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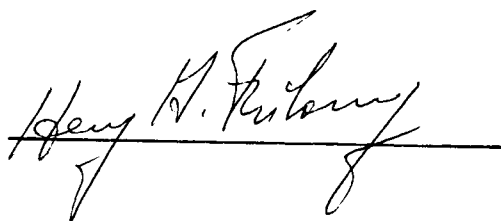


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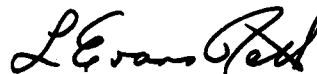


Charles E. Clark, Major Professor

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AN ORDINARY DIFFERENTIAL EQUATIONS MODEL
DESCRIBING THE DRYING RATE OF HAY

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Christie Grigsby

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ABSTRACT

An ordinary differential equations model of the form $\frac{dM(t)}{dt} = k(M(t) - M_e(t))$ is developed as a means to determine the moisture content of hay t hours after it has been cut. The drying constant k is determined with the aid of data supplied by the Plant and Soil Science Department at the University of Tennessee, Knoxville. The model is then used to determine a dew deposition parameter δ . This parameter estimates the amount of additional moisture in the hay each morning attributed to dew. The model is then used to estimate the moisture content of hay over a period of several days.

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CHAPTER I

INTRODUCTION

A vital process to many farmers is the drying of field forages to a moisture content that is safe for baling. If hay is baled before it has dried sufficiently and then stored, there is a danger of spontaneous combustion within the material which would lead to disaster. After the hay is baled, it undergoes a curing process called sweating which increases its temperature. This temperature increase is caused from heat produced by plant tissue respiration and bacterial activity. If the hay is below 28 to 30 percent moisture when baled, the hay will cool after the sweating stage. But if the moisture content is above 30 percent, the increased respiration and bacterial growth causes the production of so much heat that it cannot be dissipated from the hay by conduction. The temperature will then increase depending upon the amount of heat produced and the rate of heat loss. Spontaneous combustion can occur when a small amount of wet hay is surrounded by dry hay [2, pp. 294 - 295]. Even if combustion does not occur, overheating the hay causes a detrimental effect on the protein content such that it is not digestible by animals [8]. Thus hay must be dried until a moisture content of approximately 20 percent is attained to be considered safe for baling and storage.

The time required to dry hay is judged intuitively by most farmers using weather forecasts as a guideline. Many farmers delay the cutting of hay if the weather forecasts predict rain

within three to four days since this is the average time that it takes hay to dry adequately in Tennessee. During this three- to four-day period, the drying quality of the weather could be sufficient to ensure adequate drying of hay. This would make it possible for the farmer to cut the hay and dry it at the peak of its nutritional value, instead of delaying the cut until a better drying forecast is given. Delaying the cut would generally result in hay that is greater in fiber content and losses in nutritional value, and as such, the hay has a lower feed value.

The purpose of this work is to provide a means by which the moisture content of hay can be predicted when given a forecast of certain meteorological factors over a period of several days. The specific climatic conditions used in this study are temperature and relative humidity. The crop used in the experiments that determined the moisture content of hay was a forage sorghum (Sorghum bicolor L.)

CHAPTER II

DESCRIPTION OF EXPERIMENTS

'Superchowmaker 235', a hybrid forage sorghum, grown on the University of Tennessee Plant and Soil Science Farm in Knoxville, Tennessee during the summer of 1978 was produced and cut for the purpose of determining the moisture content of the hay during several stages of the drying process. Forty-five rectangular plots were planted, then cut nine at a time, making it possible to dry the hay at different stages of growth.

The sorghum was machine cut and conditioned at the beginning of each week, i.e., it was cut no later than Tuesday, in order to allow enough drying days before the weekend arrived. A hay that is conditioned at the time of cutting is crushed by the machine in order to squeeze some of the moisture from the plant and break the stems so it will dry faster.

The day after the sorghum was cut, it was raked into windrows to expose the bottom layer of material to sunlight. The sorghum was sampled at the time of cutting and twice daily after windrowing. The samples were taken at approximately 8:30 a.m. and at about 3:30 p.m.. To determine the percentage of moisture in the hay, a sample of the hay was taken from the windrow and placed inside a pre-weighed bag. The weight of the bag and the sample were then recorded, then the sample was put in an oven and dried until about 4 percent moisture content. The sample was weighed again after it was dried. This information makes it possible to determine the percentage of moisture in the sample.

There are two methods of describing moisture content as a percent, wet basis and dry basis. The moisture content on a wet basis is obtained by dividing the weight of the water present in the material by the total weight of the material. The moisture percentage on a dry basis is determined by dividing the weight of the water in the material by the weight of the dry material.

The moisture percentages in this experiment were determined as wet basis percents. These percentages were found using the following formula:

$$M = \frac{W_T - W_D - W_B}{W_T - W_B}$$

M = Moisture percent, wet basis, decimal

W_T = Total weight of sample

W_B = Weight of bag

W_D = Weight of dried sample

The sampling procedure described previously made it possible to obtain moisture percentages at several intervals during the drying process.

Meteorological data were also gathered during this experiment. The relative humidity of the air around the hay was determined using a psychrometer. The quantity $rh = p/p_s$ is defined to be the relative humidity of the air. The parameter p_s is the saturated vapor pressure of the air at a specific temperature and p is the vapor pressure of the moisture in the air. A psychrometer consists

of two thermometers placed in the air side by side. One of the thermometer is a wet bulb thermometer. The bulb of this thermometer is covered with a thin wet cloth. The wet bulb is cooled by evaporation of water from the cloth until a steady state is reached. This occurs when evaporation is at a constant rate, giving a constant "wet bulb" temperature. The other thermometer is a dry bulb thermometer, which is not affected by the water vapor content of the air. These two temperature values are used along with a psychrometric chart to determine the relative humidity [7].

Various temperature measurements were recorded during this experiment. Sensors were placed in the soil, inside the windrow and under the windrow. These temperature readings were recorded continuously throughout the day and night.

A hygrothermograph was used to record temperature and relative humidity values of the air throughout the day and night.

The Duvdevani method was used to estimate the amount of dew that accumulated during the night [1]. These estimates for dew were found by placing wooden blocks outside in the late afternoon. During the night dew formed on these blocks. The following morning, the amount of water droplets formed on these blocks was compared to photographs of water droplets that represent different amounts of moisture deposition. These photographs were classified according to the amount of moisture as follows: 1a, 1b, 2a, 2b, ..., 6a, 6b, 7a, 7b, rain. These classifications start with very light dew and gradually increase until heavy dew, then rain. A reading of 1a or 1b would represent light dew. A reading of 4a or 4b would represent medium dew and 7a or 7b would represent heavy dew.

CHAPTER III

THEORY OF DRYING

The moisture content of hay is usually expressed as a percent by weight on wet basis. The substance that is to be dried contains absorbed moisture. This moisture exerts a moisture vapor pressure which varies with the moisture content of the material and also from material to material. The ratio of the moisture vapor pressure exerted by the absorbed water of the material to the saturated vapor pressure of pure water at the temperature of the material is called the equilibrium relative humidity. A graph of the equilibrium relative humidity and the moisture content of the material expresses the equilibrium moisture curve [2, p. 18].

Curves for equilibrium moisture data found experimentally have been described by the following relationship [4, p. 278]:

$$1 - rh = \exp(-CTMe^n) \quad (1)$$

rh = Equilibrium relative humidity, decimal

Me = Equilibrium moisture content, dry basis, percent

C, n = Constants varying with the material

T = Temperature, absolute degrees.

The equilibrium moisture properties are important, as they have a direct relationship to the storage and drying of farm products. The moisture content of hay approaches a value determined by the relative humidity and temperature of the air surrounding the material.

If the material has a moisture content greater than the equilibrium moisture content for the prevailing temperature and relative humidity of the surrounding air, the moisture content of the material will decrease. If it has a moisture content less than the equilibrium moisture content for the prevailing temperature and relative humidity of the surrounding air the moisture content of the material will increase. In both cases, the moisture content of the material would approach the equilibrium moisture content corresponding to the particular temperature and relative humidity of the surrounding air.

The equation (1), which expresses the mathematical relationship between temperature, relative humidity, and equilibrium moisture content has not been established analytically.

Henderson [3] attempted to derive equation (1) thermodynamically but was unsuccessful. The derivation contains an algebraic mistake which would cause the temperature factor to be in the denominator of the equation. The equation would be

$$1 - rh = \exp\left(\frac{CMe^n}{T}\right) \quad (1')$$

if the mistake were to be corrected. The relationship has been shown to be valid empirically under controlled conditions. The constants C and n have been determined experimentally. For sorghum, $C = 3.40 \times 10^{-6}$, $n = 2.31$ [2, p. 19].

The equation satisfies the following characteristics of equilibrium moisture curves found experimentally:

- 1) The equilibrium relative humidity approaches 100 percent as the moisture content becomes infinite.
- 2) The equilibrium relative humidity is zero at zero moisture content.
- 3) The curve slope approaches infinity as the moisture content approaches zero or infinity [1, p. 19].

This equation is valuable as it enables the equilibrium moisture content to be calculated at several intervals throughout the day. The temperature and relative humidity change constantly throughout the day, thus the equilibrium moisture content of the hay changes also.

The drying process can be divided into two stages: the constant rate period and the falling rate period [3, pp. 279 - 282]. In the constant rate period, the material contains so much water that it dries in a manner similar to that of an open body of water. The evaporation of the dew deposition on the material would fall into this phase of drying. The more dew deposited at night, the longer the constant rate will be applicable. The constant rate period continues until all free moisture disappears. After this the moisture removal rate would decrease. If there is no dew deposited on the material, the constant rate period is so short that it can be neglected. When the hay is first cut, it is assumed to be in the constant rate period. This period is so short also, that it may be neglected. If there is a substantial amount of dew deposited, the constant rate period can no longer be neglected.

After the free moisture has dissipated, the falling rate period begins. The falling rate period is bounded by the equilibrium moisture content between zero and 100 percent relative humidity. Moisture content near the 100 percent level would be in the constant rate period. In the falling rate period, two processes are involved, the movement of moisture from within the material to the surface and the removal of moisture from the surface. Moisture movement is by liquid diffusion if the moisture content of the material is below the saturation point or within the equilibrium range. The moisture is assumed to be brought to the surface by diffusion and evaporated to a vapor at the surface by conduction through the air. The major force behind this process is the difference between the moisture content of the material at the surface and the moisture content of the material at air state, or equilibrium. This idea suggests that, as a first approximation, an ordinary differential equation might be used to describe the rate of change of the moisture content.

CHAPTER IV

THE MODEL

The model suggested from the analysis of equilibrium moisture properties is as follows

$$\frac{dM(t)}{dt} = k(M(t) - Me(t)) \quad (2)$$

where

$M(t)$ = Moisture content, decimal percent, wet basis

$Me(t)$ = Equilibrium moisture content, decimal percent, wet basis

t = Time, hours

k = Drying constant

The equilibrium moisture content, $Me(t)$, is calculated by using equation (1) solved for $Me(t)$, i.e.,

$$Me(t) = \left(\frac{-\log(1 - rh)}{CT} \right)^{1/n} \quad (3)$$

where

$Me(t)$ = Equilibrium moisture content, percent, dry basis

rh = Relative humidity, percent

T = Temperature, absolute degrees Kelvin

C, n = Constants

Since the moisture contents in the model are expressed as a wet basis percent, the equilibrium moisture content found from equation (3) must be converted from dry basis to wet basis percent. This was done by using a conversion chart given by Hall [2, p. 13].

The solution of the proposed differential equation (2) is

$$M(t) = e^{kt} \left[M(0) - k \int_0^t e^{-ks} Me(s) ds \right] \quad (4)$$

where $M(0)$ = initial moisture content and all other variables and constants are as described previously.

In order to evaluate the integral in equation (4) $Me(s)$ was assumed to be piecewise linear on subintervals. That is, $Me(s) = a_i \cdot s + b_i$ for $t_i \leq s \leq t_{i+1}$ and $t_{i+1} - t_i = 1$. Equation (4) now becomes

$$M(t) = e^{kt} \left[M(0) - k \int_0^t e^{-ks} (a_i \cdot s + b_i) ds \right] \quad (5)$$

which can be evaluated on each subinterval.

The equilibrium moisture content was calculated in hourly intervals, making it possible to obtain values for the slope of the line segment between $Me(t_i)$ and $Me(t_{i+1})$ and the intercepts which are a_i and b_i respectively.

One more modification was made to adjust the model, i.e., a parameter δ was added so that the amount of dew deposited on the material in the morning could be accounted for. The equation now becomes

$$M(t) = e^{kt} [M(0) - k \int_0^t e^{-ks} (a_i \cdot s + b_i) ds] + \delta$$

The drying constant k must now be determined. A different value for k was found for each initial moisture content that estimated the moisture content of the corresponding afternoon. The mean of all values of k was then determined and this mean value was then taken as the drying parameter k . This constant was found to be $k = -0.035$. Table 1 lists the morning moisture contents, the corresponding afternoon moisture contents and the constant k for that particular day. These constants were averaged to determine the k used in the model.

Now that a value for the parameter k has been found, it is now possible to determine the parameter δ . The parameter δ involves an estimation of the excess moisture on the hay in the mornings attributed to dew. To estimate δ , the data were separated into categories separating the days when heavy dew was deposited from the days when medium and light dew was recorded. In making an estimate for δ , the afternoon moisture content was taken as an initial condition and a morning moisture content was then found. This was accomplished by using the model and the previously found value for k and working backwards. The model started with the afternoon moisture content and proceeded to estimate the moisture content at hourly intervals until the hour that the morning moisture content was calculated. The model was taking a moisture content and essentially adding moisture to it, to estimate the morning moisture content. This process arrived at a moisture content that was greater than the actual moisture content. Thus an estimation of excess moisture on the hay could be found.

TABLE I
DAILY VALUES FOR k

PLOT NUMBER	DATE OF SAMPLE	MORNING MOISTURE CONTENT DECIMAL PERCENT	AFTERNOON MOISTURE CONTENT DECIMAL PERCENT	k
3043	7-18-78	.7169	.6083	-.035
3043	7-20-78	.4702	.2452	-.06
1011	7-18-78	.7487	.6626	-.035
1011	7-19-78	.6587	.6373	-.009
1011	7-20-78	.5882	.4174	-.05
2029	7-18-78	.6988	.6732	-.010
2029	7-19-78	.6815	.5966	-.030
2029	7-20-78	.5377	.5261	-.001
1007	7-18-78	.7533	.6849	-.015
1007	7-19-78	.5856	.5229	-.03
1007	7-20-78	.5224	.3434	-.05
1007	7-21-78	.5110	.4667	-.03
3038	7-19-78	.5889	.5529	-.012
3038	7-20-78	.6549	.4889	-.07
3038	7-21-78	.4035	.3532	-.037
2023	7-18-78	.7102	.7010	-.005
2023	7-19-78	.6449	.5211	-.045
2023	7-20-78	.4936	.4209	-.045
2023	7-21-78	.4570	.4015	-.035
2017	7-18-78	.7179	.5422	-.045
2017	7-19-78	.6127	.5541	-.020
2017	7-20-78	.5212	.4639	-.035
2017	7-21-78	.4653	.3239	-.055
1005	7-18-78	.7418	.6963	-.013
1005	7-19-78	.6748	.6707	-.001
1005	7-20-78	.6036	.5891	-.009
1005	7-21-78	.4982	.4310	-.038
3032	7-18-78	.7115	.6528	-.016
3032	7-19-78	.6042	.6037	-.006
3032	7-20-78	.5806	.4024	-.050

As a guideline to check the validity of this value for k , the model was used to obtain predicted moisture contents on afternoons when no dew was recorded that morning, i.e., $\delta = 0$. The morning moisture contents were used as the initial conditions. This choice of k provided accurate predictions on days with no dew recorded in the morning.

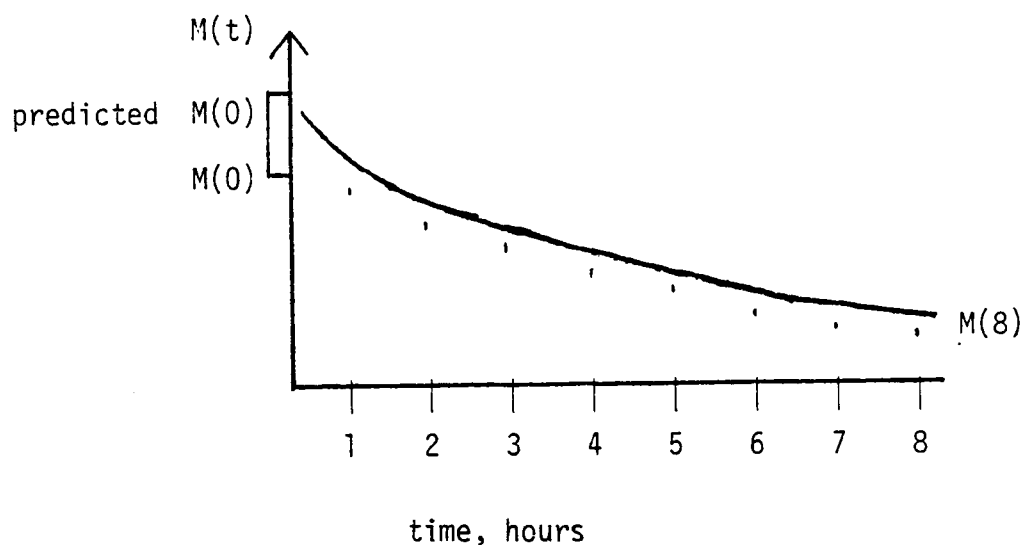


Figure 1. Illustration of actual and predicted morning moisture content

The difference between the predicted morning moisture content and the actual moisture content was taken as the value for δ for that day. This model assumes that no drying takes place during the night, so the predictions up to this point are restricted to that of a single day, morning to afternoon.

The value for δ for use when a heavy dew was recorded was $\delta = .065$. The afternoon moisture contents recorded on days with heavy dew in the morning were used to predict the respective morning moisture contents.

The model predicted morning moisture contents that were, on the average, 7 percent higher than the actual values. This indicated that approximately 7 percent of the moisture could be attributed to extra moisture in the form of dew. The factor δ is used to adjust the predicted moisture content so that the extra moisture attributed to dew is added on.

Table II shows the predicted morning moisture content, actual morning moisture content and the δ for that particular day.

TABLE II
HEAVY DEW

MORNING MOISTURE CONTENT DECIMAL PERCENT		DEW PARAMETER
ACTUAL	PREDICTED	δ
.7587	.8595	.1008
.7232	.8240	.1008
.7701	.8154	.0453
.6698	.7697	.0999
.6988	.8052	.1064
.7487	.7916	.0429
.7115	.7791	.0676
.6127	.6530	.0403
.6815	.7116	.0301
.6587	.7593	.1006
.5889	.6515	.0626
.7129	.7950	.0801
.5212	.5378	.0166

Determining a value for δ on days when light and medium dew was recorded was not possible. The data observed on these days were not constant enough to get an accurate estimate for the dew deposition parameter. It is feasible that when the moisture samples were taken, most of the dew had evaporated by the time the final sample was taken,

since at least one hour was required to take all samples and a light to medium dew could evaporate in that period of time.

The data used in the procedures for determining k and δ were restricted to two weeks during the summer. These weeks were July 11, 1978 through July 14, 1978 and July 18, 1978 through July 21, 1978. Many weeks of data were omitted because of rain. Of the two weeks that were used, some days of data were omitted because of sampling error. On these days, either the afternoon moisture content was greater than the morning moisture content or the afternoon moisture content was much less than the morning moisture content.

Tables III and IV contain comparisons of the predicted and actual moisture contents for no dew and heavy dew on the hay.

TABLE III

NO DEW

MORNING MOISTURE CONTENT DECIMAL PERCENT		AFTERNOON MOISTURE CONTENT DECIMAL PERCENT	
ACTUAL		ACTUAL	PREDICTED
.4982		.4310	.4372
.4174		.3239	.3739
.4570		.4015	.4049
.3911		.3282	.3533
.3846		.3638	.3483
.4035		.3532	.3631
.5110		.4667	.4472
.4087		.3484	.3671

TABLE IV

HEAVY DEW

MORNING MOISTURE CONTENT DECIMAL PERCENT ACTUAL	AFTERNOON MOISTURE CONTENT DECIMAL PERCENT		
	ACTUAL	PREDICTED	PREDICTED + δ
.7242	.6961	.6097	.6747
.7587	.7201	.63367	.7017
.7129	.6626	.6008	.6658
.7231	.6923	.60881	.6738
.7701	.6804	.6456	.7106
.6698	.6498	.5671	.6321
.6988	.6732	.5907	.6557
.7487	.6626	.6298	.6948
.7115	.6528	.6006	.6656
.6587	.6373	.5593	.6243
.5389	.5529	.5047	.5697

It was of interest to know whether the model indicated any drying during the night. Using the temperature and relative humidity values during a night in which no dew was formed, the model was applied with the moisture content of the hay on the preceding afternoon as the initial condition. The results of this procedure indicated that a negligible amount of drying took place during the night.

The use of this model has now been extended through an entire drying period. Previously, it had been utilized for only a single day. The moisture content of the hay on the morning after it was first cut was used as the initial condition $M(0)$. In extending the use of this model through an entire drying period, it was assumed that no drying took place during the night. Starting with the initial condition $M(0)$, the model was used to approximate the moisture

content of the hay on the corresponding afternoon, i.e., at the time of the afternoon sample. The dew parameter δ was added, if applicable, to the predicted afternoon moisture content. This moisture content was then taken as the moisture content of the hay for the next morning. This procedure was continued over a period of four days in order to determine the moisture content of the hay at the end of the drying period.

The week of July 18, 1978 through July 21, 1978 was chosen for this procedure since the dew recorded on each of the days was classified as either heavy dew or no dew. The data from five plots were chosen from the nine that were sampled to apply the model for an entire drying period. The four plots that were omitted contained obvious errors in the data on one or more days. Figures 2 through 6 show the results of the procedure described in the preceding paragraph. The particular plots used were numbers 1005, 1011, 2017, 2023, and 3038.

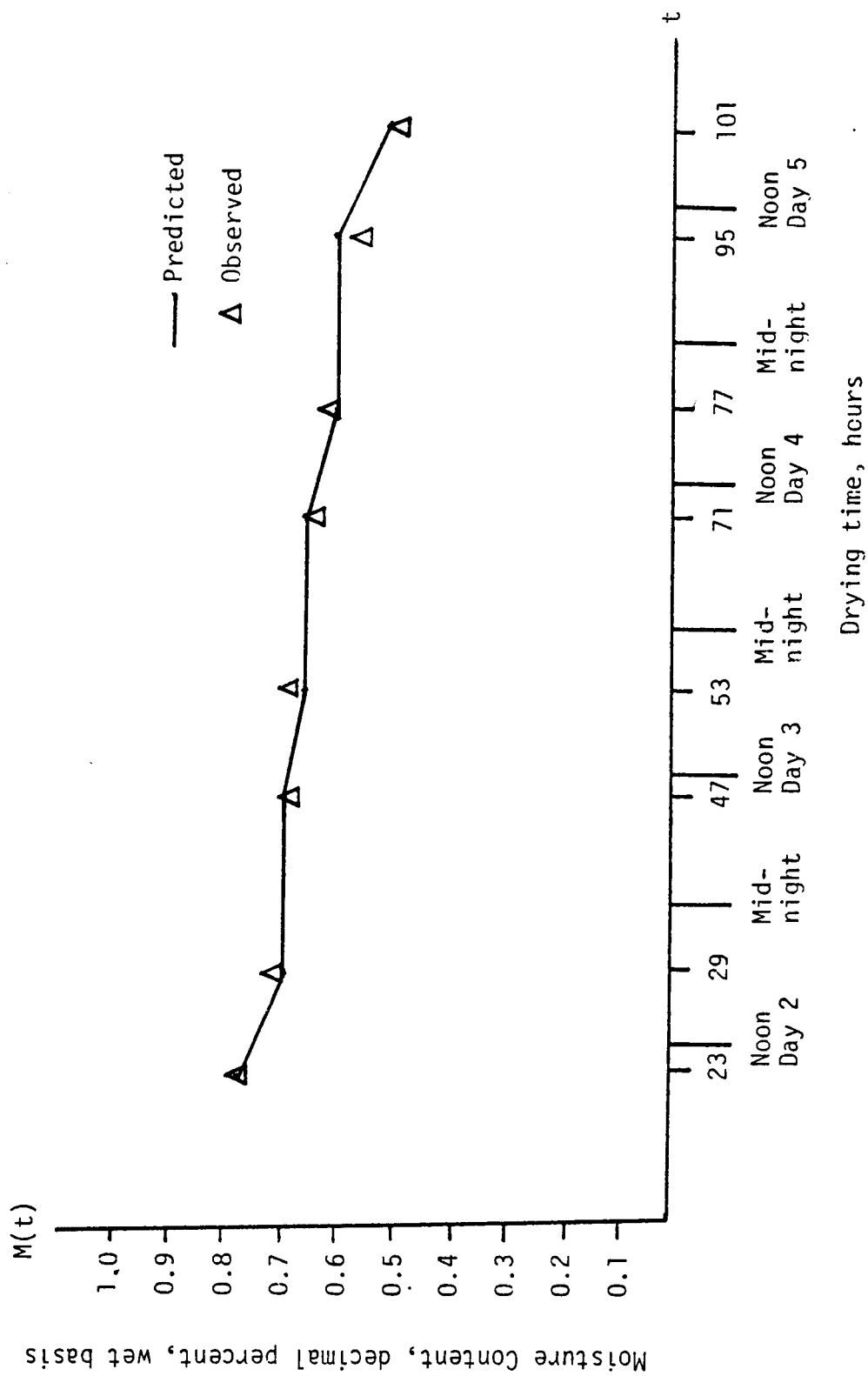


Figure 2. Observed and predicted moisture content of hay: Plot 1005

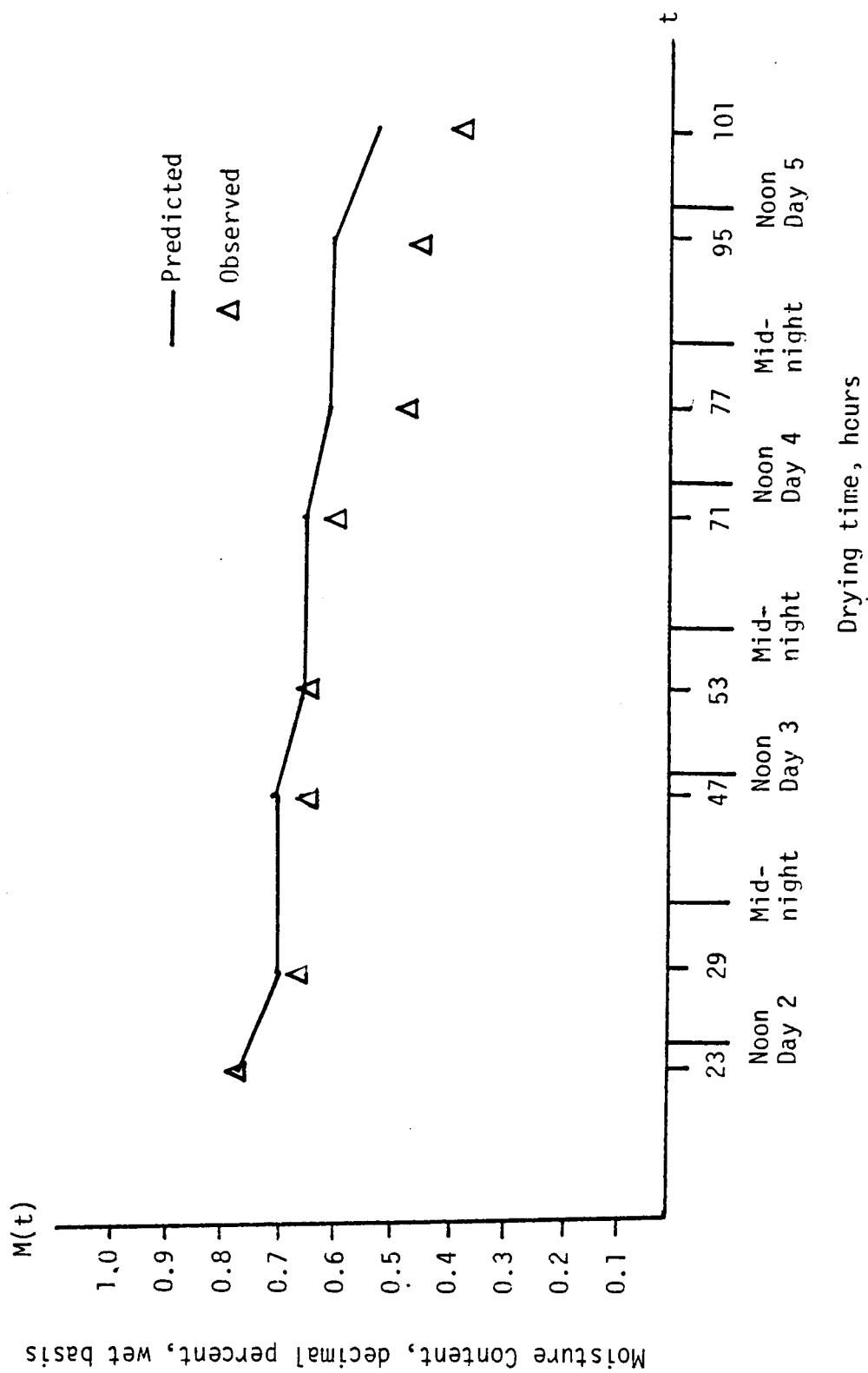


Figure 3. Observed and predicted moisture content of hay: Plot 1011

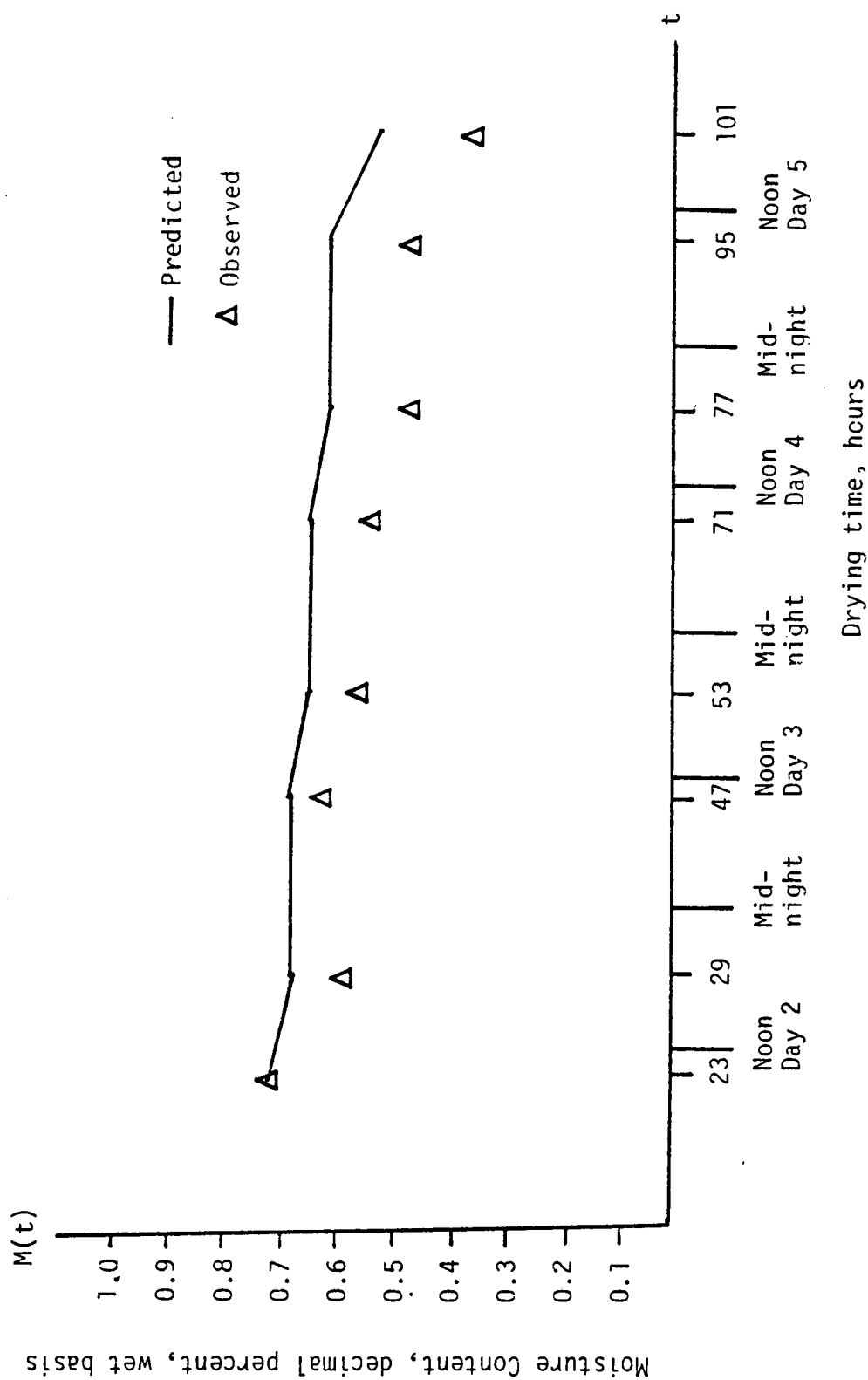


Figure 4. Observed and predicted moisture content of hay: Plot 2017

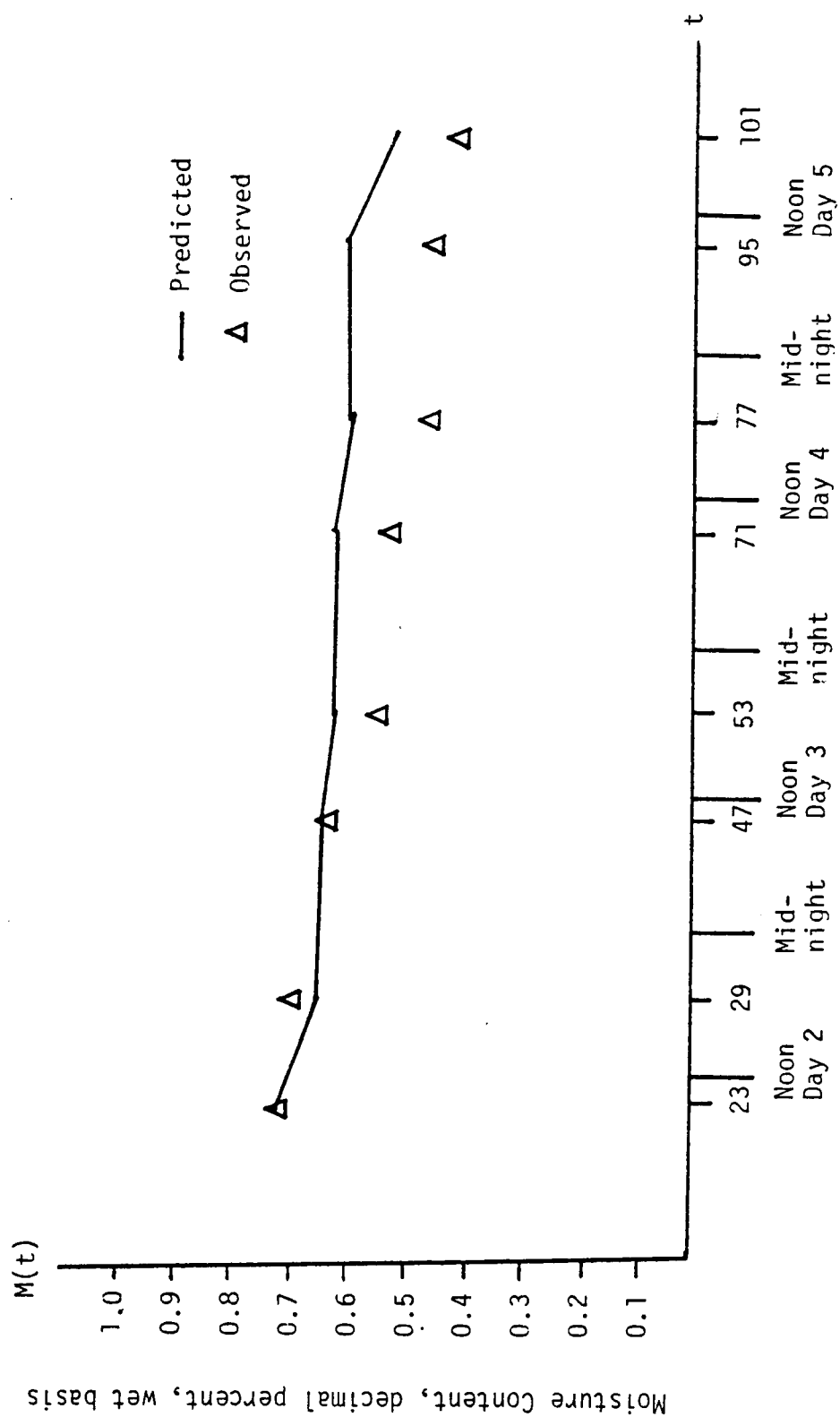
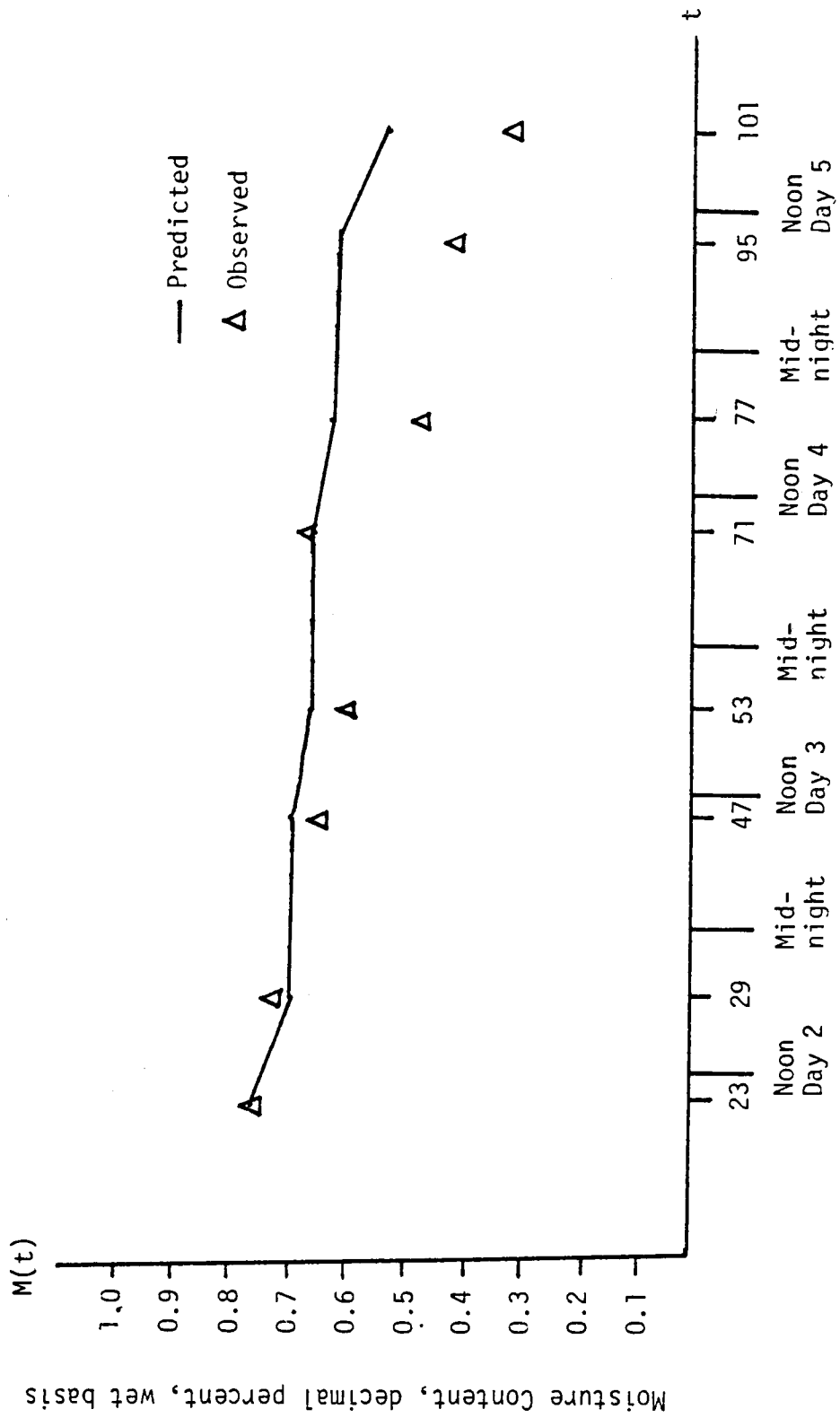


Figure 5. Observed and predicted moisture content of hay: Plot 2023



Drying time, hours

Figure 6. Observed and predicted moisture content of hay: Plot 3038

CHAPTER V

FURTHER APPLICATIONS OF THE MODEL

The previous application of the model during a full drying period resulted in an over-estimation of the moisture content at the end of the drying period. The stopping point in the afternoon in this procedure was when the afternoon sample was taken, allowing for only an eight-hour drying day. When the afternoon sample was taken, there were still up to four hours in which drying could have taken place.

This fact suggests that the model be used for a longer drying day. The same procedure that was used for a full drying period was used again, but this time the drying day was lengthened to ten, eleven, and twelve hours. Figures 7 through 21 illustrate the results of lengthening the drying day from two to four hours. The same plots were used for this application as were used previously.

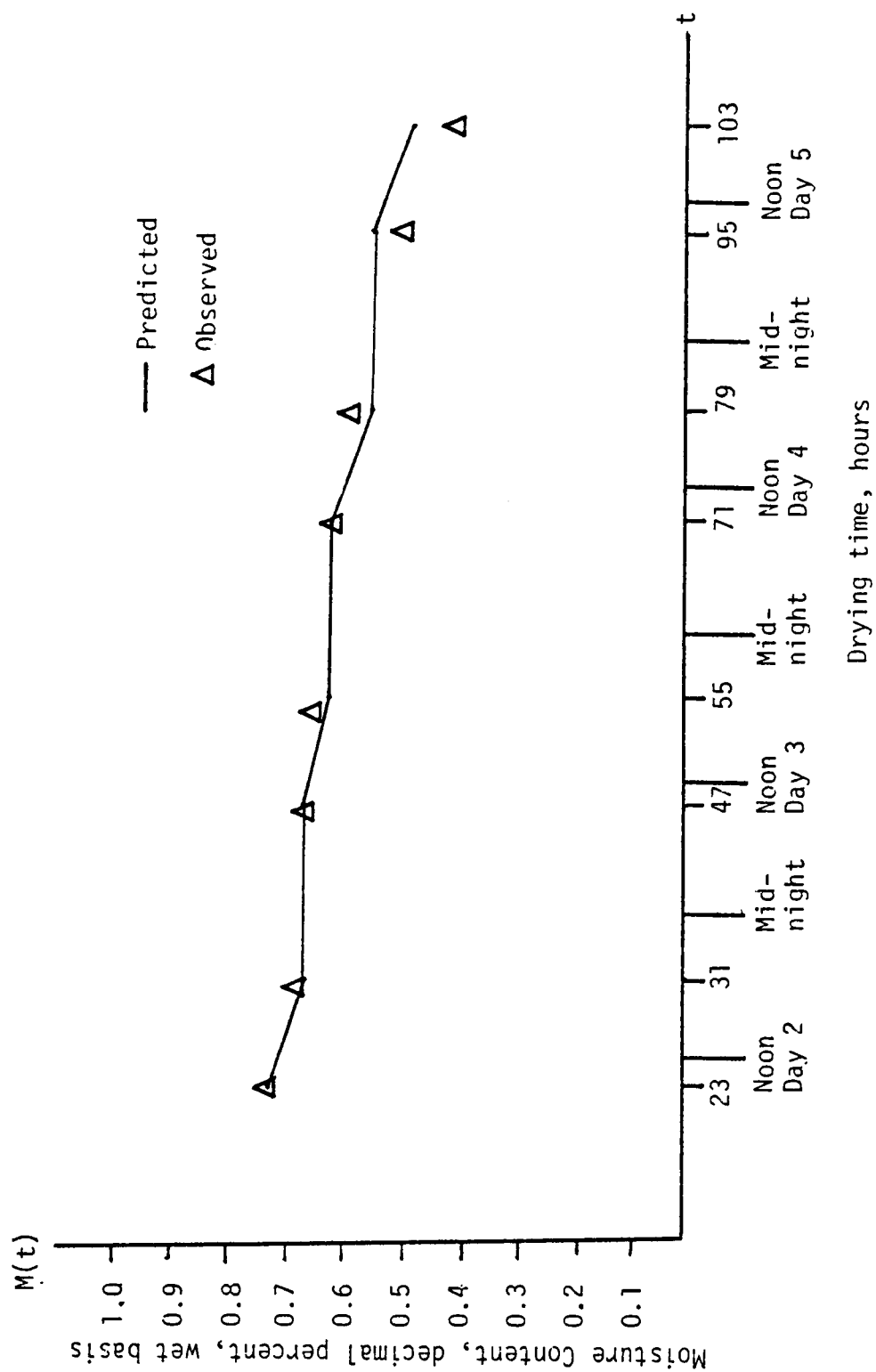


Figure 7. Observed and predicted moisture content of hay with ten-hour drying day: Plot 1005

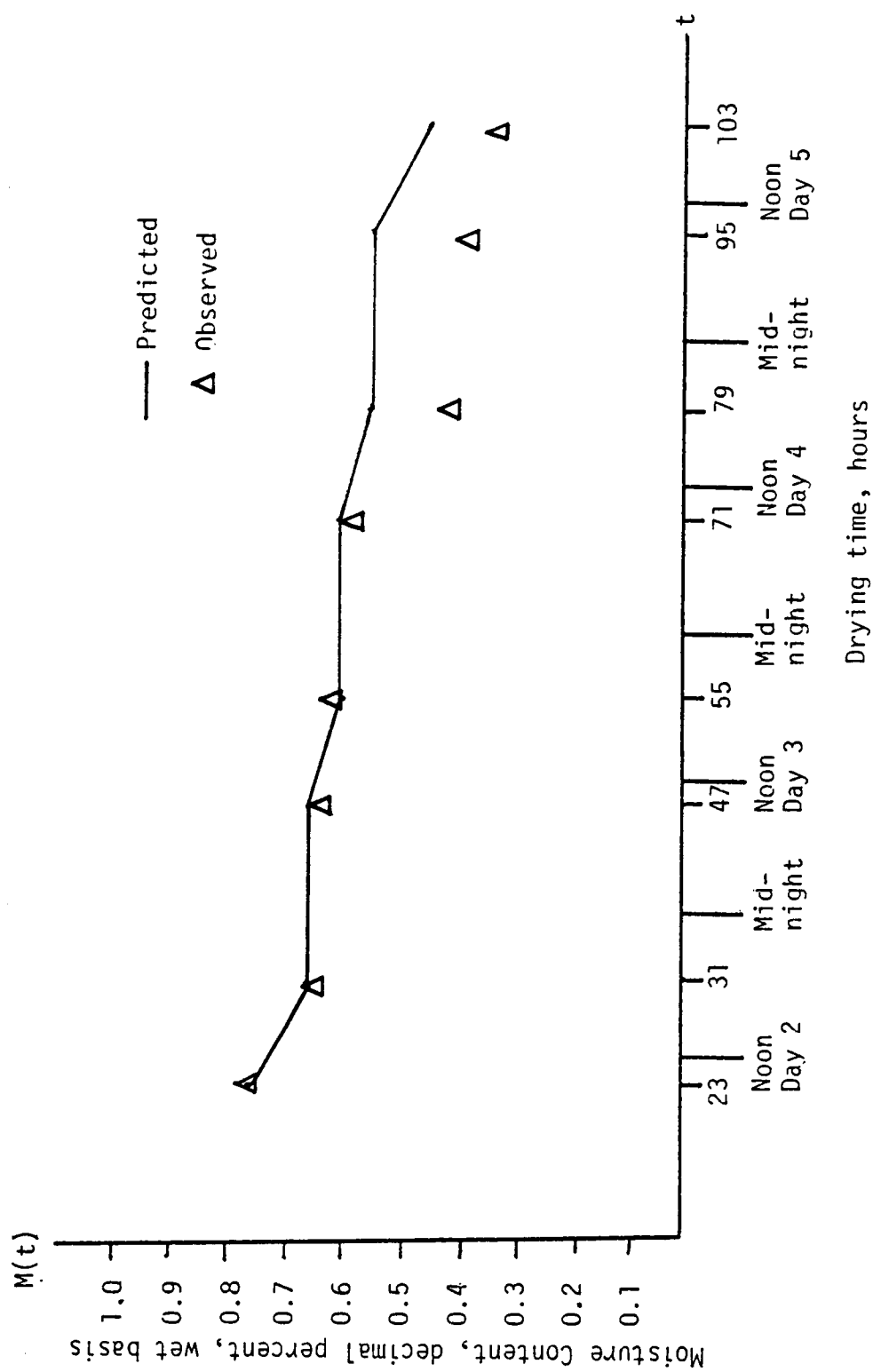


Figure 8. Observed and predicted moisture content of hay with ten-hour drying day: Plot 1011

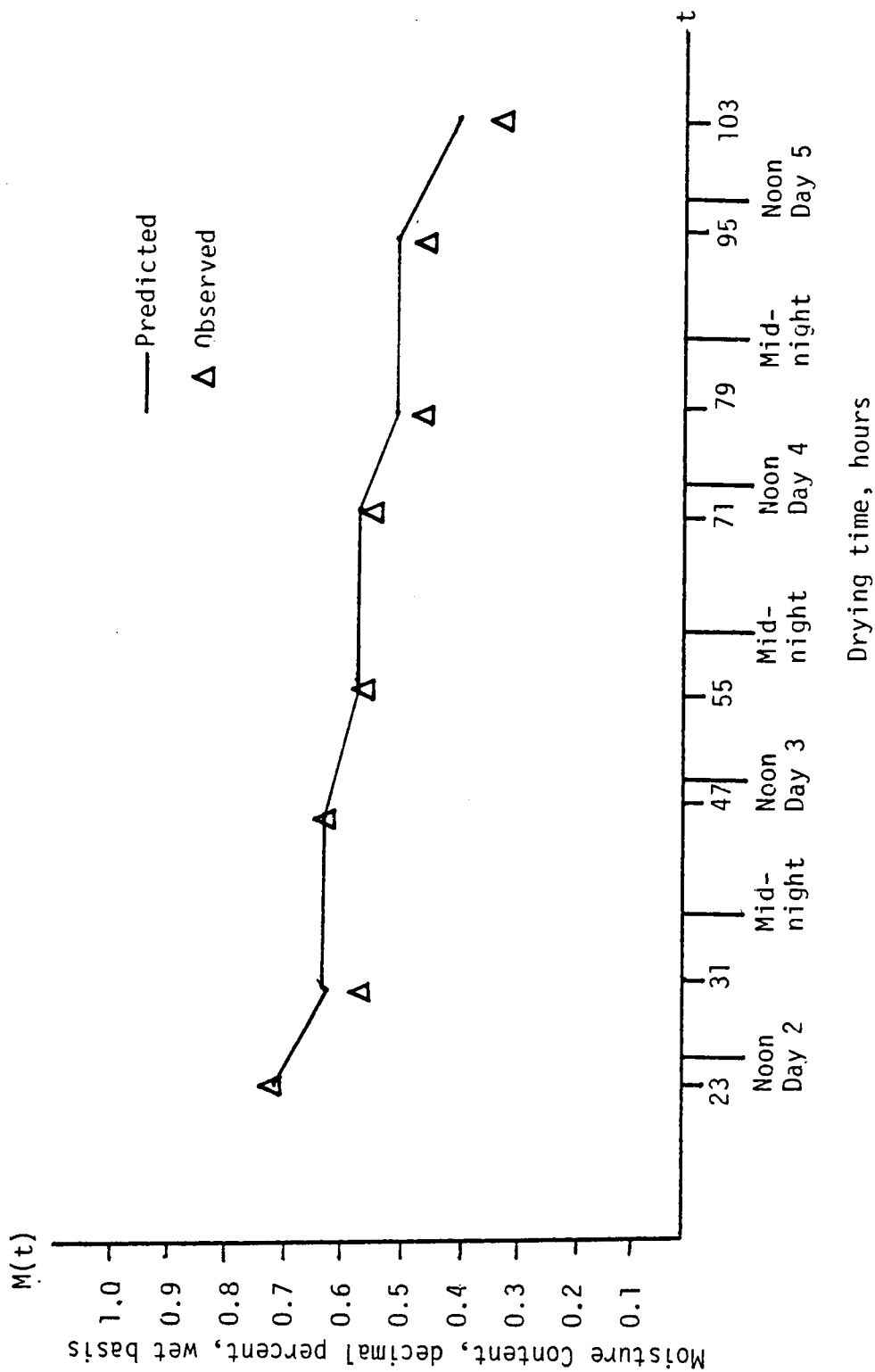


Figure 9. Observed and predicted moisture content of hay with ten-hour drying day: Plot 2017

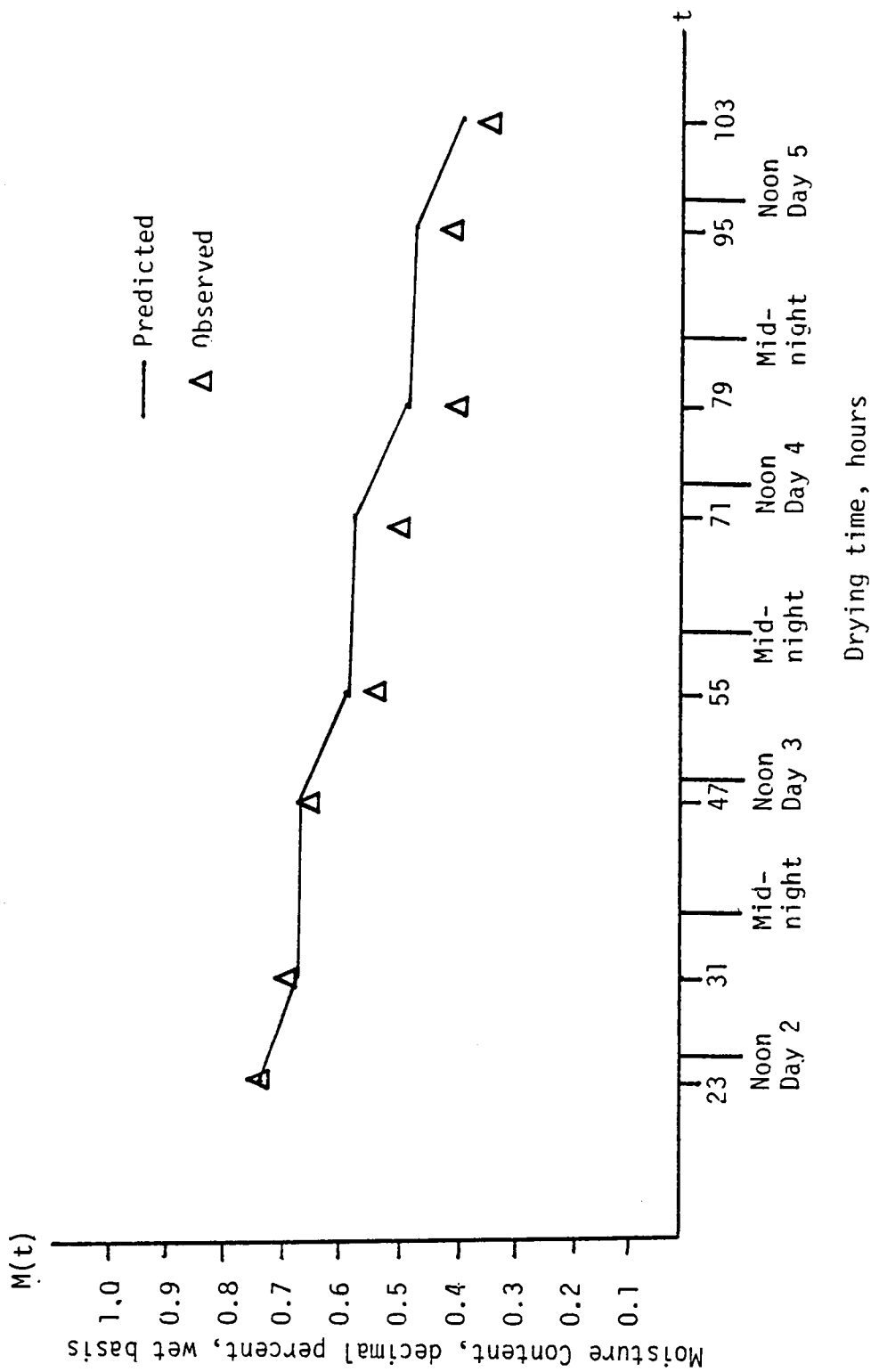


Figure 10. Observed and predicted moisture content of hay with ten-hour drying day: Plot 2023

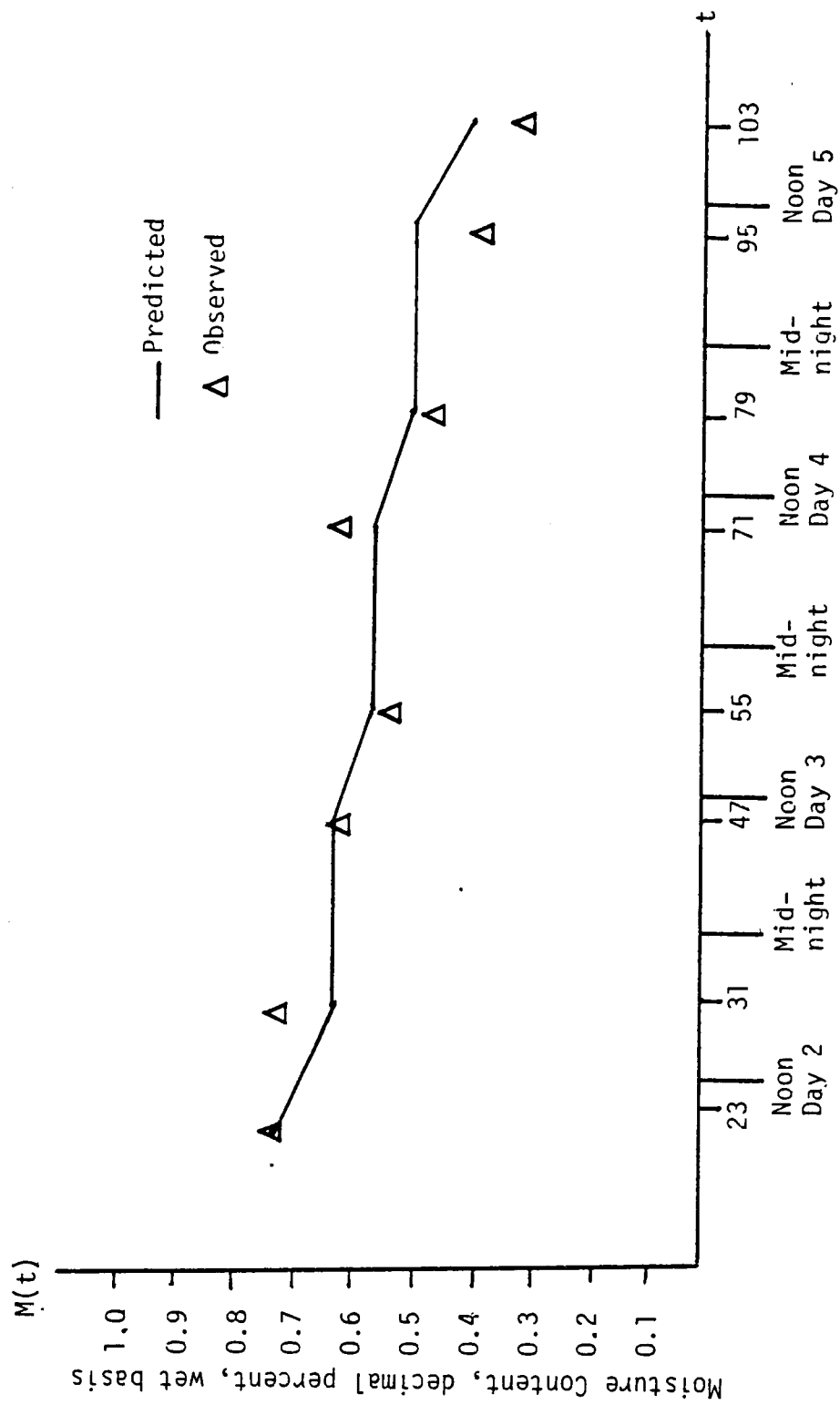


Figure 11. Observed and predicted moisture content of hay with ten-hour drying day: Plot 3038

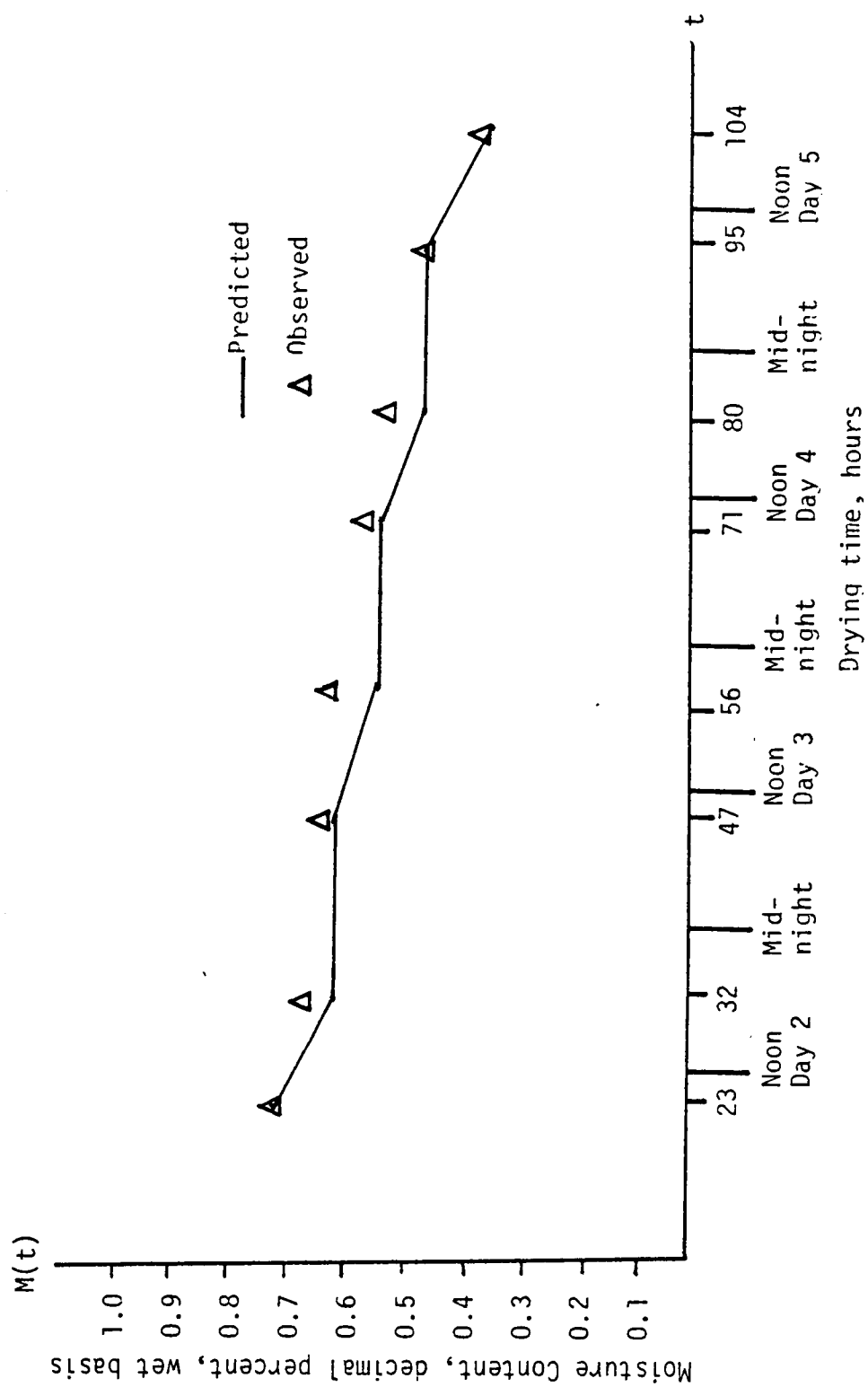


Figure 12. Observed and predicted moisture content of hay with eleven-hour drying day: Plot 1005

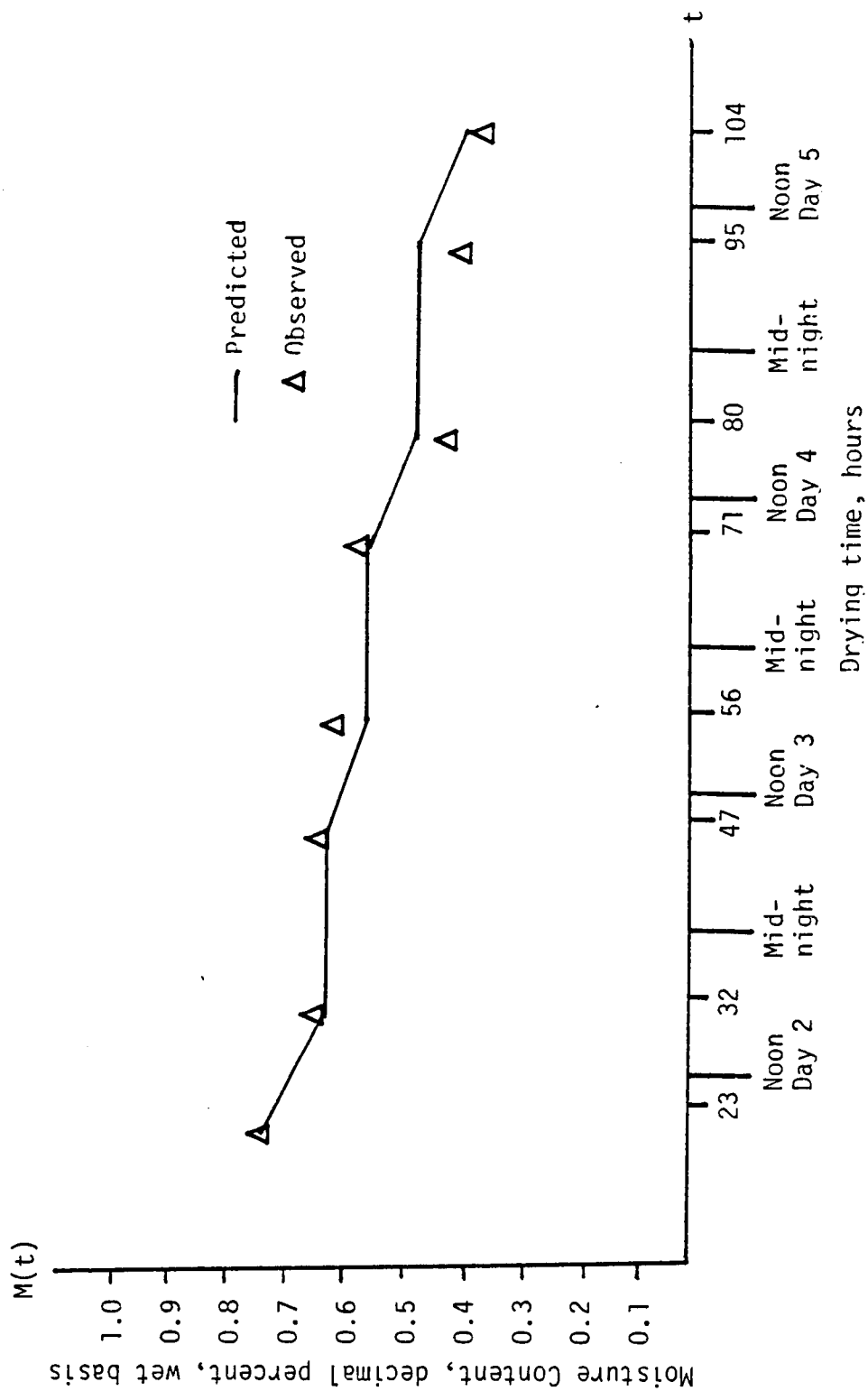


Figure 13. Observed and predicted moisture content of hay with eleven-hour drying day: Plot 1011

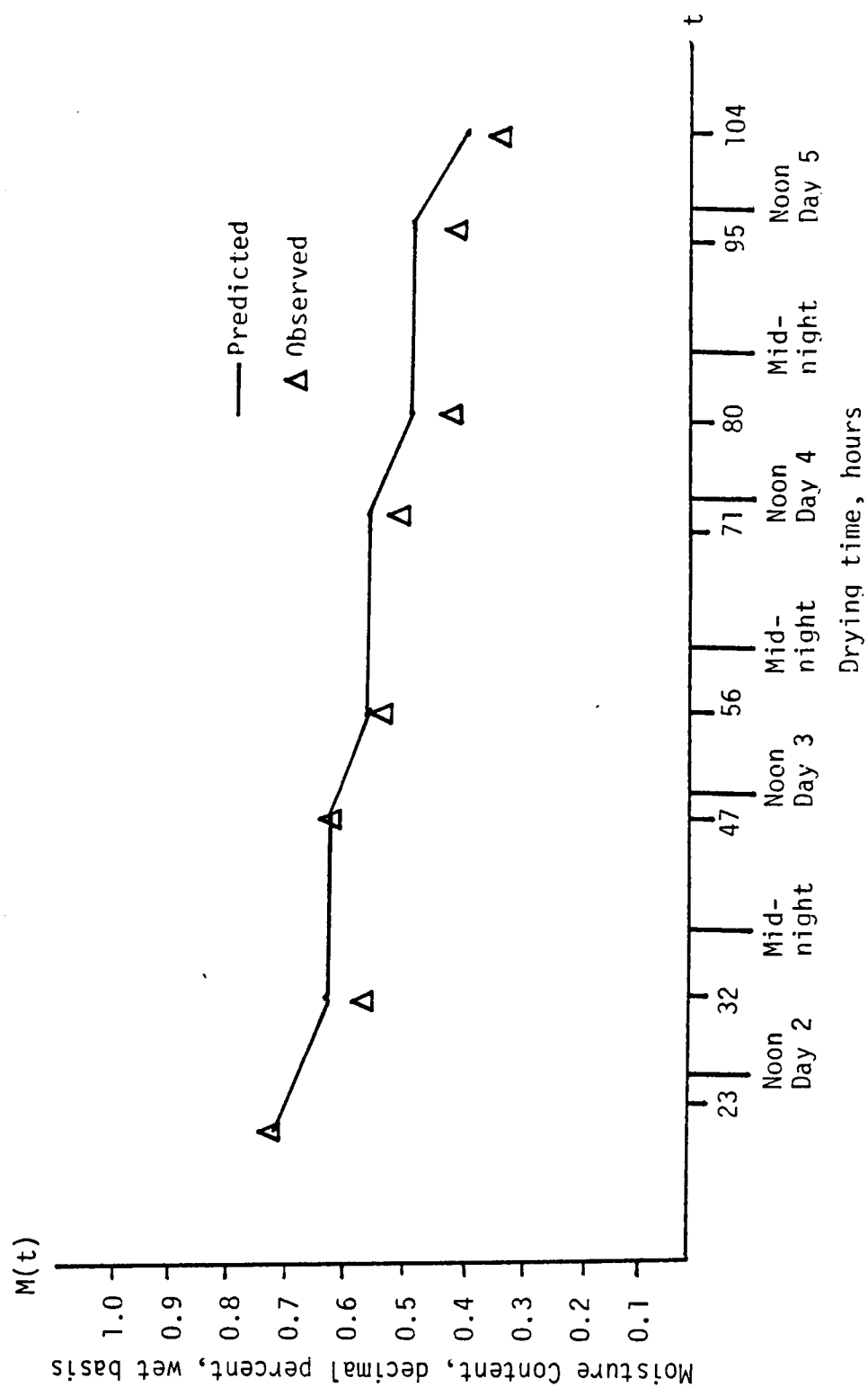


Figure 14. Observed and predicted moisture content of hay with eleven-hour drying day: Plot 2017

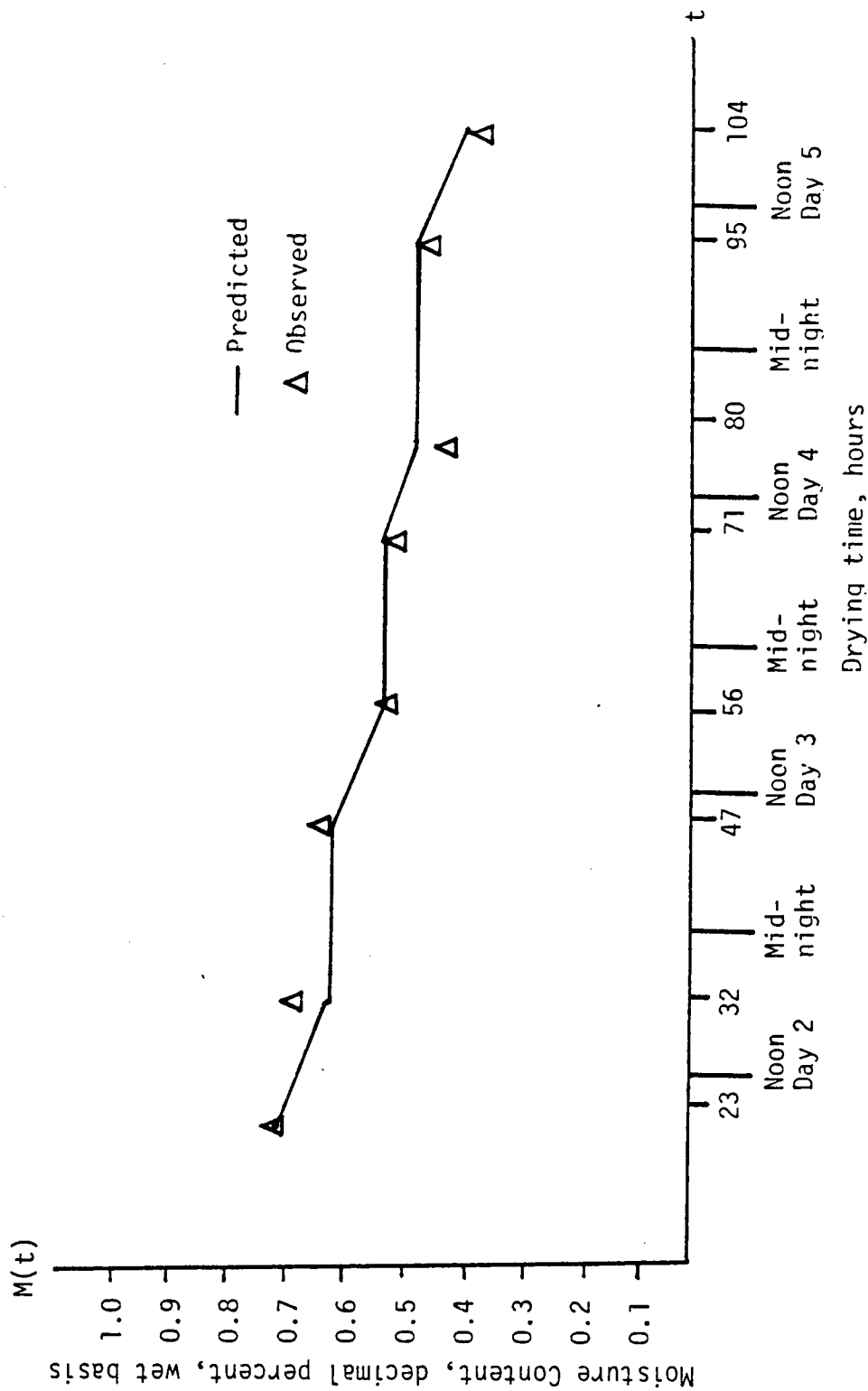


Figure 15. Observed and predicted moisture content of hay with eleven-hour drying day: Plot 2023

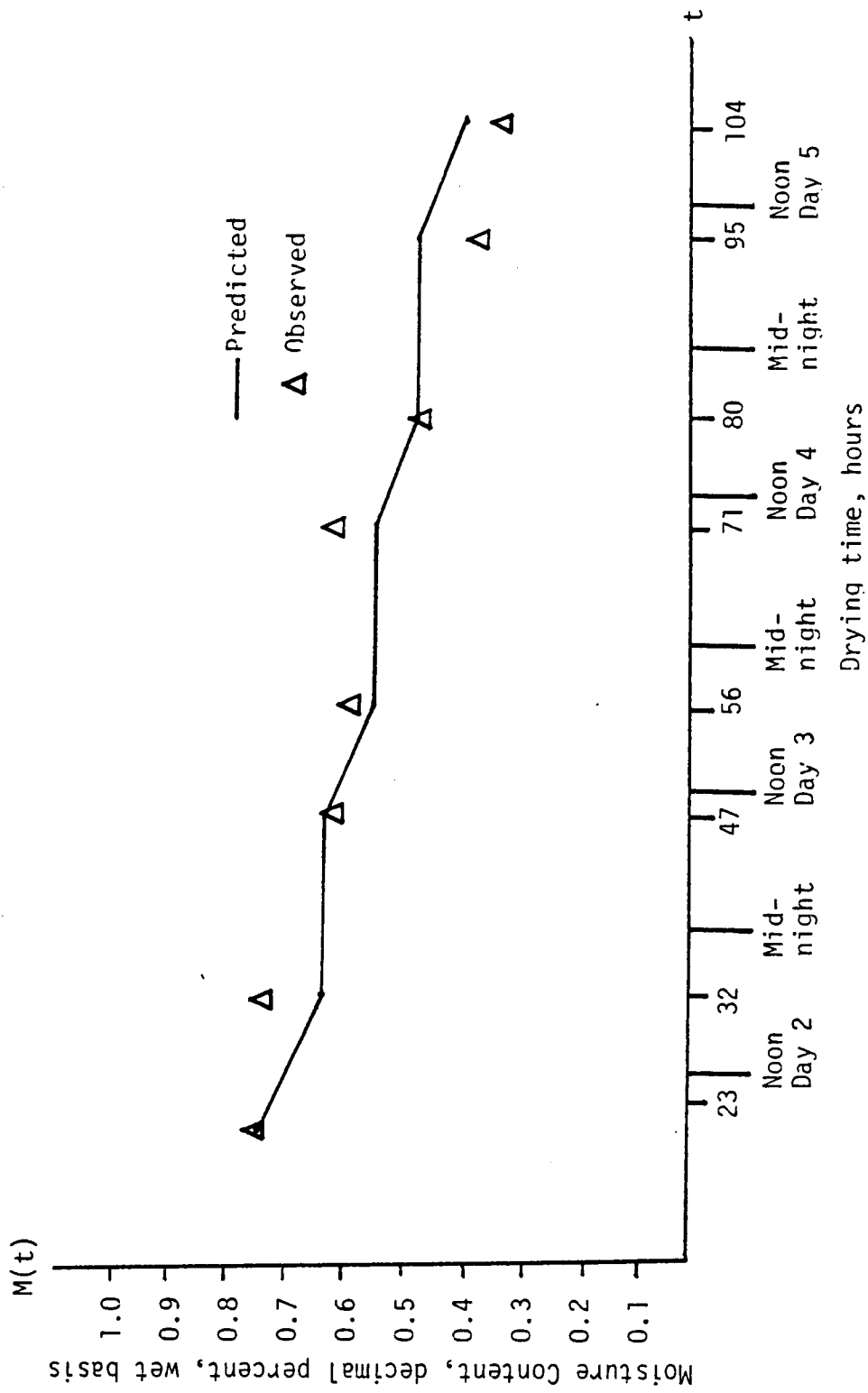


Figure 16. Observed and predicted moisture content of hay with eleven-hour drying day: Plot 3038

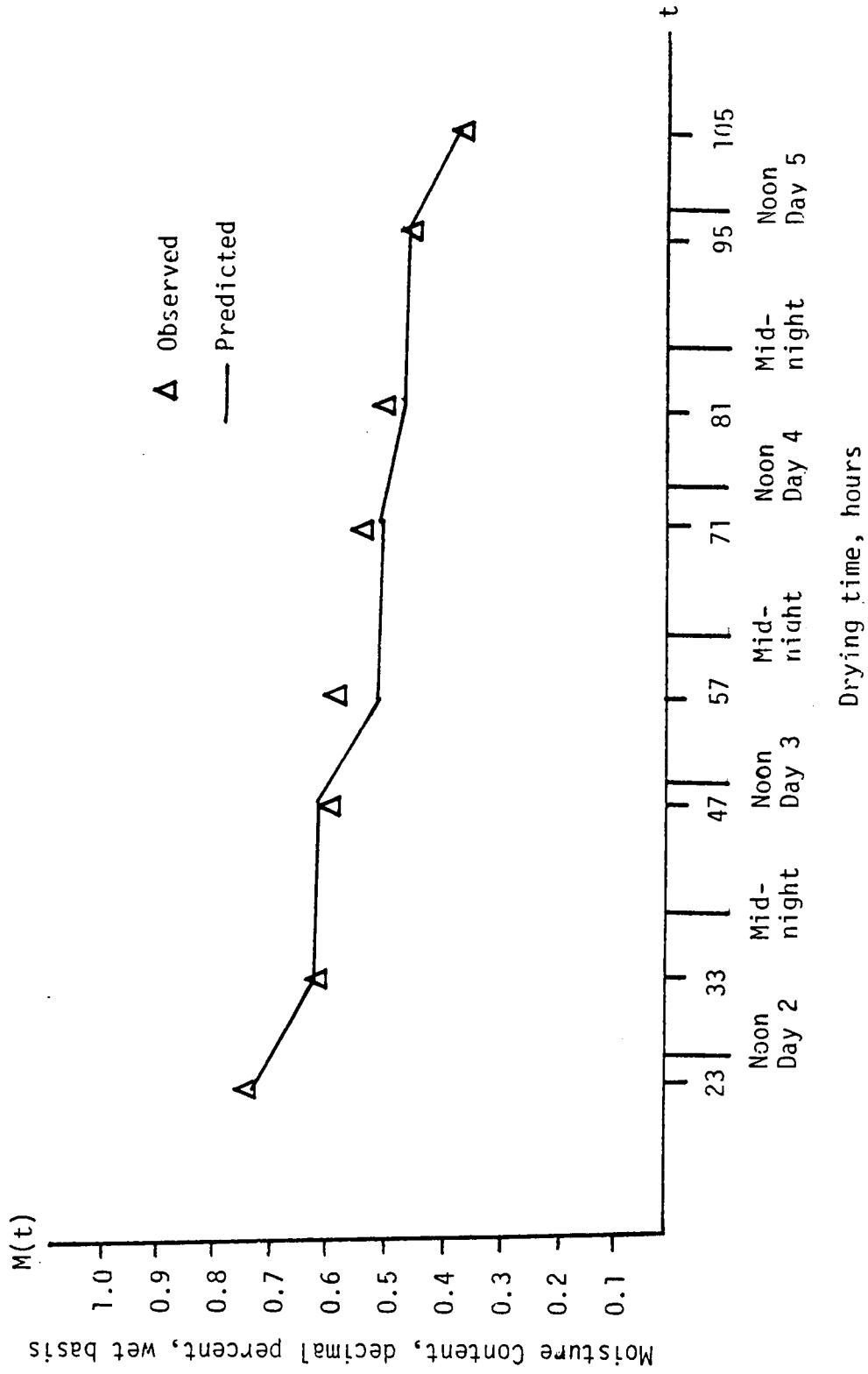


Figure 17. Observed and predicted moisture content of hay with twelve-hour drying day: Plot 1005

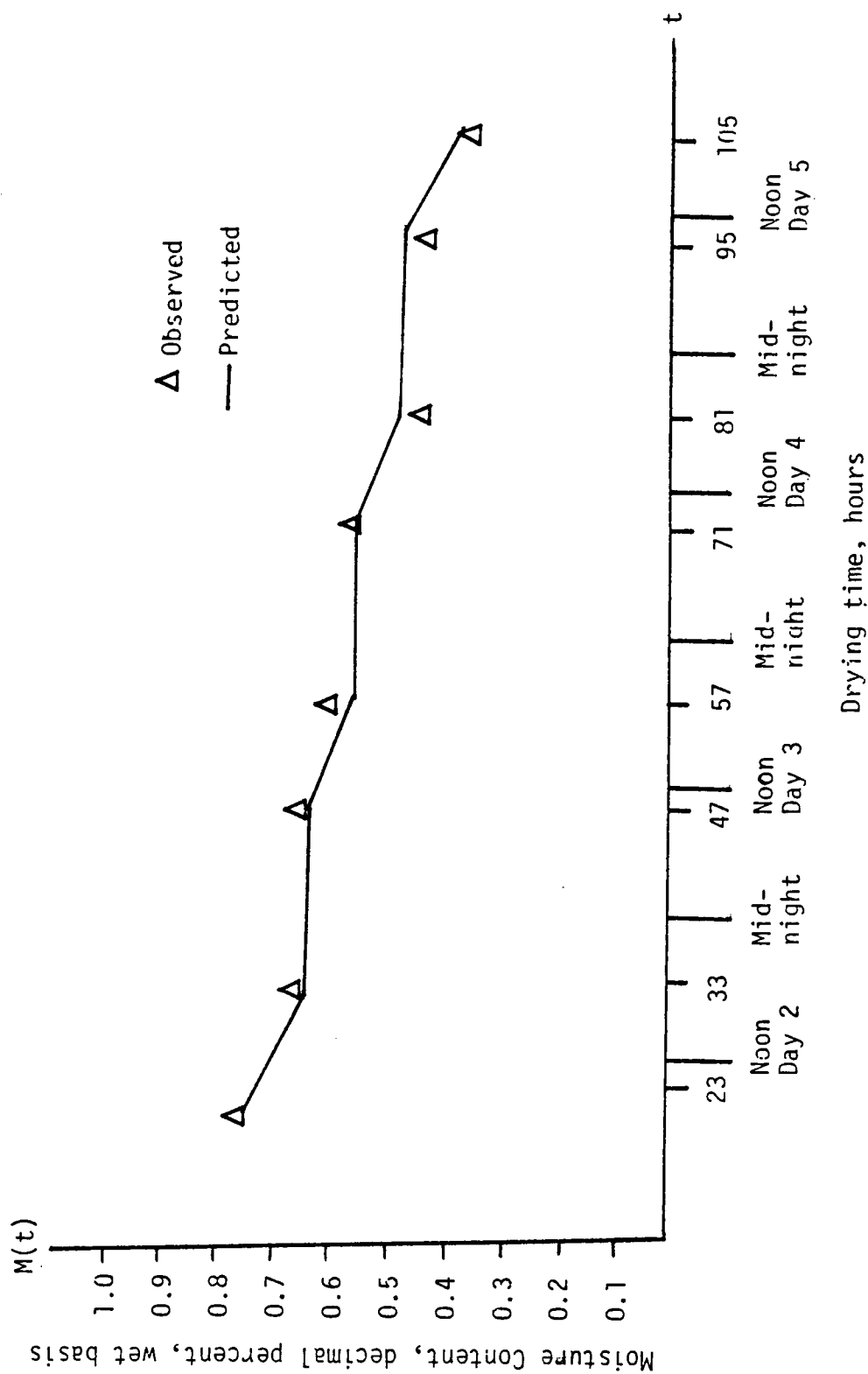


Figure 18. Observed and predicted moisture content of hay with twelve-hour drying day: Plot 1011

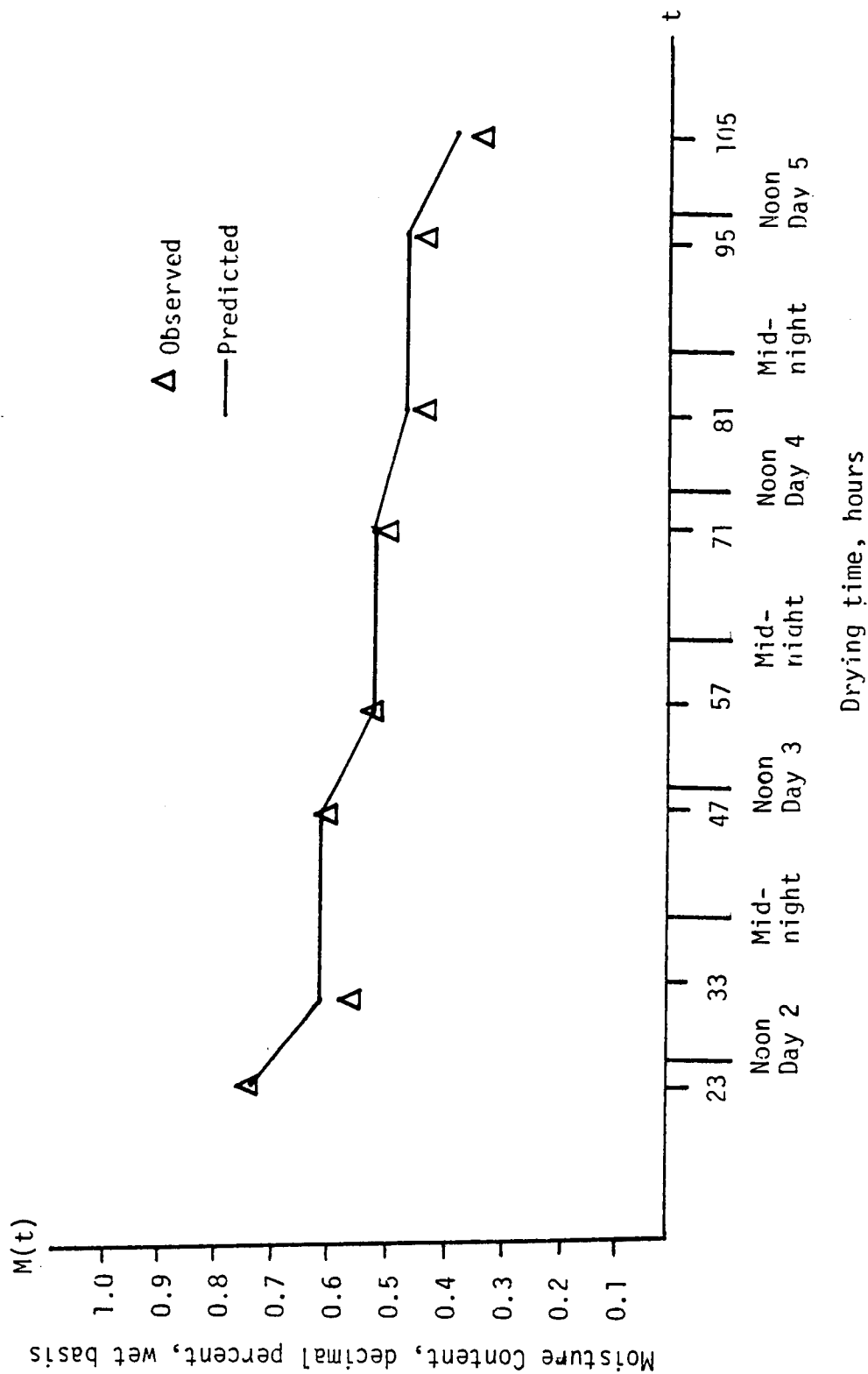
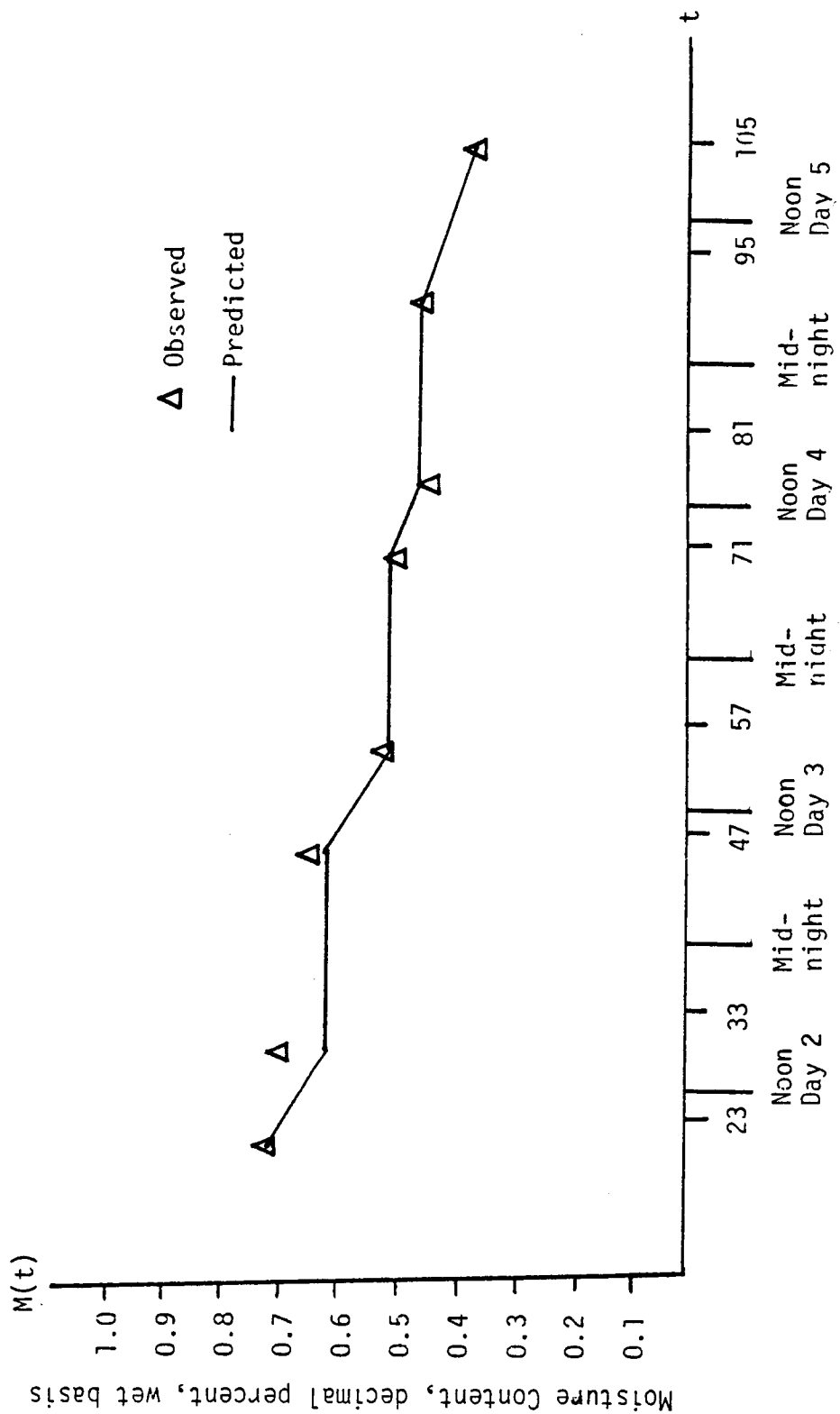


Figure 19. Observed and predicted moisture content of hay with twelve-hour drying day: Plot 2017



Drying time, hours

Figure 20. Observed and predicted moisture content of hay with twelve-hour drying day: Plot 2023

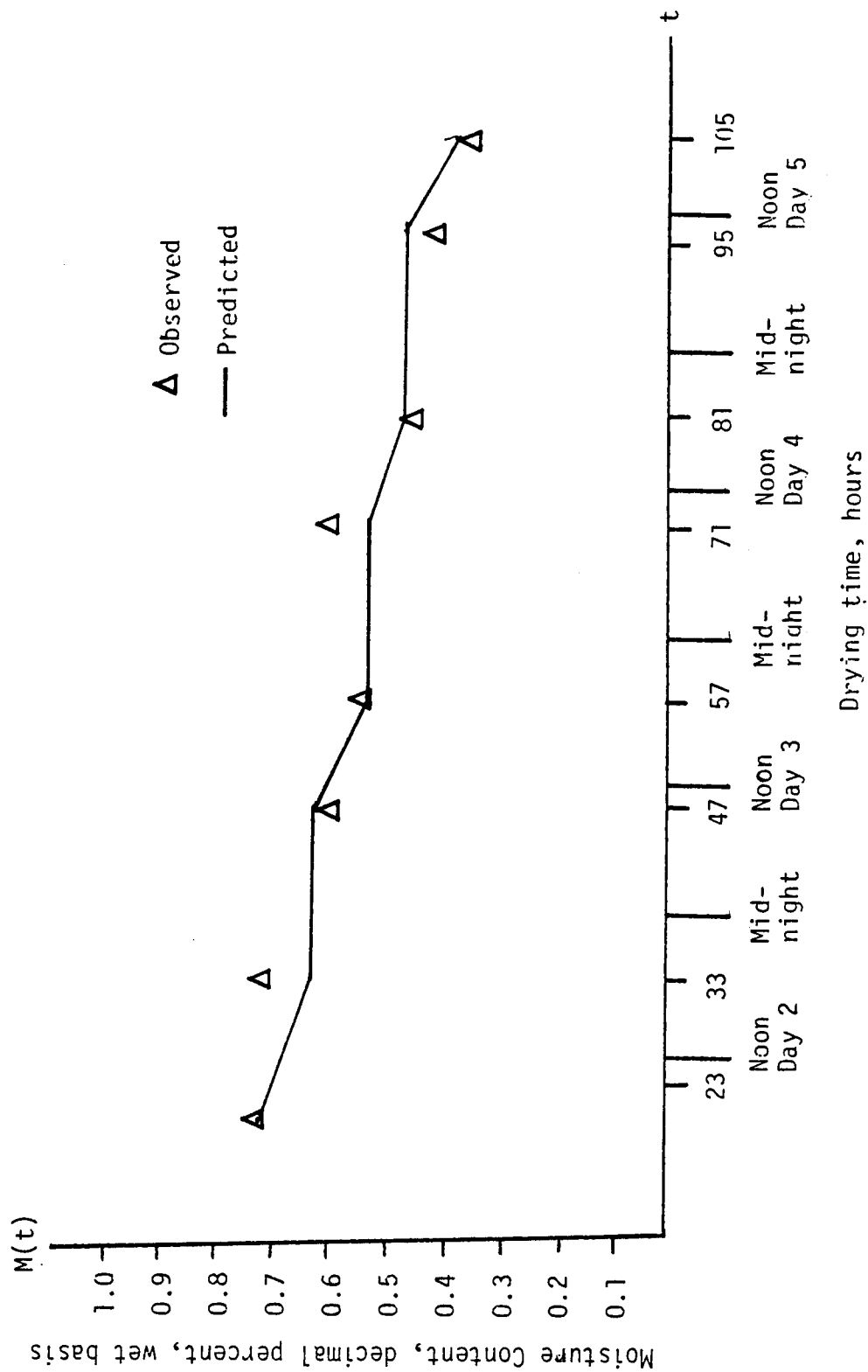


Figure 21. Observed and predicted moisture content of hay with twelve-hour drying day; Plot 3038

CHAPTER VI

DISCUSSION

The proposed model will predict the moisture content of hay given values for relative humidity, temperature and an estimation for dew deposition. This is a valuable tool for professional agriculturists to use as a guide for farmers, since the model can give an estimation of the moisture content of hay at any time. In this way, an estimation of how long it would take the hay to dry could be made, saving the farmer time and money.

Other studies have been made employing different parameters to predict the drying time of hay. One such experiment employed vapor pressure deficit along with the moisture ratio to estimate moisture content of alfalfa hay [5, pp. 2 - 7].

The moisture ratio is determined to be

$$MR = \frac{M - M_e}{M_0 - M_e} = e^{-kt}$$

M = Moisture content of hay, dry basis percent

M_e = Equilibrium moisture content, dry basis percent

M_0 = Initial moisture content, dry basis percent

k = Drying constant

Another form of this equation is

$$MR = e^{-kt^n}$$

where n and k are constants [5, pp. 2 - 7].

A nonlinear least squares program was used to fit observed data to the two exponential drying equations [5, pp. 2 - 7]. The saturation vapor pressure deficit was used as a measure of the drying potential of the air at specific temperature and relative humidity levels. With steady vapor pressure deficit, the moisture ratio could be represented as

$$MR = e^{-kt \cdot 0.03}$$

where $k = .007 (VPD) + .1164$

VPD = Mean vapor pressure deficit.

This method was found to be most accurate under conditions of high drying potential, i.e., when the vapor pressure deficit is large. This technique is useful under controlled drying conditions and could be used to make a first approximation of the drying rate of the material in a windrow. Under field conditions, the surface temperature of the drying crop would be much larger, and would have to be considered [5, pp. 2 - 7].

A parameter called latent evaporation has been used in developing an equation for drying hay [6]. Latent evaporation is a term used to describe the evaporation rate of water to the atmosphere measured by an atmometer. This instrument measures the combined efforts of solar radiation, temperature, vapor pressure deficit and wind velocity on the evaporation rate of water.

The moisture ratio equation

$$MR = e^{-C_1(L.E.)t^{C_2}}$$

where

L.E. = Latent evaporation, cc per hour

C_1 = Regression coefficient

C_2 = Constant

was used and the constants C_1 and C_2 were found using regression analysis. The constants varied as to whether the hay was conditioned or unconditioned.

The two preceding models, which are of the form

$$\frac{M(t) - Me(t)}{M(0) - Me(0)} = e^{-kt}, \text{ satisfy the following differential equation:}$$

$$\frac{d[M(t) - Me(t)]}{dt} = -k[M(t) - Me(t)]$$

Both of the models previously described employed techniques to fit the exponential curve to the data taken experimentally.

The model proposed in this paper uses temperature and relative humidity to arrive at an equation describing the drying process of hay. The data used in this experiment were taken under actual field conditions instead of in a controlled environment. So, this model should give a more realistic estimate of the moisture contents of hay dried under field conditions. This model is valid for both favorable and

unfavorable drying conditions, where the two previously described models were accurate only under conditions of high drying potential. This model depends on actual temperature and relative humidity values observed at regular intervals. This takes into consideration the variation in drying potential throughout the day. The previously described models used parameters that were a mean value, i.e., they were not allowed to vary. The values of equilibrium moisture calculated were allowed to vary in hourly intervals, depending on the actual temperature and relative humidity values at a specific time. The other models held temperature and relative humidity constant.

CHAPTER VII

CONCLUSION

An ordinary differential equation model of the form

$$\frac{dM(t)}{dt} = k(M(t) - M_e(t))$$

was suggested as a means to estimate the moisture content of hay t hours after it has been cut. Portions of data observed at the Plant and Soil Science Farm at the University of Tennessee, Knoxville were used to determine the parameter k and a dew deposition parameter δ .

An estimate for δ was found for days when a heavy dew was recorded, but determining an estimate for δ on days when a light or medium dew was recorded was not possible because of inconsistencies in the data. The following suggestions may decrease or eliminate the inconsistencies in data observed in future experiments.

Planting fewer, but larger plots for sampling would lessen the sampling time. In the previous experiments, the sampling time was about one hour. If this time were shortened, a light or medium dew might not dissipate before the last sample were taken.

In the previous experiments, the plots selected for sampling were not in any order. That is, some of the plots selected were partly shaded by plots that were not yet cut. If the plots to be sampled were cut so that none of them was shaded, the drying rate would be more uniform. Sampling the plots in the same order and at

approximately the same time each day could also eliminate some of the variability in the data.

If fewer plots were cut for sampling, it might be possible to sample the plots three times a day instead of twice a day, as was done. In order to obtain a better estimate for the parameters k and δ , the hay could be sampled in the morning before the dew had dissipated and then again later in the morning when the dew was dissipated. The hay would be sampled in the later afternoon also. Since applying the model for a twelve-hour drying day resulted in an accurate estimation of the moisture content of the hay at the end of the day, the necessity for sampling the hay a fourth time at sunset would be eliminated. This sampling procedure would differentiate between the period of drying in which the dew evaporates and the period of actual drying of the hay. In this manner, a better estimate for the dew parameter δ could be obtained, and the validity of the parameter k could be checked.

LIST OF REFERENCES

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1. Duvdevani, S., "An Optical Estimation of Dew." Royal Meteorological Society Quarterly Journal 73(1947), 282 - 296.
2. Hall, Carl W., Drying Farm Crops. Agricultural Consulting Associates, Inc., 1957.
3. Henderson, S.M., "A Basic Concept of Equilibrium Moisture." Agricultural Engineering 33(1952), 29 - 33.
4. Henderson, S.M., and Perry, R.L., Agricultural Process Engineering. John Wiley and Sons, New York, 1955.
5. Hill, J.D., et al., "The Use of Vapor Pressure Deficit to Predict Drying Time of Hay." American Society of Agricultural Engineers. Paper No. 76-3040 (1976).
6. Kemp, J.G., et al., "Development of Empirical Formulae For Drying Hay." Transactions ASAE 15(172), 723 - 725.
7. Middleton, K.W.E. and Spilhaus, A.F., Meteorological Instruments, University of Toronto, 1953.
8. Miller, L.G., et al., "Nutritive Value of Hay Baled at Various Moisture Contents." Journal of Animal Science 26(1967), 1329 - 1373.

VITA

Christie Grigsby was born in Cleveland, Tennessee, on August 28, 1954. She attended elementary schools in that city and graduated from Cleveland High School in June of 1972. The following September she entered Cleveland State Community College, and in June of 1974 she received an Associate of Science degree. In June of 1975 she entered the University of Tennessee, Knoxville, and in June of 1977 received a Bachelor of Arts degree in Mathematics. In the Fall of 1977 she accepted a teaching assistantship at The University of Tennessee, Knoxville and began study toward a Master of Science degree in Mathematics. This degree was awarded in June of 1980.