costs, except G&A), value added cost input (i.e., total cost input except G&A, material and subcontract costs), or single element cost input (e.g., direct labor dollars, direct labor hours, direct materials costs).

Labor costs, Labor Burden costs, Overhead costs and General and Administrative (G&A) costs are the typical means of allocating an organization's costs to the goods and services provided by the organization. As described above, each of these costs include numerous cost components. Included in these costs are typical costs associated with the replacement of Human Capital inventory resulting from Human Capital turnover. However, opportunity costs are not typically identified and are not included in these labor and overhead costs.

Studies by American Management Association (Gaudet, 1960) and others report a range between 25 percent and 250 percent of annual salary per exiting employee that are related to employee turnover. Entry-level, unskilled positions are at the lower end of the cost range, while executive, managerial and sales positions are at the higher end.

Turnover costs can be categorized in the following categories. The following is taken from (Al-Zayyat, 2010):

- Exit Costs
- Absence Costs
- Recruitment Costs
- Onboarding Costs

Exit Costs may include, but not limited to, the following activities or events:

1. Human Resources: HR costs associated with conducting exit interviews as well as the time of the exiting employee, costs associated with payroll and benefits changes and other administrative activities.
2. Managerial Handoff: Cost of management who will conduct their own exit interview, review the exiting employee's work-in-progress, and determine how to cover that work until a replacement is hired.
3. Involuntary Termination: If the employee was terminated involuntarily, costs of any severance package, benefits continuation and unemployment insurance premiums, if eligible.
4. Lost Customers and Contacts: Non-compete agreements are extremely difficult to enforce in practice. Particularly if the employee is in a sales or customer service position, the potential cost of customers the employee may be taking with them to their new position. For any position, the value of the contacts they have developed (of course, a new person may bring in their own contacts that offset this).
5. Lost Knowledge: During their time at your company, the exiting employee may have developed specialized knowledge and skills about your business. You may have even

invested in training for them. Training costs on either the exiting employee or the new employee should be included but not both.

Absence (shortage) Costs may include, but not limited to, costs associated with the following activities and events:

1. Coverage: The exiting employee's essential responsibilities are going to be taken up by other employees which means that some portion of their own current workload will typically be given up and their productivity reduced. Or, increased overtime costs may be necessary to cover the work. These costs should also include the management costs associated with the work-reassignment.
2. Direct Productivity Loss: Tasks that were being performed by the exiting employee simply won't be done in their absence. Costs associated with this work should be included.
3. Disruption: while it may be difficult to put an exact number on this, an exiting employee may be disruptive to essential business processes in a way that goes beyond simple coverage issues. There may also be a cost associated with the effect on the morale of other employees when a respected peer leaves.

Recruitment (Replenishment) costs may include, but not limited to, costs associated with the following activities and events:

1. Advertisements: cost associated with the advertisement for the available position. These costs may include classified ads, internet job posting, etc. Referral costs may also be incurred. External recruiter fees can run as high as 1/3 of the salary for the position.

2. Internal Recruitment: The cost of time an internal recruiter requires to understand the position requirements, develop a sourcing strategy, review resumes, prepare for and conduct interviews, conduct reference checks, make the offer to the selected candidate, and notify unsuccessful candidates.
3. Hiring Manager: the costs for the hiring manager(s) and other key stakeholders to review resumes, conduct interviews and make their selection.
4. Internal Candidates: The cost of lost productivity for any internal candidates who may apply for the position.
5. Screening: the costs of new-hire drug screens, background checks, skills assessment, personality profile testing, and any other tasks (especially outsourced ones) used to screen candidates.

Onboarding (Replenishment) Costs may include, but not be limited to, costs associated with activities and events related to the following:

1. Onboarding: The cost of both the new employee and HR staff for orientation and onboarding paperwork, including business cards, ID badge, credit card, mobile phone and so on. Cost may include the costs for IT staff to set up user accounts, telephone access, etc.
2. Training: the cost of both structured training (including materials) and the time of managers and key coworkers to train the new employee to the point of 100 percent productivity.
3. Productivity Ramp-up: Depending on the position, it may take anywhere from a few days to a few months for the new employee to be at 100 percent productivity.

The Society for Human Resource Management (Blake, 2006) estimates that employee-turnover costs 30-50% of entry-level employees' annual salaries, 150% of middle-level employees and as much as 400% for executives and employees with specialized skills.

A guide for calculating the costs of an individual leaving an organization is presented in its entirety by (Bliss, 2010). Similar formulas for determining the costs of employee turnover were developed by Wayne Cascio (Cascio, 1991). Smith and Watkins (Smith & Watkins, 1978) identified three major cost categories: separation costs, replacement costs, and training costs. Cascio added a category to include the performance differential between the employee who leaves and the replacement. Bliss added another category, *vacancy* costs, to account for the added costs/savings realized while the position is vacant.

These costs will be used to establish the cost of labor, i.e, costs associated with labor that meets demand, labor costs for excess Human Capital inventory and labor costs when a shortage of Human Capital inventory exists. Bliss recommends that the calculation of these costs include:

Cost of a person leaving may include:

1. The cost of the person(s) who fills in while the position is vacant. This can be either the cost of a temporary or the cost of existing employees performing the vacant job as well as their own. Include the cost at overtime rates. Classified as Shortage cost.
2. The cost of lost productivity at a minimum of 50% of the person's compensation and benefits cost for each week the position is vacant, even if there are people performing the work. Calculate the lost productivity at 100% if the position is completely vacant for any period of time. Classified as Shortage cost.

3. The cost of conducting an exit interview to include the time of the person conducting the interview, the time of the person leaving, the administrative costs of stopping payroll, benefit deductions, benefit enrollments, COBRA notification and administration, and the cost of the various forms needed to process a resigning employee. Classified as Replenishment costs.
4. The cost of the manager who has to understand what work remains, and how to cover that work until a replacement is found. The cost of the manager who conducts their own version of the employee exit interview. Classified as Shortage Cost.
5. The cost of training your company has invested in this employee who is leaving. Include internal training, external programs and external academic education. Include licenses or certifications the company has helped the employee obtain to do their job effectively. Classified as Replenishment cost.
6. The impact on departmental productivity because the person is leaving. Who will pick up the work, whose work will suffer, what departmental deadlines will not be met or delivered late? The cost of department staff discussing their reactions to the vacancy. Classified as Shortage Cost.
7. The cost of severance and benefits continuation provided to employees who are leaving that are eligible for coverage under these programs. Classified as Shortage cost.
8. The cost of lost knowledge, skills and contacts that the person who is leaving is taking with them out of your door. Use a formula of 50% of the person's annual salary for one year of service, increasing each year of service by 10%. Classified as Shortage cost.

9. The cost impact of unemployment insurance premiums as well as the time spent to prepare for an unemployment hearing, or the cost paid to a third party to handle the unemployment claim process on your behalf. Classified as Shortage cost.
10. The cost of losing customers that the employee is going to take with them, or the amount it will cost you to retain the customers of the sales person, or customer service representative who leaves. Classified as Shortage Cost.

(Bliss, 2010) estimates that the total of these costs can easily approach eighty percent (85%) of the based salary for an individual employee who leaves an organization. The recruitment (replenishment) costs may include:

1. The cost of advertisements (from a $200.00 classified to a $5,000.00 or more display advertisement); agency costs at 20 - 30% of annual compensation; employee referral costs of $500.00 - $2,000.00 or more; internet posting costs of $300.00 - $500.00 per listing.
2. The cost of the internal recruiter's time to understand the position requirements, develop and implement a sourcing strategy, review candidates backgrounds, prepare for interviews, conduct interviews, prepare candidate assessments, conduct reference checks, make the employment offer and notify unsuccessful candidates. This can range from a minimum of 30 hours to over 100 hours per position.
3. The cost of a recruiter's assistant who will spend 20 or more hours in basic level review of resumes, developing candidate interview schedules and making any travel arrangements for out of town candidates.

4. The cost of the hiring department (immediate supervisor, next level manager, peers and other people on the selection list) time to review and explain position requirements, review candidates background, conduct interviews, discuss their assessments and select a finalist. Also include their time to do their own sourcing of candidates from networks, contacts and other referrals. This can take upwards of 100 hours of total time.
5. The administrative cost of handling, processing and responding to the average number of resumes considered for each opening at $1.50 per resume.
6. The number of hours spend by the internal recruiter interviewing internal candidates along with the cost of those internal candidates to be away from their jobs while interviewing.
7. The cost of drug screens, educational and criminal background checks and other reference checks, especially if these tasks are outsourced. The number of times these are done per open position as some companies conduct this process for the final 2 or 3 candidates.
8. The cost of the various candidate pre-employment tests to help assess a candidates' skills, abilities, aptitude, attitude, values and behaviors.

Training (Available Inventory) Costs may include:

1. The cost of orientation in terms of the new person's salary and the cost of the person who conducts the orientation. Also include the cost of orientation materials.
2. The cost of departmental training as the actual development and delivery cost plus the cost of the salary of the new employee. Note that the cost will be significantly higher for

some positions such as sales representatives and call center agents who require 4 - 6 weeks or more of classroom training.

3. The cost of the person(s) who conduct the training.
4. The cost of various training materials needed including company or product manuals, computer or other technology equipment used in the delivery of training.
5. The cost of supervisory time spent in assigning, explaining and reviewing work assignments and output. This represents lost productivity of the supervisor. Consider the amount of time spent at 7 hours per week for at least 8 weeks.

Bliss estimates that the sum of these costs are approximately 13% of the position's base salary.

As the new employee is learning the new job, the company policies and practices, etc. they are not fully productive. The following guidelines are used to calculate the cost of this lost productivity (Available costs):

1. Upon completion of whatever training is provided, the employee is contributing at a 25% productivity level for the first 2 - 4 weeks. The cost therefore is 75% of the new employee's full salary during that time period.
2. During weeks 5 - 12, the employee is contributing at a 50% productivity level. The cost is therefore 50% of full salary during that time period.
3. During weeks 13 - 20, the employee is contributing at a 75% productivity level. The cost is therefore 25% of full salary during that time period.

4. The cost of coworkers and supervisory lost productivity due to their time spent on bringing the new employee "up to speed."
5. The cost of mistakes the new employee makes during this elongated indoctrination period.
6. The cost of lost department productivity caused by a departing member of management who is no longer available to guide and direct the remaining staff.
7. The impact cost on the completion or delivery of a critical project where the departing employee is a key participant.
8. The cost of reduced productivity of a manager or director who loses a key staff member, such as an assistant, who handled a great deal of routine, administrative tasks that the manager will now have to handle.

It is estimated that the total cost of lost productivity is approximately 150% of the base salary of the individual leaving the position. Additional costs of employee benefits and company-paid taxes on top of the 150% can range from 20 to 30 percent of the base salary.

New Hire (Replenishment) Costs may include:

1. The cost of bringing the new person on board including the cost to put the person on the payroll, establish computer and security passwords and identification cards, business cards, internal and external publicity announcements, telephone hookups, cost of establishing email accounts, costs of establishing credit card accounts, or leasing other equipment such as cell phones, automobiles, pagers.

2. The cost of a manager's time spent developing trust and building confidence in the new employee's work.

Bliss also estimates that the sum total of the hiring costs will be from 15% of this position’s base salary and could increase to about 38% of the position’s base salary if an employment agency is used.

Lost Sales (Shortage) Costs may include:

1. The lost sales until a new sales representative is fully productive. Can also be used for telemarketing and inside sales representatives.
2. For non-sales staff, the lost revenue for the position that is vacant .

Businesses and political leaders are starting to recognize that having good people who are skilled and motivated can make a significant difference in meting business objectives. Treating money spent on people as an important asset is a more appropriate mindset than treating such expenditures as an expense. However, when just as any other asset, businesses cannot maintain excess assets that are not providing economic returns on the investments of these assets. In the Human Capital inventory environment, excess Human Capital is described as those assets that exceed the demand for the Human Capital assets. As with any asset, businesses or organizations must understand the costs that are related to excess Human Capital capacity and foregone opportunity costs associated with excess Human Capital capacity. One typical method of calculating this foregone opportunity is the Minimum Acceptable Rate of Return (MARR). One

method of calculating a typical MARR is the sum of several components (Lang & Merino, 1993). Lang and Merino defines the MARR to be composed the following components:

- Traditional inflation-free rate of interest for risk-free loans (3-5%)
- Expected rate of inflation (5%)
- The anticipated change in the rate of inflation(0%)
- The risk of defaulting in a loan (0-5%)
- The risk profile of a particular venture (0-50% and higher)

Estimates of these components in calculating the MARR will be used in determining the foregone opportunity costs associated with excess Human Capital inventory.

Many organizations focus on the cash cost of employee turnover. However, organizations do not realize the entire cost and impact of employee turnover. Cost of time and lost productivity are as real as other identifiable costs such as salaries and benefits. The Human Capital Inventory Cost Model calculates a per period inventory cost associated with staffing an organization. The costs include:

- Available Inventory Cost (IC)
- Excess Inventory Costs (EC)
- Shortage Inventory Costs (SC)
- Inventory Replenishment Costs (RC)

The following rates are utilized when calculating the costs associated in the Human Capital Inventory Cost Model:

- Available Inventory Rate *(IR)*: per period rate applied to the available Human Capital inventory
- Excess Inventory Rate *(ER)*: per period rate applied to the available inventory in excess of the period demand *d(t)*
- Shortage Inventory Rate *(SR)*: per period rate applied to the amount of available inventory that is less than period demand *d(t)*. This rate is applied when *d(t)* > *I(t)*.
- Inventory Replenishment Cost (RC): per cycle cost applied to each inventory cycle associated with the costs of acquiring replacement employees.

The method of calculating the period Human Capital inventory costs follow:

- **Available Inventory Costs (IC)** – A period cost associated with available Human Capital inventory. Inventory costs are calculated for each inventory period. $IC(t) = d(t) * IR$ where $I(t) \geq d(t)$.
- **Excess Inventory Costs (EC)** – A period cost associated with having more available Human Capital inventory in a period than is necessary to meet the period demand. $EC(t) = (I(t) - d(t)) * EI \; where \; I(t) \; > d(t)$.
- **Shortage Inventory Costs (SC)** – A period cost associated with having less Human Capital inventory than is demanded in a period. $SC = \big(d(t) - I(t)\big) * SR \; where \; d(t) > I(t)$.
- **Inventory Replenishment Costs (RC)** – a cycle cost associated with the replenishment of the Human Capital inventory. The replenishment cost is applied in a cycle where the replenishment of the inventory is applied.

Cycle costs for the Human Capital Inventory Cost Model is the sum of the per period inventory costs for each period with a cycle. Human Capital inventory Cycle Cost is calculated as:

$$C_i = RC_i + \sum_{j=n_f}^{n_l} (IC_j + EC_j + SC_j)$$

where n_f is the first period in a cycle, n_l is the last period in the cycle, $1 \leq n_f \leq n_l \leq n$ and i is the cycle number.

The average Human Capital inventory Cycle Cost is calculated as $C_{i(avg)} = \frac{C_i}{(n_l - n_f + 1)}$.

3.4.1.2 The Human Capital Optimization Model

The objective of this research is the development of a Human Capital Inventory Cost Model and the development of the process to determine a minimum cost model associated with time-phased per period demand for Human Capital inventories. As with any minimum cost optimization, as discussed in Chapter 2, an objective function must first be developed. As discussed, our objective is to determine the level of Human Capital inventory that is necessary to meet a demand scenario. Within each demand scenario, we want to minimize the cost associated with the Human Capital inventory. Intuitively, most organizations want to have adequate staffing to meet customer demands. However, these organizations also do not want to incur unnecessary costs associated with overstaffing an organization. With these assumptions, our research is the development of optimized, minimum cost staffing methodology to meet an organization's staffing needs.

As with any optimization model, an objective function must be established. Based on the development of the Human Capital Inventory Cost Model, we can minimize either the average period cycle cost, as developed in Section 3.4.1.

$$C_{i(avg)} = \frac{c_i}{(n_l - n_f + 1)} \text{ or the total cycle cost } c_i.$$

or the total cycle cost. The objective functions are defined by

$$Minimize \sum_{i=1}^{m} C_{i(avg)} \; or \; Minimize \sum_{i=1}^{m} C_i$$

where *i* is the number of cycles associated with the multi-period, multi-cycle inventory analysis.

As can be seen from the calculations for the Inventory cost, Shortage cost and Excess cost, the inventory level, $I(t)$, is the control variable in the optimization function. The only constraint in the optimization function is that $I(t)$ be greater than 0.

3.4.1.2.1 The Optimization Process

The purpose of the optimization process is to determine minimum Human Capital inventory cycle costs. The optimization process is depicted in Figure 3-4. The Human Capital Inventory Cost Model is executed by the local optimization function, returning the minimum inventory cycle cost to the local optimization function. The local optimization function is 'wrapped' by the global optimization that provides the optimization variable (Cycle FTE's) to the local optimization function. Once a Global solution of determined, the optimization function terminates.

Optimization Model Functional Flow

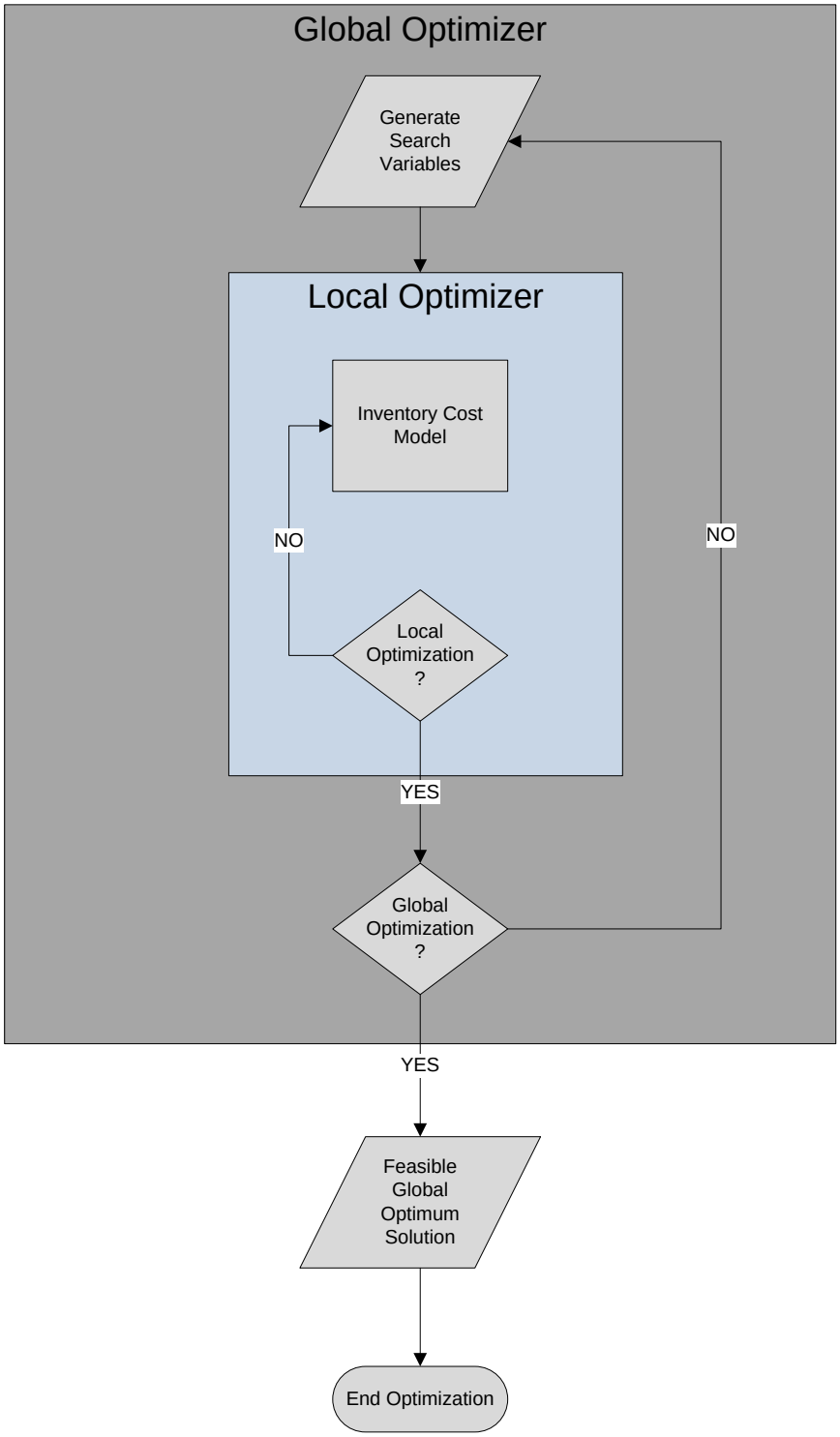


Figure 3-4 Optimization Process Flow

The assignment of the optimization variable, x_i, is provided by the global optimization function to the local optimization function. Based on the cost associated with the optimization variable, the inventory cycle cost is returned to the optimization functions.

The following steps are executed in the optimization process for each inventory cycle processed:

1.1 Initialize the Initial Inventory $I(0)$

1.2 Initialize the number of cycles to process.

1.3 Execute Optimization Function

1.3.1 Execute the Human Capital Inventory Cost Model Function

1.3.1.1 Calculate Period Costs based on optimization variable x_i

1.3.1.2 Decrement Inventory Level, $I(t+1) = I(t) - a(t)$

1.3.1.3 If end of Inventory Cycle, Calculate Average Period Cycle Cost

1.3.2 Return Inventory Cycle Cost to Optimization Function

1.4 If Minimum Cost objective not achieved, re-assign Initial Period Inventory Level to x_i, otherwise exit optimization function

1.5 Return to step 1.3.1

1.6 Results: Optimum Beginning Inventory Cycle quantity that minimizes inventory Cycle Cost.

As discussed in Chapter 2, there is a possibility of identifying multiple local optima within a stationary point. In order to attempt to avoid being *trapped* in a local optima position, Global Optimal search routines are incorporated. The following methodologies and documentation is taken from the documentation for the minimization functions within

MATLAB, a product of Mathworks. MATLAB is used throughout this research for the optimization modeling.

3.4.1.2.2 Trust-Region Methods for Nonlinear Minimization

Many of the methods used in the MATLAB Optimization Toolbox solvers are based on *trust regions*, a simple yet powerful concept in optimization.

To understand the trust-region approach to optimization, consider the unconstrained minimization problem, minimize $f(x)$, where the function takes vector arguments and returns scalars. Suppose you are at a point x in n-space and you want to improve, i.e., move to a point with a lower function value. The basic idea is to approximate f with a simpler function q, which reasonably reflects the behavior of function f in a neighborhood N around the point x. This neighborhood is the trust region. A trial step s is computed by minimizing (or approximately minimizing) over N. This is the trust-region subproblem,

$$\min\{\, q(s), s \in N\}.$$

The current point is updated to be $x+s$ if $f(x+s) < f(x)$; otherwise, the current point remains unchanged and N, the region of trust, is shrunk and the trial step computation is repeated.

The key questions in defining a specific trust-region approach to minimizing $f(x)$ are how to choose and compute the approximation q (defined at the current point x), how to choose and modify the trust region N, and how accurately to solve the trust-region subproblem. In the standard trust-region method (More & Sorensen, 1983), the quadratic approximation q is defined

by the first two terms of the Taylor approximation to F at x; the neighborhood N is usually spherical or ellipsoidal in shape. Mathematically the trust-region subproblem is typically stated

$$\min\left\{\frac{1}{2}s^T Hs + s^T g \ \text{ such that } \ \|Ds\| \le \Delta\right\},$$

where g is the gradient of f at the current point x, H is the Hessian matrix (the symmetric matrix of second derivatives), D is a diagonal scaling matrix, Δ is a positive scalar, and $\| \, . \, \|$ is the 2-norm. Such algorithms typically involve the computation of a full eigensystem and a Newton process applied to the secular equation

$$\frac{1}{\Delta} - \frac{1}{\|s\|} = 0.$$

The approximation approach followed in the MATLAB Optimization Toolbox solvers is to restrict the trust-region subproblem to a two-dimensional subspace S ((Branch, Coleman, & Li, 1999) and (Byrd, Schnabel, & shultz, 1988)).

Once the subspace S has been computed, the two-dimensional subspace S is determined with the aid of a preconditioned conjugate gradient process described below. The solver defines S as the linear space spanned by s_1 and s_2, where s_1 is in the direction of the gradient g, and s_2 is either an approximate Newton direction, i.e., a solution to

$$H \cdot s_2 = -g,$$

or a direction of negative curvature,

$$s_2^T \cdot H \cdot s_2 < 0.$$

The philosophy behind this choice of S is to force global convergence (via the steepest descent direction or negative curvature direction) and achieve fast local convergence (via the Newton step, when it exists).

A sketch of unconstrained minimization using trust-region ideas is now easy to give:

1. Formulate the two-dimensional trust-region subproblem.
2. Solve $min\left\{\frac{1}{2}s^T Hs + s^T g\right\}$ such that $\|Ds\| \leq \Delta$ to determine the trail step s
3. If $f(x + s) < f(x)$, then $x = x + s$.
4. Adjust Δ.

These four steps are repeated until convergence. The trust-region dimension Δ is adjusted according to standard rules. In particular, it is decreased if the trial step is not accepted, i.e., $f(x + s) \geq f(x)$.

The following description of Global Optimization techniques is taken directly from MATLAB documentation. This documentation is appropriate since MATLAB is used in this research for all optimization methods. Methods utilized in the search for global minima is referenced from the functionality found in MATLAB.

3.4.1.2.3 Local vs. Global Optima

Optimization is the process of finding the point that minimizes a function. More specifically, a local minimum of a function is a point where the function value is smaller than or equal to the value at nearby points, but possibly greater than at a distant point. A global

minimum is a point where the function value is smaller than or equal to the value at all other feasible points.

Generally, Optimization Toolbox solvers find a local optimum. (This local optimum can be a global optimum.) They find the optimum in the basin of attraction (Section 3.4.3.2.1) of the starting point. In contrast, Global Optimization Toolbox solvers are designed to search through more than one basin of attraction. They search in various ways:

- *GlobalSearch* and *MultiStart* generate a number of starting points. They then use a local solver to find the optima in the basins of attraction of the starting points.
- *Genetic Algorithm (GA)* uses a set of starting points (called the population) and iteratively generates better points from the population. As long as the initial population covers several basins, GA can examine several basins.
- *Simulated Annealing* (Simulannealbnd) performs a random search. Generally, Simulannealbnd accepts a point if it is better than the previous point. Simulannealbnd occasionally accepts a worse point, in order to reach a different basin.
- *Pattern Search* (Patternsearch) looks at a number of neighboring points before accepting one of them. If some neighboring points belong to different basins, Patternsearch in essence looks in a number of basins at once.

3.4.1.2.4 Basins of Attraction

If an objective function *f(x)* is smooth, the vector $-\nabla f(x)$ points in the direction where *f(x)* decreases most quickly (Figure 3-5). The equation of steepest descent, namely

$$\frac{d}{dt}x(t) = -\nabla f(x(t)),$$

yields a path *x(t)* that goes to a local minimum as *t* gets large. Generally, initial values *x(0)* that are close to each other give steepest descent paths that tend to the same minimum point. The basin of attraction for steepest descent is the set of initial values leading to the same local minimum. Figure 3-6 shows two one-dimensional minima. Figure 3-7 shows different basins of attraction with different line styles, and it shows directions of steepest descent with arrows. The black dots represent local minima. Every steepest descent path, starting at a point x(0), goes to the black dot in the basin containing x(0). Figure 3-7 shows how steepest descent paths can be more complicated in more dimensions.

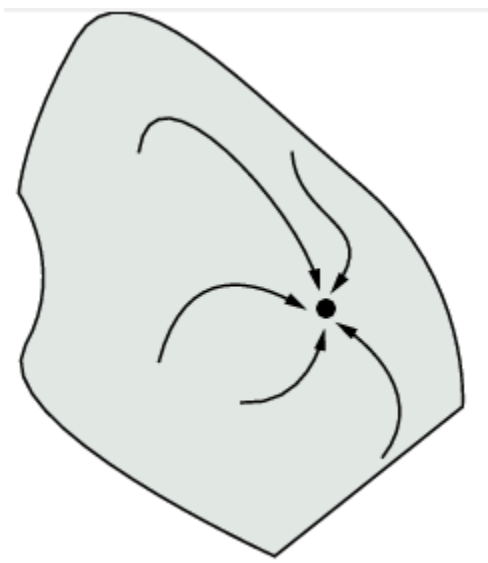


Figure 3-5 Steepest Descent with Multiple Lines of Descent

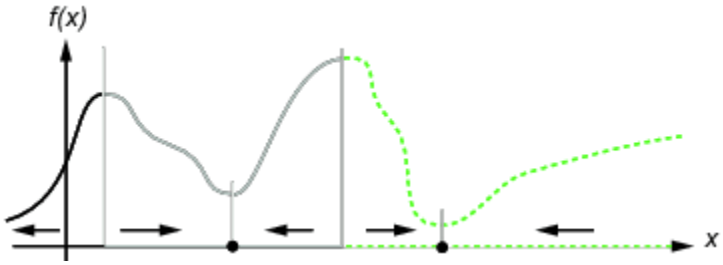


Figure 3-6 **Steepest Descent for Two One Dimensional Local Minima**

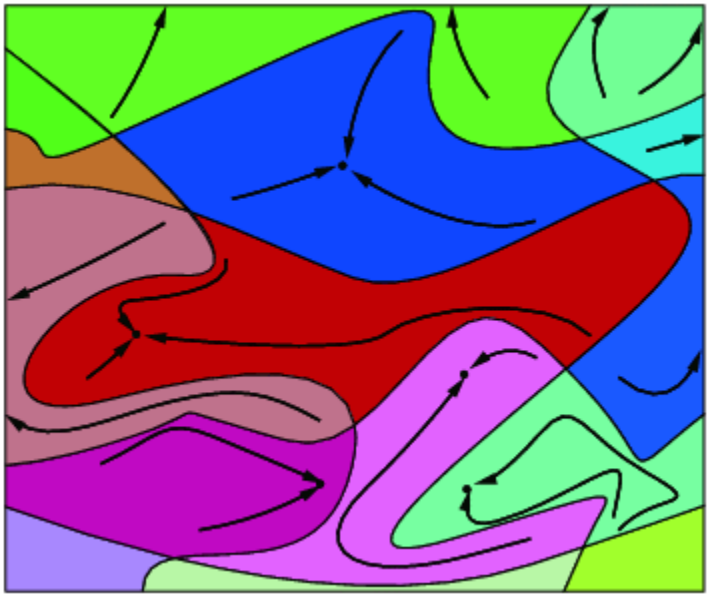


Figure 3-7 **Complex Multiple Descent Paths and Basins of Attraction**

Constraints can break up one basin of attraction into several pieces. Figure 3-8 shows two basins of attraction with the final points. The steepest descent paths are straight lines down to the constraint boundaries. From the constraint boundaries, the steepest descent paths travel down along the boundaries. The final point is either (0,0) or (11/4,11/4), depending on whether the initial x-value is above or below 2.

3.4.1.2.5 How To Choose an Optimizer

An optimizer is based on the problem characteristics and on the type of solution you want. MATLAB provides five Global Optimization solvers:

1. *ga (Genetic Algorithm)*
2. *GlobalSearch*
3. *MultiStart*
4. *Patternsearch, also called direct search*
5. *Simulannealbnd (Simulated Annealing)*

Table 3-1, obtained from the MATLAB User Documentation, recommends the global solver based on the desired optimization solution.

Table 3-2 is used to provide guidance to determine the *Desired Solution*. From Table 3-1 and Table 3-2, it is observed that there are functions that determine single local and multiple local minima as well as single global minima.

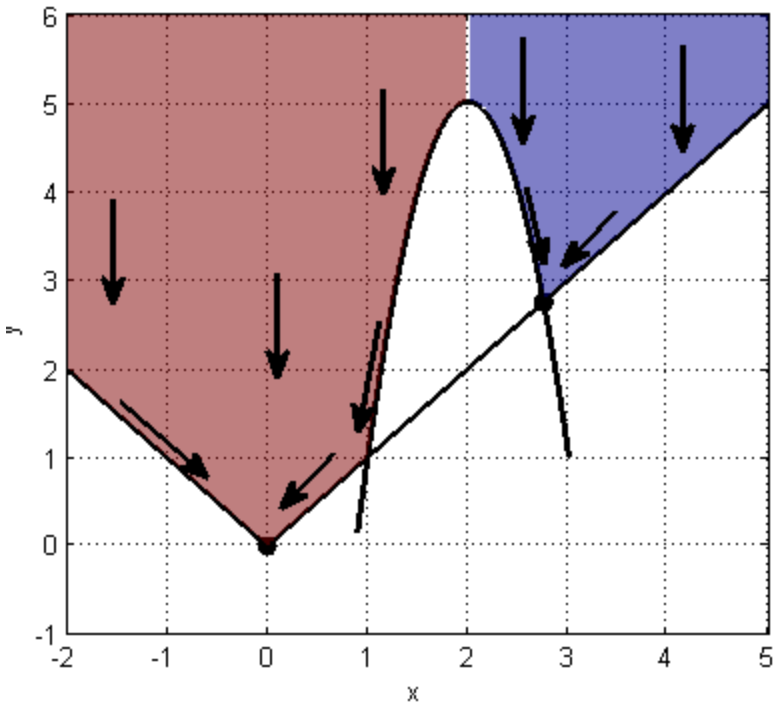


Figure 3-8 Two Basins of Attraction with Final Points

Table 3-1 MATLAB Local and Global Functions

Desired Solution	Smooth Objective and Constraints	Non-Smooth Objective or Constraints
Single Local Minimum	N/A	Fminbnd, Patternsearch, fminiseaerch,ga,simuilannealbnd
Multiple Local solutions	Globalsearch, Multistart	N/A
Single Global Solution	Globalseaerch, Multistart, Patternsearch, ga, Simulannealbnd	Patternsearch, ga, Simulannealbnd

3.4.1.3 Choosing Between Solvers for Smooth Problems

For a single global solution, MATLAB recommends using GlobalSearch first. It is the most focused on finding a global solution, and has an efficient local solver, *fmincon***.** MATLAB recommends the use of *MultiStart* second. It has efficient local solvers, and can search a wide variety of start points. Matlab recommends the use of *Patternsearch* third. It is less efficient, since it does not use gradients. However, *Patternsearch* is robust and is more efficient than the remaining local solvers. It recommends using *ga* fourth. It can handle all types of constraints, and is usually more efficient than *Simulannealbnd*. It recommends using *Simulannealbnd* last. It can handle only unconstrained or bound constraints. *Simulannealbnd* is usually the least efficient solver. However, given a slow enough cooling schedule, it can find a global solution.

Table 3-2 Definitions of Local and Global Optimal Solutions

Term	Meaning
Single Local Solution	Find one local solution, a point x where the objective function f(x) is a local minimum.
Multiple Local solutions	Find a set of local solutions.
Single Global solution	Find the point x where the objective function f(x) is a global minimum.

GlobalSearch and *MultiStart* both provide multiple local solutions. *GlobalSearch* and *MultiStart* differ in the following characteristics:

- *MultiStart* can find more local minima. This is because *GlobalSearch* rejects many generated start points (initial points for local solution). Essentially, *GlobalSearch* accepts a start point only when it determines that the point has a good chance of obtaining a global minimum. In contrast, MultiStart passes all generated start points to a local solver.
- *MultiStart* offers a choice of local solver: *fmincon, fminunc, lsqcurvefit*, or *lsqnonlin*. The *GlobalSearch* solver uses only *fmincon* as its local solver.
- *GlobalSearch* uses a scatter-search algorithm for generating start points. In contrast, *MultiStart* generates points uniformly at random within bounds, or allows you to provide your own points.
- *MultiStart* can run in parallel.

3.4.1.3.1 Choosing Between Solvers for Nonsmooth Problems

For non-smooth optimization problems, MATLAB recommends the following solvers in order to evaluate the effectiveness of the solver. Use *fminbnd* first on one-dimensional bounded problems only. *fminbnd* provably converges quickly in one dimension. Use *Patternsearch* on any other type of problem. *Patternsearch* provably converges, and handles all types of constraints. Use *fminsearch* next for low-dimensional unbounded problems. *fminsearch* is not as general as *Patternsearch* and can fail to converge. For low-dimensional problems, *fminsearch* is simple to use, since it has few tuning options. Use *ga* next. *ga* has little supporting theory and is often less efficient than *Patternsearch.* It handles all types of constraints. Use *Simulannealbnd* last for unbounded problems, or for problems with bounds. *Simulannealbnd* provably converges only for

a logarithmic cooling schedule, which is extremely slow. *Simulannealbnd* takes only bound constraints, and is often less efficient than ga.

Table 3-3 outlines the characteristics for the non-smooth solvers that are available in MATLAB and are considered for use in the Human Capital inventory optimization model.

3.4.1.3.2 Explanation of Solver Characteristics

The following are explanations of the terms used in MATLAB to define the characteristics for the various solvers.

Convergence — Solvers can fail to converge to any solution when started far from a local minimum. When started near a local minimum, gradient-based solvers converge to a local minimum quickly for smooth problems. *Patternsearch* provably converges for a wide range of problems, but the convergence is slower than gradient-based solvers. Both *ga* and *Simulannealbnd* can fail to converge in a reasonable amount of time for some problems, although they are often effective.

Iterates — Solvers iterate to find solutions. The steps in the iteration are iterates. Some solvers have deterministic iterates. Others use random numbers and have stochastic iterates.

Gradients — Some solvers use estimated or user-supplied derivatives in calculating the iterates. Other solvers do not use or estimate derivatives, but use only objective and constraint function values.

Start points — Most solvers require the user to provide a starting point for the optimization. One reason they require a start point is to obtain the dimension of the decision variables. *ga* does not

Table 3-3 Non-Smooth Solver Convergence and Characteristics

Matlab Solver	Convergence	Characteristics
fminbnd	Proven convergence to local optima for smooth problems, slower than quadratic.	Deterministic iterates No gradients User-supplied start point Only one-dimensional problems
Patternsearch	Proven convergence to local optimum, slower than gradient-based solvers.	Deterministic iterates Can run in parallel No gradients User-supplied start point
fminsearch	No convergence proof, counterexamples exist.	Deterministic iterates No gradients User-supplied start point
ga	No convergence proof.	Population-based No gradients Automatic start population, or user-supplied population, or combination of both
Simulannealbnd	Proven to converge to global optimum for bounded problems with very slow cooling schedule.	Stochastic iterates No gradients User-supplied start point Only bound constraints

require any starting points, because it takes the dimension of the decision variables as an input. *ga* can generate its start population automatically.

3.4.1.3.3 The Optimization Functions

The goal in this research is to determine a global solution for a low-dimensional optimization function for the Human Capital inventory cost function. The criteria for the Human Capital inventory cost functions are:

- Non-smooth Objective function
- Non-linear Objective function
- Low Dimension optimization problem
- Global Solutions
- Bounded Constraints
- Unknown gradients

Based on the above criteria, several optimization functions will be utilized to determine optimium minimum cost solutions. Based on the Human Capital inventory cost criteria, the *Simulannealbnd* appears to satisfy these criteria and will be utilized in this research for determining a global solution. The other solvers (fmincon, fminunc, Patternsearch, multistart, globalsearch) will be explored to investigate and compare local and global optimal solutions of the optimization functions. *Simulannealbnd* finds unconstrained or bound-constrained minimum of the objective function of several variables. The application of *Simulannealbnd* solver within MATLAB has the following input and output parameters. The other optimization functions have similar input and output parameters. These parameters are described below.

The *Simulannealbnd* format syntax is:

[x,fval,exitflag,output] = Simulannealbnd(fun,x0,lb,ub,options)

The input parameters of the *Simulannealbnd* solver are:

fun: The is the objective function that is evaluated during the optimization process. The objective function accepts either a scalar *x* or a vector *x* and returns a scalar function value.

x_0: original user selected starting points for the solver. x_0 may be scalar or a vector.

lb,ub: The *lb* and *ub* defines a set of lower and upper bounds on the design variables, *x*, so that a solution is found in the range $lb \leq x \leq ub$. Empty matrices for *lb* and *ub* can be used if no bounds exist. The lower and upper bounds can be set if no bounds exists, i.e set *lb(i)* to *-Inf* if *x(i)* is unbounded below; set *ub(i)* to *Inf* if *x(i)* is unbounded above.

The output parameters of the Simulannealbnd are:

Fval: the value returned from the objective function evaluated at *x*.

Exitflag: an integer identifying the reason the algorithm terminated. The following lists the values of exitflag and the corresponding reasons the algorithm terminated:

- 1 — Average change in the value of the objective function.
- 5 — Objective limit reached.
- 0 — Maximum number of function evaluations or iterations exceeded.
- -1 — Optimization terminated by the output or plot function.
- -2 — No feasible point found.
- -5 — Time limit exceeded.

Output: a structure that contains information about the problem and the performance of the algorithm. The output structure contains the following fields:

- problemtype — Type of problem: unconstrained or bound constrained.
- iterations — The number of iterations computed.
- funccount — The number of evaluations of the objective function.
- message — The reason the algorithm terminated.
- temperature — Temperature when the solver terminated.
- totaltime — Total time for the solver to run.
- rngstate — State of the MATLAB random number generator, just before the algorithm started. You can use the values in rngstate to reproduce the output of Simulannealbnd.

3.4.1.3.4 Simulation Model Verification, Validation, Documentation

Model verification and validation (V&V) are essential parts of the model development process if models are to be accepted and used to support decision making (Macal, 2005). Verification is concerned with building the *model right*. It is utilized in the comparison of the conceptual model to the computer representation that implements that conception. Verification asks the questions: Is the model implemented correctly in the computer? Are the input parameters and logical structure of the model correctly represented? Is the model programmed correctly? Does the model contain errors, oversights, or bugs. Verification ensures that the specification is complete and that mistakes have not been made in implementing the model. Verification does not ensure the model solves an important problem, meets a specified set of

model requirements or correctly reflects the workings of a real world process. Verification of computer models can be accomplished by:

- Making a flow diagram which includes each logically possible action a system can take when an event occurs, and follow the model logic for each action for each event type.
- Closely examining the model output for reasonableness under a variety of settings of the input parameters. Have the code print out a wide variety of output statistics.
- Having the computerized model print the input parameters at the end of the simulation, to be sure that these parameter values have not been changed inadvertently.
- Making the computer code as self-documenting as possible. Give a precise definition of every variable used, and a general description of the purpose of each major section of code.

Model *Validation* ensures that the model meets its intended requirements in terms of the methods employed and the results obtained. The ultimate goal of model *validation* is to make the model useful in the sense that the model addresses the right problem and provides accurate information about the system being modeled. Validation is concerned with building the *right model*. It is utilized to determine that a model is an accurate representation of the real system. Validation is usually achieved through the calibration of the model, an iterative process of comparing the model to actual system behavior and using the discrepancies between the two, and the insights gained, to improve the model. This process is repeated until model accuracy is judged to be acceptable. To obtain a *high* degree of confidence in a simulation model and its

results, comparisons of the model's and system's output behaviors for several different sets of experimental conditions are usually required. Thus if a system is not observable, which is often the case, it is usually not possible to obtain a high degree of confidence in the model. In this situation the model output behavior(s) should be explored as thoroughly as possible and comparisons made to other valid models whenever possible.

Documentation on model verification and validation is usually critical in convincing users of the "correctness" of a model and its results, and should be included in the simulation model documentation. Both detailed and summary documentation are desired. The detailed documentation should include specifics on the tests, evaluations made, data, results, etc. The summary documentation should contain a separate evaluation table for data validity, conceptual model validity, computer model verification, operational validity, and an overall summary.

3.5 Summary

The research conducted in this dissertation has revealed that there is going to be a significant loss of Human Capital in the public sector in the coming decades due to the large numbers of the 'baby boomers' that will be departing from the workforce. Typical methods of assessing the impact of this loss will not be adequate for Human Capital planning. New paradigms in Human Capital inventory management is necessary in order for all organizations, both public and private, to assure that an organization has the right people in the right jobs at the right time to accomplish organizational objectives. Traditionally, methods for managing the most critical physical inventories of organizations have occurred utilizing typical physical inventory models. Research associated with this dissertation has resulted in the opinion that traditional physical inventory models can be applied in a similar manner to Human Capital inventories.

Occurring simultaneously with departure of the ‘baby boomers’ is the financial stress that is occurring among all levels of government (federal, state, county, city). This research is focused on the development of methodologies and techniques for public sector organizations to address their Human Capital planning as well as ensuring that the Human Capital inventories are maintained at the least cost. In summary, this research is focused on the following:

- Identify and forecast the effects of attrition on the organization
- Determine a minimum cost Human Capital inventory management plan (minimum size Human Capital inventory and the associated cost to meet organizational demand)
- Identify when the Human Capital inventory will need to be replenished based on an organization’s staffing policies.
- Research has revealed that there are significant gaps in Human Capital inventory planning by Human Resource departments in applying comprehensive Human Capital models.

This research is an attempt to fill these gaps in the management of Human Capital inventories.

Chapter 4 - Model Development and Application

This chapter focuses on the application of cluster analysis, survival analysis and optimization functions to develop a minimum cost Human Capital Inventory Cost Model. The MATLAB software components that were developed for this study are presented in Figure 4-1.The source code for the MATLAB programs can be found in Appendix B. The process includes data preparation, cluster analysis, survival analysis and minimum cost optimization.

The data that is used in developing the Human Capital inventory attrition is presented prior to the discussion of the inventory demand and the inventory availability. Following the discussion of the data, the application of cluster analysis and survival analysis are used to develop the Human Capital replenishment model and the Human Capital inventory attrition model. Following the attrition model development, the development and application of the Human Capital inventory simulation optimization model is presented. The techniques and statistical analysis that are utilized are selected from the alternatives presented in Chapter 3. Sample scenarios applying the Human Capital inventory simulation optimization model and the results are presented in subsequent sections.

4.1 Human Capital Inventory Data

Data acquired for this research originated from Human Capital populations from two large organizations. One dataset (Dataset1) consists of 1648 records that represent employment data for the period from October 1, 2000 to January 31, 2010. This data represents 691 employees that left the organization during this time period. The remaining 957 records represent

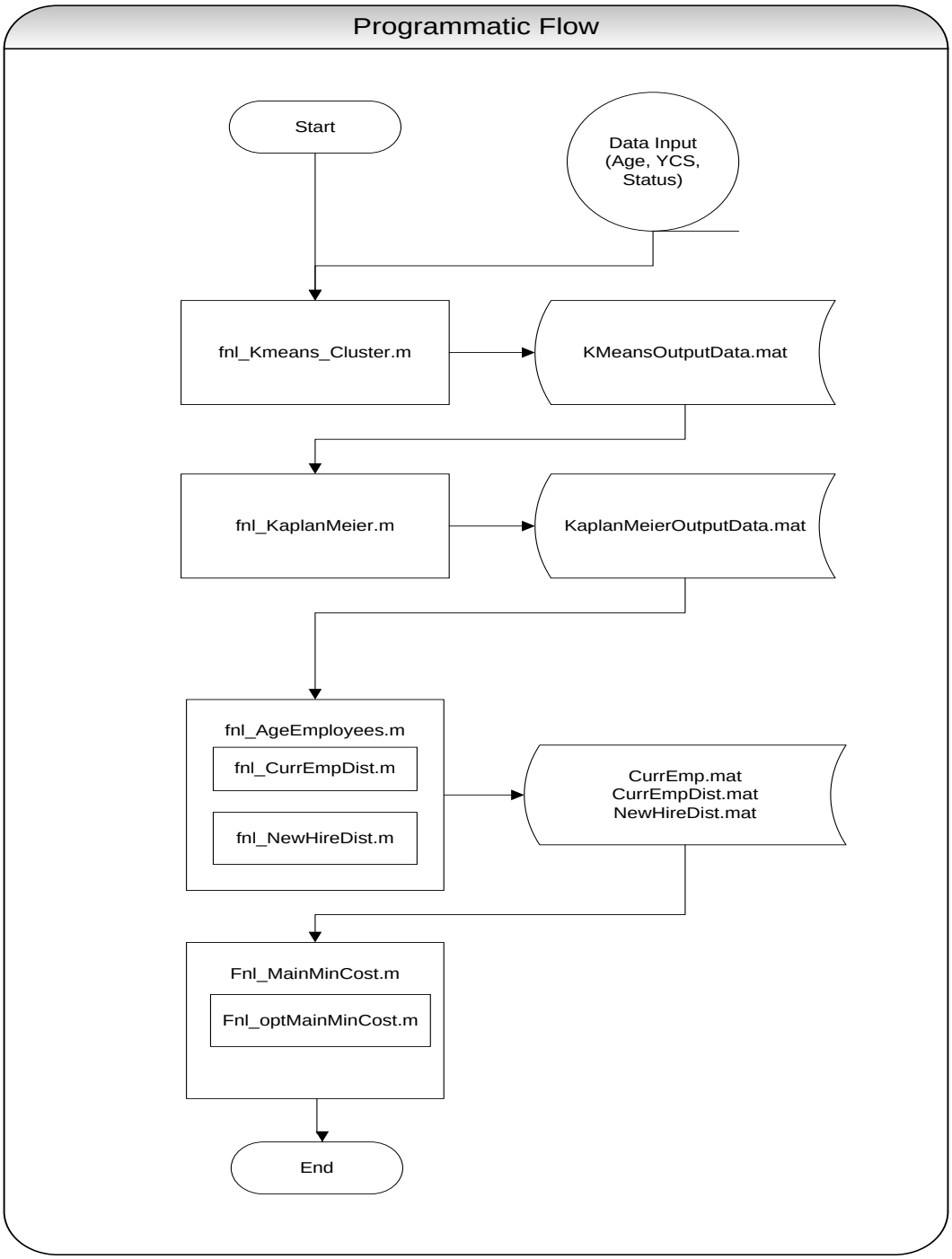


Figure 4-1 MATLAB Programmatic Flow

employees that were hired prior to or during this period and is still a member of this population at the end of the study period. The second dataset (Dataset2) consists of 9342 data records that represent employment data for the period October 1, 2000 through September 30, 2010. This dataset includes 4641 employees that left the organization during the period from October 1, 2000 to January 31, 2010 with the remaining dataset consisting of 4701 employees that remained in the organization at the end of this study period. Each employee data record in each dataset includes the following:

- Hire Date
- Age as of date of report (if current employee)
- Years of company service as of date of report (if Current Employee)
- Age at date of termination (if terminated)
- Years of company service at date of termination (if terminated)
- Date of attrition (if terminated)
- Status (1, if left the organization, 0 if remained a member)

The datasets consists of records for full-time employees. Part-time or temporary employees, such as subcontractors, interns or student coop's are not included. Inclusion of part-time employees would have the potential to bias the frequency of the terminations with a low number of years of service.

These two datasets are utilized in the development of a replenishment model and an attrition model for each organization. The replenishment model is developed utilizing the duration of employment for the last five years of the study period. An empirical distribution is created for each population dataset to be utilized for replenishing the Human Capital inventory

within the simulation model. The attrition model is developed utilizing the age and years of company service of all members of each dataset at the time of separation for individuals that left the organization. Cluster analysis and Survival analysis are the statistical techniques used to develop the attrition model that is incorporated within the Human Capital inventory simulation model. The two datasets are also used to establish the initial Human Capital inventory availability that is used in the Human Capital inventory simulation model.

4.2 Human Capital Units of Measure

A Human Capital inventory is typically represented by the number of individuals in the workforce population. However, when forecasting workforce needs in an organization, work-scopes are analyzed and the resulting workforce required to accomplish planned work-scopes are typically estimated as Full-Time-Equivalents (FTE's). The number of FTE's typically does not equal the workforce headcount. Since most Human Capital planning assumes that not all of an individual's hours in an inventory period are available during the course of a period, a calculation is made to determine the number of available hours to apply to the accomplishment of work scopes for each employee. As an example, organizations in both the public and private sector use historical labor data to determine the number of hours an individual typically utilizes to accomplish an organization's work-scope. The available hours for an organization can vary widely based on the nature of the work within an organization. Continuing our example, let's assume that the number of available hours for each individual in an organization for the period of a year is 1750 hours out of a possible annual 2080 hours (52 weeks * 40 hours/week). Assuming that the hours associated with an FTE is equal to 1750 for each individual in the organization and an organization has 100 individuals available to accomplish the organizations work-scope, the

available FTE's for a year would equal

$$100 * \left(\frac{1750}{2080}\right) = 84.3462.$$

Conversely, if 100 FTE's are needed to accomplish an organization's work-scope, the organization's workforce headcount would equal $100 * \frac{2080}{1750} = 118.8751$ individuals. Thus, the unit of measure of the inventory demand in the human resource inventory model is the number of FTE's.

In order to assure consistent units of measure are used within the Human Capital Inventory Cost Model, the Human Capital inventory availability will also use FTE's. The availability is calculated by multiplying the number of individuals in the Human Capital inventory by $\frac{1750}{2080} = .841346$ factor.

4.3 Human Capital Inventory Replenishment

All inventory systems require that inventory resources be replenished in order to sustain the resource pool to accomplish work scopes. In this research, the replenishment distribution is based on the last five years of the hiring practices of each organization. Initial analysis of the two datasets is presented in this section.

The New Hire replenishment distribution is determined by selecting from the Current Employee distribution those employees in each dataset that have years of service less than or equal to five years, i.e., those that have been hired within the last five years. The frequency distribution percentages for the New Hire ages is presented in Figure 4-2 for Dataset1 and Figure 4-3 for Dataset2. From this data, a New Hire survival distribution is determined by applying the Kaplan Meier Survival analysis for these employees. From the Kaplan-Meier analysis, New Hire

attrition rates for Dataset1 (Figure 4-4) and Dataset2 (Figure 4-5) is displayed. It is quite noticeable for Dataset1 (Figure 4-4) as for Dataset2 (Figure 4-5) that the attrition rate among the population of New Hires varies considerably over time. The attrition rate varies for Dataset1 from as low as 1.5992% to as high as 6.1415% (Figure 4-6). The attrition rate for Dataset2 (Figure 4-7) varies from a low of 1.31% to a high of 7.43%. When the replenishment of the Human Capital inventory is necessary, the quantity of FTE's determined by the optimization function at the beginning of a cycle and for the succeeding periods in the cycle will be determined by multiplying the replenishment quantity by the cumulative survival probability for the New Hire distribution. This technique reduces the available Human Capital inventory over time by attrition. As an example, using Figure 4-6 for Dataset1, if the replenishment quantity for a cycle is determined to be 100 FTE's, period 1 in this cycle will include 100% of the replenishment quantity. In period 2, only 94.2997% of the population of new hires will survive, thus lowering the number of period 2 FTE's to 94.2997. By period 12, only 50.4965% of the original cycle new hires will remain. This method of replenishing the Human Capital inventory will be applied for each cycle within the planning horizon.

4.4 Human Capital Inventory Attrition

As discussed in Chapter 3, *Attrition* is defined as the reduction in the membership of an organization. In order to appropriately determine the availability of the Human Capital inventory from one period to another, attrition must be applied to the resource inventory from period to period. In this research, attrition rates are determined utilizing cluster analysis and survival analysis. The application of these two techniques provides a better method for determining

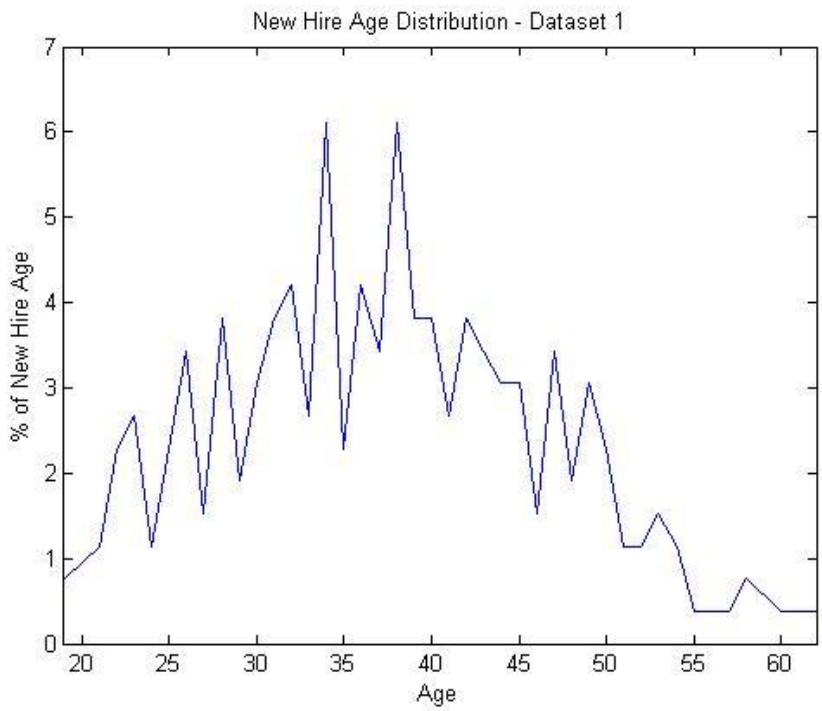


Figure 4-2 New Hire Age Frequency Distribution – Dataset1

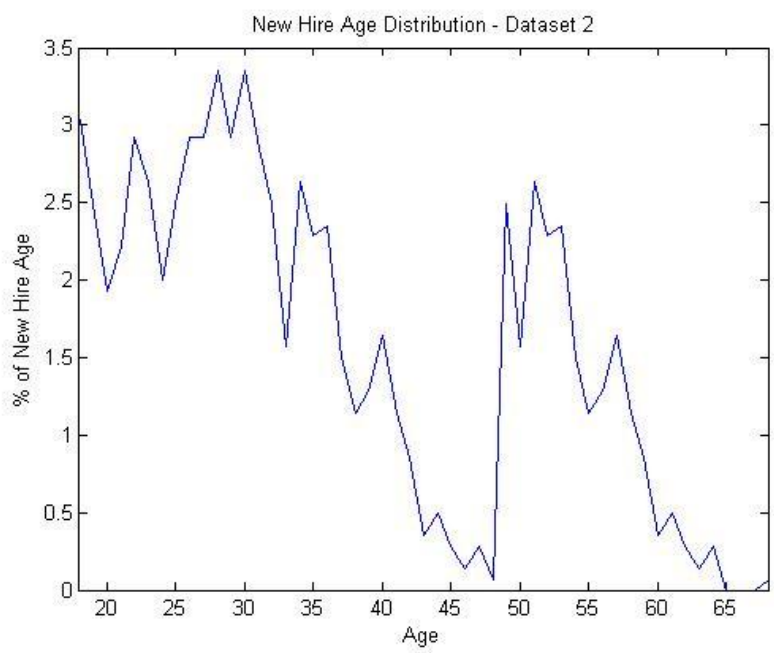


Figure 4-3 New Hire Age Frequency Distribution – Dataset2

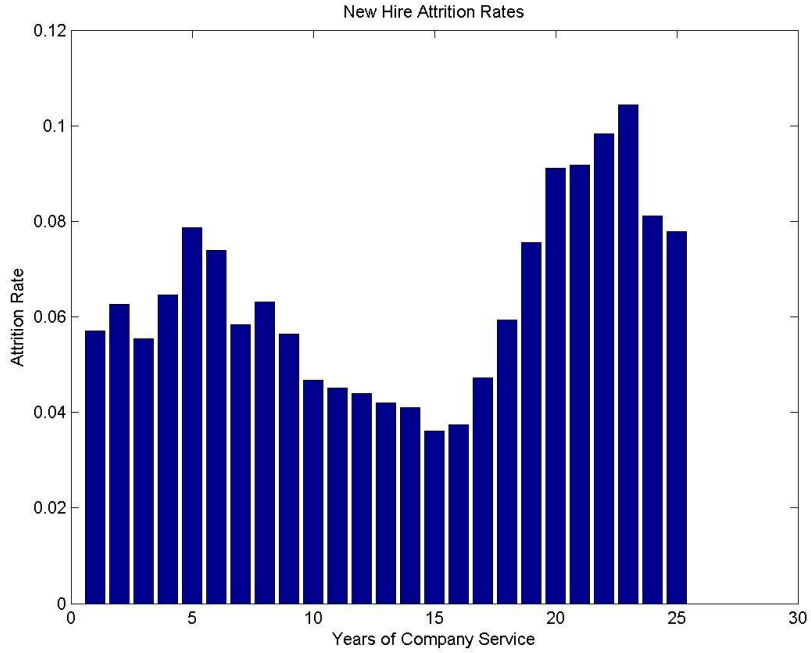


Figure 4-4 New Hire Attrition Rates – Dataset1

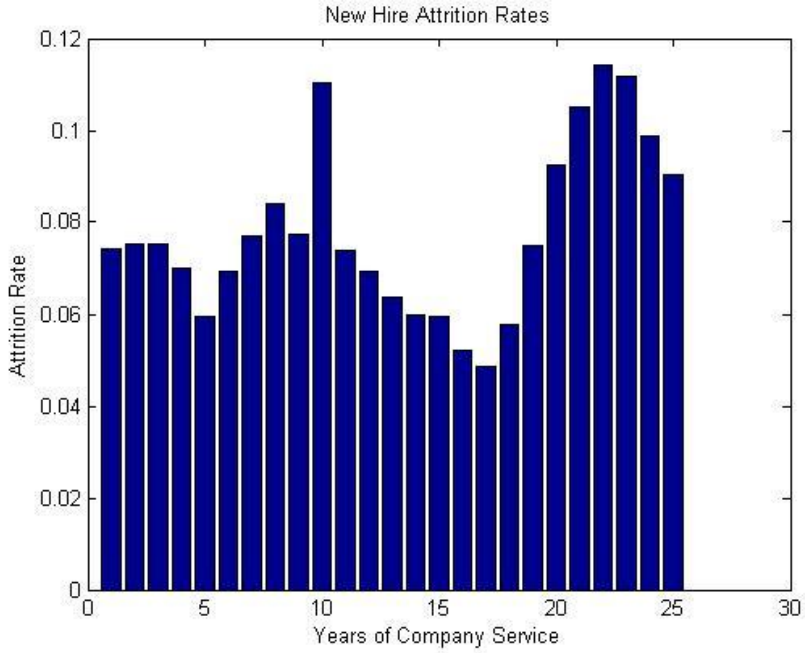


Figure 4-5 New Hire Attrition Rates – Dataset 2

New Hires (Dataset1)			
Period	Cumulative Survival Probability	Survival Probability Density	Attrition Rate
1	100%	100.0000%	0.0000%
2	94.2997%	94.2997%	5.7003%
3	88.3949%	93.7383%	5.9047%
4	83.4999%	89.0776%	4.8950%
5	78.1062%	87.6833%	5.3937%
6	71.9647%	82.0734%	6.1415%
7	66.6400%	81.1956%	5.3247%
8	62.7515%	77.2844%	3.8885%
9	58.7904%	76.0702%	3.9611%
10	55.4727%	72.9230%	3.3177%
11	52.8843%	72.5208%	2.5883%
12	50.4965%	69.6303%	2.3878%
13	48.2782%	69.3350%	2.2183%
14	46.2524%	66.7085%	2.0258%
15	44.3561%	66.4923%	1.8963%
16	42.7568%	64.3034%	1.5992%
17	41.1556%	64.0022%	1.6013%
18	39.2123%	61.2671%	1.9433%
19	36.8847%	60.2031%	2.3276%
20	34.1004%	56.6423%	2.7843%
21	30.9945%	54.7197%	3.1059%
22	28.1506%	51.4451%	2.8439%
23	25.3853%	49.3445%	2.7653%
24	22.7366%	46.0773%	2.6487%
25	20.8923%	45.3419%	1.8443%
26	19.2647%	42.4877%	1.6276%
27	0.0000%	0.0000%	0.0000%

Figure 4-6 New Hire Probabilities with Attrition Rates – Dataset1

New Hires (Dataset2)			
Period	Cumulative Survival Probability	Survival Probability Density	Attrition Rate
1	100.00%	100.00%	0.00%
2	92.57%	92.57%	7.43%
3	85.59%	92.46%	6.98%
4	79.15%	92.48%	6.44%
5	73.61%	93.00%	5.54%
6	69.24%	94.07%	4.37%
7	64.44%	93.07%	4.80%
8	59.49%	92.31%	4.95%
9	54.49%	91.59%	5.00%
10	50.27%	92.27%	4.21%
11	44.72%	88.96%	5.55%
12	41.43%	92.63%	3.30%
13	38.56%	93.08%	2.86%
14	36.10%	93.63%	2.46%
15	33.94%	94.00%	2.17%
16	31.92%	94.05%	2.02%
17	30.26%	94.81%	1.66%
18	28.80%	95.16%	1.47%
19	27.13%	94.21%	1.67%
20	25.10%	92.50%	2.04%
21	22.78%	90.75%	2.32%
22	20.38%	89.49%	2.39%
23	18.06%	88.59%	2.33%
24	16.04%	88.82%	2.02%
25	14.45%	90.12%	1.58%
26	13.15%	90.96%	1.31%
27	0.00%	0.00%	0.00%

Figure 4-7 New Hire Probabilities with Attrition Rates – Dataset2

Human Capital inventory availability than other generally accepted techniques such as previously discussed turnover rates. The process that is developed for developing the Human Capital inventory attrition model is depicted in Figure 4-8. Beginning with a population of employees and the age, years of company service and whether or not an employee has separated from the organization during the study period, all terminated employees are clustered into groups based on the 'closeness' of each individual with others. This method of grouping is used to identify clusters of individuals with similar age and years of company service. Once these clusters are determined, all non-separated members of the population are assigned to one of the 'attrited' clusters based on their current age and years of company service. Once everyone is assigned to a cluster, a survival analysis is performed for each cluster in order to determine a cluster survival distribution. These cluster survival models will be utilized in the Human Capital Inventory Cost Model to determine period-to-period attrition of employees.

4.4.1 Cluster Analysis

An initial analysis of the employee data reveals that there is a potential natural clustering of individuals that terminated employment during the study period for each dataset. Figure 4-9 and Figure 4-10 are plots of the 'attrited' employees for Dataset1 and Dataset2, respectively. A visual inspection of the data reveals a natural clustering of the attrition for the age and years-of-company service with the higher frequency of attrition clearly visual.

4.4.1.1 Initial Data Analysis: Dataset1

Figure 4.9 is a graphical representation of the distribution of the 'attrited' workforce based on the age and years of company service for each member of the workforce of Dataset1 in the study period. As can be seen in Figure 4-9, there are significant differences in the age and

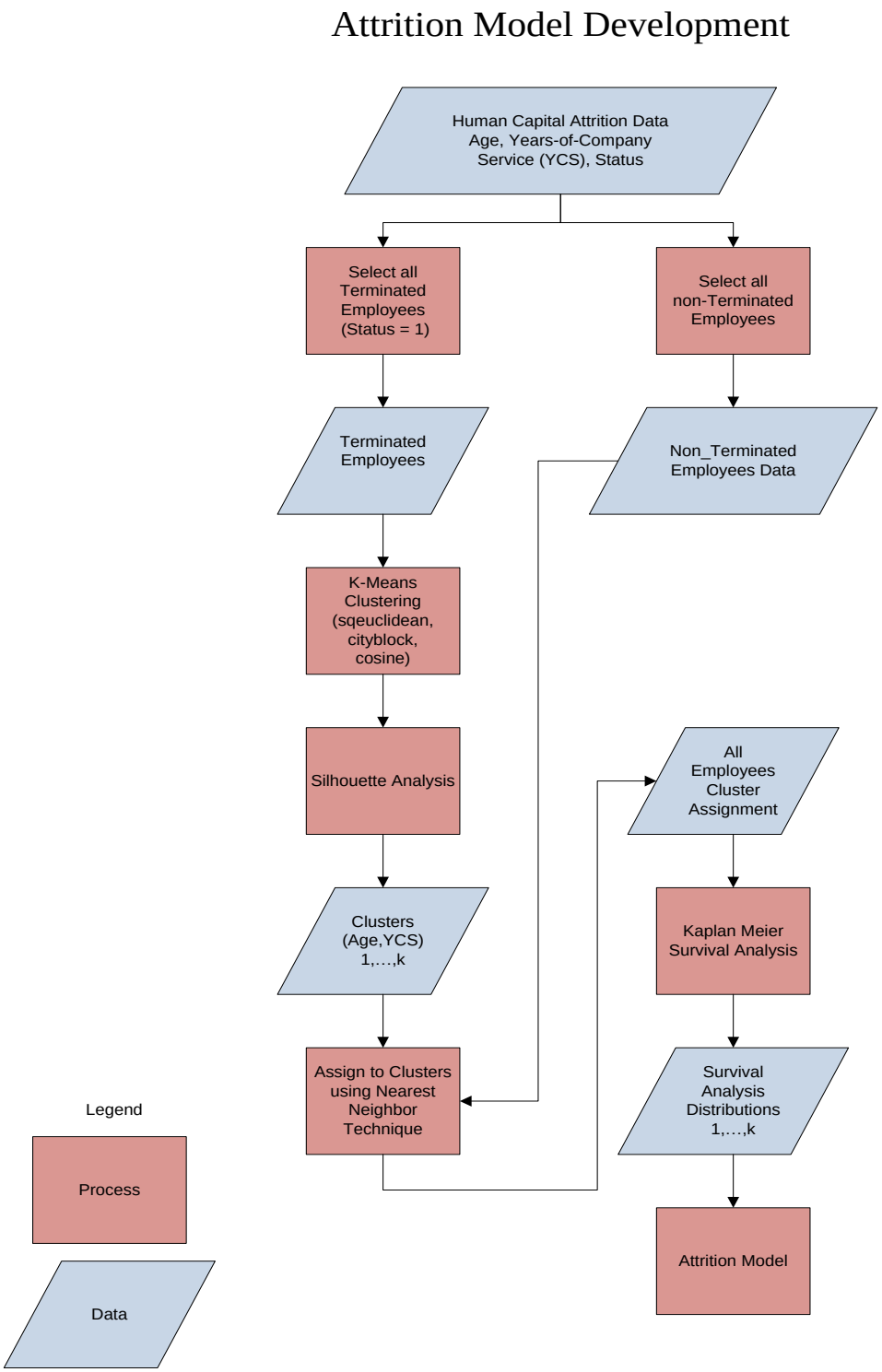


Figure 4-8 Human Capital Inventory Attrition Model

years of service frequency distribution in the workforce attrition. Intuitively, based on these observations, it would be inappropriate to estimate a single attrition rate for this population. One typical method that is used within Human Resources organizations is to calculate employee turnover (attrition) as

$$Turnover\ Rate = \frac{number\ of\ separations\ during\ a\ period}{average\ number\ of\ employees\ during\ the\ period}.$$

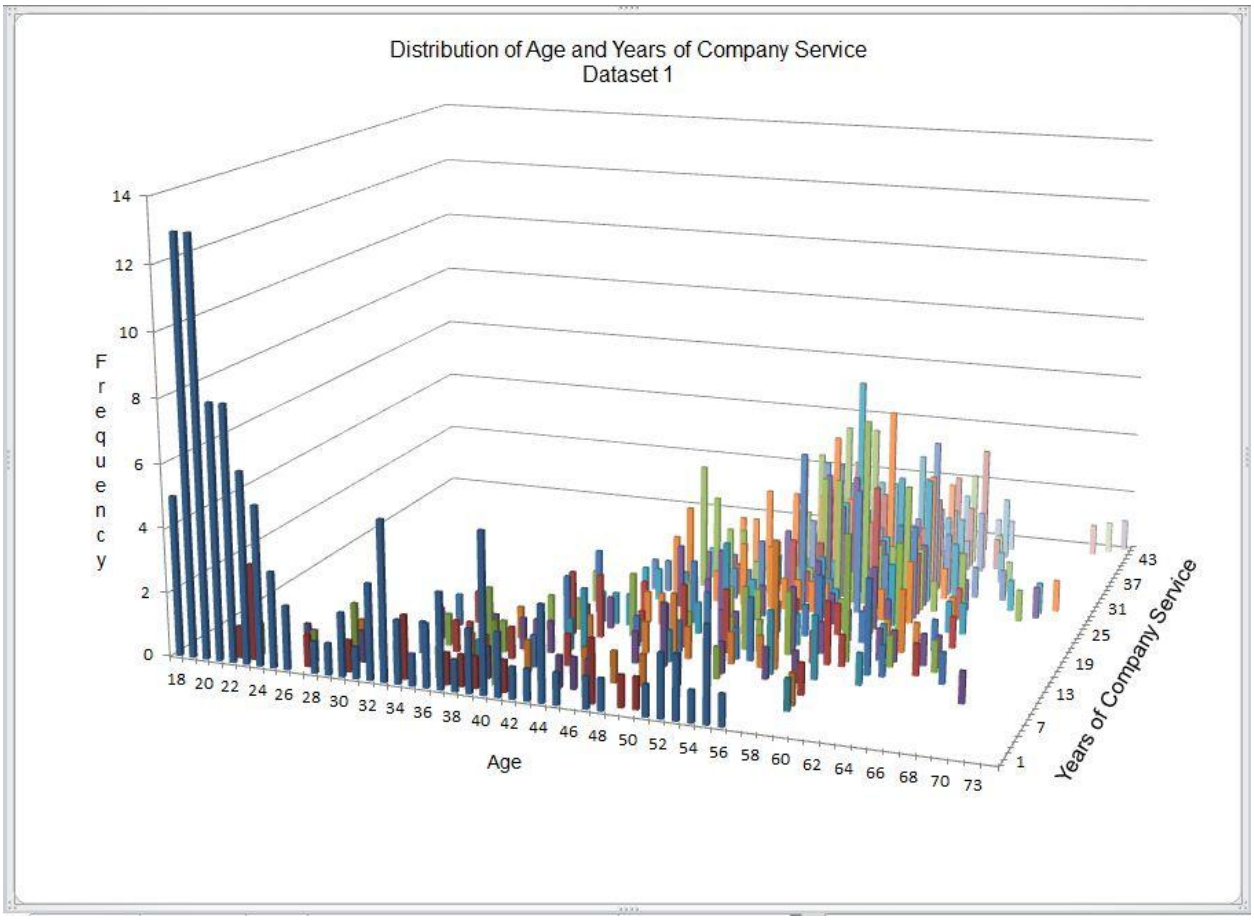


Figure 4-9 Attrition Distribution by Age and Years of Company Service – Dataset1

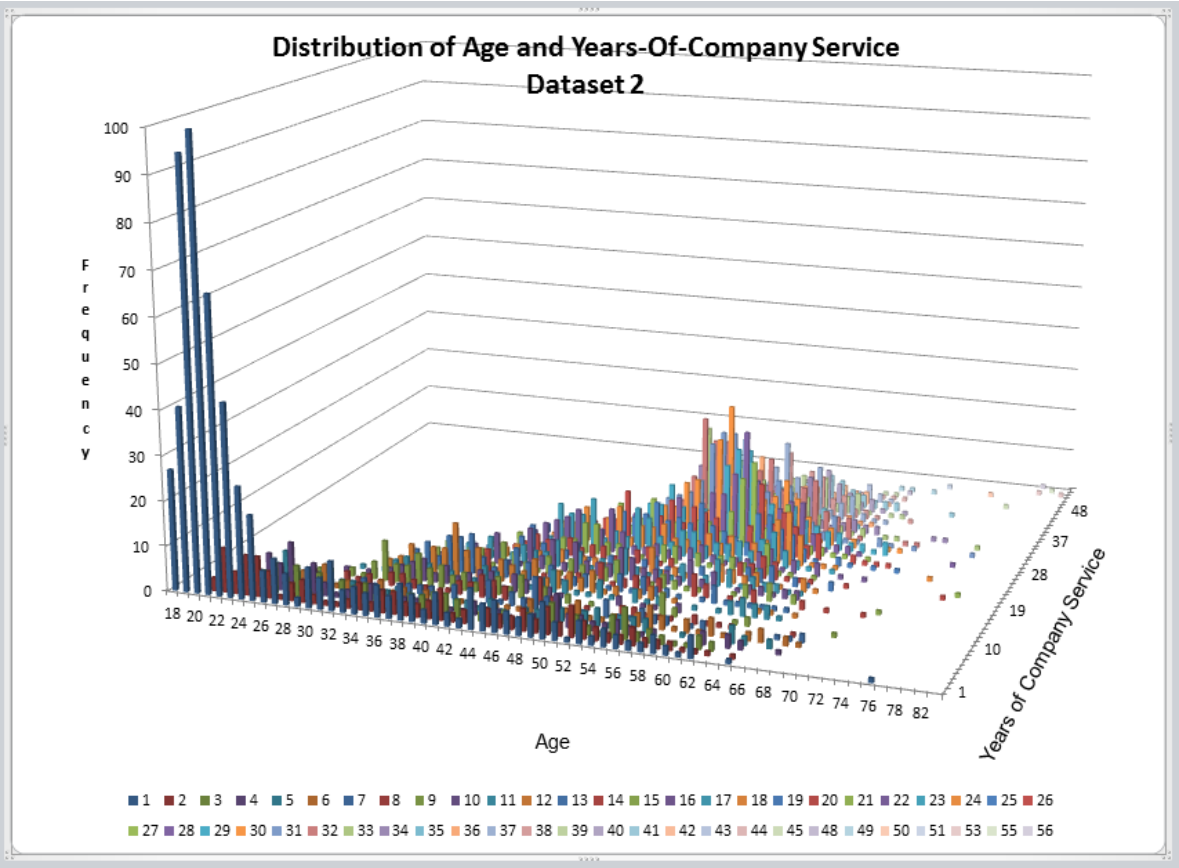


Figure 4-10 Attrition Distribution by Age and Years of Company Service – Dataset2

Turnover rates have been calculated for each calendar year for Dataset1 and Dataset 2. Results of the turnover rates are presented in Table 4-1. Several anomalies occurred in the two datasets. In Dataset 1 in the year 2010, there was insufficient data to calculate a turnover rate. Also in the year 2004, there was an offer for volunteer separation which created incentives for employees to retire. This anomaly resulted in higher than typical attrition. In Dataset2 in the year 2000, there was a change in corporate management that resulted in a distortion in the new hire numbers due to reassignment of employees to the new corporation. Also in Dataset2, in 2008, there was also a voluntary separation program with incentives that resulted in larger than normal

Table 4-1 Annual Turnover Rates (percentage) for Dataset 1 and Dataset 2

Turnover Rates (Annual Percent)											
Calendar Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Dataset1	6.5	5.4	4.8	6.3	12.4	8.7	4.9	6.2	8.2	7.1	NA
Dataset2	18.7	6.5	9.0	7.2	8.8	9.5	10.2	8.1	13.0	5.0	6.1

attrition. As can be observed, if a single turnover rate is used to forecast employee separation (attrition), significant variances would incur in a population such as in Dataset1 and Dataset2 (Figure 4-9 and Figure 4-10, respectfully). When using turnover rates, a decision must be made as to what turnover rate to use. Turnover rates have the potential to significantly overestimate or underestimate attrition.

Upon closer examination of Dataset1 (Figure 4-9), there are obvious "clusters" of data where the attrition rates are significant higher within the observed population. The same observation can be made with Dataset2 (Figure 4-10). The two most significant "clusters" in Dataset1 (Figure 4-9) are observed where the age is in the vicinity of 20 with 1 year of service and in the vicinity with an age of 54 years and 33 years of service. In Dataset2 (Figure 4-10), there visually appears to be at least three natural clusters of attrition. The utilization of the attrition data in Dataset1 and Dataset2 is used to develop two attrition models for the Human Capital inventories within each organization.

4.4.1.2 Cluster Evaluation

As discussed in Chapter 3, statistical clustering techniques are applied to the two datasets used in this research. The cluster analysis includes the application of KMEANS clustering techniques with three measures of distance. In order to evaluate the effectiveness of the clustering, the Silhouette Indexing method of measuring the clustering effectiveness is utilized.

4.4.1.3 K-MEANS Clustering

As presented in Chapter 3, *K*-Means clustering is one of the more commonly used clustering algorithms (MacQueen, 1967). The objective in clustering is to construct a partition of a dataset *D* of *N* objects into a set of *K* clusters. When using K-Means clustering, an initial

choice of *K*, the number of desirable partitions of the dataset *D*, must be chosen. K-Means Clustering is a heuristic method where each cluster is represented by the centre of the cluster (i.e. the centroid). As stated earlier, MATLAB is used to perform the Clustering Analysis in this research.

The MATLAB K-Means function partitions an n-by-p data matrix into k clusters. The iterative partitioning minimizes the sum, over all clusters, of the within-cluster sums of point-to-cluster-centroid distances. Rows of X correspond to data points, the columns of X corresponds to variables. K-Means returns an n-by-1 vector containing the cluster indices of each point. K-Means has three methods of measuring the distance between cluster centroids and other data points. These three methods include:

- sqEuclidean: (Squared Euclidean distance between the centroid (mean) of the points in the cluster and the remaining points in a cluster).
- Cityblock: (Sum of the absolute distances between the centroid (mean)of the points in the cluster and the remaining points in a cluster)
- Cosine: The distance is measured as one minus the cosine of the included angle between points (treated as vectors). Each centroid is the mean of the points in the cluster, after normalizing those points to unit Euclidean length.

The means of calculating the three distance measures follows. Given an m-by-n data matrix with row vectors $x_{1,}x_{2,} \dots x_{m,}$, the distance are calculated as:

- sqEuclidean: $d_{st}^2 = (x_s - x_t)V^{-1}(x_s - x_t)'$ where V is the n-by-n diagonal matrix whose j^{th} diagonal element is $s(j)^2$, where *s* is the vector of standard deviations.

- Cityblock: $d_{st} = \sum_{j=1}^{n} \left| x_{sj} - x_{tj} \right|$.
- Cosine: $d_{st} = 1 - \frac{x_s x_t'}{\sqrt{(x_s x_s')(x_t x_t')}}$.

The MATLAB syntax of the K-Means function that is used in partitioning the datasets in this research is:

```
[idx,ctrs,sumd,d] = kmeans(X,nk,'Distance',kmd,'Replicates',15,'Options',opts,...
'start','cluster', 'emptyaction','drop','onlinephase','on')
```

Input provided to the K-Means function includes:

- X: n-by-p data matrix that will be partitioned.
- nk: number of clusters to create.
- Distance(kmd): the distance parameter as defined above
- Replicates(15): The number of times to repeat the clustering with each replicate having a new set of initial centroids positions.
- opts: (Display, Final) displays the final output for each replication.
- start (cluster):Performs a preliminary clustering phase on a random 10% subsample of X.
- emptyaction (drop): Removes any clusters that become empty during the replication process.
- onlinephase(on): The online phase of ‘on’ guarantees a solution that is a local minimum of the distance criterion, i.e., if any point is moved to another partition, the total sum of distances will increase.

The output from the K-Means function is:

- Idx: an n-by-1 data matrix that contains the cluster indices for each data point in X.
- Ctrs: a k-by-p matric that contains the k cluster centroid locations
- Sumd: a 1-by-k vector that contains the within-cluster sums of point-to-centroid distances.
- d: an n-by-k matrix that contains the distances for each point to every centroid.

After applying the K-Means cluster analysis using the parameters outlined above, Dataset1('attrited' members) is partitioned using the three distance measures ('sqeuclidean', 'cityblock' and 'cosine') and is presented in Figure 4.11 , Figure 4.12 and Figure 4.13. K-Means clustering with 4 clusters for Dataset1 is presented in Figure 4-14, Figure 4-15 and Figure 4-16. K-Means partitioning using the 'sqeuclidean' and 'cityblock' measure of distance are visually quite similar. K-Means partitioning using the 'cosine' distance measure is quite different. The three Cluster and four cluster K-Means clustering scatter plots for Dataset2 can be found in Appendix A.

4.4.1.4 Measuring Clustering Effectiveness Using the Silhouette Index

As discussed in Chapter 3, the Silhouette Index (SI) computes for each point a width depending on its membership in any cluster. This silhouette width is an average over all observations. The Silhouette Index is calculated as:

$$\mathrm{SI_k} = \frac{1}{n}\sum_{i=1}^{n}\frac{(b_i - a_i)}{\max(a_i, b_i)}$$

where *n* is the total number of points, a_i, is the average distance between point *i* and all other

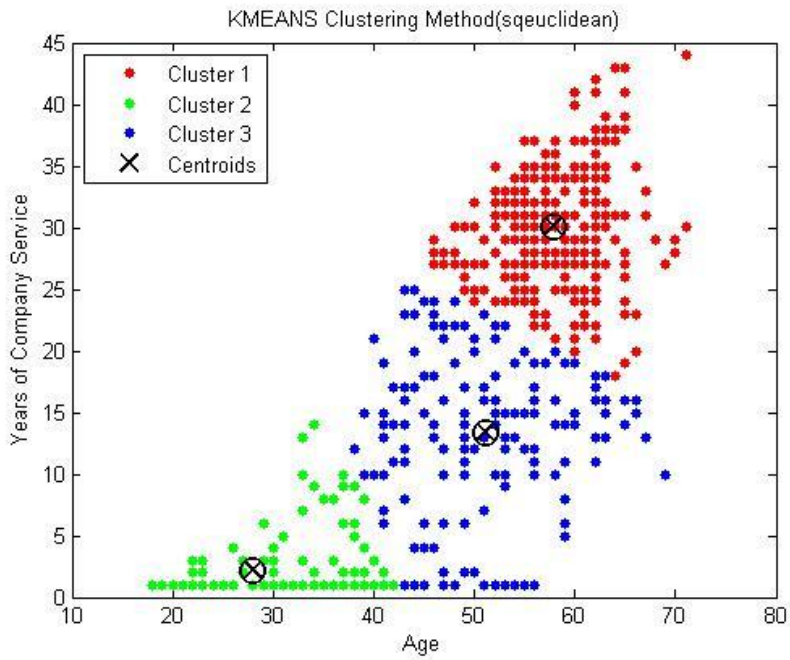


Figure 4-11 K-Means Clustering - Three Cluster Partition – Dataset1 (sqeuclidean)

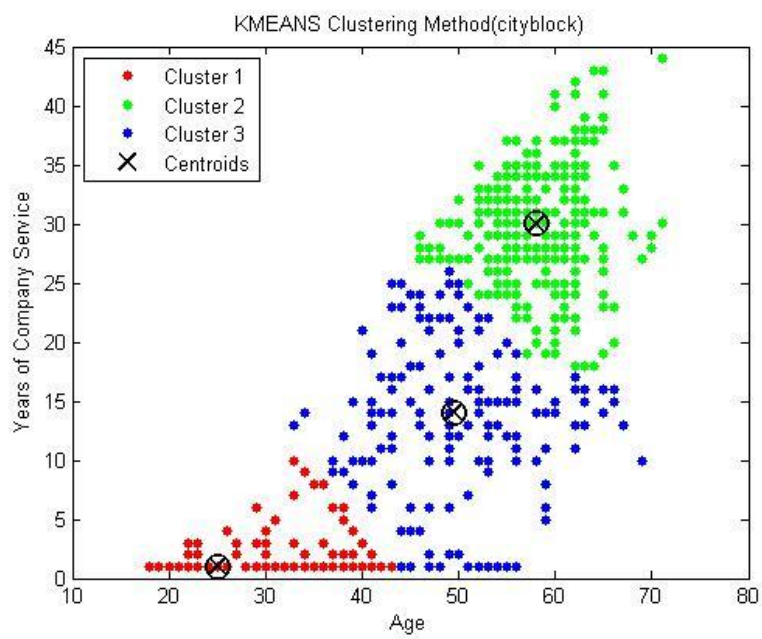


Figure 4-12 K-Means Clustering - Three Cluster Partition – Dataset1 (cityblock)

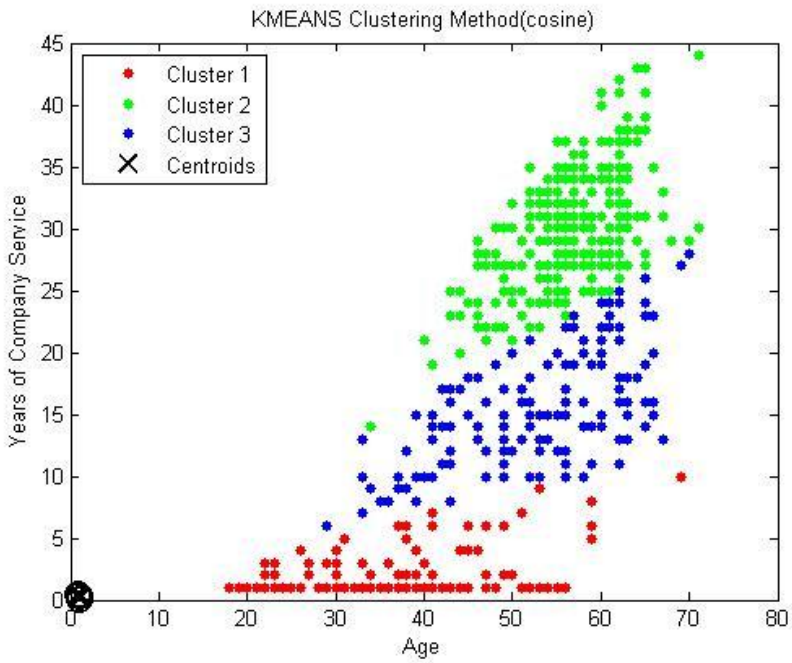


Figure 4-13 K-Means Clustering - Three Cluster Partition Dataset1 (cosine)

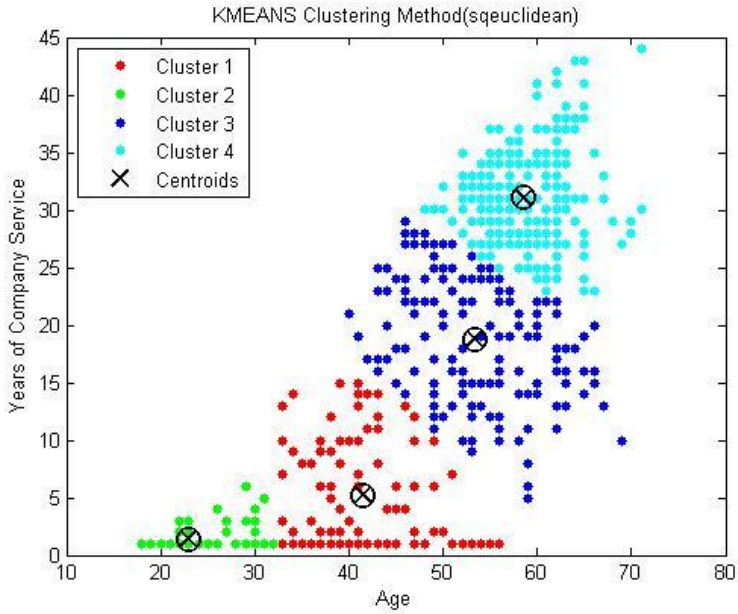


Figure 4-14 K-Means Clustering - Four Cluster Partition - Dataset1 (sqeuclidean)

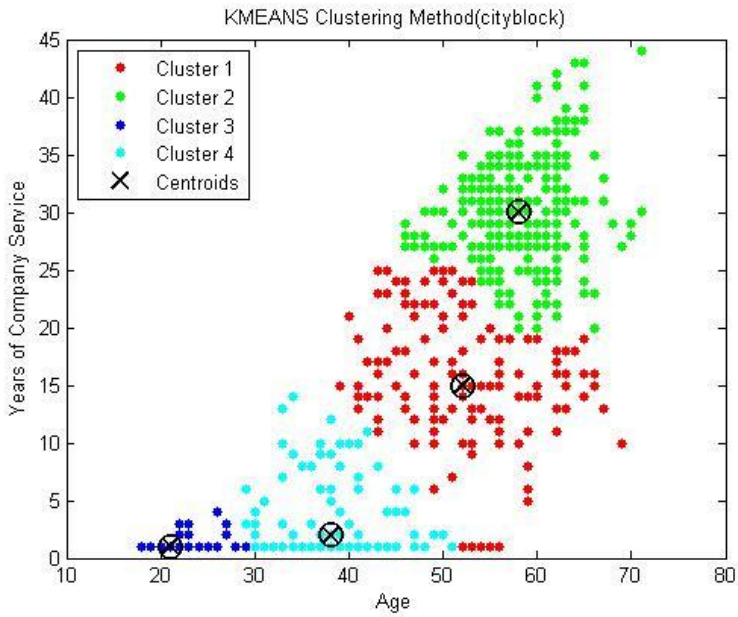


Figure 4-15 K-Means Clustering - Four Cluster Partition – Dataset1 (cityblock)

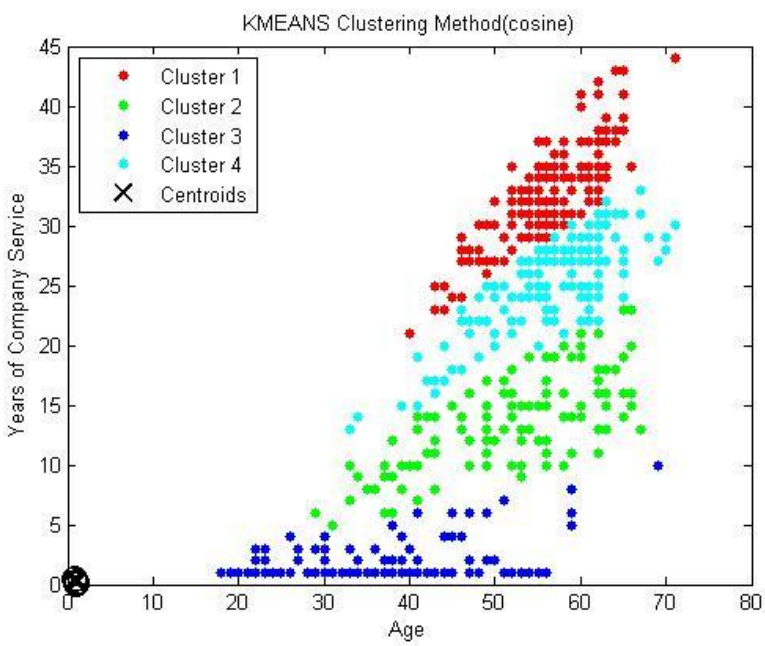


Figure 4-16 K-Means Clustering - Four Cluster Partition – Dataset1 (cosine)

points in its own cluster and b_i is the minimum of the average dissimilarities between i and points in other clusters. The partition, determined by applying clustering techniques, with the highest Silhouette Index (SI) is taken to be the optimal clustering alternative.

The syntax of the MATLAB silhouette function utilized in this study is

[silh,h] = silhouette(X,idx,kmd).

The output from the MATLAB Silhouette function creates a plot of the cluster silhouette. The Silhouette function also outputs other valuable information described below.

The Silhouette input is:

- X: the same n-by-p data matrix that is used in the KMeans function.
- Idx: an n-by-1 data matrix that contains the cluster indices for each data point in X. This matrix is the output from the K-Means function.
- Kmd: This is the same distance method that is applied in the K-Means function.

The Silhouette output is:

- Silh: an n-by-1 vector that contains the silhouette values for each data point in each cluster.
- h: h contains the silhouette figure handle. This is not used in this data study.

In order to determine the effectiveness of the K-Means clustering, Table 4-2 provides a range of Silhouette Indices to be used in measuring cluster effectiveness. The interpretation ranges from being weak or artificial to being strong. The higher the silhouette Index (SI), the better the partitioning with the index having a range $-1 \leq SI \leq 1$. The interpretation of the Silhouette Index is summarized in Table 4-2. Table 4-2 only contains positive values. Negative values are an

indication of inappropriate assignment of data points to clusters, with larger negative values indicating more significant incorrect assignment to clusters.

The silhouette indices (Table 4-3) associated with the clusters generated by the three cluster and four cluster K-Means clustering for Dataset1 varies from 0.4815 (four clusters) to 0.8200 (three clusters). Table 4-4 contains the results of the three cluster and four cluster K-Means clustering for Dataset2. The K-Means clustering using the Cosine distance with 3 clusters yielded the best Silhouette Index for both Dataset1 and Dataset2. The silhouette plots for Dataset1 for the three cluster K-Means clusters are presented in Figure 4-17, Figure 4-18 and Figure 4-19. The remaining Silhouette plots for Dataset1 and all the silhouettes plots for Dataset2 can be found in Appendix A. Based on the Silhouette Indices in Table 4-3 and Table 4-4, the best clustering of Dataset1 and Dataset2 is the partitioning of the datasets into three clusters using the cosine distance of measure. The results of the clustering on Dataset1 and Dataset2 are therefore used in the survival analysis to develop empirical survival distributions for workforce attrition in the Human Capital Inventory Cost Model. Each cluster will have its own attrition rate as determined by the Kaplan-Meier Survival Analysis.

Table 4-2 Silhouette Index Intepretation

Range of Silhouette Index	Interpretation
0.71 to 1.0	A strong partition has been found.
0.51 to 0.70	A reasonable partition has been found.
0.26 to 0.50	The partition is weak and could be artificial. Try additional methods of data analysis.
Less than 0.25	No substantial partitioning has been found.

Table 4-3 Silhouette Indices for K-Means Clustering (Dataset1)

Silhouette Indices	Number of K-Means Clusters	
Distance Measure	3	4
sqEuclidean	0.6881	0.6126
Cityblock	0.5147	0.4815
Cosine	0.8200	0.7820

Table 4-4 Silhouette Indices for K-Means Clustering (Dataset2)

Silhouette Indices	Number of K-Means Clusters	
Distance Measure	3	4
sqEuclidean	0.6864	0.6847
Cityblock	0.5160	0.4023
Cosine	0.7997	0.7598

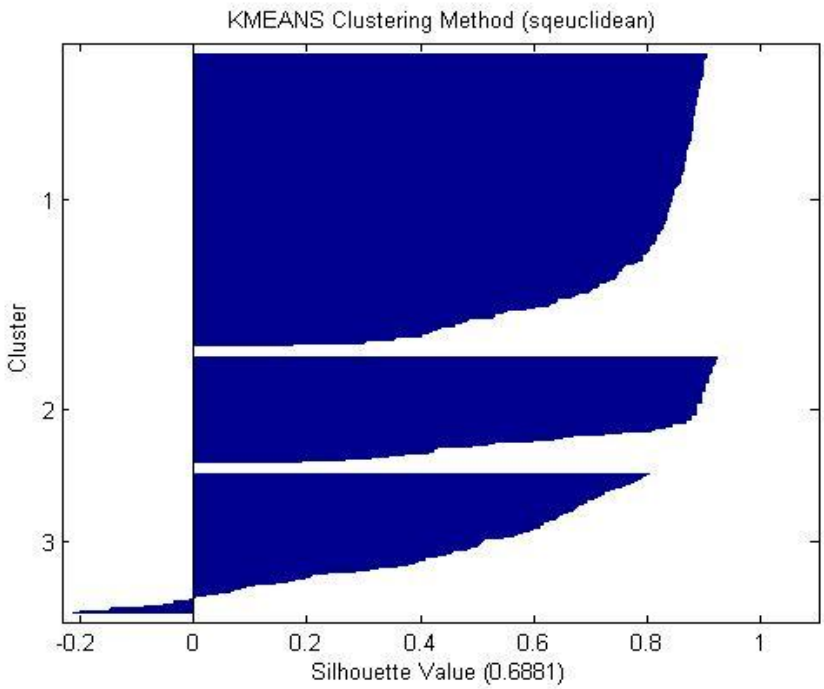


Figure 4-17 Three Cluster Silhouette – Dataset1 (sqEuclidean)

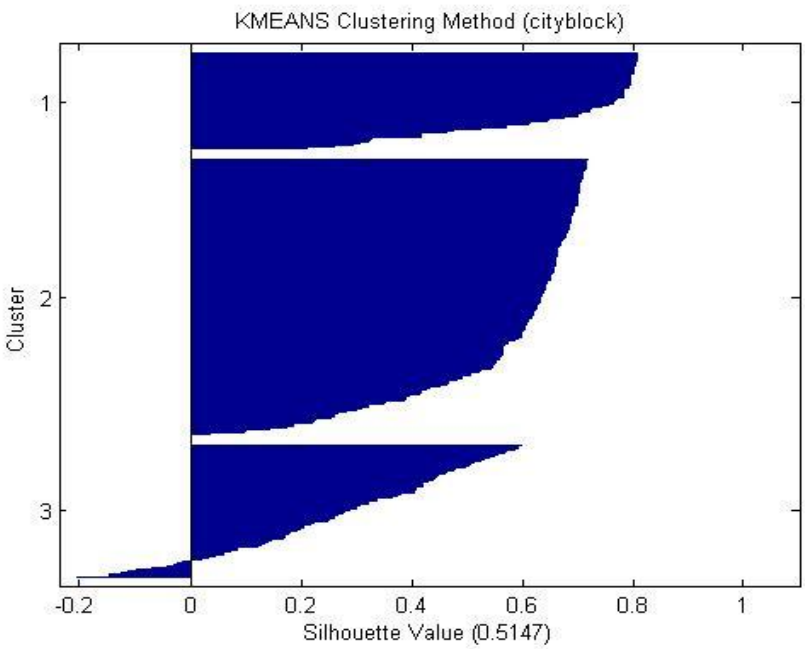


Figure 4-18 Three Cluster Silhouette – Dataset1 (cityblock)

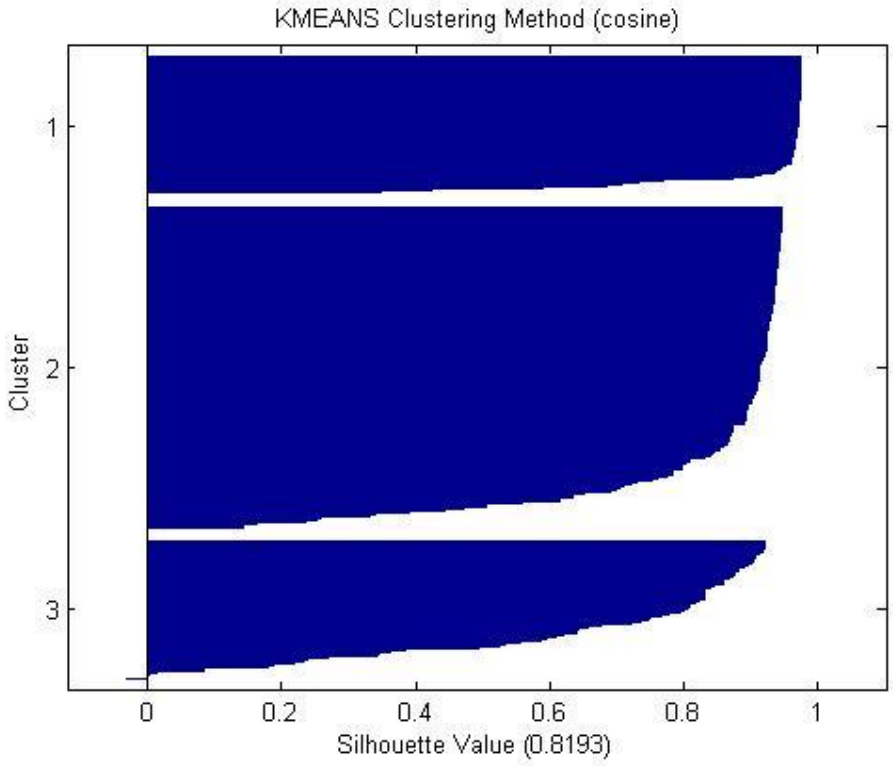


Figure 4-19 Three Cluster Silhouette – Dataset1 (Cosine)

4.4.2 Survival Analysis

Survival analysis involves the application of statistical techniques such as Life Tables, Kaplan-Meier or Cox Regression to determine the probability of survival over time. Survival Analysis in this study is used to establish attrition rates within a workforce that has been partitioned using K-Means clustering. Members of the workforce are assigned to a cluster based on their age and years of company service. The determination of the cluster attrition rates in this research utilizes the Kaplan-Meier Survival Analysis. Since the two covariates, age and years-of-company service, have been utilized in establishing the clusters, the Kaplan-Meier technique is the more appropriate method for application for survival analysis. The clustering that has been performed eliminates the need for utilizing the Cox Regression techniques for survival analysis. If more than two covariates were to be utilized, Cox Regression would have been the more appropriate. The process for the development of the attrition distributions associated with the clusters developed by using the K-Means technique for employee attrition into subgroups has been presented in a previous section. Since the clusters have been developed, Kaplan-Meier analysis is applied to each cluster to determine the attrition distribution for each cluster.

4.4.2.1 Application of Kaplan-Meir Analysis for Workforce Attrition

As presented in Chapter 3, the Kaplan-Meier method is a method for estimating survival times using non-parametric survival times and allows for variable length measures of survival time. The length of time is considered an independent variable. The Kaplan-Meier method relies on a dependent variable. This dependent variable is typically coded as a zero (0) or a one (1). The dependent variable represents whether or not an event has occurred, typically represented by a one (1). The code that is selected for the case where the event has not occurred is considered

censored an is usually coded as a zero (0). Otherwise, the occurrence of the event is considered *uncensored.* The Kaplan-Meier analysis is useful when the number of cases is small and exact survival times are known.

As presented in Chapter 3, the estimated survival function at time t is calculated as:

$$S(t) = \prod_{i=1}^{t} \left[\frac{(n-i)}{(n-i+1)}\right]^{C_i}$$

where $\prod_{i=1}^{t}()$ denotes the multiplication of the survival times across all cases less than or equal to t (the geometric mean); *t* = time, e.g. days, weeks, months, etc., *n* is the total number of cases in the sample, *i* is the number of cases surviving up to time *t*, C_i is a constant such that C_i is 0 for the uncensored or terminal case and 1 for the censored case. MATLAB requires that the code indicating that the event has occurred is 0. Other software may use 1 or a user selected value to designate whether the event has occurred.

4.4.2.2 Kaplan-Meier Survival Analysis Results

The graphical display of the empirical distributions determined by applying the Kaplan-Meier Analysis for the three cluster K-Means cluster analysis for Dataset1 is presented in Figure 4-20 and for Dataset2 in Figure 4-21. Each of the three clusters for each dataset is represented by a step function as defined in the legend. Applying survival analysis to the full Dataset1 and Dataset2, the empirical distribution is represented by the step function (color of black) in the All Data step function.

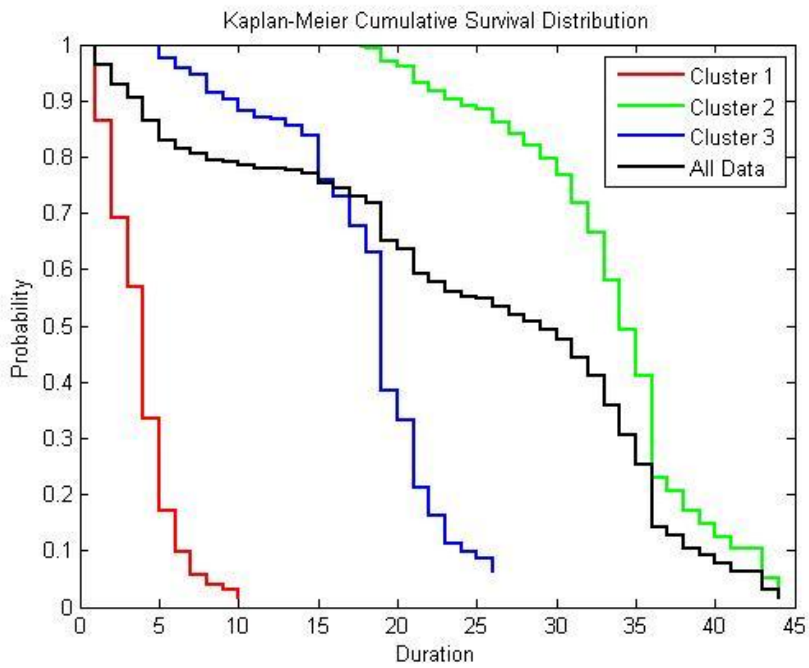


Figure 4-20 Kaplan-Meier Cumulative Survival 3-Cluster Distributions – Dataset1

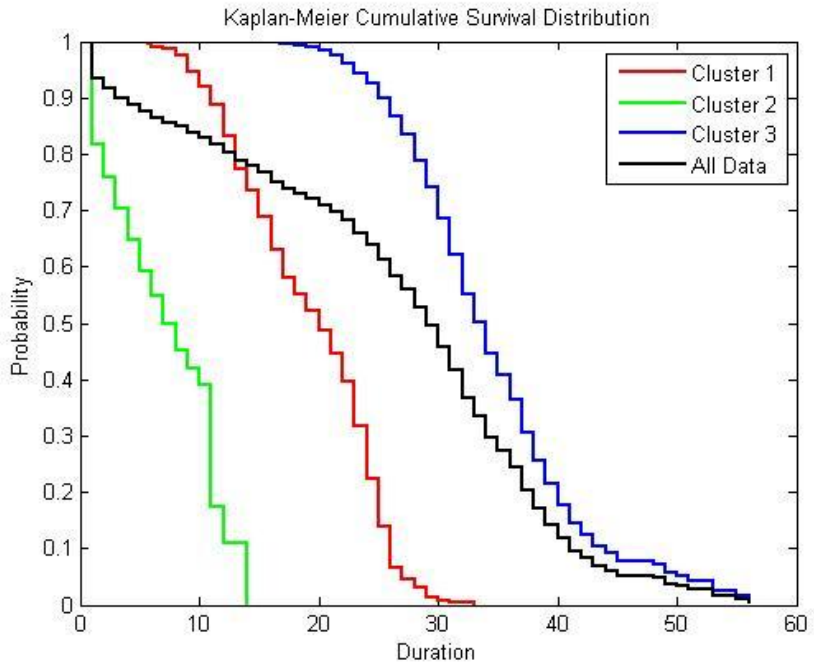


Figure 4-21 Kaplan-Meier Cumulative Survival 3-Cluster Distributions – Dataset2

One observation to make is that the survival curves do not reach zero. This occurs because of the fact that there exists survival times beyond the occurrence of the last attrition event. The differences between the cluster survival distributions demonstrates the appropriateness of developing survival distributions for each cluster rather that utilizing a single distribution and/or single turnover rate for the purpose of determining workforce attrition. Figure 4-22 is an example of the method of calculating the Kaplan-Meier Survival Distribution for Cluster 1 of Dataset1.

As can be visually observed, there appears to be significant differences between the survival function for the whole dataset when compared to each one of the three individual cluster survival curves. However, statistical tests are performed to determine the statistical significant of the difference between the survival curves. The null hypothesis that is tested is that the Kaplan-Meier Survival curves are the same. The most popular statistical tests that are used to compare survival curves are the Log Rank (mantel-Cox), the Breslow (Generalized Wilcoxon) and the Tarone-Ware statistical test. The result of the statistical analysis to test the null hypothesis is found in Table 4-5. The results of the chi-square test indicates that there is a significant difference between the survival curves, resulting in the rejection of the hypothesis that the curves are equal. Although it is not necessary to perform a pair-wise statistical test of the survival curves, the results of a pair-wise test is found in Table 4-6.

A composite of the Kaplan-Meier survival distributions for the three cluster Dataset1 analysis is presented in Figure 4-23. The three cluster Dataset2 composite Kaplan-Meier survival distributions are in Figure 4-24. For Dataset1, Figure 4-23 contains the interval start time, the cumulative survival distribution function and the probability density for each of the three

Table 4-5 Test of Significance of Survival Curves

Overall Comparisons			
	Chi-Square	df	Sig.
Log Rank (Mantel-Cox)	695.216	3	.000
Breslow (Generalized Wilcoxon)	633.000	3	.000
Tarone-Ware	665.895	3	.000

Table 4-6 Pairwise Statistical Test for Survival Curves

Pairwise Comparisons									
		0		1		2		3	
	Cluster	Chi-Square	Sig.	Chi-Square	Sig.	Chi-Square	Sig.	Chi-Square	Sig.
Log Rank (Mantel-Cox)	0			58.820	.000	242.230	.000	60.140	.000
	1	58.820	.000			236.020	.000	501.408	.000
	2	242.230	.000	236.020	.000			406.374	.000
	3	60.140	.000	501.408	.000	406.374	.000		
Breslow (Generalized Wilcoxon)	0			14.942	.000	211.242	.000	126.125	.000
	1	14.942	.000			202.201	.000	446.446	.000
	2	211.242	.000	202.201	.000			366.567	.000
	3	126.125	.000	446.446	.000	366.567	.000		
Tarone-Ware	0			32.263	.000	225.593	.000	97.923	.000
	1	32.263	.000			217.879	.000	473.141	.000
	2	225.593	.000	217.879	.000			385.415	.000
	3	97.923	.000	473.141	.000	385.415	.000		

clusters and the entire population for the three-cluster clustering of Dataset1. The interpretation of the survival tables in Figure 4-23 and Figure 4-24 is as follows:

- The Interval Start Time represents the duration to an occurrence of an event. As an example, an interval start time of ‘6’ indicates that an event has not occurred in prior time intervals.
- Cumulative Proportion Surviving at End Of interval: This is the probability that a participant will survive to the end of this interval.
- The Probability Density is the probability of survival from one interval to another interval given that a participant has survived up to some time interval.

4.4.3 Human Capital Inventory Cost Model

This section presents the development and application of the Human Capital Inventory Cost Model. The Human Capital Inventory Cost Model utilizes the replenishment model and the attrition model presented in Section 4.4.1 and Section 4.4.2., respectively. Case studies utilizing Dataset1 and Dataset2 will be developed and presented in this chapter. The Human Capital Inventory Cost Model is at the heart of the minimum cost inventory model. The cost model determines the costs associated with inventory levels for each period processed in the model. The Human Capital Continuous Review inventory model is depicted in Figure 4-25. The Human Capital Inventory Cost Model calculate inventory period costs associated with each inventory cycle.

Interval Start Time	Number Entering Interval	Number Withdrawing during Interval	Number Exposed to Risk	Number of Terminal Events	Proportion Terminating	Proportion Surviving	Cumulative Proportion Surviving at End of Interval	Std. Error of Cumulative Proportion Surviving at End of Interval	Probability Density	Std. Error of Probability Density	Hazard Rate	Std. Error of Hazard Rate
0	452	0	452.000	0	.00	1.00	1.00	.00	.000	.000	.00	.00
1	452	59	422.500	112	.27	.73	.73	.02	.265	.021	.31	.03
2	281	54	254.000	16	.06	.94	.69	.02	.046	.011	.07	.02
3	211	35	193.500	10	.05	.95	.65	.02	.036	.011	.05	.02
4	166	62	135.000	6	.04	.96	.62	.03	.029	.012	.05	.02
5	98	46	75.000	3	.04	.96	.60	.03	.025	.014	.04	.02
6	49	19	39.500	7	.18	.82	.49	.04	.106	.037	.19	.07
7	23	11	17.500	2	.11	.89	.44	.05	.056	.038	.12	.09
8	10	3	8.500	1	.12	.88	.39	.07	.051	.049	.13	.12
9	6	2	5.000	1	.20	.80	.31	.09	.077	.070	.22	.22
10	3	1	2.500	2	.80	.20	.06	.08	.247	.105	1.33	.70

Figure 4-22 Kaplan-Meier Survival Table Analysis – Dataset1, Cluster 1 Analysis

Cluster 1			Cluster 2			Cluster 3			All Data		
Interval Start Time	Cumulative Proportion Surviving At End Of Interval	Probability Density	Interval Start Time	Cumulative Proportion Surviving At End Of Interval	Probability Density	Interval Start Time	Cumulative Proportion Surviving At End Of Interval	Probability Density	Interval Start Time	Cumulative Proportion Surviving At End Of Interval	Probability Density
0	1.0000	1.0000	0	1.0000	1.0000	0	1.0000	1.0000	0	1.0000	1.0000
6	0.9976	0.9976	1	0.7522	0.7522	14	0.9987	0.9987	1	0.9320	0.9320
7	0.9951	0.9975	2	0.7094	0.9431	19	0.9961	0.9974	2	0.9219	0.9892
8	0.9853	0.9901	3	0.6758	0.9526	20	0.9948	0.9987	3	0.9154	0.9929
9	0.9751	0.9897	4	0.6513	0.9639	21	0.9908	0.9960	4	0.9114	0.9956
10	0.9469	0.9711	5	0.6314	0.9694	22	0.9783	0.9874	5	0.9092	0.9977
11	0.9232	0.9750	6	0.5412	0.8571	23	0.9699	0.9914	6	0.9034	0.9935
12	0.8912	0.9653	7	0.4941	0.9130	24	0.9499	0.9794	7	0.9011	0.9975
13	0.8511	0.9550	8	0.4447	0.9000	25	0.9166	0.9649	8	0.8973	0.9958
14	0.8103	0.9521	9	0.3706	0.8333	26	0.8990	0.9809	9	0.8935	0.9957
15	0.7603	0.9384	10	0.1235	0.3333	27	0.8345	0.9282	10	0.8835	0.9888
16	0.7263	0.9553	11	0.0000	0.0000	28	0.7975	0.9557	11	0.8765	0.9921
17	0.6974	0.9602				29	0.7486	0.9386	12	0.8671	0.9893
18	0.6730	0.9650				30	0.6802	0.9087	13	0.8554	0.9865
19	0.6392	0.9497				31	0.6309	0.9275	14	0.8429	0.9853
20	0.6072	0.9500				32	0.5835	0.9249	15	0.8287	0.9831
21	0.5622	0.9259				33	0.5398	0.9252	16	0.8197	0.9892
22	0.4748	0.8444				34	0.4962	0.9191	17	0.8124	0.9910
23	0.3868	0.8148				35	0.4414	0.8895	18	0.8065	0.9928
24	0.2381	0.6154				36	0.4245	0.9618	19	0.7972	0.9884
25	0.1488	0.6250				37	0.3445	0.8116	20	0.7916	0.9929
26	0.1116	0.7500				38	0.2742	0.7959	21	0.7829	0.9891
27	0.0558	0.5000				39	0.2388	0.8710	22	0.7665	0.9790
28	0.0000	0.0000				40	0.2285	0.9565	23	0.7548	0.9848
						41	0.1683	0.7368	24	0.7341	0.9725
						42	0.1530	0.9091	25	0.7053	0.9608
						43	0.1224	0.8000	26	0.6908	0.9794
						44	0.0918	0.7500	27	0.6402	0.9268
						51	0.0459	0.5000	28	0.6107	0.9540
						61	0.0000	0.0000	29	0.5733	0.9386
									30	0.5209	0.9087
									31	0.4832	0.9275
									32	0.4468	0.9249
									33	0.4134	0.9252
									34	0.3800	0.9191
									35	0.3380	0.8895
									36	0.3251	0.9618
									37	0.2638	0.8116
									38	0.2100	0.7959
									39	0.1829	0.8710
									40	0.1750	0.9565
									41	0.1289	0.7368
									42	0.1172	0.9091
									43	0.0938	0.8000
									44	0.0703	0.7500
									51	0.0352	0.5000
									61	0.0000	0.0000

Figure 4-23 Kaplan-Meier Survival Distributions – 3 Clusters - Dataset1

Cluster 1			Cluster 2			Cluster 3			All Data		
Interval Start Time	Cumulative Proportion Surviving At End Of Interval	Probability Density	Interval Start Time	Cumulative Proportion Surviving At End Of Interval	Probability Density	Interval Start Time	Cumulative Proportion Surviving At End Of Interval	Probability Density	Interval Start Time	Cumulative Proportion Surviving At End Of Interval	Probability Density
0	1.0000	1.0000	0	1.0000	1.0000	0	1.0000	1.0000	0	1.0000	1.0000
5	0.9972	0.9972	1	0.8163	0.8163	16	0.9982	0.9982	1	0.9343	0.9343
6	0.9903	0.9931	2	0.7590	0.9298	17	0.9956	0.9974	2	0.9165	0.9809
7	0.9857	0.9953	3	0.7031	0.9263	18	0.9935	0.9979	3	0.9006	0.9827
8	0.9761	0.9903	4	0.6465	0.9195	19	0.9887	0.9952	4	0.8868	0.9847
9	0.9447	0.9678	5	0.5928	0.9170	20	0.9840	0.9952	5	0.8750	0.9867
10	0.9187	0.9725	6	0.5487	0.9256	21	0.9762	0.9921	6	0.8652	0.9887
11	0.8873	0.9658	7	0.4976	0.9069	22	0.9616	0.9850	7	0.8566	0.9901
12	0.8314	0.9370	8	0.4519	0.9080	23	0.9426	0.9802	8	0.8487	0.9908
13	0.7740	0.9309	9	0.4209	0.9316	24	0.9249	0.9813	9	0.8372	0.9865
14	0.7357	0.9505	10	0.3909	0.9286	25	0.8981	0.9710	10	0.8292	0.9904
15	0.6900	0.9379	11	0.1752	0.4483	26	0.8684	0.9668	11	0.8184	0.9869
16	0.6287	0.9112	12	0.1095	0.6250	27	0.8354	0.9620	12	0.8030	0.9813
17	0.5810	0.9241	14	0.0000	0.0000	28	0.7892	0.9447	13	0.7880	0.9813
18	0.5500	0.9466				29	0.7420	0.9402	14	0.7780	0.9873
19	0.5226	0.9501				30	0.6871	0.9259	15	0.7664	0.9851
20	0.4867	0.9314				31	0.6222	0.9057	16	0.7502	0.9789
21	0.4463	0.9170				32	0.5517	0.8866	17	0.7375	0.9830
22	0.3963	0.8880				33	0.5022	0.9103	18	0.7291	0.9886
23	0.3170	0.8000				34	0.4457	0.8875	19	0.7201	0.9877
24	0.2233	0.7043				35	0.4082	0.9158	20	0.7099	0.9859
25	0.1396	0.6250				36	0.3639	0.8916	21	0.6984	0.9838
26	0.0659	0.4722				37	0.3056	0.8398	22	0.6818	0.9762
27	0.0461	0.7000				38	0.2563	0.8386	23	0.6596	0.9675
28	0.0316	0.6842				39	0.2141	0.8354	24	0.6378	0.9668
29	0.0126	0.4000				40	0.1764	0.8238	25	0.6115	0.9588
30	0.0063	0.5000				41	0.1438	0.8152	26	0.5844	0.9557
31	0.0032	0.5000				42	0.1242	0.8636	27	0.5607	0.9594
33	0.0000	0.0000				43	0.1024	0.8246	28	0.5287	0.9430
						44	0.0910	0.8889	29	0.4960	0.9381
						45	0.0786	0.8636	30	0.4589	0.9252
						48	0.0730	0.9286	31	0.4154	0.9053
						49	0.0561	0.7692	32	0.3684	0.8867
						50	0.0499	0.8889	33	0.3351	0.9096
						51	0.0428	0.8571	34	0.2974	0.8875
						53	0.0257	0.6000	35	0.2723	0.9158
						55	0.0171	0.6667	36	0.2428	0.8916
						56	0.0086	0.5000	37	0.2039	0.8398
						57	0.0000	0.0000	38	0.1710	0.8386
									39	0.1428	0.8354
									40	0.1177	0.8238
									41	0.0959	0.8152
									42	0.0828	0.8636
									43	0.0683	0.8246
									44	0.0607	0.8889
									45	0.0524	0.8636
									48	0.0487	0.9286
									49	0.0375	0.7692
									50	0.0333	0.8889
									51	0.0285	0.8571
									53	0.0171	0.6000
									55	0.0114	0.6667
									56	0.0057	0.5000
									57	0.0000	0.0000

Figure 4-24 Kaplan-Meier Survival Distributions – 3 Clusters – Dataset2

The replenishment point (ROP) in the inventory model is determined by comparing the ending available inventory in a period with the demand in the next inventory period. When this quantity is less than the next periods demand, a replenishment event is initiated. As can be seen in Figure 4-25, the replenishment quantity is only available after some lead time *LT*. The EOQ quantity at the beginning of a cycle is determined by the optimization function after considering the remaining available inventory in the period prior to the replenishment period.

A significant difference between the physical inventory model and the Human Capital inventory model is the method of determining the available inventory. **In the physical inventory model, the period *demand* consumes the inventory during each period. In the Human Capital inventory system, the available inventory is a function of the *attrition rate* in the Human Capital population.** Figure 4-26 is an example of the effects of attrition on a Human Capital inventory. As can be seen in Figure 4-26, the effects of the decrease in the available inventory from period to period is determined by the attrition rate with the Human Capital inventory. The bars in 'red' demonstrate that effects of attrition on the original Human Capital population as of period 1. The 'green' bars demonstrate the reduction in the Human Capital inventory associated with the first Human Capital replenishment. The 'blue' bars demonstrate the attrition of the Human Capital inventory associated with the second inventory replenishment. As stated above, the inventory available from period to period is not based on demand, but on the Human Capital attrition. Also, the Human Capital available inventory is the sum of the remaining availability from the original inventory plus the remaining inventories from each replenishment.

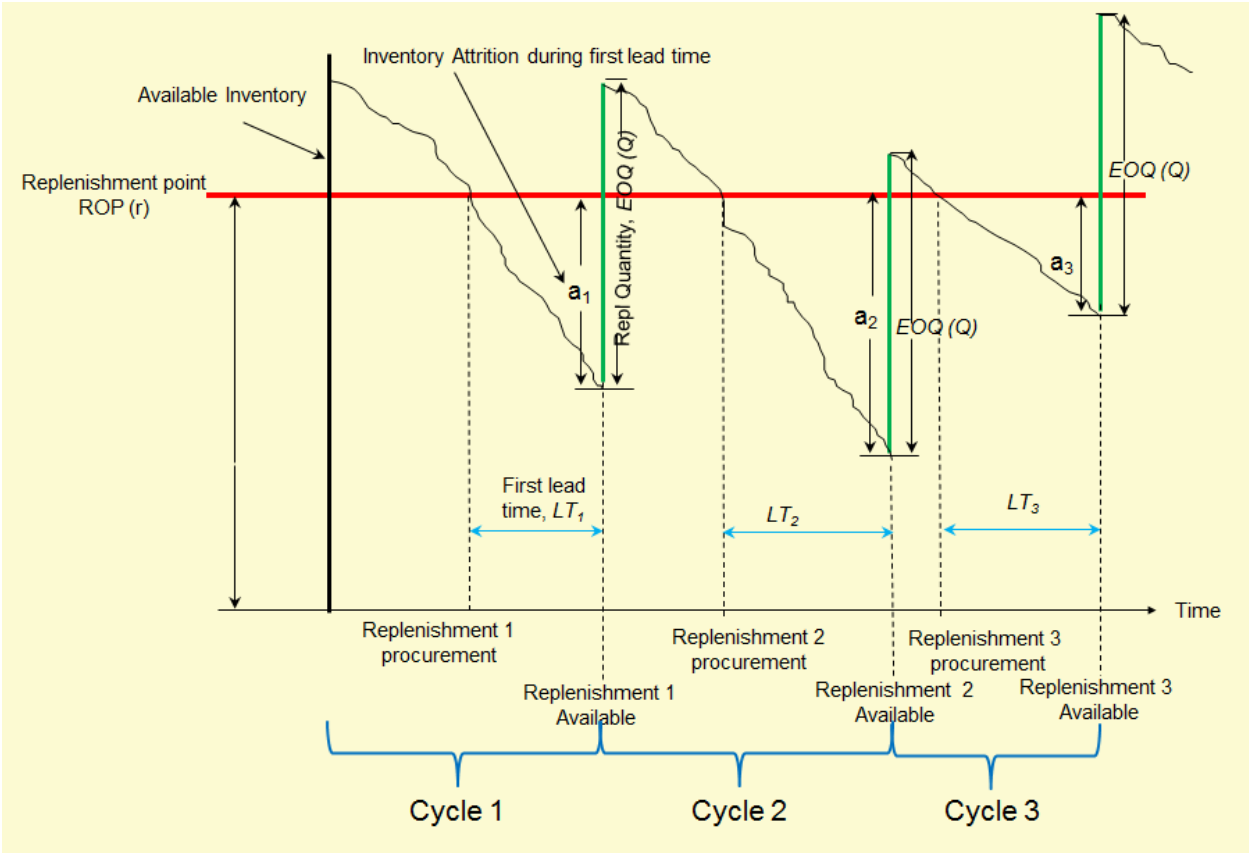


Figure 4-25 Human Capital Inventory Continuous Review

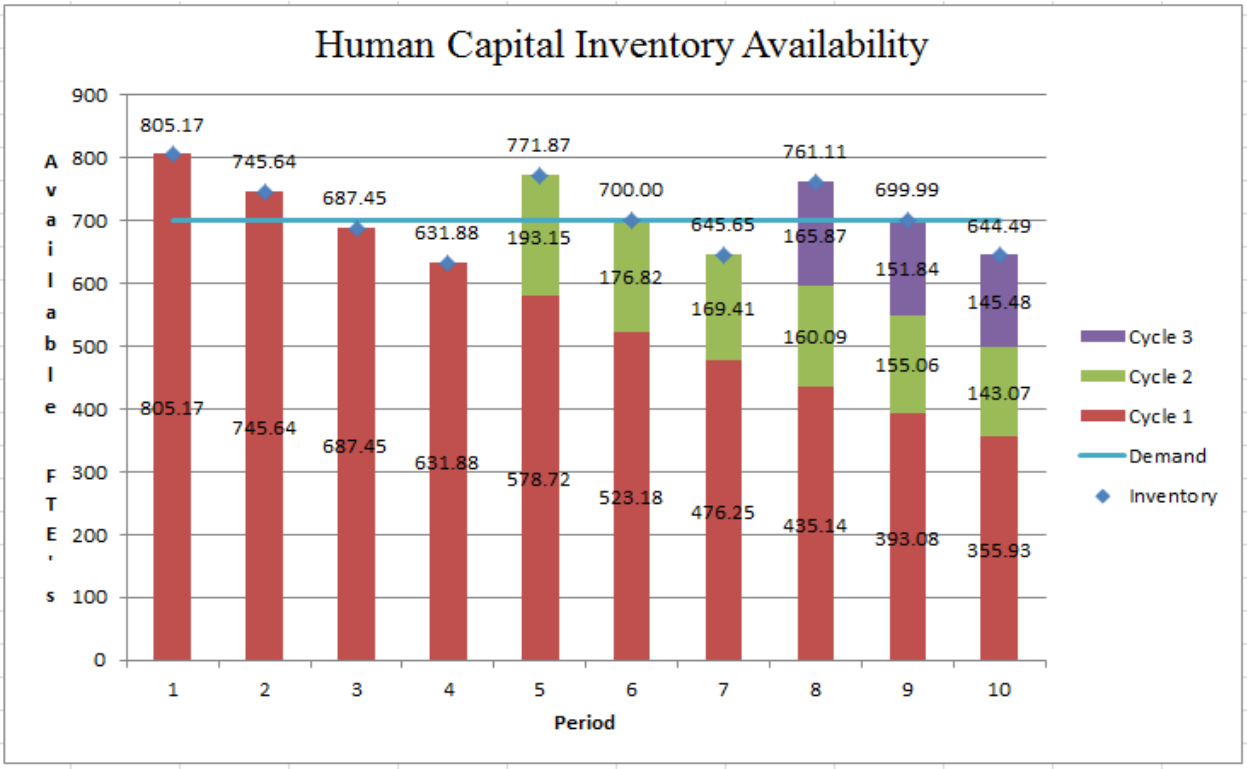


Figure 4-26 Human Capital Inventory Availability

The cost model is executed by the optimization functions as presented and exercised in the following optimization model section. A functional flow of the Human Inventory Cost Model is presented in Figure 4-27. As can be seen in Figure 4-27, the cost model accumulates cost for each period in an inventory cycle. A cycle is determined from the period that a replenishment is made through the period preceding the next replenishment period. The first period in the planning horizon is the first period in the first cycle. The last period in the planning horizon is the last period in the last cycle in the planning horizon. The model accumulates Available Human Capital Inventory Cost, Excess Human Capital Inventory Cost and Shortage Human Capital Inventory Costs. Available Human Capital Inventory Costs are costs associated with the current number of FTE's that are available in a period. Excess Human Capital Inventory Costs are associated with Human Capital inventory that exceeds the Human Capital demand during a period. Shortage Human Capital Inventory Costs are associated with available Human Capital inventory that is less than the demand for a period. The following equations are used to calculate inventory period and inventory cycle costs.

Letting I_i equal the number of available FTE's in an inventory period i and D_i equal the Human Capital inventory demand for period i,

$$Available\ Human\ Capital\ Inventory\ Costs\ (IC) = \sum_i I_i * IR$$

where IR is the rate (in dollars per period) of one available FTE.

$Human\ Capital\ Excess\ Inventory\ Costs\ (EC) = \sum_i (I_i - D_i) * ER$, for $I_i > D_i$

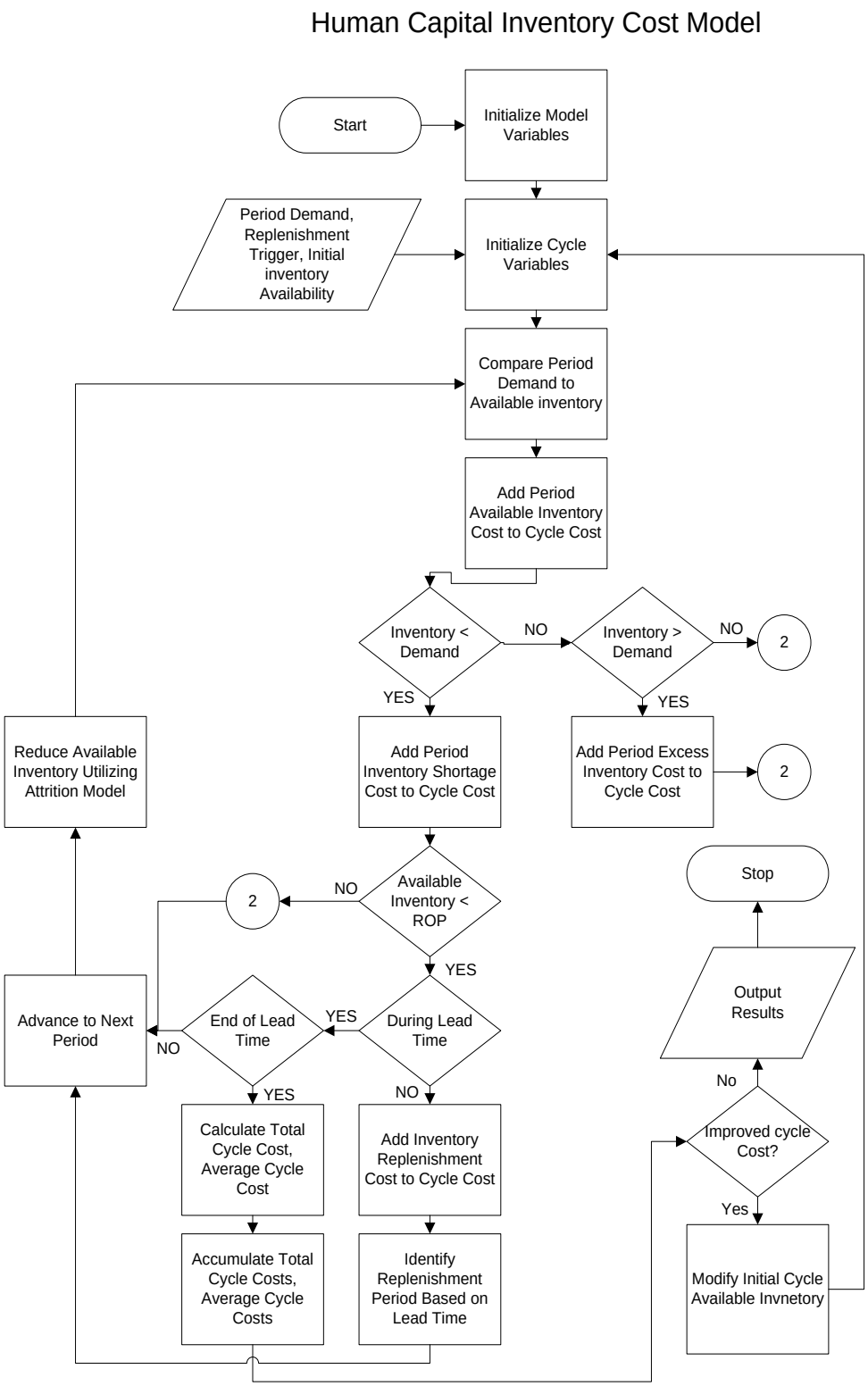


Figure 4-27 Human Capital Inventory Cost Model Flowchart

where ER is the rate (in dollars per period) for carrying Available Human Capital inventory in excess of period demand D_i.

$$Human\ Capital\ Shortage\ Inventory\ Costs\ (SC) = \sum_i (D_i - I_i) * SR, for\ \ I_i < D_i$$

where SR is the rate (in dollars per period) for Available Human Capital Inventory less than the period demand D_i.

Costs that are associated with replenishing the Human Capital inventory is also applied to the total cycle cost. These replenishment cost (RC) is assumed to be a fixed cycle cost associated with the activities of recruiting and hiring Human Capital and is applied in each cycle for each replenishment event. The Replenishment Cost is not based on the replenishment quantity. These costs are only applied to the cycle costs. Thus the Total Cycle Cost (TCC) is calculated as

$$Total\ Cycle\ Cost\ (TCC) = \sum_{i=1}^{n} (IC_i + EC_i + SC_i) + RC$$

where *i* is the number of periods in the inventory cycle.

The cycle costs determined by the Human Capital Inventory Cost Model will be utilized as the minimization functional value for the optimization functions.

4.5 Human Capital Optimization Model

As presented in previous sections of this paper, MATLAB's optimization functions are utilized in determining minimum cost utilizing the Human Capital Inventory Cost Model. Several of these routines are analyzed using a standard set of parameters. The optimization process includes the application of MATLAB optimization functions for determining both local

and global solutions. Local optimization functions utilize the Human Capital Inventory Cost Model and interact directly with the Human Capital Inventory Cost Model. The global optimization functions manage the optimization variables that are utilized by a local optimization function that interact with the Cost model. Results of local and global solutions for the minimized Human Capital inventory costs are presented in this section.

4.5.1.1 Optimization Functions

The global optimization functions manage the optimization variables for the local optimization function. When exercising a local optimizer, the optimization variables are managed by the local optimization function. The following MATLAB optimization functions are exercised in research:

- Fminunc (Local optimization function) attempts to find a minimum of a scalar function of several variables, starting at an initial estimate. This is generally referred to as unconstrained nonlinear optimization. It may only find local solutions. Other MATLAB functions are more suited for global solution identification.
- Fmincon (Local optimization function) is a gradient-based method that is designed to work on problems where the objective and constraint functions are both continuous and have continuous first derivatives. It may only find local solutions. Other MATLAB functions are more suited for global solution identification.
- Multistart (global optimization function) generates a number of starting points. It then usea a local solver to find the optima in the basins of attraction of the starting points.

- Patternsearch (Global optimization function) looks at a number of neighboring points before accepting one of them. If some neighboring points belong to different basins, Patternsearch in essence looks in a number of basins at once
- Globalsearch (Global optimization function) generate a number of starting points. It then uses a local solver to find the optima in the basins of attraction of the starting points.
- Simulannealbnd (Global optimization function) performs a random search. Generally, Simulannealbnd accepts a point if it is better than the previous point. Simulannealbnd occasionally accepts a worse point, in order to reach a different basin.

The MATLAB local and global optimization functions use the following options statement:

```
options = optimset('Display','final','Algorithm','sqp', 'MaxFunEvals',...
    200000, 'TolFun', 1.000000e-10, 'TolCon', 1.000000e-5, 'TolX', 1.0e-10,
'MaxIter', 100000);
```

Patternsearch uses the following options statement:

```
options = psoptimset('Display','iter','MaxFunEvals',...
    100000, 'TolFun', 1.0e-10, 'TolCon', 1.0e-5, 'TolX', 1.0e-10, 'MaxIter',
100000, 'TolMesh', 1e-10);
```

The Multistart and the Globalsearch function utilizes the following problem parameter:

```
problem = createOptimProblem('fmincon', 'objective',@fnl_OptMainMinCost,
'x0',x0,'lb',lb,'ub',ub,'options',options);
```

The following statements are used to exercise each of the optimization functions:

FMINCON:

```
[z, fval, exitflag, output,solutions] =
fmincon(@fnl_OptMainMinCost,x0,[],[],[],[],lb,ub,[],options
```

FMINUNC:

```
[z, fval, exitflag, output] = fminunc(@fnl_OptMainMinCost,x0,options);
```

MULTISTART:

```
problem = createOptimProblem('fmincon', 'objective',@fnl_OptMainMinCost,…
'x0',x0,'lb',lb,'ub',ub,'options',options);
ms = MultiStart;
[z,fval,exitflag,output,solutions] = run(ms, problem, 100);
```

PATTERNSEARCH:

```
[z, fval, exitflag, output] = Patternsearch(@fnl_OptMainMinCost,x0,[],[],[],[],lb,ub,[],options);
```

GLOBALSEARCH:

```
gs = GlobalSearch;
[z,fval,exitflag,output,solutions] = run(gs, problem);
```

SIMULANNEALBND:

```
[z, fval, exitflag, output] = Simulannealbnd(@fnl_OptMainMinCost,x0,lb,ub,options);
```

4.5.1.2 Human Capital Optimization Analysis

The literature search conducted for this study has identified significant costs associated with employee turnover and the associated cost of replenishing the Human Capital inventory. Most physical inventory systems classify inventory items as either Class A, Class B or Class C where the Class A inventory items are the most ‘valuable’. Value is determined by an organization based on the criticality of the resource for accomplishing the organizations mission and work scopes. When considering the management of Human Capital as a critical inventory, the cost related to Human Capital is typically significantly larger than most physical inventories. Also, the difficulty associated with replacing unique Human Capital skills suggests that Human Capital inventories should be managed with at least the same disciplines that are used to manage ‘valuable’ physical inventories.

Based on the literature search and the findings associated with labor turnover and hiring from (Gaudet, 1960), (Al-Zayyat, 2010), (Bliss, 2010), (Cascio, 1991) and (Smith & Watkins,

1978), sample rates for the Inventory Availability, Shortage and Replenishment Cost are established for use in the minimum cost Human Capital Inventory Cost Model scenarios in this study. The rate associated with excess Human Capital inventory is based on the assignment of costs provided by Lang and Merino. The Lang and Merino calculation of the MARR is the sum of the following rates as established by the Congressional Business Office (CBO, 2011) for the period CY2011-CY2016:

- Inflation-free Rate of interest for Risk free loans (based on 10-year Treasurer note): 5.6%
- Expected Rate of Inflation: 2%
- The anticipated change in the rate of inflation: 0%
- The risk of defaulting on the loan: 7% (This is based on the likelihood of an employee leaving the organization within one year utilizing the survival analysis from the entire population from dataset1)
- The Risk Profile of a particular venture: 36% (probability of termination within dataset1 of the population before 25 years, i.e., 36% of the population will leave within 25 years.)

Thus the MARR as defined by Lang and Merino equals 5.6% + 2% + 0% + 7% + 36% which equals 50.6%. Therefore, for ease of demonstration, the rate that will be applied for the excess inventories will be an additional Human Capital rate of 50% more than the available inventory rate. This represents the foregone opportunity cost associated with the cost of the excess inventory. Thus the Excess Rate is 1.50 * the IR rate which equals $30/FTE.

The assignment of rates include:

- Inventory Availability Rate (IR) per FTE: $20/hour, $41,600/year
- Inventory Excess Rate (ER) per FTE: $30/hour, $62,400/year
- Inventory Shortage Rate (SR) per FTE: $40/hour. $83,200/year
- Replenishment Cost (RC) per Inventory Cycle: 2.5 times IR = $104,000

Other parameters that are necessary for the Human Capital Inventory Cost Model includes:

- The length of time required to replace Human Capital inventory (Replenishment Lead Time) typically calculated as the time the replenishment actions are initiated and the time the inventory is available for replenishment.
- The Inventory Level that signals when actions need to take place to begin the Human Capital replenishment efforts, expressed as a percentage of the ending inventory in one period to the beginning inventory of the next period. This percentage is referenced as the Replenishment Order Point(ROP). The continuous review inventory Human Capital model requires that a reorder point or trigger be established to signal that an inventory replenishment is required. The selection of the ROP of 100% is selected for the demonstrations in this research. The selection of this ROP is arbitrary. An ROP of other values could have been selected. As in a fixed demand physical inventory system, the ROP may be calculated as the product of the demand rate per period * the lead time. This guarantees that inventory will be available during lead time. In a variable period demand as discussed in Section 2.4.7.2, the ROP could be calculated that includes the effects of an average period inventory and the variance of the period inventory.

- The number of Inventory Cycles to be processed. Each cycle is determined from the beginning of the period of the replenishment of a cycle up to the period of replenishment of the next cycle.
- Human Capital Demand: This demand is established for each period in the inventory cost model. The demand is established at the beginning of the optimization execution. The demand can be either deterministic or stochastic, i.e., randomly generated. The demands per period need not be equal. Demands in this analysis are deterministic and are the same for all periods in all cycles. The decision to use a constant demand for each inventory period was for ease of demonstration. A variable period demand capability exists within the Human Capital Inventory Cost Model. A variable period demand, along with multiple iterations of the cost model, would be useful for establishing a probabilistic model for Human Capital inventory scenarios.

Scenarios are executed and the results are recorded for each the following scenarios:

- Demand per period : Example 1 : 8000 FTE's; Example 2: 700 FTE's
- Lead Time: Example 1: One period; Example 2: Two periods
- Replenishment Point: 1.0
- Number of Cycles Processed: 3
- Optimization functions to be applied: fmincon, fminunc, Patternsearch, Simulannealbnd, multistart, globalsearch

The results include the minimum cost associated with each replenishment cycle quantity, the level of service for each period in each cycle where the level of service is defined as the available

Human Capital inventory in a period divided by the next period's demand, the available inventory costs, the excess inventory costs and the inventory shortage costs.

4.5.1.3 Human Capital Optimization Results

The results are presented in this section for the scenarios as defined in the previous section. Results include cycle replenishment quantities, cycle costs, period costs (including available inventory costs, shortage costs and excess costs), period inventory quantities (beginning available inventory, ending available inventory, inventory shortage, inventory excess) and percent of available inventory for each period. The results of the Minimum Cost Human Capital Inventory Optimization is presented in the following section.

4.5.1.4 Minimum Cost Optimization Scenarios

The results of the MATLAB optimization functions for the Human Capital Inventory Cost Model using Dataset1 with a demand of 8000 FTE's per period and varying lead times are presented in Figure 4-28. The scenario with a demand of 8000 is to demonstrate the situation where the period demand is larger than the initial beginning period available inventory. The optimization quantities as well as the minimum cost associated with this scenario are similar for the various optimization functions except for the GlobalSearch method. The Globalsearch function results varied in Cycle 3 with Lead Time of 1 and Cycle 3 with Lead Time of 2. These differences can be observed in the Cycle optimal quantities and in the minimum Cycle Cost. The minimum cycle costs with Lead Time = 1 is slightly lower than the other optimization functions. However, with the Lead Time = 2, in Cycle 3, the GlobalSearch function obtained a higher optimal cycle quantity that the other routines. However, the GlobalSearch optimization values are not consistent with the other results and are not repeatable.

Figure 4-29 presents the optimization results utilizing Dataset1 with a demand of 700 FTE's per period and a Replenishment Point of 1.0. The choice of a demand of 700 FTE's per period enabled an analysis with the beginning inventory availability greater than the demand in the inventory period. Again, it is observed that the globalsearch optimization function yields results that are not consistent with the other optimization routines. The GlobalSearch results are not repeatable and reasons for this inconsistency cannot be explained at this time and will be part of further research.

The optimization results for Dataset2 with a lead time of 0 periods and the Replenishment Order Point of 1.0 is observed in Table 4-7. It is observed that the multistart and the globalsearch optimization functions are not consistent with the other optimization functions, yielding higher optimization quantities and higher minimum costs that the other optimized functions. Reasons for this inconsistency cannot be explained at this time and will be part of further research.

Detailed Human Capital inventory cycle and period cost with optimized quantities are presented in Figures 4-30, Figure 4-31, Figure 4-32 and Figure 4-33. Each scenario in these Figures uses the Patternsearch optimization function. The scenarios for each of is as follows:

- Figure 30: Dataset1, Demand of 8000 FTE's per period, Lead time of 1 period, Replenishment Point 1.0. This scenario is evaluated with a period demand significantly higher than the first period available inventory.
- Figure 31: Dataset1, Demand of 700 FTE's per period, Lead time of 2 periods, Replenishment Point 1.0. This scenario is evaluated with a period demand slightly less than the first period available inventory.

- Figure 32: Dataset2, Demand of 8000 FTE's per period, Lead time of 1 period, Replenishment Point 1.0. This scenario is evaluated with a period demand significantly higher than the first period available inventory.
- Figure 33: Dataset2, Demand of 3500 FTE's per period, Lead time of 1 period, Replenishment Point 1.0. This scenario is evaluated with a period demand slightly less than the first period available inventory.

Each Figure is interpreted in a similar manner. As can be seen in the leftmost column, the description of the inventory values is described. The values for each period within each cycle are observed in the body of the Figure. As can be seen in Figure 4-30, Cycle 1 includes periods 1 and 2. Cycle 2 includes period 3 and 4. Cycle 3 includes periods 5 and 6. As can be observed in period 1, the beginning inventory (805.17) is the current available FTE's within the current population. The first cycle is not optimized in the Human Capital Inventory Cost Model since at the beginning of the study, it is assumed that any adjustments that are needed in the Human Capital inventory cannot be made because of the lead time associated with the replenishment lead times. Cycle 2 and Cycle 3 are optimized for minimum cycle inventory costs. Since the 'Next Inv %' in period 1 is less than the ROP of 1, a replenishment of Human Capital inventory will be in period 3 since there is a lead time of one period. Coincidently, the optimum replenishment quantity for Cycle 2 is 7312.55 FTE's since there exits an ending inventory in period 2 of 687.45 FTE's (8000 – 687.45 = 7312.55). The minimum Cycle 2 beginning inventory quantity is 8000 FTE's. The costs in each period are calculated using the previously defined rates and formula for calculating period costs. In Figure 4-31, Excess and Shortage Human Capital inventory costs are incurred. The identification of the Excess and Shortage Units and Costs are of significant value when evaluating and forecasting critical Human Capital staffing needs within

an organization. The row labeled “Next Inv %” is a measure of the service level in the next period if no replenishment takes place in that next period. As an example, in Figure 4-31, the ‘Next Inv %” in period 1 is 106.52% of the demand of 700 FTE’s in period 2. In period 4, the ‘Next Inv %” value of 578.72 FTE’s is 82.67 % of the demand of 700 in period 5.The “Next Inv %” is used to determine the ROP for the inventory cycle. The early identification of the service levels and the excess and shortage units and costs enables organizations to establish Human Capital inventory policies to mitigate these situations and costs. How an organization mitigates level of service, excess and shortage units and costs is not part of this research but should be considered for future study.

As can be seen from this technique observed in Figures 4-30 through 4-33, the calculation of the Human Capital inventory costs and the option for determining minimum optimization costs closely resemble the methods, process and techniques that are applied to typical physical inventory minimum cost optimization processes.

4.5.1.5 Human Capital Cost Model Verification and Validation

In order to assure that the Human Capital Inventory Cost Model is functioning as expected (verification) and that the results of the Human Capital Inventory Cost Model is producing the correct results (validation), Microsoft EXCEL spreadsheets were designed and created with the expected functionality as developed in the Human Capital Inventory Cost Model. As the minimum cost optimization scenarios were performed using the Human Capital Inventory Cost Model, the optimized inventory cycle quantities were input into the appropriate EXCEL inventory periods. The period costs and the cycle costs are calculated within the EXCEL spreadsheet with the expectation that all period and cycle costs are consistent with the Human

Capital Inventory Cost Model period and cycle costs. Examples of this verification and validation of the cycle and period costs using the EXCEL spreadsheets can be found in Figures 4-30, 4-31, 4-32 and 4-33. The outputs from the MATLAB Human Capital Inventory Cost Model for Dataset1 is found in Figure 4-28 and Figure 4-29. As can be seem from the EXCEL results and the MATLAB Human Capital Inventory Cost Model results, the optimized minimum cost optimization yields the same costs results.

	Cycle 1			Cycle 2			Cycle 3		
Lead Time = 0	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level
fmincon	805	632,209	10%	7,254	332,904	100%	671	332,904	100%
fminunc	805	632,209	10%	7,254	332,904	100%	671	332,904	100%
multistart	805	632,209	10%	7,254	332,904	100%	671	332,904	100%
pattersearch	805	632,209	10%	7,254	332,904	100%	671	332,904	100%
globalsearch	805	632,209	10%	7,254	332,904	100%	667	332,904	100%
simulannealbnd	805	632,209	10%	7,254	332,904	100%	671	332,904	100%

	Cycle 1			Cycle 2			Cycle 3		
Lead Time = 1	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level
fmincon	805	1,266,791	9%	7,313	693,742	92%	1,008	686,225	94%
fminunc	805	1,266,791	9%	7,313	693,742	92%	1,008	686,225	94%
multistart	805	1,266,791	9%	7,313	693,742	92%	1,008	686,225	94%
pattersearch	805	1,266,791	9%	7,313	693,742	92%	1,008	686,225	94%
globalsearch	805	1,266,791	9%	7,313	693,742	92%	993	686,207	87%
simulannealbnd	805	1,266,791	9%	7,313	693,742	92%	1,008	686,225	94%

	Cycle 1			Cycle 2			Cycle 3		
Lead Time = 2	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level
fmincon	805	1,903,793	9%	8,107	1,059,851	95%	1,154	1,044,866	93%
fminunc	805	1,903,793	9%	8,107	1,059,851	95%	1,154	1,044,866	93%
multistart	805	1,903,793	9%	8,107	1,059,851	95%	1,154	1,044,866	93%
pattersearch	805	1,903,793	9%	8,107	1,059,851	95%	1,154	1,044,866	93%
globalsearch	805	1,903,793	9%	8,107	1,059,851	95%	1,249	1,045,063	86%
simulannealbnd	805	1,903,793	9%	8,107	1,059,851	95%	1,154	1,044,866	93%

Figure 4-28 Optimization Scenarios Results – Dataset1 – Demand 8000

	Cycle 1			Cycle 2			Cycle 3		
Lead Time = 0	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level
fmincon	805	67,754	100%	13	29,224	100%	57	29,224	100%
fminunc	805	67,754	100%	14	29,224	100%	57	29,224	100%
multistart	805	67,754	100%	15	29,224	100%	57	29,224	100%
pattersearch	805	67,754	100%	16	29,224	100%	57	29,224	100%
globalsearch	805	67,754	100%	17	29,224	100%	55	29,224	100%
simulannealbnd	805	67,754	100%	18	29,224	100%	57	29,224	100%

	Cycle 1			Cycle 2			Cycle 3		
Lead Time = 1	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level
fmincon	805	97,396	98%	68	60,795	92%	117	60,845	91%
fminunc	805	97,396	98%	68	60,795	92%	117	60,845	91%
multistart	805	97,396	98%	68	60,795	92%	117	60,845	91%
pattersearch	805	97,396	98%	68	60,795	92%	117	60,845	91%
globalsearch	805	97,396	98%	68	60,795	97%	83	60,803	87%
simulannealbnd	805	97,396	98%	68	60,795	92%	117	60,845	91%

	Cycle 1			Cycle 2			Cycle 3		
Lead Time = 2	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level	Cycle Optimal Quantity	Cycle Minimum Cost	Minimum Cycle Service Level
fmincon	805	129,350	90%	193	94,209	92%	166	93,587	92%
fminunc	805	129,350	90%	193	94,209	92%	166	93,587	92%
multistart	805	129,350	90%	193	94,209	92%	166	93,587	92%
pattersearch	805	129,350	90%	193	94,209	92%	166	93,587	92%
globalsearch	805	129,350	90%	193	94,209	95%	178	93,612	86%
simulannealbnd	805	129,350	90%	193	94,209	92%	166	93,587	92%

Figure 4-29 Optimization Scenario- Dataset1 – Demand of 700

Table 4-7 Optimization Results – Dataset2

Human Capital Minimum Cost Optimization – Dataset2 – Lead Time 0 – Demand 8000			
	Cycle	Optimization Quantity	Minimized Cost
fmincon, fminunc, Patternsearch, Simulannealbnd	1	3,955	501,169
	2	4,337	332,900
	3	652	332,900
multistart, globalsearch	1	3,955	501,169
	2	8,000	2,696,700
	3	8,000	5,577,800

Demand	8000	8000	8000	8000	8000	8000
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Beg Inv	805.17	745.64	8,000.00	7,326.01	8,000.00	7,506.71
Demand	8,000.00	8,000.00	8,000.00	8,000.00	8,000.00	8,000.00
Attrition Rate	-	-	-	-	-	-
End Inv	745.64	687.45	7,326.01	6,992.31	7,506.71	7,230.64
Next Inv %	9.32%	8.59%	91.58%	87.40%	93.83%	90.38%
This Period %	10.06%	9.32%	100.00%	91.58%	100.00%	93.83%
Excess Units	-	-	-	-	-	-
Excess Cost	-	-	-	-	-	-
Periods +	1.00	1.00	1.00	1.00	1.00	1.00
Periods -	-	-	-	-	-	-
Short Units	7,194.83	7,254.36	-	673.99	-	493.29
Shortage Cost	598,610.00	603,563.09	-	56,076.30	-	41,041.98
Inv Cost	33,495.00	31,018.46	332,800.00	304,761.85	332,800.00	312,279.01
Inv Pos	805.27	745.83	8,001.09	7,327.92	8,001.92	7,508.65
setup Cost	104.00	-	104.00	-	104.00	-
Req Period	1.00	-	3.00	-	5.00	-
Period Cost	632,209.00	634,581.54	332,904.00	360,838.15	332,904.00	353,320.99
Excess Cost		-		-		-
Shortage Cost		1,202,173.09		56,076.30		41,041.98
Inv Cost		64,513.46		637,561.85		645,079.01
setup Cost		104.00		104.00		104.00
Total		1,266,790.54		693,742.15		686,224.99
# periods		2.00		2.00		2.00
Avg Period Cost		633,395.27		346,871.08		343,112.49

Figure 4-30 Human Capital Inventory Cost Calculation – Dataset1 – Demand 8000

Demand	700	700	700	700	700	700	700	700	700	700
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
Beg Inv	805.17	745.64	687.45	631.88	771.87	700.00	645.65	761.12	700.00	644.49
Demand	700.00	700.00	700.00	700.00	700.00	700.00	700.00	700.00	700.00	700.00
Attrition Rate	-	-	-	-	-	-	-	-	-	-
End Inv	745.64	687.45	631.88	578.72	700.00	645.65	595.24	700.00	644.49	596.80
Next Inv %	106.52%	98.21%	90.27%	82.67%	100.00%	92.24%	85.03%	100.00%	92.07%	85.26%
This Period %	115.02%	106.52%	98.21%	90.27%	110.27%	100.00%	92.24%	108.73%	100.00%	92.07%
Excess Units	105.17	45.64	-	-	71.87	-	-	61.12	-	-
Excess Cost	6,562.50	2,847.68	-	-	4,484.40	-	-	3,813.69	-	-
Periods +	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Periods -	-	-	-	-	-	-	-	-	-	-
Short Units	-	-	12.55	68.12	-	0.00	54.35	-	0.00	55.51
Shortage Cost	-	-	1,044.21	5,667.22	-	0.05	4,521.61	-	0.07	4,618.13
Inv Cost	29,120.00	29,120.00	28,597.90	26,286.39	29,120.00	29,119.98	26,859.19	29,120.00	29,119.97	26,810.93
Inv Pos	806.32	747.85	689.50	633.77	773.87	702.10	647.58	763.13	702.09	646.41
setup Cost	-	104.00	-	-	104.00	-	-	104.00	-	-
Req Period	-	2.00	-	-	5.00	-	-	8.00	-	-
Period Cost	35,682.50	32,071.68	29,642.10	31,953.61	33,708.40	29,120.02	31,380.81	33,037.69	29,120.03	31,429.07
Excess Cost				9,410.18			4,484.40			3,813.69
Shortage Cost				6,711.43			4,521.66			4,618.20
Inv Cost				113,124.29			85,099.17			85,050.90
setup Cost				104.00			104.00			104.00
Total				129,349.90			94,209.23			93,586.79
# periods				4.00			2.00			2.00
Avg Period Cost				32,337.47			47,104.62			46,793.40

Figure 4-31 Human Capital Inventory Cost – Dataset1 – Demand 700

Demand	8000	8000	8000	8000	8000	8000
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Beg Inv	3,955.17	3,662.73	8,000.00	7,336.07	8,000.00	7,411.10
Demand	8,000.00	8,000.00	8,000.00	8,000.00	8,000.00	8,000.00
Attrition Rate	-	-	-	-	-	-
End Inv	3,662.73	3,376.91	7,336.07	6,897.54	7,411.10	7,017.83
Next Inv %	45.78%	42.21%	91.70%	86.22%	92.64%	87.72%
This Period %	49.44%	45.78%	100.00%	91.70%	100.00%	92.64%
Excess Units	-	-	-	-	-	-
Excess Cost	-	-	-	-	-	-
Periods +	1.00	1.00	1.00	1.00	1.00	1.00
Periods -	-	-	-	-	-	-
Short Units	4,044.83	4,337.27	-	663.93	-	588.90
Shortage Cost	336,530.00	360,860.68	-	55,238.70	-	48,996.28
Inv Cost	164,535.00	152,369.66	332,800.00	305,180.65	332,800.00	308,301.86
Inv Pos	3,955.66	3,663.68	8,001.46	7,337.99	8,001.92	7,413.03
setup Cost	104.00	-	104.00	-	104.00	-
Req Period	1.00	-	3.00	-	5.00	-
Period Cost	501,169.00	513,230.34	332,904.00	360,419.35	332,904.00	357,298.14
Excess Cost		-		-		-
Shortage Cost		697,390.68		55,238.70		48,996.28
Inv Cost		316,904.66		637,980.65		641,101.86
setup Cost		104.00		104.00		104.00
Total		1,014,399.34		693,323.35		690,202.14
# periods		2.00		2.00		2.00
Avg Period Cost		507,199.67		346,661.68		345,101.07

Figure 4-32 Human Capital Inventory Cost Calculation – Dataset2 – Demand 8000

Demand	3500	3500	3500	3500	3500	3500	3500
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7
Beg Inv	3,955.17	3,662.73	3,376.91	3,500.00	3,205.33	3,500.00	3,201.07
Demand	3,500.00	3,500.00	3,500.00	3,500.00	3,500.00	3,500.00	3,500.00
Attrition Rate	-	-	-	-	-	-	-
End Inv	3,662.73	3,376.91	3,103.96	3,205.33	2,917.35	3,201.07	2,966.48
Next Inv %	104.65%	96.48%	88.68%	91.58%	83.35%	91.46%	84.76%
This Period %	113.00%	104.65%	96.48%	100.00%	91.58%	100.00%	91.46%
Excess Units	455.17	162.73	-	-	-	-	-
Excess Cost	28,402.50	10,154.49	-	-	-	-	-
Periods +	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Periods -	-	-	-	-	-	-	-
Short Units	-	-	123.09	-	294.67	-	298.93
Shortage Cost	-	-	10,241.36	-	24,516.83	-	24,870.85
Inv Cost	145,600.00	145,600.00	140,479.32	145,600.00	133,341.58	145,600.00	133,164.57
Inv Pos	3,956.30	3,664.91	3,378.92	3,501.96	3,207.24	3,501.92	3,202.99
setup Cost	-	104.00	-	-	104.00	-	-
Req Period	-	2.00	-	-	5.00	-	-
Period Cost	174,002.50	155,858.49	150,720.68	145,600.00	157,962.42	145,600.00	158,035.43
Excess Cost			38,556.99		-		-
Shortage Cost			10,241.36		24,516.83		24,870.85
Inv Cost			431,679.32		278,941.58		278,764.57
setup Cost			104.00		104		104.00
Total			480,581.67		303,562.42		303,739.43
# periods			3.00		2.00		2.00
Avg Period Cost			160,193.89		151,781.21		151,869.71

Figure 4-33 Human Capital Inventory Cost – Dataset2 – Demand 3500

Chapter 5 - Conclusions and Recommendations

5.1 Summary

The purpose of this research was to determine whether the Human Capital (Workforce) that is necessary to accomplish work in an organization could be managed by utilizing generally accepted inventory theory and models like any other traditional inventory asset. A review of current literature relating to Human Capital inventories revealed that comparable inventory policies, methodologies and tools do not exist for the management of Human Capital. This study documents the research and development of a Human Capital Inventory Cost Model as well as the application of statistical and optimization techniques that produce a minimum cost Human Capital inventory policy. Just as in the management of critical physical inventories, the Human Capital Inventory Cost Model forecasts how much Human Capital inventory is available from period to period. Using a predefined replenishment point for the available Human Capital inventory, the Human Capital Inventory Cost Model forecasts when replenishment efforts need to be initiated. Based on the lead time to acquire the Human Capital inventory, the model also determines when the Human Capital inventory will be replenished.

One of the most significant differences between the Human Capital inventory and a traditional physical inventory is the reduction in available inventories. Typical physical inventories are reduced and consumed based on the demand for the inventory item. In the Human Capital inventory, the available inventory is reduced through attrition. The development of the Human Capital inventory model required that a methodology be developed to address attrition. The application of cluster analysis and survival analysis was applied to address Human Capital

inventory attrition. This methodology for developing the attrition rates is better suited for Human Capital attrition rather than the typically used method of “turnover rates’. The Human Capital Inventory Cost Model, through the application of optimization techniques, determines minimum replenishment quantities and Human Capital inventory cycle costs, including the cost of available inventories, excess inventory costs, shortage costs and the cost of inventory replenishment.

The statistical methodologies and optimization techniques applied to Human Capital in this study have demonstrated that the application of clustering analysis, survival analysis, and minimum cost optimization provides a methodology and means for the management of Human Capital inventories in a manner that is superior to other methods and models that were found in the literature search. The application of the Human Capital Inventory Cost Model, in conjunction with the application of optimization techniques, addresses the need for the establishment of Human Capital inventory policy and its impact on Human Capital related costs with the decisions that comprise inventory policy:

- How much Human Capital inventory to have available
- When to replenish the Human Capital inventory
- How much should be replenished at the replenishment event while
    - Maintaining a target Service level
    - Minimizing Human Capital inventory related costs

The results of this study provide a quantitative framework for a comprehensive and improved methodology for workforce planning and talent management. In the workforce planning process (Figure 5-1), the application of existing statistical techniques, such as clustering

and survival analysis, and the development and demonstration of a minimum-cost Human Capital Inventory Cost model, result in improved workforce analysis for addressing gaps that exist between the workforce demands and the workforce supply. The workforce levers (Figure 5-1) are the means for addressing these gaps. The benefits of the quantification phase developed in this research and the application of workforce levers should enable an organization to meet its strategic business objectives of making intelligent workforce planning decisions resulting in reduced costs while increasing or maintaining customer satisfaction and shareholder value.

5.2 Operationalization

The term *Operationalization* is defined as the process of specifying the extension of a concept so that it may be applied and utilized in a real world environment (Dictionary.com, 2011). The datasets, model parameters and the optimization variables were selected in this research to demonstrate the concepts of inventory planning for Human Capital and for ease of demonstration. However, the principles and concepts presented in this research can readily be applied to strategic organization planning.

The major components of the Human Capital Inventory Cost Model include the Human Capital inventory demand, the Human Capital Inventory availability, the Replenishment Order point, the Replenishment Lead Time and the rates that are applied to the inventory positions. The optimization techniques utilized to determine the inventory cycle minimum replenishment quantities is based on an objective to minimize each inventory cycle cost.

The Human Capital Inventory Cost Model will allow the assignment of the demand for any period whether they are deterministic or stochastic. The Human Capital Inventory Cost Model will calculate period costs, as well as total cycle costs, regardless of the distribution of the

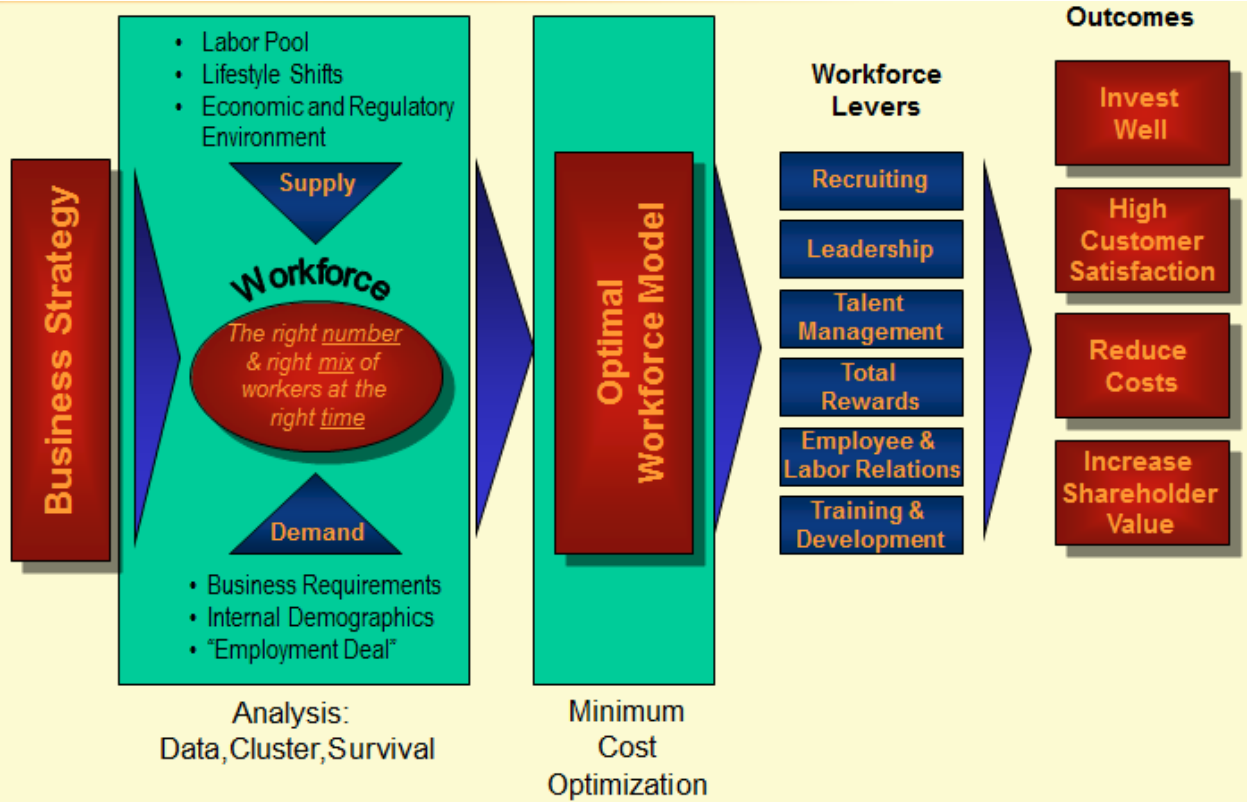


Figure 5-1 Strategic Human Capital Planning Process (Noey, 2011)

inventory period demands. The results of the cost model, and the resulting optimization quantities, are intended to be used by an organization for demand and supply analysis for its Human Capital inventory. Further statistical analysis should be conducted as appropriate that utilizes these results.

The Human Capital inventory availability in this study is determined by applying survival analysis for the two organizations represented by dataset1 and dataset2. ***The significant difference between the traditional physical inventory model and the Human Capital inventory model is that the inventory availability for the Human Capital Inventory Cost Model is based on the Human Capital population attrition, not on the period demand.*** The only population

demographics that were used in this study are an employee's age and the employee's years-of-company service during the ten-year study period. A more thorough evaluation of the Human Capital attrition should include additional employee demographics. This model developed in this study is not limited to the selection of population demographics for determining Human Capital attrition. However, when more than two attrition variables are to be used in the survival analysis, other survival analysis such as Cox Regression should be applied rather than Kaplan-Meier.

The Replenishment Order Point (ROP) in this study was arbitrarily selected to be 100%. As stated in previous sections, the ROP in this study is calculated as the ratio of the ending period inventory availability over the next periods demand. If this ratio is less than 100%, the replenishment event is initiated. An organization should run multiple scenarios utilizing the Human Capital minimum most optimization model using various ROPs in order to analyze the effects of the service level during the inventory lead times.

The replenishment lead times chosen for this study included lead times of zero, one or two periods. Lead times greater than two periods are a user defined variable in the inventory cost model and can be selected at the discretion of the user. Running multiple scenarios with varying leads times would enable an organization to analyze the effects of lead times on Human Capital availability and service levels over the planning horizon.

The selection of the rates applied to the inventory positions in the cost model were arbitrarily. However, the relative magnitude of the rates is based on the Human Capital costs as discussed in Section 3.4.1. and Section 3.4.2. The inventory position costs (availability, shortage, excess, replenishment) are the cost components applied in this study. However, the

Human Capital Inventory Cost Model is adaptable to any costs, and associated rates, that may be desired to be applied to organization Human Capital inventories.

MATLAB was utilized in this study for performing the minimum cost optimization and providing the optimum replenishment cycle quantities. MATLAB has numerous optimization functions and provides for both local and global optimization solutions. Several of the MATLAB optimization functions were demonstrated in this study in order to confirm that global optimization quantities were found for the inventory cycles that minimized each inventory cycle cost. The concepts and models developed in this study may be applied in other optimization environments. However, the results of the optimization techniques must be confirmed that global solutions have been achieved.

5.3 Future Research

The findings and limitations of this study indicate that several areas should be considered for further research. These areas include:

- Analysis of additional Human Capital workforce datasets in order to identify the effects of multiple covariates on the attrition and replenishment distributions of the Human Capital inventory.
- Application of alternate Statistical techniques related to Human Capital workforce attrition and replenishment distributions so that the best distributions can be identified.
- Expansion and enhancement of the Human Capital Inventory Cost Model so that it can be utilized for improved gap analysis between Human Capital supply and demand.

- Alternate Human Capital Inventory Cost Model optimization objective functions that reflect other single or multiple objectives other than only minimizing total cycle costs.

Detailed recommendations for these areas include:

- More than two covariates other than age and years of company service for use in Cluster Analysis and Survival Analysis (gender, occupation, etc.) for determining attrition rates within the organization's population
- The application of other clustering techniques such as Density Based Clustering for grouping an organization into more homogeneous subgroups. Although K-Means clustering is the most popular means of clustering, density based and other cluster techniques should be explored.
- An analysis of the effects of Human Capital inventory rates (Available, Excess, Shortage) on minimum cost optimization results should be explored. One of the objectives of this study was the development of the Human Inventory Cost model, not the effects of the rates applied in the model or the impact of the rates on the minimum cost optimization results.
- During the study, it was noted that there are other costs that should be considered beyond the costs included in Human Capital Inventory Cost Model. One additional cost that should be considered is the termination costs associated with employee attrition. Other additional costs may exist and should be explored.

- No constraints were applied in the optimization routines. However, cost constraints or quantity constraints could be applied to study the effects of constraining the optimization variables and costs.
- How an organization mitigates Human Capital service levels, excess and shortage units and costs are not part of this study. Additional research should address how to deal with these circumstances in the framework of a Human Capital Inventory Cost Model.
- The rates utilized in the Human Capital Inventory Cost Model are constant from period to period. Varying periodic rates can be utilized to reflect varying periodic costs.
- There was no intention to develop a 'user accessible' package with appropriate user interfaces. As currently designed, only someone with in-depth knowledge of MATLAB and knowledge of how the software functions is capable of utilizing the models.

5.4 Conclusions

The motivation for this study originated from observations by the author that much literature detailed the need for more Human Capital planning because of the impending significant loss of Human Capital within all existing workforces resulting from the attrition of 'baby boomers' from the workforce. As the Human Capital topic was initially investigated, an observation was made that the Human Capital workforce appeared to be similar in many ways to any other significantly valuable asset that an organization possesses. If the Human Capital workforce is considered an asset, the question arose "Why can't Human Capital assets be managed as any other significant valued asset by utilizing traditional physical inventories management techniques and policies". However, it was apparent that the inventory assumptions for Human Capital assets needed to be considered differently than physical assets.

These observations and the research contained in this dissertation have resulted in the development of a Human Capital inventory management methodology that offers organizations a significant degree of intelligent in their workforce planning by utilizing statistical analysis, workforce forecasting, predictive modeling and optimization. Utilizing this Human Capital inventory management methodology should provide organizations critical information for making improvements and optimizing their human capabilities needed to meet strategic business objectives and for achieving and maintaining a competitive advantage. This is significant because the literature review conducted for this dissertation did not find any Human Capital workforce planning models that include the scope of analytics provided here: forecasting, predictive modeling, and optimization. This model provides a quantification of Human Capital assets to facilitate the management of this valuable resource from planning to placement through retirement by applying traditional inventory planning models to a "different" type of asset-though very critical to the organization. In other words, if applied, it can support managers in getting the "*right number of people, with the right skills, in the right place, and at the right time"* (Drucker, 1992).

5.5 Other Areas of Applicability

The application of the methodologies and techniques developed in this study for Human Capital inventory management and policies can be applied to any inventory environment where attrition reduces available inventory. Examples of potential applications include perishable products such as medicines, tenure based organizations and organizations with retirement eligibility based on years of service. With minor modifications to account for the characteristics

of the asset and the operational conditions, this model can be applied and can provide management with useful information.

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Appendices

Appendix A: Figures

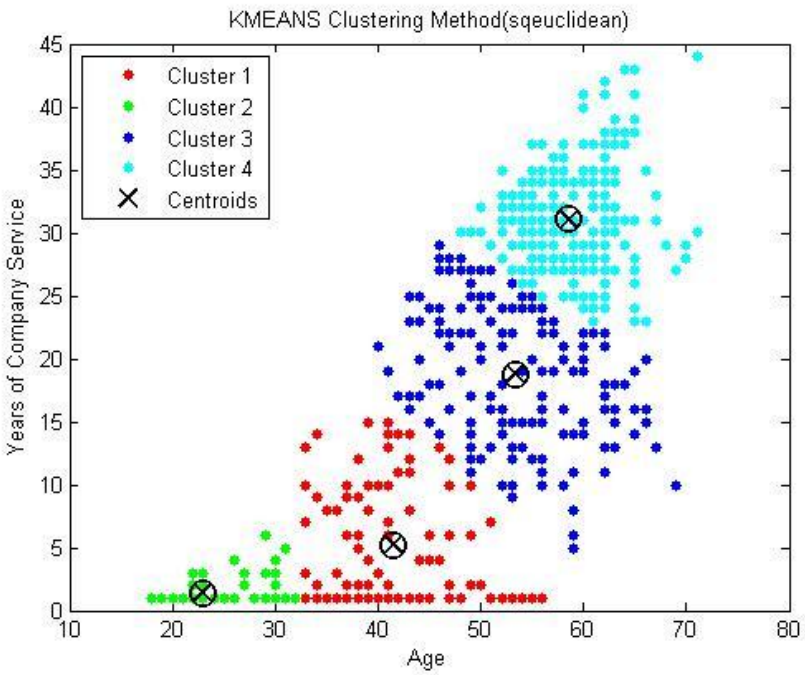

Figure A-1: KMEANS Clustering- 4 Clusters - Dataset1 – sqeuclidean

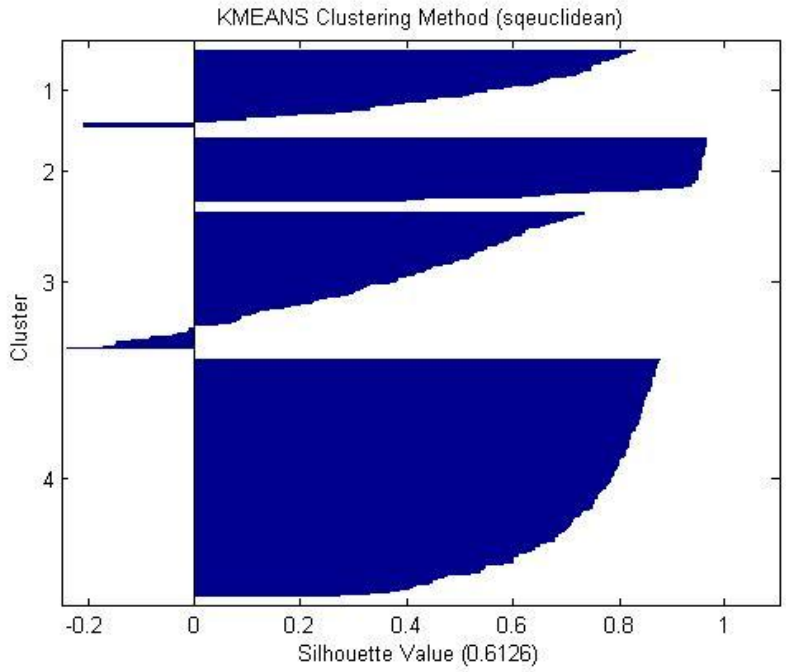

Figure A-2: Silhouette Plot – 4 Clusters – Dataset1 - sqeuclidean

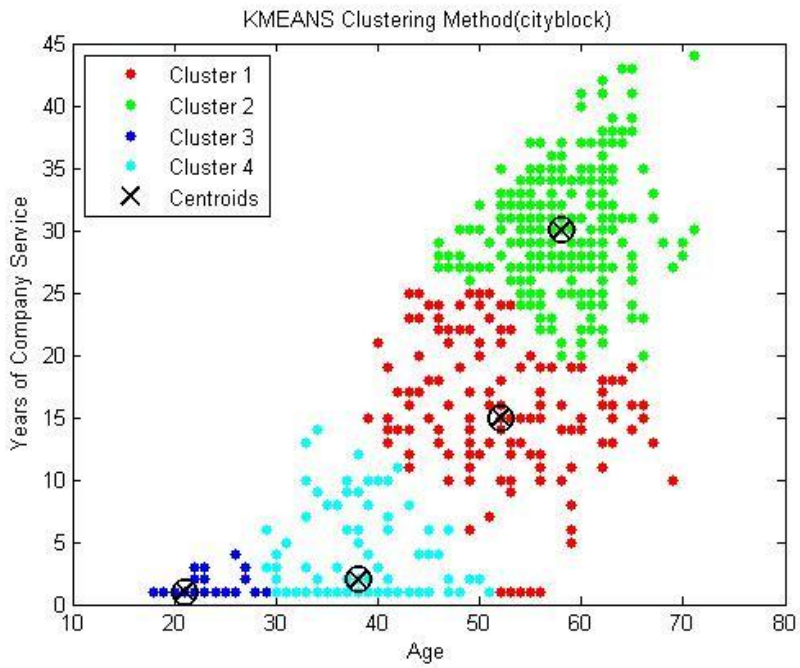

Figure A-3: KMeans Clustering – 4 Clusters – Dataset1 – cityblock

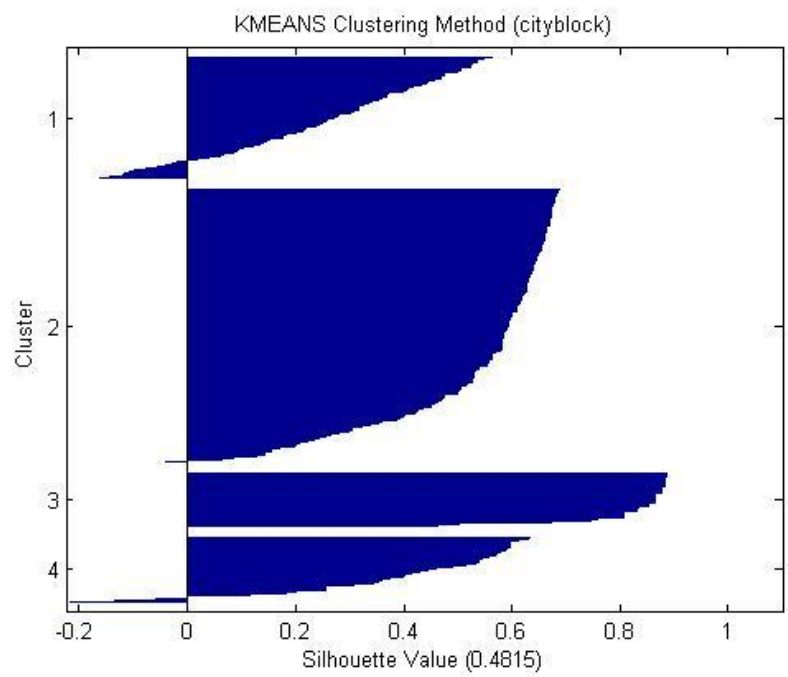

Figure A-4 Silhouette Plot – 4 Clusters – Dataset1 - cityblock

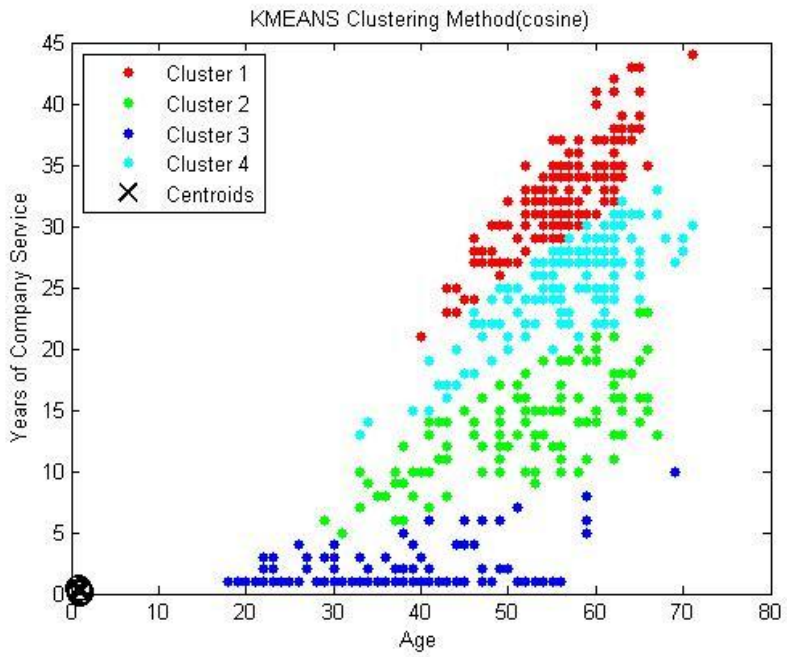

Figure A-5 KMEANS Clustering – 4 Clusters – Dataset1 – cosine

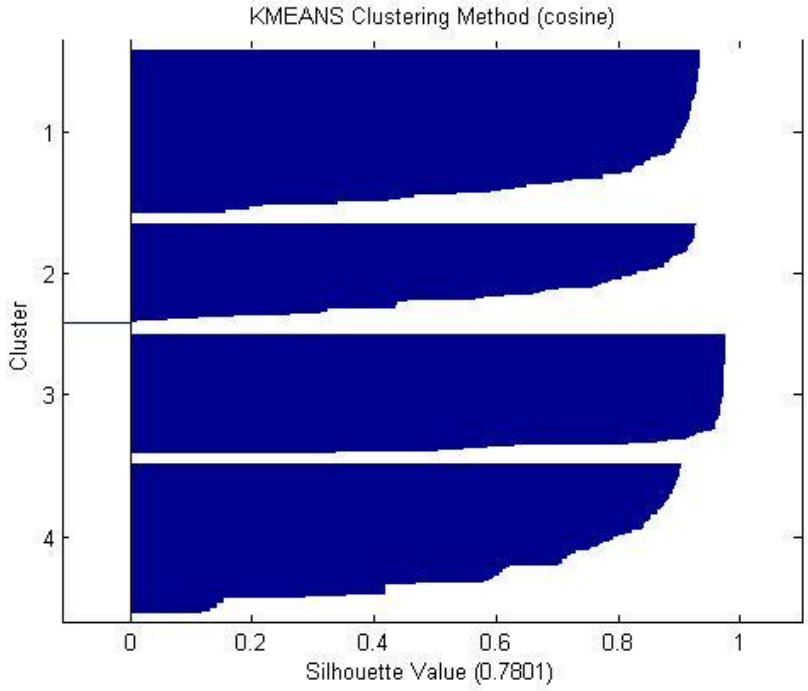

Figure A-6 Silhouette plot – 4 Clusters – Dataset1 - cosine

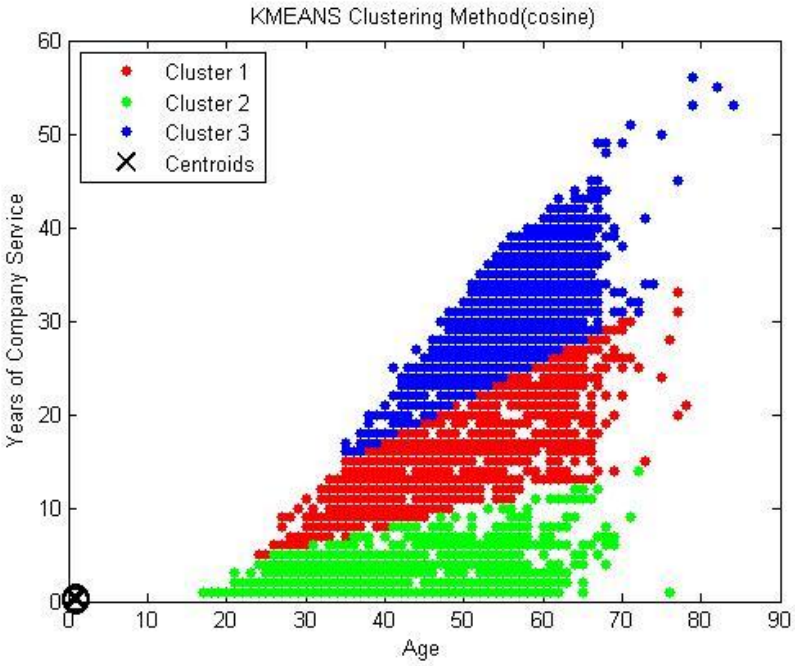

Figure A-7 KMeans Clustersing – 3 Clusters – Dataset2 (cosine)

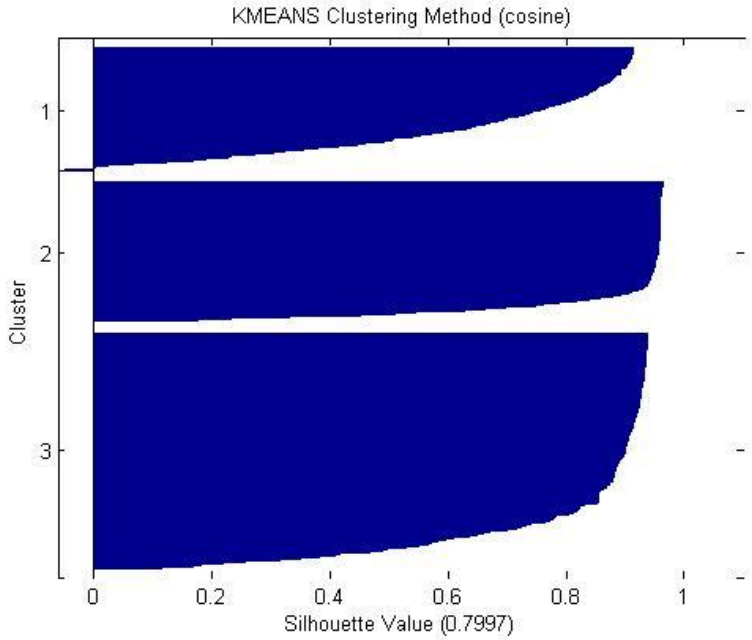

Figure A-8 Silhouette Plot – 3 Clusters – dataset2 – cosine

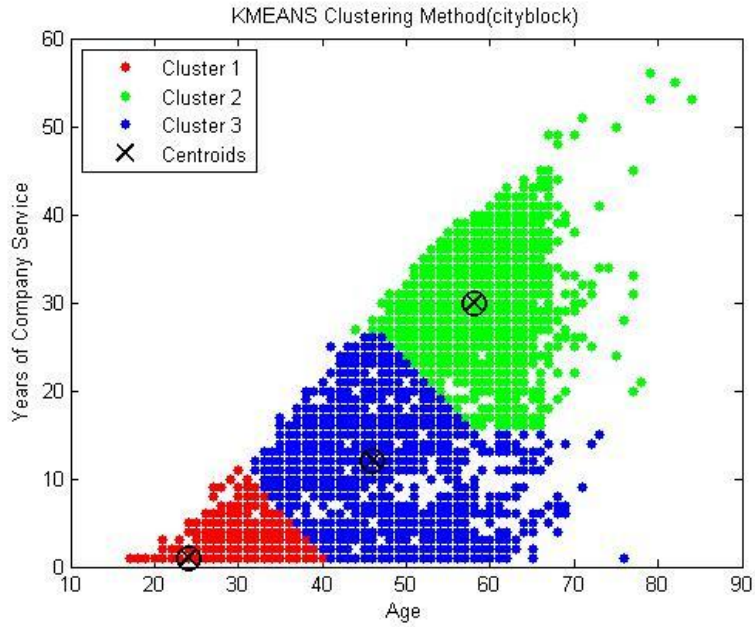

Figure A-9 KMeans Clustersing – 3 Clusters – Dataset2 (cityblock)

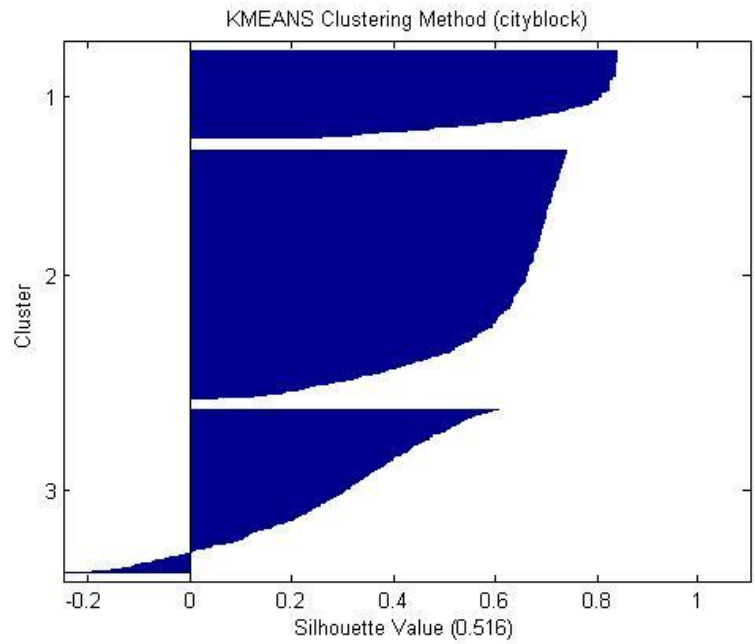

Figure A-10 Silhouette Plot – 3 Clusters – dataset2 - cityblock

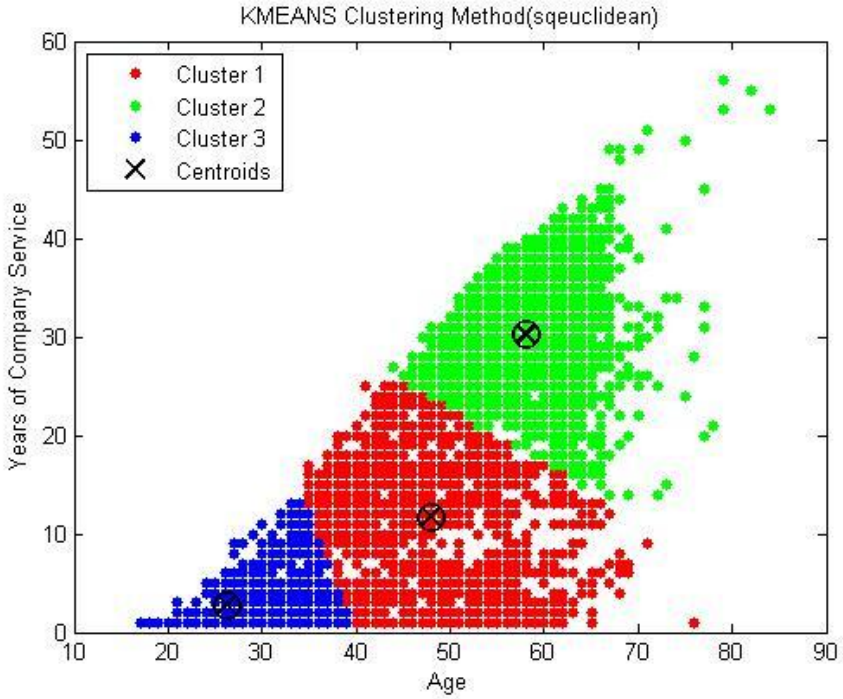

Figure A-11 KMeans Clustersing – 3 Clusters – Dataset2 (sqeuclidean)

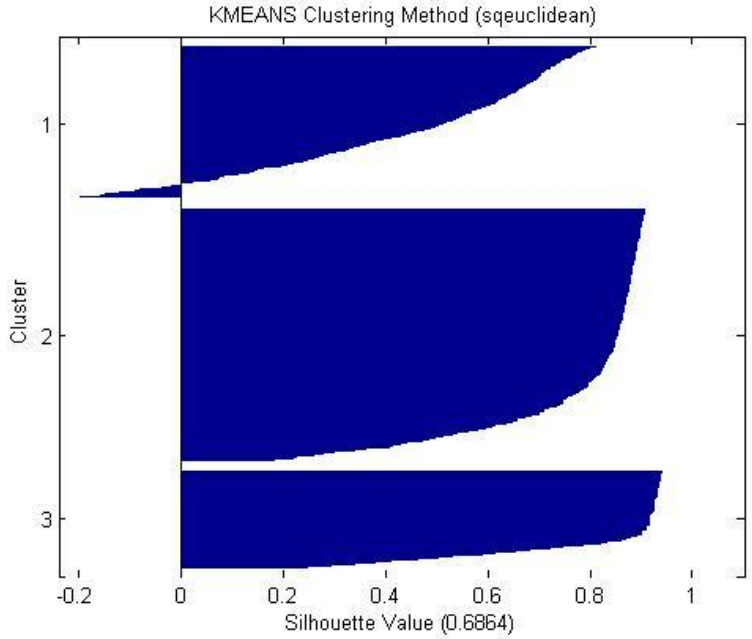

Figure A-12 Silhouette Plot – 3 Clusters – dataset2 – sqeuclidean

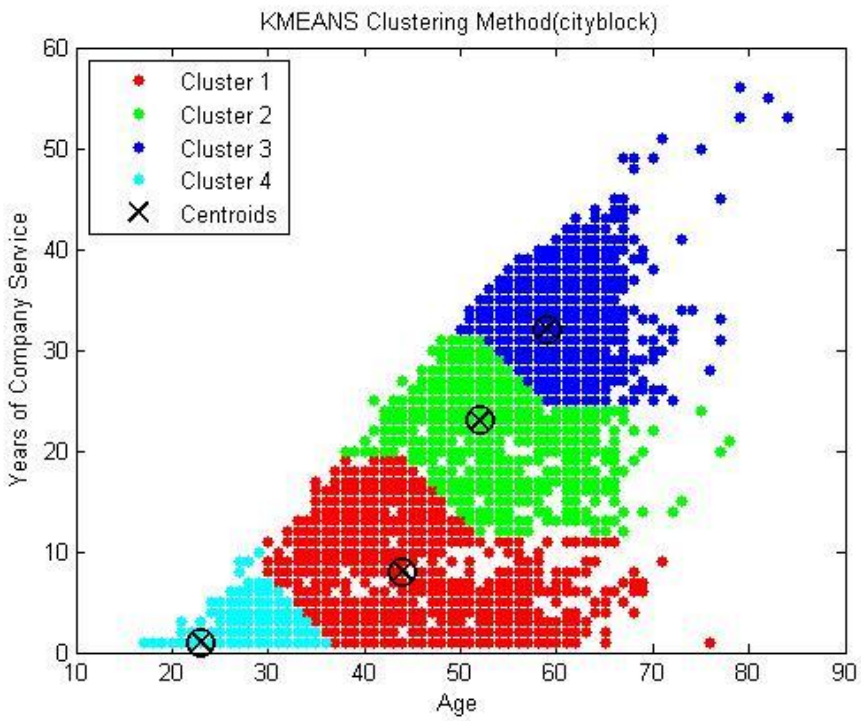

Figure A-13 KMeans Clustersing – 4 Clusters – Dataset2- cityblock

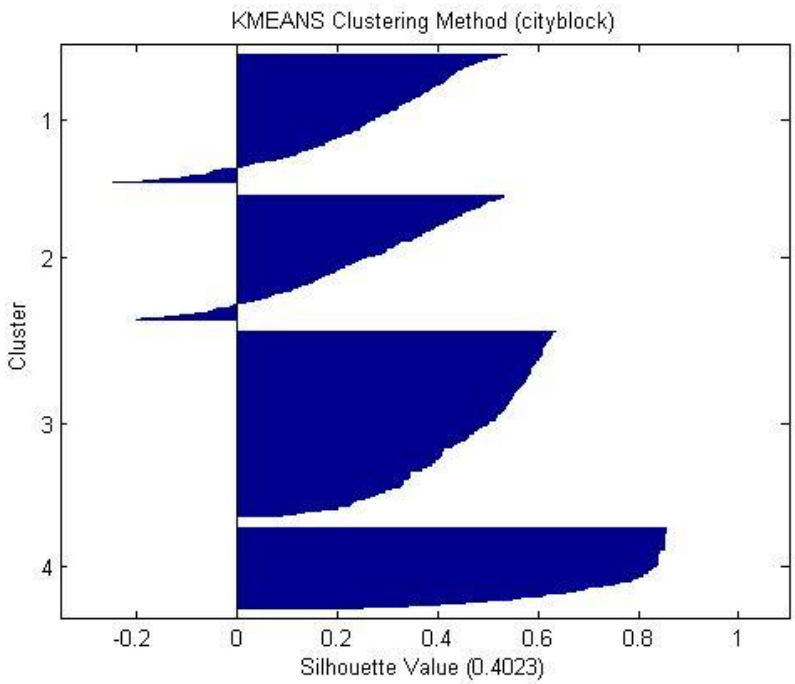

Figure A-14 Silhouette Plot – 4 Clusters – dataset2 – cityblock

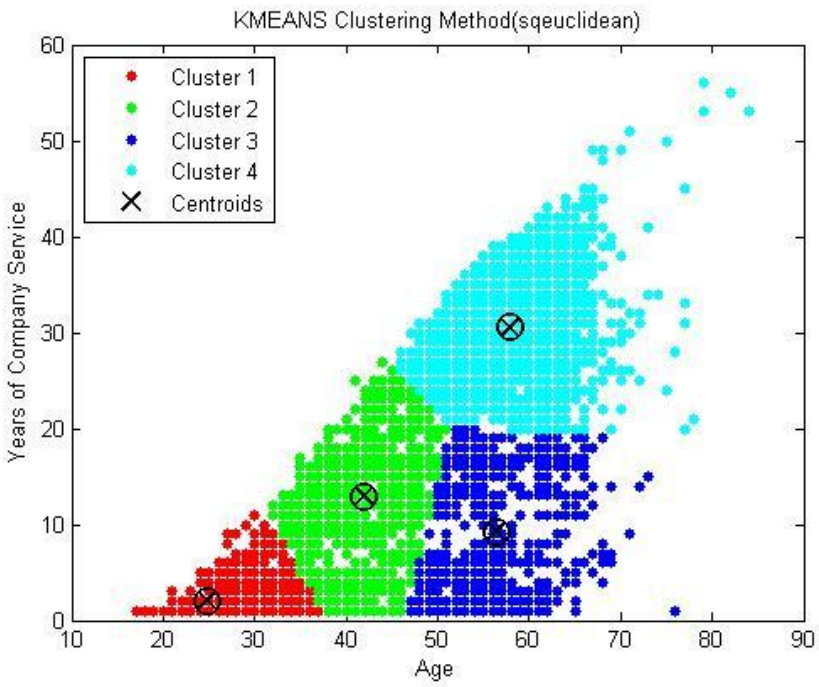

Figure A-15 KMeans Clustersing – 4 Clusters – Dataset2- sqeuclidean

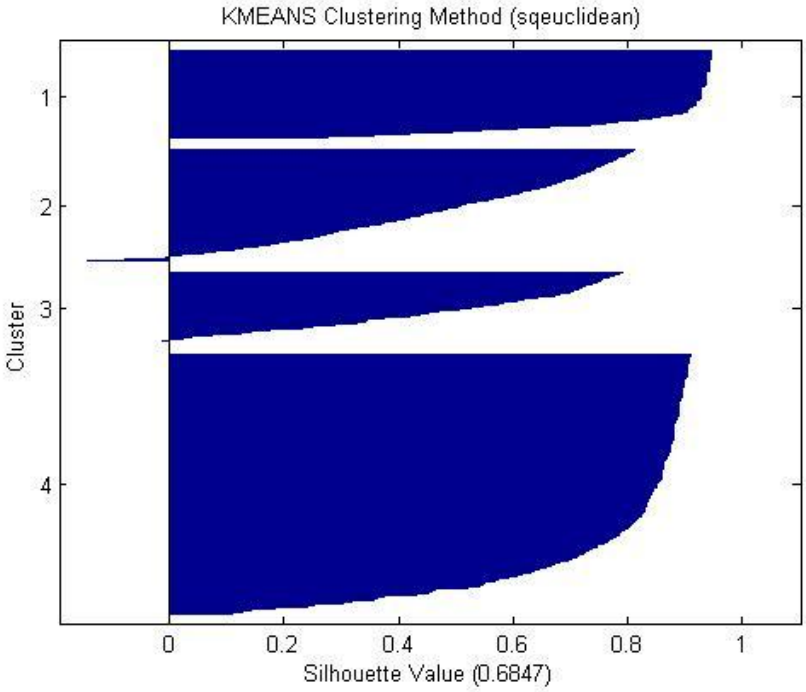

Figure A-16 Silhouette Plot – 4 Clusters – dataset2 – sqeuclidean

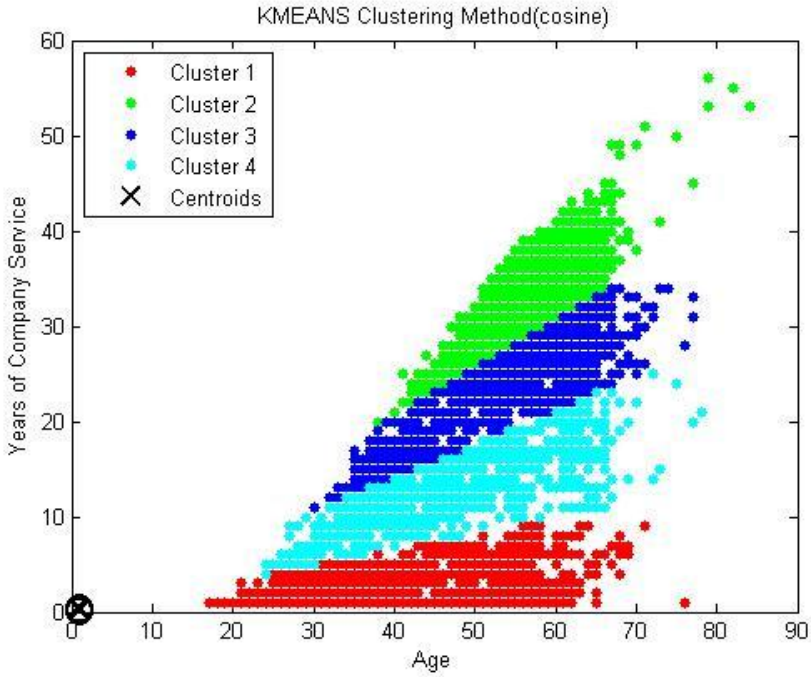

Figure A-17 KMeans Clustersing – 4 Clusters – Dataset2- sqeuclidean

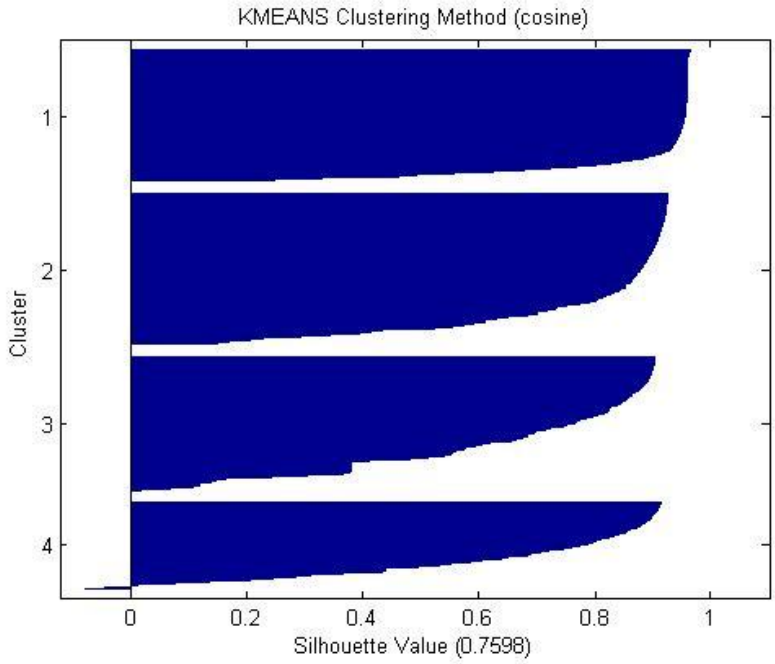

Figure A-18 Silhouette Plot – 4 Clusters – dataset2 - cosine

Appendix B: MATLAB Source Code

Appendix B-1

MATLAB Source Code: fnl_KMeans_Cluster.m

```
clear
clc

%-------------------------------------------------------------------------
------------------------
%Dataset1 Data File Field Positions
importXLSORNLFile('c:/users/bruce/dissertation/datafiles/DS1AttritionDataAll.
xlsx');
data = sortrows(data,[22,23]); % Needed for DS1 Data - Age, YCS
all = data(:,[22:23,17]); % Needed for DS1 Data - Age, YCS, Status
allStatus = data(:,17); %Needed for DS1 Data - Status
allXDur = data (:,23);% Needed for DS1 Data - YCS - Duration
%-------------------------------------------------------------------------
------------------------
%Dataset2 Data File Field Positions
%importXLSfile('c:/users/bruce/dissertation/datafiles/DS2AttritionDataAll.xls
','DS2AttritionDataAll');
%data = sortrows(data,[9:10]); % Needed for DS2 Data - Age, YCS

%all = data(:,[9:10,5]); % Needed for DS2 Data - Age, YCS, Status
%allStatus = data(:,5); %Needed for DS2 Data - Status

%allXDur = data (:,10);% Needed for DS2 Data - YCS - Duration
%-------------------------------------------------------------------------
------------------------

index = find(allStatus(:,1)==1);
XTEMP = all(index,1:3);
X = all(index,1:3);
YofX = all(index,2);
index = find(allStatus(:,1)==0);
Y = all(index,1:3);
length(all);

% Determine and Create Top Percentage of Population

XSort = sortrows(XTEMP);
xlength = length(XSort);

XHold(1,1) = XSort(1,1);
XHold(1,2) = XSort(1,2);
XHold(1,3) = XSort(1,3);
XHold(1,4) = 1;
XHold(1,5) = 0;

yidx = 1;
ytotcnt = 1;
```

```
for i = 2 : xlength;
if XSort(i,1) ~= XHold(yidx,1)
    yidx = yidx + 1;
    XHold(yidx,1) = XSort(i,1);
    XHold(yidx,2) = XSort(i,2);
    XHold(yidx,3) = XSort(i,3);
    XHold(yidx,4) = 1;
    XHold(yidx,5) = 0;
    ytotcnt = ytotcnt + 1;
    continue
end
if XSort(i,2) ~= XHold(yidx,2)
    yidx = yidx + 1;
    XHold(yidx,1) = XSort(i,1);
    XHold(yidx,2) = XSort(i,2);
    XHold(yidx,3) = XSort(i,3);
    XHold(yidx,4) = 1;
    XHold(yidx,5) = 0;
    ytotcnt = ytotcnt + 1;
    continue
end
if XSort(i,3) ~= XHold(yidx,3)
    yidx = yidx + 1;
    XHold(yidx,1) = XSort(i,1);
    XHold(yidx,2) = XSort(i,2);
    XHold(yidx,3) = XSort(i,3);
    XHold(yidx,4) = 1;
    XHold(yidx,5) = 0;
    ytotcnt = ytotcnt + 1;
    continue
end

XHold(yidx,4) = XHold(yidx,4) + 1;
ytotcnt = ytotcnt + 1;

end

XPct = sortrows(XHold,[-4 1 2 3]);;
xlength = length(XPct);

cumpct = 0;

for i = 1 : xlength;
    XPct(i,5) = XPct(i,4) / ytotcnt;
    cumpct = cumpct + XPct(i,5);
    XPct(i,6) = cumpct;
end

for i = 1 : xlength;
    if XPct(i,6) > 0.50
        break;
    end
    XFreq(i,:) = XPct(i,:);
```

```
end

xidx = 0;

xlength = length(XFreq);

for i = 1 : xlength
    for j = 1 : XFreq(i,4);
        xidx = xidx + 1
    end
end

%----- Kmeans Clustering Code Section -----

opts = statset('Display','final');

kmdistance = ['sqeuclidean';...
              'cityblock  ';...
              'cosine     '];

nk = 3;

for i = 1:3
    kmd = kmdistance(i,:);
    [idx,ctrs,sumd,d] =
kmeans(X(:,1:2),nk,'Distance',kmd,'Replicates',15,'Options',opts,...
        'start','cluster',
'emptyaction','drop','onlinephase','on','display','final');
    plot(X(idx==1,1),X(idx==1 ,2),'r.','MarkerSize',15)
    hold on
    plot(X(idx==2,1),X(idx==2,2),'g.','MarkerSize',15)
    plot(X(idx==3,1),X(idx==3,2),'b.','MarkerSize',15)

    switch nk
        case 4
            plot(X(idx==4,1),X(idx==4,2),'c.','MarkerSize',15)
        case 5
            plot(X(idx==4,1),X(idx==4,2),'c.','MarkerSize',15)
            plot(X(idx==5,1),X(idx==5,2),'m.','MarkerSize',15)
    end


plot(ctrs(:,1),ctrs(:,2),'kx',...
     'MarkerSize',12,'LineWidth',2)
plot(ctrs(:,1),ctrs(:,2),'ko',...
     'MarkerSize',12,'LineWidth',2)

switch nk
    case 3
        legend('Cluster 1','Cluster 2','Cluster 3','Centroids',...
       'Location','NW')
    case 4
```

```
        legend('Cluster 1','Cluster 2','Cluster 3','Cluster
4','Centroids',...
       'Location','NW')
    case 5
       legend('Cluster 1','Cluster 2','Cluster 3','Cluster 4','Cluster
5','Centroids',...
       'Location','NW')
end

    gphtitle = strcat('KMEANS Clustering Method(',kmd,')')
    title(gphtitle);
    xlabel('Age');
    ylabel('Years of Company Service');
    hold off

    pause

    newplot;

    scatter(X(:,1),X(:,2),25,idx,'filled');
    gphtitle = strcat('KMEANS Clustering Method(',kmd,')')
    title(gphtitle);
    xlabel('Age');
    ylabel('Years of Company Service');
    hold on
    plot(ctrs(:,1),ctrs(:,2),'kx',...
     'MarkerSize',12,'LineWidth',2)
    plot(ctrs(:,1),ctrs(:,2),'ko',...
     'MarkerSize',12,'LineWidth',2)
    pause

    [silh4,h] = silhouette(X,idx,kmd);
    silh4;
    h;
    a=mean(silh4)
    gphtitle = strcat('KMEANS Clustering Method (',kmd,')');
    xlabel(strcat('Silhouette Value (',mat2str(a,4),')'));
    title(gphtitle);
    pause
end

save
('c:\users\bruce\dissertation\matlabfiles\KMeansOutputData','X','Y','allXDur'
,'allStatus','idx', 'nk');

return
```

Appendix B-2

MATLAB Source Code: fnl_Kaplan_Meier.m

```
clc
clear

load 'c:\users\bruce\dissertation\matlabfiles\KMeansOutputData'

ilength = length(allXDur);

YCluster = knnclassify(Y(:,1:2),X(:,1:2),idx,5,'cosine','nearest');


Xdur = X(:,2);
Xcensor = ~X(:,3);

Ydur = Y(:,2);
Ycensor = ~Y(:,3);

CurrEmp = horzcat(Y, YCluster);


ClusterDur = vertcat(Xdur,Ydur);
Censor = vertcat(Xcensor, Ycensor);

for i = 1:nk

indX = find(idx(:,1) == i);
Xdur = X(indX,2);
Xcensor = ~X(indX,3);
XClusterTemp = X(indX,1:3);

indY = find(YCluster(:,1) == i);
Ydur = Y(indY,2);
Ycensor = ~Y(indY,3);
YClusterTemp = Y(indY,1:3);

switch i
    case 1
        Cluster1Dur = vertcat(Xdur,Ydur);
        Censor1 = vertcat(Xcensor, Ycensor);
        Cluster1 = vertcat(XClusterTemp, YClusterTemp);
    case 2
        Cluster2Dur = vertcat(Xdur,Ydur);
        Censor2 = vertcat(Xcensor, Ycensor);
        Cluster2 = vertcat(XClusterTemp, YClusterTemp);
    case 3
        Cluster3Dur = vertcat(Xdur,Ydur);
        Censor3 = vertcat(Xcensor, Ycensor);
```

```
        Cluster3 = vertcat(XClusterTemp, YClusterTemp);
    case 4
        Cluster4Dur = vertcat(Xdur,Ydur);
        Censor4 = vertcat(Xcensor, Ycensor);
        Cluster4 = vertcat(XClusterTemp, YClusterTemp);
    case 5
        Cluster5Dur = vertcat(Xdur,Ydur);
        Censor5 = vertcat(Xcensor, Ycensor);
        Cluster5 = vertcat(XClusterTemp, YClusterTemp);
end

end

switch nk
    case 3
        AllClusters = vertcat(Cluster1, Cluster2, Cluster3);
        AllCensor = vertcat(Censor1, Censor2, Censor3);
    case 4
        AllClusters = vertcat(Cluster1, Cluster2, Cluster3, Cluster4);
        AllCensor = vertcat(Censor1, Censor2, Censor3, Censor4);
    case 5
        AllClusters = vertcat(Cluster1, Cluster2, Cluster3,
Cluster4,Cluster5);
        AllCensor = vertcat(Censor1, Censor2, Censor3, Censor4, Censor5);
end

[fsurv1,x1,flo,fup] =
ecdf(Cluster1Dur,'censoring',Censor1,'function','survivor');
stairs(x1,fsurv1,'LineWidth',2,'color','r');
hold on
[fsurv2,x2,flo,fup] =
ecdf(Cluster2Dur,'censoring',Censor2,'function','survivor');
stairs(x2,fsurv2,'LineWidth',2,'color','g');
hold on
[fsurv3,x3,flo,fup] =
ecdf(Cluster3Dur,'censoring',Censor3,'function','survivor');
stairs(x3,fsurv3,'LineWidth',2,'color','b');
hold on

switch nk
    case 4
        [fsurv4,x4,flo,fup] =
ecdf(Cluster4Dur,'censoring',Censor4,'function','survivor');
        stairs(x4,fsurv4,'LineWidth',2,'color','c');
        hold on
    case 5
        [fsurv4,x4,flo,fup] =
ecdf(Cluster4Dur,'censoring',Censor4,'function','survivor');
        stairs(x4,fsurv4,'LineWidth',2,'color','c');
        hold on
        [fsurv5,x5,flo,fup] =
ecdf(Cluster5Dur,'censoring',Censor5,'function','survivor');
        stairs(x5,fsurv5,'LineWidth',2,'color','m');
        hold on
end
```

```
[fsurv,x,flo,fup] =
ecdf(ClusterDur,'censoring',Censor,'function','survivor');
stairs(x,fsurv,'LineWidth',2,'color','k');
hold on

switch nk
    case 3
        legend('Cluster 1','Cluster 2','Cluster 3','All
Data','Population',...
       'Location','NE')
    case 4
        legend('Cluster 1','Cluster 2','Cluster 3','Cluster 4','All
Data','Population',...
       'Location','NE')
    case 5
        legend('Cluster 1','Cluster 2','Cluster 3','Cluster 4','Cluster
5','All Data','Population',...
       'Location','NE')
end

title('Kaplan-Meier Cumulative Survival Distribution');
xlabel('Duration')
ylabel('Probability');

pause

hold off

x1(1,1) = 0;
x2(1,1) = 0;
x3(1,1) = 0;
x4(1,1) = 0;
x5(1,1) = 0;

ilength = length(fsurv1);
fsurv1(1,2) = 1;
for i = 2:ilength;
    fsurv1(i,2) = fsurv1(i,1)/fsurv1(i-1,1);
end

ilength = length(fsurv2);
fsurv2(1,2) = 1;
for i = 2:ilength;
    fsurv2(i,2) = fsurv2(i,1)/fsurv2(i-1,1);
end

ilength = length(fsurv3);
fsurv3(1,2) = 1;
for i = 2:ilength;
    fsurv3(i,2) = fsurv3(i,1)/fsurv3(i-1,1);
end

if nk == 4
```

```
    ilength = length(fsurv4);
    fsurv4(1,2) = 1;
    for i = 2:ilength;
        fsurv4(i,2) = fsurv4(i,1)/fsurv4(i-1,1);
    end
end

if nk == 5
    ilength = length(fsurv4);
    fsurv4(1,2) = 1;
    for i = 2:ilength;
        fsurv4(i,2) = fsurv4(i,1)/fsurv4(i-1,1);
    end
    ilength = length(fsurv5);
    fsurv5(1,2) = 1;
    for i = 2:ilength;
        fsurv5(i,2) = fsurv5(i,1)/fsurv5(i-1,1);
    end
end

ilength = length(fsurv);
fsurv(1,2) = 1;
for i = 2:ilength;
  fsurv(i,2) = fsurv(i,1)/fsurv(i-1,1);
end

x(1,1) = 0;
x1(1,1) = 0;
x2(1,1) = 0;
x3(1,1) = 0;
x4(1,1) = 0;
x5(1,1) = 0;

% Check for last Prob = 0 in Cluster 1

fsurv1min = min(fsurv1(:,1))

if fsurv1min > 0
   p = polyfit(x1(:,1),fsurv1(:,1),6)
   y = polyval(p,x1(:,1))
   hold off
   plot(x1(:,1),fsurv1(:,1))
   hold on
   plot(x1(:,1),y)
   [h,prob,k] = kstest2(x1(:,1),y,0.05)

   xlength = length(x1)
   xval = max(x1)

    for i = xval + 1 : xval + 50
        newx = i
        newy = polyval(p,newx)
        if newy < 0
            newx = i - 1
```

```
            break
        end
    end
    if newx == xval
        x1(xlength+1,1) = newx + 1
        fsurv1(xlength+1,1:2) = 0
    else
        x1(xlength+1,1) = newx
        fsurv1(xlength+1,1:2) = 0
    end
end

% Check for last Prob = 0 in Cluster 2

fsurv2min = min(fsurv2(:,1))

if fsurv2min > 0
   p = polyfit(x2(:,1),fsurv2(:,1),6)
   y = polyval(p,x2(:,1))
   hold off
   plot(x2(:,1),fsurv2(:,1))
   hold on
   plot(x2(:,1),y)
   [h,prob,k] = kstest2(x2(:,1),y,0.05)

   xlength = length(x2)
   xval = max(x2)

    for i = xval + 1 : xval + 50
        newx = i
        newy = polyval(p,newx)
        if newy < 0
            newx = i - 1
            break
        end
    end
    if newx == xval
        x2(xlength+1,1) = newx + 1
        fsurv2(xlength+1,1:2) = 0
    else
        x2(xlength+1,1) = newx
        fsurv2(xlength+1,1:2) = 0
    end
end

% Check for last Prob = 0 in Cluster 3


fsurv3min = min(fsurv3(:,1))

if fsurv3min > 0
   p = polyfit(x3(:,1),fsurv3(:,1),6)
   y = polyval(p,x3(:,1))
   hold off
```

```
   plot(x3(:,1),fsurv3(:,1))
   hold on
   plot(x3(:,1),y)
   [h,prob,k] = kstest2(x3(:,1),y,0.05)

   xlength = length(x3)
   xval = max(x3)

    for i = xval + 1 : xval + 50
        newx = i
        newy = polyval(p,newx)
        if newy < 0
            newx = i - 1
            break
        end
    end
    if newx == xval
        x3(xlength+1,1) = newx + 1
        fsurv3(xlength+1,1:2) = 0
    else
        x3(xlength+1,1) = newx
        fsurv3(xlength+1,1:2) = 0
    end
end

% Check for last Prob = 0 in Cluster 4

if nk >= 4

fsurv4min = min(fsurv4(:,1))

if fsurv4min > 0
   p = polyfit(x4(:,1),fsurv4(:,1),6)
   y = polyval(p,x4(:,1))
   hold off
   plot(x4(:,1),fsurv4(:,1))
   hold on
   plot(x4(:,1),y)
   [h,prob,k] = kstest2(x4(:,1),y,0.05)

   xlength = length(x4)
   xval = max(x4)

    for i = xval + 1 : xval + 50
        newx = i
        newy = polyval(p,newx)
        if newy < 0
            newx = i - 1
            break
        end
    end
    if newx == xval
        x4(xlength+1,1) = newx + 1
        fsurv4(xlength+1,1:2) = 0
```

```
    else
        x4(xlength+1,1) = newx
        fsurv4(xlength+1,1:2) = 0
    end
end
end

% Check for last Prob = 0 in Cluster 5

if nk >= 5

fsurv5min = min(fsurv5(:,1))

if fsurv5min > 0
   p = polyfit(x5(:,1),fsurv5(:,1),6)
   y = polyval(p,x5(:,1))
   hold off
   plot(x5(:,1),fsurv5(:,1))
   hold on
   plot(x5(:,1),y)
   [h,prob,k] = kstest2(x5(:,1),y,0.05)

   xlength = length(x5)
   xval = max(x5)

    for i = xval + 1 : xval + 50
        newx = i
        newy = polyval(p,newx)
        if newy < 0
            newx = i - 1
            break
        end
    end
    if newx == xval
        x5(xlength+1,1) = newx + 1
        fsurv5(xlength+1,1:2) = 0
    else
        x5(xlength+1,1) = newx
        fsurv5(xlength+1,1:2) = 0
    end
end

end

switch nk
    case 3
        save
('c:\users\bruce\dissertation\matlabfiles\KaplanMeierOutputData','CurrEmp',
'X', 'idx',...
        'x1','x2','x3','fsurv1','fsurv2','fsurv3','nk');

    case 4
```

```
        save
('c:\users\bruce\dissertation\matlabfiles\KaplanMeierOutputData','CurrEmp',
'X', 'idx',...
        'x1','x2','x3','x4','fsurv1','fsurv2','fsurv3','fsurv4','nk');

    case 5
        save
('c:\users\bruce\dissertation\matlabfiles\KaplanMeierOutputData','CurrEmp',
'X', 'idx',...

'x1','x2','x3','x4','x5','fsurv1','fsurv2','fsurv3','fsurv4','fsurv5','nk');

end


return
```

Appendix B-3

MATLAB Source Code: fnl_Age_Employees.m

```
clc;
clear;

global NewHireDist
global CurrEmp

double newage;
load 'c:\users\bruce\dissertation\matlabfiles\KaplanMeierOutputData';

ilength = length(CurrEmp)

for i = 1 : ilength
    CurrEmp(i,5) = CurrEmp(i,1);
    CurrEmp(i,6) = CurrEmp(i,2);
    CurrEmp(i,7) = CurrEmp(i,4);
    CurrEmp(i,8) = 1;
end

    for i = 1 : ilength;
    for j = 1 : 25;

        CurrEmp(i,5) = CurrEmp(i,5) + 1;
        CurrEmp(i,6) = CurrEmp(i,6) + 1;
        sample = horzcat(CurrEmp(i,5),CurrEmp(i,6),1);

        a = CurrEmp(i,5);
        b = CurrEmp(i,6);
        sample;

        knncluster = knnclassify(sample,X,idx,5,'cosine');

        if double(knncluster) ~= CurrEmp(i,7)
            'Change in Cluster';
            age = CurrEmp(i,5);
            ycs = CurrEmp(i,6);
            oldcluster = CurrEmp(i,7);
            newcluster = knncluster;
            CurrEmp(i,7) = double(knncluster);
        end

        switch CurrEmp(i,7);

                case 1;
                    pct = prd_vLookup(CurrEmp(i,6), x1, fsurv1);
                    CurrEmp(i,8) = CurrEmp(i,8) * pct;
                    CurrEmp(i,8+j) = CurrEmp(i,8);
                case 2;
```

```
                    pct = prd_vLookup(CurrEmp(i,6), x2, fsurv2);
                    CurrEmp(i,8) = CurrEmp(i,8) * pct;
                    CurrEmp(i,8+j) = CurrEmp(i,8);
                case 3;

                    pct = prd_vLookup(CurrEmp(i,6), x3, fsurv3);
                    CurrEmp(i,8) = CurrEmp(i,8) * pct;
                    CurrEmp(i,8+j) = CurrEmp(i,8);
                case 4;

                    pct = prd_vLookup(CurrEmp(i,6), x4, fsurv4);
                    CurrEmp(i,8) = CurrEmp(i,8) * pct;
                    CurrEmp(i,8+j) = CurrEmp(i,8);
                case 5;

                    pct = prd_vLookup(CurrEmp(i,6), x5, fsurv5);
                    CurrEmp(i,8) = CurrEmp(i,8) * pct;
                    CurrEmp(i,8+j) = CurrEmp(i,8);

        end;
    end;
    end

    fnl_CurrEmpDist;
    fnl_NewHireDist;

    save
('c:\users\bruce\dissertation\matlabfiles\AgeEmployeesOutputData','CurrEmp',
'CurrEmpDist', 'NewHireDist');

return;
```

Appendix B-4

MATLAB Source Code: fnl_CurrEmpDist.m

```
ilength = length(CurrEmp);
CurrEmpDist(1:27) = 0;
CurrEmpDist(:,1) = ilength;
icol = 1;
for i = 9 : 33
    icol = icol + 1;
    for j = 1 : ilength
        CurrEmpDist(1,icol) = CurrEmpDist(1,icol) + CurrEmp(j,i);
    end
    CurrEmpDist(1,icol) = CurrEmpDist(1,icol) / ilength;
end

CurrEmpDist (1,1) = 1;




%-------------------- Create and Save CurrEmp Distribution Charts

hold on

% Bar Plot for New Hire Distribution

yvector1 = CurrEmpDist;

% Create figure
figure1 = figure;

% Create axes
axes1 = axes('Parent',figure1);
box(axes1,'on');
hold(axes1,'all');

% Create bar
file_handle = bar(yvector1,'DisplayName','CurrEmpDist(1,1:27)');

% Create xlabel
xlabel({'Years of Company Service'});

% Create ylabel
ylabel({'Proportion Remaining'});

% Create title
title({'Survival Distribution for Current Employees'});
```

```
saveas
(file_handle,'c:\users\bruce\dissertation\figures\CurrEmpDist.jpg','jpg')


nhlength = length(CurrEmpDist);
for i = 2 : nhlength - 1
    CurrEmpPDF(1,i-1) =  CurrEmpDist(1,i) / CurrEmpDist(1,i-1);
    CurrEmpAttRate(1,i-1) = 1 - CurrEmpPDF(1,i-1);
end


% New Hire PDF plot

% Create figure
figure1 = figure;

% Create axes
axes1 = axes('Parent',figure1);
box(axes1,'on');
hold(axes1,'all');

% Create plot
file_handle = bar(CurrEmpPDF,'DisplayName','CurrEmpPDF(1,1:25)');

% Create xlabel
xlabel({'Year of Company Service'});

% Create ylabel
ylabel({'Proportion Surviving'});

% Create title
title({'Current Employee Probability Density Function'});
saveas
(file_handle,'c:\users\bruce\dissertation\figures\CurrEmpPDF.jpg','jpg')

% Bew Hire Attrition Rate Bar Graph

yvector1 = CurrEmpAttRate;

% Create figure
figure1 = figure;

% Create axes
axes1 = axes('Parent',figure1);
box(axes1,'on');
hold(axes1,'all');

% Create bar
file_handle = bar(yvector1,'DisplayName','CurrEmpAttRate(1,1:25)');

% Create xlabel
xlabel({'Years of Company Service'});
```

```
% Create ylabel
ylabel({'Attrition Rate'});

% Create title
title({'Current Employee Attrition Rates'});

saveas
(file_handle,'c:\users\bruce\dissertation\figures\CurrEmpAttRate.jpg','jpg')

return
```

Appendix B-5

MATLAB Source Code: FNL_NewHireDist.m

```
global NewHireDist
global CurrEmp

newhireidx = find(CurrEmp(:,2)<=5);
newhires = CurrEmp(newhireidx,:);
ilength = length(newhires);
NewHireDist(1:27) = 0;
NewHireDist(:,1) = ilength;
icol = 1;
for i = 9 : 33
    icol = icol + 1;
    for j = 1 : ilength
        NewHireDist(1,icol) = NewHireDist(1,icol) + newhires(j,i);
    end
    NewHireDist(1,icol) = NewHireDist(1,icol) / ilength;
end

NewHireDist(1,1) = 1;


hold on

% Bar Plot for New Hire Distribution

yvector1 = NewHireDist;

% Create figure
figure1 = figure;

% Create axes
axes1 = axes('Parent',figure1);
box(axes1,'on');
hold(axes1,'all');

% Create bar
file_handle = bar(yvector1,'DisplayName','NewHireDist(1,1:27)');

% Create xlabel
xlabel({'Years of Company Service'});

% Create ylabel
ylabel({'Proportion Remaining'});

% Create title
title({'Survival Distribution for New Hires'});
```

```
saveas
(file_handle,'c:\users\bruce\dissertation\figures\NewHireDist.jpg','jpg')

nhlength = length(NewHireDist);
for i = 2 : nhlength - 1
    NewHirePDF(1,i-1) =  NewHireDist(1,i) / NewHireDist(1,i-1);
    NewHireAttRate(1,i-1) = 1 - NewHirePDF(1,i-1);
end


% New Hire PDF plot

% Create figure
figure1 = figure;

% Create axes
axes1 = axes('Parent',figure1);
box(axes1,'on');
hold(axes1,'all');

% Create plot
file_handle = bar(NewHirePDF,'DisplayName','NewHirePDF(1,1:25)');

% Create xlabel
xlabel({'Year of Company Service'});

% Create ylabel
ylabel({'Proportion Surviving'});

% Create title
title({'New Hire Probability Density Function'});
saveas
(file_handle,'c:\users\bruce\dissertation\figures\NewHirePDF.jpg','jpg')

% Bew Hire Attrition Rate Bar Graph

yvector1 = NewHireAttRate;

% Create figure
figure1 = figure;

% Create axes
axes1 = axes('Parent',figure1);
box(axes1,'on');
hold(axes1,'all');

% Create bar
file_handle = bar(yvector1,'DisplayName','NewHireAttRate(1,1:25)');

% Create xlabel
xlabel({'Years of Company Service'});
```

```
% Create ylabel
ylabel({'Attrition Rate'});

% Create title
title({'New Hire Attrition Rates'});



saveas
(file_handle,'c:\users\bruce\dissertation\figures\NewHireAttRate.jpg','jpg')

return
```

Appendix B-6

MATLB Source Code: fnl_MainMinCost.m

```
clear all;

clc();

format compact
format long

global idx;
global X;
global x1  x2  x3 x4
global fsurv1 fsurv2 fsurv3 fsurv4 fsurv5
global OrigCurrEmpLength
global hiringdata
global CycleDist
global NewHireDist
global npp;
global idx1;
global nextcycle;
global cycledetailcnt;
global cycledetail;
global npp;
global idx1;
global nextcycle;
global cycledetailcnt;
global cycledetail;
global results;

CycleDist(1:5,1:27) = 0;


fid = fopen('c:\users\bruce\documents\matlab\bahtest.txt','wt+');

Init_MainMinCostModel;


Init_MainMinCostCycle;


%----- Load Data from other Routines

load 'c:\users\bruce\dissertation\matlabfiles\KaplanMeierOutputData'
%,'CurrEmp', 'X',
'idx','x1','x2','x3','x4','x5','fsurv1','fsurv2','fsurv3','fsurv4','fsurv5','
nk');
%save ('c:\users\bruce\dissertation\matlabfiles\AgeEmployees','CurrEmp');
```

```
load 'c:\users\bruce\dissertation\matlabfiles\AgeEmployeesOutputData' %This
must be loaded after KaplanMeierOutputData
%load 'c:\users\bruce\dissertation\matlabfiles\HiringData' % This is the CDF
fo the Hiring Data
%save ('c:\users\bruce\dissertation\matlabfiles\NewHireDist','NewHireDist');

load 'c:\users\bruce\dissertation\matlabfiles\NewHireDist'
%save ('c:\users\bruce\dissertation\matlabfiles\CurrEmpDist','CurrEmpDist');
load 'c:\users\bruce\dissertation\matlabfiles\CurrEmpDist'
OrigCurrEmpLength = length(CurrEmp);

%----- Set Initial Values for variables

icycle = 0;

end_inv = 0;

demand0(1:50) =8000;

demand = demand0;

total_cost = double(0);
total_inv_cost = double(0);
total_shortage_cost = double(0);
total_hold_cost = double(0);
cycle_cost = 0;
cycle_avg(1:50) = 0;
npp = 0;
nextcycle = 0;

idx1 = 0;

fprintf(fid, 'demand(1) = %f \r\n', demand(1));

start_period = 1;
last_period = 25;

avg_cycle_cost = 0;
cycle_cost = 0;
total_cost = 0;

cycle_avg = 0;

setup_cost_needed = 1;

avg_cycle_cost = 0;

icycle = 1;

end_inv_adj(1:50) = 0;
```

```
start_period = 1;
last_period = 25;
keep_start_period = start_period;
keep_last_period = last_period;

cycledetailcnt = 0;
cycledetail(100,3) = 0;

xcycle_length = 3;
lead_time = xcycle_length - 1;
ROP = 1.0;


format compact
format long

number_trials = 0;

fprintf(fid, 'In Initialize Cycle \r\n');
end_inv = 0;

unit_labor_cost = 41.6;
unit_excess_cost = 62.4;
unit_shortage_cost = 83.2;
procure_cost = 104;

max_periods = 25;

end_inv_adj(1:50) = 0;

CycleDist(10,:) = 0;


'*******************************************************'
'****************  Cycle 1  ***************************'
'*******************************************************'

for i = 1:10
    results(i,1) = i;
    for j = 2:10
        results(i,j) = 99999999999;
    end
end

CycleDist(1,1:27) = CurrEmpDist (1:27) * length(CurrEmp) * 1750/2080;


%----- Search Options
%'active-set', 'trust-region-reflective', 'interior-point', 'levenberg-
marquardt', 'trust-region-dogleg', 'lm-line-search', or 'sqp'.
%FMINCON. Choose 'sqp', 'interior-point', 'trust-region-reflective', or
'active-set'.
```

```
max_cycles = 1;

demand1 = demand0;

x0 = [ demand(1) mean(demand) mean(demand)];

lb = [demand(1) 1 1 ];

ub = [sum(demand) sum(demand) sum(demand)];

demand1 = demand0;
demand = demand1;

x0 = [demand(1)];
lb = [demand(1)];
ub = [sum(demand)];

nextcycle = 1;

z = fnl_OptMainMinCost(length(CurrEmp) * 1750/2080)

cycle_cost;

options = optimset('Display','final','Algorithm','sqp', 'MaxFunEvals',...
    200000, 'TolFun', 1.000000e-10, 'TolCon', 1.000000e-5, 'TolX', 1.0e-10,
'MaxIter', 100000);

problem = createOptimProblem('fmincon', 'objective',@fnl_OptMainMinCost,
'x0',x0,'lb',lb,'ub',ub,'options',options);


'*******************************************************'
'****************  Cycle 2  ***************************'
'*******************************************************'

for inextcycle = 2 : 6;

nextcycle = inextcycle
idx1 = find(results(:,1)==1)
start_period = results(idx1,4) + 1
newlength = length(demand1(start_period:50))
demand(1:newlength) = demand1(start_period:50);
demand2 = demand;

cyclelb = 0;
for i = 1 : nextcycle-1
    cyclelb = cyclelb + CycleDist(i,start_period)
end

xub = sum(demand(start_period:50))

x0 = [ 1 ]
```

```
lb = [ 1 ]
ub = [ sum(demand) ]

options = psoptimset('Display','iter','MaxFunEvals',...
    100000, 'TolFun', 1.0e-10, 'TolCon', 1.0e-5, 'TolX', 1.0e-10, 'MaxIter',
100000, 'TolMesh', 1e-10);

options = optimset('Display','final','Algorithm','sqp', 'MaxFunEvals',...
    200000, 'TolFun', 1.000000e-10, 'TolCon', 1.000000e-5, 'TolX', 1.0e-10,
'MaxIter', 100000);

%----- Solver Options

[z, fval, exitflag, output,solutions] =
fmincon(@fnl_OptMainMinCost,x0,[],[],[],[],lb,ub,[],options); % This one
works

%[z, fval, exitflag, output] = fminunc(@fnl_OptMainMinCost,x0,options);

%problem = createOptimProblem('fmincon', 'objective',@fnl_OptMainMinCost,
'x0',x0,'lb',lb,'ub',ub,'options',options);
%ms = MultiStart;
%[z,fval,exitflag,output,allmins] = run(ms, problem, 100);

%[z, fval, exitflag, output] =
Patternsearch(@fnl_OptMainMinCost,x0,[],[],[],[],lb,ub,[],options); % This
one works

%gs = GlobalSearch;
%[z,fval,exitflag,output,allmins] = run(gs, problem);

%[z, fval, exitflag, output] =
Simulannealbnd(@fnl_OptMainMinCost,x0,lb,ub,options);

findstartpos = find(CycleDist(nextcycle,:) > 0);
startpos = min(findstartpos);

end

return
```

Appendix B-7

MATLAB Source Code: fnl_optMainMinCost.m

```
function avg_cycle_cost = fnl_OptMainMinCost(arg)

a = '-----------------------------Entering Optimization function';

SetMainMinCostGlobalVariables;


Init_MainMinCostCycle;


global results;
global nextcycle;
global idx1;
global npp;
global OrigCurrEmpLength
global NewHireDist
global CycleDist
global cycledetailcnt;
global cycledetail;

format compact
format long

cycle_avg(1:50) = 0;

fprintf(fid, 'demand(1) = %f \r\n', demand(1));

start_period = 1;
last_period = 25;

avg_cycle_cost = 0;
cycle_cost = 0;
total_cost = 0;

cycle_avg = 0;

setup_cost_needed = 1;

avg_cycle_cost = 0;

icycle = 1;

end_inv_adj(1:50) = 0;

if nextcycle == 1
    npp = 0;
```

```
else
    npp = sum(results(1:nextcycle-1,4));
end

%CurrEmp(OrigCurrEmpLength+1:length(CurrEmp),:) = [];


while 1
    a='################################# Beginning of Cycle
################################';

    a = arg;

    nextcycle;

    icycle;
    a = arg(icycle);
    end_inv_adj (icycle);

    if nextcycle > 1
    %if icycle > 1
        a='-------------------------------------------Beg of cycledist ';
        'In New Cycle - from ARG';
        nextcycle;


        a='arg(icycle)';
        arg(icycle);
        a='end_inv_adj(icycle-1)';


        icol = 0;
        nextperiod = 0;
        for i = 1 : nextcycle-1;
            nextperiod = nextperiod + results(i,4);
        end
        nextperiod = nextperiod + 1;
        nextdistperiod = nextperiod;
        for i = nextperiod : 27
            icol = icol + 1;
            NewHireDist(1,icol);
            argicycle = arg(icycle);
            a=results(nextcycle-1,5);
            %diff = arg(icycle) - results(nextcycle-1,5);
            CycleDist(nextcycle,i) = NewHireDist(1,icol) * arg(icycle);
            a=CycleDist(nextcycle,i);
        end
        beg_inv = 0;

        a='NEW CYCLE';

         beg_inv = 0;
        optarg = arg(icycle);
```

```
        nextcycle;
        nextperiod;
        for i = 1 : nextcycle;
            cycledistvalue = CycleDist(i,nextperiod);
            beg_inv = beg_inv + CycleDist(i,nextperiod);

        end

        beg_inv;

        a = 'end of cycledist calc';



    else

        beg_inv = CycleDist(1,1);
        nextdistperiod = 1;

    end

   cycle_cost = 0;

        start_period;
        last_period;
        max_periods;

for iperiod = start_period : last_period


    a='---------------------------------- Beginning of period';
    a='Period';
    iperiod;
    a='Start Period';
    start_period;
    a='Last Period';
    last_period;
    a='Demand';
    demand(iperiod);
    a='Beg Inv';
    beg_inv;

    shortage_units = 0;
    inv_shortage_cost = 0;
    excess_units = 0;
    inv_excess_cost = 0;

    npp;

    if beg_inv >= demand(iperiod)
        inv_labor_cost = unit_labor_cost * demand(iperiod);
    else
```

```
    inv_labor_cost = unit_labor_cost * beg_inv;
end

inv_labor_cost;


if beg_inv == demand(iperiod)
  inv_excess_cost = 0;
  inv_shortage_cost = 0;
end

if beg_inv > demand(iperiod)
    excess_units = beg_inv - demand(iperiod);
    inv_excess_cost = unit_excess_cost * excess_units;
end

inv_excess_cost;

if beg_inv < demand(iperiod)
    shortage_units = demand(iperiod) - beg_inv;
    inv_shortage_cost = unit_shortage_cost * shortage_units;
end

inv_shortage_cost;

inv_labor_cost;
inv_excess_cost;
inv_shortage_cost;

period_cost = inv_labor_cost + inv_excess_cost + inv_shortage_cost;

a=' Calcualtinb BEG and Inv Inv';

nextdistperiod = nextdistperiod + 1;
if nextdistperiod > 27
    nextdistperiod = 27;
end
a='Calculating next period beginning inv';
iperiod;
beg_inv = 0;
    for sumidx = 1 : nextcycle;

    sumidx;
    CycleDist(sumidx,nextdistperiod);
    beg_inv = beg_inv + CycleDist(sumidx,nextdistperiod);
    end;

end_inv = beg_inv;

if iperiod < max_periods
    xinv_pct = end_inv / demand(iperiod+1);
else
    xinv_pct = 1.0;
```

```
    end

    end_inv;
    demand(iperiod+1);

    a='Xinv_pct';
    xinv_pct;
    a='ROP ';
    ROP;


    if xinv_pct < ROP
        a='Inv Pct below ROP';
        if setup_cost_needed
            a = 'SETUP_COST_NEEDED';
                period_cost = period_cost + procure_cost;
                setup_cost_needed = 0;
                last_period = iperiod + lead_time - 1;
                if last_period > max_periods
                    last_period = max_periods;
                end
        end
    end

    period_cost;
    cycle_cost = cycle_cost + period_cost;


    '------------------------ END OF PERIOD ----------------';

    if iperiod >= last_period
        break
    end


end
a=' ------------------------------------------- Cycle Complete -------------
----';

a='Cycle';
icycle;

idx1 = find(results(:,1) == nextcycle );

idx1 = nextcycle;

a='arg cycle = ';
arg(icycle);
a='cycle Cost';
cycle_cost;


if cycle_cost < results(idx1,3)
```

```
results(idx1,2) = arg(icycle);
v_arg = arg(icycle);

results(idx1,3) = cycle_cost;
cycle_cost;

results(idx1,4) = last_period;
last_period;

results(idx1,5) = end_inv;
end_inv ;



end

a='Cycle';
icycle;

last_period;
start_period;

a='Cycle Length';

last_period - start_period + 1;

arg;

    cycle_avg(icycle) = cycle_cost / (last_period - start_period + 1);

cycle_cost;

a='Cycle Avg';
cycle_avg(icycle);

%avg_cycle_cost = avg_cycle_cost + cycle_avg(icycle); %this is the correct
minimization value

avg_cycle_cost = avg_cycle_cost + cycle_cost;
cycle_cost;
avg_cycle_cost = cycle_cost;

%cycledetailcnt = cycledetailcnt + 1;
%cycledetail(cycledetailcnt,1) = nextcycle;
%cycledetail(cycledetailcnt,2) = arg(icycle);
%cycledetail(cycledetailcnt,3) = cycle_cost;

setup_cost_needed = 1;

start_period = last_period + 1;
```

```
last_period = max_periods;

end_inv_adj (icycle) = end_inv;

a='Cycles Processed';
icycle;

icycle = icycle + 1;
max_cycles;

%if start_period > max_periods
%       a='Exiting due to max periods exceeded'
%        start_period
%       last_period
%        break
%end

a ='$$$$$$$$$$$$$$$$$$$$$$$$$$ End of Cycle $$$$$$$$$$$$$$$$$$$';



if start_period > max_periods
    break
end

if icycle > max_cycles
    end_inv = 0;
    break
end


end




return
```

Vita

Bruce Holt is a native of Lynchburg, Virginia and has been a resident of Clinton, Tennessee since October 2000. Bruce is married and has one daughter who is currently working on a Masters degree at Liberty University in Lynchburg, Va.

Bruce's academic achievements include a Bachelor of Science Degree in Math/Computer Science from Hampden-Sydney College ('75) in Hampden-Sydney, Virginia, a Master of Science Degree from Virginia Tech ('77) in Blacksburg, Virginia, a Master of Administration Degree in Industrial Administration from Lynchburg College ('86) in Lynchburg, Virginia and a Master of Science degree in Industrial and Systems Engineering from Virginia Tech ('01). He is currently fulfilling the requirements for a Doctor of Philosophy Degree in Industrial Engineering at the University of Tennessee in Knoxville, Tennessee and is expected to complete the Ph.D. degree in Spring 2011.

During Bruce's professional career, he has held positions as a Computer Programmer and Systems Analyst, Statistician, Software Quality Assurance Manager, Software Development manager, Project Control Systems manager, Production Control manager and Integrated Planning manager.

Bruce's professional career has enabled him to participate in efforts of significant national interest. These efforts include the Naval Nuclear Reactors program, the Superconducting Super Collider Accelerator Project, the Tokamak Physics Experiment (TPX) project and is currently an employee at the Y12 National Security Complex.